# Computer Algebra
# Independent Integration Tests

Summer 2024

## 5-Inverse-trig-functions/5.3-Inverse-tangent/279-5.3.4

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# Contents

# CHAPTER 1

## Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [ **1299** ]. This is test number [ 279 ].

## 1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

## 1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

| System | % solved | % Failed |
|---|---|---|
| Rubi | 98.77 ( 1283 ) | 1.23 ( 16 ) |
| Mathematica | 98.38 ( 1278 ) | 1.62 ( 21 ) |
| Maple | 92.22 ( 1198 ) | 7.78 ( 101 ) |
| Reduce | 58.28 ( 757 ) | 41.72 ( 542 ) |
| Mupad | 57.97 ( 753 ) | 42.03 ( 546 ) |
| Giac | 48.19 ( 626 ) | 51.81 ( 673 ) |
| Fricas | 43.42 ( 564 ) | 56.58 ( 735 ) |
| Sympy | 43.34 ( 563 ) | 56.66 ( 736 ) |
| Maxima | 31.25 ( 406 ) | 68.75 ( 893 ) |

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

| grade | description |
|---|---|
| A | Integral was solved and antiderivative is optimal in quality and leaf size. |
| B | Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size. |
| C | Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons<br>1. antiderivative contains a hypergeometric function and the optimal antiderivative does not.<br>2. antiderivative contains a special function and the optimal antiderivative does not.<br>3. antiderivative contains the imaginary unit and the optimal antiderivative does not. |
| F | Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised. |

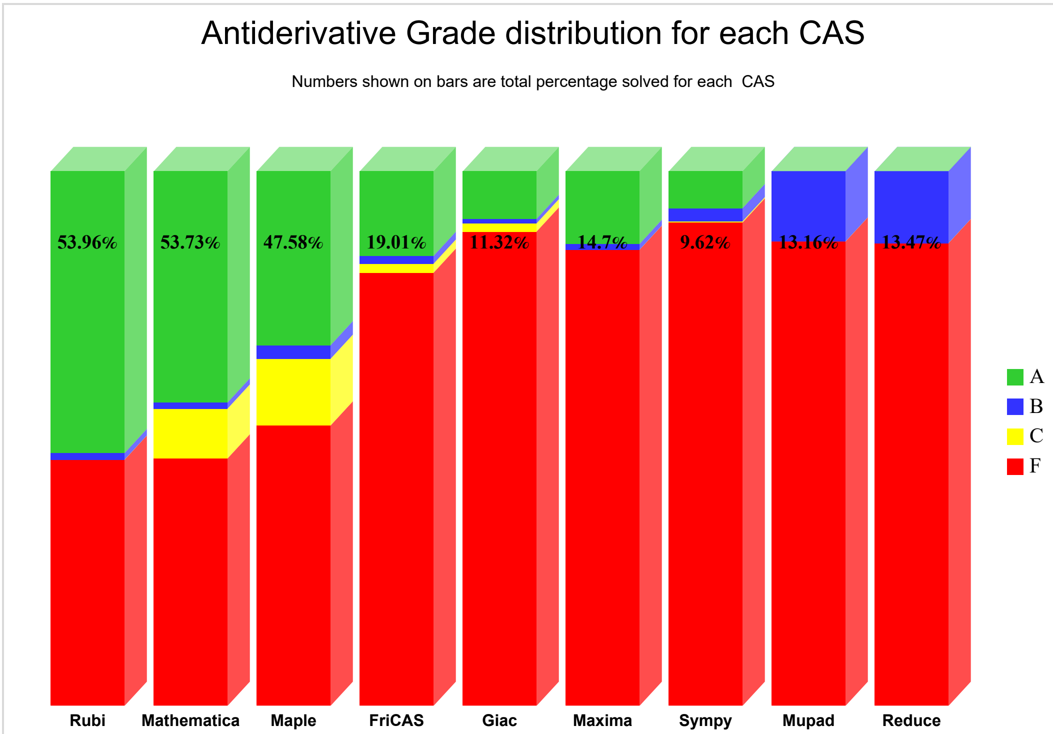
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

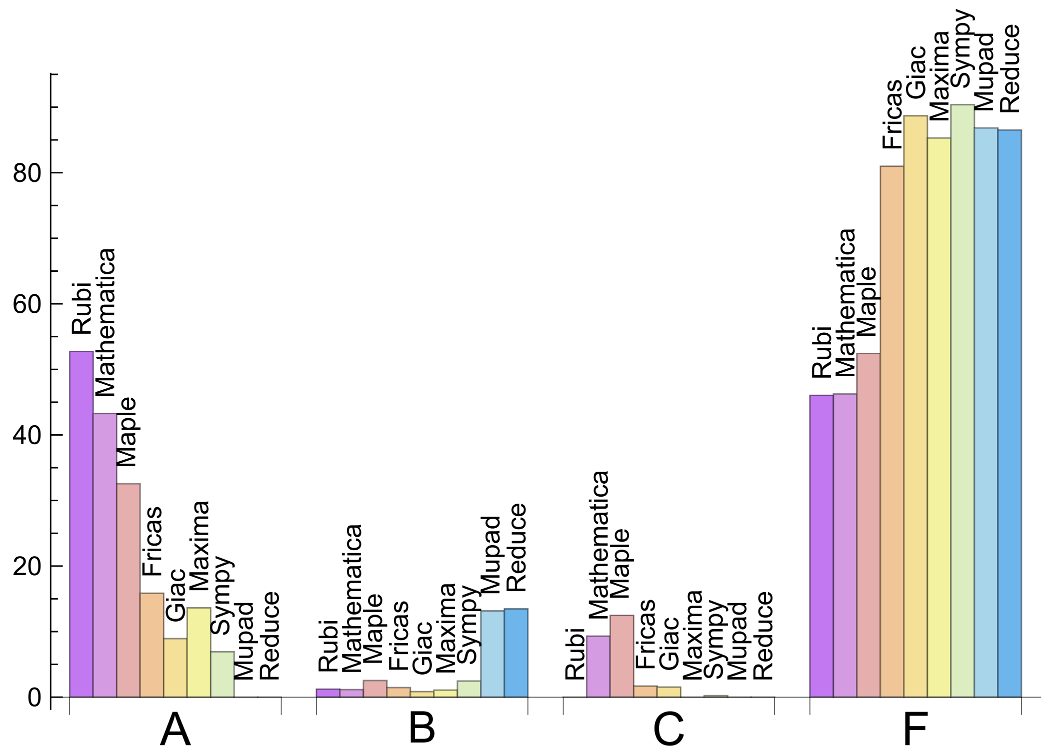
| System | % A grade | % B grade | % C grade | % F grade |
|---|---|---|---|---|
| Rubi | 52.733 | 1.232 | 0.000 | 46.035 |
| Mathematica | 43.264 | 1.155 | 9.315 | 46.266 |
| Maple | 32.564 | 2.540 | 12.471 | 52.425 |
| Fricas | 15.858 | 1.463 | 1.694 | 80.985 |
| Maxima | 13.626 | 1.078 | 0.000 | 85.296 |
| Giac | 8.930 | 0.847 | 1.540 | 88.684 |
| Sympy | 6.928 | 2.463 | 0.231 | 90.377 |
| Mupad | 0.000 | 13.164 | 0.000 | 86.836 |
| Reduce | 0.000 | 13.472 | 0.000 | 86.528 |

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.




The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

| System | Number failed | Percentage normal failure | Percentage timeout failure | Percentage exception failure |
|---|---|---|---|---|
| Rubi | 16 | 100.00 | 0.00 | 0.00 |
| Mathematica | 21 | 71.43 | 28.57 | 0.00 |
| Maple | 101 | 98.02 | 1.98 | 0.00 |
| Fricas | 735 | 52.79 | 0.00 | 47.21 |
| Maxima | 893 | 42.78 | 0.67 | 56.55 |
| Reduce | 542 | 100.00 | 0.00 | 0.00 |
| Mupad | 546 | 0.00 | 100.00 | 0.00 |
| Giac | 673 | 66.72 | 0.59 | 32.69 |
| Sympy | 736 | 62.36 | 35.73 | 1.90 |

Table 1.4: Failure statistics for each CAS

## 1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

| System | Mean time (sec) |
|---|---|
| Fricas | 0.21 |
| Giac | 0.26 |
| Maxima | 0.28 |
| Mupad | 0.77 |
| Rubi | 0.87 |
| Mathematica | 1.87 |
| Reduce | 4.59 |
| Maple | 8.93 |
| Sympy | 13.45 |

Table 1.5: Time performance for each CAS

| System | Mean size | Normalized mean | Median size | Normalized median |
|---|---|---|---|---|
| Mupad | 52.02 | 1.03 | 24.00 | 1.00 |
| Giac | 52.89 | 1.06 | 24.00 | 1.00 |
| Sympy | 72.76 | 1.50 | 29.00 | 1.21 |
| Maxima | 108.45 | 2.14 | 73.00 | 1.09 |
| Fricas | 130.76 | 1.71 | 50.00 | 1.14 |
| Rubi | 150.05 | 1.04 | 66.00 | 1.00 |
| Mathematica | 158.44 | 1.07 | 56.00 | 1.08 |
| Reduce | 416.57 | 16.49 | 63.00 | 2.00 |
| Maple | 506.38 | 2.00 | 28.00 | 0.92 |

Table 1.6: Leaf size performance for each CAS

# 1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the $y$ axis is the percentage solved which Rubi itself needed the number of rules given the $x$ axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

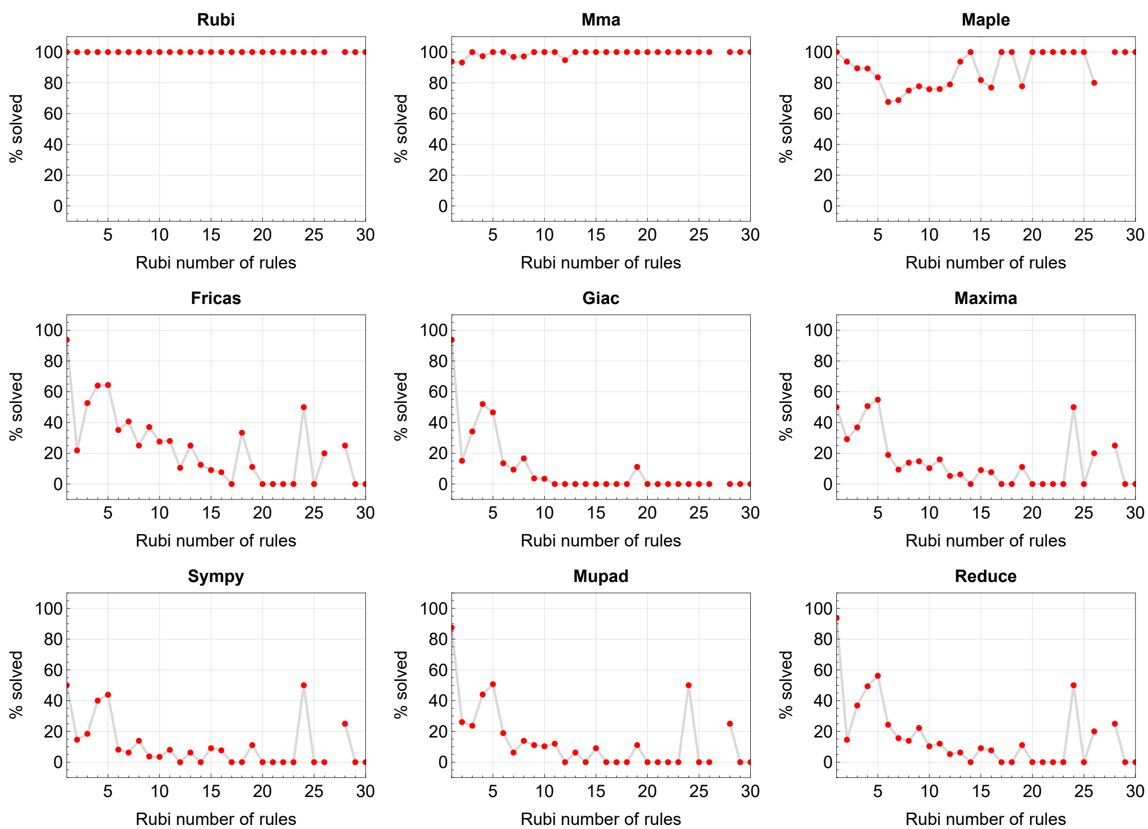



Figure 1.1: Solving statistics per number of Rubi rules used

## 1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

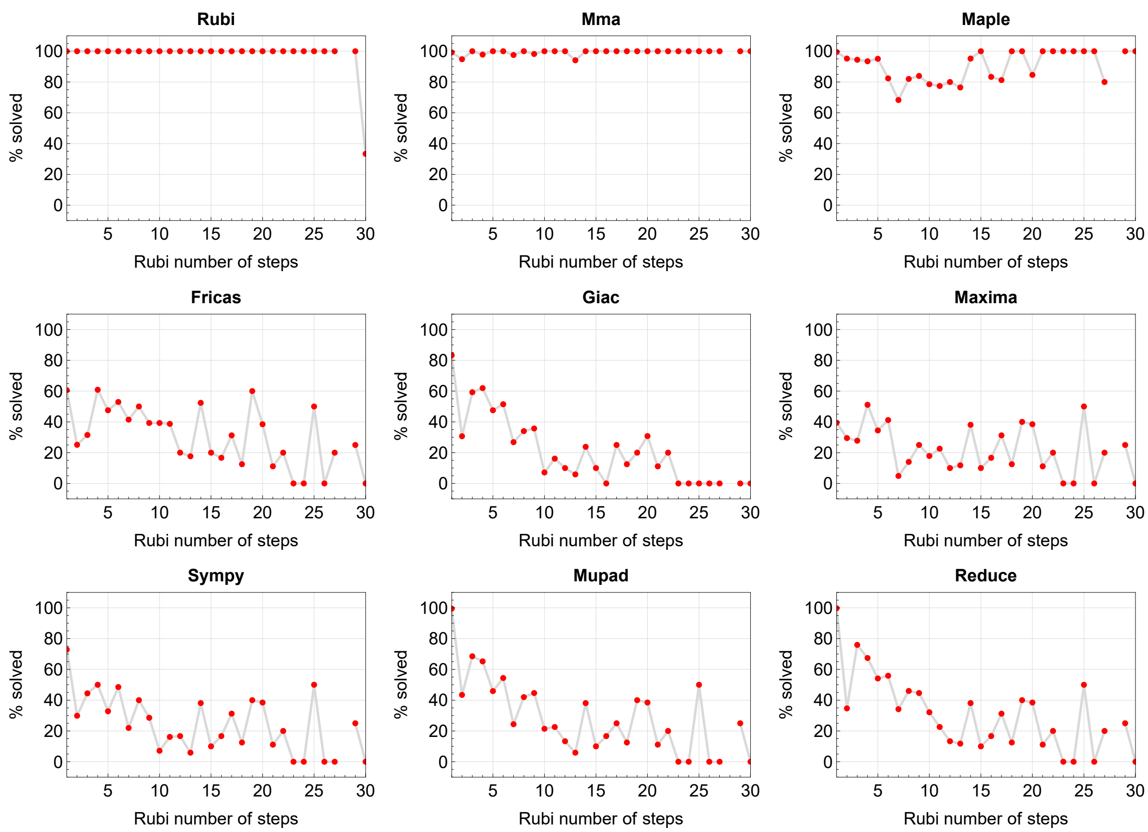



Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

## 1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

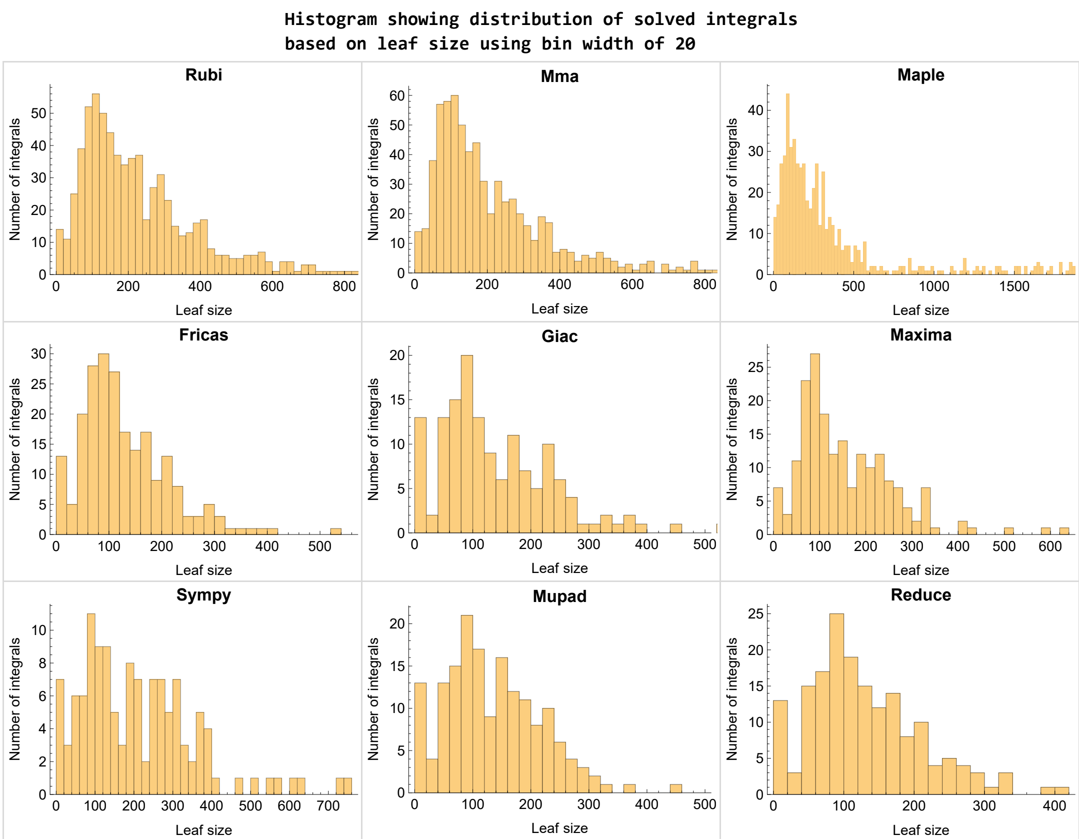



Figure 1.3: Solved integrals based on leaf size distribution

## 1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

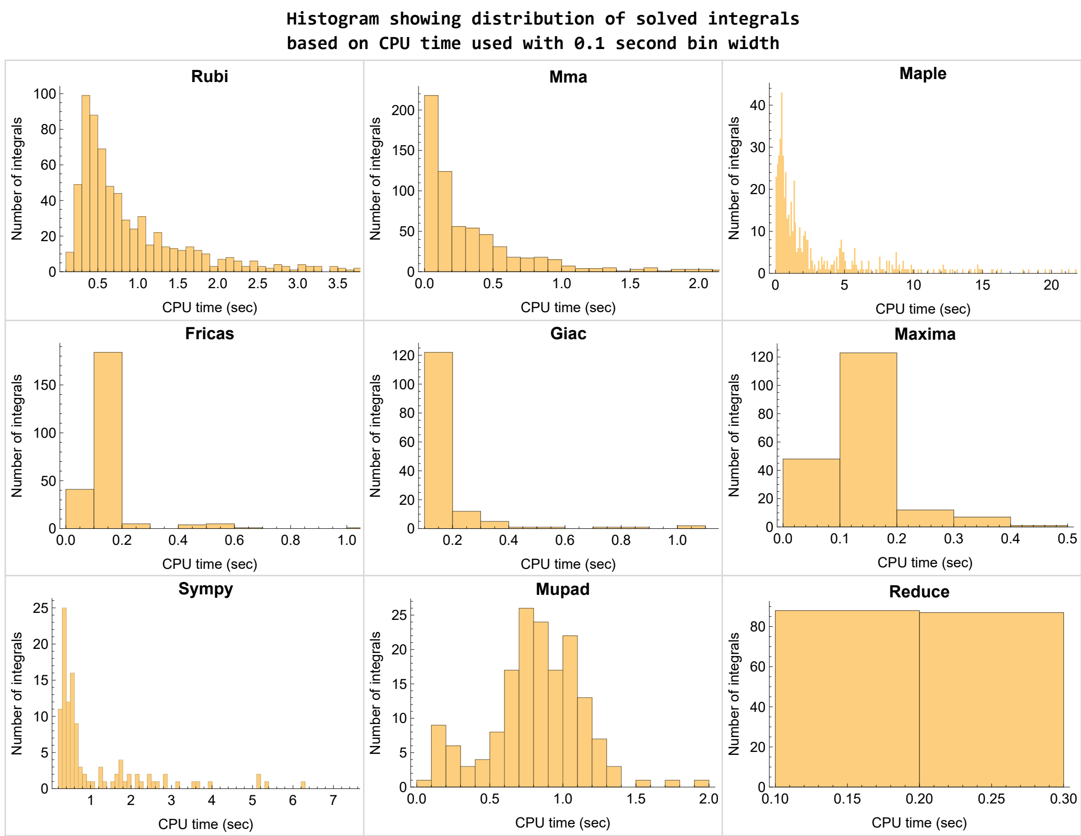



Figure 1.4: Solved integrals histogram based on CPU time used

## 1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

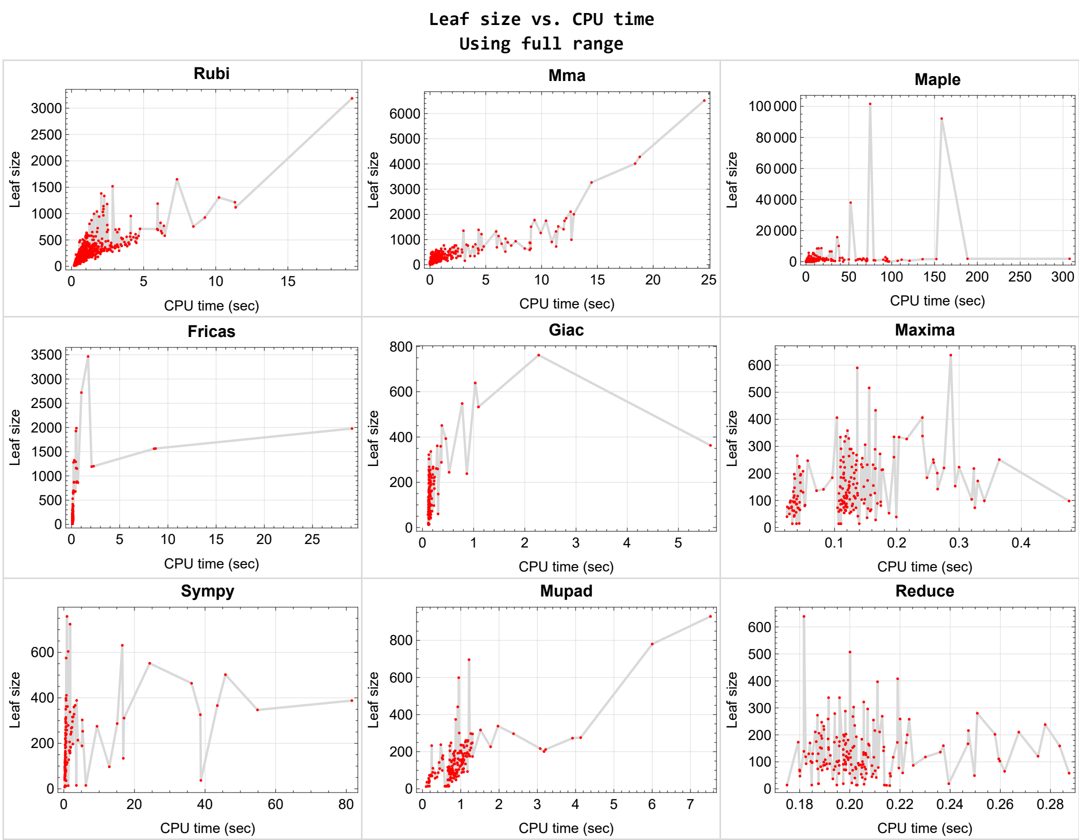



Figure 1.5: Leaf size vs. CPU time. Full range

## 1.9 list of integrals with no known antiderivative

{133, 148, 251, 252, 253, 254, 255, 256, 257, 355, 356, 357, 358, 359, 360, 361, 362, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 477, 478, 479, 480, 484, 485, 486, 487, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 511, 512, 513, 514, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 546, 547, 548, 549, 550, 554, 555, 556, 557, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 582, 583, 584, 585, 586, 587, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 622, 623, 624, 625, 626, 630, 631, 632, 633, 638, 639, 640, 641, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 661, 662, 663, 664, 665, 666, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 702, 703, 704, 705, 706, 707, 711, 712, 713, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 746, 747, 748, 749, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 778, 779, 780, 781, 782, 783, 787, 788, 789, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 825, 826, 827, 828, 829, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 859, 860, 861, 862, 863, 864, 868, 869, 870, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 905, 906, 907, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 928, 929, 930, 934, 935, 936, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 963, 964, 965, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 986, 987, 988, 989, 993, 994, 995, 996, 997, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1028, 1029, 1030, 1031, 1032, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1056, 1057, 1058, 1062, 1063, 1064, 1065, 1066, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1097, 1098, 1099, 1100, 1101, 1106, 1107, 1108, 1109, 1110, 1112, 1173, 1175, 1176, 1177, 1178, 1180, 1183, 1185, 1186, 1187, 1188, 1189, 1190, 1193, 1195, 1196, 1197, 1198, 1199, 1201, 1203, 1204, 1206, 1209, 1212, 1214, 1216, 1221, 1223, 1230, 1231, 1232, 1233, 1234, 1235, 1236, 1237, 1238, 1240, 1242, 1244, 1297}

## 1.10 List of integrals solved by CAS but has no known antiderivative

**Rubi** {}

**Mathematica** {}

**Maple** {}

**Maxima** {}

**Fricas** {}

**Sympy** {}

**Giac** {}

**Reduce** {}

**Mupad** {}

## 1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

**Rubi** {1225, 1243, 1290, 1292, 1294}

**Mathematica** {201, 209, 211, 214, 217, 219, 221, 222, 238, 307, 308, 309, 314, 315, 316, 317, 319, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 338, 345, 346, 347, 354, 412, 413, 414, 417, 418, 419, 420, 421, 422, 423, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 441, 442, 448, 449, 456, 1154, 1161, 1162, 1169, 1170, 1259, 1261, 1263, 1265, 1268, 1295, 1296, 1299}

**Maple** {72, 73, 80, 81, 82, 88, 89, 90, 91, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 116, 117, 120, 121, 122, 123, 127, 128, 129, 130, 131, 132, 141, 142, 143, 144, 145, 146, 147, 262, 264, 270, 272, 278, 280, 283, 285, 287, 289, 291, 295, 297, 303, 305, 364, 366, 368, 370, 372, 374, 376, 378, 380, 382, 384, 386, 387, 389, 390, 392, 393, 395, 396, 400, 401, 403, 408, 409, 411, 509, 510, 515, 517, 518, 1157, 1159, 1160, 1164, 1167, 1168, 1171, 1249, 1251, 1256, 1258, 1260, 1264, 1267, 1269, 1278, 1289}

**Maxima** Verification phase not currently implemented.

**Fricas** Verification phase not currently implemented.

**Sympy** Verification phase not currently implemented.

**Giac** Verification phase not currently implemented.

**Reduce** Verification phase not currently implemented.

**Mupad** Verification phase not currently implemented.

## 1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

## 1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals.

These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

# 1.14 Important notes about some of the results

## Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

## Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

## Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

## Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

# 1.15 Current tree layout of integration tests

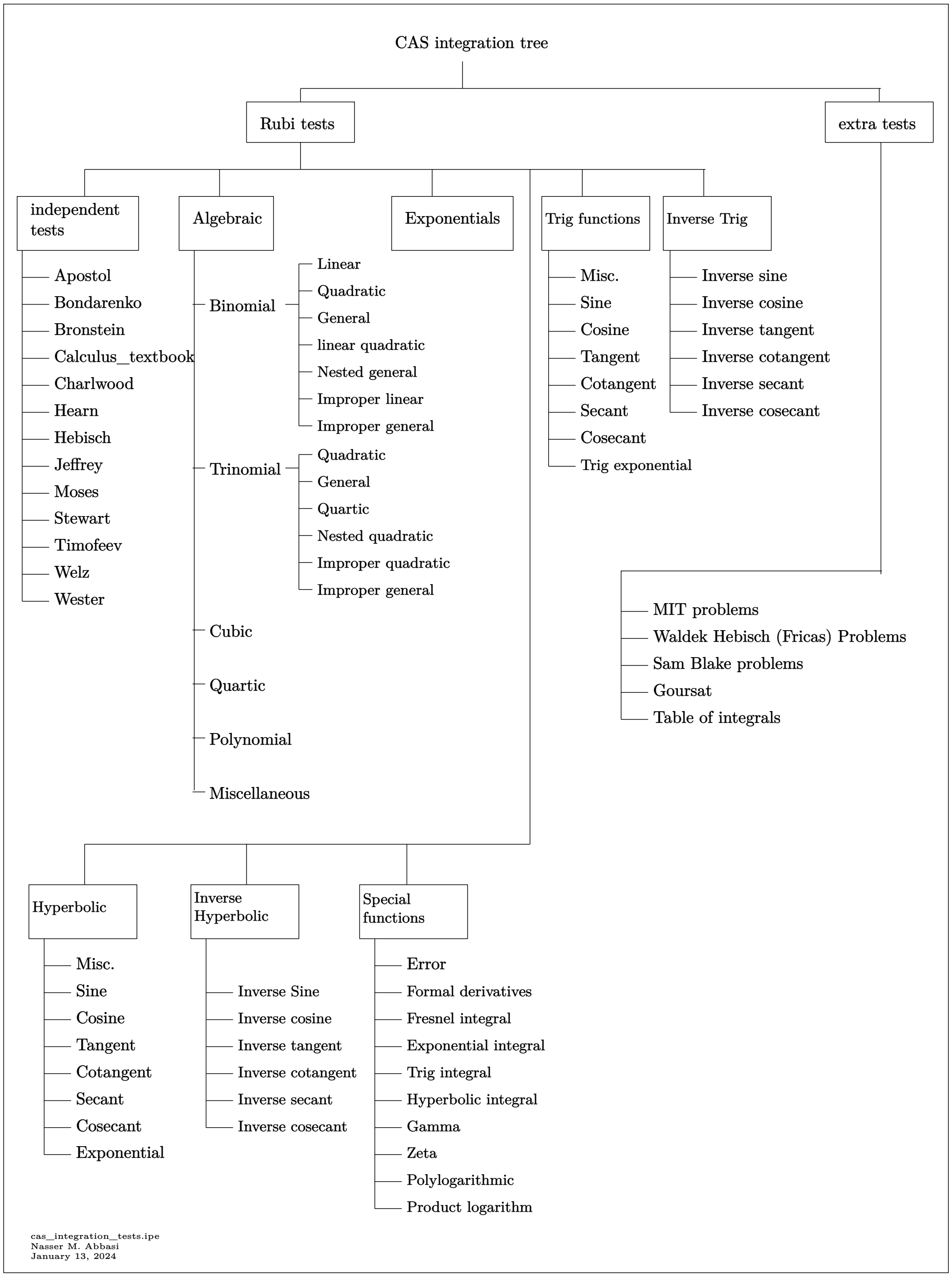
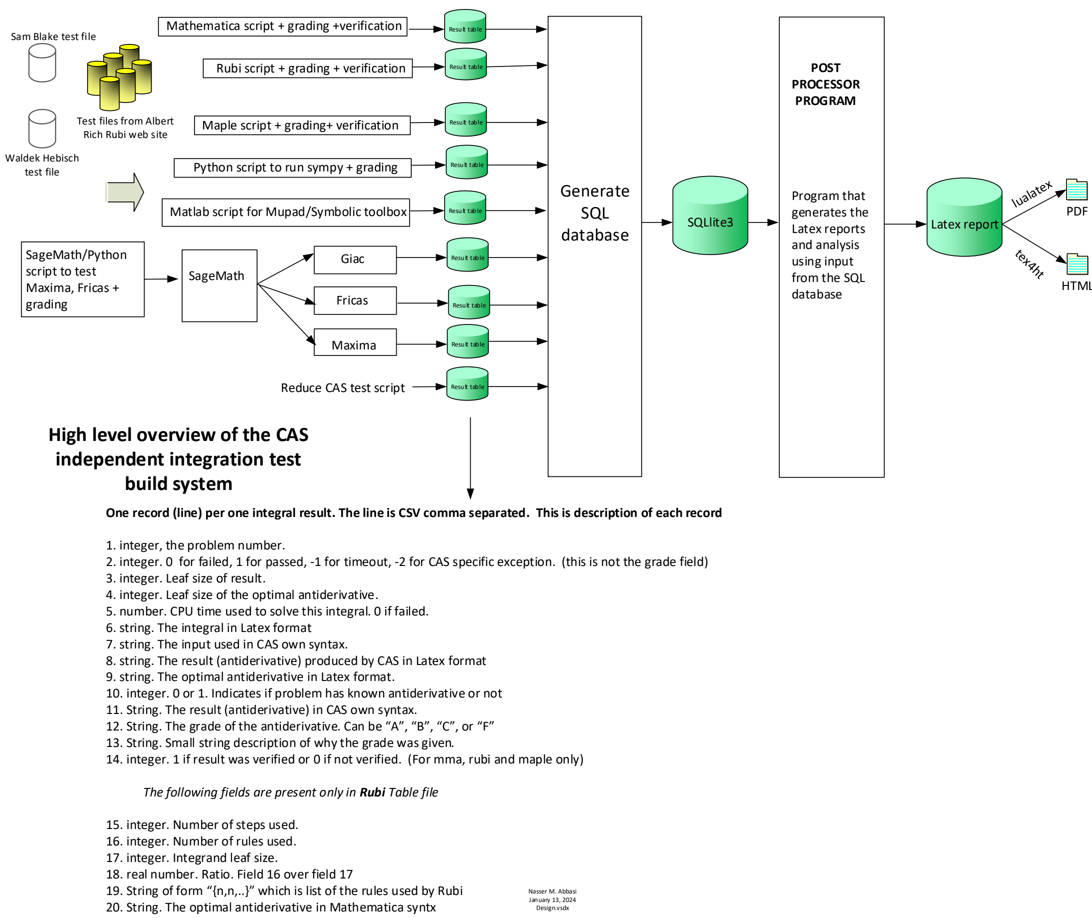



Figure 1.6: CAS integration tests tree

## 1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



# CHAPTER 2

## DETAILED SUMMARY TABLES OF RESULTS

## 2.1 List of integrals sorted by grade for each CAS



### Rubi

**A grade** { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 200, 201, 202, 203, 204, 205, 206, 207, 209, 210, 211, 212, 213, 214, 215, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 308, 309, 310, 311, 312, 313, 314, 317, 318, 319, 320, 321, 322, 325, 326, 327, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 413, 414, 415, 416, 417, 418, 419, 422, 423, 424, 425, 426, 427, 429, 430, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 476, 481, 482, 483, 488, 489, 490, 491, 492, 509, 510, 515, 516, 517, 518, 545, 551, 553, 559, 560, 561, 580, 581, 588, 589, 590, 591, 621, 627, 628, 629, 634, 635, 636, 637, 642, 659, 660, 667, 668, 669, 670, 701, 708, 709, 710, 714, 715, 716, 717, 718, 744, 745, 750, 751, 752, 753, 777, 784, 785, 786, 790, 791, 792, 793, 794, 823, 824, 830, 831, 832, 833, 858, 865, 866, 867, 871, 872, 873, 874, 875, 903, 904, 908, 909, 910, 911, 927, 931, 932, 933, 937, 938, 939, 940, 941, 961, 962, 966, 967, 968, 969, 985, 990, 991, 992, 998, 1000, 1001, 1026, 1027, 1033, 1034, 1035, 1036, 1055, 1059, 1060, 1061, 1067, 1069, 1070, 1095, 1096, 1102, 1103, 1104, 1105, 1111, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156, 1157, 1158, 1159, 1160, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1172, 1174, 1179, 1181, 1182, 1184, 1191, 1192, 1194, 1200, 1202, 1205, 1207, 1208, 1210, 1211, 1213, 1215, 1217, 1218, 1219, 1220, 1222, 1224, 1225, 1226, 1227, 1228, 1229, 1239, 1241, 1243, 1245, 1246, 1247, 1248, 1249, 1250, 1251, 1252, 1253, 1254, 1255, 1256, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1269, 1270, 1271, 1272, 1273, 1274, 1275, 1276, 1277, 1278, 1279, 1280, 1281, 1282, 1283, 1284, 1285, 1286, 1287, 1288, 1289, 1290, 1291, 1292, 1293, 1294, 1295, 1296, 1298, 1299 }

**B grade** { 198, 199, 208, 217, 258, 259, 305, 307, 315, 363, 364, 552, 558, 999, 1068, 1268 }

**C grade** { }

**F normal fail** { 216, 306, 316, 323, 324, 328, 410, 411, 412, 420, 421, 428, 431, 432, 433, 434 }

**F(-1) timedout fail** { }

**F(-2) exception fail** { }

## Mma

**A grade** { 1, 2, 3, 4, 5, 6, 10, 11, 12, 13, 14, 15, 20, 21, 22, 23, 24, 25, 26, 31, 32, 33, 34, 35, 36, 37, 38, 43, 44, 45, 46, 47, 48, 51, 52, 53, 54, 55, 56, 58, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 131, 132, 134, 135, 136, 137, 138, 139, 149, 150, 151, 152, 153, 154, 156, 157, 158, 159, 160, 161, 162, 164, 165, 166, 167, 168, 169, 170, 172, 173, 174, 175, 176, 177, 178, 179, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 324, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 414, 415, 416, 417, 418, 419, 420, 422, 424, 425, 426, 427, 429, 431, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 476, 481, 482, 483, 488, 489, 490, 491, 492, 509, 510, 515, 516, 517, 518, 545, 551, 552, 553, 558, 559, 560, 561, 580, 581, 588, 589, 590, 591, 621, 627, 628, 629, 634, 635, 636, 637, 642, 659, 660, 667, 668, 669, 670, 701, 708, 710, 745, 751, 753, 777, 785, 786, 794, 824, 858, 865, 867, 875, 904, 911, 927, 932, 941, 962, 966, 969, 985, 990, 992, 1055, 1060, 1111, 1113, 1114, 1115, 1116, 1117, 1118, 1120, 1122, 1124, 1125, 1126, 1127, 1128, 1129, 1131, 1133, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1143, 1145, 1147, 1149, 1150, 1151, 1152, 1154, 1155, 1156, 1157, 1158, 1159, 1160, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1227, 1228, 1229, 1239, 1245, 1246, 1247, 1248, 1249, 1250, 1251, 1252, 1253, 1254, 1255, 1256, 1257, 1258, 1263, 1265, 1268, 1273, 1274, 1275, 1276, 1277, 1278, 1279, 1280, 1281, 1282, 1283, 1284, 1285, 1286, 1287, 1288, 1290, 1291, 1292, 1293, 1294, 1298 }

**B grade** { 130, 217, 323, 325, 413, 421, 423, 428, 430, 432, 1259, 1261, 1295, 1296, 1299 }

**C grade** { 7, 8, 9, 16, 17, 18, 19, 27, 28, 29, 30, 39, 40, 41, 42, 49, 50, 57, 65, 140, 155, 163, 171, 180, 709, 714, 715, 716, 717, 718, 744, 750, 752, 784, 790, 791, 792, 793, 823, 830, 831, 832, 833, 866, 871, 872, 873, 874, 903, 908, 909, 910, 931, 933, 937, 938, 939, 940, 961, 967, 968, 991, 998, 999, 1000, 1001, 1026, 1027, 1033, 1034, 1035, 1036, 1059, 1061, 1067, 1068, 1069, 1070, 1095, 1096, 1102, 1103, 1104, 1105, 1119, 1121, 1123, 1130, 1132, 1134, 1142, 1144, 1146, 1148, 1153, 1172, 1174, 1179, 1181, 1182, 1184, 1191, 1192, 1194, 1200, 1202, 1205, 1207, 1208, 1210, 1211, 1213, 1215, 1217, 1218, 1219, 1220, 1222, 1224, 1225, 1226 }

**F normal fail** { 141, 142, 143, 144, 145, 146, 147, 1241, 1243, 1266, 1267, 1269, 1270, 1272, 1289 }

**F(-1) timedout fail** { 508, 514, 1260, 1262, 1264, 1271 }

**F(-2) exception fail** { }

## Maple

**A grade** { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 46, 49, 50, 51, 52, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 68, 69, 70, 76, 77, 78, 83, 84, 85, 86, 92, 93, 94, 115, 118, 125, 126, 134, 135, 136, 137, 138, 139, 140, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 177, 179, 181, 183, 185, 186, 187, 189, 191, 192, 193, 194, 195, 197, 199, 201, 203, 204, 206, 209, 211, 212, 213, 214, 217, 219, 220, 221, 222, 223, 225, 227, 228, 230, 233, 236, 238, 241, 242, 245, 246, 258, 259, 260, 261, 266, 267, 268, 269, 271, 273, 274, 275, 276, 277, 279, 281, 282, 286, 290, 292, 293, 294, 296, 298, 299, 300, 301, 302, 304, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 335, 336, 337, 338, 339, 343, 344, 345, 346, 347, 353, 354, 363, 365, 367, 369, 371, 373, 375, 377, 379, 381, 383, 385, 388, 391, 394, 397, 398, 399, 402, 404, 405, 406, 407, 410, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 439, 440, 441, 442, 447, 448, 455, 456, 476, 481, 482, 483, 488, 489, 490, 491, 492, 545, 551, 552, 553, 558, 559, 560, 561, 621, 627, 628, 629, 634, 635, 636, 637, 642, 701, 708, 709, 710, 714, 715, 716, 717, 718, 777, 784, 785, 786, 790, 791, 792, 793, 794, 858, 865, 866, 867, 871, 872, 873, 874, 875, 927, 931, 932, 933, 937, 938, 939, 940, 941, 985, 990, 991, 992, 998, 999, 1000, 1001, 1055, 1059, 1060, 1061, 1067, 1068, 1069, 1070, 1111, 1113, 1114, 1115, 1116, 1117, 1118, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1150, 1151, 1152, 1153, 1154, 1155, 1156, 1158, 1165, 1166, 1245, 1246, 1247, 1248, 1252, 1253, 1254, 1255, 1257, 1273, 1274, 1275, 1276, 1277, 1284, 1285, 1286, 1287, 1288 }

**B grade** { 45, 47, 48, 53, 67, 71, 74, 75, 79, 87, 107, 114, 119, 124, 178, 205, 215, 235, 243, 244, 263, 265, 284, 288, 1119, 1161, 1162, 1163, 1169, 1170, 1250, 1262, 1268 }

**C grade** { 72, 73, 80, 81, 82, 88, 89, 90, 91, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 116, 117, 120, 121, 122, 123, 127, 128, 129, 130, 131, 132, 141, 142, 143, 144, 145, 146, 147, 174, 176, 180, 182, 184, 188, 190, 196, 198, 200, 202, 207, 208, 210, 216, 218, 224, 226, 229, 231, 232, 234, 237, 239, 240, 247, 248, 249, 250, 262, 264, 270, 272, 278, 280, 283, 285, 287, 289, 291, 295, 297, 303, 305, 341, 342, 349, 350, 351, 352, 364, 366, 368, 370, 372, 374, 376, 378, 380, 382, 384, 386, 387, 389, 390, 392, 393, 395, 396, 400, 401, 403, 408, 409, 411, 445, 446, 451, 452, 453, 454, 509, 510, 515, 516, 517, 518, 580, 581, 588, 589, 590, 591, 659, 660, 667, 668, 669, 670, 1157, 1159, 1160, 1164, 1167, 1168, 1171, 1249, 1251, 1256, 1258, 1260, 1264, 1267, 1269, 1278, 1289 }

**F normal fail** { 334, 340, 348, 437, 438, 443, 444, 449, 450, 744, 745, 750, 751, 752, 753, 823, 824, 830, 831, 832, 833, 903, 904, 908, 909, 910, 911, 961, 962, 966, 967, 968, 969, 1026, 1027, 1033, 1034, 1035, 1036, 1095, 1096, 1102, 1103, 1104, 1105, 1172, 1174, 1179, 1181, 1182, 1184, 1191, 1192, 1194, 1200, 1202, 1205, 1207, 1208, 1210, 1211, 1213, 1215, 1217, 1218, 1219, 1220, 1222, 1224, 1225, 1226, 1227, 1228, 1229, 1239, 1241, 1243, 1259, 1261, 1263, 1265, 1266, 1270, 1271, 1272, 1279, 1280, 1281, 1282, 1283, 1290, 1291, 1292, 1293, 1294, 1295, 1296, 1298, 1299 }

**F(-1) timedout fail** { 539, 541 }

**F(-2) exception fail** { }

## Fricas

**A grade** { 1, 2, 3, 7, 8, 9, 10, 11, 12, 13, 17, 18, 19, 20, 21, 22, 23, 29, 30, 31, 32, 33, 34, 41, 42, 47, 48, 49, 50, 54, 55, 56, 57, 61, 62, 63, 64, 65, 66, 67, 99, 107, 114, 115, 118, 124, 125, 126, 149, 150, 151, 152, 154, 156, 157, 158, 159, 160, 162, 164, 165, 166, 167, 168, 170, 172, 173, 175, 177, 179, 181, 183, 185, 186, 187, 189, 191, 192, 193, 194, 195, 197, 199, 200, 202, 207, 208, 210, 216, 218, 224, 226, 229, 231, 232, 234, 235, 237, 239, 240, 242, 243, 244, 245, 247, 258, 260, 266, 268, 274, 276, 286, 292, 293, 294, 299, 300, 301, 302, 341, 342, 349, 350, 351, 352, 391, 397, 398, 399, 404, 405, 406, 407, 445, 446, 451, 452, 453, 454, 476, 545, 621, 642, 701, 777, 858, 927, 985, 1055, 1111, 1113, 1114, 1115, 1116, 1118, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1129, 1131, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1141, 1143, 1145, 1147, 1148, 1149, 1158, 1172, 1174, 1179, 1181, 1182, 1184, 1191, 1192, 1194, 1200, 1202, 1205, 1207, 1245, 1247, 1252, 1254, 1273, 1274, 1275, 1276, 1277, 1284, 1285, 1286, 1287, 1288 }

**B grade** { 4, 28, 40, 130, 1165, 1166, 1208, 1210, 1211, 1213, 1215, 1217, 1218, 1219, 1220, 1222, 1224, 1225, 1226 }

**C grade** { 481, 482, 483, 488, 489, 490, 491, 492, 551, 552, 553, 558, 559, 560, 561, 627, 628, 629, 634, 635, 636, 637 }

**F normal fail** { 5, 6, 14, 15, 16, 24, 25, 26, 27, 35, 36, 37, 38, 39, 43, 44, 45, 46, 51, 52, 53, 58, 59, 60, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 100, 101, 102, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 116, 117, 119, 120, 121, 122, 123, 127, 128, 129, 131, 132, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 153, 155, 161, 163, 169, 171, 174, 176, 178, 180, 182, 184, 188, 190, 196, 198, 201, 203, 204, 205, 206, 209, 211, 212, 213, 214, 215, 217, 219, 220, 221, 222, 223, 225, 227, 228, 230, 233, 236, 238, 241, 246, 248, 249, 250, 259, 261, 262, 263, 264, 265, 267, 269, 270, 271, 272, 273, 275, 277, 278, 279, 280, 281, 282, 283, 284, 285, 287, 288, 289, 290, 291, 295, 296, 297, 298, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 343, 344, 345, 346, 347, 348, 353, 354, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 392, 393, 394, 395, 396, 400, 401, 402, 403, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 447, 448, 449, 450, 455, 456, 509, 510, 515, 516, 517, 518, 580, 581, 588, 589, 590, 591, 659, 660, 667, 668, 669, 670, 1117, 1119, 1128, 1130, 1132, 1140, 1142, 1144, 1146, 1150, 1151, 1152, 1153, 1154, 1155, 1156, 1157, 1159, 1160, 1161, 1162, 1163, 1164, 1167, 1168, 1169, 1170, 1171, 1227, 1228, 1229, 1239, 1241, 1243, 1246, 1248, 1249, 1250, 1251, 1253, 1255, 1256, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1268, 1269, 1270, 1271, 1272, 1278, 1279, 1280, 1281, 1282, 1283, 1289, 1290, 1291, 1292, 1293, 1294, 1295, 1296, 1298, 1299 }

**F(-1) timedout fail** { }

**F(-2) exception fail** { 686, 687, 688, 690, 691, 692, 694, 695, 696, 698, 699, 700, 702, 703, 704, 705, 707, 708, 709, 710, 711, 713, 714, 715, 716, 717, 718, 719, 721, 722, 723, 725, 726, 727, 729, 730, 731, 733, 734, 735, 736, 737, 738, 739, 740, 742, 743, 744, 745, 746, 747, 749, 750, 751, 752, 753, 754, 756, 757, 758, 759, 760, 762, 763, 764, 765, 766, 768, 769, 770, 771, 772, 774, 775, 776, 778, 779, 780, 781, 783, 784, 785, 786, 787, 789, 790, 791, 792, 793, 794, 795, 797, 798, 799, 800, 802, 803, 804, 805, 807, 808, 809, 810, 812, 813, 814, 815, 816, 817, 818, 819, 821, 822, 823, 824, 825, 826, 828, 829, 830, 831, 832, 833, 834, 835, 837, 838, 839, 840, 841, 843, 844, 845, 846, 847, 849, 850, 851, 852, 853, 855, 856, 857, 859, 860, 861, 862, 864, 865, 866, 867, 868, 870, 871, 872, 873, 874, 875, 876, 878, 879, 880, 881, 883, 884, 885, 886, 888, 889, 890, 891, 893, 894, 895, 896, 897, 898, 899, 900, 902, 903, 904, 905, 907, 908, 909, 910, 911, 912, 914, 915, 916, 918, 919, 920, 922, 923, 924, 926, 928, 930, 931, 932, 933, 934, 936, 937, 938, 939, 940, 941, 942, 944, 945, 946, 948, 949, 950, 952, 953, 954, 956, 957, 958, 960, 961, 962, 963, 965, 966, 967, 968, 969, 970, 972, 973, 974, 976, 977, 978, 980, 981, 982, 984, 986, 988, 989, 990, 991, 992, 993, 994, 995, 996, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1007, 1008, 1009, 1011, 1012, 1013, 1015, 1016, 1017, 1019, 1020, 1021, 1022, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1042, 1043, 1044, 1046, 1047, 1048, 1050, 1051, 1052, 1054, 1056, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1076, 1077, 1078, 1080, 1081, 1082, 1084, 1085, 1086, 1088, 1089, 1090, 1091, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109 }

## Maxima

**A grade** { 1, 2, 3, 4, 7, 8, 9, 10, 11, 12, 14, 17, 18, 19, 20, 21, 22, 24, 25, 29, 30, 31, 35, 36, 37, 42, 58, 59, 60, 61, 62, 66, 114, 115, 118, 125, 126, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 177, 179, 181, 183, 185, 186, 187, 189, 191, 192, 193, 194, 195, 197, 199, 200, 207, 208, 216, 224, 226, 229, 231, 234, 235, 242, 243, 244, 245, 258, 260, 266, 268, 274, 276, 286, 292, 293, 294, 299, 300, 301, 302, 341, 342, 350, 352, 391, 397, 398, 399, 404, 405, 406, 407, 445, 446, 476, 545, 621, 642, 1113, 1114, 1115, 1116, 1117, 1118, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1128, 1129, 1130, 1131, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1147, 1148, 1149, 1245, 1247, 1252, 1254, 1273, 1274, 1275, 1276, 1277, 1279, 1280, 1281, 1282, 1283, 1284, 1285, 1286, 1287, 1288 }

**B grade** { 13, 23, 28, 32, 33, 34, 40, 41, 63, 65, 67, 202, 210, 218 }

**C grade** { }

**F normal fail** { 5, 6, 15, 16, 26, 27, 38, 39, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 108, 109, 111, 112, 113, 116, 119, 120, 121, 122, 123, 127, 128, 129, 130, 131, 132, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 174, 176, 178, 180, 182, 184, 188, 190, 196, 198, 201, 203, 204, 205, 206, 209, 211, 212, 213, 214, 215, 217, 219, 220, 221, 222, 223, 225, 227, 228, 230, 232, 233, 236, 237, 238, 239, 240, 241, 246, 247, 248, 249, 250, 259, 261, 262, 263, 264, 265, 267, 269, 270, 271, 272, 273, 275, 277, 278, 279, 280, 281, 282, 283, 284, 285, 287, 288, 289, 291, 295, 296, 297, 303, 305, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 343, 344, 345, 346, 347, 348, 349, 351, 353, 354, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 392, 393, 394, 396, 400, 401, 402, 403, 408, 409, 410, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 481, 482, 483, 488, 489, 490, 491, 492, 509, 510, 515, 516, 517, 518, 551, 552, 553, 558, 559, 560, 561, 580, 581, 588, 589, 590, 591, 627, 628, 629, 634, 635, 636, 637, 659, 660, 667, 668, 669, 670, 1119, 1132, 1146, 1150, 1151, 1152, 1153, 1157, 1159, 1160, 1164, 1167, 1168, 1218, 1220, 1227, 1228, 1229, 1239, 1241, 1243, 1246, 1248, 1249, 1250, 1251, 1253, 1255, 1256, 1257, 1258, 1259, 1261, 1263, 1265, 1266, 1270, 1272, 1278, 1289, 1290, 1291, 1292, 1293, 1294, 1295, 1296, 1298, 1299 }

**F(-1) timedout fail** { 290, 298, 304, 306, 395, 411 }

**F(-2) exception fail** { 54, 64, 107, 110, 117, 124, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000, 1001, 1002, 1003, 1004, 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014, 1015, 1016, 1017, 1018, 1019, 1020, 1021, 1022, 1023, 1024, 1025, 1026, 1027, 1028, 1029, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1042, 1043, 1044, 1045, 1046, 1047, 1048, 1049, 1050, 1051, 1052, 1053, 1054, 1055, 1056, 1057, 1058, 1059, 1060, 1061, 1062, 1063, 1064, 1065, 1066, 1067, 1068, 1069, 1070, 1071, 1072, 1073, 1074, 1075, 1076, 1077, 1078, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1088, 1089, 1090, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1110, 1111, 1154, 1155, 1156, 1158, 1161, 1162, 1163, 1165, 1166, 1169, 1170, 1171, 1172, 1173, 1174, 1175, 1176, 1177, 1178, 1179, 1180, 1181, 1182, 1183, 1184, 1185, 1186, 1187, 1188, 1189, 1190, 1191, 1192, 1193, 1194, 1195, 1196, 1197, 1198, 1199, 1200, 1201, 1202, 1203, 1204, 1205, 1206, 1207, 1208, 1209, 1210, 1211, 1212, 1213, 1214, 1215, 1216, 1217, 1219, 1221, 1222, 1223, 1224, 1225, 1226, 1260, 1262, 1264, 1267, 1268, 1269, 1271 }

## Giac

**A grade** { 1, 2, 3, 4, 7, 8, 9, 10, 11, 12, 13, 17, 18, 19, 20, 21, 22, 29, 30, 31, 32, 33, 41, 42, 54, 61, 62, 107, 124, 149, 150, 151, 152, 154, 156, 157, 158, 159, 160, 162, 164, 165, 166, 167, 168, 170, 172, 173, 175, 177, 186, 192, 194, 226, 234, 235, 243, 244, 245, 258, 260, 266, 268, 274, 276, 286, 341, 342, 351, 391, 445, 446, 476, 545, 621, 642, 701, 777, 858, 927, 985, 1055, 1111, 1113, 1114, 1115, 1116, 1118, 1120, 1122, 1124, 1125, 1127, 1129, 1131, 1133, 1135, 1136, 1137, 1138, 1139, 1141, 1143, 1145, 1147, 1149, 1225, 1226, 1245, 1247, 1252, 1254, 1273, 1274, 1284, 1286 }

**B grade** { 23, 28, 34, 40, 66, 1126, 1275, 1276, 1277, 1287, 1288 }

**C grade** { 708, 710, 714, 716, 718, 744, 745, 931, 933, 937, 939, 941, 962, 1121, 1123, 1134, 1148, 1158, 1165, 1166 }

**F normal fail** { 5, 6, 14, 15, 16, 24, 25, 26, 27, 35, 36, 37, 38, 39, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 63, 64, 65, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 125, 126, 127, 128, 129, 130, 131, 132, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 153, 155, 161, 163, 169, 171, 174, 176, 178, 179, 180, 181, 182, 183, 184, 185, 187, 188, 189, 190, 191, 193, 195, 196, 197, 198, 199, 201, 209, 217, 225, 227, 228, 230, 233, 236, 238, 241, 246, 248, 249, 250, 259, 261, 262, 263, 264, 265, 267, 269, 270, 271, 272, 273, 275, 277, 278, 279, 280, 281, 282, 283, 284, 285, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 308, 316, 324, 332, 333, 334, 335, 337, 340, 343, 345, 348, 353, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 413, 421, 428, 436, 437, 438, 439, 441, 444, 447, 450, 455, 481, 482, 483, 488, 489, 490, 491, 492, 510, 516, 518, 551, 552, 553, 558, 559, 560, 561, 581, 589, 591, 627, 628, 629, 634, 635, 636, 637, 660, 668, 670, 709, 715, 717, 751, 752, 753, 784, 785, 786, 790, 791, 792, 793, 794, 823, 824, 831, 832, 833, 865, 866, 867, 871, 872, 873, 874, 875, 903, 904, 909, 910, 911, 932, 938, 940, 967, 969, 990, 991, 992, 998, 999, 1000, 1001, 1027, 1034, 1036, 1059, 1060, 1061, 1067, 1068, 1069, 1070, 1096, 1103, 1105, 1117, 1119, 1128, 1130, 1132, 1140, 1142, 1144, 1146, 1150, 1151, 1152, 1153, 1154, 1155, 1156, 1157, 1159, 1160, 1161, 1162, 1163, 1164, 1167, 1168, 1169, 1170, 1171, 1179, 1181, 1191, 1200, 1202, 1205, 1207, 1208, 1210, 1211, 1213, 1215, 1217, 1218, 1219, 1220, 1222, 1224, 1227, 1228, 1229, 1239, 1241, 1243, 1246, 1248, 1249, 1250, 1251, 1253, 1255, 1256, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1268, 1269, 1270, 1271, 1272, 1278, 1279, 1280, 1281, 1282, 1283, 1289, 1290, 1291, 1292, 1293, 1294, 1295, 1296, 1298, 1299 }

**F(-1) timedout fail** { 749, 829, 907, 965 }

**F(-2) exception fail** { 200, 202, 203, 204, 205, 206, 207, 208, 210, 211, 212, 213, 214, 215, 216, 218, 219, 220, 221, 222, 223, 224, 229, 231, 232, 237, 239, 240, 242, 247, 253, 254, 255, 307, 309, 310, 311, 312, 313, 314, 315, 317, 318, 319, 320, 321, 322, 323, 325, 326, 327, 328, 329, 330, 331, 336, 338, 339, 344, 346, 347, 349, 350, 352, 354, 359, 360, 412, 414, 415, 416, 417, 418, 419, 420, 422, 423, 424, 425, 426, 427, 429, 430, 431, 432, 433, 434, 435, 440, 442, 443, 448, 449, 451, 452, 453, 454, 456, 461, 462, 507, 509, 511, 513, 515, 517, 519, 527, 528, 529, 578, 580, 582, 584, 586, 588, 590, 592, 594, 603, 604, 605, 657, 659, 661, 663, 665, 667, 669, 671, 679, 680, 681, 706, 712, 720, 722, 723, 724, 726, 727, 728, 730, 731, 733, 742, 750, 782, 788, 796, 798, 799, 800, 801, 803, 804, 805, 806, 808, 809, 810, 812, 821, 828, 830, 863, 869, 877, 879, 880, 881, 882, 884, 885, 886, 887, 889, 890, 891, 893, 908, 929, 935, 943, 947, 951, 961, 963, 966, 968, 970, 1006, 1010, 1014, 1024, 1026, 1028, 1030, 1033, 1035, 1037, 1039, 1075, 1079, 1083, 1093, 1095, 1097, 1099, 1102, 1104, 1106, 1108, 1172, 1174, 1182, 1184, 1192, 1194, 1285 }

## Mupad

**A grade** { }

**B grade** { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 177, 179, 181, 183, 185, 186, 187, 189, 191, 192, 193, 194, 195, 197, 199, 258, 260, 266, 268, 274, 276, 286, 292, 293, 294, 299, 300, 301, 302, 391, 397, 398, 399, 404, 405, 406, 407, 476, 545, 621, 642, 701, 777, 858, 927, 985, 1055, 1111, 1113, 1114, 1115, 1116, 1117, 1118, 1119, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1128, 1129, 1130, 1131, 1132, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1140, 1141, 1142, 1143, 1144, 1145, 1146, 1147, 1148, 1149, 1158, 1165, 1166, 1245, 1247, 1252, 1254, 1273, 1274, 1275, 1276, 1277, 1279, 1284, 1285, 1286, 1287, 1288 }

**C grade** { }

**F normal fail** { }

**F(-1) timedout fail** { 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 174, 176, 178, 180, 182, 184, 188, 190, 196, 198, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 259, 261, 262, 263, 264, 265, 267, 269, 270, 271, 272, 273, 275, 277, 278, 279, 280, 281, 282, 283, 284, 285, 287, 288, 289, 290, 291, 295, 296, 297, 298, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 392, 393, 394, 395, 396, 400, 401, 402, 403, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 481, 482, 483, 488, 489, 490, 491, 492, 509, 510, 515, 516, 517, 518, 551, 552, 553, 558, 559, 560, 561, 580, 581, 588, 589, 590, 591, 627, 628, 629, 634, 635, 636, 637, 659, 660, 667, 668, 669, 670, 708, 709, 710, 714, 715, 716, 717, 718, 744, 745, 750, 751, 752, 753, 784, 785, 786, 790, 791, 792, 793, 794, 823, 824, 830, 831, 832, 833, 865, 866, 867, 871, 872, 873, 874, 875, 903, 904, 908, 909, 910, 911, 931, 932, 933, 937, 938, 939, 940, 941, 961, 962, 966, 967, 968, 969, 990, 991, 992, 998, 999, 1000, 1001, 1026, 1027, 1033, 1034, 1035, 1036, 1059, 1060, 1061, 1067, 1068, 1069, 1070, 1095, 1096, 1102, 1103, 1104, 1105, 1150, 1151, 1152, 1153, 1154, 1155, 1156, 1157, 1159, 1160, 1161, 1162, 1163, 1164, 1167, 1168, 1169, 1170, 1171, 1172, 1174, 1179, 1181, 1182, 1184, 1191, 1192, 1194, 1200, 1202, 1205, 1207, 1208, 1210, 1211, 1213, 1215, 1217, 1218, 1219, 1220, 1222, 1224, 1225, 1226, 1227, 1228, 1229, 1239, 1241, 1243, 1246, 1248, 1249, 1250, 1251, 1253, 1255, 1256, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1268, 1269, 1270, 1271, 1272, 1278, 1280, 1281, 1282, 1283, 1289, 1290, 1291, 1292, 1293, 1294, 1295, 1296, 1298, 1299 }

**F(-2) exception fail** { }

## Sympy

**A grade** { 1, 2, 3, 8, 9, 10, 11, 12, 18, 19, 20, 21, 30, 31, 32, 149, 150, 151, 152, 154, 156, 157, 158, 159, 160, 162, 164, 165, 166, 167, 168, 170, 172, 173, 175, 179, 181, 186, 258, 260, 266, 268, 274, 276, 476, 545, 621, 858, 927, 985, 1055, 1113, 1114, 1115, 1116, 1118, 1120, 1121, 1122, 1123, 1124, 1125, 1127, 1129, 1131, 1133, 1134, 1135, 1136, 1137, 1139, 1141, 1143, 1145, 1147, 1149, 1245, 1247, 1252, 1254, 1273, 1274, 1275, 1276, 1277, 1284, 1285, 1286, 1287, 1288 }

**B grade** { 4, 7, 13, 17, 22, 23, 28, 29, 33, 34, 40, 41, 54, 61, 62, 66, 107, 114, 115, 118, 124, 177, 183, 189, 191, 192, 194, 197, 199, 1126, 1138, 1148 }

**C grade** { 1279, 1281, 1283 }

**F normal fail** { 5, 6, 14, 15, 16, 24, 44, 45, 46, 47, 48, 49, 53, 57, 67, 72, 73, 80, 99, 100, 101, 110, 119, 130, 131, 132, 134, 135, 136, 137, 138, 140, 141, 142, 143, 144, 145, 147, 153, 155, 161, 163, 169, 171, 174, 176, 178, 180, 182, 184, 185, 190, 193, 198, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 218, 219, 220, 221, 222, 223, 224, 225, 227, 228, 229, 230, 231, 232, 233, 235, 237, 238, 239, 240, 241, 242, 243, 245, 247, 248, 249, 250, 259, 261, 262, 263, 264, 265, 267, 269, 270, 271, 272, 273, 275, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 481, 482, 483, 488, 489, 490, 491, 492, 509, 510, 515, 516, 517, 518, 551, 552, 553, 558, 559, 560, 561, 580, 581, 588, 589, 590, 591, 627, 628, 629, 634, 635, 636, 637, 642, 659, 660, 667, 668, 669, 670, 701, 708, 709, 710, 714, 715, 716, 717, 718, 744, 745, 750, 751, 752, 753, 777, 784, 785, 786, 790, 791, 792, 793, 794, 823, 824, 830, 831, 832, 833, 865, 866, 867, 871, 872, 873, 874, 875, 904, 911, 931, 932, 933, 937, 938, 939, 940, 941, 961, 962, 966, 967, 968, 969, 990, 991, 992, 998, 999, 1000, 1001, 1026, 1027, 1059, 1060, 1061, 1067, 1068, 1069, 1070, 1111, 1117, 1119, 1128, 1130, 1132, 1140, 1142, 1144, 1146, 1150, 1151, 1152, 1153, 1154, 1155, 1156, 1172, 1174, 1179, 1181, 1182, 1184, 1191, 1192, 1194, 1200, 1202, 1205, 1207, 1208, 1210, 1211, 1213, 1215, 1217, 1219, 1225, 1226, 1227, 1228, 1229, 1246, 1248, 1249, 1250, 1251, 1253, 1255, 1256, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1278, 1280, 1282, 1291, 1293 }

**F(-1) timedout fail** { 25, 26, 27, 35, 36, 37, 38, 39, 42, 43, 50, 51, 52, 55, 56, 58, 59, 60, 64, 65, 68, 69, 70, 71, 74, 75, 76, 77, 78, 79, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 102, 103, 104, 105, 106, 108, 109, 111, 112, 113, 117, 120, 121, 122, 123, 125, 126, 127, 128, 129, 139, 146, 217, 253, 254, 359, 461, 527, 532, 603, 604, 608, 679, 680, 684, 693, 724, 728, 729, 730, 731, 748, 755, 761, 767, 788, 796, 801, 802, 803, 804, 806, 807, 808, 809, 810, 811, 820, 827, 828, 829, 835, 836, 842, 848, 849, 854, 863, 869, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 898, 899, 900, 901, 902, 903, 905, 906, 907, 908, 909, 910, 912, 921, 935, 947, 951, 952, 953, 954, 959, 964, 979, 987, 997, 1010, 1014, 1015, 1016, 1017, 1018, 1023, 1030, 1031, 1032, 1033, 1034, 1035, 1036, 1037, 1038, 1039, 1040, 1041, 1045, 1049, 1053, 1057, 1066, 1075, 1079, 1080, 1081, 1082, 1083, 1084, 1085, 1086, 1087, 1091, 1092, 1093, 1094, 1095, 1096, 1097, 1098, 1099, 1100, 1101, 1102, 1103, 1104, 1105, 1106, 1107, 1108, 1109, 1112, 1157, 1158, 1159, 1160, 1161, 1162, 1163, 1164, 1165, 1166, 1167, 1168, 1169, 1170, 1171, 1193, 1216, 1218, 1220, 1222, 1223, 1224, 1230, 1231, 1232, 1235, 1236, 1237, 1238, 1239, 1240, 1241, 1242, 1243, 1244, 1266, 1267, 1268, 1269, 1270, 1271, 1272, 1289, 1295, 1296, 1297, 1298, 1299 }

**F(-2) exception fail** { 63, 116, 187, 188, 195, 196, 226, 234, 236, 244, 246, 1290, 1292, 1294 }

## Reduce

**A grade** { }

**B grade** { 1, 2, 3, 4, 7, 8, 9, 10, 11, 12, 13, 17, 18, 19, 20, 21, 22, 23, 28, 29, 30, 31, 32, 33, 34, 40, 41, 42, 149, 150, 151, 152, 154, 156, 157, 158, 159, 160, 162, 164, 165, 166, 167, 168, 170, 172, 173, 175, 177, 179, 181, 183, 185, 186, 187, 189, 191, 192, 193, 194, 195, 197, 199, 200, 202, 207, 208, 210, 216, 218, 224, 226, 229, 231, 232, 234, 235, 237, 239, 240, 242, 243, 244, 245, 247, 258, 260, 266, 268, 274, 276, 286, 292, 293, 294, 299, 300, 301, 302, 341, 342, 349, 350, 351, 352, 391, 397, 398, 399, 404, 405, 406, 407, 445, 446, 451, 452, 453, 454, 476, 545, 621, 642, 701, 777, 858, 927, 985, 1055, 1111, 1113, 1114, 1115, 1116, 1118, 1120, 1121, 1122, 1123, 1124, 1125, 1126, 1127, 1129, 1131, 1133, 1134, 1135, 1136, 1137, 1138, 1139, 1141, 1143, 1145, 1147, 1148, 1149, 1158, 1165, 1166, 1245, 1247, 1252, 1254, 1273, 1274, 1275, 1276, 1277, 1284, 1285, 1286, 1287, 1288 }

**C grade** { }

**F normal fail** { 5, 6, 14, 15, 16, 24, 25, 26, 27, 35, 36, 37, 38, 39, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 153, 155, 161, 163, 169, 171, 174, 176, 178, 180, 182, 184, 188, 190, 196, 198, 201, 203, 204, 205, 206, 209, 211, 212, 213, 214, 215, 217, 219, 220, 221, 222, 223, 225, 227, 228, 230, 233, 236, 238, 241, 246, 248, 249, 250, 259, 261, 262, 263, 264, 265, 267, 269, 270, 271, 272, 273, 275, 277, 278, 279, 280, 281, 282, 283, 284, 285, 287, 288, 289, 290, 291, 295, 296, 297, 298, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 343, 344, 345, 346, 347, 348, 353, 354, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 392, 393, 394, 395, 396, 400, 401, 402, 403, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 447, 448, 449, 450, 455, 456, 481, 482, 483, 488, 489, 490, 491, 492, 509, 510, 515, 516, 517, 518, 551, 552, 553, 558, 559, 560, 561, 580, 581, 588, 589, 590, 591, 627, 628, 629, 634, 635, 636, 637, 659, 660, 667, 668, 669, 670, 708, 709, 710, 714, 715, 716, 717, 718, 744, 745, 750, 751, 752, 753, 784, 785, 786, 790, 791, 792, 793, 794, 823, 824, 830, 831, 832, 833, 865, 866, 867, 871, 872, 873, 874, 875, 903, 904, 908, 909, 910, 911, 931, 932, 933, 937, 938, 939, 940, 941, 961, 962, 966, 967, 968, 969, 990, 991, 992, 998, 999, 1000, 1001, 1026, 1027, 1033, 1034, 1035, 1036, 1059, 1060, 1061, 1067, 1068, 1069, 1070, 1095, 1096, 1102, 1103, 1104, 1105, 1117, 1119, 1128, 1130, 1132, 1140, 1142, 1144, 1146, 1150, 1151, 1152, 1153, 1154, 1155, 1156, 1157, 1159, 1160, 1161, 1162, 1163, 1164, 1167, 1168, 1169, 1170, 1171, 1172, 1174, 1179, 1181, 1182, 1184, 1191, 1192, 1194, 1200, 1202, 1205, 1207, 1208, 1210, 1211, 1213, 1215, 1217, 1218, 1219, 1220, 1222, 1224, 1225, 1226, 1227, 1228, 1229, 1239, 1241, 1243, 1246, 1248, 1249, 1250, 1251, 1253, 1255, 1256, 1257, 1258, 1259, 1260, 1261, 1262, 1263, 1264, 1265, 1266, 1267, 1268, 1269, 1270, 1271, 1272, 1278, 1279, 1280, 1281, 1282, 1283, 1289, 1290, 1291, 1292, 1293, 1294, 1295, 1296, 1298, 1299 }

**F(-1) timedout fail** { }

**F(-2) exception fail** { }

## 2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

| Problem 1 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 117 | 106 | 98 | 99 | 109 | 124 | 184 | 112 | 108 | 109 |
| N.S. | 1 | 0.91 | 0.84 | 0.85 | 0.93 | 1.06 | 1.57 | 0.96 | 0.92 | 0.93 |
| time (sec) | N/A | 0.305 | 0.050 | 0.450 | 0.116 | 0.157 | 1.858 | 0.133 | 0.190 | 1.075 |

| Problem 2 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 105 | 96 | 88 | 91 | 99 | 113 | 167 | 100 | 99 | 99 |
| N.S. | 1 | 0.91 | 0.84 | 0.87 | 0.94 | 1.08 | 1.59 | 0.95 | 0.94 | 0.94 |
| time (sec) | N/A | 0.307 | 0.040 | 0.338 | 0.107 | 0.111 | 1.698 | 0.164 | 0.198 | 0.516 |

| Problem 3 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 91 | 86 | 76 | 82 | 88 | 104 | 158 | 92 | 88 | 87 |
| N.S. | 1 | 0.95 | 0.84 | 0.90 | 0.97 | 1.14 | 1.74 | 1.01 | 0.97 | 0.96 |
| time (sec) | N/A | 0.285 | 0.031 | 0.346 | 0.112 | 0.140 | 1.748 | 0.131 | 0.204 | 0.412 |

| Problem 4 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | B | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 53 | 56 | 84 | 70 | 73 | 89 | 128 | 71 | 69 | 73 |
| N.S. | 1 | 1.06 | 1.58 | 1.32 | 1.38 | 1.68 | 2.42 | 1.34 | 1.30 | 1.38 |
| time (sec) | N/A | 0.231 | 0.003 | 0.252 | 0.105 | 0.101 | 1.369 | 0.123 | 0.201 | 0.842 |

| Problem 5 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 76 | 76 | 76 | 100 | 0 | 0 | 0 | 0 | 52 | 63 |
| N.S. | 1 | 1.00 | 1.00 | 1.32 | 0.00 | 0.00 | 0.00 | 0.00 | 0.68 | 0.83 |
| time (sec) | N/A | 0.270 | 0.004 | 0.292 | 0.000 | 0.000 | 0.000 | 0.000 | 0.198 | 0.379 |

| Problem 6 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 77 | 77 | 75 | 109 | 0 | 0 | 0 | 0 | 63 | 93 |
| N.S. | 1 | 1.00 | 0.97 | 1.42 | 0.00 | 0.00 | 0.00 | 0.00 | 0.82 | 1.21 |
| time (sec) | N/A | 0.305 | 0.032 | 0.341 | 0.000 | 0.000 | 0.000 | 0.000 | 0.204 | 1.175 |

| Problem 7 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 65 | 60 | 88 | 84 | 75 | 99 | 182 | 86 | 83 | 79 |
| N.S. | 1 | 0.92 | 1.35 | 1.29 | 1.15 | 1.52 | 2.80 | 1.32 | 1.28 | 1.22 |
| time (sec) | N/A | 0.255 | 0.037 | 0.346 | 0.120 | 0.115 | 1.924 | 0.131 | 0.194 | 0.855 |

| Problem 8 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 106 | 95 | 94 | 92 | 87 | 109 | 197 | 94 | 91 | 176 |
| N.S. | 1 | 0.90 | 0.89 | 0.87 | 0.82 | 1.03 | 1.86 | 0.89 | 0.86 | 1.66 |
| time (sec) | N/A | 0.315 | 0.035 | 0.318 | 0.129 | 0.119 | 2.555 | 0.137 | 0.197 | 1.236 |

| Problem 9 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 124 | 109 | 99 | 101 | 102 | 119 | 214 | 106 | 102 | 116 |
| N.S. | 1 | 0.88 | 0.80 | 0.81 | 0.82 | 0.96 | 1.73 | 0.85 | 0.82 | 0.94 |
| time (sec) | N/A | 0.328 | 0.036 | 0.388 | 0.111 | 0.118 | 3.963 | 0.130 | 0.184 | 0.660 |

| Problem 10 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 166 | 138 | 124 | 131 | 185 | 172 | 270 | 172 | 141 | 152 |
| N.S. | 1 | 0.83 | 0.75 | 0.79 | 1.11 | 1.04 | 1.63 | 1.04 | 0.85 | 0.92 |
| time (sec) | N/A | 0.444 | 0.077 | 0.614 | 0.110 | 0.127 | 2.296 | 0.135 | 0.200 | 1.077 |

| Problem 11 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 152 | 130 | 116 | 123 | 174 | 160 | 250 | 158 | 132 | 140 |
| N.S. | 1 | 0.86 | 0.76 | 0.81 | 1.14 | 1.05 | 1.64 | 1.04 | 0.87 | 0.92 |
| time (sec) | N/A | 0.418 | 0.072 | 0.521 | 0.117 | 0.137 | 2.111 | 0.164 | 0.206 | 1.058 |

| Problem 12 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 136 | 118 | 101 | 114 | 155 | 148 | 240 | 148 | 121 | 125 |
| N.S. | 1 | 0.87 | 0.74 | 0.84 | 1.14 | 1.09 | 1.76 | 1.09 | 0.89 | 0.92 |
| time (sec) | N/A | 0.406 | 0.056 | 0.520 | 0.123 | 0.118 | 1.951 | 0.136 | 0.198 | 0.998 |

| Problem 13 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 83 | 70 | 57 | 103 | 138 | 127 | 206 | 125 | 104 | 109 |
| N.S. | 1 | 0.84 | 0.69 | 1.24 | 1.66 | 1.53 | 2.48 | 1.51 | 1.25 | 1.31 |
| time (sec) | N/A | 0.248 | 0.026 | 0.330 | 0.121 | 0.126 | 1.608 | 0.137 | 0.196 | 0.451 |

| Problem 14 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | F | F | F | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 129 | 129 | 103 | 134 | 142 | 0 | 0 | 0 | 87 | 131 |
| N.S. | 1 | 1.00 | 0.80 | 1.04 | 1.10 | 0.00 | 0.00 | 0.00 | 0.67 | 1.02 |
| time (sec) | N/A | 0.343 | 0.090 | 0.425 | 0.266 | 0.000 | 0.000 | 0.000 | 0.212 | 1.030 |

| Problem 15 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 89 | 89 | 79 | 112 | 0 | 0 | 0 | 0 | 70 | 141 |
| N.S. | 1 | 1.00 | 0.89 | 1.26 | 0.00 | 0.00 | 0.00 | 0.00 | 0.79 | 1.58 |
| time (sec) | N/A | 0.340 | 0.093 | 0.459 | 0.000 | 0.000 | 0.000 | 0.000 | 0.198 | 0.909 |

| Problem 16 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F | F | F | F | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 152 | 152 | 139 | 157 | 0 | 0 | 0 | 0 | 115 | 161 |
| N.S. | 1 | 1.00 | 0.91 | 1.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.76 | 1.06 |
| time (sec) | N/A | 0.421 | 0.051 | 0.556 | 0.000 | 0.000 | 0.000 | 0.000 | 0.198 | 1.059 |

| Problem 17 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 87 | 80 | 114 | 114 | 144 | 144 | 253 | 139 | 116 | 120 |
| N.S. | 1 | 0.92 | 1.31 | 1.31 | 1.66 | 1.66 | 2.91 | 1.60 | 1.33 | 1.38 |
| time (sec) | N/A | 0.292 | 0.059 | 0.424 | 0.113 | 0.150 | 5.339 | 0.128 | 0.200 | 0.977 |

| Problem 18 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 161 | 136 | 152 | 125 | 152 | 155 | 275 | 154 | 126 | 142 |
| N.S. | 1 | 0.84 | 0.94 | 0.78 | 0.94 | 0.96 | 1.71 | 0.96 | 0.78 | 0.88 |
| time (sec) | N/A | 0.420 | 0.045 | 0.470 | 0.109 | 0.164 | 9.384 | 0.136 | 0.206 | 1.035 |

| Problem 19 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 171 | 142 | 124 | 133 | 183 | 167 | 287 | 163 | 135 | 244 |
| N.S. | 1 | 0.83 | 0.73 | 0.78 | 1.07 | 0.98 | 1.68 | 0.95 | 0.79 | 1.43 |
| time (sec) | N/A | 0.428 | 0.057 | 0.435 | 0.114 | 0.164 | 15.105 | 0.181 | 0.205 | 1.275 |

| Problem 20 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 205 | 169 | 154 | 162 | 261 | 202 | 328 | 212 | 175 | 186 |
| N.S. | 1 | 0.82 | 0.75 | 0.79 | 1.27 | 0.99 | 1.60 | 1.03 | 0.85 | 0.91 |
| time (sec) | N/A | 0.475 | 0.084 | 0.758 | 0.114 | 0.122 | 2.897 | 0.136 | 0.196 | 1.206 |

| Problem 21 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 191 | 161 | 146 | 154 | 242 | 190 | 316 | 198 | 166 | 174 |
| N.S. | 1 | 0.84 | 0.76 | 0.81 | 1.27 | 0.99 | 1.65 | 1.04 | 0.87 | 0.91 |
| time (sec) | N/A | 0.445 | 0.072 | 0.642 | 0.114 | 0.119 | 2.612 | 0.161 | 0.207 | 1.163 |

| Problem 22 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 157 | 136 | 132 | 145 | 222 | 177 | 296 | 188 | 155 | 160 |
| N.S. | 1 | 0.87 | 0.84 | 0.92 | 1.41 | 1.13 | 1.89 | 1.20 | 0.99 | 1.02 |
| time (sec) | N/A | 0.330 | 0.065 | 0.595 | 0.116 | 0.135 | 2.442 | 0.162 | 0.214 | 0.528 |

| Problem 23 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 100 | 96 | 77 | 124 | 197 | 161 | 267 | 166 | 138 | 147 |
| N.S. | 1 | 0.96 | 0.77 | 1.24 | 1.97 | 1.61 | 2.67 | 1.66 | 1.38 | 1.47 |
| time (sec) | N/A | 0.256 | 0.025 | 0.447 | 0.115 | 0.134 | 2.113 | 0.129 | 0.199 | 1.031 |

| Problem 24 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | F | F | F | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 170 | 170 | 139 | 165 | 184 | 0 | 0 | 0 | 121 | 196 |
| N.S. | 1 | 1.00 | 0.82 | 0.97 | 1.08 | 0.00 | 0.00 | 0.00 | 0.71 | 1.15 |
| time (sec) | N/A | 0.400 | 0.125 | 0.506 | 0.248 | 0.000 | 0.000 | 0.000 | 0.205 | 1.171 |

| Problem 25 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | F | F(-1) | F | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 162 | 162 | 150 | 159 | 201 | 0 | 0 | 0 | 130 | 195 |
| N.S. | 1 | 1.00 | 0.93 | 0.98 | 1.24 | 0.00 | 0.00 | 0.00 | 0.80 | 1.20 |
| time (sec) | N/A | 0.396 | 0.124 | 0.600 | 0.265 | 0.000 | 0.000 | 0.000 | 0.204 | 1.020 |

| Problem 26 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F(-1) | F | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 180 | 180 | 164 | 173 | 0 | 0 | 0 | 0 | 139 | 205 |
| N.S. | 1 | 1.00 | 0.91 | 0.96 | 0.00 | 0.00 | 0.00 | 0.00 | 0.77 | 1.14 |
| time (sec) | N/A | 0.421 | 0.122 | 0.763 | 0.000 | 0.000 | 0.000 | 0.000 | 0.200 | 1.003 |

| Problem 27 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F | F | F(-1) | F | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 189 | 189 | 170 | 183 | 0 | 0 | 0 | 0 | 148 | 221 |
| N.S. | 1 | 1.00 | 0.90 | 0.97 | 0.00 | 0.00 | 0.00 | 0.00 | 0.78 | 1.17 |
| time (sec) | N/A | 0.435 | 0.067 | 0.790 | 0.000 | 0.000 | 0.000 | 0.000 | 0.206 | 1.265 |

| Problem 28 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 103 | 86 | 165 | 147 | 202 | 174 | 311 | 182 | 150 | 154 |
| N.S. | 1 | 0.83 | 1.60 | 1.43 | 1.96 | 1.69 | 3.02 | 1.77 | 1.46 | 1.50 |
| time (sec) | N/A | 0.297 | 0.078 | 0.569 | 0.115 | 0.119 | 17.036 | 0.144 | 0.189 | 0.987 |

| Problem 29 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 150 | 121 | 185 | 153 | 224 | 185 | 326 | 192 | 159 | 174 |
| N.S. | 1 | 0.81 | 1.23 | 1.02 | 1.49 | 1.23 | 2.17 | 1.28 | 1.06 | 1.16 |
| time (sec) | N/A | 0.332 | 0.061 | 0.539 | 0.110 | 0.124 | 38.667 | 0.220 | 0.196 | 1.307 |

| Problem 30 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 209 | 179 | 188 | 164 | 248 | 198 | 347 | 206 | 169 | 192 |
| N.S. | 1 | 0.86 | 0.90 | 0.78 | 1.19 | 0.95 | 1.66 | 0.99 | 0.81 | 0.92 |
| time (sec) | N/A | 0.475 | 0.071 | 0.600 | 0.110 | 0.133 | 54.853 | 0.228 | 0.198 | 0.922 |

| Problem 31 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 238 | 198 | 290 | 190 | 337 | 230 | 389 | 252 | 206 | 217 |
| N.S. | 1 | 0.83 | 1.22 | 0.80 | 1.42 | 0.97 | 1.63 | 1.06 | 0.87 | 0.91 |
| time (sec) | N/A | 0.530 | 0.072 | 0.957 | 0.118 | 0.119 | 3.617 | 0.129 | 0.188 | 3.074 |

| Problem 32 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 193 | 169 | 276 | 182 | 318 | 218 | 367 | 238 | 197 | 205 |
| N.S. | 1 | 0.88 | 1.43 | 0.94 | 1.65 | 1.13 | 1.90 | 1.23 | 1.02 | 1.06 |
| time (sec) | N/A | 0.423 | 0.067 | 0.773 | 0.116 | 0.106 | 3.197 | 0.145 | 0.189 | 0.718 |

| Problem 33 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 178 | 153 | 264 | 173 | 290 | 206 | 360 | 228 | 187 | 191 |
| N.S. | 1 | 0.86 | 1.48 | 0.97 | 1.63 | 1.16 | 2.02 | 1.28 | 1.05 | 1.07 |
| time (sec) | N/A | 0.353 | 0.057 | 0.762 | 0.125 | 0.128 | 2.863 | 0.158 | 0.189 | 1.105 |

| Problem 34 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 125 | 115 | 77 | 145 | 264 | 188 | 316 | 206 | 170 | 175 |
| N.S. | 1 | 0.92 | 0.62 | 1.16 | 2.11 | 1.50 | 2.53 | 1.65 | 1.36 | 1.40 |
| time (sec) | N/A | 0.289 | 0.019 | 0.587 | 0.123 | 0.120 | 2.419 | 0.156 | 0.184 | 1.067 |

| Problem 35 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | F | F(-1) | F | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 203 | 203 | 174 | 193 | 220 | 0 | 0 | 0 | 153 | 248 |
| N.S. | 1 | 1.00 | 0.86 | 0.95 | 1.08 | 0.00 | 0.00 | 0.00 | 0.75 | 1.22 |
| time (sec) | N/A | 0.506 | 0.100 | 0.617 | 0.276 | 0.000 | 0.000 | 0.000 | 0.198 | 1.301 |

| Problem 36 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | F | F(-1) | F | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 190 | 190 | 181 | 189 | 240 | 0 | 0 | 0 | 162 | 253 |
| N.S. | 1 | 1.00 | 0.95 | 0.99 | 1.26 | 0.00 | 0.00 | 0.00 | 0.85 | 1.33 |
| time (sec) | N/A | 0.495 | 0.105 | 0.796 | 0.259 | 0.000 | 0.000 | 0.000 | 0.209 | 1.119 |

| Problem 37 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | F | F(-1) | F | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 173 | 173 | 163 | 178 | 251 | 0 | 0 | 0 | 135 | 258 |
| N.S. | 1 | 1.00 | 0.94 | 1.03 | 1.45 | 0.00 | 0.00 | 0.00 | 0.78 | 1.49 |
| time (sec) | N/A | 0.464 | 0.105 | 1.119 | 0.259 | 0.000 | 0.000 | 0.000 | 0.202 | 1.172 |

| Problem 38 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F(-1) | F | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 201 | 201 | 193 | 195 | 0 | 0 | 0 | 0 | 170 | 261 |
| N.S. | 1 | 1.00 | 0.96 | 0.97 | 0.00 | 0.00 | 0.00 | 0.00 | 0.85 | 1.30 |
| time (sec) | N/A | 0.474 | 0.108 | 1.092 | 0.000 | 0.000 | 0.000 | 0.000 | 0.212 | 1.134 |

| Problem 39 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F | F | F(-1) | F | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 227 | 227 | 227 | 214 | 0 | 0 | 0 | 0 | 180 | 298 |
| N.S. | 1 | 1.00 | 1.00 | 0.94 | 0.00 | 0.00 | 0.00 | 0.00 | 0.79 | 1.31 |
| time (sec) | N/A | 0.494 | 0.075 | 1.128 | 0.000 | 0.000 | 0.000 | 0.000 | 0.205 | 1.261 |

| Problem 40 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 117 | 102 | 191 | 175 | 275 | 202 | 366 | 220 | 181 | 186 |
| N.S. | 1 | 0.87 | 1.63 | 1.50 | 2.35 | 1.73 | 3.13 | 1.88 | 1.55 | 1.59 |
| time (sec) | N/A | 0.312 | 0.104 | 0.700 | 0.130 | 0.135 | 43.471 | 0.177 | 0.199 | 1.051 |

| Problem 41 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | B | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 168 | 135 | 235 | 184 | 290 | 216 | 388 | 234 | 191 | 208 |
| N.S. | 1 | 0.80 | 1.40 | 1.10 | 1.73 | 1.29 | 2.31 | 1.39 | 1.14 | 1.24 |
| time (sec) | N/A | 0.357 | 0.074 | 0.786 | 0.127 | 0.123 | 81.578 | 0.199 | 0.195 | 1.237 |

| Problem 42 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | A | A | F(-1) | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 242 | 206 | 293 | 192 | 329 | 228 | 0 | 244 | 200 | 317 |
| N.S. | 1 | 0.85 | 1.21 | 0.79 | 1.36 | 0.94 | 0.00 | 1.01 | 0.83 | 1.31 |
| time (sec) | N/A | 0.503 | 0.060 | 0.733 | 0.123 | 0.130 | 0.000 | 0.520 | 0.198 | 1.515 |

| Problem 43 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 196 | 215 | 166 | 312 | 0 | 0 | 0 | 0 | 68 | 0 |
| N.S. | 1 | 1.10 | 0.85 | 1.59 | 0.00 | 0.00 | 0.00 | 0.00 | 0.35 | 0.00 |
| time (sec) | N/A | 1.104 | 0.435 | 0.487 | 0.000 | 0.000 | 0.000 | 0.000 | 0.205 | 0.000 |

| Problem 44 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 156 | 155 | 132 | 267 | 0 | 0 | 0 | 0 | 59 | 0 |
| N.S. | 1 | 0.99 | 0.85 | 1.71 | 0.00 | 0.00 | 0.00 | 0.00 | 0.38 | 0.00 |
| time (sec) | N/A | 0.699 | 0.156 | 0.390 | 0.000 | 0.000 | 0.000 | 0.000 | 0.208 | 0.000 |

| Problem 45 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 110 | 103 | 108 | 214 | 0 | 0 | 0 | 0 | 44 | 0 |
| N.S. | 1 | 0.94 | 0.98 | 1.95 | 0.00 | 0.00 | 0.00 | 0.00 | 0.40 | 0.00 |
| time (sec) | N/A | 0.445 | 0.212 | 0.393 | 0.000 | 0.000 | 0.000 | 0.000 | 0.200 | 0.000 |

| Problem 46 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 59 | 59 | 60 | 104 | 0 | 0 | 0 | 0 | 37 | 0 |
| N.S. | 1 | 1.00 | 1.02 | 1.76 | 0.00 | 0.00 | 0.00 | 0.00 | 0.63 | 0.00 |
| time (sec) | N/A | 0.292 | 0.011 | 0.293 | 0.000 | 0.000 | 0.000 | 0.000 | 0.201 | 0.000 |

| Problem 47 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | A | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 54 | 54 | 102 | 148 | 0 | 43 | 0 | 0 | 106 | 0 |
| N.S. | 1 | 1.00 | 1.89 | 2.74 | 0.00 | 0.80 | 0.00 | 0.00 | 1.96 | 0.00 |
| time (sec) | N/A | 0.286 | 0.037 | 0.375 | 0.000 | 0.121 | 0.000 | 0.000 | 0.220 | 0.000 |

| Problem 48 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | A | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 100 | 98 | 149 | 200 | 0 | 98 | 0 | 0 | 104 | 0 |
| N.S. | 1 | 0.98 | 1.49 | 2.00 | 0.00 | 0.98 | 0.00 | 0.00 | 1.04 | 0.00 |
| time (sec) | N/A | 0.524 | 0.065 | 0.385 | 0.000 | 0.127 | 0.000 | 0.000 | 0.197 | 0.000 |

| Problem 49 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F | A | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 161 | 138 | 178 | 251 | 0 | 130 | 0 | 0 | 86 | 0 |
| N.S. | 1 | 0.86 | 1.11 | 1.56 | 0.00 | 0.81 | 0.00 | 0.00 | 0.53 | 0.00 |
| time (sec) | N/A | 0.785 | 0.118 | 0.425 | 0.000 | 0.128 | 0.000 | 0.000 | 0.199 | 0.000 |

| Problem 50 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F | A | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 197 | 194 | 254 | 278 | 0 | 155 | 0 | 0 | 226 | 0 |
| N.S. | 1 | 0.98 | 1.29 | 1.41 | 0.00 | 0.79 | 0.00 | 0.00 | 1.15 | 0.00 |
| time (sec) | N/A | 1.173 | 0.087 | 0.434 | 0.000 | 0.151 | 0.000 | 0.000 | 0.214 | 0.000 |

| Problem 51 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 203 | 203 | 186 | 317 | 0 | 0 | 0 | 0 | 59 | 0 |
| N.S. | 1 | 1.00 | 0.92 | 1.56 | 0.00 | 0.00 | 0.00 | 0.00 | 0.29 | 0.00 |
| time (sec) | N/A | 0.450 | 0.788 | 0.484 | 0.000 | 0.000 | 0.000 | 0.000 | 0.204 | 0.000 |

| Problem 52 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 167 | 167 | 153 | 271 | 0 | 0 | 0 | 0 | 59 | 0 |
| N.S. | 1 | 1.00 | 0.92 | 1.62 | 0.00 | 0.00 | 0.00 | 0.00 | 0.35 | 0.00 |
| time (sec) | N/A | 0.424 | 0.589 | 0.436 | 0.000 | 0.000 | 0.000 | 0.000 | 0.200 | 0.000 |

| Problem 53 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 122 | 122 | 128 | 252 | 0 | 0 | 0 | 0 | 55 | 0 |
| N.S. | 1 | 1.00 | 1.05 | 2.07 | 0.00 | 0.00 | 0.00 | 0.00 | 0.45 | 0.00 |
| time (sec) | N/A | 0.360 | 0.062 | 0.428 | 0.000 | 0.000 | 0.000 | 0.000 | 0.216 | 0.000 |

| Problem 54 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | A | B | A | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 69 | 66 | 42 | 68 | 0 | 50 | 116 | 87 | 52 | 0 |
| N.S. | 1 | 0.96 | 0.61 | 0.99 | 0.00 | 0.72 | 1.68 | 1.26 | 0.75 | 0.00 |
| time (sec) | N/A | 0.246 | 0.027 | 0.290 | 0.000 | 0.111 | 0.958 | 0.133 | 0.208 | 0.000 |

| Problem 55 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | A | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 150 | 150 | 128 | 207 | 0 | 127 | 0 | 0 | 60 | 0 |
| N.S. | 1 | 1.00 | 0.85 | 1.38 | 0.00 | 0.85 | 0.00 | 0.00 | 0.40 | 0.00 |
| time (sec) | N/A | 0.408 | 0.111 | 0.404 | 0.000 | 0.155 | 0.000 | 0.000 | 0.209 | 0.000 |

| Problem 56 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | A | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 194 | 194 | 165 | 257 | 0 | 182 | 0 | 0 | 64 | 0 |
| N.S. | 1 | 1.00 | 0.85 | 1.32 | 0.00 | 0.94 | 0.00 | 0.00 | 0.33 | 0.00 |
| time (sec) | N/A | 0.489 | 0.194 | 0.436 | 0.000 | 0.144 | 0.000 | 0.000 | 0.197 | 0.000 |

| Problem 57 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F | A | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 244 | 244 | 222 | 286 | 0 | 221 | 0 | 0 | 64 | 0 |
| N.S. | 1 | 1.00 | 0.91 | 1.17 | 0.00 | 0.91 | 0.00 | 0.00 | 0.26 | 0.00 |
| time (sec) | N/A | 0.537 | 0.226 | 0.447 | 0.000 | 0.131 | 0.000 | 0.000 | 0.211 | 0.000 |

| Problem 58 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 256 | 256 | 235 | 364 | 358 | 0 | 0 | 0 | 159 | 0 |
| N.S. | 1 | 1.00 | 0.92 | 1.42 | 1.40 | 0.00 | 0.00 | 0.00 | 0.62 | 0.00 |
| time (sec) | N/A | 0.550 | 0.738 | 0.493 | 0.121 | 0.000 | 0.000 | 0.000 | 0.238 | 0.000 |

| Problem 59 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 225 | 225 | 216 | 321 | 334 | 0 | 0 | 0 | 148 | 0 |
| N.S. | 1 | 1.00 | 0.96 | 1.43 | 1.48 | 0.00 | 0.00 | 0.00 | 0.66 | 0.00 |
| time (sec) | N/A | 0.511 | 0.602 | 0.454 | 0.109 | 0.000 | 0.000 | 0.000 | 0.201 | 0.000 |

| Problem 60 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 176 | 176 | 187 | 299 | 291 | 0 | 0 | 0 | 144 | 0 |
| N.S. | 1 | 1.00 | 1.06 | 1.70 | 1.65 | 0.00 | 0.00 | 0.00 | 0.82 | 0.00 |
| time (sec) | N/A | 0.439 | 0.113 | 0.447 | 0.110 | 0.000 | 0.000 | 0.000 | 0.202 | 0.000 |

| Problem 61 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 88 | 86 | 63 | 109 | 70 | 85 | 189 | 135 | 403 | 0 |
| N.S. | 1 | 0.98 | 0.72 | 1.24 | 0.80 | 0.97 | 2.15 | 1.53 | 4.58 | 0.00 |
| time (sec) | N/A | 0.289 | 0.060 | 0.461 | 0.043 | 0.111 | 5.124 | 0.116 | 0.204 | 0.000 |

| Problem 62 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 92 | 89 | 55 | 82 | 65 | 75 | 158 | 117 | 286 | 0 |
| N.S. | 1 | 0.97 | 0.60 | 0.89 | 0.71 | 0.82 | 1.72 | 1.27 | 3.11 | 0.00 |
| time (sec) | N/A | 0.260 | 0.029 | 0.319 | 0.044 | 0.091 | 1.718 | 0.124 | 0.200 | 0.000 |

| Problem 63 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 195 | 195 | 162 | 261 | 406 | 201 | 0 | 0 | 78 | 0 |
| N.S. | 1 | 1.00 | 0.83 | 1.34 | 2.08 | 1.03 | 0.00 | 0.00 | 0.40 | 0.00 |
| time (sec) | N/A | 0.482 | 0.152 | 0.458 | 0.104 | 0.128 | 0.000 | 0.000 | 0.199 | 0.000 |

| Problem 64 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | A | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 250 | 250 | 227 | 303 | 0 | 263 | 0 | 0 | 82 | 0 |
| N.S. | 1 | 1.00 | 0.91 | 1.21 | 0.00 | 1.05 | 0.00 | 0.00 | 0.33 | 0.00 |
| time (sec) | N/A | 0.545 | 0.211 | 0.464 | 0.000 | 0.126 | 0.000 | 0.000 | 0.206 | 0.000 |

| Problem 65 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | B | A | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 306 | 306 | 285 | 345 | 590 | 311 | 0 | 0 | 82 | 0 |
| N.S. | 1 | 1.00 | 0.93 | 1.13 | 1.93 | 1.02 | 0.00 | 0.00 | 0.27 | 0.00 |
| time (sec) | N/A | 0.627 | 0.347 | 0.510 | 0.136 | 0.129 | 0.000 | 0.000 | 0.211 | 0.000 |

| Problem 66 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | B | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 100 | 100 | 73 | 79 | 83 | 93 | 168 | 157 | 80 | 0 |
| N.S. | 1 | 1.00 | 0.73 | 0.79 | 0.83 | 0.93 | 1.68 | 1.57 | 0.80 | 0.00 |
| time (sec) | N/A | 0.265 | 0.036 | 0.346 | 0.053 | 0.096 | 1.509 | 0.121 | 0.196 | 0.000 |

| Problem 67 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | A | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 49 | 49 | 88 | 104 | 126 | 21 | 0 | 0 | 88 | 0 |
| N.S. | 1 | 1.00 | 1.80 | 2.12 | 2.57 | 0.43 | 0.00 | 0.00 | 1.80 | 0.00 |
| time (sec) | N/A | 0.294 | 0.018 | 0.328 | 0.107 | 0.107 | 0.000 | 0.000 | 0.211 | 0.000 |

| Problem 68 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 287 | 287 | 285 | 375 | 0 | 0 | 0 | 0 | 309 | 0 |
| N.S. | 1 | 1.00 | 0.99 | 1.31 | 0.00 | 0.00 | 0.00 | 0.00 | 1.08 | 0.00 |
| time (sec) | N/A | 0.864 | 0.536 | 0.979 | 0.000 | 0.000 | 0.000 | 0.000 | 0.200 | 0.000 |

| Problem 69 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 255 | 255 | 241 | 352 | 0 | 0 | 0 | 0 | 270 | 0 |
| N.S. | 1 | 1.00 | 0.95 | 1.38 | 0.00 | 0.00 | 0.00 | 0.00 | 1.06 | 0.00 |
| time (sec) | N/A | 0.747 | 0.401 | 0.935 | 0.000 | 0.000 | 0.000 | 0.000 | 0.190 | 0.000 |

| Problem 70 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 211 | 211 | 208 | 316 | 0 | 0 | 0 | 0 | 234 | 0 |
| N.S. | 1 | 1.00 | 0.99 | 1.50 | 0.00 | 0.00 | 0.00 | 0.00 | 1.11 | 0.00 |
| time (sec) | N/A | 0.595 | 0.492 | 0.813 | 0.000 | 0.000 | 0.000 | 0.000 | 0.193 | 0.000 |

| Problem 71 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 130 | 135 | 151 | 278 | 0 | 0 | 0 | 0 | 175 | 0 |
| N.S. | 1 | 1.04 | 1.16 | 2.14 | 0.00 | 0.00 | 0.00 | 0.00 | 1.35 | 0.00 |
| time (sec) | N/A | 0.353 | 0.261 | 0.677 | 0.000 | 0.000 | 0.000 | 0.000 | 0.192 | 0.000 |

| Problem 72 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 216 | 216 | 272 | 3480 | 0 | 0 | 0 | 0 | 86 | 0 |
| N.S. | 1 | 1.00 | 1.26 | 16.11 | 0.00 | 0.00 | 0.00 | 0.00 | 0.40 | 0.00 |
| time (sec) | N/A | 0.663 | 0.256 | 5.902 | 0.000 | 0.000 | 0.000 | 0.000 | 0.195 | 0.000 |

| Problem 73 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 228 | 228 | 289 | 5609 | 0 | 0 | 0 | 0 | 124 | 0 |
| N.S. | 1 | 1.00 | 1.27 | 24.60 | 0.00 | 0.00 | 0.00 | 0.00 | 0.54 | 0.00 |
| time (sec) | N/A | 0.718 | 0.274 | 8.257 | 0.000 | 0.000 | 0.000 | 0.000 | 0.198 | 0.000 |

| Problem 74 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 159 | 159 | 190 | 357 | 0 | 0 | 0 | 0 | 243 | 0 |
| N.S. | 1 | 1.00 | 1.19 | 2.25 | 0.00 | 0.00 | 0.00 | 0.00 | 1.53 | 0.00 |
| time (sec) | N/A | 0.561 | 0.194 | 1.176 | 0.000 | 0.000 | 0.000 | 0.000 | 0.191 | 0.000 |

| Problem 75 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 224 | 224 | 240 | 408 | 0 | 0 | 0 | 0 | 227 | 0 |
| N.S. | 1 | 1.00 | 1.07 | 1.82 | 0.00 | 0.00 | 0.00 | 0.00 | 1.01 | 0.00 |
| time (sec) | N/A | 0.654 | 0.343 | 1.261 | 0.000 | 0.000 | 0.000 | 0.000 | 0.187 | 0.000 |

| Problem 76 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 373 | 373 | 342 | 443 | 0 | 0 | 0 | 0 | 389 | 0 |
| N.S. | 1 | 1.00 | 0.92 | 1.19 | 0.00 | 0.00 | 0.00 | 0.00 | 1.04 | 0.00 |
| time (sec) | N/A | 1.198 | 0.815 | 1.397 | 0.000 | 0.000 | 0.000 | 0.000 | 0.186 | 0.000 |

| Problem 77 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 333 | 333 | 306 | 420 | 0 | 0 | 0 | 0 | 351 | 0 |
| N.S. | 1 | 1.00 | 0.92 | 1.26 | 0.00 | 0.00 | 0.00 | 0.00 | 1.05 | 0.00 |
| time (sec) | N/A | 1.062 | 0.806 | 1.320 | 0.000 | 0.000 | 0.000 | 0.000 | 0.196 | 0.000 |

| Problem 78 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 293 | 293 | 257 | 384 | 0 | 0 | 0 | 0 | 314 | 0 |
| N.S. | 1 | 1.00 | 0.88 | 1.31 | 0.00 | 0.00 | 0.00 | 0.00 | 1.07 | 0.00 |
| time (sec) | N/A | 0.863 | 0.929 | 1.135 | 0.000 | 0.000 | 0.000 | 0.000 | 0.196 | 0.000 |

| Problem 79 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 192 | 188 | 205 | 354 | 0 | 0 | 0 | 0 | 261 | 0 |
| N.S. | 1 | 0.98 | 1.07 | 1.84 | 0.00 | 0.00 | 0.00 | 0.00 | 1.36 | 0.00 |
| time (sec) | N/A | 0.399 | 0.712 | 0.829 | 0.000 | 0.000 | 0.000 | 0.000 | 0.198 | 0.000 |

| Problem 80 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 300 | 300 | 360 | 1317 | 0 | 0 | 0 | 0 | 186 | 0 |
| N.S. | 1 | 1.00 | 1.20 | 4.39 | 0.00 | 0.00 | 0.00 | 0.00 | 0.62 | 0.00 |
| time (sec) | N/A | 0.854 | 0.468 | 9.519 | 0.000 | 0.000 | 0.000 | 0.000 | 0.203 | 0.000 |

| Problem 81 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 317 | 317 | 378 | 5048 | 0 | 0 | 0 | 0 | 154 | 0 |
| N.S. | 1 | 1.00 | 1.19 | 15.92 | 0.00 | 0.00 | 0.00 | 0.00 | 0.49 | 0.00 |
| time (sec) | N/A | 0.897 | 0.585 | 8.027 | 0.000 | 0.000 | 0.000 | 0.000 | 0.220 | 0.000 |

| Problem 82 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 337 | 337 | 388 | 1381 | 0 | 0 | 0 | 0 | 268 | 0 |
| N.S. | 1 | 1.00 | 1.15 | 4.10 | 0.00 | 0.00 | 0.00 | 0.00 | 0.80 | 0.00 |
| time (sec) | N/A | 0.929 | 0.573 | 14.134 | 0.000 | 0.000 | 0.000 | 0.000 | 0.197 | 0.000 |

| Problem 83 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 267 | 251 | 253 | 445 | 0 | 0 | 0 | 0 | 308 | 0 |
| N.S. | 1 | 0.94 | 0.95 | 1.67 | 0.00 | 0.00 | 0.00 | 0.00 | 1.15 | 0.00 |
| time (sec) | N/A | 0.536 | 0.500 | 1.470 | 0.000 | 0.000 | 0.000 | 0.000 | 0.213 | 0.000 |

| Problem 84 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 438 | 438 | 408 | 511 | 0 | 0 | 0 | 0 | 473 | 0 |
| N.S. | 1 | 1.00 | 0.93 | 1.17 | 0.00 | 0.00 | 0.00 | 0.00 | 1.08 | 0.00 |
| time (sec) | N/A | 1.619 | 1.267 | 1.853 | 0.000 | 0.000 | 0.000 | 0.000 | 0.203 | 0.000 |

| Problem 85 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 402 | 402 | 369 | 488 | 0 | 0 | 0 | 0 | 435 | 0 |
| N.S. | 1 | 1.00 | 0.92 | 1.21 | 0.00 | 0.00 | 0.00 | 0.00 | 1.08 | 0.00 |
| time (sec) | N/A | 1.448 | 0.945 | 1.739 | 0.000 | 0.000 | 0.000 | 0.000 | 0.200 | 0.000 |

| Problem 86 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 307 | 307 | 325 | 452 | 0 | 0 | 0 | 0 | 398 | 0 |
| N.S. | 1 | 1.00 | 1.06 | 1.47 | 0.00 | 0.00 | 0.00 | 0.00 | 1.30 | 0.00 |
| time (sec) | N/A | 0.812 | 1.897 | 1.469 | 0.000 | 0.000 | 0.000 | 0.000 | 0.191 | 0.000 |

| Problem 87 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 226 | 215 | 267 | 412 | 0 | 0 | 0 | 0 | 348 | 0 |
| N.S. | 1 | 0.95 | 1.18 | 1.82 | 0.00 | 0.00 | 0.00 | 0.00 | 1.54 | 0.00 |
| time (sec) | N/A | 0.461 | 1.540 | 1.210 | 0.000 | 0.000 | 0.000 | 0.000 | 0.201 | 0.000 |

| Problem 88 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 385 | 385 | 465 | 1409 | 0 | 0 | 0 | 0 | 290 | 0 |
| N.S. | 1 | 1.00 | 1.21 | 3.66 | 0.00 | 0.00 | 0.00 | 0.00 | 0.75 | 0.00 |
| time (sec) | N/A | 1.078 | 0.716 | 11.952 | 0.000 | 0.000 | 0.000 | 0.000 | 0.210 | 0.000 |

| Problem 89 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 402 | 402 | 512 | 1437 | 0 | 0 | 0 | 0 | 286 | 0 |
| N.S. | 1 | 1.00 | 1.27 | 3.57 | 0.00 | 0.00 | 0.00 | 0.00 | 0.71 | 0.00 |
| time (sec) | N/A | 1.060 | 0.479 | 15.746 | 0.000 | 0.000 | 0.000 | 0.000 | 0.199 | 0.000 |

| Problem 90 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 416 | 416 | 500 | 1523 | 0 | 0 | 0 | 0 | 315 | 0 |
| N.S. | 1 | 1.00 | 1.20 | 3.66 | 0.00 | 0.00 | 0.00 | 0.00 | 0.76 | 0.00 |
| time (sec) | N/A | 1.057 | 0.796 | 14.911 | 0.000 | 0.000 | 0.000 | 0.000 | 0.209 | 0.000 |

| Problem 91 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 429 | 429 | 595 | 1655 | 0 | 0 | 0 | 0 | 367 | 0 |
| N.S. | 1 | 1.00 | 1.39 | 3.86 | 0.00 | 0.00 | 0.00 | 0.00 | 0.86 | 0.00 |
| time (sec) | N/A | 1.238 | 0.552 | 18.372 | 0.000 | 0.000 | 0.000 | 0.000 | 0.210 | 0.000 |

| Problem 92 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 293 | 276 | 322 | 501 | 0 | 0 | 0 | 0 | 390 | 0 |
| N.S. | 1 | 0.94 | 1.10 | 1.71 | 0.00 | 0.00 | 0.00 | 0.00 | 1.33 | 0.00 |
| time (sec) | N/A | 0.613 | 0.634 | 1.935 | 0.000 | 0.000 | 0.000 | 0.000 | 0.200 | 0.000 |

| Problem 93 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 384 | 362 | 363 | 541 | 0 | 0 | 0 | 0 | 428 | 0 |
| N.S. | 1 | 0.94 | 0.95 | 1.41 | 0.00 | 0.00 | 0.00 | 0.00 | 1.11 | 0.00 |
| time (sec) | N/A | 0.697 | 0.966 | 2.233 | 0.000 | 0.000 | 0.000 | 0.000 | 0.211 | 0.000 |

| Problem 94 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 513 | 483 | 401 | 564 | 0 | 0 | 0 | 0 | 465 | 0 |
| N.S. | 1 | 0.94 | 0.78 | 1.10 | 0.00 | 0.00 | 0.00 | 0.00 | 0.91 | 0.00 |
| time (sec) | N/A | 0.884 | 1.063 | 2.112 | 0.000 | 0.000 | 0.000 | 0.000 | 0.193 | 0.000 |

| Problem 95 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 356 | 469 | 421 | 1105 | 0 | 0 | 0 | 0 | 105 | 0 |
| N.S. | 1 | 1.32 | 1.18 | 3.10 | 0.00 | 0.00 | 0.00 | 0.00 | 0.29 | 0.00 |
| time (sec) | N/A | 4.430 | 0.867 | 11.441 | 0.000 | 0.000 | 0.000 | 0.000 | 0.219 | 0.000 |

| Problem 96 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 277 | 303 | 330 | 994 | 0 | 0 | 0 | 0 | 94 | 0 |
| N.S. | 1 | 1.09 | 1.19 | 3.59 | 0.00 | 0.00 | 0.00 | 0.00 | 0.34 | 0.00 |
| time (sec) | N/A | 2.716 | 0.490 | 9.485 | 0.000 | 0.000 | 0.000 | 0.000 | 0.210 | 0.000 |

| Problem 97 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 192 | 212 | 239 | 2330 | 0 | 0 | 0 | 0 | 75 | 0 |
| N.S. | 1 | 1.10 | 1.24 | 12.14 | 0.00 | 0.00 | 0.00 | 0.00 | 0.39 | 0.00 |
| time (sec) | N/A | 1.438 | 0.607 | 8.079 | 0.000 | 0.000 | 0.000 | 0.000 | 0.200 | 0.000 |

| Problem 98 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 98 | 100 | 95 | 852 | 0 | 0 | 0 | 0 | 63 | 0 |
| N.S. | 1 | 1.02 | 0.97 | 8.69 | 0.00 | 0.00 | 0.00 | 0.00 | 0.64 | 0.00 |
| time (sec) | N/A | 0.485 | 0.024 | 7.782 | 0.000 | 0.000 | 0.000 | 0.000 | 0.191 | 0.000 |

| Problem 99 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | A | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 88 | 95 | 173 | 1447 | 0 | 136 | 0 | 0 | 208 | 0 |
| N.S. | 1 | 1.08 | 1.97 | 16.44 | 0.00 | 1.55 | 0.00 | 0.00 | 2.36 | 0.00 |
| time (sec) | N/A | 0.507 | 0.390 | 9.398 | 0.000 | 0.120 | 0.000 | 0.000 | 0.209 | 0.000 |

| Problem 100 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 186 | 195 | 265 | 8468 | 0 | 0 | 0 | 0 | 195 | 0 |
| N.S. | 1 | 1.05 | 1.42 | 45.53 | 0.00 | 0.00 | 0.00 | 0.00 | 1.05 | 0.00 |
| time (sec) | N/A | 1.376 | 0.711 | 12.822 | 0.000 | 0.000 | 0.000 | 0.000 | 0.206 | 0.000 |

| Problem 101 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 273 | 276 | 372 | 1873 | 0 | 0 | 0 | 0 | 320 | 0 |
| N.S. | 1 | 1.01 | 1.36 | 6.86 | 0.00 | 0.00 | 0.00 | 0.00 | 1.17 | 0.00 |
| time (sec) | N/A | 2.835 | 0.911 | 16.049 | 0.000 | 0.000 | 0.000 | 0.000 | 0.203 | 0.000 |

| Problem 102 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 365 | 418 | 535 | 2180 | 0 | 0 | 0 | 0 | 537 | 0 |
| N.S. | 1 | 1.15 | 1.47 | 5.97 | 0.00 | 0.00 | 0.00 | 0.00 | 1.47 | 0.00 |
| time (sec) | N/A | 4.001 | 0.790 | 19.836 | 0.000 | 0.000 | 0.000 | 0.000 | 0.227 | 0.000 |

| Problem 103 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 433 | 433 | 502 | 1199 | 0 | 0 | 0 | 0 | 95 | 0 |
| N.S. | 1 | 1.00 | 1.16 | 2.77 | 0.00 | 0.00 | 0.00 | 0.00 | 0.22 | 0.00 |
| time (sec) | N/A | 1.178 | 1.665 | 15.681 | 0.000 | 0.000 | 0.000 | 0.000 | 0.204 | 0.000 |

| Problem 104 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 364 | 364 | 429 | 1104 | 0 | 0 | 0 | 0 | 95 | 0 |
| N.S. | 1 | 1.00 | 1.18 | 3.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.26 | 0.00 |
| time (sec) | N/A | 0.919 | 1.337 | 12.149 | 0.000 | 0.000 | 0.000 | 0.000 | 0.196 | 0.000 |

| Problem 105 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 292 | 292 | 362 | 4182 | 0 | 0 | 0 | 0 | 95 | 0 |
| N.S. | 1 | 1.00 | 1.24 | 14.32 | 0.00 | 0.00 | 0.00 | 0.00 | 0.33 | 0.00 |
| time (sec) | N/A | 0.774 | 0.962 | 10.080 | 0.000 | 0.000 | 0.000 | 0.000 | 0.202 | 0.000 |

| Problem 106 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 216 | 216 | 300 | 857 | 0 | 0 | 0 | 0 | 89 | 0 |
| N.S. | 1 | 1.00 | 1.39 | 3.97 | 0.00 | 0.00 | 0.00 | 0.00 | 0.41 | 0.00 |
| time (sec) | N/A | 0.592 | 0.815 | 8.262 | 0.000 | 0.000 | 0.000 | 0.000 | 0.194 | 0.000 |

| Problem 107 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F(-2) | A | B | A | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 122 | 131 | 72 | 237 | 0 | 103 | 303 | 146 | 85 | 0 |
| N.S. | 1 | 1.07 | 0.59 | 1.94 | 0.00 | 0.84 | 2.48 | 1.20 | 0.70 | 0.00 |
| time (sec) | N/A | 0.354 | 0.587 | 0.820 | 0.000 | 0.102 | 5.199 | 0.150 | 0.193 | 0.000 |

| Problem 108 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 221 | 221 | 299 | 1670 | 0 | 0 | 0 | 0 | 97 | 0 |
| N.S. | 1 | 1.00 | 1.35 | 7.56 | 0.00 | 0.00 | 0.00 | 0.00 | 0.44 | 0.00 |
| time (sec) | N/A | 0.842 | 0.699 | 12.105 | 0.000 | 0.000 | 0.000 | 0.000 | 0.196 | 0.000 |

| Problem 109 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 306 | 306 | 398 | 8556 | 0 | 0 | 0 | 0 | 103 | 0 |
| N.S. | 1 | 1.00 | 1.30 | 27.96 | 0.00 | 0.00 | 0.00 | 0.00 | 0.34 | 0.00 |
| time (sec) | N/A | 1.060 | 1.800 | 14.769 | 0.000 | 0.000 | 0.000 | 0.000 | 0.196 | 0.000 |

| Problem 110 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F(-2) | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 403 | 403 | 491 | 1967 | 0 | 0 | 0 | 0 | 103 | 0 |
| N.S. | 1 | 1.00 | 1.22 | 4.88 | 0.00 | 0.00 | 0.00 | 0.00 | 0.26 | 0.00 |
| time (sec) | N/A | 1.208 | 1.909 | 21.266 | 0.000 | 0.000 | 0.000 | 0.000 | 0.191 | 0.000 |

| Problem 111 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 462 | 462 | 578 | 1215 | 0 | 0 | 0 | 0 | 215 | 0 |
| N.S. | 1 | 1.00 | 1.25 | 2.63 | 0.00 | 0.00 | 0.00 | 0.00 | 0.47 | 0.00 |
| time (sec) | N/A | 1.095 | 1.762 | 14.612 | 0.000 | 0.000 | 0.000 | 0.000 | 0.210 | 0.000 |

| Problem 112 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 383 | 383 | 507 | 4306 | 0 | 0 | 0 | 0 | 202 | 0 |
| N.S. | 1 | 1.00 | 1.32 | 11.24 | 0.00 | 0.00 | 0.00 | 0.00 | 0.53 | 0.00 |
| time (sec) | N/A | 0.941 | 1.118 | 10.303 | 0.000 | 0.000 | 0.000 | 0.000 | 0.211 | 0.000 |

| Problem 113 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 304 | 304 | 431 | 960 | 0 | 0 | 0 | 0 | 196 | 0 |
| N.S. | 1 | 1.00 | 1.42 | 3.16 | 0.00 | 0.00 | 0.00 | 0.00 | 0.64 | 0.00 |
| time (sec) | N/A | 0.798 | 0.922 | 9.256 | 0.000 | 0.000 | 0.000 | 0.000 | 0.193 | 0.000 |

| Problem 114 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | A | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 178 | 178 | 117 | 315 | 141 | 165 | 502 | 0 | 546 | 0 |
| N.S. | 1 | 1.00 | 0.66 | 1.77 | 0.79 | 0.93 | 2.82 | 0.00 | 3.07 | 0.00 |
| time (sec) | N/A | 0.447 | 0.379 | 1.024 | 0.082 | 0.109 | 45.771 | 0.000 | 0.224 | 0.000 |

| Problem 115 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 180 | 184 | 110 | 277 | 136 | 158 | 464 | 0 | 483 | 0 |
| N.S. | 1 | 1.02 | 0.61 | 1.54 | 0.76 | 0.88 | 2.58 | 0.00 | 2.68 | 0.00 |
| time (sec) | N/A | 0.418 | 0.380 | 0.922 | 0.071 | 0.101 | 36.168 | 0.000 | 0.220 | 0.000 |

| Problem 116 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 299 | 299 | 435 | 1783 | 0 | 0 | 0 | 0 | 124 | 0 |
| N.S. | 1 | 1.00 | 1.45 | 5.96 | 0.00 | 0.00 | 0.00 | 0.00 | 0.41 | 0.00 |
| time (sec) | N/A | 1.018 | 1.042 | 12.697 | 0.000 | 0.000 | 0.000 | 0.000 | 0.204 | 0.000 |

| Problem 117 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F(-2) | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 391 | 391 | 549 | 8688 | 0 | 0 | 0 | 0 | 130 | 0 |
| N.S. | 1 | 1.00 | 1.40 | 22.22 | 0.00 | 0.00 | 0.00 | 0.00 | 0.33 | 0.00 |
| time (sec) | N/A | 1.228 | 2.324 | 17.958 | 0.000 | 0.000 | 0.000 | 0.000 | 0.215 | 0.000 |

| Problem 118 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 207 | 207 | 155 | 297 | 184 | 206 | 552 | 0 | 130 | 0 |
| N.S. | 1 | 1.00 | 0.75 | 1.43 | 0.89 | 1.00 | 2.67 | 0.00 | 0.63 | 0.00 |
| time (sec) | N/A | 0.467 | 0.296 | 1.026 | 0.096 | 0.104 | 24.311 | 0.000 | 0.200 | 0.000 |

| Problem 119 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 76 | 85 | 82 | 193 | 0 | 0 | 0 | 0 | 86 | 0 |
| N.S. | 1 | 1.12 | 1.08 | 2.54 | 0.00 | 0.00 | 0.00 | 0.00 | 1.13 | 0.00 |
| time (sec) | N/A | 0.523 | 0.193 | 2.051 | 0.000 | 0.000 | 0.000 | 0.000 | 0.190 | 0.000 |

| Problem 120 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 382 | 380 | 693 | 1513 | 0 | 0 | 0 | 0 | 631 | 0 |
| N.S. | 1 | 0.99 | 1.81 | 3.96 | 0.00 | 0.00 | 0.00 | 0.00 | 1.65 | 0.00 |
| time (sec) | N/A | 0.957 | 2.012 | 19.042 | 0.000 | 0.000 | 0.000 | 0.000 | 0.197 | 0.000 |

| Problem 121 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 298 | 307 | 528 | 1384 | 0 | 0 | 0 | 0 | 473 | 0 |
| N.S. | 1 | 1.03 | 1.77 | 4.64 | 0.00 | 0.00 | 0.00 | 0.00 | 1.59 | 0.00 |
| time (sec) | N/A | 0.752 | 1.631 | 14.092 | 0.000 | 0.000 | 0.000 | 0.000 | 0.208 | 0.000 |

| Problem 122 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 220 | 232 | 367 | 3777 | 0 | 0 | 0 | 0 | 306 | 0 |
| N.S. | 1 | 1.05 | 1.67 | 17.17 | 0.00 | 0.00 | 0.00 | 0.00 | 1.39 | 0.00 |
| time (sec) | N/A | 0.574 | 0.479 | 12.290 | 0.000 | 0.000 | 0.000 | 0.000 | 0.195 | 0.000 |

| Problem 123 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 139 | 141 | 133 | 1629 | 0 | 0 | 0 | 0 | 89 | 0 |
| N.S. | 1 | 1.01 | 0.96 | 11.72 | 0.00 | 0.00 | 0.00 | 0.00 | 0.64 | 0.00 |
| time (sec) | N/A | 0.733 | 0.053 | 11.047 | 0.000 | 0.000 | 0.000 | 0.000 | 0.211 | 0.000 |

| Problem 124 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F(-2) | A | B | A | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 182 | 186 | 121 | 337 | 0 | 175 | 631 | 255 | 118 | 0 |
| N.S. | 1 | 1.02 | 0.66 | 1.85 | 0.00 | 0.96 | 3.47 | 1.40 | 0.65 | 0.00 |
| time (sec) | N/A | 0.457 | 0.728 | 3.371 | 0.000 | 0.145 | 16.561 | 0.151 | 0.209 | 0.000 |

| Problem 125 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 271 | 276 | 183 | 425 | 233 | 265 | 0 | 0 | 684 | 0 |
| N.S. | 1 | 1.02 | 0.68 | 1.57 | 0.86 | 0.98 | 0.00 | 0.00 | 2.52 | 0.00 |
| time (sec) | N/A | 0.626 | 0.515 | 4.776 | 0.143 | 0.132 | 0.000 | 0.000 | 0.252 | 0.000 |

| Problem 126 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 360 | 358 | 269 | 518 | 327 | 359 | 0 | 0 | 184 | 0 |
| N.S. | 1 | 0.99 | 0.75 | 1.44 | 0.91 | 1.00 | 0.00 | 0.00 | 0.51 | 0.00 |
| time (sec) | N/A | 0.904 | 0.645 | 5.205 | 0.216 | 0.116 | 0.000 | 0.000 | 0.213 | 0.000 |

| Problem 127 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 410 | 451 | 541 | 1359 | 0 | 0 | 0 | 0 | 125 | 0 |
| N.S. | 1 | 1.10 | 1.32 | 3.31 | 0.00 | 0.00 | 0.00 | 0.00 | 0.30 | 0.00 |
| time (sec) | N/A | 3.952 | 0.889 | 22.690 | 0.000 | 0.000 | 0.000 | 0.000 | 0.202 | 0.000 |

| Problem 128 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 277 | 290 | 393 | 3018 | 0 | 0 | 0 | 0 | 104 | 0 |
| N.S. | 1 | 1.05 | 1.42 | 10.90 | 0.00 | 0.00 | 0.00 | 0.00 | 0.38 | 0.00 |
| time (sec) | N/A | 1.748 | 0.865 | 14.477 | 0.000 | 0.000 | 0.000 | 0.000 | 0.199 | 0.000 |

| Problem 129 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 139 | 141 | 133 | 1629 | 0 | 0 | 0 | 0 | 89 | 0 |
| N.S. | 1 | 1.01 | 0.96 | 11.72 | 0.00 | 0.00 | 0.00 | 0.00 | 0.64 | 0.00 |
| time (sec) | N/A | 0.702 | 0.043 | 0.991 | 0.000 | 0.000 | 0.000 | 0.000 | 0.190 | 0.000 |

| Problem 130 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | C | F | B | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 128 | 136 | 268 | 2791 | 0 | 246 | 0 | 0 | 312 | 0 |
| N.S. | 1 | 1.06 | 2.09 | 21.80 | 0.00 | 1.92 | 0.00 | 0.00 | 2.44 | 0.00 |
| time (sec) | N/A | 0.716 | 0.541 | 16.381 | 0.000 | 0.117 | 0.000 | 0.000 | 0.204 | 0.000 |

| Problem 131 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 263 | 281 | 436 | 10105 | 0 | 0 | 0 | 0 | 284 | 0 |
| N.S. | 1 | 1.07 | 1.66 | 38.42 | 0.00 | 0.00 | 0.00 | 0.00 | 1.08 | 0.00 |
| time (sec) | N/A | 2.043 | 1.110 | 38.247 | 0.000 | 0.000 | 0.000 | 0.000 | 0.197 | 0.000 |

| Problem 132 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 414 | 421 | 634 | 2450 | 0 | 0 | 0 | 0 | 698 | 0 |
| N.S. | 1 | 1.02 | 1.53 | 5.92 | 0.00 | 0.00 | 0.00 | 0.00 | 1.69 | 0.00 |
| time (sec) | N/A | 4.011 | 1.810 | 42.458 | 0.000 | 0.000 | 0.000 | 0.000 | 0.224 | 0.000 |

| Problem 133 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 20 | 22 | 43 | 34 | 22 | 30 | 23 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 1.00 | 1.95 | 1.55 | 1.00 | 1.36 | 1.05 |
| time (sec) | N/A | 0.213 | 2.239 | 1.057 | 0.120 | 0.115 | 5.100 | 0.117 | 0.202 | 0.740 |

| Problem 134 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 297 | 297 | 484 | 377 | 0 | 0 | 0 | 0 | 68 | 0 |
| N.S. | 1 | 1.00 | 1.63 | 1.27 | 0.00 | 0.00 | 0.00 | 0.00 | 0.23 | 0.00 |
| time (sec) | N/A | 0.555 | 2.244 | 0.852 | 0.000 | 0.000 | 0.000 | 0.000 | 0.206 | 0.000 |

| Problem 135 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 237 | 237 | 404 | 284 | 0 | 0 | 0 | 0 | 55 | 0 |
| N.S. | 1 | 1.00 | 1.70 | 1.20 | 0.00 | 0.00 | 0.00 | 0.00 | 0.23 | 0.00 |
| time (sec) | N/A | 0.459 | 1.130 | 0.750 | 0.000 | 0.000 | 0.000 | 0.000 | 0.212 | 0.000 |

| Problem 136 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 179 | 179 | 329 | 224 | 0 | 0 | 0 | 0 | 39 | 0 |
| N.S. | 1 | 1.00 | 1.84 | 1.25 | 0.00 | 0.00 | 0.00 | 0.00 | 0.22 | 0.00 |
| time (sec) | N/A | 0.395 | 1.040 | 0.678 | 0.000 | 0.000 | 0.000 | 0.000 | 0.217 | 0.000 |

| Problem 137 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 138 | 138 | 138 | 156 | 0 | 0 | 0 | 0 | 30 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 1.13 | 0.00 | 0.00 | 0.00 | 0.00 | 0.22 | 0.00 |
| time (sec) | N/A | 0.472 | 0.019 | 0.294 | 0.000 | 0.000 | 0.000 | 0.000 | 0.201 | 0.000 |

| Problem 138 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 181 | 181 | 169 | 235 | 0 | 0 | 0 | 0 | 113 | 0 |
| N.S. | 1 | 1.00 | 0.93 | 1.30 | 0.00 | 0.00 | 0.00 | 0.00 | 0.62 | 0.00 |
| time (sec) | N/A | 0.424 | 0.075 | 0.700 | 0.000 | 0.000 | 0.000 | 0.000 | 0.211 | 0.000 |

| Problem 139 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 232 | 232 | 223 | 306 | 0 | 0 | 0 | 0 | 110 | 0 |
| N.S. | 1 | 1.00 | 0.96 | 1.32 | 0.00 | 0.00 | 0.00 | 0.00 | 0.47 | 0.00 |
| time (sec) | N/A | 0.496 | 0.106 | 0.785 | 0.000 | 0.000 | 0.000 | 0.000 | 0.212 | 0.000 |

| Problem 140 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 293 | 293 | 298 | 357 | 0 | 0 | 0 | 0 | 76 | 0 |
| N.S. | 1 | 1.00 | 1.02 | 1.22 | 0.00 | 0.00 | 0.00 | 0.00 | 0.26 | 0.00 |
| time (sec) | N/A | 0.570 | 0.124 | 0.886 | 0.000 | 0.000 | 0.000 | 0.000 | 0.197 | 0.000 |

| Problem 141 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | F | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | N/A | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 598 | 598 | 0 | 2062 | 0 | 0 | 0 | 0 | 104 | 0 |
| N.S. | 1 | 1.00 | 0.00 | 3.45 | 0.00 | 0.00 | 0.00 | 0.00 | 0.17 | 0.00 |
| time (sec) | N/A | 1.030 | 0.000 | 66.735 | 0.000 | 0.000 | 0.000 | 0.000 | 0.199 | 0.000 |

| Problem 142 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | F | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | N/A | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 430 | 430 | 0 | 1710 | 0 | 0 | 0 | 0 | 89 | 0 |
| N.S. | 1 | 1.00 | 0.00 | 3.98 | 0.00 | 0.00 | 0.00 | 0.00 | 0.21 | 0.00 |
| time (sec) | N/A | 0.768 | 0.000 | 62.737 | 0.000 | 0.000 | 0.000 | 0.000 | 0.205 | 0.000 |

| Problem 143 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | F | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | N/A | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 323 | 323 | 0 | 15752 | 0 | 0 | 0 | 0 | 69 | 0 |
| N.S. | 1 | 1.00 | 0.00 | 48.77 | 0.00 | 0.00 | 0.00 | 0.00 | 0.21 | 0.00 |
| time (sec) | N/A | 0.566 | 0.000 | 36.404 | 0.000 | 0.000 | 0.000 | 0.000 | 0.198 | 0.000 |

| Problem 144 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | F | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | N/A | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 223 | 223 | 0 | 1199 | 0 | 0 | 0 | 0 | 55 | 0 |
| N.S. | 1 | 1.00 | 0.00 | 5.38 | 0.00 | 0.00 | 0.00 | 0.00 | 0.25 | 0.00 |
| time (sec) | N/A | 0.306 | 0.000 | 0.708 | 0.000 | 0.000 | 0.000 | 0.000 | 0.192 | 0.000 |

| Problem 145 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | F | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | N/A | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 369 | 369 | 0 | 2157 | 0 | 0 | 0 | 0 | 219 | 0 |
| N.S. | 1 | 1.00 | 0.00 | 5.85 | 0.00 | 0.00 | 0.00 | 0.00 | 0.59 | 0.00 |
| time (sec) | N/A | 0.722 | 0.000 | 23.734 | 0.000 | 0.000 | 0.000 | 0.000 | 0.202 | 0.000 |

| Problem 146 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | F | C | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | N/A | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 473 | 473 | 0 | 38040 | 0 | 0 | 0 | 0 | 209 | 0 |
| N.S. | 1 | 1.00 | 0.00 | 80.42 | 0.00 | 0.00 | 0.00 | 0.00 | 0.44 | 0.00 |
| time (sec) | N/A | 0.929 | 0.000 | 52.060 | 0.000 | 0.000 | 0.000 | 0.000 | 0.197 | 0.000 |

| Problem 147 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | F | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | N/A | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 591 | 591 | 0 | 2804 | 0 | 0 | 0 | 0 | 299 | 0 |
| N.S. | 1 | 1.00 | 0.00 | 4.74 | 0.00 | 0.00 | 0.00 | 0.00 | 0.51 | 0.00 |
| time (sec) | N/A | 1.129 | 0.000 | 92.652 | 0.000 | 0.000 | 0.000 | 0.000 | 0.213 | 0.000 |

| Problem 148 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 18 | 18 | 20 | 18 | 20 | 25 | 15 | 20 | 27 | 20 |
| N.S. | 1 | 1.00 | 1.11 | 1.00 | 1.11 | 1.39 | 0.83 | 1.11 | 1.50 | 1.11 |
| time (sec) | N/A | 0.208 | 0.433 | 0.986 | 0.107 | 0.089 | 1.296 | 0.115 | 0.200 | 0.777 |

| Problem 149 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 69 | 102 | 69 | 60 | 64 | 57 | 65 | 64 | 57 | 57 |
| N.S. | 1 | 1.48 | 1.00 | 0.87 | 0.93 | 0.83 | 0.94 | 0.93 | 0.83 | 0.83 |
| time (sec) | N/A | 0.346 | 0.010 | 0.582 | 0.111 | 0.101 | 0.333 | 0.124 | 0.200 | 0.330 |

| Problem 150 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 66 | 98 | 66 | 63 | 63 | 62 | 61 | 63 | 59 | 58 |
| N.S. | 1 | 1.48 | 1.00 | 0.95 | 0.95 | 0.94 | 0.92 | 0.95 | 0.89 | 0.88 |
| time (sec) | N/A | 0.349 | 0.017 | 0.497 | 0.033 | 0.125 | 0.305 | 0.122 | 0.221 | 0.764 |

| Problem 151 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 42 | 46 | 58 | 51 | 50 | 44 | 54 | 53 | 49 | 48 |
| N.S. | 1 | 1.10 | 1.38 | 1.21 | 1.19 | 1.05 | 1.29 | 1.26 | 1.17 | 1.14 |
| time (sec) | N/A | 0.224 | 0.006 | 0.608 | 0.028 | 0.097 | 0.256 | 0.125 | 0.250 | 0.721 |

| Problem 152 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 50 | 66 | 50 | 46 | 45 | 47 | 48 | 43 | 47 | 46 |
| N.S. | 1 | 1.32 | 1.00 | 0.92 | 0.90 | 0.94 | 0.96 | 0.86 | 0.94 | 0.92 |
| time (sec) | N/A | 0.252 | 0.009 | 0.204 | 0.036 | 0.107 | 0.218 | 0.121 | 0.216 | 0.177 |

| Problem 153 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | F | F | F | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 62 | 69 | 62 | 74 | 66 | 0 | 0 | 0 | 35 | 57 |
| N.S. | 1 | 1.11 | 1.00 | 1.19 | 1.06 | 0.00 | 0.00 | 0.00 | 0.56 | 0.92 |
| time (sec) | N/A | 0.351 | 0.005 | 0.277 | 0.159 | 0.000 | 0.000 | 0.000 | 0.213 | 0.825 |

| Problem 154 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 40 | 64 | 40 | 41 | 40 | 45 | 41 | 41 | 42 | 42 |
| N.S. | 1 | 1.60 | 1.00 | 1.02 | 1.00 | 1.12 | 1.02 | 1.02 | 1.05 | 1.05 |
| time (sec) | N/A | 0.329 | 0.004 | 0.251 | 0.024 | 0.101 | 0.250 | 0.127 | 0.205 | 0.176 |

| Problem 155 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | A | F | F | F | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 70 | 67 | 74 | 95 | 95 | 0 | 0 | 0 | 47 | 71 |
| N.S. | 1 | 0.96 | 1.06 | 1.36 | 1.36 | 0.00 | 0.00 | 0.00 | 0.67 | 1.01 |
| time (sec) | N/A | 0.368 | 0.006 | 0.349 | 0.174 | 0.000 | 0.000 | 0.000 | 0.212 | 0.853 |

| Problem 156 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 63 | 86 | 58 | 56 | 56 | 57 | 61 | 65 | 57 | 57 |
| N.S. | 1 | 1.37 | 0.92 | 0.89 | 0.89 | 0.90 | 0.97 | 1.03 | 0.90 | 0.90 |
| time (sec) | N/A | 0.336 | 0.016 | 0.282 | 0.038 | 0.095 | 0.321 | 0.124 | 0.196 | 0.161 |

| Problem 157 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 111 | 111 | 111 | 88 | 98 | 91 | 104 | 98 | 79 | 89 |
| N.S. | 1 | 1.00 | 1.00 | 0.79 | 0.88 | 0.82 | 0.94 | 0.88 | 0.71 | 0.80 |
| time (sec) | N/A | 0.351 | 0.032 | 0.771 | 0.113 | 0.106 | 0.434 | 0.129 | 0.211 | 0.699 |

| Problem 158 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 106 | 106 | 106 | 93 | 95 | 94 | 105 | 95 | 81 | 81 |
| N.S. | 1 | 1.00 | 1.00 | 0.88 | 0.90 | 0.89 | 0.99 | 0.90 | 0.76 | 0.76 |
| time (sec) | N/A | 0.380 | 0.042 | 0.514 | 0.036 | 0.097 | 0.375 | 0.125 | 0.206 | 0.823 |

| Problem 159 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 61 | 65 | 98 | 82 | 62 | 77 | 92 | 59 | 71 | 71 |
| N.S. | 1 | 1.07 | 1.61 | 1.34 | 1.02 | 1.26 | 1.51 | 0.97 | 1.16 | 1.16 |
| time (sec) | N/A | 0.260 | 0.024 | 0.753 | 0.035 | 0.104 | 0.317 | 0.126 | 0.198 | 0.820 |

| Problem 160 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 117 | 115 | 65 | 76 | 77 | 79 | 88 | 82 | 69 | 69 |
| N.S. | 1 | 0.98 | 0.56 | 0.65 | 0.66 | 0.68 | 0.75 | 0.70 | 0.59 | 0.59 |
| time (sec) | N/A | 0.385 | 0.020 | 0.365 | 0.031 | 0.121 | 0.299 | 0.118 | 0.189 | 0.212 |

| Problem 161 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | F | F | F | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 99 | 99 | 99 | 119 | 104 | 0 | 0 | 0 | 60 | 105 |
| N.S. | 1 | 1.00 | 1.00 | 1.20 | 1.05 | 0.00 | 0.00 | 0.00 | 0.61 | 1.06 |
| time (sec) | N/A | 0.315 | 0.021 | 0.442 | 0.149 | 0.000 | 0.000 | 0.000 | 0.194 | 1.048 |

| Problem 162 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 81 | 81 | 62 | 73 | 71 | 75 | 82 | 72 | 67 | 76 |
| N.S. | 1 | 1.00 | 0.77 | 0.90 | 0.88 | 0.93 | 1.01 | 0.89 | 0.83 | 0.94 |
| time (sec) | N/A | 0.321 | 0.027 | 0.365 | 0.027 | 0.097 | 0.354 | 0.120 | 0.210 | 0.229 |

| Problem 163 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | A | F | F | F | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 90 | 90 | 120 | 116 | 120 | 0 | 0 | 0 | 56 | 110 |
| N.S. | 1 | 1.00 | 1.33 | 1.29 | 1.33 | 0.00 | 0.00 | 0.00 | 0.62 | 1.22 |
| time (sec) | N/A | 0.323 | 0.022 | 0.640 | 0.158 | 0.000 | 0.000 | 0.000 | 0.205 | 0.778 |

| Problem 164 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 85 | 85 | 68 | 75 | 76 | 80 | 87 | 89 | 71 | 78 |
| N.S. | 1 | 1.00 | 0.80 | 0.88 | 0.89 | 0.94 | 1.02 | 1.05 | 0.84 | 0.92 |
| time (sec) | N/A | 0.340 | 0.029 | 0.419 | 0.024 | 0.124 | 0.365 | 0.118 | 0.194 | 0.748 |

| Problem 165 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 141 | 141 | 141 | 112 | 120 | 113 | 138 | 120 | 99 | 111 |
| N.S. | 1 | 1.00 | 1.00 | 0.79 | 0.85 | 0.80 | 0.98 | 0.85 | 0.70 | 0.79 |
| time (sec) | N/A | 0.401 | 0.037 | 0.754 | 0.119 | 0.102 | 0.607 | 0.123 | 0.190 | 0.710 |

| Problem 166 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 136 | 136 | 136 | 116 | 118 | 116 | 138 | 117 | 101 | 108 |
| N.S. | 1 | 1.00 | 1.00 | 0.85 | 0.87 | 0.85 | 1.01 | 0.86 | 0.74 | 0.79 |
| time (sec) | N/A | 0.436 | 0.057 | 0.678 | 0.035 | 0.110 | 0.549 | 0.117 | 0.205 | 0.696 |

| Problem 167 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 74 | 75 | 128 | 104 | 73 | 99 | 124 | 70 | 91 | 100 |
| N.S. | 1 | 1.01 | 1.73 | 1.41 | 0.99 | 1.34 | 1.68 | 0.95 | 1.23 | 1.35 |
| time (sec) | N/A | 0.257 | 0.029 | 0.879 | 0.031 | 0.094 | 0.462 | 0.116 | 0.198 | 0.667 |

| Problem 168 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 161 | 158 | 83 | 98 | 99 | 101 | 117 | 104 | 89 | 89 |
| N.S. | 1 | 0.98 | 0.52 | 0.61 | 0.61 | 0.63 | 0.73 | 0.65 | 0.55 | 0.55 |
| time (sec) | N/A | 0.495 | 0.023 | 0.445 | 0.029 | 0.088 | 0.384 | 0.128 | 0.205 | 0.266 |

| Problem 169 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | F | F | F | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 132 | 132 | 132 | 143 | 127 | 0 | 0 | 0 | 80 | 156 |
| N.S. | 1 | 1.00 | 1.00 | 1.08 | 0.96 | 0.00 | 0.00 | 0.00 | 0.61 | 1.18 |
| time (sec) | N/A | 0.364 | 0.026 | 0.596 | 0.165 | 0.000 | 0.000 | 0.000 | 0.209 | 0.965 |

| Problem 170 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 108 | 108 | 78 | 95 | 93 | 97 | 110 | 99 | 87 | 85 |
| N.S. | 1 | 1.00 | 0.72 | 0.88 | 0.86 | 0.90 | 1.02 | 0.92 | 0.81 | 0.79 |
| time (sec) | N/A | 0.365 | 0.030 | 0.579 | 0.029 | 0.102 | 0.486 | 0.130 | 0.225 | 0.838 |

| Problem 171 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | A | F | F | F | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 138 | 138 | 154 | 148 | 155 | 0 | 0 | 0 | 89 | 152 |
| N.S. | 1 | 1.00 | 1.12 | 1.07 | 1.12 | 0.00 | 0.00 | 0.00 | 0.64 | 1.10 |
| time (sec) | N/A | 0.353 | 0.028 | 1.064 | 0.162 | 0.000 | 0.000 | 0.000 | 0.208 | 0.834 |

| Problem 172 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 116 | 116 | 83 | 99 | 96 | 100 | 117 | 110 | 91 | 97 |
| N.S. | 1 | 1.00 | 0.72 | 0.85 | 0.83 | 0.86 | 1.01 | 0.95 | 0.78 | 0.84 |
| time (sec) | N/A | 0.364 | 0.033 | 0.570 | 0.033 | 0.129 | 0.497 | 0.121 | 0.205 | 0.794 |

| Problem 173 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 80 | 98 | 56 | 58 | 74 | 54 | 70 | 57 | 57 | 73 |
| N.S. | 1 | 1.22 | 0.70 | 0.72 | 0.92 | 0.68 | 0.88 | 0.71 | 0.71 | 0.91 |
| time (sec) | N/A | 0.644 | 0.063 | 0.457 | 0.115 | 0.106 | 0.369 | 0.109 | 0.207 | 0.182 |

| Problem 174 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 113 | 116 | 120 | 152 | 0 | 0 | 0 | 0 | 25 | 0 |
| N.S. | 1 | 1.03 | 1.06 | 1.35 | 0.00 | 0.00 | 0.00 | 0.00 | 0.22 | 0.00 |
| time (sec) | N/A | 0.607 | 0.023 | 0.453 | 0.000 | 0.000 | 0.000 | 0.000 | 0.196 | 0.000 |

| Problem 175 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 49 | 48 | 49 | 38 | 54 | 37 | 42 | 37 | 37 | 33 |
| N.S. | 1 | 0.98 | 1.00 | 0.78 | 1.10 | 0.76 | 0.86 | 0.76 | 0.76 | 0.67 |
| time (sec) | N/A | 0.373 | 0.045 | 0.392 | 0.115 | 0.101 | 0.278 | 0.127 | 0.193 | 0.166 |

| Problem 176 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 72 | 74 | 77 | 119 | 0 | 0 | 0 | 0 | 23 | 0 |
| N.S. | 1 | 1.03 | 1.07 | 1.65 | 0.00 | 0.00 | 0.00 | 0.00 | 0.32 | 0.00 |
| time (sec) | N/A | 0.367 | 0.006 | 0.582 | 0.000 | 0.000 | 0.000 | 0.000 | 0.207 | 0.000 |

| Problem 177 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 16 | 16 | 16 | 15 | 14 | 14 | 36 | 14 | 14 | 14 |
| N.S. | 1 | 1.00 | 1.00 | 0.94 | 0.88 | 0.88 | 2.25 | 0.88 | 0.88 | 0.88 |
| time (sec) | N/A | 0.184 | 0.004 | 0.358 | 0.107 | 0.086 | 0.632 | 0.120 | 0.203 | 0.623 |

| Problem 178 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 64 | 67 | 103 | 134 | 0 | 0 | 0 | 0 | 22 | 0 |
| N.S. | 1 | 1.05 | 1.61 | 2.09 | 0.00 | 0.00 | 0.00 | 0.00 | 0.34 | 0.00 |
| time (sec) | N/A | 0.356 | 0.017 | 0.431 | 0.000 | 0.000 | 0.000 | 0.000 | 0.192 | 0.000 |

| Problem 179 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 52 | 51 | 52 | 46 | 53 | 43 | 42 | 0 | 45 | 48 |
| N.S. | 1 | 0.98 | 1.00 | 0.88 | 1.02 | 0.83 | 0.81 | 0.00 | 0.87 | 0.92 |
| time (sec) | N/A | 0.377 | 0.007 | 0.432 | 0.124 | 0.092 | 0.395 | 0.000 | 0.209 | 0.720 |

| Problem 180 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 113 | 104 | 111 | 213 | 0 | 0 | 0 | 0 | 24 | 0 |
| N.S. | 1 | 0.92 | 0.98 | 1.88 | 0.00 | 0.00 | 0.00 | 0.00 | 0.21 | 0.00 |
| time (sec) | N/A | 0.581 | 0.065 | 0.537 | 0.000 | 0.000 | 0.000 | 0.000 | 0.201 | 0.000 |

| Problem 181 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 88 | 102 | 88 | 81 | 90 | 71 | 76 | 0 | 73 | 78 |
| N.S. | 1 | 1.16 | 1.00 | 0.92 | 1.02 | 0.81 | 0.86 | 0.00 | 0.83 | 0.89 |
| time (sec) | N/A | 0.674 | 0.014 | 0.485 | 0.115 | 0.136 | 0.551 | 0.000 | 0.206 | 0.730 |

| Problem 182 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 157 | 262 | 90 | 190 | 0 | 0 | 0 | 0 | 33 | 0 |
| N.S. | 1 | 1.67 | 0.57 | 1.21 | 0.00 | 0.00 | 0.00 | 0.00 | 0.21 | 0.00 |
| time (sec) | N/A | 1.749 | 0.186 | 0.678 | 0.000 | 0.000 | 0.000 | 0.000 | 0.200 | 0.000 |

| Problem 183 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | F | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 96 | 113 | 79 | 86 | 114 | 81 | 223 | 0 | 100 | 94 |
| N.S. | 1 | 1.18 | 0.82 | 0.90 | 1.19 | 0.84 | 2.32 | 0.00 | 1.04 | 0.98 |
| time (sec) | N/A | 0.844 | 0.052 | 0.480 | 0.120 | 0.120 | 0.467 | 0.000 | 0.195 | 0.709 |

| Problem 184 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 133 | 142 | 77 | 172 | 0 | 0 | 0 | 0 | 33 | 0 |
| N.S. | 1 | 1.07 | 0.58 | 1.29 | 0.00 | 0.00 | 0.00 | 0.00 | 0.25 | 0.00 |
| time (sec) | N/A | 0.712 | 0.095 | 0.612 | 0.000 | 0.000 | 0.000 | 0.000 | 0.200 | 0.000 |

| Problem 185 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | F | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 64 | 64 | 47 | 55 | 83 | 49 | 0 | 0 | 54 | 48 |
| N.S. | 1 | 1.00 | 0.73 | 0.86 | 1.30 | 0.77 | 0.00 | 0.00 | 0.84 | 0.75 |
| time (sec) | N/A | 0.303 | 0.044 | 0.467 | 0.118 | 0.106 | 0.000 | 0.000 | 0.205 | 0.662 |

| Problem 186 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 62 | 67 | 39 | 41 | 59 | 40 | 82 | 57 | 40 | 40 |
| N.S. | 1 | 1.08 | 0.63 | 0.66 | 0.95 | 0.65 | 1.32 | 0.92 | 0.65 | 0.65 |
| time (sec) | N/A | 0.237 | 0.034 | 0.351 | 0.107 | 0.093 | 0.387 | 0.112 | 0.202 | 0.181 |

| Problem 187 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-2) | F | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 61 | 61 | 44 | 56 | 78 | 46 | 0 | 0 | 55 | 48 |
| N.S. | 1 | 1.00 | 0.72 | 0.92 | 1.28 | 0.75 | 0.00 | 0.00 | 0.90 | 0.79 |
| time (sec) | N/A | 0.218 | 0.027 | 0.463 | 0.117 | 0.098 | 0.000 | 0.000 | 0.209 | 0.691 |

| Problem 188 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 117 | 132 | 72 | 225 | 0 | 0 | 0 | 0 | 30 | 0 |
| N.S. | 1 | 1.13 | 0.62 | 1.92 | 0.00 | 0.00 | 0.00 | 0.00 | 0.26 | 0.00 |
| time (sec) | N/A | 0.628 | 0.124 | 0.571 | 0.000 | 0.000 | 0.000 | 0.000 | 0.200 | 0.000 |

| Problem 189 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | F | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 97 | 108 | 94 | 95 | 119 | 97 | 274 | 0 | 117 | 91 |
| N.S. | 1 | 1.11 | 0.97 | 0.98 | 1.23 | 1.00 | 2.82 | 0.00 | 1.21 | 0.94 |
| time (sec) | N/A | 0.639 | 0.069 | 0.494 | 0.137 | 0.110 | 0.665 | 0.000 | 0.199 | 0.738 |

| Problem 190 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 156 | 233 | 93 | 253 | 0 | 0 | 0 | 0 | 32 | 0 |
| N.S. | 1 | 1.49 | 0.60 | 1.62 | 0.00 | 0.00 | 0.00 | 0.00 | 0.21 | 0.00 |
| time (sec) | N/A | 1.522 | 0.305 | 0.832 | 0.000 | 0.000 | 0.000 | 0.000 | 0.212 | 0.000 |

| Problem 191 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | F | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 136 | 207 | 124 | 121 | 160 | 127 | 362 | 0 | 146 | 123 |
| N.S. | 1 | 1.52 | 0.91 | 0.89 | 1.18 | 0.93 | 2.66 | 0.00 | 1.07 | 0.90 |
| time (sec) | N/A | 1.619 | 0.093 | 0.680 | 0.130 | 0.108 | 1.021 | 0.000 | 0.197 | 0.777 |

| Problem 192 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 86 | 94 | 58 | 63 | 108 | 69 | 209 | 77 | 70 | 62 |
| N.S. | 1 | 1.09 | 0.67 | 0.73 | 1.26 | 0.80 | 2.43 | 0.90 | 0.81 | 0.72 |
| time (sec) | N/A | 0.270 | 0.151 | 0.496 | 0.120 | 0.116 | 0.634 | 0.126 | 0.198 | 0.732 |

| Problem 193 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | F | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 111 | 110 | 64 | 81 | 129 | 83 | 0 | 0 | 93 | 80 |
| N.S. | 1 | 0.99 | 0.58 | 0.73 | 1.16 | 0.75 | 0.00 | 0.00 | 0.84 | 0.72 |
| time (sec) | N/A | 0.351 | 0.052 | 0.576 | 0.124 | 0.128 | 0.000 | 0.000 | 0.192 | 0.703 |

| Problem 194 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 84 | 94 | 55 | 63 | 86 | 69 | 209 | 68 | 70 | 103 |
| N.S. | 1 | 1.12 | 0.65 | 0.75 | 1.02 | 0.82 | 2.49 | 0.81 | 0.83 | 1.23 |
| time (sec) | N/A | 0.261 | 0.050 | 0.480 | 0.110 | 0.100 | 0.602 | 0.117 | 0.198 | 0.727 |

| Problem 195 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-2) | F | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 105 | 104 | 68 | 93 | 129 | 85 | 0 | 0 | 93 | 85 |
| N.S. | 1 | 0.99 | 0.65 | 0.89 | 1.23 | 0.81 | 0.00 | 0.00 | 0.89 | 0.81 |
| time (sec) | N/A | 0.313 | 0.029 | 0.596 | 0.119 | 0.097 | 0.000 | 0.000 | 0.198 | 0.728 |

| Problem 196 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 159 | 216 | 90 | 256 | 0 | 0 | 0 | 0 | 38 | 0 |
| N.S. | 1 | 1.36 | 0.57 | 1.61 | 0.00 | 0.00 | 0.00 | 0.00 | 0.24 | 0.00 |
| time (sec) | N/A | 1.046 | 0.168 | 0.756 | 0.000 | 0.000 | 0.000 | 0.000 | 0.190 | 0.000 |

| Problem 197 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | F | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 142 | 208 | 118 | 136 | 181 | 149 | 604 | 0 | 184 | 133 |
| N.S. | 1 | 1.46 | 0.83 | 0.96 | 1.27 | 1.05 | 4.25 | 0.00 | 1.30 | 0.94 |
| time (sec) | N/A | 1.221 | 0.081 | 0.631 | 0.132 | 0.150 | 1.220 | 0.000 | 0.205 | 0.821 |

| Problem 198 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | B | A | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 205 | 446 | 111 | 292 | 0 | 0 | 0 | 0 | 40 | 0 |
| N.S. | 1 | 2.18 | 0.54 | 1.42 | 0.00 | 0.00 | 0.00 | 0.00 | 0.20 | 0.00 |
| time (sec) | N/A | 2.745 | 0.433 | 1.322 | 0.000 | 0.000 | 0.000 | 0.000 | 0.202 | 0.000 |

| Problem 199 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | B | A | A | A | A | B | F | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 183 | 412 | 142 | 161 | 223 | 179 | 724 | 0 | 216 | 163 |
| N.S. | 1 | 2.25 | 0.78 | 0.88 | 1.22 | 0.98 | 3.96 | 0.00 | 1.18 | 0.89 |
| time (sec) | N/A | 3.163 | 0.104 | 0.932 | 0.145 | 0.121 | 1.756 | 0.000 | 0.210 | 0.846 |

| Problem 200 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | A | A | F | F(-2) | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 160 | 299 | 105 | 176 | 127 | 94 | 0 | 0 | 117 | 0 |
| N.S. | 1 | 1.87 | 0.66 | 1.10 | 0.79 | 0.59 | 0.00 | 0.00 | 0.73 | 0.00 |
| time (sec) | N/A | 0.799 | 0.099 | 1.346 | 0.137 | 0.118 | 0.000 | 0.000 | 0.217 | 0.000 |

| Problem 201 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 298 | 284 | 278 | 199 | 0 | 0 | 0 | 0 | 23 | 0 |
| N.S. | 1 | 0.95 | 0.93 | 0.67 | 0.00 | 0.00 | 0.00 | 0.00 | 0.08 | 0.00 |
| time (sec) | N/A | 0.802 | 2.033 | 1.425 | 0.000 | 0.000 | 0.000 | 0.000 | 0.217 | 0.000 |

| Problem 202 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | B | A | F | F(-2) | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 86 | 91 | 86 | 156 | 260 | 77 | 0 | 0 | 77 | 0 |
| N.S. | 1 | 1.06 | 1.00 | 1.81 | 3.02 | 0.90 | 0.00 | 0.00 | 0.90 | 0.00 |
| time (sec) | N/A | 0.269 | 0.087 | 1.106 | 0.195 | 0.126 | 0.000 | 0.000 | 0.220 | 0.000 |

| Problem 203 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 244 | 189 | 141 | 178 | 0 | 0 | 0 | 0 | 20 | 0 |
| N.S. | 1 | 0.77 | 0.58 | 0.73 | 0.00 | 0.00 | 0.00 | 0.00 | 0.08 | 0.00 |
| time (sec) | N/A | 0.449 | 0.463 | 1.313 | 0.000 | 0.000 | 0.000 | 0.000 | 0.195 | 0.000 |

| Problem 204 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 229 | 175 | 164 | 151 | 0 | 0 | 0 | 0 | 23 | 0 |
| N.S. | 1 | 0.76 | 0.72 | 0.66 | 0.00 | 0.00 | 0.00 | 0.00 | 0.10 | 0.00 |
| time (sec) | N/A | 0.592 | 0.199 | 1.486 | 0.000 | 0.000 | 0.000 | 0.000 | 0.199 | 0.000 |

| Problem 205 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 242 | 203 | 163 | 930 | 0 | 0 | 0 | 0 | 23 | 0 |
| N.S. | 1 | 0.84 | 0.67 | 3.84 | 0.00 | 0.00 | 0.00 | 0.00 | 0.10 | 0.00 |
| time (sec) | N/A | 0.779 | 0.408 | 1.541 | 0.000 | 0.000 | 0.000 | 0.000 | 0.200 | 0.000 |

| Problem 206 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 240 | 231 | 165 | 169 | 0 | 0 | 0 | 0 | 23 | 0 |
| N.S. | 1 | 0.96 | 0.69 | 0.70 | 0.00 | 0.00 | 0.00 | 0.00 | 0.10 | 0.00 |
| time (sec) | N/A | 0.751 | 0.876 | 1.336 | 0.000 | 0.000 | 0.000 | 0.000 | 0.228 | 0.000 |

| Problem 207 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | A | A | F | F(-2) | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 84 | 85 | 105 | 153 | 73 | 84 | 0 | 0 | 107 | 0 |
| N.S. | 1 | 1.01 | 1.25 | 1.82 | 0.87 | 1.00 | 0.00 | 0.00 | 1.27 | 0.00 |
| time (sec) | N/A | 0.298 | 0.101 | 1.273 | 0.134 | 0.129 | 0.000 | 0.000 | 0.198 | 0.000 |

| Problem 208 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | B | A | C | A | A | F | F(-2) | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 217 | 783 | 119 | 199 | 214 | 118 | 0 | 0 | 158 | 0 |
| N.S. | 1 | 3.61 | 0.55 | 0.92 | 0.99 | 0.54 | 0.00 | 0.00 | 0.73 | 0.00 |
| time (sec) | N/A | 2.479 | 0.132 | 1.395 | 0.177 | 0.185 | 0.000 | 0.000 | 0.202 | 0.000 |

| Problem 209 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 357 | 693 | 576 | 221 | 0 | 0 | 0 | 0 | 49 | 0 |
| N.S. | 1 | 1.94 | 1.61 | 0.62 | 0.00 | 0.00 | 0.00 | 0.00 | 0.14 | 0.00 |
| time (sec) | N/A | 2.527 | 4.359 | 1.357 | 0.000 | 0.000 | 0.000 | 0.000 | 0.222 | 0.000 |

| Problem 210 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | B | A | F | F(-2) | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 109 | 116 | 101 | 179 | 406 | 98 | 0 | 0 | 118 | 0 |
| N.S. | 1 | 1.06 | 0.93 | 1.64 | 3.72 | 0.90 | 0.00 | 0.00 | 1.08 | 0.00 |
| time (sec) | N/A | 0.291 | 0.121 | 1.184 | 0.241 | 0.117 | 0.000 | 0.000 | 0.230 | 0.000 |

| Problem 211 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 298 | 239 | 351 | 201 | 0 | 0 | 0 | 0 | 46 | 0 |
| N.S. | 1 | 0.80 | 1.18 | 0.67 | 0.00 | 0.00 | 0.00 | 0.00 | 0.15 | 0.00 |
| time (sec) | N/A | 0.581 | 1.928 | 1.358 | 0.000 | 0.000 | 0.000 | 0.000 | 0.228 | 0.000 |

| Problem 212 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 281 | 274 | 233 | 185 | 0 | 0 | 0 | 0 | 97 | 0 |
| N.S. | 1 | 0.98 | 0.83 | 0.66 | 0.00 | 0.00 | 0.00 | 0.00 | 0.35 | 0.00 |
| time (sec) | N/A | 1.013 | 0.229 | 1.481 | 0.000 | 0.000 | 0.000 | 0.000 | 0.226 | 0.000 |

| Problem 213 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 300 | 400 | 218 | 263 | 0 | 0 | 0 | 0 | 46 | 0 |
| N.S. | 1 | 1.33 | 0.73 | 0.88 | 0.00 | 0.00 | 0.00 | 0.00 | 0.15 | 0.00 |
| time (sec) | N/A | 1.639 | 0.770 | 1.590 | 0.000 | 0.000 | 0.000 | 0.000 | 0.221 | 0.000 |

| Problem 214 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 304 | 414 | 301 | 211 | 0 | 0 | 0 | 0 | 49 | 0 |
| N.S. | 1 | 1.36 | 0.99 | 0.69 | 0.00 | 0.00 | 0.00 | 0.00 | 0.16 | 0.00 |
| time (sec) | N/A | 1.680 | 1.300 | 1.580 | 0.000 | 0.000 | 0.000 | 0.000 | 0.258 | 0.000 |

| Problem 215 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 310 | 296 | 263 | 2335 | 0 | 0 | 0 | 0 | 171 | 0 |
| N.S. | 1 | 0.95 | 0.85 | 7.53 | 0.00 | 0.00 | 0.00 | 0.00 | 0.55 | 0.00 |
| time (sec) | N/A | 1.262 | 0.475 | 1.785 | 0.000 | 0.000 | 0.000 | 0.000 | 0.231 | 0.000 |

| Problem 216 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | F | A | C | A | A | F | F(-2) | B | F(-1) |
| verified | N/A | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 289 | 0 | 129 | 225 | 338 | 154 | 0 | 0 | 200 | 0 |
| N.S. | 1 | 0.00 | 0.45 | 0.78 | 1.17 | 0.53 | 0.00 | 0.00 | 0.69 | 0.00 |
| time (sec) | N/A | 0.000 | 0.181 | 6.510 | 0.241 | 0.172 | 0.000 | 0.000 | 0.223 | 0.000 |

| Problem 217 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | B | B | A | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 418 | 1650 | 907 | 245 | 0 | 0 | 0 | 0 | 76 | 0 |
| N.S. | 1 | 3.95 | 2.17 | 0.59 | 0.00 | 0.00 | 0.00 | 0.00 | 0.18 | 0.00 |
| time (sec) | N/A | 7.309 | 10.916 | 4.192 | 0.000 | 0.000 | 0.000 | 0.000 | 0.226 | 0.000 |

| Problem 218 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | B | A | F | F(-2) | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 134 | 141 | 111 | 205 | 637 | 130 | 0 | 0 | 160 | 0 |
| N.S. | 1 | 1.05 | 0.83 | 1.53 | 4.75 | 0.97 | 0.00 | 0.00 | 1.19 | 0.00 |
| time (sec) | N/A | 0.309 | 0.140 | 2.671 | 0.287 | 0.145 | 0.000 | 0.000 | 0.237 | 0.000 |

| Problem 219 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 348 | 289 | 643 | 225 | 0 | 0 | 0 | 0 | 73 | 0 |
| N.S. | 1 | 0.83 | 1.85 | 0.65 | 0.00 | 0.00 | 0.00 | 0.00 | 0.21 | 0.00 |
| time (sec) | N/A | 0.675 | 4.302 | 2.010 | 0.000 | 0.000 | 0.000 | 0.000 | 0.287 | 0.000 |

| Problem 220 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 329 | 398 | 283 | 198 | 0 | 0 | 0 | 0 | 139 | 0 |
| N.S. | 1 | 1.21 | 0.86 | 0.60 | 0.00 | 0.00 | 0.00 | 0.00 | 0.42 | 0.00 |
| time (sec) | N/A | 1.648 | 0.287 | 2.300 | 0.000 | 0.000 | 0.000 | 0.000 | 0.240 | 0.000 |

| Problem 221 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 355 | 647 | 491 | 265 | 0 | 0 | 0 | 0 | 73 | 0 |
| N.S. | 1 | 1.82 | 1.38 | 0.75 | 0.00 | 0.00 | 0.00 | 0.00 | 0.21 | 0.00 |
| time (sec) | N/A | 3.176 | 2.889 | 2.336 | 0.000 | 0.000 | 0.000 | 0.000 | 0.226 | 0.000 |

| Problem 222 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 364 | 696 | 376 | 256 | 0 | 0 | 0 | 0 | 132 | 0 |
| N.S. | 1 | 1.91 | 1.03 | 0.70 | 0.00 | 0.00 | 0.00 | 0.00 | 0.36 | 0.00 |
| time (sec) | N/A | 3.053 | 1.509 | 3.087 | 0.000 | 0.000 | 0.000 | 0.000 | 0.282 | 0.000 |

| Problem 223 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 372 | 704 | 313 | 270 | 0 | 0 | 0 | 0 | 198 | 0 |
| N.S. | 1 | 1.89 | 0.84 | 0.73 | 0.00 | 0.00 | 0.00 | 0.00 | 0.53 | 0.00 |
| time (sec) | N/A | 3.298 | 0.880 | 4.638 | 0.000 | 0.000 | 0.000 | 0.000 | 0.229 | 0.000 |

| Problem 224 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | A | A | F | F(-2) | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 120 | 166 | 91 | 165 | 89 | 80 | 0 | 0 | 80 | 0 |
| N.S. | 1 | 1.38 | 0.76 | 1.38 | 0.74 | 0.67 | 0.00 | 0.00 | 0.67 | 0.00 |
| time (sec) | N/A | 0.519 | 0.106 | 1.286 | 0.147 | 0.139 | 0.000 | 0.000 | 0.202 | 0.000 |

| Problem 225 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 250 | 200 | 158 | 184 | 0 | 0 | 0 | 0 | 27 | 0 |
| N.S. | 1 | 0.80 | 0.63 | 0.74 | 0.00 | 0.00 | 0.00 | 0.00 | 0.11 | 0.00 |
| time (sec) | N/A | 0.484 | 0.546 | 1.404 | 0.000 | 0.000 | 0.000 | 0.000 | 0.205 | 0.000 |

| Problem 226 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | A | A | F(-2) | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 59 | 59 | 60 | 100 | 61 | 64 | 0 | 60 | 42 | 0 |
| N.S. | 1 | 1.00 | 1.02 | 1.69 | 1.03 | 1.08 | 0.00 | 1.02 | 0.71 | 0.00 |
| time (sec) | N/A | 0.253 | 0.062 | 1.112 | 0.157 | 0.127 | 0.000 | 0.304 | 0.211 | 0.000 |

| Problem 227 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 193 | 140 | 90 | 150 | 0 | 0 | 0 | 0 | 24 | 0 |
| N.S. | 1 | 0.73 | 0.47 | 0.78 | 0.00 | 0.00 | 0.00 | 0.00 | 0.12 | 0.00 |
| time (sec) | N/A | 0.318 | 0.090 | 1.296 | 0.000 | 0.000 | 0.000 | 0.000 | 0.220 | 0.000 |

| Problem 228 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 177 | 124 | 100 | 139 | 0 | 0 | 0 | 0 | 27 | 0 |
| N.S. | 1 | 0.70 | 0.56 | 0.79 | 0.00 | 0.00 | 0.00 | 0.00 | 0.15 | 0.00 |
| time (sec) | N/A | 0.428 | 0.194 | 1.425 | 0.000 | 0.000 | 0.000 | 0.000 | 0.203 | 0.000 |

| Problem 229 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | A | A | F | F(-2) | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 56 | 56 | 62 | 125 | 36 | 68 | 0 | 0 | 65 | 0 |
| N.S. | 1 | 1.00 | 1.11 | 2.23 | 0.64 | 1.21 | 0.00 | 0.00 | 1.16 | 0.00 |
| time (sec) | N/A | 0.299 | 0.087 | 1.384 | 0.152 | 0.123 | 0.000 | 0.000 | 0.262 | 0.000 |

| Problem 230 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 242 | 184 | 165 | 175 | 0 | 0 | 0 | 0 | 27 | 0 |
| N.S. | 1 | 0.76 | 0.68 | 0.72 | 0.00 | 0.00 | 0.00 | 0.00 | 0.11 | 0.00 |
| time (sec) | N/A | 0.567 | 0.614 | 1.418 | 0.000 | 0.000 | 0.000 | 0.000 | 0.285 | 0.000 |

| Problem 231 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | A | A | F | F(-2) | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 118 | 150 | 110 | 163 | 81 | 89 | 0 | 0 | 111 | 0 |
| N.S. | 1 | 1.27 | 0.93 | 1.38 | 0.69 | 0.75 | 0.00 | 0.00 | 0.94 | 0.00 |
| time (sec) | N/A | 0.505 | 0.111 | 1.411 | 0.174 | 0.127 | 0.000 | 0.000 | 0.259 | 0.000 |

| Problem 232 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | A | F | F(-2) | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 107 | 121 | 107 | 218 | 0 | 102 | 0 | 0 | 121 | 0 |
| N.S. | 1 | 1.13 | 1.00 | 2.04 | 0.00 | 0.95 | 0.00 | 0.00 | 1.13 | 0.00 |
| time (sec) | N/A | 0.475 | 0.107 | 1.233 | 0.000 | 0.126 | 0.000 | 0.000 | 0.275 | 0.000 |

| Problem 233 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 251 | 196 | 155 | 247 | 0 | 0 | 0 | 0 | 48 | 0 |
| N.S. | 1 | 0.78 | 0.62 | 0.98 | 0.00 | 0.00 | 0.00 | 0.00 | 0.19 | 0.00 |
| time (sec) | N/A | 0.494 | 0.256 | 1.339 | 0.000 | 0.000 | 0.000 | 0.000 | 0.276 | 0.000 |

| Problem 234 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | A | A | F(-2) | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 49 | 49 | 42 | 100 | 28 | 43 | 0 | 45 | 58 | 0 |
| N.S. | 1 | 1.00 | 0.86 | 2.04 | 0.57 | 0.88 | 0.00 | 0.92 | 1.18 | 0.00 |
| time (sec) | N/A | 0.244 | 0.053 | 1.123 | 0.166 | 0.109 | 0.000 | 0.158 | 0.287 | 0.000 |

| Problem 235 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | A | F | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 45 | 45 | 38 | 94 | 41 | 40 | 0 | 41 | 103 | 0 |
| N.S. | 1 | 1.00 | 0.84 | 2.09 | 0.91 | 0.89 | 0.00 | 0.91 | 2.29 | 0.00 |
| time (sec) | N/A | 0.212 | 0.041 | 1.023 | 0.035 | 0.124 | 0.000 | 0.159 | 0.260 | 0.000 |

| Problem 236 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 229 | 182 | 141 | 232 | 0 | 0 | 0 | 0 | 47 | 0 |
| N.S. | 1 | 0.79 | 0.62 | 1.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.21 | 0.00 |
| time (sec) | N/A | 0.733 | 0.218 | 1.328 | 0.000 | 0.000 | 0.000 | 0.000 | 0.303 | 0.000 |

| Problem 237 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | A | F | F(-2) | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 103 | 111 | 122 | 219 | 0 | 104 | 0 | 0 | 159 | 0 |
| N.S. | 1 | 1.08 | 1.18 | 2.13 | 0.00 | 1.01 | 0.00 | 0.00 | 1.54 | 0.00 |
| time (sec) | N/A | 0.600 | 0.145 | 1.310 | 0.000 | 0.126 | 0.000 | 0.000 | 0.284 | 0.000 |

| Problem 238 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 300 | 376 | 258 | 273 | 0 | 0 | 0 | 0 | 49 | 0 |
| N.S. | 1 | 1.25 | 0.86 | 0.91 | 0.00 | 0.00 | 0.00 | 0.00 | 0.16 | 0.00 |
| time (sec) | N/A | 1.790 | 0.949 | 1.394 | 0.000 | 0.000 | 0.000 | 0.000 | 0.310 | 0.000 |

| Problem 239 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | A | F | F(-2) | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 165 | 271 | 143 | 273 | 0 | 129 | 0 | 0 | 210 | 0 |
| N.S. | 1 | 1.64 | 0.87 | 1.65 | 0.00 | 0.78 | 0.00 | 0.00 | 1.27 | 0.00 |
| time (sec) | N/A | 1.490 | 0.245 | 1.395 | 0.000 | 0.160 | 0.000 | 0.000 | 0.267 | 0.000 |

| Problem 240 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | A | F | F(-2) | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 170 | 252 | 131 | 327 | 0 | 140 | 0 | 0 | 202 | 0 |
| N.S. | 1 | 1.48 | 0.77 | 1.92 | 0.00 | 0.82 | 0.00 | 0.00 | 1.19 | 0.00 |
| time (sec) | N/A | 1.184 | 0.153 | 1.364 | 0.000 | 0.160 | 0.000 | 0.000 | 0.258 | 0.000 |

| Problem 241 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 308 | 290 | 177 | 389 | 0 | 0 | 0 | 0 | 66 | 0 |
| N.S. | 1 | 0.94 | 0.57 | 1.26 | 0.00 | 0.00 | 0.00 | 0.00 | 0.21 | 0.00 |
| time (sec) | N/A | 1.247 | 0.342 | 1.493 | 0.000 | 0.000 | 0.000 | 0.000 | 0.243 | 0.000 |

| Problem 242 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | F(-2) | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 112 | 118 | 65 | 154 | 65 | 74 | 0 | 0 | 117 | 0 |
| N.S. | 1 | 1.05 | 0.58 | 1.38 | 0.58 | 0.66 | 0.00 | 0.00 | 1.04 | 0.00 |
| time (sec) | N/A | 0.409 | 0.081 | 1.212 | 0.131 | 0.120 | 0.000 | 0.000 | 0.207 | 0.000 |

| Problem 243 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | A | F | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 77 | 81 | 57 | 147 | 93 | 67 | 0 | 58 | 175 | 0 |
| N.S. | 1 | 1.05 | 0.74 | 1.91 | 1.21 | 0.87 | 0.00 | 0.75 | 2.27 | 0.00 |
| time (sec) | N/A | 0.311 | 0.066 | 1.106 | 0.044 | 0.117 | 0.000 | 0.163 | 0.187 | 0.000 |

| Problem 244 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | A | F(-2) | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 79 | 81 | 51 | 144 | 66 | 64 | 0 | 58 | 95 | 0 |
| N.S. | 1 | 1.03 | 0.65 | 1.82 | 0.84 | 0.81 | 0.00 | 0.73 | 1.20 | 0.00 |
| time (sec) | N/A | 0.263 | 0.056 | 1.355 | 0.109 | 0.128 | 0.000 | 0.162 | 0.203 | 0.000 |

| Problem 245 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 101 | 103 | 63 | 114 | 86 | 72 | 0 | 70 | 193 | 0 |
| N.S. | 1 | 1.02 | 0.62 | 1.13 | 0.85 | 0.71 | 0.00 | 0.69 | 1.91 | 0.00 |
| time (sec) | N/A | 0.318 | 0.048 | 1.162 | 0.039 | 0.115 | 0.000 | 0.159 | 0.203 | 0.000 |

| Problem 246 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 279 | 273 | 168 | 370 | 0 | 0 | 0 | 0 | 65 | 0 |
| N.S. | 1 | 0.98 | 0.60 | 1.33 | 0.00 | 0.00 | 0.00 | 0.00 | 0.23 | 0.00 |
| time (sec) | N/A | 1.191 | 0.315 | 1.481 | 0.000 | 0.000 | 0.000 | 0.000 | 0.228 | 0.000 |

| Problem 247 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | A | F | F(-2) | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 158 | 224 | 151 | 325 | 0 | 142 | 0 | 0 | 258 | 0 |
| N.S. | 1 | 1.42 | 0.96 | 2.06 | 0.00 | 0.90 | 0.00 | 0.00 | 1.63 | 0.00 |
| time (sec) | N/A | 1.174 | 0.186 | 1.400 | 0.000 | 0.148 | 0.000 | 0.000 | 0.224 | 0.000 |

| Problem 248 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 270 | 270 | 234 | 600 | 0 | 0 | 0 | 0 | 1479 | 0 |
| N.S. | 1 | 1.00 | 0.87 | 2.22 | 0.00 | 0.00 | 0.00 | 0.00 | 5.48 | 0.00 |
| time (sec) | N/A | 0.501 | 0.210 | 120.870 | 0.000 | 0.000 | 0.000 | 0.000 | 0.207 | 0.000 |

| Problem 249 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 201 | 201 | 175 | 376 | 0 | 0 | 0 | 0 | 823 | 0 |
| N.S. | 1 | 1.00 | 0.87 | 1.87 | 0.00 | 0.00 | 0.00 | 0.00 | 4.09 | 0.00 |
| time (sec) | N/A | 0.422 | 0.087 | 33.821 | 0.000 | 0.000 | 0.000 | 0.000 | 0.202 | 0.000 |

| Problem 250 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 124 | 124 | 111 | 222 | 0 | 0 | 0 | 0 | 361 | 0 |
| N.S. | 1 | 1.00 | 0.90 | 1.79 | 0.00 | 0.00 | 0.00 | 0.00 | 2.91 | 0.00 |
| time (sec) | N/A | 0.338 | 0.075 | 8.491 | 0.000 | 0.000 | 0.000 | 0.000 | 0.206 | 0.000 |

| Problem 251 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 22 | 20 | 22 | 22 | 19 | 22 | 25 | 22 |
| N.S. | 1 | 1.00 | 1.10 | 1.00 | 1.10 | 1.10 | 0.95 | 1.10 | 1.25 | 1.10 |
| time (sec) | N/A | 0.222 | 0.782 | 0.868 | 0.246 | 0.098 | 1.278 | 0.112 | 0.213 | 0.608 |

| Problem 252 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 22 | 20 | 22 | 37 | 29 | 22 | 33 | 22 |
| N.S. | 1 | 1.00 | 1.10 | 1.00 | 1.10 | 1.85 | 1.45 | 1.10 | 1.65 | 1.10 |
| time (sec) | N/A | 0.228 | 0.506 | 1.293 | 0.263 | 0.109 | 4.619 | 0.126 | 0.220 | 0.631 |

| Problem 253 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | F(-1) | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 20 | 22 | 47 | 0 | 0 | 82 | 22 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 1.00 | 2.14 | 0.00 | 0.00 | 3.73 | 1.00 |
| time (sec) | N/A | 0.258 | 0.926 | 1.681 | 0.399 | 0.111 | 0.000 | 0.000 | 8.191 | 0.735 |

| Problem 254 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | F(-1) | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 20 | 22 | 22 | 0 | 0 | 52 | 22 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 1.00 | 1.00 | 0.00 | 0.00 | 2.36 | 1.00 |
| time (sec) | N/A | 0.252 | 0.482 | 1.530 | 0.276 | 0.113 | 0.000 | 0.000 | 0.372 | 0.661 |

| Problem 255 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 20 | 22 | 22 | 22 | 0 | 23 | 22 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 1.00 | 1.00 | 1.00 | 0.00 | 1.05 | 1.00 |
| time (sec) | N/A | 0.495 | 0.094 | 1.460 | 0.175 | 0.106 | 13.784 | 0.000 | 0.232 | 0.601 |

| Problem 256 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 20 | 22 | 22 | 22 | 22 | 27 | 22 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 1.00 | 1.00 | 1.00 | 1.00 | 1.23 | 1.00 |
| time (sec) | N/A | 0.270 | 0.564 | 1.349 | 0.171 | 0.113 | 8.709 | 0.143 | 0.216 | 0.688 |

| Problem 257 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 20 | 22 | 49 | 22 | 22 | 48 | 22 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 1.00 | 2.23 | 1.00 | 1.00 | 2.18 | 1.00 |
| time (sec) | N/A | 0.251 | 0.605 | 1.570 | 0.211 | 0.112 | 15.817 | 0.190 | 0.237 | 0.803 |

| Problem 258 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | B | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 124 | 290 | 89 | 108 | 116 | 97 | 121 | 105 | 103 | 102 |
| N.S. | 1 | 2.34 | 0.72 | 0.87 | 0.94 | 0.78 | 0.98 | 0.85 | 0.83 | 0.82 |
| time (sec) | N/A | 2.136 | 0.051 | 0.693 | 0.131 | 0.106 | 0.385 | 0.128 | 0.187 | 0.874 |

| Problem 259 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | B | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 156 | 318 | 104 | 223 | 0 | 0 | 0 | 0 | 96 | 0 |
| N.S. | 1 | 2.04 | 0.67 | 1.43 | 0.00 | 0.00 | 0.00 | 0.00 | 0.62 | 0.00 |
| time (sec) | N/A | 1.657 | 0.466 | 1.380 | 0.000 | 0.000 | 0.000 | 0.000 | 0.180 | 0.000 |

| Problem 260 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 96 | 97 | 64 | 87 | 87 | 74 | 94 | 83 | 82 | 83 |
| N.S. | 1 | 1.01 | 0.67 | 0.91 | 0.91 | 0.77 | 0.98 | 0.86 | 0.85 | 0.86 |
| time (sec) | N/A | 0.354 | 0.040 | 0.765 | 0.039 | 0.112 | 0.314 | 0.128 | 0.188 | 0.798 |

| Problem 261 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 128 | 134 | 82 | 200 | 0 | 0 | 0 | 0 | 75 | 0 |
| N.S. | 1 | 1.05 | 0.64 | 1.56 | 0.00 | 0.00 | 0.00 | 0.00 | 0.59 | 0.00 |
| time (sec) | N/A | 0.560 | 0.059 | 0.985 | 0.000 | 0.000 | 0.000 | 0.000 | 0.194 | 0.000 |

| Problem 262 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 169 | 209 | 183 | 1055 | 0 | 0 | 0 | 0 | 55 | 0 |
| N.S. | 1 | 1.24 | 1.08 | 6.24 | 0.00 | 0.00 | 0.00 | 0.00 | 0.33 | 0.00 |
| time (sec) | N/A | 1.325 | 0.101 | 20.722 | 0.000 | 0.000 | 0.000 | 0.000 | 0.186 | 0.000 |

| Problem 263 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 113 | 164 | 94 | 238 | 0 | 0 | 0 | 0 | 49 | 0 |
| N.S. | 1 | 1.45 | 0.83 | 2.11 | 0.00 | 0.00 | 0.00 | 0.00 | 0.43 | 0.00 |
| time (sec) | N/A | 1.098 | 0.129 | 0.477 | 0.000 | 0.000 | 0.000 | 0.000 | 0.181 | 0.000 |

| Problem 264 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 196 | 209 | 220 | 1134 | 0 | 0 | 0 | 0 | 85 | 0 |
| N.S. | 1 | 1.07 | 1.12 | 5.79 | 0.00 | 0.00 | 0.00 | 0.00 | 0.43 | 0.00 |
| time (sec) | N/A | 1.328 | 0.102 | 23.743 | 0.000 | 0.000 | 0.000 | 0.000 | 0.185 | 0.000 |

| Problem 265 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 135 | 199 | 103 | 271 | 0 | 0 | 0 | 0 | 83 | 0 |
| N.S. | 1 | 1.47 | 0.76 | 2.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.61 | 0.00 |
| time (sec) | N/A | 1.033 | 0.234 | 0.615 | 0.000 | 0.000 | 0.000 | 0.000 | 0.186 | 0.000 |

| Problem 266 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 191 | 191 | 110 | 150 | 169 | 148 | 185 | 161 | 139 | 145 |
| N.S. | 1 | 1.00 | 0.58 | 0.79 | 0.88 | 0.77 | 0.97 | 0.84 | 0.73 | 0.76 |
| time (sec) | N/A | 1.013 | 0.044 | 1.180 | 0.124 | 0.113 | 0.538 | 0.136 | 0.182 | 0.760 |

| Problem 267 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 225 | 225 | 133 | 267 | 0 | 0 | 0 | 0 | 134 | 0 |
| N.S. | 1 | 1.00 | 0.59 | 1.19 | 0.00 | 0.00 | 0.00 | 0.00 | 0.60 | 0.00 |
| time (sec) | N/A | 1.005 | 0.929 | 2.069 | 0.000 | 0.000 | 0.000 | 0.000 | 0.190 | 0.000 |

| Problem 268 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 153 | 144 | 84 | 132 | 111 | 123 | 158 | 160 | 119 | 135 |
| N.S. | 1 | 0.94 | 0.55 | 0.86 | 0.73 | 0.80 | 1.03 | 1.05 | 0.78 | 0.88 |
| time (sec) | N/A | 0.517 | 0.038 | 0.984 | 0.034 | 0.090 | 0.399 | 0.131 | 0.182 | 0.845 |

| Problem 269 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 205 | 208 | 112 | 244 | 0 | 0 | 0 | 0 | 112 | 0 |
| N.S. | 1 | 1.01 | 0.55 | 1.19 | 0.00 | 0.00 | 0.00 | 0.00 | 0.55 | 0.00 |
| time (sec) | N/A | 0.836 | 0.518 | 1.768 | 0.000 | 0.000 | 0.000 | 0.000 | 0.191 | 0.000 |

| Problem 270 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 235 | 235 | 218 | 1185 | 0 | 0 | 0 | 0 | 95 | 0 |
| N.S. | 1 | 1.00 | 0.93 | 5.04 | 0.00 | 0.00 | 0.00 | 0.00 | 0.40 | 0.00 |
| time (sec) | N/A | 0.765 | 0.218 | 25.813 | 0.000 | 0.000 | 0.000 | 0.000 | 0.185 | 0.000 |

| Problem 271 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 205 | 205 | 167 | 285 | 0 | 0 | 0 | 0 | 118 | 0 |
| N.S. | 1 | 1.00 | 0.81 | 1.39 | 0.00 | 0.00 | 0.00 | 0.00 | 0.58 | 0.00 |
| time (sec) | N/A | 0.653 | 0.278 | 0.941 | 0.000 | 0.000 | 0.000 | 0.000 | 0.182 | 0.000 |

| Problem 272 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 207 | 207 | 226 | 1184 | 0 | 0 | 0 | 0 | 80 | 0 |
| N.S. | 1 | 1.00 | 1.09 | 5.72 | 0.00 | 0.00 | 0.00 | 0.00 | 0.39 | 0.00 |
| time (sec) | N/A | 0.686 | 0.192 | 44.444 | 0.000 | 0.000 | 0.000 | 0.000 | 0.175 | 0.000 |

| Problem 273 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 216 | 216 | 189 | 290 | 0 | 0 | 0 | 0 | 101 | 0 |
| N.S. | 1 | 1.00 | 0.88 | 1.34 | 0.00 | 0.00 | 0.00 | 0.00 | 0.47 | 0.00 |
| time (sec) | N/A | 0.686 | 0.272 | 1.086 | 0.000 | 0.000 | 0.000 | 0.000 | 0.178 | 0.000 |

| Problem 274 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 240 | 240 | 126 | 188 | 202 | 181 | 241 | 198 | 173 | 178 |
| N.S. | 1 | 1.00 | 0.52 | 0.78 | 0.84 | 0.75 | 1.00 | 0.82 | 0.72 | 0.74 |
| time (sec) | N/A | 1.415 | 0.049 | 0.964 | 0.132 | 0.106 | 0.712 | 0.132 | 0.179 | 0.828 |

| Problem 275 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 274 | 274 | 157 | 306 | 0 | 0 | 0 | 0 | 168 | 0 |
| N.S. | 1 | 1.00 | 0.57 | 1.12 | 0.00 | 0.00 | 0.00 | 0.00 | 0.61 | 0.00 |
| time (sec) | N/A | 1.371 | 1.575 | 3.181 | 0.000 | 0.000 | 0.000 | 0.000 | 0.190 | 0.000 |

| Problem 276 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 200 | 187 | 100 | 168 | 133 | 156 | 207 | 223 | 153 | 156 |
| N.S. | 1 | 0.94 | 0.50 | 0.84 | 0.66 | 0.78 | 1.04 | 1.12 | 0.76 | 0.78 |
| time (sec) | N/A | 0.647 | 0.040 | 1.152 | 0.034 | 0.128 | 0.552 | 0.128 | 0.188 | 0.710 |

| Problem 277 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 268 | 290 | 137 | 280 | 0 | 0 | 0 | 0 | 147 | 0 |
| N.S. | 1 | 1.08 | 0.51 | 1.04 | 0.00 | 0.00 | 0.00 | 0.00 | 0.55 | 0.00 |
| time (sec) | N/A | 1.227 | 0.833 | 2.618 | 0.000 | 0.000 | 0.000 | 0.000 | 0.178 | 0.000 |

| Problem 278 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 287 | 287 | 252 | 1405 | 0 | 0 | 0 | 0 | 130 | 0 |
| N.S. | 1 | 1.00 | 0.88 | 4.90 | 0.00 | 0.00 | 0.00 | 0.00 | 0.45 | 0.00 |
| time (sec) | N/A | 1.063 | 0.367 | 50.002 | 0.000 | 0.000 | 0.000 | 0.000 | 0.189 | 0.000 |

| Problem 279 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 251 | 251 | 202 | 321 | 0 | 0 | 0 | 0 | 153 | 0 |
| N.S. | 1 | 1.00 | 0.80 | 1.28 | 0.00 | 0.00 | 0.00 | 0.00 | 0.61 | 0.00 |
| time (sec) | N/A | 0.929 | 0.508 | 1.325 | 0.000 | 0.000 | 0.000 | 0.000 | 0.192 | 0.000 |

| Problem 280 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 299 | 299 | 333 | 1318 | 0 | 0 | 0 | 0 | 146 | 0 |
| N.S. | 1 | 1.00 | 1.11 | 4.41 | 0.00 | 0.00 | 0.00 | 0.00 | 0.49 | 0.00 |
| time (sec) | N/A | 0.909 | 0.249 | 60.521 | 0.000 | 0.000 | 0.000 | 0.000 | 0.195 | 0.000 |

| Problem 281 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 250 | 250 | 221 | 322 | 0 | 0 | 0 | 0 | 158 | 0 |
| N.S. | 1 | 1.00 | 0.88 | 1.29 | 0.00 | 0.00 | 0.00 | 0.00 | 0.63 | 0.00 |
| time (sec) | N/A | 0.847 | 0.419 | 1.766 | 0.000 | 0.000 | 0.000 | 0.000 | 0.195 | 0.000 |

| Problem 282 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 166 | 243 | 90 | 224 | 0 | 0 | 0 | 0 | 83 | 0 |
| N.S. | 1 | 1.46 | 0.54 | 1.35 | 0.00 | 0.00 | 0.00 | 0.00 | 0.50 | 0.00 |
| time (sec) | N/A | 1.502 | 0.241 | 1.005 | 0.000 | 0.000 | 0.000 | 0.000 | 0.194 | 0.000 |

| Problem 283 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 169 | 175 | 123 | 874 | 0 | 0 | 0 | 0 | 27 | 0 |
| N.S. | 1 | 1.04 | 0.73 | 5.17 | 0.00 | 0.00 | 0.00 | 0.00 | 0.16 | 0.00 |
| time (sec) | N/A | 1.236 | 0.101 | 19.527 | 0.000 | 0.000 | 0.000 | 0.000 | 0.185 | 0.000 |

| Problem 284 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 98 | 104 | 69 | 187 | 0 | 0 | 0 | 0 | 27 | 0 |
| N.S. | 1 | 1.06 | 0.70 | 1.91 | 0.00 | 0.00 | 0.00 | 0.00 | 0.28 | 0.00 |
| time (sec) | N/A | 0.689 | 0.154 | 0.906 | 0.000 | 0.000 | 0.000 | 0.000 | 0.190 | 0.000 |

| Problem 285 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 102 | 106 | 92 | 756 | 0 | 0 | 0 | 0 | 25 | 0 |
| N.S. | 1 | 1.04 | 0.90 | 7.41 | 0.00 | 0.00 | 0.00 | 0.00 | 0.25 | 0.00 |
| time (sec) | N/A | 0.565 | 0.019 | 13.175 | 0.000 | 0.000 | 0.000 | 0.000 | 0.195 | 0.000 |

| Problem 286 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 16 | 16 | 16 | 15 | 14 | 14 | 0 | 14 | 14 | 14 |
| N.S. | 1 | 1.00 | 1.00 | 0.94 | 0.88 | 0.88 | 0.00 | 0.88 | 0.88 | 0.88 |
| time (sec) | N/A | 0.193 | 0.004 | 0.464 | 0.109 | 0.105 | 0.000 | 0.117 | 0.214 | 0.180 |

| Problem 287 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 91 | 107 | 83 | 1578 | 0 | 0 | 0 | 0 | 24 | 0 |
| N.S. | 1 | 1.18 | 0.91 | 17.34 | 0.00 | 0.00 | 0.00 | 0.00 | 0.26 | 0.00 |
| time (sec) | N/A | 0.588 | 0.104 | 60.596 | 0.000 | 0.000 | 0.000 | 0.000 | 0.186 | 0.000 |

| Problem 288 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 92 | 95 | 73 | 252 | 0 | 0 | 0 | 0 | 49 | 0 |
| N.S. | 1 | 1.03 | 0.79 | 2.74 | 0.00 | 0.00 | 0.00 | 0.00 | 0.53 | 0.00 |
| time (sec) | N/A | 0.689 | 0.159 | 0.711 | 0.000 | 0.000 | 0.000 | 0.000 | 0.191 | 0.000 |

| Problem 289 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 178 | 173 | 142 | 1826 | 0 | 0 | 0 | 0 | 26 | 0 |
| N.S. | 1 | 0.97 | 0.80 | 10.26 | 0.00 | 0.00 | 0.00 | 0.00 | 0.15 | 0.00 |
| time (sec) | N/A | 1.431 | 0.284 | 67.414 | 0.000 | 0.000 | 0.000 | 0.000 | 0.191 | 0.000 |

| Problem 290 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-1) | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 166 | 215 | 120 | 293 | 0 | 0 | 0 | 0 | 98 | 0 |
| N.S. | 1 | 1.30 | 0.72 | 1.77 | 0.00 | 0.00 | 0.00 | 0.00 | 0.59 | 0.00 |
| time (sec) | N/A | 1.398 | 0.314 | 1.325 | 0.000 | 0.000 | 0.000 | 0.000 | 0.211 | 0.000 |

| Problem 291 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 192 | 197 | 117 | 855 | 0 | 0 | 0 | 0 | 35 | 0 |
| N.S. | 1 | 1.03 | 0.61 | 4.45 | 0.00 | 0.00 | 0.00 | 0.00 | 0.18 | 0.00 |
| time (sec) | N/A | 1.243 | 0.146 | 31.129 | 0.000 | 0.000 | 0.000 | 0.000 | 0.195 | 0.000 |

| Problem 292 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | F | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 106 | 110 | 68 | 75 | 151 | 69 | 0 | 0 | 74 | 96 |
| N.S. | 1 | 1.04 | 0.64 | 0.71 | 1.42 | 0.65 | 0.00 | 0.00 | 0.70 | 0.91 |
| time (sec) | N/A | 0.386 | 0.166 | 0.727 | 0.147 | 0.099 | 0.000 | 0.000 | 0.202 | 0.678 |

| Problem 293 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | F | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 91 | 87 | 47 | 58 | 104 | 48 | 0 | 0 | 57 | 50 |
| N.S. | 1 | 0.96 | 0.52 | 0.64 | 1.14 | 0.53 | 0.00 | 0.00 | 0.63 | 0.55 |
| time (sec) | N/A | 0.327 | 0.044 | 0.600 | 0.124 | 0.132 | 0.000 | 0.000 | 0.216 | 0.681 |

| Problem 294 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | F | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 100 | 106 | 65 | 75 | 146 | 67 | 0 | 0 | 74 | 101 |
| N.S. | 1 | 1.06 | 0.65 | 0.75 | 1.46 | 0.67 | 0.00 | 0.00 | 0.74 | 1.01 |
| time (sec) | N/A | 0.344 | 0.090 | 0.686 | 0.134 | 0.117 | 0.000 | 0.000 | 0.191 | 0.788 |

| Problem 295 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 170 | 195 | 119 | 1677 | 0 | 0 | 0 | 0 | 32 | 0 |
| N.S. | 1 | 1.15 | 0.70 | 9.86 | 0.00 | 0.00 | 0.00 | 0.00 | 0.19 | 0.00 |
| time (sec) | N/A | 1.177 | 0.141 | 63.290 | 0.000 | 0.000 | 0.000 | 0.000 | 0.181 | 0.000 |

| Problem 296 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 177 | 197 | 109 | 313 | 0 | 0 | 0 | 0 | 158 | 0 |
| N.S. | 1 | 1.11 | 0.62 | 1.77 | 0.00 | 0.00 | 0.00 | 0.00 | 0.89 | 0.00 |
| time (sec) | N/A | 1.348 | 0.328 | 0.912 | 0.000 | 0.000 | 0.000 | 0.000 | 0.187 | 0.000 |

| Problem 297 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 250 | 365 | 183 | 1927 | 0 | 0 | 0 | 0 | 34 | 0 |
| N.S. | 1 | 1.46 | 0.73 | 7.71 | 0.00 | 0.00 | 0.00 | 0.00 | 0.14 | 0.00 |
| time (sec) | N/A | 3.237 | 0.420 | 71.100 | 0.000 | 0.000 | 0.000 | 0.000 | 0.191 | 0.000 |

| Problem 298 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-1) | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 242 | 409 | 166 | 351 | 0 | 0 | 0 | 0 | 182 | 0 |
| N.S. | 1 | 1.69 | 0.69 | 1.45 | 0.00 | 0.00 | 0.00 | 0.00 | 0.75 | 0.00 |
| time (sec) | N/A | 3.719 | 0.316 | 1.300 | 0.000 | 0.000 | 0.000 | 0.000 | 0.203 | 0.000 |

| Problem 299 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | F | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 140 | 145 | 74 | 93 | 185 | 87 | 0 | 0 | 93 | 85 |
| N.S. | 1 | 1.04 | 0.53 | 0.66 | 1.32 | 0.62 | 0.00 | 0.00 | 0.66 | 0.61 |
| time (sec) | N/A | 0.609 | 0.080 | 0.878 | 0.127 | 0.105 | 0.000 | 0.000 | 0.183 | 0.841 |

| Problem 300 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | F | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 181 | 312 | 95 | 123 | 232 | 114 | 0 | 0 | 130 | 150 |
| N.S. | 1 | 1.72 | 0.52 | 0.68 | 1.28 | 0.63 | 0.00 | 0.00 | 0.72 | 0.83 |
| time (sec) | N/A | 0.957 | 0.101 | 0.864 | 0.153 | 0.104 | 0.000 | 0.000 | 0.184 | 0.754 |

| Problem 301 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | F | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 138 | 133 | 71 | 93 | 163 | 87 | 0 | 0 | 93 | 85 |
| N.S. | 1 | 0.96 | 0.51 | 0.67 | 1.18 | 0.63 | 0.00 | 0.00 | 0.67 | 0.62 |
| time (sec) | N/A | 0.429 | 0.047 | 0.777 | 0.136 | 0.113 | 0.000 | 0.000 | 0.191 | 0.773 |

| Problem 302 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | F | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 169 | 220 | 98 | 123 | 232 | 113 | 0 | 0 | 130 | 157 |
| N.S. | 1 | 1.30 | 0.58 | 0.73 | 1.37 | 0.67 | 0.00 | 0.00 | 0.77 | 0.93 |
| time (sec) | N/A | 0.568 | 0.048 | 0.899 | 0.162 | 0.101 | 0.000 | 0.000 | 0.185 | 0.807 |

| Problem 303 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 236 | 327 | 156 | 1722 | 0 | 0 | 0 | 0 | 40 | 0 |
| N.S. | 1 | 1.39 | 0.66 | 7.30 | 0.00 | 0.00 | 0.00 | 0.00 | 0.17 | 0.00 |
| time (sec) | N/A | 2.120 | 0.180 | 66.500 | 0.000 | 0.000 | 0.000 | 0.000 | 0.190 | 0.000 |

| Problem 304 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-1) | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 250 | 401 | 139 | 370 | 0 | 0 | 0 | 0 | 272 | 0 |
| N.S. | 1 | 1.60 | 0.56 | 1.48 | 0.00 | 0.00 | 0.00 | 0.00 | 1.09 | 0.00 |
| time (sec) | N/A | 2.737 | 0.294 | 1.184 | 0.000 | 0.000 | 0.000 | 0.000 | 0.178 | 0.000 |

| Problem 305 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | B | A | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 322 | 689 | 226 | 1970 | 0 | 0 | 0 | 0 | 42 | 0 |
| N.S. | 1 | 2.14 | 0.70 | 6.12 | 0.00 | 0.00 | 0.00 | 0.00 | 0.13 | 0.00 |
| time (sec) | N/A | 5.988 | 0.518 | 94.585 | 0.000 | 0.000 | 0.000 | 0.000 | 0.180 | 0.000 |

| Problem 306 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | F | A | A | F(-1) | F | F | F | F | F(-1) |
| verified | N/A | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 317 | 0 | 189 | 409 | 0 | 0 | 0 | 0 | 318 | 0 |
| N.S. | 1 | 0.00 | 0.60 | 1.29 | 0.00 | 0.00 | 0.00 | 0.00 | 1.00 | 0.00 |
| time (sec) | N/A | 0.000 | 0.565 | 2.075 | 0.000 | 0.000 | 0.000 | 0.000 | 0.193 | 0.000 |

| Problem 307 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | B | A | A | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 385 | 1189 | 360 | 235 | 0 | 0 | 0 | 0 | 25 | 0 |
| N.S. | 1 | 3.09 | 0.94 | 0.61 | 0.00 | 0.00 | 0.00 | 0.00 | 0.06 | 0.00 |
| time (sec) | N/A | 5.969 | 0.907 | 2.021 | 0.000 | 0.000 | 0.000 | 0.000 | 0.210 | 0.000 |

| Problem 308 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 436 | 706 | 267 | 302 | 0 | 0 | 0 | 0 | 25 | 0 |
| N.S. | 1 | 1.62 | 0.61 | 0.69 | 0.00 | 0.00 | 0.00 | 0.00 | 0.06 | 0.00 |
| time (sec) | N/A | 5.937 | 0.912 | 2.120 | 0.000 | 0.000 | 0.000 | 0.000 | 0.215 | 0.000 |

| Problem 309 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 279 | 227 | 260 | 198 | 0 | 0 | 0 | 0 | 23 | 0 |
| N.S. | 1 | 0.81 | 0.93 | 0.71 | 0.00 | 0.00 | 0.00 | 0.00 | 0.08 | 0.00 |
| time (sec) | N/A | 0.574 | 0.449 | 1.802 | 0.000 | 0.000 | 0.000 | 0.000 | 0.228 | 0.000 |

| Problem 310 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 340 | 228 | 201 | 268 | 0 | 0 | 0 | 0 | 22 | 0 |
| N.S. | 1 | 0.67 | 0.59 | 0.79 | 0.00 | 0.00 | 0.00 | 0.00 | 0.06 | 0.00 |
| time (sec) | N/A | 0.911 | 0.282 | 1.967 | 0.000 | 0.000 | 0.000 | 0.000 | 0.207 | 0.000 |

| Problem 311 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 439 | 305 | 250 | 337 | 0 | 0 | 0 | 0 | 25 | 0 |
| N.S. | 1 | 0.69 | 0.57 | 0.77 | 0.00 | 0.00 | 0.00 | 0.00 | 0.06 | 0.00 |
| time (sec) | N/A | 1.791 | 0.254 | 2.392 | 0.000 | 0.000 | 0.000 | 0.000 | 0.204 | 0.000 |

| Problem 312 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 458 | 300 | 265 | 308 | 0 | 0 | 0 | 0 | 25 | 0 |
| N.S. | 1 | 0.66 | 0.58 | 0.67 | 0.00 | 0.00 | 0.00 | 0.00 | 0.05 | 0.00 |
| time (sec) | N/A | 2.061 | 0.605 | 2.549 | 0.000 | 0.000 | 0.000 | 0.000 | 0.196 | 0.000 |

| Problem 313 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 328 | 354 | 222 | 255 | 0 | 0 | 0 | 0 | 25 | 0 |
| N.S. | 1 | 1.08 | 0.68 | 0.78 | 0.00 | 0.00 | 0.00 | 0.00 | 0.08 | 0.00 |
| time (sec) | N/A | 3.054 | 1.339 | 2.316 | 0.000 | 0.000 | 0.000 | 0.000 | 0.205 | 0.000 |

| Problem 314 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 275 | 267 | 239 | 195 | 0 | 0 | 0 | 0 | 25 | 0 |
| N.S. | 1 | 0.97 | 0.87 | 0.71 | 0.00 | 0.00 | 0.00 | 0.00 | 0.09 | 0.00 |
| time (sec) | N/A | 0.973 | 1.222 | 2.225 | 0.000 | 0.000 | 0.000 | 0.000 | 0.216 | 0.000 |

| Problem 315 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | B | A | A | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 476 | 3185 | 797 | 271 | 0 | 0 | 0 | 0 | 53 | 0 |
| N.S. | 1 | 6.69 | 1.67 | 0.57 | 0.00 | 0.00 | 0.00 | 0.00 | 0.11 | 0.00 |
| time (sec) | N/A | 19.446 | 3.272 | 2.214 | 0.000 | 0.000 | 0.000 | 0.000 | 0.223 | 0.000 |

| Problem 316 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | F | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | N/A | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 531 | 0 | 527 | 338 | 0 | 0 | 0 | 0 | 53 | 0 |
| N.S. | 1 | 0.00 | 0.99 | 0.64 | 0.00 | 0.00 | 0.00 | 0.00 | 0.10 | 0.00 |
| time (sec) | N/A | 0.000 | 2.468 | 2.203 | 0.000 | 0.000 | 0.000 | 0.000 | 0.225 | 0.000 |

| Problem 317 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 334 | 277 | 601 | 237 | 0 | 0 | 0 | 0 | 51 | 0 |
| N.S. | 1 | 0.83 | 1.80 | 0.71 | 0.00 | 0.00 | 0.00 | 0.00 | 0.15 | 0.00 |
| time (sec) | N/A | 0.720 | 2.849 | 2.048 | 0.000 | 0.000 | 0.000 | 0.000 | 0.202 | 0.000 |

| Problem 318 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 438 | 344 | 439 | 304 | 0 | 0 | 0 | 0 | 50 | 0 |
| N.S. | 1 | 0.79 | 1.00 | 0.69 | 0.00 | 0.00 | 0.00 | 0.00 | 0.11 | 0.00 |
| time (sec) | N/A | 1.274 | 0.720 | 2.121 | 0.000 | 0.000 | 0.000 | 0.000 | 0.204 | 0.000 |

| Problem 319 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 530 | 540 | 496 | 365 | 0 | 0 | 0 | 0 | 51 | 0 |
| N.S. | 1 | 1.02 | 0.94 | 0.69 | 0.00 | 0.00 | 0.00 | 0.00 | 0.10 | 0.00 |
| time (sec) | N/A | 3.453 | 2.048 | 2.570 | 0.000 | 0.000 | 0.000 | 0.000 | 0.218 | 0.000 |

| Problem 320 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 556 | 536 | 376 | 356 | 0 | 0 | 0 | 0 | 50 | 0 |
| N.S. | 1 | 0.96 | 0.68 | 0.64 | 0.00 | 0.00 | 0.00 | 0.00 | 0.09 | 0.00 |
| time (sec) | N/A | 4.487 | 0.823 | 2.539 | 0.000 | 0.000 | 0.000 | 0.000 | 0.204 | 0.000 |

| Problem 321 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 567 | 667 | 455 | 412 | 0 | 0 | 0 | 0 | 53 | 0 |
| N.S. | 1 | 1.18 | 0.80 | 0.73 | 0.00 | 0.00 | 0.00 | 0.00 | 0.09 | 0.00 |
| time (sec) | N/A | 6.208 | 2.167 | 2.563 | 0.000 | 0.000 | 0.000 | 0.000 | 0.222 | 0.000 |

| Problem 322 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 579 | 575 | 453 | 343 | 0 | 0 | 0 | 0 | 53 | 0 |
| N.S. | 1 | 0.99 | 0.78 | 0.59 | 0.00 | 0.00 | 0.00 | 0.00 | 0.09 | 0.00 |
| time (sec) | N/A | 4.646 | 4.324 | 2.672 | 0.000 | 0.000 | 0.000 | 0.000 | 0.211 | 0.000 |

| Problem 323 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | F | B | A | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | N/A | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 578 | 0 | 1320 | 309 | 0 | 0 | 0 | 0 | 82 | 0 |
| N.S. | 1 | 0.00 | 2.28 | 0.53 | 0.00 | 0.00 | 0.00 | 0.00 | 0.14 | 0.00 |
| time (sec) | N/A | 0.000 | 5.926 | 11.800 | 0.000 | 0.000 | 0.000 | 0.000 | 0.216 | 0.000 |

| Problem 324 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | F | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | N/A | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 638 | 0 | 759 | 376 | 0 | 0 | 0 | 0 | 82 | 0 |
| N.S. | 1 | 0.00 | 1.19 | 0.59 | 0.00 | 0.00 | 0.00 | 0.00 | 0.13 | 0.00 |
| time (sec) | N/A | 0.000 | 3.399 | 7.589 | 0.000 | 0.000 | 0.000 | 0.000 | 0.244 | 0.000 |

| Problem 325 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | A | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 387 | 327 | 1039 | 275 | 0 | 0 | 0 | 0 | 80 | 0 |
| N.S. | 1 | 0.84 | 2.68 | 0.71 | 0.00 | 0.00 | 0.00 | 0.00 | 0.21 | 0.00 |
| time (sec) | N/A | 0.893 | 6.766 | 4.860 | 0.000 | 0.000 | 0.000 | 0.000 | 0.208 | 0.000 |

| Problem 326 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 516 | 485 | 771 | 342 | 0 | 0 | 0 | 0 | 79 | 0 |
| N.S. | 1 | 0.94 | 1.49 | 0.66 | 0.00 | 0.00 | 0.00 | 0.00 | 0.15 | 0.00 |
| time (sec) | N/A | 1.868 | 1.244 | 3.527 | 0.000 | 0.000 | 0.000 | 0.000 | 0.223 | 0.000 |

| Problem 327 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 605 | 825 | 889 | 404 | 0 | 0 | 0 | 0 | 80 | 0 |
| N.S. | 1 | 1.36 | 1.47 | 0.67 | 0.00 | 0.00 | 0.00 | 0.00 | 0.13 | 0.00 |
| time (sec) | N/A | 6.169 | 6.962 | 4.209 | 0.000 | 0.000 | 0.000 | 0.000 | 0.221 | 0.000 |

| Problem 328 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | F | A | A | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | N/A | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 655 | 0 | 626 | 399 | 0 | 0 | 0 | 0 | 79 | 0 |
| N.S. | 1 | 0.00 | 0.96 | 0.61 | 0.00 | 0.00 | 0.00 | 0.00 | 0.12 | 0.00 |
| time (sec) | N/A | 0.000 | 1.349 | 4.087 | 0.000 | 0.000 | 0.000 | 0.000 | 0.214 | 0.000 |

| Problem 329 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 661 | 1215 | 761 | 454 | 0 | 0 | 0 | 0 | 80 | 0 |
| N.S. | 1 | 1.84 | 1.15 | 0.69 | 0.00 | 0.00 | 0.00 | 0.00 | 0.12 | 0.00 |
| time (sec) | N/A | 11.330 | 7.279 | 4.102 | 0.000 | 0.000 | 0.000 | 0.000 | 0.238 | 0.000 |

| Problem 330 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 675 | 1119 | 644 | 401 | 0 | 0 | 0 | 0 | 79 | 0 |
| N.S. | 1 | 1.66 | 0.95 | 0.59 | 0.00 | 0.00 | 0.00 | 0.00 | 0.12 | 0.00 |
| time (sec) | N/A | 11.379 | 2.518 | 5.629 | 0.000 | 0.000 | 0.000 | 0.000 | 0.234 | 0.000 |

| Problem 331 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 315 | 420 | 279 | 206 | 0 | 0 | 0 | 0 | 29 | 0 |
| N.S. | 1 | 1.33 | 0.89 | 0.65 | 0.00 | 0.00 | 0.00 | 0.00 | 0.09 | 0.00 |
| time (sec) | N/A | 1.507 | 0.552 | 2.224 | 0.000 | 0.000 | 0.000 | 0.000 | 0.195 | 0.000 |

| Problem 332 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 344 | 242 | 175 | 274 | 0 | 0 | 0 | 0 | 29 | 0 |
| N.S. | 1 | 0.70 | 0.51 | 0.80 | 0.00 | 0.00 | 0.00 | 0.00 | 0.08 | 0.00 |
| time (sec) | N/A | 1.382 | 0.336 | 2.270 | 0.000 | 0.000 | 0.000 | 0.000 | 0.200 | 0.000 |

| Problem 333 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 220 | 172 | 126 | 180 | 0 | 0 | 0 | 0 | 27 | 0 |
| N.S. | 1 | 0.78 | 0.57 | 0.82 | 0.00 | 0.00 | 0.00 | 0.00 | 0.12 | 0.00 |
| time (sec) | N/A | 0.455 | 0.227 | 1.929 | 0.000 | 0.000 | 0.000 | 0.000 | 0.195 | 0.000 |

| Problem 334 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 256 | 143 | 140 | 0 | 0 | 0 | 0 | 0 | 26 | 0 |
| N.S. | 1 | 0.56 | 0.55 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.10 | 0.00 |
| time (sec) | N/A | 0.600 | 0.104 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.206 | 0.000 |

| Problem 335 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 227 | 126 | 145 | 197 | 0 | 0 | 0 | 0 | 29 | 0 |
| N.S. | 1 | 0.56 | 0.64 | 0.87 | 0.00 | 0.00 | 0.00 | 0.00 | 0.13 | 0.00 |
| time (sec) | N/A | 0.701 | 0.203 | 2.172 | 0.000 | 0.000 | 0.000 | 0.000 | 0.214 | 0.000 |

| Problem 336 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 208 | 155 | 128 | 171 | 0 | 0 | 0 | 0 | 29 | 0 |
| N.S. | 1 | 0.75 | 0.62 | 0.82 | 0.00 | 0.00 | 0.00 | 0.00 | 0.14 | 0.00 |
| time (sec) | N/A | 0.544 | 0.416 | 2.193 | 0.000 | 0.000 | 0.000 | 0.000 | 0.213 | 0.000 |

| Problem 337 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 328 | 221 | 231 | 261 | 0 | 0 | 0 | 0 | 29 | 0 |
| N.S. | 1 | 0.67 | 0.70 | 0.80 | 0.00 | 0.00 | 0.00 | 0.00 | 0.09 | 0.00 |
| time (sec) | N/A | 1.390 | 0.925 | 2.404 | 0.000 | 0.000 | 0.000 | 0.000 | 0.232 | 0.000 |

| Problem 338 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 311 | 382 | 228 | 206 | 0 | 0 | 0 | 0 | 29 | 0 |
| N.S. | 1 | 1.23 | 0.73 | 0.66 | 0.00 | 0.00 | 0.00 | 0.00 | 0.09 | 0.00 |
| time (sec) | N/A | 1.619 | 1.910 | 2.259 | 0.000 | 0.000 | 0.000 | 0.000 | 0.227 | 0.000 |

| Problem 339 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 305 | 264 | 209 | 294 | 0 | 0 | 0 | 0 | 50 | 0 |
| N.S. | 1 | 0.87 | 0.69 | 0.96 | 0.00 | 0.00 | 0.00 | 0.00 | 0.16 | 0.00 |
| time (sec) | N/A | 0.986 | 0.675 | 2.134 | 0.000 | 0.000 | 0.000 | 0.000 | 0.200 | 0.000 |

| Problem 340 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 349 | 224 | 228 | 0 | 0 | 0 | 0 | 0 | 50 | 0 |
| N.S. | 1 | 0.64 | 0.65 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.14 | 0.00 |
| time (sec) | N/A | 1.258 | 0.355 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.202 | 0.000 |

| Problem 341 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | A | A | F | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 78 | 79 | 50 | 116 | 73 | 51 | 0 | 72 | 127 | 0 |
| N.S. | 1 | 1.01 | 0.64 | 1.49 | 0.94 | 0.65 | 0.00 | 0.92 | 1.63 | 0.00 |
| time (sec) | N/A | 0.353 | 0.092 | 1.724 | 0.325 | 0.097 | 0.000 | 0.158 | 0.205 | 0.000 |

| Problem 342 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | A | A | F | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 72 | 72 | 49 | 114 | 53 | 51 | 0 | 72 | 79 | 0 |
| N.S. | 1 | 1.00 | 0.68 | 1.58 | 0.74 | 0.71 | 0.00 | 1.00 | 1.10 | 0.00 |
| time (sec) | N/A | 0.244 | 0.055 | 1.637 | 0.187 | 0.115 | 0.000 | 0.187 | 0.205 | 0.000 |

| Problem 343 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 310 | 214 | 204 | 306 | 0 | 0 | 0 | 0 | 49 | 0 |
| N.S. | 1 | 0.69 | 0.66 | 0.99 | 0.00 | 0.00 | 0.00 | 0.00 | 0.16 | 0.00 |
| time (sec) | N/A | 1.451 | 0.285 | 2.102 | 0.000 | 0.000 | 0.000 | 0.000 | 0.219 | 0.000 |

| Problem 344 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 293 | 237 | 226 | 279 | 0 | 0 | 0 | 0 | 51 | 0 |
| N.S. | 1 | 0.81 | 0.77 | 0.95 | 0.00 | 0.00 | 0.00 | 0.00 | 0.17 | 0.00 |
| time (sec) | N/A | 1.015 | 0.813 | 2.180 | 0.000 | 0.000 | 0.000 | 0.000 | 0.215 | 0.000 |

| Problem 345 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 422 | 445 | 371 | 376 | 0 | 0 | 0 | 0 | 51 | 0 |
| N.S. | 1 | 1.05 | 0.88 | 0.89 | 0.00 | 0.00 | 0.00 | 0.00 | 0.12 | 0.00 |
| time (sec) | N/A | 4.495 | 1.639 | 2.318 | 0.000 | 0.000 | 0.000 | 0.000 | 0.238 | 0.000 |

| Problem 346 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 397 | 629 | 270 | 318 | 0 | 0 | 0 | 0 | 51 | 0 |
| N.S. | 1 | 1.58 | 0.68 | 0.80 | 0.00 | 0.00 | 0.00 | 0.00 | 0.13 | 0.00 |
| time (sec) | N/A | 4.089 | 2.362 | 2.184 | 0.000 | 0.000 | 0.000 | 0.000 | 0.241 | 0.000 |

| Problem 347 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 400 | 482 | 229 | 454 | 0 | 0 | 0 | 0 | 68 | 0 |
| N.S. | 1 | 1.20 | 0.57 | 1.14 | 0.00 | 0.00 | 0.00 | 0.00 | 0.17 | 0.00 |
| time (sec) | N/A | 2.537 | 1.048 | 2.331 | 0.000 | 0.000 | 0.000 | 0.000 | 0.209 | 0.000 |

| Problem 348 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 444 | 391 | 239 | 0 | 0 | 0 | 0 | 0 | 68 | 0 |
| N.S. | 1 | 0.88 | 0.54 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.15 | 0.00 |
| time (sec) | N/A | 2.665 | 0.499 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.210 | 0.000 |

| Problem 349 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | A | F | F(-2) | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 172 | 205 | 81 | 276 | 0 | 92 | 0 | 0 | 235 | 0 |
| N.S. | 1 | 1.19 | 0.47 | 1.60 | 0.00 | 0.53 | 0.00 | 0.00 | 1.37 | 0.00 |
| time (sec) | N/A | 0.719 | 0.124 | 1.905 | 0.000 | 0.107 | 0.000 | 0.000 | 0.202 | 0.000 |

| Problem 350 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | A | A | F | F(-2) | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 139 | 154 | 80 | 272 | 117 | 88 | 0 | 0 | 141 | 0 |
| N.S. | 1 | 1.11 | 0.58 | 1.96 | 0.84 | 0.63 | 0.00 | 0.00 | 1.01 | 0.00 |
| time (sec) | N/A | 0.601 | 0.087 | 1.700 | 0.120 | 0.114 | 0.000 | 0.000 | 0.213 | 0.000 |

| Problem 351 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | A | F | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 137 | 141 | 71 | 276 | 0 | 82 | 0 | 97 | 211 | 0 |
| N.S. | 1 | 1.03 | 0.52 | 2.01 | 0.00 | 0.60 | 0.00 | 0.71 | 1.54 | 0.00 |
| time (sec) | N/A | 0.487 | 0.081 | 2.098 | 0.000 | 0.135 | 0.000 | 0.158 | 0.212 | 0.000 |

| Problem 352 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | A | A | F | F(-2) | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 157 | 185 | 86 | 272 | 111 | 93 | 0 | 0 | 161 | 0 |
| N.S. | 1 | 1.18 | 0.55 | 1.73 | 0.71 | 0.59 | 0.00 | 0.00 | 1.03 | 0.00 |
| time (sec) | N/A | 0.440 | 0.070 | 1.822 | 0.133 | 0.148 | 0.000 | 0.000 | 0.208 | 0.000 |

| Problem 353 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 389 | 365 | 246 | 460 | 0 | 0 | 0 | 0 | 67 | 0 |
| N.S. | 1 | 0.94 | 0.63 | 1.18 | 0.00 | 0.00 | 0.00 | 0.00 | 0.17 | 0.00 |
| time (sec) | N/A | 2.840 | 0.432 | 2.382 | 0.000 | 0.000 | 0.000 | 0.000 | 0.229 | 0.000 |

| Problem 354 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 381 | 432 | 296 | 579 | 0 | 0 | 0 | 0 | 69 | 0 |
| N.S. | 1 | 1.13 | 0.78 | 1.52 | 0.00 | 0.00 | 0.00 | 0.00 | 0.18 | 0.00 |
| time (sec) | N/A | 2.121 | 1.285 | 2.368 | 0.000 | 0.000 | 0.000 | 0.000 | 0.231 | 0.000 |

| Problem 355 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 841 | 37 | 53 | 24 | 2267 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 38.23 | 1.68 | 2.41 | 1.09 | 103.05 | 1.09 |
| time (sec) | N/A | 0.233 | 1.280 | 2.220 | 10.390 | 0.105 | 11.976 | 0.268 | 0.208 | 0.678 |

| Problem 356 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 22 | 20 | 386 | 22 | 31 | 22 | 940 | 22 |
| N.S. | 1 | 1.00 | 1.10 | 1.00 | 19.30 | 1.10 | 1.55 | 1.10 | 47.00 | 1.10 |
| time (sec) | N/A | 0.216 | 0.614 | 1.869 | 4.620 | 0.108 | 5.778 | 0.197 | 0.190 | 0.649 |

| Problem 357 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 24 | 24 | 20 | 24 | 27 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 1.09 | 1.09 | 0.91 | 1.09 | 1.23 | 1.09 |
| time (sec) | N/A | 0.235 | 0.811 | 1.135 | 0.366 | 0.100 | 1.502 | 0.120 | 0.184 | 0.605 |

| Problem 358 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 24 | 39 | 31 | 24 | 35 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 1.09 | 1.77 | 1.41 | 1.09 | 1.59 | 1.09 |
| time (sec) | N/A | 0.240 | 0.539 | 1.807 | 0.356 | 0.121 | 2.774 | 0.123 | 0.188 | 0.628 |

| Problem 359 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | F(-1) | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 24 | 24 | 0 | 0 | 56 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 1.00 | 0.00 | 0.00 | 2.33 | 1.00 |
| time (sec) | N/A | 0.277 | 0.978 | 2.431 | 0.472 | 0.101 | 0.000 | 0.000 | 0.395 | 0.715 |

| Problem 360 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 24 | 24 | 24 | 0 | 25 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 1.00 | 1.00 | 0.00 | 1.04 | 1.00 |
| time (sec) | N/A | 0.267 | 0.137 | 2.217 | 0.271 | 0.118 | 29.154 | 0.000 | 0.301 | 0.624 |

| Problem 361 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 24 | 24 | 24 | 24 | 29 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 1.00 | 1.00 | 1.00 | 1.21 | 1.00 |
| time (sec) | N/A | 0.300 | 0.575 | 2.174 | 0.231 | 0.102 | 17.114 | 0.153 | 0.291 | 0.704 |

| Problem 362 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 24 | 51 | 24 | 24 | 50 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 2.12 | 1.00 | 1.00 | 2.08 | 1.00 |
| time (sec) | N/A | 0.289 | 0.712 | 2.445 | 0.288 | 0.113 | 14.212 | 0.189 | 0.244 | 0.806 |

| Problem 363 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | B | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 219 | 711 | 135 | 273 | 0 | 0 | 0 | 0 | 147 | 0 |
| N.S. | 1 | 3.25 | 0.62 | 1.25 | 0.00 | 0.00 | 0.00 | 0.00 | 0.67 | 0.00 |
| time (sec) | N/A | 4.749 | 0.481 | 3.289 | 0.000 | 0.000 | 0.000 | 0.000 | 0.180 | 0.000 |

| Problem 364 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | B | A | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 211 | 503 | 171 | 990 | 0 | 0 | 0 | 0 | 120 | 0 |
| N.S. | 1 | 2.38 | 0.81 | 4.69 | 0.00 | 0.00 | 0.00 | 0.00 | 0.57 | 0.00 |
| time (sec) | N/A | 4.134 | 0.430 | 71.026 | 0.000 | 0.000 | 0.000 | 0.000 | 0.195 | 0.000 |

| Problem 365 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 160 | 164 | 101 | 237 | 0 | 0 | 0 | 0 | 107 | 0 |
| N.S. | 1 | 1.02 | 0.63 | 1.48 | 0.00 | 0.00 | 0.00 | 0.00 | 0.67 | 0.00 |
| time (sec) | N/A | 0.732 | 0.070 | 4.036 | 0.000 | 0.000 | 0.000 | 0.000 | 0.182 | 0.000 |

| Problem 366 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 172 | 188 | 144 | 890 | 0 | 0 | 0 | 0 | 99 | 0 |
| N.S. | 1 | 1.09 | 0.84 | 5.17 | 0.00 | 0.00 | 0.00 | 0.00 | 0.58 | 0.00 |
| time (sec) | N/A | 0.853 | 0.052 | 71.569 | 0.000 | 0.000 | 0.000 | 0.000 | 0.200 | 0.000 |

| Problem 367 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 276 | 339 | 264 | 460 | 0 | 0 | 0 | 0 | 95 | 0 |
| N.S. | 1 | 1.23 | 0.96 | 1.67 | 0.00 | 0.00 | 0.00 | 0.00 | 0.34 | 0.00 |
| time (sec) | N/A | 2.330 | 0.122 | 21.717 | 0.000 | 0.000 | 0.000 | 0.000 | 0.202 | 0.000 |

| Problem 368 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 169 | 236 | 188 | 1653 | 0 | 0 | 0 | 0 | 51 | 0 |
| N.S. | 1 | 1.40 | 1.11 | 9.78 | 0.00 | 0.00 | 0.00 | 0.00 | 0.30 | 0.00 |
| time (sec) | N/A | 1.715 | 0.134 | 152.191 | 0.000 | 0.000 | 0.000 | 0.000 | 0.182 | 0.000 |

| Problem 369 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 310 | 328 | 299 | 525 | 0 | 0 | 0 | 0 | 106 | 0 |
| N.S. | 1 | 1.06 | 0.96 | 1.69 | 0.00 | 0.00 | 0.00 | 0.00 | 0.34 | 0.00 |
| time (sec) | N/A | 2.252 | 0.258 | 29.465 | 0.000 | 0.000 | 0.000 | 0.000 | 0.187 | 0.000 |

| Problem 370 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 189 | 305 | 177 | 1848 | 0 | 0 | 0 | 0 | 121 | 0 |
| N.S. | 1 | 1.61 | 0.94 | 9.78 | 0.00 | 0.00 | 0.00 | 0.00 | 0.64 | 0.00 |
| time (sec) | N/A | 2.190 | 0.291 | 188.507 | 0.000 | 0.000 | 0.000 | 0.000 | 0.188 | 0.000 |

| Problem 371 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 313 | 313 | 165 | 331 | 0 | 0 | 0 | 0 | 197 | 0 |
| N.S. | 1 | 1.00 | 0.53 | 1.06 | 0.00 | 0.00 | 0.00 | 0.00 | 0.63 | 0.00 |
| time (sec) | N/A | 2.487 | 0.925 | 5.511 | 0.000 | 0.000 | 0.000 | 0.000 | 0.200 | 0.000 |

| Problem 372 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 321 | 321 | 233 | 1256 | 0 | 0 | 0 | 0 | 182 | 0 |
| N.S. | 1 | 1.00 | 0.73 | 3.91 | 0.00 | 0.00 | 0.00 | 0.00 | 0.57 | 0.00 |
| time (sec) | N/A | 2.009 | 0.805 | 90.177 | 0.000 | 0.000 | 0.000 | 0.000 | 0.192 | 0.000 |

| Problem 373 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 242 | 233 | 131 | 296 | 0 | 0 | 0 | 0 | 163 | 0 |
| N.S. | 1 | 0.96 | 0.54 | 1.22 | 0.00 | 0.00 | 0.00 | 0.00 | 0.67 | 0.00 |
| time (sec) | N/A | 0.962 | 0.533 | 5.710 | 0.000 | 0.000 | 0.000 | 0.000 | 0.191 | 0.000 |

| Problem 374 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 289 | 312 | 195 | 1038 | 0 | 0 | 0 | 0 | 149 | 0 |
| N.S. | 1 | 1.08 | 0.67 | 3.59 | 0.00 | 0.00 | 0.00 | 0.00 | 0.52 | 0.00 |
| time (sec) | N/A | 1.421 | 0.411 | 94.362 | 0.000 | 0.000 | 0.000 | 0.000 | 0.190 | 0.000 |

| Problem 375 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 370 | 370 | 302 | 566 | 0 | 0 | 0 | 0 | 151 | 0 |
| N.S. | 1 | 1.00 | 0.82 | 1.53 | 0.00 | 0.00 | 0.00 | 0.00 | 0.41 | 0.00 |
| time (sec) | N/A | 1.271 | 0.382 | 31.674 | 0.000 | 0.000 | 0.000 | 0.000 | 0.210 | 0.000 |

| Problem 376 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 284 | 284 | 246 | 1851 | 0 | 0 | 0 | 0 | 146 | 0 |
| N.S. | 1 | 1.00 | 0.87 | 6.52 | 0.00 | 0.00 | 0.00 | 0.00 | 0.51 | 0.00 |
| time (sec) | N/A | 1.056 | 0.269 | 307.723 | 0.000 | 0.000 | 0.000 | 0.000 | 0.192 | 0.000 |

| Problem 377 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 399 | 399 | 302 | 622 | 0 | 0 | 0 | 0 | 158 | 0 |
| N.S. | 1 | 1.00 | 0.76 | 1.56 | 0.00 | 0.00 | 0.00 | 0.00 | 0.40 | 0.00 |
| time (sec) | N/A | 1.126 | 0.345 | 71.289 | 0.000 | 0.000 | 0.000 | 0.000 | 0.197 | 0.000 |

| Problem 378 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 311 | 311 | 289 | 1924 | 0 | 0 | 0 | 0 | 139 | 0 |
| N.S. | 1 | 1.00 | 0.93 | 6.19 | 0.00 | 0.00 | 0.00 | 0.00 | 0.45 | 0.00 |
| time (sec) | N/A | 1.069 | 0.351 | 12.306 | 0.000 | 0.000 | 0.000 | 0.000 | 0.193 | 0.000 |

| Problem 379 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 381 | 381 | 191 | 383 | 0 | 0 | 0 | 0 | 245 | 0 |
| N.S. | 1 | 1.00 | 0.50 | 1.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.64 | 0.00 |
| time (sec) | N/A | 3.858 | 1.452 | 6.905 | 0.000 | 0.000 | 0.000 | 0.000 | 0.191 | 0.000 |

| Problem 380 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 389 | 389 | 281 | 1576 | 0 | 0 | 0 | 0 | 230 | 0 |
| N.S. | 1 | 1.00 | 0.72 | 4.05 | 0.00 | 0.00 | 0.00 | 0.00 | 0.59 | 0.00 |
| time (sec) | N/A | 3.187 | 1.387 | 136.069 | 0.000 | 0.000 | 0.000 | 0.000 | 0.198 | 0.000 |

| Problem 381 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 308 | 308 | 157 | 346 | 0 | 0 | 0 | 0 | 211 | 0 |
| N.S. | 1 | 1.00 | 0.51 | 1.12 | 0.00 | 0.00 | 0.00 | 0.00 | 0.69 | 0.00 |
| time (sec) | N/A | 1.310 | 0.986 | 7.210 | 0.000 | 0.000 | 0.000 | 0.000 | 0.192 | 0.000 |

| Problem 382 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 388 | 473 | 243 | 1267 | 0 | 0 | 0 | 0 | 197 | 0 |
| N.S. | 1 | 1.22 | 0.63 | 3.27 | 0.00 | 0.00 | 0.00 | 0.00 | 0.51 | 0.00 |
| time (sec) | N/A | 2.370 | 0.888 | 111.605 | 0.000 | 0.000 | 0.000 | 0.000 | 0.188 | 0.000 |

| Problem 383 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 447 | 447 | 350 | 664 | 0 | 0 | 0 | 0 | 200 | 0 |
| N.S. | 1 | 1.00 | 0.78 | 1.49 | 0.00 | 0.00 | 0.00 | 0.00 | 0.45 | 0.00 |
| time (sec) | N/A | 1.997 | 0.747 | 39.122 | 0.000 | 0.000 | 0.000 | 0.000 | 0.203 | 0.000 |

| Problem 384 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 354 | 354 | 298 | 1988 | 0 | 0 | 0 | 0 | 194 | 0 |
| N.S. | 1 | 1.00 | 0.84 | 5.62 | 0.00 | 0.00 | 0.00 | 0.00 | 0.55 | 0.00 |
| time (sec) | N/A | 1.581 | 0.523 | 10.588 | 0.000 | 0.000 | 0.000 | 0.000 | 0.208 | 0.000 |

| Problem 385 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 503 | 503 | 464 | 763 | 0 | 0 | 0 | 0 | 239 | 0 |
| N.S. | 1 | 1.00 | 0.92 | 1.52 | 0.00 | 0.00 | 0.00 | 0.00 | 0.48 | 0.00 |
| time (sec) | N/A | 1.531 | 0.506 | 80.685 | 0.000 | 0.000 | 0.000 | 0.000 | 0.201 | 0.000 |

| Problem 386 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 336 | 336 | 331 | 1992 | 0 | 0 | 0 | 0 | 206 | 0 |
| N.S. | 1 | 1.00 | 0.99 | 5.93 | 0.00 | 0.00 | 0.00 | 0.00 | 0.61 | 0.00 |
| time (sec) | N/A | 1.385 | 0.504 | 13.512 | 0.000 | 0.000 | 0.000 | 0.000 | 0.189 | 0.000 |

| Problem 387 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 217 | 332 | 154 | 919 | 0 | 0 | 0 | 0 | 109 | 0 |
| N.S. | 1 | 1.53 | 0.71 | 4.24 | 0.00 | 0.00 | 0.00 | 0.00 | 0.50 | 0.00 |
| time (sec) | N/A | 2.626 | 0.215 | 65.297 | 0.000 | 0.000 | 0.000 | 0.000 | 0.194 | 0.000 |

| Problem 388 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 260 | 269 | 162 | 259 | 0 | 0 | 0 | 0 | 27 | 0 |
| N.S. | 1 | 1.03 | 0.62 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.10 | 0.00 |
| time (sec) | N/A | 1.835 | 0.215 | 70.462 | 0.000 | 0.000 | 0.000 | 0.000 | 0.191 | 0.000 |

| Problem 389 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 130 | 136 | 93 | 785 | 0 | 0 | 0 | 0 | 27 | 0 |
| N.S. | 1 | 1.05 | 0.72 | 6.04 | 0.00 | 0.00 | 0.00 | 0.00 | 0.21 | 0.00 |
| time (sec) | N/A | 0.908 | 0.174 | 58.124 | 0.000 | 0.000 | 0.000 | 0.000 | 0.188 | 0.000 |

| Problem 390 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 138 | 142 | 119 | 789 | 0 | 0 | 0 | 0 | 25 | 0 |
| N.S. | 1 | 1.03 | 0.86 | 5.72 | 0.00 | 0.00 | 0.00 | 0.00 | 0.18 | 0.00 |
| time (sec) | N/A | 0.722 | 0.020 | 69.654 | 0.000 | 0.000 | 0.000 | 0.000 | 0.196 | 0.000 |

| Problem 391 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 16 | 16 | 16 | 15 | 14 | 14 | 0 | 14 | 14 | 14 |
| N.S. | 1 | 1.00 | 1.00 | 0.94 | 0.88 | 0.88 | 0.00 | 0.88 | 0.88 | 0.88 |
| time (sec) | N/A | 0.190 | 0.004 | 1.016 | 0.110 | 0.098 | 0.000 | 0.120 | 0.195 | 0.138 |

| Problem 392 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 124 | 143 | 100 | 1640 | 0 | 0 | 0 | 0 | 24 | 0 |
| N.S. | 1 | 1.15 | 0.81 | 13.23 | 0.00 | 0.00 | 0.00 | 0.00 | 0.19 | 0.00 |
| time (sec) | N/A | 0.749 | 0.117 | 1.935 | 0.000 | 0.000 | 0.000 | 0.000 | 0.204 | 0.000 |

| Problem 393 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 122 | 135 | 108 | 1611 | 0 | 0 | 0 | 0 | 51 | 0 |
| N.S. | 1 | 1.11 | 0.89 | 13.20 | 0.00 | 0.00 | 0.00 | 0.00 | 0.42 | 0.00 |
| time (sec) | N/A | 0.967 | 0.147 | 3.701 | 0.000 | 0.000 | 0.000 | 0.000 | 0.197 | 0.000 |

| Problem 394 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 262 | 256 | 189 | 441 | 0 | 0 | 0 | 0 | 26 | 0 |
| N.S. | 1 | 0.98 | 0.72 | 1.68 | 0.00 | 0.00 | 0.00 | 0.00 | 0.10 | 0.00 |
| time (sec) | N/A | 1.857 | 0.347 | 94.099 | 0.000 | 0.000 | 0.000 | 0.000 | 0.189 | 0.000 |

| Problem 395 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F(-1) | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 227 | 321 | 180 | 1872 | 0 | 0 | 0 | 0 | 137 | 0 |
| N.S. | 1 | 1.41 | 0.79 | 8.25 | 0.00 | 0.00 | 0.00 | 0.00 | 0.60 | 0.00 |
| time (sec) | N/A | 2.482 | 0.391 | 5.871 | 0.000 | 0.000 | 0.000 | 0.000 | 0.194 | 0.000 |

| Problem 396 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 270 | 281 | 156 | 936 | 0 | 0 | 0 | 0 | 35 | 0 |
| N.S. | 1 | 1.04 | 0.58 | 3.47 | 0.00 | 0.00 | 0.00 | 0.00 | 0.13 | 0.00 |
| time (sec) | N/A | 1.717 | 0.144 | 78.961 | 0.000 | 0.000 | 0.000 | 0.000 | 0.197 | 0.000 |

| Problem 397 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | F | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 135 | 136 | 74 | 88 | 218 | 76 | 0 | 0 | 87 | 119 |
| N.S. | 1 | 1.01 | 0.55 | 0.65 | 1.61 | 0.56 | 0.00 | 0.00 | 0.64 | 0.88 |
| time (sec) | N/A | 0.478 | 0.062 | 1.786 | 0.166 | 0.111 | 0.000 | 0.000 | 0.189 | 0.705 |

| Problem 398 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | F | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 133 | 135 | 68 | 75 | 174 | 69 | 0 | 0 | 74 | 114 |
| N.S. | 1 | 1.02 | 0.51 | 0.56 | 1.31 | 0.52 | 0.00 | 0.00 | 0.56 | 0.86 |
| time (sec) | N/A | 0.455 | 0.045 | 1.589 | 0.153 | 0.091 | 0.000 | 0.000 | 0.187 | 0.680 |

| Problem 399 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | F | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 129 | 131 | 71 | 88 | 213 | 73 | 0 | 0 | 87 | 119 |
| N.S. | 1 | 1.02 | 0.55 | 0.68 | 1.65 | 0.57 | 0.00 | 0.00 | 0.67 | 0.92 |
| time (sec) | N/A | 0.429 | 0.034 | 1.677 | 0.176 | 0.092 | 0.000 | 0.000 | 0.196 | 0.684 |

| Problem 400 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 240 | 279 | 156 | 1787 | 0 | 0 | 0 | 0 | 32 | 0 |
| N.S. | 1 | 1.16 | 0.65 | 7.45 | 0.00 | 0.00 | 0.00 | 0.00 | 0.13 | 0.00 |
| time (sec) | N/A | 1.702 | 0.151 | 7.501 | 0.000 | 0.000 | 0.000 | 0.000 | 0.187 | 0.000 |

| Problem 401 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 234 | 262 | 157 | 1731 | 0 | 0 | 0 | 0 | 178 | 0 |
| N.S. | 1 | 1.12 | 0.67 | 7.40 | 0.00 | 0.00 | 0.00 | 0.00 | 0.76 | 0.00 |
| time (sec) | N/A | 1.952 | 0.272 | 7.392 | 0.000 | 0.000 | 0.000 | 0.000 | 0.198 | 0.000 |

| Problem 402 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 374 | 532 | 243 | 531 | 0 | 0 | 0 | 0 | 34 | 0 |
| N.S. | 1 | 1.42 | 0.65 | 1.42 | 0.00 | 0.00 | 0.00 | 0.00 | 0.09 | 0.00 |
| time (sec) | N/A | 4.534 | 0.476 | 107.021 | 0.000 | 0.000 | 0.000 | 0.000 | 0.194 | 0.000 |

| Problem 403 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 332 | 580 | 243 | 1990 | 0 | 0 | 0 | 0 | 258 | 0 |
| N.S. | 1 | 1.75 | 0.73 | 5.99 | 0.00 | 0.00 | 0.00 | 0.00 | 0.78 | 0.00 |
| time (sec) | N/A | 6.462 | 0.641 | 9.989 | 0.000 | 0.000 | 0.000 | 0.000 | 0.194 | 0.000 |

| Problem 404 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | F | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 212 | 261 | 105 | 123 | 289 | 117 | 0 | 0 | 130 | 205 |
| N.S. | 1 | 1.23 | 0.50 | 0.58 | 1.36 | 0.55 | 0.00 | 0.00 | 0.61 | 0.97 |
| time (sec) | N/A | 0.811 | 0.245 | 1.862 | 0.165 | 0.142 | 0.000 | 0.000 | 0.187 | 0.853 |

| Problem 405 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | F | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 237 | 410 | 111 | 145 | 334 | 130 | 0 | 0 | 153 | 188 |
| N.S. | 1 | 1.73 | 0.47 | 0.61 | 1.41 | 0.55 | 0.00 | 0.00 | 0.65 | 0.79 |
| time (sec) | N/A | 1.641 | 0.070 | 1.682 | 0.204 | 0.102 | 0.000 | 0.000 | 0.191 | 0.809 |

| Problem 406 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | F | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 208 | 237 | 103 | 123 | 272 | 117 | 0 | 0 | 130 | 189 |
| N.S. | 1 | 1.14 | 0.50 | 0.59 | 1.31 | 0.56 | 0.00 | 0.00 | 0.62 | 0.91 |
| time (sec) | N/A | 0.669 | 0.088 | 1.671 | 0.173 | 0.107 | 0.000 | 0.000 | 0.200 | 0.810 |

| Problem 407 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | F | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 225 | 284 | 114 | 153 | 335 | 132 | 0 | 0 | 153 | 199 |
| N.S. | 1 | 1.26 | 0.51 | 0.68 | 1.49 | 0.59 | 0.00 | 0.00 | 0.68 | 0.88 |
| time (sec) | N/A | 1.046 | 0.110 | 1.788 | 0.195 | 0.120 | 0.000 | 0.000 | 0.201 | 0.826 |

| Problem 408 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 332 | 515 | 208 | 1841 | 0 | 0 | 0 | 0 | 40 | 0 |
| N.S. | 1 | 1.55 | 0.63 | 5.55 | 0.00 | 0.00 | 0.00 | 0.00 | 0.12 | 0.00 |
| time (sec) | N/A | 3.519 | 0.206 | 5.086 | 0.000 | 0.000 | 0.000 | 0.000 | 0.213 | 0.000 |

| Problem 409 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 332 | 539 | 232 | 1799 | 0 | 0 | 0 | 0 | 304 | 0 |
| N.S. | 1 | 1.62 | 0.70 | 5.42 | 0.00 | 0.00 | 0.00 | 0.00 | 0.92 | 0.00 |
| time (sec) | N/A | 4.491 | 0.389 | 5.657 | 0.000 | 0.000 | 0.000 | 0.000 | 0.192 | 0.000 |

| Problem 410 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | F | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 478 | 0 | 295 | 579 | 0 | 0 | 0 | 0 | 42 | 0 |
| N.S. | 1 | 0.00 | 0.62 | 1.21 | 0.00 | 0.00 | 0.00 | 0.00 | 0.09 | 0.00 |
| time (sec) | N/A | 0.000 | 0.720 | 92.945 | 0.000 | 0.000 | 0.000 | 0.000 | 0.191 | 0.000 |

| Problem 411 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | F | A | C | F(-1) | F | F | F | F | F(-1) |
| verified | N/A | N/A | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 432 | 0 | 301 | 2062 | 0 | 0 | 0 | 0 | 444 | 0 |
| N.S. | 1 | 0.00 | 0.70 | 4.77 | 0.00 | 0.00 | 0.00 | 0.00 | 1.03 | 0.00 |
| time (sec) | N/A | 0.000 | 0.829 | 9.276 | 0.000 | 0.000 | 0.000 | 0.000 | 0.201 | 0.000 |

| Problem 412 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | F | A | A | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | N/A | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 523 | 0 | 262 | 417 | 0 | 0 | 0 | 0 | 25 | 0 |
| N.S. | 1 | 0.00 | 0.50 | 0.80 | 0.00 | 0.00 | 0.00 | 0.00 | 0.05 | 0.00 |
| time (sec) | N/A | 0.000 | 0.900 | 4.678 | 0.000 | 0.000 | 0.000 | 0.000 | 0.212 | 0.000 |

| Problem 413 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 747 | 1306 | 1844 | 460 | 0 | 0 | 0 | 0 | 25 | 0 |
| N.S. | 1 | 1.75 | 2.47 | 0.62 | 0.00 | 0.00 | 0.00 | 0.00 | 0.03 | 0.00 |
| time (sec) | N/A | 10.223 | 12.203 | 4.995 | 0.000 | 0.000 | 0.000 | 0.000 | 0.206 | 0.000 |

| Problem 414 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 373 | 264 | 204 | 370 | 0 | 0 | 0 | 0 | 23 | 0 |
| N.S. | 1 | 0.71 | 0.55 | 0.99 | 0.00 | 0.00 | 0.00 | 0.00 | 0.06 | 0.00 |
| time (sec) | N/A | 1.141 | 0.512 | 4.223 | 0.000 | 0.000 | 0.000 | 0.000 | 0.229 | 0.000 |

| Problem 415 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 626 | 400 | 258 | 422 | 0 | 0 | 0 | 0 | 22 | 0 |
| N.S. | 1 | 0.64 | 0.41 | 0.67 | 0.00 | 0.00 | 0.00 | 0.00 | 0.04 | 0.00 |
| time (sec) | N/A | 1.611 | 0.552 | 4.774 | 0.000 | 0.000 | 0.000 | 0.000 | 0.221 | 0.000 |

| Problem 416 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 600 | 357 | 366 | 454 | 0 | 0 | 0 | 0 | 25 | 0 |
| N.S. | 1 | 0.60 | 0.61 | 0.76 | 0.00 | 0.00 | 0.00 | 0.00 | 0.04 | 0.00 |
| time (sec) | N/A | 2.990 | 0.467 | 6.241 | 0.000 | 0.000 | 0.000 | 0.000 | 0.217 | 0.000 |

| Problem 417 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 622 | 360 | 768 | 466 | 0 | 0 | 0 | 0 | 25 | 0 |
| N.S. | 1 | 0.58 | 1.23 | 0.75 | 0.00 | 0.00 | 0.00 | 0.00 | 0.04 | 0.00 |
| time (sec) | N/A | 2.763 | 2.215 | 6.155 | 0.000 | 0.000 | 0.000 | 0.000 | 0.201 | 0.000 |

| Problem 418 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 602 | 560 | 345 | 404 | 0 | 0 | 0 | 0 | 25 | 0 |
| N.S. | 1 | 0.93 | 0.57 | 0.67 | 0.00 | 0.00 | 0.00 | 0.00 | 0.04 | 0.00 |
| time (sec) | N/A | 3.820 | 3.602 | 4.764 | 0.000 | 0.000 | 0.000 | 0.000 | 0.240 | 0.000 |

| Problem 419 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 361 | 387 | 341 | 315 | 0 | 0 | 0 | 0 | 25 | 0 |
| N.S. | 1 | 1.07 | 0.94 | 0.87 | 0.00 | 0.00 | 0.00 | 0.00 | 0.07 | 0.00 |
| time (sec) | N/A | 3.477 | 2.520 | 4.831 | 0.000 | 0.000 | 0.000 | 0.000 | 0.223 | 0.000 |

| Problem 420 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | F | A | A | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | N/A | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 652 | 0 | 538 | 469 | 0 | 0 | 0 | 0 | 53 | 0 |
| N.S. | 1 | 0.00 | 0.83 | 0.72 | 0.00 | 0.00 | 0.00 | 0.00 | 0.08 | 0.00 |
| time (sec) | N/A | 0.000 | 2.293 | 4.704 | 0.000 | 0.000 | 0.000 | 0.000 | 0.221 | 0.000 |

| Problem 421 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | F | B | A | F | F | F | F | F | F(-1) |
| verified | N/A | N/A | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 882 | 0 | 4015 | 514 | 0 | 0 | 0 | 0 | 53 | 0 |
| N.S. | 1 | 0.00 | 4.55 | 0.58 | 0.00 | 0.00 | 0.00 | 0.00 | 0.06 | 0.00 |
| time (sec) | N/A | 0.000 | 18.385 | 4.961 | 0.000 | 0.000 | 0.000 | 0.000 | 0.232 | 0.000 |

| Problem 422 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 477 | 382 | 439 | 421 | 0 | 0 | 0 | 0 | 51 | 0 |
| N.S. | 1 | 0.80 | 0.92 | 0.88 | 0.00 | 0.00 | 0.00 | 0.00 | 0.11 | 0.00 |
| time (sec) | N/A | 1.593 | 2.298 | 4.441 | 0.000 | 0.000 | 0.000 | 0.000 | 0.205 | 0.000 |

| Problem 423 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | A | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 760 | 652 | 2105 | 466 | 0 | 0 | 0 | 0 | 50 | 0 |
| N.S. | 1 | 0.86 | 2.77 | 0.61 | 0.00 | 0.00 | 0.00 | 0.00 | 0.07 | 0.00 |
| time (sec) | N/A | 2.460 | 12.603 | 4.748 | 0.000 | 0.000 | 0.000 | 0.000 | 0.225 | 0.000 |

| Problem 424 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 726 | 629 | 555 | 511 | 0 | 0 | 0 | 0 | 51 | 0 |
| N.S. | 1 | 0.87 | 0.76 | 0.70 | 0.00 | 0.00 | 0.00 | 0.00 | 0.07 | 0.00 |
| time (sec) | N/A | 6.270 | 1.016 | 5.576 | 0.000 | 0.000 | 0.000 | 0.000 | 0.220 | 0.000 |

| Problem 425 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 901 | 768 | 1387 | 602 | 0 | 0 | 0 | 0 | 50 | 0 |
| N.S. | 1 | 0.85 | 1.54 | 0.67 | 0.00 | 0.00 | 0.00 | 0.00 | 0.06 | 0.00 |
| time (sec) | N/A | 6.403 | 4.350 | 5.786 | 0.000 | 0.000 | 0.000 | 0.000 | 0.206 | 0.000 |

| Problem 426 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 919 | 925 | 691 | 592 | 0 | 0 | 0 | 0 | 53 | 0 |
| N.S. | 1 | 1.01 | 0.75 | 0.64 | 0.00 | 0.00 | 0.00 | 0.00 | 0.06 | 0.00 |
| time (sec) | N/A | 9.244 | 5.993 | 5.764 | 0.000 | 0.000 | 0.000 | 0.000 | 0.242 | 0.000 |

| Problem 427 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 788 | 755 | 1508 | 557 | 0 | 0 | 0 | 0 | 53 | 0 |
| N.S. | 1 | 0.96 | 1.91 | 0.71 | 0.00 | 0.00 | 0.00 | 0.00 | 0.07 | 0.00 |
| time (sec) | N/A | 8.449 | 9.055 | 5.786 | 0.000 | 0.000 | 0.000 | 0.000 | 0.224 | 0.000 |

| Problem 428 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | F | B | A | F | F | F | F | F | F(-1) |
| verified | N/A | N/A | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 1019 | 0 | 6517 | 566 | 0 | 0 | 0 | 0 | 82 | 0 |
| N.S. | 1 | 0.00 | 6.40 | 0.56 | 0.00 | 0.00 | 0.00 | 0.00 | 0.08 | 0.00 |
| time (sec) | N/A | 0.000 | 24.584 | 14.622 | 0.000 | 0.000 | 0.000 | 0.000 | 0.266 | 0.000 |

| Problem 429 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 561 | 523 | 717 | 477 | 0 | 0 | 0 | 0 | 80 | 0 |
| N.S. | 1 | 0.93 | 1.28 | 0.85 | 0.00 | 0.00 | 0.00 | 0.00 | 0.14 | 0.00 |
| time (sec) | N/A | 2.113 | 3.849 | 9.695 | 0.000 | 0.000 | 0.000 | 0.000 | 0.223 | 0.000 |

| Problem 430 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | A | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 870 | 954 | 4281 | 518 | 0 | 0 | 0 | 0 | 79 | 0 |
| N.S. | 1 | 1.10 | 4.92 | 0.60 | 0.00 | 0.00 | 0.00 | 0.00 | 0.09 | 0.00 |
| time (sec) | N/A | 4.107 | 18.798 | 7.548 | 0.000 | 0.000 | 0.000 | 0.000 | 0.237 | 0.000 |

| Problem 431 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | F | A | A | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | N/A | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 845 | 0 | 723 | 562 | 0 | 0 | 0 | 0 | 80 | 0 |
| N.S. | 1 | 0.00 | 0.86 | 0.67 | 0.00 | 0.00 | 0.00 | 0.00 | 0.09 | 0.00 |
| time (sec) | N/A | 0.000 | 4.810 | 8.792 | 0.000 | 0.000 | 0.000 | 0.000 | 0.244 | 0.000 |

| Problem 432 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | F | B | A | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | N/A | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 1027 | 0 | 3267 | 655 | 0 | 0 | 0 | 0 | 79 | 0 |
| N.S. | 1 | 0.00 | 3.18 | 0.64 | 0.00 | 0.00 | 0.00 | 0.00 | 0.08 | 0.00 |
| time (sec) | N/A | 0.000 | 14.475 | 8.974 | 0.000 | 0.000 | 0.000 | 0.000 | 0.270 | 0.000 |

| Problem 433 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | F | A | A | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | N/A | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 1043 | 0 | 934 | 660 | 0 | 0 | 0 | 0 | 80 | 0 |
| N.S. | 1 | 0.00 | 0.90 | 0.63 | 0.00 | 0.00 | 0.00 | 0.00 | 0.08 | 0.00 |
| time (sec) | N/A | 0.000 | 7.676 | 8.631 | 0.000 | 0.000 | 0.000 | 0.000 | 0.302 | 0.000 |

| Problem 434 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | F | A | A | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | N/A | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 1061 | 0 | 1771 | 699 | 0 | 0 | 0 | 0 | 79 | 0 |
| N.S. | 1 | 0.00 | 1.67 | 0.66 | 0.00 | 0.00 | 0.00 | 0.00 | 0.07 | 0.00 |
| time (sec) | N/A | 0.000 | 9.367 | 8.790 | 0.000 | 0.000 | 0.000 | 0.000 | 0.260 | 0.000 |

| Problem 435 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 408 | 460 | 218 | 382 | 0 | 0 | 0 | 0 | 29 | 0 |
| N.S. | 1 | 1.13 | 0.53 | 0.94 | 0.00 | 0.00 | 0.00 | 0.00 | 0.07 | 0.00 |
| time (sec) | N/A | 3.664 | 0.612 | 4.652 | 0.000 | 0.000 | 0.000 | 0.000 | 0.205 | 0.000 |

| Problem 436 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 625 | 415 | 812 | 428 | 0 | 0 | 0 | 0 | 29 | 0 |
| N.S. | 1 | 0.66 | 1.30 | 0.68 | 0.00 | 0.00 | 0.00 | 0.00 | 0.05 | 0.00 |
| time (sec) | N/A | 2.129 | 4.068 | 5.016 | 0.000 | 0.000 | 0.000 | 0.000 | 0.201 | 0.000 |

| Problem 437 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 283 | 172 | 168 | 0 | 0 | 0 | 0 | 0 | 27 | 0 |
| N.S. | 1 | 0.61 | 0.59 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.10 | 0.00 |
| time (sec) | N/A | 0.788 | 0.258 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.190 | 0.000 |

| Problem 438 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 368 | 201 | 190 | 0 | 0 | 0 | 0 | 0 | 26 | 0 |
| N.S. | 1 | 0.55 | 0.52 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.07 | 0.00 |
| time (sec) | N/A | 0.759 | 0.129 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.192 | 0.000 |

| Problem 439 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 327 | 178 | 208 | 261 | 0 | 0 | 0 | 0 | 29 | 0 |
| N.S. | 1 | 0.54 | 0.64 | 0.80 | 0.00 | 0.00 | 0.00 | 0.00 | 0.09 | 0.00 |
| time (sec) | N/A | 0.833 | 0.234 | 4.784 | 0.000 | 0.000 | 0.000 | 0.000 | 0.197 | 0.000 |

| Problem 440 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 260 | 157 | 174 | 230 | 0 | 0 | 0 | 0 | 29 | 0 |
| N.S. | 1 | 0.60 | 0.67 | 0.88 | 0.00 | 0.00 | 0.00 | 0.00 | 0.11 | 0.00 |
| time (sec) | N/A | 0.924 | 0.353 | 4.701 | 0.000 | 0.000 | 0.000 | 0.000 | 0.215 | 0.000 |

| Problem 441 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 597 | 375 | 345 | 410 | 0 | 0 | 0 | 0 | 29 | 0 |
| N.S. | 1 | 0.63 | 0.58 | 0.69 | 0.00 | 0.00 | 0.00 | 0.00 | 0.05 | 0.00 |
| time (sec) | N/A | 2.089 | 2.728 | 4.694 | 0.000 | 0.000 | 0.000 | 0.000 | 0.221 | 0.000 |

| Problem 442 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 396 | 418 | 343 | 324 | 0 | 0 | 0 | 0 | 29 | 0 |
| N.S. | 1 | 1.06 | 0.87 | 0.82 | 0.00 | 0.00 | 0.00 | 0.00 | 0.07 | 0.00 |
| time (sec) | N/A | 3.758 | 3.993 | 4.852 | 0.000 | 0.000 | 0.000 | 0.000 | 0.222 | 0.000 |

| Problem 443 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 403 | 291 | 308 | 0 | 0 | 0 | 0 | 0 | 50 | 0 |
| N.S. | 1 | 0.72 | 0.76 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.12 | 0.00 |
| time (sec) | N/A | 1.641 | 0.640 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.226 | 0.000 |

| Problem 444 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 495 | 311 | 639 | 0 | 0 | 0 | 0 | 0 | 50 | 0 |
| N.S. | 1 | 0.63 | 1.29 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.10 | 0.00 |
| time (sec) | N/A | 1.593 | 0.998 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.189 | 0.000 |

| Problem 445 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | A | A | F | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 107 | 106 | 61 | 134 | 98 | 62 | 0 | 99 | 98 | 0 |
| N.S. | 1 | 0.99 | 0.57 | 1.25 | 0.92 | 0.58 | 0.00 | 0.93 | 0.92 | 0.00 |
| time (sec) | N/A | 0.385 | 0.097 | 3.668 | 0.477 | 0.108 | 0.000 | 0.174 | 0.185 | 0.000 |

| Problem 446 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | A | A | F | A | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 100 | 101 | 56 | 132 | 99 | 58 | 0 | 99 | 146 | 0 |
| N.S. | 1 | 1.01 | 0.56 | 1.32 | 0.99 | 0.58 | 0.00 | 0.99 | 1.46 | 0.00 |
| time (sec) | N/A | 0.335 | 0.061 | 3.378 | 0.341 | 0.122 | 0.000 | 0.179 | 0.192 | 0.000 |

| Problem 447 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 443 | 293 | 295 | 388 | 0 | 0 | 0 | 0 | 49 | 0 |
| N.S. | 1 | 0.66 | 0.67 | 0.88 | 0.00 | 0.00 | 0.00 | 0.00 | 0.11 | 0.00 |
| time (sec) | N/A | 1.830 | 0.358 | 4.805 | 0.000 | 0.000 | 0.000 | 0.000 | 0.232 | 0.000 |

| Problem 448 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 377 | 268 | 301 | 356 | 0 | 0 | 0 | 0 | 51 | 0 |
| N.S. | 1 | 0.71 | 0.80 | 0.94 | 0.00 | 0.00 | 0.00 | 0.00 | 0.14 | 0.00 |
| time (sec) | N/A | 1.824 | 1.082 | 4.647 | 0.000 | 0.000 | 0.000 | 0.000 | 0.219 | 0.000 |

| Problem 449 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | No | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 534 | 609 | 367 | 0 | 0 | 0 | 0 | 0 | 68 | 0 |
| N.S. | 1 | 1.14 | 0.69 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.13 | 0.00 |
| time (sec) | N/A | 4.438 | 1.622 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.221 | 0.000 |

| Problem 450 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 622 | 563 | 691 | 0 | 0 | 0 | 0 | 0 | 68 | 0 |
| N.S. | 1 | 0.91 | 1.11 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.11 | 0.00 |
| time (sec) | N/A | 4.057 | 2.140 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.214 | 0.000 |

| Problem 451 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | A | F | F(-2) | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 237 | 305 | 104 | 312 | 0 | 113 | 0 | 0 | 203 | 0 |
| N.S. | 1 | 1.29 | 0.44 | 1.32 | 0.00 | 0.48 | 0.00 | 0.00 | 0.86 | 0.00 |
| time (sec) | N/A | 1.230 | 0.135 | 3.857 | 0.000 | 0.121 | 0.000 | 0.000 | 0.198 | 0.000 |

| Problem 452 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | A | F | F(-2) | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 199 | 239 | 95 | 308 | 0 | 106 | 0 | 0 | 259 | 0 |
| N.S. | 1 | 1.20 | 0.48 | 1.55 | 0.00 | 0.53 | 0.00 | 0.00 | 1.30 | 0.00 |
| time (sec) | N/A | 0.911 | 0.106 | 3.569 | 0.000 | 0.154 | 0.000 | 0.000 | 0.191 | 0.000 |

| Problem 453 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | A | F | F(-2) | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 199 | 220 | 91 | 312 | 0 | 103 | 0 | 0 | 179 | 0 |
| N.S. | 1 | 1.11 | 0.46 | 1.57 | 0.00 | 0.52 | 0.00 | 0.00 | 0.90 | 0.00 |
| time (sec) | N/A | 0.592 | 0.092 | 4.291 | 0.000 | 0.144 | 0.000 | 0.000 | 0.198 | 0.000 |

| Problem 454 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | A | F | F(-2) | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 215 | 274 | 104 | 308 | 0 | 111 | 0 | 0 | 279 | 0 |
| N.S. | 1 | 1.27 | 0.48 | 1.43 | 0.00 | 0.52 | 0.00 | 0.00 | 1.30 | 0.00 |
| time (sec) | N/A | 0.875 | 0.079 | 3.766 | 0.000 | 0.141 | 0.000 | 0.000 | 0.194 | 0.000 |

| Problem 455 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 553 | 523 | 347 | 560 | 0 | 0 | 0 | 0 | 67 | 0 |
| N.S. | 1 | 0.95 | 0.63 | 1.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.12 | 0.00 |
| time (sec) | N/A | 3.553 | 0.584 | 5.134 | 0.000 | 0.000 | 0.000 | 0.000 | 0.231 | 0.000 |

| Problem 456 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 493 | 552 | 399 | 528 | 0 | 0 | 0 | 0 | 69 | 0 |
| N.S. | 1 | 1.12 | 0.81 | 1.07 | 0.00 | 0.00 | 0.00 | 0.00 | 0.14 | 0.00 |
| time (sec) | N/A | 4.289 | 1.696 | 4.933 | 0.000 | 0.000 | 0.000 | 0.000 | 0.231 | 0.000 |

| Problem 457 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 952 | 37 | 53 | 24 | 3236 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 43.27 | 1.68 | 2.41 | 1.09 | 147.09 | 1.09 |
| time (sec) | N/A | 0.229 | 1.303 | 4.642 | 13.904 | 0.122 | 27.589 | 0.595 | 0.205 | 0.689 |

| Problem 458 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 22 | 20 | 435 | 22 | 31 | 22 | 1269 | 22 |
| N.S. | 1 | 1.00 | 1.10 | 1.00 | 21.75 | 1.10 | 1.55 | 1.10 | 63.45 | 1.10 |
| time (sec) | N/A | 0.208 | 0.640 | 6.707 | 6.638 | 0.128 | 12.283 | 0.198 | 0.207 | 0.682 |

| Problem 459 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 24 | 24 | 20 | 24 | 27 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 1.09 | 1.09 | 0.91 | 1.09 | 1.23 | 1.09 |
| time (sec) | N/A | 0.247 | 0.825 | 2.366 | 0.463 | 0.106 | 2.934 | 0.126 | 0.188 | 0.631 |

| Problem 460 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 24 | 39 | 31 | 24 | 35 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 1.09 | 1.77 | 1.41 | 1.09 | 1.59 | 1.09 |
| time (sec) | N/A | 0.239 | 0.540 | 3.910 | 0.436 | 0.099 | 4.090 | 0.126 | 0.198 | 0.647 |

| Problem 461 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | F(-1) | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 24 | 24 | 0 | 0 | 56 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 1.00 | 0.00 | 0.00 | 2.33 | 1.00 |
| time (sec) | N/A | 0.282 | 0.988 | 8.346 | 0.571 | 0.104 | 0.000 | 0.000 | 186.864 | 0.750 |

| Problem 462 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 24 | 24 | 24 | 0 | 25 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 1.00 | 1.00 | 0.00 | 1.04 | 1.00 |
| time (sec) | N/A | 0.272 | 0.141 | 7.915 | 0.348 | 0.112 | 55.020 | 0.000 | 0.261 | 0.651 |

| Problem 463 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 24 | 24 | 24 | 24 | 24 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| time (sec) | N/A | 0.266 | 0.570 | 7.861 | 0.313 | 0.132 | 37.635 | 0.151 | 200.021 | 0.741 |

| Problem 464 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 24 | 51 | 24 | 24 | 50 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 2.12 | 1.00 | 1.00 | 2.08 | 1.00 |
| time (sec) | N/A | 0.284 | 0.722 | 8.473 | 0.333 | 0.125 | 29.597 | 0.200 | 0.437 | 0.845 |

| Problem 465 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 18 | 18 | 20 | 18 | 20 | 21 | 22 | 20 | 29 | 20 |
| N.S. | 1 | 1.00 | 1.11 | 1.00 | 1.11 | 1.17 | 1.22 | 1.11 | 1.61 | 1.11 |
| time (sec) | N/A | 0.202 | 0.317 | 39.036 | 0.119 | 0.110 | 0.598 | 0.111 | 0.175 | 0.654 |

| Problem 466 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 17 | 19 | 17 | 19 | 19 | 22 | 19 | 68 | 19 |
| N.S. | 1 | 1.00 | 1.12 | 1.00 | 1.12 | 1.12 | 1.29 | 1.12 | 4.00 | 1.12 |
| time (sec) | N/A | 0.184 | 0.277 | 17.756 | 0.109 | 0.096 | 0.435 | 0.112 | 0.199 | 0.652 |

| Problem 467 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 22 | 20 | 22 | 22 | 22 | 22 | 29 | 22 |
| N.S. | 1 | 1.00 | 1.10 | 1.00 | 1.10 | 1.10 | 1.10 | 1.10 | 1.45 | 1.10 |
| time (sec) | N/A | 0.206 | 0.449 | 58.333 | 0.121 | 0.104 | 0.714 | 0.114 | 0.180 | 0.636 |

| Problem 468 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 22 | 20 | 22 | 36 | 39 | 22 | 48 | 22 |
| N.S. | 1 | 1.00 | 1.10 | 1.00 | 1.10 | 1.80 | 1.95 | 1.10 | 2.40 | 1.10 |
| time (sec) | N/A | 0.208 | 0.394 | 43.006 | 0.133 | 0.101 | 0.849 | 0.115 | 0.204 | 0.635 |

| Problem 469 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 19 | 19 | 21 | 19 | 21 | 34 | 39 | 21 | 99 | 21 |
| N.S. | 1 | 1.00 | 1.11 | 1.00 | 1.11 | 1.79 | 2.05 | 1.11 | 5.21 | 1.11 |
| time (sec) | N/A | 0.195 | 0.295 | 87.738 | 0.144 | 0.107 | 0.787 | 0.110 | 0.169 | 0.609 |

| Problem 470 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 24 | 37 | 39 | 24 | 48 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 1.09 | 1.68 | 1.77 | 1.09 | 2.18 | 1.09 |
| time (sec) | N/A | 0.222 | 0.532 | 87.637 | 0.134 | 0.087 | 1.630 | 0.116 | 0.172 | 0.615 |

| Problem 471 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 22 | 20 | 22 | 47 | 54 | 22 | 65 | 22 |
| N.S. | 1 | 1.00 | 1.10 | 1.00 | 1.10 | 2.35 | 2.70 | 1.10 | 3.25 | 1.10 |
| time (sec) | N/A | 0.216 | 0.394 | 103.177 | 0.162 | 0.099 | 1.152 | 0.115 | 0.183 | 0.640 |

| Problem 472 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 19 | 19 | 21 | 19 | 21 | 45 | 54 | 21 | 128 | 21 |
| N.S. | 1 | 1.00 | 1.11 | 1.00 | 1.11 | 2.37 | 2.84 | 1.11 | 6.74 | 1.11 |
| time (sec) | N/A | 0.195 | 0.335 | 104.841 | 0.167 | 0.093 | 1.088 | 0.113 | 0.176 | 0.661 |

| Problem 473 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 24 | 48 | 54 | 24 | 65 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 1.09 | 2.18 | 2.45 | 1.09 | 2.95 | 1.09 |
| time (sec) | N/A | 0.232 | 0.538 | 123.218 | 0.174 | 0.103 | 2.022 | 0.115 | 0.190 | 0.651 |

| Problem 474 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 24 | 24 | 22 | 24 | 28 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 1.09 | 1.09 | 1.00 | 1.09 | 1.27 | 1.09 |
| time (sec) | N/A | 0.237 | 1.006 | 0.874 | 0.111 | 0.100 | 0.472 | 0.112 | 0.201 | 0.627 |

| Problem 475 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 22 | 20 | 22 | 22 | 20 | 22 | 26 | 22 |
| N.S. | 1 | 1.00 | 1.10 | 1.00 | 1.10 | 1.10 | 1.00 | 1.10 | 1.30 | 1.10 |
| time (sec) | N/A | 0.230 | 0.587 | 0.518 | 0.106 | 0.098 | 0.465 | 0.150 | 0.220 | 0.601 |

| Problem 476 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 12 | 12 | 12 | 13 | 15 | 12 | 8 | 13 | 12 | 12 |
| N.S. | 1 | 1.00 | 1.00 | 1.08 | 1.25 | 1.00 | 0.67 | 1.08 | 1.00 | 1.00 |
| time (sec) | N/A | 0.202 | 0.090 | 1.116 | 0.043 | 0.116 | 0.212 | 0.119 | 0.216 | 0.095 |

| Problem 477 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 24 | 23 | 22 | 24 | 26 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 1.09 | 1.05 | 1.00 | 1.09 | 1.18 | 1.09 |
| time (sec) | N/A | 0.245 | 0.130 | 0.511 | 0.107 | 0.091 | 0.657 | 0.113 | 0.235 | 0.584 |

| Problem 478 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 24 | 25 | 24 | 24 | 28 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 1.09 | 1.14 | 1.09 | 1.09 | 1.27 | 1.09 |
| time (sec) | N/A | 0.244 | 0.187 | 0.550 | 0.106 | 0.090 | 0.623 | 0.113 | 0.213 | 0.595 |

| Problem 479 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 24 | 39 | 37 | 24 | 40 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 1.09 | 1.77 | 1.68 | 1.09 | 1.82 | 1.09 |
| time (sec) | N/A | 0.236 | 2.638 | 5.697 | 0.114 | 0.113 | 0.646 | 0.116 | 0.201 | 0.613 |

| Problem 480 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 24 | 39 | 37 | 24 | 40 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 1.09 | 1.77 | 1.68 | 1.09 | 1.82 | 1.09 |
| time (sec) | N/A | 0.248 | 1.438 | 5.461 | 0.106 | 0.117 | 0.645 | 0.113 | 0.182 | 0.598 |

| Problem 481 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | C | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 33 | 28 | 25 | 24 | 0 | 74 | 0 | 0 | 48 | 0 |
| N.S. | 1 | 0.85 | 0.76 | 0.73 | 0.00 | 2.24 | 0.00 | 0.00 | 1.45 | 0.00 |
| time (sec) | N/A | 0.320 | 0.078 | 4.782 | 0.000 | 0.114 | 0.000 | 0.000 | 0.201 | 0.000 |

| Problem 482 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | C | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 17 | 17 | 16 | 0 | 67 | 0 | 0 | 38 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.94 | 0.00 | 3.94 | 0.00 | 0.00 | 2.24 | 0.00 |
| time (sec) | N/A | 0.326 | 0.046 | 4.437 | 0.000 | 0.095 | 0.000 | 0.000 | 0.172 | 0.000 |

| Problem 483 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | C | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 33 | 28 | 23 | 22 | 0 | 70 | 0 | 0 | 36 | 0 |
| N.S. | 1 | 0.85 | 0.70 | 0.67 | 0.00 | 2.12 | 0.00 | 0.00 | 1.09 | 0.00 |
| time (sec) | N/A | 0.296 | 0.029 | 4.500 | 0.000 | 0.103 | 0.000 | 0.000 | 0.174 | 0.000 |

| Problem 484 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 24 | 38 | 37 | 24 | 38 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 1.09 | 1.73 | 1.68 | 1.09 | 1.73 | 1.09 |
| time (sec) | N/A | 0.231 | 0.486 | 8.044 | 0.123 | 0.105 | 0.942 | 0.114 | 0.183 | 0.601 |

| Problem 485 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 24 | 40 | 39 | 24 | 40 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 1.09 | 1.82 | 1.77 | 1.09 | 1.82 | 1.09 |
| time (sec) | N/A | 0.244 | 0.837 | 3.012 | 0.114 | 0.088 | 0.906 | 0.117 | 0.179 | 0.614 |

| Problem 486 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 24 | 50 | 51 | 24 | 52 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 1.09 | 2.27 | 2.32 | 1.09 | 2.36 | 1.09 |
| time (sec) | N/A | 0.237 | 5.542 | 1.666 | 0.118 | 0.106 | 0.867 | 0.115 | 0.199 | 0.645 |

| Problem 487 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 24 | 50 | 51 | 24 | 52 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 1.09 | 2.27 | 2.32 | 1.09 | 2.36 | 1.09 |
| time (sec) | N/A | 0.246 | 5.206 | 4.832 | 0.118 | 0.125 | 0.890 | 0.117 | 0.178 | 0.610 |

| Problem 488 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | C | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 50 | 39 | 34 | 33 | 0 | 174 | 0 | 0 | 113 | 0 |
| N.S. | 1 | 0.78 | 0.68 | 0.66 | 0.00 | 3.48 | 0.00 | 0.00 | 2.26 | 0.00 |
| time (sec) | N/A | 0.351 | 0.093 | 5.339 | 0.000 | 0.095 | 0.000 | 0.000 | 0.191 | 0.000 |

| Problem 489 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | C | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 35 | 30 | 27 | 28 | 0 | 171 | 0 | 0 | 52 | 0 |
| N.S. | 1 | 0.86 | 0.77 | 0.80 | 0.00 | 4.89 | 0.00 | 0.00 | 1.49 | 0.00 |
| time (sec) | N/A | 0.324 | 0.099 | 4.845 | 0.000 | 0.108 | 0.000 | 0.000 | 0.179 | 0.000 |

| Problem 490 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | C | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 33 | 28 | 25 | 24 | 0 | 120 | 0 | 0 | 52 | 0 |
| N.S. | 1 | 0.85 | 0.76 | 0.73 | 0.00 | 3.64 | 0.00 | 0.00 | 1.58 | 0.00 |
| time (sec) | N/A | 0.328 | 0.066 | 4.496 | 0.000 | 0.105 | 0.000 | 0.000 | 0.200 | 0.000 |

| Problem 491 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | C | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 35 | 30 | 27 | 26 | 0 | 171 | 0 | 0 | 50 | 0 |
| N.S. | 1 | 0.86 | 0.77 | 0.74 | 0.00 | 4.89 | 0.00 | 0.00 | 1.43 | 0.00 |
| time (sec) | N/A | 0.331 | 0.079 | 4.938 | 0.000 | 0.111 | 0.000 | 0.000 | 0.179 | 0.000 |

| Problem 492 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | C | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 50 | 39 | 34 | 33 | 0 | 174 | 0 | 0 | 48 | 0 |
| N.S. | 1 | 0.78 | 0.68 | 0.66 | 0.00 | 3.48 | 0.00 | 0.00 | 0.96 | 0.00 |
| time (sec) | N/A | 0.308 | 0.031 | 5.086 | 0.000 | 0.127 | 0.000 | 0.000 | 0.186 | 0.000 |

| Problem 493 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 24 | 49 | 51 | 24 | 50 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 1.09 | 2.23 | 2.32 | 1.09 | 2.27 | 1.09 |
| time (sec) | N/A | 0.231 | 0.611 | 5.363 | 0.109 | 0.090 | 1.228 | 0.116 | 0.186 | 0.606 |

| Problem 494 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 24 | 51 | 53 | 24 | 52 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 1.09 | 2.32 | 2.41 | 1.09 | 2.36 | 1.09 |
| time (sec) | N/A | 0.239 | 0.951 | 5.366 | 0.113 | 0.099 | 1.258 | 0.124 | 0.187 | 0.616 |

| Problem 495 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 20 | 22 | 22 | 20 | 22 | 23 | 22 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 1.00 | 1.00 | 0.91 | 1.00 | 1.05 | 1.00 |
| time (sec) | N/A | 0.243 | 1.619 | 16.759 | 0.176 | 0.103 | 0.556 | 0.128 | 0.192 | 0.575 |

| Problem 496 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 21 | 21 | 23 | 19 | 21 | 21 | 19 | 21 | 22 | 21 |
| N.S. | 1 | 1.00 | 1.10 | 0.90 | 1.00 | 1.00 | 0.90 | 1.00 | 1.05 | 1.00 |
| time (sec) | N/A | 0.209 | 0.181 | 7.408 | 0.188 | 0.110 | 0.413 | 0.124 | 0.180 | 0.554 |

| Problem 497 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 24 | 24 | 20 | 24 | 25 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 1.00 | 0.83 | 1.00 | 1.04 | 1.00 |
| time (sec) | N/A | 0.273 | 1.095 | 58.277 | 0.207 | 0.095 | 0.691 | 0.123 | 0.223 | 0.588 |

| Problem 498 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 20 | 22 | 33 | 20 | 22 | 51 | 22 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 1.00 | 1.50 | 0.91 | 1.00 | 2.32 | 1.00 |
| time (sec) | N/A | 0.254 | 1.694 | 34.924 | 0.306 | 0.097 | 3.531 | 0.150 | 0.188 | 0.603 |

| Problem 499 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 21 | 21 | 23 | 19 | 21 | 21 | 19 | 21 | 50 | 21 |
| N.S. | 1 | 1.00 | 1.10 | 0.90 | 1.00 | 1.00 | 0.90 | 1.00 | 2.38 | 1.00 |
| time (sec) | N/A | 0.214 | 0.249 | 36.395 | 0.243 | 0.102 | 2.419 | 0.146 | 0.213 | 0.607 |

| Problem 500 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 24 | 24 | 20 | 24 | 51 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 1.00 | 0.83 | 1.00 | 2.12 | 1.00 |
| time (sec) | N/A | 0.281 | 1.212 | 73.000 | 0.201 | 0.118 | 4.901 | 0.145 | 0.188 | 0.603 |

| Problem 501 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 20 | 22 | 48 | 20 | 22 | 80 | 22 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 1.00 | 2.18 | 0.91 | 1.00 | 3.64 | 1.00 |
| time (sec) | N/A | 0.252 | 1.729 | 41.856 | 0.268 | 0.109 | 11.999 | 0.158 | 0.198 | 0.599 |

| Problem 502 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 21 | 21 | 23 | 19 | 21 | 46 | 19 | 21 | 79 | 21 |
| N.S. | 1 | 1.00 | 1.10 | 0.90 | 1.00 | 2.19 | 0.90 | 1.00 | 3.76 | 1.00 |
| time (sec) | N/A | 0.214 | 0.302 | 47.550 | 0.284 | 0.104 | 8.576 | 0.151 | 0.189 | 0.596 |

| Problem 503 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 24 | 49 | 20 | 24 | 80 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 2.04 | 0.83 | 1.00 | 3.33 | 1.00 |
| time (sec) | N/A | 0.286 | 1.358 | 107.113 | 0.255 | 0.094 | 11.981 | 0.151 | 0.206 | 0.614 |

| Problem 504 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 20 | 22 | 22 | 20 | 22 | 27 | 22 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 1.00 | 1.00 | 0.91 | 1.00 | 1.23 | 1.00 |
| time (sec) | N/A | 0.242 | 0.797 | 1.668 | 0.174 | 0.091 | 0.581 | 0.128 | 0.180 | 0.621 |

| Problem 505 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 21 | 21 | 23 | 19 | 21 | 21 | 20 | 21 | 26 | 21 |
| N.S. | 1 | 1.00 | 1.10 | 0.90 | 1.00 | 1.00 | 0.95 | 1.00 | 1.24 | 1.00 |
| time (sec) | N/A | 0.214 | 0.155 | 4.585 | 0.195 | 0.102 | 0.569 | 0.133 | 0.182 | 0.615 |

| Problem 506 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 24 | 35 | 22 | 24 | 29 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 1.46 | 0.92 | 1.00 | 1.21 | 1.00 |
| time (sec) | N/A | 0.275 | 0.585 | 5.279 | 0.200 | 0.097 | 0.927 | 0.123 | 0.198 | 0.629 |

| Problem 507 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 24 | 51 | 22 | 0 | 53 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 2.12 | 0.92 | 0.00 | 2.21 | 1.00 |
| time (sec) | N/A | 0.294 | 4.808 | 2.304 | 0.206 | 0.107 | 1.543 | 0.000 | 0.190 | 0.643 |

| Problem 508 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | F(-1) | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 0 | 22 | 24 | 51 | 22 | 24 | 53 | 24 |
| N.S. | 1 | 1.00 | 0.00 | 0.92 | 1.00 | 2.12 | 0.92 | 1.00 | 2.21 | 1.00 |
| time (sec) | N/A | 0.282 | 0.000 | 2.369 | 0.210 | 0.123 | 1.541 | 0.148 | 0.202 | 0.637 |

| Problem 509 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 39 | 39 | 40 | 82 | 0 | 0 | 0 | 0 | 51 | 0 |
| N.S. | 1 | 1.00 | 1.03 | 2.10 | 0.00 | 0.00 | 0.00 | 0.00 | 1.31 | 0.00 |
| time (sec) | N/A | 0.427 | 0.155 | 4.920 | 0.000 | 0.000 | 0.000 | 0.000 | 0.189 | 0.000 |

| Problem 510 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 39 | 39 | 40 | 136 | 0 | 0 | 0 | 0 | 49 | 0 |
| N.S. | 1 | 1.00 | 1.03 | 3.49 | 0.00 | 0.00 | 0.00 | 0.00 | 1.26 | 0.00 |
| time (sec) | N/A | 0.391 | 0.043 | 4.687 | 0.000 | 0.000 | 0.000 | 0.000 | 0.203 | 0.000 |

| Problem 511 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 24 | 50 | 22 | 0 | 50 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 2.08 | 0.92 | 0.00 | 2.08 | 1.00 |
| time (sec) | N/A | 0.287 | 1.095 | 2.825 | 0.199 | 0.098 | 3.056 | 0.000 | 0.190 | 0.642 |

| Problem 512 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 24 | 52 | 24 | 24 | 52 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 2.17 | 1.00 | 1.00 | 2.17 | 1.00 |
| time (sec) | N/A | 0.303 | 1.184 | 2.970 | 0.224 | 0.091 | 3.485 | 0.146 | 0.210 | 0.650 |

| Problem 513 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 24 | 62 | 22 | 0 | 75 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 2.58 | 0.92 | 0.00 | 3.12 | 1.00 |
| time (sec) | N/A | 0.296 | 5.876 | 2.573 | 0.209 | 0.090 | 4.196 | 0.000 | 0.200 | 0.656 |

| Problem 514 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | F(-1) | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 0 | 22 | 24 | 62 | 22 | 24 | 75 | 24 |
| N.S. | 1 | 1.00 | 0.00 | 0.92 | 1.00 | 2.58 | 0.92 | 1.00 | 3.12 | 1.00 |
| time (sec) | N/A | 0.285 | 0.000 | 2.486 | 0.221 | 0.093 | 4.310 | 0.151 | 0.197 | 0.652 |

| Problem 515 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 87 | 55 | 55 | 125 | 0 | 0 | 0 | 0 | 75 | 0 |
| N.S. | 1 | 0.63 | 0.63 | 1.44 | 0.00 | 0.00 | 0.00 | 0.00 | 0.86 | 0.00 |
| time (sec) | N/A | 0.536 | 0.230 | 6.220 | 0.000 | 0.000 | 0.000 | 0.000 | 0.195 | 0.000 |

| Problem 516 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 87 | 55 | 53 | 84 | 0 | 0 | 0 | 0 | 75 | 0 |
| N.S. | 1 | 0.63 | 0.61 | 0.97 | 0.00 | 0.00 | 0.00 | 0.00 | 0.86 | 0.00 |
| time (sec) | N/A | 0.593 | 0.099 | 5.401 | 0.000 | 0.000 | 0.000 | 0.000 | 0.194 | 0.000 |

| Problem 517 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 87 | 55 | 51 | 125 | 0 | 0 | 0 | 0 | 73 | 0 |
| N.S. | 1 | 0.63 | 0.59 | 1.44 | 0.00 | 0.00 | 0.00 | 0.00 | 0.84 | 0.00 |
| time (sec) | N/A | 0.497 | 0.119 | 5.513 | 0.000 | 0.000 | 0.000 | 0.000 | 0.186 | 0.000 |

| Problem 518 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 87 | 55 | 53 | 179 | 0 | 0 | 0 | 0 | 71 | 0 |
| N.S. | 1 | 0.63 | 0.61 | 2.06 | 0.00 | 0.00 | 0.00 | 0.00 | 0.82 | 0.00 |
| time (sec) | N/A | 0.437 | 0.047 | 5.764 | 0.000 | 0.000 | 0.000 | 0.000 | 0.186 | 0.000 |

| Problem 519 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 24 | 61 | 22 | 0 | 72 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 2.54 | 0.92 | 0.00 | 3.00 | 1.00 |
| time (sec) | N/A | 0.282 | 1.524 | 3.279 | 0.227 | 0.098 | 7.682 | 0.000 | 0.200 | 0.646 |

| Problem 520 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 24 | 63 | 24 | 24 | 74 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 2.62 | 1.00 | 1.00 | 3.08 | 1.00 |
| time (sec) | N/A | 0.305 | 1.752 | 3.074 | 0.227 | 0.100 | 10.761 | 0.158 | 0.194 | 0.646 |

| Problem 521 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 24 | 48 | 66 | 24 | 76 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 1.09 | 2.18 | 3.00 | 1.09 | 3.45 | 1.09 |
| time (sec) | N/A | 0.235 | 0.469 | 5.875 | 0.250 | 0.099 | 10.009 | 0.115 | 0.193 | 0.600 |

| Problem 522 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 24 | 37 | 48 | 24 | 56 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 1.09 | 1.68 | 2.18 | 1.09 | 2.55 | 1.09 |
| time (sec) | N/A | 0.237 | 0.564 | 5.474 | 0.215 | 0.101 | 4.630 | 0.123 | 0.182 | 0.602 |

| Problem 523 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 22 | 20 | 22 | 22 | 27 | 22 | 34 | 22 |
| N.S. | 1 | 1.00 | 1.10 | 1.00 | 1.10 | 1.10 | 1.35 | 1.10 | 1.70 | 1.10 |
| time (sec) | N/A | 0.209 | 0.349 | 7.832 | 0.167 | 0.095 | 2.182 | 0.119 | 0.211 | 0.611 |

| Problem 524 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 24 | 24 | 22 | 24 | 28 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 1.09 | 1.09 | 1.00 | 1.09 | 1.27 | 1.09 |
| time (sec) | N/A | 0.241 | 0.408 | 2.865 | 0.129 | 0.102 | 1.316 | 0.121 | 0.191 | 0.600 |

| Problem 525 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 24 | 39 | 37 | 24 | 40 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 1.09 | 1.77 | 1.68 | 1.09 | 1.82 | 1.09 |
| time (sec) | N/A | 0.237 | 0.335 | 4.819 | 0.129 | 0.118 | 3.488 | 0.115 | 0.202 | 0.594 |

| Problem 526 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 24 | 50 | 51 | 24 | 52 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 1.09 | 2.27 | 2.32 | 1.09 | 2.36 | 1.09 |
| time (sec) | N/A | 0.245 | 0.347 | 4.812 | 0.123 | 0.095 | 8.691 | 0.117 | 0.182 | 0.598 |

| Problem 527 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | F(-1) | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 24 | 49 | 0 | 0 | 88 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 2.04 | 0.00 | 0.00 | 3.67 | 1.00 |
| time (sec) | N/A | 0.299 | 1.136 | 7.252 | 0.304 | 0.098 | 0.000 | 0.000 | 0.219 | 0.725 |

| Problem 528 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 24 | 24 | 22 | 0 | 56 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 1.00 | 0.92 | 0.00 | 2.33 | 1.00 |
| time (sec) | N/A | 0.287 | 0.617 | 7.115 | 0.287 | 0.115 | 92.236 | 0.000 | 0.196 | 0.667 |

| Problem 529 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 24 | 24 | 22 | 0 | 25 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 1.00 | 0.92 | 0.00 | 1.04 | 1.00 |
| time (sec) | N/A | 0.268 | 0.154 | 6.911 | 0.212 | 0.100 | 5.048 | 0.000 | 0.274 | 0.644 |

| Problem 530 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 24 | 24 | 22 | 24 | 29 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 1.00 | 0.92 | 1.00 | 1.21 | 1.00 |
| time (sec) | N/A | 0.272 | 0.508 | 6.815 | 0.191 | 0.087 | 6.468 | 0.132 | 0.205 | 0.795 |

| Problem 531 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 24 | 51 | 22 | 24 | 53 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 2.12 | 0.92 | 1.00 | 2.21 | 1.00 |
| time (sec) | N/A | 0.290 | 0.633 | 7.320 | 0.208 | 0.109 | 28.732 | 0.153 | 0.219 | 0.850 |

| Problem 532 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 24 | 62 | 0 | 24 | 75 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 2.58 | 0.00 | 1.00 | 3.12 | 1.00 |
| time (sec) | N/A | 0.293 | 0.672 | 7.134 | 0.233 | 0.095 | 0.000 | 0.153 | 0.201 | 0.867 |

| Problem 533 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 18 | 18 | 20 | 18 | 67 | 21 | 26 | 20 | 29 | 20 |
| N.S. | 1 | 1.00 | 1.11 | 1.00 | 3.72 | 1.17 | 1.44 | 1.11 | 1.61 | 1.11 |
| time (sec) | N/A | 0.200 | 0.526 | 43.960 | 0.178 | 0.086 | 0.682 | 0.113 | 0.192 | 0.698 |

| Problem 534 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 17 | 19 | 17 | 60 | 19 | 26 | 19 | 27 | 19 |
| N.S. | 1 | 1.00 | 1.12 | 1.00 | 3.53 | 1.12 | 1.53 | 1.12 | 1.59 | 1.12 |
| time (sec) | N/A | 0.183 | 0.386 | 51.476 | 0.116 | 0.094 | 0.449 | 0.116 | 0.178 | 0.683 |

| Problem 535 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 22 | 20 | 74 | 22 | 26 | 22 | 29 | 22 |
| N.S. | 1 | 1.00 | 1.10 | 1.00 | 3.70 | 1.10 | 1.30 | 1.10 | 1.45 | 1.10 |
| time (sec) | N/A | 0.204 | 0.619 | 54.805 | 0.188 | 0.103 | 0.916 | 0.119 | 0.200 | 0.642 |

| Problem 536 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 22 | 20 | 101 | 36 | 44 | 22 | 48 | 22 |
| N.S. | 1 | 1.00 | 1.10 | 1.00 | 5.05 | 1.80 | 2.20 | 1.10 | 2.40 | 1.10 |
| time (sec) | N/A | 0.209 | 0.650 | 31.138 | 0.196 | 0.108 | 0.933 | 0.117 | 0.186 | 0.637 |

| Problem 537 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 19 | 19 | 21 | 19 | 92 | 34 | 44 | 21 | 46 | 21 |
| N.S. | 1 | 1.00 | 1.11 | 1.00 | 4.84 | 1.79 | 2.32 | 1.11 | 2.42 | 1.11 |
| time (sec) | N/A | 0.188 | 0.841 | 125.511 | 0.146 | 0.083 | 0.837 | 0.119 | 0.206 | 0.641 |

| Problem 538 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 108 | 37 | 44 | 24 | 48 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 4.91 | 1.68 | 2.00 | 1.09 | 2.18 | 1.09 |
| time (sec) | N/A | 0.220 | 0.809 | 197.262 | 0.195 | 0.088 | 1.706 | 0.110 | 0.180 | 0.626 |

| Problem 539 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | F(-1) | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 22 | 0 | 123 | 47 | 61 | 22 | 65 | 22 |
| N.S. | 1 | 1.00 | 1.10 | 0.00 | 6.15 | 2.35 | 3.05 | 1.10 | 3.25 | 1.10 |
| time (sec) | N/A | 0.208 | 0.697 | 0.000 | 0.207 | 0.096 | 1.273 | 0.124 | 0.183 | 0.645 |

| Problem 540 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 19 | 19 | 21 | 19 | 114 | 45 | 61 | 21 | 63 | 21 |
| N.S. | 1 | 1.00 | 1.11 | 1.00 | 6.00 | 2.37 | 3.21 | 1.11 | 3.32 | 1.11 |
| time (sec) | N/A | 0.192 | 0.544 | 178.181 | 0.193 | 0.108 | 1.131 | 0.116 | 0.191 | 0.625 |

| Problem 541 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | F(-1) | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 0 | 130 | 48 | 61 | 24 | 65 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 0.00 | 5.91 | 2.18 | 2.77 | 1.09 | 2.95 | 1.09 |
| time (sec) | N/A | 0.236 | 0.852 | 0.000 | 0.213 | 0.117 | 2.224 | 0.111 | 0.178 | 0.619 |

| Problem 542 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 36 | 24 | 26 | 24 | 32 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 1.64 | 1.09 | 1.18 | 1.09 | 1.45 | 1.09 |
| time (sec) | N/A | 0.284 | 0.787 | 74.544 | 0.089 | 0.098 | 0.509 | 0.114 | 0.191 | 0.640 |

| Problem 543 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 34 | 24 | 26 | 24 | 32 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 1.55 | 1.09 | 1.18 | 1.09 | 1.45 | 1.09 |
| time (sec) | N/A | 0.291 | 0.410 | 6.456 | 0.089 | 0.092 | 0.495 | 0.117 | 0.178 | 0.615 |

| Problem 544 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 22 | 20 | 30 | 22 | 24 | 22 | 30 | 22 |
| N.S. | 1 | 1.00 | 1.10 | 1.00 | 1.50 | 1.10 | 1.20 | 1.10 | 1.50 | 1.10 |
| time (sec) | N/A | 0.249 | 0.318 | 6.399 | 0.086 | 0.093 | 0.512 | 0.115 | 0.184 | 0.597 |

| Problem 545 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 14 | 14 | 14 | 15 | 14 | 14 | 10 | 14 | 14 | 14 |
| N.S. | 1 | 1.00 | 1.00 | 1.07 | 1.00 | 1.00 | 0.71 | 1.00 | 1.00 | 1.00 |
| time (sec) | N/A | 0.184 | 0.004 | 1.358 | 0.032 | 0.105 | 0.287 | 0.120 | 0.175 | 0.588 |

| Problem 546 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 37 | 23 | 26 | 24 | 30 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 1.68 | 1.05 | 1.18 | 1.09 | 1.36 | 1.09 |
| time (sec) | N/A | 0.284 | 0.343 | 6.454 | 0.104 | 0.088 | 0.735 | 0.121 | 0.196 | 0.588 |

| Problem 547 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 40 | 25 | 27 | 24 | 32 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 1.82 | 1.14 | 1.23 | 1.09 | 1.45 | 1.09 |
| time (sec) | N/A | 0.314 | 0.467 | 6.507 | 0.098 | 0.115 | 0.673 | 0.122 | 0.183 | 0.617 |

| Problem 548 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 40 | 25 | 27 | 24 | 32 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 1.82 | 1.14 | 1.23 | 1.09 | 1.45 | 1.09 |
| time (sec) | N/A | 0.286 | 0.742 | 71.449 | 0.085 | 0.099 | 0.717 | 0.115 | 0.197 | 0.612 |

| Problem 549 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 40 | 25 | 27 | 24 | 32 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 1.82 | 1.14 | 1.23 | 1.09 | 1.45 | 1.09 |
| time (sec) | N/A | 0.285 | 0.937 | 64.548 | 0.094 | 0.087 | 0.794 | 0.113 | 0.190 | 0.609 |

| Problem 550 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 103 | 39 | 42 | 24 | 46 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 4.68 | 1.77 | 1.91 | 1.09 | 2.09 | 1.09 |
| time (sec) | N/A | 1.238 | 2.772 | 7.967 | 0.149 | 0.090 | 0.768 | 0.128 | 0.181 | 0.654 |

| Problem 551 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | C | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 43 | 43 | 40 | 37 | 0 | 123 | 0 | 0 | 116 | 0 |
| N.S. | 1 | 1.00 | 0.93 | 0.86 | 0.00 | 2.86 | 0.00 | 0.00 | 2.70 | 0.00 |
| time (sec) | N/A | 0.446 | 0.179 | 8.293 | 0.000 | 0.112 | 0.000 | 0.000 | 0.180 | 0.000 |

| Problem 552 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | B | A | A | F | C | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 41 | 84 | 36 | 36 | 0 | 115 | 0 | 0 | 44 | 0 |
| N.S. | 1 | 2.05 | 0.88 | 0.88 | 0.00 | 2.80 | 0.00 | 0.00 | 1.07 | 0.00 |
| time (sec) | N/A | 0.850 | 0.073 | 8.490 | 0.000 | 0.099 | 0.000 | 0.000 | 0.194 | 0.000 |

| Problem 553 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | C | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 41 | 41 | 34 | 37 | 0 | 112 | 0 | 0 | 115 | 0 |
| N.S. | 1 | 1.00 | 0.83 | 0.90 | 0.00 | 2.73 | 0.00 | 0.00 | 2.80 | 0.00 |
| time (sec) | N/A | 0.430 | 0.110 | 8.053 | 0.000 | 0.107 | 0.000 | 0.000 | 0.186 | 0.000 |

| Problem 554 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 102 | 38 | 42 | 24 | 44 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 4.64 | 1.73 | 1.91 | 1.09 | 2.00 | 1.09 |
| time (sec) | N/A | 1.276 | 1.039 | 4.662 | 0.141 | 0.104 | 1.057 | 0.272 | 0.197 | 0.608 |

| Problem 555 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 107 | 40 | 44 | 24 | 46 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 4.86 | 1.82 | 2.00 | 1.09 | 2.09 | 1.09 |
| time (sec) | N/A | 0.907 | 2.166 | 5.948 | 0.179 | 0.084 | 1.075 | 0.121 | 0.180 | 0.629 |

| Problem 556 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 106 | 40 | 44 | 24 | 46 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 4.82 | 1.82 | 2.00 | 1.09 | 2.09 | 1.09 |
| time (sec) | N/A | 1.791 | 2.430 | 74.187 | 0.142 | 0.119 | 1.071 | 0.123 | 0.194 | 0.639 |

| Problem 557 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 107 | 40 | 44 | 24 | 46 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 4.86 | 1.82 | 2.00 | 1.09 | 2.09 | 1.09 |
| time (sec) | N/A | 1.432 | 2.193 | 81.240 | 0.147 | 0.109 | 1.159 | 0.125 | 0.182 | 0.633 |

| Problem 558 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | B | A | A | F | C | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 86 | 177 | 83 | 60 | 0 | 292 | 0 | 0 | 60 | 0 |
| N.S. | 1 | 2.06 | 0.97 | 0.70 | 0.00 | 3.40 | 0.00 | 0.00 | 0.70 | 0.00 |
| time (sec) | N/A | 1.674 | 0.114 | 9.821 | 0.000 | 0.109 | 0.000 | 0.000 | 0.188 | 0.000 |

| Problem 559 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | C | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 67 | 102 | 59 | 37 | 0 | 196 | 0 | 0 | 60 | 0 |
| N.S. | 1 | 1.52 | 0.88 | 0.55 | 0.00 | 2.93 | 0.00 | 0.00 | 0.90 | 0.00 |
| time (sec) | N/A | 0.759 | 0.140 | 8.567 | 0.000 | 0.099 | 0.000 | 0.000 | 0.177 | 0.000 |

| Problem 560 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | C | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 61 | 95 | 75 | 60 | 0 | 286 | 0 | 0 | 58 | 0 |
| N.S. | 1 | 1.56 | 1.23 | 0.98 | 0.00 | 4.69 | 0.00 | 0.00 | 0.95 | 0.00 |
| time (sec) | N/A | 0.869 | 0.069 | 9.214 | 0.000 | 0.105 | 0.000 | 0.000 | 0.193 | 0.000 |

| Problem 561 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | C | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 58 | 57 | 45 | 59 | 0 | 287 | 0 | 0 | 205 | 0 |
| N.S. | 1 | 0.98 | 0.78 | 1.02 | 0.00 | 4.95 | 0.00 | 0.00 | 3.53 | 0.00 |
| time (sec) | N/A | 0.394 | 0.081 | 8.731 | 0.000 | 0.108 | 0.000 | 0.000 | 0.184 | 0.000 |

| Problem 562 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 135 | 49 | 58 | 24 | 58 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 6.14 | 2.23 | 2.64 | 1.09 | 2.64 | 1.09 |
| time (sec) | N/A | 2.290 | 1.102 | 11.043 | 0.156 | 0.102 | 1.468 | 0.124 | 0.195 | 0.638 |

| Problem 563 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 140 | 51 | 60 | 24 | 60 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 6.36 | 2.32 | 2.73 | 1.09 | 2.73 | 1.09 |
| time (sec) | N/A | 1.569 | 1.644 | 5.146 | 0.163 | 0.106 | 1.486 | 0.128 | 0.177 | 0.644 |

| Problem 564 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 139 | 51 | 60 | 24 | 60 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 6.32 | 2.32 | 2.73 | 1.09 | 2.73 | 1.09 |
| time (sec) | N/A | 4.121 | 2.286 | 60.391 | 0.190 | 0.104 | 1.561 | 0.126 | 0.185 | 0.637 |

| Problem 565 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 140 | 51 | 60 | 24 | 60 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 6.36 | 2.32 | 2.73 | 1.09 | 2.73 | 1.09 |
| time (sec) | N/A | 3.035 | 2.677 | 54.385 | 0.152 | 0.103 | 1.801 | 0.128 | 0.202 | 0.636 |

| Problem 566 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 20 | 22 | 22 | 22 | 22 | 23 | 22 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 1.00 | 1.00 | 1.00 | 1.00 | 1.05 | 1.00 |
| time (sec) | N/A | 0.241 | 1.381 | 34.523 | 0.183 | 0.099 | 0.608 | 0.121 | 0.179 | 0.608 |

| Problem 567 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 21 | 21 | 23 | 19 | 21 | 21 | 20 | 21 | 22 | 21 |
| N.S. | 1 | 1.00 | 1.10 | 0.90 | 1.00 | 1.00 | 0.95 | 1.00 | 1.05 | 1.00 |
| time (sec) | N/A | 0.207 | 0.322 | 21.103 | 0.196 | 0.088 | 0.664 | 0.128 | 0.198 | 0.578 |

| Problem 568 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 24 | 24 | 22 | 24 | 25 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 1.00 | 0.92 | 1.00 | 1.04 | 1.00 |
| time (sec) | N/A | 0.265 | 2.954 | 86.256 | 0.213 | 0.089 | 1.052 | 0.121 | 0.184 | 0.612 |

| Problem 569 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 20 | 22 | 33 | 22 | 22 | 51 | 22 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 1.00 | 1.50 | 1.00 | 1.00 | 2.32 | 1.00 |
| time (sec) | N/A | 0.247 | 3.142 | 62.335 | 0.230 | 0.089 | 4.863 | 0.149 | 0.197 | 0.643 |

| Problem 570 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 21 | 21 | 23 | 19 | 21 | 21 | 20 | 21 | 50 | 21 |
| N.S. | 1 | 1.00 | 1.10 | 0.90 | 1.00 | 1.00 | 0.95 | 1.00 | 2.38 | 1.00 |
| time (sec) | N/A | 0.213 | 0.695 | 53.345 | 0.204 | 0.093 | 3.358 | 0.144 | 0.191 | 0.618 |

| Problem 571 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 24 | 24 | 22 | 24 | 51 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 1.00 | 0.92 | 1.00 | 2.12 | 1.00 |
| time (sec) | N/A | 0.276 | 3.449 | 132.993 | 0.249 | 0.112 | 5.881 | 0.144 | 0.187 | 0.654 |

| Problem 572 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 20 | 22 | 48 | 22 | 22 | 80 | 22 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 1.00 | 2.18 | 1.00 | 1.00 | 3.64 | 1.00 |
| time (sec) | N/A | 0.241 | 1.856 | 84.913 | 0.253 | 0.097 | 19.876 | 0.154 | 0.194 | 0.658 |

| Problem 573 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 21 | 21 | 23 | 19 | 21 | 46 | 20 | 21 | 79 | 21 |
| N.S. | 1 | 1.00 | 1.10 | 0.90 | 1.00 | 2.19 | 0.95 | 1.00 | 3.76 | 1.00 |
| time (sec) | N/A | 0.208 | 0.637 | 69.845 | 0.262 | 0.111 | 12.089 | 0.148 | 0.190 | 0.623 |

| Problem 574 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 24 | 49 | 22 | 24 | 80 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 2.04 | 0.92 | 1.00 | 3.33 | 1.00 |
| time (sec) | N/A | 0.275 | 1.967 | 179.767 | 0.288 | 0.107 | 14.645 | 0.148 | 0.206 | 0.664 |

| Problem 575 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 20 | 22 | 22 | 22 | 22 | 27 | 22 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 1.00 | 1.00 | 1.00 | 1.00 | 1.23 | 1.00 |
| time (sec) | N/A | 0.234 | 0.937 | 2.194 | 0.196 | 0.100 | 0.914 | 0.130 | 0.183 | 0.663 |

| Problem 576 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 21 | 21 | 23 | 19 | 21 | 21 | 22 | 21 | 26 | 21 |
| N.S. | 1 | 1.00 | 1.10 | 0.90 | 1.00 | 1.00 | 1.05 | 1.00 | 1.24 | 1.00 |
| time (sec) | N/A | 0.206 | 0.542 | 5.467 | 0.216 | 0.087 | 0.922 | 0.132 | 0.200 | 0.633 |

| Problem 577 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 24 | 35 | 24 | 24 | 29 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 1.46 | 1.00 | 1.00 | 1.21 | 1.00 |
| time (sec) | N/A | 0.441 | 1.096 | 6.424 | 0.219 | 0.092 | 1.631 | 0.126 | 0.187 | 0.665 |

| Problem 578 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 24 | 51 | 24 | 0 | 57 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 2.12 | 1.00 | 0.00 | 2.38 | 1.00 |
| time (sec) | N/A | 1.202 | 9.552 | 10.075 | 0.235 | 0.094 | 2.225 | 0.000 | 0.219 | 0.701 |

| Problem 579 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 24 | 51 | 24 | 24 | 57 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 2.12 | 1.00 | 1.00 | 2.38 | 1.00 |
| time (sec) | N/A | 1.009 | 6.045 | 10.309 | 0.230 | 0.099 | 2.211 | 0.150 | 0.206 | 0.693 |

| Problem 580 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 69 | 69 | 55 | 126 | 0 | 0 | 0 | 0 | 55 | 0 |
| N.S. | 1 | 1.00 | 0.80 | 1.83 | 0.00 | 0.00 | 0.00 | 0.00 | 0.80 | 0.00 |
| time (sec) | N/A | 0.543 | 0.101 | 6.605 | 0.000 | 0.000 | 0.000 | 0.000 | 0.211 | 0.000 |

| Problem 581 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 69 | 69 | 53 | 128 | 0 | 0 | 0 | 0 | 53 | 0 |
| N.S. | 1 | 1.00 | 0.77 | 1.86 | 0.00 | 0.00 | 0.00 | 0.00 | 0.77 | 0.00 |
| time (sec) | N/A | 0.540 | 0.134 | 6.208 | 0.000 | 0.000 | 0.000 | 0.000 | 0.259 | 0.000 |

| Problem 582 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 24 | 50 | 24 | 0 | 54 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 2.08 | 1.00 | 0.00 | 2.25 | 1.00 |
| time (sec) | N/A | 1.472 | 1.824 | 11.058 | 0.210 | 0.104 | 4.267 | 0.000 | 0.205 | 0.694 |

| Problem 583 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 24 | 52 | 26 | 24 | 56 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 2.17 | 1.08 | 1.00 | 2.33 | 1.00 |
| time (sec) | N/A | 1.390 | 3.256 | 11.997 | 0.249 | 0.089 | 5.040 | 0.150 | 0.231 | 0.701 |

| Problem 584 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 24 | 52 | 26 | 0 | 56 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 2.17 | 1.08 | 0.00 | 2.33 | 1.00 |
| time (sec) | N/A | 2.358 | 3.601 | 22.894 | 0.218 | 0.091 | 6.925 | 0.000 | 0.218 | 0.698 |

| Problem 585 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 24 | 52 | 26 | 24 | 56 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 2.17 | 1.08 | 1.00 | 2.33 | 1.00 |
| time (sec) | N/A | 2.294 | 4.135 | 36.553 | 0.232 | 0.099 | 10.751 | 0.143 | 0.250 | 0.824 |

| Problem 586 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 24 | 62 | 24 | 0 | 81 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 2.58 | 1.00 | 0.00 | 3.38 | 1.00 |
| time (sec) | N/A | 3.302 | 11.599 | 7.730 | 0.255 | 0.118 | 6.017 | 0.000 | 0.199 | 0.753 |

| Problem 587 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 24 | 62 | 24 | 24 | 81 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 2.58 | 1.00 | 1.00 | 3.38 | 1.00 |
| time (sec) | N/A | 2.307 | 10.845 | 8.029 | 0.255 | 0.102 | 6.012 | 0.155 | 0.241 | 0.707 |

| Problem 588 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 118 | 88 | 82 | 309 | 0 | 0 | 0 | 0 | 81 | 0 |
| N.S. | 1 | 0.75 | 0.69 | 2.62 | 0.00 | 0.00 | 0.00 | 0.00 | 0.69 | 0.00 |
| time (sec) | N/A | 0.740 | 0.182 | 9.813 | 0.000 | 0.000 | 0.000 | 0.000 | 0.225 | 0.000 |

| Problem 589 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 142 | 167 | 99 | 309 | 0 | 0 | 0 | 0 | 81 | 0 |
| N.S. | 1 | 1.18 | 0.70 | 2.18 | 0.00 | 0.00 | 0.00 | 0.00 | 0.57 | 0.00 |
| time (sec) | N/A | 1.221 | 0.298 | 8.700 | 0.000 | 0.000 | 0.000 | 0.000 | 0.233 | 0.000 |

| Problem 590 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 116 | 141 | 95 | 301 | 0 | 0 | 0 | 0 | 79 | 0 |
| N.S. | 1 | 1.22 | 0.82 | 2.59 | 0.00 | 0.00 | 0.00 | 0.00 | 0.68 | 0.00 |
| time (sec) | N/A | 1.648 | 0.169 | 8.836 | 0.000 | 0.000 | 0.000 | 0.000 | 0.241 | 0.000 |

| Problem 591 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 115 | 85 | 61 | 305 | 0 | 0 | 0 | 0 | 77 | 0 |
| N.S. | 1 | 0.74 | 0.53 | 2.65 | 0.00 | 0.00 | 0.00 | 0.00 | 0.67 | 0.00 |
| time (sec) | N/A | 0.587 | 0.149 | 9.150 | 0.000 | 0.000 | 0.000 | 0.000 | 0.234 | 0.000 |

| Problem 592 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 24 | 61 | 24 | 0 | 78 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 2.54 | 1.00 | 0.00 | 3.25 | 1.00 |
| time (sec) | N/A | 4.759 | 1.872 | 12.078 | 0.244 | 0.093 | 11.946 | 0.000 | 0.208 | 0.709 |

| Problem 593 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 24 | 63 | 26 | 24 | 80 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 2.62 | 1.08 | 1.00 | 3.33 | 1.00 |
| time (sec) | N/A | 2.506 | 4.232 | 8.766 | 0.234 | 0.104 | 15.340 | 0.157 | 0.236 | 0.716 |

| Problem 594 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 24 | 63 | 26 | 0 | 80 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 2.62 | 1.08 | 0.00 | 3.33 | 1.00 |
| time (sec) | N/A | 9.030 | 5.261 | 22.116 | 0.241 | 0.097 | 19.807 | 0.000 | 0.251 | 0.708 |

| Problem 595 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 24 | 63 | 26 | 24 | 80 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 2.62 | 1.08 | 1.00 | 3.33 | 1.00 |
| time (sec) | N/A | 5.152 | 6.606 | 34.590 | 0.244 | 0.110 | 26.036 | 0.152 | 0.264 | 0.721 |

| Problem 596 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 30 | 30 | 32 | 28 | 371 | 125 | 126 | 30 | 586 | 30 |
| N.S. | 1 | 1.00 | 1.07 | 0.93 | 12.37 | 4.17 | 4.20 | 1.00 | 19.53 | 1.00 |
| time (sec) | N/A | 0.306 | 49.874 | 4.806 | 1.428 | 0.088 | 100.404 | 0.123 | 0.246 | 0.707 |

| Problem 597 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 177 | 48 | 73 | 24 | 76 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 8.05 | 2.18 | 3.32 | 1.09 | 3.45 | 1.09 |
| time (sec) | N/A | 0.222 | 0.503 | 7.925 | 0.562 | 0.090 | 12.060 | 0.120 | 0.254 | 0.640 |

| Problem 598 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 144 | 37 | 53 | 24 | 56 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 6.55 | 1.68 | 2.41 | 1.09 | 2.55 | 1.09 |
| time (sec) | N/A | 0.218 | 0.583 | 10.487 | 0.474 | 0.092 | 5.683 | 0.117 | 0.207 | 0.622 |

| Problem 599 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 22 | 20 | 95 | 22 | 31 | 22 | 34 | 22 |
| N.S. | 1 | 1.00 | 1.10 | 1.00 | 4.75 | 1.10 | 1.55 | 1.10 | 1.70 | 1.10 |
| time (sec) | N/A | 0.202 | 0.368 | 6.727 | 0.403 | 0.098 | 3.464 | 0.116 | 0.220 | 0.646 |

| Problem 600 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 40 | 24 | 26 | 24 | 32 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 1.82 | 1.09 | 1.18 | 1.09 | 1.45 | 1.09 |
| time (sec) | N/A | 0.306 | 0.492 | 4.412 | 0.124 | 0.100 | 3.137 | 0.134 | 0.217 | 0.663 |

| Problem 601 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 111 | 39 | 42 | 24 | 46 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 5.05 | 1.77 | 1.91 | 1.09 | 2.09 | 1.09 |
| time (sec) | N/A | 0.237 | 0.427 | 8.921 | 0.236 | 0.098 | 8.947 | 0.126 | 0.239 | 0.659 |

| Problem 602 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 144 | 50 | 58 | 24 | 60 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 6.55 | 2.27 | 2.64 | 1.09 | 2.73 | 1.09 |
| time (sec) | N/A | 0.249 | 0.437 | 9.751 | 0.243 | 0.117 | 26.595 | 0.134 | 0.205 | 0.664 |

| Problem 603 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | F(-1) | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 24 | 49 | 0 | 0 | 88 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 2.04 | 0.00 | 0.00 | 3.67 | 1.00 |
| time (sec) | N/A | 0.283 | 1.650 | 13.622 | 0.305 | 0.113 | 0.000 | 0.000 | 15.101 | 0.754 |

| Problem 604 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | F(-1) | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 24 | 24 | 0 | 0 | 56 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 1.00 | 0.00 | 0.00 | 2.33 | 1.00 |
| time (sec) | N/A | 0.280 | 0.867 | 12.276 | 0.274 | 0.091 | 0.000 | 0.000 | 0.405 | 0.695 |

| Problem 605 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 24 | 24 | 24 | 0 | 25 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 1.00 | 1.00 | 0.00 | 1.04 | 1.00 |
| time (sec) | N/A | 0.269 | 0.232 | 11.774 | 0.207 | 0.100 | 5.883 | 0.000 | 0.228 | 0.639 |

| Problem 606 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 24 | 24 | 24 | 24 | 29 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 1.00 | 1.00 | 1.00 | 1.21 | 1.00 |
| time (sec) | N/A | 0.269 | 0.626 | 9.197 | 0.233 | 0.094 | 14.158 | 0.137 | 0.212 | 0.813 |

| Problem 607 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 24 | 51 | 24 | 24 | 57 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 2.12 | 1.00 | 1.00 | 2.38 | 1.00 |
| time (sec) | N/A | 0.283 | 0.752 | 13.198 | 0.244 | 0.118 | 52.903 | 0.180 | 0.219 | 0.846 |

| Problem 608 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 24 | 62 | 0 | 24 | 81 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 2.58 | 0.00 | 1.00 | 3.38 | 1.00 |
| time (sec) | N/A | 0.286 | 0.810 | 13.200 | 0.247 | 0.108 | 0.000 | 0.166 | 0.288 | 0.891 |

| Problem 609 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 18 | 18 | 20 | 18 | 110 | 21 | 26 | 20 | 29 | 20 |
| N.S. | 1 | 1.00 | 1.11 | 1.00 | 6.11 | 1.17 | 1.44 | 1.11 | 1.61 | 1.11 |
| time (sec) | N/A | 0.193 | 0.794 | 135.273 | 0.212 | 0.087 | 0.656 | 0.127 | 0.251 | 0.692 |

| Problem 610 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 17 | 17 | 19 | 17 | 97 | 19 | 26 | 19 | 27 | 19 |
| N.S. | 1 | 1.00 | 1.12 | 1.00 | 5.71 | 1.12 | 1.53 | 1.12 | 1.59 | 1.12 |
| time (sec) | N/A | 0.182 | 0.811 | 62.570 | 0.210 | 0.106 | 0.544 | 0.123 | 0.211 | 0.696 |

| Problem 611 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 22 | 20 | 114 | 22 | 26 | 22 | 29 | 22 |
| N.S. | 1 | 1.00 | 1.10 | 1.00 | 5.70 | 1.10 | 1.30 | 1.10 | 1.45 | 1.10 |
| time (sec) | N/A | 0.203 | 1.063 | 177.304 | 0.220 | 0.099 | 0.923 | 0.142 | 0.205 | 0.663 |

| Problem 612 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 22 | 20 | 164 | 36 | 44 | 22 | 48 | 22 |
| N.S. | 1 | 1.00 | 1.10 | 1.00 | 8.20 | 1.80 | 2.20 | 1.10 | 2.40 | 1.10 |
| time (sec) | N/A | 0.224 | 0.711 | 166.872 | 0.257 | 0.112 | 0.909 | 0.139 | 0.198 | 0.636 |

| Problem 613 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 19 | 19 | 21 | 19 | 148 | 34 | 44 | 21 | 46 | 21 |
| N.S. | 1 | 1.00 | 1.11 | 1.00 | 7.79 | 1.79 | 2.32 | 1.11 | 2.42 | 1.11 |
| time (sec) | N/A | 0.192 | 0.431 | 178.297 | 0.236 | 0.093 | 0.857 | 0.134 | 0.185 | 0.635 |

| Problem 614 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 168 | 37 | 44 | 24 | 48 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 7.64 | 1.68 | 2.00 | 1.09 | 2.18 | 1.09 |
| time (sec) | N/A | 0.225 | 0.858 | 51.720 | 0.279 | 0.096 | 1.685 | 0.136 | 0.188 | 0.637 |

| Problem 615 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 22 | 20 | 196 | 47 | 61 | 22 | 65 | 22 |
| N.S. | 1 | 1.00 | 1.10 | 1.00 | 9.80 | 2.35 | 3.05 | 1.10 | 3.25 | 1.10 |
| time (sec) | N/A | 0.212 | 0.792 | 15.169 | 0.286 | 0.112 | 1.233 | 0.133 | 0.201 | 0.650 |

| Problem 616 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 19 | 19 | 21 | 19 | 181 | 45 | 61 | 21 | 63 | 21 |
| N.S. | 1 | 1.00 | 1.11 | 1.00 | 9.53 | 2.37 | 3.21 | 1.11 | 3.32 | 1.11 |
| time (sec) | N/A | 0.198 | 0.843 | 1.675 | 0.276 | 0.112 | 1.154 | 0.126 | 0.190 | 0.648 |

| Problem 617 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 201 | 48 | 61 | 24 | 65 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 9.14 | 2.18 | 2.77 | 1.09 | 2.95 | 1.09 |
| time (sec) | N/A | 0.220 | 0.860 | 30.305 | 0.293 | 0.093 | 2.209 | 0.140 | 0.187 | 0.639 |

| Problem 618 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 65 | 24 | 26 | 24 | 32 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 2.95 | 1.09 | 1.18 | 1.09 | 1.45 | 1.09 |
| time (sec) | N/A | 0.292 | 0.642 | 7.792 | 0.128 | 0.084 | 0.560 | 0.132 | 0.179 | 0.664 |

| Problem 619 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 62 | 24 | 26 | 24 | 32 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 2.82 | 1.09 | 1.18 | 1.09 | 1.45 | 1.09 |
| time (sec) | N/A | 0.302 | 0.416 | 0.549 | 0.128 | 0.115 | 0.533 | 0.140 | 0.195 | 0.611 |

| Problem 620 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 22 | 20 | 55 | 22 | 24 | 22 | 30 | 22 |
| N.S. | 1 | 1.00 | 1.10 | 1.00 | 2.75 | 1.10 | 1.20 | 1.10 | 1.50 | 1.10 |
| time (sec) | N/A | 0.251 | 0.435 | 0.628 | 0.102 | 0.095 | 0.549 | 0.114 | 0.227 | 0.604 |

| Problem 621 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 16 | 16 | 16 | 15 | 14 | 14 | 14 | 14 | 14 | 14 |
| N.S. | 1 | 1.00 | 1.00 | 0.94 | 0.88 | 0.88 | 0.88 | 0.88 | 0.88 | 0.88 |
| time (sec) | N/A | 0.189 | 0.004 | 0.932 | 0.039 | 0.074 | 0.359 | 0.125 | 0.185 | 0.593 |

| Problem 622 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 59 | 23 | 26 | 24 | 30 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 2.68 | 1.05 | 1.18 | 1.09 | 1.36 | 1.09 |
| time (sec) | N/A | 0.313 | 0.295 | 0.500 | 0.131 | 0.084 | 0.830 | 0.122 | 0.193 | 0.594 |

| Problem 623 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 69 | 25 | 27 | 24 | 32 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 3.14 | 1.14 | 1.23 | 1.09 | 1.45 | 1.09 |
| time (sec) | N/A | 0.290 | 0.776 | 0.716 | 0.142 | 0.086 | 0.732 | 0.131 | 0.242 | 0.617 |

| Problem 624 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 70 | 25 | 27 | 24 | 32 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 3.18 | 1.14 | 1.23 | 1.09 | 1.45 | 1.09 |
| time (sec) | N/A | 0.281 | 0.985 | 7.841 | 0.136 | 0.094 | 0.771 | 0.128 | 0.196 | 0.620 |

| Problem 625 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 71 | 25 | 27 | 24 | 32 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 3.23 | 1.14 | 1.23 | 1.09 | 1.45 | 1.09 |
| time (sec) | N/A | 0.323 | 1.830 | 7.800 | 0.150 | 0.092 | 0.839 | 0.129 | 0.195 | 0.619 |

| Problem 626 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 137 | 39 | 42 | 24 | 46 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 6.23 | 1.77 | 1.91 | 1.09 | 2.09 | 1.09 |
| time (sec) | N/A | 0.863 | 5.675 | 8.104 | 0.222 | 0.122 | 0.825 | 0.127 | 0.213 | 0.665 |

| Problem 627 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | C | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 71 | 113 | 51 | 52 | 0 | 132 | 0 | 0 | 133 | 0 |
| N.S. | 1 | 1.59 | 0.72 | 0.73 | 0.00 | 1.86 | 0.00 | 0.00 | 1.87 | 0.00 |
| time (sec) | N/A | 1.037 | 0.101 | 7.813 | 0.000 | 0.137 | 0.000 | 0.000 | 0.192 | 0.000 |

| Problem 628 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | C | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 81 | 81 | 70 | 51 | 0 | 135 | 0 | 0 | 140 | 0 |
| N.S. | 1 | 1.00 | 0.86 | 0.63 | 0.00 | 1.67 | 0.00 | 0.00 | 1.73 | 0.00 |
| time (sec) | N/A | 0.500 | 0.058 | 7.509 | 0.000 | 0.132 | 0.000 | 0.000 | 0.186 | 0.000 |

| Problem 629 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | C | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 65 | 109 | 58 | 52 | 0 | 122 | 0 | 0 | 132 | 0 |
| N.S. | 1 | 1.68 | 0.89 | 0.80 | 0.00 | 1.88 | 0.00 | 0.00 | 2.03 | 0.00 |
| time (sec) | N/A | 0.985 | 0.061 | 3.378 | 0.000 | 0.127 | 0.000 | 0.000 | 0.191 | 0.000 |

| Problem 630 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 141 | 38 | 42 | 24 | 44 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 6.41 | 1.73 | 1.91 | 1.09 | 2.00 | 1.09 |
| time (sec) | N/A | 0.904 | 1.256 | 1.180 | 0.215 | 0.082 | 1.108 | 0.126 | 0.221 | 0.622 |

| Problem 631 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 142 | 40 | 44 | 24 | 46 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 6.45 | 1.82 | 2.00 | 1.09 | 2.09 | 1.09 |
| time (sec) | N/A | 1.504 | 1.906 | 1.144 | 0.221 | 0.113 | 1.114 | 0.134 | 0.220 | 0.649 |

| Problem 632 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 142 | 40 | 44 | 24 | 46 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 6.45 | 1.82 | 2.00 | 1.09 | 2.09 | 1.09 |
| time (sec) | N/A | 1.381 | 1.478 | 8.055 | 0.250 | 0.115 | 1.131 | 0.142 | 0.224 | 0.649 |

| Problem 633 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 142 | 40 | 44 | 24 | 46 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 6.45 | 1.82 | 2.00 | 1.09 | 2.09 | 1.09 |
| time (sec) | N/A | 2.095 | 5.132 | 8.724 | 0.224 | 0.093 | 1.261 | 0.147 | 0.261 | 0.655 |

| Problem 634 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | C | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 177 | 273 | 72 | 90 | 0 | 328 | 0 | 0 | 60 | 0 |
| N.S. | 1 | 1.54 | 0.41 | 0.51 | 0.00 | 1.85 | 0.00 | 0.00 | 0.34 | 0.00 |
| time (sec) | N/A | 2.045 | 0.181 | 8.787 | 0.000 | 0.134 | 0.000 | 0.000 | 0.210 | 0.000 |

| Problem 635 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | C | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 120 | 233 | 60 | 52 | 0 | 215 | 0 | 0 | 60 | 0 |
| N.S. | 1 | 1.94 | 0.50 | 0.43 | 0.00 | 1.79 | 0.00 | 0.00 | 0.50 | 0.00 |
| time (sec) | N/A | 2.001 | 0.111 | 7.663 | 0.000 | 0.114 | 0.000 | 0.000 | 0.210 | 0.000 |

| Problem 636 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | C | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 113 | 194 | 98 | 88 | 0 | 318 | 0 | 0 | 428 | 0 |
| N.S. | 1 | 1.72 | 0.87 | 0.78 | 0.00 | 2.81 | 0.00 | 0.00 | 3.79 | 0.00 |
| time (sec) | N/A | 1.327 | 0.135 | 6.333 | 0.000 | 0.110 | 0.000 | 0.000 | 0.199 | 0.000 |

| Problem 637 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | C | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 81 | 120 | 89 | 89 | 0 | 297 | 0 | 0 | 236 | 0 |
| N.S. | 1 | 1.48 | 1.10 | 1.10 | 0.00 | 3.67 | 0.00 | 0.00 | 2.91 | 0.00 |
| time (sec) | N/A | 1.007 | 0.135 | 6.796 | 0.000 | 0.100 | 0.000 | 0.000 | 0.230 | 0.000 |

| Problem 638 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 174 | 49 | 58 | 24 | 58 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 7.91 | 2.23 | 2.64 | 1.09 | 2.64 | 1.09 |
| time (sec) | N/A | 2.691 | 1.838 | 4.586 | 0.234 | 0.115 | 1.588 | 0.138 | 0.219 | 0.636 |

| Problem 639 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 175 | 51 | 60 | 24 | 60 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 7.95 | 2.32 | 2.73 | 1.09 | 2.73 | 1.09 |
| time (sec) | N/A | 2.900 | 2.696 | 0.869 | 0.255 | 0.110 | 1.633 | 0.129 | 0.218 | 0.650 |

| Problem 640 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 174 | 51 | 60 | 24 | 60 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 7.91 | 2.32 | 2.73 | 1.09 | 2.73 | 1.09 |
| time (sec) | N/A | 4.329 | 3.433 | 7.991 | 0.325 | 0.116 | 1.710 | 0.129 | 0.197 | 0.654 |

| Problem 641 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 176 | 51 | 60 | 24 | 60 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 8.00 | 2.32 | 2.73 | 1.09 | 2.73 | 1.09 |
| time (sec) | N/A | 4.965 | 7.001 | 8.460 | 0.372 | 0.125 | 1.952 | 0.128 | 0.223 | 0.652 |

| Problem 642 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 16 | 16 | 16 | 15 | 14 | 14 | 0 | 14 | 14 | 14 |
| N.S. | 1 | 1.00 | 1.00 | 0.94 | 0.88 | 0.88 | 0.00 | 0.88 | 0.88 | 0.88 |
| time (sec) | N/A | 0.223 | 0.191 | 43.356 | 0.140 | 0.085 | 0.000 | 0.126 | 0.203 | 0.691 |

| Problem 643 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 20 | 22 | 22 | 22 | 22 | 23 | 22 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 1.00 | 1.00 | 1.00 | 1.00 | 1.05 | 1.00 |
| time (sec) | N/A | 0.236 | 1.589 | 10.195 | 0.191 | 0.098 | 1.013 | 0.146 | 0.196 | 0.645 |

| Problem 644 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 21 | 21 | 23 | 19 | 21 | 21 | 20 | 21 | 22 | 21 |
| N.S. | 1 | 1.00 | 1.10 | 0.90 | 1.00 | 1.00 | 0.95 | 1.00 | 1.05 | 1.00 |
| time (sec) | N/A | 0.204 | 0.855 | 9.408 | 0.192 | 0.086 | 0.936 | 0.132 | 0.203 | 0.599 |

| Problem 645 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 24 | 24 | 22 | 24 | 25 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 1.00 | 0.92 | 1.00 | 1.04 | 1.00 |
| time (sec) | N/A | 0.259 | 2.987 | 20.566 | 0.220 | 0.097 | 1.418 | 0.123 | 0.222 | 0.645 |

| Problem 646 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 20 | 22 | 33 | 22 | 22 | 51 | 22 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 1.00 | 1.50 | 1.00 | 1.00 | 2.32 | 1.00 |
| time (sec) | N/A | 0.248 | 5.110 | 13.975 | 0.245 | 0.089 | 5.322 | 0.149 | 0.239 | 0.665 |

| Problem 647 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 21 | 21 | 23 | 19 | 21 | 21 | 20 | 21 | 50 | 21 |
| N.S. | 1 | 1.00 | 1.10 | 0.90 | 1.00 | 1.00 | 0.95 | 1.00 | 2.38 | 1.00 |
| time (sec) | N/A | 0.207 | 0.894 | 12.327 | 0.252 | 0.103 | 3.509 | 0.144 | 0.225 | 0.649 |

| Problem 648 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 24 | 24 | 22 | 24 | 51 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 1.00 | 0.92 | 1.00 | 2.12 | 1.00 |
| time (sec) | N/A | 0.285 | 3.314 | 23.784 | 0.239 | 0.100 | 7.460 | 0.156 | 0.224 | 0.687 |

| Problem 649 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 20 | 22 | 48 | 22 | 22 | 80 | 22 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 1.00 | 2.18 | 1.00 | 1.00 | 3.64 | 1.00 |
| time (sec) | N/A | 0.240 | 2.117 | 16.815 | 0.279 | 0.101 | 20.225 | 0.160 | 0.258 | 0.687 |

| Problem 650 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 21 | 21 | 23 | 19 | 21 | 46 | 20 | 21 | 79 | 21 |
| N.S. | 1 | 1.00 | 1.10 | 0.90 | 1.00 | 2.19 | 0.95 | 1.00 | 3.76 | 1.00 |
| time (sec) | N/A | 0.209 | 0.946 | 16.292 | 0.285 | 0.111 | 12.369 | 0.150 | 0.260 | 0.671 |

| Problem 651 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 24 | 49 | 22 | 24 | 80 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 2.04 | 0.92 | 1.00 | 3.33 | 1.00 |
| time (sec) | N/A | 0.275 | 2.240 | 31.804 | 0.269 | 0.102 | 15.016 | 0.147 | 0.222 | 0.697 |

| Problem 652 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 20 | 22 | 22 | 22 | 22 | 27 | 22 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 1.00 | 1.00 | 1.00 | 1.00 | 1.23 | 1.00 |
| time (sec) | N/A | 0.240 | 1.200 | 1.999 | 0.201 | 0.083 | 1.271 | 0.130 | 0.208 | 0.697 |

| Problem 653 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 21 | 21 | 23 | 19 | 21 | 21 | 22 | 21 | 26 | 21 |
| N.S. | 1 | 1.00 | 1.10 | 0.90 | 1.00 | 1.00 | 1.05 | 1.00 | 1.24 | 1.00 |
| time (sec) | N/A | 0.208 | 0.504 | 1.197 | 0.207 | 0.094 | 1.241 | 0.133 | 0.258 | 0.650 |

| Problem 654 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 24 | 35 | 24 | 24 | 29 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 1.46 | 1.00 | 1.00 | 1.21 | 1.00 |
| time (sec) | N/A | 0.432 | 3.321 | 5.394 | 0.211 | 0.101 | 2.229 | 0.124 | 0.203 | 0.689 |

| Problem 655 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 24 | 37 | 26 | 24 | 29 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 1.54 | 1.08 | 1.00 | 1.21 | 1.00 |
| time (sec) | N/A | 0.274 | 3.011 | 3.161 | 0.218 | 0.096 | 3.241 | 0.128 | 0.209 | 0.687 |

| Problem 656 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 24 | 37 | 26 | 24 | 29 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 1.54 | 1.08 | 1.00 | 1.21 | 1.00 |
| time (sec) | N/A | 0.279 | 4.540 | 10.855 | 0.209 | 0.097 | 4.582 | 0.135 | 0.207 | 0.719 |

| Problem 657 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 24 | 51 | 24 | 0 | 57 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 2.12 | 1.00 | 0.00 | 2.38 | 1.00 |
| time (sec) | N/A | 1.515 | 5.958 | 2.583 | 0.234 | 0.106 | 2.929 | 0.000 | 0.242 | 0.775 |

| Problem 658 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 24 | 51 | 24 | 24 | 57 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 2.12 | 1.00 | 1.00 | 2.38 | 1.00 |
| time (sec) | N/A | 1.247 | 4.721 | 1.434 | 0.255 | 0.106 | 2.898 | 0.142 | 0.207 | 0.788 |

| Problem 659 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 104 | 108 | 63 | 156 | 0 | 0 | 0 | 0 | 55 | 0 |
| N.S. | 1 | 1.04 | 0.61 | 1.50 | 0.00 | 0.00 | 0.00 | 0.00 | 0.53 | 0.00 |
| time (sec) | N/A | 0.774 | 0.159 | 3.951 | 0.000 | 0.000 | 0.000 | 0.000 | 0.193 | 0.000 |

| Problem 660 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 101 | 105 | 65 | 151 | 0 | 0 | 0 | 0 | 53 | 0 |
| N.S. | 1 | 1.04 | 0.64 | 1.50 | 0.00 | 0.00 | 0.00 | 0.00 | 0.52 | 0.00 |
| time (sec) | N/A | 0.663 | 0.072 | 3.503 | 0.000 | 0.000 | 0.000 | 0.000 | 0.195 | 0.000 |

| Problem 661 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 24 | 50 | 24 | 0 | 54 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 2.08 | 1.00 | 0.00 | 2.25 | 1.00 |
| time (sec) | N/A | 1.839 | 2.143 | 2.180 | 0.265 | 0.112 | 5.431 | 0.000 | 0.197 | 0.723 |

| Problem 662 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 24 | 52 | 26 | 24 | 56 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 2.17 | 1.08 | 1.00 | 2.33 | 1.00 |
| time (sec) | N/A | 1.633 | 2.347 | 2.250 | 0.251 | 0.135 | 6.828 | 0.154 | 0.197 | 0.744 |

| Problem 663 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 24 | 52 | 26 | 0 | 56 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 2.17 | 1.08 | 0.00 | 2.33 | 1.00 |
| time (sec) | N/A | 2.821 | 2.845 | 9.547 | 0.272 | 0.099 | 10.302 | 0.000 | 0.204 | 0.740 |

| Problem 664 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 24 | 52 | 26 | 24 | 56 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 2.17 | 1.08 | 1.00 | 2.33 | 1.00 |
| time (sec) | N/A | 2.693 | 5.017 | 10.572 | 0.264 | 0.103 | 14.472 | 0.146 | 0.220 | 0.776 |

| Problem 665 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 24 | 62 | 24 | 0 | 81 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 2.58 | 1.00 | 0.00 | 3.38 | 1.00 |
| time (sec) | N/A | 3.991 | 7.513 | 3.045 | 0.269 | 0.091 | 9.051 | 0.000 | 0.222 | 0.770 |

| Problem 666 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 24 | 62 | 24 | 24 | 81 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 2.58 | 1.00 | 1.00 | 3.38 | 1.00 |
| time (sec) | N/A | 5.314 | 6.844 | 2.362 | 0.291 | 0.093 | 8.991 | 0.226 | 0.206 | 0.756 |

| Problem 667 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 180 | 208 | 114 | 359 | 0 | 0 | 0 | 0 | 81 | 0 |
| N.S. | 1 | 1.16 | 0.63 | 1.99 | 0.00 | 0.00 | 0.00 | 0.00 | 0.45 | 0.00 |
| time (sec) | N/A | 1.479 | 0.332 | 6.783 | 0.000 | 0.000 | 0.000 | 0.000 | 0.237 | 0.000 |

| Problem 668 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 209 | 295 | 119 | 371 | 0 | 0 | 0 | 0 | 81 | 0 |
| N.S. | 1 | 1.41 | 0.57 | 1.78 | 0.00 | 0.00 | 0.00 | 0.00 | 0.39 | 0.00 |
| time (sec) | N/A | 3.299 | 0.178 | 5.817 | 0.000 | 0.000 | 0.000 | 0.000 | 0.210 | 0.000 |

| Problem 669 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 175 | 294 | 118 | 371 | 0 | 0 | 0 | 0 | 79 | 0 |
| N.S. | 1 | 1.68 | 0.67 | 2.12 | 0.00 | 0.00 | 0.00 | 0.00 | 0.45 | 0.00 |
| time (sec) | N/A | 2.261 | 0.298 | 6.753 | 0.000 | 0.000 | 0.000 | 0.000 | 0.196 | 0.000 |

| Problem 670 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 145 | 177 | 102 | 346 | 0 | 0 | 0 | 0 | 77 | 0 |
| N.S. | 1 | 1.22 | 0.70 | 2.39 | 0.00 | 0.00 | 0.00 | 0.00 | 0.53 | 0.00 |
| time (sec) | N/A | 1.830 | 0.163 | 5.793 | 0.000 | 0.000 | 0.000 | 0.000 | 0.243 | 0.000 |

| Problem 671 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 24 | 61 | 24 | 0 | 78 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 2.54 | 1.00 | 0.00 | 3.25 | 1.00 |
| time (sec) | N/A | 5.444 | 3.359 | 3.154 | 0.261 | 0.093 | 15.819 | 0.000 | 0.206 | 0.752 |

| Problem 672 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 24 | 63 | 26 | 24 | 80 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 2.62 | 1.08 | 1.00 | 3.33 | 1.00 |
| time (sec) | N/A | 5.586 | 3.217 | 2.546 | 0.300 | 0.111 | 20.121 | 0.150 | 0.233 | 0.757 |

| Problem 673 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 399 | 48 | 73 | 24 | 76 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 18.14 | 2.18 | 3.32 | 1.09 | 3.45 | 1.09 |
| time (sec) | N/A | 0.242 | 0.519 | 8.671 | 1.339 | 0.118 | 12.069 | 0.119 | 0.211 | 0.635 |

| Problem 674 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 333 | 37 | 53 | 24 | 56 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 15.14 | 1.68 | 2.41 | 1.09 | 2.55 | 1.09 |
| time (sec) | N/A | 0.237 | 0.593 | 4.059 | 1.617 | 0.094 | 5.991 | 0.119 | 0.206 | 0.611 |

| Problem 675 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 22 | 20 | 221 | 22 | 31 | 22 | 34 | 22 |
| N.S. | 1 | 1.00 | 1.10 | 1.00 | 11.05 | 1.10 | 1.55 | 1.10 | 1.70 | 1.10 |
| time (sec) | N/A | 0.207 | 0.384 | 3.773 | 0.998 | 0.100 | 4.446 | 0.120 | 0.234 | 0.635 |

| Problem 676 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 91 | 24 | 26 | 24 | 32 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 4.14 | 1.09 | 1.18 | 1.09 | 1.45 | 1.09 |
| time (sec) | N/A | 0.303 | 0.685 | 3.586 | 0.258 | 0.109 | 4.923 | 0.109 | 0.204 | 0.655 |

| Problem 677 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 191 | 39 | 42 | 24 | 363 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 8.68 | 1.77 | 1.91 | 1.09 | 16.50 | 1.09 |
| time (sec) | N/A | 0.238 | 0.609 | 4.711 | 0.602 | 0.097 | 13.951 | 0.122 | 0.211 | 0.658 |

| Problem 678 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 22 | 224 | 50 | 58 | 24 | 671 | 24 |
| N.S. | 1 | 1.00 | 1.09 | 1.00 | 10.18 | 2.27 | 2.64 | 1.09 | 30.50 | 1.09 |
| time (sec) | N/A | 0.259 | 0.622 | 7.996 | 0.711 | 0.095 | 39.395 | 0.123 | 0.212 | 0.655 |

| Problem 679 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | F(-1) | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 24 | 49 | 0 | 0 | 88 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 2.04 | 0.00 | 0.00 | 3.67 | 1.00 |
| time (sec) | N/A | 0.300 | 2.697 | 6.446 | 0.355 | 0.106 | 0.000 | 0.000 | 168.727 | 0.742 |

| Problem 680 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | F(-1) | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 24 | 24 | 0 | 0 | 56 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 1.00 | 0.00 | 0.00 | 2.33 | 1.00 |
| time (sec) | N/A | 0.272 | 1.401 | 5.444 | 0.296 | 0.101 | 0.000 | 0.000 | 8.147 | 0.686 |

| Problem 681 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 24 | 24 | 24 | 0 | 25 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 1.00 | 1.00 | 0.00 | 1.04 | 1.00 |
| time (sec) | N/A | 0.268 | 0.409 | 4.773 | 0.214 | 0.100 | 20.338 | 0.000 | 0.289 | 0.640 |

| Problem 682 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 24 | 24 | 24 | 24 | 29 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 1.00 | 1.00 | 1.00 | 1.21 | 1.00 |
| time (sec) | N/A | 0.277 | 0.808 | 4.985 | 0.243 | 0.097 | 28.846 | 0.126 | 0.219 | 0.795 |

| Problem 683 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 24 | 51 | 24 | 24 | 57 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 2.12 | 1.00 | 1.00 | 2.38 | 1.00 |
| time (sec) | N/A | 0.278 | 0.938 | 5.932 | 0.262 | 0.119 | 88.235 | 0.150 | 0.223 | 0.835 |

| Problem 684 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 24 | 62 | 0 | 24 | 81 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 1.00 | 2.58 | 0.00 | 1.00 | 3.38 | 1.00 |
| time (sec) | N/A | 0.283 | 0.981 | 5.881 | 0.287 | 0.103 | 0.000 | 0.160 | 0.268 | 0.856 |

| Problem 685 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 20 | 0 | 22 | 34 | 22 | 725 | 22 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 0.00 | 1.00 | 1.55 | 1.00 | 32.95 | 1.00 |
| time (sec) | N/A | 0.213 | 0.925 | 8.478 | 0.000 | 0.119 | 27.746 | 0.317 | 0.234 | 0.788 |

| Problem 686 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 22 | 18 | 0 | 0 | 29 | 20 | 93 | 20 |
| N.S. | 1 | 1.00 | 1.10 | 0.90 | 0.00 | 0.00 | 1.45 | 1.00 | 4.65 | 1.00 |
| time (sec) | N/A | 0.267 | 0.953 | 4.018 | 0.000 | 0.000 | 1.160 | 0.245 | 0.200 | 0.700 |

| Problem 687 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 19 | 19 | 21 | 17 | 0 | 0 | 27 | 19 | 89 | 19 |
| N.S. | 1 | 1.00 | 1.11 | 0.89 | 0.00 | 0.00 | 1.42 | 1.00 | 4.68 | 1.00 |
| time (sec) | N/A | 0.180 | 1.554 | 2.717 | 0.000 | 0.000 | 0.849 | 0.177 | 0.214 | 0.729 |

| Problem 688 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 20 | 0 | 0 | 27 | 22 | 64 | 22 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 0.00 | 0.00 | 1.23 | 1.00 | 2.91 | 1.00 |
| time (sec) | N/A | 0.205 | 0.847 | 3.667 | 0.000 | 0.000 | 1.585 | 0.280 | 0.229 | 0.708 |

| Problem 689 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 37 | 58 | 24 | 2086 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 1.54 | 2.42 | 1.00 | 86.92 | 1.00 |
| time (sec) | N/A | 0.229 | 0.655 | 7.950 | 0.000 | 0.112 | 92.313 | 0.397 | 0.250 | 0.744 |

| Problem 690 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 20 | 0 | 0 | 49 | 22 | 142 | 22 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 0.00 | 0.00 | 2.23 | 1.00 | 6.45 | 1.00 |
| time (sec) | N/A | 0.296 | 0.774 | 2.968 | 0.000 | 0.000 | 2.333 | 0.401 | 0.198 | 0.659 |

| Problem 691 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 21 | 21 | 23 | 19 | 0 | 0 | 48 | 21 | 138 | 21 |
| N.S. | 1 | 1.00 | 1.10 | 0.90 | 0.00 | 0.00 | 2.29 | 1.00 | 6.57 | 1.00 |
| time (sec) | N/A | 0.194 | 0.864 | 1.886 | 0.000 | 0.000 | 1.719 | 0.215 | 0.199 | 0.652 |

| Problem 692 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 48 | 24 | 113 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 2.00 | 1.00 | 4.71 | 1.00 |
| time (sec) | N/A | 0.217 | 0.685 | 2.453 | 0.000 | 0.000 | 2.529 | 0.449 | 0.223 | 0.683 |

| Problem 693 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 48 | 0 | 24 | 4495 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 2.00 | 0.00 | 1.00 | 187.29 | 1.00 |
| time (sec) | N/A | 0.222 | 0.473 | 7.965 | 0.000 | 0.099 | 0.000 | 0.481 | 0.303 | 0.726 |

| Problem 694 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 20 | 0 | 0 | 68 | 22 | 189 | 22 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 0.00 | 0.00 | 3.09 | 1.00 | 8.59 | 1.00 |
| time (sec) | N/A | 0.298 | 0.790 | 3.413 | 0.000 | 0.000 | 4.995 | 0.557 | 0.201 | 0.692 |

| Problem 695 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 21 | 21 | 23 | 19 | 0 | 0 | 66 | 21 | 185 | 21 |
| N.S. | 1 | 1.00 | 1.10 | 0.90 | 0.00 | 0.00 | 3.14 | 1.00 | 8.81 | 1.00 |
| time (sec) | N/A | 0.190 | 0.919 | 2.079 | 0.000 | 0.000 | 3.782 | 0.275 | 0.232 | 0.694 |

| Problem 696 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 66 | 24 | 160 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 2.75 | 1.00 | 6.67 | 1.00 |
| time (sec) | N/A | 0.220 | 0.712 | 3.004 | 0.000 | 0.000 | 4.207 | 0.632 | 0.200 | 0.687 |

| Problem 697 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 24 | 22 | 24 | 26 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 1.00 | 0.92 | 1.00 | 1.08 | 1.00 |
| time (sec) | N/A | 0.235 | 0.602 | 0.285 | 0.000 | 0.124 | 4.211 | 0.166 | 0.200 | 0.686 |

| Problem 698 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 22 | 24 | 26 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 0.92 | 1.00 | 1.08 | 1.00 |
| time (sec) | N/A | 0.470 | 2.444 | 0.107 | 0.000 | 0.000 | 0.648 | 0.226 | 0.196 | 0.639 |

| Problem 699 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 22 | 24 | 61 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 0.92 | 1.00 | 2.54 | 1.00 |
| time (sec) | N/A | 0.364 | 0.607 | 0.096 | 0.000 | 0.000 | 0.504 | 0.174 | 0.198 | 0.626 |

| Problem 700 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 20 | 0 | 0 | 20 | 22 | 24 | 22 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 0.00 | 0.00 | 0.91 | 1.00 | 1.09 | 1.00 |
| time (sec) | N/A | 0.255 | 0.628 | 0.059 | 0.000 | 0.000 | 0.397 | 0.138 | 0.203 | 0.633 |

| Problem 701 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | A | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 18 | 18 | 18 | 15 | 0 | 14 | 0 | 14 | 17 | 14 |
| N.S. | 1 | 1.00 | 1.00 | 0.83 | 0.00 | 0.78 | 0.00 | 0.78 | 0.94 | 0.78 |
| time (sec) | N/A | 0.185 | 0.004 | 0.044 | 0.000 | 0.085 | 0.000 | 0.118 | 0.206 | 0.637 |

| Problem 702 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 19 | 24 | 23 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 0.79 | 1.00 | 0.96 | 1.00 |
| time (sec) | N/A | 0.317 | 0.287 | 0.060 | 0.000 | 0.000 | 0.744 | 0.142 | 0.201 | 0.669 |

| Problem 703 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 20 | 24 | 61 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 0.83 | 1.00 | 2.54 | 1.00 |
| time (sec) | N/A | 0.394 | 0.657 | 0.100 | 0.000 | 0.000 | 0.904 | 0.137 | 0.203 | 0.670 |

| Problem 704 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 20 | 24 | 25 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 0.83 | 1.00 | 1.04 | 1.00 |
| time (sec) | N/A | 0.534 | 1.650 | 0.104 | 0.000 | 0.000 | 1.174 | 0.163 | 0.238 | 0.663 |

| Problem 705 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 20 | 24 | 117 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 0.83 | 1.00 | 4.88 | 1.00 |
| time (sec) | N/A | 0.615 | 4.503 | 0.143 | 0.000 | 0.000 | 1.579 | 0.143 | 0.215 | 0.669 |

| Problem 706 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 39 | 32 | 0 | 34 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 1.62 | 1.33 | 0.00 | 1.42 | 1.00 |
| time (sec) | N/A | 0.223 | 1.436 | 0.162 | 0.000 | 0.128 | 23.391 | 0.000 | 0.211 | 0.736 |

| Problem 707 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 32 | 24 | 34 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 1.33 | 1.00 | 1.42 | 1.00 |
| time (sec) | N/A | 0.235 | 2.919 | 0.084 | 0.000 | 0.000 | 0.922 | 1.126 | 0.216 | 0.674 |

| Problem 708 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | F(-2) | F | C | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 80 | 80 | 66 | 60 | 0 | 0 | 0 | 91 | 50 | 0 |
| N.S. | 1 | 1.00 | 0.82 | 0.75 | 0.00 | 0.00 | 0.00 | 1.14 | 0.62 | 0.00 |
| time (sec) | N/A | 0.525 | 0.294 | 0.074 | 0.000 | 0.000 | 0.000 | 0.130 | 0.214 | 0.000 |

| Problem 709 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F(-2) | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 79 | 74 | 136 | 46 | 0 | 0 | 0 | 0 | 120 | 0 |
| N.S. | 1 | 0.94 | 1.72 | 0.58 | 0.00 | 0.00 | 0.00 | 0.00 | 1.52 | 0.00 |
| time (sec) | N/A | 0.428 | 0.223 | 0.063 | 0.000 | 0.000 | 0.000 | 0.000 | 0.212 | 0.000 |

| Problem 710 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | F(-2) | F | C | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 77 | 77 | 68 | 60 | 0 | 0 | 0 | 91 | 31 | 0 |
| N.S. | 1 | 1.00 | 0.88 | 0.78 | 0.00 | 0.00 | 0.00 | 1.18 | 0.40 | 0.00 |
| time (sec) | N/A | 0.514 | 0.300 | 0.065 | 0.000 | 0.000 | 0.000 | 0.129 | 0.204 | 0.000 |

| Problem 711 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 29 | 24 | 31 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 1.21 | 1.00 | 1.29 | 1.00 |
| time (sec) | N/A | 0.247 | 0.714 | 0.082 | 0.000 | 0.000 | 1.355 | 0.182 | 0.221 | 0.708 |

| Problem 712 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 50 | 41 | 0 | 42 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 2.08 | 1.71 | 0.00 | 1.75 | 1.00 |
| time (sec) | N/A | 0.235 | 1.494 | 0.177 | 0.000 | 0.120 | 98.658 | 0.000 | 0.191 | 0.752 |

| Problem 713 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 41 | 24 | 42 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 1.71 | 1.00 | 1.75 | 1.00 |
| time (sec) | N/A | 0.240 | 3.851 | 0.089 | 0.000 | 0.000 | 1.713 | 2.397 | 0.192 | 0.712 |

| Problem 714 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F(-2) | F(-2) | F | C | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 139 | 116 | 181 | 102 | 0 | 0 | 0 | 179 | 380 | 0 |
| N.S. | 1 | 0.83 | 1.30 | 0.73 | 0.00 | 0.00 | 0.00 | 1.29 | 2.73 | 0.00 |
| time (sec) | N/A | 0.425 | 0.412 | 0.081 | 0.000 | 0.000 | 0.000 | 0.136 | 0.213 | 0.000 |

| Problem 715 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F(-2) | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 118 | 111 | 230 | 94 | 0 | 0 | 0 | 0 | 213 | 0 |
| N.S. | 1 | 0.94 | 1.95 | 0.80 | 0.00 | 0.00 | 0.00 | 0.00 | 1.81 | 0.00 |
| time (sec) | N/A | 0.508 | 0.511 | 0.081 | 0.000 | 0.000 | 0.000 | 0.000 | 0.192 | 0.000 |

| Problem 716 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F(-2) | F(-2) | F | C | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 83 | 72 | 141 | 66 | 0 | 0 | 0 | 103 | 329 | 0 |
| N.S. | 1 | 0.87 | 1.70 | 0.80 | 0.00 | 0.00 | 0.00 | 1.24 | 3.96 | 0.00 |
| time (sec) | N/A | 0.367 | 0.338 | 0.078 | 0.000 | 0.000 | 0.000 | 0.141 | 0.208 | 0.000 |

| Problem 717 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F(-2) | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 118 | 111 | 230 | 94 | 0 | 0 | 0 | 0 | 406 | 0 |
| N.S. | 1 | 0.94 | 1.95 | 0.80 | 0.00 | 0.00 | 0.00 | 0.00 | 3.44 | 0.00 |
| time (sec) | N/A | 0.457 | 0.453 | 0.075 | 0.000 | 0.000 | 0.000 | 0.000 | 0.232 | 0.000 |

| Problem 718 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F(-2) | F(-2) | F | C | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 139 | 116 | 192 | 102 | 0 | 0 | 0 | 179 | 39 | 0 |
| N.S. | 1 | 0.83 | 1.38 | 0.73 | 0.00 | 0.00 | 0.00 | 1.29 | 0.28 | 0.00 |
| time (sec) | N/A | 0.375 | 0.506 | 0.069 | 0.000 | 0.000 | 0.000 | 0.140 | 0.198 | 0.000 |

| Problem 719 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 37 | 24 | 39 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 1.54 | 1.00 | 1.62 | 1.00 |
| time (sec) | N/A | 0.229 | 1.101 | 0.097 | 0.000 | 0.000 | 2.675 | 0.323 | 0.194 | 0.774 |

| Problem 720 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 24 | 26 | 0 | 24 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.92 | 1.00 | 0.00 | 0.92 | 0.92 |
| time (sec) | N/A | 0.268 | 1.312 | 0.474 | 0.000 | 0.118 | 54.044 | 0.000 | 0.487 | 0.581 |

| Problem 721 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 26 | 24 | 24 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 1.00 | 0.92 | 0.92 | 0.92 |
| time (sec) | N/A | 0.296 | 4.071 | 0.267 | 0.000 | 0.000 | 5.401 | 0.249 | 0.215 | 0.576 |

| Problem 722 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 20 | 0 | 0 | 24 | 0 | 75 | 22 |
| N.S. | 1 | 1.00 | 1.08 | 0.83 | 0.00 | 0.00 | 1.00 | 0.00 | 3.12 | 0.92 |
| time (sec) | N/A | 0.346 | 3.206 | 0.203 | 0.000 | 0.000 | 2.672 | 0.000 | 0.218 | 0.585 |

| Problem 723 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 19 | 0 | 0 | 22 | 0 | 21 | 21 |
| N.S. | 1 | 1.00 | 1.09 | 0.83 | 0.00 | 0.00 | 0.96 | 0.00 | 0.91 | 0.91 |
| time (sec) | N/A | 0.205 | 0.212 | 0.168 | 0.000 | 0.000 | 1.372 | 0.000 | 0.233 | 0.591 |

| Problem 724 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | F(-1) | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 24 | 0 | 0 | 54 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.92 | 0.00 | 0.00 | 2.08 | 0.92 |
| time (sec) | N/A | 0.279 | 1.290 | 0.495 | 0.000 | 0.102 | 0.000 | 0.000 | 0.263 | 0.590 |

| Problem 725 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 26 | 24 | 51 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 1.00 | 0.92 | 1.96 | 0.92 |
| time (sec) | N/A | 0.285 | 4.327 | 0.399 | 0.000 | 0.000 | 100.357 | 0.538 | 0.230 | 0.610 |

| Problem 726 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 20 | 0 | 0 | 24 | 0 | 131 | 22 |
| N.S. | 1 | 1.00 | 1.08 | 0.83 | 0.00 | 0.00 | 1.00 | 0.00 | 5.46 | 0.92 |
| time (sec) | N/A | 0.347 | 4.112 | 0.262 | 0.000 | 0.000 | 51.818 | 0.000 | 0.256 | 0.609 |

| Problem 727 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 19 | 0 | 0 | 22 | 0 | 48 | 21 |
| N.S. | 1 | 1.00 | 1.09 | 0.83 | 0.00 | 0.00 | 0.96 | 0.00 | 2.09 | 0.91 |
| time (sec) | N/A | 0.204 | 1.619 | 0.221 | 0.000 | 0.000 | 27.982 | 0.000 | 0.210 | 0.590 |

| Problem 728 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | F(-1) | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 49 | 0 | 0 | 85 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 1.88 | 0.00 | 0.00 | 3.27 | 0.92 |
| time (sec) | N/A | 0.282 | 1.556 | 0.587 | 0.000 | 0.129 | 0.000 | 0.000 | 1.844 | 0.584 |

| Problem 729 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 0 | 24 | 79 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 0.00 | 0.92 | 3.04 | 0.92 |
| time (sec) | N/A | 0.278 | 3.607 | 0.511 | 0.000 | 0.000 | 0.000 | 1.413 | 0.251 | 0.576 |

| Problem 730 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | F(-1) | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 20 | 0 | 0 | 0 | 0 | 188 | 22 |
| N.S. | 1 | 1.00 | 1.08 | 0.83 | 0.00 | 0.00 | 0.00 | 0.00 | 7.83 | 0.92 |
| time (sec) | N/A | 0.347 | 3.666 | 0.402 | 0.000 | 0.000 | 0.000 | 0.000 | 0.318 | 0.562 |

| Problem 731 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | F(-1) | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 19 | 0 | 0 | 0 | 0 | 76 | 21 |
| N.S. | 1 | 1.00 | 1.09 | 0.83 | 0.00 | 0.00 | 0.00 | 0.00 | 3.30 | 0.91 |
| time (sec) | N/A | 0.207 | 0.346 | 0.295 | 0.000 | 0.000 | 0.000 | 0.000 | 0.227 | 0.565 |

| Problem 732 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 24 | 26 | 24 | 38 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.92 | 1.00 | 0.92 | 1.46 | 0.92 |
| time (sec) | N/A | 0.280 | 1.167 | 0.391 | 0.000 | 0.112 | 28.799 | 0.195 | 0.255 | 0.575 |

| Problem 733 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 26 | 0 | 134 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 1.00 | 0.00 | 5.15 | 0.92 |
| time (sec) | N/A | 0.751 | 4.095 | 0.268 | 0.000 | 0.000 | 7.060 | 0.000 | 0.233 | 0.570 |

| Problem 734 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 26 | 24 | 38 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 1.00 | 0.92 | 1.46 | 0.92 |
| time (sec) | N/A | 0.470 | 2.916 | 0.239 | 0.000 | 0.000 | 3.643 | 0.212 | 0.203 | 0.577 |

| Problem 735 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 20 | 0 | 0 | 24 | 22 | 67 | 22 |
| N.S. | 1 | 1.00 | 1.08 | 0.83 | 0.00 | 0.00 | 1.00 | 0.92 | 2.79 | 0.92 |
| time (sec) | N/A | 0.336 | 0.794 | 0.184 | 0.000 | 0.000 | 1.744 | 0.194 | 0.231 | 0.582 |

| Problem 736 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 19 | 0 | 0 | 22 | 21 | 35 | 21 |
| N.S. | 1 | 1.00 | 1.09 | 0.83 | 0.00 | 0.00 | 0.96 | 0.91 | 1.52 | 0.91 |
| time (sec) | N/A | 0.231 | 0.166 | 0.160 | 0.000 | 0.000 | 0.832 | 0.176 | 0.203 | 0.587 |

| Problem 737 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 24 | 24 | 35 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 0.92 | 0.92 | 1.35 | 0.92 |
| time (sec) | N/A | 0.275 | 1.345 | 0.197 | 0.000 | 0.000 | 1.752 | 0.171 | 0.211 | 0.589 |

| Problem 738 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 26 | 24 | 69 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 1.00 | 0.92 | 2.65 | 0.92 |
| time (sec) | N/A | 0.448 | 1.993 | 0.237 | 0.000 | 0.000 | 4.032 | 0.197 | 0.253 | 0.585 |

| Problem 739 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 26 | 24 | 37 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 1.00 | 0.92 | 1.42 | 0.92 |
| time (sec) | N/A | 0.542 | 7.150 | 0.268 | 0.000 | 0.000 | 7.429 | 0.366 | 0.245 | 0.592 |

| Problem 740 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 26 | 24 | 142 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 1.00 | 0.92 | 5.46 | 0.92 |
| time (sec) | N/A | 0.824 | 13.797 | 0.288 | 0.000 | 0.000 | 15.802 | 0.515 | 0.276 | 0.596 |

| Problem 741 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 51 | 26 | 24 | 46 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 1.96 | 1.00 | 0.92 | 1.77 | 0.92 |
| time (sec) | N/A | 0.279 | 1.332 | 0.357 | 0.000 | 0.114 | 72.072 | 0.217 | 0.246 | 0.593 |

| Problem 742 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 26 | 0 | 284 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 1.00 | 0.00 | 10.92 | 0.92 |
| time (sec) | N/A | 0.282 | 6.723 | 0.204 | 0.000 | 0.000 | 8.185 | 0.000 | 0.265 | 0.586 |

| Problem 743 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 26 | 24 | 46 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 1.00 | 0.92 | 1.77 | 0.92 |
| time (sec) | N/A | 0.283 | 3.814 | 0.166 | 0.000 | 0.000 | 4.416 | 0.238 | 0.204 | 0.580 |

| Problem 744 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F(-2) | F(-2) | F | C | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 93 | 93 | 121 | 0 | 0 | 0 | 0 | 79 | 144 | 0 |
| N.S. | 1 | 1.00 | 1.30 | 0.00 | 0.00 | 0.00 | 0.00 | 0.85 | 1.55 | 0.00 |
| time (sec) | N/A | 0.576 | 0.138 | 0.000 | 0.000 | 0.000 | 0.000 | 0.133 | 0.227 | 0.000 |

| Problem 745 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-2) | F(-2) | F | C | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 91 | 89 | 78 | 0 | 0 | 0 | 0 | 79 | 146 | 0 |
| N.S. | 1 | 0.98 | 0.86 | 0.00 | 0.00 | 0.00 | 0.00 | 0.87 | 1.60 | 0.00 |
| time (sec) | N/A | 0.485 | 0.087 | 0.000 | 0.000 | 0.000 | 0.000 | 0.131 | 0.218 | 0.000 |

| Problem 746 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 24 | 24 | 43 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 0.92 | 0.92 | 1.65 | 0.92 |
| time (sec) | N/A | 0.281 | 2.114 | 0.168 | 0.000 | 0.000 | 7.813 | 0.186 | 0.211 | 0.590 |

| Problem 747 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 26 | 24 | 284 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 1.00 | 0.92 | 10.92 | 0.92 |
| time (sec) | N/A | 0.282 | 4.105 | 0.206 | 0.000 | 0.000 | 14.935 | 0.203 | 0.246 | 0.588 |

| Problem 748 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 62 | 0 | 24 | 54 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 2.38 | 0.00 | 0.92 | 2.08 | 0.92 |
| time (sec) | N/A | 0.283 | 2.050 | 0.410 | 0.000 | 0.116 | 0.000 | 0.265 | 0.232 | 0.572 |

| Problem 749 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | F(-1) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 26 | 0 | 54 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 1.00 | 0.00 | 2.08 | 0.92 |
| time (sec) | N/A | 0.283 | 6.419 | 0.228 | 0.000 | 0.000 | 33.870 | 0.000 | 0.210 | 0.571 |

| Problem 750 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F(-2) | F(-2) | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 215 | 143 | 324 | 0 | 0 | 0 | 0 | 0 | 479 | 0 |
| N.S. | 1 | 0.67 | 1.51 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 2.23 | 0.00 |
| time (sec) | N/A | 0.628 | 0.392 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.298 | 0.000 |

| Problem 751 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-2) | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 163 | 135 | 133 | 0 | 0 | 0 | 0 | 0 | 255 | 0 |
| N.S. | 1 | 0.83 | 0.82 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.56 | 0.00 |
| time (sec) | N/A | 0.798 | 0.345 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.224 | 0.000 |

| Problem 752 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F(-2) | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 163 | 135 | 167 | 0 | 0 | 0 | 0 | 0 | 244 | 0 |
| N.S. | 1 | 0.83 | 1.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.50 | 0.00 |
| time (sec) | N/A | 0.655 | 0.314 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.241 | 0.000 |

| Problem 753 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-2) | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 213 | 145 | 137 | 0 | 0 | 0 | 0 | 0 | 480 | 0 |
| N.S. | 1 | 0.68 | 0.64 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 2.25 | 0.00 |
| time (sec) | N/A | 0.490 | 0.155 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.238 | 0.000 |

| Problem 754 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 24 | 24 | 51 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 0.92 | 0.92 | 1.96 | 0.92 |
| time (sec) | N/A | 0.285 | 3.243 | 0.233 | 0.000 | 0.000 | 49.310 | 3.044 | 0.217 | 0.590 |

| Problem 755 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 20 | 0 | 22 | 0 | 22 | 1635 | 22 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 0.00 | 1.00 | 0.00 | 1.00 | 74.32 | 1.00 |
| time (sec) | N/A | 0.211 | 0.792 | 0.337 | 0.000 | 0.116 | 0.000 | 0.588 | 0.221 | 0.863 |

| Problem 756 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 20 | 0 | 0 | 31 | 22 | 37 | 22 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 0.00 | 0.00 | 1.41 | 1.00 | 1.68 | 1.00 |
| time (sec) | N/A | 0.226 | 4.453 | 0.191 | 0.000 | 0.000 | 7.513 | 0.251 | 0.188 | 0.771 |

| Problem 757 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 22 | 18 | 0 | 0 | 29 | 20 | 139 | 20 |
| N.S. | 1 | 1.00 | 1.10 | 0.90 | 0.00 | 0.00 | 1.45 | 1.00 | 6.95 | 1.00 |
| time (sec) | N/A | 0.275 | 0.700 | 0.151 | 0.000 | 0.000 | 5.525 | 0.288 | 0.192 | 0.730 |

| Problem 758 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 19 | 19 | 21 | 17 | 0 | 0 | 27 | 19 | 101 | 19 |
| N.S. | 1 | 1.00 | 1.11 | 0.89 | 0.00 | 0.00 | 1.42 | 1.00 | 5.32 | 1.00 |
| time (sec) | N/A | 0.266 | 1.922 | 0.108 | 0.000 | 0.000 | 3.382 | 0.204 | 0.201 | 0.796 |

| Problem 759 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 20 | 0 | 0 | 27 | 22 | 90 | 22 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 0.00 | 0.00 | 1.23 | 1.00 | 4.09 | 1.00 |
| time (sec) | N/A | 0.210 | 0.974 | 0.145 | 0.000 | 0.000 | 3.817 | 0.391 | 0.194 | 0.757 |

| Problem 760 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 20 | 0 | 0 | 27 | 22 | 59 | 22 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 0.00 | 0.00 | 1.23 | 1.00 | 2.68 | 1.00 |
| time (sec) | N/A | 0.206 | 0.693 | 0.116 | 0.000 | 0.000 | 3.645 | 0.277 | 0.192 | 0.780 |

| Problem 761 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 37 | 0 | 24 | 5441 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 1.54 | 0.00 | 1.00 | 226.71 | 1.00 |
| time (sec) | N/A | 0.225 | 0.653 | 0.436 | 0.000 | 0.124 | 0.000 | 0.817 | 0.274 | 0.784 |

| Problem 762 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 51 | 24 | 59 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 2.12 | 1.00 | 2.46 | 1.00 |
| time (sec) | N/A | 0.228 | 3.039 | 0.351 | 0.000 | 0.000 | 14.758 | 0.316 | 0.195 | 0.731 |

| Problem 763 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 20 | 0 | 0 | 49 | 22 | 206 | 22 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 0.00 | 0.00 | 2.23 | 1.00 | 9.36 | 1.00 |
| time (sec) | N/A | 0.326 | 0.716 | 0.200 | 0.000 | 0.000 | 11.444 | 0.429 | 0.192 | 0.711 |

| Problem 764 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 21 | 21 | 23 | 19 | 0 | 0 | 48 | 21 | 167 | 21 |
| N.S. | 1 | 1.00 | 1.10 | 0.90 | 0.00 | 0.00 | 2.29 | 1.00 | 7.95 | 1.00 |
| time (sec) | N/A | 0.533 | 1.082 | 0.154 | 0.000 | 0.000 | 7.109 | 0.259 | 0.199 | 0.705 |

| Problem 765 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 48 | 24 | 155 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 2.00 | 1.00 | 6.46 | 1.00 |
| time (sec) | N/A | 0.225 | 0.964 | 0.176 | 0.000 | 0.000 | 7.110 | 0.485 | 0.196 | 0.718 |

| Problem 766 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 48 | 24 | 151 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 2.00 | 1.00 | 6.29 | 1.00 |
| time (sec) | N/A | 0.236 | 0.970 | 0.170 | 0.000 | 0.000 | 6.628 | 0.379 | 0.190 | 0.692 |

| Problem 767 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 48 | 0 | 24 | 12793 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 2.00 | 0.00 | 1.00 | 533.04 | 1.00 |
| time (sec) | N/A | 0.228 | 0.493 | 0.698 | 0.000 | 0.130 | 0.000 | 1.043 | 0.445 | 0.734 |

| Problem 768 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 70 | 24 | 79 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 2.92 | 1.00 | 3.29 | 1.00 |
| time (sec) | N/A | 0.232 | 2.963 | 0.479 | 0.000 | 0.000 | 30.688 | 0.402 | 0.205 | 0.695 |

| Problem 769 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 20 | 0 | 0 | 68 | 22 | 270 | 22 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 0.00 | 0.00 | 3.09 | 1.00 | 12.27 | 1.00 |
| time (sec) | N/A | 0.303 | 0.759 | 0.375 | 0.000 | 0.000 | 24.460 | 0.681 | 0.218 | 0.692 |

| Problem 770 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 21 | 21 | 23 | 19 | 0 | 0 | 66 | 21 | 231 | 21 |
| N.S. | 1 | 1.00 | 1.10 | 0.90 | 0.00 | 0.00 | 3.14 | 1.00 | 11.00 | 1.00 |
| time (sec) | N/A | 0.801 | 1.142 | 0.241 | 0.000 | 0.000 | 15.543 | 0.323 | 0.192 | 0.694 |

| Problem 771 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 66 | 24 | 220 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 2.75 | 1.00 | 9.17 | 1.00 |
| time (sec) | N/A | 0.220 | 0.939 | 0.322 | 0.000 | 0.000 | 14.123 | 0.732 | 0.206 | 0.705 |

| Problem 772 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 66 | 24 | 216 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 2.75 | 1.00 | 9.00 | 1.00 |
| time (sec) | N/A | 0.224 | 1.191 | 0.220 | 0.000 | 0.000 | 13.247 | 0.450 | 0.192 | 0.713 |

| Problem 773 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 24 | 22 | 24 | 30 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 1.00 | 0.92 | 1.00 | 1.25 | 1.00 |
| time (sec) | N/A | 0.230 | 0.550 | 0.153 | 0.000 | 0.099 | 59.745 | 0.189 | 0.189 | 0.732 |

| Problem 774 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 22 | 24 | 30 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 0.92 | 1.00 | 1.25 | 1.00 |
| time (sec) | N/A | 0.464 | 2.665 | 0.102 | 0.000 | 0.000 | 2.346 | 0.349 | 0.185 | 0.702 |

| Problem 775 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 22 | 24 | 30 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 0.92 | 1.00 | 1.25 | 1.00 |
| time (sec) | N/A | 0.352 | 0.567 | 0.101 | 0.000 | 0.000 | 1.559 | 0.241 | 0.190 | 0.689 |

| Problem 776 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 20 | 0 | 0 | 20 | 22 | 28 | 22 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 0.00 | 0.00 | 0.91 | 1.00 | 1.27 | 1.00 |
| time (sec) | N/A | 0.255 | 0.636 | 0.065 | 0.000 | 0.000 | 1.209 | 0.166 | 0.187 | 0.701 |

| Problem 777 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | A | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 18 | 18 | 18 | 15 | 0 | 14 | 0 | 14 | 19 | 14 |
| N.S. | 1 | 1.00 | 1.00 | 0.83 | 0.00 | 0.78 | 0.00 | 0.78 | 1.06 | 0.78 |
| time (sec) | N/A | 0.184 | 0.004 | 0.046 | 0.000 | 0.102 | 0.000 | 0.126 | 0.187 | 0.670 |

| Problem 778 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 19 | 24 | 27 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 0.79 | 1.00 | 1.12 | 1.00 |
| time (sec) | N/A | 0.329 | 0.280 | 0.063 | 0.000 | 0.000 | 1.043 | 0.168 | 0.188 | 0.708 |

| Problem 779 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 20 | 24 | 58 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 0.83 | 1.00 | 2.42 | 1.00 |
| time (sec) | N/A | 0.374 | 0.501 | 0.102 | 0.000 | 0.000 | 1.705 | 0.169 | 0.202 | 0.718 |

| Problem 780 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 20 | 24 | 29 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 0.83 | 1.00 | 1.21 | 1.00 |
| time (sec) | N/A | 0.536 | 1.346 | 0.110 | 0.000 | 0.000 | 2.289 | 0.186 | 0.190 | 0.728 |

| Problem 781 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 20 | 24 | 130 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 0.83 | 1.00 | 5.42 | 1.00 |
| time (sec) | N/A | 0.597 | 3.476 | 0.149 | 0.000 | 0.000 | 3.169 | 0.178 | 0.186 | 0.722 |

| Problem 782 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 39 | 32 | 0 | 38 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 1.62 | 1.33 | 0.00 | 1.58 | 1.00 |
| time (sec) | N/A | 0.245 | 1.297 | 0.168 | 0.000 | 0.098 | 81.615 | 0.000 | 0.185 | 0.900 |

| Problem 783 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 32 | 24 | 38 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 1.33 | 1.00 | 1.58 | 1.00 |
| time (sec) | N/A | 0.240 | 3.023 | 0.087 | 0.000 | 0.000 | 3.226 | 1.282 | 0.178 | 0.727 |

| Problem 784 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F(-2) | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 127 | 127 | 187 | 75 | 0 | 0 | 0 | 0 | 173 | 0 |
| N.S. | 1 | 1.00 | 1.47 | 0.59 | 0.00 | 0.00 | 0.00 | 0.00 | 1.36 | 0.00 |
| time (sec) | N/A | 0.588 | 0.410 | 0.082 | 0.000 | 0.000 | 0.000 | 0.000 | 0.183 | 0.000 |

| Problem 785 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 109 | 108 | 75 | 67 | 0 | 0 | 0 | 0 | 96 | 0 |
| N.S. | 1 | 0.99 | 0.69 | 0.61 | 0.00 | 0.00 | 0.00 | 0.00 | 0.88 | 0.00 |
| time (sec) | N/A | 0.652 | 0.087 | 0.069 | 0.000 | 0.000 | 0.000 | 0.000 | 0.191 | 0.000 |

| Problem 786 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 124 | 122 | 90 | 75 | 0 | 0 | 0 | 0 | 173 | 0 |
| N.S. | 1 | 0.98 | 0.73 | 0.60 | 0.00 | 0.00 | 0.00 | 0.00 | 1.40 | 0.00 |
| time (sec) | N/A | 0.574 | 0.298 | 0.073 | 0.000 | 0.000 | 0.000 | 0.000 | 0.186 | 0.000 |

| Problem 787 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 29 | 24 | 35 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 1.21 | 1.00 | 1.46 | 1.00 |
| time (sec) | N/A | 0.234 | 0.747 | 0.090 | 0.000 | 0.000 | 2.139 | 0.285 | 0.183 | 0.786 |

| Problem 788 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | F(-1) | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 50 | 0 | 0 | 46 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 2.08 | 0.00 | 0.00 | 1.92 | 1.00 |
| time (sec) | N/A | 0.242 | 1.319 | 0.186 | 0.000 | 0.116 | 0.000 | 0.000 | 0.210 | 0.788 |

| Problem 789 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 41 | 24 | 46 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 1.71 | 1.00 | 1.92 | 1.00 |
| time (sec) | N/A | 0.241 | 4.711 | 0.097 | 0.000 | 0.000 | 8.237 | 2.369 | 0.186 | 0.729 |

| Problem 790 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F(-2) | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 230 | 274 | 355 | 132 | 0 | 0 | 0 | 0 | 478 | 0 |
| N.S. | 1 | 1.19 | 1.54 | 0.57 | 0.00 | 0.00 | 0.00 | 0.00 | 2.08 | 0.00 |
| time (sec) | N/A | 1.536 | 0.631 | 0.089 | 0.000 | 0.000 | 0.000 | 0.000 | 0.219 | 0.000 |

| Problem 791 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F(-2) | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 168 | 149 | 350 | 124 | 0 | 0 | 0 | 0 | 361 | 0 |
| N.S. | 1 | 0.89 | 2.08 | 0.74 | 0.00 | 0.00 | 0.00 | 0.00 | 2.15 | 0.00 |
| time (sec) | N/A | 0.543 | 0.216 | 0.092 | 0.000 | 0.000 | 0.000 | 0.000 | 0.210 | 0.000 |

| Problem 792 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F(-2) | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 108 | 91 | 353 | 81 | 0 | 0 | 0 | 0 | 478 | 0 |
| N.S. | 1 | 0.84 | 3.27 | 0.75 | 0.00 | 0.00 | 0.00 | 0.00 | 4.43 | 0.00 |
| time (sec) | N/A | 0.383 | 0.572 | 0.084 | 0.000 | 0.000 | 0.000 | 0.000 | 0.214 | 0.000 |

| Problem 793 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F(-2) | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 168 | 149 | 347 | 124 | 0 | 0 | 0 | 0 | 163 | 0 |
| N.S. | 1 | 0.89 | 2.07 | 0.74 | 0.00 | 0.00 | 0.00 | 0.00 | 0.97 | 0.00 |
| time (sec) | N/A | 0.501 | 0.185 | 0.085 | 0.000 | 0.000 | 0.000 | 0.000 | 0.190 | 0.000 |

| Problem 794 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 219 | 258 | 142 | 132 | 0 | 0 | 0 | 0 | 478 | 0 |
| N.S. | 1 | 1.18 | 0.65 | 0.60 | 0.00 | 0.00 | 0.00 | 0.00 | 2.18 | 0.00 |
| time (sec) | N/A | 1.300 | 0.463 | 0.079 | 0.000 | 0.000 | 0.000 | 0.000 | 0.195 | 0.000 |

| Problem 795 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 37 | 24 | 43 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 1.54 | 1.00 | 1.79 | 1.00 |
| time (sec) | N/A | 0.235 | 1.304 | 0.096 | 0.000 | 0.000 | 4.883 | 0.368 | 0.192 | 0.745 |

| Problem 796 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | F(-1) | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 24 | 0 | 0 | 28 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.92 | 0.00 | 0.00 | 1.08 | 0.92 |
| time (sec) | N/A | 0.273 | 1.111 | 0.436 | 0.000 | 0.126 | 0.000 | 0.000 | 0.349 | 0.569 |

| Problem 797 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 26 | 24 | 28 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 1.00 | 0.92 | 1.08 | 0.92 |
| time (sec) | N/A | 0.274 | 3.249 | 0.375 | 0.000 | 0.000 | 70.666 | 0.331 | 0.232 | 0.566 |

| Problem 798 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 20 | 0 | 0 | 24 | 0 | 149 | 22 |
| N.S. | 1 | 1.00 | 1.08 | 0.83 | 0.00 | 0.00 | 1.00 | 0.00 | 6.21 | 0.92 |
| time (sec) | N/A | 0.367 | 5.161 | 0.206 | 0.000 | 0.000 | 39.757 | 0.000 | 0.248 | 0.554 |

| Problem 799 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 19 | 0 | 0 | 22 | 0 | 25 | 21 |
| N.S. | 1 | 1.00 | 1.09 | 0.83 | 0.00 | 0.00 | 0.96 | 0.00 | 1.09 | 0.91 |
| time (sec) | N/A | 0.387 | 0.219 | 0.189 | 0.000 | 0.000 | 20.641 | 0.000 | 0.227 | 0.593 |

| Problem 800 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 24 | 0 | 28 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 0.92 | 0.00 | 1.08 | 0.92 |
| time (sec) | N/A | 0.264 | 3.670 | 0.235 | 0.000 | 0.000 | 15.937 | 0.000 | 0.208 | 0.558 |

| Problem 801 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | F(-1) | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 24 | 0 | 0 | 62 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.92 | 0.00 | 0.00 | 2.38 | 0.92 |
| time (sec) | N/A | 0.285 | 1.302 | 0.533 | 0.000 | 0.100 | 0.000 | 0.000 | 0.707 | 0.550 |

| Problem 802 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 0 | 24 | 59 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 0.00 | 0.92 | 2.27 | 0.92 |
| time (sec) | N/A | 0.293 | 4.126 | 0.449 | 0.000 | 0.000 | 0.000 | 0.691 | 0.255 | 0.564 |

| Problem 803 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | F(-1) | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 20 | 0 | 0 | 0 | 0 | 244 | 22 |
| N.S. | 1 | 1.00 | 1.08 | 0.83 | 0.00 | 0.00 | 0.00 | 0.00 | 10.17 | 0.92 |
| time (sec) | N/A | 0.355 | 2.730 | 0.263 | 0.000 | 0.000 | 0.000 | 0.000 | 0.272 | 0.567 |

| Problem 804 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | F(-1) | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 19 | 0 | 0 | 0 | 0 | 56 | 21 |
| N.S. | 1 | 1.00 | 1.09 | 0.83 | 0.00 | 0.00 | 0.00 | 0.00 | 2.43 | 0.91 |
| time (sec) | N/A | 0.608 | 1.302 | 0.241 | 0.000 | 0.000 | 0.000 | 0.000 | 0.238 | 0.573 |

| Problem 805 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 24 | 0 | 182 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 0.92 | 0.00 | 7.00 | 0.92 |
| time (sec) | N/A | 0.281 | 2.846 | 0.263 | 0.000 | 0.000 | 107.891 | 0.000 | 0.262 | 0.602 |

| Problem 806 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | F(-1) | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 49 | 0 | 0 | 97 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 1.88 | 0.00 | 0.00 | 3.73 | 0.92 |
| time (sec) | N/A | 0.279 | 1.507 | 0.737 | 0.000 | 0.132 | 0.000 | 0.000 | 17.439 | 0.615 |

| Problem 807 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 0 | 24 | 91 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 0.00 | 0.92 | 3.50 | 0.92 |
| time (sec) | N/A | 0.282 | 3.127 | 0.556 | 0.000 | 0.000 | 0.000 | 1.605 | 0.293 | 0.575 |

| Problem 808 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | F(-1) | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 20 | 0 | 0 | 0 | 0 | 340 | 22 |
| N.S. | 1 | 1.00 | 1.08 | 0.83 | 0.00 | 0.00 | 0.00 | 0.00 | 14.17 | 0.92 |
| time (sec) | N/A | 0.375 | 5.739 | 0.428 | 0.000 | 0.000 | 0.000 | 0.000 | 0.316 | 0.572 |

| Problem 809 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | F(-1) | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 19 | 0 | 0 | 0 | 0 | 88 | 21 |
| N.S. | 1 | 1.00 | 1.09 | 0.83 | 0.00 | 0.00 | 0.00 | 0.00 | 3.83 | 0.91 |
| time (sec) | N/A | 0.898 | 0.324 | 0.385 | 0.000 | 0.000 | 0.000 | 0.000 | 0.258 | 0.571 |

| Problem 810 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | F(-1) | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 0 | 0 | 278 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 0.00 | 0.00 | 10.69 | 0.92 |
| time (sec) | N/A | 0.281 | 2.949 | 0.475 | 0.000 | 0.000 | 0.000 | 0.000 | 0.302 | 0.573 |

| Problem 811 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 24 | 0 | 24 | 42 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.92 | 0.00 | 0.92 | 1.62 | 0.92 |
| time (sec) | N/A | 0.272 | 1.223 | 0.454 | 0.000 | 0.106 | 0.000 | 0.265 | 0.469 | 0.587 |

| Problem 812 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 26 | 0 | 152 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 1.00 | 0.00 | 5.85 | 0.92 |
| time (sec) | N/A | 1.024 | 3.178 | 0.258 | 0.000 | 0.000 | 96.493 | 0.000 | 0.226 | 0.571 |

| Problem 813 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 26 | 24 | 42 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 1.00 | 0.92 | 1.62 | 0.92 |
| time (sec) | N/A | 0.610 | 2.681 | 0.252 | 0.000 | 0.000 | 50.870 | 0.320 | 0.213 | 0.588 |

| Problem 814 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 20 | 0 | 0 | 24 | 22 | 40 | 22 |
| N.S. | 1 | 1.00 | 1.08 | 0.83 | 0.00 | 0.00 | 1.00 | 0.92 | 1.67 | 0.92 |
| time (sec) | N/A | 0.337 | 0.858 | 0.186 | 0.000 | 0.000 | 28.685 | 0.241 | 0.210 | 0.582 |

| Problem 815 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 19 | 0 | 0 | 22 | 21 | 39 | 21 |
| N.S. | 1 | 1.00 | 1.09 | 0.83 | 0.00 | 0.00 | 0.96 | 0.91 | 1.70 | 0.91 |
| time (sec) | N/A | 0.203 | 0.176 | 0.167 | 0.000 | 0.000 | 10.714 | 0.234 | 0.227 | 0.583 |

| Problem 816 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 24 | 24 | 39 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 0.92 | 0.92 | 1.50 | 0.92 |
| time (sec) | N/A | 0.264 | 1.923 | 0.212 | 0.000 | 0.000 | 12.044 | 0.215 | 0.207 | 0.581 |

| Problem 817 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 26 | 24 | 65 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 1.00 | 0.92 | 2.50 | 0.92 |
| time (sec) | N/A | 0.428 | 1.954 | 0.238 | 0.000 | 0.000 | 15.648 | 0.278 | 0.257 | 0.581 |

| Problem 818 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 26 | 24 | 41 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 1.00 | 0.92 | 1.58 | 0.92 |
| time (sec) | N/A | 0.813 | 6.234 | 0.268 | 0.000 | 0.000 | 28.456 | 0.463 | 0.271 | 0.587 |

| Problem 819 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 26 | 24 | 163 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 1.00 | 0.92 | 6.27 | 0.92 |
| time (sec) | N/A | 1.213 | 15.106 | 0.301 | 0.000 | 0.000 | 49.791 | 0.896 | 0.296 | 0.575 |

| Problem 820 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 51 | 0 | 24 | 50 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 1.96 | 0.00 | 0.92 | 1.92 | 0.92 |
| time (sec) | N/A | 0.281 | 1.483 | 0.305 | 0.000 | 0.134 | 0.000 | 0.283 | 0.250 | 0.571 |

| Problem 821 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 26 | 0 | 50 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 1.00 | 0.00 | 1.92 | 0.92 |
| time (sec) | N/A | 0.300 | 5.956 | 0.221 | 0.000 | 0.000 | 102.351 | 0.000 | 0.216 | 0.580 |

| Problem 822 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 26 | 24 | 50 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 1.00 | 0.92 | 1.92 | 0.92 |
| time (sec) | N/A | 0.274 | 4.204 | 0.192 | 0.000 | 0.000 | 55.332 | 0.431 | 0.229 | 0.576 |

| Problem 823 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F(-2) | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 129 | 123 | 128 | 0 | 0 | 0 | 0 | 0 | 173 | 0 |
| N.S. | 1 | 0.95 | 0.99 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.34 | 0.00 |
| time (sec) | N/A | 0.670 | 0.149 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.245 | 0.000 |

| Problem 824 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-2) | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 125 | 125 | 86 | 0 | 0 | 0 | 0 | 0 | 169 | 0 |
| N.S. | 1 | 1.00 | 0.69 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.35 | 0.00 |
| time (sec) | N/A | 0.570 | 0.100 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.266 | 0.000 |

| Problem 825 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 24 | 24 | 47 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 0.92 | 0.92 | 1.81 | 0.92 |
| time (sec) | N/A | 0.296 | 3.087 | 0.201 | 0.000 | 0.000 | 28.125 | 0.266 | 0.227 | 0.580 |

| Problem 826 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 26 | 24 | 285 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 1.00 | 0.92 | 10.96 | 0.92 |
| time (sec) | N/A | 0.284 | 5.473 | 0.232 | 0.000 | 0.000 | 46.808 | 0.283 | 0.274 | 0.573 |

| Problem 827 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 62 | 0 | 24 | 58 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 2.38 | 0.00 | 0.92 | 2.23 | 0.92 |
| time (sec) | N/A | 0.287 | 2.107 | 0.354 | 0.000 | 0.113 | 0.000 | 0.354 | 0.299 | 0.562 |

| Problem 828 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | F(-1) | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 0 | 0 | 58 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 0.00 | 0.00 | 2.23 | 0.92 |
| time (sec) | N/A | 0.277 | 9.699 | 0.281 | 0.000 | 0.000 | 0.000 | 0.000 | 0.214 | 0.556 |

| Problem 829 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | F(-1) | F(-1) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 0 | 0 | 58 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 0.00 | 0.00 | 2.23 | 0.92 |
| time (sec) | N/A | 0.274 | 6.945 | 0.244 | 0.000 | 0.000 | 0.000 | 0.000 | 0.203 | 0.565 |

| Problem 830 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F(-2) | F(-2) | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 263 | 306 | 272 | 0 | 0 | 0 | 0 | 0 | 534 | 0 |
| N.S. | 1 | 1.16 | 1.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 2.03 | 0.00 |
| time (sec) | N/A | 2.535 | 0.841 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.248 | 0.000 |

| Problem 831 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F(-2) | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 247 | 179 | 338 | 0 | 0 | 0 | 0 | 0 | 506 | 0 |
| N.S. | 1 | 0.72 | 1.37 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 2.05 | 0.00 |
| time (sec) | N/A | 0.837 | 0.410 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.296 | 0.000 |

| Problem 832 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F(-2) | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 248 | 181 | 261 | 0 | 0 | 0 | 0 | 0 | 507 | 0 |
| N.S. | 1 | 0.73 | 1.05 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 2.04 | 0.00 |
| time (sec) | N/A | 0.670 | 0.831 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.238 | 0.000 |

| Problem 833 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F(-2) | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 252 | 297 | 345 | 0 | 0 | 0 | 0 | 0 | 529 | 0 |
| N.S. | 1 | 1.18 | 1.37 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 2.10 | 0.00 |
| time (sec) | N/A | 1.231 | 0.388 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.284 | 0.000 |

| Problem 834 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 24 | 24 | 55 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 0.92 | 0.92 | 2.12 | 0.92 |
| time (sec) | N/A | 0.303 | 3.928 | 0.259 | 0.000 | 0.000 | 86.085 | 2.332 | 0.209 | 0.557 |

| Problem 835 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 0 | 24 | 715 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 0.00 | 0.92 | 27.50 | 0.92 |
| time (sec) | N/A | 0.287 | 6.234 | 0.285 | 0.000 | 0.000 | 0.000 | 2.471 | 0.312 | 0.563 |

| Problem 836 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 20 | 0 | 22 | 0 | 22 | 1479 | 22 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 0.00 | 1.00 | 0.00 | 1.00 | 67.23 | 1.00 |
| time (sec) | N/A | 0.207 | 0.793 | 0.660 | 0.000 | 0.124 | 0.000 | 0.580 | 0.222 | 0.899 |

| Problem 837 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 20 | 0 | 0 | 31 | 22 | 41 | 22 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 0.00 | 0.00 | 1.41 | 1.00 | 1.86 | 1.00 |
| time (sec) | N/A | 0.205 | 3.385 | 0.356 | 0.000 | 0.000 | 35.965 | 0.249 | 0.196 | 0.769 |

| Problem 838 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 22 | 18 | 0 | 0 | 29 | 20 | 160 | 20 |
| N.S. | 1 | 1.00 | 1.10 | 0.90 | 0.00 | 0.00 | 1.45 | 1.00 | 8.00 | 1.00 |
| time (sec) | N/A | 0.376 | 0.910 | 0.277 | 0.000 | 0.000 | 22.366 | 0.286 | 0.196 | 0.733 |

| Problem 839 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 19 | 19 | 21 | 17 | 0 | 0 | 27 | 19 | 139 | 19 |
| N.S. | 1 | 1.00 | 1.11 | 0.89 | 0.00 | 0.00 | 1.42 | 1.00 | 7.32 | 1.00 |
| time (sec) | N/A | 0.265 | 1.927 | 0.201 | 0.000 | 0.000 | 17.791 | 0.231 | 0.183 | 0.798 |

| Problem 840 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 20 | 0 | 0 | 27 | 22 | 142 | 22 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 0.00 | 0.00 | 1.23 | 1.00 | 6.45 | 1.00 |
| time (sec) | N/A | 0.227 | 1.446 | 0.276 | 0.000 | 0.000 | 13.595 | 0.381 | 0.193 | 0.809 |

| Problem 841 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 20 | 0 | 0 | 27 | 22 | 67 | 22 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 0.00 | 0.00 | 1.23 | 1.00 | 3.05 | 1.00 |
| time (sec) | N/A | 0.209 | 0.690 | 0.234 | 0.000 | 0.000 | 17.322 | 0.277 | 0.184 | 0.785 |

| Problem 842 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 37 | 0 | 24 | 4847 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 1.54 | 0.00 | 1.00 | 201.96 | 1.00 |
| time (sec) | N/A | 0.240 | 0.658 | 0.913 | 0.000 | 0.125 | 0.000 | 0.783 | 0.294 | 0.766 |

| Problem 843 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 51 | 24 | 65 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 2.12 | 1.00 | 2.71 | 1.00 |
| time (sec) | N/A | 0.224 | 2.350 | 0.308 | 0.000 | 0.000 | 66.051 | 0.309 | 0.196 | 0.708 |

| Problem 844 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 20 | 0 | 0 | 49 | 22 | 245 | 22 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 0.00 | 0.00 | 2.23 | 1.00 | 11.14 | 1.00 |
| time (sec) | N/A | 0.676 | 0.705 | 0.222 | 0.000 | 0.000 | 40.499 | 0.409 | 0.200 | 0.690 |

| Problem 845 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 21 | 21 | 23 | 19 | 0 | 0 | 48 | 21 | 224 | 21 |
| N.S. | 1 | 1.00 | 1.10 | 0.90 | 0.00 | 0.00 | 2.29 | 1.00 | 10.67 | 1.00 |
| time (sec) | N/A | 0.551 | 1.154 | 0.320 | 0.000 | 0.000 | 33.338 | 0.249 | 0.209 | 0.705 |

| Problem 846 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 48 | 24 | 205 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 2.00 | 1.00 | 8.54 | 1.00 |
| time (sec) | N/A | 0.227 | 0.976 | 0.346 | 0.000 | 0.000 | 23.004 | 0.467 | 0.204 | 0.732 |

| Problem 847 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 48 | 24 | 193 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 2.00 | 1.00 | 8.04 | 1.00 |
| time (sec) | N/A | 0.226 | 1.253 | 0.192 | 0.000 | 0.000 | 27.280 | 0.398 | 0.197 | 0.711 |

| Problem 848 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 48 | 0 | 24 | 11573 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 2.00 | 0.00 | 1.00 | 482.21 | 1.00 |
| time (sec) | N/A | 0.230 | 0.521 | 0.935 | 0.000 | 0.130 | 0.000 | 0.982 | 0.398 | 0.778 |

| Problem 849 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 0 | 24 | 87 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 0.00 | 1.00 | 3.62 | 1.00 |
| time (sec) | N/A | 0.227 | 2.308 | 0.433 | 0.000 | 0.000 | 0.000 | 0.397 | 0.213 | 0.711 |

| Problem 850 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 20 | 0 | 0 | 68 | 22 | 328 | 22 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 0.00 | 0.00 | 3.09 | 1.00 | 14.91 | 1.00 |
| time (sec) | N/A | 0.980 | 0.740 | 0.333 | 0.000 | 0.000 | 81.814 | 0.638 | 0.203 | 0.698 |

| Problem 851 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 21 | 21 | 23 | 19 | 0 | 0 | 66 | 21 | 307 | 21 |
| N.S. | 1 | 1.00 | 1.10 | 0.90 | 0.00 | 0.00 | 3.14 | 1.00 | 14.62 | 1.00 |
| time (sec) | N/A | 0.792 | 1.152 | 0.284 | 0.000 | 0.000 | 69.237 | 0.317 | 0.207 | 0.699 |

| Problem 852 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 66 | 24 | 288 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 2.75 | 1.00 | 12.00 | 1.00 |
| time (sec) | N/A | 0.226 | 0.824 | 0.279 | 0.000 | 0.000 | 45.146 | 0.751 | 0.208 | 0.714 |

| Problem 853 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 66 | 24 | 277 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 2.75 | 1.00 | 11.54 | 1.00 |
| time (sec) | N/A | 0.222 | 1.278 | 0.256 | 0.000 | 0.000 | 51.732 | 0.467 | 0.199 | 0.716 |

| Problem 854 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 24 | 0 | 24 | 32 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 1.00 | 0.00 | 1.00 | 1.33 | 1.00 |
| time (sec) | N/A | 0.236 | 0.565 | 0.195 | 0.000 | 0.094 | 0.000 | 0.184 | 0.188 | 0.770 |

| Problem 855 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 22 | 24 | 32 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 0.92 | 1.00 | 1.33 | 1.00 |
| time (sec) | N/A | 0.463 | 2.951 | 0.128 | 0.000 | 0.000 | 8.314 | 0.349 | 0.188 | 0.698 |

| Problem 856 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 22 | 24 | 32 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 0.92 | 1.00 | 1.33 | 1.00 |
| time (sec) | N/A | 0.357 | 0.687 | 0.113 | 0.000 | 0.000 | 6.684 | 0.235 | 0.194 | 0.703 |

| Problem 857 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 20 | 0 | 0 | 20 | 22 | 30 | 22 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 0.00 | 0.00 | 0.91 | 1.00 | 1.36 | 1.00 |
| time (sec) | N/A | 0.256 | 0.644 | 0.067 | 0.000 | 0.000 | 4.451 | 0.160 | 0.191 | 0.668 |

| Problem 858 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 18 | 18 | 18 | 15 | 0 | 14 | 15 | 14 | 19 | 14 |
| N.S. | 1 | 1.00 | 1.00 | 0.83 | 0.00 | 0.78 | 0.83 | 0.78 | 1.06 | 0.78 |
| time (sec) | N/A | 0.191 | 0.004 | 1.743 | 0.000 | 0.098 | 6.229 | 0.120 | 0.188 | 0.668 |

| Problem 859 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 19 | 24 | 29 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 0.79 | 1.00 | 1.21 | 1.00 |
| time (sec) | N/A | 0.332 | 0.286 | 0.077 | 0.000 | 0.000 | 3.063 | 0.165 | 0.200 | 0.732 |

| Problem 860 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 20 | 24 | 64 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 0.83 | 1.00 | 2.67 | 1.00 |
| time (sec) | N/A | 0.370 | 0.499 | 0.121 | 0.000 | 0.000 | 4.503 | 0.165 | 0.194 | 0.724 |

| Problem 861 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 20 | 24 | 31 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 0.83 | 1.00 | 1.29 | 1.00 |
| time (sec) | N/A | 0.527 | 1.505 | 0.126 | 0.000 | 0.000 | 4.522 | 0.185 | 0.186 | 0.731 |

| Problem 862 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 20 | 24 | 170 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 0.83 | 1.00 | 7.08 | 1.00 |
| time (sec) | N/A | 0.635 | 2.840 | 0.178 | 0.000 | 0.000 | 6.192 | 0.173 | 0.196 | 0.764 |

| Problem 863 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | F(-1) | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 39 | 0 | 0 | 40 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 1.62 | 0.00 | 0.00 | 1.67 | 1.00 |
| time (sec) | N/A | 0.226 | 1.303 | 0.239 | 0.000 | 0.111 | 0.000 | 0.000 | 0.202 | 0.780 |

| Problem 864 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 32 | 24 | 40 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 1.33 | 1.00 | 1.67 | 1.00 |
| time (sec) | N/A | 0.225 | 3.334 | 0.124 | 0.000 | 0.000 | 10.915 | 1.541 | 0.194 | 0.718 |

| Problem 865 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 157 | 161 | 111 | 93 | 0 | 0 | 0 | 0 | 143 | 0 |
| N.S. | 1 | 1.03 | 0.71 | 0.59 | 0.00 | 0.00 | 0.00 | 0.00 | 0.91 | 0.00 |
| time (sec) | N/A | 0.837 | 0.325 | 95.643 | 0.000 | 0.000 | 0.000 | 0.000 | 0.197 | 0.000 |

| Problem 866 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F(-2) | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 156 | 153 | 234 | 82 | 0 | 0 | 0 | 0 | 173 | 0 |
| N.S. | 1 | 0.98 | 1.50 | 0.53 | 0.00 | 0.00 | 0.00 | 0.00 | 1.11 | 0.00 |
| time (sec) | N/A | 0.736 | 0.148 | 96.564 | 0.000 | 0.000 | 0.000 | 0.000 | 0.195 | 0.000 |

| Problem 867 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 151 | 156 | 108 | 93 | 0 | 0 | 0 | 0 | 143 | 0 |
| N.S. | 1 | 1.03 | 0.72 | 0.62 | 0.00 | 0.00 | 0.00 | 0.00 | 0.95 | 0.00 |
| time (sec) | N/A | 0.808 | 0.294 | 96.922 | 0.000 | 0.000 | 0.000 | 0.000 | 0.198 | 0.000 |

| Problem 868 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 29 | 24 | 37 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 1.21 | 1.00 | 1.54 | 1.00 |
| time (sec) | N/A | 0.240 | 0.881 | 0.123 | 0.000 | 0.000 | 3.939 | 0.266 | 0.193 | 0.791 |

| Problem 869 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | F(-1) | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 50 | 0 | 0 | 48 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 2.08 | 0.00 | 0.00 | 2.00 | 1.00 |
| time (sec) | N/A | 0.244 | 1.321 | 0.242 | 0.000 | 0.136 | 0.000 | 0.000 | 0.222 | 0.829 |

| Problem 870 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 41 | 24 | 48 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 1.71 | 1.00 | 2.00 | 1.00 |
| time (sec) | N/A | 0.242 | 6.104 | 0.132 | 0.000 | 0.000 | 24.447 | 2.686 | 0.198 | 0.762 |

| Problem 871 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F(-2) | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 310 | 346 | 287 | 168 | 0 | 0 | 0 | 0 | 448 | 0 |
| N.S. | 1 | 1.12 | 0.93 | 0.54 | 0.00 | 0.00 | 0.00 | 0.00 | 1.45 | 0.00 |
| time (sec) | N/A | 1.827 | 0.478 | 0.071 | 0.000 | 0.000 | 0.000 | 0.000 | 0.200 | 0.000 |

| Problem 872 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F(-2) | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 256 | 300 | 359 | 157 | 0 | 0 | 0 | 0 | 478 | 0 |
| N.S. | 1 | 1.17 | 1.40 | 0.61 | 0.00 | 0.00 | 0.00 | 0.00 | 1.87 | 0.00 |
| time (sec) | N/A | 1.787 | 0.524 | 0.075 | 0.000 | 0.000 | 0.000 | 0.000 | 0.210 | 0.000 |

| Problem 873 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F(-2) | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 133 | 110 | 185 | 96 | 0 | 0 | 0 | 0 | 437 | 0 |
| N.S. | 1 | 0.83 | 1.39 | 0.72 | 0.00 | 0.00 | 0.00 | 0.00 | 3.29 | 0.00 |
| time (sec) | N/A | 0.410 | 0.384 | 0.061 | 0.000 | 0.000 | 0.000 | 0.000 | 0.204 | 0.000 |

| Problem 874 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F(-2) | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 254 | 286 | 359 | 157 | 0 | 0 | 0 | 0 | 478 | 0 |
| N.S. | 1 | 1.13 | 1.41 | 0.62 | 0.00 | 0.00 | 0.00 | 0.00 | 1.88 | 0.00 |
| time (sec) | N/A | 1.508 | 0.478 | 0.069 | 0.000 | 0.000 | 0.000 | 0.000 | 0.226 | 0.000 |

| Problem 875 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 296 | 330 | 162 | 168 | 0 | 0 | 0 | 0 | 448 | 0 |
| N.S. | 1 | 1.11 | 0.55 | 0.57 | 0.00 | 0.00 | 0.00 | 0.00 | 1.51 | 0.00 |
| time (sec) | N/A | 1.727 | 0.465 | 0.066 | 0.000 | 0.000 | 0.000 | 0.000 | 0.201 | 0.000 |

| Problem 876 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 37 | 24 | 45 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 1.54 | 1.00 | 1.88 | 1.00 |
| time (sec) | N/A | 0.234 | 1.753 | 0.127 | 0.000 | 0.000 | 8.413 | 0.371 | 0.197 | 0.821 |

| Problem 877 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | F(-1) | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 24 | 0 | 0 | 24 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.92 | 0.00 | 0.00 | 0.92 | 0.92 |
| time (sec) | N/A | 0.271 | 1.127 | 0.366 | 0.000 | 0.125 | 0.000 | 0.000 | 200.027 | 0.613 |

| Problem 878 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 0 | 24 | 30 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 0.00 | 0.92 | 1.15 | 0.92 |
| time (sec) | N/A | 0.289 | 3.338 | 0.291 | 0.000 | 0.000 | 0.000 | 0.304 | 0.264 | 0.610 |

| Problem 879 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | F(-1) | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 20 | 0 | 0 | 0 | 0 | 175 | 22 |
| N.S. | 1 | 1.00 | 1.08 | 0.83 | 0.00 | 0.00 | 0.00 | 0.00 | 7.29 | 0.92 |
| time (sec) | N/A | 0.540 | 3.959 | 0.219 | 0.000 | 0.000 | 0.000 | 0.000 | 0.248 | 0.579 |

| Problem 880 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | F(-1) | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 19 | 0 | 0 | 0 | 0 | 27 | 21 |
| N.S. | 1 | 1.00 | 1.09 | 0.83 | 0.00 | 0.00 | 0.00 | 0.00 | 1.17 | 0.91 |
| time (sec) | N/A | 0.394 | 0.225 | 0.215 | 0.000 | 0.000 | 0.000 | 0.000 | 0.245 | 0.578 |

| Problem 881 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | F(-1) | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 0 | 0 | 30 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 0.00 | 0.00 | 1.15 | 0.92 |
| time (sec) | N/A | 0.276 | 3.030 | 0.244 | 0.000 | 0.000 | 0.000 | 0.000 | 0.223 | 0.563 |

| Problem 882 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | F(-1) | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 24 | 0 | 0 | 24 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.92 | 0.00 | 0.00 | 0.92 | 0.92 |
| time (sec) | N/A | 0.294 | 1.304 | 0.661 | 0.000 | 0.127 | 0.000 | 0.000 | 200.027 | 0.565 |

| Problem 883 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 0 | 24 | 63 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 0.00 | 0.92 | 2.42 | 0.92 |
| time (sec) | N/A | 0.292 | 3.976 | 0.394 | 0.000 | 0.000 | 0.000 | 0.599 | 0.267 | 0.574 |

| Problem 884 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | F(-1) | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 20 | 0 | 0 | 0 | 0 | 299 | 22 |
| N.S. | 1 | 1.00 | 1.08 | 0.83 | 0.00 | 0.00 | 0.00 | 0.00 | 12.46 | 0.92 |
| time (sec) | N/A | 0.802 | 2.615 | 0.291 | 0.000 | 0.000 | 0.000 | 0.000 | 0.305 | 0.590 |

| Problem 885 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | F(-1) | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 19 | 0 | 0 | 0 | 0 | 60 | 21 |
| N.S. | 1 | 1.00 | 1.09 | 0.83 | 0.00 | 0.00 | 0.00 | 0.00 | 2.61 | 0.91 |
| time (sec) | N/A | 0.600 | 1.529 | 0.275 | 0.000 | 0.000 | 0.000 | 0.000 | 0.250 | 0.555 |

| Problem 886 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | F(-1) | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 0 | 0 | 210 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 0.00 | 0.00 | 8.08 | 0.92 |
| time (sec) | N/A | 0.281 | 2.841 | 0.296 | 0.000 | 0.000 | 0.000 | 0.000 | 0.297 | 0.545 |

| Problem 887 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | F(-1) | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 49 | 0 | 0 | 24 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 1.88 | 0.00 | 0.00 | 0.92 | 0.92 |
| time (sec) | N/A | 0.280 | 1.535 | 0.749 | 0.000 | 0.138 | 0.000 | 0.000 | 200.023 | 0.576 |

| Problem 888 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 0 | 24 | 97 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 0.00 | 0.92 | 3.73 | 0.92 |
| time (sec) | N/A | 0.288 | 3.126 | 0.812 | 0.000 | 0.000 | 0.000 | 1.460 | 0.339 | 0.571 |

| Problem 889 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | F(-1) | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 20 | 0 | 0 | 0 | 0 | 424 | 22 |
| N.S. | 1 | 1.00 | 1.08 | 0.83 | 0.00 | 0.00 | 0.00 | 0.00 | 17.67 | 0.92 |
| time (sec) | N/A | 1.086 | 4.647 | 0.451 | 0.000 | 0.000 | 0.000 | 0.000 | 0.390 | 0.562 |

| Problem 890 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | F(-1) | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 19 | 0 | 0 | 0 | 0 | 94 | 21 |
| N.S. | 1 | 1.00 | 1.09 | 0.83 | 0.00 | 0.00 | 0.00 | 0.00 | 4.09 | 0.91 |
| time (sec) | N/A | 0.897 | 0.360 | 0.349 | 0.000 | 0.000 | 0.000 | 0.000 | 0.319 | 0.588 |

| Problem 891 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | F(-1) | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 0 | 0 | 335 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 0.00 | 0.00 | 12.88 | 0.92 |
| time (sec) | N/A | 0.296 | 3.035 | 0.625 | 0.000 | 0.000 | 0.000 | 0.000 | 0.328 | 0.585 |

| Problem 892 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 24 | 0 | 24 | 44 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.92 | 0.00 | 0.92 | 1.69 | 0.92 |
| time (sec) | N/A | 0.292 | 1.209 | 0.380 | 0.000 | 0.129 | 0.000 | 0.239 | 124.261 | 0.562 |

| Problem 893 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | F(-1) | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 0 | 0 | 178 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 0.00 | 0.00 | 6.85 | 0.92 |
| time (sec) | N/A | 1.321 | 3.410 | 0.289 | 0.000 | 0.000 | 0.000 | 0.000 | 0.280 | 0.555 |

| Problem 894 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 0 | 24 | 44 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 0.00 | 0.92 | 1.69 | 0.92 |
| time (sec) | N/A | 0.655 | 2.620 | 0.264 | 0.000 | 0.000 | 0.000 | 0.259 | 0.249 | 0.550 |

| Problem 895 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 20 | 0 | 0 | 0 | 22 | 42 | 22 |
| N.S. | 1 | 1.00 | 1.08 | 0.83 | 0.00 | 0.00 | 0.00 | 0.92 | 1.75 | 0.92 |
| time (sec) | N/A | 0.342 | 0.861 | 0.232 | 0.000 | 0.000 | 0.000 | 0.229 | 0.246 | 0.550 |

| Problem 896 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 19 | 0 | 0 | 22 | 21 | 41 | 21 |
| N.S. | 1 | 1.00 | 1.09 | 0.83 | 0.00 | 0.00 | 0.96 | 0.91 | 1.78 | 0.91 |
| time (sec) | N/A | 0.206 | 0.169 | 0.201 | 0.000 | 0.000 | 100.156 | 0.214 | 0.258 | 0.561 |

| Problem 897 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 24 | 24 | 41 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 0.92 | 0.92 | 1.58 | 0.92 |
| time (sec) | N/A | 0.266 | 1.921 | 0.238 | 0.000 | 0.000 | 113.503 | 0.213 | 0.243 | 0.556 |

| Problem 898 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 0 | 24 | 71 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 0.00 | 0.92 | 2.73 | 0.92 |
| time (sec) | N/A | 0.435 | 2.180 | 0.261 | 0.000 | 0.000 | 0.000 | 0.236 | 0.295 | 0.559 |

| Problem 899 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 0 | 24 | 43 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 0.00 | 0.92 | 1.65 | 0.92 |
| time (sec) | N/A | 0.835 | 6.973 | 0.302 | 0.000 | 0.000 | 0.000 | 0.559 | 0.274 | 0.553 |

| Problem 900 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 0 | 24 | 196 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 0.00 | 0.92 | 7.54 | 0.92 |
| time (sec) | N/A | 1.631 | 14.874 | 0.327 | 0.000 | 0.000 | 0.000 | 0.517 | 0.345 | 0.560 |

| Problem 901 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 51 | 0 | 24 | 52 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 1.96 | 0.00 | 0.92 | 2.00 | 0.92 |
| time (sec) | N/A | 0.278 | 1.504 | 0.337 | 0.000 | 0.129 | 0.000 | 0.282 | 1.260 | 0.559 |

| Problem 902 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 0 | 24 | 52 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 0.00 | 0.92 | 2.00 | 0.92 |
| time (sec) | N/A | 0.288 | 4.343 | 0.228 | 0.000 | 0.000 | 0.000 | 0.401 | 0.238 | 0.544 |

| Problem 903 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F(-2) | F(-2) | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 161 | 163 | 139 | 0 | 0 | 0 | 0 | 0 | 192 | 0 |
| N.S. | 1 | 1.01 | 0.86 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.19 | 0.00 |
| time (sec) | N/A | 0.780 | 0.164 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.287 | 0.000 |

| Problem 904 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-2) | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 155 | 152 | 97 | 0 | 0 | 0 | 0 | 0 | 198 | 0 |
| N.S. | 1 | 0.98 | 0.63 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.28 | 0.00 |
| time (sec) | N/A | 0.680 | 0.106 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.259 | 0.000 |

| Problem 905 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 0 | 24 | 49 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 0.00 | 0.92 | 1.88 | 0.92 |
| time (sec) | N/A | 0.283 | 3.306 | 0.218 | 0.000 | 0.000 | 0.000 | 0.235 | 0.272 | 0.564 |

| Problem 906 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 62 | 0 | 24 | 60 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 2.38 | 0.00 | 0.92 | 2.31 | 0.92 |
| time (sec) | N/A | 0.288 | 2.073 | 0.384 | 0.000 | 0.133 | 0.000 | 0.345 | 10.370 | 0.569 |

| Problem 907 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | F(-1) | F(-1) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 0 | 0 | 60 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 0.00 | 0.00 | 2.31 | 0.92 |
| time (sec) | N/A | 0.319 | 7.137 | 0.274 | 0.000 | 0.000 | 0.000 | 0.000 | 0.250 | 0.571 |

| Problem 908 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F(-2) | F(-2) | F(-1) | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 350 | 390 | 370 | 0 | 0 | 0 | 0 | 0 | 581 | 0 |
| N.S. | 1 | 1.11 | 1.06 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.66 | 0.00 |
| time (sec) | N/A | 3.029 | 0.430 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.319 | 0.000 |

| Problem 909 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F(-2) | F(-2) | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 295 | 344 | 287 | 0 | 0 | 0 | 0 | 0 | 563 | 0 |
| N.S. | 1 | 1.17 | 0.97 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.91 | 0.00 |
| time (sec) | N/A | 3.008 | 0.871 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.271 | 0.000 |

| Problem 910 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F(-2) | F(-2) | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 293 | 337 | 356 | 0 | 0 | 0 | 0 | 0 | 552 | 0 |
| N.S. | 1 | 1.15 | 1.22 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.88 | 0.00 |
| time (sec) | N/A | 1.443 | 0.392 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.322 | 0.000 |

| Problem 911 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-2) | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 337 | 370 | 176 | 0 | 0 | 0 | 0 | 0 | 588 | 0 |
| N.S. | 1 | 1.10 | 0.52 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.74 | 0.00 |
| time (sec) | N/A | 1.424 | 0.191 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.283 | 0.000 |

| Problem 912 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 0 | 24 | 57 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 0.00 | 0.92 | 2.19 | 0.92 |
| time (sec) | N/A | 0.313 | 5.479 | 0.264 | 0.000 | 0.000 | 0.000 | 2.168 | 0.270 | 0.559 |

| Problem 913 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 20 | 0 | 22 | 34 | 22 | 44 | 22 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 0.00 | 1.00 | 1.55 | 1.00 | 2.00 | 1.00 |
| time (sec) | N/A | 0.215 | 0.920 | 0.257 | 0.000 | 0.119 | 11.808 | 0.318 | 0.227 | 0.889 |

| Problem 914 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 22 | 18 | 0 | 0 | 29 | 20 | 39 | 20 |
| N.S. | 1 | 1.00 | 1.10 | 0.90 | 0.00 | 0.00 | 1.45 | 1.00 | 1.95 | 1.00 |
| time (sec) | N/A | 0.201 | 0.452 | 0.133 | 0.000 | 0.000 | 1.007 | 0.225 | 0.223 | 0.719 |

| Problem 915 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 19 | 19 | 21 | 17 | 0 | 0 | 29 | 19 | 38 | 19 |
| N.S. | 1 | 1.00 | 1.11 | 0.89 | 0.00 | 0.00 | 1.53 | 1.00 | 2.00 | 1.00 |
| time (sec) | N/A | 0.202 | 0.072 | 0.105 | 0.000 | 0.000 | 0.699 | 0.164 | 0.227 | 0.737 |

| Problem 916 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 20 | 0 | 0 | 29 | 22 | 39 | 22 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 0.00 | 0.00 | 1.32 | 1.00 | 1.77 | 1.00 |
| time (sec) | N/A | 0.220 | 0.900 | 0.134 | 0.000 | 0.000 | 1.109 | 0.215 | 0.222 | 0.752 |

| Problem 917 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 37 | 58 | 24 | 71 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 1.54 | 2.42 | 1.00 | 2.96 | 1.00 |
| time (sec) | N/A | 0.242 | 0.682 | 0.270 | 0.000 | 0.113 | 40.237 | 0.379 | 0.234 | 0.769 |

| Problem 918 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 20 | 0 | 0 | 49 | 22 | 63 | 22 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 0.00 | 0.00 | 2.23 | 1.00 | 2.86 | 1.00 |
| time (sec) | N/A | 0.227 | 0.555 | 0.173 | 0.000 | 0.000 | 1.799 | 0.319 | 0.233 | 0.683 |

| Problem 919 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 21 | 21 | 23 | 19 | 0 | 0 | 49 | 21 | 62 | 21 |
| N.S. | 1 | 1.00 | 1.10 | 0.90 | 0.00 | 0.00 | 2.33 | 1.00 | 2.95 | 1.00 |
| time (sec) | N/A | 0.199 | 0.326 | 0.141 | 0.000 | 0.000 | 1.383 | 0.195 | 0.221 | 0.668 |

| Problem 920 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 49 | 24 | 63 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 2.04 | 1.00 | 2.62 | 1.00 |
| time (sec) | N/A | 0.229 | 0.860 | 0.170 | 0.000 | 0.000 | 2.298 | 0.271 | 0.233 | 0.690 |

| Problem 921 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 48 | 0 | 24 | 96 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 2.00 | 0.00 | 1.00 | 4.00 | 1.00 |
| time (sec) | N/A | 0.237 | 0.473 | 0.297 | 0.000 | 0.103 | 0.000 | 0.462 | 0.261 | 0.754 |

| Problem 922 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 20 | 0 | 0 | 68 | 22 | 85 | 22 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 0.00 | 0.00 | 3.09 | 1.00 | 3.86 | 1.00 |
| time (sec) | N/A | 0.222 | 0.585 | 0.238 | 0.000 | 0.000 | 3.537 | 0.403 | 0.219 | 0.691 |

| Problem 923 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 21 | 21 | 23 | 19 | 0 | 0 | 68 | 21 | 84 | 21 |
| N.S. | 1 | 1.00 | 1.10 | 0.90 | 0.00 | 0.00 | 3.24 | 1.00 | 4.00 | 1.00 |
| time (sec) | N/A | 0.201 | 0.359 | 0.194 | 0.000 | 0.000 | 2.673 | 0.231 | 0.231 | 0.682 |

| Problem 924 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 68 | 24 | 85 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 2.83 | 1.00 | 3.54 | 1.00 |
| time (sec) | N/A | 0.237 | 0.915 | 0.219 | 0.000 | 0.000 | 3.509 | 0.362 | 0.211 | 0.697 |

| Problem 925 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 24 | 29 | 24 | 33 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 1.00 | 1.21 | 1.00 | 1.38 | 1.00 |
| time (sec) | N/A | 0.244 | 0.539 | 0.208 | 0.000 | 0.143 | 10.860 | 0.164 | 0.226 | 0.844 |

| Problem 926 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 20 | 0 | 0 | 27 | 22 | 31 | 22 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 0.00 | 0.00 | 1.23 | 1.00 | 1.41 | 1.00 |
| time (sec) | N/A | 0.265 | 0.602 | 0.075 | 0.000 | 0.000 | 0.756 | 0.148 | 0.228 | 0.655 |

| Problem 927 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 16 | 16 | 16 | 15 | 0 | 14 | 12 | 14 | 13 | 14 |
| N.S. | 1 | 1.00 | 1.00 | 0.94 | 0.00 | 0.88 | 0.75 | 0.88 | 0.81 | 0.88 |
| time (sec) | N/A | 0.186 | 0.005 | 0.109 | 0.000 | 0.101 | 0.559 | 0.116 | 0.215 | 0.598 |

| Problem 928 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 29 | 24 | 32 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 1.21 | 1.00 | 1.33 | 1.00 |
| time (sec) | N/A | 0.231 | 0.186 | 0.083 | 0.000 | 0.000 | 1.079 | 0.140 | 0.229 | 0.676 |

| Problem 929 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 39 | 48 | 0 | 45 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 1.62 | 2.00 | 0.00 | 1.88 | 1.00 |
| time (sec) | N/A | 0.236 | 1.430 | 0.210 | 0.000 | 0.126 | 58.779 | 0.000 | 0.221 | 0.854 |

| Problem 930 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 48 | 24 | 45 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 2.00 | 1.00 | 1.88 | 1.00 |
| time (sec) | N/A | 0.232 | 2.946 | 0.118 | 0.000 | 0.000 | 1.625 | 0.810 | 0.210 | 0.696 |

| Problem 931 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F(-2) | F(-2) | F | C | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 47 | 41 | 122 | 38 | 0 | 0 | 0 | 54 | 45 | 0 |
| N.S. | 1 | 0.87 | 2.60 | 0.81 | 0.00 | 0.00 | 0.00 | 1.15 | 0.96 | 0.00 |
| time (sec) | N/A | 0.327 | 0.319 | 0.155 | 0.000 | 0.000 | 0.000 | 0.128 | 0.215 | 0.000 |

| Problem 932 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 31 | 31 | 31 | 24 | 0 | 0 | 0 | 0 | 43 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.77 | 0.00 | 0.00 | 0.00 | 0.00 | 1.39 | 0.00 |
| time (sec) | N/A | 0.347 | 0.058 | 0.130 | 0.000 | 0.000 | 0.000 | 0.000 | 0.229 | 0.000 |

| Problem 933 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F(-2) | F(-2) | F | C | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 47 | 41 | 122 | 38 | 0 | 0 | 0 | 51 | 42 | 0 |
| N.S. | 1 | 0.87 | 2.60 | 0.81 | 0.00 | 0.00 | 0.00 | 1.09 | 0.89 | 0.00 |
| time (sec) | N/A | 0.297 | 0.306 | 0.151 | 0.000 | 0.000 | 0.000 | 0.136 | 0.231 | 0.000 |

| Problem 934 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 48 | 24 | 44 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 2.00 | 1.00 | 1.83 | 1.00 |
| time (sec) | N/A | 0.236 | 0.659 | 0.116 | 0.000 | 0.000 | 2.180 | 0.245 | 0.212 | 0.726 |

| Problem 935 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | F(-1) | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 50 | 0 | 0 | 57 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 2.08 | 0.00 | 0.00 | 2.38 | 1.00 |
| time (sec) | N/A | 0.245 | 1.501 | 0.223 | 0.000 | 0.122 | 0.000 | 0.000 | 0.223 | 0.903 |

| Problem 936 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 65 | 24 | 57 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 2.71 | 1.00 | 2.38 | 1.00 |
| time (sec) | N/A | 0.240 | 3.486 | 0.123 | 0.000 | 0.000 | 3.820 | 2.587 | 0.217 | 0.721 |

| Problem 937 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F(-2) | F(-2) | F | C | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 89 | 78 | 230 | 59 | 0 | 0 | 0 | 107 | 57 | 0 |
| N.S. | 1 | 0.88 | 2.58 | 0.66 | 0.00 | 0.00 | 0.00 | 1.20 | 0.64 | 0.00 |
| time (sec) | N/A | 0.373 | 0.431 | 0.158 | 0.000 | 0.000 | 0.000 | 0.137 | 0.217 | 0.000 |

| Problem 938 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F(-2) | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 71 | 66 | 131 | 47 | 0 | 0 | 0 | 0 | 57 | 0 |
| N.S. | 1 | 0.93 | 1.85 | 0.66 | 0.00 | 0.00 | 0.00 | 0.00 | 0.80 | 0.00 |
| time (sec) | N/A | 0.350 | 0.132 | 0.163 | 0.000 | 0.000 | 0.000 | 0.000 | 0.211 | 0.000 |

| Problem 939 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F(-2) | F(-2) | F | C | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 58 | 53 | 229 | 47 | 0 | 0 | 0 | 67 | 57 | 0 |
| N.S. | 1 | 0.91 | 3.95 | 0.81 | 0.00 | 0.00 | 0.00 | 1.16 | 0.98 | 0.00 |
| time (sec) | N/A | 0.335 | 0.411 | 0.160 | 0.000 | 0.000 | 0.000 | 0.138 | 0.232 | 0.000 |

| Problem 940 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F(-2) | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 71 | 66 | 133 | 46 | 0 | 0 | 0 | 0 | 55 | 0 |
| N.S. | 1 | 0.93 | 1.87 | 0.65 | 0.00 | 0.00 | 0.00 | 0.00 | 0.77 | 0.00 |
| time (sec) | N/A | 0.338 | 0.110 | 0.158 | 0.000 | 0.000 | 0.000 | 0.000 | 0.229 | 0.000 |

| Problem 941 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | F(-2) | F | C | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 89 | 78 | 72 | 59 | 0 | 0 | 0 | 107 | 54 | 0 |
| N.S. | 1 | 0.88 | 0.81 | 0.66 | 0.00 | 0.00 | 0.00 | 1.20 | 0.61 | 0.00 |
| time (sec) | N/A | 0.329 | 0.367 | 0.148 | 0.000 | 0.000 | 0.000 | 0.136 | 0.230 | 0.000 |

| Problem 942 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 65 | 24 | 56 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 2.71 | 1.00 | 2.33 | 1.00 |
| time (sec) | N/A | 0.236 | 0.844 | 0.125 | 0.000 | 0.000 | 4.600 | 0.331 | 0.209 | 0.779 |

| Problem 943 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 24 | 26 | 0 | 30 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.92 | 1.00 | 0.00 | 1.15 | 0.92 |
| time (sec) | N/A | 0.265 | 1.012 | 0.395 | 0.000 | 0.132 | 19.046 | 0.000 | 4.486 | 0.588 |

| Problem 944 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 20 | 0 | 0 | 24 | 22 | 28 | 22 |
| N.S. | 1 | 1.00 | 1.08 | 0.83 | 0.00 | 0.00 | 1.00 | 0.92 | 1.17 | 0.92 |
| time (sec) | N/A | 0.240 | 1.759 | 0.234 | 0.000 | 0.000 | 1.438 | 1.177 | 0.238 | 0.572 |

| Problem 945 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 19 | 0 | 0 | 22 | 21 | 27 | 21 |
| N.S. | 1 | 1.00 | 1.09 | 0.83 | 0.00 | 0.00 | 0.96 | 0.91 | 1.17 | 0.91 |
| time (sec) | N/A | 0.201 | 0.205 | 0.218 | 0.000 | 0.000 | 0.795 | 0.378 | 0.207 | 0.571 |

| Problem 946 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 24 | 24 | 30 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 0.92 | 0.92 | 1.15 | 0.92 |
| time (sec) | N/A | 0.264 | 4.149 | 0.249 | 0.000 | 0.000 | 1.684 | 0.305 | 0.226 | 0.571 |

| Problem 947 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | F(-1) | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 24 | 0 | 0 | 66 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.92 | 0.00 | 0.00 | 2.54 | 0.92 |
| time (sec) | N/A | 0.273 | 1.066 | 0.426 | 0.000 | 0.119 | 0.000 | 0.000 | 48.712 | 0.583 |

| Problem 948 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 20 | 0 | 0 | 24 | 22 | 61 | 22 |
| N.S. | 1 | 1.00 | 1.08 | 0.83 | 0.00 | 0.00 | 1.00 | 0.92 | 2.54 | 0.92 |
| time (sec) | N/A | 0.248 | 2.752 | 0.276 | 0.000 | 0.000 | 35.817 | 0.590 | 0.248 | 0.568 |

| Problem 949 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 19 | 0 | 0 | 22 | 21 | 60 | 21 |
| N.S. | 1 | 1.00 | 1.09 | 0.83 | 0.00 | 0.00 | 0.96 | 0.91 | 2.61 | 0.91 |
| time (sec) | N/A | 0.212 | 0.260 | 0.262 | 0.000 | 0.000 | 18.369 | 0.627 | 0.283 | 0.575 |

| Problem 950 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 24 | 24 | 61 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 0.92 | 0.92 | 2.35 | 0.92 |
| time (sec) | N/A | 0.270 | 4.719 | 0.302 | 0.000 | 0.000 | 17.343 | 0.847 | 0.250 | 0.580 |

| Problem 951 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | F(-1) | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 49 | 0 | 0 | 103 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 1.88 | 0.00 | 0.00 | 3.96 | 0.92 |
| time (sec) | N/A | 0.278 | 1.307 | 0.465 | 0.000 | 0.173 | 0.000 | 0.000 | 140.615 | 0.565 |

| Problem 952 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 20 | 0 | 0 | 0 | 22 | 95 | 22 |
| N.S. | 1 | 1.00 | 1.08 | 0.83 | 0.00 | 0.00 | 0.00 | 0.92 | 3.96 | 0.92 |
| time (sec) | N/A | 0.247 | 2.199 | 0.343 | 0.000 | 0.000 | 0.000 | 1.499 | 0.280 | 0.570 |

| Problem 953 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 19 | 0 | 0 | 0 | 21 | 94 | 21 |
| N.S. | 1 | 1.00 | 1.09 | 0.83 | 0.00 | 0.00 | 0.00 | 0.91 | 4.09 | 0.91 |
| time (sec) | N/A | 0.200 | 0.344 | 0.317 | 0.000 | 0.000 | 0.000 | 1.006 | 0.289 | 0.563 |

| Problem 954 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 0 | 24 | 95 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 0.00 | 0.92 | 3.65 | 0.92 |
| time (sec) | N/A | 0.267 | 3.948 | 0.352 | 0.000 | 0.000 | 0.000 | 3.547 | 0.286 | 0.567 |

| Problem 955 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 24 | 26 | 24 | 45 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.92 | 1.00 | 0.92 | 1.73 | 0.92 |
| time (sec) | N/A | 0.273 | 1.053 | 0.411 | 0.000 | 0.108 | 25.415 | 0.204 | 0.264 | 0.554 |

| Problem 956 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 20 | 0 | 0 | 24 | 22 | 43 | 22 |
| N.S. | 1 | 1.00 | 1.08 | 0.83 | 0.00 | 0.00 | 1.00 | 0.92 | 1.79 | 0.92 |
| time (sec) | N/A | 0.229 | 0.980 | 0.230 | 0.000 | 0.000 | 1.751 | 0.199 | 0.238 | 0.565 |

| Problem 957 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 19 | 0 | 0 | 24 | 21 | 42 | 21 |
| N.S. | 1 | 1.00 | 1.09 | 0.83 | 0.00 | 0.00 | 1.04 | 0.91 | 1.83 | 0.91 |
| time (sec) | N/A | 0.208 | 0.182 | 0.212 | 0.000 | 0.000 | 1.718 | 0.179 | 0.279 | 0.561 |

| Problem 958 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 26 | 24 | 44 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 1.00 | 0.92 | 1.69 | 0.92 |
| time (sec) | N/A | 0.265 | 1.235 | 0.258 | 0.000 | 0.000 | 3.893 | 0.182 | 0.248 | 0.560 |

| Problem 959 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 51 | 0 | 24 | 57 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 1.96 | 0.00 | 0.92 | 2.19 | 0.92 |
| time (sec) | N/A | 0.270 | 1.302 | 0.363 | 0.000 | 0.120 | 0.000 | 0.230 | 0.270 | 0.554 |

| Problem 960 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 26 | 24 | 57 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 1.00 | 0.92 | 2.19 | 0.92 |
| time (sec) | N/A | 0.271 | 10.818 | 0.254 | 0.000 | 0.000 | 7.772 | 0.201 | 0.309 | 0.565 |

| Problem 961 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F(-2) | F(-2) | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 60 | 60 | 97 | 0 | 0 | 0 | 0 | 0 | 55 | 0 |
| N.S. | 1 | 1.00 | 1.62 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.92 | 0.00 |
| time (sec) | N/A | 0.461 | 0.127 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.266 | 0.000 |

| Problem 962 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-2) | F(-2) | F | C | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 60 | 60 | 61 | 0 | 0 | 0 | 0 | 49 | 54 | 0 |
| N.S. | 1 | 1.00 | 1.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.82 | 0.90 | 0.00 |
| time (sec) | N/A | 0.405 | 0.073 | 0.000 | 0.000 | 0.000 | 0.000 | 0.134 | 0.305 | 0.000 |

| Problem 963 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 26 | 0 | 56 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 1.00 | 0.00 | 2.15 | 0.92 |
| time (sec) | N/A | 0.277 | 1.656 | 0.250 | 0.000 | 0.000 | 20.459 | 0.000 | 0.263 | 0.549 |

| Problem 964 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 62 | 0 | 24 | 69 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 2.38 | 0.00 | 0.92 | 2.65 | 0.92 |
| time (sec) | N/A | 0.268 | 1.830 | 0.424 | 0.000 | 0.149 | 0.000 | 0.300 | 0.249 | 0.560 |

| Problem 965 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | F(-1) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 26 | 0 | 69 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 1.00 | 0.00 | 2.65 | 0.92 |
| time (sec) | N/A | 0.282 | 4.599 | 0.306 | 0.000 | 0.000 | 51.600 | 0.000 | 0.384 | 0.548 |

| Problem 966 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-2) | F(-2) | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 131 | 99 | 95 | 0 | 0 | 0 | 0 | 0 | 69 | 0 |
| N.S. | 1 | 0.76 | 0.73 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.53 | 0.00 |
| time (sec) | N/A | 0.567 | 0.295 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.252 | 0.000 |

| Problem 967 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F(-2) | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 131 | 99 | 159 | 0 | 0 | 0 | 0 | 0 | 69 | 0 |
| N.S. | 1 | 0.76 | 1.21 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.53 | 0.00 |
| time (sec) | N/A | 0.548 | 0.173 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.387 | 0.000 |

| Problem 968 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F(-2) | F(-2) | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 131 | 99 | 156 | 0 | 0 | 0 | 0 | 0 | 67 | 0 |
| N.S. | 1 | 0.76 | 1.19 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.51 | 0.00 |
| time (sec) | N/A | 0.486 | 0.194 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.250 | 0.000 |

| Problem 969 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F(-2) | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 131 | 99 | 93 | 0 | 0 | 0 | 0 | 0 | 66 | 0 |
| N.S. | 1 | 0.76 | 0.71 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.50 | 0.00 |
| time (sec) | N/A | 0.456 | 0.094 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.351 | 0.000 |

| Problem 970 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 26 | 0 | 68 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 1.00 | 0.00 | 2.62 | 0.92 |
| time (sec) | N/A | 0.279 | 2.327 | 0.295 | 0.000 | 0.000 | 110.705 | 0.000 | 0.253 | 0.614 |

| Problem 971 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 20 | 0 | 22 | 34 | 22 | 44 | 22 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 0.00 | 1.00 | 1.55 | 1.00 | 2.00 | 1.00 |
| time (sec) | N/A | 0.206 | 0.865 | 0.304 | 0.000 | 0.115 | 30.333 | 0.146 | 0.266 | 1.063 |

| Problem 972 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 22 | 18 | 0 | 0 | 29 | 20 | 39 | 20 |
| N.S. | 1 | 1.00 | 1.10 | 0.90 | 0.00 | 0.00 | 1.45 | 1.00 | 1.95 | 1.00 |
| time (sec) | N/A | 0.191 | 0.672 | 0.167 | 0.000 | 0.000 | 1.458 | 0.137 | 0.234 | 0.843 |

| Problem 973 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 19 | 19 | 21 | 17 | 0 | 0 | 29 | 19 | 38 | 19 |
| N.S. | 1 | 1.00 | 1.11 | 0.89 | 0.00 | 0.00 | 1.53 | 1.00 | 2.00 | 1.00 |
| time (sec) | N/A | 0.185 | 0.610 | 0.136 | 0.000 | 0.000 | 1.283 | 0.137 | 0.237 | 0.807 |

| Problem 974 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 20 | 0 | 0 | 29 | 22 | 39 | 22 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 0.00 | 0.00 | 1.32 | 1.00 | 1.77 | 1.00 |
| time (sec) | N/A | 0.200 | 1.170 | 0.161 | 0.000 | 0.000 | 2.339 | 0.136 | 0.217 | 0.852 |

| Problem 975 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 37 | 58 | 24 | 71 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 1.54 | 2.42 | 1.00 | 2.96 | 1.00 |
| time (sec) | N/A | 0.226 | 0.692 | 0.425 | 0.000 | 0.115 | 53.307 | 0.143 | 0.234 | 0.897 |

| Problem 976 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 20 | 0 | 0 | 49 | 22 | 63 | 22 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 0.00 | 0.00 | 2.23 | 1.00 | 2.86 | 1.00 |
| time (sec) | N/A | 0.208 | 0.785 | 0.248 | 0.000 | 0.000 | 2.251 | 0.139 | 0.215 | 0.761 |

| Problem 977 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 21 | 21 | 23 | 19 | 0 | 0 | 49 | 21 | 62 | 21 |
| N.S. | 1 | 1.00 | 1.10 | 0.90 | 0.00 | 0.00 | 2.33 | 1.00 | 2.95 | 1.00 |
| time (sec) | N/A | 0.204 | 0.680 | 0.196 | 0.000 | 0.000 | 1.847 | 0.133 | 0.251 | 0.703 |

| Problem 978 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 49 | 24 | 63 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 2.04 | 1.00 | 2.62 | 1.00 |
| time (sec) | N/A | 0.239 | 1.294 | 0.215 | 0.000 | 0.000 | 2.793 | 0.139 | 0.259 | 0.756 |

| Problem 979 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 48 | 0 | 24 | 96 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 2.00 | 0.00 | 1.00 | 4.00 | 1.00 |
| time (sec) | N/A | 0.242 | 0.495 | 1.027 | 0.000 | 0.117 | 0.000 | 0.152 | 0.267 | 0.880 |

| Problem 980 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 20 | 0 | 0 | 68 | 22 | 85 | 22 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 0.00 | 0.00 | 3.09 | 1.00 | 3.86 | 1.00 |
| time (sec) | N/A | 0.206 | 0.819 | 0.385 | 0.000 | 0.000 | 4.309 | 0.137 | 0.250 | 0.763 |

| Problem 981 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 21 | 21 | 23 | 19 | 0 | 0 | 68 | 21 | 84 | 21 |
| N.S. | 1 | 1.00 | 1.10 | 0.90 | 0.00 | 0.00 | 3.24 | 1.00 | 4.00 | 1.00 |
| time (sec) | N/A | 0.206 | 0.721 | 0.296 | 0.000 | 0.000 | 3.318 | 0.136 | 0.255 | 0.718 |

| Problem 982 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 68 | 24 | 85 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 2.83 | 1.00 | 3.54 | 1.00 |
| time (sec) | N/A | 0.243 | 1.366 | 0.298 | 0.000 | 0.000 | 4.382 | 0.134 | 0.254 | 0.799 |

| Problem 983 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 24 | 29 | 24 | 37 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 1.00 | 1.21 | 1.00 | 1.54 | 1.00 |
| time (sec) | N/A | 0.305 | 0.567 | 0.217 | 0.000 | 0.114 | 49.085 | 0.145 | 0.259 | 0.850 |

| Problem 984 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 20 | 0 | 0 | 27 | 22 | 35 | 22 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 0.00 | 0.00 | 1.23 | 1.00 | 1.59 | 1.00 |
| time (sec) | N/A | 0.262 | 0.569 | 0.079 | 0.000 | 0.000 | 1.326 | 0.138 | 0.250 | 0.642 |

| Problem 985 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 16 | 16 | 16 | 15 | 0 | 14 | 14 | 14 | 19 | 14 |
| N.S. | 1 | 1.00 | 1.00 | 0.94 | 0.00 | 0.88 | 0.88 | 0.88 | 1.19 | 0.88 |
| time (sec) | N/A | 0.195 | 0.007 | 0.111 | 0.000 | 0.093 | 1.232 | 0.126 | 0.239 | 0.594 |

| Problem 986 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 29 | 24 | 36 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 1.21 | 1.00 | 1.50 | 1.00 |
| time (sec) | N/A | 0.303 | 0.158 | 0.086 | 0.000 | 0.000 | 1.750 | 0.137 | 0.244 | 0.663 |

| Problem 987 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 39 | 0 | 24 | 350 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 1.62 | 0.00 | 1.00 | 14.58 | 1.00 |
| time (sec) | N/A | 0.244 | 1.181 | 0.222 | 0.000 | 0.111 | 0.000 | 0.145 | 0.282 | 0.870 |

| Problem 988 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 48 | 24 | 51 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 2.00 | 1.00 | 2.12 | 1.00 |
| time (sec) | N/A | 0.450 | 6.174 | 0.182 | 0.000 | 0.000 | 3.158 | 0.139 | 0.236 | 0.791 |

| Problem 989 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 48 | 24 | 51 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 2.00 | 1.00 | 2.12 | 1.00 |
| time (sec) | N/A | 0.837 | 5.593 | 0.121 | 0.000 | 0.000 | 3.554 | 0.141 | 0.231 | 0.770 |

| Problem 990 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 60 | 60 | 60 | 46 | 0 | 0 | 0 | 0 | 51 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.77 | 0.00 | 0.00 | 0.00 | 0.00 | 0.85 | 0.00 |
| time (sec) | N/A | 0.510 | 0.312 | 0.168 | 0.000 | 0.000 | 0.000 | 0.000 | 0.263 | 0.000 |

| Problem 991 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F(-2) | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 138 | 139 | 158 | 46 | 0 | 0 | 0 | 0 | 158 | 0 |
| N.S. | 1 | 1.01 | 1.14 | 0.33 | 0.00 | 0.00 | 0.00 | 0.00 | 1.14 | 0.00 |
| time (sec) | N/A | 0.628 | 0.157 | 0.158 | 0.000 | 0.000 | 0.000 | 0.000 | 0.283 | 0.000 |

| Problem 992 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 57 | 57 | 52 | 47 | 0 | 0 | 0 | 0 | 151 | 0 |
| N.S. | 1 | 1.00 | 0.91 | 0.82 | 0.00 | 0.00 | 0.00 | 0.00 | 2.65 | 0.00 |
| time (sec) | N/A | 0.458 | 0.301 | 0.163 | 0.000 | 0.000 | 0.000 | 0.000 | 0.261 | 0.000 |

| Problem 993 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 48 | 24 | 50 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 2.00 | 1.00 | 2.08 | 1.00 |
| time (sec) | N/A | 0.759 | 2.058 | 0.130 | 0.000 | 0.000 | 4.066 | 0.134 | 0.253 | 0.798 |

| Problem 994 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 49 | 24 | 52 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 2.04 | 1.00 | 2.17 | 1.00 |
| time (sec) | N/A | 0.449 | 3.073 | 0.178 | 0.000 | 0.000 | 6.345 | 0.134 | 0.223 | 0.827 |

| Problem 995 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 49 | 24 | 52 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 2.04 | 1.00 | 2.17 | 1.00 |
| time (sec) | N/A | 0.462 | 6.455 | 0.150 | 0.000 | 0.000 | 8.479 | 0.130 | 0.236 | 0.804 |

| Problem 996 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 49 | 24 | 52 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 2.04 | 1.00 | 2.17 | 1.00 |
| time (sec) | N/A | 0.456 | 6.112 | 0.199 | 0.000 | 0.000 | 11.275 | 0.133 | 0.266 | 0.805 |

| Problem 997 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 50 | 0 | 24 | 631 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 2.08 | 0.00 | 1.00 | 26.29 | 1.00 |
| time (sec) | N/A | 0.243 | 1.136 | 0.243 | 0.000 | 0.129 | 0.000 | 0.150 | 0.260 | 0.931 |

| Problem 998 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F(-2) | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 96 | 164 | 148 | 86 | 0 | 0 | 0 | 0 | 65 | 0 |
| N.S. | 1 | 1.71 | 1.54 | 0.90 | 0.00 | 0.00 | 0.00 | 0.00 | 0.68 | 0.00 |
| time (sec) | N/A | 0.821 | 0.270 | 0.198 | 0.000 | 0.000 | 0.000 | 0.000 | 0.284 | 0.000 |

| Problem 999 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | B | C | A | F(-2) | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 67 | 165 | 112 | 53 | 0 | 0 | 0 | 0 | 65 | 0 |
| N.S. | 1 | 2.46 | 1.67 | 0.79 | 0.00 | 0.00 | 0.00 | 0.00 | 0.97 | 0.00 |
| time (sec) | N/A | 0.637 | 0.353 | 0.189 | 0.000 | 0.000 | 0.000 | 0.000 | 0.234 | 0.000 |

| Problem 1000 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F(-2) | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 93 | 162 | 156 | 84 | 0 | 0 | 0 | 0 | 412 | 0 |
| N.S. | 1 | 1.74 | 1.68 | 0.90 | 0.00 | 0.00 | 0.00 | 0.00 | 4.43 | 0.00 |
| time (sec) | N/A | 0.971 | 0.198 | 0.184 | 0.000 | 0.000 | 0.000 | 0.000 | 0.245 | 0.000 |

| Problem 1001 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F(-2) | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 94 | 95 | 144 | 85 | 0 | 0 | 0 | 0 | 450 | 0 |
| N.S. | 1 | 1.01 | 1.53 | 0.90 | 0.00 | 0.00 | 0.00 | 0.00 | 4.79 | 0.00 |
| time (sec) | N/A | 0.450 | 0.409 | 0.179 | 0.000 | 0.000 | 0.000 | 0.000 | 0.225 | 0.000 |

| Problem 1002 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 65 | 24 | 64 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 2.71 | 1.00 | 2.67 | 1.00 |
| time (sec) | N/A | 0.865 | 2.728 | 0.135 | 0.000 | 0.000 | 11.199 | 0.132 | 0.236 | 0.821 |

| Problem 1003 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 66 | 24 | 66 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 2.75 | 1.00 | 2.75 | 1.00 |
| time (sec) | N/A | 0.487 | 4.174 | 0.192 | 0.000 | 0.000 | 16.266 | 0.141 | 0.227 | 0.836 |

| Problem 1004 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 66 | 24 | 66 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 2.75 | 1.00 | 2.75 | 1.00 |
| time (sec) | N/A | 0.472 | 6.679 | 0.156 | 0.000 | 0.000 | 20.669 | 0.138 | 0.257 | 0.840 |

| Problem 1005 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 66 | 24 | 66 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 2.75 | 1.00 | 2.75 | 1.00 |
| time (sec) | N/A | 0.455 | 6.245 | 0.224 | 0.000 | 0.000 | 26.826 | 0.139 | 0.267 | 0.840 |

| Problem 1006 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 24 | 26 | 0 | 30 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.92 | 1.00 | 0.00 | 1.15 | 0.92 |
| time (sec) | N/A | 0.271 | 0.984 | 0.432 | 0.000 | 0.102 | 55.174 | 0.000 | 0.706 | 0.598 |

| Problem 1007 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 20 | 0 | 0 | 24 | 22 | 28 | 22 |
| N.S. | 1 | 1.00 | 1.08 | 0.83 | 0.00 | 0.00 | 1.00 | 0.92 | 1.17 | 0.92 |
| time (sec) | N/A | 0.242 | 2.046 | 0.262 | 0.000 | 0.000 | 3.390 | 0.144 | 0.245 | 0.577 |

| Problem 1008 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 19 | 0 | 0 | 22 | 21 | 27 | 21 |
| N.S. | 1 | 1.00 | 1.09 | 0.83 | 0.00 | 0.00 | 0.96 | 0.91 | 1.17 | 0.91 |
| time (sec) | N/A | 0.214 | 0.333 | 0.237 | 0.000 | 0.000 | 4.808 | 0.148 | 0.231 | 0.587 |

| Problem 1009 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 24 | 24 | 30 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 0.92 | 0.92 | 1.15 | 0.92 |
| time (sec) | N/A | 0.269 | 5.260 | 0.281 | 0.000 | 0.000 | 8.097 | 0.146 | 0.256 | 0.586 |

| Problem 1010 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | F(-1) | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 24 | 0 | 0 | 66 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.92 | 0.00 | 0.00 | 2.54 | 0.92 |
| time (sec) | N/A | 0.293 | 1.200 | 0.562 | 0.000 | 0.106 | 0.000 | 0.000 | 11.364 | 0.586 |

| Problem 1011 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 20 | 0 | 0 | 24 | 22 | 61 | 22 |
| N.S. | 1 | 1.00 | 1.08 | 0.83 | 0.00 | 0.00 | 1.00 | 0.92 | 2.54 | 0.92 |
| time (sec) | N/A | 0.246 | 6.020 | 0.345 | 0.000 | 0.000 | 49.874 | 0.179 | 0.237 | 0.584 |

| Problem 1012 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 19 | 0 | 0 | 22 | 21 | 60 | 21 |
| N.S. | 1 | 1.00 | 1.09 | 0.83 | 0.00 | 0.00 | 0.96 | 0.91 | 2.61 | 0.91 |
| time (sec) | N/A | 0.209 | 1.091 | 0.319 | 0.000 | 0.000 | 28.034 | 0.165 | 0.256 | 0.591 |

| Problem 1013 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 24 | 24 | 61 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 0.92 | 0.92 | 2.35 | 0.92 |
| time (sec) | N/A | 0.285 | 10.219 | 0.334 | 0.000 | 0.000 | 45.085 | 0.168 | 0.241 | 0.584 |

| Problem 1014 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | F(-1) | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 49 | 0 | 0 | 103 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 1.88 | 0.00 | 0.00 | 3.96 | 0.92 |
| time (sec) | N/A | 0.280 | 1.440 | 1.076 | 0.000 | 0.119 | 0.000 | 0.000 | 147.930 | 0.572 |

| Problem 1015 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 20 | 0 | 0 | 0 | 22 | 95 | 22 |
| N.S. | 1 | 1.00 | 1.08 | 0.83 | 0.00 | 0.00 | 0.00 | 0.92 | 3.96 | 0.92 |
| time (sec) | N/A | 0.255 | 3.146 | 0.454 | 0.000 | 0.000 | 0.000 | 0.197 | 0.253 | 0.572 |

| Problem 1016 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 19 | 0 | 0 | 0 | 21 | 94 | 21 |
| N.S. | 1 | 1.00 | 1.09 | 0.83 | 0.00 | 0.00 | 0.00 | 0.91 | 4.09 | 0.91 |
| time (sec) | N/A | 0.215 | 0.920 | 0.392 | 0.000 | 0.000 | 0.000 | 0.169 | 0.254 | 0.577 |

| Problem 1017 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 0 | 24 | 95 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 0.00 | 0.92 | 3.65 | 0.92 |
| time (sec) | N/A | 0.275 | 6.717 | 0.432 | 0.000 | 0.000 | 0.000 | 0.176 | 0.255 | 0.580 |

| Problem 1018 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 24 | 0 | 24 | 49 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.92 | 0.00 | 0.92 | 1.88 | 0.92 |
| time (sec) | N/A | 0.274 | 1.158 | 0.439 | 0.000 | 0.122 | 0.000 | 0.154 | 0.200 | 0.578 |

| Problem 1019 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 20 | 0 | 0 | 24 | 22 | 47 | 22 |
| N.S. | 1 | 1.00 | 1.08 | 0.83 | 0.00 | 0.00 | 1.00 | 0.92 | 1.96 | 0.92 |
| time (sec) | N/A | 0.239 | 0.875 | 0.259 | 0.000 | 0.000 | 7.695 | 0.151 | 0.203 | 0.563 |

| Problem 1020 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 19 | 0 | 0 | 24 | 21 | 79 | 21 |
| N.S. | 1 | 1.00 | 1.09 | 0.83 | 0.00 | 0.00 | 1.04 | 0.91 | 3.43 | 0.91 |
| time (sec) | N/A | 0.208 | 0.506 | 0.253 | 0.000 | 0.000 | 7.739 | 0.150 | 0.206 | 0.570 |

| Problem 1021 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 26 | 24 | 48 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 1.00 | 0.92 | 1.85 | 0.92 |
| time (sec) | N/A | 0.436 | 3.411 | 0.269 | 0.000 | 0.000 | 19.607 | 0.152 | 0.203 | 0.580 |

| Problem 1022 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 27 | 24 | 50 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 1.04 | 0.92 | 1.92 | 0.92 |
| time (sec) | N/A | 0.266 | 7.314 | 0.309 | 0.000 | 0.000 | 35.497 | 0.151 | 0.220 | 0.573 |

| Problem 1023 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 51 | 0 | 24 | 422 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 1.96 | 0.00 | 0.92 | 16.23 | 0.92 |
| time (sec) | N/A | 0.285 | 1.448 | 0.395 | 0.000 | 0.121 | 0.000 | 0.176 | 0.312 | 0.595 |

| Problem 1024 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 26 | 0 | 63 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 1.00 | 0.00 | 2.42 | 0.92 |
| time (sec) | N/A | 0.605 | 5.919 | 0.283 | 0.000 | 0.000 | 28.995 | 0.000 | 0.222 | 0.580 |

| Problem 1025 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 26 | 24 | 63 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 1.00 | 0.92 | 2.42 | 0.92 |
| time (sec) | N/A | 1.460 | 3.239 | 0.267 | 0.000 | 0.000 | 28.766 | 0.168 | 0.279 | 0.578 |

| Problem 1026 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F(-2) | F(-2) | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 93 | 93 | 116 | 0 | 0 | 0 | 0 | 0 | 158 | 0 |
| N.S. | 1 | 1.00 | 1.25 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.70 | 0.00 |
| time (sec) | N/A | 0.622 | 0.132 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.246 | 0.000 |

| Problem 1027 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F(-2) | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 92 | 92 | 107 | 0 | 0 | 0 | 0 | 0 | 161 | 0 |
| N.S. | 1 | 1.00 | 1.16 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.75 | 0.00 |
| time (sec) | N/A | 0.632 | 0.130 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.254 | 0.000 |

| Problem 1028 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 26 | 0 | 62 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 1.00 | 0.00 | 2.38 | 0.92 |
| time (sec) | N/A | 1.437 | 5.804 | 0.273 | 0.000 | 0.000 | 62.811 | 0.000 | 0.236 | 0.606 |

| Problem 1029 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 27 | 24 | 64 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 1.04 | 0.92 | 2.46 | 0.92 |
| time (sec) | N/A | 0.593 | 10.172 | 0.316 | 0.000 | 0.000 | 104.152 | 0.574 | 0.227 | 0.610 |

| Problem 1030 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | F(-1) | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 0 | 0 | 64 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 0.00 | 0.00 | 2.46 | 0.92 |
| time (sec) | N/A | 0.622 | 13.503 | 0.339 | 0.000 | 0.000 | 0.000 | 0.000 | 0.235 | 0.607 |

| Problem 1031 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 0 | 24 | 64 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 0.00 | 0.92 | 2.46 | 0.92 |
| time (sec) | N/A | 0.602 | 15.637 | 0.350 | 0.000 | 0.000 | 0.000 | 0.169 | 0.236 | 0.602 |

| Problem 1032 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 62 | 0 | 24 | 733 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 2.38 | 0.00 | 0.92 | 28.19 | 0.92 |
| time (sec) | N/A | 0.286 | 1.571 | 0.475 | 0.000 | 0.117 | 0.000 | 0.178 | 0.353 | 0.615 |

| Problem 1033 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F(-2) | F(-2) | F(-1) | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 160 | 134 | 182 | 0 | 0 | 0 | 0 | 0 | 77 | 0 |
| N.S. | 1 | 0.84 | 1.14 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.48 | 0.00 |
| time (sec) | N/A | 0.782 | 0.390 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.296 | 0.000 |

| Problem 1034 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F(-2) | F(-2) | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 281 | 234 | 241 | 0 | 0 | 0 | 0 | 0 | 77 | 0 |
| N.S. | 1 | 0.83 | 0.86 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.27 | 0.00 |
| time (sec) | N/A | 1.237 | 0.506 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.286 | 0.000 |

| Problem 1035 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F(-2) | F(-2) | F(-1) | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 280 | 232 | 299 | 0 | 0 | 0 | 0 | 0 | 484 | 0 |
| N.S. | 1 | 0.83 | 1.07 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.73 | 0.00 |
| time (sec) | N/A | 1.687 | 0.396 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.298 | 0.000 |

| Problem 1036 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F(-2) | F(-2) | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 157 | 131 | 158 | 0 | 0 | 0 | 0 | 0 | 269 | 0 |
| N.S. | 1 | 0.83 | 1.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.71 | 0.00 |
| time (sec) | N/A | 0.695 | 0.303 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.234 | 0.000 |

| Problem 1037 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | F(-1) | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 0 | 0 | 76 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 0.00 | 0.00 | 2.92 | 0.92 |
| time (sec) | N/A | 1.494 | 5.932 | 0.316 | 0.000 | 0.000 | 0.000 | 0.000 | 0.226 | 0.589 |

| Problem 1038 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 0 | 24 | 78 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 0.00 | 0.92 | 3.00 | 0.92 |
| time (sec) | N/A | 0.686 | 13.526 | 0.385 | 0.000 | 0.000 | 0.000 | 0.177 | 0.249 | 0.607 |

| Problem 1039 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | F(-1) | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 0 | 0 | 78 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 0.00 | 0.00 | 3.00 | 0.92 |
| time (sec) | N/A | 0.679 | 14.929 | 0.387 | 0.000 | 0.000 | 0.000 | 0.000 | 0.261 | 0.597 |

| Problem 1040 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 0 | 24 | 78 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 0.00 | 0.92 | 3.00 | 0.92 |
| time (sec) | N/A | 0.627 | 17.897 | 0.385 | 0.000 | 0.000 | 0.000 | 0.182 | 0.248 | 0.611 |

| Problem 1041 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 20 | 0 | 22 | 0 | 22 | 44 | 22 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 0.00 | 1.00 | 0.00 | 1.00 | 2.00 | 1.00 |
| time (sec) | N/A | 0.222 | 0.866 | 0.333 | 0.000 | 0.102 | 0.000 | 0.142 | 0.206 | 1.106 |

| Problem 1042 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 22 | 18 | 0 | 0 | 29 | 20 | 39 | 20 |
| N.S. | 1 | 1.00 | 1.10 | 0.90 | 0.00 | 0.00 | 1.45 | 1.00 | 1.95 | 1.00 |
| time (sec) | N/A | 0.210 | 1.148 | 0.176 | 0.000 | 0.000 | 5.403 | 0.136 | 0.204 | 0.907 |

| Problem 1043 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 19 | 19 | 21 | 17 | 0 | 0 | 29 | 19 | 38 | 19 |
| N.S. | 1 | 1.00 | 1.11 | 0.89 | 0.00 | 0.00 | 1.53 | 1.00 | 2.00 | 1.00 |
| time (sec) | N/A | 0.188 | 0.818 | 0.143 | 0.000 | 0.000 | 4.780 | 0.142 | 0.202 | 0.995 |

| Problem 1044 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 20 | 0 | 0 | 29 | 22 | 39 | 22 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 0.00 | 0.00 | 1.32 | 1.00 | 1.77 | 1.00 |
| time (sec) | N/A | 0.203 | 2.626 | 0.168 | 0.000 | 0.000 | 6.391 | 0.139 | 0.221 | 0.970 |

| Problem 1045 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 37 | 0 | 24 | 71 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 1.54 | 0.00 | 1.00 | 2.96 | 1.00 |
| time (sec) | N/A | 0.232 | 0.709 | 0.474 | 0.000 | 0.103 | 0.000 | 0.150 | 0.235 | 0.926 |

| Problem 1046 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 20 | 0 | 0 | 49 | 22 | 63 | 22 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 0.00 | 0.00 | 2.23 | 1.00 | 2.86 | 1.00 |
| time (sec) | N/A | 0.213 | 1.013 | 0.276 | 0.000 | 0.000 | 4.850 | 0.143 | 0.203 | 0.917 |

| Problem 1047 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 21 | 21 | 23 | 19 | 0 | 0 | 49 | 21 | 62 | 21 |
| N.S. | 1 | 1.00 | 1.10 | 0.90 | 0.00 | 0.00 | 2.33 | 1.00 | 2.95 | 1.00 |
| time (sec) | N/A | 0.197 | 0.641 | 0.203 | 0.000 | 0.000 | 5.035 | 0.140 | 0.221 | 0.813 |

| Problem 1048 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 49 | 24 | 63 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 2.04 | 1.00 | 2.62 | 1.00 |
| time (sec) | N/A | 0.224 | 1.526 | 0.217 | 0.000 | 0.000 | 7.103 | 0.144 | 0.211 | 0.884 |

| Problem 1049 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 48 | 0 | 24 | 96 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 2.00 | 0.00 | 1.00 | 4.00 | 1.00 |
| time (sec) | N/A | 0.232 | 0.500 | 1.091 | 0.000 | 0.101 | 0.000 | 0.149 | 0.222 | 0.906 |

| Problem 1050 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 20 | 0 | 0 | 68 | 22 | 85 | 22 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 0.00 | 0.00 | 3.09 | 1.00 | 3.86 | 1.00 |
| time (sec) | N/A | 0.208 | 1.063 | 0.373 | 0.000 | 0.000 | 7.918 | 0.137 | 0.209 | 0.810 |

| Problem 1051 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 21 | 21 | 23 | 19 | 0 | 0 | 68 | 21 | 84 | 21 |
| N.S. | 1 | 1.00 | 1.10 | 0.90 | 0.00 | 0.00 | 3.24 | 1.00 | 4.00 | 1.00 |
| time (sec) | N/A | 0.194 | 0.669 | 0.296 | 0.000 | 0.000 | 6.810 | 0.145 | 0.216 | 0.773 |

| Problem 1052 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 68 | 24 | 85 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 2.83 | 1.00 | 3.54 | 1.00 |
| time (sec) | N/A | 0.219 | 1.852 | 0.302 | 0.000 | 0.000 | 8.092 | 0.145 | 0.221 | 0.845 |

| Problem 1053 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 24 | 0 | 24 | 37 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 1.00 | 0.00 | 1.00 | 1.54 | 1.00 |
| time (sec) | N/A | 0.296 | 0.579 | 0.225 | 0.000 | 0.131 | 0.000 | 0.143 | 0.202 | 0.903 |

| Problem 1054 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 22 | 22 | 24 | 20 | 0 | 0 | 27 | 22 | 35 | 22 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 0.00 | 0.00 | 1.23 | 1.00 | 1.59 | 1.00 |
| time (sec) | N/A | 0.253 | 0.733 | 0.092 | 0.000 | 0.000 | 3.494 | 0.132 | 0.197 | 0.682 |

| Problem 1055 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 18 | 18 | 18 | 15 | 0 | 14 | 15 | 14 | 19 | 14 |
| N.S. | 1 | 1.00 | 1.00 | 0.83 | 0.00 | 0.78 | 0.83 | 0.78 | 1.06 | 0.78 |
| time (sec) | N/A | 0.186 | 0.006 | 0.125 | 0.000 | 0.096 | 3.550 | 0.120 | 0.197 | 0.616 |

| Problem 1056 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 29 | 24 | 36 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 1.21 | 1.00 | 1.50 | 1.00 |
| time (sec) | N/A | 0.287 | 0.455 | 0.088 | 0.000 | 0.000 | 4.278 | 0.136 | 0.199 | 0.711 |

| Problem 1057 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 39 | 0 | 24 | 859 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 1.62 | 0.00 | 1.00 | 35.79 | 1.00 |
| time (sec) | N/A | 0.236 | 1.198 | 0.236 | 0.000 | 0.134 | 0.000 | 0.145 | 0.231 | 0.906 |

| Problem 1058 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 48 | 24 | 51 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 2.00 | 1.00 | 2.12 | 1.00 |
| time (sec) | N/A | 1.778 | 6.165 | 0.140 | 0.000 | 0.000 | 10.211 | 0.141 | 0.205 | 0.877 |

| Problem 1059 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F(-2) | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 180 | 173 | 162 | 62 | 0 | 0 | 0 | 0 | 51 | 0 |
| N.S. | 1 | 0.96 | 0.90 | 0.34 | 0.00 | 0.00 | 0.00 | 0.00 | 0.28 | 0.00 |
| time (sec) | N/A | 0.793 | 0.409 | 0.195 | 0.000 | 0.000 | 0.000 | 0.000 | 0.214 | 0.000 |

| Problem 1060 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 101 | 101 | 88 | 59 | 0 | 0 | 0 | 0 | 187 | 0 |
| N.S. | 1 | 1.00 | 0.87 | 0.58 | 0.00 | 0.00 | 0.00 | 0.00 | 1.85 | 0.00 |
| time (sec) | N/A | 0.615 | 0.082 | 0.174 | 0.000 | 0.000 | 0.000 | 0.000 | 0.200 | 0.000 |

| Problem 1061 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F(-2) | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 174 | 168 | 170 | 62 | 0 | 0 | 0 | 0 | 177 | 0 |
| N.S. | 1 | 0.97 | 0.98 | 0.36 | 0.00 | 0.00 | 0.00 | 0.00 | 1.02 | 0.00 |
| time (sec) | N/A | 0.787 | 0.471 | 0.194 | 0.000 | 0.000 | 0.000 | 0.000 | 0.203 | 0.000 |

| Problem 1062 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 48 | 24 | 50 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 2.00 | 1.00 | 2.08 | 1.00 |
| time (sec) | N/A | 1.829 | 2.809 | 0.132 | 0.000 | 0.000 | 10.786 | 0.135 | 0.202 | 0.894 |

| Problem 1063 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 49 | 24 | 52 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 2.04 | 1.00 | 2.17 | 1.00 |
| time (sec) | N/A | 1.569 | 5.101 | 0.179 | 0.000 | 0.000 | 14.821 | 0.138 | 0.203 | 0.901 |

| Problem 1064 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 49 | 24 | 52 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 2.04 | 1.00 | 2.17 | 1.00 |
| time (sec) | N/A | 0.848 | 4.732 | 0.156 | 0.000 | 0.000 | 18.761 | 0.147 | 0.206 | 0.901 |

| Problem 1065 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 49 | 24 | 52 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 2.04 | 1.00 | 2.17 | 1.00 |
| time (sec) | N/A | 0.848 | 10.591 | 0.223 | 0.000 | 0.000 | 24.643 | 0.140 | 0.222 | 0.924 |

| Problem 1066 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 50 | 0 | 24 | 1536 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 2.08 | 0.00 | 1.00 | 64.00 | 1.00 |
| time (sec) | N/A | 0.243 | 1.142 | 0.258 | 0.000 | 0.095 | 0.000 | 0.155 | 0.254 | 0.975 |

| Problem 1067 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F(-2) | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 160 | 297 | 227 | 112 | 0 | 0 | 0 | 0 | 65 | 0 |
| N.S. | 1 | 1.86 | 1.42 | 0.70 | 0.00 | 0.00 | 0.00 | 0.00 | 0.41 | 0.00 |
| time (sec) | N/A | 1.230 | 0.343 | 0.242 | 0.000 | 0.000 | 0.000 | 0.000 | 0.233 | 0.000 |

| Problem 1068 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | B | C | A | F(-2) | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 129 | 359 | 259 | 68 | 0 | 0 | 0 | 0 | 65 | 0 |
| N.S. | 1 | 2.78 | 2.01 | 0.53 | 0.00 | 0.00 | 0.00 | 0.00 | 0.50 | 0.00 |
| time (sec) | N/A | 2.325 | 0.702 | 0.217 | 0.000 | 0.000 | 0.000 | 0.000 | 0.209 | 0.000 |

| Problem 1069 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F(-2) | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 155 | 292 | 220 | 110 | 0 | 0 | 0 | 0 | 701 | 0 |
| N.S. | 1 | 1.88 | 1.42 | 0.71 | 0.00 | 0.00 | 0.00 | 0.00 | 4.52 | 0.00 |
| time (sec) | N/A | 1.200 | 0.303 | 0.210 | 0.000 | 0.000 | 0.000 | 0.000 | 0.226 | 0.000 |

| Problem 1070 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F(-2) | F(-2) | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 125 | 191 | 186 | 113 | 0 | 0 | 0 | 0 | 531 | 0 |
| N.S. | 1 | 1.53 | 1.49 | 0.90 | 0.00 | 0.00 | 0.00 | 0.00 | 4.25 | 0.00 |
| time (sec) | N/A | 1.156 | 0.668 | 0.218 | 0.000 | 0.000 | 0.000 | 0.000 | 0.232 | 0.000 |

| Problem 1071 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 65 | 24 | 64 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 2.71 | 1.00 | 2.67 | 1.00 |
| time (sec) | N/A | 1.390 | 3.982 | 0.141 | 0.000 | 0.000 | 24.765 | 0.140 | 0.213 | 0.904 |

| Problem 1072 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 66 | 24 | 66 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 2.75 | 1.00 | 2.75 | 1.00 |
| time (sec) | N/A | 1.594 | 5.530 | 0.196 | 0.000 | 0.000 | 36.057 | 0.139 | 0.217 | 0.922 |

| Problem 1073 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 66 | 24 | 66 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 2.75 | 1.00 | 2.75 | 1.00 |
| time (sec) | N/A | 0.828 | 5.120 | 0.163 | 0.000 | 0.000 | 45.399 | 0.140 | 0.205 | 0.919 |

| Problem 1074 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 22 | 0 | 0 | 66 | 24 | 66 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.92 | 0.00 | 0.00 | 2.75 | 1.00 | 2.75 | 1.00 |
| time (sec) | N/A | 0.831 | 11.709 | 0.218 | 0.000 | 0.000 | 58.201 | 0.150 | 0.215 | 0.927 |

| Problem 1075 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | F(-1) | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 24 | 0 | 0 | 24 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.92 | 0.00 | 0.00 | 0.92 | 0.92 |
| time (sec) | N/A | 0.262 | 1.001 | 0.452 | 0.000 | 0.108 | 0.000 | 0.000 | 200.025 | 0.626 |

| Problem 1076 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 20 | 0 | 0 | 24 | 22 | 28 | 22 |
| N.S. | 1 | 1.00 | 1.08 | 0.83 | 0.00 | 0.00 | 1.00 | 0.92 | 1.17 | 0.92 |
| time (sec) | N/A | 0.242 | 2.672 | 0.276 | 0.000 | 0.000 | 41.135 | 0.152 | 0.249 | 0.622 |

| Problem 1077 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 19 | 0 | 0 | 22 | 21 | 27 | 21 |
| N.S. | 1 | 1.00 | 1.09 | 0.83 | 0.00 | 0.00 | 0.96 | 0.91 | 1.17 | 0.91 |
| time (sec) | N/A | 0.210 | 1.718 | 0.253 | 0.000 | 0.000 | 39.628 | 0.200 | 0.246 | 0.624 |

| Problem 1078 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 24 | 24 | 30 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 0.92 | 0.92 | 1.15 | 0.92 |
| time (sec) | N/A | 0.264 | 9.181 | 0.282 | 0.000 | 0.000 | 54.911 | 0.148 | 0.264 | 0.611 |

| Problem 1079 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | F(-1) | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 24 | 0 | 0 | 24 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.92 | 0.00 | 0.00 | 0.92 | 0.92 |
| time (sec) | N/A | 0.289 | 1.203 | 0.587 | 0.000 | 0.133 | 0.000 | 0.000 | 200.021 | 0.620 |

| Problem 1080 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 20 | 0 | 0 | 0 | 22 | 61 | 22 |
| N.S. | 1 | 1.00 | 1.08 | 0.83 | 0.00 | 0.00 | 0.00 | 0.92 | 2.54 | 0.92 |
| time (sec) | N/A | 0.252 | 2.965 | 0.349 | 0.000 | 0.000 | 0.000 | 0.174 | 0.260 | 0.620 |

| Problem 1081 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 19 | 0 | 0 | 0 | 21 | 60 | 21 |
| N.S. | 1 | 1.00 | 1.09 | 0.83 | 0.00 | 0.00 | 0.00 | 0.91 | 2.61 | 0.91 |
| time (sec) | N/A | 0.204 | 1.762 | 0.311 | 0.000 | 0.000 | 0.000 | 0.181 | 0.280 | 0.625 |

| Problem 1082 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 0 | 24 | 61 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 0.00 | 0.92 | 2.35 | 0.92 |
| time (sec) | N/A | 0.275 | 9.698 | 0.349 | 0.000 | 0.000 | 0.000 | 0.175 | 0.267 | 0.619 |

| Problem 1083 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | F(-1) | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 49 | 0 | 0 | 24 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 1.88 | 0.00 | 0.00 | 0.92 | 0.92 |
| time (sec) | N/A | 0.275 | 1.437 | 1.348 | 0.000 | 0.118 | 0.000 | 0.000 | 200.020 | 0.624 |

| Problem 1084 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 20 | 0 | 0 | 0 | 22 | 95 | 22 |
| N.S. | 1 | 1.00 | 1.08 | 0.83 | 0.00 | 0.00 | 0.00 | 0.92 | 3.96 | 0.92 |
| time (sec) | N/A | 0.249 | 3.593 | 0.458 | 0.000 | 0.000 | 0.000 | 0.183 | 0.285 | 0.629 |

| Problem 1085 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 19 | 0 | 0 | 0 | 21 | 94 | 21 |
| N.S. | 1 | 1.00 | 1.09 | 0.83 | 0.00 | 0.00 | 0.00 | 0.91 | 4.09 | 0.91 |
| time (sec) | N/A | 0.213 | 1.865 | 0.404 | 0.000 | 0.000 | 0.000 | 0.179 | 0.286 | 0.621 |

| Problem 1086 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 0 | 24 | 95 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 0.00 | 0.92 | 3.65 | 0.92 |
| time (sec) | N/A | 0.299 | 7.316 | 0.429 | 0.000 | 0.000 | 0.000 | 0.183 | 0.278 | 0.613 |

| Problem 1087 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 24 | 0 | 24 | 49 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.92 | 0.00 | 0.92 | 1.88 | 0.92 |
| time (sec) | N/A | 0.271 | 1.165 | 0.473 | 0.000 | 0.094 | 0.000 | 0.150 | 0.220 | 0.625 |

| Problem 1088 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 20 | 0 | 0 | 24 | 22 | 47 | 22 |
| N.S. | 1 | 1.00 | 1.08 | 0.83 | 0.00 | 0.00 | 1.00 | 0.92 | 1.96 | 0.92 |
| time (sec) | N/A | 0.239 | 1.763 | 0.276 | 0.000 | 0.000 | 52.215 | 0.155 | 0.201 | 0.616 |

| Problem 1089 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 19 | 0 | 0 | 24 | 21 | 85 | 21 |
| N.S. | 1 | 1.00 | 1.09 | 0.83 | 0.00 | 0.00 | 1.04 | 0.91 | 3.70 | 0.91 |
| time (sec) | N/A | 0.208 | 0.685 | 0.248 | 0.000 | 0.000 | 51.676 | 0.160 | 0.253 | 0.607 |

| Problem 1090 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 26 | 24 | 48 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 1.00 | 0.92 | 1.85 | 0.92 |
| time (sec) | N/A | 0.420 | 4.062 | 0.302 | 0.000 | 0.000 | 109.343 | 0.147 | 0.223 | 0.607 |

| Problem 1091 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 0 | 24 | 50 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 0.00 | 0.92 | 1.92 | 0.92 |
| time (sec) | N/A | 0.276 | 9.974 | 0.321 | 0.000 | 0.000 | 0.000 | 0.154 | 0.237 | 0.625 |

| Problem 1092 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 51 | 0 | 24 | 771 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 1.96 | 0.00 | 0.92 | 29.65 | 0.92 |
| time (sec) | N/A | 0.291 | 1.425 | 0.406 | 0.000 | 0.114 | 0.000 | 0.180 | 3.789 | 0.636 |

| Problem 1093 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | F(-1) | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 0 | 0 | 63 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 0.00 | 0.00 | 2.42 | 0.92 |
| time (sec) | N/A | 3.231 | 7.280 | 0.311 | 0.000 | 0.000 | 0.000 | 0.000 | 0.224 | 0.618 |

| Problem 1094 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 0 | 24 | 63 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 0.00 | 0.92 | 2.42 | 0.92 |
| time (sec) | N/A | 2.057 | 6.272 | 0.283 | 0.000 | 0.000 | 0.000 | 0.174 | 0.306 | 0.625 |

| Problem 1095 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F(-2) | F(-2) | F(-1) | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 129 | 133 | 124 | 0 | 0 | 0 | 0 | 0 | 191 | 0 |
| N.S. | 1 | 1.03 | 0.96 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.48 | 0.00 |
| time (sec) | N/A | 0.800 | 0.158 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.299 | 0.000 |

| Problem 1096 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F(-2) | F(-2) | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 126 | 131 | 120 | 0 | 0 | 0 | 0 | 0 | 187 | 0 |
| N.S. | 1 | 1.04 | 0.95 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.48 | 0.00 |
| time (sec) | N/A | 0.786 | 0.149 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.304 | 0.000 |

| Problem 1097 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | F(-1) | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 0 | 0 | 62 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 0.00 | 0.00 | 2.38 | 0.92 |
| time (sec) | N/A | 2.961 | 5.646 | 0.287 | 0.000 | 0.000 | 0.000 | 0.000 | 0.285 | 0.631 |

| Problem 1098 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 0 | 24 | 64 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 0.00 | 0.92 | 2.46 | 0.92 |
| time (sec) | N/A | 3.274 | 9.029 | 0.327 | 0.000 | 0.000 | 0.000 | 0.167 | 0.298 | 0.624 |

| Problem 1099 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | F(-1) | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 0 | 0 | 64 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 0.00 | 0.00 | 2.46 | 0.92 |
| time (sec) | N/A | 1.207 | 11.753 | 0.348 | 0.000 | 0.000 | 0.000 | 0.000 | 0.309 | 0.643 |

| Problem 1100 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 0 | 24 | 64 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 0.00 | 0.92 | 2.46 | 0.92 |
| time (sec) | N/A | 1.223 | 30.034 | 0.357 | 0.000 | 0.000 | 0.000 | 0.165 | 0.313 | 0.637 |

| Problem 1101 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 62 | 0 | 24 | 1778 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 2.38 | 0.00 | 0.92 | 68.38 | 0.92 |
| time (sec) | N/A | 0.288 | 1.603 | 0.476 | 0.000 | 0.113 | 0.000 | 0.191 | 0.671 | 0.647 |

| Problem 1102 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F(-2) | F(-2) | F(-1) | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 190 | 275 | 255 | 0 | 0 | 0 | 0 | 0 | 77 | 0 |
| N.S. | 1 | 1.45 | 1.34 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.41 | 0.00 |
| time (sec) | N/A | 1.535 | 0.726 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.367 | 0.000 |

| Problem 1103 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F(-2) | F(-2) | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 224 | 414 | 311 | 0 | 0 | 0 | 0 | 0 | 77 | 0 |
| N.S. | 1 | 1.85 | 1.39 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.34 | 0.00 |
| time (sec) | N/A | 3.410 | 0.616 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.378 | 0.000 |

| Problem 1104 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F(-2) | F(-2) | F(-1) | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 222 | 411 | 261 | 0 | 0 | 0 | 0 | 0 | 550 | 0 |
| N.S. | 1 | 1.85 | 1.18 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 2.48 | 0.00 |
| time (sec) | N/A | 2.126 | 0.714 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.353 | 0.000 |

| Problem 1105 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F(-2) | F(-2) | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 183 | 268 | 300 | 0 | 0 | 0 | 0 | 0 | 572 | 0 |
| N.S. | 1 | 1.46 | 1.64 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 3.13 | 0.00 |
| time (sec) | N/A | 1.856 | 0.400 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.327 | 0.000 |

| Problem 1106 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | F(-1) | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 0 | 0 | 76 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 0.00 | 0.00 | 2.92 | 0.92 |
| time (sec) | N/A | 2.356 | 8.745 | 0.334 | 0.000 | 0.000 | 0.000 | 0.000 | 0.255 | 0.623 |

| Problem 1107 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 0 | 24 | 78 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 0.00 | 0.92 | 3.00 | 0.92 |
| time (sec) | N/A | 3.069 | 8.676 | 0.387 | 0.000 | 0.000 | 0.000 | 0.177 | 0.240 | 0.635 |

| Problem 1108 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | F(-1) | F(-2) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 0 | 0 | 78 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 0.00 | 0.00 | 3.00 | 0.92 |
| time (sec) | N/A | 1.197 | 14.131 | 0.392 | 0.000 | 0.000 | 0.000 | 0.000 | 0.261 | 0.650 |

| Problem 1109 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | F(-2) | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 26 | 26 | 28 | 22 | 0 | 0 | 0 | 24 | 78 | 24 |
| N.S. | 1 | 1.00 | 1.08 | 0.85 | 0.00 | 0.00 | 0.00 | 0.92 | 3.00 | 0.92 |
| time (sec) | N/A | 1.197 | 36.706 | 0.402 | 0.000 | 0.000 | 0.000 | 0.182 | 0.253 | 0.634 |

| Problem 1110 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 22 | 20 | 0 | 22 | 19 | 22 | 25 | 22 |
| N.S. | 1 | 1.00 | 1.10 | 1.00 | 0.00 | 1.10 | 0.95 | 1.10 | 1.25 | 1.10 |
| time (sec) | N/A | 0.287 | 0.758 | 0.112 | 0.000 | 0.120 | 0.561 | 0.133 | 0.198 | 0.705 |

| Problem 1111 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | A | F | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 20 | 21 | 0 | 21 | 0 | 20 | 22 | 20 |
| N.S. | 1 | 1.00 | 1.00 | 1.05 | 0.00 | 1.05 | 0.00 | 1.00 | 1.10 | 1.00 |
| time (sec) | N/A | 0.202 | 0.005 | 0.313 | 0.000 | 0.107 | 0.000 | 0.114 | 0.201 | 0.669 |

| Problem 1112 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 28 | 28 | 30 | 28 | 30 | 30 | 0 | 30 | 32 | 30 |
| N.S. | 1 | 1.00 | 1.07 | 1.00 | 1.07 | 1.07 | 0.00 | 1.07 | 1.14 | 1.07 |
| time (sec) | N/A | 0.264 | 0.579 | 1.407 | 0.517 | 0.101 | 0.000 | 0.349 | 0.225 | 0.952 |

| Problem 1113 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 107 | 87 | 127 | 108 | 108 | 110 | 138 | 138 | 117 | 105 |
| N.S. | 1 | 0.81 | 1.19 | 1.01 | 1.01 | 1.03 | 1.29 | 1.29 | 1.09 | 0.98 |
| time (sec) | N/A | 0.326 | 0.009 | 0.355 | 0.133 | 0.092 | 0.383 | 0.156 | 0.210 | 0.954 |

| Problem 1114 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 94 | 93 | 119 | 104 | 105 | 107 | 128 | 115 | 115 | 101 |
| N.S. | 1 | 0.99 | 1.27 | 1.11 | 1.12 | 1.14 | 1.36 | 1.22 | 1.22 | 1.07 |
| time (sec) | N/A | 0.330 | 0.018 | 0.299 | 0.039 | 0.093 | 0.339 | 0.126 | 0.199 | 0.910 |

| Problem 1115 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 82 | 83 | 103 | 87 | 88 | 89 | 114 | 97 | 97 | 85 |
| N.S. | 1 | 1.01 | 1.26 | 1.06 | 1.07 | 1.09 | 1.39 | 1.18 | 1.18 | 1.04 |
| time (sec) | N/A | 0.282 | 0.006 | 0.416 | 0.119 | 0.089 | 0.287 | 0.223 | 0.185 | 0.329 |

| Problem 1116 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 68 | 67 | 85 | 76 | 80 | 82 | 94 | 90 | 90 | 75 |
| N.S. | 1 | 0.99 | 1.25 | 1.12 | 1.18 | 1.21 | 1.38 | 1.32 | 1.32 | 1.10 |
| time (sec) | N/A | 0.270 | 0.007 | 0.155 | 0.052 | 0.088 | 0.255 | 0.128 | 0.201 | 0.839 |

| Problem 1117 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | F | F | F | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 77 | 77 | 83 | 120 | 104 | 0 | 0 | 0 | 67 | 88 |
| N.S. | 1 | 1.00 | 1.08 | 1.56 | 1.35 | 0.00 | 0.00 | 0.00 | 0.87 | 1.14 |
| time (sec) | N/A | 0.293 | 0.001 | 0.230 | 0.320 | 0.000 | 0.000 | 0.000 | 0.207 | 1.026 |

| Problem 1118 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 57 | 61 | 73 | 74 | 73 | 74 | 80 | 86 | 86 | 69 |
| N.S. | 1 | 1.07 | 1.28 | 1.30 | 1.28 | 1.30 | 1.40 | 1.51 | 1.51 | 1.21 |
| time (sec) | N/A | 0.292 | 0.005 | 0.160 | 0.041 | 0.106 | 0.300 | 0.127 | 0.209 | 0.249 |

| Problem 1119 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | F | F | F | F | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 77 | 77 | 86 | 129 | 0 | 0 | 0 | 0 | 64 | 91 |
| N.S. | 1 | 1.00 | 1.12 | 1.68 | 0.00 | 0.00 | 0.00 | 0.00 | 0.83 | 1.18 |
| time (sec) | N/A | 0.305 | 0.006 | 0.263 | 0.000 | 0.000 | 0.000 | 0.000 | 0.206 | 1.075 |

| Problem 1120 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 83 | 79 | 98 | 97 | 93 | 85 | 116 | 101 | 101 | 92 |
| N.S. | 1 | 0.95 | 1.18 | 1.17 | 1.12 | 1.02 | 1.40 | 1.22 | 1.22 | 1.11 |
| time (sec) | N/A | 0.314 | 0.024 | 0.165 | 0.035 | 0.134 | 0.345 | 0.118 | 0.195 | 0.904 |

| Problem 1121 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | A | A | A | C | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 79 | 72 | 97 | 96 | 80 | 75 | 99 | 128 | 88 | 162 |
| N.S. | 1 | 0.91 | 1.23 | 1.22 | 1.01 | 0.95 | 1.25 | 1.62 | 1.11 | 2.05 |
| time (sec) | N/A | 0.277 | 0.007 | 0.202 | 0.128 | 0.082 | 0.265 | 0.182 | 0.190 | 0.931 |

| Problem 1122 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 110 | 105 | 131 | 119 | 116 | 111 | 153 | 124 | 124 | 111 |
| N.S. | 1 | 0.95 | 1.19 | 1.08 | 1.05 | 1.01 | 1.39 | 1.13 | 1.13 | 1.01 |
| time (sec) | N/A | 0.340 | 0.031 | 0.173 | 0.041 | 0.109 | 0.476 | 0.118 | 0.192 | 0.272 |

| Problem 1123 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | A | A | A | C | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 105 | 86 | 97 | 109 | 103 | 98 | 122 | 148 | 108 | 130 |
| N.S. | 1 | 0.82 | 0.92 | 1.04 | 0.98 | 0.93 | 1.16 | 1.41 | 1.03 | 1.24 |
| time (sec) | N/A | 0.286 | 0.007 | 0.213 | 0.144 | 0.091 | 0.342 | 0.308 | 0.199 | 0.921 |

| Problem 1124 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 185 | 182 | 173 | 197 | 184 | 201 | 260 | 220 | 220 | 374 |
| N.S. | 1 | 0.98 | 0.94 | 1.06 | 0.99 | 1.09 | 1.41 | 1.19 | 1.19 | 2.02 |
| time (sec) | N/A | 0.490 | 0.065 | 0.381 | 0.115 | 0.096 | 0.511 | 0.221 | 0.190 | 0.867 |

| Problem 1125 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 161 | 157 | 162 | 181 | 181 | 186 | 245 | 211 | 211 | 191 |
| N.S. | 1 | 0.98 | 1.01 | 1.12 | 1.12 | 1.16 | 1.52 | 1.31 | 1.31 | 1.19 |
| time (sec) | N/A | 0.492 | 0.077 | 0.348 | 0.036 | 0.116 | 0.452 | 0.126 | 0.186 | 1.157 |

| Problem 1126 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 115 | 115 | 140 | 186 | 156 | 166 | 219 | 223 | 185 | 167 |
| N.S. | 1 | 1.00 | 1.22 | 1.62 | 1.36 | 1.44 | 1.90 | 1.94 | 1.61 | 1.45 |
| time (sec) | N/A | 0.329 | 0.054 | 0.453 | 0.123 | 0.102 | 0.400 | 0.238 | 0.191 | 0.513 |

| Problem 1127 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 124 | 122 | 130 | 137 | 147 | 150 | 194 | 172 | 172 | 150 |
| N.S. | 1 | 0.98 | 1.05 | 1.10 | 1.19 | 1.21 | 1.56 | 1.39 | 1.39 | 1.21 |
| time (sec) | N/A | 0.418 | 0.047 | 0.190 | 0.036 | 0.105 | 0.355 | 0.233 | 0.219 | 1.016 |

| Problem 1128 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | F | F | F | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 137 | 137 | 123 | 175 | 172 | 0 | 0 | 0 | 141 | 157 |
| N.S. | 1 | 1.00 | 0.90 | 1.28 | 1.26 | 0.00 | 0.00 | 0.00 | 1.03 | 1.15 |
| time (sec) | N/A | 0.415 | 0.068 | 0.308 | 0.330 | 0.000 | 0.000 | 0.000 | 0.206 | 1.081 |

| Problem 1129 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 109 | 111 | 114 | 134 | 130 | 140 | 165 | 164 | 164 | 135 |
| N.S. | 1 | 1.02 | 1.05 | 1.23 | 1.19 | 1.28 | 1.51 | 1.50 | 1.50 | 1.24 |
| time (sec) | N/A | 0.441 | 0.065 | 0.223 | 0.050 | 0.107 | 0.429 | 0.123 | 0.198 | 1.033 |

| Problem 1130 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | A | F | F | F | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 128 | 128 | 118 | 178 | 153 | 0 | 0 | 0 | 139 | 157 |
| N.S. | 1 | 1.00 | 0.92 | 1.39 | 1.20 | 0.00 | 0.00 | 0.00 | 1.09 | 1.23 |
| time (sec) | N/A | 0.398 | 0.077 | 0.379 | 0.294 | 0.000 | 0.000 | 0.000 | 0.197 | 1.019 |

| Problem 1131 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 115 | 112 | 119 | 139 | 135 | 139 | 180 | 171 | 171 | 142 |
| N.S. | 1 | 0.97 | 1.03 | 1.21 | 1.17 | 1.21 | 1.57 | 1.49 | 1.49 | 1.23 |
| time (sec) | N/A | 0.426 | 0.074 | 0.238 | 0.046 | 0.103 | 0.415 | 0.128 | 0.222 | 0.434 |

| Problem 1132 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F | F | F | F | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 139 | 139 | 130 | 194 | 0 | 0 | 0 | 0 | 132 | 177 |
| N.S. | 1 | 1.00 | 0.94 | 1.40 | 0.00 | 0.00 | 0.00 | 0.00 | 0.95 | 1.27 |
| time (sec) | N/A | 0.384 | 0.071 | 0.388 | 0.000 | 0.000 | 0.000 | 0.000 | 0.237 | 1.133 |

| Problem 1133 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 150 | 143 | 149 | 167 | 166 | 160 | 235 | 195 | 195 | 179 |
| N.S. | 1 | 0.95 | 0.99 | 1.11 | 1.11 | 1.07 | 1.57 | 1.30 | 1.30 | 1.19 |
| time (sec) | N/A | 0.458 | 0.087 | 0.215 | 0.045 | 0.103 | 0.522 | 0.123 | 0.198 | 0.533 |

| Problem 1134 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | A | A | A | C | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 111 | 113 | 112 | 167 | 145 | 145 | 192 | 238 | 173 | 256 |
| N.S. | 1 | 1.02 | 1.01 | 1.50 | 1.31 | 1.31 | 1.73 | 2.14 | 1.56 | 2.31 |
| time (sec) | N/A | 0.364 | 0.065 | 0.266 | 0.123 | 0.104 | 0.389 | 0.866 | 0.194 | 1.178 |

| Problem 1135 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 186 | 178 | 177 | 202 | 197 | 194 | 289 | 232 | 232 | 232 |
| N.S. | 1 | 0.96 | 0.95 | 1.09 | 1.06 | 1.04 | 1.55 | 1.25 | 1.25 | 1.25 |
| time (sec) | N/A | 0.488 | 0.136 | 0.223 | 0.035 | 0.113 | 0.667 | 0.124 | 0.190 | 0.713 |

| Problem 1136 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 240 | 303 | 244 | 301 | 268 | 304 | 411 | 393 | 338 | 599 |
| N.S. | 1 | 1.26 | 1.02 | 1.25 | 1.12 | 1.27 | 1.71 | 1.64 | 1.41 | 2.50 |
| time (sec) | N/A | 0.661 | 0.133 | 0.399 | 0.135 | 0.120 | 0.723 | 0.455 | 0.192 | 0.947 |

| Problem 1137 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 239 | 236 | 252 | 272 | 265 | 277 | 389 | 322 | 322 | 296 |
| N.S. | 1 | 0.99 | 1.05 | 1.14 | 1.11 | 1.16 | 1.63 | 1.35 | 1.35 | 1.24 |
| time (sec) | N/A | 0.682 | 0.138 | 0.347 | 0.040 | 0.156 | 0.631 | 0.124 | 0.206 | 1.306 |

| Problem 1138 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 158 | 157 | 217 | 264 | 232 | 258 | 350 | 288 | 288 | 442 |
| N.S. | 1 | 0.99 | 1.37 | 1.67 | 1.47 | 1.63 | 2.22 | 1.82 | 1.82 | 2.80 |
| time (sec) | N/A | 0.382 | 0.076 | 0.566 | 0.134 | 0.146 | 0.529 | 0.370 | 0.198 | 0.921 |

| Problem 1139 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 188 | 185 | 192 | 210 | 222 | 229 | 306 | 268 | 268 | 238 |
| N.S. | 1 | 0.98 | 1.02 | 1.12 | 1.18 | 1.22 | 1.63 | 1.43 | 1.43 | 1.27 |
| time (sec) | N/A | 0.575 | 0.065 | 0.216 | 0.043 | 0.117 | 0.477 | 0.207 | 0.201 | 0.479 |

| Problem 1140 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | F | F | F | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 228 | 228 | 190 | 257 | 251 | 0 | 0 | 0 | 229 | 232 |
| N.S. | 1 | 1.00 | 0.83 | 1.13 | 1.10 | 0.00 | 0.00 | 0.00 | 1.00 | 1.02 |
| time (sec) | N/A | 0.469 | 0.100 | 0.338 | 0.365 | 0.000 | 0.000 | 0.000 | 0.199 | 1.158 |

| Problem 1141 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 160 | 161 | 169 | 193 | 197 | 206 | 258 | 247 | 247 | 236 |
| N.S. | 1 | 1.01 | 1.06 | 1.21 | 1.23 | 1.29 | 1.61 | 1.54 | 1.54 | 1.48 |
| time (sec) | N/A | 0.592 | 0.087 | 0.243 | 0.046 | 0.134 | 0.577 | 0.133 | 0.189 | 1.003 |

| Problem 1142 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | A | F | F | F | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 200 | 200 | 170 | 240 | 223 | 0 | 0 | 0 | 218 | 224 |
| N.S. | 1 | 1.00 | 0.85 | 1.20 | 1.12 | 0.00 | 0.00 | 0.00 | 1.09 | 1.12 |
| time (sec) | N/A | 0.465 | 0.108 | 0.525 | 0.300 | 0.000 | 0.000 | 0.000 | 0.201 | 1.090 |

| Problem 1143 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 158 | 152 | 166 | 195 | 193 | 205 | 272 | 255 | 255 | 203 |
| N.S. | 1 | 0.96 | 1.05 | 1.23 | 1.22 | 1.30 | 1.72 | 1.61 | 1.61 | 1.28 |
| time (sec) | N/A | 0.599 | 0.104 | 0.254 | 0.045 | 0.140 | 0.592 | 0.124 | 0.209 | 0.979 |

| Problem 1144 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | A | F | F | F | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 200 | 200 | 169 | 241 | 218 | 0 | 0 | 0 | 218 | 234 |
| N.S. | 1 | 1.00 | 0.84 | 1.20 | 1.09 | 0.00 | 0.00 | 0.00 | 1.09 | 1.17 |
| time (sec) | N/A | 0.461 | 0.150 | 0.537 | 0.324 | 0.000 | 0.000 | 0.000 | 0.215 | 1.068 |

| Problem 1145 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 177 | 171 | 184 | 208 | 208 | 214 | 289 | 268 | 269 | 194 |
| N.S. | 1 | 0.97 | 1.04 | 1.18 | 1.18 | 1.21 | 1.63 | 1.51 | 1.52 | 1.10 |
| time (sec) | N/A | 0.664 | 0.107 | 0.257 | 0.049 | 0.124 | 0.585 | 0.131 | 0.213 | 0.964 |

| Problem 1146 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F | F | F | F | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 228 | 228 | 175 | 258 | 0 | 0 | 0 | 0 | 217 | 261 |
| N.S. | 1 | 1.00 | 0.77 | 1.13 | 0.00 | 0.00 | 0.00 | 0.00 | 0.95 | 1.14 |
| time (sec) | N/A | 0.515 | 0.097 | 0.523 | 0.000 | 0.000 | 0.000 | 0.000 | 0.204 | 1.187 |

| Problem 1147 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 224 | 216 | 230 | 247 | 247 | 243 | 362 | 303 | 303 | 236 |
| N.S. | 1 | 0.96 | 1.03 | 1.10 | 1.10 | 1.08 | 1.62 | 1.35 | 1.35 | 1.05 |
| time (sec) | N/A | 0.690 | 0.109 | 0.243 | 0.057 | 0.127 | 0.694 | 0.127 | 0.202 | 1.015 |

| Problem 1148 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | A | A | B | C | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 152 | 150 | 154 | 249 | 218 | 228 | 309 | 363 | 273 | 301 |
| N.S. | 1 | 0.99 | 1.01 | 1.64 | 1.43 | 1.50 | 2.03 | 2.39 | 1.80 | 1.98 |
| time (sec) | N/A | 0.441 | 0.116 | 0.357 | 0.165 | 0.113 | 0.518 | 5.627 | 0.187 | 0.959 |

| Problem 1149 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 244 | 244 | 212 | 245 | 226 | 237 | 314 | 233 | 296 | 233 |
| N.S. | 1 | 1.00 | 0.87 | 1.00 | 0.93 | 0.97 | 1.29 | 0.95 | 1.21 | 0.95 |
| time (sec) | N/A | 0.674 | 0.068 | 0.326 | 0.043 | 0.095 | 0.581 | 0.124 | 0.207 | 0.240 |

| Problem 1150 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 361 | 359 | 475 | 503 | 0 | 0 | 0 | 0 | 49 | 0 |
| N.S. | 1 | 0.99 | 1.32 | 1.39 | 0.00 | 0.00 | 0.00 | 0.00 | 0.14 | 0.00 |
| time (sec) | N/A | 0.836 | 0.246 | 0.556 | 0.000 | 0.000 | 0.000 | 0.000 | 0.218 | 0.000 |

| Problem 1151 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 311 | 311 | 441 | 394 | 0 | 0 | 0 | 0 | 37 | 0 |
| N.S. | 1 | 1.00 | 1.42 | 1.27 | 0.00 | 0.00 | 0.00 | 0.00 | 0.12 | 0.00 |
| time (sec) | N/A | 0.551 | 0.082 | 0.420 | 0.000 | 0.000 | 0.000 | 0.000 | 0.215 | 0.000 |

| Problem 1152 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 353 | 353 | 484 | 435 | 0 | 0 | 0 | 0 | 44 | 0 |
| N.S. | 1 | 1.00 | 1.37 | 1.23 | 0.00 | 0.00 | 0.00 | 0.00 | 0.12 | 0.00 |
| time (sec) | N/A | 0.668 | 0.131 | 0.474 | 0.000 | 0.000 | 0.000 | 0.000 | 0.216 | 0.000 |

| Problem 1153 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 409 | 399 | 514 | 571 | 0 | 0 | 0 | 0 | 65 | 0 |
| N.S. | 1 | 0.98 | 1.26 | 1.40 | 0.00 | 0.00 | 0.00 | 0.00 | 0.16 | 0.00 |
| time (sec) | N/A | 0.937 | 0.232 | 0.593 | 0.000 | 0.000 | 0.000 | 0.000 | 0.226 | 0.000 |

| Problem 1154 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | F | F | F | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 555 | 550 | 766 | 526 | 0 | 0 | 0 | 0 | 151 | 0 |
| N.S. | 1 | 0.99 | 1.38 | 0.95 | 0.00 | 0.00 | 0.00 | 0.00 | 0.27 | 0.00 |
| time (sec) | N/A | 1.076 | 2.340 | 1.189 | 0.000 | 0.000 | 0.000 | 0.000 | 0.205 | 0.000 |

| Problem 1155 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 517 | 510 | 461 | 400 | 0 | 0 | 0 | 0 | 116 | 0 |
| N.S. | 1 | 0.99 | 0.89 | 0.77 | 0.00 | 0.00 | 0.00 | 0.00 | 0.22 | 0.00 |
| time (sec) | N/A | 0.778 | 0.163 | 0.821 | 0.000 | 0.000 | 0.000 | 0.000 | 0.221 | 0.000 |

| Problem 1156 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 561 | 558 | 467 | 537 | 0 | 0 | 0 | 0 | 157 | 0 |
| N.S. | 1 | 0.99 | 0.83 | 0.96 | 0.00 | 0.00 | 0.00 | 0.00 | 0.28 | 0.00 |
| time (sec) | N/A | 1.090 | 0.502 | 1.253 | 0.000 | 0.000 | 0.000 | 0.000 | 0.199 | 0.000 |

| Problem 1157 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 403 | 403 | 522 | 716 | 0 | 0 | 0 | 0 | 123 | 0 |
| N.S. | 1 | 1.00 | 1.30 | 1.78 | 0.00 | 0.00 | 0.00 | 0.00 | 0.31 | 0.00 |
| time (sec) | N/A | 0.786 | 6.752 | 0.588 | 0.000 | 0.000 | 0.000 | 0.000 | 0.205 | 0.000 |

| Problem 1158 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | A | F(-1) | C | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 91 | 88 | 98 | 99 | 0 | 234 | 0 | 192 | 126 | 696 |
| N.S. | 1 | 0.97 | 1.08 | 1.09 | 0.00 | 2.57 | 0.00 | 2.11 | 1.38 | 7.65 |
| time (sec) | N/A | 0.300 | 0.124 | 0.564 | 0.000 | 0.126 | 0.000 | 0.213 | 0.197 | 1.216 |

| Problem 1159 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 443 | 443 | 590 | 808 | 0 | 0 | 0 | 0 | 138 | 0 |
| N.S. | 1 | 1.00 | 1.33 | 1.82 | 0.00 | 0.00 | 0.00 | 0.00 | 0.31 | 0.00 |
| time (sec) | N/A | 0.864 | 4.656 | 0.598 | 0.000 | 0.000 | 0.000 | 0.000 | 0.228 | 0.000 |

| Problem 1160 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 489 | 489 | 643 | 851 | 0 | 0 | 0 | 0 | 168 | 0 |
| N.S. | 1 | 1.00 | 1.31 | 1.74 | 0.00 | 0.00 | 0.00 | 0.00 | 0.34 | 0.00 |
| time (sec) | N/A | 0.871 | 9.019 | 0.804 | 0.000 | 0.000 | 0.000 | 0.000 | 0.248 | 0.000 |

| Problem 1161 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F(-2) | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 1335 | 1335 | 877 | 2305 | 0 | 0 | 0 | 0 | 1308 | 0 |
| N.S. | 1 | 1.00 | 0.66 | 1.73 | 0.00 | 0.00 | 0.00 | 0.00 | 0.98 | 0.00 |
| time (sec) | N/A | 2.271 | 8.938 | 3.131 | 0.000 | 0.000 | 0.000 | 0.000 | 0.244 | 0.000 |

| Problem 1162 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F(-2) | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 819 | 806 | 861 | 2173 | 0 | 0 | 0 | 0 | 1297 | 0 |
| N.S. | 1 | 0.98 | 1.05 | 2.65 | 0.00 | 0.00 | 0.00 | 0.00 | 1.58 | 0.00 |
| time (sec) | N/A | 1.245 | 8.989 | 2.146 | 0.000 | 0.000 | 0.000 | 0.000 | 0.237 | 0.000 |

| Problem 1163 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F(-2) | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 1382 | 1382 | 992 | 2568 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.72 | 1.86 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 2.061 | 12.675 | 1.860 | 0.000 | 0.000 | 0.000 | 0.000 | 0.260 | 0.000 |

| Problem 1164 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 532 | 532 | 589 | 817 | 0 | 0 | 0 | 0 | 238 | 0 |
| N.S. | 1 | 1.00 | 1.11 | 1.54 | 0.00 | 0.00 | 0.00 | 0.00 | 0.45 | 0.00 |
| time (sec) | N/A | 1.021 | 8.930 | 0.772 | 0.000 | 0.000 | 0.000 | 0.000 | 0.220 | 0.000 |

| Problem 1165 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | B | F(-1) | C | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 130 | 151 | 158 | 211 | 0 | 697 | 0 | 762 | 408 | 273 |
| N.S. | 1 | 1.16 | 1.22 | 1.62 | 0.00 | 5.36 | 0.00 | 5.86 | 3.14 | 2.10 |
| time (sec) | N/A | 0.373 | 3.120 | 0.875 | 0.000 | 0.199 | 0.000 | 2.270 | 0.219 | 3.918 |

| Problem 1166 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F(-2) | B | F(-1) | C | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 131 | 150 | 131 | 157 | 0 | 637 | 0 | 533 | 397 | 201 |
| N.S. | 1 | 1.15 | 1.00 | 1.20 | 0.00 | 4.86 | 0.00 | 4.07 | 3.03 | 1.53 |
| time (sec) | N/A | 0.345 | 0.866 | 0.765 | 0.000 | 0.176 | 0.000 | 1.094 | 0.211 | 3.175 |

| Problem 1167 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 574 | 574 | 645 | 903 | 0 | 0 | 0 | 0 | 265 | 0 |
| N.S. | 1 | 1.00 | 1.12 | 1.57 | 0.00 | 0.00 | 0.00 | 0.00 | 0.46 | 0.00 |
| time (sec) | N/A | 0.976 | 8.489 | 0.761 | 0.000 | 0.000 | 0.000 | 0.000 | 0.237 | 0.000 |

| Problem 1168 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 629 | 629 | 723 | 951 | 0 | 0 | 0 | 0 | 299 | 0 |
| N.S. | 1 | 1.00 | 1.15 | 1.51 | 0.00 | 0.00 | 0.00 | 0.00 | 0.48 | 0.00 |
| time (sec) | N/A | 1.051 | 11.305 | 1.140 | 0.000 | 0.000 | 0.000 | 0.000 | 0.252 | 0.000 |

| Problem 1169 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F(-2) | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 966 | 966 | 1744 | 3774 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.81 | 3.91 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 2.206 | 12.056 | 3.333 | 0.000 | 0.000 | 0.000 | 0.000 | 0.306 | 0.000 |

| Problem 1170 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F(-2) | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 893 | 875 | 1745 | 4007 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.98 | 1.95 | 4.49 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.386 | 10.393 | 3.426 | 0.000 | 0.000 | 0.000 | 0.000 | 0.292 | 0.000 |

| Problem 1171 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F(-2) | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 1518 | 1518 | 2005 | 5730 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.32 | 3.77 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 2.844 | 12.886 | 3.141 | 0.000 | 0.000 | 0.000 | 0.000 | 0.322 | 0.000 |

| Problem 1172 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F(-2) | A | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 223 | 233 | 391 | 0 | 0 | 1200 | 0 | 0 | 76 | 0 |
| N.S. | 1 | 1.04 | 1.75 | 0.00 | 0.00 | 5.38 | 0.00 | 0.00 | 0.34 | 0.00 |
| time (sec) | N/A | 0.596 | 0.354 | 0.000 | 0.000 | 2.286 | 0.000 | 0.000 | 0.564 | 0.000 |

| Problem 1173 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 21 | 0 | 27 | 22 | 23 | 87 | 23 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 0.00 | 1.17 | 0.96 | 1.00 | 3.78 | 1.00 |
| time (sec) | N/A | 0.489 | 12.319 | 0.308 | 0.000 | 0.094 | 21.228 | 0.139 | 0.309 | 0.959 |

| Problem 1174 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F(-2) | A | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 140 | 148 | 279 | 0 | 0 | 879 | 0 | 0 | 51 | 0 |
| N.S. | 1 | 1.06 | 1.99 | 0.00 | 0.00 | 6.28 | 0.00 | 0.00 | 0.36 | 0.00 |
| time (sec) | N/A | 0.394 | 0.358 | 0.000 | 0.000 | 0.529 | 0.000 | 0.000 | 0.387 | 0.000 |

| Problem 1175 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 22 | 18 | 0 | 20 | 19 | 20 | 61 | 20 |
| N.S. | 1 | 1.00 | 1.10 | 0.90 | 0.00 | 1.00 | 0.95 | 1.00 | 3.05 | 1.00 |
| time (sec) | N/A | 0.208 | 5.119 | 0.293 | 0.000 | 0.093 | 6.443 | 0.129 | 0.271 | 0.880 |

| Problem 1176 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 21 | 0 | 23 | 20 | 23 | 84 | 23 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 0.00 | 1.00 | 0.87 | 1.00 | 3.65 | 1.00 |
| time (sec) | N/A | 0.396 | 9.176 | 0.310 | 0.000 | 0.114 | 5.293 | 0.122 | 0.318 | 1.102 |

| Problem 1177 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 21 | 0 | 23 | 22 | 23 | 67 | 23 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 0.00 | 1.00 | 0.96 | 1.00 | 2.91 | 1.00 |
| time (sec) | N/A | 0.403 | 9.076 | 0.298 | 0.000 | 0.117 | 2.893 | 0.123 | 0.319 | 1.330 |

| Problem 1178 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 21 | 0 | 23 | 22 | 23 | 107 | 23 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 0.00 | 1.00 | 0.96 | 1.00 | 4.65 | 1.00 |
| time (sec) | N/A | 0.405 | 10.539 | 0.311 | 0.000 | 0.091 | 3.848 | 0.120 | 0.527 | 1.222 |

| Problem 1179 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F(-2) | A | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 137 | 142 | 288 | 0 | 0 | 864 | 0 | 0 | 70 | 0 |
| N.S. | 1 | 1.04 | 2.10 | 0.00 | 0.00 | 6.31 | 0.00 | 0.00 | 0.51 | 0.00 |
| time (sec) | N/A | 0.415 | 0.424 | 0.000 | 0.000 | 0.254 | 0.000 | 0.000 | 0.430 | 0.000 |

| Problem 1180 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 21 | 0 | 23 | 22 | 23 | 131 | 23 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 0.00 | 1.00 | 0.96 | 1.00 | 5.70 | 1.00 |
| time (sec) | N/A | 0.427 | 12.277 | 0.344 | 0.000 | 0.095 | 9.133 | 0.122 | 1.135 | 1.337 |

| Problem 1181 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F(-2) | A | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 224 | 226 | 413 | 0 | 0 | 1162 | 0 | 0 | 94 | 0 |
| N.S. | 1 | 1.01 | 1.84 | 0.00 | 0.00 | 5.19 | 0.00 | 0.00 | 0.42 | 0.00 |
| time (sec) | N/A | 0.535 | 0.384 | 0.000 | 0.000 | 0.470 | 0.000 | 0.000 | 0.828 | 0.000 |

| Problem 1182 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F(-2) | A | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 279 | 294 | 418 | 0 | 0 | 1566 | 0 | 0 | 121 | 0 |
| N.S. | 1 | 1.05 | 1.50 | 0.00 | 0.00 | 5.61 | 0.00 | 0.00 | 0.43 | 0.00 |
| time (sec) | N/A | 0.673 | 0.474 | 0.000 | 0.000 | 8.723 | 0.000 | 0.000 | 2.441 | 0.000 |

| Problem 1183 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 21 | 0 | 43 | 22 | 23 | 133 | 23 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 0.00 | 1.87 | 0.96 | 1.00 | 5.78 | 1.00 |
| time (sec) | N/A | 0.505 | 12.185 | 0.356 | 0.000 | 0.093 | 59.800 | 0.161 | 0.424 | 1.020 |

| Problem 1184 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F(-2) | A | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 181 | 197 | 313 | 0 | 0 | 1192 | 0 | 0 | 96 | 0 |
| N.S. | 1 | 1.09 | 1.73 | 0.00 | 0.00 | 6.59 | 0.00 | 0.00 | 0.53 | 0.00 |
| time (sec) | N/A | 0.451 | 0.301 | 0.000 | 0.000 | 2.082 | 0.000 | 0.000 | 0.656 | 0.000 |

| Problem 1185 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 22 | 18 | 0 | 37 | 19 | 20 | 108 | 20 |
| N.S. | 1 | 1.00 | 1.10 | 0.90 | 0.00 | 1.85 | 0.95 | 1.00 | 5.40 | 1.00 |
| time (sec) | N/A | 0.203 | 5.491 | 0.370 | 0.000 | 0.093 | 27.816 | 0.152 | 0.365 | 0.950 |

| Problem 1186 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 21 | 0 | 40 | 20 | 23 | 123 | 23 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 0.00 | 1.74 | 0.87 | 1.00 | 5.35 | 1.00 |
| time (sec) | N/A | 0.446 | 9.449 | 0.397 | 0.000 | 0.112 | 21.172 | 0.156 | 0.382 | 1.218 |

| Problem 1187 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 21 | 0 | 40 | 22 | 23 | 109 | 23 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 0.00 | 1.74 | 0.96 | 1.00 | 4.74 | 1.00 |
| time (sec) | N/A | 0.456 | 10.203 | 0.369 | 0.000 | 0.097 | 14.817 | 0.138 | 0.386 | 1.610 |

| Problem 1188 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 21 | 0 | 40 | 22 | 23 | 145 | 23 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 0.00 | 1.74 | 0.96 | 1.00 | 6.30 | 1.00 |
| time (sec) | N/A | 0.470 | 10.932 | 0.394 | 0.000 | 0.109 | 11.363 | 0.164 | 0.643 | 1.435 |

| Problem 1189 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 21 | 0 | 40 | 22 | 23 | 111 | 23 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 0.00 | 1.74 | 0.96 | 1.00 | 4.83 | 1.00 |
| time (sec) | N/A | 0.461 | 32.685 | 0.414 | 0.000 | 0.105 | 11.434 | 0.135 | 0.536 | 1.353 |

| Problem 1190 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 21 | 0 | 40 | 22 | 23 | 158 | 23 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 0.00 | 1.74 | 0.96 | 1.00 | 6.87 | 1.00 |
| time (sec) | N/A | 0.466 | 11.876 | 0.403 | 0.000 | 0.093 | 19.943 | 0.139 | 1.399 | 1.515 |

| Problem 1191 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F(-2) | A | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 178 | 186 | 334 | 0 | 0 | 1151 | 0 | 0 | 120 | 0 |
| N.S. | 1 | 1.04 | 1.88 | 0.00 | 0.00 | 6.47 | 0.00 | 0.00 | 0.67 | 0.00 |
| time (sec) | N/A | 0.471 | 0.377 | 0.000 | 0.000 | 0.560 | 0.000 | 0.000 | 1.122 | 0.000 |

| Problem 1192 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F(-2) | A | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 345 | 365 | 470 | 0 | 0 | 1978 | 0 | 0 | 168 | 0 |
| N.S. | 1 | 1.06 | 1.36 | 0.00 | 0.00 | 5.73 | 0.00 | 0.00 | 0.49 | 0.00 |
| time (sec) | N/A | 0.785 | 0.589 | 0.000 | 0.000 | 29.102 | 0.000 | 0.000 | 112.040 | 0.000 |

| Problem 1193 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 21 | 0 | 67 | 0 | 23 | 180 | 23 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 0.00 | 2.91 | 0.00 | 1.00 | 7.83 | 1.00 |
| time (sec) | N/A | 0.520 | 12.666 | 0.720 | 0.000 | 0.128 | 0.000 | 0.163 | 1.175 | 1.038 |

| Problem 1194 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F(-2) | A | F | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 233 | 257 | 353 | 0 | 0 | 1562 | 0 | 0 | 143 | 0 |
| N.S. | 1 | 1.10 | 1.52 | 0.00 | 0.00 | 6.70 | 0.00 | 0.00 | 0.61 | 0.00 |
| time (sec) | N/A | 0.547 | 0.369 | 0.000 | 0.000 | 8.564 | 0.000 | 0.000 | 2.454 | 0.000 |

| Problem 1195 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 22 | 18 | 0 | 61 | 19 | 20 | 155 | 20 |
| N.S. | 1 | 1.00 | 1.10 | 0.90 | 0.00 | 3.05 | 0.95 | 1.00 | 7.75 | 1.00 |
| time (sec) | N/A | 0.203 | 5.772 | 0.351 | 0.000 | 0.099 | 91.838 | 0.166 | 0.530 | 0.987 |

| Problem 1196 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 21 | 0 | 64 | 20 | 23 | 174 | 23 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 0.00 | 2.78 | 0.87 | 1.00 | 7.57 | 1.00 |
| time (sec) | N/A | 0.471 | 9.428 | 0.409 | 0.000 | 0.118 | 48.884 | 0.154 | 0.584 | 1.240 |

| Problem 1197 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 21 | 0 | 64 | 22 | 23 | 161 | 23 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 0.00 | 2.78 | 0.96 | 1.00 | 7.00 | 1.00 |
| time (sec) | N/A | 0.468 | 10.327 | 0.400 | 0.000 | 0.113 | 69.303 | 0.160 | 0.469 | 1.851 |

| Problem 1198 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 21 | 0 | 64 | 22 | 23 | 195 | 23 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 0.00 | 2.78 | 0.96 | 1.00 | 8.48 | 1.00 |
| time (sec) | N/A | 0.463 | 11.350 | 0.430 | 0.000 | 0.118 | 47.819 | 0.169 | 0.729 | 1.540 |

| Problem 1199 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 21 | 0 | 64 | 22 | 23 | 169 | 23 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 0.00 | 2.78 | 0.96 | 1.00 | 7.35 | 1.00 |
| time (sec) | N/A | 0.479 | 10.379 | 0.767 | 0.000 | 0.118 | 47.283 | 0.152 | 0.652 | 1.609 |

| Problem 1200 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F(-2) | A | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 176 | 181 | 377 | 0 | 0 | 882 | 0 | 0 | 58 | 0 |
| N.S. | 1 | 1.03 | 2.14 | 0.00 | 0.00 | 5.01 | 0.00 | 0.00 | 0.33 | 0.00 |
| time (sec) | N/A | 0.458 | 0.376 | 0.000 | 0.000 | 0.562 | 0.000 | 0.000 | 0.428 | 0.000 |

| Problem 1201 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 21 | 0 | 27 | 22 | 23 | 69 | 23 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 0.00 | 1.17 | 0.96 | 1.00 | 3.00 | 1.00 |
| time (sec) | N/A | 0.409 | 11.148 | 0.341 | 0.000 | 0.105 | 10.642 | 0.129 | 0.270 | 1.207 |

| Problem 1202 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F(-2) | A | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 103 | 104 | 251 | 0 | 0 | 647 | 0 | 0 | 36 | 0 |
| N.S. | 1 | 1.01 | 2.44 | 0.00 | 0.00 | 6.28 | 0.00 | 0.00 | 0.35 | 0.00 |
| time (sec) | N/A | 0.319 | 0.292 | 0.000 | 0.000 | 0.177 | 0.000 | 0.000 | 0.357 | 0.000 |

| Problem 1203 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 20 | 20 | 22 | 18 | 0 | 20 | 19 | 20 | 48 | 20 |
| N.S. | 1 | 1.00 | 1.10 | 0.90 | 0.00 | 1.00 | 0.95 | 1.00 | 2.40 | 1.00 |
| time (sec) | N/A | 0.201 | 2.463 | 0.340 | 0.000 | 0.098 | 2.996 | 0.133 | 0.288 | 0.979 |

| Problem 1204 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 21 | 0 | 31 | 20 | 23 | 81 | 23 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 0.00 | 1.35 | 0.87 | 1.00 | 3.52 | 1.00 |
| time (sec) | N/A | 0.390 | 3.904 | 0.356 | 0.000 | 0.114 | 3.671 | 0.129 | 0.308 | 1.320 |

| Problem 1205 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F(-2) | A | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 100 | 108 | 247 | 0 | 0 | 666 | 0 | 0 | 49 | 0 |
| N.S. | 1 | 1.08 | 2.47 | 0.00 | 0.00 | 6.66 | 0.00 | 0.00 | 0.49 | 0.00 |
| time (sec) | N/A | 0.361 | 0.292 | 0.000 | 0.000 | 0.167 | 0.000 | 0.000 | 0.365 | 0.000 |

| Problem 1206 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 21 | 0 | 33 | 22 | 23 | 111 | 23 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 0.00 | 1.43 | 0.96 | 1.00 | 4.83 | 1.00 |
| time (sec) | N/A | 0.423 | 10.853 | 0.310 | 0.000 | 0.112 | 5.727 | 0.129 | 0.484 | 1.362 |

| Problem 1207 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F(-2) | A | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 179 | 181 | 372 | 0 | 0 | 874 | 0 | 0 | 74 | 0 |
| N.S. | 1 | 1.01 | 2.08 | 0.00 | 0.00 | 4.88 | 0.00 | 0.00 | 0.41 | 0.00 |
| time (sec) | N/A | 0.476 | 0.345 | 0.000 | 0.000 | 0.249 | 0.000 | 0.000 | 0.425 | 0.000 |

| Problem 1208 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F(-2) | B | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 137 | 135 | 321 | 0 | 0 | 1291 | 0 | 0 | 126 | 0 |
| N.S. | 1 | 0.99 | 2.34 | 0.00 | 0.00 | 9.42 | 0.00 | 0.00 | 0.92 | 0.00 |
| time (sec) | N/A | 0.406 | 0.457 | 0.000 | 0.000 | 0.435 | 0.000 | 0.000 | 0.488 | 0.000 |

| Problem 1209 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 21 | 0 | 47 | 22 | 23 | 179 | 23 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 0.00 | 2.04 | 0.96 | 1.00 | 7.78 | 1.00 |
| time (sec) | N/A | 0.433 | 20.074 | 0.371 | 0.000 | 0.099 | 50.008 | 0.133 | 0.289 | 1.043 |

| Problem 1210 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F(-2) | B | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 71 | 71 | 210 | 0 | 0 | 379 | 0 | 0 | 105 | 0 |
| N.S. | 1 | 1.00 | 2.96 | 0.00 | 0.00 | 5.34 | 0.00 | 0.00 | 1.48 | 0.00 |
| time (sec) | N/A | 0.282 | 0.276 | 0.000 | 0.000 | 0.182 | 0.000 | 0.000 | 0.475 | 0.000 |

| Problem 1211 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F(-2) | B | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 70 | 70 | 202 | 0 | 0 | 388 | 0 | 0 | 123 | 0 |
| N.S. | 1 | 1.00 | 2.89 | 0.00 | 0.00 | 5.54 | 0.00 | 0.00 | 1.76 | 0.00 |
| time (sec) | N/A | 0.284 | 0.179 | 0.000 | 0.000 | 0.156 | 0.000 | 0.000 | 0.321 | 0.000 |

| Problem 1212 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 21 | 0 | 42 | 20 | 23 | 223 | 23 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 0.00 | 1.83 | 0.87 | 1.00 | 9.70 | 1.00 |
| time (sec) | N/A | 0.437 | 8.964 | 0.394 | 0.000 | 0.090 | 37.679 | 0.125 | 0.457 | 1.421 |

| Problem 1213 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F(-2) | B | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 135 | 143 | 306 | 0 | 0 | 1323 | 0 | 0 | 147 | 0 |
| N.S. | 1 | 1.06 | 2.27 | 0.00 | 0.00 | 9.80 | 0.00 | 0.00 | 1.09 | 0.00 |
| time (sec) | N/A | 0.423 | 0.421 | 0.000 | 0.000 | 0.282 | 0.000 | 0.000 | 0.529 | 0.000 |

| Problem 1214 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 21 | 0 | 44 | 22 | 23 | 269 | 23 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 0.00 | 1.91 | 0.96 | 1.00 | 11.70 | 1.00 |
| time (sec) | N/A | 0.466 | 12.742 | 0.378 | 0.000 | 0.123 | 67.792 | 0.128 | 0.725 | 1.537 |

| Problem 1215 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F(-2) | B | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 216 | 245 | 405 | 0 | 0 | 1926 | 0 | 0 | 23 | 0 |
| N.S. | 1 | 1.13 | 1.88 | 0.00 | 0.00 | 8.92 | 0.00 | 0.00 | 0.11 | 0.00 |
| time (sec) | N/A | 1.152 | 0.483 | 0.000 | 0.000 | 0.452 | 0.000 | 0.000 | 200.027 | 0.000 |

| Problem 1216 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 21 | 0 | 58 | 0 | 23 | 336 | 23 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 0.00 | 2.52 | 0.00 | 1.00 | 14.61 | 1.00 |
| time (sec) | N/A | 0.478 | 16.434 | 0.426 | 0.000 | 0.103 | 0.000 | 0.138 | 0.325 | 1.228 |

| Problem 1217 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F(-2) | B | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 143 | 138 | 326 | 0 | 0 | 863 | 0 | 0 | 246 | 0 |
| N.S. | 1 | 0.97 | 2.28 | 0.00 | 0.00 | 6.03 | 0.00 | 0.00 | 1.72 | 0.00 |
| time (sec) | N/A | 0.403 | 0.418 | 0.000 | 0.000 | 0.663 | 0.000 | 0.000 | 0.630 | 0.000 |

| Problem 1218 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F | B | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 109 | 113 | 252 | 0 | 0 | 676 | 0 | 0 | 268 | 0 |
| N.S. | 1 | 1.04 | 2.31 | 0.00 | 0.00 | 6.20 | 0.00 | 0.00 | 2.46 | 0.00 |
| time (sec) | N/A | 0.368 | 0.677 | 0.000 | 0.000 | 0.294 | 0.000 | 0.000 | 0.402 | 0.000 |

| Problem 1219 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F(-2) | B | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 110 | 110 | 259 | 0 | 0 | 679 | 0 | 0 | 222 | 0 |
| N.S. | 1 | 1.00 | 2.35 | 0.00 | 0.00 | 6.17 | 0.00 | 0.00 | 2.02 | 0.00 |
| time (sec) | N/A | 0.308 | 0.492 | 0.000 | 0.000 | 0.404 | 0.000 | 0.000 | 0.619 | 0.000 |

| Problem 1220 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F | B | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 144 | 145 | 317 | 0 | 0 | 864 | 0 | 0 | 276 | 0 |
| N.S. | 1 | 1.01 | 2.20 | 0.00 | 0.00 | 6.00 | 0.00 | 0.00 | 1.92 | 0.00 |
| time (sec) | N/A | 0.357 | 0.362 | 0.000 | 0.000 | 0.525 | 0.000 | 0.000 | 0.438 | 0.000 |

| Problem 1221 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 21 | 0 | 53 | 20 | 23 | 433 | 23 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 0.00 | 2.30 | 0.87 | 1.00 | 18.83 | 1.00 |
| time (sec) | N/A | 0.455 | 13.176 | 0.428 | 0.000 | 0.119 | 75.661 | 0.134 | 0.829 | 1.472 |

| Problem 1222 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F(-2) | B | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 216 | 263 | 418 | 0 | 0 | 2720 | 0 | 0 | 300 | 0 |
| N.S. | 1 | 1.22 | 1.94 | 0.00 | 0.00 | 12.59 | 0.00 | 0.00 | 1.39 | 0.00 |
| time (sec) | N/A | 1.177 | 0.713 | 0.000 | 0.000 | 1.024 | 0.000 | 0.000 | 0.853 | 0.000 |

| Problem 1223 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | F(-2) | N/A | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 21 | 0 | 55 | 0 | 23 | 23 | 23 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 0.00 | 2.39 | 0.00 | 1.00 | 1.00 | 1.00 |
| time (sec) | N/A | 0.497 | 15.855 | 0.414 | 0.000 | 0.105 | 0.000 | 0.130 | 200.021 | 1.525 |

| Problem 1224 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F(-2) | B | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 294 | 398 | 510 | 0 | 0 | 3466 | 0 | 0 | 23 | 0 |
| N.S. | 1 | 1.35 | 1.73 | 0.00 | 0.00 | 11.79 | 0.00 | 0.00 | 0.08 | 0.00 |
| time (sec) | N/A | 1.416 | 1.179 | 0.000 | 0.000 | 1.719 | 0.000 | 0.000 | 200.031 | 0.000 |

| Problem 1225 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F(-2) | B | F | A | F | F(-1) |
| verified | N/A | No | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 208 | 201 | 345 | 0 | 0 | 1280 | 0 | 204 | 71 | 0 |
| N.S. | 1 | 0.97 | 1.66 | 0.00 | 0.00 | 6.15 | 0.00 | 0.98 | 0.34 | 0.00 |
| time (sec) | N/A | 1.068 | 0.506 | 0.000 | 0.000 | 0.202 | 0.000 | 0.146 | 43.882 | 0.000 |

| Problem 1226 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | F | F(-2) | B | F | A | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 293 | 291 | 450 | 0 | 0 | 1986 | 0 | 336 | 16 | 0 |
| N.S. | 1 | 0.99 | 1.54 | 0.00 | 0.00 | 6.78 | 0.00 | 1.15 | 0.05 | 0.00 |
| time (sec) | N/A | 1.277 | 0.792 | 0.000 | 0.000 | 0.507 | 0.000 | 0.163 | 200.023 | 0.000 |

| Problem 1227 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 351 | 373 | 264 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.06 | 0.75 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.879 | 0.321 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.243 | 0.000 |

| Problem 1228 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 202 | 225 | 193 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.11 | 0.96 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.579 | 0.127 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.181 | 0.000 |

| Problem 1229 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 122 | 120 | 119 | 0 | 0 | 0 | 0 | 0 | 863 | 0 |
| N.S. | 1 | 0.98 | 0.98 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 7.07 | 0.00 |
| time (sec) | N/A | 0.340 | 0.108 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.177 | 0.000 |

| Problem 1230 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 21 | 21 | 23 | 21 | 23 | 23 | 0 | 23 | 39 | 23 |
| N.S. | 1 | 1.00 | 1.10 | 1.00 | 1.10 | 1.10 | 0.00 | 1.10 | 1.86 | 1.10 |
| time (sec) | N/A | 0.373 | 2.007 | 1.020 | 0.362 | 0.114 | 0.000 | 0.112 | 0.197 | 0.951 |

| Problem 1231 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 21 | 21 | 23 | 21 | 23 | 34 | 0 | 23 | 48567 | 23 |
| N.S. | 1 | 1.00 | 1.10 | 1.00 | 1.10 | 1.62 | 0.00 | 1.10 | 2312.71 | 1.10 |
| time (sec) | N/A | 0.370 | 3.549 | 3.018 | 0.423 | 0.107 | 0.000 | 0.127 | 0.555 | 0.920 |

| Problem 1232 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 21 | 23 | 40 | 0 | 23 | 23 | 23 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 1.00 | 1.74 | 0.00 | 1.00 | 1.00 | 1.00 |
| time (sec) | N/A | 0.464 | 2.762 | 0.602 | 0.403 | 0.123 | 0.000 | 0.181 | 200.021 | 1.013 |

| Problem 1233 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 21 | 23 | 23 | 22 | 23 | 37 | 23 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 1.00 | 1.00 | 0.96 | 1.00 | 1.61 | 1.00 |
| time (sec) | N/A | 0.410 | 0.086 | 0.513 | 0.227 | 0.107 | 41.601 | 0.148 | 0.515 | 1.022 |

| Problem 1234 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 21 | 23 | 23 | 22 | 23 | 23 | 23 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 1.00 | 1.00 | 0.96 | 1.00 | 1.00 | 1.00 |
| time (sec) | N/A | 0.424 | 2.674 | 0.504 | 0.220 | 0.097 | 25.035 | 0.132 | 200.020 | 1.108 |

| Problem 1235 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 21 | 23 | 43 | 0 | 23 | 23 | 23 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 1.00 | 1.87 | 0.00 | 1.00 | 1.00 | 1.00 |
| time (sec) | N/A | 0.431 | 3.640 | 0.591 | 0.223 | 0.101 | 0.000 | 0.134 | 200.020 | 1.218 |

| Problem 1236 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 23 | 23 | 25 | 21 | 23 | 54 | 0 | 23 | 23 | 23 |
| N.S. | 1 | 1.00 | 1.09 | 0.91 | 1.00 | 2.35 | 0.00 | 1.00 | 1.00 | 1.00 |
| time (sec) | N/A | 0.430 | 5.163 | 0.630 | 0.248 | 0.131 | 0.000 | 0.141 | 200.031 | 1.210 |

| Problem 1237 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 21 | 21 | 23 | 21 | 23 | 23 | 0 | 23 | 833 | 23 |
| N.S. | 1 | 1.00 | 1.10 | 1.00 | 1.10 | 1.10 | 0.00 | 1.10 | 39.67 | 1.10 |
| time (sec) | N/A | 0.379 | 2.267 | 1.661 | 0.625 | 0.114 | 0.000 | 0.162 | 0.281 | 1.470 |

| Problem 1238 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 25 | 27 | 25 | 27 | 27 | 0 | 27 | 3643 | 29 |
| N.S. | 1 | 1.00 | 1.08 | 1.00 | 1.08 | 1.08 | 0.00 | 1.08 | 145.72 | 1.16 |
| time (sec) | N/A | 0.425 | 2.283 | 1.005 | 0.455 | 0.107 | 0.000 | 0.197 | 0.488 | 1.135 |

| Problem 1239 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 129 | 125 | 166 | 0 | 0 | 0 | 0 | 0 | 1344 | 0 |
| N.S. | 1 | 0.97 | 1.29 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 10.42 | 0.00 |
| time (sec) | N/A | 0.377 | 0.423 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.230 | 0.000 |

| Problem 1240 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 25 | 27 | 25 | 27 | 27 | 0 | 27 | 21533 | 29 |
| N.S. | 1 | 1.00 | 1.08 | 1.00 | 1.08 | 1.08 | 0.00 | 1.08 | 861.32 | 1.16 |
| time (sec) | N/A | 0.426 | 2.595 | 1.029 | 0.500 | 0.119 | 0.000 | 0.195 | 0.314 | 1.092 |

| Problem 1241 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | F | F | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 285 | 270 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.95 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.629 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.421 | 0.000 |

| Problem 1242 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 25 | 27 | 25 | 27 | 27 | 0 | 27 | 37894 | 29 |
| N.S. | 1 | 1.00 | 1.08 | 1.00 | 1.08 | 1.08 | 0.00 | 1.08 | 1515.76 | 1.16 |
| time (sec) | N/A | 0.456 | 2.768 | 1.028 | 0.495 | 0.124 | 0.000 | 0.186 | 0.678 | 1.101 |

| Problem 1243 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | F | F | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | No | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 466 | 432 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.93 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.709 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.930 | 0.000 |

| Problem 1244 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 25 | 25 | 27 | 25 | 27 | 27 | 0 | 27 | 77868 | 29 |
| N.S. | 1 | 1.00 | 1.08 | 1.00 | 1.08 | 1.08 | 0.00 | 1.08 | 3114.72 | 1.16 |
| time (sec) | N/A | 0.435 | 2.244 | 1.761 | 0.455 | 0.109 | 0.000 | 0.190 | 0.461 | 1.114 |

| Problem 1245 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 271 | 271 | 240 | 302 | 306 | 289 | 398 | 361 | 338 | 338 |
| N.S. | 1 | 1.00 | 0.89 | 1.11 | 1.13 | 1.07 | 1.47 | 1.33 | 1.25 | 1.25 |
| time (sec) | N/A | 0.876 | 0.132 | 0.971 | 0.149 | 0.113 | 0.591 | 0.288 | 0.196 | 1.972 |

| Problem 1246 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 323 | 323 | 287 | 406 | 0 | 0 | 0 | 0 | 324 | 0 |
| N.S. | 1 | 1.00 | 0.89 | 1.26 | 0.00 | 0.00 | 0.00 | 0.00 | 1.00 | 0.00 |
| time (sec) | N/A | 0.827 | 0.622 | 1.699 | 0.000 | 0.000 | 0.000 | 0.000 | 0.205 | 0.000 |

| Problem 1247 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 199 | 199 | 179 | 248 | 247 | 218 | 296 | 259 | 259 | 248 |
| N.S. | 1 | 1.00 | 0.90 | 1.25 | 1.24 | 1.10 | 1.49 | 1.30 | 1.30 | 1.25 |
| time (sec) | N/A | 0.603 | 0.096 | 1.091 | 0.136 | 0.109 | 0.491 | 0.296 | 0.220 | 0.745 |

| Problem 1248 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 231 | 231 | 208 | 321 | 0 | 0 | 0 | 0 | 220 | 0 |
| N.S. | 1 | 1.00 | 0.90 | 1.39 | 0.00 | 0.00 | 0.00 | 0.00 | 0.95 | 0.00 |
| time (sec) | N/A | 0.581 | 0.278 | 1.218 | 0.000 | 0.000 | 0.000 | 0.000 | 0.204 | 0.000 |

| Problem 1249 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 217 | 217 | 263 | 1262 | 0 | 0 | 0 | 0 | 153 | 0 |
| N.S. | 1 | 1.00 | 1.21 | 5.82 | 0.00 | 0.00 | 0.00 | 0.00 | 0.71 | 0.00 |
| time (sec) | N/A | 0.653 | 0.228 | 14.893 | 0.000 | 0.000 | 0.000 | 0.000 | 0.180 | 0.000 |

| Problem 1250 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 172 | 172 | 204 | 356 | 0 | 0 | 0 | 0 | 149 | 0 |
| N.S. | 1 | 1.00 | 1.19 | 2.07 | 0.00 | 0.00 | 0.00 | 0.00 | 0.87 | 0.00 |
| time (sec) | N/A | 0.540 | 0.178 | 1.542 | 0.000 | 0.000 | 0.000 | 0.000 | 0.193 | 0.000 |

| Problem 1251 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 220 | 220 | 273 | 1289 | 0 | 0 | 0 | 0 | 171 | 0 |
| N.S. | 1 | 1.00 | 1.24 | 5.86 | 0.00 | 0.00 | 0.00 | 0.00 | 0.78 | 0.00 |
| time (sec) | N/A | 0.708 | 0.211 | 25.942 | 0.000 | 0.000 | 0.000 | 0.000 | 0.198 | 0.000 |

| Problem 1252 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 502 | 502 | 414 | 552 | 516 | 530 | 758 | 639 | 639 | 929 |
| N.S. | 1 | 1.00 | 0.82 | 1.10 | 1.03 | 1.06 | 1.51 | 1.27 | 1.27 | 1.85 |
| time (sec) | N/A | 1.452 | 0.200 | 1.175 | 0.156 | 0.121 | 0.881 | 1.031 | 0.182 | 7.523 |

| Problem 1253 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 580 | 580 | 513 | 634 | 0 | 0 | 0 | 0 | 614 | 0 |
| N.S. | 1 | 1.00 | 0.88 | 1.09 | 0.00 | 0.00 | 0.00 | 0.00 | 1.06 | 0.00 |
| time (sec) | N/A | 1.385 | 1.092 | 2.232 | 0.000 | 0.000 | 0.000 | 0.000 | 0.191 | 0.000 |

| Problem 1254 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 380 | 380 | 317 | 492 | 433 | 416 | 575 | 548 | 507 | 780 |
| N.S. | 1 | 1.00 | 0.83 | 1.29 | 1.14 | 1.09 | 1.51 | 1.44 | 1.33 | 2.05 |
| time (sec) | N/A | 1.055 | 0.156 | 1.290 | 0.166 | 0.117 | 0.654 | 0.775 | 0.200 | 6.002 |

| Problem 1255 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 442 | 442 | 391 | 493 | 0 | 0 | 0 | 0 | 446 | 0 |
| N.S. | 1 | 1.00 | 0.88 | 1.12 | 0.00 | 0.00 | 0.00 | 0.00 | 1.01 | 0.00 |
| time (sec) | N/A | 1.028 | 0.653 | 1.730 | 0.000 | 0.000 | 0.000 | 0.000 | 0.203 | 0.000 |

| Problem 1256 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 355 | 355 | 389 | 1524 | 0 | 0 | 0 | 0 | 345 | 0 |
| N.S. | 1 | 1.00 | 1.10 | 4.29 | 0.00 | 0.00 | 0.00 | 0.00 | 0.97 | 0.00 |
| time (sec) | N/A | 0.966 | 0.439 | 29.384 | 0.000 | 0.000 | 0.000 | 0.000 | 0.185 | 0.000 |

| Problem 1257 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 343 | 343 | 349 | 524 | 0 | 0 | 0 | 0 | 365 | 0 |
| N.S. | 1 | 1.00 | 1.02 | 1.53 | 0.00 | 0.00 | 0.00 | 0.00 | 1.06 | 0.00 |
| time (sec) | N/A | 0.840 | 0.507 | 1.672 | 0.000 | 0.000 | 0.000 | 0.000 | 0.191 | 0.000 |

| Problem 1258 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 320 | 320 | 367 | 1500 | 0 | 0 | 0 | 0 | 343 | 0 |
| N.S. | 1 | 1.00 | 1.15 | 4.69 | 0.00 | 0.00 | 0.00 | 0.00 | 1.07 | 0.00 |
| time (sec) | N/A | 0.879 | 0.375 | 29.321 | 0.000 | 0.000 | 0.000 | 0.000 | 0.209 | 0.000 |

| Problem 1259 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | No | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 590 | 579 | 1520 | 0 | 0 | 0 | 0 | 0 | 83 | 0 |
| N.S. | 1 | 0.98 | 2.58 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.14 | 0.00 |
| time (sec) | N/A | 1.254 | 11.493 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.192 | 0.000 |

| Problem 1260 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | F(-1) | C | F(-2) | F | F | F | F | F(-1) |
| verified | N/A | Yes | N/A | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 554 | 568 | 0 | 92148 | 0 | 0 | 0 | 0 | 187 | 0 |
| N.S. | 1 | 1.03 | 0.00 | 166.33 | 0.00 | 0.00 | 0.00 | 0.00 | 0.34 | 0.00 |
| time (sec) | N/A | 1.678 | 0.000 | 158.363 | 0.000 | 0.000 | 0.000 | 0.000 | 0.182 | 0.000 |

| Problem 1261 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | No | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 492 | 492 | 1322 | 0 | 0 | 0 | 0 | 0 | 65 | 0 |
| N.S. | 1 | 1.00 | 2.69 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.13 | 0.00 |
| time (sec) | N/A | 0.596 | 11.295 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.192 | 0.000 |

| Problem 1262 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | F(-1) | B | F(-2) | F | F | F | F | F(-1) |
| verified | N/A | Yes | N/A | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 460 | 460 | 0 | 2344 | 0 | 0 | 0 | 0 | 220 | 0 |
| N.S. | 1 | 1.00 | 0.00 | 5.10 | 0.00 | 0.00 | 0.00 | 0.00 | 0.48 | 0.00 |
| time (sec) | N/A | 0.549 | 0.000 | 39.492 | 0.000 | 0.000 | 0.000 | 0.000 | 0.223 | 0.000 |

| Problem 1263 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | No | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 637 | 637 | 1264 | 0 | 0 | 0 | 0 | 0 | 75 | 0 |
| N.S. | 1 | 1.00 | 1.98 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.12 | 0.00 |
| time (sec) | N/A | 1.039 | 9.924 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.363 | 0.000 |

| Problem 1264 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | F(-1) | C | F(-2) | F | F | F | F | F(-1) |
| verified | N/A | Yes | N/A | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 553 | 564 | 0 | 101616 | 0 | 0 | 0 | 0 | 196 | 0 |
| N.S. | 1 | 1.02 | 0.00 | 183.75 | 0.00 | 0.00 | 0.00 | 0.00 | 0.35 | 0.00 |
| time (sec) | N/A | 1.521 | 0.000 | 74.882 | 0.000 | 0.000 | 0.000 | 0.000 | 0.210 | 0.000 |

| Problem 1265 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | No | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 745 | 724 | 1409 | 0 | 0 | 0 | 0 | 0 | 105 | 0 |
| N.S. | 1 | 0.97 | 1.89 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.14 | 0.00 |
| time (sec) | N/A | 1.810 | 11.971 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.249 | 0.000 |

| Problem 1266 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | F | F | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 943 | 943 | 0 | 0 | 0 | 0 | 0 | 0 | 215 | 0 |
| N.S. | 1 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.23 | 0.00 |
| time (sec) | N/A | 1.915 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.199 | 0.000 |

| Problem 1267 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | F | C | F(-2) | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | N/A | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 1033 | 1033 | 0 | 6565 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.00 | 6.36 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 2.171 | 0.000 | 29.833 | 0.000 | 0.000 | 0.000 | 0.000 | 0.264 | 0.000 |

| Problem 1268 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | B | A | B | F(-2) | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | No | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 457 | 996 | 836 | 1175 | 0 | 0 | 0 | 0 | 1409 | 0 |
| N.S. | 1 | 2.18 | 1.83 | 2.57 | 0.00 | 0.00 | 0.00 | 0.00 | 3.08 | 0.00 |
| time (sec) | N/A | 1.558 | 6.417 | 3.706 | 0.000 | 0.000 | 0.000 | 0.000 | 0.233 | 0.000 |

| Problem 1269 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | F | C | F(-2) | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | N/A | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 1039 | 1039 | 0 | 6565 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.00 | 6.32 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.761 | 0.000 | 27.813 | 0.000 | 0.000 | 0.000 | 0.000 | 0.274 | 0.000 |

| Problem 1270 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | F | F | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 1087 | 1087 | 0 | 0 | 0 | 0 | 0 | 0 | 232 | 0 |
| N.S. | 1 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.21 | 0.00 |
| time (sec) | N/A | 2.213 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.267 | 0.000 |

| Problem 1271 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | F(-1) | F | F(-2) | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 1141 | 1141 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 2.213 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.308 | 0.000 |

| Problem 1272 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | F | F | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 1181 | 1181 | 0 | 0 | 0 | 0 | 0 | 0 | 271 | 0 |
| N.S. | 1 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.23 | 0.00 |
| time (sec) | N/A | 2.469 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.425 | 0.000 |

| Problem 1273 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 111 | 126 | 79 | 91 | 80 | 72 | 107 | 168 | 89 | 82 |
| N.S. | 1 | 1.14 | 0.71 | 0.82 | 0.72 | 0.65 | 0.96 | 1.51 | 0.80 | 0.74 |
| time (sec) | N/A | 0.660 | 0.020 | 3.458 | 0.118 | 0.116 | 0.743 | 0.131 | 0.204 | 0.892 |

| Problem 1274 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 88 | 88 | 56 | 71 | 62 | 49 | 83 | 124 | 70 | 69 |
| N.S. | 1 | 1.00 | 0.64 | 0.81 | 0.70 | 0.56 | 0.94 | 1.41 | 0.80 | 0.78 |
| time (sec) | N/A | 0.330 | 0.017 | 2.503 | 0.118 | 0.100 | 0.566 | 0.133 | 0.180 | 0.942 |

| Problem 1275 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 82 | 97 | 64 | 68 | 65 | 55 | 78 | 135 | 66 | 65 |
| N.S. | 1 | 1.18 | 0.78 | 0.83 | 0.79 | 0.67 | 0.95 | 1.65 | 0.80 | 0.79 |
| time (sec) | N/A | 0.534 | 0.015 | 2.585 | 0.117 | 0.101 | 0.367 | 0.125 | 0.180 | 0.887 |

| Problem 1276 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 49 | 49 | 38 | 48 | 39 | 33 | 56 | 86 | 47 | 48 |
| N.S. | 1 | 1.00 | 0.78 | 0.98 | 0.80 | 0.67 | 1.14 | 1.76 | 0.96 | 0.98 |
| time (sec) | N/A | 0.250 | 0.013 | 2.043 | 0.199 | 0.105 | 0.357 | 0.127 | 0.180 | 0.875 |

| Problem 1277 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 38 | 48 | 38 | 37 | 42 | 33 | 39 | 92 | 36 | 39 |
| N.S. | 1 | 1.26 | 1.00 | 0.97 | 1.11 | 0.87 | 1.03 | 2.42 | 0.95 | 1.03 |
| time (sec) | N/A | 0.601 | 0.007 | 1.845 | 0.137 | 0.102 | 0.244 | 0.129 | 0.206 | 0.773 |

| Problem 1278 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | C | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 189 | 165 | 305 | 2966 | 0 | 0 | 0 | 0 | 14 | 0 |
| N.S. | 1 | 0.87 | 1.61 | 15.69 | 0.00 | 0.00 | 0.00 | 0.00 | 0.07 | 0.00 |
| time (sec) | N/A | 0.703 | 0.221 | 2.864 | 0.000 | 0.000 | 0.000 | 0.000 | 0.187 | 0.000 |

| Problem 1279 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | A | F | C | F | F | B |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 41 | 44 | 41 | 0 | 58 | 0 | 37 | 0 | 39 | 36 |
| N.S. | 1 | 1.07 | 1.00 | 0.00 | 1.41 | 0.00 | 0.90 | 0.00 | 0.95 | 0.88 |
| time (sec) | N/A | 0.423 | 0.008 | 0.000 | 0.127 | 0.000 | 38.809 | 0.000 | 0.188 | 0.116 |

| Problem 1280 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 69 | 76 | 49 | 0 | 70 | 0 | 0 | 0 | 14 | 0 |
| N.S. | 1 | 1.10 | 0.71 | 0.00 | 1.01 | 0.00 | 0.00 | 0.00 | 0.20 | 0.00 |
| time (sec) | N/A | 0.273 | 0.022 | 0.000 | 0.154 | 0.000 | 0.000 | 0.000 | 0.191 | 0.000 |

| Problem 1281 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | A | F | C | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 81 | 100 | 81 | 0 | 95 | 0 | 97 | 0 | 14 | 0 |
| N.S. | 1 | 1.23 | 1.00 | 0.00 | 1.17 | 0.00 | 1.20 | 0.00 | 0.17 | 0.00 |
| time (sec) | N/A | 0.702 | 0.011 | 0.000 | 0.115 | 0.000 | 12.873 | 0.000 | 0.197 | 0.000 |

| Problem 1282 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 102 | 105 | 98 | 0 | 89 | 0 | 0 | 0 | 14 | 0 |
| N.S. | 1 | 1.03 | 0.96 | 0.00 | 0.87 | 0.00 | 0.00 | 0.00 | 0.14 | 0.00 |
| time (sec) | N/A | 0.336 | 0.027 | 0.000 | 0.169 | 0.000 | 0.000 | 0.000 | 0.197 | 0.000 |

| Problem 1283 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | A | F | C | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 114 | 157 | 114 | 0 | 115 | 0 | 134 | 0 | 103 | 0 |
| N.S. | 1 | 1.38 | 1.00 | 0.00 | 1.01 | 0.00 | 1.18 | 0.00 | 0.90 | 0.00 |
| time (sec) | N/A | 1.051 | 0.016 | 0.000 | 0.115 | 0.000 | 16.833 | 0.000 | 0.243 | 0.000 |

| Problem 1284 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 278 | 279 | 214 | 289 | 256 | 220 | 338 | 451 | 280 | 276 |
| N.S. | 1 | 1.00 | 0.77 | 1.04 | 0.92 | 0.79 | 1.22 | 1.62 | 1.01 | 0.99 |
| time (sec) | N/A | 0.942 | 0.117 | 4.431 | 0.145 | 0.093 | 1.744 | 0.378 | 0.251 | 4.133 |

| Problem 1285 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | F(-2) | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 221 | 218 | 164 | 243 | 224 | 178 | 279 | 0 | 238 | 297 |
| N.S. | 1 | 0.99 | 0.74 | 1.10 | 1.01 | 0.81 | 1.26 | 0.00 | 1.08 | 1.34 |
| time (sec) | N/A | 0.510 | 0.097 | 2.743 | 0.128 | 0.095 | 1.223 | 0.000 | 0.278 | 2.378 |

| Problem 1286 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | A | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 213 | 217 | 171 | 217 | 212 | 169 | 258 | 359 | 216 | 212 |
| N.S. | 1 | 1.02 | 0.80 | 1.02 | 1.00 | 0.79 | 1.21 | 1.69 | 1.01 | 1.00 |
| time (sec) | N/A | 0.792 | 0.083 | 2.878 | 0.135 | 0.103 | 0.824 | 0.355 | 0.247 | 3.216 |

| Problem 1287 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 137 | 137 | 105 | 168 | 149 | 116 | 202 | 261 | 167 | 227 |
| N.S. | 1 | 1.00 | 0.77 | 1.23 | 1.09 | 0.85 | 1.47 | 1.91 | 1.22 | 1.66 |
| time (sec) | N/A | 0.358 | 0.059 | 2.059 | 0.109 | 0.109 | 0.591 | 0.269 | 0.247 | 1.779 |

| Problem 1288 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | A | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 100 | 112 | 138 | 137 | 153 | 105 | 148 | 261 | 136 | 134 |
| N.S. | 1 | 1.12 | 1.38 | 1.37 | 1.53 | 1.05 | 1.48 | 2.61 | 1.36 | 1.34 |
| time (sec) | N/A | 0.779 | 0.021 | 1.789 | 0.124 | 0.104 | 0.364 | 0.204 | 0.236 | 1.123 |

| Problem 1289 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | F | C | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | N/A | No | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 282 | 241 | 0 | 5420 | 0 | 0 | 0 | 0 | 161 | 0 |
| N.S. | 1 | 0.85 | 0.00 | 19.22 | 0.00 | 0.00 | 0.00 | 0.00 | 0.57 | 0.00 |
| time (sec) | N/A | 1.273 | 0.000 | 8.942 | 0.000 | 0.000 | 0.000 | 0.000 | 0.309 | 0.000 |

| Problem 1290 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | No | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 100 | 82 | 111 | 0 | 0 | 0 | 0 | 0 | 126 | 0 |
| N.S. | 1 | 0.82 | 1.11 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.26 | 0.00 |
| time (sec) | N/A | 0.686 | 0.066 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.258 | 0.000 |

| Problem 1291 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 154 | 147 | 189 | 0 | 0 | 0 | 0 | 0 | 111 | 0 |
| N.S. | 1 | 0.95 | 1.23 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.72 | 0.00 |
| time (sec) | N/A | 0.382 | 0.089 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.277 | 0.000 |

| Problem 1292 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | No | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 189 | 175 | 181 | 0 | 0 | 0 | 0 | 0 | 120 | 0 |
| N.S. | 1 | 0.93 | 0.96 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.63 | 0.00 |
| time (sec) | N/A | 1.363 | 0.110 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.254 | 0.000 |

| Problem 1293 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 225 | 216 | 260 | 0 | 0 | 0 | 0 | 0 | 131 | 0 |
| N.S. | 1 | 0.96 | 1.16 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.58 | 0.00 |
| time (sec) | N/A | 0.551 | 0.129 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.299 | 0.000 |

| Problem 1294 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F(-2) | F | F | F(-1) |
| verified | N/A | No | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 248 | 290 | 278 | 0 | 0 | 0 | 0 | 0 | 288 | 0 |
| N.S. | 1 | 1.17 | 1.12 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.16 | 0.00 |
| time (sec) | N/A | 2.460 | 0.186 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.278 | 0.000 |

| Problem 1295 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | No | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 562 | 554 | 1140 | 0 | 0 | 0 | 0 | 0 | 257 | 0 |
| N.S. | 1 | 0.99 | 2.03 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.46 | 0.00 |
| time (sec) | N/A | 0.967 | 6.141 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.262 | 0.000 |

| Problem 1296 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | No | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 656 | 653 | 1352 | 0 | 0 | 0 | 0 | 0 | 330 | 0 |
| N.S. | 1 | 1.00 | 2.06 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.50 | 0.00 |
| time (sec) | N/A | 1.768 | 2.996 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.286 | 0.000 |

| Problem 1297 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | F(-1) | N/A | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 24 | 24 | 26 | 24 | 43 | 37 | 0 | 26 | 148 | 26 |
| N.S. | 1 | 1.00 | 1.08 | 1.00 | 1.79 | 1.54 | 0.00 | 1.08 | 6.17 | 1.08 |
| time (sec) | N/A | 0.883 | 0.138 | 0.885 | 0.459 | 0.096 | 0.000 | 0.118 | 0.360 | 1.385 |

| Problem 1298 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 672 | 649 | 552 | 0 | 0 | 0 | 0 | 0 | 107 | 0 |
| N.S. | 1 | 0.97 | 0.82 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.16 | 0.00 |
| time (sec) | N/A | 1.438 | 0.698 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.283 | 0.000 |

| Problem 1299 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Reduce | Mupad |
|---|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F | F(-1) | F | F | F(-1) |
| verified | N/A | Yes | No | N/A | TBD | TBD | TBD | TBD | TBD | TBD |
| size | 528 | 544 | 1217 | 0 | 0 | 0 | 0 | 0 | 1219 | 0 |
| N.S. | 1 | 1.03 | 2.30 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 2.31 | 0.00 |
| time (sec) | N/A | 1.065 | 4.612 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.237 | 0.000 |

## 2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [199] had the largest ratio of [1.39999999999999991]

Table 2.1: Rubi specific breakdown of results for each integral

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 1 | A | 4 | 4 | 0.91 | 21 | 0.190 |
| 2 | A | 4 | 4 | 0.91 | 21 | 0.190 |
| 3 | A | 4 | 4 | 0.95 | 19 | 0.211 |
| 4 | A | 5 | 5 | 1.06 | 18 | 0.278 |
| 5 | A | 2 | 2 | 1.00 | 21 | 0.095 |
| 6 | A | 2 | 2 | 1.00 | 21 | 0.095 |
| 7 | A | 4 | 4 | 0.92 | 21 | 0.190 |
| 8 | A | 4 | 4 | 0.90 | 21 | 0.190 |
| 9 | A | 4 | 4 | 0.88 | 21 | 0.190 |
| 10 | A | 4 | 4 | 0.83 | 23 | 0.174 |
| 11 | A | 4 | 4 | 0.86 | 23 | 0.174 |
| 12 | A | 4 | 4 | 0.87 | 21 | 0.190 |
| 13 | A | 5 | 5 | 0.84 | 20 | 0.250 |
| 14 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 15 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 16 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 17 | A | 4 | 4 | 0.92 | 23 | 0.174 |
| 18 | A | 4 | 4 | 0.84 | 23 | 0.174 |
| 19 | A | 4 | 4 | 0.83 | 23 | 0.174 |
| 20 | A | 4 | 4 | 0.82 | 23 | 0.174 |
| 21 | A | 4 | 4 | 0.84 | 23 | 0.174 |

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| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 22 | A | 4 | 4 | 0.87 | 21 | 0.190 |
| 23 | A | 5 | 5 | 0.96 | 20 | 0.250 |
| 24 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 25 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 26 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 27 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 28 | A | 4 | 4 | 0.83 | 23 | 0.174 |
| 29 | A | 4 | 4 | 0.81 | 23 | 0.174 |
| 30 | A | 4 | 4 | 0.86 | 23 | 0.174 |
| 31 | A | 4 | 4 | 0.83 | 23 | 0.174 |
| 32 | A | 4 | 4 | 0.88 | 23 | 0.174 |
| 33 | A | 4 | 4 | 0.86 | 21 | 0.190 |
| 34 | A | 5 | 5 | 0.92 | 20 | 0.250 |
| 35 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 36 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 37 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 38 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 39 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 40 | A | 4 | 4 | 0.87 | 23 | 0.174 |
| 41 | A | 4 | 4 | 0.80 | 23 | 0.174 |
| 42 | A | 4 | 4 | 0.85 | 23 | 0.174 |
| 43 | A | 16 | 15 | 1.10 | 23 | 0.652 |
| 44 | A | 11 | 10 | 0.99 | 23 | 0.435 |
| 45 | A | 7 | 6 | 0.94 | 21 | 0.286 |
| 46 | A | 4 | 3 | 1.00 | 20 | 0.150 |
| 47 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 48 | A | 10 | 9 | 0.98 | 23 | 0.391 |
| 49 | A | 14 | 13 | 0.86 | 23 | 0.565 |
| 50 | A | 19 | 18 | 0.98 | 23 | 0.783 |
| 51 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 52 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 53 | A | 2 | 2 | 1.00 | 21 | 0.095 |

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| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 54 | A | 5 | 5 | 0.96 | 20 | 0.250 |
| 55 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 56 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 57 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 58 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 59 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 60 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 61 | A | 4 | 4 | 0.98 | 21 | 0.190 |
| 62 | A | 5 | 5 | 0.97 | 20 | 0.250 |
| 63 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 64 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 65 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 66 | A | 4 | 4 | 1.00 | 19 | 0.211 |
| 67 | A | 3 | 3 | 1.00 | 20 | 0.150 |
| 68 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 69 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 70 | A | 2 | 2 | 1.00 | 21 | 0.095 |
| 71 | A | 2 | 2 | 1.04 | 20 | 0.100 |
| 72 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 73 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 74 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 75 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 76 | A | 2 | 2 | 1.00 | 25 | 0.080 |
| 77 | A | 2 | 2 | 1.00 | 25 | 0.080 |
| 78 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 79 | A | 2 | 2 | 0.98 | 22 | 0.091 |
| 80 | A | 2 | 2 | 1.00 | 25 | 0.080 |
| 81 | A | 2 | 2 | 1.00 | 25 | 0.080 |
| 82 | A | 2 | 2 | 1.00 | 25 | 0.080 |
| 83 | A | 2 | 2 | 0.94 | 25 | 0.080 |
| 84 | A | 2 | 2 | 1.00 | 25 | 0.080 |
| 85 | A | 2 | 2 | 1.00 | 25 | 0.080 |

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| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 86 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 87 | A | 2 | 2 | 0.95 | 22 | 0.091 |
| 88 | A | 2 | 2 | 1.00 | 25 | 0.080 |
| 89 | A | 2 | 2 | 1.00 | 25 | 0.080 |
| 90 | A | 2 | 2 | 1.00 | 25 | 0.080 |
| 91 | A | 2 | 2 | 1.00 | 25 | 0.080 |
| 92 | A | 2 | 2 | 0.94 | 25 | 0.080 |
| 93 | A | 2 | 2 | 0.94 | 25 | 0.080 |
| 94 | A | 2 | 2 | 0.94 | 25 | 0.080 |
| 95 | A | 21 | 20 | 1.32 | 25 | 0.800 |
| 96 | A | 16 | 15 | 1.09 | 25 | 0.600 |
| 97 | A | 11 | 10 | 1.10 | 23 | 0.435 |
| 98 | A | 3 | 3 | 1.02 | 22 | 0.136 |
| 99 | A | 3 | 3 | 1.08 | 25 | 0.120 |
| 100 | A | 9 | 9 | 1.05 | 25 | 0.360 |
| 101 | A | 19 | 18 | 1.01 | 25 | 0.720 |
| 102 | A | 23 | 22 | 1.15 | 25 | 0.880 |
| 103 | A | 2 | 2 | 1.00 | 25 | 0.080 |
| 104 | A | 2 | 2 | 1.00 | 25 | 0.080 |
| 105 | A | 2 | 2 | 1.00 | 25 | 0.080 |
| 106 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 107 | A | 2 | 2 | 1.07 | 22 | 0.091 |
| 108 | A | 2 | 2 | 1.00 | 25 | 0.080 |
| 109 | A | 2 | 2 | 1.00 | 25 | 0.080 |
| 110 | A | 2 | 2 | 1.00 | 25 | 0.080 |
| 111 | A | 2 | 2 | 1.00 | 25 | 0.080 |
| 112 | A | 2 | 2 | 1.00 | 25 | 0.080 |
| 113 | A | 2 | 2 | 1.00 | 25 | 0.080 |
| 114 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 115 | A | 2 | 2 | 1.02 | 22 | 0.091 |
| 116 | A | 2 | 2 | 1.00 | 25 | 0.080 |
| 117 | A | 2 | 2 | 1.00 | 25 | 0.080 |

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| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 118 | A | 2 | 2 | 1.00 | 21 | 0.095 |
| 119 | A | 4 | 4 | 1.12 | 22 | 0.182 |
| 120 | A | 2 | 2 | 0.99 | 22 | 0.091 |
| 121 | A | 2 | 2 | 1.03 | 22 | 0.091 |
| 122 | A | 2 | 2 | 1.05 | 20 | 0.100 |
| 123 | A | 4 | 4 | 1.01 | 22 | 0.182 |
| 124 | A | 2 | 2 | 1.02 | 22 | 0.091 |
| 125 | A | 2 | 2 | 1.02 | 22 | 0.091 |
| 126 | A | 2 | 2 | 0.99 | 22 | 0.091 |
| 127 | A | 17 | 16 | 1.10 | 25 | 0.640 |
| 128 | A | 9 | 9 | 1.05 | 23 | 0.391 |
| 129 | A | 4 | 4 | 1.01 | 22 | 0.182 |
| 130 | A | 4 | 4 | 1.06 | 25 | 0.160 |
| 131 | A | 10 | 10 | 1.07 | 25 | 0.400 |
| 132 | A | 16 | 16 | 1.02 | 25 | 0.640 |
| 133 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 134 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 135 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 136 | A | 2 | 2 | 1.00 | 17 | 0.118 |
| 137 | A | 5 | 4 | 1.00 | 16 | 0.250 |
| 138 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 139 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 140 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 141 | A | 2 | 2 | 1.00 | 21 | 0.095 |
| 142 | A | 2 | 2 | 1.00 | 21 | 0.095 |
| 143 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 144 | A | 1 | 1 | 1.00 | 18 | 0.056 |
| 145 | A | 2 | 2 | 1.00 | 21 | 0.095 |
| 146 | A | 2 | 2 | 1.00 | 21 | 0.095 |
| 147 | A | 2 | 2 | 1.00 | 21 | 0.095 |
| 148 | N/A | 1 | 0 | 1.00 | 18 | 0.000 |
| 149 | A | 4 | 4 | 1.48 | 18 | 0.222 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 150 | A | 6 | 5 | 1.48 | 18 | 0.278 |
| 151 | A | 2 | 2 | 1.10 | 16 | 0.125 |
| 152 | A | 3 | 3 | 1.32 | 15 | 0.200 |
| 153 | A | 6 | 6 | 1.11 | 18 | 0.333 |
| 154 | A | 9 | 8 | 1.60 | 18 | 0.444 |
| 155 | A | 6 | 6 | 0.96 | 18 | 0.333 |
| 156 | A | 9 | 8 | 1.37 | 18 | 0.444 |
| 157 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 158 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 159 | A | 3 | 3 | 1.07 | 18 | 0.167 |
| 160 | A | 5 | 5 | 0.98 | 17 | 0.294 |
| 161 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 162 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 163 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 164 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 165 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 166 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 167 | A | 3 | 3 | 1.01 | 18 | 0.167 |
| 168 | A | 6 | 6 | 0.98 | 17 | 0.353 |
| 169 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 170 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 171 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 172 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 173 | A | 11 | 10 | 1.22 | 20 | 0.500 |
| 174 | A | 10 | 9 | 1.03 | 20 | 0.450 |
| 175 | A | 5 | 5 | 0.98 | 20 | 0.250 |
| 176 | A | 5 | 4 | 1.03 | 18 | 0.222 |
| 177 | A | 1 | 1 | 1.00 | 17 | 0.059 |
| 178 | A | 3 | 3 | 1.05 | 20 | 0.150 |
| 179 | A | 9 | 8 | 0.98 | 20 | 0.400 |
| 180 | A | 8 | 8 | 0.92 | 20 | 0.400 |
| 181 | A | 14 | 13 | 1.16 | 20 | 0.650 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 182 | A | 19 | 18 | 1.67 | 20 | 0.900 |
| 183 | A | 8 | 8 | 1.18 | 20 | 0.400 |
| 184 | A | 10 | 9 | 1.07 | 20 | 0.450 |
| 185 | A | 3 | 3 | 1.00 | 20 | 0.150 |
| 186 | A | 3 | 3 | 1.08 | 18 | 0.167 |
| 187 | A | 3 | 3 | 1.00 | 17 | 0.176 |
| 188 | A | 8 | 8 | 1.13 | 20 | 0.400 |
| 189 | A | 12 | 11 | 1.11 | 20 | 0.550 |
| 190 | A | 16 | 16 | 1.49 | 20 | 0.800 |
| 191 | A | 25 | 24 | 1.52 | 20 | 1.200 |
| 192 | A | 5 | 5 | 1.09 | 20 | 0.250 |
| 193 | A | 4 | 4 | 0.99 | 20 | 0.200 |
| 194 | A | 4 | 4 | 1.12 | 18 | 0.222 |
| 195 | A | 4 | 4 | 0.99 | 17 | 0.235 |
| 196 | A | 13 | 13 | 1.36 | 20 | 0.650 |
| 197 | A | 16 | 15 | 1.46 | 20 | 0.750 |
| 198 | B | 21 | 21 | 2.18 | 20 | 1.050 |
| 199 | B | 29 | 28 | 2.25 | 20 | 1.400 |
| 200 | A | 13 | 12 | 1.87 | 22 | 0.545 |
| 201 | A | 9 | 8 | 0.95 | 22 | 0.364 |
| 202 | A | 5 | 4 | 1.06 | 20 | 0.200 |
| 203 | A | 3 | 3 | 0.77 | 19 | 0.158 |
| 204 | A | 6 | 5 | 0.76 | 22 | 0.227 |
| 205 | A | 8 | 7 | 0.84 | 22 | 0.318 |
| 206 | A | 6 | 6 | 0.96 | 22 | 0.273 |
| 207 | A | 6 | 5 | 1.01 | 22 | 0.227 |
| 208 | B | 27 | 26 | 3.61 | 22 | 1.182 |
| 209 | A | 17 | 16 | 1.94 | 22 | 0.727 |
| 210 | A | 6 | 5 | 1.06 | 20 | 0.250 |
| 211 | A | 4 | 4 | 0.80 | 19 | 0.211 |
| 212 | A | 11 | 10 | 0.98 | 22 | 0.455 |
| 213 | A | 12 | 11 | 1.33 | 22 | 0.500 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 214 | A | 12 | 11 | 1.36 | 22 | 0.500 |
| 215 | A | 14 | 13 | 0.95 | 22 | 0.591 |
| 216 | F | 0 | 0 | N/A | 0.000 | N/A |
| 217 | B | 25 | 24 | 3.95 | 22 | 1.091 |
| 218 | A | 7 | 6 | 1.05 | 20 | 0.300 |
| 219 | A | 5 | 5 | 0.83 | 19 | 0.263 |
| 220 | A | 17 | 16 | 1.21 | 22 | 0.727 |
| 221 | A | 17 | 16 | 1.82 | 22 | 0.727 |
| 222 | A | 17 | 16 | 1.91 | 22 | 0.727 |
| 223 | A | 18 | 17 | 1.89 | 22 | 0.773 |
| 224 | A | 8 | 7 | 1.38 | 22 | 0.318 |
| 225 | A | 4 | 4 | 0.80 | 22 | 0.182 |
| 226 | A | 4 | 3 | 1.00 | 20 | 0.150 |
| 227 | A | 2 | 2 | 0.73 | 19 | 0.105 |
| 228 | A | 2 | 2 | 0.70 | 22 | 0.091 |
| 229 | A | 5 | 4 | 1.00 | 22 | 0.182 |
| 230 | A | 4 | 4 | 0.76 | 22 | 0.182 |
| 231 | A | 10 | 9 | 1.27 | 22 | 0.409 |
| 232 | A | 6 | 5 | 1.13 | 22 | 0.227 |
| 233 | A | 3 | 3 | 0.78 | 22 | 0.136 |
| 234 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 235 | A | 1 | 1 | 1.00 | 19 | 0.053 |
| 236 | A | 5 | 5 | 0.79 | 22 | 0.227 |
| 237 | A | 7 | 6 | 1.08 | 22 | 0.273 |
| 238 | A | 10 | 10 | 1.25 | 22 | 0.455 |
| 239 | A | 17 | 16 | 1.64 | 22 | 0.727 |
| 240 | A | 10 | 9 | 1.48 | 22 | 0.409 |
| 241 | A | 9 | 8 | 0.94 | 22 | 0.364 |
| 242 | A | 3 | 3 | 1.05 | 22 | 0.136 |
| 243 | A | 5 | 4 | 1.05 | 22 | 0.182 |
| 244 | A | 3 | 3 | 1.03 | 20 | 0.150 |
| 245 | A | 2 | 2 | 1.02 | 19 | 0.105 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | number of rules / integrand leaf size |
|---|---|---|---|---|---|---|
| 246 | A | 9 | 9 | 0.98 | 22 | 0.409 |
| 247 | A | 10 | 9 | 1.42 | 22 | 0.409 |
| 248 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 249 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 250 | A | 3 | 3 | 1.00 | 18 | 0.167 |
| 251 | N/A | 1 | 0 | 1.00 | 20 | 0.000 |
| 252 | N/A | 1 | 0 | 1.00 | 20 | 0.000 |
| 253 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 254 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 255 | N/A | 4 | 0 | 1.00 | 22 | 0.000 |
| 256 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 257 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 258 | B | 20 | 19 | 2.34 | 20 | 0.950 |
| 259 | B | 17 | 16 | 2.04 | 20 | 0.800 |
| 260 | A | 5 | 5 | 1.01 | 18 | 0.278 |
| 261 | A | 8 | 7 | 1.05 | 17 | 0.412 |
| 262 | A | 10 | 10 | 1.24 | 20 | 0.500 |
| 263 | A | 11 | 10 | 1.45 | 20 | 0.500 |
| 264 | A | 14 | 13 | 1.07 | 20 | 0.650 |
| 265 | A | 9 | 9 | 1.47 | 20 | 0.450 |
| 266 | A | 2 | 2 | 1.00 | 22 | 0.091 |
| 267 | A | 2 | 2 | 1.00 | 22 | 0.091 |
| 268 | A | 6 | 6 | 0.94 | 20 | 0.300 |
| 269 | A | 11 | 10 | 1.01 | 19 | 0.526 |
| 270 | A | 2 | 2 | 1.00 | 22 | 0.091 |
| 271 | A | 2 | 2 | 1.00 | 22 | 0.091 |
| 272 | A | 2 | 2 | 1.00 | 22 | 0.091 |
| 273 | A | 2 | 2 | 1.00 | 22 | 0.091 |
| 274 | A | 2 | 2 | 1.00 | 22 | 0.091 |
| 275 | A | 2 | 2 | 1.00 | 22 | 0.091 |
| 276 | A | 7 | 7 | 0.94 | 20 | 0.350 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 277 | A | 14 | 13 | 1.08 | 19 | 0.684 |
| 278 | A | 2 | 2 | 1.00 | 22 | 0.091 |
| 279 | A | 2 | 2 | 1.00 | 22 | 0.091 |
| 280 | A | 2 | 2 | 1.00 | 22 | 0.091 |
| 281 | A | 2 | 2 | 1.00 | 22 | 0.091 |
| 282 | A | 14 | 13 | 1.46 | 22 | 0.591 |
| 283 | A | 11 | 11 | 1.04 | 22 | 0.500 |
| 284 | A | 9 | 8 | 1.06 | 22 | 0.364 |
| 285 | A | 4 | 4 | 1.04 | 20 | 0.200 |
| 286 | A | 1 | 1 | 1.00 | 19 | 0.053 |
| 287 | A | 4 | 4 | 1.18 | 22 | 0.182 |
| 288 | A | 7 | 7 | 1.03 | 22 | 0.318 |
| 289 | A | 15 | 14 | 0.97 | 22 | 0.636 |
| 290 | A | 11 | 11 | 1.30 | 22 | 0.500 |
| 291 | A | 9 | 9 | 1.03 | 22 | 0.409 |
| 292 | A | 5 | 5 | 1.04 | 22 | 0.227 |
| 293 | A | 4 | 4 | 0.96 | 20 | 0.200 |
| 294 | A | 5 | 5 | 1.06 | 19 | 0.263 |
| 295 | A | 9 | 9 | 1.15 | 22 | 0.409 |
| 296 | A | 12 | 12 | 1.11 | 22 | 0.545 |
| 297 | A | 22 | 21 | 1.46 | 22 | 0.955 |
| 298 | A | 23 | 23 | 1.69 | 22 | 1.045 |
| 299 | A | 5 | 5 | 1.04 | 22 | 0.227 |
| 300 | A | 11 | 11 | 1.72 | 22 | 0.500 |
| 301 | A | 5 | 5 | 0.96 | 20 | 0.250 |
| 302 | A | 9 | 9 | 1.30 | 19 | 0.474 |
| 303 | A | 14 | 14 | 1.39 | 22 | 0.636 |
| 304 | A | 21 | 21 | 1.60 | 22 | 0.955 |
| 305 | B | 27 | 26 | 2.14 | 22 | 1.182 |
| 306 | F | 0 | 0 | N/A | 0.000 | N/A |
| 307 | B | 20 | 19 | 3.09 | 24 | 0.792 |
| 308 | A | 26 | 25 | 1.62 | 24 | 1.042 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 309 | A | 4 | 4 | 0.81 | 22 | 0.182 |
| 310 | A | 11 | 10 | 0.67 | 21 | 0.476 |
| 311 | A | 12 | 11 | 0.69 | 24 | 0.458 |
| 312 | A | 12 | 11 | 0.66 | 24 | 0.458 |
| 313 | A | 20 | 19 | 1.08 | 24 | 0.792 |
| 314 | A | 7 | 7 | 0.97 | 24 | 0.292 |
| 315 | B | 31 | 30 | 6.69 | 24 | 1.250 |
| 316 | F | 0 | 0 | N/A | 0.000 | N/A |
| 317 | A | 5 | 5 | 0.83 | 22 | 0.227 |
| 318 | A | 15 | 14 | 0.79 | 21 | 0.667 |
| 319 | A | 17 | 16 | 1.02 | 24 | 0.667 |
| 320 | A | 22 | 21 | 0.96 | 24 | 0.875 |
| 321 | A | 24 | 23 | 1.18 | 24 | 0.958 |
| 322 | A | 20 | 19 | 0.99 | 24 | 0.792 |
| 323 | F | 0 | 0 | N/A | 0.000 | N/A |
| 324 | F | 0 | 0 | N/A | 0.000 | N/A |
| 325 | A | 6 | 6 | 0.84 | 22 | 0.273 |
| 326 | A | 20 | 19 | 0.94 | 21 | 0.905 |
| 327 | A | 23 | 22 | 1.36 | 24 | 0.917 |
| 328 | F | 0 | 0 | N/A | 0.000 | N/A |
| 329 | A | 29 | 28 | 1.84 | 24 | 1.167 |
| 330 | A | 30 | 29 | 1.66 | 24 | 1.208 |
| 331 | A | 8 | 8 | 1.33 | 24 | 0.333 |
| 332 | A | 12 | 11 | 0.70 | 24 | 0.458 |
| 333 | A | 3 | 3 | 0.78 | 22 | 0.136 |
| 334 | A | 8 | 7 | 0.56 | 21 | 0.333 |
| 335 | A | 8 | 7 | 0.56 | 24 | 0.292 |
| 336 | A | 3 | 3 | 0.75 | 24 | 0.125 |
| 337 | A | 13 | 12 | 0.67 | 24 | 0.500 |
| 338 | A | 8 | 8 | 1.23 | 24 | 0.333 |
| 339 | A | 5 | 5 | 0.87 | 24 | 0.208 |
| 340 | A | 11 | 10 | 0.64 | 24 | 0.417 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 341 | A | 2 | 2 | 1.01 | 22 | 0.091 |
| 342 | A | 2 | 2 | 1.00 | 21 | 0.095 |
| 343 | A | 11 | 10 | 0.69 | 24 | 0.417 |
| 344 | A | 6 | 6 | 0.81 | 24 | 0.250 |
| 345 | A | 23 | 22 | 1.05 | 24 | 0.917 |
| 346 | A | 15 | 15 | 1.58 | 24 | 0.625 |
| 347 | A | 13 | 12 | 1.20 | 24 | 0.500 |
| 348 | A | 16 | 15 | 0.88 | 24 | 0.625 |
| 349 | A | 7 | 6 | 1.19 | 24 | 0.250 |
| 350 | A | 4 | 4 | 1.11 | 24 | 0.167 |
| 351 | A | 3 | 3 | 1.03 | 22 | 0.136 |
| 352 | A | 5 | 5 | 1.18 | 21 | 0.238 |
| 353 | A | 15 | 14 | 0.94 | 24 | 0.583 |
| 354 | A | 12 | 12 | 1.13 | 24 | 0.500 |
| 355 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 356 | N/A | 1 | 0 | 1.00 | 20 | 0.000 |
| 357 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 358 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 359 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 360 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 361 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 362 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 363 | B | 23 | 22 | 3.25 | 20 | 1.100 |
| 364 | B | 21 | 20 | 2.38 | 20 | 1.000 |
| 365 | A | 10 | 9 | 1.02 | 18 | 0.500 |
| 366 | A | 7 | 7 | 1.09 | 17 | 0.412 |
| 367 | A | 15 | 14 | 1.23 | 20 | 0.700 |
| 368 | A | 10 | 10 | 1.40 | 20 | 0.500 |
| 369 | A | 13 | 13 | 1.06 | 20 | 0.650 |
| 370 | A | 16 | 15 | 1.61 | 20 | 0.750 |
| 371 | A | 2 | 2 | 1.00 | 22 | 0.091 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 372 | A | 2 | 2 | 1.00 | 22 | 0.091 |
| 373 | A | 12 | 11 | 0.96 | 20 | 0.550 |
| 374 | A | 12 | 12 | 1.08 | 19 | 0.632 |
| 375 | A | 2 | 2 | 1.00 | 22 | 0.091 |
| 376 | A | 2 | 2 | 1.00 | 22 | 0.091 |
| 377 | A | 2 | 2 | 1.00 | 22 | 0.091 |
| 378 | A | 2 | 2 | 1.00 | 22 | 0.091 |
| 379 | A | 2 | 2 | 1.00 | 22 | 0.091 |
| 380 | A | 2 | 2 | 1.00 | 22 | 0.091 |
| 381 | A | 15 | 14 | 1.00 | 20 | 0.700 |
| 382 | A | 17 | 17 | 1.22 | 19 | 0.895 |
| 383 | A | 2 | 2 | 1.00 | 22 | 0.091 |
| 384 | A | 2 | 2 | 1.00 | 22 | 0.091 |
| 385 | A | 2 | 2 | 1.00 | 22 | 0.091 |
| 386 | A | 2 | 2 | 1.00 | 22 | 0.091 |
| 387 | A | 15 | 15 | 1.53 | 22 | 0.682 |
| 388 | A | 14 | 13 | 1.03 | 22 | 0.591 |
| 389 | A | 8 | 8 | 1.05 | 22 | 0.364 |
| 390 | A | 5 | 5 | 1.03 | 20 | 0.250 |
| 391 | A | 1 | 1 | 1.00 | 19 | 0.053 |
| 392 | A | 5 | 5 | 1.15 | 22 | 0.227 |
| 393 | A | 8 | 8 | 1.11 | 22 | 0.364 |
| 394 | A | 12 | 12 | 0.98 | 22 | 0.545 |
| 395 | A | 18 | 17 | 1.41 | 22 | 0.773 |
| 396 | A | 12 | 12 | 1.04 | 22 | 0.545 |
| 397 | A | 5 | 5 | 1.01 | 22 | 0.227 |
| 398 | A | 6 | 6 | 1.02 | 20 | 0.300 |
| 399 | A | 5 | 5 | 1.02 | 19 | 0.263 |
| 400 | A | 12 | 12 | 1.16 | 22 | 0.545 |
| 401 | A | 13 | 13 | 1.12 | 22 | 0.591 |
| 402 | A | 21 | 21 | 1.42 | 22 | 0.955 |
| 403 | A | 29 | 28 | 1.75 | 22 | 1.273 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 404 | A | 10 | 10 | 1.23 | 22 | 0.455 |
| 405 | A | 11 | 11 | 1.73 | 22 | 0.500 |
| 406 | A | 10 | 10 | 1.14 | 20 | 0.500 |
| 407 | A | 9 | 9 | 1.26 | 19 | 0.474 |
| 408 | A | 22 | 22 | 1.55 | 22 | 1.000 |
| 409 | A | 22 | 22 | 1.62 | 22 | 1.000 |
| 410 | F | 0 | 0 | N/A | 0.000 | N/A |
| 411 | F | 0 | 0 | N/A | 0.000 | N/A |
| 412 | F | 0 | 0 | N/A | 0.000 | N/A |
| 413 | A | 27 | 26 | 1.75 | 24 | 1.083 |
| 414 | A | 12 | 11 | 0.71 | 22 | 0.500 |
| 415 | A | 11 | 10 | 0.64 | 21 | 0.476 |
| 416 | A | 18 | 17 | 0.60 | 24 | 0.708 |
| 417 | A | 18 | 17 | 0.58 | 24 | 0.708 |
| 418 | A | 18 | 17 | 0.93 | 24 | 0.708 |
| 419 | A | 21 | 20 | 1.07 | 24 | 0.833 |
| 420 | F | 0 | 0 | N/A | 0.000 | N/A |
| 421 | F | 0 | 0 | N/A | 0.000 | N/A |
| 422 | A | 16 | 15 | 0.80 | 22 | 0.682 |
| 423 | A | 13 | 12 | 0.86 | 21 | 0.571 |
| 424 | A | 29 | 28 | 0.87 | 24 | 1.167 |
| 425 | A | 26 | 25 | 0.85 | 24 | 1.042 |
| 426 | A | 27 | 26 | 1.01 | 24 | 1.083 |
| 427 | A | 31 | 30 | 0.96 | 24 | 1.250 |
| 428 | F | 0 | 0 | N/A | 0.000 | N/A |
| 429 | A | 21 | 20 | 0.93 | 22 | 0.909 |
| 430 | A | 16 | 15 | 1.10 | 21 | 0.714 |
| 431 | F | 0 | 0 | N/A | 0.000 | N/A |
| 432 | F | 0 | 0 | N/A | 0.000 | N/A |
| 433 | F | 0 | 0 | N/A | 0.000 | N/A |
| 434 | F | 0 | 0 | N/A | 0.000 | N/A |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 435 | A | 20 | 19 | 1.13 | 24 | 0.792 |
| 436 | A | 13 | 12 | 0.66 | 24 | 0.500 |
| 437 | A | 9 | 8 | 0.61 | 22 | 0.364 |
| 438 | A | 9 | 8 | 0.55 | 21 | 0.381 |
| 439 | A | 9 | 8 | 0.54 | 24 | 0.333 |
| 440 | A | 9 | 8 | 0.60 | 24 | 0.333 |
| 441 | A | 12 | 11 | 0.63 | 24 | 0.458 |
| 442 | A | 21 | 20 | 1.06 | 24 | 0.833 |
| 443 | A | 12 | 11 | 0.72 | 24 | 0.458 |
| 444 | A | 12 | 11 | 0.63 | 24 | 0.458 |
| 445 | A | 3 | 3 | 0.99 | 22 | 0.136 |
| 446 | A | 2 | 2 | 1.01 | 21 | 0.095 |
| 447 | A | 13 | 12 | 0.66 | 24 | 0.500 |
| 448 | A | 12 | 11 | 0.71 | 24 | 0.458 |
| 449 | A | 20 | 19 | 1.14 | 24 | 0.792 |
| 450 | A | 20 | 19 | 0.91 | 24 | 0.792 |
| 451 | A | 7 | 7 | 1.29 | 24 | 0.292 |
| 452 | A | 8 | 7 | 1.20 | 24 | 0.292 |
| 453 | A | 6 | 6 | 1.11 | 22 | 0.273 |
| 454 | A | 5 | 5 | 1.27 | 21 | 0.238 |
| 455 | A | 20 | 19 | 0.95 | 24 | 0.792 |
| 456 | A | 18 | 17 | 1.12 | 24 | 0.708 |
| 457 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 458 | N/A | 1 | 0 | 1.00 | 20 | 0.000 |
| 459 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 460 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 461 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 462 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 463 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 464 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 465 | N/A | 1 | 0 | 1.00 | 18 | 0.000 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 466 | N/A | 1 | 0 | 1.00 | 17 | 0.000 |
| 467 | N/A | 1 | 0 | 1.00 | 20 | 0.000 |
| 468 | N/A | 1 | 0 | 1.00 | 20 | 0.000 |
| 469 | N/A | 1 | 0 | 1.00 | 19 | 0.000 |
| 470 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 471 | N/A | 1 | 0 | 1.00 | 20 | 0.000 |
| 472 | N/A | 1 | 0 | 1.00 | 19 | 0.000 |
| 473 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 474 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 475 | N/A | 1 | 0 | 1.00 | 20 | 0.000 |
| 476 | A | 1 | 1 | 1.00 | 19 | 0.053 |
| 477 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 478 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 479 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 480 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 481 | A | 5 | 4 | 0.85 | 22 | 0.182 |
| 482 | A | 6 | 5 | 1.00 | 20 | 0.250 |
| 483 | A | 5 | 4 | 0.85 | 19 | 0.211 |
| 484 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 485 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 486 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 487 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 488 | A | 5 | 4 | 0.78 | 22 | 0.182 |
| 489 | A | 4 | 3 | 0.86 | 22 | 0.136 |
| 490 | A | 4 | 3 | 0.85 | 22 | 0.136 |
| 491 | A | 4 | 3 | 0.86 | 20 | 0.150 |
| 492 | A | 5 | 4 | 0.78 | 19 | 0.211 |
| 493 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 494 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 495 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 496 | N/A | 1 | 0 | 1.00 | 21 | 0.000 |
| 497 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 498 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 499 | N/A | 1 | 0 | 1.00 | 21 | 0.000 |
| 500 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 501 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 502 | N/A | 1 | 0 | 1.00 | 21 | 0.000 |
| 503 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 504 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 505 | N/A | 1 | 0 | 1.00 | 21 | 0.000 |
| 506 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 507 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 508 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 509 | A | 5 | 4 | 1.00 | 22 | 0.182 |
| 510 | A | 5 | 4 | 1.00 | 21 | 0.190 |
| 511 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 512 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 513 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 514 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 515 | A | 6 | 5 | 0.63 | 24 | 0.208 |
| 516 | A | 5 | 4 | 0.63 | 24 | 0.167 |
| 517 | A | 5 | 4 | 0.63 | 22 | 0.182 |
| 518 | A | 6 | 5 | 0.63 | 21 | 0.238 |
| 519 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 520 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 521 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 522 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 523 | N/A | 1 | 0 | 1.00 | 20 | 0.000 |
| 524 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 525 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 526 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 527 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 528 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 529 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 530 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 531 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 532 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 533 | N/A | 1 | 0 | 1.00 | 18 | 0.000 |
| 534 | N/A | 1 | 0 | 1.00 | 17 | 0.000 |
| 535 | N/A | 1 | 0 | 1.00 | 20 | 0.000 |
| 536 | N/A | 1 | 0 | 1.00 | 20 | 0.000 |
| 537 | N/A | 1 | 0 | 1.00 | 19 | 0.000 |
| 538 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 539 | N/A | 1 | 0 | 1.00 | 20 | 0.000 |
| 540 | N/A | 1 | 0 | 1.00 | 19 | 0.000 |
| 541 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 542 | N/A | 2 | 0 | 1.00 | 22 | 0.000 |
| 543 | N/A | 2 | 0 | 1.00 | 22 | 0.000 |
| 544 | N/A | 2 | 0 | 1.00 | 20 | 0.000 |
| 545 | A | 1 | 1 | 1.00 | 19 | 0.053 |
| 546 | N/A | 2 | 0 | 1.00 | 22 | 0.000 |
| 547 | N/A | 2 | 0 | 1.00 | 22 | 0.000 |
| 548 | N/A | 2 | 0 | 1.00 | 22 | 0.000 |
| 549 | N/A | 2 | 0 | 1.00 | 22 | 0.000 |
| 550 | N/A | 14 | 0 | 1.00 | 22 | 0.000 |
| 551 | A | 8 | 7 | 1.00 | 22 | 0.318 |
| 552 | B | 11 | 10 | 2.05 | 20 | 0.500 |
| 553 | A | 8 | 7 | 1.00 | 19 | 0.368 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 554 | N/A | 14 | 0 | 1.00 | 22 | 0.000 |
| 555 | N/A | 11 | 0 | 1.00 | 22 | 0.000 |
| 556 | N/A | 17 | 0 | 1.00 | 22 | 0.000 |
| 557 | N/A | 14 | 0 | 1.00 | 22 | 0.000 |
| 558 | B | 14 | 13 | 2.06 | 22 | 0.591 |
| 559 | A | 10 | 9 | 1.52 | 22 | 0.409 |
| 560 | A | 10 | 9 | 1.56 | 20 | 0.450 |
| 561 | A | 6 | 5 | 0.98 | 19 | 0.263 |
| 562 | N/A | 17 | 0 | 1.00 | 22 | 0.000 |
| 563 | N/A | 14 | 0 | 1.00 | 22 | 0.000 |
| 564 | N/A | 20 | 0 | 1.00 | 22 | 0.000 |
| 565 | N/A | 17 | 0 | 1.00 | 22 | 0.000 |
| 566 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 567 | N/A | 1 | 0 | 1.00 | 21 | 0.000 |
| 568 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 569 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 570 | N/A | 1 | 0 | 1.00 | 21 | 0.000 |
| 571 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 572 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 573 | N/A | 1 | 0 | 1.00 | 21 | 0.000 |
| 574 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 575 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 576 | N/A | 1 | 0 | 1.00 | 21 | 0.000 |
| 577 | N/A | 2 | 0 | 1.00 | 24 | 0.000 |
| 578 | N/A | 8 | 0 | 1.00 | 24 | 0.000 |
| 579 | N/A | 8 | 0 | 1.00 | 24 | 0.000 |
| 580 | A | 6 | 5 | 1.00 | 22 | 0.227 |
| 581 | A | 6 | 5 | 1.00 | 21 | 0.238 |
| 582 | N/A | 8 | 0 | 1.00 | 24 | 0.000 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 583 | N/A | 8 | 0 | 1.00 | 24 | 0.000 |
| 584 | N/A | 9 | 0 | 1.00 | 24 | 0.000 |
| 585 | N/A | 9 | 0 | 1.00 | 24 | 0.000 |
| 586 | N/A | 14 | 0 | 1.00 | 24 | 0.000 |
| 587 | N/A | 11 | 0 | 1.00 | 24 | 0.000 |
| 588 | A | 6 | 5 | 0.75 | 24 | 0.208 |
| 589 | A | 9 | 8 | 1.18 | 24 | 0.333 |
| 590 | A | 11 | 10 | 1.22 | 22 | 0.455 |
| 591 | A | 6 | 5 | 0.74 | 21 | 0.238 |
| 592 | N/A | 19 | 0 | 1.00 | 24 | 0.000 |
| 593 | N/A | 12 | 0 | 1.00 | 24 | 0.000 |
| 594 | N/A | 20 | 0 | 1.00 | 24 | 0.000 |
| 595 | N/A | 13 | 0 | 1.00 | 24 | 0.000 |
| 596 | N/A | 1 | 0 | 1.00 | 30 | 0.000 |
| 597 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 598 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 599 | N/A | 1 | 0 | 1.00 | 20 | 0.000 |
| 600 | N/A | 2 | 0 | 1.00 | 22 | 0.000 |
| 601 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 602 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 603 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 604 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 605 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 606 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 607 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 608 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 609 | N/A | 1 | 0 | 1.00 | 18 | 0.000 |
| 610 | N/A | 1 | 0 | 1.00 | 17 | 0.000 |
| 611 | N/A | 1 | 0 | 1.00 | 20 | 0.000 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 612 | N/A | 1 | 0 | 1.00 | 20 | 0.000 |
| 613 | N/A | 1 | 0 | 1.00 | 19 | 0.000 |
| 614 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 615 | N/A | 1 | 0 | 1.00 | 20 | 0.000 |
| 616 | N/A | 1 | 0 | 1.00 | 19 | 0.000 |
| 617 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 618 | N/A | 2 | 0 | 1.00 | 22 | 0.000 |
| 619 | N/A | 2 | 0 | 1.00 | 22 | 0.000 |
| 620 | N/A | 2 | 0 | 1.00 | 20 | 0.000 |
| 621 | A | 1 | 1 | 1.00 | 19 | 0.053 |
| 622 | N/A | 2 | 0 | 1.00 | 22 | 0.000 |
| 623 | N/A | 2 | 0 | 1.00 | 22 | 0.000 |
| 624 | N/A | 2 | 0 | 1.00 | 22 | 0.000 |
| 625 | N/A | 2 | 0 | 1.00 | 22 | 0.000 |
| 626 | N/A | 11 | 0 | 1.00 | 22 | 0.000 |
| 627 | A | 12 | 11 | 1.59 | 22 | 0.500 |
| 628 | A | 8 | 7 | 1.00 | 20 | 0.350 |
| 629 | A | 12 | 11 | 1.68 | 19 | 0.579 |
| 630 | N/A | 11 | 0 | 1.00 | 22 | 0.000 |
| 631 | N/A | 15 | 0 | 1.00 | 22 | 0.000 |
| 632 | N/A | 14 | 0 | 1.00 | 22 | 0.000 |
| 633 | N/A | 18 | 0 | 1.00 | 22 | 0.000 |
| 634 | A | 14 | 13 | 1.54 | 22 | 0.591 |
| 635 | A | 15 | 14 | 1.94 | 22 | 0.636 |
| 636 | A | 12 | 11 | 1.72 | 20 | 0.550 |
| 637 | A | 11 | 10 | 1.48 | 19 | 0.526 |
| 638 | N/A | 17 | 0 | 1.00 | 22 | 0.000 |
| 639 | N/A | 19 | 0 | 1.00 | 22 | 0.000 |
| 640 | N/A | 20 | 0 | 1.00 | 22 | 0.000 |
| 641 | N/A | 22 | 0 | 1.00 | 22 | 0.000 |

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| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 642 | A | 1 | 1 | 1.00 | 38 | 0.026 |
| 643 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 644 | N/A | 1 | 0 | 1.00 | 21 | 0.000 |
| 645 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 646 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 647 | N/A | 1 | 0 | 1.00 | 21 | 0.000 |
| 648 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 649 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 650 | N/A | 1 | 0 | 1.00 | 21 | 0.000 |
| 651 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 652 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 653 | N/A | 1 | 0 | 1.00 | 21 | 0.000 |
| 654 | N/A | 2 | 0 | 1.00 | 24 | 0.000 |
| 655 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 656 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 657 | N/A | 9 | 0 | 1.00 | 24 | 0.000 |
| 658 | N/A | 9 | 0 | 1.00 | 24 | 0.000 |
| 659 | A | 7 | 6 | 1.04 | 22 | 0.273 |
| 660 | A | 7 | 6 | 1.04 | 21 | 0.286 |
| 661 | N/A | 9 | 0 | 1.00 | 24 | 0.000 |
| 662 | N/A | 9 | 0 | 1.00 | 24 | 0.000 |
| 663 | N/A | 10 | 0 | 1.00 | 24 | 0.000 |
| 664 | N/A | 10 | 0 | 1.00 | 24 | 0.000 |
| 665 | N/A | 14 | 0 | 1.00 | 24 | 0.000 |
| 666 | N/A | 20 | 0 | 1.00 | 24 | 0.000 |
| 667 | A | 10 | 9 | 1.16 | 24 | 0.375 |
| 668 | A | 18 | 17 | 1.41 | 24 | 0.708 |
| 669 | A | 11 | 10 | 1.68 | 22 | 0.455 |
| 670 | A | 12 | 11 | 1.22 | 21 | 0.524 |
| 671 | N/A | 16 | 0 | 1.00 | 24 | 0.000 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 672 | N/A | 21 | 0 | 1.00 | 24 | 0.000 |
| 673 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 674 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 675 | N/A | 1 | 0 | 1.00 | 20 | 0.000 |
| 676 | N/A | 2 | 0 | 1.00 | 22 | 0.000 |
| 677 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 678 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 679 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 680 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 681 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 682 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 683 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 684 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 685 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 686 | N/A | 3 | 0 | 1.00 | 20 | 0.000 |
| 687 | N/A | 1 | 0 | 1.00 | 19 | 0.000 |
| 688 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 689 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 690 | N/A | 3 | 0 | 1.00 | 22 | 0.000 |
| 691 | N/A | 1 | 0 | 1.00 | 21 | 0.000 |
| 692 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 693 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 694 | N/A | 3 | 0 | 1.00 | 22 | 0.000 |
| 695 | N/A | 1 | 0 | 1.00 | 21 | 0.000 |
| 696 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 697 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 698 | N/A | 5 | 0 | 1.00 | 24 | 0.000 |
| 699 | N/A | 4 | 0 | 1.00 | 24 | 0.000 |
| 700 | N/A | 2 | 0 | 1.00 | 22 | 0.000 |

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| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 701 | A | 1 | 1 | 1.00 | 21 | 0.048 |
| 702 | N/A | 2 | 0 | 1.00 | 24 | 0.000 |
| 703 | N/A | 4 | 0 | 1.00 | 24 | 0.000 |
| 704 | N/A | 5 | 0 | 1.00 | 24 | 0.000 |
| 705 | N/A | 6 | 0 | 1.00 | 24 | 0.000 |
| 706 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 707 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 708 | A | 9 | 8 | 1.00 | 24 | 0.333 |
| 709 | A | 7 | 6 | 0.94 | 22 | 0.273 |
| 710 | A | 9 | 8 | 1.00 | 21 | 0.381 |
| 711 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 712 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 713 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 714 | A | 5 | 4 | 0.83 | 24 | 0.167 |
| 715 | A | 7 | 6 | 0.94 | 24 | 0.250 |
| 716 | A | 4 | 3 | 0.87 | 24 | 0.125 |
| 717 | A | 7 | 6 | 0.94 | 22 | 0.273 |
| 718 | A | 5 | 4 | 0.83 | 21 | 0.190 |
| 719 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 720 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 721 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 722 | N/A | 2 | 0 | 1.00 | 24 | 0.000 |
| 723 | N/A | 1 | 0 | 1.00 | 23 | 0.000 |
| 724 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 725 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 726 | N/A | 2 | 0 | 1.00 | 24 | 0.000 |
| 727 | N/A | 1 | 0 | 1.00 | 23 | 0.000 |
| 728 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 729 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 730 | N/A | 2 | 0 | 1.00 | 24 | 0.000 |

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| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | number of rules / integrand leaf size |
|---|---|---|---|---|---|---|
| 731 | N/A | 1 | 0 | 1.00 | 23 | 0.000 |
| 732 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 733 | N/A | 3 | 0 | 1.00 | 26 | 0.000 |
| 734 | N/A | 2 | 0 | 1.00 | 26 | 0.000 |
| 735 | N/A | 2 | 0 | 1.00 | 24 | 0.000 |
| 736 | N/A | 1 | 0 | 1.00 | 23 | 0.000 |
| 737 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 738 | N/A | 2 | 0 | 1.00 | 26 | 0.000 |
| 739 | N/A | 2 | 0 | 1.00 | 26 | 0.000 |
| 740 | N/A | 3 | 0 | 1.00 | 26 | 0.000 |
| 741 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 742 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 743 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 744 | A | 7 | 6 | 1.00 | 24 | 0.250 |
| 745 | A | 9 | 8 | 0.98 | 23 | 0.348 |
| 746 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 747 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 748 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 749 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 750 | A | 6 | 5 | 0.67 | 26 | 0.192 |
| 751 | A | 7 | 6 | 0.83 | 26 | 0.231 |
| 752 | A | 7 | 6 | 0.83 | 24 | 0.250 |
| 753 | A | 6 | 5 | 0.68 | 23 | 0.217 |
| 754 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 755 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 756 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 757 | N/A | 3 | 0 | 1.00 | 20 | 0.000 |
| 758 | N/A | 2 | 0 | 1.00 | 19 | 0.000 |
| 759 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |

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| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 760 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 761 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 762 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 763 | N/A | 3 | 0 | 1.00 | 22 | 0.000 |
| 764 | N/A | 5 | 0 | 1.00 | 21 | 0.000 |
| 765 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 766 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 767 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 768 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 769 | N/A | 3 | 0 | 1.00 | 22 | 0.000 |
| 770 | N/A | 6 | 0 | 1.00 | 21 | 0.000 |
| 771 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 772 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 773 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 774 | N/A | 5 | 0 | 1.00 | 24 | 0.000 |
| 775 | N/A | 4 | 0 | 1.00 | 24 | 0.000 |
| 776 | N/A | 2 | 0 | 1.00 | 22 | 0.000 |
| 777 | A | 1 | 1 | 1.00 | 21 | 0.048 |
| 778 | N/A | 2 | 0 | 1.00 | 24 | 0.000 |
| 779 | N/A | 4 | 0 | 1.00 | 24 | 0.000 |
| 780 | N/A | 5 | 0 | 1.00 | 24 | 0.000 |
| 781 | N/A | 6 | 0 | 1.00 | 24 | 0.000 |
| 782 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 783 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 784 | A | 8 | 7 | 1.00 | 24 | 0.292 |
| 785 | A | 10 | 9 | 0.99 | 22 | 0.409 |
| 786 | A | 8 | 7 | 0.98 | 21 | 0.333 |
| 787 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 788 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |

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| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 789 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 790 | A | 13 | 12 | 1.19 | 24 | 0.500 |
| 791 | A | 7 | 6 | 0.89 | 24 | 0.250 |
| 792 | A | 4 | 3 | 0.84 | 24 | 0.125 |
| 793 | A | 7 | 6 | 0.89 | 22 | 0.273 |
| 794 | A | 13 | 12 | 1.18 | 21 | 0.571 |
| 795 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 796 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 797 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 798 | N/A | 2 | 0 | 1.00 | 24 | 0.000 |
| 799 | N/A | 2 | 0 | 1.00 | 23 | 0.000 |
| 800 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 801 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 802 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 803 | N/A | 2 | 0 | 1.00 | 24 | 0.000 |
| 804 | N/A | 3 | 0 | 1.00 | 23 | 0.000 |
| 805 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 806 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 807 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 808 | N/A | 2 | 0 | 1.00 | 24 | 0.000 |
| 809 | N/A | 4 | 0 | 1.00 | 23 | 0.000 |
| 810 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 811 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 812 | N/A | 4 | 0 | 1.00 | 26 | 0.000 |
| 813 | N/A | 3 | 0 | 1.00 | 26 | 0.000 |
| 814 | N/A | 2 | 0 | 1.00 | 24 | 0.000 |
| 815 | N/A | 1 | 0 | 1.00 | 23 | 0.000 |
| 816 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 817 | N/A | 2 | 0 | 1.00 | 26 | 0.000 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 818 | N/A | 3 | 0 | 1.00 | 26 | 0.000 |
| 819 | N/A | 4 | 0 | 1.00 | 26 | 0.000 |
| 820 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 821 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 822 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 823 | A | 10 | 9 | 0.95 | 24 | 0.375 |
| 824 | A | 7 | 6 | 1.00 | 23 | 0.261 |
| 825 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 826 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 827 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 828 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 829 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 830 | A | 16 | 15 | 1.16 | 26 | 0.577 |
| 831 | A | 7 | 6 | 0.72 | 26 | 0.231 |
| 832 | A | 7 | 6 | 0.73 | 24 | 0.250 |
| 833 | A | 10 | 9 | 1.18 | 23 | 0.391 |
| 834 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 835 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 836 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 837 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 838 | N/A | 4 | 0 | 1.00 | 20 | 0.000 |
| 839 | N/A | 2 | 0 | 1.00 | 19 | 0.000 |
| 840 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 841 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 842 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 843 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 844 | N/A | 6 | 0 | 1.00 | 22 | 0.000 |
| 845 | N/A | 5 | 0 | 1.00 | 21 | 0.000 |
| 846 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 847 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 848 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 849 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 850 | N/A | 7 | 0 | 1.00 | 22 | 0.000 |
| 851 | N/A | 6 | 0 | 1.00 | 21 | 0.000 |
| 852 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 853 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 854 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 855 | N/A | 5 | 0 | 1.00 | 24 | 0.000 |
| 856 | N/A | 4 | 0 | 1.00 | 24 | 0.000 |
| 857 | N/A | 2 | 0 | 1.00 | 22 | 0.000 |
| 858 | A | 1 | 1 | 1.00 | 21 | 0.048 |
| 859 | N/A | 2 | 0 | 1.00 | 24 | 0.000 |
| 860 | N/A | 4 | 0 | 1.00 | 24 | 0.000 |
| 861 | N/A | 5 | 0 | 1.00 | 24 | 0.000 |
| 862 | N/A | 6 | 0 | 1.00 | 24 | 0.000 |
| 863 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 864 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 865 | A | 11 | 10 | 1.03 | 24 | 0.417 |
| 866 | A | 9 | 8 | 0.98 | 22 | 0.364 |
| 867 | A | 11 | 10 | 1.03 | 21 | 0.476 |
| 868 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 869 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 870 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 871 | A | 15 | 14 | 1.12 | 24 | 0.583 |
| 872 | A | 14 | 13 | 1.17 | 24 | 0.542 |
| 873 | A | 4 | 3 | 0.83 | 24 | 0.125 |
| 874 | A | 14 | 13 | 1.13 | 22 | 0.591 |
| 875 | A | 16 | 15 | 1.11 | 21 | 0.714 |
| 876 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 877 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 878 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 879 | N/A | 3 | 0 | 1.00 | 24 | 0.000 |
| 880 | N/A | 2 | 0 | 1.00 | 23 | 0.000 |
| 881 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 882 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 883 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 884 | N/A | 4 | 0 | 1.00 | 24 | 0.000 |
| 885 | N/A | 3 | 0 | 1.00 | 23 | 0.000 |
| 886 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 887 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 888 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 889 | N/A | 5 | 0 | 1.00 | 24 | 0.000 |
| 890 | N/A | 4 | 0 | 1.00 | 23 | 0.000 |
| 891 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 892 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 893 | N/A | 5 | 0 | 1.00 | 26 | 0.000 |
| 894 | N/A | 3 | 0 | 1.00 | 26 | 0.000 |
| 895 | N/A | 2 | 0 | 1.00 | 24 | 0.000 |
| 896 | N/A | 1 | 0 | 1.00 | 23 | 0.000 |
| 897 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 898 | N/A | 2 | 0 | 1.00 | 26 | 0.000 |
| 899 | N/A | 3 | 0 | 1.00 | 26 | 0.000 |
| 900 | N/A | 5 | 0 | 1.00 | 26 | 0.000 |
| 901 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 902 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 903 | A | 8 | 7 | 1.01 | 24 | 0.292 |
| 904 | A | 10 | 9 | 0.98 | 23 | 0.391 |
| 905 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | number of rules / integrand leaf size |
|---|---|---|---|---|---|---|
| 906 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 907 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 908 | A | 14 | 13 | 1.11 | 26 | 0.500 |
| 909 | A | 17 | 16 | 1.17 | 26 | 0.615 |
| 910 | A | 11 | 10 | 1.15 | 24 | 0.417 |
| 911 | A | 13 | 12 | 1.10 | 23 | 0.522 |
| 912 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 913 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 914 | N/A | 1 | 0 | 1.00 | 20 | 0.000 |
| 915 | N/A | 1 | 0 | 1.00 | 19 | 0.000 |
| 916 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 917 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 918 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 919 | N/A | 1 | 0 | 1.00 | 21 | 0.000 |
| 920 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 921 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 922 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 923 | N/A | 1 | 0 | 1.00 | 21 | 0.000 |
| 924 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 925 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 926 | N/A | 2 | 0 | 1.00 | 22 | 0.000 |
| 927 | A | 1 | 1 | 1.00 | 21 | 0.048 |
| 928 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 929 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 930 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 931 | A | 5 | 4 | 0.87 | 24 | 0.167 |
| 932 | A | 7 | 6 | 1.00 | 22 | 0.273 |
| 933 | A | 5 | 4 | 0.87 | 21 | 0.190 |
| 934 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 935 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 936 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 937 | A | 5 | 4 | 0.88 | 24 | 0.167 |
| 938 | A | 4 | 3 | 0.93 | 24 | 0.125 |
| 939 | A | 4 | 3 | 0.91 | 24 | 0.125 |
| 940 | A | 4 | 3 | 0.93 | 22 | 0.136 |
| 941 | A | 5 | 4 | 0.88 | 21 | 0.190 |
| 942 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 943 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 944 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 945 | N/A | 1 | 0 | 1.00 | 23 | 0.000 |
| 946 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 947 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 948 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 949 | N/A | 1 | 0 | 1.00 | 23 | 0.000 |
| 950 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 951 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 952 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 953 | N/A | 1 | 0 | 1.00 | 23 | 0.000 |
| 954 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 955 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 956 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 957 | N/A | 1 | 0 | 1.00 | 23 | 0.000 |
| 958 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 959 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 960 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 961 | A | 6 | 5 | 1.00 | 24 | 0.208 |
| 962 | A | 6 | 5 | 1.00 | 23 | 0.217 |
| 963 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 964 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 965 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 966 | A | 6 | 5 | 0.76 | 26 | 0.192 |
| 967 | A | 5 | 4 | 0.76 | 26 | 0.154 |
| 968 | A | 5 | 4 | 0.76 | 24 | 0.167 |
| 969 | A | 6 | 5 | 0.76 | 23 | 0.217 |
| 970 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 971 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 972 | N/A | 1 | 0 | 1.00 | 20 | 0.000 |
| 973 | N/A | 1 | 0 | 1.00 | 19 | 0.000 |
| 974 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 975 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 976 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 977 | N/A | 1 | 0 | 1.00 | 21 | 0.000 |
| 978 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 979 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 980 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 981 | N/A | 1 | 0 | 1.00 | 21 | 0.000 |
| 982 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 983 | N/A | 2 | 0 | 1.00 | 24 | 0.000 |
| 984 | N/A | 2 | 0 | 1.00 | 22 | 0.000 |
| 985 | A | 1 | 1 | 1.00 | 21 | 0.048 |
| 986 | N/A | 2 | 0 | 1.00 | 24 | 0.000 |
| 987 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 988 | N/A | 3 | 0 | 1.00 | 24 | 0.000 |
| 989 | N/A | 8 | 0 | 1.00 | 24 | 0.000 |
| 990 | A | 9 | 8 | 1.00 | 24 | 0.333 |
| 991 | A | 8 | 7 | 1.01 | 22 | 0.318 |
| 992 | A | 9 | 8 | 1.00 | 21 | 0.381 |
| 993 | N/A | 8 | 0 | 1.00 | 24 | 0.000 |
| 994 | N/A | 3 | 0 | 1.00 | 24 | 0.000 |
| 995 | N/A | 3 | 0 | 1.00 | 24 | 0.000 |

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| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 996 | N/A | 3 | 0 | 1.00 | 24 | 0.000 |
| 997 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 998 | A | 9 | 8 | 1.71 | 24 | 0.333 |
| 999 | B | 6 | 5 | 2.46 | 24 | 0.208 |
| 1000 | A | 10 | 9 | 1.74 | 22 | 0.409 |
| 1001 | A | 6 | 5 | 1.01 | 21 | 0.238 |
| 1002 | N/A | 8 | 0 | 1.00 | 24 | 0.000 |
| 1003 | N/A | 3 | 0 | 1.00 | 24 | 0.000 |
| 1004 | N/A | 3 | 0 | 1.00 | 24 | 0.000 |
| 1005 | N/A | 3 | 0 | 1.00 | 24 | 0.000 |
| 1006 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 1007 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 1008 | N/A | 1 | 0 | 1.00 | 23 | 0.000 |
| 1009 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 1010 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 1011 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 1012 | N/A | 1 | 0 | 1.00 | 23 | 0.000 |
| 1013 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 1014 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 1015 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 1016 | N/A | 1 | 0 | 1.00 | 23 | 0.000 |
| 1017 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 1018 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 1019 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 1020 | N/A | 1 | 0 | 1.00 | 23 | 0.000 |
| 1021 | N/A | 2 | 0 | 1.00 | 26 | 0.000 |
| 1022 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 1023 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 1024 | N/A | 2 | 0 | 1.00 | 26 | 0.000 |

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| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 1025 | N/A | 8 | 0 | 1.00 | 26 | 0.000 |
| 1026 | A | 7 | 6 | 1.00 | 24 | 0.250 |
| 1027 | A | 7 | 6 | 1.00 | 23 | 0.261 |
| 1028 | N/A | 8 | 0 | 1.00 | 26 | 0.000 |
| 1029 | N/A | 2 | 0 | 1.00 | 26 | 0.000 |
| 1030 | N/A | 2 | 0 | 1.00 | 26 | 0.000 |
| 1031 | N/A | 2 | 0 | 1.00 | 26 | 0.000 |
| 1032 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 1033 | A | 6 | 5 | 0.84 | 26 | 0.192 |
| 1034 | A | 9 | 8 | 0.83 | 26 | 0.308 |
| 1035 | A | 11 | 10 | 0.83 | 24 | 0.417 |
| 1036 | A | 6 | 5 | 0.83 | 23 | 0.217 |
| 1037 | N/A | 8 | 0 | 1.00 | 26 | 0.000 |
| 1038 | N/A | 2 | 0 | 1.00 | 26 | 0.000 |
| 1039 | N/A | 2 | 0 | 1.00 | 26 | 0.000 |
| 1040 | N/A | 2 | 0 | 1.00 | 26 | 0.000 |
| 1041 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 1042 | N/A | 1 | 0 | 1.00 | 20 | 0.000 |
| 1043 | N/A | 1 | 0 | 1.00 | 19 | 0.000 |
| 1044 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 1045 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 1046 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 1047 | N/A | 1 | 0 | 1.00 | 21 | 0.000 |
| 1048 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 1049 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 1050 | N/A | 1 | 0 | 1.00 | 22 | 0.000 |
| 1051 | N/A | 1 | 0 | 1.00 | 21 | 0.000 |
| 1052 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 1053 | N/A | 2 | 0 | 1.00 | 24 | 0.000 |

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| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 1054 | N/A | 2 | 0 | 1.00 | 22 | 0.000 |
| 1055 | A | 1 | 1 | 1.00 | 21 | 0.048 |
| 1056 | N/A | 2 | 0 | 1.00 | 24 | 0.000 |
| 1057 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 1058 | N/A | 12 | 0 | 1.00 | 24 | 0.000 |
| 1059 | A | 9 | 8 | 0.96 | 24 | 0.333 |
| 1060 | A | 9 | 8 | 1.00 | 22 | 0.364 |
| 1061 | A | 9 | 8 | 0.97 | 21 | 0.381 |
| 1062 | N/A | 12 | 0 | 1.00 | 24 | 0.000 |
| 1063 | N/A | 9 | 0 | 1.00 | 24 | 0.000 |
| 1064 | N/A | 4 | 0 | 1.00 | 24 | 0.000 |
| 1065 | N/A | 4 | 0 | 1.00 | 24 | 0.000 |
| 1066 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 1067 | A | 8 | 7 | 1.86 | 24 | 0.292 |
| 1068 | B | 14 | 13 | 2.78 | 24 | 0.542 |
| 1069 | A | 8 | 7 | 1.88 | 22 | 0.318 |
| 1070 | A | 11 | 10 | 1.53 | 21 | 0.476 |
| 1071 | N/A | 9 | 0 | 1.00 | 24 | 0.000 |
| 1072 | N/A | 9 | 0 | 1.00 | 24 | 0.000 |
| 1073 | N/A | 4 | 0 | 1.00 | 24 | 0.000 |
| 1074 | N/A | 4 | 0 | 1.00 | 24 | 0.000 |
| 1075 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 1076 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 1077 | N/A | 1 | 0 | 1.00 | 23 | 0.000 |
| 1078 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 1079 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 1080 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 1081 | N/A | 1 | 0 | 1.00 | 23 | 0.000 |
| 1082 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 1083 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |

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| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 1084 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 1085 | N/A | 1 | 0 | 1.00 | 23 | 0.000 |
| 1086 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 1087 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 1088 | N/A | 1 | 0 | 1.00 | 24 | 0.000 |
| 1089 | N/A | 1 | 0 | 1.00 | 23 | 0.000 |
| 1090 | N/A | 2 | 0 | 1.00 | 26 | 0.000 |
| 1091 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 1092 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 1093 | N/A | 9 | 0 | 1.00 | 26 | 0.000 |
| 1094 | N/A | 10 | 0 | 1.00 | 26 | 0.000 |
| 1095 | A | 8 | 7 | 1.03 | 24 | 0.292 |
| 1096 | A | 8 | 7 | 1.04 | 23 | 0.304 |
| 1097 | N/A | 10 | 0 | 1.00 | 26 | 0.000 |
| 1098 | N/A | 9 | 0 | 1.00 | 26 | 0.000 |
| 1099 | N/A | 3 | 0 | 1.00 | 26 | 0.000 |
| 1100 | N/A | 3 | 0 | 1.00 | 26 | 0.000 |
| 1101 | N/A | 1 | 0 | 1.00 | 26 | 0.000 |
| 1102 | A | 10 | 9 | 1.45 | 26 | 0.346 |
| 1103 | A | 13 | 12 | 1.85 | 26 | 0.462 |
| 1104 | A | 11 | 10 | 1.85 | 24 | 0.417 |
| 1105 | A | 12 | 11 | 1.46 | 23 | 0.478 |
| 1106 | N/A | 9 | 0 | 1.00 | 26 | 0.000 |
| 1107 | N/A | 9 | 0 | 1.00 | 26 | 0.000 |
| 1108 | N/A | 3 | 0 | 1.00 | 26 | 0.000 |
| 1109 | N/A | 3 | 0 | 1.00 | 26 | 0.000 |
| 1110 | N/A | 2 | 0 | 1.00 | 20 | 0.000 |
| 1111 | A | 1 | 1 | 1.00 | 19 | 0.053 |
| 1112 | N/A | 1 | 0 | 1.00 | 28 | 0.000 |
| 1113 | A | 5 | 5 | 0.81 | 19 | 0.263 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | number of rules / integrand leaf size |
|---|---|---|---|---|---|---|
| 1114 | A | 6 | 5 | 0.99 | 19 | 0.263 |
| 1115 | A | 3 | 3 | 1.01 | 17 | 0.176 |
| 1116 | A | 6 | 5 | 0.99 | 16 | 0.312 |
| 1117 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 1118 | A | 6 | 5 | 1.07 | 19 | 0.263 |
| 1119 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 1120 | A | 6 | 5 | 0.95 | 19 | 0.263 |
| 1121 | A | 5 | 5 | 0.91 | 19 | 0.263 |
| 1122 | A | 6 | 5 | 0.95 | 19 | 0.263 |
| 1123 | A | 6 | 6 | 0.82 | 19 | 0.316 |
| 1124 | A | 4 | 4 | 0.98 | 21 | 0.190 |
| 1125 | A | 6 | 5 | 0.98 | 21 | 0.238 |
| 1126 | A | 3 | 3 | 1.00 | 19 | 0.158 |
| 1127 | A | 6 | 5 | 0.98 | 18 | 0.278 |
| 1128 | A | 2 | 2 | 1.00 | 21 | 0.095 |
| 1129 | A | 6 | 5 | 1.02 | 21 | 0.238 |
| 1130 | A | 2 | 2 | 1.00 | 21 | 0.095 |
| 1131 | A | 6 | 5 | 0.97 | 21 | 0.238 |
| 1132 | A | 2 | 2 | 1.00 | 21 | 0.095 |
| 1133 | A | 6 | 5 | 0.95 | 21 | 0.238 |
| 1134 | A | 4 | 4 | 1.02 | 21 | 0.190 |
| 1135 | A | 6 | 5 | 0.96 | 21 | 0.238 |
| 1136 | A | 9 | 9 | 1.26 | 21 | 0.429 |
| 1137 | A | 6 | 5 | 0.99 | 21 | 0.238 |
| 1138 | A | 3 | 3 | 0.99 | 19 | 0.158 |
| 1139 | A | 6 | 5 | 0.98 | 18 | 0.278 |
| 1140 | A | 2 | 2 | 1.00 | 21 | 0.095 |
| 1141 | A | 6 | 5 | 1.01 | 21 | 0.238 |
| 1142 | A | 2 | 2 | 1.00 | 21 | 0.095 |
| 1143 | A | 6 | 5 | 0.96 | 21 | 0.238 |
| 1144 | A | 2 | 2 | 1.00 | 21 | 0.095 |
| 1145 | A | 6 | 5 | 0.97 | 21 | 0.238 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | number of rules / integrand leaf size |
|---|---|---|---|---|---|---|
| 1146 | A | 2 | 2 | 1.00 | 21 | 0.095 |
| 1147 | A | 6 | 5 | 0.96 | 21 | 0.238 |
| 1148 | A | 4 | 4 | 0.99 | 21 | 0.190 |
| 1149 | A | 6 | 5 | 1.00 | 14 | 0.357 |
| 1150 | A | 6 | 6 | 0.99 | 21 | 0.286 |
| 1151 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 1152 | A | 2 | 2 | 1.00 | 21 | 0.095 |
| 1153 | A | 6 | 6 | 0.98 | 21 | 0.286 |
| 1154 | A | 7 | 7 | 0.99 | 21 | 0.333 |
| 1155 | A | 5 | 5 | 0.99 | 18 | 0.278 |
| 1156 | A | 12 | 11 | 0.99 | 21 | 0.524 |
| 1157 | A | 2 | 2 | 1.00 | 21 | 0.095 |
| 1158 | A | 4 | 4 | 0.97 | 19 | 0.211 |
| 1159 | A | 2 | 2 | 1.00 | 21 | 0.095 |
| 1160 | A | 2 | 2 | 1.00 | 21 | 0.095 |
| 1161 | A | 2 | 2 | 1.00 | 21 | 0.095 |
| 1162 | A | 4 | 4 | 0.98 | 18 | 0.222 |
| 1163 | A | 2 | 2 | 1.00 | 21 | 0.095 |
| 1164 | A | 2 | 2 | 1.00 | 21 | 0.095 |
| 1165 | A | 6 | 6 | 1.16 | 21 | 0.286 |
| 1166 | A | 5 | 5 | 1.15 | 19 | 0.263 |
| 1167 | A | 2 | 2 | 1.00 | 21 | 0.095 |
| 1168 | A | 2 | 2 | 1.00 | 21 | 0.095 |
| 1169 | A | 2 | 2 | 1.00 | 21 | 0.095 |
| 1170 | A | 4 | 4 | 0.98 | 18 | 0.222 |
| 1171 | A | 2 | 2 | 1.00 | 21 | 0.095 |
| 1172 | A | 10 | 9 | 1.04 | 23 | 0.391 |
| 1173 | N/A | 7 | 0 | 1.00 | 23 | 0.000 |
| 1174 | A | 8 | 7 | 1.06 | 21 | 0.333 |
| 1175 | N/A | 1 | 0 | 1.00 | 20 | 0.000 |
| 1176 | N/A | 7 | 0 | 1.00 | 23 | 0.000 |
| 1177 | N/A | 6 | 0 | 1.00 | 23 | 0.000 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 1178 | N/A | 7 | 0 | 1.00 | 23 | 0.000 |
| 1179 | A | 9 | 8 | 1.04 | 23 | 0.348 |
| 1180 | N/A | 8 | 0 | 1.00 | 23 | 0.000 |
| 1181 | A | 11 | 10 | 1.01 | 23 | 0.435 |
| 1182 | A | 12 | 11 | 1.05 | 23 | 0.478 |
| 1183 | N/A | 8 | 0 | 1.00 | 23 | 0.000 |
| 1184 | A | 9 | 8 | 1.09 | 21 | 0.381 |
| 1185 | N/A | 1 | 0 | 1.00 | 20 | 0.000 |
| 1186 | N/A | 8 | 0 | 1.00 | 23 | 0.000 |
| 1187 | N/A | 7 | 0 | 1.00 | 23 | 0.000 |
| 1188 | N/A | 8 | 0 | 1.00 | 23 | 0.000 |
| 1189 | N/A | 7 | 0 | 1.00 | 23 | 0.000 |
| 1190 | N/A | 8 | 0 | 1.00 | 23 | 0.000 |
| 1191 | A | 11 | 10 | 1.04 | 23 | 0.435 |
| 1192 | A | 13 | 12 | 1.06 | 23 | 0.522 |
| 1193 | N/A | 9 | 0 | 1.00 | 23 | 0.000 |
| 1194 | A | 11 | 10 | 1.10 | 21 | 0.476 |
| 1195 | N/A | 1 | 0 | 1.00 | 20 | 0.000 |
| 1196 | N/A | 9 | 0 | 1.00 | 23 | 0.000 |
| 1197 | N/A | 8 | 0 | 1.00 | 23 | 0.000 |
| 1198 | N/A | 9 | 0 | 1.00 | 23 | 0.000 |
| 1199 | N/A | 8 | 0 | 1.00 | 23 | 0.000 |
| 1200 | A | 9 | 8 | 1.03 | 23 | 0.348 |
| 1201 | N/A | 6 | 0 | 1.00 | 23 | 0.000 |
| 1202 | A | 7 | 6 | 1.01 | 21 | 0.286 |
| 1203 | N/A | 1 | 0 | 1.00 | 20 | 0.000 |
| 1204 | N/A | 6 | 0 | 1.00 | 23 | 0.000 |
| 1205 | A | 8 | 7 | 1.08 | 23 | 0.304 |
| 1206 | N/A | 7 | 0 | 1.00 | 23 | 0.000 |
| 1207 | A | 9 | 8 | 1.01 | 23 | 0.348 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 1208 | A | 8 | 7 | 0.99 | 23 | 0.304 |
| 1209 | N/A | 6 | 0 | 1.00 | 23 | 0.000 |
| 1210 | A | 4 | 3 | 1.00 | 21 | 0.143 |
| 1211 | A | 6 | 5 | 1.00 | 20 | 0.250 |
| 1212 | N/A | 7 | 0 | 1.00 | 23 | 0.000 |
| 1213 | A | 8 | 7 | 1.06 | 23 | 0.304 |
| 1214 | N/A | 8 | 0 | 1.00 | 23 | 0.000 |
| 1215 | A | 4 | 4 | 1.13 | 23 | 0.174 |
| 1216 | N/A | 7 | 0 | 1.00 | 23 | 0.000 |
| 1217 | A | 7 | 6 | 0.97 | 23 | 0.261 |
| 1218 | A | 7 | 6 | 1.04 | 23 | 0.261 |
| 1219 | A | 5 | 4 | 1.00 | 21 | 0.190 |
| 1220 | A | 7 | 6 | 1.01 | 20 | 0.300 |
| 1221 | N/A | 8 | 0 | 1.00 | 23 | 0.000 |
| 1222 | A | 4 | 4 | 1.22 | 23 | 0.174 |
| 1223 | N/A | 9 | 0 | 1.00 | 23 | 0.000 |
| 1224 | A | 4 | 4 | 1.35 | 23 | 0.174 |
| 1225 | A | 8 | 7 | 0.97 | 16 | 0.438 |
| 1226 | A | 6 | 5 | 0.99 | 16 | 0.312 |
| 1227 | A | 3 | 3 | 1.06 | 21 | 0.143 |
| 1228 | A | 3 | 3 | 1.11 | 21 | 0.143 |
| 1229 | A | 3 | 3 | 0.98 | 19 | 0.158 |
| 1230 | N/A | 3 | 0 | 1.00 | 21 | 0.000 |
| 1231 | N/A | 3 | 0 | 1.00 | 21 | 0.000 |
| 1232 | N/A | 4 | 0 | 1.00 | 23 | 0.000 |
| 1233 | N/A | 4 | 0 | 1.00 | 23 | 0.000 |
| 1234 | N/A | 4 | 0 | 1.00 | 23 | 0.000 |
| 1235 | N/A | 4 | 0 | 1.00 | 23 | 0.000 |
| 1236 | N/A | 4 | 0 | 1.00 | 23 | 0.000 |
| 1237 | N/A | 4 | 0 | 1.00 | 21 | 0.000 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 1238 | N/A | 4 | 0 | 1.00 | 25 | 0.000 |
| 1239 | A | 6 | 5 | 0.97 | 25 | 0.200 |
| 1240 | N/A | 4 | 0 | 1.00 | 25 | 0.000 |
| 1241 | A | 4 | 4 | 0.95 | 25 | 0.160 |
| 1242 | N/A | 4 | 0 | 1.00 | 25 | 0.000 |
| 1243 | A | 4 | 4 | 0.93 | 25 | 0.160 |
| 1244 | N/A | 4 | 0 | 1.00 | 25 | 0.000 |
| 1245 | A | 2 | 2 | 1.00 | 21 | 0.095 |
| 1246 | A | 2 | 2 | 1.00 | 21 | 0.095 |
| 1247 | A | 2 | 2 | 1.00 | 19 | 0.105 |
| 1248 | A | 2 | 2 | 1.00 | 18 | 0.111 |
| 1249 | A | 2 | 2 | 1.00 | 21 | 0.095 |
| 1250 | A | 2 | 2 | 1.00 | 21 | 0.095 |
| 1251 | A | 2 | 2 | 1.00 | 21 | 0.095 |
| 1252 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 1253 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 1254 | A | 2 | 2 | 1.00 | 21 | 0.095 |
| 1255 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 1256 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 1257 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 1258 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 1259 | A | 7 | 7 | 0.98 | 23 | 0.304 |
| 1260 | A | 9 | 8 | 1.03 | 23 | 0.348 |
| 1261 | A | 2 | 2 | 1.00 | 21 | 0.095 |
| 1262 | A | 2 | 2 | 1.00 | 20 | 0.100 |
| 1263 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 1264 | A | 7 | 7 | 1.02 | 23 | 0.304 |
| 1265 | A | 12 | 11 | 0.97 | 23 | 0.478 |
| 1266 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 1267 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 1268 | B | 3 | 3 | 2.18 | 21 | 0.143 |
| 1269 | A | 2 | 2 | 1.00 | 20 | 0.100 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 1270 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 1271 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 1272 | A | 2 | 2 | 1.00 | 23 | 0.087 |
| 1273 | A | 2 | 2 | 1.14 | 12 | 0.167 |
| 1274 | A | 2 | 2 | 1.00 | 12 | 0.167 |
| 1275 | A | 2 | 2 | 1.18 | 12 | 0.167 |
| 1276 | A | 2 | 2 | 1.00 | 10 | 0.200 |
| 1277 | A | 9 | 8 | 1.26 | 9 | 0.889 |
| 1278 | A | 8 | 7 | 0.87 | 12 | 0.583 |
| 1279 | A | 6 | 5 | 1.07 | 12 | 0.417 |
| 1280 | A | 2 | 2 | 1.10 | 12 | 0.167 |
| 1281 | A | 12 | 11 | 1.23 | 12 | 0.917 |
| 1282 | A | 2 | 2 | 1.03 | 12 | 0.167 |
| 1283 | A | 17 | 16 | 1.38 | 12 | 1.333 |
| 1284 | A | 2 | 2 | 1.00 | 26 | 0.077 |
| 1285 | A | 2 | 2 | 0.99 | 26 | 0.077 |
| 1286 | A | 2 | 2 | 1.02 | 26 | 0.077 |
| 1287 | A | 2 | 2 | 1.00 | 24 | 0.083 |
| 1288 | A | 8 | 7 | 1.12 | 23 | 0.304 |
| 1289 | A | 13 | 12 | 0.85 | 26 | 0.462 |
| 1290 | A | 9 | 8 | 0.82 | 26 | 0.308 |
| 1291 | A | 2 | 2 | 0.95 | 26 | 0.077 |
| 1292 | A | 17 | 16 | 0.93 | 26 | 0.615 |
| 1293 | A | 2 | 2 | 0.96 | 26 | 0.077 |
| 1294 | A | 27 | 26 | 1.17 | 26 | 1.000 |
| 1295 | A | 2 | 2 | 0.99 | 22 | 0.091 |
| 1296 | A | 13 | 12 | 1.00 | 21 | 0.571 |
| 1297 | N/A | 9 | 0 | 1.00 | 24 | 0.000 |
| 1298 | A | 10 | 9 | 0.97 | 24 | 0.375 |
| 1299 | A | 2 | 2 | 1.03 | 24 | 0.083 |

# CHAPTER 3

## LISTING OF INTEGRALS

# 3.1 $\int x^3(d+icdx)(a+b\arctan(cx))\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 117

$$\int x^3(d+icdx)(a+b\arctan(cx))\,dx=\frac{bdx}{4c^3}+\frac{ibdx^2}{10c^2}-\frac{bdx^3}{12c}-\frac{1}{20}ibdx^4-\frac{bd\arctan(cx)}{4c^4}+\frac{1}{4}dx^4(a+b\arctan(cx))+\frac{1}{5}icdx^5(a+b\arctan(cx))-\frac{ibd\log(1+c^2x^2)}{10c^4}$$

output
```
1/4*b*d*x/c^3+1/10*I*b*d*x^2/c^2-1/12*b*d*x^3/c-1/20*I*b*d*x^4-1/4*b*d*arc
tan(c*x)/c^4+1/4*d*x^4*(a+b*arctan(c*x))+1/5*I*c*d*x^5*(a+b*arctan(c*x))-1
/10*I*b*d*ln(c^2*x^2+1)/c^4
```

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.84

$$\int x^3(d+icdx)(a+b\arctan(cx))\,dx$$
$$=\frac{d(3ac^4x^4(5+4icx)+bcx(15+6icx-5c^2x^2-3ic^3x^3)+3b(-5+5c^4x^4+4ic^5x^5)\arctan(cx)-6ib\log(}{60c^4}$$

input
```
Integrate[x^3*(d + I*c*d*x)*(a + b*ArcTan[c*x]),x]
```

output
```
(d*(3*a*c^4*x^4*(5 + (4*I)*c*x) + b*c*x*(15 + (6*I)*c*x - 5*c^2*x^2 - (3*I
)*c^3*x^3) + 3*b*(-5 + 5*c^4*x^4 + (4*I)*c^5*x^5)*ArcTan[c*x] - (6*I)*b*Lo
g[1 + c^2*x^2]))/(60*c^4)
```

## Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.91, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {5407, 27, 523, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(d+icdx)(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5407}$$

$$-bc\int \frac{dx^4(4icx+5)}{20\left(c^2x^2+1\right)}dx+\frac{1}{5}icdx^5(a+b\arctan(cx))+\frac{1}{4}dx^4(a+b\arctan(cx))$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{20}bcd\int \frac{x^4(4icx+5)}{c^2x^2+1}dx+\frac{1}{5}icdx^5(a+b\arctan(cx))+\frac{1}{4}dx^4(a+b\arctan(cx))$$

$$\Big\downarrow \text{523}$$

$$-\frac{1}{20}bcd\int \left(\frac{4ix^3}{c}+\frac{5x^2}{c^2}-\frac{4ix}{c^3}+\frac{4icx+5}{c^4\left(c^2x^2+1\right)}-\frac{5}{c^4}\right)dx+\frac{1}{5}icdx^5(a+b\arctan(cx))+\frac{1}{4}dx^4(a+b\arctan(cx))$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}icdx^5(a+b\arctan(cx))+\frac{1}{4}dx^4(a+b\arctan(cx))-\frac{1}{20}bcd\left(\frac{5\arctan(cx)}{c^5}-\frac{5x}{c^4}-\frac{2ix^2}{c^3}+\frac{5x^3}{3c^2}+\frac{2i\log\left(c^2x^2+1\right)}{c^5}+\frac{ix^4}{c}\right)$$

input
```
Int[x^3*(d + I*c*d*x)*(a + b*ArcTan[c*x]),x]
```

output
```
(d*x^4*(a + b*ArcTan[c*x]))/4 + (I/5)*c*d*x^5*(a + b*ArcTan[c*x]) - (b*c*d
*((-5*x)/c^4 - ((2*I)*x^2)/c^3 + (5*x^3)/(3*c^2) + (I*x^4)/c + (5*ArcTan[c
*x])/c^5 + ((2*I)*Log[1 + c^2*x^2])/c^5))/20
```

**Defintions of rubi rules used**

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 523
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_)))/((a_) + (b_.)*(x_)^2), x_Symbol] :> In
t[ExpandIntegrand[x^m*((c + d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}
, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5407
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Simp[(a
 + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ
[2*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0])
)
```

## Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.85

| method | result |
|---|---|
| parts | $da\left(\frac{1}{5}ic\,x^5+\frac{1}{4}x^4\right)+\frac{db\left(\frac{i\arctan(cx)c^5x^5}{5}+\frac{c^4x^4\arctan(cx)}{4}+\frac{cx}{4}-\frac{ic^4x^4}{20}-\frac{c^3x^3}{12}+\frac{ic^2x^2}{10}-\frac{i\ln\left(c^2x^2+1\right)}{10}-\frac{\arctan(cx)}{4}\right.}{c^4}$ |
| derivativedivides | $\frac{da\left(\frac{1}{5}ic^5x^5+\frac{1}{4}c^4x^4\right)+db\left(\frac{i\arctan(cx)c^5x^5}{5}+\frac{c^4x^4\arctan(cx)}{4}+\frac{cx}{4}-\frac{ic^4x^4}{20}-\frac{c^3x^3}{12}+\frac{ic^2x^2}{10}-\frac{i\ln\left(c^2x^2+1\right)}{10}-\frac{\arctan(cx)}{4}\right)}{c^4}$ |
| default | $\frac{da\left(\frac{1}{5}ic^5x^5+\frac{1}{4}c^4x^4\right)+db\left(\frac{i\arctan(cx)c^5x^5}{5}+\frac{c^4x^4\arctan(cx)}{4}+\frac{cx}{4}-\frac{ic^4x^4}{20}-\frac{c^3x^3}{12}+\frac{ic^2x^2}{10}-\frac{i\ln\left(c^2x^2+1\right)}{10}-\frac{\arctan(cx)}{4}\right)}{c^4}$ |
| parallelrisch | $-\frac{-12ic^5bd\arctan(cx)x^5-12ix^5a\,c^5d+3ix^4b\,c^4d-15db\arctan(cx)x^4c^4-15a\,c^4d\,x^4+5b\,c^3d\,x^3-6ix^2b\,c^2d+6ibd\ln(c^2}{60c^4}$ |
| risch | $\frac{db\left(4c\,x^5-5ix^4\right)\ln(icx+1)}{40}-\frac{dcb\,x^5\ln(-icx+1)}{10}+\frac{idc\,x^5a}{5}+\frac{da\,x^4}{4}+\frac{id\,x^4b\ln(-icx+1)}{8}-\frac{ibd\,x^4}{20}-\frac{bd\,x^3}{12c}+$ |

input
```
int(x^3*(d+I*c*d*x)*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

output
```
d*a*(1/5*I*c*x^5+1/4*x^4)+d*b/c^4*(1/5*I*arctan(c*x)*c^5*x^5+1/4*c^4*x^4*a
rctan(c*x)+1/4*c*x-1/20*I*c^4*x^4-1/12*c^3*x^3+1/10*I*c^2*x^2-1/10*I*ln(c^
2*x^2+1)-1/4*arctan(c*x))
```

## Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.06

$$\int x^3(d+icdx)(a+b\arctan(cx))\,dx$$
$$=\frac{24i\,ac^5dx^5+6\,(5\,a-i\,b)c^4dx^4-10\,bc^3dx^3+12i\,bc^2dx^2+30\,bcdx-27i\,bd\log\left(\frac{cx+i}{c}\right)+3i\,bd\log\left(\frac{cx-i}{c}\right)-}{120\,c^4}$$

input
```
integrate(x^3*(d+I*c*d*x)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
1/120*(24*I*a*c^5*d*x^5 + 6*(5*a - I*b)*c^4*d*x^4 - 10*b*c^3*d*x^3 + 12*I*
b*c^2*d*x^2 + 30*b*c*d*x - 27*I*b*d*log((c*x + I)/c) + 3*I*b*d*log((c*x -
I)/c) - 3*(4*b*c^5*d*x^5 - 5*I*b*c^4*d*x^4)*log(-(c*x + I)/(c*x - I)))/c^4
```

## Sympy [A] (verification not implemented)

Time = 1.86 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.57

$$\int x^3(d+icdx)(a+b\arctan(cx))\,dx = \frac{iacdx^5}{5} - \frac{bdx^3}{12c} + \frac{ibdx^2}{10c^2} + \frac{bdx}{4c^3} + \frac{bd\left(\frac{i\log(25bcdx-25ibd)}{40} - \frac{11i\log(25bcdx+25ibd)}{60}\right)}{c^4} + x^4\left(\frac{ad}{4} - \frac{ibd}{20}\right) + \left(\frac{bcdx^5}{10} - \frac{ibdx^4}{8}\right)\log(icx+1) + \frac{(-12bc^5dx^5 + 15ibc^4dx^4 - 5ibd)\log(-icx+1)}{120c^4}$$

input
```
integrate(x**3*(d+I*c*d*x)*(a+b*atan(c*x)),x)
```

output
```
I*a*c*d*x**5/5 - b*d*x**3/(12*c) + I*b*d*x**2/(10*c**2) + b*d*x/(4*c**3) +
 b*d*(I*log(25*b*c*d*x - 25*I*b*d)/40 - 11*I*log(25*b*c*d*x + 25*I*b*d)/60
)/c**4 + x**4*(a*d/4 - I*b*d/20) + (b*c*d*x**5/10 - I*b*d*x**4/8)*log(I*c*
x + 1) + (-12*b*c**5*d*x**5 + 15*I*b*c**4*d*x**4 - 5*I*b*d)*log(-I*c*x + 1
)/(120*c**4)
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.93

$$\int x^3(d+icdx)(a+b\arctan(cx))\,dx = \frac{1}{5}i\,acdx^5 + \frac{1}{4}\,adx^4 + \frac{1}{20}i\left(4\,x^5\arctan(cx) - c\left(\frac{c^2x^4-2\,x^2}{c^4} + \frac{2\,\log(c^2x^2+1)}{c^6}\right)\right)bcd + \frac{1}{12}\left(3\,x^4\arctan(cx) - c\left(\frac{c^2x^3-3\,x}{c^4} + \frac{3\,\arctan(cx)}{c^5}\right)\right)bd$$

input
```
integrate(x^3*(d+I*c*d*x)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
1/5*I*a*c*d*x^5 + 1/4*a*d*x^4 + 1/20*I*(4*x^5*arctan(c*x) - c*((c^2*x^4 -
2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*b*c*d + 1/12*(3*x^4*arctan(c*x) - c*
((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*b*d
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.96

$$\int x^3(d+icdx)(a+b\arctan(cx))\,dx = -\frac{-24i\,bc^5dx^5\arctan(cx)-24i\,ac^5dx^5-30\,bc^4dx^4\arctan(cx)-30\,ac^4dx^4+6i\,bc^4dx^4+10\,bc^3dx^3-1}{120\,c^4}$$

input
```
integrate(x^3*(d+I*c*d*x)*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
-1/120*(-24*I*b*c^5*d*x^5*arctan(c*x) - 24*I*a*c^5*d*x^5 - 30*b*c^4*d*x^4*
arctan(c*x) - 30*a*c^4*d*x^4 + 6*I*b*c^4*d*x^4 + 10*b*c^3*d*x^3 - 12*I*b*c
^2*d*x^2 - 30*b*c*d*x - 3*I*b*d*log(I*c*x + 1) + 27*I*b*d*log(-I*c*x + 1))
/c^4
```

## Mupad [B] (verification not implemented)

Time = 1.08 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.93

$$\int x^3(d+icdx)(a+b\arctan(cx))\,dx = -\frac{\frac{d\,(15\,b\,\mathrm{atan}(c\,x)+b\,\ln(c^2\,x^2+1)\,6\mathrm{i})}{60}-\frac{b\,c\,d\,x}{4}+\frac{b\,c^3\,d\,x^3}{12}-\frac{b\,c^2\,d\,x^2\,1\mathrm{i}}{10}}{c^4} + \frac{d\,(15\,a\,x^4+15\,b\,x^4\,\mathrm{atan}(c\,x)-b\,x^4\,3\mathrm{i})}{60} + \frac{c\,d\,(a\,x^5\,12\mathrm{i}+b\,x^5\,\mathrm{atan}(c\,x)\,12\mathrm{i})}{60}$$

input
```
int(x^3*(a + b*atan(c*x))*(d + c*d*x*1i),x)
```

output
```
(d*(15*a*x^4 - b*x^4*3i + 15*b*x^4*atan(c*x)))/60 - ((d*(15*b*atan(c*x) +
b*log(c^2*x^2 + 1)*6i))/60 - (b*c*d*x)/4 - (b*c^2*d*x^2*1i)/10 + (b*c^3*d*
x^3)/12)/c^4 + (c*d*(a*x^5*12i + b*x^5*atan(c*x)*12i))/60
```

## Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.92

$$\int x^3(d+icdx)(a+b\arctan(cx))\,dx$$
$$= \frac{d(12atan(cx)\,b\,c^5i\,x^5+15atan(cx)\,b\,c^4x^4-15atan(cx)\,b-6\log(c^2x^2+1)\,bi+12a\,c^5i\,x^5+15a\,c^4x^4-3}{60c^4}$$

input
```
int(x^3*(d+I*c*d*x)*(a+b*atan(c*x)),x)
```

output
```
(d*(12*atan(c*x)*b*c**5*i*x**5 + 15*atan(c*x)*b*c**4*x**4 - 15*atan(c*x)*b
 - 6*log(c**2*x**2 + 1)*b*i + 12*a*c**5*i*x**5 + 15*a*c**4*x**4 - 3*b*c**4
*i*x**4 - 5*b*c**3*x**3 + 6*b*c**2*i*x**2 + 15*b*c*x))/(60*c**4)
```

# 3.2 $\int x^2(d+icdx)(a+b\arctan(cx))\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 105

$$\begin{aligned}\int x^2(d+icdx)(a+b\arctan(cx))\,dx=&\frac{ibdx}{4c^2}-\frac{bdx^2}{6c}-\frac{1}{12}ibdx^3-\frac{ibd\arctan(cx)}{4c^3}\\&+\frac{1}{3}dx^3(a+b\arctan(cx))\\&+\frac{1}{4}icdx^4(a+b\arctan(cx))+\frac{bd\log(1+c^2x^2)}{6c^3}\end{aligned}$$

output
```
1/4*I*b*d*x/c^2-1/6*b*d*x^2/c-1/12*I*b*d*x^3-1/4*I*b*d*arctan(c*x)/c^3+1/3
*d*x^3*(a+b*arctan(c*x))+1/4*I*c*d*x^4*(a+b*arctan(c*x))+1/6*b*d*ln(c^2*x^
2+1)/c^3
```

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.84

$$\begin{aligned}&\int x^2(d+icdx)(a+b\arctan(cx))\,dx\\&=\frac{d(ac^3x^3(4+3icx)+bcx(3i-2cx-ic^2x^2)+b(-3i+4c^3x^3+3ic^4x^4)\arctan(cx)+2b\log(1+c^2x^2))}{12c^3}\end{aligned}$$

input
```
Integrate[x^2*(d + I*c*d*x)*(a + b*ArcTan[c*x]),x]
```

output
```
(d*(a*c^3*x^3*(4 + (3*I)*c*x) + b*c*x*(3*I - 2*c*x - I*c^2*x^2) + b*(-3*I
+ 4*c^3*x^3 + (3*I)*c^4*x^4)*ArcTan[c*x] + 2*b*Log[1 + c^2*x^2]))/(12*c^3)
```

## Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.91, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {5407, 27, 523, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(d+icdx)(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5407}$$

$$-bc\int \frac{dx^3(3icx+4)}{12\left(c^2x^2+1\right)}dx+\frac{1}{4}icdx^4(a+b\arctan(cx))+\frac{1}{3}dx^3(a+b\arctan(cx))$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{12}bcd\int \frac{x^3(3icx+4)}{c^2x^2+1}dx+\frac{1}{4}icdx^4(a+b\arctan(cx))+\frac{1}{3}dx^3(a+b\arctan(cx))$$

$$\Big\downarrow \text{523}$$

$$-\frac{1}{12}bcd\int \left(\frac{3ix^2}{c}+\frac{4x}{c^2}+\frac{3i-4cx}{c^3\left(c^2x^2+1\right)}-\frac{3i}{c^3}\right)dx+\frac{1}{4}icdx^4(a+b\arctan(cx))+\frac{1}{3}dx^3(a+b\arctan(cx))$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}icdx^4(a+b\arctan(cx))+\frac{1}{3}dx^3(a+b\arctan(cx))-\frac{1}{12}bcd\left(\frac{3i\arctan(cx)}{c^4}-\frac{3ix}{c^3}+\frac{2x^2}{c^2}-\frac{2\log\left(c^2x^2+1\right)}{c^4}+\frac{ix^3}{c}\right)$$

input
```
Int[x^2*(d + I*c*d*x)*(a + b*ArcTan[c*x]),x]
```

output
```
(d*x^3*(a + b*ArcTan[c*x]))/3 + (I/4)*c*d*x^4*(a + b*ArcTan[c*x]) - (b*c*d
*(((-3*I)*x)/c^3 + (2*x^2)/c^2 + (I*x^3)/c + ((3*I)*ArcTan[c*x])/c^4 - (2*
Log[1 + c^2*x^2])/c^4))/12
```

**Defintions of rubi rules used**

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 523
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_)))/((a_) + (b_.)*(x_)^2), x_Symbol] :> In
t[ExpandIntegrand[x^m*((c + d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}
, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5407
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Simp[(a
 + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ
[2*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0])
)
```

## Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.87

| method | result |
|---|---|
| parts | $da\left(\frac{1}{4}ic\,x^4+\frac{1}{3}x^3\right)+\frac{db\left(\frac{i\arctan(cx)c^4x^4}{4}+\frac{c^3x^3\arctan(cx)}{3}+\frac{icx}{4}-\frac{ic^3x^3}{12}-\frac{c^2x^2}{6}+\frac{\ln\left(c^2x^2+1\right)}{6}-\frac{i\arctan(cx)}{4}\right)}{c^3}$ |
| derivativedivides | $\frac{da\left(\frac{1}{4}ic^4x^4+\frac{1}{3}c^3x^3\right)+db\left(\frac{i\arctan(cx)c^4x^4}{4}+\frac{c^3x^3\arctan(cx)}{3}+\frac{icx}{4}-\frac{ic^3x^3}{12}-\frac{c^2x^2}{6}+\frac{\ln\left(c^2x^2+1\right)}{6}-\frac{i\arctan(cx)}{4}\right)}{c^3}$ |
| default | $\frac{da\left(\frac{1}{4}ic^4x^4+\frac{1}{3}c^3x^3\right)+db\left(\frac{i\arctan(cx)c^4x^4}{4}+\frac{c^3x^3\arctan(cx)}{3}+\frac{icx}{4}-\frac{ic^3x^3}{12}-\frac{c^2x^2}{6}+\frac{\ln\left(c^2x^2+1\right)}{6}-\frac{i\arctan(cx)}{4}\right)}{c^3}$ |
| parallelrisch | $\frac{3ib\,c^4d\arctan(cx)x^4+3ix^4a\,c^4d-ix^3b\,c^3d+4db\arctan(cx)x^3c^3+4a\,c^3d\,x^3-2b\,c^2d\,x^2+3ibdxc-3ibd\arctan(cx)+2bd\ln}{12c^3}$ |
| risch | $\frac{db(3c\,x^4-4ix^3)\ln(icx+1)}{24}+\frac{idca\,x^4}{4}-\frac{dc\,x^4b\ln(-icx+1)}{8}+\frac{idb\,x^3\ln(-icx+1)}{6}-\frac{ibd\,x^3}{12}+\frac{d\,x^3a}{3}-\frac{bd\,x^2}{6c}+$ |

input
```
int(x^2*(d+I*c*d*x)*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

output
```
d*a*(1/4*I*c*x^4+1/3*x^3)+d*b/c^3*(1/4*I*arctan(c*x)*c^4*x^4+1/3*c^3*x^3*a
rctan(c*x)+1/4*I*c*x-1/12*I*c^3*x^3-1/6*c^2*x^2+1/6*ln(c^2*x^2+1)-1/4*I*ar
ctan(c*x))
```

## Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.08

$$\int x^2(d+icdx)(a+b\arctan(cx))\,dx$$
$$=\frac{6i\,ac^4dx^4+2\,(4\,a-i\,b)c^3dx^3-4\,bc^2dx^2+6i\,bcdx+7\,bd\log\left(\frac{cx+i}{c}\right)+bd\log\left(\frac{cx-i}{c}\right)-(3\,bc^4dx^4-4i\,bc^3d}{24\,c^3}$$

input
```
integrate(x^2*(d+I*c*d*x)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
1/24*(6*I*a*c^4*d*x^4 + 2*(4*a - I*b)*c^3*d*x^3 - 4*b*c^2*d*x^2 + 6*I*b*c*
d*x + 7*b*d*log((c*x + I)/c) + b*d*log((c*x - I)/c) - (3*b*c^4*d*x^4 - 4*I
*b*c^3*d*x^3)*log(-(c*x + I)/(c*x - I)))/c^3
```

## Sympy [A] (verification not implemented)

Time = 1.70 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.59

$$\int x^2(d+icdx)(a+b\arctan(cx))\,dx = \frac{iacdx^4}{4} - \frac{bdx^2}{6c} + \frac{ibdx}{4c^2} + \frac{bd\left(\frac{\log(11bcdx-11ibd)}{24} + \frac{9\log(11bcdx+11ibd)}{40}\right)}{c^3} + x^3\left(\frac{ad}{3} - \frac{ibd}{12}\right) + \left(\frac{bcdx^4}{8} - \frac{ibdx^3}{6}\right)\log(icx+1) + \frac{(-15bc^4dx^4+20ibc^3dx^3+8bd)\log(-icx+1)}{120c^3}$$

input
```
integrate(x**2*(d+I*c*d*x)*(a+b*atan(c*x)),x)
```

output
```
I*a*c*d*x**4/4 - b*d*x**2/(6*c) + I*b*d*x/(4*c**2) + b*d*(log(11*b*c*d*x -
 11*I*b*d)/24 + 9*log(11*b*c*d*x + 11*I*b*d)/40)/c**3 + x**3*(a*d/3 - I*b*
d/12) + (b*c*d*x**4/8 - I*b*d*x**3/6)*log(I*c*x + 1) + (-15*b*c**4*d*x**4
+ 20*I*b*c**3*d*x**3 + 8*b*d)*log(-I*c*x + 1)/(120*c**3)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.94

$$\int x^2(d+icdx)(a+b\arctan(cx))\,dx = \frac{1}{4}i\,acdx^4 + \frac{1}{3}\,adx^3 + \frac{1}{12}i\left(3\,x^4\arctan(cx) - c\left(\frac{c^2x^3-3\,x}{c^4} + \frac{3\,\arctan(cx)}{c^5}\right)\right)bcd + \frac{1}{6}\left(2\,x^3\arctan(cx) - c\left(\frac{x^2}{c^2} - \frac{\log(c^2x^2+1)}{c^4}\right)\right)bd$$

input
```
integrate(x^2*(d+I*c*d*x)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
1/4*I*a*c*d*x^4 + 1/3*a*d*x^3 + 1/12*I*(3*x^4*arctan(c*x) - c*((c^2*x^3 -
3*x)/c^4 + 3*arctan(c*x)/c^5))*b*c*d + 1/6*(2*x^3*arctan(c*x) - c*(x^2/c^2
 - log(c^2*x^2 + 1)/c^4))*b*d
```

## Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.95

$$\int x^2(d+icdx)(a+b\arctan(cx))\,dx = -\frac{-6i\,bc^4dx^4\arctan{(cx)}-6i\,ac^4dx^4-8\,bc^3dx^3\arctan{(cx)}-8\,ac^3dx^3+2i\,bc^3dx^3+4\,bc^2dx^2-6i\,bcdx}{24\,c^3}$$

input
```
integrate(x^2*(d+I*c*d*x)*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
-1/24*(-6*I*b*c^4*d*x^4*arctan(c*x) - 6*I*a*c^4*d*x^4 - 8*b*c^3*d*x^3*arct
an(c*x) - 8*a*c^3*d*x^3 + 2*I*b*c^3*d*x^3 + 4*b*c^2*d*x^2 - 6*I*b*c*d*x -
7*b*d*log(c*x + I) - b*d*log(c*x - I))/c^3
```

## Mupad [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.94

$$\int x^2(d+icdx)(a+b\arctan(cx))\,dx = -\frac{\frac{d\,\left(-2\,b\,\ln\left(c^2\,x^2+1\right)+b\,\mathrm{atan}\left(c\,x\right)\,3{}\mathrm{i}\right)}{12}+\frac{b\,c^2\,d\,x^2}{6}-\frac{b\,c\,d\,x\,1{}\mathrm{i}}{4}}{c^3} + \frac{d\,\left(4\,a\,x^3+4\,b\,x^3\,\mathrm{atan}\left(c\,x\right)-b\,x^3\,1{}\mathrm{i}\right)}{12} + \frac{c\,d\,\left(a\,x^4\,3{}\mathrm{i}+b\,x^4\,\mathrm{atan}\left(c\,x\right)\,3{}\mathrm{i}\right)}{12}$$

input
```
int(x^2*(a + b*atan(c*x))*(d + c*d*x*1i),x)
```

output
```
(d*(4*a*x^3 - b*x^3*1i + 4*b*x^3*atan(c*x)))/12 - ((d*(b*atan(c*x)*3i - 2*
b*log(c^2*x^2 + 1)))/12 - (b*c*d*x*1i)/4 + (b*c^2*d*x^2)/6)/c^3 + (c*d*(a*
x^4*3i + b*x^4*atan(c*x)*3i))/12
```

## Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.94

$$\int x^2(d+icdx)(a+b\arctan(cx))\,dx$$
$$=\frac{d(3atan(cx)\,b\,c^4i\,x^4+4atan(cx)\,b\,c^3x^3-3atan(cx)\,bi+2\log(c^2x^2+1)\,b+3a\,c^4i\,x^4+4a\,c^3x^3-b\,c^3i\,x^3}{12c^3}$$

input
```
int(x^2*(d+I*c*d*x)*(a+b*atan(c*x)),x)
```

output
```
(d*(3*atan(c*x)*b*c**4*i*x**4 + 4*atan(c*x)*b*c**3*x**3 - 3*atan(c*x)*b*i
+ 2*log(c**2*x**2 + 1)*b + 3*a*c**4*i*x**4 + 4*a*c**3*x**3 - b*c**3*i*x**3
 - 2*b*c**2*x**2 + 3*b*c*i*x))/(12*c**3)
```

## 3.3 $\int x(d+icdx)(a+b\arctan(cx))\,dx$



### Optimal result

Integrand size = 19, antiderivative size = 91

$$\int x(d+icdx)(a+b\arctan(cx))\,dx=-\frac{bdx}{2c}-\frac{1}{6}ibdx^2+\frac{bd\arctan(cx)}{2c^2}+\frac{1}{2}dx^2(a+b\arctan(cx))+\frac{1}{3}icdx^3(a+b\arctan(cx))+\frac{ibd\log(1+c^2x^2)}{6c^2}$$

output
```
-1/2*b*d*x/c-1/6*I*b*d*x^2+1/2*b*d*arctan(c*x)/c^2+1/2*d*x^2*(a+b*arctan(c
*x))+1/3*I*c*d*x^3*(a+b*arctan(c*x))+1/6*I*b*d*ln(c^2*x^2+1)/c^2
```

### Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.84

$$\int x(d+icdx)(a+b\arctan(cx))\,dx=\frac{d(cx(b(-3-icx)+acx(3+2icx))+b(3+3c^2x^2+2ic^3x^3)\arctan(cx)+ib\log(1+c^2x^2))}{6c^2}$$

input
```
Integrate[x*(d + I*c*d*x)*(a + b*ArcTan[c*x]),x]
```

output
```
(d*(c*x*(b*(-3 - I*c*x) + a*c*x*(3 + (2*I)*c*x)) + b*(3 + 3*c^2*x^2 + (2*I
)*c^3*x^3)*ArcTan[c*x] + I*b*Log[1 + c^2*x^2]))/(6*c^2)
```

## Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {5407, 27, 523, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(d+icdx)(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5407}$$

$$-bc\int \frac{dx^2(2icx+3)}{6\left(c^2x^2+1\right)}dx+\frac{1}{3}icdx^3(a+b\arctan(cx))+\frac{1}{2}dx^2(a+b\arctan(cx))$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{6}bcd\int \frac{x^2(2icx+3)}{c^2x^2+1}dx+\frac{1}{3}icdx^3(a+b\arctan(cx))+\frac{1}{2}dx^2(a+b\arctan(cx))$$

$$\Big\downarrow \text{523}$$

$$-\frac{1}{6}bcd\int \left(\frac{2ix}{c}+\frac{i(3i-2cx)}{c^2\left(c^2x^2+1\right)}+\frac{3}{c^2}\right)dx+\frac{1}{3}icdx^3(a+b\arctan(cx))+\frac{1}{2}dx^2(a+b\arctan(cx))$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}icdx^3(a+b\arctan(cx))+\frac{1}{2}dx^2(a+b\arctan(cx))-\frac{1}{6}bcd\left(-\frac{3\arctan(cx)}{c^3}+\frac{3x}{c^2}-\frac{i\log\left(c^2x^2+1\right)}{c^3}+\frac{ix^2}{c}\right)$$

input
```
Int[x*(d + I*c*d*x)*(a + b*ArcTan[c*x]),x]
```

output
```
(d*x^2*(a + b*ArcTan[c*x]))/2 + (I/3)*c*d*x^3*(a + b*ArcTan[c*x]) - (b*c*d
*((3*x)/c^2 + (I*x^2)/c - (3*ArcTan[c*x])/c^3 - (I*Log[1 + c^2*x^2])/c^3))
/6
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 523
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_)))/((a_) + (b_.)*(x_)^2), x_Symbol] :> In
t[ExpandIntegrand[x^m*((c + d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}
, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5407
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Simp[(a
 + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ
[2*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0])
)
```

## Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.90

| method | result |
|---|---|
| parts | $da\left(\frac{1}{3}ic\,x^3+\frac{1}{2}x^2\right)+\frac{db\left(\frac{i\arctan(cx)c^3x^3}{3}+\frac{c^2x^2\arctan(cx)}{2}-\frac{ic^2x^2}{6}-\frac{cx}{2}+\frac{i\ln\left(c^2x^2+1\right)}{6}+\frac{\arctan(cx)}{2}\right)}{c^2}$ |
| derivativedivides | $\frac{da\left(\frac{1}{3}ic^3x^3+\frac{1}{2}c^2x^2\right)+db\left(\frac{i\arctan(cx)c^3x^3}{3}+\frac{c^2x^2\arctan(cx)}{2}-\frac{ic^2x^2}{6}-\frac{cx}{2}+\frac{i\ln\left(c^2x^2+1\right)}{6}+\frac{\arctan(cx)}{2}\right)}{c^2}$ |
| default | $\frac{da\left(\frac{1}{3}ic^3x^3+\frac{1}{2}c^2x^2\right)+db\left(\frac{i\arctan(cx)c^3x^3}{3}+\frac{c^2x^2\arctan(cx)}{2}-\frac{ic^2x^2}{6}-\frac{cx}{2}+\frac{i\ln\left(c^2x^2+1\right)}{6}+\frac{\arctan(cx)}{2}\right)}{c^2}$ |
| parallelrisch | $\frac{2ic^3bd\arctan(cx)x^3+2ix^3a\,c^3d-ix^2b\,c^2d+3db\arctan(cx)x^2c^2+3a\,c^2d\,x^2+ibd\ln(c^2x^2+1)-3bcdx+3bd\arctan(cx)}{6c^2}$ |
| risch | $\frac{db\left(2c\,x^3-3ix^2\right)\ln(icx+1)}{12}-\frac{dcb\,x^3\ln(-icx+1)}{6}+\frac{ia\,x^3cd}{3}+\frac{da\,x^2}{2}+\frac{id\,x^2b\ln(-icx+1)}{4}-\frac{ibd\,x^2}{6}-\frac{bdx}{2c}+$ |

input
```
int(x*(d+I*c*d*x)*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

output
```
d*a*(1/3*I*c*x^3+1/2*x^2)+d*b/c^2*(1/3*I*arctan(c*x)*c^3*x^3+1/2*c^2*x^2*a
rctan(c*x)-1/6*I*c^2*x^2-1/2*c*x+1/6*I*ln(c^2*x^2+1)+1/2*arctan(c*x))
```

## Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.14

$$\int x(d+icdx)(a+b\arctan(cx))\,dx = \frac{4i\,ac^3dx^3+2\,(3\,a-i\,b)c^2dx^2-6\,bcdx+5i\,bd\log\left(\frac{cx+i}{c}\right)-i\,bd\log\left(\frac{cx-i}{c}\right)-(2\,bc^3dx^3-3i\,bc^2dx^2)\log\left(-\right.}{12\,c^2}$$

input
```
integrate(x*(d+I*c*d*x)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
1/12*(4*I*a*c^3*d*x^3 + 2*(3*a - I*b)*c^2*d*x^2 - 6*b*c*d*x + 5*I*b*d*log(
(c*x + I)/c) - I*b*d*log((c*x - I)/c) - (2*b*c^3*d*x^3 - 3*I*b*c^2*d*x^2)*
log(-(c*x + I)/(c*x - I)))/c^2
```

## Sympy [A] (verification not implemented)

Time = 1.75 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.74

$$\int x(d+icdx)(a+b\arctan(cx))\,dx = \frac{iacdx^3}{3} - \frac{bdx}{2c} + \frac{bd\left(-\frac{i\log(9bcdx-9ibd)}{12}+\frac{7i\log(9bcdx+9ibd)}{24}\right)}{c^2} + x^2\left(\frac{ad}{2}-\frac{ibd}{6}\right) + \left(\frac{bcdx^3}{6}-\frac{ibdx^2}{4}\right)\log(icx+1) + \frac{(-4bc^3dx^3+6ibc^2dx^2+3ibd)\log(-icx+1)}{24c^2}$$

input
```
integrate(x*(d+I*c*d*x)*(a+b*atan(c*x)),x)
```

output
```
I*a*c*d*x**3/3 - b*d*x/(2*c) + b*d*(-I*log(9*b*c*d*x - 9*I*b*d)/12 + 7*I*l
og(9*b*c*d*x + 9*I*b*d)/24)/c**2 + x**2*(a*d/2 - I*b*d/6) + (b*c*d*x**3/6
- I*b*d*x**2/4)*log(I*c*x + 1) + (-4*b*c**3*d*x**3 + 6*I*b*c**2*d*x**2 + 3
*I*b*d)*log(-I*c*x + 1)/(24*c**2)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.97

$$\int x(d+icdx)(a+b\arctan(cx))\,dx$$
$$=\frac{1}{3}i\,acdx^3+\frac{1}{6}i\left(2\,x^3\arctan\left(cx\right)-c\left(\frac{x^2}{c^2}-\frac{\log\left(c^2x^2+1\right)}{c^4}\right)\right)bcd$$
$$+\frac{1}{2}\,adx^2+\frac{1}{2}\left(x^2\arctan\left(cx\right)-c\left(\frac{x}{c^2}-\frac{\arctan\left(cx\right)}{c^3}\right)\right)bd$$

input
```
integrate(x*(d+I*c*d*x)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
1/3*I*a*c*d*x^3 + 1/6*I*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)
/c^4))*b*c*d + 1/2*a*d*x^2 + 1/2*(x^2*arctan(c*x) - c*(x/c^2 - arctan(c*x)
/c^3))*b*d
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.01

$$\int x(d+icdx)(a+b\arctan(cx))\,dx=$$
$$-\frac{-4i\,bc^3dx^3\arctan\left(cx\right)-4i\,ac^3dx^3-6\,bc^2dx^2\arctan\left(cx\right)-6\,ac^2dx^2+2i\,bc^2dx^2+6\,bcdx-5i\,bd\log\left(\right.}{12\,c^2}$$

input
```
integrate(x*(d+I*c*d*x)*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
-1/12*(-4*I*b*c^3*d*x^3*arctan(c*x) - 4*I*a*c^3*d*x^3 - 6*b*c^2*d*x^2*arct
an(c*x) - 6*a*c^2*d*x^2 + 2*I*b*c^2*d*x^2 + 6*b*c*d*x - 5*I*b*d*log(I*c*x
- 1) + I*b*d*log(-I*c*x - 1))/c^2
```

## Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.96

$$\int x(d+icdx)(a+b\arctan(cx))\,dx = \frac{d\,(3\,a\,x^2+3\,b\,x^2\,\mathrm{atan}(c\,x)-b\,x^2\,1\mathrm{i})}{6} + \frac{\frac{d\,(3\,b\,\mathrm{atan}(c\,x)+b\,\ln(c^2\,x^2+1)\,1\mathrm{i})}{6}-\frac{b\,c\,d\,x}{2}}{c^2} + \frac{c\,d\,(a\,x^3\,2\mathrm{i}+b\,x^3\,\mathrm{atan}(c\,x)\,2\mathrm{i})}{6}$$

input
```
int(x*(a + b*atan(c*x))*(d + c*d*x*1i),x)
```

output
```
(d*(3*a*x^2 - b*x^2*1i + 3*b*x^2*atan(c*x)))/6 + ((d*(3*b*atan(c*x) + b*lo
g(c^2*x^2 + 1)*1i))/6 - (b*c*d*x)/2)/c^2 + (c*d*(a*x^3*2i + b*x^3*atan(c*x
)*2i))/6
```

## Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.97

$$\int x(d+icdx)(a+b\arctan(cx))\,dx = \frac{d(2atan(cx)\,b\,c^3i\,x^3+3atan(cx)\,b\,c^2x^2+3atan(cx)\,b+\log(c^2x^2+1)\,bi+2a\,c^3i\,x^3+3a\,c^2x^2-b\,c^2i\,x^2}{6c^2}$$

input
```
int(x*(d+I*c*d*x)*(a+b*atan(c*x)),x)
```

output
```
(d*(2*atan(c*x)*b*c**3*i*x**3 + 3*atan(c*x)*b*c**2*x**2 + 3*atan(c*x)*b +
log(c**2*x**2 + 1)*b*i + 2*a*c**3*i*x**3 + 3*a*c**2*x**2 - b*c**2*i*x**2 -
 3*b*c*x))/(6*c**2)
```

# 3.4 $\int (d+icdx)(a+b\arctan(cx))\,dx$



## Optimal result

Integrand size = 18, antiderivative size = 53

$$\int (d+icdx)(a+b\arctan(cx))\,dx = -\frac{1}{2}ibdx - \frac{id(1+icx)^2(a+b\arctan(cx))}{2c} - \frac{bd\log(i+cx)}{c}$$

output
```
-1/2*I*b*d*x-1/2*I*d*(1+I*c*x)^2*(a+b*arctan(c*x))/c-b*d*ln(I+c*x)/c
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.58

$$\int (d+icdx)(a+b\arctan(cx))\,dx = adx - \frac{1}{2}ibdx + \frac{1}{2}iacdx^2 + \frac{ibd\arctan(cx)}{2c} + bdx\arctan(cx) + \frac{1}{2}ibcdx^2\arctan(cx) - \frac{bd\log(1+c^2x^2)}{2c}$$

input
```
Integrate[(d + I*c*d*x)*(a + b*ArcTan[c*x]),x]
```

output
```
a*d*x - (I/2)*b*d*x + (I/2)*a*c*d*x^2 + ((I/2)*b*d*ArcTan[c*x])/c + b*d*x*
ArcTan[c*x] + (I/2)*b*c*d*x^2*ArcTan[c*x] - (b*d*Log[1 + c^2*x^2])/(2*c)
```

## Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {5387, 27, 456, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+icdx)(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5387}$$

$$\frac{ib\int \frac{d^2(icx+1)^2}{c^2x^2+1}dx}{2d}-\frac{id(1+icx)^2(a+b\arctan(cx))}{2c}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}ibd\int \frac{(icx+1)^2}{c^2x^2+1}dx-\frac{id(1+icx)^2(a+b\arctan(cx))}{2c}$$

$$\Big\downarrow \text{456}$$

$$\frac{1}{2}ibd\int \frac{icx+1}{1-icx}dx-\frac{id(1+icx)^2(a+b\arctan(cx))}{2c}$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}ibd\int \left(\frac{2i}{cx+i}-1\right)dx-\frac{id(1+icx)^2(a+b\arctan(cx))}{2c}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}ibd\left(-x+\frac{2i\log(cx+i)}{c}\right)-\frac{id(1+icx)^2(a+b\arctan(cx))}{2c}$$

input
```
Int[(d + I*c*d*x)*(a + b*ArcTan[c*x]),x]
```

output
```
((-1/2*I)*d*(1 + I*c*x)^2*(a + b*ArcTan[c*x]))/c + (I/2)*b*d*(-x + ((2*I)*
Log[I + c*x])/c)
```

**Defintions of rubi rules used**

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5387
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Simp[b*(
c/(e*(q + 1)))   Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b
, c, d, e, q}, x] && NeQ[q, -1]
```

## Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.32

| method | result |
|---|---|
| parts | $-ida\left(-\frac{1}{2}c\,x^2+ix\right)+\frac{db\left(\frac{i\arctan(cx)c^2x^2}{2}+cx\arctan(cx)+\frac{i\left(-cx+i\ln\left(c^2x^2+1\right)+\arctan(cx)\right)}{2}\right)}{c}$ |
| derivativedivides | $\frac{-ida\left(-\frac{1}{2}c^2x^2+icx\right)+db\left(\frac{i\arctan(cx)c^2x^2}{2}+cx\arctan(cx)+\frac{i\left(-cx+i\ln\left(c^2x^2+1\right)+\arctan(cx)\right)}{2}\right)}{c}$ |
| default | $\frac{-ida\left(-\frac{1}{2}c^2x^2+icx\right)+db\left(\frac{i\arctan(cx)c^2x^2}{2}+cx\arctan(cx)+\frac{i\left(-cx+i\ln\left(c^2x^2+1\right)+\arctan(cx)\right)}{2}\right)}{c}$ |
| parallelrisch | $\frac{ib\,c^2d\arctan(cx)x^2+ia\,c^2d\,x^2-ibdxc+2x\arctan(cx)bcd+ibd\arctan(cx)+2acdx-bd\ln\left(c^2x^2+1\right)}{2c}$ |
| risch | $\frac{db\left(c\,x^2-2ix\right)\ln(icx+1)}{4}+\frac{ia\,x^2cd}{2}-\frac{dc\,x^2b\ln(-icx+1)}{4}+\frac{ibdx\ln(-icx+1)}{2}-\frac{ibdx}{2}+\frac{id\arctan(cx)b}{2c}+adx$ |

input
```
int((d+I*c*d*x)*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

output
```
-I*d*a*(-1/2*c*x^2+I*x)+d*b/c*(1/2*I*arctan(c*x)*c^2*x^2+c*x*arctan(c*x)+1
/2*I*(-c*x+I*ln(c^2*x^2+1)+arctan(c*x)))
```

## Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 89 vs. 2(41) = 82.

Time = 0.10 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.68

$$\int (d+icdx)(a+b\arctan(cx))\,dx$$
$$=\frac{2i\,ac^2dx^2+2\,(2\,a-i\,b)cdx-3\,bd\log\left(\frac{cx+i}{c}\right)-bd\log\left(\frac{cx-i}{c}\right)-\left(bc^2dx^2-2i\,bcdx\right)\log\left(-\frac{cx+i}{cx-i}\right)}{4\,c}$$

input
```
integrate((d+I*c*d*x)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
1/4*(2*I*a*c^2*d*x^2 + 2*(2*a - I*b)*c*d*x - 3*b*d*log((c*x + I)/c) - b*d*
log((c*x - I)/c) - (b*c^2*d*x^2 - 2*I*b*c*d*x)*log(-(c*x + I)/(c*x - I)))/
c
```

## Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(44) = 88$.

Time = 1.37 (sec) , antiderivative size = 128, normalized size of antiderivative = 2.42

$$\int (d+icdx)(a+b\arctan(cx))\,dx = \frac{iacdx^2}{2} + \frac{bd\left(-\frac{\log(bcdx-ibd)}{4} - \frac{5\log(bcdx+ibd)}{12}\right)}{c} + x\left(ad-\frac{ibd}{2}\right) + \left(\frac{bcdx^2}{4}-\frac{ibdx}{2}\right)\log(icx+1) + \frac{(-3bc^2dx^2+6ibcdx-4bd)\log(-icx+1)}{12c}$$

input
```
integrate((d+I*c*d*x)*(a+b*atan(c*x)),x)
```

output
```
I*a*c*d*x**2/2 + b*d*(-log(b*c*d*x - I*b*d)/4 - 5*log(b*c*d*x + I*b*d)/12)
/c + x*(a*d - I*b*d/2) + (b*c*d*x**2/4 - I*b*d*x/2)*log(I*c*x + 1) + (-3*b
*c**2*d*x**2 + 6*I*b*c*d*x - 4*b*d)*log(-I*c*x + 1)/(12*c)
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.38

$$\int (d+icdx)(a+b\arctan(cx))\,dx = \frac{1}{2}i\,acdx^2 + \frac{1}{2}i\left(x^2\arctan(cx) - c\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)\right)bcd + adx + \frac{(2\,cx\arctan(cx) - \log(c^2x^2+1))bd}{2\,c}$$

input
```
integrate((d+I*c*d*x)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
1/2*I*a*c*d*x^2 + 1/2*I*(x^2*arctan(c*x) - c*(x/c^2 - arctan(c*x)/c^3))*b*
c*d + a*d*x + 1/2*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b*d/c
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.34

$$\int (d+icdx)(a+b\arctan(cx))\,dx = -\frac{-2i\,bc^2dx^2\arctan{(cx)}-2i\,ac^2dx^2-4\,bcdx\arctan{(cx)}-4\,acdx+2i\,bcdx+3\,bd\log{(cx+i)}+bd\log}{4\,c}$$

input
```
integrate((d+I*c*d*x)*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
-1/4*(-2*I*b*c^2*d*x^2*arctan(c*x) - 2*I*a*c^2*d*x^2 - 4*b*c*d*x*arctan(c*
x) - 4*a*c*d*x + 2*I*b*c*d*x + 3*b*d*log(c*x + I) + b*d*log(c*x - I))/c
```

## Mupad [B] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.38

$$\int (d+icdx)(a+b\arctan(cx))\,dx = \frac{d\,(2\,a\,x+2\,b\,x\,\mathrm{atan}(c\,x)-b\,x\,1\mathrm{i})}{2} + \frac{c\,d\,(a\,x^2\,1\mathrm{i}+b\,x^2\,\mathrm{atan}(c\,x)\,1\mathrm{i})}{2} + \frac{d\,(-b\,\ln{(c^2\,x^2+1)}+b\,\mathrm{atan}(c\,x)\,1\mathrm{i})}{2\,c}$$

input
```
int((a + b*atan(c*x))*(d + c*d*x*1i),x)
```

output
```
(d*(2*a*x - b*x*1i + 2*b*x*atan(c*x)))/2 + (c*d*(a*x^2*1i + b*x^2*atan(c*x
)*1i))/2 + (d*(b*atan(c*x)*1i - b*log(c^2*x^2 + 1)))/(2*c)
```

## Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.30

$$\int (d+icdx)(a+b\arctan(cx))\,dx$$
$$=\frac{d(atan(cx)\,b\,c^2i\,x^2+2atan(cx)\,bcx+atan(cx)\,bi-\log(c^2x^2+1)\,b+a\,c^2i\,x^2+2acx-bcix)}{2c}$$

input
```
int((d+I*c*d*x)*(a+b*atan(c*x)),x)
```

output
```
(d*(atan(c*x)*b*c**2*i*x**2 + 2*atan(c*x)*b*c*x + atan(c*x)*b*i - log(c**2
*x**2 + 1)*b + a*c**2*i*x**2 + 2*a*c*x - b*c*i*x))/(2*c)
```

# 3.5 $\int \frac{(d+icdx)(a+b\arctan(cx))}{x}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 76

$$\int \frac{(d+icdx)(a+b\arctan(cx))}{x}\,dx = iacdx + ibcdx\arctan(cx) + ad\log(x) - \frac{1}{2}ibd\log\left(1+c^2x^2\right) + \frac{1}{2}ibd\,\text{PolyLog}(2,-icx) - \frac{1}{2}ibd\,\text{PolyLog}(2,icx)$$

output
```
I*a*c*d*x+I*b*c*d*x*arctan(c*x)+a*d*ln(x)-1/2*I*b*d*ln(c^2*x^2+1)+1/2*I*b*
d*polylog(2,-I*c*x)-1/2*I*b*d*polylog(2,I*c*x)
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

$$\int \frac{(d+icdx)(a+b\arctan(cx))}{x}\,dx = iacdx + ibcdx\arctan(cx) + ad\log(x) - \frac{1}{2}ibd\log\left(1+c^2x^2\right) + \frac{1}{2}ibd\,\text{PolyLog}(2,-icx) - \frac{1}{2}ibd\,\text{PolyLog}(2,icx)$$

input
```
Integrate[((d + I*c*d*x)*(a + b*ArcTan[c*x]))/x,x]
```

output
```
I*a*c*d*x + I*b*c*d*x*ArcTan[c*x] + a*d*Log[x] - (I/2)*b*d*Log[1 + c^2*x^2
] + (I/2)*b*d*PolyLog[2, (-I)*c*x] - (I/2)*b*d*PolyLog[2, I*c*x]
```

## Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)(a+b\arctan(cx))}{x}dx$$

$$\Big\downarrow \text{5411}$$

$$\int \left(\frac{d(a+b\arctan(cx))}{x}+icd(a+b\arctan(cx))\right)dx$$

$$\Big\downarrow \text{2009}$$

$$iacdx+ad\log(x)+ibcdx\arctan(cx)-\frac{1}{2}ibd\log\left(c^2x^2+1\right)+\frac{1}{2}ibd\,\text{PolyLog}(2,-icx)-\frac{1}{2}ibd\,\text{PolyLog}(2,icx)$$

input
```
Int[((d + I*c*d*x)*(a + b*ArcTan[c*x]))/x,x]
```

output
```
I*a*c*d*x + I*b*c*d*x*ArcTan[c*x] + a*d*Log[x] - (I/2)*b*d*Log[1 + c^2*x^2
] + (I/2)*b*d*PolyLog[2, (-I)*c*x] - (I/2)*b*d*PolyLog[2, I*c*x]
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

## Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.32

| method | result |
|---|---|
| parts | $da(icx+\ln(x))+db\left(i\arctan(cx)\,cx+\ln(cx)\arctan(cx)+\frac{i\ln(cx)\ln(icx+1)}{2}-\frac{i\ln(cx)\ln}{2}\right.$ |
| derivativedivides | $da(icx+\ln(cx))+db\left(i\arctan(cx)\,cx+\ln(cx)\arctan(cx)+\frac{i\ln(cx)\ln(icx+1)}{2}-\frac{i\ln(cx)}{}\right.$ |
| default | $da(icx+\ln(cx))+db\left(i\arctan(cx)\,cx+\ln(cx)\arctan(cx)+\frac{i\ln(cx)\ln(icx+1)}{2}-\frac{i\ln(cx)}{}\right.$ |
| risch | $\frac{\ln(icx+1)bcdx}{2}+\frac{i\,\mathrm{dilog}(icx+1)bd}{2}-\frac{i\ln(icx+1)bd}{2}+ibd-\frac{\ln(-icx+1)bcdx}{2}+\ln(-icx)\,ad+iacdx$ |

input
```
int((d+I*c*d*x)*(a+b*arctan(c*x))/x,x,method=_RETURNVERBOSE)
```

output
```
d*a*(I*c*x+ln(x))+d*b*(I*arctan(c*x)*c*x+ln(c*x)*arctan(c*x)+1/2*I*ln(c*x)
*ln(1+I*c*x)-1/2*I*ln(c*x)*ln(1-I*c*x)+1/2*I*dilog(1+I*c*x)-1/2*I*dilog(1-
I*c*x)-1/2*I*ln(c^2*x^2+1))
```

## Fricas [F]

$$\int \frac{(d+icdx)(a+b\arctan(cx))}{x}\,dx = \int \frac{(i\,cdx+d)(b\arctan(cx)+a)}{x}\,dx$$

input
```
integrate((d+I*c*d*x)*(a+b*arctan(c*x))/x,x, algorithm="fricas")
```

output
```
integral(1/2*(2*I*a*c*d*x + 2*a*d - (b*c*d*x - I*b*d)*log(-(c*x + I)/(c*x
- I)))/x, x)
```

## Sympy [F]

$$\int \frac{(d+icdx)(a+b\arctan(cx))}{x}\,dx = id\left(\int ac\,dx + \int \left(-\frac{ia}{x}\right)\,dx + \int bc\,\mathrm{atan}(cx)\,dx + \int \left(-\frac{ib\,\mathrm{atan}(cx)}{x}\right)\,dx\right)$$

input
```
integrate((d+I*c*d*x)*(a+b*atan(c*x))/x,x)
```

output
```
I*d*(Integral(a*c, x) + Integral(-I*a/x, x) + Integral(b*c*atan(c*x), x) +
 Integral(-I*b*atan(c*x)/x, x))
```

## Maxima [F]

$$\int \frac{(d+icdx)(a+b\arctan(cx))}{x}\,dx = \int \frac{(i\,cdx+d)(b\arctan(cx)+a)}{x}\,dx$$

input
```
integrate((d+I*c*d*x)*(a+b*arctan(c*x))/x,x, algorithm="maxima")
```

output
```
I*a*c*d*x + 1/2*I*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b*d + b*d*integra
te(arctan(c*x)/x, x) + a*d*log(x)
```

## Giac [F]

$$\int \frac{(d+icdx)(a+b\arctan(cx))}{x}\,dx = \int \frac{(i\,cdx+d)(b\arctan{(cx)}+a)}{x}\,dx$$

input
```
integrate((d+I*c*d*x)*(a+b*arctan(c*x))/x,x, algorithm="giac")
```

output
```
integrate((I*c*d*x + d)*(b*arctan(c*x) + a)/x, x)
```

## Mupad [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.83

$$\int \frac{(d+icdx)(a+b\arctan(cx))}{x}\,dx = -\frac{b\,d\,(\ln{(c^2\,x^2+1)}\;1\mathrm{i}-c\,x\,\mathrm{atan}(c\,x)\;2\mathrm{i})}{2} + a\,d\,(\ln{(x)}+c\,x\,1\mathrm{i}) - \frac{b\,d\,(\mathrm{Li}_2(1-c\,x\,1\mathrm{i})-\mathrm{Li}_2(1+c\,x\,1\mathrm{i}))\;1\mathrm{i}}{2}$$

input
```
int(((a + b*atan(c*x))*(d + c*d*x*1i))/x,x)
```

output
```
a*d*(log(x) + c*x*1i) - (b*d*(log(c^2*x^2 + 1)*1i - c*x*atan(c*x)*2i))/2 -
 (b*d*(dilog(1 - c*x*1i) - dilog(c*x*1i + 1))*1i)/2
```

## Reduce [F]

$$\int \frac{(d+icdx)(a+b\arctan(cx))}{x}\,dx = \frac{d\left(2atan(cx)\,bcix+2\left(\int \frac{atan(cx)}{x}dx\right)b-\log(c^2x^2+1)\,bi+2\log(x)\,a+2acix\right)}{2}$$

input
```
int((d+I*c*d*x)*(a+b*atan(c*x))/x,x)
```

output
```
(d*(2*atan(c*x)*b*c*i*x + 2*int(atan(c*x)/x,x)*b - log(c**2*x**2 + 1)*b*i
+ 2*log(x)*a + 2*a*c*i*x))/2
```

# 3.6 $\int \frac{(d+icdx)(a+b\arctan(cx))}{x^2}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 77

$$\int \frac{(d+icdx)(a+b\arctan(cx))}{x^2}\,dx = -\frac{d(a+b\arctan(cx))}{x} + iacd\log(x) + bcd\log(x) - \frac{1}{2}bcd\log\left(1+c^2x^2\right) - \frac{1}{2}bcd\,\mathrm{PolyLog}(2,-icx) + \frac{1}{2}bcd\,\mathrm{PolyLog}(2,icx)$$

output
```
-d*(a+b*arctan(c*x))/x+I*a*c*d*ln(x)+b*c*d*ln(x)-1/2*b*c*d*ln(c^2*x^2+1)-1
/2*b*c*d*polylog(2,-I*c*x)+1/2*b*c*d*polylog(2,I*c*x)
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.97

$$\int \frac{(d+icdx)(a+b\arctan(cx))}{x^2}\,dx = \frac{d(-2a-2b\arctan(cx)+2iacx\log(x)+2bcx\log(x)-bcx\log\left(1+c^2x^2\right)-bcx\,\mathrm{PolyLog}(2,-icx)+bcx}{2x}$$

input
```
Integrate[((d + I*c*d*x)*(a + b*ArcTan[c*x]))/x^2,x]
```

output
```
(d*(-2*a - 2*b*ArcTan[c*x] + (2*I)*a*c*x*Log[x] + 2*b*c*x*Log[x] - b*c*x*L
og[1 + c^2*x^2] - b*c*x*PolyLog[2, (-I)*c*x] + b*c*x*PolyLog[2, I*c*x]))/(
2*x)
```

## Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)(a+b\arctan(cx))}{x^2}dx$$

$$\Big\downarrow \text{5411}$$

$$\int \left(\frac{d(a+b\arctan(cx))}{x^2}+\frac{icd(a+b\arctan(cx))}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d(a+b\arctan(cx))}{x}+iacd\log(x)-\frac{1}{2}bcd\log\left(c^2x^2+1\right)-\frac{1}{2}bcd\operatorname{PolyLog}(2,-icx)+\frac{1}{2}bcd\operatorname{PolyLog}(2,icx)+bcd\log(x)$$

input
```
Int[((d + I*c*d*x)*(a + b*ArcTan[c*x]))/x^2,x]
```

output
```
-((d*(a + b*ArcTan[c*x]))/x) + I*a*c*d*Log[x] + b*c*d*Log[x] - (b*c*d*Log[
1 + c^2*x^2])/2 - (b*c*d*PolyLog[2, (-I)*c*x])/2 + (b*c*d*PolyLog[2, I*c*x
])/2
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

## Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.42

| method | result |
|---|---|
| parts | $da\left(-\frac{1}{x}+ic\ln\left(x\right)\right)+dbc\left(-\frac{\arctan(cx)}{cx}+i\arctan\left(cx\right)\ln\left(cx\right)-\frac{\ln(cx)\ln(icx+1)}{2}+\frac{\ln(cx)\ln(-}{2}\right.$ |
| derivativedivides | $c\left(da\left(-\frac{1}{cx}+i\ln\left(cx\right)\right)+db\left(-\frac{\arctan(cx)}{cx}+i\arctan\left(cx\right)\ln\left(cx\right)-\frac{\ln(cx)\ln(icx+1)}{2}+\frac{\ln(cx)\ln}{}\right.\right.$ |
| default | $c\left(da\left(-\frac{1}{cx}+i\ln\left(cx\right)\right)+db\left(-\frac{\arctan(cx)}{cx}+i\arctan\left(cx\right)\ln\left(cx\right)-\frac{\ln(cx)\ln(icx+1)}{2}+\frac{\ln(cx)\ln}{}\right.\right.$ |
| risch | $\frac{cdb\ln(icx)}{2}-\frac{cdb\ln(icx+1)}{2}+\frac{idb\ln(icx+1)}{2x}-\frac{cdb\,\mathrm{dilog}(icx+1)}{2}-\frac{da}{x}+icda\ln\left(-icx\right)+\frac{cdb\ln(-icx)}{2}$ |

input
```
int((d+I*c*d*x)*(a+b*arctan(c*x))/x^2,x,method=_RETURNVERBOSE)
```

output
```
d*a*(-1/x+I*c*ln(x))+d*b*c*(-1/c/x*arctan(c*x)+I*arctan(c*x)*ln(c*x)-1/2*l
n(c*x)*ln(1+I*c*x)+1/2*ln(c*x)*ln(1-I*c*x)-1/2*dilog(1+I*c*x)+1/2*dilog(1-
I*c*x)-1/2*ln(c^2*x^2+1)+ln(c*x))
```

## Fricas [F]

$$\int \frac{(d+icdx)(a+b\arctan(cx))}{x^2}\,dx=\int \frac{(i\,cdx+d)(b\arctan(cx)+a)}{x^2}\,dx$$

input
```
integrate((d+I*c*d*x)*(a+b*arctan(c*x))/x^2,x, algorithm="fricas")
```

output
```
integral(1/2*(2*I*a*c*d*x + 2*a*d - (b*c*d*x - I*b*d)*log(-(c*x + I)/(c*x
- I)))/x^2, x)
```

## Sympy [F]

$$\int \frac{(d+icdx)(a+b\arctan(cx))}{x^2}\,dx=id\left(\int\left(-\frac{ia}{x^2}\right)dx+\int\frac{ac}{x}\,dx+\int\left(-\frac{ib\operatorname{atan}(cx)}{x^2}\right)dx+\int\frac{bc\operatorname{atan}(cx)}{x}\,dx\right)$$

input
```
integrate((d+I*c*d*x)*(a+b*atan(c*x))/x**2,x)
```

output
```
I*d*(Integral(-I*a/x**2, x) + Integral(a*c/x, x) + Integral(-I*b*atan(c*x)
/x**2, x) + Integral(b*c*atan(c*x)/x, x))
```

## Maxima [F]

$$\int \frac{(d+icdx)(a+b\arctan(cx))}{x^2}\,dx=\int \frac{(i\,cdx+d)(b\arctan(cx)+a)}{x^2}\,dx$$

input
```
integrate((d+I*c*d*x)*(a+b*arctan(c*x))/x^2,x, algorithm="maxima")
```

output
```
I*b*c*d*integrate(arctan(c*x)/x, x) + I*a*c*d*log(x) - 1/2*(c*(log(c^2*x^2
 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b*d - a*d/x
```

## Giac [F]

$$\int \frac{(d+icdx)(a+b\arctan(cx))}{x^2}\,dx = \int \frac{(i\,cdx+d)(b\arctan(cx)+a)}{x^2}\,dx$$

input
```
integrate((d+I*c*d*x)*(a+b*arctan(c*x))/x^2,x, algorithm="giac")
```

output
```
integrate((I*c*d*x + d)*(b*arctan(c*x) + a)/x^2, x)
```

## Mupad [B] (verification not implemented)

Time = 1.18 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.21

$$\int \frac{(d+icdx)(a+b\arctan(cx))}{x^2}\,dx = \begin{cases} -\frac{a\,d}{x} & \text{if } c=0 \\ \frac{b\,d\left(c^2\,\ln(x)-\frac{c^2\,\ln\left(c^2\,x^2+1\right)}{2}\right)}{c}+\frac{b\,c\,d\,(\mathrm{Li}_2(1-c\,x\,1\mathrm{i})-\mathrm{Li}_2(1+c\,x\,1\mathrm{i}))}{2}+\frac{a\,d\,(-1+c\,x\,\ln(x)\,1\mathrm{i})}{x}-\frac{b\,d\,\mathrm{atan}(c\,x)}{x} & \text{if } c\neq 0 \end{cases}$$

input
```
int(((a + b*atan(c*x))*(d + c*d*x*1i))/x^2,x)
```

output
```
piecewise(c == 0, -(a*d)/x, c ~= 0, (b*d*(c^2*log(x) - (c^2*log(c^2*x^2 +
1))/2))/c + (b*c*d*(dilog(- c*x*1i + 1) - dilog(c*x*1i + 1)))/2 + (a*d*(c*
x*log(x)*1i - 1))/x - (b*d*atan(c*x))/x)
```

## Reduce [F]

$$\int \frac{(d+icdx)(a+b\arctan(cx))}{x^2}\,dx = \frac{d\left(-2atan(cx)\,b+2\left(\int \frac{atan(cx)}{x}dx\right)bcix-\log(c^2x^2+1)\,bcx+2\log(x)\,acix+2\log(x)\,bcx-2a\right)}{2x}$$

input
```
int((d+I*c*d*x)*(a+b*atan(c*x))/x^2,x)
```

output
```
(d*( - 2*atan(c*x)*b + 2*int(atan(c*x)/x,x)*b*c*i*x - log(c**2*x**2 + 1)*b
*c*x + 2*log(x)*a*c*i*x + 2*log(x)*b*c*x - 2*a))/(2*x)
```

# 3.7 $\int \frac{(d+icdx)(a+b\arctan(cx))}{x^3}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 65

$$\int \frac{(d+icdx)(a+b\arctan(cx))}{x^3}\,dx = -\frac{bcd}{2x}-\frac{d(1+icx)^2(a+b\arctan(cx))}{2x^2}+ibc^2d\log(x)-ibc^2d\log(i+cx)$$

output
```
-1/2*b*c*d/x-1/2*d*(1+I*c*x)^2*(a+b*arctan(c*x))/x^2+I*b*c^2*d*ln(x)-I*b*c
^2*d*ln(I+c*x)
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.35

$$\int \frac{(d+icdx)(a+b\arctan(cx))}{x^3}\,dx = -\frac{d(a+b\arctan(cx))}{2x^2}-\frac{icd(a+b\arctan(cx))}{x}-\frac{bcd\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-c^2x^2\right)}{2x}+\frac{1}{2}ibc^2d\big(2\log(x)-\log\left(1+c^2x^2\right)\big)$$

input
```
Integrate[((d + I*c*d*x)*(a + b*ArcTan[c*x]))/x^3,x]
```

output
```
-1/2*(d*(a + b*ArcTan[c*x]))/x^2 - (I*c*d*(a + b*ArcTan[c*x]))/x - (b*c*d*
Hypergeometric2F1[-1/2, 1, 1/2, -(c^2*x^2)])/(2*x) + (I/2)*b*c^2*d*(2*Log[
x] - Log[1 + c^2*x^2])
```

## Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {5407, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)(a+b\arctan(cx))}{x^3}\,dx$$

$$\Big\downarrow \text{5407}$$

$$-bc\int -\frac{d(i-cx)}{2x^2(cx+i)}dx-\frac{d(1+icx)^2(a+b\arctan(cx))}{2x^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}bcd\int \frac{i-cx}{x^2(cx+i)}dx-\frac{d(1+icx)^2(a+b\arctan(cx))}{2x^2}$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{2}bcd\int \left(-\frac{2ic^2}{cx+i}+\frac{2ic}{x}+\frac{1}{x^2}\right)dx-\frac{d(1+icx)^2(a+b\arctan(cx))}{2x^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}bcd\left(2ic\log(x)-2ic\log(cx+i)-\frac{1}{x}\right)-\frac{d(1+icx)^2(a+b\arctan(cx))}{2x^2}$$

input
```
Int[((d + I*c*d*x)*(a + b*ArcTan[c*x]))/x^3,x]
```

output
```
-1/2*(d*(1 + I*c*x)^2*(a + b*ArcTan[c*x]))/x^2 + (b*c*d*(-x^(-1) + (2*I)*c
*Log[x] - (2*I)*c*Log[I + c*x]))/2
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5407
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Simp[(a
 + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ
[2*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0])
)
```

## Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.29

| method | result |
|---|---|
| parts | $da\left(-\frac{1}{2x^2}-\frac{ic}{x}\right)+db\,c^2\left(-\frac{i\arctan(cx)}{cx}-\frac{\arctan(cx)}{2c^2x^2}-\frac{i\ln(c^2x^2+1)}{2}-\frac{\arctan(cx)}{2}+i\ln(cx)-\frac{1}{2c\cdots}\right.$ |
| derivativedivides | $c^2\left(da\left(-\frac{i}{cx}-\frac{1}{2c^2x^2}\right)+db\left(-\frac{i\arctan(cx)}{cx}-\frac{\arctan(cx)}{2c^2x^2}-\frac{i\ln(c^2x^2+1)}{2}-\frac{\arctan(cx)}{2}+i\ln(cx)-\cdots\right.\right.$ |
| default | $c^2\left(da\left(-\frac{i}{cx}-\frac{1}{2c^2x^2}\right)+db\left(-\frac{i\arctan(cx)}{cx}-\frac{\arctan(cx)}{2c^2x^2}-\frac{i\ln(c^2x^2+1)}{2}-\frac{\arctan(cx)}{2}+i\ln(cx)-\cdots\right.\right.$ |
| parallelrisch | $\frac{2ic^2bd\ln(x)x^2-ic^2bd\ln(c^2x^2+1)x^2-db\arctan(cx)x^2c^2-2ix\arctan(cx)bcd+a\,c^2d\,x^2-2iacdx-bcdx-bd\arctan(cx)-\cdots}{2x^2}$ |
| risch | $-\frac{(2bcdx-ibd)\ln(icx+1)}{4x^2}+\frac{id(4b\,c^2\ln(-35cx)x^2-3b\,c^2\ln(-7cx-7i)x^2-b\,c^2\ln(5cx-5i)x^2-4acx-2ibcx\ln(-icx+\cdots}{4x^2}$ |

input
```
int((d+I*c*d*x)*(a+b*arctan(c*x))/x^3,x,method=_RETURNVERBOSE)
```

output
```
d*a*(-1/2/x^2-I*c/x)+d*b*c^2*(-I*arctan(c*x)/c/x-1/2/c^2/x^2*arctan(c*x)-1
/2*I*ln(c^2*x^2+1)-1/2*arctan(c*x)+I*ln(c*x)-1/2/c/x)
```

## Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.52

$$\int \frac{(d+icdx)(a+b\arctan(cx))}{x^3}\,dx = \frac{4i\,bc^2dx^2\log(x) - 3i\,bc^2dx^2\log\left(\frac{cx+i}{c}\right) - i\,bc^2dx^2\log\left(\frac{cx-i}{c}\right) - 2\,(2i\,a+b)cdx - 2\,ad + (2\,bcdx - i\,bd)\log}{4\,x^2}$$

input
```
integrate((d+I*c*d*x)*(a+b*arctan(c*x))/x^3,x, algorithm="fricas")
```

output
```
1/4*(4*I*b*c^2*d*x^2*log(x) - 3*I*b*c^2*d*x^2*log((c*x + I)/c) - I*b*c^2*d
*x^2*log((c*x - I)/c) - 2*(2*I*a + b)*c*d*x - 2*a*d + (2*b*c*d*x - I*b*d)*
log(-(c*x + I)/(c*x - I)))/x^2
```

## Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(58) = 116.

Time = 1.92 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.80

$$\begin{aligned}\int \frac{(d+icdx)(a+b\arctan(cx))}{x^3}\,dx = {} & ibc^2d\log\left(35b^2c^5d^2x\right) \\ & - \frac{ibc^2d\log\left(35b^2c^5d^2x - 35ib^2c^4d^2\right)}{4} \\ & - \frac{3ibc^2d\log\left(35b^2c^5d^2x + 35ib^2c^4d^2\right)}{4} \\ & + \frac{-ad + x(-2iacd - bcd)}{2x^2} \\ & + \frac{(-2bcdx + ibd)\log\left(icx+1\right)}{4x^2} \\ & + \frac{(2bcdx - ibd)\log\left(-icx+1\right)}{4x^2}\end{aligned}$$

input
```
integrate((d+I*c*d*x)*(a+b*atan(c*x))/x**3,x)
```

output
```
I*b*c**2*d*log(35*b**2*c**5*d**2*x) - I*b*c**2*d*log(35*b**2*c**5*d**2*x -
 35*I*b**2*c**4*d**2)/4 - 3*I*b*c**2*d*log(35*b**2*c**5*d**2*x + 35*I*b**2
*c**4*d**2)/4 + (-a*d + x*(-2*I*a*c*d - b*c*d))/(2*x**2) + (-2*b*c*d*x + I
*b*d)*log(I*c*x + 1)/(4*x**2) + (2*b*c*d*x - I*b*d)*log(-I*c*x + 1)/(4*x**
2)
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.15

$$\int \frac{(d+icdx)(a+b\arctan(cx))}{x^3}\,dx = -\frac{1}{2}i\left(c\left(\log\left(c^2x^2+1\right)-\log\left(x^2\right)\right)+\frac{2\arctan\left(cx\right)}{x}\right)bcd - \frac{1}{2}\left(\left(c\arctan\left(cx\right)+\frac{1}{x}\right)c+\frac{\arctan\left(cx\right)}{x^2}\right)bd-\frac{i\,acd}{x}-\frac{ad}{2\,x^2}$$

input
```
integrate((d+I*c*d*x)*(a+b*arctan(c*x))/x^3,x, algorithm="maxima")
```

output
```
-1/2*I*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b*c*d - 1/2*((c
*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b*d - I*a*c*d/x - 1/2*a*d/x^2
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.32

$$\int \frac{(d+icdx)(a+b\arctan(cx))}{x^3}\,dx = -\frac{i\,bc^2dx^2\log\left(i\,cx+1\right)+3i\,bc^2dx^2\log\left(-i\,cx+1\right)-4i\,bc^2dx^2\log\left(x\right)+4i\,bcdx\arctan\left(cx\right)+4i\,acdx+}{4\,x^2}$$

input
```
integrate((d+I*c*d*x)*(a+b*arctan(c*x))/x^3,x, algorithm="giac")
```

output
```
-1/4*(I*b*c^2*d*x^2*log(I*c*x + 1) + 3*I*b*c^2*d*x^2*log(-I*c*x + 1) - 4*I
*b*c^2*d*x^2*log(x) + 4*I*b*c*d*x*arctan(c*x) + 4*I*a*c*d*x + 2*b*c*d*x +
2*b*d*arctan(c*x) + 2*a*d)/x^2
```

## Mupad [B] (verification not implemented)

Time = 0.86 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.22

$$\int \frac{(d+icdx)(a+b\arctan(cx))}{x^3}\,dx = -\frac{\frac{d\,(a+b\,\mathrm{atan}(c\,x))}{2}+\frac{d\,x\,(a\,c\,2\mathrm{i}+b\,c+b\,c\,\mathrm{atan}(c\,x)\,2\mathrm{i})}{2}}{x^2} - \frac{d\left(b\,c^2\,\mathrm{atan}(c\,x)+b\,c^2\,\ln\left(c^2\,x^2+1\right)\,1\mathrm{i}-b\,c^2\,\ln\left(x\right)\,2\mathrm{i}\right)}{2}$$

input
```
int(((a + b*atan(c*x))*(d + c*d*x*1i))/x^3,x)
```

output
```
- ((d*(a + b*atan(c*x)))/2 + (d*x*(a*c*2i + b*c + b*c*atan(c*x)*2i))/2)/x^
2 - (d*(b*c^2*atan(c*x) + b*c^2*log(c^2*x^2 + 1)*1i - b*c^2*log(x)*2i))/2
```

## Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.28

$$\int \frac{(d+icdx)(a+b\arctan(cx))}{x^3}\,dx = \frac{d(-atan(cx)\,b\,c^2x^2-2atan(cx)\,bcix-atan(cx)\,b-\log(c^2x^2+1)\,b\,c^2i\,x^2+2\log(x)\,b\,c^2i\,x^2-2acix-a}{2x^2}$$

input
```
int((d+I*c*d*x)*(a+b*atan(c*x))/x^3,x)
```

output
```
(d*( - atan(c*x)*b*c**2*x**2 - 2*atan(c*x)*b*c*i*x - atan(c*x)*b - log(c**
2*x**2 + 1)*b*c**2*i*x**2 + 2*log(x)*b*c**2*i*x**2 - 2*a*c*i*x - a - b*c*x
))/(2*x**2)
```

# 3.8 $\int \frac{(d+icdx)(a+b\arctan(cx))}{x^4}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 106

$$\int \frac{(d+icdx)(a+b\arctan(cx))}{x^4}\,dx = -\frac{bcd}{6x^2}-\frac{ibc^2d}{2x}-\frac{d(a+b\arctan(cx))}{3x^3} - \frac{icd(a+b\arctan(cx))}{2x^2}-\frac{1}{3}bc^3d\log(x) - \frac{1}{12}bc^3d\log(i-cx)+\frac{5}{12}bc^3d\log(i+cx)$$

output
```
-1/6*b*c*d/x^2-1/2*I*b*c^2*d/x-1/3*d*(a+b*arctan(c*x))/x^3-1/2*I*c*d*(a+b*
arctan(c*x))/x^2-1/3*b*c^3*d*ln(x)-1/12*b*c^3*d*ln(I-c*x)+5/12*b*c^3*d*ln(
I+c*x)
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.89

$$\int \frac{(d+icdx)(a+b\arctan(cx))}{x^4}\,dx = -\frac{d\left(2a+3iacx+bcx+b(2+3icx)\arctan(cx)+3ibc^2x^2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-c^2x^2\right)+2bc^3x\right.}{6x^3}$$

input
```
Integrate[((d + I*c*d*x)*(a + b*ArcTan[c*x]))/x^4,x]
```

output
```
-1/6*(d*(2*a + (3*I)*a*c*x + b*c*x + b*(2 + (3*I)*c*x)*ArcTan[c*x] + (3*I)
*b*c^2*x^2*Hypergeometric2F1[-1/2, 1, 1/2, -(c^2*x^2)] + 2*b*c^3*x^3*Log[x
] - b*c^3*x^3*Log[1 + c^2*x^2]))/x^3
```

## Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {5407, 27, 523, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)(a+b\arctan(cx))}{x^4}dx$$

$$\Big\downarrow \text{ 5407}$$

$$-bc\int -\frac{d(3icx+2)}{6x^3(c^2x^2+1)}dx-\frac{d(a+b\arctan(cx))}{3x^3}-\frac{icd(a+b\arctan(cx))}{2x^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{6}bcd\int \frac{3icx+2}{x^3(c^2x^2+1)}dx-\frac{d(a+b\arctan(cx))}{3x^3}-\frac{icd(a+b\arctan(cx))}{2x^2}$$

$$\Big\downarrow \text{ 523}$$

$$\frac{1}{6}bcd\int \left(-\frac{c^3}{2(cx-i)}+\frac{5c^3}{2(cx+i)}-\frac{2c^2}{x}+\frac{3ic}{x^2}+\frac{2}{x^3}\right)dx-\frac{d(a+b\arctan(cx))}{3x^3}-\frac{icd(a+b\arctan(cx))}{2x^2}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{d(a+b\arctan(cx))}{3x^3}-\frac{icd(a+b\arctan(cx))}{2x^2}+\frac{1}{6}bcd\left(-2c^2\log(x)-\frac{1}{2}c^2\log(-cx+i)+\frac{5}{2}c^2\log(cx+i)-\frac{3ic}{x}-\frac{1}{x^2}\right)$$

input
```
Int[((d + I*c*d*x)*(a + b*ArcTan[c*x]))/x^4,x]
```

output
```
-1/3*(d*(a + b*ArcTan[c*x]))/x^3 - ((I/2)*c*d*(a + b*ArcTan[c*x]))/x^2 + (
b*c*d*(-x^(-2) - ((3*I)*c)/x - 2*c^2*Log[x] - (c^2*Log[I - c*x])/2 + (5*c^
2*Log[I + c*x])/2))/6
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 523
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_)))/((a_) + (b_.)*(x_)^2), x_Symbol] :> In
t[ExpandIntegrand[x^m*((c + d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}
, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5407
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Simp[(a
 + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ
[2*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0])
)
```

## Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.87

| method | result |
|---|---|
| parts | $da\left(-\frac{ic}{2x^2}-\frac{1}{3x^3}\right)+db\,c^3\left(-\frac{i\arctan(cx)}{2c^2x^2}-\frac{\arctan(cx)}{3c^3x^3}+\frac{\ln(c^2x^2+1)}{6}-\frac{i\arctan(cx)}{2}-\frac{i}{2cx}-\frac{1}{6c^2x^2}\right.$ |
| derivativedivides | $c^3\left(da\left(-\frac{i}{2c^2x^2}-\frac{1}{3c^3x^3}\right)+db\left(-\frac{i\arctan(cx)}{2c^2x^2}-\frac{\arctan(cx)}{3c^3x^3}+\frac{\ln(c^2x^2+1)}{6}-\frac{i\arctan(cx)}{2}-\frac{i}{2cx}-\right.\right.$ |
| default | $c^3\left(da\left(-\frac{i}{2c^2x^2}-\frac{1}{3c^3x^3}\right)+db\left(-\frac{i\arctan(cx)}{2c^2x^2}-\frac{\arctan(cx)}{3c^3x^3}+\frac{\ln(c^2x^2+1)}{6}-\frac{i\arctan(cx)}{2}-\frac{i}{2cx}-\right.\right.$ |
| parallelrisch | $-\frac{3ic^3bd\arctan(cx)x^3-3ix^3ac^3d+2dbc^3\ln(x)x^3-dbc^3\ln(c^2x^2+1)x^3-bc^3dx^3+3ix^2bc^2d+3ix\arctan(cx)bcd+3iac}{6x^3}$ |
| risch | $-\frac{(3bcdx-2ibd)\ln(icx+1)}{12x^3}+\frac{d(5bc^3\ln(-cx-i)x^3-4bc^3\ln(-x)x^3-bc^3\ln(cx-i)x^3-6ibc^2x^2-6iacx+3bcx\ln(-icx}{12x^3}$ |

input

```
int((d+I*c*d*x)*(a+b*arctan(c*x))/x^4,x,method=_RETURNVERBOSE)
```

output

```
d*a*(-1/2*I*c/x^2-1/3/x^3)+d*b*c^3*(-1/2*I*arctan(c*x)/c^2/x^2-1/3/c^3/x^3
*arctan(c*x)+1/6*ln(c^2*x^2+1)-1/2*I*arctan(c*x)-1/2*I/c/x-1/6/c^2/x^2-1/3
*ln(c*x))
```

## Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.03

$$\int\frac{(d+icdx)(a+b\arctan(cx))}{x^4}\,dx =$$
$$-\frac{4\,bc^3dx^3\log(x)-5\,bc^3dx^3\log\left(\frac{cx+i}{c}\right)+bc^3dx^3\log\left(\frac{cx-i}{c}\right)+6i\,bc^2dx^2+2\,(3i\,a+b)cdx+4\,ad-(3\,bcd}{12\,x^3}$$

input

```
integrate((d+I*c*d*x)*(a+b*arctan(c*x))/x^4,x, algorithm="fricas")
```

output

```
-1/12*(4*b*c^3*d*x^3*log(x) - 5*b*c^3*d*x^3*log((c*x + I)/c) + b*c^3*d*x^3
*log((c*x - I)/c) + 6*I*b*c^2*d*x^2 + 2*(3*I*a + b)*c*d*x + 4*a*d - (3*b*c
*d*x - 2*I*b*d)*log(-(c*x + I)/(c*x - I)))/x^3
```

## Sympy [A] (verification not implemented)

Time = 2.55 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.86

$$\int \frac{(d+icdx)(a+b\arctan(cx))}{x^4}\,dx = -\frac{bc^3d\log(27b^2c^7d^2x)}{3} - \frac{bc^3d\log(27b^2c^7d^2x-27ib^2c^6d^2)}{12} + \frac{5bc^3d\log(27b^2c^7d^2x+27ib^2c^6d^2)}{12} + \frac{(-3bcdx+2ibd)\log(icx+1)}{12x^3} + \frac{(3bcdx-2ibd)\log(-icx+1)}{12x^3} + \frac{-2ad-3ibc^2dx^2+x(-3iacd-bcd)}{6x^3}$$

input
```
integrate((d+I*c*d*x)*(a+b*atan(c*x))/x**4,x)
```

output
```
-b*c**3*d*log(27*b**2*c**7*d**2*x)/3 - b*c**3*d*log(27*b**2*c**7*d**2*x -
27*I*b**2*c**6*d**2)/12 + 5*b*c**3*d*log(27*b**2*c**7*d**2*x + 27*I*b**2*c
**6*d**2)/12 + (-3*b*c*d*x + 2*I*b*d)*log(I*c*x + 1)/(12*x**3) + (3*b*c*d*
x - 2*I*b*d)*log(-I*c*x + 1)/(12*x**3) + (-2*a*d - 3*I*b*c**2*d*x**2 + x*(
-3*I*a*c*d - b*c*d))/(6*x**3)
```

## Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.82

$$\int \frac{(d+icdx)(a+b\arctan(cx))}{x^4}\,dx = -\frac{1}{2}i\left(\left(c\arctan(cx)+\frac{1}{x}\right)c+\frac{\arctan(cx)}{x^2}\right)bcd + \frac{1}{6}\left(\left(c^2\log(c^2x^2+1)-c^2\log(x^2)-\frac{1}{x^2}\right)c-\frac{2\arctan(cx)}{x^3}\right)bd - \frac{i\,acd}{2\,x^2} - \frac{ad}{3\,x^3}$$

input
```
integrate((d+I*c*d*x)*(a+b*arctan(c*x))/x^4,x, algorithm="maxima")
```

output
```
-1/2*I*((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b*c*d + 1/6*((c^2*log(c
^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*b*d - 1/2*I*a*c
*d/x^2 - 1/3*a*d/x^3
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.89

$$\int \frac{(d+icdx)(a+b\arctan(cx))}{x^4}\,dx$$
$$= \frac{5\,bc^3dx^3\log(cx+i) - bc^3dx^3\log(cx-i) - 4\,bc^3dx^3\log(x) - 6i\,bc^2dx^2 - 6i\,bcdx\arctan(cx) - 6i\,acdx + \cdots}{12\,x^3}$$

input
```
integrate((d+I*c*d*x)*(a+b*arctan(c*x))/x^4,x, algorithm="giac")
```

output
```
1/12*(5*b*c^3*d*x^3*log(c*x + I) - b*c^3*d*x^3*log(c*x - I) - 4*b*c^3*d*x^
3*log(x) - 6*I*b*c^2*d*x^2 - 6*I*b*c*d*x*arctan(c*x) - 6*I*a*c*d*x - 2*b*c
*d*x - 4*b*d*arctan(c*x) - 4*a*d)/x^3
```

## Mupad [B] (verification not implemented)

Time = 1.24 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.66

$$\int \frac{(d+icdx)(a+b\arctan(cx))}{x^4}\,dx = \frac{b\,c^3\,d\,\ln(c^2\,x^2+1)}{6}$$
$$-\frac{\frac{a\,d}{3} - x^5\left(\frac{b\,c^5\,d}{6} + \frac{a\,c^5\,d\,1\mathrm{i}}{2}\right) + \frac{b\,d\,\mathrm{atan}(c\,x)}{3} + \frac{c\,d\,x\,(b+a\,3\mathrm{i})}{6} + \frac{c^2\,d\,x^2\,(2\,a+b\,3\mathrm{i})}{6} + \frac{b\,c^4\,d\,x^4\,1\mathrm{i}}{2} + \frac{b\,c^2\,d\,x^2\,\mathrm{atan}(c\,x)}{3} + \frac{b\,c^3\,d\,x^3\,\cdots}{\cdots}}{c^2\,x^5+x^3}$$
$$-\frac{b\,c^3\,d\,\ln(x)}{3} - \frac{b\,d\,\mathrm{atan}\left(\frac{c^2\,x}{\sqrt{c^2}}\right)\,{\left(c^2\right)}^{3/2}\,1\mathrm{i}}{2}$$

input
```
int(((a + b*atan(c*x))*(d + c*d*x*1i))/x^4,x)
```

output
```
(b*c^3*d*log(c^2*x^2 + 1))/6 - (b*d*atan((c^2*x)/(c^2)^(1/2))*(c^2)^(3/2)*
1i)/2 - ((a*d)/3 - x^5*((a*c^5*d*1i)/2 + (b*c^5*d)/6) + (b*d*atan(c*x))/3
+ (c*d*x*(a*3i + b))/6 + (c^2*d*x^2*(2*a + b*3i))/6 + (b*c^4*d*x^4*1i)/2 +
 (b*c^2*d*x^2*atan(c*x))/3 + (b*c^3*d*x^3*atan(c*x)*1i)/2 + (b*c*d*x*atan(
c*x)*1i)/2)/(x^3 + c^2*x^5) - (b*c^3*d*log(x))/3
```

## Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.86

$$\int \frac{(d+icdx)(a+b\arctan(cx))}{x^4}\,dx$$
$$= \frac{d(-3atan(cx)\,b\,c^3i\,x^3 - 3atan(cx)\,bcix - 2atan(cx)\,b + \log(c^2x^2+1)\,b\,c^3x^3 - 2\log(x)\,b\,c^3x^3 - 3acix -}{6x^3}$$

input
```
int((d+I*c*d*x)*(a+b*atan(c*x))/x^4,x)
```

output
```
(d*( - 3*atan(c*x)*b*c**3*i*x**3 - 3*atan(c*x)*b*c*i*x - 2*atan(c*x)*b + l
og(c**2*x**2 + 1)*b*c**3*x**3 - 2*log(x)*b*c**3*x**3 - 3*a*c*i*x - 2*a - 3
*b*c**2*i*x**2 - b*c*x))/(6*x**3)
```

# 3.9 $\int \frac{(d+icdx)(a+b\arctan(cx))}{x^5}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 124

$$\int \frac{(d+icdx)(a+b\arctan(cx))}{x^5}\,dx = -\frac{bcd}{12x^3}-\frac{ibc^2d}{6x^2}+\frac{bc^3d}{4x}-\frac{d(a+b\arctan(cx))}{4x^4} -\frac{icd(a+b\arctan(cx))}{3x^3}-\frac{1}{3}ibc^4d\log(x) +\frac{1}{24}ibc^4d\log(i-cx)+\frac{7}{24}ibc^4d\log(i+cx)$$

output
```
-1/12*b*c*d/x^3-1/6*I*b*c^2*d/x^2+1/4*b*c^3*d/x-1/4*d*(a+b*arctan(c*x))/x^
4-1/3*I*c*d*(a+b*arctan(c*x))/x^3-1/3*I*b*c^4*d*ln(x)+1/24*I*b*c^4*d*ln(I-
c*x)+7/24*I*b*c^4*d*ln(I+c*x)
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.80

$$\int \frac{(d+icdx)(a+b\arctan(cx))}{x^5}\,dx = -\frac{d(a+b\arctan(cx))}{4x^4}-\frac{icd(a+b\arctan(cx))}{3x^3} -\frac{bcd\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-c^2x^2\right)}{12x^3} -\frac{1}{6}ibc^2d\left(\frac{1}{x^2}+2c^2\log(x)-c^2\log\left(1+c^2x^2\right)\right)$$

input
```
Integrate[((d + I*c*d*x)*(a + b*ArcTan[c*x]))/x^5,x]
```

output
```
-1/4*(d*(a + b*ArcTan[c*x]))/x^4 - ((I/3)*c*d*(a + b*ArcTan[c*x]))/x^3 - (
b*c*d*Hypergeometric2F1[-3/2, 1, -1/2, -(c^2*x^2)])/(12*x^3) - (I/6)*b*c^2
*d*(x^(-2) + 2*c^2*Log[x] - c^2*Log[1 + c^2*x^2])
```

## Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.88, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {5407, 27, 523, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)(a+b\arctan(cx))}{x^5}dx$$

$$\Big\downarrow \text{5407}$$

$$-bc\int -\frac{d(4icx+3)}{12x^4(c^2x^2+1)}dx-\frac{d(a+b\arctan(cx))}{4x^4}-\frac{icd(a+b\arctan(cx))}{3x^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{12}bcd\int \frac{4icx+3}{x^4(c^2x^2+1)}dx-\frac{d(a+b\arctan(cx))}{4x^4}-\frac{icd(a+b\arctan(cx))}{3x^3}$$

$$\Big\downarrow \text{523}$$

$$\frac{1}{12}bcd\int \left(\frac{ic^4}{2(cx-i)}+\frac{7ic^4}{2(cx+i)}-\frac{4ic^3}{x}-\frac{3c^2}{x^2}+\frac{4ic}{x^3}+\frac{3}{x^4}\right)dx-\frac{d(a+b\arctan(cx))}{4x^4}-\frac{icd(a+b\arctan(cx))}{3x^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d(a+b\arctan(cx))}{4x^4}-\frac{icd(a+b\arctan(cx))}{3x^3}+\frac{1}{12}bcd\left(-4ic^3\log(x)+\frac{1}{2}ic^3\log(-cx+i)+\frac{7}{2}ic^3\log(cx+i)+\frac{3c^2}{x}-\frac{2ic}{x^2}-\frac{1}{x^3}\right)$$

input
```
Int[((d + I*c*d*x)*(a + b*ArcTan[c*x]))/x^5,x]
```

output
```
-1/4*(d*(a + b*ArcTan[c*x]))/x^4 - ((I/3)*c*d*(a + b*ArcTan[c*x]))/x^3 + (
b*c*d*(-x^(-3) - ((2*I)*c)/x^2 + (3*c^2)/x - (4*I)*c^3*Log[x] + (I/2)*c^3*
Log[I - c*x] + ((7*I)/2)*c^3*Log[I + c*x]))/12
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 523
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_)))/((a_) + (b_.)*(x_)^2), x_Symbol] :> In
t[ExpandIntegrand[x^m*((c + d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}
, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5407
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Simp[(a
 + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ
[2*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0])
)
```

## Maple [A] (verified)

Time = 0.39 (sec), antiderivative size = 101, normalized size of antiderivative = 0.81

| method | result |
|---|---|
| parts | $da\left(-\frac{ic}{3x^3}-\frac{1}{4x^4}\right)+db\,c^4\left(-\frac{\arctan(cx)}{4c^4x^4}-\frac{i\arctan(cx)}{3c^3x^3}+\frac{i\ln(c^2x^2+1)}{6}+\frac{\arctan(cx)}{4}-\frac{i}{6c^2x^2}-\frac{i\ln(c}{3}\right.$ |
| derivativedivides | $c^4\left(da\left(-\frac{1}{4c^4x^4}-\frac{i}{3c^3x^3}\right)+db\left(-\frac{\arctan(cx)}{4c^4x^4}-\frac{i\arctan(cx)}{3c^3x^3}+\frac{i\ln(c^2x^2+1)}{6}+\frac{\arctan(cx)}{4}-\frac{i}{6c^2x^2}-\right.\right.$ |
| default | $c^4\left(da\left(-\frac{1}{4c^4x^4}-\frac{i}{3c^3x^3}\right)+db\left(-\frac{\arctan(cx)}{4c^4x^4}-\frac{i\arctan(cx)}{3c^3x^3}+\frac{i\ln(c^2x^2+1)}{6}+\frac{\arctan(cx)}{4}-\frac{i}{6c^2x^2}-\right.\right.$ |
| parallelrisch | $-\frac{4ic^4bd\ln(x)x^4-2ic^4bd\ln(c^2x^2+1)x^4-2ix^4bc^4d-3db\arctan(cx)x^4c^4-3bc^3dx^3+2ix^2bc^2d+4ix\arctan(cx)bcd+4i}{12x^4}$ |
| risch | $-\frac{(4bcdx-3ibd)\ln(icx+1)}{24x^4}+\frac{id(7bc^4\ln(-15cx-15i)x^4-8bc^4\ln(-45cx)x^4+bc^4\ln(9cx-9i)x^4-6ibc^3x^3-4bc^2x^2-}{24x^4}$ |

input
```
int((d+I*c*d*x)*(a+b*arctan(c*x))/x^5,x,method=_RETURNVERBOSE)
```

output
```
d*a*(-1/3*I*c/x^3-1/4/x^4)+d*b*c^4*(-1/4/c^4/x^4*arctan(c*x)-1/3*I*arctan(
c*x)/c^3/x^3+1/6*I*ln(c^2*x^2+1)+1/4*arctan(c*x)-1/6*I/c^2/x^2-1/3*I*ln(c*
x)-1/12/c^3/x^3+1/4/c/x)
```

## Fricas [A] (verification not implemented)

Time = 0.12 (sec), antiderivative size = 119, normalized size of antiderivative = 0.96

$$\int\frac{(d+icdx)(a+b\arctan(cx))}{x^5}\,dx$$
$$=\frac{-8i\,bc^4dx^4\log(x)+7i\,bc^4dx^4\log\left(\frac{cx+i}{c}\right)+i\,bc^4dx^4\log\left(\frac{cx-i}{c}\right)+6\,bc^3dx^3-4i\,bc^2dx^2-2\,(4i\,a+b)cdx-}{24\,x^4}$$

input
```
integrate((d+I*c*d*x)*(a+b*arctan(c*x))/x^5,x, algorithm="fricas")
```

output
```
1/24*(-8*I*b*c^4*d*x^4*log(x) + 7*I*b*c^4*d*x^4*log((c*x + I)/c) + I*b*c^4
*d*x^4*log((c*x - I)/c) + 6*b*c^3*d*x^3 - 4*I*b*c^2*d*x^2 - 2*(4*I*a + b)*
c*d*x - 6*a*d + (4*b*c*d*x - 3*I*b*d)*log(-(c*x + I)/(c*x - I)))/x^4
```

## Sympy [A] (verification not implemented)

Time = 3.96 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.73

$$\int \frac{(d+icdx)(a+b\arctan(cx))}{x^5}\,dx = -\frac{ibc^4d\log\left(135b^2c^9d^2x\right)}{3} + \frac{ibc^4d\log\left(135b^2c^9d^2x-135ib^2c^8d^2\right)}{24} + \frac{7ibc^4d\log\left(135b^2c^9d^2x+135ib^2c^8d^2\right)}{24} + \frac{(-4bcdx+3ibd)\log\left(icx+1\right)}{24x^4} + \frac{(4bcdx-3ibd)\log\left(-icx+1\right)}{24x^4} + \frac{-3ad+3bc^3dx^3-2ibc^2dx^2+x(-4iacd-bcd)}{12x^4}$$

input
```
integrate((d+I*c*d*x)*(a+b*atan(c*x))/x**5,x)
```

output
```
-I*b*c**4*d*log(135*b**2*c**9*d**2*x)/3 + I*b*c**4*d*log(135*b**2*c**9*d**
2*x - 135*I*b**2*c**8*d**2)/24 + 7*I*b*c**4*d*log(135*b**2*c**9*d**2*x + 1
35*I*b**2*c**8*d**2)/24 + (-4*b*c*d*x + 3*I*b*d)*log(I*c*x + 1)/(24*x**4)
+ (4*b*c*d*x - 3*I*b*d)*log(-I*c*x + 1)/(24*x**4) + (-3*a*d + 3*b*c**3*d*x
**3 - 2*I*b*c**2*d*x**2 + x*(-4*I*a*c*d - b*c*d))/(12*x**4)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.82

$$\int \frac{(d+icdx)(a+b\arctan(cx))}{x^5}\,dx = \frac{1}{6}i\left(\left(c^2\log\left(c^2x^2+1\right)-c^2\log\left(x^2\right)-\frac{1}{x^2}\right)c-\frac{2\arctan\left(cx\right)}{x^3}\right)bcd + \frac{1}{12}\left(\left(3c^3\arctan\left(cx\right)+\frac{3c^2x^2-1}{x^3}\right)c-\frac{3\arctan\left(cx\right)}{x^4}\right)bd-\frac{i\,acd}{3\,x^3}-\frac{ad}{4\,x^4}$$

input
```
integrate((d+I*c*d*x)*(a+b*arctan(c*x))/x^5,x, algorithm="maxima")
```

output
```
1/6*I*((c^2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3
)*b*c*d + 1/12*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan(c*x
)/x^4)*b*d - 1/3*I*a*c*d/x^3 - 1/4*a*d/x^4
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.85

$$\int \frac{(d+icdx)(a+b\arctan(cx))}{x^5}\,dx =$$
$$-\frac{-7i\,bc^4dx^4\log\left(i\,cx-1\right)-i\,bc^4dx^4\log\left(-i\,cx-1\right)+8i\,bc^4dx^4\log\left(x\right)-6\,bc^3dx^3+4i\,bc^2dx^2+8i\,bcdx}{24\,x^4}$$

input
```
integrate((d+I*c*d*x)*(a+b*arctan(c*x))/x^5,x, algorithm="giac")
```

output
```
-1/24*(-7*I*b*c^4*d*x^4*log(I*c*x - 1) - I*b*c^4*d*x^4*log(-I*c*x - 1) + 8
*I*b*c^4*d*x^4*log(x) - 6*b*c^3*d*x^3 + 4*I*b*c^2*d*x^2 + 8*I*b*c*d*x*arct
an(c*x) + 8*I*a*c*d*x + 2*b*c*d*x + 6*b*d*arctan(c*x) + 6*a*d)/x^4
```

## Mupad [B] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.94

$$\int \frac{(d+icdx)(a+b\arctan(cx))}{x^5}\,dx$$
$$= \frac{d\left(\frac{3\,b\,c^7\,\mathrm{atan}\left(\frac{c^2\,x}{\sqrt{c^2}}\right)}{{\left(c^2\right)}^{3/2}}+b\,c^4\,\ln\left(c^2\,x^2+1\right)\,2\mathrm{i}-b\,c^4\,\ln\left(x\right)\,4\mathrm{i}\right)}{12} - \frac{\frac{d\,\left(3\,a+3\,b\,\mathrm{atan}\left(c\,x\right)\right)}{12}+\frac{d\,x\,\left(a\,c\,4\mathrm{i}+b\,c+b\,c\,\mathrm{atan}\left(c\,x\right)\,4\mathrm{i}\right)}{12}-\frac{b\,c^3\,d\,x^3}{4}+\frac{b\,c^2\,d\,x^2\,1\mathrm{i}}{6}}{x^4}$$

input
```
int(((a + b*atan(c*x))*(d + c*d*x*1i))/x^5,x)
```

output
```
(d*(b*c^4*log(c^2*x^2 + 1)*2i - b*c^4*log(x)*4i + (3*b*c^7*atan((c^2*x)/(c
^2)^(1/2)))/(c^2)^(3/2)))/12 - ((d*(3*a + 3*b*atan(c*x)))/12 + (d*x*(a*c*4
i + b*c + b*c*atan(c*x)*4i))/12 + (b*c^2*d*x^2*1i)/6 - (b*c^3*d*x^3)/4)/x^
4
```

## Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.82

$$\int \frac{(d+icdx)(a+b\arctan(cx))}{x^5}\,dx$$
$$= \frac{d(3atan(cx)\,b\,c^4x^4 - 4atan(cx)\,bcix - 3atan(cx)\,b + 2\log(c^2x^2+1)\,b\,c^4i\,x^4 - 4\log(x)\,b\,c^4i\,x^4 - 4acix -}{12x^4}$$

input
```
int((d+I*c*d*x)*(a+b*atan(c*x))/x^5,x)
```

output
```
(d*(3*atan(c*x)*b*c**4*x**4 - 4*atan(c*x)*b*c*i*x - 3*atan(c*x)*b + 2*log(
c**2*x**2 + 1)*b*c**4*i*x**4 - 4*log(x)*b*c**4*i*x**4 - 4*a*c*i*x - 3*a +
3*b*c**3*x**3 - 2*b*c**2*i*x**2 - b*c*x))/(12*x**4)
```

# 3.10 $\int x^3(d+icdx)^2(a+b\arctan(cx))\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 166

$$\int x^3(d+icdx)^2(a+b\arctan(cx))\,dx = \frac{5bd^2x}{12c^3}+\frac{ibd^2x^2}{5c^2}-\frac{5bd^2x^3}{36c}-\frac{1}{10}ibd^2x^4+\frac{1}{30}bcd^2x^5 - \frac{5bd^2\arctan(cx)}{12c^4}+\frac{1}{4}d^2x^4(a+b\arctan(cx)) + \frac{2}{5}icd^2x^5(a+b\arctan(cx)) - \frac{1}{6}c^2d^2x^6(a+b\arctan(cx)) - \frac{ibd^2\log(1+c^2x^2)}{5c^4}$$

output
```
5/12*b*d^2*x/c^3+1/5*I*b*d^2*x^2/c^2-5/36*b*d^2*x^3/c-1/10*I*b*d^2*x^4+1/3
0*b*c*d^2*x^5-5/12*b*d^2*arctan(c*x)/c^4+1/4*d^2*x^4*(a+b*arctan(c*x))+2/5
*I*c*d^2*x^5*(a+b*arctan(c*x))-1/6*c^2*d^2*x^6*(a+b*arctan(c*x))-1/5*I*b*d
^2*ln(c^2*x^2+1)/c^4
```

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.75

$$\int x^3(d+icdx)^2(a+b\arctan(cx))\,dx$$
$$=\frac{d^2(3ac^4x^4(15+24icx-10c^2x^2)+bcx(75+36icx-25c^2x^2-18ic^3x^3+6c^4x^4)+3b(-25+15c^4x^4+24}{180c^4}$$

input
```
Integrate[x^3*(d + I*c*d*x)^2*(a + b*ArcTan[c*x]),x]
```

output
```
(d^2*(3*a*c^4*x^4*(15 + (24*I)*c*x - 10*c^2*x^2) + b*c*x*(75 + (36*I)*c*x
- 25*c^2*x^2 - (18*I)*c^3*x^3 + 6*c^4*x^4) + 3*b*(-25 + 15*c^4*x^4 + (24*I
)*c^5*x^5 - 10*c^6*x^6)*ArcTan[c*x] - (36*I)*b*Log[1 + c^2*x^2]))/(180*c^4
)
```

## Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.83, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {5407, 27, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(d+icdx)^2(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5407}$$

$$-bc\int \frac{d^2x^4(-10c^2x^2+24icx+15)}{60(c^2x^2+1)}dx-\frac{1}{6}c^2d^2x^6(a+b\arctan(cx))+\frac{2}{5}icd^2x^5(a+b\arctan(cx))+\frac{1}{4}d^2x^4(a+b\arctan(cx))$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{60}bcd^2\int \frac{x^4(-10c^2x^2+24icx+15)}{c^2x^2+1}dx-\frac{1}{6}c^2d^2x^6(a+b\arctan(cx))+\frac{2}{5}icd^2x^5(a+b\arctan(cx))+\frac{1}{4}d^2x^4(a+b\arctan(cx))$$

$$\Big\downarrow \text{2333}$$

$$-\frac{1}{60}bcd^2\int\left(-10x^4+\frac{24ix^3}{c}+\frac{25x^2}{c^2}-\frac{24ix}{c^3}+\frac{24icx+25}{c^4\left(c^2x^2+1\right)}-\frac{25}{c^4}\right)dx-\frac{1}{6}c^2d^2x^6(a+b\arctan(cx))+\frac{2}{5}icd^2x^5(a+b\arctan(cx))+\frac{1}{4}d^2x^4(a+b\arctan(cx))$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{6}c^2d^2x^6(a+b\arctan(cx))+\frac{2}{5}icd^2x^5(a+b\arctan(cx))+\frac{1}{4}d^2x^4(a+b\arctan(cx))-\frac{1}{60}bcd^2\left(\frac{25\arctan(cx)}{c^5}-\frac{25x}{c^4}-\frac{12ix^2}{c^3}+\frac{25x^3}{3c^2}+\frac{12i\log\left(c^2x^2+1\right)}{c^5}+\frac{6ix^4}{c}-2x^5\right)$$

input

```
Int[x^3*(d + I*c*d*x)^2*(a + b*ArcTan[c*x]),x]
```

output

```
(d^2*x^4*(a + b*ArcTan[c*x]))/4 + ((2*I)/5)*c*d^2*x^5*(a + b*ArcTan[c*x])
- (c^2*d^2*x^6*(a + b*ArcTan[c*x]))/6 - (b*c*d^2*((-25*x)/c^4 - ((12*I)*x^
2)/c^3 + (25*x^3)/(3*c^2) + ((6*I)*x^4)/c - 2*x^5 + (25*ArcTan[c*x])/c^5 +
 ((12*I)*Log[1 + c^2*x^2])/c^5))/60
```

## Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 5407
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Simp[(a
 + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ
[2*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0])
)
```

## Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.79

| method | result |
|---|---|
| parts | $d^2a\left(-\frac{1}{6}c^2x^6+\frac{2}{5}ic\,x^5+\frac{1}{4}x^4\right)+\frac{d^2b\left(-\frac{c^6x^6\arctan(cx)}{6}+\frac{2i\arctan(cx)c^5x^5}{5}+\frac{c^4x^4\arctan(cx)}{4}+\frac{5cx}{12}+\frac{c^5x^5}{30}-\frac{ic^4}{1}\right.}{c^4}$ |
| derivativedivides | $\frac{d^2a\left(-\frac{1}{6}c^6x^6+\frac{2}{5}ic^5x^5+\frac{1}{4}c^4x^4\right)+d^2b\left(-\frac{c^6x^6\arctan(cx)}{6}+\frac{2i\arctan(cx)c^5x^5}{5}+\frac{c^4x^4\arctan(cx)}{4}+\frac{5cx}{12}+\frac{c^5x^5}{30}-\frac{ic^4x^4}{10}-\frac{5c^3x^3}{36}\right.}{c^4}$ |
| default | $\frac{d^2a\left(-\frac{1}{6}c^6x^6+\frac{2}{5}ic^5x^5+\frac{1}{4}c^4x^4\right)+d^2b\left(-\frac{c^6x^6\arctan(cx)}{6}+\frac{2i\arctan(cx)c^5x^5}{5}+\frac{c^4x^4\arctan(cx)}{4}+\frac{5cx}{12}+\frac{c^5x^5}{30}-\frac{ic^4x^4}{10}-\frac{5c^3x^3}{36}\right.}{c^4}$ |
| parallelrisch | $-\frac{30c^6d^2b\arctan(cx)x^6-72ic^5b\,d^2\arctan(cx)x^5+30a\,c^6d^2x^6-72ix^5a\,c^5d^2-6b\,c^5d^2x^5+18ix^4b\,c^4d^2-45d^2b\arctan(c}{180c^4}$ |
| risch | $\frac{id^2b(10c^2x^6-24ic\,x^5-15x^4)\ln(icx+1)}{120}-\frac{d^2c^2a\,x^6}{6}-\frac{id^2c^2x^6b\ln(-icx+1)}{12}-\frac{d^2cb\,x^5\ln(-icx+1)}{5}+\frac{bc\,d^2x^5}{30}+$ |

input
```
int(x^3*(d+I*c*d*x)^2*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

output
```
d^2*a*(-1/6*c^2*x^6+2/5*I*c*x^5+1/4*x^4)+d^2*b/c^4*(-1/6*c^6*x^6*arctan(c*
x)+2/5*I*arctan(c*x)*c^5*x^5+1/4*c^4*x^4*arctan(c*x)+5/12*c*x+1/30*c^5*x^5
-1/10*I*c^4*x^4-5/36*c^3*x^3+1/5*I*c^2*x^2-1/5*I*ln(c^2*x^2+1)-5/12*arctan
(c*x))
```

## Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.04

$$\int x^3(d+icdx)^2(a+b\arctan(cx))\,dx = -\frac{60\,ac^6d^2x^6+12\,(-12i\,a-b)c^5d^2x^5-18\,(5\,a-2i\,b)c^4d^2x^4+50\,bc^3d^2x^3-72i\,bc^2d^2x^2-150\,bcd^2x+}{360\,c^4}$$

input
```
integrate(x^3*(d+I*c*d*x)^2*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
-1/360*(60*a*c^6*d^2*x^6 + 12*(-12*I*a - b)*c^5*d^2*x^5 - 18*(5*a - 2*I*b)
*c^4*d^2*x^4 + 50*b*c^3*d^2*x^3 - 72*I*b*c^2*d^2*x^2 - 150*b*c*d^2*x + 147
*I*b*d^2*log((c*x + I)/c) - 3*I*b*d^2*log((c*x - I)/c) + 3*(10*I*b*c^6*d^2
*x^6 + 24*b*c^5*d^2*x^5 - 15*I*b*c^4*d^2*x^4)*log(-(c*x + I)/(c*x - I)))/c
^4
```

## Sympy [A] (verification not implemented)

Time = 2.30 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.63

$$\begin{aligned}\int x^3(d+icdx)^2(a+b\arctan(cx))\,dx &= -\frac{ac^2d^2x^6}{6}-\frac{5bd^2x^3}{36c}+\frac{ibd^2x^2}{5c^2}+\frac{5bd^2x}{12c^3}\\ &\quad-\frac{bd^2\left(-\frac{i\log\left(291bcd^2x-291ibd^2\right)}{120}+\frac{71i\log\left(291bcd^2x+291ibd^2\right)}{210}\right)}{c^4}-x^5\left(-\frac{2iacd^2}{5}-\frac{bcd^2}{30}\right)\\ &\quad-x^4\left(-\frac{ad^2}{4}+\frac{ibd^2}{10}\right)+\left(\frac{ibc^2d^2x^6}{12}+\frac{bcd^2x^5}{5}-\frac{ibd^2x^4}{8}\right)\log\left(icx+1\right)\\ &\quad+\frac{\left(-70ibc^6d^2x^6-168bc^5d^2x^5+105ibc^4d^2x^4-59ibd^2\right)\log\left(-icx+1\right)}{840c^4}\end{aligned}$$

input
```
integrate(x**3*(d+I*c*d*x)**2*(a+b*atan(c*x)),x)
```

output
```
-a*c**2*d**2*x**6/6 - 5*b*d**2*x**3/(36*c) + I*b*d**2*x**2/(5*c**2) + 5*b*
d**2*x/(12*c**3) - b*d**2*(-I*log(291*b*c*d**2*x - 291*I*b*d**2)/120 + 71*
I*log(291*b*c*d**2*x + 291*I*b*d**2)/210)/c**4 - x**5*(-2*I*a*c*d**2/5 - b
*c*d**2/30) - x**4*(-a*d**2/4 + I*b*d**2/10) + (I*b*c**2*d**2*x**6/12 + b*
c*d**2*x**5/5 - I*b*d**2*x**4/8)*log(I*c*x + 1) + (-70*I*b*c**6*d**2*x**6
- 168*b*c**5*d**2*x**5 + 105*I*b*c**4*d**2*x**4 - 59*I*b*d**2)*log(-I*c*x
+ 1)/(840*c**4)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.11

$$\begin{aligned}\int x^3(d+icdx)^2(a+b\arctan(cx))\,dx &= -\frac{1}{6}\,ac^2d^2x^6+\frac{2}{5}i\,acd^2x^5+\frac{1}{4}\,ad^2x^4\\&\quad-\frac{1}{90}\left(15\,x^6\arctan\left(cx\right)-c\left(\frac{3\,c^4x^5-5\,c^2x^3+15\,x}{c^6}-\frac{15\,\arctan\left(cx\right)}{c^7}\right)\right)bc^2d^2\\&\quad+\frac{1}{10}i\left(4\,x^5\arctan\left(cx\right)-c\left(\frac{c^2x^4-2\,x^2}{c^4}+\frac{2\,\log\left(c^2x^2+1\right)}{c^6}\right)\right)bcd^2\\&\quad+\frac{1}{12}\left(3\,x^4\arctan\left(cx\right)-c\left(\frac{c^2x^3-3\,x}{c^4}+\frac{3\,\arctan\left(cx\right)}{c^5}\right)\right)bd^2\end{aligned}$$

input
```
integrate(x^3*(d+I*c*d*x)^2*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
-1/6*a*c^2*d^2*x^6 + 2/5*I*a*c*d^2*x^5 + 1/4*a*d^2*x^4 - 1/90*(15*x^6*arct
an(c*x) - c*((3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan(c*x)/c^7))*b*c
^2*d^2 + 1/10*I*(4*x^5*arctan(c*x) - c*((c^2*x^4 - 2*x^2)/c^4 + 2*log(c^2*
x^2 + 1)/c^6))*b*c*d^2 + 1/12*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4
+ 3*arctan(c*x)/c^5))*b*d^2
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.04

$$\int x^3(d+icdx)^2(a+b\arctan(cx))\,dx = -\frac{60\,bc^6d^2x^6\arctan(cx)+60\,ac^6d^2x^6-144i\,bc^5d^2x^5\arctan(cx)-144i\,ac^5d^2x^5-12\,bc^5d^2x^5-90\,bc^4d^2}{}$$

input
```
integrate(x^3*(d+I*c*d*x)^2*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
-1/360*(60*b*c^6*d^2*x^6*arctan(c*x) + 60*a*c^6*d^2*x^6 - 144*I*b*c^5*d^2*
x^5*arctan(c*x) - 144*I*a*c^5*d^2*x^5 - 12*b*c^5*d^2*x^5 - 90*b*c^4*d^2*x^
4*arctan(c*x) - 90*a*c^4*d^2*x^4 + 36*I*b*c^4*d^2*x^4 + 50*b*c^3*d^2*x^3 -
 72*I*b*c^2*d^2*x^2 - 150*b*c*d^2*x - 3*I*b*d^2*log(I*c*x + 1) + 147*I*b*d
^2*log(-I*c*x + 1))/c^4
```

## Mupad [B] (verification not implemented)

Time = 1.08 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.92

$$\int x^3(d+icdx)^2(a+b\arctan(cx))\,dx$$
$$= -\frac{\frac{d^2\,\left(75\,b\,\mathrm{atan}(c\,x)+b\,\ln\left(c^2\,x^2+1\right)\,36\mathrm{i}\right)}{180}+\frac{5\,b\,c^3\,d^2\,x^3}{36}-\frac{5\,b\,c\,d^2\,x}{12}-\frac{b\,c^2\,d^2\,x^2\,1\mathrm{i}}{5}}{c^4} + \frac{d^2\,\left(45\,a\,x^4+45\,b\,x^4\,\mathrm{atan}(c\,x)-b\,x^4\,18\mathrm{i}\right)}{180} - \frac{c^2\,d^2\,\left(30\,a\,x^6+30\,b\,x^6\,\mathrm{atan}(c\,x)\right)}{180} + \frac{c\,d^2\,\left(a\,x^5\,72\mathrm{i}+6\,b\,x^5+b\,x^5\,\mathrm{atan}(c\,x)\,72\mathrm{i}\right)}{180}$$

input
```
int(x^3*(a + b*atan(c*x))*(d + c*d*x*1i)^2,x)
```

output
```
(d^2*(45*a*x^4 - b*x^4*18i + 45*b*x^4*atan(c*x)))/180 - ((d^2*(75*b*atan(c
*x) + b*log(c^2*x^2 + 1)*36i))/180 - (b*c^2*d^2*x^2*1i)/5 + (5*b*c^3*d^2*x
^3)/36 - (5*b*c*d^2*x)/12)/c^4 - (c^2*d^2*(30*a*x^6 + 30*b*x^6*atan(c*x)))
/180 + (c*d^2*(a*x^5*72i + 6*b*x^5 + b*x^5*atan(c*x)*72i))/180
```

## Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.85

$$\int x^3(d+icdx)^2(a+b\arctan(cx))\,dx$$
$$=\frac{d^2(-30atan(cx)\,b\,c^6x^6+72atan(cx)\,b\,c^5i\,x^5+45atan(cx)\,b\,c^4x^4-75atan(cx)\,b-36\log(c^2x^2+1)\,bi-}{180c^4}$$

input
```
int(x^3*(d+I*c*d*x)^2*(a+b*atan(c*x)),x)
```

output
```
(d**2*( - 30*atan(c*x)*b*c**6*x**6 + 72*atan(c*x)*b*c**5*i*x**5 + 45*atan(
c*x)*b*c**4*x**4 - 75*atan(c*x)*b - 36*log(c**2*x**2 + 1)*b*i - 30*a*c**6*
x**6 + 72*a*c**5*i*x**5 + 45*a*c**4*x**4 + 6*b*c**5*x**5 - 18*b*c**4*i*x**
4 - 25*b*c**3*x**3 + 36*b*c**2*i*x**2 + 75*b*c*x))/(180*c**4)
```

# 3.11 $\int x^2(d+icdx)^2(a+b\arctan(cx))\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 152

$$\int x^2(d+icdx)^2(a+b\arctan(cx))\,dx = \frac{ibd^2x}{2c^2}-\frac{4bd^2x^2}{15c}-\frac{1}{6}ibd^2x^3+\frac{1}{20}bcd^2x^4 -\frac{ibd^2\arctan(cx)}{2c^3}+\frac{1}{3}d^2x^3(a+b\arctan(cx)) +\frac{1}{2}icd^2x^4(a+b\arctan(cx)) -\frac{1}{5}c^2d^2x^5(a+b\arctan(cx)) +\frac{4bd^2\log(1+c^2x^2)}{15c^3}$$

output
```
1/2*I*b*d^2*x/c^2-4/15*b*d^2*x^2/c-1/6*I*b*d^2*x^3+1/20*b*c*d^2*x^4-1/2*I*
b*d^2*arctan(c*x)/c^3+1/3*d^2*x^3*(a+b*arctan(c*x))+1/2*I*c*d^2*x^4*(a+b*a
rctan(c*x))-1/5*c^2*d^2*x^5*(a+b*arctan(c*x))+4/15*b*d^2*ln(c^2*x^2+1)/c^3
```

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.76

$$\int x^2(d+icdx)^2(a+b\arctan(cx))\,dx$$
$$=\frac{d^2(2ac^3x^3(10+15icx-6c^2x^2)+bcx(30i-16cx-10ic^2x^2+3c^3x^3)+2b(-15i+10c^3x^3+15ic^4x^4-6}{60c^3}$$

input
```
Integrate[x^2*(d + I*c*d*x)^2*(a + b*ArcTan[c*x]),x]
```

output
```
(d^2*(2*a*c^3*x^3*(10 + (15*I)*c*x - 6*c^2*x^2) + b*c*x*(30*I - 16*c*x - (
10*I)*c^2*x^2 + 3*c^3*x^3) + 2*b*(-15*I + 10*c^3*x^3 + (15*I)*c^4*x^4 - 6*
c^5*x^5)*ArcTan[c*x] + 16*b*Log[1 + c^2*x^2]))/(60*c^3)
```

## Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.86, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {5407, 27, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(d+icdx)^2(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{ 5407}$$

$$-bc\int \frac{d^2x^3\left(-6c^2x^2+15icx+10\right)}{30\left(c^2x^2+1\right)}dx-\frac{1}{5}c^2d^2x^5(a+b\arctan(cx))+\frac{1}{2}icd^2x^4(a+b\arctan(cx))+\frac{1}{3}d^2x^3(a+b\arctan(cx))$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{1}{30}bcd^2\int \frac{x^3\left(-6c^2x^2+15icx+10\right)}{c^2x^2+1}dx-\frac{1}{5}c^2d^2x^5(a+b\arctan(cx))+\frac{1}{2}icd^2x^4(a+b\arctan(cx))+\frac{1}{3}d^2x^3(a+b\arctan(cx))$$

$$\Big\downarrow \text{ 2333}$$

$$-\frac{1}{30}bcd^2\int\left(-6x^3+\frac{15ix^2}{c}+\frac{16x}{c^2}+\frac{15i-16cx}{c^3\left(c^2x^2+1\right)}-\frac{15i}{c^3}\right)dx-\frac{1}{5}c^2d^2x^5(a+b\arctan(cx))+\frac{1}{2}icd^2x^4(a+b\arctan(cx))+\frac{1}{3}d^2x^3(a+b\arctan(cx))$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{5}c^2d^2x^5(a+b\arctan(cx))+\frac{1}{2}icd^2x^4(a+b\arctan(cx))+\frac{1}{3}d^2x^3(a+b\arctan(cx))-\frac{1}{30}bcd^2\left(\frac{15i\arctan(cx)}{c^4}-\frac{15ix}{c^3}+\frac{8x^2}{c^2}-\frac{8\log\left(c^2x^2+1\right)}{c^4}+\frac{5ix^3}{c}-\frac{3x^4}{2}\right)$$

input
```
Int[x^2*(d + I*c*d*x)^2*(a + b*ArcTan[c*x]),x]
```

output
```
(d^2*x^3*(a + b*ArcTan[c*x]))/3 + (I/2)*c*d^2*x^4*(a + b*ArcTan[c*x]) - (c
^2*d^2*x^5*(a + b*ArcTan[c*x]))/5 - (b*c*d^2*(((-15*I)*x)/c^3 + (8*x^2)/c^
2 + ((5*I)*x^3)/c - (3*x^4)/2 + ((15*I)*ArcTan[c*x])/c^4 - (8*Log[1 + c^2*
x^2])/c^4))/30
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 5407
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Simp[(a
 + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ
[2*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0])
)
```

## Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.81

| method | result |
|---|---|
| parts | $d^2a\left(-\frac{1}{5}x^5c^2+\frac{1}{2}icx^4+\frac{1}{3}x^3\right)+\frac{d^2b\left(-\frac{c^5x^5\arctan(cx)}{5}+\frac{i\arctan(cx)c^4x^4}{2}+\frac{c^3x^3\arctan(cx)}{3}+\frac{icx}{2}+\frac{c^4x^4}{20}-\frac{ic^3x}{6}\right.}{c^3}$ |
| derivativedivides | $\frac{d^2a\left(-\frac{1}{5}c^5x^5+\frac{1}{2}ic^4x^4+\frac{1}{3}c^3x^3\right)+d^2b\left(-\frac{c^5x^5\arctan(cx)}{5}+\frac{i\arctan(cx)c^4x^4}{2}+\frac{c^3x^3\arctan(cx)}{3}+\frac{icx}{2}+\frac{c^4x^4}{20}-\frac{ic^3x^3}{6}-\frac{4c^2x^2}{15}\right.}{c^3}$ |
| default | $\frac{d^2a\left(-\frac{1}{5}c^5x^5+\frac{1}{2}ic^4x^4+\frac{1}{3}c^3x^3\right)+d^2b\left(-\frac{c^5x^5\arctan(cx)}{5}+\frac{i\arctan(cx)c^4x^4}{2}+\frac{c^3x^3\arctan(cx)}{3}+\frac{icx}{2}+\frac{c^4x^4}{20}-\frac{ic^3x^3}{6}-\frac{4c^2x^2}{15}\right.}{c^3}$ |
| parallelrisch | $\frac{-12c^5d^2b\arctan(cx)x^5+30ibc^4d^2\arctan(cx)x^4-12ac^5d^2x^5+30ix^4ac^4d^2+3bc^4d^2x^4-10ix^3bc^3d^2+20d^2b\arctan(cx}{60c^3}$ |
| risch | $\frac{id^2b(6x^5c^2-15icx^4-10x^3)\ln(icx+1)}{60}-\frac{id^2c^2bx^5\ln(-icx+1)}{10}-\frac{d^2c^2x^5a}{5}+\frac{id^2cax^4}{2}-\frac{d^2cx^4b\ln(-icx+1)}{4}+$ |

input
```
int(x^2*(d+I*c*d*x)^2*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

output
```
d^2*a*(-1/5*x^5*c^2+1/2*I*c*x^4+1/3*x^3)+d^2*b/c^3*(-1/5*c^5*x^5*arctan(c*
x)+1/2*I*arctan(c*x)*c^4*x^4+1/3*c^3*x^3*arctan(c*x)+1/2*I*c*x+1/20*c^4*x^
4-1/6*I*c^3*x^3-4/15*c^2*x^2+4/15*ln(c^2*x^2+1)-1/2*I*arctan(c*x))
```

## Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.05

$$\int x^2(d+icdx)^2(a+b\arctan(cx))\,dx =$$
$$-\frac{12\,ac^5d^2x^5+3\,(-10i\,a-b)c^4d^2x^4-10\,(2\,a-i\,b)c^3d^2x^3+16\,bc^2d^2x^2-30i\,bcd^2x-31\,bd^2\log\left(\frac{cx+i}{c}\right)}{60\,c^3}$$

input
```
integrate(x^2*(d+I*c*d*x)^2*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
-1/60*(12*a*c^5*d^2*x^5 + 3*(-10*I*a - b)*c^4*d^2*x^4 - 10*(2*a - I*b)*c^3
*d^2*x^3 + 16*b*c^2*d^2*x^2 - 30*I*b*c*d^2*x - 31*b*d^2*log((c*x + I)/c) -
 b*d^2*log((c*x - I)/c) - (-6*I*b*c^5*d^2*x^5 - 15*b*c^4*d^2*x^4 + 10*I*b*
c^3*d^2*x^3)*log(-(c*x + I)/(c*x - I)))/c^3
```

## Sympy [A] (verification not implemented)

Time = 2.11 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.64

$$\int x^2(d+icdx)^2(a+b\arctan(cx))\,dx$$
$$= -\frac{ac^2d^2x^5}{5} - \frac{4bd^2x^2}{15c} + \frac{ibd^2x}{2c^2} - \frac{bd^2\left(-\frac{\log\left(47bcd^2x-47ibd^2\right)}{60} - \frac{49\log\left(47bcd^2x+47ibd^2\right)}{120}\right)}{c^3}$$
$$- x^4\left(-\frac{iacd^2}{2} - \frac{bcd^2}{20}\right) - x^3\left(-\frac{ad^2}{3} + \frac{ibd^2}{6}\right)$$
$$+ \left(\frac{ibc^2d^2x^5}{10} + \frac{bcd^2x^4}{4} - \frac{ibd^2x^3}{6}\right)\log\left(icx+1\right)$$
$$+ \frac{\left(-12ibc^5d^2x^5 - 30bc^4d^2x^4 + 20ibc^3d^2x^3 + 13bd^2\right)\log\left(-icx+1\right)}{120c^3}$$

input
```
integrate(x**2*(d+I*c*d*x)**2*(a+b*atan(c*x)),x)
```

output
```
-a*c**2*d**2*x**5/5 - 4*b*d**2*x**2/(15*c) + I*b*d**2*x/(2*c**2) - b*d**2*
(-log(47*b*c*d**2*x - 47*I*b*d**2)/60 - 49*log(47*b*c*d**2*x + 47*I*b*d**2
)/120)/c**3 - x**4*(-I*a*c*d**2/2 - b*c*d**2/20) - x**3*(-a*d**2/3 + I*b*d
**2/6) + (I*b*c**2*d**2*x**5/10 + b*c*d**2*x**4/4 - I*b*d**2*x**3/6)*log(I
*c*x + 1) + (-12*I*b*c**5*d**2*x**5 - 30*b*c**4*d**2*x**4 + 20*I*b*c**3*d*
*2*x**3 + 13*b*d**2)*log(-I*c*x + 1)/(120*c**3)
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.14

$$\int x^2(d+icdx)^2(a+b\arctan(cx))\,dx$$
$$= -\frac{1}{5}\,ac^2d^2x^5 + \frac{1}{2}i\,acd^2x^4 - \frac{1}{20}\left(4\,x^5\arctan\left(cx\right) - c\left(\frac{c^2x^4-2\,x^2}{c^4} + \frac{2\,\log\left(c^2x^2+1\right)}{c^6}\right)\right)bc^2d^2 + \frac{1}{3}\,ad^2x^3 + \frac{1}{6}i\left(3\,x^4\arctan\left(cx\right) - c\left(\frac{c^2x^3-3\,x}{c^4} + \frac{3\,\arctan\left(cx\right)}{c^5}\right)\right)bcd^2 + \frac{1}{6}\left(2\,x^3\arctan\left(cx\right) - c\left(\frac{x^2}{c^2} - \frac{\log\left(c^2x^2+1\right)}{c^4}\right)\right)bd^2$$

input
```
integrate(x^2*(d+I*c*d*x)^2*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
-1/5*a*c^2*d^2*x^5 + 1/2*I*a*c*d^2*x^4 - 1/20*(4*x^5*arctan(c*x) - c*((c^2
*x^4 - 2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*b*c^2*d^2 + 1/3*a*d^2*x^3 + 1
/6*I*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*b*c
*d^2 + 1/6*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*b*d^2
```

## Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.04

$$\int x^2(d+icdx)^2(a+b\arctan(cx))\,dx = -\frac{12\,bc^5d^2x^5\arctan\left(cx\right) + 12\,ac^5d^2x^5 - 30i\,bc^4d^2x^4\arctan\left(cx\right) - 30i\,ac^4d^2x^4 - 3\,bc^4d^2x^4 - 20\,bc^3d^2x^3\,a}{60}$$

input
```
integrate(x^2*(d+I*c*d*x)^2*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
-1/60*(12*b*c^5*d^2*x^5*arctan(c*x) + 12*a*c^5*d^2*x^5 - 30*I*b*c^4*d^2*x^
4*arctan(c*x) - 30*I*a*c^4*d^2*x^4 - 3*b*c^4*d^2*x^4 - 20*b*c^3*d^2*x^3*ar
ctan(c*x) - 20*a*c^3*d^2*x^3 + 10*I*b*c^3*d^2*x^3 + 16*b*c^2*d^2*x^2 - 30*
I*b*c*d^2*x - 31*b*d^2*log(c*x + I) - b*d^2*log(c*x - I))/c^3
```

## Mupad [B] (verification not implemented)

Time = 1.06 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.92

$$\int x^2(d+icdx)^2(a+b\arctan(cx))\,dx$$
$$= -\frac{\frac{d^2\,\left(-16\,b\,\ln\left(c^2\,x^2+1\right)+b\,\mathrm{atan}\left(c\,x\right)\,30{}\mathrm{i}\right)}{60}+\frac{4\,b\,c^2\,d^2\,x^2}{15}-\frac{b\,c\,d^2\,x\,1{}\mathrm{i}}{2}}{c^3}$$
$$+\frac{d^2\,\left(20\,a\,x^3+20\,b\,x^3\,\mathrm{atan}\left(c\,x\right)-b\,x^3\,10{}\mathrm{i}\right)}{60}-\frac{c^2\,d^2\,\left(12\,a\,x^5+12\,b\,x^5\,\mathrm{atan}\left(c\,x\right)\right)}{60}$$
$$+\frac{c\,d^2\,\left(a\,x^4\,30{}\mathrm{i}+3\,b\,x^4+b\,x^4\,\mathrm{atan}\left(c\,x\right)\,30{}\mathrm{i}\right)}{60}$$

input
```
int(x^2*(a + b*atan(c*x))*(d + c*d*x*1i)^2,x)
```

output
```
(d^2*(20*a*x^3 - b*x^3*10i + 20*b*x^3*atan(c*x)))/60 - ((d^2*(b*atan(c*x)*
30i - 16*b*log(c^2*x^2 + 1)))/60 + (4*b*c^2*d^2*x^2)/15 - (b*c*d^2*x*1i)/2
)/c^3 - (c^2*d^2*(12*a*x^5 + 12*b*x^5*atan(c*x)))/60 + (c*d^2*(a*x^4*30i +
 3*b*x^4 + b*x^4*atan(c*x)*30i))/60
```

## Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.87

$$\int x^2(d+icdx)^2(a+b\arctan(cx))\,dx$$
$$= \frac{d^2(-12atan(cx)\,b\,c^5x^5+30atan(cx)\,b\,c^4i\,x^4+20atan(cx)\,b\,c^3x^3-30atan(cx)\,bi+16\log(c^2x^2+1)\,b-}{60c^3}$$

input
```
int(x^2*(d+I*c*d*x)^2*(a+b*atan(c*x)),x)
```

output
```
(d**2*( - 12*atan(c*x)*b*c**5*x**5 + 30*atan(c*x)*b*c**4*i*x**4 + 20*atan(
c*x)*b*c**3*x**3 - 30*atan(c*x)*b*i + 16*log(c**2*x**2 + 1)*b - 12*a*c**5*
x**5 + 30*a*c**4*i*x**4 + 20*a*c**3*x**3 + 3*b*c**4*x**4 - 10*b*c**3*i*x**
3 - 16*b*c**2*x**2 + 30*b*c*i*x))/(60*c**3)
```

# 3.12 $\int x(d+icdx)^2(a+b\arctan(cx))\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 136

$$\int x(d+icdx)^2(a+b\arctan(cx))\,dx = -\frac{3bd^2x}{4c}-\frac{1}{3}ibd^2x^2+\frac{1}{12}bcd^2x^3+\frac{3bd^2\arctan(cx)}{4c^2}+\frac{1}{2}d^2x^2(a+b\arctan(cx))+\frac{2}{3}icd^2x^3(a+b\arctan(cx))-\frac{1}{4}c^2d^2x^4(a+b\arctan(cx))+\frac{ibd^2\log(1+c^2x^2)}{3c^2}$$

output
```
-3/4*b*d^2*x/c-1/3*I*b*d^2*x^2+1/12*b*c*d^2*x^3+3/4*b*d^2*arctan(c*x)/c^2+
1/2*d^2*x^2*(a+b*arctan(c*x))+2/3*I*c*d^2*x^3*(a+b*arctan(c*x))-1/4*c^2*d^
2*x^4*(a+b*arctan(c*x))+1/3*I*b*d^2*ln(c^2*x^2+1)/c^2
```

## Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.74

$$\int x(d+icdx)^2(a+b\arctan(cx))\,dx$$
$$=\frac{d^2(cx(acx(6+8icx-3c^2x^2)+b(-9-4icx+c^2x^2))+b(9+6c^2x^2+8ic^3x^3-3c^4x^4)\arctan(cx)+4ib\,\mathrm{l}}{12c^2}$$

input
```
Integrate[x*(d + I*c*d*x)^2*(a + b*ArcTan[c*x]),x]
```

output
```
(d^2*(c*x*(a*c*x*(6 + (8*I)*c*x - 3*c^2*x^2) + b*(-9 - (4*I)*c*x + c^2*x^2
)) + b*(9 + 6*c^2*x^2 + (8*I)*c^3*x^3 - 3*c^4*x^4)*ArcTan[c*x] + (4*I)*b*L
og[1 + c^2*x^2]))/(12*c^2)
```

## Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.87, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {5407, 27, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(d+icdx)^2(a+b\arctan(cx))\,dx$$
$$\Big\downarrow \; 5407$$
$$-bc\int \frac{d^2x^2(-3c^2x^2+8icx+6)}{12(c^2x^2+1)}dx-\frac{1}{4}c^2d^2x^4(a+b\arctan(cx))+\frac{2}{3}icd^2x^3(a+ b\arctan(cx))+\frac{1}{2}d^2x^2(a+b\arctan(cx))$$
$$\Big\downarrow \; 27$$
$$-\frac{1}{12}bcd^2\int \frac{x^2(-3c^2x^2+8icx+6)}{c^2x^2+1}dx-\frac{1}{4}c^2d^2x^4(a+b\arctan(cx))+\frac{2}{3}icd^2x^3(a+ b\arctan(cx))+\frac{1}{2}d^2x^2(a+b\arctan(cx))$$
$$\Big\downarrow \; 2333$$

$$-\frac{1}{12}bcd^2\int\left(-3x^2+\frac{8ix}{c}+\frac{i(9i-8cx)}{c^2\left(c^2x^2+1\right)}+\frac{9}{c^2}\right)dx-\frac{1}{4}c^2d^2x^4(a+b\arctan(cx))+\frac{2}{3}icd^2x^3(a+b\arctan(cx))+\frac{1}{2}d^2x^2(a+b\arctan(cx))$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{1}{4}c^2d^2x^4(a+b\arctan(cx))+\frac{2}{3}icd^2x^3(a+b\arctan(cx))+\frac{1}{2}d^2x^2(a+b\arctan(cx))-\frac{1}{12}bcd^2\left(-\frac{9\arctan(cx)}{c^3}+\frac{9x}{c^2}-\frac{4i\log\left(c^2x^2+1\right)}{c^3}+\frac{4ix^2}{c}-x^3\right)$$

input
```
Int[x*(d + I*c*d*x)^2*(a + b*ArcTan[c*x]),x]
```

output
```
(d^2*x^2*(a + b*ArcTan[c*x]))/2 + ((2*I)/3)*c*d^2*x^3*(a + b*ArcTan[c*x])
- (c^2*d^2*x^4*(a + b*ArcTan[c*x]))/4 - (b*c*d^2*((9*x)/c^2 + ((4*I)*x^2)/
c - x^3 - (9*ArcTan[c*x])/c^3 - ((4*I)*Log[1 + c^2*x^2])/c^3))/12
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 5407
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Simp[(a
 + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ
[2*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0])
)
```

## Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.84

| method | result |
|---|---|
| parts | $d^2a\left(-\frac{1}{4}c^2x^4+\frac{2}{3}ic\,x^3+\frac{1}{2}x^2\right)+\frac{d^2b\left(-\frac{c^4x^4\arctan(cx)}{4}+\frac{2i\arctan(cx)c^3x^3}{3}+\frac{c^2x^2\arctan(cx)}{2}-\frac{3cx}{4}+\frac{c^3x^3}{12}-\frac{ic^2}{}\right.}{c^2}$ |
| derivativedivides | $\frac{d^2a\left(-\frac{1}{4}c^4x^4+\frac{2}{3}ic^3x^3+\frac{1}{2}c^2x^2\right)+d^2b\left(-\frac{c^4x^4\arctan(cx)}{4}+\frac{2i\arctan(cx)c^3x^3}{3}+\frac{c^2x^2\arctan(cx)}{2}-\frac{3cx}{4}+\frac{c^3x^3}{12}-\frac{ic^2x^2}{3}+\frac{i\ln(c}{}\right.}{c^2}$ |
| default | $\frac{d^2a\left(-\frac{1}{4}c^4x^4+\frac{2}{3}ic^3x^3+\frac{1}{2}c^2x^2\right)+d^2b\left(-\frac{c^4x^4\arctan(cx)}{4}+\frac{2i\arctan(cx)c^3x^3}{3}+\frac{c^2x^2\arctan(cx)}{2}-\frac{3cx}{4}+\frac{c^3x^3}{12}-\frac{ic^2x^2}{3}+\frac{i\ln(c}{}\right.}{c^2}$ |
| parallelrisch | $\frac{-3d^2b\arctan(cx)x^4c^4+8ic^3b\,d^2\arctan(cx)x^3-3a\,c^4d^2x^4+8ix^3a\,c^3d^2+b\,c^3d^2x^3-4ix^2b\,c^2d^2+6d^2b\arctan(cx)x^2c^2+6}{12c^2}$ |
| risch | $\frac{id^2b(3c^2x^4-8ic\,x^3-6x^2)\ln(icx+1)}{24}-\frac{d^2c^2x^4a}{4}-\frac{id^2c^2x^4b\ln(-icx+1)}{8}-\frac{d^2cb\,x^3\ln(-icx+1)}{3}+\frac{bc\,d^2x^3}{12}+\frac{2i}{}$ |

input
```
int(x*(d+I*c*d*x)^2*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

output
```
d^2*a*(-1/4*c^2*x^4+2/3*I*c*x^3+1/2*x^2)+d^2*b/c^2*(-1/4*c^4*x^4*arctan(c*
x)+2/3*I*arctan(c*x)*c^3*x^3+1/2*c^2*x^2*arctan(c*x)-3/4*c*x+1/12*c^3*x^3-
1/3*I*c^2*x^2+1/3*I*ln(c^2*x^2+1)+3/4*arctan(c*x))
```

## Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.09

$$\int x(d+icdx)^2(a+b\arctan(cx))\,dx = -\frac{6\,ac^4d^2x^4+2\,(-8i\,a-b)c^3d^2x^3-4\,(3\,a-2i\,b)c^2d^2x^2+18\,bcd^2x-17i\,bd^2\log\left(\frac{cx+i}{c}\right)+i\,bd^2\log\left(\frac{cx-i}{c}\right.}{24\,c^2}$$

input
```
integrate(x*(d+I*c*d*x)^2*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
-1/24*(6*a*c^4*d^2*x^4 + 2*(-8*I*a - b)*c^3*d^2*x^3 - 4*(3*a - 2*I*b)*c^2*
d^2*x^2 + 18*b*c*d^2*x - 17*I*b*d^2*log((c*x + I)/c) + I*b*d^2*log((c*x -
I)/c) - (-3*I*b*c^4*d^2*x^4 - 8*b*c^3*d^2*x^3 + 6*I*b*c^2*d^2*x^2)*log(-(c
*x + I)/(c*x - I)))/c^2
```

## Sympy [A] (verification not implemented)

Time = 1.95 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.76

$$\int x(d+icdx)^2(a+b\arctan(cx))\,dx$$
$$= -\frac{ac^2d^2x^4}{4} - \frac{3bd^2x}{4c} - \frac{bd^2\left(\frac{i\log\left(67bcd^2x-67ibd^2\right)}{24} - \frac{31i\log\left(67bcd^2x+67ibd^2\right)}{60}\right)}{c^2} - x^3\left(-\frac{2iacd^2}{3} - \frac{bcd^2}{12}\right) - x^2\left(-\frac{ad^2}{2} + \frac{ibd^2}{3}\right) + \left(\frac{ibc^2d^2x^4}{8} + \frac{bcd^2x^3}{3} - \frac{ibd^2x^2}{4}\right)\log\left(icx+1\right) + \frac{\left(-15ibc^4d^2x^4 - 40bc^3d^2x^3 + 30ibc^2d^2x^2 + 23ibd^2\right)\log\left(-icx+1\right)}{120c^2}$$

input
```
integrate(x*(d+I*c*d*x)**2*(a+b*atan(c*x)),x)
```

output
```
-a*c**2*d**2*x**4/4 - 3*b*d**2*x/(4*c) - b*d**2*(I*log(67*b*c*d**2*x - 67*
I*b*d**2)/24 - 31*I*log(67*b*c*d**2*x + 67*I*b*d**2)/60)/c**2 - x**3*(-2*I
*a*c*d**2/3 - b*c*d**2/12) - x**2*(-a*d**2/2 + I*b*d**2/3) + (I*b*c**2*d**
2*x**4/8 + b*c*d**2*x**3/3 - I*b*d**2*x**2/4)*log(I*c*x + 1) + (-15*I*b*c*
*4*d**2*x**4 - 40*b*c**3*d**2*x**3 + 30*I*b*c**2*d**2*x**2 + 23*I*b*d**2)*
log(-I*c*x + 1)/(120*c**2)
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.14

$$\begin{aligned}\int x(d+icdx)^2(a+b\arctan(cx))\,dx &= -\frac{1}{4}\,ac^2d^2x^4+\frac{2}{3}i\,acd^2x^3 \\ &\quad -\frac{1}{12}\left(3\,x^4\arctan\left(cx\right)-c\left(\frac{c^2x^3-3\,x}{c^4}+\frac{3\,\arctan\left(cx\right)}{c^5}\right)\right)bc^2d^2 \\ &\quad +\frac{1}{3}i\left(2\,x^3\arctan\left(cx\right)-c\left(\frac{x^2}{c^2}-\frac{\log\left(c^2x^2+1\right)}{c^4}\right)\right)bcd^2 \\ &\quad +\frac{1}{2}\,ad^2x^2+\frac{1}{2}\left(x^2\arctan\left(cx\right)-c\left(\frac{x}{c^2}-\frac{\arctan\left(cx\right)}{c^3}\right)\right)bd^2\end{aligned}$$

input
```
integrate(x*(d+I*c*d*x)^2*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
-1/4*a*c^2*d^2*x^4 + 2/3*I*a*c*d^2*x^3 - 1/12*(3*x^4*arctan(c*x) - c*((c^2
*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*b*c^2*d^2 + 1/3*I*(2*x^3*arctan(c*x)
 - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*b*c*d^2 + 1/2*a*d^2*x^2 + 1/2*(x^2*
arctan(c*x) - c*(x/c^2 - arctan(c*x)/c^3))*b*d^2
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.09

$$\int x(d+icdx)^2(a+b\arctan(cx))\,dx = -\frac{6\,bc^4d^2x^4\arctan\left(cx\right)+6\,ac^4d^2x^4-16i\,bc^3d^2x^3\arctan\left(cx\right)-16i\,ac^3d^2x^3-2\,bc^3d^2x^3-12\,bc^2d^2x^2\,\mathrm{arc}}{24\,c^2}$$

input
```
integrate(x*(d+I*c*d*x)^2*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
-1/24*(6*b*c^4*d^2*x^4*arctan(c*x) + 6*a*c^4*d^2*x^4 - 16*I*b*c^3*d^2*x^3*
arctan(c*x) - 16*I*a*c^3*d^2*x^3 - 2*b*c^3*d^2*x^3 - 12*b*c^2*d^2*x^2*arct
an(c*x) - 12*a*c^2*d^2*x^2 + 8*I*b*c^2*d^2*x^2 + 18*b*c*d^2*x - 17*I*b*d^2
*log(I*c*x - 1) + I*b*d^2*log(-I*c*x - 1))/c^2
```

## Mupad [B] (verification not implemented)

Time = 1.00 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.92

$$\int x(d+icdx)^2(a+b\arctan(cx))\,dx = \frac{\frac{d^2\,(9\,b\,\mathrm{atan}(c\,x)+b\,\ln(c^2\,x^2+1)\,4\mathrm{i})}{12}-\frac{3\,b\,c\,d^2\,x}{4}}{c^2} + \frac{d^2\,(6\,a\,x^2+6\,b\,x^2\,\mathrm{atan}(c\,x)-b\,x^2\,4\mathrm{i})}{12} - \frac{c^2\,d^2\,(3\,a\,x^4+3\,b\,x^4\,\mathrm{atan}(c\,x))}{12} + \frac{c\,d^2\,(a\,x^3\,8\mathrm{i}+b\,x^3+b\,x^3\,\mathrm{atan}(c\,x)\,8\mathrm{i})}{12}$$

input
```
int(x*(a + b*atan(c*x))*(d + c*d*x*1i)^2,x)
```

output
```
((d^2*(9*b*atan(c*x) + b*log(c^2*x^2 + 1)*4i))/12 - (3*b*c*d^2*x)/4)/c^2 +
 (d^2*(6*a*x^2 - b*x^2*4i + 6*b*x^2*atan(c*x)))/12 - (c^2*d^2*(3*a*x^4 + 3
*b*x^4*atan(c*x)))/12 + (c*d^2*(a*x^3*8i + b*x^3 + b*x^3*atan(c*x)*8i))/12
```

## Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.89

$$\int x(d+icdx)^2(a+b\arctan(cx))\,dx = \frac{d^2(-3atan(cx)\,b\,c^4x^4+8atan(cx)\,b\,c^3i\,x^3+6atan(cx)\,b\,c^2x^2+9atan(cx)\,b+4\log(c^2x^2+1)\,bi-3a\,c^4x}{12c^2}$$

input
```
int(x*(d+I*c*d*x)^2*(a+b*atan(c*x)),x)
```

output
```
(d**2*( - 3*atan(c*x)*b*c**4*x**4 + 8*atan(c*x)*b*c**3*i*x**3 + 6*atan(c*x
)*b*c**2*x**2 + 9*atan(c*x)*b + 4*log(c**2*x**2 + 1)*b*i - 3*a*c**4*x**4 +
 8*a*c**3*i*x**3 + 6*a*c**2*x**2 + b*c**3*x**3 - 4*b*c**2*i*x**2 - 9*b*c*x
))/(12*c**2)
```

# 3.13 $\int (d+icdx)^2(a+b\arctan(cx))\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 83

$$\int (d+icdx)^2(a+b\arctan(cx))\,dx = -\frac{2}{3}ibd^2x-\frac{bd^2(1+icx)^2}{6c} - \frac{id^2(1+icx)^3(a+b\arctan(cx))}{3c} - \frac{4bd^2\log(1-icx)}{3c}$$

output
```
-2/3*I*b*d^2*x-1/6*b*d^2*(1+I*c*x)^2/c-1/3*I*d^2*(1+I*c*x)^3*(a+b*arctan(c
*x))/c-4/3*b*d^2*ln(1-I*c*x)/c
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.69

$$\int (d+icdx)^2(a+b\arctan(cx))\,dx = \frac{1}{3}d^2\left(\frac{1}{2}bx(-6i+cx) - \frac{(-i+cx)^3(a+b\arctan(cx))}{c} - \frac{4b\log(i+cx)}{c}\right)$$

input
```
Integrate[(d + I*c*d*x)^2*(a + b*ArcTan[c*x]),x]
```

output
```
(d^2*((b*x*(-6*I + c*x))/2 - ((-I + c*x)^3*(a + b*ArcTan[c*x]))/c - (4*b*L
og[I + c*x])/c))/3
```

## Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.84, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5387, 27, 456, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+icdx)^2(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5387}$$

$$\frac{ib\int \frac{d^3(icx+1)^3}{c^2x^2+1}dx}{3d}-\frac{id^2(1+icx)^3(a+b\arctan(cx))}{3c}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}ibd^2\int \frac{(icx+1)^3}{c^2x^2+1}dx-\frac{id^2(1+icx)^3(a+b\arctan(cx))}{3c}$$

$$\Big\downarrow \text{456}$$

$$\frac{1}{3}ibd^2\int \frac{(icx+1)^2}{1-icx}dx-\frac{id^2(1+icx)^3(a+b\arctan(cx))}{3c}$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{3}ibd^2\int \left(-icx+\frac{4}{1-icx}-3\right)dx-\frac{id^2(1+icx)^3(a+b\arctan(cx))}{3c}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}ibd^2\left(-\frac{1}{2}icx^2+\frac{4i\log(cx+i)}{c}-3x\right)-\frac{id^2(1+icx)^3(a+b\arctan(cx))}{3c}$$

input
```
Int[(d + I*c*d*x)^2*(a + b*ArcTan[c*x]),x]
```

output
```
((-1/3*I)*d^2*(1 + I*c*x)^3*(a + b*ArcTan[c*x]))/c + (I/3)*b*d^2*(-3*x - (
I/2)*c*x^2 + ((4*I)*Log[I + c*x])/c)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5387
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Simp[b*(
c/(e*(q + 1)))   Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b
, c, d, e, q}, x] && NeQ[q, -1]
```

## Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.24

| method | result |
|---|---|
| derivativedivides | $\frac{-\frac{i d^2 a (icx+1)^3}{3}+d^2 b\left(-\frac{c^3 x^3 \arctan(cx)}{3}+i\arctan(cx)c^2x^2+cx\arctan(cx)-\frac{i\arctan(cx)}{3}+\frac{i\left(-3cx-\frac{ic^2x^2}{2}+2i\ln\left(c^2x^2+1\right)\right.}{3}\right.}{c}$ |
| default | $\frac{-\frac{i d^2 a (icx+1)^3}{3}+d^2 b\left(-\frac{c^3 x^3 \arctan(cx)}{3}+i\arctan(cx)c^2x^2+cx\arctan(cx)-\frac{i\arctan(cx)}{3}+\frac{i\left(-3cx-\frac{ic^2x^2}{2}+2i\ln\left(c^2x^2+1\right)\right.}{3}\right.}{c}$ |
| parts | $-\frac{i d^2 a (icx+1)^3}{3c}-\frac{c^2 b\, d^2 \arctan(cx) x^3}{3}+icb\, d^2 \arctan(cx)\, x^2+b\arctan(cx)\, x\, d^2+\frac{i d^2 \arctan(cx}{c}$ |
| parallelrisch | $\frac{-2d^2 b\arctan(cx)x^3c^3+6ib\,c^2d^2\arctan(cx)x^2-2a\,c^3d^2x^3+6ix^2a\,c^2d^2+b\,c^2d^2x^2-6ib\,d^2xc+6b\arctan(cx)d^2cx+6ib\,d^2}{6c}$ |
| risch | $\frac{i d^2 (cx-i)^3 b\ln(icx+1)}{6c}-\frac{i d^2 c^2 x^3 b\ln(-icx+1)}{6}-\frac{d^2 a\, c^2 x^3}{3}+i d^2 ac\, x^2-\frac{d^2 c\, x^2 b\ln(-icx+1)}{2}+\frac{ib\, d^2 x\ln(-i}{2}$ |

input
```
int((d+I*c*d*x)^2*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/c*(-1/3*I*d^2*a*(1+I*c*x)^3+d^2*b*(-1/3*c^3*x^3*arctan(c*x)+I*arctan(c*x
)*c^2*x^2+c*x*arctan(c*x)-1/3*I*arctan(c*x)+1/3*I*(-3*c*x-1/2*I*c^2*x^2+2*
I*ln(c^2*x^2+1)+4*arctan(c*x))))
```

## Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.53

$$\int (d+icdx)^2(a+b\arctan(cx))\,dx = -\frac{2\,ac^3d^2x^3-(6i\,a+b)c^2d^2x^2-6\,(a-i\,b)cd^2x+7\,bd^2\log\left(\frac{cx+i}{c}\right)+bd^2\log\left(\frac{cx-i}{c}\right)-\left(-i\,bc^3d^2x^3-3\,bc^2\right.}{6\,c}$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
-1/6*(2*a*c^3*d^2*x^3 - (6*I*a + b)*c^2*d^2*x^2 - 6*(a - I*b)*c*d^2*x + 7*
b*d^2*log((c*x + I)/c) + b*d^2*log((c*x - I)/c) - (-I*b*c^3*d^2*x^3 - 3*b*
c^2*d^2*x^2 + 3*I*b*c*d^2*x)*log(-(c*x + I)/(c*x - I)))/c
```

## Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 206 vs. $2(73) = 146$.

Time = 1.61 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.48

$$\int (d+icdx)^2(a+b\arctan(cx))\,dx = -\frac{ac^2d^2x^3}{3} - \frac{bd^2\left(\frac{\log\left(13bcd^2x-13ibd^2\right)}{6}+\frac{17\log\left(13bcd^2x+13ibd^2\right)}{24}\right)}{c} - x^2\left(-iacd^2-\frac{bcd^2}{6}\right) - x\left(-ad^2+ibd^2\right) + \left(\frac{ibc^2d^2x^3}{6}+\frac{bcd^2x^2}{2}-\frac{ibd^2x}{2}\right)\log\left(icx+1\right) + \frac{\left(-4ibc^3d^2x^3-12bc^2d^2x^2+12ibcd^2x-11bd^2\right)\log\left(-icx+1\right)}{24c}$$

input
```
integrate((d+I*c*d*x)**2*(a+b*atan(c*x)),x)
```

output
```
-a*c**2*d**2*x**3/3 - b*d**2*(log(13*b*c*d**2*x - 13*I*b*d**2)/6 + 17*log(
13*b*c*d**2*x + 13*I*b*d**2)/24)/c - x**2*(-I*a*c*d**2 - b*c*d**2/6) - x*(
-a*d**2 + I*b*d**2) + (I*b*c**2*d**2*x**3/6 + b*c*d**2*x**2/2 - I*b*d**2*x
/2)*log(I*c*x + 1) + (-4*I*b*c**3*d**2*x**3 - 12*b*c**2*d**2*x**2 + 12*I*b
*c*d**2*x - 11*b*d**2)*log(-I*c*x + 1)/(24*c)
```

## Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(65) = 130$.

Time = 0.12 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.66

$$\int (d+icdx)^2(a+b\arctan(cx))\,dx = -\frac{1}{3}\,ac^2d^2x^3 - \frac{1}{6}\left(2\,x^3\arctan\left(cx\right) - c\left(\frac{x^2}{c^2}-\frac{\log\left(c^2x^2+1\right)}{c^4}\right)\right)bc^2d^2 + i\,acd^2x^2 + i\left(x^2\arctan\left(cx\right)-c\left(\frac{x}{c^2}-\frac{\arctan\left(cx\right)}{c^3}\right)\right)bcd^2 + ad^2x + \frac{\left(2\,cx\arctan\left(cx\right)-\log\left(c^2x^2+1\right)\right)bd^2}{2\,c}$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
-1/3*a*c^2*d^2*x^3 - 1/6*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1
)/c^4))*b*c^2*d^2 + I*a*c*d^2*x^2 + I*(x^2*arctan(c*x) - c*(x/c^2 - arctan
(c*x)/c^3))*b*c*d^2 + a*d^2*x + 1/2*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))
*b*d^2/c
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.51

$$\int (d+icdx)^2(a+b\arctan(cx))\,dx = -\frac{2\,bc^3d^2x^3\arctan{(cx)}+2\,ac^3d^2x^3-6i\,bc^2d^2x^2\arctan{(cx)}-6i\,ac^2d^2x^2-bc^2d^2x^2-6\,bcd^2x\arctan{(cx}}{6\,c}$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
-1/6*(2*b*c^3*d^2*x^3*arctan(c*x) + 2*a*c^3*d^2*x^3 - 6*I*b*c^2*d^2*x^2*ar
ctan(c*x) - 6*I*a*c^2*d^2*x^2 - b*c^2*d^2*x^2 - 6*b*c*d^2*x*arctan(c*x) -
6*a*c*d^2*x + 6*I*b*c*d^2*x + 7*b*d^2*log(c*x + I) + b*d^2*log(c*x - I))/c
```

## Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.31

$$\begin{aligned}\int (d+icdx)^2(a+b\arctan(cx))\,dx &= \frac{d^2\,(6\,a\,x+6\,b\,x\,\mathrm{atan}(c\,x)-b\,x\,6\mathrm{i})}{6} \\ &\quad - \frac{c^2\,d^2\,(2\,a\,x^3+2\,b\,x^3\,\mathrm{atan}(c\,x))}{6} \\ &\quad + \frac{d^2\,(-4\,b\,\ln{(c^2\,x^2+1)}+b\,\mathrm{atan}(c\,x)\;6\mathrm{i})}{6\,c} \\ &\quad + \frac{c\,d^2\,(a\,x^2\,6\mathrm{i}+b\,x^2+b\,x^2\,\mathrm{atan}(c\,x)\;6\mathrm{i})}{6}\end{aligned}$$

input
```
int((a + b*atan(c*x))*(d + c*d*x*1i)^2,x)
```

output
```
(d^2*(6*a*x - b*x*6i + 6*b*x*atan(c*x)))/6 - (c^2*d^2*(2*a*x^3 + 2*b*x^3*a
tan(c*x)))/6 + (d^2*(b*atan(c*x)*6i - 4*b*log(c^2*x^2 + 1)))/(6*c) + (c*d^
2*(a*x^2*6i + b*x^2 + b*x^2*atan(c*x)*6i))/6
```

## Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.25

$$\int (d+icdx)^2(a+b\arctan(cx))\,dx = \frac{d^2(-2atan(cx)\,b\,c^3x^3 + 6atan(cx)\,b\,c^2i\,x^2 + 6atan(cx)\,bcx + 6atan(cx)\,bi - 4\log(c^2x^2+1)\,b - 2a\,c^3x^3 -}{6c}$$

input
```
int((d+I*c*d*x)^2*(a+b*atan(c*x)),x)
```

output
```
(d**2*( - 2*atan(c*x)*b*c**3*x**3 + 6*atan(c*x)*b*c**2*i*x**2 + 6*atan(c*x
)*b*c*x + 6*atan(c*x)*b*i - 4*log(c**2*x**2 + 1)*b - 2*a*c**3*x**3 + 6*a*c
**2*i*x**2 + 6*a*c*x + b*c**2*x**2 - 6*b*c*i*x))/(6*c)
```

# 3.14 $\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 129

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x}\,dx = 2iacd^2x+\frac{1}{2}bcd^2x-\frac{1}{2}bd^2\arctan(cx) + 2ibcd^2x\arctan(cx)-\frac{1}{2}c^2d^2x^2(a+b\arctan(cx)) + ad^2\log(x)-ibd^2\log\left(1+c^2x^2\right) + \frac{1}{2}ibd^2\operatorname{PolyLog}(2,-icx)-\frac{1}{2}ibd^2\operatorname{PolyLog}(2,icx)$$

output
```
2*I*a*c*d^2*x+1/2*b*c*d^2*x-1/2*b*d^2*arctan(c*x)+2*I*b*c*d^2*x*arctan(c*x
)-1/2*c^2*d^2*x^2*(a+b*arctan(c*x))+a*d^2*ln(x)-I*b*d^2*ln(c^2*x^2+1)+1/2*
I*b*d^2*polylog(2,-I*c*x)-1/2*I*b*d^2*polylog(2,I*c*x)
```

## Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.80

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x}\,dx = -\frac{1}{2}d^2\left(-4iacx-bcx+ac^2x^2+b\arctan(cx) - 4ibcx\arctan(cx)+bc^2x^2\arctan(cx) - 2a\log(x)+2ib\log\left(1+c^2x^2\right) - ib\operatorname{PolyLog}(2,-icx)+ib\operatorname{PolyLog}(2,icx)\right)$$

input
```
Integrate[((d + I*c*d*x)^2*(a + b*ArcTan[c*x]))/x,x]
```

output
```
-1/2*(d^2*((-4*I)*a*c*x - b*c*x + a*c^2*x^2 + b*ArcTan[c*x] - (4*I)*b*c*x*
ArcTan[c*x] + b*c^2*x^2*ArcTan[c*x] - 2*a*Log[x] + (2*I)*b*Log[1 + c^2*x^2
] - I*b*PolyLog[2, (-I)*c*x] + I*b*PolyLog[2, I*c*x]))
```

## Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x}\,dx$$

$$\Big\downarrow \text{5411}$$

$$\int \left(-c^2d^2x(a+b\arctan(cx))+2icd^2(a+b\arctan(cx))+\frac{d^2(a+b\arctan(cx))}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{2}c^2d^2x^2(a+b\arctan(cx))+2iacd^2x+ad^2\log(x)-\frac{1}{2}bd^2\arctan(cx)+2ibcd^2x\arctan(cx)-ibd^2\log\left(c^2x^2+1\right)+\frac{1}{2}ibd^2\,\text{PolyLog}(2,-icx)-\frac{1}{2}ibd^2\,\text{PolyLog}(2,icx)+\frac{1}{2}bcd^2x$$

input
```
Int[((d + I*c*d*x)^2*(a + b*ArcTan[c*x]))/x,x]
```

output
```
(2*I)*a*c*d^2*x + (b*c*d^2*x)/2 - (b*d^2*ArcTan[c*x])/2 + (2*I)*b*c*d^2*x*
ArcTan[c*x] - (c^2*d^2*x^2*(a + b*ArcTan[c*x]))/2 + a*d^2*Log[x] - I*b*d^2
*Log[1 + c^2*x^2] + (I/2)*b*d^2*PolyLog[2, (-I)*c*x] - (I/2)*b*d^2*PolyLog
[2, I*c*x]
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

## Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.04

| method | result |
|---|---|
| parts | $d^2a\left(-\frac{c^2x^2}{2}+2icx+\ln(x)\right)+d^2b\left(2i\arctan(cx)\,cx-\frac{c^2x^2\arctan(cx)}{2}+\ln(cx)\arctan(c$ |
| derivativedivides | $d^2a\left(2icx-\frac{c^2x^2}{2}+\ln(cx)\right)+d^2b\left(2i\arctan(cx)\,cx-\frac{c^2x^2\arctan(cx)}{2}+\ln(cx)\arctan(c$ |
| default | $d^2a\left(2icx-\frac{c^2x^2}{2}+\ln(cx)\right)+d^2b\left(2i\arctan(cx)\,cx-\frac{c^2x^2\arctan(cx)}{2}+\ln(cx)\arctan(c$ |
| risch | $-\frac{id^2b\ln(-icx+1)c^2x^2}{4}-d^2bcx\ln(-icx+1)-\frac{5id^2b\ln(-icx+1)}{4}+\frac{id^2b\,\mathrm{dilog}(icx+1)}{2}+\frac{bc\,d^2x}{2}-$ |

input
```
int((d+I*c*d*x)^2*(a+b*arctan(c*x))/x,x,method=_RETURNVERBOSE)
```

output
```
d^2*a*(-1/2*c^2*x^2+2*I*c*x+ln(x))+d^2*b*(2*I*arctan(c*x)*c*x-1/2*c^2*x^2*
arctan(c*x)+ln(c*x)*arctan(c*x)+1/2*c*x-I*ln(c^2*x^2+1)-1/2*arctan(c*x)+1/
2*I*ln(c*x)*ln(1+I*c*x)-1/2*I*ln(c*x)*ln(1-I*c*x)+1/2*I*dilog(1+I*c*x)-1/2
*I*dilog(1-I*c*x))
```

## Fricas [F]

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x}\,dx = \int \frac{(i\,cdx+d)^2(b\arctan(cx)+a)}{x}\,dx$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))/x,x, algorithm="fricas")
```

output
```
integral(-1/2*(2*a*c^2*d^2*x^2 - 4*I*a*c*d^2*x - 2*a*d^2 - (-I*b*c^2*d^2*x
^2 - 2*b*c*d^2*x + I*b*d^2)*log(-(c*x + I)/(c*x - I)))/x, x)
```

## Sympy [F]

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x}\,dx = -d^2\left(\int \left(-\frac{a}{x}\right)dx + \int (-2iac)\,dx + \int ac^2x\,dx + \int \left(-\frac{b\,\mathrm{atan}(cx)}{x}\right)dx + \int (-2ibc\,\mathrm{atan}(cx))\,dx + \int bc^2x\,\mathrm{atan}(cx)\,dx\right)$$

input
```
integrate((d+I*c*d*x)**2*(a+b*atan(c*x))/x,x)
```

output
```
-d**2*(Integral(-a/x, x) + Integral(-2*I*a*c, x) + Integral(a*c**2*x, x) +
 Integral(-b*atan(c*x)/x, x) + Integral(-2*I*b*c*atan(c*x), x) + Integral(
b*c**2*x*atan(c*x), x))
```

## Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.10

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x}\,dx = -\frac{1}{2}\,ac^2d^2x^2 + 2i\,acd^2x + \frac{1}{2}\,bcd^2x - \frac{1}{4}\,\pi bd^2\log\left(c^2x^2+1\right) + bd^2\arctan\left(cx\right)\log\left(cx\right) + i\left(2\,cx\arctan\left(cx\right) - \log\left(c^2x^2+1\right)\right)bd^2 - \frac{1}{2}i\,bd^2\mathrm{Li}_2(i\,cx+1) + \frac{1}{2}i\,bd^2\mathrm{Li}_2(-i\,cx+1) + ad^2\log\left(x\right) - \frac{1}{2}\left(bc^2d^2x^2 + bd^2\right)\arctan\left(cx\right)$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))/x,x, algorithm="maxima")
```

output
```
-1/2*a*c^2*d^2*x^2 + 2*I*a*c*d^2*x + 1/2*b*c*d^2*x - 1/4*pi*b*d^2*log(c^2*
x^2 + 1) + b*d^2*arctan(c*x)*log(c*x) + I*(2*c*x*arctan(c*x) - log(c^2*x^2
 + 1))*b*d^2 - 1/2*I*b*d^2*dilog(I*c*x + 1) + 1/2*I*b*d^2*dilog(-I*c*x + 1
) + a*d^2*log(x) - 1/2*(b*c^2*d^2*x^2 + b*d^2)*arctan(c*x)
```

## Giac [F]

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x}\,dx = \int \frac{(i\,cdx+d)^2(b\arctan\left(cx\right)+a)}{x}\,dx$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))/x,x, algorithm="giac")
```

output
```
integrate((I*c*d*x + d)^2*(b*arctan(c*x) + a)/x, x)
```

## Mupad [B] (verification not implemented)

Time = 1.03 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.02

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x}\,dx$$
$$=\begin{cases} a\,d^2\,\ln(x) \\ \frac{b\,c\,d^2\,x}{2}+\frac{a\,d^2\,\left(2\,\ln(x)-c^2\,x^2+c\,x\,4\mathrm{i}\right)}{2}-\frac{b\,d^2\,\mathrm{Li}_2(1-c\,x\,1\mathrm{i})\,1\mathrm{i}}{2}+\frac{b\,d^2\,\mathrm{Li}_2(1+c\,x\,1\mathrm{i})\,1\mathrm{i}}{2}-b\,d^2\,\ln\left(c^2\,x^2+1\right)\,1\mathrm{i}-b\,c^2\,d^2\,\mathrm{atan} \end{cases}$$

input
```
int(((a + b*atan(c*x))*(d + c*d*x*1i)^2)/x,x)
```

output
```
piecewise(c == 0, a*d^2*log(x), c ~= 0, - b*d^2*log(c^2*x^2 + 1)*1i + (a*d
^2*(2*log(x) + c*x*4i - c^2*x^2))/2 - (b*d^2*dilog(- c*x*1i + 1)*1i)/2 + (
b*d^2*dilog(c*x*1i + 1)*1i)/2 + (b*c*d^2*x)/2 - b*c^2*d^2*atan(c*x)*(1/(2*
c^2) + x^2/2) + b*c*d^2*x*atan(c*x)*2i)
```

## Reduce [F]

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x}\,dx$$
$$=\frac{d^2\left(-atan(cx)\,b\,c^2x^2+4atan(cx)\,bcix-atan(cx)\,b+2\left(\int\frac{atan(cx)}{x}dx\right)b-2\log(c^2x^2+1)\,bi+2\log(x)\,a\right.}{2}$$

input
```
int((d+I*c*d*x)^2*(a+b*atan(c*x))/x,x)
```

output
```
(d**2*( - atan(c*x)*b*c**2*x**2 + 4*atan(c*x)*b*c*i*x - atan(c*x)*b + 2*in
t(atan(c*x)/x,x)*b - 2*log(c**2*x**2 + 1)*b*i + 2*log(x)*a - a*c**2*x**2 +
 4*a*c*i*x + b*c*x))/2
```

# 3.15 $\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^2}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 89

$$\begin{aligned}\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^2}\,dx &= -ac^2d^2x-bc^2d^2x\arctan(cx)-\frac{d^2(a+b\arctan(cx))}{x}+2iacd^2\log(x)\\&\quad+bcd^2\log(x)-bcd^2\operatorname{PolyLog}(2,-icx)+bcd^2\operatorname{PolyLog}(2,icx)\end{aligned}$$

output
```
-a*c^2*d^2*x-b*c^2*d^2*x*arctan(c*x)-d^2*(a+b*arctan(c*x))/x+2*I*a*c*d^2*l
n(x)+b*c*d^2*ln(x)-b*c*d^2*polylog(2,-I*c*x)+b*c*d^2*polylog(2,I*c*x)
```

## Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.89

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^2}\,dx = -\frac{d^2(a+ac^2x^2+b\arctan(cx)+bc^2x^2\arctan(cx)-2iacx\log(x)-bcx\log(cx)+bcx\operatorname{PolyLog}(2,-icx)}{x}$$

input
```
Integrate[((d + I*c*d*x)^2*(a + b*ArcTan[c*x]))/x^2,x]
```

output
```
-((d^2*(a + a*c^2*x^2 + b*ArcTan[c*x] + b*c^2*x^2*ArcTan[c*x] - (2*I)*a*c*
x*Log[x] - b*c*x*Log[c*x] + b*c*x*PolyLog[2, (-I)*c*x] - b*c*x*PolyLog[2,
I*c*x]))/x)
```

## Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^2}dx$$

$$\Big\downarrow \text{5411}$$

$$\int \left(-c^2d^2(a+b\arctan(cx))+\frac{d^2(a+b\arctan(cx))}{x^2}+\frac{2icd^2(a+b\arctan(cx))}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d^2(a+b\arctan(cx))}{x}-ac^2d^2x+2iacd^2\log(x)-bc^2d^2x\arctan(cx)-bcd^2\text{PolyLog}(2,-icx)+bcd^2\text{PolyLog}(2,icx)+bcd^2\log(x)$$

input
```
Int[((d + I*c*d*x)^2*(a + b*ArcTan[c*x]))/x^2,x]
```

output
```
-(a*c^2*d^2*x) - b*c^2*d^2*x*ArcTan[c*x] - (d^2*(a + b*ArcTan[c*x]))/x + (
2*I)*a*c*d^2*Log[x] + b*c*d^2*Log[x] - b*c*d^2*PolyLog[2, (-I)*c*x] + b*c*
d^2*PolyLog[2, I*c*x]
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

## Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.26

| method | result |
|---|---|
| parts | $d^2a\left(-c^2x-\frac{1}{x}+2ic\ln(x)\right)+d^2bc\Big(-cx\arctan(cx)+2i\arctan(cx)\ln(cx)-\frac{\arctan(cx}{cx}$ |
| derivativedivides | $c\Big(d^2a\left(-cx+2i\ln(cx)-\frac{1}{cx}\right)+d^2b\Big(-cx\arctan(cx)+2i\arctan(cx)\ln(cx)-\frac{\arctan(c}{cx}$ |
| default | $c\Big(d^2a\left(-cx+2i\ln(cx)-\frac{1}{cx}\right)+d^2b\Big(-cx\arctan(cx)+2i\arctan(cx)\ln(cx)-\frac{\arctan(c}{cx}$ |
| risch | $\frac{ib\,d^2\ln(icx+1)}{2x}-bc\,d^2-\frac{ic^2d^2b\ln(-icx+1)x}{2}+\frac{c\,d^2b\ln(-icx)}{2}+\frac{ib\,c^2d^2\ln(icx+1)x}{2}+c\,d^2b\,\mathrm{dilog}(-ic$ |

input
```
int((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^2,x,method=_RETURNVERBOSE)
```

output
```
d^2*a*(-c^2*x-1/x+2*I*c*ln(x))+d^2*b*c*(-c*x*arctan(c*x)+2*I*arctan(c*x)*l
n(c*x)-1/c/x*arctan(c*x)-ln(c*x)*ln(1+I*c*x)+ln(c*x)*ln(1-I*c*x)-dilog(1+I
*c*x)+dilog(1-I*c*x)+ln(c*x))
```

## Fricas [F]

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^2}\,dx = \int \frac{(i\,cdx+d)^2(b\arctan(cx)+a)}{x^2}\,dx$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^2,x, algorithm="fricas")
```

output
```
integral(-1/2*(2*a*c^2*d^2*x^2 - 4*I*a*c*d^2*x - 2*a*d^2 - (-I*b*c^2*d^2*x
^2 - 2*b*c*d^2*x + I*b*d^2)*log(-(c*x + I)/(c*x - I)))/x^2, x)
```

## Sympy [F]

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^2}\,dx = -d^2\left(\int ac^2\,dx + \int \left(-\frac{a}{x^2}\right)dx + \int bc^2\operatorname{atan}(cx)\,dx + \int \left(-\frac{b\operatorname{atan}(cx)}{x^2}\right)dx + \int \left(-\frac{2iac}{x}\right)dx + \int \left(-\frac{2ibc\operatorname{atan}(cx)}{x}\right)dx\right)$$

input
```
integrate((d+I*c*d*x)**2*(a+b*atan(c*x))/x**2,x)
```

output
```
-d**2*(Integral(a*c**2, x) + Integral(-a/x**2, x) + Integral(b*c**2*atan(c
*x), x) + Integral(-b*atan(c*x)/x**2, x) + Integral(-2*I*a*c/x, x) + Integ
ral(-2*I*b*c*atan(c*x)/x, x))
```

## Maxima [F]

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^2}\,dx = \int \frac{(i\,cdx+d)^2(b\arctan(cx)+a)}{x^2}\,dx$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^2,x, algorithm="maxima")
```

output
```
-a*c^2*d^2*x - 1/2*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b*c*d^2 + 2*I*b*
c*d^2*integrate(arctan(c*x)/x, x) + 2*I*a*c*d^2*log(x) - 1/2*(c*(log(c^2*x
^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b*d^2 - a*d^2/x
```

## Giac [F]

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^2}\,dx = \int \frac{(i\,cdx+d)^2(b\arctan(cx)+a)}{x^2}\,dx$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^2,x, algorithm="giac")
```

output
```
integrate((I*c*d*x + d)^2*(b*arctan(c*x) + a)/x^2, x)
```

## Mupad [B] (verification not implemented)

Time = 0.91 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.58

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^2}\,dx = \begin{cases} -\frac{a\,d^2}{x} \\ \frac{b\,d^2\left(c^2\,\ln(x)-\frac{c^2\,\ln\left(c^2\,x^2+1\right)}{2}\right)}{c}+b\,c\,d^2\,\left(\mathrm{Li}_2(1-c\,x\,1\mathrm{i})-\mathrm{Li}_2(1+c\,x\,1\mathrm{i})\right)+\frac{b\,c\,d^2\,\ln\left(c^2\,x^2+1\right)}{2}-\frac{a\,d^2\,\left(c^2\,x^2+1-c\,x\,\ln(\right.}{x} \end{cases}$$

input
```
int(((a + b*atan(c*x))*(d + c*d*x*1i)^2)/x^2,x)
```

output
```
piecewise(c == 0, -(a*d^2)/x, c ~= 0, (b*d^2*(c^2*log(x) - (c^2*log(c^2*x^
2 + 1))/2))/c + b*c*d^2*(dilog(- c*x*1i + 1) - dilog(c*x*1i + 1)) + (b*c*d
^2*log(c^2*x^2 + 1))/2 - (a*d^2*(c^2*x^2 - c*x*log(x)*2i + 1))/x - (b*d^2*
atan(c*x))/x - b*c^2*d^2*x*atan(c*x))
```

## Reduce [F]

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^2}\,dx$$
$$=\frac{d^2\left(-atan(cx)\,b\,c^2x^2-atan(cx)\,b+2\left(\int \frac{atan(cx)}{x}dx\right)bcix+2\log(x)\,acix+\log(x)\,bcx-a\,c^2x^2-a\right)}{x}$$

input
```
int((d+I*c*d*x)^2*(a+b*atan(c*x))/x^2,x)
```

output
```
(d**2*( - atan(c*x)*b*c**2*x**2 - atan(c*x)*b + 2*int(atan(c*x)/x,x)*b*c*i
*x + 2*log(x)*a*c*i*x + log(x)*b*c*x - a*c**2*x**2 - a))/x
```

# 3.16 $\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^3}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 152

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^3}\,dx = -\frac{bcd^2}{2x}-\frac{1}{2}bc^2d^2\arctan(cx)-\frac{d^2(a+b\arctan(cx))}{2x^2} - \frac{2icd^2(a+b\arctan(cx))}{x}-ac^2d^2\log(x) + 2ibc^2d^2\log(x)-ibc^2d^2\log\left(1+c^2x^2\right) - \frac{1}{2}ibc^2d^2\,\mathrm{PolyLog}(2,-icx) + \frac{1}{2}ibc^2d^2\,\mathrm{PolyLog}(2,icx)$$

output

```
-1/2*b*c*d^2/x-1/2*b*c^2*d^2*arctan(c*x)-1/2*d^2*(a+b*arctan(c*x))/x^2-2*I
*c*d^2*(a+b*arctan(c*x))/x-a*c^2*d^2*ln(x)+2*I*b*c^2*d^2*ln(x)-I*b*c^2*d^2
*ln(c^2*x^2+1)-1/2*I*b*c^2*d^2*polylog(2,-I*c*x)+1/2*I*b*c^2*d^2*polylog(2
,I*c*x)
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.05 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.91

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^3}\,dx = -\frac{d^2\left(a+4iacx+b\arctan(cx)+4ibcx\arctan(cx)+bcx\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-c^2x^2\right)+2ac^2x^2\right.}{2\ldots}$$

input
```
Integrate[((d + I*c*d*x)^2*(a + b*ArcTan[c*x]))/x^3,x]
```

output
```
-1/2*(d^2*(a + (4*I)*a*c*x + b*ArcTan[c*x] + (4*I)*b*c*x*ArcTan[c*x] + b*c
*x*Hypergeometric2F1[-1/2, 1, 1/2, -(c^2*x^2)] + 2*a*c^2*x^2*Log[x] - (4*I
)*b*c^2*x^2*Log[x] + (2*I)*b*c^2*x^2*Log[1 + c^2*x^2] + I*b*c^2*x^2*PolyLo
g[2, (-I)*c*x] - I*b*c^2*x^2*PolyLog[2, I*c*x]))/x^2
```

## Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^3}\,dx$$

$$\Big\downarrow \text{5411}$$

$$\int \left(-\frac{c^2d^2(a+b\arctan(cx))}{x}+\frac{d^2(a+b\arctan(cx))}{x^3}+\frac{2icd^2(a+b\arctan(cx))}{x^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d^2(a+b\arctan(cx))}{2x^2}-\frac{2icd^2(a+b\arctan(cx))}{x}-ac^2d^2\log(x)-\frac{1}{2}bc^2d^2\arctan(cx)-\frac{1}{2}ibc^2d^2\operatorname{PolyLog}(2,-icx)+\frac{1}{2}ibc^2d^2\operatorname{PolyLog}(2,icx)-ibc^2d^2\log\left(c^2x^2+1\right)+2ibc^2d^2\log(x)-\frac{bcd^2}{2x}$$

input
```
Int[((d + I*c*d*x)^2*(a + b*ArcTan[c*x]))/x^3,x]
```

output
```
-1/2*(b*c*d^2)/x - (b*c^2*d^2*ArcTan[c*x])/2 - (d^2*(a + b*ArcTan[c*x]))/(
2*x^2) - ((2*I)*c*d^2*(a + b*ArcTan[c*x]))/x - a*c^2*d^2*Log[x] + (2*I)*b*
c^2*d^2*Log[x] - I*b*c^2*d^2*Log[1 + c^2*x^2] - (I/2)*b*c^2*d^2*PolyLog[2,
 (-I)*c*x] + (I/2)*b*c^2*d^2*PolyLog[2, I*c*x]
```

**Defintions of rubi rules used**

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

## Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.03

| method | result |
|---|---|
| parts | $d^2a\left(-\frac{1}{2x^2}-\frac{2ic}{x}-c^2\ln(x)\right)+d^2b\,c^2\left(-\frac{2i\arctan(cx)}{cx}-\frac{\arctan(cx)}{2c^2x^2}-\ln(cx)\arctan(cx)-\right.$ |
| derivativedivides | $c^2\left(d^2a\left(-\frac{2i}{cx}-\frac{1}{2c^2x^2}-\ln(cx)\right)+d^2b\left(-\frac{2i\arctan(cx)}{cx}-\frac{\arctan(cx)}{2c^2x^2}-\ln(cx)\arctan(cx)-\right.\right.$ |
| default | $c^2\left(d^2a\left(-\frac{2i}{cx}-\frac{1}{2c^2x^2}-\ln(cx)\right)+d^2b\left(-\frac{2i\arctan(cx)}{cx}-\frac{\arctan(cx)}{2c^2x^2}-\ln(cx)\arctan(cx)-\right.\right.$ |
| risch | $-\frac{bc\,d^2}{2x}-\frac{id^2b\ln(-icx+1)}{4x^2}+\frac{id^2b\ln(icx+1)}{4x^2}-\frac{5b\,c^2d^2\arctan(cx)}{4}+\frac{id^2c^2b\,\mathrm{dilog}(-icx+1)}{2}+\frac{d^2cb\ln(-icx+1)}{x}$ |

input
```
int((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^3,x,method=_RETURNVERBOSE)
```

output
```
d^2*a*(-1/2/x^2-2*I*c/x-c^2*ln(x))+d^2*b*c^2*(-2*I*arctan(c*x)/c/x-1/2/c^2
/x^2*arctan(c*x)-ln(c*x)*arctan(c*x)-1/2*I*ln(c*x)*ln(1+I*c*x)+1/2*I*ln(c*
x)*ln(1-I*c*x)-1/2*I*dilog(1+I*c*x)+1/2*I*dilog(1-I*c*x)-I*ln(c^2*x^2+1)-1
/2*arctan(c*x)-1/2/c/x+2*I*ln(c*x))
```

## Fricas [F]

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^3}\,dx = \int \frac{(i\,cdx+d)^2(b\arctan(cx)+a)}{x^3}\,dx$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^3,x, algorithm="fricas")
```

output
```
integral(-1/2*(2*a*c^2*d^2*x^2 - 4*I*a*c*d^2*x - 2*a*d^2 - (-I*b*c^2*d^2*x
^2 - 2*b*c*d^2*x + I*b*d^2)*log(-(c*x + I)/(c*x - I)))/x^3, x)
```

## Sympy [F]

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^3}\,dx = -d^2\left(\int \left(-\frac{a}{x^3}\right)dx + \int \frac{ac^2}{x}\,dx + \int \left(-\frac{b\,\mathrm{atan}(cx)}{x^3}\right)dx + \int \left(-\frac{2iac}{x^2}\right)dx + \int \frac{bc^2\,\mathrm{atan}(cx)}{x}\,dx + \int \left(-\frac{2ibc\,\mathrm{atan}(cx)}{x^2}\right)dx\right)$$

input
```
integrate((d+I*c*d*x)**2*(a+b*atan(c*x))/x**3,x)
```

output
```
-d**2*(Integral(-a/x**3, x) + Integral(a*c**2/x, x) + Integral(-b*atan(c*x
)/x**3, x) + Integral(-2*I*a*c/x**2, x) + Integral(b*c**2*atan(c*x)/x, x)
+ Integral(-2*I*b*c*atan(c*x)/x**2, x))
```

## Maxima [F]

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^3}\,dx = \int \frac{(i\,cdx+d)^2(b\arctan{(cx)}+a)}{x^3}\,dx$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^3,x, algorithm="maxima")
```

output
```
-b*c^2*d^2*integrate(arctan(c*x)/x, x) - a*c^2*d^2*log(x) - I*(c*(log(c^2*
x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b*c*d^2 - 1/2*((c*arctan(c*x) + 1/
x)*c + arctan(c*x)/x^2)*b*d^2 - 2*I*a*c*d^2/x - 1/2*a*d^2/x^2
```

## Giac [F]

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^3}\,dx = \int \frac{(i\,cdx+d)^2(b\arctan{(cx)}+a)}{x^3}\,dx$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^3,x, algorithm="giac")
```

output
```
integrate((I*c*d*x + d)^2*(b*arctan(c*x) + a)/x^3, x)
```

## Mupad [B] (verification not implemented)

Time = 1.06 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.06

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^3}\,dx$$
$$= \begin{cases} -\frac{a\,d^2}{2\,x^2} \\ b\,d^2\left(c^2\,\ln(x)-\frac{c^2\,\ln(c^2\,x^2+1)}{2}\right)\,2\mathrm{i}+\frac{b\,c^2\,d^2\,\mathrm{Li}_2(1-c\,x\,1\mathrm{i})\,1\mathrm{i}}{2}-\frac{b\,c^2\,d^2\,\mathrm{Li}_2(1+c\,x\,1\mathrm{i})\,1\mathrm{i}}{2}-\frac{b\,d^2\left(c^3\,\mathrm{atan}(c\,x)+\frac{c^2}{x}\right)}{2\,c}-\frac{a\,d^2\,(2\,c^{\cdots}}{\cdots} \end{cases}$$

input
```
int(((a + b*atan(c*x))*(d + c*d*x*1i)^2)/x^3,x)
```

output
```
piecewise(c == 0, -(a*d^2)/(2*x^2), c ~= 0, b*d^2*(c^2*log(x) - (c^2*log(c
^2*x^2 + 1))/2)*2i + (b*c^2*d^2*dilog(- c*x*1i + 1)*1i)/2 - (b*c^2*d^2*dil
og(c*x*1i + 1)*1i)/2 - (b*d^2*(c^3*atan(c*x) + c^2/x))/(2*c) - (a*d^2*(c*x
*4i + 2*c^2*x^2*log(x) + 1))/(2*x^2) - (b*d^2*atan(c*x))/(2*x^2) - (b*c*d^
2*atan(c*x)*2i)/x)
```

## Reduce [F]

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^3}\,dx$$
$$= \frac{d^2\left(-atan(cx)\,b\,c^2x^2-4atan(cx)\,bcix-atan(cx)\,b-2\left(\int\frac{atan(cx)}{x}dx\right)b\,c^2x^2-2\log(c^2x^2+1)\,b\,c^2i\,x^2-\cdots\right)}{2x^2}$$

input
```
int((d+I*c*d*x)^2*(a+b*atan(c*x))/x^3,x)
```

output
```
(d**2*( - atan(c*x)*b*c**2*x**2 - 4*atan(c*x)*b*c*i*x - atan(c*x)*b - 2*in
t(atan(c*x)/x,x)*b*c**2*x**2 - 2*log(c**2*x**2 + 1)*b*c**2*i*x**2 - 2*log(
x)*a*c**2*x**2 + 4*log(x)*b*c**2*i*x**2 - 4*a*c*i*x - a - b*c*x))/(2*x**2)
```

# 3.17 $\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^4}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 87

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^4}\,dx = -\frac{bcd^2}{6x^2}-\frac{ibc^2d^2}{x}-\frac{d^2(1+icx)^3(a+b\arctan(cx))}{3x^3} - \frac{4}{3}bc^3d^2\log(x)+\frac{4}{3}bc^3d^2\log(i+cx)$$

output
```
-1/6*b*c*d^2/x^2-I*b*c^2*d^2/x-1/3*d^2*(1+I*c*x)^3*(a+b*arctan(c*x))/x^3-4
/3*b*c^3*d^2*ln(x)+4/3*b*c^3*d^2*ln(I+c*x)
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.31

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^4}\,dx = -\frac{d^2\left(2a+6iacx+bcx-6ac^2x^2+2b(1+3icx-3c^2x^2)\arctan(cx)+6ibc^2x^2\,\mathrm{Hypergeometric2F1}\left(-\frac{1}{2},\right.\right.}{6x^3}$$

input
```
Integrate[((d + I*c*d*x)^2*(a + b*ArcTan[c*x]))/x^4,x]
```

output
```
-1/6*(d^2*(2*a + (6*I)*a*c*x + b*c*x - 6*a*c^2*x^2 + 2*b*(1 + (3*I)*c*x -
3*c^2*x^2)*ArcTan[c*x] + (6*I)*b*c^2*x^2*Hypergeometric2F1[-1/2, 1, 1/2, -
(c^2*x^2)] + 8*b*c^3*x^3*Log[x] - 4*b*c^3*x^3*Log[1 + c^2*x^2]))/x^3
```

## Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.92, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {5407, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^4}\,dx$$

$$\Big\downarrow \text{5407}$$

$$-bc\int \frac{id^2(i-cx)^2}{3x^3(cx+i)}dx-\frac{d^2(1+icx)^3(a+b\arctan(cx))}{3x^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{3}ibcd^2\int \frac{(i-cx)^2}{x^3(cx+i)}dx-\frac{d^2(1+icx)^3(a+b\arctan(cx))}{3x^3}$$

$$\Big\downarrow \text{99}$$

$$-\frac{1}{3}ibcd^2\int \left(\frac{4ic^3}{cx+i}-\frac{4ic^2}{x}-\frac{3c}{x^2}+\frac{i}{x^3}\right)dx-\frac{d^2(1+icx)^3(a+b\arctan(cx))}{3x^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d^2(1+icx)^3(a+b\arctan(cx))}{3x^3}-\frac{1}{3}ibcd^2\left(-4ic^2\log(x)+4ic^2\log(cx+i)+\frac{3c}{x}-\frac{i}{2x^2}\right)$$

input
```
Int[((d + I*c*d*x)^2*(a + b*ArcTan[c*x]))/x^4,x]
```

output
```
-1/3*(d^2*(1 + I*c*x)^3*(a + b*ArcTan[c*x]))/x^3 - (I/3)*b*c*d^2*((-1/2*I)
/x^2 + (3*c)/x - (4*I)*c^2*Log[x] + (4*I)*c^2*Log[I + c*x])
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5407
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Simp[(a
 + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ
[2*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0])
)
```

## Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.31

| method | result |
|---|---|
| parts | $d^2 a\left(-\frac{ic}{x^2}-\frac{1}{3x^3}+\frac{c^2}{x}\right)+d^2 b\, c^3\left(-\frac{\arctan(cx)}{3c^3x^3}-\frac{i\arctan(cx)}{c^2x^2}+\frac{\arctan(cx)}{cx}+\frac{2\ln(c^2x^2+1)}{3}-i\,\mathrm{arc}$ |
| derivativedivides | $c^3\left(d^2 a\left(-\frac{1}{3c^3x^3}-\frac{i}{c^2x^2}+\frac{1}{cx}\right)+d^2 b\left(-\frac{\arctan(cx)}{3c^3x^3}-\frac{i\arctan(cx)}{c^2x^2}+\frac{\arctan(cx)}{cx}+\frac{2\ln(c^2x^2+1)}{3}-i\right.\right.$ |
| default | $c^3\left(d^2 a\left(-\frac{1}{3c^3x^3}-\frac{i}{c^2x^2}+\frac{1}{cx}\right)+d^2 b\left(-\frac{\arctan(cx)}{3c^3x^3}-\frac{i\arctan(cx)}{c^2x^2}+\frac{\arctan(cx)}{cx}+\frac{2\ln(c^2x^2+1)}{3}-i\right.\right.$ |
| risch | $-\frac{i d^2 b(3c^2x^2-3icx-1)\ln(icx+1)}{6x^3}-\frac{d^2\left(8b\,c^3\ln(-x)x^3-b\,c^3\ln(cx-i)x^3-7b\,c^3\ln(-cx-i)x^3-3ib\,c^2x^2\ln(-icx+1)-\right.}{6x^3}$ |
| parallelrisch | $-\frac{6ic^3b\,d^2\arctan(cx)x^3-6ix^3a\,c^3d^2+8b\,c^3d^2\ln(x)x^3-4b\,c^3d^2\ln(c^2x^2+1)x^3-b\,c^3d^2x^3+6ix^2b\,c^2d^2-6d^2b\arctan(cx)}{6x^3}$ |

input
```
int((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^4,x,method=_RETURNVERBOSE)
```

output
```
d^2*a*(-I*c/x^2-1/3/x^3+c^2/x)+d^2*b*c^3*(-1/3/c^3/x^3*arctan(c*x)-I*arcta
n(c*x)/c^2/x^2+1/c/x*arctan(c*x)+2/3*ln(c^2*x^2+1)-I*arctan(c*x)-I/c/x-1/6
/c^2/x^2-4/3*ln(c*x))
```

## Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.66

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^4}\,dx = -\frac{8\,bc^3d^2x^3\log(x)-7\,bc^3d^2x^3\log\left(\frac{cx+i}{c}\right)-bc^3d^2x^3\log\left(\frac{cx-i}{c}\right)-6\,(a-i\,b)c^2d^2x^2-(-6i\,a-b)cd^2x+2}{6\,x^3}$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^4,x, algorithm="fricas")
```

output
```
-1/6*(8*b*c^3*d^2*x^3*log(x) - 7*b*c^3*d^2*x^3*log((c*x + I)/c) - b*c^3*d^
2*x^3*log((c*x - I)/c) - 6*(a - I*b)*c^2*d^2*x^2 - (-6*I*a - b)*c*d^2*x +
2*a*d^2 - (3*I*b*c^2*d^2*x^2 + 3*b*c*d^2*x - I*b*d^2)*log(-(c*x + I)/(c*x
- I)))/x^3
```

## Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 253 vs. 2(82) = 164.

Time = 5.34 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.91

$$\begin{aligned}\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^4}\,dx &= -\frac{4bc^3d^2\log(135b^2c^7d^4x)}{3}+\frac{bc^3d^2\log(135b^2c^7d^4x-135ib^2c^6d^4)}{6}\\ &+\frac{7bc^3d^2\log(135b^2c^7d^4x+135ib^2c^6d^4)}{6}\\ &-\frac{2ad^2+x^2(-6ac^2d^2+6ibc^2d^2)+x(6iacd^2+bcd^2)}{6x^3}\\ &+\frac{(-3ibc^2d^2x^2-3bcd^2x+ibd^2)\log(icx+1)}{6x^3}\\ &+\frac{(3ibc^2d^2x^2+3bcd^2x-ibd^2)\log(-icx+1)}{6x^3}\end{aligned}$$

input
```
integrate((d+I*c*d*x)**2*(a+b*atan(c*x))/x**4,x)
```

output
```
-4*b*c**3*d**2*log(135*b**2*c**7*d**4*x)/3 + b*c**3*d**2*log(135*b**2*c**7
*d**4*x - 135*I*b**2*c**6*d**4)/6 + 7*b*c**3*d**2*log(135*b**2*c**7*d**4*x
 + 135*I*b**2*c**6*d**4)/6 - (2*a*d**2 + x**2*(-6*a*c**2*d**2 + 6*I*b*c**2
*d**2) + x*(6*I*a*c*d**2 + b*c*d**2))/(6*x**3) + (-3*I*b*c**2*d**2*x**2 -
3*b*c*d**2*x + I*b*d**2)*log(I*c*x + 1)/(6*x**3) + (3*I*b*c**2*d**2*x**2 +
 3*b*c*d**2*x - I*b*d**2)*log(-I*c*x + 1)/(6*x**3)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.66

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^4}\,dx$$
$$= \frac{1}{2}\left(c\left(\log\left(c^2x^2+1\right)-\log\left(x^2\right)\right)+\frac{2\arctan\left(cx\right)}{x}\right)bc^2d^2 - i\left(\left(c\arctan\left(cx\right)+\frac{1}{x}\right)c+\frac{\arctan\left(cx\right)}{x^2}\right)bcd^2 + \frac{1}{6}\left(\left(c^2\log\left(c^2x^2+1\right)-c^2\log\left(x^2\right)-\frac{1}{x^2}\right)c-\frac{2\arctan\left(cx\right)}{x^3}\right)bd^2 + \frac{ac^2d^2}{x}-\frac{i\,acd^2}{x^2}-\frac{ad^2}{3\,x^3}$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^4,x, algorithm="maxima")
```

output
```
1/2*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b*c^2*d^2 - I*((c*
arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b*c*d^2 + 1/6*((c^2*log(c^2*x^2 +
1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*b*d^2 + a*c^2*d^2/x - I*
a*c*d^2/x^2 - 1/3*a*d^2/x^3
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.60

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^4}\,dx = \frac{7\,bc^3d^2x^3\log\left(cx+i\right)+bc^3d^2x^3\log\left(cx-i\right)-8\,bc^3d^2x^3\log\left(x\right)+6\,bc^2d^2x^2\arctan\left(cx\right)+6\,ac^2d^2x^2-6i\,b}{6\,x^3}$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^4,x, algorithm="giac")
```

output
```
1/6*(7*b*c^3*d^2*x^3*log(c*x + I) + b*c^3*d^2*x^3*log(c*x - I) - 8*b*c^3*d
^2*x^3*log(x) + 6*b*c^2*d^2*x^2*arctan(c*x) + 6*a*c^2*d^2*x^2 - 6*I*b*c^2*
d^2*x^2 - 6*I*b*c*d^2*x*arctan(c*x) - 6*I*a*c*d^2*x - b*c*d^2*x - 2*b*d^2*
arctan(c*x) - 2*a*d^2)/x^3
```

## Mupad [B] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.38

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^4}\,dx = -\frac{d^2\,\left(8\,b\,c^3\,\ln\left(x\right)-4\,b\,c^3\,\ln\left(c^2\,x^2+1\right)+b\,c^3\,\mathrm{atan}(c\,x)\,6\mathrm{i}\right)}{6} - \frac{\frac{d^2\,\left(2\,a+2\,b\,\mathrm{atan}(c\,x)\right)}{6}+\frac{d^2\,x\,\left(a\,c\,6\mathrm{i}+b\,c+b\,c\,\mathrm{atan}(c\,x)\,6\mathrm{i}\right)}{6}-\frac{d^2\,x^2\,\left(6\,a\,c^2+6\,b\,c^2\,\mathrm{atan}(c\,x)-b\,c^2\,6\mathrm{i}\right)}{6}}{x^3}$$

input
```
int(((a + b*atan(c*x))*(d + c*d*x*1i)^2)/x^4,x)
```

output
```
- (d^2*(b*c^3*atan(c*x)*6i - 4*b*c^3*log(c^2*x^2 + 1) + 8*b*c^3*log(x)))/6
 - ((d^2*(2*a + 2*b*atan(c*x)))/6 + (d^2*x*(a*c*6i + b*c + b*c*atan(c*x)*6
i))/6 - (d^2*x^2*(6*a*c^2 - b*c^2*6i + 6*b*c^2*atan(c*x)))/6)/x^3
```

## Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.33

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^4}\,dx$$
$$= \frac{d^2(-6atan(cx)\,bc^3i\,x^3+6atan(cx)\,bc^2x^2-6atan(cx)\,bcix-2atan(cx)\,b+4\log(c^2x^2+1)\,bc^3x^3-8\log}{6x^3}$$

input
```
int((d+I*c*d*x)^2*(a+b*atan(c*x))/x^4,x)
```

output
```
(d**2*( - 6*atan(c*x)*b*c**3*i*x**3 + 6*atan(c*x)*b*c**2*x**2 - 6*atan(c*x
)*b*c*i*x - 2*atan(c*x)*b + 4*log(c**2*x**2 + 1)*b*c**3*x**3 - 8*log(x)*b*
c**3*x**3 + 6*a*c**2*x**2 - 6*a*c*i*x - 2*a - 6*b*c**2*i*x**2 - b*c*x))/(6
*x**3)
```

# 3.18 $\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^5}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 161

$$\begin{aligned}\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^5}\,dx = &-\frac{bcd^2}{12x^3}-\frac{ibc^2d^2}{3x^2}+\frac{3bc^3d^2}{4x}-\frac{d^2(a+b\arctan(cx))}{4x^4}\\ &-\frac{2icd^2(a+b\arctan(cx))}{3x^3}\\ &+\frac{c^2d^2(a+b\arctan(cx))}{2x^2}-\frac{2}{3}ibc^4d^2\log(x)\\ &-\frac{1}{24}ibc^4d^2\log(i-cx)+\frac{17}{24}ibc^4d^2\log(i+cx)\end{aligned}$$

output
```
-1/12*b*c*d^2/x^3-1/3*I*b*c^2*d^2/x^2+3/4*b*c^3*d^2/x-1/4*d^2*(a+b*arctan(
c*x))/x^4-2/3*I*c*d^2*(a+b*arctan(c*x))/x^3+1/2*c^2*d^2*(a+b*arctan(c*x))/
x^2-2/3*I*b*c^4*d^2*ln(x)-1/24*I*b*c^4*d^2*ln(I-c*x)+17/24*I*b*c^4*d^2*ln(
I+c*x)
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.94

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^5}\,dx$$
$$= \frac{d^2(-3a-8iacx+6ac^2x^2-4ibc^2x^2-3b\arctan(cx)-8ibcx\arctan(cx)+6bc^2x^2\arctan(cx)-bcx\,\mathrm{Hype}}{}$$

input
```
Integrate[((d + I*c*d*x)^2*(a + b*ArcTan[c*x]))/x^5,x]
```

output
```
(d^2*(-3*a - (8*I)*a*c*x + 6*a*c^2*x^2 - (4*I)*b*c^2*x^2 - 3*b*ArcTan[c*x]
 - (8*I)*b*c*x*ArcTan[c*x] + 6*b*c^2*x^2*ArcTan[c*x] - b*c*x*Hypergeometri
c2F1[-3/2, 1, -1/2, -(c^2*x^2)] + 6*b*c^3*x^3*Hypergeometric2F1[-1/2, 1, 1
/2, -(c^2*x^2)] - (8*I)*b*c^4*x^4*Log[x] + (4*I)*b*c^4*x^4*Log[1 + c^2*x^2
]))/(12*x^4)
```

## Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.84, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {5407, 27, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^5}\,dx$$
$$\Big\downarrow \text{5407}$$
$$-bc\int -\frac{d^2(-6c^2x^2+8icx+3)}{12x^4(c^2x^2+1)}dx+\frac{c^2d^2(a+b\arctan(cx))}{2x^2}-\frac{d^2(a+b\arctan(cx))}{4x^4}-\frac{2icd^2(a+b\arctan(cx))}{3x^3}$$
$$\Big\downarrow \text{27}$$

$$\frac{1}{12}bcd^2\int\frac{-6c^2x^2+8icx+3}{x^4\left(c^2x^2+1\right)}dx+\frac{c^2d^2(a+b\arctan(cx))}{2x^2}-\frac{d^2(a+b\arctan(cx))}{4x^4}-\frac{2icd^2(a+b\arctan(cx))}{3x^3}$$

$$\Big\downarrow \text{2333}$$

$$\frac{1}{12}bcd^2\int\left(-\frac{ic^4}{2(cx-i)}+\frac{17ic^4}{2(cx+i)}-\frac{8ic^3}{x}-\frac{9c^2}{x^2}+\frac{8ic}{x^3}+\frac{3}{x^4}\right)dx+\frac{c^2d^2(a+b\arctan(cx))}{2x^2}-\frac{d^2(a+b\arctan(cx))}{4x^4}-\frac{2icd^2(a+b\arctan(cx))}{3x^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c^2d^2(a+b\arctan(cx))}{2x^2}-\frac{d^2(a+b\arctan(cx))}{4x^4}-\frac{2icd^2(a+b\arctan(cx))}{3x^3}+\frac{1}{12}bcd^2\left(-8ic^3\log(x)-\frac{1}{2}ic^3\log(-cx+i)+\frac{17}{2}ic^3\log(cx+i)+\frac{9c^2}{x}-\frac{4ic}{x^2}-\frac{1}{x^3}\right)$$

input
```
Int[((d + I*c*d*x)^2*(a + b*ArcTan[c*x]))/x^5,x]
```

output
```
-1/4*(d^2*(a + b*ArcTan[c*x]))/x^4 - (((2*I)/3)*c*d^2*(a + b*ArcTan[c*x]))
/x^3 + (c^2*d^2*(a + b*ArcTan[c*x]))/(2*x^2) + (b*c*d^2*(-x^(-3) - ((4*I)*
c)/x^2 + (9*c^2)/x - (8*I)*c^3*Log[x] - (I/2)*c^3*Log[I - c*x] + ((17*I)/2
)*c^3*Log[I + c*x]))/12
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 5407
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Simp[(a
 + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ
[2*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0])
)
```

## Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.78

| method | result |
|---|---|
| parts | $d^2a\left(\frac{c^2}{2x^2}-\frac{2ic}{3x^3}-\frac{1}{4x^4}\right)+d^2b\,c^4\left(-\frac{\arctan(cx)}{4c^4x^4}+\frac{\arctan(cx)}{2c^2x^2}-\frac{2i\arctan(cx)}{3c^3x^3}-\frac{i}{3c^2x^2}-\frac{2i\ln(cx)}{3}-\right.$ |
| derivativedivides | $c^4\left(d^2a\left(-\frac{1}{4c^4x^4}+\frac{1}{2c^2x^2}-\frac{2i}{3c^3x^3}\right)+d^2b\left(-\frac{\arctan(cx)}{4c^4x^4}+\frac{\arctan(cx)}{2c^2x^2}-\frac{2i\arctan(cx)}{3c^3x^3}-\frac{i}{3c^2x^2}-\frac{2i}{}\right.\right.$ |
| default | $c^4\left(d^2a\left(-\frac{1}{4c^4x^4}+\frac{1}{2c^2x^2}-\frac{2i}{3c^3x^3}\right)+d^2b\left(-\frac{\arctan(cx)}{4c^4x^4}+\frac{\arctan(cx)}{2c^2x^2}-\frac{2i\arctan(cx)}{3c^3x^3}-\frac{i}{3c^2x^2}-\frac{2i}{}\right.\right.$ |
| risch | $-\frac{id^2b(6c^2x^2-8icx-3)\ln(icx+1)}{24x^4}+\frac{id^2(17b\,c^4\ln(-99cx-99i)x^4-16b\,c^4\ln(-165cx)x^4-b\,c^4\ln(45cx-45i)x^4+6x^2}{}$ |
| parallelrisch | $-\frac{8ic^4b\,d^2\ln(x)x^4-4ic^4b\,d^2\ln(c^2x^2+1)x^4-4ix^4b\,c^4d^2-9d^2b\arctan(cx)x^4c^4+6a\,c^4d^2x^4-9b\,c^3d^2x^3+4ix^2b\,c^2d^2-6d}{12x^4}$ |

input
```
int((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^5,x,method=_RETURNVERBOSE)
```

output
```
d^2*a*(1/2*c^2/x^2-2/3*I*c/x^3-1/4/x^4)+d^2*b*c^4*(-1/4/c^4/x^4*arctan(c*x
)+1/2/c^2/x^2*arctan(c*x)-2/3*I*arctan(c*x)/c^3/x^3-1/3*I/c^2/x^2-2/3*I*ln
(c*x)-1/12/c^3/x^3+3/4/c/x+1/3*I*ln(c^2*x^2+1)+3/4*arctan(c*x))
```

## Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.96

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^5}\,dx$$
$$=\frac{-16i\,bc^4d^2x^4\log(x)+17i\,bc^4d^2x^4\log\left(\frac{cx+i}{c}\right)-i\,bc^4d^2x^4\log\left(\frac{cx-i}{c}\right)+18\,bc^3d^2x^3+4\,(3\,a-2i\,b)c^2d^2x^2-}{24\,x^4}$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^5,x, algorithm="fricas")
```

output
```
1/24*(-16*I*b*c^4*d^2*x^4*log(x) + 17*I*b*c^4*d^2*x^4*log((c*x + I)/c) - I
*b*c^4*d^2*x^4*log((c*x - I)/c) + 18*b*c^3*d^2*x^3 + 4*(3*a - 2*I*b)*c^2*d
^2*x^2 - 2*(8*I*a + b)*c*d^2*x - 6*a*d^2 + (6*I*b*c^2*d^2*x^2 + 8*b*c*d^2*
x - 3*I*b*d^2)*log(-(c*x + I)/(c*x - I)))/x^4
```

## Sympy [A] (verification not implemented)

Time = 9.38 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.71

$$\begin{aligned}
&\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^5}\,dx \\
&= -\frac{2ibc^4d^2\log\left(1485b^2c^9d^4x\right)}{3} - \frac{ibc^4d^2\log\left(1485b^2c^9d^4x-1485ib^2c^8d^4\right)}{24} \\
&\quad + \frac{17ibc^4d^2\log\left(1485b^2c^9d^4x+1485ib^2c^8d^4\right)}{24} \\
&\quad + \frac{\left(-6ibc^2d^2x^2-8bcd^2x+3ibd^2\right)\log\left(icx+1\right)}{24x^4} \\
&\quad + \frac{\left(6ibc^2d^2x^2+8bcd^2x-3ibd^2\right)\log\left(-icx+1\right)}{24x^4} \\
&\quad - \frac{3ad^2-9bc^3d^2x^3+x^2(-6ac^2d^2+4ibc^2d^2)+x(8iacd^2+bcd^2)}{12x^4}
\end{aligned}$$

input
```
integrate((d+I*c*d*x)**2*(a+b*atan(c*x))/x**5,x)
```

output
```
-2*I*b*c**4*d**2*log(1485*b**2*c**9*d**4*x)/3 - I*b*c**4*d**2*log(1485*b**
2*c**9*d**4*x - 1485*I*b**2*c**8*d**4)/24 + 17*I*b*c**4*d**2*log(1485*b**2
*c**9*d**4*x + 1485*I*b**2*c**8*d**4)/24 + (-6*I*b*c**2*d**2*x**2 - 8*b*c*
d**2*x + 3*I*b*d**2)*log(I*c*x + 1)/(24*x**4) + (6*I*b*c**2*d**2*x**2 + 8*
b*c*d**2*x - 3*I*b*d**2)*log(-I*c*x + 1)/(24*x**4) - (3*a*d**2 - 9*b*c**3*
d**2*x**3 + x**2*(-6*a*c**2*d**2 + 4*I*b*c**2*d**2) + x*(8*I*a*c*d**2 + b*
c*d**2))/(12*x**4)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.94

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^5}\,dx = \frac{1}{2}\left(\left(c\arctan(cx)+\frac{1}{x}\right)c+\frac{\arctan(cx)}{x^2}\right)bc^2d^2 + \frac{1}{3}i\left(\left(c^2\log\left(c^2x^2+1\right)-c^2\log\left(x^2\right)-\frac{1}{x^2}\right)c-\frac{2\arctan(cx)}{x^3}\right)bcd^2 + \frac{1}{12}\left(\left(3c^3\arctan(cx)+\frac{3c^2x^2-1}{x^3}\right)c-\frac{3\arctan(cx)}{x^4}\right)bd^2 + \frac{ac^2d^2}{2x^2}-\frac{2i\,acd^2}{3x^3}-\frac{ad^2}{4x^4}$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^5,x, algorithm="maxima")
```

output
```
1/2*((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b*c^2*d^2 + 1/3*I*((c^2*lo
g(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*b*c*d^2 + 1/
12*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan(c*x)/x^4)*b*d^2
 + 1/2*a*c^2*d^2/x^2 - 2/3*I*a*c*d^2/x^3 - 1/4*a*d^2/x^4
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.96

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^5}\,dx = -\frac{-17i\,bc^4d^2x^4\log\left(i\,cx-1\right)+i\,bc^4d^2x^4\log\left(-i\,cx-1\right)+16i\,bc^4d^2x^4\log\left(x\right)-18\,bc^3d^2x^3-12\,bc^2d^2x^2\,\ldots}{}$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^5,x, algorithm="giac")
```

output
```
-1/24*(-17*I*b*c^4*d^2*x^4*log(I*c*x - 1) + I*b*c^4*d^2*x^4*log(-I*c*x - 1
) + 16*I*b*c^4*d^2*x^4*log(x) - 18*b*c^3*d^2*x^3 - 12*b*c^2*d^2*x^2*arctan
(c*x) - 12*a*c^2*d^2*x^2 + 8*I*b*c^2*d^2*x^2 + 16*I*b*c*d^2*x*arctan(c*x)
+ 16*I*a*c*d^2*x + 2*b*c*d^2*x + 6*b*d^2*arctan(c*x) + 6*a*d^2)/x^4
```

## Mupad [B] (verification not implemented)

Time = 1.04 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.88

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^5}\,dx = \frac{d^2\left(9\,b\,c^3\,\mathrm{atan}\left(x\,\sqrt{c^2}\right)\,\sqrt{c^2}+b\,c^4\,\ln\left(c^2\,x^2+1\right)\,4\mathrm{i}-b\,c^4\,\ln\left(x\right)\,8\mathrm{i}\right)}{12} - \frac{\frac{d^2\,\left(3\,a+3\,b\,\mathrm{atan}(c\,x)\right)}{12}+\frac{d^2\,x\,\left(a\,c\,8\mathrm{i}+b\,c+b\,c\,\mathrm{atan}(c\,x)\,8\mathrm{i}\right)}{12}-\frac{d^2\,x^2\,\left(6\,a\,c^2+6\,b\,c^2\,\mathrm{atan}(c\,x)-b\,c^2\,4\mathrm{i}\right)}{12}-\frac{3\,b\,c^3\,d^2\,x^3}{4}}{x^4}$$

input
```
int(((a + b*atan(c*x))*(d + c*d*x*1i)^2)/x^5,x)
```

output
```
(d^2*(b*c^4*log(c^2*x^2 + 1)*4i - b*c^4*log(x)*8i + 9*b*c^3*atan(x*(c^2)^(
1/2))*(c^2)^(1/2)))/12 - ((d^2*(3*a + 3*b*atan(c*x)))/12 + (d^2*x*(a*c*8i
+ b*c + b*c*atan(c*x)*8i))/12 - (d^2*x^2*(6*a*c^2 - b*c^2*4i + 6*b*c^2*ata
n(c*x)))/12 - (3*b*c^3*d^2*x^3)/4)/x^4
```

## Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.78

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^5}\,dx = \frac{d^2(9atan(cx)\,b\,c^4x^4+6atan(cx)\,b\,c^2x^2-8atan(cx)\,bcix-3atan(cx)\,b+4\log(c^2x^2+1)\,b\,c^4i\,x^4-8\log(}{12x^4}$$

input
```
int((d+I*c*d*x)^2*(a+b*atan(c*x))/x^5,x)
```

output
```
(d**2*(9*atan(c*x)*b*c**4*x**4 + 6*atan(c*x)*b*c**2*x**2 - 8*atan(c*x)*b*c
*i*x - 3*atan(c*x)*b + 4*log(c**2*x**2 + 1)*b*c**4*i*x**4 - 8*log(x)*b*c**
4*i*x**4 + 6*a*c**2*x**2 - 8*a*c*i*x - 3*a + 9*b*c**3*x**3 - 4*b*c**2*i*x*
*2 - b*c*x))/(12*x**4)
```

# 3.19 $\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^6}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 171

$$\begin{aligned}\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^6}\,dx = & -\frac{bcd^2}{20x^4}-\frac{ibc^2d^2}{6x^3}+\frac{4bc^3d^2}{15x^2}+\frac{ibc^4d^2}{2x} \\ & -\frac{d^2(a+b\arctan(cx))}{5x^5}-\frac{icd^2(a+b\arctan(cx))}{2x^4} \\ & +\frac{c^2d^2(a+b\arctan(cx))}{3x^3}+\frac{8}{15}bc^5d^2\log(x) \\ & -\frac{1}{60}bc^5d^2\log(i-cx)-\frac{31}{60}bc^5d^2\log(i+cx)\end{aligned}$$

output
```
-1/20*b*c*d^2/x^4-1/6*I*b*c^2*d^2/x^3+4/15*b*c^3*d^2/x^2+1/2*I*b*c^4*d^2/x
-1/5*d^2*(a+b*arctan(c*x))/x^5-1/2*I*c*d^2*(a+b*arctan(c*x))/x^4+1/3*c^2*d
^2*(a+b*arctan(c*x))/x^3+8/15*b*c^5*d^2*ln(x)-1/60*b*c^5*d^2*ln(I-c*x)-31/
60*b*c^5*d^2*ln(I+c*x)
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.73

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^6}\,dx$$
$$=\frac{d^2\left(-12a-30iacx-3bcx+20ac^2x^2+16bc^3x^3+2b(-6-15icx+10c^2x^2)\arctan(cx)-10ibc^2x^2\,\text{Hype}\right.}{60x^5}$$

input
```
Integrate[((d + I*c*d*x)^2*(a + b*ArcTan[c*x]))/x^6,x]
```

output
```
(d^2*(-12*a - (30*I)*a*c*x - 3*b*c*x + 20*a*c^2*x^2 + 16*b*c^3*x^3 + 2*b*(
-6 - (15*I)*c*x + 10*c^2*x^2)*ArcTan[c*x] - (10*I)*b*c^2*x^2*Hypergeometri
c2F1[-3/2, 1, -1/2, -(c^2*x^2)] + 32*b*c^5*x^5*Log[x] - 16*b*c^5*x^5*Log[1
 + c^2*x^2]))/(60*x^5)
```

## Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.83, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {5407, 27, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^6}\,dx$$

$$\Big\downarrow \ 5407$$

$$-bc\int -\frac{d^2\left(-10c^2x^2+15icx+6\right)}{30x^5\left(c^2x^2+1\right)}dx+\frac{c^2d^2(a+b\arctan(cx))}{3x^3}-\frac{d^2(a+b\arctan(cx))}{5x^5}-\frac{icd^2(a+b\arctan(cx))}{2x^4}$$

$$\Big\downarrow \ 27$$

$$\frac{1}{30}bcd^2\int \frac{-10c^2x^2+15icx+6}{x^5\left(c^2x^2+1\right)}dx+\frac{c^2d^2(a+b\arctan(cx))}{3x^3}-\frac{d^2(a+b\arctan(cx))}{5x^5}-\frac{icd^2(a+b\arctan(cx))}{2x^4}$$

$$\Big\downarrow \text{2333}$$

$$\frac{1}{30}bcd^2\int \left(-\frac{c^5}{2(cx-i)}-\frac{31c^5}{2(cx+i)}+\frac{16c^4}{x}-\frac{15ic^3}{x^2}-\frac{16c^2}{x^3}+\frac{15ic}{x^4}+\frac{6}{x^5}\right)dx+\frac{c^2d^2(a+b\arctan(cx))}{3x^3}-\frac{d^2(a+b\arctan(cx))}{5x^5}-\frac{icd^2(a+b\arctan(cx))}{2x^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c^2d^2(a+b\arctan(cx))}{3x^3}-\frac{d^2(a+b\arctan(cx))}{5x^5}-\frac{icd^2(a+b\arctan(cx))}{2x^4}+\frac{1}{30}bcd^2\left(16c^4\log(x)-\frac{1}{2}c^4\log(-cx+i)-\frac{31}{2}c^4\log(cx+i)+\frac{15ic^3}{x}+\frac{8c^2}{x^2}-\frac{5ic}{x^3}-\frac{3}{2x^4}\right)$$

input
```
Int[((d + I*c*d*x)^2*(a + b*ArcTan[c*x]))/x^6,x]
```

output
```
-1/5*(d^2*(a + b*ArcTan[c*x]))/x^5 - ((I/2)*c*d^2*(a + b*ArcTan[c*x]))/x^4
 + (c^2*d^2*(a + b*ArcTan[c*x]))/(3*x^3) + (b*c*d^2*(-3/(2*x^4) - ((5*I)*c
)/x^3 + (8*c^2)/x^2 + ((15*I)*c^3)/x + 16*c^4*Log[x] - (c^4*Log[I - c*x])/
2 - (31*c^4*Log[I + c*x])/2))/30
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 5407
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Simp[(a
 + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ
[2*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0])
)
```

## Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.78

| method | result |
|---|---|
| parts | $d^2a\left(\frac{c^2}{3x^3}-\frac{1}{5x^5}-\frac{ic}{2x^4}\right)+d^2b\,c^5\left(\frac{\arctan(cx)}{3c^3x^3}-\frac{i\arctan(cx)}{2c^4x^4}-\frac{\arctan(cx)}{5c^5x^5}-\frac{i}{6c^3x^3}+\frac{i}{2cx}-\frac{1}{20c^4x^4}\right.$ |
| derivativedivides | $c^5\left(d^2a\left(\frac{1}{3c^3x^3}-\frac{i}{2c^4x^4}-\frac{1}{5c^5x^5}\right)+d^2b\left(\frac{\arctan(cx)}{3c^3x^3}-\frac{i\arctan(cx)}{2c^4x^4}-\frac{\arctan(cx)}{5c^5x^5}-\frac{i}{6c^3x^3}+\frac{i}{2cx}-\right.\right.$ |
| default | $c^5\left(d^2a\left(\frac{1}{3c^3x^3}-\frac{i}{2c^4x^4}-\frac{1}{5c^5x^5}\right)+d^2b\left(\frac{\arctan(cx)}{3c^3x^3}-\frac{i\arctan(cx)}{2c^4x^4}-\frac{\arctan(cx)}{5c^5x^5}-\frac{i}{6c^3x^3}+\frac{i}{2cx}-\right.\right.$ |
| parallelrisch | $\frac{30ix^5\arctan(cx)b\,c^5d^2+32b\,c^5d^2\ln(x)x^5-16b\,c^5d^2\ln(c^2x^2+1)x^5-16b\,c^5d^2x^5+30ix^4b\,c^4d^2+16b\,c^3d^2x^3-10ix^2b\,c^2d^2}{60x^5}$ |
| risch | $-\frac{id^2b(10c^2x^2-15icx-6)\ln(icx+1)}{60x^5}+\frac{d^2(32b\,c^5\ln(-x)x^5-31b\,c^5\ln(-cx-i)x^5-b\,c^5\ln(cx-i)x^5+30ib\,c^4x^4+10ib\,c}{}$ |

input
```
int((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^6,x,method=_RETURNVERBOSE)
```

output
```
d^2*a*(1/3*c^2/x^3-1/5/x^5-1/2*I*c/x^4)+d^2*b*c^5*(1/3/c^3/x^3*arctan(c*x)
-1/2*I*arctan(c*x)/c^4/x^4-1/5/c^5/x^5*arctan(c*x)-1/6*I/c^3/x^3+1/2*I/c/x
-1/20/c^4/x^4+4/15/c^2/x^2+8/15*ln(c*x)-4/15*ln(c^2*x^2+1)+1/2*I*arctan(c*
x))
```

## Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.98

$$\int\frac{(d+icdx)^2(a+b\arctan(cx))}{x^6}\,dx$$
$$=\frac{32\,bc^5d^2x^5\log(x)-31\,bc^5d^2x^5\log\left(\frac{cx+i}{c}\right)-bc^5d^2x^5\log\left(\frac{cx-i}{c}\right)+30i\,bc^4d^2x^4+16\,bc^3d^2x^3+10\,(2\,a-i\,b)}{60\,x^5}$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^6,x, algorithm="fricas")
```

output
```
1/60*(32*b*c^5*d^2*x^5*log(x) - 31*b*c^5*d^2*x^5*log((c*x + I)/c) - b*c^5*
d^2*x^5*log((c*x - I)/c) + 30*I*b*c^4*d^2*x^4 + 16*b*c^3*d^2*x^3 + 10*(2*a
 - I*b)*c^2*d^2*x^2 - 3*(10*I*a + b)*c*d^2*x - 12*a*d^2 + (10*I*b*c^2*d^2*
x^2 + 15*b*c*d^2*x - 6*I*b*d^2)*log(-(c*x + I)/(c*x - I)))/x^5
```

## Sympy [A] (verification not implemented)

Time = 15.10 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.68

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^6}\,dx = \frac{8bc^5d^2\log(10395b^2c^{11}d^4x)}{15} - \frac{bc^5d^2\log(10395b^2c^{11}d^4x-10395ib^2c^{10}d^4)}{60} - \frac{31bc^5d^2\log(10395b^2c^{11}d^4x+10395ib^2c^{10}d^4)}{60} + \frac{(-10ibc^2d^2x^2-15bcd^2x+6ibd^2)\log(icx+1)}{60x^5} + \frac{(10ibc^2d^2x^2+15bcd^2x-6ibd^2)\log(-icx+1)}{60x^5} - \frac{12ad^2-30ibc^4d^2x^4-16bc^3d^2x^3+x^2(-20ac^2d^2+10ibc^2d^2)+x(30iacd^2+3bcd^2)}{60x^5}$$

input
```
integrate((d+I*c*d*x)**2*(a+b*atan(c*x))/x**6,x)
```

output
```
8*b*c**5*d**2*log(10395*b**2*c**11*d**4*x)/15 - b*c**5*d**2*log(10395*b**2
*c**11*d**4*x - 10395*I*b**2*c**10*d**4)/60 - 31*b*c**5*d**2*log(10395*b**
2*c**11*d**4*x + 10395*I*b**2*c**10*d**4)/60 + (-10*I*b*c**2*d**2*x**2 - 1
5*b*c*d**2*x + 6*I*b*d**2)*log(I*c*x + 1)/(60*x**5) + (10*I*b*c**2*d**2*x*
*2 + 15*b*c*d**2*x - 6*I*b*d**2)*log(-I*c*x + 1)/(60*x**5) - (12*a*d**2 -
30*I*b*c**4*d**2*x**4 - 16*b*c**3*d**2*x**3 + x**2*(-20*a*c**2*d**2 + 10*I
*b*c**2*d**2) + x*(30*I*a*c*d**2 + 3*b*c*d**2))/(60*x**5)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.07

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^6}\,dx$$
$$= -\frac{1}{6}\left(\left(c^2\log\left(c^2x^2+1\right)-c^2\log\left(x^2\right)-\frac{1}{x^2}\right)c-\frac{2\,\arctan\left(cx\right)}{x^3}\right)bc^2d^2$$
$$+\frac{1}{6}i\left(\left(3\,c^3\arctan\left(cx\right)+\frac{3\,c^2x^2-1}{x^3}\right)c-\frac{3\,\arctan\left(cx\right)}{x^4}\right)bcd^2$$
$$-\frac{1}{20}\left(\left(2\,c^4\log\left(c^2x^2+1\right)-2\,c^4\log\left(x^2\right)-\frac{2\,c^2x^2-1}{x^4}\right)c+\frac{4\,\arctan\left(cx\right)}{x^5}\right)bd^2$$
$$+\frac{ac^2d^2}{3\,x^3}-\frac{i\,acd^2}{2\,x^4}-\frac{ad^2}{5\,x^5}$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^6,x, algorithm="maxima")
```

output
```
-1/6*((c^2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)
*b*c^2*d^2 + 1/6*I*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan
(c*x)/x^4)*b*c*d^2 - 1/20*((2*c^4*log(c^2*x^2 + 1) - 2*c^4*log(x^2) - (2*c
^2*x^2 - 1)/x^4)*c + 4*arctan(c*x)/x^5)*b*d^2 + 1/3*a*c^2*d^2/x^3 - 1/2*I*
a*c*d^2/x^4 - 1/5*a*d^2/x^5
```

## Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.95

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^6}\,dx =$$
$$-\frac{31\,bc^5d^2x^5\log\left(cx+i\right)+bc^5d^2x^5\log\left(cx-i\right)-32\,bc^5d^2x^5\log\left(x\right)-30i\,bc^4d^2x^4-16\,bc^3d^2x^3-20\,bc^2d}{}$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))/x^6,x, algorithm="giac")
```

output
```
-1/60*(31*b*c^5*d^2*x^5*log(c*x + I) + b*c^5*d^2*x^5*log(c*x - I) - 32*b*c
^5*d^2*x^5*log(x) - 30*I*b*c^4*d^2*x^4 - 16*b*c^3*d^2*x^3 - 20*b*c^2*d^2*x
^2*arctan(c*x) - 20*a*c^2*d^2*x^2 + 10*I*b*c^2*d^2*x^2 + 30*I*b*c*d^2*x*ar
ctan(c*x) + 30*I*a*c*d^2*x + 3*b*c*d^2*x + 12*b*d^2*arctan(c*x) + 12*a*d^2
)/x^5
```

## Mupad [B] (verification not implemented)

Time = 1.28 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.43

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^6}\,dx = \frac{8\,b\,c^5\,d^2\,\ln\left(x\right)}{15}-\frac{4\,b\,c^5\,d^2\,\ln\left(c^2\,x^2+1\right)}{15} - \frac{\frac{a\,d^2}{5}+\frac{b\,d^2\,\mathrm{atan}(c\,x)}{5}-\frac{4\,b\,c^5\,d^2\,x^5}{15}-\frac{b\,c^6\,d^2\,x^6\,1\mathrm{i}}{2}-\frac{c^4\,d^2\,x^4\,(a+b\,1\mathrm{i})}{3}+\frac{c\,d^2\,x\,(b+a\,10\mathrm{i})}{20}-\frac{c^2\,d^2\,x^2\,(4\,a-b\,5\mathrm{i})}{30}+\frac{c^3\,d^2\,x^3\,(-13\,b+c}{60}}{c^2\,x^7+x^5} + \frac{b\,c^8\,d^2\,\mathrm{atan}\left(\frac{c^2\,x}{\sqrt{c^2}}\right)\,1\mathrm{i}}{2\,{\left(c^2\right)}^{3/2}}$$

input
```
int(((a + b*atan(c*x))*(d + c*d*x*1i)^2)/x^6,x)
```

output
```
(8*b*c^5*d^2*log(x))/15 - (4*b*c^5*d^2*log(c^2*x^2 + 1))/15 - ((a*d^2)/5 +
 (b*d^2*atan(c*x))/5 - (4*b*c^5*d^2*x^5)/15 - (b*c^6*d^2*x^6*1i)/2 - (c^4*
d^2*x^4*(a + b*1i))/3 + (c*d^2*x*(a*10i + b))/20 - (c^2*d^2*x^2*(4*a - b*5
i))/30 + (c^3*d^2*x^3*(a*30i - 13*b))/60 - (2*b*c^2*d^2*x^2*atan(c*x))/15
+ (b*c^3*d^2*x^3*atan(c*x)*1i)/2 - (b*c^4*d^2*x^4*atan(c*x))/3 + (b*c*d^2*
x*atan(c*x)*1i)/2)/(x^5 + c^2*x^7) + (b*c^8*d^2*atan((c^2*x)/(c^2)^(1/2))*
1i)/(2*(c^2)^(3/2))
```

## Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.79

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))}{x^6}\,dx = \frac{d^2(30atan(cx)\,b\,c^5i\,x^5+20atan(cx)\,b\,c^2x^2-30atan(cx)\,bcix-12atan(cx)\,b-16\log(c^2x^2+1)\,b\,c^5x^5+}{60x^5}$$

input
```
int((d+I*c*d*x)^2*(a+b*atan(c*x))/x^6,x)
```

output
```
(d**2*(30*atan(c*x)*b*c**5*i*x**5 + 20*atan(c*x)*b*c**2*x**2 - 30*atan(c*x
)*b*c*i*x - 12*atan(c*x)*b - 16*log(c**2*x**2 + 1)*b*c**5*x**5 + 32*log(x)
*b*c**5*x**5 + 20*a*c**2*x**2 - 30*a*c*i*x - 12*a + 30*b*c**4*i*x**4 + 16*
b*c**3*x**3 - 10*b*c**2*i*x**2 - 3*b*c*x))/(60*x**5)
```

# 3.20 $\int x^3(d+icdx)^3(a+b\arctan(cx))\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 205

$$\begin{aligned}\int x^3(d+icdx)^3(a+b\arctan(cx))\,dx &= \frac{3bd^3x}{4c^3}+\frac{13ibd^3x^2}{35c^2}-\frac{bd^3x^3}{4c}-\frac{13}{70}ibd^3x^4+\frac{1}{10}bcd^3x^5+\frac{1}{42}ibc^2d^3x^6\\&\quad-\frac{3bd^3\arctan(cx)}{4c^4}+\frac{1}{4}d^3x^4(a+b\arctan(cx))+\frac{3}{5}icd^3x^5(a+b\arctan(cx))\\&\quad-\frac{1}{2}c^2d^3x^6(a+b\arctan(cx))-\frac{1}{7}ic^3d^3x^7(a+b\arctan(cx))-\frac{13ibd^3\log(1+c^2x^2)}{35c^4}\end{aligned}$$

output
```
3/4*b*d^3*x/c^3+13/35*I*b*d^3*x^2/c^2-1/4*b*d^3*x^3/c-13/70*I*b*d^3*x^4+1/
10*b*c*d^3*x^5+1/42*I*b*c^2*d^3*x^6-3/4*b*d^3*arctan(c*x)/c^4+1/4*d^3*x^4*
(a+b*arctan(c*x))+3/5*I*c*d^3*x^5*(a+b*arctan(c*x))-1/2*c^2*d^3*x^6*(a+b*a
rctan(c*x))-1/7*I*c^3*d^3*x^7*(a+b*arctan(c*x))-13/35*I*b*d^3*ln(c^2*x^2+1
)/c^4
```

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.75

$$\int x^3(d+icdx)^3(a+b\arctan(cx))\,dx$$
$$=\frac{d^3(3ac^4x^4(35+84icx-70c^2x^2-20ic^3x^3)+bcx(315+156icx-105c^2x^2-78ic^3x^3+42c^4x^4+10ic^5x^5}{420c^4}$$

input
```
Integrate[x^3*(d + I*c*d*x)^3*(a + b*ArcTan[c*x]),x]
```

output
```
(d^3*(3*a*c^4*x^4*(35 + (84*I)*c*x - 70*c^2*x^2 - (20*I)*c^3*x^3) + b*c*x*
(315 + (156*I)*c*x - 105*c^2*x^2 - (78*I)*c^3*x^3 + 42*c^4*x^4 + (10*I)*c^
5*x^5) + 3*b*(-105 + 35*c^4*x^4 + (84*I)*c^5*x^5 - 70*c^6*x^6 - (20*I)*c^7
*x^7)*ArcTan[c*x] - (156*I)*b*Log[1 + c^2*x^2]))/(420*c^4)
```

## Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.82, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {5407, 27, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(d+icdx)^3(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5407}$$

$$-bc\int \frac{d^3x^4\left(-20ic^3x^3-70c^2x^2+84icx+35\right)}{140\left(c^2x^2+1\right)}dx-\frac{1}{7}ic^3d^3x^7(a+b\arctan(cx))-\frac{1}{2}c^2d^3x^6(a+b\arctan(cx))+\frac{3}{5}icd^3x^5(a+b\arctan(cx))+\frac{1}{4}d^3x^4(a+b\arctan(cx))$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{140}bcd^3\int \frac{x^4\left(-20ic^3x^3-70c^2x^2+84icx+35\right)}{c^2x^2+1}dx-\frac{1}{7}ic^3d^3x^7(a+b\arctan(cx))-\frac{1}{2}c^2d^3x^6(a+b\arctan(cx))+\frac{3}{5}icd^3x^5(a+b\arctan(cx))+\frac{1}{4}d^3x^4(a+b\arctan(cx))$$

$$\Big\downarrow \text{2333}$$

$$-\frac{1}{140}bcd^3\int\left(-20icx^5-70x^4+\frac{104ix^3}{c}+\frac{105x^2}{c^2}-\frac{104ix}{c^3}+\frac{104icx+105}{c^4(c^2x^2+1)}-\frac{105}{c^4}\right)dx-\frac{1}{7}ic^3d^3x^7(a+b\arctan(cx))-\frac{1}{2}c^2d^3x^6(a+b\arctan(cx))+\frac{3}{5}icd^3x^5(a+b\arctan(cx))+\frac{1}{4}d^3x^4(a+b\arctan(cx))$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{7}ic^3d^3x^7(a+b\arctan(cx))-\frac{1}{2}c^2d^3x^6(a+b\arctan(cx))+\frac{3}{5}icd^3x^5(a+b\arctan(cx))+\frac{1}{4}d^3x^4(a+b\arctan(cx))-\frac{1}{140}bcd^3\left(\frac{105\arctan(cx)}{c^5}-\frac{105x}{c^4}-\frac{52ix^2}{c^3}+\frac{35x^3}{c^2}+\frac{52i\log(c^2x^2+1)}{c^5}-\frac{10}{3}icx^6+\frac{26ix^4}{c}-14x^5\right)$$

input
```
Int[x^3*(d + I*c*d*x)^3*(a + b*ArcTan[c*x]),x]
```

output
```
(d^3*x^4*(a + b*ArcTan[c*x]))/4 + ((3*I)/5)*c*d^3*x^5*(a + b*ArcTan[c*x])
- (c^2*d^3*x^6*(a + b*ArcTan[c*x]))/2 - (I/7)*c^3*d^3*x^7*(a + b*ArcTan[c*
x]) - (b*c*d^3*((-105*x)/c^4 - ((52*I)*x^2)/c^3 + (35*x^3)/c^2 + ((26*I)*x
^4)/c - 14*x^5 - ((10*I)/3)*c*x^6 + (105*ArcTan[c*x])/c^5 + ((52*I)*Log[1
+ c^2*x^2])/c^5))/140
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 5407
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Simp[(a
 + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ
[2*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0])
)
```

## Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.79

| method | result |
|---|---|
| parts | $d^3a\left(-\frac{1}{7}ic^3x^7-\frac{1}{2}c^2x^6+\frac{3}{5}ic\,x^5+\frac{1}{4}x^4\right)+\frac{d^3b\left(-\frac{i\arctan(cx)c^7x^7}{7}-\frac{c^6x^6\arctan(cx)}{2}+\frac{3i\arctan(cx)c^5x^5}{5}+\frac{c^4}{}\right.}{}$ |
| derivativedivides | $\frac{d^3a\left(-\frac{1}{7}ic^7x^7-\frac{1}{2}c^6x^6+\frac{3}{5}ic^5x^5+\frac{1}{4}c^4x^4\right)+d^3b\left(-\frac{i\arctan(cx)c^7x^7}{7}-\frac{c^6x^6\arctan(cx)}{2}+\frac{3i\arctan(cx)c^5x^5}{5}+\frac{c^4x^4\arctan(cx)}{4}+\right.}{c^4}$ |
| default | $\frac{d^3a\left(-\frac{1}{7}ic^7x^7-\frac{1}{2}c^6x^6+\frac{3}{5}ic^5x^5+\frac{1}{4}c^4x^4\right)+d^3b\left(-\frac{i\arctan(cx)c^7x^7}{7}-\frac{c^6x^6\arctan(cx)}{2}+\frac{3i\arctan(cx)c^5x^5}{5}+\frac{c^4x^4\arctan(cx)}{4}+\right.}{c^4}$ |
| parallelrisch | $-\frac{-252ic^5b\,d^3\arctan(cx)x^5-156ix^2b\,c^2d^3+60ix^7a\,c^7d^3+210b\,c^6d^3\arctan(cx)x^6-10ix^6b\,c^6d^3+210a\,c^6d^3x^6+78ix^4b}{}$ |
| risch | $-\frac{d^3b\left(20c^3x^7-70ic^2x^6-84c\,x^5+35ix^4\right)\ln(icx+1)}{280}+\frac{d^3c^3b\,x^7\ln(-icx+1)}{14}-\frac{13id^3b\ln\left(11025c^2x^2+11025\right)}{35c^4}-\frac{d^3}{}$ |

input
```
int(x^3*(d+I*c*d*x)^3*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

output
```
d^3*a*(-1/7*I*c^3*x^7-1/2*c^2*x^6+3/5*I*c*x^5+1/4*x^4)+d^3*b/c^4*(-1/7*I*a
rctan(c*x)*c^7*x^7-1/2*c^6*x^6*arctan(c*x)+3/5*I*arctan(c*x)*c^5*x^5+1/4*c
^4*x^4*arctan(c*x)+3/4*c*x+1/42*I*c^6*x^6+1/10*c^5*x^5-13/70*I*c^4*x^4-1/4
*c^3*x^3+13/35*I*c^2*x^2-13/35*I*ln(c^2*x^2+1)-3/4*arctan(c*x))
```

## Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.99

$$\int x^3(d+icdx)^3(a+b\arctan(cx))\,dx = \frac{-120i\,ac^7d^3x^7-20\,(21\,a-i\,b)c^6d^3x^6-84\,(-6i\,a-b)c^5d^3x^5+6\,(35\,a-26i\,b)c^4d^3x^4-210\,bc^3d^3x^3+3}{}$$

input
```
integrate(x^3*(d+I*c*d*x)^3*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
1/840*(-120*I*a*c^7*d^3*x^7 - 20*(21*a - I*b)*c^6*d^3*x^6 - 84*(-6*I*a - b
)*c^5*d^3*x^5 + 6*(35*a - 26*I*b)*c^4*d^3*x^4 - 210*b*c^3*d^3*x^3 + 312*I*
b*c^2*d^3*x^2 + 630*b*c*d^3*x - 627*I*b*d^3*log((c*x + I)/c) + 3*I*b*d^3*l
og((c*x - I)/c) + 3*(20*b*c^7*d^3*x^7 - 70*I*b*c^6*d^3*x^6 - 84*b*c^5*d^3*
x^5 + 35*I*b*c^4*d^3*x^4)*log(-(c*x + I)/(c*x - I)))/c^4
```

## Sympy [A] (verification not implemented)

Time = 2.90 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.60

$$\begin{aligned}\int x^3(d+icdx)^3(a+b\arctan(cx))\,dx &= -\frac{iac^3d^3x^7}{7}-\frac{bd^3x^3}{4c}+\frac{13ibd^3x^2}{35c^2}+\frac{3bd^3x}{4c^3} \\ &\quad-\frac{bd^3\left(-\frac{i\log\left(353bcd^3x-353ibd^3\right)}{280}+\frac{351i\log\left(353bcd^3x+353ibd^3\right)}{560}\right)}{c^4} \\ &\quad-x^6\left(\frac{ac^2d^3}{2}-\frac{ibc^2d^3}{42}\right)-x^5\left(-\frac{3iacd^3}{5}-\frac{bcd^3}{10}\right)-x^4\left(-\frac{ad^3}{4}+\frac{13ibd^3}{70}\right) \\ &\quad+\left(-\frac{bc^3d^3x^7}{14}+\frac{ibc^2d^3x^6}{4}+\frac{3bcd^3x^5}{10}-\frac{ibd^3x^4}{8}\right)\log\left(icx+1\right) \\ &\quad+\frac{\left(40bc^7d^3x^7-140ibc^6d^3x^6-168bc^5d^3x^5+70ibc^4d^3x^4-67ibd^3\right)\log\left(-icx+1\right)}{560c^4}\end{aligned}$$

input
```
integrate(x**3*(d+I*c*d*x)**3*(a+b*atan(c*x)),x)
```

output
```
-I*a*c**3*d**3*x**7/7 - b*d**3*x**3/(4*c) + 13*I*b*d**3*x**2/(35*c**2) + 3
*b*d**3*x/(4*c**3) - b*d**3*(-I*log(353*b*c*d**3*x - 353*I*b*d**3)/280 + 3
51*I*log(353*b*c*d**3*x + 353*I*b*d**3)/560)/c**4 - x**6*(a*c**2*d**3/2 -
I*b*c**2*d**3/42) - x**5*(-3*I*a*c*d**3/5 - b*c*d**3/10) - x**4*(-a*d**3/4
 + 13*I*b*d**3/70) + (-b*c**3*d**3*x**7/14 + I*b*c**2*d**3*x**6/4 + 3*b*c*
d**3*x**5/10 - I*b*d**3*x**4/8)*log(I*c*x + 1) + (40*b*c**7*d**3*x**7 - 14
0*I*b*c**6*d**3*x**6 - 168*b*c**5*d**3*x**5 + 70*I*b*c**4*d**3*x**4 - 67*I
*b*d**3)*log(-I*c*x + 1)/(560*c**4)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.27

$$\begin{aligned}
&\int x^3(d+icdx)^3(a+b\arctan(cx))\,dx \\
&= -\frac{1}{7}i\,ac^3d^3x^7 - \frac{1}{2}\,ac^2d^3x^6 + \frac{3}{5}i\,acd^3x^5 \\
&\quad - \frac{1}{84}i\left(12\,x^7\arctan(cx) - c\left(\frac{2\,c^4x^6 - 3\,c^2x^4 + 6\,x^2}{c^6} - \frac{6\,\log(c^2x^2+1)}{c^8}\right)\right)bc^3d^3 \\
&\quad + \frac{1}{4}\,ad^3x^4 \\
&\quad - \frac{1}{30}\left(15\,x^6\arctan(cx) - c\left(\frac{3\,c^4x^5 - 5\,c^2x^3 + 15\,x}{c^6} - \frac{15\,\arctan(cx)}{c^7}\right)\right)bc^2d^3 \\
&\quad + \frac{3}{20}i\left(4\,x^5\arctan(cx) - c\left(\frac{c^2x^4 - 2\,x^2}{c^4} + \frac{2\,\log(c^2x^2+1)}{c^6}\right)\right)bcd^3 \\
&\quad + \frac{1}{12}\left(3\,x^4\arctan(cx) - c\left(\frac{c^2x^3 - 3\,x}{c^4} + \frac{3\,\arctan(cx)}{c^5}\right)\right)bd^3
\end{aligned}$$

input
```
integrate(x^3*(d+I*c*d*x)^3*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
-1/7*I*a*c^3*d^3*x^7 - 1/2*a*c^2*d^3*x^6 + 3/5*I*a*c*d^3*x^5 - 1/84*I*(12*
x^7*arctan(c*x) - c*((2*c^4*x^6 - 3*c^2*x^4 + 6*x^2)/c^6 - 6*log(c^2*x^2 +
 1)/c^8))*b*c^3*d^3 + 1/4*a*d^3*x^4 - 1/30*(15*x^6*arctan(c*x) - c*((3*c^4
*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan(c*x)/c^7))*b*c^2*d^3 + 3/20*I*(4*
x^5*arctan(c*x) - c*((c^2*x^4 - 2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*b*c*
d^3 + 1/12*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5
))*b*d^3
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.03

$$\int x^3(d+icdx)^3(a+b\arctan(cx))\,dx = -\frac{120i\,bc^7d^3x^7\arctan(cx)+120i\,ac^7d^3x^7+420\,bc^6d^3x^6\arctan(cx)+420\,ac^6d^3x^6-20i\,bc^6d^3x^6-504i}{}$$

input
```
integrate(x^3*(d+I*c*d*x)^3*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
-1/840*(120*I*b*c^7*d^3*x^7*arctan(c*x) + 120*I*a*c^7*d^3*x^7 + 420*b*c^6*
d^3*x^6*arctan(c*x) + 420*a*c^6*d^3*x^6 - 20*I*b*c^6*d^3*x^6 - 504*I*b*c^5
*d^3*x^5*arctan(c*x) - 504*I*a*c^5*d^3*x^5 - 84*b*c^5*d^3*x^5 - 210*b*c^4*
d^3*x^4*arctan(c*x) - 210*a*c^4*d^3*x^4 + 156*I*b*c^4*d^3*x^4 + 210*b*c^3*
d^3*x^3 - 312*I*b*c^2*d^3*x^2 - 630*b*c*d^3*x - 3*I*b*d^3*log(I*c*x + 1) +
 627*I*b*d^3*log(-I*c*x + 1))/c^4
```

## Mupad [B] (verification not implemented)

Time = 1.21 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.91

$$\int x^3(d+icdx)^3(a+b\arctan(cx))\,dx$$
$$= -\frac{\frac{d^3\,\left(315\,b\,\mathrm{atan}(c\,x)+b\,\ln\left(c^2\,x^2+1\right)\,156\mathrm{i}\right)}{420}+\frac{b\,c^3\,d^3\,x^3}{4}-\frac{3\,b\,c\,d^3\,x}{4}-\frac{b\,c^2\,d^3\,x^2\,13\mathrm{i}}{35}}{c^4}$$
$$+\frac{d^3\,\left(105\,a\,x^4+105\,b\,x^4\,\mathrm{atan}(c\,x)-b\,x^4\,78\mathrm{i}\right)}{420}-\frac{c^3\,d^3\,\left(a\,x^7\,60\mathrm{i}+b\,x^7\,\mathrm{atan}(c\,x)\,60\mathrm{i}\right)}{420}$$
$$+\frac{c\,d^3\,\left(a\,x^5\,252\mathrm{i}+42\,b\,x^5+b\,x^5\,\mathrm{atan}(c\,x)\,252\mathrm{i}\right)}{420}$$
$$-\frac{c^2\,d^3\,\left(210\,a\,x^6+210\,b\,x^6\,\mathrm{atan}(c\,x)-b\,x^6\,10\mathrm{i}\right)}{420}$$

input
```
int(x^3*(a + b*atan(c*x))*(d + c*d*x*1i)^3,x)
```

output
```
(d^3*(105*a*x^4 - b*x^4*78i + 105*b*x^4*atan(c*x)))/420 - ((d^3*(315*b*ata
n(c*x) + b*log(c^2*x^2 + 1)*156i))/420 - (b*c^2*d^3*x^2*13i)/35 + (b*c^3*d
^3*x^3)/4 - (3*b*c*d^3*x)/4)/c^4 - (c^3*d^3*(a*x^7*60i + b*x^7*atan(c*x)*6
0i))/420 + (c*d^3*(a*x^5*252i + 42*b*x^5 + b*x^5*atan(c*x)*252i))/420 - (c
^2*d^3*(210*a*x^6 - b*x^6*10i + 210*b*x^6*atan(c*x)))/420
```

## Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.85

$$\int x^3(d+icdx)^3(a+b\arctan(cx))\,dx$$
$$= \frac{d^3(-60atan(cx)\,b\,c^7i\,x^7 - 210atan(cx)\,b\,c^6x^6 + 252atan(cx)\,b\,c^5i\,x^5 + 105atan(cx)\,b\,c^4x^4 - 315atan(cx)}{}$$

input
```
int(x^3*(d+I*c*d*x)^3*(a+b*atan(c*x)),x)
```

output
```
(d**3*( - 60*atan(c*x)*b*c**7*i*x**7 - 210*atan(c*x)*b*c**6*x**6 + 252*ata
n(c*x)*b*c**5*i*x**5 + 105*atan(c*x)*b*c**4*x**4 - 315*atan(c*x)*b - 156*l
og(c**2*x**2 + 1)*b*i - 60*a*c**7*i*x**7 - 210*a*c**6*x**6 + 252*a*c**5*i*
x**5 + 105*a*c**4*x**4 + 10*b*c**6*i*x**6 + 42*b*c**5*x**5 - 78*b*c**4*i*x
**4 - 105*b*c**3*x**3 + 156*b*c**2*i*x**2 + 315*b*c*x))/(420*c**4)
```

# 3.21 $\int x^2(d+icdx)^3(a+b\arctan(cx))\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 191

$$\begin{aligned}\int x^2(d+icdx)^3(a+b\arctan(cx))\,dx &= \frac{11ibd^3x}{12c^2}-\frac{7bd^3x^2}{15c}-\frac{11}{36}ibd^3x^3+\frac{3}{20}bcd^3x^4+\frac{1}{30}ibc^2d^3x^5\\ &\quad-\frac{11ibd^3\arctan(cx)}{12c^3}+\frac{1}{3}d^3x^3(a+b\arctan(cx))+\frac{3}{4}icd^3x^4(a+b\arctan(cx))\\ &\quad-\frac{3}{5}c^2d^3x^5(a+b\arctan(cx))-\frac{1}{6}ic^3d^3x^6(a+b\arctan(cx))+\frac{7bd^3\log(1+c^2x^2)}{15c^3}\end{aligned}$$

output
```
11/12*I*b*d^3*x/c^2-7/15*b*d^3*x^2/c-11/36*I*b*d^3*x^3+3/20*b*c*d^3*x^4+1/
30*I*b*c^2*d^3*x^5-11/12*I*b*d^3*arctan(c*x)/c^3+1/3*d^3*x^3*(a+b*arctan(c
*x))+3/4*I*c*d^3*x^4*(a+b*arctan(c*x))-3/5*c^2*d^3*x^5*(a+b*arctan(c*x))-1
/6*I*c^3*d^3*x^6*(a+b*arctan(c*x))+7/15*b*d^3*ln(c^2*x^2+1)/c^3
```

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.76

$$\int x^2(d+icdx)^3(a+b\arctan(cx))\,dx$$
$$=\frac{d^3(3ac^3x^3(20+45icx-36c^2x^2-10ic^3x^3)+bcx(165i-84cx-55ic^2x^2+27c^3x^3+6ic^4x^4)+3b(-55i}{180c^3}$$

input
```
Integrate[x^2*(d + I*c*d*x)^3*(a + b*ArcTan[c*x]),x]
```

output
```
(d^3*(3*a*c^3*x^3*(20 + (45*I)*c*x - 36*c^2*x^2 - (10*I)*c^3*x^3) + b*c*x*
(165*I - 84*c*x - (55*I)*c^2*x^2 + 27*c^3*x^3 + (6*I)*c^4*x^4) + 3*b*(-55*
I + 20*c^3*x^3 + (45*I)*c^4*x^4 - 36*c^5*x^5 - (10*I)*c^6*x^6)*ArcTan[c*x]
 + 84*b*Log[1 + c^2*x^2]))/(180*c^3)
```

## Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.84, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {5407, 27, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(d+icdx)^3(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5407}$$

$$-bc\int \frac{d^3x^3\left(-10ic^3x^3-36c^2x^2+45icx+20\right)}{60\left(c^2x^2+1\right)}dx-\frac{1}{6}ic^3d^3x^6(a+b\arctan(cx))-\frac{3}{5}c^2d^3x^5(a+b\arctan(cx))+\frac{3}{4}icd^3x^4(a+b\arctan(cx))+\frac{1}{3}d^3x^3(a+b\arctan(cx))$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{60}bcd^3\int \frac{x^3\left(-10ic^3x^3-36c^2x^2+45icx+20\right)}{c^2x^2+1}dx-\frac{1}{6}ic^3d^3x^6(a+b\arctan(cx))-\frac{3}{5}c^2d^3x^5(a+b\arctan(cx))+\frac{3}{4}icd^3x^4(a+b\arctan(cx))+\frac{1}{3}d^3x^3(a+b\arctan(cx))$$

$$\Big\downarrow \text{2333}$$

$$-\frac{1}{60}bcd^3\int\left(-10icx^4-36x^3+\frac{55ix^2}{c}+\frac{56x}{c^2}+\frac{55i-56cx}{c^3\left(c^2x^2+1\right)}-\frac{55i}{c^3}\right)dx-\frac{1}{6}ic^3d^3x^6(a+b\arctan(cx))-\frac{3}{5}c^2d^3x^5(a+b\arctan(cx))+\frac{3}{4}icd^3x^4(a+b\arctan(cx))+\frac{1}{3}d^3x^3(a+b\arctan(cx))$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{6}ic^3d^3x^6(a+b\arctan(cx))-\frac{3}{5}c^2d^3x^5(a+b\arctan(cx))+\frac{3}{4}icd^3x^4(a+b\arctan(cx))+\frac{1}{3}d^3x^3(a+b\arctan(cx))-\frac{1}{60}bcd^3\left(\frac{55i\arctan(cx)}{c^4}-\frac{55ix}{c^3}+\frac{28x^2}{c^2}-\frac{28\log\left(c^2x^2+1\right)}{c^4}-2icx^5+\frac{55ix^3}{3c}-9x^4\right)$$

input
```
Int[x^2*(d + I*c*d*x)^3*(a + b*ArcTan[c*x]),x]
```

output
```
(d^3*x^3*(a + b*ArcTan[c*x]))/3 + ((3*I)/4)*c*d^3*x^4*(a + b*ArcTan[c*x])
- (3*c^2*d^3*x^5*(a + b*ArcTan[c*x]))/5 - (I/6)*c^3*d^3*x^6*(a + b*ArcTan[
c*x]) - (b*c*d^3*(((-55*I)*x)/c^3 + (28*x^2)/c^2 + (((55*I)/3)*x^3)/c - 9*
x^4 - (2*I)*c*x^5 + ((55*I)*ArcTan[c*x])/c^4 - (28*Log[1 + c^2*x^2])/c^4))
/60
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 5407
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Simp[(a
 + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ
[2*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0])
)
```

## Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.81

| method | result |
|---|---|
| parts | $d^3a\left(-\frac{1}{6}ic^3x^6-\frac{3}{5}x^5c^2+\frac{3}{4}ic\,x^4+\frac{1}{3}x^3\right)+\frac{d^3b\left(-\frac{i\arctan(cx)c^6x^6}{6}-\frac{3c^5x^5\arctan(cx)}{5}+\frac{3i\arctan(cx)c^4x^4}{4}+\right.}{}$ |
| derivativedivides | $\frac{d^3a\left(-\frac{1}{6}ic^6x^6-\frac{3}{5}c^5x^5+\frac{3}{4}ic^4x^4+\frac{1}{3}c^3x^3\right)+d^3b\left(-\frac{i\arctan(cx)c^6x^6}{6}-\frac{3c^5x^5\arctan(cx)}{5}+\frac{3i\arctan(cx)c^4x^4}{4}+\frac{c^3x^3\arctan(cx)}{3}\right.}{c^3}$ |
| default | $\frac{d^3a\left(-\frac{1}{6}ic^6x^6-\frac{3}{5}c^5x^5+\frac{3}{4}ic^4x^4+\frac{1}{3}c^3x^3\right)+d^3b\left(-\frac{i\arctan(cx)c^6x^6}{6}-\frac{3c^5x^5\arctan(cx)}{5}+\frac{3i\arctan(cx)c^4x^4}{4}+\frac{c^3x^3\arctan(cx)}{3}\right.}{c^3}$ |
| parallelrisch | $-\frac{-135ix^4a\,c^4d^3-6ix^5b\,c^5d^3+55ix^3b\,c^3d^3+108b\,c^5d^3\arctan(cx)x^5-165ib\,d^3xc+108a\,c^5d^3x^5-135ib\,c^4d^3\arctan(cx)}{}$ |
| risch | $-\frac{d^3b\left(10c^3x^6-36ic^2x^5-45c\,x^4+20ix^3\right)\ln(icx+1)}{120}-\frac{11ib\,d^3x^3}{36}+\frac{d^3c^3x^6b\ln(-icx+1)}{12}-\frac{11ib\,d^3\arctan(cx)}{12c^3}+$ |

input
```
int(x^2*(d+I*c*d*x)^3*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

output
```
d^3*a*(-1/6*I*c^3*x^6-3/5*x^5*c^2+3/4*I*c*x^4+1/3*x^3)+d^3*b/c^3*(-1/6*I*a
rctan(c*x)*c^6*x^6-3/5*c^5*x^5*arctan(c*x)+3/4*I*arctan(c*x)*c^4*x^4+1/3*c
^3*x^3*arctan(c*x)+11/12*I*c*x+1/30*I*c^5*x^5+3/20*c^4*x^4-11/36*I*c^3*x^3
-7/15*c^2*x^2+7/15*ln(c^2*x^2+1)-11/12*I*arctan(c*x))
```

## Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.99

$$\int x^2(d+icdx)^3(a+b\arctan(cx))\,dx$$
$$= \frac{-60i\,ac^6d^3x^6 - 12\,(18\,a - i\,b)c^5d^3x^5 - 54\,(-5i\,a - b)c^4d^3x^4 + 10\,(12\,a - 11i\,b)c^3d^3x^3 - 168\,bc^2d^3x^2 + 3}{}$$

input
```
integrate(x^2*(d+I*c*d*x)^3*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
1/360*(-60*I*a*c^6*d^3*x^6 - 12*(18*a - I*b)*c^5*d^3*x^5 - 54*(-5*I*a - b)
*c^4*d^3*x^4 + 10*(12*a - 11*I*b)*c^3*d^3*x^3 - 168*b*c^2*d^3*x^2 + 330*I*
b*c*d^3*x + 333*b*d^3*log((c*x + I)/c) + 3*b*d^3*log((c*x - I)/c) + 3*(10*
b*c^6*d^3*x^6 - 36*I*b*c^5*d^3*x^5 - 45*b*c^4*d^3*x^4 + 20*I*b*c^3*d^3*x^3
)*log(-(c*x + I)/(c*x - I)))/c^3
```

## Sympy [A] (verification not implemented)

Time = 2.61 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.65

$$\int x^2(d+icdx)^3(a+b\arctan(cx))\,dx$$
$$= -\frac{iac^3d^3x^6}{6} - \frac{7bd^3x^2}{15c} + \frac{11ibd^3x}{12c^2} - \frac{bd^3\left(-\frac{\log\left(310bcd^3x-310ibd^3\right)}{120} - \frac{209\log\left(310bcd^3x+310ibd^3\right)}{280}\right)}{c^3}$$
$$- x^5\cdot\left(\frac{3ac^2d^3}{5} - \frac{ibc^2d^3}{30}\right) - x^4\left(-\frac{3iacd^3}{4} - \frac{3bcd^3}{20}\right) - x^3\left(-\frac{ad^3}{3} + \frac{11ibd^3}{36}\right)$$
$$+ \left(-\frac{bc^3d^3x^6}{12} + \frac{3ibc^2d^3x^5}{10} + \frac{3bcd^3x^4}{8} - \frac{ibd^3x^3}{6}\right)\log\left(icx+1\right)$$
$$+ \frac{\left(70bc^6d^3x^6 - 252ibc^5d^3x^5 - 315bc^4d^3x^4 + 140ibc^3d^3x^3 + 150bd^3\right)\log\left(-icx+1\right)}{840c^3}$$

input
```
integrate(x**2*(d+I*c*d*x)**3*(a+b*atan(c*x)),x)
```

output
```
-I*a*c**3*d**3*x**6/6 - 7*b*d**3*x**2/(15*c) + 11*I*b*d**3*x/(12*c**2) - b
*d**3*(-log(310*b*c*d**3*x - 310*I*b*d**3)/120 - 209*log(310*b*c*d**3*x +
310*I*b*d**3)/280)/c**3 - x**5*(3*a*c**2*d**3/5 - I*b*c**2*d**3/30) - x**4
*(-3*I*a*c*d**3/4 - 3*b*c*d**3/20) - x**3*(-a*d**3/3 + 11*I*b*d**3/36) + (
-b*c**3*d**3*x**6/12 + 3*I*b*c**2*d**3*x**5/10 + 3*b*c*d**3*x**4/8 - I*b*d
**3*x**3/6)*log(I*c*x + 1) + (70*b*c**6*d**3*x**6 - 252*I*b*c**5*d**3*x**5
 - 315*b*c**4*d**3*x**4 + 140*I*b*c**3*d**3*x**3 + 150*b*d**3)*log(-I*c*x
+ 1)/(840*c**3)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.27

$$\begin{aligned}
&\int x^2(d+icdx)^3(a+b\arctan(cx))\,dx \\
&= -\frac{1}{6}i\,ac^3d^3x^6 - \frac{3}{5}\,ac^2d^3x^5 + \frac{3}{4}i\,acd^3x^4 \\
&\quad - \frac{1}{90}i\left(15\,x^6\arctan(cx) - c\left(\frac{3\,c^4x^5 - 5\,c^2x^3 + 15\,x}{c^6} - \frac{15\,\arctan(cx)}{c^7}\right)\right)bc^3d^3 \\
&\quad - \frac{3}{20}\left(4\,x^5\arctan(cx) - c\left(\frac{c^2x^4 - 2\,x^2}{c^4} + \frac{2\,\log(c^2x^2+1)}{c^6}\right)\right)bc^2d^3 \\
&\quad + \frac{1}{3}\,ad^3x^3 + \frac{1}{4}i\left(3\,x^4\arctan(cx) - c\left(\frac{c^2x^3 - 3\,x}{c^4} + \frac{3\,\arctan(cx)}{c^5}\right)\right)bcd^3 \\
&\quad + \frac{1}{6}\left(2\,x^3\arctan(cx) - c\left(\frac{x^2}{c^2} - \frac{\log(c^2x^2+1)}{c^4}\right)\right)bd^3
\end{aligned}$$

input
```
integrate(x^2*(d+I*c*d*x)^3*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
-1/6*I*a*c^3*d^3*x^6 - 3/5*a*c^2*d^3*x^5 + 3/4*I*a*c*d^3*x^4 - 1/90*I*(15*
x^6*arctan(c*x) - c*((3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan(c*x)/c
^7))*b*c^3*d^3 - 3/20*(4*x^5*arctan(c*x) - c*((c^2*x^4 - 2*x^2)/c^4 + 2*lo
g(c^2*x^2 + 1)/c^6))*b*c^2*d^3 + 1/3*a*d^3*x^3 + 1/4*I*(3*x^4*arctan(c*x)
- c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*b*c*d^3 + 1/6*(2*x^3*arctan
(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*b*d^3
```

## Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.04

$$\int x^2(d+icdx)^3(a+b\arctan(cx))\,dx = -\frac{60i\,bc^6d^3x^6\arctan(cx)+60i\,ac^6d^3x^6+216\,bc^5d^3x^5\arctan(cx)+216\,ac^5d^3x^5-12i\,bc^5d^3x^5-270i\,bc}{}$$

input
```
integrate(x^2*(d+I*c*d*x)^3*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
-1/360*(60*I*b*c^6*d^3*x^6*arctan(c*x) + 60*I*a*c^6*d^3*x^6 + 216*b*c^5*d^
3*x^5*arctan(c*x) + 216*a*c^5*d^3*x^5 - 12*I*b*c^5*d^3*x^5 - 270*I*b*c^4*d
^3*x^4*arctan(c*x) - 270*I*a*c^4*d^3*x^4 - 54*b*c^4*d^3*x^4 - 120*b*c^3*d^
3*x^3*arctan(c*x) - 120*a*c^3*d^3*x^3 + 110*I*b*c^3*d^3*x^3 + 168*b*c^2*d^
3*x^2 - 330*I*b*c*d^3*x - 333*b*d^3*log(c*x + I) - 3*b*d^3*log(c*x - I))/c
^3
```

## Mupad [B] (verification not implemented)

Time = 1.16 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.91

$$\begin{aligned}\int x^2(d+icdx)^3(a+b\arctan(cx))\,dx &= -\frac{\frac{d^3\,\left(-84\,b\,\ln\left(c^2\,x^2+1\right)+b\,\mathrm{atan}(c\,x)\,165\mathrm{i}\right)}{180}+\frac{7\,b\,c^2\,d^3\,x^2}{15}-\frac{b\,c\,d^3\,x\,11\mathrm{i}}{12}}{c^3} \\ &+\frac{d^3\,\left(60\,a\,x^3+60\,b\,x^3\,\mathrm{atan}(c\,x)-b\,x^3\,55\mathrm{i}\right)}{180}-\frac{c^3\,d^3\,\left(a\,x^6\,30\mathrm{i}+b\,x^6\,\mathrm{atan}(c\,x)\,30\mathrm{i}\right)}{180} \\ &+\frac{c\,d^3\,\left(a\,x^4\,135\mathrm{i}+27\,b\,x^4+b\,x^4\,\mathrm{atan}(c\,x)\,135\mathrm{i}\right)}{180} \\ &-\frac{c^2\,d^3\,\left(108\,a\,x^5+108\,b\,x^5\,\mathrm{atan}(c\,x)-b\,x^5\,6\mathrm{i}\right)}{180}\end{aligned}$$

input
```
int(x^2*(a + b*atan(c*x))*(d + c*d*x*1i)^3,x)
```

output
```
(d^3*(60*a*x^3 - b*x^3*55i + 60*b*x^3*atan(c*x)))/180 - ((d^3*(b*atan(c*x)
*165i - 84*b*log(c^2*x^2 + 1)))/180 + (7*b*c^2*d^3*x^2)/15 - (b*c*d^3*x*11
i)/12)/c^3 - (c^3*d^3*(a*x^6*30i + b*x^6*atan(c*x)*30i))/180 + (c*d^3*(a*x
^4*135i + 27*b*x^4 + b*x^4*atan(c*x)*135i))/180 - (c^2*d^3*(108*a*x^5 - b*
x^5*6i + 108*b*x^5*atan(c*x)))/180
```

## Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.87

$$\int x^2(d+icdx)^3(a+b\arctan(cx))\,dx$$
$$= \frac{d^3(-30atan(cx)\,b\,c^6i\,x^6 - 108atan(cx)\,b\,c^5x^5 + 135atan(cx)\,b\,c^4i\,x^4 + 60atan(cx)\,b\,c^3x^3 - 165atan(cx)\,b$$

input
```
int(x^2*(d+I*c*d*x)^3*(a+b*atan(c*x)),x)
```

output
```
(d**3*( - 30*atan(c*x)*b*c**6*i*x**6 - 108*atan(c*x)*b*c**5*x**5 + 135*ata
n(c*x)*b*c**4*i*x**4 + 60*atan(c*x)*b*c**3*x**3 - 165*atan(c*x)*b*i + 84*l
og(c**2*x**2 + 1)*b - 30*a*c**6*i*x**6 - 108*a*c**5*x**5 + 135*a*c**4*i*x*
*4 + 60*a*c**3*x**3 + 6*b*c**5*i*x**5 + 27*b*c**4*x**4 - 55*b*c**3*i*x**3
- 84*b*c**2*x**2 + 165*b*c*i*x))/(180*c**3)
```

# 3.22 $\int x(d+icdx)^3(a+b\arctan(cx))\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 157

$$\int x(d+icdx)^3(a+b\arctan(cx))\,dx = -\frac{3bd^3x}{5c}-\frac{3ibd^3(i-cx)^2}{20c^2}-\frac{bd^3(i-cx)^3}{20c^2}+\frac{ibd^3(i-cx)^4}{20c^2}+\frac{d^3(1+icx)^4(a+b\arctan(cx))}{4c^2}-\frac{d^3(1+icx)^5(a+b\arctan(cx))}{5c^2}+\frac{6ibd^3\log(i+cx)}{5c^2}$$

output
```
-3/5*b*d^3*x/c-3/20*I*b*d^3*(I-c*x)^2/c^2-1/20*b*d^3*(I-c*x)^3/c^2+1/20*I*
b*d^3*(I-c*x)^4/c^2+1/4*d^3*(1+I*c*x)^4*(a+b*arctan(c*x))/c^2-1/5*d^3*(1+I
*c*x)^5*(a+b*arctan(c*x))/c^2+6/5*I*b*d^3*ln(I+c*x)/c^2
```

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.84

$$\int x(d+icdx)^3(a+b\arctan(cx))\,dx$$
$$=\frac{d^3(cx(b(-25-12icx+5c^2x^2+ic^3x^3)+acx(10+20icx-15c^2x^2-4ic^3x^3))+b(25+10c^2x^2+20ic^3x^3}{20c^2}$$

input
```
Integrate[x*(d + I*c*d*x)^3*(a + b*ArcTan[c*x]),x]
```

output
```
(d^3*(c*x*(b*(-25 - (12*I)*c*x + 5*c^2*x^2 + I*c^3*x^3) + a*c*x*(10 + (20*
I)*c*x - 15*c^2*x^2 - (4*I)*c^3*x^3)) + b*(25 + 10*c^2*x^2 + (20*I)*c^3*x^
3 - 15*c^4*x^4 - (4*I)*c^5*x^5)*ArcTan[c*x] + (12*I)*b*Log[1 + c^2*x^2]))/
(20*c^2)
```

## Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.87, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {5407, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(d+icdx)^3(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5407}$$

$$-bc\int -\frac{d^3(i-cx)^3(1-4icx)}{20c^2(cx+i)}dx-\frac{d^3(1+icx)^5(a+b\arctan(cx))}{5c^2}+\frac{d^3(1+icx)^4(a+b\arctan(cx))}{4c^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{bd^3\int \frac{(i-cx)^3(1-4icx)}{cx+i}dx}{20c}-\frac{d^3(1+icx)^5(a+b\arctan(cx))}{5c^2}+\frac{d^3(1+icx)^4(a+b\arctan(cx))}{4c^2}$$

$$\Big\downarrow \text{86}$$

$$\frac{bd^3 \int \left(-4i(i-cx)^3+3(cx-i)^2-6i(cx-i)+\frac{24i}{cx+i}-12\right)dx}{20c}-\frac{d^3(1+icx)^5(a+b\arctan(cx))}{5c^2}+\frac{d^3(1+icx)^4(a+b\arctan(cx))}{4c^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d^3(1+icx)^5(a+b\arctan(cx))}{5c^2}+\frac{d^3(1+icx)^4(a+b\arctan(cx))}{4c^2}+\frac{bd^3\left(\frac{i(-cx+i)^4}{c}-\frac{(-cx+i)^3}{c}-\frac{3i(-cx+i)^2}{c}+\frac{24i\log(cx+i)}{c}-12x\right)}{20c}$$

input
```
Int[x*(d + I*c*d*x)^3*(a + b*ArcTan[c*x]),x]
```

output
```
(d^3*(1 + I*c*x)^4*(a + b*ArcTan[c*x]))/(4*c^2) - (d^3*(1 + I*c*x)^5*(a +
b*ArcTan[c*x]))/(5*c^2) + (b*d^3*(-12*x - ((3*I)*(I - c*x)^2)/c - (I - c*x
)^3/c + (I*(I - c*x)^4)/c + ((24*I)*Log[I + c*x])/c))/(20*c)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5407
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Simp[(a
 + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ
[2*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0])
)
```

## Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.92

| method | result |
|---|---|
| parts | $d^3a\left(-\frac{1}{5}ic^3x^5-\frac{3}{4}c^2x^4+icx^3+\frac{1}{2}x^2\right)+\frac{d^3b\left(-\frac{i\arctan(cx)c^5x^5}{5}-\frac{3c^4x^4\arctan(cx)}{4}+i\arctan(cx)c^3x^3+\frac{c^2}{}\right.}{}$ |
| derivativedivides | $\frac{d^3a\left(-\frac{1}{5}ic^5x^5-\frac{3}{4}c^4x^4+ic^3x^3+\frac{1}{2}c^2x^2\right)+d^3b\left(-\frac{i\arctan(cx)c^5x^5}{5}-\frac{3c^4x^4\arctan(cx)}{4}+i\arctan(cx)c^3x^3+\frac{c^2x^2\arctan(cx)}{2}-\right.}{c^2}$ |
| default | $\frac{d^3a\left(-\frac{1}{5}ic^5x^5-\frac{3}{4}c^4x^4+ic^3x^3+\frac{1}{2}c^2x^2\right)+d^3b\left(-\frac{i\arctan(cx)c^5x^5}{5}-\frac{3c^4x^4\arctan(cx)}{4}+i\arctan(cx)c^3x^3+\frac{c^2x^2\arctan(cx)}{2}-\right.}{c^2}$ |
| parallelrisch | $\frac{-4ic^5bd^3\arctan(cx)x^5-4ix^5ac^5d^3+ix^4bc^4d^3-15d^3b\arctan(cx)x^4c^4+20ibd^3c^3\arctan(cx)x^3-15ac^4d^3x^4+20ix^3a}{20}$ |
| risch | $-\frac{d^3b\left(4c^3x^5-15ic^2x^4-20cx^3+10ix^2\right)\ln(icx+1)}{40}+\frac{d^3c^3bx^5\ln(-icx+1)}{10}-\frac{id^3c^3x^5a}{5}-\frac{3d^3c^2ax^4}{4}-\frac{3id^3c^2x^4}{}$ |

input
```
int(x*(d+I*c*d*x)^3*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

output
```
d^3*a*(-1/5*I*c^3*x^5-3/4*c^2*x^4+I*c*x^3+1/2*x^2)+d^3*b/c^2*(-1/5*I*arcta
n(c*x)*c^5*x^5-3/4*c^4*x^4*arctan(c*x)+I*arctan(c*x)*c^3*x^3+1/2*c^2*x^2*a
rctan(c*x)-5/4*c*x+1/20*I*c^4*x^4+1/4*c^3*x^3-3/5*I*c^2*x^2+3/5*I*ln(c^2*x
^2+1)+5/4*arctan(c*x))
```

## Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.13

$$\int x(d+icdx)^3(a+b\arctan(cx))\,dx$$
$$= \frac{-8i\,ac^5d^3x^5 - 2\,(15\,a - i\,b)c^4d^3x^4 - 10\,(-4i\,a - b)c^3d^3x^3 + 4\,(5\,a - 6i\,b)c^2d^3x^2 - 50\,bcd^3x + 49i\,bd^3\log}{40\,c^2}$$

input
```
integrate(x*(d+I*c*d*x)^3*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
1/40*(-8*I*a*c^5*d^3*x^5 - 2*(15*a - I*b)*c^4*d^3*x^4 - 10*(-4*I*a - b)*c^
3*d^3*x^3 + 4*(5*a - 6*I*b)*c^2*d^3*x^2 - 50*b*c*d^3*x + 49*I*b*d^3*log((c
*x + I)/c) - I*b*d^3*log((c*x - I)/c) + (4*b*c^5*d^3*x^5 - 15*I*b*c^4*d^3*
x^4 - 20*b*c^3*d^3*x^3 + 10*I*b*c^2*d^3*x^2)*log(-(c*x + I)/(c*x - I)))/c^
2
```

## Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 296 vs. 2(138) = 276.

Time = 2.44 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.89

$$\int x(d+icdx)^3(a+b\arctan(cx))\,dx$$
$$= -\frac{iac^3d^3x^5}{5} - \frac{5bd^3x}{4c} - \frac{bd^3\left(\frac{i\log\left(19bcd^3x-19ibd^3\right)}{40} - \frac{37i\log\left(19bcd^3x+19ibd^3\right)}{40}\right)}{c^2}$$
$$- x^4\cdot\left(\frac{3ac^2d^3}{4} - \frac{ibc^2d^3}{20}\right) - x^3\left(-iacd^3 - \frac{bcd^3}{4}\right) - x^2\left(-\frac{ad^3}{2} + \frac{3ibd^3}{5}\right)$$
$$+ \left(-\frac{bc^3d^3x^5}{10} + \frac{3ibc^2d^3x^4}{8} + \frac{bcd^3x^3}{2} - \frac{ibd^3x^2}{4}\right)\log\left(icx+1\right)$$
$$+ \frac{\left(4bc^5d^3x^5 - 15ibc^4d^3x^4 - 20bc^3d^3x^3 + 10ibc^2d^3x^2 + 12ibd^3\right)\log\left(-icx+1\right)}{40c^2}$$

input
```
integrate(x*(d+I*c*d*x)**3*(a+b*atan(c*x)),x)
```

output
```
-I*a*c**3*d**3*x**5/5 - 5*b*d**3*x/(4*c) - b*d**3*(I*log(19*b*c*d**3*x - 1
9*I*b*d**3)/40 - 37*I*log(19*b*c*d**3*x + 19*I*b*d**3)/40)/c**2 - x**4*(3*
a*c**2*d**3/4 - I*b*c**2*d**3/20) - x**3*(-I*a*c*d**3 - b*c*d**3/4) - x**2
*(-a*d**3/2 + 3*I*b*d**3/5) + (-b*c**3*d**3*x**5/10 + 3*I*b*c**2*d**3*x**4
/8 + b*c*d**3*x**3/2 - I*b*d**3*x**2/4)*log(I*c*x + 1) + (4*b*c**5*d**3*x*
*5 - 15*I*b*c**4*d**3*x**4 - 20*b*c**3*d**3*x**3 + 10*I*b*c**2*d**3*x**2 +
 12*I*b*d**3)*log(-I*c*x + 1)/(40*c**2)
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.41

$$\begin{aligned}
&\int x(d+icdx)^3(a+b\arctan(cx))\,dx \\
&= -\frac{1}{5}i\,ac^3d^3x^5 - \frac{3}{4}\,ac^2d^3x^4 \\
&\quad - \frac{1}{20}i\left(4\,x^5\arctan\left(cx\right) - c\left(\frac{c^2x^4-2\,x^2}{c^4} + \frac{2\,\log\left(c^2x^2+1\right)}{c^6}\right)\right)bc^3d^3 \\
&\quad + i\,acd^3x^3 - \frac{1}{4}\left(3\,x^4\arctan\left(cx\right) - c\left(\frac{c^2x^3-3\,x}{c^4} + \frac{3\,\arctan\left(cx\right)}{c^5}\right)\right)bc^2d^3 \\
&\quad + \frac{1}{2}i\left(2\,x^3\arctan\left(cx\right) - c\left(\frac{x^2}{c^2} - \frac{\log\left(c^2x^2+1\right)}{c^4}\right)\right)bcd^3 \\
&\quad + \frac{1}{2}\,ad^3x^2 + \frac{1}{2}\left(x^2\arctan\left(cx\right) - c\left(\frac{x}{c^2} - \frac{\arctan\left(cx\right)}{c^3}\right)\right)bd^3
\end{aligned}$$

input
```
integrate(x*(d+I*c*d*x)^3*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
-1/5*I*a*c^3*d^3*x^5 - 3/4*a*c^2*d^3*x^4 - 1/20*I*(4*x^5*arctan(c*x) - c*(
(c^2*x^4 - 2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*b*c^3*d^3 + I*a*c*d^3*x^3
 - 1/4*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*b
*c^2*d^3 + 1/2*I*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*
b*c*d^3 + 1/2*a*d^3*x^2 + 1/2*(x^2*arctan(c*x) - c*(x/c^2 - arctan(c*x)/c^
3))*b*d^3
```

## Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.20

$$\int x(d+icdx)^3(a+b\arctan(cx))\,dx = -\frac{8i\,bc^5d^3x^5\arctan(cx)+8i\,ac^5d^3x^5+30\,bc^4d^3x^4\arctan(cx)+30\,ac^4d^3x^4-2i\,bc^4d^3x^4-40i\,bc^3d^3x^3\,\mathrm{ar}}{}$$

input
```
integrate(x*(d+I*c*d*x)^3*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
-1/40*(8*I*b*c^5*d^3*x^5*arctan(c*x) + 8*I*a*c^5*d^3*x^5 + 30*b*c^4*d^3*x^
4*arctan(c*x) + 30*a*c^4*d^3*x^4 - 2*I*b*c^4*d^3*x^4 - 40*I*b*c^3*d^3*x^3*
arctan(c*x) - 40*I*a*c^3*d^3*x^3 - 10*b*c^3*d^3*x^3 - 20*b*c^2*d^3*x^2*arc
tan(c*x) - 20*a*c^2*d^3*x^2 + 24*I*b*c^2*d^3*x^2 + 50*b*c*d^3*x - 49*I*b*d
^3*log(I*c*x - 1) + I*b*d^3*log(-I*c*x - 1))/c^2
```

## Mupad [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.02

$$\int x(d+icdx)^3(a+b\arctan(cx))\,dx = \frac{\frac{d^3\,(25\,b\,\mathrm{atan}(c\,x)+b\,\ln(c^2\,x^2+1)\,12\mathrm{i})}{20}-\frac{5\,b\,c\,d^3\,x}{4}}{c^2} + \frac{d^3\,(10\,a\,x^2+10\,b\,x^2\,\mathrm{atan}(c\,x)-b\,x^2\,12\mathrm{i})}{20} - \frac{c^3\,d^3\,(a\,x^5\,4\mathrm{i}+b\,x^5\,\mathrm{atan}(c\,x)\,4\mathrm{i})}{20} + \frac{c\,d^3\,(a\,x^3\,20\mathrm{i}+5\,b\,x^3+b\,x^3\,\mathrm{atan}(c\,x)\,20\mathrm{i})}{20} - \frac{c^2\,d^3\,(15\,a\,x^4+15\,b\,x^4\,\mathrm{atan}(c\,x)-b\,x^4\,1\mathrm{i})}{20}$$

input
```
int(x*(a + b*atan(c*x))*(d + c*d*x*1i)^3,x)
```

output
```
((d^3*(25*b*atan(c*x) + b*log(c^2*x^2 + 1)*12i))/20 - (5*b*c*d^3*x)/4)/c^2
 + (d^3*(10*a*x^2 - b*x^2*12i + 10*b*x^2*atan(c*x)))/20 - (c^3*d^3*(a*x^5*
4i + b*x^5*atan(c*x)*4i))/20 + (c*d^3*(a*x^3*20i + 5*b*x^3 + b*x^3*atan(c*
x)*20i))/20 - (c^2*d^3*(15*a*x^4 - b*x^4*1i + 15*b*x^4*atan(c*x)))/20
```

## Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.99

$$\int x(d+icdx)^3(a+b\arctan(cx))\,dx$$
$$= \frac{d^3(-4atan(cx)\,b\,c^5i\,x^5-15atan(cx)\,b\,c^4x^4+20atan(cx)\,b\,c^3i\,x^3+10atan(cx)\,b\,c^2x^2+25atan(cx)\,b+1}{20}$$

input
```
int(x*(d+I*c*d*x)^3*(a+b*atan(c*x)),x)
```

output
```
(d**3*( - 4*atan(c*x)*b*c**5*i*x**5 - 15*atan(c*x)*b*c**4*x**4 + 20*atan(c
*x)*b*c**3*i*x**3 + 10*atan(c*x)*b*c**2*x**2 + 25*atan(c*x)*b + 12*log(c**
2*x**2 + 1)*b*i - 4*a*c**5*i*x**5 - 15*a*c**4*x**4 + 20*a*c**3*i*x**3 + 10
*a*c**2*x**2 + b*c**4*i*x**4 + 5*b*c**3*x**3 - 12*b*c**2*i*x**2 - 25*b*c*x
))/(20*c**2)
```

# 3.23 $\int (d+icdx)^3(a+b\arctan(cx))\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 100

$$\int (d+icdx)^3(a+b\arctan(cx))\,dx = -ibd^3x - \frac{bd^3(1+icx)^2}{4c} - \frac{bd^3(1+icx)^3}{12c} - \frac{id^3(1+icx)^4(a+b\arctan(cx))}{4c} - \frac{2bd^3\log(1-icx)}{c}$$

output
```
-I*b*d^3*x-1/4*b*d^3*(1+I*c*x)^2/c-1/12*b*d^3*(1+I*c*x)^3/c-1/4*I*d^3*(1+I
*c*x)^4*(a+b*arctan(c*x))/c-2*b*d^3*ln(1-I*c*x)/c
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.77

$$\int (d+icdx)^3(a+b\arctan(cx))\,dx = -\frac{i(3(d+icdx)^4(a+b\arctan(cx)) - bd^4(4i-21cx-6ic^2x^2+c^3x^3+24i\log(i+cx)))}{12cd}$$

input
```
Integrate[(d + I*c*d*x)^3*(a + b*ArcTan[c*x]),x]
```

output
```
((-1/12*I)*(3*(d + I*c*d*x)^4*(a + b*ArcTan[c*x]) - b*d^4*(4*I - 21*c*x -
(6*I)*c^2*x^2 + c^3*x^3 + (24*I)*Log[I + c*x])))/(c*d)
```

## Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5387, 27, 456, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+icdx)^3(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5387}$$

$$\frac{ib\int \frac{d^4(icx+1)^4}{c^2x^2+1}dx}{4d}-\frac{id^3(1+icx)^4(a+b\arctan(cx))}{4c}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}ibd^3\int \frac{(icx+1)^4}{c^2x^2+1}dx-\frac{id^3(1+icx)^4(a+b\arctan(cx))}{4c}$$

$$\Big\downarrow \text{456}$$

$$\frac{1}{4}ibd^3\int \frac{(icx+1)^3}{1-icx}dx-\frac{id^3(1+icx)^4(a+b\arctan(cx))}{4c}$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{4}ibd^3\int \left(-(icx+1)^2-2(icx+1)+\frac{8}{1-icx}-4\right)dx-\frac{id^3(1+icx)^4(a+b\arctan(cx))}{4c}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}ibd^3\left(\frac{i(1+icx)^3}{3c}+\frac{i(1+icx)^2}{c}+\frac{8i\log(cx+i)}{c}-4x\right)-\frac{id^3(1+icx)^4(a+b\arctan(cx))}{4c}$$

input
```
Int[(d + I*c*d*x)^3*(a + b*ArcTan[c*x]),x]
```

output
```
((-1/4*I)*d^3*(1 + I*c*x)^4*(a + b*ArcTan[c*x]))/c + (I/4)*b*d^3*(-4*x + (
I*(1 + I*c*x)^2)/c + ((I/3)*(1 + I*c*x)^3)/c + ((8*I)*Log[I + c*x])/c)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5387
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Simp[b*(
c/(e*(q + 1)))   Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b
, c, d, e, q}, x] && NeQ[q, -1]
```

## Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.24

| method | result |
|---|---|
| derivativedivides | $\frac{-\frac{id^3a(icx+1)^4}{4}+d^3b\left(-\frac{i\arctan(cx)c^4x^4}{4}-c^3x^3\arctan(cx)+\frac{3i\arctan(cx)c^2x^2}{2}+cx\arctan(cx)-\frac{i\arctan(cx)}{4}+\frac{i\left(-7cx+\frac{c^3}{3}\right.}{}\right.}{c}$ |
| default | $\frac{-\frac{id^3a(icx+1)^4}{4}+d^3b\left(-\frac{i\arctan(cx)c^4x^4}{4}-c^3x^3\arctan(cx)+\frac{3i\arctan(cx)c^2x^2}{2}+cx\arctan(cx)-\frac{i\arctan(cx)}{4}+\frac{i\left(-7cx+\frac{c^3}{3}\right.}{}\right.}{c}$ |
| parts | $-\frac{id^3a(icx+1)^4}{4c}+\frac{d^3b\left(-\frac{i\arctan(cx)c^4x^4}{4}-c^3x^3\arctan(cx)+\frac{3i\arctan(cx)c^2x^2}{2}+cx\arctan(cx)-\frac{i\arctan(cx)}{4}+\frac{i\left(-7cx\right.}{}\right.}{c}$ |
| parallelrisch | $-\frac{3ib\,c^4d^3\arctan(cx)x^4+3ix^4a\,c^4d^3-ix^3b\,c^3d^3+12d^3b\arctan(cx)x^3c^3-18id^3c^2b\arctan(cx)x^2+12a\,c^3d^3x^3-18ix^2a}{12c}$ |
| risch | $-\frac{d^3(cx-i)^4b\ln(icx+1)}{8c}-\frac{id^3a\,c^3x^4}{4}+\frac{d^3c^3x^4b\ln(-icx+1)}{8}-\frac{id^3c^2b\,x^3\ln(-icx+1)}{2}+\frac{id^3b\,c^2x^3}{12}-d^3a\,c^2x$ |

input
```
int((d+I*c*d*x)^3*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/c*(-1/4*I*d^3*a*(1+I*c*x)^4+d^3*b*(-1/4*I*arctan(c*x)*c^4*x^4-c^3*x^3*ar
ctan(c*x)+3/2*I*arctan(c*x)*c^2*x^2+c*x*arctan(c*x)-1/4*I*arctan(c*x)+1/4*
I*(-7*c*x+1/3*c^3*x^3-2*I*c^2*x^2+4*I*ln(c^2*x^2+1)+8*arctan(c*x))))
```

## Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.61

$$\int (d+icdx)^3(a+b\arctan(cx))\,dx$$
$$=\frac{-6i\,ac^4d^3x^4-2\,(12\,a-i\,b)c^3d^3x^3-12\,(-3i\,a-b)c^2d^3x^2+6\,(4\,a-7i\,b)cd^3x-45\,bd^3\log\left(\frac{cx+i}{c}\right)-3\,bd^3}{24\,c}$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
1/24*(-6*I*a*c^4*d^3*x^4 - 2*(12*a - I*b)*c^3*d^3*x^3 - 12*(-3*I*a - b)*c^
2*d^3*x^2 + 6*(4*a - 7*I*b)*c*d^3*x - 45*b*d^3*log((c*x + I)/c) - 3*b*d^3*
log((c*x - I)/c) + 3*(b*c^4*d^3*x^4 - 4*I*b*c^3*d^3*x^3 - 6*b*c^2*d^3*x^2
+ 4*I*b*c*d^3*x)*log(-(c*x + I)/(c*x - I)))/c
```

## Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(85) = 170$.

Time = 2.11 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.67

$$\begin{aligned}
&\int (d+icdx)^3(a+b\arctan(cx))\,dx \\
&= -\frac{iac^3d^3x^4}{4} - \frac{bd^3\left(\frac{\log(22bcd^3x-22ibd^3)}{8}+\frac{49\log(22bcd^3x+22ibd^3)}{40}\right)}{c} \\
&\quad - x^3\left(ac^2d^3-\frac{ibc^2d^3}{12}\right) - x^2\left(-\frac{3iacd^3}{2}-\frac{bcd^3}{2}\right) - x\left(-ad^3+\frac{7ibd^3}{4}\right) \\
&\quad + \left(-\frac{bc^3d^3x^4}{8}+\frac{ibc^2d^3x^3}{2}+\frac{3bcd^3x^2}{4}-\frac{ibd^3x}{2}\right)\log(icx+1) \\
&\quad + \frac{(5bc^4d^3x^4-20ibc^3d^3x^3-30bc^2d^3x^2+20ibcd^3x-26bd^3)\log(-icx+1)}{40c}
\end{aligned}$$

input
```
integrate((d+I*c*d*x)**3*(a+b*atan(c*x)),x)
```

output
```
-I*a*c**3*d**3*x**4/4 - b*d**3*(log(22*b*c*d**3*x - 22*I*b*d**3)/8 + 49*lo
g(22*b*c*d**3*x + 22*I*b*d**3)/40)/c - x**3*(a*c**2*d**3 - I*b*c**2*d**3/1
2) - x**2*(-3*I*a*c*d**3/2 - b*c*d**3/2) - x*(-a*d**3 + 7*I*b*d**3/4) + (-
b*c**3*d**3*x**4/8 + I*b*c**2*d**3*x**3/2 + 3*b*c*d**3*x**2/4 - I*b*d**3*x
/2)*log(I*c*x + 1) + (5*b*c**4*d**3*x**4 - 20*I*b*c**3*d**3*x**3 - 30*b*c*
*2*d**3*x**2 + 20*I*b*c*d**3*x - 26*b*d**3)*log(-I*c*x + 1)/(40*c)
```

## Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(82) = 164$.

Time = 0.11 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.97

$$\begin{aligned}
&\int (d+icdx)^3(a+b\arctan(cx))\,dx \\
&= -\frac{1}{4}i\,ac^3d^3x^4 - ac^2d^3x^3 \\
&\quad - \frac{1}{12}i\left(3\,x^4\arctan\left(cx\right) - c\left(\frac{c^2x^3-3\,x}{c^4} + \frac{3\,\arctan\left(cx\right)}{c^5}\right)\right)bc^3d^3 \\
&\quad - \frac{1}{2}\left(2\,x^3\arctan\left(cx\right) - c\left(\frac{x^2}{c^2} - \frac{\log\left(c^2x^2+1\right)}{c^4}\right)\right)bc^2d^3 \\
&\quad + \frac{3}{2}i\,acd^3x^2 + \frac{3}{2}i\left(x^2\arctan\left(cx\right) - c\left(\frac{x}{c^2} - \frac{\arctan\left(cx\right)}{c^3}\right)\right)bcd^3 \\
&\quad + ad^3x + \frac{\left(2\,cx\arctan\left(cx\right) - \log\left(c^2x^2+1\right)\right)bd^3}{2\,c}
\end{aligned}$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
-1/4*I*a*c^3*d^3*x^4 - a*c^2*d^3*x^3 - 1/12*I*(3*x^4*arctan(c*x) - c*((c^2
*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*b*c^3*d^3 - 1/2*(2*x^3*arctan(c*x) -
 c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*b*c^2*d^3 + 3/2*I*a*c*d^3*x^2 + 3/2*I
*(x^2*arctan(c*x) - c*(x/c^2 - arctan(c*x)/c^3))*b*c*d^3 + a*d^3*x + 1/2*(
2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b*d^3/c
```

## Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(82) = 164$.

Time = 0.13 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.66

$$\int (d+icdx)^3(a+b\arctan(cx))\,dx =$$

$$-\frac{6i\,bc^4d^3x^4\arctan\left(cx\right) + 6i\,ac^4d^3x^4 + 24\,bc^3d^3x^3\arctan\left(cx\right) + 24\,ac^3d^3x^3 - 2i\,bc^3d^3x^3 - 36i\,bc^2d^3x^2\,\mathrm{ar}}{}$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
-1/24*(6*I*b*c^4*d^3*x^4*arctan(c*x) + 6*I*a*c^4*d^3*x^4 + 24*b*c^3*d^3*x^
3*arctan(c*x) + 24*a*c^3*d^3*x^3 - 2*I*b*c^3*d^3*x^3 - 36*I*b*c^2*d^3*x^2*
arctan(c*x) - 36*I*a*c^2*d^3*x^2 - 12*b*c^2*d^3*x^2 - 24*b*c*d^3*x*arctan(
c*x) - 24*a*c*d^3*x + 42*I*b*c*d^3*x + 45*b*d^3*log(c*x + I) + 3*b*d^3*log
(c*x - I))/c
```

## Mupad [B] (verification not implemented)

Time = 1.03 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.47

$$\int (d+icdx)^3(a+b\arctan(cx))\,dx = -\frac{d^3\,(a\,x\,12\mathrm{i}+21\,b\,x+b\,x\,\mathrm{atan}(c\,x)\,12\mathrm{i})\,1\mathrm{i}}{12} - \frac{c^3\,d^3\,(3\,a\,x^4+3\,b\,x^4\,\mathrm{atan}(c\,x))\,1\mathrm{i}}{12} + \frac{d^3\,(21\,b\,\mathrm{atan}(c\,x)+b\,\ln(c^2\,x^2+1)\,12\mathrm{i})\,1\mathrm{i}}{12\,c} + \frac{c\,d^3\,(18\,a\,x^2+18\,b\,x^2\,\mathrm{atan}(c\,x)-b\,x^2\,6\mathrm{i})\,1\mathrm{i}}{12} + \frac{c^2\,d^3\,(a\,x^3\,12\mathrm{i}+b\,x^3+b\,x^3\,\mathrm{atan}(c\,x)\,12\mathrm{i})\,1\mathrm{i}}{12}$$

input
```
int((a + b*atan(c*x))*(d + c*d*x*1i)^3,x)
```

output
```
(d^3*(21*b*atan(c*x) + b*log(c^2*x^2 + 1)*12i)*1i)/(12*c) - (c^3*d^3*(3*a*
x^4 + 3*b*x^4*atan(c*x))*1i)/12 - (d^3*(a*x*12i + 21*b*x + b*x*atan(c*x)*1
2i)*1i)/12 + (c*d^3*(18*a*x^2 - b*x^2*6i + 18*b*x^2*atan(c*x))*1i)/12 + (c
^2*d^3*(a*x^3*12i + b*x^3 + b*x^3*atan(c*x)*12i)*1i)/12
```

## Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.38

$$\int (d+icdx)^3(a+b\arctan(cx))\,dx$$
$$=\frac{d^3(-3atan(cx)\,b\,c^4i\,x^4-12atan(cx)\,b\,c^3x^3+18atan(cx)\,b\,c^2i\,x^2+12atan(cx)\,bcx+21atan(cx)\,bi-12}{12c}$$

input
```
int((d+I*c*d*x)^3*(a+b*atan(c*x)),x)
```

output
```
(d**3*( - 3*atan(c*x)*b*c**4*i*x**4 - 12*atan(c*x)*b*c**3*x**3 + 18*atan(c
*x)*b*c**2*i*x**2 + 12*atan(c*x)*b*c*x + 21*atan(c*x)*b*i - 12*log(c**2*x*
*2 + 1)*b - 3*a*c**4*i*x**4 - 12*a*c**3*x**3 + 18*a*c**2*i*x**2 + 12*a*c*x
 + b*c**3*i*x**3 + 6*b*c**2*x**2 - 21*b*c*i*x))/(12*c)
```

# 3.24 $\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 170

$$\begin{aligned}\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x}\,dx &= 3iacd^3x+\frac{3}{2}bcd^3x+\frac{1}{6}ibc^2d^3x^2-\frac{3}{2}bd^3\arctan(cx)\\ &\quad+3ibcd^3x\arctan(cx)-\frac{3}{2}c^2d^3x^2(a+b\arctan(cx))\\ &\quad-\frac{1}{3}ic^3d^3x^3(a+b\arctan(cx))+ad^3\log(x)\\ &\quad-\frac{5}{3}ibd^3\log\left(1+c^2x^2\right)+\frac{1}{2}ibd^3\operatorname{PolyLog}(2,-icx)\\ &\quad-\frac{1}{2}ibd^3\operatorname{PolyLog}(2,icx)\end{aligned}$$

output
```
3*I*a*c*d^3*x+3/2*b*c*d^3*x+1/6*I*b*c^2*d^3*x^2-3/2*b*d^3*arctan(c*x)+3*I*
b*c*d^3*x*arctan(c*x)-3/2*c^2*d^3*x^2*(a+b*arctan(c*x))-1/3*I*c^3*d^3*x^3*
(a+b*arctan(c*x))+a*d^3*ln(x)-5/3*I*b*d^3*ln(c^2*x^2+1)+1/2*I*b*d^3*polylo
g(2,-I*c*x)-1/2*I*b*d^3*polylog(2,I*c*x)
```

## Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.82

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x}\,dx = -\frac{1}{6}id^3\big(-18acx+9ibcx-9iac^2x^2-bc^2x^2+2ac^3x^3-9ib\arctan(cx)-18bcx\arctan(cx)-9ibc^2x^2\arctan(cx)+2bc^3x^3\arctan(cx)+6ia\log(x)+10b\log\left(1+c^2x^2\right)-3b\,\mathrm{PolyLog}(2,-icx)+3b\,\mathrm{PolyLog}(2,icx)\big)$$

input
```
Integrate[((d + I*c*d*x)^3*(a + b*ArcTan[c*x]))/x,x]
```

output
```
(-1/6*I)*d^3*(-18*a*c*x + (9*I)*b*c*x - (9*I)*a*c^2*x^2 - b*c^2*x^2 + 2*a*
c^3*x^3 - (9*I)*b*ArcTan[c*x] - 18*b*c*x*ArcTan[c*x] - (9*I)*b*c^2*x^2*Arc
Tan[c*x] + 2*b*c^3*x^3*ArcTan[c*x] + (6*I)*a*Log[x] + 10*b*Log[1 + c^2*x^2
] - 3*b*PolyLog[2, (-I)*c*x] + 3*b*PolyLog[2, I*c*x])
```

## Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x}\,dx$$

$$\Big\downarrow \text{5411}$$

$$\int \left(-ic^3d^3x^2(a+b\arctan(cx))-3c^2d^3x(a+b\arctan(cx))+3icd^3(a+b\arctan(cx))+\frac{d^3(a+b\arctan(cx))}{x}\right)d$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{3}ic^3d^3x^3(a+b\arctan(cx))-\frac{3}{2}c^2d^3x^2(a+b\arctan(cx))+3iacd^3x+ad^3\log(x)-\frac{3}{2}bd^3\arctan(cx)+3ibcd^3x\arctan(cx)+\frac{1}{6}ibc^2d^3x^2-\frac{5}{3}ibd^3\log\left(c^2x^2+1\right)+\frac{1}{2}ibd^3\operatorname{PolyLog}(2,-icx)-\frac{1}{2}ibd^3\operatorname{PolyLog}(2,icx)+\frac{3}{2}bcd^3x$$

input
```
Int[((d + I*c*d*x)^3*(a + b*ArcTan[c*x]))/x,x]
```

output
```
(3*I)*a*c*d^3*x + (3*b*c*d^3*x)/2 + (I/6)*b*c^2*d^3*x^2 - (3*b*d^3*ArcTan[
c*x])/2 + (3*I)*b*c*d^3*x*ArcTan[c*x] - (3*c^2*d^3*x^2*(a + b*ArcTan[c*x])
)/2 - (I/3)*c^3*d^3*x^3*(a + b*ArcTan[c*x]) + a*d^3*Log[x] - ((5*I)/3)*b*d
^3*Log[1 + c^2*x^2] + (I/2)*b*d^3*PolyLog[2, (-I)*c*x] - (I/2)*b*d^3*PolyL
og[2, I*c*x]
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

## Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.97

| method | result |
|---|---|
| parts | $d^3a\left(-\frac{ic^3x^3}{3}-\frac{3c^2x^2}{2}+3icx+\ln(x)\right)+d^3b\left(3i\arctan(cx)\,cx-\frac{i\arctan(cx)c^3x^3}{3}-\frac{3c^2x^2\operatorname{arc}}{2}\right.$ |
| derivativedivides | $d^3a\left(3icx-\frac{ic^3x^3}{3}-\frac{3c^2x^2}{2}+\ln(cx)\right)+d^3b\left(3i\arctan(cx)\,cx-\frac{i\arctan(cx)c^3x^3}{3}-\frac{3c^2x^2\operatorname{arct}}{2}\right.$ |
| default | $d^3a\left(3icx-\frac{ic^3x^3}{3}-\frac{3c^2x^2}{2}+\ln(cx)\right)+d^3b\left(3i\arctan(cx)\,cx-\frac{i\arctan(cx)c^3x^3}{3}-\frac{3c^2x^2\operatorname{arct}}{2}\right.$ |
| risch | $-\frac{d^3b\ln(icx+1)c^3x^3}{6}-\frac{3id^3x^2b\ln(-icx+1)c^2}{4}+\frac{3d^3b\ln(icx+1)cx}{2}+\frac{65id^3b}{18}-\frac{11id^3b\ln(icx+1)}{12}+\frac{3bcd^3x}{2}+$ |

input
```
int((d+I*c*d*x)^3*(a+b*arctan(c*x))/x,x,method=_RETURNVERBOSE)
```

output
```
d^3*a*(-1/3*I*c^3*x^3-3/2*c^2*x^2+3*I*c*x+ln(x))+d^3*b*(3*I*arctan(c*x)*c*
x-1/3*I*arctan(c*x)*c^3*x^3-3/2*c^2*x^2*arctan(c*x)+ln(c*x)*arctan(c*x)+1/
2*I*ln(c*x)*ln(1+I*c*x)-1/2*I*ln(c*x)*ln(1-I*c*x)+1/2*I*dilog(1+I*c*x)-1/2
*I*dilog(1-I*c*x)+3/2*c*x+1/6*I*c^2*x^2-5/3*I*ln(c^2*x^2+1)-3/2*arctan(c*x
))
```

## Fricas [F]

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x}\,dx = \int \frac{(i\,cdx+d)^3(b\arctan(cx)+a)}{x}\,dx$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x,x, algorithm="fricas")
```

output
```
integral(1/2*(-2*I*a*c^3*d^3*x^3 - 6*a*c^2*d^3*x^2 + 6*I*a*c*d^3*x + 2*a*d
^3 + (b*c^3*d^3*x^3 - 3*I*b*c^2*d^3*x^2 - 3*b*c*d^3*x + I*b*d^3)*log(-(c*x
 + I)/(c*x - I)))/x, x)
```

## Sympy [F]

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x}\,dx = -id^3\left(\int(-3ac)\,dx + \int\frac{ia}{x}\,dx + \int ac^3x^2\,dx + \int(-3bc\,\mathrm{atan}(cx))\,dx + \int(-3iac^2x)\,dx + \int\frac{ib\,\mathrm{atan}(cx)}{x}\,dx + \int bc^3x^2\,\mathrm{atan}(cx)\,dx + \int(-3ibc^2x\,\mathrm{atan}(cx))\,dx\right)$$

input
```
integrate((d+I*c*d*x)**3*(a+b*atan(c*x))/x,x)
```

output
```
-I*d**3*(Integral(-3*a*c, x) + Integral(I*a/x, x) + Integral(a*c**3*x**2,
x) + Integral(-3*b*c*atan(c*x), x) + Integral(-3*I*a*c**2*x, x) + Integral
(I*b*atan(c*x)/x, x) + Integral(b*c**3*x**2*atan(c*x), x) + Integral(-3*I*
b*c**2*x*atan(c*x), x))
```

## Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.08

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x}\,dx = -\frac{1}{3}i\,ac^3d^3x^3 - \frac{3}{2}\,ac^2d^3x^2 + \frac{1}{6}i\,bc^2d^3x^2 + 3i\,acd^3x + \frac{3}{2}\,bcd^3x - \frac{1}{12}\,(3\,\pi + 2i)bd^3\log\left(c^2x^2+1\right) + bd^3\arctan(cx)\log(cx) + \frac{3}{2}i\left(2\,cx\arctan(cx) - \log\left(c^2x^2+1\right)\right)bd^3 - \frac{1}{2}i\,bd^3\mathrm{Li}_2(i\,cx+1) + \frac{1}{2}i\,bd^3\mathrm{Li}_2(-i\,cx+1) + ad^3\log(x) + \frac{1}{6}\left(-2i\,bc^3d^3x^3 - 9\,bc^2d^3x^2 - 9\,bd^3\right)\arctan(cx)$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x,x, algorithm="maxima")
```

output
```
-1/3*I*a*c^3*d^3*x^3 - 3/2*a*c^2*d^3*x^2 + 1/6*I*b*c^2*d^3*x^2 + 3*I*a*c*d
^3*x + 3/2*b*c*d^3*x - 1/12*(3*pi + 2*I)*b*d^3*log(c^2*x^2 + 1) + b*d^3*ar
ctan(c*x)*log(c*x) + 3/2*I*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b*d^3 -
1/2*I*b*d^3*dilog(I*c*x + 1) + 1/2*I*b*d^3*dilog(-I*c*x + 1) + a*d^3*log(x
) + 1/6*(-2*I*b*c^3*d^3*x^3 - 9*b*c^2*d^3*x^2 - 9*b*d^3)*arctan(c*x)
```

## Giac [F]

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x}\,dx = \int \frac{(i\,cdx+d)^3(b\arctan\left(cx\right)+a)}{x}\,dx$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x,x, algorithm="giac")
```

output
```
integrate((I*c*d*x + d)^3*(b*arctan(c*x) + a)/x, x)
```

## Mupad [B] (verification not implemented)

Time = 1.17 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.15

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x}\,dx$$
$$= \begin{cases} a\,d^3 \\ a\,d^3\,\ln\left(x\right)-\frac{b\,d^3\,\ln\left(c^2\,x^2+1\right)\,3\mathrm{i}}{2}-\frac{b\,d^3\,\mathrm{Li}_2\left(1-c\,x\,1\mathrm{i}\right)\,1\mathrm{i}}{2}+\frac{b\,d^3\,\mathrm{Li}_2\left(1+c\,x\,1\mathrm{i}\right)\,1\mathrm{i}}{2}-\frac{3\,a\,c^2\,d^3\,x^2}{2}-\frac{a\,c^3\,d^3\,x^3\,1\mathrm{i}}{3}+a\,c\,d^3\,x\,3\mathrm{i}+ \end{cases}$$

input
```
int(((a + b*atan(c*x))*(d + c*d*x*1i)^3)/x,x)
```

output
```
piecewise(c == 0, a*d^3*log(x), c ~= 0, - (b*d^3*log(c^2*x^2 + 1)*3i)/2 +
a*d^3*log(x) - (b*d^3*dilog(- c*x*1i + 1)*1i)/2 + (b*d^3*dilog(c*x*1i + 1)
*1i)/2 - (3*a*c^2*d^3*x^2)/2 - (a*c^3*d^3*x^3*1i)/3 + a*c*d^3*x*3i + (3*b*
c*d^3*x)/2 + (b*c^2*d^3*(x^2/2 - log(c^2*x^2 + 1)/(2*c^2))*1i)/3 - 3*b*c^2
*d^3*atan(c*x)*(1/(2*c^2) + x^2/2) - (b*c^3*d^3*x^3*atan(c*x)*1i)/3 + b*c*
d^3*x*atan(c*x)*3i)
```

## Reduce [F]

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x}\,dx$$
$$= \frac{d^3\left(-2atan(cx)\,b\,c^3i\,x^3-9atan(cx)\,b\,c^2x^2+18atan(cx)\,bcix-9atan(cx)\,b+6\left(\int\frac{atan(cx)}{x}dx\right)b-10\log\right.}{6}$$

input
```
int((d+I*c*d*x)^3*(a+b*atan(c*x))/x,x)
```

output
```
(d**3*( - 2*atan(c*x)*b*c**3*i*x**3 - 9*atan(c*x)*b*c**2*x**2 + 18*atan(c*
x)*b*c*i*x - 9*atan(c*x)*b + 6*int(atan(c*x)/x,x)*b - 10*log(c**2*x**2 + 1
)*b*i + 6*log(x)*a - 2*a*c**3*i*x**3 - 9*a*c**2*x**2 + 18*a*c*i*x + b*c**2
*i*x**2 + 9*b*c*x))/6
```

# 3.25 $\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^2}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 162

$$\begin{aligned}\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^2}\,dx &= -3ac^2d^3x+\frac{1}{2}ibc^2d^3x-\frac{1}{2}ibcd^3\arctan(cx)\\&\quad-3bc^2d^3x\arctan(cx)-\frac{d^3(a+b\arctan(cx))}{x}\\&\quad-\frac{1}{2}ic^3d^3x^2(a+b\arctan(cx))+3iacd^3\log(x)\\&\quad+bcd^3\log(x)+bcd^3\log\left(1+c^2x^2\right)\\&\quad-\frac{3}{2}bcd^3\operatorname{PolyLog}(2,-icx)\\&\quad+\frac{3}{2}bcd^3\operatorname{PolyLog}(2,icx)\end{aligned}$$

output
```
-3*a*c^2*d^3*x+1/2*I*b*c^2*d^3*x-1/2*I*b*c*d^3*arctan(c*x)-3*b*c^2*d^3*x*a
rctan(c*x)-d^3*(a+b*arctan(c*x))/x-1/2*I*c^3*d^3*x^2*(a+b*arctan(c*x))+3*I
*a*c*d^3*ln(x)+b*c*d^3*ln(x)+b*c*d^3*ln(c^2*x^2+1)-3/2*b*c*d^3*polylog(2,-
I*c*x)+3/2*b*c*d^3*polylog(2,I*c*x)
```

## Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.93

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^2}\,dx$$
$$= \frac{d^3(-2a-6ac^2x^2+ibc^2x^2-iac^3x^3-2b\arctan(cx)-ibcx\arctan(cx)-6bc^2x^2\arctan(cx)-ibc^3x^3\operatorname{arct}}{}$$

input
```
Integrate[((d + I*c*d*x)^3*(a + b*ArcTan[c*x]))/x^2,x]
```

output
```
(d^3*(-2*a - 6*a*c^2*x^2 + I*b*c^2*x^2 - I*a*c^3*x^3 - 2*b*ArcTan[c*x] - I
*b*c*x*ArcTan[c*x] - 6*b*c^2*x^2*ArcTan[c*x] - I*b*c^3*x^3*ArcTan[c*x] + (
6*I)*a*c*x*Log[x] + 2*b*c*x*Log[c*x] + 2*b*c*x*Log[1 + c^2*x^2] - 3*b*c*x*
PolyLog[2, (-I)*c*x] + 3*b*c*x*PolyLog[2, I*c*x]))/(2*x)
```

## Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^2}\,dx$$
$$\Big\downarrow \text{5411}$$
$$\int \left(-ic^3d^3x(a+b\arctan(cx))-3c^2d^3(a+b\arctan(cx))+\frac{d^3(a+b\arctan(cx))}{x^2}+\frac{3icd^3(a+b\arctan(cx))}{x}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{1}{2}ic^3d^3x^2(a+b\arctan(cx))-\frac{d^3(a+b\arctan(cx))}{x}-3ac^2d^3x+3iacd^3\log(x)-3bc^2d^3x\arctan(cx)-\frac{1}{2}ibcd^3\arctan(cx)+bcd^3\log\left(c^2x^2+1\right)+\frac{1}{2}ibc^2d^3x-\frac{3}{2}bcd^3\operatorname{PolyLog}(2,-icx)+\frac{3}{2}bcd^3\operatorname{PolyLog}(2,icx)+bcd^3\log(x)$$

input
```
Int[((d + I*c*d*x)^3*(a + b*ArcTan[c*x]))/x^2,x]
```

output
```
-3*a*c^2*d^3*x + (I/2)*b*c^2*d^3*x - (I/2)*b*c*d^3*ArcTan[c*x] - 3*b*c^2*d
^3*x*ArcTan[c*x] - (d^3*(a + b*ArcTan[c*x]))/x - (I/2)*c^3*d^3*x^2*(a + b*
ArcTan[c*x]) + (3*I)*a*c*d^3*Log[x] + b*c*d^3*Log[x] + b*c*d^3*Log[1 + c^2
*x^2] - (3*b*c*d^3*PolyLog[2, (-I)*c*x])/2 + (3*b*c*d^3*PolyLog[2, I*c*x])
/2
```

**Defintions of rubi rules used**

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

## Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.98

| method | result |
|---|---|
| parts | $d^3 a\left(-\frac{i c^3 x^2}{2}-3c^2 x-\frac{1}{x}+3ic\ln(x)\right)+d^3 bc\left(-3cx\arctan(cx)-\frac{i\arctan(cx)c^2x^2}{2}-\ldots\right.$ |
| derivativedivides | $c\left(d^3 a\left(-3cx-\frac{ic^2x^2}{2}-\frac{1}{cx}+3i\ln(cx)\right)+d^3 b\left(-3cx\arctan(cx)-\frac{i\arctan(cx)c^2x^2}{2}-\ldots\right.\right.$ |
| default | $c\left(d^3 a\left(-3cx-\frac{ic^2x^2}{2}-\frac{1}{cx}+3i\ln(cx)\right)+d^3 b\left(-3cx\arctan(cx)-\frac{i\arctan(cx)c^2x^2}{2}-\ldots\right.\right.$ |
| risch | $-\frac{d^3 b c^3\ln(icx+1)x^2}{4}-\frac{ic^3d^3a x^2}{2}+\frac{3d^3bc\ln(icx+1)}{4}-\frac{7icd^3a}{2}-3bcd^3+\frac{d^3bc\ln(icx)}{2}+\frac{ibc^2d^3x}{2}-\ldots$ |

input
```
int((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^2,x,method=_RETURNVERBOSE)
```

output
```
d^3*a*(-1/2*I*c^3*x^2-3*c^2*x-1/x+3*I*c*ln(x))+d^3*b*c*(-3*c*x*arctan(c*x)
-1/2*I*arctan(c*x)*c^2*x^2-1/c/x*arctan(c*x)+3*I*arctan(c*x)*ln(c*x)-3/2*l
n(c*x)*ln(1+I*c*x)+3/2*ln(c*x)*ln(1-I*c*x)-3/2*dilog(1+I*c*x)+3/2*dilog(1-
I*c*x)+1/2*I*c*x+ln(c*x)+ln(c^2*x^2+1)-1/2*I*arctan(c*x))
```

## Fricas [F]

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^2}\,dx = \int \frac{(i\,cdx+d)^3(b\arctan{(cx)}+a)}{x^2}\,dx$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^2,x, algorithm="fricas")
```

output
```
integral(1/2*(-2*I*a*c^3*d^3*x^3 - 6*a*c^2*d^3*x^2 + 6*I*a*c*d^3*x + 2*a*d
^3 + (b*c^3*d^3*x^3 - 3*I*b*c^2*d^3*x^2 - 3*b*c*d^3*x + I*b*d^3)*log(-(c*x
 + I)/(c*x - I)))/x^2, x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)**3*(a+b*atan(c*x))/x**2,x)
```

output
```
Timed out
```

## Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.24

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^2}\,dx$$
$$= -\frac{1}{2}i\,ac^3d^3x^2 - 3\,ac^2d^3x + \frac{1}{2}i\,bc^2d^3x - \frac{3}{4}i\,\pi bcd^3\log\left(c^2x^2+1\right) + 3i\,bcd^3\arctan(cx)\log(cx) - \frac{3}{2}\left(2\,cx\arctan(cx) - \log\left(c^2x^2+1\right)\right)bcd^3 + \frac{3}{2}\,bcd^3\mathrm{Li}_2(i\,cx+1) - \frac{3}{2}\,bcd^3\mathrm{Li}_2(-i\,cx+1) + 3i\,acd^3\log(x) - \frac{1}{2}\left(c\left(\log\left(c^2x^2+1\right) - \log\left(x^2\right)\right) + \frac{2\arctan(cx)}{x}\right)bd^3 - \frac{ad^3}{x} + \frac{1}{2}\left(-i\,bc^3d^3x^2 - i\,bcd^3\right)\arctan(cx)$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^2,x, algorithm="maxima")
```

output
```
-1/2*I*a*c^3*d^3*x^2 - 3*a*c^2*d^3*x + 1/2*I*b*c^2*d^3*x - 3/4*I*pi*b*c*d^
3*log(c^2*x^2 + 1) + 3*I*b*c*d^3*arctan(c*x)*log(c*x) - 3/2*(2*c*x*arctan(
c*x) - log(c^2*x^2 + 1))*b*c*d^3 + 3/2*b*c*d^3*dilog(I*c*x + 1) - 3/2*b*c*
d^3*dilog(-I*c*x + 1) + 3*I*a*c*d^3*log(x) - 1/2*(c*(log(c^2*x^2 + 1) - lo
g(x^2)) + 2*arctan(c*x)/x)*b*d^3 - a*d^3/x + 1/2*(-I*b*c^3*d^3*x^2 - I*b*c
*d^3)*arctan(c*x)
```

## Giac [F]

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^2}\,dx = \int \frac{(i\,cdx+d)^3(b\arctan(cx)+a)}{x^2}\,dx$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^2,x, algorithm="giac")
```

output
```
integrate((I*c*d*x + d)^3*(b*arctan(c*x) + a)/x^2, x)
```

## Mupad [B] (verification not implemented)

Time = 1.02 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.20

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^2}\,dx$$
$$= \begin{cases} & -\frac{a\,d^3}{x} \\ \frac{b\,d^3\left(c^2\,\ln(x)-\frac{c^2\,\ln\left(c^2\,x^2+1\right)}{2}\right)}{c} - \frac{a\,c^3\,d^3\,x^2\,1\mathrm{i}}{2} - \frac{a\,d^3}{x} + \frac{3\,b\,c\,d^3\,(\mathrm{Li}_2(1-c\,x\,1\mathrm{i})-\mathrm{Li}_2(1+c\,x\,1\mathrm{i}))}{2} + \frac{3\,b\,c\,d^3\,\ln(c^2\,x^2+1)}{2} - 3\,a\,c^2\,d^3 & \end{cases}$$

input
```
int(((a + b*atan(c*x))*(d + c*d*x*1i)^3)/x^2,x)
```

output
```
piecewise(c == 0, -(a*d^3)/x, c ~= 0, - (a*d^3)/x - (a*c^3*d^3*x^2*1i)/2 +
 (b*d^3*(c^2*log(x) - (c^2*log(c^2*x^2 + 1))/2))/c + (3*b*c*d^3*(dilog(- c
*x*1i + 1) - dilog(c*x*1i + 1)))/2 + (3*b*c*d^3*log(c^2*x^2 + 1))/2 - 3*a*
c^2*d^3*x + (b*c^2*d^3*x*1i)/2 + a*c*d^3*log(x)*3i - (b*d^3*atan(c*x))/x -
 3*b*c^2*d^3*x*atan(c*x) - b*c^3*d^3*atan(c*x)*(1/(2*c^2) + x^2/2)*1i)
```

## Reduce [F]

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^2}\,dx$$
$$= \frac{d^3\left(-atan(cx)\,b\,c^3i\,x^3 - 6atan(cx)\,b\,c^2x^2 - atan(cx)\,bcix - 2atan(cx)\,b + 6\left(\int \frac{atan(cx)}{x}dx\right)bcix + 2\log(c\right.}{2x}$$

input
```
int((d+I*c*d*x)^3*(a+b*atan(c*x))/x^2,x)
```

output
```
(d**3*( - atan(c*x)*b*c**3*i*x**3 - 6*atan(c*x)*b*c**2*x**2 - atan(c*x)*b*
c*i*x - 2*atan(c*x)*b + 6*int(atan(c*x)/x,x)*b*c*i*x + 2*log(c**2*x**2 + 1
)*b*c*x + 6*log(x)*a*c*i*x + 2*log(x)*b*c*x - a*c**3*i*x**3 - 6*a*c**2*x**
2 - 2*a + b*c**2*i*x**2))/(2*x)
```

# 3.26 $\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^3}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 180

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^3}\,dx = -\frac{bcd^3}{2x} - iac^3d^3x - \frac{1}{2}bc^2d^3\arctan(cx) - ibc^3d^3x\arctan(cx) - \frac{d^3(a+b\arctan(cx))}{2x^2} - \frac{3icd^3(a+b\arctan(cx))}{x} - 3ac^2d^3\log(x) + 3ibc^2d^3\log(x) - ibc^2d^3\log\left(1+c^2x^2\right) - \frac{3}{2}ibc^2d^3\operatorname{PolyLog}(2,-icx) + \frac{3}{2}ibc^2d^3\operatorname{PolyLog}(2,icx)$$

output
```
-1/2*b*c*d^3/x-I*a*c^3*d^3*x-1/2*b*c^2*d^3*arctan(c*x)-I*b*c^3*d^3*x*arcta
n(c*x)-1/2*d^3*(a+b*arctan(c*x))/x^2-3*I*c*d^3*(a+b*arctan(c*x))/x-3*a*c^2
*d^3*ln(x)+3*I*b*c^2*d^3*ln(x)-I*b*c^2*d^3*ln(c^2*x^2+1)-3/2*I*b*c^2*d^3*p
olylog(2,-I*c*x)+3/2*I*b*c^2*d^3*polylog(2,I*c*x)
```

## Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.91

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^3}\,dx = -\frac{id^3(-ia+6acx-ibcx+2ac^3x^3-ib\arctan(cx)+6bcx\arctan(cx)-ibc^2x^2\arctan(cx)+2bc^3x^3\arctan$$

input
```
Integrate[((d + I*c*d*x)^3*(a + b*ArcTan[c*x]))/x^3,x]
```

output
```
((-1/2*I)*d^3*((-I)*a + 6*a*c*x - I*b*c*x + 2*a*c^3*x^3 - I*b*ArcTan[c*x]
+ 6*b*c*x*ArcTan[c*x] - I*b*c^2*x^2*ArcTan[c*x] + 2*b*c^3*x^3*ArcTan[c*x]
- (6*I)*a*c^2*x^2*Log[x] - 6*b*c^2*x^2*Log[c*x] + 2*b*c^2*x^2*Log[1 + c^2*
x^2] + 3*b*c^2*x^2*PolyLog[2, (-I)*c*x] - 3*b*c^2*x^2*PolyLog[2, I*c*x]))/
x^2
```

## Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^3}\,dx$$

$$\Big\downarrow \text{5411}$$

$$\int \left(-ic^3d^3(a+b\arctan(cx))-\frac{3c^2d^3(a+b\arctan(cx))}{x}+\frac{d^3(a+b\arctan(cx))}{x^3}+\frac{3icd^3(a+b\arctan(cx))}{x^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d^3(a+b\arctan(cx))}{2x^2}-\frac{3icd^3(a+b\arctan(cx))}{x}-iac^3d^3x-3ac^2d^3\log(x)-ibc^3d^3x\arctan(cx)-\frac{1}{2}bc^2d^3\arctan(cx)-\frac{3}{2}ibc^2d^3\operatorname{PolyLog}(2,-icx)+\frac{3}{2}ibc^2d^3\operatorname{PolyLog}(2,icx)-ibc^2d^3\log\left(c^2x^2+1\right)+3ibc^2d^3\log(x)-\frac{bcd^3}{2x}$$

input
```
Int[((d + I*c*d*x)^3*(a + b*ArcTan[c*x]))/x^3,x]
```

output
```
-1/2*(b*c*d^3)/x - I*a*c^3*d^3*x - (b*c^2*d^3*ArcTan[c*x])/2 - I*b*c^3*d^3
*x*ArcTan[c*x] - (d^3*(a + b*ArcTan[c*x]))/(2*x^2) - ((3*I)*c*d^3*(a + b*A
rcTan[c*x]))/x - 3*a*c^2*d^3*Log[x] + (3*I)*b*c^2*d^3*Log[x] - I*b*c^2*d^3
*Log[1 + c^2*x^2] - ((3*I)/2)*b*c^2*d^3*PolyLog[2, (-I)*c*x] + ((3*I)/2)*b
*c^2*d^3*PolyLog[2, I*c*x]
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

## Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.96

| method | result |
|---|---|
| parts | $d^3a\left(-ic^3x-\frac{1}{2x^2}-\frac{3ic}{x}-3c^2\ln(x)\right)+d^3b\,c^2\Big(-i\arctan(cx)\,cx-3\ln(cx)\arctan(cx)$ |
| derivativedivides | $c^2\Big(d^3a\left(-icx-3\ln(cx)-\frac{1}{2c^2x^2}-\frac{3i}{cx}\right)+d^3b\Big(-i\arctan(cx)\,cx-3\ln(cx)\arctan(cx)$ |
| default | $c^2\Big(d^3a\left(-icx-3\ln(cx)-\frac{1}{2c^2x^2}-\frac{3i}{cx}\right)+d^3b\Big(-i\arctan(cx)\,cx-3\ln(cx)\arctan(cx)$ |
| risch | $-\frac{d^3b\,c^3\ln(icx+1)x}{2}-\frac{3id^3ca}{x}-ib\,c^2d^3\ln\left(c^2x^2+1\right)-\frac{bc\,d^3}{2x}-ia\,c^3d^3x-\frac{b\,c^2d^3\arctan(cx)}{2}-id^3$ |

input
```
int((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^3,x,method=_RETURNVERBOSE)
```

output
```
d^3*a*(-I*c^3*x-1/2/x^2-3*I*c/x-3*c^2*ln(x))+d^3*b*c^2*(-I*arctan(c*x)*c*x
-3*ln(c*x)*arctan(c*x)-1/2/c^2/x^2*arctan(c*x)-3*I*arctan(c*x)/c/x-3/2*I*l
n(c*x)*ln(1+I*c*x)+3/2*I*ln(c*x)*ln(1-I*c*x)-3/2*I*dilog(1+I*c*x)+3/2*I*di
log(1-I*c*x)-1/2/c/x+3*I*ln(c*x)-I*ln(c^2*x^2+1)-1/2*arctan(c*x))
```

## Fricas [F]

$$\int \frac{(d + icdx)^3(a + b\arctan(cx))}{x^3}\, dx = \int \frac{(i\,cdx + d)^3(b\arctan(cx) + a)}{x^3}\, dx$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^3,x, algorithm="fricas")
```

output
```
integral(1/2*(-2*I*a*c^3*d^3*x^3 - 6*a*c^2*d^3*x^2 + 6*I*a*c*d^3*x + 2*a*d
^3 + (b*c^3*d^3*x^3 - 3*I*b*c^2*d^3*x^2 - 3*b*c*d^3*x + I*b*d^3)*log(-(c*x
 + I)/(c*x - I)))/x^3, x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(d + icdx)^3(a + b\arctan(cx))}{x^3}\, dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)**3*(a+b*atan(c*x))/x**3,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^3}\,dx = \int \frac{(i\,cdx+d)^3(b\arctan{(cx)}+a)}{x^3}\,dx$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^3,x, algorithm="maxima")
```

output
```
-I*a*c^3*d^3*x - 1/2*I*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b*c^2*d^3 -
3*b*c^2*d^3*integrate(arctan(c*x)/x, x) - 3*a*c^2*d^3*log(x) - 3/2*I*(c*(l
og(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b*c*d^3 - 1/2*((c*arctan(c*
x) + 1/x)*c + arctan(c*x)/x^2)*b*d^3 - 3*I*a*c*d^3/x - 1/2*a*d^3/x^2
```

## Giac [F]

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^3}\,dx = \int \frac{(i\,cdx+d)^3(b\arctan{(cx)}+a)}{x^3}\,dx$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^3,x, algorithm="giac")
```

output
```
integrate((I*c*d*x + d)^3*(b*arctan(c*x) + a)/x^3, x)
```

## Mupad [B] (verification not implemented)

Time = 1.00 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.14

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^3}\,dx$$
$$= \begin{cases} & -\frac{a\,d^3}{2\,x^2} \\ b\,d^3\left(c^2\,\ln{(x)}-\frac{c^2\,\ln\left(c^2\,x^2+1\right)}{2}\right)\,3\mathrm{i}+\frac{b\,c^2\,d^3\,\ln\left(c^2\,x^2+1\right)\,1\mathrm{i}}{2}+\frac{b\,c^2\,d^3\,\mathrm{Li_2}(1-c\,x\,1\mathrm{i})\,3\mathrm{i}}{2}-\frac{b\,c^2\,d^3\,\mathrm{Li_2}(1+c\,x\,1\mathrm{i})\,3\mathrm{i}}{2}-\frac{b\,d^3\left(c^3\,\mathrm{at}\right.}{} & \end{cases}$$

input
```
int(((a + b*atan(c*x))*(d + c*d*x*1i)^3)/x^3,x)
```

output
```
piecewise(c == 0, -(a*d^3)/(2*x^2), c ~= 0, b*d^3*(c^2*log(x) - (c^2*log(c
^2*x^2 + 1))/2)*3i + (b*c^2*d^3*log(c^2*x^2 + 1)*1i)/2 + (b*c^2*d^3*dilog(
- c*x*1i + 1)*3i)/2 - (b*c^2*d^3*dilog(c*x*1i + 1)*3i)/2 - (b*d^3*(c^3*ata
n(c*x) + c^2/x))/(2*c) - (a*d^3*(c*x*6i + c^3*x^3*2i + 6*c^2*x^2*log(x) +
1))/(2*x^2) - (b*d^3*atan(c*x))/(2*x^2) - (b*c*d^3*atan(c*x)*3i)/x - b*c^3
*d^3*x*atan(c*x)*1i)
```

## Reduce [F]

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^3}\,dx$$
$$= \frac{d^3\left(-2atan(cx)\,b\,c^3i\,x^3 - atan(cx)\,b\,c^2x^2 - 6atan(cx)\,bcix - atan(cx)\,b - 6\left(\int \frac{atan(cx)}{x}dx\right)b\,c^2x^2 - 2\log\right.}{2x^2}$$

input
```
int((d+I*c*d*x)^3*(a+b*atan(c*x))/x^3,x)
```

output
```
(d**3*( - 2*atan(c*x)*b*c**3*i*x**3 - atan(c*x)*b*c**2*x**2 - 6*atan(c*x)*
b*c*i*x - atan(c*x)*b - 6*int(atan(c*x)/x,x)*b*c**2*x**2 - 2*log(c**2*x**2
 + 1)*b*c**2*i*x**2 - 6*log(x)*a*c**2*x**2 + 6*log(x)*b*c**2*i*x**2 - 2*a*
c**3*i*x**3 - 6*a*c*i*x - a - b*c*x))/(2*x**2)
```

# 3.27 $\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^4}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 189

$$\begin{aligned}\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^4}\,dx &= -\frac{bcd^3}{6x^2}-\frac{3ibc^2d^3}{2x}-\frac{3}{2}ibc^3d^3\arctan(cx)\\&\quad-\frac{d^3(a+b\arctan(cx))}{3x^3}-\frac{3icd^3(a+b\arctan(cx))}{2x^2}\\&\quad+\frac{3c^2d^3(a+b\arctan(cx))}{x}-iac^3d^3\log(x)\\&\quad-\frac{10}{3}bc^3d^3\log(x)+\frac{5}{3}bc^3d^3\log\left(1+c^2x^2\right)\\&\quad+\frac{1}{2}bc^3d^3\,\mathrm{PolyLog}(2,-icx)\\&\quad-\frac{1}{2}bc^3d^3\,\mathrm{PolyLog}(2,icx)\end{aligned}$$

output
```
-1/6*b*c*d^3/x^2-3/2*I*b*c^2*d^3/x-3/2*I*b*c^3*d^3*arctan(c*x)-1/3*d^3*(a+
b*arctan(c*x))/x^3-3/2*I*c*d^3*(a+b*arctan(c*x))/x^2+3*c^2*d^3*(a+b*arctan
(c*x))/x-I*a*c^3*d^3*ln(x)-10/3*b*c^3*d^3*ln(x)+5/3*b*c^3*d^3*ln(c^2*x^2+1
)+1/2*b*c^3*d^3*polylog(2,-I*c*x)-1/2*b*c^3*d^3*polylog(2,I*c*x)
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.07 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.90

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^4}\,dx$$
$$= \frac{d^3\left(-2a-9iacx-bcx+18ac^2x^2-2b\arctan(cx)-9ibcx\arctan(cx)+18bc^2x^2\arctan(cx)-9ibc^2x^2\,\mathrm{Hy}\right.}{}$$

input
```
Integrate[((d + I*c*d*x)^3*(a + b*ArcTan[c*x]))/x^4,x]
```

output
```
(d^3*(-2*a - (9*I)*a*c*x - b*c*x + 18*a*c^2*x^2 - 2*b*ArcTan[c*x] - (9*I)*
b*c*x*ArcTan[c*x] + 18*b*c^2*x^2*ArcTan[c*x] - (9*I)*b*c^2*x^2*Hypergeomet
ric2F1[-1/2, 1, 1/2, -(c^2*x^2)] - (6*I)*a*c^3*x^3*Log[x] - 20*b*c^3*x^3*L
og[x] + 10*b*c^3*x^3*Log[1 + c^2*x^2] + 3*b*c^3*x^3*PolyLog[2, (-I)*c*x] -
 3*b*c^3*x^3*PolyLog[2, I*c*x]))/(6*x^3)
```

## Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^4}\,dx$$
$$\Big\downarrow \text{5411}$$
$$\int \left(-\frac{ic^3d^3(a+b\arctan(cx))}{x}-\frac{3c^2d^3(a+b\arctan(cx))}{x^2}+\frac{d^3(a+b\arctan(cx))}{x^4}+\frac{3icd^3(a+b\arctan(cx))}{x^3}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{3c^2d^3(a+b\arctan(cx))}{x}-\frac{d^3(a+b\arctan(cx))}{3x^3}-\frac{3icd^3(a+b\arctan(cx))}{2x^2}-iac^3d^3\log(x)-\frac{3}{2}ibc^3d^3\arctan(cx)+\frac{1}{2}bc^3d^3\operatorname{PolyLog}(2,-icx)-\frac{1}{2}bc^3d^3\operatorname{PolyLog}(2,icx)-\frac{10}{3}bc^3d^3\log(x)-\frac{3ibc^2d^3}{2x}+\frac{5}{3}bc^3d^3\log\left(c^2x^2+1\right)-\frac{bcd^3}{6x^2}$$

input
```
Int[((d + I*c*d*x)^3*(a + b*ArcTan[c*x]))/x^4,x]
```

output
```
-1/6*(b*c*d^3)/x^2 - (((3*I)/2)*b*c^2*d^3)/x - ((3*I)/2)*b*c^3*d^3*ArcTan[
c*x] - (d^3*(a + b*ArcTan[c*x]))/(3*x^3) - (((3*I)/2)*c*d^3*(a + b*ArcTan[
c*x]))/x^2 + (3*c^2*d^3*(a + b*ArcTan[c*x]))/x - I*a*c^3*d^3*Log[x] - (10*
b*c^3*d^3*Log[x])/3 + (5*b*c^3*d^3*Log[1 + c^2*x^2])/3 + (b*c^3*d^3*PolyLo
g[2, (-I)*c*x])/2 - (b*c^3*d^3*PolyLog[2, I*c*x])/2
```

**Defintions of rubi rules used**

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

## Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.97

| method | result |
|---|---|
| parts | $d^3a\left(-\frac{3ic}{2x^2}-\frac{1}{3x^3}+\frac{3c^2}{x}-ic^3\ln\left(x\right)\right)+d^3b\,c^3\left(-\frac{\arctan(cx)}{3c^3x^3}-\frac{3i\arctan(cx)}{2c^2x^2}-i\arctan\left(cx\right)\ln\right.$ |
| derivativedivides | $c^3\left(d^3a\left(-\frac{1}{3c^3x^3}-\frac{3i}{2c^2x^2}-i\ln\left(cx\right)+\frac{3}{cx}\right)+d^3b\left(-\frac{\arctan(cx)}{3c^3x^3}-\frac{3i\arctan(cx)}{2c^2x^2}-i\arctan\left(cx\right)\right.\right.$ |
| default | $c^3\left(d^3a\left(-\frac{1}{3c^3x^3}-\frac{3i}{2c^2x^2}-i\ln\left(cx\right)+\frac{3}{cx}\right)+d^3b\left(-\frac{\arctan(cx)}{3c^3x^3}-\frac{3i\arctan(cx)}{2c^2x^2}-i\arctan\left(cx\right)\right.\right.$ |
| risch | $-\frac{id^3b\ln(-icx+1)}{6x^3}-\frac{11d^3b\,c^3\ln(icx)}{12}+\frac{5\ln(c^2x^2+1)b\,c^3d^3}{3}-id^3c^3a\ln\left(-icx\right)-\frac{3d^3bc\ln(icx+1)}{4x^2}-\frac{bc}{6x}$ |

input
```
int((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^4,x,method=_RETURNVERBOSE)
```

output
```
d^3*a*(-3/2*I*c/x^2-1/3/x^3+3*c^2/x-I*c^3*ln(x))+d^3*b*c^3*(-1/3/c^3/x^3*a
rctan(c*x)-3/2*I*arctan(c*x)/c^2/x^2-I*arctan(c*x)*ln(c*x)+3/c/x*arctan(c*
x)+1/2*ln(c*x)*ln(1+I*c*x)-1/2*ln(c*x)*ln(1-I*c*x)+1/2*dilog(1+I*c*x)-1/2*
dilog(1-I*c*x)+5/3*ln(c^2*x^2+1)-3/2*I*arctan(c*x)-3/2*I/c/x-1/6/c^2/x^2-1
0/3*ln(c*x))
```

## Fricas [F]

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^4}\,dx=\int \frac{(i\,cdx+d)^3(b\arctan\left(cx\right)+a)}{x^4}\,dx$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^4,x, algorithm="fricas")
```

output
```
integral(1/2*(-2*I*a*c^3*d^3*x^3 - 6*a*c^2*d^3*x^2 + 6*I*a*c*d^3*x + 2*a*d
^3 + (b*c^3*d^3*x^3 - 3*I*b*c^2*d^3*x^2 - 3*b*c*d^3*x + I*b*d^3)*log(-(c*x
 + I)/(c*x - I)))/x^4, x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^4}\,dx=\text{Timed out}$$

input
```
integrate((d+I*c*d*x)**3*(a+b*atan(c*x))/x**4,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^4}\,dx = \int \frac{(i\,cdx+d)^3(b\arctan(cx)+a)}{x^4}\,dx$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^4,x, algorithm="maxima")
```

output
```
-I*b*c^3*d^3*integrate(arctan(c*x)/x, x) - I*a*c^3*d^3*log(x) + 3/2*(c*(lo
g(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b*c^2*d^3 - 3/2*I*((c*arctan
(c*x) + 1/x)*c + arctan(c*x)/x^2)*b*c*d^3 + 1/6*((c^2*log(c^2*x^2 + 1) - c
^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*b*d^3 + 3*a*c^2*d^3/x - 3/2*I*
a*c*d^3/x^2 - 1/3*a*d^3/x^3
```

## Giac [F]

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^4}\,dx = \int \frac{(i\,cdx+d)^3(b\arctan(cx)+a)}{x^4}\,dx$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^4,x, algorithm="giac")
```

output
```
integrate((I*c*d*x + d)^3*(b*arctan(c*x) + a)/x^4, x)
```

## Mupad [B] (verification not implemented)

Time = 1.27 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.17

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^4}\,dx$$

$$= \left\{ \frac{b\,c^3\,d^3\,\ln\left(-\frac{3\,c^6\,x^2}{2}-\frac{3\,c^4}{2}\right)}{6} - \frac{b\,c^3\,d^3\,\ln(x)}{3} - \frac{b\,c^3\,d^3\,(\mathrm{Li}_2(1-c\,x\,1\mathrm{i})-\mathrm{Li}_2(1+c\,x\,1\mathrm{i}))}{2} - 3\,b\,c\,d^3\left(c^2\,\ln(x) - \frac{c^2\,\ln(c^2\,x^2+1)}{2}\right) - \right.$$

input
```
int(((a + b*atan(c*x))*(d + c*d*x*1i)^3)/x^4,x)
```

output
```
piecewise(c == 0, -(a*d^3)/(3*x^3), c ~= 0, - (b*d^3*(c^3*atan(c*x) + c^2/
x)*3i)/2 - (b*c^3*d^3*(dilog(- c*x*1i + 1) - dilog(c*x*1i + 1)))/2 - (b*c^
3*d^3*log(x))/3 + (b*c^3*d^3*log(- (3*c^4)/2 - (3*c^6*x^2)/2))/6 - 3*b*c*d
^3*(c^2*log(x) - (c^2*log(c^2*x^2 + 1))/2) - (b*c*d^3)/(6*x^2) - (a*d^3*(c
*x*9i - 18*c^2*x^2 + c^3*x^3*log(x)*6i + 2))/(6*x^3) - (b*d^3*atan(c*x))/(
3*x^3) - (b*c*d^3*atan(c*x)*3i)/(2*x^2) + (3*b*c^2*d^3*atan(c*x))/x)
```

## Reduce [F]

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^4}\,dx = \frac{d^3\left(-9atan(cx)\,b\,c^3i\,x^3+18atan(cx)\,b\,c^2x^2-9atan(cx)\,bcix-2atan(cx)\,b-6\left(\int\frac{atan(cx)}{x}dx\right)b\,c^3i\,x^3+\right.}{6x}$$

input
```
int((d+I*c*d*x)^3*(a+b*atan(c*x))/x^4,x)
```

output
```
(d**3*( - 9*atan(c*x)*b*c**3*i*x**3 + 18*atan(c*x)*b*c**2*x**2 - 9*atan(c*
x)*b*c*i*x - 2*atan(c*x)*b - 6*int(atan(c*x)/x,x)*b*c**3*i*x**3 + 10*log(c
**2*x**2 + 1)*b*c**3*x**3 - 6*log(x)*a*c**3*i*x**3 - 20*log(x)*b*c**3*x**3
 + 18*a*c**2*x**2 - 9*a*c*i*x - 2*a - 9*b*c**2*i*x**2 - b*c*x))/(6*x**3)
```

# 3.28 $\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^5}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 103

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^5}\,dx = -\frac{bcd^3}{12x^3}-\frac{ibc^2d^3}{2x^2}+\frac{7bc^3d^3}{4x} - \frac{d^3(1+icx)^4(a+b\arctan(cx))}{4x^4} - 2ibc^4d^3\log(x)+2ibc^4d^3\log(i+cx)$$

output
```
-1/12*b*c*d^3/x^3-1/2*I*b*c^2*d^3/x^2+7/4*b*c^3*d^3/x-1/4*d^3*(1+I*c*x)^4*
(a+b*arctan(c*x))/x^4-2*I*b*c^4*d^3*ln(x)+2*I*b*c^4*d^3*ln(I+c*x)
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.08 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.60

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^5}\,dx = \frac{d^3\left(-bcx\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-c^2x^2\right)-3i\left(-ia+4acx+6iac^2x^2+2bc^2x^2-4ac^3x^3+b(-i\right.\right.}{}$$

input
```
Integrate[((d + I*c*d*x)^3*(a + b*ArcTan[c*x]))/x^5,x]
```

output
```
(d^3*(-(b*c*x*Hypergeometric2F1[-3/2, 1, -1/2, -(c^2*x^2)]) - (3*I)*((-I)*
a + 4*a*c*x + (6*I)*a*c^2*x^2 + 2*b*c^2*x^2 - 4*a*c^3*x^3 + b*(-I + 4*c*x
+ (6*I)*c^2*x^2 - 4*c^3*x^3)*ArcTan[c*x] + (6*I)*b*c^3*x^3*Hypergeometric2
F1[-1/2, 1, 1/2, -(c^2*x^2)] + 8*b*c^4*x^4*Log[x] - 4*b*c^4*x^4*Log[1 + c^
2*x^2])))/(12*x^4)
```

## Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.83, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {5407, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^5}\,dx$$

$$\Big\downarrow \text{5407}$$

$$-bc\int \frac{d^3(i-cx)^3}{4x^4(cx+i)}dx-\frac{d^3(1+icx)^4(a+b\arctan(cx))}{4x^4}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{4}bcd^3\int \frac{(i-cx)^3}{x^4(cx+i)}dx-\frac{d^3(1+icx)^4(a+b\arctan(cx))}{4x^4}$$

$$\Big\downarrow \text{99}$$

$$-\frac{1}{4}bcd^3\int \left(-\frac{8ic^4}{cx+i}+\frac{8ic^3}{x}+\frac{7c^2}{x^2}-\frac{4ic}{x^3}-\frac{1}{x^4}\right)dx-\frac{d^3(1+icx)^4(a+b\arctan(cx))}{4x^4}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d^3(1+icx)^4(a+b\arctan(cx))}{4x^4}-\frac{1}{4}bcd^3\left(8ic^3\log(x)-8ic^3\log(cx+i)-\frac{7c^2}{x}+\frac{2ic}{x^2}+\frac{1}{3x^3}\right)$$

input
```
Int[((d + I*c*d*x)^3*(a + b*ArcTan[c*x]))/x^5,x]
```

output
```
-1/4*(d^3*(1 + I*c*x)^4*(a + b*ArcTan[c*x]))/x^4 - (b*c*d^3*(1/(3*x^3) + (
(2*I)*c)/x^2 - (7*c^2)/x + (8*I)*c^3*Log[x] - (8*I)*c^3*Log[I + c*x]))/4
```

**Defintions of rubi rules used**

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5407
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Simp[(a
 + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ
[2*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0])
)
```

## Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.43

| method | result |
|---|---|
| parts | $d^3a\left(\frac{3c^2}{2x^2}-\frac{ic}{x^3}+\frac{ic^3}{x}-\frac{1}{4x^4}\right)+d^3b\,c^4\left(\frac{3\arctan(cx)}{2c^2x^2}-\frac{\arctan(cx)}{4c^4x^4}-\frac{i\arctan(cx)}{c^3x^3}+\frac{i\arctan(cx)}{cx}+i\right.$ |
| derivativedivides | $c^4\left(d^3a\left(\frac{3}{2c^2x^2}-\frac{1}{4c^4x^4}-\frac{i}{c^3x^3}+\frac{i}{cx}\right)+d^3b\left(\frac{3\arctan(cx)}{2c^2x^2}-\frac{\arctan(cx)}{4c^4x^4}-\frac{i\arctan(cx)}{c^3x^3}+\frac{i\arctan(cx)}{cx}\right.\right.$ |
| default | $c^4\left(d^3a\left(\frac{3}{2c^2x^2}-\frac{1}{4c^4x^4}-\frac{i}{c^3x^3}+\frac{i}{cx}\right)+d^3b\left(\frac{3\arctan(cx)}{2c^2x^2}-\frac{\arctan(cx)}{4c^4x^4}-\frac{i\arctan(cx)}{c^3x^3}+\frac{i\arctan(cx)}{cx}\right.\right.$ |
| parallelrisch | $-\frac{24ic^4b\,d^3\ln(x)x^4-12ib\,d^3c^3\arctan(cx)x^3+12iac\,d^3x-21d^3b\arctan(cx)x^4c^4+6ix^2b\,c^2d^3+18a\,c^4d^3x^4-12ix^3a\,c^3d^3}{}$ |
| risch | $\frac{(4b\,c^3d^3x^3-6ix^2b\,c^2d^3-4bc\,d^3x+id^3b)\ln(icx+1)}{8x^4}-\frac{id^3(-45b\,c^4\ln(-217cx-217i)x^4+48b\,c^4\ln(-527cx)x^4-3b\,c^4\ln}{}$ |

input
```
int((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^5,x,method=_RETURNVERBOSE)
```

output
```
d^3*a*(3/2*c^2/x^2-I*c/x^3+I*c^3/x-1/4/x^4)+d^3*b*c^4*(3/2/c^2/x^2*arctan(
c*x)-1/4/c^4/x^4*arctan(c*x)-I*arctan(c*x)/c^3/x^3+I*arctan(c*x)/c/x+I*ln(
c^2*x^2+1)+7/4*arctan(c*x)-1/2*I/c^2/x^2-2*I*ln(c*x)-1/12/c^3/x^3+7/4/c/x)
```

## Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(85) = 170$.

Time = 0.12 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.69

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^5}\,dx$$
$$=\frac{-48i\,bc^4d^3x^4\log(x)+45i\,bc^4d^3x^4\log\left(\frac{cx+i}{c}\right)+3i\,bc^4d^3x^4\log\left(\frac{cx-i}{c}\right)-6\,(-4i\,a-7\,b)c^3d^3x^3+12\,(3\,a-}{24\,x^4}$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^5,x, algorithm="fricas")
```

output
```
1/24*(-48*I*b*c^4*d^3*x^4*log(x) + 45*I*b*c^4*d^3*x^4*log((c*x + I)/c) + 3
*I*b*c^4*d^3*x^4*log((c*x - I)/c) - 6*(-4*I*a - 7*b)*c^3*d^3*x^3 + 12*(3*a
 - I*b)*c^2*d^3*x^2 - 2*(12*I*a + b)*c*d^3*x - 6*a*d^3 - 3*(4*b*c^3*d^3*x^
3 - 6*I*b*c^2*d^3*x^2 - 4*b*c*d^3*x + I*b*d^3)*log(-(c*x + I)/(c*x - I)))/
x^4
```

## Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 311 vs. 2(99) = 198.

Time = 17.04 (sec) , antiderivative size = 311, normalized size of antiderivative = 3.02

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^5}\,dx$$
$$= -2ibc^4d^3\log\left(3689b^2c^9d^6x\right) + \frac{ibc^4d^3\log\left(3689b^2c^9d^6x - 3689ib^2c^8d^6\right)}{8}$$
$$+ \frac{15ibc^4d^3\log\left(3689b^2c^9d^6x + 3689ib^2c^8d^6\right)}{8}$$
$$- \frac{3ad^3 + x^3(-12iac^3d^3 - 21bc^3d^3) + x^2(-18ac^2d^3 + 6ibc^2d^3) + x(12iacd^3 + bcd^3)}{12x^4}$$
$$+ \frac{(-4bc^3d^3x^3 + 6ibc^2d^3x^2 + 4bcd^3x - ibd^3)\log(-icx+1)}{8x^4}$$
$$+ \frac{(4bc^3d^3x^3 - 6ibc^2d^3x^2 - 4bcd^3x + ibd^3)\log(icx+1)}{8x^4}$$

input
```
integrate((d+I*c*d*x)**3*(a+b*atan(c*x))/x**5,x)
```

output
```
-2*I*b*c**4*d**3*log(3689*b**2*c**9*d**6*x) + I*b*c**4*d**3*log(3689*b**2*
c**9*d**6*x - 3689*I*b**2*c**8*d**6)/8 + 15*I*b*c**4*d**3*log(3689*b**2*c*
*9*d**6*x + 3689*I*b**2*c**8*d**6)/8 - (3*a*d**3 + x**3*(-12*I*a*c**3*d**3
 - 21*b*c**3*d**3) + x**2*(-18*a*c**2*d**3 + 6*I*b*c**2*d**3) + x*(12*I*a*
c*d**3 + b*c*d**3))/(12*x**4) + (-4*b*c**3*d**3*x**3 + 6*I*b*c**2*d**3*x**
2 + 4*b*c*d**3*x - I*b*d**3)*log(-I*c*x + 1)/(8*x**4) + (4*b*c**3*d**3*x**
3 - 6*I*b*c**2*d**3*x**2 - 4*b*c*d**3*x + I*b*d**3)*log(I*c*x + 1)/(8*x**4
)
```

## Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(85) = 170.

Time = 0.11 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.96

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^5}\,dx$$
$$= \frac{1}{2}i\left(c\left(\log\left(c^2x^2+1\right)-\log\left(x^2\right)\right)+\frac{2\arctan(cx)}{x}\right)bc^3d^3 + \frac{3}{2}\left(\left(c\arctan(cx)+\frac{1}{x}\right)c+\frac{\arctan(cx)}{x^2}\right)bc^2d^3 + \frac{1}{2}i\left(\left(c^2\log\left(c^2x^2+1\right)-c^2\log\left(x^2\right)-\frac{1}{x^2}\right)c-\frac{2\arctan(cx)}{x^3}\right)bcd^3 + \frac{i\,ac^3d^3}{x}+\frac{1}{12}\left(\left(3\,c^3\arctan(cx)+\frac{3\,c^2x^2-1}{x^3}\right)c-\frac{3\arctan(cx)}{x^4}\right)bd^3 + \frac{3\,ac^2d^3}{2\,x^2}-\frac{i\,acd^3}{x^3}-\frac{ad^3}{4\,x^4}$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^5,x, algorithm="maxima")
```

output
```
1/2*I*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b*c^3*d^3 + 3/2*
((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b*c^2*d^3 + 1/2*I*((c^2*log(c^
2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*b*c*d^3 + I*a*c^
3*d^3/x + 1/12*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan(c*x
)/x^4)*b*d^3 + 3/2*a*c^2*d^3/x^2 - I*a*c*d^3/x^3 - 1/4*a*d^3/x^4
```

## Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(85) = 170.

Time = 0.14 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.77

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^5}\,dx =$$
$$-\frac{-45i\,bc^4d^3x^4\log(i\,cx-1)-3i\,bc^4d^3x^4\log(-i\,cx-1)+48i\,bc^4d^3x^4\log(x)-24i\,bc^3d^3x^3\arctan(cx)}{}$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^5,x, algorithm="giac")
```

output
```
-1/24*(-45*I*b*c^4*d^3*x^4*log(I*c*x - 1) - 3*I*b*c^4*d^3*x^4*log(-I*c*x -
 1) + 48*I*b*c^4*d^3*x^4*log(x) - 24*I*b*c^3*d^3*x^3*arctan(c*x) - 24*I*a*
c^3*d^3*x^3 - 42*b*c^3*d^3*x^3 - 36*b*c^2*d^3*x^2*arctan(c*x) - 36*a*c^2*d
^3*x^2 + 12*I*b*c^2*d^3*x^2 + 24*I*b*c*d^3*x*arctan(c*x) + 24*I*a*c*d^3*x
+ 2*b*c*d^3*x + 6*b*d^3*arctan(c*x) + 6*a*d^3)/x^4
```

## Mupad [B] (verification not implemented)

Time = 0.99 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.50

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^5}\,dx =$$

$$-\frac{\frac{d^3\,(3\,a+3\,b\,\mathrm{atan}(c\,x))}{12}+\frac{d^3\,x\,(a\,c\,12\mathrm{i}+b\,c+b\,c\,\mathrm{atan}(c\,x)\,12\mathrm{i})}{12}-\frac{d^3\,x^2\,(18\,a\,c^2+18\,b\,c^2\,\mathrm{atan}(c\,x)-b\,c^2\,6\mathrm{i})}{12}-\frac{d^3\,x^3\,(a\,c^3\,12\mathrm{i}+21\,b\,c^3+b\,c^3\,\mathrm{at}}{12}}{x^4}$$

$$+\frac{d^3\,(21\,b\,c^4\,\mathrm{atan}(c\,x)+b\,c^4\,\ln(c^2\,x^2+1)\,12\mathrm{i}-b\,c^4\,\ln(x)\,24\mathrm{i})}{12}$$

input
```
int(((a + b*atan(c*x))*(d + c*d*x*1i)^3)/x^5,x)
```

output
```
(d^3*(21*b*c^4*atan(c*x) + b*c^4*log(c^2*x^2 + 1)*12i - b*c^4*log(x)*24i))
/12 - ((d^3*(3*a + 3*b*atan(c*x)))/12 + (d^3*x*(a*c*12i + b*c + b*c*atan(c
*x)*12i))/12 - (d^3*x^2*(18*a*c^2 - b*c^2*6i + 18*b*c^2*atan(c*x)))/12 - (
d^3*x^3*(a*c^3*12i + 21*b*c^3 + b*c^3*atan(c*x)*12i))/12)/x^4
```

## Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.46

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^5}\,dx$$

$$=\frac{d^3(21atan(cx)\,b\,c^4x^4+12atan(cx)\,b\,c^3i\,x^3+18atan(cx)\,b\,c^2x^2-12atan(cx)\,bcix-3atan(cx)\,b+12\log}{12}$$

input
```
int((d+I*c*d*x)^3*(a+b*atan(c*x))/x^5,x)
```

output
```
(d**3*(21*atan(c*x)*b*c**4*x**4 + 12*atan(c*x)*b*c**3*i*x**3 + 18*atan(c*x
)*b*c**2*x**2 - 12*atan(c*x)*b*c*i*x - 3*atan(c*x)*b + 12*log(c**2*x**2 +
1)*b*c**4*i*x**4 - 24*log(x)*b*c**4*i*x**4 + 12*a*c**3*i*x**3 + 18*a*c**2*
x**2 - 12*a*c*i*x - 3*a + 21*b*c**3*x**3 - 6*b*c**2*i*x**2 - b*c*x))/(12*x
**4)
```

# 3.29 $\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^6}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 150

$$\begin{aligned}\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^6}\,dx = &-\frac{bcd^3}{20x^4}-\frac{ibc^2d^3}{4x^3}+\frac{3bc^3d^3}{5x^2}+\frac{5ibc^4d^3}{4x} \\ &-\frac{d^3(1+icx)^4(a+b\arctan(cx))}{5x^5} \\ &+\frac{icd^3(1+icx)^4(a+b\arctan(cx))}{20x^4} \\ &+\frac{6}{5}bc^5d^3\log(x)-\frac{6}{5}bc^5d^3\log(i+cx)\end{aligned}$$

output
```
-1/20*b*c*d^3/x^4-1/4*I*b*c^2*d^3/x^3+3/5*b*c^3*d^3/x^2+5/4*I*b*c^4*d^3/x-
1/5*d^3*(1+I*c*x)^4*(a+b*arctan(c*x))/x^5+1/20*I*c*d^3*(1+I*c*x)^4*(a+b*ar
ctan(c*x))/x^4+6/5*b*c^5*d^3*ln(x)-6/5*b*c^5*d^3*ln(I+c*x)
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.23

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^6}\,dx$$
$$= \frac{d^3(-4a-15iacx-bcx+20ac^2x^2+10iac^3x^3+12bc^3x^3-4b\arctan(cx)-15ibcx\arctan(cx)+20bc^2x^2}{}$$

input
```
Integrate[((d + I*c*d*x)^3*(a + b*ArcTan[c*x]))/x^6,x]
```

output
```
(d^3*(-4*a - (15*I)*a*c*x - b*c*x + 20*a*c^2*x^2 + (10*I)*a*c^3*x^3 + 12*b
*c^3*x^3 - 4*b*ArcTan[c*x] - (15*I)*b*c*x*ArcTan[c*x] + 20*b*c^2*x^2*ArcTa
n[c*x] + (10*I)*b*c^3*x^3*ArcTan[c*x] - (5*I)*b*c^2*x^2*Hypergeometric2F1[
-3/2, 1, -1/2, -(c^2*x^2)] + (10*I)*b*c^4*x^4*Hypergeometric2F1[-1/2, 1, 1
/2, -(c^2*x^2)] + 24*b*c^5*x^5*Log[x] - 12*b*c^5*x^5*Log[1 + c^2*x^2]))/(2
0*x^5)
```

## Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.81, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {5407, 27, 165, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^6}\,dx$$

$$\Big\downarrow \text{ 5407}$$

$$-bc\int -\frac{d^3(icx+1)^3(cx+4i)}{20x^5(cx+i)}dx-\frac{d^3(1+icx)^4(a+b\arctan(cx))}{5x^5}+\frac{icd^3(1+icx)^4(a+b\arctan(cx))}{20x^4}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{20}bcd^3\int\frac{(icx+1)^3(cx+4i)}{x^5(cx+i)}dx-\frac{d^3(1+icx)^4(a+b\arctan(cx))}{5x^5}+\frac{icd^3(1+icx)^4(a+b\arctan(cx))}{20x^4}$$

$$\Big\downarrow \text{165}$$

$$\frac{1}{20}bcd^3\int\left(-\frac{24c^5}{cx+i}+\frac{24c^4}{x}-\frac{25ic^3}{x^2}-\frac{24c^2}{x^3}+\frac{15ic}{x^4}+\frac{4}{x^5}\right)dx-\frac{d^3(1+icx)^4(a+b\arctan(cx))}{5x^5}+\frac{icd^3(1+icx)^4(a+b\arctan(cx))}{20x^4}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d^3(1+icx)^4(a+b\arctan(cx))}{5x^5}+\frac{icd^3(1+icx)^4(a+b\arctan(cx))}{20x^4}+\frac{1}{20}bcd^3\left(24c^4\log(x)-24c^4\log(cx+i)+\frac{25ic^3}{x}+\frac{12c^2}{x^2}-\frac{5ic}{x^3}-\frac{1}{x^4}\right)$$

input
```
Int[((d + I*c*d*x)^3*(a + b*ArcTan[c*x]))/x^6,x]
```

output
```
-1/5*(d^3*(1 + I*c*x)^4*(a + b*ArcTan[c*x]))/x^5 + ((I/20)*c*d^3*(1 + I*c*
x)^4*(a + b*ArcTan[c*x]))/x^4 + (b*c*d^3*(-x^(-4) - ((5*I)*c)/x^3 + (12*c^
2)/x^2 + ((25*I)*c^3)/x + 24*c^4*Log[x] - 24*c^4*Log[I + c*x]))/20
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 165
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*
x)^n*(e + f*x)^p*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x]
 && (IntegersQ[m, n, p] || (IGtQ[n, 0] && IGtQ[p, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5407
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Simp[(a
 + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ
[2*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0])
)
```

## Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.02

| method | result |
|---|---|
| parts | $d^3 a\left(\frac{ic^3}{2x^2}+\frac{c^2}{x^3}-\frac{1}{5x^5}-\frac{3ic}{4x^4}\right)+d^3 b\, c^5\left(\frac{i\arctan(cx)}{2c^2x^2}-\frac{3i\arctan(cx)}{4c^4x^4}-\frac{\arctan(cx)}{5c^5x^5}+\frac{\arctan(cx)}{c^3x^3}-\right.$ |
| derivativedivides | $c^5\left(d^3 a\left(\frac{i}{2c^2x^2}-\frac{3i}{4c^4x^4}-\frac{1}{5c^5x^5}+\frac{1}{c^3x^3}\right)+d^3 b\left(\frac{i\arctan(cx)}{2c^2x^2}-\frac{3i\arctan(cx)}{4c^4x^4}-\frac{\arctan(cx)}{5c^5x^5}+\frac{\arctan(}{c^3x^3}\right.\right.$ |
| default | $c^5\left(d^3 a\left(\frac{i}{2c^2x^2}-\frac{3i}{4c^4x^4}-\frac{1}{5c^5x^5}+\frac{1}{c^3x^3}\right)+d^3 b\left(\frac{i\arctan(cx)}{2c^2x^2}-\frac{3i\arctan(cx)}{4c^4x^4}-\frac{\arctan(cx)}{5c^5x^5}+\frac{\arctan(}{c^3x^3}\right.\right.$ |
| parallelrisch | $\frac{10ib\,d^3c^3\arctan(cx)x^3-5ix^2b\,c^2d^3+24c^5d^3b\ln(x)x^5-12c^5d^3b\ln(c^2x^2+1)x^5-12b\,c^5d^3x^5-15iac\,d^3x+10ix^3a\,c^3d^3+}{}$ |
| risch | $\frac{(10b\,c^3d^3x^3-20ix^2b\,c^2d^3-15bc\,d^3x+4id^3b)\ln(icx+1)}{40x^5}-\frac{d^3(49b\,c^5\ln(-cx-i)x^5-b\,c^5\ln(cx-i)x^5-48b\,c^5\ln(-x)x^5}{}$ |

input
```
int((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^6,x,method=_RETURNVERBOSE)
```

output
```
d^3*a*(1/2*I*c^3/x^2+c^2/x^3-1/5/x^5-3/4*I*c/x^4)+d^3*b*c^5*(1/2*I*arctan(
c*x)/c^2/x^2-3/4*I*arctan(c*x)/c^4/x^4-1/5/c^5/x^5*arctan(c*x)+1/c^3/x^3*a
rctan(c*x)-1/4*I/c^3/x^3+5/4*I/c/x-1/20/c^4/x^4+3/5/c^2/x^2+6/5*ln(c*x)-3/
5*ln(c^2*x^2+1)+5/4*I*arctan(c*x))
```

## Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.23

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^6}\,dx$$
$$=\frac{48\,bc^5d^3x^5\log(x)-49\,bc^5d^3x^5\log\left(\frac{cx+i}{c}\right)+bc^5d^3x^5\log\left(\frac{cx-i}{c}\right)+50i\,bc^4d^3x^4-4\,(-5i\,a-6\,b)c^3d^3x^3+10}{40}$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^6,x, algorithm="fricas")
```

output
```
1/40*(48*b*c^5*d^3*x^5*log(x) - 49*b*c^5*d^3*x^5*log((c*x + I)/c) + b*c^5*
d^3*x^5*log((c*x - I)/c) + 50*I*b*c^4*d^3*x^4 - 4*(-5*I*a - 6*b)*c^3*d^3*x
^3 + 10*(4*a - I*b)*c^2*d^3*x^2 - 2*(15*I*a + b)*c*d^3*x - 8*a*d^3 - (10*b
*c^3*d^3*x^3 - 20*I*b*c^2*d^3*x^2 - 15*b*c*d^3*x + 4*I*b*d^3)*log(-(c*x +
I)/(c*x - I)))/x^5
```

## Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(144) = 288$.

Time = 38.67 (sec) , antiderivative size = 326, normalized size of antiderivative = 2.17

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^6}\,dx = \frac{6bc^5d^3\log\left(113975b^2c^{11}d^6x\right)}{5} + \frac{bc^5d^3\log\left(113975b^2c^{11}d^6x-113975ib^2c^{10}d^6\right)}{40} - \frac{49bc^5d^3\log\left(113975b^2c^{11}d^6x+113975ib^2c^{10}d^6\right)}{40} + \frac{(-10bc^3d^3x^3+20ibc^2d^3x^2+15bcd^3x-4ibd^3)\log\left(-icx+1\right)}{40x^5} + \frac{(10bc^3d^3x^3-20ibc^2d^3x^2-15bcd^3x+4ibd^3)\log\left(icx+1\right)}{40x^5} - \frac{4ad^3-25ibc^4d^3x^4+x^3(-10iac^3d^3-12bc^3d^3)+x^2(-20ac^2d^3+5ibc^2d^3)+x(15iacd^3+bcd^3)}{20x^5}$$

input
```
integrate((d+I*c*d*x)**3*(a+b*atan(c*x))/x**6,x)
```

output
```
6*b*c**5*d**3*log(113975*b**2*c**11*d**6*x)/5 + b*c**5*d**3*log(113975*b**
2*c**11*d**6*x - 113975*I*b**2*c**10*d**6)/40 - 49*b*c**5*d**3*log(113975*
b**2*c**11*d**6*x + 113975*I*b**2*c**10*d**6)/40 + (-10*b*c**3*d**3*x**3 +
 20*I*b*c**2*d**3*x**2 + 15*b*c*d**3*x - 4*I*b*d**3)*log(-I*c*x + 1)/(40*x
**5) + (10*b*c**3*d**3*x**3 - 20*I*b*c**2*d**3*x**2 - 15*b*c*d**3*x + 4*I*
b*d**3)*log(I*c*x + 1)/(40*x**5) - (4*a*d**3 - 25*I*b*c**4*d**3*x**4 + x**
3*(-10*I*a*c**3*d**3 - 12*b*c**3*d**3) + x**2*(-20*a*c**2*d**3 + 5*I*b*c**
2*d**3) + x*(15*I*a*c*d**3 + b*c*d**3))/(20*x**5)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.49

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^6}\,dx$$
$$= \frac{1}{2}i\left(\left(c\arctan(cx)+\frac{1}{x}\right)c+\frac{\arctan(cx)}{x^2}\right)bc^3d^3$$
$$-\frac{1}{2}\left(\left(c^2\log\left(c^2x^2+1\right)-c^2\log\left(x^2\right)-\frac{1}{x^2}\right)c-\frac{2\arctan(cx)}{x^3}\right)bc^2d^3$$
$$+\frac{1}{4}i\left(\left(3c^3\arctan(cx)+\frac{3c^2x^2-1}{x^3}\right)c-\frac{3\arctan(cx)}{x^4}\right)bcd^3$$
$$-\frac{1}{20}\left(\left(2c^4\log\left(c^2x^2+1\right)-2c^4\log\left(x^2\right)-\frac{2c^2x^2-1}{x^4}\right)c+\frac{4\arctan(cx)}{x^5}\right)bd^3$$
$$+\frac{i\,ac^3d^3}{2x^2}+\frac{ac^2d^3}{x^3}-\frac{3i\,acd^3}{4x^4}-\frac{ad^3}{5x^5}$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^6,x, algorithm="maxima")
```

output
```
1/2*I*((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b*c^3*d^3 - 1/2*((c^2*lo
g(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*b*c^2*d^3 +
1/4*I*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan(c*x)/x^4)*b*
c*d^3 - 1/20*((2*c^4*log(c^2*x^2 + 1) - 2*c^4*log(x^2) - (2*c^2*x^2 - 1)/x
^4)*c + 4*arctan(c*x)/x^5)*b*d^3 + 1/2*I*a*c^3*d^3/x^2 + a*c^2*d^3/x^3 - 3
/4*I*a*c*d^3/x^4 - 1/5*a*d^3/x^5
```

## Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.28

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^6}\,dx =$$
$$-\frac{49\,bc^5d^3x^5\log(cx+i)-bc^5d^3x^5\log(cx-i)-48\,bc^5d^3x^5\log(x)-50i\,bc^4d^3x^4-20i\,bc^3d^3x^3\arctan(c}{}$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^6,x, algorithm="giac")
```

output
```
-1/40*(49*b*c^5*d^3*x^5*log(c*x + I) - b*c^5*d^3*x^5*log(c*x - I) - 48*b*c
^5*d^3*x^5*log(x) - 50*I*b*c^4*d^3*x^4 - 20*I*b*c^3*d^3*x^3*arctan(c*x) -
20*I*a*c^3*d^3*x^3 - 24*b*c^3*d^3*x^3 - 40*b*c^2*d^3*x^2*arctan(c*x) - 40*
a*c^2*d^3*x^2 + 10*I*b*c^2*d^3*x^2 + 30*I*b*c*d^3*x*arctan(c*x) + 30*I*a*c
*d^3*x + 2*b*c*d^3*x + 8*b*d^3*arctan(c*x) + 8*a*d^3)/x^5
```

## Mupad [B] (verification not implemented)

Time = 1.31 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.16

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^6}\,dx$$
$$= \frac{d^3\left(24\,b\,c^5\,\ln\left(x\right)-12\,b\,c^5\,\ln\left(c^2\,x^2+1\right)+b\,c^4\,\mathrm{atan}\left(x\,\sqrt{c^2}\right)\,\sqrt{c^2}\,25\mathrm{i}\right)}{20}$$
$$+\frac{-\frac{d^3\,\left(4\,a+4\,b\,\mathrm{atan}\left(c\,x\right)\right)}{20}-\frac{d^3\,x\,\left(a\,c\,15\mathrm{i}+b\,c+b\,c\,\mathrm{atan}\left(c\,x\right)\,15\mathrm{i}\right)}{20}+\frac{d^3\,x^3\,\left(a\,c^3\,10\mathrm{i}+12\,b\,c^3+b\,c^3\,\mathrm{atan}\left(c\,x\right)\,10\mathrm{i}\right)}{20}+\frac{d^3\,x^2\,\left(20\,a\,c^2+20\,b\,c^2\,\mathrm{atan}\right.}{20}}{x^5}$$

input
```
int(((a + b*atan(c*x))*(d + c*d*x*1i)^3)/x^6,x)
```

output
```
(d^3*(24*b*c^5*log(x) - 12*b*c^5*log(c^2*x^2 + 1) + b*c^4*atan(x*(c^2)^(1/
2))*(c^2)^(1/2)*25i))/20 + ((d^3*x^3*(a*c^3*10i + 12*b*c^3 + b*c^3*atan(c*
x)*10i))/20 - (d^3*x*(a*c*15i + b*c + b*c*atan(c*x)*15i))/20 - (d^3*(4*a +
 4*b*atan(c*x)))/20 + (d^3*x^2*(20*a*c^2 - b*c^2*5i + 20*b*c^2*atan(c*x)))
/20 + (b*c^4*d^3*x^4*5i)/4)/x^5
```

## Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.06

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^6}\,dx$$
$$= \frac{d^3(25atan(cx)\,b\,c^5i\,x^5+10atan(cx)\,b\,c^3i\,x^3+20atan(cx)\,b\,c^2x^2-15atan(cx)\,bcix-4atan(cx)\,b-12\,\mathrm{lo}}{}$$

input
```
int((d+I*c*d*x)^3*(a+b*atan(c*x))/x^6,x)
```

output
```
(d**3*(25*atan(c*x)*b*c**5*i*x**5 + 10*atan(c*x)*b*c**3*i*x**3 + 20*atan(c
*x)*b*c**2*x**2 - 15*atan(c*x)*b*c*i*x - 4*atan(c*x)*b - 12*log(c**2*x**2
+ 1)*b*c**5*x**5 + 24*log(x)*b*c**5*x**5 + 10*a*c**3*i*x**3 + 20*a*c**2*x*
*2 - 15*a*c*i*x - 4*a + 25*b*c**4*i*x**4 + 12*b*c**3*x**3 - 5*b*c**2*i*x**
2 - b*c*x))/(20*x**5)
```

# 3.30 $\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^7}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 209

$$\begin{aligned}\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^7}\,dx = & -\frac{bcd^3}{30x^5}-\frac{3ibc^2d^3}{20x^4}+\frac{11bc^3d^3}{36x^3}+\frac{7ibc^4d^3}{15x^2} \\ & -\frac{11bc^5d^3}{12x}-\frac{11}{12}bc^6d^3\arctan(cx) \\ & -\frac{d^3(a+b\arctan(cx))}{6x^6}-\frac{3icd^3(a+b\arctan(cx))}{5x^5} \\ & +\frac{3c^2d^3(a+b\arctan(cx))}{4x^4} \\ & +\frac{ic^3d^3(a+b\arctan(cx))}{3x^3} \\ & +\frac{14}{15}ibc^6d^3\log(x)-\frac{7}{15}ibc^6d^3\log\left(1+c^2x^2\right)\end{aligned}$$

output
```
-1/30*b*c*d^3/x^5-3/20*I*b*c^2*d^3/x^4+11/36*b*c^3*d^3/x^3+7/15*I*b*c^4*d^
3/x^2-11/12*b*c^5*d^3/x-11/12*b*c^6*d^3*arctan(c*x)-1/6*d^3*(a+b*arctan(c*
x))/x^6-3/5*I*c*d^3*(a+b*arctan(c*x))/x^5+3/4*c^2*d^3*(a+b*arctan(c*x))/x^
4+1/3*I*c^3*d^3*(a+b*arctan(c*x))/x^3+14/15*I*b*c^6*d^3*ln(x)-7/15*I*b*c^6
*d^3*ln(c^2*x^2+1)
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.90

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^7}\,dx$$
$$=\frac{d^3\left(-2bcx\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},-c^2x^2\right)+i\left(10ia-36acx-45iac^2x^2-9bc^2x^2+20ac^3x^3+2\right.\right.}{}$$

input
```
Integrate[((d + I*c*d*x)^3*(a + b*ArcTan[c*x]))/x^7,x]
```

output
```
(d^3*(-2*b*c*x*Hypergeometric2F1[-5/2, 1, -3/2, -(c^2*x^2)] + I*((10*I)*a
- 36*a*c*x - (45*I)*a*c^2*x^2 - 9*b*c^2*x^2 + 20*a*c^3*x^3 + 28*b*c^4*x^4
+ (10*I)*b*ArcTan[c*x] - 36*b*c*x*ArcTan[c*x] - (45*I)*b*c^2*x^2*ArcTan[c*
x] + 20*b*c^3*x^3*ArcTan[c*x] - (15*I)*b*c^3*x^3*Hypergeometric2F1[-3/2, 1
, -1/2, -(c^2*x^2)] + 56*b*c^6*x^6*Log[x] - 28*b*c^6*x^6*Log[1 + c^2*x^2])
))/(60*x^6)
```

## Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.86, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {5407, 27, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^7}\,dx$$

$$\Big\downarrow \text{5407}$$

$$-bc\int -\frac{d^3(-20ic^3x^3-45c^2x^2+36icx+10)}{60x^6(c^2x^2+1)}dx+\frac{ic^3d^3(a+b\arctan(cx))}{3x^3}+\frac{3c^2d^3(a+b\arctan(cx))}{4x^4}-\frac{d^3(a+b\arctan(cx))}{6x^6}-\frac{3icd^3(a+b\arctan(cx))}{5x^5}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{60}bcd^3\int \frac{-20ic^3x^3-45c^2x^2+36icx+10}{x^6\left(c^2x^2+1\right)}dx+\frac{ic^3d^3(a+b\arctan(cx))}{3x^3}+\frac{3c^2d^3(a+b\arctan(cx))}{4x^4}-\frac{d^3(a+b\arctan(cx))}{6x^6}-\frac{3icd^3(a+b\arctan(cx))}{5x^5}$$

$$\Big\downarrow \text{2333}$$

$$\frac{1}{60}bcd^3\int \left(-\frac{ic^6}{2(cx-i)}-\frac{111ic^6}{2(cx+i)}+\frac{56ic^5}{x}+\frac{55c^4}{x^2}-\frac{56ic^3}{x^3}-\frac{55c^2}{x^4}+\frac{36ic}{x^5}+\frac{10}{x^6}\right)dx+\frac{ic^3d^3(a+b\arctan(cx))}{3x^3}+\frac{3c^2d^3(a+b\arctan(cx))}{4x^4}-\frac{d^3(a+b\arctan(cx))}{6x^6}-\frac{3icd^3(a+b\arctan(cx))}{5x^5}$$

$$\Big\downarrow \text{2009}$$

$$\frac{ic^3d^3(a+b\arctan(cx))}{3x^3}+\frac{3c^2d^3(a+b\arctan(cx))}{4x^4}-\frac{d^3(a+b\arctan(cx))}{6x^6}-\frac{3icd^3(a+b\arctan(cx))}{5x^5}+\frac{1}{60}bcd^3\left(56ic^5\log(x)-\frac{1}{2}ic^5\log(-cx+i)-\frac{111}{2}ic^5\log(cx+i)-\frac{55c^4}{x}+\frac{28ic^3}{x^2}+\frac{55c^2}{3x^3}-\frac{9ic}{x^4}-\frac{2}{x^5}\right)$$

input
```
Int[((d + I*c*d*x)^3*(a + b*ArcTan[c*x]))/x^7,x]
```

output
```
-1/6*(d^3*(a + b*ArcTan[c*x]))/x^6 - (((3*I)/5)*c*d^3*(a + b*ArcTan[c*x]))
/x^5 + (3*c^2*d^3*(a + b*ArcTan[c*x]))/(4*x^4) + ((I/3)*c^3*d^3*(a + b*Arc
Tan[c*x]))/x^3 + (b*c*d^3*(-2/x^5 - ((9*I)*c)/x^4 + (55*c^2)/(3*x^3) + ((2
8*I)*c^3)/x^2 - (55*c^4)/x + (56*I)*c^5*Log[x] - (I/2)*c^5*Log[I - c*x] -
((111*I)/2)*c^5*Log[I + c*x]))/60
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 5407
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Simp[(a
 + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ
[2*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0])
)
```

## Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.78

| method | result |
|---|---|
| parts | $d^3 a\left(\frac{ic^3}{3x^3}-\frac{3ic}{5x^5}+\frac{3c^2}{4x^4}-\frac{1}{6x^6}\right)+d^3 b\,c^6\left(\frac{3\arctan(cx)}{4c^4x^4}-\frac{3i\arctan(cx)}{5c^5x^5}-\frac{\arctan(cx)}{6c^6x^6}+\frac{i\arctan(cx)}{3c^3x^3}+\right.$ |
| derivativedivides | $c^6\left(d^3 a\left(\frac{3}{4c^4x^4}-\frac{3i}{5c^5x^5}-\frac{1}{6c^6x^6}+\frac{i}{3c^3x^3}\right)+d^3 b\left(\frac{3\arctan(cx)}{4c^4x^4}-\frac{3i\arctan(cx)}{5c^5x^5}-\frac{\arctan(cx)}{6c^6x^6}+\frac{i\arcta}{3c^3}\right.\right.$ |
| default | $c^6\left(d^3 a\left(\frac{3}{4c^4x^4}-\frac{3i}{5c^5x^5}-\frac{1}{6c^6x^6}+\frac{i}{3c^3x^3}\right)+d^3 b\left(\frac{3\arctan(cx)}{4c^4x^4}-\frac{3i\arctan(cx)}{5c^5x^5}-\frac{\arctan(cx)}{6c^6x^6}+\frac{i\arcta}{3c^3}\right.\right.$ |
| parallelrisch | $\frac{60ib\,d^3c^3\arctan(cx)x^3-27ix^2b\,c^2d^3-84ix^6b\,c^6d^3-165b\,c^6d^3\arctan(cx)x^6-165b\,c^5d^3x^5+60ix^3a\,c^3d^3+84ix^4b\,c^4d^3+}{}$ |
| risch | $\frac{(20b\,c^3d^3x^3-45ix^2b\,c^2d^3-36bc\,d^3x+10id^3b)\ln(icx+1)}{120x^6}-\frac{id^3(3b\,c^6\ln(6215cx-6215i)x^6+333b\,c^6\ln(-12265cx-122}{}$ |

input
```
int((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^7,x,method=_RETURNVERBOSE)
```

output
```
d^3*a*(1/3*I*c^3/x^3-3/5*I*c/x^5+3/4*c^2/x^4-1/6/x^6)+d^3*b*c^6*(3/4/c^4/x
^4*arctan(c*x)-3/5*I*arctan(c*x)/c^5/x^5-1/6*arctan(c*x)/c^6/x^6+1/3*I*arc
tan(c*x)/c^3/x^3+14/15*I*ln(c*x)-3/20*I/c^4/x^4+7/15*I/c^2/x^2-1/30/c^5/x^
5+11/36/c^3/x^3-11/12/c/x-7/15*I*ln(c^2*x^2+1)-11/12*arctan(c*x))
```

## Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.95

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^7}\,dx$$
$$= \frac{336i\,bc^6d^3x^6\log(x) - 333i\,bc^6d^3x^6\log\left(\frac{cx+i}{c}\right) - 3i\,bc^6d^3x^6\log\left(\frac{cx-i}{c}\right) - 330\,bc^5d^3x^5 + 168i\,bc^4d^3x^4 - 10(}{}$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^7,x, algorithm="fricas")
```

output
```
1/360*(336*I*b*c^6*d^3*x^6*log(x) - 333*I*b*c^6*d^3*x^6*log((c*x + I)/c) -
 3*I*b*c^6*d^3*x^6*log((c*x - I)/c) - 330*b*c^5*d^3*x^5 + 168*I*b*c^4*d^3*
x^4 - 10*(-12*I*a - 11*b)*c^3*d^3*x^3 + 54*(5*a - I*b)*c^2*d^3*x^2 - 12*(1
8*I*a + b)*c*d^3*x - 60*a*d^3 - 3*(20*b*c^3*d^3*x^3 - 45*I*b*c^2*d^3*x^2 -
 36*b*c*d^3*x + 10*I*b*d^3)*log(-(c*x + I)/(c*x - I)))/x^6
```

## Sympy [A] (verification not implemented)

Time = 54.85 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.66

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^7}\,dx$$
$$= \frac{14ibc^6d^3\log(1385945b^2c^{13}d^6x)}{15} - \frac{ibc^6d^3\log(1385945b^2c^{13}d^6x - 1385945ib^2c^{12}d^6)}{120}$$
$$- \frac{37ibc^6d^3\log(1385945b^2c^{13}d^6x + 1385945ib^2c^{12}d^6)}{40}$$
$$+ \frac{(-20bc^3d^3x^3 + 45ibc^2d^3x^2 + 36bcd^3x - 10ibd^3)\log(-icx+1)}{120x^6}$$
$$+ \frac{(20bc^3d^3x^3 - 45ibc^2d^3x^2 - 36bcd^3x + 10ibd^3)\log(icx+1)}{120x^6}$$
$$- \frac{30ad^3 + 165bc^5d^3x^5 - 84ibc^4d^3x^4 + x^3(-60iac^3d^3 - 55bc^3d^3) + x^2(-135ac^2d^3 + 27ibc^2d^3) + x(108ia}{180x^6}$$

input
```
integrate((d+I*c*d*x)**3*(a+b*atan(c*x))/x**7,x)
```

output
```
14*I*b*c**6*d**3*log(1385945*b**2*c**13*d**6*x)/15 - I*b*c**6*d**3*log(138
5945*b**2*c**13*d**6*x - 1385945*I*b**2*c**12*d**6)/120 - 37*I*b*c**6*d**3
*log(1385945*b**2*c**13*d**6*x + 1385945*I*b**2*c**12*d**6)/40 + (-20*b*c*
*3*d**3*x**3 + 45*I*b*c**2*d**3*x**2 + 36*b*c*d**3*x - 10*I*b*d**3)*log(-I
*c*x + 1)/(120*x**6) + (20*b*c**3*d**3*x**3 - 45*I*b*c**2*d**3*x**2 - 36*b
*c*d**3*x + 10*I*b*d**3)*log(I*c*x + 1)/(120*x**6) - (30*a*d**3 + 165*b*c*
*5*d**3*x**5 - 84*I*b*c**4*d**3*x**4 + x**3*(-60*I*a*c**3*d**3 - 55*b*c**3
*d**3) + x**2*(-135*a*c**2*d**3 + 27*I*b*c**2*d**3) + x*(108*I*a*c*d**3 +
6*b*c*d**3))/(180*x**6)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.19

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^7}\,dx$$
$$= -\frac{1}{6}i\left(\left(c^2\log\left(c^2x^2+1\right)-c^2\log\left(x^2\right)-\frac{1}{x^2}\right)c-\frac{2\arctan\left(cx\right)}{x^3}\right)bc^3d^3$$
$$-\frac{1}{4}\left(\left(3\,c^3\arctan\left(cx\right)+\frac{3\,c^2x^2-1}{x^3}\right)c-\frac{3\arctan\left(cx\right)}{x^4}\right)bc^2d^3$$
$$-\frac{3}{20}i\left(\left(2\,c^4\log\left(c^2x^2+1\right)-2\,c^4\log\left(x^2\right)-\frac{2\,c^2x^2-1}{x^4}\right)c+\frac{4\arctan\left(cx\right)}{x^5}\right)bcd^3$$
$$-\frac{1}{90}\left(\left(15\,c^5\arctan\left(cx\right)+\frac{15\,c^4x^4-5\,c^2x^2+3}{x^5}\right)c+\frac{15\arctan\left(cx\right)}{x^6}\right)bd^3$$
$$+\frac{i\,ac^3d^3}{3\,x^3}+\frac{3\,ac^2d^3}{4\,x^4}-\frac{3i\,acd^3}{5\,x^5}-\frac{ad^3}{6\,x^6}$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^7,x, algorithm="maxima")
```

output
```
-1/6*I*((c^2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^
3)*b*c^3*d^3 - 1/4*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan
(c*x)/x^4)*b*c^2*d^3 - 3/20*I*((2*c^4*log(c^2*x^2 + 1) - 2*c^4*log(x^2) -
(2*c^2*x^2 - 1)/x^4)*c + 4*arctan(c*x)/x^5)*b*c*d^3 - 1/90*((15*c^5*arctan
(c*x) + (15*c^4*x^4 - 5*c^2*x^2 + 3)/x^5)*c + 15*arctan(c*x)/x^6)*b*d^3 +
1/3*I*a*c^3*d^3/x^3 + 3/4*a*c^2*d^3/x^4 - 3/5*I*a*c*d^3/x^5 - 1/6*a*d^3/x^
6
```

## Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.99

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^7}\,dx = -\frac{3i\,bc^6d^3x^6\log(i\,cx+1)+333i\,bc^6d^3x^6\log(-i\,cx+1)-336i\,bc^6d^3x^6\log(x)+330\,bc^5d^3x^5-168i\,bc^4d}{}$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))/x^7,x, algorithm="giac")
```

output
```
-1/360*(3*I*b*c^6*d^3*x^6*log(I*c*x + 1) + 333*I*b*c^6*d^3*x^6*log(-I*c*x
+ 1) - 336*I*b*c^6*d^3*x^6*log(x) + 330*b*c^5*d^3*x^5 - 168*I*b*c^4*d^3*x^
4 - 120*I*b*c^3*d^3*x^3*arctan(c*x) - 120*I*a*c^3*d^3*x^3 - 110*b*c^3*d^3*
x^3 - 270*b*c^2*d^3*x^2*arctan(c*x) - 270*a*c^2*d^3*x^2 + 54*I*b*c^2*d^3*x
^2 + 216*I*b*c*d^3*x*arctan(c*x) + 216*I*a*c*d^3*x + 12*b*c*d^3*x + 60*b*d
^3*arctan(c*x) + 60*a*d^3)/x^6
```

## Mupad [B] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.92

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^7}\,dx = -\frac{\frac{d^3\,(30\,a+30\,b\,\mathrm{atan}(c\,x))}{180}+\frac{d^3\,x\,(a\,c\,108\mathrm{i}+6\,b\,c+b\,c\,\mathrm{atan}(c\,x)\,108\mathrm{i})}{180}-\frac{d^3\,x^3\,(a\,c^3\,60\mathrm{i}+55\,b\,c^3+b\,c^3\,\mathrm{atan}(c\,x)\,60\mathrm{i})}{180}-\frac{d^3\,x^2\,(135\,a\,c^2+135\,b\,c}{18}}{x^6} - \frac{d^3\left(\frac{165\,b\,c^9\,\mathrm{atan}\left(\frac{c^2\,x}{\sqrt{c^2}}\right)}{(c^2)^{3/2}}+b\,c^6\,\ln\left(c^2\,x^2+1\right)\,84\mathrm{i}-b\,c^6\,\ln(x)\,168\mathrm{i}\right)}{180}$$

input
```
int(((a + b*atan(c*x))*(d + c*d*x*1i)^3)/x^7,x)
```

output
```
- ((d^3*(30*a + 30*b*atan(c*x)))/180 + (d^3*x*(a*c*108i + 6*b*c + b*c*atan
(c*x)*108i))/180 - (d^3*x^3*(a*c^3*60i + 55*b*c^3 + b*c^3*atan(c*x)*60i))/
180 - (d^3*x^2*(135*a*c^2 - b*c^2*27i + 135*b*c^2*atan(c*x)))/180 - (b*c^4
*d^3*x^4*7i)/15 + (11*b*c^5*d^3*x^5)/12)/x^6 - (d^3*(b*c^6*log(c^2*x^2 + 1
)*84i - b*c^6*log(x)*168i + (165*b*c^9*atan((c^2*x)/(c^2)^(1/2)))/(c^2)^(3
/2)))/180
```

## Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.81

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))}{x^7}\,dx$$
$$= \frac{d^3(-165atan(cx)\,b\,c^6x^6+60atan(cx)\,b\,c^3i\,x^3+135atan(cx)\,b\,c^2x^2-108atan(cx)\,bcix-30atan(cx)\,b-$$

input
```
int((d+I*c*d*x)^3*(a+b*atan(c*x))/x^7,x)
```

output
```
(d**3*( - 165*atan(c*x)*b*c**6*x**6 + 60*atan(c*x)*b*c**3*i*x**3 + 135*ata
n(c*x)*b*c**2*x**2 - 108*atan(c*x)*b*c*i*x - 30*atan(c*x)*b - 84*log(c**2*
x**2 + 1)*b*c**6*i*x**6 + 168*log(x)*b*c**6*i*x**6 + 60*a*c**3*i*x**3 + 13
5*a*c**2*x**2 - 108*a*c*i*x - 30*a - 165*b*c**5*x**5 + 84*b*c**4*i*x**4 +
55*b*c**3*x**3 - 27*b*c**2*i*x**2 - 6*b*c*x))/(180*x**6)
```

# 3.31 $\int x^3(d+icdx)^4(a+b\arctan(cx))\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 238

$$\int x^3(d+icdx)^4(a+b\arctan(cx))\,dx = \frac{11bd^4x}{8c^3}+\frac{24ibd^4x^2}{35c^2}-\frac{11bd^4x^3}{24c}-\frac{12}{35}ibd^4x^4 + \frac{9}{40}bcd^4x^5+\frac{2}{21}ibc^2d^4x^6-\frac{1}{56}bc^3d^4x^7 - \frac{11bd^4\arctan(cx)}{8c^4}+\frac{1}{4}d^4x^4(a+b\arctan(cx)) + \frac{4}{5}icd^4x^5(a+b\arctan(cx)) - c^2d^4x^6(a+b\arctan(cx)) - \frac{4}{7}ic^3d^4x^7(a+b\arctan(cx)) + \frac{1}{8}c^4d^4x^8(a+b\arctan(cx)) - \frac{24ibd^4\log(1+c^2x^2)}{35c^4}$$

output
```
11/8*b*d^4*x/c^3+24/35*I*b*d^4*x^2/c^2-11/24*b*d^4*x^3/c-12/35*I*b*d^4*x^4
+9/40*b*c*d^4*x^5+2/21*I*b*c^2*d^4*x^6-1/56*b*c^3*d^4*x^7-11/8*b*d^4*arcta
n(c*x)/c^4+1/4*d^4*x^4*(a+b*arctan(c*x))+4/5*I*c*d^4*x^5*(a+b*arctan(c*x))
-c^2*d^4*x^6*(a+b*arctan(c*x))-4/7*I*c^3*d^4*x^7*(a+b*arctan(c*x))+1/8*c^4
*d^4*x^8*(a+b*arctan(c*x))-24/35*I*b*d^4*ln(c^2*x^2+1)/c^4
```

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.22

$$\begin{aligned}\int x^3(d+icdx)^4(a+b\arctan(cx))\,dx &= \frac{11bd^4x}{8c^3}+\frac{24ibd^4x^2}{35c^2}-\frac{11bd^4x^3}{24c}+\frac{1}{4}ad^4x^4\\&-\frac{12}{35}ibd^4x^4+\frac{4}{5}iacd^4x^5+\frac{9}{40}bcd^4x^5\\&-ac^2d^4x^6+\frac{2}{21}ibc^2d^4x^6-\frac{4}{7}iac^3d^4x^7\\&-\frac{1}{56}bc^3d^4x^7+\frac{1}{8}ac^4d^4x^8-\frac{11bd^4\arctan(cx)}{8c^4}\\&+\frac{1}{4}bd^4x^4\arctan(cx)+\frac{4}{5}ibcd^4x^5\arctan(cx)\\&-bc^2d^4x^6\arctan(cx)-\frac{4}{7}ibc^3d^4x^7\arctan(cx)\\&+\frac{1}{8}bc^4d^4x^8\arctan(cx)-\frac{24ibd^4\log(1+c^2x^2)}{35c^4}\end{aligned}$$

input
```
Integrate[x^3*(d + I*c*d*x)^4*(a + b*ArcTan[c*x]),x]
```

output
```
(11*b*d^4*x)/(8*c^3) + (((24*I)/35)*b*d^4*x^2)/c^2 - (11*b*d^4*x^3)/(24*c)
 + (a*d^4*x^4)/4 - ((12*I)/35)*b*d^4*x^4 + ((4*I)/5)*a*c*d^4*x^5 + (9*b*c*
d^4*x^5)/40 - a*c^2*d^4*x^6 + ((2*I)/21)*b*c^2*d^4*x^6 - ((4*I)/7)*a*c^3*d
^4*x^7 - (b*c^3*d^4*x^7)/56 + (a*c^4*d^4*x^8)/8 - (11*b*d^4*ArcTan[c*x])/(
8*c^4) + (b*d^4*x^4*ArcTan[c*x])/4 + ((4*I)/5)*b*c*d^4*x^5*ArcTan[c*x] - b
*c^2*d^4*x^6*ArcTan[c*x] - ((4*I)/7)*b*c^3*d^4*x^7*ArcTan[c*x] + (b*c^4*d^
4*x^8*ArcTan[c*x])/8 - (((24*I)/35)*b*d^4*Log[1 + c^2*x^2])/c^4
```

## Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.83, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {5407, 27, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(d+icdx)^4(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{ 5407}$$

$$-bc\int \frac{d^4x^4\left(35c^4x^4-160ic^3x^3-280c^2x^2+224icx+70\right)}{280\left(c^2x^2+1\right)}dx+\frac{1}{8}c^4d^4x^8(a+b\arctan(cx))-\frac{4}{7}ic^3d^4x^7(a+b\arctan(cx))-c^2d^4x^6(a+b\arctan(cx))+\frac{4}{5}icd^4x^5(a+b\arctan(cx))+\frac{1}{4}d^4x^4(a+b\arctan(cx))$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{1}{280}bcd^4\int \frac{x^4\left(35c^4x^4-160ic^3x^3-280c^2x^2+224icx+70\right)}{c^2x^2+1}dx+\frac{1}{8}c^4d^4x^8(a+b\arctan(cx))-\frac{4}{7}ic^3d^4x^7(a+b\arctan(cx))-c^2d^4x^6(a+b\arctan(cx))+\frac{4}{5}icd^4x^5(a+b\arctan(cx))+\frac{1}{4}d^4x^4(a+b\arctan(cx))$$

$$\Big\downarrow \text{ 2333}$$

$$-\frac{1}{280}bcd^4\int \left(35c^2x^6-160icx^5-315x^4+\frac{384ix^3}{c}+\frac{385x^2}{c^2}-\frac{384ix}{c^3}+\frac{384icx+385}{c^4\left(c^2x^2+1\right)}-\frac{385}{c^4}\right)dx+\frac{1}{8}c^4d^4x^8(a+b\arctan(cx))-\frac{4}{7}ic^3d^4x^7(a+b\arctan(cx))-c^2d^4x^6(a+b\arctan(cx))+\frac{4}{5}icd^4x^5(a+b\arctan(cx))+\frac{1}{4}d^4x^4(a+b\arctan(cx))$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{8}c^4d^4x^8(a+b\arctan(cx))-\frac{4}{7}ic^3d^4x^7(a+b\arctan(cx))-c^2d^4x^6(a+b\arctan(cx))+\frac{4}{5}icd^4x^5(a+b\arctan(cx))+\frac{1}{4}d^4x^4(a+b\arctan(cx))-\frac{1}{280}bcd^4\left(\frac{385\arctan(cx)}{c^5}-\frac{385x}{c^4}-\frac{192ix^2}{c^3}+5c^2x^7+\frac{385x^3}{3c^2}+\frac{192i\log\left(c^2x^2+1\right)}{c^5}-\frac{80}{3}icx^6+\frac{96ix^4}{c}-63x^5\right)$$

input
```
Int[x^3*(d + I*c*d*x)^4*(a + b*ArcTan[c*x]),x]
```

output
```
(d^4*x^4*(a + b*ArcTan[c*x]))/4 + ((4*I)/5)*c*d^4*x^5*(a + b*ArcTan[c*x])
- c^2*d^4*x^6*(a + b*ArcTan[c*x]) - ((4*I)/7)*c^3*d^4*x^7*(a + b*ArcTan[c*
x]) + (c^4*d^4*x^8*(a + b*ArcTan[c*x]))/8 - (b*c*d^4*((-385*x)/c^4 - ((192
*I)*x^2)/c^3 + (385*x^3)/(3*c^2) + ((96*I)*x^4)/c - 63*x^5 - ((80*I)/3)*c*
x^6 + 5*c^2*x^7 + (385*ArcTan[c*x])/c^5 + ((192*I)*Log[1 + c^2*x^2])/c^5))
/280
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 5407
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Simp[(a
 + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ
[2*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0])
)
```

## Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.80

| method | result |
|---|---|
| parts | $d^4a\left(\frac{1}{8}c^4x^8-\frac{4}{7}ic^3x^7-c^2x^6+\frac{4}{5}ic\,x^5+\frac{1}{4}x^4\right)+\frac{d^4b\left(\frac{\arctan(cx)c^8x^8}{8}-\frac{4i\arctan(cx)c^7x^7}{7}-c^6x^6\arctan(c\right.}{}$ |
| derivativedivides | $\frac{d^4a\left(\frac{1}{8}c^8x^8-\frac{4}{7}ic^7x^7-c^6x^6+\frac{4}{5}ic^5x^5+\frac{1}{4}c^4x^4\right)+d^4b\left(\frac{\arctan(cx)c^8x^8}{8}-\frac{4i\arctan(cx)c^7x^7}{7}-c^6x^6\arctan(cx)+\frac{4i\arctan(cx)c^5}{5}\right.}{c^4}$ |
| default | $\frac{d^4a\left(\frac{1}{8}c^8x^8-\frac{4}{7}ic^7x^7-c^6x^6+\frac{4}{5}ic^5x^5+\frac{1}{4}c^4x^4\right)+d^4b\left(\frac{\arctan(cx)c^8x^8}{8}-\frac{4i\arctan(cx)c^7x^7}{7}-c^6x^6\arctan(cx)+\frac{4i\arctan(cx)c^5}{5}\right.}{c^4}$ |
| parallelrisch | $-\frac{-105b\,c^8d^4\arctan(cx)x^8+576ib\,d^4\ln(c^2x^2+1)-105a\,c^8d^4x^8+288ix^4b\,c^4d^4+15b\,c^7d^4x^7-80ix^6b\,c^6d^4+840c^6d^4b\,a}{}$ |
| risch | $-\frac{id^4c^2x^6b\ln(-icx+1)}{2}+\frac{d^4c^4a\,x^8}{8}-\frac{12ib\,d^4x^4}{35}+\frac{2d^4c^3b\,x^7\ln(-icx+1)}{7}-\frac{b\,c^3d^4x^7}{56}+\frac{id^4c^4b\,x^8\ln(-icx+1)}{16}$ |

input
```
int(x^3*(d+I*c*d*x)^4*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

output
```
d^4*a*(1/8*c^4*x^8-4/7*I*c^3*x^7-c^2*x^6+4/5*I*c*x^5+1/4*x^4)+d^4*b/c^4*(1
/8*arctan(c*x)*c^8*x^8-4/7*I*arctan(c*x)*c^7*x^7-c^6*x^6*arctan(c*x)+4/5*I
*arctan(c*x)*c^5*x^5+1/4*c^4*x^4*arctan(c*x)+11/8*c*x-1/56*c^7*x^7+2/21*I*
c^6*x^6+9/40*c^5*x^5-12/35*I*c^4*x^4-11/24*c^3*x^3+24/35*I*c^2*x^2-24/35*I
*ln(c^2*x^2+1)-11/8*arctan(c*x))
```

## Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.97

$$\int x^3(d+icdx)^4(a+b\arctan(cx))\,dx$$
$$=\frac{210\,ac^8d^4x^8-30\,(32i\,a+b)c^7d^4x^7-80\,(21\,a-2i\,b)c^6d^4x^6-42\,(-32i\,a-9\,b)c^5d^4x^5+12\,(35\,a-48i\,b)}{}$$

input
```
integrate(x^3*(d+I*c*d*x)^4*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
1/1680*(210*a*c^8*d^4*x^8 - 30*(32*I*a + b)*c^7*d^4*x^7 - 80*(21*a - 2*I*b
)*c^6*d^4*x^6 - 42*(-32*I*a - 9*b)*c^5*d^4*x^5 + 12*(35*a - 48*I*b)*c^4*d^
4*x^4 - 770*b*c^3*d^4*x^3 + 1152*I*b*c^2*d^4*x^2 + 2310*b*c*d^4*x - 2307*I
*b*d^4*log((c*x + I)/c) + 3*I*b*d^4*log((c*x - I)/c) - 3*(-35*I*b*c^8*d^4*
x^8 - 160*b*c^7*d^4*x^7 + 280*I*b*c^6*d^4*x^6 + 224*b*c^5*d^4*x^5 - 70*I*b
*c^4*d^4*x^4)*log(-(c*x + I)/(c*x - I)))/c^4
```

## Sympy [A] (verification not implemented)

Time = 3.62 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.63

$$\int x^3(d+icdx)^4(a+b\arctan(cx))\,dx = \frac{ac^4d^4x^8}{8} - \frac{11bd^4x^3}{24c} + \frac{24ibd^4x^2}{35c^2} + \frac{11bd^4x}{8c^3} + \frac{bd^4\left(\frac{i\log\left(5893bcd^4x-5893ibd^4\right)}{560} - \frac{1471i\log\left(5893bcd^4x+5893ibd^4\right)}{1260}\right)}{c^4} + x^7\left(-\frac{4iac^3d^4}{7} - \frac{bc^3d^4}{56}\right) + x^6\left(-ac^2d^4 + \frac{2ibc^2d^4}{21}\right) + x^5\cdot\left(\frac{4iacd^4}{5} + \frac{9bcd^4}{40}\right) + x^4\left(\frac{ad^4}{4} - \frac{12ibd^4}{35}\right) + \left(-\frac{ibc^4d^4x^8}{16} - \frac{2bc^3d^4x^7}{7} + \frac{ibc^2d^4x^6}{2} + \frac{2bcd^4x^5}{5} - \frac{ibd^4x^4}{8}\right)\log\left(icx+1\right) + \frac{\left(315ibc^8d^4x^8 + 1440bc^7d^4x^7 - 2520ibc^6d^4x^6 - 2016bc^5d^4x^5 + 630ibc^4d^4x^4 - 1037ibd^4\right)\log\left(-icx+1\right)}{5040c^4}$$

input
```
integrate(x**3*(d+I*c*d*x)**4*(a+b*atan(c*x)),x)
```

output
```
a*c**4*d**4*x**8/8 - 11*b*d**4*x**3/(24*c) + 24*I*b*d**4*x**2/(35*c**2) +
11*b*d**4*x/(8*c**3) + b*d**4*(I*log(5893*b*c*d**4*x - 5893*I*b*d**4)/560
- 1471*I*log(5893*b*c*d**4*x + 5893*I*b*d**4)/1260)/c**4 + x**7*(-4*I*a*c*
*3*d**4/7 - b*c**3*d**4/56) + x**6*(-a*c**2*d**4 + 2*I*b*c**2*d**4/21) + x
**5*(4*I*a*c*d**4/5 + 9*b*c*d**4/40) + x**4*(a*d**4/4 - 12*I*b*d**4/35) +
(-I*b*c**4*d**4*x**8/16 - 2*b*c**3*d**4*x**7/7 + I*b*c**2*d**4*x**6/2 + 2*
b*c*d**4*x**5/5 - I*b*d**4*x**4/8)*log(I*c*x + 1) + (315*I*b*c**8*d**4*x**
8 + 1440*b*c**7*d**4*x**7 - 2520*I*b*c**6*d**4*x**6 - 2016*b*c**5*d**4*x**
5 + 630*I*b*c**4*d**4*x**4 - 1037*I*b*d**4)*log(-I*c*x + 1)/(5040*c**4)
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.42

$$\int x^3(d+icdx)^4(a+b\arctan(cx))\,dx = \frac{1}{8}\,ac^4d^4x^8 - \frac{4}{7}i\,ac^3d^4x^7 - ac^2d^4x^6 + \frac{4}{5}i\,acd^4x^5 + \frac{1}{840}\left(105\,x^8\arctan(cx) - c\left(\frac{15\,c^6x^7 - 21\,c^4x^5 + 35\,c^2x^3 - 105\,x}{c^8} + \frac{105\,\arctan(cx)}{c^9}\right)\right)bc^4d^4 - \frac{1}{21}i\left(12\,x^7\arctan(cx) - c\left(\frac{2\,c^4x^6 - 3\,c^2x^4 + 6\,x^2}{c^6} - \frac{6\,\log(c^2x^2+1)}{c^8}\right)\right)bc^3d^4 + \frac{1}{4}\,ad^4x^4 - \frac{1}{15}\left(15\,x^6\arctan(cx) - c\left(\frac{3\,c^4x^5 - 5\,c^2x^3 + 15\,x}{c^6} - \frac{15\,\arctan(cx)}{c^7}\right)\right)bc^2d^4 + \frac{1}{5}i\left(4\,x^5\arctan(cx) - c\left(\frac{c^2x^4 - 2\,x^2}{c^4} + \frac{2\,\log(c^2x^2+1)}{c^6}\right)\right)bcd^4 + \frac{1}{12}\left(3\,x^4\arctan(cx) - c\left(\frac{c^2x^3 - 3\,x}{c^4} + \frac{3\,\arctan(cx)}{c^5}\right)\right)bd^4$$

input
```
integrate(x^3*(d+I*c*d*x)^4*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
1/8*a*c^4*d^4*x^8 - 4/7*I*a*c^3*d^4*x^7 - a*c^2*d^4*x^6 + 4/5*I*a*c*d^4*x^
5 + 1/840*(105*x^8*arctan(c*x) - c*((15*c^6*x^7 - 21*c^4*x^5 + 35*c^2*x^3
- 105*x)/c^8 + 105*arctan(c*x)/c^9))*b*c^4*d^4 - 1/21*I*(12*x^7*arctan(c*x
) - c*((2*c^4*x^6 - 3*c^2*x^4 + 6*x^2)/c^6 - 6*log(c^2*x^2 + 1)/c^8))*b*c^
3*d^4 + 1/4*a*d^4*x^4 - 1/15*(15*x^6*arctan(c*x) - c*((3*c^4*x^5 - 5*c^2*x
^3 + 15*x)/c^6 - 15*arctan(c*x)/c^7))*b*c^2*d^4 + 1/5*I*(4*x^5*arctan(c*x)
 - c*((c^2*x^4 - 2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*b*c*d^4 + 1/12*(3*x
^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*b*d^4
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.06

$$\int x^3(d+icdx)^4(a+b\arctan(cx))\,dx = \frac{210\,bc^8d^4x^8\arctan(cx)+210\,ac^8d^4x^8-960i\,bc^7d^4x^7\arctan(cx)-960i\,ac^7d^4x^7-30\,bc^7d^4x^7-1680\,bc^6}{}$$

input
```
integrate(x^3*(d+I*c*d*x)^4*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
1/1680*(210*b*c^8*d^4*x^8*arctan(c*x) + 210*a*c^8*d^4*x^8 - 960*I*b*c^7*d^
4*x^7*arctan(c*x) - 960*I*a*c^7*d^4*x^7 - 30*b*c^7*d^4*x^7 - 1680*b*c^6*d^
4*x^6*arctan(c*x) - 1680*a*c^6*d^4*x^6 + 160*I*b*c^6*d^4*x^6 + 1344*I*b*c^
5*d^4*x^5*arctan(c*x) + 1344*I*a*c^5*d^4*x^5 + 378*b*c^5*d^4*x^5 + 420*b*c
^4*d^4*x^4*arctan(c*x) + 420*a*c^4*d^4*x^4 - 576*I*b*c^4*d^4*x^4 - 770*b*c
^3*d^4*x^3 + 1152*I*b*c^2*d^4*x^2 + 2310*b*c*d^4*x + 3*I*b*d^4*log(I*c*x +
 1) - 2307*I*b*d^4*log(-I*c*x + 1))/c^4
```

## Mupad [B] (verification not implemented)

Time = 3.07 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.91

$$\int x^3(d+icdx)^4(a+b\arctan(cx))\,dx = \frac{c^4\,d^4\,(105\,a\,x^8+105\,b\,x^8\,\mathrm{atan}(c\,x))}{840}+\frac{d^4\,(210\,a\,x^4+210\,b\,x^4\,\mathrm{atan}(c\,x)-b\,x^4\,288\mathrm{i})}{840} - \frac{\frac{d^4\,(1155\,b\,\mathrm{atan}(c\,x)+b\,\ln(c^2\,x^2+1)\,576\mathrm{i})}{840}+\frac{11\,b\,c^3\,d^4\,x^3}{24}-\frac{11\,b\,c\,d^4\,x}{8}-\frac{b\,c^2\,d^4\,x^2\,24\mathrm{i}}{35}}{c^4} + \frac{c\,d^4\,(a\,x^5\,672\mathrm{i}+189\,b\,x^5+b\,x^5\,\mathrm{atan}(c\,x)\,672\mathrm{i})}{840} - \frac{c^3\,d^4\,(a\,x^7\,480\mathrm{i}+15\,b\,x^7+b\,x^7\,\mathrm{atan}(c\,x)\,480\mathrm{i})}{840} - \frac{c^2\,d^4\,(840\,a\,x^6+840\,b\,x^6\,\mathrm{atan}(c\,x)-b\,x^6\,80\mathrm{i})}{840}$$

input
```
int(x^3*(a + b*atan(c*x))*(d + c*d*x*1i)^4,x)
```

output
```
(d^4*(210*a*x^4 - b*x^4*288i + 210*b*x^4*atan(c*x)))/840 - ((d^4*(1155*b*a
tan(c*x) + b*log(c^2*x^2 + 1)*576i))/840 - (b*c^2*d^4*x^2*24i)/35 + (11*b*
c^3*d^4*x^3)/24 - (11*b*c*d^4*x)/8)/c^4 + (c^4*d^4*(105*a*x^8 + 105*b*x^8*
atan(c*x)))/840 + (c*d^4*(a*x^5*672i + 189*b*x^5 + b*x^5*atan(c*x)*672i))/
840 - (c^3*d^4*(a*x^7*480i + 15*b*x^7 + b*x^7*atan(c*x)*480i))/840 - (c^2*
d^4*(840*a*x^6 - b*x^6*80i + 840*b*x^6*atan(c*x)))/840
```

## Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.87

$$\int x^3(d+icdx)^4(a+b\arctan(cx))\,dx$$
$$= \frac{d^4(105atan(cx)\,b\,c^8x^8 - 480atan(cx)\,b\,c^7i\,x^7 - 840atan(cx)\,b\,c^6x^6 + 672atan(cx)\,b\,c^5i\,x^5 + 210atan(cx)}{}$$

input
```
int(x^3*(d+I*c*d*x)^4*(a+b*atan(c*x)),x)
```

output
```
(d**4*(105*atan(c*x)*b*c**8*x**8 - 480*atan(c*x)*b*c**7*i*x**7 - 840*atan(
c*x)*b*c**6*x**6 + 672*atan(c*x)*b*c**5*i*x**5 + 210*atan(c*x)*b*c**4*x**4
 - 1155*atan(c*x)*b - 576*log(c**2*x**2 + 1)*b*i + 105*a*c**8*x**8 - 480*a
*c**7*i*x**7 - 840*a*c**6*x**6 + 672*a*c**5*i*x**5 + 210*a*c**4*x**4 - 15*
b*c**7*x**7 + 80*b*c**6*i*x**6 + 189*b*c**5*x**5 - 288*b*c**4*i*x**4 - 385
*b*c**3*x**3 + 576*b*c**2*i*x**2 + 1155*b*c*x))/(840*c**4)
```

# 3.32 $\int x^2(d+icdx)^4(a+b\arctan(cx))\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 193

$$
\begin{aligned}
\int x^2(d+icdx)^4(a+b\arctan(cx))\,dx = {} & \frac{5ibd^4x}{3c^2}-\frac{88bd^4x^2}{105c}-\frac{5}{9}ibd^4x^3 \\
& +\frac{47}{140}bcd^4x^4+\frac{2}{15}ibc^2d^4x^5-\frac{1}{42}bc^3d^4x^6 \\
& +\frac{id^4(1+icx)^5(a+b\arctan(cx))}{5c^3} \\
& -\frac{id^4(1+icx)^6(a+b\arctan(cx))}{3c^3} \\
& +\frac{id^4(1+icx)^7(a+b\arctan(cx))}{7c^3} \\
& +\frac{176bd^4\log(i+cx)}{105c^3}
\end{aligned}
$$

output
```
5/3*I*b*d^4*x/c^2-88/105*b*d^4*x^2/c-5/9*I*b*d^4*x^3+47/140*b*c*d^4*x^4+2/
15*I*b*c^2*d^4*x^5-1/42*b*c^3*d^4*x^6+1/5*I*d^4*(1+I*c*x)^5*(a+b*arctan(c*
x))/c^3-1/3*I*d^4*(1+I*c*x)^6*(a+b*arctan(c*x))/c^3+1/7*I*d^4*(1+I*c*x)^7*
(a+b*arctan(c*x))/c^3+176/105*b*d^4*ln(I+c*x)/c^3
```

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.43

$$\int x^2(d+icdx)^4(a+b\arctan(cx))\,dx=\frac{5ibd^4x}{3c^2}-\frac{88bd^4x^2}{105c}+\frac{1}{3}ad^4x^3-\frac{5}{9}ibd^4x^3+iacd^4x^4+\frac{47}{140}bcd^4x^4-\frac{6}{5}ac^2d^4x^5+\frac{2}{15}ibc^2d^4x^5-\frac{2}{3}iac^3d^4x^6-\frac{1}{42}bc^3d^4x^6+\frac{1}{7}ac^4d^4x^7-\frac{5ibd^4\arctan(cx)}{3c^3}+\frac{1}{3}bd^4x^3\arctan(cx)+ibcd^4x^4\arctan(cx)-\frac{6}{5}bc^2d^4x^5\arctan(cx)-\frac{2}{3}ibc^3d^4x^6\arctan(cx)+\frac{1}{7}bc^4d^4x^7\arctan(cx)+\frac{88bd^4\log(1+c^2x^2)}{105c^3}$$

input
```
Integrate[x^2*(d + I*c*d*x)^4*(a + b*ArcTan[c*x]),x]
```

output
```
(((5*I)/3)*b*d^4*x)/c^2 - (88*b*d^4*x^2)/(105*c) + (a*d^4*x^3)/3 - ((5*I)/
9)*b*d^4*x^3 + I*a*c*d^4*x^4 + (47*b*c*d^4*x^4)/140 - (6*a*c^2*d^4*x^5)/5
+ ((2*I)/15)*b*c^2*d^4*x^5 - ((2*I)/3)*a*c^3*d^4*x^6 - (b*c^3*d^4*x^6)/42
+ (a*c^4*d^4*x^7)/7 - (((5*I)/3)*b*d^4*ArcTan[c*x])/c^3 + (b*d^4*x^3*ArcTa
n[c*x])/3 + I*b*c*d^4*x^4*ArcTan[c*x] - (6*b*c^2*d^4*x^5*ArcTan[c*x])/5 -
((2*I)/3)*b*c^3*d^4*x^6*ArcTan[c*x] + (b*c^4*d^4*x^7*ArcTan[c*x])/7 + (88*
b*d^4*Log[1 + c^2*x^2])/(105*c^3)
```

## Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.88, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {5407, 27, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(d+icdx)^4(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5407}$$

$$-bc\int -\frac{d^4(i-cx)^4\left(-15c^2x^2-5icx+1\right)}{105c^3(cx+i)}dx+\frac{id^4(1+icx)^7(a+b\arctan(cx))}{7c^3}-\frac{id^4(1+icx)^6(a+b\arctan(cx))}{3c^3}+\frac{id^4(1+icx)^5(a+b\arctan(cx))}{5c^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{bd^4\int\frac{(i-cx)^4\left(-15c^2x^2-5icx+1\right)}{cx+i}dx}{105c^2}+\frac{id^4(1+icx)^7(a+b\arctan(cx))}{7c^3}-\frac{id^4(1+icx)^6(a+b\arctan(cx))}{3c^3}+\frac{id^4(1+icx)^5(a+b\arctan(cx))}{5c^3}$$

$$\Big\downarrow \text{1195}$$

$$\frac{bd^4\int\left(-15c^5x^5+70ic^4x^4+141c^3x^3-175ic^2x^2-176cx+\frac{176}{cx+i}+175i\right)dx}{105c^2}+\frac{id^4(1+icx)^7(a+b\arctan(cx))}{7c^3}-\frac{id^4(1+icx)^6(a+b\arctan(cx))}{3c^3}+\frac{id^4(1+icx)^5(a+b\arctan(cx))}{5c^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{id^4(1+icx)^7(a+b\arctan(cx))}{7c^3}-\frac{id^4(1+icx)^6(a+b\arctan(cx))}{3c^3}+\frac{id^4(1+icx)^5(a+b\arctan(cx))}{5c^3}+\frac{bd^4\left(-\frac{5}{2}c^5x^6+14ic^4x^5+\frac{141c^3x^4}{4}-\frac{175}{3}ic^2x^3-88cx^2+\frac{176\log(cx+i)}{c}+175ix\right)}{105c^2}$$

input
```
Int[x^2*(d + I*c*d*x)^4*(a + b*ArcTan[c*x]),x]
```

output
```
((I/5)*d^4*(1 + I*c*x)^5*(a + b*ArcTan[c*x]))/c^3 - ((I/3)*d^4*(1 + I*c*x)
^6*(a + b*ArcTan[c*x]))/c^3 + ((I/7)*d^4*(1 + I*c*x)^7*(a + b*ArcTan[c*x])
)/c^3 + (b*d^4*((175*I)*x - 88*c*x^2 - ((175*I)/3)*c^2*x^3 + (141*c^3*x^4)
/4 + (14*I)*c^4*x^5 - (5*c^5*x^6)/2 + (176*Log[I + c*x])/c))/(105*c^2)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5407
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Simp[(a
 + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ
[2*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0])
)
```

## Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.94

| method | result |
|---|---|
| parts | $d^4a\left(\frac{1}{7}c^4x^7-\frac{2}{3}ic^3x^6-\frac{6}{5}x^5c^2+icx^4+\frac{1}{3}x^3\right)+\frac{d^4b\left(\frac{\arctan(cx)c^7x^7}{7}-\frac{2i\arctan(cx)c^6x^6}{3}-\frac{6c^5x^5\arctan(c}{5}\right.}{}$ |
| derivativedivides | $\frac{d^4a\left(\frac{1}{7}c^7x^7-\frac{2}{3}ic^6x^6-\frac{6}{5}c^5x^5+ic^4x^4+\frac{1}{3}c^3x^3\right)+d^4b\left(\frac{\arctan(cx)c^7x^7}{7}-\frac{2i\arctan(cx)c^6x^6}{3}-\frac{6c^5x^5\arctan(cx)}{5}+i\arctan(cx)c\right.}{c^3}$ |
| default | $\frac{d^4a\left(\frac{1}{7}c^7x^7-\frac{2}{3}ic^6x^6-\frac{6}{5}c^5x^5+ic^4x^4+\frac{1}{3}c^3x^3\right)+d^4b\left(\frac{\arctan(cx)c^7x^7}{7}-\frac{2i\arctan(cx)c^6x^6}{3}-\frac{6c^5x^5\arctan(cx)}{5}+i\arctan(cx)c\right.}{c^3}$ |
| parallelrisch | $-\frac{-180bc^7d^4\arctan(cx)x^7+2100ibd^4\arctan(cx)-180ac^7d^4x^7-1260ibc^4d^4\arctan(cx)x^4+30bc^6d^4x^6-1260ix^4ac^4}{}$ |
| risch | $\frac{id^4c^4bx^7\ln(-icx+1)}{14}-\frac{id^4b\left(15c^4x^7-70ic^3x^6-126x^5c^2+105icx^4+35x^3\right)\ln(icx+1)}{210}+\frac{d^4c^4ax^7}{7}-\frac{5ibd^4x^3}{9}+$ |

input
```
int(x^2*(d+I*c*d*x)^4*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

output
```
d^4*a*(1/7*c^4*x^7-2/3*I*c^3*x^6-6/5*x^5*c^2+I*c*x^4+1/3*x^3)+d^4*b/c^3*(1
/7*arctan(c*x)*c^7*x^7-2/3*I*arctan(c*x)*c^6*x^6-6/5*c^5*x^5*arctan(c*x)+I
*arctan(c*x)*c^4*x^4+1/3*c^3*x^3*arctan(c*x)+5/3*I*c*x-1/42*c^6*x^6+2/15*I
*c^5*x^5+47/140*c^4*x^4-5/9*I*c^3*x^3-88/105*c^2*x^2+88/105*ln(c^2*x^2+1)-
5/3*I*arctan(c*x))
```

## Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.13

$$\int x^2(d+icdx)^4(a+b\arctan(cx))\,dx = \frac{180\,ac^7d^4x^7 - 30\,(28i\,a+b)c^6d^4x^6 - 168\,(9\,a-i\,b)c^5d^4x^5 - 9\,(-140i\,a-47\,b)c^4d^4x^4 + 140\,(3\,a-5i\,b)c}{}$$

input
```
integrate(x^2*(d+I*c*d*x)^4*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
1/1260*(180*a*c^7*d^4*x^7 - 30*(28*I*a + b)*c^6*d^4*x^6 - 168*(9*a - I*b)*
c^5*d^4*x^5 - 9*(-140*I*a - 47*b)*c^4*d^4*x^4 + 140*(3*a - 5*I*b)*c^3*d^4*
x^3 - 1056*b*c^2*d^4*x^2 + 2100*I*b*c*d^4*x + 2106*b*d^4*log((c*x + I)/c)
+ 6*b*d^4*log((c*x - I)/c) - 6*(-15*I*b*c^7*d^4*x^7 - 70*b*c^6*d^4*x^6 + 1
26*I*b*c^5*d^4*x^5 + 105*b*c^4*d^4*x^4 - 35*I*b*c^3*d^4*x^3)*log(-(c*x + I
)/(c*x - I)))/c^3
```

## Sympy [A] (verification not implemented)

Time = 3.20 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.90

$$\int x^2(d+icdx)^4(a+b\arctan(cx))\,dx = \frac{ac^4d^4x^7}{7} - \frac{88bd^4x^2}{105c} + \frac{5ibd^4x}{3c^2} + \frac{bd^4\left(\frac{\log(2299bcd^4x-2299ibd^4)}{210} + \frac{769\log(2299bcd^4x+2299ibd^4)}{560}\right)}{c^3} + x^6\left(-\frac{2iac^3d^4}{3} - \frac{bc^3d^4}{42}\right) + x^5\left(-\frac{6ac^2d^4}{5} + \frac{2ibc^2d^4}{15}\right) + x^4\left(iacd^4 + \frac{47bcd^4}{140}\right) + x^3\left(\frac{ad^4}{3} - \frac{5ibd^4}{9}\right) + \left(-\frac{ibc^4d^4x^7}{14} - \frac{bc^3d^4x^6}{3} + \frac{3ibc^2d^4x^5}{5} + \frac{bcd^4x^4}{2} - \frac{ibd^4x^3}{6}\right)\log(icx+1) + \frac{(120ibc^7d^4x^7 + 560bc^6d^4x^6 - 1008ibc^5d^4x^5 - 840bc^4d^4x^4 + 280ibc^3d^4x^3 + 501bd^4)\log(-icx+1)}{1680c^3}$$

input
```
integrate(x**2*(d+I*c*d*x)**4*(a+b*atan(c*x)),x)
```

output
```
a*c**4*d**4*x**7/7 - 88*b*d**4*x**2/(105*c) + 5*I*b*d**4*x/(3*c**2) + b*d*
*4*(log(2299*b*c*d**4*x - 2299*I*b*d**4)/210 + 769*log(2299*b*c*d**4*x + 2
299*I*b*d**4)/560)/c**3 + x**6*(-2*I*a*c**3*d**4/3 - b*c**3*d**4/42) + x**
5*(-6*a*c**2*d**4/5 + 2*I*b*c**2*d**4/15) + x**4*(I*a*c*d**4 + 47*b*c*d**4
/140) + x**3*(a*d**4/3 - 5*I*b*d**4/9) + (-I*b*c**4*d**4*x**7/14 - b*c**3*
d**4*x**6/3 + 3*I*b*c**2*d**4*x**5/5 + b*c*d**4*x**4/2 - I*b*d**4*x**3/6)*
log(I*c*x + 1) + (120*I*b*c**7*d**4*x**7 + 560*b*c**6*d**4*x**6 - 1008*I*b
*c**5*d**4*x**5 - 840*b*c**4*d**4*x**4 + 280*I*b*c**3*d**4*x**3 + 501*b*d*
*4)*log(-I*c*x + 1)/(1680*c**3)
```

## Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 318 vs. 2(153) = 306.

Time = 0.12 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.65

$$\int x^2(d+icdx)^4(a+b\arctan(cx))\,dx$$
$$= \frac{1}{7}ac^4d^4x^7 - \frac{2}{3}i\,ac^3d^4x^6 - \frac{6}{5}ac^2d^4x^5 + \frac{1}{84}\left(12\,x^7\arctan(cx) - c\left(\frac{2\,c^4x^6 - 3\,c^2x^4 + 6\,x^2}{c^6} - \frac{6\,\log(c^2x^2+1)}{c^8}\right)\right)bc^4d^4 + i\,acd^4x^4 - \frac{2}{45}i\left(15\,x^6\arctan(cx) - c\left(\frac{3\,c^4x^5 - 5\,c^2x^3 + 15\,x}{c^6} - \frac{15\arctan(cx)}{c^7}\right)\right)bc^3d^4 - \frac{3}{10}\left(4\,x^5\arctan(cx) - c\left(\frac{c^2x^4 - 2\,x^2}{c^4} + \frac{2\,\log(c^2x^2+1)}{c^6}\right)\right)bc^2d^4 + \frac{1}{3}ad^4x^3 + \frac{1}{3}i\left(3\,x^4\arctan(cx) - c\left(\frac{c^2x^3 - 3\,x}{c^4} + \frac{3\arctan(cx)}{c^5}\right)\right)bcd^4 + \frac{1}{6}\left(2\,x^3\arctan(cx) - c\left(\frac{x^2}{c^2} - \frac{\log(c^2x^2+1)}{c^4}\right)\right)bd^4$$

input
```
integrate(x^2*(d+I*c*d*x)^4*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
1/7*a*c^4*d^4*x^7 - 2/3*I*a*c^3*d^4*x^6 - 6/5*a*c^2*d^4*x^5 + 1/84*(12*x^7
*arctan(c*x) - c*((2*c^4*x^6 - 3*c^2*x^4 + 6*x^2)/c^6 - 6*log(c^2*x^2 + 1)
/c^8))*b*c^4*d^4 + I*a*c*d^4*x^4 - 2/45*I*(15*x^6*arctan(c*x) - c*((3*c^4*
x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan(c*x)/c^7))*b*c^3*d^4 - 3/10*(4*x^5
*arctan(c*x) - c*((c^2*x^4 - 2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*b*c^2*d
^4 + 1/3*a*d^4*x^3 + 1/3*I*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3
*arctan(c*x)/c^5))*b*c*d^4 + 1/6*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2
*x^2 + 1)/c^4))*b*d^4
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.23

$$\int x^2(d+icdx)^4(a+b\arctan(cx))\,dx$$
$$= \frac{180\,bc^7d^4x^7\arctan(cx) + 180\,ac^7d^4x^7 - 840i\,bc^6d^4x^6\arctan(cx) - 840i\,ac^6d^4x^6 - 30\,bc^6d^4x^6 - 1512\,bc^5}{}$$

input
```
integrate(x^2*(d+I*c*d*x)^4*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
1/1260*(180*b*c^7*d^4*x^7*arctan(c*x) + 180*a*c^7*d^4*x^7 - 840*I*b*c^6*d^
4*x^6*arctan(c*x) - 840*I*a*c^6*d^4*x^6 - 30*b*c^6*d^4*x^6 - 1512*b*c^5*d^
4*x^5*arctan(c*x) - 1512*a*c^5*d^4*x^5 + 168*I*b*c^5*d^4*x^5 + 1260*I*b*c^
4*d^4*x^4*arctan(c*x) + 1260*I*a*c^4*d^4*x^4 + 423*b*c^4*d^4*x^4 + 420*b*c
^3*d^4*x^3*arctan(c*x) + 420*a*c^3*d^4*x^3 - 700*I*b*c^3*d^4*x^3 - 1056*b*
c^2*d^4*x^2 + 2100*I*b*c*d^4*x + 2106*b*d^4*log(c*x + I) + 6*b*d^4*log(c*x
 - I))/c^3
```

## Mupad [B] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.06

$$\int x^2(d+icdx)^4(a+b\arctan(cx))\,dx = \frac{c^4\,d^4\,\left(180\,a\,x^7+180\,b\,x^7\,\mathrm{atan}(c\,x)\right)}{1260}+\frac{d^4\,\left(420\,a\,x^3+420\,b\,x^3\,\mathrm{atan}(c\,x)-b\,x^3\,700\mathrm{i}\right)}{1260} - \frac{\frac{d^4\,\left(-1056\,b\,\ln\left(c^2\,x^2+1\right)+b\,\mathrm{atan}(c\,x)\,2100\mathrm{i}\right)}{1260}+\frac{88\,b\,c^2\,d^4\,x^2}{105}-\frac{b\,c\,d^4\,x\,5\mathrm{i}}{3}}{c^3} + \frac{c\,d^4\,\left(a\,x^4\,1260\mathrm{i}+423\,b\,x^4+b\,x^4\,\mathrm{atan}(c\,x)\,1260\mathrm{i}\right)}{1260} - \frac{c^3\,d^4\,\left(a\,x^6\,840\mathrm{i}+30\,b\,x^6+b\,x^6\,\mathrm{atan}(c\,x)\,840\mathrm{i}\right)}{1260} - \frac{c^2\,d^4\,\left(1512\,a\,x^5+1512\,b\,x^5\,\mathrm{atan}(c\,x)-b\,x^5\,168\mathrm{i}\right)}{1260}$$

input
```
int(x^2*(a + b*atan(c*x))*(d + c*d*x*1i)^4,x)
```

output
```
(d^4*(420*a*x^3 - b*x^3*700i + 420*b*x^3*atan(c*x)))/1260 - ((d^4*(b*atan(
c*x)*2100i - 1056*b*log(c^2*x^2 + 1)))/1260 + (88*b*c^2*d^4*x^2)/105 - (b*
c*d^4*x*5i)/3)/c^3 + (c^4*d^4*(180*a*x^7 + 180*b*x^7*atan(c*x)))/1260 + (c
*d^4*(a*x^4*1260i + 423*b*x^4 + b*x^4*atan(c*x)*1260i))/1260 - (c^3*d^4*(a
*x^6*840i + 30*b*x^6 + b*x^6*atan(c*x)*840i))/1260 - (c^2*d^4*(1512*a*x^5
- b*x^5*168i + 1512*b*x^5*atan(c*x)))/1260
```

## Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.02

$$\int x^2(d+icdx)^4(a+b\arctan(cx))\,dx = \frac{d^4(180atan(cx)\,b\,c^7x^7-840atan(cx)\,b\,c^6i\,x^6-1512atan(cx)\,b\,c^5x^5+1260atan(cx)\,b\,c^4i\,x^4+420atan(c$$

input
```
int(x^2*(d+I*c*d*x)^4*(a+b*atan(c*x)),x)
```

output
```
(d**4*(180*atan(c*x)*b*c**7*x**7 - 840*atan(c*x)*b*c**6*i*x**6 - 1512*atan
(c*x)*b*c**5*x**5 + 1260*atan(c*x)*b*c**4*i*x**4 + 420*atan(c*x)*b*c**3*x*
*3 - 2100*atan(c*x)*b*i + 1056*log(c**2*x**2 + 1)*b + 180*a*c**7*x**7 - 84
0*a*c**6*i*x**6 - 1512*a*c**5*x**5 + 1260*a*c**4*i*x**4 + 420*a*c**3*x**3
- 30*b*c**6*x**6 + 168*b*c**5*i*x**5 + 423*b*c**4*x**4 - 700*b*c**3*i*x**3
 - 1056*b*c**2*x**2 + 2100*b*c*i*x))/(1260*c**3)
```

# 3.33 $\int x(d+icdx)^4(a+b\arctan(cx))\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 178

$$\int x(d+icdx)^4(a+b\arctan(cx))\,dx = -\frac{16bd^4x}{15c}-\frac{4ibd^4(i-cx)^2}{15c^2}-\frac{4bd^4(i-cx)^3}{45c^2}+\frac{ibd^4(i-cx)^4}{30c^2}+\frac{bd^4(i-cx)^5}{30c^2}+\frac{d^4(1+icx)^5(a+b\arctan(cx))}{5c^2}-\frac{d^4(1+icx)^6(a+b\arctan(cx))}{6c^2}+\frac{32ibd^4\log(i+cx)}{15c^2}$$

output
```
-16/15*b*d^4*x/c-4/15*I*b*d^4*(I-c*x)^2/c^2-4/45*b*d^4*(I-c*x)^3/c^2+1/30*
I*b*d^4*(I-c*x)^4/c^2+1/30*b*d^4*(I-c*x)^5/c^2+1/5*d^4*(1+I*c*x)^5*(a+b*ar
ctan(c*x))/c^2-1/6*d^4*(1+I*c*x)^6*(a+b*arctan(c*x))/c^2+32/15*I*b*d^4*ln(
I+c*x)/c^2
```

## Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.48

$$\int x(d+icdx)^4(a+b\arctan(cx))\,dx = -\frac{13bd^4x}{6c}+\frac{1}{2}ad^4x^2-\frac{16}{15}ibd^4x^2+\frac{4}{3}iacd^4x^3 +\frac{5}{9}bcd^4x^3-\frac{3}{2}ac^2d^4x^4+\frac{1}{5}ibc^2d^4x^4-\frac{4}{5}iac^3d^4x^5 -\frac{1}{30}bc^3d^4x^5+\frac{1}{6}ac^4d^4x^6+\frac{13bd^4\arctan(cx)}{6c^2} +\frac{1}{2}bd^4x^2\arctan(cx)+\frac{4}{3}ibcd^4x^3\arctan(cx) -\frac{3}{2}bc^2d^4x^4\arctan(cx)-\frac{4}{5}ibc^3d^4x^5\arctan(cx) +\frac{1}{6}bc^4d^4x^6\arctan(cx)+\frac{16ibd^4\log(1+c^2x^2)}{15c^2}$$

input
```
Integrate[x*(d + I*c*d*x)^4*(a + b*ArcTan[c*x]),x]
```

output
```
(-13*b*d^4*x)/(6*c) + (a*d^4*x^2)/2 - ((16*I)/15)*b*d^4*x^2 + ((4*I)/3)*a*
c*d^4*x^3 + (5*b*c*d^4*x^3)/9 - (3*a*c^2*d^4*x^4)/2 + (I/5)*b*c^2*d^4*x^4
- ((4*I)/5)*a*c^3*d^4*x^5 - (b*c^3*d^4*x^5)/30 + (a*c^4*d^4*x^6)/6 + (13*b
*d^4*ArcTan[c*x])/(6*c^2) + (b*d^4*x^2*ArcTan[c*x])/2 + ((4*I)/3)*b*c*d^4*
x^3*ArcTan[c*x] - (3*b*c^2*d^4*x^4*ArcTan[c*x])/2 - ((4*I)/5)*b*c^3*d^4*x^
5*ArcTan[c*x] + (b*c^4*d^4*x^6*ArcTan[c*x])/6 + (((16*I)/15)*b*d^4*Log[1 +
 c^2*x^2])/c^2
```

## Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.86, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {5407, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(d+icdx)^4(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5407}$$

$$-bc\int \frac{d^4(i-cx)^4(5cx+i)}{30c^2(cx+i)}dx - \frac{d^4(1+icx)^6(a+b\arctan(cx))}{6c^2} + \frac{d^4(1+icx)^5(a+b\arctan(cx))}{5c^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{bd^4\int \frac{(i-cx)^4(5cx+i)}{cx+i}dx}{30c} - \frac{d^4(1+icx)^6(a+b\arctan(cx))}{6c^2} + \frac{d^4(1+icx)^5(a+b\arctan(cx))}{5c^2}$$

$$\Big\downarrow \text{86}$$

$$-\frac{bd^4\int \left(5(i-cx)^4-4i(cx-i)^3-8(cx-i)^2+16i(cx-i)-\frac{64i}{cx+i}+32\right)dx}{30c} - \frac{d^4(1+icx)^6(a+b\arctan(cx))}{6c^2} + \frac{d^4(1+icx)^5(a+b\arctan(cx))}{5c^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d^4(1+icx)^6(a+b\arctan(cx))}{6c^2} + \frac{d^4(1+icx)^5(a+b\arctan(cx))}{5c^2} - \frac{bd^4\left(-\frac{(-cx+i)^5}{c}-\frac{i(-cx+i)^4}{c}+\frac{8(-cx+i)^3}{3c}+\frac{8i(-cx+i)^2}{c}-\frac{64i\log(cx+i)}{c}+32x\right)}{30c}$$

input
```
Int[x*(d + I*c*d*x)^4*(a + b*ArcTan[c*x]),x]
```

output
```
(d^4*(1 + I*c*x)^5*(a + b*ArcTan[c*x]))/(5*c^2) - (d^4*(1 + I*c*x)^6*(a +
b*ArcTan[c*x]))/(6*c^2) - (b*d^4*(32*x + ((8*I)*(I - c*x)^2)/c + (8*(I - c
*x)^3)/(3*c) - (I*(I - c*x)^4)/c - (I - c*x)^5/c - ((64*I)*Log[I + c*x])/c
))/(30*c)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5407
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Simp[(a
 + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ
[2*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0])
)
```

## Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.97

| method | result |
|---|---|
| parts | $d^4 a\left(\frac{1}{6}x^6c^4-\frac{4}{5}ic^3x^5-\frac{3}{2}c^2x^4+\frac{4}{3}ic\,x^3+\frac{1}{2}x^2\right)+\frac{d^4 b\left(\frac{c^6x^6\arctan(cx)}{6}-\frac{4i\arctan(cx)c^5x^5}{5}-\frac{3c^4x^4\arctan}{2}\right.}{}$ |
| derivativedivides | $\frac{d^4a\left(\frac{1}{6}c^6x^6-\frac{4}{5}ic^5x^5-\frac{3}{2}c^4x^4+\frac{4}{3}ic^3x^3+\frac{1}{2}c^2x^2\right)+d^4b\left(\frac{c^6x^6\arctan(cx)}{6}-\frac{4i\arctan(cx)c^5x^5}{5}-\frac{3c^4x^4\arctan(cx)}{2}+\frac{4i\arctan(cx}{3}\right.}{c^2}$ |
| default | $\frac{d^4a\left(\frac{1}{6}c^6x^6-\frac{4}{5}ic^5x^5-\frac{3}{2}c^4x^4+\frac{4}{3}ic^3x^3+\frac{1}{2}c^2x^2\right)+d^4b\left(\frac{c^6x^6\arctan(cx)}{6}-\frac{4i\arctan(cx)c^5x^5}{5}-\frac{3c^4x^4\arctan(cx)}{2}+\frac{4i\arctan(cx}{3}\right.}{c^2}$ |
| parallelrisch | $15c^6d^4b\arctan(cx)x^6+18ix^4b\,c^4d^4+15a\,c^6d^4x^6+96ib\,d^4\ln\left(c^2x^2+1\right)-3b\,c^5d^4x^5-72ic^5b\,d^4\arctan(cx)x^5-135d^4b\,\mathrm{arc}$ |
| risch | $\frac{id^4c^2b\,x^4}{5}+\frac{d^4c^4a\,x^6}{6}-\frac{4id^4c^3x^5a}{5}+\frac{2d^4c^3b\,x^5\ln(-icx+1)}{5}-\frac{d^4c^3b\,x^5}{30}+\frac{4id^4c\,x^3a}{3}-\frac{3d^4c^2a\,x^4}{2}-\frac{3id^4c^2}{}$ |

input
```
int(x*(d+I*c*d*x)^4*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

output
```
d^4*a*(1/6*x^6*c^4-4/5*I*c^3*x^5-3/2*c^2*x^4+4/3*I*c*x^3+1/2*x^2)+d^4*b/c^
2*(1/6*c^6*x^6*arctan(c*x)-4/5*I*arctan(c*x)*c^5*x^5-3/2*c^4*x^4*arctan(c*
x)+4/3*I*arctan(c*x)*c^3*x^3+1/2*c^2*x^2*arctan(c*x)-13/6*c*x-1/30*c^5*x^5
+1/5*I*c^4*x^4+5/9*c^3*x^3-16/15*I*c^2*x^2+16/15*I*ln(c^2*x^2+1)+13/6*arct
an(c*x))
```

## Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.16

$$\int x(d+icdx)^4(a+b\arctan(cx))\,dx$$
$$= \frac{30\,ac^6d^4x^6 - 6\,(24i\,a+b)c^5d^4x^5 - 18\,(15\,a-2i\,b)c^4d^4x^4 - 20\,(-12i\,a-5\,b)c^3d^4x^3 + 6\,(15\,a-32i\,b)c^2d}{}$$

input
```
integrate(x*(d+I*c*d*x)^4*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
1/180*(30*a*c^6*d^4*x^6 - 6*(24*I*a + b)*c^5*d^4*x^5 - 18*(15*a - 2*I*b)*c
^4*d^4*x^4 - 20*(-12*I*a - 5*b)*c^3*d^4*x^3 + 6*(15*a - 32*I*b)*c^2*d^4*x^
2 - 390*b*c*d^4*x + 387*I*b*d^4*log((c*x + I)/c) - 3*I*b*d^4*log((c*x - I)
/c) - 3*(-5*I*b*c^6*d^4*x^6 - 24*b*c^5*d^4*x^5 + 45*I*b*c^4*d^4*x^4 + 40*b
*c^3*d^4*x^3 - 15*I*b*c^2*d^4*x^2)*log(-(c*x + I)/(c*x - I)))/c^2
```

## Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 360 vs. 2(156) = 312.

Time = 2.86 (sec) , antiderivative size = 360, normalized size of antiderivative = 2.02

$$\int x(d+icdx)^4(a+b\arctan(cx))\,dx = \frac{ac^4d^4x^6}{6} - \frac{13bd^4x}{6c} + \frac{bd^4\left(-\frac{i\log(709bcd^4x-709ibd^4)}{60}+\frac{117i\log(709bcd^4x+709ibd^4)}{70}\right)}{c^2} + x^5\left(-\frac{4iac^3d^4}{5}-\frac{bc^3d^4}{30}\right) + x^4\left(-\frac{3ac^2d^4}{2}+\frac{ibc^2d^4}{5}\right) + x^3\cdot\left(\frac{4iacd^4}{3}+\frac{5bcd^4}{9}\right) + x^2\left(\frac{ad^4}{2}-\frac{16ibd^4}{15}\right) + \left(-\frac{ibc^4d^4x^6}{12}-\frac{2bc^3d^4x^5}{5}+\frac{3ibc^2d^4x^4}{4}+\frac{2bcd^4x^3}{3}-\frac{ibd^4x^2}{4}\right)\log(icx+1) + \frac{(35ibc^6d^4x^6+168bc^5d^4x^5-315ibc^4d^4x^4-280bc^3d^4x^3+105ibc^2d^4x^2+201ibd^4)\log(-icx+1)}{420c^2}$$

input
```
integrate(x*(d+I*c*d*x)**4*(a+b*atan(c*x)),x)
```

output
```
a*c**4*d**4*x**6/6 - 13*b*d**4*x/(6*c) + b*d**4*(-I*log(709*b*c*d**4*x - 7
09*I*b*d**4)/60 + 117*I*log(709*b*c*d**4*x + 709*I*b*d**4)/70)/c**2 + x**5
*(-4*I*a*c**3*d**4/5 - b*c**3*d**4/30) + x**4*(-3*a*c**2*d**4/2 + I*b*c**2
*d**4/5) + x**3*(4*I*a*c*d**4/3 + 5*b*c*d**4/9) + x**2*(a*d**4/2 - 16*I*b*
d**4/15) + (-I*b*c**4*d**4*x**6/12 - 2*b*c**3*d**4*x**5/5 + 3*I*b*c**2*d**
4*x**4/4 + 2*b*c*d**4*x**3/3 - I*b*d**4*x**2/4)*log(I*c*x + 1) + (35*I*b*c
**6*d**4*x**6 + 168*b*c**5*d**4*x**5 - 315*I*b*c**4*d**4*x**4 - 280*b*c**3
*d**4*x**3 + 105*I*b*c**2*d**4*x**2 + 201*I*b*d**4)*log(-I*c*x + 1)/(420*c
**2)
```

## Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 290 vs. $2(138) = 276$.

Time = 0.12 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.63

$$\int x(d+icdx)^4(a+b\arctan(cx))\,dx$$
$$= \frac{1}{6}ac^4d^4x^6 - \frac{4}{5}i\,ac^3d^4x^5 - \frac{3}{2}ac^2d^4x^4 + \frac{1}{90}\left(15\,x^6\arctan(cx) - c\left(\frac{3\,c^4x^5 - 5\,c^2x^3 + 15\,x}{c^6} - \frac{15\,\arctan(cx)}{c^7}\right)\right)bc^4d^4 - \frac{1}{5}i\left(4\,x^5\arctan(cx) - c\left(\frac{c^2x^4 - 2\,x^2}{c^4} + \frac{2\,\log(c^2x^2+1)}{c^6}\right)\right)bc^3d^4 + \frac{4}{3}i\,acd^4x^3 - \frac{1}{2}\left(3\,x^4\arctan(cx) - c\left(\frac{c^2x^3 - 3\,x}{c^4} + \frac{3\,\arctan(cx)}{c^5}\right)\right)bc^2d^4 + \frac{2}{3}i\left(2\,x^3\arctan(cx) - c\left(\frac{x^2}{c^2} - \frac{\log(c^2x^2+1)}{c^4}\right)\right)bcd^4 + \frac{1}{2}ad^4x^2 + \frac{1}{2}\left(x^2\arctan(cx) - c\left(\frac{x}{c^2} - \frac{\arctan(cx)}{c^3}\right)\right)bd^4$$

input
```
integrate(x*(d+I*c*d*x)^4*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
1/6*a*c^4*d^4*x^6 - 4/5*I*a*c^3*d^4*x^5 - 3/2*a*c^2*d^4*x^4 + 1/90*(15*x^6
*arctan(c*x) - c*((3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan(c*x)/c^7)
)*b*c^4*d^4 - 1/5*I*(4*x^5*arctan(c*x) - c*((c^2*x^4 - 2*x^2)/c^4 + 2*log(
c^2*x^2 + 1)/c^6))*b*c^3*d^4 + 4/3*I*a*c*d^4*x^3 - 1/2*(3*x^4*arctan(c*x)
- c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*b*c^2*d^4 + 2/3*I*(2*x^3*ar
ctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*b*c*d^4 + 1/2*a*d^4*x^2 +
1/2*(x^2*arctan(c*x) - c*(x/c^2 - arctan(c*x)/c^3))*b*d^4
```

## Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.28

$$\int x(d+icdx)^4(a+b\arctan(cx))\,dx$$
$$=\frac{30\,bc^6d^4x^6\arctan(cx)+30\,ac^6d^4x^6-144i\,bc^5d^4x^5\arctan(cx)-144i\,ac^5d^4x^5-6\,bc^5d^4x^5-270\,bc^4d^4x^4}{}$$

input
```
integrate(x*(d+I*c*d*x)^4*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
1/180*(30*b*c^6*d^4*x^6*arctan(c*x) + 30*a*c^6*d^4*x^6 - 144*I*b*c^5*d^4*x
^5*arctan(c*x) - 144*I*a*c^5*d^4*x^5 - 6*b*c^5*d^4*x^5 - 270*b*c^4*d^4*x^4
*arctan(c*x) - 270*a*c^4*d^4*x^4 + 36*I*b*c^4*d^4*x^4 + 240*I*b*c^3*d^4*x^
3*arctan(c*x) + 240*I*a*c^3*d^4*x^3 + 100*b*c^3*d^4*x^3 + 90*b*c^2*d^4*x^2
*arctan(c*x) + 90*a*c^2*d^4*x^2 - 192*I*b*c^2*d^4*x^2 - 390*b*c*d^4*x + 38
7*I*b*d^4*log(I*c*x - 1) - 3*I*b*d^4*log(-I*c*x - 1))/c^2
```

## Mupad [B] (verification not implemented)

Time = 1.11 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.07

$$\int x(d+icdx)^4(a+b\arctan(cx))\,dx=\frac{\frac{d^4\,(195\,b\,\mathrm{atan}(c\,x)+b\,\ln(c^2\,x^2+1)\,96\mathrm{i})}{90}-\frac{13\,b\,c\,d^4\,x}{6}}{c^2}+\frac{d^4\,(45\,a\,x^2+45\,b\,x^2\,\mathrm{atan}(c\,x)-b\,x^2\,96\mathrm{i})}{90}+\frac{c^4\,d^4\,(15\,a\,x^6+15\,b\,x^6\,\mathrm{atan}(c\,x))}{90}+\frac{c\,d^4\,(a\,x^3\,120\mathrm{i}+50\,b\,x^3+b\,x^3\,\mathrm{atan}(c\,x)\,120\mathrm{i})}{90}-\frac{c^3\,d^4\,(a\,x^5\,72\mathrm{i}+3\,b\,x^5+b\,x^5\,\mathrm{atan}(c\,x)\,72\mathrm{i})}{90}-\frac{c^2\,d^4\,(135\,a\,x^4+135\,b\,x^4\,\mathrm{atan}(c\,x)-b\,x^4\,18\mathrm{i})}{90}$$

input
```
int(x*(a + b*atan(c*x))*(d + c*d*x*1i)^4,x)
```

output
```
((d^4*(195*b*atan(c*x) + b*log(c^2*x^2 + 1)*96i))/90 - (13*b*c*d^4*x)/6)/c
^2 + (d^4*(45*a*x^2 - b*x^2*96i + 45*b*x^2*atan(c*x)))/90 + (c^4*d^4*(15*a
*x^6 + 15*b*x^6*atan(c*x)))/90 + (c*d^4*(a*x^3*120i + 50*b*x^3 + b*x^3*ata
n(c*x)*120i))/90 - (c^3*d^4*(a*x^5*72i + 3*b*x^5 + b*x^5*atan(c*x)*72i))/9
0 - (c^2*d^4*(135*a*x^4 - b*x^4*18i + 135*b*x^4*atan(c*x)))/90
```

## Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.05

$$\int x(d+icdx)^4(a+b\arctan(cx))\,dx$$
$$= \frac{d^4(15atan(cx)\,b\,c^6x^6 - 72atan(cx)\,b\,c^5i\,x^5 - 135atan(cx)\,b\,c^4x^4 + 120atan(cx)\,b\,c^3i\,x^3 + 45atan(cx)\,b\,c^2}{}$$

input
```
int(x*(d+I*c*d*x)^4*(a+b*atan(c*x)),x)
```

output
```
(d**4*(15*atan(c*x)*b*c**6*x**6 - 72*atan(c*x)*b*c**5*i*x**5 - 135*atan(c*
x)*b*c**4*x**4 + 120*atan(c*x)*b*c**3*i*x**3 + 45*atan(c*x)*b*c**2*x**2 +
195*atan(c*x)*b + 96*log(c**2*x**2 + 1)*b*i + 15*a*c**6*x**6 - 72*a*c**5*i
*x**5 - 135*a*c**4*x**4 + 120*a*c**3*i*x**3 + 45*a*c**2*x**2 - 3*b*c**5*x*
*5 + 18*b*c**4*i*x**4 + 50*b*c**3*x**3 - 96*b*c**2*i*x**2 - 195*b*c*x))/(9
0*c**2)
```

# 3.34 $\int (d+icdx)^4(a+b\arctan(cx))\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 125

$$\int (d+icdx)^4(a+b\arctan(cx))\,dx = -\frac{8}{5}ibd^4x-\frac{2bd^4(1+icx)^2}{5c}-\frac{2bd^4(1+icx)^3}{15c}-\frac{bd^4(1+icx)^4}{20c}-\frac{id^4(1+icx)^5(a+b\arctan(cx))}{5c}-\frac{16bd^4\log(1-icx)}{5c}$$

output
```
-8/5*I*b*d^4*x-2/5*b*d^4*(1+I*c*x)^2/c-2/15*b*d^4*(1+I*c*x)^3/c-1/20*b*d^4
*(1+I*c*x)^4/c-1/5*I*d^4*(1+I*c*x)^5*(a+b*arctan(c*x))/c-16/5*b*d^4*ln(1-I
*c*x)/c
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.62

$$\int (d+icdx)^4(a+b\arctan(cx))\,dx = \frac{d^4(12(-i+cx)^5(a+b\arctan(cx))-b(35+180icx-66c^2x^2-20ic^3x^3+3c^4x^4+192\log(i+cx)))}{60c}$$

input
```
Integrate[(d + I*c*d*x)^4*(a + b*ArcTan[c*x]),x]
```

output
```
(d^4*(12*(-I + c*x)^5*(a + b*ArcTan[c*x]) - b*(35 + (180*I)*c*x - 66*c^2*x
^2 - (20*I)*c^3*x^3 + 3*c^4*x^4 + 192*Log[I + c*x])))/(60*c)
```

## Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5387, 27, 456, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+icdx)^4(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5387}$$

$$\frac{ib\int \frac{d^5(icx+1)^5}{c^2x^2+1}dx}{5d}-\frac{id^4(1+icx)^5(a+b\arctan(cx))}{5c}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}ibd^4\int \frac{(icx+1)^5}{c^2x^2+1}dx-\frac{id^4(1+icx)^5(a+b\arctan(cx))}{5c}$$

$$\Big\downarrow \text{456}$$

$$\frac{1}{5}ibd^4\int \frac{(icx+1)^4}{1-icx}dx-\frac{id^4(1+icx)^5(a+b\arctan(cx))}{5c}$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{5}ibd^4\int \left(-(icx+1)^3-2(icx+1)^2-4(icx+1)+\frac{16}{1-icx}-8\right)dx-\frac{id^4(1+icx)^5(a+b\arctan(cx))}{5c}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}ibd^4\left(\frac{i(1+icx)^4}{4c}+\frac{2i(1+icx)^3}{3c}+\frac{2i(1+icx)^2}{c}+\frac{16i\log(cx+i)}{c}-8x\right)-\frac{id^4(1+icx)^5(a+b\arctan(cx))}{5c}$$

input
```
Int[(d + I*c*d*x)^4*(a + b*ArcTan[c*x]),x]
```

output
```
((-1/5*I)*d^4*(1 + I*c*x)^5*(a + b*ArcTan[c*x]))/c + (I/5)*b*d^4*(-8*x + (
(2*I)*(1 + I*c*x)^2)/c + (((2*I)/3)*(1 + I*c*x)^3)/c + ((I/4)*(1 + I*c*x)^
4)/c + ((16*I)*Log[I + c*x])/c)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5387
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Simp[b*(
c/(e*(q + 1)))   Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b
, c, d, e, q}, x] && NeQ[q, -1]
```

## Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.16

| method | result |
|---|---|
| derivativedivides | $\frac{-\frac{i d^4 a (icx+1)^5}{5}+d^4 b\left(\frac{c^5 x^5 \arctan(cx)}{5}-i\arctan(cx)c^4x^4-2c^3x^3\arctan(cx)+2i\arctan(cx)c^2x^2+cx\arctan(cx)-\frac{i \arctan}{5}\right.}{c}$ |
| default | $\frac{-\frac{i d^4 a (icx+1)^5}{5}+d^4 b\left(\frac{c^5 x^5 \arctan(cx)}{5}-i\arctan(cx)c^4x^4-2c^3x^3\arctan(cx)+2i\arctan(cx)c^2x^2+cx\arctan(cx)-\frac{i \arctan}{5}\right.}{c}$ |
| parts | $-\frac{i d^4 a (icx+1)^5}{5c}+\frac{d^4 b\left(\frac{c^5 x^5 \arctan(cx)}{5}-i\arctan(cx)c^4x^4-2c^3x^3\arctan(cx)+2i\arctan(cx)c^2x^2+cx\arctan(cx)-\frac{i a}{}\right.}{c}$ |
| parallelrisch | $-\frac{-12c^5d^4b\arctan(cx)x^5-180ibd^4\arctan(cx)-12a c^5d^4x^5+60ibc^4d^4\arctan(cx)x^4+3bc^4d^4x^4+60ix^4ac^4d^4+120d^4}{}$ |
| risch | $2id^4acx^2+\frac{31id^4\arctan(cx)b}{10c}+\frac{d^4c^4x^5a}{5}+\frac{ibd^4x\ln(-icx+1)}{2}+\frac{d^4c^3x^4b\ln(-icx+1)}{2}-id^4c^2bx^3\ln($ |

input
```
int((d+I*c*d*x)^4*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/c*(-1/5*I*d^4*a*(1+I*c*x)^5+d^4*b*(1/5*c^5*x^5*arctan(c*x)-I*arctan(c*x)
*c^4*x^4-2*c^3*x^3*arctan(c*x)+2*I*arctan(c*x)*c^2*x^2+c*x*arctan(c*x)-1/5
*I*arctan(c*x)+1/5*I*(-15*c*x+1/4*I*c^4*x^4+5/3*c^3*x^3-11/2*I*c^2*x^2+8*I
*ln(c^2*x^2+1)+16*arctan(c*x))))
```

## Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.50

$$\int (d+icdx)^4(a+b\arctan(cx))\,dx$$
$$=\frac{12\,ac^5d^4x^5-3\,(20i\,a+b)c^4d^4x^4-20\,(6\,a-i\,b)c^3d^4x^3-6\,(-20i\,a-11\,b)c^2d^4x^2+60\,(a-3i\,b)cd^4x-1}{}$$

input
```
integrate((d+I*c*d*x)^4*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
1/60*(12*a*c^5*d^4*x^5 - 3*(20*I*a + b)*c^4*d^4*x^4 - 20*(6*a - I*b)*c^3*d
^4*x^3 - 6*(-20*I*a - 11*b)*c^2*d^4*x^2 + 60*(a - 3*I*b)*c*d^4*x - 186*b*d
^4*log((c*x + I)/c) - 6*b*d^4*log((c*x - I)/c) - 6*(-I*b*c^5*d^4*x^5 - 5*b
*c^4*d^4*x^4 + 10*I*b*c^3*d^4*x^3 + 10*b*c^2*d^4*x^2 - 5*I*b*c*d^4*x)*log(
-(c*x + I)/(c*x - I)))/c
```

## Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(110) = 220$.

Time = 2.42 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.53

$$\int (d + icdx)^4 (a + b\arctan(cx))\,dx$$
$$= \frac{ac^4d^4x^5}{5} + \frac{bd^4\left(-\frac{\log(41bcd^4x-41ibd^4)}{10} - \frac{43\log(41bcd^4x+41ibd^4)}{20}\right)}{c} + x^4\left(-iac^3d^4 - \frac{bc^3d^4}{20}\right)$$
$$+ x^3\left(-2ac^2d^4 + \frac{ibc^2d^4}{3}\right) + x^2\cdot\left(2iacd^4 + \frac{11bcd^4}{10}\right) + x(ad^4 - 3ibd^4)$$
$$+ \left(-\frac{ibc^4d^4x^5}{10} - \frac{bc^3d^4x^4}{2} + ibc^2d^4x^3 + bcd^4x^2 - \frac{ibd^4x}{2}\right)\log(icx+1)$$
$$+ \frac{(2ibc^5d^4x^5 + 10bc^4d^4x^4 - 20ibc^3d^4x^3 - 20bc^2d^4x^2 + 10ibcd^4x - 19bd^4)\log(-icx+1)}{20c}$$

input
```
integrate((d+I*c*d*x)**4*(a+b*atan(c*x)),x)
```

output
```
a*c**4*d**4*x**5/5 + b*d**4*(-log(41*b*c*d**4*x - 41*I*b*d**4)/10 - 43*log
(41*b*c*d**4*x + 41*I*b*d**4)/20)/c + x**4*(-I*a*c**3*d**4 - b*c**3*d**4/2
0) + x**3*(-2*a*c**2*d**4 + I*b*c**2*d**4/3) + x**2*(2*I*a*c*d**4 + 11*b*c
*d**4/10) + x*(a*d**4 - 3*I*b*d**4) + (-I*b*c**4*d**4*x**5/10 - b*c**3*d**
4*x**4/2 + I*b*c**2*d**4*x**3 + b*c*d**4*x**2 - I*b*d**4*x/2)*log(I*c*x +
1) + (2*I*b*c**5*d**4*x**5 + 10*b*c**4*d**4*x**4 - 20*I*b*c**3*d**4*x**3 -
 20*b*c**2*d**4*x**2 + 10*I*b*c*d**4*x - 19*b*d**4)*log(-I*c*x + 1)/(20*c)
```

## Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(99) = 198$.

Time = 0.12 (sec) , antiderivative size = 264, normalized size of antiderivative = 2.11

$$
\begin{aligned}
&\int (d+icdx)^4(a+b\arctan(cx))\,dx \\
&= \frac{1}{5}\,ac^4d^4x^5 - i\,ac^3d^4x^4 \\
&\quad + \frac{1}{20}\left(4\,x^5\arctan(cx) - c\left(\frac{c^2x^4-2\,x^2}{c^4} + \frac{2\,\log(c^2x^2+1)}{c^6}\right)\right)bc^4d^4 \\
&\quad - 2\,ac^2d^4x^3 - \frac{1}{3}i\left(3\,x^4\arctan(cx) - c\left(\frac{c^2x^3-3\,x}{c^4} + \frac{3\,\arctan(cx)}{c^5}\right)\right)bc^3d^4 \\
&\quad - \left(2\,x^3\arctan(cx) - c\left(\frac{x^2}{c^2} - \frac{\log(c^2x^2+1)}{c^4}\right)\right)bc^2d^4 \\
&\quad + 2i\,acd^4x^2 + 2i\left(x^2\arctan(cx) - c\left(\frac{x}{c^2} - \frac{\arctan(cx)}{c^3}\right)\right)bcd^4 \\
&\quad + ad^4x + \frac{(2\,cx\arctan(cx) - \log(c^2x^2+1))bd^4}{2\,c}
\end{aligned}
$$

input
```
integrate((d+I*c*d*x)^4*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
1/5*a*c^4*d^4*x^5 - I*a*c^3*d^4*x^4 + 1/20*(4*x^5*arctan(c*x) - c*((c^2*x^
4 - 2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*b*c^4*d^4 - 2*a*c^2*d^4*x^3 - 1/
3*I*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*b*c^
3*d^4 - (2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*b*c^2*d^4
 + 2*I*a*c*d^4*x^2 + 2*I*(x^2*arctan(c*x) - c*(x/c^2 - arctan(c*x)/c^3))*b
*c*d^4 + a*d^4*x + 1/2*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b*d^4/c
```

## Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 206 vs. 2(99) = 198.

Time = 0.16 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.65

$$\int (d+icdx)^4(a+b\arctan(cx))\,dx$$
$$= \frac{12\,bc^5d^4x^5\arctan(cx)+12\,ac^5d^4x^5-60i\,bc^4d^4x^4\arctan(cx)-60i\,ac^4d^4x^4-3\,bc^4d^4x^4-120\,bc^3d^4x^3\,\text{ar}}{}$$

input
```
integrate((d+I*c*d*x)^4*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
1/60*(12*b*c^5*d^4*x^5*arctan(c*x) + 12*a*c^5*d^4*x^5 - 60*I*b*c^4*d^4*x^4
*arctan(c*x) - 60*I*a*c^4*d^4*x^4 - 3*b*c^4*d^4*x^4 - 120*b*c^3*d^4*x^3*ar
ctan(c*x) - 120*a*c^3*d^4*x^3 + 20*I*b*c^3*d^4*x^3 + 120*I*b*c^2*d^4*x^2*a
rctan(c*x) + 120*I*a*c^2*d^4*x^2 + 66*b*c^2*d^4*x^2 + 60*b*c*d^4*x*arctan(
c*x) + 60*a*c*d^4*x - 180*I*b*c*d^4*x - 186*b*d^4*log(c*x + I) - 6*b*d^4*l
og(c*x - I))/c
```

## Mupad [B] (verification not implemented)

Time = 1.07 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.40

$$\begin{aligned}\int (d+icdx)^4(a+b\arctan(cx))\,dx &= \frac{d^4\,(60\,a\,x+60\,b\,x\,\mathrm{atan}(c\,x)-b\,x\,180\mathrm{i})}{60}\\ &+\frac{c^4\,d^4\,(12\,a\,x^5+12\,b\,x^5\,\mathrm{atan}(c\,x))}{60}\\ &+\frac{d^4\,(-96\,b\,\ln(c^2\,x^2+1)+b\,\mathrm{atan}(c\,x)\,180\mathrm{i})}{60\,c}\\ &+\frac{c\,d^4\,(a\,x^2\,120\mathrm{i}+66\,b\,x^2+b\,x^2\,\mathrm{atan}(c\,x)\,120\mathrm{i})}{60}\\ &-\frac{c^3\,d^4\,(a\,x^4\,60\mathrm{i}+3\,b\,x^4+b\,x^4\,\mathrm{atan}(c\,x)\,60\mathrm{i})}{60}\\ &-\frac{c^2\,d^4\,(120\,a\,x^3+120\,b\,x^3\,\mathrm{atan}(c\,x)-b\,x^3\,20\mathrm{i})}{60}\end{aligned}$$

input
```
int((a + b*atan(c*x))*(d + c*d*x*1i)^4,x)
```

output
```
(d^4*(60*a*x - b*x*180i + 60*b*x*atan(c*x)))/60 + (c^4*d^4*(12*a*x^5 + 12*
b*x^5*atan(c*x)))/60 + (d^4*(b*atan(c*x)*180i - 96*b*log(c^2*x^2 + 1)))/(6
0*c) + (c*d^4*(a*x^2*120i + 66*b*x^2 + b*x^2*atan(c*x)*120i))/60 - (c^3*d^
4*(a*x^4*60i + 3*b*x^4 + b*x^4*atan(c*x)*60i))/60 - (c^2*d^4*(120*a*x^3 -
b*x^3*20i + 120*b*x^3*atan(c*x)))/60
```

## Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.36

$$\int (d + icdx)^4 (a + b\arctan(cx))\,dx$$
$$= \frac{d^4(12atan(cx)\,b\,c^5x^5 - 60atan(cx)\,b\,c^4i\,x^4 - 120atan(cx)\,b\,c^3x^3 + 120atan(cx)\,b\,c^2i\,x^2 + 60atan(cx)\,bcx}{}$$

input
```
int((d+I*c*d*x)^4*(a+b*atan(c*x)),x)
```

output
```
(d**4*(12*atan(c*x)*b*c**5*x**5 - 60*atan(c*x)*b*c**4*i*x**4 - 120*atan(c*
x)*b*c**3*x**3 + 120*atan(c*x)*b*c**2*i*x**2 + 60*atan(c*x)*b*c*x + 180*at
an(c*x)*b*i - 96*log(c**2*x**2 + 1)*b + 12*a*c**5*x**5 - 60*a*c**4*i*x**4
- 120*a*c**3*x**3 + 120*a*c**2*i*x**2 + 60*a*c*x - 3*b*c**4*x**4 + 20*b*c*
*3*i*x**3 + 66*b*c**2*x**2 - 180*b*c*i*x))/(60*c)
```

# 3.35 $\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 203

$$\begin{aligned}\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x}\,dx &= 4iacd^4x+\frac{13}{4}bcd^4x+\frac{2}{3}ibc^2d^4x^2\\ &\quad-\frac{1}{12}bc^3d^4x^3-\frac{13}{4}bd^4\arctan(cx)\\ &\quad+4ibcd^4x\arctan(cx)-3c^2d^4x^2(a+b\arctan(cx))\\ &\quad-\frac{4}{3}ic^3d^4x^3(a+b\arctan(cx))\\ &\quad+\frac{1}{4}c^4d^4x^4(a+b\arctan(cx))+ad^4\log(x)\\ &\quad-\frac{8}{3}ibd^4\log\left(1+c^2x^2\right)+\frac{1}{2}ibd^4\operatorname{PolyLog}(2,-icx)\\ &\quad-\frac{1}{2}ibd^4\operatorname{PolyLog}(2,icx)\end{aligned}$$

output

```
4*I*a*c*d^4*x+13/4*b*c*d^4*x+2/3*I*b*c^2*d^4*x^2-1/12*b*c^3*d^4*x^3-13/4*b
*d^4*arctan(c*x)+4*I*b*c*d^4*x*arctan(c*x)-3*c^2*d^4*x^2*(a+b*arctan(c*x))
-4/3*I*c^3*d^4*x^3*(a+b*arctan(c*x))+1/4*c^4*d^4*x^4*(a+b*arctan(c*x))+a*d
^4*ln(x)-8/3*I*b*d^4*ln(c^2*x^2+1)+1/2*I*b*d^4*polylog(2,-I*c*x)-1/2*I*b*d
^4*polylog(2,I*c*x)
```

## Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.86

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x}\,dx = \frac{1}{12}d^4\big(48iacx+39bcx-36ac^2x^2+8ibc^2x^2 - 16iac^3x^3-bc^3x^3+3ac^4x^4-39b\arctan(cx) + 48ibcx\arctan(cx)-36bc^2x^2\arctan(cx) - 16ibc^3x^3\arctan(cx)+3bc^4x^4\arctan(cx) + 12a\log(x)-32ib\log\left(1+c^2x^2\right) + 6ib\,\mathrm{PolyLog}(2,-icx)-6ib\,\mathrm{PolyLog}(2,icx)\big)$$

input
```
Integrate[((d + I*c*d*x)^4*(a + b*ArcTan[c*x]))/x,x]
```

output
```
(d^4*((48*I)*a*c*x + 39*b*c*x - 36*a*c^2*x^2 + (8*I)*b*c^2*x^2 - (16*I)*a*
c^3*x^3 - b*c^3*x^3 + 3*a*c^4*x^4 - 39*b*ArcTan[c*x] + (48*I)*b*c*x*ArcTan
[c*x] - 36*b*c^2*x^2*ArcTan[c*x] - (16*I)*b*c^3*x^3*ArcTan[c*x] + 3*b*c^4*
x^4*ArcTan[c*x] + 12*a*Log[x] - (32*I)*b*Log[1 + c^2*x^2] + (6*I)*b*PolyLo
g[2, (-I)*c*x] - (6*I)*b*PolyLog[2, I*c*x]))/12
```

## Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x}\,dx$$

$$\Big\downarrow \text{5411}$$

$$\int \Big(c^4d^4x^3(a+b\arctan(cx))-4ic^3d^4x^2(a+b\arctan(cx))-6c^2d^4x(a+b\arctan(cx))+4icd^4(a+b\arctan(cx)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}c^4d^4x^4(a+b\arctan(cx))-\frac{4}{3}ic^3d^4x^3(a+b\arctan(cx))-3c^2d^4x^2(a+b\arctan(cx))+4iacd^4x+ad^4\log(x)-\frac{13}{4}bd^4\arctan(cx)+4ibcd^4x\arctan(cx)-\frac{1}{12}bc^3d^4x^3+\frac{2}{3}ibc^2d^4x^2-\frac{8}{3}ibd^4\log\left(c^2x^2+1\right)+\frac{1}{2}ibd^4\operatorname{PolyLog}(2,-icx)-\frac{1}{2}ibd^4\operatorname{PolyLog}(2,icx)+\frac{13}{4}bcd^4x$$

input
```
Int[((d + I*c*d*x)^4*(a + b*ArcTan[c*x]))/x,x]
```

output
```
(4*I)*a*c*d^4*x + (13*b*c*d^4*x)/4 + ((2*I)/3)*b*c^2*d^4*x^2 - (b*c^3*d^4*
x^3)/12 - (13*b*d^4*ArcTan[c*x])/4 + (4*I)*b*c*d^4*x*ArcTan[c*x] - 3*c^2*d
^4*x^2*(a + b*ArcTan[c*x]) - ((4*I)/3)*c^3*d^4*x^3*(a + b*ArcTan[c*x]) + (
c^4*d^4*x^4*(a + b*ArcTan[c*x]))/4 + a*d^4*Log[x] - ((8*I)/3)*b*d^4*Log[1
+ c^2*x^2] + (I/2)*b*d^4*PolyLog[2, (-I)*c*x] - (I/2)*b*d^4*PolyLog[2, I*c
*x]
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

## Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.95

| method | result |
|---|---|
| parts | $d^4a\left(\frac{c^4x^4}{4}-\frac{4ic^3x^3}{3}-3c^2x^2+4icx+\ln(x)\right)+d^4b\left(4i\arctan(cx)cx+\frac{c^4x^4\arctan(cx)}{4}-\frac{4}{}\right.$ |
| derivativedivides | $d^4a\left(4icx+\frac{c^4x^4}{4}-\frac{4ic^3x^3}{3}-3c^2x^2+\ln(cx)\right)+d^4b\left(4i\arctan(cx)cx+\frac{c^4x^4\arctan(cx)}{4}-\right.$ |
| default | $d^4a\left(4icx+\frac{c^4x^4}{4}-\frac{4ic^3x^3}{3}-3c^2x^2+\ln(cx)\right)+d^4b\left(4i\arctan(cx)cx+\frac{c^4x^4\arctan(cx)}{4}-\right.$ |
| risch | $-3a\,c^2d^4x^2-\frac{25id^4b\ln(icx+1)}{24}+\frac{id^4b\,\mathrm{dilog}(icx+1)}{2}+\frac{58id^4b}{9}-\frac{103d^4a}{12}+d^4a\ln(-icx)+\frac{2d^4bc^3x}{}$ |

input
```
int((d+I*c*d*x)^4*(a+b*arctan(c*x))/x,x,method=_RETURNVERBOSE)
```

output
```
d^4*a*(1/4*c^4*x^4-4/3*I*c^3*x^3-3*c^2*x^2+4*I*c*x+ln(x))+d^4*b*(4*I*arcta
n(c*x)*c*x+1/4*c^4*x^4*arctan(c*x)-4/3*I*arctan(c*x)*c^3*x^3-3*c^2*x^2*arc
tan(c*x)+ln(c*x)*arctan(c*x)+1/2*I*ln(c*x)*ln(1+I*c*x)-1/2*I*ln(c*x)*ln(1-
I*c*x)+1/2*I*dilog(1+I*c*x)-1/2*I*dilog(1-I*c*x)+13/4*c*x-1/12*c^3*x^3+2/3
*I*c^2*x^2-8/3*I*ln(c^2*x^2+1)-13/4*arctan(c*x))
```

## Fricas [F]

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x}\,dx = \int \frac{(i\,cdx+d)^4(b\arctan(cx)+a)}{x}\,dx$$

input
```
integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x,x, algorithm="fricas")
```

output
```
integral(1/2*(2*a*c^4*d^4*x^4 - 8*I*a*c^3*d^4*x^3 - 12*a*c^2*d^4*x^2 + 8*I
*a*c*d^4*x + 2*a*d^4 + (I*b*c^4*d^4*x^4 + 4*b*c^3*d^4*x^3 - 6*I*b*c^2*d^4*
x^2 - 4*b*c*d^4*x + I*b*d^4)*log(-(c*x + I)/(c*x - I)))/x, x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x}\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)**4*(a+b*atan(c*x))/x,x)
```

output
```
Timed out
```

## Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.08

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x}\,dx$$
$$= \frac{1}{4}\,ac^4d^4x^4 - \frac{4}{3}i\,ac^3d^4x^3 - \frac{1}{12}\,bc^3d^4x^3 - 3\,ac^2d^4x^2 + \frac{2}{3}i\,bc^2d^4x^2 + 4i\,acd^4x + \frac{13}{4}\,bcd^4x - \frac{1}{12}\,(3\,\pi + 8i)bd^4\log\left(c^2x^2 + 1\right) + bd^4\arctan\left(cx\right)\log\left(cx\right) + 2i\left(2\,cx\arctan\left(cx\right) - \log\left(c^2x^2 + 1\right)\right)bd^4 - \frac{1}{2}i\,bd^4\mathrm{Li}_2(i\,cx+1) + \frac{1}{2}i\,bd^4\mathrm{Li}_2(-i\,cx+1) + ad^4\log\left(x\right) + \frac{1}{12}\left(3\,bc^4d^4x^4 - 16i\,bc^3d^4x^3 - 36\,bc^2d^4x^2 - 39\,bd^4\right)\arctan\left(cx\right)$$

input
```
integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x,x, algorithm="maxima")
```

output
```
1/4*a*c^4*d^4*x^4 - 4/3*I*a*c^3*d^4*x^3 - 1/12*b*c^3*d^4*x^3 - 3*a*c^2*d^4
*x^2 + 2/3*I*b*c^2*d^4*x^2 + 4*I*a*c*d^4*x + 13/4*b*c*d^4*x - 1/12*(3*pi +
 8*I)*b*d^4*log(c^2*x^2 + 1) + b*d^4*arctan(c*x)*log(c*x) + 2*I*(2*c*x*arc
tan(c*x) - log(c^2*x^2 + 1))*b*d^4 - 1/2*I*b*d^4*dilog(I*c*x + 1) + 1/2*I*
b*d^4*dilog(-I*c*x + 1) + a*d^4*log(x) + 1/12*(3*b*c^4*d^4*x^4 - 16*I*b*c^
3*d^4*x^3 - 36*b*c^2*d^4*x^2 - 39*b*d^4)*arctan(c*x)
```

## Giac [F]

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x}\,dx = \int \frac{(i\,cdx + d)^4(b\arctan\left(cx\right) + a)}{x}\,dx$$

input
```
integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x,x, algorithm="giac")
```

output
```
integrate((I*c*d*x + d)^4*(b*arctan(c*x) + a)/x, x)
```

## Mupad [B] (verification not implemented)

Time = 1.30 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.22

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x}\,dx$$

$$= \begin{cases} \\ a\,d^4\,\ln(x) - b\,d^4\,\ln(c^2\,x^2+1)\,2\mathrm{i} - \frac{b\,d^4\,(3\,\mathrm{atan}(c\,x)-3\,c\,x+c^3\,x^3)}{12} - \frac{b\,d^4\,\mathrm{Li}_2(1-c\,x\,1\mathrm{i})\,1\mathrm{i}}{2} + \frac{b\,d^4\,\mathrm{Li}_2(1+c\,x\,1\mathrm{i})\,1\mathrm{i}}{2} - 3\,a\,c \end{cases}$$

input
```
int(((a + b*atan(c*x))*(d + c*d*x*1i)^4)/x,x)
```

output
```
piecewise(c == 0, a*d^4*log(x), c ~= 0, - (b*d^4*(3*atan(c*x) - 3*c*x + c^
3*x^3))/12 - b*d^4*log(c^2*x^2 + 1)*2i + a*d^4*log(x) - (b*d^4*dilog(- c*x
*1i + 1)*1i)/2 + (b*d^4*dilog(c*x*1i + 1)*1i)/2 - 3*a*c^2*d^4*x^2 - (a*c^3
*d^4*x^3*4i)/3 + (a*c^4*d^4*x^4)/4 + a*c*d^4*x*4i + 3*b*c*d^4*x + (b*c^2*d
^4*(x^2/2 - log(c^2*x^2 + 1)/(2*c^2))*4i)/3 - 6*b*c^2*d^4*atan(c*x)*(1/(2*
c^2) + x^2/2) - (b*c^3*d^4*x^3*atan(c*x)*4i)/3 + (b*c^4*d^4*x^4*atan(c*x))
/4 + b*c*d^4*x*atan(c*x)*4i)
```

## Reduce [F]

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x}\,dx$$

$$= \frac{d^4\Big(3atan(cx)\,b\,c^4x^4 - 16atan(cx)\,b\,c^3i\,x^3 - 36atan(cx)\,b\,c^2x^2 + 48atan(cx)\,bcix - 39atan(cx)\,b + 12\Big(\int}{}$$

input
```
int((d+I*c*d*x)^4*(a+b*atan(c*x))/x,x)
```

output
```
(d**4*(3*atan(c*x)*b*c**4*x**4 - 16*atan(c*x)*b*c**3*i*x**3 - 36*atan(c*x)
*b*c**2*x**2 + 48*atan(c*x)*b*c*i*x - 39*atan(c*x)*b + 12*int(atan(c*x)/x,
x)*b - 32*log(c**2*x**2 + 1)*b*i + 12*log(x)*a + 3*a*c**4*x**4 - 16*a*c**3
*i*x**3 - 36*a*c**2*x**2 + 48*a*c*i*x - b*c**3*x**3 + 8*b*c**2*i*x**2 + 39
*b*c*x))/12
```

# 3.36 $\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^2}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 190

$$\begin{aligned}\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^2}\,dx &= -6ac^2d^4x+2ibc^2d^4x-\frac{1}{6}bc^3d^4x^2\\&\quad-2ibcd^4\arctan(cx)-6bc^2d^4x\arctan(cx)\\&\quad-\frac{d^4(a+b\arctan(cx))}{x}\\&\quad-2ic^3d^4x^2(a+b\arctan(cx))\\&\quad+\frac{1}{3}c^4d^4x^3(a+b\arctan(cx))+4iacd^4\log(x)\\&\quad+bcd^4\log(x)+\frac{8}{3}bcd^4\log\left(1+c^2x^2\right)\\&\quad-2bcd^4\operatorname{PolyLog}(2,-icx)+2bcd^4\operatorname{PolyLog}(2,icx)\end{aligned}$$

output

```
-6*a*c^2*d^4*x+2*I*b*c^2*d^4*x-1/6*b*c^3*d^4*x^2-2*I*b*c*d^4*arctan(c*x)-6
*b*c^2*d^4*x*arctan(c*x)-d^4*(a+b*arctan(c*x))/x-2*I*c^3*d^4*x^2*(a+b*arct
an(c*x))+1/3*c^4*d^4*x^3*(a+b*arctan(c*x))+4*I*a*c*d^4*ln(x)+b*c*d^4*ln(x)
+8/3*b*c*d^4*ln(c^2*x^2+1)-2*b*c*d^4*polylog(2,-I*c*x)+2*b*c*d^4*polylog(2
,I*c*x)
```

## Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.95

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^2}\,dx$$
$$= \frac{d^4(-6a-36ac^2x^2+12ibc^2x^2-12iac^3x^3-bc^3x^3+2ac^4x^4-6b\arctan(cx)-12ibcx\arctan(cx)-36bc^2$$

input
```
Integrate[((d + I*c*d*x)^4*(a + b*ArcTan[c*x]))/x^2,x]
```

output
```
(d^4*(-6*a - 36*a*c^2*x^2 + (12*I)*b*c^2*x^2 - (12*I)*a*c^3*x^3 - b*c^3*x^
3 + 2*a*c^4*x^4 - 6*b*ArcTan[c*x] - (12*I)*b*c*x*ArcTan[c*x] - 36*b*c^2*x^
2*ArcTan[c*x] - (12*I)*b*c^3*x^3*ArcTan[c*x] + 2*b*c^4*x^4*ArcTan[c*x] + (
24*I)*a*c*x*Log[x] + 6*b*c*x*Log[c*x] + 16*b*c*x*Log[1 + c^2*x^2] - 12*b*c
*x*PolyLog[2, (-I)*c*x] + 12*b*c*x*PolyLog[2, I*c*x]))/(6*x)
```

## Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^2}\,dx$$
$$\Big\downarrow \text{5411}$$
$$\int \left(c^4d^4x^2(a+b\arctan(cx))-4ic^3d^4x(a+b\arctan(cx))-6c^2d^4(a+b\arctan(cx))+\frac{d^4(a+b\arctan(cx))}{x^2}+\frac{4$$
$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}c^4d^4x^3(a+b\arctan(cx))-2ic^3d^4x^2(a+b\arctan(cx))-\frac{d^4(a+b\arctan(cx))}{x}-6ac^2d^4x+4iacd^4\log(x)-6bc^2d^4x\arctan(cx)-2ibcd^4\arctan(cx)-\frac{1}{6}bc^3d^4x^2+\frac{8}{3}bcd^4\log\left(c^2x^2+1\right)+2ibc^2d^4x-2bcd^4\operatorname{PolyLog}(2,-icx)+2bcd^4\operatorname{PolyLog}(2,icx)+bcd^4\log(x)$$

input
```
Int[((d + I*c*d*x)^4*(a + b*ArcTan[c*x]))/x^2,x]
```

output
```
-6*a*c^2*d^4*x + (2*I)*b*c^2*d^4*x - (b*c^3*d^4*x^2)/6 - (2*I)*b*c*d^4*Arc
Tan[c*x] - 6*b*c^2*d^4*x*ArcTan[c*x] - (d^4*(a + b*ArcTan[c*x]))/x - (2*I)
*c^3*d^4*x^2*(a + b*ArcTan[c*x]) + (c^4*d^4*x^3*(a + b*ArcTan[c*x]))/3 + (
4*I)*a*c*d^4*Log[x] + b*c*d^4*Log[x] + (8*b*c*d^4*Log[1 + c^2*x^2])/3 - 2*
b*c*d^4*PolyLog[2, (-I)*c*x] + 2*b*c*d^4*PolyLog[2, I*c*x]
```

**Defintions of rubi rules used**

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

## Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.99

| method | result |
|---|---|
| parts | $d^4a\left(\frac{c^4x^3}{3}-2ic^3x^2-6c^2x-\frac{1}{x}+4ic\ln(x)\right)+d^4bc\left(-6cx\arctan(cx)+\frac{c^3x^3\arctan(cx)}{3}-\right.$ |
| derivativedivides | $c\left(d^4a\left(-6cx+\frac{c^3x^3}{3}-2ic^2x^2+4i\ln(cx)-\frac{1}{cx}\right)+d^4b\left(-6cx\arctan(cx)+\frac{c^3x^3\arctan(cx)}{3}\right.\right.$ |
| default | $c\left(d^4a\left(-6cx+\frac{c^3x^3}{3}-2ic^2x^2+4i\ln(cx)-\frac{1}{cx}\right)+d^4b\left(-6cx\arctan(cx)+\frac{c^3x^3\arctan(cx)}{3}\right.\right.$ |
| risch | $-b\,c^3d^4\ln(icx+1)\,x^2+\frac{11d^4cb\ln(-icx+1)}{3}+\frac{d^4c^4a\,x^3}{3}-\frac{25id^4ca}{3}+d^4c^3b\ln(-icx+1)\,x^2-$ |

input
```
int((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^2,x,method=_RETURNVERBOSE)
```

output
```
d^4*a*(1/3*c^4*x^3-2*I*c^3*x^2-6*c^2*x-1/x+4*I*c*ln(x))+d^4*b*c*(-6*c*x*ar
ctan(c*x)+1/3*c^3*x^3*arctan(c*x)-2*I*arctan(c*x)*c^2*x^2+4*I*arctan(c*x)*
ln(c*x)-1/c/x*arctan(c*x)-2*ln(c*x)*ln(1+I*c*x)+2*ln(c*x)*ln(1-I*c*x)-2*di
log(1+I*c*x)+2*dilog(1-I*c*x)+2*I*c*x-1/6*c^2*x^2+ln(c*x)+8/3*ln(c^2*x^2+1
)-2*I*arctan(c*x))
```

## Fricas [F]

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^2}\,dx = \int \frac{(i\,cdx+d)^4(b\arctan(cx)+a)}{x^2}\,dx$$

input
```
integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^2,x, algorithm="fricas")
```

output
```
integral(1/2*(2*a*c^4*d^4*x^4 - 8*I*a*c^3*d^4*x^3 - 12*a*c^2*d^4*x^2 + 8*I
*a*c*d^4*x + 2*a*d^4 + (I*b*c^4*d^4*x^4 + 4*b*c^3*d^4*x^3 - 6*I*b*c^2*d^4*
x^2 - 4*b*c*d^4*x + I*b*d^4)*log(-(c*x + I)/(c*x - I)))/x^2, x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)**4*(a+b*atan(c*x))/x**2,x)
```

output
```
Timed out
```

## Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.26

$$\begin{aligned}
&\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^2}\,dx \\
&= \frac{1}{3}\,ac^4d^4x^3 - 2i\,ac^3d^4x^2 - \frac{1}{6}\,bc^3d^4x^2 - 6\,ac^2d^4x + 2i\,bc^2d^4x \\
&\quad - \frac{1}{6}\,(6i\,\pi - 1)bcd^4\log\left(c^2x^2+1\right) + 4i\,bcd^4\arctan\left(cx\right)\log\left(cx\right) \\
&\quad - 3\left(2\,cx\arctan\left(cx\right) - \log\left(c^2x^2+1\right)\right)bcd^4 + 2\,bcd^4\mathrm{Li}_2(i\,cx+1) - 2\,bcd^4\mathrm{Li}_2(-i\,cx+1) \\
&\quad + 4i\,acd^4\log\left(x\right) - \frac{1}{2}\left(c\left(\log\left(c^2x^2+1\right) - \log\left(x^2\right)\right) + \frac{2\arctan\left(cx\right)}{x}\right)bd^4 \\
&\quad - \frac{ad^4}{x} + \frac{1}{3}\left(bc^4d^4x^3 - 6i\,bc^3d^4x^2 - 6i\,bcd^4\right)\arctan\left(cx\right)
\end{aligned}$$

input
```
integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^2,x, algorithm="maxima")
```

output
```
1/3*a*c^4*d^4*x^3 - 2*I*a*c^3*d^4*x^2 - 1/6*b*c^3*d^4*x^2 - 6*a*c^2*d^4*x
+ 2*I*b*c^2*d^4*x - 1/6*(6*I*pi - 1)*b*c*d^4*log(c^2*x^2 + 1) + 4*I*b*c*d^
4*arctan(c*x)*log(c*x) - 3*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b*c*d^4
+ 2*b*c*d^4*dilog(I*c*x + 1) - 2*b*c*d^4*dilog(-I*c*x + 1) + 4*I*a*c*d^4*l
og(x) - 1/2*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b*d^4 - a*
d^4/x + 1/3*(b*c^4*d^4*x^3 - 6*I*b*c^3*d^4*x^2 - 6*I*b*c*d^4)*arctan(c*x)
```

## Giac [F]

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^2}\,dx = \int \frac{(i\,cdx+d)^4(b\arctan\left(cx\right)+a)}{x^2}\,dx$$

input
```
integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^2,x, algorithm="giac")
```

output
```
integrate((I*c*d*x + d)^4*(b*arctan(c*x) + a)/x^2, x)
```

## Mupad [B] (verification not implemented)

Time = 1.12 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.33

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^2}\,dx$$

$$= \begin{cases} \frac{a\,c^4\,d^4\,x^3}{3} - \frac{a\,d^4}{x} + \frac{b\,d^4\left(c^2\,\ln(x) - \frac{c^2\,\ln\left(c^2\,x^2+1\right)}{2}\right)}{c} + 2\,b\,c\,d^4\,(\mathrm{Li}_2(1-c\,x\,1\mathrm{i}) - \mathrm{Li}_2(1+c\,x\,1\mathrm{i})) + 3\,b\,c\,d^4\,\ln\left(c^2\,x\right. \end{cases}$$

input
```
int(((a + b*atan(c*x))*(d + c*d*x*1i)^4)/x^2,x)
```

output
```
piecewise(c == 0, -(a*d^4)/x, c ~= 0, - (a*d^4)/x - a*c^3*d^4*x^2*2i + (a*
c^4*d^4*x^3)/3 + (b*d^4*(c^2*log(x) - (c^2*log(c^2*x^2 + 1))/2))/c + 2*b*c
*d^4*(dilog(- c*x*1i + 1) - dilog(c*x*1i + 1)) + 3*b*c*d^4*log(c^2*x^2 + 1
) - 6*a*c^2*d^4*x + b*c^2*d^4*x*2i - (b*c^3*d^4*(x^2/2 - log(c^2*x^2 + 1)/
(2*c^2)))/3 + a*c*d^4*log(x)*4i - (b*d^4*atan(c*x))/x - 6*b*c^2*d^4*x*atan
(c*x) - b*c^3*d^4*atan(c*x)*(1/(2*c^2) + x^2/2)*4i + (b*c^4*d^4*x^3*atan(c
*x))/3)
```

## Reduce [F]

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^2}\,dx$$

$$= \frac{d^4\left(2atan(cx)\,b\,c^4x^4 - 12atan(cx)\,b\,c^3i\,x^3 - 36atan(cx)\,b\,c^2x^2 - 12atan(cx)\,bcix - 6atan(cx)\,b + 24\left(\int\right.\right.}{}$$

input
```
int((d+I*c*d*x)^4*(a+b*atan(c*x))/x^2,x)
```

output
```
(d**4*(2*atan(c*x)*b*c**4*x**4 - 12*atan(c*x)*b*c**3*i*x**3 - 36*atan(c*x)
*b*c**2*x**2 - 12*atan(c*x)*b*c*i*x - 6*atan(c*x)*b + 24*int(atan(c*x)/x,x
)*b*c*i*x + 16*log(c**2*x**2 + 1)*b*c*x + 24*log(x)*a*c*i*x + 6*log(x)*b*c
*x + 2*a*c**4*x**4 - 12*a*c**3*i*x**3 - 36*a*c**2*x**2 - 6*a - b*c**3*x**3
 + 12*b*c**2*i*x**2))/(6*x)
```

# 3.37 $\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^3}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 173

$$\begin{aligned}\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^3}\,dx = &-\frac{bcd^4}{2x}-4iac^3d^4x-\frac{1}{2}bc^3d^4x-4ibc^3d^4x\arctan(cx)\\ &-\frac{d^4(a+b\arctan(cx))}{2x^2}-\frac{4icd^4(a+b\arctan(cx))}{x}\\ &+\frac{1}{2}c^4d^4x^2(a+b\arctan(cx))-6ac^2d^4\log(x)\\ &+4ibc^2d^4\log(x)-3ibc^2d^4\operatorname{PolyLog}(2,-icx)\\ &+3ibc^2d^4\operatorname{PolyLog}(2,icx)\end{aligned}$$

output

```
-1/2*b*c*d^4/x-4*I*a*c^3*d^4*x-1/2*b*c^3*d^4*x-4*I*b*c^3*d^4*x*arctan(c*x)
-1/2*d^4*(a+b*arctan(c*x))/x^2-4*I*c*d^4*(a+b*arctan(c*x))/x+1/2*c^4*d^4*x
^2*(a+b*arctan(c*x))-6*a*c^2*d^4*ln(x)+4*I*b*c^2*d^4*ln(x)-3*I*b*c^2*d^4*p
olylog(2,-I*c*x)+3*I*b*c^2*d^4*polylog(2,I*c*x)
```

## Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.94

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^3}\,dx$$
$$= \frac{d^4(-a-8iacx-bcx-8iac^3x^3-bc^3x^3+ac^4x^4-b\arctan(cx)-8ibcx\arctan(cx)-8ibc^3x^3\arctan(cx)}{}$$

input
```
Integrate[((d + I*c*d*x)^4*(a + b*ArcTan[c*x]))/x^3,x]
```

output
```
(d^4*(-a - (8*I)*a*c*x - b*c*x - (8*I)*a*c^3*x^3 - b*c^3*x^3 + a*c^4*x^4 -
 b*ArcTan[c*x] - (8*I)*b*c*x*ArcTan[c*x] - (8*I)*b*c^3*x^3*ArcTan[c*x] + b
*c^4*x^4*ArcTan[c*x] - 12*a*c^2*x^2*Log[x] + (8*I)*b*c^2*x^2*Log[c*x] - (6
*I)*b*c^2*x^2*PolyLog[2, (-I)*c*x] + (6*I)*b*c^2*x^2*PolyLog[2, I*c*x]))/(
2*x^2)
```

## Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^3}\,dx$$
$$\Big\downarrow \text{5411}$$
$$\int \left(c^4d^4x(a+b\arctan(cx))-4ic^3d^4(a+b\arctan(cx))-\frac{6c^2d^4(a+b\arctan(cx))}{x}+\frac{d^4(a+b\arctan(cx))}{x^3}+\frac{4ic}{}\right.$$
$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}c^4d^4x^2(a+b\arctan(cx))-\frac{d^4(a+b\arctan(cx))}{2x^2}-\frac{4icd^4(a+b\arctan(cx))}{x}-4iac^3d^4x-6ac^2d^4\log(x)-4ibc^3d^4x\arctan(cx)-\frac{1}{2}bc^3d^4x-3ibc^2d^4\operatorname{PolyLog}(2,-icx)+3ibc^2d^4\operatorname{PolyLog}(2,icx)+4ibc^2d^4\log(x)-\frac{bcd^4}{2x}$$

input
```
Int[((d + I*c*d*x)^4*(a + b*ArcTan[c*x]))/x^3,x]
```

output
```
-1/2*(b*c*d^4)/x - (4*I)*a*c^3*d^4*x - (b*c^3*d^4*x)/2 - (4*I)*b*c^3*d^4*x
*ArcTan[c*x] - (d^4*(a + b*ArcTan[c*x]))/(2*x^2) - ((4*I)*c*d^4*(a + b*Arc
Tan[c*x]))/x + (c^4*d^4*x^2*(a + b*ArcTan[c*x]))/2 - 6*a*c^2*d^4*Log[x] +
(4*I)*b*c^2*d^4*Log[x] - (3*I)*b*c^2*d^4*PolyLog[2, (-I)*c*x] + (3*I)*b*c^
2*d^4*PolyLog[2, I*c*x]
```

**Defintions of rubi rules used**

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

## Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.03

| method | result |
|---|---|
| parts | $d^4a\left(\frac{c^4x^2}{2}-4ic^3x-\frac{1}{2x^2}-\frac{4ic}{x}-6c^2\ln(x)\right)+d^4b\,c^2\left(-4i\arctan(cx)\,cx+\frac{c^2x^2\arctan(cx)}{2}\right.$ |
| derivativedivides | $c^2\left(d^4a\left(-4icx+\frac{c^2x^2}{2}-6\ln(cx)-\frac{1}{2c^2x^2}-\frac{4i}{cx}\right)+d^4b\left(-4i\arctan(cx)\,cx+\frac{c^2x^2\arctan(c}{2}\right.\right.$ |
| default | $c^2\left(d^4a\left(-4icx+\frac{c^2x^2}{2}-6\ln(cx)-\frac{1}{2c^2x^2}-\frac{4i}{cx}\right)+d^4b\left(-4i\arctan(cx)\,cx+\frac{c^2x^2\arctan(c}{2}\right.\right.$ |
| risch | $\frac{7ib\,d^4c^2\ln(icx)}{4}-\frac{id^4b\ln(-icx+1)}{4x^2}-\frac{d^4a}{2x^2}-3ib\,d^4c^2\operatorname{dilog}(icx+1)+\frac{ib\,d^4\ln(icx+1)}{4x^2}-4ia\,c^3d^4x$ |

input
```
int((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^3,x,method=_RETURNVERBOSE)
```

output
```
d^4*a*(1/2*c^4*x^2-4*I*c^3*x-1/2/x^2-4*I*c/x-6*c^2*ln(x))+d^4*b*c^2*(-4*I*
arctan(c*x)*c*x+1/2*c^2*x^2*arctan(c*x)-6*ln(c*x)*arctan(c*x)-1/2/c^2/x^2*
arctan(c*x)-4*I*arctan(c*x)/c/x-1/2*c*x+4*I*ln(c*x)-1/2/c/x-3*I*ln(c*x)*ln
(1+I*c*x)+3*I*ln(c*x)*ln(1-I*c*x)-3*I*dilog(1+I*c*x)+3*I*dilog(1-I*c*x))
```

## Fricas [F]

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^3}\,dx = \int \frac{(i\,cdx+d)^4(b\arctan{(cx)}+a)}{x^3}\,dx$$

input
```
integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^3,x, algorithm="fricas")
```

output
```
integral(1/2*(2*a*c^4*d^4*x^4 - 8*I*a*c^3*d^4*x^3 - 12*a*c^2*d^4*x^2 + 8*I
*a*c*d^4*x + 2*a*d^4 + (I*b*c^4*d^4*x^4 + 4*b*c^3*d^4*x^3 - 6*I*b*c^2*d^4*
x^2 - 4*b*c*d^4*x + I*b*d^4)*log(-(c*x + I)/(c*x - I)))/x^3, x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)**4*(a+b*atan(c*x))/x**3,x)
```

output
```
Timed out
```

## Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.45

$$\begin{aligned}
&\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^3}\,dx \\
&= \frac{1}{2}\,ac^4d^4x^2 - 4i\,ac^3d^4x - \frac{1}{2}\,bc^3d^4x + \frac{3}{2}\,\pi bc^2d^4\log\left(c^2x^2+1\right) \\
&\quad - 6\,bc^2d^4\arctan\left(cx\right)\log\left(cx\right) - 2i\left(2\,cx\arctan\left(cx\right) - \log\left(c^2x^2+1\right)\right)bc^2d^4 \\
&\quad + 3i\,bc^2d^4\mathrm{Li}_2(i\,cx+1) - 3i\,bc^2d^4\mathrm{Li}_2(-i\,cx+1) - 6\,ac^2d^4\log\left(x\right) \\
&\quad - 2i\left(c\left(\log\left(c^2x^2+1\right) - \log\left(x^2\right)\right) + \frac{2\,\arctan\left(cx\right)}{x}\right)bcd^4 \\
&\quad - \frac{1}{2}\left(\left(c\arctan\left(cx\right) + \frac{1}{x}\right)c + \frac{\arctan\left(cx\right)}{x^2}\right)bd^4 \\
&\quad - \frac{4i\,acd^4}{x} - \frac{ad^4}{2\,x^2} + \frac{1}{2}\left(bc^4d^4x^2 + bc^2d^4\right)\arctan\left(cx\right)
\end{aligned}$$

input
```
integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^3,x, algorithm="maxima")
```

output
```
1/2*a*c^4*d^4*x^2 - 4*I*a*c^3*d^4*x - 1/2*b*c^3*d^4*x + 3/2*pi*b*c^2*d^4*l
og(c^2*x^2 + 1) - 6*b*c^2*d^4*arctan(c*x)*log(c*x) - 2*I*(2*c*x*arctan(c*x
) - log(c^2*x^2 + 1))*b*c^2*d^4 + 3*I*b*c^2*d^4*dilog(I*c*x + 1) - 3*I*b*c
^2*d^4*dilog(-I*c*x + 1) - 6*a*c^2*d^4*log(x) - 2*I*(c*(log(c^2*x^2 + 1) -
 log(x^2)) + 2*arctan(c*x)/x)*b*c*d^4 - 1/2*((c*arctan(c*x) + 1/x)*c + arc
tan(c*x)/x^2)*b*d^4 - 4*I*a*c*d^4/x - 1/2*a*d^4/x^2 + 1/2*(b*c^4*d^4*x^2 +
 b*c^2*d^4)*arctan(c*x)
```

## Giac [F]

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^3}\,dx = \int \frac{(i\,cdx+d)^4(b\arctan\left(cx\right)+a)}{x^3}\,dx$$

input
```
integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^3,x, algorithm="giac")
```

output
```
integrate((I*c*d*x + d)^4*(b*arctan(c*x) + a)/x^3, x)
```

## Mupad [B] (verification not implemented)

Time = 1.17 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.49

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^3}\,dx$$
$$= \begin{cases} \frac{a\,c^4\,d^4\,x^2}{2} - \frac{\frac{a\,d^4}{2}+a\,c\,d^4\,x\,4\mathrm{i}}{x^2} - 6\,a\,c^2\,d^4\,\ln\left(x\right) - \frac{b\,d^4\,\left(c^3\,\mathrm{atan}(c\,x)+\frac{c^2}{x}\right)}{2\,c} - \frac{b\,c^3\,d^4\,x}{2} - \frac{b\,d^4\,\mathrm{atan}(c\,x)}{2\,x^2} + b\,c^4\,d^4\,\mathrm{atan}(c\,x) \end{cases}$$

input
```
int(((a + b*atan(c*x))*(d + c*d*x*1i)^4)/x^3,x)
```

output
```
piecewise(c == 0, -(a*d^4)/(2*x^2), c ~= 0, - ((a*d^4)/2 + a*c*d^4*x*4i)/x
^2 + b*d^4*(c^2*log(x) - (c^2*log(c^2*x^2 + 1))/2)*4i + b*c^2*d^4*log(c^2*
x^2 + 1)*2i + (a*c^4*d^4*x^2)/2 - 6*a*c^2*d^4*log(x) + b*c^2*d^4*dilog(- c
*x*1i + 1)*3i - b*c^2*d^4*dilog(c*x*1i + 1)*3i - (b*d^4*(c^3*atan(c*x) + c
^2/x))/(2*c) - a*c^3*d^4*x*4i - (b*c^3*d^4*x)/2 - (b*d^4*atan(c*x))/(2*x^2
) - (b*c*d^4*atan(c*x)*4i)/x - b*c^3*d^4*x*atan(c*x)*4i + b*c^4*d^4*atan(c
*x)*(1/(2*c^2) + x^2/2))
```

## Reduce [F]

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^3}\,dx$$
$$= \frac{d^4\left(atan(cx)\,b\,c^4x^4 - 8atan(cx)\,b\,c^3i\,x^3 - 8atan(cx)\,bcix - atan(cx)\,b - 12\left(\int \frac{atan(cx)}{x}dx\right)b\,c^2x^2 - 12\log\right.}{2x^2}$$

input
```
int((d+I*c*d*x)^4*(a+b*atan(c*x))/x^3,x)
```

output
```
(d**4*(atan(c*x)*b*c**4*x**4 - 8*atan(c*x)*b*c**3*i*x**3 - 8*atan(c*x)*b*c
*i*x - atan(c*x)*b - 12*int(atan(c*x)/x,x)*b*c**2*x**2 - 12*log(x)*a*c**2*
x**2 + 8*log(x)*b*c**2*i*x**2 + a*c**4*x**4 - 8*a*c**3*i*x**3 - 8*a*c*i*x
- a - b*c**3*x**3 - b*c*x))/(2*x**2)
```

# 3.38 $\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^4}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 201

$$\begin{aligned}\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^4}\,dx &= -\frac{bcd^4}{6x^2}-\frac{2ibc^2d^4}{x}+ac^4d^4x-2ibc^3d^4\arctan(cx)\\&\quad+bc^4d^4x\arctan(cx)-\frac{d^4(a+b\arctan(cx))}{3x^3}\\&\quad-\frac{2icd^4(a+b\arctan(cx))}{x^2}\\&\quad+\frac{6c^2d^4(a+b\arctan(cx))}{x}-4iac^3d^4\log(x)\\&\quad-\frac{19}{3}bc^3d^4\log(x)+\frac{8}{3}bc^3d^4\log\left(1+c^2x^2\right)\\&\quad+2bc^3d^4\operatorname{PolyLog}(2,-icx)\\&\quad-2bc^3d^4\operatorname{PolyLog}(2,icx)\end{aligned}$$

output
```
-1/6*b*c*d^4/x^2-2*I*b*c^2*d^4/x+a*c^4*d^4*x-2*I*b*c^3*d^4*arctan(c*x)+b*c
^4*d^4*x*arctan(c*x)-1/3*d^4*(a+b*arctan(c*x))/x^3-2*I*c*d^4*(a+b*arctan(c
*x))/x^2+6*c^2*d^4*(a+b*arctan(c*x))/x-4*I*a*c^3*d^4*ln(x)-19/3*b*c^3*d^4*
ln(x)+8/3*b*c^3*d^4*ln(c^2*x^2+1)+2*b*c^3*d^4*polylog(2,-I*c*x)-2*b*c^3*d^
4*polylog(2,I*c*x)
```

## Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.96

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^4}\,dx$$
$$= \frac{d^4(-2a-12iacx-bcx+36ac^2x^2-12ibc^2x^2+6ac^4x^4-2b\arctan(cx)-12ibcx\arctan(cx)+36bc^2x^2}{}$$

input
```
Integrate[((d + I*c*d*x)^4*(a + b*ArcTan[c*x]))/x^4,x]
```

output
```
(d^4*(-2*a - (12*I)*a*c*x - b*c*x + 36*a*c^2*x^2 - (12*I)*b*c^2*x^2 + 6*a*
c^4*x^4 - 2*b*ArcTan[c*x] - (12*I)*b*c*x*ArcTan[c*x] + 36*b*c^2*x^2*ArcTan
[c*x] - (12*I)*b*c^3*x^3*ArcTan[c*x] + 6*b*c^4*x^4*ArcTan[c*x] - (24*I)*a*
c^3*x^3*Log[x] - 38*b*c^3*x^3*Log[c*x] + 16*b*c^3*x^3*Log[1 + c^2*x^2] + 1
2*b*c^3*x^3*PolyLog[2, (-I)*c*x] - 12*b*c^3*x^3*PolyLog[2, I*c*x]))/(6*x^3
)
```

## Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^4}\,dx$$
$$\Big\downarrow \text{5411}$$
$$\int \left(c^4d^4(a+b\arctan(cx))-\frac{4ic^3d^4(a+b\arctan(cx))}{x}-\frac{6c^2d^4(a+b\arctan(cx))}{x^2}+\frac{d^4(a+b\arctan(cx))}{x^4}+\frac{4icd}{}\right.$$
$$\Big\downarrow \text{2009}$$

$$\frac{6c^2d^4(a+b\arctan(cx))}{x}-\frac{d^4(a+b\arctan(cx))}{3x^3}-\frac{2icd^4(a+b\arctan(cx))}{x^2}+ac^4d^4x-4iac^3d^4\log(x)+bc^4d^4x\arctan(cx)-2ibc^3d^4\arctan(cx)+2bc^3d^4\operatorname{PolyLog}(2,-icx)-2bc^3d^4\operatorname{PolyLog}(2,icx)-\frac{19}{3}bc^3d^4\log(x)-\frac{2ibc^2d^4}{x}+\frac{8}{3}bc^3d^4\log\left(c^2x^2+1\right)-\frac{bcd^4}{6x^2}$$

input
```
Int[((d + I*c*d*x)^4*(a + b*ArcTan[c*x]))/x^4,x]
```

output
```
-1/6*(b*c*d^4)/x^2 - ((2*I)*b*c^2*d^4)/x + a*c^4*d^4*x - (2*I)*b*c^3*d^4*A
rcTan[c*x] + b*c^4*d^4*x*ArcTan[c*x] - (d^4*(a + b*ArcTan[c*x]))/(3*x^3) -
 ((2*I)*c*d^4*(a + b*ArcTan[c*x]))/x^2 + (6*c^2*d^4*(a + b*ArcTan[c*x]))/x
 - (4*I)*a*c^3*d^4*Log[x] - (19*b*c^3*d^4*Log[x])/3 + (8*b*c^3*d^4*Log[1 +
 c^2*x^2])/3 + 2*b*c^3*d^4*PolyLog[2, (-I)*c*x] - 2*b*c^3*d^4*PolyLog[2, I
*c*x]
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

## Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.97

| method | result |
|---|---|
| parts | $d^4a\left(c^4x-\frac{2ic}{x^2}-\frac{1}{3x^3}+\frac{6c^2}{x}-4ic^3\ln(x)\right)+d^4b\,c^3\left(cx\arctan(cx)-\frac{2i\arctan(cx)}{c^2x^2}+\frac{6\arctan}{cx}\right.$ |
| derivativedivides | $c^3\left(d^4a\left(cx-\frac{2i}{c^2x^2}+\frac{6}{cx}-\frac{1}{3c^3x^3}-4i\ln(cx)\right)+d^4b\left(cx\arctan(cx)-\frac{2i\arctan(cx)}{c^2x^2}+\frac{6\arctan}{c}\right.\right.$ |
| default | $c^3\left(d^4a\left(cx-\frac{2i}{c^2x^2}+\frac{6}{cx}-\frac{1}{3c^3x^3}-4i\ln(cx)\right)+d^4b\left(cx\arctan(cx)-\frac{2i\arctan(cx)}{c^2x^2}+\frac{6\arctan}{c}\right.\right.$ |
| risch | $b\,c^3d^4-\frac{id^4b\ln(-icx+1)}{6x^3}-\frac{ib\,d^4c^4\ln(icx+1)x}{2}-2ib\,c^3d^4\arctan(cx)+\frac{3id^4c^2b\ln(-icx+1)}{x}-\frac{2id^4c}{x^2}$ |

input
```
int((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^4,x,method=_RETURNVERBOSE)
```

output
```
d^4*a*(c^4*x-2*I*c/x^2-1/3/x^3+6*c^2/x-4*I*c^3*ln(x))+d^4*b*c^3*(c*x*arcta
n(c*x)-2*I*arctan(c*x)/c^2/x^2+6/c/x*arctan(c*x)-1/3/c^3/x^3*arctan(c*x)-4
*I*arctan(c*x)*ln(c*x)+2*ln(c*x)*ln(1+I*c*x)-2*ln(c*x)*ln(1-I*c*x)+2*dilog
(1+I*c*x)-2*dilog(1-I*c*x)+8/3*ln(c^2*x^2+1)-2*I*arctan(c*x)-1/6/c^2/x^2-2
*I/c/x-19/3*ln(c*x))
```

## Fricas [F]

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^4}\,dx = \int \frac{(i\,cdx+d)^4(b\arctan{(cx)}+a)}{x^4}\,dx$$

input
```
integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^4,x, algorithm="fricas")
```

output
```
integral(1/2*(2*a*c^4*d^4*x^4 - 8*I*a*c^3*d^4*x^3 - 12*a*c^2*d^4*x^2 + 8*I
*a*c*d^4*x + 2*a*d^4 + (I*b*c^4*d^4*x^4 + 4*b*c^3*d^4*x^3 - 6*I*b*c^2*d^4*
x^2 - 4*b*c*d^4*x + I*b*d^4)*log(-(c*x + I)/(c*x - I)))/x^4, x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^4}\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)**4*(a+b*atan(c*x))/x**4,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^4}\,dx = \int \frac{(i\,cdx+d)^4(b\arctan(cx)+a)}{x^4}\,dx$$

input
```
integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^4,x, algorithm="maxima")
```

output
```
a*c^4*d^4*x + 1/2*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b*c^3*d^4 - 4*I*b
*c^3*d^4*integrate(arctan(c*x)/x, x) - 4*I*a*c^3*d^4*log(x) + 3*(c*(log(c^
2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b*c^2*d^4 - 2*I*((c*arctan(c*x)
+ 1/x)*c + arctan(c*x)/x^2)*b*c*d^4 + 1/6*((c^2*log(c^2*x^2 + 1) - c^2*log
(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*b*d^4 + 6*a*c^2*d^4/x - 2*I*a*c*d^4/
x^2 - 1/3*a*d^4/x^3
```

## Giac [F]

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^4}\,dx = \int \frac{(i\,cdx+d)^4(b\arctan(cx)+a)}{x^4}\,dx$$

input
```
integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^4,x, algorithm="giac")
```

output
```
integrate((I*c*d*x + d)^4*(b*arctan(c*x) + a)/x^4, x)
```

## Mupad [B] (verification not implemented)

Time = 1.13 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.30

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^4}\,dx$$

$$= \Bigg\{ \frac{b\,c^3\,d^4\,\ln\left(-\frac{3\,c^6\,x^2}{2}-\frac{3\,c^4}{2}\right)}{6} - \frac{b\,c^3\,d^4\,\ln(c^2\,x^2+1)}{2} - \frac{b\,c^3\,d^4\,\ln(x)}{3} - 2\,b\,c^3\,d^4\,(\mathrm{Li}_2(1-c\,x\,1\mathrm{i}) - \mathrm{Li}_2(1+c\,x\,1\mathrm{i})) - 6\,b\,c$$

input
```
int(((a + b*atan(c*x))*(d + c*d*x*1i)^4)/x^4,x)
```

output
```
piecewise(c == 0, -(a*d^4)/(3*x^3), c ~= 0, - b*d^4*(c^3*atan(c*x) + c^2/x
)*2i - 2*b*c^3*d^4*(dilog(- c*x*1i + 1) - dilog(c*x*1i + 1)) - (b*c^3*d^4*
log(c^2*x^2 + 1))/2 - (b*c^3*d^4*log(x))/3 + (b*c^3*d^4*log(- (3*c^4)/2 -
(3*c^6*x^2)/2))/6 - 6*b*c*d^4*(c^2*log(x) - (c^2*log(c^2*x^2 + 1))/2) - (b
*c*d^4)/(6*x^2) - (a*d^4*(c*x*6i - 18*c^2*x^2 - 3*c^4*x^4 + c^3*x^3*log(x)
*12i + 1))/(3*x^3) - (b*d^4*atan(c*x))/(3*x^3) - (b*c*d^4*atan(c*x)*2i)/x^
2 + b*c^4*d^4*x*atan(c*x) + (6*b*c^2*d^4*atan(c*x))/x)
```

## Reduce [F]

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^4}\,dx$$

$$= \frac{d^4\Big(6atan(cx)\,b\,c^4x^4 - 12atan(cx)\,b\,c^3i\,x^3 + 36atan(cx)\,b\,c^2x^2 - 12atan(cx)\,bcix - 2atan(cx)\,b - 24\Big(\int$$

input
```
int((d+I*c*d*x)^4*(a+b*atan(c*x))/x^4,x)
```

output
```
(d**4*(6*atan(c*x)*b*c**4*x**4 - 12*atan(c*x)*b*c**3*i*x**3 + 36*atan(c*x)
*b*c**2*x**2 - 12*atan(c*x)*b*c*i*x - 2*atan(c*x)*b - 24*int(atan(c*x)/x,x
)*b*c**3*i*x**3 + 16*log(c**2*x**2 + 1)*b*c**3*x**3 - 24*log(x)*a*c**3*i*x
**3 - 38*log(x)*b*c**3*x**3 + 6*a*c**4*x**4 + 36*a*c**2*x**2 - 12*a*c*i*x
- 2*a - 12*b*c**2*i*x**2 - b*c*x))/(6*x**3)
```

# 3.39 $\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^5}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 227

$$\begin{aligned}\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^5}\,dx = &-\frac{bcd^4}{12x^3}-\frac{2ibc^2d^4}{3x^2}+\frac{13bc^3d^4}{4x}+\frac{13}{4}bc^4d^4\arctan(cx)\\ &-\frac{d^4(a+b\arctan(cx))}{4x^4}-\frac{4icd^4(a+b\arctan(cx))}{3x^3}\\ &+\frac{3c^2d^4(a+b\arctan(cx))}{x^2}\\ &+\frac{4ic^3d^4(a+b\arctan(cx))}{x}+ac^4d^4\log(x)\\ &-\frac{16}{3}ibc^4d^4\log(x)+\frac{8}{3}ibc^4d^4\log\left(1+c^2x^2\right)\\ &+\frac{1}{2}ibc^4d^4\operatorname{PolyLog}(2,-icx)\\ &-\frac{1}{2}ibc^4d^4\operatorname{PolyLog}(2,icx)\end{aligned}$$

output
```
-1/12*b*c*d^4/x^3-2/3*I*b*c^2*d^4/x^2+13/4*b*c^3*d^4/x+13/4*b*c^4*d^4*arct
an(c*x)-1/4*d^4*(a+b*arctan(c*x))/x^4-4/3*I*c*d^4*(a+b*arctan(c*x))/x^3+3*
c^2*d^4*(a+b*arctan(c*x))/x^2+4*I*c^3*d^4*(a+b*arctan(c*x))/x+a*c^4*d^4*ln
(x)-16/3*I*b*c^4*d^4*ln(x)+8/3*I*b*c^4*d^4*ln(c^2*x^2+1)+1/2*I*b*c^4*d^4*p
olylog(2,-I*c*x)-1/2*I*b*c^4*d^4*polylog(2,I*c*x)
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.08 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.00

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^5}\,dx$$
$$= \frac{d^4(-3a-16iacx+36ac^2x^2-8ibc^2x^2+48iac^3x^3-3b\arctan(cx)-16ibcx\arctan(cx)+36bc^2x^2\arctan$$

input
```
Integrate[((d + I*c*d*x)^4*(a + b*ArcTan[c*x]))/x^5,x]
```

output
```
(d^4*(-3*a - (16*I)*a*c*x + 36*a*c^2*x^2 - (8*I)*b*c^2*x^2 + (48*I)*a*c^3*
x^3 - 3*b*ArcTan[c*x] - (16*I)*b*c*x*ArcTan[c*x] + 36*b*c^2*x^2*ArcTan[c*x
] + (48*I)*b*c^3*x^3*ArcTan[c*x] - b*c*x*Hypergeometric2F1[-3/2, 1, -1/2,
-(c^2*x^2)] + 36*b*c^3*x^3*Hypergeometric2F1[-1/2, 1, 1/2, -(c^2*x^2)] + 1
2*a*c^4*x^4*Log[x] - (64*I)*b*c^4*x^4*Log[x] + (32*I)*b*c^4*x^4*Log[1 + c^
2*x^2] + (6*I)*b*c^4*x^4*PolyLog[2, (-I)*c*x] - (6*I)*b*c^4*x^4*PolyLog[2,
 I*c*x]))/(12*x^4)
```

## Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^5}\,dx$$
$$\Big\downarrow \text{5411}$$
$$\int \left(\frac{c^4d^4(a+b\arctan(cx))}{x}-\frac{4ic^3d^4(a+b\arctan(cx))}{x^2}-\frac{6c^2d^4(a+b\arctan(cx))}{x^3}+\frac{d^4(a+b\arctan(cx))}{x^5}+\frac{4ic$$
$$\Big\downarrow \text{2009}$$

$$\frac{4ic^3d^4(a+b\arctan(cx))}{x}+\frac{3c^2d^4(a+b\arctan(cx))}{x^2}-\frac{d^4(a+b\arctan(cx))}{4x^4}-\frac{4icd^4(a+b\arctan(cx))}{3x^3}+ac^4d^4\log(x)+\frac{13}{4}bc^4d^4\arctan(cx)+\frac{1}{2}ibc^4d^4\operatorname{PolyLog}(2,-icx)-\frac{1}{2}ibc^4d^4\operatorname{PolyLog}(2,icx)-\frac{16}{3}ibc^4d^4\log(x)+\frac{13bc^3d^4}{4x}-\frac{2ibc^2d^4}{3x^2}+\frac{8}{3}ibc^4d^4\log\left(c^2x^2+1\right)-\frac{bcd^4}{12x^3}$$

input
```
Int[((d + I*c*d*x)^4*(a + b*ArcTan[c*x]))/x^5,x]
```

output
```
-1/12*(b*c*d^4)/x^3 - (((2*I)/3)*b*c^2*d^4)/x^2 + (13*b*c^3*d^4)/(4*x) + (
13*b*c^4*d^4*ArcTan[c*x])/4 - (d^4*(a + b*ArcTan[c*x]))/(4*x^4) - (((4*I)/
3)*c*d^4*(a + b*ArcTan[c*x]))/x^3 + (3*c^2*d^4*(a + b*ArcTan[c*x]))/x^2 +
((4*I)*c^3*d^4*(a + b*ArcTan[c*x]))/x + a*c^4*d^4*Log[x] - ((16*I)/3)*b*c^
4*d^4*Log[x] + ((8*I)/3)*b*c^4*d^4*Log[1 + c^2*x^2] + (I/2)*b*c^4*d^4*Poly
Log[2, (-I)*c*x] - (I/2)*b*c^4*d^4*PolyLog[2, I*c*x]
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

## Maple [A] (verified)

Time = 1.13 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.94

| method | result |
|---|---|
| parts | $d^4 a\left(\frac{3c^2}{x^2}-\frac{4ic}{3x^3}+\frac{4ic^3}{x}-\frac{1}{4x^4}+c^4\ln(x)\right)+d^4 b\, c^4\left(\frac{4i\arctan(cx)}{cx}+\frac{3\arctan(cx)}{c^2x^2}-\frac{4i\arctan(cx)}{3c^3x^3}\right.$ |
| derivativedivides | $c^4\left(d^4 a\left(\frac{4i}{cx}+\frac{3}{c^2x^2}-\frac{4i}{3c^3x^3}-\frac{1}{4c^4x^4}+\ln(cx)\right)+d^4 b\left(\frac{4i\arctan(cx)}{cx}+\frac{3\arctan(cx)}{c^2x^2}-\frac{4i\arctan(cx)}{3c^3x^3}\right.\right.$ |
| default | $c^4\left(d^4 a\left(\frac{4i}{cx}+\frac{3}{c^2x^2}-\frac{4i}{3c^3x^3}-\frac{1}{4c^4x^4}+\ln(cx)\right)+d^4 b\left(\frac{4i\arctan(cx)}{cx}+\frac{3\arctan(cx)}{c^2x^2}-\frac{4i\arctan(cx)}{3c^3x^3}\right.\right.$ |
| risch | $-\frac{id^4 b\ln(-icx+1)}{8x^4}-\frac{103id^4c^4b\ln(-icx)}{24}+\frac{13b\,c^4d^4\arctan(cx)}{4}+\frac{ib\,d^4\ln(icx+1)}{8x^4}-\frac{d^4a}{4x^4}-\frac{id^4c^4b\,\mathrm{dilog}(-icx}{2}$ |

input
```
int((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^5,x,method=_RETURNVERBOSE)
```

output
```
d^4*a*(3*c^2/x^2-4/3*I*c/x^3+4*I*c^3/x-1/4/x^4+c^4*ln(x))+d^4*b*c^4*(4*I*a
rctan(c*x)/c/x+3/c^2/x^2*arctan(c*x)-4/3*I*arctan(c*x)/c^3/x^3-1/4/c^4/x^4
*arctan(c*x)+ln(c*x)*arctan(c*x)+8/3*I*ln(c^2*x^2+1)+13/4*arctan(c*x)-2/3*
I/c^2/x^2-16/3*I*ln(c*x)-1/12/c^3/x^3+13/4/c/x+1/2*I*ln(c*x)*ln(1+I*c*x)-1
/2*I*ln(c*x)*ln(1-I*c*x)+1/2*I*dilog(1+I*c*x)-1/2*I*dilog(1-I*c*x))
```

## Fricas [F]

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^5}\,dx=\int \frac{(i\,cdx+d)^4(b\arctan(cx)+a)}{x^5}\,dx$$

input
```
integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^5,x, algorithm="fricas")
```

output
```
integral(1/2*(2*a*c^4*d^4*x^4 - 8*I*a*c^3*d^4*x^3 - 12*a*c^2*d^4*x^2 + 8*I
*a*c*d^4*x + 2*a*d^4 + (I*b*c^4*d^4*x^4 + 4*b*c^3*d^4*x^3 - 6*I*b*c^2*d^4*
x^2 - 4*b*c*d^4*x + I*b*d^4)*log(-(c*x + I)/(c*x - I)))/x^5, x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^5}\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)**4*(a+b*atan(c*x))/x**5,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^5}\,dx = \int \frac{(i\,cdx+d)^4(b\arctan(cx)+a)}{x^5}\,dx$$

input
```
integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^5,x, algorithm="maxima")
```

output
```
b*c^4*d^4*integrate(arctan(c*x)/x, x) + a*c^4*d^4*log(x) + 2*I*(c*(log(c^2
*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b*c^3*d^4 + 3*((c*arctan(c*x) + 1
/x)*c + arctan(c*x)/x^2)*b*c^2*d^4 + 2/3*I*((c^2*log(c^2*x^2 + 1) - c^2*lo
g(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*b*c*d^4 + 4*I*a*c^3*d^4/x + 1/12*((
3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan(c*x)/x^4)*b*d^4 + 3*
a*c^2*d^4/x^2 - 4/3*I*a*c*d^4/x^3 - 1/4*a*d^4/x^4
```

## Giac [F]

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^5}\,dx = \int \frac{(i\,cdx+d)^4(b\arctan(cx)+a)}{x^5}\,dx$$

input
```
integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^5,x, algorithm="giac")
```

output
```
integrate((I*c*d*x + d)^4*(b*arctan(c*x) + a)/x^5, x)
```

## Mupad [B] (verification not implemented)

Time = 1.26 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.31

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^5}\,dx$$
$$=\left\{ 3\,b\,c\,d^4\left(c^3\operatorname{atan}(c\,x)+\frac{c^2}{x}\right)-\frac{b\,d^4\left(\frac{\frac{c^2}{3}-c^4\,x^2}{x^3}-c^5\operatorname{atan}(c\,x)\right)}{4\,c}-\frac{b\,c^4\,d^4\operatorname{Li}_2(1-c\,x\,1\mathrm{i})\,1\mathrm{i}}{2}+\frac{b\,c^4\,d^4\operatorname{Li}_2(1+c\,x\,1\mathrm{i})\,1\mathrm{i}}{2}-b\,c^2\,d^4\right.$$

input
```
int(((a + b*atan(c*x))*(d + c*d*x*1i)^4)/x^5,x)
```

output
```
piecewise(c == 0, -(a*d^4)/(4*x^4), c ~= 0, - (b*d^4*(c^4*log(x) - (c^4*lo
g(- (c^4*(3*c^2*x^2 + 1))/2 - c^4))/2 + c^2/(2*x^2))*4i)/3 - (b*d^4*((c^2/
3 - c^4*x^2)/x^3 - c^5*atan(c*x)))/(4*c) - (b*c^4*d^4*dilog(- c*x*1i + 1)*
1i)/2 + (b*c^4*d^4*dilog(c*x*1i + 1)*1i)/2 - b*c^2*d^4*(c^2*log(x) - (c^2*
log(c^2*x^2 + 1))/2)*4i + (a*d^4*(- c*x*16i + 36*c^2*x^2 + c^3*x^3*48i + 1
2*c^4*x^4*log(x) - 3))/(12*x^4) - (b*d^4*atan(c*x))/(4*x^4) + 3*b*c*d^4*(c
^3*atan(c*x) + c^2/x) - (b*c*d^4*atan(c*x)*4i)/(3*x^3) + (3*b*c^2*d^4*atan
(c*x))/x^2 + (b*c^3*d^4*atan(c*x)*4i)/x)
```

## Reduce [F]

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^5}\,dx$$
$$=\frac{d^4\left(39atan(cx)\,b\,c^4x^4+48atan(cx)\,b\,c^3i\,x^3+36atan(cx)\,b\,c^2x^2-16atan(cx)\,bcix-3atan(cx)\,b+12\left(\int\right.\right.}{}$$

input
```
int((d+I*c*d*x)^4*(a+b*atan(c*x))/x^5,x)
```

output
```
(d**4*(39*atan(c*x)*b*c**4*x**4 + 48*atan(c*x)*b*c**3*i*x**3 + 36*atan(c*x
)*b*c**2*x**2 - 16*atan(c*x)*b*c*i*x - 3*atan(c*x)*b + 12*int(atan(c*x)/x,
x)*b*c**4*x**4 + 32*log(c**2*x**2 + 1)*b*c**4*i*x**4 + 12*log(x)*a*c**4*x*
*4 - 64*log(x)*b*c**4*i*x**4 + 48*a*c**3*i*x**3 + 36*a*c**2*x**2 - 16*a*c*
i*x - 3*a + 39*b*c**3*x**3 - 8*b*c**2*i*x**2 - b*c*x))/(12*x**4)
```

# 3.40 $\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^6}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 117

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^6}\,dx = -\frac{bcd^4}{20x^4}-\frac{ibc^2d^4}{3x^3}+\frac{11bc^3d^4}{10x^2}+\frac{3ibc^4d^4}{x} - \frac{d^4(1+icx)^5(a+b\arctan(cx))}{5x^5} + \frac{16}{5}bc^5d^4\log(x)-\frac{16}{5}bc^5d^4\log(i+cx)$$

output
```
-1/20*b*c*d^4/x^4-1/3*I*b*c^2*d^4/x^3+11/10*b*c^3*d^4/x^2+3*I*b*c^4*d^4/x-
1/5*d^4*(1+I*c*x)^5*(a+b*arctan(c*x))/x^5+16/5*b*c^5*d^4*ln(x)-16/5*b*c^5*
d^4*ln(I+c*x)
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.10 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.63

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^6}\,dx = -\frac{d^4\left(20ibc^2x^2\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-c^2x^2\right)+3\left(4a+20iacx+bcx-40ac^2x^2-40iac^3x^3-\right.\right.}{}$$

input
```
Integrate[((d + I*c*d*x)^4*(a + b*ArcTan[c*x]))/x^6,x]
```

output
```
-1/60*(d^4*((20*I)*b*c^2*x^2*Hypergeometric2F1[-3/2, 1, -1/2, -(c^2*x^2)]
+ 3*(4*a + (20*I)*a*c*x + b*c*x - 40*a*c^2*x^2 - (40*I)*a*c^3*x^3 - 22*b*c
^3*x^3 + 20*a*c^4*x^4 + 4*b*(1 + (5*I)*c*x - 10*c^2*x^2 - (10*I)*c^3*x^3 +
 5*c^4*x^4)*ArcTan[c*x] - (40*I)*b*c^4*x^4*Hypergeometric2F1[-1/2, 1, 1/2,
 -(c^2*x^2)] - 64*b*c^5*x^5*Log[x] + 32*b*c^5*x^5*Log[1 + c^2*x^2])))/x^5
```

## Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.87, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {5407, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^6}\,dx$$

$$\Big\downarrow \text{5407}$$

$$-bc\int -\frac{id^4(i-cx)^4}{5x^5(cx+i)}dx-\frac{d^4(1+icx)^5(a+b\arctan(cx))}{5x^5}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}ibcd^4\int \frac{(i-cx)^4}{x^5(cx+i)}dx-\frac{d^4(1+icx)^5(a+b\arctan(cx))}{5x^5}$$

$$\Big\downarrow \text{99}$$

$$\frac{1}{5}ibcd^4\int \left(\frac{16ic^5}{cx+i}-\frac{16ic^4}{x}-\frac{15c^3}{x^2}+\frac{11ic^2}{x^3}+\frac{5c}{x^4}-\frac{i}{x^5}\right)dx-\frac{d^4(1+icx)^5(a+b\arctan(cx))}{5x^5}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}ibcd^4\left(-16ic^4\log(x)+16ic^4\log(cx+i)+\frac{15c^3}{x}-\frac{11ic^2}{2x^2}-\frac{5c}{3x^3}+\frac{i}{4x^4}\right)-\frac{d^4(1+icx)^5(a+b\arctan(cx))}{5x^5}$$

input
```
Int[((d + I*c*d*x)^4*(a + b*ArcTan[c*x]))/x^6,x]
```

output
```
-1/5*(d^4*(1 + I*c*x)^5*(a + b*ArcTan[c*x]))/x^5 + (I/5)*b*c*d^4*((I/4)/x^
4 - (5*c)/(3*x^3) - (((11*I)/2)*c^2)/x^2 + (15*c^3)/x - (16*I)*c^4*Log[x]
+ (16*I)*c^4*Log[I + c*x])
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5407
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Simp[(a
 + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ
[2*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0])
)
```

## Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.50

| method | result |
|---|---|
| parts | $d^4a\left(\frac{2ic^3}{x^2}+\frac{2c^2}{x^3}-\frac{c^4}{x}-\frac{1}{5x^5}-\frac{ic}{x^4}\right)+d^4b\,c^5\left(-\frac{\arctan(cx)}{cx}-\frac{\arctan(cx)}{5c^5x^5}+\frac{2\arctan(cx)}{c^3x^3}+\frac{2i\arctan}{c^2x^2}\right.$ |
| derivativedivides | $c^5\left(d^4a\left(-\frac{1}{cx}-\frac{1}{5c^5x^5}+\frac{2}{c^3x^3}+\frac{2i}{c^2x^2}-\frac{i}{c^4x^4}\right)+d^4b\left(-\frac{\arctan(cx)}{cx}-\frac{\arctan(cx)}{5c^5x^5}+\frac{2\arctan(cx)}{c^3x^3}+\right.\right.$ |
| default | $c^5\left(d^4a\left(-\frac{1}{cx}-\frac{1}{5c^5x^5}+\frac{2}{c^3x^3}+\frac{2i}{c^2x^2}-\frac{i}{c^4x^4}\right)+d^4b\left(-\frac{\arctan(cx)}{cx}-\frac{\arctan(cx)}{5c^5x^5}+\frac{2\arctan(cx)}{c^3x^3}+\right.\right.$ |
| parallelrisch | $\frac{180ix^4b\,c^4d^4-60ix\arctan(cx)bc\,d^4+192b\,c^5d^4\ln(x)x^5-96b\,c^5d^4\ln(c^2x^2+1)x^5-66b\,c^5d^4x^5+180ic^5b\,d^4\arctan(cx)x^5}{}$ |
| risch | $\frac{id^4b(5c^4x^4-10ic^3x^3-10c^2x^2+5icx+1)\ln(icx+1)}{10x^5}-\frac{d^4(186b\,c^5\ln(-cx-i)x^5+6b\,c^5\ln(cx-i)x^5-192b\,c^5\ln(-x)x^5-}{}$ |

input
```
int((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^6,x,method=_RETURNVERBOSE)
```

output
```
d^4*a*(2*I*c^3/x^2+2*c^2/x^3-c^4/x-1/5/x^5-I*c/x^4)+d^4*b*c^5*(-1/c/x*arct
an(c*x)-1/5/c^5/x^5*arctan(c*x)+2/c^3/x^3*arctan(c*x)+2*I*arctan(c*x)/c^2/
x^2-I*arctan(c*x)/c^4/x^4-8/5*ln(c^2*x^2+1)+3*I*arctan(c*x)-1/3*I/c^3/x^3+
3*I/c/x-1/20/c^4/x^4+11/10/c^2/x^2+16/5*ln(c*x))
```

## Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(97) = 194.

Time = 0.14 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.73

$$\int\frac{(d+icdx)^4(a+b\arctan(cx))}{x^6}\,dx$$
$$=\frac{192\,bc^5d^4x^5\log(x)-186\,bc^5d^4x^5\log\left(\frac{cx+i}{c}\right)-6\,bc^5d^4x^5\log\left(\frac{cx-i}{c}\right)-60\,(a-3i\,b)c^4d^4x^4-6\,(-20i\,a-11}{}$$

input
```
integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^6,x, algorithm="fricas")
```

output
```
1/60*(192*b*c^5*d^4*x^5*log(x) - 186*b*c^5*d^4*x^5*log((c*x + I)/c) - 6*b*
c^5*d^4*x^5*log((c*x - I)/c) - 60*(a - 3*I*b)*c^4*d^4*x^4 - 6*(-20*I*a - 1
1*b)*c^3*d^4*x^3 + 20*(6*a - I*b)*c^2*d^4*x^2 - 3*(20*I*a + b)*c*d^4*x - 1
2*a*d^4 - 6*(5*I*b*c^4*d^4*x^4 + 10*b*c^3*d^4*x^3 - 10*I*b*c^2*d^4*x^2 - 5
*b*c*d^4*x + I*b*d^4)*log(-(c*x + I)/(c*x - I)))/x^5
```

## Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(114) = 228$.

Time = 43.47 (sec) , antiderivative size = 366, normalized size of antiderivative = 3.13

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^6}\,dx$$
$$= \frac{16bc^5d^4\log\left(10395b^2c^{11}d^8x\right)}{5} - \frac{bc^5d^4\log\left(10395b^2c^{11}d^8x - 10395ib^2c^{10}d^8\right)}{10}$$
$$- \frac{31bc^5d^4\log\left(10395b^2c^{11}d^8x + 10395ib^2c^{10}d^8\right)}{10}$$
$$+ \frac{-12ad^4 + x^4(-60ac^4d^4 + 180ibc^4d^4) + x^3\cdot(120iac^3d^4 + 66bc^3d^4) + x^2\cdot(120ac^2d^4 - 20ibc^2d^4) + x(-}{60x^5}$$
$$+ \frac{(-5ibc^4d^4x^4 - 10bc^3d^4x^3 + 10ibc^2d^4x^2 + 5bcd^4x - ibd^4)\log(-icx+1)}{10x^5}$$
$$+ \frac{(5ibc^4d^4x^4 + 10bc^3d^4x^3 - 10ibc^2d^4x^2 - 5bcd^4x + ibd^4)\log(icx+1)}{10x^5}$$

input
```
integrate((d+I*c*d*x)**4*(a+b*atan(c*x))/x**6,x)
```

output
```
16*b*c**5*d**4*log(10395*b**2*c**11*d**8*x)/5 - b*c**5*d**4*log(10395*b**2
*c**11*d**8*x - 10395*I*b**2*c**10*d**8)/10 - 31*b*c**5*d**4*log(10395*b**
2*c**11*d**8*x + 10395*I*b**2*c**10*d**8)/10 + (-12*a*d**4 + x**4*(-60*a*c
**4*d**4 + 180*I*b*c**4*d**4) + x**3*(120*I*a*c**3*d**4 + 66*b*c**3*d**4)
+ x**2*(120*a*c**2*d**4 - 20*I*b*c**2*d**4) + x*(-60*I*a*c*d**4 - 3*b*c*d*
*4))/(60*x**5) + (-5*I*b*c**4*d**4*x**4 - 10*b*c**3*d**4*x**3 + 10*I*b*c**
2*d**4*x**2 + 5*b*c*d**4*x - I*b*d**4)*log(-I*c*x + 1)/(10*x**5) + (5*I*b*
c**4*d**4*x**4 + 10*b*c**3*d**4*x**3 - 10*I*b*c**2*d**4*x**2 - 5*b*c*d**4*
x + I*b*d**4)*log(I*c*x + 1)/(10*x**5)
```

## Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 275 vs. $2(97) = 194$.

Time = 0.13 (sec) , antiderivative size = 275, normalized size of antiderivative = 2.35

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^6}\,dx$$
$$= -\frac{1}{2}\left(c(\log\left(c^2x^2+1\right)-\log\left(x^2\right))+\frac{2\arctan\left(cx\right)}{x}\right)bc^4d^4$$
$$+2i\left(\left(c\arctan\left(cx\right)+\frac{1}{x}\right)c+\frac{\arctan\left(cx\right)}{x^2}\right)bc^3d^4$$
$$-\left(\left(c^2\log\left(c^2x^2+1\right)-c^2\log\left(x^2\right)-\frac{1}{x^2}\right)c-\frac{2\arctan\left(cx\right)}{x^3}\right)bc^2d^4$$
$$-\frac{ac^4d^4}{x}+\frac{1}{3}i\left(\left(3\,c^3\arctan\left(cx\right)+\frac{3\,c^2x^2-1}{x^3}\right)c-\frac{3\arctan\left(cx\right)}{x^4}\right)bcd^4$$
$$-\frac{1}{20}\left(\left(2\,c^4\log\left(c^2x^2+1\right)-2\,c^4\log\left(x^2\right)-\frac{2\,c^2x^2-1}{x^4}\right)c+\frac{4\arctan\left(cx\right)}{x^5}\right)bd^4$$
$$+\frac{2i\,ac^3d^4}{x^2}+\frac{2\,ac^2d^4}{x^3}-\frac{i\,acd^4}{x^4}-\frac{ad^4}{5\,x^5}$$

input

```
integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^6,x, algorithm="maxima")
```

output

```
-1/2*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b*c^4*d^4 + 2*I*(
(c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b*c^3*d^4 - ((c^2*log(c^2*x^2 +
 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*b*c^2*d^4 - a*c^4*d^4/x
 + 1/3*I*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan(c*x)/x^4)
*b*c*d^4 - 1/20*((2*c^4*log(c^2*x^2 + 1) - 2*c^4*log(x^2) - (2*c^2*x^2 - 1
)/x^4)*c + 4*arctan(c*x)/x^5)*b*d^4 + 2*I*a*c^3*d^4/x^2 + 2*a*c^2*d^4/x^3
- I*a*c*d^4/x^4 - 1/5*a*d^4/x^5
```

## Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 220 vs. $2(97) = 194$.

Time = 0.18 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.88

$$\int \frac{(d + icdx)^4(a + b\arctan(cx))}{x^6}\,dx =$$
$$-\frac{186\,bc^5d^4x^5\log(cx+i) + 6\,bc^5d^4x^5\log(cx-i) - 192\,bc^5d^4x^5\log(x) + 60\,bc^4d^4x^4\arctan(cx) + 60\,ac}{}$$

input
```
integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^6,x, algorithm="giac")
```

output
```
-1/60*(186*b*c^5*d^4*x^5*log(c*x + I) + 6*b*c^5*d^4*x^5*log(c*x - I) - 192
*b*c^5*d^4*x^5*log(x) + 60*b*c^4*d^4*x^4*arctan(c*x) + 60*a*c^4*d^4*x^4 -
180*I*b*c^4*d^4*x^4 - 120*I*b*c^3*d^4*x^3*arctan(c*x) - 120*I*a*c^3*d^4*x^
3 - 66*b*c^3*d^4*x^3 - 120*b*c^2*d^4*x^2*arctan(c*x) - 120*a*c^2*d^4*x^2 +
 20*I*b*c^2*d^4*x^2 + 60*I*b*c*d^4*x*arctan(c*x) + 60*I*a*c*d^4*x + 3*b*c*
d^4*x + 12*b*d^4*arctan(c*x) + 12*a*d^4)/x^5
```

## Mupad [B] (verification not implemented)

Time = 1.05 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.59

$$\int \frac{(d + icdx)^4(a + b\arctan(cx))}{x^6}\,dx$$
$$= \frac{d^4\,(192\,b\,c^5\,\ln(x) - 96\,b\,c^5\,\ln(c^2\,x^2+1) + b\,c^5\,\mathrm{atan}(c\,x)\,180\mathrm{i})}{60}$$
$$-\frac{\frac{d^4\,(12\,a+12\,b\,\mathrm{atan}(c\,x))}{60} + \frac{d^4\,x\,(a\,c\,60\mathrm{i}+3\,b\,c+b\,c\,\mathrm{atan}(c\,x)\,60\mathrm{i})}{60} - \frac{d^4\,x^2\,(120\,a\,c^2+120\,b\,c^2\,\mathrm{atan}(c\,x)-b\,c^2\,20\mathrm{i})}{60} + \frac{d^4\,x^4\,(60\,a\,c^4+60\,b\,c^4\,\mathrm{a}}{60}}{x^5}$$

input
```
int(((a + b*atan(c*x))*(d + c*d*x*1i)^4)/x^6,x)
```

output
```
(d^4*(b*c^5*atan(c*x)*180i - 96*b*c^5*log(c^2*x^2 + 1) + 192*b*c^5*log(x))
)/60 - ((d^4*(12*a + 12*b*atan(c*x)))/60 + (d^4*x*(a*c*60i + 3*b*c + b*c*a
tan(c*x)*60i))/60 - (d^4*x^2*(120*a*c^2 - b*c^2*20i + 120*b*c^2*atan(c*x))
)/60 + (d^4*x^4*(60*a*c^4 - b*c^4*180i + 60*b*c^4*atan(c*x)))/60 - (d^4*x^
3*(a*c^3*120i + 66*b*c^3 + b*c^3*atan(c*x)*120i))/60)/x^5
```

## Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.55

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^6}\,dx$$
$$= \frac{d^4(180atan(cx)\,b\,c^5i\,x^5-60atan(cx)\,b\,c^4x^4+120atan(cx)\,b\,c^3i\,x^3+120atan(cx)\,b\,c^2x^2-60atan(cx)\,bc}{}$$

input
```
int((d+I*c*d*x)^4*(a+b*atan(c*x))/x^6,x)
```

output
```
(d**4*(180*atan(c*x)*b*c**5*i*x**5 - 60*atan(c*x)*b*c**4*x**4 + 120*atan(c
*x)*b*c**3*i*x**3 + 120*atan(c*x)*b*c**2*x**2 - 60*atan(c*x)*b*c*i*x - 12*
atan(c*x)*b - 96*log(c**2*x**2 + 1)*b*c**5*x**5 + 192*log(x)*b*c**5*x**5 -
 60*a*c**4*x**4 + 120*a*c**3*i*x**3 + 120*a*c**2*x**2 - 60*a*c*i*x - 12*a
+ 180*b*c**4*i*x**4 + 66*b*c**3*x**3 - 20*b*c**2*i*x**2 - 3*b*c*x))/(60*x*
*5)
```

# 3.41 $\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^7}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 168

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^7}\,dx = -\frac{bcd^4}{30x^5}-\frac{ibc^2d^4}{5x^4}+\frac{5bc^3d^4}{9x^3}+\frac{16ibc^4d^4}{15x^2} -\frac{13bc^5d^4}{6x}-\frac{d^4(1+icx)^5(a+b\arctan(cx))}{6x^6} +\frac{icd^4(1+icx)^5(a+b\arctan(cx))}{30x^5} +\frac{32}{15}ibc^6d^4\log(x)-\frac{32}{15}ibc^6d^4\log(i+cx)$$

output
```
-1/30*b*c*d^4/x^5-1/5*I*b*c^2*d^4/x^4+5/9*b*c^3*d^4/x^3+16/15*I*b*c^4*d^4/
x^2-13/6*b*c^5*d^4/x-1/6*d^4*(1+I*c*x)^5*(a+b*arctan(c*x))/x^6+1/30*I*c*d^
4*(1+I*c*x)^5*(a+b*arctan(c*x))/x^5+32/15*I*b*c^6*d^4*ln(x)-32/15*I*b*c^6*
d^4*ln(I+c*x)
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.40

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^7}\,dx =$$
$$-\frac{d^4\left(5a+24iacx-45ac^2x^2+6ibc^2x^2-40iac^3x^3+15ac^4x^4-32ibc^4x^4+5b\arctan(cx)+24ibcx\operatorname{arcta}\right.}{}$$

input
```
Integrate[((d + I*c*d*x)^4*(a + b*ArcTan[c*x]))/x^7,x]
```

output
```
-1/30*(d^4*(5*a + (24*I)*a*c*x - 45*a*c^2*x^2 + (6*I)*b*c^2*x^2 - (40*I)*a
*c^3*x^3 + 15*a*c^4*x^4 - (32*I)*b*c^4*x^4 + 5*b*ArcTan[c*x] + (24*I)*b*c*
x*ArcTan[c*x] - 45*b*c^2*x^2*ArcTan[c*x] - (40*I)*b*c^3*x^3*ArcTan[c*x] +
15*b*c^4*x^4*ArcTan[c*x] + b*c*x*Hypergeometric2F1[-5/2, 1, -3/2, -(c^2*x^
2)] - 15*b*c^3*x^3*Hypergeometric2F1[-3/2, 1, -1/2, -(c^2*x^2)] + 15*b*c^5
*x^5*Hypergeometric2F1[-1/2, 1, 1/2, -(c^2*x^2)] - (64*I)*b*c^6*x^6*Log[x]
 + (32*I)*b*c^6*x^6*Log[1 + c^2*x^2]))/x^6
```

## Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.80, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {5407, 27, 165, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^7}\,dx$$
$$\Big\downarrow \; 5407$$
$$-bc\int -\frac{d^4(i-cx)^4(cx+5i)}{30x^6(cx+i)}dx-\frac{d^4(1+icx)^5(a+b\arctan(cx))}{6x^6}+\frac{icd^4(1+icx)^5(a+b\arctan(cx))}{30x^5}$$

$$\downarrow 27$$

$$\frac{1}{30}bcd^4\int \frac{(i-cx)^4(cx+5i)}{x^6(cx+i)}dx-\frac{d^4(1+icx)^5(a+b\arctan(cx))}{6x^6}+\frac{icd^4(1+icx)^5(a+b\arctan(cx))}{30x^5}$$

$$\downarrow 165$$

$$\frac{1}{30}bcd^4\int \left(-\frac{64ic^6}{cx+i}+\frac{64ic^5}{x}+\frac{65c^4}{x^2}-\frac{64ic^3}{x^3}-\frac{50c^2}{x^4}+\frac{24ic}{x^5}+\frac{5}{x^6}\right)dx-\frac{d^4(1+icx)^5(a+b\arctan(cx))}{6x^6}+\frac{icd^4(1+icx)^5(a+b\arctan(cx))}{30x^5}$$

$$\downarrow 2009$$

$$-\frac{d^4(1+icx)^5(a+b\arctan(cx))}{6x^6}+\frac{icd^4(1+icx)^5(a+b\arctan(cx))}{30x^5}+\frac{1}{30}bcd^4\left(64ic^5\log(x)-64ic^5\log(cx+i)-\frac{65c^4}{x}+\frac{32ic^3}{x^2}+\frac{50c^2}{3x^3}-\frac{6ic}{x^4}-\frac{1}{x^5}\right)$$

input
```
Int[((d + I*c*d*x)^4*(a + b*ArcTan[c*x]))/x^7,x]
```

output
```
-1/6*(d^4*(1 + I*c*x)^5*(a + b*ArcTan[c*x]))/x^6 + ((I/30)*c*d^4*(1 + I*c*
x)^5*(a + b*ArcTan[c*x]))/x^5 + (b*c*d^4*(-x^(-5) - ((6*I)*c)/x^4 + (50*c^
2)/(3*x^3) + ((32*I)*c^3)/x^2 - (65*c^4)/x + (64*I)*c^5*Log[x] - (64*I)*c^
5*Log[I + c*x]))/30
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 165
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*
x)^n*(e + f*x)^p*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m}, x]
 && (IntegersQ[m, n, p] || (IGtQ[n, 0] && IGtQ[p, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5407
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Simp[(a
 + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ
[2*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0])
)
```

## Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.10

| method | result |
|---|---|
| parts | $d^4 a\left(-\frac{c^4}{2x^2}+\frac{4ic^3}{3x^3}-\frac{1}{6x^6}-\frac{4ic}{5x^5}+\frac{3c^2}{2x^4}\right)+d^4 b\, c^6\left(-\frac{\arctan(cx)}{6c^6x^6}-\frac{\arctan(cx)}{2c^2x^2}-\frac{4i\arctan(cx)}{5c^5x^5}+\frac{4i}{}\right.$ |
| derivativedivides | $c^6\left(d^4 a\left(-\frac{1}{6c^6x^6}-\frac{1}{2c^2x^2}-\frac{4i}{5c^5x^5}+\frac{4i}{3c^3x^3}+\frac{3}{2c^4x^4}\right)+d^4 b\left(-\frac{\arctan(cx)}{6c^6x^6}-\frac{\arctan(cx)}{2c^2x^2}-\frac{4i\arctan(}{5c^5x^5}\right.\right.$ |
| default | $c^6\left(d^4 a\left(-\frac{1}{6c^6x^6}-\frac{1}{2c^2x^2}-\frac{4i}{5c^5x^5}+\frac{4i}{3c^3x^3}+\frac{3}{2c^4x^4}\right)+d^4 b\left(-\frac{\arctan(cx)}{6c^6x^6}-\frac{\arctan(cx)}{2c^2x^2}-\frac{4i\arctan(}{5c^5x^5}\right.\right.$ |
| parallelrisch | $\frac{-96ib\,c^6d^4\ln(c^2x^2+1)x^6+96ix^4b\,c^4d^4-72ix\arctan(cx)bc\,d^4-195c^6d^4b\arctan(cx)x^6+45a\,c^6d^4x^6-195b\,c^5d^4x^5+192}{}$ |
| risch | $\frac{id^4b(15c^4x^4-40ic^3x^3-45c^2x^2+24icx+5)\ln(icx+1)}{60x^6}+\frac{id^4(384b\,c^6\ln(-32639cx)x^6-387b\,c^6\ln(-16705cx-16705i)}{}$ |

input
```
int((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^7,x,method=_RETURNVERBOSE)
```

output
```
d^4*a*(-1/2*c^4/x^2+4/3*I*c^3/x^3-1/6/x^6-4/5*I*c/x^5+3/2*c^2/x^4)+d^4*b*c
^6*(-1/6*arctan(c*x)/c^6/x^6-1/2/c^2/x^2*arctan(c*x)-4/5*I*arctan(c*x)/c^5
/x^5+4/3*I*arctan(c*x)/c^3/x^3+3/2/c^4/x^4*arctan(c*x)-16/15*I*ln(c^2*x^2+
1)-13/6*arctan(c*x)+32/15*I*ln(c*x)-1/5*I/c^4/x^4+16/15*I/c^2/x^2-1/30/c^5
/x^5+5/9/c^3/x^3-13/6/c/x)
```

## Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.29

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^7}\,dx$$
$$= \frac{384i\,bc^6d^4x^6\log(x) - 387i\,bc^6d^4x^6\log\left(\frac{cx+i}{c}\right) + 3i\,bc^6d^4x^6\log\left(\frac{cx-i}{c}\right) - 390\,bc^5d^4x^5 - 6\,(15\,a-32i\,b)c^4d^4}{}$$

input
```
integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^7,x, algorithm="fricas")
```

output
```
1/180*(384*I*b*c^6*d^4*x^6*log(x) - 387*I*b*c^6*d^4*x^6*log((c*x + I)/c) +
 3*I*b*c^6*d^4*x^6*log((c*x - I)/c) - 390*b*c^5*d^4*x^5 - 6*(15*a - 32*I*b
)*c^4*d^4*x^4 - 20*(-12*I*a - 5*b)*c^3*d^4*x^3 + 18*(15*a - 2*I*b)*c^2*d^4
*x^2 - 6*(24*I*a + b)*c*d^4*x - 30*a*d^4 - 3*(15*I*b*c^4*d^4*x^4 + 40*b*c^
3*d^4*x^3 - 45*I*b*c^2*d^4*x^2 - 24*b*c*d^4*x + 5*I*b*d^4)*log(-(c*x + I)/
(c*x - I)))/x^6
```

## Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 388 vs. 2(163) = 326.

Time = 81.58 (sec) , antiderivative size = 388, normalized size of antiderivative = 2.31

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^7}\,dx$$
$$= \frac{32ibc^6d^4\log(2121535b^2c^{13}d^8x)}{15} + \frac{ibc^6d^4\log(2121535b^2c^{13}d^8x - 2121535ib^2c^{12}d^8)}{60}$$
$$- \frac{43ibc^6d^4\log(2121535b^2c^{13}d^8x + 2121535ib^2c^{12}d^8)}{20}$$
$$+ \frac{(-15ibc^4d^4x^4 - 40bc^3d^4x^3 + 45ibc^2d^4x^2 + 24bcd^4x - 5ibd^4)\log(-icx+1)}{60x^6}$$
$$+ \frac{(15ibc^4d^4x^4 + 40bc^3d^4x^3 - 45ibc^2d^4x^2 - 24bcd^4x + 5ibd^4)\log(icx+1)}{60x^6}$$
$$+ \frac{-15ad^4 - 195bc^5d^4x^5 + x^4(-45ac^4d^4 + 96ibc^4d^4) + x^3\cdot(120iac^3d^4 + 50bc^3d^4) + x^2\cdot(135ac^2d^4 - 18i}{90x^6}$$

input
```
integrate((d+I*c*d*x)**4*(a+b*atan(c*x))/x**7,x)
```

output
```
32*I*b*c**6*d**4*log(2121535*b**2*c**13*d**8*x)/15 + I*b*c**6*d**4*log(212
1535*b**2*c**13*d**8*x - 2121535*I*b**2*c**12*d**8)/60 - 43*I*b*c**6*d**4*
log(2121535*b**2*c**13*d**8*x + 2121535*I*b**2*c**12*d**8)/20 + (-15*I*b*c
**4*d**4*x**4 - 40*b*c**3*d**4*x**3 + 45*I*b*c**2*d**4*x**2 + 24*b*c*d**4*
x - 5*I*b*d**4)*log(-I*c*x + 1)/(60*x**6) + (15*I*b*c**4*d**4*x**4 + 40*b*
c**3*d**4*x**3 - 45*I*b*c**2*d**4*x**2 - 24*b*c*d**4*x + 5*I*b*d**4)*log(I
*c*x + 1)/(60*x**6) + (-15*a*d**4 - 195*b*c**5*d**4*x**5 + x**4*(-45*a*c**
4*d**4 + 96*I*b*c**4*d**4) + x**3*(120*I*a*c**3*d**4 + 50*b*c**3*d**4) + x
**2*(135*a*c**2*d**4 - 18*I*b*c**2*d**4) + x*(-72*I*a*c*d**4 - 3*b*c*d**4)
)/(90*x**6)
```

## Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 290 vs. 2(134) = 268.

Time = 0.13 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.73

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^7}\,dx$$
$$= -\frac{1}{2}\left(\left(c\arctan(cx)+\frac{1}{x}\right)c+\frac{\arctan(cx)}{x^2}\right)bc^4d^4 - \frac{2}{3}i\left(\left(c^2\log\left(c^2x^2+1\right)-c^2\log\left(x^2\right)-\frac{1}{x^2}\right)c-\frac{2\arctan(cx)}{x^3}\right)bc^3d^4 - \frac{1}{2}\left(\left(3c^3\arctan(cx)+\frac{3c^2x^2-1}{x^3}\right)c-\frac{3\arctan(cx)}{x^4}\right)bc^2d^4 - \frac{1}{5}i\left(\left(2c^4\log\left(c^2x^2+1\right)-2c^4\log\left(x^2\right)-\frac{2c^2x^2-1}{x^4}\right)c+\frac{4\arctan(cx)}{x^5}\right)bcd^4 - \frac{ac^4d^4}{2x^2} - \frac{1}{90}\left(\left(15c^5\arctan(cx)+\frac{15c^4x^4-5c^2x^2+3}{x^5}\right)c+\frac{15\arctan(cx)}{x^6}\right)bd^4 + \frac{4i\,ac^3d^4}{3x^3} + \frac{3ac^2d^4}{2x^4} - \frac{4i\,acd^4}{5x^5} - \frac{ad^4}{6x^6}$$

input
```
integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^7,x, algorithm="maxima")
```

output
```
-1/2*((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b*c^4*d^4 - 2/3*I*((c^2*l
og(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*b*c^3*d^4 -
 1/2*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan(c*x)/x^4)*b*c
^2*d^4 - 1/5*I*((2*c^4*log(c^2*x^2 + 1) - 2*c^4*log(x^2) - (2*c^2*x^2 - 1)
/x^4)*c + 4*arctan(c*x)/x^5)*b*c*d^4 - 1/2*a*c^4*d^4/x^2 - 1/90*((15*c^5*a
rctan(c*x) + (15*c^4*x^4 - 5*c^2*x^2 + 3)/x^5)*c + 15*arctan(c*x)/x^6)*b*d
^4 + 4/3*I*a*c^3*d^4/x^3 + 3/2*a*c^2*d^4/x^4 - 4/5*I*a*c*d^4/x^5 - 1/6*a*d
^4/x^6
```

## Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.39

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^7}\,dx = -\frac{-3i\,bc^6d^4x^6\log(i\,cx+1)+387i\,bc^6d^4x^6\log(-i\,cx+1)-384i\,bc^6d^4x^6\log(x)+390\,bc^5d^4x^5+90\,bc^4c$$

input
```
integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^7,x, algorithm="giac")
```

output
```
-1/180*(-3*I*b*c^6*d^4*x^6*log(I*c*x + 1) + 387*I*b*c^6*d^4*x^6*log(-I*c*x
 + 1) - 384*I*b*c^6*d^4*x^6*log(x) + 390*b*c^5*d^4*x^5 + 90*b*c^4*d^4*x^4*
arctan(c*x) + 90*a*c^4*d^4*x^4 - 192*I*b*c^4*d^4*x^4 - 240*I*b*c^3*d^4*x^3
*arctan(c*x) - 240*I*a*c^3*d^4*x^3 - 100*b*c^3*d^4*x^3 - 270*b*c^2*d^4*x^2
*arctan(c*x) - 270*a*c^2*d^4*x^2 + 36*I*b*c^2*d^4*x^2 + 144*I*b*c*d^4*x*ar
ctan(c*x) + 144*I*a*c*d^4*x + 6*b*c*d^4*x + 30*b*d^4*arctan(c*x) + 30*a*d^
4)/x^6
```

## Mupad [B] (verification not implemented)

Time = 1.24 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.24

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^7}\,dx$$

$$= -\frac{d^4\left(195\,b\,c^5\,\mathrm{atan}\left(x\sqrt{c^2}\right)\sqrt{c^2}+b\,c^6\,\ln\left(c^2\,x^2+1\right)\,96\mathrm{i}-b\,c^6\,\ln\left(x\right)\,192\mathrm{i}\right)}{90} - \frac{\frac{d^4\,(15\,a+15\,b\,\mathrm{atan}(c\,x))}{90}+\frac{d^4\,x\,(a\,c\,72\mathrm{i}+3\,b\,c+b\,c\,\mathrm{atan}(c\,x)\,72\mathrm{i})}{90}+\frac{d^4\,x^4\,\left(45\,a\,c^4+45\,b\,c^4\,\mathrm{atan}(c\,x)-b\,c^4\,96\mathrm{i}\right)}{90}-\frac{d^4\,x^2\,\left(135\,a\,c^2+135\,b\,c^2\,a\right.}{90}}{x^6}$$

input
```
int(((a + b*atan(c*x))*(d + c*d*x*1i)^4)/x^7,x)
```

output
```
- (d^4*(b*c^6*log(c^2*x^2 + 1)*96i - b*c^6*log(x)*192i + 195*b*c^5*atan(x*
(c^2)^(1/2))*(c^2)^(1/2)))/90 - ((d^4*(15*a + 15*b*atan(c*x)))/90 + (d^4*x
*(a*c*72i + 3*b*c + b*c*atan(c*x)*72i))/90 + (d^4*x^4*(45*a*c^4 - b*c^4*96
i + 45*b*c^4*atan(c*x)))/90 - (d^4*x^2*(135*a*c^2 - b*c^2*18i + 135*b*c^2*
atan(c*x)))/90 - (d^4*x^3*(a*c^3*120i + 50*b*c^3 + b*c^3*atan(c*x)*120i))/
90 + (13*b*c^5*d^4*x^5)/6)/x^6
```

## Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.14

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^7}\,dx$$
$$= \frac{d^4(-195atan(cx)\,bc^6x^6 - 45atan(cx)\,bc^4x^4 + 120atan(cx)\,bc^3i\,x^3 + 135atan(cx)\,bc^2x^2 - 72atan(cx)\,bc$$

input
```
int((d+I*c*d*x)^4*(a+b*atan(c*x))/x^7,x)
```

output
```
(d**4*( - 195*atan(c*x)*b*c**6*x**6 - 45*atan(c*x)*b*c**4*x**4 + 120*atan(
c*x)*b*c**3*i*x**3 + 135*atan(c*x)*b*c**2*x**2 - 72*atan(c*x)*b*c*i*x - 15
*atan(c*x)*b - 96*log(c**2*x**2 + 1)*b*c**6*i*x**6 + 192*log(x)*b*c**6*i*x
**6 - 45*a*c**4*x**4 + 120*a*c**3*i*x**3 + 135*a*c**2*x**2 - 72*a*c*i*x -
15*a - 195*b*c**5*x**5 + 96*b*c**4*i*x**4 + 50*b*c**3*x**3 - 18*b*c**2*i*x
**2 - 3*b*c*x))/(90*x**6)
```

# 3.42 $\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^8}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 242

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^8}\,dx = -\frac{bcd^4}{42x^6}-\frac{2ibc^2d^4}{15x^5}+\frac{47bc^3d^4}{140x^4}+\frac{5ibc^4d^4}{9x^3}-\frac{88bc^5d^4}{105x^2}-\frac{5ibc^6d^4}{3x}-\frac{5}{3}ibc^7d^4\arctan(cx)-\frac{d^4(a+b\arctan(cx))}{7x^7}-\frac{2icd^4(a+b\arctan(cx))}{3x^6}+\frac{6c^2d^4(a+b\arctan(cx))}{5x^5}+\frac{ic^3d^4(a+b\arctan(cx))}{x^4}-\frac{c^4d^4(a+b\arctan(cx))}{3x^3}-\frac{176}{105}bc^7d^4\log(x)+\frac{88}{105}bc^7d^4\log\left(1+c^2x^2\right)$$

output

```
-1/42*b*c*d^4/x^6-2/15*I*b*c^2*d^4/x^5+47/140*b*c^3*d^4/x^4+5/9*I*b*c^4*d^
4/x^3-88/105*b*c^5*d^4/x^2-5/3*I*b*c^6*d^4/x-5/3*I*b*c^7*d^4*arctan(c*x)-1
/7*d^4*(a+b*arctan(c*x))/x^7-2/3*I*c*d^4*(a+b*arctan(c*x))/x^6+6/5*c^2*d^4
*(a+b*arctan(c*x))/x^5+I*c^3*d^4*(a+b*arctan(c*x))/x^4-1/3*c^4*d^4*(a+b*ar
ctan(c*x))/x^3-176/105*b*c^7*d^4*ln(x)+88/105*b*c^7*d^4*ln(c^2*x^2+1)
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.21

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^8}\,dx = -\frac{ad^4}{7x^7}-\frac{2iacd^4}{3x^6}-\frac{bcd^4}{42x^6}+\frac{6ac^2d^4}{5x^5}+\frac{iac^3d^4}{x^4} +\frac{47bc^3d^4}{140x^4}-\frac{ac^4d^4}{3x^3}-\frac{88bc^5d^4}{105x^2}-\frac{bd^4\arctan(cx)}{7x^7} -\frac{2ibcd^4\arctan(cx)}{3x^6}+\frac{6bc^2d^4\arctan(cx)}{5x^5} +\frac{ibc^3d^4\arctan(cx)}{x^4}-\frac{bc^4d^4\arctan(cx)}{3x^3} -\frac{2ibc^2d^4\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},-c^2x^2\right)}{15x^5} +\frac{ibc^4d^4\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-c^2x^2\right)}{3x^3} -\frac{176}{105}bc^7d^4\log(x)+\frac{88}{105}bc^7d^4\log\left(1+c^2x^2\right)$$

input
```
Integrate[((d + I*c*d*x)^4*(a + b*ArcTan[c*x]))/x^8,x]
```

output
```
-1/7*(a*d^4)/x^7 - (((2*I)/3)*a*c*d^4)/x^6 - (b*c*d^4)/(42*x^6) + (6*a*c^2
*d^4)/(5*x^5) + (I*a*c^3*d^4)/x^4 + (47*b*c^3*d^4)/(140*x^4) - (a*c^4*d^4)
/(3*x^3) - (88*b*c^5*d^4)/(105*x^2) - (b*d^4*ArcTan[c*x])/(7*x^7) - (((2*I
)/3)*b*c*d^4*ArcTan[c*x])/x^6 + (6*b*c^2*d^4*ArcTan[c*x])/(5*x^5) + (I*b*c
^3*d^4*ArcTan[c*x])/x^4 - (b*c^4*d^4*ArcTan[c*x])/(3*x^3) - (((2*I)/15)*b*
c^2*d^4*Hypergeometric2F1[-5/2, 1, -3/2, -(c^2*x^2)])/x^5 + ((I/3)*b*c^4*d
^4*Hypergeometric2F1[-3/2, 1, -1/2, -(c^2*x^2)])/x^3 - (176*b*c^7*d^4*Log[
x])/105 + (88*b*c^7*d^4*Log[1 + c^2*x^2])/105
```

## Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.85, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {5407, 27, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^8}dx$$

$$\Big\downarrow \text{5407}$$

$$-bc\int -\frac{d^4(35c^4x^4-105ic^3x^3-126c^2x^2+70icx+15)}{105x^7(c^2x^2+1)}dx-\frac{c^4d^4(a+b\arctan(cx))}{3x^3}+\frac{ic^3d^4(a+b\arctan(cx))}{x^4}+\frac{6c^2d^4(a+b\arctan(cx))}{5x^5}-\frac{d^4(a+b\arctan(cx))}{7x^7}-\frac{2icd^4(a+b\arctan(cx))}{3x^6}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{105}bcd^4\int \frac{35c^4x^4-105ic^3x^3-126c^2x^2+70icx+15}{x^7(c^2x^2+1)}dx-\frac{c^4d^4(a+b\arctan(cx))}{3x^3}+\frac{ic^3d^4(a+b\arctan(cx))}{x^4}+\frac{6c^2d^4(a+b\arctan(cx))}{5x^5}-\frac{d^4(a+b\arctan(cx))}{7x^7}-\frac{2icd^4(a+b\arctan(cx))}{3x^6}$$

$$\Big\downarrow \text{2333}$$

$$\frac{1}{105}bcd^4\int \left(\frac{c^7}{2(cx-i)}+\frac{351c^7}{2(cx+i)}-\frac{176c^6}{x}+\frac{175ic^5}{x^2}+\frac{176c^4}{x^3}-\frac{175ic^3}{x^4}-\frac{141c^2}{x^5}+\frac{70ic}{x^6}+\frac{15}{x^7}\right)dx-\frac{c^4d^4(a+b\arctan(cx))}{3x^3}+\frac{ic^3d^4(a+b\arctan(cx))}{x^4}+\frac{6c^2d^4(a+b\arctan(cx))}{5x^5}-\frac{d^4(a+b\arctan(cx))}{7x^7}-\frac{2icd^4(a+b\arctan(cx))}{3x^6}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c^4d^4(a+b\arctan(cx))}{3x^3}+\frac{ic^3d^4(a+b\arctan(cx))}{x^4}+\frac{6c^2d^4(a+b\arctan(cx))}{5x^5}-\frac{d^4(a+b\arctan(cx))}{7x^7}-\frac{2icd^4(a+b\arctan(cx))}{3x^6}+\frac{1}{105}bcd^4\left(-176c^6\log(x)+\frac{1}{2}c^6\log(-cx+i)+\frac{351}{2}c^6\log(cx+i)-\frac{175ic^5}{x}-\frac{88c^4}{x^2}+\frac{175ic^3}{3x^3}+\frac{141c^2}{4x^4}-\frac{14ic}{x^5}-\right.$$

input
```
Int[((d + I*c*d*x)^4*(a + b*ArcTan[c*x]))/x^8,x]
```

output
```
-1/7*(d^4*(a + b*ArcTan[c*x]))/x^7 - (((2*I)/3)*c*d^4*(a + b*ArcTan[c*x]))
/x^6 + (6*c^2*d^4*(a + b*ArcTan[c*x]))/(5*x^5) + (I*c^3*d^4*(a + b*ArcTan[
c*x]))/x^4 - (c^4*d^4*(a + b*ArcTan[c*x]))/(3*x^3) + (b*c*d^4*(-5/(2*x^6)
- ((14*I)*c)/x^5 + (141*c^2)/(4*x^4) + (((175*I)/3)*c^3)/x^3 - (88*c^4)/x^
2 - ((175*I)*c^5)/x - 176*c^6*Log[x] + (c^6*Log[I - c*x])/2 + (351*c^6*Log
[I + c*x])/2))/105
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 5407
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Simp[(a
 + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ
[2*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0])
)
```

## Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.79

| method | result |
|---|---|
| parts | $d^4a\left(-\frac{c^4}{3x^3}-\frac{1}{7x^7}-\frac{2ic}{3x^6}+\frac{6c^2}{5x^5}+\frac{ic^3}{x^4}\right)+d^4b\,c^7\Big(-\frac{2i\arctan(cx)}{3c^6x^6}-\frac{\arctan(cx)}{3c^3x^3}+\frac{i\arctan(cx)}{c^4x^4}-$ arc |
| derivativedivides | $c^7\Big(d^4a\left(-\frac{2i}{3c^6x^6}-\frac{1}{3c^3x^3}+\frac{i}{c^4x^4}-\frac{1}{7c^7x^7}+\frac{6}{5c^5x^5}\right)+d^4b\Big(-\frac{2i\arctan(cx)}{3c^6x^6}-\frac{\arctan(cx)}{3c^3x^3}+\frac{i\arctan(}{c^4x^4}$ |
| default | $c^7\Big(d^4a\left(-\frac{2i}{3c^6x^6}-\frac{1}{3c^3x^3}+\frac{i}{c^4x^4}-\frac{1}{7c^7x^7}+\frac{6}{5c^5x^5}\right)+d^4b\Big(-\frac{2i\arctan(cx)}{3c^6x^6}-\frac{\arctan(cx)}{3c^3x^3}+\frac{i\arctan(}{c^4x^4}$ |
| parallelrisch | $-\frac{-700ix^4b\,c^4d^4+2112b\,c^7d^4\ln(x)x^7-1056b\,c^7d^4\ln(c^2x^2+1)x^7-1056b\,c^7d^4x^7+840ix\arctan(cx)bc\,d^4+1056b\,c^5d^4x}{}$ |
| risch | $\frac{id^4b(35c^4x^4-105ic^3x^3-126c^2x^2+70icx+15)\ln(icx+1)}{210x^7}+\frac{d^4(6b\,c^7\ln(cx-i)x^7+2106b\,c^7\ln(-cx-i)x^7-2112b\,c^7\ln(}{}$ |

input
```
int((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^8,x,method=_RETURNVERBOSE)
```

output
```
d^4*a*(-1/3*c^4/x^3-1/7/x^7-2/3*I*c/x^6+6/5*c^2/x^5+I*c^3/x^4)+d^4*b*c^7*(
-2/3*I*arctan(c*x)/c^6/x^6-1/3/c^3/x^3*arctan(c*x)+I*arctan(c*x)/c^4/x^4-1
/7*arctan(c*x)/c^7/x^7+6/5/c^5/x^5*arctan(c*x)+88/105*ln(c^2*x^2+1)-5/3*I*
arctan(c*x)-5/3*I/c/x-2/15*I/c^5/x^5+5/9*I/c^3/x^3-1/42/c^6/x^6+47/140/c^4
/x^4-88/105/c^2/x^2-176/105*ln(c*x))
```

## Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.94

$$\int\frac{(d+icdx)^4(a+b\arctan(cx))}{x^8}\,dx=$$
$$-\frac{2112\,bc^7d^4x^7\log{(x)}-2106\,bc^7d^4x^7\log{\left(\frac{cx+i}{c}\right)}-6\,bc^7d^4x^7\log{\left(\frac{cx-i}{c}\right)}+2100i\,bc^6d^4x^6+1056\,bc^5d^4x^5+}{}$$

input
```
integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^8,x, algorithm="fricas")
```

output
```
-1/1260*(2112*b*c^7*d^4*x^7*log(x) - 2106*b*c^7*d^4*x^7*log((c*x + I)/c) -
 6*b*c^7*d^4*x^7*log((c*x - I)/c) + 2100*I*b*c^6*d^4*x^6 + 1056*b*c^5*d^4*
x^5 + 140*(3*a - 5*I*b)*c^4*d^4*x^4 + 9*(-140*I*a - 47*b)*c^3*d^4*x^3 - 16
8*(9*a - I*b)*c^2*d^4*x^2 + 30*(28*I*a + b)*c*d^4*x + 180*a*d^4 + 6*(35*I*
b*c^4*d^4*x^4 + 105*b*c^3*d^4*x^3 - 126*I*b*c^2*d^4*x^2 - 70*b*c*d^4*x + 1
5*I*b*d^4)*log(-(c*x + I)/(c*x - I)))/x^7
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^8}\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)**4*(a+b*atan(c*x))/x**8,x)
```

output
```
Timed out
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.36

$$\begin{aligned}
&\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^8}\,dx \\
&= \frac{1}{6}\left(\left(c^2\log\left(c^2x^2+1\right)-c^2\log\left(x^2\right)-\frac{1}{x^2}\right)c-\frac{2\arctan\left(cx\right)}{x^3}\right)bc^4d^4 \\
&\quad -\frac{1}{3}i\left(\left(3\,c^3\arctan\left(cx\right)+\frac{3\,c^2x^2-1}{x^3}\right)c-\frac{3\arctan\left(cx\right)}{x^4}\right)bc^3d^4 \\
&\quad +\frac{3}{10}\left(\left(2\,c^4\log\left(c^2x^2+1\right)-2\,c^4\log\left(x^2\right)-\frac{2\,c^2x^2-1}{x^4}\right)c+\frac{4\arctan\left(cx\right)}{x^5}\right)bc^2d^4 \\
&\quad -\frac{2}{45}i\left(\left(15\,c^5\arctan\left(cx\right)+\frac{15\,c^4x^4-5\,c^2x^2+3}{x^5}\right)c+\frac{15\arctan\left(cx\right)}{x^6}\right)bcd^4 \\
&\quad +\frac{1}{84}\left(\left(6\,c^6\log\left(c^2x^2+1\right)-6\,c^6\log\left(x^2\right)-\frac{6\,c^4x^4-3\,c^2x^2+2}{x^6}\right)c-\frac{12\arctan\left(cx\right)}{x^7}\right)bd^4 \\
&\quad -\frac{ac^4d^4}{3\,x^3}+\frac{i\,ac^3d^4}{x^4}+\frac{6\,ac^2d^4}{5\,x^5}-\frac{2i\,acd^4}{3\,x^6}-\frac{ad^4}{7\,x^7}
\end{aligned}$$

input
```
integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^8,x, algorithm="maxima")
```

output
```
1/6*((c^2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*
b*c^4*d^4 - 1/3*I*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan(
c*x)/x^4)*b*c^3*d^4 + 3/10*((2*c^4*log(c^2*x^2 + 1) - 2*c^4*log(x^2) - (2*
c^2*x^2 - 1)/x^4)*c + 4*arctan(c*x)/x^5)*b*c^2*d^4 - 2/45*I*((15*c^5*arcta
n(c*x) + (15*c^4*x^4 - 5*c^2*x^2 + 3)/x^5)*c + 15*arctan(c*x)/x^6)*b*c*d^4
 + 1/84*((6*c^6*log(c^2*x^2 + 1) - 6*c^6*log(x^2) - (6*c^4*x^4 - 3*c^2*x^2
 + 2)/x^6)*c - 12*arctan(c*x)/x^7)*b*d^4 - 1/3*a*c^4*d^4/x^3 + I*a*c^3*d^4
/x^4 + 6/5*a*c^2*d^4/x^5 - 2/3*I*a*c*d^4/x^6 - 1/7*a*d^4/x^7
```

## Giac [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.01

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^8}\,dx = \frac{2106\,bc^7d^4x^7\log(cx+i)+6\,bc^7d^4x^7\log(cx-i)-2112\,bc^7d^4x^7\log(x)-2100i\,bc^6d^4x^6-1056\,bc^5d^4x^5 -}{}$$

input
```
integrate((d+I*c*d*x)^4*(a+b*arctan(c*x))/x^8,x, algorithm="giac")
```

output
```
1/1260*(2106*b*c^7*d^4*x^7*log(c*x + I) + 6*b*c^7*d^4*x^7*log(c*x - I) - 2
112*b*c^7*d^4*x^7*log(x) - 2100*I*b*c^6*d^4*x^6 - 1056*b*c^5*d^4*x^5 - 420
*b*c^4*d^4*x^4*arctan(c*x) - 420*a*c^4*d^4*x^4 + 700*I*b*c^4*d^4*x^4 + 126
0*I*b*c^3*d^4*x^3*arctan(c*x) + 1260*I*a*c^3*d^4*x^3 + 423*b*c^3*d^4*x^3 +
 1512*b*c^2*d^4*x^2*arctan(c*x) + 1512*a*c^2*d^4*x^2 - 168*I*b*c^2*d^4*x^2
 - 840*I*b*c*d^4*x*arctan(c*x) - 840*I*a*c*d^4*x - 30*b*c*d^4*x - 180*b*d^
4*arctan(c*x) - 180*a*d^4)/x^7
```

## Mupad [B] (verification not implemented)

Time = 1.52 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.31

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^8}\,dx = \frac{88\,b\,c^7\,d^4\,\ln\left(c^2\,x^2+1\right)}{105}$$
$$-\frac{\frac{a\,d^4}{7}+\frac{b\,d^4\,\mathrm{atan}(c\,x)}{7}+\frac{88\,b\,c^7\,d^4\,x^7}{105}+\frac{b\,c^8\,d^4\,x^8\,5\mathrm{i}}{3}+\frac{c\,d^4\,x\,(b+a\,28\mathrm{i})}{42}+\frac{c^6\,d^4\,x^6\,(3\,a+b\,10\mathrm{i})}{9}-\frac{c^4\,d^4\,x^4\,(39\,a+b\,19\mathrm{i})}{45}-\frac{c^2\,d^4\,x^2\,(11}{10}}{}$$
$$-\frac{176\,b\,c^7\,d^4\,\ln(x)}{105}-\frac{b\,c^{10}\,d^4\,\mathrm{atan}\left(\frac{c^2\,x}{\sqrt{c^2}}\right)\,5\mathrm{i}}{3\,\left(c^2\right)^{3/2}}$$

input
```
int(((a + b*atan(c*x))*(d + c*d*x*1i)^4)/x^8,x)
```

output
```
(88*b*c^7*d^4*log(c^2*x^2 + 1))/105 - ((a*d^4)/7 + (b*d^4*atan(c*x))/7 + (
88*b*c^7*d^4*x^7)/105 + (b*c^8*d^4*x^8*5i)/3 + (c*d^4*x*(a*28i + b))/42 +
(c^6*d^4*x^6*(3*a + b*10i))/9 - (c^4*d^4*x^4*(39*a + b*19i))/45 - (c^2*d^4
*x^2*(111*a - b*14i))/105 - (c^3*d^4*x^3*(a*140i + 131*b))/420 - c^5*d^4*x
^5*(a*1i - (211*b)/420) - (37*b*c^2*d^4*x^2*atan(c*x))/35 - (b*c^3*d^4*x^3
*atan(c*x)*1i)/3 - (13*b*c^4*d^4*x^4*atan(c*x))/15 - b*c^5*d^4*x^5*atan(c*
x)*1i + (b*c^6*d^4*x^6*atan(c*x))/3 + (b*c*d^4*x*atan(c*x)*2i)/3)/(x^7 + c
^2*x^9) - (176*b*c^7*d^4*log(x))/105 - (b*c^10*d^4*atan((c^2*x)/(c^2)^(1/2
))*5i)/(3*(c^2)^(3/2))
```

## Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.83

$$\int \frac{(d+icdx)^4(a+b\arctan(cx))}{x^8}\,dx$$
$$=\frac{d^4(-2100atan(cx)\,b\,c^7i\,x^7-420atan(cx)\,b\,c^4x^4+1260atan(cx)\,b\,c^3i\,x^3+1512atan(cx)\,b\,c^2x^2-840ata}{}$$

input
```
int((d+I*c*d*x)^4*(a+b*atan(c*x))/x^8,x)
```

output
```
(d**4*( - 2100*atan(c*x)*b*c**7*i*x**7 - 420*atan(c*x)*b*c**4*x**4 + 1260*
atan(c*x)*b*c**3*i*x**3 + 1512*atan(c*x)*b*c**2*x**2 - 840*atan(c*x)*b*c*i
*x - 180*atan(c*x)*b + 1056*log(c**2*x**2 + 1)*b*c**7*x**7 - 2112*log(x)*b
*c**7*x**7 - 420*a*c**4*x**4 + 1260*a*c**3*i*x**3 + 1512*a*c**2*x**2 - 840
*a*c*i*x - 180*a - 2100*b*c**6*i*x**6 - 1056*b*c**5*x**5 + 700*b*c**4*i*x*
*4 + 423*b*c**3*x**3 - 168*b*c**2*i*x**2 - 30*b*c*x))/(1260*x**7)
```

# 3.43 $\int \frac{x^3(a+b\arctan(cx))}{d+icdx}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 196

$$\begin{aligned}\int \frac{x^3(a+b\arctan(cx))}{d+icdx}\,dx =& \frac{iax}{c^3d}-\frac{bx}{2c^3d}+\frac{ibx^2}{6c^2d}+\frac{b\arctan(cx)}{2c^4d}\\&+\frac{ibx\arctan(cx)}{c^3d}+\frac{x^2(a+b\arctan(cx))}{2c^2d}\\&-\frac{ix^3(a+b\arctan(cx))}{3cd}+\frac{(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c^4d}\\&-\frac{2ib\log(1+c^2x^2)}{3c^4d}+\frac{ib\,\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2c^4d}\end{aligned}$$

output
```
I*a*x/c^3/d-1/2*b*x/c^3/d+1/6*I*b*x^2/c^2/d+1/2*b*arctan(c*x)/c^4/d+I*b*x*
arctan(c*x)/c^3/d+1/2*x^2*(a+b*arctan(c*x))/c^2/d-1/3*I*x^3*(a+b*arctan(c*
x))/c/d+(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^4/d-2/3*I*b*ln(c^2*x^2+1)/c^4/
d+1/2*I*b*polylog(2,1-2/(1+I*c*x))/c^4/d
```

## Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.85

$$\int \frac{x^3(a+b\arctan(cx))}{d+icdx}\,dx =$$
$$-\frac{i\left(-b-6acx-3ibcx+3iac^2x^2-bc^2x^2+2ac^3x^3+6b\arctan(cx)^2+\arctan(cx)\left(6a+b(3i-6cx+3\right.\right.}{}$$

input
```
Integrate[(x^3*(a + b*ArcTan[c*x]))/(d + I*c*d*x),x]
```

output
```
((-1/6*I)*(-b - 6*a*c*x - (3*I)*b*c*x + (3*I)*a*c^2*x^2 - b*c^2*x^2 + 2*a*
c^3*x^3 + 6*b*ArcTan[c*x]^2 + ArcTan[c*x]*(6*a + b*(3*I - 6*c*x + (3*I)*c^
2*x^2 + 2*c^3*x^3) + (6*I)*b*Log[1 + E^((2*I)*ArcTan[c*x])]) - (3*I)*a*Log
[1 + c^2*x^2] + 4*b*Log[1 + c^2*x^2] + 3*b*PolyLog[2, -E^((2*I)*ArcTan[c*x
])]))/(c^4*d)
```

## Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.10, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {5401, 27, 5361, 243, 49, 2009, 5401, 5361, 262, 216, 5401, 2009, 5379, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\arctan(cx))}{d+icdx}dx$$
$$\Big\downarrow \text{ 5401}$$
$$\frac{i\int \frac{x^2(a+b\arctan(cx))}{d(icx+1)}dx}{c}-\frac{i\int x^2(a+b\arctan(cx))dx}{cd}$$
$$\Big\downarrow \text{ 27}$$
$$\frac{i\int \frac{x^2(a+b\arctan(cx))}{icx+1}dx}{cd}-\frac{i\int x^2(a+b\arctan(cx))dx}{cd}$$

$$\Big\downarrow \text{5361}$$

$$\frac{i\int \frac{x^2(a+b\arctan(cx))}{icx+1}dx}{cd}-\frac{i\left(\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{3}bc\int \frac{x^3}{c^2x^2+1}dx\right)}{cd}$$

$$\Big\downarrow \text{243}$$

$$\frac{i\int \frac{x^2(a+b\arctan(cx))}{icx+1}dx}{cd}-\frac{i\left(\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{6}bc\int \frac{x^2}{c^2x^2+1}dx^2\right)}{cd}$$

$$\Big\downarrow \text{49}$$

$$\frac{i\int \frac{x^2(a+b\arctan(cx))}{icx+1}dx}{cd}-\frac{i\left(\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{6}bc\int \left(\frac{1}{c^2}-\frac{1}{c^2(c^2x^2+1)}\right)dx^2\right)}{cd}$$

$$\Big\downarrow \text{2009}$$

$$\frac{i\int \frac{x^2(a+b\arctan(cx))}{icx+1}dx}{cd}-\frac{i\left(\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{6}bc\left(\frac{x^2}{c^2}-\frac{\log(c^2x^2+1)}{c^4}\right)\right)}{cd}$$

$$\Big\downarrow \text{5401}$$

$$\frac{i\left(\frac{i\int \frac{x(a+b\arctan(cx))}{icx+1}dx}{c}-\frac{i\int x(a+b\arctan(cx))dx}{c}\right)}{cd}-\frac{i\left(\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{6}bc\left(\frac{x^2}{c^2}-\frac{\log(c^2x^2+1)}{c^4}\right)\right)}{cd}$$

$$\Big\downarrow \text{5361}$$

$$\frac{i\left(\frac{i\int \frac{x(a+b\arctan(cx))}{icx+1}dx}{c}-\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\int \frac{x^2}{c^2x^2+1}dx\right)}{c}\right)}{cd}-\frac{i\left(\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{6}bc\left(\frac{x^2}{c^2}-\frac{\log(c^2x^2+1)}{c^4}\right)\right)}{cd}$$

$$\Big\downarrow \text{262}$$

$$\frac{i\left(\frac{i\int \frac{x(a+b\arctan(cx))}{icx+1}dx}{c}-\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\int \frac{1}{c^2x^2+1}dx}{c^2}\right)\right)}{c}\right)}{cd}-\frac{i\left(\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{6}bc\left(\frac{x^2}{c^2}-\frac{\log(c^2x^2+1)}{c^4}\right)\right)}{cd}$$

$$\Big\downarrow \text{216}$$

$$\frac{i\left(\frac{i\int \frac{x(a+b\arctan(cx))}{icx+1}dx}{c}-\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)\right)}{c}\right)}{cd}-\frac{i\left(\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{6}bc\left(\frac{x^2}{c^2}-\frac{\log(c^2x^2+1)}{c^4}\right)\right)}{cd}$$

$$\Big\downarrow \text{ 5401}$$

$$\frac{i\left(\frac{i\left(\frac{i\int \frac{a+b\arctan(cx)}{icx+1}dx}{c}-\frac{i\int (a+b\arctan(cx))dx}{c}\right)}{c}-\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)\right)}{c}\right)}{cd}-\frac{i\left(\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{6}bc\left(\frac{x^2}{c^2}-\frac{\log(c^2x^2+1)}{c^4}\right)\right)}{cd}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{i\left(\frac{i\left(\frac{i\int \frac{a+b\arctan(cx)}{icx+1}dx}{c}-\frac{i\left(ax+bx\arctan(cx)-\frac{b\log\left(c^2x^2+1\right)}{2c}\right)}{c}\right)}{c}-\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)\right)}{c}\right)}{cd}-\frac{i\left(\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{6}bc\left(\frac{x^2}{c^2}-\frac{\log(c^2x^2+1)}{c^4}\right)\right)}{cd}$$

$$\Big\downarrow \text{ 5379}$$

$$\frac{i\left(\frac{i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}-ib\int \frac{\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{c}-\frac{i\left(ax+bx\arctan(cx)-\frac{b\log\left(c^2x^2+1\right)}{2c}\right)}{c}\right)}{c}-\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(}{c^3}\right.\right.}{c}\right.}{cd}-\frac{i\left(\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{6}bc\left(\frac{x^2}{c^2}-\frac{\log(c^2x^2+1)}{c^4}\right)\right)}{cd}$$

$$\Big\downarrow \text{ 2849}$$

$$\frac{i\left(\frac{i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}-\frac{b\int\frac{\log\left(\frac{2}{icx+1}\right)}{1-\frac{2}{icx+1}}d\frac{1}{icx+1}}{c}\right)}{c}-\frac{i\left(ax+bx\arctan(cx)-\frac{b\log\left(c^2x^2+1\right)}{2c}\right)}{c}\right)}{c}-\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\text{ar}}{}\right.\right.}{c}}{cd}$$

$$\frac{i\left(\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{6}bc\left(\frac{x^2}{c^2}-\frac{\log(c^2x^2+1)}{c^4}\right)\right)}{cd}$$

$\big\downarrow$ 2752

$$\frac{i\left(\frac{i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}-\frac{b\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{2c}\right)}{c}-\frac{i\left(ax+bx\arctan(cx)-\frac{b\log\left(c^2x^2+1\right)}{2c}\right)}{c}\right)}{c}-\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\text{arc}}{}\right.\right.}{c}}{cd}$$

$$\frac{i\left(\frac{1}{3}x^3(a+b\arctan(cx))-\frac{1}{6}bc\left(\frac{x^2}{c^2}-\frac{\log(c^2x^2+1)}{c^4}\right)\right)}{cd}$$

input
```
Int[(x^3*(a + b*ArcTan[c*x]))/(d + I*c*d*x),x]
```

output
```
((-I)*((x^3*(a + b*ArcTan[c*x]))/3 - (b*c*(x^2/c^2 - Log[1 + c^2*x^2]/c^4)
)/6))/(c*d) + (I*(((-I)*((x^2*(a + b*ArcTan[c*x]))/2 - (b*c*(x/c^2 - ArcTa
n[c*x]/c^3))/2))/c + (I*(((-I)*(a*x + b*x*ArcTan[c*x] - (b*Log[1 + c^2*x^2
])/(2*c)))/c + (I*((I*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/c - (b*PolyL
og[2, 1 - 2/(1 + I*c*x)])/(2*c)))/c))/c))/(c*d)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5401
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_) + (
e_.)*(x_)), x_Symbol] :> Simp[f/e   Int[(f*x)^(m - 1)*(a + b*ArcTan[c*x])^p
, x], x] - Simp[d*(f/e)   Int[(f*x)^(m - 1)*((a + b*ArcTan[c*x])^p/(d + e*x
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e
^2, 0] && GtQ[m, 0]
```

## Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.59

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{2ib}{3d}+\frac{ib\ln(cx-i)\ln\left(-\frac{i(cx+i)}{2}\right)}{2d}+\frac{a\,c^2x^2}{2d}-\frac{a\ln\left(c^2x^2+1\right)}{2d}+\frac{ib\,\mathrm{dilog}\left(-\frac{i(cx+i)}{2}\right)}{2d}+\frac{iacx}{d}+\frac{ib\,c^2x^2}{6d}+\frac{b\arctan(cx)c^2x^2}{2d}-\frac{b\arctan(c}{}}{}$ |
| default | $\frac{\frac{2ib}{3d}+\frac{ib\ln(cx-i)\ln\left(-\frac{i(cx+i)}{2}\right)}{2d}+\frac{a\,c^2x^2}{2d}-\frac{a\ln\left(c^2x^2+1\right)}{2d}+\frac{ib\,\mathrm{dilog}\left(-\frac{i(cx+i)}{2}\right)}{2d}+\frac{iacx}{d}+\frac{ib\,c^2x^2}{6d}+\frac{b\arctan(cx)c^2x^2}{2d}-\frac{b\arctan(c}{}}{}$ |
| risch | $\frac{ib\,\mathrm{dilog}\left(\frac{1}{2}-\frac{icx}{2}\right)}{2d\,c^4}-\frac{b\left(\frac{1}{3}x^3c^2+\frac{1}{2}ic\,x^2-x\right)\ln(icx+1)}{2c^3d}-\frac{ia\arctan(cx)}{d\,c^4}-\frac{bx}{2c^3d}-\frac{5a}{6d\,c^4}+\frac{iax}{c^3d}+\frac{b\ln(-icx+1)x^3}{6dc}$ |
| parts | $-\frac{ia\arctan(cx)}{d\,c^4}+\frac{a\,x^2}{2d\,c^2}-\frac{ib\ln(cx-i)^2}{4d\,c^4}-\frac{a\ln\left(c^2x^2+1\right)}{2d\,c^4}-\frac{5ib\ln\left(c^4x^4+10c^2x^2+9\right)}{48d\,c^4}+\frac{iax}{c^3d}+\frac{ib\,\mathrm{dilog}\left(-\frac{i(cx-}{2}\right)}{2d\,c^4}$ |

input
```
int(x^3*(a+b*arctan(c*x))/(d+I*c*d*x),x,method=_RETURNVERBOSE)
```

output
```
1/c^4*(2/3*I*b/d+1/2*I*b/d*ln(c*x-I)*ln(-1/2*I*(c*x+I))+1/2*a/d*c^2*x^2-1/
2*a/d*ln(c^2*x^2+1)+1/2*I*b/d*dilog(-1/2*I*(c*x+I))+I*a/d*c*x+1/6*I*b/d*c^
2*x^2+1/2*b/d*arctan(c*x)*c^2*x^2-b/d*arctan(c*x)*ln(c*x-I)-1/3*I*a/d*c^3*
x^3-I*a/d*arctan(c*x)-11/24*I*b/d*ln(c^2*x^2+1)-1/2*b/d*c*x+I*b/d*arctan(c
*x)*c*x-1/3*I*b/d*arctan(c*x)*c^3*x^3-1/4*I*b/d*ln(c*x-I)^2+5/24*b/d*arcta
n(1/2*c*x)-5/24*b/d*arctan(1/6*c^3*x^3+7/6*c*x)-5/12*b/d*arctan(1/2*c*x-1/
2*I)-5/48*I*b/d*ln(c^4*x^4+10*c^2*x^2+9)+11/12*b/d*arctan(c*x))
```

## Fricas [F]

$$\int \frac{x^3(a+b\arctan(cx))}{d+icdx}\,dx=\int \frac{(b\arctan(cx)+a)x^3}{i\,cdx+d}\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))/(d+I*c*d*x),x, algorithm="fricas")
```

output
```
integral(1/2*(b*x^3*log(-(c*x + I)/(c*x - I)) - 2*I*a*x^3)/(c*d*x - I*d),
x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\arctan(cx))}{d+icdx}\,dx=\text{Timed out}$$

input
```
integrate(x**3*(a+b*atan(c*x))/(d+I*c*d*x),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{x^3(a + b\arctan(cx))}{d + icdx}\, dx = \int \frac{(b\arctan(cx) + a)x^3}{i\, cdx + d}\, dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))/(d+I*c*d*x),x, algorithm="maxima")
```

output
```
-1/6*a*(I*(2*c^2*x^3 + 3*I*c*x^2 - 6*x)/(c^3*d) + 6*log(I*c*x + 1)/(c^4*d)
) - 1/72*(432*I*c^8*d*integrate(1/12*x^4*arctan(c*x)/(c^5*d*x^2 + c^3*d),
x) + 216*c^8*d*integrate(1/12*x^4*log(c^2*x^2 + 1)/(c^5*d*x^2 + c^3*d), x)
 - 432*c^7*d*integrate(1/12*x^3*arctan(c*x)/(c^5*d*x^2 + c^3*d), x) + 216*
I*c^7*d*integrate(1/12*x^3*log(c^2*x^2 + 1)/(c^5*d*x^2 + c^3*d), x) + 432*
c^5*d*integrate(1/12*x*arctan(c*x)/(c^5*d*x^2 + c^3*d), x) - 216*I*c^5*d*i
ntegrate(1/12*x*log(c^2*x^2 + 1)/(c^5*d*x^2 + c^3*d), x) + 4*c^3*x^3 - 216
*c^4*d*integrate(1/12*log(c^2*x^2 + 1)/(c^5*d*x^2 + c^3*d), x) + 3*I*c^2*x
^2 - 30*c*x - 6*(-2*I*c^3*x^3 + 3*c^2*x^2 + 6*I*c*x - 5)*arctan(c*x) + 18*
I*arctan(c*x)^2 - 3*(2*c^3*x^3 + 3*I*c^2*x^2 - 6*c*x + I)*log(c^2*x^2 + 1)
 + 9*I*log(c^2*x^2 + 1)^2 + 18*I*log(12*c^5*d*x^2 + 12*c^3*d))*b/(c^4*d)
```

## Giac [F]

$$\int \frac{x^3(a + b\arctan(cx))}{d + icdx}\, dx = \int \frac{(b\arctan(cx) + a)x^3}{i\, cdx + d}\, dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))/(d+I*c*d*x),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)*x^3/(I*c*d*x + d), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\arctan(cx))}{d+icdx}\,dx=\int \frac{x^3\,(a+b\,\mathrm{atan}(c\,x))}{d+c\,d\,x\,1\mathrm{i}}\,dx$$

input
```
int((x^3*(a + b*atan(c*x)))/(d + c*d*x*1i),x)
```

output
```
int((x^3*(a + b*atan(c*x)))/(d + c*d*x*1i), x)
```

## Reduce [F]

$$\int \frac{x^3(a+b\arctan(cx))}{d+icdx}\,dx = \frac{6\left(\int \frac{atan(cx)x^3}{cix+1}dx\right)b\,c^4-6\log(cix+1)\,a-2a\,c^3i\,x^3+3a\,c^2x^2+6acix}{6c^4d}$$

input
```
int(x^3*(a+b*atan(c*x))/(d+I*c*d*x),x)
```

output
```
(6*int((atan(c*x)*x**3)/(c*i*x + 1),x)*b*c**4 - 6*log(c*i*x + 1)*a - 2*a*c
**3*i*x**3 + 3*a*c**2*x**2 + 6*a*c*i*x)/(6*c**4*d)
```

# 3.44 $\int \frac{x^2(a+b\arctan(cx))}{d+icdx}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 156

$$\int \frac{x^2(a+b\arctan(cx))}{d+icdx}\,dx = \frac{ax}{c^2d}+\frac{ibx}{2c^2d}-\frac{ib\arctan(cx)}{2c^3d}+\frac{bx\arctan(cx)}{c^2d} - \frac{ix^2(a+b\arctan(cx))}{2cd}-\frac{i(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c^3d} - \frac{b\log(1+c^2x^2)}{2c^3d}+\frac{b\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2c^3d}$$

output
```
a*x/c^2/d+1/2*I*b*x/c^2/d-1/2*I*b*arctan(c*x)/c^3/d+b*x*arctan(c*x)/c^2/d-
1/2*I*x^2*(a+b*arctan(c*x))/c/d-I*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^3/d-
1/2*b*ln(c^2*x^2+1)/c^3/d+1/2*b*polylog(2,1-2/(1+I*c*x))/c^3/d
```

## Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.85

$$\int \frac{x^2(a+b\arctan(cx))}{d+icdx}\,dx = -\frac{-2acx-ibcx+iac^2x^2+2b\arctan(cx)^2+i\arctan(cx)\left(-2ia+b+2ibcx+bc^2x^2+2b\log\left(1+e^{2i\,\mathrm{arc}}\right.\right.}{2c^3d}$$

input
```
Integrate[(x^2*(a + b*ArcTan[c*x]))/(d + I*c*d*x),x]
```

output
```
-1/2*(-2*a*c*x - I*b*c*x + I*a*c^2*x^2 + 2*b*ArcTan[c*x]^2 + I*ArcTan[c*x]
*((-2*I)*a + b + (2*I)*b*c*x + b*c^2*x^2 + 2*b*Log[1 + E^((2*I)*ArcTan[c*x
])]) - I*a*Log[1 + c^2*x^2] + b*Log[1 + c^2*x^2] + b*PolyLog[2, -E^((2*I)*
ArcTan[c*x])])/(c^3*d)
```

## Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.99, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {5401, 27, 5361, 262, 216, 5401, 2009, 5379, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\arctan(cx))}{d+icdx}dx$$

$$\Big\downarrow \text{5401}$$

$$\frac{i\int \frac{x(a+b\arctan(cx))}{d(icx+1)}dx}{c}-\frac{i\int x(a+b\arctan(cx))dx}{cd}$$

$$\Big\downarrow \text{27}$$

$$\frac{i\int \frac{x(a+b\arctan(cx))}{icx+1}dx}{cd}-\frac{i\int x(a+b\arctan(cx))dx}{cd}$$

$$\Big\downarrow \text{5361}$$

$$\frac{i\int \frac{x(a+b\arctan(cx))}{icx+1}dx}{cd}-\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\int \frac{x^2}{c^2x^2+1}dx\right)}{cd}$$

$$\Big\downarrow \text{262}$$

$$\frac{i\int \frac{x(a+b\arctan(cx))}{icx+1}dx}{cd}-\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\int \frac{1}{c^2x^2+1}dx}{c^2}\right)\right)}{cd}$$

$$\Big\downarrow \text{216}$$

$$\frac{i\int \frac{x(a+b\arctan(cx))}{icx+1}dx}{cd}-\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)\right)}{cd}$$

$\Big\downarrow$ 5401

$$\frac{i\left(\frac{i\int \frac{a+b\arctan(cx)}{icx+1}dx}{c}-\frac{i\int(a+b\arctan(cx))dx}{c}\right)}{cd}-\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)\right)}{cd}$$

$\Big\downarrow$ 2009

$$\frac{i\left(\frac{i\int \frac{a+b\arctan(cx)}{icx+1}dx}{c}-\frac{i\left(ax+bx\arctan(cx)-\frac{b\log(c^2x^2+1)}{2c}\right)}{c}\right)}{cd}-\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)\right)}{cd}$$

$\Big\downarrow$ 5379

$$\frac{i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}-ib\int\frac{\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{c}-\frac{i\left(ax+bx\arctan(cx)-\frac{b\log(c^2x^2+1)}{2c}\right)}{c}\right)}{cd}-\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)\right)}{cd}$$

$\Big\downarrow$ 2849

$$\frac{i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}-\frac{b\int\frac{\log\left(\frac{2}{icx+1}\right)}{1-\frac{2}{icx+1}}d\frac{1}{icx+1}}{c}\right)}{c}-\frac{i\left(ax+bx\arctan(cx)-\frac{b\log(c^2x^2+1)}{2c}\right)}{c}\right)}{cd}-\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)\right)}{cd}$$

$\Big\downarrow$ 2752

$$\frac{i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}-\frac{b\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{2c}\right)}{c}-\frac{i\left(ax+bx\arctan(cx)-\frac{b\log\left(c^2x^2+1\right)}{2c}\right)}{c}\right)}{cd}-\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)\right)}{cd}$$

input
```
Int[(x^2*(a + b*ArcTan[c*x]))/(d + I*c*d*x),x]
```

output
```
((-I)*((x^2*(a + b*ArcTan[c*x]))/2 - (b*c*(x/c^2 - ArcTan[c*x]/c^3))/2))/(
c*d) + (I*(((-I)*(a*x + b*x*ArcTan[c*x] - (b*Log[1 + c^2*x^2])/(2*c)))/c +
 (I*((I*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/c - (b*PolyLog[2, 1 - 2/(1
 + I*c*x)])/(2*c)))/c))/(c*d)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5401
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_) + (
e_.)*(x_)), x_Symbol] :> Simp[f/e   Int[(f*x)^(m - 1)*(a + b*ArcTan[c*x])^p
, x], x] - Simp[d*(f/e)   Int[(f*x)^(m - 1)*((a + b*ArcTan[c*x])^p/(d + e*x
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e
^2, 0] && GtQ[m, 0]
```

## Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.71

| method | result |
|---|---|
| derivativedivides | $\frac{acx}{d}+\frac{ibcx}{2d}+\frac{ia\ln\left(c^2x^2+1\right)}{2d}-\frac{a\arctan(cx)}{d}+\frac{b\arctan(cx)cx}{d}-\frac{ia\,c^2x^2}{2d}+\frac{ib\arctan\left(\frac{cx}{2}-\frac{i}{2}\right)}{4d}+\frac{b\ln(cx-i)\ln\left(-\frac{i(cx+i)}{2}\right)}{2d}+\frac{b\,\mathrm{dilog}\left(\cdots\right.}{}$ |
| default | $\frac{acx}{d}+\frac{ibcx}{2d}+\frac{ia\ln\left(c^2x^2+1\right)}{2d}-\frac{a\arctan(cx)}{d}+\frac{b\arctan(cx)cx}{d}-\frac{ia\,c^2x^2}{2d}+\frac{ib\arctan\left(\frac{cx}{2}-\frac{i}{2}\right)}{4d}+\frac{b\ln(cx-i)\ln\left(-\frac{i(cx+i)}{2}\right)}{2d}+\frac{b\,\mathrm{dilog}\left(\cdots\right.}{}$ |
| risch | $\frac{b\ln(icx+1)^2}{4c^3d}-\frac{b\left(\frac{1}{2}c\,x^2+ix\right)\ln(icx+1)}{2c^2d}+\frac{ibx}{2c^2d}+\frac{ax}{c^2d}-\frac{a\arctan(cx)}{d\,c^3}+\frac{ia}{2d\,c^3}+\frac{ib\ln(-icx+1)x}{2d\,c^2}+\frac{x^2b\ln(-\cdots}{4c\cdots}$ |
| parts | $\frac{ibx}{2c^2d}+\frac{ax}{c^2d}+\frac{ib\arctan\left(\frac{1}{6}c^3x^3+\frac{7}{6}cx\right)}{8c^3d}-\frac{a\arctan(cx)}{d\,c^3}+\frac{bx\arctan(cx)}{c^2d}-\frac{ib\arctan\left(\frac{cx}{2}\right)}{8c^3d}+\frac{ib\arctan(cx)\ln(cx\cdots}{c^3d}$ |

input
```
int(x^2*(a+b*arctan(c*x))/(d+I*c*d*x),x,method=_RETURNVERBOSE)
```

output
```
1/c^3*(a/d*c*x+1/2*I*b/d*c*x+1/2*I*a/d*ln(c^2*x^2+1)-a/d*arctan(c*x)+b/d*a
rctan(c*x)*c*x-1/2*I*a/d*c^2*x^2+1/4*I*b/d*arctan(1/2*c*x-1/2*I)+1/2*b/d*l
n(c*x-I)*ln(-1/2*I*(c*x+I))+1/2*b/d*dilog(-1/2*I*(c*x+I))-1/4*b/d*ln(c*x-I
)^2+1/2*b/d-3/4*I*b/d*arctan(c*x)-1/16*b/d*ln(c^4*x^4+10*c^2*x^2+9)-1/2*I*
b/d*arctan(c*x)*c^2*x^2+I*b/d*arctan(c*x)*ln(c*x-I)-1/8*I*b/d*arctan(1/2*c
*x)-3/8*b/d*ln(c^2*x^2+1)+1/8*I*b/d*arctan(1/6*c^3*x^3+7/6*c*x))
```

## Fricas [F]

$$\int \frac{x^2(a+b\arctan(cx))}{d+icdx}\,dx = \int \frac{(b\arctan(cx)+a)x^2}{i\,cdx+d}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))/(d+I*c*d*x),x, algorithm="fricas")
```

output
```
integral(1/2*(b*x^2*log(-(c*x + I)/(c*x - I)) - 2*I*a*x^2)/(c*d*x - I*d),
x)
```

## Sympy [F]

$$\int \frac{x^2(a+b\arctan(cx))}{d+icdx}\,dx =$$
$$-\frac{i\left(\int \frac{2b\log(icx+1)}{c^2x^2+1}\,dx + \int \frac{4ac^3x^3}{c^2x^2+1}\,dx + \int \frac{bc^2x^2}{c^2x^2+1}\,dx + \int \frac{4iac^2x^2}{c^2x^2+1}\,dx + \int \left(-\frac{2ibcx}{c^2x^2+1}\right)\,dx + \int \left(-\frac{ibc^3x^3}{c^2x^2+1}\right)\,dx + \int\right.}{4c^2d}$$
$$+\frac{(bc^2x^2+2ibcx-2b\log(icx+1))\log(-icx+1)}{4c^3d}$$

input
```
integrate(x**2*(a+b*atan(c*x))/(d+I*c*d*x),x)
```

output
```
-I*(Integral(2*b*log(I*c*x + 1)/(c**2*x**2 + 1), x) + Integral(4*a*c**3*x**3/(c**2*x**2 + 1), x) + Integral(b*c**2*x**2/(c**2*x**2 + 1), x) + Integral(4*I*a*c**2*x**2/(c**2*x**2 + 1), x) + Integral(-2*I*b*c*x/(c**2*x**2 + 1), x) + Integral(-I*b*c**3*x**3/(c**2*x**2 + 1), x) + Integral(2*b*c**2*x**2*log(I*c*x + 1)/(c**2*x**2 + 1), x) + Integral(2*I*b*c*x*log(I*c*x + 1)/(c**2*x**2 + 1), x) + Integral(-2*I*b*c**3*x**3*log(I*c*x + 1)/(c**2*x**2 + 1), x))/(4*c**2*d) + (b*c**2*x**2 + 2*I*b*c*x - 2*b*log(I*c*x + 1))*log(-I*c*x + 1)/(4*c**3*d)
```

## Maxima [F]

$$\int \frac{x^2(a+b\arctan(cx))}{d+icdx}\,dx = \int \frac{(b\arctan(cx)+a)x^2}{i\,cdx+d}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))/(d+I*c*d*x),x, algorithm="maxima")
```

output
```
-1/2*a*((I*c*x^2 - 2*x)/(c^2*d) - 2*I*log(I*c*x + 1)/(c^3*d)) - 1/8*(32*I*c^6*d*integrate(1/8*x^3*arctan(c*x)/(c^4*d*x^2 + c^2*d), x) + 16*c^6*d*integrate(1/8*x^3*log(c^2*x^2 + 1)/(c^4*d*x^2 + c^2*d), x) - 32*c^5*d*integrate(1/8*x^2*arctan(c*x)/(c^4*d*x^2 + c^2*d), x) + 16*I*c^5*d*integrate(1/8*x^2*log(c^2*x^2 + 1)/(c^4*d*x^2 + c^2*d), x) - 32*I*c^4*d*integrate(1/8*x*arctan(c*x)/(c^4*d*x^2 + c^2*d), x) - 16*c^4*d*integrate(1/8*x*log(c^2*x^2 + 1)/(c^4*d*x^2 + c^2*d), x) + 16*I*c^3*d*integrate(1/8*log(c^2*x^2 + 1)/(c^4*d*x^2 + c^2*d), x) + c^2*x^2 + 2*I*c*x - 2*(-I*c^2*x^2 + 2*c*x + I)*arctan(c*x) + 2*arctan(c*x)^2 - (c^2*x^2 + 2*I*c*x + 1)*log(c^2*x^2 + 1) + log(c^2*x^2 + 1)^2 + 2*log(8*c^4*d*x^2 + 8*c^2*d))*b/(c^3*d)
```

## Giac [F]

$$\int \frac{x^2(a+b\arctan(cx))}{d+icdx}\,dx = \int \frac{(b\arctan(cx)+a)x^2}{i\,cdx+d}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))/(d+I*c*d*x),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)*x^2/(I*c*d*x + d), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\arctan(cx))}{d+icdx}\,dx=\int \frac{x^2\,(a+b\,\mathrm{atan}(c\,x))}{d+c\,d\,x\,1\mathrm{i}}\,dx$$

input
```
int((x^2*(a + b*atan(c*x)))/(d + c*d*x*1i),x)
```

output
```
int((x^2*(a + b*atan(c*x)))/(d + c*d*x*1i), x)
```

## Reduce [F]

$$\int \frac{x^2(a+b\arctan(cx))}{d+icdx}\,dx=\frac{2\left(\int \frac{atan(cx)x^2}{cix+1}dx\right)b\,c^3+2\log(cix+1)\,ai-a\,c^2i\,x^2+2acx}{2c^3d}$$

input
```
int(x^2*(a+b*atan(c*x))/(d+I*c*d*x),x)
```

output
```
(2*int((atan(c*x)*x**2)/(c*i*x + 1),x)*b*c**3 + 2*log(c*i*x + 1)*a*i - a*c
**2*i*x**2 + 2*a*c*x)/(2*c**3*d)
```

# 3.45 $\int \frac{x(a+b\arctan(cx))}{d+icdx}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 110

$$\int \frac{x(a+b\arctan(cx))}{d+icdx}\,dx = -\frac{iax}{cd}-\frac{ibx\arctan(cx)}{cd}-\frac{(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c^2d}+\frac{ib\log(1+c^2x^2)}{2c^2d}-\frac{ib\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2c^2d}$$

output
```
-I*a*x/c/d-I*b*x*arctan(c*x)/c/d-(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^2/d+1
/2*I*b*ln(c^2*x^2+1)/c^2/d-1/2*I*b*polylog(2,1-2/(1+I*c*x))/c^2/d
```

## Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.98

$$\int \frac{x(a+b\arctan(cx))}{d+icdx}\,dx = \frac{-2iacx+2ib\arctan(cx)^2+2i\arctan(cx)\left(a-bcx+ib\log\left(1+e^{2i\arctan(cx)}\right)\right)+a\log(1+c^2x^2)+ib\log}{2c^2d}$$

input
```
Integrate[(x*(a + b*ArcTan[c*x]))/(d + I*c*d*x),x]
```

output
```
((-2*I)*a*c*x + (2*I)*b*ArcTan[c*x]^2 + (2*I)*ArcTan[c*x]*(a - b*c*x + I*b
*Log[1 + E^((2*I)*ArcTan[c*x])]) + a*Log[1 + c^2*x^2] + I*b*Log[1 + c^2*x^
2] + I*b*PolyLog[2, -E^((2*I)*ArcTan[c*x])])/(2*c^2*d)
```

## Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5401, 27, 2009, 5379, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\arctan(cx))}{d+icdx}\,dx$$

$$\Big\downarrow \text{5401}$$

$$\frac{i\int \frac{a+b\arctan(cx)}{d(icx+1)}dx}{c}-\frac{i\int (a+b\arctan(cx))dx}{cd}$$

$$\Big\downarrow \text{27}$$

$$\frac{i\int \frac{a+b\arctan(cx)}{icx+1}dx}{cd}-\frac{i\int (a+b\arctan(cx))dx}{cd}$$

$$\Big\downarrow \text{2009}$$

$$\frac{i\int \frac{a+b\arctan(cx)}{icx+1}dx}{cd}-\frac{i\left(ax+bx\arctan(cx)-\frac{b\log(c^2x^2+1)}{2c}\right)}{cd}$$

$$\Big\downarrow \text{5379}$$

$$\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}-ib\int \frac{\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{cd}-\frac{i\left(ax+bx\arctan(cx)-\frac{b\log(c^2x^2+1)}{2c}\right)}{cd}$$

$$\Big\downarrow \text{2849}$$

$$\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}-\frac{b\int \frac{\log\left(\frac{2}{icx+1}\right)}{1-\frac{2}{icx+1}}d\frac{1}{icx+1}}{c}\right)}{cd}-\frac{i\left(ax+bx\arctan(cx)-\frac{b\log(c^2x^2+1)}{2c}\right)}{cd}$$

$$\Big\downarrow \text{2752}$$

$$\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}-\frac{b\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{2c}\right)}{cd}-\frac{i\left(ax+bx\arctan(cx)-\frac{b\log(c^2x^2+1)}{2c}\right)}{cd}$$

input
```
Int[(x*(a + b*ArcTan[c*x]))/(d + I*c*d*x),x]
```

output
```
((-I)*(a*x + b*x*ArcTan[c*x] - (b*Log[1 + c^2*x^2])/(2*c)))/(c*d) + (I*((I
*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/c - (b*PolyLog[2, 1 - 2/(1 + I*c*
x)])/(2*c)))/(c*d)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5401
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_) + (
e_.)*(x_)), x_Symbol] :> Simp[f/e   Int[(f*x)^(m - 1)*(a + b*ArcTan[c*x])^p
, x], x] - Simp[d*(f/e)   Int[(f*x)^(m - 1)*((a + b*ArcTan[c*x])^p/(d + e*x
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e
^2, 0] && GtQ[m, 0]
```

## Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 213 vs. 2(100) = 200.

Time = 0.39 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.95

| method | result |
|---|---|
| derivativedivides | $\frac{-\frac{iacx}{d}+\frac{a\ln\left(c^2x^2+1\right)}{2d}+\frac{ia\arctan(cx)}{d}-\frac{ib\arctan(cx)cx}{d}+\frac{b\arctan(cx)\ln(cx-i)}{d}+\frac{ib\ln(cx-i)^2}{4d}-\frac{ib\ln(cx-i)\ln\left(-\frac{i(cx+i)}{2}\right)}{2d}-\frac{ib}{}\cdots}{c^2}$ |
| default | $\frac{-\frac{iacx}{d}+\frac{a\ln\left(c^2x^2+1\right)}{2d}+\frac{ia\arctan(cx)}{d}-\frac{ib\arctan(cx)cx}{d}+\frac{b\arctan(cx)\ln(cx-i)}{d}+\frac{ib\ln(cx-i)^2}{4d}-\frac{ib\ln(cx-i)\ln\left(-\frac{i(cx+i)}{2}\right)}{2d}-\frac{ib}{}\cdots}{c^2}$ |
| risch | $\frac{ib\ln(c^2x^2+1)}{4d\,c^2}-\frac{bx\ln(icx+1)}{2cd}+\frac{ia\arctan(cx)}{d\,c^2}-\frac{b\arctan(cx)}{2d\,c^2}+\frac{\ln(-icx+1)bx}{2dc}+\frac{a\ln(c^2x^2+1)}{2d\,c^2}+i\ln\left(\frac{1}{2}+\cdots\right.$ |
| parts | $-\frac{iax}{cd}+\frac{a\ln(c^2x^2+1)}{2d\,c^2}+\frac{ia\arctan(cx)}{d\,c^2}-\frac{ibx\arctan(cx)}{cd}+\frac{b\arctan(cx)\ln(cx-i)}{d\,c^2}-\frac{ib\ln(cx-i)\ln\left(-\frac{i(cx+i)}{2}\right)}{2d\,c^2}\cdots$ |

input
```
int(x*(a+b*arctan(c*x))/(d+I*c*d*x),x,method=_RETURNVERBOSE)
```

output
```
1/c^2*(-I*a/d*c*x+1/2*a/d*ln(c^2*x^2+1)+I*a/d*arctan(c*x)-I*b/d*arctan(c*x
)*c*x+b/d*arctan(c*x)*ln(c*x-I)+1/4*I*b/d*ln(c*x-I)^2-1/2*I*b/d*ln(c*x-I)*
ln(-1/2*I*(c*x+I))-1/2*I*b/d*dilog(-1/2*I*(c*x+I))+1/8*I*b/d*ln(c^8*x^8+12
*c^6*x^6+30*c^4*x^4+28*c^2*x^2+9)-1/4*b/d*arctan(1/12*c^3*x^3+13/12*c*x)-1
/4*b/d*arctan(1/4*c*x)+1/2*b/d*arctan(1/2*c*x-1/2*I))
```

## Fricas [F]

$$\int \frac{x(a+b\arctan(cx))}{d+icdx}\,dx = \int \frac{(b\arctan(cx)+a)x}{i\,cdx+d}\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))/(d+I*c*d*x),x, algorithm="fricas")
```

output
```
integral(1/2*(b*x*log(-(c*x + I)/(c*x - I)) - 2*I*a*x)/(c*d*x - I*d), x)
```

## Sympy [F]

$$\int \frac{x(a+b\arctan(cx))}{d+icdx}\,dx =$$
$$-\frac{i\left(\int\left(-\frac{ib\log(icx+1)}{c^2x^2+1}\right)dx + \int\frac{2ac^2x^2}{c^2x^2+1}\,dx + \int\left(-\frac{bcx}{c^2x^2+1}\right)dx + \int\frac{2iacx}{c^2x^2+1}\,dx + \int\left(-\frac{ibc^2x^2}{c^2x^2+1}\right)dx + \int\frac{2bcx\log(ic}{c^2x^2+}\right.}{2cd}$$
$$+\frac{(bcx+ib\log(icx+1))\log(-icx+1)}{2c^2d}$$

input
```
integrate(x*(a+b*atan(c*x))/(d+I*c*d*x),x)
```

output
```
-I*(Integral(-I*b*log(I*c*x + 1)/(c**2*x**2 + 1), x) + Integral(2*a*c**2*x
**2/(c**2*x**2 + 1), x) + Integral(-b*c*x/(c**2*x**2 + 1), x) + Integral(2
*I*a*c*x/(c**2*x**2 + 1), x) + Integral(-I*b*c**2*x**2/(c**2*x**2 + 1), x)
 + Integral(2*b*c*x*log(I*c*x + 1)/(c**2*x**2 + 1), x) + Integral(-I*b*c**
2*x**2*log(I*c*x + 1)/(c**2*x**2 + 1), x))/(2*c*d) + (b*c*x + I*b*log(I*c*
x + 1))*log(-I*c*x + 1)/(2*c**2*d)
```

## Maxima [F]

$$\int \frac{x(a+b\arctan(cx))}{d+icdx}\,dx = \int \frac{(b\arctan(cx)+a)x}{i\,cdx+d}\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))/(d+I*c*d*x),x, algorithm="maxima")
```

output
```
a*(-I*x/(c*d) + log(I*c*x + 1)/(c^2*d)) - 1/8*(8*I*c^4*d*integrate(1/2*x^2
*arctan(c*x)/(c^3*d*x^2 + c*d), x) + 4*c^4*d*integrate(1/2*x^2*log(c^2*x^2
 + 1)/(c^3*d*x^2 + c*d), x) - 16*c^3*d*integrate(1/2*x*arctan(c*x)/(c^3*d*
x^2 + c*d), x) + 8*I*c^3*d*integrate(1/2*x*log(c^2*x^2 + 1)/(c^3*d*x^2 + c
*d), x) + 4*c^2*d*integrate(1/2*log(c^2*x^2 + 1)/(c^3*d*x^2 + c*d), x) - 2
*c*x*log(c^2*x^2 + 1) + 4*c*x - 4*(-I*c*x + 1)*arctan(c*x) - 2*I*arctan(c*
x)^2 - I*log(c^2*x^2 + 1)^2 - 2*I*log(2*c^3*d*x^2 + 2*c*d))*b/(c^2*d)
```

## Giac [F]

$$\int \frac{x(a+b\arctan(cx))}{d+icdx}\,dx = \int \frac{(b\arctan(cx)+a)x}{i\,cdx+d}\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))/(d+I*c*d*x),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)*x/(I*c*d*x + d), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\arctan(cx))}{d+icdx}\,dx = \int \frac{x\,(a+b\,\mathrm{atan}(c\,x))}{d+c\,d\,x\,1\mathrm{i}}\,dx$$

input
```
int((x*(a + b*atan(c*x)))/(d + c*d*x*1i),x)
```

output
```
int((x*(a + b*atan(c*x)))/(d + c*d*x*1i), x)
```

## Reduce [F]

$$\int \frac{x(a+b\arctan(cx))}{d+icdx}\,dx = \frac{\left(\int \frac{atan(cx)x}{cix+1}dx\right)b\,c^2 + \log(cix+1)\,a - acix}{c^2d}$$

input
```
int(x*(a+b*atan(c*x))/(d+I*c*d*x),x)
```

output
```
(int((atan(c*x)*x)/(c*i*x + 1),x)*b*c**2 + log(c*i*x + 1)*a - a*c*i*x)/(c*
*2*d)
```

# 3.46 $\int \frac{a+b\arctan(cx)}{d+icdx}\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 59

$$\int \frac{a+b\arctan(cx)}{d+icdx}\,dx=\frac{i(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{cd}-\frac{b\,\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2cd}$$

output
```
I*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c/d-1/2*b*polylog(2,1-2/(1+I*c*x))/c/d
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.02

$$\int \frac{a+b\arctan(cx)}{d+icdx}\,dx=\frac{2i(a+b\arctan(cx))\log\left(\frac{2d}{d+icdx}\right)-b\,\text{PolyLog}\left(2,\frac{i+cx}{-i+cx}\right)}{2cd}$$

input
```
Integrate[(a + b*ArcTan[c*x])/(d + I*c*d*x),x]
```

output
```
((2*I)*(a + b*ArcTan[c*x])*Log[(2*d)/(d + I*c*d*x)] - b*PolyLog[2, (I + c*
x)/(-I + c*x)])/(2*c*d)
```

## Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {5379, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{d+icdx}\,dx$$

$$\Big\downarrow \text{5379}$$

$$\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{cd}-\frac{ib\int\frac{\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx}{d}$$

$$\Big\downarrow \text{2849}$$

$$\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{cd}-\frac{b\int\frac{\log\left(\frac{2}{icx+1}\right)}{1-\frac{2}{icx+1}}d\frac{1}{icx+1}}{cd}$$

$$\Big\downarrow \text{2752}$$

$$\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{cd}-\frac{b\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{2cd}$$

input

```
Int[(a + b*ArcTan[c*x])/(d + I*c*d*x),x]
```

output

```
(I*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(c*d) - (b*PolyLog[2, 1 - 2/(1
+ I*c*x)])/(2*c*d)
```

### Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

## Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.76

| method | result |
|---|---|
| derivativedivides | $\frac{-\frac{ia\ln\left(c^2x^2+1\right)}{2d}+\frac{a\arctan(cx)}{d}+\frac{b\left(-i\ln(icx+1)\arctan(cx)+\frac{\ln(icx+1)^2}{4}-\frac{\left(\ln(icx+1)-\ln\left(\frac{1}{2}+\frac{icx}{2}\right)\right)\ln\left(\frac{1}{2}-\frac{icx}{2}\right)}{2}+\frac{\mathrm{dilog}\left(\frac{1}{2}+\frac{ic}{2}\right.}{2}\right.}{d}}{c}$ |
| default | $\frac{-\frac{ia\ln\left(c^2x^2+1\right)}{2d}+\frac{a\arctan(cx)}{d}+\frac{b\left(-i\ln(icx+1)\arctan(cx)+\frac{\ln(icx+1)^2}{4}-\frac{\left(\ln(icx+1)-\ln\left(\frac{1}{2}+\frac{icx}{2}\right)\right)\ln\left(\frac{1}{2}-\frac{icx}{2}\right)}{2}+\frac{\mathrm{dilog}\left(\frac{1}{2}+\frac{ic}{2}\right.}{2}\right.}{d}}{c}$ |
| parts | $-\frac{ia\ln\left(c^2x^2+1\right)}{2cd}+\frac{a\arctan(cx)}{cd}+\frac{b\left(-i\ln(icx+1)\arctan(cx)+\frac{\ln(icx+1)^2}{4}-\frac{\left(\ln(icx+1)-\ln\left(\frac{1}{2}+\frac{icx}{2}\right)\right)\ln\left(\frac{1}{2}-\frac{icx}{2}\right)}{2}+\right.}{dc}$ |
| risch | $-\frac{b\ln(icx+1)^2}{4cd}-\frac{ia\ln\left(c^2x^2+1\right)}{2cd}-\frac{\ln\left(\frac{1}{2}-\frac{icx}{2}\right)\ln\left(\frac{1}{2}+\frac{icx}{2}\right)b}{2cd}+\frac{\ln\left(\frac{1}{2}+\frac{icx}{2}\right)\ln(-icx+1)b}{2cd}+\frac{a\arctan(cx)}{cd}-\mathrm{dilo}$ |

input
```
int((a+b*arctan(c*x))/(d+I*c*d*x),x,method=_RETURNVERBOSE)
```

output
```
1/c*(-1/2*I*a/d*ln(c^2*x^2+1)+a/d*arctan(c*x)+b/d*(-I*ln(1+I*c*x)*arctan(c
*x)+1/4*ln(1+I*c*x)^2-1/2*(ln(1+I*c*x)-ln(1/2+1/2*I*c*x))*ln(1/2-1/2*I*c*x
)+1/2*dilog(1/2+1/2*I*c*x)))
```

## Fricas [F]

$$\int \frac{a+b\arctan(cx)}{d+icdx}\,dx = \int \frac{b\arctan(cx)+a}{i\,cdx+d}\,dx$$

input
```
integrate((a+b*arctan(c*x))/(d+I*c*d*x),x, algorithm="fricas")
```

output
```
integral(1/2*(b*log(-(c*x + I)/(c*x - I)) - 2*I*a)/(c*d*x - I*d), x)
```

## Sympy [F]

$$\int \frac{a+b\arctan(cx)}{d+icdx}\,dx = \frac{b\log(-icx+1)\log(icx+1)}{2cd} - \frac{i\left(\int \frac{ia}{c^2x^2+1}\,dx + \int \frac{acx}{c^2x^2+1}\,dx + \int \left(-\frac{ibcx\log(icx+1)}{c^2x^2+1}\right)\,dx\right)}{d}$$

input
```
integrate((a+b*atan(c*x))/(d+I*c*d*x),x)
```

output
```
b*log(-I*c*x + 1)*log(I*c*x + 1)/(2*c*d) - I*(Integral(I*a/(c**2*x**2 + 1)
, x) + Integral(a*c*x/(c**2*x**2 + 1), x) + Integral(-I*b*c*x*log(I*c*x +
1)/(c**2*x**2 + 1), x))/d
```

## Maxima [F]

$$\int \frac{a+b\arctan(cx)}{d+icdx}\,dx = \int \frac{b\arctan(cx)+a}{i\,cdx+d}\,dx$$

input
```
integrate((a+b*arctan(c*x))/(d+I*c*d*x),x, algorithm="maxima")
```

output
```
-1/8*(8*I*c^2*d*integrate(x*arctan(c*x)/(c^2*d*x^2 + d), x) + 4*c^2*d*inte
grate(x*log(c^2*x^2 + 1)/(c^2*d*x^2 + d), x) - 4*arctan(c*x)^2 - log(c^2*x
^2 + 1)^2)*b/(c*d) - I*a*log(I*c*d*x + d)/(c*d)
```

## Giac [F]

$$\int \frac{a+b\arctan(cx)}{d+icdx}\,dx=\int \frac{b\arctan{(cx)}+a}{i\,cdx+d}\,dx$$

input
```
integrate((a+b*arctan(c*x))/(d+I*c*d*x),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)/(I*c*d*x + d), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{d+icdx}\,dx=\int \frac{a+b\,\mathrm{atan}(c\,x)}{d+c\,d\,x\,1\mathrm{i}}\,dx$$

input
```
int((a + b*atan(c*x))/(d + c*d*x*1i),x)
```

output
```
int((a + b*atan(c*x))/(d + c*d*x*1i), x)
```

## Reduce [F]

$$\int \frac{a+b\arctan(cx)}{d+icdx}\,dx=\frac{\left(\int \frac{atan(cx)}{cix+1}dx\right)bc-\log(cix+1)\,ai}{cd}$$

input
```
int((a+b*atan(c*x))/(d+I*c*d*x),x)
```

output
```
(int(atan(c*x)/(c*i*x + 1),x)*b*c - log(c*i*x + 1)*a*i)/(c*d)
```

# 3.47 $\int \frac{a+b\arctan(cx)}{x(d+icdx)}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 54

$$\int \frac{a+b\arctan(cx)}{x(d+icdx)}\,dx=\frac{(a+b\arctan(cx))\log\left(2-\frac{2}{1+icx}\right)}{d}+\frac{ib\,\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)}{2d}$$

output
```
(a+b*arctan(c*x))*ln(2-2/(1+I*c*x))/d+1/2*I*b*polylog(2,-1+2/(1+I*c*x))/d
```

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.89

$$\int \frac{a+b\arctan(cx)}{x(d+icdx)}\,dx=\frac{a\log(x)}{d}+\frac{(a+b\arctan(cx))\log\left(\frac{2i}{i-cx}\right)}{d}+\frac{ib\,\text{PolyLog}(2,-icx)}{2d}-\frac{ib\,\text{PolyLog}(2,icx)}{2d}+\frac{ib\,\text{PolyLog}\left(2,-\frac{i+cx}{i-cx}\right)}{2d}$$

input
```
Integrate[(a + b*ArcTan[c*x])/(x*(d + I*c*d*x)),x]
```

output
```
(a*Log[x])/d + ((a + b*ArcTan[c*x])*Log[(2*I)/(I - c*x)])/d + ((I/2)*b*Pol
yLog[2, (-I)*c*x])/d - ((I/2)*b*PolyLog[2, I*c*x])/d + ((I/2)*b*PolyLog[2,
 -((I + c*x)/(I - c*x))])/d
```

## Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5403, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x(d+icdx)}\,dx$$
$$\Big\downarrow \text{5403}$$
$$\frac{\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))}{d}-\frac{bc\int \frac{\log\left(2-\frac{2}{icx+1}\right)}{c^2x^2+1}dx}{d}$$
$$\Big\downarrow \text{2897}$$
$$\frac{\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))}{d}+\frac{ib\,\text{PolyLog}\left(2,\frac{2}{icx+1}-1\right)}{2d}$$

input
```
Int[(a + b*ArcTan[c*x])/(x*(d + I*c*d*x)),x]
```

output
```
((a + b*ArcTan[c*x])*Log[2 - 2/(1 + I*c*x)])/d + ((I/2)*b*PolyLog[2, -1 +
2/(1 + I*c*x)])/d
```

### Defintions of rubi rules used

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

## Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 147 vs. $2(49) = 98$.

Time = 0.38 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.74

| method | result |
|---|---|
| risch | $\frac{ib\ln(icx+1)^2}{4d}+\frac{ib\,\mathrm{dilog}(icx+1)}{2d}+\frac{a\ln(-icx)}{d}-\frac{a\ln(c^2x^2+1)}{2d}-\frac{i\ln(-icx+1)\ln(\frac{1}{2}+\frac{icx}{2})b}{2d}+\frac{i\ln(\frac{1}{2}-\frac{icx}{2})\ln(\ldots}{2d}$ |
| parts | $-\frac{a\ln(c^2x^2+1)}{2d}-\frac{ia\arctan(cx)}{d}+\frac{a\ln(x)}{d}+\frac{b\left(\ln(cx)\arctan(cx)-\arctan(cx)\ln(cx-i)+\frac{i\ln(cx)\ln(icx+1)}{2}-\frac{i\ln(c\ldots}{}\right.}{}$ |
| derivativedivides | $\frac{a\ln(cx)}{d}-\frac{a\ln(c^2x^2+1)}{2d}-\frac{ia\arctan(cx)}{d}+\frac{b\left(\ln(cx)\arctan(cx)-\arctan(cx)\ln(cx-i)+\frac{i\ln(cx)\ln(icx+1)}{2}-\frac{i\ln(cx)\ldots}{}\right.}{}$ |
| default | $\frac{a\ln(cx)}{d}-\frac{a\ln(c^2x^2+1)}{2d}-\frac{ia\arctan(cx)}{d}+\frac{b\left(\ln(cx)\arctan(cx)-\arctan(cx)\ln(cx-i)+\frac{i\ln(cx)\ln(icx+1)}{2}-\frac{i\ln(cx)\ldots}{}\right.}{}$ |

input
```
int((a+b*arctan(c*x))/x/(d+I*c*d*x),x,method=_RETURNVERBOSE)
```

output
```
1/4*I*b/d*ln(1+I*c*x)^2+1/2*I*b/d*dilog(1+I*c*x)+1/d*a*ln(-I*c*x)-1/2*a/d*
ln(c^2*x^2+1)-1/2*I/d*ln(1-I*c*x)*ln(1/2+1/2*I*c*x)*b+1/2*I/d*ln(1/2-1/2*I
*c*x)*ln(1/2+1/2*I*c*x)*b-I*a/d*arctan(c*x)+1/2*I/d*dilog(1/2-1/2*I*c*x)*b
-1/2*I/d*dilog(1-I*c*x)*b
```

## Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.80

$$\int \frac{a + b\arctan(cx)}{x(d + icdx)}\, dx = \frac{-i\, b\mathrm{Li}_2\left(\frac{cx+i}{cx-i} + 1\right) + 2\, a\log\left(x\right) - 2\, a\log\left(\frac{cx-i}{c}\right)}{2\, d}$$

input
```
integrate((a+b*arctan(c*x))/x/(d+I*c*d*x),x, algorithm="fricas")
```

output
```
1/2*(-I*b*dilog((c*x + I)/(c*x - I) + 1) + 2*a*log(x) - 2*a*log((c*x - I)/
c))/d
```

## Sympy [F]

$$\int \frac{a + b\arctan(cx)}{x(d + icdx)}\, dx = -\frac{i\left(\int \frac{a}{cx^2 - ix}\, dx + \int \frac{b\,\mathrm{atan}\,(cx)}{cx^2 - ix}\, dx\right)}{d}$$

input
```
integrate((a+b*atan(c*x))/x/(d+I*c*d*x),x)
```

output
```
-I*(Integral(a/(c*x**2 - I*x), x) + Integral(b*atan(c*x)/(c*x**2 - I*x), x
))/d
```

## Maxima [F]

$$\int \frac{a + b\arctan(cx)}{x(d + icdx)}\, dx = \int \frac{b\arctan\left(cx\right) + a}{(i\, cdx + d)x}\, dx$$

input
```
integrate((a+b*arctan(c*x))/x/(d+I*c*d*x),x, algorithm="maxima")
```

output
```
-1/2*b*(I*arctan(c*x)^2/d - 2*integrate(arctan(c*x)/(c^2*d*x^3 + d*x), x))
 - a*(log(I*c*x + 1)/d - log(x)/d)
```

## Giac [F]

$$\int \frac{a+b\arctan(cx)}{x(d+icdx)}\,dx=\int \frac{b\arctan{(cx)}+a}{(i\,cdx+d)x}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x/(d+I*c*d*x),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)/((I*c*d*x + d)*x), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{x(d+icdx)}\,dx=\int \frac{a+b\,\mathrm{atan}(c\,x)}{x\,\left(d+c\,d\,x\,1\mathrm{i}\right)}\,dx$$

input
```
int((a + b*atan(c*x))/(x*(d + c*d*x*1i)),x)
```

output
```
int((a + b*atan(c*x))/(x*(d + c*d*x*1i)), x)
```

## Reduce [F]

$$\int \frac{a+b\arctan(cx)}{x(d+icdx)}\,dx = \frac{-atan(cx)^2\,bi-2\left(\int \frac{atan(cx)}{c^3x^4-c^2i\,x^3+c\,x^2-ix}dx\right)bi+2\left(\int \frac{atan(cx)}{c^3x^3-c^2i\,x^2+cx-i}dx\right)bc-2\log(cix+1)\,a+2\log(x)\,a}{2d}$$

input
```
int((a+b*atan(c*x))/x/(d+I*c*d*x),x)
```

output
```
( - atan(c*x)**2*b*i - 2*int(atan(c*x)/(c**3*x**4 - c**2*i*x**3 + c*x**2 -
 i*x),x)*b*i + 2*int(atan(c*x)/(c**3*x**3 - c**2*i*x**2 + c*x - i),x)*b*c
- 2*log(c*i*x + 1)*a + 2*log(x)*a)/(2*d)
```

# 3.48 $\int \frac{a+b\arctan(cx)}{x^2(d+icdx)}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 100

$$\int \frac{a+b\arctan(cx)}{x^2(d+icdx)}\,dx = -\frac{a+b\arctan(cx)}{dx}+\frac{bc\log(x)}{d}-\frac{bc\log(1+c^2x^2)}{2d} - \frac{ic(a+b\arctan(cx))\log\left(2-\frac{2}{1+icx}\right)}{d} + \frac{bc\,\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)}{2d}$$

output
```
-(a+b*arctan(c*x))/d/x+b*c*ln(x)/d-1/2*b*c*ln(c^2*x^2+1)/d-I*c*(a+b*arctan
(c*x))*ln(2-2/(1+I*c*x))/d+1/2*b*c*polylog(2,-1+2/(1+I*c*x))/d
```

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.49

$$\int \frac{a+b\arctan(cx)}{x^2(d+icdx)}\,dx = -\frac{a+b\arctan(cx)}{dx}-\frac{iac\log(x)}{d}-\frac{ic(a+b\arctan(cx))\log\left(\frac{2i}{i-cx}\right)}{d}+\frac{bc(2\log(x)-\log(1+c^2x^2))}{2d}+\frac{bc\,\mathrm{PolyLog}(2,-icx)}{2d}-\frac{bc\,\mathrm{PolyLog}(2,icx)}{2d}+\frac{bc\,\mathrm{PolyLog}\left(2,-\frac{i+cx}{i-cx}\right)}{2d}$$

input
```
Integrate[(a + b*ArcTan[c*x])/(x^2*(d + I*c*d*x)),x]
```

output
```
-((a + b*ArcTan[c*x])/(d*x)) - (I*a*c*Log[x])/d - (I*c*(a + b*ArcTan[c*x])
*Log[(2*I)/(I - c*x)])/d + (b*c*(2*Log[x] - Log[1 + c^2*x^2]))/(2*d) + (b*
c*PolyLog[2, (-I)*c*x])/(2*d) - (b*c*PolyLog[2, I*c*x])/(2*d) + (b*c*PolyL
og[2, -((I + c*x)/(I - c*x))])/(2*d)
```

## Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {5405, 27, 5361, 243, 47, 14, 16, 5403, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x^2(d+icdx)}\,dx$$

$$\Big\downarrow \text{5405}$$

$$\frac{\int \frac{a+b\arctan(cx)}{x^2}dx}{d}-ic\int \frac{a+b\arctan(cx)}{dx(icx+1)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{a+b\arctan(cx)}{x^2}dx}{d}-\frac{ic\int \frac{a+b\arctan(cx)}{x(icx+1)}dx}{d}$$

$\downarrow$ 5361

$$\frac{bc\int \frac{1}{x(c^2x^2+1)}dx-\frac{a+b\arctan(cx)}{x}}{d}-\frac{ic\int \frac{a+b\arctan(cx)}{x(icx+1)}dx}{d}$$

$\downarrow$ 243

$$\frac{\frac{1}{2}bc\int \frac{1}{x^2(c^2x^2+1)}dx^2-\frac{a+b\arctan(cx)}{x}}{d}-\frac{ic\int \frac{a+b\arctan(cx)}{x(icx+1)}dx}{d}$$

$\downarrow$ 47

$$\frac{\frac{1}{2}bc\left(\int \frac{1}{x^2}dx^2-c^2\int \frac{1}{c^2x^2+1}dx^2\right)-\frac{a+b\arctan(cx)}{x}}{d}-\frac{ic\int \frac{a+b\arctan(cx)}{x(icx+1)}dx}{d}$$

$\downarrow$ 14

$$\frac{\frac{1}{2}bc\left(\log\left(x^2\right)-c^2\int \frac{1}{c^2x^2+1}dx^2\right)-\frac{a+b\arctan(cx)}{x}}{d}-\frac{ic\int \frac{a+b\arctan(cx)}{x(icx+1)}dx}{d}$$

$\downarrow$ 16

$$\frac{\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right)\right)-\frac{a+b\arctan(cx)}{x}}{d}-\frac{ic\int \frac{a+b\arctan(cx)}{x(icx+1)}dx}{d}$$

$\downarrow$ 5403

$$\frac{\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right)\right)-\frac{a+b\arctan(cx)}{x}}{d}-\frac{ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))-bc\int \frac{\log\left(2-\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{d}$$

$\downarrow$ 2897

$$\frac{\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right)\right)-\frac{a+b\arctan(cx)}{x}}{d}-\frac{ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))+\frac{1}{2}ib\,\text{PolyLog}\left(2,\frac{2}{icx+1}-1\right)\right)}{d}$$

input
```
Int[(a + b*ArcTan[c*x])/(x^2*(d + I*c*d*x)),x]
```

output
```
(-((a + b*ArcTan[c*x])/x) + (b*c*(Log[x^2] - Log[1 + c^2*x^2]))/2)/d - (I*
c*((a + b*ArcTan[c*x])*Log[2 - 2/(1 + I*c*x)] + (I/2)*b*PolyLog[2, -1 + 2/
(1 + I*c*x)]))/d
```

## Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5405
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x], x
] - Simp[e/(d*f)   Int[(f*x)^(m + 1)*((a + b*ArcTan[c*x])^p/(d + e*x)), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
&& LtQ[m, -1]
```

## Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 199 vs. 2(93) = 186.

Time = 0.38 (sec) , antiderivative size = 200, normalized size of antiderivative = 2.00

| method | result |
|---|---|
| parts | $\frac{ica\ln(c^2x^2+1)}{2d} - \frac{ca\arctan(cx)}{d} - \frac{a}{dx} - \frac{iac\ln(x)}{d} + \frac{bc\left(-\frac{\arctan(cx)}{cx} - i\arctan(cx)\ln(cx) + i\arctan(cx)\ln(cx-i\right.}{}$ |
| derivativedivides | $c\left(-\frac{a}{dcx} - \frac{ia\ln(cx)}{d} + \frac{ia\ln(c^2x^2+1)}{2d} - \frac{a\arctan(cx)}{d} + \frac{b\left(-\frac{\arctan(cx)}{cx} - i\arctan(cx)\ln(cx) + i\arctan(cx)\ln(\right.}{}\right.$ |
| default | $c\left(-\frac{a}{dcx} - \frac{ia\ln(cx)}{d} + \frac{ia\ln(c^2x^2+1)}{2d} - \frac{a\arctan(cx)}{d} + \frac{b\left(-\frac{\arctan(cx)}{cx} - i\arctan(cx)\ln(cx) + i\arctan(cx)\ln(\right.}{}\right.$ |
| risch | $\frac{bc\ln(icx)}{2d} - \frac{bc\ln(icx+1)}{2d} - \frac{ica\ln(-icx)}{d} + \frac{bc\ln(icx+1)^2}{4d} + \frac{bc\,\mathrm{dilog}(icx+1)}{2d} + \frac{ib\ln(icx+1)}{2dx} - \frac{ca\arctan(cx)}{d}$ |

input
```
int((a+b*arctan(c*x))/x^2/(d+I*c*d*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*I/d*c*a*ln(c^2*x^2+1)-1/d*c*a*arctan(c*x)-1/d*a/x-I*a/d*c*ln(x)+b/d*c*
(-1/c/x*arctan(c*x)-I*arctan(c*x)*ln(c*x)+I*arctan(c*x)*ln(c*x-I)+1/2*ln(c
*x)*ln(1+I*c*x)-1/2*ln(c*x)*ln(1-I*c*x)+1/2*dilog(1+I*c*x)-1/2*dilog(1-I*c
*x)+1/2*ln(c*x-I)*ln(-1/2*I*(c*x+I))+1/2*dilog(-1/2*I*(c*x+I))-1/4*ln(c*x-
I)^2-1/2*ln(c^2*x^2+1)+ln(c*x))
```

## Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.98

$$\int \frac{a+b\arctan(cx)}{x^2(d+icdx)}\,dx = -\frac{bcx\mathrm{Li}_2\left(\frac{cx+i}{cx-i}+1\right)+2\,(i\,a-b)cx\log\left(x\right)+bcx\log\left(\frac{cx+i}{c}\right)-(2i\,a-b)cx\log\left(\frac{cx-i}{c}\right)+i\,b\log\left(-\frac{cx+i}{cx-i}\right)+2}{2\,dx}$$

input
```
integrate((a+b*arctan(c*x))/x^2/(d+I*c*d*x),x, algorithm="fricas")
```

output
```
-1/2*(b*c*x*dilog((c*x + I)/(c*x - I) + 1) + 2*(I*a - b)*c*x*log(x) + b*c*
x*log((c*x + I)/c) - (2*I*a - b)*c*x*log((c*x - I)/c) + I*b*log(-(c*x + I)
/(c*x - I)) + 2*a)/(d*x)
```

## Sympy [F]

$$\int \frac{a+b\arctan(cx)}{x^2(d+icdx)}\,dx = -\frac{i\left(\int \frac{a}{cx^3-ix^2}\,dx+\int \frac{b\,\mathrm{atan}\,(cx)}{cx^3-ix^2}\,dx\right)}{d}$$

input
```
integrate((a+b*atan(c*x))/x**2/(d+I*c*d*x),x)
```

output
```
-I*(Integral(a/(c*x**3 - I*x**2), x) + Integral(b*atan(c*x)/(c*x**3 - I*x*
*2), x))/d
```

## Maxima [F]

$$\int \frac{a + b\arctan(cx)}{x^2(d + icdx)} dx = \int \frac{b\arctan(cx) + a}{(i\,cdx + d)x^2} dx$$

input
```
integrate((a+b*arctan(c*x))/x^2/(d+I*c*d*x),x, algorithm="maxima")
```

output
```
(-I*c*integrate(arctan(c*x)/(c^2*d*x^3 + d*x), x) + integrate(arctan(c*x)/
(c^2*d*x^4 + d*x^2), x))*b + a*(I*c*log(I*c*x + 1)/d - I*c*log(x)/d - 1/(d
*x))
```

## Giac [F]

$$\int \frac{a + b\arctan(cx)}{x^2(d + icdx)} dx = \int \frac{b\arctan(cx) + a}{(i\,cdx + d)x^2} dx$$

input
```
integrate((a+b*arctan(c*x))/x^2/(d+I*c*d*x),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)/((I*c*d*x + d)*x^2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\arctan(cx)}{x^2(d + icdx)} dx = \int \frac{a + b\,\mathrm{atan}(c\,x)}{x^2\,(d + c\,d\,x\,1\mathrm{i})} dx$$

input
```
int((a + b*atan(c*x))/(x^2*(d + c*d*x*1i)),x)
```

output
```
int((a + b*atan(c*x))/(x^2*(d + c*d*x*1i)), x)
```

## Reduce [F]

$$\int \frac{a+b\arctan(cx)}{x^2(d+icdx)}\,dx$$
$$=\frac{\left(\int \frac{atan(cx)}{c^3i\,x^5+c^2x^4+ci\,x^3+x^2}dx\right)bx+\left(\int \frac{atan(cx)}{c^3i\,x^3+c^2x^2+cix+1}dx\right)b\,c^2x+\log(cix+1)\,acix-\log(x)\,acix-a}{dx}$$

input
```
int((a+b*atan(c*x))/x^2/(d+I*c*d*x),x)
```

output
```
(int(atan(c*x)/(c**3*i*x**5 + c**2*x**4 + c*i*x**3 + x**2),x)*b*x + int(at
an(c*x)/(c**3*i*x**3 + c**2*x**2 + c*i*x + 1),x)*b*c**2*x + log(c*i*x + 1)
*a*c*i*x - log(x)*a*c*i*x - a)/(d*x)
```

# 3.49 $\int \frac{a+b\arctan(cx)}{x^3(d+icdx)}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 161

$$\int \frac{a+b\arctan(cx)}{x^3(d+icdx)}\,dx = -\frac{bc}{2dx} - \frac{bc^2\arctan(cx)}{2d} - \frac{a+b\arctan(cx)}{2dx^2} + \frac{ic(a+b\arctan(cx))}{dx} - \frac{ibc^2\log(x)}{d} + \frac{ibc^2\log(1+c^2x^2)}{2d} - \frac{c^2(a+b\arctan(cx))\log\left(2-\frac{2}{1+icx}\right)}{d} - \frac{ibc^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)}{2d}$$

output
```
-1/2*b*c/d/x-1/2*b*c^2*arctan(c*x)/d-1/2*(a+b*arctan(c*x))/d/x^2+I*c*(a+b*
arctan(c*x))/d/x-I*b*c^2*ln(x)/d+1/2*I*b*c^2*ln(c^2*x^2+1)/d-c^2*(a+b*arct
an(c*x))*ln(2-2/(1+I*c*x))/d-1/2*I*b*c^2*polylog(2,-1+2/(1+I*c*x))/d
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.12 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.11

$$\int \frac{a+b\arctan(cx)}{x^3(d+icdx)}\,dx =$$

$$-\frac{\frac{a+b\arctan(cx)}{x^2}-\frac{2ic(a+b\arctan(cx))}{x}+\frac{bc\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-c^2x^2\right)}{x}+2ac^2\log(x)+2c^2(a+b\arctan(cx))\,\text{l}}{}$$

input
```
Integrate[(a + b*ArcTan[c*x])/(x^3*(d + I*c*d*x)),x]
```

output
```
-1/2*((a + b*ArcTan[c*x])/x^2 - ((2*I)*c*(a + b*ArcTan[c*x]))/x + (b*c*Hyp
ergeometric2F1[-1/2, 1, 1/2, -(c^2*x^2)]])/x + 2*a*c^2*Log[x] + 2*c^2*(a +
b*ArcTan[c*x])*Log[(2*I)/(I - c*x)] + I*b*c^2*(2*Log[x] - Log[1 + c^2*x^2]
) + I*b*c^2*PolyLog[2, (-I)*c*x] - I*b*c^2*PolyLog[2, I*c*x] + I*b*c^2*Pol
yLog[2, (I + c*x)/(-I + c*x)])/d
```

## Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.86, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {5405, 27, 5361, 264, 216, 5405, 5361, 243, 47, 14, 16, 5403, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x^3(d+icdx)}\,dx$$

$$\Big\downarrow \ 5405$$

$$\frac{\int \frac{a+b\arctan(cx)}{x^3}dx}{d}-ic\int \frac{a+b\arctan(cx)}{dx^2(icx+1)}dx$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{a+b\arctan(cx)}{x^3}dx}{d}-\frac{ic\int \frac{a+b\arctan(cx)}{x^2(icx+1)}dx}{d}$$

$$\Big\downarrow \text{5361}$$

$$\frac{\frac{1}{2}bc\int \frac{1}{x^2(c^2x^2+1)}dx-\frac{a+b\arctan(cx)}{2x^2}}{d}-\frac{ic\int \frac{a+b\arctan(cx)}{x^2(icx+1)}dx}{d}$$

$$\Big\downarrow \text{264}$$

$$\frac{\frac{1}{2}bc\left(c^2\left(-\int \frac{1}{c^2x^2+1}dx\right)-\frac{1}{x}\right)-\frac{a+b\arctan(cx)}{2x^2}}{d}-\frac{ic\int \frac{a+b\arctan(cx)}{x^2(icx+1)}dx}{d}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{1}{2}bc\left(-c\arctan(cx)-\frac{1}{x}\right)-\frac{a+b\arctan(cx)}{2x^2}}{d}-\frac{ic\int \frac{a+b\arctan(cx)}{x^2(icx+1)}dx}{d}$$

$$\Big\downarrow \text{5405}$$

$$\frac{\frac{1}{2}bc\left(-c\arctan(cx)-\frac{1}{x}\right)-\frac{a+b\arctan(cx)}{2x^2}}{d}-\frac{ic\left(\int \frac{a+b\arctan(cx)}{x^2}dx-ic\int \frac{a+b\arctan(cx)}{x(icx+1)}dx\right)}{d}$$

$$\Big\downarrow \text{5361}$$

$$\frac{\frac{1}{2}bc\left(-c\arctan(cx)-\frac{1}{x}\right)-\frac{a+b\arctan(cx)}{2x^2}}{d}-\frac{ic\left(-ic\int \frac{a+b\arctan(cx)}{x(icx+1)}dx+bc\int \frac{1}{x(c^2x^2+1)}dx-\frac{a+b\arctan(cx)}{x}\right)}{d}$$

$$\Big\downarrow \text{243}$$

$$\frac{\frac{1}{2}bc\left(-c\arctan(cx)-\frac{1}{x}\right)-\frac{a+b\arctan(cx)}{2x^2}}{d}-\frac{ic\left(-ic\int \frac{a+b\arctan(cx)}{x(icx+1)}dx+\frac{1}{2}bc\int \frac{1}{x^2(c^2x^2+1)}dx^2-\frac{a+b\arctan(cx)}{x}\right)}{d}$$

$$\Big\downarrow \text{47}$$

$$\frac{\frac{1}{2}bc\left(-c\arctan(cx)-\frac{1}{x}\right)-\frac{a+b\arctan(cx)}{2x^2}}{d}-\frac{ic\left(-ic\int \frac{a+b\arctan(cx)}{x(icx+1)}dx+\frac{1}{2}bc\left(\int \frac{1}{x^2}dx^2-c^2\int \frac{1}{c^2x^2+1}dx^2\right)-\frac{a+b\arctan(cx)}{x}\right)}{d}$$

$$\Big\downarrow \text{14}$$

$$\frac{\frac{1}{2}bc\left(-c\arctan(cx)-\frac{1}{x}\right)-\frac{a+b\arctan(cx)}{2x^2}}{d}-\frac{ic\left(-ic\int \frac{a+b\arctan(cx)}{x(icx+1)}dx+\frac{1}{2}bc\left(\log\left(x^2\right)-c^2\int \frac{1}{c^2x^2+1}dx^2\right)-\frac{a+b\arctan(cx)}{x}\right)}{d}$$

$$\Big\downarrow \, 16$$

$$\frac{\frac{1}{2}bc\left(-c\arctan(cx)-\frac{1}{x}\right)-\frac{a+b\arctan(cx)}{2x^2}}{d}-\frac{ic\left(-ic\int\frac{a+b\arctan(cx)}{x(icx+1)}dx-\frac{a+b\arctan(cx)}{x}+\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right)\right)\right)}{d}$$

$$\Big\downarrow \, 5403$$

$$\frac{\frac{1}{2}bc\left(-c\arctan(cx)-\frac{1}{x}\right)-\frac{a+b\arctan(cx)}{2x^2}}{d}-\frac{ic\left(-ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))-bc\int\frac{\log\left(2-\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)-\frac{a+b\arctan(cx)}{x}+\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(c^2x^2+\right.\right.\right.}{d}$$

$$\Big\downarrow \, 2897$$

$$\frac{\frac{1}{2}bc\left(-c\arctan(cx)-\frac{1}{x}\right)-\frac{a+b\arctan(cx)}{2x^2}}{d}-\frac{ic\left(-ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))+\frac{1}{2}ib\,\text{PolyLog}\left(2,\frac{2}{icx+1}-1\right)\right)-\frac{a+b\arctan(cx)}{x}+\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(\right.\right.\right.}{d}$$

input
```
Int[(a + b*ArcTan[c*x])/(x^3*(d + I*c*d*x)),x]
```

output
```
(-1/2*(a + b*ArcTan[c*x])/x^2 + (b*c*(-x^(-1) - c*ArcTan[c*x]))/2)/d - (I*
c*(-((a + b*ArcTan[c*x])/x) + (b*c*(Log[x^2] - Log[1 + c^2*x^2]))/2 - I*c*
((a + b*ArcTan[c*x])*Log[2 - 2/(1 + I*c*x)] + (I/2)*b*PolyLog[2, -1 + 2/(1
 + I*c*x)])))/d
```

## Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5405
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x], x
] - Simp[e/(d*f)   Int[(f*x)^(m + 1)*((a + b*ArcTan[c*x])^p/(d + e*x)), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
&& LtQ[m, -1]
```

## Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.56

| method | result |
|---|---|
| derivativedivides | $c^2\left(-\frac{a}{2d\,c^2x^2}+\frac{ia}{dcx}-\frac{a\ln(cx)}{d}+\frac{a\ln(c^2x^2+1)}{2d}+\frac{ia\arctan(cx)}{d}+\frac{b\left(-\frac{\arctan(cx)}{2c^2x^2}+\frac{i\arctan(cx)}{cx}-\ln(cx)\,\mathrm{arct}\right.}{}\right.$ |
| default | $c^2\left(-\frac{a}{2d\,c^2x^2}+\frac{ia}{dcx}-\frac{a\ln(cx)}{d}+\frac{a\ln(c^2x^2+1)}{2d}+\frac{ia\arctan(cx)}{d}+\frac{b\left(-\frac{\arctan(cx)}{2c^2x^2}+\frac{i\arctan(cx)}{cx}-\ln(cx)\,\mathrm{arct}\right.}{}\right.$ |
| parts | $\frac{c^2a\ln(c^2x^2+1)}{2d}+\frac{ic^2a\arctan(cx)}{d}-\frac{a}{2d\,x^2}+\frac{ica}{dx}-\frac{a\,c^2\ln(x)}{d}+\frac{b\,c^2\left(-\frac{\arctan(cx)}{2c^2x^2}+\frac{i\arctan(cx)}{cx}-\ln(cx)\arctan\right.}{}$ |
| risch | $-\frac{ic^2b\ln(-icx)}{4d}+\frac{ica}{dx}-\frac{3b\,c^2\arctan(cx)}{4d}+\frac{bc\ln(icx+1)}{2dx}-\frac{bc}{2dx}-\frac{ib\,c^2\,\mathrm{dilog}(icx+1)}{2d}+\frac{ic^2b\,\mathrm{dilog}(-icx+1)}{2d}$ |

input
```
int((a+b*arctan(c*x))/x^3/(d+I*c*d*x),x,method=_RETURNVERBOSE)
```

output
```
c^2*(-1/2*a/d/c^2/x^2+I*a/d/c/x-a/d*ln(c*x)+1/2*a/d*ln(c^2*x^2+1)+I*a/d*ar
ctan(c*x)+b/d*(-1/2/c^2/x^2*arctan(c*x)+I*arctan(c*x)/c/x-ln(c*x)*arctan(c
*x)+arctan(c*x)*ln(c*x-I)+1/2*I*ln(c^2*x^2+1)-1/2*arctan(c*x)-I*ln(c*x)-1/
2/c/x-1/2*I*ln(c*x)*ln(1+I*c*x)+1/2*I*ln(c*x)*ln(1-I*c*x)-1/2*I*dilog(1+I*
c*x)+1/2*I*dilog(1-I*c*x)-1/2*I*(dilog(-1/2*I*(c*x+I))+ln(c*x-I)*ln(-1/2*I
*(c*x+I)))+1/4*I*ln(c*x-I)^2))
```

## Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.81

$$\int \frac{a+b\arctan(cx)}{x^3(d+icdx)}\,dx = \frac{2i\,bc^2x^2\text{Li}_2\left(\frac{cx+i}{cx-i}+1\right)-4\,(a+i\,b)c^2x^2\log\left(x\right)+i\,bc^2x^2\log\left(\frac{cx+i}{c}\right)+(4\,a+3i\,b)c^2x^2\log\left(\frac{cx-i}{c}\right)-2\,(-2i\,a}{4\,dx^2}$$

input
```
integrate((a+b*arctan(c*x))/x^3/(d+I*c*d*x),x, algorithm="fricas")
```

output
```
1/4*(2*I*b*c^2*x^2*dilog((c*x + I)/(c*x - I) + 1) - 4*(a + I*b)*c^2*x^2*lo
g(x) + I*b*c^2*x^2*log((c*x + I)/c) + (4*a + 3*I*b)*c^2*x^2*log((c*x - I)/
c) - 2*(-2*I*a + b)*c*x - (2*b*c*x + I*b)*log(-(c*x + I)/(c*x - I)) - 2*a)
/(d*x^2)
```

## Sympy [F]

$$\int \frac{a+b\arctan(cx)}{x^3(d+icdx)}\,dx = -\frac{i\left(\int \frac{a}{cx^4-ix^3}\,dx+\int \frac{b\operatorname{atan}(cx)}{cx^4-ix^3}\,dx\right)}{d}$$

input
```
integrate((a+b*atan(c*x))/x**3/(d+I*c*d*x),x)
```

output
```
-I*(Integral(a/(c*x**4 - I*x**3), x) + Integral(b*atan(c*x)/(c*x**4 - I*x*
*3), x))/d
```

## Maxima [F]

$$\int \frac{a+b\arctan(cx)}{x^3(d+icdx)}\,dx=\int \frac{b\arctan(cx)+a}{(i\,cdx+d)x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^3/(d+I*c*d*x),x, algorithm="maxima")
```

output
```
1/2*(2*c^2*log(I*c*x + 1)/d - 2*c^2*log(x)/d + (2*I*c*x - 1)/(d*x^2))*a +
(-I*c*integrate(arctan(c*x)/(c^2*d*x^4 + d*x^2), x) + integrate(arctan(c*x
)/(c^2*d*x^5 + d*x^3), x))*b
```

## Giac [F]

$$\int \frac{a+b\arctan(cx)}{x^3(d+icdx)}\,dx=\int \frac{b\arctan(cx)+a}{(i\,cdx+d)x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^3/(d+I*c*d*x),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)/((I*c*d*x + d)*x^3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{x^3(d+icdx)}\,dx=\int \frac{a+b\,\mathrm{atan}(c\,x)}{x^3\,\left(d+c\,d\,x\,1\mathrm{i}\right)}\,dx$$

input
```
int((a + b*atan(c*x))/(x^3*(d + c*d*x*1i)),x)
```

output
```
int((a + b*atan(c*x))/(x^3*(d + c*d*x*1i)), x)
```

## Reduce [F]

$$\int \frac{a+b\arctan(cx)}{x^3(d+icdx)}\,dx$$
$$=\frac{2atan(cx)\,a\,c^2i\,x^2+2\left(\int\frac{atan(cx)}{ci\,x^4+x^3}dx\right)b\,x^2+\log(c^2x^2+1)\,a\,c^2x^2-2\log(x)\,a\,c^2x^2+2acix-a}{2d\,x^2}$$

input
```
int((a+b*atan(c*x))/x^3/(d+I*c*d*x),x)
```

output
```
(2*atan(c*x)*a*c**2*i*x**2 + 2*int(atan(c*x)/(c*i*x**4 + x**3),x)*b*x**2 +
 log(c**2*x**2 + 1)*a*c**2*x**2 - 2*log(x)*a*c**2*x**2 + 2*a*c*i*x - a)/(2
*d*x**2)
```

# 3.50 $\int \frac{a+b\arctan(cx)}{x^4(d+icdx)}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 197

$$\int \frac{a+b\arctan(cx)}{x^4(d+icdx)}\,dx = -\frac{bc}{6dx^2}+\frac{ibc^2}{2dx}+\frac{ibc^3\arctan(cx)}{2d}-\frac{a+b\arctan(cx)}{3dx^3}+\frac{ic(a+b\arctan(cx))}{2dx^2}+\frac{c^2(a+b\arctan(cx))}{dx}-\frac{4bc^3\log(x)}{3d}+\frac{2bc^3\log(1+c^2x^2)}{3d}+\frac{ic^3(a+b\arctan(cx))\log\left(2-\frac{2}{1+icx}\right)}{d}-\frac{bc^3\,\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)}{2d}$$

output

```
-1/6*b*c/d/x^2+1/2*I*b*c^2/d/x+1/2*I*b*c^3*arctan(c*x)/d-1/3*(a+b*arctan(c
*x))/d/x^3+1/2*I*c*(a+b*arctan(c*x))/d/x^2+c^2*(a+b*arctan(c*x))/d/x-4/3*b
*c^3*ln(x)/d+2/3*b*c^3*ln(c^2*x^2+1)/d+I*c^3*(a+b*arctan(c*x))*ln(2-2/(1+I
*c*x))/d-1/2*b*c^3*polylog(2,-1+2/(1+I*c*x))/d
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.09 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.29

$$\int \frac{a+b\arctan(cx)}{x^4(d+icdx)}\,dx$$
$$= \frac{-2a+3iacx-bcx+6ac^2x^2-2b\arctan(cx)+3ibcx\arctan(cx)+6bc^2x^2\arctan(cx)+3ibc^2x^2\,\text{Hyperge}}{}$$

input
```
Integrate[(a + b*ArcTan[c*x])/(x^4*(d + I*c*d*x)),x]
```

output
```
(-2*a + (3*I)*a*c*x - b*c*x + 6*a*c^2*x^2 - 2*b*ArcTan[c*x] + (3*I)*b*c*x*
ArcTan[c*x] + 6*b*c^2*x^2*ArcTan[c*x] + (3*I)*b*c^2*x^2*Hypergeometric2F1[
-1/2, 1, 1/2, -(c^2*x^2)] + (6*I)*a*c^3*x^3*Log[x] - 8*b*c^3*x^3*Log[x] +
(6*I)*a*c^3*x^3*Log[(2*I)/(I - c*x)] + (6*I)*b*c^3*x^3*ArcTan[c*x]*Log[(2*
I)/(I - c*x)] + 4*b*c^3*x^3*Log[1 + c^2*x^2] - 3*b*c^3*x^3*PolyLog[2, (-I)
*c*x] + 3*b*c^3*x^3*PolyLog[2, I*c*x] - 3*b*c^3*x^3*PolyLog[2, (I + c*x)/(
-I + c*x)])/(6*d*x^3)
```

## Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.98, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.783$, Rules used = {5405, 27, 5361, 243, 54, 2009, 5405, 5361, 264, 216, 5405, 5361, 243, 47, 14, 16, 5403, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x^4(d+icdx)}\,dx$$
$$\Big\downarrow \; 5405$$
$$\frac{\int \frac{a+b\arctan(cx)}{x^4}dx}{d} - ic\int \frac{a+b\arctan(cx)}{dx^3(icx+1)}dx$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{a+b\arctan(cx)}{x^4}dx}{d}-\frac{ic\int \frac{a+b\arctan(cx)}{x^3(icx+1)}dx}{d}$$

$$\Big\downarrow \ 5361$$

$$\frac{\frac{1}{3}bc\int \frac{1}{x^3(c^2x^2+1)}dx-\frac{a+b\arctan(cx)}{3x^3}}{d}-\frac{ic\int \frac{a+b\arctan(cx)}{x^3(icx+1)}dx}{d}$$

$$\Big\downarrow \ 243$$

$$\frac{\frac{1}{6}bc\int \frac{1}{x^4(c^2x^2+1)}dx^2-\frac{a+b\arctan(cx)}{3x^3}}{d}-\frac{ic\int \frac{a+b\arctan(cx)}{x^3(icx+1)}dx}{d}$$

$$\Big\downarrow \ 54$$

$$\frac{\frac{1}{6}bc\int \left(\frac{c^4}{c^2x^2+1}-\frac{c^2}{x^2}+\frac{1}{x^4}\right)dx^2-\frac{a+b\arctan(cx)}{3x^3}}{d}-\frac{ic\int \frac{a+b\arctan(cx)}{x^3(icx+1)}dx}{d}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{1}{6}bc\left(c^2\left(-\log\left(x^2\right)\right)+c^2\log\left(c^2x^2+1\right)-\frac{1}{x^2}\right)-\frac{a+b\arctan(cx)}{3x^3}}{d}-\frac{ic\int \frac{a+b\arctan(cx)}{x^3(icx+1)}dx}{d}$$

$$\Big\downarrow \ 5405$$

$$\frac{\frac{1}{6}bc\left(c^2\left(-\log\left(x^2\right)\right)+c^2\log\left(c^2x^2+1\right)-\frac{1}{x^2}\right)-\frac{a+b\arctan(cx)}{3x^3}}{d}-\frac{ic\left(\int \frac{a+b\arctan(cx)}{x^3}dx-ic\int \frac{a+b\arctan(cx)}{x^2(icx+1)}dx\right)}{d}$$

$$\Big\downarrow \ 5361$$

$$\frac{\frac{1}{6}bc\left(c^2\left(-\log\left(x^2\right)\right)+c^2\log\left(c^2x^2+1\right)-\frac{1}{x^2}\right)-\frac{a+b\arctan(cx)}{3x^3}}{d}-\frac{ic\left(-ic\int \frac{a+b\arctan(cx)}{x^2(icx+1)}dx+\frac{1}{2}bc\int \frac{1}{x^2(c^2x^2+1)}dx-\frac{a+b\arctan(cx)}{2x^2}\right)}{d}$$

$$\Big\downarrow \ 264$$

$$\frac{\frac{1}{6}bc\left(c^2\left(-\log\left(x^2\right)\right)+c^2\log\left(c^2x^2+1\right)-\frac{1}{x^2}\right)-\frac{a+b\arctan(cx)}{3x^3}}{d}-\frac{ic\left(-ic\int \frac{a+b\arctan(cx)}{x^2(icx+1)}dx+\frac{1}{2}bc\left(c^2\left(-\int \frac{1}{c^2x^2+1}dx\right)-\frac{1}{x}\right)-\frac{a+b\arctan(cx)}{2x^2}\right)}{d}$$

$$\Big\downarrow \ 216$$

$$\frac{\frac{1}{6} b c \left(c^2 \left(-\log \left(x^2\right)\right)+c^2 \log \left(c^2 x^2+1\right)-\frac{1}{x^2}\right)-\frac{a+b \arctan (c x)}{3 x^3}}{d}-\frac{i c \left(-i c \int \frac{a+b \arctan (c x)}{x^2 (i c x+1)}dx-\frac{a+b \arctan (c x)}{2 x^2}+\frac{1}{2} b c \left(-c \arctan (c x)-\frac{1}{x}\right)\right)}{d}$$

$\Big\downarrow$ 5405

$$\frac{\frac{1}{6} b c \left(c^2 \left(-\log \left(x^2\right)\right)+c^2 \log \left(c^2 x^2+1\right)-\frac{1}{x^2}\right)-\frac{a+b \arctan (c x)}{3 x^3}}{d}-\frac{i c \left(-i c \left(\int \frac{a+b \arctan (c x)}{x^2}dx-i c \int \frac{a+b \arctan (c x)}{x (i c x+1)}dx\right)-\frac{a+b \arctan (c x)}{2 x^2}+\frac{1}{2} b c \left(-c \arctan (c x)-\frac{1}{x}\right)\right)}{d}$$

$\Big\downarrow$ 5361

$$\frac{\frac{1}{6} b c \left(c^2 \left(-\log \left(x^2\right)\right)+c^2 \log \left(c^2 x^2+1\right)-\frac{1}{x^2}\right)-\frac{a+b \arctan (c x)}{3 x^3}}{d}-\frac{i c \left(-i c \left(-i c \int \frac{a+b \arctan (c x)}{x (i c x+1)}dx+b c \int \frac{1}{x \left(c^2 x^2+1\right)}dx-\frac{a+b \arctan (c x)}{x}\right)-\frac{a+b \arctan (c x)}{2 x^2}+\frac{1}{2} b c \left(-c \arctan (c x)-\frac{1}{x}\right)\right)}{d}$$

$\Big\downarrow$ 243

$$\frac{\frac{1}{6} b c \left(c^2 \left(-\log \left(x^2\right)\right)+c^2 \log \left(c^2 x^2+1\right)-\frac{1}{x^2}\right)-\frac{a+b \arctan (c x)}{3 x^3}}{d}-\frac{i c \left(-i c \left(-i c \int \frac{a+b \arctan (c x)}{x (i c x+1)}dx+\frac{1}{2} b c \int \frac{1}{x^2 \left(c^2 x^2+1\right)}dx^2-\frac{a+b \arctan (c x)}{x}\right)-\frac{a+b \arctan (c x)}{2 x^2}+\frac{1}{2} b c \left(-c \arctan (c x)-\frac{1}{x}\right)\right)}{d}$$

$\Big\downarrow$ 47

$$\frac{\frac{1}{6} b c \left(c^2 \left(-\log \left(x^2\right)\right)+c^2 \log \left(c^2 x^2+1\right)-\frac{1}{x^2}\right)-\frac{a+b \arctan (c x)}{3 x^3}}{d}-\frac{i c \left(-i c \left(-i c \int \frac{a+b \arctan (c x)}{x (i c x+1)}dx+\frac{1}{2} b c \left(\int \frac{1}{x^2}dx^2-c^2 \int \frac{1}{c^2 x^2+1}dx^2\right)-\frac{a+b \arctan (c x)}{x}\right)-\frac{a+b \arctan (c x)}{2 x^2}+\frac{1}{2} b c (-c \,\mathrm{arctar}\ldots}{d}$$

$\Big\downarrow$ 14

$$\frac{\frac{1}{6} b c \left(c^2 \left(-\log \left(x^2\right)\right)+c^2 \log \left(c^2 x^2+1\right)-\frac{1}{x^2}\right)-\frac{a+b \arctan (c x)}{3 x^3}}{d}-\frac{i c \left(-i c \left(-i c \int \frac{a+b \arctan (c x)}{x (i c x+1)}dx+\frac{1}{2} b c \left(\log \left(x^2\right)-c^2 \int \frac{1}{c^2 x^2+1}dx^2\right)-\frac{a+b \arctan (c x)}{x}\right)-\frac{a+b \arctan (c x)}{2 x^2}+\frac{1}{2} b c (-c \,\mathrm{arctar}\ldots}{d}$$

$\Big\downarrow$ 16

$$\frac{\frac{1}{6} b c \left(c^2 \left(-\log \left(x^2\right)\right)+c^2 \log \left(c^2 x^2+1\right)-\frac{1}{x^2}\right)-\frac{a+b \arctan (c x)}{3 x^3}}{d}-\frac{i c \left(-i c \left(-i c \int \frac{a+b \arctan (c x)}{x (i c x+1)}dx-\frac{a+b \arctan (c x)}{x}+\frac{1}{2} b c \left(\log \left(x^2\right)-\log \left(c^2 x^2+1\right)\right)\right)-\frac{a+b \arctan (c x)}{2 x^2}+\frac{1}{2} b c (-c \,\mathrm{arctan}\ldots}{d}$$

$\Big\downarrow$ 5403

$$\frac{\frac{1}{6}bc(c^2(-\log(x^2))+c^2\log(c^2x^2+1)-\frac{1}{x^2})-\frac{a+b\arctan(cx)}{3x^3}}{d}-$$

$$\frac{ic\left(-ic\left(-ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))-bc\int\frac{\log\left(2-\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)-\frac{a+b\arctan(cx)}{x}+\frac{1}{2}bc(\log(x^2)-\log(c^2\right.\right.}{d}$$

$\Big\downarrow$ 2897

$$\frac{\frac{1}{6}bc(c^2(-\log(x^2))+c^2\log(c^2x^2+1)-\frac{1}{x^2})-\frac{a+b\arctan(cx)}{3x^3}}{d}-$$

$$\frac{ic\left(-ic\left(-ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))+\frac{1}{2}ib\,\text{PolyLog}\left(2,\frac{2}{icx+1}-1\right)\right)-\frac{a+b\arctan(cx)}{x}+\frac{1}{2}bc(\log(x^2)-\right.\right.}{d}$$

input
```
Int[(a + b*ArcTan[c*x])/(x^4*(d + I*c*d*x)),x]
```

output
```
(-1/3*(a + b*ArcTan[c*x])/x^3 + (b*c*(-x^(-2) - c^2*Log[x^2] + c^2*Log[1 +
 c^2*x^2]))/6)/d - (I*c*(-1/2*(a + b*ArcTan[c*x])/x^2 + (b*c*(-x^(-1) - c*
ArcTan[c*x]))/2 - I*c*(-((a + b*ArcTan[c*x])/x) + (b*c*(Log[x^2] - Log[1 +
 c^2*x^2]))/2 - I*c*((a + b*ArcTan[c*x])*Log[2 - 2/(1 + I*c*x)] + (I/2)*b*
PolyLog[2, -1 + 2/(1 + I*c*x)]))))/d
```

### Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5405
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x], x
] - Simp[e/(d*f)   Int[(f*x)^(m + 1)*((a + b*ArcTan[c*x])^p/(d + e*x)), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
&& LtQ[m, -1]
```

## Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.41

| method | result |
|---|---|
| derivativedivides | $c^3\left(-\frac{a}{3d\,c^3x^3}+\frac{ia}{2d\,c^2x^2}+\frac{ia\ln(cx)}{d}+\frac{a}{dcx}-\frac{ia\ln(c^2x^2+1)}{2d}+\frac{a\arctan(cx)}{d}+\frac{b\left(-\frac{\arctan(cx)}{3c^3x^3}-i\arctan(c\right.}{}\right.$ |
| default | $c^3\left(-\frac{a}{3d\,c^3x^3}+\frac{ia}{2d\,c^2x^2}+\frac{ia\ln(cx)}{d}+\frac{a}{dcx}-\frac{ia\ln(c^2x^2+1)}{2d}+\frac{a\arctan(cx)}{d}+\frac{b\left(-\frac{\arctan(cx)}{3c^3x^3}-i\arctan(c\right.}{}\right.$ |
| parts | $-\frac{ic^3a\ln(c^2x^2+1)}{2d}+\frac{c^3a\arctan(cx)}{d}-\frac{a}{3d\,x^3}+\frac{ica}{2d\,x^2}+\frac{ia\,c^3\ln(x)}{d}+\frac{c^2a}{dx}+\frac{b\,c^3\left(-\frac{\arctan(cx)}{3c^3x^3}-i\arctan(cx)\right.}{}$ |
| risch | $-\frac{a}{3d\,x^3}+\frac{c^2a}{dx}+\frac{c^3a\arctan(cx)}{d}-\frac{ic^3a\ln(c^2x^2+1)}{2d}+\frac{ica}{2d\,x^2}-\frac{ib\,c^2\ln(icx+1)}{2dx}+\frac{ic^2b\ln(-icx+1)}{2dx}+\frac{ib\ln(i}{6d}$ |

input
```
int((a+b*arctan(c*x))/x^4/(d+I*c*d*x),x,method=_RETURNVERBOSE)
```

output
```
c^3*(-1/3*a/d/c^3/x^3+1/2*I*a/d/c^2/x^2+I*a/d*ln(c*x)+a/d/c/x-1/2*I*a/d*ln
(c^2*x^2+1)+a/d*arctan(c*x)+b/d*(-1/3/c^3/x^3*arctan(c*x)-I*arctan(c*x)*ln
(c*x-I)+1/2*I*arctan(c*x)+1/c/x*arctan(c*x)+1/2*I*arctan(c*x)/c^2/x^2+1/4*
ln(c*x-I)^2-1/2*ln(c*x-I)*ln(-1/2*I*(c*x+I))-1/2*dilog(-1/2*I*(c*x+I))-1/2
*ln(c*x)*ln(1+I*c*x)+1/2*ln(c*x)*ln(1-I*c*x)-1/2*dilog(1+I*c*x)+1/2*dilog(
1-I*c*x)+2/3*ln(c^2*x^2+1)+I*arctan(c*x)*ln(c*x)+1/2*I/c/x-1/6/c^2/x^2-4/3
*ln(c*x)))
```

## Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.79

$$\int \frac{a+b\arctan(cx)}{x^4(d+icdx)}\,dx = \frac{6\,bc^3x^3\mathrm{Li}_2\left(\frac{cx+i}{cx-i}+1\right)-4\,(-3i\,a+4\,b)c^3x^3\log\left(x\right)+5\,bc^3x^3\log\left(\frac{cx+i}{c}\right)+(-12i\,a+11\,b)c^3x^3\log\left(\frac{cx-i}{c}\right)+}{12\,dx^3}$$

input
```
integrate((a+b*arctan(c*x))/x^4/(d+I*c*d*x),x, algorithm="fricas")
```

output
```
1/12*(6*b*c^3*x^3*dilog((c*x + I)/(c*x - I) + 1) - 4*(-3*I*a + 4*b)*c^3*x^
3*log(x) + 5*b*c^3*x^3*log((c*x + I)/c) + (-12*I*a + 11*b)*c^3*x^3*log((c*
x - I)/c) + 6*(2*a + I*b)*c^2*x^2 - 2*(-3*I*a + b)*c*x + (6*I*b*c^2*x^2 -
3*b*c*x - 2*I*b)*log(-(c*x + I)/(c*x - I)) - 4*a)/(d*x^3)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{x^4(d+icdx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*atan(c*x))/x**4/(d+I*c*d*x),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{a+b\arctan(cx)}{x^4(d+icdx)}\,dx=\int \frac{b\arctan(cx)+a}{(i\,cdx+d)x^4}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^4/(d+I*c*d*x),x, algorithm="maxima")
```

output
```
-1/6*(6*I*c^3*log(I*c*x + 1)/d - 6*I*c^3*log(x)/d - (6*c^2*x^2 + 3*I*c*x -
 2)/(d*x^3))*a + (-I*c*integrate(arctan(c*x)/(c^2*d*x^5 + d*x^3), x) + int
egrate(arctan(c*x)/(c^2*d*x^6 + d*x^4), x))*b
```

## Giac [F]

$$\int \frac{a+b\arctan(cx)}{x^4(d+icdx)}\,dx=\int \frac{b\arctan(cx)+a}{(i\,cdx+d)x^4}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^4/(d+I*c*d*x),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)/((I*c*d*x + d)*x^4), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{x^4(d+icdx)}\,dx=\int \frac{a+b\,\mathrm{atan}(c\,x)}{x^4\,\left(d+c\,d\,x\,1\mathrm{i}\right)}\,dx$$

input
```
int((a + b*atan(c*x))/(x^4*(d + c*d*x*1i)),x)
```

output
```
int((a + b*atan(c*x))/(x^4*(d + c*d*x*1i)), x)
```

## Reduce [F]

$$\int \frac{a+b\arctan(cx)}{x^4(d+icdx)}\,dx$$
$$= \frac{-3atan(cx)^2\,b\,c^3x^3-6atan(cx)\,b\,c^2x^2+6\left(\int \frac{atan(cx)}{c^3i\,x^7+c^2x^6+ci\,x^5+x^4}dx\right)b\,x^3-6\left(\int \frac{atan(cx)}{c^3i\,x^4+c^2x^3+ci\,x^2+x}dx\right)b\,c^3i}{}$$

input
```
int((a+b*atan(c*x))/x^4/(d+I*c*d*x),x)
```

output
```
( - 3*atan(c*x)**2*b*c**3*x**3 - 6*atan(c*x)*b*c**2*x**2 + 6*int(atan(c*x)
/(c**3*i*x**7 + c**2*x**6 + c*i*x**5 + x**4),x)*b*x**3 - 6*int(atan(c*x)/(
c**3*i*x**4 + c**2*x**3 + c*i*x**2 + x),x)*b*c**3*i*x**3 - 6*int(1/(c**3*i
*x**6 + c**2*x**5 + c*i*x**4 + x**3),x)*a*c*i*x**3 - 6*int(1/(c**3*i*x**4
 + c**2*x**3 + c*i*x**2 + x),x)*a*c**3*i*x**3 - 3*log(c**2*x**2 + 1)*b*c**3
*x**3 + 6*log(x)*b*c**3*x**3 - 2*a)/(6*d*x**3)
```

# 3.51 $\int \frac{x^3(a+b\arctan(cx))}{(d+icdx)^2}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 203

$$\int \frac{x^3(a+b\arctan(cx))}{(d+icdx)^2}\,dx = -\frac{2iax}{c^3d^2}+\frac{bx}{2c^3d^2}+\frac{b}{2c^4d^2(i-cx)}-\frac{b\arctan(cx)}{c^4d^2}-\frac{2ibx\arctan(cx)}{c^3d^2}-\frac{x^2(a+b\arctan(cx))}{2c^2d^2}+\frac{i(a+b\arctan(cx))}{c^4d^2(i-cx)}-\frac{3(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c^4d^2}+\frac{ib\log(1+c^2x^2)}{c^4d^2}-\frac{3ib\,\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2c^4d^2}$$

output
```
-2*I*a*x/c^3/d^2+1/2*b*x/c^3/d^2+1/2*b/c^4/d^2/(I-c*x)-b*arctan(c*x)/c^4/d
^2-2*I*b*x*arctan(c*x)/c^3/d^2-1/2*x^2*(a+b*arctan(c*x))/c^2/d^2+I*(a+b*ar
ctan(c*x))/c^4/d^2/(I-c*x)-3*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^4/d^2+I*b
*ln(c^2*x^2+1)/c^4/d^2-3/2*I*b*polylog(2,1-2/(1+I*c*x))/c^4/d^2
```

## Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.92

$$\int \frac{x^3(a+b\arctan(cx))}{(d+icdx)^2}\,dx =$$
$$-\frac{8iacx+2ac^2x^2+\frac{4ia}{-i+cx}-12ia\arctan(cx)-6a\log\left(1+c^2x^2\right)+b\big(-2cx-12i\arctan(cx)^2+i\cos(2\,\mathrm{a}}{}$$

input
```
Integrate[(x^3*(a + b*ArcTan[c*x]))/(d + I*c*d*x)^2,x]
```

output
```
-1/4*((8*I)*a*c*x + 2*a*c^2*x^2 + ((4*I)*a)/(-I + c*x) - (12*I)*a*ArcTan[c
*x] - 6*a*Log[1 + c^2*x^2] + b*(-2*c*x - (12*I)*ArcTan[c*x]^2 + I*Cos[2*Ar
cTan[c*x]] - (4*I)*Log[1 + c^2*x^2] - (6*I)*PolyLog[2, -E^((2*I)*ArcTan[c*
x])] + 2*ArcTan[c*x]*(1 + (4*I)*c*x + c^2*x^2 - Cos[2*ArcTan[c*x]] + 6*Log
[1 + E^((2*I)*ArcTan[c*x])] + I*Sin[2*ArcTan[c*x]]) + Sin[2*ArcTan[c*x]]))
/(c^4*d^2)
```

## Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\arctan(cx))}{(d+icdx)^2}\,dx$$

$$\Big\downarrow \; 5411$$

$$\int \left(\frac{3(a+b\arctan(cx))}{c^3d^2(cx-i)}-\frac{2i(a+b\arctan(cx))}{c^3d^2}+\frac{i(a+b\arctan(cx))}{c^3d^2(cx-i)^2}-\frac{x(a+b\arctan(cx))}{c^2d^2}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{i(a+b\arctan(cx))}{c^4d^2(-cx+i)}-\frac{3\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c^4d^2}-\frac{x^2(a+b\arctan(cx))}{2c^2d^2}-\frac{2iax}{c^3d^2}-\frac{b\arctan(cx)}{c^4d^2}-\frac{2ibx\arctan(cx)}{c^3d^2}-\frac{3ib\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{2c^4d^2}+\frac{b}{2c^4d^2(-cx+i)}+\frac{bx}{2c^3d^2}+\frac{ib\log\left(c^2x^2+1\right)}{c^4d^2}$$

input
```
Int[(x^3*(a + b*ArcTan[c*x]))/(d + I*c*d*x)^2,x]
```

output
```
((-2*I)*a*x)/(c^3*d^2) + (b*x)/(2*c^3*d^2) + b/(2*c^4*d^2*(I - c*x)) - (b*
ArcTan[c*x])/(c^4*d^2) - ((2*I)*b*x*ArcTan[c*x])/(c^3*d^2) - (x^2*(a + b*A
rcTan[c*x]))/(2*c^2*d^2) + (I*(a + b*ArcTan[c*x]))/(c^4*d^2*(I - c*x)) - (
3*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(c^4*d^2) + (I*b*Log[1 + c^2*x^2
])/(c^4*d^2) - (((3*I)/2)*b*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c^4*d^2)
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

## Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.56

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{3ib\ln\left(c^2x^2+1\right)}{4d^2}-\frac{a c^2x^2}{2d^2}+\frac{3a\ln\left(c^2x^2+1\right)}{2d^2}-\frac{2iacx}{d^2}-\frac{ia}{d^2(cx-i)}-\frac{3ib\ln\left(-\frac{i(cx+i)}{2}\right)\ln(cx-i)}{2d^2}-\frac{b\arctan(cx)c^2x^2}{2d^2}+\frac{3b\arctan(cx)\ln}{d^2}}{}$ |
| default | $\frac{\frac{3ib\ln\left(c^2x^2+1\right)}{4d^2}-\frac{a c^2x^2}{2d^2}+\frac{3a\ln\left(c^2x^2+1\right)}{2d^2}-\frac{2iacx}{d^2}-\frac{ia}{d^2(cx-i)}-\frac{3ib\ln\left(-\frac{i(cx+i)}{2}\right)\ln(cx-i)}{2d^2}-\frac{b\arctan(cx)c^2x^2}{2d^2}+\frac{3b\arctan(cx)\ln}{d^2}}{}$ |
| parts | $-\frac{a x^2}{2d^2c^2}-\frac{3ib\ln(cx-i)\ln\left(-\frac{i(cx+i)}{2}\right)}{2c^4d^2}+\frac{3a\ln(c^2x^2+1)}{2d^2c^4}+\frac{3ia\arctan(cx)}{d^2c^4}-\frac{ib}{2d^2c^4}-\frac{2iax}{c^3d^2}-\frac{b\arctan(cx)x^2}{2c^2d^2}$ |
| risch | $\frac{bx\ln(-icx+1)}{d^2c^3}+\frac{3ib\ln(\frac{1}{2}+\frac{icx}{2})\ln(-icx+1)}{2d^2c^4}-\frac{3ib\ln(icx+1)^2}{4c^4d^2}+\frac{3ib\ln(-icx+1)}{4d^2c^4}+\Bigg(\frac{ib(\frac{1}{2}cx^2+2ix)}{2d^2c^3}-\frac{}{2c^4d^2}$ |

input
```
int(x^3*(a+b*arctan(c*x))/(d+I*c*d*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c^4*(3/4*I*b/d^2*ln(c^2*x^2+1)-1/2*a/d^2*c^2*x^2+3/2*a/d^2*ln(c^2*x^2+1)
-2*I*a/d^2*c*x-I*a/d^2/(c*x-I)-3/2*I*b/d^2*ln(-1/2*I*(c*x+I))*ln(c*x-I)-1/
2*b/d^2*arctan(c*x)*c^2*x^2+3*b/d^2*arctan(c*x)*ln(c*x-I)-I*b/d^2*arctan(c
*x)/(c*x-I)+1/8*I*b/d^2*ln(c^4*x^4+10*c^2*x^2+9)-1/2*I*b/d^2-3/2*I*b/d^2*d
ilog(-1/2*I*(c*x+I))+1/2*b/d^2*c*x+3/4*I*b/d^2*ln(c*x-I)^2-1/2*b/d^2/(c*x-
I)-2*I*b/d^2*arctan(c*x)*c*x-3/2*b*arctan(c*x)/d^2+3*I*a/d^2*arctan(c*x)-1
/4*b/d^2*arctan(1/2*c*x)+1/4*b/d^2*arctan(1/6*c^3*x^3+7/6*c*x)+1/2*b/d^2*a
rctan(1/2*c*x-1/2*I))
```

## Fricas [F]

$$\int \frac{x^3(a+b\arctan(cx))}{(d+icdx)^2}\,dx=\int \frac{(b\arctan{(cx)}+a)x^3}{(i\,cdx+d)^2}\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))/(d+I*c*d*x)^2,x, algorithm="fricas")
```

output
```
integral(1/2*(-I*b*x^3*log(-(c*x + I)/(c*x - I)) - 2*a*x^3)/(c^2*d^2*x^2 -
 2*I*c*d^2*x - d^2), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\arctan(cx))}{(d+icdx)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(a+b*atan(c*x))/(d+I*c*d*x)**2,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{x^3(a+b\arctan(cx))}{(d+icdx)^2}\,dx = \int \frac{(b\arctan(cx)+a)x^3}{(i\,cdx+d)^2}\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))/(d+I*c*d*x)^2,x, algorithm="maxima")
```

output
```
-1/2*a*(2*I/(c^5*d^2*x - I*c^4*d^2) + (c*x^2 + 4*I*x)/(c^3*d^2) - 6*log(c*
x - I)/(c^4*d^2)) + 1/8*(I*c^3*x^3 - 5*c^2*x^2 - 2*c*x*(arctan2(1, c*x) -
3*I) - 12*(-I*c*x - 1)*arctan(c*x)^2 - 3*(-I*c*x - 1)*log(c^2*x^2 + 1)^2 -
 3*(c^5*d^2*x - I*c^4*d^2)*((c*(x/(c^7*d^2*x^2 + c^5*d^2) + arctan(c*x)/(c
^6*d^2)) - 2*arctan(c*x)/(c^7*d^2*x^2 + c^5*d^2))*c + 16*integrate(1/8*log
(c^2*x^2 + 1)/(c^7*d^2*x^4 + 2*c^5*d^2*x^2 + c^3*d^2), x)) + 3*(-I*c^5*d^2
*x - c^4*d^2)*(c*(c^2/(c^9*d^2*x^2 + c^7*d^2) + log(c^2*x^2 + 1)/(c^7*d^2*
x^2 + c^5*d^2)) + 32*integrate(1/8*arctan(c*x)/(c^7*d^2*x^4 + 2*c^5*d^2*x^
2 + c^3*d^2), x)) + 6*(c^6*d^2*x - I*c^5*d^2)*(c*(x/(c^7*d^2*x^2 + c^5*d^2
) + arctan(c*x)/(c^6*d^2)) - 16*c*integrate(1/8*x^2*log(c^2*x^2 + 1)/(c^7*
d^2*x^4 + 2*c^5*d^2*x^2 + c^3*d^2), x) - 2*arctan(c*x)/(c^7*d^2*x^2 + c^5*
d^2)) - 6*(I*c^6*d^2*x + c^5*d^2)*(32*c*integrate(1/8*x^2*arctan(c*x)/(c^7
*d^2*x^4 + 2*c^5*d^2*x^2 + c^3*d^2), x) - c^2/(c^9*d^2*x^2 + c^7*d^2) - lo
g(c^2*x^2 + 1)/(c^7*d^2*x^2 + c^5*d^2)) - 2*(c^3*x^3 + 3*I*c^2*x^2 + 4*c*x
 + 2*I)*arctan(c*x) - 16*(c^9*d^2*x - I*c^8*d^2)*integrate(1/8*(2*c*x^5*ar
ctan(c*x) + x^4*log(c^2*x^2 + 1))/(c^7*d^2*x^4 + 2*c^5*d^2*x^2 + c^3*d^2),
 x) - 16*(-I*c^9*d^2*x - c^8*d^2)*integrate(1/8*(c*x^5*log(c^2*x^2 + 1) -
2*x^4*arctan(c*x))/(c^7*d^2*x^4 + 2*c^5*d^2*x^2 + c^3*d^2), x) - 16*(I*c^8
*d^2*x + c^7*d^2)*integrate(1/8*(2*c*x^4*arctan(c*x) + x^3*log(c^2*x^2 + 1
))/(c^7*d^2*x^4 + 2*c^5*d^2*x^2 + c^3*d^2), x) - 16*(c^8*d^2*x - I*c^7*...
```

## Giac [F]

$$\int \frac{x^3(a+b\arctan(cx))}{(d+icdx)^2}\,dx=\int \frac{(b\arctan{(cx)}+a)x^3}{(i\,cdx+d)^2}\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))/(d+I*c*d*x)^2,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)*x^3/(I*c*d*x + d)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\arctan(cx))}{(d+icdx)^2}\,dx=\int \frac{x^3\,(a+b\,\mathrm{atan}(c\,x))}{(d+c\,d\,x\,1\mathrm{i})^2}\,dx$$

input
```
int((x^3*(a + b*atan(c*x)))/(d + c*d*x*1i)^2,x)
```

output
```
int((x^3*(a + b*atan(c*x)))/(d + c*d*x*1i)^2, x)
```

## Reduce [F]

$$\int \frac{x^3(a+b\arctan(cx))}{(d+icdx)^2}\,dx=\frac{-\left(\int \frac{x^3}{c^2x^2-2cix-1}dx\right)a-\left(\int \frac{atan(cx)x^3}{c^2x^2-2cix-1}dx\right)b}{d^2}$$

input
```
int(x^3*(a+b*atan(c*x))/(d+I*c*d*x)^2,x)
```

output
```
( - (int(x**3/(c**2*x**2 - 2*c*i*x - 1),x)*a + int((atan(c*x)*x**3)/(c**2*
x**2 - 2*c*i*x - 1),x)*b))/d**2
```

# 3.52 $\int \frac{x^2(a+b\arctan(cx))}{(d+icdx)^2}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 167

$$\int \frac{x^2(a+b\arctan(cx))}{(d+icdx)^2}\,dx = -\frac{ax}{c^2d^2}-\frac{ib}{2c^3d^2(i-cx)}+\frac{ib\arctan(cx)}{2c^3d^2}-\frac{bx\arctan(cx)}{c^2d^2}+\frac{a+b\arctan(cx)}{c^3d^2(i-cx)}+\frac{2i(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c^3d^2}+\frac{b\log(1+c^2x^2)}{2c^3d^2}-\frac{b\,\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^3d^2}$$

output
```
-a*x/c^2/d^2-1/2*I*b/c^3/d^2/(I-c*x)+1/2*I*b*arctan(c*x)/c^3/d^2-b*x*arcta
n(c*x)/c^2/d^2+(a+b*arctan(c*x))/c^3/d^2/(I-c*x)+2*I*(a+b*arctan(c*x))*ln(
2/(1+I*c*x))/c^3/d^2+1/2*b*ln(c^2*x^2+1)/c^3/d^2-b*polylog(2,1-2/(1+I*c*x)
)/c^3/d^2
```

## Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.92

$$\int \frac{x^2(a+b\arctan(cx))}{(d+icdx)^2}\,dx =$$

$$-\frac{4acx+\frac{4a}{-i+cx}-8a\arctan(cx)+4ia\log(1+c^2x^2)+b\big(-8\arctan(cx)^2+\cos(2\arctan(cx))-2\log(1}{}$$

input
```
Integrate[(x^2*(a + b*ArcTan[c*x]))/(d + I*c*d*x)^2,x]
```

output
```
-1/4*(4*a*c*x + (4*a)/(-I + c*x) - 8*a*ArcTan[c*x] + (4*I)*a*Log[1 + c^2*x
^2] + b*(-8*ArcTan[c*x]^2 + Cos[2*ArcTan[c*x]] - 2*Log[1 + c^2*x^2] - 4*Po
lyLog[2, -E^((2*I)*ArcTan[c*x])] - I*Sin[2*ArcTan[c*x]] + 2*ArcTan[c*x]*(2
*c*x + I*Cos[2*ArcTan[c*x]] - (4*I)*Log[1 + E^((2*I)*ArcTan[c*x])] + Sin[2
*ArcTan[c*x]])))/(c^3*d^2)
```

## Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\arctan(cx))}{(d+icdx)^2}\,dx$$

$$\Big\downarrow \text{5411}$$

$$\int \left(-\frac{2i(a+b\arctan(cx))}{c^2d^2(cx-i)}-\frac{a+b\arctan(cx)}{c^2d^2}+\frac{a+b\arctan(cx)}{c^2d^2(cx-i)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a+b\arctan(cx)}{c^3d^2(-cx+i)}+\frac{2i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c^3d^2}-\frac{ax}{c^2d^2}+\frac{ib\arctan(cx)}{2c^3d^2}-\frac{bx\arctan(cx)}{c^2d^2}-\frac{b\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{c^3d^2}-\frac{ib}{2c^3d^2(-cx+i)}+\frac{b\log\left(c^2x^2+1\right)}{2c^3d^2}$$

input
```
Int[(x^2*(a + b*ArcTan[c*x]))/(d + I*c*d*x)^2,x]
```

output
```
-((a*x)/(c^2*d^2)) - ((I/2)*b)/(c^3*d^2*(I - c*x)) + ((I/2)*b*ArcTan[c*x])
/(c^3*d^2) - (b*x*ArcTan[c*x])/(c^2*d^2) + (a + b*ArcTan[c*x])/(c^3*d^2*(I
 - c*x)) + ((2*I)*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(c^3*d^2) + (b*L
og[1 + c^2*x^2])/(2*c^3*d^2) - (b*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c^3*d^2)
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

## Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.62

| method | result |
|---|---|
| derivativedivides | $-\frac{acx}{d^2}+\frac{2a\arctan(cx)}{d^2}-\frac{ia\ln\left(c^2x^2+1\right)}{d^2}-\frac{a}{d^2(cx-i)}-\frac{b\arctan(cx)cx}{d^2}+\frac{ib}{2d^2(cx-i)}-\frac{b\arctan(cx)}{d^2(cx-i)}-\frac{b\ln(cx-i)\ln\left(-\frac{i(cx+i)}{2}\right)}{d^2}-\frac{b\,\mathrm{di}}{}$ |
| default | $-\frac{acx}{d^2}+\frac{2a\arctan(cx)}{d^2}-\frac{ia\ln\left(c^2x^2+1\right)}{d^2}-\frac{a}{d^2(cx-i)}-\frac{b\arctan(cx)cx}{d^2}+\frac{ib}{2d^2(cx-i)}-\frac{b\arctan(cx)}{d^2(cx-i)}-\frac{b\ln(cx-i)\ln\left(-\frac{i(cx+i)}{2}\right)}{d^2}-\frac{b\,\mathrm{di}}{}$ |
| parts | $-\frac{ax}{c^2d^2}-\frac{ib\arctan\left(\frac{cx}{2}-\frac{i}{2}\right)}{4c^3d^2}+\frac{2a\arctan(cx)}{d^2c^3}+\frac{a}{d^2c^3(-cx+i)}-\frac{bx\arctan(cx)}{c^2d^2}-\frac{ia\ln\left(c^2x^2+1\right)}{d^2c^3}-\frac{b\arctan(c}{c^3d^2(cx-}$ |
| risch | $-\frac{b\ln(icx+1)^2}{2c^3d^2}+\left(\frac{ixb}{2c^2d^2}+\frac{ib}{2c^3d^2(cx-i)}\right)\ln\left(icx+1\right)+\frac{ib\ln(-icx+1)x}{4d^2c^2(-icx-1)}-\frac{b\,\mathrm{dilog}\left(\frac{1}{2}-\frac{icx}{2}\right)}{d^2c^3}+\frac{2a\arctan}{d^2c^3}$ |

input
```
int(x^2*(a+b*arctan(c*x))/(d+I*c*d*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c^3*(-a/d^2*c*x+2*a/d^2*arctan(c*x)-I*a/d^2*ln(c^2*x^2+1)-a/d^2/(c*x-I)-
b/d^2*arctan(c*x)*c*x+1/2*I*b/d^2/(c*x-I)-b/d^2*arctan(c*x)/(c*x-I)-b/d^2*
ln(c*x-I)*ln(-1/2*I*(c*x+I))-b/d^2*dilog(-1/2*I*(c*x+I))+1/2*b/d^2*ln(c*x-
I)^2+1/16*b/d^2*ln(c^4*x^4+10*c^2*x^2+9)-1/8*I*b/d^2*arctan(1/6*c^3*x^3+7/
6*c*x)-2*I*b/d^2*arctan(c*x)*ln(c*x-I)-1/4*I*b/d^2*arctan(1/2*c*x-1/2*I)+3
/4*I*b/d^2*arctan(c*x)+3/8*b/d^2*ln(c^2*x^2+1)+1/8*I*b/d^2*arctan(1/2*c*x)
)
```

## Fricas [F]

$$\int \frac{x^2(a+b\arctan(cx))}{(d+icdx)^2}\,dx = \int \frac{(b\arctan(cx)+a)x^2}{(i\,cdx+d)^2}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))/(d+I*c*d*x)^2,x, algorithm="fricas")
```

output
```
integral(1/2*(-I*b*x^2*log(-(c*x + I)/(c*x - I)) - 2*a*x^2)/(c^2*d^2*x^2 -
 2*I*c*d^2*x - d^2), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\arctan(cx))}{(d+icdx)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(a+b*atan(c*x))/(d+I*c*d*x)**2,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{x^2(a+b\arctan(cx))}{(d+icdx)^2}\,dx=\int \frac{(b\arctan(cx)+a)x^2}{(i\,cdx+d)^2}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))/(d+I*c*d*x)^2,x, algorithm="maxima")
```

output
```
-a*(1/(c^4*d^2*x - I*c^3*d^2) + x/(c^2*d^2) + 2*I*log(c*x - I)/(c^3*d^2))
+ 1/4*(2*I*c^2*x^2 + 4*(c*x - I)*arctan(c*x)^2 + (c*x - I)*log(c^2*x^2 + 1
)^2 - (-I*c^4*d^2*x - c^3*d^2)*((c*(x/(c^6*d^2*x^2 + c^4*d^2) + arctan(c*x
)/(c^5*d^2)) - 2*arctan(c*x)/(c^6*d^2*x^2 + c^4*d^2))*c + 8*integrate(1/4*
log(c^2*x^2 + 1)/(c^6*d^2*x^4 + 2*c^4*d^2*x^2 + c^2*d^2), x)) - (c^4*d^2*x
 - I*c^3*d^2)*(c*(c^2/(c^8*d^2*x^2 + c^6*d^2) + log(c^2*x^2 + 1)/(c^6*d^2*
x^2 + c^4*d^2)) + 16*integrate(1/4*arctan(c*x)/(c^6*d^2*x^4 + 2*c^4*d^2*x^
2 + c^2*d^2), x)) + 2*(-I*c^5*d^2*x - c^4*d^2)*(c*(x/(c^6*d^2*x^2 + c^4*d^
2) + arctan(c*x)/(c^5*d^2)) - 8*c*integrate(1/4*x^2*log(c^2*x^2 + 1)/(c^6*
d^2*x^4 + 2*c^4*d^2*x^2 + c^2*d^2), x) - 2*arctan(c*x)/(c^6*d^2*x^2 + c^4*
d^2)) - 2*(c^5*d^2*x - I*c^4*d^2)*(16*c*integrate(1/4*x^2*arctan(c*x)/(c^6
*d^2*x^4 + 2*c^4*d^2*x^2 + c^2*d^2), x) - c^2/(c^8*d^2*x^2 + c^6*d^2) - lo
g(c^2*x^2 + 1)/(c^6*d^2*x^2 + c^4*d^2)) + 2*c*x - 2*(c^2*x^2 - I*c*x + 1)*
arctan(c*x) - 4*(c^7*d^2*x - I*c^6*d^2)*integrate(1/4*(2*c*x^4*arctan(c*x)
 + x^3*log(c^2*x^2 + 1))/(c^6*d^2*x^4 + 2*c^4*d^2*x^2 + c^2*d^2), x) - 4*(
-I*c^7*d^2*x - c^6*d^2)*integrate(1/4*(c*x^4*log(c^2*x^2 + 1) - 2*x^3*arct
an(c*x))/(c^6*d^2*x^4 + 2*c^4*d^2*x^2 + c^2*d^2), x) - 12*(I*c^6*d^2*x + c
^5*d^2)*integrate(1/4*(2*c*x^3*arctan(c*x) + x^2*log(c^2*x^2 + 1))/(c^6*d^
2*x^4 + 2*c^4*d^2*x^2 + c^2*d^2), x) - 12*(c^6*d^2*x - I*c^5*d^2)*integrat
e(1/4*(c*x^3*log(c^2*x^2 + 1) - 2*x^2*arctan(c*x))/(c^6*d^2*x^4 + 2*c^4...
```

## Giac [F]

$$\int \frac{x^2(a+b\arctan(cx))}{(d+icdx)^2}\,dx=\int \frac{(b\arctan(cx)+a)x^2}{(i\,cdx+d)^2}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))/(d+I*c*d*x)^2,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)*x^2/(I*c*d*x + d)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\arctan(cx))}{(d+icdx)^2}\,dx=\int \frac{x^2\,(a+b\,\mathrm{atan}(c\,x))}{(d+c\,d\,x\,1\mathrm{i})^2}\,dx$$

input
```
int((x^2*(a + b*atan(c*x)))/(d + c*d*x*1i)^2,x)
```

output
```
int((x^2*(a + b*atan(c*x)))/(d + c*d*x*1i)^2, x)
```

## Reduce [F]

$$\int \frac{x^2(a+b\arctan(cx))}{(d+icdx)^2}\,dx=\frac{-\left(\int \frac{x^2}{c^2x^2-2cix-1}dx\right)a-\left(\int \frac{atan(cx)x^2}{c^2x^2-2cix-1}dx\right)b}{d^2}$$

input
```
int(x^2*(a+b*atan(c*x))/(d+I*c*d*x)^2,x)
```

output
```
( - (int(x**2/(c**2*x**2 - 2*c*i*x - 1),x)*a + int((atan(c*x)*x**2)/(c**2*
x**2 - 2*c*i*x - 1),x)*b))/d**2
```

# 3.53 $\int \frac{x(a+b\arctan(cx))}{(d+icdx)^2}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 122

$$\int \frac{x(a+b\arctan(cx))}{(d+icdx)^2}\,dx = -\frac{b}{2c^2d^2(i-cx)}+\frac{b\arctan(cx)}{2c^2d^2}-\frac{i(a+b\arctan(cx))}{c^2d^2(i-cx)}+\frac{(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c^2d^2}+\frac{ib\,\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2c^2d^2}$$

output
```
-1/2*b/c^2/d^2/(I-c*x)+1/2*b*arctan(c*x)/c^2/d^2-I*(a+b*arctan(c*x))/c^2/d
^2/(I-c*x)+(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^2/d^2+1/2*I*b*polylog(2,1-2
/(1+I*c*x))/c^2/d^2
```

## Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.05

$$\int \frac{x(a+b\arctan(cx))}{(d+icdx)^2}\,dx = -\frac{i(a+b\arctan(cx))}{c^2d^2(i-cx)}-\frac{b\left(\frac{1}{c(i-cx)}-\frac{\arctan(cx)}{c}\right)}{2cd^2}+\frac{(a+b\arctan(cx))\log\left(\frac{2i}{i-cx}\right)}{c^2d^2}+\frac{ib\,\text{PolyLog}\left(2,-\frac{i+cx}{i-cx}\right)}{2c^2d^2}$$

input
```
Integrate[(x*(a + b*ArcTan[c*x]))/(d + I*c*d*x)^2,x]
```

output
```
((-I)*(a + b*ArcTan[c*x]))/(c^2*d^2*(I - c*x)) - (b*(1/(c*(I - c*x)) - Arc
Tan[c*x]/c))/(2*c*d^2) + ((a + b*ArcTan[c*x])*Log[(2*I)/(I - c*x)])/(c^2*d
^2) + ((I/2)*b*PolyLog[2, -((I + c*x)/(I - c*x))])/(c^2*d^2)
```

## Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\arctan(cx))}{(d+icdx)^2}\,dx$$

$$\Big\downarrow \text{5411}$$

$$\int \left(-\frac{a+b\arctan(cx)}{cd^2(cx-i)}-\frac{i(a+b\arctan(cx))}{cd^2(cx-i)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i(a+b\arctan(cx))}{c^2d^2(-cx+i)}+\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c^2d^2}+\frac{b\arctan(cx)}{2c^2d^2}+\frac{ib\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{2c^2d^2}-\frac{b}{2c^2d^2(-cx+i)}$$

input
```
Int[(x*(a + b*ArcTan[c*x]))/(d + I*c*d*x)^2,x]
```

output
```
-1/2*b/(c^2*d^2*(I - c*x)) + (b*ArcTan[c*x])/(2*c^2*d^2) - (I*(a + b*ArcTa
n[c*x]))/(c^2*d^2*(I - c*x)) + ((a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(c
^2*d^2) + ((I/2)*b*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c^2*d^2)
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

## Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 251 vs. 2(110) = 220.

Time = 0.43 (sec) , antiderivative size = 252, normalized size of antiderivative = 2.07

| method | result |
|---|---|
| derivativedivides | $\frac{-\frac{a\ln\left(c^2x^2+1\right)}{2d^2}-\frac{ia\arctan(cx)}{d^2}+\frac{ia}{d^2(cx-i)}-\frac{b\arctan(cx)\ln(cx-i)}{d^2}+\frac{ib\arctan(cx)}{d^2(cx-i)}+\frac{b}{2d^2(cx-i)}-\frac{ib\ln\left(c^2x^2+1\right)}{8d^2}+\frac{b\arctan(cx)}{4d^2}+}{}$ |
| default | $\frac{-\frac{a\ln\left(c^2x^2+1\right)}{2d^2}-\frac{ia\arctan(cx)}{d^2}+\frac{ia}{d^2(cx-i)}-\frac{b\arctan(cx)\ln(cx-i)}{d^2}+\frac{ib\arctan(cx)}{d^2(cx-i)}+\frac{b}{2d^2(cx-i)}-\frac{ib\ln\left(c^2x^2+1\right)}{8d^2}+\frac{b\arctan(cx)}{4d^2}+}{}$ |
| risch | $\frac{ib\ln(icx+1)^2}{4c^2d^2}+\frac{b\ln(icx+1)}{2c^2d^2(cx-i)}-\frac{ib\ln(c^2x^2+1)}{8d^2c^2}+\frac{b\arctan(cx)}{4c^2d^2}+\frac{b\ln(-icx+1)x}{4d^2c(-icx-1)}+\frac{ib\ln(-icx+1)}{4d^2c^2(-icx-1)}-\frac{ib\ln\left(\frac{1}{2}\right.}{}$ |
| parts | $-\frac{a\ln(c^2x^2+1)}{2d^2c^2}-\frac{ia\arctan(cx)}{d^2c^2}-\frac{ia}{d^2c^2(-cx+i)}-\frac{b\arctan(cx)\ln(cx-i)}{c^2d^2}+\frac{ib\arctan(cx)}{c^2d^2(cx-i)}+\frac{ib\ln(c^4x^4+10c^2}{16c^2d^2}$ |

input
```
int(x*(a+b*arctan(c*x))/(d+I*c*d*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c^2*(-1/2*a/d^2*ln(c^2*x^2+1)-I*a/d^2*arctan(c*x)+I*a/d^2/(c*x-I)-b/d^2*
arctan(c*x)*ln(c*x-I)+I*b/d^2*arctan(c*x)/(c*x-I)+1/2*b/d^2/(c*x-I)-1/8*I*
b/d^2*ln(c^2*x^2+1)+1/4*b*arctan(c*x)/d^2+1/16*I*b/d^2*ln(c^4*x^4+10*c^2*x
^2+9)-1/8*b/d^2*arctan(1/2*c*x)+1/8*b/d^2*arctan(1/6*c^3*x^3+7/6*c*x)+1/4*
b/d^2*arctan(1/2*c*x-1/2*I)-1/4*I*b/d^2*ln(c*x-I)^2+1/2*I*b/d^2*ln(-1/2*I*
(c*x+I))*ln(c*x-I)+1/2*I*b/d^2*dilog(-1/2*I*(c*x+I)))
```

## Fricas [F]

$$\int \frac{x(a+b\arctan(cx))}{(d+icdx)^2}\,dx = \int \frac{(b\arctan(cx)+a)x}{(i\,cdx+d)^2}\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))/(d+I*c*d*x)^2,x, algorithm="fricas")
```

output
```
integral(1/2*(-I*b*x*log(-(c*x + I)/(c*x - I)) - 2*a*x)/(c^2*d^2*x^2 - 2*I
*c*d^2*x - d^2), x)
```

## Sympy [F]

$$\int \frac{x(a+b\arctan(cx))}{(d+icdx)^2}\,dx = \frac{(-ibcx\log(icx+1)-b\log(icx+1)-b)\log(-icx+1)}{2c^3d^2x-2ic^2d^2}$$
$$-\frac{\int \frac{ib}{c^3x^3-ic^2x^2+cx-i}\,dx+\int \frac{ib\log(icx+1)}{c^3x^3-ic^2x^2+cx-i}\,dx+\int \frac{2ac^2x^2}{c^3x^3-ic^2x^2+cx-i}\,dx+\int\left(-\frac{bcx}{c^3x^3-ic^2x^2+cx-i}\right)dx+\int \frac{2ic}{c^3x^3-ic^2}}{2cd^2}$$

input
```
integrate(x*(a+b*atan(c*x))/(d+I*c*d*x)**2,x)
```

output
```
(-I*b*c*x*log(I*c*x + 1) - b*log(I*c*x + 1) - b)*log(-I*c*x + 1)/(2*c**3*d
**2*x - 2*I*c**2*d**2) - (Integral(I*b/(c**3*x**3 - I*c**2*x**2 + c*x - I)
, x) + Integral(I*b*log(I*c*x + 1)/(c**3*x**3 - I*c**2*x**2 + c*x - I), x)
 + Integral(2*a*c**2*x**2/(c**3*x**3 - I*c**2*x**2 + c*x - I), x) + Integr
al(-b*c*x/(c**3*x**3 - I*c**2*x**2 + c*x - I), x) + Integral(2*I*a*c*x/(c*
*3*x**3 - I*c**2*x**2 + c*x - I), x) + Integral(-b*c*x*log(I*c*x + 1)/(c**
3*x**3 - I*c**2*x**2 + c*x - I), x) + Integral(-2*I*b*c**2*x**2*log(I*c*x
+ 1)/(c**3*x**3 - I*c**2*x**2 + c*x - I), x))/(2*c*d**2)
```

## Maxima [F]

$$\int \frac{x(a+b\arctan(cx))}{(d+icdx)^2}\,dx = \int \frac{(b\arctan{(cx)}+a)x}{(i\,cdx+d)^2}\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))/(d+I*c*d*x)^2,x, algorithm="maxima")
```

output
```
a*(I/(c^3*d^2*x - I*c^2*d^2) - log(c*x - I)/(c^2*d^2)) - 1/8*(4*(I*c*x + 1
)*arctan(c*x)^2 + 4*c*x*arctan2(1, c*x) - (-I*c*x - 1)*log(c^2*x^2 + 1)^2
- (c^3*d^2*x - I*c^2*d^2)*((c*(x/(c^5*d^2*x^2 + c^3*d^2) + arctan(c*x)/(c^
4*d^2)) - 2*arctan(c*x)/(c^5*d^2*x^2 + c^3*d^2))*c + 8*integrate(1/4*log(c
^2*x^2 + 1)/(c^5*d^2*x^4 + 2*c^3*d^2*x^2 + c*d^2), x)) - (I*c^3*d^2*x + c^
2*d^2)*(c*(c^2/(c^7*d^2*x^2 + c^5*d^2) + log(c^2*x^2 + 1)/(c^5*d^2*x^2 + c
^3*d^2)) + 16*integrate(1/4*arctan(c*x)/(c^5*d^2*x^4 + 2*c^3*d^2*x^2 + c*d
^2), x)) + (c^4*d^2*x - I*c^3*d^2)*(c*(x/(c^5*d^2*x^2 + c^3*d^2) + arctan(
c*x)/(c^4*d^2)) - 8*c*integrate(1/4*x^2*log(c^2*x^2 + 1)/(c^5*d^2*x^4 + 2*
c^3*d^2*x^2 + c*d^2), x) - 2*arctan(c*x)/(c^5*d^2*x^2 + c^3*d^2)) + (-I*c^
4*d^2*x - c^3*d^2)*(16*c*integrate(1/4*x^2*arctan(c*x)/(c^5*d^2*x^4 + 2*c^
3*d^2*x^2 + c*d^2), x) - c^2/(c^7*d^2*x^2 + c^5*d^2) - log(c^2*x^2 + 1)/(c
^5*d^2*x^2 + c^3*d^2)) + 16*(c^5*d^2*x - I*c^4*d^2)*integrate(1/4*(2*c*x^3
*arctan(c*x) + x^2*log(c^2*x^2 + 1))/(c^5*d^2*x^4 + 2*c^3*d^2*x^2 + c*d^2)
, x) + 16*(-I*c^5*d^2*x - c^4*d^2)*integrate(1/4*(c*x^3*log(c^2*x^2 + 1) -
 2*x^2*arctan(c*x))/(c^5*d^2*x^4 + 2*c^3*d^2*x^2 + c*d^2), x) - 4*I*arctan
(c*x) - 4*I*arctan2(1, c*x) + 2*log(c^2*x^2 + 1))*b/(c^3*d^2*x - I*c^2*d^2
)
```

## Giac [F]

$$\int \frac{x(a+b\arctan(cx))}{(d+icdx)^2}\,dx = \int \frac{(b\arctan{(cx)}+a)x}{(i\,cdx+d)^2}\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))/(d+I*c*d*x)^2,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)*x/(I*c*d*x + d)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\arctan(cx))}{(d+icdx)^2}\,dx=\int \frac{x\,(a+b\,\mathrm{atan}(c\,x))}{(d+c\,d\,x\,1\mathrm{i})^2}\,dx$$

input
```
int((x*(a + b*atan(c*x)))/(d + c*d*x*1i)^2,x)
```

output
```
int((x*(a + b*atan(c*x)))/(d + c*d*x*1i)^2, x)
```

## Reduce [F]

$$\int \frac{x(a+b\arctan(cx))}{(d+icdx)^2}\,dx=\frac{-\left(\int \frac{atan(cx)x}{c^2x^2-2cix-1}dx\right)b-\left(\int \frac{x}{c^2x^2-2cix-1}dx\right)a}{d^2}$$

input
```
int(x*(a+b*atan(c*x))/(d+I*c*d*x)^2,x)
```

output
```
( - (int((atan(c*x)*x)/(c**2*x**2 - 2*c*i*x - 1),x)*b + int(x/(c**2*x**2 -
 2*c*i*x - 1),x)*a))/d**2
```

# 3.54 $\int \frac{a+b\arctan(cx)}{(d+icdx)^2} dx$



## Optimal result

Integrand size = 20, antiderivative size = 69

$$\int \frac{a+b\arctan(cx)}{(d+icdx)^2} dx = \frac{ib}{2cd^2(i-cx)} - \frac{ib\arctan(cx)}{2cd^2} + \frac{i(a+b\arctan(cx))}{cd^2(1+icx)}$$

output
```
1/2*I*b/c/d^2/(I-c*x)-1/2*I*b*arctan(c*x)/c/d^2+I*(a+b*arctan(c*x))/c/d^2/
(1+I*c*x)
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.61

$$\int \frac{a+b\arctan(cx)}{(d+icdx)^2} dx = \frac{2a-ib+(b-ibcx)\arctan(cx)}{2cd^2(-i+cx)}$$

input
```
Integrate[(a + b*ArcTan[c*x])/(d + I*c*d*x)^2,x]
```

output
```
(2*a - I*b + (b - I*b*c*x)*ArcTan[c*x])/(2*c*d^2*(-I + c*x))
```

## Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5387, 27, 456, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{(d+icdx)^2}dx$$

$$\Big\downarrow \text{5387}$$

$$\frac{i(a+b\arctan(cx))}{cd^2(1+icx)}-\frac{ib\int \frac{1}{d(icx+1)(c^2x^2+1)}dx}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{i(a+b\arctan(cx))}{cd^2(1+icx)}-\frac{ib\int \frac{1}{(icx+1)(c^2x^2+1)}dx}{d^2}$$

$$\Big\downarrow \text{456}$$

$$\frac{i(a+b\arctan(cx))}{cd^2(1+icx)}-\frac{ib\int \frac{1}{(1-icx)(icx+1)^2}dx}{d^2}$$

$$\Big\downarrow \text{54}$$

$$\frac{i(a+b\arctan(cx))}{cd^2(1+icx)}-\frac{ib\int \left(\frac{1}{2(c^2x^2+1)}-\frac{1}{2(cx-i)^2}\right)dx}{d^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{i(a+b\arctan(cx))}{cd^2(1+icx)}-\frac{ib\left(\frac{\arctan(cx)}{2c}-\frac{1}{2c(-cx+i)}\right)}{d^2}$$

input
```
Int[(a + b*ArcTan[c*x])/(d + I*c*d*x)^2,x]
```

output
```
(I*(a + b*ArcTan[c*x]))/(c*d^2*(1 + I*c*x)) - (I*b*(-1/2*1/(c*(I - c*x)) +
 ArcTan[c*x]/(2*c)))/d^2
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5387
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Simp[b*(
c/(e*(q + 1)))   Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b
, c, d, e, q}, x] && NeQ[q, -1]
```

## Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.99

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\frac{ia}{d^2(icx+1)}+\frac{ib\arctan(cx)}{d^2(icx+1)}-\frac{ib\arctan(cx)}{2d^2}-\frac{ib}{2d^2(cx-i)}}{c}$ | 68 |
| default | $\frac{\frac{ia}{d^2(icx+1)}+\frac{ib\arctan(cx)}{d^2(icx+1)}-\frac{ib\arctan(cx)}{2d^2}-\frac{ib}{2d^2(cx-i)}}{c}$ | 68 |
| parts | $\frac{ia}{d^2(icx+1)c}+\frac{ib\arctan(cx)}{c\,d^2(icx+1)}-\frac{ib\arctan(cx)}{2c\,d^2}-\frac{ib}{2c\,d^2(cx-i)}$ | 76 |
| risch | $-\frac{ib\ln(icx+1)}{2c\,d^2(cx-i)}+\frac{2ib\ln(-icx+1)-\ln(cx-i)bcx+\ln(-cx-i)bcx+i\ln(cx-i)b-i\ln(-cx-i)b+4a-2ib}{4d^2(cx-i)c}$ | 111 |
| orering | $\frac{(c^3x^3+2ic^2x^2+cx+2i)(a+b\arctan(cx))}{2c(icdx+d)^2}+\frac{(c^2x^2+1)^2\left(\frac{bc}{(c^2x^2+1)(icdx+d)^2}-\frac{2i(a+b\arctan(cx))cd}{(icdx+d)^3}\right)}{4c^2}$ | 111 |

input
```
int((a+b*arctan(c*x))/(d+I*c*d*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*(I*a/d^2/(1+I*c*x)+I*b/d^2/(1+I*c*x)*arctan(c*x)-1/2*I*b/d^2*arctan(c*
x)-1/2*I*b/d^2/(c*x-I))
```

## Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.72

$$\int \frac{a+b\arctan(cx)}{(d+icdx)^2}\,dx=\frac{(bcx+i\,b)\log\left(-\frac{cx+i}{cx-i}\right)+4\,a-2i\,b}{4\,(c^2d^2x-i\,cd^2)}$$

input
```
integrate((a+b*arctan(c*x))/(d+I*c*d*x)^2,x, algorithm="fricas")
```

output
```
1/4*((b*c*x + I*b)*log(-(c*x + I)/(c*x - I)) + 4*a - 2*I*b)/(c^2*d^2*x - I
*c*d^2)
```

## Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(51) = 102.

Time = 0.96 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.68

$$\int \frac{a+b\arctan(cx)}{(d+icdx)^2}\,dx=\frac{ib\log(-icx+1)}{2c^2d^2x-2icd^2}-\frac{ib\log(icx+1)}{2c^2d^2x-2icd^2}-\frac{b\left(\frac{\log\left(bx-\frac{ib}{c}\right)}{4}-\frac{\log\left(bx+\frac{ib}{c}\right)}{4}\right)}{cd^2}-\frac{-2a+ib}{2c^2d^2x-2icd^2}$$

input
```
integrate((a+b*atan(c*x))/(d+I*c*d*x)**2,x)
```

output
```
I*b*log(-I*c*x + 1)/(2*c**2*d**2*x - 2*I*c*d**2) - I*b*log(I*c*x + 1)/(2*c
**2*d**2*x - 2*I*c*d**2) - b*(log(b*x - I*b/c)/4 - log(b*x + I*b/c)/4)/(c*
d**2) - (-2*a + I*b)/(2*c**2*d**2*x - 2*I*c*d**2)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\arctan(cx)}{(d+icdx)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arctan(c*x))/(d+I*c*d*x)^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.26

$$\int \frac{a+b\arctan(cx)}{(d+icdx)^2}\,dx = -\frac{b\left(2i\arctan\left(-\frac{i\,(i\,cdx+d)\left(\frac{d}{i\,cdx+d}-1\right)}{d}\right)-1\right)e^{\left(2i\arctan\left(-\frac{i\,(i\,cdx+d)\left(\frac{d}{i\,cdx+d}-1\right)}{d}\right)\right)}}{4\,cd^2} + \frac{i\,a}{(i\,cdx+d)cd}$$

input
```
integrate((a+b*arctan(c*x))/(d+I*c*d*x)^2,x, algorithm="giac")
```

output
```
-1/4*b*(2*I*arctan(-I*(I*c*d*x + d)*(d/(I*c*d*x + d) - 1)/d) - 1)*e^(2*I*a
rctan(-I*(I*c*d*x + d)*(d/(I*c*d*x + d) - 1)/d))/(c*d^2) + I*a/((I*c*d*x +
 d)*c*d)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{(d+icdx)^2}\,dx=\int \frac{a+b\,\mathrm{atan}(c\,x)}{\left(d+c\,d\,x\,1\mathrm{i}\right)^2}\,dx$$

input
```
int((a + b*atan(c*x))/(d + c*d*x*1i)^2,x)
```

output
```
int((a + b*atan(c*x))/(d + c*d*x*1i)^2, x)
```

## Reduce [F]

$$\int \frac{a+b\arctan(cx)}{(d+icdx)^2}\,dx=\frac{-\left(\int \frac{atan(cx)}{c^2x^2-2cix-1}dx\right)b-\left(\int \frac{1}{c^2x^2-2cix-1}dx\right)a}{d^2}$$

input
```
int((a+b*atan(c*x))/(d+I*c*d*x)^2,x)
```

output
```
( - (int(atan(c*x)/(c**2*x**2 - 2*c*i*x - 1),x)*b + int(1/(c**2*x**2 - 2*c
*i*x - 1),x)*a))/d**2
```

# 3.55 $\int \frac{a+b\arctan(cx)}{x(d+icdx)^2}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 150

$$\int \frac{a+b\arctan(cx)}{x(d+icdx)^2}\,dx = \frac{b}{2d^2(i-cx)} - \frac{b\arctan(cx)}{2d^2} + \frac{i(a+b\arctan(cx))}{d^2(i-cx)} + \frac{a\log(x)}{d^2} + \frac{(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{d^2} + \frac{ib\operatorname{PolyLog}(2,-icx)}{2d^2} - \frac{ib\operatorname{PolyLog}(2,icx)}{2d^2} + \frac{ib\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2d^2}$$

output
```
1/2*b/d^2/(I-c*x)-1/2*b*arctan(c*x)/d^2+I*(a+b*arctan(c*x))/d^2/(I-c*x)+a*
ln(x)/d^2+(a+b*arctan(c*x))*ln(2/(1+I*c*x))/d^2+1/2*I*b*polylog(2,-I*c*x)/
d^2-1/2*I*b*polylog(2,I*c*x)/d^2+1/2*I*b*polylog(2,1-2/(1+I*c*x))/d^2
```

## Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.85

$$\int \frac{a+b\arctan(cx)}{x(d+icdx)^2}\,dx = \frac{b\left(\frac{1}{i-cx}-\arctan(cx)\right) - \frac{2i(a+b\arctan(cx))}{-i+cx} + 2a\log(x) + 2(a+b\arctan(cx))\log\left(\frac{2i}{i-cx}\right) + ib\operatorname{PolyLog}(2,-ic}{2d^2}$$

input
```
Integrate[(a + b*ArcTan[c*x])/(x*(d + I*c*d*x)^2),x]
```

output
```
(b*((I - c*x)^(-1) - ArcTan[c*x]) - ((2*I)*(a + b*ArcTan[c*x]))/(-I + c*x)
 + 2*a*Log[x] + 2*(a + b*ArcTan[c*x])*Log[(2*I)/(I - c*x)] + I*b*PolyLog[2
, (-I)*c*x] - I*b*PolyLog[2, I*c*x] + I*b*PolyLog[2, (I + c*x)/(-I + c*x)]
)/(2*d^2)
```

## Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x(d+icdx)^2}\,dx$$

$$\Big\downarrow \text{5411}$$

$$\int \left(\frac{a+b\arctan(cx)}{d^2x}-\frac{c(a+b\arctan(cx))}{d^2(cx-i)}+\frac{ic(a+b\arctan(cx))}{d^2(cx-i)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{i(a+b\arctan(cx))}{d^2(-cx+i)}+\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{d^2}+\frac{a\log(x)}{d^2}-\frac{b\arctan(cx)}{2d^2}+\frac{ib\operatorname{PolyLog}(2,-icx)}{2d^2}-\frac{ib\operatorname{PolyLog}(2,icx)}{2d^2}+\frac{ib\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{2d^2}+\frac{b}{2d^2(-cx+i)}$$

input
```
Int[(a + b*ArcTan[c*x])/(x*(d + I*c*d*x)^2),x]
```

output
```
b/(2*d^2*(I - c*x)) - (b*ArcTan[c*x])/(2*d^2) + (I*(a + b*ArcTan[c*x]))/(d
^2*(I - c*x)) + (a*Log[x])/d^2 + ((a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/
d^2 + ((I/2)*b*PolyLog[2, (-I)*c*x])/d^2 - ((I/2)*b*PolyLog[2, I*c*x])/d^2
 + ((I/2)*b*PolyLog[2, 1 - 2/(1 + I*c*x)])/d^2
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

## Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.38

| method | result |
|---|---|
| parts | $\frac{ia}{d^2(-cx+i)} - \frac{a\ln(c^2x^2+1)}{2d^2} - \frac{ia\arctan(cx)}{d^2} + \frac{a\ln(x)}{d^2} + \frac{b\left(\ln(cx)\arctan(cx) - \frac{i\arctan(cx)}{cx-i} - \arctan(cx)\ln(cx-i\right.}{}$ |
| derivativedivides | $\frac{a\ln(cx)}{d^2} - \frac{ia}{d^2(cx-i)} - \frac{a\ln(c^2x^2+1)}{2d^2} - \frac{ia\arctan(cx)}{d^2} + \frac{b\left(\ln(cx)\arctan(cx) - \frac{i\arctan(cx)}{cx-i} - \arctan(cx)\ln(cx-i)\right.}{}$ |
| default | $\frac{a\ln(cx)}{d^2} - \frac{ia}{d^2(cx-i)} - \frac{a\ln(c^2x^2+1)}{2d^2} - \frac{ia\arctan(cx)}{d^2} + \frac{b\left(\ln(cx)\arctan(cx) - \frac{i\arctan(cx)}{cx-i} - \arctan(cx)\ln(cx-i)\right.}{}$ |
| risch | $-\frac{ib}{2d^2(icx+1)} - \frac{b\arctan(cx)}{4d^2} - \frac{b\ln(-icx+1)cx}{4d^2(-icx-1)} + \frac{ib\ln(icx+1)^2}{4d^2} + \frac{ib\ln(c^2x^2+1)}{8d^2} - \frac{ib\ln(icx+1)}{2d^2(icx+1)} + \frac{ib\,\mathrm{dilog}(}{2d}$ |

input
```
int((a+b*arctan(c*x))/x/(d+I*c*d*x)^2,x,method=_RETURNVERBOSE)
```

output
```
I*a/d^2/(-c*x+I)-1/2*a/d^2*ln(c^2*x^2+1)-I*a/d^2*arctan(c*x)+a*ln(x)/d^2+b
/d^2*(ln(c*x)*arctan(c*x)-I*arctan(c*x)/(c*x-I)-arctan(c*x)*ln(c*x-I)+1/2*
I*ln(c*x)*ln(1+I*c*x)-1/2*I*ln(c*x)*ln(1-I*c*x)+1/2*I*dilog(1+I*c*x)-1/2*I
*dilog(1-I*c*x)-1/2*arctan(c*x)-1/2/(c*x-I)-1/4*I*ln(c*x-I)^2+1/2*I*(dilog
(-1/2*I*(c*x+I))+ln(c*x-I)*ln(-1/2*I*(c*x+I))))
```

## Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.85

$$\int \frac{a+b\arctan(cx)}{x(d+icdx)^2}\,dx = -\frac{2\,(i\,bcx+b)\mathrm{Li}_2\left(\frac{cx+i}{cx-i}+1\right)-4\,(acx-i\,a)\log\left(x\right)-2\,b\log\left(-\frac{cx+i}{cx-i}\right)-\left(-i\,bcx-b\right)\log\left(\frac{cx+i}{c}\right)+((4\,a-}{4\,(cd^2x-i\,d^2)}$$

input
```
integrate((a+b*arctan(c*x))/x/(d+I*c*d*x)^2,x, algorithm="fricas")
```

output
```
-1/4*(2*(I*b*c*x + b)*dilog((c*x + I)/(c*x - I) + 1) - 4*(a*c*x - I*a)*log
(x) - 2*b*log(-(c*x + I)/(c*x - I)) - (-I*b*c*x - b)*log((c*x + I)/c) + ((
4*a - I*b)*c*x - 4*I*a - b)*log((c*x - I)/c) + 4*I*a + 2*b)/(c*d^2*x - I*d
^2)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{x(d+icdx)^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*atan(c*x))/x/(d+I*c*d*x)**2,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{a+b\arctan(cx)}{x(d+icdx)^2}\,dx = \int \frac{b\arctan\left(cx\right)+a}{\left(i\,cdx+d\right)^2x}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x/(d+I*c*d*x)^2,x, algorithm="maxima")
```

output
```
(-2*I*c*integrate(arctan(c*x)/(c^4*d^2*x^4 + 2*c^2*d^2*x^2 + d^2), x) - in
tegrate((c^2*x^2 - 1)*arctan(c*x)/(c^4*d^2*x^5 + 2*c^2*d^2*x^3 + d^2*x), x
))*b + a*(-I/(c*d^2*x - I*d^2) - log(c*x - I)/d^2 + log(x)/d^2)
```

## Giac [F]

$$\int \frac{a+b\arctan(cx)}{x(d+icdx)^2}\,dx=\int \frac{b\arctan(cx)+a}{(i\,cdx+d)^2x}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x/(d+I*c*d*x)^2,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)/((I*c*d*x + d)^2*x), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{x(d+icdx)^2}\,dx=\int \frac{a+b\,\mathrm{atan}(c\,x)}{x\,(d+c\,d\,x\,1\mathrm{i})^2}\,dx$$

input
```
int((a + b*atan(c*x))/(x*(d + c*d*x*1i)^2),x)
```

output
```
int((a + b*atan(c*x))/(x*(d + c*d*x*1i)^2), x)
```

## Reduce [F]

$$\int \frac{a+b\arctan(cx)}{x(d+icdx)^2}\,dx=\frac{-\left(\int \frac{atan(cx)}{c^2x^3-2ci\,x^2-x}dx\right)b-\left(\int \frac{1}{c^2x^3-2ci\,x^2-x}dx\right)a}{d^2}$$

input
```
int((a+b*atan(c*x))/x/(d+I*c*d*x)^2,x)
```

output
```
( - (int(atan(c*x)/(c**2*x**3 - 2*c*i*x**2 - x),x)*b + int(1/(c**2*x**3 -
2*c*i*x**2 - x),x)*a))/d**2
```

# 3.56 $\int \frac{a+b\arctan(cx)}{x^2(d+icdx)^2}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 194

$$\begin{aligned}\int \frac{a+b\arctan(cx)}{x^2(d+icdx)^2}\,dx =& -\frac{ibc}{2d^2(i-cx)}+\frac{ibc\arctan(cx)}{2d^2}-\frac{a+b\arctan(cx)}{d^2x} \\ &+\frac{c(a+b\arctan(cx))}{d^2(i-cx)}-\frac{2iac\log(x)}{d^2} \\ &+\frac{bc\log(x)}{d^2}-\frac{2ic(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{d^2} \\ &-\frac{bc\log(1+c^2x^2)}{2d^2}+\frac{bc\,\mathrm{PolyLog}(2,-icx)}{d^2} \\ &-\frac{bc\,\mathrm{PolyLog}(2,icx)}{d^2}+\frac{bc\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{d^2}\end{aligned}$$

output
```
-1/2*I*b*c/d^2/(I-c*x)+1/2*I*b*c*arctan(c*x)/d^2-(a+b*arctan(c*x))/d^2/x+c
*(a+b*arctan(c*x))/d^2/(I-c*x)-2*I*a*c*ln(x)/d^2+b*c*ln(x)/d^2-2*I*c*(a+b*
arctan(c*x))*ln(2/(1+I*c*x))/d^2-1/2*b*c*ln(c^2*x^2+1)/d^2+b*c*polylog(2,-
I*c*x)/d^2-b*c*polylog(2,I*c*x)/d^2+b*c*polylog(2,1-2/(1+I*c*x))/d^2
```

## Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.85

$$\int \frac{a+b\arctan(cx)}{x^2(d+icdx)^2}\,dx =$$
$$-\frac{ibc\left(\frac{1}{i-cx}-\arctan(cx)\right)+\frac{2(a+b\arctan(cx))}{x}+\frac{2c(a+b\arctan(cx))}{-i+cx}+4iac\log(x)+4ic(a+b\arctan(cx))\log\left(\frac{\cdots}{i-\cdots}\right)\cdots}{\cdots}$$

input
```
Integrate[(a + b*ArcTan[c*x])/(x^2*(d + I*c*d*x)^2),x]
```

output
```
-1/2*(I*b*c*((I - c*x)^(-1) - ArcTan[c*x]) + (2*(a + b*ArcTan[c*x]))/x + (
2*c*(a + b*ArcTan[c*x]))/(-I + c*x) + (4*I)*a*c*Log[x] + (4*I)*c*(a + b*Ar
cTan[c*x])*Log[(2*I)/(I - c*x)] + b*c*(-2*Log[x] + Log[1 + c^2*x^2]) - 2*b
*c*PolyLog[2, (-I)*c*x] + 2*b*c*PolyLog[2, I*c*x] - 2*b*c*PolyLog[2, (I +
c*x)/(-I + c*x)])/d^2
```

## Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x^2(d+icdx)^2}\,dx$$
$$\Big\downarrow \text{5411}$$
$$\int \left(\frac{2ic^2(a+b\arctan(cx))}{d^2(cx-i)}+\frac{c^2(a+b\arctan(cx))}{d^2(cx-i)^2}+\frac{a+b\arctan(cx)}{d^2x^2}-\frac{2ic(a+b\arctan(cx))}{d^2x}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{c(a+b\arctan(cx))}{d^2(-cx+i)}-\frac{a+b\arctan(cx)}{d^2x}-\frac{2ic\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{d^2}-\frac{2iac\log(x)}{d^2}+\frac{ibc\arctan(cx)}{2d^2}-\frac{bc\log\left(c^2x^2+1\right)}{2d^2}+\frac{bc\,\text{PolyLog}(2,-icx)}{d^2}-\frac{bc\,\text{PolyLog}(2,icx)}{d^2}+\frac{bc\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{d^2}-\frac{ibc}{2d^2(-cx+i)}+\frac{bc\log(x)}{d^2}$$

input
```
Int[(a + b*ArcTan[c*x])/(x^2*(d + I*c*d*x)^2),x]
```

output
```
((-1/2*I)*b*c)/(d^2*(I - c*x)) + ((I/2)*b*c*ArcTan[c*x])/d^2 - (a + b*ArcT
an[c*x])/(d^2*x) + (c*(a + b*ArcTan[c*x]))/(d^2*(I - c*x)) - ((2*I)*a*c*Lo
g[x])/d^2 + (b*c*Log[x])/d^2 - ((2*I)*c*(a + b*ArcTan[c*x])*Log[2/(1 + I*c
*x)])/d^2 - (b*c*Log[1 + c^2*x^2])/(2*d^2) + (b*c*PolyLog[2, (-I)*c*x])/d^
2 - (b*c*PolyLog[2, I*c*x])/d^2 + (b*c*PolyLog[2, 1 - 2/(1 + I*c*x)])/d^2
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

## Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.32

| method | result |
|---|---|
| parts | $\frac{ac}{d^2(-cx+i)}-\frac{2ca\arctan(cx)}{d^2}+\frac{ica\ln(c^2x^2+1)}{d^2}-\frac{a}{x\,d^2}-\frac{2iac\ln(x)}{d^2}+\frac{bc\left(-\frac{\arctan(cx)}{cx-i}+2i\arctan(cx)\ln(cx-i\right.}{}$ |
| derivativedivides | $c\left(-\frac{a}{d^2(cx-i)}-\frac{2a\arctan(cx)}{d^2}+\frac{ia\ln(c^2x^2+1)}{d^2}-\frac{a}{d^2cx}-\frac{2ia\ln(cx)}{d^2}+\frac{b\left(-\frac{\arctan(cx)}{cx-i}+2i\arctan(cx)\ln(cx\right.}{}\right.$ |
| default | $c\left(-\frac{a}{d^2(cx-i)}-\frac{2a\arctan(cx)}{d^2}+\frac{ia\ln(c^2x^2+1)}{d^2}-\frac{a}{d^2cx}-\frac{2ia\ln(cx)}{d^2}+\frac{b\left(-\frac{\arctan(cx)}{cx-i}+2i\arctan(cx)\ln(cx\right.}{}\right.$ |
| risch | $-\frac{cb\,\mathrm{dilog}(-icx+1)}{d^2}+\frac{ic^2b\ln(-icx+1)x}{4d^2(-icx-1)}+\frac{icb\arctan(cx)}{4d^2}-\frac{a}{x\,d^2}+\frac{cb\ln(-icx)}{2d^2}+\frac{cb\ln\left(\frac{1}{2}+\frac{icx}{2}\right)\ln\left(\frac{1}{2}-\frac{icx}{2}\right)}{d^2}-$ |

input
```
int((a+b*arctan(c*x))/x^2/(d+I*c*d*x)^2,x,method=_RETURNVERBOSE)
```

output
```
a/d^2*c/(-c*x+I)-2/d^2*c*a*arctan(c*x)+I/d^2*c*a*ln(c^2*x^2+1)-a/x/d^2-2*I
*a*c*ln(x)/d^2+b/d^2*c*(-arctan(c*x)/(c*x-I)+2*I*arctan(c*x)*ln(c*x-I)-1/c
/x*arctan(c*x)-2*I*arctan(c*x)*ln(c*x)-1/2*ln(c^2*x^2+1)+1/2*I*arctan(c*x)
+1/2*I/(c*x-I)+ln(c*x)-dilog(-I*(c*x+I))-ln(c*x)*ln(-I*(c*x+I))+(ln(c*x)-l
n(-I*c*x))*ln(-I*(-c*x+I))-dilog(-I*c*x)+dilog(-1/2*I*(c*x+I))+ln(c*x-I)*l
n(-1/2*I*(c*x+I))-1/2*ln(c*x-I)^2)
```

## Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.94

$$\int \frac{a+b\arctan(cx)}{x^2(d+icdx)^2}\,dx =$$
$$-\frac{2\,(4\,a-i\,b)cx+4\,(bc^2x^2-i\,bcx)\mathrm{Li}_2\left(\frac{cx+i}{cx-i}+1\right)+4\,((2i\,a-b)c^2x^2+(2\,a+i\,b)cx)\log(x)+2\,(2i\,bcx+}{4\,(cd^2x^2-i}$$

input
```
integrate((a+b*arctan(c*x))/x^2/(d+I*c*d*x)^2,x, algorithm="fricas")
```

output
```
-1/4*(2*(4*a - I*b)*c*x + 4*(b*c^2*x^2 - I*b*c*x)*dilog((c*x + I)/(c*x - I
) + 1) + 4*((2*I*a - b)*c^2*x^2 + (2*a + I*b)*c*x)*log(x) + 2*(2*I*b*c*x +
 b)*log(-(c*x + I)/(c*x - I)) + 3*(b*c^2*x^2 - I*b*c*x)*log((c*x + I)/c) -
 ((8*I*a - b)*c^2*x^2 + (8*a + I*b)*c*x)*log((c*x - I)/c) - 4*I*a)/(c*d^2*
x^2 - I*d^2*x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{a + b\arctan(cx)}{x^2(d + icdx)^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*atan(c*x))/x**2/(d+I*c*d*x)**2,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{a + b\arctan(cx)}{x^2(d + icdx)^2}\,dx = \int \frac{b\arctan(cx) + a}{(i\,cdx + d)^2 x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^2/(d+I*c*d*x)^2,x, algorithm="maxima")
```

output
```
(-2*I*c*integrate(arctan(c*x)/(c^4*d^2*x^5 + 2*c^2*d^2*x^3 + d^2*x), x) -
integrate((c^2*x^2 - 1)*arctan(c*x)/(c^4*d^2*x^6 + 2*c^2*d^2*x^4 + d^2*x^2
), x))*b - a*(c/(c*d^2*x - I*d^2) - 2*I*c*log(c*x - I)/d^2 + 2*I*c*log(x)/
d^2 + 1/(d^2*x))
```

## Giac [F]

$$\int \frac{a+b\arctan(cx)}{x^2(d+icdx)^2}\,dx = \int \frac{b\arctan{(cx)}+a}{(i\,cdx+d)^2x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^2/(d+I*c*d*x)^2,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)/((I*c*d*x + d)^2*x^2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{x^2(d+icdx)^2}\,dx = \int \frac{a+b\,\mathrm{atan}(c\,x)}{x^2\,{(d+c\,d\,x\,1\mathrm{i})}^2}\,dx$$

input
```
int((a + b*atan(c*x))/(x^2*(d + c*d*x*1i)^2),x)
```

output
```
int((a + b*atan(c*x))/(x^2*(d + c*d*x*1i)^2), x)
```

## Reduce [F]

$$\int \frac{a+b\arctan(cx)}{x^2(d+icdx)^2}\,dx = \frac{-\left(\int \frac{atan(cx)}{c^2x^4-2ci\,x^3-x^2}dx\right)b-\left(\int \frac{1}{c^2x^4-2ci\,x^3-x^2}dx\right)a}{d^2}$$

input
```
int((a+b*atan(c*x))/x^2/(d+I*c*d*x)^2,x)
```

output
```
( - (int(atan(c*x)/(c**2*x**4 - 2*c*i*x**3 - x**2),x)*b + int(1/(c**2*x**4
 - 2*c*i*x**3 - x**2),x)*a))/d**2
```

# 3.57 $\int \frac{a+b\arctan(cx)}{x^3(d+icdx)^2}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 244

$$\begin{aligned}\int \frac{a+b\arctan(cx)}{x^3(d+icdx)^2}\,dx = &-\frac{bc}{2d^2x}-\frac{bc^2}{2d^2(i-cx)}-\frac{a+b\arctan(cx)}{2d^2x^2}\\&+\frac{2ic(a+b\arctan(cx))}{d^2x}-\frac{ic^2(a+b\arctan(cx))}{d^2(i-cx)}-\frac{3ac^2\log(x)}{d^2}\\&-\frac{2ibc^2\log(x)}{d^2}-\frac{3c^2(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{d^2}\\&+\frac{ibc^2\log(1+c^2x^2)}{d^2}-\frac{3ibc^2\operatorname{PolyLog}(2,-icx)}{2d^2}\\&+\frac{3ibc^2\operatorname{PolyLog}(2,icx)}{2d^2}-\frac{3ibc^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2d^2}\end{aligned}$$

output

```
-1/2*b*c/d^2/x-1/2*b*c^2/d^2/(I-c*x)-1/2*(a+b*arctan(c*x))/d^2/x^2+2*I*c*(
a+b*arctan(c*x))/d^2/x-I*c^2*(a+b*arctan(c*x))/d^2/(I-c*x)-3*a*c^2*ln(x)/d
^2-2*I*b*c^2*ln(x)/d^2-3*c^2*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/d^2+I*b*c^2
*ln(c^2*x^2+1)/d^2-3/2*I*b*c^2*polylog(2,-I*c*x)/d^2+3/2*I*b*c^2*polylog(2
,I*c*x)/d^2-3/2*I*b*c^2*polylog(2,1-2/(1+I*c*x))/d^2
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.23 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.91

$$\int \frac{a+b\arctan(cx)}{x^3(d+icdx)^2}\,dx = -\frac{-bc^2\left(\frac{1}{-i+cx}+\arctan(cx)\right)+\frac{a+b\arctan(cx)}{x^2}-\frac{4ic(a+b\arctan(cx))}{x}-\frac{2ic^2(a+b\arctan(cx))}{-i+cx}+\frac{bc\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1\right.}{x}}{}$$

input
```
Integrate[(a + b*ArcTan[c*x])/(x^3*(d + I*c*d*x)^2),x]
```

output
```
-1/2*(-(b*c^2*((-I + c*x)^(-1) + ArcTan[c*x])) + (a + b*ArcTan[c*x])/x^2 -
 ((4*I)*c*(a + b*ArcTan[c*x]))/x - ((2*I)*c^2*(a + b*ArcTan[c*x]))/(-I + c
*x) + (b*c*Hypergeometric2F1[-1/2, 1, 1/2, -(c^2*x^2)])/x + 6*a*c^2*Log[x]
 + 6*c^2*(a + b*ArcTan[c*x])*Log[(2*I)/(I - c*x)] + (2*I)*b*c^2*(2*Log[x]
- Log[1 + c^2*x^2]) + (3*I)*b*c^2*PolyLog[2, (-I)*c*x] - (3*I)*b*c^2*PolyL
og[2, I*c*x] + (3*I)*b*c^2*PolyLog[2, (I + c*x)/(-I + c*x)])/d^2
```

## Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x^3(d+icdx)^2}\,dx$$

$$\Big\downarrow \; 5411$$

$$\int \left(\frac{3c^3(a+b\arctan(cx))}{d^2(cx-i)}-\frac{ic^3(a+b\arctan(cx))}{d^2(cx-i)^2}-\frac{3c^2(a+b\arctan(cx))}{d^2x}+\frac{a+b\arctan(cx)}{d^2x^3}-\frac{2ic(a+b\,\text{arcta}}{d^2x^2}\right.$$

$$\Big\downarrow \; 2009$$

$$-\frac{ic^2(a+b\arctan(cx))}{d^2(-cx+i)}-\frac{3c^2\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{d^2}-\frac{a+b\arctan(cx)}{2d^2x^2}+\frac{2ic(a+b\arctan(cx))}{d^2x}-\frac{3ac^2\log(x)}{d^2}-\frac{3ibc^2\operatorname{PolyLog}(2,-icx)}{2d^2}+\frac{3ibc^2\operatorname{PolyLog}(2,icx)}{2d^2}-\frac{3ibc^2\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{2d^2}+\frac{ibc^2\log\left(c^2x^2+1\right)}{d^2}-\frac{bc^2}{2d^2(-cx+i)}-\frac{2ibc^2\log(x)}{d^2}-\frac{bc}{2d^2x}$$

input
```
Int[(a + b*ArcTan[c*x])/(x^3*(d + I*c*d*x)^2),x]
```

output
```
-1/2*(b*c)/(d^2*x) - (b*c^2)/(2*d^2*(I - c*x)) - (a + b*ArcTan[c*x])/(2*d^
2*x^2) + ((2*I)*c*(a + b*ArcTan[c*x]))/(d^2*x) - (I*c^2*(a + b*ArcTan[c*x]
))/(d^2*(I - c*x)) - (3*a*c^2*Log[x])/d^2 - ((2*I)*b*c^2*Log[x])/d^2 - (3*
c^2*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/d^2 + (I*b*c^2*Log[1 + c^2*x^2
])/d^2 - (((3*I)/2)*b*c^2*PolyLog[2, (-I)*c*x])/d^2 + (((3*I)/2)*b*c^2*Pol
yLog[2, I*c*x])/d^2 - (((3*I)/2)*b*c^2*PolyLog[2, 1 - 2/(1 + I*c*x)])/d^2
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

## Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.17

| method | result |
|---|---|
| derivativedivides | $c^2\left(-\frac{a}{2d^2c^2x^2}+\frac{2ia}{d^2cx}-\frac{3a\ln(cx)}{d^2}+\frac{ia}{d^2(cx-i)}+\frac{3a\ln(c^2x^2+1)}{2d^2}+\frac{3ia\arctan(cx)}{d^2}+\frac{b\left(-\frac{\arctan(cx)}{2c^2x^2}+\frac{2i\text{ar}}{}\right.}{}\right.$ |
| default | $c^2\left(-\frac{a}{2d^2c^2x^2}+\frac{2ia}{d^2cx}-\frac{3a\ln(cx)}{d^2}+\frac{ia}{d^2(cx-i)}+\frac{3a\ln(c^2x^2+1)}{2d^2}+\frac{3ia\arctan(cx)}{d^2}+\frac{b\left(-\frac{\arctan(cx)}{2c^2x^2}+\frac{2i\text{ar}}{}\right.}{}\right.$ |
| parts | $-\frac{ia\,c^2}{d^2(-cx+i)}+\frac{3c^2a\ln(c^2x^2+1)}{2d^2}+\frac{3ic^2a\arctan(cx)}{d^2}-\frac{a}{2x^2d^2}+\frac{2ica}{d^2x}-\frac{3a\,c^2\ln(x)}{d^2}+\frac{b\,c^2\left(-\frac{\arctan(cx)}{2c^2x^2}+\frac{2i}{}\right.}{}$ |
| risch | $\frac{c^2a}{d^2(-icx-1)}-\frac{3c^2a\ln(-icx)}{d^2}-\frac{a}{2x^2d^2}+\frac{3c^2a\ln(c^2x^2+1)}{2d^2}+\frac{3ic^2b\ln\left(\frac{1}{2}+\frac{icx}{2}\right)\ln(-icx+1)}{2d^2}+\frac{ic^2b\ln(-icx+1)}{4d^2(-icx-1)}$ |

input
```
int((a+b*arctan(c*x))/x^3/(d+I*c*d*x)^2,x,method=_RETURNVERBOSE)
```

output
```
c^2*(-1/2*a/d^2/c^2/x^2+2*I*a/d^2/c/x-3*a/d^2*ln(c*x)+I*a/d^2/(c*x-I)+3/2*
a/d^2*ln(c^2*x^2+1)+3*I*a/d^2*arctan(c*x)+b/d^2*(-1/2/c^2/x^2*arctan(c*x)+
2*I*arctan(c*x)/c/x-3*ln(c*x)*arctan(c*x)+I*arctan(c*x)/(c*x-I)+3*arctan(c
*x)*ln(c*x-I)-3/2*I*ln(c*x)*ln(1+I*c*x)+3/2*I*ln(c*x)*ln(1-I*c*x)-3/2*I*di
log(1+I*c*x)+3/2*I*dilog(1-I*c*x)-3/2*I*(dilog(-1/2*I*(c*x+I))+ln(c*x-I)*l
n(-1/2*I*(c*x+I)))+3/4*I*ln(c*x-I)^2-1/2/c/x-2*I*ln(c*x)+I*ln(c^2*x^2+1)+1
/2/(c*x-I)))
```

## Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.91

$$\int\frac{a+b\arctan(cx)}{x^3(d+icdx)^2}\,dx$$
$$=\frac{12i\,ac^2x^2+2\,(3\,a+i\,b)cx-6\,(-i\,bc^3x^3-bc^2x^2)\text{Li}_2\left(\frac{cx+i}{cx-i}+1\right)-4\,((3\,a+2i\,b)c^3x^3+(-3i\,a+2\,b)c^2x^2)}{}$$

input
```
integrate((a+b*arctan(c*x))/x^3/(d+I*c*d*x)^2,x, algorithm="fricas")
```

output
```
1/4*(12*I*a*c^2*x^2 + 2*(3*a + I*b)*c*x - 6*(-I*b*c^3*x^3 - b*c^2*x^2)*dil
og((c*x + I)/(c*x - I) + 1) - 4*((3*a + 2*I*b)*c^3*x^3 + (-3*I*a + 2*b)*c^
2*x^2)*log(x) - (6*b*c^2*x^2 - 3*I*b*c*x + b)*log(-(c*x + I)/(c*x - I)) -
4*(-I*b*c^3*x^3 - b*c^2*x^2)*log((c*x + I)/c) + 4*((3*a + I*b)*c^3*x^3 - (
3*I*a - b)*c^2*x^2)*log((c*x - I)/c) + 2*I*a)/(c*d^2*x^3 - I*d^2*x^2)
```

## Sympy [F]

$$\int \frac{a + b\arctan(cx)}{x^3(d + icdx)^2}\, dx = -\frac{\int \frac{a}{c^2x^5 - 2icx^4 - x^3}\, dx + \int \frac{b\,\mathrm{atan}\,(cx)}{c^2x^5 - 2icx^4 - x^3}\, dx}{d^2}$$

input
```
integrate((a+b*atan(c*x))/x**3/(d+I*c*d*x)**2,x)
```

output
```
-(Integral(a/(c**2*x**5 - 2*I*c*x**4 - x**3), x) + Integral(b*atan(c*x)/(c
**2*x**5 - 2*I*c*x**4 - x**3), x))/d**2
```

## Maxima [F]

$$\int \frac{a + b\arctan(cx)}{x^3(d + icdx)^2}\, dx = \int \frac{b\arctan(cx) + a}{(i\,cdx + d)^2x^3}\, dx$$

input
```
integrate((a+b*arctan(c*x))/x^3/(d+I*c*d*x)^2,x, algorithm="maxima")
```

output
```
(-2*I*c*integrate(arctan(c*x)/(c^4*d^2*x^6 + 2*c^2*d^2*x^4 + d^2*x^2), x)
- integrate((c^2*x^2 - 1)*arctan(c*x)/(c^4*d^2*x^7 + 2*c^2*d^2*x^5 + d^2*x
^3), x))*b - 1/2*a*(-2*I*c^2/(c*d^2*x - I*d^2) - 6*c^2*log(c*x - I)/d^2 +
6*c^2*log(x)/d^2 - (4*I*c*x - 1)/(d^2*x^2))
```

## Giac [F]

$$\int \frac{a+b\arctan(cx)}{x^3(d+icdx)^2}\,dx=\int \frac{b\arctan\left(cx\right)+a}{\left(i\,cdx+d\right)^2x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^3/(d+I*c*d*x)^2,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)/((I*c*d*x + d)^2*x^3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{x^3(d+icdx)^2}\,dx=\int \frac{a+b\,\mathrm{atan}(c\,x)}{x^3\,\left(d+c\,d\,x\,1\mathrm{i}\right)^2}\,dx$$

input
```
int((a + b*atan(c*x))/(x^3*(d + c*d*x*1i)^2),x)
```

output
```
int((a + b*atan(c*x))/(x^3*(d + c*d*x*1i)^2), x)
```

## Reduce [F]

$$\int \frac{a+b\arctan(cx)}{x^3(d+icdx)^2}\,dx=\frac{-\left(\int \frac{atan(cx)}{c^2x^5-2ci\,x^4-x^3}dx\right)b-\left(\int \frac{1}{c^2x^5-2ci\,x^4-x^3}dx\right)a}{d^2}$$

input
```
int((a+b*atan(c*x))/x^3/(d+I*c*d*x)^2,x)
```

output
```
( - (int(atan(c*x)/(c**2*x**5 - 2*c*i*x**4 - x**3),x)*b + int(1/(c**2*x**5
 - 2*c*i*x**4 - x**3),x)*a))/d**2
```

# 3.58 $\int \frac{x^4(a+b\arctan(cx))}{(d+icdx)^3}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 256

$$\begin{aligned}\int \frac{x^4(a+b\arctan(cx))}{(d+icdx)^3}\,dx = & -\frac{3ax}{c^4d^3}-\frac{ibx}{2c^4d^3}-\frac{b}{8c^5d^3(i-cx)^2}-\frac{15ib}{8c^5d^3(i-cx)} \\ & +\frac{19ib\arctan(cx)}{8c^5d^3}-\frac{3bx\arctan(cx)}{c^4d^3} \\ & +\frac{ix^2(a+b\arctan(cx))}{2c^3d^3}-\frac{i(a+b\arctan(cx))}{2c^5d^3(i-cx)^2} \\ & +\frac{4(a+b\arctan(cx))}{c^5d^3(i-cx)}+\frac{6i(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c^5d^3} \\ & +\frac{3b\log(1+c^2x^2)}{2c^5d^3}-\frac{3b\,\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^5d^3}\end{aligned}$$

output
```
-3*a*x/c^4/d^3-1/2*I*b*x/c^4/d^3-1/8*b/c^5/d^3/(I-c*x)^2-15/8*I*b/c^5/d^3/
(I-c*x)+19/8*I*b*arctan(c*x)/c^5/d^3-3*b*x*arctan(c*x)/c^4/d^3+1/2*I*x^2*(
a+b*arctan(c*x))/c^3/d^3-1/2*I*(a+b*arctan(c*x))/c^5/d^3/(I-c*x)^2+4*(a+b*
arctan(c*x))/c^5/d^3/(I-c*x)+6*I*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^5/d^3
+3/2*b*ln(c^2*x^2+1)/c^5/d^3-3*b*polylog(2,1-2/(1+I*c*x))/c^5/d^3
```

## Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.92

$$\int \frac{x^4(a+b\arctan(cx))}{(d+icdx)^3}\,dx$$
$$= \frac{-96acx+16iac^2x^2-\frac{16ia}{(-i+cx)^2}-\frac{128a}{-i+cx}+192a\arctan(cx)-96ia\log(1+c^2x^2)+b\big(-16icx+192\arctan$$

input
```
Integrate[(x^4*(a + b*ArcTan[c*x]))/(d + I*c*d*x)^3,x]
```

output
```
(-96*a*c*x + (16*I)*a*c^2*x^2 - ((16*I)*a)/(-I + c*x)^2 - (128*a)/(-I + c*
x) + 192*a*ArcTan[c*x] - (96*I)*a*Log[1 + c^2*x^2] + b*((-16*I)*c*x + 192*
ArcTan[c*x]^2 - 28*Cos[2*ArcTan[c*x]] + Cos[4*ArcTan[c*x]] + 48*Log[1 + c^
2*x^2] + 96*PolyLog[2, -E^((2*I)*ArcTan[c*x])] + (28*I)*Sin[2*ArcTan[c*x]]
 + (4*I)*ArcTan[c*x]*(4 + (24*I)*c*x + 4*c^2*x^2 - 14*Cos[2*ArcTan[c*x]] +
 Cos[4*ArcTan[c*x]] + 48*Log[1 + E^((2*I)*ArcTan[c*x])] + (14*I)*Sin[2*Arc
Tan[c*x]] - I*Sin[4*ArcTan[c*x]]) - I*Sin[4*ArcTan[c*x]]))/(32*c^5*d^3)
```

## Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a+b\arctan(cx))}{(d+icdx)^3}\,dx$$
$$\Big\downarrow \text{5411}$$
$$\int \left(-\frac{6i(a+b\arctan(cx))}{c^4d^3(cx-i)}+\frac{4(a+b\arctan(cx))}{c^4d^3(cx-i)^2}-\frac{3(a+b\arctan(cx))}{c^4d^3}+\frac{i(a+b\arctan(cx))}{c^4d^3(cx-i)^3}+\frac{ix(a+b\arctan}{c^3d^3}\right.$$
$$\Big\downarrow \text{2009}$$

$$\frac{4(a+b\arctan(cx))}{c^5d^3(-cx+i)}-\frac{i(a+b\arctan(cx))}{2c^5d^3(-cx+i)^2}+\frac{6i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c^5d^3}+\frac{ix^2(a+b\arctan(cx))}{2c^3d^3}-\frac{3ax}{c^4d^3}+\frac{19ib\arctan(cx)}{8c^5d^3}-\frac{3bx\arctan(cx)}{c^4d^3}-\frac{3b\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{c^5d^3}-\frac{15ib}{8c^5d^3(-cx+i)}-\frac{b}{8c^5d^3(-cx+i)^2}-\frac{ibx}{2c^4d^3}+\frac{3b\log\left(c^2x^2+1\right)}{2c^5d^3}$$

input
```
Int[(x^4*(a + b*ArcTan[c*x]))/(d + I*c*d*x)^3,x]
```

output
```
(-3*a*x)/(c^4*d^3) - ((I/2)*b*x)/(c^4*d^3) - b/(8*c^5*d^3*(I - c*x)^2) - (
((15*I)/8)*b)/(c^5*d^3*(I - c*x)) + (((19*I)/8)*b*ArcTan[c*x])/(c^5*d^3) -
 (3*b*x*ArcTan[c*x])/(c^4*d^3) + ((I/2)*x^2*(a + b*ArcTan[c*x]))/(c^3*d^3)
 - ((I/2)*(a + b*ArcTan[c*x]))/(c^5*d^3*(I - c*x)^2) + (4*(a + b*ArcTan[c*
x]))/(c^5*d^3*(I - c*x)) + ((6*I)*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/
(c^5*d^3) + (3*b*Log[1 + c^2*x^2])/(2*c^5*d^3) - (3*b*PolyLog[2, 1 - 2/(1
+ I*c*x)])/(c^5*d^3)
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

## Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.42

| method | result |
|---|---|
| derivativedivides | $\frac{-\frac{3acx}{d^3}-\frac{ib\arctan(cx)}{2d^3(cx-i)^2}+\frac{6a\arctan(cx)}{d^3}-\frac{5ib\arctan\left(\frac{cx}{2}-\frac{i}{2}\right)}{16d^3}-\frac{ia}{2d^3(cx-i)^2}-\frac{4a}{d^3(cx-i)}-\frac{3b\arctan(cx)cx}{d^3}+\frac{ib\arctan(cx)c^2x^2}{2d^3}+43\cdots}{\cdots}$ |
| default | $\frac{-\frac{3acx}{d^3}-\frac{ib\arctan(cx)}{2d^3(cx-i)^2}+\frac{6a\arctan(cx)}{d^3}-\frac{5ib\arctan\left(\frac{cx}{2}-\frac{i}{2}\right)}{16d^3}-\frac{ia}{2d^3(cx-i)^2}-\frac{4a}{d^3(cx-i)}-\frac{3b\arctan(cx)cx}{d^3}+\frac{ib\arctan(cx)c^2x^2}{2d^3}+43\cdots}{\cdots}$ |
| parts | $-\frac{6ib\arctan(cx)\ln(cx-i)}{c^5d^3}-\frac{3ax}{c^4d^3}-\frac{5ib\arctan\left(\frac{cx}{2}-\frac{i}{2}\right)}{16c^5d^3}+\frac{6a\arctan(cx)}{d^3c^5}+\frac{4a}{d^3c^5(-cx+i)}+\frac{ia\,x^2}{2d^3c^3}-\frac{3bx\,\mathrm{arct}\cdots}{c^4\cdots}$ |
| risch | $-\frac{2ib}{c^5d^3(-cx+i)}-\frac{x^2b\ln(-icx+1)}{4d^3c^3}-\frac{3b\ln(-icx+1)}{16d^3c^5(-icx-1)^2}+\frac{3b\ln\left(\frac{1}{2}+\frac{icx}{2}\right)\ln(-icx+1)}{d^3c^5}-\frac{3b\ln\left(\frac{1}{2}+\frac{icx}{2}\right)\ln\left(\frac{1}{2}-\frac{ic\cdots}{2}\right)\cdots}{d^3c^5}$ |

input
```
int(x^4*(a+b*arctan(c*x))/(d+I*c*d*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^5*(-3*a/d^3*c*x-1/2*I*b/d^3*arctan(c*x)/(c*x-I)^2+6*a/d^3*arctan(c*x)-
5/16*I*b/d^3*arctan(1/2*c*x-1/2*I)-1/2*I*a/d^3/(c*x-I)^2-4*a/d^3/(c*x-I)-3
*b/d^3*arctan(c*x)*c*x+1/2*I*b/d^3*arctan(c*x)*c^2*x^2+43/16*I*b/d^3*arcta
n(c*x)-5/32*I*b/d^3*arctan(1/6*c^3*x^3+7/6*c*x)-4*b/d^3*arctan(c*x)/(c*x-I
)-1/2*b/d^3+5/32*I*b/d^3*arctan(1/2*c*x)+5/64*b/d^3*ln(c^4*x^4+10*c^2*x^2+
9)-6*I*b/d^3*arctan(c*x)*ln(c*x-I)-1/2*I*b/d^3*c*x+15/8*I*b/d^3/(c*x-I)-3*
I*a/d^3*ln(c^2*x^2+1)-1/8*b/d^3/(c*x-I)^2+43/32*b/d^3*ln(c^2*x^2+1)+1/2*I*
a/d^3*c^2*x^2-3*b/d^3*ln(c*x-I)*ln(-1/2*I*(c*x+I))+3/2*b/d^3*ln(c*x-I)^2-3
*b/d^3*dilog(-1/2*I*(c*x+I)))
```

## Fricas [F]

$$\int \frac{x^4(a+b\arctan(cx))}{(d+icdx)^3}\,dx=\int \frac{(b\arctan(cx)+a)x^4}{(i\,cdx+d)^3}\,dx$$

input
```
integrate(x^4*(a+b*arctan(c*x))/(d+I*c*d*x)^3,x, algorithm="fricas")
```

output
```
integral(-1/2*(b*x^4*log(-(c*x + I)/(c*x - I)) - 2*I*a*x^4)/(c^3*d^3*x^3 -
 3*I*c^2*d^3*x^2 - 3*c*d^3*x + I*d^3), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\arctan(cx))}{(d+icdx)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**4*(a+b*atan(c*x))/(d+I*c*d*x)**3,x)
```

output
```
Timed out
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.40

$$\int \frac{x^4(a+b\arctan(cx))}{(d+icdx)^3}\,dx = \frac{8i\,ac^4x^4 - 8\,(4\,a+i\,b)c^3x^3 + (b(5i\,\arctan(1,cx)-16)+88i\,a)c^2x^2 + 2\,(b(5\,\arctan(1,cx)+19i)-8\,a)}{}$$

input
```
integrate(x^4*(a+b*arctan(c*x))/(d+I*c*d*x)^3,x, algorithm="maxima")
```

output
```
1/16*(8*I*a*c^4*x^4 - 8*(4*a + I*b)*c^3*x^3 + (b*(5*I*arctan2(1, c*x) - 16
) + 88*I*a)*c^2*x^2 + 2*(b*(5*arctan2(1, c*x) + 19*I) - 8*a)*c*x + 24*(b*c
^2*x^2 - 2*I*b*c*x - b)*arctan(c*x)^2 + 6*(b*c^2*x^2 - 2*I*b*c*x - b)*log(
c^2*x^2 + 1)^2 - 24*(I*b*c^2*x^2 + 2*b*c*x - I*b)*arctan(c*x)*log(1/4*c^2*
x^2 + 1/4) + b*(-5*I*arctan2(1, c*x) + 28) + (8*I*b*c^4*x^4 - 32*b*c^3*x^3
 + (96*a + 131*I*b)*c^2*x^2 - 2*(96*I*a - 35*b)*c*x - 96*a + 13*I*b)*arcta
n(c*x) - 48*(b*c^2*x^2 - 2*I*b*c*x - b)*dilog(1/2*I*c*x + 1/2) - 12*(2*(2*
I*a - b)*c^2*x^2 + 4*(2*a + I*b)*c*x + (b*c^2*x^2 - 2*I*b*c*x - b)*log(1/4
*c^2*x^2 + 1/4) - 4*I*a + 2*b)*log(c^2*x^2 + 1) + 56*I*a)/(c^7*d^3*x^2 - 2
*I*c^6*d^3*x - c^5*d^3)
```

## Giac [F]

$$\int \frac{x^4(a+b\arctan(cx))}{(d+icdx)^3}\,dx=\int \frac{(b\arctan(cx)+a)x^4}{(i\,cdx+d)^3}\,dx$$

input
```
integrate(x^4*(a+b*arctan(c*x))/(d+I*c*d*x)^3,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)*x^4/(I*c*d*x + d)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\arctan(cx))}{(d+icdx)^3}\,dx=\int \frac{x^4\,(a+b\,\mathrm{atan}(c\,x))}{(d+c\,d\,x\,1\mathrm{i})^3}\,dx$$

input
```
int((x^4*(a + b*atan(c*x)))/(d + c*d*x*1i)^3,x)
```

output
```
int((x^4*(a + b*atan(c*x)))/(d + c*d*x*1i)^3, x)
```

## Reduce [F]

$$\int \frac{x^4(a+b\arctan(cx))}{(d+icdx)^3}\,dx = \frac{-2\left(\int \frac{atan(cx)x^4}{c^3i\,x^3+3c^2x^2-3cix-1}dx\right)b\,c^5+8\left(\int \frac{x}{c^3i\,x^3+3c^2x^2-3cix-1}dx\right)a\,c^2i+6\left(\int \frac{1}{c^3i\,x^3+3c^2x^2-3cix-1}dx\right)ac-4\log(c}{2c^5d^3}$$

input
```
int(x^4*(a+b*atan(c*x))/(d+I*c*d*x)^3,x)
```

output
```
( - 2*int((atan(c*x)*x**4)/(c**3*i*x**3 + 3*c**2*x**2 - 3*c*i*x - 1),x)*b*
c**5 + 8*int(x/(c**3*i*x**3 + 3*c**2*x**2 - 3*c*i*x - 1),x)*a*c**2*i + 6*i
nt(1/(c**3*i*x**3 + 3*c**2*x**2 - 3*c*i*x - 1),x)*a*c - 4*log(c**3*i*x**3
+ 3*c**2*x**2 - 3*c*i*x - 1)*a*i + a*c**2*i*x**2 - 6*a*c*x)/(2*c**5*d**3)
```

# 3.59 $\int \frac{x^3(a+b\arctan(cx))}{(d+icdx)^3}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 225

$$\int \frac{x^3(a+b\arctan(cx))}{(d+icdx)^3}\,dx = \frac{iax}{c^3d^3} + \frac{ib}{8c^4d^3(i-cx)^2} - \frac{11b}{8c^4d^3(i-cx)} + \frac{11b\arctan(cx)}{8c^4d^3} + \frac{ibx\arctan(cx)}{c^3d^3} - \frac{a+b\arctan(cx)}{2c^4d^3(i-cx)^2} - \frac{3i(a+b\arctan(cx))}{c^4d^3(i-cx)} + \frac{3(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c^4d^3} - \frac{ib\log(1+c^2x^2)}{2c^4d^3} + \frac{3ib\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2c^4d^3}$$

output

```
I*a*x/c^3/d^3+1/8*I*b/c^4/d^3/(I-c*x)^2-11/8*b/c^4/d^3/(I-c*x)+11/8*b*arct
an(c*x)/c^4/d^3+I*b*x*arctan(c*x)/c^3/d^3-1/2*(a+b*arctan(c*x))/c^4/d^3/(I
-c*x)^2-3*I*(a+b*arctan(c*x))/c^4/d^3/(I-c*x)+3*(a+b*arctan(c*x))*ln(2/(1+
I*c*x))/c^4/d^3-1/2*I*b*ln(c^2*x^2+1)/c^4/d^3+3/2*I*b*polylog(2,1-2/(1+I*c
*x))/c^4/d^3
```

## Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.96

$$\int \frac{x^3(a+b\arctan(cx))}{(d+icdx)^3}\,dx = \frac{32iacx - \frac{16a}{(-i+cx)^2} + \frac{96ia}{-i+cx} - 96ia\arctan(cx) - 48a\log(1+c^2x^2) + ib\big(-96\arctan(cx)^2 + 20\cos(2\arcta$$

input
```
Integrate[(x^3*(a + b*ArcTan[c*x]))/(d + I*c*d*x)^3,x]
```

output
```
((32*I)*a*c*x - (16*a)/(-I + c*x)^2 + ((96*I)*a)/(-I + c*x) - (96*I)*a*Arc
Tan[c*x] - 48*a*Log[1 + c^2*x^2] + I*b*(-96*ArcTan[c*x]^2 + 20*Cos[2*ArcTa
n[c*x]] - Cos[4*ArcTan[c*x]] - 16*Log[1 + c^2*x^2] - 48*PolyLog[2, -E^((2*
I)*ArcTan[c*x])] - (20*I)*Sin[2*ArcTan[c*x]] + 4*ArcTan[c*x]*(8*c*x + (10*
I)*Cos[2*ArcTan[c*x]] - I*Cos[4*ArcTan[c*x]] - (24*I)*Log[1 + E^((2*I)*Arc
Tan[c*x])] + 10*Sin[2*ArcTan[c*x]] - Sin[4*ArcTan[c*x]]) + I*Sin[4*ArcTan[
c*x]]))/(32*c^4*d^3)
```

## Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\arctan(cx))}{(d+icdx)^3}\,dx$$

$$\Big\downarrow \text{5411}$$

$$\int \left(-\frac{3(a+b\arctan(cx))}{c^3d^3(cx-i)} - \frac{3i(a+b\arctan(cx))}{c^3d^3(cx-i)^2} + \frac{i(a+b\arctan(cx))}{c^3d^3} + \frac{a+b\arctan(cx)}{c^3d^3(cx-i)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3i(a+b\arctan(cx))}{c^4d^3(-cx+i)}-\frac{a+b\arctan(cx)}{2c^4d^3(-cx+i)^2}+\frac{3\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c^4d^3}+\frac{iax}{c^3d^3}+\frac{11b\arctan(cx)}{8c^4d^3}+\frac{ibx\arctan(cx)}{c^3d^3}+\frac{3ib\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{2c^4d^3}-\frac{11b}{8c^4d^3(-cx+i)}+\frac{ib}{8c^4d^3(-cx+i)^2}-\frac{ib\log\left(c^2x^2+1\right)}{2c^4d^3}$$

input
```
Int[(x^3*(a + b*ArcTan[c*x]))/(d + I*c*d*x)^3,x]
```

output
```
(I*a*x)/(c^3*d^3) + ((I/8)*b)/(c^4*d^3*(I - c*x)^2) - (11*b)/(8*c^4*d^3*(I
 - c*x)) + (11*b*ArcTan[c*x])/(8*c^4*d^3) + (I*b*x*ArcTan[c*x])/(c^3*d^3)
- (a + b*ArcTan[c*x])/(2*c^4*d^3*(I - c*x)^2) - ((3*I)*(a + b*ArcTan[c*x])
)/(c^4*d^3*(I - c*x)) + (3*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(c^4*d^
3) - ((I/2)*b*Log[1 + c^2*x^2])/(c^4*d^3) + (((3*I)/2)*b*PolyLog[2, 1 - 2/
(1 + I*c*x)])/(c^4*d^3)
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

## Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.43

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{3ib\ln\left(-\frac{i(cx+i)}{2}\right)\ln(cx-i)}{2d^3}-\frac{3a\ln\left(c^2x^2+1\right)}{2d^3}+\frac{ib\arctan(cx)cx}{d^3}+\frac{iacx}{d^3}-\frac{a}{2d^3(cx-i)^2}-\frac{3ia\arctan(cx)}{d^3}-\frac{3b\arctan(cx)\ln(cx-i)}{d^3}+\cdots}{}$ |
| default | $\frac{\frac{3ib\ln\left(-\frac{i(cx+i)}{2}\right)\ln(cx-i)}{2d^3}-\frac{3a\ln\left(c^2x^2+1\right)}{2d^3}+\frac{ib\arctan(cx)cx}{d^3}+\frac{iacx}{d^3}-\frac{a}{2d^3(cx-i)^2}-\frac{3ia\arctan(cx)}{d^3}-\frac{3b\arctan(cx)\ln(cx-i)}{d^3}+\cdots}{}$ |
| parts | $\frac{3ib\,\mathrm{dilog}\left(-\frac{i(cx+i)}{2}\right)}{2c^4d^3}-\frac{a}{2d^3c^4(-cx+i)^2}-\frac{3a\ln(c^2x^2+1)}{2d^3c^4}+\frac{3ib\arctan(cx)}{c^4d^3(cx-i)}-\frac{19ib\ln(c^2x^2+1)}{32c^4d^3}+\frac{ibx\arctan(c}{c^3d^3}$ |
| risch | $\left(\frac{bx}{2c^3d^3}-\frac{-3d^3bx+\frac{5ib\,d^3}{2c}}{2c^3d^6(cx-i)^2}\right)\ln(icx+1)+\frac{b\ln(-icx+1)x}{8d^3c^3(-icx-1)^2}+\frac{3ib\ln(icx+1)^2}{4c^4d^3}+\frac{ib\ln(-icx+1)x^2}{16d^3c^2(-icx-1)^2}-\frac{}{d^3}$ |

input
```
int(x^3*(a+b*arctan(c*x))/(d+I*c*d*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^4*(3/2*I*b/d^3*ln(-1/2*I*(c*x+I))*ln(c*x-I)-3/2*a/d^3*ln(c^2*x^2+1)+I*
b/d^3*arctan(c*x)*c*x+I*a/d^3*c*x-1/2*a/d^3/(c*x-I)^2-3*I*a/d^3*arctan(c*x
)-3*b/d^3*arctan(c*x)*ln(c*x-I)+1/8*I*b/d^3/(c*x-I)^2-1/2*b/d^3*arctan(c*x
)/(c*x-I)^2-19/32*I*b/d^3*ln(c^2*x^2+1)+3*I*a/d^3/(c*x-I)+19/16*b*arctan(c
*x)/d^3+11/8*b/d^3/(c*x-I)-3/4*I*b/d^3*ln(c*x-I)^2+3/32*b/d^3*arctan(1/6*c
^3*x^3+7/6*c*x)-3/32*b/d^3*arctan(1/2*c*x)+3/16*b/d^3*arctan(1/2*c*x-1/2*I
)+3/64*I*b/d^3*ln(c^4*x^4+10*c^2*x^2+9)+3/2*I*b/d^3*dilog(-1/2*I*(c*x+I))+
3*I*b/d^3*arctan(c*x)/(c*x-I))
```

## Fricas [F]

$$\int\frac{x^3(a+b\arctan(cx))}{(d+icdx)^3}\,dx=\int\frac{(b\arctan(cx)+a)x^3}{(i\,cdx+d)^3}\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))/(d+I*c*d*x)^3,x, algorithm="fricas")
```

output
```
integral(-1/2*(b*x^3*log(-(c*x + I)/(c*x - I)) - 2*I*a*x^3)/(c^3*d^3*x^3 -
 3*I*c^2*d^3*x^2 - 3*c*d^3*x + I*d^3), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\arctan(cx))}{(d+icdx)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(a+b*atan(c*x))/(d+I*c*d*x)**3,x)
```

output
```
Timed out
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.48

$$\int \frac{x^3(a+b\arctan(cx))}{(d+icdx)^3}\,dx = -\frac{-16i\,ac^3x^3-32\,ac^2x^2-2\,(16i\,a+11\,b)cx-12\,(-i\,bc^2x^2-2\,bcx+i\,b)\arctan(cx)^2-3\,(-i\,bc^2x^2-2$$

input
```
integrate(x^3*(a+b*arctan(c*x))/(d+I*c*d*x)^3,x, algorithm="maxima")
```

output
```
-1/16*(-16*I*a*c^3*x^3 - 32*a*c^2*x^2 - 2*(16*I*a + 11*b)*c*x - 12*(-I*b*c
^2*x^2 - 2*b*c*x + I*b)*arctan(c*x)^2 - 3*(-I*b*c^2*x^2 - 2*b*c*x + I*b)*l
og(c^2*x^2 + 1)^2 + 12*(b*c^2*x^2 - 2*I*b*c*x - b)*arctan(c*x)*log(1/4*c^2
*x^2 + 1/4) + (-16*I*b*c^3*x^3 - 3*(-16*I*a + 17*b)*c^2*x^2 + 6*(16*a + I*
b)*c*x - 48*I*a - 21*b)*arctan(c*x) + 3*(b*c^2*x^2 - 2*I*b*c*x - b)*arctan
2(c*x, -1) - 24*(I*b*c^2*x^2 + 2*b*c*x - I*b)*dilog(1/2*I*c*x + 1/2) + 2*(
4*(3*a + I*b)*c^2*x^2 - 8*(3*I*a - b)*c*x - 3*(I*b*c^2*x^2 + 2*b*c*x - I*b
)*log(1/4*c^2*x^2 + 1/4) - 12*a - 4*I*b)*log(c^2*x^2 + 1) - 40*a + 20*I*b)
/(c^6*d^3*x^2 - 2*I*c^5*d^3*x - c^4*d^3)
```

## Giac [F]

$$\int \frac{x^3(a+b\arctan(cx))}{(d+icdx)^3}\,dx=\int \frac{(b\arctan(cx)+a)x^3}{(i\,cdx+d)^3}\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))/(d+I*c*d*x)^3,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)*x^3/(I*c*d*x + d)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\arctan(cx))}{(d+icdx)^3}\,dx=\int \frac{x^3\,(a+b\,\mathrm{atan}(c\,x))}{(d+c\,d\,x\,1\mathrm{i})^3}\,dx$$

input
```
int((x^3*(a + b*atan(c*x)))/(d + c*d*x*1i)^3,x)
```

output
```
int((x^3*(a + b*atan(c*x)))/(d + c*d*x*1i)^3, x)
```

## Reduce [F]

$$\int \frac{x^3(a+b\arctan(cx))}{(d+icdx)^3}\,dx = \frac{-\left(\int \frac{atan(cx)x^3}{c^3i\,x^3+3c^2x^2-3cix-1}dx\right)b\,c^4+3\left(\int \frac{x}{c^3i\,x^3+3c^2x^2-3cix-1}dx\right)a\,c^2-2\left(\int \frac{1}{c^3i\,x^3+3c^2x^2-3cix-1}dx\right)aci-\log(c^3i\,x^3 \cdots}{c^4d^3}$$

input
```
int(x^3*(a+b*atan(c*x))/(d+I*c*d*x)^3,x)
```

output
```
( - int((atan(c*x)*x**3)/(c**3*i*x**3 + 3*c**2*x**2 - 3*c*i*x - 1),x)*b*c*
*4 + 3*int(x/(c**3*i*x**3 + 3*c**2*x**2 - 3*c*i*x - 1),x)*a*c**2 - 2*int(1
/(c**3*i*x**3 + 3*c**2*x**2 - 3*c*i*x - 1),x)*a*c*i - log(c**3*i*x**3 + 3*
c**2*x**2 - 3*c*i*x - 1)*a + a*c*i*x)/(c**4*d**3)
```

# 3.60 $\int \frac{x^2(a+b\arctan(cx))}{(d+icdx)^3}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 176

$$\begin{aligned}\int \frac{x^2(a+b\arctan(cx))}{(d+icdx)^3}\,dx = {} & \frac{b}{8c^3d^3(i-cx)^2}+\frac{7ib}{8c^3d^3(i-cx)}-\frac{7ib\arctan(cx)}{8c^3d^3} \\ & +\frac{i(a+b\arctan(cx))}{2c^3d^3(i-cx)^2}-\frac{2(a+b\arctan(cx))}{c^3d^3(i-cx)} \\ & -\frac{i(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c^3d^3} \\ & +\frac{b\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2c^3d^3}\end{aligned}$$

output
```
1/8*b/c^3/d^3/(I-c*x)^2+7/8*I*b/c^3/d^3/(I-c*x)-7/8*I*b*arctan(c*x)/c^3/d^
3+1/2*I*(a+b*arctan(c*x))/c^3/d^3/(I-c*x)^2-2*(a+b*arctan(c*x))/c^3/d^3/(I
-c*x)-I*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^3/d^3+1/2*b*polylog(2,1-2/(1+I
*c*x))/c^3/d^3
```

## Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.06

$$\int \frac{x^2(a+b\arctan(cx))}{(d+icdx)^3}\,dx =$$
$$-\frac{i\left(12a-6ib+16iacx+7bcx-8a\log\left(\frac{2i}{i-cx}\right)-16iacx\log\left(\frac{2i}{i-cx}\right)+8ac^2x^2\log\left(\frac{2i}{i-cx}\right)+b\arctan(cx)\left(5\right.\right.}{8c^3d^3(-i+cx)^2}$$

input
```
Integrate[(x^2*(a + b*ArcTan[c*x]))/(d + I*c*d*x)^3,x]
```

output
```
((-1/8*I)*(12*a - (6*I)*b + (16*I)*a*c*x + 7*b*c*x - 8*a*Log[(2*I)/(I - c*
x)] - (16*I)*a*c*x*Log[(2*I)/(I - c*x)] + 8*a*c^2*x^2*Log[(2*I)/(I - c*x)]
 + b*ArcTan[c*x]*(5 + (2*I)*c*x + 7*c^2*x^2 + 8*(-I + c*x)^2*Log[(2*I)/(I
- c*x)]) + (4*I)*b*(-I + c*x)^2*PolyLog[2, (I + c*x)/(-I + c*x)]))/(c^3*d^
3*(-I + c*x)^2)
```

## Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\arctan(cx))}{(d+icdx)^3}\,dx$$
$$\Big\downarrow \text{5411}$$
$$\int \left(\frac{i(a+b\arctan(cx))}{c^2d^3(cx-i)}-\frac{2(a+b\arctan(cx))}{c^2d^3(cx-i)^2}-\frac{i(a+b\arctan(cx))}{c^2d^3(cx-i)^3}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{2(a+b\arctan(cx))}{c^3d^3(-cx+i)}+\frac{i(a+b\arctan(cx))}{2c^3d^3(-cx+i)^2}-\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c^3d^3}-\frac{7ib\arctan(cx)}{8c^3d^3}+\frac{b\,\mathrm{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{2c^3d^3}+\frac{7ib}{8c^3d^3(-cx+i)}+\frac{b}{8c^3d^3(-cx+i)^2}$$

input
```
Int[(x^2*(a + b*ArcTan[c*x]))/(d + I*c*d*x)^3,x]
```

output
```
b/(8*c^3*d^3*(I - c*x)^2) + (((7*I)/8)*b)/(c^3*d^3*(I - c*x)) - (((7*I)/8)
*b*ArcTan[c*x])/(c^3*d^3) + ((I/2)*(a + b*ArcTan[c*x]))/(c^3*d^3*(I - c*x)
^2) - (2*(a + b*ArcTan[c*x]))/(c^3*d^3*(I - c*x)) - (I*(a + b*ArcTan[c*x])
*Log[2/(1 + I*c*x)])/(c^3*d^3) + (b*PolyLog[2, 1 - 2/(1 + I*c*x)])/(2*c^3*
d^3)
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

## Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.70

| method | result |
|---|---|
| derivativedivides | $\frac{ia}{2d^3(cx-i)^2}-\frac{a\arctan(cx)}{d^3}+\frac{ia\ln\left(c^2x^2+1\right)}{2d^3}+\frac{2a}{d^3(cx-i)}-\frac{7ib}{8d^3(cx-i)}-\frac{7ib\arctan\left(\frac{1}{6}c^3x^3+\frac{7}{6}cx\right)}{32d^3}+\frac{2b\arctan(cx)}{d^3(cx-i)}+\frac{7b\ln\left(c^4x^4+10\cdots\right.}{64d^3}$ |
| default | $\frac{ia}{2d^3(cx-i)^2}-\frac{a\arctan(cx)}{d^3}+\frac{ia\ln\left(c^2x^2+1\right)}{2d^3}+\frac{2a}{d^3(cx-i)}-\frac{7ib}{8d^3(cx-i)}-\frac{7ib\arctan\left(\frac{1}{6}c^3x^3+\frac{7}{6}cx\right)}{32d^3}+\frac{2b\arctan(cx)}{d^3(cx-i)}+\frac{7b\ln\left(c^4x^4+10\cdots\right.}{64d^3}$ |
| parts | $\frac{ia\ln(c^2x^2+1)}{2d^3c^3}-\frac{a\arctan(cx)}{d^3c^3}-\frac{2a}{d^3c^3(-cx+i)}+\frac{ib\arctan(cx)}{2c^3d^3(cx-i)^2}-\frac{7ib\arctan\left(\frac{cx}{2}-\frac{i}{2}\right)}{16c^3d^3}+\frac{ib\arctan(cx)\ln(cx-i\cdots}{c^3d^3}$ |
| risch | $\frac{b\ln(icx+1)^2}{4c^3d^3}+\frac{\left(-\frac{ibx}{c^2}-\frac{3b}{4c^3}\right)\ln(icx+1)}{d^3(cx-i)^2}-\frac{ib\ln(-icx+1)x}{2d^3c^2(-icx-1)}-\frac{ib\ln(-icx+1)x}{8d^3c^2(-icx-1)^2}-\frac{a\arctan(cx)}{d^3c^3}+\frac{ia\ln(c^2x^2+\cdots}{2d^3c^3}$ |

input
```
int(x^2*(a+b*arctan(c*x))/(d+I*c*d*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^3*(1/2*I*a/d^3/(c*x-I)^2-a/d^3*arctan(c*x)+1/2*I*a/d^3*ln(c^2*x^2+1)+2
*a/d^3/(c*x-I)-7/8*I*b/d^3/(c*x-I)-7/32*I*b/d^3*arctan(1/6*c^3*x^3+7/6*c*x
)+2*b/d^3*arctan(c*x)/(c*x-I)+7/64*b/d^3*ln(c^4*x^4+10*c^2*x^2+9)-7/16*I*b
/d^3*arctan(c*x)-7/16*I*b/d^3*arctan(1/2*c*x-1/2*I)+1/2*I*b/d^3*arctan(c*x
)/(c*x-I)^2+7/32*I*b/d^3*arctan(1/2*c*x)+1/8*b/d^3/(c*x-I)^2-7/32*b/d^3*ln
(c^2*x^2+1)+I*b/d^3*arctan(c*x)*ln(c*x-I)+1/2*b/d^3*ln(c*x-I)*ln(-1/2*I*(c
*x+I))+1/2*b/d^3*dilog(-1/2*I*(c*x+I))-1/4*b/d^3*ln(c*x-I)^2)
```

## Fricas [F]

$$\int \frac{x^2(a+b\arctan(cx))}{(d+icdx)^3}\,dx=\int \frac{(b\arctan(cx)+a)x^2}{(i\,cdx+d)^3}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))/(d+I*c*d*x)^3,x, algorithm="fricas")
```

output
```
integral(-1/2*(b*x^2*log(-(c*x + I)/(c*x - I)) - 2*I*a*x^2)/(c^3*d^3*x^3 -
 3*I*c^2*d^3*x^2 - 3*c*d^3*x + I*d^3), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\arctan(cx))}{(d+icdx)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(a+b*atan(c*x))/(d+I*c*d*x)**3,x)
```

output
```
Timed out
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.65

$$\int \frac{x^2(a+b\arctan(cx))}{(d+icdx)^3}\,dx =$$
$$-\frac{-7i\,bc^2x^2\arctan(1,cx) - 2\,(7\,b(\arctan(1,cx) - i) + 16\,a)cx + 4\,(bc^2x^2 - 2i\,bcx - b)\arctan(cx)^2 + (}{}$$

input
```
integrate(x^2*(a+b*arctan(c*x))/(d+I*c*d*x)^3,x, algorithm="maxima")
```

output
```
-1/16*(-7*I*b*c^2*x^2*arctan2(1, c*x) - 2*(7*b*(arctan2(1, c*x) - I) + 16*
a)*c*x + 4*(b*c^2*x^2 - 2*I*b*c*x - b)*arctan(c*x)^2 + (b*c^2*x^2 - 2*I*b*
c*x - b)*log(c^2*x^2 + 1)^2 - 4*(I*b*c^2*x^2 + 2*b*c*x - I*b)*arctan(c*x)*
log(1/4*c^2*x^2 + 1/4) + b*(7*I*arctan2(1, c*x) + 12) + ((16*a + 7*I*b)*c^
2*x^2 - 2*(16*I*a + 9*b)*c*x - 16*a + 17*I*b)*arctan(c*x) - 8*(b*c^2*x^2 -
 2*I*b*c*x - b)*dilog(1/2*I*c*x + 1/2) - 2*(4*I*a*c^2*x^2 + 8*a*c*x + (b*c
^2*x^2 - 2*I*b*c*x - b)*log(1/4*c^2*x^2 + 1/4) - 4*I*a)*log(c^2*x^2 + 1) +
 24*I*a)/(c^5*d^3*x^2 - 2*I*c^4*d^3*x - c^3*d^3)
```

## Giac [F]

$$\int \frac{x^2(a+b\arctan(cx))}{(d+icdx)^3}\,dx=\int \frac{(b\arctan(cx)+a)x^2}{(i\,cdx+d)^3}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))/(d+I*c*d*x)^3,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)*x^2/(I*c*d*x + d)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\arctan(cx))}{(d+icdx)^3}\,dx=\int \frac{x^2\,(a+b\,\mathrm{atan}(c\,x))}{(d+c\,d\,x\,1\mathrm{i})^3}\,dx$$

input
```
int((x^2*(a + b*atan(c*x)))/(d + c*d*x*1i)^3,x)
```

output
```
int((x^2*(a + b*atan(c*x)))/(d + c*d*x*1i)^3, x)
```

## Reduce [F]

$$\int \frac{x^2(a+b\arctan(cx))}{(d+icdx)^3}\,dx$$
$$=\frac{-3\left(\int \frac{atan(cx)x^2}{c^3i\,x^3+3c^2x^2-3cix-1}dx\right)b\,c^3-6\left(\int \frac{x}{c^3i\,x^3+3c^2x^2-3cix-1}dx\right)a\,c^2i-3\left(\int \frac{1}{c^3i\,x^3+3c^2x^2-3cix-1}dx\right)ac+\log(c^3i\ldots}{3c^3d^3}$$

input
```
int(x^2*(a+b*atan(c*x))/(d+I*c*d*x)^3,x)
```

output
```
( - 3*int((atan(c*x)*x**2)/(c**3*i*x**3 + 3*c**2*x**2 - 3*c*i*x - 1),x)*b*
c**3 - 6*int(x/(c**3*i*x**3 + 3*c**2*x**2 - 3*c*i*x - 1),x)*a*c**2*i - 3*i
nt(1/(c**3*i*x**3 + 3*c**2*x**2 - 3*c*i*x - 1),x)*a*c + log(c**3*i*x**3 +
3*c**2*x**2 - 3*c*i*x - 1)*a*i)/(3*c**3*d**3)
```

# 3.61 $\int \frac{x(a+b\arctan(cx))}{(d+icdx)^3}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 88

$$\int \frac{x(a+b\arctan(cx))}{(d+icdx)^3}\,dx = -\frac{ib}{8c^2d^3(i-cx)^2}+\frac{3b}{8c^2d^3(i-cx)}+\frac{b\arctan(cx)}{8c^2d^3}+\frac{x^2(a+b\arctan(cx))}{2d^3(1+icx)^2}$$

output
```
-1/8*I*b/c^2/d^3/(I-c*x)^2+3/8*b/c^2/d^3/(I-c*x)+1/8*b*arctan(c*x)/c^2/d^3
+1/2*x^2*(a+b*arctan(c*x))/d^3/(1+I*c*x)^2
```

## Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.72

$$\int \frac{x(a+b\arctan(cx))}{(d+icdx)^3}\,dx = \frac{b(2i-3cx)+a(-4-8icx)-b(1+2icx+3c^2x^2)\arctan(cx)}{8c^2d^3(-i+cx)^2}$$

input
```
Integrate[(x*(a + b*ArcTan[c*x]))/(d + I*c*d*x)^3,x]
```

output
```
(b*(2*I - 3*c*x) + a*(-4 - (8*I)*c*x) - b*(1 + (2*I)*c*x + 3*c^2*x^2)*ArcT
an[c*x])/(8*c^2*d^3*(-I + c*x)^2)
```

## Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {5407, 27, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\arctan(cx))}{(d+icdx)^3}dx$$

$$\Big\downarrow \text{5407}$$

$$\frac{x^2(a+b\arctan(cx))}{2d^3(1+icx)^2}-bc\int \frac{x^2}{2d^3(i-cx)^3(cx+i)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{x^2(a+b\arctan(cx))}{2d^3(1+icx)^2}-\frac{bc\int \frac{x^2}{(i-cx)^3(cx+i)}dx}{2d^3}$$

$$\Big\downarrow \text{99}$$

$$\frac{x^2(a+b\arctan(cx))}{2d^3(1+icx)^2}-\frac{bc\int \left(-\frac{3}{4c^2(cx-i)^2}-\frac{i}{2c^2(cx-i)^3}-\frac{1}{4c^2(c^2x^2+1)}\right)dx}{2d^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^2(a+b\arctan(cx))}{2d^3(1+icx)^2}-\frac{bc\left(-\frac{\arctan(cx)}{4c^3}-\frac{3}{4c^3(-cx+i)}+\frac{i}{4c^3(-cx+i)^2}\right)}{2d^3}$$

input
```
Int[(x*(a + b*ArcTan[c*x]))/(d + I*c*d*x)^3,x]
```

output
```
(x^2*(a + b*ArcTan[c*x]))/(2*d^3*(1 + I*c*x)^2) - (b*c*((I/4)/(c^3*(I - c*
x)^2) - 3/(4*c^3*(I - c*x)) - ArcTan[c*x]/(4*c^3)))/(2*d^3)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5407
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_))^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Simp[(a
 + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2
), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[q, -1] && IntegerQ
[2*m] && ((IGtQ[m, 0] && IGtQ[q, 0]) || (ILtQ[m + q + 1, 0] && LtQ[m*q, 0])
)
```

## Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.24

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{a\left(\frac{1}{2(cx-i)^2}-\frac{i}{cx-i}\right)}{d^3}+\frac{b\arctan(cx)}{2d^3(cx-i)^2}-\frac{ib\arctan(cx)}{d^3(cx-i)}-\frac{3b\arctan(cx)}{8d^3}-\frac{ib}{8d^3(cx-i)^2}-\frac{3b}{8d^3(cx-i)}}{c^2}$ |
| default | $\frac{\frac{a\left(\frac{1}{2(cx-i)^2}-\frac{i}{cx-i}\right)}{d^3}+\frac{b\arctan(cx)}{2d^3(cx-i)^2}-\frac{ib\arctan(cx)}{d^3(cx-i)}-\frac{3b\arctan(cx)}{8d^3}-\frac{ib}{8d^3(cx-i)^2}-\frac{3b}{8d^3(cx-i)}}{c^2}$ |
| parts | $\frac{a\left(\frac{i}{c^2(-cx+i)}+\frac{1}{2c^2(-cx+i)^2}\right)}{d^3}+\frac{b\arctan(cx)}{2c^2d^3(cx-i)^2}-\frac{ib\arctan(cx)}{c^2d^3(cx-i)}-\frac{3b\arctan(cx)}{8c^2d^3}-\frac{ib}{8c^2d^3(cx-i)^2}-\frac{3b}{8c^2d^3(cx-}$ |
| orering | $\frac{(10c^5x^5+21ic^4x^4+18c^3x^3+24ic^2x^2+8cx+3i)(a+b\arctan(cx))}{32c^3x(icdx+d)^3}-\frac{(-5cx+3i)(c^2x^2+1)^2\left(\frac{a+b\arctan(cx)}{(icdx+d)^3}+\frac{x}{(c^2x^2+1)}\right.}{32c^3x}$ |
| risch | $-\frac{b(2cx-i)\ln(icx+1)}{4c^2d^3(cx-i)^2}-\frac{i(-3\ln(-cx+i)bc^2x^2+3\ln(cx+i)bc^2x^2+6i\ln(-cx+i)bcx-6i\ln(cx+i)bcx+8ibcx\ln(-icx}{16c^2d^3(cx-i)^2}$ |

input
```
int(x*(a+b*arctan(c*x))/(d+I*c*d*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^2*(a/d^3*(1/2/(c*x-I)^2-I/(c*x-I))+1/2*b/d^3*arctan(c*x)/(c*x-I)^2-I*b
/d^3*arctan(c*x)/(c*x-I)-3/8*b*arctan(c*x)/d^3-1/8*I*b/d^3/(c*x-I)^2-3/8*b
/d^3/(c*x-I))
```

## Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.97

$$\int \frac{x(a+b\arctan(cx))}{(d+icdx)^3}\,dx = -\frac{2\,(8i\,a+3\,b)cx-(-3i\,bc^2x^2+2\,bcx-i\,b)\log\left(-\frac{cx+i}{cx-i}\right)+8\,a-4i\,b}{16\,(c^4d^3x^2-2i\,c^3d^3x-c^2d^3)}$$

input
```
integrate(x*(a+b*arctan(c*x))/(d+I*c*d*x)^3,x, algorithm="fricas")
```

output
```
-1/16*(2*(8*I*a + 3*b)*c*x - (-3*I*b*c^2*x^2 + 2*b*c*x - I*b)*log(-(c*x +
I)/(c*x - I)) + 8*a - 4*I*b)/(c^4*d^3*x^2 - 2*I*c^3*d^3*x - c^2*d^3)
```

## Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(75) = 150$.

Time = 5.12 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.15

$$\int \frac{x(a+b\arctan(cx))}{(d+icdx)^3}\,dx = \frac{b\left(\frac{3i\log\left(x-\frac{i}{c}\right)}{16}-\frac{3i\log\left(x+\frac{i}{c}\right)}{16}\right)}{c^2d^3}+\frac{(-2bcx+ib)\log(icx+1)}{4c^4d^3x^2-8ic^3d^3x-4c^2d^3}+\frac{(2bcx-ib)\log(-icx+1)}{4c^4d^3x^2-8ic^3d^3x-4c^2d^3}+\frac{-4a+2ib+x(-8iac-3bc)}{8c^4d^3x^2-16ic^3d^3x-8c^2d^3}$$

input
```
integrate(x*(a+b*atan(c*x))/(d+I*c*d*x)**3,x)
```

output
```
b*(3*I*log(x - I/c)/16 - 3*I*log(x + I/c)/16)/(c**2*d**3) + (-2*b*c*x + I*
b)*log(I*c*x + 1)/(4*c**4*d**3*x**2 - 8*I*c**3*d**3*x - 4*c**2*d**3) + (2*
b*c*x - I*b)*log(-I*c*x + 1)/(4*c**4*d**3*x**2 - 8*I*c**3*d**3*x - 4*c**2*
d**3) + (-4*a + 2*I*b + x*(-8*I*a*c - 3*b*c))/(8*c**4*d**3*x**2 - 16*I*c**
3*d**3*x - 8*c**2*d**3)
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.80

$$\int \frac{x(a+b\arctan(cx))}{(d+icdx)^3}\,dx = -\frac{(8i\,a+3\,b)cx+(3\,bc^2x^2+2i\,bcx+b)\arctan(cx)+4\,a-2i\,b}{8\,(c^4d^3x^2-2i\,c^3d^3x-c^2d^3)}$$

input
```
integrate(x*(a+b*arctan(c*x))/(d+I*c*d*x)^3,x, algorithm="maxima")
```

output
```
-1/8*((8*I*a + 3*b)*c*x + (3*b*c^2*x^2 + 2*I*b*c*x + b)*arctan(c*x) + 4*a
- 2*I*b)/(c^4*d^3*x^2 - 2*I*c^3*d^3*x - c^2*d^3)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.53

$$\int \frac{x(a+b\arctan(cx))}{(d+icdx)^3}\,dx = \frac{3\,bc^2x^2\log(cx+i)-3\,bc^2x^2\log(cx-i)+16\,bcx\arctan(cx)-6i\,bcx\log(cx+i)+6i\,bcx\log(cx-i)+}{16i\,c^4d^3x^2+32\,c^3d^3x-16i}$$

input
```
integrate(x*(a+b*arctan(c*x))/(d+I*c*d*x)^3,x, algorithm="giac")
```

output
```
(3*b*c^2*x^2*log(c*x + I) - 3*b*c^2*x^2*log(c*x - I) + 16*b*c*x*arctan(c*x
) - 6*I*b*c*x*log(c*x + I) + 6*I*b*c*x*log(c*x - I) + 16*a*c*x - 6*I*b*c*x
 - 8*I*b*arctan(c*x) - 3*b*log(c*x + I) + 3*b*log(c*x - I) - 8*I*a - 4*b)/
(16*I*c^4*d^3*x^2 + 32*c^3*d^3*x - 16*I*c^2*d^3)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\arctan(cx))}{(d+icdx)^3}\,dx = \int \frac{x\,(a+b\,\mathrm{atan}(c\,x))}{(d+c\,d\,x\,1\mathrm{i})^3}\,dx$$

input
```
int((x*(a + b*atan(c*x)))/(d + c*d*x*1i)^3,x)
```

output
```
int((x*(a + b*atan(c*x)))/(d + c*d*x*1i)^3, x)
```

## Reduce [F]

$$\int \frac{x(a+b\arctan(cx))}{(d+icdx)^3}\,dx = \frac{atan(cx)^2\,bi + 2atan(cx)\,ai + 2\left(\int \frac{atan(cx)}{c^5i\,x^5+3c^4x^4-2c^3i\,x^3+2c^2x^2-3cix-1}dx\right)bci + 6\left(\int -\frac{x^2}{c^5x^5-3c^4i\,x^4-2c^3x^3-2c^2i\,x^2-}\right.}{}$$

input
```
int(x*(a+b*atan(c*x))/(d+I*c*d*x)^3,x)
```

output
```
(atan(c*x)**2*b*i + 2*atan(c*x)*a*i + 2*int(atan(c*x)/(c**5*i*x**5 + 3*c**
4*x**4 - 2*c**3*i*x**3 + 2*c**2*x**2 - 3*c*i*x - 1),x)*b*c*i + 6*int(( - x
**2)/(c**5*x**5 - 3*c**4*i*x**4 - 2*c**3*x**3 - 2*c**2*i*x**2 - 3*c*x + i)
,x)*a*c**3 - 6*int(( - x)/(c**5*x**5 - 3*c**4*i*x**4 - 2*c**3*x**3 - 2*c**
2*i*x**2 - 3*c*x + i),x)*a*c**2*i - 2*int(( - 1)/(c**5*x**5 - 3*c**4*i*x**
4 - 2*c**3*x**3 - 2*c**2*i*x**2 - 3*c*x + i),x)*a*c - 6*int((atan(c*x)*x**
2)/(c**5*i*x**5 + 3*c**4*x**4 - 2*c**3*i*x**3 + 2*c**2*x**2 - 3*c*i*x - 1)
,x)*b*c**3*i - 8*int((atan(c*x)*x)/(c**5*i*x**5 + 3*c**4*x**4 - 2*c**3*i*x
**3 + 2*c**2*x**2 - 3*c*i*x - 1),x)*b*c**2 - 2*int(x/(c**5*i*x**5 + 3*c**4
*x**4 - 2*c**3*i*x**3 + 2*c**2*x**2 - 3*c*i*x - 1),x)*a*c**2)/(2*c**2*d**3
)
```

# 3.62 $\int \frac{a+b\arctan(cx)}{(d+icdx)^3}\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 92

$$\int \frac{a+b\arctan(cx)}{(d+icdx)^3}\,dx = -\frac{b}{8cd^3(i-cx)^2}+\frac{ib}{8cd^3(i-cx)} - \frac{ib\arctan(cx)}{8cd^3}+\frac{i(a+b\arctan(cx))}{2cd^3(1+icx)^2}$$

output
```
-1/8*b/c/d^3/(I-c*x)^2+1/8*I*b/c/d^3/(I-c*x)-1/8*I*b*arctan(c*x)/c/d^3+1/2
*I*(a+b*arctan(c*x))/c/d^3/(1+I*c*x)^2
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.60

$$\int \frac{a+b\arctan(cx)}{(d+icdx)^3}\,dx = -\frac{i(4a+b(-2i+cx)+b(3-2icx+c^2x^2)\arctan(cx))}{8cd^3(-i+cx)^2}$$

input
```
Integrate[(a + b*ArcTan[c*x])/(d + I*c*d*x)^3,x]
```

output
```
((-1/8*I)*(4*a + b*(-2*I + c*x) + b*(3 - (2*I)*c*x + c^2*x^2)*ArcTan[c*x])
)/(c*d^3*(-I + c*x)^2)
```

## Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5387, 27, 456, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{(d+icdx)^3}\,dx$$

$$\Big\downarrow \text{5387}$$

$$\frac{i(a+b\arctan(cx))}{2cd^3(1+icx)^2}-\frac{ib\int \frac{1}{d^2(icx+1)^2(c^2x^2+1)}dx}{2d}$$

$$\Big\downarrow \text{27}$$

$$\frac{i(a+b\arctan(cx))}{2cd^3(1+icx)^2}-\frac{ib\int \frac{1}{(icx+1)^2(c^2x^2+1)}dx}{2d^3}$$

$$\Big\downarrow \text{456}$$

$$\frac{i(a+b\arctan(cx))}{2cd^3(1+icx)^2}-\frac{ib\int \frac{1}{(1-icx)(icx+1)^3}dx}{2d^3}$$

$$\Big\downarrow \text{54}$$

$$\frac{i(a+b\arctan(cx))}{2cd^3(1+icx)^2}-\frac{ib\int \left(-\frac{1}{4(cx-i)^2}+\frac{i}{2(cx-i)^3}+\frac{1}{4(c^2x^2+1)}\right)dx}{2d^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{i(a+b\arctan(cx))}{2cd^3(1+icx)^2}-\frac{ib\left(\frac{\arctan(cx)}{4c}-\frac{1}{4c(-cx+i)}-\frac{i}{4c(-cx+i)^2}\right)}{2d^3}$$

input
```
Int[(a + b*ArcTan[c*x])/(d + I*c*d*x)^3,x]
```

output
```
((I/2)*(a + b*ArcTan[c*x]))/(c*d^3*(1 + I*c*x)^2) - ((I/2)*b*((-1/4*I)/(c*
(I - c*x)^2) - 1/(4*c*(I - c*x)) + ArcTan[c*x]/(4*c)))/d^3
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5387
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Simp[b*(
c/(e*(q + 1)))   Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b
, c, d, e, q}, x] && NeQ[q, -1]
```

## Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.89

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{ia}{2d^3(icx+1)^2}+\frac{ib\arctan(cx)}{2d^3(icx+1)^2}-\frac{ib\arctan(cx)}{8d^3}-\frac{b}{8d^3(cx-i)^2}-\frac{ib}{8d^3(cx-i)}}{c}$ |
| default | $\frac{\frac{ia}{2d^3(icx+1)^2}+\frac{ib\arctan(cx)}{2d^3(icx+1)^2}-\frac{ib\arctan(cx)}{8d^3}-\frac{b}{8d^3(cx-i)^2}-\frac{ib}{8d^3(cx-i)}}{c}$ |
| parts | $\frac{ia}{2d^3(icx+1)^2c}+\frac{ib\arctan(cx)}{2c\,d^3(icx+1)^2}-\frac{ib\arctan(cx)}{8c\,d^3}-\frac{b}{8c\,d^3(cx-i)^2}-\frac{ib}{8c\,d^3(cx-i)}$ |
| orering | $-\frac{i(-9c^4x^4+2ic^3x^3-36c^2x^2+2icx-27)(a+b\arctan(cx))}{32c(icdx+d)^3}-\frac{i(-3cx+5i)(c^2x^2+1)^2\left(\frac{bc}{(c^2x^2+1)(icdx+d)^3}-\frac{3i(a+b\,\mathrm{ar}}{(icd}\right.}{32c^2}$ |
| risch | $-\frac{b\ln(icx+1)}{4c\,d^3(cx-i)^2}+\frac{4b\ln(-icx+1)-\ln(-cx+i)b\,c^2x^2+\ln(cx+i)b\,c^2x^2+2i\ln(-cx+i)bcx-2i\ln(cx+i)bcx+b\ln(-cx+i}{16d^3(cx-i)^2c}$ |

input
```
int((a+b*arctan(c*x))/(d+I*c*d*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/c*(1/2*I*a/d^3/(1+I*c*x)^2+1/2*I*b/d^3/(1+I*c*x)^2*arctan(c*x)-1/8*I*b/d
^3*arctan(c*x)-1/8*b/d^3/(c*x-I)^2-1/8*I*b/d^3/(c*x-I))
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.82

$$\int \frac{a+b\arctan(cx)}{(d+icdx)^3}\,dx = \frac{-2i\,bcx+(bc^2x^2-2i\,bcx+3\,b)\log\left(-\frac{cx+i}{cx-i}\right)-8i\,a-4\,b}{16\,(c^3d^3x^2-2i\,c^2d^3x-cd^3)}$$

input
```
integrate((a+b*arctan(c*x))/(d+I*c*d*x)^3,x, algorithm="fricas")
```

output
```
1/16*(-2*I*b*c*x + (b*c^2*x^2 - 2*I*b*c*x + 3*b)*log(-(c*x + I)/(c*x - I))
 - 8*I*a - 4*b)/(c^3*d^3*x^2 - 2*I*c^2*d^3*x - c*d^3)
```

## Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 158 vs. 2(70) = 140.

Time = 1.72 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.72

$$\int \frac{a+b\arctan(cx)}{(d+icdx)^3}\,dx = \frac{b\log(-icx+1)}{4c^3d^3x^2-8ic^2d^3x-4cd^3} - \frac{b\log(icx+1)}{4c^3d^3x^2-8ic^2d^3x-4cd^3} + \frac{b\left(-\frac{\log\left(bx-\frac{ib}{c}\right)}{16}+\frac{\log\left(bx+\frac{ib}{c}\right)}{16}\right)}{cd^3} + \frac{-4ia-ibcx-2b}{8c^3d^3x^2-16ic^2d^3x-8cd^3}$$

input
```
integrate((a+b*atan(c*x))/(d+I*c*d*x)**3,x)
```

output
```
b*log(-I*c*x + 1)/(4*c**3*d**3*x**2 - 8*I*c**2*d**3*x - 4*c*d**3) - b*log(
I*c*x + 1)/(4*c**3*d**3*x**2 - 8*I*c**2*d**3*x - 4*c*d**3) + b*(-log(b*x -
 I*b/c)/16 + log(b*x + I*b/c)/16)/(c*d**3) + (-4*I*a - I*b*c*x - 2*b)/(8*c
**3*d**3*x**2 - 16*I*c**2*d**3*x - 8*c*d**3)
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.71

$$\int \frac{a+b\arctan(cx)}{(d+icdx)^3}\,dx = -\frac{i\,bcx+\left(i\,bc^2x^2+2\,bcx+3i\,b\right)\arctan\left(cx\right)+4i\,a+2\,b}{8\left(c^3d^3x^2-2i\,c^2d^3x-cd^3\right)}$$

input
```
integrate((a+b*arctan(c*x))/(d+I*c*d*x)^3,x, algorithm="maxima")
```

output
```
-1/8*(I*b*c*x + (I*b*c^2*x^2 + 2*b*c*x + 3*I*b)*arctan(c*x) + 4*I*a + 2*b)
/(c^3*d^3*x^2 - 2*I*c^2*d^3*x - c*d^3)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.27

$$\int \frac{a+b\arctan(cx)}{(d+icdx)^3}\,dx$$
$$= \frac{bc^2x^2\log\left(cx+i\right)-bc^2x^2\log\left(cx-i\right)-2i\,bcx\log\left(cx+i\right)+2i\,bcx\log\left(cx-i\right)-2i\,bcx-8i\,b\arctan\left(cx\right.}{16\left(c^3d^3x^2-2i\,c^2d^3x-cd^3\right)}$$

input
```
integrate((a+b*arctan(c*x))/(d+I*c*d*x)^3,x, algorithm="giac")
```

output
```
1/16*(b*c^2*x^2*log(c*x + I) - b*c^2*x^2*log(c*x - I) - 2*I*b*c*x*log(c*x
+ I) + 2*I*b*c*x*log(c*x - I) - 2*I*b*c*x - 8*I*b*arctan(c*x) - b*log(c*x
+ I) + b*log(c*x - I) - 8*I*a - 4*b)/(c^3*d^3*x^2 - 2*I*c^2*d^3*x - c*d^3)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{(d+icdx)^3}\,dx = \int \frac{a+b\,\mathrm{atan}(c\,x)}{\left(d+c\,d\,x\,1\mathrm{i}\right)^3}\,dx$$

input
```
int((a + b*atan(c*x))/(d + c*d*x*1i)^3,x)
```

output
```
int((a + b*atan(c*x))/(d + c*d*x*1i)^3, x)
```

## Reduce [F]

$$\int \frac{a+b\arctan(cx)}{(d+icdx)^3}\,dx = \frac{-atan(cx)^2 b+8\left(\int \frac{atan(cx)}{c^5x^5-3c^4i\,x^4-2c^3x^3-2c^2i\,x^2-3cx+i}dx\right)bci+6\left(\int \frac{x^2}{c^5x^5-3c^4i\,x^4-2c^3x^3-2c^2i\,x^2-3cx+i}dx\right)a\,c^3i-2}{}$$

input
```
int((a+b*atan(c*x))/(d+I*c*d*x)^3,x)
```

output
```
( - atan(c*x)**2*b + 8*int(atan(c*x)/(c**5*x**5 - 3*c**4*i*x**4 - 2*c**3*x
**3 - 2*c**2*i*x**2 - 3*c*x + i),x)*b*c*i + 6*int(x**2/(c**5*x**5 - 3*c**4
*i*x**4 - 2*c**3*x**3 - 2*c**2*i*x**2 - 3*c*x + i),x)*a*c**3*i - 2*int(( -
 atan(c*x)*x**3)/(c**5*x**5 - 3*c**4*i*x**4 - 2*c**3*x**3 - 2*c**2*i*x**2
- 3*c*x + i),x)*b*c**4 - 6*int((atan(c*x)*x)/(c**5*x**5 - 3*c**4*i*x**4 -
2*c**3*x**3 - 2*c**2*i*x**2 - 3*c*x + i),x)*b*c**2 + 6*int(1/(c**5*x**5 -
3*c**4*i*x**4 - 2*c**3*x**3 - 2*c**2*i*x**2 - 3*c*x + i),x)*a*c*i)/(6*c*d*
*3)
```

# 3.63 $\int \frac{a+b\arctan(cx)}{x(d+icdx)^3}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 195

$$\begin{aligned}\int \frac{a+b\arctan(cx)}{x(d+icdx)^3}\,dx &= \frac{ib}{8d^3(i-cx)^2}+\frac{5b}{8d^3(i-cx)}-\frac{5b\arctan(cx)}{8d^3}\\&\quad-\frac{a+b\arctan(cx)}{2d^3(i-cx)^2}+\frac{i(a+b\arctan(cx))}{d^3(i-cx)}+\frac{a\log(x)}{d^3}\\&\quad+\frac{(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{d^3}+\frac{ib\operatorname{PolyLog}(2,-icx)}{2d^3}\\&\quad-\frac{ib\operatorname{PolyLog}(2,icx)}{2d^3}+\frac{ib\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2d^3}\end{aligned}$$

output
```
1/8*I*b/d^3/(I-c*x)^2+5/8*b/d^3/(I-c*x)-5/8*b*arctan(c*x)/d^3-1/2*(a+b*arc
tan(c*x))/d^3/(I-c*x)^2+I*(a+b*arctan(c*x))/d^3/(I-c*x)+a*ln(x)/d^3+(a+b*a
rctan(c*x))*ln(2/(1+I*c*x))/d^3+1/2*I*b*polylog(2,-I*c*x)/d^3-1/2*I*b*poly
log(2,I*c*x)/d^3+1/2*I*b*polylog(2,1-2/(1+I*c*x))/d^3
```

## Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.83

$$\int \frac{a+b\arctan(cx)}{x(d+icdx)^3}\,dx$$
$$= \frac{\frac{5b}{i-cx}+\frac{ib}{(-i+cx)^2}-5b\arctan(cx)-\frac{4(a+b\arctan(cx))}{(-i+cx)^2}-\frac{8i(a+b\arctan(cx))}{-i+cx}+8a\log(x)+8(a+b\arctan(cx))\log\left(\right.}{8d^3}$$

input
```
Integrate[(a + b*ArcTan[c*x])/(x*(d + I*c*d*x)^3),x]
```

output
```
((5*b)/(I - c*x) + (I*b)/(-I + c*x)^2 - 5*b*ArcTan[c*x] - (4*(a + b*ArcTan
[c*x]))/(-I + c*x)^2 - ((8*I)*(a + b*ArcTan[c*x]))/(-I + c*x) + 8*a*Log[x]
 + 8*(a + b*ArcTan[c*x])*Log[(2*I)/(I - c*x)] + (4*I)*b*PolyLog[2, (-I)*c*
x] - (4*I)*b*PolyLog[2, I*c*x] + (4*I)*b*PolyLog[2, (I + c*x)/(-I + c*x)])
/(8*d^3)
```

## Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x(d+icdx)^3}\,dx$$
$$\Big\downarrow \text{5411}$$
$$\int \left(\frac{a+b\arctan(cx)}{d^3x}-\frac{c(a+b\arctan(cx))}{d^3(cx-i)}+\frac{ic(a+b\arctan(cx))}{d^3(cx-i)^2}+\frac{c(a+b\arctan(cx))}{d^3(cx-i)^3}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{i(a+b\arctan(cx))}{d^3(-cx+i)}-\frac{a+b\arctan(cx)}{2d^3(-cx+i)^2}+\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{d^3}+\frac{a\log(x)}{d^3}-\frac{5b\arctan(cx)}{8d^3}+\frac{ib\,\mathrm{PolyLog}(2,-icx)}{2d^3}-\frac{ib\,\mathrm{PolyLog}(2,icx)}{2d^3}+\frac{ib\,\mathrm{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{2d^3}+\frac{5b}{8d^3(-cx+i)}+\frac{ib}{8d^3(-cx+i)^2}$$

input
```
Int[(a + b*ArcTan[c*x])/(x*(d + I*c*d*x)^3),x]
```

output
```
((I/8)*b)/(d^3*(I - c*x)^2) + (5*b)/(8*d^3*(I - c*x)) - (5*b*ArcTan[c*x])/
(8*d^3) - (a + b*ArcTan[c*x])/(2*d^3*(I - c*x)^2) + (I*(a + b*ArcTan[c*x])
)/(d^3*(I - c*x)) + (a*Log[x])/d^3 + ((a + b*ArcTan[c*x])*Log[2/(1 + I*c*x
)])/d^3 + ((I/2)*b*PolyLog[2, (-I)*c*x])/d^3 - ((I/2)*b*PolyLog[2, I*c*x])
/d^3 + ((I/2)*b*PolyLog[2, 1 - 2/(1 + I*c*x)])/d^3
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

## Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.34

| method | result |
|---|---|
| derivativedivides | $-\frac{a}{2d^3(cx-i)^2}-\frac{ia}{d^3(cx-i)}-\frac{a\ln(c^2x^2+1)}{2d^3}-\frac{ia\arctan(cx)}{d^3}+\frac{a\ln(cx)}{d^3}+\frac{b\left(-\frac{\arctan(cx)}{2(cx-i)^2}-\frac{i\arctan(cx)}{cx-i}-\text{arctan}\right.}{}$ |
| default | $-\frac{a}{2d^3(cx-i)^2}-\frac{ia}{d^3(cx-i)}-\frac{a\ln(c^2x^2+1)}{2d^3}-\frac{ia\arctan(cx)}{d^3}+\frac{a\ln(cx)}{d^3}+\frac{b\left(-\frac{\arctan(cx)}{2(cx-i)^2}-\frac{i\arctan(cx)}{cx-i}-\text{arctan}\right.}{}$ |
| parts | $-\frac{a}{2d^3(-cx+i)^2}+\frac{ia}{d^3(-cx+i)}-\frac{a\ln(c^2x^2+1)}{2d^3}-\frac{ia\arctan(cx)}{d^3}+\frac{a\ln(x)}{d^3}+\frac{b\left(-\frac{\arctan(cx)}{2(cx-i)^2}-\frac{i\arctan(cx)}{cx-i}-\text{arct}\right.}{}$ |
| risch | $\frac{ib\ln(icx+1)^2}{4d^3}-\frac{ib\ln\left(\frac{1}{2}+\frac{icx}{2}\right)\ln(-icx+1)}{2d^3}+\frac{3ib\ln(-icx+1)}{16d^3(-icx-1)^2}-\frac{ib\ln(icx+1)}{4d^3(icx+1)^2}+\frac{ib\,\text{dilog}(icx+1)}{2d^3}-\frac{ib\ln(icx+1}{2d^3(icx+1}$ |

input
```
int((a+b*arctan(c*x))/x/(d+I*c*d*x)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/d^3/(c*x-I)^2-I*a/d^3/(c*x-I)-1/2*a/d^3*ln(c^2*x^2+1)-I*a/d^3*arcta
n(c*x)+a/d^3*ln(c*x)+b/d^3*(-1/2*arctan(c*x)/(c*x-I)^2-I*arctan(c*x)/(c*x-
I)-arctan(c*x)*ln(c*x-I)+ln(c*x)*arctan(c*x)+1/8*I/(c*x-I)^2-5/8*arctan(c*
x)-5/8/(c*x-I)+1/2*I*((ln(c*x)-ln(-I*c*x))*ln(-I*(-c*x+I))-dilog(-I*c*x))-
1/2*I*(dilog(-I*(c*x+I))+ln(c*x)*ln(-I*(c*x+I)))+1/2*I*(dilog(-1/2*I*(c*x+
I))+ln(c*x-I)*ln(-1/2*I*(c*x+I)))-1/4*I*ln(c*x-I)^2)
```

## Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.03

$$\int \frac{a+b\arctan(cx)}{x(d+icdx)^3}\,dx = -\frac{2\,(8i\,a+5\,b)cx+8\,(i\,bc^2x^2+2\,bcx-i\,b)\text{Li}_2\left(\frac{cx+i}{cx-i}+1\right)-16\,(ac^2x^2-2i\,acx-a)\log{(x)}-4\,(2\,bcx-3}{}$$

input
```
integrate((a+b*arctan(c*x))/x/(d+I*c*d*x)^3,x, algorithm="fricas")
```

output
```
-1/16*(2*(8*I*a + 5*b)*c*x + 8*(I*b*c^2*x^2 + 2*b*c*x - I*b)*dilog((c*x +
I)/(c*x - I) + 1) - 16*(a*c^2*x^2 - 2*I*a*c*x - a)*log(x) - 4*(2*b*c*x - 3
*I*b)*log(-(c*x + I)/(c*x - I)) + 5*(I*b*c^2*x^2 + 2*b*c*x - I*b)*log((c*x
 + I)/c) + ((16*a - 5*I*b)*c^2*x^2 + 2*(-16*I*a - 5*b)*c*x - 16*a + 5*I*b)
*log((c*x - I)/c) + 24*a - 12*I*b)/(c^2*d^3*x^2 - 2*I*c*d^3*x - d^3)
```

## Sympy [F(-2)]

Exception generated.

$$\int \frac{a + b\arctan(cx)}{x(d + icdx)^3}\, dx = \text{Exception raised: RecursionError}$$

input
```
integrate((a+b*atan(c*x))/x/(d+I*c*d*x)**3,x)
```

output
```
Exception raised: RecursionError >> maximum recursion depth exceeded in co
mparison
```

## Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 406 vs. $2(151) = 302$.

Time = 0.10 (sec) , antiderivative size = 406, normalized size of antiderivative = 2.08

$$\int \frac{a + b\arctan(cx)}{x(d + icdx)^3}\, dx$$
$$= \frac{2\,(-8i\,a - 5\,b)cx + 4\,(-i\,bc^2x^2 - 2\,bcx + i\,b)\arctan{(cx)}^2 - (i\,bc^2x^2 + 2\,bcx - i\,b)\log{(c^2x^2 + 1)}^2 - 4\,(bc}{}$$

input
```
integrate((a+b*arctan(c*x))/x/(d+I*c*d*x)^3,x, algorithm="maxima")
```

output
```
1/16*(2*(-8*I*a - 5*b)*c*x + 4*(-I*b*c^2*x^2 - 2*b*c*x + I*b)*arctan(c*x)^
2 - (I*b*c^2*x^2 + 2*b*c*x - I*b)*log(c^2*x^2 + 1)^2 - 4*(b*c^2*x^2 - 2*I*
b*c*x - b)*arctan(c*x)*log(1/4*c^2*x^2 + 1/4) + 16*(b*c^2*x^2 - 2*I*b*c*x
- b)*arctan(c*x)*log(c*x) - ((16*I*a + 5*b)*c^2*x^2 + 2*(16*a + 3*I*b)*c*x
 - 16*I*a + 19*b)*arctan(c*x) + 5*(b*c^2*x^2 - 2*I*b*c*x - b)*arctan2(c*x,
 -1) + 8*(-I*b*c^2*x^2 - 2*b*c*x + I*b)*dilog(I*c*x + 1) + 8*(I*b*c^2*x^2
+ 2*b*c*x - I*b)*dilog(1/2*I*c*x + 1/2) + 8*(I*b*c^2*x^2 + 2*b*c*x - I*b)*
dilog(-I*c*x + 1) - 2*(2*(pi*b + 2*a)*c^2*x^2 - 4*(I*pi*b + 2*I*a)*c*x - 2
*pi*b - (I*b*c^2*x^2 + 2*b*c*x - I*b)*log(1/4*c^2*x^2 + 1/4) - 4*a)*log(c^
2*x^2 + 1) + 16*(a*c^2*x^2 - 2*I*a*c*x - a)*log(x) - 24*a + 12*I*b)/(c^2*d
^3*x^2 - 2*I*c*d^3*x - d^3)
```

## Giac [F]

$$\int \frac{a+b\arctan(cx)}{x(d+icdx)^3}\,dx=\int \frac{b\arctan\left(cx\right)+a}{\left(i\,cdx+d\right)^3x}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x/(d+I*c*d*x)^3,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)/((I*c*d*x + d)^3*x), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{x(d+icdx)^3}\,dx=\int \frac{a+b\,\mathrm{atan}(c\,x)}{x\,\left(d+c\,d\,x\,1\mathrm{i}\right)^3}\,dx$$

input
```
int((a + b*atan(c*x))/(x*(d + c*d*x*1i)^3),x)
```

output
```
int((a + b*atan(c*x))/(x*(d + c*d*x*1i)^3), x)
```

## Reduce [F]

$$\int \frac{a+b\arctan(cx)}{x(d+icdx)^3}\,dx = \frac{-\left(\int \frac{atan(cx)}{c^3i\,x^4+3c^2x^3-3ci\,x^2-x}dx\right)b-\left(\int \frac{1}{c^3i\,x^4+3c^2x^3-3ci\,x^2-x}dx\right)a}{d^3}$$

input
```
int((a+b*atan(c*x))/x/(d+I*c*d*x)^3,x)
```

output
```
( - (int(atan(c*x)/(c**3*i*x**4 + 3*c**2*x**3 - 3*c*i*x**2 - x),x)*b + int
(1/(c**3*i*x**4 + 3*c**2*x**3 - 3*c*i*x**2 - x),x)*a))/d**3
```

# 3.64 $\int \frac{a+b\arctan(cx)}{x^2(d+icdx)^3}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 250

$$\begin{aligned}\int \frac{a+b\arctan(cx)}{x^2(d+icdx)^3}\,dx &= \frac{bc}{8d^3(i-cx)^2}-\frac{9ibc}{8d^3(i-cx)}+\frac{9ibc\arctan(cx)}{8d^3}\\ &\quad-\frac{a+b\arctan(cx)}{d^3x}+\frac{ic(a+b\arctan(cx))}{2d^3(i-cx)^2}\\ &\quad+\frac{2c(a+b\arctan(cx))}{d^3(i-cx)}-\frac{3iac\log(x)}{d^3}\\ &\quad+\frac{bc\log(x)}{d^3}-\frac{3ic(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{d^3}\\ &\quad-\frac{bc\log(1+c^2x^2)}{2d^3}+\frac{3bc\,\mathrm{PolyLog}(2,-icx)}{2d^3}\\ &\quad-\frac{3bc\,\mathrm{PolyLog}(2,icx)}{2d^3}+\frac{3bc\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2d^3}\end{aligned}$$

output

```
1/8*b*c/d^3/(I-c*x)^2-9/8*I*b*c/d^3/(I-c*x)+9/8*I*b*c*arctan(c*x)/d^3-(a+b
*arctan(c*x))/d^3/x+1/2*I*c*(a+b*arctan(c*x))/d^3/(I-c*x)^2+2*c*(a+b*arcta
n(c*x))/d^3/(I-c*x)-3*I*a*c*ln(x)/d^3+b*c*ln(x)/d^3-3*I*c*(a+b*arctan(c*x)
)*ln(2/(1+I*c*x))/d^3-1/2*b*c*ln(c^2*x^2+1)/d^3+3/2*b*c*polylog(2,-I*c*x)/
d^3-3/2*b*c*polylog(2,I*c*x)/d^3+3/2*b*c*polylog(2,1-2/(1+I*c*x))/d^3
```

## Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.91

$$\int \frac{a+b\arctan(cx)}{x^2(d+icdx)^3}\,dx = \frac{-8ibc\left(\frac{1}{i-cx}-\arctan(cx)\right)-\frac{8(a+b\arctan(cx))}{x}+\frac{4ic(a+b\arctan(cx))}{(-i+cx)^2}-\frac{16c(a+b\arctan(cx))}{-i+cx}+\frac{bc(2+icx+i(-i+cx)^2\arctan(c}{(-i+cx)^2}}{}$$

input
```
Integrate[(a + b*ArcTan[c*x])/(x^2*(d + I*c*d*x)^3),x]
```

output
```
((-8*I)*b*c*((I - c*x)^(-1) - ArcTan[c*x]) - (8*(a + b*ArcTan[c*x]))/x + (
(4*I)*c*(a + b*ArcTan[c*x]))/(-I + c*x)^2 - (16*c*(a + b*ArcTan[c*x]))/(-I
 + c*x) + (b*c*(2 + I*c*x + I*(-I + c*x)^2*ArcTan[c*x]))/(-I + c*x)^2 - (2
4*I)*a*c*Log[x] - (24*I)*c*(a + b*ArcTan[c*x])*Log[(2*I)/(I - c*x)] + 4*b*
c*(2*Log[x] - Log[1 + c^2*x^2]) + 12*b*c*PolyLog[2, (-I)*c*x] - 12*b*c*Pol
yLog[2, I*c*x] + 12*b*c*PolyLog[2, (I + c*x)/(-I + c*x)])/(8*d^3)
```

## Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x^2(d+icdx)^3}\,dx$$

$$\Big\downarrow \text{5411}$$

$$\int \left(\frac{3ic^2(a+b\arctan(cx))}{d^3(cx-i)}+\frac{2c^2(a+b\arctan(cx))}{d^3(cx-i)^2}-\frac{ic^2(a+b\arctan(cx))}{d^3(cx-i)^3}+\frac{a+b\arctan(cx)}{d^3x^2}-\frac{3ic(a+b\arct}{d^3x}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{2c(a+b\arctan(cx))}{d^3(-cx+i)}+\frac{ic(a+b\arctan(cx))}{2d^3(-cx+i)^2}-\frac{a+b\arctan(cx)}{d^3x}-\frac{3ic\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{d^3}-\frac{3iac\log(x)}{d^3}+\frac{9ibc\arctan(cx)}{8d^3}-\frac{bc\log\left(c^2x^2+1\right)}{2d^3}+\frac{3bc\,\text{PolyLog}(2,-icx)}{2d^3}-\frac{3bc\,\text{PolyLog}(2,icx)}{2d^3}+\frac{3bc\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{2d^3}-\frac{9ibc}{8d^3(-cx+i)}+\frac{bc}{8d^3(-cx+i)^2}+\frac{bc\log(x)}{d^3}$$

input
```
Int[(a + b*ArcTan[c*x])/(x^2*(d + I*c*d*x)^3),x]
```

output
```
(b*c)/(8*d^3*(I - c*x)^2) - (((9*I)/8)*b*c)/(d^3*(I - c*x)) + (((9*I)/8)*b
*c*ArcTan[c*x])/d^3 - (a + b*ArcTan[c*x])/(d^3*x) + ((I/2)*c*(a + b*ArcTan
[c*x]))/(d^3*(I - c*x)^2) + (2*c*(a + b*ArcTan[c*x]))/(d^3*(I - c*x)) - ((
3*I)*a*c*Log[x])/d^3 + (b*c*Log[x])/d^3 - ((3*I)*c*(a + b*ArcTan[c*x])*Log
[2/(1 + I*c*x)])/d^3 - (b*c*Log[1 + c^2*x^2])/(2*d^3) + (3*b*c*PolyLog[2,
(-I)*c*x])/(2*d^3) - (3*b*c*PolyLog[2, I*c*x])/(2*d^3) + (3*b*c*PolyLog[2,
 1 - 2/(1 + I*c*x)])/(2*d^3)
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

## Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.21

| method | result |
|---|---|
| derivativedivides | $c\left(\frac{ia}{2d^3(cx-i)^2}+\frac{3ia\ln(c^2x^2+1)}{2d^3}-\frac{3a\arctan(cx)}{d^3}-\frac{2a}{d^3(cx-i)}-\frac{a}{d^3cx}-\frac{3ia\ln(cx)}{d^3}+\frac{b\left(\frac{9i\arctan(cx)}{8}+3ia\cdots\right.}{\cdots}\right.$ |
| default | $c\left(\frac{ia}{2d^3(cx-i)^2}+\frac{3ia\ln(c^2x^2+1)}{2d^3}-\frac{3a\arctan(cx)}{d^3}-\frac{2a}{d^3(cx-i)}-\frac{a}{d^3cx}-\frac{3ia\ln(cx)}{d^3}+\frac{b\left(\frac{9i\arctan(cx)}{8}+3ia\cdots\right.}{\cdots}\right.$ |
| parts | $\frac{iac}{2d^3(-cx+i)^2}+\frac{3ica\ln(c^2x^2+1)}{2d^3}-\frac{3ca\arctan(cx)}{d^3}+\frac{2ac}{d^3(-cx+i)}-\frac{a}{d^3x}-\frac{3iac\ln(x)}{d^3}+\frac{bc\left(\frac{9i\arctan(cx)}{8}+3i\cdots\right.}{\cdots}$ |
| risch | $\frac{3ica\ln(c^2x^2+1)}{2d^3}+\frac{9icb\arctan(cx)}{16d^3}-\frac{3ica\ln(-icx)}{d^3}-\frac{a}{d^3x}-\frac{3ca\arctan(cx)}{d^3}+\frac{c^3b\ln(-icx+1)x^2}{16d^3(-icx-1)^2}+\frac{ic^2b\ln(\cdots}{2d^3(-\cdots}$ |

input
```
int((a+b*arctan(c*x))/x^2/(d+I*c*d*x)^3,x,method=_RETURNVERBOSE)
```

output
```
c*(1/2*I*a/d^3/(c*x-I)^2+3/2*I*a/d^3*ln(c^2*x^2+1)-3*a/d^3*arctan(c*x)-2*a
/d^3/(c*x-I)-a/d^3/c/x-3*I*a/d^3*ln(c*x)+b/d^3*(9/8*I*arctan(c*x)+3*I*arct
an(c*x)*ln(c*x-I)-2*arctan(c*x)/(c*x-I)-1/c/x*arctan(c*x)-3*I*arctan(c*x)*
ln(c*x)-1/2*ln(c^2*x^2+1)+1/2*I*arctan(c*x)/(c*x-I)^2+9/8*I/(c*x-I)+1/8/(c
*x-I)^2+ln(c*x)-3/2*dilog(-I*(c*x+I))-3/2*ln(c*x)*ln(-I*(c*x+I))+3/2*(ln(c
*x)-ln(-I*c*x))*ln(-I*(-c*x+I))-3/2*dilog(-I*c*x)+3/2*dilog(-1/2*I*(c*x+I)
)+3/2*ln(c*x-I)*ln(-1/2*I*(c*x+I))-3/4*ln(c*x-I)^2))
```

## Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.05

$$\int \frac{a+b\arctan(cx)}{x^2(d+icdx)^3}\,dx =$$
$$-\frac{6\,(8\,a-3i\,b)c^2x^2+4\,(-18i\,a-5\,b)cx+24\,(bc^3x^3-2i\,bc^2x^2-bcx)\mathrm{Li}_2\left(\frac{cx+i}{cx-i}+1\right)+16\,((3i\,a-b)c^3x^3\cdots}{\cdots}$$

input
```
integrate((a+b*arctan(c*x))/x^2/(d+I*c*d*x)^3,x, algorithm="fricas")
```

output
```
-1/16*(6*(8*a - 3*I*b)*c^2*x^2 + 4*(-18*I*a - 5*b)*c*x + 24*(b*c^3*x^3 - 2
*I*b*c^2*x^2 - b*c*x)*dilog((c*x + I)/(c*x - I) + 1) + 16*((3*I*a - b)*c^3
*x^3 + 2*(3*a + I*b)*c^2*x^2 + (-3*I*a + b)*c*x)*log(x) + 4*(6*I*b*c^2*x^2
 + 9*b*c*x - 2*I*b)*log(-(c*x + I)/(c*x - I)) + 17*(b*c^3*x^3 - 2*I*b*c^2*
x^2 - b*c*x)*log((c*x + I)/c) - ((48*I*a + b)*c^3*x^3 + 2*(48*a - I*b)*c^2
*x^2 + (-48*I*a - b)*c*x)*log((c*x - I)/c) - 16*a)/(c^2*d^3*x^3 - 2*I*c*d^
3*x^2 - d^3*x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{a + b\arctan(cx)}{x^2(d + icdx)^3} dx = \text{Timed out}$$

input
```
integrate((a+b*atan(c*x))/x**2/(d+I*c*d*x)**3,x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\arctan(cx)}{x^2(d + icdx)^3} dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arctan(c*x))/x^2/(d+I*c*d*x)^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

## Giac [F]

$$\int \frac{a+b\arctan(cx)}{x^2(d+icdx)^3}\,dx=\int \frac{b\arctan(cx)+a}{(i\,cdx+d)^3x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^2/(d+I*c*d*x)^3,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)/((I*c*d*x + d)^3*x^2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{x^2(d+icdx)^3}\,dx=\int \frac{a+b\,\mathrm{atan}(c\,x)}{x^2\,(d+c\,d\,x\,1\mathrm{i})^3}\,dx$$

input
```
int((a + b*atan(c*x))/(x^2*(d + c*d*x*1i)^3),x)
```

output
```
int((a + b*atan(c*x))/(x^2*(d + c*d*x*1i)^3), x)
```

## Reduce [F]

$$\int \frac{a+b\arctan(cx)}{x^2(d+icdx)^3}\,dx=\frac{-\left(\int \frac{atan(cx)}{c^3i\,x^5+3c^2x^4-3ci\,x^3-x^2}dx\right)b-\left(\int \frac{1}{c^3i\,x^5+3c^2x^4-3ci\,x^3-x^2}dx\right)a}{d^3}$$

input
```
int((a+b*atan(c*x))/x^2/(d+I*c*d*x)^3,x)
```

output
```
( - (int(atan(c*x)/(c**3*i*x**5 + 3*c**2*x**4 - 3*c*i*x**3 - x**2),x)*b +
int(1/(c**3*i*x**5 + 3*c**2*x**4 - 3*c*i*x**3 - x**2),x)*a))/d**3
```

# 3.65 $\int \frac{a+b\arctan(cx)}{x^3(d+icdx)^3}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 306

$$\begin{aligned}\int \frac{a+b\arctan(cx)}{x^3(d+icdx)^3}\,dx =& -\frac{bc}{2d^3x}-\frac{ibc^2}{8d^3(i-cx)^2}-\frac{13bc^2}{8d^3(i-cx)}+\frac{9bc^2\arctan(cx)}{8d^3}\\ &-\frac{a+b\arctan(cx)}{2d^3x^2}+\frac{3ic(a+b\arctan(cx))}{d^3x}\\ &+\frac{c^2(a+b\arctan(cx))}{2d^3(i-cx)^2}-\frac{3ic^2(a+b\arctan(cx))}{d^3(i-cx)}-\frac{6ac^2\log(x)}{d^3}\\ &-\frac{3ibc^2\log(x)}{d^3}-\frac{6c^2(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{d^3}\\ &+\frac{3ibc^2\log(1+c^2x^2)}{2d^3}-\frac{3ibc^2\,\mathrm{PolyLog}(2,-icx)}{d^3}\\ &+\frac{3ibc^2\,\mathrm{PolyLog}(2,icx)}{d^3}-\frac{3ibc^2\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{d^3}\end{aligned}$$

output

```
-1/2*b*c/d^3/x-1/8*I*b*c^2/d^3/(I-c*x)^2-13/8*b*c^2/d^3/(I-c*x)+9/8*b*c^2*
arctan(c*x)/d^3-1/2*(a+b*arctan(c*x))/d^3/x^2+3*I*c*(a+b*arctan(c*x))/d^3/
x+1/2*c^2*(a+b*arctan(c*x))/d^3/(I-c*x)^2-3*I*c^2*(a+b*arctan(c*x))/d^3/(I
-c*x)-6*a*c^2*ln(x)/d^3-3*I*b*c^2*ln(x)/d^3-6*c^2*(a+b*arctan(c*x))*ln(2/(
1+I*c*x))/d^3+3/2*I*b*c^2*ln(c^2*x^2+1)/d^3-3*I*b*c^2*polylog(2,-I*c*x)/d^
3+3*I*b*c^2*polylog(2,I*c*x)/d^3-3*I*b*c^2*polylog(2,1-2/(1+I*c*x))/d^3
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.35 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.93

$$\int \frac{a+b\arctan(cx)}{x^3(d+icdx)^3}\,dx =$$
$$-\frac{12bc^2\left(\frac{1}{i-cx}-\arctan(cx)\right)+\frac{4(a+b\arctan(cx))}{x^2}-\frac{24ic(a+b\arctan(cx))}{x}-\frac{4c^2(a+b\arctan(cx))}{(-i+cx)^2}-\frac{24ic^2(a+b\arctan(cx))}{-i+cx}-$$

input
```
Integrate[(a + b*ArcTan[c*x])/(x^3*(d + I*c*d*x)^3),x]
```

output
```
-1/8*(12*b*c^2*((I - c*x)^(-1) - ArcTan[c*x]) + (4*(a + b*ArcTan[c*x]))/x^
2 - ((24*I)*c*(a + b*ArcTan[c*x]))/x - (4*c^2*(a + b*ArcTan[c*x]))/(-I + c
*x)^2 - ((24*I)*c^2*(a + b*ArcTan[c*x]))/(-I + c*x) - (b*c^2*(-2*I + c*x +
 (-I + c*x)^2*ArcTan[c*x]))/(-I + c*x)^2 + (4*b*c*Hypergeometric2F1[-1/2,
1, 1/2, -(c^2*x^2)])/x + 48*a*c^2*Log[x] + 48*c^2*(a + b*ArcTan[c*x])*Log[
(2*I)/(I - c*x)] + (12*I)*b*c^2*(2*Log[x] - Log[1 + c^2*x^2]) + (24*I)*b*c
^2*PolyLog[2, (-I)*c*x] - (24*I)*b*c^2*PolyLog[2, I*c*x] + (24*I)*b*c^2*Po
lyLog[2, (I + c*x)/(-I + c*x)])/d^3
```

## Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x^3(d+icdx)^3}\,dx$$
$$\Big\downarrow \ 5411$$
$$\int \left(\frac{6c^3(a+b\arctan(cx))}{d^3(cx-i)}-\frac{3ic^3(a+b\arctan(cx))}{d^3(cx-i)^2}-\frac{c^3(a+b\arctan(cx))}{d^3(cx-i)^3}-\frac{6c^2(a+b\arctan(cx))}{d^3x}+\frac{a+b\arc}{d^3x}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3ic^2(a+b\arctan(cx))}{d^3(-cx+i)}+\frac{c^2(a+b\arctan(cx))}{2d^3(-cx+i)^2}-\frac{6c^2\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{d^3}-\frac{a+b\arctan(cx)}{2d^3x^2}+\frac{3ic(a+b\arctan(cx))}{d^3x}-\frac{6ac^2\log(x)}{d^3}+\frac{9bc^2\arctan(cx)}{8d^3}-\frac{3ibc^2\operatorname{PolyLog}(2,-icx)}{d^3}+\frac{3ibc^2\operatorname{PolyLog}(2,icx)}{d^3}-\frac{3ibc^2\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{d^3}+\frac{3ibc^2\log\left(c^2x^2+1\right)}{2d^3}-\frac{13bc^2}{8d^3(-cx+i)}-\frac{ibc^2}{8d^3(-cx+i)^2}-\frac{3ibc^2\log(x)}{d^3}-\frac{bc}{2d^3x}$$

input
```
Int[(a + b*ArcTan[c*x])/(x^3*(d + I*c*d*x)^3),x]
```

output
```
-1/2*(b*c)/(d^3*x) - ((I/8)*b*c^2)/(d^3*(I - c*x)^2) - (13*b*c^2)/(8*d^3*(
I - c*x)) + (9*b*c^2*ArcTan[c*x])/(8*d^3) - (a + b*ArcTan[c*x])/(2*d^3*x^2
) + ((3*I)*c*(a + b*ArcTan[c*x]))/(d^3*x) + (c^2*(a + b*ArcTan[c*x]))/(2*d
^3*(I - c*x)^2) - ((3*I)*c^2*(a + b*ArcTan[c*x]))/(d^3*(I - c*x)) - (6*a*c
^2*Log[x])/d^3 - ((3*I)*b*c^2*Log[x])/d^3 - (6*c^2*(a + b*ArcTan[c*x])*Log
[2/(1 + I*c*x)])/d^3 + (((3*I)/2)*b*c^2*Log[1 + c^2*x^2])/d^3 - ((3*I)*b*c
^2*PolyLog[2, (-I)*c*x])/d^3 + ((3*I)*b*c^2*PolyLog[2, I*c*x])/d^3 - ((3*I
)*b*c^2*PolyLog[2, 1 - 2/(1 + I*c*x)])/d^3
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

## Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.13

| method | result |
|---|---|
| derivativedivides | $c^2\left(-\frac{a}{2d^3c^2x^2}+\frac{3ia}{d^3cx}-\frac{6a\ln(cx)}{d^3}+\frac{3ia}{d^3(cx-i)}+\frac{a}{2d^3(cx-i)^2}+\frac{3a\ln(c^2x^2+1)}{d^3}+\frac{6ia\arctan(cx)}{d^3}+\frac{b\left(-\right.}{}\right.$ |
| default | $c^2\left(-\frac{a}{2d^3c^2x^2}+\frac{3ia}{d^3cx}-\frac{6a\ln(cx)}{d^3}+\frac{3ia}{d^3(cx-i)}+\frac{a}{2d^3(cx-i)^2}+\frac{3a\ln(c^2x^2+1)}{d^3}+\frac{6ia\arctan(cx)}{d^3}+\frac{b\left(-\right.}{}\right.$ |
| parts | $-\frac{3ia\,c^2}{d^3(-cx+i)}+\frac{a\,c^2}{2d^3(-cx+i)^2}+\frac{6ic^2a\arctan(cx)}{d^3}+\frac{3c^2a\ln(c^2x^2+1)}{d^3}-\frac{a}{2d^3x^2}+\frac{3ica}{d^3x}-\frac{6a\,c^2\ln(x)}{d^3}+\frac{b\,c^2}{}$ |
| risch | $-\frac{a}{2d^3x^2}+\frac{3c^3b\ln(-icx+1)x}{4d^3(-icx-1)}+\frac{3ic^2b\ln(-icx+1)}{4d^3(-icx-1)}+\frac{3ic^2b\ln\left(\frac{1}{2}+\frac{icx}{2}\right)\ln(-icx+1)}{d^3}-\frac{3ic^2b\ln\left(\frac{1}{2}+\frac{icx}{2}\right)\ln\left(\frac{1}{2}-\frac{ic}{2}\right.}{d^3}$ |

input
```
int((a+b*arctan(c*x))/x^3/(d+I*c*d*x)^3,x,method=_RETURNVERBOSE)
```

output
```
c^2*(-1/2*a/d^3/c^2/x^2+3*I*a/d^3/c/x-6*a/d^3*ln(c*x)+3*I*a/d^3/(c*x-I)+1/
2*a/d^3/(c*x-I)^2+3*a/d^3*ln(c^2*x^2+1)+6*I*a/d^3*arctan(c*x)+b/d^3*(-1/2/
c^2/x^2*arctan(c*x)-3*I*(dilog(-1/2*I*(c*x+I))+ln(c*x-I)*ln(-1/2*I*(c*x+I)
))-6*ln(c*x)*arctan(c*x)-1/8*I/(c*x-I)^2+1/2*arctan(c*x)/(c*x-I)^2+6*arcta
n(c*x)*ln(c*x-I)-3*I*ln(c*x)-1/2/c/x+3/2*I*ln(c*x-I)^2+9/8*arctan(c*x)+3*I
*arctan(c*x)/c/x+13/8/(c*x-I)+3*I*(dilog(-I*(c*x+I))+ln(c*x)*ln(-I*(c*x+I)
))+3/2*I*ln(c^2*x^2+1)+3*I*arctan(c*x)/(c*x-I)-3*I*((ln(c*x)-ln(-I*c*x))*l
n(-I*(-c*x+I))-dilog(-I*c*x))))
```

## Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.02

$$\int \frac{a+b\arctan(cx)}{x^3(d+icdx)^3}\,dx =$$
$$-\frac{6\,(-16i\,a-3\,b)c^3x^3-12\,(12\,a-i\,b)c^2x^2+8\,(4i\,a-b)cx+48\,(-i\,bc^4x^4-2\,bc^3x^3+i\,bc^2x^2)\mathrm{Li}_2\left(\frac{cx+i}{cx-i}\right.}{}$$

input
```
integrate((a+b*arctan(c*x))/x^3/(d+I*c*d*x)^3,x, algorithm="fricas")
```

output
```
-1/16*(6*(-16*I*a - 3*b)*c^3*x^3 - 12*(12*a - I*b)*c^2*x^2 + 8*(4*I*a - b)
*c*x + 48*(-I*b*c^4*x^4 - 2*b*c^3*x^3 + I*b*c^2*x^2)*dilog((c*x + I)/(c*x
- I) + 1) + 48*((2*a + I*b)*c^4*x^4 + 2*(-2*I*a + b)*c^3*x^3 - (2*a + I*b)
*c^2*x^2)*log(x) + 4*(12*b*c^3*x^3 - 18*I*b*c^2*x^2 - 4*b*c*x - I*b)*log(-
(c*x + I)/(c*x - I)) + 33*(-I*b*c^4*x^4 - 2*b*c^3*x^3 + I*b*c^2*x^2)*log((
c*x + I)/c) - 3*((32*a + 5*I*b)*c^4*x^4 - 2*(32*I*a - 5*b)*c^3*x^3 - (32*a
 + 5*I*b)*c^2*x^2)*log((c*x - I)/c) - 8*a)/(c^2*d^3*x^4 - 2*I*c*d^3*x^3 -
d^3*x^2)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{a + b \arctan(cx)}{x^3(d + icdx)^3} \, dx = \text{Timed out}$$

input
```
integrate((a+b*atan(c*x))/x**3/(d+I*c*d*x)**3,x)
```

output
```
Timed out
```

## Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 590 vs. $2(253) = 506$.

Time = 0.14 (sec) , antiderivative size = 590, normalized size of antiderivative = 1.93

$$\int \frac{a + b \arctan(cx)}{x^3(d + icdx)^3} \, dx = \text{Too large to display}$$

input
```
integrate((a+b*arctan(c*x))/x^3/(d+I*c*d*x)^3,x, algorithm="maxima")
```

output
```
-1/16*(33*b*c^4*x^4*arctan2(1, c*x) + 6*(b*(-11*I*arctan2(1, c*x) - 3) - 1
6*I*a)*c^3*x^3 - 3*(b*(11*arctan2(1, c*x) - 4*I) + 48*a)*c^2*x^2 + 8*(4*I*
a - b)*c*x + 24*(-I*b*c^4*x^4 - 2*b*c^3*x^3 + I*b*c^2*x^2)*arctan(c*x)^2 +
 6*(-I*b*c^4*x^4 - 2*b*c^3*x^3 + I*b*c^2*x^2)*log(c^2*x^2 + 1)^2 - 24*(b*c
^4*x^4 - 2*I*b*c^3*x^3 - b*c^2*x^2)*arctan(c*x)*log(1/4*c^2*x^2 + 1/4) + 9
6*(b*c^4*x^4 - 2*I*b*c^3*x^3 - b*c^2*x^2)*arctan(c*x)*log(c*x) + (3*(-32*I
*a + 5*b)*c^4*x^4 - 6*(32*a + 21*I*b)*c^3*x^3 + 3*(32*I*a - 53*b)*c^2*x^2
+ 32*I*b*c*x - 8*b)*arctan(c*x) + 48*(-I*b*c^4*x^4 - 2*b*c^3*x^3 + I*b*c^2
*x^2)*dilog(I*c*x + 1) + 48*(I*b*c^4*x^4 + 2*b*c^3*x^3 - I*b*c^2*x^2)*dilo
g(1/2*I*c*x + 1/2) + 48*(I*b*c^4*x^4 + 2*b*c^3*x^3 - I*b*c^2*x^2)*dilog(-I
*c*x + 1) - 12*(2*((pi + I)*b + 2*a)*c^4*x^4 - 4*((I*pi - 1)*b + 2*I*a)*c^
3*x^3 - 2*((pi + I)*b + 2*a)*c^2*x^2 - (I*b*c^4*x^4 + 2*b*c^3*x^3 - I*b*c^
2*x^2)*log(1/4*c^2*x^2 + 1/4))*log(c^2*x^2 + 1) + 48*((2*a + I*b)*c^4*x^4
+ 2*(-2*I*a + b)*c^3*x^3 - (2*a + I*b)*c^2*x^2)*log(x) - 8*a)/(c^2*d^3*x^4
 - 2*I*c*d^3*x^3 - d^3*x^2)
```

## Giac [F]

$$\int \frac{a+b\arctan(cx)}{x^3(d+icdx)^3}\,dx = \int \frac{b\arctan(cx)+a}{(i\,cdx+d)^3x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^3/(d+I*c*d*x)^3,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)/((I*c*d*x + d)^3*x^3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{x^3(d+icdx)^3}\,dx = \int \frac{a+b\,\mathrm{atan}(c\,x)}{x^3\,(d+c\,d\,x\,1\mathrm{i})^3}\,dx$$

input
```
int((a + b*atan(c*x))/(x^3*(d + c*d*x*1i)^3),x)
```

output
```
int((a + b*atan(c*x))/(x^3*(d + c*d*x*1i)^3), x)
```

## Reduce [F]

$$\int \frac{a+b\arctan(cx)}{x^3(d+icdx)^3}\,dx = \frac{-\left(\int \frac{atan(cx)}{c^3i\,x^6+3c^2x^5-3ci\,x^4-x^3}dx\right)b-\left(\int \frac{1}{c^3i\,x^6+3c^2x^5-3ci\,x^4-x^3}dx\right)a}{d^3}$$

input
```
int((a+b*atan(c*x))/x^3/(d+I*c*d*x)^3,x)
```

output
```
( - (int(atan(c*x)/(c**3*i*x**6 + 3*c**2*x**5 - 3*c*i*x**4 - x**3),x)*b +
int(1/(c**3*i*x**6 + 3*c**2*x**5 - 3*c*i*x**4 - x**3),x)*a))/d**3
```

# 3.66 $\int \frac{a+b\arctan(cx)}{(1+icx)^4}\,dx$



## Optimal result

Integrand size = 19, antiderivative size = 100

$$\int \frac{a+b\arctan(cx)}{(1+icx)^4}\,dx = -\frac{ib}{18c(i-cx)^3}-\frac{b}{24c(i-cx)^2}+\frac{ib}{24c(i-cx)} -\frac{ib\arctan(cx)}{24c}+\frac{i(a+b\arctan(cx))}{3c(1+icx)^3}$$

output
```
-1/18*I*b/c/(I-c*x)^3-1/24*b/c/(I-c*x)^2+1/24*I*b/c/(I-c*x)-1/24*I*b*arcta
n(c*x)/c+1/3*I*(a+b*arctan(c*x))/c/(1+I*c*x)^3
```

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.73

$$\int \frac{a+b\arctan(cx)}{(1+icx)^4}\,dx = \frac{-24a+b(10i-9cx-3ic^2x^2)+3b(-7+3icx-3c^2x^2-ic^3x^3)\arctan(cx)}{72c(-i+cx)^3}$$

input
```
Integrate[(a + b*ArcTan[c*x])/(1 + I*c*x)^4,x]
```

output
```
(-24*a + b*(10*I - 9*c*x - (3*I)*c^2*x^2) + 3*b*(-7 + (3*I)*c*x - 3*c^2*x^
2 - I*c^3*x^3)*ArcTan[c*x])/(72*c*(-I + c*x)^3)
```

## Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {5387, 456, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{(1+icx)^4}dx$$

$$\Big\downarrow \text{5387}$$

$$\frac{i(a+b\arctan(cx))}{3c(1+icx)^3}-\frac{1}{3}ib\int \frac{1}{(icx+1)^3(c^2x^2+1)}dx$$

$$\Big\downarrow \text{456}$$

$$\frac{i(a+b\arctan(cx))}{3c(1+icx)^3}-\frac{1}{3}ib\int \frac{1}{(1-icx)(icx+1)^4}dx$$

$$\Big\downarrow \text{54}$$

$$\frac{i(a+b\arctan(cx))}{3c(1+icx)^3}-\frac{1}{3}ib\int \left(-\frac{1}{8(cx-i)^2}+\frac{i}{4(cx-i)^3}+\frac{1}{2(cx-i)^4}+\frac{1}{8(c^2x^2+1)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{i(a+b\arctan(cx))}{3c(1+icx)^3}-\frac{1}{3}ib\left(\frac{\arctan(cx)}{8c}-\frac{1}{8c(-cx+i)}-\frac{i}{8c(-cx+i)^2}+\frac{1}{6c(-cx+i)^3}\right)$$

input
```
Int[(a + b*ArcTan[c*x])/(1 + I*c*x)^4,x]
```

output
```
((I/3)*(a + b*ArcTan[c*x]))/(c*(1 + I*c*x)^3) - (I/3)*b*(1/(6*c*(I - c*x)^
3) - (I/8)/(c*(I - c*x)^2) - 1/(8*c*(I - c*x)) + ArcTan[c*x]/(8*c))
```

### Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 456
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}, x] &&
EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&  !Integ
erQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5387
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_))^(q_.), x_Symbol]
 :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])/(e*(q + 1))), x] - Simp[b*(
c/(e*(q + 1)))   Int[(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x] /; FreeQ[{a, b
, c, d, e, q}, x] && NeQ[q, -1]
```

## Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.79

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{ia}{3(icx+1)^{3}}+\frac{ib\arctan(cx)}{3(icx+1)^{3}}-\frac{ib\arctan(cx)}{24}-\frac{b}{24(cx-i)^{2}}+\frac{ib}{18(cx-i)^{3}}-\frac{ib}{24(cx-i)}}{c}$ |
| default | $\frac{\frac{ia}{3(icx+1)^{3}}+\frac{ib\arctan(cx)}{3(icx+1)^{3}}-\frac{ib\arctan(cx)}{24}-\frac{b}{24(cx-i)^{2}}+\frac{ib}{18(cx-i)^{3}}-\frac{ib}{24(cx-i)}}{c}$ |
| parts | $\frac{ia}{3(icx+1)^{3}c}+\frac{ib\arctan(cx)}{3c(icx+1)^{3}}-\frac{ib\arctan(cx)}{24c}-\frac{b}{24c(cx-i)^{2}}+\frac{ib}{18c(cx-i)^{3}}-\frac{ib}{24c(cx-i)}$ |
| orering | $\frac{(-11c^{5}x^{5}+18ic^{4}x^{4}-26c^{3}x^{3}+68ic^{2}x^{2}-15cx+50i)(a+b\arctan(cx))}{72c(icx+1)^{4}}+\frac{(-11c^{2}x^{2}+32icx+29)(c^{2}x^{2}+1)^{2}\left(\frac{b\cdots}{(c^{2}x^{2}+1)\cdots}\cdots\right.}{288c^{2}\cdots}$ |
| risch | $\frac{ib\ln(icx+1)}{6c(cx-i)^{3}}-\frac{24ib\ln(-icx+1)+3bc^{3}\ln(cx-i)x^{3}-3bc^{3}\ln(-cx-i)x^{3}-9i\ln(cx-i)bc^{2}x^{2}+9i\ln(-cx-i)bc^{2}x^{2}-9\ln\cdots}{144(cx-i)^{3}c\cdots}$ |

input
```
int((a+b*arctan(c*x))/(1+I*c*x)^4,x,method=_RETURNVERBOSE)
```

output
```
1/c*(1/3*I*a/(1+I*c*x)^3+1/3*I*b/(1+I*c*x)^3*arctan(c*x)-1/24*I*b*arctan(c
*x)-1/24*b/(c*x-I)^2+1/18*I*b/(c*x-I)^3-1/24*I*b/(c*x-I))
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.93

$$\int \frac{a+b\arctan(cx)}{(1+icx)^4}\,dx = \frac{-6i\,bc^2x^2-18\,bcx+3\,(bc^3x^3-3i\,bc^2x^2-3\,bcx-7i\,b)\log\left(-\frac{cx+i}{cx-i}\right)-48\,a+20i\,b}{144\,(c^4x^3-3i\,c^3x^2-3\,c^2x+i\,c)}$$

input
```
integrate((a+b*arctan(c*x))/(1+I*c*x)^4,x, algorithm="fricas")
```

output
```
1/144*(-6*I*b*c^2*x^2 - 18*b*c*x + 3*(b*c^3*x^3 - 3*I*b*c^2*x^2 - 3*b*c*x
- 7*I*b)*log(-(c*x + I)/(c*x - I)) - 48*a + 20*I*b)/(c^4*x^3 - 3*I*c^3*x^2
 - 3*c^2*x + I*c)
```

## Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(70) = 140.

Time = 1.51 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.68

$$\int \frac{a+b\arctan(cx)}{(1+icx)^4}\,dx = -\frac{ib\log(-icx+1)}{6c^4x^3-18ic^3x^2-18c^2x+6ic} + \frac{ib\log(icx+1)}{6c^4x^3-18ic^3x^2-18c^2x+6ic} + \frac{b\left(-\frac{\log\left(bx-\frac{ib}{c}\right)}{48}+\frac{\log\left(bx+\frac{ib}{c}\right)}{48}\right)}{c} + \frac{-24a-3ibc^2x^2-9bcx+10ib}{72c^4x^3-216ic^3x^2-216c^2x+72ic}$$

input
```
integrate((a+b*atan(c*x))/(1+I*c*x)**4,x)
```

output
```
-I*b*log(-I*c*x + 1)/(6*c**4*x**3 - 18*I*c**3*x**2 - 18*c**2*x + 6*I*c) +
I*b*log(I*c*x + 1)/(6*c**4*x**3 - 18*I*c**3*x**2 - 18*c**2*x + 6*I*c) + b*
(-log(b*x - I*b/c)/48 + log(b*x + I*b/c)/48)/c + (-24*a - 3*I*b*c**2*x**2
- 9*b*c*x + 10*I*b)/(72*c**4*x**3 - 216*I*c**3*x**2 - 216*c**2*x + 72*I*c)
```

## Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.83

$$\int \frac{a+b\arctan(cx)}{(1+icx)^4}\,dx = -\frac{3i\,bc^2x^2+9\,bcx-3\left(-i\,bc^3x^3-3\,bc^2x^2+3i\,bcx-7\,b\right)\arctan\left(cx\right)+24\,a-10i\,b}{72\left(c^4x^3-3i\,c^3x^2-3\,c^2x+i\,c\right)}$$

input
```
integrate((a+b*arctan(c*x))/(1+I*c*x)^4,x, algorithm="maxima")
```

output
```
-1/72*(3*I*b*c^2*x^2 + 9*b*c*x - 3*(-I*b*c^3*x^3 - 3*b*c^2*x^2 + 3*I*b*c*x
 - 7*b)*arctan(c*x) + 24*a - 10*I*b)/(c^4*x^3 - 3*I*c^3*x^2 - 3*c^2*x + I*
c)
```

## Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 157 vs. 2(74) = 148.

Time = 0.12 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.57

$$\int \frac{a+b\arctan(cx)}{(1+icx)^4}\,dx = \frac{3\,bc^3x^3\log\left(cx+i\right)-3\,bc^3x^3\log\left(cx-i\right)-9i\,bc^2x^2\log\left(cx+i\right)+9i\,bc^2x^2\log\left(cx-i\right)-6i\,bc^2x^2-9\,bcx}{144\left(c^4x^3-3\right.}$$

input
```
integrate((a+b*arctan(c*x))/(1+I*c*x)^4,x, algorithm="giac")
```

output
```
1/144*(3*b*c^3*x^3*log(c*x + I) - 3*b*c^3*x^3*log(c*x - I) - 9*I*b*c^2*x^2
*log(c*x + I) + 9*I*b*c^2*x^2*log(c*x - I) - 6*I*b*c^2*x^2 - 9*b*c*x*log(c
*x + I) + 9*b*c*x*log(c*x - I) - 18*b*c*x - 48*b*arctan(c*x) + 3*I*b*log(c
*x + I) - 3*I*b*log(c*x - I) - 48*a + 20*I*b)/(c^4*x^3 - 3*I*c^3*x^2 - 3*c
^2*x + I*c)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{(1+icx)^4}\,dx=\int \frac{a+b\,\mathrm{atan}(c\,x)}{(1+c\,x\,1\mathrm{i})^4}\,dx$$

input
```
int((a + b*atan(c*x))/(c*x*1i + 1)^4,x)
```

output
```
int((a + b*atan(c*x))/(c*x*1i + 1)^4, x)
```

## Reduce [F]

$$\int \frac{a+b\arctan(cx)}{(1+icx)^4}\,dx=\left(\int \frac{atan(cx)}{c^4x^4-4c^3i\,x^3-6c^2x^2+4cix+1}dx\right)b+\left(\int \frac{1}{c^4x^4-4c^3i\,x^3-6c^2x^2+4cix+1}dx\right)a$$

input
```
int((a+b*atan(c*x))/(1+I*c*x)^4,x)
```

output
```
int(atan(c*x)/(c**4*x**4 - 4*c**3*i*x**3 - 6*c**2*x**2 + 4*c*i*x + 1),x)*b
 + int(1/(c**4*x**4 - 4*c**3*i*x**3 - 6*c**2*x**2 + 4*c*i*x + 1),x)*a
```

# 3.67 $\int \frac{\arctan(ax)}{cx+iacx^2} dx$



## Optimal result

Integrand size = 20, antiderivative size = 49

$$\int \frac{\arctan(ax)}{cx+iacx^2} dx = \frac{\arctan(ax)\log\left(2-\frac{2}{1+iax}\right)}{c}+\frac{i\,\text{PolyLog}\left(2,-1+\frac{2}{1+iax}\right)}{2c}$$

output
```
arctan(a*x)*ln(2-2/(1+I*a*x))/c+1/2*I*polylog(2,-1+2/(1+I*a*x))/c
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.80

$$\int \frac{\arctan(ax)}{cx+iacx^2} dx = \frac{\arctan(ax)\log\left(\frac{2i}{i-ax}\right)}{c}+\frac{i\,\text{PolyLog}(2,-iax)}{2c} - \frac{i\,\text{PolyLog}(2,iax)}{2c}+\frac{i\,\text{PolyLog}\left(2,-\frac{i+ax}{i-ax}\right)}{2c}$$

input
```
Integrate[ArcTan[a*x]/(c*x + I*a*c*x^2),x]
```

output
```
(ArcTan[a*x]*Log[(2*I)/(I - a*x)])/c + ((I/2)*PolyLog[2, (-I)*a*x])/c - ((
I/2)*PolyLog[2, I*a*x])/c + ((I/2)*PolyLog[2, -((I + a*x)/(I - a*x))])/c
```

## Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2026, 5403, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)}{cx + iacx^2} \, dx$$

$$\Big\downarrow \text{ 2026}$$

$$\int \frac{\arctan(ax)}{x(c + iacx)} dx$$

$$\Big\downarrow \text{ 5403}$$

$$\frac{\arctan(ax) \log\left(2 - \frac{2}{1+iax}\right)}{c} - \frac{a \int \frac{\log\left(2 - \frac{2}{iax+1}\right)}{a^2x^2+1} dx}{c}$$

$$\Big\downarrow \text{ 2897}$$

$$\frac{\arctan(ax) \log\left(2 - \frac{2}{1+iax}\right)}{c} + \frac{i \operatorname{PolyLog}\left(2, \frac{2}{iax+1} - 1\right)}{2c}$$

input
```
Int[ArcTan[a*x]/(c*x + I*a*c*x^2),x]
```

output
```
(ArcTan[a*x]*Log[2 - 2/(1 + I*a*x)])/c + ((I/2)*PolyLog[2, -1 + 2/(1 + I*a
*x)])/c
```

### Defintions of rubi rules used

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

## Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(44) = 88$.

Time = 0.33 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.12

| method | result |
|---|---|
| risch | $\frac{i\ln(iax+1)^2}{4c}+\frac{i\,\mathrm{dilog}(iax+1)}{2c}-\frac{i\ln(-iax+1)\ln\left(\frac{1}{2}+\frac{iax}{2}\right)}{2c}+\frac{i\ln\left(\frac{1}{2}-\frac{iax}{2}\right)\ln\left(\frac{1}{2}+\frac{iax}{2}\right)}{2c}-\frac{i\,\mathrm{dilog}(-iax+1)}{2c}+\frac{i}{}$ |
| derivativedivides | $\frac{\frac{a\arctan(ax)\ln(ax)}{c}-\frac{a\arctan(ax)\ln(ax-i)}{c}-\frac{a\left(-\frac{i\ln(ax)\ln(iax+1)}{2}+\frac{i\ln(ax)\ln(-iax+1)}{2}-\frac{i\,\mathrm{dilog}(iax+1)}{2}+\frac{i\,\mathrm{dilog}(-iax+1)}{2}+\frac{i}{}\right.}{c}}{a}$ |
| default | $\frac{\frac{a\arctan(ax)\ln(ax)}{c}-\frac{a\arctan(ax)\ln(ax-i)}{c}-\frac{a\left(-\frac{i\ln(ax)\ln(iax+1)}{2}+\frac{i\ln(ax)\ln(-iax+1)}{2}-\frac{i\,\mathrm{dilog}(iax+1)}{2}+\frac{i\,\mathrm{dilog}(-iax+1)}{2}+\frac{i}{}\right.}{c}}{a}$ |
| parts | $-\frac{\arctan(ax)\ln(-ax+i)}{c}+\frac{\arctan(ax)\ln(x)}{c}-\frac{a\left(-\frac{i\ln(x)(-\ln(-iax+1)+\ln(iax+1))}{2a}-\frac{i(\mathrm{dilog}(iax+1)-\mathrm{dilog}(-iax+1))}{2a}\right.}{}$ |

input
```
int(arctan(a*x)/(c*x+I*a*c*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/4*I/c*ln(1+I*a*x)^2+1/2*I/c*dilog(1+I*a*x)-1/2*I/c*ln(1-I*a*x)*ln(1/2+1/
2*I*a*x)+1/2*I/c*ln(1/2-1/2*I*a*x)*ln(1/2+1/2*I*a*x)-1/2*I/c*dilog(1-I*a*x
)+1/2*I/c*dilog(1/2-1/2*I*a*x)
```

## Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.43

$$\int \frac{\arctan(ax)}{cx + iacx^2}\,dx = -\frac{i\operatorname{Li}_2\left(\frac{ax+i}{ax-i} + 1\right)}{2\,c}$$

input
```
integrate(arctan(a*x)/(c*x+I*a*c*x^2),x, algorithm="fricas")
```

output
```
-1/2*I*dilog((a*x + I)/(a*x - I) + 1)/c
```

## Sympy [F]

$$\int \frac{\arctan(ax)}{cx + iacx^2}\,dx = -\frac{i\int \frac{\operatorname{atan}(ax)}{ax^2 - ix}\,dx}{c}$$

input
```
integrate(atan(a*x)/(c*x+I*a*c*x**2),x)
```

output
```
-I*Integral(atan(a*x)/(a*x**2 - I*x), x)/c
```

## Maxima [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 126 vs. 2(40) = 80.

Time = 0.11 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.57

$$\int \frac{\arctan(ax)}{cx + iacx^2}\,dx$$
$$= \frac{1}{4}a\left(-\frac{i\log(i\,ax+1)^2}{ac} + \frac{2i\left(\log(i\,ax+1)\log\left(-\frac{1}{2}i\,ax+\frac{1}{2}\right) + \operatorname{Li}_2\left(\frac{1}{2}i\,ax+\frac{1}{2}\right)\right)}{ac} + \frac{2i\left(\log(i\,ax+1)\log\left(\right.\right.}{ac}\right.$$
$$- \left(\frac{\log(i\,ax+1)}{c} - \frac{\log(x)}{c}\right)\arctan(ax)$$

input
```
integrate(arctan(a*x)/(c*x+I*a*c*x^2),x, algorithm="maxima")
```

output
```
1/4*a*(-I*log(I*a*x + 1)^2/(a*c) + 2*I*(log(I*a*x + 1)*log(-1/2*I*a*x + 1/
2) + dilog(1/2*I*a*x + 1/2))/(a*c) + 2*I*(log(I*a*x + 1)*log(x) + dilog(-I
*a*x))/(a*c) - 2*I*(log(-I*a*x + 1)*log(x) + dilog(I*a*x))/(a*c)) - (log(I
*a*x + 1)/c - log(x)/c)*arctan(a*x)
```

## Giac [F]

$$\int \frac{\arctan(ax)}{cx + iacx^2} \, dx = \int \frac{\arctan{(ax)}}{i\,acx^2 + cx} \, dx$$

input
```
integrate(arctan(a*x)/(c*x+I*a*c*x^2),x, algorithm="giac")
```

output
```
integrate(arctan(a*x)/(I*a*c*x^2 + c*x), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)}{cx + iacx^2} \, dx = \int \frac{\mathrm{atan}(a\,x)}{1\mathrm{i}\,a\,c\,x^2 + c\,x} \, dx$$

input
```
int(atan(a*x)/(c*x + a*c*x^2*1i),x)
```

output
```
int(atan(a*x)/(c*x + a*c*x^2*1i), x)
```

## Reduce [F]

$$\int \frac{\arctan(ax)}{cx + iacx^2}\,dx$$
$$= \frac{-atan(ax)^2 i - 2\left(\int \frac{atan(ax)}{a^3x^4 - a^2i\,x^3 + a\,x^2 - ix}dx\right) i + 2\left(\int \frac{atan(ax)}{a^3x^3 - a^2i\,x^2 + ax - i}dx\right) a}{2c}$$

input
```
int(atan(a*x)/(c*x+I*a*c*x^2),x)
```

output
```
( - atan(a*x)**2*i - 2*int(atan(a*x)/(a**3*x**4 - a**2*i*x**3 + a*x**2 - i
*x),x)*i + 2*int(atan(a*x)/(a**3*x**3 - a**2*i*x**2 + a*x - i),x)*a)/(2*c)
```

# 3.68 $\int x^3(d+icdx)(a+b\arctan(cx))^2\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 287

$$\begin{aligned}\int x^3(d+icdx)(a+b\arctan(cx))^2\,dx &= \frac{abdx}{2c^3}-\frac{3ib^2dx}{10c^3}+\frac{b^2dx^2}{12c^2}+\frac{ib^2dx^3}{30c}\\ &+\frac{3ib^2d\arctan(cx)}{10c^4}+\frac{b^2dx\arctan(cx)}{2c^3}\\ &+\frac{ibdx^2(a+b\arctan(cx))}{5c^2}\\ &-\frac{bdx^3(a+b\arctan(cx))}{6c}\\ &-\frac{1}{10}ibdx^4(a+b\arctan(cx))\\ &-\frac{9d(a+b\arctan(cx))^2}{20c^4}\\ &+\frac{1}{4}dx^4(a+b\arctan(cx))^2\\ &+\frac{1}{5}icdx^5(a+b\arctan(cx))^2\\ &+\frac{2ibd(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{5c^4}\\ &-\frac{b^2d\log(1+c^2x^2)}{3c^4}\\ &-\frac{b^2d\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{5c^4}\end{aligned}$$

output
```
1/2*a*b*d*x/c^3-3/10*I*b^2*d*x/c^3+1/12*b^2*d*x^2/c^2+1/30*I*b^2*d*x^3/c+3
/10*I*b^2*d*arctan(c*x)/c^4+1/2*b^2*d*x*arctan(c*x)/c^3+1/5*I*b*d*x^2*(a+b
*arctan(c*x))/c^2-1/6*b*d*x^3*(a+b*arctan(c*x))/c-1/10*I*b*d*x^4*(a+b*arct
an(c*x))-9/20*d*(a+b*arctan(c*x))^2/c^4+1/4*d*x^4*(a+b*arctan(c*x))^2+1/5*
I*c*d*x^5*(a+b*arctan(c*x))^2+2/5*I*b*d*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/
c^4-1/3*b^2*d*ln(c^2*x^2+1)/c^4-1/5*b^2*d*polylog(2,1-2/(1+I*c*x))/c^4
```

## Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 285, normalized size of antiderivative = 0.99

$$\int x^3(d+icdx)(a+b\arctan(cx))^2\,dx$$
$$=\frac{d\left(18iab+5b^2+30abcx-18ib^2cx+12iabc^2x^2+5b^2c^2x^2-10abc^3x^3+2ib^2c^3x^3+15a^2c^4x^4-6iabc^4x^4\right.}{}$$

input
```
Integrate[x^3*(d + I*c*d*x)*(a + b*ArcTan[c*x])^2,x]
```

output
```
(d*((18*I)*a*b + 5*b^2 + 30*a*b*c*x - (18*I)*b^2*c*x + (12*I)*a*b*c^2*x^2
+ 5*b^2*c^2*x^2 - 10*a*b*c^3*x^3 + (2*I)*b^2*c^3*x^3 + 15*a^2*c^4*x^4 - (6
*I)*a*b*c^4*x^4 + (12*I)*a^2*c^5*x^5 + 3*b^2*(-1 + 5*c^4*x^4 + (4*I)*c^5*x
^5)*ArcTan[c*x]^2 + 2*b*ArcTan[c*x]*(b*(9*I + 15*c*x + (6*I)*c^2*x^2 - 5*c
^3*x^3 - (3*I)*c^4*x^4) + 3*a*(-5 + 5*c^4*x^4 + (4*I)*c^5*x^5) + (12*I)*b*
Log[1 + E^((2*I)*ArcTan[c*x])]) - (12*I)*a*b*Log[1 + c^2*x^2] - 20*b^2*Log
[1 + c^2*x^2] + 12*b^2*PolyLog[2, -E^((2*I)*ArcTan[c*x])]))/(60*c^4)
```

## Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(d+icdx)(a+b\arctan(cx))^2\,dx$$
$$\Big\downarrow \text{5411}$$
$$\int \left(dx^3(a+b\arctan(cx))^2+icdx^4(a+b\arctan(cx))^2\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{9d(a+b\arctan(cx))^2}{20c^4}+\frac{2ibd\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{5c^4}+\frac{ibdx^2(a+b\arctan(cx))}{5c^2}+\frac{1}{5}icdx^5(a+b\arctan(cx))^2+\frac{1}{4}dx^4(a+b\arctan(cx))^2-\frac{1}{10}ibdx^4(a+b\arctan(cx))-\frac{bdx^3(a+b\arctan(cx))}{6c}+\frac{abdx}{2c^3}+\frac{3ib^2d\arctan(cx)}{10c^4}+\frac{b^2dx\arctan(cx)}{2c^3}-\frac{b^2d\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{5c^4}-\frac{3ib^2dx}{10c^3}+\frac{b^2dx^2}{12c^2}-\frac{b^2d\log\left(c^2x^2+1\right)}{3c^4}+\frac{ib^2dx^3}{30c}$$

input
```
Int[x^3*(d + I*c*d*x)*(a + b*ArcTan[c*x])^2,x]
```

output
```
(a*b*d*x)/(2*c^3) - (((3*I)/10)*b^2*d*x)/c^3 + (b^2*d*x^2)/(12*c^2) + ((I/
30)*b^2*d*x^3)/c + (((3*I)/10)*b^2*d*ArcTan[c*x])/c^4 + (b^2*d*x*ArcTan[c*
x])/(2*c^3) + ((I/5)*b*d*x^2*(a + b*ArcTan[c*x]))/c^2 - (b*d*x^3*(a + b*Ar
cTan[c*x]))/(6*c) - (I/10)*b*d*x^4*(a + b*ArcTan[c*x]) - (9*d*(a + b*ArcTa
n[c*x])^2)/(20*c^4) + (d*x^4*(a + b*ArcTan[c*x])^2)/4 + (I/5)*c*d*x^5*(a +
 b*ArcTan[c*x])^2 + (((2*I)/5)*b*d*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])
/c^4 - (b^2*d*Log[1 + c^2*x^2])/(3*c^4) - (b^2*d*PolyLog[2, 1 - 2/(1 + I*c
*x)])/(5*c^4)
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

## Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.31

| method | result |
|---|---|
| parts | $d\,a^2\left(\frac{1}{5}ic\,x^5+\frac{1}{4}x^4\right)+\frac{d\,b^2\left(\frac{3i\arctan(cx)}{10}+\frac{c^4x^4\arctan(cx)^2}{4}+\frac{cx\arctan(cx)}{2}+\frac{i\arctan(cx)^2c^5x^5}{5}-\frac{c^3x^3\arctan(cx)}{6}-\right.}{}$ |
| derivativedivides | $\frac{d\,a^2\left(\frac{1}{5}ic^5x^5+\frac{1}{4}c^4x^4\right)+d\,b^2\left(\frac{3i\arctan(cx)}{10}+\frac{c^4x^4\arctan(cx)^2}{4}+\frac{cx\arctan(cx)}{2}+\frac{i\arctan(cx)^2c^5x^5}{5}-\frac{c^3x^3\arctan(cx)}{6}-\frac{i\arctan}{}\right.}{}$ |
| default | $\frac{d\,a^2\left(\frac{1}{5}ic^5x^5+\frac{1}{4}c^4x^4\right)+d\,b^2\left(\frac{3i\arctan(cx)}{10}+\frac{c^4x^4\arctan(cx)^2}{4}+\frac{cx\arctan(cx)}{2}+\frac{i\arctan(cx)^2c^5x^5}{5}-\frac{c^3x^3\arctan(cx)}{6}-\frac{i\arctan}{}\right.}{}$ |
| risch | $\frac{idc\,a^2x^5}{5}+\frac{29idab}{30c^4}-\frac{idba\,x^4}{10}-\frac{d\,b^2\ln(-icx+1)x^2}{10c^2}+\frac{d\,b^2\ln\left(\frac{1}{2}+\frac{icx}{2}\right)\ln(-icx+1)}{5c^4}-\frac{d\,b^2\ln\left(\frac{1}{2}+\frac{icx}{2}\right)\ln\left(\frac{1}{2}-\frac{icx}{2}\right.}{5c^4}$ |

input
```
int(x^3*(d+I*c*d*x)*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
d*a^2*(1/5*I*c*x^5+1/4*x^4)+d*b^2/c^4*(3/10*I*arctan(c*x)+1/4*c^4*x^4*arct
an(c*x)^2+1/2*c*x*arctan(c*x)+1/5*I*arctan(c*x)^2*c^5*x^5-1/6*c^3*x^3*arct
an(c*x)-1/5*I*arctan(c*x)*ln(c^2*x^2+1)-1/10*I*arctan(c*x)*c^4*x^4-1/4*arc
tan(c*x)^2+1/10*ln(c*x-I)*ln(c^2*x^2+1)-1/20*ln(c*x-I)^2-1/10*dilog(-1/2*I
*(c*x+I))-1/10*ln(c*x-I)*ln(-1/2*I*(c*x+I))-1/10*ln(c*x+I)*ln(c^2*x^2+1)+1
/20*ln(c*x+I)^2+1/10*dilog(1/2*I*(c*x-I))+1/10*ln(c*x+I)*ln(1/2*I*(c*x-I))
+1/30*I*c^3*x^3+1/5*I*arctan(c*x)*c^2*x^2+1/12*c^2*x^2-1/3*ln(c^2*x^2+1)-3
/10*I*c*x)+2*d*a*b/c^4*(1/5*I*arctan(c*x)*c^5*x^5+1/4*c^4*x^4*arctan(c*x)+
1/4*c*x-1/20*I*c^4*x^4-1/12*c^3*x^3+1/10*I*c^2*x^2-1/10*I*ln(c^2*x^2+1)-1/
4*arctan(c*x))
```

## Fricas [F]

$$\int x^3(d+icdx)(a+b\arctan(cx))^2\,dx=\int(i\,cdx+d)(b\arctan(cx)+a)^2x^3\,dx$$

input
```
integrate(x^3*(d+I*c*d*x)*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

output
```
1/80*(-4*I*b^2*c*d*x^5 - 5*b^2*d*x^4)*log(-(c*x + I)/(c*x - I))^2 + integr
al(1/20*(20*I*a^2*c^3*d*x^6 + 20*a^2*c^2*d*x^5 + 20*I*a^2*c*d*x^4 + 20*a^2
*d*x^3 - (20*a*b*c^3*d*x^6 + 4*(-5*I*a*b - b^2)*c^2*d*x^5 + 5*(4*a*b + I*b
^2)*c*d*x^4 - 20*I*a*b*d*x^3)*log(-(c*x + I)/(c*x - I)))/(c^2*x^2 + 1), x)
```

## Sympy [F(-1)]

Timed out.

$$\int x^3(d + icdx)(a + b\arctan(cx))^2\,dx = \text{Timed out}$$

input
```
integrate(x**3*(d+I*c*d*x)*(a+b*atan(c*x))**2,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int x^3(d + icdx)(a + b\arctan(cx))^2\,dx = \int (i\,cdx + d)(b\arctan(cx) + a)^2x^3\,dx$$

input
```
integrate(x^3*(d+I*c*d*x)*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

output
```
1/5*I*a^2*c*d*x^5 + 1/4*b^2*d*x^4*arctan(c*x)^2 + 1/4*a^2*d*x^4 + 1/10*I*(
4*x^5*arctan(c*x) - c*((c^2*x^4 - 2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*a*
b*c*d + 1/80*I*(4*x^5*arctan(c*x)^2 - x^5*log(c^2*x^2 + 1)^2 + 80*integrat
e(1/80*(4*c^2*x^6*log(c^2*x^2 + 1) - 8*c*x^5*arctan(c*x) + 60*(c^2*x^6 + x
^4)*arctan(c*x)^2 + 5*(c^2*x^6 + x^4)*log(c^2*x^2 + 1)^2)/(c^2*x^2 + 1), x
))*b^2*c*d + 1/6*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*
x)/c^5))*a*b*d - 1/12*(2*c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5)*arcta
n(c*x) - (c^2*x^2 + 3*arctan(c*x)^2 - 4*log(c^2*x^2 + 1))/c^4)*b^2*d
```

## Giac [F]

$$\int x^3(d+icdx)(a+b\arctan(cx))^2\,dx=\int(i\,cdx+d)(b\arctan{(cx)}+a)^2x^3\,dx$$

input
```
integrate(x^3*(d+I*c*d*x)*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

output
```
integrate((I*c*d*x + d)*(b*arctan(c*x) + a)^2*x^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int x^3(d+icdx)(a+b\arctan(cx))^2\,dx=\int x^3\,(a+b\,\mathrm{atan}(c\,x))^2\,(d+c\,d\,x\,1\mathrm{i})\,dx$$

input
```
int(x^3*(a + b*atan(c*x))^2*(d + c*d*x*1i),x)
```

output
```
int(x^3*(a + b*atan(c*x))^2*(d + c*d*x*1i), x)
```

## Reduce [F]

$$\int x^3(d+icdx)(a+b\arctan(cx))^2\,dx$$
$$=\frac{d\big(12atan(cx)^2\,b^2c^5i\,x^5+15atan(cx)^2\,b^2c^4x^4-12atan(cx)^2\,b^2cix-15atan(cx)^2\,b^2+24atan(cx)\,ab\,c^5i\,x}{}$$

input
```
int(x^3*(d+I*c*d*x)*(a+b*atan(c*x))^2,x)
```

output
```
(d*(12*atan(c*x)**2*b**2*c**5*i*x**5 + 15*atan(c*x)**2*b**2*c**4*x**4 - 12
*atan(c*x)**2*b**2*c*i*x - 15*atan(c*x)**2*b**2 + 24*atan(c*x)*a*b*c**5*i*
x**5 + 30*atan(c*x)*a*b*c**4*x**4 - 30*atan(c*x)*a*b - 6*atan(c*x)*b**2*c*
*4*i*x**4 - 10*atan(c*x)*b**2*c**3*x**3 + 12*atan(c*x)*b**2*c**2*i*x**2 +
30*atan(c*x)*b**2*c*x + 18*atan(c*x)*b**2*i + 12*int(atan(c*x)**2,x)*b**2*
c*i - 12*log(c**2*x**2 + 1)*a*b*i - 20*log(c**2*x**2 + 1)*b**2 + 12*a**2*c
**5*i*x**5 + 15*a**2*c**4*x**4 - 6*a*b*c**4*i*x**4 - 10*a*b*c**3*x**3 + 12
*a*b*c**2*i*x**2 + 30*a*b*c*x + 2*b**2*c**3*i*x**3 + 5*b**2*c**2*x**2 - 18
*b**2*c*i*x))/(60*c**4)
```

# 3.69 $\int x^2(d+icdx)(a+b\arctan(cx))^2\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 255

$$\begin{aligned}\int x^2(d+icdx)(a+b\arctan(cx))^2\,dx &= \frac{iabdx}{2c^2}+\frac{b^2dx}{3c^2}+\frac{ib^2dx^2}{12c}-\frac{b^2d\arctan(cx)}{3c^3}+\frac{ib^2dx\arctan(cx)}{2c^2}-\frac{bdx^2(a+b\arctan(cx))}{3c}\\&-\frac{1}{6}ibdx^3(a+b\arctan(cx))-\frac{7id(a+b\arctan(cx))^2}{12c^3}+\frac{1}{3}dx^3(a+b\arctan(cx))^2\\&+\frac{1}{4}icdx^4(a+b\arctan(cx))^2-\frac{2bd(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{3c^3}\\&-\frac{ib^2d\log(1+c^2x^2)}{3c^3}-\frac{ib^2d\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{3c^3}\end{aligned}$$

output
```
1/2*I*a*b*d*x/c^2+1/3*b^2*d*x/c^2+1/12*I*b^2*d*x^2/c-1/3*b^2*d*arctan(c*x)
/c^3+1/2*I*b^2*d*x*arctan(c*x)/c^2-1/3*b*d*x^2*(a+b*arctan(c*x))/c-1/6*I*b
*d*x^3*(a+b*arctan(c*x))-7/12*I*d*(a+b*arctan(c*x))^2/c^3+1/3*d*x^3*(a+b*a
rctan(c*x))^2+1/4*I*c*d*x^4*(a+b*arctan(c*x))^2-2/3*b*d*(a+b*arctan(c*x))*
ln(2/(1+I*c*x))/c^3-1/3*I*b^2*d*ln(c^2*x^2+1)/c^3-1/3*I*b^2*d*polylog(2,1-
2/(1+I*c*x))/c^3
```

## Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.95

$$\int x^2(d+icdx)(a+b\arctan(cx))^2\,dx$$
$$= \frac{id(b^2+6abcx-4ib^2cx+4iabc^2x^2+b^2c^2x^2-4ia^2c^3x^3-2abc^3x^3+3a^2c^4x^4+b^2(1-4ic^3x^3+3c^4x^4)\,\text{ar}}{}$$

input
```
Integrate[x^2*(d + I*c*d*x)*(a + b*ArcTan[c*x])^2,x]
```

output
```
((I/12)*d*(b^2 + 6*a*b*c*x - (4*I)*b^2*c*x + (4*I)*a*b*c^2*x^2 + b^2*c^2*x
^2 - (4*I)*a^2*c^3*x^3 - 2*a*b*c^3*x^3 + 3*a^2*c^4*x^4 + b^2*(1 - (4*I)*c^
3*x^3 + 3*c^4*x^4)*ArcTan[c*x]^2 + 2*b*ArcTan[c*x]*(b*(2*I + 3*c*x + (2*I)
*c^2*x^2 - c^3*x^3) + a*(-3 - (4*I)*c^3*x^3 + 3*c^4*x^4) + (4*I)*b*Log[1 +
 E^((2*I)*ArcTan[c*x])]) - (4*I)*a*b*Log[1 + c^2*x^2] - 4*b^2*Log[1 + c^2*
x^2] + 4*b^2*PolyLog[2, -E^((2*I)*ArcTan[c*x])]))/c^3
```

## Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(d+icdx)(a+b\arctan(cx))^2\,dx$$

$$\Big\downarrow \text{5411}$$

$$\int \left(dx^2(a+b\arctan(cx))^2+icdx^3(a+b\arctan(cx))^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{7id(a+b\arctan(cx))^2}{12c^3}-\frac{2bd\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{3c^3}+\frac{1}{4}icdx^4(a+b\arctan(cx))^2+\frac{1}{3}dx^3(a+b\arctan(cx))^2-\frac{1}{6}ibdx^3(a+b\arctan(cx))-\frac{bdx^2(a+b\arctan(cx))}{3c}+\frac{iabdx}{2c^2}-\frac{b^2d\arctan(cx)}{3c^3}+\frac{ib^2dx\arctan(cx)}{2c^2}-\frac{ib^2d\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{3c^3}+\frac{b^2dx}{3c^2}-\frac{ib^2d\log(c^2x^2+1)}{3c^3}+\frac{ib^2dx^2}{12c}$$

input
```
Int[x^2*(d + I*c*d*x)*(a + b*ArcTan[c*x])^2,x]
```

output
```
((I/2)*a*b*d*x)/c^2 + (b^2*d*x)/(3*c^2) + ((I/12)*b^2*d*x^2)/c - (b^2*d*Ar
cTan[c*x])/(3*c^3) + ((I/2)*b^2*d*x*ArcTan[c*x])/c^2 - (b*d*x^2*(a + b*Arc
Tan[c*x]))/(3*c) - (I/6)*b*d*x^3*(a + b*ArcTan[c*x]) - (((7*I)/12)*d*(a +
b*ArcTan[c*x])^2)/c^3 + (d*x^3*(a + b*ArcTan[c*x])^2)/3 + (I/4)*c*d*x^4*(a
 + b*ArcTan[c*x])^2 - (2*b*d*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(3*c^
3) - ((I/3)*b^2*d*Log[1 + c^2*x^2])/c^3 - ((I/3)*b^2*d*PolyLog[2, 1 - 2/(1
 + I*c*x)])/c^3
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

## Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.38

| method | result |
|---|---|
| parts | $d\,a^2\left(\frac{1}{4}ic\,x^4+\frac{1}{3}x^3\right)+\frac{d\,b^2\left(\frac{i\arctan(cx)^2c^4x^4}{4}+\frac{c^3x^3\arctan(cx)^2}{3}-\frac{i\arctan(cx)c^3x^3}{6}-\frac{i\arctan(cx)^2}{4}+\frac{i\arctan(cx)cx}{2}-\right.}{}$ |
| derivativedivides | $\frac{d\,a^2\left(\frac{1}{4}ic^4x^4+\frac{1}{3}c^3x^3\right)+d\,b^2\left(\frac{i\arctan(cx)^2c^4x^4}{4}+\frac{c^3x^3\arctan(cx)^2}{3}-\frac{i\arctan(cx)c^3x^3}{6}-\frac{i\arctan(cx)^2}{4}+\frac{i\arctan(cx)cx}{2}-\frac{c^2x^2}{}\right.}{}$ |
| default | $\frac{d\,a^2\left(\frac{1}{4}ic^4x^4+\frac{1}{3}c^3x^3\right)+d\,b^2\left(\frac{i\arctan(cx)^2c^4x^4}{4}+\frac{c^3x^3\arctan(cx)^2}{3}-\frac{i\arctan(cx)c^3x^3}{6}-\frac{i\arctan(cx)^2}{4}+\frac{i\arctan(cx)cx}{2}-\frac{c^2x^2}{}\right.}{}$ |
| risch | $-\frac{dab\,x^2}{3c}+\frac{id\,b^2\ln\left(\frac{1}{2}+\frac{icx}{2}\right)\ln(-icx+1)}{3c^3}+\frac{d\,x^3a^2}{3}+\frac{iabdx}{2c^2}-\frac{id\,b^2\ln(-icx+1)x^2}{6c}-\frac{idba\arctan(cx)}{2c^3}-\frac{idc\,b^2}{}$ |

input
```
int(x^2*(d+I*c*d*x)*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
d*a^2*(1/4*I*c*x^4+1/3*x^3)+d*b^2/c^3*(1/4*I*arctan(c*x)^2*c^4*x^4+1/3*c^3
*x^3*arctan(c*x)^2-1/6*I*arctan(c*x)*c^3*x^3-1/4*I*arctan(c*x)^2+1/2*I*arc
tan(c*x)*c*x-1/3*c^2*x^2*arctan(c*x)+1/3*arctan(c*x)*ln(c^2*x^2+1)+1/3*c*x
+1/12*I*c^2*x^2-1/3*I*ln(c^2*x^2+1)-1/3*arctan(c*x)+1/6*I*(ln(c*x-I)*ln(c^
2*x^2+1)-1/2*ln(c*x-I)^2-dilog(-1/2*I*(c*x+I))-ln(c*x-I)*ln(-1/2*I*(c*x+I)
))-1/6*I*(ln(c*x+I)*ln(c^2*x^2+1)-1/2*ln(c*x+I)^2-dilog(1/2*I*(c*x-I))-ln(
c*x+I)*ln(1/2*I*(c*x-I))))+2*d*a*b/c^3*(1/4*I*arctan(c*x)*c^4*x^4+1/3*c^3*
x^3*arctan(c*x)+1/4*I*c*x-1/12*I*c^3*x^3-1/6*c^2*x^2+1/6*ln(c^2*x^2+1)-1/4
*I*arctan(c*x))
```

## Fricas [F]

$$\int x^2(d+icdx)(a+b\arctan(cx))^2\,dx=\int(i\,cdx+d)(b\arctan(cx)+a)^2x^2\,dx$$

input
```
integrate(x^2*(d+I*c*d*x)*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

output
```
1/48*(-3*I*b^2*c*d*x^4 - 4*b^2*d*x^3)*log(-(c*x + I)/(c*x - I))^2 + integr
al(1/12*(12*I*a^2*c^3*d*x^5 + 12*a^2*c^2*d*x^4 + 12*I*a^2*c*d*x^3 + 12*a^2
*d*x^2 - (12*a*b*c^3*d*x^5 + 3*(-4*I*a*b - b^2)*c^2*d*x^4 + 4*(3*a*b + I*b
^2)*c*d*x^3 - 12*I*a*b*d*x^2)*log(-(c*x + I)/(c*x - I)))/(c^2*x^2 + 1), x)
```

## Sympy [F(-1)]

Timed out.

$$\int x^2(d+icdx)(a+b\arctan(cx))^2\,dx=\text{Timed out}$$

input
```
integrate(x**2*(d+I*c*d*x)*(a+b*atan(c*x))**2,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int x^2(d+icdx)(a+b\arctan(cx))^2\,dx=\int(i\,cdx+d)(b\arctan(cx)+a)^2x^2\,dx$$

input
```
integrate(x^2*(d+I*c*d*x)*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

output
```
1/4*I*a^2*c*d*x^4 + 1/3*a^2*d*x^3 + 1/6*I*(3*x^4*arctan(c*x) - c*((c^2*x^3
 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*a*b*c*d + 1/3*(2*x^3*arctan(c*x) - c*(x^
2/c^2 - log(c^2*x^2 + 1)/c^4))*a*b*d - 1/48*(-3*I*b^2*c*d*x^4 - 4*b^2*d*x^
3)*arctan(c*x)^2 - 1/48*(3*b^2*c*d*x^4 - 4*I*b^2*d*x^3)*arctan(c*x)*log(c^
2*x^2 + 1) + 1/192*(-3*I*b^2*c*d*x^4 - 4*b^2*d*x^3)*log(c^2*x^2 + 1)^2 + I
*integrate(-1/48*(14*b^2*c^2*d*x^4*arctan(c*x) - 36*(b^2*c^3*d*x^5 + b^2*c
*d*x^3)*arctan(c*x)^2 - 3*(b^2*c^3*d*x^5 + b^2*c*d*x^3)*log(c^2*x^2 + 1)^2
 - (3*b^2*c^3*d*x^5 - 4*b^2*c*d*x^3 - 12*(b^2*c^2*d*x^4 + b^2*d*x^2)*arcta
n(c*x))*log(c^2*x^2 + 1))/(c^2*x^2 + 1), x) + integrate(1/48*(36*(b^2*c^2*
d*x^4 + b^2*d*x^2)*arctan(c*x)^2 + 3*(b^2*c^2*d*x^4 + b^2*d*x^2)*log(c^2*x
^2 + 1)^2 + 2*(3*b^2*c^3*d*x^5 - 4*b^2*c*d*x^3)*arctan(c*x) + (7*b^2*c^2*d
*x^4 + 12*(b^2*c^3*d*x^5 + b^2*c*d*x^3)*arctan(c*x))*log(c^2*x^2 + 1))/(c^
2*x^2 + 1), x)
```

## Giac [F]

$$\int x^2(d+icdx)(a+b\arctan(cx))^2\,dx = \int (i\,cdx+d)(b\arctan(cx)+a)^2x^2\,dx$$

input
```
integrate(x^2*(d+I*c*d*x)*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

output
```
integrate((I*c*d*x + d)*(b*arctan(c*x) + a)^2*x^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int x^2(d+icdx)(a+b\arctan(cx))^2\,dx = \int x^2\,(a+b\,\mathrm{atan}(c\,x))^2\,(d+c\,d\,x\,1\mathrm{i})\,dx$$

input
```
int(x^2*(a + b*atan(c*x))^2*(d + c*d*x*1i),x)
```

output
```
int(x^2*(a + b*atan(c*x))^2*(d + c*d*x*1i), x)
```

## Reduce [F]

$$\int x^2(d+icdx)(a+b\arctan(cx))^2\,dx = \frac{d\big(3atan(cx)^2\,b^2c^4i\,x^4+4atan(cx)^2\,b^2c^3x^3+4atan(cx)^2\,b^2cx-3atan(cx)^2\,b^2i+6atan(cx)\,ab\,c^4i\,x^4+8a}{}$$

input
```
int(x^2*(d+I*c*d*x)*(a+b*atan(c*x))^2,x)
```

output
```
(d*(3*atan(c*x)**2*b**2*c**4*i*x**4 + 4*atan(c*x)**2*b**2*c**3*x**3 + 4*at
an(c*x)**2*b**2*c*x - 3*atan(c*x)**2*b**2*i + 6*atan(c*x)*a*b*c**4*i*x**4
+ 8*atan(c*x)*a*b*c**3*x**3 - 6*atan(c*x)*a*b*i - 2*atan(c*x)*b**2*c**3*i*
x**3 - 4*atan(c*x)*b**2*c**2*x**2 + 6*atan(c*x)*b**2*c*i*x - 4*atan(c*x)*b
**2 - 4*int(atan(c*x)**2,x)*b**2*c + 4*log(c**2*x**2 + 1)*a*b - 4*log(c**2
*x**2 + 1)*b**2*i + 3*a**2*c**4*i*x**4 + 4*a**2*c**3*x**3 - 2*a*b*c**3*i*x
**3 - 4*a*b*c**2*x**2 + 6*a*b*c*i*x + b**2*c**2*i*x**2 + 4*b**2*c*x))/(12*
c**3)
```

# 3.70 $\int x(d+icdx)(a+b\arctan(cx))^2\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 211

$$\int x(d+icdx)(a+b\arctan(cx))^2\,dx=-\frac{abdx}{c}+\frac{ib^2dx}{3c}-\frac{ib^2d\arctan(cx)}{3c^2}-\frac{b^2dx\arctan(cx)}{c}-\frac{1}{3}ibdx^2(a+b\arctan(cx))+\frac{5d(a+b\arctan(cx))^2}{6c^2}+\frac{1}{2}dx^2(a+b\arctan(cx))^2+\frac{1}{3}icdx^3(a+b\arctan(cx))^2-\frac{2ibd(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{3c^2}+\frac{b^2d\log(1+c^2x^2)}{2c^2}+\frac{b^2d\,\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{3c^2}$$

output
```
-a*b*d*x/c+1/3*I*b^2*d*x/c-1/3*I*b^2*d*arctan(c*x)/c^2-b^2*d*x*arctan(c*x)
/c-1/3*I*b*d*x^2*(a+b*arctan(c*x))+5/6*d*(a+b*arctan(c*x))^2/c^2+1/2*d*x^2
*(a+b*arctan(c*x))^2+1/3*I*c*d*x^3*(a+b*arctan(c*x))^2-2/3*I*b*d*(a+b*arct
an(c*x))*ln(2/(1+I*c*x))/c^2+1/2*b^2*d*ln(c^2*x^2+1)/c^2+1/3*b^2*d*polylog
(2,1-2/(1+I*c*x))/c^2
```

## Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.99

$$\int x(d+icdx)(a+b\arctan(cx))^2\,dx$$
$$=\frac{d\left(-6abcx+2ib^2cx+3a^2c^2x^2-2iabc^2x^2+2ia^2c^3x^3+b^2(1+3c^2x^2+2ic^3x^3)\arctan(cx)^2+2b\arctan(c\right.}{}$$

input
```
Integrate[x*(d + I*c*d*x)*(a + b*ArcTan[c*x])^2,x]
```

output
```
(d*(-6*a*b*c*x + (2*I)*b^2*c*x + 3*a^2*c^2*x^2 - (2*I)*a*b*c^2*x^2 + (2*I)
*a^2*c^3*x^3 + b^2*(1 + 3*c^2*x^2 + (2*I)*c^3*x^3)*ArcTan[c*x]^2 + 2*b*Arc
Tan[c*x]*((-I)*b*(1 - (3*I)*c*x + c^2*x^2) + a*(3 + 3*c^2*x^2 + (2*I)*c^3*
x^3) - (2*I)*b*Log[1 + E^((2*I)*ArcTan[c*x])]) + (2*I)*a*b*Log[1 + c^2*x^2
] + 3*b^2*Log[1 + c^2*x^2] - 2*b^2*PolyLog[2, -E^((2*I)*ArcTan[c*x])]))/(6
*c^2)
```

## Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(d+icdx)(a+b\arctan(cx))^2\,dx$$
$$\Big\downarrow \text{5411}$$
$$\int \left(dx(a+b\arctan(cx))^2+icdx^2(a+b\arctan(cx))^2\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{5d(a+b\arctan(cx))^2}{6c^2}-\frac{2ibd\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{3c^2}+\frac{1}{3}icdx^3(a+b\arctan(cx))^2+\frac{1}{2}dx^2(a+b\arctan(cx))^2-\frac{1}{3}ibdx^2(a+b\arctan(cx))-\frac{abdx}{c}-\frac{ib^2d\arctan(cx)}{3c^2}-\frac{b^2dx\arctan(cx)}{c}+\frac{b^2d\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{3c^2}+\frac{b^2d\log(c^2x^2+1)}{2c^2}+\frac{ib^2dx}{3c}$$

input
```
Int[x*(d + I*c*d*x)*(a + b*ArcTan[c*x])^2,x]
```

output
```
-((a*b*d*x)/c) + ((I/3)*b^2*d*x)/c - ((I/3)*b^2*d*ArcTan[c*x])/c^2 - (b^2*
d*x*ArcTan[c*x])/c - (I/3)*b*d*x^2*(a + b*ArcTan[c*x]) + (5*d*(a + b*ArcTa
n[c*x])^2)/(6*c^2) + (d*x^2*(a + b*ArcTan[c*x])^2)/2 + (I/3)*c*d*x^3*(a +
b*ArcTan[c*x])^2 - (((2*I)/3)*b*d*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/
c^2 + (b^2*d*Log[1 + c^2*x^2])/(2*c^2) + (b^2*d*PolyLog[2, 1 - 2/(1 + I*c*
x)])/(3*c^2)
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

## Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.50

| method | result |
|---|---|
| parts | $d\,a^2\left(\frac{1}{3}ic\,x^3+\frac{1}{2}x^2\right)+\frac{d\,b^2\left(-\frac{i\arctan(cx)}{3}+\frac{c^2x^2\arctan(cx)^2}{2}+\frac{i\arctan(cx)^2c^3x^3}{3}+\frac{i\arctan(cx)\ln\left(c^2x^2+1\right)}{3}+\frac{\arctan(}{2}\right.}{}$ |
| derivativedivides | $\frac{d\,a^2\left(\frac{1}{3}ic^3x^3+\frac{1}{2}c^2x^2\right)+d\,b^2\left(-\frac{i\arctan(cx)}{3}+\frac{c^2x^2\arctan(cx)^2}{2}+\frac{i\arctan(cx)^2c^3x^3}{3}+\frac{i\arctan(cx)\ln\left(c^2x^2+1\right)}{3}+\frac{\arctan(cx)^2}{2}-\right.}{}$ |
| default | $\frac{d\,a^2\left(\frac{1}{3}ic^3x^3+\frac{1}{2}c^2x^2\right)+d\,b^2\left(-\frac{i\arctan(cx)}{3}+\frac{c^2x^2\arctan(cx)^2}{2}+\frac{i\arctan(cx)^2c^3x^3}{3}+\frac{i\arctan(cx)\ln\left(c^2x^2+1\right)}{3}+\frac{\arctan(cx)^2}{2}-\right.}{}$ |
| risch | $-\frac{d\,b^2}{3c^2}+\frac{d\,x^2a^2}{2}+\frac{5d\,a^2}{6c^2}-\frac{5d\,b^2\ln(-icx+1)^2}{24c^2}-\frac{abdx}{c}-\frac{id\,b^2\left(2c^3x^3-3ic^2x^2-i\right)\ln(icx+1)^2}{24c^2}-\frac{dcab\ln(-icx}{3}$ |

input
```
int(x*(d+I*c*d*x)*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
d*a^2*(1/3*I*c*x^3+1/2*x^2)+d*b^2/c^2*(-1/3*I*arctan(c*x)+1/2*c^2*x^2*arct
an(c*x)^2+1/3*I*arctan(c*x)^2*c^3*x^3+1/3*I*ln(c^2*x^2+1)*arctan(c*x)+1/2*
arctan(c*x)^2-c*x*arctan(c*x)-1/6*ln(c*x-I)*ln(c^2*x^2+1)+1/6*dilog(-1/2*I
*(c*x+I))+1/6*ln(c*x-I)*ln(-1/2*I*(c*x+I))+1/12*ln(c*x-I)^2+1/6*ln(c*x+I)*
ln(c^2*x^2+1)-1/6*dilog(1/2*I*(c*x-I))-1/6*ln(c*x+I)*ln(1/2*I*(c*x-I))-1/1
2*ln(c*x+I)^2-1/3*I*arctan(c*x)*c^2*x^2+1/2*ln(c^2*x^2+1)+1/3*I*c*x)+2*d*a
*b/c^2*(1/3*I*arctan(c*x)*c^3*x^3+1/2*c^2*x^2*arctan(c*x)-1/6*I*c^2*x^2-1/
2*c*x+1/6*I*ln(c^2*x^2+1)+1/2*arctan(c*x))
```

## Fricas [F]

$$\int x(d+icdx)(a+b\arctan(cx))^2\,dx=\int(i\,cdx+d)(b\arctan\left(cx\right)+a)^2x\,dx$$

input
```
integrate(x*(d+I*c*d*x)*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

output
```
1/24*(-2*I*b^2*c*d*x^3 - 3*b^2*d*x^2)*log(-(c*x + I)/(c*x - I))^2 + integr
al(1/6*(6*I*a^2*c^3*d*x^4 + 6*a^2*c^2*d*x^3 + 6*I*a^2*c*d*x^2 + 6*a^2*d*x
- (6*a*b*c^3*d*x^4 + 2*(-3*I*a*b - b^2)*c^2*d*x^3 + 3*(2*a*b + I*b^2)*c*d*
x^2 - 6*I*a*b*d*x)*log(-(c*x + I)/(c*x - I)))/(c^2*x^2 + 1), x)
```

## Sympy [F(-1)]

Timed out.

$$\int x(d+icdx)(a+b\arctan(cx))^2\,dx = \text{Timed out}$$

input
```
integrate(x*(d+I*c*d*x)*(a+b*atan(c*x))**2,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int x(d+icdx)(a+b\arctan(cx))^2\,dx = \int (i\,cdx+d)(b\arctan(cx)+a)^2x\,dx$$

input
```
integrate(x*(d+I*c*d*x)*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

output
```
1/3*I*a^2*c*d*x^3 + 1/2*b^2*d*x^2*arctan(c*x)^2 + 1/3*I*(2*x^3*arctan(c*x)
 - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*a*b*c*d + 1/48*I*(4*x^3*arctan(c*x)
^2 - x^3*log(c^2*x^2 + 1)^2 + 48*integrate(1/48*(4*c^2*x^4*log(c^2*x^2 + 1
) - 8*c*x^3*arctan(c*x) + 36*(c^2*x^4 + x^2)*arctan(c*x)^2 + 3*(c^2*x^4 +
x^2)*log(c^2*x^2 + 1)^2)/(c^2*x^2 + 1), x))*b^2*c*d + 1/2*a^2*d*x^2 + (x^2
*arctan(c*x) - c*(x/c^2 - arctan(c*x)/c^3))*a*b*d - 1/2*(2*c*(x/c^2 - arct
an(c*x)/c^3)*arctan(c*x) + (arctan(c*x)^2 - log(c^2*x^2 + 1))/c^2)*b^2*d
```

## Giac [F]

$$\int x(d+icdx)(a+b\arctan(cx))^2\,dx=\int (i\,cdx+d)(b\arctan{(cx)}+a)^2x\,dx$$

input
```
integrate(x*(d+I*c*d*x)*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

output
```
integrate((I*c*d*x + d)*(b*arctan(c*x) + a)^2*x, x)
```

## Mupad [F(-1)]

Timed out.

$$\int x(d+icdx)(a+b\arctan(cx))^2\,dx=\int x\,{\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}^2\,\left(d+c\,d\,x\,1{}\mathrm{i}\right)\,dx$$

input
```
int(x*(a + b*atan(c*x))^2*(d + c*d*x*1i),x)
```

output
```
int(x*(a + b*atan(c*x))^2*(d + c*d*x*1i), x)
```

## Reduce [F]

$$\int x(d+icdx)(a+b\arctan(cx))^2\,dx$$
$$=\frac{d\left(2atan(cx)^2b^2c^3i\,x^3+3atan(cx)^2b^2c^2x^2+2atan(cx)^2b^2cix+3atan(cx)^2b^2+4atan(cx)\,ab\,c^3i\,x^3+6a\cdots\right.}{}$$

input
```
int(x*(d+I*c*d*x)*(a+b*atan(c*x))^2,x)
```

output
```
(d*(2*atan(c*x)**2*b**2*c**3*i*x**3 + 3*atan(c*x)**2*b**2*c**2*x**2 + 2*at
an(c*x)**2*b**2*c*i*x + 3*atan(c*x)**2*b**2 + 4*atan(c*x)*a*b*c**3*i*x**3
+ 6*atan(c*x)*a*b*c**2*x**2 + 6*atan(c*x)*a*b - 2*atan(c*x)*b**2*c**2*i*x*
*2 - 6*atan(c*x)*b**2*c*x - 2*atan(c*x)*b**2*i - 2*int(atan(c*x)**2,x)*b**
2*c*i + 2*log(c**2*x**2 + 1)*a*b*i + 3*log(c**2*x**2 + 1)*b**2 + 2*a**2*c*
*3*i*x**3 + 3*a**2*c**2*x**2 - 2*a*b*c**2*i*x**2 - 6*a*b*c*x + 2*b**2*c*i*
x))/(6*c**2)
```

# 3.71 $\int (d + icdx)(a + b\arctan(cx))^2\, dx$



## Optimal result

Integrand size = 20, antiderivative size = 130

$$\int (d + icdx)(a + b\arctan(cx))^2\, dx = -iabdx - ib^2dx\arctan(cx) - \frac{id(1 + icx)^2(a + b\arctan(cx))^2}{2c} + \frac{2bd(a + b\arctan(cx))\log\left(\frac{2}{1-icx}\right)}{c} + \frac{ib^2d\log(1 + c^2x^2)}{2c} - \frac{ib^2d\,\text{PolyLog}\left(2, 1 - \frac{2}{1-icx}\right)}{c}$$

output
```
-I*a*b*d*x-I*b^2*d*x*arctan(c*x)-1/2*I*d*(1+I*c*x)^2*(a+b*arctan(c*x))^2/c
+2*b*d*(a+b*arctan(c*x))*ln(2/(1-I*c*x))/c+1/2*I*b^2*d*ln(c^2*x^2+1)/c-I*b
^2*d*polylog(2,1-2/(1-I*c*x))/c
```

## Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.16

$$\int (d + icdx)(a + b\arctan(cx))^2\,dx$$
$$= \frac{id\left(-2ia^2cx - 2abcx + a^2c^2x^2 + b^2(-i+cx)^2\arctan(cx)^2 + 2b\arctan(cx)\left(a - 2iacx - bcx + ac^2x^2 - 2\right.\right.}{2c}$$

input
```
Integrate[(d + I*c*d*x)*(a + b*ArcTan[c*x])^2,x]
```

output
```
((I/2)*d*((-2*I)*a^2*c*x - 2*a*b*c*x + a^2*c^2*x^2 + b^2*(-I + c*x)^2*ArcT
an[c*x]^2 + 2*b*ArcTan[c*x]*(a - (2*I)*a*c*x - b*c*x + a*c^2*x^2 - (2*I)*b
*Log[1 + E^((2*I)*ArcTan[c*x])]) + (2*I)*a*b*Log[1 + c^2*x^2] + b^2*Log[1
+ c^2*x^2] - 2*b^2*PolyLog[2, -E^((2*I)*ArcTan[c*x])]))/c
```

## Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.04, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {5389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d + icdx)(a + b\arctan(cx))^2\,dx$$

$$\Big\downarrow \; 5389$$

$$\frac{ib\int \left(-\frac{2i(i-cx)(a+b\arctan(cx))d^2}{c^2x^2+1} - (a + b\arctan(cx))d^2\right)dx}{d} - \frac{id(1+icx)^2(a+b\arctan(cx))^2}{2c}$$

$$\Big\downarrow \; 2009$$

$$\frac{ib\left(-\frac{2id^2\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))}{c}-ad^2x-bd^2x\arctan(cx)+\frac{bd^2\log(c^2x^2+1)}{2c}-\frac{bd^2\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{c}\right)}{d}-\frac{id(1+icx)^2(a+b\arctan(cx))^2}{2c}$$

input
```
Int[(d + I*c*d*x)*(a + b*ArcTan[c*x])^2,x]
```

output
```
((-1/2*I)*d*(1 + I*c*x)^2*(a + b*ArcTan[c*x])^2)/c + (I*b*(-(a*d^2*x) - b*
d^2*x*ArcTan[c*x] - ((2*I)*d^2*(a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/c +
 (b*d^2*Log[1 + c^2*x^2])/(2*c) - (b*d^2*PolyLog[2, 1 - 2/(1 - I*c*x)])/c)
)/d
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5389
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - S
imp[b*c*(p/(e*(q + 1)))   Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1),
(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

## Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(118) = 236$.

Time = 0.68 (sec) , antiderivative size = 278, normalized size of antiderivative = 2.14

| method | result |
|---|---|
| parts | $-id\,a^2\left(-\frac{1}{2}c\,x^2+ix\right)+\frac{d\,b^2\left(\frac{i\arctan(cx)^2c^2x^2}{2}+cx\arctan(cx)^2+i\left(i\arctan(cx)\ln(c^2x^2+1)+\frac{\arctan(cx)^2}{2}-cx\right.\right.}{}$ |
| derivativedivides | $\frac{-id\,a^2\left(-\frac{1}{2}c^2x^2+icx\right)+d\,b^2\left(\frac{i\arctan(cx)^2c^2x^2}{2}+cx\arctan(cx)^2+i\left(i\arctan(cx)\ln(c^2x^2+1)+\frac{\arctan(cx)^2}{2}-cx\arctan(\right.\right.}{}$ |
| default | $\frac{-id\,a^2\left(-\frac{1}{2}c^2x^2+icx\right)+d\,b^2\left(\frac{i\arctan(cx)^2c^2x^2}{2}+cx\arctan(cx)^2+i\left(i\arctan(cx)\ln(c^2x^2+1)+\frac{\arctan(cx)^2}{2}-cx\arctan(\right.\right.}{}$ |
| risch | $\frac{3id\,a^2}{2c}+a^2dx-\frac{iabd\arctan(cx)}{2c}+\frac{ib^2\operatorname{dilog}\left(\frac{1}{2}-\frac{icx}{2}\right)d}{c}+\frac{ib^2\ln\left(\frac{1}{2}-\frac{icx}{2}\right)\ln\left(\frac{1}{2}+\frac{icx}{2}\right)d}{c}-\frac{3\ln(-icx+1)abd}{2c}-$ |

input
```
int((d+I*c*d*x)*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
-I*d*a^2*(-1/2*c*x^2+I*x)+d*b^2/c*(1/2*I*arctan(c*x)^2*c^2*x^2+c*x*arctan(
c*x)^2+I*(I*arctan(c*x)*ln(c^2*x^2+1)+1/2*arctan(c*x)^2-c*x*arctan(c*x)-1/
2*ln(c*x-I)*ln(c^2*x^2+1)+1/2*dilog(-1/2*I*(c*x+I))+1/2*ln(c*x-I)*ln(-1/2*
I*(c*x+I))+1/4*ln(c*x-I)^2+1/2*ln(c*x+I)*ln(c^2*x^2+1)-1/2*dilog(1/2*I*(c*
x-I))-1/2*ln(c*x+I)*ln(1/2*I*(c*x-I))-1/4*ln(c*x+I)^2+1/2*ln(c^2*x^2+1)))+
2*d*a*b/c*(1/2*I*arctan(c*x)*c^2*x^2+c*x*arctan(c*x)+1/2*I*(-c*x+I*ln(c^2*
x^2+1)+arctan(c*x)))
```

## Fricas [F]

$$\int (d+icdx)(a+b\arctan(cx))^2\,dx=\int (i\,cdx+d)(b\arctan{(cx)}+a)^2\,dx$$

input
```
integrate((d+I*c*d*x)*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

output
```
1/8*(-I*b^2*c*d*x^2 - 2*b^2*d*x)*log(-(c*x + I)/(c*x - I))^2 + integral(1/
2*(2*I*a^2*c^3*d*x^3 + 2*a^2*c^2*d*x^2 + 2*I*a^2*c*d*x + 2*a^2*d - (2*a*b*
c^3*d*x^3 - (2*I*a*b + b^2)*c^2*d*x^2 + 2*(a*b + I*b^2)*c*d*x - 2*I*a*b*d)
*log(-(c*x + I)/(c*x - I)))/(c^2*x^2 + 1), x)
```

## Sympy [F(-1)]

Timed out.

$$\int (d+icdx)(a+b\arctan(cx))^2\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)*(a+b*atan(c*x))**2,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int (d+icdx)(a+b\arctan(cx))^2\,dx = \int (i\,cdx+d)(b\arctan(cx)+a)^2\,dx$$

input
```
integrate((d+I*c*d*x)*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

output
```
4*b^2*c^3*d*integrate(1/16*x^3*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^2 + 1),
 x) + 4*b^2*c^3*d*integrate(1/16*x^3*arctan(c*x)/(c^2*x^2 + 1), x) + 1/2*I
*a^2*c*d*x^2 + 12*b^2*c^2*d*integrate(1/16*x^2*arctan(c*x)^2/(c^2*x^2 + 1)
, x) + b^2*c^2*d*integrate(1/16*x^2*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) +
 6*b^2*c^2*d*integrate(1/16*x^2*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) + I*(x^
2*arctan(c*x) - c*(x/c^2 - arctan(c*x)/c^3))*a*b*c*d + 1/4*b^2*d*arctan(c*
x)^3/c + 4*b^2*c*d*integrate(1/16*x*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^2
+ 1), x) - 8*b^2*c*d*integrate(1/16*x*arctan(c*x)/(c^2*x^2 + 1), x) + a^2*
d*x + b^2*d*integrate(1/16*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + (2*c*x*a
rctan(c*x) - log(c^2*x^2 + 1))*a*b*d/c - 1/8*(-I*b^2*c*d*x^2 - 2*b^2*d*x)*
arctan(c*x)^2 - 1/8*(b^2*c*d*x^2 - 2*I*b^2*d*x)*arctan(c*x)*log(c^2*x^2 +
1) + 1/32*(-I*b^2*c*d*x^2 - 2*b^2*d*x)*log(c^2*x^2 + 1)^2 + I*integrate(-1
/16*(12*b^2*c^2*d*x^2*arctan(c*x) - 12*(b^2*c^3*d*x^3 + b^2*c*d*x)*arctan(
c*x)^2 - (b^2*c^3*d*x^3 + b^2*c*d*x)*log(c^2*x^2 + 1)^2 - 2*(b^2*c^3*d*x^3
 - 2*b^2*c*d*x - 2*(b^2*c^2*d*x^2 + b^2*d)*arctan(c*x))*log(c^2*x^2 + 1))/
(c^2*x^2 + 1), x)
```

## Giac [F]

$$\int (d+icdx)(a+b\arctan(cx))^2\,dx=\int (i\,cdx+d)(b\arctan(cx)+a)^2\,dx$$

input
```
integrate((d+I*c*d*x)*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

output
```
integrate((I*c*d*x + d)*(b*arctan(c*x) + a)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (d+icdx)(a+b\arctan(cx))^2\,dx=\int (a+b\,\mathrm{atan}(c\,x))^2\,(d+c\,d\,x\,1\mathrm{i})\,dx$$

input
```
int((a + b*atan(c*x))^2*(d + c*d*x*1i),x)
```

output
```
int((a + b*atan(c*x))^2*(d + c*d*x*1i), x)
```

## Reduce [F]

$$\int (d+icdx)(a+b\arctan(cx))^2\,dx$$
$$=\frac{d\Big(atan(cx)^2\,b^2c^2i\,x^2+2atan(cx)^2\,b^2cx+atan(cx)^2\,b^2i+2atan(cx)\,ab\,c^2i\,x^2+4atan(cx)\,abcx+2atan(}{}$$

input
```
int((d+I*c*d*x)*(a+b*atan(c*x))^2,x)
```

output
```
(d*(atan(c*x)**2*b**2*c**2*i*x**2 + 2*atan(c*x)**2*b**2*c*x + atan(c*x)**2
*b**2*i + 2*atan(c*x)*a*b*c**2*i*x**2 + 4*atan(c*x)*a*b*c*x + 2*atan(c*x)*
a*b*i - 2*atan(c*x)*b**2*c*i*x - 4*int((atan(c*x)*x)/(c**2*x**2 + 1),x)*b*
*2*c**2 - 2*log(c**2*x**2 + 1)*a*b + log(c**2*x**2 + 1)*b**2*i + a**2*c**2
*i*x**2 + 2*a**2*c*x - 2*a*b*c*i*x))/(2*c)
```

# 3.72 $\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 216

$$\begin{aligned}\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x}\,dx &= -d(a+b\arctan(cx))^2+icdx(a+b\arctan(cx))^2\\ &\quad+2d(a+b\arctan(cx))^2\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right)\\ &\quad+2ibd(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)\\ &\quad-b^2d\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\\ &\quad-ibd(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\\ &\quad+ibd(a+b\arctan(cx))\operatorname{PolyLog}\left(2,-1\right.\\ &\qquad\left.+\frac{2}{1+icx}\right)-\frac{1}{2}b^2d\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)\\ &\quad+\frac{1}{2}b^2d\operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)\end{aligned}$$

output
```
-d*(a+b*arctan(c*x))^2+I*c*d*x*(a+b*arctan(c*x))^2-2*d*(a+b*arctan(c*x))^2
*arctanh(-1+2/(1+I*c*x))+2*I*b*d*(a+b*arctan(c*x))*ln(2/(1+I*c*x))-b^2*d*p
olylog(2,1-2/(1+I*c*x))-I*b*d*(a+b*arctan(c*x))*polylog(2,1-2/(1+I*c*x))+I
*b*d*(a+b*arctan(c*x))*polylog(2,-1+2/(1+I*c*x))-1/2*b^2*d*polylog(3,1-2/(
1+I*c*x))+1/2*b^2*d*polylog(3,-1+2/(1+I*c*x))
```

## Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.26

$$\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x}\,dx$$
$$= d\Bigg(ia^2cx + a^2\log(cx) + iab\left(2cx\arctan(cx) - \log\left(1+c^2x^2\right)\right) + b^2\left(\arctan(cx)\left((1+icx)\arctan(cx) + 2i\log\left(1+e^{2i\arctan(cx)}\right)\right) + \text{PolyLog}\left(2,-e^{2i\arctan(cx)}\right)\right) + iab(\text{PolyLog}(2,-icx) - \text{PolyLog}(2,icx)) + b^2\Bigg(-\frac{i\pi^3}{24} + \frac{2}{3}i\arctan(cx)^3 + \arctan(cx)^2\log\left(1-e^{-2i\arctan(cx)}\right) - \arctan(cx)^2\log\left(1+e^{2i\arctan(cx)}\right) + i\arctan(cx)\,\text{PolyLog}\left(2,e^{-2i\arctan(cx)}\right) + i\arctan(cx)\,\text{PolyLog}\left(2,-e^{2i\arctan(cx)}\right) + \frac{1}{2}\text{PolyLog}\left(3,e^{-2i\arctan(cx)}\right) - \frac{1}{2}\text{PolyLog}\left(3,-e^{2i\arctan(cx)}\right)\Bigg)\Bigg)$$

input
```
Integrate[((d + I*c*d*x)*(a + b*ArcTan[c*x])^2)/x,x]
```

output
```
d*(I*a^2*c*x + a^2*Log[c*x] + I*a*b*(2*c*x*ArcTan[c*x] - Log[1 + c^2*x^2])
 + b^2*(ArcTan[c*x]*((1 + I*c*x)*ArcTan[c*x] + (2*I)*Log[1 + E^((2*I)*ArcT
an[c*x])]) + PolyLog[2, -E^((2*I)*ArcTan[c*x])]) + I*a*b*(PolyLog[2, (-I)*
c*x] - PolyLog[2, I*c*x]) + b^2*((-1/24*I)*Pi^3 + ((2*I)/3)*ArcTan[c*x]^3
+ ArcTan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] - ArcTan[c*x]^2*Log[1 + E^
((2*I)*ArcTan[c*x])] + I*ArcTan[c*x]*PolyLog[2, E^((-2*I)*ArcTan[c*x])] +
I*ArcTan[c*x]*PolyLog[2, -E^((2*I)*ArcTan[c*x])] + PolyLog[3, E^((-2*I)*Ar
cTan[c*x])]/2 - PolyLog[3, -E^((2*I)*ArcTan[c*x])]/2))
```

## Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x}dx$$

$$\Big\downarrow \text{5411}$$

$$\int \left(\frac{d(a+b\arctan(cx))^2}{x}+icd(a+b\arctan(cx))^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$2d\text{arctanh}\left(1-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2-ibd\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))+ibd\,\text{PolyLog}\left(2,\frac{2}{icx+1}-1\right)(a+b\arctan(cx))-d(a+b\arctan(cx))^2+icdx(a+b\arctan(cx))^2+2ibd\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))+b^2(-d)\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)-\frac{1}{2}b^2d\,\text{PolyLog}\left(3,1-\frac{2}{icx+1}\right)+\frac{1}{2}b^2d\,\text{PolyLog}\left(3,\frac{2}{icx+1}-1\right)$$

input
```
Int[((d + I*c*d*x)*(a + b*ArcTan[c*x])^2)/x,x]
```

output
```
-(d*(a + b*ArcTan[c*x])^2) + I*c*d*x*(a + b*ArcTan[c*x])^2 + 2*d*(a + b*Ar
cTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)] + (2*I)*b*d*(a + b*ArcTan[c*x])*Lo
g[2/(1 + I*c*x)] - b^2*d*PolyLog[2, 1 - 2/(1 + I*c*x)] - I*b*d*(a + b*ArcT
an[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)] + I*b*d*(a + b*ArcTan[c*x])*PolyLog
[2, -1 + 2/(1 + I*c*x)] - (b^2*d*PolyLog[3, 1 - 2/(1 + I*c*x)])/2 + (b^2*d
*PolyLog[3, -1 + 2/(1 + I*c*x)])/2
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.90 (sec) , antiderivative size = 3480, normalized size of antiderivative = 16.11

| method | result | size |
|---|---|---|
| parts | Expression too large to display | 3480 |
| derivativedivides | Expression too large to display | 3482 |
| default | Expression too large to display | 3482 |

input
```
int((d+I*c*d*x)*(a+b*arctan(c*x))^2/x,x,method=_RETURNVERBOSE)
```

output
```
d*a^2*(I*c*x+ln(x))+d*b^2*(1/4*I*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*cs
gn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1
+I*c*x)^2/(c^2*x^2+1)))*(2*I*arctan(c*x)*ln(1+(1+I*c*x)^2/(c^2*x^2+1))+2*a
rctan(c*x)^2+polylog(2,-(1+I*c*x)^2/(c^2*x^2+1)))-1/4*I*Pi*csgn(I*((1+I*c*
x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(((1+I*c*x)^2/(c^2*x^
2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*(2*I*arctan(c*x)*ln(1+(1+I*c*x)^2/(
c^2*x^2+1))+2*arctan(c*x)^2+polylog(2,-(1+I*c*x)^2/(c^2*x^2+1)))-1/2*Pi*ar
ctan(c*x)*ln(1+(1+I*c*x)^2/(c^2*x^2+1))+1/2*Pi*arctan(c*x)*ln(1+I*(1+I*c*x
)/(c^2*x^2+1)^(1/2))+1/2*Pi*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2)
)+I*arctan(c*x)*ln(1+(1+I*c*x)^2/(c^2*x^2+1))+arctan(c*x)^2+1/2*polylog(2,
-(1+I*c*x)^2/(c^2*x^2+1))+dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+dilog(1-I
*(1+I*c*x)/(c^2*x^2+1)^(1/2))+ln(c*x)*arctan(c*x)^2-arctan(c*x)^2*ln((1+I*
c*x)^2/(c^2*x^2+1)-1)+arctan(c*x)^2*ln(1-(1+I*c*x)/(c^2*x^2+1)^(1/2))+arct
an(c*x)^2*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))+I*arctan(c*x)*polylog(2,-(1+I*
c*x)^2/(c^2*x^2+1))-2*I*arctan(c*x)*polylog(2,(1+I*c*x)/(c^2*x^2+1)^(1/2))
-2*I*arctan(c*x)*polylog(2,-(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/2*polylog(3,-(1
+I*c*x)^2/(c^2*x^2+1))+2*polylog(3,(1+I*c*x)/(c^2*x^2+1)^(1/2))+2*polylog(
3,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+I*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)
^(1/2))+I*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/4*I*Pi*csgn(I*
((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(((1+I*c*x...
```

## Fricas [F]

$$\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x}\,dx=\int \frac{(i\,cdx+d)(b\arctan{(cx)}+a)^2}{x}\,dx$$

input
```
integrate((d+I*c*d*x)*(a+b*arctan(c*x))^2/x,x, algorithm="fricas")
```

output
```
integral(1/4*(4*I*a^2*c*d*x + 4*a^2*d + (-I*b^2*c*d*x - b^2*d)*log(-(c*x +
 I)/(c*x - I))^2 - 4*(a*b*c*d*x - I*a*b*d)*log(-(c*x + I)/(c*x - I)))/x, x
)
```

## Sympy [F]

$$\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x}\,dx = id\left(\int a^2c\,dx + \int \left(-\frac{ia^2}{x}\right)\,dx + \int b^2c\,\mathrm{atan}^2\,(cx)\,dx + \int \left(-\frac{ib^2\,\mathrm{atan}^2\,(cx)}{x}\right)\,dx + \int 2abc\,\mathrm{atan}\,(cx)\,dx + \int \left(-\frac{2iab\,\mathrm{atan}\,(cx)}{x}\right)\,dx\right)$$

input
```
integrate((d+I*c*d*x)*(a+b*atan(c*x))**2/x,x)
```

output
```
I*d*(Integral(a**2*c, x) + Integral(-I*a**2/x, x) + Integral(b**2*c*atan(c
*x)**2, x) + Integral(-I*b**2*atan(c*x)**2/x, x) + Integral(2*a*b*c*atan(c
*x), x) + Integral(-2*I*a*b*atan(c*x)/x, x))
```

## Maxima [F]

$$\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x}\,dx = \int \frac{(i\,cdx+d)(b\arctan\,(cx)+a)^2}{x}\,dx$$

input
```
integrate((d+I*c*d*x)*(a+b*arctan(c*x))^2/x,x, algorithm="maxima")
```

output
```
1/4*I*b^2*c*d*x*arctan(c*x)^2 + 12*I*b^2*c^3*d*integrate(1/16*x^3*arctan(c
*x)^2/(c^2*x^3 + x), x) + 4*b^2*c^3*d*integrate(1/16*x^3*arctan(c*x)*log(c
^2*x^2 + 1)/(c^2*x^3 + x), x) + I*b^2*c^3*d*integrate(1/16*x^3*log(c^2*x^2
 + 1)^2/(c^2*x^3 + x), x) + 8*b^2*c^3*d*integrate(1/16*x^3*arctan(c*x)/(c^
2*x^3 + x), x) + 4*I*b^2*c^3*d*integrate(1/16*x^3*log(c^2*x^2 + 1)/(c^2*x^
3 + x), x) - 1/4*b^2*c*d*x*arctan(c*x)*log(c^2*x^2 + 1) - 1/16*I*b^2*c*d*x
*log(c^2*x^2 + 1)^2 + 1/4*I*b^2*d*arctan(c*x)^3 + 12*b^2*c^2*d*integrate(1
/16*x^2*arctan(c*x)^2/(c^2*x^3 + x), x) - 4*I*b^2*c^2*d*integrate(1/16*x^2
*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^3 + x), x) + 32*a*b*c^2*d*integrate(1
/16*x^2*arctan(c*x)/(c^2*x^3 + x), x) - 8*I*b^2*c^2*d*integrate(1/16*x^2*a
rctan(c*x)/(c^2*x^3 + x), x) + 1/96*b^2*d*log(c^2*x^2 + 1)^3 + I*a^2*c*d*x
 + 4*b^2*c*d*integrate(1/16*x*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^3 + x),
x) + I*b^2*c*d*integrate(1/16*x*log(c^2*x^2 + 1)^2/(c^2*x^3 + x), x) + 1/1
6*b^2*d*log(c^2*x^2 + 1)^2 + I*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*a*b*
d + 12*b^2*d*integrate(1/16*arctan(c*x)^2/(c^2*x^3 + x), x) - 4*I*b^2*d*in
tegrate(1/16*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^3 + x), x) + b^2*d*integr
ate(1/16*log(c^2*x^2 + 1)^2/(c^2*x^3 + x), x) + 32*a*b*d*integrate(1/16*ar
ctan(c*x)/(c^2*x^3 + x), x) + a^2*d*log(x)
```

## Giac [F]

$$\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x}\,dx=\int \frac{(i\,cdx+d)(b\arctan{(cx)}+a)^2}{x}\,dx$$

input
```
integrate((d+I*c*d*x)*(a+b*arctan(c*x))^2/x,x, algorithm="giac")
```

output
```
integrate((I*c*d*x + d)*(b*arctan(c*x) + a)^2/x, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x}\,dx=\int \frac{(a+b\operatorname{atan}(c\,x))^2\,(d+c\,d\,x\,1\mathrm{i})}{x}\,dx$$

input
```
int(((a + b*atan(c*x))^2*(d + c*d*x*1i))/x,x)
```

output
```
int(((a + b*atan(c*x))^2*(d + c*d*x*1i))/x, x)
```

## Reduce [F]

$$\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x}\,dx=d\left(2atan(cx)\,abcix+\left(\int atan(cx)^2\,dx\right)b^2ci+2\left(\int \frac{atan(cx)}{x}dx\right)ab+\left(\int \frac{atan(cx)^2}{x}dx\right)b^2-\log\left(c^2x^2+1\right)abi+\log(x)\,a^2+a^2cix\right)$$

input
```
int((d+I*c*d*x)*(a+b*atan(c*x))^2/x,x)
```

output
```
d*(2*atan(c*x)*a*b*c*i*x + int(atan(c*x)**2,x)*b**2*c*i + 2*int(atan(c*x)/
x,x)*a*b + int(atan(c*x)**2/x,x)*b**2 - log(c**2*x**2 + 1)*a*b*i + log(x)*
a**2 + a**2*c*i*x)
```

# 3.73 $\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x^2}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 228

$$\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x^2}\,dx = -icd(a+b\arctan(cx))^2 - \frac{d(a+b\arctan(cx))^2}{x} + 2icd(a+b\arctan(cx))^2\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right) + 2bcd(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right) - ib^2cd\,\operatorname{PolyLog}\left(2,-1+\frac{2}{1-icx}\right) + bcd(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right) - bcd(a+b\arctan(cx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx}\right) - \frac{1}{2}ib^2cd\,\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right) + \frac{1}{2}ib^2cd\,\operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)$$

output
```
-I*c*d*(a+b*arctan(c*x))^2-d*(a+b*arctan(c*x))^2/x-2*I*c*d*(a+b*arctan(c*x
))^2*arctanh(-1+2/(1+I*c*x))+2*b*c*d*(a+b*arctan(c*x))*ln(2-2/(1-I*c*x))-I
*b^2*c*d*polylog(2,-1+2/(1-I*c*x))+b*c*d*(a+b*arctan(c*x))*polylog(2,1-2/(
1+I*c*x))-b*c*d*(a+b*arctan(c*x))*polylog(2,-1+2/(1+I*c*x))-1/2*I*b^2*c*d*
polylog(3,1-2/(1+I*c*x))+1/2*I*b^2*c*d*polylog(3,-1+2/(1+I*c*x))
```

## Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.27

$$\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x^2}\,dx$$
$$= \frac{id(ia^2+a^2cx\log(x)+iab(2\arctan(cx)+cx(-2\log(cx)+\log\left(1+c^2x^2\right)))+ib^2\left(\arctan(cx)^2-2cx\operatorname{arct}\right.}{}$$

input
```
Integrate[((d + I*c*d*x)*(a + b*ArcTan[c*x])^2)/x^2,x]
```

output
```
(I*d*(I*a^2 + a^2*c*x*Log[x] + I*a*b*(2*ArcTan[c*x] + c*x*(-2*Log[c*x] + L
og[1 + c^2*x^2])) + I*b^2*(ArcTan[c*x]^2 - 2*c*x*ArcTan[c*x]*Log[1 - E^((2
*I)*ArcTan[c*x])] + I*c*x*(ArcTan[c*x]^2 + PolyLog[2, E^((2*I)*ArcTan[c*x]
)])) + I*a*b*c*x*(PolyLog[2, (-I)*c*x] - PolyLog[2, I*c*x]) + (b^2*c*x*((-
I)*Pi^3 + (16*I)*ArcTan[c*x]^3 + 24*ArcTan[c*x]^2*Log[1 - E^((-2*I)*ArcTan
[c*x])] - 24*ArcTan[c*x]^2*Log[1 + E^((2*I)*ArcTan[c*x])] + (24*I)*ArcTan[
c*x]*PolyLog[2, E^((-2*I)*ArcTan[c*x])] + (24*I)*ArcTan[c*x]*PolyLog[2, -E
^((2*I)*ArcTan[c*x])] + 12*PolyLog[3, E^((-2*I)*ArcTan[c*x])] - 12*PolyLog
[3, -E^((2*I)*ArcTan[c*x])]))/24))/x
```

## Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x^2}\,dx$$

$$\Big\downarrow \text{5411}$$

$$\int \left(\frac{d(a+b\arctan(cx))^2}{x^2}+\frac{icd(a+b\arctan(cx))^2}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$2icd\text{arctanh}\left(1-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2+bcd\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))-bcd\,\text{PolyLog}\left(2,\frac{2}{icx+1}-1\right)(a+b\arctan(cx))-icd(a+b\arctan(cx))^2-\frac{d(a+b\arctan(cx))^2}{x}+2bcd\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))-ib^2cd\,\text{PolyLog}\left(2,\frac{2}{1-icx}-1\right)-\frac{1}{2}ib^2cd\,\text{PolyLog}\left(3,1-\frac{2}{icx+1}\right)+\frac{1}{2}ib^2cd\,\text{PolyLog}\left(3,\frac{2}{icx+1}-1\right)$$

input
```
Int[((d + I*c*d*x)*(a + b*ArcTan[c*x])^2)/x^2,x]
```

output
```
(-I)*c*d*(a + b*ArcTan[c*x])^2 - (d*(a + b*ArcTan[c*x])^2)/x + (2*I)*c*d*(
a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)] + 2*b*c*d*(a + b*ArcTan[c*
x])*Log[2 - 2/(1 - I*c*x)] - I*b^2*c*d*PolyLog[2, -1 + 2/(1 - I*c*x)] + b*
c*d*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)] - b*c*d*(a + b*ArcTa
n[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)] - (I/2)*b^2*c*d*PolyLog[3, 1 - 2/(1
 + I*c*x)] + (I/2)*b^2*c*d*PolyLog[3, -1 + 2/(1 + I*c*x)]
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 8.26 (sec) , antiderivative size = 5609, normalized size of antiderivative = 24.60

| method | result | size |
|---|---|---|
| parts | Expression too large to display | 5609 |
| derivativedivides | Expression too large to display | 5613 |
| default | Expression too large to display | 5613 |

input
```
int((d+I*c*d*x)*(a+b*arctan(c*x))^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

## Fricas [F]

$$\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x^2}\,dx=\int \frac{(i\,cdx+d)(b\arctan{(cx)}+a)^2}{x^2}\,dx$$

input
```
integrate((d+I*c*d*x)*(a+b*arctan(c*x))^2/x^2,x, algorithm="fricas")
```

output
```
integral(1/4*(4*I*a^2*c*d*x + 4*a^2*d + (-I*b^2*c*d*x - b^2*d)*log(-(c*x +
 I)/(c*x - I))^2 - 4*(a*b*c*d*x - I*a*b*d)*log(-(c*x + I)/(c*x - I)))/x^2,
 x)
```

## Sympy [F]

$$\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x^2}\,dx = id\left(\int\left(-\frac{ia^2}{x^2}\right)dx + \int\frac{a^2c}{x}\,dx + \int\left(-\frac{ib^2\,\mathrm{atan}^2(cx)}{x^2}\right)dx + \int\frac{b^2c\,\mathrm{atan}^2(cx)}{x}\,dx + \int\left(-\frac{2iab\,\mathrm{atan}(cx)}{x^2}\right)dx + \int\frac{2abc\,\mathrm{atan}(cx)}{x}\,dx\right)$$

input
```
integrate((d+I*c*d*x)*(a+b*atan(c*x))**2/x**2,x)
```

output
```
I*d*(Integral(-I*a**2/x**2, x) + Integral(a**2*c/x, x) + Integral(-I*b**2*
atan(c*x)**2/x**2, x) + Integral(b**2*c*atan(c*x)**2/x, x) + Integral(-2*I
*a*b*atan(c*x)/x**2, x) + Integral(2*a*b*c*atan(c*x)/x, x))
```

## Maxima [F]

$$\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x^2}\,dx = \int \frac{(i\,cdx+d)(b\arctan(cx)+a)^2}{x^2}\,dx$$

input
```
integrate((d+I*c*d*x)*(a+b*arctan(c*x))^2/x^2,x, algorithm="maxima")
```

output
```
I*a^2*c*d*log(x) - (c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*a*b
*d - a^2*d/x - 1/96*(24*b^2*d*arctan(c*x)^2 + 24*I*b^2*d*arctan(c*x)*log(c
^2*x^2 + 1) - 6*b^2*d*log(c^2*x^2 + 1)^2 - 24*(b^2*c*d*arctan(c*x)^3 + 16*
b^2*c^3*d*integrate(1/16*x^3*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^4 + x^2),
 x) + 4*b^2*c^2*d*integrate(1/16*x^2*log(c^2*x^2 + 1)^2/(c^2*x^4 + x^2), x
) - 16*b^2*c^2*d*integrate(1/16*x^2*log(c^2*x^2 + 1)/(c^2*x^4 + x^2), x) +
 16*b^2*c*d*integrate(1/16*x*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^4 + x^2),
 x) + 32*b^2*c*d*integrate(1/16*x*arctan(c*x)/(c^2*x^4 + x^2), x) + 48*b^2
*d*integrate(1/16*arctan(c*x)^2/(c^2*x^4 + x^2), x) + 4*b^2*d*integrate(1/
16*log(c^2*x^2 + 1)^2/(c^2*x^4 + x^2), x))*x - I*(1152*b^2*c^3*d*integrate
(1/16*x^3*arctan(c*x)^2/(c^2*x^4 + x^2), x) + 3072*a*b*c^3*d*integrate(1/1
6*x^3*arctan(c*x)/(c^2*x^4 + x^2), x) + b^2*c*d*log(c^2*x^2 + 1)^3 + 24*b^
2*c*d*arctan(c*x)^2 - 384*b^2*c^2*d*integrate(1/16*x^2*arctan(c*x)*log(c^2
*x^2 + 1)/(c^2*x^4 + x^2), x) + 1152*b^2*c*d*integrate(1/16*x*arctan(c*x)^
2/(c^2*x^4 + x^2), x) + 96*b^2*c*d*integrate(1/16*x*log(c^2*x^2 + 1)^2/(c^
2*x^4 + x^2), x) + 3072*a*b*c*d*integrate(1/16*x*arctan(c*x)/(c^2*x^4 + x^
2), x) + 384*b^2*c*d*integrate(1/16*x*log(c^2*x^2 + 1)/(c^2*x^4 + x^2), x)
 - 384*b^2*d*integrate(1/16*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^4 + x^2),
x))*x)/x
```

## Giac [F]

$$\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x^2}\,dx = \int \frac{(i\,cdx+d)(b\arctan{(cx)}+a)^2}{x^2}\,dx$$

input
```
integrate((d+I*c*d*x)*(a+b*arctan(c*x))^2/x^2,x, algorithm="giac")
```

output
```
integrate((I*c*d*x + d)*(b*arctan(c*x) + a)^2/x^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x^2}\,dx=\int \frac{(a+b\,\mathrm{atan}(c\,x))^2\,(d+c\,d\,x\,1\mathrm{i})}{x^2}\,dx$$

input
```
int(((a + b*atan(c*x))^2*(d + c*d*x*1i))/x^2,x)
```

output
```
int(((a + b*atan(c*x))^2*(d + c*d*x*1i))/x^2, x)
```

## Reduce [F]

$$\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x^2}\,dx$$
$$=\frac{d\left(-atan(cx)^2\,b^2-2atan(cx)\,ab+2\left(\int \frac{atan(cx)}{c^2x^3+x}dx\right)b^2cx+2\left(\int \frac{atan(cx)}{x}dx\right)abcix+\left(\int \frac{atan(cx)^2}{x}dx\right)b^2cix}{x}$$

input
```
int((d+I*c*d*x)*(a+b*atan(c*x))^2/x^2,x)
```

output
```
(d*( - atan(c*x)**2*b**2 - 2*atan(c*x)*a*b + 2*int(atan(c*x)/(c**2*x**3 +
x),x)*b**2*c*x + 2*int(atan(c*x)/x,x)*a*b*c*i*x + int(atan(c*x)**2/x,x)*b*
*2*c*i*x - log(c**2*x**2 + 1)*a*b*c*x + log(x)*a**2*c*i*x + 2*log(x)*a*b*c
*x - a**2))/x
```

# 3.74 $\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x^3}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 159

$$\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x^3}\,dx = -\frac{bcd(a+b\arctan(cx))}{x}+\frac{1}{2}c^2d(a+b\arctan(cx))^2 -\frac{d(a+b\arctan(cx))^2}{2x^2}-\frac{icd(a+b\arctan(cx))^2}{x} +b^2c^2d\log(x)-\frac{1}{2}b^2c^2d\log\left(1+c^2x^2\right) +2ibc^2d(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right) +b^2c^2d\,\mathrm{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)$$

output

```
-b*c*d*(a+b*arctan(c*x))/x+1/2*c^2*d*(a+b*arctan(c*x))^2-1/2*d*(a+b*arctan
(c*x))^2/x^2-I*c*d*(a+b*arctan(c*x))^2/x+b^2*c^2*d*ln(x)-1/2*b^2*c^2*d*ln(
c^2*x^2+1)+2*I*b*c^2*d*(a+b*arctan(c*x))*ln(2-2/(1-I*c*x))+b^2*c^2*d*polyl
og(2,-1+2/(1-I*c*x))
```

## Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.19

$$\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x^3}\,dx =$$
$$-\frac{d\Big(a^2+2ia^2cx+2abcx-b^2(-i+cx)^2\arctan(cx)^2+2b\arctan(cx)\left(a+2iacx+bcx+ac^2x^2-2ibc^2\right.}{}$$

input
```
Integrate[((d + I*c*d*x)*(a + b*ArcTan[c*x])^2)/x^3,x]
```

output
```
-1/2*(d*(a^2 + (2*I)*a^2*c*x + 2*a*b*c*x - b^2*(-I + c*x)^2*ArcTan[c*x]^2
+ 2*b*ArcTan[c*x]*(a + (2*I)*a*c*x + b*c*x + a*c^2*x^2 - (2*I)*b*c^2*x^2*L
og[1 - E^((2*I)*ArcTan[c*x])]) - (4*I)*a*b*c^2*x^2*Log[c*x] - 2*b^2*c^2*x^
2*Log[(c*x)/Sqrt[1 + c^2*x^2]] + (2*I)*a*b*c^2*x^2*Log[1 + c^2*x^2] - 2*b^
2*c^2*x^2*PolyLog[2, E^((2*I)*ArcTan[c*x])]))/x^2
```

## Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x^3}\,dx$$
$$\Big\downarrow \text{5411}$$
$$\int \left(\frac{d(a+b\arctan(cx))^2}{x^3}+\frac{icd(a+b\arctan(cx))^2}{x^2}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}c^2d(a+b\arctan(cx))^2+2ibc^2d\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))-\frac{d(a+b\arctan(cx))^2}{2x^2}-\frac{icd(a+b\arctan(cx))^2}{x}-\frac{bcd(a+b\arctan(cx))}{x}+b^2c^2d\,\mathrm{PolyLog}\left(2,\frac{2}{1-icx}-1\right)-\frac{1}{2}b^2c^2d\log\left(c^2x^2+1\right)+b^2c^2d\log(x)$$

input
```
Int[((d + I*c*d*x)*(a + b*ArcTan[c*x])^2)/x^3,x]
```

output
```
-((b*c*d*(a + b*ArcTan[c*x]))/x) + (c^2*d*(a + b*ArcTan[c*x])^2)/2 - (d*(a
 + b*ArcTan[c*x])^2)/(2*x^2) - (I*c*d*(a + b*ArcTan[c*x])^2)/x + b^2*c^2*d
*Log[x] - (b^2*c^2*d*Log[1 + c^2*x^2])/2 + (2*I)*b*c^2*d*(a + b*ArcTan[c*x
])*Log[2 - 2/(1 - I*c*x)] + b^2*c^2*d*PolyLog[2, -1 + 2/(1 - I*c*x)]
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

## Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 356 vs. 2(149) = 298.

Time = 1.18 (sec) , antiderivative size = 357, normalized size of antiderivative = 2.25

| method | result |
|---|---|
| parts | $d\,a^2\left(-\frac{1}{2x^2}-\frac{ic}{x}\right)+d\,b^2c^2\left(-\frac{i\arctan(cx)^2}{cx}-\frac{\arctan(cx)^2}{2c^2x^2}-\frac{\arctan(cx)}{cx}+2i\arctan\left(cx\right)\ln\left(cx\right)\right.$ |
| derivativedivides | $c^2\left(d\,a^2\left(-\frac{i}{cx}-\frac{1}{2c^2x^2}\right)+d\,b^2\left(-\frac{i\arctan(cx)^2}{cx}-\frac{\arctan(cx)^2}{2c^2x^2}-\frac{\arctan(cx)}{cx}+2i\arctan\left(cx\right)\ln\left(\right.\right.\right.$ |
| default | $c^2\left(d\,a^2\left(-\frac{i}{cx}-\frac{1}{2c^2x^2}\right)+d\,b^2\left(-\frac{i\arctan(cx)^2}{cx}-\frac{\arctan(cx)^2}{2c^2x^2}-\frac{\arctan(cx)}{cx}+2i\arctan\left(cx\right)\ln\left(\right.\right.\right.$ |

input
```
int((d+I*c*d*x)*(a+b*arctan(c*x))^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
d*a^2*(-1/2/x^2-I*c/x)+d*b^2*c^2*(-I*arctan(c*x)^2/c/x-1/2/c^2/x^2*arctan(
c*x)^2-1/c/x*arctan(c*x)+2*I*arctan(c*x)*ln(c*x)-I*arctan(c*x)*ln(c^2*x^2+
1)-1/2*arctan(c*x)^2+ln(c*x)-1/2*ln(c^2*x^2+1)-ln(c*x)*ln(1+I*c*x)+ln(c*x)
*ln(1-I*c*x)-dilog(1+I*c*x)+dilog(1-I*c*x)+1/2*ln(c*x-I)*ln(c^2*x^2+1)-1/2
*dilog(-1/2*I*(c*x+I))-1/2*ln(c*x-I)*ln(-1/2*I*(c*x+I))-1/4*ln(c*x-I)^2-1/
2*ln(c*x+I)*ln(c^2*x^2+1)+1/2*dilog(1/2*I*(c*x-I))+1/2*ln(c*x+I)*ln(1/2*I*
(c*x-I))+1/4*ln(c*x+I)^2)+2*d*a*b*c^2*(-I*arctan(c*x)/c/x-1/2/c^2/x^2*arct
an(c*x)-1/2/c/x+I*ln(c*x)-1/2*I*ln(c^2*x^2+1)-1/2*arctan(c*x))
```

## Fricas [F]

$$\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x^3}\,dx=\int \frac{(i\,cdx+d)(b\arctan\left(cx\right)+a)^2}{x^3}\,dx$$

input
```
integrate((d+I*c*d*x)*(a+b*arctan(c*x))^2/x^3,x, algorithm="fricas")
```

output
```
1/8*(8*x^2*integral(1/2*(2*I*a^2*c^3*d*x^3 + 2*a^2*c^2*d*x^2 + 2*I*a^2*c*d
*x + 2*a^2*d - (2*a*b*c^3*d*x^3 + 2*(-I*a*b + b^2)*c^2*d*x^2 + (2*a*b - I*
b^2)*c*d*x - 2*I*a*b*d)*log(-(c*x + I)/(c*x - I)))/(c^2*x^5 + x^3), x) + (
2*I*b^2*c*d*x + b^2*d)*log(-(c*x + I)/(c*x - I))^2)/x^2
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)*(a+b*atan(c*x))**2/x**3,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x^3}\,dx = \int \frac{(i\,cdx+d)(b\arctan(cx)+a)^2}{x^3}\,dx$$

input
```
integrate((d+I*c*d*x)*(a+b*arctan(c*x))^2/x^3,x, algorithm="maxima")
```

output
```
-I*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*a*b*c*d - ((c*arcta
n(c*x) + 1/x)*c + arctan(c*x)/x^2)*a*b*d + 1/2*((arctan(c*x)^2 - log(c^2*x
^2 + 1) + 2*log(x))*c^2 - 2*(c*arctan(c*x) + 1/x)*c*arctan(c*x))*b^2*d + 1
/16*I*(4*(c*arctan(c*x)^3 + 4*c^2*integrate(1/16*x^2*log(c^2*x^2 + 1)^2/(c
^2*x^4 + x^2), x) - 16*c^2*integrate(1/16*x^2*log(c^2*x^2 + 1)/(c^2*x^4 +
x^2), x) + 32*c*integrate(1/16*x*arctan(c*x)/(c^2*x^4 + x^2), x) + 48*inte
grate(1/16*arctan(c*x)^2/(c^2*x^4 + x^2), x) + 4*integrate(1/16*log(c^2*x^
2 + 1)^2/(c^2*x^4 + x^2), x))*x - 4*arctan(c*x)^2 + log(c^2*x^2 + 1)^2)*b^
2*c*d/x - I*a^2*c*d/x - 1/2*b^2*d*arctan(c*x)^2/x^2 - 1/2*a^2*d/x^2
```

## Giac [F]

$$\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x^3}\,dx=\int \frac{(i\,cdx+d)(b\arctan{(cx)}+a)^2}{x^3}\,dx$$

input
```
integrate((d+I*c*d*x)*(a+b*arctan(c*x))^2/x^3,x, algorithm="giac")
```

output
```
integrate((I*c*d*x + d)*(b*arctan(c*x) + a)^2/x^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x^3}\,dx=\int \frac{(a+b\,\mathrm{atan}(c\,x))^2\,(d+c\,d\,x\,1\mathrm{i})}{x^3}\,dx$$

input
```
int(((a + b*atan(c*x))^2*(d + c*d*x*1i))/x^3,x)
```

output
```
int(((a + b*atan(c*x))^2*(d + c*d*x*1i))/x^3, x)
```

## Reduce [F]

$$\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x^3}\,dx$$
$$=\frac{d\Big(-atan(cx)^2\,b^2c^2x^2-2atan(cx)^2\,b^2cix-atan(cx)^2\,b^2-2atan(cx)\,ab\,c^2x^2-4atan(cx)\,abcix-2atan}{}$$

input
```
int((d+I*c*d*x)*(a+b*atan(c*x))^2/x^3,x)
```

output
```
(d*( - atan(c*x)**2*b**2*c**2*x**2 - 2*atan(c*x)**2*b**2*c*i*x - atan(c*x)
**2*b**2 - 2*atan(c*x)*a*b*c**2*x**2 - 4*atan(c*x)*a*b*c*i*x - 2*atan(c*x)
*a*b - 2*atan(c*x)*b**2*c**2*i*x**2 - 2*atan(c*x)*b**2*c*x - 2*atan(c*x)*b
**2*i - 4*int(atan(c*x)/(c**2*x**5 + x**3),x)*b**2*i*x**2 - 2*log(c**2*x**
2 + 1)*a*b*c**2*i*x**2 - log(c**2*x**2 + 1)*b**2*c**2*x**2 + 4*log(x)*a*b*
c**2*i*x**2 + 2*log(x)*b**2*c**2*x**2 - 2*a**2*c*i*x - a**2 - 2*a*b*c*x -
2*b**2*c*i*x))/(2*x**2)
```

# 3.75 $\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x^4}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 224

$$\begin{aligned}\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x^4}\,dx = & -\frac{b^2c^2d}{3x}-\frac{1}{3}b^2c^3d\arctan(cx)\\ & -\frac{bcd(a+b\arctan(cx))}{3x^2}\\ & -\frac{ibc^2d(a+b\arctan(cx))}{x}\\ & -\frac{1}{6}ic^3d(a+b\arctan(cx))^2\\ & -\frac{d(a+b\arctan(cx))^2}{3x^3}-\frac{icd(a+b\arctan(cx))^2}{2x^2}\\ & +ib^2c^3d\log(x)-\frac{1}{2}ib^2c^3d\log\left(1+c^2x^2\right)\\ & -\frac{2}{3}bc^3d(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right)\\ & +\frac{1}{3}ib^2c^3d\,\text{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)\end{aligned}$$

output
```
-1/3*b^2*c^2*d/x-1/3*b^2*c^3*d*arctan(c*x)-1/3*b*c*d*(a+b*arctan(c*x))/x^2
-I*b*c^2*d*(a+b*arctan(c*x))/x-1/6*I*c^3*d*(a+b*arctan(c*x))^2-1/3*d*(a+b*
arctan(c*x))^2/x^3-1/2*I*c*d*(a+b*arctan(c*x))^2/x^2+I*b^2*c^3*d*ln(x)-1/2
*I*b^2*c^3*d*ln(c^2*x^2+1)-2/3*b*c^3*d*(a+b*arctan(c*x))*ln(2-2/(1-I*c*x))
+1/3*I*b^2*c^3*d*polylog(2,-1+2/(1-I*c*x))
```

## Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.07

$$\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x^4}\,dx$$
$$= \frac{d\Big(-2a^2-3ia^2cx-2abcx-6iabc^2x^2-2b^2c^2x^2-ib^2(-2i+3cx+c^3x^3)\arctan(cx)^2-2b\arctan(cx)\,(b}{}$$

input
```
Integrate[((d + I*c*d*x)*(a + b*ArcTan[c*x])^2)/x^4,x]
```

output
```
(d*(-2*a^2 - (3*I)*a^2*c*x - 2*a*b*c*x - (6*I)*a*b*c^2*x^2 - 2*b^2*c^2*x^2
 - I*b^2*(-2*I + 3*c*x + c^3*x^3)*ArcTan[c*x]^2 - 2*b*ArcTan[c*x]*(b*c*x*(
1 + (3*I)*c*x + c^2*x^2) + a*(2 + (3*I)*c*x + (3*I)*c^3*x^3) + 2*b*c^3*x^3
*Log[1 - E^((2*I)*ArcTan[c*x])]) - 4*a*b*c^3*x^3*Log[c*x] + (6*I)*b^2*c^3*
x^3*Log[(c*x)/Sqrt[1 + c^2*x^2]] + 2*a*b*c^3*x^3*Log[1 + c^2*x^2] + (2*I)*
b^2*c^3*x^3*PolyLog[2, E^((2*I)*ArcTan[c*x])]))/(6*x^3)
```

## Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x^4}\,dx$$
$$\Big\downarrow \text{5411}$$
$$\int \left(\frac{d(a+b\arctan(cx))^2}{x^4}+\frac{icd(a+b\arctan(cx))^2}{x^3}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{1}{6}ic^3d(a+b\arctan(cx))^2-\frac{2}{3}bc^3d\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))-\frac{ibc^2d(a+b\arctan(cx))}{x}-\frac{d(a+b\arctan(cx))^2}{3x^3}-\frac{icd(a+b\arctan(cx))^2}{2x^2}-\frac{bcd(a+b\arctan(cx))}{3x^2}-\frac{1}{3}b^2c^3d\arctan(cx)+\frac{1}{3}ib^2c^3d\operatorname{PolyLog}\left(2,\frac{2}{1-icx}-1\right)+ib^2c^3d\log(x)-\frac{b^2c^2d}{3x}-\frac{1}{2}ib^2c^3d\log\left(c^2x^2+1\right)$$

input
```
Int[((d + I*c*d*x)*(a + b*ArcTan[c*x])^2)/x^4,x]
```

output
```
-1/3*(b^2*c^2*d)/x - (b^2*c^3*d*ArcTan[c*x])/3 - (b*c*d*(a + b*ArcTan[c*x]
))/(3*x^2) - (I*b*c^2*d*(a + b*ArcTan[c*x]))/x - (I/6)*c^3*d*(a + b*ArcTan
[c*x])^2 - (d*(a + b*ArcTan[c*x])^2)/(3*x^3) - ((I/2)*c*d*(a + b*ArcTan[c*
x])^2)/x^2 + I*b^2*c^3*d*Log[x] - (I/2)*b^2*c^3*d*Log[1 + c^2*x^2] - (2*b*
c^3*d*(a + b*ArcTan[c*x])*Log[2 - 2/(1 - I*c*x)])/3 + (I/3)*b^2*c^3*d*Poly
Log[2, -1 + 2/(1 - I*c*x)]
```

**Defintions of rubi rules used**

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

## Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 407 vs. $2(198) = 396$.

Time = 1.26 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.82

| method | result |
|---|---|
| parts | $d\,a^2\left(-\frac{ic}{2x^2}-\frac{1}{3x^3}\right)+d\,b^2c^3\left(-\frac{\arctan(cx)^2}{3c^3x^3}+\frac{i\,\mathrm{dilog}(-icx+1)}{3}+\frac{\arctan(cx)\ln(c^2x^2+1)}{3}+\frac{i\left(\ln(cx-i)\right.}{}\right.$ |
| derivativedivides | $c^3\left(d\,a^2\left(-\frac{i}{2c^2x^2}-\frac{1}{3c^3x^3}\right)+d\,b^2\left(-\frac{\arctan(cx)^2}{3c^3x^3}+\frac{i\,\mathrm{dilog}(-icx+1)}{3}+\frac{\arctan(cx)\ln(c^2x^2+1)}{3}+\frac{i\left(\ln\right.}{}\right.\right.$ |
| default | $c^3\left(d\,a^2\left(-\frac{i}{2c^2x^2}-\frac{1}{3c^3x^3}\right)+d\,b^2\left(-\frac{\arctan(cx)^2}{3c^3x^3}+\frac{i\,\mathrm{dilog}(-icx+1)}{3}+\frac{\arctan(cx)\ln(c^2x^2+1)}{3}+\frac{i\left(\ln\right.}{}\right.\right.$ |

input
```
int((d+I*c*d*x)*(a+b*arctan(c*x))^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
d*a^2*(-1/2*I*c/x^2-1/3/x^3)+d*b^2*c^3*(-1/3/c^3/x^3*arctan(c*x)^2+1/3*I*d
ilog(1-I*c*x)+1/3*arctan(c*x)*ln(c^2*x^2+1)+1/6*I*(ln(c*x-I)*ln(c^2*x^2+1)
-1/2*ln(c*x-I)^2-dilog(-1/2*I*(c*x+I))-ln(c*x-I)*ln(-1/2*I*(c*x+I)))-1/3*I
*dilog(1+I*c*x)-1/3/c^2/x^2*arctan(c*x)-2/3*ln(c*x)*arctan(c*x)-1/3*I*ln(c
*x)*ln(1+I*c*x)+I*ln(c*x)-1/3*arctan(c*x)-1/2*I*arctan(c*x)^2-1/3/c/x-1/6*
I*(ln(c*x+I)*ln(c^2*x^2+1)-1/2*ln(c*x+I)^2-dilog(1/2*I*(c*x-I))-ln(c*x+I)*
ln(1/2*I*(c*x-I)))+1/3*I*ln(c*x)*ln(1-I*c*x)-1/2*I*ln(c^2*x^2+1)-I*arctan(
c*x)/c/x-1/2*I*arctan(c*x)^2/c^2/x^2)+2*d*a*b*c^3*(-1/3/c^3/x^3*arctan(c*x
)-1/2*I*arctan(c*x)/c^2/x^2+1/6*ln(c^2*x^2+1)-1/2*I*arctan(c*x)-1/2*I/c/x-
1/6/c^2/x^2-1/3*ln(c*x))
```

## Fricas [F]

$$\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x^4}\,dx=\int \frac{(i\,cdx+d)(b\arctan\left(cx\right)+a)^2}{x^4}\,dx$$

input
```
integrate((d+I*c*d*x)*(a+b*arctan(c*x))^2/x^4,x, algorithm="fricas")
```

output
```
1/24*(24*x^3*integral(1/6*(6*I*a^2*c^3*d*x^3 + 6*a^2*c^2*d*x^2 + 6*I*a^2*c
*d*x + 6*a^2*d - (6*a*b*c^3*d*x^3 + 3*(-2*I*a*b + b^2)*c^2*d*x^2 + 2*(3*a*
b - I*b^2)*c*d*x - 6*I*a*b*d)*log(-(c*x + I)/(c*x - I)))/(c^2*x^6 + x^4),
x) + (3*I*b^2*c*d*x + 2*b^2*d)*log(-(c*x + I)/(c*x - I))^2)/x^3
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x^4}\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)*(a+b*atan(c*x))**2/x**4,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x^4}\,dx = \int \frac{(i\,cdx+d)(b\arctan{(cx)}+a)^2}{x^4}\,dx$$

input
```
integrate((d+I*c*d*x)*(a+b*arctan(c*x))^2/x^4,x, algorithm="maxima")
```

output
```
-I*((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*a*b*c*d + 1/3*((c^2*log(c^2
*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*a*b*d - 1/2*I*a^2
*c*d/x^2 - 1/3*a^2*d/x^3 + 1/96*(96*I*x^3*integrate(1/48*(20*b^2*c^2*d*x^2
*arctan(c*x) + 36*(b^2*c^3*d*x^3 + b^2*c*d*x)*arctan(c*x)^2 + 3*(b^2*c^3*d
*x^3 + b^2*c*d*x)*log(c^2*x^2 + 1)^2 - 2*(3*b^2*c^3*d*x^3 - 2*b^2*c*d*x +
6*(b^2*c^2*d*x^2 + b^2*d)*arctan(c*x))*log(c^2*x^2 + 1))/(c^2*x^6 + x^4),
x) + 96*x^3*integrate(1/48*(36*(b^2*c^2*d*x^2 + b^2*d)*arctan(c*x)^2 + 3*(
b^2*c^2*d*x^2 + b^2*d)*log(c^2*x^2 + 1)^2 - 4*(3*b^2*c^3*d*x^3 - 2*b^2*c*d
*x)*arctan(c*x) - 2*(5*b^2*c^2*d*x^2 - 6*(b^2*c^3*d*x^3 + b^2*c*d*x)*arcta
n(c*x))*log(c^2*x^2 + 1))/(c^2*x^6 + x^4), x) - 4*(3*I*b^2*c*d*x + 2*b^2*d
)*arctan(c*x)^2 + 4*(3*b^2*c*d*x - 2*I*b^2*d)*arctan(c*x)*log(c^2*x^2 + 1)
 + (3*I*b^2*c*d*x + 2*b^2*d)*log(c^2*x^2 + 1)^2)/x^3
```

## Giac [F]

$$\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x^4}\,dx = \int \frac{(i\,cdx+d)(b\arctan(cx)+a)^2}{x^4}\,dx$$

input
```
integrate((d+I*c*d*x)*(a+b*arctan(c*x))^2/x^4,x, algorithm="giac")
```

output
```
integrate((I*c*d*x + d)*(b*arctan(c*x) + a)^2/x^4, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x^4}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^2\,(d+c\,d\,x\,1\mathrm{i})}{x^4}\,dx$$

input
```
int(((a + b*atan(c*x))^2*(d + c*d*x*1i))/x^4,x)
```

output
```
int(((a + b*atan(c*x))^2*(d + c*d*x*1i))/x^4, x)
```

## Reduce [F]

$$\int \frac{(d+icdx)(a+b\arctan(cx))^2}{x^4}\,dx$$
$$= \frac{d\Big(-3atan(cx)^2\,b^2c^3i\,x^3 - 3atan(cx)^2\,b^2cix - 2atan(cx)^2\,b^2 - 6atan(cx)\,ab\,c^3i\,x^3 - 6atan(cx)\,abcix - 4}{}$$

input
```
int((d+I*c*d*x)*(a+b*atan(c*x))^2/x^4,x)
```

output

```
(d*( - 3*atan(c*x)**2*b**2*c**3*i*x**3 - 3*atan(c*x)**2*b**2*c*i*x - 2*ata
n(c*x)**2*b**2 - 6*atan(c*x)*a*b*c**3*i*x**3 - 6*atan(c*x)*a*b*c*i*x - 4*a
tan(c*x)*a*b - 6*atan(c*x)*b**2*c**2*i*x**2 + 4*int(atan(c*x)/(c**2*x**5 +
 x**3),x)*b**2*c*x**3 + 2*log(c**2*x**2 + 1)*a*b*c**3*x**3 - 3*log(c**2*x*
*2 + 1)*b**2*c**3*i*x**3 - 4*log(x)*a*b*c**3*x**3 + 6*log(x)*b**2*c**3*i*x
**3 - 3*a**2*c*i*x - 2*a**2 - 6*a*b*c**2*i*x**2 - 2*a*b*c*x))/(6*x**3)
```

# 3.76 $\int x^3(d+icdx)^2(a+b\arctan(cx))^2\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 373

$$\begin{aligned}\int x^3(d+icdx)^2(a+b\arctan(cx))^2\,dx &= \frac{5abd^2x}{6c^3}-\frac{3ib^2d^2x}{5c^3}+\frac{31b^2d^2x^2}{180c^2}+\frac{ib^2d^2x^3}{15c}-\frac{1}{60}b^2d^2x^4+\frac{3ib^2d^2\arctan(cx)}{5c^4}\\&\quad+\frac{5b^2d^2x\arctan(cx)}{6c^3}+\frac{2ibd^2x^2(a+b\arctan(cx))}{5c^2}-\frac{5bd^2x^3(a+b\arctan(cx))}{18c}\\&\quad-\frac{1}{5}ibd^2x^4(a+b\arctan(cx))+\frac{1}{15}bcd^2x^5(a+b\arctan(cx))\\&\quad-\frac{49d^2(a+b\arctan(cx))^2}{60c^4}+\frac{1}{4}d^2x^4(a+b\arctan(cx))^2+\frac{2}{5}icd^2x^5(a+b\arctan(cx))^2\\&\quad-\frac{1}{6}c^2d^2x^6(a+b\arctan(cx))^2+\frac{4ibd^2(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{5c^4}\\&\quad-\frac{53b^2d^2\log(1+c^2x^2)}{90c^4}-\frac{2b^2d^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{5c^4}\end{aligned}$$

output
```
5/6*a*b*d^2*x/c^3+1/15*I*b^2*d^2*x^3/c+31/180*b^2*d^2*x^2/c^2+2/5*I*b*d^2*
x^2*(a+b*arctan(c*x))/c^2-1/60*b^2*d^2*x^4+4/5*I*b*d^2*(a+b*arctan(c*x))*l
n(2/(1+I*c*x))/c^4+5/6*b^2*d^2*x*arctan(c*x)/c^3-3/5*I*b^2*d^2*x/c^3-5/18*
b*d^2*x^3*(a+b*arctan(c*x))/c+3/5*I*b^2*d^2*arctan(c*x)/c^4+1/15*b*c*d^2*x
^5*(a+b*arctan(c*x))-49/60*d^2*(a+b*arctan(c*x))^2/c^4+1/4*d^2*x^4*(a+b*ar
ctan(c*x))^2-1/5*I*b*d^2*x^4*(a+b*arctan(c*x))-1/6*c^2*d^2*x^6*(a+b*arctan
(c*x))^2+2/5*I*c*d^2*x^5*(a+b*arctan(c*x))^2-53/90*b^2*d^2*ln(c^2*x^2+1)/c
^4-2/5*b^2*d^2*polylog(2,1-2/(1+I*c*x))/c^4
```

## Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 342, normalized size of antiderivative = 0.92

$$\int x^3(d+icdx)^2(a+b\arctan(cx))^2\,dx$$
$$= \frac{d^2\big(108iab + 34b^2 + 150abcx - 108ib^2cx + 72iabc^2x^2 + 31b^2c^2x^2 - 50abc^3x^3 + 12ib^2c^3x^3 + 45a^2c^4x^4 - 3}{}$$

input
```
Integrate[x^3*(d + I*c*d*x)^2*(a + b*ArcTan[c*x])^2,x]
```

output
```
(d^2*((108*I)*a*b + 34*b^2 + 150*a*b*c*x - (108*I)*b^2*c*x + (72*I)*a*b*c^
2*x^2 + 31*b^2*c^2*x^2 - 50*a*b*c^3*x^3 + (12*I)*b^2*c^3*x^3 + 45*a^2*c^4*
x^4 - (36*I)*a*b*c^4*x^4 - 3*b^2*c^4*x^4 + (72*I)*a^2*c^5*x^5 + 12*a*b*c^5
*x^5 - 30*a^2*c^6*x^6 - 3*b^2*(1 - 15*c^4*x^4 - (24*I)*c^5*x^5 + 10*c^6*x^
6)*ArcTan[c*x]^2 + 2*b*ArcTan[c*x]*(b*(54*I + 75*c*x + (36*I)*c^2*x^2 - 25
*c^3*x^3 - (18*I)*c^4*x^4 + 6*c^5*x^5) + a*(-75 + 45*c^4*x^4 + (72*I)*c^5*
x^5 - 30*c^6*x^6) + (72*I)*b*Log[1 + E^((2*I)*ArcTan[c*x])]) - (72*I)*a*b*
Log[1 + c^2*x^2] - 106*b^2*Log[1 + c^2*x^2] + 72*b^2*PolyLog[2, -E^((2*I)*
ArcTan[c*x])]))/(180*c^4)
```

## Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(d+icdx)^2(a+b\arctan(cx))^2\,dx$$

$$\Big\downarrow \text{ 5411}$$

$$\int \left(-c^2d^2x^5(a+b\arctan(cx))^2+2icd^2x^4(a+b\arctan(cx))^2+d^2x^3(a+b\arctan(cx))^2\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{49d^2(a+b\arctan(cx))^2}{60c^4}+\frac{4ibd^2\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{5c^4}-\frac{1}{6}c^2d^2x^6(a+b\arctan(cx))^2+\frac{2ibd^2x^2(a+b\arctan(cx))}{5c^2}+\frac{2}{5}icd^2x^5(a+b\arctan(cx))^2+\frac{1}{15}bcd^2x^5(a+b\arctan(cx))+\frac{1}{4}d^2x^4(a+b\arctan(cx))^2-\frac{1}{5}ibd^2x^4(a+b\arctan(cx))-\frac{5bd^2x^3(a+b\arctan(cx))}{18c}+\frac{5abd^2x}{6c^3}+\frac{3ib^2d^2\arctan(cx)}{5c^4}+\frac{5b^2d^2x\arctan(cx)}{6c^3}-\frac{2b^2d^2\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{5c^4}-\frac{3ib^2d^2x}{5c^3}+\frac{31b^2d^2x^2}{180c^2}-\frac{53b^2d^2\log\left(c^2x^2+1\right)}{90c^4}+\frac{ib^2d^2x^3}{15c}-\frac{1}{60}b^2d^2x^4$$

input
```
Int[x^3*(d + I*c*d*x)^2*(a + b*ArcTan[c*x])^2,x]
```

output
```
(5*a*b*d^2*x)/(6*c^3) - (((3*I)/5)*b^2*d^2*x)/c^3 + (31*b^2*d^2*x^2)/(180*
c^2) + ((I/15)*b^2*d^2*x^3)/c - (b^2*d^2*x^4)/60 + (((3*I)/5)*b^2*d^2*ArcT
an[c*x])/c^4 + (5*b^2*d^2*x*ArcTan[c*x])/(6*c^3) + (((2*I)/5)*b*d^2*x^2*(a
 + b*ArcTan[c*x]))/c^2 - (5*b*d^2*x^3*(a + b*ArcTan[c*x]))/(18*c) - (I/5)*
b*d^2*x^4*(a + b*ArcTan[c*x]) + (b*c*d^2*x^5*(a + b*ArcTan[c*x]))/15 - (49
*d^2*(a + b*ArcTan[c*x])^2)/(60*c^4) + (d^2*x^4*(a + b*ArcTan[c*x])^2)/4 +
 ((2*I)/5)*c*d^2*x^5*(a + b*ArcTan[c*x])^2 - (c^2*d^2*x^6*(a + b*ArcTan[c*
x])^2)/6 + (((4*I)/5)*b*d^2*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/c^4 -
(53*b^2*d^2*Log[1 + c^2*x^2])/(90*c^4) - (2*b^2*d^2*PolyLog[2, 1 - 2/(1 +
I*c*x)])/(5*c^4)
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

## Maple [A] (verified)

Time = 1.40 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.19

| method | result |
|---|---|
| parts | $d^2a^2\left(-\frac{1}{6}c^2x^6+\frac{2}{5}icx^5+\frac{1}{4}x^4\right)+\frac{d^2b^2\left(-\frac{c^6x^6\arctan(cx)^2}{6}+\frac{3i\arctan(cx)}{5}+\frac{c^4x^4\arctan(cx)^2}{4}+\frac{5cx\arctan(cx)}{6}+\right.}{}$ |
| derivativedivides | $\frac{d^2a^2\left(-\frac{1}{6}c^6x^6+\frac{2}{5}ic^5x^5+\frac{1}{4}c^4x^4\right)+d^2b^2\left(-\frac{c^6x^6\arctan(cx)^2}{6}+\frac{3i\arctan(cx)}{5}+\frac{c^4x^4\arctan(cx)^2}{4}+\frac{5cx\arctan(cx)}{6}+\frac{c^5x^5\operatorname{arcta}}{15}\right.}{}$ |
| default | $\frac{d^2a^2\left(-\frac{1}{6}c^6x^6+\frac{2}{5}ic^5x^5+\frac{1}{4}c^4x^4\right)+d^2b^2\left(-\frac{c^6x^6\arctan(cx)^2}{6}+\frac{3i\arctan(cx)}{5}+\frac{c^4x^4\arctan(cx)^2}{4}+\frac{5cx\arctan(cx)}{6}+\frac{c^5x^5\operatorname{arcta}}{15}\right.}{}$ |
| risch | $\frac{d^2b^2\ln(-icx+1)x^4}{10}-\frac{d^2b^2\ln(-icx+1)^2x^4}{16}+\frac{233d^2b^2\ln(-icx+1)}{7200c^4}+\frac{49d^2b^2\ln(-icx+1)^2}{240c^4}-\frac{2d^2b^2\operatorname{dilog}\left(\frac{1}{2}-\frac{icx}{2}\right.}{5c^4}$ |

input
```
int(x^3*(d+I*c*d*x)^2*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
d^2*a^2*(-1/6*c^2*x^6+2/5*I*c*x^5+1/4*x^4)+d^2*b^2/c^4*(-1/6*c^6*x^6*arcta
n(c*x)^2+3/5*I*arctan(c*x)+1/4*c^4*x^4*arctan(c*x)^2+5/6*c*x*arctan(c*x)+1
/15*c^5*x^5*arctan(c*x)+2/5*I*arctan(c*x)^2*c^5*x^5-5/18*c^3*x^3*arctan(c*
x)-1/5*I*arctan(c*x)*c^4*x^4-2/5*I*arctan(c*x)*ln(c^2*x^2+1)-5/12*arctan(c
*x)^2-1/5*ln(c*x+I)*ln(c^2*x^2+1)+1/5*ln(c*x-I)*ln(c^2*x^2+1)+1/10*ln(c*x+
I)^2+1/5*ln(c*x+I)*ln(1/2*I*(c*x-I))-1/10*ln(c*x-I)^2-1/5*ln(c*x-I)*ln(-1/
2*I*(c*x+I))+1/5*dilog(1/2*I*(c*x-I))-1/5*dilog(-1/2*I*(c*x+I))+1/15*I*c^3
*x^3-1/60*c^4*x^4+2/5*I*arctan(c*x)*c^2*x^2+31/180*c^2*x^2-53/90*ln(c^2*x^
2+1)-3/5*I*c*x)+2*d^2*a*b/c^4*(-1/6*c^6*x^6*arctan(c*x)+2/5*I*arctan(c*x)*
c^5*x^5+1/4*c^4*x^4*arctan(c*x)+5/12*c*x+1/30*c^5*x^5-1/10*I*c^4*x^4-5/36*
c^3*x^3+1/5*I*c^2*x^2-1/5*I*ln(c^2*x^2+1)-5/12*arctan(c*x))
```

## Fricas [F]

$$\int x^3(d+icdx)^2(a+b\arctan(cx))^2\,dx=\int(i\,cdx+d)^2(b\arctan(cx)+a)^2x^3\,dx$$

input
```
integrate(x^3*(d+I*c*d*x)^2*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

output
```
1/240*(10*b^2*c^2*d^2*x^6 - 24*I*b^2*c*d^2*x^5 - 15*b^2*d^2*x^4)*log(-(c*x
 + I)/(c*x - I))^2 + integral(-1/60*(60*a^2*c^4*d^2*x^7 - 120*I*a^2*c^3*d^
2*x^6 - 120*I*a^2*c*d^2*x^4 - 60*a^2*d^2*x^3 - (-60*I*a*b*c^4*d^2*x^7 - 10
*(12*a*b - I*b^2)*c^3*d^2*x^6 + 24*b^2*c^2*d^2*x^5 - 15*(8*a*b + I*b^2)*c*
d^2*x^4 + 60*I*a*b*d^2*x^3)*log(-(c*x + I)/(c*x - I)))/(c^2*x^2 + 1), x)
```

## Sympy [F(-1)]

Timed out.

$$\int x^3(d+icdx)^2(a+b\arctan(cx))^2\,dx = \text{Timed out}$$

input
```
integrate(x**3*(d+I*c*d*x)**2*(a+b*atan(c*x))**2,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int x^3(d+icdx)^2(a+b\arctan(cx))^2\,dx = \int (i\,cdx+d)^2(b\arctan(cx)+a)^2x^3\,dx$$

input
```
integrate(x^3*(d+I*c*d*x)^2*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

output
```
-1/6*a^2*c^2*d^2*x^6 + 2/5*I*a^2*c*d^2*x^5 + 1/4*b^2*d^2*x^4*arctan(c*x)^2
 + 1/4*a^2*d^2*x^4 - 1/45*(15*x^6*arctan(c*x) - c*((3*c^4*x^5 - 5*c^2*x^3
+ 15*x)/c^6 - 15*arctan(c*x)/c^7))*a*b*c^2*d^2 + 1/5*I*(4*x^5*arctan(c*x)
- c*((c^2*x^4 - 2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*a*b*c*d^2 + 1/6*(3*x
^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*a*b*d^2 - 1/
12*(2*c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5)*arctan(c*x) - (c^2*x^2 +
 3*arctan(c*x)^2 - 4*log(c^2*x^2 + 1))/c^4)*b^2*d^2 - 1/120*(5*b^2*c^2*d^2
*x^6 - 12*I*b^2*c*d^2*x^5)*arctan(c*x)^2 + 1/120*(-5*I*b^2*c^2*d^2*x^6 - 1
2*b^2*c*d^2*x^5)*arctan(c*x)*log(c^2*x^2 + 1) + 1/480*(5*b^2*c^2*d^2*x^6 -
 12*I*b^2*c*d^2*x^5)*log(c^2*x^2 + 1)^2 - integrate(-1/240*(68*b^2*c^3*d^2
*x^6*arctan(c*x) - 180*(b^2*c^4*d^2*x^7 + b^2*c^2*d^2*x^5)*arctan(c*x)^2 -
 15*(b^2*c^4*d^2*x^7 + b^2*c^2*d^2*x^5)*log(c^2*x^2 + 1)^2 - 2*(5*b^2*c^4*
d^2*x^7 - 12*b^2*c^2*d^2*x^5 - 60*(b^2*c^3*d^2*x^6 + b^2*c*d^2*x^4)*arctan
(c*x))*log(c^2*x^2 + 1))/(c^2*x^2 + 1), x) + I*integrate(1/120*(180*(b^2*c
^3*d^2*x^6 + b^2*c*d^2*x^4)*arctan(c*x)^2 + 15*(b^2*c^3*d^2*x^6 + b^2*c*d^
2*x^4)*log(c^2*x^2 + 1)^2 + 2*(5*b^2*c^4*d^2*x^7 - 12*b^2*c^2*d^2*x^5)*arc
tan(c*x) + (17*b^2*c^3*d^2*x^6 + 30*(b^2*c^4*d^2*x^7 + b^2*c^2*d^2*x^5)*ar
ctan(c*x))*log(c^2*x^2 + 1))/(c^2*x^2 + 1), x)
```

## Giac [F]

$$\int x^3(d+icdx)^2(a+b\arctan(cx))^2\,dx=\int (i\,cdx+d)^2(b\arctan{(cx)}+a)^2x^3\,dx$$

input
```
integrate(x^3*(d+I*c*d*x)^2*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

output
```
integrate((I*c*d*x + d)^2*(b*arctan(c*x) + a)^2*x^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int x^3(d+icdx)^2(a+b\arctan(cx))^2\,dx=\int x^3\,{\left(a+b\,\mathrm{atan}(c\,x)\right)}^2\,{\left(d+c\,d\,x\,1{}\mathrm{i}\right)}^2\,dx$$

input
```
int(x^3*(a + b*atan(c*x))^2*(d + c*d*x*1i)^2,x)
```

output
```
int(x^3*(a + b*atan(c*x))^2*(d + c*d*x*1i)^2, x)
```

## Reduce [F]

$$\int x^3(d+icdx)^2(a+b\arctan(cx))^2\,dx$$
$$=\frac{d^2\left(90atan(cx)\,ab\,c^4x^4+72atan(cx)^2\,b^2c^5i\,x^5-72atan(cx)^2\,b^2cix-36atan(cx)\,b^2c^4i\,x^4+72atan(cx)\,b^2\right.}{}$$

input
```
int(x^3*(d+I*c*d*x)^2*(a+b*atan(c*x))^2,x)
```

output
```
(d**2*( - 30*atan(c*x)**2*b**2*c**6*x**6 + 72*atan(c*x)**2*b**2*c**5*i*x**
5 + 45*atan(c*x)**2*b**2*c**4*x**4 - 72*atan(c*x)**2*b**2*c*i*x - 75*atan(
c*x)**2*b**2 - 60*atan(c*x)*a*b*c**6*x**6 + 144*atan(c*x)*a*b*c**5*i*x**5
+ 90*atan(c*x)*a*b*c**4*x**4 - 150*atan(c*x)*a*b + 12*atan(c*x)*b**2*c**5*
x**5 - 36*atan(c*x)*b**2*c**4*i*x**4 - 50*atan(c*x)*b**2*c**3*x**3 + 72*at
an(c*x)*b**2*c**2*i*x**2 + 150*atan(c*x)*b**2*c*x + 108*atan(c*x)*b**2*i +
 72*int(atan(c*x)**2,x)*b**2*c*i - 72*log(c**2*x**2 + 1)*a*b*i - 106*log(c
**2*x**2 + 1)*b**2 - 30*a**2*c**6*x**6 + 72*a**2*c**5*i*x**5 + 45*a**2*c**
4*x**4 + 12*a*b*c**5*x**5 - 36*a*b*c**4*i*x**4 - 50*a*b*c**3*x**3 + 72*a*b
*c**2*i*x**2 + 150*a*b*c*x - 3*b**2*c**4*x**4 + 12*b**2*c**3*i*x**3 + 31*b
**2*c**2*x**2 - 108*b**2*c*i*x))/(180*c**4)
```

# 3.77 $\int x^2(d+icdx)^2(a+b\arctan(cx))^2\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 333

$$\begin{aligned}\int x^2(d+icdx)^2(a+b\arctan(cx))^2\,dx &= \frac{iabd^2x}{c^2}+\frac{19b^2d^2x}{30c^2}+\frac{ib^2d^2x^2}{6c}-\frac{1}{30}b^2d^2x^3-\frac{19b^2d^2\arctan(cx)}{30c^3}+\frac{ib^2d^2x\arctan(cx)}{c^2}\\&\quad-\frac{8bd^2x^2(a+b\arctan(cx))}{15c}-\frac{1}{3}ibd^2x^3(a+b\arctan(cx))+\frac{1}{10}bcd^2x^4(a+b\arctan(cx))\\&\quad-\frac{31id^2(a+b\arctan(cx))^2}{30c^3}+\frac{1}{3}d^2x^3(a+b\arctan(cx))^2+\frac{1}{2}icd^2x^4(a+b\arctan(cx))^2\\&\quad-\frac{1}{5}c^2d^2x^5(a+b\arctan(cx))^2-\frac{16bd^2(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{15c^3}\\&\quad-\frac{2ib^2d^2\log(1+c^2x^2)}{3c^3}-\frac{8ib^2d^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{15c^3}\end{aligned}$$

output

```
-8/15*I*b^2*d^2*polylog(2,1-2/(1+I*c*x))/c^3+19/30*b^2*d^2*x/c^2+I*b^2*d^2
*x*arctan(c*x)/c^2-1/30*b^2*d^2*x^3-19/30*b^2*d^2*arctan(c*x)/c^3-31/30*I*
d^2*(a+b*arctan(c*x))^2/c^3-8/15*b*d^2*x^2*(a+b*arctan(c*x))/c-2/3*I*b^2*d
^2*ln(c^2*x^2+1)/c^3+1/10*b*c*d^2*x^4*(a+b*arctan(c*x))+1/2*I*c*d^2*x^4*(a
+b*arctan(c*x))^2+1/3*d^2*x^3*(a+b*arctan(c*x))^2+I*a*b*d^2*x/c^2-1/5*c^2*
d^2*x^5*(a+b*arctan(c*x))^2-16/15*b*d^2*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/
c^3+1/6*I*b^2*d^2*x^2/c-1/3*I*b*d^2*x^3*(a+b*arctan(c*x))
```

## Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 306, normalized size of antiderivative = 0.92

$$\int x^2(d+icdx)^2(a+b\arctan(cx))^2\,dx =$$
$$-\frac{d^2\left(9ab-5ib^2-30iabcx-19b^2cx+16abc^2x^2-5ib^2c^2x^2-10a^2c^3x^3+10iabc^3x^3+b^2c^3x^3-15ia^2c^4x^4\ldots\right.}{}$$

input
```
Integrate[x^2*(d + I*c*d*x)^2*(a + b*ArcTan[c*x])^2,x]
```

output
```
-1/30*(d^2*(9*a*b - (5*I)*b^2 - (30*I)*a*b*c*x - 19*b^2*c*x + 16*a*b*c^2*x
^2 - (5*I)*b^2*c^2*x^2 - 10*a^2*c^3*x^3 + (10*I)*a*b*c^3*x^3 + b^2*c^3*x^3
 - (15*I)*a^2*c^4*x^4 - 3*a*b*c^4*x^4 + 6*a^2*c^5*x^5 + b^2*(-I + c*x)^3*(
-1 + (3*I)*c*x + 6*c^2*x^2)*ArcTan[c*x]^2 + b*ArcTan[c*x]*(b*(19 - (30*I)*
c*x + 16*c^2*x^2 + (10*I)*c^3*x^3 - 3*c^4*x^4) + 2*a*(15*I - 10*c^3*x^3 -
(15*I)*c^4*x^4 + 6*c^5*x^5) + 32*b*Log[1 + E^((2*I)*ArcTan[c*x])]) - 16*a*
b*Log[1 + c^2*x^2] + (20*I)*b^2*Log[1 + c^2*x^2] - (16*I)*b^2*PolyLog[2, -
E^((2*I)*ArcTan[c*x])]))/c^3
```

## Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(d+icdx)^2(a+b\arctan(cx))^2\,dx$$

$$\Big\downarrow \text{5411}$$

$$\int \left(-c^2d^2x^4(a+b\arctan(cx))^2+2icd^2x^3(a+b\arctan(cx))^2+d^2x^2(a+b\arctan(cx))^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{31id^2(a+b\arctan(cx))^2}{30c^3}-\frac{16bd^2\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{15c^3}-\frac{1}{5}c^2d^2x^5(a+b\arctan(cx))^2+\frac{1}{2}icd^2x^4(a+b\arctan(cx))^2+\frac{1}{10}bcd^2x^4(a+b\arctan(cx))+\frac{1}{3}d^2x^3(a+b\arctan(cx))^2-\frac{1}{3}ibd^2x^3(a+b\arctan(cx))-\frac{8bd^2x^2(a+b\arctan(cx))}{15c}+\frac{iabd^2x}{c^2}-\frac{19b^2d^2\arctan(cx)}{30c^3}+\frac{ib^2d^2x\arctan(cx)}{c^2}-\frac{8ib^2d^2\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{15c^3}+\frac{19b^2d^2x}{30c^2}-\frac{2ib^2d^2\log(c^2x^2+1)}{3c^3}+\frac{ib^2d^2x^2}{6c}-\frac{1}{30}b^2d^2x^3$$

input
```
Int[x^2*(d + I*c*d*x)^2*(a + b*ArcTan[c*x])^2,x]
```

output
```
(I*a*b*d^2*x)/c^2 + (19*b^2*d^2*x)/(30*c^2) + ((I/6)*b^2*d^2*x^2)/c - (b^2
*d^2*x^3)/30 - (19*b^2*d^2*ArcTan[c*x])/(30*c^3) + (I*b^2*d^2*x*ArcTan[c*x
])/c^2 - (8*b*d^2*x^2*(a + b*ArcTan[c*x]))/(15*c) - (I/3)*b*d^2*x^3*(a + b
*ArcTan[c*x]) + (b*c*d^2*x^4*(a + b*ArcTan[c*x]))/10 - (((31*I)/30)*d^2*(a
 + b*ArcTan[c*x])^2)/c^3 + (d^2*x^3*(a + b*ArcTan[c*x])^2)/3 + (I/2)*c*d^2
*x^4*(a + b*ArcTan[c*x])^2 - (c^2*d^2*x^5*(a + b*ArcTan[c*x])^2)/5 - (16*b
*d^2*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(15*c^3) - (((2*I)/3)*b^2*d^2
*Log[1 + c^2*x^2])/c^3 - (((8*I)/15)*b^2*d^2*PolyLog[2, 1 - 2/(1 + I*c*x)]
)/c^3
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

## Maple [A] (verified)

Time = 1.32 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.26

| method | result |
|---|---|
| parts | $d^2a^2\left(-\frac{1}{5}x^5c^2+\frac{1}{2}ic\,x^4+\frac{1}{3}x^3\right)+\frac{d^2b^2\left(-\frac{c^5x^5\arctan(cx)^2}{5}+\frac{ic^2x^2}{6}+\frac{c^3x^3\arctan(cx)^2}{3}+\frac{i\arctan(cx)^2c^4x^4}{2}+\frac{c^4}{}\right.}{}$ |
| derivativedivides | $\frac{d^2a^2\left(-\frac{1}{5}c^5x^5+\frac{1}{2}ic^4x^4+\frac{1}{3}c^3x^3\right)+d^2b^2\left(-\frac{c^5x^5\arctan(cx)^2}{5}+\frac{ic^2x^2}{6}+\frac{c^3x^3\arctan(cx)^2}{3}+\frac{i\arctan(cx)^2c^4x^4}{2}+\frac{c^4x^4\arctan(c}{10}\right.}{}$ |
| default | $\frac{d^2a^2\left(-\frac{1}{5}c^5x^5+\frac{1}{2}ic^4x^4+\frac{1}{3}c^3x^3\right)+d^2b^2\left(-\frac{c^5x^5\arctan(cx)^2}{5}+\frac{ic^2x^2}{6}+\frac{c^3x^3\arctan(cx)^2}{3}+\frac{i\arctan(cx)^2c^4x^4}{2}+\frac{c^4x^4\arctan(c}{10}\right.}{}$ |
| risch | $-\frac{8d^2ab\,x^2}{15c}+\frac{d^2cba\,x^4}{10}-\frac{id^2c^2ba\ln(-icx+1)x^5}{5}-\frac{d^2c^2a^2x^5}{5}+\frac{8b\,d^2\ln(c^2x^2+1)a}{15c^3}-\frac{59d^2ba}{30c^3}-\frac{b^2d^2x^3}{30}+$ |

input
```
int(x^2*(d+I*c*d*x)^2*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
d^2*a^2*(-1/5*x^5*c^2+1/2*I*c*x^4+1/3*x^3)+d^2*b^2/c^3*(-1/5*c^5*x^5*arcta
n(c*x)^2+1/6*I*c^2*x^2+1/3*c^3*x^3*arctan(c*x)^2+1/2*I*arctan(c*x)^2*c^4*x
^4+1/10*c^4*x^4*arctan(c*x)+I*arctan(c*x)*c*x-8/15*c^2*x^2*arctan(c*x)+8/1
5*arctan(c*x)*ln(c^2*x^2+1)-4/15*I*(ln(c*x+I)*ln(c^2*x^2+1)-1/2*ln(c*x+I)^
2-dilog(1/2*I*(c*x-I))-ln(c*x+I)*ln(1/2*I*(c*x-I)))+19/30*c*x-1/30*c^3*x^3
+4/15*I*(ln(c*x-I)*ln(c^2*x^2+1)-1/2*ln(c*x-I)^2-dilog(-1/2*I*(c*x+I))-ln(
c*x-I)*ln(-1/2*I*(c*x+I)))-1/3*I*arctan(c*x)*c^3*x^3-19/30*arctan(c*x)-2/3
*I*ln(c^2*x^2+1)-1/2*I*arctan(c*x)^2)+2*d^2*a*b/c^3*(-1/5*c^5*x^5*arctan(c
*x)+1/2*I*arctan(c*x)*c^4*x^4+1/3*c^3*x^3*arctan(c*x)+1/2*I*c*x+1/20*c^4*x
^4-1/6*I*c^3*x^3-4/15*c^2*x^2+4/15*ln(c^2*x^2+1)-1/2*I*arctan(c*x))
```

## Fricas [F]

$$\int x^2(d+icdx)^2(a+b\arctan(cx))^2\,dx=\int(i\,cdx+d)^2(b\arctan(cx)+a)^2x^2\,dx$$

input
```
integrate(x^2*(d+I*c*d*x)^2*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

output
```
1/120*(6*b^2*c^2*d^2*x^5 - 15*I*b^2*c*d^2*x^4 - 10*b^2*d^2*x^3)*log(-(c*x
+ I)/(c*x - I))^2 + integral(-1/30*(30*a^2*c^4*d^2*x^6 - 60*I*a^2*c^3*d^2*
x^5 - 60*I*a^2*c*d^2*x^3 - 30*a^2*d^2*x^2 - (-30*I*a*b*c^4*d^2*x^6 - 6*(10
*a*b - I*b^2)*c^3*d^2*x^5 + 15*b^2*c^2*d^2*x^4 - 10*(6*a*b + I*b^2)*c*d^2*
x^3 + 30*I*a*b*d^2*x^2)*log(-(c*x + I)/(c*x - I)))/(c^2*x^2 + 1), x)
```

## Sympy [F(-1)]

Timed out.

$$\int x^2(d+icdx)^2(a+b\arctan(cx))^2\,dx=\text{Timed out}$$

input
```
integrate(x**2*(d+I*c*d*x)**2*(a+b*atan(c*x))**2,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int x^2(d+icdx)^2(a+b\arctan(cx))^2\,dx=\int (i\,cdx+d)^2(b\arctan{(cx)}+a)^2x^2\,dx$$

input
```
integrate(x^2*(d+I*c*d*x)^2*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

output
```
-1/5*a^2*c^2*d^2*x^5 + 1/2*I*a^2*c*d^2*x^4 - 1/10*(4*x^5*arctan(c*x) - c*(
(c^2*x^4 - 2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*a*b*c^2*d^2 + 1/3*a^2*d^2
*x^3 + 1/3*I*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c
^5))*a*b*c*d^2 + 1/3*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^
4))*a*b*d^2 - 1/120*(6*b^2*c^2*d^2*x^5 - 15*I*b^2*c*d^2*x^4 - 10*b^2*d^2*x
^3)*arctan(c*x)^2 + 1/120*(-6*I*b^2*c^2*d^2*x^5 - 15*b^2*c*d^2*x^4 + 10*I*
b^2*d^2*x^3)*arctan(c*x)*log(c^2*x^2 + 1) + 1/480*(6*b^2*c^2*d^2*x^5 - 15*
I*b^2*c*d^2*x^4 - 10*b^2*d^2*x^3)*log(c^2*x^2 + 1)^2 - integrate(1/240*(18
0*(b^2*c^4*d^2*x^6 - b^2*d^2*x^2)*arctan(c*x)^2 + 15*(b^2*c^4*d^2*x^6 - b^
2*d^2*x^2)*log(c^2*x^2 + 1)^2 - 4*(21*b^2*c^3*d^2*x^5 - 10*b^2*c*d^2*x^3)*
arctan(c*x) + 2*(6*b^2*c^4*d^2*x^6 - 25*b^2*c^2*d^2*x^4 - 60*(b^2*c^3*d^2*
x^5 + b^2*c*d^2*x^3)*arctan(c*x))*log(c^2*x^2 + 1))/(c^2*x^2 + 1), x) + I*
integrate(1/120*(180*(b^2*c^3*d^2*x^5 + b^2*c*d^2*x^3)*arctan(c*x)^2 + 15*
(b^2*c^3*d^2*x^5 + b^2*c*d^2*x^3)*log(c^2*x^2 + 1)^2 + 2*(6*b^2*c^4*d^2*x^
6 - 25*b^2*c^2*d^2*x^4)*arctan(c*x) + (21*b^2*c^3*d^2*x^5 - 10*b^2*c*d^2*x
^3 + 30*(b^2*c^4*d^2*x^6 - b^2*d^2*x^2)*arctan(c*x))*log(c^2*x^2 + 1))/(c^
2*x^2 + 1), x)
```

## Giac [F]

$$\int x^2(d+icdx)^2(a+b\arctan(cx))^2\,dx=\int(i\,cdx+d)^2(b\arctan(cx)+a)^2x^2\,dx$$

input
```
integrate(x^2*(d+I*c*d*x)^2*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

output
```
integrate((I*c*d*x + d)^2*(b*arctan(c*x) + a)^2*x^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int x^2(d+icdx)^2(a+b\arctan(cx))^2\,dx=\int x^2\,(a+b\,\mathrm{atan}(c\,x))^2\,(d+c\,d\,x\,1\mathrm{i})^2\,dx$$

input
```
int(x^2*(a + b*atan(c*x))^2*(d + c*d*x*1i)^2,x)
```

output
```
int(x^2*(a + b*atan(c*x))^2*(d + c*d*x*1i)^2, x)
```

## Reduce [F]

$$\int x^2(d+icdx)^2(a+b\arctan(cx))^2\,dx$$
$$=\frac{d^2\left(16\log(c^2x^2+1)\,ab-6a^2c^5x^5+16atan(cx)^2\,b^2cx+3atan(cx)\,b^2c^4x^4-16atan(cx)\,b^2c^2x^2+3ab\,c^4x^4\right.}{}$$

input
```
int(x^2*(d+I*c*d*x)^2*(a+b*atan(c*x))^2,x)
```

output
```
(d**2*( - 6*atan(c*x)**2*b**2*c**5*x**5 + 15*atan(c*x)**2*b**2*c**4*i*x**4
 + 10*atan(c*x)**2*b**2*c**3*x**3 + 16*atan(c*x)**2*b**2*c*x - 15*atan(c*x
)**2*b**2*i - 12*atan(c*x)*a*b*c**5*x**5 + 30*atan(c*x)*a*b*c**4*i*x**4 +
20*atan(c*x)*a*b*c**3*x**3 - 30*atan(c*x)*a*b*i + 3*atan(c*x)*b**2*c**4*x*
*4 - 10*atan(c*x)*b**2*c**3*i*x**3 - 16*atan(c*x)*b**2*c**2*x**2 + 30*atan
(c*x)*b**2*c*i*x - 19*atan(c*x)*b**2 - 16*int(atan(c*x)**2,x)*b**2*c + 16*
log(c**2*x**2 + 1)*a*b - 20*log(c**2*x**2 + 1)*b**2*i - 6*a**2*c**5*x**5 +
 15*a**2*c**4*i*x**4 + 10*a**2*c**3*x**3 + 3*a*b*c**4*x**4 - 10*a*b*c**3*i
*x**3 - 16*a*b*c**2*x**2 + 30*a*b*c*i*x - b**2*c**3*x**3 + 5*b**2*c**2*i*x
**2 + 19*b**2*c*x))/(30*c**3)
```

# 3.78 $\int x(d+icdx)^2(a+b\arctan(cx))^2\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 293

$$\begin{aligned}\int x(d+icdx)^2(a+b\arctan(cx))^2\,dx &= -\frac{3abd^2x}{2c}+\frac{2ib^2d^2x}{3c}-\frac{1}{12}b^2d^2x^2-\frac{2ib^2d^2\arctan(cx)}{3c^2}-\frac{3b^2d^2x\arctan(cx)}{2c}\\&\quad-\frac{2}{3}ibd^2x^2(a+b\arctan(cx))+\frac{1}{6}bcd^2x^3(a+b\arctan(cx))\\&\quad+\frac{17d^2(a+b\arctan(cx))^2}{12c^2}+\frac{1}{2}d^2x^2(a+b\arctan(cx))^2+\frac{2}{3}icd^2x^3(a+b\arctan(cx))^2\\&\quad-\frac{1}{4}c^2d^2x^4(a+b\arctan(cx))^2-\frac{4ibd^2(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{3c^2}\\&\quad+\frac{5b^2d^2\log(1+c^2x^2)}{6c^2}+\frac{2b^2d^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{3c^2}\end{aligned}$$

output
```
-3/2*a*b*d^2*x/c+2/3*I*b^2*d^2*x/c-1/12*b^2*d^2*x^2-2/3*I*b^2*d^2*arctan(c
*x)/c^2-3/2*b^2*d^2*x*arctan(c*x)/c-2/3*I*b*d^2*x^2*(a+b*arctan(c*x))+1/6*
b*c*d^2*x^3*(a+b*arctan(c*x))+17/12*d^2*(a+b*arctan(c*x))^2/c^2+1/2*d^2*x^
2*(a+b*arctan(c*x))^2+2/3*I*c*d^2*x^3*(a+b*arctan(c*x))^2-1/4*c^2*d^2*x^4*
(a+b*arctan(c*x))^2-4/3*I*b*d^2*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^2+5/6*
b^2*d^2*ln(c^2*x^2+1)/c^2+2/3*b^2*d^2*polylog(2,1-2/(1+I*c*x))/c^2
```

## Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.88

$$\int x(d+icdx)^2(a+b\arctan(cx))^2\,dx =$$
$$-\frac{d^2\left(b^2+18abcx-8ib^2cx-6a^2c^2x^2+8iabc^2x^2+b^2c^2x^2-8ia^2c^3x^3-2abc^3x^3+3a^2c^4x^4+b^2(-i+cx\right.}{}$$

input
```
Integrate[x*(d + I*c*d*x)^2*(a + b*ArcTan[c*x])^2,x]
```

output
```
-1/12*(d^2*(b^2 + 18*a*b*c*x - (8*I)*b^2*c*x - 6*a^2*c^2*x^2 + (8*I)*a*b*c
^2*x^2 + b^2*c^2*x^2 - (8*I)*a^2*c^3*x^3 - 2*a*b*c^3*x^3 + 3*a^2*c^4*x^4 +
 b^2*(-I + c*x)^3*(I + 3*c*x)*ArcTan[c*x]^2 + 2*b*ArcTan[c*x]*(b*(4*I + 9*
c*x + (4*I)*c^2*x^2 - c^3*x^3) + a*(-9 - 6*c^2*x^2 - (8*I)*c^3*x^3 + 3*c^4
*x^4) + (8*I)*b*Log[1 + E^((2*I)*ArcTan[c*x])]) - (8*I)*a*b*Log[1 + c^2*x^
2] - 10*b^2*Log[1 + c^2*x^2] + 8*b^2*PolyLog[2, -E^((2*I)*ArcTan[c*x])]))/
c^2
```

## Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(d+icdx)^2(a+b\arctan(cx))^2\,dx$$
$$\Big\downarrow \; 5411$$
$$\int \left(-c^2d^2x^3(a+b\arctan(cx))^2+2icd^2x^2(a+b\arctan(cx))^2+d^2x(a+b\arctan(cx))^2\right)dx$$
$$\Big\downarrow \; 2009$$

$$-\frac{1}{4}c^2d^2x^4(a+b\arctan(cx))^2+\frac{17d^2(a+b\arctan(cx))^2}{12c^2}-\frac{4ibd^2\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{3c^2}+\frac{2}{3}icd^2x^3(a+b\arctan(cx))^2+\frac{1}{6}bcd^2x^3(a+b\arctan(cx))+\frac{1}{2}d^2x^2(a+b\arctan(cx))^2-\frac{2}{3}ibd^2x^2(a+b\arctan(cx))-\frac{3abd^2x}{2c}-\frac{2ib^2d^2\arctan(cx)}{3c^2}-\frac{3b^2d^2x\arctan(cx)}{2c}+\frac{2b^2d^2\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{3c^2}+\frac{5b^2d^2\log(c^2x^2+1)}{6c^2}+\frac{2ib^2d^2x}{3c}-\frac{1}{12}b^2d^2x^2$$

input
```
Int[x*(d + I*c*d*x)^2*(a + b*ArcTan[c*x])^2,x]
```

output
```
(-3*a*b*d^2*x)/(2*c) + (((2*I)/3)*b^2*d^2*x)/c - (b^2*d^2*x^2)/12 - (((2*I
)/3)*b^2*d^2*ArcTan[c*x])/c^2 - (3*b^2*d^2*x*ArcTan[c*x])/(2*c) - ((2*I)/3
)*b*d^2*x^2*(a + b*ArcTan[c*x]) + (b*c*d^2*x^3*(a + b*ArcTan[c*x]))/6 + (1
7*d^2*(a + b*ArcTan[c*x])^2)/(12*c^2) + (d^2*x^2*(a + b*ArcTan[c*x])^2)/2
+ ((2*I)/3)*c*d^2*x^3*(a + b*ArcTan[c*x])^2 - (c^2*d^2*x^4*(a + b*ArcTan[c
*x])^2)/4 - (((4*I)/3)*b*d^2*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/c^2 +
 (5*b^2*d^2*Log[1 + c^2*x^2])/(6*c^2) + (2*b^2*d^2*PolyLog[2, 1 - 2/(1 + I
*c*x)])/(3*c^2)
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

## Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.31

| method | result |
|---|---|
| parts | $d^2a^2\left(-\frac{1}{4}c^2x^4+\frac{2}{3}icx^3+\frac{1}{2}x^2\right)+\frac{d^2b^2\left(-\frac{c^4x^4\arctan(cx)^2}{4}-\frac{2i\arctan(cx)}{3}+\frac{c^2x^2\arctan(cx)^2}{2}+\frac{2i\arctan(cx)^2c^3}{3}\right.}{}$ |
| derivativedivides | $\frac{d^2a^2\left(-\frac{1}{4}c^4x^4+\frac{2}{3}ic^3x^3+\frac{1}{2}c^2x^2\right)+d^2b^2\left(-\frac{c^4x^4\arctan(cx)^2}{4}-\frac{2i\arctan(cx)}{3}+\frac{c^2x^2\arctan(cx)^2}{2}+\frac{2i\arctan(cx)^2c^3x^3}{3}+\frac{c^3x^3}{}\right.}{}$ |
| default | $\frac{d^2a^2\left(-\frac{1}{4}c^4x^4+\frac{2}{3}ic^3x^3+\frac{1}{2}c^2x^2\right)+d^2b^2\left(-\frac{c^4x^4\arctan(cx)^2}{4}-\frac{2i\arctan(cx)}{3}+\frac{c^2x^2\arctan(cx)^2}{2}+\frac{2i\arctan(cx)^2c^3x^3}{3}+\frac{c^3x^3}{}\right.}{}$ |
| risch | $\left(-\frac{d^2b^2(3c^2x^4-8icx^3-6x^2)\ln(-icx+1)}{24}-\frac{d^2b(-6iac^4x^4+2ibc^3x^3-16c^3x^3a+12iac^2x^2+8bc^2x^2-18ixbc-17b\ln}{24c^2}\right.$ |

input
```
int(x*(d+I*c*d*x)^2*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
d^2*a^2*(-1/4*c^2*x^4+2/3*I*c*x^3+1/2*x^2)+d^2*b^2/c^2*(-1/4*c^4*x^4*arcta
n(c*x)^2-2/3*I*arctan(c*x)+1/2*c^2*x^2*arctan(c*x)^2+2/3*I*arctan(c*x)^2*c
^3*x^3+1/6*c^3*x^3*arctan(c*x)+2/3*I*ln(c^2*x^2+1)*arctan(c*x)+3/4*arctan(
c*x)^2-3/2*c*x*arctan(c*x)+1/3*ln(c*x+I)*ln(c^2*x^2+1)-1/3*ln(c*x-I)*ln(c^
2*x^2+1)-1/6*ln(c*x+I)^2-1/3*ln(c*x+I)*ln(1/2*I*(c*x-I))+1/6*ln(c*x-I)^2+1
/3*ln(c*x-I)*ln(-1/2*I*(c*x+I))-1/3*dilog(1/2*I*(c*x-I))+1/3*dilog(-1/2*I*
(c*x+I))-2/3*I*arctan(c*x)*c^2*x^2-1/12*c^2*x^2+5/6*ln(c^2*x^2+1)+2/3*I*c*
x)+2*d^2*a*b/c^2*(-1/4*c^4*x^4*arctan(c*x)+2/3*I*arctan(c*x)*c^3*x^3+1/2*c
^2*x^2*arctan(c*x)-3/4*c*x+1/12*c^3*x^3-1/3*I*c^2*x^2+1/3*I*ln(c^2*x^2+1)+
3/4*arctan(c*x))
```

## Fricas [F]

$$\int x(d+icdx)^2(a+b\arctan(cx))^2\,dx=\int(i\,cdx+d)^2(b\arctan(cx)+a)^2x\,dx$$

input
```
integrate(x*(d+I*c*d*x)^2*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

output
```
1/48*(3*b^2*c^2*d^2*x^4 - 8*I*b^2*c*d^2*x^3 - 6*b^2*d^2*x^2)*log(-(c*x + I
)/(c*x - I))^2 + integral(-1/12*(12*a^2*c^4*d^2*x^5 - 24*I*a^2*c^3*d^2*x^4
 - 24*I*a^2*c*d^2*x^2 - 12*a^2*d^2*x - (-12*I*a*b*c^4*d^2*x^5 - 3*(8*a*b -
 I*b^2)*c^3*d^2*x^4 + 8*b^2*c^2*d^2*x^3 - 6*(4*a*b + I*b^2)*c*d^2*x^2 + 12
*I*a*b*d^2*x)*log(-(c*x + I)/(c*x - I)))/(c^2*x^2 + 1), x)
```

## Sympy [F(-1)]

Timed out.

$$\int x(d+icdx)^2(a+b\arctan(cx))^2\,dx=\text{Timed out}$$

input
```
integrate(x*(d+I*c*d*x)**2*(a+b*atan(c*x))**2,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int x(d+icdx)^2(a+b\arctan(cx))^2\,dx=\int(i\,cdx+d)^2(b\arctan(cx)+a)^2x\,dx$$

input
```
integrate(x*(d+I*c*d*x)^2*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

output
```
-1/4*a^2*c^2*d^2*x^4 + 2/3*I*a^2*c*d^2*x^3 + 1/2*b^2*d^2*x^2*arctan(c*x)^2
 - 1/6*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*a
*b*c^2*d^2 + 2/3*I*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4)
)*a*b*c*d^2 + 1/2*a^2*d^2*x^2 + (x^2*arctan(c*x) - c*(x/c^2 - arctan(c*x)/
c^3))*a*b*d^2 - 1/2*(2*c*(x/c^2 - arctan(c*x)/c^3)*arctan(c*x) + (arctan(c
*x)^2 - log(c^2*x^2 + 1))/c^2)*b^2*d^2 - 1/48*(3*b^2*c^2*d^2*x^4 - 8*I*b^2
*c*d^2*x^3)*arctan(c*x)^2 + 1/48*(-3*I*b^2*c^2*d^2*x^4 - 8*b^2*c*d^2*x^3)*
arctan(c*x)*log(c^2*x^2 + 1) + 1/192*(3*b^2*c^2*d^2*x^4 - 8*I*b^2*c*d^2*x^
3)*log(c^2*x^2 + 1)^2 - integrate(-1/48*(22*b^2*c^3*d^2*x^4*arctan(c*x) -
36*(b^2*c^4*d^2*x^5 + b^2*c^2*d^2*x^3)*arctan(c*x)^2 - 3*(b^2*c^4*d^2*x^5
+ b^2*c^2*d^2*x^3)*log(c^2*x^2 + 1)^2 - (3*b^2*c^4*d^2*x^5 - 8*b^2*c^2*d^2
*x^3 - 24*(b^2*c^3*d^2*x^4 + b^2*c*d^2*x^2)*arctan(c*x))*log(c^2*x^2 + 1))
/(c^2*x^2 + 1), x) + I*integrate(1/48*(72*(b^2*c^3*d^2*x^4 + b^2*c*d^2*x^2
)*arctan(c*x)^2 + 6*(b^2*c^3*d^2*x^4 + b^2*c*d^2*x^2)*log(c^2*x^2 + 1)^2 +
 2*(3*b^2*c^4*d^2*x^5 - 8*b^2*c^2*d^2*x^3)*arctan(c*x) + (11*b^2*c^3*d^2*x
^4 + 12*(b^2*c^4*d^2*x^5 + b^2*c^2*d^2*x^3)*arctan(c*x))*log(c^2*x^2 + 1))
/(c^2*x^2 + 1), x)
```

## Giac [F]

$$\int x(d+icdx)^2(a+b\arctan(cx))^2\,dx=\int(i\,cdx+d)^2(b\arctan(cx)+a)^2x\,dx$$

input
```
integrate(x*(d+I*c*d*x)^2*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

output
```
integrate((I*c*d*x + d)^2*(b*arctan(c*x) + a)^2*x, x)
```

## Mupad [F(-1)]

Timed out.

$$\int x(d+icdx)^2(a+b\arctan(cx))^2\,dx=\int x\,(a+b\,\mathrm{atan}(c\,x))^2\,(d+c\,d\,x\,1\mathrm{i})^2\,dx$$

input
```
int(x*(a + b*atan(c*x))^2*(d + c*d*x*1i)^2,x)
```

output
```
int(x*(a + b*atan(c*x))^2*(d + c*d*x*1i)^2, x)
```

## Reduce [F]

$$\int x(d+icdx)^2(a+b\arctan(cx))^2\,dx$$
$$=\frac{d^2\left(-3atan(cx)^2\,b^2c^4x^4+8atan(cx)^2\,b^2c^3i\,x^3+6atan(cx)^2\,b^2c^2x^2+8atan(cx)^2\,b^2cix+9atan(cx)^2\,b^2-\right.}{}$$

input
```
int(x*(d+I*c*d*x)^2*(a+b*atan(c*x))^2,x)
```

output
```
(d**2*( - 3*atan(c*x)**2*b**2*c**4*x**4 + 8*atan(c*x)**2*b**2*c**3*i*x**3
+ 6*atan(c*x)**2*b**2*c**2*x**2 + 8*atan(c*x)**2*b**2*c*i*x + 9*atan(c*x)*
*2*b**2 - 6*atan(c*x)*a*b*c**4*x**4 + 16*atan(c*x)*a*b*c**3*i*x**3 + 12*at
an(c*x)*a*b*c**2*x**2 + 18*atan(c*x)*a*b + 2*atan(c*x)*b**2*c**3*x**3 - 8*
atan(c*x)*b**2*c**2*i*x**2 - 18*atan(c*x)*b**2*c*x - 8*atan(c*x)*b**2*i -
8*int(atan(c*x)**2,x)*b**2*c*i + 8*log(c**2*x**2 + 1)*a*b*i + 10*log(c**2*
x**2 + 1)*b**2 - 3*a**2*c**4*x**4 + 8*a**2*c**3*i*x**3 + 6*a**2*c**2*x**2
+ 2*a*b*c**3*x**3 - 8*a*b*c**2*i*x**2 - 18*a*b*c*x - b**2*c**2*x**2 + 8*b*
*2*c*i*x))/(12*c**2)
```

# 3.79 $\int (d+icdx)^2(a+b\arctan(cx))^2\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 192

$$\int (d+icdx)^2(a+b\arctan(cx))^2\,dx = -2iabd^2x-\frac{1}{3}b^2d^2x+\frac{b^2d^2\arctan(cx)}{3c} - 2ib^2d^2x\arctan(cx)+\frac{1}{3}bcd^2x^2(a+b\arctan(cx)) - \frac{id^2(1+icx)^3(a+b\arctan(cx))^2}{3c} + \frac{8bd^2(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)}{3c} + \frac{ib^2d^2\log(1+c^2x^2)}{c} - \frac{4ib^2d^2\,\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{3c}$$

output
```
-2*I*a*b*d^2*x-1/3*b^2*d^2*x+1/3*b^2*d^2*arctan(c*x)/c-2*I*b^2*d^2*x*arcta
n(c*x)+1/3*b*c*d^2*x^2*(a+b*arctan(c*x))-1/3*I*d^2*(1+I*c*x)^3*(a+b*arctan
(c*x))^2/c+8/3*b*d^2*(a+b*arctan(c*x))*ln(2/(1-I*c*x))/c+I*b^2*d^2*ln(c^2*
x^2+1)/c-4/3*I*b^2*d^2*polylog(2,1-2/(1-I*c*x))/c
```

## Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.07

$$\int (d+icdx)^2(a+b\arctan(cx))^2\,dx = -\frac{d^2\left(-3a^2cx+6iabcx+b^2cx-3ia^2c^2x^2-abc^2x^2+a^2c^3x^3+b^2(-i+cx)^3\arctan(cx)^2-b\arctan(cx)\right.}{}$$

input
```
Integrate[(d + I*c*d*x)^2*(a + b*ArcTan[c*x])^2,x]
```

output
```
-1/3*(d^2*(-3*a^2*c*x + (6*I)*a*b*c*x + b^2*c*x - (3*I)*a^2*c^2*x^2 - a*b*
c^2*x^2 + a^2*c^3*x^3 + b^2*(-I + c*x)^3*ArcTan[c*x]^2 - b*ArcTan[c*x]*(b*
(1 - (6*I)*c*x + c^2*x^2) + a*(6*I + 6*c*x + (6*I)*c^2*x^2 - 2*c^3*x^3) +
8*b*Log[1 + E^((2*I)*ArcTan[c*x])]) + 4*a*b*Log[1 + c^2*x^2] - (3*I)*b^2*L
og[1 + c^2*x^2] + (4*I)*b^2*PolyLog[2, -E^((2*I)*ArcTan[c*x])]))/c
```

## Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.98, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {5389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+icdx)^2(a+b\arctan(cx))^2\,dx$$

$$\Big\downarrow \text{5389}$$

$$\frac{2ib\int \left(-icx(a+b\arctan(cx))d^3-\frac{4i(i-cx)(a+b\arctan(cx))d^3}{c^2x^2+1}-3(a+b\arctan(cx))d^3\right)dx}{3d}-\frac{id^2(1+icx)^3(a+b\arctan(cx))^2}{3c}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2ib\left(-\frac{1}{2}icd^3x^2(a+b\arctan(cx))-\frac{4id^3\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))}{c}-3ad^3x-3bd^3x\arctan(cx)-\frac{ibd^3\arctan(cx)}{2c}+\frac{3bd}{}\right.}{3d}$$

$$\frac{id^2(1+icx)^3(a+b\arctan(cx))^2}{3c}$$

input
```
Int[(d + I*c*d*x)^2*(a + b*ArcTan[c*x])^2,x]
```

output
```
((-1/3*I)*d^2*(1 + I*c*x)^3*(a + b*ArcTan[c*x])^2)/c + (((2*I)/3)*b*(-3*a*
d^3*x + (I/2)*b*d^3*x - ((I/2)*b*d^3*ArcTan[c*x])/c - 3*b*d^3*x*ArcTan[c*x
] - (I/2)*c*d^3*x^2*(a + b*ArcTan[c*x]) - ((4*I)*d^3*(a + b*ArcTan[c*x])*L
og[2/(1 - I*c*x)])/c + (3*b*d^3*Log[1 + c^2*x^2])/(2*c) - (2*b*d^3*PolyLog
[2, 1 - 2/(1 - I*c*x)])/c))/d
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5389
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - S
imp[b*c*(p/(e*(q + 1)))   Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1),
(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

## Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 353 vs. $2(172) = 344$.

Time $= 0.83$ (sec) , antiderivative size $= 354$, normalized size of antiderivative $= 1.84$

| method | result |
|---|---|
| derivativedivides | $\frac{-\frac{id^2a^2(icx+1)^3}{3}+d^2b^2\left(-\frac{c^3x^3\arctan(cx)^2}{3}+i\arctan(cx)^2c^2x^2+cx\arctan(cx)^2-\frac{i\arctan(cx)^2}{3}+\frac{2i\left(-3cx\arctan(cx)-\frac{i\,\mathrm{ar}}{}\right.}{}\right.}{}$ |
| default | $\frac{-\frac{id^2a^2(icx+1)^3}{3}+d^2b^2\left(-\frac{c^3x^3\arctan(cx)^2}{3}+i\arctan(cx)^2c^2x^2+cx\arctan(cx)^2-\frac{i\arctan(cx)^2}{3}+\frac{2i\left(-3cx\arctan(cx)-\frac{i\,\mathrm{ar}}{}\right.}{}\right.}{}$ |
| parts | $-\frac{id^2a^2(icx+1)^3}{3c}+\frac{d^2b^2\left(-\frac{c^3x^3\arctan(cx)^2}{3}+i\arctan(cx)^2c^2x^2+cx\arctan(cx)^2-\frac{i\arctan(cx)^2}{3}+\frac{2i\left(-3cx\arctan(cx)\right.}{}\right.}{}$ |
| risch | $x\,d^2a^2+\frac{id^2cb^2\ln(-icx+1)x^2}{6}+\frac{2iab\,d^2\arctan(cx)}{c}-\frac{id^2cb^2\ln(-icx+1)^2x^2}{4}+\frac{b\,d^2ac\,x^2}{3}-\frac{7i\ln(-icx+1)^2}{12c}$ |

input
```
int((d+I*c*d*x)^2*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*(-1/3*I*d^2*a^2*(1+I*c*x)^3+d^2*b^2*(-1/3*c^3*x^3*arctan(c*x)^2+I*arct
an(c*x)^2*c^2*x^2+c*x*arctan(c*x)^2-1/3*I*arctan(c*x)^2+2/3*I*(-3*c*x*arct
an(c*x)-1/2*I*arctan(c*x)*c^2*x^2+1/2*I*c*x+2*arctan(c*x)^2+ln(c*x+I)*ln(c
^2*x^2+1)-ln(c*x-I)*ln(c^2*x^2+1)-ln(c*x+I)*ln(1/2*I*(c*x-I))+ln(c*x-I)*ln
(-1/2*I*(c*x+I))-1/2*ln(c*x+I)^2+1/2*ln(c*x-I)^2-dilog(1/2*I*(c*x-I))+dilo
g(-1/2*I*(c*x+I))+2*I*arctan(c*x)*ln(c^2*x^2+1)+3/2*ln(c^2*x^2+1)-1/2*I*ar
ctan(c*x)))+2*d^2*a*b*(-1/3*c^3*x^3*arctan(c*x)+I*arctan(c*x)*c^2*x^2+c*x*
arctan(c*x)-1/3*I*arctan(c*x)+1/3*I*(-3*c*x-1/2*I*c^2*x^2+2*I*ln(c^2*x^2+1
)+4*arctan(c*x))))
```

## Fricas [F]

$$\int (d+icdx)^2(a+b\arctan(cx))^2\,dx=\int (i\,cdx+d)^2(b\arctan(cx)+a)^2\,dx$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

output
```
1/12*(b^2*c^2*d^2*x^3 - 3*I*b^2*c*d^2*x^2 - 3*b^2*d^2*x)*log(-(c*x + I)/(c
*x - I))^2 + integral(-1/3*(3*a^2*c^4*d^2*x^4 - 6*I*a^2*c^3*d^2*x^3 - 6*I*
a^2*c*d^2*x - 3*a^2*d^2 - (-3*I*a*b*c^4*d^2*x^4 - (6*a*b - I*b^2)*c^3*d^2*
x^3 + 3*b^2*c^2*d^2*x^2 - 3*(2*a*b + I*b^2)*c*d^2*x + 3*I*a*b*d^2)*log(-(c
*x + I)/(c*x - I)))/(c^2*x^2 + 1), x)
```

## Sympy [F(-1)]

Timed out.

$$\int (d + icdx)^2 (a + b\arctan(cx))^2 \, dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)**2*(a+b*atan(c*x))**2,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int (d + icdx)^2 (a + b\arctan(cx))^2 \, dx = \int (i\,cdx + d)^2 (b\arctan(cx) + a)^2 \, dx$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

output
```
-1/3*a^2*c^2*d^2*x^3 - 36*b^2*c^4*d^2*integrate(1/48*x^4*arctan(c*x)^2/(c^
2*x^2 + 1), x) - 3*b^2*c^4*d^2*integrate(1/48*x^4*log(c^2*x^2 + 1)^2/(c^2*
x^2 + 1), x) - 4*b^2*c^4*d^2*integrate(1/48*x^4*log(c^2*x^2 + 1)/(c^2*x^2
+ 1), x) + 24*b^2*c^3*d^2*integrate(1/48*x^3*arctan(c*x)*log(c^2*x^2 + 1)/
(c^2*x^2 + 1), x) + 32*b^2*c^3*d^2*integrate(1/48*x^3*arctan(c*x)/(c^2*x^2
 + 1), x) - 1/3*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*a
*b*c^2*d^2 + I*a^2*c*d^2*x^2 + 24*b^2*c^2*d^2*integrate(1/48*x^2*log(c^2*x
^2 + 1)/(c^2*x^2 + 1), x) + 2*I*(x^2*arctan(c*x) - c*(x/c^2 - arctan(c*x)/
c^3))*a*b*c*d^2 + 1/4*b^2*d^2*arctan(c*x)^3/c + 24*b^2*c*d^2*integrate(1/4
8*x*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) - 24*b^2*c*d^2*integrat
e(1/48*x*arctan(c*x)/(c^2*x^2 + 1), x) + a^2*d^2*x + 3*b^2*d^2*integrate(1
/48*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + (2*c*x*arctan(c*x) - log(c^2*x^
2 + 1))*a*b*d^2/c - 1/12*(b^2*c^2*d^2*x^3 - 3*I*b^2*c*d^2*x^2 - 3*b^2*d^2*
x)*arctan(c*x)^2 + 1/12*(-I*b^2*c^2*d^2*x^3 - 3*b^2*c*d^2*x^2 + 3*I*b^2*d^
2*x)*arctan(c*x)*log(c^2*x^2 + 1) + 1/48*(b^2*c^2*d^2*x^3 - 3*I*b^2*c*d^2*
x^2 - 3*b^2*d^2*x)*log(c^2*x^2 + 1)^2 + I*integrate(1/24*(36*(b^2*c^3*d^2*
x^3 + b^2*c*d^2*x)*arctan(c*x)^2 + 3*(b^2*c^3*d^2*x^3 + b^2*c*d^2*x)*log(c
^2*x^2 + 1)^2 + 4*(b^2*c^4*d^2*x^4 - 6*b^2*c^2*d^2*x^2)*arctan(c*x) + 2*(4
*b^2*c^3*d^2*x^3 - 3*b^2*c*d^2*x + 3*(b^2*c^4*d^2*x^4 - b^2*d^2)*arctan(c*
x))*log(c^2*x^2 + 1))/(c^2*x^2 + 1), x)
```

## Giac [F]

$$\int (d+icdx)^2(a+b\arctan(cx))^2\,dx=\int (i\,cdx+d)^2(b\arctan{(cx)}+a)^2\,dx$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

output
```
integrate((I*c*d*x + d)^2*(b*arctan(c*x) + a)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (d+icdx)^2(a+b\arctan(cx))^2\,dx=\int (a+b\,\mathrm{atan}(c\,x))^2\,(d+c\,d\,x\,1\mathrm{i})^2\,dx$$

input
```
int((a + b*atan(c*x))^2*(d + c*d*x*1i)^2,x)
```

output
```
int((a + b*atan(c*x))^2*(d + c*d*x*1i)^2, x)
```

## Reduce [F]

$$\int (d+icdx)^2(a+b\arctan(cx))^2\,dx$$
$$=\frac{d^2\left(-atan(cx)^2\,b^2c^3x^3+3atan(cx)^2\,b^2c^2i\,x^2+3atan(cx)^2\,b^2cx+3atan(cx)^2\,b^2i-2atan(cx)\,ab\,c^3x^3+6\right.}{}$$

input
```
int((d+I*c*d*x)^2*(a+b*atan(c*x))^2,x)
```

output
```
(d**2*( - atan(c*x)**2*b**2*c**3*x**3 + 3*atan(c*x)**2*b**2*c**2*i*x**2 +
3*atan(c*x)**2*b**2*c*x + 3*atan(c*x)**2*b**2*i - 2*atan(c*x)*a*b*c**3*x**
3 + 6*atan(c*x)*a*b*c**2*i*x**2 + 6*atan(c*x)*a*b*c*x + 6*atan(c*x)*a*b*i
+ atan(c*x)*b**2*c**2*x**2 - 6*atan(c*x)*b**2*c*i*x + atan(c*x)*b**2 - 8*i
nt((atan(c*x)*x)/(c**2*x**2 + 1),x)*b**2*c**2 - 4*log(c**2*x**2 + 1)*a*b +
 3*log(c**2*x**2 + 1)*b**2*i - a**2*c**3*x**3 + 3*a**2*c**2*i*x**2 + 3*a**
2*c*x + a*b*c**2*x**2 - 6*a*b*c*i*x - b**2*c*x))/(3*c)
```

## 3.80 $\int \frac{(d+icdx)^2(a+b\arctan(cx))^2}{x}\,dx$

## Optimal result

Integrand size = 25, antiderivative size = 300

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))^2}{x}\,dx = abcd^2x+b^2cd^2x\arctan(cx) - \frac{5}{2}d^2(a+b\arctan(cx))^2 + 2icd^2x(a+b\arctan(cx))^2 - \frac{1}{2}c^2d^2x^2(a+b\arctan(cx))^2 + 2d^2(a+b\arctan(cx))^2\text{arctanh}\left(1-\frac{2}{1+icx}\right) + 4ibd^2(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right) - \frac{1}{2}b^2d^2\log\left(1+c^2x^2\right) - 2b^2d^2\,\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right) - ibd^2(a+b\arctan(cx))\,\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right) + ibd^2(a+b\arctan(cx))\,\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right) - \frac{1}{2}b^2d^2\,\text{PolyLog}\left(3,1-\frac{2}{1+icx}\right) + \frac{1}{2}b^2d^2\,\text{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)$$

output
```
a*b*c*d^2*x+b^2*c*d^2*x*arctan(c*x)-5/2*d^2*(a+b*arctan(c*x))^2+2*I*c*d^2*
x*(a+b*arctan(c*x))^2-1/2*c^2*d^2*x^2*(a+b*arctan(c*x))^2-2*d^2*(a+b*arcta
n(c*x))^2*arctanh(-1+2/(1+I*c*x))+4*I*b*d^2*(a+b*arctan(c*x))*ln(2/(1+I*c*
x))-1/2*b^2*d^2*ln(c^2*x^2+1)-2*b^2*d^2*polylog(2,1-2/(1+I*c*x))-I*b*d^2*(
a+b*arctan(c*x))*polylog(2,1-2/(1+I*c*x))+I*b*d^2*(a+b*arctan(c*x))*polylo
g(2,-1+2/(1+I*c*x))-1/2*b^2*d^2*polylog(3,1-2/(1+I*c*x))+1/2*b^2*d^2*polyl
og(3,-1+2/(1+I*c*x))
```

## Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.20

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))^2}{x}\,dx$$
$$=\frac{1}{2}d^2\Bigg(4ia^2cx-a^2c^2x^2+2b^2cx\arctan(cx)-b^2(1+c^2x^2)\arctan(cx)^2 -2ab(-cx+(1+c^2x^2)\arctan(cx))+2a^2\log(cx) +4iab(2cx\arctan(cx)-\log(1+c^2x^2))-b^2\log(1+c^2x^2) +4b^2\left(\arctan(cx)\left((1+icx)\arctan(cx)+2i\log\left(1+e^{2i\arctan(cx)}\right)\right) +\operatorname{PolyLog}\left(2,-e^{2i\arctan(cx)}\right)\right)+2iab(\operatorname{PolyLog}(2,-icx)-\operatorname{PolyLog}(2,icx)) +2b^2\Bigg(-\frac{i\pi^3}{24}+\frac{2}{3}i\arctan(cx)^3+\arctan(cx)^2\log\left(1-e^{-2i\arctan(cx)}\right) -\arctan(cx)^2\log\left(1+e^{2i\arctan(cx)}\right)+i\arctan(cx)\operatorname{PolyLog}\left(2,e^{-2i\arctan(cx)}\right) +i\arctan(cx)\operatorname{PolyLog}\left(2,-e^{2i\arctan(cx)}\right)+\frac{1}{2}\operatorname{PolyLog}\left(3,e^{-2i\arctan(cx)}\right) -\frac{1}{2}\operatorname{PolyLog}\left(3,-e^{2i\arctan(cx)}\right)\Bigg)\Bigg)$$

input
```
Integrate[((d + I*c*d*x)^2*(a + b*ArcTan[c*x])^2)/x,x]
```

output
```
(d^2*((4*I)*a^2*c*x - a^2*c^2*x^2 + 2*b^2*c*x*ArcTan[c*x] - b^2*(1 + c^2*x
^2)*ArcTan[c*x]^2 - 2*a*b*(-(c*x) + (1 + c^2*x^2)*ArcTan[c*x]) + 2*a^2*Log
[c*x] + (4*I)*a*b*(2*c*x*ArcTan[c*x] - Log[1 + c^2*x^2]) - b^2*Log[1 + c^2
*x^2] + 4*b^2*(ArcTan[c*x]*((1 + I*c*x)*ArcTan[c*x] + (2*I)*Log[1 + E^((2*
I)*ArcTan[c*x])]) + PolyLog[2, -E^((2*I)*ArcTan[c*x])]) + (2*I)*a*b*(PolyL
og[2, (-I)*c*x] - PolyLog[2, I*c*x]) + 2*b^2*((-1/24*I)*Pi^3 + ((2*I)/3)*A
rcTan[c*x]^3 + ArcTan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] - ArcTan[c*x]
^2*Log[1 + E^((2*I)*ArcTan[c*x])] + I*ArcTan[c*x]*PolyLog[2, E^((-2*I)*Arc
Tan[c*x])] + I*ArcTan[c*x]*PolyLog[2, -E^((2*I)*ArcTan[c*x])] + PolyLog[3,
 E^((-2*I)*ArcTan[c*x])]/2 - PolyLog[3, -E^((2*I)*ArcTan[c*x])]/2)))/2
```

## Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))^2}{x}\,dx$$

$$\Big\downarrow \text{5411}$$

$$\int \left(-c^2d^2x(a+b\arctan(cx))^2+2icd^2(a+b\arctan(cx))^2+\frac{d^2(a+b\arctan(cx))^2}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$2d^2\text{arctanh}\left(1-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2-\frac{1}{2}c^2d^2x^2(a+b\arctan(cx))^2-ibd^2\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))+ibd^2\,\text{PolyLog}\left(2,\frac{2}{icx+1}-1\right)(a+b\arctan(cx))+2icd^2x(a+b\arctan(cx))^2-\frac{5}{2}d^2(a+b\arctan(cx))^2+4ibd^2\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))+abcd^2x+b^2cd^2x\arctan(cx)-\frac{1}{2}b^2d^2\log\left(c^2x^2+1\right)-2b^2d^2\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)-\frac{1}{2}b^2d^2\,\text{PolyLog}\left(3,1-\frac{2}{icx+1}\right)+\frac{1}{2}b^2d^2\,\text{PolyLog}\left(3,\frac{2}{icx+1}-1\right)$$

input
```
Int[((d + I*c*d*x)^2*(a + b*ArcTan[c*x])^2)/x,x]
```

output
```
a*b*c*d^2*x + b^2*c*d^2*x*ArcTan[c*x] - (5*d^2*(a + b*ArcTan[c*x])^2)/2 +
(2*I)*c*d^2*x*(a + b*ArcTan[c*x])^2 - (c^2*d^2*x^2*(a + b*ArcTan[c*x])^2)/
2 + 2*d^2*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)] + (4*I)*b*d^2*(
a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)] - (b^2*d^2*Log[1 + c^2*x^2])/2 - 2*b
^2*d^2*PolyLog[2, 1 - 2/(1 + I*c*x)] - I*b*d^2*(a + b*ArcTan[c*x])*PolyLog
[2, 1 - 2/(1 + I*c*x)] + I*b*d^2*(a + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1
+ I*c*x)] - (b^2*d^2*PolyLog[3, 1 - 2/(1 + I*c*x)])/2 + (b^2*d^2*PolyLog[3
, -1 + 2/(1 + I*c*x)])/2
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 9.52 (sec) , antiderivative size = 1317, normalized size of antiderivative = 4.39

| method | result | size |
|---|---|---|
| parts | Expression too large to display | 1317 |
| derivativedivides | Expression too large to display | 1319 |
| default | Expression too large to display | 1319 |

input
```
int((d+I*c*d*x)^2*(a+b*arctan(c*x))^2/x,x,method=_RETURNVERBOSE)
```

output
```
d^2*a^2*(-1/2*c^2*x^2+2*I*c*x+ln(x))+d^2*b^2*(1/2*I*Pi*csgn(I*((1+I*c*x)^2
/(c^2*x^2+1)-1))*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I*((1+I*c*x)^2/(
c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*arctan(c*x)^2-1/2*c^2*x^2*arcta
n(c*x)^2+3/2*arctan(c*x)^2+ln(1+(1+I*c*x)^2/(c^2*x^2+1))+4*dilog(1+I*(1+I*
c*x)/(c^2*x^2+1)^(1/2))+4*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+ln(c*x)*a
rctan(c*x)^2-arctan(c*x)^2*ln((1+I*c*x)^2/(c^2*x^2+1)-1)+arctan(c*x)^2*ln(
1-(1+I*c*x)/(c^2*x^2+1)^(1/2))+arctan(c*x)^2*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1
/2))+I*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))-2*I*arctan(c*x)*pol
ylog(2,(1+I*c*x)/(c^2*x^2+1)^(1/2))-2*I*arctan(c*x)*polylog(2,-(1+I*c*x)/(
c^2*x^2+1)^(1/2))-1/2*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))+2*polylog(3,(1+I
*c*x)/(c^2*x^2+1)^(1/2))+2*polylog(3,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+arctan(
c*x)*(c*x-I)-1/2*I*Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I*((1+I*c*x
)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*arctan(c*x)^2-1/2*I*Pi*c
sgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(((1+I*
c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*arctan(c*x)^2-1/2*I*P
i*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(
1+(1+I*c*x)^2/(c^2*x^2+1)))^2*arctan(c*x)^2+1/2*I*Pi*csgn(I*((1+I*c*x)^2/(
c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1
)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*arctan(c*x)^2+2*I*arctan(c*x)^2*c*x+1/2*I*c
sgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^3*Pi*arc...
```

## Fricas [F]

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))^2}{x}\,dx=\int \frac{(i\,cdx+d)^2(b\arctan{(cx)}+a)^2}{x}\,dx$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))^2/x,x, algorithm="fricas")
```

output
```
integral(-1/4*(4*a^2*c^2*d^2*x^2 - 8*I*a^2*c*d^2*x - 4*a^2*d^2 - (b^2*c^2*
d^2*x^2 - 2*I*b^2*c*d^2*x - b^2*d^2)*log(-(c*x + I)/(c*x - I))^2 + 4*(I*a*
b*c^2*d^2*x^2 + 2*a*b*c*d^2*x - I*a*b*d^2)*log(-(c*x + I)/(c*x - I)))/x, x
)
```

## Sympy [F]

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))^2}{x}\,dx = -d^2\left(\int\left(-\frac{a^2}{x}\right)dx+\int\left(-2ia^2c\right)dx+\int a^2c^2x\,dx+\int\left(-\frac{b^2\operatorname{atan}^2(cx)}{x}\right)dx+\int\left(-2ib^2c\operatorname{atan}^2(cx)\right)dx+\int\left(-\frac{2ab\operatorname{atan}(cx)}{x}\right)dx+\int b^2c^2x\operatorname{atan}^2(cx)\,dx+\int\left(-4iabc\operatorname{atan}(cx)\right)dx+\int 2abc^2x\operatorname{atan}(cx)\,dx\right)$$

input
```
integrate((d+I*c*d*x)**2*(a+b*atan(c*x))**2/x,x)
```

output
```
-d**2*(Integral(-a**2/x, x) + Integral(-2*I*a**2*c, x) + Integral(a**2*c**
2*x, x) + Integral(-b**2*atan(c*x)**2/x, x) + Integral(-2*I*b**2*c*atan(c*
x)**2, x) + Integral(-2*a*b*atan(c*x)/x, x) + Integral(b**2*c**2*x*atan(c*
x)**2, x) + Integral(-4*I*a*b*c*atan(c*x), x) + Integral(2*a*b*c**2*x*atan
(c*x), x))
```

## Maxima [F]

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))^2}{x}\,dx = \int \frac{(i\,cdx+d)^2(b\arctan(cx)+a)^2}{x}\,dx$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))^2/x,x, algorithm="maxima")
```

output
```
-12*b^2*c^4*d^2*integrate(1/16*x^4*arctan(c*x)^2/(c^2*x^3 + x), x) + 2*I*b
^2*c^4*d^2*integrate(1/8*x^4*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^3 + x), x
) - b^2*c^4*d^2*integrate(1/16*x^4*log(c^2*x^2 + 1)^2/(c^2*x^3 + x), x) +
2*I*b^2*c^4*d^2*integrate(1/8*x^4*arctan(c*x)/(c^2*x^3 + x), x) - 32*a*b*c
^4*d^2*integrate(1/16*x^4*arctan(c*x)/(c^2*x^3 + x), x) - 2*b^2*c^4*d^2*in
tegrate(1/16*x^4*log(c^2*x^2 + 1)/(c^2*x^3 + x), x) - 1/2*a^2*c^2*d^2*x^2
+ 12*I*b^2*c^3*d^2*integrate(1/8*x^3*arctan(c*x)^2/(c^2*x^3 + x), x) + 8*b
^2*c^3*d^2*integrate(1/16*x^3*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^3 + x),
x) + I*b^2*c^3*d^2*integrate(1/8*x^3*log(c^2*x^2 + 1)^2/(c^2*x^3 + x), x)
+ 20*b^2*c^3*d^2*integrate(1/16*x^3*arctan(c*x)/(c^2*x^3 + x), x) + 5*I*b^
2*c^3*d^2*integrate(1/8*x^3*log(c^2*x^2 + 1)/(c^2*x^3 + x), x) + 1/2*I*b^2
*d^2*arctan(c*x)^3 - 8*I*b^2*c^2*d^2*integrate(1/8*x^2*arctan(c*x)/(c^2*x^
3 + x), x) + 2*I*a^2*c*d^2*x + 8*b^2*c*d^2*integrate(1/16*x*arctan(c*x)*lo
g(c^2*x^2 + 1)/(c^2*x^3 + x), x) + I*b^2*c*d^2*integrate(1/8*x*log(c^2*x^2
 + 1)^2/(c^2*x^3 + x), x) + 1/8*b^2*d^2*log(c^2*x^2 + 1)^2 + 2*I*(2*c*x*ar
ctan(c*x) - log(c^2*x^2 + 1))*a*b*d^2 + 12*b^2*d^2*integrate(1/16*arctan(c
*x)^2/(c^2*x^3 + x), x) - 2*I*b^2*d^2*integrate(1/8*arctan(c*x)*log(c^2*x^
2 + 1)/(c^2*x^3 + x), x) + b^2*d^2*integrate(1/16*log(c^2*x^2 + 1)^2/(c^2*
x^3 + x), x) + 32*a*b*d^2*integrate(1/16*arctan(c*x)/(c^2*x^3 + x), x) + a
^2*d^2*log(x) - 1/8*(b^2*c^2*d^2*x^2 - 4*I*b^2*c*d^2*x)*arctan(c*x)^2 +...
```

## Giac [F]

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))^2}{x}\,dx=\int \frac{(i\,cdx+d)^2(b\arctan{(cx)}+a)^2}{x}\,dx$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))^2/x,x, algorithm="giac")
```

output
```
integrate((I*c*d*x + d)^2*(b*arctan(c*x) + a)^2/x, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))^2}{x}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^2\,(d+c\,d\,x\,1\mathrm{i})^2}{x}\,dx$$

input
```
int(((a + b*atan(c*x))^2*(d + c*d*x*1i)^2)/x,x)
```

output
```
int(((a + b*atan(c*x))^2*(d + c*d*x*1i)^2)/x, x)
```

## Reduce [F]

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))^2}{x}\,dx = \frac{d^2\left(-atan(cx)^2\,b^2c^2x^2 - atan(cx)^2\,b^2 - 2atan(cx)\,ab\,c^2x^2 + 8atan(cx)\,abcix - 2atan(cx)\,ab + 2atan(cx\right.}{}$$

input
```
int((d+I*c*d*x)^2*(a+b*atan(c*x))^2/x,x)
```

output
```
(d**2*( - atan(c*x)**2*b**2*c**2*x**2 - atan(c*x)**2*b**2 - 2*atan(c*x)*a*
b*c**2*x**2 + 8*atan(c*x)*a*b*c*i*x - 2*atan(c*x)*a*b + 2*atan(c*x)*b**2*c
*x + 4*int(atan(c*x)**2,x)*b**2*c*i + 4*int(atan(c*x)/x,x)*a*b + 2*int(ata
n(c*x)**2/x,x)*b**2 - 4*log(c**2*x**2 + 1)*a*b*i - log(c**2*x**2 + 1)*b**2
 + 2*log(x)*a**2 - a**2*c**2*x**2 + 4*a**2*c*i*x + 2*a*b*c*x))/2
```

# 3.81 $\int \frac{(d+icdx)^2(a+b\arctan(cx))^2}{x^2}\,dx$

## Optimal result

Integrand size = 25, antiderivative size = 317

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))^2}{x^2}\,dx = -2icd^2(a+b\arctan(cx))^2 - \frac{d^2(a+b\arctan(cx))^2}{x} - c^2d^2x(a+b\arctan(cx))^2 + 4icd^2(a+b\arctan(cx))^2\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right) - 2bcd^2(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right) + 2bcd^2(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right) - ib^2cd^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1-icx}\right) - ib^2cd^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right) + 2bcd^2(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right) - 2bcd^2(a+b\arctan(cx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx}\right) - ib^2cd^2\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right) + ib^2cd^2\operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)$$

output
```
-2*I*c*d^2*(a+b*arctan(c*x))^2-d^2*(a+b*arctan(c*x))^2/x-c^2*d^2*x*(a+b*ar
ctan(c*x))^2-4*I*c*d^2*(a+b*arctan(c*x))^2*arctanh(-1+2/(1+I*c*x))-2*b*c*d
^2*(a+b*arctan(c*x))*ln(2/(1+I*c*x))+2*b*c*d^2*(a+b*arctan(c*x))*ln(2-2/(1
-I*c*x))-I*b^2*c*d^2*polylog(2,-1+2/(1-I*c*x))-I*b^2*c*d^2*polylog(2,1-2/(
1+I*c*x))+2*b*c*d^2*(a+b*arctan(c*x))*polylog(2,1-2/(1+I*c*x))-2*b*c*d^2*(
a+b*arctan(c*x))*polylog(2,-1+2/(1+I*c*x))-I*b^2*c*d^2*polylog(3,1-2/(1+I*
c*x))+I*b^2*c*d^2*polylog(3,-1+2/(1+I*c*x))
```

## Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.19

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))^2}{x^2}\,dx =$$
$$-\frac{d^2\left(12a^2-b^2c\pi^3x+12a^2c^2x^2+24ab\arctan(cx)+24abc^2x^2\arctan(cx)+12b^2\arctan(cx)^2+12b^2c^2x^2\right.}{}$$

input
```
Integrate[((d + I*c*d*x)^2*(a + b*ArcTan[c*x])^2)/x^2,x]
```

output
```
-1/12*(d^2*(12*a^2 - b^2*c*Pi^3*x + 12*a^2*c^2*x^2 + 24*a*b*ArcTan[c*x] +
24*a*b*c^2*x^2*ArcTan[c*x] + 12*b^2*ArcTan[c*x]^2 + 12*b^2*c^2*x^2*ArcTan[
c*x]^2 + 16*b^2*c*x*ArcTan[c*x]^3 - (24*I)*b^2*c*x*ArcTan[c*x]^2*Log[1 - E
^((-2*I)*ArcTan[c*x])] - 24*b^2*c*x*ArcTan[c*x]*Log[1 - E^((2*I)*ArcTan[c*
x])] + 24*b^2*c*x*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] + (24*I)*b^2*
c*x*ArcTan[c*x]^2*Log[1 + E^((2*I)*ArcTan[c*x])] - (24*I)*a^2*c*x*Log[c*x]
 - 24*a*b*c*x*Log[c*x] + 24*b^2*c*x*ArcTan[c*x]*PolyLog[2, E^((-2*I)*ArcTa
n[c*x])] + 12*b^2*c*x*(-I + 2*ArcTan[c*x])*PolyLog[2, -E^((2*I)*ArcTan[c*x
])] + (12*I)*b^2*c*x*PolyLog[2, E^((2*I)*ArcTan[c*x])] + 24*a*b*c*x*PolyLo
g[2, (-I)*c*x] - 24*a*b*c*x*PolyLog[2, I*c*x] - (12*I)*b^2*c*x*PolyLog[3,
E^((-2*I)*ArcTan[c*x])] + (12*I)*b^2*c*x*PolyLog[3, -E^((2*I)*ArcTan[c*x])
]))/x
```

## Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))^2}{x^2}\,dx$$

$$\Big\downarrow \text{ 5411}$$

$$\int \left(-c^2d^2(a+b\arctan(cx))^2+\frac{d^2(a+b\arctan(cx))^2}{x^2}+\frac{2icd^2(a+b\arctan(cx))^2}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$4icd^2\text{arctanh}\left(1-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2-c^2d^2x(a+b\arctan(cx))^2+2bcd^2\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))-2bcd^2\text{PolyLog}\left(2,\frac{2}{icx+1}-1\right)(a+b\arctan(cx))-2icd^2(a+b\arctan(cx))^2-\frac{d^2(a+b\arctan(cx))^2}{x}-2bcd^2\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))+2bcd^2\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))-ib^2cd^2\text{PolyLog}\left(2,\frac{2}{1-icx}-1\right)-ib^2cd^2\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)-ib^2cd^2\text{PolyLog}\left(3,1-\frac{2}{icx+1}\right)+ib^2cd^2\text{PolyLog}\left(3,\frac{2}{icx+1}-1\right)$$

input
```
Int[((d + I*c*d*x)^2*(a + b*ArcTan[c*x])^2)/x^2,x]
```

output
```
(-2*I)*c*d^2*(a + b*ArcTan[c*x])^2 - (d^2*(a + b*ArcTan[c*x])^2)/x - c^2*d
^2*x*(a + b*ArcTan[c*x])^2 + (4*I)*c*d^2*(a + b*ArcTan[c*x])^2*ArcTanh[1 -
 2/(1 + I*c*x)] - 2*b*c*d^2*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)] + 2*b*c
*d^2*(a + b*ArcTan[c*x])*Log[2 - 2/(1 - I*c*x)] - I*b^2*c*d^2*PolyLog[2, -
1 + 2/(1 - I*c*x)] - I*b^2*c*d^2*PolyLog[2, 1 - 2/(1 + I*c*x)] + 2*b*c*d^2
*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)] - 2*b*c*d^2*(a + b*ArcT
an[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)] - I*b^2*c*d^2*PolyLog[3, 1 - 2/(1
+ I*c*x)] + I*b^2*c*d^2*PolyLog[3, -1 + 2/(1 + I*c*x)]
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 8.03 (sec) , antiderivative size = 5048, normalized size of antiderivative = 15.92

| method | result | size |
|---|---|---|
| parts | Expression too large to display | 5048 |
| derivativedivides | Expression too large to display | 5050 |
| default | Expression too large to display | 5050 |

input
```
int((d+I*c*d*x)^2*(a+b*arctan(c*x))^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

## Fricas [F]

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))^2}{x^2}\,dx = \int \frac{(i\,cdx+d)^2(b\arctan{(cx)}+a)^2}{x^2}\,dx$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))^2/x^2,x, algorithm="fricas")
```

output
```
integral(-1/4*(4*a^2*c^2*d^2*x^2 - 8*I*a^2*c*d^2*x - 4*a^2*d^2 - (b^2*c^2*
d^2*x^2 - 2*I*b^2*c*d^2*x - b^2*d^2)*log(-(c*x + I)/(c*x - I))^2 + 4*(I*a*
b*c^2*d^2*x^2 + 2*a*b*c*d^2*x - I*a*b*d^2)*log(-(c*x + I)/(c*x - I)))/x^2,
 x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))^2}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)**2*(a+b*atan(c*x))**2/x**2,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))^2}{x^2}\,dx = \int \frac{(i\,cdx+d)^2(b\arctan(cx)+a)^2}{x^2}\,dx$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))^2/x^2,x, algorithm="maxima")
```

output
```
-a^2*c^2*d^2*x - (2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*a*b*c*d^2 + 2*I*a^
2*c*d^2*log(x) - (c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*a*b*d
^2 - a^2*d^2/x + 1/16*(8*b^2*c^2*d^2*x^2 - 2*b^2*c^2*d^2*x*integrate(4*arc
tan(c*x)^2 + log(c^2*x^2 + 1)^2, x) + 4*I*b^2*c^2*d^2*x*integrate(-1/4*(8*
(c^2*x^2 + 1)*c*x*arctan(c*x)^2 - 2*(c^2*x^2 + 1)*c*x*log(c^2*x^2 + 1)^2 +
 8*(c^2*x^2 + 1)*arctan(c*x)*log(c^2*x^2 + 1) - (4*(c^2*x^2 + 1)^(3/2)*arc
tan(c*x)*cos(2*arctan(c*x))*log(c^2*x^2 + 1) + 4*sqrt(c^2*x^2 + 1)*arctan(
c*x)*log(c^2*x^2 + 1) + (4*(c^2*x^2 + 1)^(3/2)*arctan(c*x)^2 - (c^2*x^2 +
1)^(3/2)*log(c^2*x^2 + 1)^2)*sin(2*arctan(c*x)))*sqrt(c^2*x^2 + 1))/((c^2*
x^2 + 1)^3*cos(2*arctan(c*x))^2 + (c^2*x^2 + 1)^3*sin(2*arctan(c*x))^2 + c
^2*x^2 + 2*(c^2*x^2 + 1)^2*cos(2*arctan(c*x)) + 4*(c^2*x^2 + 1)^2 - 4*((c^
2*x^2 + 1)^(3/2)*c*x*sin(2*arctan(c*x)) + (c^2*x^2 + 1)^(3/2)*cos(2*arctan
(c*x)) + sqrt(c^2*x^2 + 1))*sqrt(c^2*x^2 + 1) + 1), x) - 4*b^2*c^2*d^2*x*i
ntegrate(1/4*(8*(c^2*x^2 + 1)*c*x*arctan(c*x)*log(c^2*x^2 + 1) - 8*(c^2*x^
2 + 1)*arctan(c*x)^2 + 2*(c^2*x^2 + 1)*log(c^2*x^2 + 1)^2 - (4*(c^2*x^2 +
1)^(3/2)*arctan(c*x)*log(c^2*x^2 + 1)*sin(2*arctan(c*x)) - 4*sqrt(c^2*x^2
+ 1)*arctan(c*x)^2 + sqrt(c^2*x^2 + 1)*log(c^2*x^2 + 1)^2 - (4*(c^2*x^2 +
1)^(3/2)*arctan(c*x)^2 - (c^2*x^2 + 1)^(3/2)*log(c^2*x^2 + 1)^2)*cos(2*arc
tan(c*x)))*sqrt(c^2*x^2 + 1))/((c^2*x^2 + 1)^3*cos(2*arctan(c*x))^2 + (c^2
*x^2 + 1)^3*sin(2*arctan(c*x))^2 + c^2*x^2 + 2*(c^2*x^2 + 1)^2*cos(2*ar...
```

## Giac [F]

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))^2}{x^2}\,dx=\int \frac{(i\,cdx+d)^2(b\arctan(cx)+a)^2}{x^2}\,dx$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))^2/x^2,x, algorithm="giac")
```

output
```
integrate((I*c*d*x + d)^2*(b*arctan(c*x) + a)^2/x^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))^2}{x^2}\,dx=\int \frac{\left(a+b\,\mathrm{atan}(c\,x)\right)^2\,\left(d+c\,d\,x\,1\mathrm{i}\right)^2}{x^2}\,dx$$

input
```
int(((a + b*atan(c*x))^2*(d + c*d*x*1i)^2)/x^2,x)
```

output
```
int(((a + b*atan(c*x))^2*(d + c*d*x*1i)^2)/x^2, x)
```

## Reduce [F]

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))^2}{x^2}\,dx = \frac{d^2\left(-atan(cx)^2 b^2-2atan(cx)\,ab\,c^2x^2-2atan(cx)\,ab-\left(\int atan(cx)^2\,dx\right)b^2c^2x+2\left(\int \frac{atan(cx)}{c^2x^3+x}dx\right)b^2cx -\right.}{x}$$

input
```
int((d+I*c*d*x)^2*(a+b*atan(c*x))^2/x^2,x)
```

output
```
(d**2*( - atan(c*x)**2*b**2 - 2*atan(c*x)*a*b*c**2*x**2 - 2*atan(c*x)*a*b
- int(atan(c*x)**2,x)*b**2*c**2*x + 2*int(atan(c*x)/(c**2*x**3 + x),x)*b**
2*c*x + 4*int(atan(c*x)/x,x)*a*b*c*i*x + 2*int(atan(c*x)**2/x,x)*b**2*c*i*
x + 2*log(x)*a**2*c*i*x + 2*log(x)*a*b*c*x - a**2*c**2*x**2 - a**2))/x
```

# 3.82 $\int \frac{(d+icdx)^2(a+b\arctan(cx))^2}{x^3}\,dx$

## Optimal result

Integrand size = 25, antiderivative size = 337

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))^2}{x^3}\,dx = -\frac{bcd^2(a+b\arctan(cx))}{x} + \frac{3}{2}c^2d^2(a+b\arctan(cx))^2 - \frac{d^2(a+b\arctan(cx))^2}{2x^2} - \frac{2icd^2(a+b\arctan(cx))^2}{x} - 2c^2d^2(a+b\arctan(cx))^2\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right) + b^2c^2d^2\log(x) - \frac{1}{2}b^2c^2d^2\log\left(1+c^2x^2\right) + 4ibc^2d^2(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right) + 2b^2c^2d^2\,\text{PolyLog}\left(2,-1+\frac{2}{1-icx}\right) + ibc^2d^2(a+b\arctan(cx))\,\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right) - ibc^2d^2(a+b\arctan(cx))\,\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right) + \frac{1}{2}b^2c^2d^2\,\text{PolyLog}\left(3,1-\frac{2}{1+icx}\right) - \frac{1}{2}b^2c^2d^2\,\text{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)$$

output

```
-b*c*d^2*(a+b*arctan(c*x))/x+3/2*c^2*d^2*(a+b*arctan(c*x))^2-1/2*d^2*(a+b*
arctan(c*x))^2/x^2-2*I*c*d^2*(a+b*arctan(c*x))^2/x+2*c^2*d^2*(a+b*arctan(c
*x))^2*arctanh(-1+2/(1+I*c*x))+b^2*c^2*d^2*ln(x)-1/2*b^2*c^2*d^2*ln(c^2*x^
2+1)+4*I*b*c^2*d^2*(a+b*arctan(c*x))*ln(2-2/(1-I*c*x))+2*b^2*c^2*d^2*polyl
og(2,-1+2/(1-I*c*x))+I*b*c^2*d^2*(a+b*arctan(c*x))*polylog(2,1-2/(1+I*c*x)
)-I*b*c^2*d^2*(a+b*arctan(c*x))*polylog(2,-1+2/(1+I*c*x))+1/2*b^2*c^2*d^2*
polylog(3,1-2/(1+I*c*x))-1/2*b^2*c^2*d^2*polylog(3,-1+2/(1+I*c*x))
```

## Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.15

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))^2}{x^3}\,dx =$$

$$-\frac{d^2\Big(a^2+4ia^2cx+2ab(\arctan(cx)+cx(1+cx\arctan(cx)))+2a^2c^2x^2\log(x)+b^2\Big(2cx\arctan(cx)+(\cdots}{\cdots}$$

input
```
Integrate[((d + I*c*d*x)^2*(a + b*ArcTan[c*x])^2)/x^3,x]
```

output
```
-1/2*(d^2*(a^2 + (4*I)*a^2*c*x + 2*a*b*(ArcTan[c*x] + c*x*(1 + c*x*ArcTan[
c*x])) + 2*a^2*c^2*x^2*Log[x] + b^2*(2*c*x*ArcTan[c*x] + (1 + c^2*x^2)*Arc
Tan[c*x]^2 - 2*c^2*x^2*Log[(c*x)/Sqrt[1 + c^2*x^2]]) + (4*I)*a*b*c*x*(2*Ar
cTan[c*x] + c*x*(-2*Log[c*x] + Log[1 + c^2*x^2])) + (4*I)*b^2*c*x*(ArcTan[
c*x]^2 - 2*c*x*ArcTan[c*x]*Log[1 - E^((2*I)*ArcTan[c*x])] + I*c*x*(ArcTan[
c*x]^2 + PolyLog[2, E^((2*I)*ArcTan[c*x])])) + (2*I)*a*b*c^2*x^2*(PolyLog[
2, (-I)*c*x] - PolyLog[2, I*c*x]) + (b^2*c^2*x^2*((-I)*Pi^3 + (16*I)*ArcTa
n[c*x]^3 + 24*ArcTan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] - 24*ArcTan[c*
x]^2*Log[1 + E^((2*I)*ArcTan[c*x])] + (24*I)*ArcTan[c*x]*PolyLog[2, E^((-2
*I)*ArcTan[c*x])] + (24*I)*ArcTan[c*x]*PolyLog[2, -E^((2*I)*ArcTan[c*x])]
+ 12*PolyLog[3, E^((-2*I)*ArcTan[c*x])] - 12*PolyLog[3, -E^((2*I)*ArcTan[c
*x])]))/12))/x^2
```

## Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))^2}{x^3}\,dx$$

$$\Big\downarrow \text{5411}$$

$$\int \left(-\frac{c^2d^2(a+b\arctan(cx))^2}{x}+\frac{d^2(a+b\arctan(cx))^2}{x^3}+\frac{2icd^2(a+b\arctan(cx))^2}{x^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-2c^2d^2\text{arctanh}\left(1-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2+ibc^2d^2\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))-ibc^2d^2\,\text{PolyLog}\left(2,\frac{2}{icx+1}-1\right)(a+b\arctan(cx))+\frac{3}{2}c^2d^2(a+b\arctan(cx))^2+4ibc^2d^2\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))-\frac{d^2(a+b\arctan(cx))^2}{2x^2}-\frac{2icd^2(a+b\arctan(cx))^2}{x}-\frac{bcd^2(a+b\arctan(cx))}{x}+2b^2c^2d^2\,\text{PolyLog}\left(2,\frac{2}{1-icx}-1\right)+\frac{1}{2}b^2c^2d^2\,\text{PolyLog}\left(3,1-\frac{2}{icx+1}\right)-\frac{1}{2}b^2c^2d^2\,\text{PolyLog}\left(3,\frac{2}{icx+1}-1\right)-\frac{1}{2}b^2c^2d^2\log\left(c^2x^2+1\right)+b^2c^2d^2\log(x)$$

input
```
Int[((d + I*c*d*x)^2*(a + b*ArcTan[c*x])^2)/x^3,x]
```

output
```
-((b*c*d^2*(a + b*ArcTan[c*x]))/x) + (3*c^2*d^2*(a + b*ArcTan[c*x])^2)/2 -
 (d^2*(a + b*ArcTan[c*x])^2)/(2*x^2) - ((2*I)*c*d^2*(a + b*ArcTan[c*x])^2)
/x - 2*c^2*d^2*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)] + b^2*c^2*
d^2*Log[x] - (b^2*c^2*d^2*Log[1 + c^2*x^2])/2 + (4*I)*b*c^2*d^2*(a + b*Arc
Tan[c*x])*Log[2 - 2/(1 - I*c*x)] + 2*b^2*c^2*d^2*PolyLog[2, -1 + 2/(1 - I*
c*x)] + I*b*c^2*d^2*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)] - I*
b*c^2*d^2*(a + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)] + (b^2*c^2*d^
2*PolyLog[3, 1 - 2/(1 + I*c*x)])/2 - (b^2*c^2*d^2*PolyLog[3, -1 + 2/(1 + I
*c*x)])/2
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 14.13 (sec) , antiderivative size = 1381, normalized size of antiderivative = 4.10

| method | result | size |
|---|---|---|
| parts | Expression too large to display | 1381 |
| derivativedivides | Expression too large to display | 1383 |
| default | Expression too large to display | 1383 |

input
```
int((d+I*c*d*x)^2*(a+b*arctan(c*x))^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
d^2*a^2*(-1/2/x^2-2*I*c/x-c^2*ln(x))+d^2*b^2*c^2*(-1/2/c^2/x^2*arctan(c*x)
^2-1/2*I*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I/(1+(1+I*c*x)^2/(c^2
*x^2+1)))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*
arctan(c*x)^2+3/2*arctan(c*x)^2-1/2*arctan(c*x)*(I*c*x-(c^2*x^2+1)^(1/2)+1
)/c/x-ln(c*x)*arctan(c*x)^2+arctan(c*x)^2*ln((1+I*c*x)^2/(c^2*x^2+1)-1)-ar
ctan(c*x)^2*ln(1-(1+I*c*x)/(c^2*x^2+1)^(1/2))-arctan(c*x)^2*ln(1+(1+I*c*x)
/(c^2*x^2+1)^(1/2))-1/2*I*Pi*arctan(c*x)^2+2*I*arctan(c*x)*polylog(2,(1+I*
c*x)/(c^2*x^2+1)^(1/2))+2*I*arctan(c*x)*polylog(2,-(1+I*c*x)/(c^2*x^2+1)^(
1/2))+ln((1+I*c*x)/(c^2*x^2+1)^(1/2)-1)+1/2*polylog(3,-(1+I*c*x)^2/(c^2*x^
2+1))-2*polylog(3,(1+I*c*x)/(c^2*x^2+1)^(1/2))+ln(1+(1+I*c*x)/(c^2*x^2+1)^
(1/2))-2*polylog(3,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/2*I*Pi*csgn(I*((1+I*c*x
)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(((1+I*c*x)^2/(c^2*x^2
+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*arctan(c*x)^2+1/2*I*Pi*csgn(I*((1+I*
c*x)^2/(c^2*x^2+1)-1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(
c^2*x^2+1)))^2*arctan(c*x)^2+1/2*I*Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*
csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*arctan(c
*x)^2-1/2*I*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+
1)))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*arctan(
c*x)^2-1/2*arctan(c*x)*(I*c*x+(c^2*x^2+1)^(1/2)+1)/c/x-1/2*I*Pi*csgn(I*((1
+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^3*arctan(c*x)^2+1...
```

## Fricas [F]

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))^2}{x^3}\,dx = \int \frac{(i\,cdx+d)^2(b\arctan{(cx)}+a)^2}{x^3}\,dx$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))^2/x^3,x, algorithm="fricas")
```

output
```
integral(-1/4*(4*a^2*c^2*d^2*x^2 - 8*I*a^2*c*d^2*x - 4*a^2*d^2 - (b^2*c^2*
d^2*x^2 - 2*I*b^2*c*d^2*x - b^2*d^2)*log(-(c*x + I)/(c*x - I))^2 + 4*(I*a*
b*c^2*d^2*x^2 + 2*a*b*c*d^2*x - I*a*b*d^2)*log(-(c*x + I)/(c*x - I)))/x^3,
 x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))^2}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)**2*(a+b*atan(c*x))**2/x**3,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))^2}{x^3}\,dx = \int \frac{(i\,cdx+d)^2(b\arctan{(cx)}+a)^2}{x^3}\,dx$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))^2/x^3,x, algorithm="maxima")
```

output
```
-a^2*c^2*d^2*log(x) - 2*I*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)
/x)*a*b*c*d^2 - ((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*a*b*d^2 - 2*I*
a^2*c*d^2/x - 1/2*a^2*d^2/x^2 + 1/96*(48*I*(b^2*c^2*d^2*arctan(c*x)^3 + 4*
b^2*c^4*d^2*integrate(1/8*x^4*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^5 + x^3)
, x) + 2*b^2*c^3*d^2*integrate(1/8*x^3*log(c^2*x^2 + 1)^2/(c^2*x^5 + x^3),
 x) - 8*b^2*c^3*d^2*integrate(1/8*x^3*log(c^2*x^2 + 1)/(c^2*x^5 + x^3), x)
 + 20*b^2*c^2*d^2*integrate(1/8*x^2*arctan(c*x)/(c^2*x^5 + x^3), x) + 24*b
^2*c*d^2*integrate(1/8*x*arctan(c*x)^2/(c^2*x^5 + x^3), x) + 2*b^2*c*d^2*i
ntegrate(1/8*x*log(c^2*x^2 + 1)^2/(c^2*x^5 + x^3), x) + 2*b^2*c*d^2*integr
ate(1/8*x*log(c^2*x^2 + 1)/(c^2*x^5 + x^3), x) - 4*b^2*d^2*integrate(1/8*a
rctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^5 + x^3), x))*x^2 - (1152*b^2*c^4*d^2*i
ntegrate(1/16*x^4*arctan(c*x)^2/(c^2*x^5 + x^3), x) + 3072*a*b*c^4*d^2*int
egrate(1/16*x^4*arctan(c*x)/(c^2*x^5 + x^3), x) + b^2*c^2*d^2*log(c^2*x^2
+ 1)^3 + 48*b^2*c^2*d^2*arctan(c*x)^2 - 768*b^2*c^3*d^2*integrate(1/16*x^3
*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^5 + x^3), x) + 3072*a*b*c^2*d^2*integ
rate(1/16*x^2*arctan(c*x)/(c^2*x^5 + x^3), x) + 960*b^2*c^2*d^2*integrate(
1/16*x^2*log(c^2*x^2 + 1)/(c^2*x^5 + x^3), x) - 768*b^2*c*d^2*integrate(1/
16*x*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^5 + x^3), x) - 384*b^2*c*d^2*inte
grate(1/16*x*arctan(c*x)/(c^2*x^5 + x^3), x) - 1152*b^2*d^2*integrate(1/16
*arctan(c*x)^2/(c^2*x^5 + x^3), x) - 96*b^2*d^2*integrate(1/16*log(c^2*...
```

## Giac [F]

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))^2}{x^3}\,dx = \int \frac{(i\,cdx+d)^2(b\arctan(cx)+a)^2}{x^3}\,dx$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))^2/x^3,x, algorithm="giac")
```

output
```
integrate((I*c*d*x + d)^2*(b*arctan(c*x) + a)^2/x^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))^2}{x^3}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^2\,(d+c\,d\,x\,1\mathrm{i})^2}{x^3}\,dx$$

input
```
int(((a + b*atan(c*x))^2*(d + c*d*x*1i)^2)/x^3,x)
```

output
```
int(((a + b*atan(c*x))^2*(d + c*d*x*1i)^2)/x^3, x)
```

## Reduce [F]

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))^2}{x^3}\,dx = \frac{d^2\left(-atan(cx)^2\,b^2c^2x^2-4atan(cx)^2\,b^2cix-atan(cx)^2\,b^2-2atan(cx)\,ab\,c^2x^2-8atan(cx)\,abcix-2atan\right.}{}$$

input
```
int((d+I*c*d*x)^2*(a+b*atan(c*x))^2/x^3,x)
```

output
```
(d**2*( - atan(c*x)**2*b**2*c**2*x**2 - 4*atan(c*x)**2*b**2*c*i*x - atan(c
*x)**2*b**2 - 2*atan(c*x)*a*b*c**2*x**2 - 8*atan(c*x)*a*b*c*i*x - 2*atan(c
*x)*a*b - 2*atan(c*x)*b**2*c*x + 8*int(atan(c*x)/(c**2*x**3 + x),x)*b**2*c
**2*i*x**2 - 4*int(atan(c*x)/x,x)*a*b*c**2*x**2 - 2*int(atan(c*x)**2/x,x)*
b**2*c**2*x**2 - 4*log(c**2*x**2 + 1)*a*b*c**2*i*x**2 - log(c**2*x**2 + 1)
*b**2*c**2*x**2 - 2*log(x)*a**2*c**2*x**2 + 8*log(x)*a*b*c**2*i*x**2 + 2*l
og(x)*b**2*c**2*x**2 - 4*a**2*c*i*x - a**2 - 2*a*b*c*x))/(2*x**2)
```

# 3.83 $\int \frac{(d+icdx)^2(a+b\arctan(cx))^2}{x^4}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 267

$$\begin{aligned}\int \frac{(d+icdx)^2(a+b\arctan(cx))^2}{x^4}\,dx &= -\frac{b^2c^2d^2}{3x}-\frac{1}{3}b^2c^3d^2\arctan(cx)-\frac{bcd^2(a+b\arctan(cx))}{3x^2}\\ &\quad-\frac{2ibc^2d^2(a+b\arctan(cx))}{x}-\frac{d^2(1+icx)^3(a+b\arctan(cx))^2}{3x^3}\\ &\quad-\frac{8}{3}abc^3d^2\log(x)+2ib^2c^3d^2\log(x)-\frac{8}{3}bc^3d^2(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)\\ &\quad-ib^2c^3d^2\log\left(1+c^2x^2\right)-\frac{4}{3}ib^2c^3d^2\,\mathrm{PolyLog}(2,-icx)\\ &\quad+\frac{4}{3}ib^2c^3d^2\,\mathrm{PolyLog}(2,icx)+\frac{4}{3}ib^2c^3d^2\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-icx}\right)\end{aligned}$$

output
```
-1/3*b^2*c^2*d^2/x-1/3*b^2*c^3*d^2*arctan(c*x)-1/3*b*c*d^2*(a+b*arctan(c*x
))/x^2-2*I*b*c^2*d^2*(a+b*arctan(c*x))/x-1/3*d^2*(1+I*c*x)^3*(a+b*arctan(c
*x))^2/x^3-8/3*a*b*c^3*d^2*ln(x)+2*I*b^2*c^3*d^2*ln(x)-8/3*b*c^3*d^2*(a+b*
arctan(c*x))*ln(2/(1-I*c*x))-I*b^2*c^3*d^2*ln(c^2*x^2+1)-4/3*I*b^2*c^3*d^2
*polylog(2,-I*c*x)+4/3*I*b^2*c^3*d^2*polylog(2,I*c*x)+4/3*I*b^2*c^3*d^2*po
lylog(2,1-2/(1-I*c*x))
```

## Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.95

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))^2}{x^4}\,dx$$
$$= \frac{d^2\left(-a^2-3ia^2cx-abcx+3a^2c^2x^2-6iabc^2x^2-b^2c^2x^2+b^2(-1-icx)^3\arctan(cx)^2-b\arctan(cx)\left(bc\right.\right.}{}$$

input
```
Integrate[((d + I*c*d*x)^2*(a + b*ArcTan[c*x])^2)/x^4,x]
```

output
```
(d^2*(-a^2 - (3*I)*a^2*c*x - a*b*c*x + 3*a^2*c^2*x^2 - (6*I)*a*b*c^2*x^2 -
 b^2*c^2*x^2 + b^2*(-1 - I*c*x)^3*ArcTan[c*x]^2 - b*ArcTan[c*x]*(b*c*x*(1
+ (6*I)*c*x + c^2*x^2) + a*(2 + (6*I)*c*x - 6*c^2*x^2 + (6*I)*c^3*x^3) + 8
*b*c^3*x^3*Log[1 - E^((2*I)*ArcTan[c*x])]) - 8*a*b*c^3*x^3*Log[c*x] + (6*I
)*b^2*c^3*x^3*Log[(c*x)/Sqrt[1 + c^2*x^2]] + 4*a*b*c^3*x^3*Log[1 + c^2*x^2
] + (4*I)*b^2*c^3*x^3*PolyLog[2, E^((2*I)*ArcTan[c*x])]))/(3*x^3)
```

## Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.94, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {5409, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))^2}{x^4}\,dx$$
$$\Big\downarrow \; 5409$$
$$-2bc\int \left(-\frac{4d^2(a+b\arctan(cx))c^3}{3(cx+i)}+\frac{4d^2(a+b\arctan(cx))c^2}{3x}-\frac{id^2(a+b\arctan(cx))c}{x^2}-\frac{d^2(a+b\arctan(cx))}{3x^3}\right.$$
$$\frac{d^2(1+icx)^3(a+b\arctan(cx))^2}{3x^3}$$
$$\Big\downarrow \; 2009$$

$$-2bc\left(\frac{4}{3}c^2d^2\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))+\frac{d^2(a+b\arctan(cx))}{6x^2}+\frac{icd^2(a+b\arctan(cx))}{x}+\frac{4}{3}ac^2d^2\log(x)\right.$$
$$\frac{d^2(1+icx)^3(a+b\arctan(cx))^2}{3x^3}$$

input
```
Int[((d + I*c*d*x)^2*(a + b*ArcTan[c*x])^2)/x^4,x]
```

output
```
-1/3*(d^2*(1 + I*c*x)^3*(a + b*ArcTan[c*x])^2)/x^3 - 2*b*c*((b*c*d^2)/(6*x
) + (b*c^2*d^2*ArcTan[c*x])/6 + (d^2*(a + b*ArcTan[c*x]))/(6*x^2) + (I*c*d
^2*(a + b*ArcTan[c*x]))/x + (4*a*c^2*d^2*Log[x])/3 - I*b*c^2*d^2*Log[x] +
(4*c^2*d^2*(a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/3 + (I/2)*b*c^2*d^2*Log
[1 + c^2*x^2] + ((2*I)/3)*b*c^2*d^2*PolyLog[2, (-I)*c*x] - ((2*I)/3)*b*c^2
*d^2*PolyLog[2, I*c*x] - ((2*I)/3)*b*c^2*d^2*PolyLog[2, 1 - 2/(1 - I*c*x)]
)
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5409
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_.) + (e_
.)*(x_))^(q_), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Sim
p[(a + b*ArcTan[c*x])^p   u, x] - Simp[b*c*p   Int[ExpandIntegrand[(a + b*A
rcTan[c*x])^(p - 1), u/(1 + c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e,
f, q}, x] && IGtQ[p, 1] && EqQ[c^2*d^2 + e^2, 0] && IntegersQ[m, q] && NeQ[
m, -1] && NeQ[q, -1] && ILtQ[m + q + 1, 0] && LtQ[m*q, 0]
```

## Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.67

| method | result |
|---|---|
| parts | $d^2a^2\left(-\frac{ic}{x^2}-\frac{1}{3x^3}+\frac{c^2}{x}\right)+d^2b^2c^3\left(-\frac{\arctan(cx)^2}{3c^3x^3}+\frac{\arctan(cx)^2}{cx}+\frac{4i\,\mathrm{dilog}(-icx+1)}{3}-\frac{\arctan(cx)}{3c^2x^2}-\right.$ |
| derivativedivides | $c^3\left(d^2a^2\left(-\frac{1}{3c^3x^3}+\frac{1}{cx}-\frac{i}{c^2x^2}\right)+d^2b^2\left(-\frac{\arctan(cx)^2}{3c^3x^3}+\frac{\arctan(cx)^2}{cx}+\frac{4i\,\mathrm{dilog}(-icx+1)}{3}-\frac{\arctan(}{3c^2x}\right.\right.$ |
| default | $c^3\left(d^2a^2\left(-\frac{1}{3c^3x^3}+\frac{1}{cx}-\frac{i}{c^2x^2}\right)+d^2b^2\left(-\frac{\arctan(cx)^2}{3c^3x^3}+\frac{\arctan(cx)^2}{cx}+\frac{4i\,\mathrm{dilog}(-icx+1)}{3}-\frac{\arctan(}{3c^2x}\right.\right.$ |

input
```
int((d+I*c*d*x)^2*(a+b*arctan(c*x))^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
d^2*a^2*(-I*c/x^2-1/3/x^3+c^2/x)+d^2*b^2*c^3*(-1/3/c^3/x^3*arctan(c*x)^2+1
/c/x*arctan(c*x)^2+4/3*I*dilog(1-I*c*x)-1/3/c^2/x^2*arctan(c*x)-4/3*I*dilo
g(1+I*c*x)-8/3*ln(c*x)*arctan(c*x)+4/3*arctan(c*x)*ln(c^2*x^2+1)+2/3*I*(ln
(c*x-I)*ln(c^2*x^2+1)-1/2*ln(c*x-I)^2-dilog(-1/2*I*(c*x+I))-ln(c*x-I)*ln(-
1/2*I*(c*x+I)))-4/3*I*ln(c*x)*ln(1+I*c*x)-1/3/c/x+2*I*ln(c*x)-I*arctan(c*x
)^2-1/3*arctan(c*x)-2/3*I*(ln(c*x+I)*ln(c^2*x^2+1)-1/2*ln(c*x+I)^2-dilog(1
/2*I*(c*x-I))-ln(c*x+I)*ln(1/2*I*(c*x-I)))+4/3*I*ln(c*x)*ln(1-I*c*x)-2*I*a
rctan(c*x)/c/x-I*ln(c^2*x^2+1)-I*arctan(c*x)^2/c^2/x^2)+2*d^2*a*b*c^3*(-1/
3/c^3/x^3*arctan(c*x)+1/c/x*arctan(c*x)-I*arctan(c*x)/c^2/x^2-1/6/c^2/x^2-
I/c/x-4/3*ln(c*x)+2/3*ln(c^2*x^2+1)-I*arctan(c*x))
```

## Fricas [F]

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))^2}{x^4}\,dx=\int \frac{(i\,cdx+d)^2(b\arctan(cx)+a)^2}{x^4}\,dx$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))^2/x^4,x, algorithm="fricas")
```

output
```
1/12*(12*x^3*integral(-1/3*(3*a^2*c^4*d^2*x^4 - 6*I*a^2*c^3*d^2*x^3 - 6*I*
a^2*c*d^2*x - 3*a^2*d^2 - (-3*I*a*b*c^4*d^2*x^4 - 3*(2*a*b + I*b^2)*c^3*d^
2*x^3 - 3*b^2*c^2*d^2*x^2 - (6*a*b - I*b^2)*c*d^2*x + 3*I*a*b*d^2)*log(-(c
*x + I)/(c*x - I)))/(c^2*x^6 + x^4), x) - (3*b^2*c^2*d^2*x^2 - 3*I*b^2*c*d
^2*x - b^2*d^2)*log(-(c*x + I)/(c*x - I))^2)/x^3
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))^2}{x^4}\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)**2*(a+b*atan(c*x))**2/x**4,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))^2}{x^4}\,dx = \int \frac{(i\,cdx+d)^2(b\arctan(cx)+a)^2}{x^4}\,dx$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))^2/x^4,x, algorithm="maxima")
```

output
```
(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*a*b*c^2*d^2 - 2*I*((c*
arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*a*b*c*d^2 + 1/3*((c^2*log(c^2*x^2
+ 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*a*b*d^2 + a^2*c^2*d^2/
x - I*a^2*c*d^2/x^2 - 1/3*a^2*d^2/x^3 - 1/48*(12*(b^2*c^3*d^2*arctan(c*x)^
3 + 12*b^2*c^4*d^2*integrate(1/48*x^4*log(c^2*x^2 + 1)^2/(c^2*x^6 + x^4),
x) - 48*b^2*c^4*d^2*integrate(1/48*x^4*log(c^2*x^2 + 1)/(c^2*x^6 + x^4), x
) - 96*b^2*c^3*d^2*integrate(1/48*x^3*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^
6 + x^4), x) + 192*b^2*c^3*d^2*integrate(1/48*x^3*arctan(c*x)/(c^2*x^6 + x
^4), x) + 64*b^2*c^2*d^2*integrate(1/48*x^2*log(c^2*x^2 + 1)/(c^2*x^6 + x^
4), x) - 96*b^2*c*d^2*integrate(1/48*x*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x
^6 + x^4), x) - 32*b^2*c*d^2*integrate(1/48*x*arctan(c*x)/(c^2*x^6 + x^4),
 x) - 144*b^2*d^2*integrate(1/48*arctan(c*x)^2/(c^2*x^6 + x^4), x) - 12*b^
2*d^2*integrate(1/48*log(c^2*x^2 + 1)^2/(c^2*x^6 + x^4), x))*x^3 + 12*I*(b
^2*c^3*d^2*arctan(c*x)^2 - 24*b^2*c^4*d^2*integrate(1/24*x^4*arctan(c*x)*l
og(c^2*x^2 + 1)/(c^2*x^6 + x^4), x) - 144*b^2*c^3*d^2*integrate(1/24*x^3*a
rctan(c*x)^2/(c^2*x^6 + x^4), x) - 12*b^2*c^3*d^2*integrate(1/24*x^3*log(c
^2*x^2 + 1)^2/(c^2*x^6 + x^4), x) + 48*b^2*c^3*d^2*integrate(1/24*x^3*log(
c^2*x^2 + 1)/(c^2*x^6 + x^4), x) - 64*b^2*c^2*d^2*integrate(1/24*x^2*arcta
n(c*x)/(c^2*x^6 + x^4), x) - 144*b^2*c*d^2*integrate(1/24*x*arctan(c*x)^2/
(c^2*x^6 + x^4), x) - 12*b^2*c*d^2*integrate(1/24*x*log(c^2*x^2 + 1)^2/...
```

## Giac [F]

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))^2}{x^4}\,dx=\int \frac{(i\,cdx+d)^2(b\arctan{(cx)}+a)^2}{x^4}\,dx$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))^2/x^4,x, algorithm="giac")
```

output
```
integrate((I*c*d*x + d)^2*(b*arctan(c*x) + a)^2/x^4, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))^2}{x^4}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^2\,(d+c\,d\,x\,1\mathrm{i})^2}{x^4}\,dx$$

input
```
int(((a + b*atan(c*x))^2*(d + c*d*x*1i)^2)/x^4,x)
```

output
```
int(((a + b*atan(c*x))^2*(d + c*d*x*1i)^2)/x^4, x)
```

## Reduce [F]

$$\int \frac{(d+icdx)^2(a+b\arctan(cx))^2}{x^4}\,dx = \frac{d^2\left(-3atan(cx)^2\,b^2c^3i\,x^3+3atan(cx)^2\,b^2c^2x^2-3atan(cx)^2\,b^2cix-atan(cx)^2\,b^2-6atan(cx)\,ab\,c^3i\,x^3+\right.}{}$$

input
```
int((d+I*c*d*x)^2*(a+b*atan(c*x))^2/x^4,x)
```

output
```
(d**2*( - 3*atan(c*x)**2*b**2*c**3*i*x**3 + 3*atan(c*x)**2*b**2*c**2*x**2
- 3*atan(c*x)**2*b**2*c*i*x - atan(c*x)**2*b**2 - 6*atan(c*x)*a*b*c**3*i*x
**3 + 6*atan(c*x)*a*b*c**2*x**2 - 6*atan(c*x)*a*b*c*i*x - 2*atan(c*x)*a*b
+ 3*atan(c*x)*b**2*c**3*x**3 - 6*atan(c*x)*b**2*c**2*i*x**2 + 3*atan(c*x)*
b**2*c*x + 8*int(atan(c*x)/(c**2*x**5 + x**3),x)*b**2*c*x**3 + 4*log(c**2*
x**2 + 1)*a*b*c**3*x**3 - 3*log(c**2*x**2 + 1)*b**2*c**3*i*x**3 - 8*log(x)
*a*b*c**3*x**3 + 6*log(x)*b**2*c**3*i*x**3 + 3*a**2*c**2*x**2 - 3*a**2*c*i
*x - a**2 - 6*a*b*c**2*i*x**2 - a*b*c*x + 3*b**2*c**2*x**2))/(3*x**3)
```

# 3.84 $\int x^3(d+icdx)^3(a+b\arctan(cx))^2\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 438

$$\begin{aligned}
&\int x^3(d+icdx)^3(a+b\arctan(cx))^2\,dx\\
&=\frac{3abd^3x}{2c^3}-\frac{122ib^2d^3x}{105c^3}+\frac{7b^2d^3x^2}{20c^2}+\frac{44ib^2d^3x^3}{315c}-\frac{1}{20}b^2d^3x^4-\frac{1}{105}ib^2cd^3x^5\\
&\quad+\frac{122ib^2d^3\arctan(cx)}{105c^4}+\frac{3b^2d^3x\arctan(cx)}{2c^3}+\frac{26ibd^3x^2(a+b\arctan(cx))}{35c^2}\\
&\quad-\frac{bd^3x^3(a+b\arctan(cx))}{2c}-\frac{13}{35}ibd^3x^4(a+b\arctan(cx))+\frac{1}{5}bcd^3x^5(a+b\arctan(cx))\\
&\quad+\frac{1}{21}ibc^2d^3x^6(a+b\arctan(cx))-\frac{209d^3(a+b\arctan(cx))^2}{140c^4}\\
&\quad+\frac{1}{4}d^3x^4(a+b\arctan(cx))^2+\frac{3}{5}icd^3x^5(a+b\arctan(cx))^2-\frac{1}{2}c^2d^3x^6(a+b\arctan(cx))^2\\
&\quad-\frac{1}{7}ic^3d^3x^7(a+b\arctan(cx))^2+\frac{52ibd^3(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{35c^4}\\
&\quad-\frac{11b^2d^3\log(1+c^2x^2)}{10c^4}-\frac{26b^2d^3\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{35c^4}
\end{aligned}$$

output
```
3/2*a*b*d^3*x/c^3+122/105*I*b^2*d^3*arctan(c*x)/c^4+7/20*b^2*d^3*x^2/c^2+5
2/35*I*b*d^3*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^4-1/20*b^2*d^3*x^4-122/10
5*I*b^2*d^3*x/c^3+1/21*I*b*c^2*d^3*x^6*(a+b*arctan(c*x))+3/2*b^2*d^3*x*arc
tan(c*x)/c^3+3/5*I*c*d^3*x^5*(a+b*arctan(c*x))^2-1/2*b*d^3*x^3*(a+b*arctan
(c*x))/c-13/35*I*b*d^3*x^4*(a+b*arctan(c*x))+1/5*b*c*d^3*x^5*(a+b*arctan(c
*x))-1/7*I*c^3*d^3*x^7*(a+b*arctan(c*x))^2-209/140*d^3*(a+b*arctan(c*x))^2
/c^4+1/4*d^3*x^4*(a+b*arctan(c*x))^2+26/35*I*b*d^3*x^2*(a+b*arctan(c*x))/c
^2-1/2*c^2*d^3*x^6*(a+b*arctan(c*x))^2+44/315*I*b^2*d^3*x^3/c-1/105*I*b^2*
c*d^3*x^5-11/10*b^2*d^3*ln(c^2*x^2+1)/c^4-26/35*b^2*d^3*polylog(2,1-2/(1+I
*c*x))/c^4
```

## Mathematica [A] (verified)

Time = 1.27 (sec) , antiderivative size = 408, normalized size of antiderivative = 0.93

$$\int x^3(d+icdx)^3(a+b\arctan(cx))^2\,dx$$
$$=\frac{d^3\left(1464iab+504b^2+1890abcx-1464ib^2cx+936iabc^2x^2+441b^2c^2x^2-630abc^3x^3+176ib^2c^3x^3+315\right.}{}$$

input
```
Integrate[x^3*(d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2,x]
```

output
```
(d^3*((1464*I)*a*b + 504*b^2 + 1890*a*b*c*x - (1464*I)*b^2*c*x + (936*I)*a
*b*c^2*x^2 + 441*b^2*c^2*x^2 - 630*a*b*c^3*x^3 + (176*I)*b^2*c^3*x^3 + 315
*a^2*c^4*x^4 - (468*I)*a*b*c^4*x^4 - 63*b^2*c^4*x^4 + (756*I)*a^2*c^5*x^5
+ 252*a*b*c^5*x^5 - (12*I)*b^2*c^5*x^5 - 630*a^2*c^6*x^6 + (60*I)*a*b*c^6*
x^6 - (180*I)*a^2*c^7*x^7 + 9*b^2*(-I + c*x)^4*(-1 + (4*I)*c*x + 10*c^2*x^
2 - (20*I)*c^3*x^3)*ArcTan[c*x]^2 + 6*b*ArcTan[c*x]*(b*(244*I + 315*c*x +
(156*I)*c^2*x^2 - 105*c^3*x^3 - (78*I)*c^4*x^4 + 42*c^5*x^5 + (10*I)*c^6*x
^6) + 3*a*(-105 + 35*c^4*x^4 + (84*I)*c^5*x^5 - 70*c^6*x^6 - (20*I)*c^7*x^
7) + (312*I)*b*Log[1 + E^((2*I)*ArcTan[c*x])]) - (936*I)*a*b*Log[1 + c^2*x
^2] - 1386*b^2*Log[1 + c^2*x^2] + 936*b^2*PolyLog[2, -E^((2*I)*ArcTan[c*x]
)]))/(1260*c^4)
```

## Rubi [A] (verified)

Time = 1.62 (sec) , antiderivative size = 438, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(d+icdx)^3(a+b\arctan(cx))^2\,dx$$

$$\Big\downarrow \text{5411}$$

$$\int \left(-ic^3d^3x^6(a+b\arctan(cx))^2-3c^2d^3x^5(a+b\arctan(cx))^2+3icd^3x^4(a+b\arctan(cx))^2+d^3x^3(a+b\arctan$$

$$\Big\downarrow \text{2009}$$

$$-\frac{209d^3(a+b\arctan(cx))^2}{140c^4}+\frac{52ibd^3\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{35c^4}-\frac{1}{7}ic^3d^3x^7(a+b\arctan(cx))^2-\frac{1}{2}c^2d^3x^6(a+b\arctan(cx))^2+\frac{1}{21}ibc^2d^3x^6(a+b\arctan(cx))+\frac{26ibd^3x^2(a+b\arctan(cx))}{35c^2}+\frac{3}{5}icd^3x^5(a+b\arctan(cx))^2+\frac{1}{5}bcd^3x^5(a+b\arctan(cx))+\frac{1}{4}d^3x^4(a+b\arctan(cx))^2-\frac{13}{35}ibd^3x^4(a+b\arctan(cx))-\frac{bd^3x^3(a+b\arctan(cx))}{2c}+\frac{3abd^3x}{2c^3}+\frac{122ib^2d^3\arctan(cx)}{105c^4}+\frac{3b^2d^3x\arctan(cx)}{2c^3}-\frac{26b^2d^3\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{35c^4}-\frac{122ib^2d^3x}{105c^3}+\frac{7b^2d^3x^2}{20c^2}-\frac{11b^2d^3\log\left(c^2x^2+1\right)}{10c^4}-\frac{1}{105}ib^2cd^3x^5+\frac{44ib^2d^3x^3}{315c}-\frac{1}{20}b^2d^3x^4$$

input
```
Int[x^3*(d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2,x]
```

output
```
(3*a*b*d^3*x)/(2*c^3) - (((122*I)/105)*b^2*d^3*x)/c^3 + (7*b^2*d^3*x^2)/(2
0*c^2) + (((44*I)/315)*b^2*d^3*x^3)/c - (b^2*d^3*x^4)/20 - (I/105)*b^2*c*d
^3*x^5 + (((122*I)/105)*b^2*d^3*ArcTan[c*x])/c^4 + (3*b^2*d^3*x*ArcTan[c*x
])/(2*c^3) + (((26*I)/35)*b*d^3*x^2*(a + b*ArcTan[c*x]))/c^2 - (b*d^3*x^3*
(a + b*ArcTan[c*x]))/(2*c) - ((13*I)/35)*b*d^3*x^4*(a + b*ArcTan[c*x]) + (
b*c*d^3*x^5*(a + b*ArcTan[c*x]))/5 + (I/21)*b*c^2*d^3*x^6*(a + b*ArcTan[c*
x]) - (209*d^3*(a + b*ArcTan[c*x])^2)/(140*c^4) + (d^3*x^4*(a + b*ArcTan[c
*x])^2)/4 + ((3*I)/5)*c*d^3*x^5*(a + b*ArcTan[c*x])^2 - (c^2*d^3*x^6*(a +
b*ArcTan[c*x])^2)/2 - (I/7)*c^3*d^3*x^7*(a + b*ArcTan[c*x])^2 + (((52*I)/3
5)*b*d^3*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/c^4 - (11*b^2*d^3*Log[1 +
 c^2*x^2])/(10*c^4) - (26*b^2*d^3*PolyLog[2, 1 - 2/(1 + I*c*x)])/(35*c^4)
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

## Maple [A] (verified)

Time = 1.85 (sec) , antiderivative size = 511, normalized size of antiderivative = 1.17

| method | result |
|---|---|
| parts | $d^3a^2\left(-\frac{1}{7}ic^3x^7-\frac{1}{2}c^2x^6+\frac{3}{5}ic\,x^5+\frac{1}{4}x^4\right)+\frac{d^3b^2\left(\frac{3cx\arctan(cx)}{2}+\frac{c^5x^5\arctan(cx)}{5}-\frac{c^3x^3\arctan(cx)}{2}+\frac{122i}{}\right.}{}$ |
| derivativedivides | $\frac{d^3a^2\left(-\frac{1}{7}ic^7x^7-\frac{1}{2}c^6x^6+\frac{3}{5}ic^5x^5+\frac{1}{4}c^4x^4\right)+d^3b^2\left(\frac{3cx\arctan(cx)}{2}+\frac{c^5x^5\arctan(cx)}{5}-\frac{c^3x^3\arctan(cx)}{2}+\frac{122i\arctan(cx)}{105}+\frac{c^4}{}\right.}{}$ |
| default | $\frac{d^3a^2\left(-\frac{1}{7}ic^7x^7-\frac{1}{2}c^6x^6+\frac{3}{5}ic^5x^5+\frac{1}{4}c^4x^4\right)+d^3b^2\left(\frac{3cx\arctan(cx)}{2}+\frac{c^5x^5\arctan(cx)}{5}-\frac{c^3x^3\arctan(cx)}{2}+\frac{122i\arctan(cx)}{105}+\frac{c^4}{}\right.}{}$ |
| risch | $\frac{77d^3b^2}{45c^4}+\frac{id^3b\,c^2x^6a}{21}+\frac{26id^3b\,x^2a}{35c^2}+\frac{id^3b^2c\ln(-icx+1)x^5}{10}-\frac{id^3b^2\ln(-icx+1)x^3}{4c}+\frac{3id^3b^2\ln(-icx+1)x}{4c^3}-$ |

input
```
int(x^3*(d+I*c*d*x)^3*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
d^3*a^2*(-1/7*I*c^3*x^7-1/2*c^2*x^6+3/5*I*c*x^5+1/4*x^4)+d^3*b^2/c^4*(3/2*
c*x*arctan(c*x)+1/5*c^5*x^5*arctan(c*x)-1/2*c^3*x^3*arctan(c*x)+1/4*c^4*x^
4*arctan(c*x)^2-3/4*arctan(c*x)^2-11/10*ln(c^2*x^2+1)+7/20*c^2*x^2-1/2*c^6
*x^6*arctan(c*x)^2-1/20*c^4*x^4-13/70*ln(c*x-I)^2-13/35*dilog(-1/2*I*(c*x+
I))+13/70*ln(c*x+I)^2+13/35*dilog(1/2*I*(c*x-I))+13/35*ln(c*x-I)*ln(c^2*x^
2+1)-13/35*ln(c*x-I)*ln(-1/2*I*(c*x+I))-13/35*ln(c*x+I)*ln(c^2*x^2+1)+13/3
5*ln(c*x+I)*ln(1/2*I*(c*x-I))-26/35*I*arctan(c*x)*ln(c^2*x^2+1)-1/105*I*c^
5*x^5-122/105*I*c*x+44/315*I*c^3*x^3-1/7*I*arctan(c*x)^2*c^7*x^7+3/5*I*arc
tan(c*x)^2*c^5*x^5+1/21*I*arctan(c*x)*c^6*x^6+26/35*I*arctan(c*x)*c^2*x^2-
13/35*I*arctan(c*x)*c^4*x^4+122/105*I*arctan(c*x))+2*d^3*a*b/c^4*(-1/7*I*a
rctan(c*x)*c^7*x^7-1/2*c^6*x^6*arctan(c*x)+3/5*I*arctan(c*x)*c^5*x^5+1/4*c
^4*x^4*arctan(c*x)+3/4*c*x+1/42*I*c^6*x^6+1/10*c^5*x^5-13/70*I*c^4*x^4-1/4
*c^3*x^3+13/35*I*c^2*x^2-13/35*I*ln(c^2*x^2+1)-3/4*arctan(c*x))
```

## Fricas [F]

$$\int x^3(d+icdx)^3(a+b\arctan(cx))^2\,dx=\int (i\,cdx+d)^3(b\arctan{(cx)}+a)^2x^3\,dx$$

input
```
integrate(x^3*(d+I*c*d*x)^3*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

output
```
1/560*(20*I*b^2*c^3*d^3*x^7 + 70*b^2*c^2*d^3*x^6 - 84*I*b^2*c*d^3*x^5 - 35
*b^2*d^3*x^4)*log(-(c*x + I)/(c*x - I))^2 + integral(1/140*(-140*I*a^2*c^5
*d^3*x^8 - 420*a^2*c^4*d^3*x^7 + 280*I*a^2*c^3*d^3*x^6 - 280*a^2*c^2*d^3*x
^5 + 420*I*a^2*c*d^3*x^4 + 140*a^2*d^3*x^3 + (140*a*b*c^5*d^3*x^8 - 20*(21
*I*a*b + b^2)*c^4*d^3*x^7 - 70*(4*a*b - I*b^2)*c^3*d^3*x^6 - 28*(10*I*a*b
- 3*b^2)*c^2*d^3*x^5 - 35*(12*a*b + I*b^2)*c*d^3*x^4 + 140*I*a*b*d^3*x^3)*
log(-(c*x + I)/(c*x - I)))/(c^2*x^2 + 1), x)
```

## Sympy [F(-1)]

Timed out.

$$\int x^3(d+icdx)^3(a+b\arctan(cx))^2\,dx=\text{Timed out}$$

input
```
integrate(x**3*(d+I*c*d*x)**3*(a+b*atan(c*x))**2,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int x^3(d+icdx)^3(a+b\arctan(cx))^2\,dx=\int (i\,cdx+d)^3(b\arctan(cx)+a)^2x^3\,dx$$

input
```
integrate(x^3*(d+I*c*d*x)^3*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

output
```
-1/7*I*a^2*c^3*d^3*x^7 - 1/2*a^2*c^2*d^3*x^6 + 3/5*I*a^2*c*d^3*x^5 + 1/4*b
^2*d^3*x^4*arctan(c*x)^2 - 1/42*I*(12*x^7*arctan(c*x) - c*((2*c^4*x^6 - 3*
c^2*x^4 + 6*x^2)/c^6 - 6*log(c^2*x^2 + 1)/c^8))*a*b*c^3*d^3 + 1/4*a^2*d^3*
x^4 - 1/15*(15*x^6*arctan(c*x) - c*((3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 1
5*arctan(c*x)/c^7))*a*b*c^2*d^3 + 3/10*I*(4*x^5*arctan(c*x) - c*((c^2*x^4
- 2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*a*b*c*d^3 + 1/6*(3*x^4*arctan(c*x)
 - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*a*b*d^3 - 1/12*(2*c*((c^2*
x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5)*arctan(c*x) - (c^2*x^2 + 3*arctan(c*x)
^2 - 4*log(c^2*x^2 + 1))/c^4)*b^2*d^3 + 1/280*(-10*I*b^2*c^3*d^3*x^7 - 35*
b^2*c^2*d^3*x^6 + 42*I*b^2*c*d^3*x^5)*arctan(c*x)^2 + 1/280*(10*b^2*c^3*d^
3*x^7 - 35*I*b^2*c^2*d^3*x^6 - 42*b^2*c*d^3*x^5)*arctan(c*x)*log(c^2*x^2 +
 1) - 1/1120*(-10*I*b^2*c^3*d^3*x^7 - 35*b^2*c^2*d^3*x^6 + 42*I*b^2*c*d^3*
x^5)*log(c^2*x^2 + 1)^2 - I*integrate(1/560*(420*(b^2*c^5*d^3*x^8 - 2*b^2*
c^3*d^3*x^6 - 3*b^2*c*d^3*x^4)*arctan(c*x)^2 + 35*(b^2*c^5*d^3*x^8 - 2*b^2
*c^3*d^3*x^6 - 3*b^2*c*d^3*x^4)*log(c^2*x^2 + 1)^2 - 12*(15*b^2*c^4*d^3*x^
7 - 14*b^2*c^2*d^3*x^5)*arctan(c*x) + 2*(10*b^2*c^5*d^3*x^8 - 77*b^2*c^3*d
^3*x^6 - 210*(b^2*c^4*d^3*x^7 + b^2*c^2*d^3*x^5)*arctan(c*x))*log(c^2*x^2
+ 1))/(c^2*x^2 + 1), x) - integrate(1/560*(1260*(b^2*c^4*d^3*x^7 + b^2*c^2
*d^3*x^5)*arctan(c*x)^2 + 105*(b^2*c^4*d^3*x^7 + b^2*c^2*d^3*x^5)*log(c^2*
x^2 + 1)^2 + 4*(10*b^2*c^5*d^3*x^8 - 77*b^2*c^3*d^3*x^6)*arctan(c*x) + ...
```

## Giac [F]

$$\int x^3(d+icdx)^3(a+b\arctan(cx))^2\,dx=\int(i\,cdx+d)^3(b\arctan{(cx)}+a)^2x^3\,dx$$

input
```
integrate(x^3*(d+I*c*d*x)^3*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

output
```
integrate((I*c*d*x + d)^3*(b*arctan(c*x) + a)^2*x^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int x^3(d+icdx)^3(a+b\arctan(cx))^2\,dx=\int x^3\,(a+b\,\mathrm{atan}(c\,x))^2\,(d+c\,d\,x\,1\mathrm{i})^3\,dx$$

input
```
int(x^3*(a + b*atan(c*x))^2*(d + c*d*x*1i)^3,x)
```

output
```
int(x^3*(a + b*atan(c*x))^2*(d + c*d*x*1i)^3, x)
```

## Reduce [F]

$$\int x^3(d+icdx)^3(a+b\arctan(cx))^2\,dx$$
$$=\frac{d^3\left(630atan(cx)\,ab\,c^4x^4+756atan(cx)^2\,b^2c^5i\,x^5-936atan(cx)^2\,b^2cix-468atan(cx)\,b^2c^4i\,x^4+936atan(\right.}{}$$

input
```
int(x^3*(d+I*c*d*x)^3*(a+b*atan(c*x))^2,x)
```

output
```
(d**3*( - 180*atan(c*x)**2*b**2*c**7*i*x**7 - 630*atan(c*x)**2*b**2*c**6*x
**6 + 756*atan(c*x)**2*b**2*c**5*i*x**5 + 315*atan(c*x)**2*b**2*c**4*x**4
- 936*atan(c*x)**2*b**2*c*i*x - 945*atan(c*x)**2*b**2 - 360*atan(c*x)*a*b*
c**7*i*x**7 - 1260*atan(c*x)*a*b*c**6*x**6 + 1512*atan(c*x)*a*b*c**5*i*x**
5 + 630*atan(c*x)*a*b*c**4*x**4 - 1890*atan(c*x)*a*b + 60*atan(c*x)*b**2*c
**6*i*x**6 + 252*atan(c*x)*b**2*c**5*x**5 - 468*atan(c*x)*b**2*c**4*i*x**4
 - 630*atan(c*x)*b**2*c**3*x**3 + 936*atan(c*x)*b**2*c**2*i*x**2 + 1890*at
an(c*x)*b**2*c*x + 1464*atan(c*x)*b**2*i + 936*int(atan(c*x)**2,x)*b**2*c*
i - 936*log(c**2*x**2 + 1)*a*b*i - 1386*log(c**2*x**2 + 1)*b**2 - 180*a**2
*c**7*i*x**7 - 630*a**2*c**6*x**6 + 756*a**2*c**5*i*x**5 + 315*a**2*c**4*x
**4 + 60*a*b*c**6*i*x**6 + 252*a*b*c**5*x**5 - 468*a*b*c**4*i*x**4 - 630*a
*b*c**3*x**3 + 936*a*b*c**2*i*x**2 + 1890*a*b*c*x - 12*b**2*c**5*i*x**5 -
63*b**2*c**4*x**4 + 176*b**2*c**3*i*x**3 + 441*b**2*c**2*x**2 - 1464*b**2*
c*i*x))/(1260*c**4)
```

# 3.85 $\int x^2(d+icdx)^3(a+b\arctan(cx))^2\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 402

$$\begin{aligned}\int x^2(d+icdx)^3(a+b\arctan(cx))^2\,dx &= \frac{11iabd^3x}{6c^2}+\frac{37b^2d^3x}{30c^2}+\frac{61ib^2d^3x^2}{180c}-\frac{1}{10}b^2d^3x^3-\frac{1}{60}ib^2cd^3x^4-\frac{37b^2d^3\arctan(cx)}{30c^3}\\&\quad+\frac{11ib^2d^3x\arctan(cx)}{6c^2}-\frac{14bd^3x^2(a+b\arctan(cx))}{15c}-\frac{11}{18}ibd^3x^3(a+b\arctan(cx))\\&\quad+\frac{3}{10}bcd^3x^4(a+b\arctan(cx))+\frac{1}{15}ibc^2d^3x^5(a+b\arctan(cx))\\&\quad-\frac{37id^3(a+b\arctan(cx))^2}{20c^3}+\frac{1}{3}d^3x^3(a+b\arctan(cx))^2\\&\quad+\frac{3}{4}icd^3x^4(a+b\arctan(cx))^2-\frac{3}{5}c^2d^3x^5(a+b\arctan(cx))^2\\&\quad-\frac{1}{6}ic^3d^3x^6(a+b\arctan(cx))^2-\frac{28bd^3(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{15c^3}\\&\quad-\frac{113ib^2d^3\log(1+c^2x^2)}{90c^3}-\frac{14ib^2d^3\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{15c^3}\end{aligned}$$

output
```
-1/6*I*c^3*d^3*x^6*(a+b*arctan(c*x))^2+37/30*b^2*d^3*x/c^2+11/6*I*a*b*d^3*
x/c^2-1/10*b^2*d^3*x^3-113/90*I*b^2*d^3*ln(c^2*x^2+1)/c^3-37/30*b^2*d^3*ar
ctan(c*x)/c^3-14/15*I*b^2*d^3*polylog(2,1-2/(1+I*c*x))/c^3-14/15*b*d^3*x^2
*(a+b*arctan(c*x))/c+61/180*I*b^2*d^3*x^2/c+3/10*b*c*d^3*x^4*(a+b*arctan(c
*x))-1/60*I*b^2*c*d^3*x^4-37/20*I*d^3*(a+b*arctan(c*x))^2/c^3+1/3*d^3*x^3*
(a+b*arctan(c*x))^2-11/18*I*b*d^3*x^3*(a+b*arctan(c*x))-3/5*c^2*d^3*x^5*(a
+b*arctan(c*x))^2+11/6*I*b^2*d^3*x*arctan(c*x)/c^2-28/15*b*d^3*(a+b*arctan
(c*x))*ln(2/(1+I*c*x))/c^3+1/15*I*b*c^2*d^3*x^5*(a+b*arctan(c*x))+3/4*I*c*
d^3*x^4*(a+b*arctan(c*x))^2
```

## Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 369, normalized size of antiderivative = 0.92

$$\int x^2(d+icdx)^3(a+b\arctan(cx))^2\,dx = \frac{d^3\left(-162ab+64ib^2+330iabcx+222b^2cx-168abc^2x^2+61ib^2c^2x^2+60a^2c^3x^3-110iabc^3x^3-18b^2c^3x^3\right.}{}$$

input
```
Integrate[x^2*(d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2,x]
```

output
```
(d^3*(-162*a*b + (64*I)*b^2 + (330*I)*a*b*c*x + 222*b^2*c*x - 168*a*b*c^2*
x^2 + (61*I)*b^2*c^2*x^2 + 60*a^2*c^3*x^3 - (110*I)*a*b*c^3*x^3 - 18*b^2*c
^3*x^3 + (135*I)*a^2*c^4*x^4 + 54*a*b*c^4*x^4 - (3*I)*b^2*c^4*x^4 - 108*a^
2*c^5*x^5 + (12*I)*a*b*c^5*x^5 - (30*I)*a^2*c^6*x^6 + 3*b^2*(-I + c*x)^4*(
I + 4*c*x - (10*I)*c^2*x^2)*ArcTan[c*x]^2 + 2*b*ArcTan[c*x]*(b*(-111 + (16
5*I)*c*x - 84*c^2*x^2 - (55*I)*c^3*x^3 + 27*c^4*x^4 + (6*I)*c^5*x^5) + 3*a
*(-55*I + 20*c^3*x^3 + (45*I)*c^4*x^4 - 36*c^5*x^5 - (10*I)*c^6*x^6) - 168
*b*Log[1 + E^((2*I)*ArcTan[c*x])]) + 168*a*b*Log[1 + c^2*x^2] - (226*I)*b^
2*Log[1 + c^2*x^2] + (168*I)*b^2*PolyLog[2, -E^((2*I)*ArcTan[c*x])]))/(180
*c^3)
```

## Rubi [A] (verified)

Time = 1.45 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(d+icdx)^3(a+b\arctan(cx))^2\,dx$$

$$\Big\downarrow \text{5411}$$

$$\int \left(-ic^3d^3x^5(a+b\arctan(cx))^2-3c^2d^3x^4(a+b\arctan(cx))^2+3icd^3x^3(a+b\arctan(cx))^2+d^3x^2(a+b\arctan\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{6}ic^3d^3x^6(a+b\arctan(cx))^2-\frac{37id^3(a+b\arctan(cx))^2}{20c^3}-\frac{28bd^3\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{15c^3}-\frac{3}{5}c^2d^3x^5(a+b\arctan(cx))^2+\frac{1}{15}ibc^2d^3x^5(a+b\arctan(cx))+\frac{3}{4}icd^3x^4(a+b\arctan(cx))^2+\frac{3}{10}bcd^3x^4(a+b\arctan(cx))+\frac{1}{3}d^3x^3(a+b\arctan(cx))^2-\frac{11}{18}ibd^3x^3(a+b\arctan(cx))-\frac{14bd^3x^2(a+b\arctan(cx))}{15c}+\frac{11iabd^3x}{6c^2}-\frac{37b^2d^3\arctan(cx)}{30c^3}+\frac{11ib^2d^3x\arctan(cx)}{6c^2}-\frac{14ib^2d^3\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{15c^3}+\frac{37b^2d^3x}{30c^2}-\frac{113ib^2d^3\log\left(c^2x^2+1\right)}{90c^3}-\frac{1}{60}ib^2cd^3x^4+\frac{61ib^2d^3x^2}{180c}-\frac{1}{10}b^2d^3x^3$$

input
```
Int[x^2*(d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2,x]
```

output

```
(((11*I)/6)*a*b*d^3*x)/c^2 + (37*b^2*d^3*x)/(30*c^2) + (((61*I)/180)*b^2*d
^3*x^2)/c - (b^2*d^3*x^3)/10 - (I/60)*b^2*c*d^3*x^4 - (37*b^2*d^3*ArcTan[c
*x])/(30*c^3) + (((11*I)/6)*b^2*d^3*x*ArcTan[c*x])/c^2 - (14*b*d^3*x^2*(a
+ b*ArcTan[c*x]))/(15*c) - ((11*I)/18)*b*d^3*x^3*(a + b*ArcTan[c*x]) + (3*
b*c*d^3*x^4*(a + b*ArcTan[c*x]))/10 + (I/15)*b*c^2*d^3*x^5*(a + b*ArcTan[c
*x]) - (((37*I)/20)*d^3*(a + b*ArcTan[c*x])^2)/c^3 + (d^3*x^3*(a + b*ArcTa
n[c*x])^2)/3 + ((3*I)/4)*c*d^3*x^4*(a + b*ArcTan[c*x])^2 - (3*c^2*d^3*x^5*
(a + b*ArcTan[c*x])^2)/5 - (I/6)*c^3*d^3*x^6*(a + b*ArcTan[c*x])^2 - (28*b
*d^3*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(15*c^3) - (((113*I)/90)*b^2*
d^3*Log[1 + c^2*x^2])/c^3 - (((14*I)/15)*b^2*d^3*PolyLog[2, 1 - 2/(1 + I*c
*x)])/c^3
```

**Defintions of rubi rules used**

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

## Maple [A] (verified)

Time = 1.74 (sec) , antiderivative size = 488, normalized size of antiderivative = 1.21

| method | result |
|---|---|
| parts | $d^3a^2\left(-\frac{1}{6}ic^3x^6-\frac{3}{5}x^5c^2+\frac{3}{4}ic\,x^4+\frac{1}{3}x^3\right)+\frac{d^3b^2\left(\frac{61ic^2x^2}{180}-\frac{3c^5x^5\arctan(cx)^2}{5}+\frac{11i\arctan(cx)cx}{6}+\frac{c^3x^3\,\mathrm{arc}}{}\right.}{}$ |
| derivativedivides | $\frac{d^3a^2\left(-\frac{1}{6}ic^6x^6-\frac{3}{5}c^5x^5+\frac{3}{4}ic^4x^4+\frac{1}{3}c^3x^3\right)+d^3b^2\left(\frac{61ic^2x^2}{180}-\frac{3c^5x^5\arctan(cx)^2}{5}+\frac{11i\arctan(cx)cx}{6}+\frac{c^3x^3\arctan(cx)^2}{3}+\frac{7i(}{}\right.}{}$ |
| default | $\frac{d^3a^2\left(-\frac{1}{6}ic^6x^6-\frac{3}{5}c^5x^5+\frac{3}{4}ic^4x^4+\frac{1}{3}c^3x^3\right)+d^3b^2\left(\frac{61ic^2x^2}{180}-\frac{3c^5x^5\arctan(cx)^2}{5}+\frac{11i\arctan(cx)cx}{6}+\frac{c^3x^3\arctan(cx)^2}{3}+\frac{7i(}{}\right.}{}$ |
| risch | $-\frac{3id^3c^2ab\ln(-icx+1)x^5}{5}+\frac{3d^3cba\,x^4}{10}-\frac{337d^3ba}{90c^3}+\frac{14d^3ba\ln(c^2x^2+1)}{15c^3}-\frac{3d^3c^2a^2x^5}{5}-\frac{3d^3cba\ln(-icx+1)x}{4}$ |

input
```
int(x^2*(d+I*c*d*x)^3*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
d^3*a^2*(-1/6*I*c^3*x^6-3/5*x^5*c^2+3/4*I*c*x^4+1/3*x^3)+d^3*b^2/c^3*(61/1
80*I*c^2*x^2-3/5*c^5*x^5*arctan(c*x)^2+11/6*I*arctan(c*x)*c*x+1/3*c^3*x^3*
arctan(c*x)^2+7/15*I*(ln(c*x-I)*ln(c^2*x^2+1)-1/2*ln(c*x-I)^2-dilog(-1/2*I
*(c*x+I))-ln(c*x-I)*ln(-1/2*I*(c*x+I)))-11/12*I*arctan(c*x)^2+3/10*c^4*x^4
*arctan(c*x)-1/6*I*arctan(c*x)^2*c^6*x^6-14/15*c^2*x^2*arctan(c*x)+14/15*a
rctan(c*x)*ln(c^2*x^2+1)+1/15*I*arctan(c*x)*c^5*x^5+3/4*I*arctan(c*x)^2*c^
4*x^4-1/60*I*c^4*x^4-7/15*I*(ln(c*x+I)*ln(c^2*x^2+1)-1/2*ln(c*x+I)^2-dilog
(1/2*I*(c*x-I))-ln(c*x+I)*ln(1/2*I*(c*x-I)))+37/30*c*x-11/18*I*arctan(c*x)
*c^3*x^3-1/10*c^3*x^3-113/90*I*ln(c^2*x^2+1)-37/30*arctan(c*x))+2*d^3*a*b/
c^3*(-1/6*I*arctan(c*x)*c^6*x^6-3/5*c^5*x^5*arctan(c*x)+3/4*I*arctan(c*x)*
c^4*x^4+1/3*c^3*x^3*arctan(c*x)+11/12*I*c*x+1/30*I*c^5*x^5+3/20*c^4*x^4-11
/36*I*c^3*x^3-7/15*c^2*x^2+7/15*ln(c^2*x^2+1)-11/12*I*arctan(c*x))
```

## Fricas [F]

$$\int x^2(d+icdx)^3(a+b\arctan(cx))^2\,dx=\int (i\,cdx+d)^3(b\arctan(cx)+a)^2x^2\,dx$$

input
```
integrate(x^2*(d+I*c*d*x)^3*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

output
```
1/240*(10*I*b^2*c^3*d^3*x^6 + 36*b^2*c^2*d^3*x^5 - 45*I*b^2*c*d^3*x^4 - 20
*b^2*d^3*x^3)*log(-(c*x + I)/(c*x - I))^2 + integral(1/60*(-60*I*a^2*c^5*d
^3*x^7 - 180*a^2*c^4*d^3*x^6 + 120*I*a^2*c^3*d^3*x^5 - 120*a^2*c^2*d^3*x^4
 + 180*I*a^2*c*d^3*x^3 + 60*a^2*d^3*x^2 + (60*a*b*c^5*d^3*x^7 - 10*(18*I*a
*b + b^2)*c^4*d^3*x^6 - 12*(10*a*b - 3*I*b^2)*c^3*d^3*x^5 - 15*(8*I*a*b -
3*b^2)*c^2*d^3*x^4 - 20*(9*a*b + I*b^2)*c*d^3*x^3 + 60*I*a*b*d^3*x^2)*log(
-(c*x + I)/(c*x - I)))/(c^2*x^2 + 1), x)
```

## Sympy [F(-1)]

Timed out.

$$\int x^2(d+icdx)^3(a+b\arctan(cx))^2\,dx=\text{Timed out}$$

input
```
integrate(x**2*(d+I*c*d*x)**3*(a+b*atan(c*x))**2,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int x^2(d+icdx)^3(a+b\arctan(cx))^2\,dx=\int(i\,cdx+d)^3(b\arctan(cx)+a)^2x^2\,dx$$

input
```
integrate(x^2*(d+I*c*d*x)^3*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

output
```
-1/6*I*a^2*c^3*d^3*x^6 - 3/5*a^2*c^2*d^3*x^5 + 3/4*I*a^2*c*d^3*x^4 - 1/45*
I*(15*x^6*arctan(c*x) - c*((3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan(
c*x)/c^7))*a*b*c^3*d^3 - 3/10*(4*x^5*arctan(c*x) - c*((c^2*x^4 - 2*x^2)/c^
4 + 2*log(c^2*x^2 + 1)/c^6))*a*b*c^2*d^3 + 1/3*a^2*d^3*x^3 + 1/2*I*(3*x^4*
arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*a*b*c*d^3 + 1/3
*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*a*b*d^3 + 1/240*
(-10*I*b^2*c^3*d^3*x^6 - 36*b^2*c^2*d^3*x^5 + 45*I*b^2*c*d^3*x^4 + 20*b^2*
d^3*x^3)*arctan(c*x)^2 + 1/240*(10*b^2*c^3*d^3*x^6 - 36*I*b^2*c^2*d^3*x^5
- 45*b^2*c*d^3*x^4 + 20*I*b^2*d^3*x^3)*arctan(c*x)*log(c^2*x^2 + 1) - 1/96
0*(-10*I*b^2*c^3*d^3*x^6 - 36*b^2*c^2*d^3*x^5 + 45*I*b^2*c*d^3*x^4 + 20*b^
2*d^3*x^3)*log(c^2*x^2 + 1)^2 - I*integrate(1/240*(180*(b^2*c^5*d^3*x^7 -
2*b^2*c^3*d^3*x^5 - 3*b^2*c*d^3*x^3)*arctan(c*x)^2 + 15*(b^2*c^5*d^3*x^7 -
 2*b^2*c^3*d^3*x^5 - 3*b^2*c*d^3*x^3)*log(c^2*x^2 + 1)^2 - 2*(46*b^2*c^4*d
^3*x^6 - 65*b^2*c^2*d^3*x^4)*arctan(c*x) + (10*b^2*c^5*d^3*x^7 - 81*b^2*c^
3*d^3*x^5 + 20*b^2*c*d^3*x^3 - 60*(3*b^2*c^4*d^3*x^6 + 2*b^2*c^2*d^3*x^4 -
 b^2*d^3*x^2)*arctan(c*x))*log(c^2*x^2 + 1))/(c^2*x^2 + 1), x) - integrate
(1/240*(180*(3*b^2*c^4*d^3*x^6 + 2*b^2*c^2*d^3*x^4 - b^2*d^3*x^2)*arctan(c
*x)^2 + 15*(3*b^2*c^4*d^3*x^6 + 2*b^2*c^2*d^3*x^4 - b^2*d^3*x^2)*log(c^2*x
^2 + 1)^2 + 2*(10*b^2*c^5*d^3*x^7 - 81*b^2*c^3*d^3*x^5 + 20*b^2*c*d^3*x^3)
*arctan(c*x) + (46*b^2*c^4*d^3*x^6 - 65*b^2*c^2*d^3*x^4 + 60*(b^2*c^5*d...
```

## Giac [F]

$$\int x^2(d+icdx)^3(a+b\arctan(cx))^2\,dx=\int (i\,cdx+d)^3(b\arctan{(cx)}+a)^2x^2\,dx$$

input
```
integrate(x^2*(d+I*c*d*x)^3*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

output
```
integrate((I*c*d*x + d)^3*(b*arctan(c*x) + a)^2*x^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int x^2(d+icdx)^3(a+b\arctan(cx))^2\,dx=\int x^2\,(a+b\,\mathrm{atan}(c\,x))^2\,(d+c\,d\,x\,1\mathrm{i})^3\,dx$$

input
```
int(x^2*(a + b*atan(c*x))^2*(d + c*d*x*1i)^3,x)
```

output
```
int(x^2*(a + b*atan(c*x))^2*(d + c*d*x*1i)^3, x)
```

## Reduce [F]

$$\int x^2(d+icdx)^3(a+b\arctan(cx))^2\,dx$$
$$=\frac{d^3\left(168\log(c^2x^2+1)\,ab-108a^2c^5x^5+168atan(cx)^2\,b^2cx+54atan(cx)\,b^2c^4x^4-168atan(cx)\,b^2c^2x^2+5\right.}{}$$

input
```
int(x^2*(d+I*c*d*x)^3*(a+b*atan(c*x))^2,x)
```

output
```
(d**3*( - 30*atan(c*x)**2*b**2*c**6*i*x**6 - 108*atan(c*x)**2*b**2*c**5*x*
*5 + 135*atan(c*x)**2*b**2*c**4*i*x**4 + 60*atan(c*x)**2*b**2*c**3*x**3 +
168*atan(c*x)**2*b**2*c*x - 165*atan(c*x)**2*b**2*i - 60*atan(c*x)*a*b*c**
6*i*x**6 - 216*atan(c*x)*a*b*c**5*x**5 + 270*atan(c*x)*a*b*c**4*i*x**4 + 1
20*atan(c*x)*a*b*c**3*x**3 - 330*atan(c*x)*a*b*i + 12*atan(c*x)*b**2*c**5*
i*x**5 + 54*atan(c*x)*b**2*c**4*x**4 - 110*atan(c*x)*b**2*c**3*i*x**3 - 16
8*atan(c*x)*b**2*c**2*x**2 + 330*atan(c*x)*b**2*c*i*x - 222*atan(c*x)*b**2
 - 168*int(atan(c*x)**2,x)*b**2*c + 168*log(c**2*x**2 + 1)*a*b - 226*log(c
**2*x**2 + 1)*b**2*i - 30*a**2*c**6*i*x**6 - 108*a**2*c**5*x**5 + 135*a**2
*c**4*i*x**4 + 60*a**2*c**3*x**3 + 12*a*b*c**5*i*x**5 + 54*a*b*c**4*x**4 -
 110*a*b*c**3*i*x**3 - 168*a*b*c**2*x**2 + 330*a*b*c*i*x - 3*b**2*c**4*i*x
**4 - 18*b**2*c**3*x**3 + 61*b**2*c**2*i*x**2 + 222*b**2*c*x))/(180*c**3)
```

# 3.86 $\int x(d+icdx)^3(a+b\arctan(cx))^2\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 307

$$\begin{aligned}\int x(d+icdx)^3(a+b\arctan(cx))^2\,dx = & -\frac{5abd^3x}{2c}+\frac{13ib^2d^3x}{10c}-\frac{1}{4}b^2d^3x^2-\frac{1}{30}ib^2cd^3x^3 \\ & -\frac{13ib^2d^3\arctan(cx)}{10c^2}-\frac{5b^2d^3x\arctan(cx)}{2c} \\ & -\frac{6}{5}ibd^3x^2(a+b\arctan(cx)) \\ & +\frac{1}{2}bcd^3x^3(a+b\arctan(cx)) \\ & +\frac{1}{10}ibc^2d^3x^4(a+b\arctan(cx)) \\ & +\frac{d^3(1+icx)^4(a+b\arctan(cx))^2}{4c^2} \\ & -\frac{d^3(1+icx)^5(a+b\arctan(cx))^2}{5c^2} \\ & -\frac{12ibd^3(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)}{5c^2} \\ & +\frac{3b^2d^3\log(1+c^2x^2)}{2c^2} \\ & -\frac{6b^2d^3\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{5c^2}\end{aligned}$$

output
```
-5/2*a*b*d^3*x/c+13/10*I*b^2*d^3*x/c-1/4*b^2*d^3*x^2-1/30*I*b^2*c*d^3*x^3-
13/10*I*b^2*d^3*arctan(c*x)/c^2-5/2*b^2*d^3*x*arctan(c*x)/c-6/5*I*b*d^3*x^
2*(a+b*arctan(c*x))+1/2*b*c*d^3*x^3*(a+b*arctan(c*x))+1/10*I*b*c^2*d^3*x^4
*(a+b*arctan(c*x))+1/4*d^3*(1+I*c*x)^4*(a+b*arctan(c*x))^2/c^2-1/5*d^3*(1+
I*c*x)^5*(a+b*arctan(c*x))^2/c^2-12/5*I*b*d^3*(a+b*arctan(c*x))*ln(2/(1-I*
c*x))/c^2+3/2*b^2*d^3*ln(c^2*x^2+1)/c^2-6/5*b^2*d^3*polylog(2,1-2/(1-I*c*x
))/c^2
```

## Mathematica [A] (verified)

Time = 1.90 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.06

$$\int x(d+icdx)^3(a+b\arctan(cx))^2\,dx$$
$$= \frac{d^3\left(-18iab-15b^2-150abcx+78ib^2cx+30a^2c^2x^2-72iabc^2x^2-15b^2c^2x^2+60ia^2c^3x^3+30abc^3x^3-2\right.}{}$$

input
```
Integrate[x*(d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2,x]
```

output
```
(d^3*((-18*I)*a*b - 15*b^2 - 150*a*b*c*x + (78*I)*b^2*c*x + 30*a^2*c^2*x^2
 - (72*I)*a*b*c^2*x^2 - 15*b^2*c^2*x^2 + (60*I)*a^2*c^3*x^3 + 30*a*b*c^3*x
^3 - (2*I)*b^2*c^3*x^3 - 45*a^2*c^4*x^4 + (6*I)*a*b*c^4*x^4 - (12*I)*a^2*c
^5*x^5 + 3*b^2*(1 - (4*I)*c*x)*(-I + c*x)^4*ArcTan[c*x]^2 + 6*b*ArcTan[c*x
]*(b*(-13*I - 25*c*x - (12*I)*c^2*x^2 + 5*c^3*x^3 + I*c^4*x^4) + a*(25 + 1
0*c^2*x^2 + (20*I)*c^3*x^3 - 15*c^4*x^4 - (4*I)*c^5*x^5) - (24*I)*b*Log[1
+ E^((2*I)*ArcTan[c*x])]) + (72*I)*a*b*Log[1 + c^2*x^2] + 90*b^2*Log[1 + c
^2*x^2] - 72*b^2*PolyLog[2, -E^((2*I)*ArcTan[c*x])]))/(60*c^2)
```

## Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(d+icdx)^3(a+b\arctan(cx))^2\,dx$$

$$\Big\downarrow \text{5411}$$

$$\int \left(\frac{i(d+icdx)^3(a+b\arctan(cx))^2}{c}-\frac{i(d+icdx)^4(a+b\arctan(cx))^2}{cd}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{10}ibc^2d^3x^4(a+b\arctan(cx))-\frac{d^3(1+icx)^5(a+b\arctan(cx))^2}{5c^2}+\frac{d^3(1+icx)^4(a+b\arctan(cx))^2}{4c^2}-\frac{12ibd^3\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))}{5c^2}+\frac{1}{2}bcd^3x^3(a+b\arctan(cx))-\frac{6}{5}ibd^3x^2(a+b\arctan(cx))-\frac{5abd^3x}{2c}-\frac{13ib^2d^3\arctan(cx)}{10c^2}-\frac{5b^2d^3x\arctan(cx)}{2c}-\frac{6b^2d^3\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{5c^2}+\frac{3b^2d^3\log\left(c^2x^2+1\right)}{2c^2}-\frac{1}{30}ib^2cd^3x^3+\frac{13ib^2d^3x}{10c}-\frac{1}{4}b^2d^3x^2$$

input

```
Int[x*(d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2,x]
```

output

```
(-5*a*b*d^3*x)/(2*c) + (((13*I)/10)*b^2*d^3*x)/c - (b^2*d^3*x^2)/4 - (I/30
)*b^2*c*d^3*x^3 - (((13*I)/10)*b^2*d^3*ArcTan[c*x])/c^2 - (5*b^2*d^3*x*Arc
Tan[c*x])/(2*c) - ((6*I)/5)*b*d^3*x^2*(a + b*ArcTan[c*x]) + (b*c*d^3*x^3*(
a + b*ArcTan[c*x]))/2 + (I/10)*b*c^2*d^3*x^4*(a + b*ArcTan[c*x]) + (d^3*(1
 + I*c*x)^4*(a + b*ArcTan[c*x])^2)/(4*c^2) - (d^3*(1 + I*c*x)^5*(a + b*Arc
Tan[c*x])^2)/(5*c^2) - (((12*I)/5)*b*d^3*(a + b*ArcTan[c*x])*Log[2/(1 - I*
c*x)])/c^2 + (3*b^2*d^3*Log[1 + c^2*x^2])/(2*c^2) - (6*b^2*d^3*PolyLog[2,
1 - 2/(1 - I*c*x)])/(5*c^2)
```

## Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

## Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 452, normalized size of antiderivative = 1.47

| method | result |
|---|---|
| parts | $d^3a^2\left(-\frac{1}{5}ic^3x^5-\frac{3}{4}c^2x^4+icx^3+\frac{1}{2}x^2\right)+\frac{d^3b^2\left(i\arctan(cx)^2c^3x^3-\frac{3c^4x^4\arctan(cx)^2}{4}-\frac{13i\arctan(cx)}{10}+\right.}{}$ |
| derivativedivides | $\frac{d^3a^2\left(-\frac{1}{5}ic^5x^5-\frac{3}{4}c^4x^4+ic^3x^3+\frac{1}{2}c^2x^2\right)+d^3b^2\left(i\arctan(cx)^2c^3x^3-\frac{3c^4x^4\arctan(cx)^2}{4}-\frac{13i\arctan(cx)}{10}+\frac{c^2x^2\arctan(cx)^2}{2}\right.}{}$ |
| default | $\frac{d^3a^2\left(-\frac{1}{5}ic^5x^5-\frac{3}{4}c^4x^4+ic^3x^3+\frac{1}{2}c^2x^2\right)+d^3b^2\left(i\arctan(cx)^2c^3x^3-\frac{3c^4x^4\arctan(cx)^2}{4}-\frac{13i\arctan(cx)}{10}+\frac{c^2x^2\arctan(cx)^2}{2}\right.}{}$ |
| risch | $\frac{5d^3ba\arctan(cx)}{2c^2}+\frac{49d^3a^2}{20c^2}+\frac{id^3ba\ln(-icx+1)x^2}{2}-d^3cba\ln(-icx+1)x^3+\frac{d^3c^3ba\ln(-icx+1)x^5}{5}+$ |

input
```
int(x*(d+I*c*d*x)^3*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
d^3*a^2*(-1/5*I*c^3*x^5-3/4*c^2*x^4+I*c*x^3+1/2*x^2)+d^3*b^2/c^2*(I*arctan
(c*x)^2*c^3*x^3-3/4*c^4*x^4*arctan(c*x)^2-13/10*I*arctan(c*x)+1/2*c^2*x^2*
arctan(c*x)^2-1/5*I*arctan(c*x)^2*c^5*x^5+6/5*I*ln(c^2*x^2+1)*arctan(c*x)+
1/2*c^3*x^3*arctan(c*x)+1/10*I*arctan(c*x)*c^4*x^4+5/4*arctan(c*x)^2-5/2*c
*x*arctan(c*x)+3/5*ln(c*x+I)*ln(c^2*x^2+1)-3/5*ln(c*x-I)*ln(c^2*x^2+1)-3/1
0*ln(c*x+I)^2-3/5*ln(c*x+I)*ln(1/2*I*(c*x-I))+3/5*ln(c*x-I)*ln(-1/2*I*(c*x
+I))+3/10*ln(c*x-I)^2-3/5*dilog(1/2*I*(c*x-I))+3/5*dilog(-1/2*I*(c*x+I))-1
/30*I*c^3*x^3-6/5*I*arctan(c*x)*c^2*x^2-1/4*c^2*x^2+3/2*ln(c^2*x^2+1)+13/1
0*I*c*x)+2*d^3*a*b/c^2*(-1/5*I*arctan(c*x)*c^5*x^5-3/4*c^4*x^4*arctan(c*x)
+I*arctan(c*x)*c^3*x^3+1/2*c^2*x^2*arctan(c*x)-5/4*c*x+1/20*I*c^4*x^4+1/4*
c^3*x^3-3/5*I*c^2*x^2+3/5*I*ln(c^2*x^2+1)+5/4*arctan(c*x))
```

## Fricas [F]

$$\int x(d+icdx)^3(a+b\arctan(cx))^2\,dx=\int(i\,cdx+d)^3(b\arctan(cx)+a)^2x\,dx$$

input
```
integrate(x*(d+I*c*d*x)^3*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

output
```
1/80*(4*I*b^2*c^3*d^3*x^5 + 15*b^2*c^2*d^3*x^4 - 20*I*b^2*c*d^3*x^3 - 10*b
^2*d^3*x^2)*log(-(c*x + I)/(c*x - I))^2 + integral(1/20*(-20*I*a^2*c^5*d^3
*x^6 - 60*a^2*c^4*d^3*x^5 + 40*I*a^2*c^3*d^3*x^4 - 40*a^2*c^2*d^3*x^3 + 60
*I*a^2*c*d^3*x^2 + 20*a^2*d^3*x + (20*a*b*c^5*d^3*x^6 - 4*(15*I*a*b + b^2)
*c^4*d^3*x^5 - 5*(8*a*b - 3*I*b^2)*c^3*d^3*x^4 - 20*(2*I*a*b - b^2)*c^2*d^
3*x^3 - 10*(6*a*b + I*b^2)*c*d^3*x^2 + 20*I*a*b*d^3*x)*log(-(c*x + I)/(c*x
 - I)))/(c^2*x^2 + 1), x)
```

## Sympy [F(-1)]

Timed out.

$$\int x(d+icdx)^3(a+b\arctan(cx))^2\,dx=\text{Timed out}$$

input
```
integrate(x*(d+I*c*d*x)**3*(a+b*atan(c*x))**2,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int x(d+icdx)^3(a+b\arctan(cx))^2\,dx=\int(i\,cdx+d)^3(b\arctan(cx)+a)^2x\,dx$$

input
```
integrate(x*(d+I*c*d*x)^3*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

output
```
-1/5*I*a^2*c^3*d^3*x^5 - 3/4*a^2*c^2*d^3*x^4 - 1/10*I*(4*x^5*arctan(c*x) -
 c*((c^2*x^4 - 2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*a*b*c^3*d^3 + I*a^2*c
*d^3*x^3 + 1/2*b^2*d^3*x^2*arctan(c*x)^2 - 1/2*(3*x^4*arctan(c*x) - c*((c^
2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*a*b*c^2*d^3 + I*(2*x^3*arctan(c*x)
- c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*a*b*c*d^3 + 1/2*a^2*d^3*x^2 + (x^2*a
rctan(c*x) - c*(x/c^2 - arctan(c*x)/c^3))*a*b*d^3 - 1/2*(2*c*(x/c^2 - arct
an(c*x)/c^3)*arctan(c*x) + (arctan(c*x)^2 - log(c^2*x^2 + 1))/c^2)*b^2*d^3
 + 1/80*(-4*I*b^2*c^3*d^3*x^5 - 15*b^2*c^2*d^3*x^4 + 20*I*b^2*c*d^3*x^3)*a
rctan(c*x)^2 + 1/80*(4*b^2*c^3*d^3*x^5 - 15*I*b^2*c^2*d^3*x^4 - 20*b^2*c*d
^3*x^3)*arctan(c*x)*log(c^2*x^2 + 1) - 1/320*(-4*I*b^2*c^3*d^3*x^5 - 15*b^
2*c^2*d^3*x^4 + 20*I*b^2*c*d^3*x^3)*log(c^2*x^2 + 1)^2 - I*integrate(1/80*
(60*(b^2*c^5*d^3*x^6 - 2*b^2*c^3*d^3*x^4 - 3*b^2*c*d^3*x^2)*arctan(c*x)^2
+ 5*(b^2*c^5*d^3*x^6 - 2*b^2*c^3*d^3*x^4 - 3*b^2*c*d^3*x^2)*log(c^2*x^2 +
1)^2 - 2*(19*b^2*c^4*d^3*x^5 - 20*b^2*c^2*d^3*x^3)*arctan(c*x) + (4*b^2*c^
5*d^3*x^6 - 35*b^2*c^3*d^3*x^4 - 60*(b^2*c^4*d^3*x^5 + b^2*c^2*d^3*x^3)*ar
ctan(c*x))*log(c^2*x^2 + 1))/(c^2*x^2 + 1), x) - integrate(1/80*(180*(b^2*
c^4*d^3*x^5 + b^2*c^2*d^3*x^3)*arctan(c*x)^2 + 15*(b^2*c^4*d^3*x^5 + b^2*c
^2*d^3*x^3)*log(c^2*x^2 + 1)^2 + 2*(4*b^2*c^5*d^3*x^6 - 35*b^2*c^3*d^3*x^4
)*arctan(c*x) + (19*b^2*c^4*d^3*x^5 - 20*b^2*c^2*d^3*x^3 + 20*(b^2*c^5*d^3
*x^6 - 2*b^2*c^3*d^3*x^4 - 3*b^2*c*d^3*x^2)*arctan(c*x))*log(c^2*x^2 + ...
```

## Giac [F]

$$\int x(d+icdx)^3(a+b\arctan(cx))^2\,dx=\int (i\,cdx+d)^3(b\arctan(cx)+a)^2x\,dx$$

input
```
integrate(x*(d+I*c*d*x)^3*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

output
```
integrate((I*c*d*x + d)^3*(b*arctan(c*x) + a)^2*x, x)
```

## Mupad [F(-1)]

Timed out.

$$\int x(d+icdx)^3(a+b\arctan(cx))^2\,dx=\int x\,(a+b\,\mathrm{atan}(c\,x))^2\,(d+c\,d\,x\,1\mathrm{i})^3\,dx$$

input
```
int(x*(a + b*atan(c*x))^2*(d + c*d*x*1i)^3,x)
```

output
```
int(x*(a + b*atan(c*x))^2*(d + c*d*x*1i)^3, x)
```

## Reduce [F]

$$\int x(d+icdx)^3(a+b\arctan(cx))^2\,dx$$
$$=\frac{d^3\left(60atan(cx)^2\,b^2c^3i\,x^3-90atan(cx)\,ab\,c^4x^4-12atan(cx)^2\,b^2c^5i\,x^5+72atan(cx)^2\,b^2cix+6atan(cx)\,b^2\right.}{}$$

input
```
int(x*(d+I*c*d*x)^3*(a+b*atan(c*x))^2,x)
```

output
```
(d**3*( - 12*atan(c*x)**2*b**2*c**5*i*x**5 - 45*atan(c*x)**2*b**2*c**4*x**
4 + 60*atan(c*x)**2*b**2*c**3*i*x**3 + 30*atan(c*x)**2*b**2*c**2*x**2 + 72
*atan(c*x)**2*b**2*c*i*x + 75*atan(c*x)**2*b**2 - 24*atan(c*x)*a*b*c**5*i*
x**5 - 90*atan(c*x)*a*b*c**4*x**4 + 120*atan(c*x)*a*b*c**3*i*x**3 + 60*ata
n(c*x)*a*b*c**2*x**2 + 150*atan(c*x)*a*b + 6*atan(c*x)*b**2*c**4*i*x**4 +
30*atan(c*x)*b**2*c**3*x**3 - 72*atan(c*x)*b**2*c**2*i*x**2 - 150*atan(c*x
)*b**2*c*x - 78*atan(c*x)*b**2*i - 72*int(atan(c*x)**2,x)*b**2*c*i + 72*lo
g(c**2*x**2 + 1)*a*b*i + 90*log(c**2*x**2 + 1)*b**2 - 12*a**2*c**5*i*x**5
- 45*a**2*c**4*x**4 + 60*a**2*c**3*i*x**3 + 30*a**2*c**2*x**2 + 6*a*b*c**4
*i*x**4 + 30*a*b*c**3*x**3 - 72*a*b*c**2*i*x**2 - 150*a*b*c*x - 2*b**2*c**
3*i*x**3 - 15*b**2*c**2*x**2 + 78*b**2*c*i*x))/(60*c**2)
```

# 3.87 $\int (d+icdx)^3(a+b\arctan(cx))^2\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 226

$$\begin{aligned}\int (d+icdx)^3(a+b\arctan(cx))^2\,dx = &-\frac{7}{2}iabd^3x-b^2d^3x-\frac{1}{12}ib^2cd^3x^2+\frac{b^2d^3\arctan(cx)}{c}\\&-\frac{7}{2}ib^2d^3x\arctan(cx)+bcd^3x^2(a+b\arctan(cx))\\&+\frac{1}{6}ibc^2d^3x^3(a+b\arctan(cx))\\&-\frac{id^3(1+icx)^4(a+b\arctan(cx))^2}{4c}\\&+\frac{4bd^3(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)}{c}\\&+\frac{11ib^2d^3\log(1+c^2x^2)}{6c}\\&-\frac{2ib^2d^3\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{c}\end{aligned}$$

output

```
-7/2*I*a*b*d^3*x-b^2*d^3*x-1/12*I*b^2*c*d^3*x^2+b^2*d^3*arctan(c*x)/c-7/2*
I*b^2*d^3*x*arctan(c*x)+b*c*d^3*x^2*(a+b*arctan(c*x))+1/6*I*b*c^2*d^3*x^3*
(a+b*arctan(c*x))-1/4*I*d^3*(1+I*c*x)^4*(a+b*arctan(c*x))^2/c+4*b*d^3*(a+b
*arctan(c*x))*ln(2/(1-I*c*x))/c+11/6*I*b^2*d^3*ln(c^2*x^2+1)/c-2*I*b^2*d^3
*polylog(2,1-2/(1-I*c*x))/c
```

## Mathematica [A] (verified)

Time = 1.54 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.18

$$\int (d+icdx)^3(a+b\arctan(cx))^2\,dx = -\frac{id^3\left(b^2+12ia^2cx+42abcx-12ib^2cx-18a^2c^2x^2+12iabc^2x^2+b^2c^2x^2-12ia^2c^3x^3-2abc^3x^3+3a^2c\right.}{}$$

input
```
Integrate[(d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2,x]
```

output
```
((-1/12*I)*d^3*(b^2 + (12*I)*a^2*c*x + 42*a*b*c*x - (12*I)*b^2*c*x - 18*a^
2*c^2*x^2 + (12*I)*a*b*c^2*x^2 + b^2*c^2*x^2 - (12*I)*a^2*c^3*x^3 - 2*a*b*
c^3*x^3 + 3*a^2*c^4*x^4 + 3*b^2*(-I + c*x)^4*ArcTan[c*x]^2 + 2*b*ArcTan[c*
x]*(b*(6*I + 21*c*x + (6*I)*c^2*x^2 - c^3*x^3) + 3*a*(-7 + (4*I)*c*x - 6*c
^2*x^2 - (4*I)*c^3*x^3 + c^4*x^4) + (24*I)*b*Log[1 + E^((2*I)*ArcTan[c*x])
]) - (24*I)*a*b*Log[1 + c^2*x^2] - 22*b^2*Log[1 + c^2*x^2] + 24*b^2*PolyLo
g[2, -E^((2*I)*ArcTan[c*x])]))/c
```

## Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.95, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {5389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+icdx)^3(a+b\arctan(cx))^2\,dx$$

$$\Big\downarrow \text{5389}$$

$$\frac{ib\int \left(c^2x^2(a+b\arctan(cx))d^4-4icx(a+b\arctan(cx))d^4-\frac{8i(i-cx)(a+b\arctan(cx))d^4}{c^2x^2+1}-7(a+b\arctan(cx))d^4\right)dx}{2d}$$
$$\frac{id^3(1+icx)^4(a+b\arctan(cx))^2}{4c}$$

$$\Big\downarrow \text{2009}$$

$$\frac{ib\left(\frac{1}{3}c^2d^4x^3(a+b\arctan(cx))-2icd^4x^2(a+b\arctan(cx))-\frac{8id^4\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))}{c}-7ad^4x-7bd^4x\arctan\right.}{2d}$$

$$\frac{id^3(1+icx)^4(a+b\arctan(cx))^2}{4c}$$

input
```
Int[(d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2,x]
```

output
```
((-1/4*I)*d^3*(1 + I*c*x)^4*(a + b*ArcTan[c*x])^2)/c + ((I/2)*b*(-7*a*d^4*
x + (2*I)*b*d^4*x - (b*c*d^4*x^2)/6 - ((2*I)*b*d^4*ArcTan[c*x])/c - 7*b*d^
4*x*ArcTan[c*x] - (2*I)*c*d^4*x^2*(a + b*ArcTan[c*x]) + (c^2*d^4*x^3*(a +
b*ArcTan[c*x]))/3 - ((8*I)*d^4*(a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/c +
 (11*b*d^4*Log[1 + c^2*x^2])/(3*c) - (4*b*d^4*PolyLog[2, 1 - 2/(1 - I*c*x)
])/c))/d
```

**Defintions of rubi rules used**

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5389
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - S
imp[b*c*(p/(e*(q + 1)))   Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1),
(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

## Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 411 vs. 2(204) = 408.

Time = 1.21 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.82

| method | result |
|---|---|
| derivativedivides | $\frac{-\frac{i d^3 a^2 (icx+1)^4}{4}+d^3 b^2 \left(-\frac{i \arctan(cx)^2 c^4 x^4}{4}-c^3 x^3 \arctan(cx)^2+\frac{3 i \arctan(cx)^2 c^2 x^2}{2}+cx \arctan(cx)^2-\frac{i \arctan(cx)^2}{4}+\frac{i(-}{}\right.}{}$ |
| default | $\frac{-\frac{i d^3 a^2 (icx+1)^4}{4}+d^3 b^2 \left(-\frac{i \arctan(cx)^2 c^4 x^4}{4}-c^3 x^3 \arctan(cx)^2+\frac{3 i \arctan(cx)^2 c^2 x^2}{2}+cx \arctan(cx)^2-\frac{i \arctan(cx)^2}{4}+\frac{i(-}{}\right.}{}$ |
| parts | $-\frac{i d^3 a^2 (icx+1)^4}{4c}+\frac{d^3 b^2 \left(-\frac{i \arctan(cx)^2 c^4 x^4}{4}-c^3 x^3 \arctan(cx)^2+\frac{3 i \arctan(cx)^2 c^2 x^2}{2}+cx \arctan(cx)^2-\frac{i \arctan(cx)^2}{4}+\right.}{}$ |
| risch | $x d^3 a^2+\frac{14 a b d^3}{3c}+d^3 b a c x^2+\frac{i d^3 (cx-i)^4 b^2 \ln(icx+1)^2}{16c}-b^2 d^3 x-\frac{2 a b d^3 \ln(c^2x^2+1)}{c}-d^3 c^2 a^2 x^3 -$ |

input
```
int((d+I*c*d*x)^3*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*(-1/4*I*d^3*a^2*(1+I*c*x)^4+d^3*b^2*(-1/4*I*arctan(c*x)^2*c^4*x^4-c^3*
x^3*arctan(c*x)^2+3/2*I*arctan(c*x)^2*c^2*x^2+c*x*arctan(c*x)^2-1/4*I*arct
an(c*x)^2+1/2*I*(-7*c*x*arctan(c*x)+1/3*c^3*x^3*arctan(c*x)-2*I*arctan(c*x
)+4*I*arctan(c*x)*ln(c^2*x^2+1)+4*arctan(c*x)^2+2*ln(c*x+I)*ln(c^2*x^2+1)-
2*ln(c*x-I)*ln(c^2*x^2+1)-ln(c*x+I)^2-2*ln(c*x+I)*ln(1/2*I*(c*x-I))+ln(c*x
-I)^2+2*ln(c*x-I)*ln(-1/2*I*(c*x+I))-2*dilog(1/2*I*(c*x-I))+2*dilog(-1/2*I
*(c*x+I))-2*I*arctan(c*x)*c^2*x^2-1/6*c^2*x^2+11/3*ln(c^2*x^2+1)+2*I*c*x))
+2*d^3*a*b*(-1/4*I*arctan(c*x)*c^4*x^4-c^3*x^3*arctan(c*x)+3/2*I*arctan(c*
x)*c^2*x^2+c*x*arctan(c*x)-1/4*I*arctan(c*x)+1/4*I*(-7*c*x+1/3*c^3*x^3-2*I
*c^2*x^2+4*I*ln(c^2*x^2+1)+8*arctan(c*x))))
```

## Fricas [F]

$$\int (d+icdx)^3(a+b\arctan(cx))^2\,dx=\int (i\,cdx+d)^3(b\arctan(cx)+a)^2\,dx$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

output
```
1/16*(I*b^2*c^3*d^3*x^4 + 4*b^2*c^2*d^3*x^3 - 6*I*b^2*c*d^3*x^2 - 4*b^2*d^
3*x)*log(-(c*x + I)/(c*x - I))^2 + integral(1/4*(-4*I*a^2*c^5*d^3*x^5 - 12
*a^2*c^4*d^3*x^4 + 8*I*a^2*c^3*d^3*x^3 - 8*a^2*c^2*d^3*x^2 + 12*I*a^2*c*d^
3*x + 4*a^2*d^3 + (4*a*b*c^5*d^3*x^5 + (-12*I*a*b - b^2)*c^4*d^3*x^4 - 4*(
2*a*b - I*b^2)*c^3*d^3*x^3 - 2*(4*I*a*b - 3*b^2)*c^2*d^3*x^2 - 4*(3*a*b +
I*b^2)*c*d^3*x + 4*I*a*b*d^3)*log(-(c*x + I)/(c*x - I)))/(c^2*x^2 + 1), x)
```

## Sympy [F(-1)]

Timed out.

$$\int (d+icdx)^3(a+b\arctan(cx))^2\,dx=\text{Timed out}$$

input
```
integrate((d+I*c*d*x)**3*(a+b*atan(c*x))**2,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int (d+icdx)^3(a+b\arctan(cx))^2\,dx=\int (i\,cdx+d)^3(b\arctan(cx)+a)^2\,dx$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

output
```
-1/4*I*a^2*c^3*d^3*x^4 - 4*b^2*c^5*d^3*integrate(1/16*x^5*arctan(c*x)*log(
c^2*x^2 + 1)/(c^2*x^2 + 1), x) - 2*b^2*c^5*d^3*integrate(1/16*x^5*arctan(c
*x)/(c^2*x^2 + 1), x) - a^2*c^2*d^3*x^3 - 36*b^2*c^4*d^3*integrate(1/16*x^
4*arctan(c*x)^2/(c^2*x^2 + 1), x) - 3*b^2*c^4*d^3*integrate(1/16*x^4*log(c
^2*x^2 + 1)^2/(c^2*x^2 + 1), x) - 5*b^2*c^4*d^3*integrate(1/16*x^4*log(c^2
*x^2 + 1)/(c^2*x^2 + 1), x) - 1/6*I*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x
)/c^4 + 3*arctan(c*x)/c^5))*a*b*c^3*d^3 + 8*b^2*c^3*d^3*integrate(1/16*x^3
*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) + 20*b^2*c^3*d^3*integrate
(1/16*x^3*arctan(c*x)/(c^2*x^2 + 1), x) - (2*x^3*arctan(c*x) - c*(x^2/c^2
- log(c^2*x^2 + 1)/c^4))*a*b*c^2*d^3 + 3/2*I*a^2*c*d^3*x^2 - 24*b^2*c^2*d^
3*integrate(1/16*x^2*arctan(c*x)^2/(c^2*x^2 + 1), x) - 2*b^2*c^2*d^3*integ
rate(1/16*x^2*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + 10*b^2*c^2*d^3*integr
ate(1/16*x^2*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) + 3*I*(x^2*arctan(c*x) - c
*(x/c^2 - arctan(c*x)/c^3))*a*b*c*d^3 + 1/4*b^2*d^3*arctan(c*x)^3/c + 12*b
^2*c*d^3*integrate(1/16*x*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) -
 8*b^2*c*d^3*integrate(1/16*x*arctan(c*x)/(c^2*x^2 + 1), x) + a^2*d^3*x +
b^2*d^3*integrate(1/16*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + (2*c*x*arcta
n(c*x) - log(c^2*x^2 + 1))*a*b*d^3/c + 1/16*(-I*b^2*c^3*d^3*x^4 - 4*b^2*c^
2*d^3*x^3 + 6*I*b^2*c*d^3*x^2 + 4*b^2*d^3*x)*arctan(c*x)^2 + 1/16*(b^2*c^3
*d^3*x^4 - 4*I*b^2*c^2*d^3*x^3 - 6*b^2*c*d^3*x^2 + 4*I*b^2*d^3*x)*arcta...
```

## Giac [F]

$$\int (d+icdx)^3(a+b\arctan(cx))^2\,dx=\int (i\,cdx+d)^3(b\arctan(cx)+a)^2\,dx$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

output
```
integrate((I*c*d*x + d)^3*(b*arctan(c*x) + a)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (d+icdx)^3(a+b\arctan(cx))^2\,dx = \int (a+b\,\mathrm{atan}(c\,x))^2\,(d+c\,d\,x\,1\mathrm{i})^3\,dx$$

input
```
int((a + b*atan(c*x))^2*(d + c*d*x*1i)^3,x)
```

output
```
int((a + b*atan(c*x))^2*(d + c*d*x*1i)^3, x)
```

## Reduce [F]

$$\int (d+icdx)^3(a+b\arctan(cx))^2\,dx = \frac{d^3\left(36atan(cx)\,ab\,c^2i\,x^2-48\left(\int\frac{atan(cx)x}{c^2x^2+1}dx\right)b^2c^2+12a^2cx-24\log(c^2x^2+1)\,ab+12atan(cx)^2\,b^2cx+1\right.}{}$$

input
```
int((d+I*c*d*x)^3*(a+b*atan(c*x))^2,x)
```

output
```
(d**3*( - 3*atan(c*x)**2*b**2*c**4*i*x**4 - 12*atan(c*x)**2*b**2*c**3*x**3
 + 18*atan(c*x)**2*b**2*c**2*i*x**2 + 12*atan(c*x)**2*b**2*c*x + 21*atan(c
*x)**2*b**2*i - 6*atan(c*x)*a*b*c**4*i*x**4 - 24*atan(c*x)*a*b*c**3*x**3 +
 36*atan(c*x)*a*b*c**2*i*x**2 + 24*atan(c*x)*a*b*c*x + 42*atan(c*x)*a*b*i
+ 2*atan(c*x)*b**2*c**3*i*x**3 + 12*atan(c*x)*b**2*c**2*x**2 - 42*atan(c*x
)*b**2*c*i*x + 12*atan(c*x)*b**2 - 48*int((atan(c*x)*x)/(c**2*x**2 + 1),x)
*b**2*c**2 - 24*log(c**2*x**2 + 1)*a*b + 22*log(c**2*x**2 + 1)*b**2*i - 3*
a**2*c**4*i*x**4 - 12*a**2*c**3*x**3 + 18*a**2*c**2*i*x**2 + 12*a**2*c*x +
 2*a*b*c**3*i*x**3 + 12*a*b*c**2*x**2 - 42*a*b*c*i*x - b**2*c**2*i*x**2 -
12*b**2*c*x))/(12*c)
```

## 3.88 $\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x}\,dx$

## Optimal result

Integrand size = 25, antiderivative size = 385

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x}\,dx = 3abcd^3x-\frac{1}{3}ib^2cd^3x+\frac{1}{3}ib^2d^3\arctan(cx)+3b^2cd^3x\arctan(cx)+\frac{1}{3}ibc^2d^3x^2(a+b\arctan(cx))-\frac{29}{6}d^3(a+b\arctan(cx))^2+3icd^3x(a+b\arctan(cx))^2-\frac{3}{2}c^2d^3x^2(a+b\arctan(cx))^2-\frac{1}{3}ic^3d^3x^3(a+b\arctan(cx))^2+2d^3(a+b\arctan(cx))^2\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right)+\frac{20}{3}ibd^3(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)-\frac{3}{2}b^2d^3\log\left(1+c^2x^2\right)-\frac{10}{3}b^2d^3\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)-ibd^3(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)+ibd^3(a+b\arctan(cx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)-\frac{1}{2}b^2d^3\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)+\frac{1}{2}b^2d^3\operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)$$

output
```
3*a*b*c*d^3*x+1/3*I*b^2*d^3*arctan(c*x)+20/3*I*b*d^3*(a+b*arctan(c*x))*ln(
2/(1+I*c*x))+3*b^2*c*d^3*x*arctan(c*x)-1/3*I*b^2*c*d^3*x-29/6*d^3*(a+b*arc
tan(c*x))^2+I*b*d^3*(a+b*arctan(c*x))*polylog(2,-1+2/(1+I*c*x))-3/2*c^2*d^
3*x^2*(a+b*arctan(c*x))^2-I*b*d^3*(a+b*arctan(c*x))*polylog(2,1-2/(1+I*c*x
))-2*d^3*(a+b*arctan(c*x))^2*arctanh(-1+2/(1+I*c*x))-1/3*I*c^3*d^3*x^3*(a+
b*arctan(c*x))^2-3/2*b^2*d^3*ln(c^2*x^2+1)-10/3*b^2*d^3*polylog(2,1-2/(1+I
*c*x))+1/3*I*b*c^2*d^3*x^2*(a+b*arctan(c*x))+3*I*c*d^3*x*(a+b*arctan(c*x))
^2-1/2*b^2*d^3*polylog(3,1-2/(1+I*c*x))+1/2*b^2*d^3*polylog(3,-1+2/(1+I*c*
x))
```

## Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 465, normalized size of antiderivative = 1.21

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x}\,dx = -\frac{1}{24}id^3\big(b^2\pi^3 - 72a^2cx + 72iabcx + 8b^2cx - 36ia^2c^2x^2 - 8abc^2x^2 + 8a^2c^3x^3 - 72iab\arctan(cx) - 8b^2\arctan(cx) - 144abcx\arctan(cx) + 72ib^2cx\arctan(cx) - 72iabc^2x^2\arctan(cx) - 8b^2c^2x^2\arctan(cx) + 16abc^3x^3\arctan(cx) + 44ib^2\arctan(cx)^2 - 72b^2cx\arctan(cx)^2 - 36ib^2c^2x^2\arctan(cx)^2 + 8b^2c^3x^3\arctan(cx)^2 - 16b^2\arctan(cx)^3 + 24ib^2\arctan(cx)^2\log\left(1-e^{-2i\arctan(cx)}\right) - 160b^2\arctan(cx)\log\left(1+e^{2i\arctan(cx)}\right) - 24ib^2\arctan(cx)^2\log\left(1+e^{2i\arctan(cx)}\right) + 24ia^2\log(cx) + 80ab\log\left(1+c^2x^2\right) - 36ib^2\log\left(1+c^2x^2\right) - 24b^2\arctan(cx)\operatorname{PolyLog}\left(2,e^{-2i\arctan(cx)}\right) - 8b^2(-10i+3\arctan(cx))\operatorname{PolyLog}\left(2,-e^{2i\arctan(cx)}\right) - 24ab\operatorname{PolyLog}(2,-icx) + 24ab\operatorname{PolyLog}(2,icx) + 12ib^2\operatorname{PolyLog}\left(3,e^{-2i\arctan(cx)}\right) - 12ib^2\operatorname{PolyLog}\left(3,-e^{2i\arctan(cx)}\right)\big)$$

input
```
Integrate[((d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2)/x,x]
```

output
```
(-1/24*I)*d^3*(b^2*Pi^3 - 72*a^2*c*x + (72*I)*a*b*c*x + 8*b^2*c*x - (36*I)
*a^2*c^2*x^2 - 8*a*b*c^2*x^2 + 8*a^2*c^3*x^3 - (72*I)*a*b*ArcTan[c*x] - 8*
b^2*ArcTan[c*x] - 144*a*b*c*x*ArcTan[c*x] + (72*I)*b^2*c*x*ArcTan[c*x] - (
72*I)*a*b*c^2*x^2*ArcTan[c*x] - 8*b^2*c^2*x^2*ArcTan[c*x] + 16*a*b*c^3*x^3
*ArcTan[c*x] + (44*I)*b^2*ArcTan[c*x]^2 - 72*b^2*c*x*ArcTan[c*x]^2 - (36*I
)*b^2*c^2*x^2*ArcTan[c*x]^2 + 8*b^2*c^3*x^3*ArcTan[c*x]^2 - 16*b^2*ArcTan[
c*x]^3 + (24*I)*b^2*ArcTan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] - 160*b^
2*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] - (24*I)*b^2*ArcTan[c*x]^2*Lo
g[1 + E^((2*I)*ArcTan[c*x])] + (24*I)*a^2*Log[c*x] + 80*a*b*Log[1 + c^2*x^
2] - (36*I)*b^2*Log[1 + c^2*x^2] - 24*b^2*ArcTan[c*x]*PolyLog[2, E^((-2*I)
*ArcTan[c*x])] - 8*b^2*(-10*I + 3*ArcTan[c*x])*PolyLog[2, -E^((2*I)*ArcTan
[c*x])] - 24*a*b*PolyLog[2, (-I)*c*x] + 24*a*b*PolyLog[2, I*c*x] + (12*I)*
b^2*PolyLog[3, E^((-2*I)*ArcTan[c*x])] - (12*I)*b^2*PolyLog[3, -E^((2*I)*A
rcTan[c*x])])
```

## Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x}\,dx$$

$$\Big\downarrow \text{5411}$$

$$\int \left(-ic^3d^3x^2(a+b\arctan(cx))^2-3c^2d^3x(a+b\arctan(cx))^2+3icd^3(a+b\arctan(cx))^2+\frac{d^3(a+b\arctan(cx))}{x}\right.$$

$$\Big\downarrow \text{2009}$$

$$2d^3\text{arctanh}\left(1-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2-\frac{1}{3}ic^3d^3x^3(a+b\arctan(cx))^2-\frac{3}{2}c^2d^3x^2(a+b\arctan(cx))^2+\frac{1}{3}ibc^2d^3x^2(a+b\arctan(cx))-ibd^3\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))+ibd^3\operatorname{PolyLog}\left(2,\frac{2}{icx+1}-1\right)(a+b\arctan(cx))+3icd^3x(a+b\arctan(cx))^2-\frac{29}{6}d^3(a+b\arctan(cx))^2+\frac{20}{3}ibd^3\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))+3abcd^3x+\frac{1}{3}ib^2d^3\arctan(cx)+3b^2cd^3x\arctan(cx)-\frac{3}{2}b^2d^3\log\left(c^2x^2+1\right)-\frac{10}{3}b^2d^3\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)-\frac{1}{2}b^2d^3\operatorname{PolyLog}\left(3,1-\frac{2}{icx+1}\right)+\frac{1}{2}b^2d^3\operatorname{PolyLog}\left(3,\frac{2}{icx+1}-1\right)-\frac{1}{3}ib^2cd^3x$$

input
```
Int[((d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2)/x,x]
```

output
```
3*a*b*c*d^3*x - (I/3)*b^2*c*d^3*x + (I/3)*b^2*d^3*ArcTan[c*x] + 3*b^2*c*d^
3*x*ArcTan[c*x] + (I/3)*b*c^2*d^3*x^2*(a + b*ArcTan[c*x]) - (29*d^3*(a + b
*ArcTan[c*x])^2)/6 + (3*I)*c*d^3*x*(a + b*ArcTan[c*x])^2 - (3*c^2*d^3*x^2*
(a + b*ArcTan[c*x])^2)/2 - (I/3)*c^3*d^3*x^3*(a + b*ArcTan[c*x])^2 + 2*d^3
*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)] + ((20*I)/3)*b*d^3*(a +
b*ArcTan[c*x])*Log[2/(1 + I*c*x)] - (3*b^2*d^3*Log[1 + c^2*x^2])/2 - (10*b
^2*d^3*PolyLog[2, 1 - 2/(1 + I*c*x)])/3 - I*b*d^3*(a + b*ArcTan[c*x])*Poly
Log[2, 1 - 2/(1 + I*c*x)] + I*b*d^3*(a + b*ArcTan[c*x])*PolyLog[2, -1 + 2/
(1 + I*c*x)] - (b^2*d^3*PolyLog[3, 1 - 2/(1 + I*c*x)])/2 + (b^2*d^3*PolyLo
g[3, -1 + 2/(1 + I*c*x)])/2
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 11.95 (sec) , antiderivative size = 1409, normalized size of antiderivative = 3.66

| method | result | size |
|---|---|---|
| parts | Expression too large to display | 1409 |
| derivativedivides | Expression too large to display | 1411 |
| default | Expression too large to display | 1411 |

input
```
int((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x,x,method=_RETURNVERBOSE)
```

output
```
d^3*a^2*(-1/3*I*c^3*x^3-3/2*c^2*x^2+3*I*c*x+ln(x))+d^3*b^2*(1/2*I*Pi*csgn(
I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I*
((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*arctan(c*x)^2-3/2
*c^2*x^2*arctan(c*x)^2+11/6*arctan(c*x)^2+3*ln(1+(1+I*c*x)^2/(c^2*x^2+1))+
20/3*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+20/3*dilog(1-I*(1+I*c*x)/(c^2*
x^2+1)^(1/2))+ln(c*x)*arctan(c*x)^2-arctan(c*x)^2*ln((1+I*c*x)^2/(c^2*x^2+
1)-1)+arctan(c*x)^2*ln(1-(1+I*c*x)/(c^2*x^2+1)^(1/2))+arctan(c*x)^2*ln(1+(
1+I*c*x)/(c^2*x^2+1)^(1/2))+I*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+
1))-2*I*arctan(c*x)*polylog(2,(1+I*c*x)/(c^2*x^2+1)^(1/2))-2*I*arctan(c*x)
*polylog(2,-(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/2*polylog(3,-(1+I*c*x)^2/(c^2*x
^2+1))+2*polylog(3,(1+I*c*x)/(c^2*x^2+1)^(1/2))+2*polylog(3,-(1+I*c*x)/(c^
2*x^2+1)^(1/2))+11/3*arctan(c*x)*(c*x-I)-1/3*I*(c*x+I)-1/2*I*Pi*csgn(I/(1+
(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^
2/(c^2*x^2+1)))^2*arctan(c*x)^2-1/2*I*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1
)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*
x)^2/(c^2*x^2+1)))^2*arctan(c*x)^2-1/2*I*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1
)-1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*ar
ctan(c*x)^2+1/2*I*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^
2*x^2+1)))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*a
rctan(c*x)^2+1/2*I*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c...
```

## Fricas [F]

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x}\,dx = \int \frac{(i\,cdx+d)^3(b\arctan(cx)+a)^2}{x}\,dx$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x,x, algorithm="fricas")
```

output
```
integral(1/4*(-4*I*a^2*c^3*d^3*x^3 - 12*a^2*c^2*d^3*x^2 + 12*I*a^2*c*d^3*x
 + 4*a^2*d^3 + (I*b^2*c^3*d^3*x^3 + 3*b^2*c^2*d^3*x^2 - 3*I*b^2*c*d^3*x -
b^2*d^3)*log(-(c*x + I)/(c*x - I))^2 + 4*(a*b*c^3*d^3*x^3 - 3*I*a*b*c^2*d^
3*x^2 - 3*a*b*c*d^3*x + I*a*b*d^3)*log(-(c*x + I)/(c*x - I)))/x, x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x}\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)**3*(a+b*atan(c*x))**2/x,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x}\,dx = \int \frac{(i\,cdx+d)^3(b\arctan(cx)+a)^2}{x}\,dx$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x,x, algorithm="maxima")
```

output
```
-1/3*I*a^2*c^3*d^3*x^3 - 36*I*b^2*c^5*d^3*integrate(1/48*x^5*arctan(c*x)^2
/(c^2*x^3 + x), x) - 12*b^2*c^5*d^3*integrate(1/48*x^5*arctan(c*x)*log(c^2
*x^2 + 1)/(c^2*x^3 + x), x) - 3*I*b^2*c^5*d^3*integrate(1/48*x^5*log(c^2*x
^2 + 1)^2/(c^2*x^3 + x), x) - 96*I*a*b*c^5*d^3*integrate(1/48*x^5*arctan(c
*x)/(c^2*x^3 + x), x) - 8*b^2*c^5*d^3*integrate(1/48*x^5*arctan(c*x)/(c^2*
x^3 + x), x) - 4*I*b^2*c^5*d^3*integrate(1/48*x^5*log(c^2*x^2 + 1)/(c^2*x^
3 + x), x) - 108*b^2*c^4*d^3*integrate(1/48*x^4*arctan(c*x)^2/(c^2*x^3 + x
), x) + 36*I*b^2*c^4*d^3*integrate(1/48*x^4*arctan(c*x)*log(c^2*x^2 + 1)/(
c^2*x^3 + x), x) - 9*b^2*c^4*d^3*integrate(1/48*x^4*log(c^2*x^2 + 1)^2/(c^
2*x^3 + x), x) - 288*a*b*c^4*d^3*integrate(1/48*x^4*arctan(c*x)/(c^2*x^3 +
 x), x) + 44*I*b^2*c^4*d^3*integrate(1/48*x^4*arctan(c*x)/(c^2*x^3 + x), x
) - 22*b^2*c^4*d^3*integrate(1/48*x^4*log(c^2*x^2 + 1)/(c^2*x^3 + x), x) -
 3/2*a^2*c^2*d^3*x^2 + 72*I*b^2*c^3*d^3*integrate(1/48*x^3*arctan(c*x)^2/(
c^2*x^3 + x), x) + 24*b^2*c^3*d^3*integrate(1/48*x^3*arctan(c*x)*log(c^2*x
^2 + 1)/(c^2*x^3 + x), x) + 6*I*b^2*c^3*d^3*integrate(1/48*x^3*log(c^2*x^2
 + 1)^2/(c^2*x^3 + x), x) - 96*I*a*b*c^3*d^3*integrate(1/48*x^3*arctan(c*x
)/(c^2*x^3 + x), x) + 108*b^2*c^3*d^3*integrate(1/48*x^3*arctan(c*x)/(c^2*
x^3 + x), x) + 54*I*b^2*c^3*d^3*integrate(1/48*x^3*log(c^2*x^2 + 1)/(c^2*x
^3 + x), x) + 3/4*I*b^2*d^3*arctan(c*x)^3 - 72*b^2*c^2*d^3*integrate(1/48*
x^2*arctan(c*x)^2/(c^2*x^3 + x), x) + 24*I*b^2*c^2*d^3*integrate(1/48*x...
```

## Giac [F]

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x}\,dx=\int \frac{(i\,cdx+d)^3(b\arctan(cx)+a)^2}{x}\,dx$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x,x, algorithm="giac")
```

output
```
integrate((I*c*d*x + d)^3*(b*arctan(c*x) + a)^2/x, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^2\,(d+c\,d\,x\,1\mathrm{i})^3}{x}\,dx$$

input
```
int(((a + b*atan(c*x))^2*(d + c*d*x*1i)^3)/x,x)
```

output
```
int(((a + b*atan(c*x))^2*(d + c*d*x*1i)^3)/x, x)
```

## Reduce [F]

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x}\,dx$$
$$= \frac{d^3\left(-2atan(cx)^2\,b^2c^3i\,x^3-9atan(cx)^2\,b^2c^2x^2-2atan(cx)^2\,b^2cix-9atan(cx)^2\,b^2-4atan(cx)\,ab\,c^3i\,x^3-\right.}{}$$

input
```
int((d+I*c*d*x)^3*(a+b*atan(c*x))^2/x,x)
```

output
```
(d**3*( - 2*atan(c*x)**2*b**2*c**3*i*x**3 - 9*atan(c*x)**2*b**2*c**2*x**2
- 2*atan(c*x)**2*b**2*c*i*x - 9*atan(c*x)**2*b**2 - 4*atan(c*x)*a*b*c**3*i
*x**3 - 18*atan(c*x)*a*b*c**2*x**2 + 36*atan(c*x)*a*b*c*i*x - 18*atan(c*x)
*a*b + 2*atan(c*x)*b**2*c**2*i*x**2 + 18*atan(c*x)*b**2*c*x + 2*atan(c*x)*
b**2*i + 20*int(atan(c*x)**2,x)*b**2*c*i + 12*int(atan(c*x)/x,x)*a*b + 6*i
nt(atan(c*x)**2/x,x)*b**2 - 20*log(c**2*x**2 + 1)*a*b*i - 9*log(c**2*x**2
+ 1)*b**2 + 6*log(x)*a**2 - 2*a**2*c**3*i*x**3 - 9*a**2*c**2*x**2 + 18*a**
2*c*i*x + 2*a*b*c**2*i*x**2 + 18*a*b*c*x - 2*b**2*c*i*x))/6
```

# 3.89 $\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^2}\,dx$

## Optimal result

Integrand size = 25, antiderivative size = 402

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^2}\,dx = iabc^2d^3x + ib^2c^2d^3x\arctan(cx) - \frac{9}{2}icd^3(a+b\arctan(cx))^2 - \frac{d^3(a+b\arctan(cx))^2}{x} - 3c^2d^3x(a+b\arctan(cx))^2 - \frac{1}{2}ic^3d^3x^2(a+b\arctan(cx))^2 + 6icd^3(a+b\arctan(cx))^2\text{arctanh}\left(1-\frac{2}{1+icx}\right) - 6bcd^3(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right) - \frac{1}{2}ib^2cd^3\log\left(1+c^2x^2\right) + 2bcd^3(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right) - ib^2cd^3\,\text{PolyLog}\left(2,-1+\frac{2}{1-icx}\right) - 3ib^2cd^3\,\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right) + 3bcd^3(a+b\arctan(cx))\,\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right) - 3bcd^3(a+b\arctan(cx))\,\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right) - \frac{3}{2}ib^2cd^3\,\text{PolyLog}\left(3,1-\frac{2}{1+icx}\right) + \frac{3}{2}ib^2cd^3\,\text{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)$$

output
```
I*b^2*c^2*d^3*x*arctan(c*x)-I*b^2*c*d^3*polylog(2,-1+2/(1-I*c*x))-3*I*b^2*
c*d^3*polylog(2,1-2/(1+I*c*x))-d^3*(a+b*arctan(c*x))^2/x-3*c^2*d^3*x*(a+b*
arctan(c*x))^2+I*a*b*c^2*d^3*x-9/2*I*c*d^3*(a+b*arctan(c*x))^2-6*b*c*d^3*(
a+b*arctan(c*x))*ln(2/(1+I*c*x))-6*I*c*d^3*(a+b*arctan(c*x))^2*arctanh(-1+
2/(1+I*c*x))+2*b*c*d^3*(a+b*arctan(c*x))*ln(2-2/(1-I*c*x))-1/2*I*c^3*d^3*x
^2*(a+b*arctan(c*x))^2+3/2*I*b^2*c*d^3*polylog(3,-1+2/(1+I*c*x))+3*b*c*d^3
*(a+b*arctan(c*x))*polylog(2,1-2/(1+I*c*x))-3*b*c*d^3*(a+b*arctan(c*x))*po
lylog(2,-1+2/(1+I*c*x))-3/2*I*b^2*c*d^3*polylog(3,1-2/(1+I*c*x))-1/2*I*b^2
*c*d^3*ln(c^2*x^2+1)
```

## Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 512, normalized size of antiderivative = 1.27

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^2}\,dx$$
$$= \frac{d^3\left(-8a^2+b^2c\pi^3x-24a^2c^2x^2+8iabc^2x^2-4ia^2c^3x^3-16ab\arctan(cx)-8iabcx\arctan(cx)-48abc^2x^2\right.}{}$$

input
```
Integrate[((d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2)/x^2,x]
```

output
```
(d^3*(-8*a^2 + b^2*c*Pi^3*x - 24*a^2*c^2*x^2 + (8*I)*a*b*c^2*x^2 - (4*I)*a
^2*c^3*x^3 - 16*a*b*ArcTan[c*x] - (8*I)*a*b*c*x*ArcTan[c*x] - 48*a*b*c^2*x
^2*ArcTan[c*x] + (8*I)*b^2*c^2*x^2*ArcTan[c*x] - (8*I)*a*b*c^3*x^3*ArcTan[
c*x] - 8*b^2*ArcTan[c*x]^2 + (12*I)*b^2*c*x*ArcTan[c*x]^2 - 24*b^2*c^2*x^2
*ArcTan[c*x]^2 - (4*I)*b^2*c^3*x^3*ArcTan[c*x]^2 - 16*b^2*c*x*ArcTan[c*x]^
3 + (24*I)*b^2*c*x*ArcTan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] + 16*b^2*
c*x*ArcTan[c*x]*Log[1 - E^((2*I)*ArcTan[c*x])] - 48*b^2*c*x*ArcTan[c*x]*Lo
g[1 + E^((2*I)*ArcTan[c*x])] - (24*I)*b^2*c*x*ArcTan[c*x]^2*Log[1 + E^((2*
I)*ArcTan[c*x])] + (24*I)*a^2*c*x*Log[x] + 16*a*b*c*x*Log[c*x] + 16*a*b*c*
x*Log[1 + c^2*x^2] - (4*I)*b^2*c*x*Log[1 + c^2*x^2] - 24*b^2*c*x*ArcTan[c*
x]*PolyLog[2, E^((-2*I)*ArcTan[c*x])] - 24*b^2*c*x*(-I + ArcTan[c*x])*Poly
Log[2, -E^((2*I)*ArcTan[c*x])] - (8*I)*b^2*c*x*PolyLog[2, E^((2*I)*ArcTan[
c*x])] - 24*a*b*c*x*PolyLog[2, (-I)*c*x] + 24*a*b*c*x*PolyLog[2, I*c*x] +
(12*I)*b^2*c*x*PolyLog[3, E^((-2*I)*ArcTan[c*x])] - (12*I)*b^2*c*x*PolyLog
[3, -E^((2*I)*ArcTan[c*x])]))/(8*x)
```

## Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^2}\,dx$$

$$\Big\downarrow \; 5411$$

$$\int \left(-ic^3d^3x(a+b\arctan(cx))^2-3c^2d^3(a+b\arctan(cx))^2+\frac{d^3(a+b\arctan(cx))^2}{x^2}+\frac{3icd^3(a+b\arctan(cx))^2}{x}\right.$$

$$\Big\downarrow \; 2009$$

$$6icd^3\text{arctanh}\left(1-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2-\frac{1}{2}ic^3d^3x^2(a+b\arctan(cx))^2-3c^2d^3x(a+b\arctan(cx))^2+3bcd^3\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))-3bcd^3\,\text{PolyLog}\left(2,\frac{2}{icx+1}-1\right)(a+b\arctan(cx))-\frac{9}{2}icd^3(a+b\arctan(cx))^2-\frac{d^3(a+b\arctan(cx))^2}{x}-6bcd^3\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))+2bcd^3\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))+iabc^2d^3x+ib^2c^2d^3x\arctan(cx)-\frac{1}{2}ib^2cd^3\log\left(c^2x^2+1\right)-ib^2cd^3\,\text{PolyLog}\left(2,\frac{2}{1-icx}-1\right)-3ib^2cd^3\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)-\frac{3}{2}ib^2cd^3\,\text{PolyLog}\left(3,1-\frac{2}{icx+1}\right)+\frac{3}{2}ib^2cd^3\,\text{PolyLog}\left(3,\frac{2}{icx+1}-1\right)$$

input
```
Int[((d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2)/x^2,x]
```

output
```
I*a*b*c^2*d^3*x + I*b^2*c^2*d^3*x*ArcTan[c*x] - ((9*I)/2)*c*d^3*(a + b*Arc
Tan[c*x])^2 - (d^3*(a + b*ArcTan[c*x])^2)/x - 3*c^2*d^3*x*(a + b*ArcTan[c*
x])^2 - (I/2)*c^3*d^3*x^2*(a + b*ArcTan[c*x])^2 + (6*I)*c*d^3*(a + b*ArcTa
n[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)] - 6*b*c*d^3*(a + b*ArcTan[c*x])*Log[2
/(1 + I*c*x)] - (I/2)*b^2*c*d^3*Log[1 + c^2*x^2] + 2*b*c*d^3*(a + b*ArcTan
[c*x])*Log[2 - 2/(1 - I*c*x)] - I*b^2*c*d^3*PolyLog[2, -1 + 2/(1 - I*c*x)]
 - (3*I)*b^2*c*d^3*PolyLog[2, 1 - 2/(1 + I*c*x)] + 3*b*c*d^3*(a + b*ArcTan
[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)] - 3*b*c*d^3*(a + b*ArcTan[c*x])*PolyL
og[2, -1 + 2/(1 + I*c*x)] - ((3*I)/2)*b^2*c*d^3*PolyLog[3, 1 - 2/(1 + I*c*
x)] + ((3*I)/2)*b^2*c*d^3*PolyLog[3, -1 + 2/(1 + I*c*x)]
```

**Defintions of rubi rules used**

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 15.75 (sec) , antiderivative size = 1437, normalized size of antiderivative = 3.57

| method | result | size |
|---|---|---|
| parts | Expression too large to display | 1437 |
| derivativedivides | Expression too large to display | 1439 |
| default | Expression too large to display | 1439 |

input
```
int((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
d^3*a^2*(-1/2*I*c^3*x^2-3*c^2*x-1/x+3*I*c*ln(x))+d^3*b^2*c*(-1/2*I*arctan(
c*x)^2*c^2*x^2-3*c*x*arctan(c*x)^2-1/c/x*arctan(c*x)^2-3/2*I*polylog(3,-(1
+I*c*x)^2/(c^2*x^2+1))+3/2*I*arctan(c*x)^2-2*I*dilog(1+(1+I*c*x)/(c^2*x^2+
1)^(1/2))+2*I*dilog((1+I*c*x)/(c^2*x^2+1)^(1/2))+6*I*dilog(1+I*(1+I*c*x)/(
c^2*x^2+1)^(1/2))+6*I*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+6*I*polylog(3
,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+6*I*polylog(3,(1+I*c*x)/(c^2*x^2+1)^(1/2))+
I*ln(1+(1+I*c*x)^2/(c^2*x^2+1))-3/2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))
*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1
+(1+I*c*x)^2/(c^2*x^2+1)))*arctan(c*x)^2-3/2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x
^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+
(1+I*c*x)^2/(c^2*x^2+1)))*arctan(c*x)^2+3/2*Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*
x^2+1)))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2
*arctan(c*x)^2-3/2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c
^2*x^2+1)))^3*arctan(c*x)^2+3*I*arctan(c*x)^2*ln(1-(1+I*c*x)/(c^2*x^2+1)^(
1/2))+3*I*arctan(c*x)^2*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))+3*I*arctan(c*x)^
2*ln(c*x)-6*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-6*arctan(c*x)*
ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+6*arctan(c*x)*polylog(2,-(1+I*c*x)/(c^
2*x^2+1)^(1/2))+6*arctan(c*x)*polylog(2,(1+I*c*x)/(c^2*x^2+1)^(1/2))-3*arc
tan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))-3*I*arctan(c*x)^2*ln((1+I*c*x
)^2/(c^2*x^2+1)-1)+I*arctan(c*x)*(c*x-I)-3/2*Pi*csgn(((1+I*c*x)^2/(c^2*...
```

## Fricas [F]

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^2}\,dx = \int \frac{(i\,cdx+d)^3(b\arctan(cx)+a)^2}{x^2}\,dx$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^2,x, algorithm="fricas")
```

output
```
integral(1/4*(-4*I*a^2*c^3*d^3*x^3 - 12*a^2*c^2*d^3*x^2 + 12*I*a^2*c*d^3*x
 + 4*a^2*d^3 + (I*b^2*c^3*d^3*x^3 + 3*b^2*c^2*d^3*x^2 - 3*I*b^2*c*d^3*x -
b^2*d^3)*log(-(c*x + I)/(c*x - I))^2 + 4*(a*b*c^3*d^3*x^3 - 3*I*a*b*c^2*d^
3*x^2 - 3*a*b*c*d^3*x + I*a*b*d^3)*log(-(c*x + I)/(c*x - I)))/x^2, x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)**3*(a+b*atan(c*x))**2/x**2,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^2}\,dx = \int \frac{(i\,cdx+d)^3(b\arctan(cx)+a)^2}{x^2}\,dx$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^2,x, algorithm="maxima")
```

output
```
-1/2*I*a^2*c^3*d^3*x^2 - 3*a^2*c^2*d^3*x - 3*(2*c*x*arctan(c*x) - log(c^2*
x^2 + 1))*a*b*c*d^3 + 3*I*a^2*c*d^3*log(x) - (c*(log(c^2*x^2 + 1) - log(x^
2)) + 2*arctan(c*x)/x)*a*b*d^3 - a^2*d^3/x + 1/96*(12*(-I*b^2*c^3*d^3*x^3
- 6*b^2*c^2*d^3*x^2 - 2*b^2*d^3)*arctan(c*x)^2 + 12*(b^2*c^3*d^3*x^3 - 6*I
*b^2*c^2*d^3*x^2 - 2*I*b^2*d^3)*arctan(c*x)*log(c^2*x^2 + 1) - 3*(-I*b^2*c
^3*d^3*x^3 - 6*b^2*c^2*d^3*x^2 - 2*b^2*d^3)*log(c^2*x^2 + 1)^2 - 2*I*(576*
b^2*c^5*d^3*integrate(1/16*x^5*arctan(c*x)^2/(c^2*x^4 + x^2), x) + 48*b^2*
c^5*d^3*integrate(1/16*x^5*log(c^2*x^2 + 1)^2/(c^2*x^4 + x^2), x) + 1536*a
*b*c^5*d^3*integrate(1/16*x^5*arctan(c*x)/(c^2*x^4 + x^2), x) + 96*b^2*c^5
*d^3*integrate(1/16*x^5*log(c^2*x^2 + 1)/(c^2*x^4 + x^2), x) - 576*b^2*c^4
*d^3*integrate(1/16*x^4*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^4 + x^2), x) -
 1344*b^2*c^4*d^3*integrate(1/16*x^4*arctan(c*x)/(c^2*x^4 + x^2), x) - 115
2*b^2*c^3*d^3*integrate(1/16*x^3*arctan(c*x)^2/(c^2*x^4 + x^2), x) - 3072*
a*b*c^3*d^3*integrate(1/16*x^3*arctan(c*x)/(c^2*x^4 + x^2), x) - b^2*c*d^3
*log(c^2*x^2 + 1)^3 - 12*b^2*c*d^3*arctan(c*x)^2 - 384*b^2*c^2*d^3*integra
te(1/16*x^2*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^4 + x^2), x) - 9*b^2*c*d^3
*log(c^2*x^2 + 1)^2 - 1728*b^2*c*d^3*integrate(1/16*x*arctan(c*x)^2/(c^2*x
^4 + x^2), x) - 144*b^2*c*d^3*integrate(1/16*x*log(c^2*x^2 + 1)^2/(c^2*x^4
 + x^2), x) - 4608*a*b*c*d^3*integrate(1/16*x*arctan(c*x)/(c^2*x^4 + x^2),
 x) - 192*b^2*c*d^3*integrate(1/16*x*log(c^2*x^2 + 1)/(c^2*x^4 + x^2), ...
```

## Giac [F]

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^2}\,dx=\int \frac{(i\,cdx+d)^3(b\arctan{(cx)}+a)^2}{x^2}\,dx$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^2,x, algorithm="giac")
```

output
```
integrate((I*c*d*x + d)^3*(b*arctan(c*x) + a)^2/x^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^2}\,dx = \int \frac{\left(a+b\,\mathrm{atan}(c\,x)\right)^2\,\left(d+c\,d\,x\,1\mathrm{i}\right)^3}{x^2}\,dx$$

input
```
int(((a + b*atan(c*x))^2*(d + c*d*x*1i)^3)/x^2,x)
```

output
```
int(((a + b*atan(c*x))^2*(d + c*d*x*1i)^3)/x^2, x)
```

## Reduce [F]

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^2}\,dx = \frac{d^3\left(-atan(cx)^2\,b^2c^3i\,x^3 - atan(cx)^2\,b^2cix - 2atan(cx)^2\,b^2 - 2atan(cx)\,ab\,c^3i\,x^3 - 12atan(cx)\,ab\,c^2x^2 - \right.}{}$$

input
```
int((d+I*c*d*x)^3*(a+b*atan(c*x))^2/x^2,x)
```

output
```
(d**3*( - atan(c*x)**2*b**2*c**3*i*x**3 - atan(c*x)**2*b**2*c*i*x - 2*atan
(c*x)**2*b**2 - 2*atan(c*x)*a*b*c**3*i*x**3 - 12*atan(c*x)*a*b*c**2*x**2 -
 2*atan(c*x)*a*b*c*i*x - 4*atan(c*x)*a*b + 2*atan(c*x)*b**2*c**2*i*x**2 -
6*int(atan(c*x)**2,x)*b**2*c**2*x + 4*int(atan(c*x)/(c**2*x**3 + x),x)*b**
2*c*x + 12*int(atan(c*x)/x,x)*a*b*c*i*x + 6*int(atan(c*x)**2/x,x)*b**2*c*i
*x + 4*log(c**2*x**2 + 1)*a*b*c*x - log(c**2*x**2 + 1)*b**2*c*i*x + 6*log(
x)*a**2*c*i*x + 4*log(x)*a*b*c*x - a**2*c**3*i*x**3 - 6*a**2*c**2*x**2 - 2
*a**2 + 2*a*b*c**2*i*x**2))/(2*x)
```

# 3.90 $\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^3}\,dx$

## Optimal result

Integrand size = 25, antiderivative size = 416

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^3}\,dx = -\frac{bcd^3(a+b\arctan(cx))}{x} + \frac{7}{2}c^2d^3(a+b\arctan(cx))^2 - \frac{d^3(a+b\arctan(cx))^2}{2x^2} - \frac{3icd^3(a+b\arctan(cx))^2}{x} - ic^3d^3x(a+b\arctan(cx))^2 - 6c^2d^3(a+b\arctan(cx))^2\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right) + b^2c^2d^3\log(x) - 2ibc^2d^3(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right) - \frac{1}{2}b^2c^2d^3\log\left(1+c^2x^2\right) + 6ibc^2d^3(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right) + 3b^2c^2d^3\operatorname{PolyLog}\left(2,-1+\frac{2}{1-icx}\right) + b^2c^2d^3\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right) + 3ibc^2d^3(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right) - 3ibc^2d^3(a+b\arctan(cx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx}\right) + \frac{3}{2}b^2c^2d^3\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right) - \frac{3}{2}b^2c^2d^3\operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)$$

output
```
-b*c*d^3*(a+b*arctan(c*x))/x+7/2*c^2*d^3*(a+b*arctan(c*x))^2-1/2*d^3*(a+b*
arctan(c*x))^2/x^2-2*I*b*c^2*d^3*(a+b*arctan(c*x))*ln(2/(1+I*c*x))-3*I*b*c
^2*d^3*(a+b*arctan(c*x))*polylog(2,-1+2/(1+I*c*x))+6*c^2*d^3*(a+b*arctan(c
*x))^2*arctanh(-1+2/(1+I*c*x))+b^2*c^2*d^3*ln(x)+3*I*b*c^2*d^3*(a+b*arctan
(c*x))*polylog(2,1-2/(1+I*c*x))-1/2*b^2*c^2*d^3*ln(c^2*x^2+1)-I*c^3*d^3*x*
(a+b*arctan(c*x))^2+3*b^2*c^2*d^3*polylog(2,-1+2/(1-I*c*x))+b^2*c^2*d^3*po
lylog(2,1-2/(1+I*c*x))-3*I*c*d^3*(a+b*arctan(c*x))^2/x+6*I*b*c^2*d^3*(a+b*
arctan(c*x))*ln(2-2/(1-I*c*x))+3/2*b^2*c^2*d^3*polylog(3,1-2/(1+I*c*x))-3/
2*b^2*c^2*d^3*polylog(3,-1+2/(1+I*c*x))
```

## Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 500, normalized size of antiderivative = 1.20

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^3}\,dx$$
$$= \frac{1}{2}d^3\left(-\frac{a^2}{x^2}-\frac{6ia^2c}{x}-2ia^2c^3x-\frac{2ab(\arctan(cx)+cx(1+cx\arctan(cx)))}{x^2}\right.$$
$$-6a^2c^2\log(x)-\frac{b^2\left(2cx\arctan(cx)+(1+c^2x^2)\arctan(cx)^2-2c^2x^2\log\left(\frac{cx}{\sqrt{1+c^2x^2}}\right)\right)}{x^2}$$
$$-2iabc^2\left(2cx\arctan(cx)-\log\left(1+c^2x^2\right)\right)$$
$$-\frac{6iabc(2\arctan(cx)+cx(-2\log(cx)+\log(1+c^2x^2)))}{x}$$
$$-2ib^2c^2\left(\arctan(cx)\left((-i+cx)\arctan(cx)+2\log\left(1+e^{2i\arctan(cx)}\right)\right)\right.$$
$$\left.-i\,\mathrm{PolyLog}\left(2,-e^{2i\arctan(cx)}\right)\right)$$
$$+\frac{6b^2c\left(\arctan(cx)\left((-i+cx)\arctan(cx)+2icx\log\left(1-e^{2i\arctan(cx)}\right)\right)+cx\,\mathrm{PolyLog}\left(2,e^{2i\arctan(cx)}\right)\right)}{x}$$
$$-6iabc^2(\mathrm{PolyLog}(2,-icx)-\mathrm{PolyLog}(2,icx))+6b^2c^2\left(\frac{i\pi^3}{24}-\frac{2}{3}i\arctan(cx)^3\right.$$
$$-\arctan(cx)^2\log\left(1-e^{-2i\arctan(cx)}\right)+\arctan(cx)^2\log\left(1+e^{2i\arctan(cx)}\right)$$
$$-i\arctan(cx)\,\mathrm{PolyLog}\left(2,e^{-2i\arctan(cx)}\right)-i\arctan(cx)\,\mathrm{PolyLog}\left(2,-e^{2i\arctan(cx)}\right)$$
$$\left.\left.-\frac{1}{2}\mathrm{PolyLog}\left(3,e^{-2i\arctan(cx)}\right)+\frac{1}{2}\mathrm{PolyLog}\left(3,-e^{2i\arctan(cx)}\right)\right)\right)$$

input
```
Integrate[((d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2)/x^3,x]
```

output
```
(d^3*(-(a^2/x^2) - ((6*I)*a^2*c)/x - (2*I)*a^2*c^3*x - (2*a*b*(ArcTan[c*x]
 + c*x*(1 + c*x*ArcTan[c*x])))/x^2 - 6*a^2*c^2*Log[x] - (b^2*(2*c*x*ArcTan
[c*x] + (1 + c^2*x^2)*ArcTan[c*x]^2 - 2*c^2*x^2*Log[(c*x)/Sqrt[1 + c^2*x^2
]]))/x^2 - (2*I)*a*b*c^2*(2*c*x*ArcTan[c*x] - Log[1 + c^2*x^2]) - ((6*I)*a
*b*c*(2*ArcTan[c*x] + c*x*(-2*Log[c*x] + Log[1 + c^2*x^2])))/x - (2*I)*b^2
*c^2*(ArcTan[c*x]*((-I + c*x)*ArcTan[c*x] + 2*Log[1 + E^((2*I)*ArcTan[c*x]
)]) - I*PolyLog[2, -E^((2*I)*ArcTan[c*x])]) + (6*b^2*c*(ArcTan[c*x]*((-I +
 c*x)*ArcTan[c*x] + (2*I)*c*x*Log[1 - E^((2*I)*ArcTan[c*x])]) + c*x*PolyLo
g[2, E^((2*I)*ArcTan[c*x])]))/x - (6*I)*a*b*c^2*(PolyLog[2, (-I)*c*x] - Po
lyLog[2, I*c*x]) + 6*b^2*c^2*((I/24)*Pi^3 - ((2*I)/3)*ArcTan[c*x]^3 - ArcT
an[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] + ArcTan[c*x]^2*Log[1 + E^((2*I)
*ArcTan[c*x])] - I*ArcTan[c*x]*PolyLog[2, E^((-2*I)*ArcTan[c*x])] - I*ArcT
an[c*x]*PolyLog[2, -E^((2*I)*ArcTan[c*x])] - PolyLog[3, E^((-2*I)*ArcTan[c
*x])]/2 + PolyLog[3, -E^((2*I)*ArcTan[c*x])]/2)))/2
```

## Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^3}\,dx$$

$$\Big\downarrow \text{5411}$$

$$\int \left(-ic^3d^3(a+b\arctan(cx))^2-\frac{3c^2d^3(a+b\arctan(cx))^2}{x}+\frac{d^3(a+b\arctan(cx))^2}{x^3}+\frac{3icd^3(a+b\arctan(cx))^2}{x^2}\right)$$

$$\Big\downarrow \text{2009}$$

$$-6c^2d^3\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2-ic^3d^3x(a+b\arctan(cx))^2+3ibc^2d^3\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))-3ibc^2d^3\operatorname{PolyLog}\left(2,\frac{2}{icx+1}-1\right)(a+b\arctan(cx))+\frac{7}{2}c^2d^3(a+b\arctan(cx))^2-2ibc^2d^3\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))+6ibc^2d^3\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))-\frac{d^3(a+b\arctan(cx))^2}{2x^2}-\frac{3icd^3(a+b\arctan(cx))^2}{x}-\frac{bcd^3(a+b\arctan(cx))}{x}+3b^2c^2d^3\operatorname{PolyLog}\left(2,\frac{2}{1-icx}-1\right)+b^2c^2d^3\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)+\frac{3}{2}b^2c^2d^3\operatorname{PolyLog}\left(3,1-\frac{2}{icx+1}\right)-\frac{3}{2}b^2c^2d^3\operatorname{PolyLog}\left(3,\frac{2}{icx+1}-1\right)-\frac{1}{2}b^2c^2d^3\log\left(c^2x^2+1\right)+b^2c^2d^3\log(x)$$

input
```
Int[((d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2)/x^3,x]
```

output
```
-((b*c*d^3*(a + b*ArcTan[c*x]))/x) + (7*c^2*d^3*(a + b*ArcTan[c*x])^2)/2 -
 (d^3*(a + b*ArcTan[c*x])^2)/(2*x^2) - ((3*I)*c*d^3*(a + b*ArcTan[c*x])^2)
/x - I*c^3*d^3*x*(a + b*ArcTan[c*x])^2 - 6*c^2*d^3*(a + b*ArcTan[c*x])^2*A
rcTanh[1 - 2/(1 + I*c*x)] + b^2*c^2*d^3*Log[x] - (2*I)*b*c^2*d^3*(a + b*Ar
cTan[c*x])*Log[2/(1 + I*c*x)] - (b^2*c^2*d^3*Log[1 + c^2*x^2])/2 + (6*I)*b
*c^2*d^3*(a + b*ArcTan[c*x])*Log[2 - 2/(1 - I*c*x)] + 3*b^2*c^2*d^3*PolyLo
g[2, -1 + 2/(1 - I*c*x)] + b^2*c^2*d^3*PolyLog[2, 1 - 2/(1 + I*c*x)] + (3*
I)*b*c^2*d^3*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)] - (3*I)*b*c
^2*d^3*(a + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)] + (3*b^2*c^2*d^3
*PolyLog[3, 1 - 2/(1 + I*c*x)])/2 - (3*b^2*c^2*d^3*PolyLog[3, -1 + 2/(1 +
I*c*x)])/2
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 14.91 (sec) , antiderivative size = 1523, normalized size of antiderivative = 3.66

| method | result | size |
|---|---|---|
| derivativedivides | Expression too large to display | 1523 |
| default | Expression too large to display | 1523 |
| parts | Expression too large to display | 1523 |

input
```
int((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
c^2*(d^3*a^2*(-I*c*x-3*ln(c*x)-1/2/c^2/x^2-3*I/c/x)+d^3*b^2*(-1/2/c^2/x^2*
arctan(c*x)^2-3/2*I*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I/(1+(1+I*
c*x)^2/(c^2*x^2+1)))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^
2*x^2+1)))*arctan(c*x)^2+3/2*arctan(c*x)^2-1/2*arctan(c*x)*(I*c*x-(c^2*x^2
+1)^(1/2)+1)/c/x-3*I*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))-2*dil
og(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-2*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2
))-3*ln(c*x)*arctan(c*x)^2+3*arctan(c*x)^2*ln((1+I*c*x)^2/(c^2*x^2+1)-1)-3
*arctan(c*x)^2*ln(1-(1+I*c*x)/(c^2*x^2+1)^(1/2))-3*arctan(c*x)^2*ln(1+(1+I
*c*x)/(c^2*x^2+1)^(1/2))+ln((1+I*c*x)/(c^2*x^2+1)^(1/2)-1)+3/2*polylog(3,-
(1+I*c*x)^2/(c^2*x^2+1))-6*polylog(3,(1+I*c*x)/(c^2*x^2+1)^(1/2))+ln(1+(1+
I*c*x)/(c^2*x^2+1)^(1/2))-6*polylog(3,-(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/2*ar
ctan(c*x)*(I*c*x+(c^2*x^2+1)^(1/2)+1)/c/x-I*arctan(c*x)^2*c*x-3*I*arctan(c
*x)^2/c/x+3/2*I*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^
2+1)))^2*arctan(c*x)^2-3/2*I*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c
*x)^2/(c^2*x^2+1)))^3*arctan(c*x)^2-3/2*I*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+
1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^3*arctan(c*x)^2+6*I*arctan(c*x)*polylog
(2,-(1+I*c*x)/(c^2*x^2+1)^(1/2))-2*I*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2
+1)^(1/2))+6*I*arctan(c*x)*polylog(2,(1+I*c*x)/(c^2*x^2+1)^(1/2))-3/2*I*Pi
*arctan(c*x)^2+6*I*arctan(c*x)*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))-2*I*arcta
n(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+3/2*I*Pi*csgn(I*((1+I*c*x)^2...
```

## Fricas [F]

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^3}\,dx = \int \frac{(i\,cdx+d)^3(b\arctan(cx)+a)^2}{x^3}\,dx$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^3,x, algorithm="fricas")
```

output
```
integral(1/4*(-4*I*a^2*c^3*d^3*x^3 - 12*a^2*c^2*d^3*x^2 + 12*I*a^2*c*d^3*x
 + 4*a^2*d^3 + (I*b^2*c^3*d^3*x^3 + 3*b^2*c^2*d^3*x^2 - 3*I*b^2*c*d^3*x -
b^2*d^3)*log(-(c*x + I)/(c*x - I))^2 + 4*(a*b*c^3*d^3*x^3 - 3*I*a*b*c^2*d^
3*x^2 - 3*a*b*c*d^3*x + I*a*b*d^3)*log(-(c*x + I)/(c*x - I)))/x^3, x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)**3*(a+b*atan(c*x))**2/x**3,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^3}\,dx = \int \frac{(i\,cdx+d)^3(b\arctan(cx)+a)^2}{x^3}\,dx$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^3,x, algorithm="maxima")
```

output
```
-I*a^2*c^3*d^3*x - I*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*a*b*c^2*d^3 -
3*a^2*c^2*d^3*log(x) - 3*I*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x
)/x)*a*b*c*d^3 - ((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*a*b*d^3 - 3*I
*a^2*c*d^3/x - 1/2*a^2*d^3/x^2 - 1/32*(16*I*(24*b^2*c^5*d^3*integrate(1/16
*x^5*arctan(c*x)^2/(c^2*x^5 + x^3), x) + 2*b^2*c^5*d^3*integrate(1/16*x^5*
log(c^2*x^2 + 1)^2/(c^2*x^5 + x^3), x) + 8*b^2*c^5*d^3*integrate(1/16*x^5*
log(c^2*x^2 + 1)/(c^2*x^5 + x^3), x) - b^2*c^2*d^3*arctan(c*x)^3 - 24*b^2*
c^4*d^3*integrate(1/16*x^4*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^5 + x^3), x
) - 16*b^2*c^4*d^3*integrate(1/16*x^4*arctan(c*x)/(c^2*x^5 + x^3), x) - 4*
b^2*c^3*d^3*integrate(1/16*x^3*log(c^2*x^2 + 1)^2/(c^2*x^5 + x^3), x) + 24
*b^2*c^3*d^3*integrate(1/16*x^3*log(c^2*x^2 + 1)/(c^2*x^5 + x^3), x) - 16*
b^2*c^2*d^3*integrate(1/16*x^2*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^5 + x^3
), x) - 56*b^2*c^2*d^3*integrate(1/16*x^2*arctan(c*x)/(c^2*x^5 + x^3), x)
- 72*b^2*c*d^3*integrate(1/16*x*arctan(c*x)^2/(c^2*x^5 + x^3), x) - 6*b^2*
c*d^3*integrate(1/16*x*log(c^2*x^2 + 1)^2/(c^2*x^5 + x^3), x) - 4*b^2*c*d^
3*integrate(1/16*x*log(c^2*x^2 + 1)/(c^2*x^5 + x^3), x) + 8*b^2*d^3*integr
ate(1/16*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^5 + x^3), x))*x^2 + (128*b^2*
c^5*d^3*integrate(1/16*x^5*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^5 + x^3), x
) + 256*b^2*c^5*d^3*integrate(1/16*x^5*arctan(c*x)/(c^2*x^5 + x^3), x) + 1
152*b^2*c^4*d^3*integrate(1/16*x^4*arctan(c*x)^2/(c^2*x^5 + x^3), x) + ...
```

## Giac [F]

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^3}\,dx=\int \frac{(i\,cdx+d)^3(b\arctan\left(cx\right)+a)^2}{x^3}\,dx$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^3,x, algorithm="giac")
```

output
```
integrate((I*c*d*x + d)^3*(b*arctan(c*x) + a)^2/x^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^3}\,dx=\int \frac{(a+b\,\mathrm{atan}(c\,x))^2\,(d+c\,d\,x\,1\mathrm{i})^3}{x^3}\,dx$$

input
```
int(((a + b*atan(c*x))^2*(d + c*d*x*1i)^3)/x^3,x)
```

output
```
int(((a + b*atan(c*x))^2*(d + c*d*x*1i)^3)/x^3, x)
```

## Reduce [F]

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^3}\,dx = \frac{d^3\left(-atan(cx)^2\,b^2c^2x^2-6atan(cx)^2\,b^2cix-atan(cx)^2\,b^2-4atan(cx)\,ab\,c^3i\,x^3-2atan(cx)\,ab\,c^2x^2-12\right.}{}$$

input
```
int((d+I*c*d*x)^3*(a+b*atan(c*x))^2/x^3,x)
```

output
```
(d**3*( - atan(c*x)**2*b**2*c**2*x**2 - 6*atan(c*x)**2*b**2*c*i*x - atan(c
*x)**2*b**2 - 4*atan(c*x)*a*b*c**3*i*x**3 - 2*atan(c*x)*a*b*c**2*x**2 - 12
*atan(c*x)*a*b*c*i*x - 2*atan(c*x)*a*b - 2*atan(c*x)*b**2*c*x - 2*int(atan
(c*x)**2,x)*b**2*c**3*i*x**2 + 12*int(atan(c*x)/(c**2*x**3 + x),x)*b**2*c*
*2*i*x**2 - 12*int(atan(c*x)/x,x)*a*b*c**2*x**2 - 6*int(atan(c*x)**2/x,x)*
b**2*c**2*x**2 - 4*log(c**2*x**2 + 1)*a*b*c**2*i*x**2 - log(c**2*x**2 + 1)
*b**2*c**2*x**2 - 6*log(x)*a**2*c**2*x**2 + 12*log(x)*a*b*c**2*i*x**2 + 2*
log(x)*b**2*c**2*x**2 - 2*a**2*c**3*i*x**3 - 6*a**2*c*i*x - a**2 - 2*a*b*c
*x))/(2*x**2)
```

# 3.91 $\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^4} dx$

## Optimal result

Integrand size = 25, antiderivative size = 429

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^4}\,dx = -\frac{b^2c^2d^3}{3x} - \frac{1}{3}b^2c^3d^3\arctan(cx) - \frac{bcd^3(a+b\arctan(cx))}{3x^2} - \frac{3ibc^2d^3(a+b\arctan(cx))}{x} + \frac{11}{6}ic^3d^3(a+b\arctan(cx))^2 - \frac{d^3(a+b\arctan(cx))^2}{3x^3} - \frac{3icd^3(a+b\arctan(cx))^2}{2x^2} + \frac{3c^2d^3(a+b\arctan(cx))^2}{x} - 2ic^3d^3(a+b\arctan(cx))^2\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right) + 3ib^2c^3d^3\log(x) - \frac{3}{2}ib^2c^3d^3\log\left(1+c^2x^2\right) - \frac{20}{3}bc^3d^3(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right) + \frac{10}{3}ib^2c^3d^3\operatorname{PolyLog}\left(2,-1+\frac{2}{1-icx}\right) - bc^3d^3(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right) + bc^3d^3(a+b\arctan(cx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx}\right) + \frac{1}{2}ib^2c^3d^3\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right) - \frac{1}{2}ib^2c^3d^3\operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)$$

output
```
-1/3*b^2*c^2*d^3/x-1/3*b^2*c^3*d^3*arctan(c*x)-1/3*b*c*d^3*(a+b*arctan(c*x
))/x^2-3*I*b*c^2*d^3*(a+b*arctan(c*x))/x+10/3*I*b^2*c^3*d^3*polylog(2,-1+2
/(1-I*c*x))-1/3*d^3*(a+b*arctan(c*x))^2/x^3-3/2*I*c*d^3*(a+b*arctan(c*x))^
2/x^2+3*c^2*d^3*(a+b*arctan(c*x))^2/x-1/2*I*b^2*c^3*d^3*polylog(3,-1+2/(1+
I*c*x))+3*I*b^2*c^3*d^3*ln(x)+2*I*c^3*d^3*(a+b*arctan(c*x))^2*arctanh(-1+2
/(1+I*c*x))-20/3*b*c^3*d^3*(a+b*arctan(c*x))*ln(2-2/(1-I*c*x))-3/2*I*b^2*c
^3*d^3*ln(c^2*x^2+1)-b*c^3*d^3*(a+b*arctan(c*x))*polylog(2,1-2/(1+I*c*x))+
b*c^3*d^3*(a+b*arctan(c*x))*polylog(2,-1+2/(1+I*c*x))+11/6*I*c^3*d^3*(a+b*
arctan(c*x))^2+1/2*I*b^2*c^3*d^3*polylog(3,1-2/(1+I*c*x))
```

## Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 595, normalized size of antiderivative = 1.39

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^4}\,dx$$
$$= \frac{d^3\Big(-8a^2-36ia^2cx-8abcx+72a^2c^2x^2-72iabc^2x^2-8b^2c^2x^2-b^2c^3\pi^3x^3-16ab\arctan(cx)-72iabcx$$

input
```
Integrate[((d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2)/x^4,x]
```

output
```
(d^3*(-8*a^2 - (36*I)*a^2*c*x - 8*a*b*c*x + 72*a^2*c^2*x^2 - (72*I)*a*b*c^
2*x^2 - 8*b^2*c^2*x^2 - b^2*c^3*Pi^3*x^3 - 16*a*b*ArcTan[c*x] - (72*I)*a*b
*c*x*ArcTan[c*x] - 8*b^2*c*x*ArcTan[c*x] + 144*a*b*c^2*x^2*ArcTan[c*x] - (
72*I)*b^2*c^2*x^2*ArcTan[c*x] - (72*I)*a*b*c^3*x^3*ArcTan[c*x] - 8*b^2*c^3
*x^3*ArcTan[c*x] - 8*b^2*ArcTan[c*x]^2 - (36*I)*b^2*c*x*ArcTan[c*x]^2 + 72
*b^2*c^2*x^2*ArcTan[c*x]^2 + (44*I)*b^2*c^3*x^3*ArcTan[c*x]^2 + 16*b^2*c^3
*x^3*ArcTan[c*x]^3 - (24*I)*b^2*c^3*x^3*ArcTan[c*x]^2*Log[1 - E^((-2*I)*Ar
cTan[c*x])] - 160*b^2*c^3*x^3*ArcTan[c*x]*Log[1 - E^((2*I)*ArcTan[c*x])] +
 (24*I)*b^2*c^3*x^3*ArcTan[c*x]^2*Log[1 + E^((2*I)*ArcTan[c*x])] - (24*I)*
a^2*c^3*x^3*Log[x] - 160*a*b*c^3*x^3*Log[c*x] + (72*I)*b^2*c^3*x^3*Log[(c*
x)/Sqrt[1 + c^2*x^2]] + 80*a*b*c^3*x^3*Log[1 + c^2*x^2] + 24*b^2*c^3*x^3*A
rcTan[c*x]*PolyLog[2, E^((-2*I)*ArcTan[c*x])] + 24*b^2*c^3*x^3*ArcTan[c*x]
*PolyLog[2, -E^((2*I)*ArcTan[c*x])] + (80*I)*b^2*c^3*x^3*PolyLog[2, E^((2*
I)*ArcTan[c*x])] + 24*a*b*c^3*x^3*PolyLog[2, (-I)*c*x] - 24*a*b*c^3*x^3*Po
lyLog[2, I*c*x] - (12*I)*b^2*c^3*x^3*PolyLog[3, E^((-2*I)*ArcTan[c*x])] +
(12*I)*b^2*c^3*x^3*PolyLog[3, -E^((2*I)*ArcTan[c*x])]))/(24*x^3)
```

## Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^4}\,dx$$

$$\Big\downarrow \text{5411}$$

$$\int \left(-\frac{ic^3d^3(a+b\arctan(cx))^2}{x}-\frac{3c^2d^3(a+b\arctan(cx))^2}{x^2}+\frac{d^3(a+b\arctan(cx))^2}{x^4}+\frac{3icd^3(a+b\arctan(cx))^2}{x^3}\right)$$

$$\Big\downarrow \text{2009}$$

$$-2ic^3d^3\text{arctanh}\left(1-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2-bc^3d^3\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))+bc^3d^3\,\text{PolyLog}\left(2,\frac{2}{icx+1}-1\right)(a+b\arctan(cx))+\frac{11}{6}ic^3d^3(a+b\arctan(cx))^2-\frac{20}{3}bc^3d^3\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))+\frac{3c^2d^3(a+b\arctan(cx))^2}{x}-\frac{3ibc^2d^3(a+b\arctan(cx))}{x}-\frac{d^3(a+b\arctan(cx))^2}{3x^3}-\frac{3icd^3(a+b\arctan(cx))^2}{2x^2}-\frac{bcd^3(a+b\arctan(cx))}{3x^2}-\frac{1}{3}b^2c^3d^3\arctan(cx)+\frac{10}{3}ib^2c^3d^3\,\text{PolyLog}\left(2,\frac{2}{1-icx}-1\right)+\frac{1}{2}ib^2c^3d^3\,\text{PolyLog}\left(3,1-\frac{2}{icx+1}\right)-\frac{1}{2}ib^2c^3d^3\,\text{PolyLog}\left(3,\frac{2}{icx+1}-1\right)+3ib^2c^3d^3\log(x)-\frac{b^2c^2d^3}{3x}-\frac{3}{2}ib^2c^3d^3\log\left(c^2x^2+1\right)$$

input
```
Int[((d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2)/x^4,x]
```

output
```
-1/3*(b^2*c^2*d^3)/x - (b^2*c^3*d^3*ArcTan[c*x])/3 - (b*c*d^3*(a + b*ArcTa
n[c*x]))/(3*x^2) - ((3*I)*b*c^2*d^3*(a + b*ArcTan[c*x]))/x + ((11*I)/6)*c^
3*d^3*(a + b*ArcTan[c*x])^2 - (d^3*(a + b*ArcTan[c*x])^2)/(3*x^3) - (((3*I
)/2)*c*d^3*(a + b*ArcTan[c*x])^2)/x^2 + (3*c^2*d^3*(a + b*ArcTan[c*x])^2)/
x - (2*I)*c^3*d^3*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)] + (3*I)
*b^2*c^3*d^3*Log[x] - ((3*I)/2)*b^2*c^3*d^3*Log[1 + c^2*x^2] - (20*b*c^3*d
^3*(a + b*ArcTan[c*x])*Log[2 - 2/(1 - I*c*x)])/3 + ((10*I)/3)*b^2*c^3*d^3*
PolyLog[2, -1 + 2/(1 - I*c*x)] - b*c^3*d^3*(a + b*ArcTan[c*x])*PolyLog[2,
1 - 2/(1 + I*c*x)] + b*c^3*d^3*(a + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 +
I*c*x)] + (I/2)*b^2*c^3*d^3*PolyLog[3, 1 - 2/(1 + I*c*x)] - (I/2)*b^2*c^3*
d^3*PolyLog[3, -1 + 2/(1 + I*c*x)]
```

**Defintions of rubi rules used**

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 18.37 (sec) , antiderivative size = 1655, normalized size of antiderivative = 3.86

| method | result | size |
|---|---|---|
| derivativedivides | Expression too large to display | 1655 |
| default | Expression too large to display | 1655 |
| parts | Expression too large to display | 1727 |

input
```
int((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
c^3*(-3/2*I*d^3*b^2*arctan(c*x)^2/c^2/x^2+d^3*a^2*(-1/3/c^3/x^3-I*ln(c*x)-
3/2*I/c^2/x^2+3/c/x)-20/3*d^3*b^2*arctan(c*x)*ln(1+(1+I*c*x)/(c^2*x^2+1)^(
1/2))-2*d^3*b^2*arctan(c*x)*polylog(2,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/2*d^
3*b^2*Pi*arctan(c*x)^2+d^3*b^2*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2
+1))-2*d^3*b^2*arctan(c*x)*polylog(2,(1+I*c*x)/(c^2*x^2+1)^(1/2))-2*I*d^3*
b^2*polylog(3,(1+I*c*x)/(c^2*x^2+1)^(1/2))+3*I*d^3*b^2*ln((1+I*c*x)/(c^2*x
^2+1)^(1/2)-1)+3*I*d^3*b^2*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))-2*I*d^3*b^2*p
olylog(3,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/2*I*d^3*b^2*polylog(3,-(1+I*c*x)^
2/(c^2*x^2+1))+11/6*I*d^3*b^2*arctan(c*x)^2-20/3*I*d^3*b^2*dilog((1+I*c*x)
/(c^2*x^2+1)^(1/2))+20/3*I*d^3*b^2*dilog(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))+2*
d^3*a*b*(-1/3/c^3/x^3*arctan(c*x)-I*arctan(c*x)*ln(c*x)-3/2*I*arctan(c*x)/
c^2/x^2+3/c/x*arctan(c*x)+1/2*ln(c*x)*ln(1+I*c*x)-1/2*ln(c*x)*ln(1-I*c*x)+
1/2*dilog(1+I*c*x)-1/2*dilog(1-I*c*x)+5/3*ln(c^2*x^2+1)-3/2*I*arctan(c*x)-
3/2*I/c/x-1/6/c^2/x^2-10/3*ln(c*x))+8/3*d^3*b^2*arctan(c*x)+1/2*d^3*b^2*Pi
*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*c
sgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*arctan(c*x)
^2-I*d^3*b^2*arctan(c*x)^2*ln(c*x)-I*d^3*b^2*arctan(c*x)^2*ln(1-(1+I*c*x)/
(c^2*x^2+1)^(1/2))-I*d^3*b^2*arctan(c*x)^2*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2
))+1/3*I*d^3*b^2/(I*c*x+(c^2*x^2+1)^(1/2)+1)*(c^2*x^2+1)^(1/2)-1/3*I*d^3*b
^2/(I*c*x-(c^2*x^2+1)^(1/2)+1)*(c^2*x^2+1)^(1/2)+I*d^3*b^2*arctan(c*x)^...
```

## Fricas [F]

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^4}\,dx=\int \frac{(i\,cdx+d)^3(b\arctan(cx)+a)^2}{x^4}\,dx$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^4,x, algorithm="fricas")
```

output
```
integral(1/4*(-4*I*a^2*c^3*d^3*x^3 - 12*a^2*c^2*d^3*x^2 + 12*I*a^2*c*d^3*x
 + 4*a^2*d^3 + (I*b^2*c^3*d^3*x^3 + 3*b^2*c^2*d^3*x^2 - 3*I*b^2*c*d^3*x -
b^2*d^3)*log(-(c*x + I)/(c*x - I))^2 + 4*(a*b*c^3*d^3*x^3 - 3*I*a*b*c^2*d^
3*x^2 - 3*a*b*c*d^3*x + I*a*b*d^3)*log(-(c*x + I)/(c*x - I)))/x^4, x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^4}\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)**3*(a+b*atan(c*x))**2/x**4,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^4}\,dx = \int \frac{(i\,cdx+d)^3(b\arctan(cx)+a)^2}{x^4}\,dx$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^4,x, algorithm="maxima")
```

output
```
-I*a^2*c^3*d^3*log(x) + 3*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)
/x)*a*b*c^2*d^3 - 3*I*((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*a*b*c*d^
3 + 1/3*((c^2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x
^3)*a*b*d^3 + 3*a^2*c^2*d^3/x - 3/2*I*a^2*c*d^3/x^2 - 1/3*a^2*d^3/x^3 - 1/
96*(24*(3*b^2*c^3*d^3*arctan(c*x)^3 + 48*b^2*c^5*d^3*integrate(1/48*x^5*ar
ctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^6 + x^4), x) + 36*b^2*c^4*d^3*integrate(
1/48*x^4*log(c^2*x^2 + 1)^2/(c^2*x^6 + x^4), x) - 144*b^2*c^4*d^3*integrat
e(1/48*x^4*log(c^2*x^2 + 1)/(c^2*x^6 + x^4), x) - 96*b^2*c^3*d^3*integrate
(1/48*x^3*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^6 + x^4), x) + 432*b^2*c^3*d
^3*integrate(1/48*x^3*arctan(c*x)/(c^2*x^6 + x^4), x) + 288*b^2*c^2*d^3*in
tegrate(1/48*x^2*arctan(c*x)^2/(c^2*x^6 + x^4), x) + 24*b^2*c^2*d^3*integr
ate(1/48*x^2*log(c^2*x^2 + 1)^2/(c^2*x^6 + x^4), x) + 88*b^2*c^2*d^3*integ
rate(1/48*x^2*log(c^2*x^2 + 1)/(c^2*x^6 + x^4), x) - 144*b^2*c*d^3*integra
te(1/48*x*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^6 + x^4), x) - 32*b^2*c*d^3*
integrate(1/48*x*arctan(c*x)/(c^2*x^6 + x^4), x) - 144*b^2*d^3*integrate(1
/48*arctan(c*x)^2/(c^2*x^6 + x^4), x) - 12*b^2*d^3*integrate(1/48*log(c^2*
x^2 + 1)^2/(c^2*x^6 + x^4), x))*x^3 + I*(3456*b^2*c^5*d^3*integrate(1/48*x
^5*arctan(c*x)^2/(c^2*x^6 + x^4), x) + 9216*a*b*c^5*d^3*integrate(1/48*x^5
*arctan(c*x)/(c^2*x^6 + x^4), x) + b^2*c^3*d^3*log(c^2*x^2 + 1)^3 + 72*b^2
*c^3*d^3*arctan(c*x)^2 - 3456*b^2*c^4*d^3*integrate(1/48*x^4*arctan(c*x...
```

## Giac [F]

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^4}\,dx=\int \frac{(i\,cdx+d)^3(b\arctan(cx)+a)^2}{x^4}\,dx$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^4,x, algorithm="giac")
```

output
```
integrate((I*c*d*x + d)^3*(b*arctan(c*x) + a)^2/x^4, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^4}\,dx=\int \frac{(a+b\,\mathrm{atan}(c\,x))^2\,(d+c\,d\,x\,1\mathrm{i})^3}{x^4}\,dx$$

input
```
int(((a + b*atan(c*x))^2*(d + c*d*x*1i)^3)/x^4,x)
```

output
```
int(((a + b*atan(c*x))^2*(d + c*d*x*1i)^3)/x^4, x)
```

## Reduce [F]

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^4}\,dx$$
$$=\frac{d^3\Big(-18atan(cx)\,abcix-9atan(cx)^2\,b^2c^3i\,x^3+20\log(c^2x^2+1)\,ab\,c^3x^3-9\log(c^2x^2+1)\,b^2c^3i\,x^3-40\,\mathrm{lo}}{}$$

input
```
int((d+I*c*d*x)^3*(a+b*atan(c*x))^2/x^4,x)
```

output
```
(d**3*( - 9*atan(c*x)**2*b**2*c**3*i*x**3 + 18*atan(c*x)**2*b**2*c**2*x**2
 - 9*atan(c*x)**2*b**2*c*i*x - 2*atan(c*x)**2*b**2 - 18*atan(c*x)*a*b*c**3
*i*x**3 + 36*atan(c*x)*a*b*c**2*x**2 - 18*atan(c*x)*a*b*c*i*x - 4*atan(c*x
)*a*b - 2*atan(c*x)*b**2*c**3*x**3 - 18*atan(c*x)*b**2*c**2*i*x**2 - 2*ata
n(c*x)*b**2*c*x - 40*int(atan(c*x)/(c**2*x**3 + x),x)*b**2*c**3*x**3 - 12*
int(atan(c*x)/x,x)*a*b*c**3*i*x**3 - 6*int(atan(c*x)**2/x,x)*b**2*c**3*i*x
**3 + 20*log(c**2*x**2 + 1)*a*b*c**3*x**3 - 9*log(c**2*x**2 + 1)*b**2*c**3
*i*x**3 - 6*log(x)*a**2*c**3*i*x**3 - 40*log(x)*a*b*c**3*x**3 + 18*log(x)*
b**2*c**3*i*x**3 + 18*a**2*c**2*x**2 - 9*a**2*c*i*x - 2*a**2 - 18*a*b*c**2
*i*x**2 - 2*a*b*c*x - 2*b**2*c**2*x**2))/(6*x**3)
```

# 3.92 $\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^5}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 293

$$\begin{aligned}
&\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^5}\,dx\\
&= -\frac{b^2c^2d^3}{12x^2}-\frac{ib^2c^3d^3}{x}-ib^2c^4d^3\arctan(cx)-\frac{bcd^3(a+b\arctan(cx))}{6x^3}\\
&\quad-\frac{ibc^2d^3(a+b\arctan(cx))}{x^2}+\frac{7bc^3d^3(a+b\arctan(cx))}{2x}\\
&\quad-\frac{d^3(1+icx)^4(a+b\arctan(cx))^2}{4x^4}-4iabc^4d^3\log(x)\\
&\quad-\frac{11}{3}b^2c^4d^3\log(x)-4ibc^4d^3(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)\\
&\quad+\frac{11}{6}b^2c^4d^3\log\left(1+c^2x^2\right)+2b^2c^4d^3\operatorname{PolyLog}(2,-icx)\\
&\quad-2b^2c^4d^3\operatorname{PolyLog}(2,icx)-2b^2c^4d^3\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)
\end{aligned}$$

output

```
-1/12*b^2*c^2*d^3/x^2-I*b^2*c^3*d^3/x-I*b^2*c^4*d^3*arctan(c*x)-1/6*b*c*d^
3*(a+b*arctan(c*x))/x^3-I*b*c^2*d^3*(a+b*arctan(c*x))/x^2+7/2*b*c^3*d^3*(a
+b*arctan(c*x))/x-1/4*d^3*(1+I*c*x)^4*(a+b*arctan(c*x))^2/x^4-4*I*a*b*c^4*
d^3*ln(x)-11/3*b^2*c^4*d^3*ln(x)-4*I*b*c^4*d^3*(a+b*arctan(c*x))*ln(2/(1-I
*c*x))+11/6*b^2*c^4*d^3*ln(c^2*x^2+1)+2*b^2*c^4*d^3*polylog(2,-I*c*x)-2*b^
2*c^4*d^3*polylog(2,I*c*x)-2*b^2*c^4*d^3*polylog(2,1-2/(1-I*c*x))
```

## Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.10

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^5}\,dx$$
$$= \frac{d^3\left(-3a^2-12ia^2cx-2abcx+18a^2c^2x^2-12iabc^2x^2-b^2c^2x^2+12ia^2c^3x^3+42abc^3x^3-12ib^2c^3x^3-b^2c\right.}{}$$

input
```
Integrate[((d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2)/x^5,x]
```

output
```
(d^3*(-3*a^2 - (12*I)*a^2*c*x - 2*a*b*c*x + 18*a^2*c^2*x^2 - (12*I)*a*b*c^
2*x^2 - b^2*c^2*x^2 + (12*I)*a^2*c^3*x^3 + 42*a*b*c^3*x^3 - (12*I)*b^2*c^3
*x^3 - b^2*c^4*x^4 - 3*b^2*(-I + c*x)^4*ArcTan[c*x]^2 + 2*b*ArcTan[c*x]*(b
*c*x*(-1 - (6*I)*c*x + 21*c^2*x^2 - (6*I)*c^3*x^3) + 3*a*(-1 - (4*I)*c*x +
 6*c^2*x^2 + (4*I)*c^3*x^3 + 7*c^4*x^4) - (24*I)*b*c^4*x^4*Log[1 - E^((2*I
)*ArcTan[c*x])]) - (48*I)*a*b*c^4*x^4*Log[c*x] - 44*b^2*c^4*x^4*Log[(c*x)/
Sqrt[1 + c^2*x^2]] + (24*I)*a*b*c^4*x^4*Log[1 + c^2*x^2] - 24*b^2*c^4*x^4*
PolyLog[2, E^((2*I)*ArcTan[c*x])]))/(12*x^4)
```

## Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.94, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {5409, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^5}\,dx$$
$$\Big\downarrow \text{5409}$$
$$-2bc\int\left(-\frac{2id^3(a+b\arctan(cx))c^4}{cx+i}+\frac{2id^3(a+b\arctan(cx))c^3}{x}+\frac{7d^3(a+b\arctan(cx))c^2}{4x^2}-\frac{id^3(a+b\arctan(c\right.$$
$$\frac{d^3(1+icx)^4(a+b\arctan(cx))^2}{4x^4}$$

$\downarrow$ 2009

$$-2bc\left(2ic^3d^3\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))-\frac{7c^2d^3(a+b\arctan(cx))}{4x}+\frac{d^3(a+b\arctan(cx))}{12x^3}+\frac{icd^3(a+b\,\mathrm{a}}{2x}\right.$$
$$\frac{d^3(1+icx)^4(a+b\arctan(cx))^2}{4x^4}$$

input
```
Int[((d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2)/x^5,x]
```

output
```
-1/4*(d^3*(1 + I*c*x)^4*(a + b*ArcTan[c*x])^2)/x^4 - 2*b*c*((b*c*d^3)/(24*
x^2) + ((I/2)*b*c^2*d^3)/x + (I/2)*b*c^3*d^3*ArcTan[c*x] + (d^3*(a + b*Arc
Tan[c*x]))/(12*x^3) + ((I/2)*c*d^3*(a + b*ArcTan[c*x]))/x^2 - (7*c^2*d^3*(
a + b*ArcTan[c*x]))/(4*x) + (2*I)*a*c^3*d^3*Log[x] + (11*b*c^3*d^3*Log[x])
/6 + (2*I)*c^3*d^3*(a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)] - (11*b*c^3*d^3*
Log[1 + c^2*x^2])/12 - b*c^3*d^3*PolyLog[2, (-I)*c*x] + b*c^3*d^3*PolyLog[
2, I*c*x] + b*c^3*d^3*PolyLog[2, 1 - 2/(1 - I*c*x)])
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5409
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_.) + (e_
.)*(x_))^(q_), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Sim
p[(a + b*ArcTan[c*x])^p   u, x] - Simp[b*c*p   Int[ExpandIntegrand[(a + b*A
rcTan[c*x])^(p - 1), u/(1 + c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e,
f, q}, x] && IGtQ[p, 1] && EqQ[c^2*d^2 + e^2, 0] && IntegersQ[m, q] && NeQ[
m, -1] && NeQ[q, -1] && ILtQ[m + q + 1, 0] && LtQ[m*q, 0]
```

## Maple [A] (verified)

Time = 1.94 (sec) , antiderivative size = 501, normalized size of antiderivative = 1.71

| method | result |
|---|---|
| parts | $d^3a^2\left(\frac{3c^2}{2x^2}-\frac{ic}{x^3}+\frac{ic^3}{x}-\frac{1}{4x^4}\right)+d^3b^2c^4\left(\frac{7\arctan(cx)}{2cx}+\frac{3\arctan(cx)^2}{2c^2x^2}-\frac{\arctan(cx)}{6c^3x^3}-i\arctan(c\right.$ |
| derivativedivides | $c^4\left(d^3a^2\left(\frac{3}{2c^2x^2}-\frac{1}{4c^4x^4}-\frac{i}{c^3x^3}+\frac{i}{cx}\right)+d^3b^2\left(\frac{7\arctan(cx)}{2cx}+\frac{3\arctan(cx)^2}{2c^2x^2}-\frac{\arctan(cx)}{6c^3x^3}-i\,\mathrm{arcta}\right.\right.$ |
| default | $c^4\left(d^3a^2\left(\frac{3}{2c^2x^2}-\frac{1}{4c^4x^4}-\frac{i}{c^3x^3}+\frac{i}{cx}\right)+d^3b^2\left(\frac{7\arctan(cx)}{2cx}+\frac{3\arctan(cx)^2}{2c^2x^2}-\frac{\arctan(cx)}{6c^3x^3}-i\,\mathrm{arcta}\right.\right.$ |

input
```
int((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^5,x,method=_RETURNVERBOSE)
```

output
```
d^3*a^2*(3/2*c^2/x^2-I*c/x^3+I*c^3/x-1/4/x^4)+d^3*b^2*c^4*(7/2/c/x*arctan(
c*x)+3/2/c^2/x^2*arctan(c*x)^2-1/6/c^3/x^3*arctan(c*x)+2*dilog(1+I*c*x)-2*
dilog(1-I*c*x)-11/3*ln(c*x)-1/12/c^2/x^2+2*I*arctan(c*x)*ln(c^2*x^2+1)-4*I
*arctan(c*x)*ln(c*x)+7/4*arctan(c*x)^2+11/6*ln(c^2*x^2+1)-1/4/c^4/x^4*arct
an(c*x)^2+1/2*ln(c*x-I)^2+dilog(-1/2*I*(c*x+I))-1/2*ln(c*x+I)^2-dilog(1/2*
I*(c*x-I))-ln(c*x-I)*ln(c^2*x^2+1)+ln(c*x-I)*ln(-1/2*I*(c*x+I))+ln(c*x+I)*
ln(c^2*x^2+1)-ln(c*x+I)*ln(1/2*I*(c*x-I))+I*arctan(c*x)^2/c/x-I*arctan(c*x
)^2/c^3/x^3-I*arctan(c*x)/c^2/x^2-I/c/x-I*arctan(c*x)+2*ln(c*x)*ln(1+I*c*x
)-2*ln(c*x)*ln(1-I*c*x))+2*d^3*a*b*c^4*(3/2/c^2/x^2*arctan(c*x)-1/4/c^4/x^
4*arctan(c*x)-I*arctan(c*x)/c^3/x^3+I*arctan(c*x)/c/x+I*ln(c^2*x^2+1)+7/4*
arctan(c*x)-1/2*I/c^2/x^2-2*I*ln(c*x)-1/12/c^3/x^3+7/4/c/x)
```

## Fricas [F]

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^5}\,dx=\int \frac{(i\,cdx+d)^3(b\arctan(cx)+a)^2}{x^5}\,dx$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^5,x, algorithm="fricas")
```

output
```
1/16*(16*x^4*integral(1/4*(-4*I*a^2*c^5*d^3*x^5 - 12*a^2*c^4*d^3*x^4 + 8*I
*a^2*c^3*d^3*x^3 - 8*a^2*c^2*d^3*x^2 + 12*I*a^2*c*d^3*x + 4*a^2*d^3 + (4*a
*b*c^5*d^3*x^5 - 4*(3*I*a*b - b^2)*c^4*d^3*x^4 - 2*(4*a*b + 3*I*b^2)*c^3*d
^3*x^3 - 4*(2*I*a*b + b^2)*c^2*d^3*x^2 - (12*a*b - I*b^2)*c*d^3*x + 4*I*a*
b*d^3)*log(-(c*x + I)/(c*x - I)))/(c^2*x^7 + x^5), x) + (-4*I*b^2*c^3*d^3*
x^3 - 6*b^2*c^2*d^3*x^2 + 4*I*b^2*c*d^3*x + b^2*d^3)*log(-(c*x + I)/(c*x -
 I))^2)/x^4
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^5}\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)**3*(a+b*atan(c*x))**2/x**5,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^5}\,dx = \int \frac{(i\,cdx+d)^3(b\arctan(cx)+a)^2}{x^5}\,dx$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^5,x, algorithm="maxima")
```

output
```
I*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*a*b*c^3*d^3 + 3*((c*
arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*a*b*c^2*d^3 + I*((c^2*log(c^2*x^2
+ 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*a*b*c*d^3 + I*a^2*c^3*
d^3/x + 1/6*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan(c*x)/x
^4)*a*b*d^3 + 1/12*(2*(3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c*arctan(c
*x) - (3*c^2*x^2*arctan(c*x)^2 - 4*c^2*x^2*log(c^2*x^2 + 1) + 8*c^2*x^2*lo
g(x) + 1)*c^2/x^2)*b^2*d^3 + 3/2*a^2*c^2*d^3/x^2 - I*a^2*c*d^3/x^3 - 1/4*b
^2*d^3*arctan(c*x)^2/x^4 - 1/4*a^2*d^3/x^4 - 1/32*(8*I*(b^2*c^4*d^3*arctan
(c*x)^3 + 4*b^2*c^5*d^3*integrate(1/16*x^4*log(c^2*x^2 + 1)^2/(c^2*x^6 + x
^4), x) - 16*b^2*c^5*d^3*integrate(1/16*x^4*log(c^2*x^2 + 1)/(c^2*x^6 + x^
4), x) - 48*b^2*c^4*d^3*integrate(1/16*x^3*arctan(c*x)*log(c^2*x^2 + 1)/(c
^2*x^6 + x^4), x) + 80*b^2*c^4*d^3*integrate(1/16*x^3*arctan(c*x)/(c^2*x^6
 + x^4), x) - 96*b^2*c^3*d^3*integrate(1/16*x^2*arctan(c*x)^2/(c^2*x^6 + x
^4), x) - 8*b^2*c^3*d^3*integrate(1/16*x^2*log(c^2*x^2 + 1)^2/(c^2*x^6 + x
^4), x) + 40*b^2*c^3*d^3*integrate(1/16*x^2*log(c^2*x^2 + 1)/(c^2*x^6 + x^
4), x) - 48*b^2*c^2*d^3*integrate(1/16*x*arctan(c*x)*log(c^2*x^2 + 1)/(c^2
*x^6 + x^4), x) - 32*b^2*c^2*d^3*integrate(1/16*x*arctan(c*x)/(c^2*x^6 + x
^4), x) - 144*b^2*c*d^3*integrate(1/16*arctan(c*x)^2/(c^2*x^6 + x^4), x) -
 12*b^2*c*d^3*integrate(1/16*log(c^2*x^2 + 1)^2/(c^2*x^6 + x^4), x))*x^3 -
 8*(b^2*c^4*d^3*arctan(c*x)^2 - 16*b^2*c^5*d^3*integrate(1/16*x^4*arcta...
```

## Giac [F]

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^5}\,dx=\int \frac{(i\,cdx+d)^3(b\arctan{(cx)}+a)^2}{x^5}\,dx$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^5,x, algorithm="giac")
```

output
```
integrate((I*c*d*x + d)^3*(b*arctan(c*x) + a)^2/x^5, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^5}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^2\,(d+c\,d\,x\,1\mathrm{i})^3}{x^5}\,dx$$

input
```
int(((a + b*atan(c*x))^2*(d + c*d*x*1i)^3)/x^5,x)
```

output
```
int(((a + b*atan(c*x))^2*(d + c*d*x*1i)^3)/x^5, x)
```

## Reduce [F]

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^5}\,dx$$
$$= \frac{d^3\Big(-24atan(cx)\,abcix+12atan(cx)^2\,b^2c^3i\,x^3+22\log(c^2x^2+1)\,b^2c^4x^4-44\log(x)\,b^2c^4x^4+42atan(cx)}{}$$

input
```
int((d+I*c*d*x)^3*(a+b*atan(c*x))^2/x^5,x)
```

output
```
(d**3*(21*atan(c*x)**2*b**2*c**4*x**4 + 12*atan(c*x)**2*b**2*c**3*i*x**3 +
 18*atan(c*x)**2*b**2*c**2*x**2 - 12*atan(c*x)**2*b**2*c*i*x - 3*atan(c*x)
**2*b**2 + 42*atan(c*x)*a*b*c**4*x**4 + 24*atan(c*x)*a*b*c**3*i*x**3 + 36*
atan(c*x)*a*b*c**2*x**2 - 24*atan(c*x)*a*b*c*i*x - 6*atan(c*x)*a*b - 12*at
an(c*x)*b**2*c**4*i*x**4 + 42*atan(c*x)*b**2*c**3*x**3 - 12*atan(c*x)*b**2
*c**2*i*x**2 - 2*atan(c*x)*b**2*c*x - 48*int(atan(c*x)/(c**2*x**3 + x),x)*
b**2*c**4*i*x**4 + 24*log(c**2*x**2 + 1)*a*b*c**4*i*x**4 + 22*log(c**2*x**
2 + 1)*b**2*c**4*x**4 - 48*log(x)*a*b*c**4*i*x**4 - 44*log(x)*b**2*c**4*x*
*4 + 12*a**2*c**3*i*x**3 + 18*a**2*c**2*x**2 - 12*a**2*c*i*x - 3*a**2 + 42
*a*b*c**3*x**3 - 12*a*b*c**2*i*x**2 - 2*a*b*c*x - 12*b**2*c**3*i*x**3 - b*
*2*c**2*x**2))/(12*x**4)
```

## 3.93 $\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^6}\,dx$

## Optimal result

Integrand size = 25, antiderivative size = 384

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^6}\,dx = -\frac{b^2c^2d^3}{30x^3}-\frac{ib^2c^3d^3}{4x^2}+\frac{13b^2c^4d^3}{10x}+\frac{13}{10}b^2c^5d^3\arctan(cx)-\frac{bcd^3(a+b\arctan(cx))}{10x^4}-\frac{ibc^2d^3(a+b\arctan(cx))}{2x^3}+\frac{6bc^3d^3(a+b\arctan(cx))}{5x^2}+\frac{5ibc^4d^3(a+b\arctan(cx))}{2x}-\frac{d^3(1+icx)^4(a+b\arctan(cx))^2}{5x^5}+\frac{icd^3(1+icx)^4(a+b\arctan(cx))^2}{20x^4}+\frac{12}{5}abc^5d^3\log(x)-3ib^2c^5d^3\log(x)+\frac{12}{5}bc^5d^3(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)+\frac{3}{2}ib^2c^5d^3\log\left(1+c^2x^2\right)+\frac{6}{5}ib^2c^5d^3\,\text{PolyLog}(2,-icx)-\frac{6}{5}ib^2c^5d^3\,\text{PolyLog}(2,icx)-\frac{6}{5}ib^2c^5d^3\,\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)$$

output
```
-1/30*b^2*c^2*d^3/x^3-6/5*I*b^2*c^5*d^3*polylog(2,I*c*x)+13/10*b^2*c^4*d^3
/x+13/10*b^2*c^5*d^3*arctan(c*x)-1/10*b*c*d^3*(a+b*arctan(c*x))/x^4-1/4*I*
b^2*c^3*d^3/x^2+6/5*b*c^3*d^3*(a+b*arctan(c*x))/x^2-3*I*b^2*c^5*d^3*ln(x)-
1/5*d^3*(1+I*c*x)^4*(a+b*arctan(c*x))^2/x^5+3/2*I*b^2*c^5*d^3*ln(c^2*x^2+1
)+12/5*a*b*c^5*d^3*ln(x)+6/5*I*b^2*c^5*d^3*polylog(2,-I*c*x)+12/5*b*c^5*d^
3*(a+b*arctan(c*x))*ln(2/(1-I*c*x))-1/2*I*b*c^2*d^3*(a+b*arctan(c*x))/x^3-
6/5*I*b^2*c^5*d^3*polylog(2,1-2/(1-I*c*x))+5/2*I*b*c^4*d^3*(a+b*arctan(c*x
))/x+1/20*I*c*d^3*(1+I*c*x)^4*(a+b*arctan(c*x))^2/x^4
```

## Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 363, normalized size of antiderivative = 0.95

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^6}\,dx$$
$$= \frac{d^3\left(-12a^2-45ia^2cx-6abcx+60a^2c^2x^2-30iabc^2x^2-2b^2c^2x^2+30ia^2c^3x^3+72abc^3x^3-15ib^2c^3x^3+\right.}{}$$

input
```
Integrate[((d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2)/x^6,x]
```

output
```
(d^3*(-12*a^2 - (45*I)*a^2*c*x - 6*a*b*c*x + 60*a^2*c^2*x^2 - (30*I)*a*b*c
^2*x^2 - 2*b^2*c^2*x^2 + (30*I)*a^2*c^3*x^3 + 72*a*b*c^3*x^3 - (15*I)*b^2*
c^3*x^3 + (150*I)*a*b*c^4*x^4 + 78*b^2*c^4*x^4 - (15*I)*b^2*c^5*x^5 + (3*I
)*b^2*(-I + c*x)^4*(4*I + c*x)*ArcTan[c*x]^2 + 6*b*ArcTan[c*x]*(b*c*x*(-1
- (5*I)*c*x + 12*c^2*x^2 + (25*I)*c^3*x^3 + 13*c^4*x^4) + a*(-4 - (15*I)*c
*x + 20*c^2*x^2 + (10*I)*c^3*x^3 + (25*I)*c^5*x^5) + 24*b*c^5*x^5*Log[1 -
E^((2*I)*ArcTan[c*x])]) + 144*a*b*c^5*x^5*Log[c*x] - (180*I)*b^2*c^5*x^5*L
og[(c*x)/Sqrt[1 + c^2*x^2]] - 72*a*b*c^5*x^5*Log[1 + c^2*x^2] - (72*I)*b^2
*c^5*x^5*PolyLog[2, E^((2*I)*ArcTan[c*x])]))/(60*x^5)
```

## Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 362, normalized size of antiderivative = 0.94, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {5409, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^6}\,dx$$
$$\Big\downarrow \; 5409$$

$$-2bc\int\left(\frac{6d^3(a+b\arctan(cx))c^5}{5(cx+i)}-\frac{6d^3(a+b\arctan(cx))c^4}{5x}+\frac{5id^3(a+b\arctan(cx))c^3}{4x^2}+\frac{6d^3(a+b\arctan(cx)}{5x^3}\right.$$
$$\frac{d^3(1+icx)^4(a+b\arctan(cx))^2}{5x^5}+\frac{icd^3(1+icx)^4(a+b\arctan(cx))^2}{20x^4}$$

$\big\downarrow$ 2009

$$-2bc\left(-\frac{6}{5}c^4d^3\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))-\frac{5ic^3d^3(a+b\arctan(cx))}{4x}-\frac{3c^2d^3(a+b\arctan(cx))}{5x^2}+\frac{d^3(a+\right.$$
$$\frac{d^3(1+icx)^4(a+b\arctan(cx))^2}{5x^5}+\frac{icd^3(1+icx)^4(a+b\arctan(cx))^2}{20x^4}$$

input
```
Int[((d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2)/x^6,x]
```

output
```
-1/5*(d^3*(1 + I*c*x)^4*(a + b*ArcTan[c*x])^2)/x^5 + ((I/20)*c*d^3*(1 + I*
c*x)^4*(a + b*ArcTan[c*x])^2)/x^4 - 2*b*c*((b*c*d^3)/(60*x^3) + ((I/8)*b*c
^2*d^3)/x^2 - (13*b*c^3*d^3)/(20*x) - (13*b*c^4*d^3*ArcTan[c*x])/20 + (d^3
*(a + b*ArcTan[c*x]))/(20*x^4) + ((I/4)*c*d^3*(a + b*ArcTan[c*x]))/x^3 - (
3*c^2*d^3*(a + b*ArcTan[c*x]))/(5*x^2) - (((5*I)/4)*c^3*d^3*(a + b*ArcTan[
c*x]))/x - (6*a*c^4*d^3*Log[x])/5 + ((3*I)/2)*b*c^4*d^3*Log[x] - (6*c^4*d^
3*(a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/5 - ((3*I)/4)*b*c^4*d^3*Log[1 +
c^2*x^2] - ((3*I)/5)*b*c^4*d^3*PolyLog[2, (-I)*c*x] + ((3*I)/5)*b*c^4*d^3*
PolyLog[2, I*c*x] + ((3*I)/5)*b*c^4*d^3*PolyLog[2, 1 - 2/(1 - I*c*x)])
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5409
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_.) + (e_
.)*(x_))^(q_), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Sim
p[(a + b*ArcTan[c*x])^p   u, x] - Simp[b*c*p   Int[ExpandIntegrand[(a + b*A
rcTan[c*x])^(p - 1), u/(1 + c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e,
f, q}, x] && IGtQ[p, 1] && EqQ[c^2*d^2 + e^2, 0] && IntegersQ[m, q] && NeQ[
m, -1] && NeQ[q, -1] && ILtQ[m + q + 1, 0] && LtQ[m*q, 0]
```

## Maple [A] (verified)

Time = 2.23 (sec) , antiderivative size = 541, normalized size of antiderivative = 1.41

| method | result |
|---|---|
| parts | $d^3a^2\left(\frac{ic^3}{2x^2}+\frac{c^2}{x^3}-\frac{1}{5x^5}-\frac{3ic}{4x^4}\right)+d^3b^2c^5\left(\frac{\arctan(cx)^2}{c^3x^3}-\frac{6i\,\mathrm{dilog}(-icx+1)}{5}+\frac{6i\,\mathrm{dilog}(icx+1)}{5}-\frac{\mathrm{arcta}}{5c}\right.$ |
| derivativedivides | $c^5\left(d^3a^2\left(\frac{1}{c^3x^3}-\frac{3i}{4c^4x^4}+\frac{i}{2c^2x^2}-\frac{1}{5c^5x^5}\right)+d^3b^2\left(\frac{\arctan(cx)^2}{c^3x^3}-\frac{6i\,\mathrm{dilog}(-icx+1)}{5}+\frac{6i\,\mathrm{dilog}(icx+1)}{5}\right.\right.$ |
| default | $c^5\left(d^3a^2\left(\frac{1}{c^3x^3}-\frac{3i}{4c^4x^4}+\frac{i}{2c^2x^2}-\frac{1}{5c^5x^5}\right)+d^3b^2\left(\frac{\arctan(cx)^2}{c^3x^3}-\frac{6i\,\mathrm{dilog}(-icx+1)}{5}+\frac{6i\,\mathrm{dilog}(icx+1)}{5}\right.\right.$ |

input
```
int((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^6,x,method=_RETURNVERBOSE)
```

output
```
d^3*a^2*(1/2*I*c^3/x^2+c^2/x^3-1/5/x^5-3/4*I*c/x^4)+d^3*b^2*c^5*(1/c^3/x^3
*arctan(c*x)^2-6/5*I*dilog(1-I*c*x)+6/5*I*dilog(1+I*c*x)-1/5*arctan(c*x)^2
/c^5/x^5-6/5*arctan(c*x)*ln(c^2*x^2+1)-3/5*I*(ln(c*x-I)*ln(c^2*x^2+1)-1/2*
ln(c*x-I)^2-dilog(-1/2*I*(c*x+I))-ln(c*x-I)*ln(-1/2*I*(c*x+I)))-3*I*ln(c*x
)+5/4*I*arctan(c*x)^2-1/10/c^4/x^4*arctan(c*x)+6/5/c^2/x^2*arctan(c*x)+12/
5*ln(c*x)*arctan(c*x)-1/4*I/c^2/x^2-1/2*I*arctan(c*x)/c^3/x^3+6/5*I*ln(c*x
)*ln(1+I*c*x)+3/5*I*(ln(c*x+I)*ln(c^2*x^2+1)-1/2*ln(c*x+I)^2-dilog(1/2*I*(
c*x-I))-ln(c*x+I)*ln(1/2*I*(c*x-I)))-6/5*I*ln(c*x)*ln(1-I*c*x)+3/2*I*ln(c^
2*x^2+1)+5/2*I*arctan(c*x)/c/x-3/4*I*arctan(c*x)^2/c^4/x^4+13/10*arctan(c*
x)+1/2*I*arctan(c*x)^2/c^2/x^2-1/30/c^3/x^3+13/10/c/x)+2*d^3*a*b*c^5*(1/c^
3/x^3*arctan(c*x)-3/4*I*arctan(c*x)/c^4/x^4+1/2*I*arctan(c*x)/c^2/x^2-1/5/
c^5/x^5*arctan(c*x)-3/5*ln(c^2*x^2+1)+5/4*I*arctan(c*x)-1/4*I/c^3/x^3+5/4*
I/c/x-1/20/c^4/x^4+3/5/c^2/x^2+6/5*ln(c*x))
```

## Fricas [F]

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^6}\,dx=\int \frac{(i\,cdx+d)^3(b\arctan\left(cx\right)+a)^2}{x^6}\,dx$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^6,x, algorithm="fricas")
```

output
```
1/80*(80*x^5*integral(1/20*(-20*I*a^2*c^5*d^3*x^5 - 60*a^2*c^4*d^3*x^4 + 4
0*I*a^2*c^3*d^3*x^3 - 40*a^2*c^2*d^3*x^2 + 60*I*a^2*c*d^3*x + 20*a^2*d^3 +
 (20*a*b*c^5*d^3*x^5 - 10*(6*I*a*b - b^2)*c^4*d^3*x^4 - 20*(2*a*b + I*b^2)
*c^3*d^3*x^3 - 5*(8*I*a*b + 3*b^2)*c^2*d^3*x^2 - 4*(15*a*b - I*b^2)*c*d^3*
x + 20*I*a*b*d^3)*log(-(c*x + I)/(c*x - I)))/(c^2*x^8 + x^6), x) + (-10*I*
b^2*c^3*d^3*x^3 - 20*b^2*c^2*d^3*x^2 + 15*I*b^2*c*d^3*x + 4*b^2*d^3)*log(-
(c*x + I)/(c*x - I))^2)/x^5
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^6}\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)**3*(a+b*atan(c*x))**2/x**6,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^6}\,dx = \int \frac{(i\,cdx+d)^3(b\arctan(cx)+a)^2}{x^6}\,dx$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^6,x, algorithm="maxima")
```

output
```
I*((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*a*b*c^3*d^3 - ((c^2*log(c^2*
x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*a*b*c^2*d^3 + 1/2*
I*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan(c*x)/x^4)*a*b*c*
d^3 - 1/10*((2*c^4*log(c^2*x^2 + 1) - 2*c^4*log(x^2) - (2*c^2*x^2 - 1)/x^4
)*c + 4*arctan(c*x)/x^5)*a*b*d^3 + 1/2*I*a^2*c^3*d^3/x^2 + a^2*c^2*d^3/x^3
 - 3/4*I*a^2*c*d^3/x^4 - 1/5*a^2*d^3/x^5 - 1/320*(320*I*x^5*integrate(1/80
*(60*(b^2*c^5*d^3*x^5 - 2*b^2*c^3*d^3*x^3 - 3*b^2*c*d^3*x)*arctan(c*x)^2 +
 5*(b^2*c^5*d^3*x^5 - 2*b^2*c^3*d^3*x^3 - 3*b^2*c*d^3*x)*log(c^2*x^2 + 1)^
2 + 2*(30*b^2*c^4*d^3*x^4 - 19*b^2*c^2*d^3*x^2)*arctan(c*x) - (10*b^2*c^5*
d^3*x^5 - 35*b^2*c^3*d^3*x^3 + 4*b^2*c*d^3*x + 20*(3*b^2*c^4*d^3*x^4 + 2*b
^2*c^2*d^3*x^2 - b^2*d^3)*arctan(c*x))*log(c^2*x^2 + 1))/(c^2*x^8 + x^6),
x) + 320*x^5*integrate(1/80*(60*(3*b^2*c^4*d^3*x^4 + 2*b^2*c^2*d^3*x^2 - b
^2*d^3)*arctan(c*x)^2 + 5*(3*b^2*c^4*d^3*x^4 + 2*b^2*c^2*d^3*x^2 - b^2*d^3
)*log(c^2*x^2 + 1)^2 - 2*(10*b^2*c^5*d^3*x^5 - 35*b^2*c^3*d^3*x^3 + 4*b^2*
c*d^3*x)*arctan(c*x) - (30*b^2*c^4*d^3*x^4 - 19*b^2*c^2*d^3*x^2 - 20*(b^2*
c^5*d^3*x^5 - 2*b^2*c^3*d^3*x^3 - 3*b^2*c*d^3*x)*arctan(c*x))*log(c^2*x^2
+ 1))/(c^2*x^8 + x^6), x) - 4*(10*I*b^2*c^3*d^3*x^3 + 20*b^2*c^2*d^3*x^2 -
 15*I*b^2*c*d^3*x - 4*b^2*d^3)*arctan(c*x)^2 + 4*(10*b^2*c^3*d^3*x^3 - 20*
I*b^2*c^2*d^3*x^2 - 15*b^2*c*d^3*x + 4*I*b^2*d^3)*arctan(c*x)*log(c^2*x^2
+ 1) + (10*I*b^2*c^3*d^3*x^3 + 20*b^2*c^2*d^3*x^2 - 15*I*b^2*c*d^3*x - ...
```

## Giac [F]

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^6}\,dx=\int \frac{(i\,cdx+d)^3(b\arctan{(cx)}+a)^2}{x^6}\,dx$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^6,x, algorithm="giac")
```

output
```
integrate((I*c*d*x + d)^3*(b*arctan(c*x) + a)^2/x^6, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^6}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^2\,(d+c\,d\,x\,1\mathrm{i})^3}{x^6}\,dx$$

input
```
int(((a + b*atan(c*x))^2*(d + c*d*x*1i)^3)/x^6,x)
```

output
```
int(((a + b*atan(c*x))^2*(d + c*d*x*1i)^3)/x^6, x)
```

## Reduce [F]

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^6}\,dx = \frac{d^3\left(-90atan(cx)\,abcix+30atan(cx)^2\,b^2c^3i\,x^3+75atan(cx)^2\,b^2c^5i\,x^5-45atan(cx)^2\,b^2cix+150atan(cx)\right.}{}$$

input
```
int((d+I*c*d*x)^3*(a+b*atan(c*x))^2/x^6,x)
```

output
```
(d**3*(75*atan(c*x)**2*b**2*c**5*i*x**5 + 30*atan(c*x)**2*b**2*c**3*i*x**3
 + 60*atan(c*x)**2*b**2*c**2*x**2 - 45*atan(c*x)**2*b**2*c*i*x - 12*atan(c
*x)**2*b**2 + 150*atan(c*x)*a*b*c**5*i*x**5 + 60*atan(c*x)*a*b*c**3*i*x**3
 + 120*atan(c*x)*a*b*c**2*x**2 - 90*atan(c*x)*a*b*c*i*x - 24*atan(c*x)*a*b
 + 78*atan(c*x)*b**2*c**5*x**5 + 150*atan(c*x)*b**2*c**4*i*x**4 + 72*atan(
c*x)*b**2*c**3*x**3 - 30*atan(c*x)*b**2*c**2*i*x**2 - 6*atan(c*x)*b**2*c*x
 + 144*int(atan(c*x)/(c**2*x**3 + x),x)*b**2*c**5*x**5 - 72*log(c**2*x**2
+ 1)*a*b*c**5*x**5 + 90*log(c**2*x**2 + 1)*b**2*c**5*i*x**5 + 144*log(x)*a
*b*c**5*x**5 - 180*log(x)*b**2*c**5*i*x**5 + 30*a**2*c**3*i*x**3 + 60*a**2
*c**2*x**2 - 45*a**2*c*i*x - 12*a**2 + 150*a*b*c**4*i*x**4 + 72*a*b*c**3*x
**3 - 30*a*b*c**2*i*x**2 - 6*a*b*c*x + 78*b**2*c**4*x**4 - 15*b**2*c**3*i*
x**3 - 2*b**2*c**2*x**2))/(60*x**5)
```

# 3.94 $\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^7}\,dx$

## Optimal result

Integrand size = 25, antiderivative size = 513

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^7}\,dx = -\frac{b^2c^2d^3}{60x^4}-\frac{ib^2c^3d^3}{10x^3}+\frac{61b^2c^4d^3}{180x^2}+\frac{37ib^2c^5d^3}{30x}+\frac{37}{30}ib^2c^6d^3\arctan(cx)-\frac{bcd^3(a+b\arctan(cx))}{15x^5}-\frac{3ibc^2d^3(a+b\arctan(cx))}{10x^4}+\frac{11bc^3d^3(a+b\arctan(cx))}{18x^3}+\frac{14ibc^4d^3(a+b\arctan(cx))}{15x^2}-\frac{11bc^5d^3(a+b\arctan(cx))}{6x}-\frac{d^3(a+b\arctan(cx))^2}{6x^6}-\frac{3icd^3(a+b\arctan(cx))^2}{5x^5}+\frac{3c^2d^3(a+b\arctan(cx))^2}{4x^4}+\frac{ic^3d^3(a+b\arctan(cx))^2}{3x^3}+\frac{28}{15}iabc^6d^3\log(x)+\frac{113}{45}b^2c^6d^3\log(x)+\frac{37}{20}ibc^6d^3(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)+\frac{1}{60}ibc^6d^3(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)-\frac{113}{90}b^2c^6d^3\log\left(1+c^2x^2\right)-\frac{14}{15}b^2c^6d^3\,\mathrm{PolyLog}(2,-icx)+\frac{14}{15}b^2c^6d^3\,\mathrm{PolyLog}(2,icx)+\frac{37}{40}b^2c^6d^3\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-icx}\right)-\frac{1}{120}b^2c^6d^3\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+icx}\right)$$

output
```
-1/60*b^2*c^2*d^3/x^4-1/10*I*b^2*c^3*d^3/x^3+61/180*b^2*c^4*d^3/x^2+28/15*
I*a*b*c^6*d^3*ln(x)-3/5*I*c*d^3*(a+b*arctan(c*x))^2/x^5-1/15*b*c*d^3*(a+b*
arctan(c*x))/x^5+1/3*I*c^3*d^3*(a+b*arctan(c*x))^2/x^3+11/18*b*c^3*d^3*(a+
b*arctan(c*x))/x^3+37/30*I*b^2*c^6*d^3*arctan(c*x)-11/6*b*c^5*d^3*(a+b*arc
tan(c*x))/x-1/6*d^3*(a+b*arctan(c*x))^2/x^6+14/15*I*b*c^4*d^3*(a+b*arctan(
c*x))/x^2+3/4*c^2*d^3*(a+b*arctan(c*x))^2/x^4+37/30*I*b^2*c^5*d^3/x+1/60*I
*b*c^6*d^3*(a+b*arctan(c*x))*ln(2/(1+I*c*x))+113/45*b^2*c^6*d^3*ln(x)+37/2
0*I*b*c^6*d^3*(a+b*arctan(c*x))*ln(2/(1-I*c*x))-3/10*I*b*c^2*d^3*(a+b*arct
an(c*x))/x^4-113/90*b^2*c^6*d^3*ln(c^2*x^2+1)-14/15*b^2*c^6*d^3*polylog(2,
-I*c*x)+14/15*b^2*c^6*d^3*polylog(2,I*c*x)+37/40*b^2*c^6*d^3*polylog(2,1-2
/(1-I*c*x))-1/120*b^2*c^6*d^3*polylog(2,1-2/(1+I*c*x))
```

## Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 401, normalized size of antiderivative = 0.78

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^7}\,dx = \frac{d^3\left(-30a^2-108ia^2cx-12abcx+135a^2c^2x^2-54iabc^2x^2-3b^2c^2x^2+60ia^2c^3x^3+110abc^3x^3-18ib^2c^3\right.}{}$$

input
```
Integrate[((d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2)/x^7,x]
```

output
```
(d^3*(-30*a^2 - (108*I)*a^2*c*x - 12*a*b*c*x + 135*a^2*c^2*x^2 - (54*I)*a*
b*c^2*x^2 - 3*b^2*c^2*x^2 + (60*I)*a^2*c^3*x^3 + 110*a*b*c^3*x^3 - (18*I)*
b^2*c^3*x^3 + (168*I)*a*b*c^4*x^4 + 61*b^2*c^4*x^4 - 330*a*b*c^5*x^5 + (22
2*I)*b^2*c^5*x^5 + 64*b^2*c^6*x^6 + 3*b^2*(-I + c*x)^4*(-10 + (4*I)*c*x +
c^2*x^2)*ArcTan[c*x]^2 + 2*b*ArcTan[c*x]*(b*c*x*(-6 - (27*I)*c*x + 55*c^2*
x^2 + (84*I)*c^3*x^3 - 165*c^4*x^4 + (111*I)*c^5*x^5) - 3*a*(10 + (36*I)*c
*x - 45*c^2*x^2 - (20*I)*c^3*x^3 + 55*c^6*x^6) + (168*I)*b*c^6*x^6*Log[1 -
 E^((2*I)*ArcTan[c*x])]) + (336*I)*a*b*c^6*x^6*Log[c*x] + 452*b^2*c^6*x^6*
Log[(c*x)/Sqrt[1 + c^2*x^2]] - (168*I)*a*b*c^6*x^6*Log[1 + c^2*x^2] + 168*
b^2*c^6*x^6*PolyLog[2, E^((2*I)*ArcTan[c*x])]))/(180*x^6)
```

## Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 483, normalized size of antiderivative = 0.94, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {5409, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^7}\,dx$$

$$\Big\downarrow \ 5409$$

$$-2bc\int \Bigg(-\frac{id^3(a+b\arctan(cx))c^6}{120(i-cx)}+\frac{37id^3(a+b\arctan(cx))c^6}{40(cx+i)}-\frac{14id^3(a+b\arctan(cx))c^5}{15x}-\frac{11d^3(a+b\arcta}{12x^2}$$
$$\frac{ic^3d^3(a+b\arctan(cx))^2}{3x^3}+\frac{3c^2d^3(a+b\arctan(cx))^2}{4x^4}-\frac{d^3(a+b\arctan(cx))^2}{6x^6}-\frac{3icd^3(a+b\arctan(cx))^2}{5x^5}$$

$$\Big\downarrow \ 2009$$

$$\frac{ic^3d^3(a+b\arctan(cx))^2}{3x^3}+\frac{3c^2d^3(a+b\arctan(cx))^2}{4x^4}-$$
$$2bc\left(-\frac{37}{40}ic^5d^3\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))-\frac{1}{120}ic^5d^3\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))+\frac{11c^4d^3(a+b\,\mathrm{ar}}{12x}\right.$$
$$\frac{d^3(a+b\arctan(cx))^2}{6x^6}-\frac{3icd^3(a+b\arctan(cx))^2}{5x^5}$$

input
```
Int[((d + I*c*d*x)^3*(a + b*ArcTan[c*x])^2)/x^7,x]
```

output
```
-1/6*(d^3*(a + b*ArcTan[c*x])^2)/x^6 - (((3*I)/5)*c*d^3*(a + b*ArcTan[c*x]
)^2)/x^5 + (3*c^2*d^3*(a + b*ArcTan[c*x])^2)/(4*x^4) + ((I/3)*c^3*d^3*(a +
 b*ArcTan[c*x])^2)/x^3 - 2*b*c*((b*c*d^3)/(120*x^4) + ((I/20)*b*c^2*d^3)/x
^3 - (61*b*c^3*d^3)/(360*x^2) - (((37*I)/60)*b*c^4*d^3)/x - ((37*I)/60)*b*
c^5*d^3*ArcTan[c*x] + (d^3*(a + b*ArcTan[c*x]))/(30*x^5) + (((3*I)/20)*c*d
^3*(a + b*ArcTan[c*x]))/x^4 - (11*c^2*d^3*(a + b*ArcTan[c*x]))/(36*x^3) -
(((7*I)/15)*c^3*d^3*(a + b*ArcTan[c*x]))/x^2 + (11*c^4*d^3*(a + b*ArcTan[c
*x]))/(12*x) - ((14*I)/15)*a*c^5*d^3*Log[x] - (113*b*c^5*d^3*Log[x])/90 -
((37*I)/40)*c^5*d^3*(a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)] - (I/120)*c^5*d
^3*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)] + (113*b*c^5*d^3*Log[1 + c^2*x^2
])/180 + (7*b*c^5*d^3*PolyLog[2, (-I)*c*x])/15 - (7*b*c^5*d^3*PolyLog[2, I
*c*x])/15 - (37*b*c^5*d^3*PolyLog[2, 1 - 2/(1 - I*c*x)])/80 + (b*c^5*d^3*P
olyLog[2, 1 - 2/(1 + I*c*x)])/240)
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5409
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_.) + (e_
.)*(x_))^(q_), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Sim
p[(a + b*ArcTan[c*x])^p   u, x] - Simp[b*c*p   Int[ExpandIntegrand[(a + b*A
rcTan[c*x])^(p - 1), u/(1 + c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e,
f, q}, x] && IGtQ[p, 1] && EqQ[c^2*d^2 + e^2, 0] && IntegersQ[m, q] && NeQ[
m, -1] && NeQ[q, -1] && ILtQ[m + q + 1, 0] && LtQ[m*q, 0]
```

## Maple [A] (verified)

Time = 2.11 (sec) , antiderivative size = 564, normalized size of antiderivative = 1.10

| method | result |
|---|---|
| parts | $d^3a^2\left(\frac{ic^3}{3x^3}-\frac{1}{6x^6}-\frac{3ic}{5x^5}+\frac{3c^2}{4x^4}\right)+d^3b^2c^6\left(-\frac{11\arctan(cx)}{6cx}+\frac{11\arctan(cx)}{18c^3x^3}-\frac{\arctan(cx)}{15c^5x^5}+\frac{37i\,\mathrm{arcta}}{30}\right.$ |
| derivativedivides | $c^6\left(d^3a^2\left(\frac{3}{4c^4x^4}-\frac{3i}{5c^5x^5}+\frac{i}{3c^3x^3}-\frac{1}{6c^6x^6}\right)+d^3b^2\left(-\frac{11\arctan(cx)}{6cx}+\frac{11\arctan(cx)}{18c^3x^3}-\frac{\arctan(cx)}{15c^5x^5}+\right.\right.$ |
| default | $c^6\left(d^3a^2\left(\frac{3}{4c^4x^4}-\frac{3i}{5c^5x^5}+\frac{i}{3c^3x^3}-\frac{1}{6c^6x^6}\right)+d^3b^2\left(-\frac{11\arctan(cx)}{6cx}+\frac{11\arctan(cx)}{18c^3x^3}-\frac{\arctan(cx)}{15c^5x^5}+\right.\right.$ |

input
```
int((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^7,x,method=_RETURNVERBOSE)
```

output
```
d^3*a^2*(1/3*I*c^3/x^3-1/6/x^6-3/5*I*c/x^5+3/4*c^2/x^4)+d^3*b^2*c^6*(-11/6
/c/x*arctan(c*x)+11/18/c^3/x^3*arctan(c*x)-1/15/c^5/x^5*arctan(c*x)-14/15*
dilog(1+I*c*x)+14/15*dilog(1-I*c*x)+113/45*ln(c*x)-1/60/c^4/x^4+61/180/c^2
/x^2-11/12*arctan(c*x)^2-113/90*ln(c^2*x^2+1)+3/4/c^4/x^4*arctan(c*x)^2-14
/15*I*arctan(c*x)*ln(c^2*x^2+1)+28/15*I*arctan(c*x)*ln(c*x)-1/6*arctan(c*x
)^2/c^6/x^6+37/30*I/c/x-1/10*I/c^3/x^3-7/30*ln(c*x-I)^2-7/15*dilog(-1/2*I*
(c*x+I))+7/30*ln(c*x+I)^2+7/15*dilog(1/2*I*(c*x-I))+7/15*ln(c*x-I)*ln(c^2*
x^2+1)-7/15*ln(c*x-I)*ln(-1/2*I*(c*x+I))-7/15*ln(c*x+I)*ln(c^2*x^2+1)+7/15
*ln(c*x+I)*ln(1/2*I*(c*x-I))+37/30*I*arctan(c*x)+1/3*I*arctan(c*x)^2/c^3/x
^3-3/5*I*arctan(c*x)^2/c^5/x^5-3/10*I*arctan(c*x)/c^4/x^4+14/15*I*arctan(c
*x)/c^2/x^2-14/15*ln(c*x)*ln(1+I*c*x)+14/15*ln(c*x)*ln(1-I*c*x))+2*d^3*a*b
*c^6*(3/4/c^4/x^4*arctan(c*x)-3/5*I*arctan(c*x)/c^5/x^5+1/3*I*arctan(c*x)/
c^3/x^3-1/6*arctan(c*x)/c^6/x^6-7/15*I*ln(c^2*x^2+1)-11/12*arctan(c*x)+14/
15*I*ln(c*x)-3/20*I/c^4/x^4+7/15*I/c^2/x^2-1/30/c^5/x^5+11/36/c^3/x^3-11/1
2/c/x)
```

## Fricas [F]

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^7}\,dx = \int \frac{(i\,cdx+d)^3(b\arctan\left(cx\right)+a)^2}{x^7}\,dx$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^7,x, algorithm="fricas")
```

output
```
1/240*(240*x^6*integral(1/60*(-60*I*a^2*c^5*d^3*x^5 - 180*a^2*c^4*d^3*x^4
+ 120*I*a^2*c^3*d^3*x^3 - 120*a^2*c^2*d^3*x^2 + 180*I*a^2*c*d^3*x + 60*a^2
*d^3 + (60*a*b*c^5*d^3*x^5 - 20*(9*I*a*b - b^2)*c^4*d^3*x^4 - 15*(8*a*b +
3*I*b^2)*c^3*d^3*x^3 - 12*(10*I*a*b + 3*b^2)*c^2*d^3*x^2 - 10*(18*a*b - I*
b^2)*c*d^3*x + 60*I*a*b*d^3)*log(-(c*x + I)/(c*x - I)))/(c^2*x^9 + x^7), x
) + (-20*I*b^2*c^3*d^3*x^3 - 45*b^2*c^2*d^3*x^2 + 36*I*b^2*c*d^3*x + 10*b^
2*d^3)*log(-(c*x + I)/(c*x - I))^2)/x^6
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^7}\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)**3*(a+b*atan(c*x))**2/x**7,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^7}\,dx = \int \frac{(i\,cdx+d)^3(b\arctan(cx)+a)^2}{x^7}\,dx$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^7,x, algorithm="maxima")
```

output
```
-1/3*I*((c^2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^
3)*a*b*c^3*d^3 - 1/2*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arct
an(c*x)/x^4)*a*b*c^2*d^3 - 3/10*I*((2*c^4*log(c^2*x^2 + 1) - 2*c^4*log(x^2
) - (2*c^2*x^2 - 1)/x^4)*c + 4*arctan(c*x)/x^5)*a*b*c*d^3 - 1/45*((15*c^5*
arctan(c*x) + (15*c^4*x^4 - 5*c^2*x^2 + 3)/x^5)*c + 15*arctan(c*x)/x^6)*a*
b*d^3 - 1/180*(4*(15*c^5*arctan(c*x) + (15*c^4*x^4 - 5*c^2*x^2 + 3)/x^5)*c
*arctan(c*x) - (30*c^4*x^4*arctan(c*x)^2 - 46*c^4*x^4*log(c^2*x^2 + 1) + 9
2*c^4*x^4*log(x) + 16*c^2*x^2 - 3)*c^2/x^4)*b^2*d^3 + 1/3*I*a^2*c^3*d^3/x^
3 + 3/4*a^2*c^2*d^3/x^4 - 3/5*I*a^2*c*d^3/x^5 - 1/6*b^2*d^3*arctan(c*x)^2/
x^6 - 1/6*a^2*d^3/x^6 - 1/960*(960*I*x^5*integrate(1/240*(180*(b^2*c^5*d^3
*x^4 - 2*b^2*c^3*d^3*x^2 - 3*b^2*c*d^3)*arctan(c*x)^2 + 15*(b^2*c^5*d^3*x^
4 - 2*b^2*c^3*d^3*x^2 - 3*b^2*c*d^3)*log(c^2*x^2 + 1)^2 + 2*(65*b^2*c^4*d^
3*x^3 - 36*b^2*c^2*d^3*x)*arctan(c*x) - (20*b^2*c^5*d^3*x^4 - 81*b^2*c^3*d
^3*x^2 + 180*(b^2*c^4*d^3*x^3 + b^2*c^2*d^3*x)*arctan(c*x))*log(c^2*x^2 +
1))/(c^2*x^8 + x^6), x) + 960*x^5*integrate(1/240*(540*(b^2*c^4*d^3*x^3 +
b^2*c^2*d^3*x)*arctan(c*x)^2 + 45*(b^2*c^4*d^3*x^3 + b^2*c^2*d^3*x)*log(c^
2*x^2 + 1)^2 - 2*(20*b^2*c^5*d^3*x^4 - 81*b^2*c^3*d^3*x^2)*arctan(c*x) - (
65*b^2*c^4*d^3*x^3 - 36*b^2*c^2*d^3*x - 60*(b^2*c^5*d^3*x^4 - 2*b^2*c^3*d^
3*x^2 - 3*b^2*c*d^3)*arctan(c*x))*log(c^2*x^2 + 1))/(c^2*x^8 + x^6), x) -
4*(20*I*b^2*c^3*d^3*x^2 + 45*b^2*c^2*d^3*x - 36*I*b^2*c*d^3)*arctan(c*x...
```

## Giac [F]

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^7}\,dx = \int \frac{(i\,cdx+d)^3(b\arctan(cx)+a)^2}{x^7}\,dx$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^2/x^7,x, algorithm="giac")
```

output
```
integrate((I*c*d*x + d)^3*(b*arctan(c*x) + a)^2/x^7, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^7}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^2\,(d+c\,d\,x\,1\mathrm{i})^3}{x^7}\,dx$$

input
```
int(((a + b*atan(c*x))^2*(d + c*d*x*1i)^3)/x^7,x)
```

output
```
int(((a + b*atan(c*x))^2*(d + c*d*x*1i)^3)/x^7, x)
```

## Reduce [F]

$$\int \frac{(d+icdx)^3(a+b\arctan(cx))^2}{x^7}\,dx$$
$$= \frac{d^3\left(-216atan(cx)\,abcix - 168\log(c^2x^2+1)\,ab\,c^6i\,x^6 + 336\log(x)\,ab\,c^6i\,x^6 + 60atan(cx)^2\,b^2c^3i\,x^3 - 108\right.}{}$$

input
```
int((d+I*c*d*x)^3*(a+b*atan(c*x))^2/x^7,x)
```

output
```
(d**3*( - 165*atan(c*x)**2*b**2*c**6*x**6 + 60*atan(c*x)**2*b**2*c**3*i*x**
*3 + 135*atan(c*x)**2*b**2*c**2*x**2 - 108*atan(c*x)**2*b**2*c*i*x - 30*at
an(c*x)**2*b**2 - 330*atan(c*x)*a*b*c**6*x**6 + 120*atan(c*x)*a*b*c**3*i*x
**3 + 270*atan(c*x)*a*b*c**2*x**2 - 216*atan(c*x)*a*b*c*i*x - 60*atan(c*x)
*a*b + 222*atan(c*x)*b**2*c**6*i*x**6 - 330*atan(c*x)*b**2*c**5*x**5 + 168
*atan(c*x)*b**2*c**4*i*x**4 + 110*atan(c*x)*b**2*c**3*x**3 - 54*atan(c*x)*
b**2*c**2*i*x**2 - 12*atan(c*x)*b**2*c*x + 336*int(atan(c*x)/(c**2*x**3 +
x),x)*b**2*c**6*i*x**6 - 168*log(c**2*x**2 + 1)*a*b*c**6*i*x**6 - 226*log(
c**2*x**2 + 1)*b**2*c**6*x**6 + 336*log(x)*a*b*c**6*i*x**6 + 452*log(x)*b*
*2*c**6*x**6 + 60*a**2*c**3*i*x**3 + 135*a**2*c**2*x**2 - 108*a**2*c*i*x -
 30*a**2 - 330*a*b*c**5*x**5 + 168*a*b*c**4*i*x**4 + 110*a*b*c**3*x**3 - 5
4*a*b*c**2*i*x**2 - 12*a*b*c*x + 222*b**2*c**5*i*x**5 + 61*b**2*c**4*x**4
- 18*b**2*c**3*i*x**3 - 3*b**2*c**2*x**2))/(180*x**6)
```

# 3.95 $\int \frac{x^3(a+b\arctan(cx))^2}{d+icdx}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 356

$$\begin{aligned}\int \frac{x^3(a+b\arctan(cx))^2}{d+icdx}\,dx = &-\frac{abx}{c^3d}-\frac{ib^2x}{3c^3d}+\frac{ib^2\arctan(cx)}{3c^4d}\\ &-\frac{b^2x\arctan(cx)}{c^3d}+\frac{ibx^2(a+b\arctan(cx))}{3c^2d}\\ &-\frac{5(a+b\arctan(cx))^2}{6c^4d}+\frac{ix(a+b\arctan(cx))^2}{c^3d}\\ &+\frac{x^2(a+b\arctan(cx))^2}{2c^2d}-\frac{ix^3(a+b\arctan(cx))^2}{3cd}\\ &+\frac{8ib(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{3c^4d}\\ &+\frac{(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{c^4d}\\ &+\frac{b^2\log(1+c^2x^2)}{2c^4d}-\frac{4b^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{3c^4d}\\ &+\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^4d}\\ &+\frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2c^4d}\end{aligned}$$

output
```
-a*b*x/c^3/d-1/3*I*b^2*x/c^3/d+1/3*I*b^2*arctan(c*x)/c^4/d-b^2*x*arctan(c*
x)/c^3/d+1/3*I*b*x^2*(a+b*arctan(c*x))/c^2/d-5/6*(a+b*arctan(c*x))^2/c^4/d
+I*x*(a+b*arctan(c*x))^2/c^3/d+1/2*x^2*(a+b*arctan(c*x))^2/c^2/d-1/3*I*x^3
*(a+b*arctan(c*x))^2/c/d+8/3*I*b*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^4/d+(
a+b*arctan(c*x))^2*ln(2/(1+I*c*x))/c^4/d+1/2*b^2*ln(c^2*x^2+1)/c^4/d-4/3*b
^2*polylog(2,1-2/(1+I*c*x))/c^4/d+I*b*(a+b*arctan(c*x))*polylog(2,1-2/(1+I
*c*x))/c^4/d+1/2*b^2*polylog(3,1-2/(1+I*c*x))/c^4/d
```

## Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.18

$$\int \frac{x^3(a+b\arctan(cx))^2}{d+icdx}\,dx = \frac{ia^2x}{c^3d}+\frac{a^2x^2}{2c^2d}-\frac{ia^2x^3}{3cd}-\frac{ia^2\arctan(cx)}{c^4d}-\frac{a^2\log(1+c^2x^2)}{2c^4d}$$
$$-\frac{iab\Big(-3icx-8cx\arctan(cx)+6\arctan(cx)^2+(1+c^2x^2)\left(-1+3i\arctan(cx)+2cx\arctan(cx)\right)+6i}{3c^4d}$$
$$-\frac{ib^2\Big(2cx-6icx\arctan(cx)-2(1+c^2x^2)\arctan(cx)+8i\arctan(cx)^2-8cx\arctan(cx)^2+3i(1+c^2x^2}{}$$

input
```
Integrate[(x^3*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x),x]
```

output
```
(I*a^2*x)/(c^3*d) + (a^2*x^2)/(2*c^2*d) - ((I/3)*a^2*x^3)/(c*d) - (I*a^2*A
rcTan[c*x])/(c^4*d) - (a^2*Log[1 + c^2*x^2])/(2*c^4*d) - ((I/3)*a*b*((-3*I
)*c*x - 8*c*x*ArcTan[c*x] + 6*ArcTan[c*x]^2 + (1 + c^2*x^2)*(-1 + (3*I)*Ar
cTan[c*x] + 2*c*x*ArcTan[c*x]) + (6*I)*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan
[c*x])] - 8*Log[1/Sqrt[1 + c^2*x^2]] + 3*PolyLog[2, -E^((2*I)*ArcTan[c*x])
]))/(c^4*d) - ((I/6)*b^2*(2*c*x - (6*I)*c*x*ArcTan[c*x] - 2*(1 + c^2*x^2)*
ArcTan[c*x] + (8*I)*ArcTan[c*x]^2 - 8*c*x*ArcTan[c*x]^2 + (3*I)*(1 + c^2*x
^2)*ArcTan[c*x]^2 + 2*c*x*(1 + c^2*x^2)*ArcTan[c*x]^2 + 4*ArcTan[c*x]^3 -
16*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] + (6*I)*ArcTan[c*x]^2*Log[1
+ E^((2*I)*ArcTan[c*x])] - (6*I)*Log[1/Sqrt[1 + c^2*x^2]] + (8*I + 6*ArcTa
n[c*x])*PolyLog[2, -E^((2*I)*ArcTan[c*x])] + (3*I)*PolyLog[3, -E^((2*I)*Ar
cTan[c*x])]))/(c^4*d)
```

## Rubi [A] (verified)

Time = 4.43 (sec) , antiderivative size = 469, normalized size of antiderivative = 1.32, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {5401, 27, 5361, 5401, 5361, 5401, 5345, 5379, 5451, 2009, 5361, 262, 216, 5419, 5455, 5379, 2849, 2752, 5529, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\arctan(cx))^2}{d+icdx}dx$$

$$\Big\downarrow \text{5401}$$

$$\frac{i\int \frac{x^2(a+b\arctan(cx))^2}{d(icx+1)}dx}{c}-\frac{i\int x^2(a+b\arctan(cx))^2dx}{cd}$$

$$\Big\downarrow \text{27}$$

$$\frac{i\int \frac{x^2(a+b\arctan(cx))^2}{icx+1}dx}{cd}-\frac{i\int x^2(a+b\arctan(cx))^2dx}{cd}$$

$$\Big\downarrow \text{5361}$$

$$\frac{i\int \frac{x^2(a+b\arctan(cx))^2}{icx+1}dx}{cd}-\frac{i\left(\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2}{3}bc\int \frac{x^3(a+b\arctan(cx))}{c^2x^2+1}dx\right)}{cd}$$

$$\Big\downarrow \text{5401}$$

$$\frac{i\left(\frac{i\int \frac{x(a+b\arctan(cx))^2}{icx+1}dx}{c}-\frac{i\int x(a+b\arctan(cx))^2dx}{c}\right)}{cd}-\frac{i\left(\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2}{3}bc\int \frac{x^3(a+b\arctan(cx))}{c^2x^2+1}dx\right)}{cd}$$

$$\Big\downarrow \text{5361}$$

$$\frac{i\left(\frac{i\int \frac{x(a+b\arctan(cx))^2}{icx+1}dx}{c}-\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))^2-bc\int \frac{x^2(a+b\arctan(cx))}{c^2x^2+1}dx\right)}{c}\right)}{cd}-\frac{i\left(\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2}{3}bc\int \frac{x^3(a+b\arctan(cx))}{c^2x^2+1}dx\right)}{cd}$$

$$\Big\downarrow \text{5401}$$

$$\frac{i\left(\frac{i\left(\frac{i\int\frac{(a+b\arctan(cx))^2}{icx+1}dx}{c}-\frac{i\int(a+b\arctan(cx))^2dx}{c}\right)}{c}-\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))^2-bc\int\frac{x^2(a+b\arctan(cx))}{c^2x^2+1}dx\right)}{c}\right)}{cd}-\frac{i\left(\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2}{3}bc\int\frac{x^3(a+b\arctan(cx))}{c^2x^2+1}dx\right)}{cd}$$

$$\Big\downarrow \text{5345}$$

$$\frac{i\left(\frac{i\left(\frac{i\int\frac{(a+b\arctan(cx))^2}{icx+1}dx}{c}-\frac{i\left(x(a+b\arctan(cx))^2-2bc\int\frac{x(a+b\arctan(cx))}{c^2x^2+1}dx\right)}{c}\right)}{c}-\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))^2-bc\int\frac{x^2(a+b\arctan(cx))}{c^2x^2+1}dx\right)}{c}\right)}{cd}-\frac{i\left(\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2}{3}bc\int\frac{x^3(a+b\arctan(cx))}{c^2x^2+1}dx\right)}{cd}$$

$$\Big\downarrow \text{5379}$$

$$\frac{i\left(\frac{i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2ib\int\frac{(a+b\arctan(cx))\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{c}-\frac{i\left(x(a+b\arctan(cx))^2-2bc\int\frac{x(a+b\arctan(cx))}{c^2x^2+1}dx\right)}{c}\right)}{c}-\frac{i\left(\frac{1}{2}x^2(a+b\text{a}\right.}{}}{cd}-\frac{i\left(\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2}{3}bc\int\frac{x^3(a+b\arctan(cx))}{c^2x^2+1}dx\right)}{cd}$$

$$\Big\downarrow \text{5451}$$

$$\frac{i\left(\frac{i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2ib\int\frac{(a+b\arctan(cx))\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{c}-\frac{i\left(x(a+b\arctan(cx))^2-2bc\int\frac{x(a+b\arctan(cx))}{c^2x^2+1}dx\right)}{c}\right)}{c}-\frac{i\left(\frac{1}{2}x^2(a+b\text{a}\right.}{}}{cd}-\frac{i\left(\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2}{3}bc\left(\frac{\int x(a+b\arctan(cx))dx}{c^2}-\frac{\int\frac{x(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)\right)}{cd}$$

$\Big\downarrow$ 2009

$$\frac{i\left(\frac{i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2ib\int\frac{(a+b\arctan(cx))\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{c}-\frac{i\left(x(a+b\arctan(cx))^2-2bc\int\frac{x(a+b\arctan(cx))}{c^2x^2+1}dx\right)}{c}\right)}{c}-\frac{i\left(\frac{1}{2}x^2(a+b\,a\cdots\right.}{\cdots}\right.}{cd}$$

$$\frac{i\left(\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2}{3}bc\left(\frac{\int x(a+b\arctan(cx))dx}{c^2}-\frac{\int\frac{x(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)\right)}{cd}$$

$\Big\downarrow$ 5361

$$\frac{i\left(\frac{i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2ib\int\frac{(a+b\arctan(cx))\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{c}-\frac{i\left(x(a+b\arctan(cx))^2-2bc\int\frac{x(a+b\arctan(cx))}{c^2x^2+1}dx\right)}{c}\right)}{c}-\frac{i\left(\frac{1}{2}x^2(a+b\,a\cdots\right.}{\cdots}\right.}{cd}$$

$$\frac{i\left(\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2}{3}bc\left(\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\int\frac{x^2}{c^2x^2+1}dx}{c^2}-\frac{\int\frac{x(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)\right)}{cd}$$

$\Big\downarrow$ 262

$$\frac{i\left(\frac{i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2ib\int\frac{(a+b\arctan(cx))\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{c}-\frac{i\left(x(a+b\arctan(cx))^2-2bc\int\frac{x(a+b\arctan(cx))}{c^2x^2+1}dx\right)}{c}\right)}{c}-\frac{i\left(\frac{1}{2}x^2(a+b\,a\cdots\right.}{\cdots}\right.}{cd}$$

$$\frac{i\left(\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2}{3}bc\left(\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\int\frac{1}{c^2x^2+1}dx}{c^2}\right)}{c^2}-\frac{\int\frac{x(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)\right)}{cd}$$

$\Big\downarrow$ 216

$$i\left(\frac{i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2ib\int\frac{(a+b\arctan(cx))\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{c}-\frac{i\left(x(a+b\arctan(cx))^2-2bc\int\frac{x(a+b\arctan(cx))}{c^2x^2+1}dx\right)}{c}\right)}{c}-\frac{i\left(\frac{1}{2}x^2(a+b\,a\right.}{}\right.$$

$$\frac{i\left(\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2}{3}bc\left(\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)}{c^2}-\frac{\int\frac{x(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)\right)}{cd}$$

$$\Big\downarrow \text{5419}$$

$$i\left(\frac{i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2ib\int\frac{(a+b\arctan(cx))\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{c}-\frac{i\left(x(a+b\arctan(cx))^2-2bc\int\frac{x(a+b\arctan(cx))}{c^2x^2+1}dx\right)}{c}\right)}{c}-\frac{i\left(\frac{1}{2}x^2(a+b\,a\right.}{}\right.$$

$$\frac{i\left(\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2}{3}bc\left(\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)}{c^2}-\frac{\int\frac{x(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}\right)\right)}{cd}$$

$$\Big\downarrow \text{5455}$$

$$i\left(\frac{i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2ib\int\frac{(a+b\arctan(cx))\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{c}-\frac{i\left(x(a+b\arctan(cx))^2-2bc\left(-\frac{\int\frac{a+b\arctan(cx)}{i-cx}dx}{c}-\frac{i(a+b\arctan(cx))^2}{2bc^2}\right)\right.}{c}\right.}{c}\right.$$

$$\frac{i\left(\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2}{3}bc\left(\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)}{c^2}-\frac{-\frac{\int\frac{a+b\arctan(cx)}{i-cx}dx}{c}-\frac{i(a+b\arctan(cx))^2}{2bc^2}}{c^2}\right)\right)}{cd}$$

$$\Big\downarrow \text{5379}$$

$$i\left(\frac{i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2ib\int\frac{(a+b\arctan(cx))\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{c}-\frac{i\left(x(a+b\arctan(cx))^2-2bc\left(-\frac{\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}-b\int\frac{\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}\cdots}{c}\cdots\right)\right)}{c}\cdots\right)}{c}\cdots}{cd}\cdots\right)$$

$$\frac{i\left(\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2}{3}bc\left(\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)}{c^2}-\frac{-\frac{\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}-b\int\frac{\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx}{c}-\frac{i(\cdots}{\cdots}}{c^2}\cdots\right)\cdots\right)}{cd}$$

$\downarrow$ 2849

$$i\left(\frac{i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2ib\int\frac{(a+b\arctan(cx))\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{c}-\frac{i\left(x(a+b\arctan(cx))^2-2bc\left(-\frac{\frac{ib\int\frac{\log\left(\frac{2}{icx+1}\right)}{1-\frac{2}{icx+1}}d\frac{1}{icx+1}}{c}+\frac{\log\left(\frac{2}{1+icx}\right)(a\cdots}{c}}{c}\cdots\right)\right)}{c}\cdots\right)}{c}\cdots}{cd}\cdots\right)$$

$$\frac{i\left(\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2}{3}bc\left(\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)}{c^2}-\frac{-\frac{\frac{ib\int\frac{\log\left(\frac{2}{icx+1}\right)}{1-\frac{2}{icx+1}}d\frac{1}{icx+1}}{c}+\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx\cdots}{c}}{c}\cdots}{c^2}\cdots\right)\cdots\right)}{cd}$$

$\downarrow$ 2752

$$\frac{i\left(\frac{i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2ib\int\frac{(a+b\arctan(cx))\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{c}-\frac{i\left(x(a+b\arctan(cx))^2-2bc\left(-\frac{i(a+b\arctan(cx))^2}{2bc^2}-\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arct}{c}\right.\right.}{c}}{c}\right.}{cd}$$

$$\frac{i\left(\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2}{3}bc\left(\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)}{c^2}-\frac{-\frac{i(a+b\arctan(cx))^2}{2bc^2}-\frac{\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}+\frac{ib}{}}{c}}{c^2}\right.\right.}{cd}$$

$\downarrow$ 5529

$$\frac{i\left(\frac{i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2ib\left(\frac{1}{2}ib\int\frac{\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{c^2x^2+1}dx-\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{2c}\right)\right)}{c}-\frac{i\left(x(a+b\arctan(cx))^2-2bc\left(-\right.\right.}{}}{c}\right.}{}$$

$$\frac{i\left(\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2}{3}bc\left(\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)}{c^2}-\frac{-\frac{i(a+b\arctan(cx))^2}{2bc^2}-\frac{\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}+\frac{ib}{}}{c}}{c^2}\right.\right.}{cd}$$

$\downarrow$ 7164

$$\frac{i\left(\frac{i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2ib\left(-\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{2c}-\frac{b\,\text{PolyLog}\left(3,1-\frac{2}{icx+1}\right)}{4c}\right)\right)}{c}-\frac{i\left(x(a+b\arctan(cx))^2-2bc\left(-\frac{i(a+b\cdots\right.\right.}{\cdots}}{c}\right.\right.}{\cdots}$$

$$\frac{i\left(\frac{1}{3}x^3(a+b\arctan(cx))^2-\frac{2}{3}bc\left(\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)}{c^2}-\frac{-\frac{i(a+b\arctan(cx))^2}{2bc^2}-\frac{\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}+\frac{ib\cdots}{\cdots}}{c}}{c^2}\cdots\right.\right.}{cd}$$

input
```
Int[(x^3*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x),x]
```

output
```
((-I)*((x^3*(a + b*ArcTan[c*x])^2)/3 - (2*b*c*(((x^2*(a + b*ArcTan[c*x]))/
2 - (b*c*(x/c^2 - ArcTan[c*x]/c^3))/2)/c^2 - (((-1/2*I)*(a + b*ArcTan[c*x]
)^2)/(b*c^2) - (((a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/c + ((I/2)*b*Poly
Log[2, 1 - 2/(1 + I*c*x)])/c)/c)/c^2))/3))/(c*d) + (I*(((-I)*((x^2*(a + b*
ArcTan[c*x])^2)/2 - b*c*(-1/2*(a + b*ArcTan[c*x])^2/(b*c^3) + (a*x + b*x*A
rcTan[c*x] - (b*Log[1 + c^2*x^2])/(2*c))/c^2)))/c + (I*(((-I)*(x*(a + b*Ar
cTan[c*x])^2 - 2*b*c*(((-1/2*I)*(a + b*ArcTan[c*x])^2)/(b*c^2) - (((a + b*
ArcTan[c*x])*Log[2/(1 + I*c*x)])/c + ((I/2)*b*PolyLog[2, 1 - 2/(1 + I*c*x)
])/c)/c)))/c + (I*((I*(a + b*ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/c - (2*I)*
b*(((-1/2*I)*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)])/c - (b*Pol
yLog[3, 1 - 2/(1 + I*c*x)])/(4*c))))/c))/c))/(c*d)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5401
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_) + (
e_.)*(x_)), x_Symbol] :> Simp[f/e   Int[(f*x)^(m - 1)*(a + b*ArcTan[c*x])^p
, x], x] - Simp[d*(f/e)   Int[(f*x)^(m - 1)*((a + b*ArcTan[c*x])^p/(d + e*x
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e
^2, 0] && GtQ[m, 0]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 11.44 (sec) , antiderivative size = 1105, normalized size of antiderivative = 3.10

| method | result | size |
|---|---|---|
| derivativedivides | Expression too large to display | 1105 |
| default | Expression too large to display | 1105 |
| parts | Expression too large to display | 1149 |

input
```
int(x^3*(a+b*arctan(c*x))^2/(d+I*c*d*x),x,method=_RETURNVERBOSE)
```

output
```
1/c^4*(I*a*b/d*ln(c*x-I)*ln(-1/2*I*(c*x+I))+2*I*a*b/d*arctan(c*x)*c*x+1/2*
a^2/d*c^2*x^2-1/2*a^2/d*ln(c^2*x^2+1)+1/3*I*a*b/d*c^2*x^2+b^2/d*(-1/3*I*ar
ctan(c*x)^2*c^3*x^3-1/3*I*(c*x+I)+1/2*c^2*x^2*arctan(c*x)^2-arctan(c*x)^2*
ln(c*x-I)+arctan(c*x)^2*ln(2*I*(1+I*c*x)^2/(c^2*x^2+1))+8/3*I*arctan(c*x)*
ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+I*arctan(c*x)^2*c*x+1/2*polylog(3,-(1+
I*c*x)^2/(c^2*x^2+1))+1/2*I*Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn((1
+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*arctan(c*x)^2-ln(1+(1
+I*c*x)^2/(c^2*x^2+1))-1/2*I*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*
x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*arctan(c*x)^2-1/2*I*Pi*csg
n((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^3*arctan(c*x)^2-2/3
*I*arctan(c*x)^3+11/6*arctan(c*x)^2+8/3*I*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^
2*x^2+1)^(1/2))-1/2*I*Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn((1+I*c*x
)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))
*arctan(c*x)^2+I*Pi*arctan(c*x)^2-I*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^
2*x^2+1))-1/3*I*arctan(c*x)*(c*x-I)^2-1/3*arctan(c*x)*(c*x-I)+2/3*I*arctan
(c*x)*(c*x-I)*(c*x+I)-I*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^
2*x^2+1)))^2*arctan(c*x)^2+8/3*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+8/3*
dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2)))-1/3*I*a^2/d*c^3*x^3-I*a^2/d*arctan
(c*x)+a*b/d*arctan(c*x)*c^2*x^2-2*a*b/d*arctan(c*x)*ln(c*x-I)-5/24*I*a*b/d
*ln(c^4*x^4+10*c^2*x^2+9)-1/2*I*a*b/d*ln(c*x-I)^2-11/12*I*a*b/d*ln(c^2*...
```

## Fricas [F]

$$\int \frac{x^3(a+b\arctan(cx))^2}{d+icdx}\,dx = \int \frac{(b\arctan(cx)+a)^2x^3}{i\,cdx+d}\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))^2/(d+I*c*d*x),x, algorithm="fricas")
```

output
```
integral(1/4*(I*b^2*x^3*log(-(c*x + I)/(c*x - I))^2 + 4*a*b*x^3*log(-(c*x
+ I)/(c*x - I)) - 4*I*a^2*x^3)/(c*d*x - I*d), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\arctan(cx))^2}{d+icdx}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(a+b*atan(c*x))**2/(d+I*c*d*x),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{x^3(a+b\arctan(cx))^2}{d+icdx}\,dx = \int \frac{(b\arctan(cx)+a)^2x^3}{i\,cdx+d}\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))^2/(d+I*c*d*x),x, algorithm="maxima")
```

output
```
-1/6*a^2*(I*(2*c^2*x^3 + 3*I*c*x^2 - 6*x)/(c^3*d) + 6*log(I*c*x + 1)/(c^4*
d)) - 1/96*(16*I*(216*b^2*c^4*integrate(1/48*x^4*arctan(c*x)^2/(c^5*d*x^2
+ c^3*d), x) + 18*b^2*c^4*integrate(1/48*x^4*log(c^2*x^2 + 1)^2/(c^5*d*x^2
 + c^3*d), x) + 576*a*b*c^4*integrate(1/48*x^4*arctan(c*x)/(c^5*d*x^2 + c^
3*d), x) + 24*b^2*c^4*integrate(1/48*x^4*log(c^2*x^2 + 1)/(c^5*d*x^2 + c^3
*d), x) + 72*b^2*c^3*integrate(1/48*x^3*arctan(c*x)*log(c^2*x^2 + 1)/(c^5*
d*x^2 + c^3*d), x) + 24*b^2*c^3*integrate(1/48*x^3*arctan(c*x)/(c^5*d*x^2
+ c^3*d), x) - 36*b^2*c^2*integrate(1/48*x^2*log(c^2*x^2 + 1)/(c^5*d*x^2 +
 c^3*d), x) + 144*b^2*c*integrate(1/48*x*arctan(c*x)/(c^5*d*x^2 + c^3*d),
x) - 36*b^2*integrate(1/48*log(c^2*x^2 + 1)^2/(c^5*d*x^2 + c^3*d), x) - b^
2*arctan(c*x)^3/(c^4*d))*c^4*d - 96*c^4*d*integrate(1/48*(12*(3*b^2*c^2*x^
3 - 2*b^2*x)*arctan(c*x)^2 + 3*(b^2*c^2*x^3 - 2*b^2*x)*log(c^2*x^2 + 1)^2
- 4*(2*b^2*c^3*x^4 - 24*a*b*c^2*x^3 - 3*b^2*c*x^2)*arctan(c*x) - 2*(6*b^2*
c^3*x^4*arctan(c*x) - b^2*c^2*x^3 - 6*b^2*x)*log(c^2*x^2 + 1))/(c^4*d*x^2
+ c^2*d), x) + 24*I*b^2*arctan(c*x)^3 - 3*b^2*log(c^2*x^2 + 1)^3 - 4*(-2*I
*b^2*c^3*x^3 + 3*b^2*c^2*x^2 + 6*I*b^2*c*x)*arctan(c*x)^2 + (-2*I*b^2*c^3*
x^3 + 3*b^2*c^2*x^2 + 6*I*b^2*c*x + 6*I*b^2*arctan(c*x))*log(c^2*x^2 + 1)^
2 - 4*(3*b^2*arctan(c*x)^2 + (2*b^2*c^3*x^3 + 3*I*b^2*c^2*x^2 - 6*b^2*c*x)
*arctan(c*x))*log(c^2*x^2 + 1))/(c^4*d)
```

## Giac [F]

$$\int \frac{x^3(a+b\arctan(cx))^2}{d+icdx}\,dx = \int \frac{(b\arctan(cx)+a)^2x^3}{i\,cdx+d}\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))^2/(d+I*c*d*x),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)^2*x^3/(I*c*d*x + d), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\arctan(cx))^2}{d+icdx}\,dx = \int \frac{x^3\,(a+b\,\mathrm{atan}(c\,x))^2}{d+c\,d\,x\,1\mathrm{i}}\,dx$$

input
```
int((x^3*(a + b*atan(c*x))^2)/(d + c*d*x*1i),x)
```

output
```
int((x^3*(a + b*atan(c*x))^2)/(d + c*d*x*1i), x)
```

## Reduce [F]

$$\int \frac{x^3(a+b\arctan(cx))^2}{d+icdx}\,dx = \frac{12\left(\int \frac{atan(cx)x^3}{cix+1}dx\right)ab\,c^4+6\left(\int \frac{atan(cx)^2x^3}{cix+1}dx\right)b^2c^4-6\log(cix+1)\,a^2-2a^2c^3i\,x^3+3a^2c^2x^2+6a^2cix}{6c^4d}$$

input
```
int(x^3*(a+b*atan(c*x))^2/(d+I*c*d*x),x)
```

output
```
(12*int((atan(c*x)*x**3)/(c*i*x + 1),x)*a*b*c**4 + 6*int((atan(c*x)**2*x**
3)/(c*i*x + 1),x)*b**2*c**4 - 6*log(c*i*x + 1)*a**2 - 2*a**2*c**3*i*x**3 +
 3*a**2*c**2*x**2 + 6*a**2*c*i*x)/(6*c**4*d)
```

# 3.96 $\int \frac{x^2(a+b\arctan(cx))^2}{d+icdx}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 277

$$\begin{aligned}\int \frac{x^2(a+b\arctan(cx))^2}{d+icdx}\,dx &= \frac{iabx}{c^2d}+\frac{ib^2x\arctan(cx)}{c^2d}+\frac{i(a+b\arctan(cx))^2}{2c^3d}\\&\quad+\frac{x(a+b\arctan(cx))^2}{c^2d}-\frac{ix^2(a+b\arctan(cx))^2}{2cd}\\&\quad+\frac{2b(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c^3d}\\&\quad-\frac{i(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{c^3d}\\&\quad-\frac{ib^2\log(1+c^2x^2)}{2c^3d}+\frac{ib^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^3d}\\&\quad+\frac{b(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^3d}\\&\quad-\frac{ib^2\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2c^3d}\end{aligned}$$

output
```
I*a*b*x/c^2/d+I*b^2*x*arctan(c*x)/c^2/d+1/2*I*(a+b*arctan(c*x))^2/c^3/d+x*
(a+b*arctan(c*x))^2/c^2/d-1/2*I*x^2*(a+b*arctan(c*x))^2/c/d+2*b*(a+b*arcta
n(c*x))*ln(2/(1+I*c*x))/c^3/d-I*(a+b*arctan(c*x))^2*ln(2/(1+I*c*x))/c^3/d-
1/2*I*b^2*ln(c^2*x^2+1)/c^3/d+I*b^2*polylog(2,1-2/(1+I*c*x))/c^3/d+b*(a+b*
arctan(c*x))*polylog(2,1-2/(1+I*c*x))/c^3/d-1/2*I*b^2*polylog(3,1-2/(1+I*c
*x))/c^3/d
```

## Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.19

$$\int \frac{x^2(a+b\arctan(cx))^2}{d+icdx}\,dx =$$
$$-\frac{i\left(6ia^2cx-6abcx+3a^2c^2x^2-6ia^2\arctan(cx)+6ab\arctan(cx)+12iabcx\arctan(cx)-6b^2cx\arctan\right.}{}$$

input
```
Integrate[(x^2*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x),x]
```

output
```
((-1/6*I)*((6*I)*a^2*c*x - 6*a*b*c*x + 3*a^2*c^2*x^2 - (6*I)*a^2*ArcTan[c*
x] + 6*a*b*ArcTan[c*x] + (12*I)*a*b*c*x*ArcTan[c*x] - 6*b^2*c*x*ArcTan[c*x
] + 6*a*b*c^2*x^2*ArcTan[c*x] - (12*I)*a*b*ArcTan[c*x]^2 + 9*b^2*ArcTan[c*
x]^2 + (6*I)*b^2*c*x*ArcTan[c*x]^2 + 3*b^2*c^2*x^2*ArcTan[c*x]^2 - (4*I)*b
^2*ArcTan[c*x]^3 + 12*a*b*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] + (12
*I)*b^2*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] + 6*b^2*ArcTan[c*x]^2*L
og[1 + E^((2*I)*ArcTan[c*x])] - 3*a^2*Log[1 + c^2*x^2] - (6*I)*a*b*Log[1 +
 c^2*x^2] + 3*b^2*Log[1 + c^2*x^2] + 6*b*((-I)*a + b - I*b*ArcTan[c*x])*Po
lyLog[2, -E^((2*I)*ArcTan[c*x])] + 3*b^2*PolyLog[3, -E^((2*I)*ArcTan[c*x])
]))/(c^3*d)
```

## Rubi [A] (verified)

Time = 2.72 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.09, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {5401, 27, 5361, 5401, 5345, 5379, 5451, 2009, 5419, 5455, 5379, 2849, 2752, 5529, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\arctan(cx))^2}{d+icdx}\,dx$$
$$\Big\downarrow \; 5401$$
$$\frac{i\int \frac{x(a+b\arctan(cx))^2}{d(icx+1)}dx}{c}-\frac{i\int x(a+b\arctan(cx))^2dx}{cd}$$

$$\Big\downarrow \; 27$$

$$\frac{i\int \frac{x(a+b\arctan(cx))^2}{icx+1}dx}{cd}-\frac{i\int x(a+b\arctan(cx))^2dx}{cd}$$

$$\Big\downarrow \; 5361$$

$$\frac{i\int \frac{x(a+b\arctan(cx))^2}{icx+1}dx}{cd}-\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))^2-bc\int \frac{x^2(a+b\arctan(cx))}{c^2x^2+1}dx\right)}{cd}$$

$$\Big\downarrow \; 5401$$

$$\frac{i\left(\frac{i\int \frac{(a+b\arctan(cx))^2}{icx+1}dx}{c}-\frac{i\int (a+b\arctan(cx))^2dx}{c}\right)}{cd}-\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))^2-bc\int \frac{x^2(a+b\arctan(cx))}{c^2x^2+1}dx\right)}{cd}$$

$$\Big\downarrow \; 5345$$

$$\frac{i\left(\frac{i\int \frac{(a+b\arctan(cx))^2}{icx+1}dx}{c}-\frac{i\left(x(a+b\arctan(cx))^2-2bc\int \frac{x(a+b\arctan(cx))}{c^2x^2+1}dx\right)}{c}\right)}{cd}-\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))^2-bc\int \frac{x^2(a+b\arctan(cx))}{c^2x^2+1}dx\right)}{cd}$$

$$\Big\downarrow \; 5379$$

$$\frac{i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2ib\int \frac{(a+b\arctan(cx))\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{c}-\frac{i\left(x(a+b\arctan(cx))^2-2bc\int \frac{x(a+b\arctan(cx))}{c^2x^2+1}dx\right)}{c}\right)}{cd}-\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))^2-bc\int \frac{x^2(a+b\arctan(cx))}{c^2x^2+1}dx\right)}{cd}$$

$$\Big\downarrow \; 5451$$

$$\frac{i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2ib\int \frac{(a+b\arctan(cx))\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{c}-\frac{i\left(x(a+b\arctan(cx))^2-2bc\int \frac{x(a+b\arctan(cx))}{c^2x^2+1}dx\right)}{c}\right)}{cd}-\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))^2-bc\left(\frac{\int (a+b\arctan(cx))dx}{c^2}-\frac{\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx}{c^2}\right)\right)}{cd}$$

$\downarrow$ 2009

$$\frac{i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2ib\int\frac{(a+b\arctan(cx))\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{c}-\frac{i\left(x(a+b\arctan(cx))^2-2bc\int\frac{x(a+b\arctan(cx))}{c^2x^2+1}dx\right)}{c}\right)}{cd}-\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))^2-bc\left(\frac{ax+bx\arctan(cx)-\frac{b\log\left(c^2x^2+1\right)}{2c}}{c^2}-\frac{\int\frac{a+b\arctan(cx)}{c^2x^2+1}dx}{c^2}\right)\right)}{cd}$$

$\downarrow$ 5419

$$\frac{i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2ib\int\frac{(a+b\arctan(cx))\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{c}-\frac{i\left(x(a+b\arctan(cx))^2-2bc\int\frac{x(a+b\arctan(cx))}{c^2x^2+1}dx\right)}{c}\right)}{cd}-\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))^2-bc\left(\frac{ax+bx\arctan(cx)-\frac{b\log\left(c^2x^2+1\right)}{2c}}{c^2}-\frac{(a+b\arctan(cx))^2}{2bc^3}\right)\right)}{cd}$$

$\downarrow$ 5455

$$\frac{i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2ib\int\frac{(a+b\arctan(cx))\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{c}-\frac{i\left(x(a+b\arctan(cx))^2-2bc\left(-\frac{\int\frac{a+b\arctan(cx)}{i-cx}dx}{c}-\frac{i(a+b\arctan(cx}{2bc^2}\right.\right.}{c}\right.}{cd}-\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))^2-bc\left(\frac{ax+bx\arctan(cx)-\frac{b\log\left(c^2x^2+1\right)}{2c}}{c^2}-\frac{(a+b\arctan(cx))^2}{2bc^3}\right)\right)}{cd}$$

$\downarrow$ 5379

$$\frac{i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2ib\int\frac{(a+b\arctan(cx))\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{c}-\frac{i\left(x(a+b\arctan(cx))^2-2bc\left(-\frac{\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}-b\int\frac{\log}{c}}{c}\right.\right.}{c}\right.}{cd}-\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))^2-bc\left(\frac{ax+bx\arctan(cx)-\frac{b\log\left(c^2x^2+1\right)}{2c}}{c^2}-\frac{(a+b\arctan(cx))^2}{2bc^3}\right)\right)}{cd}$$

$\downarrow$ 2849

$$\frac{i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2ib\int\frac{(a+b\arctan(cx))\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{c}-\frac{i\left(x(a+b\arctan(cx))^2-2bc\left(-\frac{\frac{ib\int\frac{\log\left(\frac{2}{icx+1}\right)}{1-\frac{2}{icx+1}}d\frac{1}{icx+1}}{c}}{c}+\frac{\log\left(\frac{2}{1+icx}\right.}{c}\right.\right.}{c}\right.}{cd}$$

$$\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))^2-bc\left(\frac{ax+bx\arctan(cx)-\frac{b\log(c^2x^2+1)}{2c}}{c^2}-\frac{(a+b\arctan(cx))^2}{2bc^3}\right)\right)}{cd}$$

$\downarrow$ 2752

$$\frac{i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2ib\int\frac{(a+b\arctan(cx))\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{c}-\frac{i\left(x(a+b\arctan(cx))^2-2bc\left(-\frac{i(a+b\arctan(cx))^2}{2bc^2}-\frac{\log\left(\frac{2}{1+icx}\right)(a+}{c}\right.\right.}{c}\right.}{cd}$$

$$\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))^2-bc\left(\frac{ax+bx\arctan(cx)-\frac{b\log(c^2x^2+1)}{2c}}{c^2}-\frac{(a+b\arctan(cx))^2}{2bc^3}\right)\right)}{cd}$$

$\downarrow$ 5529

$$\frac{i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2ib\left(\frac{1}{2}ib\int\frac{\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{c^2x^2+1}dx-\frac{i\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{2c}\right)\right)}{c}-\frac{i\left(x(a+b\arctan(cx))^2-2b\right.}{}\right.}{cd}$$

$$\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))^2-bc\left(\frac{ax+bx\arctan(cx)-\frac{b\log(c^2x^2+1)}{2c}}{c^2}-\frac{(a+b\arctan(cx))^2}{2bc^3}\right)\right)}{cd}$$

$\downarrow$ 7164

$$\frac{i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2ib\left(-\frac{i\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{2c}-\frac{b\operatorname{PolyLog}\left(3,1-\frac{2}{icx+1}\right)}{4c}\right)\right)}{c}-\frac{i\left(x(a+b\arctan(cx))^2-2bc\left(-\right.\right.}{}\right.}{cd}$$

$$\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))^2-bc\left(\frac{ax+bx\arctan(cx)-\frac{b\log\left(c^2x^2+1\right)}{2c}}{c^2}-\frac{(a+b\arctan(cx))^2}{2bc^3}\right)\right)}{cd}$$

input
```
Int[(x^2*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x),x]
```

output
```
((-I)*((x^2*(a + b*ArcTan[c*x])^2)/2 - b*c*(-1/2*(a + b*ArcTan[c*x])^2/(b*
c^3) + (a*x + b*x*ArcTan[c*x] - (b*Log[1 + c^2*x^2])/(2*c))/c^2)))/(c*d) +
 (I*(((-I)*(x*(a + b*ArcTan[c*x])^2 - 2*b*c*(((-1/2*I)*(a + b*ArcTan[c*x])
^2)/(b*c^2) - (((a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/c + ((I/2)*b*PolyL
og[2, 1 - 2/(1 + I*c*x)])/c)/c)))/c + (I*((I*(a + b*ArcTan[c*x])^2*Log[2/(
1 + I*c*x)])/c - (2*I)*b*(((-1/2*I)*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(
1 + I*c*x)])/c - (b*PolyLog[3, 1 - 2/(1 + I*c*x)])/(4*c))))/c))/(c*d)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5401
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_) + (
e_.)*(x_)), x_Symbol] :> Simp[f/e   Int[(f*x)^(m - 1)*(a + b*ArcTan[c*x])^p
, x], x] - Simp[d*(f/e)   Int[(f*x)^(m - 1)*((a + b*ArcTan[c*x])^p/(d + e*x
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e
^2, 0] && GtQ[m, 0]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 9.48 (sec) , antiderivative size = 994, normalized size of antiderivative = 3.59

| method | result | size |
|---|---|---|
| derivativedivides | Expression too large to display | 994 |
| default | Expression too large to display | 994 |
| parts | Expression too large to display | 1038 |

input
```
int(x^2*(a+b*arctan(c*x))^2/(d+I*c*d*x),x,method=_RETURNVERBOSE)
```

output
```
1/c^3*(a^2/d*c*x-3/2*I*a*b/d*arctan(c*x)+1/4*I*a*b/d*arctan(1/6*c^3*x^3+7/
6*c*x)-a^2/d*arctan(c*x)+b^2/d*(c*x*arctan(c*x)^2-2*I*dilog(1+I*(1+I*c*x)/
(c^2*x^2+1)^(1/2))-I*arctan(c*x)^2*ln(2*I*(1+I*c*x)^2/(c^2*x^2+1))-1/2*Pi*
csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+
I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*arctan(c*x)^2+I*arctan(c
*x)*(c*x-I)-Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2
*arctan(c*x)^2-2*I*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+I*arctan(c*x)^2*
ln(c*x-I)-1/2*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))
^3*arctan(c*x)^2+2*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+2*arcta
n(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/2*I*polylog(3,-(1+I*c*x)^2/(c
^2*x^2+1))+Pi*arctan(c*x)^2-3/2*I*arctan(c*x)^2+1/2*Pi*csgn(I/(1+(1+I*c*x)
^2/(c^2*x^2+1)))*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))
^2*arctan(c*x)^2-1/2*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^
2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*arctan(c*x)^2-1/2*I*arctan(c*x)^2*
c^2*x^2-2/3*arctan(c*x)^3-arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))+
I*ln(1+(1+I*c*x)^2/(c^2*x^2+1)))+2*a*b/d*arctan(c*x)*c*x+I*a*b/d*c*x-I*a*b
/d*arctan(c*x)*c^2*x^2+a*b/d*ln(c*x-I)*ln(-1/2*I*(c*x+I))+a*b/d*dilog(-1/2
*I*(c*x+I))-1/2*a*b/d*ln(c*x-I)^2-1/4*I*a*b/d*arctan(1/2*c*x)+a*b/d-1/8*a*
b/d*ln(c^4*x^4+10*c^2*x^2+9)+2*I*a*b/d*arctan(c*x)*ln(c*x-I)-1/2*I*a^2/d*c
^2*x^2+1/2*I*a*b/d*arctan(1/2*c*x-1/2*I)-3/4*a*b/d*ln(c^2*x^2+1)+1/2*I*...
```

## Fricas [F]

$$\int \frac{x^2(a+b\arctan(cx))^2}{d+icdx}\,dx=\int \frac{\left(b\arctan\left(cx\right)+a\right)^2x^2}{i\,cdx+d}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))^2/(d+I*c*d*x),x, algorithm="fricas")
```

output
```
integral(1/4*(I*b^2*x^2*log(-(c*x + I)/(c*x - I))^2 + 4*a*b*x^2*log(-(c*x
+ I)/(c*x - I)) - 4*I*a^2*x^2)/(c*d*x - I*d), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(a + b\arctan(cx))^2}{d + icdx}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(a+b*atan(c*x))**2/(d+I*c*d*x),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{x^2(a + b\arctan(cx))^2}{d + icdx}\,dx = \int \frac{(b\arctan(cx) + a)^2 x^2}{i\,cdx + d}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))^2/(d+I*c*d*x),x, algorithm="maxima")
```

output
```
-1/2*a^2*((I*c*x^2 - 2*x)/(c^2*d) - 2*I*log(I*c*x + 1)/(c^3*d)) - 1/96*(16
*(24*b^2*c^3*integrate(1/16*x^3*arctan(c*x)*log(c^2*x^2 + 1)/(c^4*d*x^2 +
c^2*d), x) + 24*b^2*c^3*integrate(1/16*x^3*arctan(c*x)/(c^4*d*x^2 + c^2*d)
, x) - 72*b^2*c^2*integrate(1/16*x^2*arctan(c*x)^2/(c^4*d*x^2 + c^2*d), x)
 - 6*b^2*c^2*integrate(1/16*x^2*log(c^2*x^2 + 1)^2/(c^4*d*x^2 + c^2*d), x)
 - 192*a*b*c^2*integrate(1/16*x^2*arctan(c*x)/(c^4*d*x^2 + c^2*d), x) - 12
*b^2*c^2*integrate(1/16*x^2*log(c^2*x^2 + 1)/(c^4*d*x^2 + c^2*d), x) + 48*
b^2*c*integrate(1/16*x*arctan(c*x)/(c^4*d*x^2 + c^2*d), x) - 12*b^2*integr
ate(1/16*log(c^2*x^2 + 1)^2/(c^4*d*x^2 + c^2*d), x) - b^2*arctan(c*x)^3/(c
^3*d))*c^3*d + 24*b^2*arctan(c*x)^3 + 96*I*c^3*d*integrate(1/16*(4*(3*b^2*
c^2*x^3 - 2*b^2*x)*arctan(c*x)^2 + (b^2*c^2*x^3 - 2*b^2*x)*log(c^2*x^2 + 1
)^2 + 4*(8*a*b*c^2*x^3 + b^2*c*x^2)*arctan(c*x) + 2*(b^2*c^2*x^3 + 2*b^2*c
*x^2*arctan(c*x) + 2*b^2*x)*log(c^2*x^2 + 1))/(c^3*d*x^2 + c*d), x) + 3*I*
b^2*log(c^2*x^2 + 1)^3 - 12*(-I*b^2*c^2*x^2 + 2*b^2*c*x)*arctan(c*x)^2 + 3
*(-I*b^2*c^2*x^2 + 2*b^2*c*x + 2*b^2*arctan(c*x))*log(c^2*x^2 + 1)^2 - 12*
(-I*b^2*arctan(c*x)^2 + (b^2*c^2*x^2 + 2*I*b^2*c*x)*arctan(c*x))*log(c^2*x
^2 + 1))/(c^3*d)
```

## Giac [F]

$$\int \frac{x^2(a+b\arctan(cx))^2}{d+icdx}\,dx=\int \frac{(b\arctan(cx)+a)^2x^2}{i\,cdx+d}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))^2/(d+I*c*d*x),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)^2*x^2/(I*c*d*x + d), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\arctan(cx))^2}{d+icdx}\,dx=\int \frac{x^2\,(a+b\,\mathrm{atan}(c\,x))^2}{d+c\,d\,x\,1\mathrm{i}}\,dx$$

input
```
int((x^2*(a + b*atan(c*x))^2)/(d + c*d*x*1i),x)
```

output
```
int((x^2*(a + b*atan(c*x))^2)/(d + c*d*x*1i), x)
```

## Reduce [F]

$$\int \frac{x^2(a+b\arctan(cx))^2}{d+icdx}\,dx$$
$$=\frac{4\left(\int \frac{atan(cx)x^2}{cix+1}dx\right)ab\,c^3+2\left(\int \frac{atan(cx)^2x^2}{cix+1}dx\right)b^2c^3+2\log(cix+1)\,a^2i-a^2c^2i\,x^2+2a^2cx}{2c^3d}$$

input
```
int(x^2*(a+b*atan(c*x))^2/(d+I*c*d*x),x)
```

output
```
(4*int((atan(c*x)*x**2)/(c*i*x + 1),x)*a*b*c**3 + 2*int((atan(c*x)**2*x**2
)/(c*i*x + 1),x)*b**2*c**3 + 2*log(c*i*x + 1)*a**2*i - a**2*c**2*i*x**2 +
2*a**2*c*x)/(2*c**3*d)
```

# 3.97 $\int \frac{x(a+b\arctan(cx))^2}{d+icdx}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 192

$$\begin{aligned}\int \frac{x(a+b\arctan(cx))^2}{d+icdx}\,dx =& \frac{(a+b\arctan(cx))^2}{c^2d}-\frac{ix(a+b\arctan(cx))^2}{cd}\\&-\frac{2ib(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c^2d}\\&-\frac{(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{c^2d}\\&+\frac{b^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^2d}\\&-\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^2d}\\&-\frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2c^2d}\end{aligned}$$

output
```
(a+b*arctan(c*x))^2/c^2/d-I*x*(a+b*arctan(c*x))^2/c/d-2*I*b*(a+b*arctan(c*
x))*ln(2/(1+I*c*x))/c^2/d-(a+b*arctan(c*x))^2*ln(2/(1+I*c*x))/c^2/d+b^2*po
lylog(2,1-2/(1+I*c*x))/c^2/d-I*b*(a+b*arctan(c*x))*polylog(2,1-2/(1+I*c*x)
)/c^2/d-1/2*b^2*polylog(3,1-2/(1+I*c*x))/c^2/d
```

## Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.24

$$\int \frac{x(a+b\arctan(cx))^2}{d+icdx}\,dx = -\frac{i\left(6a^2cx-6a^2\arctan(cx)+12abcx\arctan(cx)-12ab\arctan(cx)^2-6ib^2\arctan(cx)^2+6b^2cx\arctan(\right.}{}$$

input
```
Integrate[(x*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x),x]
```

output
```
((-1/6*I)*(6*a^2*c*x - 6*a^2*ArcTan[c*x] + 12*a*b*c*x*ArcTan[c*x] - 12*a*b
*ArcTan[c*x]^2 - (6*I)*b^2*ArcTan[c*x]^2 + 6*b^2*c*x*ArcTan[c*x]^2 - 4*b^2
*ArcTan[c*x]^3 - (12*I)*a*b*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] + 1
2*b^2*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] - (6*I)*b^2*ArcTan[c*x]^2
*Log[1 + E^((2*I)*ArcTan[c*x])] + (3*I)*a^2*Log[1 + c^2*x^2] - 6*a*b*Log[1
 + c^2*x^2] - 6*b*(a + I*b + b*ArcTan[c*x])*PolyLog[2, -E^((2*I)*ArcTan[c*
x])] - (3*I)*b^2*PolyLog[3, -E^((2*I)*ArcTan[c*x])]))/(c^2*d)
```

## Rubi [A] (verified)

Time = 1.44 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {5401, 27, 5345, 5379, 5455, 5379, 2849, 2752, 5529, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\arctan(cx))^2}{d+icdx}\,dx$$

$$\Big\downarrow \ 5401$$

$$\frac{i\int \frac{(a+b\arctan(cx))^2}{d(icx+1)}dx}{c}-\frac{i\int (a+b\arctan(cx))^2dx}{cd}$$

$$\Big\downarrow \ 27$$

$$\frac{i\int \frac{(a+b\arctan(cx))^2}{icx+1}dx}{cd}-\frac{i\int (a+b\arctan(cx))^2dx}{cd}$$

$$\Big\downarrow \text{5345}$$

$$\frac{i\int \frac{(a+b\arctan(cx))^2}{icx+1}dx}{cd}-\frac{i\left(x(a+b\arctan(cx))^2-2bc\int \frac{x(a+b\arctan(cx))}{c^2x^2+1}dx\right)}{cd}$$

$$\Big\downarrow \text{5379}$$

$$\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2ib\int \frac{(a+b\arctan(cx))\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{cd}-\frac{i\left(x(a+b\arctan(cx))^2-2bc\int \frac{x(a+b\arctan(cx))}{c^2x^2+1}dx\right)}{cd}$$

$$\Big\downarrow \text{5455}$$

$$\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2ib\int \frac{(a+b\arctan(cx))\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{cd}-\frac{i\left(x(a+b\arctan(cx))^2-2bc\left(-\frac{\int \frac{a+b\arctan(cx)}{i-cx}dx}{c}-\frac{i(a+b\arctan(cx))^2}{2bc^2}\right)\right)}{cd}$$

$$\Big\downarrow \text{5379}$$

$$\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2ib\int \frac{(a+b\arctan(cx))\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{cd}-\frac{i\left(x(a+b\arctan(cx))^2-2bc\left(-\frac{\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}-b\int \frac{\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx}{c}-\frac{i(a+b\arctan(cx))^2}{2bc^2}\right)\right)}{cd}$$

$$\Big\downarrow \text{2849}$$

$$\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2ib\int \frac{(a+b\arctan(cx))\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{cd}-\frac{i\left(x(a+b\arctan(cx))^2-2bc\left(-\frac{\frac{ib\int \frac{\log\left(\frac{2}{icx+1}\right)}{1-\frac{2}{icx+1}}d\frac{1}{icx+1}}{c}+\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}}{c}-\frac{i(a+b\arctan(cx))^2}{2bc^2}\right)\right)}{cd}$$

$$\Big\downarrow \text{2752}$$

$$\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2ib\int\frac{(a+b\arctan(cx))\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{cd}-\frac{i\left(x(a+b\arctan(cx))^2-2bc\left(-\frac{i(a+b\arctan(cx))^2}{2bc^2}-\frac{\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}+\frac{ib\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{2c}}{c}\right)\right)}{cd}$$

$$\Big\downarrow \; 5529$$

$$\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2ib\left(\frac{1}{2}ib\int\frac{\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{c^2x^2+1}dx-\frac{i\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{2c}\right)\right)}{cd}-\frac{i\left(x(a+b\arctan(cx))^2-2bc\left(-\frac{i(a+b\arctan(cx))^2}{2bc^2}-\frac{\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}+\frac{ib\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{2c}}{c}\right)\right)}{cd}$$

$$\Big\downarrow \; 7164$$

$$\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2ib\left(-\frac{i\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{2c}-\frac{b\operatorname{PolyLog}\left(3,1-\frac{2}{icx+1}\right)}{4c}\right)\right)}{cd}-\frac{i\left(x(a+b\arctan(cx))^2-2bc\left(-\frac{i(a+b\arctan(cx))^2}{2bc^2}-\frac{\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}+\frac{ib\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{2c}}{c}\right)\right)}{cd}$$

input
```
Int[(x*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x),x]
```

output
```
((-I)*(x*(a + b*ArcTan[c*x])^2 - 2*b*c*(((-1/2*I)*(a + b*ArcTan[c*x])^2)/(
b*c^2) - (((a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/c + ((I/2)*b*PolyLog[2,
 1 - 2/(1 + I*c*x)])/c)/c)))/(c*d) + (I*((I*(a + b*ArcTan[c*x])^2*Log[2/(1
 + I*c*x)])/c - (2*I)*b*(((-1/2*I)*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1
 + I*c*x)])/c - (b*PolyLog[3, 1 - 2/(1 + I*c*x)])/(4*c))))/(c*d)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5401
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_) + (
e_.)*(x_)), x_Symbol] :> Simp[f/e   Int[(f*x)^(m - 1)*(a + b*ArcTan[c*x])^p
, x], x] - Simp[d*(f/e)   Int[(f*x)^(m - 1)*((a + b*ArcTan[c*x])^p/(d + e*x
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e
^2, 0] && GtQ[m, 0]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 8.08 (sec) , antiderivative size = 2330, normalized size of antiderivative = 12.14

| method | result | size |
|---|---|---|
| derivativedivides | Expression too large to display | 2330 |
| default | Expression too large to display | 2330 |
| parts | Expression too large to display | 2363 |

input
```
int(x*(a+b*arctan(c*x))^2/(d+I*c*d*x),x,method=_RETURNVERBOSE)
```

output
```
1/c^2*(-I*a^2/d*c*x+1/2*a^2/d*ln(c^2*x^2+1)+I*a^2/d*arctan(c*x)+b^2/d*(1/4
*I*Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn((1+I*c*x)^2/(c^2*x^2+1))*cs
gn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*(2*I*arctan(c*x)*l
n(1+(1+I*c*x)^2/(c^2*x^2+1))+2*arctan(c*x)^2+polylog(2,-(1+I*c*x)^2/(c^2*x
^2+1)))-Pi*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-Pi*arctan(c*x)*
ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+Pi*arctan(c*x)*ln(1+(1+I*c*x)^2/(c^2*x
^2+1))-arctan(c*x)^2-1/2*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))-dilog(1+I*(1+
I*c*x)/(c^2*x^2+1)^(1/2))-dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+I*arctan(
c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))-1/2*polylog(3,-(1+I*c*x)^2/(c^2*x
^2+1))+1/4*I*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^
3*(2*I*arctan(c*x)*ln(1+(1+I*c*x)^2/(c^2*x^2+1))+2*arctan(c*x)^2+polylog(2
,-(1+I*c*x)^2/(c^2*x^2+1)))-I*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)
^2/(c^2*x^2+1)))^2*(I*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+I*ar
ctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+dilog(1+I*(1+I*c*x)/(c^2*x^2
+1)^(1/2))+dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2)))-1/2*I*Pi*csgn((1+I*c*x)
^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^3*(I*arctan(c*x)*ln(1+I*(1+I*c
*x)/(c^2*x^2+1)^(1/2))+I*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+d
ilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2
)))+1/2*I*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*(
2*I*arctan(c*x)*ln(1+(1+I*c*x)^2/(c^2*x^2+1))+2*arctan(c*x)^2+polylog(2...
```

## Fricas [F]

$$\int \frac{x(a+b\arctan(cx))^2}{d+icdx}\,dx = \int \frac{(b\arctan{(cx)}+a)^2x}{i\,cdx+d}\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))^2/(d+I*c*d*x),x, algorithm="fricas")
```

output
```
integral(1/4*(I*b^2*x*log(-(c*x + I)/(c*x - I))^2 + 4*a*b*x*log(-(c*x + I)
/(c*x - I)) - 4*I*a^2*x)/(c*d*x - I*d), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x(a+b\arctan(cx))^2}{d+icdx}\,dx=\text{Timed out}$$

input
```
integrate(x*(a+b*atan(c*x))**2/(d+I*c*d*x),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{x(a+b\arctan(cx))^2}{d+icdx}\,dx=\int \frac{(b\arctan(cx)+a)^2x}{i\,cdx+d}\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))^2/(d+I*c*d*x),x, algorithm="maxima")
```

output
```
a^2*(-I*x/(c*d) + log(I*c*x + 1)/(c^2*d)) + 1/96*(-24*I*b^2*c*x*arctan(c*x
)^2 + 24*I*b^2*arctan(c*x)^3 - 3*b^2*log(c^2*x^2 + 1)^3 - 16*I*(72*b^2*c^2
*integrate(1/16*x^2*arctan(c*x)^2/(c^3*d*x^2 + c*d), x) + 6*b^2*c^2*integr
ate(1/16*x^2*log(c^2*x^2 + 1)^2/(c^3*d*x^2 + c*d), x) + 192*a*b*c^2*integr
ate(1/16*x^2*arctan(c*x)/(c^3*d*x^2 + c*d), x) + 24*b^2*c^2*integrate(1/16
*x^2*log(c^2*x^2 + 1)/(c^3*d*x^2 + c*d), x) + 24*b^2*c*integrate(1/16*x*ar
ctan(c*x)*log(c^2*x^2 + 1)/(c^3*d*x^2 + c*d), x) - 48*b^2*c*integrate(1/16
*x*arctan(c*x)/(c^3*d*x^2 + c*d), x) + 12*b^2*integrate(1/16*log(c^2*x^2 +
 1)^2/(c^3*d*x^2 + c*d), x) + b^2*arctan(c*x)^3/(c^2*d))*c^2*d + 96*c^2*d*
integrate(1/16*(20*b^2*x*arctan(c*x)^2 + 3*b^2*x*log(c^2*x^2 + 1)^2 - 8*(b
^2*c*x^2 - 4*a*b*x)*arctan(c*x) - 4*(b^2*c*x^2*arctan(c*x) + b^2*x)*log(c^
2*x^2 + 1))/(c^2*d*x^2 + d), x) + 6*(I*b^2*c*x + I*b^2*arctan(c*x))*log(c^
2*x^2 + 1)^2 + 12*(2*b^2*c*x*arctan(c*x) - b^2*arctan(c*x)^2)*log(c^2*x^2
+ 1))/(c^2*d)
```

## Giac [F]

$$\int \frac{x(a+b\arctan(cx))^2}{d+icdx}\,dx = \int \frac{(b\arctan(cx)+a)^2x}{i\,cdx+d}\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))^2/(d+I*c*d*x),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)^2*x/(I*c*d*x + d), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\arctan(cx))^2}{d+icdx}\,dx = \int \frac{x\,(a+b\,\mathrm{atan}(c\,x))^2}{d+c\,d\,x\,1\mathrm{i}}\,dx$$

input
```
int((x*(a + b*atan(c*x))^2)/(d + c*d*x*1i),x)
```

output
```
int((x*(a + b*atan(c*x))^2)/(d + c*d*x*1i), x)
```

## Reduce [F]

$$\int \frac{x(a+b\arctan(cx))^2}{d+icdx}\,dx = \frac{2\left(\int \frac{atan(cx)x}{cix+1}dx\right)ab\,c^2+\left(\int \frac{atan(cx)^2x}{cix+1}dx\right)b^2c^2+\log(cix+1)\,a^2-a^2cix}{c^2d}$$

input
```
int(x*(a+b*atan(c*x))^2/(d+I*c*d*x),x)
```

output
```
(2*int((atan(c*x)*x)/(c*i*x + 1),x)*a*b*c**2 + int((atan(c*x)**2*x)/(c*i*x
 + 1),x)*b**2*c**2 + log(c*i*x + 1)*a**2 - a**2*c*i*x)/(c**2*d)
```

# 3.98 $\int \frac{(a+b\arctan(cx))^2}{d+icdx}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 98

$$\int \frac{(a+b\arctan(cx))^2}{d+icdx}\,dx=\frac{i(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{cd}-\frac{b(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{cd}+\frac{ib^2\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2cd}$$

output
```
I*(a+b*arctan(c*x))^2*ln(2/(1+I*c*x))/c/d-b*(a+b*arctan(c*x))*polylog(2,1-
2/(1+I*c*x))/c/d+1/2*I*b^2*polylog(3,1-2/(1+I*c*x))/c/d
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.97

$$\int \frac{(a+b\arctan(cx))^2}{d+icdx}\,dx=\frac{i\left(2(a+b\arctan(cx))^2\log\left(\frac{2d}{d+icdx}\right)+2ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,\frac{i+cx}{-i+cx}\right)+b^2\operatorname{PolyLog}\left(3,\frac{i+cx}{-i+cx}\right)\right)}{2cd}$$

input
```
Integrate[(a + b*ArcTan[c*x])^2/(d + I*c*d*x),x]
```

output
```
((I/2)*(2*(a + b*ArcTan[c*x])^2*Log[(2*d)/(d + I*c*d*x)] + (2*I)*b*(a + b*
ArcTan[c*x])*PolyLog[2, (I + c*x)/(-I + c*x)] + b^2*PolyLog[3, (I + c*x)/(
-I + c*x)]))/(c*d)
```

## Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {5379, 5529, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^2}{d+icdx}\,dx$$

$$\Big\downarrow \text{5379}$$

$$\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{cd}-\frac{2ib\int \frac{(a+b\arctan(cx))\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx}{d}$$

$$\Big\downarrow \text{5529}$$

$$\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{cd}-\frac{2ib\left(\frac{1}{2}ib\int \frac{\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{c^2x^2+1}dx-\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{2c}\right)}{d}$$

$$\Big\downarrow \text{7164}$$

$$\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{cd}-\frac{2ib\left(-\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{2c}-\frac{b\,\text{PolyLog}\left(3,1-\frac{2}{icx+1}\right)}{4c}\right)}{d}$$

input
```
Int[(a + b*ArcTan[c*x])^2/(d + I*c*d*x),x]
```

output
```
(I*(a + b*ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/(c*d) - ((2*I)*b*(((-1/2*I)*(
a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)])/c - (b*PolyLog[3, 1 - 2/
(1 + I*c*x)])/(4*c)))/d
```

**Defintions of rubi rules used**

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 7.78 (sec) , antiderivative size = 852, normalized size of antiderivative = 8.69

| method | result |
|---|---|
| derivativedivides | $-\frac{i a^2 \ln\left(c^2x^2+1\right)}{2d}+\frac{a^2 \arctan(cx)}{d}+\frac{b^2\left(-i\ln(icx+1)\arctan(cx)^2+2i\left(\frac{\arctan(cx)^2\ln\left(\frac{2i(icx+1)^2}{c^2x^2+1}\right)}{2}-\frac{i\arctan(cx)^3}{3}-\frac{i\pi\left(\operatorname{csgn}\left(\cdots\right)\cdots\right)}{\cdots}\cdots\right)\cdots\right)}{\cdots}$ |
| default | $-\frac{i a^2 \ln\left(c^2x^2+1\right)}{2d}+\frac{a^2 \arctan(cx)}{d}+\frac{b^2\left(-i\ln(icx+1)\arctan(cx)^2+2i\left(\frac{\arctan(cx)^2\ln\left(\frac{2i(icx+1)^2}{c^2x^2+1}\right)}{2}-\frac{i\arctan(cx)^3}{3}-\frac{i\pi\left(\operatorname{csgn}\left(\cdots\right)\cdots\right)}{\cdots}\cdots\right)\cdots\right)}{\cdots}$ |
| parts | $-\frac{i a^2 \ln(c^2x^2+1)}{2dc}+\frac{a^2\arctan(cx)}{dc}+\frac{b^2\left(-i\ln(icx+1)\arctan(cx)^2+2i\left(\frac{\arctan(cx)^2\ln\left(\frac{2i(icx+1)^2}{c^2x^2+1}\right)}{2}-\frac{i\arctan(cx)^3}{3}\cdots\right)\cdots\right)}{\cdots}$ |

input
```
int((a+b*arctan(c*x))^2/(d+I*c*d*x),x,method=_RETURNVERBOSE)
```

output
```
1/c*(-1/2*I*a^2/d*ln(c^2*x^2+1)+a^2/d*arctan(c*x)+b^2/d*(-I*ln(1+I*c*x)*ar
ctan(c*x)^2+2*I*(1/2*arctan(c*x)^2*ln(2*I*(1+I*c*x)^2/(c^2*x^2+1))-1/3*I*a
rctan(c*x)^3-1/4*I*Pi*(csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn((1+I*c*x)^
2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))-c
sgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*
x)^2/(c^2*x^2+1)))^2+csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x
^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2-csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*
c*x)^2/(c^2*x^2+1)))^3+csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^
2+1)))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2-csgn(
(1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I*(1+I*c*x)^2/(c
^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))+csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/(1+(
1+I*c*x)^2/(c^2*x^2+1)))^3-csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(
c^2*x^2+1)))^2+1)*arctan(c*x)^2-1/2*I*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(
c^2*x^2+1))+1/4*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))))+2*a*b/d*(-I*ln(1+I*c
*x)*arctan(c*x)-1/2*(ln(1+I*c*x)-ln(1/2+1/2*I*c*x))*ln(1/2-1/2*I*c*x)+1/2*
dilog(1/2+1/2*I*c*x)+1/4*ln(1+I*c*x)^2))
```

## Fricas [F]

$$\int \frac{(a+b\arctan(cx))^2}{d+icdx}\,dx = \int \frac{(b\arctan(cx)+a)^2}{i\,cdx+d}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/(d+I*c*d*x),x, algorithm="fricas")
```

output
```
integral(1/4*(I*b^2*log(-(c*x + I)/(c*x - I))^2 + 4*a*b*log(-(c*x + I)/(c*
x - I)) - 4*I*a^2)/(c*d*x - I*d), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{d+icdx}\,dx = \text{Timed out}$$

input
```
integrate((a+b*atan(c*x))**2/(d+I*c*d*x),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(a+b\arctan(cx))^2}{d+icdx}\,dx = \int \frac{(b\arctan(cx)+a)^2}{i\,cdx+d}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/(d+I*c*d*x),x, algorithm="maxima")
```

output
```
-I*a^2*log(I*c*d*x + d)/(c*d) + 1/96*(24*b^2*arctan(c*x)^3 + 12*I*b^2*arct
an(c*x)^2*log(c^2*x^2 + 1) + 6*b^2*arctan(c*x)*log(c^2*x^2 + 1)^2 + 3*I*b^
2*log(c^2*x^2 + 1)^3 - 8*(48*b^2*c*integrate(1/16*x*arctan(c*x)*log(c^2*x^
2 + 1)/(c^2*d*x^2 + d), x) - b^2*arctan(c*x)^3/(c*d) + 12*b^2*integrate(1/
16*log(c^2*x^2 + 1)^2/(c^2*d*x^2 + d), x) - 12*a*b*arctan(c*x)^2/(c*d))*c*
d - 96*I*c*d*integrate(1/16*(20*b^2*c*x*arctan(c*x)^2 + 3*b^2*c*x*log(c^2*
x^2 + 1)^2 + 32*a*b*c*x*arctan(c*x) + 4*b^2*arctan(c*x)*log(c^2*x^2 + 1))/
(c^2*d*x^2 + d), x))/(c*d)
```

## Giac [F]

$$\int \frac{(a+b\arctan(cx))^2}{d+icdx}\,dx=\int \frac{(b\arctan(cx)+a)^2}{i\,cdx+d}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/(d+I*c*d*x),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)^2/(I*c*d*x + d), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{d+icdx}\,dx=\int \frac{(a+b\,\mathrm{atan}(c\,x))^2}{d+c\,d\,x\,1\mathrm{i}}\,dx$$

input
```
int((a + b*atan(c*x))^2/(d + c*d*x*1i),x)
```

output
```
int((a + b*atan(c*x))^2/(d + c*d*x*1i), x)
```

## Reduce [F]

$$\int \frac{(a+b\arctan(cx))^2}{d+icdx}\,dx = \frac{2\left(\int \frac{atan(cx)}{cix+1}dx\right)abc+\left(\int \frac{atan(cx)^2}{cix+1}dx\right)b^2c-\log(cix+1)\,a^2i}{cd}$$

input
```
int((a+b*atan(c*x))^2/(d+I*c*d*x),x)
```

output
```
(2*int(atan(c*x)/(c*i*x + 1),x)*a*b*c + int(atan(c*x)**2/(c*i*x + 1),x)*b*
*2*c - log(c*i*x + 1)*a**2*i)/(c*d)
```

# 3.99 $\int \frac{(a+b\arctan(cx))^2}{x(d+icdx)}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 88

$$\int \frac{(a+b\arctan(cx))^2}{x(d+icdx)}\,dx = \frac{(a+b\arctan(cx))^2\log\left(2-\frac{2}{1+icx}\right)}{d} + \frac{ib(a+b\arctan(cx))\,\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)}{d} + \frac{b^2\,\text{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)}{2d}$$

output
```
(a+b*arctan(c*x))^2*ln(2-2/(1+I*c*x))/d+I*b*(a+b*arctan(c*x))*polylog(2,-1
+2/(1+I*c*x))/d+1/2*b^2*polylog(3,-1+2/(1+I*c*x))/d
```

## Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.97

$$\int \frac{(a+b\arctan(cx))^2}{x(d+icdx)}\,dx = -\frac{i\left(b^2\pi^3+24a^2\arctan(cx)+48ab\arctan(cx)^2+24ib^2\arctan(cx)^2\log\left(1-e^{-2i\arctan(cx)}\right)+48iab\,\text{arcta}\right.}{}$$

input
```
Integrate[(a + b*ArcTan[c*x])^2/(x*(d + I*c*d*x)),x]
```

output
```
((-1/24*I)*(b^2*Pi^3 + 24*a^2*ArcTan[c*x] + 48*a*b*ArcTan[c*x]^2 + (24*I)*
b^2*ArcTan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] + (48*I)*a*b*ArcTan[c*x]
*Log[1 - E^((2*I)*ArcTan[c*x])] + (24*I)*a^2*Log[c*x] - (12*I)*a^2*Log[1 +
 c^2*x^2] - 24*b^2*ArcTan[c*x]*PolyLog[2, E^((-2*I)*ArcTan[c*x])] + 24*a*b
*PolyLog[2, E^((2*I)*ArcTan[c*x])] + (12*I)*b^2*PolyLog[3, E^((-2*I)*ArcTa
n[c*x])]))/d
```

## Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {5403, 5529, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^2}{x(d+icdx)}\,dx$$

$$\Big\downarrow \text{5403}$$

$$\frac{\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{d}-\frac{2bc\int \frac{(a+b\arctan(cx))\log\left(2-\frac{2}{icx+1}\right)}{c^2x^2+1}dx}{d}$$

$$\Big\downarrow \text{5529}$$

$$\frac{\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{d}-\frac{2bc\left(\frac{1}{2}ib\int \frac{\text{PolyLog}\left(2,\frac{2}{icx+1}-1\right)}{c^2x^2+1}dx-\frac{i\,\text{PolyLog}\left(2,\frac{2}{icx+1}-1\right)(a+b\arctan(cx))}{2c}\right)}{d}$$

$$\Big\downarrow \text{7164}$$

$$\frac{\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{d}-\frac{2bc\left(-\frac{i\,\text{PolyLog}\left(2,\frac{2}{icx+1}-1\right)(a+b\arctan(cx))}{2c}-\frac{b\,\text{PolyLog}\left(3,\frac{2}{icx+1}-1\right)}{4c}\right)}{d}$$

input
```
Int[(a + b*ArcTan[c*x])^2/(x*(d + I*c*d*x)),x]
```

output
```
((a + b*ArcTan[c*x])^2*Log[2 - 2/(1 + I*c*x)])/d - (2*b*c*(((-1/2*I)*(a +
b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)])/c - (b*PolyLog[3, -1 + 2/(1
 + I*c*x)])/(4*c)))/d
```

**Defintions of rubi rules used**

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 9.40 (sec) , antiderivative size = 1447, normalized size of antiderivative = 16.44

| method | result | size |
|---|---|---|
| parts | Expression too large to display | 1447 |
| derivativedivides | Expression too large to display | 1449 |
| default | Expression too large to display | 1449 |

input
```
int((a+b*arctan(c*x))^2/x/(d+I*c*d*x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^2/d*ln(c^2*x^2+1)-I*a^2/d*arctan(c*x)+a^2/d*ln(x)+b^2/d*(ln(c*x)*ar
ctan(c*x)^2-ln(c*x-I)*arctan(c*x)^2+arctan(c*x)^2*ln(2*I*(1+I*c*x)^2/(c^2*
x^2+1))-2/3*I*arctan(c*x)^3-arctan(c*x)^2*ln((1+I*c*x)^2/(c^2*x^2+1)-1)+ar
ctan(c*x)^2*ln(1-(1+I*c*x)/(c^2*x^2+1)^(1/2))-2*I*arctan(c*x)*polylog(2,(1
+I*c*x)/(c^2*x^2+1)^(1/2))+2*polylog(3,(1+I*c*x)/(c^2*x^2+1)^(1/2))+arctan
(c*x)^2*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))-2*I*arctan(c*x)*polylog(2,-(1+I*
c*x)/(c^2*x^2+1)^(1/2))+2*polylog(3,-(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/2*I*Pi
*(csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^3-csgn(I/(1+(1
+I*c*x)^2/(c^2*x^2+1)))*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x
^2+1)))^2+csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1
+I*c*x)^2/(c^2*x^2+1)))^2+csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn((1+I*c*
x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1))
)-csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*
csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))+csgn(I*((1
+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^
2/(c^2*x^2+1)))^2+csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I*((1+I*c*x)^2/
(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2-csgn(I*((1+I*c*x)^2/(c^2*x^2
+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^3+csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(
1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^
2/(c^2*x^2+1)))^2-csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*...
```

## Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.55

$$\int \frac{(a+b\arctan(cx))^2}{x(d+icdx)}\,dx = -\frac{b^2\log\left(\frac{2\,cx}{cx-i}\right)\log\left(-\frac{cx+i}{cx-i}\right)^2+2\,b^2\mathrm{Li}_2\left(-\frac{2\,cx}{cx-i}+1\right)\log\left(-\frac{cx+i}{cx-i}\right)+4i\,ab\mathrm{Li}_2\left(\frac{cx+i}{cx-i}+1\right)-4\,a^2\log\left(x\right)+4\,a^2\,\mathrm{l}}{4\,d}$$

input
```
integrate((a+b*arctan(c*x))^2/x/(d+I*c*d*x),x, algorithm="fricas")
```

output
```
-1/4*(b^2*log(2*c*x/(c*x - I))*log(-(c*x + I)/(c*x - I))^2 + 2*b^2*dilog(-
2*c*x/(c*x - I) + 1)*log(-(c*x + I)/(c*x - I)) + 4*I*a*b*dilog((c*x + I)/(
c*x - I) + 1) - 4*a^2*log(x) + 4*a^2*log((c*x - I)/c) - 2*b^2*polylog(3, -
(c*x + I)/(c*x - I)))/d
```

## Sympy [F]

$$\int \frac{(a+b\arctan(cx))^2}{x(d+icdx)}\,dx = -\frac{i\left(\int \frac{a^2}{cx^2-ix}\,dx + \int \frac{b^2\operatorname{atan}^2(cx)}{cx^2-ix}\,dx + \int \frac{2ab\operatorname{atan}(cx)}{cx^2-ix}\,dx\right)}{d}$$

input
```
integrate((a+b*atan(c*x))**2/x/(d+I*c*d*x),x)
```

output
```
-I*(Integral(a**2/(c*x**2 - I*x), x) + Integral(b**2*atan(c*x)**2/(c*x**2
- I*x), x) + Integral(2*a*b*atan(c*x)/(c*x**2 - I*x), x))/d
```

## Maxima [F]

$$\int \frac{(a+b\arctan(cx))^2}{x(d+icdx)}\,dx = \int \frac{(b\arctan(cx)+a)^2}{(i\,cdx+d)x}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x/(d+I*c*d*x),x, algorithm="maxima")
```

output
```
-a^2*(log(I*c*x + 1)/d - log(x)/d) + 1/96*(-24*I*b^2*arctan(c*x)^3 + 12*b^
2*arctan(c*x)^2*log(c^2*x^2 + 1) - 6*I*b^2*arctan(c*x)*log(c^2*x^2 + 1)^2
+ 3*b^2*log(c^2*x^2 + 1)^3 - 2*(384*b^2*c^2*integrate(1/16*x^2*arctan(c*x)
^2/(c^2*d*x^3 + d*x), x) + 192*b^2*c*integrate(1/16*x*arctan(c*x)*log(c^2*
x^2 + 1)/(c^2*d*x^3 + d*x), x) + b^2*log(c^2*x^2 + 1)^3/d - 576*b^2*integr
ate(1/16*arctan(c*x)^2/(c^2*d*x^3 + d*x), x) - 48*b^2*integrate(1/16*log(c
^2*x^2 + 1)^2/(c^2*d*x^3 + d*x), x) - 1536*a*b*integrate(1/16*arctan(c*x)/
(c^2*d*x^3 + d*x), x))*d - 8*I*(b^2*arctan(c*x)^3/d - 12*b^2*c*integrate(1
/16*x*log(c^2*x^2 + 1)^2/(c^2*d*x^3 + d*x), x) + 12*a*b*arctan(c*x)^2/d +
48*b^2*integrate(1/16*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*d*x^3 + d*x), x))*
d)/d
```

## Giac [F]

$$\int \frac{(a+b\arctan(cx))^2}{x(d+icdx)}\,dx = \int \frac{(b\arctan(cx)+a)^2}{(i\,cdx+d)x}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x/(d+I*c*d*x),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)^2/((I*c*d*x + d)*x), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{x(d+icdx)}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^2}{x\,(d+c\,d\,x\,1\mathrm{i})}\,dx$$

input
```
int((a + b*atan(c*x))^2/(x*(d + c*d*x*1i)),x)
```

output
```
int((a + b*atan(c*x))^2/(x*(d + c*d*x*1i)), x)
```

## Reduce [F]

$$\int \frac{(a+b\arctan(cx))^2}{x(d+icdx)}\,dx$$
$$= \frac{-atan(cx)^3\,b^2i - 3atan(cx)^2\,abi - 6\left(\int \frac{atan(cx)}{c^3x^4-c^2i\,x^3+c\,x^2-ix}dx\right)abi + 6\left(\int \frac{atan(cx)}{c^3x^3-c^2i\,x^2+cx-i}dx\right)abc - 3\left(\int \frac{}{c^3x}\right.}{3d}$$

input
```
int((a+b*atan(c*x))^2/x/(d+I*c*d*x),x)
```

output

```
( - atan(c*x)**3*b**2*i - 3*atan(c*x)**2*a*b*i - 6*int(atan(c*x)/(c**3*x**
4 - c**2*i*x**3 + c*x**2 - i*x),x)*a*b*i + 6*int(atan(c*x)/(c**3*x**3 - c*
*2*i*x**2 + c*x - i),x)*a*b*c - 3*int(atan(c*x)**2/(c**3*x**4 - c**2*i*x**
3 + c*x**2 - i*x),x)*b**2*i + 3*int(atan(c*x)**2/(c**3*x**3 - c**2*i*x**2
+ c*x - i),x)*b**2*c - 3*log(c*i*x + 1)*a**2 + 3*log(x)*a**2)/(3*d)
```

# 3.100 $\int \frac{(a+b\arctan(cx))^2}{x^2(d+icdx)}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 186

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+icdx)}\,dx = -\frac{ic(a+b\arctan(cx))^2}{d}-\frac{(a+b\arctan(cx))^2}{dx}+\frac{2bc(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right)}{d}-\frac{ic(a+b\arctan(cx))^2\log\left(2-\frac{2}{1+icx}\right)}{d}-\frac{ib^2c\,\text{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)}{d}+\frac{bc(a+b\arctan(cx))\,\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)}{d}-\frac{ib^2c\,\text{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)}{2d}$$

output
```
-I*c*(a+b*arctan(c*x))^2/d-(a+b*arctan(c*x))^2/d/x+2*b*c*(a+b*arctan(c*x))
*ln(2-2/(1-I*c*x))/d-I*c*(a+b*arctan(c*x))^2*ln(2-2/(1+I*c*x))/d-I*b^2*c*p
olylog(2,-1+2/(1-I*c*x))/d+b*c*(a+b*arctan(c*x))*polylog(2,-1+2/(1+I*c*x))
/d-1/2*I*b^2*c*polylog(3,-1+2/(1+I*c*x))/d
```

## Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.42

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+icdx)}\,dx =$$

$$-\frac{\frac{2a^2}{x}+2a^2c\arctan(cx)+2ia^2c\log(x)-ia^2c\log\left(1+c^2x^2\right)+2abc\Big(2\Big(\arctan(cx)^2+\arctan(cx)\left(\frac{1}{cx}+i\right.}{}$$

input
```
Integrate[(a + b*ArcTan[c*x])^2/(x^2*(d + I*c*d*x)),x]
```

output
```
-1/2*((2*a^2)/x + 2*a^2*c*ArcTan[c*x] + (2*I)*a^2*c*Log[x] - I*a^2*c*Log[1
 + c^2*x^2] + 2*a*b*c*(2*(ArcTan[c*x]^2 + ArcTan[c*x]*(1/(c*x) + I*Log[1 -
 E^((2*I)*ArcTan[c*x])]) - Log[(c*x)/Sqrt[1 + c^2*x^2]]) + PolyLog[2, E^((
2*I)*ArcTan[c*x])]) + (2*I)*b^2*c*((-1/24*I)*Pi^3 + ArcTan[c*x]^2 - (I*Arc
Tan[c*x]^2)/(c*x) + ArcTan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] + (2*I)*
ArcTan[c*x]*Log[1 - E^((2*I)*ArcTan[c*x])] + I*ArcTan[c*x]*PolyLog[2, E^((
-2*I)*ArcTan[c*x])] + PolyLog[2, E^((2*I)*ArcTan[c*x])] + PolyLog[3, E^((-
2*I)*ArcTan[c*x])]/2))/d
```

## Rubi [A] (verified)

Time = 1.38 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {5405, 27, 5361, 5403, 5459, 5403, 2897, 5529, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+icdx)}\,dx$$

$$\Big\downarrow \text{ 5405}$$

$$\frac{\int \frac{(a+b\arctan(cx))^2}{x^2}dx}{d}-ic\int \frac{(a+b\arctan(cx))^2}{dx(icx+1)}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{(a+b\arctan(cx))^2}{x^2}dx}{d}-\frac{ic\int \frac{(a+b\arctan(cx))^2}{x(icx+1)}dx}{d}$$

$\Big\downarrow$ 5361

$$\frac{2bc\int \frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx-\frac{(a+b\arctan(cx))^2}{x}}{d}-\frac{ic\int \frac{(a+b\arctan(cx))^2}{x(icx+1)}dx}{d}$$

$\Big\downarrow$ 5403

$$\frac{2bc\int \frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx-\frac{(a+b\arctan(cx))^2}{x}}{d}-\frac{ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2-2bc\int \frac{(a+b\arctan(cx))\log\left(2-\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{d}$$

$\Big\downarrow$ 5459

$$\frac{-\frac{(a+b\arctan(cx))^2}{x}+2bc\left(i\int \frac{a+b\arctan(cx)}{x(cx+i)}dx-\frac{i(a+b\arctan(cx))^2}{2b}\right)}{d}-\frac{ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2-2bc\int \frac{(a+b\arctan(cx))\log\left(2-\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{d}$$

$\Big\downarrow$ 5403

$$\frac{-\frac{(a+b\arctan(cx))^2}{x}+2bc\left(i\left(ibc\int \frac{\log\left(2-\frac{2}{1-icx}\right)}{c^2x^2+1}dx-i\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))\right)-\frac{i(a+b\arctan(cx))^2}{2b}\right)}{d}-\frac{ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2-2bc\int \frac{(a+b\arctan(cx))\log\left(2-\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{d}$$

$\Big\downarrow$ 2897

$$\frac{-\frac{(a+b\arctan(cx))^2}{x}+2bc\left(i\left(-i\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{2}{1-icx}-1\right)\right)-\frac{i(a+b\arctan(cx))^2}{2b}\right)}{d}-\frac{ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2-2bc\int \frac{(a+b\arctan(cx))\log\left(2-\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{d}$$

$\Big\downarrow$ 5529

$$\frac{-\frac{(a+b\arctan(cx))^2}{x}+2bc\left(i\left(-i\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))-\frac{1}{2}b\operatorname{PolyLog}\left(2,\frac{2}{1-icx}-1\right)\right)-\frac{i(a+b\arctan(cx))^2}{2b}\right)}{d}$$

$$\frac{ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2-2bc\left(\frac{1}{2}ib\int\frac{\operatorname{PolyLog}\left(2,\frac{2}{icx+1}-1\right)}{c^2x^2+1}dx-\frac{i\operatorname{PolyLog}\left(2,\frac{2}{icx+1}-1\right)(a+b\arctan(cx))}{2c}\right)\right)}{d}$$

$\Big\downarrow$ 7164

$$\frac{-\frac{(a+b\arctan(cx))^2}{x}+2bc\left(i\left(-i\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))-\frac{1}{2}b\operatorname{PolyLog}\left(2,\frac{2}{1-icx}-1\right)\right)-\frac{i(a+b\arctan(cx))^2}{2b}\right)}{d}$$

$$\frac{ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2-2bc\left(-\frac{i\operatorname{PolyLog}\left(2,\frac{2}{icx+1}-1\right)(a+b\arctan(cx))}{2c}-\frac{b\operatorname{PolyLog}\left(3,\frac{2}{icx+1}-1\right)}{4c}\right)\right)}{d}$$

input
```
Int[(a + b*ArcTan[c*x])^2/(x^2*(d + I*c*d*x)),x]
```

output
```
(-((a + b*ArcTan[c*x])^2/x) + 2*b*c*(((-1/2*I)*(a + b*ArcTan[c*x])^2)/b +
I*((-I)*(a + b*ArcTan[c*x])*Log[2 - 2/(1 - I*c*x)] - (b*PolyLog[2, -1 + 2/
(1 - I*c*x)])/2)))/d - (I*c*((a + b*ArcTan[c*x])^2*Log[2 - 2/(1 + I*c*x)]
- 2*b*c*(((-1/2*I)*(a + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)])/c -
 (b*PolyLog[3, -1 + 2/(1 + I*c*x)])/(4*c))))/d
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5405
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x], x
] - Simp[e/(d*f)   Int[(f*x)^(m + 1)*((a + b*ArcTan[c*x])^p/(d + e*x)), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
&& LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 12.82 (sec) , antiderivative size = 8468, normalized size of antiderivative = 45.53

| method | result | size |
|---|---|---|
| parts | Expression too large to display | 8468 |
| derivativedivides | Expression too large to display | 8470 |
| default | Expression too large to display | 8470 |

input
```
int((a+b*arctan(c*x))^2/x^2/(d+I*c*d*x),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

## Fricas [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+icdx)}\,dx=\int \frac{(b\arctan{(cx)}+a)^2}{(i\,cdx+d)x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^2/(d+I*c*d*x),x, algorithm="fricas")
```

output
```
1/4*(I*b^2*c*x*log(2*c*x/(c*x - I))*log(-(c*x + I)/(c*x - I))^2 + 2*I*b^2*
c*x*dilog(-2*c*x/(c*x - I) + 1)*log(-(c*x + I)/(c*x - I)) - 2*I*b^2*c*x*po
lylog(3, -(c*x + I)/(c*x - I)) + b^2*log(-(c*x + I)/(c*x - I))^2 + 4*d*x*i
ntegral((-I*a^2*c*x + a^2 + ((a*b + I*b^2)*c*x + I*a*b)*log(-(c*x + I)/(c*
x - I)))/(c^2*d*x^4 + d*x^2), x))/(d*x)
```

## Sympy [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+icdx)}\,dx = -\frac{i\left(\int \frac{a^2}{cx^3-ix^2}\,dx + \int \frac{b^2\operatorname{atan}^2(cx)}{cx^3-ix^2}\,dx + \int \frac{2ab\operatorname{atan}(cx)}{cx^3-ix^2}\,dx\right)}{d}$$

input
```
integrate((a+b*atan(c*x))**2/x**2/(d+I*c*d*x),x)
```

output
```
-I*(Integral(a**2/(c*x**3 - I*x**2), x) + Integral(b**2*atan(c*x)**2/(c*x*
*3 - I*x**2), x) + Integral(2*a*b*atan(c*x)/(c*x**3 - I*x**2), x))/d
```

## Maxima [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+icdx)}\,dx = \int \frac{(b\arctan(cx)+a)^2}{(i\,cdx+d)x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^2/(d+I*c*d*x),x, algorithm="maxima")
```

output
```
a^2*(I*c*log(I*c*x + 1)/d - I*c*log(x)/d - 1/(d*x)) - 1/96*(24*b^2*c*x*arc
tan(c*x)^3 + 3*I*b^2*c*x*log(c^2*x^2 + 1)^3 + 24*b^2*arctan(c*x)^2 - 2*I*(
384*b^2*c^3*integrate(1/16*x^3*arctan(c*x)^2/(c^2*d*x^4 + d*x^2), x) + b^2
*c*log(c^2*x^2 + 1)^3/d + 12*b^2*c*arctan(c*x)^2/d - 576*b^2*c*integrate(1
/16*x*arctan(c*x)^2/(c^2*d*x^4 + d*x^2), x) - 48*b^2*c*integrate(1/16*x*lo
g(c^2*x^2 + 1)^2/(c^2*d*x^4 + d*x^2), x) - 1536*a*b*c*integrate(1/16*x*arc
tan(c*x)/(c^2*d*x^4 + d*x^2), x) + 192*b^2*c*integrate(1/16*x*log(c^2*x^2
+ 1)/(c^2*d*x^4 + d*x^2), x) - 192*b^2*integrate(1/16*arctan(c*x)*log(c^2*
x^2 + 1)/(c^2*d*x^4 + d*x^2), x))*d*x - 16*(b^2*c*arctan(c*x)^3/d + 12*b^2
*c^2*integrate(1/16*x^2*log(c^2*x^2 + 1)^2/(c^2*d*x^4 + d*x^2), x) - 24*b^
2*c^2*integrate(1/16*x^2*log(c^2*x^2 + 1)/(c^2*d*x^4 + d*x^2), x) - 24*b^2
*c*integrate(1/16*x*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*d*x^4 + d*x^2), x) +
 48*b^2*c*integrate(1/16*x*arctan(c*x)/(c^2*d*x^4 + d*x^2), x) + 72*b^2*in
tegrate(1/16*arctan(c*x)^2/(c^2*d*x^4 + d*x^2), x) + 6*b^2*integrate(1/16*
log(c^2*x^2 + 1)^2/(c^2*d*x^4 + d*x^2), x) + 192*a*b*integrate(1/16*arctan
(c*x)/(c^2*d*x^4 + d*x^2), x))*d*x + 6*(b^2*c*x*arctan(c*x) - b^2)*log(c^2
*x^2 + 1)^2 + 12*(I*b^2*c*x*arctan(c*x)^2 + 2*I*b^2*arctan(c*x))*log(c^2*x
^2 + 1))/(d*x)
```

## Giac [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+icdx)}\,dx = \int \frac{(b\arctan{(cx)}+a)^2}{(i\,cdx+d)x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^2/(d+I*c*d*x),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)^2/((I*c*d*x + d)*x^2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+icdx)}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^2}{x^2\,(d+c\,d\,x\,1\mathrm{i})}\,dx$$

input
```
int((a + b*atan(c*x))^2/(x^2*(d + c*d*x*1i)),x)
```

output
```
int((a + b*atan(c*x))^2/(x^2*(d + c*d*x*1i)), x)
```

## Reduce [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+icdx)}\,dx$$
$$= \frac{2\left(\int \frac{atan(cx)}{c^3i\,x^5+c^2x^4+ci\,x^3+x^2}dx\right)abx+2\left(\int \frac{atan(cx)}{c^3i\,x^3+c^2x^2+cix+1}dx\right)ab\,c^2x+\left(\int \frac{atan(cx)^2}{c^3i\,x^5+c^2x^4+ci\,x^3+x^2}dx\right)b^2x+\left(\int \frac{}{c^3i}\right.}{dx}$$

input
```
int((a+b*atan(c*x))^2/x^2/(d+I*c*d*x),x)
```

output

```
(2*int(atan(c*x)/(c**3*i*x**5 + c**2*x**4 + c*i*x**3 + x**2),x)*a*b*x + 2*
int(atan(c*x)/(c**3*i*x**3 + c**2*x**2 + c*i*x + 1),x)*a*b*c**2*x + int(at
an(c*x)**2/(c**3*i*x**5 + c**2*x**4 + c*i*x**3 + x**2),x)*b**2*x + int(ata
n(c*x)**2/(c**3*i*x**3 + c**2*x**2 + c*i*x + 1),x)*b**2*c**2*x + log(c*i*x
 + 1)*a**2*c*i*x - log(x)*a**2*c*i*x - a**2)/(d*x)
```

# 3.101 $\int \frac{(a+b\arctan(cx))^2}{x^3(d+icdx)}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 273

$$\begin{aligned}\int \frac{(a+b\arctan(cx))^2}{x^3(d+icdx)}\,dx = & -\frac{bc(a+b\arctan(cx))}{dx}-\frac{3c^2(a+b\arctan(cx))^2}{2d} \\ & -\frac{(a+b\arctan(cx))^2}{2dx^2}+\frac{ic(a+b\arctan(cx))^2}{dx} \\ & +\frac{b^2c^2\log(x)}{d}-\frac{b^2c^2\log(1+c^2x^2)}{2d} \\ & -\frac{2ibc^2(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right)}{d} \\ & -\frac{c^2(a+b\arctan(cx))^2\log\left(2-\frac{2}{1+icx}\right)}{d} \\ & -\frac{b^2c^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)}{d} \\ & -\frac{ibc^2(a+b\arctan(cx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)}{d} \\ & -\frac{b^2c^2\operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)}{2d}\end{aligned}$$

output
```
-b*c*(a+b*arctan(c*x))/d/x-3/2*c^2*(a+b*arctan(c*x))^2/d-1/2*(a+b*arctan(c
*x))^2/d/x^2+I*c*(a+b*arctan(c*x))^2/d/x+b^2*c^2*ln(x)/d-1/2*b^2*c^2*ln(c^
2*x^2+1)/d-2*I*b*c^2*(a+b*arctan(c*x))*ln(2-2/(1-I*c*x))/d-c^2*(a+b*arctan
(c*x))^2*ln(2-2/(1+I*c*x))/d-b^2*c^2*polylog(2,-1+2/(1-I*c*x))/d-I*b*c^2*(
a+b*arctan(c*x))*polylog(2,-1+2/(1+I*c*x))/d-1/2*b^2*c^2*polylog(3,-1+2/(1
+I*c*x))/d
```

## Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.36

$$\int \frac{(a+b\arctan(cx))^2}{x^3(d+icdx)}\,dx$$

$$= \frac{-\frac{a^2}{x^2}+\frac{2ia^2c}{x}+2ia^2c^2\arctan(cx)-2a^2c^2\log(x)+a^2c^2\log(1+c^2x^2)+\frac{2iab\left(2c^2x^2\arctan(cx)^2+\arctan(cx)(i+2cx+\right.}{}}{}$$

input
```
Integrate[(a + b*ArcTan[c*x])^2/(x^3*(d + I*c*d*x)),x]
```

output
```
(-(a^2/x^2) + ((2*I)*a^2*c)/x + (2*I)*a^2*c^2*ArcTan[c*x] - 2*a^2*c^2*Log[
x] + a^2*c^2*Log[1 + c^2*x^2] + ((2*I)*a*b*(2*c^2*x^2*ArcTan[c*x]^2 + ArcT
an[c*x]*(I + 2*c*x + I*c^2*x^2 + (2*I)*c^2*x^2*Log[1 - E^((2*I)*ArcTan[c*x
])]) + c*x*(I - 2*c*x*Log[(c*x)/Sqrt[1 + c^2*x^2]]) + c^2*x^2*PolyLog[2, E
^((2*I)*ArcTan[c*x])]))/x^2 + 2*b^2*c^2*((I/24)*Pi^3 - ArcTan[c*x]/(c*x) -
 (3*ArcTan[c*x]^2)/2 - ArcTan[c*x]^2/(2*c^2*x^2) + (I*ArcTan[c*x]^2)/(c*x)
 - ArcTan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] - (2*I)*ArcTan[c*x]*Log[1
 - E^((2*I)*ArcTan[c*x])] + Log[(c*x)/Sqrt[1 + c^2*x^2]] - I*ArcTan[c*x]*P
olyLog[2, E^((-2*I)*ArcTan[c*x])] - PolyLog[2, E^((2*I)*ArcTan[c*x])] - Po
lyLog[3, E^((-2*I)*ArcTan[c*x])]/2))/(2*d)
```

## Rubi [A] (verified)

Time = 2.84 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.01, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.720, Rules used = {5405, 27, 5361, 5405, 5361, 5403, 5453, 5361, 243, 47, 14, 16, 5419, 5459, 5403, 2897, 5529, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^2}{x^3(d+icdx)}dx$$

$$\Big\downarrow \text{5405}$$

$$\frac{\int \frac{(a+b\arctan(cx))^2}{x^3}dx}{d}-ic\int \frac{(a+b\arctan(cx))^2}{dx^2(icx+1)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(a+b\arctan(cx))^2}{x^3}dx}{d}-\frac{ic\int \frac{(a+b\arctan(cx))^2}{x^2(icx+1)}dx}{d}$$

$$\Big\downarrow \text{5361}$$

$$\frac{bc\int \frac{a+b\arctan(cx)}{x^2(c^2x^2+1)}dx-\frac{(a+b\arctan(cx))^2}{2x^2}}{d}-\frac{ic\int \frac{(a+b\arctan(cx))^2}{x^2(icx+1)}dx}{d}$$

$$\Big\downarrow \text{5405}$$

$$\frac{bc\int \frac{a+b\arctan(cx)}{x^2(c^2x^2+1)}dx-\frac{(a+b\arctan(cx))^2}{2x^2}}{d}-\frac{ic\left(\int \frac{(a+b\arctan(cx))^2}{x^2}dx-ic\int \frac{(a+b\arctan(cx))^2}{x(icx+1)}dx\right)}{d}$$

$$\Big\downarrow \text{5361}$$

$$\frac{bc\int \frac{a+b\arctan(cx)}{x^2(c^2x^2+1)}dx-\frac{(a+b\arctan(cx))^2}{2x^2}}{d}-\frac{ic\left(2bc\int \frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx-ic\int \frac{(a+b\arctan(cx))^2}{x(icx+1)}dx-\frac{(a+b\arctan(cx))^2}{x}\right)}{d}$$

$$\Big\downarrow \text{5403}$$

$$\frac{bc\int \frac{a+b\arctan(cx)}{x^2(c^2x^2+1)}dx-\frac{(a+b\arctan(cx))^2}{2x^2}}{d}-\frac{ic\left(2bc\int \frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx-ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2-2bc\int \frac{(a+b\arctan(cx))\log\left(2-\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)-\frac{(a+b}{}\right.}{d}$$

$\downarrow$ 5453

$$\frac{bc\left(\int \frac{a+b\arctan(cx)}{x^2}dx-c^2\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)-\frac{(a+b\arctan(cx))^2}{2x^2}}{d}-\frac{ic\left(2bc\int \frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx-ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2-2bc\int \frac{(a+b\arctan(cx))\log\left(2-\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)-\frac{(a+b}{}\right.}{d}$$

$\downarrow$ 5361

$$\frac{bc\left(c^2\left(-\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)+bc\int \frac{1}{x(c^2x^2+1)}dx-\frac{a+b\arctan(cx)}{x}\right)-\frac{(a+b\arctan(cx))^2}{2x^2}}{d}-\frac{ic\left(2bc\int \frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx-ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2-2bc\int \frac{(a+b\arctan(cx))\log\left(2-\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)-\frac{(a+b}{}\right.}{d}$$

$\downarrow$ 243

$$\frac{bc\left(c^2\left(-\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)+\frac{1}{2}bc\int \frac{1}{x^2(c^2x^2+1)}dx^2-\frac{a+b\arctan(cx)}{x}\right)-\frac{(a+b\arctan(cx))^2}{2x^2}}{d}-\frac{ic\left(2bc\int \frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx-ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2-2bc\int \frac{(a+b\arctan(cx))\log\left(2-\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)-\frac{(a+b}{}\right.}{d}$$

$\downarrow$ 47

$$\frac{bc\left(c^2\left(-\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)+\frac{1}{2}bc\left(\int \frac{1}{x^2}dx^2-c^2\int \frac{1}{c^2x^2+1}dx^2\right)-\frac{a+b\arctan(cx)}{x}\right)-\frac{(a+b\arctan(cx))^2}{2x^2}}{d}-\frac{ic\left(2bc\int \frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx-ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2-2bc\int \frac{(a+b\arctan(cx))\log\left(2-\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)-\frac{(a+b}{}\right.}{d}$$

$\downarrow$ 14

$$\frac{bc\left(c^2\left(-\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)+\frac{1}{2}bc\left(\log\left(x^2\right)-c^2\int \frac{1}{c^2x^2+1}dx^2\right)-\frac{a+b\arctan(cx)}{x}\right)-\frac{(a+b\arctan(cx))^2}{2x^2}}{d}-$$

$$\frac{ic\left(2bc\int \frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx-ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2-2bc\int \frac{(a+b\arctan(cx))\log\left(2-\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)-\frac{(a+b}{}\right.}{d}$$

$\Big\downarrow$ 16

$$\frac{bc\left(c^2\left(-\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)-\frac{a+b\arctan(cx)}{x}+\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right)\right)\right)-\frac{(a+b\arctan(cx))^2}{2x^2}}{d}-$$

$$\frac{ic\left(2bc\int \frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx-ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2-2bc\int \frac{(a+b\arctan(cx))\log\left(2-\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)-\frac{(a+b}{}\right.}{d}$$

$\Big\downarrow$ 5419

$$\frac{bc\left(-\frac{c(a+b\arctan(cx))^2}{2b}-\frac{a+b\arctan(cx)}{x}+\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right)\right)\right)-\frac{(a+b\arctan(cx))^2}{2x^2}}{d}-$$

$$\frac{ic\left(2bc\int \frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx-ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2-2bc\int \frac{(a+b\arctan(cx))\log\left(2-\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)-\frac{(a+b}{}\right.}{d}$$

$\Big\downarrow$ 5459

$$\frac{bc\left(-\frac{c(a+b\arctan(cx))^2}{2b}-\frac{a+b\arctan(cx)}{x}+\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right)\right)\right)-\frac{(a+b\arctan(cx))^2}{2x^2}}{d}-$$

$$\frac{ic\left(-ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2-2bc\int \frac{(a+b\arctan(cx))\log\left(2-\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)+2bc\left(i\int \frac{a+b\arctan(cx)}{x(cx+i)}dx-\right.\right.}{d}$$

$\Big\downarrow$ 5403

$$\frac{bc\left(-\frac{c(a+b\arctan(cx))^2}{2b}-\frac{a+b\arctan(cx)}{x}+\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right)\right)\right)-\frac{(a+b\arctan(cx))^2}{2x^2}}{d}-$$

$$\frac{ic\left(2bc\left(i\left(ibc\int \frac{\log\left(2-\frac{2}{1-icx}\right)}{c^2x^2+1}dx-i\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))\right)-\frac{i(a+b\arctan(cx))^2}{2b}\right)-ic\left(\log\left(2-\frac{2}{1+icx}\right.\right.\right.}{d}$$

$\Big\downarrow$ 2897

$$\frac{bc\left(-\frac{c(a+b\arctan(cx))^2}{2b}-\frac{a+b\arctan(cx)}{x}+\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right)\right)\right)-\frac{(a+b\arctan(cx))^2}{2x^2}}{d}-$$

$$\frac{ic\left(-ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2-2bc\int\frac{(a+b\arctan(cx))\log\left(2-\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)+2bc\left(i\left(-i\log\left(2-\frac{2}{1-icx}\right)\right.\right.\right.}{d}$$

$\Big\downarrow$ 5529

$$\frac{bc\left(-\frac{c(a+b\arctan(cx))^2}{2b}-\frac{a+b\arctan(cx)}{x}+\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right)\right)\right)-\frac{(a+b\arctan(cx))^2}{2x^2}}{d}-$$

$$ic\left(-ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2-2bc\left(\frac{1}{2}ib\int\frac{\operatorname{PolyLog}\left(2,\frac{2}{icx+1}-1\right)}{c^2x^2+1}dx-\frac{i\operatorname{PolyLog}\left(2,\frac{2}{icx+1}-1\right)(a+b\arctan(cx)}{2c}\right.\right.\right.$$

$\Big\downarrow$ 7164

$$\frac{bc\left(-\frac{c(a+b\arctan(cx))^2}{2b}-\frac{a+b\arctan(cx)}{x}+\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right)\right)\right)-\frac{(a+b\arctan(cx))^2}{2x^2}}{d}-$$

$$ic\left(2bc\left(i\left(-i\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))-\frac{1}{2}b\operatorname{PolyLog}\left(2,\frac{2}{1-icx}-1\right)\right)-\frac{i(a+b\arctan(cx))^2}{2b}\right)-ic\left(\log\left(2-\right.\right.\right.$$

input
```
Int[(a + b*ArcTan[c*x])^2/(x^3*(d + I*c*d*x)),x]
```

output
```
(-1/2*(a + b*ArcTan[c*x])^2/x^2 + b*c*(-((a + b*ArcTan[c*x])/x) - (c*(a +
b*ArcTan[c*x])^2)/(2*b) + (b*c*(Log[x^2] - Log[1 + c^2*x^2]))/2))/d - (I*c
*(-((a + b*ArcTan[c*x])^2/x) + 2*b*c*(((-1/2*I)*(a + b*ArcTan[c*x])^2)/b +
 I*((-I)*(a + b*ArcTan[c*x])*Log[2 - 2/(1 - I*c*x)] - (b*PolyLog[2, -1 + 2
/(1 - I*c*x)])/2)) - I*c*((a + b*ArcTan[c*x])^2*Log[2 - 2/(1 + I*c*x)] - 2
*b*c*(((-1/2*I)*(a + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)])/c - (b
*PolyLog[3, -1 + 2/(1 + I*c*x)])/(4*c)))))/d
```

### Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5405
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x], x
] - Simp[e/(d*f)   Int[(f*x)^(m + 1)*((a + b*ArcTan[c*x])^p/(d + e*x)), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
&& LtQ[m, -1]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 16.05 (sec) , antiderivative size = 1873, normalized size of antiderivative = 6.86

| method | result | size |
|---|---|---|
| derivativedivides | Expression too large to display | 1873 |
| default | Expression too large to display | 1873 |
| parts | Expression too large to display | 1877 |

input
```
int((a+b*arctan(c*x))^2/x^3/(d+I*c*d*x),x,method=_RETURNVERBOSE)
```

output
```
c^2*(-1/2*a^2/d/c^2/x^2+I*a^2/d/c/x-a^2/d*ln(c*x)+1/2*a^2/d*ln(c^2*x^2+1)+
I*a^2/d*arctan(c*x)+b^2/d*(-1/2/c^2/x^2*arctan(c*x)^2-1/2*I*Pi*csgn(I*((1+
I*c*x)^2/(c^2*x^2+1)-1))*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I*((1+I*
c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*arctan(c*x)^2-3/2*arcta
n(c*x)^2-1/2*arctan(c*x)*(I*c*x-(c^2*x^2+1)^(1/2)+1)/c/x-ln(c*x)*arctan(c*
x)^2+arctan(c*x)^2*ln((1+I*c*x)^2/(c^2*x^2+1)-1)-arctan(c*x)^2*ln(1-(1+I*c
*x)/(c^2*x^2+1)^(1/2))-arctan(c*x)^2*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))+2*I
*arctan(c*x)*polylog(2,(1+I*c*x)/(c^2*x^2+1)^(1/2))+2*I*arctan(c*x)*polylo
g(2,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+ln((1+I*c*x)/(c^2*x^2+1)^(1/2)-1)-2*poly
log(3,(1+I*c*x)/(c^2*x^2+1)^(1/2))+ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))-2*pol
ylog(3,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/2*I*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2
+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1
+I*c*x)^2/(c^2*x^2+1)))^2*arctan(c*x)^2+1/2*I*Pi*csgn(I*((1+I*c*x)^2/(c^2*
x^2+1)-1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))
^2*arctan(c*x)^2+1/2*I*Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I*((1+I
*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*arctan(c*x)^2-1/2*I*
Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(((
1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*arctan(c*x)^2-1/2*a
rctan(c*x)*(I*c*x+(c^2*x^2+1)^(1/2)+1)/c/x-1/2*I*Pi*csgn(I*((1+I*c*x)^2/(c
^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^3*arctan(c*x)^2+1/2*I*Pi*csgn...
```

## Fricas [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^3(d+icdx)}\,dx=\int \frac{(b\arctan(cx)+a)^2}{(i\,cdx+d)x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^3/(d+I*c*d*x),x, algorithm="fricas")
```

output
```
1/8*(2*b^2*c^2*x^2*log(2*c*x/(c*x - I))*log(-(c*x + I)/(c*x - I))^2 + 4*b^
2*c^2*x^2*dilog(-2*c*x/(c*x - I) + 1)*log(-(c*x + I)/(c*x - I)) - 4*b^2*c^
2*x^2*polylog(3, -(c*x + I)/(c*x - I)) + 8*d*x^2*integral(1/2*(-2*I*a^2*c*
x + 2*a^2 + (2*b^2*c^2*x^2 + (2*a*b + I*b^2)*c*x + 2*I*a*b)*log(-(c*x + I)
/(c*x - I)))/(c^2*d*x^5 + d*x^3), x) + (-2*I*b^2*c*x + b^2)*log(-(c*x + I)
/(c*x - I))^2)/(d*x^2)
```

## Sympy [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^3(d+icdx)}\,dx=-\frac{i\left(\int \frac{a^2}{cx^4-ix^3}\,dx+\int \frac{b^2\operatorname{atan}^2(cx)}{cx^4-ix^3}\,dx+\int \frac{2ab\operatorname{atan}(cx)}{cx^4-ix^3}\,dx\right)}{d}$$

input
```
integrate((a+b*atan(c*x))**2/x**3/(d+I*c*d*x),x)
```

output
```
-I*(Integral(a**2/(c*x**4 - I*x**3), x) + Integral(b**2*atan(c*x)**2/(c*x*
*4 - I*x**3), x) + Integral(2*a*b*atan(c*x)/(c*x**4 - I*x**3), x))/d
```

## Maxima [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^3(d+icdx)}\,dx=\int \frac{(b\arctan(cx)+a)^2}{(i\,cdx+d)x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^3/(d+I*c*d*x),x, algorithm="maxima")
```

output
```
1/2*(2*c^2*log(I*c*x + 1)/d - 2*c^2*log(x)/d + (2*I*c*x - 1)/(d*x^2))*a^2
- 1/96*(-24*I*b^2*c^2*x^2*arctan(c*x)^3 + 3*b^2*c^2*x^2*log(c^2*x^2 + 1)^3
 - 2*(384*b^2*c^4*integrate(1/16*x^4*arctan(c*x)^2/(c^2*d*x^5 + d*x^3), x)
 + b^2*c^2*log(c^2*x^2 + 1)^3/d + 12*b^2*c^2*arctan(c*x)^2/d + 96*b^2*c^2*
integrate(1/16*x^2*log(c^2*x^2 + 1)/(c^2*d*x^5 + d*x^3), x) - 192*b^2*c*in
tegrate(1/16*x*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*d*x^5 + d*x^3), x) + 192*
b^2*c*integrate(1/16*x*arctan(c*x)/(c^2*d*x^5 + d*x^3), x) + 576*b^2*integ
rate(1/16*arctan(c*x)^2/(c^2*d*x^5 + d*x^3), x) + 48*b^2*integrate(1/16*lo
g(c^2*x^2 + 1)^2/(c^2*d*x^5 + d*x^3), x) + 1536*a*b*integrate(1/16*arctan(
c*x)/(c^2*d*x^5 + d*x^3), x))*d*x^2 + 16*I*(b^2*c^2*arctan(c*x)^3/d + 12*b
^2*c^3*integrate(1/16*x^3*log(c^2*x^2 + 1)^2/(c^2*d*x^5 + d*x^3), x) - 24*
b^2*c^3*integrate(1/16*x^3*log(c^2*x^2 + 1)/(c^2*d*x^5 + d*x^3), x) + 24*b
^2*c^2*integrate(1/16*x^2*arctan(c*x)/(c^2*d*x^5 + d*x^3), x) + 72*b^2*c*i
ntegrate(1/16*x*arctan(c*x)^2/(c^2*d*x^5 + d*x^3), x) + 6*b^2*c*integrate(
1/16*x*log(c^2*x^2 + 1)^2/(c^2*d*x^5 + d*x^3), x) + 192*a*b*c*integrate(1/
16*x*arctan(c*x)/(c^2*d*x^5 + d*x^3), x) - 12*b^2*c*integrate(1/16*x*log(c
^2*x^2 + 1)/(c^2*d*x^5 + d*x^3), x) + 24*b^2*integrate(1/16*arctan(c*x)*lo
g(c^2*x^2 + 1)/(c^2*d*x^5 + d*x^3), x))*d*x^2 + 12*(-2*I*b^2*c*x + b^2)*ar
ctan(c*x)^2 - 3*(2*I*b^2*c^2*x^2*arctan(c*x) - 2*I*b^2*c*x + b^2)*log(c^2*
x^2 + 1)^2 + 12*(b^2*c^2*x^2*arctan(c*x)^2 + (2*b^2*c*x + I*b^2)*arctan...
```

## Giac [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^3(d+icdx)}\,dx=\int \frac{(b\arctan(cx)+a)^2}{(i\,cdx+d)x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^3/(d+I*c*d*x),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)^2/((I*c*d*x + d)*x^3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{x^3(d+icdx)}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^2}{x^3\,(d+c\,d\,x\,1\mathrm{i})}\,dx$$

input
```
int((a + b*atan(c*x))^2/(x^3*(d + c*d*x*1i)),x)
```

output
```
int((a + b*atan(c*x))^2/(x^3*(d + c*d*x*1i)), x)
```

## Reduce [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^3(d+icdx)}\,dx$$
$$= \frac{2atan(cx)^2\,ab\,c^2i\,x^2+2atan(cx)\,a^2c^2i\,x^2+4atan(cx)\,abcix-4\left(\int \frac{atan(cx)}{c^3x^6-c^2i\,x^5+c\,x^4-i\,x^3}dx\right)abi\,x^2+4\left(\int \frac{}{c^3}\right.}{}$$

input
```
int((a+b*atan(c*x))^2/x^3/(d+I*c*d*x),x)
```

output
```
(2*atan(c*x)**2*a*b*c**2*i*x**2 + 2*atan(c*x)*a**2*c**2*i*x**2 + 4*atan(c*
x)*a*b*c*i*x - 4*int(atan(c*x)/(c**3*x**6 - c**2*i*x**5 + c*x**4 - i*x**3)
,x)*a*b*i*x**2 + 4*int(atan(c*x)/(c**3*x**5 - c**2*i*x**4 + c*x**3 - i*x**
2),x)*a*b*c*x**2 - 2*int(atan(c*x)**2/(c**3*x**6 - c**2*i*x**5 + c*x**4 -
i*x**3),x)*b**2*i*x**2 - 2*int(atan(c*x)**2/(c**3*x**4 - c**2*i*x**3 + c*x
**2 - i*x),x)*b**2*c**2*i*x**2 + log(c**2*x**2 + 1)*a**2*c**2*x**2 + 2*log
(c**2*x**2 + 1)*a*b*c**2*i*x**2 - 2*log(x)*a**2*c**2*x**2 - 4*log(x)*a*b*c
**2*i*x**2 + 2*a**2*c*i*x - a**2)/(2*d*x**2)
```

# 3.102 $\int \frac{(a+b\arctan(cx))^2}{x^4(d+icdx)}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 365

$$\begin{aligned}\int \frac{(a+b\arctan(cx))^2}{x^4(d+icdx)}\,dx = &-\frac{b^2c^2}{3dx}-\frac{b^2c^3\arctan(cx)}{3d}-\frac{bc(a+b\arctan(cx))}{3dx^2}\\&+\frac{ibc^2(a+b\arctan(cx))}{dx}+\frac{11ic^3(a+b\arctan(cx))^2}{6d}\\&-\frac{(a+b\arctan(cx))^2}{3dx^3}+\frac{ic(a+b\arctan(cx))^2}{2dx^2}\\&+\frac{c^2(a+b\arctan(cx))^2}{dx}-\frac{ib^2c^3\log(x)}{d}+\frac{ib^2c^3\log(1+c^2x^2)}{2d}\\&-\frac{8bc^3(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right)}{3d}\\&+\frac{ic^3(a+b\arctan(cx))^2\log\left(2-\frac{2}{1+icx}\right)}{d}\\&+\frac{4ib^2c^3\operatorname{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)}{3d}\\&-\frac{bc^3(a+b\arctan(cx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)}{d}\\&+\frac{ib^2c^3\operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)}{2d}\end{aligned}$$

output
```
-1/3*b^2*c^2/d/x-1/3*b^2*c^3*arctan(c*x)/d-1/3*b*c*(a+b*arctan(c*x))/d/x^2
+I*b*c^2*(a+b*arctan(c*x))/d/x+11/6*I*c^3*(a+b*arctan(c*x))^2/d-1/3*(a+b*a
rctan(c*x))^2/d/x^3+1/2*I*c*(a+b*arctan(c*x))^2/d/x^2+c^2*(a+b*arctan(c*x)
)^2/d/x-I*b^2*c^3*ln(x)/d+1/2*I*b^2*c^3*ln(c^2*x^2+1)/d-8/3*b*c^3*(a+b*arc
tan(c*x))*ln(2-2/(1-I*c*x))/d+I*c^3*(a+b*arctan(c*x))^2*ln(2-2/(1+I*c*x))/
d+4/3*I*b^2*c^3*polylog(2,-1+2/(1-I*c*x))/d-b*c^3*(a+b*arctan(c*x))*polylo
g(2,-1+2/(1+I*c*x))/d+1/2*I*b^2*c^3*polylog(3,-1+2/(1+I*c*x))/d
```

## Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 535, normalized size of antiderivative = 1.47

$$\int \frac{(a+b\arctan(cx))^2}{x^4(d+icdx)}\,dx$$
$$= -\frac{a^2}{3dx^3}+\frac{ia^2c}{2dx^2}+\frac{a^2c^2}{dx}+\frac{a^2c^3\arctan(cx)}{d}+\frac{ia^2c^3\log(x)}{d}-\frac{ia^2c^3\log(1+c^2x^2)}{2d}$$
$$-\frac{2iabc^3\left(-\frac{1}{2cx}-\frac{i(1+c^2x^2)}{6c^2x^2}+\frac{4i\arctan(cx)}{3cx}-\frac{i(1+c^2x^2)\arctan(cx)}{3c^3x^3}-\frac{(1+c^2x^2)\arctan(cx)}{2c^2x^2}+\frac{1}{2}i\arctan(cx)^2-\arctan\right.}{d}$$
$$+\frac{b^2c^3\left(\pi^3-\frac{8}{cx}+\frac{24i\arctan(cx)}{cx}-\frac{8(1+c^2x^2)\arctan(cx)}{c^2x^2}+32i\arctan(cx)^2+\frac{32\arctan(cx)^2}{cx}-\frac{8(1+c^2x^2)\arctan(cx)^2}{c^3x^3}+\right.}{}$$

input
```
Integrate[(a + b*ArcTan[c*x])^2/(x^4*(d + I*c*d*x)),x]
```

output
```
-1/3*a^2/(d*x^3) + ((I/2)*a^2*c)/(d*x^2) + (a^2*c^2)/(d*x) + (a^2*c^3*ArcT
an[c*x])/d + (I*a^2*c^3*Log[x])/d - ((I/2)*a^2*c^3*Log[1 + c^2*x^2])/d - (
(2*I)*a*b*c^3*(-1/2*1/(c*x) - ((I/6)*(1 + c^2*x^2))/(c^2*x^2) + (((4*I)/3)
*ArcTan[c*x])/(c*x) - ((I/3)*(1 + c^2*x^2)*ArcTan[c*x])/(c^3*x^3) - ((1 +
c^2*x^2)*ArcTan[c*x])/(2*c^2*x^2) + (I/2)*ArcTan[c*x]^2 - ArcTan[c*x]*Log[
1 - E^((2*I)*ArcTan[c*x])] - ((4*I)/3)*Log[(c*x)/Sqrt[1 + c^2*x^2]] + (I/2
)*(ArcTan[c*x]^2 + PolyLog[2, E^((2*I)*ArcTan[c*x])])))/d + (b^2*c^3*(Pi^3
 - 8/(c*x) + ((24*I)*ArcTan[c*x])/(c*x) - (8*(1 + c^2*x^2)*ArcTan[c*x])/(c
^2*x^2) + (32*I)*ArcTan[c*x]^2 + (32*ArcTan[c*x]^2)/(c*x) - (8*(1 + c^2*x^
2)*ArcTan[c*x]^2)/(c^3*x^3) + ((12*I)*(1 + c^2*x^2)*ArcTan[c*x]^2)/(c^2*x^
2) + (24*I)*ArcTan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] - 64*ArcTan[c*x]
*Log[1 - E^((2*I)*ArcTan[c*x])] - (24*I)*Log[(c*x)/Sqrt[1 + c^2*x^2]] - 24
*ArcTan[c*x]*PolyLog[2, E^((-2*I)*ArcTan[c*x])] + (32*I)*PolyLog[2, E^((2*
I)*ArcTan[c*x])] + (12*I)*PolyLog[3, E^((-2*I)*ArcTan[c*x])]))/(24*d)
```

## Rubi [A] (verified)

Time = 4.00 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.15, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.880$, Rules used = {5405, 27, 5361, 5405, 5361, 5405, 5361, 5403, 5453, 5361, 243, 47, 14, 16, 264, 216, 5419, 5459, 5403, 2897, 5529, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^2}{x^4(d+icdx)}\,dx$$

$$\Big\downarrow \text{5405}$$

$$\frac{\int \frac{(a+b\arctan(cx))^2}{x^4}dx}{d} - ic\int \frac{(a+b\arctan(cx))^2}{dx^3(icx+1)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(a+b\arctan(cx))^2}{x^4}dx}{d} - \frac{ic\int \frac{(a+b\arctan(cx))^2}{x^3(icx+1)}dx}{d}$$

$$\Big\downarrow \text{5361}$$

$$\frac{\frac{2}{3}bc\int \frac{a+b\arctan(cx)}{x^3(c^2x^2+1)}dx - \frac{(a+b\arctan(cx))^2}{3x^3}}{d} - \frac{ic\int \frac{(a+b\arctan(cx))^2}{x^3(icx+1)}dx}{d}$$

$$\Big\downarrow \text{5405}$$

$$\frac{\frac{2}{3}bc\int \frac{a+b\arctan(cx)}{x^3(c^2x^2+1)}dx - \frac{(a+b\arctan(cx))^2}{3x^3}}{d} - \frac{ic\left(\int \frac{(a+b\arctan(cx))^2}{x^3}dx - ic\int \frac{(a+b\arctan(cx))^2}{x^2(icx+1)}dx\right)}{d}$$

$$\Big\downarrow \text{5361}$$

$$\frac{\frac{2}{3}bc\int \frac{a+b\arctan(cx)}{x^3(c^2x^2+1)}dx - \frac{(a+b\arctan(cx))^2}{3x^3}}{d} - \frac{ic\left(bc\int \frac{a+b\arctan(cx)}{x^2(c^2x^2+1)}dx - ic\int \frac{(a+b\arctan(cx))^2}{x^2(icx+1)}dx - \frac{(a+b\arctan(cx))^2}{2x^2}\right)}{d}$$

$$\Big\downarrow \text{5405}$$

$$\frac{\frac{2}{3}bc\int \frac{a+b\arctan(cx)}{x^3(c^2x^2+1)}dx - \frac{(a+b\arctan(cx))^2}{3x^3}}{d} - \frac{ic\left(bc\int \frac{a+b\arctan(cx)}{x^2(c^2x^2+1)}dx - ic\left(\int \frac{(a+b\arctan(cx))^2}{x^2}dx - ic\int \frac{(a+b\arctan(cx))^2}{x(icx+1)}dx\right) - \frac{(a+b\arctan(cx))^2}{2x^2}\right)}{d}$$

$\Big\downarrow$ 5361

$$\frac{\frac{2}{3}bc\int \frac{a+b\arctan(cx)}{x^3(c^2x^2+1)}dx-\frac{(a+b\arctan(cx))^2}{3x^3}}{d}-\frac{ic\left(bc\int \frac{a+b\arctan(cx)}{x^2(c^2x^2+1)}dx-ic\left(2bc\int \frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx-ic\int \frac{(a+b\arctan(cx))^2}{x(icx+1)}dx-\frac{(a+b\arctan(cx))^2}{x}\right)-\frac{(a+b\arctan(cx))^2}{2x^2}\right)}{d}$$

$\Big\downarrow$ 5403

$$\frac{\frac{2}{3}bc\int \frac{a+b\arctan(cx)}{x^3(c^2x^2+1)}dx-\frac{(a+b\arctan(cx))^2}{3x^3}}{d}-\frac{ic\left(bc\int \frac{a+b\arctan(cx)}{x^2(c^2x^2+1)}dx-ic\left(2bc\int \frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx-ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2-2bc\int \frac{(a+b\arctan(cx}{c^2}\right.\right.\right.}{d}$$

$\Big\downarrow$ 5453

$$\frac{\frac{2}{3}bc\left(\int \frac{a+b\arctan(cx)}{x^3}dx-c^2\int \frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx\right)-\frac{(a+b\arctan(cx))^2}{3x^3}}{d}-\frac{ic\left(bc\left(\int \frac{a+b\arctan(cx)}{x^2}dx-c^2\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)-ic\left(2bc\int \frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx-ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(c\right.\right.\right.}{d}$$

$\Big\downarrow$ 5361

$$\frac{\frac{2}{3}bc\left(c^2\left(-\int \frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx\right)+\frac{1}{2}bc\int \frac{1}{x^2(c^2x^2+1)}dx-\frac{a+b\arctan(cx)}{2x^2}\right)-\frac{(a+b\arctan(cx))^2}{3x^3}}{d}-\frac{ic\left(bc\left(c^2\left(-\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)+bc\int \frac{1}{x(c^2x^2+1)}dx-\frac{a+b\arctan(cx)}{x}\right)-ic\left(2bc\int \frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx-ic\left(\log\left(2-\frac{2}{1+i}\right.\right.\right.\right.}{d}$$

$\Big\downarrow$ 243

$$\frac{\frac{2}{3}bc\left(c^2\left(-\int \frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx\right)+\frac{1}{2}bc\int \frac{1}{x^2(c^2x^2+1)}dx-\frac{a+b\arctan(cx)}{2x^2}\right)-\frac{(a+b\arctan(cx))^2}{3x^3}}{d}-\frac{ic\left(bc\left(c^2\left(-\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)+\frac{1}{2}bc\int \frac{1}{x^2(c^2x^2+1)}dx^2-\frac{a+b\arctan(cx)}{x}\right)-ic\left(2bc\int \frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx-ic\left(\log\left(2-\right.\right.\right.\right.}{d}$$

$\Big\downarrow$ 47

$$\frac{\frac{2}{3}bc\left(c^2\left(-\int\frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx\right)+\frac{1}{2}bc\int\frac{1}{x^2(c^2x^2+1)}dx-\frac{a+b\arctan(cx)}{2x^2}\right)-\frac{(a+b\arctan(cx))^2}{3x^3}}{d}-$$
$$ic\left(bc\left(c^2\left(-\int\frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)+\frac{1}{2}bc\left(\int\frac{1}{x^2}dx^2-c^2\int\frac{1}{c^2x^2+1}dx^2\right)-\frac{a+b\arctan(cx)}{x}\right)-ic\left(2bc\int\frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx-\right.\right.$$

$$\downarrow 14$$

$$\frac{\frac{2}{3}bc\left(c^2\left(-\int\frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx\right)+\frac{1}{2}bc\int\frac{1}{x^2(c^2x^2+1)}dx-\frac{a+b\arctan(cx)}{2x^2}\right)-\frac{(a+b\arctan(cx))^2}{3x^3}}{d}-$$
$$ic\left(bc\left(c^2\left(-\int\frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)+\frac{1}{2}bc\left(\log\left(x^2\right)-c^2\int\frac{1}{c^2x^2+1}dx^2\right)-\frac{a+b\arctan(cx)}{x}\right)-ic\left(2bc\int\frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx-\right.\right.$$

$$\downarrow 16$$

$$\frac{\frac{2}{3}bc\left(c^2\left(-\int\frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx\right)+\frac{1}{2}bc\int\frac{1}{x^2(c^2x^2+1)}dx-\frac{a+b\arctan(cx)}{2x^2}\right)-\frac{(a+b\arctan(cx))^2}{3x^3}}{d}-$$
$$ic\left(bc\left(c^2\left(-\int\frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)-\frac{a+b\arctan(cx)}{x}+\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right)\right)\right)-ic\left(2bc\int\frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx-i\right.\right.$$

$$\downarrow 264$$

$$\frac{\frac{2}{3}bc\left(c^2\left(-\int\frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx\right)+\frac{1}{2}bc\left(c^2\left(-\int\frac{1}{c^2x^2+1}dx\right)-\frac{1}{x}\right)-\frac{a+b\arctan(cx)}{2x^2}\right)-\frac{(a+b\arctan(cx))^2}{3x^3}}{d}-$$
$$ic\left(bc\left(c^2\left(-\int\frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)-\frac{a+b\arctan(cx)}{x}+\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right)\right)\right)-ic\left(2bc\int\frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx-i\right.\right.$$

$$\downarrow 216$$

$$\frac{\frac{2}{3}bc\left(c^2\left(-\int\frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx\right)-\frac{a+b\arctan(cx)}{2x^2}+\frac{1}{2}bc\left(-c\arctan(cx)-\frac{1}{x}\right)\right)-\frac{(a+b\arctan(cx))^2}{3x^3}}{d}-$$
$$ic\left(bc\left(c^2\left(-\int\frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)-\frac{a+b\arctan(cx)}{x}+\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right)\right)\right)-ic\left(2bc\int\frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx-i\right.\right.$$

$$\downarrow 5419$$

$$\frac{\frac{2}{3}bc\left(c^2\left(-\int \frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx\right)-\frac{a+b\arctan(cx)}{2x^2}+\frac{1}{2}bc\left(-c\arctan(cx)-\frac{1}{x}\right)\right)-\frac{(a+b\arctan(cx))^2}{3x^3}}{d}-$$

$$\frac{ic\left(-ic\left(2bc\int \frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx-ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2-2bc\int \frac{(a+b\arctan(cx))\log\left(2-\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)-\right.\right.}{d}$$

$\downarrow$ 5459

$$\frac{-\frac{(a+b\arctan(cx))^2}{3x^3}+\frac{2}{3}bc\left(-\left(c^2\left(i\int \frac{a+b\arctan(cx)}{x(cx+i)}dx-\frac{i(a+b\arctan(cx))^2}{2b}\right)\right)-\frac{a+b\arctan(cx)}{2x^2}+\frac{1}{2}bc\left(-c\arctan(cx)-\frac{1}{x}\right)\right.}{d}$$

$$ic\left(-ic\left(-ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2-2bc\int \frac{(a+b\arctan(cx))\log\left(2-\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)+2bc\left(i\int \frac{a+b\arctan(cx)}{x(cx+i)}\right.\right.\right.$$

$\downarrow$ 5403

$$\frac{-\frac{(a+b\arctan(cx))^2}{3x^3}+\frac{2}{3}bc\left(-\left(c^2\left(i\left(ibc\int \frac{\log\left(2-\frac{2}{1-icx}\right)}{c^2x^2+1}dx-i\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))\right)-\frac{i(a+b\arctan(cx))}{2b}\right.\right.\right.}{d}$$

$$ic\left(-ic\left(2bc\left(i\left(ibc\int \frac{\log\left(2-\frac{2}{1-icx}\right)}{c^2x^2+1}dx-i\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))\right)-\frac{i(a+b\arctan(cx))^2}{2b}\right)-ic\left(\log\left(2-\right.\right.\right.\right.$$

$\downarrow$ 2897

$$\frac{-\frac{(a+b\arctan(cx))^2}{3x^3}+\frac{2}{3}bc\left(-\left(c^2\left(i\left(-i\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))-\frac{1}{2}b\,\mathrm{PolyLog}\left(2,\frac{2}{1-icx}-1\right)\right)\right)-\frac{i(a+b\arcta}{2b}\right.\right.}{d}$$

$$ic\left(-ic\left(-ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2-2bc\int \frac{(a+b\arctan(cx))\log\left(2-\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)+2bc\left(i\left(-i\log\left(2-\frac{}{1-}\right.\right.\right.\right.\right.$$

$\downarrow$ 5529

$$\frac{-\frac{(a+b\arctan(cx))^2}{3x^3}+\frac{2}{3}bc\left(-\left(c^2\left(i\left(-i\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))-\frac{1}{2}b\,\mathrm{PolyLog}\left(2,\frac{2}{1-icx}-1\right)\right)\right)-\frac{i(a+b\arcta}{2b}\right.\right.}{d}$$

$$ic\left(-ic\left(-ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2-2bc\left(\frac{1}{2}ib\int \frac{\mathrm{PolyLog}\left(2,\frac{2}{icx+1}-1\right)}{c^2x^2+1}dx-\frac{i\,\mathrm{PolyLog}\left(2,\frac{2}{icx+1}-1\right)(a+b\arct}{2c}\right.\right.\right.\right.$$

$\downarrow$ 7164

$$\frac{-\frac{(a+b\arctan(cx))^2}{3x^3}+\frac{2}{3}bc\left(-\left(c^2\left(i\left(-i\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{2}{1-icx}-1\right)\right)-\frac{i(a+b\arctan}{2b}\right.\right.\right.}{d}$$

$$\frac{ic\left(bc\left(-\frac{c(a+b\arctan(cx))^2}{2b}-\frac{a+b\arctan(cx)}{x}+\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right)\right)\right)-ic\left(2bc\left(i\left(-i\log\left(2-\frac{2}{1-icx}\right)(a+\right.\right.\right.\right.}{}$$

input
```
Int[(a + b*ArcTan[c*x])^2/(x^4*(d + I*c*d*x)),x]
```

output
```
(-1/3*(a + b*ArcTan[c*x])^2/x^3 + (2*b*c*(-1/2*(a + b*ArcTan[c*x])/x^2 + (
b*c*(-x^(-1) - c*ArcTan[c*x]))/2 - c^2*(((-1/2*I)*(a + b*ArcTan[c*x])^2)/b
 + I*((-I)*(a + b*ArcTan[c*x])*Log[2 - 2/(1 - I*c*x)] - (b*PolyLog[2, -1 +
 2/(1 - I*c*x)])/2))))/3)/d - (I*c*(-1/2*(a + b*ArcTan[c*x])^2/x^2 + b*c*(
-((a + b*ArcTan[c*x])/x) - (c*(a + b*ArcTan[c*x])^2)/(2*b) + (b*c*(Log[x^2
] - Log[1 + c^2*x^2]))/2) - I*c*(-((a + b*ArcTan[c*x])^2/x) + 2*b*c*(((-1/
2*I)*(a + b*ArcTan[c*x])^2)/b + I*((-I)*(a + b*ArcTan[c*x])*Log[2 - 2/(1 -
 I*c*x)] - (b*PolyLog[2, -1 + 2/(1 - I*c*x)])/2)) - I*c*((a + b*ArcTan[c*x
])^2*Log[2 - 2/(1 + I*c*x)] - 2*b*c*(((-1/2*I)*(a + b*ArcTan[c*x])*PolyLog
[2, -1 + 2/(1 + I*c*x)])/c - (b*PolyLog[3, -1 + 2/(1 + I*c*x)])/(4*c))))))
/d
```

### Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5405
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x], x
] - Simp[e/(d*f)   Int[(f*x)^(m + 1)*((a + b*ArcTan[c*x])^p/(d + e*x)), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
&& LtQ[m, -1]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 19.84 (sec) , antiderivative size = 2180, normalized size of antiderivative = 5.97

| method | result | size |
|---|---|---|
| derivativedivides | Expression too large to display | 2180 |
| default | Expression too large to display | 2180 |
| parts | Expression too large to display | 2276 |

input
```
int((a+b*arctan(c*x))^2/x^4/(d+I*c*d*x),x,method=_RETURNVERBOSE)
```

output
```
c^3*(1/2*b^2/d*Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I*((1+I*c*x)^2/
(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*arctan(c*x)^2+a^2/d*arctan(c
*x)-1/2*I*a^2/d*ln(c^2*x^2+1)+1/2*b^2/d*Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+
1)))*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*
x)^2/(c^2*x^2+1)))*arctan(c*x)^2-1/2*b^2/d*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2
+1)-1))*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1
)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*arctan(c*x)^2+1/2*b^2/d*Pi*csgn((1+I*c*x
)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))
^2*arctan(c*x)^2+1/2*b^2/d*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c
*x)^2/(c^2*x^2+1)))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x
^2+1)))^2*arctan(c*x)^2+1/2*b^2/d*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*c
sgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*arctan(c*
x)^2-1/2*b^2/d*Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn((1+I*c*x)^2/(c^
2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*arctan(c*x)^2-1/2*b^2/d*Pi*csgn(I*
((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(((1+I*c*x)^2
/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*arctan(c*x)^2+I*b^2/d*arctan(
c*x)/c/x+1/2*I*b^2/d*arctan(c*x)^2/c^2/x^2+1/2*I*a^2/d/c^2/x^2-1/3*b^2/d*a
rctan(c*x)^2/c^3/x^3+b^2/d*arctan(c*x)^2/c/x-1/3*b^2/d*arctan(c*x)/c^2/x^2
+I*b^2/d*arctan(c*x)^2*ln(c*x)+I*b^2/d*arctan(c*x)^2*ln(2*I*(1+I*c*x)^2/(c
^2*x^2+1))+1/2*b^2/d*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2...
```

## Fricas [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^4(d+icdx)}\,dx = \int \frac{(b\arctan(cx)+a)^2}{(i\,cdx+d)x^4}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^4/(d+I*c*d*x),x, algorithm="fricas")
```

output
```
1/24*(-6*I*b^2*c^3*x^3*log(2*c*x/(c*x - I))*log(-(c*x + I)/(c*x - I))^2 -
12*I*b^2*c^3*x^3*dilog(-2*c*x/(c*x - I) + 1)*log(-(c*x + I)/(c*x - I)) + 1
2*I*b^2*c^3*x^3*polylog(3, -(c*x + I)/(c*x - I)) + 24*d*x^3*integral(1/6*(
-6*I*a^2*c*x + 6*a^2 + (-6*I*b^2*c^3*x^3 + 3*b^2*c^2*x^2 + 2*(3*a*b + I*b^
2)*c*x + 6*I*a*b)*log(-(c*x + I)/(c*x - I)))/(c^2*d*x^6 + d*x^4), x) - (6*
b^2*c^2*x^2 + 3*I*b^2*c*x - 2*b^2)*log(-(c*x + I)/(c*x - I))^2)/(d*x^3)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{x^4(d+icdx)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*atan(c*x))**2/x**4/(d+I*c*d*x),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^4(d+icdx)}\,dx = \int \frac{(b\arctan(cx)+a)^2}{(i\,cdx+d)x^4}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^4/(d+I*c*d*x),x, algorithm="maxima")
```

output
```
-1/6*(6*I*c^3*log(I*c*x + 1)/d - 6*I*c^3*log(x)/d - (6*c^2*x^2 + 3*I*c*x -
 2)/(d*x^3))*a^2 + 1/96*(24*b^2*c^3*x^3*arctan(c*x)^3 + 3*I*b^2*c^3*x^3*lo
g(c^2*x^2 + 1)^3 - 2*I*(1152*b^2*c^5*integrate(1/48*x^5*arctan(c*x)^2/(c^2
*d*x^6 + d*x^4), x) + b^2*c^3*log(c^2*x^2 + 1)^3/d + 12*b^2*c^3*arctan(c*x
)^2/d + 288*b^2*c^3*integrate(1/48*x^3*log(c^2*x^2 + 1)/(c^2*d*x^6 + d*x^4
), x) + 192*b^2*c^2*integrate(1/48*x^2*arctan(c*x)/(c^2*d*x^6 + d*x^4), x)
 + 1728*b^2*c*integrate(1/48*x*arctan(c*x)^2/(c^2*d*x^6 + d*x^4), x) + 144
*b^2*c*integrate(1/48*x*log(c^2*x^2 + 1)^2/(c^2*d*x^6 + d*x^4), x) + 4608*
a*b*c*integrate(1/48*x*arctan(c*x)/(c^2*d*x^6 + d*x^4), x) - 192*b^2*c*int
egrate(1/48*x*log(c^2*x^2 + 1)/(c^2*d*x^6 + d*x^4), x) + 576*b^2*integrate
(1/48*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*d*x^6 + d*x^4), x))*d*x^3 - 16*(b^
2*c^3*arctan(c*x)^3/d + 36*b^2*c^4*integrate(1/48*x^4*log(c^2*x^2 + 1)^2/(
c^2*d*x^6 + d*x^4), x) - 72*b^2*c^4*integrate(1/48*x^4*log(c^2*x^2 + 1)/(c
^2*d*x^6 + d*x^4), x) + 72*b^2*c^3*integrate(1/48*x^3*arctan(c*x)/(c^2*d*x
^6 + d*x^4), x) - 12*b^2*c^2*integrate(1/48*x^2*log(c^2*x^2 + 1)/(c^2*d*x^
6 + d*x^4), x) + 72*b^2*c*integrate(1/48*x*arctan(c*x)*log(c^2*x^2 + 1)/(c
^2*d*x^6 + d*x^4), x) - 48*b^2*c*integrate(1/48*x*arctan(c*x)/(c^2*d*x^6 +
 d*x^4), x) - 216*b^2*integrate(1/48*arctan(c*x)^2/(c^2*d*x^6 + d*x^4), x)
 - 18*b^2*integrate(1/48*log(c^2*x^2 + 1)^2/(c^2*d*x^6 + d*x^4), x) - 576*
a*b*integrate(1/48*arctan(c*x)/(c^2*d*x^6 + d*x^4), x))*d*x^3 + 4*(6*b^...
```

## Giac [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^4(d+icdx)}\,dx=\int \frac{(b\arctan(cx)+a)^2}{(i\,cdx+d)x^4}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^4/(d+I*c*d*x),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)^2/((I*c*d*x + d)*x^4), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{x^4(d+icdx)}\,dx=\int \frac{\left(a+b\,\mathrm{atan}(c\,x)\right)^2}{x^4\,\left(d+c\,d\,x\,1\mathrm{i}\right)}\,dx$$

input
```
int((a + b*atan(c*x))^2/(x^4*(d + c*d*x*1i)),x)
```

output
```
int((a + b*atan(c*x))^2/(x^4*(d + c*d*x*1i)), x)
```

## Reduce [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^4(d+icdx)}\,dx = \frac{-atan(cx)^3 b^2c^3x^3+3atan(cx)^2 b^2c^3i\,x^3-3atan(cx)^2 b^2c^2x^2-3atan(cx)\,b^2c^3x^3+6atan(cx)\,b^2c^2i\,x^2-}{}$$

input
```
int((a+b*atan(c*x))^2/x^4/(d+I*c*d*x),x)
```

output
```
( - atan(c*x)**3*b**2*c**3*x**3 + 3*atan(c*x)**2*b**2*c**3*i*x**3 - 3*atan
(c*x)**2*b**2*c**2*x**2 - 3*atan(c*x)*b**2*c**3*x**3 + 6*atan(c*x)*b**2*c*
*2*i*x**2 - 3*atan(c*x)*b**2*c*x - 2*atan(c*x)*b**2*i - 6*int(atan(c*x)/(c
**3*x**7 - c**2*i*x**6 + c*x**5 - i*x**4),x)*a*b*i*x**3 - 6*int(atan(c*x)/
(c**3*x**7 - c**2*i*x**6 + c*x**5 - i*x**4),x)*b**2*x**3 - 6*int(atan(c*x)
/(c**3*x**5 - c**2*i*x**4 + c*x**3 - i*x**2),x)*a*b*c**2*i*x**3 - 6*int(at
an(c*x)/(c**3*x**5 - c**2*i*x**4 + c*x**3 - i*x**2),x)*b**2*c**2*x**3 - 3*
int(atan(c*x)**2/(c**3*x**7 - c**2*i*x**6 + c*x**5 - i*x**4),x)*b**2*i*x**
3 - 3*int(atan(c*x)**2/(c**3*x**4 - c**2*i*x**3 + c*x**2 - i*x),x)*b**2*c*
*3*x**3 - 3*int(1/(c**3*x**6 - c**2*i*x**5 + c*x**4 - i*x**3),x)*a**2*c*x*
*3 - 3*int(1/(c**3*x**4 - c**2*i*x**3 + c*x**2 - i*x),x)*a**2*c**3*x**3 +
4*log(c**2*x**2 + 1)*b**2*c**3*i*x**3 - 8*log(x)*b**2*c**3*i*x**3 - a**2 -
 3*b**2*c**2*x**2 - b**2*c*i*x)/(3*d*x**3)
```

# 3.103 $\int \frac{x^4(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 433

$$\begin{aligned}\int \frac{x^4(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx &= \frac{2iabx}{c^4d^2}-\frac{b^2x}{3c^4d^2}+\frac{b^2}{2c^5d^2(i-cx)}-\frac{b^2\arctan(cx)}{6c^5d^2}\\
&+\frac{2ib^2x\arctan(cx)}{c^4d^2}+\frac{bx^2(a+b\arctan(cx))}{3c^3d^2}\\
&+\frac{ib(a+b\arctan(cx))}{c^5d^2(i-cx)}+\frac{11i(a+b\arctan(cx))^2}{6c^5d^2}\\
&+\frac{3x(a+b\arctan(cx))^2}{c^4d^2}-\frac{ix^2(a+b\arctan(cx))^2}{c^3d^2}\\
&-\frac{x^3(a+b\arctan(cx))^2}{3c^2d^2}-\frac{(a+b\arctan(cx))^2}{c^5d^2(i-cx)}\\
&+\frac{20b(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{3c^5d^2}\\
&-\frac{4i(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{c^5d^2}\\
&-\frac{ib^2\log(1+c^2x^2)}{c^5d^2}+\frac{10ib^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{3c^5d^2}\\
&+\frac{4b(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^5d^2}\\
&-\frac{2ib^2\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{c^5d^2}\end{aligned}$$

output
```
-2*I*b^2*polylog(3,1-2/(1+I*c*x))/c^5/d^2-1/3*b^2*x/c^4/d^2+1/2*b^2/c^5/d^
2/(I-c*x)-1/6*b^2*arctan(c*x)/c^5/d^2+10/3*I*b^2*polylog(2,1-2/(1+I*c*x))/
c^5/d^2+1/3*b*x^2*(a+b*arctan(c*x))/c^3/d^2-I*x^2*(a+b*arctan(c*x))^2/c^3/
d^2+2*I*a*b*x/c^4/d^2+3*x*(a+b*arctan(c*x))^2/c^4/d^2+11/6*I*(a+b*arctan(c
*x))^2/c^5/d^2-1/3*x^3*(a+b*arctan(c*x))^2/c^2/d^2-(a+b*arctan(c*x))^2/c^5
/d^2/(I-c*x)+20/3*b*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^5/d^2-4*I*(a+b*arc
tan(c*x))^2*ln(2/(1+I*c*x))/c^5/d^2-I*b^2*ln(c^2*x^2+1)/c^5/d^2+2*I*b^2*x*
arctan(c*x)/c^4/d^2+4*b*(a+b*arctan(c*x))*polylog(2,1-2/(1+I*c*x))/c^5/d^2
+I*b*(a+b*arctan(c*x))/c^5/d^2/(I-c*x)
```

## Mathematica [A] (verified)

Time = 1.67 (sec) , antiderivative size = 502, normalized size of antiderivative = 1.16

$$\int \frac{x^4(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx =$$
$$-\frac{-36a^2cx+12ia^2c^2x^2+4a^2c^3x^3-\frac{12a^2}{-i+cx}+48a^2\arctan(cx)-24ia^2\log\left(1+c^2x^2\right)+2ab(-2-12icx\cdots}{\cdots}$$

input
```
Integrate[(x^4*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x)^2,x]
```

output
```
-1/12*(-36*a^2*c*x + (12*I)*a^2*c^2*x^2 + 4*a^2*c^3*x^3 - (12*a^2)/(-I + c
*x) + 48*a^2*ArcTan[c*x] - (24*I)*a^2*Log[1 + c^2*x^2] + 2*a*b*(-2 - (12*I
)*c*x - 2*c^2*x^2 + 48*ArcTan[c*x]^2 - 3*Cos[2*ArcTan[c*x]] + 20*Log[1 + c
^2*x^2] + 24*PolyLog[2, -E^((2*I)*ArcTan[c*x])] + 2*ArcTan[c*x]*(6*I - 18*
c*x + (6*I)*c^2*x^2 + 2*c^3*x^3 - (3*I)*Cos[2*ArcTan[c*x]] + (24*I)*Log[1
+ E^((2*I)*ArcTan[c*x])] - 3*Sin[2*ArcTan[c*x]]) + (3*I)*Sin[2*ArcTan[c*x]
]) + b^2*(4*c*x - 4*ArcTan[c*x] - (24*I)*c*x*ArcTan[c*x] - 4*c^2*x^2*ArcTa
n[c*x] + (52*I)*ArcTan[c*x]^2 - 36*c*x*ArcTan[c*x]^2 + (12*I)*c^2*x^2*ArcT
an[c*x]^2 + 4*c^3*x^3*ArcTan[c*x]^2 + 32*ArcTan[c*x]^3 + (3*I)*Cos[2*ArcTa
n[c*x]] - 6*ArcTan[c*x]*Cos[2*ArcTan[c*x]] - (6*I)*ArcTan[c*x]^2*Cos[2*Arc
Tan[c*x]] - 80*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] + (48*I)*ArcTan[
c*x]^2*Log[1 + E^((2*I)*ArcTan[c*x])] + (12*I)*Log[1 + c^2*x^2] + 8*(5*I +
 6*ArcTan[c*x])*PolyLog[2, -E^((2*I)*ArcTan[c*x])] + (24*I)*PolyLog[3, -E^
((2*I)*ArcTan[c*x])] + 3*Sin[2*ArcTan[c*x]] + (6*I)*ArcTan[c*x]*Sin[2*ArcT
an[c*x]] - 6*ArcTan[c*x]^2*Sin[2*ArcTan[c*x]]))/(c^5*d^2)
```

## Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 433, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a+b\arctan(cx))^2}{(d+icdx)^2}dx$$

$$\Big\downarrow \text{5411}$$

$$\int \left(\frac{4i(a+b\arctan(cx))^2}{c^4d^2(cx-i)}+\frac{3(a+b\arctan(cx))^2}{c^4d^2}-\frac{(a+b\arctan(cx))^2}{c^4d^2(cx-i)^2}-\frac{2ix(a+b\arctan(cx))^2}{c^3d^2}-\frac{x^2(a+b\operatorname{ar}}{c^2}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{4b\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{c^5d^2}-\frac{(a+b\arctan(cx))^2}{c^5d^2(-cx+i)}+\frac{11i(a+b\arctan(cx))^2}{6c^5d^2}+\frac{ib(a+b\arctan(cx))}{c^5d^2(-cx+i)}-\frac{4i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c^5d^2}+\frac{20b\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{3c^5d^2}+\frac{3x(a+b\arctan(cx))^2}{c^4d^2}-\frac{ix^2(a+b\arctan(cx))^2}{c^3d^2}+\frac{bx^2(a+b\arctan(cx))}{3c^3d^2}-\frac{x^3(a+b\arctan(cx))^2}{3c^2d^2}+\frac{2iabx}{c^4d^2}-\frac{b^2\arctan(cx)}{6c^5d^2}+\frac{2ib^2x\arctan(cx)}{c^4d^2}+\frac{10ib^2\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{3c^5d^2}-\frac{2ib^2\operatorname{PolyLog}\left(3,1-\frac{2}{icx+1}\right)}{c^5d^2}+\frac{b^2}{2c^5d^2(-cx+i)}-\frac{b^2x}{3c^4d^2}-\frac{ib^2\log\left(c^2x^2+1\right)}{c^5d^2}$$

input
```
Int[(x^4*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x)^2,x]
```

output
```
((2*I)*a*b*x)/(c^4*d^2) - (b^2*x)/(3*c^4*d^2) + b^2/(2*c^5*d^2*(I - c*x))
- (b^2*ArcTan[c*x])/(6*c^5*d^2) + ((2*I)*b^2*x*ArcTan[c*x])/(c^4*d^2) + (b
*x^2*(a + b*ArcTan[c*x]))/(3*c^3*d^2) + (I*b*(a + b*ArcTan[c*x]))/(c^5*d^2
*(I - c*x)) + (((11*I)/6)*(a + b*ArcTan[c*x])^2)/(c^5*d^2) + (3*x*(a + b*A
rcTan[c*x])^2)/(c^4*d^2) - (I*x^2*(a + b*ArcTan[c*x])^2)/(c^3*d^2) - (x^3*
(a + b*ArcTan[c*x])^2)/(3*c^2*d^2) - (a + b*ArcTan[c*x])^2/(c^5*d^2*(I - c
*x)) + (20*b*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(3*c^5*d^2) - ((4*I)*
(a + b*ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/(c^5*d^2) - (I*b^2*Log[1 + c^2*x
^2])/(c^5*d^2) + (((10*I)/3)*b^2*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c^5*d^2)
+ (4*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c^5*d^2) - ((2*
I)*b^2*PolyLog[3, 1 - 2/(1 + I*c*x)])/(c^5*d^2)
```

**Defintions of rubi rules used**

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 15.68 (sec) , antiderivative size = 1199, normalized size of antiderivative = 2.77

| method | result | size |
|---|---|---|
| derivativedivides | Expression too large to display | 1199 |
| default | Expression too large to display | 1199 |
| parts | Expression too large to display | 1254 |

input
```
int(x^4*(a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c^5*(2*I*a*b/d^2*c*x+a^2/d^2/(c*x-I)+b^2/d^2*(-1/2*(c*x+I)*arctan(c*x)/(
c*x-I)+4/3*I*arctan(c*x)*(c*x-I)+2*I*(c*x+I)/(8*c*x-8*I)+2/3*arctan(c*x)*(
c*x-I)*(c*x+I)-1/3*c^3*x^3*arctan(c*x)^2-1/3*I+3*c*x*arctan(c*x)^2-1/3*c*x
-8/3*arctan(c*x)^3+arctan(c*x)^2/(c*x-I)+4*I*arctan(c*x)^2*ln(c*x-I)-4*I*a
rctan(c*x)^2*ln(2*I*(1+I*c*x)^2/(c^2*x^2+1))-4*Pi*csgn((1+I*c*x)^2/(c^2*x^
2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*arctan(c*x)^2-2*Pi*csgn((1+I*c*x)^2/(c
^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^3*arctan(c*x)^2-2*Pi*csgn(I/(1+(1+I
*c*x)^2/(c^2*x^2+1)))*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*
x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*arctan(c*x)^2-1/3*arctan(c*x)*(c*x-I)^
2-2*I*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))-20/3*I*dilog(1-I*(1+I*c*x)/(c^2*
x^2+1)^(1/2))+2*I*ln(1+(1+I*c*x)^2/(c^2*x^2+1))-20/3*I*dilog(1+I*(1+I*c*x)
/(c^2*x^2+1)^(1/2))-29/6*I*arctan(c*x)^2+20/3*arctan(c*x)*ln(1+I*(1+I*c*x)
/(c^2*x^2+1)^(1/2))+20/3*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+4
*Pi*arctan(c*x)^2-4*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))+2*Pi*c
sgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*
x)^2/(c^2*x^2+1)))^2*arctan(c*x)^2-2*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn
((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*arctan(c*x)^2-I*ar
ctan(c*x)^2*c^2*x^2)-4*a^2/d^2*arctan(c*x)+8*I*a*b/d^2*arctan(c*x)*ln(c*x-
I)+6*a*b/d^2*arctan(c*x)*c*x-2/3*a*b/d^2*arctan(c*x)*c^3*x^3+7/3*a*b/d^2-1
/3*a^2/d^2*c^3*x^3+3*a^2/d^2*c*x-2*a*b/d^2*ln(c*x-I)^2+4*a*b/d^2*dilog(...
```

## Fricas [F]

$$\int \frac{x^4(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx=\int \frac{(b\arctan(cx)+a)^2x^4}{(i\,cdx+d)^2}\,dx$$

input
```
integrate(x^4*(a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x, algorithm="fricas")
```

output
```
integral(1/4*(b^2*x^4*log(-(c*x + I)/(c*x - I))^2 - 4*I*a*b*x^4*log(-(c*x
+ I)/(c*x - I)) - 4*a^2*x^4)/(c^2*d^2*x^2 - 2*I*c*d^2*x - d^2), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**4*(a+b*atan(c*x))**2/(d+I*c*d*x)**2,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{x^4(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx = \int \frac{(b\arctan(cx)+a)^2x^4}{(i\,cdx+d)^2}\,dx$$

input
```
integrate(x^4*(a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x, algorithm="maxima")
```

output
```
1/3*a^2*(3/(c^6*d^2*x - I*c^5*d^2) - (c^2*x^3 + 3*I*c*x^2 - 9*x)/(c^4*d^2)
 + 12*I*log(c*x - I)/(c^5*d^2)) - 1/48*(48*(b^2*c*x - I*b^2)*arctan(c*x)^3
 - 6*(-I*b^2*c*x - b^2)*log(c^2*x^2 + 1)^3 + 4*(b^2*c^4*x^4 + 2*I*b^2*c^3*
x^3 - 6*b^2*c^2*x^2 + 9*I*b^2*c*x - 3*b^2)*arctan(c*x)^2 - (b^2*c^4*x^4 +
2*I*b^2*c^3*x^3 - 6*b^2*c^2*x^2 + 9*I*b^2*c*x - 3*b^2 - 12*(b^2*c*x - I*b^
2)*arctan(c*x))*log(c^2*x^2 + 1)^2 + 6*(c^6*d^2*x - I*c^5*d^2)*(288*b^2*c^
6*integrate(1/48*x^6*arctan(c*x)^2/(c^8*d^2*x^4 + 2*c^6*d^2*x^2 + c^4*d^2)
, x) + 24*b^2*c^6*integrate(1/48*x^6*log(c^2*x^2 + 1)^2/(c^8*d^2*x^4 + 2*c
^6*d^2*x^2 + c^4*d^2), x) + 768*a*b*c^6*integrate(1/48*x^6*arctan(c*x)/(c^
8*d^2*x^4 + 2*c^6*d^2*x^2 + c^4*d^2), x) + 32*b^2*c^6*integrate(1/48*x^6*l
og(c^2*x^2 + 1)/(c^8*d^2*x^4 + 2*c^6*d^2*x^2 + c^4*d^2), x) + 192*b^2*c^5*
integrate(1/48*x^5*arctan(c*x)*log(c^2*x^2 + 1)/(c^8*d^2*x^4 + 2*c^6*d^2*x
^2 + c^4*d^2), x) + 128*b^2*c^5*integrate(1/48*x^5*arctan(c*x)/(c^8*d^2*x^
4 + 2*c^6*d^2*x^2 + c^4*d^2), x) - 288*b^2*c^4*integrate(1/48*x^4*arctan(c
*x)^2/(c^8*d^2*x^4 + 2*c^6*d^2*x^2 + c^4*d^2), x) - 24*b^2*c^4*integrate(1
/48*x^4*log(c^2*x^2 + 1)^2/(c^8*d^2*x^4 + 2*c^6*d^2*x^2 + c^4*d^2), x) - 7
68*a*b*c^4*integrate(1/48*x^4*arctan(c*x)/(c^8*d^2*x^4 + 2*c^6*d^2*x^2 + c
^4*d^2), x) - 160*b^2*c^4*integrate(1/48*x^4*log(c^2*x^2 + 1)/(c^8*d^2*x^4
 + 2*c^6*d^2*x^2 + c^4*d^2), x) + 704*b^2*c^3*integrate(1/48*x^3*arctan(c*
x)/(c^8*d^2*x^4 + 2*c^6*d^2*x^2 + c^4*d^2), x) - 768*b^2*c^2*integrate(...
```

## Giac [F]

$$\int \frac{x^4(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx=\int \frac{(b\arctan(cx)+a)^2x^4}{(i\,cdx+d)^2}\,dx$$

input
```
integrate(x^4*(a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)^2*x^4/(I*c*d*x + d)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx=\int \frac{x^4\,{\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}^2}{{\left(d+c\,d\,x\,1{}\mathrm{i}\right)}^2}\,dx$$

input
```
int((x^4*(a + b*atan(c*x))^2)/(d + c*d*x*1i)^2,x)
```

output
```
int((x^4*(a + b*atan(c*x))^2)/(d + c*d*x*1i)^2, x)
```

## Reduce [F]

$$\int \frac{x^4(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx = \frac{-\left(\int \frac{x^4}{c^2x^2-2cix-1}dx\right)a^2-2\left(\int \frac{atan(cx)\,x^4}{c^2x^2-2cix-1}dx\right)ab-\left(\int \frac{atan(cx)^2x^4}{c^2x^2-2cix-1}dx\right)b^2}{d^2}$$

input
```
int(x^4*(a+b*atan(c*x))^2/(d+I*c*d*x)^2,x)
```

output
```
( - int(x**4/(c**2*x**2 - 2*c*i*x - 1),x)*a**2 - 2*int((atan(c*x)*x**4)/(c
**2*x**2 - 2*c*i*x - 1),x)*a*b - int((atan(c*x)**2*x**4)/(c**2*x**2 - 2*c*
i*x - 1),x)*b**2)/d**2
```

# 3.104 $\int \frac{x^3(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 364

$$\begin{aligned}\int \frac{x^3(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx &= \frac{abx}{c^3d^2}-\frac{ib^2}{2c^4d^2(i-cx)}+\frac{ib^2\arctan(cx)}{2c^4d^2}+\frac{b^2x\arctan(cx)}{c^3d^2}\\
&+\frac{b(a+b\arctan(cx))}{c^4d^2(i-cx)}+\frac{(a+b\arctan(cx))^2}{c^4d^2}\\
&-\frac{2ix(a+b\arctan(cx))^2}{c^3d^2}-\frac{x^2(a+b\arctan(cx))^2}{2c^2d^2}\\
&+\frac{i(a+b\arctan(cx))^2}{c^4d^2(i-cx)}-\frac{4ib(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c^4d^2}\\
&-\frac{3(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{c^4d^2}\\
&-\frac{b^2\log(1+c^2x^2)}{2c^4d^2}+\frac{2b^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^4d^2}\\
&-\frac{3ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^4d^2}\\
&-\frac{3b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2c^4d^2}\end{aligned}$$

output
```
a*b*x/c^3/d^2-1/2*I*b^2/c^4/d^2/(I-c*x)+1/2*I*b^2*arctan(c*x)/c^4/d^2+b^2*
x*arctan(c*x)/c^3/d^2+b*(a+b*arctan(c*x))/c^4/d^2/(I-c*x)+(a+b*arctan(c*x)
)^2/c^4/d^2-2*I*x*(a+b*arctan(c*x))^2/c^3/d^2-1/2*x^2*(a+b*arctan(c*x))^2/
c^2/d^2+I*(a+b*arctan(c*x))^2/c^4/d^2/(I-c*x)-4*I*b*(a+b*arctan(c*x))*ln(2
/(1+I*c*x))/c^4/d^2-3*(a+b*arctan(c*x))^2*ln(2/(1+I*c*x))/c^4/d^2-1/2*b^2*
ln(c^2*x^2+1)/c^4/d^2+2*b^2*polylog(2,1-2/(1+I*c*x))/c^4/d^2-3*I*b*(a+b*ar
ctan(c*x))*polylog(2,1-2/(1+I*c*x))/c^4/d^2-3/2*b^2*polylog(3,1-2/(1+I*c*x
))/c^4/d^2
```

## Mathematica [A] (verified)

Time = 1.34 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.18

$$\int \frac{x^3(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx =$$

$$-\frac{8ia^2cx+2a^2c^2x^2+\frac{4ia^2}{-i+cx}-12ia^2\arctan(cx)-6a^2\log\left(1+c^2x^2\right)+2ab\left(-2cx-12i\arctan(cx)^2+i\,c\right.}{}$$

input
```
Integrate[(x^3*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x)^2,x]
```

output
```
-1/4*((8*I)*a^2*c*x + 2*a^2*c^2*x^2 + ((4*I)*a^2)/(-I + c*x) - (12*I)*a^2*
ArcTan[c*x] - 6*a^2*Log[1 + c^2*x^2] + 2*a*b*(-2*c*x - (12*I)*ArcTan[c*x]^
2 + I*Cos[2*ArcTan[c*x]] - (4*I)*Log[1 + c^2*x^2] - (6*I)*PolyLog[2, -E^((
2*I)*ArcTan[c*x])] + 2*ArcTan[c*x]*(1 + (4*I)*c*x + c^2*x^2 - Cos[2*ArcTan
[c*x]] + 6*Log[1 + E^((2*I)*ArcTan[c*x])] + I*Sin[2*ArcTan[c*x]]) + Sin[2*
ArcTan[c*x]]) + b^2*(-4*c*x*ArcTan[c*x] + 10*ArcTan[c*x]^2 + (8*I)*c*x*Arc
Tan[c*x]^2 + 2*c^2*x^2*ArcTan[c*x]^2 - (8*I)*ArcTan[c*x]^3 + Cos[2*ArcTan[
c*x]] + (2*I)*ArcTan[c*x]*Cos[2*ArcTan[c*x]] - 2*ArcTan[c*x]^2*Cos[2*ArcTa
n[c*x]] + (16*I)*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] + 12*ArcTan[c*
x]^2*Log[1 + E^((2*I)*ArcTan[c*x])] + 2*Log[1 + c^2*x^2] + 4*(2 - (3*I)*Ar
cTan[c*x])*PolyLog[2, -E^((2*I)*ArcTan[c*x])] + 6*PolyLog[3, -E^((2*I)*Arc
Tan[c*x])] - I*Sin[2*ArcTan[c*x]] + 2*ArcTan[c*x]*Sin[2*ArcTan[c*x]] + (2*
I)*ArcTan[c*x]^2*Sin[2*ArcTan[c*x]]))/(c^4*d^2)
```

## Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 364, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\arctan(cx))^2}{(d+icdx)^2}dx$$

$$\Big\downarrow \text{5411}$$

$$\int \left(\frac{3(a+b\arctan(cx))^2}{c^3d^2(cx-i)}-\frac{2i(a+b\arctan(cx))^2}{c^3d^2}+\frac{i(a+b\arctan(cx))^2}{c^3d^2(cx-i)^2}-\frac{x(a+b\arctan(cx))^2}{c^2d^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3ib\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{c^4d^2}+\frac{b(a+b\arctan(cx))}{c^4d^2(-cx+i)}+\frac{i(a+b\arctan(cx))^2}{c^4d^2(-cx+i)}+\frac{(a+b\arctan(cx))^2}{c^4d^2}-\frac{4ib\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c^4d^2}-\frac{3\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c^4d^2}-\frac{2ix(a+b\arctan(cx))^2}{c^3d^2}-\frac{x^2(a+b\arctan(cx))^2}{2c^2d^2}+\frac{abx}{c^3d^2}+\frac{ib^2\arctan(cx)}{2c^4d^2}+\frac{b^2x\arctan(cx)}{c^3d^2}+\frac{2b^2\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{c^4d^2}-\frac{3b^2\operatorname{PolyLog}\left(3,1-\frac{2}{icx+1}\right)}{2c^4d^2}-\frac{ib^2}{2c^4d^2(-cx+i)}-\frac{b^2\log\left(c^2x^2+1\right)}{2c^4d^2}$$

input
```
Int[(x^3*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x)^2,x]
```

output
```
(a*b*x)/(c^3*d^2) - ((I/2)*b^2)/(c^4*d^2*(I - c*x)) + ((I/2)*b^2*ArcTan[c*
x])/(c^4*d^2) + (b^2*x*ArcTan[c*x])/(c^3*d^2) + (b*(a + b*ArcTan[c*x]))/(c
^4*d^2*(I - c*x)) + (a + b*ArcTan[c*x])^2/(c^4*d^2) - ((2*I)*x*(a + b*ArcT
an[c*x])^2)/(c^3*d^2) - (x^2*(a + b*ArcTan[c*x])^2)/(2*c^2*d^2) + (I*(a +
b*ArcTan[c*x])^2)/(c^4*d^2*(I - c*x)) - ((4*I)*b*(a + b*ArcTan[c*x])*Log[2
/(1 + I*c*x)])/(c^4*d^2) - (3*(a + b*ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/(c
^4*d^2) - (b^2*Log[1 + c^2*x^2])/(2*c^4*d^2) + (2*b^2*PolyLog[2, 1 - 2/(1
+ I*c*x)])/(c^4*d^2) - ((3*I)*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 +
I*c*x)])/(c^4*d^2) - (3*b^2*PolyLog[3, 1 - 2/(1 + I*c*x)])/(2*c^4*d^2)
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 12.15 (sec) , antiderivative size = 1104, normalized size of antiderivative = 3.03

| method | result | size |
|---|---|---|
| derivativedivides | Expression too large to display | 1104 |
| default | Expression too large to display | 1104 |
| parts | Expression too large to display | 1158 |

input
```
int(x^3*(a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c^4*(-I*a*b/d^2-1/2*a^2/d^2*c^2*x^2-2*I*a^2/d^2*c*x+3/2*a^2/d^2*ln(c^2*x
^2+1)+3/2*I*a*b/d^2*ln(c*x-I)^2+b^2/d^2*(3/2*I*Pi*csgn((1+I*c*x)^2/(c^2*x^
2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*arctan(c
*x)^2-1/2*c^2*x^2*arctan(c*x)^2-4*I*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+
1)^(1/2))+3*ln(c*x-I)*arctan(c*x)^2+1/4*(c*x+I)/(c*x-I)-2*I*arctan(c*x)^2*
c*x+I*(c*x+I)*arctan(c*x)/(2*c*x-2*I)+ln(1+(1+I*c*x)^2/(c^2*x^2+1))+3/2*I*
Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^3*arctan(c*x)
^2-4*I*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+3*I*arctan(c*x)*pol
ylog(2,-(1+I*c*x)^2/(c^2*x^2+1))+2*I*arctan(c*x)^3+arctan(c*x)*(c*x-I)-I*a
rctan(c*x)^2/(c*x-I)-3*I*Pi*arctan(c*x)^2-3/2*I*Pi*csgn(I/(1+(1+I*c*x)^2/(
c^2*x^2+1)))*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*a
rctan(c*x)^2+3*I*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1
)))^2*arctan(c*x)^2+3/2*I*Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn((1+I
*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+
1)))*arctan(c*x)^2-3*arctan(c*x)^2-4*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2)
)-4*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-3*arctan(c*x)^2*ln(2*I*(1+I*c*x
)^2/(c^2*x^2+1))-3/2*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1)))+3*I*a^2/d^2*arct
an(c*x)-a*b/d^2*arctan(c*x)*c^2*x^2-3*I*a*b/d^2*dilog(-1/2*I*(c*x+I))+6*a*
b/d^2*arctan(c*x)*ln(c*x-I)+3/2*I*a*b/d^2*ln(c^2*x^2+1)+1/4*I*a*b/d^2*ln(c
^4*x^4+10*c^2*x^2+9)-I*a^2/d^2/(c*x-I)+a*b/d^2*c*x-4*I*a*b/d^2*arctan(c...
```

## Fricas [F]

$$\int \frac{x^3(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx=\int \frac{\left(b\arctan\left(cx\right)+a\right)^2x^3}{\left(i\,cdx+d\right)^2}\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x, algorithm="fricas")
```

output
```
integral(1/4*(b^2*x^3*log(-(c*x + I)/(c*x - I))^2 - 4*I*a*b*x^3*log(-(c*x
+ I)/(c*x - I)) - 4*a^2*x^3)/(c^2*d^2*x^2 - 2*I*c*d^2*x - d^2), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(a+b*atan(c*x))**2/(d+I*c*d*x)**2,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{x^3(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx = \int \frac{(b\arctan(cx)+a)^2x^3}{(i\,cdx+d)^2}\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x, algorithm="maxima")
```

output
```
-1/2*a^2*(2*I/(c^5*d^2*x - I*c^4*d^2) + (c*x^2 + 4*I*x)/(c^3*d^2) - 6*log(
c*x - I)/(c^4*d^2)) + 1/32*(24*(I*b^2*c*x + b^2)*arctan(c*x)^3 - 3*(b^2*c*
x - I*b^2)*log(c^2*x^2 + 1)^3 - 4*(b^2*c^3*x^3 + 3*I*b^2*c^2*x^2 + 4*b^2*c
*x + 2*I*b^2)*arctan(c*x)^2 + (b^2*c^3*x^3 + 3*I*b^2*c^2*x^2 + 4*b^2*c*x +
 2*I*b^2 + 6*(I*b^2*c*x + b^2)*arctan(c*x))*log(c^2*x^2 + 1)^2 - 2*(c^5*d^
2*x - I*c^4*d^2)*(192*b^2*c^5*integrate(1/16*x^5*arctan(c*x)^2/(c^7*d^2*x^
4 + 2*c^5*d^2*x^2 + c^3*d^2), x) + 16*b^2*c^5*integrate(1/16*x^5*log(c^2*x
^2 + 1)^2/(c^7*d^2*x^4 + 2*c^5*d^2*x^2 + c^3*d^2), x) + 512*a*b*c^5*integr
ate(1/16*x^5*arctan(c*x)/(c^7*d^2*x^4 + 2*c^5*d^2*x^2 + c^3*d^2), x) + 32*
b^2*c^5*integrate(1/16*x^5*log(c^2*x^2 + 1)/(c^7*d^2*x^4 + 2*c^5*d^2*x^2 +
 c^3*d^2), x) + 128*b^2*c^4*integrate(1/16*x^4*arctan(c*x)*log(c^2*x^2 + 1
)/(c^7*d^2*x^4 + 2*c^5*d^2*x^2 + c^3*d^2), x) + 192*b^2*c^4*integrate(1/16
*x^4*arctan(c*x)/(c^7*d^2*x^4 + 2*c^5*d^2*x^2 + c^3*d^2), x) + 6*b^2*c^3*(
c^2/(c^11*d^2*x^2 + c^9*d^2) + log(c^2*x^2 + 1)/(c^9*d^2*x^2 + c^7*d^2)) -
 576*b^2*c^3*integrate(1/16*x^3*arctan(c*x)^2/(c^7*d^2*x^4 + 2*c^5*d^2*x^2
 + c^3*d^2), x) - 112*b^2*c^3*integrate(1/16*x^3*log(c^2*x^2 + 1)^2/(c^7*d
^2*x^4 + 2*c^5*d^2*x^2 + c^3*d^2), x) - 512*a*b*c^3*integrate(1/16*x^3*arc
tan(c*x)/(c^7*d^2*x^4 + 2*c^5*d^2*x^2 + c^3*d^2), x) + 160*b^2*c^3*integra
te(1/16*x^3*log(c^2*x^2 + 1)/(c^7*d^2*x^4 + 2*c^5*d^2*x^2 + c^3*d^2), x) +
 320*b^2*c^2*integrate(1/16*x^2*arctan(c*x)/(c^7*d^2*x^4 + 2*c^5*d^2*x^...
```

## Giac [F]

$$\int \frac{x^3(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx=\int \frac{(b\arctan(cx)+a)^2x^3}{(i\,cdx+d)^2}\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)^2*x^3/(I*c*d*x + d)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx=\int \frac{x^3\,\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)^2}{{\left(d+c\,d\,x\,1{}\mathrm{i}\right)}^2}\,dx$$

input
```
int((x^3*(a + b*atan(c*x))^2)/(d + c*d*x*1i)^2,x)
```

output
```
int((x^3*(a + b*atan(c*x))^2)/(d + c*d*x*1i)^2, x)
```

## Reduce [F]

$$\int \frac{x^3(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx = \frac{-\left(\int \frac{x^3}{c^2x^2-2cix-1}dx\right)a^2-2\left(\int \frac{atan(cx)x^3}{c^2x^2-2cix-1}dx\right)ab-\left(\int \frac{atan(cx)^2x^3}{c^2x^2-2cix-1}dx\right)b^2}{d^2}$$

input
```
int(x^3*(a+b*atan(c*x))^2/(d+I*c*d*x)^2,x)
```

output
```
( - int(x**3/(c**2*x**2 - 2*c*i*x - 1),x)*a**2 - 2*int((atan(c*x)*x**3)/(c
**2*x**2 - 2*c*i*x - 1),x)*a*b - int((atan(c*x)**2*x**3)/(c**2*x**2 - 2*c*
i*x - 1),x)*b**2)/d**2
```

# 3.105 $\int \frac{x^2(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 292

$$\int \frac{x^2(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx = -\frac{b^2}{2c^3d^2(i-cx)}+\frac{b^2\arctan(cx)}{2c^3d^2}-\frac{ib(a+b\arctan(cx))}{c^3d^2(i-cx)} -\frac{i(a+b\arctan(cx))^2}{2c^3d^2}-\frac{x(a+b\arctan(cx))^2}{c^2d^2} +\frac{(a+b\arctan(cx))^2}{c^3d^2(i-cx)}-\frac{2b(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c^3d^2} +\frac{2i(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{c^3d^2} -\frac{ib^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^3d^2} -\frac{2b(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^3d^2} +\frac{ib^2\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{c^3d^2}$$

output
```
-1/2*b^2/c^3/d^2/(I-c*x)+1/2*b^2*arctan(c*x)/c^3/d^2-I*b*(a+b*arctan(c*x))
/c^3/d^2/(I-c*x)-1/2*I*(a+b*arctan(c*x))^2/c^3/d^2-x*(a+b*arctan(c*x))^2/c
^2/d^2+(a+b*arctan(c*x))^2/c^3/d^2/(I-c*x)-2*b*(a+b*arctan(c*x))*ln(2/(1+I
*c*x))/c^3/d^2+2*I*(a+b*arctan(c*x))^2*ln(2/(1+I*c*x))/c^3/d^2-I*b^2*polyl
og(2,1-2/(1+I*c*x))/c^3/d^2-2*b*(a+b*arctan(c*x))*polylog(2,1-2/(1+I*c*x))
/c^3/d^2+I*b^2*polylog(3,1-2/(1+I*c*x))/c^3/d^2
```

## Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.24

$$\int \frac{x^2(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx =$$
$$-\frac{12a^2cx+\frac{12a^2}{-i+cx}-24a^2\arctan(cx)+12ia^2\log\left(1+c^2x^2\right)+b^2\left(-12i\arctan(cx)^2+12cx\arctan(cx)^2-\right.}{}$$

input
```
Integrate[(x^2*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x)^2,x]
```

output
```
-1/12*(12*a^2*c*x + (12*a^2)/(-I + c*x) - 24*a^2*ArcTan[c*x] + (12*I)*a^2*
Log[1 + c^2*x^2] + b^2*((-12*I)*ArcTan[c*x]^2 + 12*c*x*ArcTan[c*x]^2 - 16*
ArcTan[c*x]^3 - (3*I)*Cos[2*ArcTan[c*x]] + 6*ArcTan[c*x]*Cos[2*ArcTan[c*x]
] + (6*I)*ArcTan[c*x]^2*Cos[2*ArcTan[c*x]] + 24*ArcTan[c*x]*Log[1 + E^((2*
I)*ArcTan[c*x])] - (24*I)*ArcTan[c*x]^2*Log[1 + E^((2*I)*ArcTan[c*x])] - 1
2*(I + 2*ArcTan[c*x])*PolyLog[2, -E^((2*I)*ArcTan[c*x])] - (12*I)*PolyLog[
3, -E^((2*I)*ArcTan[c*x])] - 3*Sin[2*ArcTan[c*x]] - (6*I)*ArcTan[c*x]*Sin[
2*ArcTan[c*x]] + 6*ArcTan[c*x]^2*Sin[2*ArcTan[c*x]]) + 6*a*b*(-8*ArcTan[c*
x]^2 + Cos[2*ArcTan[c*x]] - 2*Log[1 + c^2*x^2] - 4*PolyLog[2, -E^((2*I)*Ar
cTan[c*x])] - I*Sin[2*ArcTan[c*x]] + 2*ArcTan[c*x]*(2*c*x + I*Cos[2*ArcTan
[c*x]] - (4*I)*Log[1 + E^((2*I)*ArcTan[c*x])] + Sin[2*ArcTan[c*x]])))/(c^3
*d^2)
```

## Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx$$
$$\Big\downarrow \; 5411$$

$$\int \left(-\frac{2i(a+b\arctan(cx))^2}{c^2d^2(cx-i)}-\frac{(a+b\arctan(cx))^2}{c^2d^2}+\frac{(a+b\arctan(cx))^2}{c^2d^2(cx-i)^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2b\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{c^3d^2}-\frac{ib(a+b\arctan(cx))}{c^3d^2(-cx+i)}+\frac{(a+b\arctan(cx))^2}{c^3d^2(-cx+i)}-\frac{i(a+b\arctan(cx))^2}{2c^3d^2}-\frac{2b\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c^3d^2}+\frac{2i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c^3d^2}-\frac{x(a+b\arctan(cx))^2}{c^2d^2}+\frac{b^2\arctan(cx)}{2c^3d^2}-\frac{ib^2\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{c^3d^2}+\frac{ib^2\,\text{PolyLog}\left(3,1-\frac{2}{icx+1}\right)}{c^3d^2}-\frac{b^2}{2c^3d^2(-cx+i)}$$

input
```
Int[(x^2*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x)^2,x]
```

output
```
-1/2*b^2/(c^3*d^2*(I - c*x)) + (b^2*ArcTan[c*x])/(2*c^3*d^2) - (I*b*(a + b
*ArcTan[c*x]))/(c^3*d^2*(I - c*x)) - ((I/2)*(a + b*ArcTan[c*x])^2)/(c^3*d^
2) - (x*(a + b*ArcTan[c*x])^2)/(c^2*d^2) + (a + b*ArcTan[c*x])^2/(c^3*d^2*
(I - c*x)) - (2*b*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(c^3*d^2) + ((2*
I)*(a + b*ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/(c^3*d^2) - (I*b^2*PolyLog[2,
 1 - 2/(1 + I*c*x)])/(c^3*d^2) - (2*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2
/(1 + I*c*x)])/(c^3*d^2) + (I*b^2*PolyLog[3, 1 - 2/(1 + I*c*x)])/(c^3*d^2)
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 10.08 (sec) , antiderivative size = 4182, normalized size of antiderivative = 14.32

| method | result | size |
|---|---|---|
| derivativedivides | Expression too large to display | 4182 |
| default | Expression too large to display | 4182 |
| parts | Expression too large to display | 4230 |

input
```
int(x^2*(a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c^3*(-a^2/d^2*c*x-a^2/d^2/(c*x-I)+2*a^2/d^2*arctan(c*x)-I*a^2/d^2*ln(c^2
*x^2+1)+b^2/d^2*(-2*I*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*
x^2+1)))^2*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-2*I*Pi*csgn((1+
I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*arctan(c*x)*ln(1-I*(1+
I*c*x)/(c^2*x^2+1)^(1/2))+I*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2
/(c^2*x^2+1)))^3*arctan(c*x)*ln(1+(1+I*c*x)^2/(c^2*x^2+1))+1/2*Pi*csgn(I/(
1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2
/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*polylog(2,-(1+I*c*x)^2/(c^2*x^2+
1))-Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn((1+I*c*x)^2/(c^2*x^2+1))*c
sgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*dilog(1+I*(1+I*c*
x)/(c^2*x^2+1)^(1/2))-Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn((1+I*c*x
)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))
*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+2*I*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1
)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*arctan(c*x)*ln(1+(1+I*c*x)^2/(c^2*x^2+1))
-I*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^3*arctan(c
*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-I*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/
(1+(1+I*c*x)^2/(c^2*x^2+1)))^3*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1
/2))+1/2*(c*x+I)*arctan(c*x)/(c*x-I)-c*x*arctan(c*x)^2+4/3*arctan(c*x)^3-3
/2*arctan(c*x)*ln(1+(1+I*c*x)^2/(c^2*x^2+1))-I*Pi*csgn(I/(1+(1+I*c*x)^2/(c
^2*x^2+1)))*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2...
```

## Fricas [F]

$$\int \frac{x^2(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx=\int \frac{(b\arctan(cx)+a)^2x^2}{(i\,cdx+d)^2}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x, algorithm="fricas")
```

output
```
integral(1/4*(b^2*x^2*log(-(c*x + I)/(c*x - I))^2 - 4*I*a*b*x^2*log(-(c*x
+ I)/(c*x - I)) - 4*a^2*x^2)/(c^2*d^2*x^2 - 2*I*c*d^2*x - d^2), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx=\text{Timed out}$$

input
```
integrate(x**2*(a+b*atan(c*x))**2/(d+I*c*d*x)**2,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{x^2(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx=\int \frac{(b\arctan(cx)+a)^2x^2}{(i\,cdx+d)^2}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x, algorithm="maxima")
```

output
```
-a^2*(1/(c^4*d^2*x - I*c^3*d^2) + x/(c^2*d^2) + 2*I*log(c*x - I)/(c^3*d^2)
) + 1/16*(8*(b^2*c*x - I*b^2)*arctan(c*x)^3 - (-I*b^2*c*x - b^2)*log(c^2*x
^2 + 1)^3 - 4*(b^2*c^2*x^2 - I*b^2*c*x + b^2)*arctan(c*x)^2 + (b^2*c^2*x^2
 - I*b^2*c*x + b^2 + 2*(b^2*c*x - I*b^2)*arctan(c*x))*log(c^2*x^2 + 1)^2 -
 2*(c^4*d^2*x - I*c^3*d^2)*(96*b^2*c^4*integrate(1/16*x^4*arctan(c*x)^2/(c
^6*d^2*x^4 + 2*c^4*d^2*x^2 + c^2*d^2), x) + 8*b^2*c^4*integrate(1/16*x^4*l
og(c^2*x^2 + 1)^2/(c^6*d^2*x^4 + 2*c^4*d^2*x^2 + c^2*d^2), x) + 256*a*b*c^
4*integrate(1/16*x^4*arctan(c*x)/(c^6*d^2*x^4 + 2*c^4*d^2*x^2 + c^2*d^2),
x) + 32*b^2*c^4*integrate(1/16*x^4*log(c^2*x^2 + 1)/(c^6*d^2*x^4 + 2*c^4*d
^2*x^2 + c^2*d^2), x) + 64*b^2*c^3*integrate(1/16*x^3*arctan(c*x)*log(c^2*
x^2 + 1)/(c^6*d^2*x^4 + 2*c^4*d^2*x^2 + c^2*d^2), x) - 64*b^2*c^3*integrat
e(1/16*x^3*arctan(c*x)/(c^6*d^2*x^4 + 2*c^4*d^2*x^2 + c^2*d^2), x) + 32*b^
2*c^2*integrate(1/16*x^2*arctan(c*x)^2/(c^6*d^2*x^4 + 2*c^4*d^2*x^2 + c^2*
d^2), x) + 24*b^2*c^2*integrate(1/16*x^2*log(c^2*x^2 + 1)^2/(c^6*d^2*x^4 +
 2*c^4*d^2*x^2 + c^2*d^2), x) - 256*a*b*c^2*integrate(1/16*x^2*arctan(c*x)
/(c^6*d^2*x^4 + 2*c^4*d^2*x^2 + c^2*d^2), x) + 64*b^2*c^2*integrate(1/16*x
^2*log(c^2*x^2 + 1)/(c^6*d^2*x^4 + 2*c^4*d^2*x^2 + c^2*d^2), x) - (c*(x/(c
^6*d^2*x^2 + c^4*d^2) + arctan(c*x)/(c^5*d^2)) - 2*arctan(c*x)/(c^6*d^2*x^
2 + c^4*d^2))*b^2*c + 128*b^2*integrate(1/16*arctan(c*x)^2/(c^6*d^2*x^4 +
2*c^4*d^2*x^2 + c^2*d^2), x) + 32*b^2*integrate(1/16*log(c^2*x^2 + 1)^2...
```

## Giac [F]

$$\int \frac{x^2(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx=\int \frac{(b\arctan(cx)+a)^2x^2}{(i\,cdx+d)^2}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)^2*x^2/(I*c*d*x + d)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx = \int \frac{x^2\,(a+b\,\mathrm{atan}(c\,x))^2}{(d+c\,d\,x\,1\mathrm{i})^2}\,dx$$

input
```
int((x^2*(a + b*atan(c*x))^2)/(d + c*d*x*1i)^2,x)
```

output
```
int((x^2*(a + b*atan(c*x))^2)/(d + c*d*x*1i)^2, x)
```

## Reduce [F]

$$\int \frac{x^2(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx = \frac{-\left(\int \frac{x^2}{c^2x^2-2cix-1}dx\right)a^2-2\left(\int \frac{atan(cx)\,x^2}{c^2x^2-2cix-1}dx\right)ab-\left(\int \frac{atan(cx)^2x^2}{c^2x^2-2cix-1}dx\right)b^2}{d^2}$$

input
```
int(x^2*(a+b*atan(c*x))^2/(d+I*c*d*x)^2,x)
```

output
```
( - int(x**2/(c**2*x**2 - 2*c*i*x - 1),x)*a**2 - 2*int((atan(c*x)*x**2)/(c
**2*x**2 - 2*c*i*x - 1),x)*a*b - int((atan(c*x)**2*x**2)/(c**2*x**2 - 2*c*
i*x - 1),x)*b**2)/d**2
```

# 3.106 $\int \frac{x(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 216

$$\begin{aligned}\int \frac{x(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx = {} & \frac{ib^2}{2c^2d^2(i-cx)} - \frac{ib^2\arctan(cx)}{2c^2d^2} - \frac{b(a+b\arctan(cx))}{c^2d^2(i-cx)} \\ & + \frac{(a+b\arctan(cx))^2}{2c^2d^2} - \frac{i(a+b\arctan(cx))^2}{c^2d^2(i-cx)} \\ & + \frac{(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{c^2d^2} \\ & + \frac{ib(a+b\arctan(cx))\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^2d^2} \\ & + \frac{b^2\,\mathrm{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2c^2d^2}\end{aligned}$$

output

```
1/2*I*b^2/c^2/d^2/(I-c*x)-1/2*I*b^2*arctan(c*x)/c^2/d^2-b*(a+b*arctan(c*x)
)/c^2/d^2/(I-c*x)+1/2*(a+b*arctan(c*x))^2/c^2/d^2-I*(a+b*arctan(c*x))^2/c^
2/d^2/(I-c*x)+(a+b*arctan(c*x))^2*ln(2/(1+I*c*x))/c^2/d^2+I*b*(a+b*arctan(
c*x))*polylog(2,1-2/(1+I*c*x))/c^2/d^2+1/2*b^2*polylog(3,1-2/(1+I*c*x))/c^
2/d^2
```

## Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.39

$$\int \frac{x(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx$$
$$= \frac{\frac{12ia^2}{-i+cx} - 12ia^2\arctan(cx) - 6a^2\log\left(1+c^2x^2\right) - 6iab\big(4\arctan(cx)^2 - \cos(2\arctan(cx)) + 2\,\mathrm{PolyLog}\left(2,\right.}{}$$

input
```
Integrate[(x*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x)^2,x]
```

output
```
(((12*I)*a^2)/(-I + c*x) - (12*I)*a^2*ArcTan[c*x] - 6*a^2*Log[1 + c^2*x^2]
 - (6*I)*a*b*(4*ArcTan[c*x]^2 - Cos[2*ArcTan[c*x]] + 2*PolyLog[2, -E^((2*I
)*ArcTan[c*x])] - (2*I)*ArcTan[c*x]*(Cos[2*ArcTan[c*x]] - 2*Log[1 + E^((2*
I)*ArcTan[c*x])] - I*Sin[2*ArcTan[c*x]]) + I*Sin[2*ArcTan[c*x]]) + b^2*((-
8*I)*ArcTan[c*x]^3 + 3*Cos[2*ArcTan[c*x]] + (6*I)*ArcTan[c*x]*Cos[2*ArcTan
[c*x]] - 6*ArcTan[c*x]^2*Cos[2*ArcTan[c*x]] + 12*ArcTan[c*x]^2*Log[1 + E^(
(2*I)*ArcTan[c*x])] - (12*I)*ArcTan[c*x]*PolyLog[2, -E^((2*I)*ArcTan[c*x])
] + 6*PolyLog[3, -E^((2*I)*ArcTan[c*x])] - (3*I)*Sin[2*ArcTan[c*x]] + 6*Ar
cTan[c*x]*Sin[2*ArcTan[c*x]] + (6*I)*ArcTan[c*x]^2*Sin[2*ArcTan[c*x]]))/(1
2*c^2*d^2)
```

## Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx$$
$$\Big\downarrow \text{5411}$$
$$\int \left(-\frac{(a+b\arctan(cx))^2}{cd^2(cx-i)} - \frac{i(a+b\arctan(cx))^2}{cd^2(cx-i)^2}\right)dx$$

$$\big\downarrow \text{2009}$$

$$\frac{ib\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{c^2d^2}-\frac{b(a+b\arctan(cx))}{c^2d^2(-cx+i)}-\frac{i(a+b\arctan(cx))^2}{c^2d^2(-cx+i)}+\frac{(a+b\arctan(cx))^2}{2c^2d^2}+\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c^2d^2}-\frac{ib^2\arctan(cx)}{2c^2d^2}+\frac{b^2\,\text{PolyLog}\left(3,1-\frac{2}{icx+1}\right)}{2c^2d^2}+\frac{ib^2}{2c^2d^2(-cx+i)}$$

input
```
Int[(x*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x)^2,x]
```

output
```
((I/2)*b^2)/(c^2*d^2*(I - c*x)) - ((I/2)*b^2*ArcTan[c*x])/(c^2*d^2) - (b*(
a + b*ArcTan[c*x]))/(c^2*d^2*(I - c*x)) + (a + b*ArcTan[c*x])^2/(2*c^2*d^2
) - (I*(a + b*ArcTan[c*x])^2)/(c^2*d^2*(I - c*x)) + ((a + b*ArcTan[c*x])^2
*Log[2/(1 + I*c*x)])/(c^2*d^2) + (I*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2
/(1 + I*c*x)])/(c^2*d^2) + (b^2*PolyLog[3, 1 - 2/(1 + I*c*x)])/(2*c^2*d^2)
```

**Defintions of rubi rules used**

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 8.26 (sec) , antiderivative size = 857, normalized size of antiderivative = 3.97

| method | result |
|---|---|
| derivativedivides | $-\frac{a^2\ln(c^2x^2+1)}{2d^2}-\frac{iab\ln(cx-i)^2}{2d^2}+\frac{iab\,\mathrm{dilog}\left(-\frac{i(cx+i)}{2}\right)}{d^2}+\frac{b^2\left(-\ln(cx-i)\arctan(cx)^2+\frac{i\arctan(cx)^2}{cx-i}+\arctan(cx)^2\ln\left(\frac{2i(icx+1)}{c^2x^2+1}\right.\right.}{}$ |
| default | $-\frac{a^2\ln(c^2x^2+1)}{2d^2}-\frac{iab\ln(cx-i)^2}{2d^2}+\frac{iab\,\mathrm{dilog}\left(-\frac{i(cx+i)}{2}\right)}{d^2}+\frac{b^2\left(-\ln(cx-i)\arctan(cx)^2+\frac{i\arctan(cx)^2}{cx-i}+\arctan(cx)^2\ln\left(\frac{2i(icx+1)}{c^2x^2+1}\right.\right.}{}$ |
| parts | $-\frac{a^2\ln(c^2x^2+1)}{2d^2c^2}-\frac{ia^2}{d^2c^2(-cx+i)}+\frac{iab\ln(c^4x^4+10c^2x^2+9)}{8d^2c^2}+\frac{b^2\left(-\ln(cx-i)\arctan(cx)^2+\frac{i\arctan(cx)^2}{cx-i}+\mathrm{arcta}\right.}{}$ |

input
```
int(x*(a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c^2*(-1/2*a^2/d^2*ln(c^2*x^2+1)-1/2*I*a*b/d^2*ln(c*x-I)^2+I*a*b/d^2*dilo
g(-1/2*I*(c*x+I))+b^2/d^2*(-ln(c*x-I)*arctan(c*x)^2+I*arctan(c*x)^2/(c*x-I
)+arctan(c*x)^2*ln(2*I*(1+I*c*x)^2/(c^2*x^2+1))-2/3*I*arctan(c*x)^3-1/2*I*
Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^3*arctan(c*x)
^2+1/2*I*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*cs
gn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*arctan(c*x)^2-1/2*I*Pi*csgn((1+I*c*x)^2/
(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*csgn((1+I*c*x)^2/(c^2*x^2+1))*a
rctan(c*x)^2-1/2*I*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2
+1)))*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn((1+I*c*x)^2/(c^2*x^2+1))*ar
ctan(c*x)^2-I*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))
^2*arctan(c*x)^2+1/2*arctan(c*x)^2+I*Pi*arctan(c*x)^2-2*I*arctan(c*x)*(c*x
+I)/(4*c*x-4*I)-1/4*(c*x+I)/(c*x-I)-I*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(
c^2*x^2+1))+1/2*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1)))-2*a*b/d^2*arctan(c*x)
*ln(c*x-I)-I*a^2/d^2*arctan(c*x)+a*b/d^2/(c*x-I)-1/4*I*a*b/d^2*ln(c^2*x^2+
1)+1/2*a*b/d^2*arctan(c*x)+1/8*I*a*b/d^2*ln(c^4*x^4+10*c^2*x^2+9)-1/4*a*b/
d^2*arctan(1/2*c*x)+1/4*a*b/d^2*arctan(1/6*c^3*x^3+7/6*c*x)+1/2*a*b/d^2*ar
ctan(1/2*c*x-1/2*I)+I*a^2/d^2/(c*x-I)+2*I*a*b/d^2*arctan(c*x)/(c*x-I)+I*a*
b/d^2*ln(c*x-I)*ln(-1/2*I*(c*x+I)))
```

## Fricas [F]

$$\int \frac{x(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx = \int \frac{\left(b\arctan\left(cx\right)+a\right)^2 x}{\left(i\,cdx+d\right)^2}\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x, algorithm="fricas")
```

output
```
integral(1/4*(b^2*x*log(-(c*x + I)/(c*x - I))^2 - 4*I*a*b*x*log(-(c*x + I)
/(c*x - I)) - 4*a^2*x)/(c^2*d^2*x^2 - 2*I*c*d^2*x - d^2), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx = \text{Timed out}$$

input
```
integrate(x*(a+b*atan(c*x))**2/(d+I*c*d*x)**2,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{x(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx = \int \frac{\left(b\arctan\left(cx\right)+a\right)^2 x}{\left(i\,cdx+d\right)^2}\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x, algorithm="maxima")
```

output
```
a^2*(I/(c^3*d^2*x - I*c^2*d^2) - log(c*x - I)/(c^2*d^2)) - 1/32*(-8*I*b^2*
arctan(c*x)^2 - 8*(-I*b^2*c*x - b^2)*arctan(c*x)^3 - (b^2*c*x - I*b^2)*log
(c^2*x^2 + 1)^3 - 2*(-I*b^2 + (-I*b^2*c*x - b^2)*arctan(c*x))*log(c^2*x^2
+ 1)^2 - (2*b^2*c^3*(c^2/(c^9*d^2*x^2 + c^7*d^2) + log(c^2*x^2 + 1)/(c^7*d
^2*x^2 + c^5*d^2)) - 640*b^2*c^3*integrate(1/16*x^3*arctan(c*x)^2/(c^5*d^2
*x^4 + 2*c^3*d^2*x^2 + c*d^2), x) - 96*b^2*c^3*integrate(1/16*x^3*log(c^2*
x^2 + 1)^2/(c^5*d^2*x^4 + 2*c^3*d^2*x^2 + c*d^2), x) - 1024*a*b*c^3*integr
ate(1/16*x^3*arctan(c*x)/(c^5*d^2*x^4 + 2*c^3*d^2*x^2 + c*d^2), x) - 256*b
^2*c^2*integrate(1/16*x^2*arctan(c*x)*log(c^2*x^2 + 1)/(c^5*d^2*x^4 + 2*c^
3*d^2*x^2 + c*d^2), x) + 256*b^2*c^2*integrate(1/16*x^2*arctan(c*x)/(c^5*d
^2*x^4 + 2*c^3*d^2*x^2 + c*d^2), x) + 16*(c*(x/(c^5*d^2*x^2 + c^3*d^2) + a
rctan(c*x)/(c^4*d^2)) - 2*arctan(c*x)/(c^5*d^2*x^2 + c^3*d^2))*a*b*c + 128
*b^2*c*integrate(1/16*x*arctan(c*x)^2/(c^5*d^2*x^4 + 2*c^3*d^2*x^2 + c*d^2
), x) + b^2*c*log(c^2*x^2 + 1)^2/(c^5*d^2*x^2 + c^3*d^2) + 256*b^2*integra
te(1/16*arctan(c*x)/(c^5*d^2*x^4 + 2*c^3*d^2*x^2 + c*d^2), x))*(c^3*d^2*x
- I*c^2*d^2) - 32*(I*c^3*d^2*x + c^2*d^2)*integrate(-1/8*(32*a*b*c^2*x^2*a
rctan(c*x) - b^2*log(c^2*x^2 + 1)^2 + 4*(2*b^2*c^2*x^2 - b^2)*arctan(c*x)^
2 - 2*(b^2*c^2*x^2 + b^2 + (b^2*c^3*x^3 - b^2*c*x)*arctan(c*x))*log(c^2*x^
2 + 1))/(c^5*d^2*x^4 + 2*c^3*d^2*x^2 + c*d^2), x) + 4*(2*b^2*arctan(c*x) -
 (b^2*c*x - I*b^2)*arctan(c*x)^2)*log(c^2*x^2 + 1))/(c^3*d^2*x - I*c^2*...
```

## Giac [F]

$$\int \frac{x(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx = \int \frac{(b\arctan(cx)+a)^2x}{(i\,cdx+d)^2}\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)^2*x/(I*c*d*x + d)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx=\int \frac{x\,(a+b\,\mathrm{atan}(c\,x))^2}{(d+c\,d\,x\,1\mathrm{i})^2}\,dx$$

input
```
int((x*(a + b*atan(c*x))^2)/(d + c*d*x*1i)^2,x)
```

output
```
int((x*(a + b*atan(c*x))^2)/(d + c*d*x*1i)^2, x)
```

## Reduce [F]

$$\int \frac{x(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx = \frac{-2\left(\int \frac{atan(cx)x}{c^2x^2-2cix-1}dx\right)ab-\left(\int \frac{atan(cx)^2x}{c^2x^2-2cix-1}dx\right)b^2-\left(\int \frac{x}{c^2x^2-2cix-1}dx\right)a^2}{d^2}$$

input
```
int(x*(a+b*atan(c*x))^2/(d+I*c*d*x)^2,x)
```

output
```
( - 2*int((atan(c*x)*x)/(c**2*x**2 - 2*c*i*x - 1),x)*a*b - int((atan(c*x)*
*2*x)/(c**2*x**2 - 2*c*i*x - 1),x)*b**2 - int(x/(c**2*x**2 - 2*c*i*x - 1),
x)*a**2)/d**2
```

# 3.107 $\int \frac{(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 122

$$\int \frac{(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx = \frac{b^2}{2cd^2(i-cx)} - \frac{b^2\arctan(cx)}{2cd^2} + \frac{ib(a+b\arctan(cx))}{cd^2(i-cx)} - \frac{i(a+b\arctan(cx))^2}{2cd^2} + \frac{i(a+b\arctan(cx))^2}{cd^2(1+icx)}$$

output
```
1/2*b^2/c/d^2/(I-c*x)-1/2*b^2*arctan(c*x)/c/d^2+I*b*(a+b*arctan(c*x))/c/d^
2/(I-c*x)-1/2*I*(a+b*arctan(c*x))^2/c/d^2+I*(a+b*arctan(c*x))^2/c/d^2/(1+I
*c*x)
```

## Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.59

$$\int \frac{(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx = -\frac{-2a^2+2iab+b^2+b(2ia+b)(i+cx)\arctan(cx)+b^2(-1+icx)\arctan(cx)^2}{2cd^2(-i+cx)}$$

input
```
Integrate[(a + b*ArcTan[c*x])^2/(d + I*c*d*x)^2,x]
```

output
```
-1/2*(-2*a^2 + (2*I)*a*b + b^2 + b*((2*I)*a + b)*(I + c*x)*ArcTan[c*x] + b
^2*(-1 + I*c*x)*ArcTan[c*x]^2)/(c*d^2*(-I + c*x))
```

## Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.07, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {5389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^2}{(d+icdx)^2}dx$$

$$\Big\downarrow \text{5389}$$

$$\frac{i(a+b\arctan(cx))^2}{cd^2(1+icx)} - \frac{2ib\int \left(\frac{a+b\arctan(cx)}{2d(c^2x^2+1)} - \frac{a+b\arctan(cx)}{2d(i-cx)^2}\right)dx}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{i(a+b\arctan(cx))^2}{cd^2(1+icx)} - \frac{2ib\left(\frac{(a+b\arctan(cx))^2}{4bcd} - \frac{a+b\arctan(cx)}{2cd(-cx+i)} - \frac{ib\arctan(cx)}{4cd} + \frac{ib}{4cd(-cx+i)}\right)}{d}$$

input
```
Int[(a + b*ArcTan[c*x])^2/(d + I*c*d*x)^2,x]
```

output
```
(I*(a + b*ArcTan[c*x])^2)/(c*d^2*(1 + I*c*x)) - ((2*I)*b*(((I/4)*b)/(c*d*(
I - c*x)) - ((I/4)*b*ArcTan[c*x])/(c*d) - (a + b*ArcTan[c*x])/(2*c*d*(I -
c*x)) + (a + b*ArcTan[c*x])^2/(4*b*c*d)))/d
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5389
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - S
imp[b*c*(p/(e*(q + 1)))   Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1),
(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

## Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(110) = 220$.

Time = 0.82 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.94

| method | result |
|---|---|
| derivativedivides | $\frac{ia^2}{d^2(icx+1)} + \frac{b^2\left(\frac{i\arctan(cx)^2}{icx+1} - 2i\left(\frac{i\arctan(cx)\ln(cx+i)}{4} - \frac{i\arctan(cx)\ln(cx-i)}{4} + \frac{\arctan(cx)}{2cx-2i} + \frac{\ln(cx-i)^2}{16} - \frac{\ln(cx-i)\ln\left(-\frac{i(cx+i)}{2}\right)}{8} \cdots\right)\right)}{d^2}$ |
| default | $\frac{ia^2}{d^2(icx+1)} + \frac{b^2\left(\frac{i\arctan(cx)^2}{icx+1} - 2i\left(\frac{i\arctan(cx)\ln(cx+i)}{4} - \frac{i\arctan(cx)\ln(cx-i)}{4} + \frac{\arctan(cx)}{2cx-2i} + \frac{\ln(cx-i)^2}{16} - \frac{\ln(cx-i)\ln\left(-\frac{i(cx+i)}{2}\right)}{8} \cdots\right)\right)}{d^2}$ |
| parts | $\frac{ia^2}{d^2(icx+1)c} + \frac{b^2\left(\frac{i\arctan(cx)^2}{icx+1} - 2i\left(\frac{i\arctan(cx)\ln(cx+i)}{4} - \frac{i\arctan(cx)\ln(cx-i)}{4} + \frac{\arctan(cx)}{2cx-2i} + \frac{\ln(cx-i)^2}{16} - \frac{\ln(cx-i)\ln\left(- \cdots\right)}{8}\right)\right)}{d^2c}$ |
| orering | $-\frac{i(3c^4x^4+6ic^3x^3-2c^2x^2+6icx-5)(a+b\arctan(cx))^2}{4c(icdx+d)^2} - \frac{3i(c^5x^5+ic^4x^4+2c^3x^3+2ic^2x^2+cx+i)\left(\frac{2(a+b\arctan(cx))}{(icdx+d)^2(c^2x^2+\cdots}\right)}{4c^2}$ |
| risch | $\frac{ib^2(cx+i)\ln(icx+1)^2}{8d^2(cx-i)c} - \frac{ib(bcx\ln(-icx+1)+ib\ln(-icx+1)-2ib+4a)\ln(icx+1)}{4d^2(cx-i)c} + \frac{8iab\ln(-icx+1)+4b^2\ln(-icx+\cdots}{}$ |

input
```
int((a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*(I*a^2/d^2/(1+I*c*x)+b^2/d^2*(I/(1+I*c*x)*arctan(c*x)^2-2*I*(1/4*I*arc
tan(c*x)*ln(c*x+I)-1/4*I*arctan(c*x)*ln(c*x-I)+1/2*arctan(c*x)/(c*x-I)+1/1
6*ln(c*x-I)^2-1/8*ln(c*x-I)*ln(-1/2*I*(c*x+I))-1/4*I*arctan(c*x)-1/4*I/(c*
x-I)+1/16*ln(c*x+I)^2-1/8*(ln(c*x+I)-ln(-1/2*I*(c*x+I)))*ln(-1/2*I*(-c*x+I
))))+2*I*a*b/d^2/(1+I*c*x)*arctan(c*x)-I*a*b/d^2*arctan(c*x)-I*a*b/d^2/(c*
x-I))
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.84

$$\int \frac{(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx = \frac{(i\,b^2cx-b^2)\log\left(-\frac{cx+i}{cx-i}\right)^2+8\,a^2-8i\,ab-4\,b^2+2\left((2\,ab-i\,b^2)cx+2i\,ab+b^2\right)\log\left(-\frac{cx+i}{cx-i}\right)}{8\left(c^2d^2x-i\,cd^2\right)}$$

input
```
integrate((a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x, algorithm="fricas")
```

output
```
1/8*((I*b^2*c*x - b^2)*log(-(c*x + I)/(c*x - I))^2 + 8*a^2 - 8*I*a*b - 4*b
^2 + 2*((2*a*b - I*b^2)*c*x + 2*I*a*b + b^2)*log(-(c*x + I)/(c*x - I)))/(c
^2*d^2*x - I*c*d^2)
```

## Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(94) = 188$.

Time = 5.20 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.48

$$\int \frac{(a + b \arctan(cx))^2}{(d + icdx)^2}\, dx$$
$$= -\frac{b(2a - ib) \log \left(-\frac{ib(2a-ib)}{c} + x(2ab - ib^2)\right)}{4cd^2} + \frac{b(2a - ib) \log \left(\frac{ib(2a-ib)}{c} + x(2ab - ib^2)\right)}{4cd^2} + \frac{(-2iab - b^2) \log (icx + 1)}{2c^2d^2x - 2icd^2} + \frac{(ib^2cx - b^2) \log (-icx + 1)^2}{8c^2d^2x - 8icd^2} + \frac{(ib^2cx - b^2) \log (icx + 1)^2}{8c^2d^2x - 8icd^2} + \frac{(4iab - ib^2cx \log (icx + 1) + b^2 \log (icx + 1) + 2b^2) \log (-icx + 1)}{4c^2d^2x - 4icd^2} - \frac{-2a^2 + 2iab + b^2}{2c^2d^2x - 2icd^2}$$

input
```
integrate((a+b*atan(c*x))**2/(d+I*c*d*x)**2,x)
```

output
```
-b*(2*a - I*b)*log(-I*b*(2*a - I*b)/c + x*(2*a*b - I*b**2))/(4*c*d**2) + b
*(2*a - I*b)*log(I*b*(2*a - I*b)/c + x*(2*a*b - I*b**2))/(4*c*d**2) + (-2*
I*a*b - b**2)*log(I*c*x + 1)/(2*c**2*d**2*x - 2*I*c*d**2) + (I*b**2*c*x -
b**2)*log(-I*c*x + 1)**2/(8*c**2*d**2*x - 8*I*c*d**2) + (I*b**2*c*x - b**2
)*log(I*c*x + 1)**2/(8*c**2*d**2*x - 8*I*c*d**2) + (4*I*a*b - I*b**2*c*x*l
og(I*c*x + 1) + b**2*log(I*c*x + 1) + 2*b**2)*log(-I*c*x + 1)/(4*c**2*d**2
*x - 4*I*c*d**2) - (-2*a**2 + 2*I*a*b + b**2)/(2*c**2*d**2*x - 2*I*c*d**2)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.20

$$\int \frac{(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx = -\frac{\left(-2i\,b^2\arctan\left(-\frac{i\,(i\,cdx+d)\left(\frac{d}{i\,cdx+d}-1\right)}{d}\right)^2+4i\,ab\arctan\left(-\frac{i\,(i\,cdx+d)\left(\frac{d}{i\,cdx+d}-1\right)}{d}\right)+2\,b^2\arctan\left(-\frac{i\,(i\,cdx\right.\right.}{4\,cd^2}$$

input
```
integrate((a+b*arctan(c*x))^2/(d+I*c*d*x)^2,x, algorithm="giac")
```

output
```
-1/4*(-2*I*b^2*arctan(-I*(I*c*d*x + d)*(d/(I*c*d*x + d) - 1)/d)^2 + 4*I*a*
b*arctan(-I*(I*c*d*x + d)*(d/(I*c*d*x + d) - 1)/d) + 2*b^2*arctan(-I*(I*c*
d*x + d)*(d/(I*c*d*x + d) - 1)/d) - 2*I*a^2 - 2*a*b + I*b^2)*e^(2*I*arctan
(-I*(I*c*d*x + d)*(d/(I*c*d*x + d) - 1)/d))/(c*d^2)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^2}{(d+c\,d\,x\,1\mathrm{i})^2}\,dx$$

input
```
int((a + b*atan(c*x))^2/(d + c*d*x*1i)^2,x)
```

output
```
int((a + b*atan(c*x))^2/(d + c*d*x*1i)^2, x)
```

## Reduce [F]

$$\int \frac{(a+b\arctan(cx))^2}{(d+icdx)^2}\,dx = \frac{-2\left(\int \frac{atan(cx)}{c^2x^2-2cix-1}dx\right)ab-\left(\int \frac{atan(cx)^2}{c^2x^2-2cix-1}dx\right)b^2-\left(\int \frac{1}{c^2x^2-2cix-1}dx\right)a^2}{d^2}$$

input
```
int((a+b*atan(c*x))^2/(d+I*c*d*x)^2,x)
```

output
```
( - 2*int(atan(c*x)/(c**2*x**2 - 2*c*i*x - 1),x)*a*b - int(atan(c*x)**2/(c
**2*x**2 - 2*c*i*x - 1),x)*b**2 - int(1/(c**2*x**2 - 2*c*i*x - 1),x)*a**2)
/d**2
```

# 3.108 $\int \frac{(a+b\arctan(cx))^2}{x(d+icdx)^2}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 221

$$\begin{aligned}\int \frac{(a+b\arctan(cx))^2}{x(d+icdx)^2}\,dx =& -\frac{ib^2}{2d^2(i-cx)}+\frac{ib^2\arctan(cx)}{2d^2}+\frac{b(a+b\arctan(cx))}{d^2(i-cx)} \\ &-\frac{(a+b\arctan(cx))^2}{2d^2}+\frac{i(a+b\arctan(cx))^2}{d^2(i-cx)} \\ &+\frac{2(a+b\arctan(cx))^2\text{arctanh}\left(1-\frac{2}{1+icx}\right)}{d^2} \\ &+\frac{(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{d^2} \\ &+\frac{ib(a+b\arctan(cx))\,\text{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)}{d^2} \\ &+\frac{b^2\,\text{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)}{2d^2}\end{aligned}$$

output

```
-1/2*I*b^2/d^2/(I-c*x)+1/2*I*b^2*arctan(c*x)/d^2+b*(a+b*arctan(c*x))/d^2/(
I-c*x)-1/2*(a+b*arctan(c*x))^2/d^2+I*(a+b*arctan(c*x))^2/d^2/(I-c*x)-2*(a+
b*arctan(c*x))^2*arctanh(-1+2/(1+I*c*x))/d^2+(a+b*arctan(c*x))^2*ln(2/(1+I
*c*x))/d^2+I*b*(a+b*arctan(c*x))*polylog(2,-1+2/(1+I*c*x))/d^2+1/2*b^2*pol
ylog(3,-1+2/(1+I*c*x))/d^2
```

## Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.35

$$\int \frac{(a+b\arctan(cx))^2}{x(d+icdx)^2}\,dx$$
$$= \frac{-\frac{24ia^2}{-i+cx} - 24ia^2\arctan(cx) + 24a^2\log(cx) - 12a^2\log(1+c^2x^2) - 12ab\big(4i\arctan(cx)^2 + i\cos(2\arctan(\cdots}{\cdots}$$

input
```
Integrate[(a + b*ArcTan[c*x])^2/(x*(d + I*c*d*x)^2),x]
```

output
```
(((-24*I)*a^2)/(-I + c*x) - (24*I)*a^2*ArcTan[c*x] + 24*a^2*Log[c*x] - 12*
a^2*Log[1 + c^2*x^2] - 12*a*b*((4*I)*ArcTan[c*x]^2 + I*Cos[2*ArcTan[c*x]]
+ (2*I)*PolyLog[2, E^((2*I)*ArcTan[c*x])] - 2*ArcTan[c*x]*(Cos[2*ArcTan[c*
x]] + 2*Log[1 - E^((2*I)*ArcTan[c*x])] - I*Sin[2*ArcTan[c*x]]) + Sin[2*Arc
Tan[c*x]]) + b^2*((-I)*Pi^3 - 6*Cos[2*ArcTan[c*x]] - (12*I)*ArcTan[c*x]*Co
s[2*ArcTan[c*x]] + 12*ArcTan[c*x]^2*Cos[2*ArcTan[c*x]] + 24*ArcTan[c*x]^2*
Log[1 - E^((-2*I)*ArcTan[c*x])] + (24*I)*ArcTan[c*x]*PolyLog[2, E^((-2*I)*
ArcTan[c*x])] + 12*PolyLog[3, E^((-2*I)*ArcTan[c*x])] + (6*I)*Sin[2*ArcTan
[c*x]] - 12*ArcTan[c*x]*Sin[2*ArcTan[c*x]] - (12*I)*ArcTan[c*x]^2*Sin[2*Ar
cTan[c*x]]))/(24*d^2)
```

## Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^2}{x(d+icdx)^2}\,dx$$
$$\Big\downarrow \; 5411$$
$$\int \left(\frac{(a+b\arctan(cx))^2}{d^2x} - \frac{c(a+b\arctan(cx))^2}{d^2(cx-i)} + \frac{ic(a+b\arctan(cx))^2}{d^2(cx-i)^2}\right)dx$$

$$\downarrow \text{2009}$$

$$\frac{2\text{arctanh}\left(1-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{d^2}+\frac{ib\,\text{PolyLog}\left(2,\frac{2}{icx+1}-1\right)(a+b\arctan(cx))}{d^2}+\frac{b(a+b\arctan(cx))}{d^2(-cx+i)}+\frac{i(a+b\arctan(cx))^2}{d^2(-cx+i)}-\frac{(a+b\arctan(cx))^2}{2d^2}+\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{d^2}+\frac{ib^2\arctan(cx)}{2d^2}+\frac{b^2\,\text{PolyLog}\left(3,\frac{2}{icx+1}-1\right)}{2d^2}-\frac{ib^2}{2d^2(-cx+i)}$$

input
```
Int[(a + b*ArcTan[c*x])^2/(x*(d + I*c*d*x)^2),x]
```

output
```
((-1/2*I)*b^2)/(d^2*(I - c*x)) + ((I/2)*b^2*ArcTan[c*x])/d^2 + (b*(a + b*A
rcTan[c*x]))/(d^2*(I - c*x)) - (a + b*ArcTan[c*x])^2/(2*d^2) + (I*(a + b*A
rcTan[c*x])^2)/(d^2*(I - c*x)) + (2*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1
 + I*c*x)])/d^2 + ((a + b*ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/d^2 + (I*b*(a
 + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)])/d^2 + (b^2*PolyLog[3, -1
 + 2/(1 + I*c*x)])/(2*d^2)
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 12.10 (sec) , antiderivative size = 1670, normalized size of antiderivative = 7.56

| method | result | size |
|---|---|---|
| parts | Expression too large to display | 1670 |
| derivativedivides | Expression too large to display | 1671 |
| default | Expression too large to display | 1671 |

input
```
int((a+b*arctan(c*x))^2/x/(d+I*c*d*x)^2,x,method=_RETURNVERBOSE)
```

output
```
I*a^2/d^2/(-c*x+I)-1/2*a^2/d^2*ln(c^2*x^2+1)-I*a^2/d^2*arctan(c*x)+a^2/d^2
*ln(x)+b^2/d^2*(-1/2*I*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I*((1+I
*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*arctan(c*x)^2-1/2*I*
Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/
(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*arctan(c*x)^2-1/2*I*Pi*csgn(I*((1+I*c*x)^2/
(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-
1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*arctan(c*x)^2-1/2*arctan(c*x)^2-1/2*I*Pi
*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I/(1+(1+I*
c*x)^2/(c^2*x^2+1)))*csgn((1+I*c*x)^2/(c^2*x^2+1))*arctan(c*x)^2+ln(c*x)*a
rctan(c*x)^2-arctan(c*x)^2*ln((1+I*c*x)^2/(c^2*x^2+1)-1)+arctan(c*x)^2*ln(
1-(1+I*c*x)/(c^2*x^2+1)^(1/2))+arctan(c*x)^2*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1
/2))-2*I*arctan(c*x)*polylog(2,(1+I*c*x)/(c^2*x^2+1)^(1/2))-2*I*arctan(c*x
)*polylog(2,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+2*polylog(3,(1+I*c*x)/(c^2*x^2+1
)^(1/2))+2*polylog(3,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/2*I*Pi*csgn(I*((1+I*c
*x)^2/(c^2*x^2+1)-1))*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I*((1+I*c*x
)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*arctan(c*x)^2-1/2*I*Pi*csg
n((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*csgn((1+I*c*x)^2/
(c^2*x^2+1))*arctan(c*x)^2+1/2*I*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c
*x)^2/(c^2*x^2+1)))^2*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*arctan(c*x)^2-1/
2*I*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^3*arct...
```

## Fricas [F]

$$\int \frac{(a+b\arctan(cx))^2}{x(d+icdx)^2}\,dx=\int \frac{(b\arctan(cx)+a)^2}{(i\,cdx+d)^2x}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x/(d+I*c*d*x)^2,x, algorithm="fricas")
```

output
```
1/4*(I*b^2*log(-(c*x + I)/(c*x - I))^2 - (b^2*c*x - I*b^2)*log(2*c*x/(c*x
- I))*log(-(c*x + I)/(c*x - I))^2 - 2*(b^2*c*x - I*b^2)*dilog(-2*c*x/(c*x
- I) + 1)*log(-(c*x + I)/(c*x - I)) + 4*(c*d^2*x - I*d^2)*integral(-(a^2*c
*x + I*a^2 - ((-I*a*b - b^2)*c*x + a*b)*log(-(c*x + I)/(c*x - I)))/(c^3*d^
2*x^4 - I*c^2*d^2*x^3 + c*d^2*x^2 - I*d^2*x), x) + 2*(b^2*c*x - I*b^2)*pol
ylog(3, -(c*x + I)/(c*x - I)))/(c*d^2*x - I*d^2)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{x(d+icdx)^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*atan(c*x))**2/x/(d+I*c*d*x)**2,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(a+b\arctan(cx))^2}{x(d+icdx)^2}\,dx = \int \frac{(b\arctan(cx)+a)^2}{(i\,cdx+d)^2x}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x/(d+I*c*d*x)^2,x, algorithm="maxima")
```

output
```
a^2*(-I/(c*d^2*x - I*d^2) - log(c*x - I)/d^2 + log(x)/d^2) - 1/32*(8*I*b^2
*arctan(c*x)^2 - 8*(-I*b^2*c*x - b^2)*arctan(c*x)^3 - (b^2*c*x - I*b^2)*lo
g(c^2*x^2 + 1)^3 - 2*(I*b^2 + (-I*b^2*c*x - b^2)*arctan(c*x))*log(c^2*x^2
+ 1)^2 - (6*b^2*c^4*(c^2/(c^8*d^2*x^2 + c^6*d^2) + log(c^2*x^2 + 1)/(c^6*d
^2*x^2 + c^4*d^2)) - 256*b^2*c^4*integrate(1/16*x^4*arctan(c*x)^2/(c^4*d^2
*x^5 + 2*c^2*d^2*x^3 + d^2*x), x) - 64*b^2*c^4*integrate(1/16*x^4*log(c^2*
x^2 + 1)^2/(c^4*d^2*x^5 + 2*c^2*d^2*x^3 + d^2*x), x) - 256*b^2*c^3*integra
te(1/16*x^3*arctan(c*x)/(c^4*d^2*x^5 + 2*c^2*d^2*x^3 + d^2*x), x) - 16*(c*
(x/(c^4*d^2*x^2 + c^2*d^2) + arctan(c*x)/(c^3*d^2)) - 2*arctan(c*x)/(c^4*d
^2*x^2 + c^2*d^2))*a*b*c^2 - 640*b^2*c^2*integrate(1/16*x^2*arctan(c*x)^2/
(c^4*d^2*x^5 + 2*c^2*d^2*x^3 + d^2*x), x) + 3*b^2*c^2*log(c^2*x^2 + 1)^2/(
c^4*d^2*x^2 + c^2*d^2) - 256*b^2*c*integrate(1/16*x*arctan(c*x)*log(c^2*x^
2 + 1)/(c^4*d^2*x^5 + 2*c^2*d^2*x^3 + d^2*x), x) - 256*b^2*c*integrate(1/1
6*x*arctan(c*x)/(c^4*d^2*x^5 + 2*c^2*d^2*x^3 + d^2*x), x) + 384*b^2*integr
ate(1/16*arctan(c*x)^2/(c^4*d^2*x^5 + 2*c^2*d^2*x^3 + d^2*x), x) + 32*b^2*
integrate(1/16*log(c^2*x^2 + 1)^2/(c^4*d^2*x^5 + 2*c^2*d^2*x^3 + d^2*x), x
) + 1024*a*b*integrate(1/16*arctan(c*x)/(c^4*d^2*x^5 + 2*c^2*d^2*x^3 + d^2
*x), x))*(c*d^2*x - I*d^2) - 32*(I*c*d^2*x + d^2)*integrate(1/8*(b^2*c^3*x
^3*log(c^2*x^2 + 1)^2 - 32*a*b*c*x*arctan(c*x) + 4*(b^2*c^3*x^3 - 2*b^2*c*
x)*arctan(c*x)^2 - 2*(b^2*c^3*x^3 + b^2*c*x - (b^2*c^2*x^2 - b^2)*arcta...
```

## Giac [F]

$$\int \frac{(a+b\arctan(cx))^2}{x(d+icdx)^2}\,dx=\int \frac{(b\arctan{(cx)}+a)^2}{(i\,cdx+d)^2x}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x/(d+I*c*d*x)^2,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)^2/((I*c*d*x + d)^2*x), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{x(d+icdx)^2}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^2}{x\,(d+c\,d\,x\,1\mathrm{i})^2}\,dx$$

input
```
int((a + b*atan(c*x))^2/(x*(d + c*d*x*1i)^2),x)
```

output
```
int((a + b*atan(c*x))^2/(x*(d + c*d*x*1i)^2), x)
```

## Reduce [F]

$$\int \frac{(a+b\arctan(cx))^2}{x(d+icdx)^2}\,dx = \frac{-2\left(\int \frac{atan(cx)}{c^2x^3-2ci\,x^2-x}dx\right)ab-\left(\int \frac{atan(cx)^2}{c^2x^3-2ci\,x^2-x}dx\right)b^2-\left(\int \frac{1}{c^2x^3-2ci\,x^2-x}dx\right)a^2}{d^2}$$

input
```
int((a+b*atan(c*x))^2/x/(d+I*c*d*x)^2,x)
```

output
```
( - 2*int(atan(c*x)/(c**2*x**3 - 2*c*i*x**2 - x),x)*a*b - int(atan(c*x)**2
/(c**2*x**3 - 2*c*i*x**2 - x),x)*b**2 - int(1/(c**2*x**3 - 2*c*i*x**2 - x)
,x)*a**2)/d**2
```

# 3.109 $\int \frac{(a+b\arctan(cx))^2}{x^2(d+icdx)^2}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 306

$$\begin{aligned}\int \frac{(a+b\arctan(cx))^2}{x^2(d+icdx)^2}\,dx &= -\frac{b^2c}{2d^2(i-cx)}+\frac{b^2c\arctan(cx)}{2d^2}\\
&\quad-\frac{ibc(a+b\arctan(cx))}{d^2(i-cx)}-\frac{ic(a+b\arctan(cx))^2}{2d^2}\\
&\quad-\frac{(a+b\arctan(cx))^2}{d^2x}+\frac{c(a+b\arctan(cx))^2}{d^2(i-cx)}\\
&\quad-\frac{4ic(a+b\arctan(cx))^2\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right)}{d^2}\\
&\quad-\frac{2ic(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{d^2}\\
&\quad+\frac{2bc(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right)}{d^2}\\
&\quad-\frac{ib^2c\,\mathrm{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)}{d^2}\\
&\quad+\frac{2bc(a+b\arctan(cx))\,\mathrm{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)}{d^2}\\
&\quad-\frac{ib^2c\,\mathrm{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)}{d^2}\end{aligned}$$

output
```
-1/2*b^2*c/d^2/(I-c*x)+1/2*b^2*c*arctan(c*x)/d^2-I*b*c*(a+b*arctan(c*x))/d
^2/(I-c*x)-1/2*I*c*(a+b*arctan(c*x))^2/d^2-(a+b*arctan(c*x))^2/d^2/x+c*(a+
b*arctan(c*x))^2/d^2/(I-c*x)+4*I*c*(a+b*arctan(c*x))^2*arctanh(-1+2/(1+I*c
*x))/d^2-2*I*c*(a+b*arctan(c*x))^2*ln(2/(1+I*c*x))/d^2+2*b*c*(a+b*arctan(c
*x))*ln(2-2/(1-I*c*x))/d^2-I*b^2*c*polylog(2,-1+2/(1-I*c*x))/d^2+2*b*c*(a+
b*arctan(c*x))*polylog(2,-1+2/(1+I*c*x))/d^2-I*b^2*c*polylog(3,-1+2/(1+I*c
*x))/d^2
```

## Mathematica [A] (verified)

Time = 1.80 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.30

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+icdx)^2}\,dx = -\frac{\frac{12a^2}{x}+\frac{12a^2c}{-i+cx}+24a^2c\arctan(cx)+24ia^2c\log(cx)-12ia^2c\log\left(1+c^2x^2\right)+b^2c\Big(\pi^3+12i\arctan(cx)^2 \cdots}{\cdots}$$

input
```
Integrate[(a + b*ArcTan[c*x])^2/(x^2*(d + I*c*d*x)^2),x]
```

output
```
-1/12*((12*a^2)/x + (12*a^2*c)/(-I + c*x) + 24*a^2*c*ArcTan[c*x] + (24*I)*
a^2*c*Log[c*x] - (12*I)*a^2*c*Log[1 + c^2*x^2] + b^2*c*(Pi^3 + (12*I)*ArcT
an[c*x]^2 + (12*ArcTan[c*x]^2)/(c*x) - (3*I)*Cos[2*ArcTan[c*x]] + 6*ArcTan
[c*x]*Cos[2*ArcTan[c*x]] + (6*I)*ArcTan[c*x]^2*Cos[2*ArcTan[c*x]] + (24*I)
*ArcTan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] - 24*ArcTan[c*x]*Log[1 - E^
((2*I)*ArcTan[c*x])] - 24*ArcTan[c*x]*PolyLog[2, E^((-2*I)*ArcTan[c*x])] +
 (12*I)*PolyLog[2, E^((2*I)*ArcTan[c*x])] + (12*I)*PolyLog[3, E^((-2*I)*Ar
cTan[c*x])] - 3*Sin[2*ArcTan[c*x]] - (6*I)*ArcTan[c*x]*Sin[2*ArcTan[c*x]]
 + 6*ArcTan[c*x]^2*Sin[2*ArcTan[c*x]]) + 6*a*b*c*(8*ArcTan[c*x]^2 + Cos[2*A
rcTan[c*x]] - 4*Log[(c*x)/Sqrt[1 + c^2*x^2]] + 4*PolyLog[2, E^((2*I)*ArcTa
n[c*x])] - I*Sin[2*ArcTan[c*x]] + ArcTan[c*x]*(4/(c*x) + (2*I)*Cos[2*ArcTa
n[c*x]] + (8*I)*Log[1 - E^((2*I)*ArcTan[c*x])] + 2*Sin[2*ArcTan[c*x]])))/d
^2
```

## Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+icdx)^2}\,dx$$

$$\Big\downarrow \text{ 5411}$$

$$\int \left(\frac{2ic^2(a+b\arctan(cx))^2}{d^2(cx-i)}+\frac{c^2(a+b\arctan(cx))^2}{d^2(cx-i)^2}+\frac{(a+b\arctan(cx))^2}{d^2x^2}-\frac{2ic(a+b\arctan(cx))^2}{d^2x}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{4ic\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{d^2}+\frac{2bc\operatorname{PolyLog}\left(2,\frac{2}{icx+1}-1\right)(a+b\arctan(cx))}{d^2}-\frac{ibc(a+b\arctan(cx))}{d^2(-cx+i)}-\frac{(a+b\arctan(cx))^2}{d^2x}+\frac{c(a+b\arctan(cx))^2}{d^2(-cx+i)}-\frac{ic(a+b\arctan(cx))^2}{2d^2}+\frac{2bc\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))}{d^2}-\frac{2ic\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{d^2}+\frac{b^2c\arctan(cx)}{2d^2}-\frac{ib^2c\operatorname{PolyLog}\left(2,\frac{2}{1-icx}-1\right)}{d^2}-\frac{ib^2c\operatorname{PolyLog}\left(3,\frac{2}{icx+1}-1\right)}{d^2}-\frac{b^2c}{2d^2(-cx+i)}$$

input
```
Int[(a + b*ArcTan[c*x])^2/(x^2*(d + I*c*d*x)^2),x]
```

output
```
-1/2*(b^2*c)/(d^2*(I - c*x)) + (b^2*c*ArcTan[c*x])/(2*d^2) - (I*b*c*(a + b
*ArcTan[c*x]))/(d^2*(I - c*x)) - ((I/2)*c*(a + b*ArcTan[c*x])^2)/d^2 - (a
+ b*ArcTan[c*x])^2/(d^2*x) + (c*(a + b*ArcTan[c*x])^2)/(d^2*(I - c*x)) - (
(4*I)*c*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)])/d^2 - ((2*I)*c*(
a + b*ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/d^2 + (2*b*c*(a + b*ArcTan[c*x])*
Log[2 - 2/(1 - I*c*x)])/d^2 - (I*b^2*c*PolyLog[2, -1 + 2/(1 - I*c*x)])/d^2
 + (2*b*c*(a + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)])/d^2 - (I*b^2
*c*PolyLog[3, -1 + 2/(1 + I*c*x)])/d^2
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 14.77 (sec) , antiderivative size = 8556, normalized size of antiderivative = 27.96

| method | result | size |
|---|---|---|
| parts | Expression too large to display | 8556 |
| derivativedivides | Expression too large to display | 8557 |
| default | Expression too large to display | 8557 |

input
```
int((a+b*arctan(c*x))^2/x^2/(d+I*c*d*x)^2,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

## Fricas [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+icdx)^2}\,dx = \int \frac{(b\arctan(cx)+a)^2}{(i\,cdx+d)^2x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^2/(d+I*c*d*x)^2,x, algorithm="fricas")
```

output
```
-1/4*(2*(-I*b^2*c^2*x^2 - b^2*c*x)*log(2*c*x/(c*x - I))*log(-(c*x + I)/(c*
x - I))^2 + 4*(-I*b^2*c^2*x^2 - b^2*c*x)*dilog(-2*c*x/(c*x - I) + 1)*log(-
(c*x + I)/(c*x - I)) - (2*b^2*c*x - I*b^2)*log(-(c*x + I)/(c*x - I))^2 - 4
*(c*d^2*x^2 - I*d^2*x)*integral(-(a^2*c*x + I*a^2 - (2*I*b^2*c^2*x^2 + (-I
*a*b + b^2)*c*x + a*b)*log(-(c*x + I)/(c*x - I)))/(c^3*d^2*x^5 - I*c^2*d^2
*x^4 + c*d^2*x^3 - I*d^2*x^2), x) + 4*(I*b^2*c^2*x^2 + b^2*c*x)*polylog(3,
 -(c*x + I)/(c*x - I)))/(c*d^2*x^2 - I*d^2*x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+icdx)^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*atan(c*x))**2/x**2/(d+I*c*d*x)**2,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+icdx)^2}\,dx = \int \frac{(b\arctan(cx)+a)^2}{(i\,cdx+d)^2x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^2/(d+I*c*d*x)^2,x, algorithm="maxima")
```

output
```
-a^2*(c/(c*d^2*x - I*d^2) - 2*I*c*log(c*x - I)/d^2 + 2*I*c*log(x)/d^2 + 1/
(d^2*x)) - 1/16*(8*(b^2*c^2*x^2 - I*b^2*c*x)*arctan(c*x)^3 - (-I*b^2*c^2*x
^2 - b^2*c*x)*log(c^2*x^2 + 1)^3 + 4*(2*b^2*c*x - I*b^2)*arctan(c*x)^2 - (
2*b^2*c*x - I*b^2 - 2*(b^2*c^2*x^2 - I*b^2*c*x)*arctan(c*x))*log(c^2*x^2 +
 1)^2 - 2*(128*b^2*c^4*integrate(1/16*x^4*arctan(c*x)^2/(c^4*d^2*x^6 + 2*c
^2*d^2*x^4 + d^2*x^2), x) + 32*b^2*c^4*integrate(1/16*x^4*log(c^2*x^2 + 1)
^2/(c^4*d^2*x^6 + 2*c^2*d^2*x^4 + d^2*x^2), x) - 64*b^2*c^4*integrate(1/16
*x^4*log(c^2*x^2 + 1)/(c^4*d^2*x^6 + 2*c^2*d^2*x^4 + d^2*x^2), x) + (c*(x/
(c^4*d^2*x^2 + c^2*d^2) + arctan(c*x)/(c^3*d^2)) - 2*arctan(c*x)/(c^4*d^2*
x^2 + c^2*d^2))*b^2*c^3 + 32*b^2*c^2*integrate(1/16*x^2*arctan(c*x)^2/(c^4
*d^2*x^6 + 2*c^2*d^2*x^4 + d^2*x^2), x) + 24*b^2*c^2*integrate(1/16*x^2*lo
g(c^2*x^2 + 1)^2/(c^4*d^2*x^6 + 2*c^2*d^2*x^4 + d^2*x^2), x) - 256*a*b*c^2
*integrate(1/16*x^2*arctan(c*x)/(c^4*d^2*x^6 + 2*c^2*d^2*x^4 + d^2*x^2), x
) - 64*b^2*c^2*integrate(1/16*x^2*log(c^2*x^2 + 1)/(c^4*d^2*x^6 + 2*c^2*d^
2*x^4 + d^2*x^2), x) - 64*b^2*c*integrate(1/16*x*arctan(c*x)*log(c^2*x^2 +
 1)/(c^4*d^2*x^6 + 2*c^2*d^2*x^4 + d^2*x^2), x) + 64*b^2*c*integrate(1/16*
x*arctan(c*x)/(c^4*d^2*x^6 + 2*c^2*d^2*x^4 + d^2*x^2), x) + 96*b^2*integra
te(1/16*arctan(c*x)^2/(c^4*d^2*x^6 + 2*c^2*d^2*x^4 + d^2*x^2), x) + 8*b^2*
integrate(1/16*log(c^2*x^2 + 1)^2/(c^4*d^2*x^6 + 2*c^2*d^2*x^4 + d^2*x^2),
 x) + 256*a*b*integrate(1/16*arctan(c*x)/(c^4*d^2*x^6 + 2*c^2*d^2*x^4 +...
```

## Giac [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+icdx)^2}\,dx=\int \frac{(b\arctan{(cx)}+a)^2}{(i\,cdx+d)^2x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^2/(d+I*c*d*x)^2,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)^2/((I*c*d*x + d)^2*x^2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+icdx)^2}\,dx = \int \frac{\left(a+b\,\mathrm{atan}(c\,x)\right)^2}{x^2\,\left(d+c\,d\,x\,1\mathrm{i}\right)^2}\,dx$$

input
```
int((a + b*atan(c*x))^2/(x^2*(d + c*d*x*1i)^2),x)
```

output
```
int((a + b*atan(c*x))^2/(x^2*(d + c*d*x*1i)^2), x)
```

## Reduce [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+icdx)^2}\,dx = \frac{-2\left(\int \frac{atan(cx)}{c^2x^4-2ci\,x^3-x^2}dx\right)ab-\left(\int \frac{atan(cx)^2}{c^2x^4-2ci\,x^3-x^2}dx\right)b^2-\left(\int \frac{1}{c^2x^4-2ci\,x^3-x^2}dx\right)a^2}{d^2}$$

input
```
int((a+b*atan(c*x))^2/x^2/(d+I*c*d*x)^2,x)
```

output
```
( - 2*int(atan(c*x)/(c**2*x**4 - 2*c*i*x**3 - x**2),x)*a*b - int(atan(c*x)
**2/(c**2*x**4 - 2*c*i*x**3 - x**2),x)*b**2 - int(1/(c**2*x**4 - 2*c*i*x**
3 - x**2),x)*a**2)/d**2
```

# 3.110 $\int \frac{(a+b\arctan(cx))^2}{x^3(d+icdx)^2}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 403

$$\begin{aligned}\int \frac{(a+b\arctan(cx))^2}{x^3(d+icdx)^2}\,dx &= \frac{ib^2c^2}{2d^2(i-cx)}-\frac{ib^2c^2\arctan(cx)}{2d^2}\\ &-\frac{bc(a+b\arctan(cx))}{d^2x}-\frac{bc^2(a+b\arctan(cx))}{d^2(i-cx)}\\ &-\frac{2c^2(a+b\arctan(cx))^2}{d^2}-\frac{(a+b\arctan(cx))^2}{2d^2x^2}\\ &+\frac{2ic(a+b\arctan(cx))^2}{d^2x}-\frac{ic^2(a+b\arctan(cx))^2}{d^2(i-cx)}\\ &-\frac{6c^2(a+b\arctan(cx))^2\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right)}{d^2}+\frac{b^2c^2\log(x)}{d^2}\\ &-\frac{3c^2(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{d^2}-\frac{b^2c^2\log\left(1+c^2x^2\right)}{2d^2}\\ &-\frac{4ibc^2(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right)}{d^2}\\ &-\frac{2b^2c^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)}{d^2}\\ &-\frac{3ibc^2(a+b\arctan(cx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)}{d^2}\\ &-\frac{3b^2c^2\operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)}{2d^2}\end{aligned}$$

output
```
-1/2*I*b^2*c^2*arctan(c*x)/d^2+2*I*c*(a+b*arctan(c*x))^2/d^2/x-b*c*(a+b*ar
ctan(c*x))/d^2/x-b*c^2*(a+b*arctan(c*x))/d^2/(I-c*x)-2*c^2*(a+b*arctan(c*x
))^2/d^2-1/2*(a+b*arctan(c*x))^2/d^2/x^2-4*I*b*c^2*(a+b*arctan(c*x))*ln(2-
2/(1-I*c*x))/d^2+1/2*I*b^2*c^2/d^2/(I-c*x)+6*c^2*(a+b*arctan(c*x))^2*arcta
nh(-1+2/(1+I*c*x))/d^2+b^2*c^2*ln(x)/d^2-3*c^2*(a+b*arctan(c*x))^2*ln(2/(1
+I*c*x))/d^2-1/2*b^2*c^2*ln(c^2*x^2+1)/d^2-I*c^2*(a+b*arctan(c*x))^2/d^2/(
I-c*x)-2*b^2*c^2*polylog(2,-1+2/(1-I*c*x))/d^2-3*I*b*c^2*(a+b*arctan(c*x))
*polylog(2,-1+2/(1+I*c*x))/d^2-3/2*b^2*c^2*polylog(3,-1+2/(1+I*c*x))/d^2
```

## Mathematica [A] (verified)

Time = 1.91 (sec) , antiderivative size = 491, normalized size of antiderivative = 1.22

$$\int \frac{(a+b\arctan(cx))^2}{x^3(d+icdx)^2}\,dx = \frac{-\frac{4a^2}{x^2}+\frac{16ia^2c}{x}+\frac{8ia^2c^2}{-i+cx}+24ia^2c^2\arctan(cx)-24a^2c^2\log(x)+12a^2c^2\log\left(1+c^2x^2\right)-b^2c^2\left(-i\pi^3+\frac{8\,\mathrm{arcta}\ldots}{c\ldots}\right.}{\ldots}$$

input
```
Integrate[(a + b*ArcTan[c*x])^2/(x^3*(d + I*c*d*x)^2),x]
```

output
```
((-4*a^2)/x^2 + ((16*I)*a^2*c)/x + ((8*I)*a^2*c^2)/(-I + c*x) + (24*I)*a^2
*c^2*ArcTan[c*x] - 24*a^2*c^2*Log[x] + 12*a^2*c^2*Log[1 + c^2*x^2] - b^2*c
^2*((-I)*Pi^3 + (8*ArcTan[c*x])/(c*x) + 20*ArcTan[c*x]^2 + (4*ArcTan[c*x]^
2)/(c^2*x^2) - ((16*I)*ArcTan[c*x]^2)/(c*x) - 2*Cos[2*ArcTan[c*x]] - (4*I)
*ArcTan[c*x]*Cos[2*ArcTan[c*x]] + 4*ArcTan[c*x]^2*Cos[2*ArcTan[c*x]] + 24*
ArcTan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] + (32*I)*ArcTan[c*x]*Log[1 -
 E^((2*I)*ArcTan[c*x])] - 8*Log[(c*x)/Sqrt[1 + c^2*x^2]] + (24*I)*ArcTan[c
*x]*PolyLog[2, E^((-2*I)*ArcTan[c*x])] + 16*PolyLog[2, E^((2*I)*ArcTan[c*x
])] + 12*PolyLog[3, E^((-2*I)*ArcTan[c*x])] + (2*I)*Sin[2*ArcTan[c*x]] - 4
*ArcTan[c*x]*Sin[2*ArcTan[c*x]] - (4*I)*ArcTan[c*x]^2*Sin[2*ArcTan[c*x]])
+ (4*I)*a*b*c^2*((2*I)/(c*x) + 12*ArcTan[c*x]^2 + Cos[2*ArcTan[c*x]] - 8*L
og[(c*x)/Sqrt[1 + c^2*x^2]] + 6*PolyLog[2, E^((2*I)*ArcTan[c*x])] - I*Sin[
2*ArcTan[c*x]] + 2*ArcTan[c*x]*(I + I/(c^2*x^2) + 4/(c*x) + I*Cos[2*ArcTan
[c*x]] + (6*I)*Log[1 - E^((2*I)*ArcTan[c*x])] + Sin[2*ArcTan[c*x]])))/(8*d
^2)
```

## Rubi [A] (verified)

Time = 1.21 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^2}{x^3(d+icdx)^2}dx$$

$$\Big\downarrow \text{5411}$$

$$\int \left(\frac{3c^3(a+b\arctan(cx))^2}{d^2(cx-i)}-\frac{ic^3(a+b\arctan(cx))^2}{d^2(cx-i)^2}-\frac{3c^2(a+b\arctan(cx))^2}{d^2x}+\frac{(a+b\arctan(cx))^2}{d^2x^3}-\frac{2ic(a+\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{6c^2\text{arctanh}\left(1-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{d^2}-\frac{3ibc^2\text{PolyLog}\left(2,\frac{2}{icx+1}-1\right)(a+b\arctan(cx))}{d^2}-\frac{ic^2(a+b\arctan(cx))^2}{d^2(-cx+i)}-\frac{2c^2(a+b\arctan(cx))^2}{d^2}-\frac{bc^2(a+b\arctan(cx))}{d^2(-cx+i)}-\frac{3c^2\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{d^2}-\frac{4ibc^2\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))}{d^2}-\frac{(a+b\arctan(cx))^2}{2d^2x^2}+\frac{2ic(a+b\arctan(cx))^2}{d^2x}-\frac{bc(a+b\arctan(cx))}{d^2x}-\frac{ib^2c^2\arctan(cx)}{2d^2}-\frac{2b^2c^2\text{PolyLog}\left(2,\frac{2}{1-icx}-1\right)}{d^2}-\frac{3b^2c^2\text{PolyLog}\left(3,\frac{2}{icx+1}-1\right)}{2d^2}-\frac{b^2c^2\log\left(c^2x^2+1\right)}{2d^2}+\frac{ib^2c^2}{2d^2(-cx+i)}+\frac{b^2c^2\log(x)}{d^2}$$

input
```
Int[(a + b*ArcTan[c*x])^2/(x^3*(d + I*c*d*x)^2),x]
```

output
```
((I/2)*b^2*c^2)/(d^2*(I - c*x)) - ((I/2)*b^2*c^2*ArcTan[c*x])/d^2 - (b*c*(
a + b*ArcTan[c*x]))/(d^2*x) - (b*c^2*(a + b*ArcTan[c*x]))/(d^2*(I - c*x))
- (2*c^2*(a + b*ArcTan[c*x])^2)/d^2 - (a + b*ArcTan[c*x])^2/(2*d^2*x^2) +
((2*I)*c*(a + b*ArcTan[c*x])^2)/(d^2*x) - (I*c^2*(a + b*ArcTan[c*x])^2)/(d
^2*(I - c*x)) - (6*c^2*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)])/d
^2 + (b^2*c^2*Log[x])/d^2 - (3*c^2*(a + b*ArcTan[c*x])^2*Log[2/(1 + I*c*x)
])/d^2 - (b^2*c^2*Log[1 + c^2*x^2])/(2*d^2) - ((4*I)*b*c^2*(a + b*ArcTan[c
*x])*Log[2 - 2/(1 - I*c*x)])/d^2 - (2*b^2*c^2*PolyLog[2, -1 + 2/(1 - I*c*x
)])/d^2 - ((3*I)*b*c^2*(a + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)])
/d^2 - (3*b^2*c^2*PolyLog[3, -1 + 2/(1 + I*c*x)])/(2*d^2)
```

**Defintions of rubi rules used**

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 21.27 (sec) , antiderivative size = 1967, normalized size of antiderivative = 4.88

| method | result | size |
|---|---|---|
| derivativedivides | Expression too large to display | 1967 |
| default | Expression too large to display | 1967 |
| parts | Expression too large to display | 1975 |

input
```
int((a+b*arctan(c*x))^2/x^3/(d+I*c*d*x)^2,x,method=_RETURNVERBOSE)
```

output
```
c^2*(-1/2*a^2/d^2/c^2/x^2+2*I*a^2/d^2/c/x-3*a^2/d^2*ln(c*x)+I*a^2/d^2/(c*x
-I)+3/2*a^2/d^2*ln(c^2*x^2+1)+3*I*a^2/d^2*arctan(c*x)+b^2/d^2*(-1/2/c^2/x^
2*arctan(c*x)^2+3/2*I*Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn((1+I*c*x
)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))
*arctan(c*x)^2-2*arctan(c*x)^2-1/2*arctan(c*x)*(I*c*x-(c^2*x^2+1)^(1/2)+1)
/c/x-3*ln(c*x)*arctan(c*x)^2+3*arctan(c*x)^2*ln((1+I*c*x)^2/(c^2*x^2+1)-1)
-3*arctan(c*x)^2*ln(1-(1+I*c*x)/(c^2*x^2+1)^(1/2))-3*arctan(c*x)^2*ln(1+(1
+I*c*x)/(c^2*x^2+1)^(1/2))+ln((1+I*c*x)/(c^2*x^2+1)^(1/2)-1)-6*polylog(3,(
1+I*c*x)/(c^2*x^2+1)^(1/2))+ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))-6*polylog(3,
-(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/2*arctan(c*x)*(I*c*x+(c^2*x^2+1)^(1/2)+1)/
c/x+2*I*arctan(c*x)^2/c/x-I*(c*x+I)*arctan(c*x)/(2*c*x-2*I)-3/2*I*Pi*csgn(
I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^3*arctan(c*x)^2
-3/2*I*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^3*
arctan(c*x)^2+3/2*I*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^
2*x^2+1)))^2*arctan(c*x)^2-3/2*I*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*cs
gn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1
+I*c*x)^2/(c^2*x^2+1)))*arctan(c*x)^2+3/2*I*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1
)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^3*arctan(c*x)^2+3*I*Pi*csgn((1+I*c*x)^2/(c^
2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*arctan(c*x)^2+6*I*arctan(c*x)*poly
log(2,(1+I*c*x)/(c^2*x^2+1)^(1/2))-4*I*arctan(c*x)*ln(1+(1+I*c*x)/(c^2*...
```

## Fricas [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^3(d+icdx)^2}\,dx = \int \frac{(b\arctan(cx)+a)^2}{(i\,cdx+d)^2x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^3/(d+I*c*d*x)^2,x, algorithm="fricas")
```

output
```
1/8*(6*(b^2*c^3*x^3 - I*b^2*c^2*x^2)*log(2*c*x/(c*x - I))*log(-(c*x + I)/(
c*x - I))^2 + 12*(b^2*c^3*x^3 - I*b^2*c^2*x^2)*dilog(-2*c*x/(c*x - I) + 1)
*log(-(c*x + I)/(c*x - I)) + (-6*I*b^2*c^2*x^2 - 3*b^2*c*x - I*b^2)*log(-(
c*x + I)/(c*x - I))^2 + 8*(c*d^2*x^3 - I*d^2*x^2)*integral(-1/2*(2*a^2*c*x
 + 2*I*a^2 - (6*b^2*c^3*x^3 - 3*I*b^2*c^2*x^2 + (-2*I*a*b + b^2)*c*x + 2*a
*b)*log(-(c*x + I)/(c*x - I)))/(c^3*d^2*x^6 - I*c^2*d^2*x^5 + c*d^2*x^4 -
I*d^2*x^3), x) - 12*(b^2*c^3*x^3 - I*b^2*c^2*x^2)*polylog(3, -(c*x + I)/(c
*x - I)))/(c*d^2*x^3 - I*d^2*x^2)
```

## Sympy [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^3(d+icdx)^2}\,dx = -\frac{\int \frac{a^2}{c^2x^5-2icx^4-x^3}\,dx + \int \frac{b^2\,\mathrm{atan}^2(cx)}{c^2x^5-2icx^4-x^3}\,dx + \int \frac{2ab\,\mathrm{atan}(cx)}{c^2x^5-2icx^4-x^3}\,dx}{d^2}$$

input
```
integrate((a+b*atan(c*x))**2/x**3/(d+I*c*d*x)**2,x)
```

output
```
-(Integral(a**2/(c**2*x**5 - 2*I*c*x**4 - x**3), x) + Integral(b**2*atan(c
*x)**2/(c**2*x**5 - 2*I*c*x**4 - x**3), x) + Integral(2*a*b*atan(c*x)/(c**
2*x**5 - 2*I*c*x**4 - x**3), x))/d**2
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\arctan(cx))^2}{x^3(d+icdx)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arctan(c*x))^2/x^3/(d+I*c*d*x)^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

## Giac [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^3(d+icdx)^2}\,dx = \int \frac{(b\arctan(cx)+a)^2}{(i\,cdx+d)^2x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^3/(d+I*c*d*x)^2,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)^2/((I*c*d*x + d)^2*x^3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{x^3(d+icdx)^2}\,dx=\int \frac{{\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}^2}{x^3\,{\left(d+c\,d\,x\,1{}\mathrm{i}\right)}^2}\,dx$$

input
```
int((a + b*atan(c*x))^2/(x^3*(d + c*d*x*1i)^2),x)
```

output
```
int((a + b*atan(c*x))^2/(x^3*(d + c*d*x*1i)^2), x)
```

## Reduce [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^3(d+icdx)^2}\,dx = \frac{-2\left(\int \frac{atan(cx)}{c^2x^5-2ci\,x^4-x^3}dx\right)ab-\left(\int \frac{atan(cx)^2}{c^2x^5-2ci\,x^4-x^3}dx\right)b^2-\left(\int \frac{1}{c^2x^5-2ci\,x^4-x^3}dx\right)a^2}{d^2}$$

input
```
int((a+b*atan(c*x))^2/x^3/(d+I*c*d*x)^2,x)
```

output
```
( - 2*int(atan(c*x)/(c**2*x**5 - 2*c*i*x**4 - x**3),x)*a*b - int(atan(c*x)
**2/(c**2*x**5 - 2*c*i*x**4 - x**3),x)*b**2 - int(1/(c**2*x**5 - 2*c*i*x**
4 - x**3),x)*a**2)/d**2
```

# 3.111 $\int \frac{x^4(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 462

$$\begin{aligned}\int \frac{x^4(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx =& -\frac{iabx}{c^4d^3}+\frac{ib^2}{16c^5d^3(i-cx)^2}-\frac{29b^2}{16c^5d^3(i-cx)} \\ &+\frac{29b^2\arctan(cx)}{16c^5d^3}-\frac{ib^2x\arctan(cx)}{c^4d^3} \\ &-\frac{b(a+b\arctan(cx))}{4c^5d^3(i-cx)^2}-\frac{15ib(a+b\arctan(cx))}{4c^5d^3(i-cx)} \\ &-\frac{5i(a+b\arctan(cx))^2}{8c^5d^3}-\frac{3x(a+b\arctan(cx))^2}{c^4d^3} \\ &+\frac{ix^2(a+b\arctan(cx))^2}{2c^3d^3}-\frac{i(a+b\arctan(cx))^2}{2c^5d^3(i-cx)^2} \\ &+\frac{4(a+b\arctan(cx))^2}{c^5d^3(i-cx)}-\frac{6b(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c^5d^3} \\ &+\frac{6i(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{c^5d^3} \\ &+\frac{ib^2\log(1+c^2x^2)}{2c^5d^3}-\frac{3ib^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^5d^3} \\ &-\frac{6b(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^5d^3} \\ &+\frac{3ib^2\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{c^5d^3}\end{aligned}$$

output
```
-I*a*b*x/c^4/d^3+1/2*I*b^2*ln(c^2*x^2+1)/c^5/d^3-29/16*b^2/c^5/d^3/(I-c*x)
+29/16*b^2*arctan(c*x)/c^5/d^3+3*I*b^2*polylog(3,1-2/(1+I*c*x))/c^5/d^3-1/
4*b*(a+b*arctan(c*x))/c^5/d^3/(I-c*x)^2-5/8*I*(a+b*arctan(c*x))^2/c^5/d^3-
3*I*b^2*polylog(2,1-2/(1+I*c*x))/c^5/d^3-3*x*(a+b*arctan(c*x))^2/c^4/d^3+1
/2*I*x^2*(a+b*arctan(c*x))^2/c^3/d^3-I*b^2*x*arctan(c*x)/c^4/d^3+4*(a+b*ar
ctan(c*x))^2/c^5/d^3/(I-c*x)-6*b*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^5/d^3
+1/16*I*b^2/c^5/d^3/(I-c*x)^2-15/4*I*b*(a+b*arctan(c*x))/c^5/d^3/(I-c*x)-1
/2*I*(a+b*arctan(c*x))^2/c^5/d^3/(I-c*x)^2-6*b*(a+b*arctan(c*x))*polylog(2
,1-2/(1+I*c*x))/c^5/d^3+6*I*(a+b*arctan(c*x))^2*ln(2/(1+I*c*x))/c^5/d^3
```

## Mathematica [A] (verified)

Time = 1.76 (sec) , antiderivative size = 578, normalized size of antiderivative = 1.25

$$\int \frac{x^4(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx$$
$$= \frac{-48a^2cx + 8ia^2c^2x^2 - \frac{8ia^2}{(-i+cx)^2} - \frac{64a^2}{-i+cx} + 96a^2\arctan(cx) - 48ia^2\log(1+c^2x^2) + ab\big(-16icx + 192\,\mathrm{arc}}{}$$

input
```
Integrate[(x^4*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x)^3,x]
```

output
```
(-48*a^2*c*x + (8*I)*a^2*c^2*x^2 - ((8*I)*a^2)/(-I + c*x)^2 - (64*a^2)/(-I
 + c*x) + 96*a^2*ArcTan[c*x] - (48*I)*a^2*Log[1 + c^2*x^2] + a*b*((-16*I)*
c*x + 192*ArcTan[c*x]^2 - 28*Cos[2*ArcTan[c*x]] + Cos[4*ArcTan[c*x]] + 48*
Log[1 + c^2*x^2] + 96*PolyLog[2, -E^((2*I)*ArcTan[c*x])] + (28*I)*Sin[2*Ar
cTan[c*x]] + (4*I)*ArcTan[c*x]*(4 + (24*I)*c*x + 4*c^2*x^2 - 14*Cos[2*ArcT
an[c*x]] + Cos[4*ArcTan[c*x]] + 48*Log[1 + E^((2*I)*ArcTan[c*x])] + (14*I)
*Sin[2*ArcTan[c*x]] - I*Sin[4*ArcTan[c*x]]) - I*Sin[4*ArcTan[c*x]]) + (16*
I)*b^2*(-(c*x*ArcTan[c*x]) + 3*ArcTan[c*x]^2 + (3*I)*c*x*ArcTan[c*x]^2 + (
(1 + c^2*x^2)*ArcTan[c*x]^2)/2 - (4*I)*ArcTan[c*x]^3 - (7*(-1 - (2*I)*ArcT
an[c*x] + 2*ArcTan[c*x]^2)*Cos[2*ArcTan[c*x]])/8 - Cos[4*ArcTan[c*x]]/64 -
 (I/16)*ArcTan[c*x]*Cos[4*ArcTan[c*x]] + (ArcTan[c*x]^2*Cos[4*ArcTan[c*x]]
)/8 + (6*I)*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] + 6*ArcTan[c*x]^2*L
og[1 + E^((2*I)*ArcTan[c*x])] + Log[1 + c^2*x^2]/2 + (3 - (6*I)*ArcTan[c*x
])*PolyLog[2, -E^((2*I)*ArcTan[c*x])] + 3*PolyLog[3, -E^((2*I)*ArcTan[c*x]
)] - ((7*I)/8)*Sin[2*ArcTan[c*x]] + (7*ArcTan[c*x]*Sin[2*ArcTan[c*x]])/4 +
 ((7*I)/4)*ArcTan[c*x]^2*Sin[2*ArcTan[c*x]] + (I/64)*Sin[4*ArcTan[c*x]] -
(ArcTan[c*x]*Sin[4*ArcTan[c*x]])/16 - (I/8)*ArcTan[c*x]^2*Sin[4*ArcTan[c*x
]]))/(16*c^5*d^3)
```

## Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 462, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx$$

$$\Big\downarrow \text{5411}$$

$$\int \left(-\frac{6i(a+b\arctan(cx))^2}{c^4d^3(cx-i)}+\frac{4(a+b\arctan(cx))^2}{c^4d^3(cx-i)^2}-\frac{3(a+b\arctan(cx))^2}{c^4d^3}+\frac{i(a+b\arctan(cx))^2}{c^4d^3(cx-i)^3}+\frac{ix(a+b\,\mathrm{ar}}{c^3}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{6b\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{c^5d^3}-\frac{15ib(a+b\arctan(cx))}{4c^5d^3(-cx+i)}-\frac{b(a+b\arctan(cx))}{4c^5d^3(-cx+i)^2}+\frac{4(a+b\arctan(cx))^2}{c^5d^3(-cx+i)}-\frac{i(a+b\arctan(cx))^2}{2c^5d^3(-cx+i)^2}-\frac{5i(a+b\arctan(cx))^2}{8c^5d^3}-\frac{6b\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c^5d^3}+\frac{6i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c^5d^3}-\frac{3x(a+b\arctan(cx))^2}{c^4d^3}+\frac{ix^2(a+b\arctan(cx))^2}{2c^3d^3}-\frac{iabx}{c^4d^3}+\frac{29b^2\arctan(cx)}{16c^5d^3}-\frac{ib^2x\arctan(cx)}{c^4d^3}-\frac{3ib^2\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{c^5d^3}+\frac{3ib^2\,\text{PolyLog}\left(3,1-\frac{2}{icx+1}\right)}{c^5d^3}-\frac{29b^2}{16c^5d^3(-cx+i)}+\frac{ib^2}{16c^5d^3(-cx+i)^2}+\frac{ib^2\log\left(c^2x^2+1\right)}{2c^5d^3}$$

input
```
Int[(x^4*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x)^3,x]
```

output
```
((-I)*a*b*x)/(c^4*d^3) + ((I/16)*b^2)/(c^5*d^3*(I - c*x)^2) - (29*b^2)/(16
*c^5*d^3*(I - c*x)) + (29*b^2*ArcTan[c*x])/(16*c^5*d^3) - (I*b^2*x*ArcTan[
c*x])/(c^4*d^3) - (b*(a + b*ArcTan[c*x]))/(4*c^5*d^3*(I - c*x)^2) - (((15*
I)/4)*b*(a + b*ArcTan[c*x]))/(c^5*d^3*(I - c*x)) - (((5*I)/8)*(a + b*ArcTa
n[c*x])^2)/(c^5*d^3) - (3*x*(a + b*ArcTan[c*x])^2)/(c^4*d^3) + ((I/2)*x^2*
(a + b*ArcTan[c*x])^2)/(c^3*d^3) - ((I/2)*(a + b*ArcTan[c*x])^2)/(c^5*d^3*
(I - c*x)^2) + (4*(a + b*ArcTan[c*x])^2)/(c^5*d^3*(I - c*x)) - (6*b*(a + b
*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(c^5*d^3) + ((6*I)*(a + b*ArcTan[c*x])^2
*Log[2/(1 + I*c*x)])/(c^5*d^3) + ((I/2)*b^2*Log[1 + c^2*x^2])/(c^5*d^3) -
((3*I)*b^2*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c^5*d^3) - (6*b*(a + b*ArcTan[c
*x])*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c^5*d^3) + ((3*I)*b^2*PolyLog[3, 1 -
2/(1 + I*c*x)])/(c^5*d^3)
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 14.61 (sec) , antiderivative size = 1215, normalized size of antiderivative = 2.63

| method | result | size |
|---|---|---|
| derivativedivides | Expression too large to display | 1215 |
| default | Expression too large to display | 1215 |
| parts | Expression too large to display | 1279 |

input
```
int(x^4*(a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^5*(I*a*b/d^3*arctan(c*x)*c^2*x^2-12*I*a*b/d^3*arctan(c*x)*ln(c*x-I)-I*
a*b/d^3*arctan(c*x)/(c*x-I)^2-6*a*b/d^3*arctan(c*x)*c*x-I*a*b/d^3*c*x-a*b/
d^3-6*a*b/d^3*ln(c*x-I)*ln(-1/2*I*(c*x+I))-5/8*I*a*b/d^3*arctan(1/2*c*x-1/
2*I)+5/16*I*a*b/d^3*arctan(1/2*c*x)+15/4*I*a*b/d^3/(c*x-I)+43/8*I*a*b/d^3*
arctan(c*x)-5/16*I*a*b/d^3*arctan(1/6*c^3*x^3+7/6*c*x)-3*I*a^2/d^3*ln(c^2*
x^2+1)-1/2*I*a^2/d^3/(c*x-I)^2-3*a^2/d^3*c*x+5/32*a*b/d^3*ln(c^4*x^4+10*c^
2*x^2+9)-1/4*a*b/d^3/(c*x-I)^2+43/16*a*b/d^3*ln(c^2*x^2+1)+3*a*b/d^3*ln(c*
x-I)^2-6*a*b/d^3*dilog(-1/2*I*(c*x+I))+b^2/d^3*(7/4*(c*x+I)*arctan(c*x)/(c
*x-I)-3*c*x*arctan(c*x)^2+4*arctan(c*x)^3-4*arctan(c*x)^2/(c*x-I)+6*Pi*csg
n((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*arctan(c*x)^2+3*P
i*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^3*arctan(c*x)^
2+1/2*I*arctan(c*x)^2*c^2*x^2+3*Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csg
n((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^
2*x^2+1)))*arctan(c*x)^2-6*Pi*arctan(c*x)^2-3*Pi*csgn(I/(1+(1+I*c*x)^2/(c^
2*x^2+1)))*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*arc
tan(c*x)^2+3*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)
/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*arctan(c*x)^2+6*I*arctan(c*x)^2*ln(2*I*(1+
I*c*x)^2/(c^2*x^2+1))-I*arctan(c*x)*(c*x-I)-6*I*arctan(c*x)^2*ln(c*x-I)-1/
2*I*arctan(c*x)^2/(c*x-I)^2+43/8*I*arctan(c*x)^2+6*I*dilog(1+I*(1+I*c*x)/(
c^2*x^2+1)^(1/2))-I*ln(1+(1+I*c*x)^2/(c^2*x^2+1))+6*arctan(c*x)*polylog...
```

## Fricas [F]

$$\int \frac{x^4(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx = \int \frac{(b\arctan(cx)+a)^2x^4}{(i\,cdx+d)^3}\,dx$$

input
```
integrate(x^4*(a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x, algorithm="fricas")
```

output
```
integral(1/4*(-I*b^2*x^4*log(-(c*x + I)/(c*x - I))^2 - 4*a*b*x^4*log(-(c*x
 + I)/(c*x - I)) + 4*I*a^2*x^4)/(c^3*d^3*x^3 - 3*I*c^2*d^3*x^2 - 3*c*d^3*x
 + I*d^3), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**4*(a+b*atan(c*x))**2/(d+I*c*d*x)**3,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{x^4(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx = \int \frac{(b\arctan(cx)+a)^2x^4}{(i\,cdx+d)^3}\,dx$$

input
```
integrate(x^4*(a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x, algorithm="maxima")
```

output
```
1/128*(64*I*a^2*c^4*x^4 - 256*a^2*c^3*x^3 - 32*a^2*c^2*x^2*(15*arctan2(1,
c*x) - 22*I) + 64*a^2*c*x*(15*I*arctan2(1, c*x) - 2) + 192*(b^2*c^2*x^2 -
2*I*b^2*c*x - b^2)*arctan(c*x)^3 + 24*(I*b^2*c^2*x^2 + 2*b^2*c*x - I*b^2)*
log(c^2*x^2 + 1)^3 + 32*a^2*(15*arctan2(1, c*x) + 14*I) + 16*(I*b^2*c^4*x^
4 - 4*b^2*c^3*x^3 + 11*I*b^2*c^2*x^2 - 2*b^2*c*x + 7*I*b^2)*arctan(c*x)^2
- 4*(I*b^2*c^4*x^4 - 4*b^2*c^3*x^3 + 11*I*b^2*c^2*x^2 - 2*b^2*c*x + 7*I*b^
2 - 12*(b^2*c^2*x^2 - 2*I*b^2*c*x - b^2)*arctan(c*x))*log(c^2*x^2 + 1)^2 -
 36*(I*b^2*c^8*d^3*x^2 + 2*b^2*c^7*d^3*x - I*b^2*c^6*d^3)*(((8*c^2*x^2 + 7
)*c^2/(c^16*d^3*x^4 + 2*c^14*d^3*x^2 + c^12*d^3) + 2*(4*c^2*x^2 + 3)*log(c
^2*x^2 + 1)/(c^14*d^3*x^4 + 2*c^12*d^3*x^2 + c^10*d^3))*c^4 + 2*(2*c^2*x^2
 + 1)*c^2*log(c^2*x^2 + 1)^2/(c^12*d^3*x^4 + 2*c^10*d^3*x^2 + c^8*d^3) - c
^2*(c^2/(c^14*d^3*x^4 + 2*c^12*d^3*x^2 + c^10*d^3) + 2*log(c^2*x^2 + 1)/(c
^12*d^3*x^4 + 2*c^10*d^3*x^2 + c^8*d^3)) - 2048*c^2*integrate(1/64*x^3*arc
tan(c*x)^2/(c^10*d^3*x^6 + 3*c^8*d^3*x^4 + 3*c^6*d^3*x^2 + c^4*d^3), x) -
2*log(c^2*x^2 + 1)^2/(c^10*d^3*x^4 + 2*c^8*d^3*x^2 + c^6*d^3) + 2048*integ
rate(1/64*x*arctan(c*x)^2/(c^10*d^3*x^6 + 3*c^8*d^3*x^4 + 3*c^6*d^3*x^2 +
c^4*d^3), x)) + 12*(-I*b^2*c^10*d^3*x^2 - 2*b^2*c^9*d^3*x + I*b^2*c^8*d^3)
*(((8*c^2*x^2 + 7)*c^2/(c^16*d^3*x^4 + 2*c^14*d^3*x^2 + c^12*d^3) + 2*(4*c
^2*x^2 + 3)*log(c^2*x^2 + 1)/(c^14*d^3*x^4 + 2*c^12*d^3*x^2 + c^10*d^3))*c
^2 + 2048*c^2*integrate(1/64*x^5*arctan(c*x)^2/(c^10*d^3*x^6 + 3*c^8*d^...
```

## Giac [F]

$$\int \frac{x^4(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx=\int \frac{(b\arctan{(cx)}+a)^2x^4}{(i\,cdx+d)^3}\,dx$$

input
```
integrate(x^4*(a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)^2*x^4/(I*c*d*x + d)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx = \int \frac{x^4\,(a+b\,\mathrm{atan}(c\,x))^2}{(d+c\,d\,x\,1\mathrm{i})^3}\,dx$$

input
```
int((x^4*(a + b*atan(c*x))^2)/(d + c*d*x*1i)^3,x)
```

output
```
int((x^4*(a + b*atan(c*x))^2)/(d + c*d*x*1i)^3, x)
```

## Reduce [F]

$$\int \frac{x^4(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx$$
$$= \frac{-4\left(\int \frac{atan(cx)x^4}{c^3i\,x^3+3c^2x^2-3cix-1}dx\right)ab\,c^5 - 2\left(\int \frac{atan(cx)^2x^4}{c^3i\,x^3+3c^2x^2-3cix-1}dx\right)b^2c^5 + 8\left(\int \frac{x}{c^3i\,x^3+3c^2x^2-3cix-1}dx\right)a^2c^2i + 6(}{2c^5d^3}$$

input
```
int(x^4*(a+b*atan(c*x))^2/(d+I*c*d*x)^3,x)
```

output
```
( - 4*int((atan(c*x)*x**4)/(c**3*i*x**3 + 3*c**2*x**2 - 3*c*i*x - 1),x)*a*
b*c**5 - 2*int((atan(c*x)**2*x**4)/(c**3*i*x**3 + 3*c**2*x**2 - 3*c*i*x -
1),x)*b**2*c**5 + 8*int(x/(c**3*i*x**3 + 3*c**2*x**2 - 3*c*i*x - 1),x)*a**
2*c**2*i + 6*int(1/(c**3*i*x**3 + 3*c**2*x**2 - 3*c*i*x - 1),x)*a**2*c - 4
*log(c**3*i*x**3 + 3*c**2*x**2 - 3*c*i*x - 1)*a**2*i + a**2*c**2*i*x**2 -
6*a**2*c*x)/(2*c**5*d**3)
```

# 3.112 $\int \frac{x^3(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 383

$$\begin{aligned}\int \frac{x^3(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx &= \frac{b^2}{16c^4d^3(i-cx)^2}+\frac{21ib^2}{16c^4d^3(i-cx)}-\frac{21ib^2\arctan(cx)}{16c^4d^3}\\
&+\frac{ib(a+b\arctan(cx))}{4c^4d^3(i-cx)^2}-\frac{11b(a+b\arctan(cx))}{4c^4d^3(i-cx)}\\
&+\frac{3(a+b\arctan(cx))^2}{8c^4d^3}+\frac{ix(a+b\arctan(cx))^2}{c^3d^3}\\
&-\frac{(a+b\arctan(cx))^2}{2c^4d^3(i-cx)^2}-\frac{3i(a+b\arctan(cx))^2}{c^4d^3(i-cx)}\\
&+\frac{2ib(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c^4d^3}\\
&+\frac{3(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{c^4d^3}\\
&-\frac{b^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^4d^3}\\
&+\frac{3ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^4d^3}\\
&+\frac{3b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2c^4d^3}\end{aligned}$$

output
```
1/16*b^2/c^4/d^3/(I-c*x)^2+21/16*I*b^2/c^4/d^3/(I-c*x)-21/16*I*b^2*arctan(
c*x)/c^4/d^3+1/4*I*b*(a+b*arctan(c*x))/c^4/d^3/(I-c*x)^2-11/4*b*(a+b*arcta
n(c*x))/c^4/d^3/(I-c*x)+3/8*(a+b*arctan(c*x))^2/c^4/d^3+I*x*(a+b*arctan(c*
x))^2/c^3/d^3-1/2*(a+b*arctan(c*x))^2/c^4/d^3/(I-c*x)^2-3*I*(a+b*arctan(c*
x))^2/c^4/d^3/(I-c*x)+2*I*b*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^4/d^3+3*(a
+b*arctan(c*x))^2*ln(2/(1+I*c*x))/c^4/d^3-b^2*polylog(2,1-2/(1+I*c*x))/c^4
/d^3+3*I*b*(a+b*arctan(c*x))*polylog(2,1-2/(1+I*c*x))/c^4/d^3+3/2*b^2*poly
log(3,1-2/(1+I*c*x))/c^4/d^3
```

## Mathematica [A] (verified)

Time = 1.12 (sec) , antiderivative size = 507, normalized size of antiderivative = 1.32

$$\int \frac{x^3(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx = \frac{64ia^2cx - \frac{32a^2}{(-i+cx)^2} + \frac{192ia^2}{-i+cx} - 192ia^2\arctan(cx) - 96a^2\log(1+c^2x^2) + 4iab\big(-96\arctan(cx)^2 + 20\cos(2$$

input
```
Integrate[(x^3*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x)^3,x]
```

output
```
((64*I)*a^2*c*x - (32*a^2)/(-I + c*x)^2 + ((192*I)*a^2)/(-I + c*x) - (192*
I)*a^2*ArcTan[c*x] - 96*a^2*Log[1 + c^2*x^2] + (4*I)*a*b*(-96*ArcTan[c*x]^
2 + 20*Cos[2*ArcTan[c*x]] - Cos[4*ArcTan[c*x]] - 16*Log[1 + c^2*x^2] - 48*
PolyLog[2, -E^((2*I)*ArcTan[c*x])] - (20*I)*Sin[2*ArcTan[c*x]] + 4*ArcTan[
c*x]*(8*c*x + (10*I)*Cos[2*ArcTan[c*x]] - I*Cos[4*ArcTan[c*x]] - (24*I)*Lo
g[1 + E^((2*I)*ArcTan[c*x])] + 10*Sin[2*ArcTan[c*x]] - Sin[4*ArcTan[c*x]])
 + I*Sin[4*ArcTan[c*x]]) + I*b^2*((-64*I)*ArcTan[c*x]^2 + 64*c*x*ArcTan[c*
x]^2 - 128*ArcTan[c*x]^3 - (40*I)*Cos[2*ArcTan[c*x]] + 80*ArcTan[c*x]*Cos[
2*ArcTan[c*x]] + (80*I)*ArcTan[c*x]^2*Cos[2*ArcTan[c*x]] + I*Cos[4*ArcTan[
c*x]] - 4*ArcTan[c*x]*Cos[4*ArcTan[c*x]] - (8*I)*ArcTan[c*x]^2*Cos[4*ArcTa
n[c*x]] + 128*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] - (192*I)*ArcTan[
c*x]^2*Log[1 + E^((2*I)*ArcTan[c*x])] - 64*(I + 3*ArcTan[c*x])*PolyLog[2,
-E^((2*I)*ArcTan[c*x])] - (96*I)*PolyLog[3, -E^((2*I)*ArcTan[c*x])] - 40*S
in[2*ArcTan[c*x]] - (80*I)*ArcTan[c*x]*Sin[2*ArcTan[c*x]] + 80*ArcTan[c*x]
^2*Sin[2*ArcTan[c*x]] + Sin[4*ArcTan[c*x]] + (4*I)*ArcTan[c*x]*Sin[4*ArcTa
n[c*x]] - 8*ArcTan[c*x]^2*Sin[4*ArcTan[c*x]]))/(64*c^4*d^3)
```

## Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx$$

$$\Big\downarrow \text{5411}$$

$$\int \left(-\frac{3(a+b\arctan(cx))^2}{c^3d^3(cx-i)}-\frac{3i(a+b\arctan(cx))^2}{c^3d^3(cx-i)^2}+\frac{i(a+b\arctan(cx))^2}{c^3d^3}+\frac{(a+b\arctan(cx))^2}{c^3d^3(cx-i)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3ib\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{c^4d^3}-\frac{11b(a+b\arctan(cx))}{4c^4d^3(-cx+i)}+\frac{ib(a+b\arctan(cx))}{4c^4d^3(-cx+i)^2}-\frac{3i(a+b\arctan(cx))^2}{c^4d^3(-cx+i)}-\frac{(a+b\arctan(cx))^2}{2c^4d^3(-cx+i)^2}+\frac{3(a+b\arctan(cx))^2}{8c^4d^3}+\frac{2ib\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c^4d^3}+\frac{3\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c^4d^3}+\frac{ix(a+b\arctan(cx))^2}{c^3d^3}-\frac{21ib^2\arctan(cx)}{16c^4d^3}-\frac{b^2\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{c^4d^3}+\frac{3b^2\,\text{PolyLog}\left(3,1-\frac{2}{icx+1}\right)}{2c^4d^3}+\frac{21ib^2}{16c^4d^3(-cx+i)}+\frac{b^2}{16c^4d^3(-cx+i)^2}$$

input
```
Int[(x^3*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x)^3,x]
```

output
```
b^2/(16*c^4*d^3*(I - c*x)^2) + (((21*I)/16)*b^2)/(c^4*d^3*(I - c*x)) - (((
21*I)/16)*b^2*ArcTan[c*x])/(c^4*d^3) + ((I/4)*b*(a + b*ArcTan[c*x]))/(c^4*
d^3*(I - c*x)^2) - (11*b*(a + b*ArcTan[c*x]))/(4*c^4*d^3*(I - c*x)) + (3*(
a + b*ArcTan[c*x])^2)/(8*c^4*d^3) + (I*x*(a + b*ArcTan[c*x])^2)/(c^3*d^3)
- (a + b*ArcTan[c*x])^2/(2*c^4*d^3*(I - c*x)^2) - ((3*I)*(a + b*ArcTan[c*x
])^2)/(c^4*d^3*(I - c*x)) + ((2*I)*b*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)
])/(c^4*d^3) + (3*(a + b*ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/(c^4*d^3) - (b
^2*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c^4*d^3) + ((3*I)*b*(a + b*ArcTan[c*x])
*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c^4*d^3) + (3*b^2*PolyLog[3, 1 - 2/(1 + I
*c*x)])/(2*c^4*d^3)
```

**Defintions of rubi rules used**

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 10.30 (sec) , antiderivative size = 4306, normalized size of antiderivative = 11.24

| method | result | size |
|---|---|---|
| derivativedivides | Expression too large to display | 4306 |
| default | Expression too large to display | 4306 |
| parts | Expression too large to display | 4365 |

input
```
int(x^3*(a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^4*(2*I*a*b/d^3*arctan(c*x)*c*x-3/2*a^2/d^3*ln(c^2*x^2+1)+3*I*a*b/d^3*l
n(c*x-I)*ln(-1/2*I*(c*x+I))+3*I*a^2/d^3/(c*x-I)-1/2*a^2/d^3/(c*x-I)^2+b^2/
d^3*(-3*I*Pi*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+19/8*arctan(c*x)^2+19/
16*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))-3*I*arctan(c*x)*polylog(2,-(1+I*c*x
)^2/(c^2*x^2+1))-3*I*Pi*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-3/8*I*arcta
n(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-3/8*I*arctan(c*x)*ln(1-I*(1+I*c
*x)/(c^2*x^2+1)^(1/2))+19/8*I*arctan(c*x)*ln(1+(1+I*c*x)^2/(c^2*x^2+1))+3*
I*Pi*arctan(c*x)^2+3*Pi*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+3*
Pi*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-3*Pi*arctan(c*x)*ln(1+(
1+I*c*x)^2/(c^2*x^2+1))-3/8*dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-3/8*dil
og(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-2*I*arctan(c*x)^3+3/2*polylog(3,-(1+I*
c*x)^2/(c^2*x^2+1))+3/2*I*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^
2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*dilog(1+I*(1+I*c*x)/(c^2*x^2+
1)^(1/2))-3/2*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1
)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^
(1/2))-3/2*Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn((1+I*c*x)^2/(c^2*x^
2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*arctan(c*x)*ln(1+(1+I*c*x)^2/(c^2*x^2+
1))+3/2*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(
1+I*c*x)^2/(c^2*x^2+1)))^2*arctan(c*x)*ln(1+(1+I*c*x)^2/(c^2*x^2+1))+3/2*P
i*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(...
```

## Fricas [F]

$$\int \frac{x^3(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx=\int \frac{(b\arctan{(cx)}+a)^2x^3}{(i\,cdx+d)^3}\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x, algorithm="fricas")
```

output
```
integral(1/4*(-I*b^2*x^3*log(-(c*x + I)/(c*x - I))^2 - 4*a*b*x^3*log(-(c*x
 + I)/(c*x - I)) + 4*I*a^2*x^3)/(c^3*d^3*x^3 - 3*I*c^2*d^3*x^2 - 3*c*d^3*x
 + I*d^3), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(a+b*atan(c*x))**2/(d+I*c*d*x)**3,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{x^3(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx = \int \frac{(b\arctan(cx)+a)^2x^3}{(i\,cdx+d)^3}\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x, algorithm="maxima")
```

output
```
1/128*(128*I*a^2*c^3*x^3 + 32*a^2*c^2*x^2*(3*I*arctan2(1, c*x) + 8) + 64*a
^2*c*x*(3*arctan2(1, c*x) + 4*I) + 96*(-I*b^2*c^2*x^2 - 2*b^2*c*x + I*b^2)
*arctan(c*x)^3 + 12*(b^2*c^2*x^2 - 2*I*b^2*c*x - b^2)*log(c^2*x^2 + 1)^3 +
 32*a^2*(-3*I*arctan2(1, c*x) + 10) + 16*(2*I*b^2*c^3*x^3 + 4*b^2*c^2*x^2
+ 4*I*b^2*c*x + 5*b^2)*arctan(c*x)^2 - 4*(2*I*b^2*c^3*x^3 + 4*b^2*c^2*x^2
+ 4*I*b^2*c*x + 5*b^2 - 6*(-I*b^2*c^2*x^2 - 2*b^2*c*x + I*b^2)*arctan(c*x)
)*log(c^2*x^2 + 1)^2 - 18*(b^2*c^7*d^3*x^2 - 2*I*b^2*c^6*d^3*x - b^2*c^5*d
^3)*(((8*c^2*x^2 + 7)*c^2/(c^15*d^3*x^4 + 2*c^13*d^3*x^2 + c^11*d^3) + 2*(
4*c^2*x^2 + 3)*log(c^2*x^2 + 1)/(c^13*d^3*x^4 + 2*c^11*d^3*x^2 + c^9*d^3))
*c^4 + 2*(2*c^2*x^2 + 1)*c^2*log(c^2*x^2 + 1)^2/(c^11*d^3*x^4 + 2*c^9*d^3*
x^2 + c^7*d^3) - c^2*(c^2/(c^13*d^3*x^4 + 2*c^11*d^3*x^2 + c^9*d^3) + 2*lo
g(c^2*x^2 + 1)/(c^11*d^3*x^4 + 2*c^9*d^3*x^2 + c^7*d^3)) - 4096*c^2*integr
ate(1/128*x^3*arctan(c*x)^2/(c^9*d^3*x^6 + 3*c^7*d^3*x^4 + 3*c^5*d^3*x^2 +
 c^3*d^3), x) - 2*log(c^2*x^2 + 1)^2/(c^9*d^3*x^4 + 2*c^7*d^3*x^2 + c^5*d^
3) + 4096*integrate(1/128*x*arctan(c*x)^2/(c^9*d^3*x^6 + 3*c^7*d^3*x^4 + 3
*c^5*d^3*x^2 + c^3*d^3), x)) - 8*(b^2*c^9*d^3*x^2 - 2*I*b^2*c^8*d^3*x - b^
2*c^7*d^3)*(((8*c^2*x^2 + 7)*c^2/(c^15*d^3*x^4 + 2*c^13*d^3*x^2 + c^11*d^3
) + 2*(4*c^2*x^2 + 3)*log(c^2*x^2 + 1)/(c^13*d^3*x^4 + 2*c^11*d^3*x^2 + c^
9*d^3))*c^2 + 4096*c^2*integrate(1/128*x^5*arctan(c*x)^2/(c^9*d^3*x^6 + 3*
c^7*d^3*x^4 + 3*c^5*d^3*x^2 + c^3*d^3), x) + 1024*c^2*integrate(1/128*x...
```

## Giac [F]

$$\int \frac{x^3(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx=\int \frac{(b\arctan(cx)+a)^2x^3}{(i\,cdx+d)^3}\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)^2*x^3/(I*c*d*x + d)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx=\int \frac{x^3\,{\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}^2}{{\left(d+c\,d\,x\,1{}\mathrm{i}\right)}^3}\,dx$$

input
```
int((x^3*(a + b*atan(c*x))^2)/(d + c*d*x*1i)^3,x)
```

output
```
int((x^3*(a + b*atan(c*x))^2)/(d + c*d*x*1i)^3, x)
```

## Reduce [F]

$$\int \frac{x^3(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx = \frac{-2\left(\int \frac{atan(cx)x^3}{c^3i\,x^3+3c^2x^2-3cix-1}dx\right)ab\,c^4-\left(\int \frac{atan(cx)^2x^3}{c^3i\,x^3+3c^2x^2-3cix-1}dx\right)b^2c^4+3\left(\int \frac{x}{c^3i\,x^3+3c^2x^2-3cix-1}dx\right)a^2c^2-2\left(\int\right.}{c^4d^3}$$

input
```
int(x^3*(a+b*atan(c*x))^2/(d+I*c*d*x)^3,x)
```

output
```
( - 2*int((atan(c*x)*x**3)/(c**3*i*x**3 + 3*c**2*x**2 - 3*c*i*x - 1),x)*a*
b*c**4 - int((atan(c*x)**2*x**3)/(c**3*i*x**3 + 3*c**2*x**2 - 3*c*i*x - 1)
,x)*b**2*c**4 + 3*int(x/(c**3*i*x**3 + 3*c**2*x**2 - 3*c*i*x - 1),x)*a**2*
c**2 - 2*int(1/(c**3*i*x**3 + 3*c**2*x**2 - 3*c*i*x - 1),x)*a**2*c*i - log
(c**3*i*x**3 + 3*c**2*x**2 - 3*c*i*x - 1)*a**2 + a**2*c*i*x)/(c**4*d**3)
```

# 3.113 $\int \frac{x^2(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 304

$$\int \frac{x^2(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx = -\frac{ib^2}{16c^3d^3(i-cx)^2}+\frac{13b^2}{16c^3d^3(i-cx)}-\frac{13b^2\arctan(cx)}{16c^3d^3}+\frac{b(a+b\arctan(cx))}{4c^3d^3(i-cx)^2}+\frac{7ib(a+b\arctan(cx))}{4c^3d^3(i-cx)}-\frac{7i(a+b\arctan(cx))^2}{8c^3d^3}+\frac{i(a+b\arctan(cx))^2}{2c^3d^3(i-cx)^2}-\frac{2(a+b\arctan(cx))^2}{c^3d^3(i-cx)}-\frac{i(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{c^3d^3}+\frac{b(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^3d^3}-\frac{ib^2\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2c^3d^3}$$

output

```
-1/16*I*b^2/c^3/d^3/(I-c*x)^2+13/16*b^2/c^3/d^3/(I-c*x)-13/16*b^2*arctan(c
*x)/c^3/d^3+1/4*b*(a+b*arctan(c*x))/c^3/d^3/(I-c*x)^2+7/4*I*b*(a+b*arctan(
c*x))/c^3/d^3/(I-c*x)-7/8*I*(a+b*arctan(c*x))^2/c^3/d^3+1/2*I*(a+b*arctan(
c*x))^2/c^3/d^3/(I-c*x)^2-2*(a+b*arctan(c*x))^2/c^3/d^3/(I-c*x)-I*(a+b*arc
tan(c*x))^2*ln(2/(1+I*c*x))/c^3/d^3+b*(a+b*arctan(c*x))*polylog(2,1-2/(1+I
*c*x))/c^3/d^3-1/2*I*b^2*polylog(3,1-2/(1+I*c*x))/c^3/d^3
```

## Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 431, normalized size of antiderivative = 1.42

$$\int \frac{x^2(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx$$
$$= \frac{\frac{96ia^2}{(-i+cx)^2}+\frac{384a^2}{-i+cx}-192a^2\arctan(cx)+96ia^2\log(1+c^2x^2)-b^2\left(128\arctan(cx)^3+72i\cos(2\arctan(cx))\right)}{}$$

input
```
Integrate[(x^2*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x)^3,x]
```

output
```
(((96*I)*a^2)/(-I + c*x)^2 + (384*a^2)/(-I + c*x) - 192*a^2*ArcTan[c*x] +
(96*I)*a^2*Log[1 + c^2*x^2] - b^2*(128*ArcTan[c*x]^3 + (72*I)*Cos[2*ArcTan
[c*x]] - 144*ArcTan[c*x]*Cos[2*ArcTan[c*x]] - (144*I)*ArcTan[c*x]^2*Cos[2*
ArcTan[c*x]] - (3*I)*Cos[4*ArcTan[c*x]] + 12*ArcTan[c*x]*Cos[4*ArcTan[c*x]
] + (24*I)*ArcTan[c*x]^2*Cos[4*ArcTan[c*x]] + (192*I)*ArcTan[c*x]^2*Log[1
+ E^((2*I)*ArcTan[c*x])] + 192*ArcTan[c*x]*PolyLog[2, -E^((2*I)*ArcTan[c*x
])] + (96*I)*PolyLog[3, -E^((2*I)*ArcTan[c*x])] + 72*Sin[2*ArcTan[c*x]] +
(144*I)*ArcTan[c*x]*Sin[2*ArcTan[c*x]] - 144*ArcTan[c*x]^2*Sin[2*ArcTan[c*
x]] - 3*Sin[4*ArcTan[c*x]] - (12*I)*ArcTan[c*x]*Sin[4*ArcTan[c*x]] + 24*Ar
cTan[c*x]^2*Sin[4*ArcTan[c*x]]) - 12*a*b*(32*ArcTan[c*x]^2 - 12*Cos[2*ArcT
an[c*x]] + Cos[4*ArcTan[c*x]] + 16*PolyLog[2, -E^((2*I)*ArcTan[c*x])] + (1
2*I)*Sin[2*ArcTan[c*x]] - I*Sin[4*ArcTan[c*x]] + 4*ArcTan[c*x]*((-6*I)*Cos
[2*ArcTan[c*x]] + I*Cos[4*ArcTan[c*x]] + (8*I)*Log[1 + E^((2*I)*ArcTan[c*x
])] - 6*Sin[2*ArcTan[c*x]] + Sin[4*ArcTan[c*x]])))/(192*c^3*d^3)
```

## Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx$$

$$\downarrow \text{5411}$$

$$\int \left(\frac{i(a+b\arctan(cx))^2}{c^2d^3(cx-i)}-\frac{2(a+b\arctan(cx))^2}{c^2d^3(cx-i)^2}-\frac{i(a+b\arctan(cx))^2}{c^2d^3(cx-i)^3}\right)dx$$

$$\downarrow \text{2009}$$

$$\frac{b\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{c^3d^3}+\frac{7ib(a+b\arctan(cx))}{4c^3d^3(-cx+i)}+\frac{b(a+b\arctan(cx))}{4c^3d^3(-cx+i)^2}-\frac{2(a+b\arctan(cx))^2}{c^3d^3(-cx+i)}+\frac{i(a+b\arctan(cx))^2}{2c^3d^3(-cx+i)^2}-\frac{7i(a+b\arctan(cx))^2}{8c^3d^3}-\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c^3d^3}-\frac{13b^2\arctan(cx)}{16c^3d^3}-\frac{ib^2\,\text{PolyLog}\left(3,1-\frac{2}{icx+1}\right)}{2c^3d^3}+\frac{13b^2}{16c^3d^3(-cx+i)}-\frac{ib^2}{16c^3d^3(-cx+i)^2}$$

input
```
Int[(x^2*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x)^3,x]
```

output
```
((-1/16*I)*b^2)/(c^3*d^3*(I - c*x)^2) + (13*b^2)/(16*c^3*d^3*(I - c*x)) -
(13*b^2*ArcTan[c*x])/(16*c^3*d^3) + (b*(a + b*ArcTan[c*x]))/(4*c^3*d^3*(I
- c*x)^2) + (((7*I)/4)*b*(a + b*ArcTan[c*x]))/(c^3*d^3*(I - c*x)) - (((7*I
)/8)*(a + b*ArcTan[c*x])^2)/(c^3*d^3) + ((I/2)*(a + b*ArcTan[c*x])^2)/(c^3
*d^3*(I - c*x)^2) - (2*(a + b*ArcTan[c*x])^2)/(c^3*d^3*(I - c*x)) - (I*(a
+ b*ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/(c^3*d^3) + (b*(a + b*ArcTan[c*x])*
PolyLog[2, 1 - 2/(1 + I*c*x)])/(c^3*d^3) - ((I/2)*b^2*PolyLog[3, 1 - 2/(1
+ I*c*x)])/(c^3*d^3)
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 9.26 (sec) , antiderivative size = 960, normalized size of antiderivative = 3.16

| method | result | size |
|---|---|---|
| derivativedivides | Expression too large to display | 960 |
| default | Expression too large to display | 960 |
| parts | Expression too large to display | 1015 |

input
```
int(x^2*(a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^3*(1/2*I*a^2/d^3*ln(c^2*x^2+1)-a^2/d^3*arctan(c*x)+I*a*b/d^3*arctan(c*
x)/(c*x-I)^2+2*a^2/d^3/(c*x-I)+b^2/d^3*(1/64*I*(c*x+I)^2/(c*x-I)^2+3*I*(c*
x+I)/(8*c*x-8*I)+2*arctan(c*x)^2/(c*x-I)+1/2*I*arctan(c*x)^2/(c*x-I)^2-2/3
*arctan(c*x)^3-1/16*(c*x+I)^2*arctan(c*x)/(c*x-I)^2-1/2*I*polylog(3,-(1+I*
c*x)^2/(c^2*x^2+1))-3/4*(c*x+I)*arctan(c*x)/(c*x-I)-7/8*I*arctan(c*x)^2-ar
ctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))-I*arctan(c*x)^2*ln(2*I*(1+I*
c*x)^2/(c^2*x^2+1))-1/2*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^
2*x^2+1)))^3*arctan(c*x)^2+1/2*Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn
((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*arctan(c*x)^2-1/2*
Pi*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)
^2/(c^2*x^2+1)))^2*arctan(c*x)^2-1/2*Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1))
)*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^
2/(c^2*x^2+1)))*arctan(c*x)^2-Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)
^2/(c^2*x^2+1)))^2*arctan(c*x)^2+Pi*arctan(c*x)^2+I*arctan(c*x)^2*ln(c*x-I
))-7/4*I*a*b/d^3/(c*x-I)-7/8*I*a*b/d^3*arctan(1/2*c*x-1/2*I)+4*a*b/d^3*arc
tan(c*x)/(c*x-I)+7/32*a*b/d^3*ln(c^4*x^4+10*c^2*x^2+9)-7/16*I*a*b/d^3*arct
an(1/6*c^3*x^3+7/6*c*x)-7/8*I*a*b/d^3*arctan(c*x)+2*I*a*b/d^3*arctan(c*x)*
ln(c*x-I)+1/2*I*a^2/d^3/(c*x-I)^2+1/4*a*b/d^3/(c*x-I)^2-7/16*a*b/d^3*ln(c^
2*x^2+1)+7/16*I*a*b/d^3*arctan(1/2*c*x)+a*b/d^3*ln(c*x-I)*ln(-1/2*I*(c*x+I
))+a*b/d^3*dilog(-1/2*I*(c*x+I))-1/2*a*b/d^3*ln(c*x-I)^2)
```

## Fricas [F]

$$\int \frac{x^2(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx=\int \frac{(b\arctan(cx)+a)^2x^2}{(i\,cdx+d)^3}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x, algorithm="fricas")
```

output
```
integral(1/4*(-I*b^2*x^2*log(-(c*x + I)/(c*x - I))^2 - 4*a*b*x^2*log(-(c*x
 + I)/(c*x - I)) + 4*I*a^2*x^2)/(c^3*d^3*x^3 - 3*I*c^2*d^3*x^2 - 3*c*d^3*x
 + I*d^3), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx=\text{Timed out}$$

input
```
integrate(x**2*(a+b*atan(c*x))**2/(d+I*c*d*x)**3,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{x^2(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx=\int \frac{(b\arctan(cx)+a)^2x^2}{(i\,cdx+d)^3}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x, algorithm="maxima")
```

output
```
1/128*(144*a^2*c^2*x^2*arctan2(1, c*x) - 32*a^2*c*x*(9*I*arctan2(1, c*x) -
 8) - 32*(b^2*c^2*x^2 - 2*I*b^2*c*x - b^2)*arctan(c*x)^3 + 4*(-I*b^2*c^2*x
^2 - 2*b^2*c*x + I*b^2)*log(c^2*x^2 + 1)^3 - 48*a^2*(3*arctan2(1, c*x) + 4
*I) + 16*(4*b^2*c*x - 3*I*b^2)*arctan(c*x)^2 - 4*(4*b^2*c*x - 3*I*b^2 + 2*
(b^2*c^2*x^2 - 2*I*b^2*c*x - b^2)*arctan(c*x))*log(c^2*x^2 + 1)^2 + 6*(I*b
^2*c^6*d^3*x^2 + 2*b^2*c^5*d^3*x - I*b^2*c^4*d^3)*(((8*c^2*x^2 + 7)*c^2/(c
^14*d^3*x^4 + 2*c^12*d^3*x^2 + c^10*d^3) + 2*(4*c^2*x^2 + 3)*log(c^2*x^2 +
 1)/(c^12*d^3*x^4 + 2*c^10*d^3*x^2 + c^8*d^3))*c^4 + 2*(2*c^2*x^2 + 1)*c^2
*log(c^2*x^2 + 1)^2/(c^10*d^3*x^4 + 2*c^8*d^3*x^2 + c^6*d^3) - c^2*(c^2/(c
^12*d^3*x^4 + 2*c^10*d^3*x^2 + c^8*d^3) + 2*log(c^2*x^2 + 1)/(c^10*d^3*x^4
 + 2*c^8*d^3*x^2 + c^6*d^3)) - 512*c^2*integrate(1/16*x^3*arctan(c*x)^2/(c
^8*d^3*x^6 + 3*c^6*d^3*x^4 + 3*c^4*d^3*x^2 + c^2*d^3), x) - 2*log(c^2*x^2
+ 1)^2/(c^8*d^3*x^4 + 2*c^6*d^3*x^2 + c^4*d^3) + 512*integrate(1/16*x*arct
an(c*x)^2/(c^8*d^3*x^6 + 3*c^6*d^3*x^4 + 3*c^4*d^3*x^2 + c^2*d^3), x)) - 4
*(-I*b^2*c^8*d^3*x^2 - 2*b^2*c^7*d^3*x + I*b^2*c^6*d^3)*(((8*c^2*x^2 + 7)*
c^2/(c^14*d^3*x^4 + 2*c^12*d^3*x^2 + c^10*d^3) + 2*(4*c^2*x^2 + 3)*log(c^2
*x^2 + 1)/(c^12*d^3*x^4 + 2*c^10*d^3*x^2 + c^8*d^3))*c^2 + 512*c^2*integra
te(1/16*x^5*arctan(c*x)^2/(c^8*d^3*x^6 + 3*c^6*d^3*x^4 + 3*c^4*d^3*x^2 + c
^2*d^3), x) + 128*c^2*integrate(1/16*x^5*log(c^2*x^2 + 1)^2/(c^8*d^3*x^6 +
 3*c^6*d^3*x^4 + 3*c^4*d^3*x^2 + c^2*d^3), x) + 2*(2*c^2*x^2 + 1)*log(c...
```

## Giac [F]

$$\int \frac{x^2(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx=\int \frac{(b\arctan(cx)+a)^2x^2}{(i\,cdx+d)^3}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)^2*x^2/(I*c*d*x + d)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx = \int \frac{x^2\,(a+b\,\mathrm{atan}(c\,x))^2}{(d+c\,d\,x\,1\mathrm{i})^3}\,dx$$

input
```
int((x^2*(a + b*atan(c*x))^2)/(d + c*d*x*1i)^3,x)
```

output
```
int((x^2*(a + b*atan(c*x))^2)/(d + c*d*x*1i)^3, x)
```

## Reduce [F]

$$\int \frac{x^2(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx = \frac{-6\left(\int \frac{atan(cx)x^2}{c^3i\,x^3+3c^2x^2-3cix-1}dx\right)ab\,c^3-3\left(\int \frac{atan(cx)^2x^2}{c^3i\,x^3+3c^2x^2-3cix-1}dx\right)b^2c^3-6\left(\int \frac{x}{c^3i\,x^3+3c^2x^2-3cix-1}dx\right)a^2c^2i-3(}{3c^3d^3}$$

input
```
int(x^2*(a+b*atan(c*x))^2/(d+I*c*d*x)^3,x)
```

output
```
( - 6*int((atan(c*x)*x**2)/(c**3*i*x**3 + 3*c**2*x**2 - 3*c*i*x - 1),x)*a*
b*c**3 - 3*int((atan(c*x)**2*x**2)/(c**3*i*x**3 + 3*c**2*x**2 - 3*c*i*x -
1),x)*b**2*c**3 - 6*int(x/(c**3*i*x**3 + 3*c**2*x**2 - 3*c*i*x - 1),x)*a**
2*c**2*i - 3*int(1/(c**3*i*x**3 + 3*c**2*x**2 - 3*c*i*x - 1),x)*a**2*c + l
og(c**3*i*x**3 + 3*c**2*x**2 - 3*c*i*x - 1)*a**2*i)/(3*c**3*d**3)
```

# 3.114 $\int \frac{x(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 178

$$\int \frac{x(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx = -\frac{b^2}{16c^2d^3(i-cx)^2} - \frac{5ib^2}{16c^2d^3(i-cx)} + \frac{5ib^2\arctan(cx)}{16c^2d^3} - \frac{ib(a+b\arctan(cx))}{4c^2d^3(i-cx)^2} + \frac{3b(a+b\arctan(cx))}{4c^2d^3(i-cx)} + \frac{(a+b\arctan(cx))^2}{8c^2d^3} + \frac{x^2(a+b\arctan(cx))^2}{2d^3(1+icx)^2}$$

output
```
-1/16*b^2/c^2/d^3/(I-c*x)^2-5/16*I*b^2/c^2/d^3/(I-c*x)+5/16*I*b^2*arctan(c
*x)/c^2/d^3-1/4*I*b*(a+b*arctan(c*x))/c^2/d^3/(I-c*x)^2+3/4*b*(a+b*arctan(
c*x))/c^2/d^3/(I-c*x)+1/8*(a+b*arctan(c*x))^2/c^2/d^3+1/2*x^2*(a+b*arctan(
c*x))^2/d^3/(1+I*c*x)^2
```

## Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.66

$$\int \frac{x(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx = \frac{4ab(2i-3cx)+b^2(4+5icx)+a^2(-8-16icx)+b(i+cx)(a(4i-12cx)+b(3+5icx))\arctan(cx)-2b}{16c^2d^3(-i+cx)^2}$$

input
```
Integrate[(x*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x)^3,x]
```

output
```
(4*a*b*(2*I - 3*c*x) + b^2*(4 + (5*I)*c*x) + a^2*(-8 - (16*I)*c*x) + b*(I
+ c*x)*(a*(4*I - 12*c*x) + b*(3 + (5*I)*c*x))*ArcTan[c*x] - 2*b^2*(1 + (2*
I)*c*x + 3*c^2*x^2)*ArcTan[c*x]^2)/(16*c^2*d^3*(-I + c*x)^2)
```

## Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {5409, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx$$

$$\Big\downarrow \ 5409$$

$$\frac{x^2(a+b\arctan(cx))^2}{2d^3(1+icx)^2} - 2bc\int \left(-\frac{a+b\arctan(cx)}{8c^2d^3\,(c^2x^2+1)} - \frac{3(a+b\arctan(cx))}{8c^2d^3(i-cx)^2} + \frac{i(a+b\arctan(cx))}{4c^2d^3(i-cx)^3}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{x^2(a+b\arctan(cx))^2}{2d^3(1+icx)^2} - 2bc\left(-\frac{(a+b\arctan(cx))^2}{16bc^3d^3} - \frac{3(a+b\arctan(cx))}{8c^3d^3(-cx+i)} + \frac{i(a+b\arctan(cx))}{8c^3d^3(-cx+i)^2} - \frac{5ib\arctan(cx)}{32c^3d^3} + \frac{5ib}{32c^3d^3(-cx+i)} + \right.$$

input
```
Int[(x*(a + b*ArcTan[c*x])^2)/(d + I*c*d*x)^3,x]
```

output
```
(x^2*(a + b*ArcTan[c*x])^2)/(2*d^3*(1 + I*c*x)^2) - 2*b*c*(b/(32*c^3*d^3*(
I - c*x)^2) + (((5*I)/32)*b)/(c^3*d^3*(I - c*x)) - (((5*I)/32)*b*ArcTan[c*
x])/(c^3*d^3) + ((I/8)*(a + b*ArcTan[c*x]))/(c^3*d^3*(I - c*x)^2) - (3*(a
+ b*ArcTan[c*x]))/(8*c^3*d^3*(I - c*x)) - (a + b*ArcTan[c*x])^2/(16*b*c^3*
d^3))
```

**Defintions of rubi rules used**

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5409
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_.) + (e_
.)*(x_))^(q_), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x)^q, x]}, Sim
p[(a + b*ArcTan[c*x])^p   u, x] - Simp[b*c*p   Int[ExpandIntegrand[(a + b*A
rcTan[c*x])^(p - 1), u/(1 + c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e,
f, q}, x] && IGtQ[p, 1] && EqQ[c^2*d^2 + e^2, 0] && IntegersQ[m, q] && NeQ[
m, -1] && NeQ[q, -1] && ILtQ[m + q + 1, 0] && LtQ[m*q, 0]
```

## Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(156) = 312$.

Time = 1.02 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.77

| method | result |
|---|---|
| derivativedivides | $\frac{a^2\left(-\frac{i}{cx-i}+\frac{1}{2(cx-i)^2}\right)}{d^3}+\frac{b^2\left(-\frac{i\arctan(cx)^2}{cx-i}+\frac{\arctan(cx)^2}{2(cx-i)^2}-\frac{3i\arctan(cx)\ln(cx+i)}{8}-\frac{i\arctan(cx)}{4(cx-i)^2}+\frac{3i\arctan(cx)\ln(cx-i)}{8}-\frac{3\,\mathrm{ar}}{4(}\right.}{}$ |
| default | $\frac{a^2\left(-\frac{i}{cx-i}+\frac{1}{2(cx-i)^2}\right)}{d^3}+\frac{b^2\left(-\frac{i\arctan(cx)^2}{cx-i}+\frac{\arctan(cx)^2}{2(cx-i)^2}-\frac{3i\arctan(cx)\ln(cx+i)}{8}-\frac{i\arctan(cx)}{4(cx-i)^2}+\frac{3i\arctan(cx)\ln(cx-i)}{8}-\frac{3\,\mathrm{ar}}{4(}\right.}{}$ |
| parts | $\frac{a^2\left(\frac{i}{c^2(-cx+i)}+\frac{1}{2c^2(-cx+i)^2}\right)}{d^3}+\frac{b^2\left(-\frac{i\arctan(cx)^2}{cx-i}+\frac{\arctan(cx)^2}{2(cx-i)^2}-\frac{3i\arctan(cx)\ln(cx+i)}{8}-\frac{i\arctan(cx)}{4(cx-i)^2}+\frac{3i\arctan(cx)\ln(}{8}\right.}{}$ |
| orering | $-\frac{i(27c^7x^7+53ic^6x^6-18c^5x^5+77ic^4x^4-63c^3x^3+17ic^2x^2-18cx-7i)(a+b\arctan(cx))^2}{64c^4x^2(icdx+d)^3}-\frac{i(27c^7x^7+16ic^6x^6+72c^5}{}$ |
| risch | $\frac{(3b^2c^2x^2+2ib^2cx+b^2)\ln(icx+1)^2}{32c^2d^3(cx-i)^2}-\frac{(2i\ln(-icx+1)b^2cx+b^2\ln(-icx+1)+3\ln(-icx+1)b^2c^2x^2-6ib^2cx+16abcx-8}{16c^2d^3(cx-i)^2}$ |

input
```
int(x*(a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^2*(a^2/d^3*(-I/(c*x-I)+1/2/(c*x-I)^2)+b^2/d^3*(-I*arctan(c*x)^2/(c*x-I
)+1/2*arctan(c*x)^2/(c*x-I)^2-3/8*I*arctan(c*x)*ln(c*x+I)-1/4*I*arctan(c*x
)/(c*x-I)^2+3/8*I*arctan(c*x)*ln(c*x-I)-3/4*arctan(c*x)/(c*x-I)+5/16*I*arc
tan(c*x)+5/16*I/(c*x-I)-1/16/(c*x-I)^2-3/32*ln(c*x-I)^2+3/16*ln(c*x-I)*ln(
-1/2*I*(c*x+I))+3/16*(ln(c*x+I)-ln(-1/2*I*(c*x+I)))*ln(-1/2*I*(-c*x+I))-3/
32*ln(c*x+I)^2)-2*I*a*b/d^3*arctan(c*x)/(c*x-I)+a*b/d^3*arctan(c*x)/(c*x-I
)^2-3/4*a*b/d^3*arctan(c*x)-1/4*I*a*b/d^3/(c*x-I)^2-3/4*a*b/d^3/(c*x-I))
```

## Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.93

$$\int\frac{x(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx=$$

$$-\frac{2\,(16i\,a^2+12\,ab-5i\,b^2)cx-(3\,b^2c^2x^2+2i\,b^2cx+b^2)\log\left(-\frac{cx+i}{cx-i}\right)^2+16\,a^2-16i\,ab-8\,b^2-((-12i\,a}{32\,(c^4d^3x^2-2i\,c^3d^3x-c^2d^3)}$$

input
```
integrate(x*(a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x, algorithm="fricas")
```

output
```
-1/32*(2*(16*I*a^2 + 12*a*b - 5*I*b^2)*c*x - (3*b^2*c^2*x^2 + 2*I*b^2*c*x
+ b^2)*log(-(c*x + I)/(c*x - I))^2 + 16*a^2 - 16*I*a*b - 8*b^2 - ((-12*I*a
*b - 5*b^2)*c^2*x^2 + 2*(4*a*b - I*b^2)*c*x - 4*I*a*b - 3*b^2)*log(-(c*x +
 I)/(c*x - I)))/(c^4*d^3*x^2 - 2*I*c^3*d^3*x - c^2*d^3)
```

## Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 502 vs. $2(156) = 312$.

Time = 45.77 (sec) , antiderivative size = 502, normalized size of antiderivative = 2.82

$$\int \frac{x(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx = \frac{ib(12a-5ib)\log\left(-\frac{ib(12a-5ib)}{c}+x(12ab-5ib^2)\right)}{32c^2d^3} - \frac{ib(12a-5ib)\log\left(\frac{ib(12a-5ib)}{c}+x(12ab-5ib^2)\right)}{32c^2d^3} + \frac{(3b^2c^2x^2+2ib^2cx+b^2)\log(-icx+1)^2}{32c^4d^3x^2-64ic^3d^3x-32c^2d^3} + \frac{(3b^2c^2x^2+2ib^2cx+b^2)\log(icx+1)^2}{32c^4d^3x^2-64ic^3d^3x-32c^2d^3} + \frac{-8a^2+8iab+4b^2+x(-16ia^2c-12abc+5ib^2c)}{16c^4d^3x^2-32ic^3d^3x-16c^2d^3} + \frac{(16abcx-8iab-3b^2c^2x^2\log(icx+1)-2ib^2cx\log(icx+1)-6ib^2cx-b^2\log(icx+1)-4b^2)\log(-}{16c^4d^3x^2-32ic^3d^3x-16c^2d^3} + \frac{(-8abcx+4iab+3ib^2cx+2b^2)\log(icx+1)}{8c^4d^3x^2-16ic^3d^3x-8c^2d^3}$$

input
```
integrate(x*(a+b*atan(c*x))**2/(d+I*c*d*x)**3,x)
```

output
```
I*b*(12*a - 5*I*b)*log(-I*b*(12*a - 5*I*b)/c + x*(12*a*b - 5*I*b**2))/(32*
c**2*d**3) - I*b*(12*a - 5*I*b)*log(I*b*(12*a - 5*I*b)/c + x*(12*a*b - 5*I
*b**2))/(32*c**2*d**3) + (3*b**2*c**2*x**2 + 2*I*b**2*c*x + b**2)*log(-I*c
*x + 1)**2/(32*c**4*d**3*x**2 - 64*I*c**3*d**3*x - 32*c**2*d**3) + (3*b**2
*c**2*x**2 + 2*I*b**2*c*x + b**2)*log(I*c*x + 1)**2/(32*c**4*d**3*x**2 - 6
4*I*c**3*d**3*x - 32*c**2*d**3) + (-8*a**2 + 8*I*a*b + 4*b**2 + x*(-16*I*a
**2*c - 12*a*b*c + 5*I*b**2*c))/(16*c**4*d**3*x**2 - 32*I*c**3*d**3*x - 16
*c**2*d**3) + (16*a*b*c*x - 8*I*a*b - 3*b**2*c**2*x**2*log(I*c*x + 1) - 2*
I*b**2*c*x*log(I*c*x + 1) - 6*I*b**2*c*x - b**2*log(I*c*x + 1) - 4*b**2)*l
og(-I*c*x + 1)/(16*c**4*d**3*x**2 - 32*I*c**3*d**3*x - 16*c**2*d**3) + (-8
*a*b*c*x + 4*I*a*b + 3*I*b**2*c*x + 2*b**2)*log(I*c*x + 1)/(8*c**4*d**3*x*
*2 - 16*I*c**3*d**3*x - 8*c**2*d**3)
```

## Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.79

$$\int \frac{x(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx = -\frac{(16i\,a^2+12\,ab-5i\,b^2)cx+2\,(3\,b^2c^2x^2+2i\,b^2cx+b^2)\arctan{(cx)}^2+8\,a^2-8i\,ab-4\,b^2+((12\,ab-5\cdots}{16\,(c^4d^3x^2-2i\,c^3d^3x-c^2d^3)}$$

input
```
integrate(x*(a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x, algorithm="maxima")
```

output
```
-1/16*((16*I*a^2 + 12*a*b - 5*I*b^2)*c*x + 2*(3*b^2*c^2*x^2 + 2*I*b^2*c*x
+ b^2)*arctan(c*x)^2 + 8*a^2 - 8*I*a*b - 4*b^2 + ((12*a*b - 5*I*b^2)*c^2*x
^2 - 2*(-4*I*a*b - b^2)*c*x + 4*a*b - 3*I*b^2)*arctan(c*x))/(c^4*d^3*x^2 -
 2*I*c^3*d^3*x - c^2*d^3)
```

## Giac [F]

$$\int \frac{x(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx = \int \frac{(b\arctan(cx)+a)^2x}{(i\,cdx+d)^3}\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)^2*x/(I*c*d*x + d)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx = \int \frac{x\,(a+b\,\mathrm{atan}(c\,x))^2}{(d+c\,d\,x\,1\mathrm{i})^3}\,dx$$

input
```
int((x*(a + b*atan(c*x))^2)/(d + c*d*x*1i)^3,x)
```

output
```
int((x*(a + b*atan(c*x))^2)/(d + c*d*x*1i)^3, x)
```

## Reduce [F]

$$\int \frac{x(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx = \frac{atan(cx)^3\,b^2i+3atan(cx)^2\,abi+3atan(cx)\,a^2i+6\left(\int \frac{atan(cx)}{c^5x^5-3c^4i\,x^4-2c^3x^3-2c^2i\,x^2-3cx+i}dx\right)abc+3\left(\int \frac{}{c^5x^5-3c}\right.}{}$$

input
```
int(x*(a+b*atan(c*x))^2/(d+I*c*d*x)^3,x)
```

output
```
(atan(c*x)**3*b**2*i + 3*atan(c*x)**2*a*b*i + 3*atan(c*x)*a**2*i + 6*int(a
tan(c*x)/(c**5*x**5 - 3*c**4*i*x**4 - 2*c**3*x**3 - 2*c**2*i*x**2 - 3*c*x
+ i),x)*a*b*c + 3*int(atan(c*x)**2/(c**5*x**5 - 3*c**4*i*x**4 - 2*c**3*x**
3 - 2*c**2*i*x**2 - 3*c*x + i),x)*b**2*c - 9*int(x**2/(c**5*x**5 - 3*c**4*
i*x**4 - 2*c**3*x**3 - 2*c**2*i*x**2 - 3*c*x + i),x)*a**2*c**3 + 18*int((
- atan(c*x)*x**2)/(c**5*x**5 - 3*c**4*i*x**4 - 2*c**3*x**3 - 2*c**2*i*x**2
 - 3*c*x + i),x)*a*b*c**3 + 9*int(( - atan(c*x)**2*x**2)/(c**5*x**5 - 3*c*
*4*i*x**4 - 2*c**3*x**3 - 2*c**2*i*x**2 - 3*c*x + i),x)*b**2*c**3 + 24*int
((atan(c*x)*x)/(c**5*x**5 - 3*c**4*i*x**4 - 2*c**3*x**3 - 2*c**2*i*x**2 -
3*c*x + i),x)*a*b*c**2*i + 12*int((atan(c*x)**2*x)/(c**5*x**5 - 3*c**4*i*x
**4 - 2*c**3*x**3 - 2*c**2*i*x**2 - 3*c*x + i),x)*b**2*c**2*i + 12*int(x/(
c**5*x**5 - 3*c**4*i*x**4 - 2*c**3*x**3 - 2*c**2*i*x**2 - 3*c*x + i),x)*a*
*2*c**2*i + 3*int(1/(c**5*x**5 - 3*c**4*i*x**4 - 2*c**3*x**3 - 2*c**2*i*x*
*2 - 3*c*x + i),x)*a**2*c)/(3*c**2*d**3)
```

# 3.115 $\int \frac{(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 180

$$\begin{aligned}\int \frac{(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx &= \frac{ib^2}{16cd^3(i-cx)^2}+\frac{3b^2}{16cd^3(i-cx)}-\frac{3b^2\arctan(cx)}{16cd^3}\\ &\quad-\frac{b(a+b\arctan(cx))}{4cd^3(i-cx)^2}+\frac{ib(a+b\arctan(cx))}{4cd^3(i-cx)}\\ &\quad-\frac{i(a+b\arctan(cx))^2}{8cd^3}+\frac{i(a+b\arctan(cx))^2}{2cd^3(1+icx)^2}\end{aligned}$$

output
```
1/16*I*b^2/c/d^3/(I-c*x)^2+3/16*b^2/c/d^3/(I-c*x)-3/16*b^2*arctan(c*x)/c/d
^3-1/4*b*(a+b*arctan(c*x))/c/d^3/(I-c*x)^2+1/4*I*b*(a+b*arctan(c*x))/c/d^3
/(I-c*x)-1/8*I*(a+b*arctan(c*x))^2/c/d^3+1/2*I*(a+b*arctan(c*x))^2/c/d^3/(
1+I*c*x)^2
```

## Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.61

$$\int \frac{(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx = -\frac{i(8a^2+b^2(-4-3icx)+4ab(-2i+cx)+b(i+cx)(b(-5-3icx)+4a(-3i+cx))\arctan(cx)+2b^2(3}{16cd^3(-i+cx)^2}$$

input
```
Integrate[(a + b*ArcTan[c*x])^2/(d + I*c*d*x)^3,x]
```

output
```
((-1/16*I)*(8*a^2 + b^2*(-4 - (3*I)*c*x) + 4*a*b*(-2*I + c*x) + b*(I + c*x
)*(b*(-5 - (3*I)*c*x) + 4*a*(-3*I + c*x))*ArcTan[c*x] + 2*b^2*(3 - (2*I)*c
*x + c^2*x^2)*ArcTan[c*x]^2))/(c*d^3*(-I + c*x)^2)
```

## Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx$$

$$\Big\downarrow \text{5389}$$

$$\frac{i(a+b\arctan(cx))^2}{2cd^3(1+icx)^2}-\frac{ib\int \left(\frac{a+b\arctan(cx)}{4d^2(c^2x^2+1)}-\frac{a+b\arctan(cx)}{4d^2(i-cx)^2}-\frac{i(a+b\arctan(cx))}{2d^2(i-cx)^3}\right)dx}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{i(a+b\arctan(cx))^2}{2cd^3(1+icx)^2}-\frac{ib\left(\frac{(a+b\arctan(cx))^2}{8bcd^2}-\frac{a+b\arctan(cx)}{4cd^2(-cx+i)}-\frac{i(a+b\arctan(cx))}{4cd^2(-cx+i)^2}-\frac{3ib\arctan(cx)}{16cd^2}+\frac{3ib}{16cd^2(-cx+i)}-\frac{b}{16cd^2(-cx+i)^2}\right)}{d}$$

input
```
Int[(a + b*ArcTan[c*x])^2/(d + I*c*d*x)^3,x]
```

output
```
((I/2)*(a + b*ArcTan[c*x])^2)/(c*d^3*(1 + I*c*x)^2) - (I*b*(-1/16*b/(c*d^2
*(I - c*x)^2) + (((3*I)/16)*b)/(c*d^2*(I - c*x)) - (((3*I)/16)*b*ArcTan[c*
x])/(c*d^2) - ((I/4)*(a + b*ArcTan[c*x]))/(c*d^2*(I - c*x)^2) - (a + b*Arc
Tan[c*x])/(4*c*d^2*(I - c*x)) + (a + b*ArcTan[c*x])^2/(8*b*c*d^2)))/d
```

**Defintions of rubi rules used**

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5389
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - S
imp[b*c*(p/(e*(q + 1)))   Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1),
(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

## Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.54

| method | result |
|---|---|
| derivativedivides | $\frac{ia^2}{2d^3(icx+1)^2}+\frac{b^2\left(\frac{i\arctan(cx)^2}{2(icx+1)^2}-i\left(\frac{i\arctan(cx)\ln(cx+i)}{8}-\frac{i\arctan(cx)\ln(cx-i)}{8}-\frac{i\arctan(cx)}{4(cx-i)^2}+\frac{\arctan(cx)}{4cx-4i}-\frac{\left(\ln(cx+i)-\ln\left(-\frac{i}{\cdots}\right.\right.}{\cdots}\right.\right.}{\cdots}$ |
| default | $\frac{ia^2}{2d^3(icx+1)^2}+\frac{b^2\left(\frac{i\arctan(cx)^2}{2(icx+1)^2}-i\left(\frac{i\arctan(cx)\ln(cx+i)}{8}-\frac{i\arctan(cx)\ln(cx-i)}{8}-\frac{i\arctan(cx)}{4(cx-i)^2}+\frac{\arctan(cx)}{4cx-4i}-\frac{\left(\ln(cx+i)-\ln\left(-\frac{i}{\cdots}\right.\right.}{\cdots}\right.\right.}{\cdots}$ |
| parts | $\frac{ia^2}{2d^3(icx+1)^2c}+\frac{b^2\left(\frac{i\arctan(cx)^2}{2(icx+1)^2}-i\left(\frac{i\arctan(cx)\ln(cx+i)}{8}-\frac{i\arctan(cx)\ln(cx-i)}{8}-\frac{i\arctan(cx)}{4(cx-i)^2}+\frac{\arctan(cx)}{4cx-4i}-\frac{\left(\ln(cx+i)-\cdots\right.}{\cdots}\right.\right.}{\cdots}$ |
| orering | $\frac{(42c^5x^5+27ic^4x^4+106c^3x^3+108ic^2x^2+64cx+81i)(a+b\arctan(cx))^2}{64c(icdx+d)^3}-\frac{(-28c^6x^6+9ic^5x^5-93c^4x^4+18ic^3x^3-102c\cdots}{\cdots}$ |
| risch | $\frac{ib^2(c^2x^2-2icx+3)\ln(icx+1)^2}{32d^3(cx-i)^2c}-\frac{(3ib^2\ln(-icx+1)+i\ln(-icx+1)b^2c^2x^2+2\ln(-icx+1)b^2cx+2b^2cx-4ib^2+8ab)\ln(\cdots}{16d^3(cx-i)^2c}$ |

input
```
int((a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/c*(1/2*I*a^2/d^3/(1+I*c*x)^2+b^2/d^3*(1/2*I/(1+I*c*x)^2*arctan(c*x)^2-I*
(1/8*I*arctan(c*x)*ln(c*x+I)-1/8*I*arctan(c*x)*ln(c*x-I)-1/4*I*arctan(c*x)
/(c*x-I)^2+1/4*arctan(c*x)/(c*x-I)-1/16*(ln(c*x+I)-ln(-1/2*I*(c*x+I)))*ln(
-1/2*I*(-c*x+I))+1/32*ln(c*x+I)^2-3/16*I*arctan(c*x)-3/16*I/(c*x-I)-1/16/(
c*x-I)^2-1/16*ln(c*x-I)*ln(-1/2*I*(c*x+I))+1/32*ln(c*x-I)^2))+I*a*b/d^3/(1
+I*c*x)^2*arctan(c*x)-1/4*I*a*b/d^3*arctan(c*x)-1/4*a*b/d^3/(c*x-I)^2-1/4*
I*a*b/d^3/(c*x-I))
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.88

$$\int \frac{(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx = -\frac{2\,(4i\,ab+3\,b^2)cx-(i\,b^2c^2x^2+2\,b^2cx+3i\,b^2)\log\left(-\frac{cx+i}{cx-i}\right)^2+16i\,a^2+16\,ab-8i\,b^2-((4\,ab-3i\,b^2)c^2\cdots}{32\,(c^3d^3x^2-2i\,c^2d^3x-cd^3)}$$

input
```
integrate((a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x, algorithm="fricas")
```

output
```
-1/32*(2*(4*I*a*b + 3*b^2)*c*x - (I*b^2*c^2*x^2 + 2*b^2*c*x + 3*I*b^2)*log
(-(c*x + I)/(c*x - I))^2 + 16*I*a^2 + 16*a*b - 8*I*b^2 - ((4*a*b - 3*I*b^2
)*c^2*x^2 - 2*(4*I*a*b + b^2)*c*x + 12*a*b - 5*I*b^2)*log(-(c*x + I)/(c*x
- I)))/(c^3*d^3*x^2 - 2*I*c^2*d^3*x - c*d^3)
```

## Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 464 vs. $2(144) = 288$.

Time = 36.17 (sec) , antiderivative size = 464, normalized size of antiderivative = 2.58

$$\int \frac{(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx = -\frac{b(4a-3ib)\log\left(-\frac{ib(4a-3ib)}{c}+x(4ab-3ib^2)\right)}{32cd^3} + \frac{b(4a-3ib)\log\left(\frac{ib(4a-3ib)}{c}+x(4ab-3ib^2)\right)}{32cd^3} + \frac{(-4ab-b^2cx+2ib^2)\log(icx+1)}{8c^3d^3x^2-16ic^2d^3x-8cd^3} + \frac{(ib^2c^2x^2+2b^2cx+3ib^2)\log(-icx+1)^2}{32c^3d^3x^2-64ic^2d^3x-32cd^3} + \frac{(ib^2c^2x^2+2b^2cx+3ib^2)\log(icx+1)^2}{32c^3d^3x^2-64ic^2d^3x-32cd^3} + \frac{-8ia^2-8ab+4ib^2+x(-4iabc-3b^2c)}{16c^3d^3x^2-32ic^2d^3x-16cd^3} + \frac{(8ab-ib^2c^2x^2\log(icx+1)-2b^2cx\log(icx+1)+2b^2cx-3ib^2\log(icx+1)-4ib^2)\log(-icx+1)}{16c^3d^3x^2-32ic^2d^3x-16cd^3}$$

input
```
integrate((a+b*atan(c*x))**2/(d+I*c*d*x)**3,x)
```

output
```
-b*(4*a - 3*I*b)*log(-I*b*(4*a - 3*I*b)/c + x*(4*a*b - 3*I*b**2))/(32*c*d*
*3) + b*(4*a - 3*I*b)*log(I*b*(4*a - 3*I*b)/c + x*(4*a*b - 3*I*b**2))/(32*
c*d**3) + (-4*a*b - b**2*c*x + 2*I*b**2)*log(I*c*x + 1)/(8*c**3*d**3*x**2
- 16*I*c**2*d**3*x - 8*c*d**3) + (I*b**2*c**2*x**2 + 2*b**2*c*x + 3*I*b**2
)*log(-I*c*x + 1)**2/(32*c**3*d**3*x**2 - 64*I*c**2*d**3*x - 32*c*d**3) +
(I*b**2*c**2*x**2 + 2*b**2*c*x + 3*I*b**2)*log(I*c*x + 1)**2/(32*c**3*d**3
*x**2 - 64*I*c**2*d**3*x - 32*c*d**3) + (-8*I*a**2 - 8*a*b + 4*I*b**2 + x*
(-4*I*a*b*c - 3*b**2*c))/(16*c**3*d**3*x**2 - 32*I*c**2*d**3*x - 16*c*d**3
) + (8*a*b - I*b**2*c**2*x**2*log(I*c*x + 1) - 2*b**2*c*x*log(I*c*x + 1) +
 2*b**2*c*x - 3*I*b**2*log(I*c*x + 1) - 4*I*b**2)*log(-I*c*x + 1)/(16*c**3
*d**3*x**2 - 32*I*c**2*d**3*x - 16*c*d**3)
```

## Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.76

$$\int \frac{(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx = -\frac{(4i\,ab+3\,b^2)cx-2\left(-i\,b^2c^2x^2-2\,b^2cx-3i\,b^2\right)\arctan\left(cx\right)^2+8i\,a^2+8\,ab-4i\,b^2+\left((4i\,ab+3\,b^2)c^2x\right.}{16\left(c^3d^3x^2-2i\,c^2d^3x-cd^3\right)}$$

input
```
integrate((a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x, algorithm="maxima")
```

output
```
-1/16*((4*I*a*b + 3*b^2)*c*x - 2*(-I*b^2*c^2*x^2 - 2*b^2*c*x - 3*I*b^2)*ar
ctan(c*x)^2 + 8*I*a^2 + 8*a*b - 4*I*b^2 + ((4*I*a*b + 3*b^2)*c^2*x^2 + 2*(
4*a*b - I*b^2)*c*x + 12*I*a*b + 5*b^2)*arctan(c*x))/(c^3*d^3*x^2 - 2*I*c^2
*d^3*x - c*d^3)
```

## Giac [F]

$$\int \frac{(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx = \int \frac{\left(b\arctan\left(cx\right)+a\right)^2}{\left(i\,cdx+d\right)^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/(d+I*c*d*x)^3,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)^2/(I*c*d*x + d)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx = \int \frac{\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)^2}{\left(d+c\,d\,x\,1\mathrm{i}\right)^3}\,dx$$

input
```
int((a + b*atan(c*x))^2/(d + c*d*x*1i)^3,x)
```

output
```
int((a + b*atan(c*x))^2/(d + c*d*x*1i)^3, x)
```

## Reduce [F]

$$\int \frac{(a+b\arctan(cx))^2}{(d+icdx)^3}\,dx = \frac{-atan(cx)^3 b^2 - 3atan(cx)^2 ab + 24\left(\int \frac{atan(cx)}{c^5x^5-3c^4i\,x^4-2c^3x^3-2c^2i\,x^2-3cx+i}dx\right) abci + 12\left(\int \frac{atan(cx)^2}{c^5x^5-3c^4i\,x^4-2c^3x^3-2c^2}\right.}{}$$

input
```
int((a+b*atan(c*x))^2/(d+I*c*d*x)^3,x)
```

output
```
( - atan(c*x)**3*b**2 - 3*atan(c*x)**2*a*b + 24*int(atan(c*x)/(c**5*x**5 -
 3*c**4*i*x**4 - 2*c**3*x**3 - 2*c**2*i*x**2 - 3*c*x + i),x)*a*b*c*i + 12*
int(atan(c*x)**2/(c**5*x**5 - 3*c**4*i*x**4 - 2*c**3*x**3 - 2*c**2*i*x**2
- 3*c*x + i),x)*b**2*c*i + 9*int(x**2/(c**5*x**5 - 3*c**4*i*x**4 - 2*c**3*
x**3 - 2*c**2*i*x**2 - 3*c*x + i),x)*a**2*c**3*i - 6*int(( - atan(c*x)*x**
3)/(c**5*x**5 - 3*c**4*i*x**4 - 2*c**3*x**3 - 2*c**2*i*x**2 - 3*c*x + i),x
)*a*b*c**4 - 3*int(( - atan(c*x)**2*x**3)/(c**5*x**5 - 3*c**4*i*x**4 - 2*c
**3*x**3 - 2*c**2*i*x**2 - 3*c*x + i),x)*b**2*c**4 - 18*int((atan(c*x)*x)/
(c**5*x**5 - 3*c**4*i*x**4 - 2*c**3*x**3 - 2*c**2*i*x**2 - 3*c*x + i),x)*a
*b*c**2 - 9*int((atan(c*x)**2*x)/(c**5*x**5 - 3*c**4*i*x**4 - 2*c**3*x**3
- 2*c**2*i*x**2 - 3*c*x + i),x)*b**2*c**2 + 9*int(1/(c**5*x**5 - 3*c**4*i*
x**4 - 2*c**3*x**3 - 2*c**2*i*x**2 - 3*c*x + i),x)*a**2*c*i)/(9*c*d**3)
```

# 3.116 $\int \frac{(a+b\arctan(cx))^2}{x(d+icdx)^3}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 299

$$\begin{aligned}\int \frac{(a+b\arctan(cx))^2}{x(d+icdx)^3}\,dx &= \frac{b^2}{16d^3(i-cx)^2}-\frac{11ib^2}{16d^3(i-cx)} \\ &+\frac{11ib^2\arctan(cx)}{16d^3}+\frac{ib(a+b\arctan(cx))}{4d^3(i-cx)^2} \\ &+\frac{5b(a+b\arctan(cx))}{4d^3(i-cx)}-\frac{5(a+b\arctan(cx))^2}{8d^3} \\ &-\frac{(a+b\arctan(cx))^2}{2d^3(i-cx)^2}+\frac{i(a+b\arctan(cx))^2}{d^3(i-cx)} \\ &+\frac{2(a+b\arctan(cx))^2\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right)}{d^3} \\ &+\frac{(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{d^3} \\ &+\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)}{d^3} \\ &+\frac{b^2\operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)}{2d^3}\end{aligned}$$

output
```
1/16*b^2/d^3/(I-c*x)^2-11/16*I*b^2/d^3/(I-c*x)+11/16*I*b^2*arctan(c*x)/d^3
+1/4*I*b*(a+b*arctan(c*x))/d^3/(I-c*x)^2+5/4*b*(a+b*arctan(c*x))/d^3/(I-c*
x)-5/8*(a+b*arctan(c*x))^2/d^3-1/2*(a+b*arctan(c*x))^2/d^3/(I-c*x)^2+I*(a+
b*arctan(c*x))^2/d^3/(I-c*x)-2*(a+b*arctan(c*x))^2*arctanh(-1+2/(1+I*c*x))
/d^3+(a+b*arctan(c*x))^2*ln(2/(1+I*c*x))/d^3+I*b*(a+b*arctan(c*x))*polylog
(2,-1+2/(1+I*c*x))/d^3+1/2*b^2*polylog(3,-1+2/(1+I*c*x))/d^3
```

## Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.45

$$\int \frac{(a+b\arctan(cx))^2}{x(d+icdx)^3}\,dx$$
$$= \frac{-\frac{96a^2}{(-i+cx)^2}-\frac{192ia^2}{-i+cx}-192ia^2\arctan(cx)+192a^2\log(cx)-96a^2\log\left(1+c^2x^2\right)+12iab\big(-32\arctan(cx)^2 -}{}$$

input
```
Integrate[(a + b*ArcTan[c*x])^2/(x*(d + I*c*d*x)^3),x]
```

output
```
((-96*a^2)/(-I + c*x)^2 - ((192*I)*a^2)/(-I + c*x) - (192*I)*a^2*ArcTan[c*
x] + 192*a^2*Log[c*x] - 96*a^2*Log[1 + c^2*x^2] + (12*I)*a*b*(-32*ArcTan[c
*x]^2 - 12*Cos[2*ArcTan[c*x]] - Cos[4*ArcTan[c*x]] - 16*PolyLog[2, E^((2*I
)*ArcTan[c*x])] + (12*I)*Sin[2*ArcTan[c*x]] - (4*I)*ArcTan[c*x]*(6*Cos[2*A
rcTan[c*x]] + Cos[4*ArcTan[c*x]] + 8*Log[1 - E^((2*I)*ArcTan[c*x])] - (6*I
)*Sin[2*ArcTan[c*x]] - I*Sin[4*ArcTan[c*x]]) + I*Sin[4*ArcTan[c*x]]) + b^2
*((-8*I)*Pi^3 - 72*Cos[2*ArcTan[c*x]] - (144*I)*ArcTan[c*x]*Cos[2*ArcTan[c
*x]] + 144*ArcTan[c*x]^2*Cos[2*ArcTan[c*x]] - 3*Cos[4*ArcTan[c*x]] - (12*I
)*ArcTan[c*x]*Cos[4*ArcTan[c*x]] + 24*ArcTan[c*x]^2*Cos[4*ArcTan[c*x]] + 1
92*ArcTan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] + (192*I)*ArcTan[c*x]*Pol
yLog[2, E^((-2*I)*ArcTan[c*x])] + 96*PolyLog[3, E^((-2*I)*ArcTan[c*x])] +
(72*I)*Sin[2*ArcTan[c*x]] - 144*ArcTan[c*x]*Sin[2*ArcTan[c*x]] - (144*I)*A
rcTan[c*x]^2*Sin[2*ArcTan[c*x]] + (3*I)*Sin[4*ArcTan[c*x]] - 12*ArcTan[c*x
]*Sin[4*ArcTan[c*x]] - (24*I)*ArcTan[c*x]^2*Sin[4*ArcTan[c*x]]))/(192*d^3)
```

## Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^2}{x(d+icdx)^3}\,dx$$

$$\Big\downarrow \text{5411}$$

$$\int \left(\frac{(a+b\arctan(cx))^2}{d^3x}-\frac{c(a+b\arctan(cx))^2}{d^3(cx-i)}+\frac{ic(a+b\arctan(cx))^2}{d^3(cx-i)^2}+\frac{c(a+b\arctan(cx))^2}{d^3(cx-i)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\text{arctanh}\left(1-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{d^3}+\frac{ib\,\text{PolyLog}\left(2,\frac{2}{icx+1}-1\right)(a+b\arctan(cx))}{d^3}+\frac{5b(a+b\arctan(cx))}{4d^3(-cx+i)}+\frac{ib(a+b\arctan(cx))}{4d^3(-cx+i)^2}+\frac{i(a+b\arctan(cx))^2}{d^3(-cx+i)}-\frac{(a+b\arctan(cx))^2}{2d^3(-cx+i)^2}-\frac{5(a+b\arctan(cx))^2}{8d^3}+\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{d^3}+\frac{11ib^2\arctan(cx)}{16d^3}+\frac{b^2\,\text{PolyLog}\left(3,\frac{2}{icx+1}-1\right)}{2d^3}-\frac{11ib^2}{16d^3(-cx+i)}+\frac{b^2}{16d^3(-cx+i)^2}$$

input
```
Int[(a + b*ArcTan[c*x])^2/(x*(d + I*c*d*x)^3),x]
```

output
```
b^2/(16*d^3*(I - c*x)^2) - (((11*I)/16)*b^2)/(d^3*(I - c*x)) + (((11*I)/16
)*b^2*ArcTan[c*x])/d^3 + ((I/4)*b*(a + b*ArcTan[c*x]))/(d^3*(I - c*x)^2) +
 (5*b*(a + b*ArcTan[c*x]))/(4*d^3*(I - c*x)) - (5*(a + b*ArcTan[c*x])^2)/(
8*d^3) - (a + b*ArcTan[c*x])^2/(2*d^3*(I - c*x)^2) + (I*(a + b*ArcTan[c*x]
)^2)/(d^3*(I - c*x)) + (2*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)]
)/d^3 + ((a + b*ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/d^3 + (I*b*(a + b*ArcTa
n[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)])/d^3 + (b^2*PolyLog[3, -1 + 2/(1 +
I*c*x)])/(2*d^3)
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 12.70 (sec) , antiderivative size = 1783, normalized size of antiderivative = 5.96

| method | result | size |
|---|---|---|
| derivativedivides | Expression too large to display | 1783 |
| default | Expression too large to display | 1783 |
| parts | Expression too large to display | 1783 |

input
```
int((a+b*arctan(c*x))^2/x/(d+I*c*d*x)^3,x,method=_RETURNVERBOSE)
```

output
```
a^2/d^3*ln(c*x)-1/2*a^2/d^3/(c*x-I)^2-I*a^2/d^3/(c*x-I)-1/2*a^2/d^3*ln(c^2
*x^2+1)-I*a^2/d^3*arctan(c*x)+b^2/d^3*(1/2*I*Pi*csgn(I*((1+I*c*x)^2/(c^2*x
^2+1)-1))*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I*((1+I*c*x)^2/(c^2*x^2
+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*arctan(c*x)^2-5/8*arctan(c*x)^2+ln(c*x
)*arctan(c*x)^2-arctan(c*x)^2*ln((1+I*c*x)^2/(c^2*x^2+1)-1)+arctan(c*x)^2*
ln(1-(1+I*c*x)/(c^2*x^2+1)^(1/2))+arctan(c*x)^2*ln(1+(1+I*c*x)/(c^2*x^2+1)
^(1/2))-2*I*arctan(c*x)*polylog(2,(1+I*c*x)/(c^2*x^2+1)^(1/2))-2*I*arctan(
c*x)*polylog(2,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+2*polylog(3,(1+I*c*x)/(c^2*x^
2+1)^(1/2))+2*polylog(3,-(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/2*I*Pi*csgn((1+I*c
*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I/(1+(1+I*c*x)^2/(c^2*
x^2+1)))*csgn((1+I*c*x)^2/(c^2*x^2+1))*arctan(c*x)^2+3*I*arctan(c*x)*(c*x+
I)/(4*c*x-4*I)-I*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1
)))^2*arctan(c*x)^2-1/2*I*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(
c^2*x^2+1)))^3*arctan(c*x)^2+1/2*I*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(
1+(1+I*c*x)^2/(c^2*x^2+1)))^3*arctan(c*x)^2+1/2*I*Pi*csgn(((1+I*c*x)^2/(c^
2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^3*arctan(c*x)^2-1/2*I*Pi*csgn(((1
+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*arctan(c*x)^2-ln(c
*x-I)*arctan(c*x)^2-1/64*(c*x+I)^2/(c*x-I)^2-I*arctan(c*x)^2/(c*x-I)-1/2*a
rctan(c*x)^2/(c*x-I)^2+3/8*(c*x+I)/(c*x-I)+arctan(c*x)^2*ln(2*I*(1+I*c*x)^
2/(c^2*x^2+1))-1/2*I*Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I*((1+...
```

## Fricas [F]

$$\int \frac{(a+b\arctan(cx))^2}{x(d+icdx)^3}\,dx=\int \frac{(b\arctan(cx)+a)^2}{(i\,cdx+d)^3x}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x/(d+I*c*d*x)^3,x, algorithm="fricas")
```

output
```
-1/8*(2*(b^2*c^2*x^2 - 2*I*b^2*c*x - b^2)*log(2*c*x/(c*x - I))*log(-(c*x +
 I)/(c*x - I))^2 + 4*(b^2*c^2*x^2 - 2*I*b^2*c*x - b^2)*dilog(-2*c*x/(c*x -
 I) + 1)*log(-(c*x + I)/(c*x - I)) - (2*I*b^2*c*x + 3*b^2)*log(-(c*x + I)/
(c*x - I))^2 - 8*(c^2*d^3*x^2 - 2*I*c*d^3*x - d^3)*integral(1/2*(2*I*a^2*c
*x - 2*a^2 - (2*b^2*c^2*x^2 + (2*a*b - 3*I*b^2)*c*x + 2*I*a*b)*log(-(c*x +
 I)/(c*x - I)))/(c^4*d^3*x^5 - 2*I*c^3*d^3*x^4 - 2*I*c*d^3*x^2 - d^3*x), x
) - 4*(b^2*c^2*x^2 - 2*I*b^2*c*x - b^2)*polylog(3, -(c*x + I)/(c*x - I)))/
(c^2*d^3*x^2 - 2*I*c*d^3*x - d^3)
```

## Sympy [F(-2)]

Exception generated.

$$\int \frac{(a+b\arctan(cx))^2}{x(d+icdx)^3}\,dx = \text{Exception raised: RecursionError}$$

input
```
integrate((a+b*atan(c*x))**2/x/(d+I*c*d*x)**3,x)
```

output
```
Exception raised: RecursionError >> maximum recursion depth exceeded in co
mparison
```

## Maxima [F]

$$\int \frac{(a+b\arctan(cx))^2}{x(d+icdx)^3}\,dx = \int \frac{(b\arctan(cx)+a)^2}{(i\,cdx+d)^3x}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x/(d+I*c*d*x)^3,x, algorithm="maxima")
```

output
```
-1/128*(-16*I*a^2*c^2*x^2*arctan2(1, c*x) - 32*a^2*c*x*(arctan2(1, c*x) -
4*I) + 32*(I*b^2*c^2*x^2 + 2*b^2*c*x - I*b^2)*arctan(c*x)^3 - 4*(b^2*c^2*x
^2 - 2*I*b^2*c*x - b^2)*log(c^2*x^2 + 1)^3 + 16*a^2*(I*arctan2(1, c*x) + 1
2) + 16*(2*I*b^2*c*x + 3*b^2)*arctan(c*x)^2 - 4*(2*I*b^2*c*x + 3*b^2 - 2*(
I*b^2*c^2*x^2 + 2*b^2*c*x - I*b^2)*arctan(c*x))*log(c^2*x^2 + 1)^2 + 6*(b^
2*c^4*d^3*x^2 - 2*I*b^2*c^3*d^3*x - b^2*c^2*d^3)*(((8*c^2*x^2 + 7)*c^2/(c^
12*d^3*x^4 + 2*c^10*d^3*x^2 + c^8*d^3) + 2*(4*c^2*x^2 + 3)*log(c^2*x^2 + 1
)/(c^10*d^3*x^4 + 2*c^8*d^3*x^2 + c^6*d^3))*c^4 + 2*(2*c^2*x^2 + 1)*c^2*lo
g(c^2*x^2 + 1)^2/(c^8*d^3*x^4 + 2*c^6*d^3*x^2 + c^4*d^3) - c^2*(c^2/(c^10*
d^3*x^4 + 2*c^8*d^3*x^2 + c^6*d^3) + 2*log(c^2*x^2 + 1)/(c^8*d^3*x^4 + 2*c
^6*d^3*x^2 + c^4*d^3)) - 512*c^2*integrate(1/16*x^3*arctan(c*x)^2/(c^6*d^3
*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x) - 2*log(c^2*x^2 + 1)^2/(c^
6*d^3*x^4 + 2*c^4*d^3*x^2 + c^2*d^3) + 512*integrate(1/16*x*arctan(c*x)^2/
(c^6*d^3*x^6 + 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 + d^3), x)) - 2*(b^2*c^2*d^3*
x^2 - 2*I*b^2*c*d^3*x - b^2*d^3)*(c^4*(c^2/(c^10*d^3*x^4 + 2*c^8*d^3*x^2 +
 c^6*d^3) + 2*log(c^2*x^2 + 1)/(c^8*d^3*x^4 + 2*c^6*d^3*x^2 + c^4*d^3)) -
512*c^2*integrate(1/16*x^2*arctan(c*x)^2/(c^6*d^3*x^7 + 3*c^4*d^3*x^5 + 3*
c^2*d^3*x^3 + d^3*x), x) + 2*c^2*log(c^2*x^2 + 1)^2/(c^6*d^3*x^4 + 2*c^4*d
^3*x^2 + c^2*d^3) + 512*integrate(1/16*arctan(c*x)^2/(c^6*d^3*x^7 + 3*c^4*
d^3*x^5 + 3*c^2*d^3*x^3 + d^3*x), x) + 128*integrate(1/16*log(c^2*x^2 +...
```

## Giac [F]

$$\int \frac{(a+b\arctan(cx))^2}{x(d+icdx)^3}\,dx=\int \frac{(b\arctan(cx)+a)^2}{(i\,cdx+d)^3x}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x/(d+I*c*d*x)^3,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)^2/((I*c*d*x + d)^3*x), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{x(d+icdx)^3}\,dx=\int \frac{{\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}^2}{x\,{\left(d+c\,d\,x\,1{}\mathrm{i}\right)}^3}\,dx$$

input
```
int((a + b*atan(c*x))^2/(x*(d + c*d*x*1i)^3),x)
```

output
```
int((a + b*atan(c*x))^2/(x*(d + c*d*x*1i)^3), x)
```

## Reduce [F]

$$\int \frac{(a+b\arctan(cx))^2}{x(d+icdx)^3}\,dx = \frac{-2\left(\int \frac{atan(cx)}{c^3i\,x^4+3c^2x^3-3ci\,x^2-x}dx\right)ab-\left(\int \frac{atan(cx)^2}{c^3i\,x^4+3c^2x^3-3ci\,x^2-x}dx\right)b^2-\left(\int \frac{1}{c^3i\,x^4+3c^2x^3-3ci\,x^2-x}dx\right)a^2}{d^3}$$

input
```
int((a+b*atan(c*x))^2/x/(d+I*c*d*x)^3,x)
```

output
```
( - 2*int(atan(c*x)/(c**3*i*x**4 + 3*c**2*x**3 - 3*c*i*x**2 - x),x)*a*b -
int(atan(c*x)**2/(c**3*i*x**4 + 3*c**2*x**3 - 3*c*i*x**2 - x),x)*b**2 - in
t(1/(c**3*i*x**4 + 3*c**2*x**3 - 3*c*i*x**2 - x),x)*a**2)/d**3
```

# 3.117 $\int \frac{(a+b\arctan(cx))^2}{x^2(d+icdx)^3}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 391

$$\begin{aligned}\int \frac{(a+b\arctan(cx))^2}{x^2(d+icdx)^3}\,dx = &-\frac{ib^2c}{16d^3(i-cx)^2}-\frac{19b^2c}{16d^3(i-cx)}+\frac{19b^2c\arctan(cx)}{16d^3}\\
&+\frac{bc(a+b\arctan(cx))}{4d^3(i-cx)^2}-\frac{9ibc(a+b\arctan(cx))}{4d^3(i-cx)}\\
&+\frac{ic(a+b\arctan(cx))^2}{8d^3}-\frac{(a+b\arctan(cx))^2}{d^3x}\\
&+\frac{ic(a+b\arctan(cx))^2}{2d^3(i-cx)^2}+\frac{2c(a+b\arctan(cx))^2}{d^3(i-cx)}\\
&-\frac{6ic(a+b\arctan(cx))^2\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right)}{d^3}\\
&-\frac{3ic(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{d^3}\\
&+\frac{2bc(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right)}{d^3}\\
&-\frac{ib^2c\operatorname{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)}{d^3}\\
&+\frac{3bc(a+b\arctan(cx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)}{d^3}\\
&-\frac{3ib^2c\operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)}{2d^3}\end{aligned}$$

output
```
-1/16*I*b^2*c/d^3/(I-c*x)^2-19/16*b^2*c/d^3/(I-c*x)+19/16*b^2*c*arctan(c*x
)/d^3+1/4*b*c*(a+b*arctan(c*x))/d^3/(I-c*x)^2-9/4*I*b*c*(a+b*arctan(c*x))/
d^3/(I-c*x)+1/8*I*c*(a+b*arctan(c*x))^2/d^3-(a+b*arctan(c*x))^2/d^3/x+1/2*
I*c*(a+b*arctan(c*x))^2/d^3/(I-c*x)^2+2*c*(a+b*arctan(c*x))^2/d^3/(I-c*x)+
6*I*c*(a+b*arctan(c*x))^2*arctanh(-1+2/(1+I*c*x))/d^3-3*I*c*(a+b*arctan(c*
x))^2*ln(2/(1+I*c*x))/d^3+2*b*c*(a+b*arctan(c*x))*ln(2-2/(1-I*c*x))/d^3-I*
b^2*c*polylog(2,-1+2/(1-I*c*x))/d^3+3*b*c*(a+b*arctan(c*x))*polylog(2,-1+2
/(1+I*c*x))/d^3-3/2*I*b^2*c*polylog(3,-1+2/(1+I*c*x))/d^3
```

## Mathematica [A] (verified)

Time = 2.32 (sec) , antiderivative size = 549, normalized size of antiderivative = 1.40

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+icdx)^3}\,dx =$$

$$-\frac{\frac{64a^2}{x}-\frac{32ia^2c}{(-i+cx)^2}+\frac{128a^2c}{-i+cx}+192a^2c\arctan(cx)+192ia^2c\log(x)-96ia^2c\log\left(1+c^2x^2\right)-ib^2c\Big(8i\pi^3-6\cdots}{\cdots}$$

input
```
Integrate[(a + b*ArcTan[c*x])^2/(x^2*(d + I*c*d*x)^3),x]
```

output
```
-1/64*((64*a^2)/x - ((32*I)*a^2*c)/(-I + c*x)^2 + (128*a^2*c)/(-I + c*x) +
 192*a^2*c*ArcTan[c*x] + (192*I)*a^2*c*Log[x] - (96*I)*a^2*c*Log[1 + c^2*x
^2] - I*b^2*c*((8*I)*Pi^3 - 64*ArcTan[c*x]^2 + ((64*I)*ArcTan[c*x]^2)/(c*x
) + 40*Cos[2*ArcTan[c*x]] + (80*I)*ArcTan[c*x]*Cos[2*ArcTan[c*x]] - 80*Arc
Tan[c*x]^2*Cos[2*ArcTan[c*x]] + Cos[4*ArcTan[c*x]] + (4*I)*ArcTan[c*x]*Cos
[4*ArcTan[c*x]] - 8*ArcTan[c*x]^2*Cos[4*ArcTan[c*x]] - 192*ArcTan[c*x]^2*L
og[1 - E^((-2*I)*ArcTan[c*x])] - (128*I)*ArcTan[c*x]*Log[1 - E^((2*I)*ArcT
an[c*x])] - (192*I)*ArcTan[c*x]*PolyLog[2, E^((-2*I)*ArcTan[c*x])] - 64*Po
lyLog[2, E^((2*I)*ArcTan[c*x])] - 96*PolyLog[3, E^((-2*I)*ArcTan[c*x])] -
(40*I)*Sin[2*ArcTan[c*x]] + 80*ArcTan[c*x]*Sin[2*ArcTan[c*x]] + (80*I)*Arc
Tan[c*x]^2*Sin[2*ArcTan[c*x]] - I*Sin[4*ArcTan[c*x]] + 4*ArcTan[c*x]*Sin[4
*ArcTan[c*x]] + (8*I)*ArcTan[c*x]^2*Sin[4*ArcTan[c*x]]) + (4*a*b*(96*c*x*A
rcTan[c*x]^2 + 48*c*x*PolyLog[2, E^((2*I)*ArcTan[c*x])] + c*x*(20*Cos[2*Ar
cTan[c*x]] + Cos[4*ArcTan[c*x]] - 32*Log[(c*x)/Sqrt[1 + c^2*x^2]] - (20*I)
*Sin[2*ArcTan[c*x]] - I*Sin[4*ArcTan[c*x]]) + 4*ArcTan[c*x]*(8 + (10*I)*c*
x*Cos[2*ArcTan[c*x]] + I*c*x*Cos[4*ArcTan[c*x]] + (24*I)*c*x*Log[1 - E^((2
*I)*ArcTan[c*x])] + 10*c*x*Sin[2*ArcTan[c*x]] + c*x*Sin[4*ArcTan[c*x]])))/
x)/d^3
```

## Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+icdx)^3}\,dx$$

$$\Big\downarrow \text{5411}$$

$$\int \left(\frac{3ic^2(a+b\arctan(cx))^2}{d^3(cx-i)}+\frac{2c^2(a+b\arctan(cx))^2}{d^3(cx-i)^2}-\frac{ic^2(a+b\arctan(cx))^2}{d^3(cx-i)^3}+\frac{(a+b\arctan(cx))^2}{d^3x^2}-\frac{3ic(a+\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{6ic\,\text{arctanh}\left(1-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{d^3}+\frac{3bc\,\text{PolyLog}\left(2,\frac{2}{icx+1}-1\right)(a+b\arctan(cx))}{d^3}-\frac{9ibc(a+b\arctan(cx))}{4d^3(-cx+i)}+\frac{bc(a+b\arctan(cx))}{4d^3(-cx+i)^2}-\frac{(a+b\arctan(cx))^2}{d^3x}+\frac{2c(a+b\arctan(cx))^2}{d^3(-cx+i)}+\frac{ic(a+b\arctan(cx))^2}{2d^3(-cx+i)^2}+\frac{ic(a+b\arctan(cx))^2}{8d^3}+\frac{2bc\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))}{d^3}-\frac{3ic\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{d^3}+\frac{19b^2c\arctan(cx)}{16d^3}-\frac{ib^2c\,\text{PolyLog}\left(2,\frac{2}{1-icx}-1\right)}{d^3}-\frac{3ib^2c\,\text{PolyLog}\left(3,\frac{2}{icx+1}-1\right)}{2d^3}-\frac{19b^2c}{16d^3(-cx+i)}-\frac{ib^2c}{16d^3(-cx+i)^2}$$

input
```
Int[(a + b*ArcTan[c*x])^2/(x^2*(d + I*c*d*x)^3),x]
```

output
```
((-1/16*I)*b^2*c)/(d^3*(I - c*x)^2) - (19*b^2*c)/(16*d^3*(I - c*x)) + (19*
b^2*c*ArcTan[c*x])/(16*d^3) + (b*c*(a + b*ArcTan[c*x]))/(4*d^3*(I - c*x)^2
) - (((9*I)/4)*b*c*(a + b*ArcTan[c*x]))/(d^3*(I - c*x)) + ((I/8)*c*(a + b*
ArcTan[c*x])^2)/d^3 - (a + b*ArcTan[c*x])^2/(d^3*x) + ((I/2)*c*(a + b*ArcT
an[c*x])^2)/(d^3*(I - c*x)^2) + (2*c*(a + b*ArcTan[c*x])^2)/(d^3*(I - c*x)
) - ((6*I)*c*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)])/d^3 - ((3*I
)*c*(a + b*ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/d^3 + (2*b*c*(a + b*ArcTan[c
*x])*Log[2 - 2/(1 - I*c*x)])/d^3 - (I*b^2*c*PolyLog[2, -1 + 2/(1 - I*c*x)]
)/d^3 + (3*b*c*(a + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)])/d^3 - (
((3*I)/2)*b^2*c*PolyLog[3, -1 + 2/(1 + I*c*x)])/d^3
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 17.96 (sec) , antiderivative size = 8688, normalized size of antiderivative = 22.22

| method | result | size |
|---|---|---|
| derivativedivides | Expression too large to display | 8688 |
| default | Expression too large to display | 8688 |
| parts | Expression too large to display | 8690 |

input
```
int((a+b*arctan(c*x))^2/x^2/(d+I*c*d*x)^3,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

## Fricas [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+icdx)^3}\,dx = \int \frac{(b\arctan(cx)+a)^2}{(i\,cdx+d)^3x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^2/(d+I*c*d*x)^3,x, algorithm="fricas")
```

output
```
-1/8*(6*(-I*b^2*c^3*x^3 - 2*b^2*c^2*x^2 + I*b^2*c*x)*log(2*c*x/(c*x - I))*
log(-(c*x + I)/(c*x - I))^2 + 12*(-I*b^2*c^3*x^3 - 2*b^2*c^2*x^2 + I*b^2*c
*x)*dilog(-2*c*x/(c*x - I) + 1)*log(-(c*x + I)/(c*x - I)) - (6*b^2*c^2*x^2
 - 9*I*b^2*c*x - 2*b^2)*log(-(c*x + I)/(c*x - I))^2 - 8*(c^2*d^3*x^3 - 2*I
*c*d^3*x^2 - d^3*x)*integral(1/2*(2*I*a^2*c*x - 2*a^2 + (6*I*b^2*c^3*x^3 +
 9*b^2*c^2*x^2 - 2*(a*b + I*b^2)*c*x - 2*I*a*b)*log(-(c*x + I)/(c*x - I)))
/(c^4*d^3*x^6 - 2*I*c^3*d^3*x^5 - 2*I*c*d^3*x^3 - d^3*x^2), x) + 12*(I*b^2
*c^3*x^3 + 2*b^2*c^2*x^2 - I*b^2*c*x)*polylog(3, -(c*x + I)/(c*x - I)))/(c
^2*d^3*x^3 - 2*I*c*d^3*x^2 - d^3*x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+icdx)^3}\,dx = \text{Timed out}$$

input
```
integrate((a+b*atan(c*x))**2/x**2/(d+I*c*d*x)**3,x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+icdx)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arctan(c*x))^2/x^2/(d+I*c*d*x)^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

## Giac [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+icdx)^3}\,dx = \int \frac{(b\arctan(cx)+a)^2}{(i\,cdx+d)^3x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^2/(d+I*c*d*x)^3,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)^2/((I*c*d*x + d)^3*x^2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+icdx)^3}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^2}{x^2\,(d+c\,d\,x\,1\mathrm{i})^3}\,dx$$

input
```
int((a + b*atan(c*x))^2/(x^2*(d + c*d*x*1i)^3),x)
```

output
```
int((a + b*atan(c*x))^2/(x^2*(d + c*d*x*1i)^3), x)
```

## Reduce [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+icdx)^3}\,dx = \frac{-2\left(\int \frac{atan(cx)}{c^3i\,x^5+3c^2x^4-3ci\,x^3-x^2}dx\right)ab+\left(\int -\frac{atan(cx)^2}{c^3i\,x^5+3c^2x^4-3ci\,x^3-x^2}dx\right)b^2-\left(\int \frac{1}{c^3i\,x^5+3c^2x^4-3ci\,x^3-x^2}dx\right)a^2}{d^3}$$

input
```
int((a+b*atan(c*x))^2/x^2/(d+I*c*d*x)^3,x)
```

output
```
( - 2*int(atan(c*x)/(c**3*i*x**5 + 3*c**2*x**4 - 3*c*i*x**3 - x**2),x)*a*b
 + int(( - atan(c*x)**2)/(c**3*i*x**5 + 3*c**2*x**4 - 3*c*i*x**3 - x**2),x
)*b**2 - int(1/(c**3*i*x**5 + 3*c**2*x**4 - 3*c*i*x**3 - x**2),x)*a**2)/d*
*3
```

# 3.118 $\int \frac{(a+b\arctan(cx))^2}{(1+icx)^4} dx$



## Optimal result

Integrand size = 21, antiderivative size = 207

$$\int \frac{(a+b\arctan(cx))^2}{(1+icx)^4} dx = -\frac{b^2}{54c(i-cx)^3} + \frac{5ib^2}{144c(i-cx)^2} + \frac{11b^2}{144c(i-cx)} - \frac{11b^2\arctan(cx)}{144c} - \frac{ib(a+b\arctan(cx))}{9c(i-cx)^3} - \frac{b(a+b\arctan(cx))}{12c(i-cx)^2} + \frac{ib(a+b\arctan(cx))}{12c(i-cx)} - \frac{i(a+b\arctan(cx))^2}{24c} + \frac{i(a+b\arctan(cx))^2}{3c(1+icx)^3}$$

output

```
-1/54*b^2/c/(I-c*x)^3+5/144*I*b^2/c/(I-c*x)^2+11/144*b^2/c/(I-c*x)-11/144*
b^2*arctan(c*x)/c-1/9*I*b*(a+b*arctan(c*x))/c/(I-c*x)^3-1/12*b*(a+b*arctan
(c*x))/c/(I-c*x)^2+1/12*I*b*(a+b*arctan(c*x))/c/(I-c*x)-1/24*I*(a+b*arctan
(c*x))^2/c+1/3*I*(a+b*arctan(c*x))^2/c/(1+I*c*x)^3
```

## Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.75

$$\int \frac{(a+b\arctan(cx))^2}{(1+icx)^4}\,dx = -\frac{144a^2+12ab(-10i+9cx+3ic^2x^2)+b^2(-56-81icx+33c^2x^2)+3b(i+cx)\left(12a(-7i+4cx+ic^2x^2\right.}{432c(-i+cx)}$$

input
```
Integrate[(a + b*ArcTan[c*x])^2/(1 + I*c*x)^4,x]
```

output
```
-1/432*(144*a^2 + 12*a*b*(-10*I + 9*c*x + (3*I)*c^2*x^2) + b^2*(-56 - (81*
I)*c*x + 33*c^2*x^2) + 3*b*(I + c*x)*(12*a*(-7*I + 4*c*x + I*c^2*x^2) + b*
(-29 - (32*I)*c*x + 11*c^2*x^2))*ArcTan[c*x] + 18*b^2*(7 - (3*I)*c*x + 3*c
^2*x^2 + I*c^3*x^3)*ArcTan[c*x]^2)/(c*(-I + c*x)^3)
```

## Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^2}{(1+icx)^4}\,dx$$

$$\Big\downarrow \text{5389}$$

$$\frac{i(a+b\arctan(cx))^2}{3c(1+icx)^3}-\frac{2}{3}ib\int \left(\frac{a+b\arctan(cx)}{8\left(c^2x^2+1\right)}-\frac{a+b\arctan(cx)}{8(i-cx)^2}-\frac{i(a+b\arctan(cx))}{4(i-cx)^3}+\frac{a+b\arctan(cx)}{2(i-cx)^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{i(a+b\arctan(cx))^2}{3c(1+icx)^3} -$$
$$\frac{2}{3}ib\left(\frac{(a+b\arctan(cx))^2}{16bc} - \frac{a+b\arctan(cx)}{8c(-cx+i)} - \frac{i(a+b\arctan(cx))}{8c(-cx+i)^2} + \frac{a+b\arctan(cx)}{6c(-cx+i)^3} - \frac{11ib\arctan(cx)}{96c} + \frac{}{96c}\right.$$

input
```
Int[(a + b*ArcTan[c*x])^2/(1 + I*c*x)^4,x]
```

output
```
((I/3)*(a + b*ArcTan[c*x])^2)/(c*(1 + I*c*x)^3) - ((2*I)/3)*b*(((-1/36*I)*
b)/(c*(I - c*x)^3) - (5*b)/(96*c*(I - c*x)^2) + (((11*I)/96)*b)/(c*(I - c*
x)) - (((11*I)/96)*b*ArcTan[c*x])/c + (a + b*ArcTan[c*x])/(6*c*(I - c*x)^3
) - ((I/8)*(a + b*ArcTan[c*x]))/(c*(I - c*x)^2) - (a + b*ArcTan[c*x])/(8*c
*(I - c*x)) + (a + b*ArcTan[c*x])^2/(16*b*c))
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5389
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - S
imp[b*c*(p/(e*(q + 1)))   Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1),
(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

## Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.43

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{ia^2}{3(icx+1)^3}+b^2\left(\frac{i\arctan(cx)^2}{3(icx+1)^3}-\frac{2i\left(-\frac{i\arctan(cx)\ln(cx-i)}{16}-\frac{i\arctan(cx)}{8(cx-i)^2}-\frac{\arctan(cx)}{6(cx-i)^3}+\frac{\arctan(cx)}{8cx-8i}+\frac{i\arctan(cx)\ln(cx+i)}{16}+\frac{}{36(c}\right.}{}\right.}{}$ |
| default | $\frac{\frac{ia^2}{3(icx+1)^3}+b^2\left(\frac{i\arctan(cx)^2}{3(icx+1)^3}-\frac{2i\left(-\frac{i\arctan(cx)\ln(cx-i)}{16}-\frac{i\arctan(cx)}{8(cx-i)^2}-\frac{\arctan(cx)}{6(cx-i)^3}+\frac{\arctan(cx)}{8cx-8i}+\frac{i\arctan(cx)\ln(cx+i)}{16}+\frac{}{36(c}\right.}{}\right.}{}$ |
| parts | $\frac{ia^2}{3(icx+1)^3c}+\frac{b^2\left(\frac{i\arctan(cx)^2}{3(icx+1)^3}-\frac{2i\left(-\frac{i\arctan(cx)\ln(cx-i)}{16}-\frac{i\arctan(cx)}{8(cx-i)^2}-\frac{\arctan(cx)}{6(cx-i)^3}+\frac{\arctan(cx)}{8cx-8i}+\frac{i\arctan(cx)\ln(cx+i)}{16}+\right.}{}\right.}{}$ |
| orering | $-\frac{i(-425c^6x^6+228ic^5x^5-1355c^4x^4+536ic^3x^3-1947c^2x^2+308icx-1017)(a+b\arctan(cx))^2}{864c(icx+1)^4}-\frac{i(-425c^7x^7+634ic^{}}{}$ |
| risch | $\frac{ib^2(c^3x^3-3ic^2x^2-3cx-7i)\ln(icx+1)^2}{96(cx-i)^3c}+\frac{ib(-3bc^3x^3\ln(-icx+1)+9i\ln(-icx+1)bc^2x^2+6ibc^2x^2+9bcx\ln(-icx+1}{144(cx-i)^3c}$ |

input
```
int((a+b*arctan(c*x))^2/(1+I*c*x)^4,x,method=_RETURNVERBOSE)
```

output
```
1/c*(1/3*I*a^2/(1+I*c*x)^3+b^2*(1/3*I/(1+I*c*x)^3*arctan(c*x)^2-2/3*I*(-1/
16*I*arctan(c*x)*ln(c*x-I)-1/8*I*arctan(c*x)/(c*x-I)^2-1/6*arctan(c*x)/(c*
x-I)^3+1/8*arctan(c*x)/(c*x-I)+1/16*I*arctan(c*x)*ln(c*x+I)+1/36*I/(c*x-I)
^3-11/96*I/(c*x-I)-5/96/(c*x-I)^2-11/96*I*arctan(c*x)+1/64*ln(c*x-I)^2-1/3
2*ln(c*x-I)*ln(-1/2*I*(c*x+I))-1/32*(ln(c*x+I)-ln(-1/2*I*(c*x+I)))*ln(-1/2
*I*(-c*x+I))+1/64*ln(c*x+I)^2))+2/3*I*a*b/(1+I*c*x)^3*arctan(c*x)-1/12*I*a
*b*arctan(c*x)-1/12*a*b/(c*x-I)^2+1/9*I*a*b/(c*x-I)^3-1/12*I*a*b/(c*x-I))
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\arctan(cx))^2}{(1+icx)^4}\,dx = -\frac{6\,(12i\,ab+11\,b^2)c^2x^2+54\,(4\,ab-3i\,b^2)cx+9\,(-i\,b^2c^3x^3-3\,b^2c^2x^2+3i\,b^2cx-7\,b^2)\log\left(-\frac{cx+i}{cx-i}\right)^2+2}{864\,(c}$$

input

```
integrate((a+b*arctan(c*x))^2/(1+I*c*x)^4,x, algorithm="fricas")
```

output

```
-1/864*(6*(12*I*a*b + 11*b^2)*c^2*x^2 + 54*(4*a*b - 3*I*b^2)*c*x + 9*(-I*b
^2*c^3*x^3 - 3*b^2*c^2*x^2 + 3*I*b^2*c*x - 7*b^2)*log(-(c*x + I)/(c*x - I)
)^2 + 288*a^2 - 240*I*a*b - 112*b^2 - 3*((12*a*b - 11*I*b^2)*c^3*x^3 - 3*(
12*I*a*b + 7*b^2)*c^2*x^2 - 3*(12*a*b + I*b^2)*c*x - 84*I*a*b - 29*b^2)*lo
g(-(c*x + I)/(c*x - I)))/(c^4*x^3 - 3*I*c^3*x^2 - 3*c^2*x + I*c)
```

## Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 552 vs. 2(158) = 316.

Time = 24.31 (sec) , antiderivative size = 552, normalized size of antiderivative = 2.67

$$\begin{aligned}\int \frac{(a+b\arctan(cx))^2}{(1+icx)^4}\,dx = &-\frac{b(12a-11ib)\log\left(-\frac{ib(12a-11ib)}{c}+x(12ab-11ib^2)\right)}{288c} \\ &+\frac{b(12a-11ib)\log\left(\frac{ib(12a-11ib)}{c}+x(12ab-11ib^2)\right)}{288c} \\ &+\frac{-144a^2+120iab+56b^2+x^2(-36iabc^2-33b^2c^2)+x(-108abc+81ib^2c)}{432c^4x^3-1296ic^3x^2-1296c^2x+432ic} \\ &+\frac{(-48iab-3ib^2c^3x^3\log(icx+1)-9b^2c^2x^2\log(icx+1)+6b^2c^2x^2+9ib^2cx\log(icx+1)-18ib^2cx-}{144c^4x^3-432ic^3x^2-432c^2x+144ic} \\ &+\frac{(ib^2c^3x^3+3b^2c^2x^2-3ib^2cx+7b^2)\log(-icx+1)^2}{96c^4x^3-288ic^3x^2-288c^2x+96ic} \\ &+\frac{(ib^2c^3x^3+3b^2c^2x^2-3ib^2cx+7b^2)\log(icx+1)^2}{96c^4x^3-288ic^3x^2-288c^2x+96ic} \\ &+\frac{(24iab-3b^2c^2x^2+9ib^2cx+10b^2)\log(icx+1)}{72c^4x^3-216ic^3x^2-216c^2x+72ic}\end{aligned}$$

input
```
integrate((a+b*atan(c*x))**2/(1+I*c*x)**4,x)
```

output
```
-b*(12*a - 11*I*b)*log(-I*b*(12*a - 11*I*b)/c + x*(12*a*b - 11*I*b**2))/(2
88*c) + b*(12*a - 11*I*b)*log(I*b*(12*a - 11*I*b)/c + x*(12*a*b - 11*I*b**
2))/(288*c) + (-144*a**2 + 120*I*a*b + 56*b**2 + x**2*(-36*I*a*b*c**2 - 33
*b**2*c**2) + x*(-108*a*b*c + 81*I*b**2*c))/(432*c**4*x**3 - 1296*I*c**3*x
**2 - 1296*c**2*x + 432*I*c) + (-48*I*a*b - 3*I*b**2*c**3*x**3*log(I*c*x +
 1) - 9*b**2*c**2*x**2*log(I*c*x + 1) + 6*b**2*c**2*x**2 + 9*I*b**2*c*x*lo
g(I*c*x + 1) - 18*I*b**2*c*x - 21*b**2*log(I*c*x + 1) - 20*b**2)*log(-I*c*
x + 1)/(144*c**4*x**3 - 432*I*c**3*x**2 - 432*c**2*x + 144*I*c) + (I*b**2*
c**3*x**3 + 3*b**2*c**2*x**2 - 3*I*b**2*c*x + 7*b**2)*log(-I*c*x + 1)**2/(
96*c**4*x**3 - 288*I*c**3*x**2 - 288*c**2*x + 96*I*c) + (I*b**2*c**3*x**3
+ 3*b**2*c**2*x**2 - 3*I*b**2*c*x + 7*b**2)*log(I*c*x + 1)**2/(96*c**4*x**
3 - 288*I*c**3*x**2 - 288*c**2*x + 96*I*c) + (24*I*a*b - 3*b**2*c**2*x**2
+ 9*I*b**2*c*x + 10*b**2)*log(I*c*x + 1)/(72*c**4*x**3 - 216*I*c**3*x**2 -
 216*c**2*x + 72*I*c)
```

## Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.89

$$\int \frac{(a+b\arctan(cx))^2}{(1+icx)^4}\,dx = \frac{3\,(-12i\,ab-11\,b^2)c^2x^2-27\,(4\,ab-3i\,b^2)cx+18\,(-i\,b^2c^3x^3-3\,b^2c^2x^2+3i\,b^2cx-7\,b^2)\arctan{(cx)}^2-}{432\,(c^4}$$

input
```
integrate((a+b*arctan(c*x))^2/(1+I*c*x)^4,x, algorithm="maxima")
```

output
```
1/432*(3*(-12*I*a*b - 11*b^2)*c^2*x^2 - 27*(4*a*b - 3*I*b^2)*c*x + 18*(-I*
b^2*c^3*x^3 - 3*b^2*c^2*x^2 + 3*I*b^2*c*x - 7*b^2)*arctan(c*x)^2 - 144*a^2
 + 120*I*a*b + 56*b^2 + 3*((-12*I*a*b - 11*b^2)*c^3*x^3 - 3*(12*a*b - 7*I*
b^2)*c^2*x^2 + 3*(12*I*a*b - b^2)*c*x - 84*a*b + 29*I*b^2)*arctan(c*x))/(c
^4*x^3 - 3*I*c^3*x^2 - 3*c^2*x + I*c)
```

## Giac [F]

$$\int \frac{(a+b\arctan(cx))^2}{(1+icx)^4}\,dx = \int \frac{(b\arctan(cx)+a)^2}{(i\,cx+1)^4}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/(1+I*c*x)^4,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)^2/(I*c*x + 1)^4, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{(1+icx)^4}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^2}{(1+c\,x\,1\mathrm{i})^4}\,dx$$

input
```
int((a + b*atan(c*x))^2/(c*x*1i + 1)^4,x)
```

output
```
int((a + b*atan(c*x))^2/(c*x*1i + 1)^4, x)
```

## Reduce [F]

$$\int \frac{(a+b\arctan(cx))^2}{(1+icx)^4}\,dx = 2\left(\int \frac{atan(cx)}{c^4x^4-4c^3i\,x^3-6c^2x^2+4cix+1}dx\right)ab + \left(\int \frac{atan(cx)^2}{c^4x^4-4c^3i\,x^3-6c^2x^2+4cix+1}dx\right)b^2 + \left(\int \frac{1}{c^4x^4-4c^3i\,x^3-6c^2x^2+4cix+1}dx\right)a^2$$

input
```
int((a+b*atan(c*x))^2/(1+I*c*x)^4,x)
```

output

```
2*int(atan(c*x)/(c**4*x**4 - 4*c**3*i*x**3 - 6*c**2*x**2 + 4*c*i*x + 1),x)
*a*b + int(atan(c*x)**2/(c**4*x**4 - 4*c**3*i*x**3 - 6*c**2*x**2 + 4*c*i*x
 + 1),x)*b**2 + int(1/(c**4*x**4 - 4*c**3*i*x**3 - 6*c**2*x**2 + 4*c*i*x +
 1),x)*a**2
```

# 3.119 $\int \frac{\arctan(ax)^2}{cx-iacx^2}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 76

$$\int \frac{\arctan(ax)^2}{cx-iacx^2}\,dx = \frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{c} - \frac{i\arctan(ax)\,\mathrm{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{c} + \frac{\mathrm{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)}{2c}$$

output
```
arctan(a*x)^2*ln(2-2/(1-I*a*x))/c-I*arctan(a*x)*polylog(2,-1+2/(1-I*a*x))/
c+1/2*polylog(3,-1+2/(1-I*a*x))/c
```

## Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.08

$$\int \frac{\arctan(ax)^2}{cx-iacx^2}\,dx = \frac{-i\pi^3+16i\arctan(ax)^3+24\arctan(ax)^2\log\left(1-e^{-2i\arctan(ax)}\right)+24i\arctan(ax)\,\mathrm{PolyLog}\left(2,e^{-2i\arctan(a}\right.}{24c}$$

input
```
Integrate[ArcTan[a*x]^2/(c*x - I*a*c*x^2),x]
```

output
```
((-I)*Pi^3 + (16*I)*ArcTan[a*x]^3 + 24*ArcTan[a*x]^2*Log[1 - E^((-2*I)*Arc
Tan[a*x])] + (24*I)*ArcTan[a*x]*PolyLog[2, E^((-2*I)*ArcTan[a*x])] + 12*Po
lyLog[3, E^((-2*I)*ArcTan[a*x])])/(24*c)
```

## Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2026, 5403, 5527, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2}{cx - iacx^2} \, dx$$

$$\Big\downarrow \text{ 2026}$$

$$\int \frac{\arctan(ax)^2}{x(c - iacx)} dx$$

$$\Big\downarrow \text{ 5403}$$

$$\frac{\arctan(ax)^2 \log\left(2 - \frac{2}{1-iax}\right)}{c} - \frac{2a \int \frac{\arctan(ax) \log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1} dx}{c}$$

$$\Big\downarrow \text{ 5527}$$

$$\frac{\arctan(ax)^2 \log\left(2 - \frac{2}{1-iax}\right)}{c} - \frac{2a \left(\frac{i \arctan(ax) \operatorname{PolyLog}\left(2, \frac{2}{1-iax}-1\right)}{2a} - \frac{1}{2} i \int \frac{\operatorname{PolyLog}\left(2, \frac{2}{1-iax}-1\right)}{a^2x^2+1} dx\right)}{c}$$

$$\Big\downarrow \text{ 7164}$$

$$\frac{\arctan(ax)^2 \log\left(2 - \frac{2}{1-iax}\right)}{c} - \frac{2a \left(\frac{i \arctan(ax) \operatorname{PolyLog}\left(2, \frac{2}{1-iax}-1\right)}{2a} - \frac{\operatorname{PolyLog}\left(3, \frac{2}{1-iax}-1\right)}{4a}\right)}{c}$$

input
```
Int[ArcTan[a*x]^2/(c*x - I*a*c*x^2),x]
```

output
```
(ArcTan[a*x]^2*Log[2 - 2/(1 - I*a*x)])/c - (2*a*(((I/2)*ArcTan[a*x]*PolyLo
g[2, -1 + 2/(1 - I*a*x)])/a - PolyLog[3, -1 + 2/(1 - I*a*x)]/(4*a)))/c
```

### Defintions of rubi rules used

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5527
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

## Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 192 vs. $2(70) = 140$.

Time = 2.05 (sec) , antiderivative size = 193, normalized size of antiderivative = 2.54

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{a \arctan(ax)^2 \ln\left(1-\frac{iax+1}{\sqrt{a^2x^2+1}}\right)}{c} - \frac{2ia \arctan(ax) \operatorname{polylog}\left(2,\frac{iax+1}{\sqrt{a^2x^2+1}}\right)}{c} + \frac{2a \operatorname{polylog}\left(3,\frac{iax+1}{\sqrt{a^2x^2+1}}\right)}{c} + \frac{a \arctan(ax)^2 \ln\left(1+\frac{iax}{\sqrt{a^2}}\right.}{c}}{a}$ |
| default | $\frac{\frac{a \arctan(ax)^2 \ln\left(1-\frac{iax+1}{\sqrt{a^2x^2+1}}\right)}{c} - \frac{2ia \arctan(ax) \operatorname{polylog}\left(2,\frac{iax+1}{\sqrt{a^2x^2+1}}\right)}{c} + \frac{2a \operatorname{polylog}\left(3,\frac{iax+1}{\sqrt{a^2x^2+1}}\right)}{c} + \frac{a \arctan(ax)^2 \ln\left(1+\frac{iax}{\sqrt{a^2}}\right.}{c}}{a}$ |

input
```
int(arctan(a*x)^2/(c*x-I*a*c*x^2),x,method=_RETURNVERBOSE)
```

output
```
1/a*(a/c*arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*I*a/c*arctan(a*
x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*a/c*polylog(3,(1+I*a*x)/(a^2*x
^2+1)^(1/2))+a/c*arctan(a*x)^2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*I*a/c*a
rctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*a/c*polylog(3,-(1+I*a
*x)/(a^2*x^2+1)^(1/2)))
```

## Fricas [F]

$$\int \frac{\arctan(ax)^2}{cx - iacx^2}\,dx = \int \frac{\arctan(ax)^2}{-i\,acx^2 + cx}\,dx$$

input
```
integrate(arctan(a*x)^2/(c*x-I*a*c*x^2),x, algorithm="fricas")
```

output
```
integral(-1/4*I*log(-(a*x + I)/(a*x - I))^2/(a*c*x^2 + I*c*x), x)
```

## Sympy [F]

$$\int \frac{\arctan(ax)^2}{cx - iacx^2}\,dx = \frac{i\int \frac{\operatorname{atan}^2(ax)}{ax^2+ix}\,dx}{c}$$

input
```
integrate(atan(a*x)**2/(c*x-I*a*c*x**2),x)
```

output
```
I*Integral(atan(a*x)**2/(a*x**2 + I*x), x)/c
```

## Maxima [F]

$$\int \frac{\arctan(ax)^2}{cx - iacx^2}\,dx = \int \frac{\arctan(ax)^2}{-i\,acx^2 + cx}\,dx$$

input
```
integrate(arctan(a*x)^2/(c*x-I*a*c*x^2),x, algorithm="maxima")
```

output
```
1/96*(8*I*arctan(a*x)^3 - 12*arctan(a*x)^2*log(a^2*x^2 + 1) - 6*I*arctan(a
*x)*log(a^2*x^2 + 1)^2 + log(a^2*x^2 + 1)^3 + 24*I*(arctan(a*x)^3/c + 4*a*
integrate(1/16*x*log(a^2*x^2 + 1)^2/(a^2*c*x^3 + c*x), x) - 16*integrate(1
/16*arctan(a*x)*log(a^2*x^2 + 1)/(a^2*c*x^3 + c*x), x))*c + 96*c*integrate
(1/16*(4*a*x*arctan(a*x)*log(a^2*x^2 + 1) + 12*arctan(a*x)^2 + log(a^2*x^2
 + 1)^2)/(a^2*c*x^3 + c*x), x))/c
```

## Giac [F]

$$\int \frac{\arctan(ax)^2}{cx - iacx^2}\,dx = \int \frac{\arctan(ax)^2}{-i\,acx^2 + cx}\,dx$$

input
```
integrate(arctan(a*x)^2/(c*x-I*a*c*x^2),x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^2/(-I*a*c*x^2 + c*x), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^2}{cx - iacx^2}\,dx = \int \frac{\operatorname{atan}(a\,x)^2}{c\,x - a\,c\,x^2\,1\mathrm{i}}\,dx$$

input
```
int(atan(a*x)^2/(c*x - a*c*x^2*1i),x)
```

output
```
int(atan(a*x)^2/(c*x - a*c*x^2*1i), x)
```

## Reduce [F]

$$\int \frac{\arctan(ax)^2}{cx - iacx^2}\,dx = \frac{atan(ax)^3\,i + 3\left(\int \frac{atan(ax)^2}{a^3x^4 + a^2i\,x^3 + a\,x^2 + ix}dx\right)i + 3\left(\int \frac{atan(ax)^2}{a^3x^3 + a^2i\,x^2 + ax + i}dx\right)a}{3c}$$

input
```
int(atan(a*x)^2/(c*x-I*a*c*x^2),x)
```

output
```
(atan(a*x)**3*i + 3*int(atan(a*x)**2/(a**3*x**4 + a**2*i*x**3 + a*x**2 + i
*x),x)*i + 3*int(atan(a*x)**2/(a**3*x**3 + a**2*i*x**2 + a*x + i),x)*a)/(3
*c)
```

# 3.120 $\int (d+icdx)^3(a+b\arctan(cx))^3\,dx$

## Optimal result

Integrand size = 22, antiderivative size = 382

$$\int (d+icdx)^3(a+b\arctan(cx))^3\,dx = -3ab^2d^3x+\frac{1}{4}ib^3d^3x-\frac{ib^3d^3\arctan(cx)}{4c} -3b^3d^3x\arctan(cx)-\frac{1}{4}ib^2cd^3x^2(a+b\arctan(cx)) +\frac{7bd^3(a+b\arctan(cx))^2}{c} -\frac{21}{4}ibd^3x(a+b\arctan(cx))^2 +\frac{3}{2}bcd^3x^2(a+b\arctan(cx))^2 +\frac{1}{4}ibc^2d^3x^3(a+b\arctan(cx))^2 -\frac{id^3(1+icx)^4(a+b\arctan(cx))^3}{4c} +\frac{6bd^3(a+b\arctan(cx))^2\log\left(\frac{2}{1-icx}\right)}{c} -\frac{11ib^2d^3(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c} +\frac{3b^3d^3\log(1+c^2x^2)}{2c} -\frac{6ib^2d^3(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{c} +\frac{11b^3d^3\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2c} +\frac{3b^3d^3\operatorname{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{c}$$

output

```
-3*a*b^2*d^3*x-11*I*b^2*d^3*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c-6*I*b^2*d^
3*(a+b*arctan(c*x))*polylog(2,1-2/(1-I*c*x))/c-3*b^3*d^3*x*arctan(c*x)+1/4
*I*b*c^2*d^3*x^3*(a+b*arctan(c*x))^2+7*b*d^3*(a+b*arctan(c*x))^2/c+1/4*I*b
^3*d^3*x+3/2*b*c*d^3*x^2*(a+b*arctan(c*x))^2-1/4*I*b^2*c*d^3*x^2*(a+b*arct
an(c*x))-1/4*I*b^3*d^3*arctan(c*x)/c+6*b*d^3*(a+b*arctan(c*x))^2*ln(2/(1-I
*c*x))/c-21/4*I*b*d^3*x*(a+b*arctan(c*x))^2+3/2*b^3*d^3*ln(c^2*x^2+1)/c-1/
4*I*d^3*(1+I*c*x)^4*(a+b*arctan(c*x))^3/c+11/2*b^3*d^3*polylog(2,1-2/(1+I*
c*x))/c+3*b^3*d^3*polylog(3,1-2/(1-I*c*x))/c
```

## Mathematica [A] (verified)

Time = 2.01 (sec) , antiderivative size = 693, normalized size of antiderivative = 1.81

$$\int (d+icdx)^3(a+b\arctan(cx))^3\,dx = -\frac{id^3\left(ab^2+4ia^3cx+21a^2bcx-12iab^2cx-b^3cx-6a^3c^2x^2+6ia^2bc^2x^2+ab^2c^2x^2-4ia^3c^3x^3-a^2bc^3x^3\right.}{}$$

input
```
Integrate[(d + I*c*d*x)^3*(a + b*ArcTan[c*x])^3,x]
```

output
```
((-1/4*I)*d^3*(a*b^2 + (4*I)*a^3*c*x + 21*a^2*b*c*x - (12*I)*a*b^2*c*x - b
^3*c*x - 6*a^3*c^2*x^2 + (6*I)*a^2*b*c^2*x^2 + a*b^2*c^2*x^2 - (4*I)*a^3*c
^3*x^3 - a^2*b*c^3*x^3 + a^3*c^4*x^4 - 21*a^2*b*ArcTan[c*x] + (12*I)*a*b^2
*ArcTan[c*x] + b^3*ArcTan[c*x] + (12*I)*a^2*b*c*x*ArcTan[c*x] + 42*a*b^2*c
*x*ArcTan[c*x] - (12*I)*b^3*c*x*ArcTan[c*x] - 18*a^2*b*c^2*x^2*ArcTan[c*x]
 + (12*I)*a*b^2*c^2*x^2*ArcTan[c*x] + b^3*c^2*x^2*ArcTan[c*x] - (12*I)*a^2
*b*c^3*x^3*ArcTan[c*x] - 2*a*b^2*c^3*x^3*ArcTan[c*x] + 3*a^2*b*c^4*x^4*Arc
Tan[c*x] + 3*a*b^2*ArcTan[c*x]^2 - (16*I)*b^3*ArcTan[c*x]^2 + (12*I)*a*b^2
*c*x*ArcTan[c*x]^2 + 21*b^3*c*x*ArcTan[c*x]^2 - 18*a*b^2*c^2*x^2*ArcTan[c*
x]^2 + (6*I)*b^3*c^2*x^2*ArcTan[c*x]^2 - (12*I)*a*b^2*c^3*x^3*ArcTan[c*x]^
2 - b^3*c^3*x^3*ArcTan[c*x]^2 + 3*a*b^2*c^4*x^4*ArcTan[c*x]^2 + b^3*ArcTan
[c*x]^3 + (4*I)*b^3*c*x*ArcTan[c*x]^3 - 6*b^3*c^2*x^2*ArcTan[c*x]^3 - (4*I
)*b^3*c^3*x^3*ArcTan[c*x]^3 + b^3*c^4*x^4*ArcTan[c*x]^3 + (48*I)*a*b^2*Arc
Tan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] + 44*b^3*ArcTan[c*x]*Log[1 + E^((2
*I)*ArcTan[c*x])] + (24*I)*b^3*ArcTan[c*x]^2*Log[1 + E^((2*I)*ArcTan[c*x])
] - (12*I)*a^2*b*Log[1 + c^2*x^2] - 22*a*b^2*Log[1 + c^2*x^2] + (6*I)*b^3*
Log[1 + c^2*x^2] + 2*b^2*(12*a - (11*I)*b + 12*b*ArcTan[c*x])*PolyLog[2, -
E^((2*I)*ArcTan[c*x])] + (12*I)*b^3*PolyLog[3, -E^((2*I)*ArcTan[c*x])]))/c
```

## Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 380, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {5389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+icdx)^3(a+b\arctan(cx))^3\,dx$$

$$\Big\downarrow \text{5389}$$

$$\frac{3ib\int \left(c^2x^2(a+b\arctan(cx))^2d^4-4icx(a+b\arctan(cx))^2d^4-\frac{8i(i-cx)(a+b\arctan(cx))^2d^4}{c^2x^2+1}-7(a+b\arctan(cx))^2d\right.}{4d}$$
$$\frac{id^3(1+icx)^4(a+b\arctan(cx))^3}{4c}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3ib\left(\frac{1}{3}c^2d^4x^3(a+b\arctan(cx))^2-\frac{8bd^4\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)(a+b\arctan(cx))}{c}-2icd^4x^2(a+b\arctan(cx))^2-\frac{1}{3}bcd^4x^2(a\right.}{}$$
$$\frac{id^3(1+icx)^4(a+b\arctan(cx))^3}{4c}$$

input

```
Int[(d + I*c*d*x)^3*(a + b*ArcTan[c*x])^3,x]
```

output

```
((-1/4*I)*d^3*(1 + I*c*x)^4*(a + b*ArcTan[c*x])^3)/c + (((3*I)/4)*b*((4*I)
*a*b*d^4*x + (b^2*d^4*x)/3 - (b^2*d^4*ArcTan[c*x])/(3*c) + (4*I)*b^2*d^4*x
*ArcTan[c*x] - (b*c*d^4*x^2*(a + b*ArcTan[c*x]))/3 - (((28*I)/3)*d^4*(a +
b*ArcTan[c*x])^2)/c - 7*d^4*x*(a + b*ArcTan[c*x])^2 - (2*I)*c*d^4*x^2*(a +
 b*ArcTan[c*x])^2 + (c^2*d^4*x^3*(a + b*ArcTan[c*x])^2)/3 - ((8*I)*d^4*(a
+ b*ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/c - (44*b*d^4*(a + b*ArcTan[c*x])*L
og[2/(1 + I*c*x)])/(3*c) - ((2*I)*b^2*d^4*Log[1 + c^2*x^2])/c - (8*b*d^4*(
a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 - I*c*x)])/c - (((22*I)/3)*b^2*d^4*
PolyLog[2, 1 - 2/(1 + I*c*x)])/c - ((4*I)*b^2*d^4*PolyLog[3, 1 - 2/(1 - I*
c*x)])/c))/d
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5389
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - S
imp[b*c*(p/(e*(q + 1)))   Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1),
(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 19.04 (sec) , antiderivative size = 1513, normalized size of antiderivative = 3.96

| method | result | size |
|---|---|---|
| derivativedivides | Expression too large to display | 1513 |
| default | Expression too large to display | 1513 |
| parts | Expression too large to display | 1521 |

input
```
int((d+I*c*d*x)^3*(a+b*arctan(c*x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/c*(-1/4*I*d^3*a^3*(1+I*c*x)^4+d^3*b^3*(-1/4*I*arctan(c*x)^3*c^4*x^4-c^3*
x^3*arctan(c*x)^3+3/2*I*arctan(c*x)^3*c^2*x^2+arctan(c*x)^3*c*x-1/4*I*arct
an(c*x)^3+3/4*I*(1/3*c^3*x^3*arctan(c*x)^2+1/3*I-7*c*x*arctan(c*x)^2+1/3*c
*x+2*Pi*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1))^2)^3*arctan(c*x)^2+1/3*arctan(c
*x)*(c*x-I)^2+4*Pi*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))*csgn(I*(1+I*c*x)^2/
(c^2*x^2+1))^2*arctan(c*x)^2-2*Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)*cs
gn(I*(1+I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^
2/(c^2*x^2+1))^2)*arctan(c*x)^2-44/3*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2
+1)^(1/2))-44/3*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+2*Pi*csgn(
I/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*
x)^2/(c^2*x^2+1))^2)^2*arctan(c*x)^2-4*Pi*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1
)))*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1))^2)^2*arctan(c*x)^2+2*Pi*csgn(I*(1+I
*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^
2+1))^2)^2*arctan(c*x)^2-2*Pi*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))^2*csgn(I
*(1+I*c*x)^2/(c^2*x^2+1))*arctan(c*x)^2-2*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1
)/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)^3*arctan(c*x)^2-2*Pi*csgn(I*(1+I*c*x)^2/(
c^2*x^2+1))^3*arctan(c*x)^2-2*I*arctan(c*x)^2*c^2*x^2+2*Pi*csgn(I*(1+(1+I*
c*x)^2/(c^2*x^2+1)))^2*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1))^2)*arctan(c*x)^2
-8*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))+14/3*I*arctan(c*x)*(c*x
-I)+4*I*arctan(c*x)^2*ln(c^2*x^2+1)-8*I*arctan(c*x)^2*ln((1+I*c*x)/(c^2...
```

## Fricas [F]

$$\int (d+icdx)^3(a+b\arctan(cx))^3\,dx=\int (i\,cdx+d)^3(b\arctan{(cx)}+a)^3\,dx$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^3,x, algorithm="fricas")
```

output
```
-1/32*(b^3*c^3*d^3*x^4 - 4*I*b^3*c^2*d^3*x^3 - 6*b^3*c*d^3*x^2 + 4*I*b^3*d
^3*x)*log(-(c*x + I)/(c*x - I))^3 + integral(1/16*(-16*I*a^3*c^5*d^3*x^5 -
 48*a^3*c^4*d^3*x^4 + 32*I*a^3*c^3*d^3*x^3 - 32*a^3*c^2*d^3*x^2 + 48*I*a^3
*c*d^3*x + 16*a^3*d^3 - 3*(-4*I*a*b^2*c^5*d^3*x^5 - (12*a*b^2 - I*b^3)*c^4
*d^3*x^4 + 4*(2*I*a*b^2 + b^3)*c^3*d^3*x^3 - 2*(4*a*b^2 + 3*I*b^3)*c^2*d^3
*x^2 + 4*a*b^2*d^3 + 4*(3*I*a*b^2 - b^3)*c*d^3*x)*log(-(c*x + I)/(c*x - I)
)^2 + 24*(a^2*b*c^5*d^3*x^5 - 3*I*a^2*b*c^4*d^3*x^4 - 2*a^2*b*c^3*d^3*x^3
- 2*I*a^2*b*c^2*d^3*x^2 - 3*a^2*b*c*d^3*x + I*a^2*b*d^3)*log(-(c*x + I)/(c
*x - I)))/(c^2*x^2 + 1), x)
```

## Sympy [F(-1)]

Timed out.

$$\int (d+icdx)^3(a+b\arctan(cx))^3\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)**3*(a+b*atan(c*x))**3,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int (d+icdx)^3(a+b\arctan(cx))^3\,dx = \int (i\,cdx+d)^3(b\arctan(cx)+a)^3\,dx$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^3,x, algorithm="maxima")
```

output
```
-1/4*I*a^3*c^3*d^3*x^4 - 24*b^3*c^5*d^3*integrate(1/128*x^5*arctan(c*x)^2*
log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) + 2*b^3*c^5*d^3*integrate(1/128*x^5*log
(c^2*x^2 + 1)^3/(c^2*x^2 + 1), x) - 12*b^3*c^5*d^3*integrate(1/128*x^5*arc
tan(c*x)^2/(c^2*x^2 + 1), x) + 3*b^3*c^5*d^3*integrate(1/128*x^5*log(c^2*x
^2 + 1)^2/(c^2*x^2 + 1), x) - a^3*c^2*d^3*x^3 - 336*b^3*c^4*d^3*integrate(
1/128*x^4*arctan(c*x)^3/(c^2*x^2 + 1), x) - 36*b^3*c^4*d^3*integrate(1/128
*x^4*arctan(c*x)*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) - 1152*a*b^2*c^4*d^3
*integrate(1/128*x^4*arctan(c*x)^2/(c^2*x^2 + 1), x) - 60*b^3*c^4*d^3*inte
grate(1/128*x^4*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) - 1/4*I*(3*
x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*a^2*b*c^3*d
^3 + 48*b^3*c^3*d^3*integrate(1/128*x^3*arctan(c*x)^2*log(c^2*x^2 + 1)/(c^
2*x^2 + 1), x) - 4*b^3*c^3*d^3*integrate(1/128*x^3*log(c^2*x^2 + 1)^3/(c^2
*x^2 + 1), x) + 120*b^3*c^3*d^3*integrate(1/128*x^3*arctan(c*x)^2/(c^2*x^2
 + 1), x) - 30*b^3*c^3*d^3*integrate(1/128*x^3*log(c^2*x^2 + 1)^2/(c^2*x^2
 + 1), x) - 3/2*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*a
^2*b*c^2*d^3 + 3/2*I*a^3*c*d^3*x^2 + 7/32*b^3*d^3*arctan(c*x)^4/c - 224*b^
3*c^2*d^3*integrate(1/128*x^2*arctan(c*x)^3/(c^2*x^2 + 1), x) - 24*b^3*c^2
*d^3*integrate(1/128*x^2*arctan(c*x)*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x)
- 768*a*b^2*c^2*d^3*integrate(1/128*x^2*arctan(c*x)^2/(c^2*x^2 + 1), x) +
120*b^3*c^2*d^3*integrate(1/128*x^2*arctan(c*x)*log(c^2*x^2 + 1)/(c^2*x...
```

## Giac [F]

$$\int (d+icdx)^3(a+b\arctan(cx))^3\,dx=\int (i\,cdx+d)^3(b\arctan(cx)+a)^3\,dx$$

input
```
integrate((d+I*c*d*x)^3*(a+b*arctan(c*x))^3,x, algorithm="giac")
```

output
```
integrate((I*c*d*x + d)^3*(b*arctan(c*x) + a)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (d+icdx)^3(a+b\arctan(cx))^3\,dx = \int (a+b\,\mathrm{atan}(c\,x))^3\,(d+c\,d\,x\,1\mathrm{i})^3\,dx$$

input
```
int((a + b*atan(c*x))^3*(d + c*d*x*1i)^3,x)
```

output
```
int((a + b*atan(c*x))^3*(d + c*d*x*1i)^3, x)
```

## Reduce [F]

$$\int (d+icdx)^3(a+b\arctan(cx))^3\,dx = \text{Too large to display}$$

input
```
int((d+I*c*d*x)^3*(a+b*atan(c*x))^3,x)
```

output
```
(d**3*( - atan(c*x)**3*b**3*c**4*i*x**4 - 4*atan(c*x)**3*b**3*c**3*x**3 +
6*atan(c*x)**3*b**3*c**2*i*x**2 + 4*atan(c*x)**3*b**3*c*x + 7*atan(c*x)**3
*b**3*i - 3*atan(c*x)**2*a*b**2*c**4*i*x**4 - 12*atan(c*x)**2*a*b**2*c**3*
x**3 + 18*atan(c*x)**2*a*b**2*c**2*i*x**2 + 12*atan(c*x)**2*a*b**2*c*x + 2
1*atan(c*x)**2*a*b**2*i + atan(c*x)**2*b**3*c**3*i*x**3 + 6*atan(c*x)**2*b
**3*c**2*x**2 - 21*atan(c*x)**2*b**3*c*i*x + 6*atan(c*x)**2*b**3 - 3*atan(
c*x)*a**2*b*c**4*i*x**4 - 12*atan(c*x)*a**2*b*c**3*x**3 + 18*atan(c*x)*a**
2*b*c**2*i*x**2 + 12*atan(c*x)*a**2*b*c*x + 21*atan(c*x)*a**2*b*i + 2*atan
(c*x)*a*b**2*c**3*i*x**3 + 12*atan(c*x)*a*b**2*c**2*x**2 - 42*atan(c*x)*a*
b**2*c*i*x + 12*atan(c*x)*a*b**2 - atan(c*x)*b**3*c**2*i*x**2 - 12*atan(c*
x)*b**3*c*x - atan(c*x)*b**3*i - 48*int((atan(c*x)*x)/(c**2*x**2 + 1),x)*a
*b**2*c**2 + 44*int((atan(c*x)*x)/(c**2*x**2 + 1),x)*b**3*c**2*i - 24*int(
(atan(c*x)**2*x)/(c**2*x**2 + 1),x)*b**3*c**2 - 12*log(c**2*x**2 + 1)*a**2
*b + 22*log(c**2*x**2 + 1)*a*b**2*i + 6*log(c**2*x**2 + 1)*b**3 - a**3*c**
4*i*x**4 - 4*a**3*c**3*x**3 + 6*a**3*c**2*i*x**2 + 4*a**3*c*x + a**2*b*c**
3*i*x**3 + 6*a**2*b*c**2*x**2 - 21*a**2*b*c*i*x - a*b**2*c**2*i*x**2 - 12*
a*b**2*c*x + b**3*c*i*x))/(4*c)
```

# 3.121 $\int (d+icdx)^2(a+b\arctan(cx))^3\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 298

$$\begin{aligned}\int (d+icdx)^2(a+b\arctan(cx))^3\,dx = {} & -ab^2d^2x-b^3d^2x\arctan(cx) \\ & +\frac{7bd^2(a+b\arctan(cx))^2}{2c} \\ & -3ibd^2x(a+b\arctan(cx))^2 \\ & +\frac{1}{2}bcd^2x^2(a+b\arctan(cx))^2 \\ & -\frac{id^2(1+icx)^3(a+b\arctan(cx))^3}{3c} \\ & +\frac{4bd^2(a+b\arctan(cx))^2\log\left(\frac{2}{1-icx}\right)}{c} \\ & -\frac{6ib^2d^2(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c} \\ & +\frac{b^3d^2\log(1+c^2x^2)}{2c} \\ & -\frac{4ib^2d^2(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{c} \\ & +\frac{3b^3d^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c} \\ & +\frac{2b^3d^2\operatorname{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{c}\end{aligned}$$

output
```
-a*b^2*d^2*x-b^3*d^2*x*arctan(c*x)+7/2*b*d^2*(a+b*arctan(c*x))^2/c-3*I*b*d
^2*x*(a+b*arctan(c*x))^2+1/2*b*c*d^2*x^2*(a+b*arctan(c*x))^2-1/3*I*d^2*(1+
I*c*x)^3*(a+b*arctan(c*x))^3/c+4*b*d^2*(a+b*arctan(c*x))^2*ln(2/(1-I*c*x))
/c-6*I*b^2*d^2*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c+1/2*b^3*d^2*ln(c^2*x^2+
1)/c-4*I*b^2*d^2*(a+b*arctan(c*x))*polylog(2,1-2/(1-I*c*x))/c+3*b^3*d^2*po
lylog(2,1-2/(1+I*c*x))/c+2*b^3*d^2*polylog(3,1-2/(1-I*c*x))/c
```

## Mathematica [A] (verified)

Time = 1.63 (sec) , antiderivative size = 528, normalized size of antiderivative = 1.77

$$\int (d+icdx)^2(a+b\arctan(cx))^3\,dx = -\frac{d^2\left(-6a^3cx+18ia^2bcx+6ab^2cx-6ia^3c^2x^2-3a^2bc^2x^2+2a^3c^3x^3-18ia^2b\arctan(cx)-6ab^2\arctan(\right.}{}$$

input
```
Integrate[(d + I*c*d*x)^2*(a + b*ArcTan[c*x])^3,x]
```

output
```
-1/6*(d^2*(-6*a^3*c*x + (18*I)*a^2*b*c*x + 6*a*b^2*c*x - (6*I)*a^3*c^2*x^2
 - 3*a^2*b*c^2*x^2 + 2*a^3*c^3*x^3 - (18*I)*a^2*b*ArcTan[c*x] - 6*a*b^2*Ar
cTan[c*x] - 18*a^2*b*c*x*ArcTan[c*x] + (36*I)*a*b^2*c*x*ArcTan[c*x] + 6*b^
3*c*x*ArcTan[c*x] - (18*I)*a^2*b*c^2*x^2*ArcTan[c*x] - 6*a*b^2*c^2*x^2*Arc
Tan[c*x] + 6*a^2*b*c^3*x^3*ArcTan[c*x] + (6*I)*a*b^2*ArcTan[c*x]^2 + 15*b^
3*ArcTan[c*x]^2 - 18*a*b^2*c*x*ArcTan[c*x]^2 + (18*I)*b^3*c*x*ArcTan[c*x]^
2 - (18*I)*a*b^2*c^2*x^2*ArcTan[c*x]^2 - 3*b^3*c^2*x^2*ArcTan[c*x]^2 + 6*a
*b^2*c^3*x^3*ArcTan[c*x]^2 + (2*I)*b^3*ArcTan[c*x]^3 - 6*b^3*c*x*ArcTan[c*
x]^3 - (6*I)*b^3*c^2*x^2*ArcTan[c*x]^3 + 2*b^3*c^3*x^3*ArcTan[c*x]^3 - 48*
a*b^2*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] + (36*I)*b^3*ArcTan[c*x]*
Log[1 + E^((2*I)*ArcTan[c*x])] - 24*b^3*ArcTan[c*x]^2*Log[1 + E^((2*I)*Arc
Tan[c*x])] + 12*a^2*b*Log[1 + c^2*x^2] - (18*I)*a*b^2*Log[1 + c^2*x^2] - 3
*b^3*Log[1 + c^2*x^2] + 6*b^2*((4*I)*a + 3*b + (4*I)*b*ArcTan[c*x])*PolyLo
g[2, -E^((2*I)*ArcTan[c*x])] - 12*b^3*PolyLog[3, -E^((2*I)*ArcTan[c*x])]))
/c
```

## Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.03, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {5389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+icdx)^2(a+b\arctan(cx))^3\,dx$$

$$\Big\downarrow \text{ 5389}$$

$$\frac{ib\int \left(-icx(a+b\arctan(cx))^2d^3-\frac{4i(i-cx)(a+b\arctan(cx))^2d^3}{c^2x^2+1}-3(a+b\arctan(cx))^2d^3\right)dx}{d}-\frac{id^2(1+icx)^3(a+b\arctan(cx))^3}{3c}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{ib\left(-\frac{4bd^3\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)(a+b\arctan(cx))}{c}-\frac{1}{2}icd^3x^2(a+b\arctan(cx))^2-3d^3x(a+b\arctan(cx))^2-\frac{7id^3(a+b\arcta}{2c}\right.}{}$$

$$\frac{id^2(1+icx)^3(a+b\arctan(cx))^3}{3c}$$

input
```
Int[(d + I*c*d*x)^2*(a + b*ArcTan[c*x])^3,x]
```

output
```
((-1/3*I)*d^2*(1 + I*c*x)^3*(a + b*ArcTan[c*x])^3)/c + (I*b*(I*a*b*d^3*x +
 I*b^2*d^3*x*ArcTan[c*x] - (((7*I)/2)*d^3*(a + b*ArcTan[c*x])^2)/c - 3*d^3
*x*(a + b*ArcTan[c*x])^2 - (I/2)*c*d^3*x^2*(a + b*ArcTan[c*x])^2 - ((4*I)*
d^3*(a + b*ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/c - (6*b*d^3*(a + b*ArcTan[c
*x])*Log[2/(1 + I*c*x)])/c - ((I/2)*b^2*d^3*Log[1 + c^2*x^2])/c - (4*b*d^3
*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 - I*c*x)])/c - ((3*I)*b^2*d^3*Pol
yLog[2, 1 - 2/(1 + I*c*x)])/c - ((2*I)*b^2*d^3*PolyLog[3, 1 - 2/(1 - I*c*x
)])/c))/d
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5389
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - S
imp[b*c*(p/(e*(q + 1)))   Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1),
(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 14.09 (sec) , antiderivative size = 1384, normalized size of antiderivative = 4.64

| method | result | size |
|---|---|---|
| derivativedivides | Expression too large to display | 1384 |
| default | Expression too large to display | 1384 |
| parts | Expression too large to display | 1392 |

input
```
int((d+I*c*d*x)^2*(a+b*arctan(c*x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/c*(-1/3*I*d^2*a^3*(1+I*c*x)^3+d^2*b^3*(-1/3*c^3*x^3*arctan(c*x)^3+I*arct
an(c*x)^3*c^2*x^2+arctan(c*x)^3*c*x-1/3*I*arctan(c*x)^3+I*(Pi*csgn(I/(1+(1
+I*c*x)^2/(c^2*x^2+1))^2)*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c
^2*x^2+1))^2)^2*arctan(c*x)^2+2*Pi*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))*csg
n(I*(1+I*c*x)^2/(c^2*x^2+1))^2*arctan(c*x)^2+6*I*dilog(1+I*(1+I*c*x)/(c^2*
x^2+1)^(1/2))+I*arctan(c*x)*(c*x-I)+6*I*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1
/2))-2*I*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))-3*c*x*arctan(c*x)^2+5/2*I*arc
tan(c*x)^2-4*I*ln(2)*arctan(c*x)^2+2*I*arctan(c*x)^2*ln(c^2*x^2+1)-1/2*I*a
rctan(c*x)^2*c^2*x^2-6*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-6*a
rctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-4*arctan(c*x)*polylog(2,-(1
+I*c*x)^2/(c^2*x^2+1))-Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)*csgn(I*(1+
I*c*x)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x
^2+1))^2)*arctan(c*x)^2-Pi*csgn(I*(1+I*c*x)/(c^2*x^2+1)^(1/2))^2*csgn(I*(1
+I*c*x)^2/(c^2*x^2+1))*arctan(c*x)^2+Pi*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1))
)^2*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1))^2)*arctan(c*x)^2+Pi*csgn(I*(1+I*c*x
)^2/(c^2*x^2+1))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)
)^2)^2*arctan(c*x)^2-4*I*arctan(c*x)^2*ln((1+I*c*x)/(c^2*x^2+1)^(1/2))-2*P
i*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1))^2
)^2*arctan(c*x)^2+I*ln(1+(1+I*c*x)^2/(c^2*x^2+1))-Pi*csgn(I*(1+I*c*x)^2/(c
^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)^3*arctan(c*x)^2-Pi*csgn(I*(1+I...
```

## Fricas [F]

$$\int (d+icdx)^2(a+b\arctan(cx))^3\,dx=\int (i\,cdx+d)^2(b\arctan\left(cx\right)+a)^3\,dx$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))^3,x, algorithm="fricas")
```

output
```
1/24*(I*b^3*c^2*d^2*x^3 + 3*b^3*c*d^2*x^2 - 3*I*b^3*d^2*x)*log(-(c*x + I)/
(c*x - I))^3 + integral(-1/4*(4*a^3*c^4*d^2*x^4 - 8*I*a^3*c^3*d^2*x^3 - 8*
I*a^3*c*d^2*x - 4*a^3*d^2 - (3*a*b^2*c^4*d^2*x^4 + 3*I*b^3*c^2*d^2*x^2 + (
-6*I*a*b^2 - b^3)*c^3*d^2*x^3 - 3*a*b^2*d^2 - 3*(2*I*a*b^2 - b^3)*c*d^2*x)
*log(-(c*x + I)/(c*x - I))^2 + 6*(I*a^2*b*c^4*d^2*x^4 + 2*a^2*b*c^3*d^2*x^
3 + 2*a^2*b*c*d^2*x - I*a^2*b*d^2)*log(-(c*x + I)/(c*x - I)))/(c^2*x^2 + 1
), x)
```

## Sympy [F(-1)]

Timed out.

$$\int (d+icdx)^2(a+b\arctan(cx))^3\,dx = \text{Timed out}$$

input
```
integrate((d+I*c*d*x)**2*(a+b*atan(c*x))**3,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int (d+icdx)^2(a+b\arctan(cx))^3\,dx = \int (i\,cdx+d)^2(b\arctan(cx)+a)^3\,dx$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))^3,x, algorithm="maxima")
```

output
```
-1/3*a^3*c^2*d^2*x^3 - 28*b^3*c^4*d^2*integrate(1/32*x^4*arctan(c*x)^3/(c^
2*x^2 + 1), x) - 3*b^3*c^4*d^2*integrate(1/32*x^4*arctan(c*x)*log(c^2*x^2
+ 1)^2/(c^2*x^2 + 1), x) - 96*a*b^2*c^4*d^2*integrate(1/32*x^4*arctan(c*x)
^2/(c^2*x^2 + 1), x) - 4*b^3*c^4*d^2*integrate(1/32*x^4*arctan(c*x)*log(c^
2*x^2 + 1)/(c^2*x^2 + 1), x) + 12*b^3*c^3*d^2*integrate(1/32*x^3*arctan(c*
x)^2*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) - b^3*c^3*d^2*integrate(1/32*x^3*l
og(c^2*x^2 + 1)^3/(c^2*x^2 + 1), x) + 16*b^3*c^3*d^2*integrate(1/32*x^3*ar
ctan(c*x)^2/(c^2*x^2 + 1), x) - 4*b^3*c^3*d^2*integrate(1/32*x^3*log(c^2*x
^2 + 1)^2/(c^2*x^2 + 1), x) - 1/2*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^
2*x^2 + 1)/c^4))*a^2*b*c^2*d^2 + I*a^3*c*d^2*x^2 + 7/32*b^3*d^2*arctan(c*x
)^4/c + 24*b^3*c^2*d^2*integrate(1/32*x^2*arctan(c*x)*log(c^2*x^2 + 1)/(c^
2*x^2 + 1), x) + 3*I*(x^2*arctan(c*x) - c*(x/c^2 - arctan(c*x)/c^3))*a^2*b
*c*d^2 + a*b^2*d^2*arctan(c*x)^3/c + 12*b^3*c*d^2*integrate(1/32*x*arctan(
c*x)^2*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) - b^3*c*d^2*integrate(1/32*x*log
(c^2*x^2 + 1)^3/(c^2*x^2 + 1), x) - 12*b^3*c*d^2*integrate(1/32*x*arctan(c
*x)^2/(c^2*x^2 + 1), x) + 3*b^3*c*d^2*integrate(1/32*x*log(c^2*x^2 + 1)^2/
(c^2*x^2 + 1), x) + a^3*d^2*x + 3*b^3*d^2*integrate(1/32*arctan(c*x)*log(c
^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + 3/2*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1
))*a^2*b*d^2/c - 1/24*(b^3*c^2*d^2*x^3 - 3*I*b^3*c*d^2*x^2 - 3*b^3*d^2*x)*
arctan(c*x)^3 + 1/16*(-I*b^3*c^2*d^2*x^3 - 3*b^3*c*d^2*x^2 + 3*I*b^3*d^...
```

## Giac [F]

$$\int (d+icdx)^2(a+b\arctan(cx))^3\,dx=\int (i\,cdx+d)^2(b\arctan{(cx)}+a)^3\,dx$$

input
```
integrate((d+I*c*d*x)^2*(a+b*arctan(c*x))^3,x, algorithm="giac")
```

output
```
integrate((I*c*d*x + d)^2*(b*arctan(c*x) + a)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (d+icdx)^2(a+b\arctan(cx))^3\,dx=\int (a+b\,\mathrm{atan}(c\,x))^3\,(d+c\,d\,x\,1\mathrm{i})^2\,dx$$

input
```
int((a + b*atan(c*x))^3*(d + c*d*x*1i)^2,x)
```

output
```
int((a + b*atan(c*x))^3*(d + c*d*x*1i)^2, x)
```

## Reduce [F]

$$\int (d+icdx)^2(a+b\arctan(cx))^3\,dx = \frac{d^2\left(18atan(cx)\,a^2bcx-48\left(\int \frac{atan(cx)x}{c^2x^2+1}dx\right)a\,b^2c^2+36\left(\int \frac{atan(cx)x}{c^2x^2+1}dx\right)b^3c^2i+18atan(cx)^2\,a\,b^2c^2i\,x^2+18a\cdots\right.}{\cdots}$$

input
```
int((d+I*c*d*x)^2*(a+b*atan(c*x))^3,x)
```

output
```
(d**2*( - 2*atan(c*x)**3*b**3*c**3*x**3 + 6*atan(c*x)**3*b**3*c**2*i*x**2
+ 6*atan(c*x)**3*b**3*c*x + 6*atan(c*x)**3*b**3*i - 6*atan(c*x)**2*a*b**2*
c**3*x**3 + 18*atan(c*x)**2*a*b**2*c**2*i*x**2 + 18*atan(c*x)**2*a*b**2*c*
x + 18*atan(c*x)**2*a*b**2*i + 3*atan(c*x)**2*b**3*c**2*x**2 - 18*atan(c*x
)**2*b**3*c*i*x + 3*atan(c*x)**2*b**3 - 6*atan(c*x)*a**2*b*c**3*x**3 + 18*
atan(c*x)*a**2*b*c**2*i*x**2 + 18*atan(c*x)*a**2*b*c*x + 18*atan(c*x)*a**2
*b*i + 6*atan(c*x)*a*b**2*c**2*x**2 - 36*atan(c*x)*a*b**2*c*i*x + 6*atan(c
*x)*a*b**2 - 6*atan(c*x)*b**3*c*x - 48*int((atan(c*x)*x)/(c**2*x**2 + 1),x
)*a*b**2*c**2 + 36*int((atan(c*x)*x)/(c**2*x**2 + 1),x)*b**3*c**2*i - 24*i
nt((atan(c*x)**2*x)/(c**2*x**2 + 1),x)*b**3*c**2 - 12*log(c**2*x**2 + 1)*a
**2*b + 18*log(c**2*x**2 + 1)*a*b**2*i + 3*log(c**2*x**2 + 1)*b**3 - 2*a**
3*c**3*x**3 + 6*a**3*c**2*i*x**2 + 6*a**3*c*x + 3*a**2*b*c**2*x**2 - 18*a*
*2*b*c*i*x - 6*a*b**2*c*x))/(6*c)
```

# 3.122 $\int (d+icdx)(a+b\arctan(cx))^3\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 220

$$\int (d+icdx)(a+b\arctan(cx))^3\,dx = \frac{3bd(a+b\arctan(cx))^2}{2c} - \frac{3}{2}ibdx(a+b\arctan(cx))^2 - \frac{id(1+icx)^2(a+b\arctan(cx))^3}{2c} + \frac{3bd(a+b\arctan(cx))^2\log\left(\frac{2}{1-icx}\right)}{c} - \frac{3ib^2d(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c} - \frac{3ib^2d(a+b\arctan(cx))\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{c} + \frac{3b^3d\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2c} + \frac{3b^3d\,\mathrm{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{2c}$$

output
```
3/2*b*d*(a+b*arctan(c*x))^2/c-3/2*I*b*d*x*(a+b*arctan(c*x))^2-1/2*I*d*(1+I
*c*x)^2*(a+b*arctan(c*x))^3/c+3*b*d*(a+b*arctan(c*x))^2*ln(2/(1-I*c*x))/c-
3*I*b^2*d*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c-3*I*b^2*d*(a+b*arctan(c*x))*
polylog(2,1-2/(1-I*c*x))/c+3/2*b^3*d*polylog(2,1-2/(1+I*c*x))/c+3/2*b^3*d*
polylog(3,1-2/(1-I*c*x))/c
```

## Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.67

$$\int (d+icdx)(a+b\arctan(cx))^3\,dx$$
$$= \frac{id(-2ia^3cx-3a^2bcx+a^3c^2x^2+3a^2b\arctan(cx)-6ia^2bcx\arctan(cx)-6ab^2cx\arctan(cx)+3a^2bc^2x^2}{}$$

input
```
Integrate[(d + I*c*d*x)*(a + b*ArcTan[c*x])^3,x]
```

output
```
((I/2)*d*((-2*I)*a^3*c*x - 3*a^2*b*c*x + a^3*c^2*x^2 + 3*a^2*b*ArcTan[c*x]
 - (6*I)*a^2*b*c*x*ArcTan[c*x] - 6*a*b^2*c*x*ArcTan[c*x] + 3*a^2*b*c^2*x^2
*ArcTan[c*x] - 3*a*b^2*ArcTan[c*x]^2 + (3*I)*b^3*ArcTan[c*x]^2 - (6*I)*a*b
^2*c*x*ArcTan[c*x]^2 - 3*b^3*c*x*ArcTan[c*x]^2 + 3*a*b^2*c^2*x^2*ArcTan[c*
x]^2 - b^3*ArcTan[c*x]^3 - (2*I)*b^3*c*x*ArcTan[c*x]^3 + b^3*c^2*x^2*ArcTa
n[c*x]^3 - (12*I)*a*b^2*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] - 6*b^3
*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] - (6*I)*b^3*ArcTan[c*x]^2*Log[
1 + E^((2*I)*ArcTan[c*x])] + (3*I)*a^2*b*Log[1 + c^2*x^2] + 3*a*b^2*Log[1
+ c^2*x^2] - 3*b^2*(2*a - I*b + 2*b*ArcTan[c*x])*PolyLog[2, -E^((2*I)*ArcT
an[c*x])] - (3*I)*b^3*PolyLog[3, -E^((2*I)*ArcTan[c*x])]))/c
```

## Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.05, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {5389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+icdx)(a+b\arctan(cx))^3\,dx$$
$$\Big\downarrow \; 5389$$
$$\frac{3ib\int \left(-d^2(a+b\arctan(cx))^2-\frac{2id^2(i-cx)(a+b\arctan(cx))^2}{c^2x^2+1}\right)dx}{2d}-\frac{id(1+icx)^2(a+b\arctan(cx))^3}{2c}$$

$$\Big\downarrow \; 2009$$

$$\frac{3ib\left(-\frac{2bd^2\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)(a+b\arctan(cx))}{c}+d^2(-x)(a+b\arctan(cx))^2-\frac{id^2(a+b\arctan(cx))^2}{c}-\frac{2id^2\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))}{c}\right.}{2d}$$

$$\frac{id(1+icx)^2(a+b\arctan(cx))^3}{2c}$$

input
```
Int[(d + I*c*d*x)*(a + b*ArcTan[c*x])^3,x]
```

output
```
((-1/2*I)*d*(1 + I*c*x)^2*(a + b*ArcTan[c*x])^3)/c + (((3*I)/2)*b*(((-I)*d
^2*(a + b*ArcTan[c*x])^2)/c - d^2*x*(a + b*ArcTan[c*x])^2 - ((2*I)*d^2*(a
+ b*ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/c - (2*b*d^2*(a + b*ArcTan[c*x])*Lo
g[2/(1 + I*c*x)])/c - (2*b*d^2*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 - I
*c*x)])/c - (I*b^2*d^2*PolyLog[2, 1 - 2/(1 + I*c*x)])/c - (I*b^2*d^2*PolyL
og[3, 1 - 2/(1 - I*c*x)])/c))/d
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5389
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - S
imp[b*c*(p/(e*(q + 1)))   Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1),
(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 12.29 (sec) , antiderivative size = 3777, normalized size of antiderivative = 17.17

| method | result | size |
|---|---|---|
| derivativedivides | Expression too large to display | 3777 |
| default | Expression too large to display | 3777 |
| parts | Expression too large to display | 3779 |

input
```
int((d+I*c*d*x)*(a+b*arctan(c*x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/c*(-I*d*a^3*(-1/2*c^2*x^2+I*c*x)+d*b^3*(1/2*I*arctan(c*x)^3*c^2*x^2+arct
an(c*x)^3*c*x+3/2*I*(-c*x*arctan(c*x)^2-1/3*arctan(c*x)^3-arctan(c*x)*ln(1
+(1+I*c*x)^2/(c^2*x^2+1))+1/2*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*
x)^2/(c^2*x^2+1))^2)^3*(I*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+
I*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+dilog(1+I*(1+I*c*x)/(c^2
*x^2+1)^(1/2))+dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2)))-1/4*Pi*csgn(I*(1+I*
c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1))^2)^3*(2*I*arctan(c*x)*ln(1+
(1+I*c*x)^2/(c^2*x^2+1))+2*arctan(c*x)^2+polylog(2,-(1+I*c*x)^2/(c^2*x^2+1
)))+1/2*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^3*(I*arctan(c*x)*ln(1+I*(1+I*c*
x)/(c^2*x^2+1)^(1/2))+I*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+di
log(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2)
))+1/4*Pi*csgn(I*(1+(1+I*c*x)^2/(c^2*x^2+1))^2)^3*(2*I*arctan(c*x)*ln(1+(1
+I*c*x)^2/(c^2*x^2+1))+2*arctan(c*x)^2+polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))
)-1/4*Pi*csgn(I*(1+I*c*x)^2/(c^2*x^2+1))^3*(2*I*arctan(c*x)*ln(1+(1+I*c*x)
^2/(c^2*x^2+1))+2*arctan(c*x)^2+polylog(2,-(1+I*c*x)^2/(c^2*x^2+1)))-I*ln(
2)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))+2*I*ln(2)*dilog(1+I*(1+I*c*x)/(c^2*
x^2+1)^(1/2))+2*I*ln(2)*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-2*I*arctan(
c*x)^2*ln((1+I*c*x)/(c^2*x^2+1)^(1/2))+2*ln(2)*arctan(c*x)*ln(1+(1+I*c*x)^
2/(c^2*x^2+1))-2*ln(2)*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-2*l
n(2)*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-2*I*ln(2)*arctan(c...
```

## Fricas [F]

$$\int (d + icdx)(a + b \arctan(cx))^3 \, dx = \int (i \, cdx + d)(b \arctan(cx) + a)^3 \, dx$$

input
```
integrate((d+I*c*d*x)*(a+b*arctan(c*x))^3,x, algorithm="fricas")
```

output
```
1/16*(b^3*c*d*x^2 - 2*I*b^3*d*x)*log(-(c*x + I)/(c*x - I))^3 + integral(1/
8*(8*I*a^3*c^3*d*x^3 + 8*a^3*c^2*d*x^2 + 8*I*a^3*c*d*x + 8*a^3*d - 3*(2*I*
a*b^2*c^3*d*x^3 + (2*a*b^2 - I*b^3)*c^2*d*x^2 + 2*a*b^2*d + 2*(I*a*b^2 - b
^3)*c*d*x)*log(-(c*x + I)/(c*x - I))^2 - 12*(a^2*b*c^3*d*x^3 - I*a^2*b*c^2
*d*x^2 + a^2*b*c*d*x - I*a^2*b*d)*log(-(c*x + I)/(c*x - I)))/(c^2*x^2 + 1)
, x)
```

## Sympy [F(-1)]

Timed out.

$$\int (d+icdx)(a+b\arctan(cx))^3\,dx=\text{Timed out}$$

input
```
integrate((d+I*c*d*x)*(a+b*atan(c*x))**3,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int (d+icdx)(a+b\arctan(cx))^3\,dx=\int (i\,cdx+d)(b\arctan(cx)+a)^3\,dx$$

input
```
integrate((d+I*c*d*x)*(a+b*arctan(c*x))^3,x, algorithm="maxima")
```

output
```
12*b^3*c^3*d*integrate(1/64*x^3*arctan(c*x)^2*log(c^2*x^2 + 1)/(c^2*x^2 +
1), x) - b^3*c^3*d*integrate(1/64*x^3*log(c^2*x^2 + 1)^3/(c^2*x^2 + 1), x)
 + 12*b^3*c^3*d*integrate(1/64*x^3*arctan(c*x)^2/(c^2*x^2 + 1), x) - 3*b^3
*c^3*d*integrate(1/64*x^3*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + 1/2*I*a^3
*c*d*x^2 + 7/32*b^3*d*arctan(c*x)^4/c + 56*b^3*c^2*d*integrate(1/64*x^2*ar
ctan(c*x)^3/(c^2*x^2 + 1), x) + 6*b^3*c^2*d*integrate(1/64*x^2*arctan(c*x)
*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + 192*a*b^2*c^2*d*integrate(1/64*x^2
*arctan(c*x)^2/(c^2*x^2 + 1), x) + 36*b^3*c^2*d*integrate(1/64*x^2*arctan(
c*x)*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) + 3/2*I*(x^2*arctan(c*x) - c*(x/c^
2 - arctan(c*x)/c^3))*a^2*b*c*d + a*b^2*d*arctan(c*x)^3/c + 12*b^3*c*d*int
egrate(1/64*x*arctan(c*x)^2*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) - b^3*c*d*i
ntegrate(1/64*x*log(c^2*x^2 + 1)^3/(c^2*x^2 + 1), x) - 24*b^3*c*d*integrat
e(1/64*x*arctan(c*x)^2/(c^2*x^2 + 1), x) + 6*b^3*c*d*integrate(1/64*x*log(
c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + a^3*d*x + 6*b^3*d*integrate(1/64*arctan
(c*x)*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + 3/2*(2*c*x*arctan(c*x) - log(
c^2*x^2 + 1))*a^2*b*d/c + 1/16*(I*b^3*c*d*x^2 + 2*b^3*d*x)*arctan(c*x)^3 -
 3/32*(b^3*c*d*x^2 - 2*I*b^3*d*x)*arctan(c*x)^2*log(c^2*x^2 + 1) + 3/64*(-
I*b^3*c*d*x^2 - 2*b^3*d*x)*arctan(c*x)*log(c^2*x^2 + 1)^2 + 1/128*(b^3*c*d
*x^2 - 2*I*b^3*d*x)*log(c^2*x^2 + 1)^3 + I*integrate(1/64*(56*(b^3*c^3*d*x
^3 + b^3*c*d*x)*arctan(c*x)^3 + (b^3*c^2*d*x^2 + b^3*d)*log(c^2*x^2 + 1...
```

## Giac [F]

$$\int (d+icdx)(a+b\arctan(cx))^3\,dx=\int (i\,cdx+d)(b\arctan\left(cx\right)+a)^3\,dx$$

input
```
integrate((d+I*c*d*x)*(a+b*arctan(c*x))^3,x, algorithm="giac")
```

output
```
integrate((I*c*d*x + d)*(b*arctan(c*x) + a)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (d+icdx)(a+b\arctan(cx))^3\,dx = \int (a+b\,\mathrm{atan}(c\,x))^3\,(d+c\,d\,x\,1\mathrm{i})\,dx$$

input
```
int((a + b*atan(c*x))^3*(d + c*d*x*1i),x)
```

output
```
int((a + b*atan(c*x))^3*(d + c*d*x*1i), x)
```

## Reduce [F]

$$\int (d+icdx)(a+b\arctan(cx))^3\,dx$$
$$= \frac{d\Big(atan(cx)^3\,b^3c^2i\,x^2 + 2atan(cx)^3\,b^3cx + atan(cx)^3\,b^3i + 3atan(cx)^2\,a\,b^2c^2i\,x^2 + 6atan(cx)^2\,a\,b^2cx + 3a}{}$$

input
```
int((d+I*c*d*x)*(a+b*atan(c*x))^3,x)
```

output
```
(d*(atan(c*x)**3*b**3*c**2*i*x**2 + 2*atan(c*x)**3*b**3*c*x + atan(c*x)**3
*b**3*i + 3*atan(c*x)**2*a*b**2*c**2*i*x**2 + 6*atan(c*x)**2*a*b**2*c*x +
3*atan(c*x)**2*a*b**2*i - 3*atan(c*x)**2*b**3*c*i*x + 3*atan(c*x)*a**2*b*c
**2*i*x**2 + 6*atan(c*x)*a**2*b*c*x + 3*atan(c*x)*a**2*b*i - 6*atan(c*x)*a
*b**2*c*i*x - 12*int((atan(c*x)*x)/(c**2*x**2 + 1),x)*a*b**2*c**2 + 6*int(
(atan(c*x)*x)/(c**2*x**2 + 1),x)*b**3*c**2*i - 6*int((atan(c*x)**2*x)/(c**
2*x**2 + 1),x)*b**3*c**2 - 3*log(c**2*x**2 + 1)*a**2*b + 3*log(c**2*x**2 +
 1)*a*b**2*i + a**3*c**2*i*x**2 + 2*a**3*c*x - 3*a**2*b*c*i*x))/(2*c)
```

# 3.123 $\int \frac{(a+b\arctan(cx))^3}{d+icdx}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 139

$$\int \frac{(a+b\arctan(cx))^3}{d+icdx}\,dx = \frac{i(a+b\arctan(cx))^3\log\left(\frac{2}{1+icx}\right)}{cd} - \frac{3b(a+b\arctan(cx))^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2cd} + \frac{3ib^2(a+b\arctan(cx))\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2cd} + \frac{3b^3\operatorname{PolyLog}\left(4,1-\frac{2}{1+icx}\right)}{4cd}$$

output
```
I*(a+b*arctan(c*x))^3*ln(2/(1+I*c*x))/c/d-3/2*b*(a+b*arctan(c*x))^2*polylo
g(2,1-2/(1+I*c*x))/c/d+3/2*I*b^2*(a+b*arctan(c*x))*polylog(3,1-2/(1+I*c*x)
)/c/d+3/4*b^3*polylog(4,1-2/(1+I*c*x))/c/d
```

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.96

$$\int \frac{(a+b\arctan(cx))^3}{d+icdx}\,dx$$
$$= \frac{i\left(4(a+b\arctan(cx))^3\log\left(\frac{2d}{d+icdx}\right)+3ib\left(2(a+b\arctan(cx))^2\operatorname{PolyLog}\left(2,\frac{i+cx}{-i+cx}\right)-b\left(2i(a+b\arctan(cx\right.\right.\right.}{4cd}$$

input
```
Integrate[(a + b*ArcTan[c*x])^3/(d + I*c*d*x),x]
```

output
```
((I/4)*(4*(a + b*ArcTan[c*x])^3*Log[(2*d)/(d + I*c*d*x)] + (3*I)*b*(2*(a +
 b*ArcTan[c*x])^2*PolyLog[2, (I + c*x)/(-I + c*x)] - b*((2*I)*(a + b*ArcTa
n[c*x])*PolyLog[3, (I + c*x)/(-I + c*x)] + b*PolyLog[4, (I + c*x)/(-I + c*
x)]))))/(c*d)
```

## Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {5379, 5529, 5533, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^3}{d+icdx}\,dx$$
$$\Big\downarrow \ 5379$$
$$\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^3}{cd}-\frac{3ib\int \frac{(a+b\arctan(cx))^2\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx}{d}$$
$$\Big\downarrow \ 5529$$

$$\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^3}{cd}-\frac{3ib\left(ib\int\frac{(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{c^2x^2+1}dx-\frac{i\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))^2}{2c}\right)}{d}$$

$$\Big\downarrow \text{ 5533}$$

$$\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^3}{cd}-\frac{3ib\left(ib\left(\frac{i\operatorname{PolyLog}\left(3,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{2c}-\frac{1}{2}ib\int\frac{\operatorname{PolyLog}\left(3,1-\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)-\frac{i\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))^2}{2c}\right)}{d}$$

$$\Big\downarrow \text{ 7164}$$

$$\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^3}{cd}-\frac{3ib\left(ib\left(\frac{i\operatorname{PolyLog}\left(3,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{2c}+\frac{b\operatorname{PolyLog}\left(4,1-\frac{2}{icx+1}\right)}{4c}\right)-\frac{i\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))^2}{2c}\right)}{d}$$

input
```
Int[(a + b*ArcTan[c*x])^3/(d + I*c*d*x),x]
```

output
```
(I*(a + b*ArcTan[c*x])^3*Log[2/(1 + I*c*x)])/(c*d) - ((3*I)*b*(((-1/2*I)*(
a + b*ArcTan[c*x])^2*PolyLog[2, 1 - 2/(1 + I*c*x)])/c + I*b*(((I/2)*(a + b
*ArcTan[c*x])*PolyLog[3, 1 - 2/(1 + I*c*x)])/c + (b*PolyLog[4, 1 - 2/(1 +
I*c*x)])/(4*c))))/d
```

## Defintions of rubi rules used

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 5533
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k + 1
, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &&
EqQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 11.05 (sec) , antiderivative size = 1629, normalized size of antiderivative = 11.72

| method | result | size |
|---|---|---|
| derivativedivides | Expression too large to display | 1629 |
| default | Expression too large to display | 1629 |
| parts | Expression too large to display | 1640 |

input
```
int((a+b*arctan(c*x))^3/(d+I*c*d*x),x,method=_RETURNVERBOSE)
```

output
```
1/c*(-1/2*I*a^3/d*ln(c^2*x^2+1)+a^3/d*arctan(c*x)+b^3/d*(-I*ln(1+I*c*x)*ar
ctan(c*x)^3+3*I*(1/3*arctan(c*x)^3*ln(2*I*(1+I*c*x)^2/(c^2*x^2+1))-1/6*I*a
rctan(c*x)^4+1/6*I*Pi*(csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^
2+1)))^3+csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn((1+I*c*x)^2/(c^2*x^2+1)/
(1+(1+I*c*x)^2/(c^2*x^2+1)))^2-csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x
)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2-csgn((1+I*c*x)^2/(c^2*x^2+1
)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)
^2/(c^2*x^2+1)))^2-csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn((1+I*c*x)^2/(c
^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))-csgn(
I*(1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^3+csgn((1+I*c*x)^2/
(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/(1
+(1+I*c*x)^2/(c^2*x^2+1)))+csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(
c^2*x^2+1)))^2-1)*arctan(c*x)^3-1/2*I*arctan(c*x)^2*polylog(2,-(1+I*c*x)^2
/(c^2*x^2+1))+1/2*arctan(c*x)*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))+1/4*I*po
lylog(4,-(1+I*c*x)^2/(c^2*x^2+1))))+3*a*b^2/d*(-I*ln(1+I*c*x)*arctan(c*x)^
2+2*I*(1/2*arctan(c*x)^2*ln(2*I*(1+I*c*x)^2/(c^2*x^2+1))+1/4*I*Pi*(csgn((1
+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^3+csgn(I/(1+(1+I*c*x)^2
/(c^2*x^2+1)))*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2
-csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2
/(c^2*x^2+1)))^2-csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1...
```

## Fricas [F]

$$\int \frac{(a+b\arctan(cx))^3}{d+icdx}\,dx = \int \frac{(b\arctan(cx)+a)^3}{i\,cdx+d}\,dx$$

input
```
integrate((a+b*arctan(c*x))^3/(d+I*c*d*x),x, algorithm="fricas")
```

output
```
integral(-1/8*(b^3*log(-(c*x + I)/(c*x - I))^3 - 6*I*a*b^2*log(-(c*x + I)/
(c*x - I))^2 - 12*a^2*b*log(-(c*x + I)/(c*x - I)) + 8*I*a^3)/(c*d*x - I*d)
, x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^3}{d+icdx}\,dx = \text{Timed out}$$

input
```
integrate((a+b*atan(c*x))**3/(d+I*c*d*x),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(a+b\arctan(cx))^3}{d+icdx}\,dx = \int \frac{(b\arctan(cx)+a)^3}{i\,cdx+d}\,dx$$

input
```
integrate((a+b*arctan(c*x))^3/(d+I*c*d*x),x, algorithm="maxima")
```

output
```
-I*a^3*log(I*c*d*x + d)/(c*d) + 1/128*(16*b^3*arctan(c*x)^4 + 16*I*b^3*arc
tan(c*x)^3*log(c^2*x^2 + 1) + 4*I*b^3*arctan(c*x)*log(c^2*x^2 + 1)^3 - b^3
*log(c^2*x^2 + 1)^4 + 16*(b^3*arctan(c*x)^4/(c*d) + 8*b^3*c*integrate(1/16
*x*log(c^2*x^2 + 1)^3/(c^2*d*x^2 + d), x) + 8*a*b^2*arctan(c*x)^3/(c*d) +
12*a^2*b*arctan(c*x)^2/(c*d))*c*d - 128*I*c*d*integrate(1/32*(40*b^3*c*x*a
rctan(c*x)^3 + 6*b^3*c*x*arctan(c*x)*log(c^2*x^2 + 1)^2 + 96*a*b^2*c*x*arc
tan(c*x)^2 + 96*a^2*b*c*x*arctan(c*x) + 12*b^3*arctan(c*x)^2*log(c^2*x^2 +
 1) + b^3*log(c^2*x^2 + 1)^3)/(c^2*d*x^2 + d), x))/(c*d)
```

## Giac [F]

$$\int \frac{(a+b\arctan(cx))^3}{d+icdx}\,dx = \int \frac{(b\arctan(cx)+a)^3}{i\,cdx+d}\,dx$$

input
```
integrate((a+b*arctan(c*x))^3/(d+I*c*d*x),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)^3/(I*c*d*x + d), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^3}{d+icdx}\,dx=\int \frac{(a+b\,\mathrm{atan}(c\,x))^3}{d+c\,d\,x\,1\mathrm{i}}\,dx$$

input
```
int((a + b*atan(c*x))^3/(d + c*d*x*1i),x)
```

output
```
int((a + b*atan(c*x))^3/(d + c*d*x*1i), x)
```

## Reduce [F]

$$\int \frac{(a+b\arctan(cx))^3}{d+icdx}\,dx = \frac{3\left(\int \frac{atan(cx)}{cix+1}dx\right)a^2bc+\left(\int \frac{atan(cx)^3}{cix+1}dx\right)b^3c+3\left(\int \frac{atan(cx)^2}{cix+1}dx\right)a\,b^2c-\log(cix+1)\,a^3i}{cd}$$

input
```
int((a+b*atan(c*x))^3/(d+I*c*d*x),x)
```

output
```
(3*int(atan(c*x)/(c*i*x + 1),x)*a**2*b*c + int(atan(c*x)**3/(c*i*x + 1),x)
*b**3*c + 3*int(atan(c*x)**2/(c*i*x + 1),x)*a*b**2*c - log(c*i*x + 1)*a**3
*i)/(c*d)
```

# 3.124 $\int \frac{(a+b\arctan(cx))^3}{(d+icdx)^2}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 182

$$\int \frac{(a+b\arctan(cx))^3}{(d+icdx)^2}\,dx = -\frac{3ib^3}{4cd^2(i-cx)}+\frac{3ib^3\arctan(cx)}{4cd^2}+\frac{3b^2(a+b\arctan(cx))}{2cd^2(i-cx)} - \frac{3b(a+b\arctan(cx))^2}{4cd^2}+\frac{3ib(a+b\arctan(cx))^2}{2cd^2(i-cx)} - \frac{i(a+b\arctan(cx))^3}{2cd^2}+\frac{i(a+b\arctan(cx))^3}{cd^2(1+icx)}$$

output
```
-3/4*I*b^3/c/d^2/(I-c*x)+3/4*I*b^3*arctan(c*x)/c/d^2+3/2*b^2*(a+b*arctan(c
*x))/c/d^2/(I-c*x)-3/4*b*(a+b*arctan(c*x))^2/c/d^2+3/2*I*b*(a+b*arctan(c*x
))^2/c/d^2/(I-c*x)-1/2*I*(a+b*arctan(c*x))^3/c/d^2+I*(a+b*arctan(c*x))^3/c
/d^2/(1+I*c*x)
```

## Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.66

$$\int \frac{(a+b\arctan(cx))^3}{(d+icdx)^2}\,dx = \frac{4a^3-6ia^2b-6ab^2+3ib^3+3ib(-2a^2+2iab+b^2)(i+cx)\arctan(cx)-3b^2(2ia+b)(i+cx)\arctan(cx}{4cd^2(-i+cx)}$$

input
```
Integrate[(a + b*ArcTan[c*x])^3/(d + I*c*d*x)^2,x]
```

output
```
(4*a^3 - (6*I)*a^2*b - 6*a*b^2 + (3*I)*b^3 + (3*I)*b*(-2*a^2 + (2*I)*a*b +
 b^2)*(I + c*x)*ArcTan[c*x] - 3*b^2*((2*I)*a + b)*(I + c*x)*ArcTan[c*x]^2
+ 2*b^3*(1 - I*c*x)*ArcTan[c*x]^3)/(4*c*d^2*(-I + c*x))
```

## Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {5389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^3}{(d+icdx)^2}\,dx$$

$$\Big\downarrow \text{5389}$$

$$\frac{i(a+b\arctan(cx))^3}{cd^2(1+icx)} - \frac{3ib\int \left(\frac{(a+b\arctan(cx))^2}{2d(c^2x^2+1)} - \frac{(a+b\arctan(cx))^2}{2d(i-cx)^2}\right)dx}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{i(a+b\arctan(cx))^3}{cd^2(1+icx)} - \frac{3ib\left(\frac{(a+b\arctan(cx))^3}{6bcd} - \frac{i(a+b\arctan(cx))^2}{4cd} - \frac{(a+b\arctan(cx))^2}{2cd(-cx+i)} + \frac{ib(a+b\arctan(cx))}{2cd(-cx+i)} - \frac{b^2\arctan(cx)}{4cd} + \frac{b^2}{4cd(-cx+i)}\right)}{d}$$

input
```
Int[(a + b*ArcTan[c*x])^3/(d + I*c*d*x)^2,x]
```

output
```
(I*(a + b*ArcTan[c*x])^3)/(c*d^2*(1 + I*c*x)) - ((3*I)*b*(b^2/(4*c*d*(I -
c*x)) - (b^2*ArcTan[c*x])/(4*c*d) + ((I/2)*b*(a + b*ArcTan[c*x]))/(c*d*(I
- c*x)) - ((I/4)*(a + b*ArcTan[c*x])^2)/(c*d) - (a + b*ArcTan[c*x])^2/(2*c
*d*(I - c*x)) + (a + b*ArcTan[c*x])^3/(6*b*c*d)))/d
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5389
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - S
imp[b*c*(p/(e*(q + 1)))   Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1),
(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

## Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 336 vs. $2(161) = 322$.

Time = 3.37 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.85

| method | result |
|---|---|
| derivativedivides | $\frac{ia^3}{d^2(icx+1)} + \frac{b^3\left(\frac{i\arctan(cx)^3}{icx+1} - \frac{i\left(-2i\arctan(cx)^3+2\arctan(cx)^3cx-3i\arctan(cx)^2cx+3\arctan(cx)^2-3i\arctan(cx)-3cx\arctan(c\right.}{4(cx-i)}\right.}{d^2}$ |
| default | $\frac{ia^3}{d^2(icx+1)} + \frac{b^3\left(\frac{i\arctan(cx)^3}{icx+1} - \frac{i\left(-2i\arctan(cx)^3+2\arctan(cx)^3cx-3i\arctan(cx)^2cx+3\arctan(cx)^2-3i\arctan(cx)-3cx\arctan(c\right.}{4(cx-i)}\right.}{d^2}$ |
| parts | $\frac{ia^3}{d^2(icx+1)c} + \frac{b^3\left(\frac{i\arctan(cx)^3}{icx+1} - \frac{i\left(-2i\arctan(cx)^3+2\arctan(cx)^3cx-3i\arctan(cx)^2cx+3\arctan(cx)^2-3i\arctan(cx)-3cx\,\mathrm{arc}\right.}{4(cx-i)}\right.}{d^2c}$ |
| risch | $\frac{(b^3cx+ib^3)\ln(icx+1)^3}{16d^2(cx-i)c} + \frac{3ib^2(i\ln(-icx+1)bcx-ibcx+2acx+2ia-b\ln(-icx+1)+b)\ln(icx+1)^2}{16d^2(cx-i)c} - \frac{3ib\left(ib^2cx\ln(-\right.}{}$ |

input
```
int((a+b*arctan(c*x))^3/(d+I*c*d*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*(I*a^3/d^2/(1+I*c*x)+b^3/d^2*(I/(1+I*c*x)*arctan(c*x)^3-1/4*I*(-2*I*ar
ctan(c*x)^3+2*arctan(c*x)^3*c*x-3*I*arctan(c*x)^2*c*x+3*arctan(c*x)^2-3*I*
arctan(c*x)-3*c*x*arctan(c*x)-3)/(c*x-I))+3*a*b^2/d^2*(I/(1+I*c*x)*arctan(
c*x)^2-2*I*(-1/4*I*arctan(c*x)*ln(c*x-I)+1/2*arctan(c*x)/(c*x-I)+1/4*I*arc
tan(c*x)*ln(c*x+I)+1/16*ln(c*x-I)^2-1/8*ln(c*x-I)*ln(-1/2*I*(c*x+I))-1/4*I
*arctan(c*x)-1/4*I/(c*x-I)+1/16*ln(c*x+I)^2-1/8*(ln(c*x+I)-ln(-1/2*I*(c*x+
I)))*ln(-1/2*I*(-c*x+I))))+3*I*a^2*b/d^2/(1+I*c*x)*arctan(c*x)-3/2*I*a^2*b
/d^2*arctan(c*x)-3/2*I*a^2*b/d^2/(c*x-I))
```

## Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.96

$$\int \frac{(a+b\arctan(cx))^3}{(d+icdx)^2}\,dx = -\frac{(b^3cx+i\,b^3)\log\left(-\frac{cx+i}{cx-i}\right)^3-16\,a^3+24i\,a^2b+24\,ab^2-12i\,b^3+3\,(2\,ab^2-i\,b^3+(-2i\,ab^2-b^3)cx)\log\left(\right.}{16\,(c^2d^2x-i\,cd^2)}$$

input
```
integrate((a+b*arctan(c*x))^3/(d+I*c*d*x)^2,x, algorithm="fricas")
```

output
```
-1/16*((b^3*c*x + I*b^3)*log(-(c*x + I)/(c*x - I))^3 - 16*a^3 + 24*I*a^2*b
 + 24*a*b^2 - 12*I*b^3 + 3*(2*a*b^2 - I*b^3 + (-2*I*a*b^2 - b^3)*c*x)*log(
-(c*x + I)/(c*x - I))^2 + 6*(-2*I*a^2*b - 2*a*b^2 + I*b^3 - (2*a^2*b - 2*I
*a*b^2 - b^3)*c*x)*log(-(c*x + I)/(c*x - I)))/(c^2*d^2*x - I*c*d^2)
```

## Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 631 vs. 2(151) = 302.

Time = 16.56 (sec) , antiderivative size = 631, normalized size of antiderivative = 3.47

$$\int \frac{(a+b\arctan(cx))^3}{(d+icdx)^2}\,dx$$
$$= \frac{3ib(a(1-i)-b)\left(a(1-i)-ib\right)\log\left(-\frac{3b(a(1-i)-b)(a(1-i)-ib)}{c}+x(6a^2b-6iab^2-3b^3)\right)}{8cd^2}$$
$$-\frac{3ib(a(1-i)-b)\left(a(1-i)-ib\right)\log\left(\frac{3b(a(1-i)-b)(a(1-i)-ib)}{c}+x(6a^2b-6iab^2-3b^3)\right)}{8cd^2}$$
$$+\frac{(-b^3cx-ib^3)\log\left(-icx+1\right)^3}{16c^2d^2x-16icd^2}+\frac{(b^3cx+ib^3)\log\left(icx+1\right)^3}{16c^2d^2x-16icd^2}$$
$$+\frac{(6iab^2cx-6ab^2+3b^3cx+3ib^3)\log\left(icx+1\right)^2}{16c^2d^2x-16icd^2}$$
$$+\frac{(6iab^2cx-6ab^2+3b^3cx\log\left(icx+1\right)+3b^3cx+3ib^3\log\left(icx+1\right)+3ib^3)\log\left(-icx+1\right)^2}{16c^2d^2x-16icd^2}$$
$$+\frac{(24ia^2b-12iab^2cx\log\left(icx+1\right)+12ab^2\log\left(icx+1\right)+24ab^2-3b^3cx\log\left(icx+1\right)^2-6b^3cx\log\left(icx\right.}{16c^2d^2x-16icd^2}$$
$$+\frac{(-6ia^2b-6ab^2+3ib^3)\log\left(icx+1\right)}{4c^2d^2x-4icd^2}-\frac{-4a^3+6ia^2b+6ab^2-3ib^3}{4c^2d^2x-4icd^2}$$

input
```
integrate((a+b*atan(c*x))**3/(d+I*c*d*x)**2,x)
```

output
```
3*I*b*(a*(1 - I) - b)*(a*(1 - I) - I*b)*log(-3*b*(a*(1 - I) - b)*(a*(1 - I
) - I*b)/c + x*(6*a**2*b - 6*I*a*b**2 - 3*b**3))/(8*c*d**2) - 3*I*b*(a*(1
- I) - b)*(a*(1 - I) - I*b)*log(3*b*(a*(1 - I) - b)*(a*(1 - I) - I*b)/c +
x*(6*a**2*b - 6*I*a*b**2 - 3*b**3))/(8*c*d**2) + (-b**3*c*x - I*b**3)*log(
-I*c*x + 1)**3/(16*c**2*d**2*x - 16*I*c*d**2) + (b**3*c*x + I*b**3)*log(I*
c*x + 1)**3/(16*c**2*d**2*x - 16*I*c*d**2) + (6*I*a*b**2*c*x - 6*a*b**2 +
3*b**3*c*x + 3*I*b**3)*log(I*c*x + 1)**2/(16*c**2*d**2*x - 16*I*c*d**2) +
(6*I*a*b**2*c*x - 6*a*b**2 + 3*b**3*c*x*log(I*c*x + 1) + 3*b**3*c*x + 3*I*
b**3*log(I*c*x + 1) + 3*I*b**3)*log(-I*c*x + 1)**2/(16*c**2*d**2*x - 16*I*
c*d**2) + (24*I*a**2*b - 12*I*a*b**2*c*x*log(I*c*x + 1) + 12*a*b**2*log(I*
c*x + 1) + 24*a*b**2 - 3*b**3*c*x*log(I*c*x + 1)**2 - 6*b**3*c*x*log(I*c*x
 + 1) - 3*I*b**3*log(I*c*x + 1)**2 - 6*I*b**3*log(I*c*x + 1) - 12*I*b**3)*
log(-I*c*x + 1)/(16*c**2*d**2*x - 16*I*c*d**2) + (-6*I*a**2*b - 6*a*b**2 +
 3*I*b**3)*log(I*c*x + 1)/(4*c**2*d**2*x - 4*I*c*d**2) - (-4*a**3 + 6*I*a*
*2*b + 6*a*b**2 - 3*I*b**3)/(4*c**2*d**2*x - 4*I*c*d**2)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\arctan(cx))^3}{(d+icdx)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a+b*arctan(c*x))^3/(d+I*c*d*x)^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.40

$$\int \frac{(a+b\arctan(cx))^3}{(d+icdx)^2}\,dx =$$

$$-\frac{\left(4i\,b^3\arctan\left(-\frac{i\,(i\,cdx+d)\left(\frac{d}{i\,cdx+d}-1\right)}{d}\right)^3 - 12i\,ab^2\arctan\left(-\frac{i\,(i\,cdx+d)\left(\frac{d}{i\,cdx+d}-1\right)}{d}\right)^2 - 6\,b^3\arctan\left(-\frac{i\,(i\,cd}{}\right.\right.}{}$$

input
```
integrate((a+b*arctan(c*x))^3/(d+I*c*d*x)^2,x, algorithm="giac")
```

output
```
-1/8*(4*I*b^3*arctan(-I*(I*c*d*x + d)*(d/(I*c*d*x + d) - 1)/d)^3 - 12*I*a*
b^2*arctan(-I*(I*c*d*x + d)*(d/(I*c*d*x + d) - 1)/d)^2 - 6*b^3*arctan(-I*(
I*c*d*x + d)*(d/(I*c*d*x + d) - 1)/d)^2 + 12*I*a^2*b*arctan(-I*(I*c*d*x +
d)*(d/(I*c*d*x + d) - 1)/d) + 12*a*b^2*arctan(-I*(I*c*d*x + d)*(d/(I*c*d*x
 + d) - 1)/d) - 6*I*b^3*arctan(-I*(I*c*d*x + d)*(d/(I*c*d*x + d) - 1)/d) -
 4*I*a^3 - 6*a^2*b + 6*I*a*b^2 + 3*b^3)*e^(2*I*arctan(-I*(I*c*d*x + d)*(d/
(I*c*d*x + d) - 1)/d))/(c*d^2)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^3}{(d+icdx)^2}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^3}{(d+c\,d\,x\,1\mathrm{i})^2}\,dx$$

input
```
int((a + b*atan(c*x))^3/(d + c*d*x*1i)^2,x)
```

output
```
int((a + b*atan(c*x))^3/(d + c*d*x*1i)^2, x)
```

## Reduce [F]

$$\int \frac{(a+b\arctan(cx))^3}{(d+icdx)^2}\,dx = \frac{-3\left(\int \frac{atan(cx)}{c^2x^2-2cix-1}dx\right)a^2b-\left(\int \frac{atan(cx)^3}{c^2x^2-2cix-1}dx\right)b^3-3\left(\int \frac{atan(cx)^2}{c^2x^2-2cix-1}dx\right)a\,b^2-\left(\int \frac{1}{c^2x^2-2cix-1}dx\right)a^3}{d^2}$$

input
```
int((a+b*atan(c*x))^3/(d+I*c*d*x)^2,x)
```

output
```
( - 3*int(atan(c*x)/(c**2*x**2 - 2*c*i*x - 1),x)*a**2*b - int(atan(c*x)**3
/(c**2*x**2 - 2*c*i*x - 1),x)*b**3 - 3*int(atan(c*x)**2/(c**2*x**2 - 2*c*i
*x - 1),x)*a*b**2 - int(1/(c**2*x**2 - 2*c*i*x - 1),x)*a**3)/d**2
```

# 3.125 $\int \frac{(a+b\arctan(cx))^3}{(d+icdx)^3}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 271

$$\begin{aligned}\int \frac{(a+b\arctan(cx))^3}{(d+icdx)^3}\,dx &= \frac{3b^3}{64cd^3(i-cx)^2}-\frac{21ib^3}{64cd^3(i-cx)} \\ &\quad+\frac{21ib^3\arctan(cx)}{64cd^3}+\frac{3ib^2(a+b\arctan(cx))}{16cd^3(i-cx)^2} \\ &\quad+\frac{9b^2(a+b\arctan(cx))}{16cd^3(i-cx)}-\frac{9b(a+b\arctan(cx))^2}{32cd^3} \\ &\quad-\frac{3b(a+b\arctan(cx))^2}{8cd^3(i-cx)^2}+\frac{3ib(a+b\arctan(cx))^2}{8cd^3(i-cx)} \\ &\quad-\frac{i(a+b\arctan(cx))^3}{8cd^3}+\frac{i(a+b\arctan(cx))^3}{2cd^3(1+icx)^2}\end{aligned}$$

output

```
3/64*b^3/c/d^3/(I-c*x)^2-21/64*I*b^3/c/d^3/(I-c*x)+21/64*I*b^3*arctan(c*x)
/c/d^3+3/16*I*b^2*(a+b*arctan(c*x))/c/d^3/(I-c*x)^2+9/16*b^2*(a+b*arctan(c
*x))/c/d^3/(I-c*x)-9/32*b*(a+b*arctan(c*x))^2/c/d^3-3/8*b*(a+b*arctan(c*x)
)^2/c/d^3/(I-c*x)^2+3/8*I*b*(a+b*arctan(c*x))^2/c/d^3/(I-c*x)-1/8*I*(a+b*a
rctan(c*x))^3/c/d^3+1/2*I*(a+b*arctan(c*x))^3/c/d^3/(1+I*c*x)^2
```

## Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.68

$$\int \frac{(a+b\arctan(cx))^3}{(d+icdx)^3}\,dx =$$
$$-\frac{i(32a^3+3b^3(8i-7cx)+12ab^2(-4-3icx)+24a^2b(-2i+cx)+3b(i+cx)\left(b^2(9i-7cx)+4ab(-5-\right.}{}$$

input
```
Integrate[(a + b*ArcTan[c*x])^3/(d + I*c*d*x)^3,x]
```

output
```
((-1/64*I)*(32*a^3 + 3*b^3*(8*I - 7*c*x) + 12*a*b^2*(-4 - (3*I)*c*x) + 24*
a^2*b*(-2*I + c*x) + 3*b*(I + c*x)*(b^2*(9*I - 7*c*x) + 4*a*b*(-5 - (3*I)*
c*x) + 8*a^2*(-3*I + c*x))*ArcTan[c*x] + 6*b^2*(I + c*x)*(b*(-5 - (3*I)*c*
x) + 4*a*(-3*I + c*x))*ArcTan[c*x]^2 + 8*b^3*(3 - (2*I)*c*x + c^2*x^2)*Arc
Tan[c*x]^3))/(c*d^3*(-I + c*x)^2)
```

## Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {5389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^3}{(d+icdx)^3}\,dx$$

$$\Big\downarrow \text{5389}$$

$$\frac{i(a+b\arctan(cx))^3}{2cd^3(1+icx)^2}-\frac{3ib\int \left(\frac{(a+b\arctan(cx))^2}{4d^2(c^2x^2+1)}-\frac{(a+b\arctan(cx))^2}{4d^2(i-cx)^2}-\frac{i(a+b\arctan(cx))^2}{2d^2(i-cx)^3}\right)dx}{2d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{i(a+b\arctan(cx))^3}{2cd^3(1+icx)^2} - \frac{3ib\left(\frac{(a+b\arctan(cx))^3}{12bcd^2} - \frac{(a+b\arctan(cx))^2}{4cd^2(-cx+i)} - \frac{3i(a+b\arctan(cx))^2}{16cd^2} - \frac{i(a+b\arctan(cx))^2}{4cd^2(-cx+i)^2} + \frac{3ib(a+b\arctan(cx))}{8cd^2(-cx+i)} - \frac{b(a+b\arctan(cx))}{8cd^2(-cx+i)^2} - \right.}{2d}$$

input
```
Int[(a + b*ArcTan[c*x])^3/(d + I*c*d*x)^3,x]
```

output
```
((I/2)*(a + b*ArcTan[c*x])^3)/(c*d^3*(1 + I*c*x)^2) - (((3*I)/2)*b*(((I/32
)*b^2)/(c*d^2*(I - c*x)^2) + (7*b^2)/(32*c*d^2*(I - c*x)) - (7*b^2*ArcTan[
c*x])/(32*c*d^2) - (b*(a + b*ArcTan[c*x]))/(8*c*d^2*(I - c*x)^2) + (((3*I)
/8)*b*(a + b*ArcTan[c*x]))/(c*d^2*(I - c*x)) - (((3*I)/16)*(a + b*ArcTan[c
*x])^2)/(c*d^2) - ((I/4)*(a + b*ArcTan[c*x])^2)/(c*d^2*(I - c*x)^2) - (a +
 b*ArcTan[c*x])^2/(4*c*d^2*(I - c*x)) + (a + b*ArcTan[c*x])^3/(12*b*c*d^2)
))/d
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5389
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - S
imp[b*c*(p/(e*(q + 1)))   Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1),
(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

## Maple [A] (verified)

Time = 4.78 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.57

| method | result |
|---|---|
| derivativedivides | $\frac{ia^3}{2d^3(icx+1)^2}+\frac{b^3\left(\frac{i\arctan(cx)^3}{2(icx+1)^2}-\frac{i\left(-21cx+24i-16i\arctan(cx)^3cx+8\arctan(cx)^3c^2x^2-8\arctan(cx)^3-18i\arctan(cx)^2c^2x^2-30i\right.}{64(cx-i)^2}\right.}{d^3}$ |
| default | $\frac{ia^3}{2d^3(icx+1)^2}+\frac{b^3\left(\frac{i\arctan(cx)^3}{2(icx+1)^2}-\frac{i\left(-21cx+24i-16i\arctan(cx)^3cx+8\arctan(cx)^3c^2x^2-8\arctan(cx)^3-18i\arctan(cx)^2c^2x^2-30i\right.}{64(cx-i)^2}\right.}{d^3}$ |
| parts | $\frac{ia^3}{2d^3(icx+1)^2c}+\frac{b^3\left(\frac{i\arctan(cx)^3}{2(icx+1)^2}-\frac{i\left(-21cx+24i-16i\arctan(cx)^3cx+8\arctan(cx)^3c^2x^2-8\arctan(cx)^3-18i\arctan(cx)^2c^2x^2\right.}{64(cx\right.}\right.}{d^3c}$ |
| risch | Expression too large to display |

input

```
int((a+b*arctan(c*x))^3/(d+I*c*d*x)^3,x,method=_RETURNVERBOSE)
```

output

```
1/c*(1/2*I*a^3/d^3/(1+I*c*x)^2+b^3/d^3*(1/2*I/(1+I*c*x)^2*arctan(c*x)^3-1/
64*I*(-21*c*x+24*I-16*I*arctan(c*x)^3*c*x+8*arctan(c*x)^3*c^2*x^2-8*arctan
(c*x)^3-18*I*arctan(c*x)^2*c^2*x^2-30*I*arctan(c*x)^2-12*c*x*arctan(c*x)^2
+6*I*arctan(c*x)*c*x-21*c^2*x^2*arctan(c*x)-27*arctan(c*x))/(c*x-I)^2)+3*a
*b^2/d^3*(1/2*I/(1+I*c*x)^2*arctan(c*x)^2-I*(1/8*I*arctan(c*x)*ln(c*x+I)-1
/8*I*arctan(c*x)*ln(c*x-I)-1/4*I/(c*x-I)^2*arctan(c*x)+1/4*arctan(c*x)/(c*
x-I)+1/32*ln(c*x+I)^2-1/16*(ln(c*x+I)-ln(-1/2*I*(c*x+I)))*ln(-1/2*I*(-c*x+
I))-3/16*I*arctan(c*x)-3/16*I/(c*x-I)-1/16/(c*x-I)^2+1/32*ln(c*x-I)^2-1/16
*ln(c*x-I)*ln(-1/2*I*(c*x+I))))+3/2*I*a^2*b/d^3/(1+I*c*x)^2*arctan(c*x)-3/
8*I*a^2*b/d^3*arctan(c*x)-3/8*a^2*b/d^3/(c*x-I)^2-3/8*I*a^2*b/d^3/(c*x-I))
```

## Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.98

$$\int \frac{(a+b\arctan(cx))^3}{(d+icdx)^3}\,dx =$$

$$-\frac{2\left(b^3c^2x^2-2i\,b^3cx+3\,b^3\right)\log\left(-\frac{cx+i}{cx-i}\right)^3+64i\,a^3+96\,a^2b-96i\,ab^2-48\,b^3+6\left(8i\,a^2b+12\,ab^2-7i\,b^3\right)}{}$$

input
```
integrate((a+b*arctan(c*x))^3/(d+I*c*d*x)^3,x, algorithm="fricas")
```

output
```
-1/128*(2*(b^3*c^2*x^2 - 2*I*b^3*c*x + 3*b^3)*log(-(c*x + I)/(c*x - I))^3
+ 64*I*a^3 + 96*a^2*b - 96*I*a*b^2 - 48*b^3 + 6*(8*I*a^2*b + 12*a*b^2 - 7*
I*b^3)*c*x + 3*((-4*I*a*b^2 - 3*b^3)*c^2*x^2 - 12*I*a*b^2 - 5*b^3 - 2*(4*a
*b^2 - I*b^3)*c*x)*log(-(c*x + I)/(c*x - I))^2 - 3*((8*a^2*b - 12*I*a*b^2
- 7*b^3)*c^2*x^2 + 24*a^2*b - 20*I*a*b^2 - 9*b^3 - 2*(8*I*a^2*b + 4*a*b^2
- I*b^3)*c*x)*log(-(c*x + I)/(c*x - I)))/(c^3*d^3*x^2 - 2*I*c^2*d^3*x - c*
d^3)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^3}{(d+icdx)^3}\,dx = \text{Timed out}$$

input
```
integrate((a+b*atan(c*x))**3/(d+I*c*d*x)**3,x)
```

output
```
Timed out
```

## Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.86

$$\int \frac{(a+b\arctan(cx))^3}{(d+icdx)^3}\,dx$$
$$= \frac{8\left(-i\,b^3c^2x^2-2\,b^3cx-3i\,b^3\right)\arctan\left(cx\right)^3-32i\,a^3-48\,a^2b+48i\,ab^2+24\,b^3+3\left(-8i\,a^2b-12\,ab^2+7i\right.}{}$$

input
```
integrate((a+b*arctan(c*x))^3/(d+I*c*d*x)^3,x, algorithm="maxima")
```

output
```
1/64*(8*(-I*b^3*c^2*x^2 - 2*b^3*c*x - 3*I*b^3)*arctan(c*x)^3 - 32*I*a^3 -
48*a^2*b + 48*I*a*b^2 + 24*b^3 + 3*(-8*I*a^2*b - 12*a*b^2 + 7*I*b^3)*c*x +
 6*((-4*I*a*b^2 - 3*b^3)*c^2*x^2 - 12*I*a*b^2 - 5*b^3 - 2*(4*a*b^2 - I*b^3
)*c*x)*arctan(c*x)^2 + 3*((-8*I*a^2*b - 12*a*b^2 + 7*I*b^3)*c^2*x^2 - 24*I
*a^2*b - 20*a*b^2 + 9*I*b^3 - 2*(8*a^2*b - 4*I*a*b^2 - b^3)*c*x)*arctan(c*
x))/(c^3*d^3*x^2 - 2*I*c^2*d^3*x - c*d^3)
```

## Giac [F]

$$\int \frac{(a+b\arctan(cx))^3}{(d+icdx)^3}\,dx=\int \frac{(b\arctan(cx)+a)^3}{(i\,cdx+d)^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))^3/(d+I*c*d*x)^3,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)^3/(I*c*d*x + d)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^3}{(d+icdx)^3}\,dx=\int \frac{(a+b\,\mathrm{atan}(c\,x))^3}{(d+c\,d\,x\,1\mathrm{i})^3}\,dx$$

input
```
int((a + b*atan(c*x))^3/(d + c*d*x*1i)^3,x)
```

output
```
int((a + b*atan(c*x))^3/(d + c*d*x*1i)^3, x)
```

## Reduce [F]

$$\int \frac{(a+b\arctan(cx))^3}{(d+icdx)^3}\,dx = \frac{-atan(cx)^4 b^3 - 4atan(cx)^3 a b^2 - 6atan(cx)^2 a^2 b + 48\left(\int \frac{atan(cx)}{c^5x^5-3c^4i x^4-2c^3x^3-2c^2i x^2-3cx+i}dx\right) a^2bci + 16\left(\int\right.}{}$$

input
```
int((a+b*atan(c*x))^3/(d+I*c*d*x)^3,x)
```

output
```
( - atan(c*x)**4*b**3 - 4*atan(c*x)**3*a*b**2 - 6*atan(c*x)**2*a**2*b + 48
*int(atan(c*x)/(c**5*x**5 - 3*c**4*i*x**4 - 2*c**3*x**3 - 2*c**2*i*x**2 -
3*c*x + i),x)*a**2*b*c*i + 16*int(atan(c*x)**3/(c**5*x**5 - 3*c**4*i*x**4
- 2*c**3*x**3 - 2*c**2*i*x**2 - 3*c*x + i),x)*b**3*c*i + 48*int(atan(c*x)*
*2/(c**5*x**5 - 3*c**4*i*x**4 - 2*c**3*x**3 - 2*c**2*i*x**2 - 3*c*x + i),x
)*a*b**2*c*i + 12*int(x**2/(c**5*x**5 - 3*c**4*i*x**4 - 2*c**3*x**3 - 2*c*
*2*i*x**2 - 3*c*x + i),x)*a**3*c**3*i - 12*int(( - atan(c*x)*x**3)/(c**5*x
**5 - 3*c**4*i*x**4 - 2*c**3*x**3 - 2*c**2*i*x**2 - 3*c*x + i),x)*a**2*b*c
**4 - 4*int(( - atan(c*x)**3*x**3)/(c**5*x**5 - 3*c**4*i*x**4 - 2*c**3*x**
3 - 2*c**2*i*x**2 - 3*c*x + i),x)*b**3*c**4 - 12*int(( - atan(c*x)**2*x**3
)/(c**5*x**5 - 3*c**4*i*x**4 - 2*c**3*x**3 - 2*c**2*i*x**2 - 3*c*x + i),x)
*a*b**2*c**4 - 36*int((atan(c*x)*x)/(c**5*x**5 - 3*c**4*i*x**4 - 2*c**3*x*
*3 - 2*c**2*i*x**2 - 3*c*x + i),x)*a**2*b*c**2 - 12*int((atan(c*x)**3*x)/(
c**5*x**5 - 3*c**4*i*x**4 - 2*c**3*x**3 - 2*c**2*i*x**2 - 3*c*x + i),x)*b*
*3*c**2 - 36*int((atan(c*x)**2*x)/(c**5*x**5 - 3*c**4*i*x**4 - 2*c**3*x**3
 - 2*c**2*i*x**2 - 3*c*x + i),x)*a*b**2*c**2 + 12*int(1/(c**5*x**5 - 3*c**
4*i*x**4 - 2*c**3*x**3 - 2*c**2*i*x**2 - 3*c*x + i),x)*a**3*c*i)/(12*c*d**
3)
```

# 3.126 $\int \frac{(a+b\arctan(cx))^3}{(d+icdx)^4}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 360

$$\begin{aligned}\int \frac{(a+b\arctan(cx))^3}{(d+icdx)^4}\,dx &= \frac{ib^3}{108cd^4(i-cx)^3}+\frac{19b^3}{576cd^4(i-cx)^2}-\frac{85ib^3}{576cd^4(i-cx)} \\ &\quad+\frac{85ib^3\arctan(cx)}{576cd^4}-\frac{b^2(a+b\arctan(cx))}{18cd^4(i-cx)^3} \\ &\quad+\frac{5ib^2(a+b\arctan(cx))}{48cd^4(i-cx)^2}+\frac{11b^2(a+b\arctan(cx))}{48cd^4(i-cx)} \\ &\quad-\frac{11b(a+b\arctan(cx))^2}{96cd^4}-\frac{ib(a+b\arctan(cx))^2}{6cd^4(i-cx)^3} \\ &\quad-\frac{b(a+b\arctan(cx))^2}{8cd^4(i-cx)^2}+\frac{ib(a+b\arctan(cx))^2}{8cd^4(i-cx)} \\ &\quad-\frac{i(a+b\arctan(cx))^3}{24cd^4}+\frac{i(a+b\arctan(cx))^3}{3cd^4(1+icx)^3}\end{aligned}$$

output
```
1/108*I*b^3/c/d^4/(I-c*x)^3+19/576*b^3/c/d^4/(I-c*x)^2-85/576*I*b^3/c/d^4/
(I-c*x)+85/576*I*b^3*arctan(c*x)/c/d^4-1/18*b^2*(a+b*arctan(c*x))/c/d^4/(I
-c*x)^3+5/48*I*b^2*(a+b*arctan(c*x))/c/d^4/(I-c*x)^2+11/48*b^2*(a+b*arctan
(c*x))/c/d^4/(I-c*x)-11/96*b*(a+b*arctan(c*x))^2/c/d^4-1/6*I*b*(a+b*arctan
(c*x))^2/c/d^4/(I-c*x)^3-1/8*b*(a+b*arctan(c*x))^2/c/d^4/(I-c*x)^2+1/8*I*b
*(a+b*arctan(c*x))^2/c/d^4/(I-c*x)-1/24*I*(a+b*arctan(c*x))^3/c/d^4+1/3*I*
(a+b*arctan(c*x))^3/c/d^4/(1+I*c*x)^3
```

## Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.75

$$\int \frac{(a+b\arctan(cx))^3}{(d+icdx)^4}\,dx = \frac{-576a^3+12ab^2(56+81icx-33c^2x^2)+b^3(-328i+567cx+255ic^2x^2)-72ia^2b(-10-9icx+3c^2x^2)}{}$$

input
```
Integrate[(a + b*ArcTan[c*x])^3/(d + I*c*d*x)^4,x]
```

output
```
(-576*a^3 + 12*a*b^2*(56 + (81*I)*c*x - 33*c^2*x^2) + b^3*(-328*I + 567*c*
x + (255*I)*c^2*x^2) - (72*I)*a^2*b*(-10 - (9*I)*c*x + 3*c^2*x^2) + 3*b*(I
 + c*x)*(12*a*b*(29 + (32*I)*c*x - 11*c^2*x^2) + b^2*(-139*I + 208*c*x + (
85*I)*c^2*x^2) - (72*I)*a^2*(-7 - (4*I)*c*x + c^2*x^2))*ArcTan[c*x] - (18*
I)*b^2*(I + c*x)*(b*(29*I - 32*c*x - (11*I)*c^2*x^2) + 12*a*(-7 - (4*I)*c*
x + c^2*x^2))*ArcTan[c*x]^2 - (72*I)*b^3*(-7*I - 3*c*x - (3*I)*c^2*x^2 + c
^3*x^3)*ArcTan[c*x]^3)/(1728*c*d^4*(-I + c*x)^3)
```

## Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 358, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {5389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^3}{(d+icdx)^4}\,dx$$

$$\Big\downarrow \text{5389}$$

$$\frac{i(a+b\arctan(cx))^3}{3cd^4(1+icx)^3}-\frac{ib\int\left(\frac{(a+b\arctan(cx))^2}{8d^3(c^2x^2+1)}-\frac{(a+b\arctan(cx))^2}{8d^3(i-cx)^2}-\frac{i(a+b\arctan(cx))^2}{4d^3(i-cx)^3}+\frac{(a+b\arctan(cx))^2}{2d^3(i-cx)^4}\right)dx}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{i(a+b\arctan(cx))^3}{3cd^4(1+icx)^3}-$$
$$\frac{ib\left(\frac{(a+b\arctan(cx))^3}{24bcd^3}-\frac{(a+b\arctan(cx))^2}{8cd^3(-cx+i)}-\frac{i(a+b\arctan(cx))^2}{8cd^3(-cx+i)^2}-\frac{11i(a+b\arctan(cx))^2}{96cd^3}+\frac{(a+b\arctan(cx))^2}{6cd^3(-cx+i)^3}+\frac{11ib(a+b\arctan(cx))}{48cd^3(-cx+i)}\right.}{d}$$

input
```
Int[(a + b*ArcTan[c*x])^3/(d + I*c*d*x)^4,x]
```

output
```
((I/3)*(a + b*ArcTan[c*x])^3)/(c*d^4*(1 + I*c*x)^3) - (I*b*(-1/108*b^2/(c*
d^3*(I - c*x)^3) + (((19*I)/576)*b^2)/(c*d^3*(I - c*x)^2) + (85*b^2)/(576*
c*d^3*(I - c*x)) - (85*b^2*ArcTan[c*x])/(576*c*d^3) - ((I/18)*b*(a + b*Arc
Tan[c*x]))/(c*d^3*(I - c*x)^3) - (5*b*(a + b*ArcTan[c*x]))/(48*c*d^3*(I -
c*x)^2) + (((11*I)/48)*b*(a + b*ArcTan[c*x]))/(c*d^3*(I - c*x)) - (((11*I)
/96)*(a + b*ArcTan[c*x])^2)/(c*d^3) + (a + b*ArcTan[c*x])^2/(6*c*d^3*(I -
c*x)^3) - ((I/8)*(a + b*ArcTan[c*x])^2)/(c*d^3*(I - c*x)^2) - (a + b*ArcTa
n[c*x])^2/(8*c*d^3*(I - c*x)) + (a + b*ArcTan[c*x])^3/(24*b*c*d^3)))/d
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5389
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - S
imp[b*c*(p/(e*(q + 1)))   Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1),
(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

## Maple [A] (verified)

Time = 5.20 (sec) , antiderivative size = 518, normalized size of antiderivative = 1.44

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{ia^3}{3d^4(icx+1)^3}+\frac{b^3\left(\frac{i\arctan(cx)^3}{3(icx+1)^3}-\frac{i\left(-216i\arctan(cx)^3c^2x^2+72c^3x^3\arctan(cx)^3-198i\arctan(cx)^2c^3x^3+72i\arctan(cx)^3-216\,\mathrm{arc}\right.}{}\right.}{}}{}$ |
| default | $\frac{\frac{ia^3}{3d^4(icx+1)^3}+\frac{b^3\left(\frac{i\arctan(cx)^3}{3(icx+1)^3}-\frac{i\left(-216i\arctan(cx)^3c^2x^2+72c^3x^3\arctan(cx)^3-198i\arctan(cx)^2c^3x^3+72i\arctan(cx)^3-216\,\mathrm{arc}\right.}{}\right.}{}}{}$ |
| parts | $\frac{ia^3}{3d^4(icx+1)^3c}+\frac{b^3\left(\frac{i\arctan(cx)^3}{3(icx+1)^3}-\frac{i\left(-216i\arctan(cx)^3c^2x^2+72c^3x^3\arctan(cx)^3-198i\arctan(cx)^2c^3x^3+72i\arctan(cx)^3-2\right.}{}\right.}{}$ |
| risch | Expression too large to display |

input
```
int((a+b*arctan(c*x))^3/(d+I*c*d*x)^4,x,method=_RETURNVERBOSE)
```

output
```
1/c*(1/3*I*a^3/d^4/(1+I*c*x)^3+b^3/d^4*(1/3*I/(1+I*c*x)^3*arctan(c*x)^3-1/
1728*I*(-216*I*arctan(c*x)^3*c^2*x^2+72*c^3*x^3*arctan(c*x)^3-198*I*arctan
(c*x)^2*c^3*x^3+72*I*arctan(c*x)^3-216*arctan(c*x)^3*c*x-54*I*arctan(c*x)^
2*c*x-378*c^2*x^2*arctan(c*x)^2+369*I*c^2*x^2*arctan(c*x)-255*c^3*x^3*arct
an(c*x)-522*arctan(c*x)^2+417*I*arctan(c*x)-207*c*x*arctan(c*x)+567*I*c*x-
255*c^2*x^2+328)/(c*x-I)^3)+3*a*b^2/d^4*(1/3*I/(1+I*c*x)^3*arctan(c*x)^2-2
/3*I*(-1/16*I*arctan(c*x)*ln(c*x-I)-1/8*I*arctan(c*x)/(c*x-I)^2-1/6*arctan
(c*x)/(c*x-I)^3+1/8*arctan(c*x)/(c*x-I)+1/16*I*arctan(c*x)*ln(c*x+I)+1/36*
I/(c*x-I)^3-11/96*I/(c*x-I)-5/96/(c*x-I)^2-11/96*I*arctan(c*x)+1/64*ln(c*x
-I)^2-1/32*ln(c*x-I)*ln(-1/2*I*(c*x+I))-1/32*(ln(c*x+I)-ln(-1/2*I*(c*x+I))
)*ln(-1/2*I*(-c*x+I))+1/64*ln(c*x+I)^2))+I*a^2*b/d^4/(1+I*c*x)^3*arctan(c*
x)-1/8*I*a^2*b/d^4*arctan(c*x)-1/8*a^2*b/d^4/(c*x-I)^2+1/6*I*a^2*b/d^4/(c*
x-I)^3-1/8*I*a^2*b/d^4/(c*x-I))
```

## Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\arctan(cx))^3}{(d+icdx)^4}\,dx =$$
$$-\frac{6\,(72i\,a^2b+132\,ab^2-85i\,b^3)c^2x^2+18\,(b^3c^3x^3-3i\,b^3c^2x^2-3\,b^3cx-7i\,b^3)\log\left(-\frac{cx+i}{cx-i}\right)^3+1152\,a^3-1}{}$$

input
```
integrate((a+b*arctan(c*x))^3/(d+I*c*d*x)^4,x, algorithm="fricas")
```

output
```
-1/3456*(6*(72*I*a^2*b + 132*a*b^2 - 85*I*b^3)*c^2*x^2 + 18*(b^3*c^3*x^3 -
 3*I*b^3*c^2*x^2 - 3*b^3*c*x - 7*I*b^3)*log(-(c*x + I)/(c*x - I))^3 + 1152
*a^3 - 1440*I*a^2*b - 1344*a*b^2 + 656*I*b^3 + 162*(8*a^2*b - 12*I*a*b^2 -
 7*b^3)*c*x + 9*((-12*I*a*b^2 - 11*b^3)*c^3*x^3 - 3*(12*a*b^2 - 7*I*b^3)*c
^2*x^2 - 84*a*b^2 + 29*I*b^3 + 3*(12*I*a*b^2 - b^3)*c*x)*log(-(c*x + I)/(c
*x - I))^2 - 3*((72*a^2*b - 132*I*a*b^2 - 85*b^3)*c^3*x^3 - 3*(72*I*a^2*b
+ 84*a*b^2 - 41*I*b^3)*c^2*x^2 - 504*I*a^2*b - 348*a*b^2 + 139*I*b^3 - 3*(
72*a^2*b + 12*I*a*b^2 + 23*b^3)*c*x)*log(-(c*x + I)/(c*x - I)))/(c^4*d^4*x
^3 - 3*I*c^3*d^4*x^2 - 3*c^2*d^4*x + I*c*d^4)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^3}{(d+icdx)^4}\,dx = \text{Timed out}$$

input
```
integrate((a+b*atan(c*x))**3/(d+I*c*d*x)**4,x)
```

output
```
Timed out
```

## Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 327, normalized size of antiderivative = 0.91

$$\int \frac{(a+b\arctan(cx))^3}{(d+icdx)^4}\,dx$$
$$=\frac{3\left(-72i\,a^2b-132\,ab^2+85i\,b^3\right)c^2x^2+72\left(-i\,b^3c^3x^3-3\,b^3c^2x^2+3i\,b^3cx-7\,b^3\right)\arctan\left(cx\right)^3-576\,a^3+7}{}$$

input
```
integrate((a+b*arctan(c*x))^3/(d+I*c*d*x)^4,x, algorithm="maxima")
```

output
```
1/1728*(3*(-72*I*a^2*b - 132*a*b^2 + 85*I*b^3)*c^2*x^2 + 72*(-I*b^3*c^3*x^
3 - 3*b^3*c^2*x^2 + 3*I*b^3*c*x - 7*b^3)*arctan(c*x)^3 - 576*a^3 + 720*I*a
^2*b + 672*a*b^2 - 328*I*b^3 - 81*(8*a^2*b - 12*I*a*b^2 - 7*b^3)*c*x + 18*
((-12*I*a*b^2 - 11*b^3)*c^3*x^3 - 3*(12*a*b^2 - 7*I*b^3)*c^2*x^2 - 84*a*b^
2 + 29*I*b^3 + 3*(12*I*a*b^2 - b^3)*c*x)*arctan(c*x)^2 + 3*((-72*I*a^2*b -
 132*a*b^2 + 85*I*b^3)*c^3*x^3 - 3*(72*a^2*b - 84*I*a*b^2 - 41*b^3)*c^2*x^
2 - 504*a^2*b + 348*I*a*b^2 + 139*b^3 + 3*(72*I*a^2*b - 12*a*b^2 + 23*I*b^
3)*c*x)*arctan(c*x))/(c^4*d^4*x^3 - 3*I*c^3*d^4*x^2 - 3*c^2*d^4*x + I*c*d^
4)
```

## Giac [F]

$$\int \frac{(a+b\arctan(cx))^3}{(d+icdx)^4}\,dx=\int \frac{\left(b\arctan\left(cx\right)+a\right)^3}{\left(i\,cdx+d\right)^4}\,dx$$

input
```
integrate((a+b*arctan(c*x))^3/(d+I*c*d*x)^4,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)^3/(I*c*d*x + d)^4, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^3}{(d+icdx)^4}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^3}{(d+c\,d\,x\,1\mathrm{i})^4}\,dx$$

input
```
int((a + b*atan(c*x))^3/(d + c*d*x*1i)^4,x)
```

output
```
int((a + b*atan(c*x))^3/(d + c*d*x*1i)^4, x)
```

## Reduce [F]

$$\int \frac{(a+b\arctan(cx))^3}{(d+icdx)^4}\,dx = \frac{3\left(\int \frac{atan(cx)}{c^4x^4-4c^3i\,x^3-6c^2x^2+4cix+1}dx\right)a^2b+\left(\int \frac{atan(cx)^3}{c^4x^4-4c^3i\,x^3-6c^2x^2+4cix+1}dx\right)b^3+3\left(\int \frac{atan(cx)^2}{c^4x^4-4c^3i\,x^3-6c^2x^2+4cix+1}dx\right)}{d^4}$$

input
```
int((a+b*atan(c*x))^3/(d+I*c*d*x)^4,x)
```

output
```
(3*int(atan(c*x)/(c**4*x**4 - 4*c**3*i*x**3 - 6*c**2*x**2 + 4*c*i*x + 1),x
)*a**2*b + int(atan(c*x)**3/(c**4*x**4 - 4*c**3*i*x**3 - 6*c**2*x**2 + 4*c
*i*x + 1),x)*b**3 + 3*int(atan(c*x)**2/(c**4*x**4 - 4*c**3*i*x**3 - 6*c**2
*x**2 + 4*c*i*x + 1),x)*a*b**2 + int(1/(c**4*x**4 - 4*c**3*i*x**3 - 6*c**2
*x**2 + 4*c*i*x + 1),x)*a**3)/d**4
```

# 3.127 $\int \frac{x^2(a+b\arctan(cx))^3}{d+icdx}\,dx$

## Optimal result

Integrand size = 25, antiderivative size = 410

$$\int \frac{x^2(a+b\arctan(cx))^3}{d+icdx}\,dx = -\frac{3b(a+b\arctan(cx))^2}{2c^3d}+\frac{3ibx(a+b\arctan(cx))^2}{2c^2d} + \frac{i(a+b\arctan(cx))^3}{2c^3d}+\frac{x(a+b\arctan(cx))^3}{c^2d} - \frac{ix^2(a+b\arctan(cx))^3}{2cd} + \frac{3ib^2(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c^3d} + \frac{3b(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{c^3d} - \frac{i(a+b\arctan(cx))^3\log\left(\frac{2}{1+icx}\right)}{c^3d} - \frac{3b^3\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2c^3d} + \frac{3ib^2(a+b\arctan(cx))\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^3d} + \frac{3b(a+b\arctan(cx))^2\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2c^3d} + \frac{3b^3\,\mathrm{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2c^3d} - \frac{3ib^2(a+b\arctan(cx))\,\mathrm{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2c^3d} - \frac{3b^3\,\mathrm{PolyLog}\left(4,1-\frac{2}{1+icx}\right)}{4c^3d}$$

output

```
-3/2*b*(a+b*arctan(c*x))^2/c^3/d+3/2*I*b*x*(a+b*arctan(c*x))^2/c^2/d+1/2*I
*(a+b*arctan(c*x))^3/c^3/d+x*(a+b*arctan(c*x))^3/c^2/d-1/2*I*x^2*(a+b*arct
an(c*x))^3/c/d+3*I*b^2*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^3/d+3*b*(a+b*ar
ctan(c*x))^2*ln(2/(1+I*c*x))/c^3/d-I*(a+b*arctan(c*x))^3*ln(2/(1+I*c*x))/c
^3/d-3/2*b^3*polylog(2,1-2/(1+I*c*x))/c^3/d+3*I*b^2*(a+b*arctan(c*x))*poly
log(2,1-2/(1+I*c*x))/c^3/d+3/2*b*(a+b*arctan(c*x))^2*polylog(2,1-2/(1+I*c*
x))/c^3/d+3/2*b^3*polylog(3,1-2/(1+I*c*x))/c^3/d-3/2*I*b^2*(a+b*arctan(c*x
))*polylog(3,1-2/(1+I*c*x))/c^3/d-3/4*b^3*polylog(4,1-2/(1+I*c*x))/c^3/d
```

## Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 541, normalized size of antiderivative = 1.32

$$\int \frac{x^2(a+b\arctan(cx))^3}{d+icdx}\,dx = -\frac{i\left(4ia^3cx-6a^2bcx+2a^3c^2x^2-4ia^3\arctan(cx)+6a^2b\arctan(cx)+12ia^2bcx\arctan(cx)-12ab^2cx\,\mathrm{ar}\right.}{}$$

input
```
Integrate[(x^2*(a + b*ArcTan[c*x])^3)/(d + I*c*d*x),x]
```

output
```
((-1/4*I)*((4*I)*a^3*c*x - 6*a^2*b*c*x + 2*a^3*c^2*x^2 - (4*I)*a^3*ArcTan[
c*x] + 6*a^2*b*ArcTan[c*x] + (12*I)*a^2*b*c*x*ArcTan[c*x] - 12*a*b^2*c*x*A
rcTan[c*x] + 6*a^2*b*c^2*x^2*ArcTan[c*x] - (12*I)*a^2*b*ArcTan[c*x]^2 + 18
*a*b^2*ArcTan[c*x]^2 + (6*I)*b^3*ArcTan[c*x]^2 + (12*I)*a*b^2*c*x*ArcTan[c
*x]^2 - 6*b^3*c*x*ArcTan[c*x]^2 + 6*a*b^2*c^2*x^2*ArcTan[c*x]^2 - (8*I)*a*
b^2*ArcTan[c*x]^3 + 6*b^3*ArcTan[c*x]^3 + (4*I)*b^3*c*x*ArcTan[c*x]^3 + 2*
b^3*c^2*x^2*ArcTan[c*x]^3 - (2*I)*b^3*ArcTan[c*x]^4 + 12*a^2*b*ArcTan[c*x]
*Log[1 + E^((2*I)*ArcTan[c*x])] + (24*I)*a*b^2*ArcTan[c*x]*Log[1 + E^((2*I
)*ArcTan[c*x])] - 12*b^3*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] + 12*a
*b^2*ArcTan[c*x]^2*Log[1 + E^((2*I)*ArcTan[c*x])] + (12*I)*b^3*ArcTan[c*x]
^2*Log[1 + E^((2*I)*ArcTan[c*x])] + 4*b^3*ArcTan[c*x]^3*Log[1 + E^((2*I)*A
rcTan[c*x])] - 2*a^3*Log[1 + c^2*x^2] - (6*I)*a^2*b*Log[1 + c^2*x^2] + 6*a
*b^2*Log[1 + c^2*x^2] - (6*I)*b*(a + I*b + b*ArcTan[c*x])^2*PolyLog[2, -E^
((2*I)*ArcTan[c*x])] + 6*b^2*(a + I*b + b*ArcTan[c*x])*PolyLog[3, -E^((2*I
)*ArcTan[c*x])] + (3*I)*b^3*PolyLog[4, -E^((2*I)*ArcTan[c*x])]))/(c^3*d)
```

## Rubi [A] (verified)

Time = 3.95 (sec) , antiderivative size = 451, normalized size of antiderivative = 1.10, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.640$, Rules used = {5401, 27, 5361, 5401, 5345, 5379, 5451, 5345, 5419, 5455, 5379, 2849, 2752, 5529, 5533, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\arctan(cx))^3}{d+icdx}\,dx$$

$\Big\downarrow$ 5401

$$\frac{i\int \frac{x(a+b\arctan(cx))^3}{d(icx+1)}dx}{c}-\frac{i\int x(a+b\arctan(cx))^3dx}{cd}$$

$\Big\downarrow$ 27

$$\frac{i\int \frac{x(a+b\arctan(cx))^3}{icx+1}dx}{cd}-\frac{i\int x(a+b\arctan(cx))^3dx}{cd}$$

$\Big\downarrow$ 5361

$$\frac{i\int \frac{x(a+b\arctan(cx))^3}{icx+1}dx}{cd}-\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))^3-\frac{3}{2}bc\int \frac{x^2(a+b\arctan(cx))^2}{c^2x^2+1}dx\right)}{cd}$$

$\Big\downarrow$ 5401

$$\frac{i\left(\frac{i\int \frac{(a+b\arctan(cx))^3}{icx+1}dx}{c}-\frac{i\int (a+b\arctan(cx))^3dx}{c}\right)}{cd}-\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))^3-\frac{3}{2}bc\int \frac{x^2(a+b\arctan(cx))^2}{c^2x^2+1}dx\right)}{cd}$$

$\Big\downarrow$ 5345

$$\frac{i\left(\frac{i\int \frac{(a+b\arctan(cx))^3}{icx+1}dx}{c}-\frac{i\left(x(a+b\arctan(cx))^3-3bc\int \frac{x(a+b\arctan(cx))^2}{c^2x^2+1}dx\right)}{c}\right)}{cd}-\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))^3-\frac{3}{2}bc\int \frac{x^2(a+b\arctan(cx))^2}{c^2x^2+1}dx\right)}{cd}$$

$\Big\downarrow$ 5379

$$\frac{i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^3}{c}-3ib\int \frac{(a+b\arctan(cx))^2\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{c}-\frac{i\left(x(a+b\arctan(cx))^3-3bc\int \frac{x(a+b\arctan(cx))^2}{c^2x^2+1}dx\right)}{c}\right)}{cd}-\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))^3-\frac{3}{2}bc\int \frac{x^2(a+b\arctan(cx))^2}{c^2x^2+1}dx\right)}{cd}$$

$\Big\downarrow$ 5451

$$\frac{i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^3}{c}-3ib\int\frac{(a+b\arctan(cx))^2\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{c}-\frac{i\left(x(a+b\arctan(cx))^3-3bc\int\frac{x(a+b\arctan(cx))^2}{c^2x^2+1}dx\right)}{c}\right)}{cd}-\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))^3-\frac{3}{2}bc\left(\frac{\int(a+b\arctan(cx))^2dx}{c^2}-\frac{\int\frac{(a+b\arctan(cx))^2}{c^2x^2+1}dx}{c^2}\right)\right)}{cd}$$

$$\Big\downarrow \text{5345}$$

$$\frac{i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^3}{c}-3ib\int\frac{(a+b\arctan(cx))^2\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{c}-\frac{i\left(x(a+b\arctan(cx))^3-3bc\int\frac{x(a+b\arctan(cx))^2}{c^2x^2+1}dx\right)}{c}\right)}{cd}-\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))^3-\frac{3}{2}bc\left(\frac{x(a+b\arctan(cx))^2-2bc\int\frac{x(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}-\frac{\int\frac{(a+b\arctan(cx))^2}{c^2x^2+1}dx}{c^2}\right)\right)}{cd}$$

$$\Big\downarrow \text{5419}$$

$$\frac{i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^3}{c}-3ib\int\frac{(a+b\arctan(cx))^2\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{c}-\frac{i\left(x(a+b\arctan(cx))^3-3bc\int\frac{x(a+b\arctan(cx))^2}{c^2x^2+1}dx\right)}{c}\right)}{cd}-\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))^3-\frac{3}{2}bc\left(\frac{x(a+b\arctan(cx))^2-2bc\int\frac{x(a+b\arctan(cx))}{c^2x^2+1}dx}{c^2}-\frac{(a+b\arctan(cx))^3}{3bc^3}\right)\right)}{cd}$$

$$\Big\downarrow \text{5455}$$

$$\frac{i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^3}{c}-3ib\int\frac{(a+b\arctan(cx))^2\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{c}-\frac{i\left(x(a+b\arctan(cx))^3-3bc\left(-\frac{\int\frac{(a+b\arctan(cx))^2}{i-cx}dx}{c}-\frac{i(a+b\arcta}{3bc^2}\right.\right.}{c}\right.}{cd}$$

$$-\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))^3-\frac{3}{2}bc\left(-\frac{(a+b\arctan(cx))^3}{3bc^3}+\frac{x(a+b\arctan(cx))^2-2bc\left(-\frac{\int\frac{a+b\arctan(cx)}{i-cx}dx}{c}-\frac{i(a+b\arctan(cx))^2}{2bc^2}\right)}{c^2}\right)\right)}{cd}$$

$$\Big\downarrow \text{5379}$$

$$\frac{i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^3}{c}-3ib\int\frac{(a+b\arctan(cx))^2\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{c}-\frac{i\left(x(a+b\arctan(cx))^3-3bc\left(-\frac{\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2b\int \cdots}{c}\right.\right.}{c}\cdots\right.}{cd}$$

$$\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))^3-\frac{3}{2}bc\left(-\frac{(a+b\arctan(cx))^3}{3bc^3}+\frac{x(a+b\arctan(cx))^2-2bc\left(-\frac{\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}-b\int\frac{\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx}{c}\cdots\right.}{c^2}\cdots\right.\right.}{cd}$$

$$\Big\downarrow \; 2849$$

$$\frac{i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^3}{c}-3ib\int\frac{(a+b\arctan(cx))^2\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{c}-\frac{i\left(x(a+b\arctan(cx))^3-3bc\left(-\frac{\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2b\int \cdots}{c}\right.\right.}{c}\cdots\right.}{cd}$$

$$\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))^3-\frac{3}{2}bc\left(-\frac{(a+b\arctan(cx))^3}{3bc^3}+\frac{x(a+b\arctan(cx))^2-2bc\left(-\frac{\frac{ib\int\frac{\log\left(\frac{2}{icx+1}\right)}{1-\frac{2}{icx+1}}d\frac{1}{icx+1}}{c}+\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan \cdots}{c}}{c}\cdots\right.}{c^2}\cdots\right.\right.}{cd}$$

$$\Big\downarrow \; 2752$$

$$\frac{i\Bigg(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^3}{c}-3ib\int\frac{(a+b\arctan(cx))^2\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{c}-\frac{i\Big(x(a+b\arctan(cx))^3-3bc\Big(-\frac{\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2b\int}{c}}{c}}{cd}$$

$$\frac{i\Bigg(\frac{1}{2}x^2(a+b\arctan(cx))^3-\frac{3}{2}bc\Bigg(-\frac{(a+b\arctan(cx))^3}{3bc^3}+\frac{x(a+b\arctan(cx))^2-2bc\Big(-\frac{i(a+b\arctan(cx))^2}{2bc^2}-\frac{\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}}{c}}{c^2}}{cd}$$

$\downarrow$ 5529

$$\frac{i\Bigg(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^3}{c}-3ib\left(ib\int\frac{(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{c^2x^2+1}dx-\frac{i\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))^2}{2c}\right)\right)}{c}-\frac{i\Big(x(a+b\operatorname{arc}}{}}{cd}$$

$$\frac{i\Bigg(\frac{1}{2}x^2(a+b\arctan(cx))^3-\frac{3}{2}bc\Bigg(-\frac{(a+b\arctan(cx))^3}{3bc^3}+\frac{x(a+b\arctan(cx))^2-2bc\Big(-\frac{i(a+b\arctan(cx))^2}{2bc^2}-\frac{\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}}{c}}{c^2}}{cd}$$

$\downarrow$ 5533

$$\frac{i\Bigg(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^3}{c}-3ib\left(ib\left(\frac{i\operatorname{PolyLog}\left(3,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{2c}-\frac{1}{2}ib\int\frac{\operatorname{PolyLog}\left(3,1-\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)-\frac{i\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\operatorname{a}}{2c}}{c}}{cd}$$

$$\frac{i\Bigg(\frac{1}{2}x^2(a+b\arctan(cx))^3-\frac{3}{2}bc\Bigg(-\frac{(a+b\arctan(cx))^3}{3bc^3}+\frac{x(a+b\arctan(cx))^2-2bc\Big(-\frac{i(a+b\arctan(cx))^2}{2bc^2}-\frac{\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}}{c}}{c^2}}{cd}$$

$\downarrow$ 7164

$$i\left(\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^3}{c}-3ib\left(ib\left(\frac{i\,\text{PolyLog}\left(3,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{2c}+\frac{b\,\text{PolyLog}\left(4,1-\frac{2}{icx+1}\right)}{4c}\right)-\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx}{2c}\right.\right.}{c}\right.$$

$$\frac{i\left(\frac{1}{2}x^2(a+b\arctan(cx))^3-\frac{3}{2}bc\left(-\frac{(a+b\arctan(cx))^3}{3bc^3}+\frac{x(a+b\arctan(cx))^2-2bc\left(-\frac{i(a+b\arctan(cx))^2}{2bc^2}-\frac{\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}}{c}\right.}{c^2}\right.\right.}{cd}$$

input
```
Int[(x^2*(a + b*ArcTan[c*x])^3)/(d + I*c*d*x),x]
```

output
```
((-I)*((x^2*(a + b*ArcTan[c*x])^3)/2 - (3*b*c*(-1/3*(a + b*ArcTan[c*x])^3/
(b*c^3) + (x*(a + b*ArcTan[c*x])^2 - 2*b*c*(((-1/2*I)*(a + b*ArcTan[c*x])^
2)/(b*c^2) - (((a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/c + ((I/2)*b*PolyLo
g[2, 1 - 2/(1 + I*c*x)])/c)/c))/c^2))/2))/(c*d) + (I*(((-I)*(x*(a + b*ArcT
an[c*x])^3 - 3*b*c*(((-1/3*I)*(a + b*ArcTan[c*x])^3)/(b*c^2) - (((a + b*Ar
cTan[c*x])^2*Log[2/(1 + I*c*x)])/c - 2*b*(((-1/2*I)*(a + b*ArcTan[c*x])*Po
lyLog[2, 1 - 2/(1 + I*c*x)])/c - (b*PolyLog[3, 1 - 2/(1 + I*c*x)])/(4*c)))
/c)))/c + (I*((I*(a + b*ArcTan[c*x])^3*Log[2/(1 + I*c*x)])/c - (3*I)*b*(((
-1/2*I)*(a + b*ArcTan[c*x])^2*PolyLog[2, 1 - 2/(1 + I*c*x)])/c + I*b*(((I/
2)*(a + b*ArcTan[c*x])*PolyLog[3, 1 - 2/(1 + I*c*x)])/c + (b*PolyLog[4, 1
- 2/(1 + I*c*x)])/(4*c)))))/c))/(c*d)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5401
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_) + (
e_.)*(x_)), x_Symbol] :> Simp[f/e   Int[(f*x)^(m - 1)*(a + b*ArcTan[c*x])^p
, x], x] - Simp[d*(f/e)   Int[(f*x)^(m - 1)*((a + b*ArcTan[c*x])^p/(d + e*x
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e
^2, 0] && GtQ[m, 0]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 5533
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k + 1
, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &&
EqQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 22.69 (sec) , antiderivative size = 1359, normalized size of antiderivative = 3.31

| method | result | size |
|---|---|---|
| derivativedivides | Expression too large to display | 1359 |
| default | Expression too large to display | 1359 |
| parts | Expression too large to display | 1406 |

input
```
int(x^2*(a+b*arctan(c*x))^3/(d+I*c*d*x),x,method=_RETURNVERBOSE)
```

output
```
1/c^3*(a^3/d*c*x+3/2*I*a^2*b/d*c*x+3*I*a^2*b/d*arctan(c*x)*ln(c*x-I)-a^3/d
*arctan(c*x)+b^3/d*(-1/2*I*arctan(c*x)^2*(I*arctan(c*x)+c*x*arctan(c*x)-3)
*(c*x+I)-1/2*arctan(c*x)^4-I*arctan(c*x)^3*ln(1+(1+I*c*x)^2/(c^2*x^2+1))-3
/2*arctan(c*x)^2*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))-3/2*I*arctan(c*x)*pol
ylog(3,-(1+I*c*x)^2/(c^2*x^2+1))+3/4*polylog(4,-(1+I*c*x)^2/(c^2*x^2+1))+3
*arctan(c*x)^2+3*I*arctan(c*x)*ln(1+(1+I*c*x)^2/(c^2*x^2+1))+3/2*polylog(2
,-(1+I*c*x)^2/(c^2*x^2+1))-2*I*arctan(c*x)^3+3*arctan(c*x)^2*ln(1+(1+I*c*x
)^2/(c^2*x^2+1))-3*I*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))+3/2*p
olylog(3,-(1+I*c*x)^2/(c^2*x^2+1)))+3*a*b^2/d*(c*x*arctan(c*x)^2-2*I*dilog
(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+I*arctan(c*x)*(c*x-I)-1/2*Pi*csgn((1+I*c
*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^3*arctan(c*x)^2-I*arctan(c*
x)^2*ln(2*I*(1+I*c*x)^2/(c^2*x^2+1))-Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1
+I*c*x)^2/(c^2*x^2+1)))^2*arctan(c*x)^2-1/2*Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*
x^2+1)))*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+
I*c*x)^2/(c^2*x^2+1)))*arctan(c*x)^2-1/2*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1))*
csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*arctan(c*x)^2-
2*I*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/2*I*polylog(3,-(1+I*c*x)^2/(c
^2*x^2+1))-3/2*I*arctan(c*x)^2-1/2*I*arctan(c*x)^2*c^2*x^2+Pi*arctan(c*x)^
2+2*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+2*arctan(c*x)*ln(1-I*(
1+I*c*x)/(c^2*x^2+1)^(1/2))+1/2*Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*...
```

## Fricas [F]

$$\int \frac{x^2(a+b\arctan(cx))^3}{d+icdx}\,dx = \int \frac{(b\arctan(cx)+a)^3x^2}{i\,cdx+d}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))^3/(d+I*c*d*x),x, algorithm="fricas")
```

output
```
integral(-1/8*(b^3*x^2*log(-(c*x + I)/(c*x - I))^3 - 6*I*a*b^2*x^2*log(-(c
*x + I)/(c*x - I))^2 - 12*a^2*b*x^2*log(-(c*x + I)/(c*x - I)) + 8*I*a^3*x^
2)/(c*d*x - I*d), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\arctan(cx))^3}{d+icdx}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(a+b*atan(c*x))**3/(d+I*c*d*x),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{x^2(a+b\arctan(cx))^3}{d+icdx}\,dx = \int \frac{(b\arctan(cx)+a)^3x^2}{i\,cdx+d}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))^3/(d+I*c*d*x),x, algorithm="maxima")
```

output
```
-1/2*a^3*((I*c*x^2 - 2*x)/(c^2*d) - 2*I*log(I*c*x + 1)/(c^3*d)) - 1/128*(1
6*b^3*arctan(c*x)^4 - b^3*log(c^2*x^2 + 1)^4 + 4*(384*b^3*c^3*integrate(1/
64*x^3*arctan(c*x)^2*log(c^2*x^2 + 1)/(c^4*d*x^2 + c^2*d), x) - 32*b^3*c^3
*integrate(1/64*x^3*log(c^2*x^2 + 1)^3/(c^4*d*x^2 + c^2*d), x) + 384*b^3*c
^3*integrate(1/64*x^3*arctan(c*x)^2/(c^4*d*x^2 + c^2*d), x) - 96*b^3*c^3*i
ntegrate(1/64*x^3*log(c^2*x^2 + 1)^2/(c^4*d*x^2 + c^2*d), x) - 1792*b^3*c^
2*integrate(1/64*x^2*arctan(c*x)^3/(c^4*d*x^2 + c^2*d), x) - 192*b^3*c^2*i
ntegrate(1/64*x^2*arctan(c*x)*log(c^2*x^2 + 1)^2/(c^4*d*x^2 + c^2*d), x) -
 6144*a*b^2*c^2*integrate(1/64*x^2*arctan(c*x)^2/(c^4*d*x^2 + c^2*d), x) -
 384*b^3*c^2*integrate(1/64*x^2*arctan(c*x)*log(c^2*x^2 + 1)/(c^4*d*x^2 +
c^2*d), x) - 6144*a^2*b*c^2*integrate(1/64*x^2*arctan(c*x)/(c^4*d*x^2 + c^
2*d), x) + 384*b^3*c*integrate(1/64*x*arctan(c*x)^2*log(c^2*x^2 + 1)/(c^4*
d*x^2 + c^2*d), x) + 96*b^3*c*integrate(1/64*x*log(c^2*x^2 + 1)^3/(c^4*d*x
^2 + c^2*d), x) + 768*b^3*c*integrate(1/64*x*arctan(c*x)^2/(c^4*d*x^2 + c^
2*d), x) - 192*b^3*c*integrate(1/64*x*log(c^2*x^2 + 1)^2/(c^4*d*x^2 + c^2*
d), x) - 192*b^3*integrate(1/64*arctan(c*x)*log(c^2*x^2 + 1)^2/(c^4*d*x^2
+ c^2*d), x) - 3*b^3*arctan(c*x)^4/(c^3*d))*c^3*d - 128*I*c^3*d*integrate(
-1/64*(192*a^2*b*c^3*x^3*arctan(c*x) + 8*(7*b^3*c^3*x^3 - 3*b^3*c*x)*arcta
n(c*x)^3 - (b^3*c^2*x^2 + 3*b^3)*log(c^2*x^2 + 1)^3 + 12*(16*a*b^2*c^3*x^3
 + b^3*c^2*x^2)*arctan(c*x)^2 - 3*(b^3*c^2*x^2 - 2*(b^3*c^3*x^3 - b^3*c...
```

## Giac [F]

$$\int \frac{x^2(a+b\arctan(cx))^3}{d+icdx}\,dx=\int \frac{(b\arctan{(cx)}+a)^3x^2}{i\,cdx+d}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))^3/(d+I*c*d*x),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)^3*x^2/(I*c*d*x + d), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\arctan(cx))^3}{d+icdx}\,dx = \int \frac{x^2\,(a+b\,\mathrm{atan}(c\,x))^3}{d+c\,d\,x\,1\mathrm{i}}\,dx$$

input
```
int((x^2*(a + b*atan(c*x))^3)/(d + c*d*x*1i),x)
```

output
```
int((x^2*(a + b*atan(c*x))^3)/(d + c*d*x*1i), x)
```

## Reduce [F]

$$\int \frac{x^2(a+b\arctan(cx))^3}{d+icdx}\,dx = \frac{6\left(\int \frac{atan(cx)x^2}{cix+1}dx\right)a^2b\,c^3+2\left(\int \frac{atan(cx)^3x^2}{cix+1}dx\right)b^3c^3+6\left(\int \frac{atan(cx)^2x^2}{cix+1}dx\right)a\,b^2c^3+2\log(cix+1)\,a^3i-a^3c^2i}{2c^3d}$$

input
```
int(x^2*(a+b*atan(c*x))^3/(d+I*c*d*x),x)
```

output
```
(6*int((atan(c*x)*x**2)/(c*i*x + 1),x)*a**2*b*c**3 + 2*int((atan(c*x)**3*x
**2)/(c*i*x + 1),x)*b**3*c**3 + 6*int((atan(c*x)**2*x**2)/(c*i*x + 1),x)*a
*b**2*c**3 + 2*log(c*i*x + 1)*a**3*i - a**3*c**2*i*x**2 + 2*a**3*c*x)/(2*c
**3*d)
```

# 3.128 $\int \frac{x(a+b\arctan(cx))^3}{d+icdx}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 277

$$\begin{aligned}\int \frac{x(a+b\arctan(cx))^3}{d+icdx}\,dx &= \frac{(a+b\arctan(cx))^3}{c^2d}-\frac{ix(a+b\arctan(cx))^3}{cd}\\&\quad-\frac{3ib(a+b\arctan(cx))^2\log\left(\frac{2}{1+icx}\right)}{c^2d}\\&\quad-\frac{(a+b\arctan(cx))^3\log\left(\frac{2}{1+icx}\right)}{c^2d}\\&\quad+\frac{3b^2(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c^2d}\\&\quad-\frac{3ib(a+b\arctan(cx))^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2c^2d}\\&\quad-\frac{3ib^3\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2c^2d}\\&\quad-\frac{3b^2(a+b\arctan(cx))\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2c^2d}\\&\quad+\frac{3ib^3\operatorname{PolyLog}\left(4,1-\frac{2}{1+icx}\right)}{4c^2d}\end{aligned}$$

output
```
(a+b*arctan(c*x))^3/c^2/d-I*x*(a+b*arctan(c*x))^3/c/d-3*I*b*(a+b*arctan(c*
x))^2*ln(2/(1+I*c*x))/c^2/d-(a+b*arctan(c*x))^3*ln(2/(1+I*c*x))/c^2/d+3*b^
2*(a+b*arctan(c*x))*polylog(2,1-2/(1+I*c*x))/c^2/d-3/2*I*b*(a+b*arctan(c*x
))^2*polylog(2,1-2/(1+I*c*x))/c^2/d-3/2*I*b^3*polylog(3,1-2/(1+I*c*x))/c^2
/d-3/2*b^2*(a+b*arctan(c*x))*polylog(3,1-2/(1+I*c*x))/c^2/d+3/4*I*b^3*poly
log(4,1-2/(1+I*c*x))/c^2/d
```

## Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.42

$$\int \frac{x(a+b\arctan(cx))^3}{d+icdx}\,dx = -\frac{i\left(4a^3cx-4a^3\arctan(cx)+12a^2bcx\arctan(cx)-12a^2b\arctan(cx)^2-12iab^2\arctan(cx)^2+12ab^2cx\,a\right.}{}$$

input
```
Integrate[(x*(a + b*ArcTan[c*x])^3)/(d + I*c*d*x),x]
```

output
```
((-1/4*I)*(4*a^3*c*x - 4*a^3*ArcTan[c*x] + 12*a^2*b*c*x*ArcTan[c*x] - 12*a
^2*b*ArcTan[c*x]^2 - (12*I)*a*b^2*ArcTan[c*x]^2 + 12*a*b^2*c*x*ArcTan[c*x]
^2 - 8*a*b^2*ArcTan[c*x]^3 - (4*I)*b^3*ArcTan[c*x]^3 + 4*b^3*c*x*ArcTan[c*
x]^3 - 2*b^3*ArcTan[c*x]^4 - (12*I)*a^2*b*ArcTan[c*x]*Log[1 + E^((2*I)*Arc
Tan[c*x])] + 24*a*b^2*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] - (12*I)*
a*b^2*ArcTan[c*x]^2*Log[1 + E^((2*I)*ArcTan[c*x])] + 12*b^3*ArcTan[c*x]^2*
Log[1 + E^((2*I)*ArcTan[c*x])] - (4*I)*b^3*ArcTan[c*x]^3*Log[1 + E^((2*I)*
ArcTan[c*x])] + (2*I)*a^3*Log[1 + c^2*x^2] - 6*a^2*b*Log[1 + c^2*x^2] - 6*
b*(a*(a + (2*I)*b) + 2*(a + I*b)*b*ArcTan[c*x] + b^2*ArcTan[c*x]^2)*PolyLo
g[2, -E^((2*I)*ArcTan[c*x])] + 6*b^2*((-I)*a + b - I*b*ArcTan[c*x])*PolyLo
g[3, -E^((2*I)*ArcTan[c*x])] + 3*b^3*PolyLog[4, -E^((2*I)*ArcTan[c*x])]))/
(c^2*d)
```

## Rubi [A] (verified)

Time = 1.75 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {5401, 27, 5345, 5379, 5455, 5379, 5529, 5533, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\arctan(cx))^3}{d+icdx}dx$$

$$\Big\downarrow \text{5401}$$

$$\frac{i\int \frac{(a+b\arctan(cx))^3}{d(icx+1)}dx}{c}-\frac{i\int (a+b\arctan(cx))^3dx}{cd}$$

$$\Big\downarrow \text{27}$$

$$\frac{i\int \frac{(a+b\arctan(cx))^3}{icx+1}dx}{cd}-\frac{i\int (a+b\arctan(cx))^3dx}{cd}$$

$$\Big\downarrow \text{5345}$$

$$\frac{i\int \frac{(a+b\arctan(cx))^3}{icx+1}dx}{cd}-\frac{i\left(x(a+b\arctan(cx))^3-3bc\int \frac{x(a+b\arctan(cx))^2}{c^2x^2+1}dx\right)}{cd}$$

$$\Big\downarrow \text{5379}$$

$$\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^3}{c}-3ib\int \frac{(a+b\arctan(cx))^2\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{cd}-\frac{i\left(x(a+b\arctan(cx))^3-3bc\int \frac{x(a+b\arctan(cx))^2}{c^2x^2+1}dx\right)}{cd}$$

$$\Big\downarrow \text{5455}$$

$$\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^3}{c}-3ib\int \frac{(a+b\arctan(cx))^2\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{cd}-\frac{i\left(x(a+b\arctan(cx))^3-3bc\left(-\frac{\int \frac{(a+b\arctan(cx))^2}{i-cx}dx}{c}-\frac{i(a+b\arctan(cx))^3}{3bc^2}\right)\right)}{cd}$$

$$\Big\downarrow \text{5379}$$

$$\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^3}{c}-3ib\int\frac{(a+b\arctan(cx))^2\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{cd}-$$

$$\frac{i\left(x(a+b\arctan(cx))^3-3bc\left(-\frac{\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2b\int\frac{(a+b\arctan(cx))\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx}{c}-\frac{i(a+b\arctan(cx))^3}{3bc^2}\right)\right)}{cd}$$

$$\Big\downarrow \text{5529}$$

$$\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^3}{c}-3ib\left(ib\int\frac{(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{c^2x^2+1}dx-\frac{i\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))^2}{2c}\right)\right)}{cd}$$

$$\frac{i\left(x(a+b\arctan(cx))^3-3bc\left(-\frac{\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2b\left(\frac{1}{2}ib\int\frac{\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{c^2x^2+1}dx-\frac{i\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{2c}\right.}{c}\right.\right.}{cd}$$

$$\Big\downarrow \text{5533}$$

$$\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^3}{c}-3ib\left(ib\left(\frac{i\operatorname{PolyLog}\left(3,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{2c}-\frac{1}{2}ib\int\frac{\operatorname{PolyLog}\left(3,1-\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)-\frac{i\operatorname{PolyLog}}{}\right.\right.}{cd}$$

$$\frac{i\left(x(a+b\arctan(cx))^3-3bc\left(-\frac{\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2b\left(\frac{1}{2}ib\int\frac{\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{c^2x^2+1}dx-\frac{i\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{2c}\right.}{c}\right.\right.}{cd}$$

$$\Big\downarrow \text{7164}$$

$$\frac{i\left(\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^3}{c}-3ib\left(ib\left(\frac{i\operatorname{PolyLog}\left(3,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{2c}+\frac{b\operatorname{PolyLog}\left(4,1-\frac{2}{icx+1}\right)}{4c}\right)-\frac{i\operatorname{PolyLog}\left(2,1-\frac{2}{icx}\right.}{}\right.\right.}{cd}$$

$$\frac{i\left(x(a+b\arctan(cx))^3-3bc\left(-\frac{i(a+b\arctan(cx))^3}{3bc^2}-\frac{\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{c}-2b\left(-\frac{i\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{2c}-\frac{b\operatorname{Po}}{}\right.}{c}\right.\right.}{cd}$$

input

```
Int[(x*(a + b*ArcTan[c*x])^3)/(d + I*c*d*x),x]
```

output
```
((-I)*(x*(a + b*ArcTan[c*x])^3 - 3*b*c*(((-1/3*I)*(a + b*ArcTan[c*x])^3)/(
b*c^2) - (((a + b*ArcTan[c*x])^2*Log[2/(1 + I*c*x)])/c - 2*b*(((-1/2*I)*(a
 + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)])/c - (b*PolyLog[3, 1 - 2/(
1 + I*c*x)])/(4*c))/c)))/(c*d) + (I*((I*(a + b*ArcTan[c*x])^3*Log[2/(1 +
I*c*x)])/c - (3*I)*b*(((-1/2*I)*(a + b*ArcTan[c*x])^2*PolyLog[2, 1 - 2/(1
+ I*c*x)])/c + I*b*(((I/2)*(a + b*ArcTan[c*x])*PolyLog[3, 1 - 2/(1 + I*c*x
)])/c + (b*PolyLog[4, 1 - 2/(1 + I*c*x)])/(4*c)))))/(c*d)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5401
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.))/((d_) + (
e_.)*(x_)), x_Symbol] :> Simp[f/e   Int[(f*x)^(m - 1)*(a + b*ArcTan[c*x])^p
, x], x] - Simp[d*(f/e)   Int[(f*x)^(m - 1)*((a + b*ArcTan[c*x])^p/(d + e*x
)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e
^2, 0] && GtQ[m, 0]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 5533
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k + 1
, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &&
EqQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 14.48 (sec) , antiderivative size = 3018, normalized size of antiderivative = 10.90

| method | result | size |
|---|---|---|
| derivativedivides | Expression too large to display | 3018 |
| default | Expression too large to display | 3018 |
| parts | Expression too large to display | 3054 |

input
```
int(x*(a+b*arctan(c*x))^3/(d+I*c*d*x),x,method=_RETURNVERBOSE)
```

output
```
1/c^2*(-I*a^3/d*c*x+1/2*a^3/d*ln(c^2*x^2+1)+I*a^3/d*arctan(c*x)+b^3/d*(1/2
*I*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c
*x)^2/(c^2*x^2+1)))^2*arctan(c*x)^3+ln(c*x-I)*arctan(c*x)^3-arctan(c*x)^3*
ln(2*I*(1+I*c*x)^2/(c^2*x^2+1))+3/2*I*arctan(c*x)^2*polylog(2,-(1+I*c*x)^2
/(c^2*x^2+1))-3/2*I*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))-3/2*arctan(c*x)*po
lylog(3,-(1+I*c*x)^2/(c^2*x^2+1))+1/2*I*Pi*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1
+(1+I*c*x)^2/(c^2*x^2+1)))^3*arctan(c*x)^3+1/2*I*arctan(c*x)^4-1/2*I*Pi*cs
gn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x
)^2/(c^2*x^2+1)))^2*arctan(c*x)^3-3*I*arctan(c*x)^2*ln(1+(1+I*c*x)^2/(c^2*
x^2+1))-3/4*I*polylog(4,-(1+I*c*x)^2/(c^2*x^2+1))-3*arctan(c*x)*polylog(2,
-(1+I*c*x)^2/(c^2*x^2+1))-I*Pi*arctan(c*x)^3+I*Pi*csgn((1+I*c*x)^2/(c^2*x^
2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*arctan(c*x)^3-arctan(c*x)^3-I*arctan(c
*x)^3*c*x+1/2*I*Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn((1+I*c*x)^2/(c
^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*arcta
n(c*x)^3)+3*a*b^2/d*(-arctan(c*x)^2-1/2*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1)
)-dilog(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))-dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(
1/2))+I*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))-1/2*polylog(3,-(1+
I*c*x)^2/(c^2*x^2+1))-1/2*I*Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn((1
+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^
2+1)))*(I*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+I*arctan(c*x)...
```

## Fricas [F]

$$\int \frac{x(a+b\arctan(cx))^3}{d+icdx}\,dx = \int \frac{(b\arctan{(cx)}+a)^3x}{i\,cdx+d}\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))^3/(d+I*c*d*x),x, algorithm="fricas")
```

output
```
integral(-1/8*(b^3*x*log(-(c*x + I)/(c*x - I))^3 - 6*I*a*b^2*x*log(-(c*x +
 I)/(c*x - I))^2 - 12*a^2*b*x*log(-(c*x + I)/(c*x - I)) + 8*I*a^3*x)/(c*d*
x - I*d), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x(a+b\arctan(cx))^3}{d+icdx}\,dx = \text{Timed out}$$

input
```
integrate(x*(a+b*atan(c*x))**3/(d+I*c*d*x),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{x(a+b\arctan(cx))^3}{d+icdx}\,dx = \int \frac{(b\arctan(cx)+a)^3x}{i\,cdx+d}\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))^3/(d+I*c*d*x),x, algorithm="maxima")
```

output
```
a^3*(-I*x/(c*d) + log(I*c*x + 1)/(c^2*d)) + 1/128*(-16*I*b^3*c*x*arctan(c*
x)^3 + 12*I*b^3*c*x*arctan(c*x)*log(c^2*x^2 + 1)^2 + 16*I*b^3*arctan(c*x)^
4 - I*b^3*log(c^2*x^2 + 1)^4 - 4*I*(896*b^3*c^2*integrate(1/32*x^2*arctan(
c*x)^3/(c^3*d*x^2 + c*d), x) + 96*b^3*c^2*integrate(1/32*x^2*arctan(c*x)*l
og(c^2*x^2 + 1)^2/(c^3*d*x^2 + c*d), x) + 3072*a*b^2*c^2*integrate(1/32*x^
2*arctan(c*x)^2/(c^3*d*x^2 + c*d), x) + 384*b^3*c^2*integrate(1/32*x^2*arc
tan(c*x)*log(c^2*x^2 + 1)/(c^3*d*x^2 + c*d), x) + 3072*a^2*b*c^2*integrate
(1/32*x^2*arctan(c*x)/(c^3*d*x^2 + c*d), x) - 64*b^3*c*integrate(1/32*x*lo
g(c^2*x^2 + 1)^3/(c^3*d*x^2 + c*d), x) - 384*b^3*c*integrate(1/32*x*arctan
(c*x)^2/(c^3*d*x^2 + c*d), x) + 96*b^3*c*integrate(1/32*x*log(c^2*x^2 + 1)
^2/(c^3*d*x^2 + c*d), x) + 3*b^3*arctan(c*x)^4/(c^2*d) + 96*b^3*integrate(
1/32*arctan(c*x)*log(c^2*x^2 + 1)^2/(c^3*d*x^2 + c*d), x))*c^2*d + 128*c^2
*d*integrate(1/64*(80*b^3*c*x*arctan(c*x)^3 + 192*a^2*b*c*x*arctan(c*x) +
(b^3*c^2*x^2 + 3*b^3)*log(c^2*x^2 + 1)^3 - 24*(b^3*c^2*x^2 - 8*a*b^2*c*x)*
arctan(c*x)^2 + 6*(b^3*c^2*x^2 + 2*b^3*c*x*arctan(c*x))*log(c^2*x^2 + 1)^2
 - 12*(2*b^3*c*x*arctan(c*x) + (b^3*c^2*x^2 - b^3)*arctan(c*x)^2)*log(c^2*
x^2 + 1))/(c^3*d*x^2 + c*d), x) - 2*(b^3*c*x + 2*b^3*arctan(c*x))*log(c^2*
x^2 + 1)^3 + 8*(3*b^3*c*x*arctan(c*x)^2 - 2*b^3*arctan(c*x)^3)*log(c^2*x^2
 + 1))/(c^2*d)
```

## Giac [F]

$$\int \frac{x(a+b\arctan(cx))^3}{d+icdx}\,dx = \int \frac{(b\arctan(cx)+a)^3 x}{i\,cdx+d}\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))^3/(d+I*c*d*x),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)^3*x/(I*c*d*x + d), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\arctan(cx))^3}{d+icdx}\,dx = \int \frac{x\,(a+b\,\mathrm{atan}(c\,x))^3}{d+c\,d\,x\,1\mathrm{i}}\,dx$$

input
```
int((x*(a + b*atan(c*x))^3)/(d + c*d*x*1i),x)
```

output
```
int((x*(a + b*atan(c*x))^3)/(d + c*d*x*1i), x)
```

## Reduce [F]

$$\int \frac{x(a+b\arctan(cx))^3}{d+icdx}\,dx = \frac{3\left(\int \frac{atan(cx)x}{cix+1}dx\right)a^2b\,c^2+\left(\int \frac{atan(cx)^3x}{cix+1}dx\right)b^3c^2+3\left(\int \frac{atan(cx)^2x}{cix+1}dx\right)a\,b^2c^2+\log(cix+1)\,a^3-a^3cix}{c^2d}$$

input
```
int(x*(a+b*atan(c*x))^3/(d+I*c*d*x),x)
```

output
```
(3*int((atan(c*x)*x)/(c*i*x + 1),x)*a**2*b*c**2 + int((atan(c*x)**3*x)/(c*
i*x + 1),x)*b**3*c**2 + 3*int((atan(c*x)**2*x)/(c*i*x + 1),x)*a*b**2*c**2
+ log(c*i*x + 1)*a**3 - a**3*c*i*x)/(c**2*d)
```

# 3.129 $\int \frac{(a+b\arctan(cx))^3}{d+icdx}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 139

$$\int \frac{(a+b\arctan(cx))^3}{d+icdx}\,dx = \frac{i(a+b\arctan(cx))^3\log\left(\frac{2}{1+icx}\right)}{cd} - \frac{3b(a+b\arctan(cx))^2\,\text{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{2cd} + \frac{3ib^2(a+b\arctan(cx))\,\text{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2cd} + \frac{3b^3\,\text{PolyLog}\left(4,1-\frac{2}{1+icx}\right)}{4cd}$$

output
```
I*(a+b*arctan(c*x))^3*ln(2/(1+I*c*x))/c/d-3/2*b*(a+b*arctan(c*x))^2*polylo
g(2,1-2/(1+I*c*x))/c/d+3/2*I*b^2*(a+b*arctan(c*x))*polylog(3,1-2/(1+I*c*x)
)/c/d+3/4*b^3*polylog(4,1-2/(1+I*c*x))/c/d
```

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.96

$$\int \frac{(a+b\arctan(cx))^3}{d+icdx}\,dx$$
$$= \frac{i\left(4(a+b\arctan(cx))^3\log\left(\frac{2d}{d+icdx}\right)+3ib\left(2(a+b\arctan(cx))^2\operatorname{PolyLog}\left(2,\frac{i+cx}{-i+cx}\right)-b\left(2i(a+b\arctan(cx\right.\right.\right.}{4cd}$$

input
```
Integrate[(a + b*ArcTan[c*x])^3/(d + I*c*d*x),x]
```

output
```
((I/4)*(4*(a + b*ArcTan[c*x])^3*Log[(2*d)/(d + I*c*d*x)] + (3*I)*b*(2*(a +
 b*ArcTan[c*x])^2*PolyLog[2, (I + c*x)/(-I + c*x)] - b*((2*I)*(a + b*ArcTa
n[c*x])*PolyLog[3, (I + c*x)/(-I + c*x)] + b*PolyLog[4, (I + c*x)/(-I + c*
x)]))))/(c*d)
```

## Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {5379, 5529, 5533, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^3}{d+icdx}\,dx$$
$$\Big\downarrow \; 5379$$
$$\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^3}{cd}-\frac{3ib\int \frac{(a+b\arctan(cx))^2\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx}{d}$$
$$\Big\downarrow \; 5529$$

$$\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^3}{cd}-\frac{3ib\left(ib\int\frac{(a+b\arctan(cx))\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{c^2x^2+1}dx-\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))^2}{2c}\right)}{d}$$

$\Big\downarrow$ 5533

$$\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^3}{cd}-\frac{3ib\left(ib\left(\frac{i\,\text{PolyLog}\left(3,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{2c}-\frac{1}{2}ib\int\frac{\text{PolyLog}\left(3,1-\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)-\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))^2}{2c}\right)}{d}$$

$\Big\downarrow$ 7164

$$\frac{i\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))^3}{cd}-\frac{3ib\left(ib\left(\frac{i\,\text{PolyLog}\left(3,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{2c}+\frac{b\,\text{PolyLog}\left(4,1-\frac{2}{icx+1}\right)}{4c}\right)-\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))^2}{2c}\right)}{d}$$

input
```
Int[(a + b*ArcTan[c*x])^3/(d + I*c*d*x),x]
```

output
```
(I*(a + b*ArcTan[c*x])^3*Log[2/(1 + I*c*x)])/(c*d) - ((3*I)*b*(((-1/2*I)*(
a + b*ArcTan[c*x])^2*PolyLog[2, 1 - 2/(1 + I*c*x)])/c + I*b*(((I/2)*(a + b
*ArcTan[c*x])*PolyLog[3, 1 - 2/(1 + I*c*x)])/c + (b*PolyLog[4, 1 - 2/(1 +
I*c*x)])/(4*c))))/d
```

## Defintions of rubi rules used

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 5533
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k + 1
, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &&
EqQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.99 (sec) , antiderivative size = 1629, normalized size of antiderivative = 11.72

| method | result | size |
|---|---|---|
| derivativedivides | Expression too large to display | 1629 |
| default | Expression too large to display | 1629 |
| parts | Expression too large to display | 1640 |

input
```
int((a+b*arctan(c*x))^3/(d+I*c*d*x),x,method=_RETURNVERBOSE)
```

output
```
1/c*(-1/2*I*a^3/d*ln(c^2*x^2+1)+a^3/d*arctan(c*x)+b^3/d*(-I*ln(1+I*c*x)*ar
ctan(c*x)^3+3*I*(1/3*arctan(c*x)^3*ln(2*I*(1+I*c*x)^2/(c^2*x^2+1))-1/6*I*a
rctan(c*x)^4+1/6*I*Pi*(csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^
2+1)))^3+csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn((1+I*c*x)^2/(c^2*x^2+1)/
(1+(1+I*c*x)^2/(c^2*x^2+1)))^2-csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x
)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2-csgn((1+I*c*x)^2/(c^2*x^2+1
)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)
^2/(c^2*x^2+1)))^2-csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn((1+I*c*x)^2/(c
^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))-csgn(
I*(1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^3+csgn((1+I*c*x)^2/
(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/(1
+(1+I*c*x)^2/(c^2*x^2+1)))+csgn(I*(1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(
c^2*x^2+1)))^2-1)*arctan(c*x)^3-1/2*I*arctan(c*x)^2*polylog(2,-(1+I*c*x)^2
/(c^2*x^2+1))+1/2*arctan(c*x)*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))+1/4*I*po
lylog(4,-(1+I*c*x)^2/(c^2*x^2+1))))+3*a*b^2/d*(-I*ln(1+I*c*x)*arctan(c*x)^
2+2*I*(1/2*arctan(c*x)^2*ln(2*I*(1+I*c*x)^2/(c^2*x^2+1))+1/4*I*Pi*(csgn((1
+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^3+csgn(I/(1+(1+I*c*x)^2
/(c^2*x^2+1)))*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2
-csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2
/(c^2*x^2+1)))^2-csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1...
```

## Fricas [F]

$$\int \frac{(a+b\arctan(cx))^3}{d+icdx}\,dx=\int \frac{(b\arctan(cx)+a)^3}{i\,cdx+d}\,dx$$

input
```
integrate((a+b*arctan(c*x))^3/(d+I*c*d*x),x, algorithm="fricas")
```

output
```
integral(-1/8*(b^3*log(-(c*x + I)/(c*x - I))^3 - 6*I*a*b^2*log(-(c*x + I)/
(c*x - I))^2 - 12*a^2*b*log(-(c*x + I)/(c*x - I)) + 8*I*a^3)/(c*d*x - I*d)
, x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^3}{d+icdx}\,dx = \text{Timed out}$$

input
```
integrate((a+b*atan(c*x))**3/(d+I*c*d*x),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(a+b\arctan(cx))^3}{d+icdx}\,dx = \int \frac{(b\arctan(cx)+a)^3}{i\,cdx+d}\,dx$$

input
```
integrate((a+b*arctan(c*x))^3/(d+I*c*d*x),x, algorithm="maxima")
```

output
```
-I*a^3*log(I*c*d*x + d)/(c*d) + 1/128*(16*b^3*arctan(c*x)^4 + 16*I*b^3*arc
tan(c*x)^3*log(c^2*x^2 + 1) + 4*I*b^3*arctan(c*x)*log(c^2*x^2 + 1)^3 - b^3
*log(c^2*x^2 + 1)^4 + 16*(b^3*arctan(c*x)^4/(c*d) + 8*b^3*c*integrate(1/16
*x*log(c^2*x^2 + 1)^3/(c^2*d*x^2 + d), x) + 8*a*b^2*arctan(c*x)^3/(c*d) +
12*a^2*b*arctan(c*x)^2/(c*d))*c*d - 128*I*c*d*integrate(1/32*(40*b^3*c*x*a
rctan(c*x)^3 + 6*b^3*c*x*arctan(c*x)*log(c^2*x^2 + 1)^2 + 96*a*b^2*c*x*arc
tan(c*x)^2 + 96*a^2*b*c*x*arctan(c*x) + 12*b^3*arctan(c*x)^2*log(c^2*x^2 +
 1) + b^3*log(c^2*x^2 + 1)^3)/(c^2*d*x^2 + d), x))/(c*d)
```

## Giac [F]

$$\int \frac{(a+b\arctan(cx))^3}{d+icdx}\,dx = \int \frac{(b\arctan(cx)+a)^3}{i\,cdx+d}\,dx$$

input
```
integrate((a+b*arctan(c*x))^3/(d+I*c*d*x),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)^3/(I*c*d*x + d), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^3}{d+icdx}\,dx=\int \frac{(a+b\,\mathrm{atan}(c\,x))^3}{d+c\,d\,x\,1\mathrm{i}}\,dx$$

input
```
int((a + b*atan(c*x))^3/(d + c*d*x*1i),x)
```

output
```
int((a + b*atan(c*x))^3/(d + c*d*x*1i), x)
```

## Reduce [F]

$$\int \frac{(a+b\arctan(cx))^3}{d+icdx}\,dx = \frac{3\left(\int \frac{atan(cx)}{cix+1}dx\right)a^2bc+\left(\int \frac{atan(cx)^3}{cix+1}dx\right)b^3c+3\left(\int \frac{atan(cx)^2}{cix+1}dx\right)a\,b^2c-\log(cix+1)\,a^3i}{cd}$$

input
```
int((a+b*atan(c*x))^3/(d+I*c*d*x),x)
```

output
```
(3*int(atan(c*x)/(c*i*x + 1),x)*a**2*b*c + int(atan(c*x)**3/(c*i*x + 1),x)
*b**3*c + 3*int(atan(c*x)**2/(c*i*x + 1),x)*a*b**2*c - log(c*i*x + 1)*a**3
*i)/(c*d)
```

# 3.130 $\int \frac{(a+b\arctan(cx))^3}{x(d+icdx)}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 128

$$\int \frac{(a+b\arctan(cx))^3}{x(d+icdx)}\,dx = \frac{(a+b\arctan(cx))^3\log\left(2-\frac{2}{1+icx}\right)}{d} + \frac{3ib(a+b\arctan(cx))^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)}{2d} + \frac{3b^2(a+b\arctan(cx))\operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)}{2d} - \frac{3ib^3\operatorname{PolyLog}\left(4,-1+\frac{2}{1+icx}\right)}{4d}$$

output
```
(a+b*arctan(c*x))^3*ln(2-2/(1+I*c*x))/d+3/2*I*b*(a+b*arctan(c*x))^2*polylo
g(2,-1+2/(1+I*c*x))/d+3/2*b^2*(a+b*arctan(c*x))*polylog(3,-1+2/(1+I*c*x))/
d-3/4*I*b^3*polylog(4,-1+2/(1+I*c*x))/d
```

## Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 268 vs. $2(128) = 256$.

Time = 0.54 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.09

$$\int \frac{(a+b\arctan(cx))^3}{x(d+icdx)}\,dx =$$
$$-\frac{i\left(8ab^2\pi^3+b^3\pi^4+64a^3\arctan(cx)+192a^2b\arctan(cx)^2+192iab^2\arctan(cx)^2\log\left(1-e^{-2i\arctan(cx)}\right)\right.}{}$$

input
```
Integrate[(a + b*ArcTan[c*x])^3/(x*(d + I*c*d*x)),x]
```

output
```
((-1/64*I)*(8*a*b^2*Pi^3 + b^3*Pi^4 + 64*a^3*ArcTan[c*x] + 192*a^2*b*ArcTa
n[c*x]^2 + (192*I)*a*b^2*ArcTan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] + (
64*I)*b^3*ArcTan[c*x]^3*Log[1 - E^((-2*I)*ArcTan[c*x])] + (192*I)*a^2*b*Ar
cTan[c*x]*Log[1 - E^((2*I)*ArcTan[c*x])] + (64*I)*a^3*Log[c*x] - (32*I)*a^
3*Log[1 + c^2*x^2] - 96*b^2*ArcTan[c*x]*(2*a + b*ArcTan[c*x])*PolyLog[2, E
^((-2*I)*ArcTan[c*x])] + 96*a^2*b*PolyLog[2, E^((2*I)*ArcTan[c*x])] + (96*
I)*a*b^2*PolyLog[3, E^((-2*I)*ArcTan[c*x])] + (96*I)*b^3*ArcTan[c*x]*PolyL
og[3, E^((-2*I)*ArcTan[c*x])] + 48*b^3*PolyLog[4, E^((-2*I)*ArcTan[c*x])])
)/d
```

## Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {5403, 5529, 5533, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^3}{x(d+icdx)}\,dx$$
$$\Big\downarrow \; 5403$$

$$\frac{\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^3}{d}-\frac{3bc\int\frac{(a+b\arctan(cx))^2\log\left(2-\frac{2}{icx+1}\right)}{c^2x^2+1}dx}{d}$$

$\Big\downarrow$ 5529

$$\frac{\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^3}{d}-\frac{3bc\left(ib\int\frac{(a+b\arctan(cx))\operatorname{PolyLog}\left(2,\frac{2}{icx+1}-1\right)}{c^2x^2+1}dx-\frac{i\operatorname{PolyLog}\left(2,\frac{2}{icx+1}-1\right)(a+b\arctan(cx))^2}{2c}\right)}{d}$$

$\Big\downarrow$ 5533

$$\frac{\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^3}{d}-\frac{3bc\left(ib\left(\frac{i\operatorname{PolyLog}\left(3,\frac{2}{icx+1}-1\right)(a+b\arctan(cx))}{2c}-\frac{1}{2}ib\int\frac{\operatorname{PolyLog}\left(3,\frac{2}{icx+1}-1\right)}{c^2x^2+1}dx\right)-\frac{i\operatorname{PolyLog}\left(2,\frac{2}{icx+1}-1\right)(a+b\arctan(cx))^2}{2c}\right)}{d}$$

$\Big\downarrow$ 7164

$$\frac{\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^3}{d}-\frac{3bc\left(ib\left(\frac{i\operatorname{PolyLog}\left(3,\frac{2}{icx+1}-1\right)(a+b\arctan(cx))}{2c}+\frac{b\operatorname{PolyLog}\left(4,\frac{2}{icx+1}-1\right)}{4c}\right)-\frac{i\operatorname{PolyLog}\left(2,\frac{2}{icx+1}-1\right)(a+b\arctan(cx))^2}{2c}\right)}{d}$$

input
```
Int[(a + b*ArcTan[c*x])^3/(x*(d + I*c*d*x)),x]
```

output
```
((a + b*ArcTan[c*x])^3*Log[2 - 2/(1 + I*c*x)])/d - (3*b*c*(((-1/2*I)*(a +
b*ArcTan[c*x])^2*PolyLog[2, -1 + 2/(1 + I*c*x)])/c + I*b*(((I/2)*(a + b*Ar
cTan[c*x])*PolyLog[3, -1 + 2/(1 + I*c*x)])/c + (b*PolyLog[4, -1 + 2/(1 + I
*c*x)])/(4*c))))/d
```

### Defintions of rubi rules used

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 5533
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k + 1
, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &&
EqQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 16.38 (sec) , antiderivative size = 2791, normalized size of antiderivative = 21.80

| method | result | size |
|---|---|---|
| parts | Expression too large to display | 2791 |
| derivativedivides | Expression too large to display | 2793 |
| default | Expression too large to display | 2793 |

input
```
int((a+b*arctan(c*x))^3/x/(d+I*c*d*x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^3/d*ln(c^2*x^2+1)-I*a^3/d*arctan(c*x)+a^3/d*ln(x)+b^3/d*(arctan(c*x
)^3*ln(c*x)-ln(c*x-I)*arctan(c*x)^3+arctan(c*x)^3*ln(2*I*(1+I*c*x)^2/(c^2*
x^2+1))-1/2*I*arctan(c*x)^4-arctan(c*x)^3*ln((1+I*c*x)^2/(c^2*x^2+1)-1)+ar
ctan(c*x)^3*ln(1-(1+I*c*x)/(c^2*x^2+1)^(1/2))-3*I*arctan(c*x)^2*polylog(2,
(1+I*c*x)/(c^2*x^2+1)^(1/2))+6*arctan(c*x)*polylog(3,(1+I*c*x)/(c^2*x^2+1)
^(1/2))+6*I*polylog(4,(1+I*c*x)/(c^2*x^2+1)^(1/2))+arctan(c*x)^3*ln(1+(1+I
*c*x)/(c^2*x^2+1)^(1/2))-3*I*arctan(c*x)^2*polylog(2,-(1+I*c*x)/(c^2*x^2+1
)^(1/2))+6*arctan(c*x)*polylog(3,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+6*I*polylog
(4,-(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/2*I*Pi*(csgn((1+I*c*x)^2/(c^2*x^2+1)/(1
+(1+I*c*x)^2/(c^2*x^2+1)))^3-csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn((1+I
*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2+csgn((1+I*c*x)^2/(c^2*x
^2+1))*csgn((1+I*c*x)^2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2+csgn(I/
(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn((1+I*c*x)^2/(c^2*x^2+1))*csgn((1+I*c*x)^
2/(c^2*x^2+1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))-csgn(I*((1+I*c*x)^2/(c^2*x^2+1)
-1))*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1
)/(1+(1+I*c*x)^2/(c^2*x^2+1)))+csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I*
((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2+csgn(I/(1+(1+I*
c*x)^2/(c^2*x^2+1)))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^
2*x^2+1)))^2-csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)
))^3+csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*cs...
```

## Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 246 vs. 2(109) = 218.

Time = 0.12 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.92

$$\int \frac{(a+b\arctan(cx))^3}{x(d+icdx)}\,dx$$
$$= \frac{-3i\,b^3\mathrm{Li}_2\left(-\frac{2\,cx}{cx-i}+1\right)\log\left(-\frac{cx+i}{cx-i}\right)^2 - 12\,ab^2\mathrm{Li}_2\left(-\frac{2\,cx}{cx-i}+1\right)\log\left(-\frac{cx+i}{cx-i}\right) - 12i\,a^2b\mathrm{Li}_2\left(\frac{cx+i}{cx-i}+1\right) + 8\,a^3\log\cdots}{\cdots}$$

input
```
integrate((a+b*arctan(c*x))^3/x/(d+I*c*d*x),x, algorithm="fricas")
```

output
```
1/8*(-3*I*b^3*dilog(-2*c*x/(c*x - I) + 1)*log(-(c*x + I)/(c*x - I))^2 - 12
*a*b^2*dilog(-2*c*x/(c*x - I) + 1)*log(-(c*x + I)/(c*x - I)) - 12*I*a^2*b*
dilog((c*x + I)/(c*x - I) + 1) + 8*a^3*log(x) - 8*a^3*log((c*x - I)/c) - 6
*I*b^3*polylog(4, -(c*x + I)/(c*x - I)) + (-I*b^3*log(-(c*x + I)/(c*x - I)
)^3 - 6*a*b^2*log(-(c*x + I)/(c*x - I))^2)*log(2*c*x/(c*x - I)) - 6*(-I*b^
3*log(-(c*x + I)/(c*x - I)) - 2*a*b^2)*polylog(3, -(c*x + I)/(c*x - I)))/d
```

## Sympy [F]

$$\int \frac{(a+b\arctan(cx))^3}{x(d+icdx)}\,dx = -\frac{i\left(\int \frac{a^3}{cx^2-ix}\,dx + \int \frac{b^3\operatorname{atan}^3(cx)}{cx^2-ix}\,dx + \int \frac{3ab^2\operatorname{atan}^2(cx)}{cx^2-ix}\,dx + \int \frac{3a^2b\operatorname{atan}(cx)}{cx^2-ix}\,dx\right)}{d}$$

input
```
integrate((a+b*atan(c*x))**3/x/(d+I*c*d*x),x)
```

output
```
-I*(Integral(a**3/(c*x**2 - I*x), x) + Integral(b**3*atan(c*x)**3/(c*x**2
- I*x), x) + Integral(3*a*b**2*atan(c*x)**2/(c*x**2 - I*x), x) + Integral(
3*a**2*b*atan(c*x)/(c*x**2 - I*x), x))/d
```

## Maxima [F]

$$\int \frac{(a+b\arctan(cx))^3}{x(d+icdx)}\,dx = \int \frac{(b\arctan(cx)+a)^3}{(i\,cdx+d)x}\,dx$$

input
```
integrate((a+b*arctan(c*x))^3/x/(d+I*c*d*x),x, algorithm="maxima")
```

output
```
-a^3*(log(I*c*x + 1)/d - log(x)/d) + 1/512*(-64*I*b^3*arctan(c*x)^4 + 64*b
^3*arctan(c*x)^3*log(c^2*x^2 + 1) + 16*b^3*arctan(c*x)*log(c^2*x^2 + 1)^3
+ 4*I*b^3*log(c^2*x^2 + 1)^4 - I*(64*b^3*arctan(c*x)^4/d + 6144*b^3*c^2*in
tegrate(1/64*x^2*arctan(c*x)^2*log(c^2*x^2 + 1)/(c^2*d*x^3 + d*x), x) + 3*
b^3*log(c^2*x^2 + 1)^4/d + 512*a*b^2*arctan(c*x)^3/d + 768*a^2*b*arctan(c*
x)^2/d + 6144*b^3*integrate(1/64*arctan(c*x)^2*log(c^2*x^2 + 1)/(c^2*d*x^3
 + d*x), x) - 512*b^3*integrate(1/64*log(c^2*x^2 + 1)^3/(c^2*d*x^3 + d*x),
 x))*d - 512*d*integrate(1/32*(12*b^3*c*x*arctan(c*x)^2*log(c^2*x^2 + 1) +
 b^3*c*x*log(c^2*x^2 + 1)^3 - 96*a*b^2*arctan(c*x)^2 - 96*a^2*b*arctan(c*x
) + 4*(3*b^3*c^2*x^2 - 7*b^3)*arctan(c*x)^3 + 3*(b^3*c^2*x^2 - b^3)*arctan
(c*x)*log(c^2*x^2 + 1)^2)/(c^2*d*x^3 + d*x), x))/d
```

## Giac [F]

$$\int \frac{(a+b\arctan(cx))^3}{x(d+icdx)}\,dx=\int \frac{(b\arctan(cx)+a)^3}{(i\,cdx+d)x}\,dx$$

input
```
integrate((a+b*arctan(c*x))^3/x/(d+I*c*d*x),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)^3/((I*c*d*x + d)*x), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^3}{x(d+icdx)}\,dx=\int \frac{(a+b\,\mathrm{atan}(c\,x))^3}{x\,\left(d+c\,d\,x\,1\mathrm{i}\right)}\,dx$$

input
```
int((a + b*atan(c*x))^3/(x*(d + c*d*x*1i)),x)
```

output
```
int((a + b*atan(c*x))^3/(x*(d + c*d*x*1i)), x)
```

## Reduce [F]

$$\int \frac{(a+b\arctan(cx))^3}{x(d+icdx)}\,dx$$
$$= \frac{-atan(cx)^4 b^3i - 4atan(cx)^3 a b^2i - 6atan(cx)^2 a^2bi - 12\left(\int \frac{atan(cx)}{c^3x^4-c^2i x^3+c x^2-ix}dx\right) a^2bi + 12\left(\int \frac{atan(cx)}{c^3x^3-c^2i x^2 \cdots}\right.}{}$$

input
```
int((a+b*atan(c*x))^3/x/(d+I*c*d*x),x)
```

output
```
( - atan(c*x)**4*b**3*i - 4*atan(c*x)**3*a*b**2*i - 6*atan(c*x)**2*a**2*b*
i - 12*int(atan(c*x)/(c**3*x**4 - c**2*i*x**3 + c*x**2 - i*x),x)*a**2*b*i
+ 12*int(atan(c*x)/(c**3*x**3 - c**2*i*x**2 + c*x - i),x)*a**2*b*c - 4*int
(atan(c*x)**3/(c**3*x**4 - c**2*i*x**3 + c*x**2 - i*x),x)*b**3*i + 4*int(a
tan(c*x)**3/(c**3*x**3 - c**2*i*x**2 + c*x - i),x)*b**3*c - 12*int(atan(c*
x)**2/(c**3*x**4 - c**2*i*x**3 + c*x**2 - i*x),x)*a*b**2*i + 12*int(atan(c
*x)**2/(c**3*x**3 - c**2*i*x**2 + c*x - i),x)*a*b**2*c - 4*log(c*i*x + 1)*
a**3 + 4*log(x)*a**3)/(4*d)
```

# 3.131 $\int \frac{(a+b\arctan(cx))^3}{x^2(d+icdx)}\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 263

$$\begin{aligned}\int \frac{(a+b\arctan(cx))^3}{x^2(d+icdx)}\,dx =& -\frac{ic(a+b\arctan(cx))^3}{d}-\frac{(a+b\arctan(cx))^3}{dx}\\
&+\frac{3bc(a+b\arctan(cx))^2\log\left(2-\frac{2}{1-icx}\right)}{d}\\
&-\frac{ic(a+b\arctan(cx))^3\log\left(2-\frac{2}{1+icx}\right)}{d}\\
&-\frac{3ib^2c(a+b\arctan(cx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)}{d}\\
&+\frac{3bc(a+b\arctan(cx))^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)}{2d}\\
&+\frac{3b^3c\operatorname{PolyLog}\left(3,-1+\frac{2}{1-icx}\right)}{2d}\\
&-\frac{3ib^2c(a+b\arctan(cx))\operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)}{2d}\\
&-\frac{3b^3c\operatorname{PolyLog}\left(4,-1+\frac{2}{1+icx}\right)}{4d}\end{aligned}$$

output
```
-I*c*(a+b*arctan(c*x))^3/d-(a+b*arctan(c*x))^3/d/x+3*b*c*(a+b*arctan(c*x))
^2*ln(2-2/(1-I*c*x))/d-I*c*(a+b*arctan(c*x))^3*ln(2-2/(1+I*c*x))/d-3*I*b^2
*c*(a+b*arctan(c*x))*polylog(2,-1+2/(1-I*c*x))/d+3/2*b*c*(a+b*arctan(c*x))
^2*polylog(2,-1+2/(1+I*c*x))/d+3/2*b^3*c*polylog(3,-1+2/(1-I*c*x))/d-3/2*I
*b^2*c*(a+b*arctan(c*x))*polylog(3,-1+2/(1+I*c*x))/d-3/4*b^3*c*polylog(4,-
1+2/(1+I*c*x))/d
```

## Mathematica [A] (verified)

Time = 1.11 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.66

$$\int \frac{(a+b\arctan(cx))^3}{x^2(d+icdx)}\,dx =$$

$$-\frac{\frac{2a^3}{x}+2a^3c\arctan(cx)+2ia^3c\log(x)-ia^3c\log\left(1+c^2x^2\right)+3a^2bc\Big(2\Big(\arctan(cx)^2+\arctan(cx)\left(\frac{1}{cx}+\right.}{}$$

input
```
Integrate[(a + b*ArcTan[c*x])^3/(x^2*(d + I*c*d*x)),x]
```

output
```
-1/2*((2*a^3)/x + 2*a^3*c*ArcTan[c*x] + (2*I)*a^3*c*Log[x] - I*a^3*c*Log[1
 + c^2*x^2] + 3*a^2*b*c*(2*(ArcTan[c*x]^2 + ArcTan[c*x]*(1/(c*x) + I*Log[1
 - E^((2*I)*ArcTan[c*x])]) - Log[(c*x)/Sqrt[1 + c^2*x^2]]) + PolyLog[2, E^
((2*I)*ArcTan[c*x])]) + (6*I)*a*b^2*c*((-1/24*I)*Pi^3 + ArcTan[c*x]^2 - (I
*ArcTan[c*x]^2)/(c*x) + ArcTan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] + (2
*I)*ArcTan[c*x]*Log[1 - E^((2*I)*ArcTan[c*x])] + I*ArcTan[c*x]*PolyLog[2,
E^((-2*I)*ArcTan[c*x])] + PolyLog[2, E^((2*I)*ArcTan[c*x])] + PolyLog[3, E
^((-2*I)*ArcTan[c*x])]/2) + (2*I)*b^3*c*(Pi^3/8 - (I/64)*Pi^4 - ArcTan[c*x
]^3 - (I*ArcTan[c*x]^3)/(c*x) + (3*I)*ArcTan[c*x]^2*Log[1 - E^((-2*I)*ArcT
an[c*x])] + ArcTan[c*x]^3*Log[1 - E^((-2*I)*ArcTan[c*x])] + ((3*I)/2)*ArcT
an[c*x]*(2*I + ArcTan[c*x])*PolyLog[2, E^((-2*I)*ArcTan[c*x])] + (3*(I + A
rcTan[c*x])*PolyLog[3, E^((-2*I)*ArcTan[c*x])])/2 - ((3*I)/4)*PolyLog[4, E
^((-2*I)*ArcTan[c*x])]))/d
```

## Rubi [A] (verified)

Time = 2.04 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5405, 27, 5361, 5403, 5459, 5403, 5527, 5529, 5533, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^3}{x^2(d+icdx)}dx$$

$$\Big\downarrow \text{ 5405}$$

$$\frac{\int \frac{(a+b\arctan(cx))^3}{x^2}dx}{d}-ic\int \frac{(a+b\arctan(cx))^3}{dx(icx+1)}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{(a+b\arctan(cx))^3}{x^2}dx}{d}-\frac{ic\int \frac{(a+b\arctan(cx))^3}{x(icx+1)}dx}{d}$$

$$\Big\downarrow \text{ 5361}$$

$$\frac{3bc\int \frac{(a+b\arctan(cx))^2}{x(c^2x^2+1)}dx-\frac{(a+b\arctan(cx))^3}{x}}{d}-\frac{ic\int \frac{(a+b\arctan(cx))^3}{x(icx+1)}dx}{d}$$

$$\Big\downarrow \text{ 5403}$$

$$\frac{3bc\int \frac{(a+b\arctan(cx))^2}{x(c^2x^2+1)}dx-\frac{(a+b\arctan(cx))^3}{x}}{d}-\frac{ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^3-3bc\int \frac{(a+b\arctan(cx))^2\log\left(2-\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{d}$$

$$\Big\downarrow \text{ 5459}$$

$$\frac{-\frac{(a+b\arctan(cx))^3}{x}+3bc\left(i\int \frac{(a+b\arctan(cx))^2}{x(cx+i)}dx-\frac{i(a+b\arctan(cx))^3}{3b}\right)}{d}-\frac{ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^3-3bc\int \frac{(a+b\arctan(cx))^2\log\left(2-\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{d}$$

$$\Big\downarrow \text{ 5403}$$

$$\frac{-\frac{(a+b\arctan(cx))^3}{x}+3bc\left(i\left(2ibc\int\frac{(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right)}{c^2x^2+1}dx-i\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))^2\right)-\frac{i(a+b\,\mathrm{ar}}{}\right.}{d}$$

$$\frac{ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^3-3bc\int\frac{(a+b\arctan(cx))^2\log\left(2-\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{d}$$

$\big\downarrow$ 5527

$$\frac{-\frac{(a+b\arctan(cx))^3}{x}+3bc\left(i\left(2ibc\left(\frac{i\,\mathrm{PolyLog}\left(2,\frac{2}{1-icx}-1\right)(a+b\arctan(cx))}{2c}-\frac{1}{2}ib\int\frac{\mathrm{PolyLog}\left(2,\frac{2}{1-icx}-1\right)}{c^2x^2+1}dx\right)-i\log\left(2-\frac{2}{1-i\cdots}\right.\right.\right.}{d}$$

$$\frac{ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^3-3bc\int\frac{(a+b\arctan(cx))^2\log\left(2-\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)}{d}$$

$\big\downarrow$ 5529

$$\frac{-\frac{(a+b\arctan(cx))^3}{x}+3bc\left(i\left(2ibc\left(\frac{i\,\mathrm{PolyLog}\left(2,\frac{2}{1-icx}-1\right)(a+b\arctan(cx))}{2c}-\frac{1}{2}ib\int\frac{\mathrm{PolyLog}\left(2,\frac{2}{1-icx}-1\right)}{c^2x^2+1}dx\right)-i\log\left(2-\frac{2}{1-i\cdots}\right.\right.\right.}{d}$$

$$\frac{ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^3-3bc\left(ib\int\frac{(a+b\arctan(cx))\,\mathrm{PolyLog}\left(2,\frac{2}{icx+1}-1\right)}{c^2x^2+1}dx-\frac{i\,\mathrm{PolyLog}\left(2,\frac{2}{icx+1}-1\right)(a+b\,\mathrm{ar}\cdots}{2c}\right.\right.}{d}$$

$\big\downarrow$ 5533

$$\frac{-\frac{(a+b\arctan(cx))^3}{x}+3bc\left(i\left(2ibc\left(\frac{i\,\mathrm{PolyLog}\left(2,\frac{2}{1-icx}-1\right)(a+b\arctan(cx))}{2c}-\frac{1}{2}ib\int\frac{\mathrm{PolyLog}\left(2,\frac{2}{1-icx}-1\right)}{c^2x^2+1}dx\right)-i\log\left(2-\frac{2}{1-i\cdots}\right.\right.\right.}{d}$$

$$\frac{ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^3-3bc\left(ib\left(\frac{i\,\mathrm{PolyLog}\left(3,\frac{2}{icx+1}-1\right)(a+b\arctan(cx))}{2c}-\frac{1}{2}ib\int\frac{\mathrm{PolyLog}\left(3,\frac{2}{icx+1}-1\right)}{c^2x^2+1}dx\right)\right.\right.}{d}$$

$\big\downarrow$ 7164

$$-\frac{(a+b\arctan(cx))^3}{x}+3bc\left(i\left(2ibc\left(\frac{i\,\mathrm{PolyLog}\left(2,\frac{2}{1-icx}-1\right)(a+b\arctan(cx))}{2c}-\frac{b\,\mathrm{PolyLog}\left(3,\frac{2}{1-icx}-1\right)}{4c}\right)-i\log\left(2-\frac{2}{1-icx}\right)(a\right.\right.\right.$$
$$\overline{\qquad\qquad d\qquad\qquad}$$

$$ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^3-3bc\left(ib\left(\frac{i\,\mathrm{PolyLog}\left(3,\frac{2}{icx+1}-1\right)(a+b\arctan(cx))}{2c}+\frac{b\,\mathrm{PolyLog}\left(4,\frac{2}{icx+1}-1\right)}{4c}\right)-\frac{i\,\mathrm{Po}}{}\right.\right.$$
$$\overline{\qquad\qquad d\qquad\qquad}$$

input
```
Int[(a + b*ArcTan[c*x])^3/(x^2*(d + I*c*d*x)),x]
```

output
```
(-((a + b*ArcTan[c*x])^3/x) + 3*b*c*(((-1/3*I)*(a + b*ArcTan[c*x])^3)/b +
I*((-I)*(a + b*ArcTan[c*x])^2*Log[2 - 2/(1 - I*c*x)] + (2*I)*b*c*(((I/2)*(
a + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 - I*c*x)])/c - (b*PolyLog[3, -1 +
2/(1 - I*c*x)])/(4*c)))))/d - (I*c*((a + b*ArcTan[c*x])^3*Log[2 - 2/(1 + I
*c*x)] - 3*b*c*(((-1/2*I)*(a + b*ArcTan[c*x])^2*PolyLog[2, -1 + 2/(1 + I*c
*x)])/c + I*b*(((I/2)*(a + b*ArcTan[c*x])*PolyLog[3, -1 + 2/(1 + I*c*x)])/
c + (b*PolyLog[4, -1 + 2/(1 + I*c*x)])/(4*c)))))/d
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5405
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x], x
] - Simp[e/(d*f)   Int[(f*x)^(m + 1)*((a + b*ArcTan[c*x])^p/(d + e*x)), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
&& LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5527
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 5533
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k + 1
, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &&
EqQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 38.25 (sec) , antiderivative size = 10105, normalized size of antiderivative = 38.42

| method | result | size |
|---|---|---|
| parts | Expression too large to display | 10105 |
| derivativedivides | Expression too large to display | 10106 |
| default | Expression too large to display | 10106 |

input
```
int((a+b*arctan(c*x))^3/x^2/(d+I*c*d*x),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

## Fricas [F]

$$\int \frac{(a+b\arctan(cx))^3}{x^2(d+icdx)}\,dx=\int \frac{(b\arctan{(cx)}+a)^3}{(i\,cdx+d)x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))^3/x^2/(d+I*c*d*x),x, algorithm="fricas")
```

output
```
-1/8*(b^3*c*x*log(2*c*x/(c*x - I))*log(-(c*x + I)/(c*x - I))^3 + 3*b^3*c*x
*dilog(-2*c*x/(c*x - I) + 1)*log(-(c*x + I)/(c*x - I))^2 - 6*b^3*c*x*log(-
(c*x + I)/(c*x - I))*polylog(3, -(c*x + I)/(c*x - I)) - I*b^3*log(-(c*x +
I)/(c*x - I))^3 + 6*b^3*c*x*polylog(4, -(c*x + I)/(c*x - I)) - 8*d*x*integ
ral(1/4*(-4*I*a^3*c*x + 4*a^3 - 3*(a*b^2 + (-I*a*b^2 + b^3)*c*x)*log(-(c*x
 + I)/(c*x - I))^2 + 6*(a^2*b*c*x + I*a^2*b)*log(-(c*x + I)/(c*x - I)))/(c
^2*d*x^4 + d*x^2), x))/(d*x)
```

## Sympy [F]

$$\int \frac{(a+b\arctan(cx))^3}{x^2(d+icdx)}\,dx = -\frac{i\left(\int \frac{a^3}{cx^3-ix^2}\,dx+\int \frac{b^3\,\mathrm{atan}^3\,(cx)}{cx^3-ix^2}\,dx+\int \frac{3ab^2\,\mathrm{atan}^2\,(cx)}{cx^3-ix^2}\,dx+\int \frac{3a^2b\,\mathrm{atan}\,(cx)}{cx^3-ix^2}\,dx\right)}{d}$$

input
```
integrate((a+b*atan(c*x))**3/x**2/(d+I*c*d*x),x)
```

output
```
-I*(Integral(a**3/(c*x**3 - I*x**2), x) + Integral(b**3*atan(c*x)**3/(c*x*
*3 - I*x**2), x) + Integral(3*a*b**2*atan(c*x)**2/(c*x**3 - I*x**2), x) +
Integral(3*a**2*b*atan(c*x)/(c*x**3 - I*x**2), x))/d
```

## Maxima [F]

$$\int \frac{(a+b\arctan(cx))^3}{x^2(d+icdx)}\,dx = \int \frac{(b\arctan(cx)+a)^3}{(i\,cdx+d)x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))^3/x^2/(d+I*c*d*x),x, algorithm="maxima")
```

output
```
a^3*(I*c*log(I*c*x + 1)/d - I*c*log(x)/d - 1/(d*x)) - 1/512*(64*b^3*c*x*ar
ctan(c*x)^4 - 4*b^3*c*x*log(c^2*x^2 + 1)^4 + 64*b^3*arctan(c*x)^3 - 48*b^3
*arctan(c*x)*log(c^2*x^2 + 1)^2 - 8*(-2*I*b^3*c*x*arctan(c*x) + I*b^3)*log
(c^2*x^2 + 1)^3 - (48*b^3*c*arctan(c*x)^4/d - 6144*b^3*c^3*integrate(1/64*
x^3*arctan(c*x)^2*log(c^2*x^2 + 1)/(c^2*d*x^4 + d*x^2), x) - 3*b^3*c*log(c
^2*x^2 + 1)^4/d + 3072*b^3*c^2*integrate(1/64*x^2*arctan(c*x)*log(c^2*x^2
+ 1)^2/(c^2*d*x^4 + d*x^2), x) - 12288*b^3*c^2*integrate(1/64*x^2*arctan(c
*x)*log(c^2*x^2 + 1)/(c^2*d*x^4 + d*x^2), x) - 6144*b^3*c*integrate(1/64*x
*arctan(c*x)^2*log(c^2*x^2 + 1)/(c^2*d*x^4 + d*x^2), x) + 512*b^3*c*integr
ate(1/64*x*log(c^2*x^2 + 1)^3/(c^2*d*x^4 + d*x^2), x) + 12288*b^3*c*integr
ate(1/64*x*arctan(c*x)^2/(c^2*d*x^4 + d*x^2), x) - 3072*b^3*c*integrate(1/
64*x*log(c^2*x^2 + 1)^2/(c^2*d*x^4 + d*x^2), x) + 28672*b^3*integrate(1/64
*arctan(c*x)^3/(c^2*d*x^4 + d*x^2), x) + 3072*b^3*integrate(1/64*arctan(c*
x)*log(c^2*x^2 + 1)^2/(c^2*d*x^4 + d*x^2), x) + 98304*a*b^2*integrate(1/64
*arctan(c*x)^2/(c^2*d*x^4 + d*x^2), x) + 98304*a^2*b*integrate(1/64*arctan
(c*x)/(c^2*d*x^4 + d*x^2), x))*d*x - 64*I*(192*b^3*c^3*integrate(1/64*x^3*
arctan(c*x)^3/(c^2*d*x^4 + d*x^2), x) + 48*b^3*c^3*integrate(1/64*x^3*arct
an(c*x)*log(c^2*x^2 + 1)^2/(c^2*d*x^4 + d*x^2), x) + b^3*c*arctan(c*x)^3/d
 + 96*b^3*c^2*integrate(1/64*x^2*arctan(c*x)^2*log(c^2*x^2 + 1)/(c^2*d*x^4
 + d*x^2), x) + 24*b^3*c^2*integrate(1/64*x^2*log(c^2*x^2 + 1)^3/(c^2*d...
```

## Giac [F]

$$\int \frac{(a+b\arctan(cx))^3}{x^2(d+icdx)}\,dx=\int \frac{(b\arctan(cx)+a)^3}{(i\,cdx+d)x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))^3/x^2/(d+I*c*d*x),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)^3/((I*c*d*x + d)*x^2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^3}{x^2(d+icdx)}\,dx=\int \frac{{\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}^3}{x^2\,\left(d+c\,d\,x\,1{}\mathrm{i}\right)}\,dx$$

input
```
int((a + b*atan(c*x))^3/(x^2*(d + c*d*x*1i)),x)
```

output
```
int((a + b*atan(c*x))^3/(x^2*(d + c*d*x*1i)), x)
```

## Reduce [F]

$$\int \frac{(a+b\arctan(cx))^3}{x^2(d+icdx)}\,dx = \frac{3\left(\int \frac{atan(cx)}{c^3i\,x^5+c^2x^4+ci\,x^3+x^2}dx\right)a^2bx+3\left(\int \frac{atan(cx)}{c^3i\,x^3+c^2x^2+cix+1}dx\right)a^2b\,c^2x+\left(\int \frac{atan(cx)^3}{c^3i\,x^5+c^2x^4+ci\,x^3+x^2}dx\right)b^3x+\left(\int \frac{}{c}\right.}{}$$

input
```
int((a+b*atan(c*x))^3/x^2/(d+I*c*d*x),x)
```

output
```
(3*int(atan(c*x)/(c**3*i*x**5 + c**2*x**4 + c*i*x**3 + x**2),x)*a**2*b*x +
 3*int(atan(c*x)/(c**3*i*x**3 + c**2*x**2 + c*i*x + 1),x)*a**2*b*c**2*x +
int(atan(c*x)**3/(c**3*i*x**5 + c**2*x**4 + c*i*x**3 + x**2),x)*b**3*x + i
nt(atan(c*x)**3/(c**3*i*x**3 + c**2*x**2 + c*i*x + 1),x)*b**3*c**2*x + 3*i
nt(atan(c*x)**2/(c**3*i*x**5 + c**2*x**4 + c*i*x**3 + x**2),x)*a*b**2*x +
3*int(atan(c*x)**2/(c**3*i*x**3 + c**2*x**2 + c*i*x + 1),x)*a*b**2*c**2*x
+ log(c*i*x + 1)*a**3*c*i*x - log(x)*a**3*c*i*x - a**3)/(d*x)
```

# 3.132 $\int \frac{(a+b\arctan(cx))^3}{x^3(d+icdx)}\,dx$

## Optimal result

Integrand size = 25, antiderivative size = 414

$$\int \frac{(a+b\arctan(cx))^3}{x^3(d+icdx)}\,dx = -\frac{3ibc^2(a+b\arctan(cx))^2}{2d} - \frac{3bc(a+b\arctan(cx))^2}{2dx} - \frac{3c^2(a+b\arctan(cx))^3}{2d} - \frac{(a+b\arctan(cx))^3}{2dx^2} + \frac{ic(a+b\arctan(cx))^3}{dx} + \frac{3b^2c^2(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right)}{d} - \frac{3ibc^2(a+b\arctan(cx))^2\log\left(2-\frac{2}{1-icx}\right)}{d} - \frac{c^2(a+b\arctan(cx))^3\log\left(2-\frac{2}{1+icx}\right)}{d} - \frac{3ib^3c^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)}{2d} - \frac{3b^2c^2(a+b\arctan(cx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)}{d} - \frac{3ibc^2(a+b\arctan(cx))^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)}{2d} - \frac{3ib^3c^2\operatorname{PolyLog}\left(3,-1+\frac{2}{1-icx}\right)}{2d} - \frac{3b^2c^2(a+b\arctan(cx))\operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)}{2d} + \frac{3ib^3c^2\operatorname{PolyLog}\left(4,-1+\frac{2}{1+icx}\right)}{4d}$$

output
```
-3/2*I*b*c^2*(a+b*arctan(c*x))^2/d-3/2*b*c*(a+b*arctan(c*x))^2/d/x-3/2*c^2
*(a+b*arctan(c*x))^3/d-1/2*(a+b*arctan(c*x))^3/d/x^2+I*c*(a+b*arctan(c*x))
^3/d/x+3*b^2*c^2*(a+b*arctan(c*x))*ln(2-2/(1-I*c*x))/d-3*I*b*c^2*(a+b*arct
an(c*x))^2*ln(2-2/(1-I*c*x))/d-c^2*(a+b*arctan(c*x))^3*ln(2-2/(1+I*c*x))/d
-3/2*I*b^3*c^2*polylog(2,-1+2/(1-I*c*x))/d-3*b^2*c^2*(a+b*arctan(c*x))*pol
ylog(2,-1+2/(1-I*c*x))/d-3/2*I*b*c^2*(a+b*arctan(c*x))^2*polylog(2,-1+2/(1
+I*c*x))/d-3/2*I*b^3*c^2*polylog(3,-1+2/(1-I*c*x))/d-3/2*b^2*c^2*(a+b*arct
an(c*x))*polylog(3,-1+2/(1+I*c*x))/d+3/4*I*b^3*c^2*polylog(4,-1+2/(1+I*c*x
))/d
```

## Mathematica [A] (verified)

Time = 1.81 (sec) , antiderivative size = 634, normalized size of antiderivative = 1.53

$$\int \frac{(a+b\arctan(cx))^3}{x^3(d+icdx)}\,dx$$

$$= \frac{-\frac{a^3}{x^2}+\frac{2ia^3c}{x}+2ia^3c^2\arctan(cx)-2a^3c^2\log(x)+a^3c^2\log\left(1+c^2x^2\right)+\frac{3ia^2b\left(2c^2x^2\arctan(cx)^2+\arctan(cx)(i+2cx\right.}{}}{}$$

input
```
Integrate[(a + b*ArcTan[c*x])^3/(x^3*(d + I*c*d*x)),x]
```

output
```
(-(a^3/x^2) + ((2*I)*a^3*c)/x + (2*I)*a^3*c^2*ArcTan[c*x] - 2*a^3*c^2*Log[
x] + a^3*c^2*Log[1 + c^2*x^2] + ((3*I)*a^2*b*(2*c^2*x^2*ArcTan[c*x]^2 + Ar
cTan[c*x]*(I + 2*c*x + I*c^2*x^2 + (2*I)*c^2*x^2*Log[1 - E^((2*I)*ArcTan[c
*x])]) + c*x*(I - 2*c*x*Log[(c*x)/Sqrt[1 + c^2*x^2]]) + c^2*x^2*PolyLog[2,
 E^((2*I)*ArcTan[c*x])]))/x^2 + 6*a*b^2*c^2*((I/24)*Pi^3 - ArcTan[c*x]/(c*
x) - (3*ArcTan[c*x]^2)/2 - ArcTan[c*x]^2/(2*c^2*x^2) + (I*ArcTan[c*x]^2)/(
c*x) - ArcTan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] - (2*I)*ArcTan[c*x]*L
og[1 - E^((2*I)*ArcTan[c*x])] + Log[(c*x)/Sqrt[1 + c^2*x^2]] - I*ArcTan[c*
x]*PolyLog[2, E^((-2*I)*ArcTan[c*x])] - PolyLog[2, E^((2*I)*ArcTan[c*x])]
- PolyLog[3, E^((-2*I)*ArcTan[c*x])]/2) + 2*b^3*c^2*(-1/8*Pi^3 + (I/64)*Pi
^4 - ((3*I)/2)*ArcTan[c*x]^2 - (3*ArcTan[c*x]^2)/(2*c*x) + ArcTan[c*x]^3 +
 (I*ArcTan[c*x]^3)/(c*x) - ((1 + c^2*x^2)*ArcTan[c*x]^3)/(2*c^2*x^2) - (3*
I)*ArcTan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] - ArcTan[c*x]^3*Log[1 - E
^((-2*I)*ArcTan[c*x])] + 3*ArcTan[c*x]*Log[1 - E^((2*I)*ArcTan[c*x])] + (3
*(2 - I*ArcTan[c*x])*ArcTan[c*x]*PolyLog[2, E^((-2*I)*ArcTan[c*x])])/2 - (
(3*I)/2)*PolyLog[2, E^((2*I)*ArcTan[c*x])] - (3*(I + ArcTan[c*x])*PolyLog[
3, E^((-2*I)*ArcTan[c*x])])/2 + ((3*I)/4)*PolyLog[4, E^((-2*I)*ArcTan[c*x]
)]))/(2*d)
```

## Rubi [A] (verified)

Time = 4.01 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.02, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.640, Rules used = {5405, 27, 5361, 5405, 5361, 5403, 5453, 5361, 5419, 5459, 5403, 2897, 5527, 5529, 5533, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^3}{x^3(d+icdx)}dx$$

$$\Big\downarrow \text{5405}$$

$$\frac{\int \frac{(a+b\arctan(cx))^3}{x^3}dx}{d}-ic\int \frac{(a+b\arctan(cx))^3}{dx^2(icx+1)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(a+b\arctan(cx))^3}{x^3}dx}{d}-\frac{ic\int \frac{(a+b\arctan(cx))^3}{x^2(icx+1)}dx}{d}$$

$$\Big\downarrow \text{5361}$$

$$\frac{\frac{3}{2}bc\int \frac{(a+b\arctan(cx))^2}{x^2(c^2x^2+1)}dx-\frac{(a+b\arctan(cx))^3}{2x^2}}{d}-\frac{ic\int \frac{(a+b\arctan(cx))^3}{x^2(icx+1)}dx}{d}$$

$$\Big\downarrow \text{5405}$$

$$\frac{\frac{3}{2}bc\int \frac{(a+b\arctan(cx))^2}{x^2(c^2x^2+1)}dx-\frac{(a+b\arctan(cx))^3}{2x^2}}{d}-\frac{ic\left(\int \frac{(a+b\arctan(cx))^3}{x^2}dx-ic\int \frac{(a+b\arctan(cx))^3}{x(icx+1)}dx\right)}{d}$$

$$\Big\downarrow \text{5361}$$

$$\frac{\frac{3}{2}bc\int \frac{(a+b\arctan(cx))^2}{x^2(c^2x^2+1)}dx-\frac{(a+b\arctan(cx))^3}{2x^2}}{d}-\frac{ic\left(3bc\int \frac{(a+b\arctan(cx))^2}{x(c^2x^2+1)}dx-ic\int \frac{(a+b\arctan(cx))^3}{x(icx+1)}dx-\frac{(a+b\arctan(cx))^3}{x}\right)}{d}$$

$$\Big\downarrow \text{5403}$$

$$\frac{\frac{3}{2}bc\int \frac{(a+b\arctan(cx))^2}{x^2(c^2x^2+1)}dx-\frac{(a+b\arctan(cx))^3}{2x^2}}{d}-$$

$$\frac{ic\left(3bc\int \frac{(a+b\arctan(cx))^2}{x(c^2x^2+1)}dx-ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^3-3bc\int \frac{(a+b\arctan(cx))^2\log\left(2-\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)-(a\right.}{d}$$

$\downarrow$ 5453

$$\frac{\frac{3}{2}bc\left(\int \frac{(a+b\arctan(cx))^2}{x^2}dx-c^2\int \frac{(a+b\arctan(cx))^2}{c^2x^2+1}dx\right)-\frac{(a+b\arctan(cx))^3}{2x^2}}{d}-$$

$$\frac{ic\left(3bc\int \frac{(a+b\arctan(cx))^2}{x(c^2x^2+1)}dx-ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^3-3bc\int \frac{(a+b\arctan(cx))^2\log\left(2-\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)-(a\right.}{d}$$

$\downarrow$ 5361

$$\frac{\frac{3}{2}bc\left(c^2\left(-\int \frac{(a+b\arctan(cx))^2}{c^2x^2+1}dx\right)+2bc\int \frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx-\frac{(a+b\arctan(cx))^2}{x}\right)-\frac{(a+b\arctan(cx))^3}{2x^2}}{d}-$$

$$\frac{ic\left(3bc\int \frac{(a+b\arctan(cx))^2}{x(c^2x^2+1)}dx-ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^3-3bc\int \frac{(a+b\arctan(cx))^2\log\left(2-\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)-(a\right.}{d}$$

$\downarrow$ 5419

$$\frac{\frac{3}{2}bc\left(2bc\int \frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx-\frac{c(a+b\arctan(cx))^3}{3b}-\frac{(a+b\arctan(cx))^2}{x}\right)-\frac{(a+b\arctan(cx))^3}{2x^2}}{d}-$$

$$\frac{ic\left(3bc\int \frac{(a+b\arctan(cx))^2}{x(c^2x^2+1)}dx-ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^3-3bc\int \frac{(a+b\arctan(cx))^2\log\left(2-\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)-(a\right.}{d}$$

$\downarrow$ 5459

$$\frac{-\frac{(a+b\arctan(cx))^3}{2x^2}+\frac{3}{2}bc\left(2bc\left(i\int \frac{a+b\arctan(cx)}{x(cx+i)}dx-\frac{i(a+b\arctan(cx))^2}{2b}\right)-\frac{c(a+b\arctan(cx))^3}{3b}-\frac{(a+b\arctan(cx))^2}{x}\right)}{d}-$$

$$\frac{ic\left(-ic\left(\log\left(2-\frac{2}{1+icx}\right)(a+b\arctan(cx))^3-3bc\int \frac{(a+b\arctan(cx))^2\log\left(2-\frac{2}{icx+1}\right)}{c^2x^2+1}dx\right)+3bc\left(i\int \frac{(a+b\arctan(cx))^2}{x(cx+i)}dx\right.\right.}{d}$$

$\downarrow$ 5403

$$\frac{-\frac{(a+b\arctan(cx))^3}{2x^2}+\frac{3}{2}bc\left(2bc\left(i\left(ibc\int\frac{\log\left(2-\frac{2}{1-icx}\right)}{c^2x^2+1}dx-i\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))\right)-\frac{i(a+b\arctan(cx))^2}{2b}\right)\right.}{d}$$

$$\frac{ic\left(3bc\left(i\left(2ibc\int\frac{(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right)}{c^2x^2+1}dx-i\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))^2\right)-\frac{i(a+b\arctan(cx))^3}{3b}\right)-ic\right.}{d}$$

$\downarrow$ 2897

$$\frac{-\frac{(a+b\arctan(cx))^3}{2x^2}+\frac{3}{2}bc\left(2bc\left(i\left(-i\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{2}{1-icx}-1\right)\right)-\frac{i(a+b\arctan(}{2b}\right.\right.}{d}$$

$$\frac{ic\left(3bc\left(i\left(2ibc\int\frac{(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right)}{c^2x^2+1}dx-i\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))^2\right)-\frac{i(a+b\arctan(cx))^3}{3b}\right)-ic\right.}{d}$$

$\downarrow$ 5527

$$\frac{-\frac{(a+b\arctan(cx))^3}{2x^2}+\frac{3}{2}bc\left(2bc\left(i\left(-i\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{2}{1-icx}-1\right)\right)-\frac{i(a+b\arctan(}{2b}\right.\right.}{d}$$

$$ic\left(3bc\left(i\left(2ibc\left(\frac{i\,\text{PolyLog}\left(2,\frac{2}{1-icx}-1\right)(a+b\arctan(cx))}{2c}-\frac{1}{2}ib\int\frac{\text{PolyLog}\left(2,\frac{2}{1-icx}-1\right)}{c^2x^2+1}dx\right)-i\log\left(2-\frac{2}{1-icx}\right)(a+b\arcta\right.\right.\right.$$

$\downarrow$ 5529

$$\frac{-\frac{(a+b\arctan(cx))^3}{2x^2}+\frac{3}{2}bc\left(2bc\left(i\left(-i\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{2}{1-icx}-1\right)\right)-\frac{i(a+b\arctan(}{2b}\right.\right.}{d}$$

$$ic\left(3bc\left(i\left(2ibc\left(\frac{i\,\text{PolyLog}\left(2,\frac{2}{1-icx}-1\right)(a+b\arctan(cx))}{2c}-\frac{1}{2}ib\int\frac{\text{PolyLog}\left(2,\frac{2}{1-icx}-1\right)}{c^2x^2+1}dx\right)-i\log\left(2-\frac{2}{1-icx}\right)(a+b\arcta\right.\right.\right.$$

$\downarrow$ 5533

$$\frac{-\frac{(a+b\arctan(cx))^3}{2x^2}+\frac{3}{2}bc\left(2bc\left(i\left(-i\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{2}{1-icx}-1\right)\right)-\frac{i(a+b\arctan(}{2b}\right.\right.}{d}$$

$$ic\left(3bc\left(i\left(2ibc\left(\frac{i\,\text{PolyLog}\left(2,\frac{2}{1-icx}-1\right)(a+b\arctan(cx))}{2c}-\frac{1}{2}ib\int\frac{\text{PolyLog}\left(2,\frac{2}{1-icx}-1\right)}{c^2x^2+1}dx\right)-i\log\left(2-\frac{2}{1-icx}\right)(a+b\arcta\right.\right.\right.$$

$\downarrow$ 7164

$$\frac{-\frac{(a+b\arctan(cx))^3}{2x^2}+\frac{3}{2}bc\left(2bc\left(i\left(-i\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))-\frac{1}{2}b\operatorname{PolyLog}\left(2,\frac{2}{1-icx}-1\right)\right)-\frac{i(a+b\arctan(}{2b}\right.\right.}{d}$$

$$ic\left(3bc\left(i\left(2ibc\left(\frac{i\operatorname{PolyLog}\left(2,\frac{2}{1-icx}-1\right)(a+b\arctan(cx))}{2c}-\frac{b\operatorname{PolyLog}\left(3,\frac{2}{1-icx}-1\right)}{4c}\right)-i\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))^2\right.\right.\right.$$

input
```
Int[(a + b*ArcTan[c*x])^3/(x^3*(d + I*c*d*x)),x]
```

output
```
(-1/2*(a + b*ArcTan[c*x])^3/x^2 + (3*b*c*(-((a + b*ArcTan[c*x])^2/x) - (c*
(a + b*ArcTan[c*x])^3)/(3*b) + 2*b*c*(((-1/2*I)*(a + b*ArcTan[c*x])^2)/b +
 I*((-I)*(a + b*ArcTan[c*x])*Log[2 - 2/(1 - I*c*x)] - (b*PolyLog[2, -1 + 2
/(1 - I*c*x)])/2))))/2)/d - (I*c*(-((a + b*ArcTan[c*x])^3/x) + 3*b*c*(((-1
/3*I)*(a + b*ArcTan[c*x])^3)/b + I*((-I)*(a + b*ArcTan[c*x])^2*Log[2 - 2/(
1 - I*c*x)] + (2*I)*b*c*(((I/2)*(a + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 -
 I*c*x)])/c - (b*PolyLog[3, -1 + 2/(1 - I*c*x)])/(4*c)))) - I*c*((a + b*Ar
cTan[c*x])^3*Log[2 - 2/(1 + I*c*x)] - 3*b*c*(((-1/2*I)*(a + b*ArcTan[c*x])
^2*PolyLog[2, -1 + 2/(1 + I*c*x)])/c + I*b*(((I/2)*(a + b*ArcTan[c*x])*Pol
yLog[3, -1 + 2/(1 + I*c*x)])/c + (b*PolyLog[4, -1 + 2/(1 + I*c*x)])/(4*c))
))))/d
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5405
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x], x
] - Simp[e/(d*f)   Int[(f*x)^(m + 1)*((a + b*ArcTan[c*x])^p/(d + e*x)), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0]
&& LtQ[m, -1]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5527
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 5533
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k + 1
, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &&
EqQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 42.46 (sec) , antiderivative size = 2450, normalized size of antiderivative = 5.92

| method | result | size |
|---|---|---|
| derivativedivides | Expression too large to display | 2450 |
| default | Expression too large to display | 2450 |
| parts | Expression too large to display | 2457 |

input
```
int((a+b*arctan(c*x))^3/x^3/(d+I*c*d*x),x,method=_RETURNVERBOSE)
```

output
```
c^2*(1/2*a^3/d*ln(c^2*x^2+1)+I*a^3/d*arctan(c*x)-1/2*a^3/d/c^2/x^2+I*a^3/d
/c/x-a^3/d*ln(c*x)+b^3/d*(-6*I*polylog(4,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+3*I
*arctan(c*x)^2*polylog(2,-(1+I*c*x)/(c^2*x^2+1)^(1/2))-2*arctan(c*x)^3-3*I
*polylog(2,-(1+I*c*x)/(c^2*x^2+1)^(1/2))-arctan(c*x)^3*ln(1+(1+I*c*x)/(c^2
*x^2+1)^(1/2))-3*I*polylog(2,(1+I*c*x)/(c^2*x^2+1)^(1/2))-6*arctan(c*x)*po
lylog(3,-(1+I*c*x)/(c^2*x^2+1)^(1/2))-6*I*polylog(3,-(1+I*c*x)/(c^2*x^2+1)
^(1/2))+3*arctan(c*x)*ln(1-(1+I*c*x)/(c^2*x^2+1)^(1/2))-6*I*polylog(3,(1+I
*c*x)/(c^2*x^2+1)^(1/2))-3*I*arctan(c*x)^2*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2
))-6*arctan(c*x)*polylog(3,(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/2*arctan(c*x)^2*
(I*arctan(c*x)+3*I*c*x+c*x*arctan(c*x))*(c*x+I)/c^2/x^2-6*arctan(c*x)*poly
log(2,-(1+I*c*x)/(c^2*x^2+1)^(1/2))-3*I*arctan(c*x)^2-6*arctan(c*x)*polylo
g(2,(1+I*c*x)/(c^2*x^2+1)^(1/2))-6*I*polylog(4,(1+I*c*x)/(c^2*x^2+1)^(1/2)
)+3*arctan(c*x)*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/2*I*arctan(c*x)^4-3*I*
arctan(c*x)^2*ln(1-(1+I*c*x)/(c^2*x^2+1)^(1/2))-arctan(c*x)^3*ln(1-(1+I*c*
x)/(c^2*x^2+1)^(1/2))+3*I*arctan(c*x)^2*polylog(2,(1+I*c*x)/(c^2*x^2+1)^(1
/2)))+3*a*b^2/d*(-1/2/c^2/x^2*arctan(c*x)^2-1/2*I*Pi*csgn(I*((1+I*c*x)^2/(
c^2*x^2+1)-1))*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I*((1+I*c*x)^2/(c^
2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*arctan(c*x)^2-3/2*arctan(c*x)^2-1
/2*arctan(c*x)*(I*c*x-(c^2*x^2+1)^(1/2)+1)/c/x-ln(c*x)*arctan(c*x)^2+arcta
n(c*x)^2*ln((1+I*c*x)^2/(c^2*x^2+1)-1)-arctan(c*x)^2*ln(1-(1+I*c*x)/(c^...
```

## Fricas [F]

$$\int \frac{(a+b\arctan(cx))^3}{x^3(d+icdx)}\,dx=\int \frac{(b\arctan{(cx)}+a)^3}{(i\,cdx+d)x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))^3/x^3/(d+I*c*d*x),x, algorithm="fricas")
```

output
```
1/16*(2*I*b^3*c^2*x^2*log(2*c*x/(c*x - I))*log(-(c*x + I)/(c*x - I))^3 + 6
*I*b^3*c^2*x^2*dilog(-2*c*x/(c*x - I) + 1)*log(-(c*x + I)/(c*x - I))^2 - 1
2*I*b^3*c^2*x^2*log(-(c*x + I)/(c*x - I))*polylog(3, -(c*x + I)/(c*x - I))
 + 12*I*b^3*c^2*x^2*polylog(4, -(c*x + I)/(c*x - I)) + 16*d*x^2*integral(1
/8*(-8*I*a^3*c*x + 8*a^3 - 3*(-2*I*b^3*c^2*x^2 + 2*a*b^2 + (-2*I*a*b^2 + b
^3)*c*x)*log(-(c*x + I)/(c*x - I))^2 + 12*(a^2*b*c*x + I*a^2*b)*log(-(c*x
+ I)/(c*x - I)))/(c^2*d*x^5 + d*x^3), x) + (2*b^3*c*x + I*b^3)*log(-(c*x +
 I)/(c*x - I))^3)/(d*x^2)
```

## Sympy [F]

$$\int \frac{(a+b\arctan(cx))^3}{x^3(d+icdx)}\,dx = -\frac{i\left(\int \frac{a^3}{cx^4-ix^3}\,dx+\int \frac{b^3\operatorname{atan}^3(cx)}{cx^4-ix^3}\,dx+\int \frac{3ab^2\operatorname{atan}^2(cx)}{cx^4-ix^3}\,dx+\int \frac{3a^2b\operatorname{atan}(cx)}{cx^4-ix^3}\,dx\right)}{d}$$

input
```
integrate((a+b*atan(c*x))**3/x**3/(d+I*c*d*x),x)
```

output
```
-I*(Integral(a**3/(c*x**4 - I*x**3), x) + Integral(b**3*atan(c*x)**3/(c*x*
*4 - I*x**3), x) + Integral(3*a*b**2*atan(c*x)**2/(c*x**4 - I*x**3), x) +
Integral(3*a**2*b*atan(c*x)/(c*x**4 - I*x**3), x))/d
```

## Maxima [F]

$$\int \frac{(a+b\arctan(cx))^3}{x^3(d+icdx)}\,dx = \int \frac{(b\arctan(cx)+a)^3}{(i\,cdx+d)x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))^3/x^3/(d+I*c*d*x),x, algorithm="maxima")
```

output
```
1/2*(2*c^2*log(I*c*x + 1)/d - 2*c^2*log(x)/d + (2*I*c*x - 1)/(d*x^2))*a^3
- 1/512*(-64*I*b^3*c^2*x^2*arctan(c*x)^4 + 4*I*b^3*c^2*x^2*log(c^2*x^2 + 1
)^4 + I*(48*b^3*c^2*arctan(c*x)^4/d - 6144*b^3*c^4*integrate(1/64*x^4*arct
an(c*x)^2*log(c^2*x^2 + 1)/(c^2*d*x^5 + d*x^3), x) - 3*b^3*c^2*log(c^2*x^2
 + 1)^4/d + 3072*b^3*c^3*integrate(1/64*x^3*arctan(c*x)*log(c^2*x^2 + 1)^2
/(c^2*d*x^5 + d*x^3), x) - 12288*b^3*c^3*integrate(1/64*x^3*arctan(c*x)*lo
g(c^2*x^2 + 1)/(c^2*d*x^5 + d*x^3), x) + 6144*b^3*c^2*integrate(1/64*x^2*a
rctan(c*x)^2/(c^2*d*x^5 + d*x^3), x) - 1536*b^3*c^2*integrate(1/64*x^2*log
(c^2*x^2 + 1)^2/(c^2*d*x^5 + d*x^3), x) + 28672*b^3*c*integrate(1/64*x*arc
tan(c*x)^3/(c^2*d*x^5 + d*x^3), x) + 3072*b^3*c*integrate(1/64*x*arctan(c*
x)*log(c^2*x^2 + 1)^2/(c^2*d*x^5 + d*x^3), x) + 98304*a*b^2*c*integrate(1/
64*x*arctan(c*x)^2/(c^2*d*x^5 + d*x^3), x) - 6144*b^3*c*integrate(1/64*x*a
rctan(c*x)*log(c^2*x^2 + 1)/(c^2*d*x^5 + d*x^3), x) + 98304*a^2*b*c*integr
ate(1/64*x*arctan(c*x)/(c^2*d*x^5 + d*x^3), x) + 6144*b^3*integrate(1/64*a
rctan(c*x)^2*log(c^2*x^2 + 1)/(c^2*d*x^5 + d*x^3), x) - 512*b^3*integrate(
1/64*log(c^2*x^2 + 1)^3/(c^2*d*x^5 + d*x^3), x))*d*x^2 - 64*(192*b^3*c^4*i
ntegrate(1/64*x^4*arctan(c*x)^3/(c^2*d*x^5 + d*x^3), x) + 48*b^3*c^4*integ
rate(1/64*x^4*arctan(c*x)*log(c^2*x^2 + 1)^2/(c^2*d*x^5 + d*x^3), x) + b^3
*c^2*arctan(c*x)^3/d + 96*b^3*c^3*integrate(1/64*x^3*arctan(c*x)^2*log(c^2
*x^2 + 1)/(c^2*d*x^5 + d*x^3), x) + 24*b^3*c^3*integrate(1/64*x^3*log(c...
```

## Giac [F]

$$\int \frac{(a+b\arctan(cx))^3}{x^3(d+icdx)}\,dx = \int \frac{(b\arctan(cx)+a)^3}{(i\,cdx+d)x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))^3/x^3/(d+I*c*d*x),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)^3/((I*c*d*x + d)*x^3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^3}{x^3(d+icdx)}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^3}{x^3\,(d+c\,d\,x\,1\mathrm{i})}\,dx$$

input
```
int((a + b*atan(c*x))^3/(x^3*(d + c*d*x*1i)),x)
```

output
```
int((a + b*atan(c*x))^3/(x^3*(d + c*d*x*1i)), x)
```

## Reduce [F]

$$\int \frac{(a+b\arctan(cx))^3}{x^3(d+icdx)}\,dx = \text{Too large to display}$$

input
```
int((a+b*atan(c*x))^3/x^3/(d+I*c*d*x),x)
```

output
```
(2*atan(c*x)**3*a*b**2*c**2*i*x**2 - 3*atan(c*x)**2*a**2*b*c**2*i*x**2 + 1
2*atan(c*x)**2*a*b**2*c**2*x**2 + 6*atan(c*x)**2*a*b**2*c*i*x + 2*atan(c*x
)*a**3*c**2*i*x**2 - 3*atan(c*x)*a**2*b*c**2*x**2 - 3*atan(c*x)*a**2*b + 1
2*atan(c*x)*a*b**2*c*x + 6*int(atan(c*x)/(c**3*i*x**5 + c**2*x**4 + c*i*x*
*3 + x**2),x)*a**2*b*c*i*x**2 - 12*int(atan(c*x)/(c**3*i*x**5 + c**2*x**4
+ c*i*x**3 + x**2),x)*a*b**2*c*x**2 + 2*int(atan(c*x)**3/(c**3*i*x**4 + c*
*2*x**3 + c*i*x**2 + x),x)*b**3*c**2*x**2 + 6*int(atan(c*x)**2/(c**3*i*x**
5 + c**2*x**4 + c*i*x**3 + x**2),x)*a*b**2*c*i*x**2 + 12*int(( - atan(c*x)
)/(c**3*x**5 - c**2*i*x**4 + c*x**3 - i*x**2),x)*a**2*b*c*x**2 + 24*int((
- atan(c*x))/(c**3*x**5 - c**2*i*x**4 + c*x**3 - i*x**2),x)*a*b**2*c*i*x**
2 - 2*int(( - atan(c*x)**3)/(c**3*d*i*x**6 + c**2*d*x**5 + c*d*i*x**4 + d*
x**3),x)*b**3*d*x**2 - 6*int(( - atan(c*x)**2)/(c**3*d*i*x**6 + c**2*d*x**
5 + c*d*i*x**4 + d*x**3),x)*a*b**2*d*x**2 - 6*int(( - atan(c*x)*x)/(c**3*x
**3 - c**2*i*x**2 + c*x - i),x)*a**2*b*c**4*i*x**2 + 12*int(( - atan(c*x)*
x)/(c**3*x**3 - c**2*i*x**2 + c*x - i),x)*a*b**2*c**4*x**2 + log(c**2*x**2
 + 1)*a**3*c**2*x**2 + 6*log(c**2*x**2 + 1)*a*b**2*c**2*x**2 - 2*log(x)*a*
*3*c**2*x**2 - 12*log(x)*a*b**2*c**2*x**2 + 2*a**3*c*i*x - a**3 - 3*a**2*b
*c*x)/(2*d*x**2)
```

# 3.133 $\int \frac{1}{(d+icdx)(a+b\arctan(cx))}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{(d+icdx)(a+b\arctan(cx))}\,dx=\operatorname{Int}\left(\frac{1}{(d+icdx)(a+b\arctan(cx))},x\right)$$

output
```
Defer(Int)(1/(d+I*c*d*x)/(a+b*arctan(c*x)),x)
```

## Mathematica [N/A]

Not integrable

Time = 2.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+icdx)(a+b\arctan(cx))}\,dx=\int \frac{1}{(d+icdx)(a+b\arctan(cx))}\,dx$$

input
```
Integrate[1/((d + I*c*d*x)*(a + b*ArcTan[c*x])),x]
```

output
```
Integrate[1/((d + I*c*d*x)*(a + b*ArcTan[c*x])), x]
```

## Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+icdx)(a+b\arctan(cx))}\,dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int \frac{1}{(d+icdx)(a+b\arctan(cx))}dx$$

input
```
Int[1/((d + I*c*d*x)*(a + b*ArcTan[c*x])),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 1.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{(icdx+d)\,(a+b\arctan\,(cx))}dx$$

input
```
int(1/(d+I*c*d*x)/(a+b*arctan(c*x)),x)
```

output
```
int(1/(d+I*c*d*x)/(a+b*arctan(c*x)),x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.95

$$\int \frac{1}{(d+icdx)(a+b\arctan(cx))}\,dx = \int \frac{1}{(i\,cdx+d)(b\arctan{(cx)}+a)}\,dx$$

input
```
integrate(1/(d+I*c*d*x)/(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
integral(-2/(-2*I*a*c*d*x - 2*a*d + (b*c*d*x - I*b*d)*log(-(c*x + I)/(c*x
- I))), x)
```

## Sympy [N/A]

Not integrable

Time = 5.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.55

$$\int \frac{1}{(d+icdx)(a+b\arctan(cx))}\,dx = -\frac{i\int \frac{1}{acx-ia+bcx\operatorname{atan}{(cx)}-ib\operatorname{atan}{(cx)}}\,dx}{d}$$

input
```
integrate(1/(d+I*c*d*x)/(a+b*atan(c*x)),x)
```

output
```
-I*Integral(1/(a*c*x - I*a + b*c*x*atan(c*x) - I*b*atan(c*x)), x)/d
```

## Maxima [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+icdx)(a+b\arctan(cx))}\,dx = \int \frac{1}{(i\,cdx+d)(b\arctan{(cx)}+a)}\,dx$$

input
```
integrate(1/(d+I*c*d*x)/(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/((I*c*d*x + d)*(b*arctan(c*x) + a)), x)
```

## Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+icdx)(a+b\arctan(cx))}\,dx=\int \frac{1}{(i\,cdx+d)(b\arctan{(cx)}+a)}\,dx$$

input
```
integrate(1/(d+I*c*d*x)/(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
integrate(1/((I*c*d*x + d)*(b*arctan(c*x) + a)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{1}{(d+icdx)(a+b\arctan(cx))}\,dx=\int \frac{1}{(a+b\,\mathrm{atan}\left(c\,x\right))\,\left(d+c\,d\,x\,1\mathrm{i}\right)}\,dx$$

input
```
int(1/((a + b*atan(c*x))*(d + c*d*x*1i)),x)
```

output
```
int(1/((a + b*atan(c*x))*(d + c*d*x*1i)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.36

$$\int \frac{1}{(d+icdx)(a+b\arctan(cx))}\,dx = \frac{\int \frac{1}{atan(cx)bcix+atan(cx)b+acix+a}dx}{d}$$

input
```
int(1/(d+I*c*d*x)/(a+b*atan(c*x)),x)
```

output
```
int(1/(atan(c*x)*b*c*i*x + atan(c*x)*b + a*c*i*x + a),x)/d
```

# 3.134 $\int \frac{x^3(a+b\arctan(cx))}{d+ex}\,dx$



## Optimal result

Integrand size = 19, antiderivative size = 297

$$\begin{aligned}\int \frac{x^3(a+b\arctan(cx))}{d+ex}\,dx =\ & \frac{ad^2x}{e^3}+\frac{bdx}{2ce^2}-\frac{bx^2}{6ce}-\frac{bd\arctan(cx)}{2c^2e^2}+\frac{bd^2x\arctan(cx)}{e^3} \\ & -\frac{dx^2(a+b\arctan(cx))}{2e^2}+\frac{x^3(a+b\arctan(cx))}{3e} \\ & +\frac{d^3(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)}{e^4} \\ & -\frac{d^3(a+b\arctan(cx))\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{e^4} \\ & -\frac{bd^2\log(1+c^2x^2)}{2ce^3}+\frac{b\log(1+c^2x^2)}{6c^3e} \\ & -\frac{ibd^3\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2e^4} \\ & +\frac{ibd^3\operatorname{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2e^4}\end{aligned}$$

output
```
a*d^2*x/e^3+1/2*b*d*x/c/e^2-1/6*b*x^2/c/e-1/2*b*d*arctan(c*x)/c^2/e^2+b*d^
2*x*arctan(c*x)/e^3-1/2*d*x^2*(a+b*arctan(c*x))/e^2+1/3*x^3*(a+b*arctan(c*
x))/e+d^3*(a+b*arctan(c*x))*ln(2/(1-I*c*x))/e^4-d^3*(a+b*arctan(c*x))*ln(2
*c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/e^4-1/2*b*d^2*ln(c^2*x^2+1)/c/e^3+1/6*b*ln
(c^2*x^2+1)/c^3/e-1/2*I*b*d^3*polylog(2,1-2/(1-I*c*x))/e^4+1/2*I*b*d^3*pol
ylog(2,1-2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/e^4
```

## Mathematica [A] (verified)

Time = 2.24 (sec) , antiderivative size = 484, normalized size of antiderivative = 1.63

$$\int \frac{x^3(a+b\arctan(cx))}{d+ex}\,dx = -\frac{\frac{be^3}{c^3}-6ad^2ex-\frac{3bde^2x}{c}+3ade^2x^2+\frac{be^3x^2}{c}-2ae^3x^3+\frac{3bde^2\arctan(cx)}{c^2}+3ibd^3\pi\arctan(cx)-6bd^2ex\,\text{arcta}}{}$$

input
```
Integrate[(x^3*(a + b*ArcTan[c*x]))/(d + e*x),x]
```

output
```
-1/6*((b*e^3)/c^3 - 6*a*d^2*e*x - (3*b*d*e^2*x)/c + 3*a*d*e^2*x^2 + (b*e^3
*x^2)/c - 2*a*e^3*x^3 + (3*b*d*e^2*ArcTan[c*x])/c^2 + (3*I)*b*d^3*Pi*ArcTa
n[c*x] - 6*b*d^2*e*x*ArcTan[c*x] + 3*b*d*e^2*x^2*ArcTan[c*x] - 2*b*e^3*x^3
*ArcTan[c*x] - (6*I)*b*d^3*ArcTan[(c*d)/e]*ArcTan[c*x] + (3*I)*b*d^3*ArcTa
n[c*x]^2 + (3*b*d^2*e*ArcTan[c*x]^2)/c - (3*b*d^2*Sqrt[1 + (c^2*d^2)/e^2]*
e*E^(I*ArcTan[(c*d)/e])*ArcTan[c*x]^2)/c + 3*b*d^3*Pi*Log[1 + E^((-2*I)*Ar
cTan[c*x])] - 6*b*d^3*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] + 6*b*d^3
*ArcTan[(c*d)/e]*Log[1 - E^((2*I)*(ArcTan[(c*d)/e] + ArcTan[c*x]))] + 6*b*
d^3*ArcTan[c*x]*Log[1 - E^((2*I)*(ArcTan[(c*d)/e] + ArcTan[c*x]))] + 6*a*d
^3*Log[d + e*x] + (3*b*d^2*e*Log[1 + c^2*x^2])/c - (b*e^3*Log[1 + c^2*x^2]
)/c^3 + (3*b*d^3*Pi*Log[1 + c^2*x^2])/2 - 6*b*d^3*ArcTan[(c*d)/e]*Log[Sin[
ArcTan[(c*d)/e] + ArcTan[c*x]]] + (3*I)*b*d^3*PolyLog[2, -E^((2*I)*ArcTan[
c*x])] - (3*I)*b*d^3*PolyLog[2, E^((2*I)*(ArcTan[(c*d)/e] + ArcTan[c*x]))]
)/e^4
```

## Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\arctan(cx))}{d+ex}\,dx$$

$$\Big\downarrow \text{5411}$$

$$\int \left(-\frac{d^3(a+b\arctan(cx))}{e^3(d+ex)}+\frac{d^2(a+b\arctan(cx))}{e^3}-\frac{dx(a+b\arctan(cx))}{e^2}+\frac{x^2(a+b\arctan(cx))}{e}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{d^3\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))}{e^4}-\frac{d^3(a+b\arctan(cx))\log\left(\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{e^4}-\frac{dx^2(a+b\arctan(cx))}{2e^2}+\frac{x^3(a+b\arctan(cx))}{3e}+\frac{ad^2x}{e^3}-\frac{bd\arctan(cx)}{2c^2e^2}+\frac{bd^2x\arctan(cx)}{e^3}-\frac{bd^2\log\left(c^2x^2+1\right)}{2ce^3}+\frac{b\log\left(c^2x^2+1\right)}{6c^3e}-\frac{ibd^3\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2e^4}+\frac{ibd^3\operatorname{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2e^4}+\frac{bdx}{2ce^2}-\frac{bx^2}{6ce}$$

input
```
Int[(x^3*(a + b*ArcTan[c*x]))/(d + e*x),x]
```

output
```
(a*d^2*x)/e^3 + (b*d*x)/(2*c*e^2) - (b*x^2)/(6*c*e) - (b*d*ArcTan[c*x])/(2
*c^2*e^2) + (b*d^2*x*ArcTan[c*x])/e^3 - (d*x^2*(a + b*ArcTan[c*x]))/(2*e^2
) + (x^3*(a + b*ArcTan[c*x]))/(3*e) + (d^3*(a + b*ArcTan[c*x])*Log[2/(1 -
I*c*x)])/e^4 - (d^3*(a + b*ArcTan[c*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(
1 - I*c*x))])/e^4 - (b*d^2*Log[1 + c^2*x^2])/(2*c*e^3) + (b*Log[1 + c^2*x^
2])/(6*c^3*e) - ((I/2)*b*d^3*PolyLog[2, 1 - 2/(1 - I*c*x)])/e^4 + ((I/2)*b
*d^3*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e^4
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

## Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.27

| method | result |
|---|---|
| parts | $\frac{a\,x^3}{3e}-\frac{ad\,x^2}{2e^2}+\frac{a\,d^2x}{e^3}-\frac{a\,d^3\ln(ex+d)}{e^4}+\frac{b\left(\frac{c^4\arctan(cx)x^3}{3e}-\frac{c^4\arctan(cx)x^2d}{2e^2}+\frac{c^4\arctan(cx)x\,d^2}{e^3}-\frac{c^4\arctan(cx)d^3}{e^4}\right.}{}$ |
| derivativedivides | $\frac{\frac{a\,c^4d^2x}{e^3}-\frac{a\,c^4d\,x^2}{2e^2}+\frac{a\,c^4x^3}{3e}-\frac{a\,c^4d^3\ln(cex+cd)}{e^4}+bc\left(\frac{\arctan(cx)c^3d^2x}{e^3}-\frac{\arctan(cx)c^3d\,x^2}{2e^2}+\frac{\arctan(cx)c^3x^3}{3e}-\frac{\arctan(cx)c^3d^3\ln\right.}{}$ |
| default | $\frac{\frac{a\,c^4d^2x}{e^3}-\frac{a\,c^4d\,x^2}{2e^2}+\frac{a\,c^4x^3}{3e}-\frac{a\,c^4d^3\ln(cex+cd)}{e^4}+bc\left(\frac{\arctan(cx)c^3d^2x}{e^3}-\frac{\arctan(cx)c^3d\,x^2}{2e^2}+\frac{\arctan(cx)c^3x^3}{3e}-\frac{\arctan(cx)c^3d^3\ln\right.}{}$ |
| risch | $-\frac{ad}{2c^2e^2}-\frac{a\,d^3\ln(icd-(-icx+1)e+e)}{e^4}+\frac{ib\,d^3\ln(icx+1)\ln\left(\frac{icd+(icx+1)e-e}{icd-e}\right)}{2e^4}-\frac{ibd\ln(-icx+1)x^2}{4e^2}-\frac{ib\,d^3\ln(-}{}$ |

input
```
int(x^3*(a+b*arctan(c*x))/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/3*a/e*x^3-1/2*a/e^2*d*x^2+a*d^2*x/e^3-a*d^3/e^4*ln(e*x+d)+b/c^4*(1/3*c^4
*arctan(c*x)/e*x^3-1/2*c^4*arctan(c*x)/e^2*x^2*d+c^4*arctan(c*x)/e^3*x*d^2
-c^4*arctan(c*x)*d^3/e^4*ln(c*e*x+c*d)-c/e*(1/2/e^2*ln(c^2*d^2-2*c*d*(c*e*
x+c*d)+e^2+(c*e*x+c*d)^2)*c^2*d^2+1/2/e*arctan(c*x)*c*d-1/6*ln(c^2*d^2-2*c
*d*(c*e*x+c*d)+e^2+(c*e*x+c*d)^2)-5/6/e^2*c*d*(c*e*x+c*d)+1/6/e^2*(c*e*x+c
*d)^2-1/e^2*c^3*d^3*(-1/2*I*ln(c*e*x+c*d)*(ln((I*e-c*e*x)/(c*d+I*e))-ln((I
*e+c*e*x)/(I*e-c*d)))/e-1/2*I*(dilog((I*e-c*e*x)/(c*d+I*e))-dilog((I*e+c*e
*x)/(I*e-c*d)))/e)))
```

## Fricas [F]

$$\int \frac{x^3(a+b\arctan(cx))}{d+ex}\,dx = \int \frac{(b\arctan(cx)+a)x^3}{ex+d}\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))/(e*x+d),x, algorithm="fricas")
```

output
```
integral((b*x^3*arctan(c*x) + a*x^3)/(e*x + d), x)
```

## Sympy [F]

$$\int \frac{x^3(a+b\arctan(cx))}{d+ex}\,dx = \int \frac{x^3(a+b\operatorname{atan}(cx))}{d+ex}\,dx$$

input
```
integrate(x**3*(a+b*atan(c*x))/(e*x+d),x)
```

output
```
Integral(x**3*(a + b*atan(c*x))/(d + e*x), x)
```

## Maxima [F]

$$\int \frac{x^3(a+b\arctan(cx))}{d+ex}\,dx = \int \frac{(b\arctan{(cx)}+a)x^3}{ex+d}\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))/(e*x+d),x, algorithm="maxima")
```

output
```
-1/6*a*(6*d^3*log(e*x + d)/e^4 - (2*e^2*x^3 - 3*d*e*x^2 + 6*d^2*x)/e^3) +
2*b*integrate(1/2*x^3*arctan(c*x)/(e*x + d), x)
```

## Giac [F]

$$\int \frac{x^3(a+b\arctan(cx))}{d+ex}\,dx = \int \frac{(b\arctan{(cx)}+a)x^3}{ex+d}\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))/(e*x+d),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)*x^3/(e*x + d), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\arctan(cx))}{d+ex}\,dx = \int \frac{x^3\,(a+b\,\mathrm{atan}(c\,x))}{d+e\,x}\,dx$$

input
```
int((x^3*(a + b*atan(c*x)))/(d + e*x),x)
```

output
```
int((x^3*(a + b*atan(c*x)))/(d + e*x), x)
```

## Reduce [F]

$$\int \frac{x^3(a+b\arctan(cx))}{d+ex}\,dx = \frac{6\left(\int \frac{atan(cx)x^3}{ex+d}dx\right)b\,e^4-6\log(ex+d)\,a\,d^3+6a\,d^2ex-3ad\,e^2x^2+2a\,e^3x^3}{6e^4}$$

input
```
int(x^3*(a+b*atan(c*x))/(e*x+d),x)
```

output
```
(6*int((atan(c*x)*x**3)/(d + e*x),x)*b*e**4 - 6*log(d + e*x)*a*d**3 + 6*a*
d**2*e*x - 3*a*d*e**2*x**2 + 2*a*e**3*x**3)/(6*e**4)
```

# 3.135 $\int \frac{x^2(a+b\arctan(cx))}{d+ex}\,dx$



## Optimal result

Integrand size = 19, antiderivative size = 237

$$\int \frac{x^2(a+b\arctan(cx))}{d+ex}\,dx = -\frac{adx}{e^2}-\frac{bx}{2ce}+\frac{b\arctan(cx)}{2c^2e}-\frac{bdx\arctan(cx)}{e^2}+\frac{x^2(a+b\arctan(cx))}{2e}-\frac{d^2(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)}{e^3}+\frac{d^2(a+b\arctan(cx))\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{e^3}+\frac{bd\log(1+c^2x^2)}{2ce^2}+\frac{ibd^2\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2e^3}-\frac{ibd^2\operatorname{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2e^3}$$

output
```
-a*d*x/e^2-1/2*b*x/c/e+1/2*b*arctan(c*x)/c^2/e-b*d*x*arctan(c*x)/e^2+1/2*x
^2*(a+b*arctan(c*x))/e-d^2*(a+b*arctan(c*x))*ln(2/(1-I*c*x))/e^3+d^2*(a+b*
arctan(c*x))*ln(2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/e^3+1/2*b*d*ln(c^2*x^2+1)
/c/e^2+1/2*I*b*d^2*polylog(2,1-2/(1-I*c*x))/e^3-1/2*I*b*d^2*polylog(2,1-2*
c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/e^3
```

## Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 404, normalized size of antiderivative = 1.70

$$\int \frac{x^2(a+b\arctan(cx))}{d+ex}\,dx$$

$$= \frac{-2adex - \frac{be^2x}{c} + ae^2x^2 + \frac{be^2\arctan(cx)}{c^2} + ibd^2\pi\arctan(cx) - 2bdex\arctan(cx) + be^2x^2\arctan(cx) - 2ibd^2}{}$$

input
```
Integrate[(x^2*(a + b*ArcTan[c*x]))/(d + e*x),x]
```

output
```
(-2*a*d*e*x - (b*e^2*x)/c + a*e^2*x^2 + (b*e^2*ArcTan[c*x])/c^2 + I*b*d^2*
Pi*ArcTan[c*x] - 2*b*d*e*x*ArcTan[c*x] + b*e^2*x^2*ArcTan[c*x] - (2*I)*b*d
^2*ArcTan[(c*d)/e]*ArcTan[c*x] + I*b*d^2*ArcTan[c*x]^2 + (b*d*e*ArcTan[c*x
]^2)/c - (b*d*Sqrt[1 + (c^2*d^2)/e^2]*e*E^(I*ArcTan[(c*d)/e])*ArcTan[c*x]^
2)/c + b*d^2*Pi*Log[1 + E^((-2*I)*ArcTan[c*x])] - 2*b*d^2*ArcTan[c*x]*Log[
1 + E^((2*I)*ArcTan[c*x])] + 2*b*d^2*ArcTan[(c*d)/e]*Log[1 - E^((2*I)*(Arc
Tan[(c*d)/e] + ArcTan[c*x]))] + 2*b*d^2*ArcTan[c*x]*Log[1 - E^((2*I)*(ArcT
an[(c*d)/e] + ArcTan[c*x]))] + 2*a*d^2*Log[d + e*x] + (b*d*e*Log[1 + c^2*x
^2])/c + (b*d^2*Pi*Log[1 + c^2*x^2])/2 - 2*b*d^2*ArcTan[(c*d)/e]*Log[Sin[A
rcTan[(c*d)/e] + ArcTan[c*x]]] + I*b*d^2*PolyLog[2, -E^((2*I)*ArcTan[c*x])
] - I*b*d^2*PolyLog[2, E^((2*I)*(ArcTan[(c*d)/e] + ArcTan[c*x]))])/(2*e^3)
```

## Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\arctan(cx))}{d+ex}\,dx$$

$$\Big\downarrow \text{5411}$$

$$\int \left(\frac{d^2(a+b\arctan(cx))}{e^2(d+ex)}-\frac{d(a+b\arctan(cx))}{e^2}+\frac{x(a+b\arctan(cx))}{e}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d^2\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))}{e^3}+\frac{d^2(a+b\arctan(cx))\log\left(\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{e^3}+\frac{x^2(a+b\arctan(cx))}{2e}-\frac{adx}{e^2}+\frac{b\arctan(cx)}{2c^2e}-\frac{bdx\arctan(cx)}{e^2}+\frac{bd\log(c^2x^2+1)}{2ce^2}+\frac{ibd^2\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2e^3}-\frac{ibd^2\operatorname{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2e^3}-\frac{bx}{2ce}$$

input
```
Int[(x^2*(a + b*ArcTan[c*x]))/(d + e*x),x]
```

output
```
-((a*d*x)/e^2) - (b*x)/(2*c*e) + (b*ArcTan[c*x])/(2*c^2*e) - (b*d*x*ArcTan
[c*x])/e^2 + (x^2*(a + b*ArcTan[c*x]))/(2*e) - (d^2*(a + b*ArcTan[c*x])*Lo
g[2/(1 - I*c*x)])/e^3 + (d^2*(a + b*ArcTan[c*x])*Log[(2*c*(d + e*x))/((c*d
 + I*e)*(1 - I*c*x))])/e^3 + (b*d*Log[1 + c^2*x^2])/(2*c*e^2) + ((I/2)*b*d
^2*PolyLog[2, 1 - 2/(1 - I*c*x)])/e^3 - ((I/2)*b*d^2*PolyLog[2, 1 - (2*c*(
d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e^3
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

## Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.20

| method | result |
|---|---|
| parts | $\frac{a\,x^{2}}{2e}-\frac{adx}{e^{2}}+\frac{a\,d^{2}\ln(ex+d)}{e^{3}}+\frac{b\left(\frac{c^{3}\arctan(cx)x^{2}}{2e}-\frac{c^{3}\arctan(cx)dx}{e^{2}}+\frac{c^{3}\arctan(cx)d^{2}\ln(cex+cd)}{e^{3}}-\frac{c\left(\frac{c^{2}d^{2}\left(-\frac{i\ln(cex\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$ |
| derivativedivides | $\frac{-\frac{a\,c^{3}dx}{e^{2}}+\frac{a\,c^{3}x^{2}}{2e}+\frac{a\,c^{3}d^{2}\ln(cex+cd)}{e^{3}}+bc\left(-\frac{\arctan(cx)c^{2}dx}{e^{2}}+\frac{\arctan(cx)c^{2}x^{2}}{2e}+\frac{\arctan(cx)c^{2}d^{2}\ln(cex+cd)}{e^{3}}-\frac{c^{2}d^{2}\left(-\frac{i\ln(cex\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}$ |
| default | $\frac{-\frac{a\,c^{3}dx}{e^{2}}+\frac{a\,c^{3}x^{2}}{2e}+\frac{a\,c^{3}d^{2}\ln(cex+cd)}{e^{3}}+bc\left(-\frac{\arctan(cx)c^{2}dx}{e^{2}}+\frac{\arctan(cx)c^{2}x^{2}}{2e}+\frac{\arctan(cx)c^{2}d^{2}\ln(cex+cd)}{e^{3}}-\frac{c^{2}d^{2}\left(-\frac{i\ln(cex\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}$ |
| risch | $\frac{ib\ln(c^{2}x^{2}+1)}{8c^{2}e}-\frac{ib\,d^{2}\ln(icx+1)\ln\left(\frac{icd+(icx+1)e-e}{icd-e}\right)}{2e^{3}}+\frac{a\,d^{2}\ln(icd-(-icx+1)e+e)}{e^{3}}+\frac{ib\,d^{2}\ln(-icx+1)\ln\left(\frac{-icd+\cdots}{\cdots}\right)}{2e^{3}}\cdots$ |

input
```
int(x^2*(a+b*arctan(c*x))/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/2*a/e*x^2-a*d*x/e^2+a*d^2/e^3*ln(e*x+d)+b/c^3*(1/2*c^3*arctan(c*x)/e*x^2
-c^3*arctan(c*x)/e^2*d*x+c^3*arctan(c*x)*d^2/e^3*ln(c*e*x+c*d)-c/e*(1/e*c^
2*d^2*(-1/2*I*ln(c*e*x+c*d)*(ln((I*e-c*e*x)/(c*d+I*e))-ln((I*e+c*e*x)/(I*e
-c*d)))/e+1/2*I*(-dilog((I*e-c*e*x)/(c*d+I*e))+dilog((I*e+c*e*x)/(I*e-c*d)
))/e)-1/2/e*c*d*ln(c^2*d^2-2*c*d*(c*e*x+c*d)+e^2+(c*e*x+c*d)^2)-1/2*arctan
(c*x)+1/2/e*(c*e*x+c*d)))
```

## Fricas [F]

$$\int \frac{x^2(a+b\arctan(cx))}{d+ex}\,dx = \int \frac{(b\arctan(cx)+a)x^2}{ex+d}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))/(e*x+d),x, algorithm="fricas")
```

output
```
integral((b*x^2*arctan(c*x) + a*x^2)/(e*x + d), x)
```

## Sympy [F]

$$\int \frac{x^2(a+b\arctan(cx))}{d+ex}\,dx = \int \frac{x^2(a+b\operatorname{atan}(cx))}{d+ex}\,dx$$

input
```
integrate(x**2*(a+b*atan(c*x))/(e*x+d),x)
```

output
```
Integral(x**2*(a + b*atan(c*x))/(d + e*x), x)
```

## Maxima [F]

$$\int \frac{x^2(a+b\arctan(cx))}{d+ex}\,dx = \int \frac{(b\arctan(cx)+a)x^2}{ex+d}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))/(e*x+d),x, algorithm="maxima")
```

output
```
1/2*a*(2*d^2*log(e*x + d)/e^3 + (e*x^2 - 2*d*x)/e^2) + 2*b*integrate(1/2*x
^2*arctan(c*x)/(e*x + d), x)
```

## Giac [F]

$$\int \frac{x^2(a+b\arctan(cx))}{d+ex}\,dx=\int \frac{(b\arctan(cx)+a)x^2}{ex+d}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))/(e*x+d),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)*x^2/(e*x + d), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\arctan(cx))}{d+ex}\,dx=\int \frac{x^2\,(a+b\,\mathrm{atan}(c\,x))}{d+e\,x}\,dx$$

input
```
int((x^2*(a + b*atan(c*x)))/(d + e*x),x)
```

output
```
int((x^2*(a + b*atan(c*x)))/(d + e*x), x)
```

## Reduce [F]

$$\int \frac{x^2(a+b\arctan(cx))}{d+ex}\,dx=\frac{2\left(\int \frac{atan(cx)x^2}{ex+d}dx\right)b\,e^3+2\log(ex+d)\,a\,d^2-2adex+a\,e^2x^2}{2e^3}$$

input
```
int(x^2*(a+b*atan(c*x))/(e*x+d),x)
```

output
```
(2*int((atan(c*x)*x**2)/(d + e*x),x)*b*e**3 + 2*log(d + e*x)*a*d**2 - 2*a*
d*e*x + a*e**2*x**2)/(2*e**3)
```

# 3.136 $\int \frac{x(a+b\arctan(cx))}{d+ex}\,dx$



## Optimal result

Integrand size = 17, antiderivative size = 179

$$\int \frac{x(a+b\arctan(cx))}{d+ex}\,dx = \frac{ax}{e}+\frac{bx\arctan(cx)}{e}+\frac{d(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)}{e^2} - \frac{d(a+b\arctan(cx))\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{e^2} - \frac{b\log(1+c^2x^2)}{2ce}-\frac{ibd\,\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2e^2} + \frac{ibd\,\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2e^2}$$

output
```
a*x/e+b*x*arctan(c*x)/e+d*(a+b*arctan(c*x))*ln(2/(1-I*c*x))/e^2-d*(a+b*arc
tan(c*x))*ln(2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/e^2-1/2*b*ln(c^2*x^2+1)/c/e-
1/2*I*b*d*polylog(2,1-2/(1-I*c*x))/e^2+1/2*I*b*d*polylog(2,1-2*c*(e*x+d)/(
c*d+I*e)/(1-I*c*x))/e^2
```

## Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.84

$$\int \frac{x(a+b\arctan(cx))}{d+ex}\,dx$$
$$= \frac{2aex-2ad\log(d+ex)+\frac{b\left(-icd\pi\arctan(cx)+2cex\arctan(cx)+2icd\arctan\left(\frac{cd}{e}\right)\arctan(cx)-icd\arctan(cx)^2-e\arctan(cx)^2+\sqrt{1+\cdots}\right.}{\cdots}}{\cdots}$$

input
```
Integrate[(x*(a + b*ArcTan[c*x]))/(d + e*x),x]
```

output
```
(2*a*e*x - 2*a*d*Log[d + e*x] + (b*((-I)*c*d*Pi*ArcTan[c*x] + 2*c*e*x*ArcT
an[c*x] + (2*I)*c*d*ArcTan[(c*d)/e]*ArcTan[c*x] - I*c*d*ArcTan[c*x]^2 - e*
ArcTan[c*x]^2 + Sqrt[1 + (c^2*d^2)/e^2]*e*E^(I*ArcTan[(c*d)/e])*ArcTan[c*x
]^2 - c*d*Pi*Log[1 + E^((-2*I)*ArcTan[c*x])] + 2*c*d*ArcTan[c*x]*Log[1 + E
^((2*I)*ArcTan[c*x])] - 2*c*d*ArcTan[(c*d)/e]*Log[1 - E^((2*I)*(ArcTan[(c*
d)/e] + ArcTan[c*x]))] - 2*c*d*ArcTan[c*x]*Log[1 - E^((2*I)*(ArcTan[(c*d)/
e] + ArcTan[c*x]))] - e*Log[1 + c^2*x^2] - (c*d*Pi*Log[1 + c^2*x^2])/2 + 2
*c*d*ArcTan[(c*d)/e]*Log[Sin[ArcTan[(c*d)/e] + ArcTan[c*x]]] - I*c*d*PolyL
og[2, -E^((2*I)*ArcTan[c*x])] + I*c*d*PolyLog[2, E^((2*I)*(ArcTan[(c*d)/e]
 + ArcTan[c*x]))]))/c)/(2*e^2)
```

## Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\arctan(cx))}{d+ex}\,dx$$
$$\Big\downarrow \text{ 5411}$$
$$\int \left(\frac{a+b\arctan(cx)}{e}-\frac{d(a+b\arctan(cx))}{e(d+ex)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{d\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))}{e^2}-\frac{d(a+b\arctan(cx))\log\left(\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{e^2}+\frac{ax}{e}+\frac{bx\arctan(cx)}{e}-\frac{b\log\left(c^2x^2+1\right)}{2ce}-\frac{ibd\,\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2e^2}+\frac{ibd\,\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2e^2}$$

input
```
Int[(x*(a + b*ArcTan[c*x]))/(d + e*x),x]
```

output
```
(a*x)/e + (b*x*ArcTan[c*x])/e + (d*(a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])
/e^2 - (d*(a + b*ArcTan[c*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x)
)])/e^2 - (b*Log[1 + c^2*x^2])/(2*c*e) - ((I/2)*b*d*PolyLog[2, 1 - 2/(1 -
I*c*x)])/e^2 + ((I/2)*b*d*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 -
 I*c*x))])/e^2
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

## Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.25

| method | result |
|---|---|
| parts | $\frac{ax}{e}-\frac{ad\ln(ex+d)}{e^2}+\frac{b\left(\frac{c^2\arctan(cx)x}{e}-\frac{c^2\arctan(cx)d\ln(cex+cd)}{e^2}-\frac{c\left(\frac{\ln\left(c^2d^2-2cd(cex+cd)+e^2+(cex+cd)^2\right)}{2}-cd\left(-\frac{i\ln}{}\right.\right.}{}\right.}{c^2}$ |
| derivativedivides | $\frac{\frac{a\,c^2x}{e}-\frac{a\,c^2d\ln(cex+cd)}{e^2}+bc\left(\frac{\arctan(cx)cx}{e}-\frac{\arctan(cx)dc\ln(cex+cd)}{e^2}-\frac{\frac{\ln\left(c^2d^2-2cd(cex+cd)+e^2+(cex+cd)^2\right)}{2}-cd\left(-\frac{i\ln(cex}{}\right.}{}\right.}{c^2}$ |
| default | $\frac{\frac{a\,c^2x}{e}-\frac{a\,c^2d\ln(cex+cd)}{e^2}+bc\left(\frac{\arctan(cx)cx}{e}-\frac{\arctan(cx)dc\ln(cex+cd)}{e^2}-\frac{\frac{\ln\left(c^2d^2-2cd(cex+cd)+e^2+(cex+cd)^2\right)}{2}-cd\left(-\frac{i\ln(cex}{}\right.}{}\right.}{c^2}$ |
| risch | $-\frac{ibd\,\mathrm{dilog}\left(\frac{-icd+(-icx+1)e-e}{-icd-e}\right)}{2e^2}-\frac{ib\ln(icx+1)x}{2e}-\frac{b\ln(c^2x^2+1)}{4ce}+\frac{ibd\,\mathrm{dilog}\left(\frac{icd+(icx+1)e-e}{icd-e}\right)}{2e^2}+\frac{b}{ce}+\frac{ib\,\mathrm{arc}}{2}$ |

input
```
int(x*(a+b*arctan(c*x))/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
a*x/e-a*d/e^2*ln(e*x+d)+b/c^2*(c^2*arctan(c*x)/e*x-c^2*arctan(c*x)*d/e^2*l
n(c*e*x+c*d)-c/e*(1/2*ln(c^2*d^2-2*c*d*(c*e*x+c*d)+e^2+(c*e*x+c*d)^2)-c*d*
(-1/2*I*ln(c*e*x+c*d)*(ln((I*e-c*e*x)/(c*d+I*e))-ln((I*e+c*e*x)/(I*e-c*d))
)/e-1/2*I*(dilog((I*e-c*e*x)/(c*d+I*e))-dilog((I*e+c*e*x)/(I*e-c*d)))/e)))
```

## Fricas [F]

$$\int \frac{x(a+b\arctan(cx))}{d+ex}\,dx=\int \frac{(b\arctan(cx)+a)x}{ex+d}\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))/(e*x+d),x, algorithm="fricas")
```

output
```
integral((b*x*arctan(c*x) + a*x)/(e*x + d), x)
```

## Sympy [F]

$$\int \frac{x(a+b\arctan(cx))}{d+ex}\,dx = \int \frac{x(a+b\,\mathrm{atan}\,(cx))}{d+ex}\,dx$$

input
```
integrate(x*(a+b*atan(c*x))/(e*x+d),x)
```

output
```
Integral(x*(a + b*atan(c*x))/(d + e*x), x)
```

## Maxima [F]

$$\int \frac{x(a+b\arctan(cx))}{d+ex}\,dx = \int \frac{(b\arctan{(cx)}+a)x}{ex+d}\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))/(e*x+d),x, algorithm="maxima")
```

output
```
a*(x/e - d*log(e*x + d)/e^2) + 2*b*integrate(1/2*x*arctan(c*x)/(e*x + d),
x)
```

## Giac [F]

$$\int \frac{x(a+b\arctan(cx))}{d+ex}\,dx = \int \frac{(b\arctan{(cx)}+a)x}{ex+d}\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))/(e*x+d),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)*x/(e*x + d), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\arctan(cx))}{d+ex}\,dx=\int \frac{x\,(a+b\,\mathrm{atan}(c\,x))}{d+e\,x}\,dx$$

input
```
int((x*(a + b*atan(c*x)))/(d + e*x),x)
```

output
```
int((x*(a + b*atan(c*x)))/(d + e*x), x)
```

## Reduce [F]

$$\int \frac{x(a+b\arctan(cx))}{d+ex}\,dx=\frac{\left(\int \frac{atan(cx)x}{ex+d}dx\right)b\,e^2-\log(ex+d)\,ad+aex}{e^2}$$

input
```
int(x*(a+b*atan(c*x))/(e*x+d),x)
```

output
```
(int((atan(c*x)*x)/(d + e*x),x)*b*e**2 - log(d + e*x)*a*d + a*e*x)/e**2
```

# 3.137 $\int \frac{a+b\arctan(cx)}{d+ex}\,dx$



## Optimal result

Integrand size = 16, antiderivative size = 138

$$\int \frac{a+b\arctan(cx)}{d+ex}\,dx = -\frac{(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)}{e} + \frac{(a+b\arctan(cx))\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{e} + \frac{ib\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2e} - \frac{ib\operatorname{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2e}$$

output
```
-(a+b*arctan(c*x))*ln(2/(1-I*c*x))/e+(a+b*arctan(c*x))*ln(2*c*(e*x+d)/(c*d
+I*e)/(1-I*c*x))/e+1/2*I*b*polylog(2,1-2/(1-I*c*x))/e-1/2*I*b*polylog(2,1-
2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/e
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00

$$\int \frac{a+b\arctan(cx)}{d+ex}\,dx = \frac{2a\log(d+ex)+ib\log(1-icx)\log\left(\frac{c(d+ex)}{cd-ie}\right)-ib\log(1+icx)\log\left(\frac{c(d+ex)}{cd+ie}\right)+ib\operatorname{PolyLog}\left(2,\frac{e(1-icx)}{icd+e}\right)-}{2e}$$

input
```
Integrate[(a + b*ArcTan[c*x])/(d + e*x),x]
```

output
```
(2*a*Log[d + e*x] + I*b*Log[1 - I*c*x]*Log[(c*(d + e*x))/(c*d - I*e)] - I*
b*Log[1 + I*c*x]*Log[(c*(d + e*x))/(c*d + I*e)] + I*b*PolyLog[2, (e*(1 - I
*c*x))/(I*c*d + e)] - I*b*PolyLog[2, -((e*(-I + c*x))/(c*d + I*e))])/(2*e)
```

## Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5381, 2849, 2752, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{d+ex}dx$$

$$\Big\downarrow \text{5381}$$

$$-\frac{bc\int \frac{\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{c^2x^2+1}dx}{e}+\frac{bc\int \frac{\log\left(\frac{2}{1-icx}\right)}{c^2x^2+1}dx}{e}+\frac{(a+b\arctan(cx))\log\left(\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{e}-\frac{\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))}{e}$$

$$\Big\downarrow \text{2849}$$

$$-\frac{bc\int \frac{\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{c^2x^2+1}dx}{e}+\frac{ib\int \frac{\log\left(\frac{2}{1-icx}\right)}{1-\frac{2}{1-icx}}d\frac{1}{1-icx}}{e}+\frac{(a+b\arctan(cx))\log\left(\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{e}-\frac{\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))}{e}$$

$$\Big\downarrow \text{2752}$$

$$-\frac{bc\int \frac{\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{c^2x^2+1}dx}{e}+\frac{(a+b\arctan(cx))\log\left(\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{e}-\frac{\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))}{e}+\frac{ib\,\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2e}$$

$$\Big\downarrow \text{2897}$$

$$\frac{(a+b\arctan(cx))\log\left(\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{e}-\frac{\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))}{e}-\frac{ib\,\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2e}+\frac{ib\,\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2e}$$

input
```
Int[(a + b*ArcTan[c*x])/(d + e*x),x]
```

output
```
-(((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e) + ((a + b*ArcTan[c*x])*Log[(
2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e + ((I/2)*b*PolyLog[2, 1 - 2/(
1 - I*c*x)])/e - ((I/2)*b*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 -
 I*c*x))])/e
```

## Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5381
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)), x_Symbol] :> Si
mp[(-(a + b*ArcTan[c*x]))*(Log[2/(1 - I*c*x)]/e), x] + (Simp[(a + b*ArcTan[
c*x])*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x] + Simp[b*(c/e)
   Int[Log[2/(1 - I*c*x)]/(1 + c^2*x^2), x], x] - Simp[b*(c/e)   Int[Log[2*
c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/(1 + c^2*x^2), x], x]) /; FreeQ[{a
, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

## Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.13

| method | result |
|---|---|
| parts | $\frac{a\ln(ex+d)}{e}+\frac{b\left(\frac{c\ln(cex+cd)\arctan(cx)}{e}-c\left(-\frac{i\ln(cex+cd)\left(\ln\left(\frac{-cex+ie}{cd+ie}\right)-\ln\left(\frac{cex+ie}{-cd+ie}\right)\right)}{2e}-\frac{i\left(\operatorname{dilog}\left(\frac{-cex+ie}{cd+ie}\right)-\operatorname{dilog}\left(\frac{cex\cdots}{-cd\cdots}\right)\cdots\right)}{2e}\cdots\right)\right)}{c}$ |
| derivativedivides | $\frac{\frac{ac\ln(cex+cd)}{e}+bc\left(\frac{\ln(cex+cd)\arctan(cx)}{e}+\frac{i\ln(cex+cd)\left(\ln\left(\frac{-cex+ie}{cd+ie}\right)-\ln\left(\frac{cex+ie}{-cd+ie}\right)\right)}{2e}+\frac{i\left(\operatorname{dilog}\left(\frac{-cex+ie}{cd+ie}\right)-\operatorname{dilog}\left(\frac{cex+ie}{-cd+ie}\right)\right)}{2e}\cdots\right)}{c}$ |
| default | $\frac{\frac{ac\ln(cex+cd)}{e}+bc\left(\frac{\ln(cex+cd)\arctan(cx)}{e}+\frac{i\ln(cex+cd)\left(\ln\left(\frac{-cex+ie}{cd+ie}\right)-\ln\left(\frac{cex+ie}{-cd+ie}\right)\right)}{2e}+\frac{i\left(\operatorname{dilog}\left(\frac{-cex+ie}{cd+ie}\right)-\operatorname{dilog}\left(\frac{cex+ie}{-cd+ie}\right)\right)}{2e}\cdots\right)}{c}$ |
| risch | $\frac{ib\operatorname{dilog}\left(\frac{-icd+(-icx+1)e-e}{-icd-e}\right)}{2e}+\frac{ib\ln(-icx+1)\ln\left(\frac{-icd+(-icx+1)e-e}{-icd-e}\right)}{2e}+\frac{a\ln(icd-(-icx+1)e+e)}{e}-\frac{ib\operatorname{dilog}\left(\frac{icd\cdots}{\cdots}\right)}{2\cdots}$ |

input
```
int((a+b*arctan(c*x))/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
a*ln(e*x+d)/e+b/c*(c*ln(c*e*x+c*d)/e*arctan(c*x)-c*(-1/2*I*ln(c*e*x+c*d)*(
ln((I*e-c*e*x)/(c*d+I*e))-ln((I*e+c*e*x)/(I*e-c*d)))/e-1/2*I*(dilog((I*e-c
*e*x)/(c*d+I*e))-dilog((I*e+c*e*x)/(I*e-c*d)))/e))
```

## Fricas [F]

$$\int \frac{a+b\arctan(cx)}{d+ex}\,dx=\int \frac{b\arctan(cx)+a}{ex+d}\,dx$$

input
```
integrate((a+b*arctan(c*x))/(e*x+d),x, algorithm="fricas")
```

output
```
integral((b*arctan(c*x) + a)/(e*x + d), x)
```

## Sympy [F]

$$\int \frac{a+b\arctan(cx)}{d+ex}\,dx=\int \frac{a+b\,\mathrm{atan}\,(cx)}{d+ex}\,dx$$

input
```
integrate((a+b*atan(c*x))/(e*x+d),x)
```

output
```
Integral((a + b*atan(c*x))/(d + e*x), x)
```

## Maxima [F]

$$\int \frac{a+b\arctan(cx)}{d+ex}\,dx=\int \frac{b\arctan\,(cx)+a}{ex+d}\,dx$$

input
```
integrate((a+b*arctan(c*x))/(e*x+d),x, algorithm="maxima")
```

output
```
2*b*integrate(1/2*arctan(c*x)/(e*x + d), x) + a*log(e*x + d)/e
```

## Giac [F]

$$\int \frac{a+b\arctan(cx)}{d+ex}\,dx=\int \frac{b\arctan\,(cx)+a}{ex+d}\,dx$$

input
```
integrate((a+b*arctan(c*x))/(e*x+d),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)/(e*x + d), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{d+ex}\,dx = \int \frac{a+b\,\mathrm{atan}(c\,x)}{d+e\,x}\,dx$$

input
```
int((a + b*atan(c*x))/(d + e*x),x)
```

output
```
int((a + b*atan(c*x))/(d + e*x), x)
```

## Reduce [F]

$$\int \frac{a+b\arctan(cx)}{d+ex}\,dx = \frac{\left(\int \frac{atan(cx)}{ex+d}dx\right)be+\log(ex+d)\,a}{e}$$

input
```
int((a+b*atan(c*x))/(e*x+d),x)
```

output
```
(int(atan(c*x)/(d + e*x),x)*b*e + log(d + e*x)*a)/e
```

# 3.138 $\int \frac{a+b\arctan(cx)}{x(d+ex)}\,dx$



## Optimal result

Integrand size = 19, antiderivative size = 181

$$\int \frac{a+b\arctan(cx)}{x(d+ex)}\,dx = \frac{a\log(x)}{d}+\frac{(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)}{d}-\frac{(a+b\arctan(cx))\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{d}+\frac{ib\,\text{PolyLog}(2,-icx)}{2d}-\frac{ib\,\text{PolyLog}(2,icx)}{2d}-\frac{ib\,\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2d}+\frac{ib\,\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2d}$$

output
```
a*ln(x)/d+(a+b*arctan(c*x))*ln(2/(1-I*c*x))/d-(a+b*arctan(c*x))*ln(2*c*(e*
x+d)/(c*d+I*e)/(1-I*c*x))/d+1/2*I*b*polylog(2,-I*c*x)/d-1/2*I*b*polylog(2,
I*c*x)/d-1/2*I*b*polylog(2,1-2/(1-I*c*x))/d+1/2*I*b*polylog(2,1-2*c*(e*x+d
)/(c*d+I*e)/(1-I*c*x))/d
```

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.93

$$\int \frac{a+b\arctan(cx)}{x(d+ex)}\,dx = \frac{2a\log(x)-2a\log(d+ex)-ib\log(1-icx)\log\left(\frac{c(d+ex)}{cd-ie}\right)+ib\log(1+icx)\log\left(\frac{c(d+ex)}{cd+ie}\right)+ib\,\mathrm{PolyLog}(2,}{2d}$$

input
```
Integrate[(a + b*ArcTan[c*x])/(x*(d + e*x)),x]
```

output
```
(2*a*Log[x] - 2*a*Log[d + e*x] - I*b*Log[1 - I*c*x]*Log[(c*(d + e*x))/(c*d
 - I*e)] + I*b*Log[1 + I*c*x]*Log[(c*(d + e*x))/(c*d + I*e)] + I*b*PolyLog
[2, (-I)*c*x] - I*b*PolyLog[2, I*c*x] - I*b*PolyLog[2, (e*(1 - I*c*x))/(I*
c*d + e)] + I*b*PolyLog[2, -((e*(-I + c*x))/(c*d + I*e))])/(2*d)
```

## Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x(d+ex)}\,dx$$

$$\Big\downarrow \text{5411}$$

$$\int \left(\frac{a+b\arctan(cx)}{dx}-\frac{e(a+b\arctan(cx))}{d(d+ex)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(a+b\arctan(cx))\log\left(\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{d}+\frac{\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))}{d}+\frac{a\log(x)}{d}+\frac{ib\operatorname{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2d}+\frac{ib\operatorname{PolyLog}(2,-icx)}{2d}-\frac{ib\operatorname{PolyLog}(2,icx)}{2d}-\frac{ib\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2d}$$

input
```
Int[(a + b*ArcTan[c*x])/(x*(d + e*x)),x]
```

output
```
(a*Log[x])/d + ((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/d - ((a + b*ArcTan
[c*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/d + ((I/2)*b*PolyLo
g[2, (-I)*c*x])/d - ((I/2)*b*PolyLog[2, I*c*x])/d - ((I/2)*b*PolyLog[2, 1
- 2/(1 - I*c*x)])/d + ((I/2)*b*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)
*(1 - I*c*x))])/d
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

## Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.30

| method | result |
|---|---|
| risch | $-\frac{ib\,\mathrm{dilog}\left(\frac{-icd+(-icx+1)e-e}{-icd-e}\right)}{2d}-\frac{ib\ln(-icx+1)\ln\left(\frac{-icd+(-icx+1)e-e}{-icd-e}\right)}{2d}-\frac{ib\,\mathrm{dilog}(-icx+1)}{2d}-\frac{a\ln(icd-(-icx+$ |
| parts | $-\frac{a\ln(ex+d)}{d}+\frac{a\ln(x)}{d}+b\left(\frac{\arctan(cx)\ln(cx)}{d}-\frac{\arctan(cx)\ln(cex+cd)}{d}-c\left(\frac{-\frac{i\ln(cx)\ln(icx+1)}{2}+\frac{i\ln(cx)\ln(}{2}}{}\right.\right.$ |
| derivativedivides | $\frac{a\ln(cx)}{d}-\frac{a\ln(cex+cd)}{d}+bc\left(\frac{\arctan(cx)\ln(cx)}{dc}-\frac{\arctan(cx)\ln(cex+cd)}{dc}-\frac{-\frac{i\ln(cx)\ln(icx+1)}{2}+\frac{i\ln(cx)\ln(-i}{2}}{}\right.$ |
| default | $\frac{a\ln(cx)}{d}-\frac{a\ln(cex+cd)}{d}+bc\left(\frac{\arctan(cx)\ln(cx)}{dc}-\frac{\arctan(cx)\ln(cex+cd)}{dc}-\frac{-\frac{i\ln(cx)\ln(icx+1)}{2}+\frac{i\ln(cx)\ln(-i}{2}}{}\right.$ |

input
```
int((a+b*arctan(c*x))/x/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*b/d*dilog((-I*c*d+(1-I*c*x)*e-e)/(-I*c*d-e))-1/2*I*b/d*ln(1-I*c*x)*
ln((-I*c*d+(1-I*c*x)*e-e)/(-I*c*d-e))-1/2*I*b/d*dilog(1-I*c*x)-a/d*ln(I*c*
d-(1-I*c*x)*e+e)+a/d*ln(-I*c*x)+1/2*I*b/d*dilog((I*c*d+(1+I*c*x)*e-e)/(I*c
*d-e))+1/2*I*b/d*ln(1+I*c*x)*ln((I*c*d+(1+I*c*x)*e-e)/(I*c*d-e))+1/2*I*b/d
*dilog(1+I*c*x)
```

## Fricas [F]

$$\int \frac{a+b\arctan(cx)}{x(d+ex)}\,dx=\int \frac{b\arctan(cx)+a}{(ex+d)x}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x/(e*x+d),x, algorithm="fricas")
```

output
```
integral((b*arctan(c*x) + a)/(e*x^2 + d*x), x)
```

## Sympy [F]

$$\int \frac{a+b\arctan(cx)}{x(d+ex)}\,dx = \int \frac{a+b\,\mathrm{atan}\,(cx)}{x\,(d+ex)}\,dx$$

input
```
integrate((a+b*atan(c*x))/x/(e*x+d),x)
```

output
```
Integral((a + b*atan(c*x))/(x*(d + e*x)), x)
```

## Maxima [F]

$$\int \frac{a+b\arctan(cx)}{x(d+ex)}\,dx = \int \frac{b\arctan\,(cx)+a}{(ex+d)x}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x/(e*x+d),x, algorithm="maxima")
```

output
```
-a*(log(e*x + d)/d - log(x)/d) + 2*b*integrate(1/2*arctan(c*x)/(e*x^2 + d*
x), x)
```

## Giac [F]

$$\int \frac{a+b\arctan(cx)}{x(d+ex)}\,dx = \int \frac{b\arctan\,(cx)+a}{(ex+d)x}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x/(e*x+d),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)/((e*x + d)*x), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{x(d+ex)}\,dx = \int \frac{a+b\,\mathrm{atan}(c\,x)}{x\,(d+e\,x)}\,dx$$

input
```
int((a + b*atan(c*x))/(x*(d + e*x)),x)
```

output
```
int((a + b*atan(c*x))/(x*(d + e*x)), x)
```

## Reduce [F]

$$\int \frac{a+b\arctan(cx)}{x(d+ex)}\,dx = \frac{atan(cx)^2\,bcd + 2\left(\int \frac{atan(cx)}{c^2e\,x^4+c^2d\,x^3+e\,x^2+dx}dx\right)bde - 2\left(\int \frac{atan(cx)}{c^2e\,x^3+c^2d\,x^2+ex+d}dx\right)b\,c^2d^2 - 2\log(ex+d)\,ae + 2}{2de}$$

input
```
int((a+b*atan(c*x))/x/(e*x+d),x)
```

output
```
(atan(c*x)**2*b*c*d + 2*int(atan(c*x)/(c**2*d*x**3 + c**2*e*x**4 + d*x + e
*x**2),x)*b*d*e - 2*int(atan(c*x)/(c**2*d*x**2 + c**2*e*x**3 + d + e*x),x)
*b*c**2*d**2 - 2*log(d + e*x)*a*e + 2*log(x)*a*e)/(2*d*e)
```

# 3.139 $\int \frac{a+b\arctan(cx)}{x^2(d+ex)}\,dx$



## Optimal result

Integrand size = 19, antiderivative size = 232

$$\begin{aligned}\int \frac{a+b\arctan(cx)}{x^2(d+ex)}\,dx = &-\frac{a+b\arctan(cx)}{dx}+\frac{bc\log(x)}{d}-\frac{ae\log(x)}{d^2}\\&-\frac{e(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)}{d^2}\\&+\frac{e(a+b\arctan(cx))\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{d^2}\\&-\frac{bc\log\left(1+c^2x^2\right)}{2d}-\frac{ibe\operatorname{PolyLog}(2,-icx)}{2d^2}\\&+\frac{ibe\operatorname{PolyLog}(2,icx)}{2d^2}+\frac{ibe\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2d^2}\\&-\frac{ibe\operatorname{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2d^2}\end{aligned}$$

output
```
-(a+b*arctan(c*x))/d/x+b*c*ln(x)/d-a*e*ln(x)/d^2-e*(a+b*arctan(c*x))*ln(2/
(1-I*c*x))/d^2+e*(a+b*arctan(c*x))*ln(2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/d^2
-1/2*b*c*ln(c^2*x^2+1)/d-1/2*I*b*e*polylog(2,-I*c*x)/d^2+1/2*I*b*e*polylog
(2,I*c*x)/d^2+1/2*I*b*e*polylog(2,1-2/(1-I*c*x))/d^2-1/2*I*b*e*polylog(2,1
-2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/d^2
```

## Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.96

$$\int \frac{a+b\arctan(cx)}{x^2(d+ex)}\,dx =$$

$$-\frac{2ad+2bd\arctan(cx)-2bcdx\log(x)+2aex\log(x)-2aex\log(d+ex)-ibex\log(1-icx)\log\left(\frac{c(d+ex)}{cd-ie}\right)}{}$$

input
```
Integrate[(a + b*ArcTan[c*x])/(x^2*(d + e*x)),x]
```

output
```
-1/2*(2*a*d + 2*b*d*ArcTan[c*x] - 2*b*c*d*x*Log[x] + 2*a*e*x*Log[x] - 2*a*
e*x*Log[d + e*x] - I*b*e*x*Log[1 - I*c*x]*Log[(c*(d + e*x))/(c*d - I*e)] +
 I*b*e*x*Log[1 + I*c*x]*Log[(c*(d + e*x))/(c*d + I*e)] + b*c*d*x*Log[1 + c
^2*x^2] + I*b*e*x*PolyLog[2, (-I)*c*x] - I*b*e*x*PolyLog[2, I*c*x] - I*b*e
*x*PolyLog[2, (e*(1 - I*c*x))/(I*c*d + e)] + I*b*e*x*PolyLog[2, -((e*(-I +
 c*x))/(c*d + I*e))])/(d^2*x)
```

## Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x^2(d+ex)}\,dx$$

$$\Big\downarrow \text{5411}$$

$$\int \left(\frac{e^2(a+b\arctan(cx))}{d^2(d+ex)}-\frac{e(a+b\arctan(cx))}{d^2x}+\frac{a+b\arctan(cx)}{dx^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{e\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))}{d^2}+\frac{e(a+b\arctan(cx))\log\left(\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{d^2}-\frac{a+b\arctan(cx)}{dx}-\frac{ae\log(x)}{d^2}-\frac{bc\log\left(c^2x^2+1\right)}{2d}-\frac{ibe\operatorname{PolyLog}(2,-icx)}{2d^2}+\frac{ibe\operatorname{PolyLog}(2,icx)}{2d^2}+\frac{ibe\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2d^2}-\frac{ibe\operatorname{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2d^2}+\frac{bc\log(x)}{d}$$

input
```
Int[(a + b*ArcTan[c*x])/(x^2*(d + e*x)),x]
```

output
```
-((a + b*ArcTan[c*x])/(d*x)) + (b*c*Log[x])/d - (a*e*Log[x])/d^2 - (e*(a +
 b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/d^2 + (e*(a + b*ArcTan[c*x])*Log[(2*c*
(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/d^2 - (b*c*Log[1 + c^2*x^2])/(2*d)
- ((I/2)*b*e*PolyLog[2, (-I)*c*x])/d^2 + ((I/2)*b*e*PolyLog[2, I*c*x])/d^2
 + ((I/2)*b*e*PolyLog[2, 1 - 2/(1 - I*c*x)])/d^2 - ((I/2)*b*e*PolyLog[2, 1
 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/d^2
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

## Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.32

| method | result |
|---|---|
| parts | $a\left(\frac{e\ln(ex+d)}{d^2}-\frac{1}{dx}-\frac{e\ln(x)}{d^2}\right)+bc\left(-\frac{\arctan(cx)}{dcx}-\frac{\arctan(cx)e\ln(cx)}{c\,d^2}+\frac{\arctan(cx)e\ln(cex+cd)}{c\,d^2}-c\right.$ |
| derivativedivides | $c\left(-\frac{a}{dcx}-\frac{ae\ln(cx)}{c\,d^2}+\frac{ae\ln(cex+cd)}{c\,d^2}+bc\left(-\frac{\arctan(cx)}{d\,c^2x}-\frac{\arctan(cx)e\ln(cx)}{d^2c^2}+\frac{\arctan(cx)e\ln(cex+cd}{d^2c^2}\right.\right.$ |
| default | $c\left(-\frac{a}{dcx}-\frac{ae\ln(cx)}{c\,d^2}+\frac{ae\ln(cex+cd)}{c\,d^2}+bc\left(-\frac{\arctan(cx)}{d\,c^2x}-\frac{\arctan(cx)e\ln(cx)}{d^2c^2}+\frac{\arctan(cx)e\ln(cex+cd}{d^2c^2}\right.\right.$ |
| risch | $\frac{ibe\,\mathrm{dilog}\left(\frac{-icd+(-icx+1)e-e}{-icd-e}\right)}{2d^2}+\frac{ibe\ln(-icx+1)\ln\left(\frac{-icd+(-icx+1)e-e}{-icd-e}\right)}{2d^2}+\frac{ibe\,\mathrm{dilog}(-icx+1)}{2d^2}+\frac{cb\ln(-icx)}{2d}-$ |

input

```
int((a+b*arctan(c*x))/x^2/(e*x+d),x,method=_RETURNVERBOSE)
```

output

```
a*(e/d^2*ln(e*x+d)-1/d/x-e/d^2*ln(x))+b*c*(-arctan(c*x)/d/c/x-arctan(c*x)/
c*e/d^2*ln(c*x)+arctan(c*x)/c*e/d^2*ln(c*e*x+c*d)-c*(1/d^2/c^2*e^2*(-1/2*I
*ln(c*e*x+c*d)*(ln((I*e-c*e*x)/(c*d+I*e))-ln((I*e+c*e*x)/(I*e-c*d)))/e-1/2
*I*(dilog((I*e-c*e*x)/(c*d+I*e))-dilog((I*e+c*e*x)/(I*e-c*d)))/e)-1/d/c*(l
n(c*x)-1/2*ln(c^2*x^2+1))-1/d^2/c^2*e*(-1/2*I*ln(c*x)*ln(1+I*c*x)+1/2*I*ln
(c*x)*ln(1-I*c*x)-1/2*I*dilog(1+I*c*x)+1/2*I*dilog(1-I*c*x))))
```

## Fricas [F]

$$\int \frac{a+b\arctan(cx)}{x^2(d+ex)}\,dx=\int \frac{b\arctan{(cx)}+a}{(ex+d)x^2}\,dx$$

input

```
integrate((a+b*arctan(c*x))/x^2/(e*x+d),x, algorithm="fricas")
```

output

```
integral((b*arctan(c*x) + a)/(e*x^3 + d*x^2), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{x^2(d+ex)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*atan(c*x))/x**2/(e*x+d),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{a+b\arctan(cx)}{x^2(d+ex)}\,dx = \int \frac{b\arctan(cx)+a}{(ex+d)x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^2/(e*x+d),x, algorithm="maxima")
```

output
```
a*(e*log(e*x + d)/d^2 - e*log(x)/d^2 - 1/(d*x)) + 2*b*integrate(1/2*arctan
(c*x)/(e*x^3 + d*x^2), x)
```

## Giac [F]

$$\int \frac{a+b\arctan(cx)}{x^2(d+ex)}\,dx = \int \frac{b\arctan(cx)+a}{(ex+d)x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^2/(e*x+d),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)/((e*x + d)*x^2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{x^2(d+ex)}\,dx = \int \frac{a+b\,\mathrm{atan}(c\,x)}{x^2\,\left(d+e\,x\right)}\,dx$$

input
```
int((a + b*atan(c*x))/(x^2*(d + e*x)),x)
```

output
```
int((a + b*atan(c*x))/(x^2*(d + e*x)), x)
```

## Reduce [F]

$$\int \frac{a+b\arctan(cx)}{x^2(d+ex)}\,dx$$
$$= \frac{\left(\int \frac{atan(cx)}{c^2e\,x^5+c^2d\,x^4+e\,x^3+d\,x^2}dx\right) b\,d^2x + \left(\int \frac{atan(cx)}{c^2e\,x^3+c^2d\,x^2+ex+d}dx\right) b\,c^2d^2x + \log(ex+d)\,aex - \log(x)\,aex - ad}{d^2x}$$

input
```
int((a+b*atan(c*x))/x^2/(e*x+d),x)
```

output
```
(int(atan(c*x)/(c**2*d*x**4 + c**2*e*x**5 + d*x**2 + e*x**3),x)*b*d**2*x +
 int(atan(c*x)/(c**2*d*x**2 + c**2*e*x**3 + d + e*x),x)*b*c**2*d**2*x + lo
g(d + e*x)*a*e*x - log(x)*a*e*x - a*d)/(d**2*x)
```

# 3.140 $\int \frac{a+b\arctan(cx)}{x^3(d+ex)}\,dx$



## Optimal result

Integrand size = 19, antiderivative size = 293

$$\begin{aligned}\int \frac{a+b\arctan(cx)}{x^3(d+ex)}\,dx = &-\frac{bc}{2dx}-\frac{bc^2\arctan(cx)}{2d}-\frac{a+b\arctan(cx)}{2dx^2}\\ &+\frac{e(a+b\arctan(cx))}{d^2x}-\frac{bce\log(x)}{d^2}\\ &+\frac{ae^2\log(x)}{d^3}+\frac{e^2(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)}{d^3}\\ &-\frac{e^2(a+b\arctan(cx))\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{d^3}\\ &+\frac{bce\log\left(1+c^2x^2\right)}{2d^2}+\frac{ibe^2\operatorname{PolyLog}(2,-icx)}{2d^3}\\ &-\frac{ibe^2\operatorname{PolyLog}(2,icx)}{2d^3}-\frac{ibe^2\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2d^3}\\ &+\frac{ibe^2\operatorname{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2d^3}\end{aligned}$$

output
```
-1/2*b*c/d/x-1/2*b*c^2*arctan(c*x)/d-1/2*(a+b*arctan(c*x))/d/x^2+e*(a+b*ar
ctan(c*x))/d^2/x-b*c*e*ln(x)/d^2+a*e^2*ln(x)/d^3+e^2*(a+b*arctan(c*x))*ln(
2/(1-I*c*x))/d^3-e^2*(a+b*arctan(c*x))*ln(2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x))
/d^3+1/2*b*c*e*ln(c^2*x^2+1)/d^2+1/2*I*b*e^2*polylog(2,-I*c*x)/d^3-1/2*I*b
*e^2*polylog(2,I*c*x)/d^3-1/2*I*b*e^2*polylog(2,1-2/(1-I*c*x))/d^3+1/2*I*b
*e^2*polylog(2,1-2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/d^3
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.12 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.02

$$\int \frac{a+b\arctan(cx)}{x^3(d+ex)}\,dx = -\frac{a+b\arctan(cx)}{2dx^2}+\frac{e(a+b\arctan(cx))}{d^2x} - \frac{bc\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-c^2x^2\right)}{2dx}+\frac{ae^2\log(x)}{d^3} - \frac{ae^2\log(d+ex)}{d^3}-\frac{bce(2\log(x)-\log(1+c^2x^2))}{2d^2} + \frac{ibe^2\,\text{PolyLog}(2,-icx)}{2d^3}-\frac{ibe^2\,\text{PolyLog}(2,icx)}{2d^3} - \frac{ib\left(e^2\log(1-icx)\log\left(\frac{c(d+ex)}{cd-ie}\right)+e^2\,\text{PolyLog}\left(2,\frac{e(1-icx)}{icd+e}\right)\right)}{2d^3} + \frac{ib\left(e^2\log(1+icx)\log\left(\frac{c(d+ex)}{cd+ie}\right)+e^2\,\text{PolyLog}\left(2,-\frac{e(1+icx)}{icd-e}\right)\right)}{2d^3}$$

input
```
Integrate[(a + b*ArcTan[c*x])/(x^3*(d + e*x)),x]
```

output
```
-1/2*(a + b*ArcTan[c*x])/(d*x^2) + (e*(a + b*ArcTan[c*x]))/(d^2*x) - (b*c*
Hypergeometric2F1[-1/2, 1, 1/2, -(c^2*x^2)])/(2*d*x) + (a*e^2*Log[x])/d^3
- (a*e^2*Log[d + e*x])/d^3 - (b*c*e*(2*Log[x] - Log[1 + c^2*x^2]))/(2*d^2)
 + ((I/2)*b*e^2*PolyLog[2, (-I)*c*x])/d^3 - ((I/2)*b*e^2*PolyLog[2, I*c*x]
)/d^3 - ((I/2)*b*(e^2*Log[1 - I*c*x]*Log[(c*(d + e*x))/(c*d - I*e)] + e^2*
PolyLog[2, (e*(1 - I*c*x))/(I*c*d + e)]))/d^3 + ((I/2)*b*(e^2*Log[1 + I*c*
x]*Log[(c*(d + e*x))/(c*d + I*e)] + e^2*PolyLog[2, -((e*(1 + I*c*x))/(I*c*
d - e))]))/d^3
```

## Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x^3(d+ex)}\,dx$$

$$\Big\downarrow \text{5411}$$

$$\int \left(-\frac{e^3(a+b\arctan(cx))}{d^3(d+ex)}+\frac{e^2(a+b\arctan(cx))}{d^3x}-\frac{e(a+b\arctan(cx))}{d^2x^2}+\frac{a+b\arctan(cx)}{dx^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^2\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))}{d^3}-\frac{e^2(a+b\arctan(cx))\log\left(\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{d^3}+\frac{e(a+b\arctan(cx))}{d^2x}-\frac{a+b\arctan(cx)}{2dx^2}+\frac{ae^2\log(x)}{d^3}-\frac{bc^2\arctan(cx)}{2d}+\frac{bce\log\left(c^2x^2+1\right)}{2d^2}+\frac{ibe^2\,\text{PolyLog}(2,-icx)}{2d^3}-\frac{ibe^2\,\text{PolyLog}(2,icx)}{2d^3}-\frac{ibe^2\,\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2d^3}+\frac{ibe^2\,\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2d^3}-\frac{bce\log(x)}{d^2}-\frac{bc}{2dx}$$

input
```
Int[(a + b*ArcTan[c*x])/(x^3*(d + e*x)),x]
```

output
```
-1/2*(b*c)/(d*x) - (b*c^2*ArcTan[c*x])/(2*d) - (a + b*ArcTan[c*x])/(2*d*x^
2) + (e*(a + b*ArcTan[c*x]))/(d^2*x) - (b*c*e*Log[x])/d^2 + (a*e^2*Log[x])
/d^3 + (e^2*(a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/d^3 - (e^2*(a + b*ArcT
an[c*x])*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/d^3 + (b*c*e*Log[
1 + c^2*x^2])/(2*d^2) + ((I/2)*b*e^2*PolyLog[2, (-I)*c*x])/d^3 - ((I/2)*b*
e^2*PolyLog[2, I*c*x])/d^3 - ((I/2)*b*e^2*PolyLog[2, 1 - 2/(1 - I*c*x)])/d
^3 + ((I/2)*b*e^2*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))
])/d^3
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

## Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.22

| method | result |
|---|---|
| parts | $a\left(-\frac{e^2\ln(ex+d)}{d^3}-\frac{1}{2d\,x^2}+\frac{e^2\ln(x)}{d^3}+\frac{e}{d^2x}\right)+b\,c^2\left(-\frac{\arctan(cx)}{2d\,c^2x^2}+\frac{\arctan(cx)e^2\ln(cx)}{c^2d^3}+\frac{\arctan(cx)e}{c^2d^2x}\right.$ |
| derivativedivides | $c^2\left(-\frac{a}{2d\,c^2x^2}+\frac{a\,e^2\ln(cx)}{c^2d^3}+\frac{ae}{c^2d^2x}-\frac{a\,e^2\ln(cex+cd)}{c^2d^3}+bc\left(-\frac{\arctan(cx)}{2d\,c^3x^2}+\frac{\arctan(cx)e^2\ln(cx)}{d^3c^3}+\mathrm{arc}\right.\right.$ |
| default | $c^2\left(-\frac{a}{2d\,c^2x^2}+\frac{a\,e^2\ln(cx)}{c^2d^3}+\frac{ae}{c^2d^2x}-\frac{a\,e^2\ln(cex+cd)}{c^2d^3}+bc\left(-\frac{\arctan(cx)}{2d\,c^3x^2}+\frac{\arctan(cx)e^2\ln(cx)}{d^3c^3}+\mathrm{arc}\right.\right.$ |
| risch | $-\frac{bc}{2dx}+\frac{ibe\ln(-icx+1)}{2d^2x}+\frac{ib\,e^2\,\mathrm{dilog}\left(\frac{icd+(icx+1)e-e}{icd-e}\right)}{2d^3}+\frac{ib\ln(icx+1)}{4d\,x^2}+\frac{ib\,c^2\ln(icx+1)}{4d}+\frac{ib\,e^2\ln(icx+1)\ln}{2}$ |

input
```
int((a+b*arctan(c*x))/x^3/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
a*(-e^2/d^3*ln(e*x+d)-1/2/d/x^2+e^2/d^3*ln(x)+e/d^2/x)+b*c^2*(-1/2*arctan(
c*x)/d/c^2/x^2+arctan(c*x)/c^2*e^2/d^3*ln(c*x)+arctan(c*x)/c^2*e/d^2/x-arc
tan(c*x)/c^2*e^2/d^3*ln(c*e*x+c*d)-1/2*c*(1/c^2/d^2*(-e*ln(c^2*x^2+1)+d*c*
arctan(c*x)+2*e*ln(c*x)+d/x)+2/d^3/c^3*e^2*(-1/2*I*ln(c*x)*ln(1+I*c*x)+1/2
*I*ln(c*x)*ln(1-I*c*x)-1/2*I*dilog(1+I*c*x)+1/2*I*dilog(1-I*c*x))-2/d^3/c^
3*e^3*(-1/2*I*ln(c*e*x+c*d)*(ln((I*e-c*e*x)/(c*d+I*e))-ln((I*e+c*e*x)/(I*e
-c*d)))/e-1/2*I*(dilog((I*e-c*e*x)/(c*d+I*e))-dilog((I*e+c*e*x)/(I*e-c*d))
)/e)))
```

## Fricas [F]

$$\int \frac{a+b\arctan(cx)}{x^3(d+ex)}\,dx=\int \frac{b\arctan{(cx)}+a}{(ex+d)x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^3/(e*x+d),x, algorithm="fricas")
```

output
```
integral((b*arctan(c*x) + a)/(e*x^4 + d*x^3), x)
```

## Sympy [F]

$$\int \frac{a+b\arctan(cx)}{x^3(d+ex)}\,dx=\int \frac{a+b\,\mathrm{atan}\,(cx)}{x^3\,(d+ex)}\,dx$$

input
```
integrate((a+b*atan(c*x))/x**3/(e*x+d),x)
```

output
```
Integral((a + b*atan(c*x))/(x**3*(d + e*x)), x)
```

## Maxima [F]

$$\int \frac{a+b\arctan(cx)}{x^3(d+ex)}\,dx = \int \frac{b\arctan(cx)+a}{(ex+d)x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^3/(e*x+d),x, algorithm="maxima")
```

output
```
-1/2*a*(2*e^2*log(e*x + d)/d^3 - 2*e^2*log(x)/d^3 - (2*e*x - d)/(d^2*x^2))
 + 2*b*integrate(1/2*arctan(c*x)/(e*x^4 + d*x^3), x)
```

## Giac [F]

$$\int \frac{a+b\arctan(cx)}{x^3(d+ex)}\,dx = \int \frac{b\arctan(cx)+a}{(ex+d)x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^3/(e*x+d),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)/((e*x + d)*x^3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{x^3(d+ex)}\,dx = \int \frac{a+b\,\mathrm{atan}(c\,x)}{x^3\,(d+e\,x)}\,dx$$

input
```
int((a + b*atan(c*x))/(x^3*(d + e*x)),x)
```

output
```
int((a + b*atan(c*x))/(x^3*(d + e*x)), x)
```

## Reduce [F]

$$\int \frac{a+b\arctan(cx)}{x^3(d+ex)}\,dx$$
$$= \frac{2\left(\int \frac{atan(cx)}{e\,x^4+d\,x^3}dx\right)b\,d^3x^2-2\log(ex+d)\,a\,e^2x^2+2\log(x)\,a\,e^2x^2-a\,d^2+2adex}{2d^3x^2}$$

input
```
int((a+b*atan(c*x))/x^3/(e*x+d),x)
```

output
```
(2*int(atan(c*x)/(d*x**3 + e*x**4),x)*b*d**3*x**2 - 2*log(d + e*x)*a*e**2*
x**2 + 2*log(x)*a*e**2*x**2 - a*d**2 + 2*a*d*e*x)/(2*d**3*x**2)
```

# 3.141 $\int \frac{x^3(a+b\arctan(cx))^2}{d+ex}\,dx$

## Optimal result

Integrand size = 21, antiderivative size = 598

$$\int \frac{x^3(a+b\arctan(cx))^2}{d+ex}\,dx = \frac{abdx}{ce^2}+\frac{b^2x}{3c^2e}-\frac{b^2\arctan(cx)}{3c^3e} + \frac{b^2dx\arctan(cx)}{ce^2}-\frac{bx^2(a+b\arctan(cx))}{3ce} + \frac{id^2(a+b\arctan(cx))^2}{ce^3}-\frac{d(a+b\arctan(cx))^2}{2c^2e^2} - \frac{i(a+b\arctan(cx))^2}{3c^3e}+\frac{d^2x(a+b\arctan(cx))^2}{e^3} - \frac{dx^2(a+b\arctan(cx))^2}{2e^2}+\frac{x^3(a+b\arctan(cx))^2}{3e} + \frac{d^3(a+b\arctan(cx))^2\log\left(\frac{2}{1-icx}\right)}{e^4} + \frac{2bd^2(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{ce^3} - \frac{2b(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{3c^3e} - \frac{d^3(a+b\arctan(cx))^2\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{e^4} - \frac{b^2d\log(1+c^2x^2)}{2c^2e^2} - \frac{ibd^3(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{e^4} + \frac{ib^2d^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{ce^3} - \frac{ib^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{3c^3e} + \frac{ibd^3(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{e^4} + \frac{b^2d^3\operatorname{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{2e^4} - \frac{b^2d^3\operatorname{PolyLog}\left(3,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2e^4}$$

output
```
a*b*d*x/c/e^2+1/3*b^2*x/c^2/e-1/3*b^2*arctan(c*x)/c^3/e+b^2*d*x*arctan(c*x
)/c/e^2-1/3*b*x^2*(a+b*arctan(c*x))/c/e-I*b*d^3*(a+b*arctan(c*x))*polylog(
2,1-2/(1-I*c*x))/e^4-1/2*d*(a+b*arctan(c*x))^2/c^2/e^2+I*b^2*d^2*polylog(2
,1-2/(1+I*c*x))/c/e^3+d^2*x*(a+b*arctan(c*x))^2/e^3-1/2*d*x^2*(a+b*arctan(
c*x))^2/e^2+1/3*x^3*(a+b*arctan(c*x))^2/e+d^3*(a+b*arctan(c*x))^2*ln(2/(1-
I*c*x))/e^4+2*b*d^2*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c/e^3-2/3*b*(a+b*arc
tan(c*x))*ln(2/(1+I*c*x))/c^3/e-d^3*(a+b*arctan(c*x))^2*ln(2*c*(e*x+d)/(c*
d+I*e)/(1-I*c*x))/e^4-1/2*b^2*d*ln(c^2*x^2+1)/c^2/e^2-1/3*I*(a+b*arctan(c*
x))^2/c^3/e+I*d^2*(a+b*arctan(c*x))^2/c/e^3-1/3*I*b^2*polylog(2,1-2/(1+I*c
*x))/c^3/e+I*b*d^3*(a+b*arctan(c*x))*polylog(2,1-2*c*(e*x+d)/(c*d+I*e)/(1-
I*c*x))/e^4+1/2*b^2*d^3*polylog(3,1-2/(1-I*c*x))/e^4-1/2*b^2*d^3*polylog(3
,1-2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/e^4
```

## Mathematica [F]

$$\int \frac{x^3(a+b\arctan(cx))^2}{d+ex}\,dx=\int \frac{x^3(a+b\arctan(cx))^2}{d+ex}\,dx$$

input
```
Integrate[(x^3*(a + b*ArcTan[c*x])^2)/(d + e*x),x]
```

output
```
Integrate[(x^3*(a + b*ArcTan[c*x])^2)/(d + e*x), x]
```

## Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 598, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\arctan(cx))^2}{d+ex}\,dx$$

$$\Big\downarrow \; 5411$$

$$\int \left(-\frac{d^3(a+b\arctan(cx))^2}{e^3(d+ex)}+\frac{d^2(a+b\arctan(cx))^2}{e^3}-\frac{dx(a+b\arctan(cx))^2}{e^2}+\frac{x^2(a+b\arctan(cx))^2}{e}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{i(a+b\arctan(cx))^2}{3c^3e}-\frac{2b\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{3c^3e}-\frac{d(a+b\arctan(cx))^2}{2c^2e^2}-\frac{ibd^3\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)(a+b\arctan(cx))}{e^4}+\frac{ibd^3(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{e^4}+\frac{d^3\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))^2}{e^4}-\frac{d^3(a+b\arctan(cx))^2\log\left(\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{e^4}+\frac{d^2x(a+b\arctan(cx))^2}{e^3}+\frac{id^2(a+b\arctan(cx))^2}{ce^3}+\frac{2bd^2\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{ce^3}-\frac{dx^2(a+b\arctan(cx))^2}{2e^2}+\frac{x^3(a+b\arctan(cx))^2}{3e}-\frac{bx^2(a+b\arctan(cx))}{3ce}+\frac{abdx}{ce^2}-\frac{b^2\arctan(cx)}{3c^3e}+\frac{b^2dx\arctan(cx)}{ce^2}-\frac{ib^2\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{3c^3e}-\frac{b^2d\log\left(c^2x^2+1\right)}{2c^2e^2}+\frac{b^2x}{3c^2e}+\frac{b^2d^3\operatorname{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{2e^4}-\frac{b^2d^3\operatorname{PolyLog}\left(3,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2e^4}+\frac{ib^2d^2\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{ce^3}$$

input
```
Int[(x^3*(a + b*ArcTan[c*x])^2)/(d + e*x),x]
```

output
```
(a*b*d*x)/(c*e^2) + (b^2*x)/(3*c^2*e) - (b^2*ArcTan[c*x])/(3*c^3*e) + (b^2
*d*x*ArcTan[c*x])/(c*e^2) - (b*x^2*(a + b*ArcTan[c*x]))/(3*c*e) + (I*d^2*(
a + b*ArcTan[c*x])^2)/(c*e^3) - (d*(a + b*ArcTan[c*x])^2)/(2*c^2*e^2) - ((
I/3)*(a + b*ArcTan[c*x])^2)/(c^3*e) + (d^2*x*(a + b*ArcTan[c*x])^2)/e^3 -
(d*x^2*(a + b*ArcTan[c*x])^2)/(2*e^2) + (x^3*(a + b*ArcTan[c*x])^2)/(3*e)
+ (d^3*(a + b*ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/e^4 + (2*b*d^2*(a + b*Arc
Tan[c*x])*Log[2/(1 + I*c*x)])/(c*e^3) - (2*b*(a + b*ArcTan[c*x])*Log[2/(1
+ I*c*x)])/(3*c^3*e) - (d^3*(a + b*ArcTan[c*x])^2*Log[(2*c*(d + e*x))/((c*
d + I*e)*(1 - I*c*x))])/e^4 - (b^2*d*Log[1 + c^2*x^2])/(2*c^2*e^2) - (I*b*
d^3*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 - I*c*x)])/e^4 + (I*b^2*d^2*Po
lyLog[2, 1 - 2/(1 + I*c*x)])/(c*e^3) - ((I/3)*b^2*PolyLog[2, 1 - 2/(1 + I*
c*x)])/(c^3*e) + (I*b*d^3*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(d + e*x
))/((c*d + I*e)*(1 - I*c*x))])/e^4 + (b^2*d^3*PolyLog[3, 1 - 2/(1 - I*c*x)
])/(2*e^4) - (b^2*d^3*PolyLog[3, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c
*x))])/(2*e^4)
```

**Defintions of rubi rules used**

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 66.74 (sec) , antiderivative size = 2062, normalized size of antiderivative = 3.45

| method | result | size |
|---|---|---|
| derivativedivides | Expression too large to display | 2062 |
| default | Expression too large to display | 2062 |
| parts | Expression too large to display | 2107 |

input
```
int(x^3*(a+b*arctan(c*x))^2/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/c^4*(a^2*c^4/e^3*d^2*x-1/2*a^2*c^4/e^2*d*x^2+1/3*a^2*c^4/e*x^3-a^2*c^4*d
^3/e^4*ln(c*e*x+c*d)+b^2*c*(-arctan(c*x)^2*c^3*d^3/e^4*ln(c*e*x+c*d)-1/2*I
/e^4*c^3*d^3*Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I*(-I*e*(1+I*c*x)
^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+c*d))*csgn(I*(-I*e*(1+I*c*x
)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+c*d)/(1+(1+I*c*x)^2/(c^2*x
^2+1)))*arctan(c*x)^2+1/2*I/e^4*c^3*d^3*Pi*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+
1)))*csgn(I*(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+
c*d)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*arctan(c*x)^2+1/2*I/e^4*c^3*d^3*Pi*csg
n(I*(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+c*d))*cs
gn(I*(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+c*d)/(1
+(1+I*c*x)^2/(c^2*x^2+1)))^2*arctan(c*x)^2+arctan(c*x)^2/e^3*c^3*d^2*x-1/2
*arctan(c*x)^2/e^2*c^3*d*x^2-2*I/e^3*c^2*d^2*dilog(1-I*(1+I*c*x)/(c^2*x^2+
1)^(1/2))-I/e^3*c^2*d^2*arctan(c*x)^2+2/3*I/e*dilog(1+I*(1+I*c*x)/(c^2*x^2
+1)^(1/2))+1/3*arctan(c*x)^2/e*c^3*x^3-2/3/e*arctan(c*x)*ln(1+I*(1+I*c*x)/
(c^2*x^2+1)^(1/2))-2/3/e*arctan(c*x)*ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+1
/3*arctan(c*x)*(c*x-I)^2/e+2/3*I/e*dilog(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+
1/3*I/e*arctan(c*x)^2-1/2*I/e^4*c^3*d^3*Pi*csgn(I*(-I*e*(1+I*c*x)^2/(c^2*x
^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+c*d)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^3*
arctan(c*x)^2+I*d^3*c^3/e^3/(c*d-I*e)*arctan(c*x)^2*ln(1-(I*e-c*d)/(c*d+I*
e)*(1+I*c*x)^2/(c^2*x^2+1))+I*d^4*c^4/e^4/(c*d-I*e)*arctan(c*x)*polylog...
```

## Fricas [F]

$$\int \frac{x^3(a+b\arctan(cx))^2}{d+ex}\,dx = \int \frac{(b\arctan(cx)+a)^2x^3}{ex+d}\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))^2/(e*x+d),x, algorithm="fricas")
```

output
```
integral((b^2*x^3*arctan(c*x)^2 + 2*a*b*x^3*arctan(c*x) + a^2*x^3)/(e*x +
d), x)
```

## Sympy [F]

$$\int \frac{x^3(a+b\arctan(cx))^2}{d+ex}\,dx = \int \frac{x^3(a+b\operatorname{atan}(cx))^2}{d+ex}\,dx$$

input
```
integrate(x**3*(a+b*atan(c*x))**2/(e*x+d),x)
```

output
```
Integral(x**3*(a + b*atan(c*x))**2/(d + e*x), x)
```

## Maxima [F]

$$\int \frac{x^3(a+b\arctan(cx))^2}{d+ex}\,dx = \int \frac{(b\arctan(cx)+a)^2x^3}{ex+d}\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))^2/(e*x+d),x, algorithm="maxima")
```

output
```
-1/6*a^2*(6*d^3*log(e*x + d)/e^4 - (2*e^2*x^3 - 3*d*e*x^2 + 6*d^2*x)/e^3)
+ 1/96*(96*e^3*integrate(1/48*(36*(b^2*c^2*e^3*x^5 + b^2*e^3*x^3)*arctan(c
*x)^2 + 3*(b^2*c^2*e^3*x^5 + b^2*e^3*x^3)*log(c^2*x^2 + 1)^2 + 4*(24*a*b*c
^2*e^3*x^5 - 2*b^2*c*e^3*x^4 - 3*b^2*c*d^2*e*x^2 - 6*b^2*c*d^3*x + (b^2*c*
d*e^2 + 24*a*b*e^3)*x^3)*arctan(c*x) + 2*(2*b^2*c^2*e^3*x^5 - b^2*c^2*d*e^
2*x^4 + 3*b^2*c^2*d^2*e*x^3 + 6*b^2*c^2*d^3*x^2)*log(c^2*x^2 + 1))/(c^2*e^
4*x^3 + c^2*d*e^3*x^2 + e^4*x + d*e^3), x) + 4*(2*b^2*e^2*x^3 - 3*b^2*d*e*
x^2 + 6*b^2*d^2*x)*arctan(c*x)^2 - (2*b^2*e^2*x^3 - 3*b^2*d*e*x^2 + 6*b^2*
d^2*x)*log(c^2*x^2 + 1)^2)/e^3
```

## Giac [F]

$$\int \frac{x^3(a+b\arctan(cx))^2}{d+ex}\,dx = \int \frac{(b\arctan(cx)+a)^2x^3}{ex+d}\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))^2/(e*x+d),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)^2*x^3/(e*x + d), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\arctan(cx))^2}{d+ex}\,dx = \int \frac{x^3\,(a+b\,\mathrm{atan}(c\,x))^2}{d+e\,x}\,dx$$

input
```
int((x^3*(a + b*atan(c*x))^2)/(d + e*x),x)
```

output
```
int((x^3*(a + b*atan(c*x))^2)/(d + e*x), x)
```

## Reduce [F]

$$\int \frac{x^3(a+b\arctan(cx))^2}{d+ex}\,dx = \frac{12\left(\int \frac{atan(cx)x^3}{ex+d}dx\right)ab\,e^4+6\left(\int \frac{atan(cx)^2x^3}{ex+d}dx\right)b^2e^4-6\log(ex+d)\,a^2d^3+6a^2d^2ex-3a^2d\,e^2x^2+2a^2e^3x^3}{6e^4}$$

input
```
int(x^3*(a+b*atan(c*x))^2/(e*x+d),x)
```

output
```
(12*int((atan(c*x)*x**3)/(d + e*x),x)*a*b*e**4 + 6*int((atan(c*x)**2*x**3)
/(d + e*x),x)*b**2*e**4 - 6*log(d + e*x)*a**2*d**3 + 6*a**2*d**2*e*x - 3*a
**2*d*e**2*x**2 + 2*a**2*e**3*x**3)/(6*e**4)
```

# 3.142 $\int \frac{x^2(a+b\arctan(cx))^2}{d+ex}\,dx$

## Optimal result

Integrand size = 21, antiderivative size = 430

$$\int \frac{x^2(a+b\arctan(cx))^2}{d+ex}\,dx = -\frac{abx}{ce}-\frac{b^2x\arctan(cx)}{ce}-\frac{id(a+b\arctan(cx))^2}{ce^2} + \frac{(a+b\arctan(cx))^2}{2c^2e}-\frac{dx(a+b\arctan(cx))^2}{e^2} + \frac{x^2(a+b\arctan(cx))^2}{2e} - \frac{d^2(a+b\arctan(cx))^2\log\left(\frac{2}{1-icx}\right)}{e^3} - \frac{2bd(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{ce^2} + \frac{d^2(a+b\arctan(cx))^2\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{e^3} + \frac{b^2\log(1+c^2x^2)}{2c^2e} + \frac{ibd^2(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{e^3} - \frac{ib^2d\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{ce^2} - \frac{ibd^2(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{e^3} - \frac{b^2d^2\operatorname{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{2e^3} + \frac{b^2d^2\operatorname{PolyLog}\left(3,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2e^3}$$

output
```
-a*b*x/c/e-b^2*x*arctan(c*x)/c/e-I*d*(a+b*arctan(c*x))^2/c/e^2+1/2*(a+b*ar
ctan(c*x))^2/c^2/e-d*x*(a+b*arctan(c*x))^2/e^2+1/2*x^2*(a+b*arctan(c*x))^2
/e-d^2*(a+b*arctan(c*x))^2*ln(2/(1-I*c*x))/e^3-2*b*d*(a+b*arctan(c*x))*ln(
2/(1+I*c*x))/c/e^2+d^2*(a+b*arctan(c*x))^2*ln(2*c*(e*x+d)/(c*d+I*e)/(1-I*c
*x))/e^3+1/2*b^2*ln(c^2*x^2+1)/c^2/e+I*b*d^2*(a+b*arctan(c*x))*polylog(2,1
-2/(1-I*c*x))/e^3-I*b^2*d*polylog(2,1-2/(1+I*c*x))/c/e^2-I*b*d^2*(a+b*arct
an(c*x))*polylog(2,1-2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/e^3-1/2*b^2*d^2*poly
log(3,1-2/(1-I*c*x))/e^3+1/2*b^2*d^2*polylog(3,1-2*c*(e*x+d)/(c*d+I*e)/(1-
I*c*x))/e^3
```

## Mathematica [F]

$$\int \frac{x^2(a+b\arctan(cx))^2}{d+ex}\,dx = \int \frac{x^2(a+b\arctan(cx))^2}{d+ex}\,dx$$

input
```
Integrate[(x^2*(a + b*ArcTan[c*x])^2)/(d + e*x),x]
```

output
```
Integrate[(x^2*(a + b*ArcTan[c*x])^2)/(d + e*x), x]
```

## Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 430, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\arctan(cx))^2}{d+ex}\,dx$$

$$\Big\downarrow \text{ 5411}$$

$$\int \left(\frac{d^2(a+b\arctan(cx))^2}{e^2(d+ex)}-\frac{d(a+b\arctan(cx))^2}{e^2}+\frac{x(a+b\arctan(cx))^2}{e}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(a+b\arctan(cx))^2}{2c^2e}+\frac{ibd^2\,\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)(a+b\arctan(cx))}{e^3}-\frac{ibd^2(a+b\arctan(cx))\,\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{e^3}-\frac{d^2\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))^2}{e^3}+\frac{d^2(a+b\arctan(cx))^2\log\left(\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{e^3}-\frac{dx(a+b\arctan(cx))^2}{e^2}-\frac{id(a+b\arctan(cx))^2}{ce^2}-\frac{2bd\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{ce^2}+\frac{x^2(a+b\arctan(cx))^2}{2e}-\frac{abx}{ce}-\frac{b^2x\arctan(cx)}{ce}+\frac{b^2\log\left(c^2x^2+1\right)}{2c^2e}-\frac{b^2d^2\,\text{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{2e^3}+\frac{b^2d^2\,\text{PolyLog}\left(3,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2e^3}-\frac{ib^2d\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{ce^2}$$

input
```
Int[(x^2*(a + b*ArcTan[c*x])^2)/(d + e*x),x]
```

output
```
-((a*b*x)/(c*e)) - (b^2*x*ArcTan[c*x])/(c*e) - (I*d*(a + b*ArcTan[c*x])^2)
/(c*e^2) + (a + b*ArcTan[c*x])^2/(2*c^2*e) - (d*x*(a + b*ArcTan[c*x])^2)/e
^2 + (x^2*(a + b*ArcTan[c*x])^2)/(2*e) - (d^2*(a + b*ArcTan[c*x])^2*Log[2/
(1 - I*c*x)])/e^3 - (2*b*d*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(c*e^2)
 + (d^2*(a + b*ArcTan[c*x])^2*Log[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x)
)])/e^3 + (b^2*Log[1 + c^2*x^2])/(2*c^2*e) + (I*b*d^2*(a + b*ArcTan[c*x])*
PolyLog[2, 1 - 2/(1 - I*c*x)])/e^3 - (I*b^2*d*PolyLog[2, 1 - 2/(1 + I*c*x)
])/(c*e^2) - (I*b*d^2*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/(
(c*d + I*e)*(1 - I*c*x))])/e^3 - (b^2*d^2*PolyLog[3, 1 - 2/(1 - I*c*x)])/(
2*e^3) + (b^2*d^2*PolyLog[3, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))
])/(2*e^3)
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 62.74 (sec) , antiderivative size = 1710, normalized size of antiderivative = 3.98

| method | result | size |
|---|---|---|
| parts | Expression too large to display | 1710 |
| derivativedivides | Expression too large to display | 1718 |
| default | Expression too large to display | 1718 |

input
```
int(x^2*(a+b*arctan(c*x))^2/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/2*a^2/e*x^2-a^2/e^2*x*d+a^2*d^2/e^3*ln(e*x+d)+b^2/c^3*(1/2*arctan(c*x)^2
/e*c^3*x^2-arctan(c*x)^2/e^2*c^3*d*x+c^3*arctan(c*x)^2*d^2/e^3*ln(c*e*x+c*
d)-2*c*(1/2/e*ln(1+(1+I*c*x)^2/(c^2*x^2+1))-1/4/e*arctan(c*x)^2+1/4*I/e^3*
c^2*d^2*Pi*csgn(I*(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1
)+I*e+c*d))*csgn(I*(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+
1)+I*e+c*d)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*arctan(c*x)^2-1/4*I/e^3*c^2*d^2
*Pi*csgn(I*(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+c
*d)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^3*arctan(c*x)^2+1/4*I/e^3*c^2*d^2*Pi*csgn
(I*(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+c*d)/(1+(
1+I*c*x)^2/(c^2*x^2+1)))^2*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*arctan(c*x)
^2-1/2*I/e^2*c*d*arctan(c*x)^2+1/2*I*d^2*c^2/e^2/(c*d-I*e)*arctan(c*x)^2*l
n(1-(I*e-c*d)/(c*d+I*e)*(1+I*c*x)^2/(c^2*x^2+1))-1/4*I/e^3*Pi*c^2*d^2*csgn
(I*(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+c*d))*csg
n(I*(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+c*d)/(1+
(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*arctan(c*x)^
2-1/2*I/e^3*c^2*d^2*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))+1/e^2*
c*d*arctan(c*x)*ln(1+I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/e^2*c*d*arctan(c*x)*
ln(1-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/2*I*d^3*c^3/e^3/(c*d-I*e)*arctan(c*x
)*polylog(2,(I*e-c*d)/(c*d+I*e)*(1+I*c*x)^2/(c^2*x^2+1))-I/e^2*c*d*dilog(1
-I*(1+I*c*x)/(c^2*x^2+1)^(1/2))+1/4/e^3*c^2*d^2*polylog(3,-(1+I*c*x)^2/...
```

## Fricas [F]

$$\int \frac{x^2(a+b\arctan(cx))^2}{d+ex}\,dx = \int \frac{(b\arctan(cx)+a)^2x^2}{ex+d}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))^2/(e*x+d),x, algorithm="fricas")
```

output
```
integral((b^2*x^2*arctan(c*x)^2 + 2*a*b*x^2*arctan(c*x) + a^2*x^2)/(e*x +
d), x)
```

## Sympy [F]

$$\int \frac{x^2(a+b\arctan(cx))^2}{d+ex}\,dx = \int \frac{x^2(a+b\operatorname{atan}(cx))^2}{d+ex}\,dx$$

input
```
integrate(x**2*(a+b*atan(c*x))**2/(e*x+d),x)
```

output
```
Integral(x**2*(a + b*atan(c*x))**2/(d + e*x), x)
```

## Maxima [F]

$$\int \frac{x^2(a+b\arctan(cx))^2}{d+ex}\,dx = \int \frac{(b\arctan(cx)+a)^2x^2}{ex+d}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))^2/(e*x+d),x, algorithm="maxima")
```

output
```
1/2*a^2*(2*d^2*log(e*x + d)/e^3 + (e*x^2 - 2*d*x)/e^2) + 1/32*(4*(b^2*e*x^
2 - 2*b^2*d*x)*arctan(c*x)^2 + 32*e^2*integrate(1/16*(12*(b^2*c^2*e^2*x^4
+ b^2*e^2*x^2)*arctan(c*x)^2 + (b^2*c^2*e^2*x^4 + b^2*e^2*x^2)*log(c^2*x^2
 + 1)^2 + 4*(8*a*b*c^2*e^2*x^4 - b^2*c*e^2*x^3 + 2*b^2*c*d^2*x + (b^2*c*d*
e + 8*a*b*e^2)*x^2)*arctan(c*x) + 2*(b^2*c^2*e^2*x^4 - b^2*c^2*d*e*x^3 - 2
*b^2*c^2*d^2*x^2)*log(c^2*x^2 + 1))/(c^2*e^3*x^3 + c^2*d*e^2*x^2 + e^3*x +
 d*e^2), x) - (b^2*e*x^2 - 2*b^2*d*x)*log(c^2*x^2 + 1)^2)/e^2
```

## Giac [F]

$$\int \frac{x^2(a+b\arctan(cx))^2}{d+ex}\,dx=\int \frac{(b\arctan(cx)+a)^2x^2}{ex+d}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))^2/(e*x+d),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)^2*x^2/(e*x + d), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\arctan(cx))^2}{d+ex}\,dx=\int \frac{x^2\,(a+b\,\mathrm{atan}(c\,x))^2}{d+e\,x}\,dx$$

input
```
int((x^2*(a + b*atan(c*x))^2)/(d + e*x),x)
```

output
```
int((x^2*(a + b*atan(c*x))^2)/(d + e*x), x)
```

## Reduce [F]

$$\int \frac{x^2(a+b\arctan(cx))^2}{d+ex}\,dx$$
$$= \frac{4\left(\int \frac{atan(cx)x^2}{ex+d}dx\right)ab\,e^3 + 2\left(\int \frac{atan(cx)^2x^2}{ex+d}dx\right)b^2e^3 + 2\log(ex+d)\,a^2d^2 - 2a^2dex + a^2e^2x^2}{2e^3}$$

input
```
int(x^2*(a+b*atan(c*x))^2/(e*x+d),x)
```

output
```
(4*int((atan(c*x)*x**2)/(d + e*x),x)*a*b*e**3 + 2*int((atan(c*x)**2*x**2)/
(d + e*x),x)*b**2*e**3 + 2*log(d + e*x)*a**2*d**2 - 2*a**2*d*e*x + a**2*e*
*2*x**2)/(2*e**3)
```

# 3.143 $\int \frac{x(a+b\arctan(cx))^2}{d+ex}\,dx$



## Optimal result

Integrand size = 19, antiderivative size = 323

$$\begin{aligned}\int \frac{x(a+b\arctan(cx))^2}{d+ex}\,dx &= \frac{i(a+b\arctan(cx))^2}{ce}+\frac{x(a+b\arctan(cx))^2}{e} \\ &+\frac{d(a+b\arctan(cx))^2\log\left(\frac{2}{1-icx}\right)}{e^2} \\ &+\frac{2b(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{ce} \\ &-\frac{d(a+b\arctan(cx))^2\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{e^2} \\ &-\frac{ibd(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{e^2} \\ &+\frac{ib^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{ce} \\ &+\frac{ibd(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{e^2} \\ &+\frac{b^2d\operatorname{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{2e^2} \\ &-\frac{b^2d\operatorname{PolyLog}\left(3,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2e^2}\end{aligned}$$

output
```
I*(a+b*arctan(c*x))^2/c/e+x*(a+b*arctan(c*x))^2/e+d*(a+b*arctan(c*x))^2*ln
(2/(1-I*c*x))/e^2+2*b*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c/e-d*(a+b*arctan(
c*x))^2*ln(2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/e^2-I*b*d*(a+b*arctan(c*x))*po
lylog(2,1-2/(1-I*c*x))/e^2+I*b^2*polylog(2,1-2/(1+I*c*x))/c/e+I*b*d*(a+b*a
rctan(c*x))*polylog(2,1-2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/e^2+1/2*b^2*d*pol
ylog(3,1-2/(1-I*c*x))/e^2-1/2*b^2*d*polylog(3,1-2*c*(e*x+d)/(c*d+I*e)/(1-I
*c*x))/e^2
```

## Mathematica [F]

$$\int \frac{x(a+b\arctan(cx))^2}{d+ex}\,dx=\int \frac{x(a+b\arctan(cx))^2}{d+ex}\,dx$$

input
```
Integrate[(x*(a + b*ArcTan[c*x])^2)/(d + e*x),x]
```

output
```
Integrate[(x*(a + b*ArcTan[c*x])^2)/(d + e*x), x]
```

## Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\arctan(cx))^2}{d+ex}\,dx$$

$$\Big\downarrow \text{5411}$$

$$\int \left(\frac{(a+b\arctan(cx))^2}{e}-\frac{d(a+b\arctan(cx))^2}{e(d+ex)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ibd\,\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)(a+b\arctan(cx))}{e^2}+\frac{ibd(a+b\arctan(cx))\,\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{e^2}+\frac{d\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))^2}{e^2}-\frac{d(a+b\arctan(cx))^2\log\left(\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{e^2}+\frac{x(a+b\arctan(cx))^2}{e}+\frac{i(a+b\arctan(cx))^2}{ce}+\frac{2b\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{ce}+\frac{b^2d\,\text{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{2e^2}-\frac{b^2d\,\text{PolyLog}\left(3,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2e^2}+\frac{ib^2\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{ce}$$

input
```
Int[(x*(a + b*ArcTan[c*x])^2)/(d + e*x),x]
```

output
```
(I*(a + b*ArcTan[c*x])^2)/(c*e) + (x*(a + b*ArcTan[c*x])^2)/e + (d*(a + b*
ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/e^2 + (2*b*(a + b*ArcTan[c*x])*Log[2/(1
 + I*c*x)])/(c*e) - (d*(a + b*ArcTan[c*x])^2*Log[(2*c*(d + e*x))/((c*d + I
*e)*(1 - I*c*x))])/e^2 - (I*b*d*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 -
I*c*x)])/e^2 + (I*b^2*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c*e) + (I*b*d*(a + b
*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e
^2 + (b^2*d*PolyLog[3, 1 - 2/(1 - I*c*x)])/(2*e^2) - (b^2*d*PolyLog[3, 1 -
 (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/(2*e^2)
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 36.40 (sec) , antiderivative size = 15752, normalized size of antiderivative = 48.77

| method | result | size |
|---|---|---|
| derivativedivides | Expression too large to display | 15752 |
| default | Expression too large to display | 15752 |
| parts | Expression too large to display | 15757 |

input
```
int(x*(a+b*arctan(c*x))^2/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

## Fricas [F]

$$\int \frac{x(a+b\arctan(cx))^2}{d+ex}\,dx = \int \frac{(b\arctan(cx)+a)^2x}{ex+d}\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))^2/(e*x+d),x, algorithm="fricas")
```

output
```
integral((b^2*x*arctan(c*x)^2 + 2*a*b*x*arctan(c*x) + a^2*x)/(e*x + d), x)
```

## Sympy [F]

$$\int \frac{x(a+b\arctan(cx))^2}{d+ex}\,dx = \int \frac{x(a+b\operatorname{atan}(cx))^2}{d+ex}\,dx$$

input
```
integrate(x*(a+b*atan(c*x))**2/(e*x+d),x)
```

output
```
Integral(x*(a + b*atan(c*x))**2/(d + e*x), x)
```

## Maxima [F]

$$\int \frac{x(a+b\arctan(cx))^2}{d+ex}\,dx = \int \frac{(b\arctan(cx)+a)^2 x}{ex+d}\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))^2/(e*x+d),x, algorithm="maxima")
```

output
```
a^2*(x/e - d*log(e*x + d)/e^2) + 1/16*(4*b^2*x*arctan(c*x)^2 - b^2*x*log(c
^2*x^2 + 1)^2 + 16*e*integrate(1/16*(12*(b^2*c^2*e*x^3 + b^2*e*x)*arctan(c
*x)^2 + (b^2*c^2*e*x^3 + b^2*e*x)*log(c^2*x^2 + 1)^2 + 8*(4*a*b*c^2*e*x^3
- b^2*c*e*x^2 - (b^2*c*d - 4*a*b*e)*x)*arctan(c*x) + 4*(b^2*c^2*e*x^3 + b^
2*c^2*d*x^2)*log(c^2*x^2 + 1))/(c^2*e^2*x^3 + c^2*d*e*x^2 + e^2*x + d*e),
x))/e
```

## Giac [F]

$$\int \frac{x(a+b\arctan(cx))^2}{d+ex}\,dx = \int \frac{(b\arctan(cx)+a)^2 x}{ex+d}\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))^2/(e*x+d),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)^2*x/(e*x + d), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\arctan(cx))^2}{d+ex}\,dx = \int \frac{x\,(a+b\,\mathrm{atan}(c\,x))^2}{d+e\,x}\,dx$$

input
```
int((x*(a + b*atan(c*x))^2)/(d + e*x),x)
```

output
```
int((x*(a + b*atan(c*x))^2)/(d + e*x), x)
```

## Reduce [F]

$$\int \frac{x(a+b\arctan(cx))^2}{d+ex}\,dx$$
$$=\frac{2\left(\int \frac{atan(cx)x}{ex+d}dx\right)ab\,e^2+\left(\int \frac{atan(cx)^2x}{ex+d}dx\right)b^2e^2-\log(ex+d)\,a^2d+a^2ex}{e^2}$$

input
```
int(x*(a+b*atan(c*x))^2/(e*x+d),x)
```

output
```
(2*int((atan(c*x)*x)/(d + e*x),x)*a*b*e**2 + int((atan(c*x)**2*x)/(d + e*x
),x)*b**2*e**2 - log(d + e*x)*a**2*d + a**2*e*x)/e**2
```

# 3.144 $\int \frac{(a+b\arctan(cx))^2}{d+ex}\,dx$



## Optimal result

Integrand size = 18, antiderivative size = 223

$$\int \frac{(a+b\arctan(cx))^2}{d+ex}\,dx = -\frac{(a+b\arctan(cx))^2\log\left(\frac{2}{1-icx}\right)}{e} + \frac{(a+b\arctan(cx))^2\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{e} + \frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{e} - \frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{e} - \frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{2e} + \frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2e}$$

output

```
-(a+b*arctan(c*x))^2*ln(2/(1-I*c*x))/e+(a+b*arctan(c*x))^2*ln(2*c*(e*x+d)/
(c*d+I*e)/(1-I*c*x))/e+I*b*(a+b*arctan(c*x))*polylog(2,1-2/(1-I*c*x))/e-I*
b*(a+b*arctan(c*x))*polylog(2,1-2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/e-1/2*b^2
*polylog(3,1-2/(1-I*c*x))/e+1/2*b^2*polylog(3,1-2*c*(e*x+d)/(c*d+I*e)/(1-I
*c*x))/e
```

## Mathematica [F]

$$\int \frac{(a+b\arctan(cx))^2}{d+ex}\,dx = \int \frac{(a+b\arctan(cx))^2}{d+ex}\,dx$$

input
```
Integrate[(a + b*ArcTan[c*x])^2/(d + e*x),x]
```

output
```
Integrate[(a + b*ArcTan[c*x])^2/(d + e*x), x]
```

## Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {5383}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^2}{d+ex}\,dx$$

$$\Big\downarrow \text{5383}$$

$$-\frac{ib(a+b\arctan(cx))\,\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{e}+\frac{(a+b\arctan(cx))^2\log\left(\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{e}+\frac{ib\,\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)(a+b\arctan(cx))}{e}-\frac{\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))^2}{e}+\frac{b^2\,\text{PolyLog}\left(3,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2e}-\frac{b^2\,\text{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{2e}$$

input
```
Int[(a + b*ArcTan[c*x])^2/(d + e*x),x]
```

output
```
-(((a + b*ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/e) + ((a + b*ArcTan[c*x])^2*L
og[(2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e + (I*b*(a + b*ArcTan[c*x]
)*PolyLog[2, 1 - 2/(1 - I*c*x)])/e - (I*b*(a + b*ArcTan[c*x])*PolyLog[2, 1
 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/e - (b^2*PolyLog[3, 1 - 2/(
1 - I*c*x)])/(2*e) + (b^2*PolyLog[3, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 -
 I*c*x))])/(2*e)
```

### Defintions of rubi rules used

rule 5383
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^2/((d_) + (e_.)*(x_)), x_Symbol] :>
Simp[(-(a + b*ArcTan[c*x])^2)*(Log[2/(1 - I*c*x)]/e), x] + (Simp[(a + b*Arc
Tan[c*x])^2*(Log[2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/e), x] + Simp[I
*b*(a + b*ArcTan[c*x])*(PolyLog[2, 1 - 2/(1 - I*c*x)]/e), x] - Simp[I*b*(a
+ b*ArcTan[c*x])*(PolyLog[2, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]
/e), x] - Simp[b^2*(PolyLog[3, 1 - 2/(1 - I*c*x)]/(2*e)), x] + Simp[b^2*(Po
lyLog[3, 1 - 2*c*((d + e*x)/((c*d + I*e)*(1 - I*c*x)))]/(2*e)), x]) /; Free
Q[{a, b, c, d, e}, x] && NeQ[c^2*d^2 + e^2, 0]
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.71 (sec) , antiderivative size = 1199, normalized size of antiderivative = 5.38

| method | result | size |
|---|---|---|
| derivativedivides | Expression too large to display | 1199 |
| default | Expression too large to display | 1199 |
| parts | Expression too large to display | 1202 |

input
```
int((a+b*arctan(c*x))^2/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/c*(a^2*c*ln(c*e*x+c*d)/e+b^2*c*(ln(c*e*x+c*d)/e*arctan(c*x)^2-2/e*(1/2*a
rctan(c*x)^2*ln(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I
*e+c*d)-1/2*I*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))+1/4*polylog(
3,-(1+I*c*x)^2/(c^2*x^2+1))-1/4*I*Pi*csgn(I*(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+
c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+c*d)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*(csgn(I*
(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+c*d)/(1+(1+I
*c*x)^2/(c^2*x^2+1)))^2-csgn(I*(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)
^2/(c^2*x^2+1)+I*e+c*d))*csgn(I*(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x
)^2/(c^2*x^2+1)+I*e+c*d)/(1+(1+I*c*x)^2/(c^2*x^2+1)))-csgn(I*(-I*e*(1+I*c*
x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+c*d)/(1+(1+I*c*x)^2/(c^2*
x^2+1)))*csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))+csgn(I*(-I*e*(1+I*c*x)^2/(c^2
*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+c*d))*csgn(I/(1+(1+I*c*x)^2/(c^2*x
^2+1))))*arctan(c*x)^2-1/2*c*d/(c*d-I*e)*arctan(c*x)^2*ln(1-(I*e-c*d)/(c*d
+I*e)*(1+I*c*x)^2/(c^2*x^2+1))+1/2*I*c*d/(c*d-I*e)*arctan(c*x)*polylog(2,(
I*e-c*d)/(c*d+I*e)*(1+I*c*x)^2/(c^2*x^2+1))-1/4*c*d/(c*d-I*e)*polylog(3,(I
*e-c*d)/(c*d+I*e)*(1+I*c*x)^2/(c^2*x^2+1))-1/2*e*arctan(c*x)^2*ln(1-(I*e-c
*d)/(c*d+I*e)*(1+I*c*x)^2/(c^2*x^2+1))/(e+I*c*d)+1/2*I*e*arctan(c*x)*polyl
og(2,(I*e-c*d)/(c*d+I*e)*(1+I*c*x)^2/(c^2*x^2+1))/(e+I*c*d)-1/4*e*polylog(
3,(I*e-c*d)/(c*d+I*e)*(1+I*c*x)^2/(c^2*x^2+1))/(e+I*c*d)))+2*a*b*c*(ln(c*e
*x+c*d)/e*arctan(c*x)+1/2*I*ln(c*e*x+c*d)*(ln((I*e-c*e*x)/(c*d+I*e))-ln...
```

## Fricas [F]

$$\int \frac{(a+b\arctan(cx))^2}{d+ex}\,dx = \int \frac{\left(b\arctan\left(cx\right)+a\right)^2}{ex+d}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/(e*x+d),x, algorithm="fricas")
```

output
```
integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)/(e*x + d), x)
```

## Sympy [F]

$$\int \frac{(a+b\arctan(cx))^2}{d+ex}\,dx = \int \frac{(a+b\,\mathrm{atan}\,(cx))^2}{d+ex}\,dx$$

input
```
integrate((a+b*atan(c*x))**2/(e*x+d),x)
```

output
```
Integral((a + b*atan(c*x))**2/(d + e*x), x)
```

## Maxima [F]

$$\int \frac{(a+b\arctan(cx))^2}{d+ex}\,dx = \int \frac{(b\arctan(cx)+a)^2}{ex+d}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/(e*x+d),x, algorithm="maxima")
```

output
```
a^2*log(e*x + d)/e + integrate(1/16*(12*b^2*arctan(c*x)^2 + b^2*log(c^2*x^
2 + 1)^2 + 32*a*b*arctan(c*x))/(e*x + d), x)
```

## Giac [F]

$$\int \frac{(a+b\arctan(cx))^2}{d+ex}\,dx = \int \frac{(b\arctan(cx)+a)^2}{ex+d}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/(e*x+d),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)^2/(e*x + d), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{d+ex}\,dx=\int \frac{\left(a+b\,\mathrm{atan}(c\,x)\right)^2}{d+e\,x}\,dx$$

input
```
int((a + b*atan(c*x))^2/(d + e*x),x)
```

output
```
int((a + b*atan(c*x))^2/(d + e*x), x)
```

## Reduce [F]

$$\int \frac{(a+b\arctan(cx))^2}{d+ex}\,dx=\frac{2\left(\int \frac{atan(cx)}{ex+d}dx\right)abe+\left(\int \frac{atan(cx)^2}{ex+d}dx\right)b^2e+\log(ex+d)\,a^2}{e}$$

input
```
int((a+b*atan(c*x))^2/(e*x+d),x)
```

output
```
(2*int(atan(c*x)/(d + e*x),x)*a*b*e + int(atan(c*x)**2/(d + e*x),x)*b**2*e
 + log(d + e*x)*a**2)/e
```

# 3.145 $\int \frac{(a+b\arctan(cx))^2}{x(d+ex)}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 369

$$
\begin{aligned}
\int \frac{(a+b\arctan(cx))^2}{x(d+ex)}\,dx &= \frac{2(a+b\arctan(cx))^2\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right)}{d} \\
&+ \frac{(a+b\arctan(cx))^2\log\left(\frac{2}{1-icx}\right)}{d} \\
&- \frac{(a+b\arctan(cx))^2\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{d} \\
&- \frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{d} \\
&- \frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{d} \\
&+ \frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)}{d} \\
&+ \frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{d} \\
&+ \frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{2d} - \frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2d} \\
&+ \frac{b^2\operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)}{2d} \\
&- \frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2d}
\end{aligned}
$$

output
```
-2*(a+b*arctan(c*x))^2*arctanh(-1+2/(1+I*c*x))/d+(a+b*arctan(c*x))^2*ln(2/
(1-I*c*x))/d-(a+b*arctan(c*x))^2*ln(2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/d-I*b
*(a+b*arctan(c*x))*polylog(2,1-2/(1-I*c*x))/d-I*b*(a+b*arctan(c*x))*polylo
g(2,1-2/(1+I*c*x))/d+I*b*(a+b*arctan(c*x))*polylog(2,-1+2/(1+I*c*x))/d+I*b
*(a+b*arctan(c*x))*polylog(2,1-2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/d+1/2*b^2*
polylog(3,1-2/(1-I*c*x))/d-1/2*b^2*polylog(3,1-2/(1+I*c*x))/d+1/2*b^2*poly
log(3,-1+2/(1+I*c*x))/d-1/2*b^2*polylog(3,1-2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x
))/d
```

## Mathematica [F]

$$\int \frac{(a+b\arctan(cx))^2}{x(d+ex)}\,dx = \int \frac{(a+b\arctan(cx))^2}{x(d+ex)}\,dx$$

input
```
Integrate[(a + b*ArcTan[c*x])^2/(x*(d + e*x)),x]
```

output
```
Integrate[(a + b*ArcTan[c*x])^2/(x*(d + e*x)), x]
```

## Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^2}{x(d+ex)}\,dx$$

$$\Big\downarrow \text{ 5411}$$

$$\int \left(\frac{(a+b\arctan(cx))^2}{dx}-\frac{e(a+b\arctan(cx))^2}{d(d+ex)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{d}+\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{d}-\frac{(a+b\arctan(cx))^2\log\left(\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{d}-\frac{ib\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)(a+b\arctan(cx))}{d}-\frac{ib\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{d}+\frac{ib\operatorname{PolyLog}\left(2,\frac{2}{icx+1}-1\right)(a+b\arctan(cx))}{d}+\frac{\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))^2}{d}-\frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2d}+\frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{2d}-\frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2}{icx+1}\right)}{2d}+\frac{b^2\operatorname{PolyLog}\left(3,\frac{2}{icx+1}-1\right)}{2d}$$

input
```
Int[(a + b*ArcTan[c*x])^2/(x*(d + e*x)),x]
```

output
```
(2*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)])/d + ((a + b*ArcTan[c*
x])^2*Log[2/(1 - I*c*x)])/d - ((a + b*ArcTan[c*x])^2*Log[(2*c*(d + e*x))/(
(c*d + I*e)*(1 - I*c*x))])/d - (I*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(
1 - I*c*x)])/d - (I*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)])/d
 + (I*b*(a + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)])/d + (I*b*(a +
b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/
d + (b^2*PolyLog[3, 1 - 2/(1 - I*c*x)])/(2*d) - (b^2*PolyLog[3, 1 - 2/(1 +
 I*c*x)])/(2*d) + (b^2*PolyLog[3, -1 + 2/(1 + I*c*x)])/(2*d) - (b^2*PolyLo
g[3, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/(2*d)
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 23.73 (sec) , antiderivative size = 2157, normalized size of antiderivative = 5.85

| method | result | size |
|---|---|---|
| parts | Expression too large to display | 2157 |
| derivativedivides | Expression too large to display | 2165 |
| default | Expression too large to display | 2165 |

input
```
int((a+b*arctan(c*x))^2/x/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-a^2/d*ln(e*x+d)+a^2/d*ln(x)+b^2*(arctan(c*x)^2/d*ln(c*x)-arctan(c*x)^2/d*
ln(c*e*x+c*d)-2*c*(-1/2/d/c*arctan(c*x)^2*ln(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+
c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+c*d)+1/2/d/c*arctan(c*x)^2*ln((1+I*c*x)^2/
(c^2*x^2+1)-1)-1/2/d/c*arctan(c*x)^2*ln(1+(1+I*c*x)/(c^2*x^2+1)^(1/2))+I/c
/d*arctan(c*x)*polylog(2,-(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/d/c*polylog(3,-(1
+I*c*x)/(c^2*x^2+1)^(1/2))-1/2/d/c*arctan(c*x)^2*ln(1-(1+I*c*x)/(c^2*x^2+1
)^(1/2))+I/c/d*arctan(c*x)*polylog(2,(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/d/c*po
lylog(3,(1+I*c*x)/(c^2*x^2+1)^(1/2))-1/4*I/c/d*Pi*(csgn(I*((1+I*c*x)^2/(c^
2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/
(1+(1+I*c*x)^2/(c^2*x^2+1)))-csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)
^2/(c^2*x^2+1)))^2+csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I/(1+(1+I*c*x)
^2/(c^2*x^2+1)))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^
2+1)))-csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)
-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2-csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csg
n(I*(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+c*d))*cs
gn(I*(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+c*d)/(1
+(1+I*c*x)^2/(c^2*x^2+1)))-csgn(I/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(I*((1+
I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2+csgn(I/(1+(1+I*c*x)
^2/(c^2*x^2+1)))*csgn(I*(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2
*x^2+1)+I*e+c*d)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2+csgn(I*(-I*e*(1+I*c*x)^...
```

## Fricas [F]

$$\int \frac{(a+b\arctan(cx))^2}{x(d+ex)}\,dx = \int \frac{(b\arctan(cx)+a)^2}{(ex+d)x}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x/(e*x+d),x, algorithm="fricas")
```

output
```
integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)/(e*x^2 + d*x), x)
```

## Sympy [F]

$$\int \frac{(a+b\arctan(cx))^2}{x(d+ex)}\,dx = \int \frac{(a+b\,\mathrm{atan}\,(cx))^2}{x\,(d+ex)}\,dx$$

input
```
integrate((a+b*atan(c*x))**2/x/(e*x+d),x)
```

output
```
Integral((a + b*atan(c*x))**2/(x*(d + e*x)), x)
```

## Maxima [F]

$$\int \frac{(a+b\arctan(cx))^2}{x(d+ex)}\,dx = \int \frac{(b\arctan(cx)+a)^2}{(ex+d)x}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x/(e*x+d),x, algorithm="maxima")
```

output
```
-a^2*(log(e*x + d)/d - log(x)/d) + integrate(1/16*(12*b^2*arctan(c*x)^2 +
b^2*log(c^2*x^2 + 1)^2 + 32*a*b*arctan(c*x))/(e*x^2 + d*x), x)
```

## Giac [F]

$$\int \frac{(a+b\arctan(cx))^2}{x(d+ex)}\,dx=\int \frac{(b\arctan(cx)+a)^2}{(ex+d)x}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x/(e*x+d),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)^2/((e*x + d)*x), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{x(d+ex)}\,dx=\int \frac{(a+b\,\mathrm{atan}(c\,x))^2}{x\,(d+e\,x)}\,dx$$

input
```
int((a + b*atan(c*x))^2/(x*(d + e*x)),x)
```

output
```
int((a + b*atan(c*x))^2/(x*(d + e*x)), x)
```

## Reduce [F]

$$\int \frac{(a+b\arctan(cx))^2}{x(d+ex)}\,dx$$
$$=\frac{atan(cx)^3 b^2cd+3atan(cx)^2 abcd+6\left(\int \frac{atan(cx)}{c^2e\,x^4+c^2d\,x^3+e\,x^2+dx}dx\right)abde-6\left(\int \frac{atan(cx)}{c^2e\,x^3+c^2d\,x^2+ex+d}dx\right)ab\,c^2d^2}{3d}$$

input
```
int((a+b*atan(c*x))^2/x/(e*x+d),x)
```

output
```
(atan(c*x)**3*b**2*c*d + 3*atan(c*x)**2*a*b*c*d + 6*int(atan(c*x)/(c**2*d*
x**3 + c**2*e*x**4 + d*x + e*x**2),x)*a*b*d*e - 6*int(atan(c*x)/(c**2*d*x*
*2 + c**2*e*x**3 + d + e*x),x)*a*b*c**2*d**2 + 3*int(atan(c*x)**2/(c**2*d*
x**3 + c**2*e*x**4 + d*x + e*x**2),x)*b**2*d*e - 3*int(atan(c*x)**2/(c**2*
d*x**2 + c**2*e*x**3 + d + e*x),x)*b**2*c**2*d**2 - 3*log(d + e*x)*a**2*e
+ 3*log(x)*a**2*e)/(3*d*e)
```

# 3.146 $\int \frac{(a+b\arctan(cx))^2}{x^2(d+ex)}\,dx$

## Optimal result

Integrand size = 21, antiderivative size = 473

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+ex)}\,dx = -\frac{ic(a+b\arctan(cx))^2}{d} - \frac{(a+b\arctan(cx))^2}{dx} - \frac{2e(a+b\arctan(cx))^2\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right)}{d^2} - \frac{e(a+b\arctan(cx))^2\log\left(\frac{2}{1-icx}\right)}{d^2} + \frac{e(a+b\arctan(cx))^2\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{d^2} + \frac{2bc(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right)}{d} + \frac{ibe(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{d^2} - \frac{ib^2c\operatorname{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)}{d} + \frac{ibe(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{d^2} - \frac{ibe(a+b\arctan(cx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)}{d^2} - \frac{ibe(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{d^2} - \frac{b^2e\operatorname{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{2d^2} + \frac{b^2e\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2d^2} - \frac{b^2e\operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)}{2d^2} + \frac{b^2e\operatorname{PolyLog}\left(3,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2d^2}$$

output
```
-I*c*(a+b*arctan(c*x))^2/d-(a+b*arctan(c*x))^2/d/x+2*e*(a+b*arctan(c*x))^2
*arctanh(-1+2/(1+I*c*x))/d^2-e*(a+b*arctan(c*x))^2*ln(2/(1-I*c*x))/d^2+e*(
a+b*arctan(c*x))^2*ln(2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/d^2+2*b*c*(a+b*arct
an(c*x))*ln(2-2/(1-I*c*x))/d+I*b*e*(a+b*arctan(c*x))*polylog(2,1-2/(1-I*c*
x))/d^2-I*b^2*c*polylog(2,-1+2/(1-I*c*x))/d+I*b*e*(a+b*arctan(c*x))*polylo
g(2,1-2/(1+I*c*x))/d^2-I*b*e*(a+b*arctan(c*x))*polylog(2,-1+2/(1+I*c*x))/d
^2-I*b*e*(a+b*arctan(c*x))*polylog(2,1-2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/d^
2-1/2*b^2*e*polylog(3,1-2/(1-I*c*x))/d^2+1/2*b^2*e*polylog(3,1-2/(1+I*c*x)
)/d^2-1/2*b^2*e*polylog(3,-1+2/(1+I*c*x))/d^2+1/2*b^2*e*polylog(3,1-2*c*(e
*x+d)/(c*d+I*e)/(1-I*c*x))/d^2
```

## Mathematica [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+ex)}\,dx=\int \frac{(a+b\arctan(cx))^2}{x^2(d+ex)}\,dx$$

input
```
Integrate[(a + b*ArcTan[c*x])^2/(x^2*(d + e*x)),x]
```

output
```
Integrate[(a + b*ArcTan[c*x])^2/(x^2*(d + e*x)), x]
```

## Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 473, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+ex)}\,dx$$

$$\Big\downarrow \text{5411}$$

$$\int \left(\frac{e^2(a+b\arctan(cx))^2}{d^2(d+ex)}-\frac{e(a+b\arctan(cx))^2}{d^2x}+\frac{(a+b\arctan(cx))^2}{dx^2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{2e\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{d^2}+\frac{ibe\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)(a+b\arctan(cx))}{d^2}+\frac{ibe\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{d^2}-\frac{ibe\operatorname{PolyLog}\left(2,\frac{2}{icx+1}-1\right)(a+b\arctan(cx))}{d^2}-\frac{ibe(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{d^2}-\frac{e\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))^2}{d^2}+\frac{e(a+b\arctan(cx))^2\log\left(\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{d^2}-\frac{ic(a+b\arctan(cx))^2}{d}-\frac{(a+b\arctan(cx))^2}{dx}+\frac{2bc\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))}{d}-\frac{b^2e\operatorname{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{2d^2}+\frac{b^2e\operatorname{PolyLog}\left(3,1-\frac{2}{icx+1}\right)}{2d^2}-\frac{b^2e\operatorname{PolyLog}\left(3,\frac{2}{icx+1}-1\right)}{2d^2}+\frac{b^2e\operatorname{PolyLog}\left(3,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2d^2}-\frac{ib^2c\operatorname{PolyLog}\left(2,\frac{2}{1-icx}-1\right)}{d}$$

input
```
Int[(a + b*ArcTan[c*x])^2/(x^2*(d + e*x)),x]
```

output
```
((-I)*c*(a + b*ArcTan[c*x])^2)/d - (a + b*ArcTan[c*x])^2/(d*x) - (2*e*(a +
 b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)])/d^2 - (e*(a + b*ArcTan[c*x])
^2*Log[2/(1 - I*c*x)])/d^2 + (e*(a + b*ArcTan[c*x])^2*Log[(2*c*(d + e*x))/
((c*d + I*e)*(1 - I*c*x))])/d^2 + (2*b*c*(a + b*ArcTan[c*x])*Log[2 - 2/(1
- I*c*x)])/d + (I*b*e*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 - I*c*x)])/d
^2 - (I*b^2*c*PolyLog[2, -1 + 2/(1 - I*c*x)])/d + (I*b*e*(a + b*ArcTan[c*x
])*PolyLog[2, 1 - 2/(1 + I*c*x)])/d^2 - (I*b*e*(a + b*ArcTan[c*x])*PolyLog
[2, -1 + 2/(1 + I*c*x)])/d^2 - (I*b*e*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (
2*c*(d + e*x))/((c*d + I*e)*(1 - I*c*x))])/d^2 - (b^2*e*PolyLog[3, 1 - 2/(
1 - I*c*x)])/(2*d^2) + (b^2*e*PolyLog[3, 1 - 2/(1 + I*c*x)])/(2*d^2) - (b^
2*e*PolyLog[3, -1 + 2/(1 + I*c*x)])/(2*d^2) + (b^2*e*PolyLog[3, 1 - (2*c*(
d + e*x))/((c*d + I*e)*(1 - I*c*x))])/(2*d^2)
```

**Defintions of rubi rules used**

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 52.06 (sec) , antiderivative size = 38040, normalized size of antiderivative = 80.42

| method | result | size |
|---|---|---|
| parts | Expression too large to display | 38040 |
| derivativedivides | Expression too large to display | 38050 |
| default | Expression too large to display | 38050 |

input
```
int((a+b*arctan(c*x))^2/x^2/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

## Fricas [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+ex)}\,dx = \int \frac{(b\arctan(cx)+a)^2}{(ex+d)x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^2/(e*x+d),x, algorithm="fricas")
```

output
```
integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)/(e*x^3 + d*x^2), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(a + b\arctan(cx))^2}{x^2(d + ex)}\,dx = \text{Timed out}$$

input
```
integrate((a+b*atan(c*x))**2/x**2/(e*x+d),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(a + b\arctan(cx))^2}{x^2(d + ex)}\,dx = \int \frac{(b\arctan(cx) + a)^2}{(ex + d)x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^2/(e*x+d),x, algorithm="maxima")
```

output
```
a^2*(e*log(e*x + d)/d^2 - e*log(x)/d^2 - 1/(d*x)) - 1/16*(4*b^2*arctan(c*x
)^2 - b^2*log(c^2*x^2 + 1)^2 - 16*d*x*integrate(1/16*(12*(b^2*c^2*d*x^2 +
b^2*d)*arctan(c*x)^2 + (b^2*c^2*d*x^2 + b^2*d)*log(c^2*x^2 + 1)^2 + 8*(b^2
*c*d*x + 4*a*b*d + (4*a*b*c^2*d + b^2*c*e)*x^2)*arctan(c*x) - 4*(b^2*c^2*e
*x^3 + b^2*c^2*d*x^2)*log(c^2*x^2 + 1))/(c^2*d*e*x^5 + c^2*d^2*x^4 + d*e*x
^3 + d^2*x^2), x))/(d*x)
```

## Giac [F]

$$\int \frac{(a + b\arctan(cx))^2}{x^2(d + ex)}\,dx = \int \frac{(b\arctan(cx) + a)^2}{(ex + d)x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^2/(e*x+d),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)^2/((e*x + d)*x^2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+ex)}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^2}{x^2\,(d+e\,x)}\,dx$$

input
```
int((a + b*atan(c*x))^2/(x^2*(d + e*x)),x)
```

output
```
int((a + b*atan(c*x))^2/(x^2*(d + e*x)), x)
```

## Reduce [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+ex)}\,dx = \frac{2\left(\int \frac{atan(cx)}{c^2e\,x^5+c^2d\,x^4+e\,x^3+d\,x^2}dx\right)ab\,d^2x+2\left(\int \frac{atan(cx)}{c^2e\,x^3+c^2d\,x^2+ex+d}dx\right)ab\,c^2d^2x+\left(\int \frac{atan(cx)^2}{c^2e\,x^5+c^2d\,x^4+e\,x^3+d\,x^2}dx\right)b^2d}{d^2x}$$

input
```
int((a+b*atan(c*x))^2/x^2/(e*x+d),x)
```

output
```
(2*int(atan(c*x)/(c**2*d*x**4 + c**2*e*x**5 + d*x**2 + e*x**3),x)*a*b*d**2
*x + 2*int(atan(c*x)/(c**2*d*x**2 + c**2*e*x**3 + d + e*x),x)*a*b*c**2*d**
2*x + int(atan(c*x)**2/(c**2*d*x**4 + c**2*e*x**5 + d*x**2 + e*x**3),x)*b*
*2*d**2*x + int(atan(c*x)**2/(c**2*d*x**2 + c**2*e*x**3 + d + e*x),x)*b**2
*c**2*d**2*x + log(d + e*x)*a**2*e*x - log(x)*a**2*e*x - a**2*d)/(d**2*x)
```

# 3.147 $\int \frac{(a+b\arctan(cx))^2}{x^3(d+ex)}\,dx$

## Optimal result

Integrand size = 21, antiderivative size = 591

$$\int \frac{(a+b\arctan(cx))^2}{x^3(d+ex)}\,dx = -\frac{bc(a+b\arctan(cx))}{dx} - \frac{c^2(a+b\arctan(cx))^2}{2d} + \frac{ice(a+b\arctan(cx))^2}{d^2} - \frac{(a+b\arctan(cx))^2}{2dx^2} + \frac{e(a+b\arctan(cx))^2}{d^2x} + \frac{2e^2(a+b\arctan(cx))^2\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right)}{d^3} + \frac{b^2c^2\log(x)}{d} + \frac{e^2(a+b\arctan(cx))^2\log\left(\frac{2}{1-icx}\right)}{d^3} - \frac{e^2(a+b\arctan(cx))^2\log\left(\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{d^3} - \frac{b^2c^2\log(1+c^2x^2)}{2d} - \frac{2bce(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right)}{d^2} - \frac{ibe^2(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{d^3} + \frac{ib^2ce\operatorname{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)}{d^2} - \frac{ibe^2(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{d^3} + \frac{ibe^2(a+b\arctan(cx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)}{d^3} + \frac{ibe^2(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{d^3} + \frac{b^2e^2\operatorname{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{2d^3} - \frac{b^2e^2\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2d^3} + \frac{b^2e^2\operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)}{2d^3} - \frac{b^2e^2\operatorname{PolyLog}\left(3,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2d^3}$$

output
```
-b*c*(a+b*arctan(c*x))/d/x-1/2*c^2*(a+b*arctan(c*x))^2/d-I*b*e^2*(a+b*arct
an(c*x))*polylog(2,1-2/(1+I*c*x))/d^3-1/2*(a+b*arctan(c*x))^2/d/x^2+e*(a+b
*arctan(c*x))^2/d^2/x-2*e^2*(a+b*arctan(c*x))^2*arctanh(-1+2/(1+I*c*x))/d^
3+b^2*c^2*ln(x)/d+e^2*(a+b*arctan(c*x))^2*ln(2/(1-I*c*x))/d^3-e^2*(a+b*arc
tan(c*x))^2*ln(2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/d^3-1/2*b^2*c^2*ln(c^2*x^2
+1)/d-2*b*c*e*(a+b*arctan(c*x))*ln(2-2/(1-I*c*x))/d^2+I*c*e*(a+b*arctan(c*
x))^2/d^2+I*b*e^2*(a+b*arctan(c*x))*polylog(2,-1+2/(1+I*c*x))/d^3+I*b*e^2*
(a+b*arctan(c*x))*polylog(2,1-2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/d^3+I*b^2*c
*e*polylog(2,-1+2/(1-I*c*x))/d^2-I*b*e^2*(a+b*arctan(c*x))*polylog(2,1-2/(
1-I*c*x))/d^3+1/2*b^2*e^2*polylog(3,1-2/(1-I*c*x))/d^3-1/2*b^2*e^2*polylog
(3,1-2/(1+I*c*x))/d^3+1/2*b^2*e^2*polylog(3,-1+2/(1+I*c*x))/d^3-1/2*b^2*e^
2*polylog(3,1-2*c*(e*x+d)/(c*d+I*e)/(1-I*c*x))/d^3
```

## Mathematica [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^3(d+ex)}\,dx=\int \frac{(a+b\arctan(cx))^2}{x^3(d+ex)}\,dx$$

input
```
Integrate[(a + b*ArcTan[c*x])^2/(x^3*(d + e*x)),x]
```

output
```
Integrate[(a + b*ArcTan[c*x])^2/(x^3*(d + e*x)), x]
```

## Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 591, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5411, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^2}{x^3(d+ex)}\,dx$$

$$\Big\downarrow \text{ 5411}$$

$$\int \left(-\frac{e^3(a+b\arctan(cx))^2}{d^3(d+ex)}+\frac{e^2(a+b\arctan(cx))^2}{d^3x}-\frac{e(a+b\arctan(cx))^2}{d^2x^2}+\frac{(a+b\arctan(cx))^2}{dx^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2e^2\text{arctanh}\left(1-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{d^3}-\frac{c^2(a+b\arctan(cx))^2}{2d}-\frac{ibe^2\,\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)(a+b\arctan(cx))}{d^3}-\frac{ibe^2\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{d^3}+\frac{ibe^2\,\text{PolyLog}\left(2,\frac{2}{icx+1}-1\right)(a+b\arctan(cx))}{d^3}+\frac{ibe^2(a+b\arctan(cx))\,\text{PolyLog}\left(2,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{d^3}+\frac{e^2\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))^2}{d^3}-\frac{e^2(a+b\arctan(cx))^2\log\left(\frac{2c(d+ex)}{(1-icx)(cd+ie)}\right)}{d^3}+\frac{ice(a+b\arctan(cx))^2}{d^2}+\frac{e(a+b\arctan(cx))^2}{d^2x}-\frac{2bce\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))}{d^2}-\frac{(a+b\arctan(cx))^2}{2dx^2}-\frac{bc(a+b\arctan(cx))}{dx}-\frac{b^2c^2\log\left(c^2x^2+1\right)}{2d}+\frac{b^2c^2\log(x)}{d}+\frac{b^2e^2\,\text{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{2d^3}-\frac{b^2e^2\,\text{PolyLog}\left(3,1-\frac{2}{icx+1}\right)}{2d^3}+\frac{b^2e^2\,\text{PolyLog}\left(3,\frac{2}{icx+1}-1\right)}{2d^3}-\frac{b^2e^2\,\text{PolyLog}\left(3,1-\frac{2c(d+ex)}{(cd+ie)(1-icx)}\right)}{2d^3}+\frac{ib^2ce\,\text{PolyLog}\left(2,\frac{2}{1-icx}-1\right)}{d^2}$$

input
```
Int[(a + b*ArcTan[c*x])^2/(x^3*(d + e*x)),x]
```

output
```
-((b*c*(a + b*ArcTan[c*x]))/(d*x)) - (c^2*(a + b*ArcTan[c*x])^2)/(2*d) + (
I*c*e*(a + b*ArcTan[c*x])^2)/d^2 - (a + b*ArcTan[c*x])^2/(2*d*x^2) + (e*(a
 + b*ArcTan[c*x])^2)/(d^2*x) + (2*e^2*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/
(1 + I*c*x)])/d^3 + (b^2*c^2*Log[x])/d + (e^2*(a + b*ArcTan[c*x])^2*Log[2/
(1 - I*c*x)])/d^3 - (e^2*(a + b*ArcTan[c*x])^2*Log[(2*c*(d + e*x))/((c*d +
 I*e)*(1 - I*c*x))])/d^3 - (b^2*c^2*Log[1 + c^2*x^2])/(2*d) - (2*b*c*e*(a
+ b*ArcTan[c*x])*Log[2 - 2/(1 - I*c*x)])/d^2 - (I*b*e^2*(a + b*ArcTan[c*x]
)*PolyLog[2, 1 - 2/(1 - I*c*x)])/d^3 + (I*b^2*c*e*PolyLog[2, -1 + 2/(1 - I
*c*x)])/d^2 - (I*b*e^2*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)])/
d^3 + (I*b*e^2*(a + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)])/d^3 + (
I*b*e^2*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(d + e*x))/((c*d + I*e)*(1
 - I*c*x))])/d^3 + (b^2*e^2*PolyLog[3, 1 - 2/(1 - I*c*x)])/(2*d^3) - (b^2*
e^2*PolyLog[3, 1 - 2/(1 + I*c*x)])/(2*d^3) + (b^2*e^2*PolyLog[3, -1 + 2/(1
 + I*c*x)])/(2*d^3) - (b^2*e^2*PolyLog[3, 1 - (2*c*(d + e*x))/((c*d + I*e)
*(1 - I*c*x))])/(2*d^3)
```

**Defintions of rubi rules used**

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5411
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (f*
x)^m*(d + e*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[p, 0] &
& IntegerQ[q] && (GtQ[q, 0] || NeQ[a, 0] || IntegerQ[m])
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 92.65 (sec) , antiderivative size = 2804, normalized size of antiderivative = 4.74

| method | result | size |
|---|---|---|
| parts | Expression too large to display | 2804 |
| derivativedivides | Expression too large to display | 2915 |
| default | Expression too large to display | 2915 |

input
```
int((a+b*arctan(c*x))^2/x^3/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
b^2*arctan(c*x)^2*e^2/d^3*ln(c*x)-b^2/d^3*e^2*arctan(c*x)^2*ln((1+I*c*x)^2
/(c^2*x^2+1)-1)-1/2*b^2*e^3*polylog(3,(I*e-c*d)/(c*d+I*e)*(1+I*c*x)^2/(c^2
*x^2+1))/d^3/(e+I*c*d)+b^2/d^3*e^2*arctan(c*x)^2*ln(1+(1+I*c*x)/(c^2*x^2+1
)^(1/2))+b^2/d^3*e^2*arctan(c*x)^2*ln(1-(1+I*c*x)/(c^2*x^2+1)^(1/2))+b^2*e
^2/d^3*arctan(c*x)^2*ln(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*
x^2+1)+I*e+c*d)-b^2*arctan(c*x)^2*e^2/d^3*ln(c*e*x+c*d)+b^2*arctan(c*x)^2*
e/d^2/x-b^2*c*arctan(c*x)/d/x-I*b^2*c^2*arctan(c*x)/d+1/2*I*b^2/d^3*e^2*Pi
*csgn(I*(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+c*d)
)*csgn(I*(-I*e*(1+I*c*x)^2/(c^2*x^2+1)+c*d*(1+I*c*x)^2/(c^2*x^2+1)+I*e+c*d
)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*arctan(c*x)^2+1/2*I*b^2/d^3*e^2*Pi*csgn(I
*((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*csgn(((1+I*c*x)^
2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))*arctan(c*x)^2-I*b^2*c*e^2*ar
ctan(c*x)^2*ln(1-(I*e-c*d)/(c*d+I*e)*(1+I*c*x)^2/(c^2*x^2+1))/d^2/(e+I*c*d
)-b^2*c*e^2*arctan(c*x)*polylog(2,(I*e-c*d)/(c*d+I*e)*(1+I*c*x)^2/(c^2*x^2
+1))/d^2/(e+I*c*d)+I*b^2*e^3*arctan(c*x)*polylog(2,(I*e-c*d)/(c*d+I*e)*(1+
I*c*x)^2/(c^2*x^2+1))/d^3/(e+I*c*d)-1/2*I*b^2*c*e^2*polylog(3,(I*e-c*d)/(c
*d+I*e)*(1+I*c*x)^2/(c^2*x^2+1))/d^2/(e+I*c*d)-1/2*I*b^2/d^3*e^2*Pi*csgn((
(1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^2+1)))^2*arctan(c*x)^2+1/
2*I*b^2/d^3*e^2*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/(1+(1+I*c*x)^2/(c^2*x^
2+1)))^3*arctan(c*x)^2+1/2*I*b^2/d^3*e^2*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^...
```

## Fricas [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^3(d+ex)}\,dx = \int \frac{(b\arctan(cx)+a)^2}{(ex+d)x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^3/(e*x+d),x, algorithm="fricas")
```

output
```
integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)/(e*x^4 + d*x^3), x)
```

## Sympy [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^3(d+ex)}\,dx = \int \frac{(a+b\,\mathrm{atan}\,(cx))^2}{x^3\,(d+ex)}\,dx$$

input
```
integrate((a+b*atan(c*x))**2/x**3/(e*x+d),x)
```

output
```
Integral((a + b*atan(c*x))**2/(x**3*(d + e*x)), x)
```

## Maxima [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^3(d+ex)}\,dx = \int \frac{(b\arctan(cx)+a)^2}{(ex+d)x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^3/(e*x+d),x, algorithm="maxima")
```

output
```
-1/2*a^2*(2*e^2*log(e*x + d)/d^3 - 2*e^2*log(x)/d^3 - (2*e*x - d)/(d^2*x^2
)) + 1/32*(32*d^2*x^2*integrate(1/16*(12*(b^2*c^2*d^2*x^2 + b^2*d^2)*arcta
n(c*x)^2 + (b^2*c^2*d^2*x^2 + b^2*d^2)*log(c^2*x^2 + 1)^2 - 4*(2*b^2*c*e^2
*x^3 - b^2*c*d^2*x - 8*a*b*d^2 - (8*a*b*c^2*d^2 - b^2*c*d*e)*x^2)*arctan(c
*x) + 2*(2*b^2*c^2*e^2*x^4 + b^2*c^2*d*e*x^3 - b^2*c^2*d^2*x^2)*log(c^2*x^
2 + 1))/(c^2*d^2*e*x^6 + c^2*d^3*x^5 + d^2*e*x^4 + d^3*x^3), x) + 4*(2*b^2
*e*x - b^2*d)*arctan(c*x)^2 - (2*b^2*e*x - b^2*d)*log(c^2*x^2 + 1)^2)/(d^2
*x^2)
```

## Giac [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^3(d+ex)}\,dx = \int \frac{(b\arctan(cx)+a)^2}{(ex+d)x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^3/(e*x+d),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)^2/((e*x + d)*x^3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{x^3(d+ex)}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^2}{x^3\,(d+e\,x)}\,dx$$

input
```
int((a + b*atan(c*x))^2/(x^3*(d + e*x)),x)
```

output
```
int((a + b*atan(c*x))^2/(x^3*(d + e*x)), x)
```

## Reduce [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^3(d+ex)}\,dx = \frac{-2atan(cx)^2\,abcde\,x^2-2atan(cx)\,ab\,c^2d^2x^2-2atan(cx)\,ab\,d^2-4\left(\int \frac{atan(cx)}{c^2e\,x^5+c^2d\,x^4+e\,x^3+d\,x^2}dx\right)ab\,d^2e\,x^2+}{}$$

input
```
int((a+b*atan(c*x))^2/x^3/(e*x+d),x)
```

output
```
( - 2*atan(c*x)**2*a*b*c*d*e*x**2 - 2*atan(c*x)*a*b*c**2*d**2*x**2 - 2*ata
n(c*x)*a*b*d**2 - 4*int(atan(c*x)/(c**2*d*x**4 + c**2*e*x**5 + d*x**2 + e*
x**3),x)*a*b*d**2*e*x**2 + 2*int(atan(c*x)**2/(c**2*d*x**5 + c**2*e*x**6 +
 d*x**3 + e*x**4),x)*b**2*d**3*x**2 + 2*int(atan(c*x)**2/(c**2*d*x**3 + c*
*2*e*x**4 + d*x + e*x**2),x)*b**2*c**2*d**3*x**2 + 4*int((atan(c*x)*x)/(c*
*2*d*x**2 + c**2*e*x**3 + d + e*x),x)*a*b*c**2*d*e**2*x**2 - 2*log(d + e*x
)*a**2*e**2*x**2 + 2*log(x)*a**2*e**2*x**2 - a**2*d**2 + 2*a**2*d*e*x - 2*
a*b*c*d**2*x)/(2*d**3*x**2)
```

# 3.148 $\int \frac{1}{(d+ex)(a+b\arctan(cx))}\,dx$



## Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{(d+ex)(a+b\arctan(cx))}\,dx = \text{Int}\left(\frac{1}{(d+ex)(a+b\arctan(cx))},x\right)$$

output
```
Defer(Int)(1/(e*x+d)/(a+b*arctan(c*x)),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{(d+ex)(a+b\arctan(cx))}\,dx = \int \frac{1}{(d+ex)(a+b\arctan(cx))}\,dx$$

input
```
Integrate[1/((d + e*x)*(a + b*ArcTan[c*x])),x]
```

output
```
Integrate[1/((d + e*x)*(a + b*ArcTan[c*x])), x]
```

## Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d+ex)(a+b\arctan(cx))}dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{1}{(d+ex)(a+b\arctan(cx))}dx$$

input
```
Int[1/((d + e*x)*(a + b*ArcTan[c*x])),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.99 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ex+d)\,(a+b\arctan(cx))}dx$$

input
```
int(1/(e*x+d)/(a+b*arctan(c*x)),x)
```

output
```
int(1/(e*x+d)/(a+b*arctan(c*x)),x)
```

## Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.39

$$\int \frac{1}{(d+ex)(a+b\arctan(cx))}\,dx = \int \frac{1}{(ex+d)(b\arctan(cx)+a)}\,dx$$

input
```
integrate(1/(e*x+d)/(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
integral(1/(a*e*x + a*d + (b*e*x + b*d)*arctan(c*x)), x)
```

## Sympy [N/A]

Not integrable

Time = 1.30 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{1}{(d+ex)(a+b\arctan(cx))}\,dx = \int \frac{1}{(a+b\,\mathrm{atan}(cx))\,(d+ex)}\,dx$$

input
```
integrate(1/(e*x+d)/(a+b*atan(c*x)),x)
```

output
```
Integral(1/((a + b*atan(c*x))*(d + e*x)), x)
```

## Maxima [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{(d+ex)(a+b\arctan(cx))}\,dx = \int \frac{1}{(ex+d)(b\arctan(cx)+a)}\,dx$$

input
```
integrate(1/(e*x+d)/(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
integrate(1/((e*x + d)*(b*arctan(c*x) + a)), x)
```

## Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{(d+ex)(a+b\arctan(cx))}\,dx = \int \frac{1}{(ex+d)(b\arctan(cx)+a)}\,dx$$

input
```
integrate(1/(e*x+d)/(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
integrate(1/((e*x + d)*(b*arctan(c*x) + a)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{(d+ex)(a+b\arctan(cx))}\,dx = \int \frac{1}{(a+b\,\mathrm{atan}(c\,x))\,(d+e\,x)}\,dx$$

input
```
int(1/((a + b*atan(c*x))*(d + e*x)),x)
```

output
```
int(1/((a + b*atan(c*x))*(d + e*x)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.50

$$\int \frac{1}{(d+ex)(a+b\arctan(cx))}\,dx = \int \frac{1}{atan\,(cx)\,bd + atan\,(cx)\,bex + ad + aex}dx$$

input
```
int(1/(e*x+d)/(a+b*atan(c*x)),x)
```

output
```
int(1/(atan(c*x)*b*d + atan(c*x)*b*e*x + a*d + a*e*x),x)
```

# 3.149 $\int x^3(c+a^2cx^2)\arctan(ax)\,dx$



## Optimal result

Integrand size = 18, antiderivative size = 69

$$\int x^3(c+a^2cx^2)\arctan(ax)\,dx=\frac{cx}{12a^3}-\frac{cx^3}{36a}-\frac{1}{30}acx^5-\frac{c\arctan(ax)}{12a^4}+\frac{1}{4}cx^4\arctan(ax)+\frac{1}{6}a^2cx^6\arctan(ax)$$

output
```
1/12*c*x/a^3-1/36*c*x^3/a-1/30*a*c*x^5-1/12*c*arctan(a*x)/a^4+1/4*c*x^4*ar
ctan(a*x)+1/6*a^2*c*x^6*arctan(a*x)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00

$$\int x^3(c+a^2cx^2)\arctan(ax)\,dx=\frac{cx}{12a^3}-\frac{cx^3}{36a}-\frac{1}{30}acx^5-\frac{c\arctan(ax)}{12a^4}+\frac{1}{4}cx^4\arctan(ax)+\frac{1}{6}a^2cx^6\arctan(ax)$$

input
```
Integrate[x^3*(c + a^2*c*x^2)*ArcTan[a*x],x]
```

output
```
(c*x)/(12*a^3) - (c*x^3)/(36*a) - (a*c*x^5)/30 - (c*ArcTan[a*x])/(12*a^4)
+ (c*x^4*ArcTan[a*x])/4 + (a^2*c*x^6*ArcTan[a*x])/6
```

## Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.48, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5485, 5361, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \arctan(ax) \left(a^2cx^2+c\right) dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\int x^5 \arctan(ax)dx+c\int x^3 \arctan(ax)dx$$

$$\Big\downarrow \text{5361}$$

$$a^2c\left(\frac{1}{6}x^6 \arctan(ax)-\frac{1}{6}a\int \frac{x^6}{a^2x^2+1}dx\right)+c\left(\frac{1}{4}x^4 \arctan(ax)-\frac{1}{4}a\int \frac{x^4}{a^2x^2+1}dx\right)$$

$$\Big\downarrow \text{254}$$

$$c\left(\frac{1}{4}x^4 \arctan(ax)-\frac{1}{4}a\int \left(\frac{x^2}{a^2}+\frac{1}{a^4\left(a^2x^2+1\right)}-\frac{1}{a^4}\right)dx\right)+ a^2c\left(\frac{1}{6}x^6 \arctan(ax)-\frac{1}{6}a\int \left(\frac{x^4}{a^2}-\frac{x^2}{a^4}-\frac{1}{a^6\left(a^2x^2+1\right)}+\frac{1}{a^6}\right)dx\right)$$

$$\Big\downarrow \text{2009}$$

$$c\left(\frac{1}{4}x^4 \arctan(ax)-\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)\right)+ a^2c\left(\frac{1}{6}x^6 \arctan(ax)-\frac{1}{6}a\left(-\frac{\arctan(ax)}{a^7}+\frac{x}{a^6}-\frac{x^3}{3a^4}+\frac{x^5}{5a^2}\right)\right)$$

input
```
Int[x^3*(c + a^2*c*x^2)*ArcTan[a*x],x]
```

output
```
a^2*c*((x^6*ArcTan[a*x])/6 - (a*(x/a^6 - x^3/(3*a^4) + x^5/(5*a^2) - ArcTa
n[a*x]/a^7))/6) + c*((x^4*ArcTan[a*x])/4 - (a*(-(x/a^4) + x^3/(3*a^2) + Ar
cTan[a*x]/a^5))/4)
```

### Defintions of rubi rules used

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

## Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.87

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{c\arctan(ax)a^6x^6}{6}+\frac{c\arctan(ax)a^4x^4}{4}-\frac{c\left(\frac{2a^5x^5}{5}+\frac{a^3x^3}{3}-ax+\arctan(ax)\right)}{12}}{a^4}$ |
| default | $\frac{\frac{c\arctan(ax)a^6x^6}{6}+\frac{c\arctan(ax)a^4x^4}{4}-\frac{c\left(\frac{2a^5x^5}{5}+\frac{a^3x^3}{3}-ax+\arctan(ax)\right)}{12}}{a^4}$ |
| parts | $\frac{a^2c\,x^6\arctan(ax)}{6}+\frac{c\,x^4\arctan(ax)}{4}-\frac{ac\left(\frac{\frac{2}{5}a^4x^5+\frac{1}{3}a^2x^3-x}{a^4}+\frac{\arctan(ax)}{a^5}\right)}{12}$ |
| parallelrisch | $\frac{30c\arctan(ax)a^6x^6-6a^5c\,x^5+45c\arctan(ax)a^4x^4-5a^3c\,x^3+15acx-15c\arctan(ax)}{180a^4}$ |
| risch | $-\frac{ic\,x^4(2a^2x^2+3)\ln(iax+1)}{24}+\frac{ic\,a^2x^6\ln(-iax+1)}{12}-\frac{ac\,x^5}{30}+\frac{ic\,x^4\ln(-iax+1)}{8}-\frac{c\,x^3}{36a}+\frac{cx}{12a^3}-\frac{c\arctan(}{12a^4}$ |
| meijerg | $\frac{c\left(-\frac{2xa\left(21a^4x^4-35a^2x^2+105\right)}{315}+\frac{2xa\left(7a^6x^6+7\right)\arctan\left(\sqrt{a^2x^2}\right)}{21\sqrt{a^2x^2}}\right)}{4a^4}+\frac{c\left(\frac{ax\left(-5a^2x^2+15\right)}{15}-\frac{ax\left(-5a^4x^4+5\right)\arctan\left(\sqrt{a^2x^2}\right.}{5\sqrt{a^2x^2}}\right.}{4a^4}$ |
| orering | $\frac{(15a^6x^6+22a^4x^4-15a^2x^2-15)(a^2c\,x^2+c)\arctan(ax)}{45a^4(a^2x^2+1)}-\frac{(6a^4x^4+5a^2x^2-15)\left(3x^2(a^2c\,x^2+c)\arctan(ax)+2x^4c\,a^2\right.}{180x^2a^4}$ |

input
```
int(x^3*(a^2*c*x^2+c)*arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/a^4*(1/6*c*arctan(a*x)*a^6*x^6+1/4*c*arctan(a*x)*a^4*x^4-1/12*c*(2/5*a^5
*x^5+1/3*a^3*x^3-a*x+arctan(a*x)))
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int x^3(c+a^2cx^2)\arctan(ax)\,dx$$
$$=-\frac{6\,a^5cx^5+5\,a^3cx^3-15\,acx-15\,(2\,a^6cx^6+3\,a^4cx^4-c)\arctan(ax)}{180\,a^4}$$

input
```
integrate(x^3*(a^2*c*x^2+c)*arctan(a*x),x, algorithm="fricas")
```

output
```
-1/180*(6*a^5*c*x^5 + 5*a^3*c*x^3 - 15*a*c*x - 15*(2*a^6*c*x^6 + 3*a^4*c*x
^4 - c)*arctan(a*x))/a^4
```

## Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.94

$$\int x^3(c+a^2cx^2)\arctan(ax)\,dx = \begin{cases} \frac{a^2cx^6\,\mathrm{atan}\,(ax)}{6} - \frac{acx^5}{30} + \frac{cx^4\,\mathrm{atan}\,(ax)}{4} - \frac{cx^3}{36a} + \frac{cx}{12a^3} - \frac{c\,\mathrm{atan}\,(ax)}{12a^4} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*(a**2*c*x**2+c)*atan(a*x),x)
```

output
```
Piecewise((a**2*c*x**6*atan(a*x)/6 - a*c*x**5/30 + c*x**4*atan(a*x)/4 - c*
x**3/(36*a) + c*x/(12*a**3) - c*atan(a*x)/(12*a**4), Ne(a, 0)), (0, True))
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.93

$$\int x^3(c+a^2cx^2)\arctan(ax)\,dx = -\frac{1}{180}\,a\left(\frac{6\,a^4cx^5+5\,a^2cx^3-15\,cx}{a^4}+\frac{15\,c\arctan\,(ax)}{a^5}\right) + \frac{1}{12}\left(2\,a^2cx^6+3\,cx^4\right)\arctan\,(ax)$$

input
```
integrate(x^3*(a^2*c*x^2+c)*arctan(a*x),x, algorithm="maxima")
```

output
```
-1/180*a*((6*a^4*c*x^5 + 5*a^2*c*x^3 - 15*c*x)/a^4 + 15*c*arctan(a*x)/a^5)
 + 1/12*(2*a^2*c*x^6 + 3*c*x^4)*arctan(a*x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.93

$$\int x^3(c+a^2cx^2)\arctan(ax)\,dx = \frac{1}{12}\left(2\,a^2cx^6+3\,cx^4\right)\arctan\left(ax\right) - \frac{c\arctan\left(ax\right)}{12\,a^4} - \frac{6\,a^{11}cx^5+5\,a^9cx^3-15\,a^7cx}{180\,a^{10}}$$

input
```
integrate(x^3*(a^2*c*x^2+c)*arctan(a*x),x, algorithm="giac")
```

output
```
1/12*(2*a^2*c*x^6 + 3*c*x^4)*arctan(a*x) - 1/12*c*arctan(a*x)/a^4 - 1/180*
(6*a^11*c*x^5 + 5*a^9*c*x^3 - 15*a^7*c*x)/a^10
```

## Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int x^3(c+a^2cx^2)\arctan(ax)\,dx = -\frac{c\,(15\,\mathrm{atan}(a\,x)-15\,a\,x+5\,a^3\,x^3+6\,a^5\,x^5-45\,a^4\,x^4\,\mathrm{atan}(a\,x)-30\,a^6\,x^6\,\mathrm{atan}(a\,x))}{180\,a^4}$$

input
```
int(x^3*atan(a*x)*(c + a^2*c*x^2),x)
```

output
```
-(c*(15*atan(a*x) - 15*a*x + 5*a^3*x^3 + 6*a^5*x^5 - 45*a^4*x^4*atan(a*x)
- 30*a^6*x^6*atan(a*x)))/(180*a^4)
```

## Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int x^3(c+a^2cx^2)\arctan(ax)\,dx = \frac{c(30atan(ax)\,a^6x^6+45atan(ax)\,a^4x^4-15atan(ax)-6a^5x^5-5a^3x^3+15ax)}{180a^4}$$

input
```
int(x^3*(a^2*c*x^2+c)*atan(a*x),x)
```

output
```
(c*(30*atan(a*x)*a**6*x**6 + 45*atan(a*x)*a**4*x**4 - 15*atan(a*x) - 6*a**
5*x**5 - 5*a**3*x**3 + 15*a*x))/(180*a**4)
```

# 3.150 $\int x^2(c+a^2cx^2)\arctan(ax)\,dx$



## Optimal result

Integrand size = 18, antiderivative size = 66

$$\int x^2(c+a^2cx^2)\arctan(ax)\,dx = -\frac{cx^2}{15a}-\frac{1}{20}acx^4+\frac{1}{3}cx^3\arctan(ax)+\frac{1}{5}a^2cx^5\arctan(ax)+\frac{c\log(1+a^2x^2)}{15a^3}$$

output
```
-1/15*c*x^2/a-1/20*a*c*x^4+1/3*c*x^3*arctan(a*x)+1/5*a^2*c*x^5*arctan(a*x)
+1/15*c*ln(a^2*x^2+1)/a^3
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00

$$\int x^2(c+a^2cx^2)\arctan(ax)\,dx = -\frac{cx^2}{15a}-\frac{1}{20}acx^4+\frac{1}{3}cx^3\arctan(ax)+\frac{1}{5}a^2cx^5\arctan(ax)+\frac{c\log(1+a^2x^2)}{15a^3}$$

input
```
Integrate[x^2*(c + a^2*c*x^2)*ArcTan[a*x],x]
```

output
```
-1/15*(c*x^2)/a - (a*c*x^4)/20 + (c*x^3*ArcTan[a*x])/3 + (a^2*c*x^5*ArcTan
[a*x])/5 + (c*Log[1 + a^2*x^2])/(15*a^3)
```

## Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.48, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5485, 5361, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arctan(ax) \left(a^2cx^2+c\right) \, dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\int x^4\arctan(ax)dx+c\int x^2\arctan(ax)dx$$

$$\Big\downarrow \text{5361}$$

$$a^2c\left(\frac{1}{5}x^5\arctan(ax)-\frac{1}{5}a\int \frac{x^5}{a^2x^2+1}dx\right)+c\left(\frac{1}{3}x^3\arctan(ax)-\frac{1}{3}a\int \frac{x^3}{a^2x^2+1}dx\right)$$

$$\Big\downarrow \text{243}$$

$$c\left(\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\int \frac{x^2}{a^2x^2+1}dx^2\right)+a^2c\left(\frac{1}{5}x^5\arctan(ax)-\frac{1}{10}a\int \frac{x^4}{a^2x^2+1}dx^2\right)$$

$$\Big\downarrow \text{49}$$

$$c\left(\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\int \left(\frac{1}{a^2}-\frac{1}{a^2\left(a^2x^2+1\right)}\right)dx^2\right)+ a^2c\left(\frac{1}{5}x^5\arctan(ax)-\frac{1}{10}a\int \left(\frac{x^2}{a^2}+\frac{1}{a^4\left(a^2x^2+1\right)}-\frac{1}{a^4}\right)dx^2\right)$$

$$\Big\downarrow \text{2009}$$

$$c\left(\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log\left(a^2x^2+1\right)}{a^4}\right)\right)+ a^2c\left(\frac{1}{5}x^5\arctan(ax)-\frac{1}{10}a\left(-\frac{x^2}{a^4}+\frac{x^4}{2a^2}+\frac{\log\left(a^2x^2+1\right)}{a^6}\right)\right)$$

input
```
Int[x^2*(c + a^2*c*x^2)*ArcTan[a*x],x]
```

output
```
a^2*c*((x^5*ArcTan[a*x])/5 - (a*(-(x^2/a^4) + x^4/(2*a^2) + Log[1 + a^2*x^
2]/a^6))/10) + c*((x^3*ArcTan[a*x])/3 - (a*(x^2/a^2 - Log[1 + a^2*x^2]/a^4
))/6)
```

### Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

## Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.95

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{c\arctan(ax)a^5x^5}{5}+\frac{c\arctan(ax)a^3x^3}{3}-\frac{c\left(\frac{3a^4x^4}{4}+a^2x^2-\ln\left(a^2x^2+1\right)\right)}{15}}{a^3}$ |
| default | $\frac{\frac{c\arctan(ax)a^5x^5}{5}+\frac{c\arctan(ax)a^3x^3}{3}-\frac{c\left(\frac{3a^4x^4}{4}+a^2x^2-\ln\left(a^2x^2+1\right)\right)}{15}}{a^3}$ |
| parallelrisch | $\frac{12c\arctan(ax)a^5x^5-3a^4c\,x^4+20c\arctan(ax)a^3x^3-4a^2c\,x^2+4c\ln(a^2x^2+1)}{60a^3}$ |
| parts | $\frac{a^2c\,x^5\arctan(ax)}{5}+\frac{c\,x^3\arctan(ax)}{3}-\frac{ac\left(\frac{\frac{3}{2}a^2x^4+2x^2}{2a^2}-\frac{\ln\left(a^2x^2+1\right)}{a^4}\right)}{15}$ |
| risch | $-\frac{ic\,x^3(3a^2x^2+5)\ln(iax+1)}{30}+\frac{ic\,a^2x^5\ln(-iax+1)}{10}-\frac{x^4ac}{20}+\frac{ic\,x^3\ln(-iax+1)}{6}-\frac{c\,x^2}{15a}+\frac{c\ln(-a^2x^2-1)}{15a^3}-$ |
| meijerg | $\frac{c\left(\frac{a^2x^2\left(-3a^2x^2+6\right)}{15}+\frac{4a^6x^6\arctan\left(\sqrt{a^2x^2}\right)}{5\sqrt{a^2x^2}}-\frac{2\ln\left(a^2x^2+1\right)}{5}\right)}{4a^3}+\frac{c\left(-\frac{2a^2x^2}{3}+\frac{4a^4x^4\arctan\left(\sqrt{a^2x^2}\right)}{3\sqrt{a^2x^2}}+\frac{2\ln\left(a^2x^2+1\right)}{3}\right)}{4a^3}$ |

input
```
int(x^2*(a^2*c*x^2+c)*arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(1/5*c*arctan(a*x)*a^5*x^5+1/3*c*arctan(a*x)*a^3*x^3-1/15*c*(3/4*a^4
*x^4+a^2*x^2-ln(a^2*x^2+1)))
```

## Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.94

$$\int x^2\left(c+a^2cx^2\right)\arctan(ax)\,dx$$
$$=-\frac{3\,a^4cx^4+4\,a^2cx^2-4\,(3\,a^5cx^5+5\,a^3cx^3)\arctan\left(ax\right)-4\,c\log\left(a^2x^2+1\right)}{60\,a^3}$$

input
```
integrate(x^2*(a^2*c*x^2+c)*arctan(a*x),x, algorithm="fricas")
```

output
```
-1/60*(3*a^4*c*x^4 + 4*a^2*c*x^2 - 4*(3*a^5*c*x^5 + 5*a^3*c*x^3)*arctan(a*
x) - 4*c*log(a^2*x^2 + 1))/a^3
```

## Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.92

$$\int x^2\left(c+a^2cx^2\right)\arctan(ax)\,dx = \begin{cases} \frac{a^2cx^5\operatorname{atan}(ax)}{5}-\frac{acx^4}{20}+\frac{cx^3\operatorname{atan}(ax)}{3}-\frac{cx^2}{15a}+\frac{c\log\left(x^2+\frac{1}{a^2}\right)}{15a^3} & \text{for } a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

input
```
integrate(x**2*(a**2*c*x**2+c)*atan(a*x),x)
```

output
```
Piecewise((a**2*c*x**5*atan(a*x)/5 - a*c*x**4/20 + c*x**3*atan(a*x)/3 - c*
x**2/(15*a) + c*log(x**2 + a**(-2))/(15*a**3), Ne(a, 0)), (0, True))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.95

$$\int x^2\left(c+a^2cx^2\right)\arctan(ax)\,dx = -\frac{1}{60}\,a\left(\frac{3\,a^2cx^4+4\,cx^2}{a^2}-\frac{4\,c\log\left(a^2x^2+1\right)}{a^4}\right)+\frac{1}{15}\left(3\,a^2cx^5+5\,cx^3\right)\arctan\left(ax\right)$$

input
```
integrate(x^2*(a^2*c*x^2+c)*arctan(a*x),x, algorithm="maxima")
```

output
```
-1/60*a*((3*a^2*c*x^4 + 4*c*x^2)/a^2 - 4*c*log(a^2*x^2 + 1)/a^4) + 1/15*(3
*a^2*c*x^5 + 5*c*x^3)*arctan(a*x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.95

$$\int x^2(c+a^2cx^2)\arctan(ax)\,dx = \frac{1}{15}\left(3\,a^2cx^5+5\,cx^3\right)\arctan\left(ax\right) + \frac{c\log\left(a^2x^2+1\right)}{15\,a^3} - \frac{3\,a^5cx^4+4\,a^3cx^2}{60\,a^4}$$

input
```
integrate(x^2*(a^2*c*x^2+c)*arctan(a*x),x, algorithm="giac")
```

output
```
1/15*(3*a^2*c*x^5 + 5*c*x^3)*arctan(a*x) + 1/15*c*log(a^2*x^2 + 1)/a^3 - 1
/60*(3*a^5*c*x^4 + 4*a^3*c*x^2)/a^4
```

## Mupad [B] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.88

$$\int x^2(c+a^2cx^2)\arctan(ax)\,dx = \frac{\frac{c\,\ln\left(a^2\,x^2+1\right)}{15}-\frac{a^2\,c\,x^2}{15}}{a^3} + \frac{c\,x^3\,\mathrm{atan}(a\,x)}{3} - \frac{a\,c\,x^4}{20} + \frac{a^2\,c\,x^5\,\mathrm{atan}(a\,x)}{5}$$

input
```
int(x^2*atan(a*x)*(c + a^2*c*x^2),x)
```

output
```
((c*log(a^2*x^2 + 1))/15 - (a^2*c*x^2)/15)/a^3 + (c*x^3*atan(a*x))/3 - (a*
c*x^4)/20 + (a^2*c*x^5*atan(a*x))/5
```

## Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.89

$$\int x^2(c+a^2cx^2)\arctan(ax)\,dx$$
$$=\frac{c(12atan(ax)\,a^5x^5+20atan(ax)\,a^3x^3+4\log(a^2x^2+1)-3a^4x^4-4a^2x^2)}{60a^3}$$

input
```
int(x^2*(a^2*c*x^2+c)*atan(a*x),x)
```

output
```
(c*(12*atan(a*x)*a**5*x**5 + 20*atan(a*x)*a**3*x**3 + 4*log(a**2*x**2 + 1)
 - 3*a**4*x**4 - 4*a**2*x**2))/(60*a**3)
```

# 3.151 $\int x(c+a^2cx^2)\arctan(ax)\,dx$



## Optimal result

Integrand size = 16, antiderivative size = 42

$$\int x(c+a^2cx^2)\arctan(ax)\,dx = -\frac{cx}{4a}-\frac{1}{12}acx^3+\frac{c(1+a^2x^2)^2\arctan(ax)}{4a^2}$$

output
```
-1/4*c*x/a-1/12*a*c*x^3+1/4*c*(a^2*x^2+1)^2*arctan(a*x)/a^2
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.38

$$\int x(c+a^2cx^2)\arctan(ax)\,dx = -\frac{cx}{4a}-\frac{1}{12}acx^3+\frac{c\arctan(ax)}{4a^2}+\frac{1}{2}cx^2\arctan(ax)+\frac{1}{4}a^2cx^4\arctan(ax)$$

input
```
Integrate[x*(c + a^2*c*x^2)*ArcTan[a*x],x]
```

output
```
-1/4*(c*x)/a - (a*c*x^3)/12 + (c*ArcTan[a*x])/(4*a^2) + (c*x^2*ArcTan[a*x]
)/2 + (a^2*c*x^4*ArcTan[a*x])/4
```

## Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.10, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {5465, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \arctan(ax) \left(a^2cx^2+c\right) \, dx$$

$$\Big\downarrow \text{ 5465}$$

$$\frac{c\left(a^2x^2+1\right)^2\arctan(ax)}{4a^2}-\frac{\int \left(a^2cx^2+c\right)dx}{4a}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{c\left(a^2x^2+1\right)^2\arctan(ax)}{4a^2}-\frac{\frac{1}{3}a^2cx^3+cx}{4a}$$

input
```
Int[x*(c + a^2*c*x^2)*ArcTan[a*x],x]
```

output
```
-1/4*(c*x + (a^2*c*x^3)/3)/a + (c*(1 + a^2*x^2)^2*ArcTan[a*x])/(4*a^2)
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

## Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.21

| method | result |
|---|---|
| parts | $\frac{x^4 c\, a^2 \arctan(ax)}{4}+\frac{c \arctan(ax) x^2}{2}+\frac{c \arctan(ax)}{4a^2}-\frac{c\left(\frac{1}{3}a^2x^3+x\right)}{4a}$ |
| derivativedivides | $\frac{\frac{c \arctan(ax) a^4 x^4}{4}+\frac{a^2 c\, x^2 \arctan(ax)}{2}+\frac{c \arctan(ax)}{4}-\frac{c\left(\frac{1}{3}a^3x^3+ax\right)}{4}}{a^2}$ |
| default | $\frac{\frac{c \arctan(ax) a^4 x^4}{4}+\frac{a^2 c\, x^2 \arctan(ax)}{2}+\frac{c \arctan(ax)}{4}-\frac{c\left(\frac{1}{3}a^3x^3+ax\right)}{4}}{a^2}$ |
| parallelrisch | $\frac{3c \arctan(ax) a^4 x^4-a^3 c\, x^3+6a^2 c\, x^2 \arctan(ax)-3acx+3c \arctan(ax)}{12a^2}$ |
| meijerg | $\frac{c\left(\frac{ax\left(-5a^2x^2+15\right)}{15}-\frac{ax\left(-5a^4x^4+5\right)\arctan\left(\sqrt{a^2x^2}\right)}{5\sqrt{a^2x^2}}\right)}{4a^2}+\frac{c\left(-2ax+\frac{2\left(3a^2x^2+3\right)\arctan(ax)}{3}\right)}{4a^2}$ |
| risch | $-\frac{ic\left(a^2x^2+1\right)^2 \ln(iax+1)}{8a^2}+\frac{ic\, a^2x^4 \ln(-iax+1)}{8}-\frac{ac\, x^3}{12}+\frac{ic\, x^2 \ln(-iax+1)}{4}-\frac{cx}{4a}+\frac{ic \ln(a^2x^2+1)}{16a^2}+\frac{c \arc}{}$ |
| orering | $\frac{\left(3a^4x^4+8a^2x^2+3\right)\left(a^2c\,x^2+c\right)\arctan(ax)}{6a^2\left(a^2x^2+1\right)}-\frac{\left(a^2x^2+3\right)\left(\left(a^2c\,x^2+c\right)\arctan(ax)+2a^2c\,x^2\arctan(ax)+\frac{x\left(a^2c\,x^2+c\right)a}{a^2x^2+1}\right.}{12a^2}$ |

input
```
int(x*(a^2*c*x^2+c)*arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4*c*a^2*arctan(a*x)+1/2*c*arctan(a*x)*x^2+1/4*c/a^2*arctan(a*x)-1/4*
c/a*(1/3*a^2*x^3+x)
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.05

$$\int x\left(c+a^2cx^2\right)\arctan(ax)\,dx=-\frac{a^3cx^3+3\,acx-3\left(a^4cx^4+2\,a^2cx^2+c\right)\arctan\left(ax\right)}{12\,a^2}$$

input
```
integrate(x*(a^2*c*x^2+c)*arctan(a*x),x, algorithm="fricas")
```

output
```
-1/12*(a^3*c*x^3 + 3*a*c*x - 3*(a^4*c*x^4 + 2*a^2*c*x^2 + c)*arctan(a*x))/
a^2
```

## Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.29

$$\int x(c+a^2cx^2)\arctan(ax)\,dx = \begin{cases} \frac{a^2cx^4\operatorname{atan}(ax)}{4} - \frac{acx^3}{12} + \frac{cx^2\operatorname{atan}(ax)}{2} - \frac{cx}{4a} + \frac{c\operatorname{atan}(ax)}{4a^2} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x*(a**2*c*x**2+c)*atan(a*x),x)
```

output
```
Piecewise((a**2*c*x**4*atan(a*x)/4 - a*c*x**3/12 + c*x**2*atan(a*x)/2 - c*
x/(4*a) + c*atan(a*x)/(4*a**2), Ne(a, 0)), (0, True))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.19

$$\int x(c+a^2cx^2)\arctan(ax)\,dx = \frac{(a^2cx^2+c)^2\arctan(ax)}{4\,a^2c} - \frac{a^2c^2x^3+3\,c^2x}{12\,ac}$$

input
```
integrate(x*(a^2*c*x^2+c)*arctan(a*x),x, algorithm="maxima")
```

output
```
1/4*(a^2*c*x^2 + c)^2*arctan(a*x)/(a^2*c) - 1/12*(a^2*c^2*x^3 + 3*c^2*x)/(
a*c)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.26

$$\int x(c+a^2cx^2)\arctan(ax)\,dx = \frac{1}{4}\left(a^2cx^4+2\,cx^2\right)\arctan(ax) + \frac{c\arctan(ax)}{4\,a^2} - \frac{a^7cx^3+3\,a^5cx}{12\,a^6}$$

input
```
integrate(x*(a^2*c*x^2+c)*arctan(a*x),x, algorithm="giac")
```

output
```
1/4*(a^2*c*x^4 + 2*c*x^2)*arctan(a*x) + 1/4*c*arctan(a*x)/a^2 - 1/12*(a^7*
c*x^3 + 3*a^5*c*x)/a^6
```

## Mupad [B] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.14

$$\int x(c+a^2cx^2)\arctan(ax)\,dx = \frac{\frac{c\,\mathrm{atan}(a\,x)}{4}-\frac{a\,c\,x}{4}}{a^2}+\frac{c\,x^2\,\mathrm{atan}(a\,x)}{2}-\frac{a\,c\,x^3}{12}+\frac{a^2\,c\,x^4\,\mathrm{atan}(a\,x)}{4}$$

input
```
int(x*atan(a*x)*(c + a^2*c*x^2),x)
```

output
```
((c*atan(a*x))/4 - (a*c*x)/4)/a^2 + (c*x^2*atan(a*x))/2 - (a*c*x^3)/12 + (
a^2*c*x^4*atan(a*x))/4
```

## Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.17

$$\int x(c+a^2cx^2)\arctan(ax)\,dx = \frac{c(3atan(ax)\,a^4x^4+6atan(ax)\,a^2x^2+3atan(ax)-a^3x^3-3ax)}{12a^2}$$

input
```
int(x*(a^2*c*x^2+c)*atan(a*x),x)
```

output
```
(c*(3*atan(a*x)*a**4*x**4 + 6*atan(a*x)*a**2*x**2 + 3*atan(a*x) - a**3*x**
3 - 3*a*x))/(12*a**2)
```

# 3.152 $\int (c+a^2cx^2)\arctan(ax)\,dx$



## Optimal result

Integrand size = 15, antiderivative size = 50

$$\int (c+a^2cx^2)\arctan(ax)\,dx = -\frac{1}{6}acx^2+cx\arctan(ax)+\frac{1}{3}a^2cx^3\arctan(ax)-\frac{c\log(1+a^2x^2)}{3a}$$

output
```
-1/6*a*c*x^2+c*x*arctan(a*x)+1/3*a^2*c*x^3*arctan(a*x)-1/3*c*ln(a^2*x^2+1)
/a
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int (c+a^2cx^2)\arctan(ax)\,dx = -\frac{1}{6}acx^2+cx\arctan(ax)+\frac{1}{3}a^2cx^3\arctan(ax)-\frac{c\log(1+a^2x^2)}{3a}$$

input
```
Integrate[(c + a^2*c*x^2)*ArcTan[a*x],x]
```

output
```
-1/6*(a*c*x^2) + c*x*ArcTan[a*x] + (a^2*c*x^3*ArcTan[a*x])/3 - (c*Log[1 +
a^2*x^2])/(3*a)
```

## Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.32, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5413, 5345, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(ax)\left(a^2cx^2+c\right)\,dx$$

$$\Big\downarrow \text{5413}$$

$$\frac{2}{3}c\int \arctan(ax)dx+\frac{1}{3}cx\left(a^2x^2+1\right)\arctan(ax)-\frac{c(a^2x^2+1)}{6a}$$

$$\Big\downarrow \text{5345}$$

$$\frac{2}{3}c\left(x\arctan(ax)-a\int \frac{x}{a^2x^2+1}dx\right)+\frac{1}{3}cx\left(a^2x^2+1\right)\arctan(ax)-\frac{c(a^2x^2+1)}{6a}$$

$$\Big\downarrow \text{240}$$

$$\frac{1}{3}cx\left(a^2x^2+1\right)\arctan(ax)+\frac{2}{3}c\left(x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}\right)-\frac{c(a^2x^2+1)}{6a}$$

input
```
Int[(c + a^2*c*x^2)*ArcTan[a*x],x]
```

output
```
-1/6*(c*(1 + a^2*x^2))/a + (c*x*(1 + a^2*x^2)*ArcTan[a*x])/3 + (2*c*(x*Arc
Tan[a*x] - Log[1 + a^2*x^2]/(2*a)))/3
```

### Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5413
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Simp[x*(d + e*x^2)
^q*((a + b*ArcTan[c*x])/(2*q + 1)), x] + Simp[2*d*(q/(2*q + 1))   Int[(d +
e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0]
```

## Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

| method | result | size |
|---|---|---|
| parts | $\frac{a^{2} c \,x^{3} \arctan \left(a x \right)}{3}+c x \arctan \left(a x \right)-\frac{a c \left(\frac{x^{2}}{2}+\frac{\ln \left(a^{2} x^{2}+1\right)}{a^{2}}\right)}{3}$ | 46 |
| derivativedivides | $\frac{\frac{c \arctan \left(a x \right) a^{3} x^{3}}{3}+a c x \arctan \left(a x \right)-\frac{c \left(\frac{a^{2} x^{2}}{2}+\ln \left(a^{2} x^{2}+1\right)\right)}{3}}{a}$ | 49 |
| default | $\frac{\frac{c \arctan \left(a x \right) a^{3} x^{3}}{3}+a c x \arctan \left(a x \right)-\frac{c \left(\frac{a^{2} x^{2}}{2}+\ln \left(a^{2} x^{2}+1\right)\right)}{3}}{a}$ | 49 |
| parallelrisch | $-\frac{-2 c \arctan \left(a x \right) a^{3} x^{3}+a^{2} c \,x^{2}-6 a c x \arctan \left(a x \right)+2 c \ln \left(a^{2} x^{2}+1\right)}{6 a}$ | 50 |
| risch | $-\frac{i c x \left(a^{2} x^{2}+3\right) \ln \left(i a x +1\right)}{6}+\frac{i c \,a^{2} x^{3} \ln \left(-i a x +1\right)}{6}-\frac{a c \,x^{2}}{6}+\frac{i c x \ln \left(-i a x +1\right)}{2}-\frac{c \ln \left(-a^{2} x^{2}-1\right)}{3 a}$ | 79 |
| meijerg | $\frac{c \left(-\frac{2 a^{2} x^{2}}{3}+\frac{4 a^{4} x^{4} \arctan \left(\sqrt{a^{2} x^{2}}\right)}{3 \sqrt{a^{2} x^{2}}}+\frac{2 \ln \left(a^{2} x^{2}+1\right)}{3}\right)}{4 a}+\frac{c \left(\frac{4 a^{2} x^{2} \arctan \left(\sqrt{a^{2} x^{2}}\right)}{\sqrt{a^{2} x^{2}}}-2 \ln \left(a^{2} x^{2}+1\right)\right)}{4 a}$ | 102 |

input
```
int((a^2*c*x^2+c)*arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/3*a^2*c*x^3*arctan(a*x)+c*x*arctan(a*x)-1/3*a*c*(1/2*x^2+1/a^2*ln(a^2*x^
2+1))
```

## Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94

$$\int (c + a^2 c x^2) \arctan(ax) \, dx$$
$$= -\frac{a^2 c x^2 - 2 (a^3 c x^3 + 3 \, a c x) \arctan (ax) + 2 \, c \log (a^2 x^2 + 1)}{6 \, a}$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x),x, algorithm="fricas")
```

output
```
-1/6*(a^2*c*x^2 - 2*(a^3*c*x^3 + 3*a*c*x)*arctan(a*x) + 2*c*log(a^2*x^2 +
1))/a
```

## Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int (c + a^2 c x^2) \arctan(ax) \, dx$$
$$= \begin{cases} \frac{a^2 c x^3 \operatorname{atan}(ax)}{3} - \frac{a c x^2}{6} + c x \operatorname{atan}(ax) - \frac{c \log \left(x^2 + \frac{1}{a^2}\right)}{3a} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate((a**2*c*x**2+c)*atan(a*x),x)
```

output
```
Piecewise((a**2*c*x**3*atan(a*x)/3 - a*c*x**2/6 + c*x*atan(a*x) - c*log(x*
*2 + a**(-2))/(3*a), Ne(a, 0)), (0, True))
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90

$$\int (c+a^2cx^2) \arctan(ax) \, dx = -\frac{1}{6} \left( cx^2 + \frac{2 \, c \log{(a^2x^2+1)}}{a^2} \right) a + \frac{1}{3} \left( a^2cx^3 + 3 \, cx \right) \arctan{(ax)}$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x),x, algorithm="maxima")
```

output
```
-1/6*(c*x^2 + 2*c*log(a^2*x^2 + 1)/a^2)*a + 1/3*(a^2*c*x^3 + 3*c*x)*arctan
(a*x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.86

$$\int (c+a^2cx^2) \arctan(ax) \, dx = -\frac{1}{6} \, acx^2 + \frac{1}{3} \left( a^2cx^3 + 3 \, cx \right) \arctan{(ax)} - \frac{c \log{(a^2x^2+1)}}{3 \, a}$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x),x, algorithm="giac")
```

output
```
-1/6*a*c*x^2 + 1/3*(a^2*c*x^3 + 3*c*x)*arctan(a*x) - 1/3*c*log(a^2*x^2 + 1
)/a
```

## Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int (c+a^2cx^2) \arctan(ax) \, dx = -\frac{c \left(2 \, \ln{(a^2 \, x^2 + 1)} + a^2 \, x^2 - 2 \, a^3 \, x^3 \, \mathrm{atan}(a \, x) - 6 \, a \, x \, \mathrm{atan}(a \, x)\right)}{6 \, a}$$

input
```
int(atan(a*x)*(c + a^2*c*x^2),x)
```

output
```
-(c*(2*log(a^2*x^2 + 1) + a^2*x^2 - 2*a^3*x^3*atan(a*x) - 6*a*x*atan(a*x))
)/(6*a)
```

## Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94

$$\int (c + a^2cx^2) \arctan(ax) \, dx = \frac{c(2atan(ax)\, a^3x^3 + 6atan(ax)\, ax - 2\log(a^2x^2 + 1) - a^2x^2)}{6a}$$

input
```
int((a^2*c*x^2+c)*atan(a*x),x)
```

output
```
(c*(2*atan(a*x)*a**3*x**3 + 6*atan(a*x)*a*x - 2*log(a**2*x**2 + 1) - a**2*
x**2))/(6*a)
```

# 3.153 $\int \frac{(c+a^2cx^2)\arctan(ax)}{x}\,dx$



## Optimal result

Integrand size = 18, antiderivative size = 62

$$\int \frac{(c+a^2cx^2)\arctan(ax)}{x}\,dx = -\frac{1}{2}acx + \frac{1}{2}c\arctan(ax) + \frac{1}{2}a^2cx^2\arctan(ax) + \frac{1}{2}ic\,\text{PolyLog}(2,-iax) - \frac{1}{2}ic\,\text{PolyLog}(2,iax)$$

output
```
-1/2*a*c*x+1/2*c*arctan(a*x)+1/2*a^2*c*x^2*arctan(a*x)+1/2*I*c*polylog(2,-
I*a*x)-1/2*I*c*polylog(2,I*a*x)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)\arctan(ax)}{x}\,dx = -\frac{1}{2}acx + \frac{1}{2}c\arctan(ax) + \frac{1}{2}a^2cx^2\arctan(ax) + \frac{1}{2}ic\,\text{PolyLog}(2,-iax) - \frac{1}{2}ic\,\text{PolyLog}(2,iax)$$

input
```
Integrate[((c + a^2*c*x^2)*ArcTan[a*x])/x,x]
```

output
```
-1/2*(a*c*x) + (c*ArcTan[a*x])/2 + (a^2*c*x^2*ArcTan[a*x])/2 + (I/2)*c*Pol
yLog[2, (-I)*a*x] - (I/2)*c*PolyLog[2, I*a*x]
```

## Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5485, 5355, 2838, 5361, 262, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)\left(a^2cx^2+c\right)}{x}dx$$

$$\Big\downarrow \text{ 5485}$$

$$a^2c\int x\arctan(ax)dx+c\int \frac{\arctan(ax)}{x}dx$$

$$\Big\downarrow \text{ 5355}$$

$$a^2c\int x\arctan(ax)dx+c\left(\frac{1}{2}i\int \frac{\log(1-iax)}{x}dx-\frac{1}{2}i\int \frac{\log(iax+1)}{x}dx\right)$$

$$\Big\downarrow \text{ 2838}$$

$$a^2c\int x\arctan(ax)dx+c\left(\frac{1}{2}i\,\text{PolyLog}(2,-iax)-\frac{1}{2}i\,\text{PolyLog}(2,iax)\right)$$

$$\Big\downarrow \text{ 5361}$$

$$a^2c\left(\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\int \frac{x^2}{a^2x^2+1}dx\right)+c\left(\frac{1}{2}i\,\text{PolyLog}(2,-iax)-\frac{1}{2}i\,\text{PolyLog}(2,iax)\right)$$

$$\Big\downarrow \text{ 262}$$

$$a^2c\left(\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\int \frac{1}{a^2x^2+1}dx}{a^2}\right)\right)+c\left(\frac{1}{2}i\,\text{PolyLog}(2,-iax)-\frac{1}{2}i\,\text{PolyLog}(2,iax)\right)$$

$$\Big\downarrow \text{ 216}$$

$$a^2c\left(\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)\right)+
c\left(\frac{1}{2}i\operatorname{PolyLog}(2,-iax)-\frac{1}{2}i\operatorname{PolyLog}(2,iax)\right)$$

input
```
Int[((c + a^2*c*x^2)*ArcTan[a*x])/x,x]
```

output
```
a^2*c*((x^2*ArcTan[a*x])/2 - (a*(x/a^2 - ArcTan[a*x]/a^3))/2) + c*((I/2)*P
olyLog[2, (-I)*a*x] - (I/2)*PolyLog[2, I*a*x])
```

## Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5355
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Simp[I*(b/2)   Int[Log[1 - I*c*x]/x, x], x] - Simp[I*(b/2)   Int[Log[1
+ I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

## Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.19

| method | result |
|---|---|
| risch | $\frac{ic\ln(-iax+1)x^2a^2}{4}+\frac{c\arctan(ax)}{2}-\frac{acx}{2}-\frac{ic\,\mathrm{dilog}(-iax+1)}{2}-\frac{ic\ln(iax+1)x^2a^2}{4}+\frac{ic\,\mathrm{dilog}(iax+1)}{2}$ |
| meijerg | $\frac{c\left(-2ax+\frac{2\left(3a^2x^2+3\right)\arctan(ax)}{3}\right)}{4}+\frac{c\left(-\frac{2iax\,\mathrm{polylog}\left(2,i\sqrt{a^2x^2}\right)}{\sqrt{a^2x^2}}+\frac{2iax\,\mathrm{polylog}\left(2,-i\sqrt{a^2x^2}\right)}{\sqrt{a^2x^2}}\right)}{4}$ |
| derivativedivides | $\frac{a^2cx^2\arctan(ax)}{2}+c\arctan(ax)\ln(ax)-\frac{c(ax-\arctan(ax)-i\ln(ax)\ln(iax+1)+i\ln(ax)\ln(-iax+1)-i\,\mathrm{d}$ |
| default | $\frac{a^2cx^2\arctan(ax)}{2}+c\arctan(ax)\ln(ax)-\frac{c(ax-\arctan(ax)-i\ln(ax)\ln(iax+1)+i\ln(ax)\ln(-iax+1)-i\,\mathrm{d}$ |
| parts | $\frac{a^2cx^2\arctan(ax)}{2}+c\arctan(ax)\ln(x)-\frac{ac\left(x-\frac{\arctan(ax)}{a}-\frac{i\ln(x)(-\ln(-iax+1)+\ln(iax+1))}{a}-\frac{i(\mathrm{dilog}(iax+1)}{}\right.}{2}$ |

input
```
int((a^2*c*x^2+c)*arctan(a*x)/x,x,method=_RETURNVERBOSE)
```

output
```
1/4*I*c*ln(1-I*a*x)*x^2*a^2+1/2*c*arctan(a*x)-1/2*a*c*x-1/2*I*c*dilog(1-I*
a*x)-1/4*I*c*ln(1+I*a*x)*x^2*a^2+1/2*I*c*dilog(1+I*a*x)
```

## Fricas [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)}{x}\,dx = \int \frac{(a^2cx^2+c)\arctan(ax)}{x}\,dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)/x,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)*arctan(a*x)/x, x)
```

## Sympy [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)}{x}\,dx = c\left(\int \frac{\operatorname{atan}(ax)}{x}\,dx + \int a^2x\operatorname{atan}(ax)\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)*atan(a*x)/x,x)
```

output
```
c*(Integral(atan(a*x)/x, x) + Integral(a**2*x*atan(a*x), x))
```

## Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.06

$$\begin{aligned}\int \frac{(c+a^2cx^2)\arctan(ax)}{x}\,dx = &-\frac{1}{2}acx-\frac{1}{4}\pi c\log\left(a^2x^2+1\right)\\&+c\arctan(ax)\log(ax)+\frac{1}{2}\left(a^2cx^2+c\right)\arctan(ax)\\&-\frac{1}{2}ic\operatorname{Li}_2(iax+1)+\frac{1}{2}ic\operatorname{Li}_2(-iax+1)\end{aligned}$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)/x,x, algorithm="maxima")
```

output
```
-1/2*a*c*x - 1/4*pi*c*log(a^2*x^2 + 1) + c*arctan(a*x)*log(a*x) + 1/2*(a^2
*c*x^2 + c)*arctan(a*x) - 1/2*I*c*dilog(I*a*x + 1) + 1/2*I*c*dilog(-I*a*x
+ 1)
```

## Giac [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)}{x}\,dx = \int \frac{(a^2cx^2+c)\arctan(ax)}{x}\,dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)/x,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)*arctan(a*x)/x, x)
```

## Mupad [B] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.92

$$\int \frac{(c+a^2cx^2)\arctan(ax)}{x}\,dx = \begin{cases} 0 & \text{if } a=0 \\ a^2\,c\,\mathrm{atan}(a\,x)\,\left(\frac{1}{2\,a^2}+\frac{x^2}{2}\right)-\frac{a\,c\,x}{2}-\frac{c\,(\mathrm{Li}_2(1-a\,x\,1\mathrm{i})-\mathrm{Li}_2(1+a\,x\,1\mathrm{i}))\,1\mathrm{i}}{2} & \text{if } a\neq 0 \end{cases}$$

input
```
int((atan(a*x)*(c + a^2*c*x^2))/x,x)
```

output
```
piecewise(a == 0, 0, a ~= 0, - (c*(dilog(- a*x*1i + 1) - dilog(a*x*1i + 1)
)*1i)/2 - (a*c*x)/2 + a^2*c*atan(a*x)*(1/(2*a^2) + x^2/2))
```

## Reduce [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)}{x}\,dx = \frac{c\left(atan(ax)\,a^2x^2+atan(ax)+2\left(\int \frac{atan(ax)}{x}dx\right)-ax\right)}{2}$$

input
```
int((a^2*c*x^2+c)*atan(a*x)/x,x)
```

output
```
(c*(atan(a*x)*a**2*x**2 + atan(a*x) + 2*int(atan(a*x)/x,x) - a*x))/2
```

# 3.154 $\int \frac{(c+a^2cx^2)\arctan(ax)}{x^2}\,dx$



## Optimal result

Integrand size = 18, antiderivative size = 40

$$\int \frac{(c+a^2cx^2)\arctan(ax)}{x^2}\,dx = -\frac{c\arctan(ax)}{x} + a^2cx\arctan(ax) + ac\log(x) - ac\log\left(1+a^2x^2\right)$$

output
```
-c*arctan(a*x)/x+a^2*c*x*arctan(a*x)+a*c*ln(x)-a*c*ln(a^2*x^2+1)
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)\arctan(ax)}{x^2}\,dx = -\frac{c\arctan(ax)}{x} + a^2cx\arctan(ax) + ac\log(x) - ac\log\left(1+a^2x^2\right)$$

input
```
Integrate[((c + a^2*c*x^2)*ArcTan[a*x])/x^2,x]
```

output
```
-((c*ArcTan[a*x])/x) + a^2*c*x*ArcTan[a*x] + a*c*Log[x] - a*c*Log[1 + a^2*
x^2]
```

## Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.60, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {5485, 5345, 240, 5361, 243, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)\left(a^2cx^2+c\right)}{x^2}dx$$

$$\Big\downarrow \ 5485$$

$$a^2c\int \arctan(ax)dx+c\int \frac{\arctan(ax)}{x^2}dx$$

$$\Big\downarrow \ 5345$$

$$a^2c\left(x\arctan(ax)-a\int \frac{x}{a^2x^2+1}dx\right)+c\int \frac{\arctan(ax)}{x^2}dx$$

$$\Big\downarrow \ 240$$

$$c\int \frac{\arctan(ax)}{x^2}dx+a^2c\left(x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}\right)$$

$$\Big\downarrow \ 5361$$

$$c\left(a\int \frac{1}{x\left(a^2x^2+1\right)}dx-\frac{\arctan(ax)}{x}\right)+a^2c\left(x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}\right)$$

$$\Big\downarrow \ 243$$

$$c\left(\frac{1}{2}a\int \frac{1}{x^2\left(a^2x^2+1\right)}dx^2-\frac{\arctan(ax)}{x}\right)+a^2c\left(x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}\right)$$

$$\Big\downarrow \ 47$$

$$c\left(\frac{1}{2}a\left(\int \frac{1}{x^2}dx^2-a^2\int \frac{1}{a^2x^2+1}dx^2\right)-\frac{\arctan(ax)}{x}\right)+a^2c\left(x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}\right)$$

$$\Big\downarrow \ 14$$

$$c\left(\frac{1}{2}a\left(\log\left(x^2\right)-a^2\int \frac{1}{a^2x^2+1}dx^2\right)-\frac{\arctan(ax)}{x}\right)+a^2c\left(x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}\right)$$

$$\Big\downarrow \textcolor{blue}{16}$$

$$a^2c\left(x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}\right)+c\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{\arctan(ax)}{x}\right)$$

input
```
Int[((c + a^2*c*x^2)*ArcTan[a*x])/x^2,x]
```

output
```
c*(-(ArcTan[a*x]/x) + (a*(Log[x^2] - Log[1 + a^2*x^2]))/2) + a^2*c*(x*ArcT
an[a*x] - Log[1 + a^2*x^2]/(2*a))
```

## Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

## Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

| method | result | size |
|---|---|---|
| parts | $a^{2} c x \arctan \left(a x \right)-\frac{c \arctan \left(a x \right)}{x}-a c \left(\ln \left(a^{2} x^{2}+1\right)-\ln \left(x \right)\right)$ | 41 |
| derivativedivides | $a \left(a c x \arctan \left(a x \right)-\frac{c \arctan \left(a x \right)}{a x}-c \left(-\ln \left(a x \right)+\ln \left(a^{2} x^{2}+1\right)\right)\right)$ | 45 |
| default | $a \left(a c x \arctan \left(a x \right)-\frac{c \arctan \left(a x \right)}{a x}-c \left(-\ln \left(a x \right)+\ln \left(a^{2} x^{2}+1\right)\right)\right)$ | 45 |
| parallelrisch | $\frac{a^{2} c \,x^{2} \arctan \left(a x \right)+a c \ln \left(x \right) x -a c \ln \left(a^{2} x^{2}+1\right) x -c \arctan \left(a x \right)}{x}$ | 46 |
| risch | $-\frac{i c \left(a^{2} x^{2}-1\right) \ln \left(i a x +1\right)}{2 x}+\frac{i c \left(a^{2} x^{2} \ln \left(-i a x +1\right)-2 i a \ln \left(x \right) x +2 i a \ln \left(-2 a^{2} x^{2}-2\right) x -\ln \left(-i a x +1\right)\right)}{2 x}$ | 82 |
| meijerg | $\frac{a c \left(\frac{4 a^{2} x^{2} \arctan \left(\sqrt{a^{2} x^{2}}\right)}{\sqrt{a^{2} x^{2}}}-2 \ln \left(a^{2} x^{2}+1\right)\right)}{4}+\frac{a c \left(4 \ln \left(x \right)+4 \ln \left(a \right)-\frac{4 \arctan \left(\sqrt{a^{2} x^{2}}\right)}{\sqrt{a^{2} x^{2}}}-2 \ln \left(a^{2} x^{2}+1\right)\right)}{4}$ | 92 |

input
```
int((a^2*c*x^2+c)*arctan(a*x)/x^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*c*x*arctan(a*x)-c*arctan(a*x)/x-a*c*(ln(a^2*x^2+1)-ln(x))
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.12

$$\int \frac{(c+a^2cx^2)\arctan(ax)}{x^2}\,dx$$
$$= -\frac{acx\log(a^2x^2+1) - acx\log(x) - (a^2cx^2-c)\arctan(ax)}{x}$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)/x^2,x, algorithm="fricas")
```

output
```
-(a*c*x*log(a^2*x^2 + 1) - a*c*x*log(x) - (a^2*c*x^2 - c)*arctan(a*x))/x
```

## Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

$$\int \frac{(c+a^2cx^2)\arctan(ax)}{x^2}\,dx$$
$$= \begin{cases} a^2cx\,\mathrm{atan}\,(ax) + ac\log(x) - ac\log\left(x^2+\frac{1}{a^2}\right) - \frac{c\,\mathrm{atan}\,(ax)}{x} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate((a**2*c*x**2+c)*atan(a*x)/x**2,x)
```

output
```
Piecewise((a**2*c*x*atan(a*x) + a*c*log(x) - a*c*log(x**2 + a**(-2)) - c*a
tan(a*x)/x, Ne(a, 0)), (0, True))
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)\arctan(ax)}{x^2}\,dx$$
$$= -\left(c\log\left(a^2x^2+1\right)-c\log\left(x\right)\right)a+\left(a^2cx-\frac{c}{x}\right)\arctan\left(ax\right)$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)/x^2,x, algorithm="maxima")
```

output
```
-(c*log(a^2*x^2 + 1) - c*log(x))*a + (a^2*c*x - c/x)*arctan(a*x)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

$$\int \frac{(c+a^2cx^2)\arctan(ax)}{x^2}\,dx = -ac\log\left(a^2x^2+1\right)+\frac{1}{2}\,ac\log\left(x^2\right)$$
$$+\left(a^2cx-\frac{c}{x}\right)\arctan\left(ax\right)$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)/x^2,x, algorithm="giac")
```

output
```
-a*c*log(a^2*x^2 + 1) + 1/2*a*c*log(x^2) + (a^2*c*x - c/x)*arctan(a*x)
```

## Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{(c+a^2cx^2)\arctan(ax)}{x^2}\,dx = a^2\,c\,x\,\mathrm{atan}(a\,x)-\frac{c\,\mathrm{atan}(a\,x)}{x}$$
$$-c\left(a\,\ln\left(a^2\,x^2+1\right)-a\,\ln\left(x\right)\right)$$

input
```
int((atan(a*x)*(c + a^2*c*x^2))/x^2,x)
```

output
```
a^2*c*x*atan(a*x) - (c*atan(a*x))/x - c*(a*log(a^2*x^2 + 1) - a*log(x))
```

## Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05

$$\int \frac{(c + a^2cx^2)\arctan(ax)}{x^2}\,dx = \frac{c(atan(ax)\,a^2x^2 - atan(ax) - \log(a^2x^2+1)\,ax + \log(x)\,ax)}{x}$$

input
```
int((a^2*c*x^2+c)*atan(a*x)/x^2,x)
```

output
```
(c*(atan(a*x)*a**2*x**2 - atan(a*x) - log(a**2*x**2 + 1)*a*x + log(x)*a*x)
)/x
```

# 3.155 $\int \frac{(c+a^2cx^2)\arctan(ax)}{x^3}\,dx$



## Optimal result

Integrand size = 18, antiderivative size = 70

$$\int \frac{(c+a^2cx^2)\arctan(ax)}{x^3}\,dx = -\frac{ac}{2x} - \frac{1}{2}a^2c\arctan(ax) - \frac{c\arctan(ax)}{2x^2} + \frac{1}{2}ia^2c\,\mathrm{PolyLog}(2,-iax) - \frac{1}{2}ia^2c\,\mathrm{PolyLog}(2,iax)$$

output
```
-1/2*a*c/x-1/2*a^2*c*arctan(a*x)-1/2*c*arctan(a*x)/x^2+1/2*I*a^2*c*polylog
(2,-I*a*x)-1/2*I*a^2*c*polylog(2,I*a*x)
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.01 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.06

$$\int \frac{(c+a^2cx^2)\arctan(ax)}{x^3}\,dx = -\frac{c\arctan(ax)}{2x^2} - \frac{ac\,\mathrm{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-a^2x^2\right)}{2x} + \frac{1}{2}ia^2c\,\mathrm{PolyLog}(2,-iax) - \frac{1}{2}ia^2c\,\mathrm{PolyLog}(2,iax)$$

input
```
Integrate[((c + a^2*c*x^2)*ArcTan[a*x])/x^3,x]
```

output
```
-1/2*(c*ArcTan[a*x])/x^2 - (a*c*Hypergeometric2F1[-1/2, 1, 1/2, -(a^2*x^2)
])/(2*x) + (I/2)*a^2*c*PolyLog[2, (-I)*a*x] - (I/2)*a^2*c*PolyLog[2, I*a*x
]
```

## Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5485, 5355, 2838, 5361, 264, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)\left(a^2cx^2+c\right)}{x^3}dx$$

$$\Big\downarrow \text{ 5485}$$

$$a^2c\int \frac{\arctan(ax)}{x}dx+c\int \frac{\arctan(ax)}{x^3}dx$$

$$\Big\downarrow \text{ 5355}$$

$$c\int \frac{\arctan(ax)}{x^3}dx+a^2c\left(\frac{1}{2}i\int \frac{\log(1-iax)}{x}dx-\frac{1}{2}i\int \frac{\log(iax+1)}{x}dx\right)$$

$$\Big\downarrow \text{ 2838}$$

$$c\int \frac{\arctan(ax)}{x^3}dx+a^2c\left(\frac{1}{2}i\operatorname{PolyLog}(2,-iax)-\frac{1}{2}i\operatorname{PolyLog}(2,iax)\right)$$

$$\Big\downarrow \text{ 5361}$$

$$c\left(\frac{1}{2}a\int \frac{1}{x^2\left(a^2x^2+1\right)}dx-\frac{\arctan(ax)}{2x^2}\right)+a^2c\left(\frac{1}{2}i\operatorname{PolyLog}(2,-iax)-\frac{1}{2}i\operatorname{PolyLog}(2,iax)\right)$$

$$\Big\downarrow \text{ 264}$$

$$c\left(\frac{1}{2}a\left(a^2\left(-\int \frac{1}{a^2x^2+1}dx\right)-\frac{1}{x}\right)-\frac{\arctan(ax)}{2x^2}\right)+a^2c\left(\frac{1}{2}i\operatorname{PolyLog}(2,-iax)-\frac{1}{2}i\operatorname{PolyLog}(2,iax)\right)$$

$$\downarrow \text{216}$$

$$c\left(\frac{1}{2}a\left(-a\arctan(ax)-\frac{1}{x}\right)-\frac{\arctan(ax)}{2x^2}\right)+ a^2c\left(\frac{1}{2}i\operatorname{PolyLog}(2,-iax)-\frac{1}{2}i\operatorname{PolyLog}(2,iax)\right)$$

input
```
Int[((c + a^2*c*x^2)*ArcTan[a*x])/x^3,x]
```

output
```
c*(-1/2*ArcTan[a*x]/x^2 + (a*(-x^(-1) - a*ArcTan[a*x]))/2) + a^2*c*((I/2)*
PolyLog[2, (-I)*a*x] - (I/2)*PolyLog[2, I*a*x])
```

### Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 5355
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Simp[I*(b/2)   Int[Log[1 - I*c*x]/x, x], x] - Simp[I*(b/2)   Int[Log[1
+ I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

## Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.36

| method | result |
|---|---|
| parts | $-\frac{c\arctan(ax)}{2x^2}+c\arctan(ax)\,a^2\ln(x)-\frac{ac\left(a\arctan(ax)+\frac{1}{x}+2a^2\left(-\frac{i\ln(x)(-\ln(-iax+1)+\ln(iax+1))}{2a}-\frac{i(\mathrm{d}\cdots\right.\right.}{2}$ |
| derivativedivides | $a^2\left(c\arctan(ax)\ln(ax)-\frac{c\arctan(ax)}{2a^2x^2}-\frac{c\left(\frac{1}{ax}+\arctan(ax)-i\ln(ax)\ln(iax+1)+i\ln(ax)\ln(-iax+1)-i\,\mathrm{d}\cdots\right.}{2}\right.$ |
| default | $a^2\left(c\arctan(ax)\ln(ax)-\frac{c\arctan(ax)}{2a^2x^2}-\frac{c\left(\frac{1}{ax}+\arctan(ax)-i\ln(ax)\ln(iax+1)+i\ln(ax)\ln(-iax+1)-i\,\mathrm{d}\cdots\right.}{2}\right.$ |
| meijerg | $\frac{c\,a^2\left(-\frac{2iax\,\mathrm{polylog}\left(2,i\sqrt{a^2x^2}\right)}{\sqrt{a^2x^2}}+\frac{2iax\,\mathrm{polylog}\left(2,-i\sqrt{a^2x^2}\right)}{\sqrt{a^2x^2}}\right)}{4}+\frac{c\,a^2\left(-\frac{2}{ax}-\frac{2\left(a^2x^2+1\right)\arctan(ax)}{a^2x^2}\right)}{4}$ |
| risch | $-\frac{ca}{2x}+\frac{ic\,a^2\ln(-iax)}{4}-\frac{ic\,a^2\ln(-iax+1)}{4}-\frac{ic\ln(-iax+1)}{4x^2}-\frac{ic\,a^2\,\mathrm{dilog}(-iax+1)}{2}-\frac{ic\,a^2\ln(iax)}{4}+\frac{ic\,a^2\ln\cdots}{}$ |

input
```
int((a^2*c*x^2+c)*arctan(a*x)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*c*arctan(a*x)/x^2+c*arctan(a*x)*a^2*ln(x)-1/2*a*c*(a*arctan(a*x)+1/x+
2*a^2*(-1/2*I*ln(x)*(-ln(1-I*a*x)+ln(1+I*a*x))/a-1/2*I*(dilog(1+I*a*x)-dil
og(1-I*a*x))/a))
```

## Fricas [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)}{x^3}\,dx=\int \frac{(a^2cx^2+c)\arctan{(ax)}}{x^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)/x^3,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)*arctan(a*x)/x^3, x)
```

## Sympy [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)}{x^3}\,dx=c\left(\int \frac{\operatorname{atan}{(ax)}}{x^3}\,dx+\int \frac{a^2\operatorname{atan}{(ax)}}{x}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)*atan(a*x)/x**3,x)
```

output
```
c*(Integral(atan(a*x)/x**3, x) + Integral(a**2*atan(a*x)/x, x))
```

## Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.36

$$\int \frac{(c+a^2cx^2)\arctan(ax)}{x^3}\,dx=-\frac{\pi a^2cx^2\log{(a^2x^2+1)}-4\,a^2cx^2\arctan{(ax)}\log{(ax)}+2i\,a^2cx^2\mathrm{Li}_2(i\,ax+1)-2i\,a^2cx^2\mathrm{Li}_2(-i\,ax+1)}{4\,x^2}$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)/x^3,x, algorithm="maxima")
```

output
```
-1/4*(pi*a^2*c*x^2*log(a^2*x^2 + 1) - 4*a^2*c*x^2*arctan(a*x)*log(a*x) + 2
*I*a^2*c*x^2*dilog(I*a*x + 1) - 2*I*a^2*c*x^2*dilog(-I*a*x + 1) + 2*a*c*x
+ 2*(a^2*c*x^2 + c)*arctan(a*x))/x^2
```

## Giac [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)}{x^3}\,dx = \int \frac{(a^2cx^2+c)\arctan(ax)}{x^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)/x^3,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)*arctan(a*x)/x^3, x)
```

## Mupad [B] (verification not implemented)

Time = 0.85 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.01

$$\int \frac{(c+a^2cx^2)\arctan(ax)}{x^3}\,dx = \begin{cases} 0 & \text{if } a=0 \\ -\frac{c\,\mathrm{atan}(a\,x)}{2\,x^2}-\frac{c\left(a^3\,\mathrm{atan}(a\,x)+\frac{a^2}{x}\right)}{2\,a}-\frac{a^2\,c\,\mathrm{Li_2}(1-a\,x\,1\mathrm{i})\,1\mathrm{i}}{2}+\frac{a^2\,c\,\mathrm{Li_2}(1+a\,x\,1\mathrm{i})\,1\mathrm{i}}{2} & \text{if } a\neq 0 \end{cases}$$

input
```
int((atan(a*x)*(c + a^2*c*x^2))/x^3,x)
```

output
```
piecewise(a == 0, 0, a ~= 0, - (c*atan(a*x))/(2*x^2) - (a^2*c*dilog(- a*x*
1i + 1)*1i)/2 + (a^2*c*dilog(a*x*1i + 1)*1i)/2 - (c*(a^3*atan(a*x) + a^2/x
))/(2*a))
```

## Reduce [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)}{x^3}\,dx = \frac{c\left(-atan(ax)\,a^2x^2-atan(ax)+2\left(\int \frac{atan(ax)}{x}dx\right)a^2x^2-ax\right)}{2x^2}$$

input
```
int((a^2*c*x^2+c)*atan(a*x)/x^3,x)
```

output
```
(c*( - atan(a*x)*a**2*x**2 - atan(a*x) + 2*int(atan(a*x)/x,x)*a**2*x**2 -
a*x))/(2*x**2)
```

# 3.156 $\int \frac{(c+a^2cx^2)\arctan(ax)}{x^4}\,dx$



## Optimal result

Integrand size = 18, antiderivative size = 63

$$\int \frac{(c+a^2cx^2)\arctan(ax)}{x^4}\,dx = -\frac{ac}{6x^2} - \frac{c\arctan(ax)}{3x^3} - \frac{a^2c\arctan(ax)}{x} + \frac{2}{3}a^3c\log(x) - \frac{1}{3}a^3c\log\left(1+a^2x^2\right)$$

output
```
-1/6*a*c/x^2-1/3*c*arctan(a*x)/x^3-a^2*c*arctan(a*x)/x+2/3*a^3*c*ln(x)-1/3
*a^3*c*ln(a^2*x^2+1)
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

$$\int \frac{(c+a^2cx^2)\arctan(ax)}{x^4}\,dx = \frac{c(-2(1+3a^2x^2)\arctan(ax)+ax(-1+4a^2x^2\log(x)-2a^2x^2\log\left(1+a^2x^2\right)))}{6x^3}$$

input
```
Integrate[((c + a^2*c*x^2)*ArcTan[a*x])/x^4,x]
```

output
```
(c*(-2*(1 + 3*a^2*x^2)*ArcTan[a*x] + a*x*(-1 + 4*a^2*x^2*Log[x] - 2*a^2*x^
2*Log[1 + a^2*x^2])))/(6*x^3)
```

## Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.37, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {5485, 5361, 243, 47, 14, 16, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan (a x) \left(a^2 c x^2+c\right)}{x^4} \, dx$$

$$\Big\downarrow \text{5485}$$

$$a^2 c \int \frac{\arctan (a x)}{x^2}dx+c \int \frac{\arctan (a x)}{x^4}dx$$

$$\Big\downarrow \text{5361}$$

$$a^2 c \left(a \int \frac{1}{x \left(a^2 x^2+1\right)}dx-\frac{\arctan (a x)}{x}\right)+c \left(\frac{1}{3} a \int \frac{1}{x^3 \left(a^2 x^2+1\right)}dx-\frac{\arctan (a x)}{3 x^3}\right)$$

$$\Big\downarrow \text{243}$$

$$a^2 c \left(\frac{1}{2} a \int \frac{1}{x^2 \left(a^2 x^2+1\right)}dx^2-\frac{\arctan (a x)}{x}\right)+c \left(\frac{1}{6} a \int \frac{1}{x^4 \left(a^2 x^2+1\right)}dx^2-\frac{\arctan (a x)}{3 x^3}\right)$$

$$\Big\downarrow \text{47}$$

$$a^2 c \left(\frac{1}{2} a \left(\int \frac{1}{x^2}dx^2-a^2 \int \frac{1}{a^2 x^2+1}dx^2\right)-\frac{\arctan (a x)}{x}\right)+c \left(\frac{1}{6} a \int \frac{1}{x^4 \left(a^2 x^2+1\right)}dx^2-\frac{\arctan (a x)}{3 x^3}\right)$$

$$\Big\downarrow \text{14}$$

$$a^2 c \left(\frac{1}{2} a \left(\log \left(x^2\right)-a^2 \int \frac{1}{a^2 x^2+1}dx^2\right)-\frac{\arctan (a x)}{x}\right)+c \left(\frac{1}{6} a \int \frac{1}{x^4 \left(a^2 x^2+1\right)}dx^2-\frac{\arctan (a x)}{3 x^3}\right)$$

$$\Big\downarrow \text{16}$$

$$c\left(\frac{1}{6}a\int \frac{1}{x^4\left(a^2x^2+1\right)}dx^2-\frac{\arctan(ax)}{3x^3}\right)+$$
$$a^2c\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{\arctan(ax)}{x}\right)$$

$$\Big\downarrow \; 54$$

$$c\left(\frac{1}{6}a\int \left(\frac{a^4}{a^2x^2+1}-\frac{a^2}{x^2}+\frac{1}{x^4}\right)dx^2-\frac{\arctan(ax)}{3x^3}\right)+$$
$$a^2c\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{\arctan(ax)}{x}\right)$$

$$\Big\downarrow \; 2009$$

$$a^2c\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{\arctan(ax)}{x}\right)+$$
$$c\left(\frac{1}{6}a\left(a^2\left(-\log\left(x^2\right)\right)+a^2\log\left(a^2x^2+1\right)-\frac{1}{x^2}\right)-\frac{\arctan(ax)}{3x^3}\right)$$

input
```
Int[((c + a^2*c*x^2)*ArcTan[a*x])/x^4,x]
```

output
```
a^2*c*(-(ArcTan[a*x]/x) + (a*(Log[x^2] - Log[1 + a^2*x^2]))/2) + c*(-1/3*A
rcTan[a*x]/x^3 + (a*(-x^(-2) - a^2*Log[x^2] + a^2*Log[1 + a^2*x^2]))/6)
```

### Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

## Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

| method | result |
|---|---|
| parts | $-\frac{c\arctan(ax)}{3x^3}-\frac{a^2c\arctan(ax)}{x}-\frac{ac\left(a^2\ln(a^2x^2+1)+\frac{1}{2x^2}-2a^2\ln(x)\right)}{3}$ |
| derivativedivides | $a^3\left(-\frac{c\arctan(ax)}{ax}-\frac{c\arctan(ax)}{3a^3x^3}-\frac{c\left(\frac{1}{2a^2x^2}-2\ln(ax)+\ln(a^2x^2+1)\right)}{3}\right)$ |
| default | $a^3\left(-\frac{c\arctan(ax)}{ax}-\frac{c\arctan(ax)}{3a^3x^3}-\frac{c\left(\frac{1}{2a^2x^2}-2\ln(ax)+\ln(a^2x^2+1)\right)}{3}\right)$ |
| parallelrisch | $\frac{4c\,a^3\ln(x)x^3-2c\,a^3\ln(a^2x^2+1)x^3+a^3c\,x^3-6a^2c\,x^2\arctan(ax)-acx-2c\arctan(ax)}{6x^3}$ |
| risch | $\frac{ic(3a^2x^2+1)\ln(iax+1)}{6x^3}+\frac{c(4\ln(x)a^3x^3-2\ln(-3a^2x^2-3)a^3x^3-3ia^2x^2\ln(-iax+1)-ax-i\ln(-iax+1))}{6x^3}$ |
| meijerg | $\frac{a^3c\left(4\ln(x)+4\ln(a)-\frac{4\arctan\left(\sqrt{a^2x^2}\right)}{\sqrt{a^2x^2}}-2\ln(a^2x^2+1)\right)}{4}+\frac{a^3c\left(-\frac{2}{a^2x^2}+\frac{4}{9}-\frac{4\ln(x)}{3}-\frac{4\ln(a)}{3}+\frac{-\frac{4a^2x^2}{9}+\frac{4}{3}}{a^2x^2}-\frac{4\arctan(}{3a^2x^2}\right.}{4}$ |

input
```
int((a^2*c*x^2+c)*arctan(a*x)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*c*arctan(a*x)/x^3-a^2*c*arctan(a*x)/x-1/3*a*c*(a^2*ln(a^2*x^2+1)+1/2/
x^2-2*a^2*ln(x))
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

$$\int \frac{(c+a^2cx^2)\arctan(ax)}{x^4}\,dx$$
$$= -\frac{2\,a^3cx^3\log\left(a^2x^2+1\right)-4\,a^3cx^3\log\left(x\right)+acx+2\left(3\,a^2cx^2+c\right)\arctan\left(ax\right)}{6\,x^3}$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)/x^4,x, algorithm="fricas")
```

output
```
-1/6*(2*a^3*c*x^3*log(a^2*x^2 + 1) - 4*a^3*c*x^3*log(x) + a*c*x + 2*(3*a^2
*c*x^2 + c)*arctan(a*x))/x^3
```

## Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.97

$$\int \frac{(c + a^2cx^2)\arctan(ax)}{x^4}\,dx = \begin{cases} \frac{2a^3c\log(x)}{3} - \frac{a^3c\log\left(x^2+\frac{1}{a^2}\right)}{3} - \frac{a^2c\,\mathrm{atan}\,(ax)}{x} - \frac{ac}{6x^2} - \frac{c\,\mathrm{atan}\,(ax)}{3x^3} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate((a**2*c*x**2+c)*atan(a*x)/x**4,x)
```

output
```
Piecewise((2*a**3*c*log(x)/3 - a**3*c*log(x**2 + a**(-2))/3 - a**2*c*atan(
a*x)/x - a*c/(6*x**2) - c*atan(a*x)/(3*x**3), Ne(a, 0)), (0, True))
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int \frac{(c + a^2cx^2)\arctan(ax)}{x^4}\,dx = -\frac{1}{6}\left(2\,a^2c\log\left(a^2x^2+1\right) - 2\,a^2c\log\left(x^2\right) + \frac{c}{x^2}\right)a - \frac{(3\,a^2cx^2 + c)\arctan(ax)}{3\,x^3}$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)/x^4,x, algorithm="maxima")
```

output
```
-1/6*(2*a^2*c*log(a^2*x^2 + 1) - 2*a^2*c*log(x^2) + c/x^2)*a - 1/3*(3*a^2*
c*x^2 + c)*arctan(a*x)/x^3
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.03

$$\int \frac{(c+a^2cx^2)\arctan(ax)}{x^4}\,dx = -\frac{1}{3}\,a^3c\log\left(a^2x^2+1\right)+\frac{1}{3}\,a^3c\log\left(x^2\right) - \frac{2\,a^3cx^2+ac}{6\,x^2}-\frac{(3\,a^2cx^2+c)\arctan\left(ax\right)}{3\,x^3}$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)/x^4,x, algorithm="giac")
```

output
```
-1/3*a^3*c*log(a^2*x^2 + 1) + 1/3*a^3*c*log(x^2) - 1/6*(2*a^3*c*x^2 + a*c)
/x^2 - 1/3*(3*a^2*c*x^2 + c)*arctan(a*x)/x^3
```

## Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

$$\int \frac{(c+a^2cx^2)\arctan(ax)}{x^4}\,dx = \frac{c\left(4\,a^3\,\ln\left(x\right)-2\,a^3\,\ln\left(a^2\,x^2+1\right)\right)}{6} - \frac{\frac{c\,\mathrm{atan}(a\,x)}{3}+\frac{a\,c\,x}{6}+a^2\,c\,x^2\,\mathrm{atan}(a\,x)}{x^3}$$

input
```
int((atan(a*x)*(c + a^2*c*x^2))/x^4,x)
```

output
```
(c*(4*a^3*log(x) - 2*a^3*log(a^2*x^2 + 1)))/6 - ((c*atan(a*x))/3 + (a*c*x)
/6 + a^2*c*x^2*atan(a*x))/x^3
```

## Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

$$\int \frac{(c + a^2cx^2) \arctan(ax)}{x^4} \, dx = \frac{c(-6atan(ax)\, a^2x^2 - 2atan(ax) - 2\log(a^2x^2 + 1)\, a^3x^3 + 4\log(x)\, a^3x^3 - ax)}{6x^3}$$

input
```
int((a^2*c*x^2+c)*atan(a*x)/x^4,x)
```

output
```
(c*( - 6*atan(a*x)*a**2*x**2 - 2*atan(a*x) - 2*log(a**2*x**2 + 1)*a**3*x**
3 + 4*log(x)*a**3*x**3 - a*x))/(6*x**3)
```

# 3.157 $\int x^3(c+a^2cx^2)^2\arctan(ax)\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 111

$$\int x^3(c+a^2cx^2)^2\arctan(ax)\,dx = \frac{c^2x}{24a^3}-\frac{c^2x^3}{72a}-\frac{1}{24}ac^2x^5-\frac{1}{56}a^3c^2x^7-\frac{c^2\arctan(ax)}{24a^4}+\frac{1}{4}c^2x^4\arctan(ax)+\frac{1}{3}a^2c^2x^6\arctan(ax)+\frac{1}{8}a^4c^2x^8\arctan(ax)$$

output
```
1/24*c^2*x/a^3-1/72*c^2*x^3/a-1/24*a*c^2*x^5-1/56*a^3*c^2*x^7-1/24*c^2*arc
tan(a*x)/a^4+1/4*c^2*x^4*arctan(a*x)+1/3*a^2*c^2*x^6*arctan(a*x)+1/8*a^4*c
^2*x^8*arctan(a*x)
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00

$$\int x^3(c+a^2cx^2)^2\arctan(ax)\,dx = \frac{c^2x}{24a^3}-\frac{c^2x^3}{72a}-\frac{1}{24}ac^2x^5-\frac{1}{56}a^3c^2x^7-\frac{c^2\arctan(ax)}{24a^4}+\frac{1}{4}c^2x^4\arctan(ax)+\frac{1}{3}a^2c^2x^6\arctan(ax)+\frac{1}{8}a^4c^2x^8\arctan(ax)$$

input
```
Integrate[x^3*(c + a^2*c*x^2)^2*ArcTan[a*x],x]
```

output
```
(c^2*x)/(24*a^3) - (c^2*x^3)/(72*a) - (a*c^2*x^5)/24 - (a^3*c^2*x^7)/56 -
(c^2*ArcTan[a*x])/(24*a^4) + (c^2*x^4*ArcTan[a*x])/4 + (a^2*c^2*x^6*ArcTan
[a*x])/3 + (a^4*c^2*x^8*ArcTan[a*x])/8
```

## Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \arctan(ax) \left(a^2cx^2+c\right)^2 dx$$

$$\Big\downarrow \text{5483}$$

$$\int \left(a^4c^2x^7 \arctan(ax)+2a^2c^2x^5 \arctan(ax)+c^2x^3 \arctan(ax)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{8}a^4c^2x^8 \arctan(ax)-\frac{c^2 \arctan(ax)}{24a^4}-\frac{1}{56}a^3c^2x^7+\frac{c^2x}{24a^3}+\frac{1}{3}a^2c^2x^6 \arctan(ax)+\frac{1}{4}c^2x^4 \arctan(ax)-\frac{1}{24}ac^2x^5-\frac{c^2x^3}{72a}$$

input
```
Int[x^3*(c + a^2*c*x^2)^2*ArcTan[a*x],x]
```

output
```
(c^2*x)/(24*a^3) - (c^2*x^3)/(72*a) - (a*c^2*x^5)/24 - (a^3*c^2*x^7)/56 -
(c^2*ArcTan[a*x])/(24*a^4) + (c^2*x^4*ArcTan[a*x])/4 + (a^2*c^2*x^6*ArcTan
[a*x])/3 + (a^4*c^2*x^8*ArcTan[a*x])/8
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

## Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.79

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{c^2\arctan(ax)a^8x^8}{8}+\frac{c^2\arctan(ax)a^6x^6}{3}+\frac{a^4c^2x^4\arctan(ax)}{4}-\frac{c^2\left(\frac{3a^7x^7}{7}+a^5x^5+\frac{a^3x^3}{3}-ax+\arctan(ax)\right)}{24}}{a^4}$ |
| default | $\frac{\frac{c^2\arctan(ax)a^8x^8}{8}+\frac{c^2\arctan(ax)a^6x^6}{3}+\frac{a^4c^2x^4\arctan(ax)}{4}-\frac{c^2\left(\frac{3a^7x^7}{7}+a^5x^5+\frac{a^3x^3}{3}-ax+\arctan(ax)\right)}{24}}{a^4}$ |
| parts | $\frac{a^4c^2x^8\arctan(ax)}{8}+\frac{a^2c^2x^6\arctan(ax)}{3}+\frac{c^2x^4\arctan(ax)}{4}-\frac{a\,c^2\left(\frac{\frac{3}{7}a^6x^7+a^4x^5+\frac{1}{3}a^2x^3-x}{a^4}+\frac{\arctan(ax)}{a^5}\right)}{24}$ |
| parallelrisch | $\frac{63c^2\arctan(ax)a^8x^8-9a^7c^2x^7+168c^2\arctan(ax)a^6x^6-21a^5c^2x^5+126a^4c^2x^4\arctan(ax)-7a^3c^2x^3+21ax\,c^2-21c^2\,a}{504a^4}$ |
| risch | $-\frac{ic^2x^4(3a^4x^4+8a^2x^2+6)\ln(iax+1)}{48}+\frac{ic^2a^4x^8\ln(-iax+1)}{16}-\frac{a^3c^2x^7}{56}+\frac{ic^2a^2x^6\ln(-iax+1)}{6}-\frac{a\,c^2x^5}{24}+ic$ |
| orering | $\frac{(63a^8x^8+171a^6x^6+119a^4x^4-63a^2x^2-42)(a^2c\,x^2+c)^2\arctan(ax)}{252a^4(a^2x^2+1)^2}-\frac{(9a^6x^6+21a^4x^4+7a^2x^2-21)\left(3x^2(a^2c\,x^2+c\right.}{}$ |
| meijerg | $\frac{c^2\left(\frac{xa\left(-45a^6x^6+63a^4x^4-105a^2x^2+315\right)}{630}-\frac{xa\left(-9a^8x^8+9\right)\arctan\left(\sqrt{a^2x^2}\right)}{18\sqrt{a^2x^2}}\right)}{4a^4}+\frac{c^2\left(-\frac{2ax\left(21a^4x^4-35a^2x^2+105\right)}{315}+\frac{2ax(7\right.}{}}{2a^4}$ |

input
```
int(x^3*(a^2*c*x^2+c)^2*arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/a^4*(1/8*c^2*arctan(a*x)*a^8*x^8+1/3*c^2*arctan(a*x)*a^6*x^6+1/4*a^4*c^2
*x^4*arctan(a*x)-1/24*c^2*(3/7*a^7*x^7+a^5*x^5+1/3*a^3*x^3-a*x+arctan(a*x)
))
```

## Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.82

$$\int x^3 \left(c+a^2 c x^2\right)^2 \arctan(ax)\,dx = -\frac{9\,a^7c^2x^7+21\,a^5c^2x^5+7\,a^3c^2x^3-21\,ac^2x-21\,(3\,a^8c^2x^8+8\,a^6c^2x^6+6\,a^4c^2x^4-c^2)\arctan{(ax)}}{504\,a^4}$$

input
```
integrate(x^3*(a^2*c*x^2+c)^2*arctan(a*x),x, algorithm="fricas")
```

output
```
-1/504*(9*a^7*c^2*x^7 + 21*a^5*c^2*x^5 + 7*a^3*c^2*x^3 - 21*a*c^2*x - 21*(
3*a^8*c^2*x^8 + 8*a^6*c^2*x^6 + 6*a^4*c^2*x^4 - c^2)*arctan(a*x))/a^4
```

## Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.94

$$\int x^3 \left(c+a^2 c x^2\right)^2 \arctan(ax)\,dx = \begin{cases} \frac{a^4c^2x^8\operatorname{atan}{(ax)}}{8}-\frac{a^3c^2x^7}{56}+\frac{a^2c^2x^6\operatorname{atan}{(ax)}}{3}-\frac{ac^2x^5}{24}+\frac{c^2x^4\operatorname{atan}{(ax)}}{4}-\frac{c^2x^3}{72a}+\frac{c^2x}{24a^3}-\frac{c^2\operatorname{atan}{(ax)}}{24a^4} & \text{for}\ a\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*(a**2*c*x**2+c)**2*atan(a*x),x)
```

output
```
Piecewise((a**4*c**2*x**8*atan(a*x)/8 - a**3*c**2*x**7/56 + a**2*c**2*x**6
*atan(a*x)/3 - a*c**2*x**5/24 + c**2*x**4*atan(a*x)/4 - c**2*x**3/(72*a) +
 c**2*x/(24*a**3) - c**2*atan(a*x)/(24*a**4), Ne(a, 0)), (0, True))
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.88

$$\int x^3(c+a^2cx^2)^2\arctan(ax)\,dx$$
$$= -\frac{1}{504}a\left(\frac{21\,c^2\arctan(ax)}{a^5}+\frac{9\,a^6c^2x^7+21\,a^4c^2x^5+7\,a^2c^2x^3-21\,c^2x}{a^4}\right)$$
$$+\frac{1}{24}\left(3\,a^4c^2x^8+8\,a^2c^2x^6+6\,c^2x^4\right)\arctan(ax)$$

input
```
integrate(x^3*(a^2*c*x^2+c)^2*arctan(a*x),x, algorithm="maxima")
```

output
```
-1/504*a*(21*c^2*arctan(a*x)/a^5 + (9*a^6*c^2*x^7 + 21*a^4*c^2*x^5 + 7*a^2
*c^2*x^3 - 21*c^2*x)/a^4) + 1/24*(3*a^4*c^2*x^8 + 8*a^2*c^2*x^6 + 6*c^2*x^
4)*arctan(a*x)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.88

$$\int x^3(c+a^2cx^2)^2\arctan(ax)\,dx = \frac{1}{24}\left(3\,a^4c^2x^8+8\,a^2c^2x^6+6\,c^2x^4\right)\arctan(ax)$$
$$-\frac{c^2\arctan(ax)}{24\,a^4}$$
$$-\frac{9\,a^{17}c^2x^7+21\,a^{15}c^2x^5+7\,a^{13}c^2x^3-21\,a^{11}c^2x}{504\,a^{14}}$$

input
```
integrate(x^3*(a^2*c*x^2+c)^2*arctan(a*x),x, algorithm="giac")
```

output
```
1/24*(3*a^4*c^2*x^8 + 8*a^2*c^2*x^6 + 6*c^2*x^4)*arctan(a*x) - 1/24*c^2*ar
ctan(a*x)/a^4 - 1/504*(9*a^17*c^2*x^7 + 21*a^15*c^2*x^5 + 7*a^13*c^2*x^3 -
 21*a^11*c^2*x)/a^14
```

## Mupad [B] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.80

$$\int x^3 (c + a^2 c x^2)^2 \arctan(ax)\, dx = \operatorname{atan}(a\,x)\left(\frac{a^4\,c^2\,x^8}{8} + \frac{a^2\,c^2\,x^6}{3} + \frac{c^2\,x^4}{4}\right) + \frac{c^2\,x}{24\,a^3} - \frac{a\,c^2\,x^5}{24} - \frac{c^2\operatorname{atan}(a\,x)}{24\,a^4} - \frac{c^2\,x^3}{72\,a} - \frac{a^3\,c^2\,x^7}{56}$$

input
```
int(x^3*atan(a*x)*(c + a^2*c*x^2)^2,x)
```

output
```
atan(a*x)*((c^2*x^4)/4 + (a^2*c^2*x^6)/3 + (a^4*c^2*x^8)/8) + (c^2*x)/(24*
a^3) - (a*c^2*x^5)/24 - (c^2*atan(a*x))/(24*a^4) - (c^2*x^3)/(72*a) - (a^3
*c^2*x^7)/56
```

## Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.71

$$\int x^3 (c + a^2 c x^2)^2 \arctan(ax)\, dx = \frac{c^2(63 atan(ax)\, a^8 x^8 + 168 atan(ax)\, a^6 x^6 + 126 atan(ax)\, a^4 x^4 - 21 atan(ax) - 9a^7x^7 - 21a^5x^5 - 7a^3x^3 +}{504a^4}$$

input
```
int(x^3*(a^2*c*x^2+c)^2*atan(a*x),x)
```

output
```
(c**2*(63*atan(a*x)*a**8*x**8 + 168*atan(a*x)*a**6*x**6 + 126*atan(a*x)*a*
*4*x**4 - 21*atan(a*x) - 9*a**7*x**7 - 21*a**5*x**5 - 7*a**3*x**3 + 21*a*x
))/(504*a**4)
```

# 3.158 $\int x^2(c+a^2cx^2)^2 \arctan(ax)\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 106

$$\int x^2(c+a^2cx^2)^2 \arctan(ax)\,dx = -\frac{4c^2x^2}{105a} - \frac{9}{140}ac^2x^4 - \frac{1}{42}a^3c^2x^6 + \frac{1}{3}c^2x^3\arctan(ax) + \frac{2}{5}a^2c^2x^5\arctan(ax) + \frac{1}{7}a^4c^2x^7\arctan(ax) + \frac{4c^2\log(1+a^2x^2)}{105a^3}$$

output
```
-4/105*c^2*x^2/a-9/140*a*c^2*x^4-1/42*a^3*c^2*x^6+1/3*c^2*x^3*arctan(a*x)+
2/5*a^2*c^2*x^5*arctan(a*x)+1/7*a^4*c^2*x^7*arctan(a*x)+4/105*c^2*ln(a^2*x
^2+1)/a^3
```

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00

$$\int x^2(c+a^2cx^2)^2 \arctan(ax)\,dx = -\frac{4c^2x^2}{105a} - \frac{9}{140}ac^2x^4 - \frac{1}{42}a^3c^2x^6 + \frac{1}{3}c^2x^3\arctan(ax) + \frac{2}{5}a^2c^2x^5\arctan(ax) + \frac{1}{7}a^4c^2x^7\arctan(ax) + \frac{4c^2\log(1+a^2x^2)}{105a^3}$$

input
```
Integrate[x^2*(c + a^2*c*x^2)^2*ArcTan[a*x],x]
```

output
```
(-4*c^2*x^2)/(105*a) - (9*a*c^2*x^4)/140 - (a^3*c^2*x^6)/42 + (c^2*x^3*Arc
Tan[a*x])/3 + (2*a^2*c^2*x^5*ArcTan[a*x])/5 + (a^4*c^2*x^7*ArcTan[a*x])/7
+ (4*c^2*Log[1 + a^2*x^2])/(105*a^3)
```

## Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arctan(ax) \left(a^2cx^2+c\right)^2 dx$$

$$\Big\downarrow \ 5483$$

$$\int \left(a^4c^2x^6\arctan(ax)+2a^2c^2x^4\arctan(ax)+c^2x^2\arctan(ax)\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{7}a^4c^2x^7\arctan(ax)-\frac{1}{42}a^3c^2x^6+\frac{2}{5}a^2c^2x^5\arctan(ax)+\frac{4c^2\log\left(a^2x^2+1\right)}{105a^3}+\frac{1}{3}c^2x^3\arctan(ax)-\frac{9}{140}ac^2x^4-\frac{4c^2x^2}{105a}$$

input
```
Int[x^2*(c + a^2*c*x^2)^2*ArcTan[a*x],x]
```

output

```
(-4*c^2*x^2)/(105*a) - (9*a*c^2*x^4)/140 - (a^3*c^2*x^6)/42 + (c^2*x^3*Arc
Tan[a*x])/3 + (2*a^2*c^2*x^5*ArcTan[a*x])/5 + (a^4*c^2*x^7*ArcTan[a*x])/7
+ (4*c^2*Log[1 + a^2*x^2])/(105*a^3)
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

## Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.88

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{c^2\arctan(ax)a^7x^7}{7}+\frac{2c^2\arctan(ax)a^5x^5}{5}+\frac{a^3c^2x^3\arctan(ax)}{3}-\frac{c^2\left(\frac{5a^6x^6}{2}+\frac{27a^4x^4}{4}+4a^2x^2-4\ln\left(a^2x^2+1\right)\right)}{105}}{a^3}$ |
| default | $\frac{\frac{c^2\arctan(ax)a^7x^7}{7}+\frac{2c^2\arctan(ax)a^5x^5}{5}+\frac{a^3c^2x^3\arctan(ax)}{3}-\frac{c^2\left(\frac{5a^6x^6}{2}+\frac{27a^4x^4}{4}+4a^2x^2-4\ln\left(a^2x^2+1\right)\right)}{105}}{a^3}$ |
| parts | $\frac{a^4c^2x^7\arctan(ax)}{7}+\frac{2a^2c^2x^5\arctan(ax)}{5}+\frac{c^2x^3\arctan(ax)}{3}-\frac{a\,c^2\left(\frac{5a^4x^6+\frac{27}{2}a^2x^4+8x^2}{2a^2}-\frac{4\ln\left(a^2x^2+1\right)}{a^4}\right)}{105}$ |
| parallelrisch | $\frac{60c^2\arctan(ax)a^7x^7-10a^6c^2x^6+168c^2\arctan(ax)a^5x^5-27a^4c^2x^4+140a^3c^2x^3\arctan(ax)-16x^2c^2a^2+16c^2\ln(a^2x^2\cdots}{420a^3}$ |
| risch | $-\frac{ic^2x^3\left(15a^4x^4+42a^2x^2+35\right)\ln(iax+1)}{210}+\frac{ic^2a^4x^7\ln(-iax+1)}{14}-\frac{a^3c^2x^6}{42}+\frac{ic^2a^2x^5\ln(-iax+1)}{5}-\frac{9a\,c^2x^4}{140}\cdots$ |
| meijerg | $\frac{c^2\left(-\frac{a^2x^2\left(4a^4x^4-6a^2x^2+12\right)}{42}+\frac{4a^8x^8\arctan\left(\sqrt{a^2x^2}\right)}{7\sqrt{a^2x^2}}+\frac{2\ln\left(a^2x^2+1\right)}{7}\right)}{4a^3}+\frac{c^2\left(\frac{a^2x^2\left(-3a^2x^2+6\right)}{15}+\frac{4a^6x^6\arctan\left(\sqrt{a^2x^2}\cdots\right.\right.}{5\sqrt{a^2x^2}}\cdots\right)}{2a^3}$ |

input
```
int(x^2*(a^2*c*x^2+c)^2*arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(1/7*c^2*arctan(a*x)*a^7*x^7+2/5*c^2*arctan(a*x)*a^5*x^5+1/3*a^3*c^2
*x^3*arctan(a*x)-1/105*c^2*(5/2*a^6*x^6+27/4*a^4*x^4+4*a^2*x^2-4*ln(a^2*x^
2+1)))
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.89

$$\int x^2 \left(c+a^2cx^2\right)^2 \arctan(ax)\,dx = -\frac{10\,a^6c^2x^6+27\,a^4c^2x^4+16\,a^2c^2x^2-16\,c^2\log\left(a^2x^2+1\right)-4\left(15\,a^7c^2x^7+42\,a^5c^2x^5+35\,a^3c^2x^3\right)\text{arcta}}{420\,a^3}$$

input
```
integrate(x^2*(a^2*c*x^2+c)^2*arctan(a*x),x, algorithm="fricas")
```

output
```
-1/420*(10*a^6*c^2*x^6 + 27*a^4*c^2*x^4 + 16*a^2*c^2*x^2 - 16*c^2*log(a^2*
x^2 + 1) - 4*(15*a^7*c^2*x^7 + 42*a^5*c^2*x^5 + 35*a^3*c^2*x^3)*arctan(a*x
))/a^3
```

## Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.99

$$\int x^2 \left(c+a^2cx^2\right)^2 \arctan(ax)\,dx = \begin{cases} \frac{a^4c^2x^7\operatorname{atan}(ax)}{7}-\frac{a^3c^2x^6}{42}+\frac{2a^2c^2x^5\operatorname{atan}(ax)}{5}-\frac{9ac^2x^4}{140}+\frac{c^2x^3\operatorname{atan}(ax)}{3}-\frac{4c^2x^2}{105a}+\frac{4c^2\log\left(x^2+\frac{1}{a^2}\right)}{105a^3} & \text{for } a\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*(a**2*c*x**2+c)**2*atan(a*x),x)
```

output
```
Piecewise((a**4*c**2*x**7*atan(a*x)/7 - a**3*c**2*x**6/42 + 2*a**2*c**2*x*
*5*atan(a*x)/5 - 9*a*c**2*x**4/140 + c**2*x**3*atan(a*x)/3 - 4*c**2*x**2/(
105*a) + 4*c**2*log(x**2 + a**(-2))/(105*a**3), Ne(a, 0)), (0, True))
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.90

$$\int x^2\left(c+a^2cx^2\right)^2\arctan(ax)\,dx$$
$$=-\frac{1}{420}\,a\left(\frac{10\,a^4c^2x^6+27\,a^2c^2x^4+16\,c^2x^2}{a^2}-\frac{16\,c^2\log\left(a^2x^2+1\right)}{a^4}\right)$$
$$+\frac{1}{105}\left(15\,a^4c^2x^7+42\,a^2c^2x^5+35\,c^2x^3\right)\arctan\left(ax\right)$$

input
```
integrate(x^2*(a^2*c*x^2+c)^2*arctan(a*x),x, algorithm="maxima")
```

output
```
-1/420*a*((10*a^4*c^2*x^6 + 27*a^2*c^2*x^4 + 16*c^2*x^2)/a^2 - 16*c^2*log(
a^2*x^2 + 1)/a^4) + 1/105*(15*a^4*c^2*x^7 + 42*a^2*c^2*x^5 + 35*c^2*x^3)*a
rctan(a*x)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.90

$$\int x^2\left(c+a^2cx^2\right)^2\arctan(ax)\,dx=\frac{1}{105}\left(15\,a^4c^2x^7+42\,a^2c^2x^5+35\,c^2x^3\right)\arctan\left(ax\right)$$
$$+\frac{4\,c^2\log\left(a^2x^2+1\right)}{105\,a^3}$$
$$-\frac{10\,a^9c^2x^6+27\,a^7c^2x^4+16\,a^5c^2x^2}{420\,a^6}$$

input
```
integrate(x^2*(a^2*c*x^2+c)^2*arctan(a*x),x, algorithm="giac")
```

output
```
1/105*(15*a^4*c^2*x^7 + 42*a^2*c^2*x^5 + 35*c^2*x^3)*arctan(a*x) + 4/105*c
^2*log(a^2*x^2 + 1)/a^3 - 1/420*(10*a^9*c^2*x^6 + 27*a^7*c^2*x^4 + 16*a^5*
c^2*x^2)/a^6
```

## Mupad [B] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.76

$$\int x^2(c+a^2cx^2)^2\arctan(ax)\,dx$$
$$=\frac{c^2\,(16\,\ln\left(a^2\,x^2+1\right)-16\,a^2\,x^2-27\,a^4\,x^4-10\,a^6\,x^6+140\,a^3\,x^3\,\mathrm{atan}(a\,x)+168\,a^5\,x^5\,\mathrm{atan}(a\,x)+60\,a^7\,x}{420\,a^3}$$

input
```
int(x^2*atan(a*x)*(c + a^2*c*x^2)^2,x)
```

output
```
(c^2*(16*log(a^2*x^2 + 1) - 16*a^2*x^2 - 27*a^4*x^4 - 10*a^6*x^6 + 140*a^3
*x^3*atan(a*x) + 168*a^5*x^5*atan(a*x) + 60*a^7*x^7*atan(a*x)))/(420*a^3)
```

## Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.76

$$\int x^2(c+a^2cx^2)^2\arctan(ax)\,dx$$
$$=\frac{c^2(60atan(ax)\,a^7x^7+168atan(ax)\,a^5x^5+140atan(ax)\,a^3x^3+16\log(a^2x^2+1)-10a^6x^6-27a^4x^4-16}{420a^3}$$

input
```
int(x^2*(a^2*c*x^2+c)^2*atan(a*x),x)
```

output
```
(c**2*(60*atan(a*x)*a**7*x**7 + 168*atan(a*x)*a**5*x**5 + 140*atan(a*x)*a*
*3*x**3 + 16*log(a**2*x**2 + 1) - 10*a**6*x**6 - 27*a**4*x**4 - 16*a**2*x*
*2))/(420*a**3)
```

# 3.159 $\int x(c+a^2cx^2)^2 \arctan(ax)\,dx$



## Optimal result

Integrand size = 18, antiderivative size = 61

$$\int x(c+a^2cx^2)^2 \arctan(ax)\,dx = -\frac{c^2x}{6a} - \frac{1}{9}ac^2x^3 - \frac{1}{30}a^3c^2x^5 + \frac{c^2(1+a^2x^2)^3\arctan(ax)}{6a^2}$$

output
```
-1/6*c^2*x/a-1/9*a*c^2*x^3-1/30*a^3*c^2*x^5+1/6*c^2*(a^2*x^2+1)^3*arctan(a
*x)/a^2
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.61

$$\int x(c+a^2cx^2)^2 \arctan(ax)\,dx = -\frac{c^2x}{6a} - \frac{1}{9}ac^2x^3 - \frac{1}{30}a^3c^2x^5 + \frac{c^2\arctan(ax)}{6a^2} + \frac{1}{2}c^2x^2\arctan(ax) + \frac{1}{2}a^2c^2x^4\arctan(ax) + \frac{1}{6}a^4c^2x^6\arctan(ax)$$

input
```
Integrate[x*(c + a^2*c*x^2)^2*ArcTan[a*x],x]
```

output
```
-1/6*(c^2*x)/a - (a*c^2*x^3)/9 - (a^3*c^2*x^5)/30 + (c^2*ArcTan[a*x])/(6*a
^2) + (c^2*x^2*ArcTan[a*x])/2 + (a^2*c^2*x^4*ArcTan[a*x])/2 + (a^4*c^2*x^6
*ArcTan[a*x])/6
```

## Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5465, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \arctan(ax) \left(a^2cx^2+c\right)^2 dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{c^2\left(a^2x^2+1\right)^3\arctan(ax)}{6a^2}-\frac{\int \left(a^2cx^2+c\right)^2dx}{6a}$$

$$\Big\downarrow \text{210}$$

$$\frac{c^2\left(a^2x^2+1\right)^3\arctan(ax)}{6a^2}-\frac{\int \left(a^4c^2x^4+2a^2c^2x^2+c^2\right)dx}{6a}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c^2\left(a^2x^2+1\right)^3\arctan(ax)}{6a^2}-\frac{\frac{1}{5}a^4c^2x^5+\frac{2}{3}a^2c^2x^3+c^2x}{6a}$$

input
```
Int[x*(c + a^2*c*x^2)^2*ArcTan[a*x],x]
```

output
```
-1/6*(c^2*x + (2*a^2*c^2*x^3)/3 + (a^4*c^2*x^5)/5)/a + (c^2*(1 + a^2*x^2)^
3*ArcTan[a*x])/(6*a^2)
```

### Defintions of rubi rules used

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

## Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.34

| method | result |
|---|---|
| parts | $\frac{c^2 \arctan(ax) a^4 x^6}{6}+\frac{c^2 \arctan(ax) a^2 x^4}{2}+\frac{c^2 \arctan(ax) x^2}{2}+\frac{c^2 \arctan(ax)}{6a^2}-\frac{c^2\left(\frac{1}{5}a^4x^5+\frac{2}{3}a^2x^3+x\right)}{6a}$ |
| derivativedivides | $\frac{\frac{c^2 \arctan(ax) a^6 x^6}{6}+\frac{a^4 c^2 x^4 \arctan(ax)}{2}+\frac{a^2 c^2 x^2 \arctan(ax)}{2}+\frac{c^2 \arctan(ax)}{6}-\frac{c^2\left(\frac{1}{5}a^5x^5+\frac{2}{3}a^3x^3+ax\right)}{6}}{a^2}$ |
| default | $\frac{\frac{c^2 \arctan(ax) a^6 x^6}{6}+\frac{a^4 c^2 x^4 \arctan(ax)}{2}+\frac{a^2 c^2 x^2 \arctan(ax)}{2}+\frac{c^2 \arctan(ax)}{6}-\frac{c^2\left(\frac{1}{5}a^5x^5+\frac{2}{3}a^3x^3+ax\right)}{6}}{a^2}$ |
| parallelrisch | $\frac{15c^2 \arctan(ax) a^6 x^6-3a^5c^2x^5+45a^4c^2x^4 \arctan(ax)-10a^3c^2x^3+45a^2c^2x^2 \arctan(ax)-15ax\,c^2+15c^2 \arctan(ax)}{90a^2}$ |
| risch | $-\frac{ic^2\left(a^2x^2+1\right)^3 \ln(iax+1)}{12a^2}+\frac{ic^2a^4x^6 \ln(-iax+1)}{12}-\frac{a^3c^2x^5}{30}+\frac{ic^2a^2x^4 \ln(-iax+1)}{4}-\frac{a\,c^2x^3}{9}+\frac{ic^2x^2 \ln(-ia}{4}$ |
| meijerg | $\frac{c^2\left(-\frac{2ax\left(21a^4x^4-35a^2x^2+105\right)}{315}+\frac{2ax\left(7a^6x^6+7\right)\arctan\left(\sqrt{a^2x^2}\right)}{21\sqrt{a^2x^2}}\right)}{4a^2}+\frac{c^2\left(\frac{ax\left(-5a^2x^2+15\right)}{15}-\frac{ax\left(-5a^4x^4+5\right)\arctan\left(\sqrt{a^2}\right.}{5\sqrt{a^2x^2}}\right.}{2a^2}$ |
| orering | $\frac{\left(15a^6x^6+49a^4x^4+65a^2x^2+15\right)\left(a^2c\,x^2+c\right)^2 \arctan(ax)}{45a^2(a^2x^2+1)^2}-\frac{\left(3a^4x^4+10a^2x^2+15\right)\left(\left(a^2c\,x^2+c\right)^2 \arctan(ax)+4x^2\left(a^2c\right.\right.}{90a^2(a^2x^2+1)}$ |

input
```
int(x*(a^2*c*x^2+c)^2*arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/6*c^2*arctan(a*x)*a^4*x^6+1/2*c^2*arctan(a*x)*a^2*x^4+1/2*c^2*arctan(a*x
)*x^2+1/6*c^2/a^2*arctan(a*x)-1/6*c^2/a*(1/5*a^4*x^5+2/3*a^2*x^3+x)
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.26

$$\int x(c+a^2cx^2)^2 \arctan(ax)\,dx = -\frac{3\,a^5c^2x^5+10\,a^3c^2x^3+15\,ac^2x-15\,(a^6c^2x^6+3\,a^4c^2x^4+3\,a^2c^2x^2+c^2)\arctan(ax)}{90\,a^2}$$

input
```
integrate(x*(a^2*c*x^2+c)^2*arctan(a*x),x, algorithm="fricas")
```

output
```
-1/90*(3*a^5*c^2*x^5 + 10*a^3*c^2*x^3 + 15*a*c^2*x - 15*(a^6*c^2*x^6 + 3*a
^4*c^2*x^4 + 3*a^2*c^2*x^2 + c^2)*arctan(a*x))/a^2
```

## Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.51

$$\int x(c+a^2cx^2)^2 \arctan(ax)\,dx = \begin{cases} \frac{a^4c^2x^6\operatorname{atan}(ax)}{6}-\frac{a^3c^2x^5}{30}+\frac{a^2c^2x^4\operatorname{atan}(ax)}{2}-\frac{ac^2x^3}{9}+\frac{c^2x^2\operatorname{atan}(ax)}{2}-\frac{c^2x}{6a}+\frac{c^2\operatorname{atan}(ax)}{6a^2} & \text{for } a\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x*(a**2*c*x**2+c)**2*atan(a*x),x)
```

output
```
Piecewise((a**4*c**2*x**6*atan(a*x)/6 - a**3*c**2*x**5/30 + a**2*c**2*x**4
*atan(a*x)/2 - a*c**2*x**3/9 + c**2*x**2*atan(a*x)/2 - c**2*x/(6*a) + c**2
*atan(a*x)/(6*a**2), Ne(a, 0)), (0, True))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.02

$$\int x(c+a^2cx^2)^2 \arctan(ax)\,dx = \frac{(a^2cx^2+c)^3 \arctan{(ax)}}{6\,a^2c} - \frac{3\,a^4c^3x^5 + 10\,a^2c^3x^3 + 15\,c^3x}{90\,ac}$$

input
```
integrate(x*(a^2*c*x^2+c)^2*arctan(a*x),x, algorithm="maxima")
```

output
```
1/6*(a^2*c*x^2 + c)^3*arctan(a*x)/(a^2*c) - 1/90*(3*a^4*c^3*x^5 + 10*a^2*c
^3*x^3 + 15*c^3*x)/(a*c)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.97

$$\int x(c+a^2cx^2)^2 \arctan(ax)\,dx = \frac{(a^2cx^2+c)^3 \arctan{(ax)}}{6\,a^2c} - \frac{3\,a^4c^2x^5 + 10\,a^2c^2x^3 + 15\,c^2x}{90\,a}$$

input
```
integrate(x*(a^2*c*x^2+c)^2*arctan(a*x),x, algorithm="giac")
```

output
```
1/6*(a^2*c*x^2 + c)^3*arctan(a*x)/(a^2*c) - 1/90*(3*a^4*c^2*x^5 + 10*a^2*c
^2*x^3 + 15*c^2*x)/a
```

## Mupad [B] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.16

$$\int x(c+a^2cx^2)^2 \arctan(ax)\,dx$$
$$= \frac{c^2\,(15\,\mathrm{atan}(a\,x) - 15\,a\,x - 10\,a^3\,x^3 - 3\,a^5\,x^5 + 45\,a^2\,x^2\,\mathrm{atan}(a\,x) + 45\,a^4\,x^4\,\mathrm{atan}(a\,x) + 15\,a^6\,x^6\,\mathrm{atan}(a\,x}{90\,a^2}$$

input
```
int(x*atan(a*x)*(c + a^2*c*x^2)^2,x)
```

output
```
(c^2*(15*atan(a*x) - 15*a*x - 10*a^3*x^3 - 3*a^5*x^5 + 45*a^2*x^2*atan(a*x
) + 45*a^4*x^4*atan(a*x) + 15*a^6*x^6*atan(a*x)))/(90*a^2)
```

## Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.16

$$\int x(c+a^2cx^2)^2\arctan(ax)\,dx$$
$$=\frac{c^2(15atan(ax)\,a^6x^6+45atan(ax)\,a^4x^4+45atan(ax)\,a^2x^2+15atan(ax)-3a^5x^5-10a^3x^3-15ax)}{90a^2}$$

input
```
int(x*(a^2*c*x^2+c)^2*atan(a*x),x)
```

output
```
(c**2*(15*atan(a*x)*a**6*x**6 + 45*atan(a*x)*a**4*x**4 + 45*atan(a*x)*a**2
*x**2 + 15*atan(a*x) - 3*a**5*x**5 - 10*a**3*x**3 - 15*a*x))/(90*a**2)
```

# 3.160 $\int (c+a^2cx^2)^2 \arctan(ax)\,dx$



## Optimal result

Integrand size = 17, antiderivative size = 117

$$\int (c+a^2cx^2)^2 \arctan(ax)\,dx = -\frac{2c^2(1+a^2x^2)}{15a} - \frac{c^2(1+a^2x^2)^2}{20a} + \frac{8}{15}c^2x\arctan(ax) + \frac{4}{15}c^2x(1+a^2x^2)\arctan(ax) + \frac{1}{5}c^2x(1+a^2x^2)^2\arctan(ax) - \frac{4c^2\log(1+a^2x^2)}{15a}$$

output
```
-2/15*c^2*(a^2*x^2+1)/a-1/20*c^2*(a^2*x^2+1)^2/a+8/15*c^2*x*arctan(a*x)+4/
15*c^2*x*(a^2*x^2+1)*arctan(a*x)+1/5*c^2*x*(a^2*x^2+1)^2*arctan(a*x)-4/15*
c^2*ln(a^2*x^2+1)/a
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.56

$$\int (c+a^2cx^2)^2 \arctan(ax)\,dx = \frac{c^2(-14a^2x^2-3a^4x^4+4ax(15+10a^2x^2+3a^4x^4)\arctan(ax)-16\log(1+a^2x^2))}{60a}$$

input
```
Integrate[(c + a^2*c*x^2)^2*ArcTan[a*x],x]
```

output
```
(c^2*(-14*a^2*x^2 - 3*a^4*x^4 + 4*a*x*(15 + 10*a^2*x^2 + 3*a^4*x^4)*ArcTan
[a*x] - 16*Log[1 + a^2*x^2]))/(60*a)
```

## Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {5413, 27, 5413, 5345, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(ax)\left(a^2cx^2+c\right)^2dx$$

$$\Big\downarrow \text{5413}$$

$$\frac{4}{5}c\int c\left(a^2x^2+1\right)\arctan(ax)dx+\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\arctan(ax)-\frac{c^2\left(a^2x^2+1\right)^2}{20a}$$

$$\Big\downarrow \text{27}$$

$$\frac{4}{5}c^2\int \left(a^2x^2+1\right)\arctan(ax)dx+\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\arctan(ax)-\frac{c^2\left(a^2x^2+1\right)^2}{20a}$$

$$\Big\downarrow \text{5413}$$

$$\frac{4}{5}c^2\left(\frac{2}{3}\int \arctan(ax)dx+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)-\frac{a^2x^2+1}{6a}\right)+\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\arctan(ax)-\frac{c^2\left(a^2x^2+1\right)^2}{20a}$$

$$\Big\downarrow \text{5345}$$

$$\frac{4}{5}c^2\left(\frac{2}{3}\left(x\arctan(ax)-a\int \frac{x}{a^2x^2+1}dx\right)+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)-\frac{a^2x^2+1}{6a}\right)+\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\arctan(ax)-\frac{c^2\left(a^2x^2+1\right)^2}{20a}$$

$$\Big\downarrow \text{240}$$

$$\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\arctan(ax)+$$
$$\frac{4}{5}c^2\left(\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)+\frac{2}{3}\left(x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}\right)-\frac{a^2x^2+1}{6a}\right)-$$
$$\frac{c^2\left(a^2x^2+1\right)^2}{20a}$$

input
```
Int[(c + a^2*c*x^2)^2*ArcTan[a*x],x]
```

output
```
-1/20*(c^2*(1 + a^2*x^2)^2)/a + (c^2*x*(1 + a^2*x^2)^2*ArcTan[a*x])/5 + (4
*c^2*(-1/6*(1 + a^2*x^2)/a + (x*(1 + a^2*x^2)*ArcTan[a*x])/3 + (2*(x*ArcTa
n[a*x] - Log[1 + a^2*x^2]/(2*a)))/3))/5
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5413
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Simp[x*(d + e*x^2)
^q*((a + b*ArcTan[c*x]))/(2*q + 1)), x] + Simp[2*d*(q/(2*q + 1))   Int[(d +
e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0]
```

## Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.65

| method | result |
|---|---|
| parts | $\frac{c^2 \arctan(ax) a^4 x^5}{5}+\frac{2c^2 \arctan(ax) a^2 x^3}{3}+c^2 x \arctan(ax)-\frac{a c^2 \left(\frac{3a^2 x^4}{4}+\frac{7x^2}{2}+\frac{4\ln(a^2x^2+1)}{a^2}\right)}{15}$ |
| derivativedivides | $\frac{\frac{c^2 \arctan(ax) a^5 x^5}{5}+\frac{2a^3c^2x^3\arctan(ax)}{3}+a c^2 x \arctan(ax)-\frac{c^2\left(\frac{3a^4x^4}{4}+\frac{7a^2x^2}{2}+4\ln(a^2x^2+1)\right)}{15}}{a}$ |
| default | $\frac{\frac{c^2 \arctan(ax) a^5 x^5}{5}+\frac{2a^3c^2x^3\arctan(ax)}{3}+a c^2 x \arctan(ax)-\frac{c^2\left(\frac{3a^4x^4}{4}+\frac{7a^2x^2}{2}+4\ln(a^2x^2+1)\right)}{15}}{a}$ |
| parallelrisch | $-\frac{-12c^2\arctan(ax)a^5x^5+3a^4c^2x^4-40a^3c^2x^3\arctan(ax)+14x^2c^2a^2-60a c^2x\arctan(ax)+16c^2\ln(a^2x^2+1)}{60a}$ |
| risch | $-\frac{ic^2x(3a^4x^4+10a^2x^2+15)\ln(iax+1)}{30}+\frac{ic^2a^4x^5\ln(-iax+1)}{10}-\frac{a^3c^2x^4}{20}+\frac{ic^2a^2x^3\ln(-iax+1)}{3}-\frac{7a c^2x^2}{30}+$ |
| meijerg | $\frac{c^2\left(\frac{a^2x^2(-3a^2x^2+6)}{15}+\frac{4a^6x^6\arctan(\sqrt{a^2x^2})}{5\sqrt{a^2x^2}}-\frac{2\ln(a^2x^2+1)}{5}\right)}{4a}+\frac{c^2\left(-\frac{2a^2x^2}{3}+\frac{4a^4x^4\arctan(\sqrt{a^2x^2})}{3\sqrt{a^2x^2}}+\frac{2\ln(a^2x^2+1)}{3}\right)}{2a}$ |

input
```
int((a^2*c*x^2+c)^2*arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/5*c^2*arctan(a*x)*a^4*x^5+2/3*c^2*arctan(a*x)*a^2*x^3+c^2*x*arctan(a*x)-
1/15*a*c^2*(3/4*a^2*x^4+7/2*x^2+4/a^2*ln(a^2*x^2+1))
```

## Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.68

$$\int \left(c+a^2cx^2\right)^2 \arctan(ax)\,dx = -\frac{3\,a^4c^2x^4+14\,a^2c^2x^2+16\,c^2\log\left(a^2x^2+1\right)-4\left(3\,a^5c^2x^5+10\,a^3c^2x^3+15\,ac^2x\right)\arctan\left(ax\right)}{60\,a}$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x),x, algorithm="fricas")
```

output
```
-1/60*(3*a^4*c^2*x^4 + 14*a^2*c^2*x^2 + 16*c^2*log(a^2*x^2 + 1) - 4*(3*a^5
*c^2*x^5 + 10*a^3*c^2*x^3 + 15*a*c^2*x)*arctan(a*x))/a
```

## Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.75

$$\int (c+a^2cx^2)^2 \arctan(ax)\,dx = \begin{cases} \frac{a^4c^2x^5 \operatorname{atan}(ax)}{5} - \frac{a^3c^2x^4}{20} + \frac{2a^2c^2x^3 \operatorname{atan}(ax)}{3} - \frac{7ac^2x^2}{30} + c^2x \operatorname{atan}(ax) - \frac{4c^2 \log\left(x^2+\frac{1}{a^2}\right)}{15a} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate((a**2*c*x**2+c)**2*atan(a*x),x)
```

output
```
Piecewise((a**4*c**2*x**5*atan(a*x)/5 - a**3*c**2*x**4/20 + 2*a**2*c**2*x**
*3*atan(a*x)/3 - 7*a*c**2*x**2/30 + c**2*x*atan(a*x) - 4*c**2*log(x**2 + a
**(-2))/(15*a), Ne(a, 0)), (0, True))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.66

$$\int (c+a^2cx^2)^2 \arctan(ax)\,dx = -\frac{1}{60}\left(3a^2c^2x^4 + 14c^2x^2 + \frac{16c^2 \log(a^2x^2+1)}{a^2}\right)a + \frac{1}{15}\left(3a^4c^2x^5 + 10a^2c^2x^3 + 15c^2x\right)\arctan(ax)$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x),x, algorithm="maxima")
```

output
```
-1/60*(3*a^2*c^2*x^4 + 14*c^2*x^2 + 16*c^2*log(a^2*x^2 + 1)/a^2)*a + 1/15*
(3*a^4*c^2*x^5 + 10*a^2*c^2*x^3 + 15*c^2*x)*arctan(a*x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.70

$$\int (c+a^2cx^2)^2 \arctan(ax)\,dx = \frac{1}{15}\left(3\,a^4c^2x^5+10\,a^2c^2x^3+15\,c^2x\right)\arctan(ax) - \frac{4\,c^2\log(a^2x^2+1)}{15\,a} - \frac{3\,a^7c^2x^4+14\,a^5c^2x^2}{60\,a^4}$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x),x, algorithm="giac")
```

output
```
1/15*(3*a^4*c^2*x^5 + 10*a^2*c^2*x^3 + 15*c^2*x)*arctan(a*x) - 4/15*c^2*lo
g(a^2*x^2 + 1)/a - 1/60*(3*a^7*c^2*x^4 + 14*a^5*c^2*x^2)/a^4
```

## Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.59

$$\int (c+a^2cx^2)^2 \arctan(ax)\,dx = -\frac{c^2\left(16\,\ln(a^2\,x^2+1)+14\,a^2\,x^2+3\,a^4\,x^4-40\,a^3\,x^3\,\mathrm{atan}(a\,x)-12\,a^5\,x^5\,\mathrm{atan}(a\,x)-60\,a\,x\,\mathrm{atan}(a\,x)\right)}{60\,a}$$

input
```
int(atan(a*x)*(c + a^2*c*x^2)^2,x)
```

output
```
-(c^2*(16*log(a^2*x^2 + 1) + 14*a^2*x^2 + 3*a^4*x^4 - 40*a^3*x^3*atan(a*x)
 - 12*a^5*x^5*atan(a*x) - 60*a*x*atan(a*x)))/(60*a)
```

## Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.59

$$\int (c + a^2 c x^2)^2 \arctan(ax)\, dx$$
$$= \frac{c^2(12atan(ax)\, a^5x^5 + 40atan(ax)\, a^3x^3 + 60atan(ax)\, ax - 16\log(a^2x^2 + 1) - 3a^4x^4 - 14a^2x^2)}{60a}$$

input
```
int((a^2*c*x^2+c)^2*atan(a*x),x)
```

output
```
(c**2*(12*atan(a*x)*a**5*x**5 + 40*atan(a*x)*a**3*x**3 + 60*atan(a*x)*a*x
- 16*log(a**2*x**2 + 1) - 3*a**4*x**4 - 14*a**2*x**2))/(60*a)
```

# 3.161 $\int \frac{(c+a^2cx^2)^2 \arctan(ax)}{x} dx$



## Optimal result

Integrand size = 20, antiderivative size = 99

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)}{x} dx = -\frac{3}{4}ac^2x - \frac{1}{12}a^3c^2x^3 + \frac{3}{4}c^2\arctan(ax) + a^2c^2x^2\arctan(ax) + \frac{1}{4}a^4c^2x^4\arctan(ax) + \frac{1}{2}ic^2\,\mathrm{PolyLog}(2,-iax) - \frac{1}{2}ic^2\,\mathrm{PolyLog}(2,iax)$$

output
```
-3/4*a*c^2*x-1/12*a^3*c^2*x^3+3/4*c^2*arctan(a*x)+a^2*c^2*x^2*arctan(a*x)+
1/4*a^4*c^2*x^4*arctan(a*x)+1/2*I*c^2*polylog(2,-I*a*x)-1/2*I*c^2*polylog(
2,I*a*x)
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)}{x} dx = -\frac{3}{4}ac^2x - \frac{1}{12}a^3c^2x^3 + \frac{3}{4}c^2\arctan(ax) + a^2c^2x^2\arctan(ax) + \frac{1}{4}a^4c^2x^4\arctan(ax) + \frac{1}{2}ic^2\,\mathrm{PolyLog}(2,-iax) - \frac{1}{2}ic^2\,\mathrm{PolyLog}(2,iax)$$

input
```
Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x])/x,x]
```

output
```
(-3*a*c^2*x)/4 - (a^3*c^2*x^3)/12 + (3*c^2*ArcTan[a*x])/4 + a^2*c^2*x^2*Ar
cTan[a*x] + (a^4*c^2*x^4*ArcTan[a*x])/4 + (I/2)*c^2*PolyLog[2, (-I)*a*x] -
 (I/2)*c^2*PolyLog[2, I*a*x]
```

## Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)\left(a^2cx^2+c\right)^2}{x}dx$$
$$\Big\downarrow \text{5483}$$
$$\int \left(a^4c^2x^3\arctan(ax)+2a^2c^2x\arctan(ax)+\frac{c^2\arctan(ax)}{x}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{4}a^4c^2x^4\arctan(ax)-\frac{1}{12}a^3c^2x^3+a^2c^2x^2\arctan(ax)+\frac{3}{4}c^2\arctan(ax)+\frac{1}{2}ic^2\operatorname{PolyLog}(2,-iax)-\frac{1}{2}ic^2\operatorname{PolyLog}(2,iax)-\frac{3}{4}ac^2x$$

input
```
Int[((c + a^2*c*x^2)^2*ArcTan[a*x])/x,x]
```

output
```
(-3*a*c^2*x)/4 - (a^3*c^2*x^3)/12 + (3*c^2*ArcTan[a*x])/4 + a^2*c^2*x^2*Ar
cTan[a*x] + (a^4*c^2*x^4*ArcTan[a*x])/4 + (I/2)*c^2*PolyLog[2, (-I)*a*x] -
 (I/2)*c^2*PolyLog[2, I*a*x]
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

## Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.20

| method | result |
|---|---|
| derivativedivides | $\frac{a^4c^2x^4\arctan(ax)}{4}+a^2c^2x^2\arctan(ax)+c^2\arctan(ax)\ln(ax)-\frac{c^2\left(\frac{a^3x^3}{3}+3ax-3\arctan(ax)-2\right.}{}$ |
| default | $\frac{a^4c^2x^4\arctan(ax)}{4}+a^2c^2x^2\arctan(ax)+c^2\arctan(ax)\ln(ax)-\frac{c^2\left(\frac{a^3x^3}{3}+3ax-3\arctan(ax)-2\right.}{}$ |
| parts | $\frac{a^4c^2x^4\arctan(ax)}{4}+a^2c^2x^2\arctan(ax)+c^2\arctan(ax)\ln(x)-\frac{a\,c^2\left(\frac{a^2x^3}{3}+3x-\frac{3\arctan(ax)}{a}-\frac{2i\ln}{}\right.}{}$ |
| risch | $\frac{ic^2\ln(-iax+1)x^4a^4}{8}+\frac{ic^2\ln(-iax+1)x^2a^2}{2}+\frac{3c^2\arctan(ax)}{4}-\frac{a^3c^2x^3}{12}-\frac{3ax\,c^2}{4}-\frac{ic^2\operatorname{dilog}(-iax+1)}{2}-\frac{ic}{}$ |
| meijerg | $\frac{c^2\left(\frac{ax(-5a^2x^2+15)}{15}-\frac{ax(-5a^4x^4+5)\arctan\left(\sqrt{a^2x^2}\right)}{5\sqrt{a^2x^2}}\right)}{4}+\frac{c^2\left(-2ax+\frac{2(3a^2x^2+3)\arctan(ax)}{3}\right)}{2}+\frac{c^2\left(-\frac{2iax\operatorname{polylog}(}{\sqrt{a^2}}\right.}{}$ |

input
```
int((a^2*c*x^2+c)^2*arctan(a*x)/x,x,method=_RETURNVERBOSE)
```

output
```
1/4*a^4*c^2*x^4*arctan(a*x)+a^2*c^2*x^2*arctan(a*x)+c^2*arctan(a*x)*ln(a*x
)-1/4*c^2*(1/3*a^3*x^3+3*a*x-3*arctan(a*x)-2*I*ln(a*x)*ln(1+I*a*x)+2*I*ln(
a*x)*ln(1-I*a*x)-2*I*dilog(1+I*a*x)+2*I*dilog(1-I*a*x))
```

## Fricas [F]

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)}{x}\,dx = \int \frac{(a^2cx^2+c)^2 \arctan(ax)}{x}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)/x,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)/x, x)
```

## Sympy [F]

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)}{x}\,dx = c^2\left(\int \frac{\operatorname{atan}(ax)}{x}\,dx + \int 2a^2x\operatorname{atan}(ax)\,dx + \int a^4x^3\operatorname{atan}(ax)\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**2*atan(a*x)/x,x)
```

output
```
c**2*(Integral(atan(a*x)/x, x) + Integral(2*a**2*x*atan(a*x), x) + Integra
l(a**4*x**3*atan(a*x), x))
```

## Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.05

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)}{x}\,dx = -\frac{1}{12}a^3c^2x^3 - \frac{3}{4}ac^2x - \frac{1}{4}\pi c^2\log\left(a^2x^2+1\right) + c^2\arctan(ax)\log(ax) - \frac{1}{2}ic^2\mathrm{Li}_2(iax+1) + \frac{1}{2}ic^2\mathrm{Li}_2(-iax+1) + \frac{1}{4}\left(a^4c^2x^4 + 4a^2c^2x^2 + 3c^2\right)\arctan(ax)$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)/x,x, algorithm="maxima")
```

output
```
-1/12*a^3*c^2*x^3 - 3/4*a*c^2*x - 1/4*pi*c^2*log(a^2*x^2 + 1) + c^2*arctan
(a*x)*log(a*x) - 1/2*I*c^2*dilog(I*a*x + 1) + 1/2*I*c^2*dilog(-I*a*x + 1)
+ 1/4*(a^4*c^2*x^4 + 4*a^2*c^2*x^2 + 3*c^2)*arctan(a*x)
```

## Giac [F]

$$\int \frac{(c+a^2cx^2)^2\arctan(ax)}{x}\,dx = \int \frac{(a^2cx^2+c)^2\arctan(ax)}{x}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)/x,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^2*arctan(a*x)/x, x)
```

## Mupad [B] (verification not implemented)

Time = 1.05 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.06

$$\int \frac{(c+a^2cx^2)^2\arctan(ax)}{x}\,dx = \begin{cases} 0 \\ 2\,a^2\,c^2\,\mathrm{atan}(a\,x)\,\left(\frac{1}{2\,a^2}+\frac{x^2}{2}\right)-a\,c^2\,x-\frac{c^2\,\left(3\,\mathrm{atan}(a\,x)-3\,a\,x+a^3\,x^3\right)}{12}+\frac{a^4\,c^2\,x^4\,\mathrm{atan}(a\,x)}{4}-\frac{c^2\,\mathrm{Li_2}(1-a\,x\,1\mathrm{i})\,1\mathrm{i}}{2}+\frac{c^2\,\mathrm{Li}}{}\end{cases}$$

input
```
int((atan(a*x)*(c + a^2*c*x^2)^2)/x,x)
```

output
```
piecewise(a == 0, 0, a ~= 0, - (c^2*dilog(- a*x*1i + 1)*1i)/2 + (c^2*dilog
(a*x*1i + 1)*1i)/2 - (c^2*(3*atan(a*x) - 3*a*x + a^3*x^3))/12 - a*c^2*x +
2*a^2*c^2*atan(a*x)*(1/(2*a^2) + x^2/2) + (a^4*c^2*x^4*atan(a*x))/4)
```

## Reduce [F]

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)}{x} \, dx$$
$$= \frac{c^2\left(3atan(ax)\,a^4x^4 + 12atan(ax)\,a^2x^2 + 9atan(ax) + 12\left(\int \frac{atan(ax)}{x}dx\right) - a^3x^3 - 9ax\right)}{12}$$

input
```
int((a^2*c*x^2+c)^2*atan(a*x)/x,x)
```

output
```
(c**2*(3*atan(a*x)*a**4*x**4 + 12*atan(a*x)*a**2*x**2 + 9*atan(a*x) + 12*i
nt(atan(a*x)/x,x) - a**3*x**3 - 9*a*x))/12
```

# 3.162 $\int \frac{(c+a^2cx^2)^2\arctan(ax)}{x^2}\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 81

$$\int \frac{(c+a^2cx^2)^2\arctan(ax)}{x^2}\,dx = -\frac{1}{6}a^3c^2x^2-\frac{c^2\arctan(ax)}{x}+2a^2c^2x\arctan(ax)+\frac{1}{3}a^4c^2x^3\arctan(ax)+ac^2\log(x)-\frac{4}{3}ac^2\log\left(1+a^2x^2\right)$$

output
```
-1/6*a^3*c^2*x^2-c^2*arctan(a*x)/x+2*a^2*c^2*x*arctan(a*x)+1/3*a^4*c^2*x^3
*arctan(a*x)+a*c^2*ln(x)-4/3*a*c^2*ln(a^2*x^2+1)
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.77

$$\int \frac{(c+a^2cx^2)^2\arctan(ax)}{x^2}\,dx = \frac{c^2(2(-3+6a^2x^2+a^4x^4)\arctan(ax)-ax(a^2x^2-6\log(x)+8\log\left(1+a^2x^2\right)))}{6x}$$

input
```
Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x])/x^2,x]
```

output
```
(c^2*(2*(-3 + 6*a^2*x^2 + a^4*x^4)*ArcTan[a*x] - a*x*(a^2*x^2 - 6*Log[x] +
 8*Log[1 + a^2*x^2])))/(6*x)
```

## Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)\left(a^2cx^2+c\right)^2}{x^2}\,dx$$

$$\Big\downarrow \text{ 5483}$$

$$\int \left(a^4c^2x^2\arctan(ax)+2a^2c^2\arctan(ax)+\frac{c^2\arctan(ax)}{x^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}a^4c^2x^3\arctan(ax)-\frac{1}{6}a^3c^2x^2+2a^2c^2x\arctan(ax)-\frac{4}{3}ac^2\log\left(a^2x^2+1\right)-\frac{c^2\arctan(ax)}{x}+ac^2\log(x)$$

input
```
Int[((c + a^2*c*x^2)^2*ArcTan[a*x])/x^2,x]
```

output
```
-1/6*(a^3*c^2*x^2) - (c^2*ArcTan[a*x])/x + 2*a^2*c^2*x*ArcTan[a*x] + (a^4*
c^2*x^3*ArcTan[a*x])/3 + a*c^2*Log[x] - (4*a*c^2*Log[1 + a^2*x^2])/3
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

## Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.90

| method | result |
|---|---|
| parts | $\frac{a^4c^2x^3\arctan(ax)}{3}+2a^2c^2x\arctan(ax)-\frac{c^2\arctan(ax)}{x}-\frac{ac^2\left(\frac{a^2x^2}{2}+4\ln(a^2x^2+1)-3\ln(x)\right)}{3}$ |
| derivativedivides | $a\left(\frac{a^3c^2x^3\arctan(ax)}{3}+2ac^2x\arctan(ax)-\frac{c^2\arctan(ax)}{ax}-\frac{c^2\left(\frac{a^2x^2}{2}+4\ln(a^2x^2+1)-3\ln(ax)\right)}{3}\right)$ |
| default | $a\left(\frac{a^3c^2x^3\arctan(ax)}{3}+2ac^2x\arctan(ax)-\frac{c^2\arctan(ax)}{ax}-\frac{c^2\left(\frac{a^2x^2}{2}+4\ln(a^2x^2+1)-3\ln(ax)\right)}{3}\right)$ |
| parallelrisch | $\frac{2a^4c^2x^4\arctan(ax)-a^3c^2x^3+12a^2c^2x^2\arctan(ax)+6ac^2\ln(x)x-8ac^2\ln(a^2x^2+1)x-6c^2\arctan(ax)}{6x}$ |
| risch | $-\frac{ic^2(a^4x^4+6a^2x^2-3)\ln(iax+1)}{6x}+\frac{ic^2(x^4\ln(-iax+1)a^4+ia^3x^3+6a^2x^2\ln(-iax+1)-6ia\ln(x)x+8ia\ln(7a^2x^2+7$ |
| meijerg | $\frac{ac^2\left(-\frac{2a^2x^2}{3}+\frac{4a^4x^4\arctan\left(\sqrt{a^2x^2}\right)}{3\sqrt{a^2x^2}}+\frac{2\ln(a^2x^2+1)}{3}\right)}{4}+\frac{ac^2\left(\frac{4a^2x^2\arctan\left(\sqrt{a^2x^2}\right)}{\sqrt{a^2x^2}}-2\ln(a^2x^2+1)\right)}{2}+\frac{ac^2\left(4\ln(\right.}{}$ |

input
```
int((a^2*c*x^2+c)^2*arctan(a*x)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*a^4*c^2*x^3*arctan(a*x)+2*a^2*c^2*x*arctan(a*x)-c^2*arctan(a*x)/x-1/3*
a*c^2*(1/2*a^2*x^2+4*ln(a^2*x^2+1)-3*ln(x))
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.93

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)}{x^2} \, dx = -\frac{a^3c^2x^3 + 8\,ac^2x\log\left(a^2x^2+1\right) - 6\,ac^2x\log\left(x\right) - 2\left(a^4c^2x^4 + 6\,a^2c^2x^2 - 3\,c^2\right)\arctan\left(ax\right)}{6\,x}$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)/x^2,x, algorithm="fricas")
```

output
```
-1/6*(a^3*c^2*x^3 + 8*a*c^2*x*log(a^2*x^2 + 1) - 6*a*c^2*x*log(x) - 2*(a^4
*c^2*x^4 + 6*a^2*c^2*x^2 - 3*c^2)*arctan(a*x))/x
```

## Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.01

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)}{x^2} \, dx = \begin{cases} \frac{a^4c^2x^3\,\mathrm{atan}\,(ax)}{3} - \frac{a^3c^2x^2}{6} + 2a^2c^2x\,\mathrm{atan}\,(ax) + ac^2\log\left(x\right) - \frac{4ac^2\log\left(x^2+\frac{1}{a^2}\right)}{3} - \frac{c^2\,\mathrm{atan}\,(ax)}{x} & \text{for}\ a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate((a**2*c*x**2+c)**2*atan(a*x)/x**2,x)
```

output
```
Piecewise((a**4*c**2*x**3*atan(a*x)/3 - a**3*c**2*x**2/6 + 2*a**2*c**2*x*a
tan(a*x) + a*c**2*log(x) - 4*a*c**2*log(x**2 + a**(-2))/3 - c**2*atan(a*x)
/x, Ne(a, 0)), (0, True))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.88

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)}{x^2} dx = -\frac{1}{6} \left(a^2c^2x^2 + 8\,c^2 \log\left(a^2x^2+1\right) - 6\,c^2 \log\left(x\right)\right)a + \frac{1}{3} \left(a^4c^2x^3 + 6\,a^2c^2x - \frac{3\,c^2}{x}\right) \arctan\left(ax\right)$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)/x^2,x, algorithm="maxima")
```

output
```
-1/6*(a^2*c^2*x^2 + 8*c^2*log(a^2*x^2 + 1) - 6*c^2*log(x))*a + 1/3*(a^4*c^
2*x^3 + 6*a^2*c^2*x - 3*c^2/x)*arctan(a*x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.89

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)}{x^2} dx = -\frac{1}{6}\,a^3c^2x^2 - \frac{4}{3}\,ac^2 \log\left(a^2x^2+1\right) + \frac{1}{2}\,ac^2 \log\left(x^2\right) + \frac{1}{3} \left(a^4c^2x^3 + 6\,a^2c^2x - \frac{3\,c^2}{x}\right) \arctan\left(ax\right)$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)/x^2,x, algorithm="giac")
```

output
```
-1/6*a^3*c^2*x^2 - 4/3*a*c^2*log(a^2*x^2 + 1) + 1/2*a*c^2*log(x^2) + 1/3*(
a^4*c^2*x^3 + 6*a^2*c^2*x - 3*c^2/x)*arctan(a*x)
```

## Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.94

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)}{x^2} \, dx = \frac{a^4\,c^2\,x^3\,\mathrm{atan}(a\,x)}{3} - \frac{c^2\,\mathrm{atan}(a\,x)}{x} - \frac{a^3\,c^2\,x^2}{6} - \frac{c^2\,(8\,a\,\ln(a^2\,x^2+1) - 6\,a\,\ln(x))}{6} + 2\,a^2\,c^2\,x\,\mathrm{atan}(a\,x)$$

input
```
int((atan(a*x)*(c + a^2*c*x^2)^2)/x^2,x)
```

output
```
(a^4*c^2*x^3*atan(a*x))/3 - (c^2*atan(a*x))/x - (a^3*c^2*x^2)/6 - (c^2*(8*
a*log(a^2*x^2 + 1) - 6*a*log(x)))/6 + 2*a^2*c^2*x*atan(a*x)
```

## Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.83

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)}{x^2} \, dx = \frac{c^2(2atan(ax)\,a^4x^4 + 12atan(ax)\,a^2x^2 - 6atan(ax) - 8\log(a^2x^2+1)\,ax + 6\log(x)\,ax - a^3x^3)}{6x}$$

input
```
int((a^2*c*x^2+c)^2*atan(a*x)/x^2,x)
```

output
```
(c**2*(2*atan(a*x)*a**4*x**4 + 12*atan(a*x)*a**2*x**2 - 6*atan(a*x) - 8*lo
g(a**2*x**2 + 1)*a*x + 6*log(x)*a*x - a**3*x**3))/(6*x)
```

# 3.163 $\int \frac{(c+a^2cx^2)^2\arctan(ax)}{x^3}\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 90

$$\int \frac{(c+a^2cx^2)^2\arctan(ax)}{x^3}\,dx = -\frac{ac^2}{2x}-\frac{1}{2}a^3c^2x-\frac{c^2\arctan(ax)}{2x^2}+\frac{1}{2}a^4c^2x^2\arctan(ax) + ia^2c^2\,\mathrm{PolyLog}(2,-iax)-ia^2c^2\,\mathrm{PolyLog}(2,iax)$$

output
```
-1/2*a*c^2/x-1/2*a^3*c^2*x-1/2*c^2*arctan(a*x)/x^2+1/2*a^4*c^2*x^2*arctan(
a*x)+I*a^2*c^2*polylog(2,-I*a*x)-I*a^2*c^2*polylog(2,I*a*x)
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.02 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.33

$$\int \frac{(c+a^2cx^2)^2\arctan(ax)}{x^3}\,dx = -\frac{1}{2}a^3c^2x+\frac{1}{2}a^2c^2\arctan(ax) - \frac{c^2\arctan(ax)}{2x^2}+\frac{1}{2}a^4c^2x^2\arctan(ax) - \frac{ac^2\,\mathrm{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-a^2x^2\right)}{2x} + ia^2c^2\,\mathrm{PolyLog}(2,-iax)-ia^2c^2\,\mathrm{PolyLog}(2,iax)$$

input
```
Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x])/x^3,x]
```

output
```
-1/2*(a^3*c^2*x) + (a^2*c^2*ArcTan[a*x])/2 - (c^2*ArcTan[a*x])/(2*x^2) + (
a^4*c^2*x^2*ArcTan[a*x])/2 - (a*c^2*Hypergeometric2F1[-1/2, 1, 1/2, -(a^2*
x^2)])/(2*x) + I*a^2*c^2*PolyLog[2, (-I)*a*x] - I*a^2*c^2*PolyLog[2, I*a*x
]
```

## Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)\left(a^2cx^2+c\right)^2}{x^3}dx$$

$$\Big\downarrow \text{5483}$$

$$\int \left(a^4c^2x\arctan(ax)+\frac{2a^2c^2\arctan(ax)}{x}+\frac{c^2\arctan(ax)}{x^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}a^4c^2x^2\arctan(ax)-\frac{1}{2}a^3c^2x+ia^2c^2\,\text{PolyLog}(2,-iax)-ia^2c^2\,\text{PolyLog}(2,iax)-\frac{c^2\arctan(ax)}{2x^2}-\frac{ac^2}{2x}$$

input
```
Int[((c + a^2*c*x^2)^2*ArcTan[a*x])/x^3,x]
```

output
```
-1/2*(a*c^2)/x - (a^3*c^2*x)/2 - (c^2*ArcTan[a*x])/(2*x^2) + (a^4*c^2*x^2*
ArcTan[a*x])/2 + I*a^2*c^2*PolyLog[2, (-I)*a*x] - I*a^2*c^2*PolyLog[2, I*a
*x]
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

## Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.29

| method | result |
|---|---|
| parts | $\frac{a^4c^2x^2\arctan(ax)}{2}-\frac{c^2\arctan(ax)}{2x^2}+2c^2\arctan(ax)\,a^2\ln(x)-\frac{a\,c^2\left(a^2x+\frac{1}{x}+4a^2\left(-\frac{i\ln(x)(-\ln(-iax+1}{2a}\right.\right.}{}$ |
| derivativedivides | $a^2\left(\frac{a^2c^2x^2\arctan(ax)}{2}-\frac{c^2\arctan(ax)}{2a^2x^2}+2c^2\arctan(ax)\ln(ax)-\frac{c^2\left(ax+\frac{1}{ax}-2i\ln(ax)\ln(iax+1)+2i\right.}{}\right.$ |
| default | $a^2\left(\frac{a^2c^2x^2\arctan(ax)}{2}-\frac{c^2\arctan(ax)}{2a^2x^2}+2c^2\arctan(ax)\ln(ax)-\frac{c^2\left(ax+\frac{1}{ax}-2i\ln(ax)\ln(iax+1)+2i\right.}{}\right.$ |
| meijerg | $\frac{a^2c^2\left(-2ax+\frac{2(3a^2x^2+3)\arctan(ax)}{3}\right)}{4}+\frac{a^2c^2\left(-\frac{2iax\,\mathrm{polylog}\left(2,i\sqrt{a^2x^2}\right)}{\sqrt{a^2x^2}}+\frac{2iax\,\mathrm{polylog}\left(2,-i\sqrt{a^2x^2}\right)}{\sqrt{a^2x^2}}\right)}{2}+\frac{a^2c^2\left(-\frac{2}{ax}\right.}{}$ |
| risch | $\frac{ic^2a^4\ln(-iax+1)x^2}{4}-\frac{a^3c^2x}{2}-\frac{a\,c^2}{2x}+\frac{ic^2a^2\ln(-iax)}{4}-\frac{ic^2\ln(-iax+1)}{4x^2}-ic^2a^2\,\mathrm{dilog}(-iax+1)-$ |

input
```
int((a^2*c*x^2+c)^2*arctan(a*x)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*a^4*c^2*x^2*arctan(a*x)-1/2*c^2*arctan(a*x)/x^2+2*c^2*arctan(a*x)*a^2*
ln(x)-1/2*a*c^2*(a^2*x+1/x+4*a^2*(-1/2*I*ln(x)*(-ln(1-I*a*x)+ln(1+I*a*x))/
a-1/2*I*(dilog(1+I*a*x)-dilog(1-I*a*x))/a))
```

## Fricas [F]

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)}{x^3} \, dx = \int \frac{(a^2cx^2+c)^2 \arctan(ax)}{x^3} \, dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)/x^3,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)/x^3, x)
```

## Sympy [F]

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)}{x^3} \, dx = c^2 \left( \int \frac{\operatorname{atan}(ax)}{x^3} \, dx + \int \frac{2a^2 \operatorname{atan}(ax)}{x} \, dx + \int a^4 x \operatorname{atan}(ax) \, dx \right)$$

input
```
integrate((a**2*c*x**2+c)**2*atan(a*x)/x**3,x)
```

output
```
c**2*(Integral(atan(a*x)/x**3, x) + Integral(2*a**2*atan(a*x)/x, x) + Inte
gral(a**4*x*atan(a*x), x))
```

## Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.33

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)}{x^3} \, dx = -\frac{a^3c^2x^3 + \pi a^2c^2x^2 \log(a^2x^2+1) - 4\,a^2c^2x^2 \arctan(ax) \log(ax) + 2i\,a^2c^2x^2 \mathrm{Li}_2(i\,ax+1) - 2i\,a^2c^2x^2 \mathrm{Li}}{2\,x^2}$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)/x^3,x, algorithm="maxima")
```

output
```
-1/2*(a^3*c^2*x^3 + pi*a^2*c^2*x^2*log(a^2*x^2 + 1) - 4*a^2*c^2*x^2*arctan
(a*x)*log(a*x) + 2*I*a^2*c^2*x^2*dilog(I*a*x + 1) - 2*I*a^2*c^2*x^2*dilog(
-I*a*x + 1) + a*c^2*x - (a^4*c^2*x^4 - c^2)*arctan(a*x))/x^2
```

## Giac [F]

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)}{x^3}\,dx = \int \frac{(a^2cx^2+c)^2 \arctan(ax)}{x^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)/x^3,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^2*arctan(a*x)/x^3, x)
```

## Mupad [B] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.22

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)}{x^3}\,dx$$
$$= \begin{cases} 0 \\ a^4\,c^2\,\mathrm{atan}(a\,x)\,\left(\frac{1}{2\,a^2}+\frac{x^2}{2}\right)-\frac{c^2\,\mathrm{atan}(a\,x)}{2\,x^2}-\frac{c^2\,\left(a^3\,\mathrm{atan}(a\,x)+\frac{a^2}{x}\right)}{2\,a}-\frac{a^3\,c^2\,x}{2}-a^2\,c^2\,\mathrm{Li}_2(1-a\,x\,1\mathrm{i})\,1\mathrm{i}+a^2\,c^2\,\mathrm{L} \end{cases}$$

input
```
int((atan(a*x)*(c + a^2*c*x^2)^2)/x^3,x)
```

output
```
piecewise(a == 0, 0, a ~= 0, - (a^3*c^2*x)/2 - (c^2*atan(a*x))/(2*x^2) - a
^2*c^2*dilog(- a*x*1i + 1)*1i + a^2*c^2*dilog(a*x*1i + 1)*1i - (c^2*(a^3*a
tan(a*x) + a^2/x))/(2*a) + a^4*c^2*atan(a*x)*(1/(2*a^2) + x^2/2))
```

## Reduce [F]

$$\int \frac{(c+a^2cx^2)^2\arctan(ax)}{x^3}\,dx = \frac{c^2\left(atan(ax)\,a^4x^4 - atan(ax) + 4\left(\int \frac{atan(ax)}{x}dx\right)a^2x^2 - a^3x^3 - ax\right)}{2x^2}$$

input
```
int((a^2*c*x^2+c)^2*atan(a*x)/x^3,x)
```

output
```
(c**2*(atan(a*x)*a**4*x**4 - atan(a*x) + 4*int(atan(a*x)/x,x)*a**2*x**2 -
a**3*x**3 - a*x))/(2*x**2)
```

# 3.164 $\int \frac{(c+a^2cx^2)^2 \arctan(ax)}{x^4} \, dx$



## Optimal result

Integrand size = 20, antiderivative size = 85

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)}{x^4} \, dx = -\frac{ac^2}{6x^2} - \frac{c^2 \arctan(ax)}{3x^3} - \frac{2a^2c^2 \arctan(ax)}{x} + a^4c^2x \arctan(ax) + \frac{5}{3}a^3c^2 \log(x) - \frac{4}{3}a^3c^2 \log(1+a^2x^2)$$

output
```
-1/6*a*c^2/x^2-1/3*c^2*arctan(a*x)/x^3-2*a^2*c^2*arctan(a*x)/x+a^4*c^2*x*a
rctan(a*x)+5/3*a^3*c^2*ln(x)-4/3*a^3*c^2*ln(a^2*x^2+1)
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.80

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)}{x^4} \, dx = \frac{c^2(2(-1-6a^2x^2+3a^4x^4) \arctan(ax) + ax(-1+10a^2x^2 \log(x) - 8a^2x^2 \log(1+a^2x^2)))}{6x^3}$$

input
```
Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x])/x^4,x]
```

output
```
(c^2*(2*(-1 - 6*a^2*x^2 + 3*a^4*x^4)*ArcTan[a*x] + a*x*(-1 + 10*a^2*x^2*Lo
g[x] - 8*a^2*x^2*Log[1 + a^2*x^2])))/(6*x^3)
```

## Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)\left(a^2cx^2+c\right)^2}{x^4}dx$$

$$\Big\downarrow \text{ 5483}$$

$$\int \left(a^4c^2\arctan(ax)+\frac{2a^2c^2\arctan(ax)}{x^2}+\frac{c^2\arctan(ax)}{x^4}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$a^4c^2x\arctan(ax)+\frac{5}{3}a^3c^2\log(x)-\frac{2a^2c^2\arctan(ax)}{x}-\frac{4}{3}a^3c^2\log\left(a^2x^2+1\right)-\frac{c^2\arctan(ax)}{3x^3}-\frac{ac^2}{6x^2}$$

input
```
Int[((c + a^2*c*x^2)^2*ArcTan[a*x])/x^4,x]
```

output
```
-1/6*(a*c^2)/x^2 - (c^2*ArcTan[a*x])/(3*x^3) - (2*a^2*c^2*ArcTan[a*x])/x +
 a^4*c^2*x*ArcTan[a*x] + (5*a^3*c^2*Log[x])/3 - (4*a^3*c^2*Log[1 + a^2*x^2
])/3
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

## Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.88

| method | result |
|---|---|
| parts | $a^{4} c^{2} x \arctan \left(a x \right)-\frac{c^{2} \arctan \left(a x \right)}{3 x^{3}}-\frac{2 a^{2} c^{2} \arctan \left(a x \right)}{x}-\frac{a \,c^{2} \left(4 a^{2} \ln \left(a^{2} x^{2}+1\right)+\frac{1}{2 x^{2}}-5 a^{2} \ln \left(x \right)\right)}{3}$ |
| derivativedivides | $a^{3} \left(a \,c^{2} x \arctan \left(a x \right)-\frac{2 c^{2} \arctan \left(a x \right)}{a x}-\frac{c^{2} \arctan \left(a x \right)}{3 a^{3} x^{3}}-\frac{c^{2} \left(\frac{1}{2 a^{2} x^{2}}-5 \ln \left(a x \right)+4 \ln \left(a^{2} x^{2}+1\right)\right)}{3}\right)$ |
| default | $a^{3} \left(a \,c^{2} x \arctan \left(a x \right)-\frac{2 c^{2} \arctan \left(a x \right)}{a x}-\frac{c^{2} \arctan \left(a x \right)}{3 a^{3} x^{3}}-\frac{c^{2} \left(\frac{1}{2 a^{2} x^{2}}-5 \ln \left(a x \right)+4 \ln \left(a^{2} x^{2}+1\right)\right)}{3}\right)$ |
| parallelrisch | $\frac{6 a^{4} c^{2} x^{4} \arctan \left(a x \right)+10 a^{3} c^{2} \ln \left(x \right) x^{3}-8 a^{3} c^{2} \ln \left(a^{2} x^{2}+1\right) x^{3}+a^{3} c^{2} x^{3}-12 a^{2} c^{2} x^{2} \arctan \left(a x \right)-a x \,c^{2}-2 c^{2} \arctan \left(a x \right)}{6 x^{3}}$ |
| risch | $-\frac{i c^{2} \left(3 a^{4} x^{4}-6 a^{2} x^{2}-1\right) \ln \left(i a x +1\right)}{6 x^{3}}+\frac{i c^{2} \left(3 x^{4} \ln \left(-i a x +1\right) a^{4}-10 i \ln \left(x \right) a^{3} x^{3}+8 i \ln \left(-9 a^{2} x^{2}-9\right) a^{3} x^{3}-6 a^{2} x^{2} \ln (-ia}{6 x^{3}}$ |
| meijerg | $\frac{a^{3} c^{2} \left(\frac{4 a^{2} x^{2} \arctan \left(\sqrt{a^{2} x^{2}}\right)}{\sqrt{a^{2} x^{2}}}-2 \ln \left(a^{2} x^{2}+1\right)\right)}{4}+\frac{a^{3} c^{2} \left(4 \ln \left(x \right)+4 \ln \left(a \right)-\frac{4 \arctan \left(\sqrt{a^{2} x^{2}}\right)}{\sqrt{a^{2} x^{2}}}-2 \ln \left(a^{2} x^{2}+1\right)\right)}{2}+\frac{a^{3} c^{2} (}{}$ |

input
```
int((a^2*c*x^2+c)^2*arctan(a*x)/x^4,x,method=_RETURNVERBOSE)
```

output
```
a^4*c^2*x*arctan(a*x)-1/3*c^2*arctan(a*x)/x^3-2*a^2*c^2*arctan(a*x)/x-1/3*
a*c^2*(4*a^2*ln(a^2*x^2+1)+1/2/x^2-5*a^2*ln(x))
```

## Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.94

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)}{x^4} \, dx = -\frac{8\,a^3c^2x^3\log\left(a^2x^2+1\right)-10\,a^3c^2x^3\log\left(x\right)+ac^2x-2\left(3\,a^4c^2x^4-6\,a^2c^2x^2-c^2\right)\arctan\left(ax\right)}{6\,x^3}$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)/x^4,x, algorithm="fricas")
```

output
```
-1/6*(8*a^3*c^2*x^3*log(a^2*x^2 + 1) - 10*a^3*c^2*x^3*log(x) + a*c^2*x - 2
*(3*a^4*c^2*x^4 - 6*a^2*c^2*x^2 - c^2)*arctan(a*x))/x^3
```

## Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.02

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)}{x^4} \, dx = \begin{cases} a^4c^2x\operatorname{atan}\left(ax\right)+\frac{5a^3c^2\log\left(x\right)}{3}-\frac{4a^3c^2\log\left(x^2+\frac{1}{a^2}\right)}{3}-\frac{2a^2c^2\operatorname{atan}\left(ax\right)}{x}-\frac{ac^2}{6x^2}-\frac{c^2\operatorname{atan}\left(ax\right)}{3x^3} & \text{for}\: a\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate((a**2*c*x**2+c)**2*atan(a*x)/x**4,x)
```

output
```
Piecewise((a**4*c**2*x*atan(a*x) + 5*a**3*c**2*log(x)/3 - 4*a**3*c**2*log(
x**2 + a**(-2))/3 - 2*a**2*c**2*atan(a*x)/x - a*c**2/(6*x**2) - c**2*atan(
a*x)/(3*x**3), Ne(a, 0)), (0, True))
```

## Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)}{x^4} dx = -\frac{1}{6}\left(8\,a^2c^2\log\left(a^2x^2+1\right)-10\,a^2c^2\log\left(x\right)+\frac{c^2}{x^2}\right)a + \frac{1}{3}\left(3\,a^4c^2x-\frac{6\,a^2c^2x^2+c^2}{x^3}\right)\arctan\left(ax\right)$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)/x^4,x, algorithm="maxima")
```

output
```
-1/6*(8*a^2*c^2*log(a^2*x^2 + 1) - 10*a^2*c^2*log(x) + c^2/x^2)*a + 1/3*(3
*a^4*c^2*x - (6*a^2*c^2*x^2 + c^2)/x^3)*arctan(a*x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.05

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)}{x^4} dx = -\frac{4}{3}\,a^3c^2\log\left(a^2x^2+1\right)+\frac{5}{6}\,a^3c^2\log\left(x^2\right) + \frac{1}{3}\left(3\,a^4c^2x-\frac{6\,a^2c^2x^2+c^2}{x^3}\right)\arctan\left(ax\right) - \frac{5\,a^3c^2x^2+ac^2}{6\,x^2}$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)/x^4,x, algorithm="giac")
```

output
```
-4/3*a^3*c^2*log(a^2*x^2 + 1) + 5/6*a^3*c^2*log(x^2) + 1/3*(3*a^4*c^2*x -
(6*a^2*c^2*x^2 + c^2)/x^3)*arctan(a*x) - 1/6*(5*a^3*c^2*x^2 + a*c^2)/x^2
```

## Mupad [B] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)}{x^4} dx = \frac{c^2 (10 a^3 \ln(x) - 8 a^3 \ln(a^2 x^2 + 1))}{6} - \frac{\frac{c^2 \operatorname{atan}(a x)}{3} + \frac{a c^2 x}{6} + 2 a^2 c^2 x^2 \operatorname{atan}(a x)}{x^3} + a^4 c^2 x \operatorname{atan}(a x)$$

input
```
int((atan(a*x)*(c + a^2*c*x^2)^2)/x^4,x)
```

output
```
(c^2*(10*a^3*log(x) - 8*a^3*log(a^2*x^2 + 1)))/6 - ((c^2*atan(a*x))/3 + (a
*c^2*x)/6 + 2*a^2*c^2*x^2*atan(a*x))/x^3 + a^4*c^2*x*atan(a*x)
```

## Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.84

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)}{x^4} dx = \frac{c^2(6atan(ax) a^4x^4 - 12atan(ax) a^2x^2 - 2atan(ax) - 8\log(a^2x^2+1) a^3x^3 + 10\log(x) a^3x^3 - ax)}{6x^3}$$

input
```
int((a^2*c*x^2+c)^2*atan(a*x)/x^4,x)
```

output
```
(c**2*(6*atan(a*x)*a**4*x**4 - 12*atan(a*x)*a**2*x**2 - 2*atan(a*x) - 8*lo
g(a**2*x**2 + 1)*a**3*x**3 + 10*log(x)*a**3*x**3 - a*x))/(6*x**3)
```

# 3.165 $\int x^3(c+a^2cx^2)^3\arctan(ax)\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 141

$$\begin{aligned}\int x^3(c+a^2cx^2)^3\arctan(ax)\,dx &= \frac{c^3x}{40a^3}-\frac{c^3x^3}{120a}-\frac{9}{200}ac^3x^5-\frac{11}{280}a^3c^3x^7\\&\quad-\frac{1}{90}a^5c^3x^9-\frac{c^3\arctan(ax)}{40a^4}\\&\quad+\frac{1}{4}c^3x^4\arctan(ax)+\frac{1}{2}a^2c^3x^6\arctan(ax)\\&\quad+\frac{3}{8}a^4c^3x^8\arctan(ax)+\frac{1}{10}a^6c^3x^{10}\arctan(ax)\end{aligned}$$

output
```
1/40*c^3*x/a^3-1/120*c^3*x^3/a-9/200*a*c^3*x^5-11/280*a^3*c^3*x^7-1/90*a^5
*c^3*x^9-1/40*c^3*arctan(a*x)/a^4+1/4*c^3*x^4*arctan(a*x)+1/2*a^2*c^3*x^6*
arctan(a*x)+3/8*a^4*c^3*x^8*arctan(a*x)+1/10*a^6*c^3*x^10*arctan(a*x)
```

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00

$$\int x^3 \left(c+a^2 c x^2\right)^3 \arctan(ax)\, dx = \frac{c^3 x}{40a^3} - \frac{c^3 x^3}{120a} - \frac{9}{200} a c^3 x^5 - \frac{11}{280} a^3 c^3 x^7 - \frac{1}{90} a^5 c^3 x^9 - \frac{c^3 \arctan(ax)}{40a^4} + \frac{1}{4} c^3 x^4 \arctan(ax) + \frac{1}{2} a^2 c^3 x^6 \arctan(ax) + \frac{3}{8} a^4 c^3 x^8 \arctan(ax) + \frac{1}{10} a^6 c^3 x^{10} \arctan(ax)$$

input
```
Integrate[x^3*(c + a^2*c*x^2)^3*ArcTan[a*x],x]
```

output
```
(c^3*x)/(40*a^3) - (c^3*x^3)/(120*a) - (9*a*c^3*x^5)/200 - (11*a^3*c^3*x^7
)/280 - (a^5*c^3*x^9)/90 - (c^3*ArcTan[a*x])/(40*a^4) + (c^3*x^4*ArcTan[a*
x])/4 + (a^2*c^3*x^6*ArcTan[a*x])/2 + (3*a^4*c^3*x^8*ArcTan[a*x])/8 + (a^6
*c^3*x^10*ArcTan[a*x])/10
```

## Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \arctan(ax) \left(a^2 c x^2 + c\right)^3 dx$$

$$\Big\downarrow \text{5483}$$

$$\int \left(a^6 c^3 x^9 \arctan(ax) + 3a^4 c^3 x^7 \arctan(ax) + 3a^2 c^3 x^5 \arctan(ax) + c^3 x^3 \arctan(ax)\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{10}a^6c^3x^{10}\arctan(ax)-\frac{1}{90}a^5c^3x^9+\frac{3}{8}a^4c^3x^8\arctan(ax)-\frac{c^3\arctan(ax)}{40a^4}-\frac{11}{280}a^3c^3x^7+\frac{c^3x}{40a^3}+\frac{1}{2}a^2c^3x^6\arctan(ax)+\frac{1}{4}c^3x^4\arctan(ax)-\frac{9}{200}ac^3x^5-\frac{c^3x^3}{120a}$$

input
```
Int[x^3*(c + a^2*c*x^2)^3*ArcTan[a*x],x]
```

output
```
(c^3*x)/(40*a^3) - (c^3*x^3)/(120*a) - (9*a*c^3*x^5)/200 - (11*a^3*c^3*x^7
)/280 - (a^5*c^3*x^9)/90 - (c^3*ArcTan[a*x])/(40*a^4) + (c^3*x^4*ArcTan[a*
x])/4 + (a^2*c^3*x^6*ArcTan[a*x])/2 + (3*a^4*c^3*x^8*ArcTan[a*x])/8 + (a^6
*c^3*x^10*ArcTan[a*x])/10
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

## Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.79

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{c^3\arctan(ax)a^{10}x^{10}}{10}+\frac{3c^3\arctan(ax)a^8x^8}{8}+\frac{a^6c^3x^6\arctan(ax)}{2}+\frac{a^4c^3x^4\arctan(ax)}{4}-\frac{c^3\left(\frac{4a^9x^9}{9}+\frac{11a^7x^7}{7}+\frac{9a^5x^5}{5}+\frac{a^3x^3}{3}-ax\right.}{40}}{a^4}$ |
| default | $\frac{\frac{c^3\arctan(ax)a^{10}x^{10}}{10}+\frac{3c^3\arctan(ax)a^8x^8}{8}+\frac{a^6c^3x^6\arctan(ax)}{2}+\frac{a^4c^3x^4\arctan(ax)}{4}-\frac{c^3\left(\frac{4a^9x^9}{9}+\frac{11a^7x^7}{7}+\frac{9a^5x^5}{5}+\frac{a^3x^3}{3}-ax\right.}{40}}{a^4}$ |
| parts | $\frac{a^6c^3x^{10}\arctan(ax)}{10}+\frac{3a^4c^3x^8\arctan(ax)}{8}+\frac{a^2c^3x^6\arctan(ax)}{2}+\frac{c^3x^4\arctan(ax)}{4}-\frac{a\,c^3\left(\frac{4}{9}a^8x^9+\frac{11}{7}a^6x^7+\frac{9}{5}\right.}{a^4}$ |
| parallelrisch | $\frac{1260c^3\arctan(ax)a^{10}x^{10}-140a^9c^3x^9+4725c^3\arctan(ax)a^8x^8-495a^7c^3x^7+6300a^6c^3x^6\arctan(ax)-567a^5c^3x^5+315}{12600a^4}$ |
| risch | $-\frac{ic^3x^4(4a^6x^6+15a^4x^4+20a^2x^2+10)\ln(iax+1)}{80}+\frac{ic^3a^6x^{10}\ln(-iax+1)}{20}-\frac{a^5c^3x^9}{90}+\frac{3ic^3a^4x^8\ln(-iax+1)}{16}-$ |
| orering | $\frac{(210a^{10}x^{10}+800a^8x^8+1074a^6x^6+483a^4x^4-210a^2x^2-105)(a^2cx^2+c)^3\arctan(ax)}{1050a^4(a^2x^2+1)^3}-\frac{(140a^8x^8+495a^6x^6+567a^4x}{}$ |
| meijerg | $\frac{c^3\left(-\frac{2xa(385a^8x^8-495a^6x^6+693a^4x^4-1155a^2x^2+3465)}{17325}+\frac{2xa(11a^{10}x^{10}+11)\arctan(\sqrt{a^2x^2})}{55\sqrt{a^2x^2}}\right)}{4a^4}+\frac{3c^3\left(\frac{xa(-45a^6x^6+63a}{}\right.}{}$ |

input
```
int(x^3*(a^2*c*x^2+c)^3*arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/a^4*(1/10*c^3*arctan(a*x)*a^10*x^10+3/8*c^3*arctan(a*x)*a^8*x^8+1/2*a^6*
c^3*x^6*arctan(a*x)+1/4*a^4*c^3*x^4*arctan(a*x)-1/40*c^3*(4/9*a^9*x^9+11/7
*a^7*x^7+9/5*a^5*x^5+1/3*a^3*x^3-a*x+arctan(a*x)))
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.80

$$\int x^3(c+a^2cx^2)^3\arctan(ax)\,dx = -\frac{140\,a^9c^3x^9+495\,a^7c^3x^7+567\,a^5c^3x^5+105\,a^3c^3x^3-315\,ac^3x-315\,(4\,a^{10}c^3x^{10}+15\,a^8c^3x^8+20\,a^6c^3}{12600\,a^4}$$

input
```
integrate(x^3*(a^2*c*x^2+c)^3*arctan(a*x),x, algorithm="fricas")
```

output
```
-1/12600*(140*a^9*c^3*x^9 + 495*a^7*c^3*x^7 + 567*a^5*c^3*x^5 + 105*a^3*c^
3*x^3 - 315*a*c^3*x - 315*(4*a^10*c^3*x^10 + 15*a^8*c^3*x^8 + 20*a^6*c^3*x
^6 + 10*a^4*c^3*x^4 - c^3)*arctan(a*x))/a^4
```

## Sympy [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.98

$$\int x^3 (c + a^2 cx^2)^3 \arctan(ax)\,dx$$
$$= \begin{cases} \frac{a^6c^3x^{10}\operatorname{atan}(ax)}{10} - \frac{a^5c^3x^9}{90} + \frac{3a^4c^3x^8\operatorname{atan}(ax)}{8} - \frac{11a^3c^3x^7}{280} + \frac{a^2c^3x^6\operatorname{atan}(ax)}{2} - \frac{9ac^3x^5}{200} + \frac{c^3x^4\operatorname{atan}(ax)}{4} - \frac{c^3x^3}{120a} + \frac{c^3x}{40a^3} - \\ 0 \end{cases}$$

input
```
integrate(x**3*(a**2*c*x**2+c)**3*atan(a*x),x)
```

output
```
Piecewise((a**6*c**3*x**10*atan(a*x)/10 - a**5*c**3*x**9/90 + 3*a**4*c**3*
x**8*atan(a*x)/8 - 11*a**3*c**3*x**7/280 + a**2*c**3*x**6*atan(a*x)/2 - 9*
a*c**3*x**5/200 + c**3*x**4*atan(a*x)/4 - c**3*x**3/(120*a) + c**3*x/(40*a
**3) - c**3*atan(a*x)/(40*a**4), Ne(a, 0)), (0, True))
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.85

$$\int x^3 (c + a^2 cx^2)^3 \arctan(ax)\,dx =$$
$$-\frac{1}{12600}\,a\left(\frac{315\,c^3\arctan(ax)}{a^5} + \frac{140\,a^8c^3x^9 + 495\,a^6c^3x^7 + 567\,a^4c^3x^5 + 105\,a^2c^3x^3 - 315\,c^3x}{a^4}\right) + \frac{1}{40}\left(4\,a^6c^3x^{10} + 15\,a^4c^3x^8 + 20\,a^2c^3x^6 + 10\,c^3x^4\right)\arctan(ax)$$

input
```
integrate(x^3*(a^2*c*x^2+c)^3*arctan(a*x),x, algorithm="maxima")
```

output
```
-1/12600*a*(315*c^3*arctan(a*x)/a^5 + (140*a^8*c^3*x^9 + 495*a^6*c^3*x^7 +
 567*a^4*c^3*x^5 + 105*a^2*c^3*x^3 - 315*c^3*x)/a^4) + 1/40*(4*a^6*c^3*x^1
0 + 15*a^4*c^3*x^8 + 20*a^2*c^3*x^6 + 10*c^3*x^4)*arctan(a*x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.85

$$\int x^3(c+a^2cx^2)^3\arctan(ax)\,dx = \frac{1}{40}\left(4\,a^6c^3x^{10}+15\,a^4c^3x^8+20\,a^2c^3x^6+10\,c^3x^4\right)\arctan(ax) - \frac{c^3\arctan(ax)}{40\,a^4} - \frac{140\,a^{23}c^3x^9+495\,a^{21}c^3x^7+567\,a^{19}c^3x^5+105\,a^{17}c^3x^3-315\,a^{15}c^3x}{12600\,a^{18}}$$

input
```
integrate(x^3*(a^2*c*x^2+c)^3*arctan(a*x),x, algorithm="giac")
```

output
```
1/40*(4*a^6*c^3*x^10 + 15*a^4*c^3*x^8 + 20*a^2*c^3*x^6 + 10*c^3*x^4)*arcta
n(a*x) - 1/40*c^3*arctan(a*x)/a^4 - 1/12600*(140*a^23*c^3*x^9 + 495*a^21*c
^3*x^7 + 567*a^19*c^3*x^5 + 105*a^17*c^3*x^3 - 315*a^15*c^3*x)/a^18
```

## Mupad [B] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.79

$$\int x^3(c+a^2cx^2)^3\arctan(ax)\,dx = \mathrm{atan}(a\,x)\left(\frac{a^6\,c^3\,x^{10}}{10}+\frac{3\,a^4\,c^3\,x^8}{8}+\frac{a^2\,c^3\,x^6}{2}+\frac{c^3\,x^4}{4}\right) + \frac{c^3\,x}{40\,a^3} - \frac{9\,a\,c^3\,x^5}{200} - \frac{c^3\,\mathrm{atan}(a\,x)}{40\,a^4} - \frac{c^3\,x^3}{120\,a} - \frac{11\,a^3\,c^3\,x^7}{280} - \frac{a^5\,c^3\,x^9}{90}$$

input
```
int(x^3*atan(a*x)*(c + a^2*c*x^2)^3,x)
```

output
```
atan(a*x)*((c^3*x^4)/4 + (a^2*c^3*x^6)/2 + (3*a^4*c^3*x^8)/8 + (a^6*c^3*x^
10)/10) + (c^3*x)/(40*a^3) - (9*a*c^3*x^5)/200 - (c^3*atan(a*x))/(40*a^4)
- (c^3*x^3)/(120*a) - (11*a^3*c^3*x^7)/280 - (a^5*c^3*x^9)/90
```

## Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.70

$$\int x^3(c+a^2cx^2)^3\arctan(ax)\,dx$$
$$= \frac{c^3(1260atan(ax)\,a^{10}x^{10}+4725atan(ax)\,a^8x^8+6300atan(ax)\,a^6x^6+3150atan(ax)\,a^4x^4-315atan(ax)}{12600a^4}$$

input
```
int(x^3*(a^2*c*x^2+c)^3*atan(a*x),x)
```

output
```
(c**3*(1260*atan(a*x)*a**10*x**10 + 4725*atan(a*x)*a**8*x**8 + 6300*atan(a
*x)*a**6*x**6 + 3150*atan(a*x)*a**4*x**4 - 315*atan(a*x) - 140*a**9*x**9 -
 495*a**7*x**7 - 567*a**5*x**5 - 105*a**3*x**3 + 315*a*x))/(12600*a**4)
```

# 3.166 $\int x^2(c+a^2cx^2)^3 \arctan(ax)\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 136

$$\int x^2(c+a^2cx^2)^3 \arctan(ax)\,dx = -\frac{8c^3x^2}{315a}-\frac{89ac^3x^4}{1260}-\frac{10}{189}a^3c^3x^6-\frac{1}{72}a^5c^3x^8+\frac{1}{3}c^3x^3\arctan(ax)+\frac{3}{5}a^2c^3x^5\arctan(ax)+\frac{3}{7}a^4c^3x^7\arctan(ax)+\frac{1}{9}a^6c^3x^9\arctan(ax)+\frac{8c^3\log(1+a^2x^2)}{315a^3}$$

output

```
-8/315*c^3*x^2/a-89/1260*a*c^3*x^4-10/189*a^3*c^3*x^6-1/72*a^5*c^3*x^8+1/3
*c^3*x^3*arctan(a*x)+3/5*a^2*c^3*x^5*arctan(a*x)+3/7*a^4*c^3*x^7*arctan(a*
x)+1/9*a^6*c^3*x^9*arctan(a*x)+8/315*c^3*ln(a^2*x^2+1)/a^3
```

## Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00

$$\int x^2\left(c+a^2cx^2\right)^3\arctan(ax)\,dx = -\frac{8c^3x^2}{315a}-\frac{89ac^3x^4}{1260}-\frac{10}{189}a^3c^3x^6-\frac{1}{72}a^5c^3x^8+\frac{1}{3}c^3x^3\arctan(ax)+\frac{3}{5}a^2c^3x^5\arctan(ax)+\frac{3}{7}a^4c^3x^7\arctan(ax)+\frac{1}{9}a^6c^3x^9\arctan(ax)+\frac{8c^3\log\left(1+a^2x^2\right)}{315a^3}$$

input
```
Integrate[x^2*(c + a^2*c*x^2)^3*ArcTan[a*x],x]
```

output
```
(-8*c^3*x^2)/(315*a) - (89*a*c^3*x^4)/1260 - (10*a^3*c^3*x^6)/189 - (a^5*c
^3*x^8)/72 + (c^3*x^3*ArcTan[a*x])/3 + (3*a^2*c^3*x^5*ArcTan[a*x])/5 + (3*
a^4*c^3*x^7*ArcTan[a*x])/7 + (a^6*c^3*x^9*ArcTan[a*x])/9 + (8*c^3*Log[1 +
a^2*x^2])/(315*a^3)
```

## Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\arctan(ax)\left(a^2cx^2+c\right)^3\,dx$$

$$\Big\downarrow \text{5483}$$

$$\int \left(a^6c^3x^8\arctan(ax)+3a^4c^3x^6\arctan(ax)+3a^2c^3x^4\arctan(ax)+c^3x^2\arctan(ax)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{9}a^6c^3x^9\arctan(ax) - \frac{1}{72}a^5c^3x^8 + \frac{3}{7}a^4c^3x^7\arctan(ax) - \frac{10}{189}a^3c^3x^6 + \frac{3}{5}a^2c^3x^5\arctan(ax) + \frac{8c^3\log(a^2x^2+1)}{315a^3} + \frac{1}{3}c^3x^3\arctan(ax) - \frac{89ac^3x^4}{1260} - \frac{8c^3x^2}{315a}$$

input
```
Int[x^2*(c + a^2*c*x^2)^3*ArcTan[a*x],x]
```

output
```
(-8*c^3*x^2)/(315*a) - (89*a*c^3*x^4)/1260 - (10*a^3*c^3*x^6)/189 - (a^5*c
^3*x^8)/72 + (c^3*x^3*ArcTan[a*x])/3 + (3*a^2*c^3*x^5*ArcTan[a*x])/5 + (3*
a^4*c^3*x^7*ArcTan[a*x])/7 + (a^6*c^3*x^9*ArcTan[a*x])/9 + (8*c^3*Log[1 +
a^2*x^2])/(315*a^3)
```

**Defintions of rubi rules used**

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

## Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.85

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{c^3\arctan(ax)a^9x^9}{9}+\frac{3c^3\arctan(ax)a^7x^7}{7}+\frac{3a^5c^3x^5\arctan(ax)}{5}+\frac{a^3c^3x^3\arctan(ax)}{3}-\frac{c^3\left(\frac{35a^8x^8}{8}+\frac{50a^6x^6}{3}+\frac{89a^4x^4}{4}+8a^2x^2-8\right.}{315}}{a^3}$ |
| default | $\frac{\frac{c^3\arctan(ax)a^9x^9}{9}+\frac{3c^3\arctan(ax)a^7x^7}{7}+\frac{3a^5c^3x^5\arctan(ax)}{5}+\frac{a^3c^3x^3\arctan(ax)}{3}-\frac{c^3\left(\frac{35a^8x^8}{8}+\frac{50a^6x^6}{3}+\frac{89a^4x^4}{4}+8a^2x^2-8\right.}{315}}{a^3}$ |
| parts | $\frac{a^6c^3x^9\arctan(ax)}{9}+\frac{3a^4c^3x^7\arctan(ax)}{7}+\frac{3a^2c^3x^5\arctan(ax)}{5}+\frac{c^3x^3\arctan(ax)}{3}-\frac{a\,c^3\left(\frac{\frac{35}{4}a^6x^8+\frac{100}{3}a^4x^6+}{2a^2}\right.}{}$ |
| parallelrisch | $\frac{840c^3\arctan(ax)a^9x^9-105a^8c^3x^8+3240c^3\arctan(ax)a^7x^7-400a^6c^3x^6+4536a^5c^3x^5\arctan(ax)-534a^4c^3x^4+2520a}{7560a^3}$ |
| risch | $-\frac{ic^3x^3(35a^6x^6+135a^4x^4+189a^2x^2+105)\ln(iax+1)}{630}+\frac{ic^3a^6x^9\ln(-iax+1)}{18}-\frac{a^5c^3x^8}{72}+\frac{3ic^3a^4x^7\ln(-iax+1)}{14}$ |
| meijerg | $\frac{c^3\left(\frac{a^2x^2(-15a^6x^6+20a^4x^4-30a^2x^2+60)}{270}+\frac{4a^{10}x^{10}\arctan\left(\sqrt{a^2x^2}\right)}{9\sqrt{a^2x^2}}-\frac{2\ln(a^2x^2+1)}{9}\right)}{4a^3}+\frac{3c^3\left(-\frac{a^2x^2\left(4a^4x^4-6a^2x^2+12\right)}{42}\right.}{}$ |

input
```
int(x^2*(a^2*c*x^2+c)^3*arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(1/9*c^3*arctan(a*x)*a^9*x^9+3/7*c^3*arctan(a*x)*a^7*x^7+3/5*a^5*c^3
*x^5*arctan(a*x)+1/3*a^3*c^3*x^3*arctan(a*x)-1/315*c^3*(35/8*a^8*x^8+50/3*
a^6*x^6+89/4*a^4*x^4+8*a^2*x^2-8*ln(a^2*x^2+1)))
```

## Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.85

$$\int x^2(c+a^2cx^2)^3\arctan(ax)\,dx =$$
$$-\frac{105\,a^8c^3x^8+400\,a^6c^3x^6+534\,a^4c^3x^4+192\,a^2c^3x^2-192\,c^3\log\left(a^2x^2+1\right)-24\left(35\,a^9c^3x^9+135\,a^7c^3x\right.}{7560\,a^3}$$

input
```
integrate(x^2*(a^2*c*x^2+c)^3*arctan(a*x),x, algorithm="fricas")
```

output
```
-1/7560*(105*a^8*c^3*x^8 + 400*a^6*c^3*x^6 + 534*a^4*c^3*x^4 + 192*a^2*c^3
*x^2 - 192*c^3*log(a^2*x^2 + 1) - 24*(35*a^9*c^3*x^9 + 135*a^7*c^3*x^7 + 1
89*a^5*c^3*x^5 + 105*a^3*c^3*x^3)*arctan(a*x))/a^3
```

## Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.01

$$\int x^2(c+a^2cx^2)^3\arctan(ax)\,dx$$
$$=\begin{cases}\frac{a^6c^3x^9\operatorname{atan}(ax)}{9}-\frac{a^5c^3x^8}{72}+\frac{3a^4c^3x^7\operatorname{atan}(ax)}{7}-\frac{10a^3c^3x^6}{189}+\frac{3a^2c^3x^5\operatorname{atan}(ax)}{5}-\frac{89ac^3x^4}{1260}+\frac{c^3x^3\operatorname{atan}(ax)}{3}-\frac{8c^3x^2}{315a}+\frac{8c^3\mathrm{l}}{} \\ 0\end{cases}$$

input
```
integrate(x**2*(a**2*c*x**2+c)**3*atan(a*x),x)
```

output
```
Piecewise((a**6*c**3*x**9*atan(a*x)/9 - a**5*c**3*x**8/72 + 3*a**4*c**3*x*
*7*atan(a*x)/7 - 10*a**3*c**3*x**6/189 + 3*a**2*c**3*x**5*atan(a*x)/5 - 89
*a*c**3*x**4/1260 + c**3*x**3*atan(a*x)/3 - 8*c**3*x**2/(315*a) + 8*c**3*l
og(x**2 + a**(-2))/(315*a**3), Ne(a, 0)), (0, True))
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.87

$$\int x^2(c+a^2cx^2)^3\arctan(ax)\,dx$$
$$=\frac{1}{7560}a\left(\frac{192\,c^3\log(a^2x^2+1)}{a^4}-\frac{105\,a^6c^3x^8+400\,a^4c^3x^6+534\,a^2c^3x^4+192\,c^3x^2}{a^2}\right)$$
$$+\frac{1}{315}\left(35\,a^6c^3x^9+135\,a^4c^3x^7+189\,a^2c^3x^5+105\,c^3x^3\right)\arctan(ax)$$

input
```
integrate(x^2*(a^2*c*x^2+c)^3*arctan(a*x),x, algorithm="maxima")
```

output
```
1/7560*a*(192*c^3*log(a^2*x^2 + 1)/a^4 - (105*a^6*c^3*x^8 + 400*a^4*c^3*x^
6 + 534*a^2*c^3*x^4 + 192*c^3*x^2)/a^2) + 1/315*(35*a^6*c^3*x^9 + 135*a^4*
c^3*x^7 + 189*a^2*c^3*x^5 + 105*c^3*x^3)*arctan(a*x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.86

$$\int x^2 (c + a^2 c x^2)^3 \arctan(ax)\,dx$$
$$= \frac{1}{315}\left(35\,a^6 c^3 x^9 + 135\,a^4 c^3 x^7 + 189\,a^2 c^3 x^5 + 105\,c^3 x^3\right)\arctan(ax) + \frac{8\,c^3 \log(a^2 x^2 + 1)}{315\,a^3} - \frac{105\,a^{13} c^3 x^8 + 400\,a^{11} c^3 x^6 + 534\,a^9 c^3 x^4 + 192\,a^7 c^3 x^2}{7560\,a^8}$$

input
```
integrate(x^2*(a^2*c*x^2+c)^3*arctan(a*x),x, algorithm="giac")
```

output
```
1/315*(35*a^6*c^3*x^9 + 135*a^4*c^3*x^7 + 189*a^2*c^3*x^5 + 105*c^3*x^3)*a
rctan(a*x) + 8/315*c^3*log(a^2*x^2 + 1)/a^3 - 1/7560*(105*a^13*c^3*x^8 + 4
00*a^11*c^3*x^6 + 534*a^9*c^3*x^4 + 192*a^7*c^3*x^2)/a^8
```

## Mupad [B] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.79

$$\int x^2 (c + a^2 c x^2)^3 \arctan(ax)\,dx = \operatorname{atan}(a\,x)\left(\frac{a^6\,c^3\,x^9}{9} + \frac{3\,a^4\,c^3\,x^7}{7} + \frac{3\,a^2\,c^3\,x^5}{5} + \frac{c^3\,x^3}{3}\right) - \frac{89\,a\,c^3\,x^4}{1260} + \frac{8\,c^3\,\ln(a^2\,x^2 + 1)}{315\,a^3} - \frac{8\,c^3\,x^2}{315\,a} - \frac{10\,a^3\,c^3\,x^6}{189} - \frac{a^5\,c^3\,x^8}{72}$$

input
```
int(x^2*atan(a*x)*(c + a^2*c*x^2)^3,x)
```

output
```
atan(a*x)*((c^3*x^3)/3 + (3*a^2*c^3*x^5)/5 + (3*a^4*c^3*x^7)/7 + (a^6*c^3*
x^9)/9) - (89*a*c^3*x^4)/1260 + (8*c^3*log(a^2*x^2 + 1))/(315*a^3) - (8*c^
3*x^2)/(315*a) - (10*a^3*c^3*x^6)/189 - (a^5*c^3*x^8)/72
```

## Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.74

$$\int x^2(c+a^2cx^2)^3 \arctan(ax)\,dx$$
$$= \frac{c^3(840atan(ax)\,a^9x^9 + 3240atan(ax)\,a^7x^7 + 4536atan(ax)\,a^5x^5 + 2520atan(ax)\,a^3x^3 + 192\log(a^2x^2 +}{7560a^3}$$

input
```
int(x^2*(a^2*c*x^2+c)^3*atan(a*x),x)
```

output
```
(c**3*(840*atan(a*x)*a**9*x**9 + 3240*atan(a*x)*a**7*x**7 + 4536*atan(a*x)
*a**5*x**5 + 2520*atan(a*x)*a**3*x**3 + 192*log(a**2*x**2 + 1) - 105*a**8*
x**8 - 400*a**6*x**6 - 534*a**4*x**4 - 192*a**2*x**2))/(7560*a**3)
```

# 3.167 $\int x(c+a^2cx^2)^3 \arctan(ax)\,dx$



## Optimal result

Integrand size = 18, antiderivative size = 74

$$\int x(c+a^2cx^2)^3 \arctan(ax)\,dx = -\frac{c^3x}{8a}-\frac{1}{8}ac^3x^3-\frac{3}{40}a^3c^3x^5-\frac{1}{56}a^5c^3x^7+\frac{c^3(1+a^2x^2)^4\arctan(ax)}{8a^2}$$

output
```
-1/8*c^3*x/a-1/8*a*c^3*x^3-3/40*a^3*c^3*x^5-1/56*a^5*c^3*x^7+1/8*c^3*(a^2*
x^2+1)^4*arctan(a*x)/a^2
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.73

$$\int x(c+a^2cx^2)^3 \arctan(ax)\,dx = -\frac{c^3x}{8a}-\frac{1}{8}ac^3x^3-\frac{3}{40}a^3c^3x^5-\frac{1}{56}a^5c^3x^7+\frac{c^3\arctan(ax)}{8a^2}+\frac{1}{2}c^3x^2\arctan(ax)+\frac{3}{4}a^2c^3x^4\arctan(ax)+\frac{1}{2}a^4c^3x^6\arctan(ax)+\frac{1}{8}a^6c^3x^8\arctan(ax)$$

input
```
Integrate[x*(c + a^2*c*x^2)^3*ArcTan[a*x],x]
```

output
```
-1/8*(c^3*x)/a - (a*c^3*x^3)/8 - (3*a^3*c^3*x^5)/40 - (a^5*c^3*x^7)/56 + (
c^3*ArcTan[a*x])/(8*a^2) + (c^3*x^2*ArcTan[a*x])/2 + (3*a^2*c^3*x^4*ArcTan
[a*x])/4 + (a^4*c^3*x^6*ArcTan[a*x])/2 + (a^6*c^3*x^8*ArcTan[a*x])/8
```

## Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5465, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \arctan(ax) \left(a^2cx^2+c\right)^3 dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{c^3\left(a^2x^2+1\right)^4\arctan(ax)}{8a^2}-\frac{\int \left(a^2cx^2+c\right)^3dx}{8a}$$

$$\Big\downarrow \text{210}$$

$$\frac{c^3\left(a^2x^2+1\right)^4\arctan(ax)}{8a^2}-\frac{\int \left(a^6c^3x^6+3a^4c^3x^4+3a^2c^3x^2+c^3\right)dx}{8a}$$

$$\Big\downarrow \text{2009}$$

$$\frac{c^3\left(a^2x^2+1\right)^4\arctan(ax)}{8a^2}-\frac{\frac{1}{7}a^6c^3x^7+\frac{3}{5}a^4c^3x^5+a^2c^3x^3+c^3x}{8a}$$

input
```
Int[x*(c + a^2*c*x^2)^3*ArcTan[a*x],x]
```

output
```
-1/8*(c^3*x + a^2*c^3*x^3 + (3*a^4*c^3*x^5)/5 + (a^6*c^3*x^7)/7)/a + (c^3*
(1 + a^2*x^2)^4*ArcTan[a*x])/(8*a^2)
```

### Defintions of rubi rules used

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

## Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.41

| method | result |
|---|---|
| parts | $\frac{c^3\arctan(ax)a^6x^8}{8}+\frac{c^3\arctan(ax)a^4x^6}{2}+\frac{3c^3\arctan(ax)a^2x^4}{4}+\frac{c^3\arctan(ax)x^2}{2}+\frac{c^3\arctan(ax)}{8a^2}-\frac{c^3\left(\frac{1}{7}a^6\right.}{}$ |
| derivativedivides | $\frac{\frac{c^3\arctan(ax)a^8x^8}{8}+\frac{a^6c^3x^6\arctan(ax)}{2}+\frac{3a^4c^3x^4\arctan(ax)}{4}+\frac{a^2c^3x^2\arctan(ax)}{2}+\frac{c^3\arctan(ax)}{8}-\frac{c^3\left(\frac{1}{7}a^7x^7+\frac{3}{5}a^5x^5+a^3x^3+\right.}{8}}{a^2}$ |
| default | $\frac{\frac{c^3\arctan(ax)a^8x^8}{8}+\frac{a^6c^3x^6\arctan(ax)}{2}+\frac{3a^4c^3x^4\arctan(ax)}{4}+\frac{a^2c^3x^2\arctan(ax)}{2}+\frac{c^3\arctan(ax)}{8}-\frac{c^3\left(\frac{1}{7}a^7x^7+\frac{3}{5}a^5x^5+a^3x^3+\right.}{8}}{a^2}$ |
| parallelrisch | $\frac{35c^3\arctan(ax)a^8x^8-5a^7c^3x^7+140a^6c^3x^6\arctan(ax)-21a^5c^3x^5+210a^4c^3x^4\arctan(ax)-35c^3x^3a^3+140a^2c^3x^2\,\mathrm{arc}}{280a^2}$ |
| risch | $-\frac{ic^3\left(a^2x^2+1\right)^4\ln(iax+1)}{16a^2}+\frac{ic^3a^6x^8\ln(-iax+1)}{16}-\frac{a^5c^3x^7}{56}+\frac{ic^3a^4x^6\ln(-iax+1)}{4}-\frac{3a^3c^3x^5}{40}+\frac{3ic^3a^2x^4\ln}{}$ |
| orering | $\frac{\left(35a^8x^8+146a^6x^6+238a^4x^4+210a^2x^2+35\right)\left(a^2c\,x^2+c\right)^3\arctan(ax)}{140a^2\left(a^2x^2+1\right)^3}-\frac{\left(5a^6x^6+21a^4x^4+35a^2x^2+35\right)\left(\left(a^2c\,x^2+c\right)^3\right.}{}$ |
| meijerg | $\frac{c^3\left(\frac{xa\left(-45a^6x^6+63a^4x^4-105a^2x^2+315\right)}{630}-\frac{xa\left(-9a^8x^8+9\right)\arctan\left(\sqrt{a^2x^2}\right)}{18\sqrt{a^2x^2}}\right)}{4a^2}+\frac{3c^3\left(-\frac{2ax\left(21a^4x^4-35a^2x^2+105\right)}{315}+\frac{2ax(}{}\right.}{4a^2}$ |

input
```
int(x*(a^2*c*x^2+c)^3*arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/8*c^3*arctan(a*x)*a^6*x^8+1/2*c^3*arctan(a*x)*a^4*x^6+3/4*c^3*arctan(a*x
)*a^2*x^4+1/2*c^3*arctan(a*x)*x^2+1/8*c^3/a^2*arctan(a*x)-1/8*c^3/a*(1/7*a
^6*x^7+3/5*a^4*x^5+a^2*x^3+x)
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.34

$$\int x(c+a^2cx^2)^3\arctan(ax)\,dx = -\frac{5\,a^7c^3x^7+21\,a^5c^3x^5+35\,a^3c^3x^3+35\,ac^3x-35\,(a^8c^3x^8+4\,a^6c^3x^6+6\,a^4c^3x^4+4\,a^2c^3x^2+c^3)\arctan}{280\,a^2}$$

input
```
integrate(x*(a^2*c*x^2+c)^3*arctan(a*x),x, algorithm="fricas")
```

output
```
-1/280*(5*a^7*c^3*x^7 + 21*a^5*c^3*x^5 + 35*a^3*c^3*x^3 + 35*a*c^3*x - 35*
(a^8*c^3*x^8 + 4*a^6*c^3*x^6 + 6*a^4*c^3*x^4 + 4*a^2*c^3*x^2 + c^3)*arctan
(a*x))/a^2
```

## Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.68

$$\int x(c+a^2cx^2)^3\arctan(ax)\,dx = \begin{cases} \frac{a^6c^3x^8\operatorname{atan}(ax)}{8}-\frac{a^5c^3x^7}{56}+\frac{a^4c^3x^6\operatorname{atan}(ax)}{2}-\frac{3a^3c^3x^5}{40}+\frac{3a^2c^3x^4\operatorname{atan}(ax)}{4}-\frac{ac^3x^3}{8}+\frac{c^3x^2\operatorname{atan}(ax)}{2}-\frac{c^3x}{8a}+\frac{c^3\operatorname{atan}(ax)}{8a^2} \\ 0 \end{cases}$$

input
```
integrate(x*(a**2*c*x**2+c)**3*atan(a*x),x)
```

output
```
Piecewise((a**6*c**3*x**8*atan(a*x)/8 - a**5*c**3*x**7/56 + a**4*c**3*x**6
*atan(a*x)/2 - 3*a**3*c**3*x**5/40 + 3*a**2*c**3*x**4*atan(a*x)/4 - a*c**3
*x**3/8 + c**3*x**2*atan(a*x)/2 - c**3*x/(8*a) + c**3*atan(a*x)/(8*a**2),
Ne(a, 0)), (0, True))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.99

$$\int x\left(c+a^2cx^2\right)^3\arctan(ax)\,dx=\frac{\left(a^2cx^2+c\right)^4\arctan\left(ax\right)}{8\,a^2c}-\frac{5\,a^6c^4x^7+21\,a^4c^4x^5+35\,a^2c^4x^3+35\,c^4x}{280\,ac}$$

input
```
integrate(x*(a^2*c*x^2+c)^3*arctan(a*x),x, algorithm="maxima")
```

output
```
1/8*(a^2*c*x^2 + c)^4*arctan(a*x)/(a^2*c) - 1/280*(5*a^6*c^4*x^7 + 21*a^4*
c^4*x^5 + 35*a^2*c^4*x^3 + 35*c^4*x)/(a*c)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.95

$$\int x\left(c+a^2cx^2\right)^3\arctan(ax)\,dx=\frac{\left(a^2cx^2+c\right)^4\arctan\left(ax\right)}{8\,a^2c}-\frac{5\,a^6c^3x^7+21\,a^4c^3x^5+35\,a^2c^3x^3+35\,c^3x}{280\,a}$$

input
```
integrate(x*(a^2*c*x^2+c)^3*arctan(a*x),x, algorithm="giac")
```

output
```
1/8*(a^2*c*x^2 + c)^4*arctan(a*x)/(a^2*c) - 1/280*(5*a^6*c^3*x^7 + 21*a^4*
c^3*x^5 + 35*a^2*c^3*x^3 + 35*c^3*x)/a
```

## Mupad [B] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.35

$$\int x(c+a^2cx^2)^3\arctan(ax)\,dx = \operatorname{atan}(a\,x)\,\left(\frac{a^6\,c^3\,x^8}{8}+\frac{a^4\,c^3\,x^6}{2}+\frac{3\,a^2\,c^3\,x^4}{4}+\frac{c^3\,x^2}{2}\right) - \frac{c^3\,x}{8\,a}-\frac{a\,c^3\,x^3}{8}+\frac{c^3\operatorname{atan}(a\,x)}{8\,a^2}-\frac{3\,a^3\,c^3\,x^5}{40}-\frac{a^5\,c^3\,x^7}{56}$$

input
```
int(x*atan(a*x)*(c + a^2*c*x^2)^3,x)
```

output
```
atan(a*x)*((c^3*x^2)/2 + (3*a^2*c^3*x^4)/4 + (a^4*c^3*x^6)/2 + (a^6*c^3*x^
8)/8) - (c^3*x)/(8*a) - (a*c^3*x^3)/8 + (c^3*atan(a*x))/(8*a^2) - (3*a^3*c
^3*x^5)/40 - (a^5*c^3*x^7)/56
```

## Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.23

$$\int x(c+a^2cx^2)^3\arctan(ax)\,dx = \frac{c^3(35atan(ax)\,a^8x^8+140atan(ax)\,a^6x^6+210atan(ax)\,a^4x^4+140atan(ax)\,a^2x^2+35atan(ax)-5a^7x^7}{280a^2}$$

input
```
int(x*(a^2*c*x^2+c)^3*atan(a*x),x)
```

output
```
(c**3*(35*atan(a*x)*a**8*x**8 + 140*atan(a*x)*a**6*x**6 + 210*atan(a*x)*a*
*4*x**4 + 140*atan(a*x)*a**2*x**2 + 35*atan(a*x) - 5*a**7*x**7 - 21*a**5*x
**5 - 35*a**3*x**3 - 35*a*x))/(280*a**2)
```

# 3.168 $\int (c+a^2cx^2)^3 \arctan(ax)\,dx$



## Optimal result

Integrand size = 17, antiderivative size = 161

$$\begin{aligned}\int (c+a^2cx^2)^3 \arctan(ax)\,dx = &-\frac{4c^3(1+a^2x^2)}{35a}-\frac{3c^3(1+a^2x^2)^2}{70a}-\frac{c^3(1+a^2x^2)^3}{42a}\\&+\frac{16}{35}c^3x\arctan(ax)+\frac{8}{35}c^3x(1+a^2x^2)\arctan(ax)\\&+\frac{6}{35}c^3x(1+a^2x^2)^2\arctan(ax)\\&+\frac{1}{7}c^3x(1+a^2x^2)^3\arctan(ax)-\frac{8c^3\log(1+a^2x^2)}{35a}\end{aligned}$$

output
```
-4/35*c^3*(a^2*x^2+1)/a-3/70*c^3*(a^2*x^2+1)^2/a-1/42*c^3*(a^2*x^2+1)^3/a+
16/35*c^3*x*arctan(a*x)+8/35*c^3*x*(a^2*x^2+1)*arctan(a*x)+6/35*c^3*x*(a^2
*x^2+1)^2*arctan(a*x)+1/7*c^3*x*(a^2*x^2+1)^3*arctan(a*x)-8/35*c^3*ln(a^2*
x^2+1)/a
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.52

$$\int (c+a^2cx^2)^3 \arctan(ax)\,dx = \frac{c^3(-a^2x^2(57+24a^2x^2+5a^4x^4)+6ax(35+35a^2x^2+21a^4x^4+5a^6x^6)\arctan(ax)-48\log(1+a^2x^2))}{210a}$$

input
```
Integrate[(c + a^2*c*x^2)^3*ArcTan[a*x],x]
```

output
```
(c^3*(-(a^2*x^2*(57 + 24*a^2*x^2 + 5*a^4*x^4)) + 6*a*x*(35 + 35*a^2*x^2 +
21*a^4*x^4 + 5*a^6*x^6)*ArcTan[a*x] - 48*Log[1 + a^2*x^2]))/(210*a)
```

## Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {5413, 27, 5413, 5413, 5345, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(ax)\left(a^2cx^2+c\right)^3\,dx$$

$$\Big\downarrow \text{ 5413}$$

$$\frac{6}{7}c\int c^2\left(a^2x^2+1\right)^2\arctan(ax)dx+\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)-\frac{c^3\left(a^2x^2+1\right)^3}{42a}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{6}{7}c^3\int \left(a^2x^2+1\right)^2\arctan(ax)dx+\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)-\frac{c^3\left(a^2x^2+1\right)^3}{42a}$$

$$\Big\downarrow \text{ 5413}$$

$$\frac{6}{7}c^3\left(\frac{4}{5}\int \left(a^2x^2+1\right)\arctan(ax)dx+\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan(ax)-\frac{\left(a^2x^2+1\right)^2}{20a}\right)+$$
$$\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)-\frac{c^3\left(a^2x^2+1\right)^3}{42a}$$

$\downarrow$ 5413

$$\frac{6}{7}c^3\left(\frac{4}{5}\left(\frac{2}{3}\int \arctan(ax)dx+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)-\frac{a^2x^2+1}{6a}\right)+\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan(ax)-\frac{\left(a^2x^2+1\right)}{20a}\right.$$
$$\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)-\frac{c^3\left(a^2x^2+1\right)^3}{42a}$$

$\downarrow$ 5345

$$\frac{6}{7}c^3\left(\frac{4}{5}\left(\frac{2}{3}\left(x\arctan(ax)-a\int \frac{x}{a^2x^2+1}dx\right)+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)-\frac{a^2x^2+1}{6a}\right)+\frac{1}{5}x\left(a^2x^2+1\right)^2\text{arctan}\right.$$
$$\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)-\frac{c^3\left(a^2x^2+1\right)^3}{42a}$$

$\downarrow$ 240

$$\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)+$$
$$\frac{6}{7}c^3\left(\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan(ax)+\frac{4}{5}\left(\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)+\frac{2}{3}\left(x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}\right)-\frac{a^2x^2+}{6a}\right.\right.$$
$$\frac{c^3\left(a^2x^2+1\right)^3}{42a}$$

input
```
Int[(c + a^2*c*x^2)^3*ArcTan[a*x],x]
```

output
```
-1/42*(c^3*(1 + a^2*x^2)^3)/a + (c^3*x*(1 + a^2*x^2)^3*ArcTan[a*x])/7 + (6
*c^3*(-1/20*(1 + a^2*x^2)^2/a + (x*(1 + a^2*x^2)^2*ArcTan[a*x])/5 + (4*(-1
/6*(1 + a^2*x^2)/a + (x*(1 + a^2*x^2)*ArcTan[a*x])/3 + (2*(x*ArcTan[a*x] -
 Log[1 + a^2*x^2]/(2*a)))/3))/5))/7
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5413
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Simp[x*(d + e*x^2)
^q*((a + b*ArcTan[c*x])/(2*q + 1)), x] + Simp[2*d*(q/(2*q + 1))   Int[(d +
e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0]
```

## Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.61

| method | result |
|---|---|
| parts | $\frac{c^3 \arctan(ax) a^6 x^7}{7}+\frac{3c^3 \arctan(ax) a^4 x^5}{5}+c^3 \arctan(ax) a^2 x^3+c^3 x \arctan(ax)-\frac{a c^3 \left(\frac{5a^4x^6}{6}+4 \right.}{}$ |
| derivativedivides | $\frac{\frac{c^3 \arctan(ax) a^7 x^7}{7}+\frac{3a^5c^3x^5 \arctan(ax)}{5}+a^3c^3x^3 \arctan(ax)+a c^3 x \arctan(ax)-\frac{c^3\left(\frac{5a^6x^6}{6}+4a^4x^4+\frac{19a^2x^2}{2}+8\ln\left(a^2x^2+1\right.\right.}{35}}{a}$ |
| default | $\frac{\frac{c^3 \arctan(ax) a^7 x^7}{7}+\frac{3a^5c^3x^5 \arctan(ax)}{5}+a^3c^3x^3 \arctan(ax)+a c^3 x \arctan(ax)-\frac{c^3\left(\frac{5a^6x^6}{6}+4a^4x^4+\frac{19a^2x^2}{2}+8\ln\left(a^2x^2+1\right.\right.}{35}}{a}$ |
| parallelrisch | $-\frac{-30c^3 \arctan(ax) a^7 x^7+5a^6c^3x^6-126a^5c^3x^5 \arctan(ax)+24a^4c^3x^4-210a^3c^3x^3 \arctan(ax)+57a^2c^3x^2-210a c^3 x a}{210a}$ |
| risch | $-\frac{i c^3 x\left(5a^6x^6+21a^4x^4+35a^2x^2+35\right)\ln(iax+1)}{70}+\frac{i c^3 a^6 x^7 \ln(-iax+1)}{14}-\frac{a^5c^3x^6}{42}+\frac{3i c^3 a^4 x^5 \ln(-iax+1)}{10}-4$ |
| meijerg | $\frac{c^3\left(-\frac{a^2x^2\left(4a^4x^4-6a^2x^2+12\right)}{42}+\frac{4a^8x^8 \arctan\left(\sqrt{a^2x^2}\right)}{7\sqrt{a^2x^2}}+\frac{2\ln\left(a^2x^2+1\right)}{7}\right)}{4a}+\frac{3c^3\left(\frac{a^2x^2\left(-3a^2x^2+6\right)}{15}+\frac{4a^6x^6 \arctan\left(\sqrt{a^2x\right.\right.}{5\sqrt{a^2x^2}}}{4a}$ |

input
```
int((a^2*c*x^2+c)^3*arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/7*c^3*arctan(a*x)*a^6*x^7+3/5*c^3*arctan(a*x)*a^4*x^5+c^3*arctan(a*x)*a^
2*x^3+c^3*x*arctan(a*x)-1/35*a*c^3*(5/6*a^4*x^6+4*a^2*x^4+19/2*x^2+8/a^2*l
n(a^2*x^2+1))
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.63

$$\int (c+a^2cx^2)^3 \arctan(ax)\,dx = -\frac{5\,a^6c^3x^6+24\,a^4c^3x^4+57\,a^2c^3x^2+48\,c^3\log\left(a^2x^2+1\right)-6\left(5\,a^7c^3x^7+21\,a^5c^3x^5+35\,a^3c^3x^3+35\,ac^3}{210\,a}$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x),x, algorithm="fricas")
```

output
```
-1/210*(5*a^6*c^3*x^6 + 24*a^4*c^3*x^4 + 57*a^2*c^3*x^2 + 48*c^3*log(a^2*x
^2 + 1) - 6*(5*a^7*c^3*x^7 + 21*a^5*c^3*x^5 + 35*a^3*c^3*x^3 + 35*a*c^3*x)
*arctan(a*x))/a
```

## Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.73

$$\int (c+a^2cx^2)^3 \arctan(ax)\,dx = \begin{cases} \frac{a^6c^3x^7\operatorname{atan}(ax)}{7} - \frac{a^5c^3x^6}{42} + \frac{3a^4c^3x^5\operatorname{atan}(ax)}{5} - \frac{4a^3c^3x^4}{35} + a^2c^3x^3\operatorname{atan}(ax) - \frac{19ac^3x^2}{70} + c^3x\operatorname{atan}(ax) - \frac{8c^3\log(}{35} \\ 0 \end{cases}$$

input
```
integrate((a**2*c*x**2+c)**3*atan(a*x),x)
```

output
```
Piecewise((a**6*c**3*x**7*atan(a*x)/7 - a**5*c**3*x**6/42 + 3*a**4*c**3*x*
*5*atan(a*x)/5 - 4*a**3*c**3*x**4/35 + a**2*c**3*x**3*atan(a*x) - 19*a*c**
3*x**2/70 + c**3*x*atan(a*x) - 8*c**3*log(x**2 + a**(-2))/(35*a), Ne(a, 0)
), (0, True))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.61

$$\int (c+a^2cx^2)^3 \arctan(ax)\,dx = -\frac{1}{210}\left(5\,a^4c^3x^6+24\,a^2c^3x^4+57\,c^3x^2+\frac{48\,c^3\log\left(a^2x^2+1\right)}{a^2}\right)a + \frac{1}{35}\left(5\,a^6c^3x^7+21\,a^4c^3x^5+35\,a^2c^3x^3+35\,c^3x\right)\arctan\left(ax\right)$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x),x, algorithm="maxima")
```

output
```
-1/210*(5*a^4*c^3*x^6 + 24*a^2*c^3*x^4 + 57*c^3*x^2 + 48*c^3*log(a^2*x^2 +
 1)/a^2)*a + 1/35*(5*a^6*c^3*x^7 + 21*a^4*c^3*x^5 + 35*a^2*c^3*x^3 + 35*c^
3*x)*arctan(a*x)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.65

$$\int (c+a^2cx^2)^3 \arctan(ax)\,dx = -\frac{8\,c^3\log\left(a^2x^2+1\right)}{35\,a}+\frac{1}{35}\left(5\,a^6c^3x^7+21\,a^4c^3x^5+35\,a^2c^3x^3+35\,c^3x\right)\arctan\left(ax\right) - \frac{5\,a^{11}c^3x^6+24\,a^9c^3x^4+57\,a^7c^3x^2}{210\,a^6}$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x),x, algorithm="giac")
```

output
```
-8/35*c^3*log(a^2*x^2 + 1)/a + 1/35*(5*a^6*c^3*x^7 + 21*a^4*c^3*x^5 + 35*a
^2*c^3*x^3 + 35*c^3*x)*arctan(a*x) - 1/210*(5*a^11*c^3*x^6 + 24*a^9*c^3*x^
4 + 57*a^7*c^3*x^2)/a^6
```

## Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.55

$$\int (c+a^2cx^2)^3 \arctan(ax)\,dx = -\frac{c^3\left(48\ln\left(a^2x^2+1\right)+57a^2x^2+24a^4x^4+5a^6x^6-210a^3x^3\,\mathrm{atan}(a\,x)-126a^5x^5\,\mathrm{atan}(a\,x)-30\,a^7\right.}{210\,a}$$

input
```
int(atan(a*x)*(c + a^2*c*x^2)^3,x)
```

output
```
-(c^3*(48*log(a^2*x^2 + 1) + 57*a^2*x^2 + 24*a^4*x^4 + 5*a^6*x^6 - 210*a^3
*x^3*atan(a*x) - 126*a^5*x^5*atan(a*x) - 30*a^7*x^7*atan(a*x) - 210*a*x*at
an(a*x)))/(210*a)
```

## Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.55

$$\int (c+a^2cx^2)^3 \arctan(ax)\,dx = \frac{c^3(30atan(ax)\,a^7x^7+126atan(ax)\,a^5x^5+210atan(ax)\,a^3x^3+210atan(ax)\,ax-48\log(a^2x^2+1)-5a^6}{210a}$$

input
```
int((a^2*c*x^2+c)^3*atan(a*x),x)
```

output
```
(c**3*(30*atan(a*x)*a**7*x**7 + 126*atan(a*x)*a**5*x**5 + 210*atan(a*x)*a*
*3*x**3 + 210*atan(a*x)*a*x - 48*log(a**2*x**2 + 1) - 5*a**6*x**6 - 24*a**
4*x**4 - 57*a**2*x**2))/(210*a)
```

# 3.169 $\int \frac{(c+a^2cx^2)^3 \arctan(ax)}{x} dx$



## Optimal result

Integrand size = 20, antiderivative size = 132

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)}{x} dx = -\frac{11}{12}ac^3x - \frac{7}{36}a^3c^3x^3 - \frac{1}{30}a^5c^3x^5 + \frac{11}{12}c^3\arctan(ax) + \frac{3}{2}a^2c^3x^2\arctan(ax) + \frac{3}{4}a^4c^3x^4\arctan(ax) + \frac{1}{6}a^6c^3x^6\arctan(ax) + \frac{1}{2}ic^3\,\text{PolyLog}(2,-iax) - \frac{1}{2}ic^3\,\text{PolyLog}(2,iax)$$

output
```
-11/12*a*c^3*x-7/36*a^3*c^3*x^3-1/30*a^5*c^3*x^5+11/12*c^3*arctan(a*x)+3/2
*a^2*c^3*x^2*arctan(a*x)+3/4*a^4*c^3*x^4*arctan(a*x)+1/6*a^6*c^3*x^6*arcta
n(a*x)+1/2*I*c^3*polylog(2,-I*a*x)-1/2*I*c^3*polylog(2,I*a*x)
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)}{x} dx = -\frac{11}{12}ac^3x - \frac{7}{36}a^3c^3x^3 - \frac{1}{30}a^5c^3x^5 + \frac{11}{12}c^3\arctan(ax) + \frac{3}{2}a^2c^3x^2\arctan(ax) + \frac{3}{4}a^4c^3x^4\arctan(ax) + \frac{1}{6}a^6c^3x^6\arctan(ax) + \frac{1}{2}ic^3\operatorname{PolyLog}(2,-iax) - \frac{1}{2}ic^3\operatorname{PolyLog}(2,iax)$$

input
```
Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x])/x,x]
```

output
```
(-11*a*c^3*x)/12 - (7*a^3*c^3*x^3)/36 - (a^5*c^3*x^5)/30 + (11*c^3*ArcTan[
a*x])/12 + (3*a^2*c^3*x^2*ArcTan[a*x])/2 + (3*a^4*c^3*x^4*ArcTan[a*x])/4 +
 (a^6*c^3*x^6*ArcTan[a*x])/6 + (I/2)*c^3*PolyLog[2, (-I)*a*x] - (I/2)*c^3*
PolyLog[2, I*a*x]
```

## Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)\left(a^2cx^2+c\right)^3}{x} dx$$

$$\Big\downarrow \text{5483}$$

$$\int \left(a^6c^3x^5\arctan(ax) + 3a^4c^3x^3\arctan(ax) + 3a^2c^3x\arctan(ax) + \frac{c^3\arctan(ax)}{x}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{6}a^6c^3x^6\arctan(ax)-\frac{1}{30}a^5c^3x^5+\frac{3}{4}a^4c^3x^4\arctan(ax)-\frac{7}{36}a^3c^3x^3+\frac{3}{2}a^2c^3x^2\arctan(ax)+\frac{11}{12}c^3\arctan(ax)+\frac{1}{2}ic^3\operatorname{PolyLog}(2,-iax)-\frac{1}{2}ic^3\operatorname{PolyLog}(2,iax)-\frac{11}{12}ac^3x$$

input

```
Int[((c + a^2*c*x^2)^3*ArcTan[a*x])/x,x]
```

output

```
(-11*a*c^3*x)/12 - (7*a^3*c^3*x^3)/36 - (a^5*c^3*x^5)/30 + (11*c^3*ArcTan[
a*x])/12 + (3*a^2*c^3*x^2*ArcTan[a*x])/2 + (3*a^4*c^3*x^4*ArcTan[a*x])/4 +
 (a^6*c^3*x^6*ArcTan[a*x])/6 + (I/2)*c^3*PolyLog[2, (-I)*a*x] - (I/2)*c^3*
PolyLog[2, I*a*x]
```

### Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

## Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.08

| method | result |
|---|---|
| derivativedivides | $\frac{a^6c^3x^6\arctan(ax)}{6}+\frac{3a^4c^3x^4\arctan(ax)}{4}+\frac{3a^2c^3x^2\arctan(ax)}{2}+c^3\arctan(ax)\ln(ax)-\frac{c^3\left(\frac{2a^5x^5}{5}+\right.}{}$ |
| default | $\frac{a^6c^3x^6\arctan(ax)}{6}+\frac{3a^4c^3x^4\arctan(ax)}{4}+\frac{3a^2c^3x^2\arctan(ax)}{2}+c^3\arctan(ax)\ln(ax)-\frac{c^3\left(\frac{2a^5x^5}{5}+\right.}{}$ |
| parts | $\frac{a^6c^3x^6\arctan(ax)}{6}+\frac{3a^4c^3x^4\arctan(ax)}{4}+\frac{3a^2c^3x^2\arctan(ax)}{2}+c^3\arctan(ax)\ln(x)-\frac{a\,c^3\left(\frac{2a^4x^5}{5}+\right.}{}$ |
| risch | $-\frac{11a\,c^3x}{12}-\frac{7c^3x^3a^3}{36}-\frac{a^5c^3x^5}{30}-\frac{ic^3\operatorname{dilog}(-iax+1)}{2}+\frac{11c^3\arctan(ax)}{12}+\frac{ic^3\ln(-iax+1)x^6a^6}{12}+\frac{3ic^3\ln(-}{}$ |
| meijerg | $\frac{c^3\left(-\frac{2ax\left(21a^4x^4-35a^2x^2+105\right)}{315}+\frac{2ax\left(7a^6x^6+7\right)\arctan\left(\sqrt{a^2x^2}\right)}{21\sqrt{a^2x^2}}\right)}{4}+\frac{3c^3\left(\frac{ax\left(-5a^2x^2+15\right)}{15}-\frac{ax\left(-5a^4x^4+5\right)\arctan\left(\sqrt{a\right.}{5\sqrt{a^2x^2}}\right.}{4}$ |

input
```
int((a^2*c*x^2+c)^3*arctan(a*x)/x,x,method=_RETURNVERBOSE)
```

output
```
1/6*a^6*c^3*x^6*arctan(a*x)+3/4*a^4*c^3*x^4*arctan(a*x)+3/2*a^2*c^3*x^2*ar
ctan(a*x)+c^3*arctan(a*x)*ln(a*x)-1/12*c^3*(2/5*a^5*x^5+7/3*a^3*x^3+11*a*x
-11*arctan(a*x)-6*I*ln(a*x)*ln(1+I*a*x)+6*I*ln(a*x)*ln(1-I*a*x)-6*I*dilog(
1+I*a*x)+6*I*dilog(1-I*a*x))
```

## Fricas [F]

$$\int \frac{(c+a^2cx^2)^3\arctan(ax)}{x}\,dx = \int \frac{(a^2cx^2+c)^3\arctan(ax)}{x}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)/x,x, algorithm="fricas")
```

output
```
integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a*x)/x
, x)
```

## Sympy [F]

$$\int \frac{(c+a^2cx^2)^3\arctan(ax)}{x}\,dx = c^3\left(\int \frac{\operatorname{atan}(ax)}{x}\,dx + \int 3a^2x\operatorname{atan}(ax)\,dx + \int 3a^4x^3\operatorname{atan}(ax)\,dx + \int a^6x^5\operatorname{atan}(ax)\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**3*atan(a*x)/x,x)
```

output
```
c**3*(Integral(atan(a*x)/x, x) + Integral(3*a**2*x*atan(a*x), x) + Integra
l(3*a**4*x**3*atan(a*x), x) + Integral(a**6*x**5*atan(a*x), x))
```

## Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.96

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)}{x} \, dx$$
$$= -\frac{1}{30} a^5c^3x^5 - \frac{7}{36} a^3c^3x^3 - \frac{11}{12} ac^3x - \frac{1}{4} \pi c^3 \log\left(a^2x^2+1\right)$$
$$+ c^3 \arctan(ax) \log(ax) - \frac{1}{2} i c^3 \text{Li}_2(i ax+1) + \frac{1}{2} i c^3 \text{Li}_2(-i ax+1)$$
$$+ \frac{1}{12} \left(2 a^6c^3x^6 + 9 a^4c^3x^4 + 18 a^2c^3x^2 + 11 c^3\right) \arctan(ax)$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)/x,x, algorithm="maxima")
```

output
```
-1/30*a^5*c^3*x^5 - 7/36*a^3*c^3*x^3 - 11/12*a*c^3*x - 1/4*pi*c^3*log(a^2*
x^2 + 1) + c^3*arctan(a*x)*log(a*x) - 1/2*I*c^3*dilog(I*a*x + 1) + 1/2*I*c
^3*dilog(-I*a*x + 1) + 1/12*(2*a^6*c^3*x^6 + 9*a^4*c^3*x^4 + 18*a^2*c^3*x^
2 + 11*c^3)*arctan(a*x)
```

## Giac [F]

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)}{x} \, dx = \int \frac{(a^2cx^2+c)^3 \arctan(ax)}{x} \, dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)/x,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^3*arctan(a*x)/x, x)
```

## Mupad [B] (verification not implemented)

Time = 0.97 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.18

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)}{x}\,dx$$
$$= \begin{cases} 0 \\ 3\,a^2\,c^3\,\mathrm{atan}(a\,x)\,\left(\frac{1}{2\,a^2}+\frac{x^2}{2}\right)-\frac{a^5\,c^3\,\left(\frac{x}{a^4}-\frac{\mathrm{atan}(a\,x)}{a^5}+\frac{x^5}{5}-\frac{x^3}{3\,a^2}\right)}{6}-\frac{3\,a\,c^3\,x}{2}-\frac{c^3\,\left(3\,\mathrm{atan}(a\,x)-3\,a\,x+a^3\,x^3\right)}{4}+\frac{3\,a^4\,c^3\,x^4\,\mathrm{ata}}{4} \end{cases}$$

input
```
int((atan(a*x)*(c + a^2*c*x^2)^3)/x,x)
```

output
```
piecewise(a == 0, 0, a ~= 0, - (c^3*dilog(- a*x*1i + 1)*1i)/2 + (c^3*dilog
(a*x*1i + 1)*1i)/2 - (c^3*(3*atan(a*x) - 3*a*x + a^3*x^3))/4 - (a^5*c^3*(x
/a^4 - atan(a*x)/a^5 + x^5/5 - x^3/(3*a^2)))/6 - (3*a*c^3*x)/2 + 3*a^2*c^3
*atan(a*x)*(1/(2*a^2) + x^2/2) + (3*a^4*c^3*x^4*atan(a*x))/4 + (a^6*c^3*x^
6*atan(a*x))/6)
```

## Reduce [F]

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)}{x}\,dx$$
$$= \frac{c^3\left(30atan(ax)\,a^6x^6+135atan(ax)\,a^4x^4+270atan(ax)\,a^2x^2+165atan(ax)+180\left(\int\frac{atan(ax)}{x}dx\right)-6a^5\right.}{180}$$

input
```
int((a^2*c*x^2+c)^3*atan(a*x)/x,x)
```

output
```
(c**3*(30*atan(a*x)*a**6*x**6 + 135*atan(a*x)*a**4*x**4 + 270*atan(a*x)*a*
*2*x**2 + 165*atan(a*x) + 180*int(atan(a*x)/x,x) - 6*a**5*x**5 - 35*a**3*x
**3 - 165*a*x))/180
```

# 3.170 $\int \frac{(c+a^2cx^2)^3 \arctan(ax)}{x^2}\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 108

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)}{x^2}\,dx = -\frac{2}{5}a^3c^3x^2 - \frac{1}{20}a^5c^3x^4 - \frac{c^3\arctan(ax)}{x} + 3a^2c^3x\arctan(ax) + a^4c^3x^3\arctan(ax) + \frac{1}{5}a^6c^3x^5\arctan(ax) + ac^3\log(x) - \frac{8}{5}ac^3\log\left(1+a^2x^2\right)$$

output
```
-2/5*a^3*c^3*x^2-1/20*a^5*c^3*x^4-c^3*arctan(a*x)/x+3*a^2*c^3*x*arctan(a*x
)+a^4*c^3*x^3*arctan(a*x)+1/5*a^6*c^3*x^5*arctan(a*x)+a*c^3*ln(x)-8/5*a*c^
3*ln(a^2*x^2+1)
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.72

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)}{x^2}\,dx = \frac{c^3(4(-5+15a^2x^2+5a^4x^4+a^6x^6)\arctan(ax) - ax(8a^2x^2+a^4x^4-20\log(x)+32\log(1+a^2x^2)))}{20x}$$

input
```
Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x])/x^2,x]
```

output
```
(c^3*(4*(-5 + 15*a^2*x^2 + 5*a^4*x^4 + a^6*x^6)*ArcTan[a*x] - a*x*(8*a^2*x
^2 + a^4*x^4 - 20*Log[x] + 32*Log[1 + a^2*x^2])))/(20*x)
```

## Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)\left(a^2cx^2+c\right)^3}{x^2}dx$$

$$\Big\downarrow \text{5483}$$

$$\int \left(a^6c^3x^4\arctan(ax)+3a^4c^3x^2\arctan(ax)+3a^2c^3\arctan(ax)+\frac{c^3\arctan(ax)}{x^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}a^6c^3x^5\arctan(ax)-\frac{1}{20}a^5c^3x^4+a^4c^3x^3\arctan(ax)-\frac{2}{5}a^3c^3x^2+3a^2c^3x\arctan(ax)-\frac{8}{5}ac^3\log\left(a^2x^2+1\right)-\frac{c^3\arctan(ax)}{x}+ac^3\log(x)$$

input
```
Int[((c + a^2*c*x^2)^3*ArcTan[a*x])/x^2,x]
```

output
```
(-2*a^3*c^3*x^2)/5 - (a^5*c^3*x^4)/20 - (c^3*ArcTan[a*x])/x + 3*a^2*c^3*x*
ArcTan[a*x] + a^4*c^3*x^3*ArcTan[a*x] + (a^6*c^3*x^5*ArcTan[a*x])/5 + a*c^
3*Log[x] - (8*a*c^3*Log[1 + a^2*x^2])/5
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

## Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.88

| method | result |
|---|---|
| parts | $\frac{a^6c^3x^5\arctan(ax)}{5}+a^4c^3x^3\arctan(ax)+3a^2c^3x\arctan(ax)-\frac{c^3\arctan(ax)}{x}-\frac{a\,c^3\left(\frac{a^4x^4}{4}+2a^2x\right.}{}$ |
| derivativedivides | $a\left(\frac{a^5c^3x^5\arctan(ax)}{5}+a^3c^3x^3\arctan(ax)+3a\,c^3x\arctan(ax)-\frac{c^3\arctan(ax)}{ax}-\frac{c^3\left(\frac{a^4x^4}{4}+2a^2\right.}{}\right.$ |
| default | $a\left(\frac{a^5c^3x^5\arctan(ax)}{5}+a^3c^3x^3\arctan(ax)+3a\,c^3x\arctan(ax)-\frac{c^3\arctan(ax)}{ax}-\frac{c^3\left(\frac{a^4x^4}{4}+2a^2\right.}{}\right.$ |
| parallelrisch | $\frac{4a^6c^3x^6\arctan(ax)-a^5c^3x^5+20a^4c^3x^4\arctan(ax)-8c^3x^3a^3+60a^2c^3x^2\arctan(ax)+20a\,c^3\ln(x)x-32a\,c^3\ln(a^2x^2+}{20x}$ |
| risch | $-\frac{ic^3(a^6x^6+5a^4x^4+15a^2x^2-5)\ln(iax+1)}{10x}+\frac{ic^3(2a^6x^6\ln(-iax+1)+ia^5x^5+10x^4\ln(-iax+1)a^4+8ia^3x^3+30a^2x^2}{20}$ |
| meijerg | $\frac{a\,c^3\left(\frac{a^2x^2(-3a^2x^2+6)}{15}+\frac{4a^6x^6\arctan\left(\sqrt{a^2x^2}\right)}{5\sqrt{a^2x^2}}-\frac{2\ln(a^2x^2+1)}{5}\right)}{4}+\frac{3a\,c^3\left(-\frac{2a^2x^2}{3}+\frac{4a^4x^4\arctan\left(\sqrt{a^2x^2}\right)}{3\sqrt{a^2x^2}}+\frac{2\ln(a^2x^2-}{3}\right.}{4}$ |

input
```
int((a^2*c*x^2+c)^3*arctan(a*x)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/5*a^6*c^3*x^5*arctan(a*x)+a^4*c^3*x^3*arctan(a*x)+3*a^2*c^3*x*arctan(a*x
)-c^3*arctan(a*x)/x-1/5*a*c^3*(1/4*a^4*x^4+2*a^2*x^2+8*ln(a^2*x^2+1)-5*ln(
x))
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.90

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)}{x^2}\,dx = -\frac{a^5c^3x^5 + 8\,a^3c^3x^3 + 32\,ac^3x\log\left(a^2x^2+1\right) - 20\,ac^3x\log\left(x\right) - 4\left(a^6c^3x^6 + 5\,a^4c^3x^4 + 15\,a^2c^3x^2 - 5\,c^3\right)}{20\,x}$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)/x^2,x, algorithm="fricas")
```

output
```
-1/20*(a^5*c^3*x^5 + 8*a^3*c^3*x^3 + 32*a*c^3*x*log(a^2*x^2 + 1) - 20*a*c^
3*x*log(x) - 4*(a^6*c^3*x^6 + 5*a^4*c^3*x^4 + 15*a^2*c^3*x^2 - 5*c^3)*arct
an(a*x))/x
```

## Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.02

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)}{x^2}\,dx = \begin{cases} \frac{a^6c^3x^5\operatorname{atan}(ax)}{5} - \frac{a^5c^3x^4}{20} + a^4c^3x^3\operatorname{atan}(ax) - \frac{2a^3c^3x^2}{5} + 3a^2c^3x\operatorname{atan}(ax) + ac^3\log(x) - \frac{8ac^3\log\left(x^2+\frac{1}{a^2}\right)}{5} - \\ 0 \end{cases}$$

input
```
integrate((a**2*c*x**2+c)**3*atan(a*x)/x**2,x)
```

output
```
Piecewise((a**6*c**3*x**5*atan(a*x)/5 - a**5*c**3*x**4/20 + a**4*c**3*x**3
*atan(a*x) - 2*a**3*c**3*x**2/5 + 3*a**2*c**3*x*atan(a*x) + a*c**3*log(x)
- 8*a*c**3*log(x**2 + a**(-2))/5 - c**3*atan(a*x)/x, Ne(a, 0)), (0, True))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.86

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)}{x^2} dx = -\frac{1}{20} \left(a^4c^3x^4 + 8 a^2c^3x^2 + 32 c^3 \log \left(a^2x^2+1\right) - 20 c^3 \log (x)\right)a + \frac{1}{5} \left(a^6c^3x^5 + 5 a^4c^3x^3 + 15 a^2c^3x - \frac{5 c^3}{x}\right) \arctan (ax)$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)/x^2,x, algorithm="maxima")
```

output
```
-1/20*(a^4*c^3*x^4 + 8*a^2*c^3*x^2 + 32*c^3*log(a^2*x^2 + 1) - 20*c^3*log(
x))*a + 1/5*(a^6*c^3*x^5 + 5*a^4*c^3*x^3 + 15*a^2*c^3*x - 5*c^3/x)*arctan(
a*x)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.92

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)}{x^2} dx = -\frac{8}{5} ac^3 \log \left(a^2x^2+1\right) + \frac{1}{2} ac^3 \log \left(x^2\right) + \frac{1}{5} \left(a^6c^3x^5 + 5 a^4c^3x^3 + 15 a^2c^3x - \frac{5 c^3}{x}\right) \arctan (ax) - \frac{a^9c^3x^4 + 8 a^7c^3x^2}{20 a^4}$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)/x^2,x, algorithm="giac")
```

output
```
-8/5*a*c^3*log(a^2*x^2 + 1) + 1/2*a*c^3*log(x^2) + 1/5*(a^6*c^3*x^5 + 5*a^
4*c^3*x^3 + 15*a^2*c^3*x - 5*c^3/x)*arctan(a*x) - 1/20*(a^9*c^3*x^4 + 8*a^
7*c^3*x^2)/a^4
```

## Mupad [B] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.79

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)}{x^2} dx = -\frac{c^3 \left(\operatorname{atan}(a\,x) + \frac{2\,a^3\,x^3}{5} + \frac{a^5\,x^5}{20} - a\,x\,\ln(x) - 3\,a^2\,x^2\operatorname{atan}(a\,x) - a^4\,x^4\operatorname{atan}(a\,x) - \frac{a^6\,x^6\operatorname{atan}(a\,x)}{5} + \frac{8\,a\,x\,\ln(\dots}{\dots}\right.}{x}$$

input
```
int((atan(a*x)*(c + a^2*c*x^2)^3)/x^2,x)
```

output
```
-(c^3*(atan(a*x) + (2*a^3*x^3)/5 + (a^5*x^5)/20 - a*x*log(x) - 3*a^2*x^2*a
tan(a*x) - a^4*x^4*atan(a*x) - (a^6*x^6*atan(a*x))/5 + (8*a*x*log(a^2*x^2
+ 1))/5))/x
```

## Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.81

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)}{x^2} dx = \frac{c^3(4atan(ax)\,a^6x^6 + 20atan(ax)\,a^4x^4 + 60atan(ax)\,a^2x^2 - 20atan(ax) - 32\log(a^2x^2+1)\,ax + 20\log(x\dots}{20x}$$

input
```
int((a^2*c*x^2+c)^3*atan(a*x)/x^2,x)
```

output
```
(c**3*(4*atan(a*x)*a**6*x**6 + 20*atan(a*x)*a**4*x**4 + 60*atan(a*x)*a**2*
x**2 - 20*atan(a*x) - 32*log(a**2*x**2 + 1)*a*x + 20*log(x)*a*x - a**5*x**
5 - 8*a**3*x**3))/(20*x)
```

# 3.171 $\int \frac{(c+a^2cx^2)^3\arctan(ax)}{x^3}\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 138

$$\begin{aligned}\int \frac{(c+a^2cx^2)^3\arctan(ax)}{x^3}\,dx = &-\frac{ac^3}{2x}-\frac{5}{4}a^3c^3x-\frac{1}{12}a^5c^3x^3\\&+\frac{3}{4}a^2c^3\arctan(ax)-\frac{c^3\arctan(ax)}{2x^2}\\&+\frac{3}{2}a^4c^3x^2\arctan(ax)+\frac{1}{4}a^6c^3x^4\arctan(ax)\\&+\frac{3}{2}ia^2c^3\,\mathrm{PolyLog}(2,-iax)-\frac{3}{2}ia^2c^3\,\mathrm{PolyLog}(2,iax)\end{aligned}$$

output
```
-1/2*a*c^3/x-5/4*a^3*c^3*x-1/12*a^5*c^3*x^3+3/4*a^2*c^3*arctan(a*x)-1/2*c^
3*arctan(a*x)/x^2+3/2*a^4*c^3*x^2*arctan(a*x)+1/4*a^6*c^3*x^4*arctan(a*x)+
3/2*I*a^2*c^3*polylog(2,-I*a*x)-3/2*I*a^2*c^3*polylog(2,I*a*x)
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.03 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.12

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)}{x^3} dx = -\frac{5}{4}a^3c^3x - \frac{1}{12}a^5c^3x^3 + \frac{5}{4}a^2c^3 \arctan(ax) - \frac{c^3 \arctan(ax)}{2x^2} + \frac{3}{2}a^4c^3x^2 \arctan(ax) + \frac{1}{4}a^6c^3x^4 \arctan(ax) - \frac{ac^3 \text{Hypergeometric2F1}\left(-\frac{1}{2}, 1, \frac{1}{2}, -a^2x^2\right)}{2x} + \frac{3}{2}ia^2c^3 \text{PolyLog}(2, -iax) - \frac{3}{2}ia^2c^3 \text{PolyLog}(2, iax)$$

input
```
Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x])/x^3,x]
```

output
```
(-5*a^3*c^3*x)/4 - (a^5*c^3*x^3)/12 + (5*a^2*c^3*ArcTan[a*x])/4 - (c^3*Arc
Tan[a*x])/(2*x^2) + (3*a^4*c^3*x^2*ArcTan[a*x])/2 + (a^6*c^3*x^4*ArcTan[a*
x])/4 - (a*c^3*Hypergeometric2F1[-1/2, 1, 1/2, -(a^2*x^2)])/(2*x) + ((3*I)
/2)*a^2*c^3*PolyLog[2, (-I)*a*x] - ((3*I)/2)*a^2*c^3*PolyLog[2, I*a*x]
```

## Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)\left(a^2cx^2+c\right)^3}{x^3} dx$$

$$\Big\downarrow \text{ 5483}$$

$$\int \left(a^6c^3x^3 \arctan(ax) + 3a^4c^3x \arctan(ax) + \frac{3a^2c^3 \arctan(ax)}{x} + \frac{c^3 \arctan(ax)}{x^3}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4}a^6c^3x^4\arctan(ax) - \frac{1}{12}a^5c^3x^3 + \frac{3}{2}a^4c^3x^2\arctan(ax) - \frac{5}{4}a^3c^3x + \frac{3}{4}a^2c^3\arctan(ax) + \frac{3}{2}ia^2c^3\,\text{PolyLog}(2,-iax) - \frac{3}{2}ia^2c^3\,\text{PolyLog}(2,iax) - \frac{c^3\arctan(ax)}{2x^2} - \frac{ac^3}{2x}$$

input
```
Int[((c + a^2*c*x^2)^3*ArcTan[a*x])/x^3,x]
```

output
```
-1/2*(a*c^3)/x - (5*a^3*c^3*x)/4 - (a^5*c^3*x^3)/12 + (3*a^2*c^3*ArcTan[a*
x])/4 - (c^3*ArcTan[a*x])/(2*x^2) + (3*a^4*c^3*x^2*ArcTan[a*x])/2 + (a^6*c
^3*x^4*ArcTan[a*x])/4 + ((3*I)/2)*a^2*c^3*PolyLog[2, (-I)*a*x] - ((3*I)/2)
*a^2*c^3*PolyLog[2, I*a*x]
```

**Defintions of rubi rules used**

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

## Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.07

| method | result |
|---|---|
| derivativedivides | $a^2\left(\frac{a^4c^3x^4\arctan(ax)}{4}+\frac{3a^2c^3x^2\arctan(ax)}{2}+3c^3\arctan(ax)\ln(ax)-\frac{c^3\arctan(ax)}{2a^2x^2}-\frac{c^3\left(\frac{a^3x^3}{3}+5\right.}{}\right.$ |
| default | $a^2\left(\frac{a^4c^3x^4\arctan(ax)}{4}+\frac{3a^2c^3x^2\arctan(ax)}{2}+3c^3\arctan(ax)\ln(ax)-\frac{c^3\arctan(ax)}{2a^2x^2}-\frac{c^3\left(\frac{a^3x^3}{3}+5\right.}{}\right.$ |
| parts | $\frac{a^6c^3x^4\arctan(ax)}{4}+\frac{3a^4c^3x^2\arctan(ax)}{2}-\frac{c^3\arctan(ax)}{2x^2}+3c^3\arctan(ax)a^2\ln(x)-\frac{a\,c^3\left(\frac{x^3a^4}{3}+5a\right.}{}$ |
| meijerg | $\frac{a^2c^3\left(\frac{ax\left(-5a^2x^2+15\right)}{15}-\frac{ax\left(-5a^4x^4+5\right)\arctan\left(\sqrt{a^2x^2}\right)}{5\sqrt{a^2x^2}}\right)}{4}+\frac{3a^2c^3\left(-2ax+\frac{2\left(3a^2x^2+3\right)\arctan(ax)}{3}\right)}{4}+\frac{3a^2c^3\left(-\frac{2ia}{}\right.}{}$ |
| risch | $-\frac{ic^3a^6\ln(iax+1)x^4}{8}+\frac{3ic^3a^4\ln(-iax+1)x^2}{4}-\frac{3ic^3a^4\ln(iax+1)x^2}{4}+\frac{3a^2c^3\arctan(ax)}{4}-\frac{a^5c^3x^3}{12}-\frac{5a^3c^3x}{4}$ |

input
```
int((a^2*c*x^2+c)^3*arctan(a*x)/x^3,x,method=_RETURNVERBOSE)
```

output
```
a^2*(1/4*a^4*c^3*x^4*arctan(a*x)+3/2*a^2*c^3*x^2*arctan(a*x)+3*c^3*arctan(
a*x)*ln(a*x)-1/2*c^3*arctan(a*x)/a^2/x^2-1/4*c^3*(1/3*a^3*x^3+5*a*x+2/a/x-
3*arctan(a*x)-6*I*ln(a*x)*ln(1+I*a*x)+6*I*ln(a*x)*ln(1-I*a*x)-6*I*dilog(1+
I*a*x)+6*I*dilog(1-I*a*x)))
```

## Fricas [F]

$$\int\frac{\left(c+a^2cx^2\right)^3\arctan(ax)}{x^3}\,dx=\int\frac{\left(a^2cx^2+c\right)^3\arctan\left(ax\right)}{x^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)/x^3,x, algorithm="fricas")
```

output
```
integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a*x)/x
^3, x)
```

## Sympy [F]

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)}{x^3} \, dx = c^3 \left( \int \frac{\operatorname{atan}(ax)}{x^3} \, dx + \int \frac{3a^2 \operatorname{atan}(ax)}{x} \, dx + \int 3a^4 x \operatorname{atan}(ax) \, dx + \int a^6 x^3 \operatorname{atan}(ax) \, dx \right)$$

input
```
integrate((a**2*c*x**2+c)**3*atan(a*x)/x**3,x)
```

output
```
c**3*(Integral(atan(a*x)/x**3, x) + Integral(3*a**2*atan(a*x)/x, x) + Inte
gral(3*a**4*x*atan(a*x), x) + Integral(a**6*x**3*atan(a*x), x))
```

## Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.12

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)}{x^3} \, dx = -\frac{a^5c^3x^5 + 15\,a^3c^3x^3 + 9\,\pi a^2c^3x^2 \log(a^2x^2+1) - 36\,a^2c^3x^2 \arctan(ax)\log(ax) + 18i\,a^2c^3x^2 \mathrm{Li}_2(i\,ax +}{12\,x^2}$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)/x^3,x, algorithm="maxima")
```

output
```
-1/12*(a^5*c^3*x^5 + 15*a^3*c^3*x^3 + 9*pi*a^2*c^3*x^2*log(a^2*x^2 + 1) -
36*a^2*c^3*x^2*arctan(a*x)*log(a*x) + 18*I*a^2*c^3*x^2*dilog(I*a*x + 1) -
18*I*a^2*c^3*x^2*dilog(-I*a*x + 1) + 6*a*c^3*x - 3*(a^6*c^3*x^6 + 6*a^4*c^
3*x^4 + 3*a^2*c^3*x^2 - 2*c^3)*arctan(a*x))/x^2
```

## Giac [F]

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)}{x^3} \, dx = \int \frac{(a^2cx^2+c)^3 \arctan(ax)}{x^3} \, dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)/x^3,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^3*arctan(a*x)/x^3, x)
```

## Mupad [B] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.10

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)}{x^3} \, dx = \begin{cases} 0 \\ 3a^4c^3\operatorname{atan}(a\,x)\left(\frac{1}{2a^2}+\frac{x^2}{2}\right) - \frac{a^2c^3\left(3\operatorname{atan}(a\,x)-3\,a\,x+a^3x^3\right)}{12} - \frac{c^3\operatorname{atan}(a\,x)}{2\,x^2} - \frac{c^3\left(a^3\operatorname{atan}(a\,x)+\frac{a^2}{x}\right)}{2\,a} - \frac{3\,a^3c^3x}{2} + a^6 \end{cases}$$

input
```
int((atan(a*x)*(c + a^2*c*x^2)^3)/x^3,x)
```

output
```
piecewise(a == 0, 0, a ~= 0, - (3*a^3*c^3*x)/2 - (a^2*c^3*(3*atan(a*x) - 3
*a*x + a^3*x^3))/12 - (c^3*atan(a*x))/(2*x^2) - (a^2*c^3*dilog(- a*x*1i +
1)*3i)/2 + (a^2*c^3*dilog(a*x*1i + 1)*3i)/2 - (c^3*(a^3*atan(a*x) + a^2/x)
)/(2*a) + 3*a^4*c^3*atan(a*x)*(1/(2*a^2) + x^2/2) + (a^6*c^3*x^4*atan(a*x)
)/4)
```

## Reduce [F]

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)}{x^3}\,dx$$
$$= \frac{c^3\Big(3atan(ax)\,a^6x^6 + 18atan(ax)\,a^4x^4 + 9atan(ax)\,a^2x^2 - 6atan(ax) + 36\Big(\int \frac{atan(ax)}{x}dx\Big)\,a^2x^2 - a^5x^5 -}{12x^2}$$

input
```
int((a^2*c*x^2+c)^3*atan(a*x)/x^3,x)
```

output
```
(c**3*(3*atan(a*x)*a**6*x**6 + 18*atan(a*x)*a**4*x**4 + 9*atan(a*x)*a**2*x
**2 - 6*atan(a*x) + 36*int(atan(a*x)/x,x)*a**2*x**2 - a**5*x**5 - 15*a**3*
x**3 - 6*a*x))/(12*x**2)
```

# 3.172 $\int \frac{(c+a^2cx^2)^3\arctan(ax)}{x^4}\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 116

$$\int \frac{(c+a^2cx^2)^3\arctan(ax)}{x^4}\,dx = -\frac{ac^3}{6x^2}-\frac{1}{6}a^5c^3x^2-\frac{c^3\arctan(ax)}{3x^3}-\frac{3a^2c^3\arctan(ax)}{x}+3a^4c^3x\arctan(ax)+\frac{1}{3}a^6c^3x^3\arctan(ax)+\frac{8}{3}a^3c^3\log(x)-\frac{8}{3}a^3c^3\log\left(1+a^2x^2\right)$$

output
```
-1/6*a*c^3/x^2-1/6*a^5*c^3*x^2-1/3*c^3*arctan(a*x)/x^3-3*a^2*c^3*arctan(a*
x)/x+3*a^4*c^3*x*arctan(a*x)+1/3*a^6*c^3*x^3*arctan(a*x)+8/3*a^3*c^3*ln(x)
-8/3*a^3*c^3*ln(a^2*x^2+1)
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.72

$$\int \frac{(c+a^2cx^2)^3\arctan(ax)}{x^4}\,dx = \frac{c^3(2(-1-9a^2x^2+9a^4x^4+a^6x^6)\arctan(ax)-ax(1+a^4x^4-16a^2x^2\log(x)+16a^2x^2\log(1+a^2x^2)))}{6x^3}$$

input
```
Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x])/x^4,x]
```

output
```
(c^3*(2*(-1 - 9*a^2*x^2 + 9*a^4*x^4 + a^6*x^6)*ArcTan[a*x] - a*x*(1 + a^4*
x^4 - 16*a^2*x^2*Log[x] + 16*a^2*x^2*Log[1 + a^2*x^2])))/(6*x^3)
```

## Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)\left(a^2cx^2+c\right)^3}{x^4}dx$$

$$\Big\downarrow \text{ 5483}$$

$$\int \left(a^6c^3x^2\arctan(ax)+3a^4c^3\arctan(ax)+\frac{3a^2c^3\arctan(ax)}{x^2}+\frac{c^3\arctan(ax)}{x^4}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}a^6c^3x^3\arctan(ax)-\frac{1}{6}a^5c^3x^2+3a^4c^3x\arctan(ax)+\frac{8}{3}a^3c^3\log(x)-\frac{3a^2c^3\arctan(ax)}{x}-\frac{8}{3}a^3c^3\log\left(a^2x^2+1\right)-\frac{c^3\arctan(ax)}{3x^3}-\frac{ac^3}{6x^2}$$

input
```
Int[((c + a^2*c*x^2)^3*ArcTan[a*x])/x^4,x]
```

output
```
-1/6*(a*c^3)/x^2 - (a^5*c^3*x^2)/6 - (c^3*ArcTan[a*x])/(3*x^3) - (3*a^2*c^
3*ArcTan[a*x])/x + 3*a^4*c^3*x*ArcTan[a*x] + (a^6*c^3*x^3*ArcTan[a*x])/3 +
 (8*a^3*c^3*Log[x])/3 - (8*a^3*c^3*Log[1 + a^2*x^2])/3
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

## Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.85

| method | result |
|---|---|
| parts | $\frac{a^6c^3x^3\arctan(ax)}{3}+3a^4c^3x\arctan(ax)-\frac{c^3\arctan(ax)}{3x^3}-\frac{3a^2c^3\arctan(ax)}{x}-\frac{a\,c^3\left(\frac{x^2a^4}{2}+8a^2\ln(a^2x^2\right.}{3}$ |
| derivativedivides | $a^3\left(\frac{a^3c^3x^3\arctan(ax)}{3}+3a\,c^3x\arctan(ax)-\frac{3c^3\arctan(ax)}{ax}-\frac{c^3\arctan(ax)}{3a^3x^3}-\frac{c^3\left(\frac{a^2x^2}{2}+8\ln(a^2x^2+\right.}{3}\right.$ |
| default | $a^3\left(\frac{a^3c^3x^3\arctan(ax)}{3}+3a\,c^3x\arctan(ax)-\frac{3c^3\arctan(ax)}{ax}-\frac{c^3\arctan(ax)}{3a^3x^3}-\frac{c^3\left(\frac{a^2x^2}{2}+8\ln(a^2x^2+\right.}{3}\right.$ |
| parallelrisch | $\frac{2a^6c^3x^6\arctan(ax)-a^5c^3x^5+18a^4c^3x^4\arctan(ax)+16a^3c^3\ln(x)x^3-16a^3c^3\ln(a^2x^2+1)x^3+c^3x^3a^3-18a^2c^3x^2\arctan}{6x^3}$ |
| risch | $-\frac{ic^3(a^6x^6+9a^4x^4-9a^2x^2-1)\ln(iax+1)}{6x^3}+\frac{ic^3(a^6x^6\ln(-iax+1)+ia^5x^5+9x^4\ln(-iax+1)a^4-16i\ln(x)a^3x^3+16i}{6x^3}$ |
| meijerg | $\frac{a^3c^3\left(-\frac{2a^2x^2}{3}+\frac{4a^4x^4\arctan(\sqrt{a^2x^2})}{3\sqrt{a^2x^2}}+\frac{2\ln(a^2x^2+1)}{3}\right)}{4}+\frac{3a^3c^3\left(\frac{4a^2x^2\arctan(\sqrt{a^2x^2})}{\sqrt{a^2x^2}}-2\ln(a^2x^2+1)\right)}{4}+\frac{3a^3c^3\left(\right.}{}$ |

input
```
int((a^2*c*x^2+c)^3*arctan(a*x)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/3*a^6*c^3*x^3*arctan(a*x)+3*a^4*c^3*x*arctan(a*x)-1/3*c^3*arctan(a*x)/x^
3-3*a^2*c^3*arctan(a*x)/x-1/3*a*c^3*(1/2*x^2*a^4+8*a^2*ln(a^2*x^2+1)+1/2/x
^2-8*a^2*ln(x))
```

## Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.86

$$\int \frac{(c+a^2cx^2)^3\arctan(ax)}{x^4}\,dx = -\frac{a^5c^3x^5+16\,a^3c^3x^3\log\left(a^2x^2+1\right)-16\,a^3c^3x^3\log\left(x\right)+ac^3x-2\left(a^6c^3x^6+9\,a^4c^3x^4-9\,a^2c^3x^2-c^3\right)\text{ar}}{6\,x^3}$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)/x^4,x, algorithm="fricas")
```

output
```
-1/6*(a^5*c^3*x^5 + 16*a^3*c^3*x^3*log(a^2*x^2 + 1) - 16*a^3*c^3*x^3*log(x
) + a*c^3*x - 2*(a^6*c^3*x^6 + 9*a^4*c^3*x^4 - 9*a^2*c^3*x^2 - c^3)*arctan
(a*x))/x^3
```

## Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.01

$$\int \frac{(c+a^2cx^2)^3\arctan(ax)}{x^4}\,dx = \begin{cases} \frac{a^6c^3x^3\,\text{atan}\,(ax)}{3}-\frac{a^5c^3x^2}{6}+3a^4c^3x\,\text{atan}\,(ax)+\frac{8a^3c^3\log{(x)}}{3}-\frac{8a^3c^3\log{\left(x^2+\frac{1}{a^2}\right)}}{3}-\frac{3a^2c^3\,\text{atan}\,(ax)}{x}-\frac{ac^3}{6x^2}-\frac{c^3\,\text{atan}\,(a}{3x^3} \\ 0 \end{cases}$$

input
```
integrate((a**2*c*x**2+c)**3*atan(a*x)/x**4,x)
```

output
```
Piecewise((a**6*c**3*x**3*atan(a*x)/3 - a**5*c**3*x**2/6 + 3*a**4*c**3*x*a
tan(a*x) + 8*a**3*c**3*log(x)/3 - 8*a**3*c**3*log(x**2 + a**(-2))/3 - 3*a*
*2*c**3*atan(a*x)/x - a*c**3/(6*x**2) - c**3*atan(a*x)/(3*x**3), Ne(a, 0))
, (0, True))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.83

$$\int \frac{(c + a^2cx^2)^3 \arctan(ax)}{x^4} dx = -\frac{1}{6}\left(a^4c^3x^2 + 16\,a^2c^3\log\left(a^2x^2 + 1\right) - 16\,a^2c^3\log\left(x\right) + \frac{c^3}{x^2}\right)a + \frac{1}{3}\left(a^6c^3x^3 + 9\,a^4c^3x - \frac{9\,a^2c^3x^2 + c^3}{x^3}\right)\arctan\left(ax\right)$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)/x^4,x, algorithm="maxima")
```

output
```
-1/6*(a^4*c^3*x^2 + 16*a^2*c^3*log(a^2*x^2 + 1) - 16*a^2*c^3*log(x) + c^3/
x^2)*a + 1/3*(a^6*c^3*x^3 + 9*a^4*c^3*x - (9*a^2*c^3*x^2 + c^3)/x^3)*arcta
n(a*x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.95

$$\int \frac{(c + a^2cx^2)^3 \arctan(ax)}{x^4} dx = -\frac{1}{6}\,a^5c^3x^2 - \frac{8}{3}\,a^3c^3\log\left(a^2x^2 + 1\right) + \frac{4}{3}\,a^3c^3\log\left(x^2\right) + \frac{1}{3}\left(a^6c^3x^3 + 9\,a^4c^3x - \frac{9\,a^2c^3x^2 + c^3}{x^3}\right)\arctan\left(ax\right) - \frac{8\,a^3c^3x^2 + ac^3}{6\,x^2}$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)/x^4,x, algorithm="giac")
```

output
```
-1/6*a^5*c^3*x^2 - 8/3*a^3*c^3*log(a^2*x^2 + 1) + 4/3*a^3*c^3*log(x^2) + 1
/3*(a^6*c^3*x^3 + 9*a^4*c^3*x - (9*a^2*c^3*x^2 + c^3)/x^3)*arctan(a*x) - 1
/6*(8*a^3*c^3*x^2 + a*c^3)/x^2
```

## Mupad [B] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.84

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)}{x^4} \, dx = -\frac{c^3 \left(2 \operatorname{atan}(a\,x) + a\,x - a^3\,x^3 + a^5\,x^5 + 18\,a^2\,x^2 \operatorname{atan}(a\,x) - 18\,a^4\,x^4 \operatorname{atan}(a\,x) - 2\,a^6\,x^6 \operatorname{atan}(a\,x) + 16\right.}{6\,x^3}$$

input
```
int((atan(a*x)*(c + a^2*c*x^2)^3)/x^4,x)
```

output
```
-(c^3*(2*atan(a*x) + a*x - a^3*x^3 + a^5*x^5 + 18*a^2*x^2*atan(a*x) - 18*a
^4*x^4*atan(a*x) - 2*a^6*x^6*atan(a*x) + 16*a^3*x^3*log(a^2*x^2 + 1) - 16*
a^3*x^3*log(x)))/(6*x^3)
```

## Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.78

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)}{x^4} \, dx = \frac{c^3 (2atan(ax)\,a^6x^6 + 18atan(ax)\,a^4x^4 - 18atan(ax)\,a^2x^2 - 2atan(ax) - 16\log(a^2x^2+1)\,a^3x^3 + 16\log(}{6x^3}$$

input
```
int((a^2*c*x^2+c)^3*atan(a*x)/x^4,x)
```

output
```
(c**3*(2*atan(a*x)*a**6*x**6 + 18*atan(a*x)*a**4*x**4 - 18*atan(a*x)*a**2*
x**2 - 2*atan(a*x) - 16*log(a**2*x**2 + 1)*a**3*x**3 + 16*log(x)*a**3*x**3
 - a**5*x**5 - a*x))/(6*x**3)
```

# 3.173 $\int \frac{x^4 \arctan(ax)}{c+a^2cx^2} \, dx$



## Optimal result

Integrand size = 20, antiderivative size = 80

$$\int \frac{x^4 \arctan(ax)}{c+a^2cx^2} \, dx = -\frac{x^2}{6a^3c} - \frac{x \arctan(ax)}{a^4c} + \frac{x^3 \arctan(ax)}{3a^2c} + \frac{\arctan(ax)^2}{2a^5c} + \frac{2\log(1+a^2x^2)}{3a^5c}$$

output
```
-1/6*x^2/a^3/c-x*arctan(a*x)/a^4/c+1/3*x^3*arctan(a*x)/a^2/c+1/2*arctan(a*
x)^2/a^5/c+2/3*ln(a^2*x^2+1)/a^5/c
```

## Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.70

$$\int \frac{x^4 \arctan(ax)}{c+a^2cx^2} \, dx = \frac{-a^2x^2 + 2ax(-3+a^2x^2)\arctan(ax) + 3\arctan(ax)^2 + 4\log(1+a^2x^2)}{6a^5c}$$

input
```
Integrate[(x^4*ArcTan[a*x])/(c + a^2*c*x^2),x]
```

output
```
(-(a^2*x^2) + 2*a*x*(-3 + a^2*x^2)*ArcTan[a*x] + 3*ArcTan[a*x]^2 + 4*Log[1
 + a^2*x^2])/(6*a^5*c)
```

## Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.22, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5451, 27, 5361, 243, 49, 2009, 5451, 5345, 240, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \arctan(ax)}{a^2cx^2+c} \, dx$$

$$\Big\downarrow \; 5451$$

$$\frac{\int x^2 \arctan(ax)dx}{a^2c} - \frac{\int \frac{x^2 \arctan(ax)}{c(a^2x^2+1)}dx}{a^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\int x^2 \arctan(ax)dx}{a^2c} - \frac{\int \frac{x^2 \arctan(ax)}{a^2x^2+1}dx}{a^2c}$$

$$\Big\downarrow \; 5361$$

$$\frac{\frac{1}{3}x^3 \arctan(ax) - \frac{1}{3}a \int \frac{x^3}{a^2x^2+1}dx}{a^2c} - \frac{\int \frac{x^2 \arctan(ax)}{a^2x^2+1}dx}{a^2c}$$

$$\Big\downarrow \; 243$$

$$\frac{\frac{1}{3}x^3 \arctan(ax) - \frac{1}{6}a \int \frac{x^2}{a^2x^2+1}dx^2}{a^2c} - \frac{\int \frac{x^2 \arctan(ax)}{a^2x^2+1}dx}{a^2c}$$

$$\Big\downarrow \; 49$$

$$\frac{\frac{1}{3}x^3 \arctan(ax) - \frac{1}{6}a \int \left(\frac{1}{a^2} - \frac{1}{a^2(a^2x^2+1)}\right)dx^2}{a^2c} - \frac{\int \frac{x^2 \arctan(ax)}{a^2x^2+1}dx}{a^2c}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{1}{3}x^3 \arctan(ax) - \frac{1}{6}a \left(\frac{x^2}{a^2} - \frac{\log(a^2x^2+1)}{a^4}\right)}{a^2c} - \frac{\int \frac{x^2 \arctan(ax)}{a^2x^2+1}dx}{a^2c}$$

$$\Big\downarrow \; 5451$$

$$\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log(a^2x^2+1)}{a^4}\right)}{a^2c}-\frac{\frac{\int \arctan(ax)dx}{a^2}-\frac{\int \frac{\arctan(ax)}{a^2x^2+1}dx}{a^2}}{a^2c}$$

$$\Big\downarrow \; 5345$$

$$\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log(a^2x^2+1)}{a^4}\right)}{a^2c}-\frac{\frac{x\arctan(ax)-a\int \frac{x}{a^2x^2+1}dx}{a^2}-\frac{\int \frac{\arctan(ax)}{a^2x^2+1}dx}{a^2}}{a^2c}$$

$$\Big\downarrow \; 240$$

$$\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log(a^2x^2+1)}{a^4}\right)}{a^2c}-\frac{\frac{x\arctan(ax)-\frac{\log(a^2x^2+1)}{2a}}{a^2}-\frac{\int \frac{\arctan(ax)}{a^2x^2+1}dx}{a^2}}{a^2c}$$

$$\Big\downarrow \; 5419$$

$$\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log(a^2x^2+1)}{a^4}\right)}{a^2c}-\frac{\frac{x\arctan(ax)-\frac{\log(a^2x^2+1)}{2a}}{a^2}-\frac{\arctan(ax)^2}{2a^3}}{a^2c}$$

input
```
Int[(x^4*ArcTan[a*x])/(c + a^2*c*x^2),x]
```

output
```
((x^3*ArcTan[a*x])/3 - (a*(x^2/a^2 - Log[1 + a^2*x^2]/a^4))/6)/(a^2*c) - (
-1/2*ArcTan[a*x]^2/a^3 + (x*ArcTan[a*x] - Log[1 + a^2*x^2]/(2*a))/a^2)/(a^
2*c)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

## Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.72

| method | result |
|---|---|
| parallelrisch | $\frac{2\arctan(ax)x^3a^3-a^2x^2-6\arctan(ax)ax+3\arctan(ax)^2+4\ln(a^2x^2+1)}{6c\,a^5}$ |
| derivativedivides | $\frac{\frac{\arctan(ax)a^3x^3}{3c}-\frac{\arctan(ax)ax}{c}+\frac{\arctan(ax)^2}{c}-\frac{\frac{a^2x^2}{2}-2\ln\left(a^2x^2+1\right)+\frac{3\arctan(ax)^2}{2}}{3c}}{a^5}$ |
| default | $\frac{\frac{\arctan(ax)a^3x^3}{3c}-\frac{\arctan(ax)ax}{c}+\frac{\arctan(ax)^2}{c}-\frac{\frac{a^2x^2}{2}-2\ln\left(a^2x^2+1\right)+\frac{3\arctan(ax)^2}{2}}{3c}}{a^5}$ |
| parts | $\frac{x^3\arctan(ax)}{3a^2c}-\frac{x\arctan(ax)}{a^4c}+\frac{\arctan(ax)^2}{a^5c}-\frac{\frac{\arctan(ax)^2}{2a^5}+\frac{\frac{a^2x^2}{2}-2\ln\left(a^2x^2+1\right)}{3a^5}}{c}$ |
| risch | $-\frac{\ln(iax+1)^2}{8a^5c}-\frac{i(2a^3x^3+3i\ln(-iax+1)-6ax)\ln(iax+1)}{12c\,a^5}+\frac{ix^3\ln(-iax+1)}{6a^2c}-\frac{ix\ln(-iax+1)}{2a^4c}-\frac{x^2}{6a^3c}-\frac{\ln(}{}$ |

input
```
int(x^4*arctan(a*x)/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*arctan(a*x)*x^3*a^3-a^2*x^2-6*arctan(a*x)*a*x+3*arctan(a*x)^2+4*ln(
a^2*x^2+1))/c/a^5
```

## Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.68

$$\int \frac{x^4\arctan(ax)}{c+a^2cx^2}\,dx = -\frac{a^2x^2-2\left(a^3x^3-3\,ax\right)\arctan\left(ax\right)-3\,\arctan\left(ax\right)^2-4\,\log\left(a^2x^2+1\right)}{6\,a^5c}$$

input
```
integrate(x^4*arctan(a*x)/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
-1/6*(a^2*x^2 - 2*(a^3*x^3 - 3*a*x)*arctan(a*x) - 3*arctan(a*x)^2 - 4*log(
a^2*x^2 + 1))/(a^5*c)
```

## Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.88

$$\int \frac{x^4 \arctan(ax)}{c + a^2cx^2} \, dx = \begin{cases} \frac{x^3 \operatorname{atan}(ax)}{3a^2c} - \frac{x^2}{6a^3c} - \frac{x \operatorname{atan}(ax)}{a^4c} + \frac{2 \log\left(x^2 + \frac{1}{a^2}\right)}{3a^5c} + \frac{\operatorname{atan}^2(ax)}{2a^5c} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x**4*atan(a*x)/(a**2*c*x**2+c),x)
```

output
```
Piecewise((x**3*atan(a*x)/(3*a**2*c) - x**2/(6*a**3*c) - x*atan(a*x)/(a**4
*c) + 2*log(x**2 + a**(-2))/(3*a**5*c) + atan(a*x)**2/(2*a**5*c), Ne(a, 0)
), (0, True))
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.92

$$\int \frac{x^4 \arctan(ax)}{c + a^2cx^2} \, dx = \frac{1}{3} \left( \frac{a^2x^3 - 3x}{a^4c} + \frac{3 \arctan(ax)}{a^5c} \right) \arctan(ax) - \frac{a^2x^2 + 3 \arctan(ax)^2 - 4 \log(a^2x^2 + 1)}{6a^5c}$$

input
```
integrate(x^4*arctan(a*x)/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
1/3*((a^2*x^3 - 3*x)/(a^4*c) + 3*arctan(a*x)/(a^5*c))*arctan(a*x) - 1/6*(a
^2*x^2 + 3*arctan(a*x)^2 - 4*log(a^2*x^2 + 1))/(a^5*c)
```

## Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.71

$$\int \frac{x^4 \arctan(ax)}{c+a^2cx^2}\,dx = \frac{2\,a^3x^3 \arctan{(ax)} - a^2x^2 - 6\,ax \arctan{(ax)} + 3\,\arctan{(ax)}^2 + 4\,\log{(a^2x^2+1)}}{6\,a^5c}$$

input
```
integrate(x^4*arctan(a*x)/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
1/6*(2*a^3*x^3*arctan(a*x) - a^2*x^2 - 6*a*x*arctan(a*x) + 3*arctan(a*x)^2
 + 4*log(a^2*x^2 + 1))/(a^5*c)
```

## Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.91

$$\int \frac{x^4 \arctan(ax)}{c+a^2cx^2}\,dx = \frac{2\,\ln{(a^2\,x^2+1)}}{3\,a^5\,c} - a^2\,\mathrm{atan}(a\,x)\,\left(\frac{x}{a^6\,c} - \frac{x^3}{3\,a^4\,c}\right) - \frac{x^2}{6\,a^3\,c} + \frac{\mathrm{atan}(a\,x)^2}{2\,a^5\,c}$$

input
```
int((x^4*atan(a*x))/(c + a^2*c*x^2),x)
```

output
```
(2*log(a^2*x^2 + 1))/(3*a^5*c) - a^2*atan(a*x)*(x/(a^6*c) - x^3/(3*a^4*c))
 - x^2/(6*a^3*c) + atan(a*x)^2/(2*a^5*c)
```

## Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.71

$$\int \frac{x^4 \arctan(ax)}{c+a^2cx^2}\,dx = \frac{3atan(ax)^2 + 2atan(ax)\,a^3x^3 - 6atan(ax)\,ax + 4\log(a^2x^2+1) - a^2x^2}{6a^5c}$$

input
```
int(x^4*atan(a*x)/(a^2*c*x^2+c),x)
```

output
```
(3*atan(a*x)**2 + 2*atan(a*x)*a**3*x**3 - 6*atan(a*x)*a*x + 4*log(a**2*x**
2 + 1) - a**2*x**2)/(6*a**5*c)
```

# 3.174 $\int \frac{x^3 \arctan(ax)}{c+a^2cx^2} \, dx$



## Optimal result

Integrand size = 20, antiderivative size = 113

$$\int \frac{x^3 \arctan(ax)}{c+a^2cx^2} \, dx = -\frac{x}{2a^3c} + \frac{\arctan(ax)}{2a^4c} + \frac{x^2\arctan(ax)}{2a^2c} + \frac{i\arctan(ax)^2}{2a^4c} + \frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a^4c} + \frac{i\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{2a^4c}$$

output
```
-1/2*x/a^3/c+1/2*arctan(a*x)/a^4/c+1/2*x^2*arctan(a*x)/a^2/c+1/2*I*arctan(
a*x)^2/a^4/c+arctan(a*x)*ln(2/(1+I*a*x))/a^4/c+1/2*I*polylog(2,1-2/(1+I*a*
x))/a^4/c
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.06

$$\int \frac{x^3 \arctan(ax)}{c+a^2cx^2} \, dx = -\frac{x}{2a^3c} + \frac{\arctan(ax)}{2a^4c} + \frac{x^2\arctan(ax)}{2a^2c} + \frac{i\arctan(ax)^2}{2a^4c} + \frac{\arctan(ax)\log\left(\frac{2i}{i-ax}\right)}{a^4c} + \frac{i\operatorname{PolyLog}\left(2,-\frac{i+ax}{i-ax}\right)}{2a^4c}$$

input
```
Integrate[(x^3*ArcTan[a*x])/(c + a^2*c*x^2),x]
```

output
```
-1/2*x/(a^3*c) + ArcTan[a*x]/(2*a^4*c) + (x^2*ArcTan[a*x])/(2*a^2*c) + ((I
/2)*ArcTan[a*x]^2)/(a^4*c) + (ArcTan[a*x]*Log[(2*I)/(I - a*x)])/(a^4*c) +
((I/2)*PolyLog[2, -((I + a*x)/(I - a*x))])/(a^4*c)
```

## Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {5451, 27, 5361, 262, 216, 5455, 5379, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \arctan(ax)}{a^2cx^2+c} \, dx$$

$$\Big\downarrow \text{5451}$$

$$\frac{\int x\arctan(ax)dx}{a^2c} - \frac{\int \frac{x\arctan(ax)}{c(a^2x^2+1)}dx}{a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int x\arctan(ax)dx}{a^2c} - \frac{\int \frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2c}$$

$$\Big\downarrow \text{5361}$$

$$\frac{\frac{1}{2}x^2\arctan(ax) - \frac{1}{2}a\int \frac{x^2}{a^2x^2+1}dx}{a^2c} - \frac{\int \frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2c}$$

$$\Big\downarrow \text{262}$$

$$\frac{\frac{1}{2}x^2\arctan(ax) - \frac{1}{2}a\left(\frac{x}{a^2} - \frac{\int \frac{1}{a^2x^2+1}dx}{a^2}\right)}{a^2c} - \frac{\int \frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2c}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{1}{2}x^2\arctan(ax) - \frac{1}{2}a\left(\frac{x}{a^2} - \frac{\arctan(ax)}{a^3}\right)}{a^2c} - \frac{\int \frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2c}$$

$$\Big\downarrow \text{5455}$$

$$\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2c}-\frac{-\frac{\int\frac{\arctan(ax)}{i-ax}dx}{a}-\frac{i\arctan(ax)^2}{2a^2}}{a^2c}$$

$\Big\downarrow$ 5379

$$\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2c}-\frac{-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}-\int\frac{\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^2}{2a^2}}{a^2c}$$

$\Big\downarrow$ 2849

$$\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2c}-\frac{-\frac{\frac{i\int\frac{\log\left(\frac{2}{iax+1}\right)}{1-\frac{2}{iax+1}}d\frac{1}{iax+1}}{a}+\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}}{a}-\frac{i\arctan(ax)^2}{2a^2}}{a^2c}$$

$\Big\downarrow$ 2752

$$\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2c}-\frac{-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}}{a^2c}$$

input
```
Int[(x^3*ArcTan[a*x])/(c + a^2*c*x^2),x]
```

output
```
((x^2*ArcTan[a*x])/2 - (a*(x/a^2 - ArcTan[a*x]/a^3))/2)/(a^2*c) - (((-1/2*
I)*ArcTan[a*x]^2)/a^2 - ((ArcTan[a*x]*Log[2/(1 + I*a*x)])/a + ((I/2)*PolyL
og[2, 1 - 2/(1 + I*a*x)])/a)/a)/(a^2*c)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

## Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.45 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.35

| method | result |
|---|---|
| parts | $\frac{x^2\arctan(ax)}{2a^2c}-\frac{\arctan(ax)\ln(a^2x^2+1)}{2c\,a^4}-\frac{a\left(\frac{x}{a^4}-\frac{\arctan(ax)}{a^5}-\frac{\sum_{\_\alpha=\mathrm{RootOf}(a^2\_Z^2+1)}\frac{2\ln(x-\_\alpha)\ln(a^2x^2+1)-a^2 \cdots}{\cdots}}{\cdots}\right)}{2c \cdots}$ |
| derivativedivides | $\frac{\frac{\arctan(ax)a^2x^2}{2c}-\frac{\arctan(ax)\ln(a^2x^2+1)}{2c}-\frac{ax-\arctan(ax)+\frac{i\left(\ln(ax-i)\ln(a^2x^2+1)-\frac{\ln(ax-i)^2}{2}-\mathrm{dilog}\left(-\frac{i(ax+i)}{2}\right)-\ln(ax-i) \cdots\right.}{2}}{\cdots}}{a^4}$ |
| default | $\frac{\frac{\arctan(ax)a^2x^2}{2c}-\frac{\arctan(ax)\ln(a^2x^2+1)}{2c}-\frac{ax-\arctan(ax)+\frac{i\left(\ln(ax-i)\ln(a^2x^2+1)-\frac{\ln(ax-i)^2}{2}-\mathrm{dilog}\left(-\frac{i(ax+i)}{2}\right)-\ln(ax-i) \cdots\right.}{2}}{\cdots}}{a^4}$ |
| risch | $-\frac{i\ln\left(\frac{1}{2}+\frac{iax}{2}\right)\ln(-iax+1)}{4c\,a^4}-\frac{i\ln(-iax+1)^2}{8c\,a^4}-\frac{i\,\mathrm{dilog}\left(\frac{1}{2}+\frac{iax}{2}\right)}{4c\,a^4}-\frac{x}{2a^3c}+\frac{i\ln\left(\frac{1}{2}-\frac{iax}{2}\right)\ln(iax+1)}{4c\,a^4}+\frac{i\,\mathrm{dilog}\left(\frac{1}{2} \cdots\right)}{4c \cdots}$ |

input
```
int(x^3*arctan(a*x)/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2*arctan(a*x)/a^2/c-1/2/c*arctan(a*x)/a^4*ln(a^2*x^2+1)-1/2/c*a*(1/a
^4*x-1/a^5*arctan(a*x)-1/4/a^6*sum(1/_alpha*(2*ln(x-_alpha)*ln(a^2*x^2+1)-
a^2*(1/a^2/_alpha*ln(x-_alpha)^2+2*_alpha*ln(x-_alpha)*ln(1/2*(x+_alpha)/_
alpha)+2*_alpha*dilog(1/2*(x+_alpha)/_alpha))),_alpha=RootOf(_Z^2*a^2+1)))
```

## Fricas [F]

$$\int \frac{x^3\arctan(ax)}{c+a^2cx^2}\,dx=\int \frac{x^3\arctan(ax)}{a^2cx^2+c}\,dx$$

input
```
integrate(x^3*arctan(a*x)/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
integral(x^3*arctan(a*x)/(a^2*c*x^2 + c), x)
```

## Sympy [F]

$$\int \frac{x^3 \arctan(ax)}{c + a^2cx^2} \, dx = \frac{\int \frac{x^3 \operatorname{atan}(ax)}{a^2x^2+1} \, dx}{c}$$

input
```
integrate(x**3*atan(a*x)/(a**2*c*x**2+c),x)
```

output
```
Integral(x**3*atan(a*x)/(a**2*x**2 + 1), x)/c
```

## Maxima [F]

$$\int \frac{x^3 \arctan(ax)}{c + a^2cx^2} \, dx = \int \frac{x^3 \arctan(ax)}{a^2cx^2 + c} \, dx$$

input
```
integrate(x^3*arctan(a*x)/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
integrate(x^3*arctan(a*x)/(a^2*c*x^2 + c), x)
```

## Giac [F]

$$\int \frac{x^3 \arctan(ax)}{c + a^2cx^2} \, dx = \int \frac{x^3 \arctan(ax)}{a^2cx^2 + c} \, dx$$

input
```
integrate(x^3*arctan(a*x)/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
integrate(x^3*arctan(a*x)/(a^2*c*x^2 + c), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \arctan(ax)}{c+a^2cx^2} \, dx = \int \frac{x^3 \operatorname{atan}(a\,x)}{c\,a^2\,x^2+c} \, dx$$

input
```
int((x^3*atan(a*x))/(c + a^2*c*x^2),x)
```

output
```
int((x^3*atan(a*x))/(c + a^2*c*x^2), x)
```

## Reduce [F]

$$\int \frac{x^3 \arctan(ax)}{c+a^2cx^2} \, dx = \frac{\int \frac{atan(ax)x^3}{a^2x^2+1}dx}{c}$$

input
```
int(x^3*atan(a*x)/(a^2*c*x^2+c),x)
```

output
```
int((atan(a*x)*x**3)/(a**2*x**2 + 1),x)/c
```

# 3.175 $\int \frac{x^2 \arctan(ax)}{c+a^2cx^2}\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 49

$$\int \frac{x^2 \arctan(ax)}{c+a^2cx^2}\,dx = \frac{x\arctan(ax)}{a^2c} - \frac{\arctan(ax)^2}{2a^3c} - \frac{\log(1+a^2x^2)}{2a^3c}$$

output
```
x*arctan(a*x)/a^2/c-1/2*arctan(a*x)^2/a^3/c-1/2*ln(a^2*x^2+1)/a^3/c
```

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{x^2 \arctan(ax)}{c+a^2cx^2}\,dx = \frac{x\arctan(ax)}{a^2c} - \frac{\arctan(ax)^2}{2a^3c} - \frac{\log(1+a^2x^2)}{2a^3c}$$

input
```
Integrate[(x^2*ArcTan[a*x])/(c + a^2*c*x^2),x]
```

output
```
(x*ArcTan[a*x])/(a^2*c) - ArcTan[a*x]^2/(2*a^3*c) - Log[1 + a^2*x^2]/(2*a^
3*c)
```

## Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5451, 27, 5345, 240, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \arctan(ax)}{a^2cx^2+c} \, dx$$

$$\Big\downarrow \text{ 5451}$$

$$\frac{\int \arctan(ax)dx}{a^2c} - \frac{\int \frac{\arctan(ax)}{c(a^2x^2+1)}dx}{a^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \arctan(ax)dx}{a^2c} - \frac{\int \frac{\arctan(ax)}{a^2x^2+1}dx}{a^2c}$$

$$\Big\downarrow \text{ 5345}$$

$$\frac{x\arctan(ax) - a\int \frac{x}{a^2x^2+1}dx}{a^2c} - \frac{\int \frac{\arctan(ax)}{a^2x^2+1}dx}{a^2c}$$

$$\Big\downarrow \text{ 240}$$

$$\frac{x\arctan(ax) - \frac{\log(a^2x^2+1)}{2a}}{a^2c} - \frac{\int \frac{\arctan(ax)}{a^2x^2+1}dx}{a^2c}$$

$$\Big\downarrow \text{ 5419}$$

$$\frac{x\arctan(ax) - \frac{\log(a^2x^2+1)}{2a}}{a^2c} - \frac{\arctan(ax)^2}{2a^3c}$$

input
```
Int[(x^2*ArcTan[a*x])/(c + a^2*c*x^2),x]
```

output
```
-1/2*ArcTan[a*x]^2/(a^3*c) + (x*ArcTan[a*x] - Log[1 + a^2*x^2]/(2*a))/(a^2
*c)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

## Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.78

| method | result | size |
|---|---|---|
| parallelrisch | $\frac{2\arctan(ax)ax-\arctan(ax)^2-\ln(a^2x^2+1)}{2c\,a^3}$ | 38 |
| derivativedivides | $\frac{\frac{\arctan(ax)ax}{c}-\frac{\arctan(ax)^2}{c}-\frac{\frac{\ln\left(a^2x^2+1\right)}{2}-\frac{\arctan(ax)^2}{2}}{c}}{a^3}$ | 53 |
| default | $\frac{\frac{\arctan(ax)ax}{c}-\frac{\arctan(ax)^2}{c}-\frac{\frac{\ln\left(a^2x^2+1\right)}{2}-\frac{\arctan(ax)^2}{2}}{c}}{a^3}$ | 53 |
| parts | $\frac{x\arctan(ax)}{a^2c}-\frac{\arctan(ax)^2}{a^3c}-\frac{\frac{\ln\left(a^2x^2+1\right)}{2a^3}-\frac{\arctan(ax)^2}{2a^3}}{c}$ | 60 |
| risch | $\frac{\ln(iax+1)^2}{8a^3c}-\frac{i(-i\ln(-iax+1)+2ax)\ln(iax+1)}{4c\,a^3}+\frac{\ln(-iax+1)^2}{8c\,a^3}+\frac{ix\ln(-iax+1)}{2c\,a^2}-\frac{\ln(-a^2x^2-1)}{2c\,a^3}$ | 108 |

input
```
int(x^2*arctan(a*x)/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
1/2*(2*arctan(a*x)*a*x-arctan(a*x)^2-ln(a^2*x^2+1))/c/a^3
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

$$\int \frac{x^2\arctan(ax)}{c+a^2cx^2}\,dx=\frac{2\,ax\arctan(ax)-\arctan(ax)^2-\log(a^2x^2+1)}{2\,a^3c}$$

input
```
integrate(x^2*arctan(a*x)/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
1/2*(2*a*x*arctan(a*x) - arctan(a*x)^2 - log(a^2*x^2 + 1))/(a^3*c)
```

## Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

$$\int \frac{x^2\arctan(ax)}{c+a^2cx^2}\,dx=\begin{cases}\frac{x\,\mathrm{atan}\,(ax)}{a^2c}-\frac{\log\left(x^2+\frac{1}{a^2}\right)}{2a^3c}-\frac{\mathrm{atan}^2\,(ax)}{2a^3c} & \text{for } a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

input
```
integrate(x**2*atan(a*x)/(a**2*c*x**2+c),x)
```

output
```
Piecewise((x*atan(a*x)/(a**2*c) - log(x**2 + a**(-2))/(2*a**3*c) - atan(a*
x)**2/(2*a**3*c), Ne(a, 0)), (0, True))
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.10

$$\int \frac{x^2 \arctan(ax)}{c + a^2cx^2}\, dx = \left(\frac{x}{a^2c} - \frac{\arctan(ax)}{a^3c}\right)\arctan(ax) + \frac{\arctan(ax)^2 - \log(a^2x^2 + 1)}{2\,a^3c}$$

input
```
integrate(x^2*arctan(a*x)/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
(x/(a^2*c) - arctan(a*x)/(a^3*c))*arctan(a*x) + 1/2*(arctan(a*x)^2 - log(a
^2*x^2 + 1))/(a^3*c)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

$$\int \frac{x^2 \arctan(ax)}{c + a^2cx^2}\, dx = \frac{2\,ax\arctan(ax) - \arctan(ax)^2 - \log(a^2x^2 + 1)}{2\,a^3c}$$

input
```
integrate(x^2*arctan(a*x)/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
1/2*(2*a*x*arctan(a*x) - arctan(a*x)^2 - log(a^2*x^2 + 1))/(a^3*c)
```

## Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.67

$$\int \frac{x^2 \arctan(ax)}{c+a^2cx^2} \, dx = -\frac{\operatorname{atan}(a\,x)^2 - 2\,a\,x\,\operatorname{atan}(a\,x) + \ln\left(a^2\,x^2+1\right)}{2\,a^3\,c}$$

input
```
int((x^2*atan(a*x))/(c + a^2*c*x^2),x)
```

output
```
-(log(a^2*x^2 + 1) + atan(a*x)^2 - 2*a*x*atan(a*x))/(2*a^3*c)
```

## Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

$$\int \frac{x^2 \arctan(ax)}{c+a^2cx^2} \, dx = \frac{-atan(ax)^2 + 2atan(ax)\,ax - \log(a^2x^2+1)}{2a^3c}$$

input
```
int(x^2*atan(a*x)/(a^2*c*x^2+c),x)
```

output
```
( - atan(a*x)**2 + 2*atan(a*x)*a*x - log(a**2*x**2 + 1))/(2*a**3*c)
```

# 3.176 $\int \frac{x\arctan(ax)}{c+a^2cx^2}\,dx$



## Optimal result

Integrand size = 18, antiderivative size = 72

$$\int \frac{x\arctan(ax)}{c+a^2cx^2}\,dx = -\frac{i\arctan(ax)^2}{2a^2c}-\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a^2c}-\frac{i\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{2a^2c}$$

output
```
-1/2*I*arctan(a*x)^2/a^2/c-arctan(a*x)*ln(2/(1+I*a*x))/a^2/c-1/2*I*polylog
(2,1-2/(1+I*a*x))/a^2/c
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.07

$$\int \frac{x\arctan(ax)}{c+a^2cx^2}\,dx = -\frac{i\arctan(ax)^2}{2a^2c}-\frac{\arctan(ax)\log\left(\frac{2i}{i-ax}\right)}{a^2c}-\frac{i\,\mathrm{PolyLog}\left(2,\frac{i+ax}{-i+ax}\right)}{2a^2c}$$

input
```
Integrate[(x*ArcTan[a*x])/(c + a^2*c*x^2),x]
```

output
```
((-1/2*I)*ArcTan[a*x]^2)/(a^2*c) - (ArcTan[a*x]*Log[(2*I)/(I - a*x)])/(a^2
*c) - ((I/2)*PolyLog[2, (I + a*x)/(-I + a*x)])/(a^2*c)
```

## Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5455, 5379, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\arctan(ax)}{a^2cx^2+c}dx$$

$$\Big\downarrow \text{5455}$$

$$-\frac{\int \frac{\arctan(ax)}{i-ax}dx}{ac}-\frac{i\arctan(ax)^2}{2a^2c}$$

$$\Big\downarrow \text{5379}$$

$$-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}-\int \frac{\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{ac}-\frac{i\arctan(ax)^2}{2a^2c}$$

$$\Big\downarrow \text{2849}$$

$$-\frac{\frac{i\int \frac{\log\left(\frac{2}{iax+1}\right)}{1-\frac{2}{iax+1}}d\frac{1}{iax+1}}{a}+\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}}{ac}-\frac{i\arctan(ax)^2}{2a^2c}$$

$$\Big\downarrow \text{2752}$$

$$-\frac{i\arctan(ax)^2}{2a^2c}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{ac}$$

input
```
Int[(x*ArcTan[a*x])/(c + a^2*c*x^2),x]
```

output
```
((-1/2*I)*ArcTan[a*x]^2)/(a^2*c) - ((ArcTan[a*x]*Log[2/(1 + I*a*x)])/a + (
(I/2)*PolyLog[2, 1 - 2/(1 + I*a*x)])/a)/(a*c)
```

### Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

## Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.58 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.65

| method | result |
|---|---|
| parts | $\frac{\ln(a^2x^2+1)\arctan(ax)}{2a^2c} - \frac{\sum_{\_\alpha=\mathrm{RootOf}\left(a^2\_Z^2+1\right)} \frac{2\ln(x-\_\alpha)\ln\left(a^2x^2+1\right)-a^2\left(\frac{\ln(x-\_\alpha)^2}{a^2\_\alpha}+2\_\alpha\ln(x-\_\alpha)\ln\left(\frac{x+\_\alpha}{2\_\alpha}\right)-\cdots\right.}{\_\alpha}}{8a^3c}$ |
| risch | $\frac{i\ln\left(\frac{1}{2}+\frac{iax}{2}\right)\ln(-iax+1)}{4c\,a^2} - \frac{i\,\mathrm{dilog}\left(\frac{1}{2}-\frac{iax}{2}\right)}{4c\,a^2} + \frac{i\ln(-iax+1)^2}{8c\,a^2} - \frac{i\ln\left(\frac{1}{2}-\frac{iax}{2}\right)\ln(iax+1)}{4c\,a^2} + \frac{i\,\mathrm{dilog}\left(\frac{1}{2}+\frac{iax}{2}\right)}{4c\,a^2} - \cdots$ |
| derivativedivides | $\frac{\frac{\arctan(ax)\ln\left(a^2x^2+1\right)}{2c} - \frac{-\frac{i\left(\ln(ax-i)\ln\left(a^2x^2+1\right)-\frac{\ln(ax-i)^2}{2}-\mathrm{dilog}\left(-\frac{i(ax+i)}{2}\right)-\ln(ax-i)\ln\left(-\frac{i(ax+i)}{2}\right)\right)}{2}+\frac{i\left(\ln(ax+i)\ln(\cdots\right.}{\cdots}}{2c}}{a^2}$ |
| default | $\frac{\frac{\arctan(ax)\ln\left(a^2x^2+1\right)}{2c} - \frac{-\frac{i\left(\ln(ax-i)\ln\left(a^2x^2+1\right)-\frac{\ln(ax-i)^2}{2}-\mathrm{dilog}\left(-\frac{i(ax+i)}{2}\right)-\ln(ax-i)\ln\left(-\frac{i(ax+i)}{2}\right)\right)}{2}+\frac{i\left(\ln(ax+i)\ln(\cdots\right.}{\cdots}}{2c}}{a^2}$ |

input
```
int(x*arctan(a*x)/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
1/2/a^2/c*ln(a^2*x^2+1)*arctan(a*x)-1/8/a^3/c*sum(1/_alpha*(2*ln(x-_alpha)
*ln(a^2*x^2+1)-a^2*(1/a^2/_alpha*ln(x-_alpha)^2+2*_alpha*ln(x-_alpha)*ln(1
/2*(x+_alpha)/_alpha)+2*_alpha*dilog(1/2*(x+_alpha)/_alpha))),_alpha=RootO
f(_Z^2*a^2+1))
```

## Fricas [F]

$$\int \frac{x\arctan(ax)}{c+a^2cx^2}\,dx = \int \frac{x\arctan(ax)}{a^2cx^2+c}\,dx$$

input
```
integrate(x*arctan(a*x)/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
integral(x*arctan(a*x)/(a^2*c*x^2 + c), x)
```

## Sympy [F]

$$\int \frac{x\arctan(ax)}{c+a^2cx^2}\,dx = \frac{\int \frac{x\,\mathrm{atan}\,(ax)}{a^2x^2+1}\,dx}{c}$$

input
```
integrate(x*atan(a*x)/(a**2*c*x**2+c),x)
```

output
```
Integral(x*atan(a*x)/(a**2*x**2 + 1), x)/c
```

## Maxima [F]

$$\int \frac{x\arctan(ax)}{c+a^2cx^2}\,dx = \int \frac{x\arctan(ax)}{a^2cx^2+c}\,dx$$

input
```
integrate(x*arctan(a*x)/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
integrate(x*arctan(a*x)/(a^2*c*x^2 + c), x)
```

## Giac [F]

$$\int \frac{x\arctan(ax)}{c+a^2cx^2}\,dx = \int \frac{x\arctan(ax)}{a^2cx^2+c}\,dx$$

input
```
integrate(x*arctan(a*x)/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
integrate(x*arctan(a*x)/(a^2*c*x^2 + c), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x\arctan(ax)}{c+a^2cx^2}\,dx = \int \frac{x\,\mathrm{atan}(a\,x)}{c\,a^2\,x^2+c}\,dx$$

input
```
int((x*atan(a*x))/(c + a^2*c*x^2),x)
```

output
```
int((x*atan(a*x))/(c + a^2*c*x^2), x)
```

## Reduce [F]

$$\int \frac{x\arctan(ax)}{c+a^2cx^2}\,dx = \frac{\int \frac{atan(ax)x}{a^2x^2+1}dx}{c}$$

input
```
int(x*atan(a*x)/(a^2*c*x^2+c),x)
```

output
```
int((atan(a*x)*x)/(a**2*x**2 + 1),x)/c
```

# 3.177 $\int \frac{\arctan(ax)}{c+a^2cx^2} dx$



## Optimal result

Integrand size = 17, antiderivative size = 16

$$\int \frac{\arctan(ax)}{c+a^2cx^2} dx = \frac{\arctan(ax)^2}{2ac}$$

output
```
1/2*arctan(a*x)^2/a/c
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax)}{c+a^2cx^2} dx = \frac{\arctan(ax)^2}{2ac}$$

input
```
Integrate[ArcTan[a*x]/(c + a^2*c*x^2),x]
```

output
```
ArcTan[a*x]^2/(2*a*c)
```

## Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)}{a^2cx^2+c}\,dx$$
$$\Big\downarrow \; 5419$$
$$\frac{\arctan(ax)^2}{2ac}$$

input
```
Int[ArcTan[a*x]/(c + a^2*c*x^2),x]
```

output
```
ArcTan[a*x]^2/(2*a*c)
```

### Defintions of rubi rules used

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

## Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\arctan(ax)^2}{2ac}$ | 15 |
| default | $\frac{\arctan(ax)^2}{2ac}$ | 15 |
| parallelrisch | $\frac{\arctan(ax)^2}{2ac}$ | 15 |
| parts | $\frac{\arctan(ax)^2}{2ac}$ | 15 |
| risch | $-\frac{\ln(iax+1)^2}{8ca}+\frac{\ln(-iax+1)\ln(iax+1)}{4ca}-\frac{\ln(-iax+1)^2}{8ca}$ | 62 |

input
```
int(arctan(a*x)/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
1/2*arctan(a*x)^2/a/c
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\arctan(ax)}{c+a^2cx^2}\,dx=\frac{\arctan{(ax)}^2}{2\,ac}$$

input
```
integrate(arctan(a*x)/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
1/2*arctan(a*x)^2/(a*c)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(10) = 20$.

Time = 0.63 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.25

$$\int \frac{\arctan(ax)}{c + a^2cx^2}\, dx = \begin{cases} 0 & \text{for}\: a = 0 \\ \tilde{\infty} \left(\begin{cases} 0 & \text{for}\: a = 0 \\ \frac{ax \operatorname{atan}{(ax)} - \frac{\log{\left(a^2x^2+1 \right)}}{2}}{a} & \text{otherwise} \end{cases}\right) & \text{for}\: c = 0 \\ \frac{\operatorname{atan}^2{(ax)}}{2ac} & \text{otherwise} \end{cases}$$

input
```
integrate(atan(a*x)/(a**2*c*x**2+c),x)
```

output
```
Piecewise((0, Eq(a, 0)), (zoo*Piecewise((0, Eq(a, 0)), ((a*x*atan(a*x) - l
og(a**2*x**2 + 1)/2)/a, True)), Eq(c, 0)), (atan(a*x)**2/(2*a*c), True))
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\arctan(ax)}{c + a^2cx^2}\, dx = \frac{\arctan{(ax)}^2}{2\,ac}$$

input
```
integrate(arctan(a*x)/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
1/2*arctan(a*x)^2/(a*c)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\arctan(ax)}{c + a^2cx^2}\,dx = \frac{\arctan{(ax)}^2}{2\,ac}$$

input
```
integrate(arctan(a*x)/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
1/2*arctan(a*x)^2/(a*c)
```

## Mupad [B] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\arctan(ax)}{c + a^2cx^2}\,dx = \frac{\mathrm{atan}(a\,x)^2}{2\,a\,c}$$

input
```
int(atan(a*x)/(c + a^2*c*x^2),x)
```

output
```
atan(a*x)^2/(2*a*c)
```

## Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\arctan(ax)}{c + a^2cx^2}\,dx = \frac{atan(ax)^2}{2ac}$$

input
```
int(atan(a*x)/(a^2*c*x^2+c),x)
```

output
```
atan(a*x)**2/(2*a*c)
```

# 3.178 $\int \frac{\arctan(ax)}{x(c+a^2cx^2)}\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 64

$$\int \frac{\arctan(ax)}{x(c+a^2cx^2)}\,dx = -\frac{i\arctan(ax)^2}{2c} + \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{c} - \frac{i\,\mathrm{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{2c}$$

output
```
-1/2*I*arctan(a*x)^2/c+arctan(a*x)*ln(2-2/(1-I*a*x))/c-1/2*I*polylog(2,-1+
2/(1-I*a*x))/c
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.61

$$\int \frac{\arctan(ax)}{x(c+a^2cx^2)}\,dx = \frac{i\arctan(ax)^2}{2c} + \frac{\arctan(ax)\log\left(\frac{2i}{i-ax}\right)}{c} + \frac{i\,\mathrm{PolyLog}(2,-iax)}{2c} - \frac{i\,\mathrm{PolyLog}(2,iax)}{2c} + \frac{i\,\mathrm{PolyLog}\left(2,-\frac{i+ax}{i-ax}\right)}{2c}$$

input
```
Integrate[ArcTan[a*x]/(x*(c + a^2*c*x^2)),x]
```

output
```
((I/2)*ArcTan[a*x]^2)/c + (ArcTan[a*x]*Log[(2*I)/(I - a*x)])/c + ((I/2)*Po
lyLog[2, (-I)*a*x])/c - ((I/2)*PolyLog[2, I*a*x])/c + ((I/2)*PolyLog[2, -(
(I + a*x)/(I - a*x))])/c
```

## Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {5459, 5403, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)}{x\left(a^2cx^2+c\right)}dx$$

$$\Big\downarrow \text{5459}$$

$$\frac{i\int \frac{\arctan(ax)}{x(ax+i)}dx}{c}-\frac{i\arctan(ax)^2}{2c}$$

$$\Big\downarrow \text{5403}$$

$$\frac{i\left(ia\int \frac{\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)\right)}{c}-\frac{i\arctan(ax)^2}{2c}$$

$$\Big\downarrow \text{2897}$$

$$\frac{i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)}{c}-\frac{i\arctan(ax)^2}{2c}$$

input
```
Int[ArcTan[a*x]/(x*(c + a^2*c*x^2)),x]
```

output
```
((-1/2*I)*ArcTan[a*x]^2)/c + (I*((-I)*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)] -
 PolyLog[2, -1 + 2/(1 - I*a*x)]/2))/c
```

### Defintions of rubi rules used

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

## Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 133 vs. 2(56) = 112.

Time = 0.43 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.09

| method | result |
|---|---|
| risch | $-\frac{i\ln\left(\frac{1}{2}+\frac{iax}{2}\right)\ln(-iax+1)}{4c}+\frac{i\,\mathrm{dilog}\left(\frac{1}{2}-\frac{iax}{2}\right)}{4c}-\frac{i\ln(-iax+1)^2}{8c}-\frac{i\,\mathrm{dilog}(-iax+1)}{2c}+\frac{i\ln\left(\frac{1}{2}-\frac{iax}{2}\right)\ln(iax+1)}{4c}$ |
| parts | $-\frac{\arctan(ax)\ln(a^2x^2+1)}{2c}+\frac{\arctan(ax)\ln(x)}{c}-\frac{a\left(-\frac{i\ln(x)(-\ln(-iax+1)+\ln(iax+1))}{a}-\frac{i(\mathrm{dilog}(iax+1)-\mathrm{dilog}(-iax+1}{a}\right.}{}$ |
| derivativedivides | $\frac{\arctan(ax)\ln(ax)}{c}-\frac{\arctan(ax)\ln(a^2x^2+1)}{2c}-\frac{-i\ln(ax)\ln(iax+1)+i\ln(ax)\ln(-iax+1)-i\,\mathrm{dilog}(iax+1)+i\,\mathrm{dilog}(}{}$ |
| default | $\frac{\arctan(ax)\ln(ax)}{c}-\frac{\arctan(ax)\ln(a^2x^2+1)}{2c}-\frac{-i\ln(ax)\ln(iax+1)+i\ln(ax)\ln(-iax+1)-i\,\mathrm{dilog}(iax+1)+i\,\mathrm{dilog}(}{}$ |

input
```
int(arctan(a*x)/x/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
-1/4*I/c*ln(1/2+1/2*I*a*x)*ln(1-I*a*x)+1/4*I/c*dilog(1/2-1/2*I*a*x)-1/8*I/
c*ln(1-I*a*x)^2-1/2*I/c*dilog(1-I*a*x)+1/4*I/c*ln(1/2-1/2*I*a*x)*ln(1+I*a*
x)-1/4*I/c*dilog(1/2+1/2*I*a*x)+1/8*I/c*ln(1+I*a*x)^2+1/2*I/c*dilog(1+I*a*
x)
```

## Fricas [F]

$$\int \frac{\arctan(ax)}{x\left(c+a^2cx^2\right)}\,dx=\int \frac{\arctan\left(ax\right)}{(a^2cx^2+c)x}\,dx$$

input
```
integrate(arctan(a*x)/x/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
integral(arctan(a*x)/(a^2*c*x^3 + c*x), x)
```

## Sympy [F]

$$\int \frac{\arctan(ax)}{x\left(c+a^2cx^2\right)}\,dx=\frac{\int \frac{\operatorname{atan}\left(ax\right)}{a^2x^3+x}\,dx}{c}$$

input
```
integrate(atan(a*x)/x/(a**2*c*x**2+c),x)
```

output
```
Integral(atan(a*x)/(a**2*x**3 + x), x)/c
```

## Maxima [F]

$$\int \frac{\arctan(ax)}{x(c+a^2cx^2)}\,dx = \int \frac{\arctan(ax)}{(a^2cx^2+c)x}\,dx$$

input
```
integrate(arctan(a*x)/x/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)/((a^2*c*x^2 + c)*x), x)
```

## Giac [F]

$$\int \frac{\arctan(ax)}{x(c+a^2cx^2)}\,dx = \int \frac{\arctan(ax)}{(a^2cx^2+c)x}\,dx$$

input
```
integrate(arctan(a*x)/x/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
integrate(arctan(a*x)/((a^2*c*x^2 + c)*x), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)}{x(c+a^2cx^2)}\,dx = \int \frac{\operatorname{atan}(a\,x)}{x\,\left(c\,a^2\,x^2+c\right)}\,dx$$

input
```
int(atan(a*x)/(x*(c + a^2*c*x^2)),x)
```

output
```
int(atan(a*x)/(x*(c + a^2*c*x^2)), x)
```

## Reduce [F]

$$\int \frac{\arctan(ax)}{x\,(c+a^2cx^2)}\,dx = \frac{\int \frac{atan(ax)}{a^2x^3+x}dx}{c}$$

input
```
int(atan(a*x)/x/(a^2*c*x^2+c),x)
```

output
```
int(atan(a*x)/(a**2*x**3 + x),x)/c
```

# 3.179 $\int \frac{\arctan(ax)}{x^2(c+a^2cx^2)}\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 52

$$\int \frac{\arctan(ax)}{x^2(c+a^2cx^2)}\,dx = -\frac{\arctan(ax)}{cx} - \frac{a\arctan(ax)^2}{2c} + \frac{a\log(x)}{c} - \frac{a\log(1+a^2x^2)}{2c}$$

output
```
-arctan(a*x)/c/x-1/2*a*arctan(a*x)^2/c+a*ln(x)/c-1/2*a*ln(a^2*x^2+1)/c
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax)}{x^2(c+a^2cx^2)}\,dx = -\frac{\arctan(ax)}{cx} - \frac{a\arctan(ax)^2}{2c} + \frac{a\log(x)}{c} - \frac{a\log(1+a^2x^2)}{2c}$$

input
```
Integrate[ArcTan[a*x]/(x^2*(c + a^2*c*x^2)),x]
```

output
```
-(ArcTan[a*x]/(c*x)) - (a*ArcTan[a*x]^2)/(2*c) + (a*Log[x])/c - (a*Log[1 +
 a^2*x^2])/(2*c)
```

## Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5453, 27, 5361, 243, 47, 14, 16, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)}{x^2 (a^2cx^2+c)} \, dx$$

$$\Big\downarrow \text{ 5453}$$

$$\frac{\int \frac{\arctan(ax)}{x^2}dx}{c}-a^2\int \frac{\arctan(ax)}{c(a^2x^2+1)}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{\arctan(ax)}{x^2}dx}{c}-\frac{a^2\int \frac{\arctan(ax)}{a^2x^2+1}dx}{c}$$

$$\Big\downarrow \text{ 5361}$$

$$\frac{a\int \frac{1}{x(a^2x^2+1)}dx-\frac{\arctan(ax)}{x}}{c}-\frac{a^2\int \frac{\arctan(ax)}{a^2x^2+1}dx}{c}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{\frac{1}{2}a\int \frac{1}{x^2(a^2x^2+1)}dx^2-\frac{\arctan(ax)}{x}}{c}-\frac{a^2\int \frac{\arctan(ax)}{a^2x^2+1}dx}{c}$$

$$\Big\downarrow \text{ 47}$$

$$\frac{\frac{1}{2}a\left(\int \frac{1}{x^2}dx^2-a^2\int \frac{1}{a^2x^2+1}dx^2\right)-\frac{\arctan(ax)}{x}}{c}-\frac{a^2\int \frac{\arctan(ax)}{a^2x^2+1}dx}{c}$$

$$\Big\downarrow \text{ 14}$$

$$\frac{\frac{1}{2}a\left(\log(x^2)-a^2\int \frac{1}{a^2x^2+1}dx^2\right)-\frac{\arctan(ax)}{x}}{c}-\frac{a^2\int \frac{\arctan(ax)}{a^2x^2+1}dx}{c}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{\frac{1}{2}a(\log(x^2)-\log(a^2x^2+1))-\frac{\arctan(ax)}{x}}{c}-\frac{a^2\int \frac{\arctan(ax)}{a^2x^2+1}dx}{c}$$

$$\Big\downarrow \text{ 5419}$$

$$\frac{\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{\arctan(ax)}{x}}{c}-\frac{a\arctan(ax)^2}{2c}$$

input
```
Int[ArcTan[a*x]/(x^2*(c + a^2*c*x^2)),x]
```

output
```
-1/2*(a*ArcTan[a*x]^2)/c + (-(ArcTan[a*x]/x) + (a*(Log[x^2] - Log[1 + a^2*
x^2]))/2)/c
```

## Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

## Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.88

| method | result |
|---|---|
| parallelrisch | $\frac{-a\arctan(ax)^2x+2ax\ln(x)-a\ln(a^2x^2+1)x-2\arctan(ax)}{2cx}$ |
| parts | $-\frac{a\arctan(ax)^2}{c}-\frac{\arctan(ax)}{cx}-\frac{-\frac{a\arctan(ax)^2}{2}-a\left(-\frac{\ln\left(a^2x^2+1\right)}{2}+\ln(ax)\right)}{c}$ |
| derivativedivides | $a\left(-\frac{\arctan(ax)^2}{c}-\frac{\arctan(ax)}{cax}-\frac{\frac{\ln\left(a^2x^2+1\right)}{2}-\ln(ax)-\frac{\arctan(ax)^2}{2}}{c}\right)$ |
| default | $a\left(-\frac{\arctan(ax)^2}{c}-\frac{\arctan(ax)}{cax}-\frac{\frac{\ln\left(a^2x^2+1\right)}{2}-\ln(ax)-\frac{\arctan(ax)^2}{2}}{c}\right)$ |
| risch | $\frac{a\ln(iax+1)^2}{8c}-\frac{(ax\ln(-iax+1)-2i)\ln(iax+1)}{4cx}-\frac{-a\ln(-iax+1)^2x-8ax\ln(x)+4ax\ln\left(-3a^2x^2-3\right)+4i\ln(-iax+}{8cx}$ |

input
```
int(arctan(a*x)/x^2/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-a*arctan(a*x)^2*x+2*a*x*ln(x)-a*ln(a^2*x^2+1)*x-2*arctan(a*x))/c/x
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.83

$$\int \frac{\arctan(ax)}{x^2(c+a^2cx^2)}\,dx = -\frac{ax\arctan(ax)^2 + ax\log(a^2x^2+1) - 2\,ax\log(x) + 2\,\arctan(ax)}{2\,cx}$$

input
```
integrate(arctan(a*x)/x^2/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
-1/2*(a*x*arctan(a*x)^2 + a*x*log(a^2*x^2 + 1) - 2*a*x*log(x) + 2*arctan(a
*x))/(c*x)
```

## Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.81

$$\int \frac{\arctan(ax)}{x^2(c+a^2cx^2)}\,dx = \begin{cases} \frac{a\log(x)}{c} - \frac{a\log\left(x^2+\frac{1}{a^2}\right)}{2c} - \frac{a\,\mathrm{atan}^2(ax)}{2c} - \frac{\mathrm{atan}(ax)}{cx} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(atan(a*x)/x**2/(a**2*c*x**2+c),x)
```

output
```
Piecewise((a*log(x)/c - a*log(x**2 + a**(-2))/(2*c) - a*atan(a*x)**2/(2*c)
 - atan(a*x)/(c*x), Ne(a, 0)), (0, True))
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.02

$$\int \frac{\arctan(ax)}{x^2(c+a^2cx^2)}\,dx = -\left(\frac{a\arctan(ax)}{c} + \frac{1}{cx}\right)\arctan(ax) + \frac{\left(\arctan(ax)^2 - \log(a^2x^2+1) + 2\,\log(x)\right)a}{2\,c}$$

input
```
integrate(arctan(a*x)/x^2/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
-(a*arctan(a*x)/c + 1/(c*x))*arctan(a*x) + 1/2*(arctan(a*x)^2 - log(a^2*x^
2 + 1) + 2*log(x))*a/c
```

## Giac [F]

$$\int \frac{\arctan(ax)}{x^2\,(c+a^2cx^2)}\,dx = \int \frac{\arctan\,(ax)}{(a^2cx^2+c)x^2}\,dx$$

input
```
integrate(arctan(a*x)/x^2/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
integrate(arctan(a*x)/((a^2*c*x^2 + c)*x^2), x)
```

## Mupad [B] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.92

$$\int \frac{\arctan(ax)}{x^2\,(c+a^2cx^2)}\,dx = \frac{a\,\ln\,(x)}{c} - \frac{a\,\ln\,(a^2\,x^2+1)}{2\,c} - \frac{a\,\mathrm{atan}(a\,x)^2}{2\,c} - \frac{\mathrm{atan}(a\,x)}{c\,x}$$

input
```
int(atan(a*x)/(x^2*(c + a^2*c*x^2)),x)
```

output
```
(a*log(x))/c - (a*log(a^2*x^2 + 1))/(2*c) - (a*atan(a*x)^2)/(2*c) - atan(a
*x)/(c*x)
```

## Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.87

$$\int \frac{\arctan(ax)}{x^2 (c + a^2cx^2)} \, dx = \frac{-atan(ax)^2 ax - 2atan(ax) - \log(a^2x^2 + 1) ax + 2 \log(x) ax}{2cx}$$

input
```
int(atan(a*x)/x^2/(a^2*c*x^2+c),x)
```

output
```
( - atan(a*x)**2*a*x - 2*atan(a*x) - log(a**2*x**2 + 1)*a*x + 2*log(x)*a*x
)/(2*c*x)
```

# 3.180 $\int \frac{\arctan(ax)}{x^3(c+a^2cx^2)}\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 113

$$\int \frac{\arctan(ax)}{x^3\left(c+a^2cx^2\right)}\,dx = -\frac{a}{2cx}-\frac{a^2\arctan(ax)}{2c}-\frac{\arctan(ax)}{2cx^2}+\frac{ia^2\arctan(ax)^2}{2c}-\frac{a^2\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{c}+\frac{ia^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{2c}$$

output
```
-1/2*a/c/x-1/2*a^2*arctan(a*x)/c-1/2*arctan(a*x)/c/x^2+1/2*I*a^2*arctan(a*
x)^2/c-a^2*arctan(a*x)*ln(2-2/(1-I*a*x))/c+1/2*I*a^2*polylog(2,-1+2/(1-I*a
*x))/c
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.06 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.98

$$\int \frac{\arctan(ax)}{x^3\left(c+a^2cx^2\right)}\,dx = -\frac{\frac{\arctan(ax)}{x^2}+\frac{a\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-a^2x^2\right)}{x}+ia^2\left(\arctan(ax)^2-2i\arctan(ax)\log\left(\frac{2i}{i-ax}\right)+\operatorname{PolyLog}(2,-\right.}{2c}$$

input
```
Integrate[ArcTan[a*x]/(x^3*(c + a^2*c*x^2)),x]
```

output
```
-1/2*(ArcTan[a*x]/x^2 + (a*Hypergeometric2F1[-1/2, 1, 1/2, -(a^2*x^2)])/x
+ I*a^2*(ArcTan[a*x]^2 - (2*I)*ArcTan[a*x]*Log[(2*I)/(I - a*x)] + PolyLog[
2, (-I)*a*x] - PolyLog[2, I*a*x] + PolyLog[2, (I + a*x)/(-I + a*x)]))/c
```

## Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.92, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5453, 27, 5361, 264, 216, 5459, 5403, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)}{x^3\left(a^2cx^2+c\right)}dx$$

$$\Big\downarrow \text{5453}$$

$$\frac{\int \frac{\arctan(ax)}{x^3}dx}{c}-a^2\int \frac{\arctan(ax)}{cx\left(a^2x^2+1\right)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\arctan(ax)}{x^3}dx}{c}-\frac{a^2\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \text{5361}$$

$$\frac{\frac{1}{2}a\int \frac{1}{x^2(a^2x^2+1)}dx-\frac{\arctan(ax)}{2x^2}}{c}-\frac{a^2\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \text{264}$$

$$\frac{\frac{1}{2}a\left(a^2\left(-\int \frac{1}{a^2x^2+1}dx\right)-\frac{1}{x}\right)-\frac{\arctan(ax)}{2x^2}}{c}-\frac{a^2\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{1}{2}a\left(-a\arctan(ax)-\frac{1}{x}\right)-\frac{\arctan(ax)}{2x^2}}{c}-\frac{a^2\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \text{5459}$$

$$\frac{\frac{1}{2}a\left(-a\arctan(ax)-\frac{1}{x}\right)-\frac{\arctan(ax)}{2x^2}}{c}-\frac{a^2\left(i\int\frac{\arctan(ax)}{x(ax+i)}dx-\frac{1}{2}i\arctan(ax)^2\right)}{c}$$

$$\Big\downarrow \text{5403}$$

$$\frac{\frac{1}{2}a\left(-a\arctan(ax)-\frac{1}{x}\right)-\frac{\arctan(ax)}{2x^2}}{c}-\frac{a^2\left(i\left(ia\int\frac{\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)}{c}$$

$$\Big\downarrow \text{2897}$$

$$\frac{\frac{1}{2}a\left(-a\arctan(ax)-\frac{1}{x}\right)-\frac{\arctan(ax)}{2x^2}}{c}-\frac{a^2\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)}{c}$$

input
```
Int[ArcTan[a*x]/(x^3*(c + a^2*c*x^2)),x]
```

output
```
(-1/2*ArcTan[a*x]/x^2 + (a*(-x^(-1) - a*ArcTan[a*x]))/2)/c - (a^2*((-1/2*I
)*ArcTan[a*x]^2 + I*((-I)*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)] - PolyLog[2,
-1 + 2/(1 - I*a*x)]/2)))/c
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

## Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.54 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.88

| method | result |
|---|---|
| parts | $\frac{\arctan(ax)a^2\ln(a^2x^2+1)}{2c}-\frac{\arctan(ax)}{2cx^2}-\frac{\arctan(ax)a^2\ln(x)}{c}-\frac{a\left(\left(\sum_{\_\alpha=\mathrm{RootOf}(a^2\_Z^2+1)}\frac{2\ln(x-\_\alpha)\ln(a^2x^2 \ldots}{\ldots}\right.\right.}{\ldots}$ |
| derivativedivides | $a^2\left(-\frac{\arctan(ax)}{2c\,a^2x^2}-\frac{\arctan(ax)\ln(ax)}{c}+\frac{\arctan(ax)\ln(a^2x^2+1)}{2c}-\frac{-\frac{i\left(\ln(ax-i)\ln(a^2x^2+1)-\frac{\ln(ax-i)^2}{2}-\mathrm{dilog}(\ldots\right.}{2}}{\ldots}\right.$ |
| default | $a^2\left(-\frac{\arctan(ax)}{2c\,a^2x^2}-\frac{\arctan(ax)\ln(ax)}{c}+\frac{\arctan(ax)\ln(a^2x^2+1)}{2c}-\frac{-\frac{i\left(\ln(ax-i)\ln(a^2x^2+1)-\frac{\ln(ax-i)^2}{2}-\mathrm{dilog}(\ldots\right.}{2}}{\ldots}\right.$ |
| risch | $\frac{ia^2\ln(iax+1)}{4c}-\frac{ia^2\ln(-iax+1)}{4c}-\frac{ia^2\,\mathrm{dilog}(\frac{1}{2}-\frac{iax}{2})}{4c}-\frac{a}{2cx}+\frac{i\ln(iax+1)}{4c\,x^2}-\frac{i\ln(-iax+1)}{4c\,x^2}-\frac{ia^2\ln(iax+1)\ldots}{8c}$ |

input
```
int(arctan(a*x)/x^3/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
1/2/c*arctan(a*x)*a^2*ln(a^2*x^2+1)-1/2*arctan(a*x)/c/x^2-1/c*arctan(a*x)*
a^2*ln(x)-1/2*a/c*(1/4*sum(1/_alpha*(2*ln(x-_alpha)*ln(a^2*x^2+1)-a^2*(1/a
^2/_alpha*ln(x-_alpha)^2+2*_alpha*ln(x-_alpha)*ln(1/2*(x+_alpha)/_alpha)+2
*_alpha*dilog(1/2*(x+_alpha)/_alpha))),_alpha=RootOf(_Z^2*a^2+1))+a*arctan
(a*x)+1/x-2*a^2*(-1/2*I*ln(x)*(-ln(1-I*a*x)+ln(1+I*a*x))/a-1/2*I*(dilog(1+
I*a*x)-dilog(1-I*a*x))/a))
```

## Fricas [F]

$$\int \frac{\arctan(ax)}{x^3\,(c+a^2cx^2)}\,dx = \int \frac{\arctan(ax)}{(a^2cx^2+c)x^3}\,dx$$

input
```
integrate(arctan(a*x)/x^3/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
integral(arctan(a*x)/(a^2*c*x^5 + c*x^3), x)
```

## Sympy [F]

$$\int \frac{\arctan(ax)}{x^3\,(c+a^2cx^2)}\,dx = \frac{\int \frac{\operatorname{atan}(ax)}{a^2x^5+x^3}\,dx}{c}$$

input
```
integrate(atan(a*x)/x**3/(a**2*c*x**2+c),x)
```

output
```
Integral(atan(a*x)/(a**2*x**5 + x**3), x)/c
```

## Maxima [F]

$$\int \frac{\arctan(ax)}{x^3\,(c+a^2cx^2)}\,dx = \int \frac{\arctan(ax)}{(a^2cx^2+c)x^3}\,dx$$

input
```
integrate(arctan(a*x)/x^3/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)/((a^2*c*x^2 + c)*x^3), x)
```

## Giac [F]

$$\int \frac{\arctan(ax)}{x^3(c+a^2cx^2)}\,dx = \int \frac{\arctan(ax)}{(a^2cx^2+c)x^3}\,dx$$

input
```
integrate(arctan(a*x)/x^3/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
integrate(arctan(a*x)/((a^2*c*x^2 + c)*x^3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)}{x^3(c+a^2cx^2)}\,dx = \int \frac{\mathrm{atan}(a\,x)}{x^3\,(c\,a^2\,x^2+c)}\,dx$$

input
```
int(atan(a*x)/(x^3*(c + a^2*c*x^2)),x)
```

output
```
int(atan(a*x)/(x^3*(c + a^2*c*x^2)), x)
```

## Reduce [F]

$$\int \frac{\arctan(ax)}{x^3(c+a^2cx^2)}\,dx = \frac{\int \frac{atan(ax)}{a^2x^5+x^3}dx}{c}$$

input
```
int(atan(a*x)/x^3/(a^2*c*x^2+c),x)
```

output
```
int(atan(a*x)/(a**2*x**5 + x**3),x)/c
```

# 3.181 $\int \frac{\arctan(ax)}{x^4(c+a^2cx^2)}\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 88

$$\int \frac{\arctan(ax)}{x^4\left(c+a^2cx^2\right)}\,dx = -\frac{a}{6cx^2}-\frac{\arctan(ax)}{3cx^3}+\frac{a^2\arctan(ax)}{cx}+\frac{a^3\arctan(ax)^2}{2c}-\frac{4a^3\log(x)}{3c}+\frac{2a^3\log\left(1+a^2x^2\right)}{3c}$$

output
```
-1/6*a/c/x^2-1/3*arctan(a*x)/c/x^3+a^2*arctan(a*x)/c/x+1/2*a^3*arctan(a*x)
^2/c-4/3*a^3*ln(x)/c+2/3*a^3*ln(a^2*x^2+1)/c
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax)}{x^4\left(c+a^2cx^2\right)}\,dx = -\frac{a}{6cx^2}-\frac{\arctan(ax)}{3cx^3}+\frac{a^2\arctan(ax)}{cx}+\frac{a^3\arctan(ax)^2}{2c}-\frac{4a^3\log(x)}{3c}+\frac{2a^3\log\left(1+a^2x^2\right)}{3c}$$

input
```
Integrate[ArcTan[a*x]/(x^4*(c + a^2*c*x^2)),x]
```

output
```
-1/6*a/(c*x^2) - ArcTan[a*x]/(3*c*x^3) + (a^2*ArcTan[a*x])/(c*x) + (a^3*Ar
cTan[a*x]^2)/(2*c) - (4*a^3*Log[x])/(3*c) + (2*a^3*Log[1 + a^2*x^2])/(3*c)
```

## Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.16, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.650, Rules used = {5453, 27, 5361, 243, 54, 2009, 5453, 5361, 243, 47, 14, 16, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)}{x^4\left(a^2cx^2+c\right)}dx$$

$$\Big\downarrow \ 5453$$

$$\frac{\int \frac{\arctan(ax)}{x^4}dx}{c}-a^2\int \frac{\arctan(ax)}{cx^2\left(a^2x^2+1\right)}dx$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{\arctan(ax)}{x^4}dx}{c}-\frac{a^2\int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \ 5361$$

$$\frac{\frac{1}{3}a\int \frac{1}{x^3(a^2x^2+1)}dx-\frac{\arctan(ax)}{3x^3}}{c}-\frac{a^2\int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \ 243$$

$$\frac{\frac{1}{6}a\int \frac{1}{x^4(a^2x^2+1)}dx^2-\frac{\arctan(ax)}{3x^3}}{c}-\frac{a^2\int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \ 54$$

$$\frac{\frac{1}{6}a\int \left(\frac{a^4}{a^2x^2+1}-\frac{a^2}{x^2}+\frac{1}{x^4}\right)dx^2-\frac{\arctan(ax)}{3x^3}}{c}-\frac{a^2\int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{1}{6}a\left(a^2\left(-\log\left(x^2\right)\right)+a^2\log\left(a^2x^2+1\right)-\frac{1}{x^2}\right)-\frac{\arctan(ax)}{3x^3}}{c}-\frac{a^2\int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \; 5453$$

$$\frac{\frac{1}{6}a\left(a^2\left(-\log\left(x^2\right)\right)+a^2\log\left(a^2x^2+1\right)-\frac{1}{x^2}\right)-\frac{\arctan(ax)}{3x^3}}{c}-\frac{a^2\left(\int\frac{\arctan(ax)}{x^2}dx-a^2\int\frac{\arctan(ax)}{a^2x^2+1}dx\right)}{c}$$

$$\Big\downarrow \; 5361$$

$$\frac{\frac{1}{6}a\left(a^2\left(-\log\left(x^2\right)\right)+a^2\log\left(a^2x^2+1\right)-\frac{1}{x^2}\right)-\frac{\arctan(ax)}{3x^3}}{c}-\frac{a^2\left(a^2\left(-\int\frac{\arctan(ax)}{a^2x^2+1}dx\right)+a\int\frac{1}{x(a^2x^2+1)}dx-\frac{\arctan(ax)}{x}\right)}{c}$$

$$\Big\downarrow \; 243$$

$$\frac{\frac{1}{6}a\left(a^2\left(-\log\left(x^2\right)\right)+a^2\log\left(a^2x^2+1\right)-\frac{1}{x^2}\right)-\frac{\arctan(ax)}{3x^3}}{c}-\frac{a^2\left(a^2\left(-\int\frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{2}a\int\frac{1}{x^2(a^2x^2+1)}dx^2-\frac{\arctan(ax)}{x}\right)}{c}$$

$$\Big\downarrow \; 47$$

$$\frac{\frac{1}{6}a\left(a^2\left(-\log\left(x^2\right)\right)+a^2\log\left(a^2x^2+1\right)-\frac{1}{x^2}\right)-\frac{\arctan(ax)}{3x^3}}{c}-\frac{a^2\left(a^2\left(-\int\frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{2}a\left(\int\frac{1}{x^2}dx^2-a^2\int\frac{1}{a^2x^2+1}dx^2\right)-\frac{\arctan(ax)}{x}\right)}{c}$$

$$\Big\downarrow \; 14$$

$$\frac{\frac{1}{6}a\left(a^2\left(-\log\left(x^2\right)\right)+a^2\log\left(a^2x^2+1\right)-\frac{1}{x^2}\right)-\frac{\arctan(ax)}{3x^3}}{c}-\frac{a^2\left(a^2\left(-\int\frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{2}a\left(\log\left(x^2\right)-a^2\int\frac{1}{a^2x^2+1}dx^2\right)-\frac{\arctan(ax)}{x}\right)}{c}$$

$$\Big\downarrow \; 16$$

$$\frac{\frac{1}{6}a\left(a^2\left(-\log\left(x^2\right)\right)+a^2\log\left(a^2x^2+1\right)-\frac{1}{x^2}\right)-\frac{\arctan(ax)}{3x^3}}{c}-\frac{a^2\left(a^2\left(-\int\frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{\arctan(ax)}{x}\right)}{c}$$

$$\Big\downarrow \; 5419$$

$$\frac{\frac{1}{6}a\left(a^2\left(-\log\left(x^2\right)\right)+a^2\log\left(a^2x^2+1\right)-\frac{1}{x^2}\right)-\frac{\arctan(ax)}{3x^3}}{c}-\frac{a^2\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{1}{2}a\arctan(ax)^2-\frac{\arctan(ax)}{x}\right)}{c}$$

input
```
Int[ArcTan[a*x]/(x^4*(c + a^2*c*x^2)),x]
```

output
```
-((a^2*(-(ArcTan[a*x]/x) - (a*ArcTan[a*x]^2)/2 + (a*(Log[x^2] - Log[1 + a^
2*x^2]))/2))/c) + (-1/3*ArcTan[a*x]/x^3 + (a*(-x^(-2) - a^2*Log[x^2] + a^2
*Log[1 + a^2*x^2]))/6)/c
```

## Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

## Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.92

| method | result |
|---|---|
| parallelrisch | $-\frac{-3a^3\arctan(ax)^2x^3+8\ln(x)a^3x^3-4a^3\ln(a^2x^2+1)x^3-2a^3x^3-6x^2a^2\arctan(ax)+ax+2\arctan(ax)}{6c\,x^3}$ |
| derivativedivides | $a^3\left(\frac{\arctan(ax)^2}{c}-\frac{\arctan(ax)}{3c\,a^3x^3}+\frac{\arctan(ax)}{cax}-\frac{-2\ln(a^2x^2+1)+\frac{1}{2a^2x^2}+4\ln(ax)+\frac{3\arctan(ax)^2}{2}}{3c}\right)$ |
| default | $a^3\left(\frac{\arctan(ax)^2}{c}-\frac{\arctan(ax)}{3c\,a^3x^3}+\frac{\arctan(ax)}{cax}-\frac{-2\ln(a^2x^2+1)+\frac{1}{2a^2x^2}+4\ln(ax)+\frac{3\arctan(ax)^2}{2}}{3c}\right)$ |
| parts | $\frac{a^3\arctan(ax)^2}{c}-\frac{\arctan(ax)}{3c\,x^3}+\frac{a^2\arctan(ax)}{cx}-\frac{a^3\left(-2\ln(a^2x^2+1)+\frac{1}{2a^2x^2}+4\ln(ax)\right)+\frac{3a^3\arctan(ax)^2}{2}}{3c}$ |
| risch | $\frac{a^3\ln\left(\frac{1}{2}+\frac{iax}{2}\right)\ln(-iax+1)}{4c}-\frac{a^3\ln\left(\frac{1}{2}+\frac{iax}{2}\right)\ln\left(\frac{1}{2}-\frac{iax}{2}\right)}{2c}-\frac{a^3\operatorname{dilog}\left(\frac{1}{2}-\frac{iax}{2}\right)}{4c}-\frac{2a^3\ln(-iax)}{3c}+\frac{2a^3\ln(-iax+1)}{3c}$ |

input
```
int(arctan(a*x)/x^4/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(-3*a^3*arctan(a*x)^2*x^3+8*ln(x)*a^3*x^3-4*a^3*ln(a^2*x^2+1)*x^3-2*a
^3*x^3-6*x^2*a^2*arctan(a*x)+a*x+2*arctan(a*x))/c/x^3
```

## Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.81

$$\int \frac{\arctan(ax)}{x^4 (c + a^2cx^2)} dx = \frac{3 a^3x^3 \arctan(ax)^2 + 4 a^3x^3 \log(a^2x^2 + 1) - 8 a^3x^3 \log(x) - ax + 2 (3 a^2x^2 - 1) \arctan(ax)}{6 cx^3}$$

input
```
integrate(arctan(a*x)/x^4/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
1/6*(3*a^3*x^3*arctan(a*x)^2 + 4*a^3*x^3*log(a^2*x^2 + 1) - 8*a^3*x^3*log(
x) - a*x + 2*(3*a^2*x^2 - 1)*arctan(a*x))/(c*x^3)
```

## Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.86

$$\int \frac{\arctan(ax)}{x^4 (c + a^2cx^2)} dx = \begin{cases} -\frac{4a^3 \log(x)}{3c} + \frac{2a^3 \log\left(x^2 + \frac{1}{a^2}\right)}{3c} + \frac{a^3 \operatorname{atan}^2(ax)}{2c} + \frac{a^2 \operatorname{atan}(ax)}{cx} - \frac{a}{6cx^2} - \frac{\operatorname{atan}(ax)}{3cx^3} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(atan(a*x)/x**4/(a**2*c*x**2+c),x)
```

output
```
Piecewise((-4*a**3*log(x)/(3*c) + 2*a**3*log(x**2 + a**(-2))/(3*c) + a**3*
atan(a*x)**2/(2*c) + a**2*atan(a*x)/(c*x) - a/(6*c*x**2) - atan(a*x)/(3*c*
x**3), Ne(a, 0)), (0, True))
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.02

$$\int \frac{\arctan(ax)}{x^4\left(c+a^2cx^2\right)}\,dx = \frac{1}{3}\left(\frac{3\,a^3\arctan\left(ax\right)}{c}+\frac{3\,a^2x^2-1}{cx^3}\right)\arctan\left(ax\right) - \frac{\left(3\,a^2x^2\arctan\left(ax\right)^2-4\,a^2x^2\log\left(a^2x^2+1\right)+8\,a^2x^2\log\left(x\right)+1\right)a}{6\,cx^2}$$

input
```
integrate(arctan(a*x)/x^4/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
1/3*(3*a^3*arctan(a*x)/c + (3*a^2*x^2 - 1)/(c*x^3))*arctan(a*x) - 1/6*(3*a
^2*x^2*arctan(a*x)^2 - 4*a^2*x^2*log(a^2*x^2 + 1) + 8*a^2*x^2*log(x) + 1)*
a/(c*x^2)
```

## Giac [F]

$$\int \frac{\arctan(ax)}{x^4\left(c+a^2cx^2\right)}\,dx = \int \frac{\arctan\left(ax\right)}{\left(a^2cx^2+c\right)x^4}\,dx$$

input
```
integrate(arctan(a*x)/x^4/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
integrate(arctan(a*x)/((a^2*c*x^2 + c)*x^4), x)
```

## Mupad [B] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.89

$$\int \frac{\arctan(ax)}{x^4\left(c+a^2cx^2\right)}\,dx = \frac{2\,a^3\,\ln\left(a^2\,x^2+1\right)}{3\,c} - \frac{\mathrm{atan}(a\,x)}{3\,c\,x^3} - \frac{a}{6\,c\,x^2} - \frac{4\,a^3\,\ln\left(x\right)}{3\,c} + \frac{a^3\,\mathrm{atan}(a\,x)^2}{2\,c} + \frac{a^2\,\mathrm{atan}(a\,x)}{c\,x}$$

input
```
int(atan(a*x)/(x^4*(c + a^2*c*x^2)),x)
```

output
```
(2*a^3*log(a^2*x^2 + 1))/(3*c) - atan(a*x)/(3*c*x^3) - a/(6*c*x^2) - (4*a^
3*log(x))/(3*c) + (a^3*atan(a*x)^2)/(2*c) + (a^2*atan(a*x))/(c*x)
```

## Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.83

$$\int \frac{\arctan(ax)}{x^4 (c + a^2cx^2)}\,dx = \frac{3atan(ax)^2 a^3x^3 + 6atan(ax)\,a^2x^2 - 2atan(ax) + 4\log(a^2x^2+1)\,a^3x^3 - 8\log(x)\,a^3x^3 - ax}{6c\,x^3}$$

input
```
int(atan(a*x)/x^4/(a^2*c*x^2+c),x)
```

output
```
(3*atan(a*x)**2*a**3*x**3 + 6*atan(a*x)*a**2*x**2 - 2*atan(a*x) + 4*log(a*
*2*x**2 + 1)*a**3*x**3 - 8*log(x)*a**3*x**3 - a*x)/(6*c*x**3)
```

# 3.182 $\int \frac{x^5 \arctan(ax)}{(c+a^2cx^2)^2}\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 157

$$\int \frac{x^5 \arctan(ax)}{(c+a^2cx^2)^2}\,dx = -\frac{x}{2a^5c^2}+\frac{x}{4a^5c^2(1+a^2x^2)}+\frac{3\arctan(ax)}{4a^6c^2}+\frac{x^2\arctan(ax)}{2a^4c^2}-\frac{\arctan(ax)}{2a^6c^2(1+a^2x^2)}+\frac{i\arctan(ax)^2}{a^6c^2}+\frac{2\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a^6c^2}+\frac{i\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{a^6c^2}$$

output
```
-1/2*x/a^5/c^2+1/4*x/a^5/c^2/(a^2*x^2+1)+3/4*arctan(a*x)/a^6/c^2+1/2*x^2*a
rctan(a*x)/a^4/c^2-1/2*arctan(a*x)/a^6/c^2/(a^2*x^2+1)+I*arctan(a*x)^2/a^6
/c^2+2*arctan(a*x)*ln(2/(1+I*a*x))/a^6/c^2+I*polylog(2,1-2/(1+I*a*x))/a^6/
c^2
```

## Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.57

$$\int \frac{x^5 \arctan(ax)}{(c+a^2cx^2)^2} dx = \frac{-4ax-8i\arctan(ax)^2+2\arctan(ax)\left(2+2a^2x^2-\cos(2\arctan(ax))+8\log\left(1+e^{2i\arctan(ax)}\right)\right)-8i\text{ Po}}{8a^6c^2}$$

input
```
Integrate[(x^5*ArcTan[a*x])/(c + a^2*c*x^2)^2,x]
```

output
```
(-4*a*x - (8*I)*ArcTan[a*x]^2 + 2*ArcTan[a*x]*(2 + 2*a^2*x^2 - Cos[2*ArcTa
n[a*x]] + 8*Log[1 + E^((2*I)*ArcTan[a*x])]) - (8*I)*PolyLog[2, -E^((2*I)*A
rcTan[a*x])] + Sin[2*ArcTan[a*x]])/(8*a^6*c^2)
```

## Rubi [A] (verified)

Time = 1.75 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.67, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.900$, Rules used = {5499, 27, 5451, 5361, 262, 216, 5455, 5379, 2849, 2752, 5499, 5455, 5379, 2849, 2752, 5465, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 \arctan(ax)}{(a^2cx^2+c)^2} dx$$

$$\Big\downarrow \text{ 5499}$$

$$\frac{\int \frac{x^3 \arctan(ax)}{c(a^2x^2+1)}dx}{a^2c} - \frac{\int \frac{x^3 \arctan(ax)}{c^2(a^2x^2+1)^2}dx}{a^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{x^3 \arctan(ax)}{a^2x^2+1}dx}{a^2c^2} - \frac{\int \frac{x^3 \arctan(ax)}{(a^2x^2+1)^2}dx}{a^2c^2}$$

$$\Big\downarrow \text{ 5451}$$

$$\frac{\frac{\int x\arctan(ax)dx}{a^2}-\frac{\int \frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}}{a^2c^2}-\frac{\int \frac{x^3\arctan(ax)}{(a^2x^2+1)^2}dx}{a^2c^2}$$

$$\Big\downarrow \text{5361}$$

$$\frac{\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\int \frac{x^2}{a^2x^2+1}dx}{a^2}-\frac{\int \frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}}{a^2c^2}-\frac{\int \frac{x^3\arctan(ax)}{(a^2x^2+1)^2}dx}{a^2c^2}$$

$$\Big\downarrow \text{262}$$

$$\frac{\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\int \frac{1}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\frac{\int \frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}}{a^2c^2}-\frac{\int \frac{x^3\arctan(ax)}{(a^2x^2+1)^2}dx}{a^2c^2}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{\int \frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}}{a^2c^2}-\frac{\int \frac{x^3\arctan(ax)}{(a^2x^2+1)^2}dx}{a^2c^2}$$

$$\Big\downarrow \text{5455}$$

$$-\frac{\int \frac{x^3\arctan(ax)}{(a^2x^2+1)^2}dx}{a^2c^2}+\frac{\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{-\frac{\int \frac{\arctan(ax)}{i-ax}dx}{a}-\frac{i\arctan(ax)^2}{2a^2}}{a^2}}{a^2c^2}$$

$$\Big\downarrow \text{5379}$$

$$-\frac{\int \frac{x^3\arctan(ax)}{(a^2x^2+1)^2}dx}{a^2c^2}+\frac{\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}-\int \frac{\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^2}{2a^2}}{a^2}}{a^2c^2}$$

$$\Big\downarrow \text{2849}$$

$$-\frac{\int \frac{x^3\arctan(ax)}{(a^2x^2+1)^2}dx}{a^2c^2}+\frac{\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{-\frac{\frac{i\int \frac{\log\left(\frac{2}{iax+1}\right)}{1-\frac{2}{iax+1}}d\frac{1}{iax+1}}{a}+\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}}{a}-\frac{i\arctan(ax)^2}{2a^2}}{a^2}}{a^2c^2}$$

$$\Big\downarrow \text{2752}$$

$$-\frac{\int \frac{x^3\arctan(ax)}{(a^2x^2+1)^2}dx}{a^2c^2}+\frac{\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}}{a^2}}{a^2c^2}$$

$\Big\downarrow$ 5499

$$-\frac{\frac{\int \frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}-\frac{\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx}{a^2}}{a^2c^2}+\frac{\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}}{a^2}}{a^2c^2}$$

$\Big\downarrow$ 5455

$$\frac{\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}}{a^2}}{a^2c^2}-\frac{-\frac{\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx}{a^2}+\frac{-\frac{\int \frac{\arctan(ax)}{i-ax}dx}{a}-\frac{i\arctan(ax)^2}{2a^2}}{a^2}}{a^2c^2}$$

$\Big\downarrow$ 5379

$$\frac{\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}}{a^2}}{a^2c^2}-\frac{-\frac{\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx}{a^2}+\frac{-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}-\int \frac{\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^2}{2a^2}}{a^2}}{a^2c^2}$$

$\Big\downarrow$ 2849

$$\frac{\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}}{a^2}}{a^2c^2}-\frac{-\frac{\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx}{a^2}+\frac{-\frac{\frac{i\int \frac{\log\left(\frac{2}{iax+1}\right)}{1-\frac{2}{iax+1}}d\frac{1}{iax+1}}{a}+\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}}{a}-\frac{i\arctan(ax)^2}{2a^2}}{a^2}}{a^2c^2}$$

$\Big\downarrow$ 2752

$$\frac{\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}}{a^2}}{a^2c^2}-\frac{-\frac{\int\frac{x\arctan(ax)}{\left(a^2x^2+1\right)^2}dx}{a^2}+\frac{-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}}{a^2}}{a^2c^2}$$

$$\Big\downarrow \; 5465$$

$$\frac{\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}}{a^2}}{a^2c^2}-\frac{-\frac{\frac{\int\frac{1}{\left(a^2x^2+1\right)^2}dx}{2a}-\frac{\arctan(ax)}{2a^2\left(a^2x^2+1\right)}}{a^2}+\frac{-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}}{a^2}}{a^2c^2}$$

$$\Big\downarrow \; 215$$

$$\frac{\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}}{a^2}}{a^2c^2}-\frac{-\frac{\frac{\frac{1}{2}\int\frac{1}{a^2x^2+1}dx+\frac{x}{2\left(a^2x^2+1\right)}}{2a}-\frac{\arctan(ax)}{2a^2\left(a^2x^2+1\right)}}{a^2}+\frac{-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}}{a^2}}{a^2c^2}$$

$$\Big\downarrow \; 216$$

$$\frac{\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}}{a^2}}{a^2c^2}-\frac{-\frac{\frac{\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\arctan(ax)}{2a^2\left(a^2x^2+1\right)}}{a^2}+\frac{-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}}{a^2}}{a^2c^2}$$

input

```
Int[(x^5*ArcTan[a*x])/(c + a^2*c*x^2)^2,x]
```

output
```
(((x^2*ArcTan[a*x])/2 - (a*(x/a^2 - ArcTan[a*x]/a^3))/2)/a^2 - (((-1/2*I)*
ArcTan[a*x]^2)/a^2 - ((ArcTan[a*x]*Log[2/(1 + I*a*x)])/a + ((I/2)*PolyLog[
2, 1 - 2/(1 + I*a*x)])/a)/a)/a^2)/(a^2*c^2) - (-((-1/2*ArcTan[a*x]/(a^2*(1
 + a^2*x^2)) + (x/(2*(1 + a^2*x^2)) + ArcTan[a*x]/(2*a))/(2*a))/a^2) + (((
-1/2*I)*ArcTan[a*x]^2)/a^2 - ((ArcTan[a*x]*Log[2/(1 + I*a*x)])/a + ((I/2)*
PolyLog[2, 1 - 2/(1 + I*a*x)])/a)/a)/a^2)/(a^2*c^2)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5499
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ
[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

## Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.68 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.21

| method | result |
|---|---|
| parts | $\frac{x^2\arctan(ax)}{2a^4c^2}-\frac{\arctan(ax)}{2a^6c^2(a^2x^2+1)}-\frac{\arctan(ax)\ln(a^2x^2+1)}{c^2a^6}-\frac{a\left(\frac{x-\frac{x}{2\left(a^2x^2+1\right)}-\frac{3\arctan(ax)}{2a}}{a^6}-\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(a^2\_\right.}}{}\right.}{}$ |
| derivativedivides | $\frac{\frac{\arctan(ax)a^2x^2}{2c^2}-\frac{\arctan(ax)}{2c^2\left(a^2x^2+1\right)}-\frac{\arctan(ax)\ln\left(a^2x^2+1\right)}{c^2}-\frac{ax-\frac{ax}{2\left(a^2x^2+1\right)}-\frac{3\arctan(ax)}{2}+i\left(\ln(ax-i)\ln\left(a^2x^2+1\right)-\frac{\ln(ax-i)}{2}\right.}{}}{}$ |
| default | $\frac{\frac{\arctan(ax)a^2x^2}{2c^2}-\frac{\arctan(ax)}{2c^2\left(a^2x^2+1\right)}-\frac{\arctan(ax)\ln\left(a^2x^2+1\right)}{c^2}-\frac{ax-\frac{ax}{2\left(a^2x^2+1\right)}-\frac{3\arctan(ax)}{2}+i\left(\ln(ax-i)\ln\left(a^2x^2+1\right)-\frac{\ln(ax-i)}{2}\right.}{}}{}$ |
| risch | $\frac{\ln(iax+1)x}{16c^2a^5(iax-1)}+\frac{i\ln(-iax+1)}{16c^2a^6(-iax-1)}-\frac{i\ln\left(\frac{1}{2}+\frac{iax}{2}\right)\ln(-iax+1)}{2c^2a^6}-\frac{i\,\mathrm{dilog}\left(\frac{1}{2}+\frac{iax}{2}\right)}{2c^2a^6}+\frac{i\ln(-iax+1)x^2}{4c^2a^4}+\frac{\ln(\cdot}{16c^2a}$ |

input
```
int(x^5*arctan(a*x)/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2*arctan(a*x)/a^4/c^2-1/2*arctan(a*x)/a^6/c^2/(a^2*x^2+1)-1/c^2*arct
an(a*x)/a^6*ln(a^2*x^2+1)-1/2/c^2*a*(1/a^6*(x-1/2*x/(a^2*x^2+1)-3/2/a*arct
an(a*x))-1/2/a^8*sum(1/_alpha*(2*ln(x-_alpha)*ln(a^2*x^2+1)-a^2*(1/a^2/_al
pha*ln(x-_alpha)^2+2*_alpha*ln(x-_alpha)*ln(1/2*(x+_alpha)/_alpha)+2*_alph
a*dilog(1/2*(x+_alpha)/_alpha))),_alpha=RootOf(_Z^2*a^2+1))
```

## Fricas [F]

$$\int \frac{x^5\arctan(ax)}{(c+a^2cx^2)^2}\,dx=\int \frac{x^5\arctan(ax)}{(a^2cx^2+c)^2}\,dx$$

input
```
integrate(x^5*arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
integral(x^5*arctan(a*x)/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2), x)
```

## Sympy [F]

$$\int \frac{x^5 \arctan(ax)}{(c+a^2cx^2)^2}\,dx = \frac{\int \frac{x^5 \operatorname{atan}(ax)}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

input
```
integrate(x**5*atan(a*x)/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(x**5*atan(a*x)/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

## Maxima [F]

$$\int \frac{x^5 \arctan(ax)}{(c+a^2cx^2)^2}\,dx = \int \frac{x^5 \arctan(ax)}{(a^2cx^2+c)^2}\,dx$$

input
```
integrate(x^5*arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
integrate(x^5*arctan(a*x)/(a^2*c*x^2 + c)^2, x)
```

## Giac [F]

$$\int \frac{x^5 \arctan(ax)}{(c+a^2cx^2)^2}\,dx = \int \frac{x^5 \arctan(ax)}{(a^2cx^2+c)^2}\,dx$$

input
```
integrate(x^5*arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate(x^5*arctan(a*x)/(a^2*c*x^2 + c)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^5 \arctan(ax)}{(c+a^2cx^2)^2}\,dx = \int \frac{x^5\,\mathrm{atan}(a\,x)}{(c\,a^2\,x^2+c)^2}\,dx$$

input
```
int((x^5*atan(a*x))/(c + a^2*c*x^2)^2,x)
```

output
```
int((x^5*atan(a*x))/(c + a^2*c*x^2)^2, x)
```

## Reduce [F]

$$\int \frac{x^5 \arctan(ax)}{(c+a^2cx^2)^2}\,dx = \frac{\int \frac{atan(ax)x^5}{a^4x^4+2a^2x^2+1}dx}{c^2}$$

input
```
int(x^5*atan(a*x)/(a^2*c*x^2+c)^2,x)
```

output
```
int((atan(a*x)*x**5)/(a**4*x**4 + 2*a**2*x**2 + 1),x)/c**2
```

# 3.183 $\int \frac{x^4 \arctan(ax)}{(c+a^2cx^2)^2} dx$



## Optimal result

Integrand size = 20, antiderivative size = 96

$$\int \frac{x^4 \arctan(ax)}{(c+a^2cx^2)^2} dx = \frac{1}{4a^5c^2(1+a^2x^2)} + \frac{x \arctan(ax)}{a^4c^2} + \frac{x \arctan(ax)}{2a^4c^2(1+a^2x^2)} - \frac{3\arctan(ax)^2}{4a^5c^2} - \frac{\log(1+a^2x^2)}{2a^5c^2}$$

output
```
1/4/a^5/c^2/(a^2*x^2+1)+x*arctan(a*x)/a^4/c^2+1/2*x*arctan(a*x)/a^4/c^2/(a
^2*x^2+1)-3/4*arctan(a*x)^2/a^5/c^2-1/2*ln(a^2*x^2+1)/a^5/c^2
```

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.82

$$\int \frac{x^4 \arctan(ax)}{(c+a^2cx^2)^2} dx = \frac{1+(6ax+4a^3x^3)\arctan(ax) - 3(1+a^2x^2)\arctan(ax)^2 - 2(1+a^2x^2)\log(1+a^2x^2)}{4a^5c^2(1+a^2x^2)}$$

input
```
Integrate[(x^4*ArcTan[a*x])/(c + a^2*c*x^2)^2,x]
```

output
```
(1 + (6*a*x + 4*a^3*x^3)*ArcTan[a*x] - 3*(1 + a^2*x^2)*ArcTan[a*x]^2 - 2*(
1 + a^2*x^2)*Log[1 + a^2*x^2])/(4*a^5*c^2*(1 + a^2*x^2))
```

## Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.18, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5499, 27, 5451, 5345, 240, 5419, 5469, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \arctan(ax)}{(a^2cx^2+c)^2}dx$$

$$\Big\downarrow \text{5499}$$

$$\frac{\int \frac{x^2\arctan(ax)}{c(a^2x^2+1)}dx}{a^2c}-\frac{\int \frac{x^2\arctan(ax)}{c^2(a^2x^2+1)^2}dx}{a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{x^2\arctan(ax)}{a^2x^2+1}dx}{a^2c^2}-\frac{\int \frac{x^2\arctan(ax)}{(a^2x^2+1)^2}dx}{a^2c^2}$$

$$\Big\downarrow \text{5451}$$

$$\frac{\frac{\int \arctan(ax)dx}{a^2}-\frac{\int \frac{\arctan(ax)}{a^2x^2+1}dx}{a^2}}{a^2c^2}-\frac{\int \frac{x^2\arctan(ax)}{(a^2x^2+1)^2}dx}{a^2c^2}$$

$$\Big\downarrow \text{5345}$$

$$\frac{\frac{x\arctan(ax)-a\int \frac{x}{a^2x^2+1}dx}{a^2}-\frac{\int \frac{\arctan(ax)}{a^2x^2+1}dx}{a^2}}{a^2c^2}-\frac{\int \frac{x^2\arctan(ax)}{(a^2x^2+1)^2}dx}{a^2c^2}$$

$$\Big\downarrow \text{240}$$

$$\frac{\frac{x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}}{a^2}-\frac{\int \frac{\arctan(ax)}{a^2x^2+1}dx}{a^2}}{a^2c^2}-\frac{\int \frac{x^2\arctan(ax)}{(a^2x^2+1)^2}dx}{a^2c^2}$$

$$\Big\downarrow \text{5419}$$

$$\frac{\frac{x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}}{a^2}-\frac{\arctan(ax)^2}{2a^3}}{a^2c^2}-\frac{\int\frac{x^2\arctan(ax)}{\left(a^2x^2+1\right)^2}dx}{a^2c^2}$$

$$\Big\downarrow \text{5469}$$

$$\frac{\frac{x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}}{a^2}-\frac{\arctan(ax)^2}{2a^3}}{a^2c^2}-\frac{\frac{\int\frac{\arctan(ax)}{a^2x^2+1}dx}{2a^2}-\frac{x\arctan(ax)}{2a^2(a^2x^2+1)}-\frac{1}{4a^3(a^2x^2+1)}}{a^2c^2}$$

$$\Big\downarrow \text{5419}$$

$$\frac{\frac{x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}}{a^2}-\frac{\arctan(ax)^2}{2a^3}}{a^2c^2}-\frac{\frac{\arctan(ax)^2}{4a^3}-\frac{x\arctan(ax)}{2a^2(a^2x^2+1)}-\frac{1}{4a^3(a^2x^2+1)}}{a^2c^2}$$

input
```
Int[(x^4*ArcTan[a*x])/(c + a^2*c*x^2)^2,x]
```

output
```
-((-1/4*1/(a^3*(1 + a^2*x^2)) - (x*ArcTan[a*x])/(2*a^2*(1 + a^2*x^2)) + Ar
cTan[a*x]^2/(4*a^3))/(a^2*c^2)) + (-1/2*ArcTan[a*x]^2/a^3 + (x*ArcTan[a*x]
 - Log[1 + a^2*x^2]/(2*a))/a^2)/(a^2*c^2)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5469
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^2*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c^3*d*(q + 1)^2)), x] + (Simp
[x*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*c^2*d*(q + 1))), x] - Simp[1
/(2*c^2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x]) /
; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -5/2]
```

rule 5499
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ
[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

## Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.90

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{\arctan(ax)ax}{c^2}+\frac{ax\arctan(ax)}{2c^2\left(a^2x^2+1\right)}-\frac{3\arctan(ax)^2}{2c^2}-\frac{\ln\left(a^2x^2+1\right)-\frac{1}{2\left(a^2x^2+1\right)}-\frac{3\arctan(ax)^2}{2}}{2c^2}}{a^5}$ |
| default | $\frac{\frac{\arctan(ax)ax}{c^2}+\frac{ax\arctan(ax)}{2c^2\left(a^2x^2+1\right)}-\frac{3\arctan(ax)^2}{2c^2}-\frac{\ln\left(a^2x^2+1\right)-\frac{1}{2\left(a^2x^2+1\right)}-\frac{3\arctan(ax)^2}{2}}{2c^2}}{a^5}$ |
| parts | $\frac{x\arctan(ax)}{a^4c^2}+\frac{x\arctan(ax)}{2a^4c^2(a^2x^2+1)}-\frac{3\arctan(ax)^2}{2a^5c^2}-\frac{-\frac{3\arctan(ax)^2}{4a^5}+\frac{\ln\left(a^2x^2+1\right)-\frac{1}{2\left(a^2x^2+1\right)}}{2a^5}}{c^2}$ |
| parallelrisch | $\frac{4\arctan(ax)x^3a^3-3\arctan(ax)^2x^2a^2-2a^2\ln(a^2x^2+1)x^2-a^2x^2+6\arctan(ax)ax-3\arctan(ax)^2-2\ln(a^2x^2+1)}{4c^2(a^2x^2+1)a^5}$ |
| risch | $\frac{3\ln(iax+1)^2}{16a^5c^2}-\frac{i(-3ia^2x^2\ln(-iax+1)+4a^3x^3-3i\ln(-iax+1)+6ax)\ln(iax+1)}{8a^5c^2(a^2x^2+1)}+\frac{i\left(-3ia^2x^2\ln(-iax+1)^2-3i\ln\right.}{}$ |

input
```
int(x^4*arctan(a*x)/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^5*(1/c^2*arctan(a*x)*a*x+1/2*a*x*arctan(a*x)/c^2/(a^2*x^2+1)-3/2*arcta
n(a*x)^2/c^2-1/2/c^2*(ln(a^2*x^2+1)-1/2/(a^2*x^2+1)-3/2*arctan(a*x)^2))
```

## Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.84

$$\int \frac{x^4 \arctan(ax)}{(c+a^2cx^2)^2}\,dx = -\frac{3\,(a^2x^2+1)\arctan{(ax)}^2 - 2\,(2\,a^3x^3+3\,ax)\arctan{(ax)} + 2\,(a^2x^2+1)\log{(a^2x^2+1)} - 1}{4\,(a^7c^2x^2+a^5c^2)}$$

input
```
integrate(x^4*arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
-1/4*(3*(a^2*x^2 + 1)*arctan(a*x)^2 - 2*(2*a^3*x^3 + 3*a*x)*arctan(a*x) +
2*(a^2*x^2 + 1)*log(a^2*x^2 + 1) - 1)/(a^7*c^2*x^2 + a^5*c^2)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. 2(90) = 180.

Time = 0.47 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.32

$$\int \frac{x^4 \arctan(ax)}{(c+a^2cx^2)^2}\,dx = \begin{cases} \frac{4a^3x^3 \operatorname{atan}{(ax)}}{4a^7c^2x^2+4a^5c^2} - \frac{2a^2x^2\log{\left(x^2+\frac{1}{a^2}\right)}}{4a^7c^2x^2+4a^5c^2} - \frac{3a^2x^2\operatorname{atan}^2{(ax)}}{4a^7c^2x^2+4a^5c^2} + \frac{6ax\operatorname{atan}{(ax)}}{4a^7c^2x^2+4a^5c^2} - \frac{2\log{\left(x^2+\frac{1}{a^2}\right)}}{4a^7c^2x^2+4a^5c^2} - \frac{3\operatorname{atan}^2{(ax)}}{4a^7c^2x^2+4a^5c^2} + \frac{1}{4a^7c^2x^2+4} \\ 0 \end{cases}$$

input
```
integrate(x**4*atan(a*x)/(a**2*c*x**2+c)**2,x)
```

output
```
Piecewise((4*a**3*x**3*atan(a*x)/(4*a**7*c**2*x**2 + 4*a**5*c**2) - 2*a**2
*x**2*log(x**2 + a**(-2))/(4*a**7*c**2*x**2 + 4*a**5*c**2) - 3*a**2*x**2*a
tan(a*x)**2/(4*a**7*c**2*x**2 + 4*a**5*c**2) + 6*a*x*atan(a*x)/(4*a**7*c**
2*x**2 + 4*a**5*c**2) - 2*log(x**2 + a**(-2))/(4*a**7*c**2*x**2 + 4*a**5*c
**2) - 3*atan(a*x)**2/(4*a**7*c**2*x**2 + 4*a**5*c**2) + 1/(4*a**7*c**2*x*
*2 + 4*a**5*c**2), Ne(a, 0)), (0, True))
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.19

$$\int \frac{x^4 \arctan(ax)}{(c+a^2cx^2)^2}\,dx = \frac{1}{2}\left(\frac{x}{a^6c^2x^2+a^4c^2}+\frac{2\,x}{a^4c^2}-\frac{3\,\arctan(ax)}{a^5c^2}\right)\arctan(ax) + \frac{\left(3\,(a^2x^2+1)\arctan(ax)^2-2\,(a^2x^2+1)\log(a^2x^2+1)+1\right)a}{4\,(a^8c^2x^2+a^6c^2)}$$

input
```
integrate(x^4*arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
1/2*(x/(a^6*c^2*x^2 + a^4*c^2) + 2*x/(a^4*c^2) - 3*arctan(a*x)/(a^5*c^2))*
arctan(a*x) + 1/4*(3*(a^2*x^2 + 1)*arctan(a*x)^2 - 2*(a^2*x^2 + 1)*log(a^2
*x^2 + 1) + 1)*a/(a^8*c^2*x^2 + a^6*c^2)
```

## Giac [F]

$$\int \frac{x^4 \arctan(ax)}{(c+a^2cx^2)^2}\,dx = \int \frac{x^4\arctan(ax)}{(a^2cx^2+c)^2}\,dx$$

input
```
integrate(x^4*arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate(x^4*arctan(a*x)/(a^2*c*x^2 + c)^2, x)
```

## Mupad [B] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.98

$$\int \frac{x^4 \arctan(ax)}{(c+a^2cx^2)^2}\,dx = \frac{1}{2\,a^2\,\left(2\,a^5\,c^2\,x^2+2\,a^3\,c^2\right)} - \frac{\ln\left(a^2\,x^2+1\right)}{2\,a^5\,c^2} + \frac{\mathrm{atan}(a\,x)\,\left(\frac{3\,x}{2\,a^6\,c^2}+\frac{x^3}{a^4\,c^2}\right)}{\frac{1}{a^2}+x^2} - \frac{3\,\mathrm{atan}(a\,x)^2}{4\,a^5\,c^2}$$

input
```
int((x^4*atan(a*x))/(c + a^2*c*x^2)^2,x)
```

output
```
1/(2*a^2*(2*a^3*c^2 + 2*a^5*c^2*x^2)) - log(a^2*x^2 + 1)/(2*a^5*c^2) + (at
an(a*x)*((3*x)/(2*a^6*c^2) + x^3/(a^4*c^2)))/(1/a^2 + x^2) - (3*atan(a*x)^
2)/(4*a^5*c^2)
```

## Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.04

$$\int \frac{x^4 \arctan(ax)}{(c+a^2cx^2)^2}\,dx = \frac{-3atan(ax)^2\,a^2x^2-3atan(ax)^2+4atan(ax)\,a^3x^3+6atan(ax)\,ax-2\log(a^2x^2+1)\,a^2x^2-2\log(a^2x^2+}{4a^5c^2\,(a^2x^2+1)}$$

input
```
int(x^4*atan(a*x)/(a^2*c*x^2+c)^2,x)
```

output
```
( - 3*atan(a*x)**2*a**2*x**2 - 3*atan(a*x)**2 + 4*atan(a*x)*a**3*x**3 + 6*
atan(a*x)*a*x - 2*log(a**2*x**2 + 1)*a**2*x**2 - 2*log(a**2*x**2 + 1) - a*
*2*x**2)/(4*a**5*c**2*(a**2*x**2 + 1))
```

# 3.184 $\int \frac{x^3 \arctan(ax)}{(c+a^2cx^2)^2} dx$



## Optimal result

Integrand size = 20, antiderivative size = 133

$$\int \frac{x^3 \arctan(ax)}{(c+a^2cx^2)^2} dx = -\frac{x}{4a^3c^2(1+a^2x^2)} - \frac{\arctan(ax)}{4a^4c^2} + \frac{\arctan(ax)}{2a^4c^2(1+a^2x^2)} - \frac{i\arctan(ax)^2}{2a^4c^2} - \frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a^4c^2} - \frac{i\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{2a^4c^2}$$

output
```
-1/4*x/a^3/c^2/(a^2*x^2+1)-1/4*arctan(a*x)/a^4/c^2+1/2*arctan(a*x)/a^4/c^2
/(a^2*x^2+1)-1/2*I*arctan(a*x)^2/a^4/c^2-arctan(a*x)*ln(2/(1+I*a*x))/a^4/c
^2-1/2*I*polylog(2,1-2/(1+I*a*x))/a^4/c^2
```

## Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.58

$$\int \frac{x^3 \arctan(ax)}{(c+a^2cx^2)^2} dx = \frac{4i\arctan(ax)^2 + 2\arctan(ax)\left(\cos(2\arctan(ax)) - 4\log\left(1+e^{2i\arctan(ax)}\right)\right) + 4i\,\mathrm{PolyLog}\left(2,-e^{2i\arctan(a}\right.}{8a^4c^2}$$

input
```
Integrate[(x^3*ArcTan[a*x])/(c + a^2*c*x^2)^2,x]
```

output
```
((4*I)*ArcTan[a*x]^2 + 2*ArcTan[a*x]*(Cos[2*ArcTan[a*x]] - 4*Log[1 + E^((2
*I)*ArcTan[a*x])]) + (4*I)*PolyLog[2, -E^((2*I)*ArcTan[a*x])] - Sin[2*ArcT
an[a*x]])/(8*a^4*c^2)
```

## Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {5499, 27, 5455, 5379, 2849, 2752, 5465, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \arctan(ax)}{(a^2cx^2+c)^2}dx$$

$$\Big\downarrow \; 5499$$

$$\frac{\int \frac{x\arctan(ax)}{c(a^2x^2+1)}dx}{a^2c}-\frac{\int \frac{x\arctan(ax)}{c^2(a^2x^2+1)^2}dx}{a^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2c^2}-\frac{\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx}{a^2c^2}$$

$$\Big\downarrow \; 5455$$

$$-\frac{\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx}{a^2c^2}+\frac{-\frac{\int \frac{\arctan(ax)}{i-ax}dx}{a}-\frac{i\arctan(ax)^2}{2a^2}}{a^2c^2}$$

$$\Big\downarrow \; 5379$$

$$-\frac{\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx}{a^2c^2}+\frac{-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}-\int \frac{\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^2}{2a^2}}{a^2c^2}$$

$$\Big\downarrow \; 2849$$

$$-\frac{\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx}{a^2c^2}+\frac{-\frac{\frac{i\int \frac{\log\left(\frac{2}{iax+1}\right)}{1-\frac{2}{iax+1}}d\frac{1}{iax+1}}{a}+\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}}{a}-\frac{i\arctan(ax)^2}{2a^2}}{a^2c^2}$$

$\downarrow$ 2752

$$-\frac{\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx}{a^2c^2}+\frac{-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}}{a^2c^2}$$

$\downarrow$ 5465

$$-\frac{\frac{\int \frac{1}{(a^2x^2+1)^2}dx}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}}{a^2c^2}+\frac{-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}}{a^2c^2}$$

$\downarrow$ 215

$$-\frac{\frac{\frac{1}{2}\int \frac{1}{a^2x^2+1}dx+\frac{x}{2(a^2x^2+1)}}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}}{a^2c^2}+\frac{-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}}{a^2c^2}$$

$\downarrow$ 216

$$-\frac{\frac{\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}}{a^2c^2}+\frac{-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}}{a^2c^2}$$

input
```
Int[(x^3*ArcTan[a*x])/(c + a^2*c*x^2)^2,x]
```

output
```
-((-1/2*ArcTan[a*x]/(a^2*(1 + a^2*x^2)) + (x/(2*(1 + a^2*x^2)) + ArcTan[a*
x]/(2*a))/(2*a))/(a^2*c^2)) + (((-1/2*I)*ArcTan[a*x]^2)/a^2 - ((ArcTan[a*x
]*Log[2/(1 + I*a*x)])/a + ((I/2)*PolyLog[2, 1 - 2/(1 + I*a*x)])/a)/a)/(a^2
*c^2)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5499
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ
[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

## Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.61 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.29

| method | result |
|---|---|
| parts | $\frac{\arctan(ax)}{2a^4c^2(a^2x^2+1)}+\frac{\arctan(ax)\ln(a^2x^2+1)}{2c^2a^4}-\frac{a\left(\frac{x}{2a^4\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a^5}+\frac{\sum_{\_\alpha=\mathrm{RootOf}\left(a^2\_Z^2+1\right)}\frac{2\ln(x-\_\alpha)\ln\left(\cdots\right.}{\cdots}}{\cdots}\cdots\right)}{\cdots}$ |
| derivativedivides | $\frac{\frac{\arctan(ax)}{2c^2\left(a^2x^2+1\right)}+\frac{\arctan(ax)\ln\left(a^2x^2+1\right)}{2c^2}-\frac{\frac{ax}{2a^2x^2+2}+\frac{\arctan(ax)}{2}-\frac{i\left(\ln(ax-i)\ln\left(a^2x^2+1\right)-\frac{\ln(ax-i)^2}{2}-\mathrm{dilog}\left(-\frac{i(ax+i)}{2}\right)-\ln(\cdots\right.}{2}\cdots}{\cdots}}{a^4}$ |
| default | $\frac{\frac{\arctan(ax)}{2c^2\left(a^2x^2+1\right)}+\frac{\arctan(ax)\ln\left(a^2x^2+1\right)}{2c^2}-\frac{\frac{ax}{2a^2x^2+2}+\frac{\arctan(ax)}{2}-\frac{i\left(\ln(ax-i)\ln\left(a^2x^2+1\right)-\frac{\ln(ax-i)^2}{2}-\mathrm{dilog}\left(-\frac{i(ax+i)}{2}\right)-\ln(\cdots\right.}{2}\cdots}{\cdots}}{a^4}$ |
| risch | $\frac{i\ln(-iax+1)}{8c^2a^4(-iax+1)}-\frac{i\ln(-iax+1)}{16c^2a^4(-iax-1)}-\frac{i}{8c^2a^4(iax+1)}+\frac{i\ln(iax+1)}{16c^2a^4(iax-1)}+\frac{i\ln(-iax+1)^2}{8c^2a^4}-\frac{i\ln(iax+1)}{8c^2a^4(iax+1)}-\cdots$ |

input
```
int(x^3*arctan(a*x)/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*arctan(a*x)/a^4/c^2/(a^2*x^2+1)+1/2/c^2*arctan(a*x)/a^4*ln(a^2*x^2+1)-
1/2/c^2*a*(1/2/a^4*x/(a^2*x^2+1)+1/2/a^5*arctan(a*x)+1/4/a^6*sum(1/_alpha*
(2*ln(x-_alpha)*ln(a^2*x^2+1)-a^2*(1/a^2/_alpha*ln(x-_alpha)^2+2*_alpha*ln
(x-_alpha)*ln(1/2*(x+_alpha)/_alpha)+2*_alpha*dilog(1/2*(x+_alpha)/_alpha)
)),_alpha=RootOf(_Z^2*a^2+1)))
```

## Fricas [F]

$$\int \frac{x^3 \arctan(ax)}{(c+a^2cx^2)^2}\,dx = \int \frac{x^3 \arctan(ax)}{(a^2cx^2+c)^2}\,dx$$

input
```
integrate(x^3*arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
integral(x^3*arctan(a*x)/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2), x)
```

## Sympy [F]

$$\int \frac{x^3 \arctan(ax)}{(c+a^2cx^2)^2}\,dx = \frac{\int \frac{x^3 \operatorname{atan}(ax)}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

input
```
integrate(x**3*atan(a*x)/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(x**3*atan(a*x)/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

## Maxima [F]

$$\int \frac{x^3 \arctan(ax)}{(c+a^2cx^2)^2}\,dx = \int \frac{x^3 \arctan(ax)}{(a^2cx^2+c)^2}\,dx$$

input
```
integrate(x^3*arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
integrate(x^3*arctan(a*x)/(a^2*c*x^2 + c)^2, x)
```

## Giac [F]

$$\int \frac{x^3 \arctan(ax)}{(c+a^2cx^2)^2}\,dx = \int \frac{x^3 \arctan(ax)}{(a^2cx^2+c)^2}\,dx$$

input
```
integrate(x^3*arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate(x^3*arctan(a*x)/(a^2*c*x^2 + c)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \arctan(ax)}{(c+a^2cx^2)^2}\,dx = \int \frac{x^3 \operatorname{atan}(a\,x)}{(c\,a^2\,x^2+c)^2}\,dx$$

input
```
int((x^3*atan(a*x))/(c + a^2*c*x^2)^2,x)
```

output
```
int((x^3*atan(a*x))/(c + a^2*c*x^2)^2, x)
```

## Reduce [F]

$$\int \frac{x^3 \arctan(ax)}{(c+a^2cx^2)^2}\,dx = \frac{\int \frac{atan(ax)x^3}{a^4x^4+2a^2x^2+1}dx}{c^2}$$

input
```
int(x^3*atan(a*x)/(a^2*c*x^2+c)^2,x)
```

output
```
int((atan(a*x)*x**3)/(a**4*x**4 + 2*a**2*x**2 + 1),x)/c**2
```

# 3.185 $\int \frac{x^2 \arctan(ax)}{(c+a^2cx^2)^2} \, dx$



## Optimal result

Integrand size = 20, antiderivative size = 64

$$\int \frac{x^2 \arctan(ax)}{(c+a^2cx^2)^2} \, dx = -\frac{1}{4a^3c^2(1+a^2x^2)} - \frac{x \arctan(ax)}{2a^2c^2(1+a^2x^2)} + \frac{\arctan(ax)^2}{4a^3c^2}$$

output
```
-1/4/a^3/c^2/(a^2*x^2+1)-1/2*x*arctan(a*x)/a^2/c^2/(a^2*x^2+1)+1/4*arctan(
a*x)^2/a^3/c^2
```

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.73

$$\int \frac{x^2 \arctan(ax)}{(c+a^2cx^2)^2} \, dx = \frac{-1-2ax \arctan(ax)+(1+a^2x^2)\arctan(ax)^2}{4a^3c^2(1+a^2x^2)}$$

input
```
Integrate[(x^2*ArcTan[a*x])/(c + a^2*c*x^2)^2,x]
```

output
```
(-1 - 2*a*x*ArcTan[a*x] + (1 + a^2*x^2)*ArcTan[a*x]^2)/(4*a^3*c^2*(1 + a^2
*x^2))
```

## Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {5469, 27, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \arctan(ax)}{(a^2cx^2+c)^2}dx$$

$$\Big\downarrow \text{5469}$$

$$\frac{\int \frac{\arctan(ax)}{c(a^2x^2+1)}dx}{2a^2c}-\frac{x\arctan(ax)}{2a^2c^2(a^2x^2+1)}-\frac{1}{4a^3c^2(a^2x^2+1)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\arctan(ax)}{a^2x^2+1}dx}{2a^2c^2}-\frac{x\arctan(ax)}{2a^2c^2(a^2x^2+1)}-\frac{1}{4a^3c^2(a^2x^2+1)}$$

$$\Big\downarrow \text{5419}$$

$$\frac{\arctan(ax)^2}{4a^3c^2}-\frac{x\arctan(ax)}{2a^2c^2(a^2x^2+1)}-\frac{1}{4a^3c^2(a^2x^2+1)}$$

input
```
Int[(x^2*ArcTan[a*x])/(c + a^2*c*x^2)^2,x]
```

output
```
-1/4*1/(a^3*c^2*(1 + a^2*x^2)) - (x*ArcTan[a*x])/(2*a^2*c^2*(1 + a^2*x^2))
 + ArcTan[a*x]^2/(4*a^3*c^2)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5469
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^2*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c^3*d*(q + 1)^2)), x] + (Simp
[x*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*c^2*d*(q + 1))), x] - Simp[1
/(2*c^2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x]) /
; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -5/2]
```

## Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.86

| method | result |
|---|---|
| parallelrisch | $\frac{\arctan(ax)^2x^2a^2+a^2x^2-2\arctan(ax)ax+\arctan(ax)^2}{4c^2(a^2x^2+1)a^3}$ |
| derivativedivides | $\frac{-\frac{ax\arctan(ax)}{2c^2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^2}{2c^2}-\frac{\frac{\arctan(ax)^2}{2}+\frac{1}{2a^2x^2+2}}{2c^2}}{a^3}$ |
| default | $\frac{-\frac{ax\arctan(ax)}{2c^2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^2}{2c^2}-\frac{\frac{\arctan(ax)^2}{2}+\frac{1}{2a^2x^2+2}}{2c^2}}{a^3}$ |
| parts | $-\frac{x\arctan(ax)}{2a^2c^2(a^2x^2+1)}+\frac{\arctan(ax)^2}{2a^3c^2}-\frac{\frac{\arctan(ax)^2}{2a^3}+\frac{1}{2a^3\left(a^2x^2+1\right)}}{2c^2}$ |
| risch | $-\frac{\ln(iax+1)^2}{16a^3c^2}+\frac{(a^2x^2\ln(-iax+1)+\ln(-iax+1)+2iax)\ln(iax+1)}{8a^3c^2(a^2x^2+1)}-\frac{a^2x^2\ln(-iax+1)^2+\ln(-iax+1)^2+4iax\ln(}{16(ax+i)c^2(ax-i)a^3}$ |

input
```
int(x^2*arctan(a*x)/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*(arctan(a*x)^2*x^2*a^2+a^2*x^2-2*arctan(a*x)*a*x+arctan(a*x)^2)/c^2/(a
^2*x^2+1)/a^3
```

## Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.77

$$\int \frac{x^2 \arctan(ax)}{(c+a^2cx^2)^2}\,dx = -\frac{2\,ax\arctan{(ax)} - (a^2x^2+1)\arctan{(ax)}^2 + 1}{4\,(a^5c^2x^2 + a^3c^2)}$$

input
```
integrate(x^2*arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
-1/4*(2*a*x*arctan(a*x) - (a^2*x^2 + 1)*arctan(a*x)^2 + 1)/(a^5*c^2*x^2 +
a^3*c^2)
```

## Sympy [F]

$$\int \frac{x^2 \arctan(ax)}{(c+a^2cx^2)^2}\,dx = \frac{\int \frac{x^2\,\mathrm{atan}\,(ax)}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

input
```
integrate(x**2*atan(a*x)/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(x**2*atan(a*x)/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.30

$$\int \frac{x^2 \arctan(ax)}{(c+a^2cx^2)^2}\,dx = -\frac{1}{2}\left(\frac{x}{a^4c^2x^2 + a^2c^2} - \frac{\arctan{(ax)}}{a^3c^2}\right)\arctan{(ax)} - \frac{\left((a^2x^2+1)\arctan{(ax)}^2 + 1\right)a}{4\,(a^6c^2x^2 + a^4c^2)}$$

input
```
integrate(x^2*arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
-1/2*(x/(a^4*c^2*x^2 + a^2*c^2) - arctan(a*x)/(a^3*c^2))*arctan(a*x) - 1/4
*((a^2*x^2 + 1)*arctan(a*x)^2 + 1)*a/(a^6*c^2*x^2 + a^4*c^2)
```

## Giac [F]

$$\int \frac{x^2 \arctan(ax)}{(c+a^2cx^2)^2}\,dx = \int \frac{x^2 \arctan{(ax)}}{(a^2cx^2+c)^2}\,dx$$

input
```
integrate(x^2*arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate(x^2*arctan(a*x)/(a^2*c*x^2 + c)^2, x)
```

## Mupad [B] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.75

$$\int \frac{x^2 \arctan(ax)}{(c+a^2cx^2)^2}\,dx = \frac{a^2\,x^2\,\mathrm{atan}(a\,x)^2 - 2\,a\,x\,\mathrm{atan}(a\,x) + \mathrm{atan}(a\,x)^2 - 1}{4\,a^3\,c^2\,\left(a^2\,x^2+1\right)}$$

input
```
int((x^2*atan(a*x))/(c + a^2*c*x^2)^2,x)
```

output
```
(atan(a*x)^2 - 2*a*x*atan(a*x) + a^2*x^2*atan(a*x)^2 - 1)/(4*a^3*c^2*(a^2*
x^2 + 1))
```

## Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.84

$$\int \frac{x^2 \arctan(ax)}{(c+a^2cx^2)^2}\,dx = \frac{atan(ax)^2\,a^2x^2 + atan(ax)^2 - 2atan(ax)\,ax + a^2x^2}{4a^3c^2\,(a^2x^2+1)}$$

input
```
int(x^2*atan(a*x)/(a^2*c*x^2+c)^2,x)
```

output
```
(atan(a*x)**2*a**2*x**2 + atan(a*x)**2 - 2*atan(a*x)*a*x + a**2*x**2)/(4*a
**3*c**2*(a**2*x**2 + 1))
```

# 3.186 $\int \frac{x\arctan(ax)}{(c+a^2cx^2)^2}\,dx$



## Optimal result

Integrand size = 18, antiderivative size = 62

$$\int \frac{x\arctan(ax)}{(c+a^2cx^2)^2}\,dx = \frac{x}{4ac^2(1+a^2x^2)} + \frac{\arctan(ax)}{4a^2c^2} - \frac{\arctan(ax)}{2a^2c^2(1+a^2x^2)}$$

output
```
1/4*x/a/c^2/(a^2*x^2+1)+1/4*arctan(a*x)/a^2/c^2-1/2*arctan(a*x)/a^2/c^2/(a
^2*x^2+1)
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.63

$$\int \frac{x\arctan(ax)}{(c+a^2cx^2)^2}\,dx = \frac{ax + (-1+a^2x^2)\arctan(ax)}{4a^2c^2(1+a^2x^2)}$$

input
```
Integrate[(x*ArcTan[a*x])/(c + a^2*c*x^2)^2,x]
```

output
```
(a*x + (-1 + a^2*x^2)*ArcTan[a*x])/(4*a^2*c^2*(1 + a^2*x^2))
```

## Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5465, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\arctan(ax)}{(a^2cx^2+c)^2}dx$$

$$\Big\downarrow \ 5465$$

$$\frac{\int \frac{1}{(a^2cx^2+c)^2}dx}{2a}-\frac{\arctan(ax)}{2a^2c^2(a^2x^2+1)}$$

$$\Big\downarrow \ 215$$

$$\frac{\frac{\int \frac{1}{a^2cx^2+c}dx}{2c}+\frac{x}{2c^2(a^2x^2+1)}}{2a}-\frac{\arctan(ax)}{2a^2c^2(a^2x^2+1)}$$

$$\Big\downarrow \ 218$$

$$\frac{\frac{x}{2c^2(a^2x^2+1)}+\frac{\arctan(ax)}{2ac^2}}{2a}-\frac{\arctan(ax)}{2a^2c^2(a^2x^2+1)}$$

input
```
Int[(x*ArcTan[a*x])/(c + a^2*c*x^2)^2,x]
```

output
```
-1/2*ArcTan[a*x]/(a^2*c^2*(1 + a^2*x^2)) + (x/(2*c^2*(1 + a^2*x^2)) + ArcT
an[a*x]/(2*a*c^2))/(2*a)
```

### Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

## Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.66

| method | result | size |
|---|---|---|
| parallelrisch | $\frac{x^2a^2\arctan(ax)+ax-\arctan(ax)}{4c^2(a^2x^2+1)a^2}$ | 41 |
| derivativedivides | $\frac{-\frac{\arctan(ax)}{2c^2\left(a^2x^2+1\right)}+\frac{\frac{ax}{2a^2x^2+2}+\frac{\arctan(ax)}{2}}{2c^2}}{a^2}$ | 53 |
| default | $\frac{-\frac{\arctan(ax)}{2c^2\left(a^2x^2+1\right)}+\frac{\frac{ax}{2a^2x^2+2}+\frac{\arctan(ax)}{2}}{2c^2}}{a^2}$ | 53 |
| parts | $-\frac{\arctan(ax)}{2a^2c^2(a^2x^2+1)}+\frac{\frac{x}{2a^2x^2+2}+\frac{\arctan(ax)}{2a}}{2a\,c^2}$ | 57 |
| risch | $\frac{i\ln(iax+1)}{4a^2c^2(a^2x^2+1)}-\frac{i(2\ln(-iax+1)-\ln(-ax-i)a^2x^2-\ln(-ax-i)+\ln(ax-i)a^2x^2+\ln(ax-i)+2iax)}{8(ax+i)a^2c^2(ax-i)}$ | 118 |
| orering | $\frac{(2a^4x^4+a^2x^2-1)\arctan(ax)}{2a^2(a^2c\,x^2+c)^2}+\frac{(a^2x^2+1)^2\left(\frac{\arctan(ax)}{\left(a^2c\,x^2+c\right)^2}+\frac{xa}{\left(a^2x^2+1\right)\left(a^2c\,x^2+c\right)^2}-\frac{4x^2\arctan(ax)c\,a^2}{\left(a^2c\,x^2+c\right)^3}\right)}{4a^2}$ | 125 |

input
```
int(x*arctan(a*x)/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*(x^2*a^2*arctan(a*x)+a*x-arctan(a*x))/c^2/(a^2*x^2+1)/a^2
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.65

$$\int \frac{x \arctan(ax)}{(c + a^2cx^2)^2} \, dx = \frac{ax + (a^2x^2 - 1) \arctan(ax)}{4(a^4c^2x^2 + a^2c^2)}$$

input
```
integrate(x*arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
1/4*(a*x + (a^2*x^2 - 1)*arctan(a*x))/(a^4*c^2*x^2 + a^2*c^2)
```

## Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.32

$$\int \frac{x \arctan(ax)}{(c + a^2cx^2)^2} \, dx = \begin{cases} \frac{a^2x^2 \operatorname{atan}(ax)}{4a^4c^2x^2+4a^2c^2} + \frac{ax}{4a^4c^2x^2+4a^2c^2} - \frac{\operatorname{atan}(ax)}{4a^4c^2x^2+4a^2c^2} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(x*atan(a*x)/(a**2*c*x**2+c)**2,x)
```

output
```
Piecewise((a**2*x**2*atan(a*x)/(4*a**4*c**2*x**2 + 4*a**2*c**2) + a*x/(4*a
**4*c**2*x**2 + 4*a**2*c**2) - atan(a*x)/(4*a**4*c**2*x**2 + 4*a**2*c**2),
 Ne(a, 0)), (0, True))
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.95

$$\int \frac{x \arctan(ax)}{(c+a^2cx^2)^2}\,dx = \frac{\frac{x}{a^2cx^2+c} + \frac{\arctan(ax)}{ac}}{4\,ac} - \frac{\arctan(ax)}{2\,(a^2cx^2+c)a^2c}$$

input
```
integrate(x*arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
1/4*(x/(a^2*c*x^2 + c) + arctan(a*x)/(a*c))/(a*c) - 1/2*arctan(a*x)/((a^2*
c*x^2 + c)*a^2*c)
```

## Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.92

$$\int \frac{x \arctan(ax)}{(c+a^2cx^2)^2}\,dx = \frac{x}{4\,(a^2x^2+1)ac^2} + \frac{\arctan(ax)}{4\,a^2c^2} - \frac{\arctan(ax)}{2\,(a^2cx^2+c)a^2c}$$

input
```
integrate(x*arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
1/4*x/((a^2*x^2 + 1)*a*c^2) + 1/4*arctan(a*x)/(a^2*c^2) - 1/2*arctan(a*x)/
((a^2*c*x^2 + c)*a^2*c)
```

## Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.65

$$\int \frac{x \arctan(ax)}{(c+a^2cx^2)^2}\,dx = \frac{a\,x - \mathrm{atan}(a\,x) + a^2\,x^2\,\mathrm{atan}(a\,x)}{4\,a^2\,c^2\,\left(a^2\,x^2+1\right)}$$

input
```
int((x*atan(a*x))/(c + a^2*c*x^2)^2,x)
```

output
```
(a*x - atan(a*x) + a^2*x^2*atan(a*x))/(4*a^2*c^2*(a^2*x^2 + 1))
```

## Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.65

$$\int \frac{x \arctan(ax)}{(c+a^2cx^2)^2}\,dx = \frac{atan(ax)\,a^2x^2 - atan(ax) + ax}{4a^2c^2\,(a^2x^2+1)}$$

input
```
int(x*atan(a*x)/(a^2*c*x^2+c)^2,x)
```

output
```
(atan(a*x)*a**2*x**2 - atan(a*x) + a*x)/(4*a**2*c**2*(a**2*x**2 + 1))
```

# 3.187 $\int \frac{\arctan(ax)}{(c+a^2cx^2)^2}\,dx$



## Optimal result

Integrand size = 17, antiderivative size = 61

$$\int \frac{\arctan(ax)}{(c+a^2cx^2)^2}\,dx = \frac{1}{4ac^2(1+a^2x^2)} + \frac{x\arctan(ax)}{2c^2(1+a^2x^2)} + \frac{\arctan(ax)^2}{4ac^2}$$

output
```
1/4/a/c^2/(a^2*x^2+1)+1/2*x*arctan(a*x)/c^2/(a^2*x^2+1)+1/4*arctan(a*x)^2/
a/c^2
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.72

$$\int \frac{\arctan(ax)}{(c+a^2cx^2)^2}\,dx = \frac{1+2ax\arctan(ax)+(1+a^2x^2)\arctan(ax)^2}{4c^2(a+a^3x^2)}$$

input
```
Integrate[ArcTan[a*x]/(c + a^2*c*x^2)^2,x]
```

output
```
(1 + 2*a*x*ArcTan[a*x] + (1 + a^2*x^2)*ArcTan[a*x]^2)/(4*c^2*(a + a^3*x^2)
)
```

## Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {5427, 27, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)}{(a^2cx^2+c)^2}dx$$

$$\Big\downarrow \text{5427}$$

$$-\frac{1}{2}a\int \frac{x}{c^2(a^2x^2+1)^2}dx+\frac{x\arctan(ax)}{2c^2(a^2x^2+1)}+\frac{\arctan(ax)^2}{4ac^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\int \frac{x}{(a^2x^2+1)^2}dx}{2c^2}+\frac{x\arctan(ax)}{2c^2(a^2x^2+1)}+\frac{\arctan(ax)^2}{4ac^2}$$

$$\Big\downarrow \text{241}$$

$$\frac{x\arctan(ax)}{2c^2(a^2x^2+1)}+\frac{1}{4ac^2(a^2x^2+1)}+\frac{\arctan(ax)^2}{4ac^2}$$

input
```
Int[ArcTan[a*x]/(c + a^2*c*x^2)^2,x]
```

output
```
1/(4*a*c^2*(1 + a^2*x^2)) + (x*ArcTan[a*x])/(2*c^2*(1 + a^2*x^2)) + ArcTan
[a*x]^2/(4*a*c^2)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

## Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.92

| method | result |
|---|---|
| parallelrisch | $\frac{\arctan(ax)^2 x^2 a^2 - a^2 x^2 + 2\arctan(ax) ax + \arctan(ax)^2}{4c^2(a^2x^2+1)a}$ |
| derivativedivides | $\frac{\frac{ax\arctan(ax)}{2c^2\left(a^2x^2+1\right)} + \frac{\arctan(ax)^2}{2c^2} - \frac{-\frac{1}{2\left(a^2x^2+1\right)} + \frac{\arctan(ax)^2}{2}}{2c^2}}{a}$ |
| default | $\frac{\frac{ax\arctan(ax)}{2c^2\left(a^2x^2+1\right)} + \frac{\arctan(ax)^2}{2c^2} - \frac{-\frac{1}{2\left(a^2x^2+1\right)} + \frac{\arctan(ax)^2}{2}}{2c^2}}{a}$ |
| parts | $\frac{x\arctan(ax)}{2c^2(a^2x^2+1)} + \frac{\arctan(ax)^2}{2a\,c^2} - \frac{\frac{\arctan(ax)^2}{2a} - \frac{1}{2a\left(a^2x^2+1\right)}}{2c^2}$ |
| risch | $-\frac{\ln(iax+1)^2}{16a\,c^2} + \frac{\left(a^2x^2\ln(-iax+1)+\ln(-iax+1)-2iax\right)\ln(iax+1)}{8a\,c^2(a^2x^2+1)} - \frac{a^2x^2\ln(-iax+1)^2+\ln(-iax+1)^2-4iax\ln(}{16c^2(ax+i)(ax-i)a}$ |

input
```
int(arctan(a*x)/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*(arctan(a*x)^2*x^2*a^2-a^2*x^2+2*arctan(a*x)*a*x+arctan(a*x)^2)/c^2/(a
^2*x^2+1)/a
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.75

$$\int \frac{\arctan(ax)}{(c+a^2cx^2)^2}\,dx = \frac{2\,ax\arctan{(ax)} + (a^2x^2+1)\arctan{(ax)}^2 + 1}{4\,(a^3c^2x^2+ac^2)}$$

input
```
integrate(arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
1/4*(2*a*x*arctan(a*x) + (a^2*x^2 + 1)*arctan(a*x)^2 + 1)/(a^3*c^2*x^2 + a
*c^2)
```

## Sympy [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)}{(c+a^2cx^2)^2}\,dx = \text{Exception raised: RecursionError}$$

input
```
integrate(atan(a*x)/(a**2*c*x**2+c)**2,x)
```

output
```
Exception raised: RecursionError >> maximum recursion depth exceeded while
 calling a Python object
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.28

$$\int \frac{\arctan(ax)}{(c+a^2cx^2)^2}\,dx = \frac{1}{2}\left(\frac{x}{a^2c^2x^2+c^2} + \frac{\arctan{(ax)}}{ac^2}\right)\arctan{(ax)} - \frac{((a^2x^2+1)\arctan{(ax)}^2-1)a}{4\,(a^4c^2x^2+a^2c^2)}$$

input
```
integrate(arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
1/2*(x/(a^2*c^2*x^2 + c^2) + arctan(a*x)/(a*c^2))*arctan(a*x) - 1/4*((a^2*
x^2 + 1)*arctan(a*x)^2 - 1)*a/(a^4*c^2*x^2 + a^2*c^2)
```

## Giac [F]

$$\int \frac{\arctan(ax)}{(c+a^2cx^2)^2}\,dx = \int \frac{\arctan(ax)}{(a^2cx^2+c)^2}\,dx$$

input
```
integrate(arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate(arctan(a*x)/(a^2*c*x^2 + c)^2, x)
```

## Mupad [B] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.79

$$\int \frac{\arctan(ax)}{(c+a^2cx^2)^2}\,dx = \frac{a^2\,x^2\,\mathrm{atan}(a\,x)^2+2\,a\,x\,\mathrm{atan}(a\,x)+\mathrm{atan}(a\,x)^2+1}{4\,a\,c^2\,\left(a^2\,x^2+1\right)}$$

input
```
int(atan(a*x)/(c + a^2*c*x^2)^2,x)
```

output
```
(atan(a*x)^2 + 2*a*x*atan(a*x) + a^2*x^2*atan(a*x)^2 + 1)/(4*a*c^2*(a^2*x^
2 + 1))
```

## Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.90

$$\int \frac{\arctan(ax)}{(c+a^2cx^2)^2}\,dx = \frac{atan(ax)^2\,a^2x^2+atan(ax)^2+2atan(ax)\,ax-a^2x^2}{4a\,c^2\,(a^2x^2+1)}$$

input
```
int(atan(a*x)/(a^2*c*x^2+c)^2,x)
```

output
```
(atan(a*x)**2*a**2*x**2 + atan(a*x)**2 + 2*atan(a*x)*a*x - a**2*x**2)/(4*a
*c**2*(a**2*x**2 + 1))
```

# 3.188 $\int \frac{\arctan(ax)}{x(c+a^2cx^2)^2} dx$



## Optimal result

Integrand size = 20, antiderivative size = 117

$$\int \frac{\arctan(ax)}{x(c+a^2cx^2)^2} dx = -\frac{ax}{4c^2(1+a^2x^2)} - \frac{\arctan(ax)}{4c^2} + \frac{\arctan(ax)}{2c^2(1+a^2x^2)} - \frac{i\arctan(ax)^2}{2c^2} + \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{c^2} - \frac{i\,\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{2c^2}$$

output
```
-1/4*a*x/c^2/(a^2*x^2+1)-1/4*arctan(a*x)/c^2+1/2*arctan(a*x)/c^2/(a^2*x^2+
1)-1/2*I*arctan(a*x)^2/c^2+arctan(a*x)*ln(2-2/(1-I*a*x))/c^2-1/2*I*polylog
(2,-1+2/(1-I*a*x))/c^2
```

## Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.62

$$\int \frac{\arctan(ax)}{x(c+a^2cx^2)^2} dx = -\frac{4i\arctan(ax)^2 - 2\arctan(ax)\left(\cos(2\arctan(ax)) + 4\log\left(1-e^{2i\arctan(ax)}\right)\right) + 4i\,\text{PolyLog}\left(2, e^{2i\arctan(}\right.}{8c^2}$$

input
```
Integrate[ArcTan[a*x]/(x*(c + a^2*c*x^2)^2),x]
```

output
```
-1/8*((4*I)*ArcTan[a*x]^2 - 2*ArcTan[a*x]*(Cos[2*ArcTan[a*x]] + 4*Log[1 -
E^((2*I)*ArcTan[a*x])]) + (4*I)*PolyLog[2, E^((2*I)*ArcTan[a*x])] + Sin[2*
ArcTan[a*x]])/c^2
```

## Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5501, 27, 5459, 5403, 2897, 5465, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)}{x\left(a^2cx^2+c\right)^2}dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{\arctan(ax)}{cx(a^2x^2+1)}dx}{c}-a^2\int \frac{x\arctan(ax)}{c^2\left(a^2x^2+1\right)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx}{c^2}-\frac{a^2\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \text{5459}$$

$$-\frac{a^2\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx}{c^2}+\frac{i\int \frac{\arctan(ax)}{x(ax+i)}dx-\frac{1}{2}i\arctan(ax)^2}{c^2}$$

$$\Big\downarrow \text{5403}$$

$$-\frac{a^2\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx}{c^2}+\frac{i\left(ia\int \frac{\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{2}i\arctan(ax)^2}{c^2}$$

$$\Big\downarrow \text{2897}$$

$$-\frac{a^2\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx}{c^2}+\frac{i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2}{c^2}$$

$$\Big\downarrow \text{5465}$$

$$-\frac{a^2\left(\frac{\int \frac{1}{\left(a^2x^2+1\right)^2}dx}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)}{c^2}+\frac{i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2}{c^2}$$

$$\Big\downarrow \text{215}$$

$$-\frac{a^2\left(\frac{\frac{1}{2}\int \frac{1}{a^2x^2+1}dx+\frac{x}{2\left(a^2x^2+1\right)}}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)}{c^2}+\frac{i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2}{c^2}$$

$$\Big\downarrow \text{216}$$

$$-\frac{a^2\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)}{c^2}+\frac{i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2}{c^2}$$

input
```
Int[ArcTan[a*x]/(x*(c + a^2*c*x^2)^2),x]
```

output
```
-((a^2*(-1/2*ArcTan[a*x]/(a^2*(1 + a^2*x^2)) + (x/(2*(1 + a^2*x^2)) + ArcT
an[a*x]/(2*a))/(2*a)))/c^2) + ((-1/2*I)*ArcTan[a*x]^2 + I*((-I)*ArcTan[a*x
]*Log[2 - 2/(1 - I*a*x)] - PolyLog[2, -1 + 2/(1 - I*a*x)]/2))/c^2
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

## Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.57 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.92

| method | result |
|---|---|
| parts | $\frac{\arctan(ax)}{2c^2(a^2x^2+1)} - \frac{\arctan(ax)\ln(a^2x^2+1)}{2c^2} + \frac{\arctan(ax)\ln(x)}{c^2} - \frac{a\left(\frac{x}{2a^2x^2+2} + \frac{\arctan(ax)}{2a} - \frac{i\ln(x)(-\ln(-iax+1)+\ln(i}{a}\right.}{}$ |
| derivativedivides | $-\frac{\arctan(ax)\ln(a^2x^2+1)}{2c^2} + \frac{\arctan(ax)}{2c^2(a^2x^2+1)} + \frac{\arctan(ax)\ln(ax)}{c^2} - \frac{\frac{ax}{2a^2x^2+2} + \frac{\arctan(ax)}{2} - i\ln(ax)\ln(iax+1) + i\ln}{}$ |
| default | $-\frac{\arctan(ax)\ln(a^2x^2+1)}{2c^2} + \frac{\arctan(ax)}{2c^2(a^2x^2+1)} + \frac{\arctan(ax)\ln(ax)}{c^2} - \frac{\frac{ax}{2a^2x^2+2} + \frac{\arctan(ax)}{2} - i\ln(ax)\ln(iax+1) + i\ln}{}$ |
| risch | $\frac{i\ln(-iax+1)}{8c^2(-iax+1)} + \frac{i\,\mathrm{dilog}(iax+1)}{2c^2} + \frac{i\ln(iax+1)}{16c^2(iax-1)} + \frac{i\,\mathrm{dilog}\left(\frac{1}{2}-\frac{iax}{2}\right)}{4c^2} - \frac{i\ln(-iax+1)}{16c^2(-iax-1)} - \frac{i\ln\left(\frac{1}{2}+\frac{iax}{2}\right)\ln(-iax+}{4c^2}$ |

input
```
int(arctan(a*x)/x/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*arctan(a*x)/c^2/(a^2*x^2+1)-1/2/c^2*arctan(a*x)*ln(a^2*x^2+1)+1/c^2*ar
ctan(a*x)*ln(x)-1/2/c^2*a*(1/2*x/(a^2*x^2+1)+1/2/a*arctan(a*x)-I*ln(x)*(-l
n(1-I*a*x)+ln(1+I*a*x))/a-I*(dilog(1+I*a*x)-dilog(1-I*a*x))/a-1/4/a^2*sum(
1/_alpha*(2*ln(x-_alpha)*ln(a^2*x^2+1)-a^2*(1/a^2/_alpha*ln(x-_alpha)^2+2*
_alpha*ln(x-_alpha)*ln(1/2*(x+_alpha)/_alpha)+2*_alpha*dilog(1/2*(x+_alpha
)/_alpha))),_alpha=RootOf(_Z^2*a^2+1)))
```

## Fricas [F]

$$\int \frac{\arctan(ax)}{x\left(c+a^2cx^2\right)^2}\,dx = \int \frac{\arctan\left(ax\right)}{\left(a^2cx^2+c\right)^2x}\,dx$$

input
```
integrate(arctan(a*x)/x/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
integral(arctan(a*x)/(a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^2*x), x)
```

## Sympy [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)}{x\left(c+a^2cx^2\right)^2}\,dx = \text{Exception raised: RecursionError}$$

input
```
integrate(atan(a*x)/x/(a**2*c*x**2+c)**2,x)
```

output
```
Exception raised: RecursionError >> maximum recursion depth exceeded
```

## Maxima [F]

$$\int \frac{\arctan(ax)}{x\left(c+a^2cx^2\right)^2}\,dx = \int \frac{\arctan\left(ax\right)}{\left(a^2cx^2+c\right)^2x}\,dx$$

input
```
integrate(arctan(a*x)/x/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)/((a^2*c*x^2 + c)^2*x), x)
```

## Giac [F]

$$\int \frac{\arctan(ax)}{x\left(c+a^2cx^2\right)^2}\,dx=\int \frac{\arctan(ax)}{\left(a^2cx^2+c\right)^2x}\,dx$$

input
```
integrate(arctan(a*x)/x/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate(arctan(a*x)/((a^2*c*x^2 + c)^2*x), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)}{x\left(c+a^2cx^2\right)^2}\,dx=\int \frac{\mathrm{atan}(a\,x)}{x\,\left(c\,a^2\,x^2+c\right)^2}\,dx$$

input
```
int(atan(a*x)/(x*(c + a^2*c*x^2)^2),x)
```

output
```
int(atan(a*x)/(x*(c + a^2*c*x^2)^2), x)
```

## Reduce [F]

$$\int \frac{\arctan(ax)}{x\left(c+a^2cx^2\right)^2}\,dx=\frac{\int \frac{atan(ax)}{a^4x^5+2a^2x^3+x}dx}{c^2}$$

input
```
int(atan(a*x)/x/(a^2*c*x^2+c)^2,x)
```

output
```
int(atan(a*x)/(a**4*x**5 + 2*a**2*x**3 + x),x)/c**2
```

# 3.189 $\int \frac{\arctan(ax)}{x^2(c+a^2cx^2)^2}\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 97

$$\int \frac{\arctan(ax)}{x^2(c+a^2cx^2)^2}\,dx = -\frac{a}{4c^2(1+a^2x^2)} - \frac{\arctan(ax)}{c^2x} - \frac{a^2x\arctan(ax)}{2c^2(1+a^2x^2)} - \frac{3a\arctan(ax)^2}{4c^2} + \frac{a\log(x)}{c^2} - \frac{a\log(1+a^2x^2)}{2c^2}$$

output
```
-1/4*a/c^2/(a^2*x^2+1)-arctan(a*x)/c^2/x-1/2*a^2*x*arctan(a*x)/c^2/(a^2*x^
2+1)-3/4*a*arctan(a*x)^2/c^2+a*ln(x)/c^2-1/2*a*ln(a^2*x^2+1)/c^2
```

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.97

$$\int \frac{\arctan(ax)}{x^2(c+a^2cx^2)^2}\,dx = -\frac{a}{4c^2(1+a^2x^2)} - \frac{(2+3a^2x^2)\arctan(ax)}{2c^2x(1+a^2x^2)} - \frac{3a\arctan(ax)^2}{4c^2} + \frac{a\log(x)}{c^2} - \frac{a\log(1+a^2x^2)}{2c^2}$$

input
```
Integrate[ArcTan[a*x]/(x^2*(c + a^2*c*x^2)^2),x]
```

output
```
-1/4*a/(c^2*(1 + a^2*x^2)) - ((2 + 3*a^2*x^2)*ArcTan[a*x])/(2*c^2*x*(1 + a
^2*x^2)) - (3*a*ArcTan[a*x]^2)/(4*c^2) + (a*Log[x])/c^2 - (a*Log[1 + a^2*x
^2])/(2*c^2)
```

## Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.11, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {5501, 27, 5427, 241, 5453, 5361, 243, 47, 14, 16, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)}{x^2 (a^2cx^2+c)^2}dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{\arctan(ax)}{cx^2(a^2x^2+1)}dx}{c}-a^2\int \frac{\arctan(ax)}{c^2 (a^2x^2+1)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx}{c^2}-\frac{a^2\int \frac{\arctan(ax)}{(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \text{5427}$$

$$\frac{\int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx}{c^2}-\frac{a^2\left(-\frac{1}{2}a\int \frac{x}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)}{c^2}$$

$$\Big\downarrow \text{241}$$

$$\frac{\int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx}{c^2}-\frac{a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)}{c^2}$$

$$\Big\downarrow \text{5453}$$

$$\frac{\int \frac{\arctan(ax)}{x^2}dx-a^2\int \frac{\arctan(ax)}{a^2x^2+1}dx}{c^2}-\frac{a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)}{c^2}$$

$$\Big\downarrow \text{5361}$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+a\int \frac{1}{x(a^2x^2+1)}dx-\frac{\arctan(ax)}{x}}{c^2}-\frac{a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)}{c^2}$$

$$\Big\downarrow \; 243$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{2}a\int \frac{1}{x^2(a^2x^2+1)}dx^2-\frac{\arctan(ax)}{x}}{c^2}-\frac{a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)}{c^2}$$

$$\Big\downarrow \; 47$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{2}a\left(\int \frac{1}{x^2}dx^2-a^2\int \frac{1}{a^2x^2+1}dx^2\right)-\frac{\arctan(ax)}{x}}{c^2}-\frac{a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)}{c^2}$$

$$\Big\downarrow \; 14$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{2}a\left(\log\left(x^2\right)-a^2\int \frac{1}{a^2x^2+1}dx^2\right)-\frac{\arctan(ax)}{x}}{c^2}-\frac{a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)}{c^2}$$

$$\Big\downarrow \; 16$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{\arctan(ax)}{x}}{c^2}-\frac{a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)}{c^2}$$

$$\Big\downarrow \; 5419$$

$$\frac{\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{1}{2}a\arctan(ax)^2-\frac{\arctan(ax)}{x}}{c^2}-\frac{a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)}{c^2}$$

input
```
Int[ArcTan[a*x]/(x^2*(c + a^2*c*x^2)^2),x]
```

output
```
-((a^2*(1/(4*a*(1 + a^2*x^2)) + (x*ArcTan[a*x])/(2*(1 + a^2*x^2)) + ArcTan
[a*x]^2/(4*a)))/c^2) + (-(ArcTan[a*x]/x) - (a*ArcTan[a*x]^2)/2 + (a*(Log[x
^2] - Log[1 + a^2*x^2]))/2)/c^2
```

## Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

## Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.98

| method | result |
|---|---|
| derivativedivides | $a\left(-\frac{ax\arctan(ax)}{2c^2(a^2x^2+1)}-\frac{3\arctan(ax)^2}{2c^2}-\frac{\arctan(ax)}{c^2ax}-\frac{-\frac{3\arctan(ax)^2}{2}+\frac{1}{2a^2x^2+2}+\ln(a^2x^2+1)-2\ln(ax)}{2c^2}\right)$ |
| default | $a\left(-\frac{ax\arctan(ax)}{2c^2(a^2x^2+1)}-\frac{3\arctan(ax)^2}{2c^2}-\frac{\arctan(ax)}{c^2ax}-\frac{-\frac{3\arctan(ax)^2}{2}+\frac{1}{2a^2x^2+2}+\ln(a^2x^2+1)-2\ln(ax)}{2c^2}\right)$ |
| parts | $-\frac{a^2x\arctan(ax)}{2c^2(a^2x^2+1)}-\frac{3a\arctan(ax)^2}{2c^2}-\frac{\arctan(ax)}{c^2x}-\frac{-\frac{3a\arctan(ax)^2}{4}-\frac{a\left(-\frac{1}{2\left(a^2x^2+1\right)}-\ln\left(a^2x^2+1\right)+2\ln(ax)\right)}{2}}{c^2}$ |
| parallelrisch | $\frac{-3a^3\arctan(ax)^2x^3+4\ln(x)a^3x^3-2a^3\ln(a^2x^2+1)x^3+a^3x^3-6x^2a^2\arctan(ax)-3a\arctan(ax)^2x+4ax\ln(x)-2a\ln(}{4x\,c^2(a^2x^2+1)}$ |
| risch | $-\frac{a\ln(-iax+1)}{16c^2(-iax-1)}-\frac{a\ln(-iax+1)}{8c^2(-iax+1)}-\frac{3a\ln\left(\frac{1}{2}+\frac{iax}{2}\right)\ln(-iax+1)}{8c^2}+\frac{3a\ln\left(\frac{1}{2}+\frac{iax}{2}\right)\ln\left(\frac{1}{2}-\frac{iax}{2}\right)}{4c^2}+\frac{i\ln(iax+1)}{2c^2x}-$ |

input
```
int(arctan(a*x)/x^2/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
a*(-1/2*a*x*arctan(a*x)/c^2/(a^2*x^2+1)-3/2*arctan(a*x)^2/c^2-1/c^2*arctan
(a*x)/a/x-1/2/c^2*(-3/2*arctan(a*x)^2+1/2/(a^2*x^2+1)+ln(a^2*x^2+1)-2*ln(a
*x)))
```

## Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00

$$\int\frac{\arctan(ax)}{x^2\left(c+a^2cx^2\right)^2}\,dx=$$
$$-\frac{3\left(a^3x^3+ax\right)\arctan\left(ax\right)^2+ax+2\left(3\,a^2x^2+2\right)\arctan\left(ax\right)+2\left(a^3x^3+ax\right)\log\left(a^2x^2+1\right)-4\left(a^3x^3\right.}{4\left(a^2c^2x^3+c^2x\right)}$$

input
```
integrate(arctan(a*x)/x^2/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
-1/4*(3*(a^3*x^3 + a*x)*arctan(a*x)^2 + a*x + 2*(3*a^2*x^2 + 2)*arctan(a*x
) + 2*(a^3*x^3 + a*x)*log(a^2*x^2 + 1) - 4*(a^3*x^3 + a*x)*log(x))/(a^2*c^
2*x^3 + c^2*x)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(88) = 176$.

Time = 0.66 (sec) , antiderivative size = 274, normalized size of antiderivative = 2.82

$$\int \frac{\arctan(ax)}{x^2 (c + a^2cx^2)^2} \, dx$$
$$= \begin{cases} \frac{4a^3x^3\log(x)}{4a^2c^2x^3+4c^2x} - \frac{2a^3x^3\log\left(x^2+\frac{1}{a^2}\right)}{4a^2c^2x^3+4c^2x} - \frac{3a^3x^3\operatorname{atan}^2(ax)}{4a^2c^2x^3+4c^2x} - \frac{6a^2x^2\operatorname{atan}(ax)}{4a^2c^2x^3+4c^2x} + \frac{4ax\log(x)}{4a^2c^2x^3+4c^2x} - \frac{2ax\log\left(x^2+\frac{1}{a^2}\right)}{4a^2c^2x^3+4c^2x} - \frac{3ax\operatorname{atan}^2(\ldots}{4a^2c^2x^3+4\ldots} \\ 0 \end{cases}$$

input
```
integrate(atan(a*x)/x**2/(a**2*c*x**2+c)**2,x)
```

output
```
Piecewise((4*a**3*x**3*log(x)/(4*a**2*c**2*x**3 + 4*c**2*x) - 2*a**3*x**3*
log(x**2 + a**(-2))/(4*a**2*c**2*x**3 + 4*c**2*x) - 3*a**3*x**3*atan(a*x)*
*2/(4*a**2*c**2*x**3 + 4*c**2*x) - 6*a**2*x**2*atan(a*x)/(4*a**2*c**2*x**3
 + 4*c**2*x) + 4*a*x*log(x)/(4*a**2*c**2*x**3 + 4*c**2*x) - 2*a*x*log(x**2
 + a**(-2))/(4*a**2*c**2*x**3 + 4*c**2*x) - 3*a*x*atan(a*x)**2/(4*a**2*c**
2*x**3 + 4*c**2*x) - a*x/(4*a**2*c**2*x**3 + 4*c**2*x) - 4*atan(a*x)/(4*a*
*2*c**2*x**3 + 4*c**2*x), Ne(a, 0)), (0, True))
```

## Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.23

$$\int \frac{\arctan(ax)}{x^2 (c + a^2cx^2)^2} \, dx = -\frac{1}{2}\left(\frac{3\,a^2x^2+2}{a^2c^2x^3+c^2x} + \frac{3\,a\arctan(ax)}{c^2}\right)\arctan(ax)$$
$$+ \frac{\left(3\,(a^2x^2+1)\arctan(ax)^2 - 2\,(a^2x^2+1)\log(a^2x^2+1) + 4\,(a^2x^2+1)\log(x) - 1\right)a}{4\,(a^2c^2x^2+c^2)}$$

input
```
integrate(arctan(a*x)/x^2/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
-1/2*((3*a^2*x^2 + 2)/(a^2*c^2*x^3 + c^2*x) + 3*a*arctan(a*x)/c^2)*arctan(
a*x) + 1/4*(3*(a^2*x^2 + 1)*arctan(a*x)^2 - 2*(a^2*x^2 + 1)*log(a^2*x^2 +
1) + 4*(a^2*x^2 + 1)*log(x) - 1)*a/(a^2*c^2*x^2 + c^2)
```

## Giac [F]

$$\int \frac{\arctan(ax)}{x^2 (c + a^2cx^2)^2} \, dx = \int \frac{\arctan(ax)}{(a^2cx^2 + c)^2 x^2} \, dx$$

input
```
integrate(arctan(a*x)/x^2/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate(arctan(a*x)/((a^2*c*x^2 + c)^2*x^2), x)
```

## Mupad [B] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.94

$$\int \frac{\arctan(ax)}{x^2 (c + a^2cx^2)^2} \, dx = \frac{a \ln(x)}{c^2} - \frac{a \ln(a^2 x^2 + 1)}{2 c^2} - \frac{\operatorname{atan}(a x) \left(\frac{1}{a^2 c^2} + \frac{3 x^2}{2 c^2}\right)}{\frac{x}{a^2} + x^3} - \frac{a}{2 (2 a^2 c^2 x^2 + 2 c^2)} - \frac{3 a \operatorname{atan}(a x)^2}{4 c^2}$$

input
```
int(atan(a*x)/(x^2*(c + a^2*c*x^2)^2),x)
```

output
```
(a*log(x))/c^2 - (a*log(a^2*x^2 + 1))/(2*c^2) - (atan(a*x)*(1/(a^2*c^2) +
(3*x^2)/(2*c^2)))/(x/a^2 + x^3) - a/(2*(2*c^2 + 2*a^2*c^2*x^2)) - (3*a*ata
n(a*x)^2)/(4*c^2)
```

## Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.21

$$\int \frac{\arctan(ax)}{x^2 (c + a^2cx^2)^2} \, dx = \frac{-3atan(ax)^2 a^3x^3 - 3atan(ax)^2 ax - 6atan(ax) a^2x^2 - 4atan(ax) - 2\log(a^2x^2 + 1) a^3x^3 - 2\log(a^2x^2 -}{4c^2x (a^2x^2 + 1)}$$

input
```
int(atan(a*x)/x^2/(a^2*c*x^2+c)^2,x)
```

output
```
( - 3*atan(a*x)**2*a**3*x**3 - 3*atan(a*x)**2*a*x - 6*atan(a*x)*a**2*x**2
- 4*atan(a*x) - 2*log(a**2*x**2 + 1)*a**3*x**3 - 2*log(a**2*x**2 + 1)*a*x
+ 4*log(x)*a**3*x**3 + 4*log(x)*a*x + a**3*x**3)/(4*c**2*x*(a**2*x**2 + 1)
)
```

# 3.190 $\int \frac{\arctan(ax)}{x^3(c+a^2cx^2)^2}\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 156

$$\int \frac{\arctan(ax)}{x^3(c+a^2cx^2)^2}\,dx = -\frac{a}{2c^2x}+\frac{a^3x}{4c^2(1+a^2x^2)}-\frac{a^2\arctan(ax)}{4c^2} - \frac{\arctan(ax)}{2c^2x^2}-\frac{a^2\arctan(ax)}{2c^2(1+a^2x^2)}+\frac{ia^2\arctan(ax)^2}{c^2} - \frac{2a^2\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{c^2}+\frac{ia^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{c^2}$$

output
```
-1/2*a/c^2/x+1/4*a^3*x/c^2/(a^2*x^2+1)-1/4*a^2*arctan(a*x)/c^2-1/2*arctan(
a*x)/c^2/x^2-1/2*a^2*arctan(a*x)/c^2/(a^2*x^2+1)+I*a^2*arctan(a*x)^2/c^2-2
*a^2*arctan(a*x)*ln(2-2/(1-I*a*x))/c^2+I*a^2*polylog(2,-1+2/(1-I*a*x))/c^2
```

## Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.60

$$\int \frac{\arctan(ax)}{x^3(c+a^2cx^2)^2}\,dx = \frac{a^2\left(-\frac{4}{ax}+8i\arctan(ax)^2+\arctan(ax)\left(-4-\frac{4}{a^2x^2}-2\cos(2\arctan(ax))-16\log\left(1-e^{2i\arctan(ax)}\right)\right)+8}{8c^2}$$

input
```
Integrate[ArcTan[a*x]/(x^3*(c + a^2*c*x^2)^2),x]
```

output
```
(a^2*(-4/(a*x) + (8*I)*ArcTan[a*x]^2 + ArcTan[a*x]*(-4 - 4/(a^2*x^2) - 2*C
os[2*ArcTan[a*x]] - 16*Log[1 - E^((2*I)*ArcTan[a*x])]) + (8*I)*PolyLog[2,
E^((2*I)*ArcTan[a*x])] + Sin[2*ArcTan[a*x]]))/(8*c^2)
```

## Rubi [A] (verified)

Time = 1.52 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.49, number of steps used = 16, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {5501, 27, 5453, 5361, 264, 216, 5459, 5403, 2897, 5501, 5459, 5403, 2897, 5465, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)}{x^3 (a^2cx^2+c)^2} \, dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{\arctan(ax)}{cx^3(a^2x^2+1)}dx}{c} - a^2 \int \frac{\arctan(ax)}{c^2x(a^2x^2+1)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\arctan(ax)}{x^3(a^2x^2+1)}dx}{c^2} - \frac{a^2 \int \frac{\arctan(ax)}{x(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \text{5453}$$

$$\frac{\int \frac{\arctan(ax)}{x^3}dx - a^2 \int \frac{\arctan(ax)}{x(a^2x^2+1)}dx}{c^2} - \frac{a^2 \int \frac{\arctan(ax)}{x(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \text{5361}$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx\right) + \frac{1}{2}a \int \frac{1}{x^2(a^2x^2+1)}dx - \frac{\arctan(ax)}{2x^2}}{c^2} - \frac{a^2 \int \frac{\arctan(ax)}{x(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \text{264}$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx\right)+\frac{1}{2}a\left(a^2\left(-\int \frac{1}{a^2x^2+1}dx\right)-\frac{1}{x}\right)-\frac{\arctan(ax)}{2x^2}}{c^2}-\frac{a^2\int \frac{\arctan(ax)}{x(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \text{216}$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx\right)-\frac{\arctan(ax)}{2x^2}+\frac{1}{2}a\left(-a\arctan(ax)-\frac{1}{x}\right)}{c^2}-\frac{a^2\int \frac{\arctan(ax)}{x(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \text{5459}$$

$$-\frac{a^2\int \frac{\arctan(ax)}{x(a^2x^2+1)^2}dx}{c^2}+\frac{-\left(a^2\left(i\int \frac{\arctan(ax)}{x(ax+i)}dx-\frac{1}{2}i\arctan(ax)^2\right)\right)-\frac{\arctan(ax)}{2x^2}+\frac{1}{2}a\left(-a\arctan(ax)-\frac{1}{x}\right)}{c^2}$$

$$\Big\downarrow \text{5403}$$

$$-\frac{a^2\int \frac{\arctan(ax)}{x(a^2x^2+1)^2}dx}{c^2}+\frac{-\left(a^2\left(i\left(ia\int \frac{\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)\right)-\frac{\arctan(ax)}{2x^2}+\frac{1}{2}a(-a\arctan}{c^2}$$

$$\Big\downarrow \text{2897}$$

$$-\frac{a^2\int \frac{\arctan(ax)}{x(a^2x^2+1)^2}dx}{c^2}+\frac{-\left(a^2\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)\right)-\frac{\arctan(ax)}{2x^2}+\frac{1}{2}a(-a\,a}{c^2}$$

$$\Big\downarrow \text{5501}$$

$$-\frac{a^2\left(\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx-a^2\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx\right)}{c^2}+\frac{-\left(a^2\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)\right)-\frac{\arctan(ax)}{2x^2}+\frac{1}{2}a(-a\,a}{c^2}$$

$$\Big\downarrow \text{5459}$$

$$\frac{-\left(a^2\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)\right)-\frac{\arctan(ax)}{2x^2}+\frac{1}{2}a(-a\,a}{c^2}$$

$$\frac{a^2\left(a^2\left(-\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx\right)+i\int \frac{\arctan(ax)}{x(ax+i)}dx-\frac{1}{2}i\arctan(ax)^2\right)}{c^2}$$

$\Big\downarrow$ 5403

$$\frac{-\left(a^2\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)\right)-\frac{\arctan(ax)}{2x^2}+\frac{1}{2}a(-a\,\ldots}{c^2}$$

$$\frac{a^2\left(a^2\left(-\int\frac{x\arctan(ax)}{(a^2x^2+1)^2}dx\right)+i\left(ia\int\frac{\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)}{c^2}$$

$\Big\downarrow$ 2897

$$\frac{-\left(a^2\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)\right)-\frac{\arctan(ax)}{2x^2}+\frac{1}{2}a(-a\,\ldots}{c^2}$$

$$\frac{a^2\left(a^2\left(-\int\frac{x\arctan(ax)}{(a^2x^2+1)^2}dx\right)+i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)}{c^2}$$

$\Big\downarrow$ 5465

$$\frac{-\left(a^2\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)\right)-\frac{\arctan(ax)}{2x^2}+\frac{1}{2}a(-a\,\ldots}{c^2}$$

$$\frac{a^2\left(-\left(a^2\left(\frac{\int\frac{1}{(a^2x^2+1)^2}dx}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)\right)+i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}\ldots\right.}{c^2}$$

$\Big\downarrow$ 215

$$\frac{-\left(a^2\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)\right)-\frac{\arctan(ax)}{2x^2}+\frac{1}{2}a(-a\,\ldots}{c^2}$$

$$\frac{a^2\left(-\left(a^2\left(\frac{\frac{1}{2}\int\frac{1}{a^2x^2+1}dx+\frac{x}{2(a^2x^2+1)}}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)\right)+i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-\ldots\right.\right.}{c^2}$$

$\Big\downarrow$ 216

$$\frac{-\left(a^2\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)\right)-\frac{\arctan(ax)}{2x^2}+\frac{1}{2}a(-a\,\ldots}{c^2}$$

$$\frac{a^2\left(-\left(a^2\left(\frac{\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)\right)+i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\ldots\right.\right.}{c^2}$$

input
```
Int[ArcTan[a*x]/(x^3*(c + a^2*c*x^2)^2),x]
```

output
```
(-1/2*ArcTan[a*x]/x^2 + (a*(-x^(-1) - a*ArcTan[a*x]))/2 - a^2*((-1/2*I)*Ar
cTan[a*x]^2 + I*((-I)*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)] - PolyLog[2, -1 +
 2/(1 - I*a*x)]/2)))/c^2 - (a^2*((-1/2*I)*ArcTan[a*x]^2 - a^2*(-1/2*ArcTan
[a*x]/(a^2*(1 + a^2*x^2)) + (x/(2*(1 + a^2*x^2)) + ArcTan[a*x]/(2*a))/(2*a
)) + I*((-I)*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)] - PolyLog[2, -1 + 2/(1 - I
*a*x)]/2)))/c^2
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

## Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.83 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.62

| method | result |
|---|---|
| parts | $-\frac{a^2\arctan(ax)}{2c^2(a^2x^2+1)}+\frac{\arctan(ax)a^2\ln(a^2x^2+1)}{c^2}-\frac{\arctan(ax)}{2c^2x^2}-\frac{2\arctan(ax)a^2\ln(x)}{c^2}-\frac{a\left(-4a^2\left(-\frac{i\ln(x)(-\ln(-\right.\right.}{}$ |
| derivativedivides | $a^2\left(-\frac{\arctan(ax)}{2c^2a^2x^2}-\frac{2\arctan(ax)\ln(ax)}{c^2}+\frac{\arctan(ax)\ln(a^2x^2+1)}{c^2}-\frac{\arctan(ax)}{2c^2(a^2x^2+1)}-\frac{2i\ln(ax)\ln(iax+1)-2i}{}\right.$ |
| default | $a^2\left(-\frac{\arctan(ax)}{2c^2a^2x^2}-\frac{2\arctan(ax)\ln(ax)}{c^2}+\frac{\arctan(ax)\ln(a^2x^2+1)}{c^2}-\frac{\arctan(ax)}{2c^2(a^2x^2+1)}-\frac{2i\ln(ax)\ln(iax+1)-2i}{}\right.$ |
| risch | $\frac{ia^2\,\text{dilog}(-iax+1)}{c^2}+\frac{a^3\ln(iax+1)x}{16c^2(iax-1)}+\frac{ia^2\ln(iax+1)}{8c^2(iax+1)}-\frac{ia^2\ln\left(\frac{1}{2}-\frac{iax}{2}\right)\ln(iax+1)}{2c^2}-\frac{ia^2\ln(iax+1)}{16c^2(iax-1)}+\frac{a^3\ln(-}{16c^2(-}$ |

input
```
int(arctan(a*x)/x^3/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^2*arctan(a*x)/c^2/(a^2*x^2+1)+1/c^2*arctan(a*x)*a^2*ln(a^2*x^2+1)-1
/2*arctan(a*x)/c^2/x^2-2/c^2*arctan(a*x)*a^2*ln(x)-1/2/c^2*a*(-4*a^2*(-1/2
*I*ln(x)*(-ln(1-I*a*x)+ln(1+I*a*x))/a-1/2*I*(dilog(1+I*a*x)-dilog(1-I*a*x)
)/a)-1/2*x*a^2/(a^2*x^2+1)+1/2*a*arctan(a*x)+1/x+1/2*sum(1/_alpha*(2*ln(x-
_alpha)*ln(a^2*x^2+1)-a^2*(1/a^2/_alpha*ln(x-_alpha)^2+2*_alpha*ln(x-_alph
a)*ln(1/2*(x+_alpha)/_alpha)+2*_alpha*dilog(1/2*(x+_alpha)/_alpha))),_alph
a=RootOf(_Z^2*a^2+1)))
```

## Fricas [F]

$$\int \frac{\arctan(ax)}{x^3\left(c+a^2cx^2\right)^2}\,dx=\int \frac{\arctan\left(ax\right)}{\left(a^2cx^2+c\right)^2x^3}\,dx$$

input
```
integrate(arctan(a*x)/x^3/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
integral(arctan(a*x)/(a^4*c^2*x^7 + 2*a^2*c^2*x^5 + c^2*x^3), x)
```

## Sympy [F]

$$\int \frac{\arctan(ax)}{x^3\left(c+a^2cx^2\right)^2}\,dx=\frac{\int \frac{\operatorname{atan}\left(ax\right)}{a^4x^7+2a^2x^5+x^3}\,dx}{c^2}$$

input
```
integrate(atan(a*x)/x**3/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(atan(a*x)/(a**4*x**7 + 2*a**2*x**5 + x**3), x)/c**2
```

## Maxima [F]

$$\int \frac{\arctan(ax)}{x^3\left(c+a^2cx^2\right)^2}\,dx=\int \frac{\arctan\left(ax\right)}{\left(a^2cx^2+c\right)^2x^3}\,dx$$

input
```
integrate(arctan(a*x)/x^3/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)/((a^2*c*x^2 + c)^2*x^3), x)
```

## Giac [F]

$$\int \frac{\arctan(ax)}{x^3 (c + a^2cx^2)^2} \, dx = \int \frac{\arctan(ax)}{(a^2cx^2 + c)^2 x^3} \, dx$$

input
```
integrate(arctan(a*x)/x^3/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate(arctan(a*x)/((a^2*c*x^2 + c)^2*x^3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)}{x^3 (c + a^2cx^2)^2} \, dx = \int \frac{\mathrm{atan}(a\,x)}{x^3\,(c\,a^2\,x^2 + c)^2} \, dx$$

input
```
int(atan(a*x)/(x^3*(c + a^2*c*x^2)^2),x)
```

output
```
int(atan(a*x)/(x^3*(c + a^2*c*x^2)^2), x)
```

## Reduce [F]

$$\int \frac{\arctan(ax)}{x^3 (c + a^2cx^2)^2} \, dx = \frac{\int \frac{atan(ax)}{a^4x^7+2a^2x^5+x^3}dx}{c^2}$$

input
```
int(atan(a*x)/x^3/(a^2*c*x^2+c)^2,x)
```

output
```
int(atan(a*x)/(a**4*x**7 + 2*a**2*x**5 + x**3),x)/c**2
```

# 3.191 $\int \frac{\arctan(ax)}{x^4(c+a^2cx^2)^2} dx$



## Optimal result

Integrand size = 20, antiderivative size = 136

$$\int \frac{\arctan(ax)}{x^4(c+a^2cx^2)^2} dx = -\frac{a}{6c^2x^2} + \frac{a^3}{4c^2(1+a^2x^2)} - \frac{\arctan(ax)}{3c^2x^3} + \frac{2a^2\arctan(ax)}{c^2x} + \frac{a^4x\arctan(ax)}{2c^2(1+a^2x^2)} + \frac{5a^3\arctan(ax)^2}{4c^2} - \frac{7a^3\log(x)}{3c^2} + \frac{7a^3\log(1+a^2x^2)}{6c^2}$$

output
```
-1/6*a/c^2/x^2+1/4*a^3/c^2/(a^2*x^2+1)-1/3*arctan(a*x)/c^2/x^3+2*a^2*arcta
n(a*x)/c^2/x+1/2*a^4*x*arctan(a*x)/c^2/(a^2*x^2+1)+5/4*a^3*arctan(a*x)^2/c
^2-7/3*a^3*ln(x)/c^2+7/6*a^3*ln(a^2*x^2+1)/c^2
```

## Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.91

$$\int \frac{\arctan(ax)}{x^4(c+a^2cx^2)^2} dx = -\frac{a}{6c^2x^2} + \frac{a^3}{4c^2(1+a^2x^2)} + \frac{(-2+10a^2x^2+15a^4x^4)\arctan(ax)}{6c^2x^3(1+a^2x^2)} + \frac{5a^3\arctan(ax)^2}{4c^2} - \frac{7a^3\log(x)}{3c^2} + \frac{7a^3\log(1+a^2x^2)}{6c^2}$$

input
```
Integrate[ArcTan[a*x]/(x^4*(c + a^2*c*x^2)^2),x]
```

output
```
-1/6*a/(c^2*x^2) + a^3/(4*c^2*(1 + a^2*x^2)) + ((-2 + 10*a^2*x^2 + 15*a^4*
x^4)*ArcTan[a*x])/(6*c^2*x^3*(1 + a^2*x^2)) + (5*a^3*ArcTan[a*x]^2)/(4*c^2
) - (7*a^3*Log[x])/(3*c^2) + (7*a^3*Log[1 + a^2*x^2])/(6*c^2)
```

## Rubi [A] (verified)

Time = 1.62 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.52, number of steps used = 25, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 1.200$, Rules used = {5501, 27, 5453, 5361, 243, 54, 2009, 5453, 5361, 243, 47, 14, 16, 5419, 5501, 5427, 241, 5453, 5361, 243, 47, 14, 16, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)}{x^4 \left(a^2cx^2+c\right)^2}dx$$

$$\Big\downarrow \text{ 5501}$$

$$\frac{\int \frac{\arctan(ax)}{cx^4(a^2x^2+1)}dx}{c}-a^2\int \frac{\arctan(ax)}{c^2x^2\left(a^2x^2+1\right)^2}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{\arctan(ax)}{x^4(a^2x^2+1)}dx}{c^2}-\frac{a^2\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \text{ 5453}$$

$$\frac{\int \frac{\arctan(ax)}{x^4}dx-a^2\int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx}{c^2}-\frac{a^2\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \text{ 5361}$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx\right)+\frac{1}{3}a\int \frac{1}{x^3(a^2x^2+1)}dx-\frac{\arctan(ax)}{3x^3}}{c^2}-\frac{a^2\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx\right)+\frac{1}{6}a\int \frac{1}{x^4(a^2x^2+1)}dx^2-\frac{\arctan(ax)}{3x^3}}{c^2}-\frac{a^2\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \ 54$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx\right)+\frac{1}{6}a\int \left(\frac{a^4}{a^2x^2+1}-\frac{a^2}{x^2}+\frac{1}{x^4}\right)dx^2-\frac{\arctan(ax)}{3x^3}}{c^2}-\frac{a^2\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \ 2009$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx\right)+\frac{1}{6}a\left(a^2\left(-\log\left(x^2\right)\right)+a^2\log\left(a^2x^2+1\right)-\frac{1}{x^2}\right)-\frac{\arctan(ax)}{3x^3}}{c^2}-\frac{a^2\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \ 5453$$

$$\frac{-\left(a^2\left(\int \frac{\arctan(ax)}{x^2}dx-a^2\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)\right)+\frac{1}{6}a\left(a^2\left(-\log\left(x^2\right)\right)+a^2\log\left(a^2x^2+1\right)-\frac{1}{x^2}\right)-\frac{\arctan(ax)}{3x^3}}{c^2}-\frac{a^2\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \ 5361$$

$$\frac{-\left(a^2\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+a\int \frac{1}{x(a^2x^2+1)}dx-\frac{\arctan(ax)}{x}\right)\right)+\frac{1}{6}a\left(a^2\left(-\log\left(x^2\right)\right)+a^2\log\left(a^2x^2+1\right)-\frac{1}{x^2}\right)-\cdots}{c^2}$$
$$\frac{a^2\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \ 243$$

$$\frac{-\left(a^2\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{2}a\int \frac{1}{x^2(a^2x^2+1)}dx^2-\frac{\arctan(ax)}{x}\right)\right)+\frac{1}{6}a\left(a^2\left(-\log\left(x^2\right)\right)+a^2\log\left(a^2x^2+1\right)-\frac{1}{x^2}\right)\cdots}{c^2}$$
$$\frac{a^2\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \ 47$$

$$\frac{-\left(a^2\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{2}a\left(\int \frac{1}{x^2}dx^2-a^2\int \frac{1}{a^2x^2+1}dx^2\right)-\frac{\arctan(ax)}{x}\right)\right)+\frac{1}{6}a\left(a^2\left(-\log\left(x^2\right)\right)+a^2\log\left(a^2x^2\cdots\right.\right.}{c^2}$$
$$\frac{a^2\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \text{14}$$

$$\frac{-\left(a^2\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{2}a\left(\log\left(x^2\right)-a^2\int \frac{1}{a^2x^2+1}dx^2\right)-\frac{\arctan(ax)}{x}\right)\right)+\frac{1}{6}a\left(a^2\left(-\log\left(x^2\right)\right)+a^2\log\left(a^2x^2\right.\right.}{c^2}$$

$$\frac{a^2\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \text{16}$$

$$\frac{-\left(a^2\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{\arctan(ax)}{x}\right)\right)+\frac{1}{6}a\left(a^2\left(-\log\left(x^2\right)\right)+a^2\log\left(a^2x^2\right.\right.}{c^2}$$

$$\frac{a^2\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \text{5419}$$

$$\frac{-\left(a^2\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{1}{2}a\arctan(ax)^2-\frac{\arctan(ax)}{x}\right)\right)+\frac{1}{6}a\left(a^2\left(-\log\left(x^2\right)\right)+a^2\log\left(a^2x^2+1\right)-\right.}{c^2}$$

$$\frac{a^2\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \text{5501}$$

$$\frac{-\left(a^2\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{1}{2}a\arctan(ax)^2-\frac{\arctan(ax)}{x}\right)\right)+\frac{1}{6}a\left(a^2\left(-\log\left(x^2\right)\right)+a^2\log\left(a^2x^2+1\right)-\right.}{c^2}$$

$$\frac{a^2\left(\int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx-a^2\int \frac{\arctan(ax)}{(a^2x^2+1)^2}dx\right)}{c^2}$$

$$\Big\downarrow \text{5427}$$

$$\frac{-\left(a^2\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{1}{2}a\arctan(ax)^2-\frac{\arctan(ax)}{x}\right)\right)+\frac{1}{6}a\left(a^2\left(-\log\left(x^2\right)\right)+a^2\log\left(a^2x^2+1\right)-\right.}{c^2}$$

$$\frac{a^2\left(\int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx-a^2\left(-\frac{1}{2}a\int \frac{x}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)\right)}{c^2}$$

$$\Big\downarrow \text{241}$$

$$\frac{-\left(a^2\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{1}{2}a\arctan(ax)^2-\frac{\arctan(ax)}{x}\right)\right)+\frac{1}{6}a\left(a^2\left(-\log\left(x^2\right)\right)+a^2\log\left(a^2x^2+1\right)-\right.}{c^2}$$

$$\frac{a^2\left(\int\frac{\arctan(ax)}{x^2(a^2x^2+1)}dx-a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)\right)}{c^2}$$

$$\Big\downarrow \ 5453$$

$$\frac{-\left(a^2\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{1}{2}a\arctan(ax)^2-\frac{\arctan(ax)}{x}\right)\right)+\frac{1}{6}a\left(a^2\left(-\log\left(x^2\right)\right)+a^2\log\left(a^2x^2+1\right)-\right.}{c^2}$$

$$\frac{a^2\left(-a^2\int\frac{\arctan(ax)}{a^2x^2+1}dx+\int\frac{\arctan(ax)}{x^2}dx-\left(a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)\right)\right)}{c^2}$$

$$\Big\downarrow \ 5361$$

$$\frac{-\left(a^2\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{1}{2}a\arctan(ax)^2-\frac{\arctan(ax)}{x}\right)\right)+\frac{1}{6}a\left(a^2\left(-\log\left(x^2\right)\right)+a^2\log\left(a^2x^2+1\right)-\right.}{c^2}$$

$$\frac{a^2\left(-a^2\int\frac{\arctan(ax)}{a^2x^2+1}dx+a\int\frac{1}{x(a^2x^2+1)}dx-\left(a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)\right)-\frac{\arctan(ax)}{x}\right)}{c^2}$$

$$\Big\downarrow \ 243$$

$$\frac{-\left(a^2\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{1}{2}a\arctan(ax)^2-\frac{\arctan(ax)}{x}\right)\right)+\frac{1}{6}a\left(a^2\left(-\log\left(x^2\right)\right)+a^2\log\left(a^2x^2+1\right)-\right.}{c^2}$$

$$\frac{a^2\left(-a^2\int\frac{\arctan(ax)}{a^2x^2+1}dx+\frac{1}{2}a\int\frac{1}{x^2(a^2x^2+1)}dx^2-\left(a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)\right)-\frac{\arctan(ax)}{x}\right)}{c^2}$$

$$\Big\downarrow \ 47$$

$$\frac{-\left(a^2\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{1}{2}a\arctan(ax)^2-\frac{\arctan(ax)}{x}\right)\right)+\frac{1}{6}a\left(a^2\left(-\log\left(x^2\right)\right)+a^2\log\left(a^2x^2+1\right)-\right.}{c^2}$$

$$\frac{a^2\left(-a^2\int\frac{\arctan(ax)}{a^2x^2+1}dx+\frac{1}{2}a\left(\int\frac{1}{x^2}dx^2-a^2\int\frac{1}{a^2x^2+1}dx^2\right)-\left(a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)\right)-\frac{\arctan(\right.}{x}}{c^2}$$

$$\Big\downarrow \ 14$$

$$\frac{-\left(a^2\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{1}{2}a\arctan(ax)^2-\frac{\arctan(ax)}{x}\right)\right)+\frac{1}{6}a\left(a^2\left(-\log\left(x^2\right)\right)+a^2\log\left(a^2x^2+1\right)-\right.}{c^2}$$

$$\frac{a^2\left(-a^2\int\frac{\arctan(ax)}{a^2x^2+1}dx+\frac{1}{2}a\left(\log\left(x^2\right)-a^2\int\frac{1}{a^2x^2+1}dx^2\right)-\left(a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)\right)-\frac{\arctan(\right.}{x}}{c^2}$$

$$\Big\downarrow 16$$

$$\frac{-\left(a^2\left(\frac{1}{2}a(\log(x^2)-\log(a^2x^2+1))-\frac{1}{2}a\arctan(ax)^2-\frac{\arctan(ax)}{x}\right)\right)+\frac{1}{6}a(a^2(-\log(x^2))+a^2\log(a^2x^2+1)-}{c^2}$$

$$\frac{a^2\left(-a^2\int\frac{\arctan(ax)}{a^2x^2+1}dx-\left(a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)\right)+\frac{1}{2}a(\log(x^2)-\log(a^2x^2+1))-\frac{\arctan(ax}{x}\right.}{c^2}$$

$$\Big\downarrow 5419$$

$$\frac{-\left(a^2\left(\frac{1}{2}a(\log(x^2)-\log(a^2x^2+1))-\frac{1}{2}a\arctan(ax)^2-\frac{\arctan(ax)}{x}\right)\right)+\frac{1}{6}a(a^2(-\log(x^2))+a^2\log(a^2x^2+1)-}{c^2}$$

$$\frac{a^2\left(-\left(a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)\right)+\frac{1}{2}a(\log(x^2)-\log(a^2x^2+1))-\frac{1}{2}a\arctan(ax)^2-\frac{\arctan(ax)}{x}\right.}{c^2}$$

input
```
Int[ArcTan[a*x]/(x^4*(c + a^2*c*x^2)^2),x]
```

output
```
-((a^2*(-(ArcTan[a*x]/x) - (a*ArcTan[a*x]^2)/2 - a^2*(1/(4*a*(1 + a^2*x^2)
) + (x*ArcTan[a*x])/(2*(1 + a^2*x^2)) + ArcTan[a*x]^2/(4*a)) + (a*(Log[x^2
] - Log[1 + a^2*x^2]))/2))/c^2) + (-1/3*ArcTan[a*x]/x^3 - a^2*(-(ArcTan[a*
x]/x) - (a*ArcTan[a*x]^2)/2 + (a*(Log[x^2] - Log[1 + a^2*x^2]))/2) + (a*(-
x^(-2) - a^2*Log[x^2] + a^2*Log[1 + a^2*x^2]))/6)/c^2
```

## Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

## Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.89

| method | result |
|---|---|
| derivativedivides | $a^3\left(\frac{ax\arctan(ax)}{2c^2(a^2x^2+1)}+\frac{5\arctan(ax)^2}{2c^2}-\frac{\arctan(ax)}{3c^2a^3x^3}+\frac{2\arctan(ax)}{c^2ax}-\frac{-7\ln(a^2x^2+1)-\frac{3}{2(a^2x^2+1)}+\frac{1}{a^2x^2}+14\ln\right.$ / $6c^2$ |
| default | $a^3\left(\frac{ax\arctan(ax)}{2c^2(a^2x^2+1)}+\frac{5\arctan(ax)^2}{2c^2}-\frac{\arctan(ax)}{3c^2a^3x^3}+\frac{2\arctan(ax)}{c^2ax}-\frac{-7\ln(a^2x^2+1)-\frac{3}{2(a^2x^2+1)}+\frac{1}{a^2x^2}+14\ln\right.$ / $6c^2$ |
| parts | $\frac{a^4x\arctan(ax)}{2c^2(a^2x^2+1)}+\frac{5a^3\arctan(ax)^2}{2c^2}-\frac{\arctan(ax)}{3c^2x^3}+\frac{2a^2\arctan(ax)}{c^2x}-\frac{\frac{15a^3\arctan(ax)^2}{4}+\frac{a^3\left(-7\ln\left(a^2x^2+1\right)-\frac{}{2(a}\right.}{}}{3c^2}$ |
| parallelrisch | $-\frac{-15a^5\arctan(ax)^2x^5+28\ln(x)a^5x^5-14\ln(a^2x^2+1)x^5a^5-3a^5x^5-30x^4\arctan(ax)a^4-15a^3\arctan(ax)^2x^3+28\ln(}{12x^3c^2(a^2x^2+1)}$ |
| risch | $-\frac{5a^3\ln(iax+1)^2}{16c^2}+\frac{(15a^5x^5\ln(-iax+1)-30ia^4x^4+15a^3x^3\ln(-iax+1)-20ia^2x^2+4i)\ln(iax+1)}{24x^3c^2(a^2x^2+1)}-\frac{15a^5x^5\ln(-}{}$ |

input
```
int(arctan(a*x)/x^4/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
a^3*(1/2*a*x*arctan(a*x)/c^2/(a^2*x^2+1)+5/2*arctan(a*x)^2/c^2-1/3/c^2*arc
tan(a*x)/a^3/x^3+2/c^2*arctan(a*x)/a/x-1/6/c^2*(-7*ln(a^2*x^2+1)-3/2/(a^2*
x^2+1)+1/a^2/x^2+14*ln(a*x)+15/2*arctan(a*x)^2))
```

## Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.93

$$\int \frac{\arctan(ax)}{x^4 (c + a^2cx^2)^2} \, dx$$
$$= \frac{a^3x^3 + 15\,(a^5x^5 + a^3x^3)\arctan{(ax)}^2 - 2\,ax + 2\,(15\,a^4x^4 + 10\,a^2x^2 - 2)\arctan{(ax)} + 14\,(a^5x^5 + a^3x^3)\,\mathrm{l}}{12\,(a^2c^2x^5 + c^2x^3)}$$

input
```
integrate(arctan(a*x)/x^4/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
1/12*(a^3*x^3 + 15*(a^5*x^5 + a^3*x^3)*arctan(a*x)^2 - 2*a*x + 2*(15*a^4*x
^4 + 10*a^2*x^2 - 2)*arctan(a*x) + 14*(a^5*x^5 + a^3*x^3)*log(a^2*x^2 + 1)
 - 28*(a^5*x^5 + a^3*x^3)*log(x))/(a^2*c^2*x^5 + c^2*x^3)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 362 vs. $2(129) = 258$.

Time = 1.02 (sec) , antiderivative size = 362, normalized size of antiderivative = 2.66

$$\int \frac{\arctan(ax)}{x^4 (c + a^2cx^2)^2} \, dx$$
$$= \begin{cases} -\frac{28a^5x^5\log{(x)}}{12a^2c^2x^5+12c^2x^3} + \frac{14a^5x^5\log{\left(x^2+\frac{1}{a^2}\right)}}{12a^2c^2x^5+12c^2x^3} + \frac{15a^5x^5\operatorname{atan}^2{(ax)}}{12a^2c^2x^5+12c^2x^3} + \frac{30a^4x^4\operatorname{atan}{(ax)}}{12a^2c^2x^5+12c^2x^3} - \frac{28a^3x^3\log{(x)}}{12a^2c^2x^5+12c^2x^3} + \frac{14a^3x^3\log{\left(x^2+\frac{1}{a^2}\right.}}{12a^2c^2x^5+12c^2x^3} \\ 0 \end{cases}$$

input
```
integrate(atan(a*x)/x**4/(a**2*c*x**2+c)**2,x)
```

output
```
Piecewise((-28*a**5*x**5*log(x)/(12*a**2*c**2*x**5 + 12*c**2*x**3) + 14*a*
*5*x**5*log(x**2 + a**(-2))/(12*a**2*c**2*x**5 + 12*c**2*x**3) + 15*a**5*x
**5*atan(a*x)**2/(12*a**2*c**2*x**5 + 12*c**2*x**3) + 30*a**4*x**4*atan(a*
x)/(12*a**2*c**2*x**5 + 12*c**2*x**3) - 28*a**3*x**3*log(x)/(12*a**2*c**2*
x**5 + 12*c**2*x**3) + 14*a**3*x**3*log(x**2 + a**(-2))/(12*a**2*c**2*x**5
 + 12*c**2*x**3) + 15*a**3*x**3*atan(a*x)**2/(12*a**2*c**2*x**5 + 12*c**2*
x**3) + a**3*x**3/(12*a**2*c**2*x**5 + 12*c**2*x**3) + 20*a**2*x**2*atan(a
*x)/(12*a**2*c**2*x**5 + 12*c**2*x**3) - 2*a*x/(12*a**2*c**2*x**5 + 12*c**
2*x**3) - 4*atan(a*x)/(12*a**2*c**2*x**5 + 12*c**2*x**3), Ne(a, 0)), (0, T
rue))
```

## Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.18

$$\int \frac{\arctan(ax)}{x^4(c+a^2cx^2)^2}\,dx = \frac{1}{6}\left(\frac{15\,a^3\arctan(ax)}{c^2} + \frac{15\,a^4x^4 + 10\,a^2x^2 - 2}{a^2c^2x^5 + c^2x^3}\right)\arctan(ax) + \frac{\left(a^2x^2 - 15\,(a^4x^4 + a^2x^2)\arctan(ax)^2 + 14\,(a^4x^4 + a^2x^2)\log(a^2x^2 + 1) - 28\,(a^4x^4 + a^2x^2)\log(x) - 2\right.}{12\,(a^2c^2x^4 + c^2x^2)}$$

input
```
integrate(arctan(a*x)/x^4/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
1/6*(15*a^3*arctan(a*x)/c^2 + (15*a^4*x^4 + 10*a^2*x^2 - 2)/(a^2*c^2*x^5 +
 c^2*x^3))*arctan(a*x) + 1/12*(a^2*x^2 - 15*(a^4*x^4 + a^2*x^2)*arctan(a*x
)^2 + 14*(a^4*x^4 + a^2*x^2)*log(a^2*x^2 + 1) - 28*(a^4*x^4 + a^2*x^2)*log
(x) - 2)*a/(a^2*c^2*x^4 + c^2*x^2)
```

## Giac [F]

$$\int \frac{\arctan(ax)}{x^4(c+a^2cx^2)^2}\,dx = \int \frac{\arctan(ax)}{(a^2cx^2+c)^2x^4}\,dx$$

input
```
integrate(arctan(a*x)/x^4/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate(arctan(a*x)/((a^2*c*x^2 + c)^2*x^4), x)
```

## Mupad [B] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.90

$$\int \frac{\arctan(ax)}{x^4 (c+a^2cx^2)^2}\,dx = \frac{\operatorname{atan}(a\,x)\,\left(\frac{5\,x^2}{3\,c^2}-\frac{1}{3\,a^2\,c^2}+\frac{5\,a^2\,x^4}{2\,c^2}\right)}{x^5+\frac{x^3}{a^2}}-\frac{a-\frac{a^3\,x^2}{2}}{6\,a^2\,c^2\,x^4+6\,c^2\,x^2}+\frac{7\,a^3\,\ln\left(a^2\,x^2+1\right)}{6\,c^2}-\frac{7\,a^3\,\ln\left(x\right)}{3\,c^2}+\frac{5\,a^3\,{\operatorname{atan}(a\,x)}^2}{4\,c^2}$$

input
```
int(atan(a*x)/(x^4*(c + a^2*c*x^2)^2),x)
```

output
```
(atan(a*x)*((5*x^2)/(3*c^2) - 1/(3*a^2*c^2) + (5*a^2*x^4)/(2*c^2)))/(x^5 +
 x^3/a^2) - (a - (a^3*x^2)/2)/(6*c^2*x^2 + 6*a^2*c^2*x^4) + (7*a^3*log(a^2
*x^2 + 1))/(6*c^2) - (7*a^3*log(x))/(3*c^2) + (5*a^3*atan(a*x)^2)/(4*c^2)
```

## Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.07

$$\int \frac{\arctan(ax)}{x^4 (c+a^2cx^2)^2}\,dx = \frac{15 atan(ax)^2 a^5x^5+15 atan(ax)^2 a^3x^3+30 atan(ax)\, a^4x^4+20 atan(ax)\, a^2x^2-4 atan(ax)+14\log(a^2x^2}{12c^2x^3\,(a^2x^2+1)}$$

input
```
int(atan(a*x)/x^4/(a^2*c*x^2+c)^2,x)
```

output
```
(15*atan(a*x)**2*a**5*x**5 + 15*atan(a*x)**2*a**3*x**3 + 30*atan(a*x)*a**4
*x**4 + 20*atan(a*x)*a**2*x**2 - 4*atan(a*x) + 14*log(a**2*x**2 + 1)*a**5*
x**5 + 14*log(a**2*x**2 + 1)*a**3*x**3 - 28*log(x)*a**5*x**5 - 28*log(x)*a
**3*x**3 - a**5*x**5 - 2*a*x)/(12*c**2*x**3*(a**2*x**2 + 1))
```

# 3.192 $\int \frac{x^3 \arctan(ax)}{(c+a^2cx^2)^3} \, dx$



## Optimal result

Integrand size = 20, antiderivative size = 86

$$\int \frac{x^3 \arctan(ax)}{(c+a^2cx^2)^3} \, dx = \frac{x^3}{16ac^3(1+a^2x^2)^2} + \frac{3x}{32a^3c^3(1+a^2x^2)} - \frac{3\arctan(ax)}{32a^4c^3} + \frac{x^4\arctan(ax)}{4c^3(1+a^2x^2)^2}$$

output
```
1/16*x^3/a/c^3/(a^2*x^2+1)^2+3/32*x/a^3/c^3/(a^2*x^2+1)-3/32*arctan(a*x)/a
^4/c^3+1/4*x^4*arctan(a*x)/c^3/(a^2*x^2+1)^2
```

## Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.67

$$\int \frac{x^3 \arctan(ax)}{(c+a^2cx^2)^3} \, dx = \frac{ax(3+5a^2x^2) + (-3-6a^2x^2+5a^4x^4)\arctan(ax)}{32a^4c^3(1+a^2x^2)^2}$$

input
```
Integrate[(x^3*ArcTan[a*x])/(c + a^2*c*x^2)^3,x]
```

output
```
(a*x*(3 + 5*a^2*x^2) + (-3 - 6*a^2*x^2 + 5*a^4*x^4)*ArcTan[a*x])/(32*a^4*c
^3*(1 + a^2*x^2)^2)
```

## Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5479, 27, 252, 252, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \arctan(ax)}{\left(a^2cx^2+c\right)^3}dx$$

$$\Big\downarrow \text{5479}$$

$$\frac{x^4 \arctan(ax)}{4c^3\left(a^2x^2+1\right)^2}-\frac{1}{4}a\int \frac{x^4}{c^3\left(a^2x^2+1\right)^3}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{x^4 \arctan(ax)}{4c^3\left(a^2x^2+1\right)^2}-\frac{a\int \frac{x^4}{\left(a^2x^2+1\right)^3}dx}{4c^3}$$

$$\Big\downarrow \text{252}$$

$$\frac{x^4 \arctan(ax)}{4c^3\left(a^2x^2+1\right)^2}-\frac{a\left(\frac{3\int \frac{x^2}{\left(a^2x^2+1\right)^2}dx}{4a^2}-\frac{x^3}{4a^2\left(a^2x^2+1\right)^2}\right)}{4c^3}$$

$$\Big\downarrow \text{252}$$

$$\frac{x^4 \arctan(ax)}{4c^3\left(a^2x^2+1\right)^2}-\frac{a\left(\frac{3\left(\frac{\int \frac{1}{a^2x^2+1}dx}{2a^2}-\frac{x}{2a^2\left(a^2x^2+1\right)}\right)}{4a^2}-\frac{x^3}{4a^2\left(a^2x^2+1\right)^2}\right)}{4c^3}$$

$$\Big\downarrow \text{216}$$

$$\frac{x^4 \arctan(ax)}{4c^3 (a^2x^2+1)^2} - \frac{a\left(\frac{3\left(\frac{\arctan(ax)}{2a^3} - \frac{x}{2a^2(a^2x^2+1)}\right)}{4a^2} - \frac{x^3}{4a^2(a^2x^2+1)^2}\right)}{4c^3}$$

input
```
Int[(x^3*ArcTan[a*x])/(c + a^2*c*x^2)^3,x]
```

output
```
(x^4*ArcTan[a*x])/(4*c^3*(1 + a^2*x^2)^2) - (a*(-1/4*x^3/(a^2*(1 + a^2*x^2
)^2) + (3*(-1/2*x/(a^2*(1 + a^2*x^2)) + ArcTan[a*x]/(2*a^3)))/(4*a^2)))/(4
*c^3)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

## Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.73

| method | result |
|---|---|
| parallelrisch | $\frac{5x^4 \arctan(ax)a^4+5a^3x^3-6x^2a^2\arctan(ax)+3ax-3\arctan(ax)}{32c^3(a^2x^2+1)^2a^4}$ |
| derivativedivides | $\frac{-\frac{\arctan(ax)}{2c^3\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{4c^3\left(a^2x^2+1\right)^2}-\frac{-\frac{\frac{5}{8}a^3x^3+\frac{3}{8}ax}{\left(a^2x^2+1\right)^2}-\frac{5\arctan(ax)}{8}}{4c^3}}{a^4}$ |
| default | $\frac{-\frac{\arctan(ax)}{2c^3\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{4c^3\left(a^2x^2+1\right)^2}-\frac{-\frac{\frac{5}{8}a^3x^3+\frac{3}{8}ax}{\left(a^2x^2+1\right)^2}-\frac{5\arctan(ax)}{8}}{4c^3}}{a^4}$ |
| parts | $\frac{\arctan(ax)}{4c^3a^4(a^2x^2+1)^2}-\frac{\arctan(ax)}{2a^4c^3(a^2x^2+1)}-\frac{-\frac{\frac{5}{8}a^2x^3+\frac{3}{8}x}{\left(a^2x^2+1\right)^2}-\frac{5\arctan(ax)}{8a}}{4c^3a^3}$ |
| orering | $\frac{(a^2x^2+1)\left(5a^4x^4-3a^2x^2-3\right)\arctan(ax)}{8a^4(a^2cx^2+c)^3}+\frac{(5a^2x^2+3)\left(a^2x^2+1\right)^2\left(\frac{3x^2\arctan(ax)}{\left(a^2cx^2+c\right)^3}+\frac{x^3a}{\left(a^2x^2+1\right)\left(a^2cx^2+c\right)^3}-\frac{6x^4 \operatorname{arcta}}{\left(a^2c\right.}\right.}{32x^2a^4}$ |
| risch | $\frac{i(2a^2x^2+1)\ln(iax+1)}{8a^4c^3(a^2x^2+1)^2}-\frac{i(16a^2x^2\ln(-iax+1)+8\ln(-iax+1)-5\ln(ax+i)a^4x^4-10\ln(ax+i)a^2x^2-5\ln(ax+i)+5\ln}{64a^4(ax+i)^2(ax-i)^2c^3}$ |

input
```
int(x^3*arctan(a*x)/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/32*(5*x^4*arctan(a*x)*a^4+5*a^3*x^3-6*x^2*a^2*arctan(a*x)+3*a*x-3*arctan
(a*x))/c^3/(a^2*x^2+1)^2/a^4
```

## Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.80

$$\int \frac{x^3\arctan(ax)}{\left(c+a^2cx^2\right)^3}\,dx=\frac{5\,a^3x^3+3\,ax+\left(5\,a^4x^4-6\,a^2x^2-3\right)\arctan\left(ax\right)}{32\,\left(a^8c^3x^4+2\,a^6c^3x^2+a^4c^3\right)}$$

input
```
integrate(x^3*arctan(a*x)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
1/32*(5*a^3*x^3 + 3*a*x + (5*a^4*x^4 - 6*a^2*x^2 - 3)*arctan(a*x))/(a^8*c^
3*x^4 + 2*a^6*c^3*x^2 + a^4*c^3)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(78) = 156$.

Time = 0.63 (sec) , antiderivative size = 209, normalized size of antiderivative = 2.43

$$\int \frac{x^3 \arctan(ax)}{\left(c+a^2cx^2\right)^3} \, dx = \begin{cases} \frac{5a^4x^4 \operatorname{atan}(ax)}{32a^8c^3x^4+64a^6c^3x^2+32a^4c^3} + \frac{5a^3x^3}{32a^8c^3x^4+64a^6c^3x^2+32a^4c^3} - \frac{6a^2x^2 \operatorname{atan}(ax)}{32a^8c^3x^4+64a^6c^3x^2+32a^4c^3} + \frac{3ax}{32a^8c^3x^4+64a^6c^3x^2+32a^4c^3} - \frac{}{32a} \\ 0 \end{cases}$$

input
```
integrate(x**3*atan(a*x)/(a**2*c*x**2+c)**3,x)
```

output
```
Piecewise((5*a**4*x**4*atan(a*x)/(32*a**8*c**3*x**4 + 64*a**6*c**3*x**2 +
32*a**4*c**3) + 5*a**3*x**3/(32*a**8*c**3*x**4 + 64*a**6*c**3*x**2 + 32*a*
*4*c**3) - 6*a**2*x**2*atan(a*x)/(32*a**8*c**3*x**4 + 64*a**6*c**3*x**2 +
32*a**4*c**3) + 3*a*x/(32*a**8*c**3*x**4 + 64*a**6*c**3*x**2 + 32*a**4*c**
3) - 3*atan(a*x)/(32*a**8*c**3*x**4 + 64*a**6*c**3*x**2 + 32*a**4*c**3), N
e(a, 0)), (0, True))
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.26

$$\int \frac{x^3 \arctan(ax)}{\left(c+a^2cx^2\right)^3} \, dx = \frac{1}{32} \, a \left( \frac{5 \, a^2 x^3 + 3 \, x}{a^8 c^3 x^4 + 2 \, a^6 c^3 x^2 + a^4 c^3} + \frac{5 \, \arctan\left(ax\right)}{a^5 c^3} \right) - \frac{\left(2 \, a^2 x^2 + 1\right) \arctan\left(ax\right)}{4 \, \left(a^8 c^3 x^4 + 2 \, a^6 c^3 x^2 + a^4 c^3\right)}$$

input
```
integrate(x^3*arctan(a*x)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
1/32*a*((5*a^2*x^3 + 3*x)/(a^8*c^3*x^4 + 2*a^6*c^3*x^2 + a^4*c^3) + 5*arct
an(a*x)/(a^5*c^3)) - 1/4*(2*a^2*x^2 + 1)*arctan(a*x)/(a^8*c^3*x^4 + 2*a^6*
c^3*x^2 + a^4*c^3)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.90

$$\int \frac{x^3 \arctan(ax)}{(c+a^2cx^2)^3}\,dx = \frac{5\arctan(ax)}{32\,a^4c^3} + \frac{5\,a^2x^3+3\,x}{32\,(a^2x^2+1)^2a^3c^3} - \frac{(2\,a^2x^2+1)\arctan(ax)}{4\,(a^2x^2+1)^2a^4c^3}$$

input
```
integrate(x^3*arctan(a*x)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
5/32*arctan(a*x)/(a^4*c^3) + 1/32*(5*a^2*x^3 + 3*x)/((a^2*x^2 + 1)^2*a^3*c
^3) - 1/4*(2*a^2*x^2 + 1)*arctan(a*x)/((a^2*x^2 + 1)^2*a^4*c^3)
```

## Mupad [B] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.72

$$\int \frac{x^3 \arctan(ax)}{(c+a^2cx^2)^3}\,dx = \frac{3\,a\,x-3\,\mathrm{atan}(a\,x)+5\,a^3\,x^3-6\,a^2\,x^2\,\mathrm{atan}(a\,x)+5\,a^4\,x^4\,\mathrm{atan}(a\,x)}{32\,a^4\,c^3\,(a^2\,x^2+1)^2}$$

input
```
int((x^3*atan(a*x))/(c + a^2*c*x^2)^3,x)
```

output
```
(3*a*x - 3*atan(a*x) + 5*a^3*x^3 - 6*a^2*x^2*atan(a*x) + 5*a^4*x^4*atan(a*
x))/(32*a^4*c^3*(a^2*x^2 + 1)^2)
```

## Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.81

$$\int \frac{x^3 \arctan(ax)}{(c+a^2cx^2)^3}\,dx = \frac{5atan(ax)\,a^4x^4 - 6atan(ax)\,a^2x^2 - 3atan(ax) + 5a^3x^3 + 3ax}{32a^4c^3\,(a^4x^4 + 2a^2x^2 + 1)}$$

input
```
int(x^3*atan(a*x)/(a^2*c*x^2+c)^3,x)
```

output
```
(5*atan(a*x)*a**4*x**4 - 6*atan(a*x)*a**2*x**2 - 3*atan(a*x) + 5*a**3*x**3
 + 3*a*x)/(32*a**4*c**3*(a**4*x**4 + 2*a**2*x**2 + 1))
```

# 3.193 $\int \frac{x^2 \arctan(ax)}{(c+a^2cx^2)^3} dx$



## Optimal result

Integrand size = 20, antiderivative size = 111

$$\int \frac{x^2 \arctan(ax)}{(c+a^2cx^2)^3} dx = -\frac{1}{16a^3c^3(1+a^2x^2)^2} + \frac{1}{16a^3c^3(1+a^2x^2)} - \frac{x\arctan(ax)}{4a^2c^3(1+a^2x^2)^2} + \frac{x\arctan(ax)}{8a^2c^3(1+a^2x^2)} + \frac{\arctan(ax)^2}{16a^3c^3}$$

output
```
-1/16/a^3/c^3/(a^2*x^2+1)^2+1/16/a^3/c^3/(a^2*x^2+1)-1/4*x*arctan(a*x)/a^2
/c^3/(a^2*x^2+1)^2+1/8*x*arctan(a*x)/a^2/c^3/(a^2*x^2+1)+1/16*arctan(a*x)^
2/a^3/c^3
```

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.58

$$\int \frac{x^2 \arctan(ax)}{(c+a^2cx^2)^3} dx = \frac{a^2x^2 + 2ax(-1+a^2x^2)\arctan(ax) + (1+a^2x^2)^2\arctan(ax)^2}{16a^3c^3(1+a^2x^2)^2}$$

input
```
Integrate[(x^2*ArcTan[a*x])/(c + a^2*c*x^2)^3,x]
```

output
```
(a^2*x^2 + 2*a*x*(-1 + a^2*x^2)*ArcTan[a*x] + (1 + a^2*x^2)^2*ArcTan[a*x]^
2)/(16*a^3*c^3*(1 + a^2*x^2)^2)
```

## Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5469, 27, 5427, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \arctan(ax)}{(a^2cx^2+c)^3}dx$$

$$\Big\downarrow \text{5469}$$

$$\frac{\int \frac{\arctan(ax)}{c^2(a^2x^2+1)^2}dx}{4a^2c} - \frac{x\arctan(ax)}{4a^2c^3(a^2x^2+1)^2} - \frac{1}{16a^3c^3(a^2x^2+1)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\arctan(ax)}{(a^2x^2+1)^2}dx}{4a^2c^3} - \frac{x\arctan(ax)}{4a^2c^3(a^2x^2+1)^2} - \frac{1}{16a^3c^3(a^2x^2+1)^2}$$

$$\Big\downarrow \text{5427}$$

$$\frac{-\frac{1}{2}a\int \frac{x}{(a^2x^2+1)^2}dx + \frac{x\arctan(ax)}{2(a^2x^2+1)} + \frac{\arctan(ax)^2}{4a}}{4a^2c^3} - \frac{x\arctan(ax)}{4a^2c^3(a^2x^2+1)^2} - \frac{1}{16a^3c^3(a^2x^2+1)^2}$$

$$\Big\downarrow \text{241}$$

$$-\frac{x\arctan(ax)}{4a^2c^3(a^2x^2+1)^2} + \frac{\frac{x\arctan(ax)}{2(a^2x^2+1)} + \frac{1}{4a(a^2x^2+1)} + \frac{\arctan(ax)^2}{4a}}{4a^2c^3} - \frac{1}{16a^3c^3(a^2x^2+1)^2}$$

input
```
Int[(x^2*ArcTan[a*x])/(c + a^2*c*x^2)^3,x]
```

output
```
-1/16*1/(a^3*c^3*(1 + a^2*x^2)^2) - (x*ArcTan[a*x])/(4*a^2*c^3*(1 + a^2*x^
2)^2) + (1/(4*a*(1 + a^2*x^2)) + (x*ArcTan[a*x])/(2*(1 + a^2*x^2)) + ArcTa
n[a*x]^2/(4*a))/(4*a^2*c^3)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5469
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^2*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c^3*d*(q + 1)^2)), x] + (Simp
[x*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*c^2*d*(q + 1))), x] - Simp[1
/(2*c^2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x]) /
; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -5/2]
```

## Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.73

| method | result |
|---|---|
| parallelrisch | $\frac{a^{4} \arctan \left(a x \right)^{2} x^{4}+2 \arctan \left(a x \right) x^{3} a^{3}+2 \arctan \left(a x \right)^{2} x^{2} a^{2}+a^{2} x^{2}-2 \arctan \left(a x \right) a x +\arctan \left(a x \right)^{2}}{16 c^{3} \left(a^{2} x^{2}+1\right)^{2} a^{3}}$ |
| derivativedivides | $\frac{\frac{\arctan \left(a x \right) a^{3} x^{3}}{8 c^{3} \left(a^{2} x^{2}+1\right)^{2}}-\frac{a x \arctan \left(a x \right)}{8 c^{3} \left(a^{2} x^{2}+1\right)^{2}}+\frac{\arctan \left(a x \right)^{2}}{8 c^{3}}-\frac{\frac{\arctan \left(a x \right)^{2}}{2}+\frac{1}{2 \left(a^{2} x^{2}+1\right)^{2}}-\frac{1}{2 \left(a^{2} x^{2}+1\right)}}{8 c^{3}}}{a^{3}}$ |
| default | $\frac{\frac{\arctan \left(a x \right) a^{3} x^{3}}{8 c^{3} \left(a^{2} x^{2}+1\right)^{2}}-\frac{a x \arctan \left(a x \right)}{8 c^{3} \left(a^{2} x^{2}+1\right)^{2}}+\frac{\arctan \left(a x \right)^{2}}{8 c^{3}}-\frac{\frac{\arctan \left(a x \right)^{2}}{2}+\frac{1}{2 \left(a^{2} x^{2}+1\right)^{2}}-\frac{1}{2 \left(a^{2} x^{2}+1\right)}}{8 c^{3}}}{a^{3}}$ |
| parts | $\frac{\arctan \left(a x \right) x^{3}}{8 c^{3} \left(a^{2} x^{2}+1\right)^{2}}-\frac{x \arctan \left(a x \right)}{8 a^{2} c^{3} \left(a^{2} x^{2}+1\right)^{2}}+\frac{\arctan \left(a x \right)^{2}}{8 a^{3} c^{3}}-\frac{\frac{\arctan \left(a x \right)^{2}}{2 a^{3}}+\frac{\frac{1}{2 \left(a^{2} x^{2}+1\right)^{2}}-\frac{1}{2 \left(a^{2} x^{2}+1\right)}}{a^{3}}}{8 c^{3}}$ |
| risch | $-\frac{\ln \left(i a x +1\right)^{2}}{64 a^{3} c^{3}}+\frac{\left(x^{4} \ln \left(-i a x +1\right) a^{4}+2 a^{2} x^{2} \ln \left(-i a x +1\right)-2 i a^{3} x^{3}+\ln \left(-i a x +1\right)+2 i a x \right) \ln \left(i a x +1\right)}{32 a^{3} c^{3} \left(a^{2} x^{2}+1\right)^{2}}-\frac{a^{4} x^{4} \ln(-iax+$ |

input
```
int(x^2*arctan(a*x)/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/16*(a^4*arctan(a*x)^2*x^4+2*arctan(a*x)*x^3*a^3+2*arctan(a*x)^2*x^2*a^2+
a^2*x^2-2*arctan(a*x)*a*x+arctan(a*x)^2)/c^3/(a^2*x^2+1)^2/a^3
```

## Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.75

$$\int \frac{x^2 \arctan(ax)}{(c+a^2cx^2)^3} \, dx = \frac{a^{2} x^{2} + {\left(a^{4} x^{4} + 2 \, a^{2} x^{2} + 1\right)} \arctan \left(a x\right)^{2} + 2 \, {\left(a^{3} x^{3} - a x\right)} \arctan \left(a x\right)}{16 \, {\left(a^{7} c^{3} x^{4} + 2 \, a^{5} c^{3} x^{2} + a^{3} c^{3}\right)}}$$

input
```
integrate(x^2*arctan(a*x)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
1/16*(a^2*x^2 + (a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x)^2 + 2*(a^3*x^3 - a*x
)*arctan(a*x))/(a^7*c^3*x^4 + 2*a^5*c^3*x^2 + a^3*c^3)
```

## Sympy [F]

$$\int \frac{x^2 \arctan(ax)}{(c+a^2cx^2)^3} \, dx = \frac{\int \frac{x^2 \operatorname{atan}(ax)}{a^6x^6+3a^4x^4+3a^2x^2+1} \, dx}{c^3}$$

input
```
integrate(x**2*atan(a*x)/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(x**2*atan(a*x)/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x)/c*
*3
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.16

$$\int \frac{x^2 \arctan(ax)}{(c+a^2cx^2)^3} \, dx = \frac{1}{8} \left( \frac{a^2x^3 - x}{a^6c^3x^4 + 2\,a^4c^3x^2 + a^2c^3} + \frac{\arctan(ax)}{a^3c^3} \right) \arctan(ax) + \frac{\left(a^2x^2 - (a^4x^4 + 2\,a^2x^2 + 1)\arctan(ax)^2\right)a}{16\,(a^8c^3x^4 + 2\,a^6c^3x^2 + a^4c^3)}$$

input
```
integrate(x^2*arctan(a*x)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
1/8*((a^2*x^3 - x)/(a^6*c^3*x^4 + 2*a^4*c^3*x^2 + a^2*c^3) + arctan(a*x)/(
a^3*c^3))*arctan(a*x) + 1/16*(a^2*x^2 - (a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a
*x)^2)*a/(a^8*c^3*x^4 + 2*a^6*c^3*x^2 + a^4*c^3)
```

## Giac [F]

$$\int \frac{x^2 \arctan(ax)}{(c+a^2cx^2)^3} \, dx = \int \frac{x^2 \arctan(ax)}{(a^2cx^2+c)^3} \, dx$$

input
```
integrate(x^2*arctan(a*x)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
integrate(x^2*arctan(a*x)/(a^2*c*x^2 + c)^3, x)
```

## Mupad [B] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.72

$$\int \frac{x^2 \arctan(ax)}{(c+a^2cx^2)^3} dx = \frac{a^4 x^4 \operatorname{atan}(a x)^2 + 2 a^3 x^3 \operatorname{atan}(a x) + 2 a^2 x^2 \operatorname{atan}(a x)^2 + a^2 x^2 - 2 a x \operatorname{atan}(a x) + \operatorname{atan}(a x)^2}{16 a^3 c^3 (a^2 x^2 + 1)^2}$$

input
```
int((x^2*atan(a*x))/(c + a^2*c*x^2)^3,x)
```

output
```
(a^2*x^2 + atan(a*x)^2 + 2*a^3*x^3*atan(a*x) - 2*a*x*atan(a*x) + 2*a^2*x^2
*atan(a*x)^2 + a^4*x^4*atan(a*x)^2)/(16*a^3*c^3*(a^2*x^2 + 1)^2)
```

## Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.84

$$\int \frac{x^2 \arctan(ax)}{(c+a^2cx^2)^3} dx = \frac{2atan(ax)^2 a^4x^4 + 4atan(ax)^2 a^2x^2 + 2atan(ax)^2 + 4atan(ax) a^3x^3 - 4atan(ax) ax - a^4x^4 - 1}{32a^3c^3 (a^4x^4 + 2a^2x^2 + 1)}$$

input
```
int(x^2*atan(a*x)/(a^2*c*x^2+c)^3,x)
```

output
```
(2*atan(a*x)**2*a**4*x**4 + 4*atan(a*x)**2*a**2*x**2 + 2*atan(a*x)**2 + 4*
atan(a*x)*a**3*x**3 - 4*atan(a*x)*a*x - a**4*x**4 - 1)/(32*a**3*c**3*(a**4
*x**4 + 2*a**2*x**2 + 1))
```

# 3.194 $\int \frac{x\arctan(ax)}{(c+a^2cx^2)^3}\,dx$



## Optimal result

Integrand size = 18, antiderivative size = 84

$$\int \frac{x\arctan(ax)}{(c+a^2cx^2)^3}\,dx = \frac{x}{16ac^3(1+a^2x^2)^2} + \frac{3x}{32ac^3(1+a^2x^2)} + \frac{3\arctan(ax)}{32a^2c^3} - \frac{\arctan(ax)}{4a^2c^3(1+a^2x^2)^2}$$

output
```
1/16*x/a/c^3/(a^2*x^2+1)^2+3/32*x/a/c^3/(a^2*x^2+1)+3/32*arctan(a*x)/a^2/c
^3-1/4*arctan(a*x)/a^2/c^3/(a^2*x^2+1)^2
```

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.65

$$\int \frac{x\arctan(ax)}{(c+a^2cx^2)^3}\,dx = \frac{ax(5+3a^2x^2)+(-5+6a^2x^2+3a^4x^4)\arctan(ax)}{32c^3(a+a^3x^2)^2}$$

input
```
Integrate[(x*ArcTan[a*x])/(c + a^2*c*x^2)^3,x]
```

output
```
(a*x*(5 + 3*a^2*x^2) + (-5 + 6*a^2*x^2 + 3*a^4*x^4)*ArcTan[a*x])/(32*c^3*(
a + a^3*x^2)^2)
```

## Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5465, 215, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \arctan(ax)}{(a^2cx^2+c)^3} \, dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{\int \frac{1}{(a^2cx^2+c)^3}dx}{4a} - \frac{\arctan(ax)}{4a^2c^3(a^2x^2+1)^2}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{3\int \frac{1}{(a^2cx^2+c)^2}dx}{4c} + \frac{x}{4c^3(a^2x^2+1)^2}}{4a} - \frac{\arctan(ax)}{4a^2c^3(a^2x^2+1)^2}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{3\left(\frac{\int \frac{1}{a^2cx^2+c}dx}{2c} + \frac{x}{2c^2(a^2x^2+1)}\right)}{4c} + \frac{x}{4c^3(a^2x^2+1)^2}}{4a} - \frac{\arctan(ax)}{4a^2c^3(a^2x^2+1)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{3\left(\frac{x}{2c^2(a^2x^2+1)} + \frac{\arctan(ax)}{2ac^2}\right)}{4c} + \frac{x}{4c^3(a^2x^2+1)^2}}{4a} - \frac{\arctan(ax)}{4a^2c^3(a^2x^2+1)^2}$$

input
```
Int[(x*ArcTan[a*x])/(c + a^2*c*x^2)^3,x]
```

output
```
-1/4*ArcTan[a*x]/(a^2*c^3*(1 + a^2*x^2)^2) + (x/(4*c^3*(1 + a^2*x^2)^2) +
(3*(x/(2*c^2*(1 + a^2*x^2)) + ArcTan[a*x]/(2*a*c^2)))/(4*c))/(4*a)
```

**Defintions of rubi rules used**

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

## Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.75

| method | result |
|---|---|
| parallelrisch | $\frac{3x^4 \arctan(ax)a^4+3a^3x^3+6x^2a^2\arctan(ax)+5ax-5\arctan(ax)}{32c^3(a^2x^2+1)^2a^2}$ |
| derivativedivides | $\frac{-\frac{\arctan(ax)}{4c^3\left(a^2x^2+1\right)^2}+\frac{\frac{ax}{4\left(a^2x^2+1\right)^2}+\frac{3ax}{8\left(a^2x^2+1\right)}+\frac{3\arctan(ax)}{8}}{4c^3}}{a^2}$ |
| default | $\frac{-\frac{\arctan(ax)}{4c^3\left(a^2x^2+1\right)^2}+\frac{\frac{ax}{4\left(a^2x^2+1\right)^2}+\frac{3ax}{8\left(a^2x^2+1\right)}+\frac{3\arctan(ax)}{8}}{4c^3}}{a^2}$ |
| parts | $-\frac{\arctan(ax)}{4a^2c^3(a^2x^2+1)^2}+\frac{\frac{x}{4\left(a^2x^2+1\right)^2}+\frac{3x}{8\left(a^2x^2+1\right)}+\frac{3\arctan(ax)}{8a}}{4a\,c^3}$ |
| orering | $\frac{(9a^6x^6+23a^4x^4+9a^2x^2-5)\arctan(ax)}{16a^2(a^2c\,x^2+c)^3}+\frac{(3a^2x^2+5)\,(a^2x^2+1)^2\left(\frac{\arctan(ax)}{\left(a^2c\,x^2+c\right)^3}+\frac{xa}{\left(a^2x^2+1\right)\left(a^2c\,x^2+c\right)^3}-\frac{6x^2\arctan(}{\left(a^2c\,x^2\right.}\right.}{32a^2}$ |
| risch | $\frac{i\ln(iax+1)}{8a^2c^3(a^2x^2+1)^2}-\frac{i(8\ln(-iax+1)-3\ln(ax+i)a^4x^4-6\ln(ax+i)a^2x^2-3\ln(ax+i)+3\ln(-ax+i)a^4x^4+6\ln(-ax+i)}{64a^2(ax+i)^2(ax-i)^2c^3}$ |

input
```
int(x*arctan(a*x)/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/32*(3*x^4*arctan(a*x)*a^4+3*a^3*x^3+6*x^2*a^2*arctan(a*x)+5*a*x-5*arctan
(a*x))/c^3/(a^2*x^2+1)^2/a^2
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.82

$$\int \frac{x\arctan(ax)}{(c+a^2cx^2)^3}\,dx=\frac{3\,a^3x^3+5\,ax+(3\,a^4x^4+6\,a^2x^2-5)\arctan\left(ax\right)}{32\left(a^6c^3x^4+2\,a^4c^3x^2+a^2c^3\right)}$$

input
```
integrate(x*arctan(a*x)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
1/32*(3*a^3*x^3 + 5*a*x + (3*a^4*x^4 + 6*a^2*x^2 - 5)*arctan(a*x))/(a^6*c^
3*x^4 + 2*a^4*c^3*x^2 + a^2*c^3)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(75) = 150$.

Time = 0.60 (sec) , antiderivative size = 209, normalized size of antiderivative = 2.49

$$\int \frac{x\arctan(ax)}{(c+a^2cx^2)^3}\,dx = \begin{cases} \frac{3a^4x^4\operatorname{atan}(ax)}{32a^6c^3x^4+64a^4c^3x^2+32a^2c^3} + \frac{3a^3x^3}{32a^6c^3x^4+64a^4c^3x^2+32a^2c^3} + \frac{6a^2x^2\operatorname{atan}(ax)}{32a^6c^3x^4+64a^4c^3x^2+32a^2c^3} + \frac{5ax}{32a^6c^3x^4+64a^4c^3x^2+32a^2c^3} - \\ 0 \end{cases}$$

input
```
integrate(x*atan(a*x)/(a**2*c*x**2+c)**3,x)
```

output
```
Piecewise((3*a**4*x**4*atan(a*x)/(32*a**6*c**3*x**4 + 64*a**4*c**3*x**2 +
32*a**2*c**3) + 3*a**3*x**3/(32*a**6*c**3*x**4 + 64*a**4*c**3*x**2 + 32*a*
*2*c**3) + 6*a**2*x**2*atan(a*x)/(32*a**6*c**3*x**4 + 64*a**4*c**3*x**2 +
32*a**2*c**3) + 5*a*x/(32*a**6*c**3*x**4 + 64*a**4*c**3*x**2 + 32*a**2*c**
3) - 5*atan(a*x)/(32*a**6*c**3*x**4 + 64*a**4*c**3*x**2 + 32*a**2*c**3), N
e(a, 0)), (0, True))
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.02

$$\int \frac{x\arctan(ax)}{(c+a^2cx^2)^3}\,dx = \frac{\frac{3a^2x^3+5x}{a^4c^2x^4+2a^2c^2x^2+c^2} + \frac{3\arctan(ax)}{ac^2}}{32ac} - \frac{\arctan(ax)}{4(a^2cx^2+c)^2a^2c}$$

input
```
integrate(x*arctan(a*x)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
1/32*((3*a^2*x^3 + 5*x)/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2) + 3*arctan(a*x
)/(a*c^2))/(a*c) - 1/4*arctan(a*x)/((a^2*c*x^2 + c)^2*a^2*c)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.81

$$\int \frac{x \arctan(ax)}{(c+a^2cx^2)^3}\,dx = \frac{3\arctan(ax)}{32\,a^2c^3} - \frac{\arctan(ax)}{4\,(a^2cx^2+c)^2a^2c} + \frac{3\,a^2x^3+5\,x}{32\,(a^2x^2+1)^2ac^3}$$

input
```
integrate(x*arctan(a*x)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
3/32*arctan(a*x)/(a^2*c^3) - 1/4*arctan(a*x)/((a^2*c*x^2 + c)^2*a^2*c) + 1
/32*(3*a^2*x^3 + 5*x)/((a^2*x^2 + 1)^2*a*c^3)
```

## Mupad [B] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.23

$$\int \frac{x \arctan(ax)}{(c+a^2cx^2)^3}\,dx = \frac{\frac{5\,x}{32\,a}+\frac{a\,x^3}{4}-\frac{\mathrm{atan}(a\,x)}{4\,a^2}-\frac{x^2\,\mathrm{atan}(a\,x)}{4}+\frac{3\,a^3\,x^5}{32}}{a^6\,c^3\,x^6+3\,a^4\,c^3\,x^4+3\,a^2\,c^3\,x^2+c^3} + \frac{3\,\mathrm{atan}\left(\frac{a^2\,x}{\sqrt{a^2}}\right)}{32\,a\,c^3\,\sqrt{a^2}}$$

input
```
int((x*atan(a*x))/(c + a^2*c*x^2)^3,x)
```

output
```
((5*x)/(32*a) + (a*x^3)/4 - atan(a*x)/(4*a^2) - (x^2*atan(a*x))/4 + (3*a^3
*x^5)/32)/(c^3 + 3*a^2*c^3*x^2 + 3*a^4*c^3*x^4 + a^6*c^3*x^6) + (3*atan((a
^2*x)/(a^2)^(1/2)))/(32*a*c^3*(a^2)^(1/2))
```

## Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.83

$$\int \frac{x \arctan(ax)}{(c+a^2cx^2)^3}\,dx = \frac{3atan(ax)\,a^4x^4+6atan(ax)\,a^2x^2-5atan(ax)+3a^3x^3+5ax}{32a^2c^3\,(a^4x^4+2a^2x^2+1)}$$

input
```
int(x*atan(a*x)/(a^2*c*x^2+c)^3,x)
```

output
```
(3*atan(a*x)*a**4*x**4 + 6*atan(a*x)*a**2*x**2 - 5*atan(a*x) + 3*a**3*x**3
 + 5*a*x)/(32*a**2*c**3*(a**4*x**4 + 2*a**2*x**2 + 1))
```

# 3.195 $\int \frac{\arctan(ax)}{(c+a^2cx^2)^3}\,dx$



## Optimal result

Integrand size = 17, antiderivative size = 105

$$\int \frac{\arctan(ax)}{(c+a^2cx^2)^3}\,dx = \frac{1}{16ac^3(1+a^2x^2)^2}+\frac{3}{16ac^3(1+a^2x^2)}+\frac{x\arctan(ax)}{4c^3(1+a^2x^2)^2}+\frac{3x\arctan(ax)}{8c^3(1+a^2x^2)}+\frac{3\arctan(ax)^2}{16ac^3}$$

output
```
1/16/a/c^3/(a^2*x^2+1)^2+3/16/a/c^3/(a^2*x^2+1)+1/4*x*arctan(a*x)/c^3/(a^2
*x^2+1)^2+3/8*x*arctan(a*x)/c^3/(a^2*x^2+1)+3/16*arctan(a*x)^2/a/c^3
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.65

$$\int \frac{\arctan(ax)}{(c+a^2cx^2)^3}\,dx = \frac{4+3a^2x^2+2ax(5+3a^2x^2)\arctan(ax)+3(1+a^2x^2)^2\arctan(ax)^2}{16ac^3(1+a^2x^2)^2}$$

input
```
Integrate[ArcTan[a*x]/(c + a^2*c*x^2)^3,x]
```

output
```
(4 + 3*a^2*x^2 + 2*a*x*(5 + 3*a^2*x^2)*ArcTan[a*x] + 3*(1 + a^2*x^2)^2*Arc
Tan[a*x]^2)/(16*a*c^3*(1 + a^2*x^2)^2)
```

## Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {5431, 27, 5427, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)}{(a^2cx^2+c)^3}\,dx$$

$$\Big\downarrow \text{5431}$$

$$\frac{3\int \frac{\arctan(ax)}{c^2(a^2x^2+1)^2}dx}{4c}+\frac{x\arctan(ax)}{4c^3(a^2x^2+1)^2}+\frac{1}{16ac^3(a^2x^2+1)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\int \frac{\arctan(ax)}{(a^2x^2+1)^2}dx}{4c^3}+\frac{x\arctan(ax)}{4c^3(a^2x^2+1)^2}+\frac{1}{16ac^3(a^2x^2+1)^2}$$

$$\Big\downarrow \text{5427}$$

$$\frac{3\left(-\frac{1}{2}a\int \frac{x}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)}{4c^3}+\frac{x\arctan(ax)}{4c^3(a^2x^2+1)^2}+\frac{1}{16ac^3(a^2x^2+1)^2}$$

$$\Big\downarrow \text{241}$$

$$\frac{x\arctan(ax)}{4c^3(a^2x^2+1)^2}+\frac{3\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)}{4c^3}+\frac{1}{16ac^3(a^2x^2+1)^2}$$

input
```
Int[ArcTan[a*x]/(c + a^2*c*x^2)^3,x]
```

output
```
1/(16*a*c^3*(1 + a^2*x^2)^2) + (x*ArcTan[a*x])/(4*c^3*(1 + a^2*x^2)^2) + (
3*(1/(4*a*(1 + a^2*x^2)) + (x*ArcTan[a*x])/(2*(1 + a^2*x^2)) + ArcTan[a*x]
^2/(4*a)))/(4*c^3)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5431
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbol
] :> Simp[b*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (-Simp[x*(d + e*x
^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*d*(q + 1))), x] + Simp[(2*q + 3)/(2*d*(
q + 1))   Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a,
 b, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -3/2]
```

## Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.89

| method | result |
|---|---|
| parallelrisch | $\frac{3a^4\arctan(ax)^2x^4-4a^4x^4+6\arctan(ax)x^3a^3+6\arctan(ax)^2x^2a^2-5a^2x^2+10\arctan(ax)ax+3\arctan(ax)^2}{16c^3(a^2x^2+1)^2a}$ |
| derivativedivides | $\frac{\frac{ax\arctan(ax)}{4c^3(a^2x^2+1)^2}+\frac{3ax\arctan(ax)}{8c^3(a^2x^2+1)}+\frac{3\arctan(ax)^2}{8c^3}-\frac{-\frac{3}{2(a^2x^2+1)}-\frac{1}{2(a^2x^2+1)^2}+\frac{3\arctan(ax)^2}{2}}{8c^3}}{a}$ |
| default | $\frac{\frac{ax\arctan(ax)}{4c^3(a^2x^2+1)^2}+\frac{3ax\arctan(ax)}{8c^3(a^2x^2+1)}+\frac{3\arctan(ax)^2}{8c^3}-\frac{-\frac{3}{2(a^2x^2+1)}-\frac{1}{2(a^2x^2+1)^2}+\frac{3\arctan(ax)^2}{2}}{8c^3}}{a}$ |
| parts | $\frac{x\arctan(ax)}{4c^3(a^2x^2+1)^2}+\frac{3x\arctan(ax)}{8c^3(a^2x^2+1)}+\frac{3\arctan(ax)^2}{8a\,c^3}-\frac{\frac{-\frac{3}{2(a^2x^2+1)}-\frac{1}{2(a^2x^2+1)^2}}{a}+\frac{3\arctan(ax)^2}{2a}}{8c^3}$ |
| risch | $-\frac{3\ln(iax+1)^2}{64a\,c^3}+\frac{(3x^4\ln(-iax+1)a^4+6a^2x^2\ln(-iax+1)-6ia^3x^3+3\ln(-iax+1)-10iax)\ln(iax+1)}{32a(a^2x^2+1)^2c^3}-\frac{3a^4x^4\ln(}{}$ |

input
```
int(arctan(a*x)/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/16*(3*a^4*arctan(a*x)^2*x^4-4*a^4*x^4+6*arctan(a*x)*x^3*a^3+6*arctan(a*x
)^2*x^2*a^2-5*a^2*x^2+10*arctan(a*x)*a*x+3*arctan(a*x)^2)/c^3/(a^2*x^2+1)^
2/a
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.81

$$\int \frac{\arctan(ax)}{(c+a^2cx^2)^3}\,dx = \frac{3\,a^2x^2+3\,(a^4x^4+2\,a^2x^2+1)\arctan(ax)^2+2\,(3\,a^3x^3+5\,ax)\arctan(ax)+4}{16\,(a^5c^3x^4+2\,a^3c^3x^2+ac^3)}$$

input
```
integrate(arctan(a*x)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
1/16*(3*a^2*x^2 + 3*(a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x)^2 + 2*(3*a^3*x^3
 + 5*a*x)*arctan(a*x) + 4)/(a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)
```

## Sympy [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)}{(c+a^2cx^2)^3}\,dx = \text{Exception raised: RecursionError}$$

input
```
integrate(atan(a*x)/(a**2*c*x**2+c)**3,x)
```

output
```
Exception raised: RecursionError >> maximum recursion depth exceeded in co
mparison
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.23

$$\int \frac{\arctan(ax)}{(c+a^2cx^2)^3}\,dx = \frac{1}{8}\left(\frac{3\,a^2x^3+5\,x}{a^4c^3x^4+2\,a^2c^3x^2+c^3}+\frac{3\,\arctan\left(ax\right)}{ac^3}\right)\arctan\left(ax\right) + \frac{\left(3\,a^2x^2-3\,\left(a^4x^4+2\,a^2x^2+1\right)\arctan\left(ax\right)^2+4\right)a}{16\,\left(a^6c^3x^4+2\,a^4c^3x^2+a^2c^3\right)}$$

input
```
integrate(arctan(a*x)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
1/8*((3*a^2*x^3 + 5*x)/(a^4*c^3*x^4 + 2*a^2*c^3*x^2 + c^3) + 3*arctan(a*x)
/(a*c^3))*arctan(a*x) + 1/16*(3*a^2*x^2 - 3*(a^4*x^4 + 2*a^2*x^2 + 1)*arct
an(a*x)^2 + 4)*a/(a^6*c^3*x^4 + 2*a^4*c^3*x^2 + a^2*c^3)
```

## Giac [F]

$$\int \frac{\arctan(ax)}{(c+a^2cx^2)^3}\,dx = \int \frac{\arctan(ax)}{(a^2cx^2+c)^3}\,dx$$

input
```
integrate(arctan(a*x)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
integrate(arctan(a*x)/(a^2*c*x^2 + c)^3, x)
```

## Mupad [B] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.81

$$\int \frac{\arctan(ax)}{(c+a^2cx^2)^3}\,dx = \frac{3\,a^4\,x^4\,\mathrm{atan}(a\,x)^2+6\,a^3\,x^3\,\mathrm{atan}(a\,x)+6\,a^2\,x^2\,\mathrm{atan}(a\,x)^2+3\,a^2\,x^2+10\,a\,x\,\mathrm{atan}(a\,x)+3\,\mathrm{atan}(a\,x)^2+4}{16\,a\,c^3\,(a^2\,x^2+1)^2}$$

input
```
int(atan(a*x)/(c + a^2*c*x^2)^3,x)
```

output
```
(3*a^2*x^2 + 3*atan(a*x)^2 + 6*a^3*x^3*atan(a*x) + 10*a*x*atan(a*x) + 6*a^
2*x^2*atan(a*x)^2 + 3*a^4*x^4*atan(a*x)^2 + 4)/(16*a*c^3*(a^2*x^2 + 1)^2)
```

## Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.89

$$\int \frac{\arctan(ax)}{(c+a^2cx^2)^3}\,dx = \frac{6atan(ax)^2\,a^4x^4+12atan(ax)^2\,a^2x^2+6atan(ax)^2+12atan(ax)\,a^3x^3+20atan(ax)\,ax-3a^4x^4+5}{32a\,c^3\,(a^4x^4+2a^2x^2+1)}$$

input
```
int(atan(a*x)/(a^2*c*x^2+c)^3,x)
```

output
```
(6*atan(a*x)**2*a**4*x**4 + 12*atan(a*x)**2*a**2*x**2 + 6*atan(a*x)**2 + 1
2*atan(a*x)*a**3*x**3 + 20*atan(a*x)*a*x - 3*a**4*x**4 + 5)/(32*a*c**3*(a*
*4*x**4 + 2*a**2*x**2 + 1))
```

# 3.196 $\int \frac{\arctan(ax)}{x(c+a^2cx^2)^3}\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 159

$$\begin{aligned}\int \frac{\arctan(ax)}{x\left(c+a^2cx^2\right)^3}\,dx = & -\frac{ax}{16c^3\left(1+a^2x^2\right)^2}-\frac{11ax}{32c^3\left(1+a^2x^2\right)}-\frac{11\arctan(ax)}{32c^3} \\ & +\frac{\arctan(ax)}{4c^3\left(1+a^2x^2\right)^2}+\frac{\arctan(ax)}{2c^3\left(1+a^2x^2\right)}-\frac{i\arctan(ax)^2}{2c^3} \\ & +\frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{c^3}-\frac{i\,\mathrm{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{2c^3}\end{aligned}$$

output
```
-1/16*a*x/c^3/(a^2*x^2+1)^2-11/32*a*x/c^3/(a^2*x^2+1)-11/32*arctan(a*x)/c^
3+1/4*arctan(a*x)/c^3/(a^2*x^2+1)^2+1/2*arctan(a*x)/c^3/(a^2*x^2+1)-1/2*I*
arctan(a*x)^2/c^3+arctan(a*x)*ln(2-2/(1-I*a*x))/c^3-1/2*I*polylog(2,-1+2/(
1-I*a*x))/c^3
```

## Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.57

$$\int \frac{\arctan(ax)}{x\left(c+a^2cx^2\right)^3}\,dx = -\frac{64i\arctan(ax)^2-4\arctan(ax)\left(12\cos(2\arctan(ax))+\cos(4\arctan(ax))+32\log\left(1-e^{2i\arctan(ax)}\right)\right)}{128c^3}$$

input
```
Integrate[ArcTan[a*x]/(x*(c + a^2*c*x^2)^3),x]
```

output
```
-1/128*((64*I)*ArcTan[a*x]^2 - 4*ArcTan[a*x]*(12*Cos[2*ArcTan[a*x]] + Cos[
4*ArcTan[a*x]] + 32*Log[1 - E^((2*I)*ArcTan[a*x])]) + (64*I)*PolyLog[2, E^
((2*I)*ArcTan[a*x])] + 24*Sin[2*ArcTan[a*x]] + Sin[4*ArcTan[a*x]])/c^3
```

## Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.36, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.650$, Rules used = {5501, 27, 5465, 215, 215, 216, 5501, 5459, 5403, 2897, 5465, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)}{x\left(a^2cx^2+c\right)^3}dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{\arctan(ax)}{c^2x\left(a^2x^2+1\right)^2}dx}{c}-a^2\int \frac{x\arctan(ax)}{c^3\left(a^2x^2+1\right)^3}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\arctan(ax)}{x\left(a^2x^2+1\right)^2}dx}{c^3}-\frac{a^2\int \frac{x\arctan(ax)}{\left(a^2x^2+1\right)^3}dx}{c^3}$$

$$\Big\downarrow \text{5465}$$

$$\frac{\int \frac{\arctan(ax)}{x(a^2x^2+1)^2}dx}{c^3}-\frac{a^2\left(\frac{\int \frac{1}{\left(a^2x^2+1\right)^3}dx}{4a}-\frac{\arctan(ax)}{4a^2(a^2x^2+1)^2}\right)}{c^3}$$

$$\Big\downarrow \; 215$$

$$\frac{\int \frac{\arctan(ax)}{x(a^2x^2+1)^2}dx}{c^3}-\frac{a^2\left(\frac{\frac{3}{4}\int \frac{1}{\left(a^2x^2+1\right)^2}dx+\frac{x}{4\left(a^2x^2+1\right)^2}}{4a}-\frac{\arctan(ax)}{4a^2(a^2x^2+1)^2}\right)}{c^3}$$

$$\Big\downarrow \; 215$$

$$\frac{\int \frac{\arctan(ax)}{x(a^2x^2+1)^2}dx}{c^3}-\frac{a^2\left(\frac{\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{a^2x^2+1}dx+\frac{x}{2\left(a^2x^2+1\right)}\right)+\frac{x}{4\left(a^2x^2+1\right)^2}}{4a}-\frac{\arctan(ax)}{4a^2(a^2x^2+1)^2}\right)}{c^3}$$

$$\Big\downarrow \; 216$$

$$\frac{\int \frac{\arctan(ax)}{x(a^2x^2+1)^2}dx}{c^3}-\frac{a^2\left(\frac{\frac{3}{4}\left(\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}\right)+\frac{x}{4\left(a^2x^2+1\right)^2}}{4a}-\frac{\arctan(ax)}{4a^2(a^2x^2+1)^2}\right)}{c^3}$$

$$\Big\downarrow \; 5501$$

$$\frac{\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx-a^2\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx}{c^3}-\frac{a^2\left(\frac{\frac{3}{4}\left(\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}\right)+\frac{x}{4\left(a^2x^2+1\right)^2}}{4a}-\frac{\arctan(ax)}{4a^2(a^2x^2+1)^2}\right)}{c^3}$$

$$\Big\downarrow \; 5459$$

$$-\frac{a^2\left(\frac{\frac{3}{4}\left(\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}\right)+\frac{x}{4\left(a^2x^2+1\right)^2}}{4a}-\frac{\arctan(ax)}{4a^2(a^2x^2+1)^2}\right)}{c^3}+\frac{a^2\left(-\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx\right)+i\int \frac{\arctan(ax)}{x(ax+i)}dx-\frac{1}{2}i\arctan(ax)^2}{c^3}$$

$$\Big\downarrow \; 5403$$

$$-\frac{a^2\left(\frac{\frac{3}{4}\left(\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}\right)+\frac{x}{4\left(a^2x^2+1\right)^2}}{4a}-\frac{\arctan(ax)}{4a^2\left(a^2x^2+1\right)^2}\right)}{c^3}+\frac{a^2\left(-\int\frac{x\arctan(ax)}{\left(a^2x^2+1\right)^2}dx\right)+i\left(ia\int\frac{\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{2}i\arctan(ax)^2}{c^3}$$

$$\Big\downarrow \ 2897$$

$$-\frac{a^2\left(\frac{\frac{3}{4}\left(\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}\right)+\frac{x}{4\left(a^2x^2+1\right)^2}}{4a}-\frac{\arctan(ax)}{4a^2\left(a^2x^2+1\right)^2}\right)}{c^3}+\frac{a^2\left(-\int\frac{x\arctan(ax)}{\left(a^2x^2+1\right)^2}dx\right)+i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2}{c^3}$$

$$\Big\downarrow \ 5465$$

$$-\frac{a^2\left(\frac{\frac{3}{4}\left(\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}\right)+\frac{x}{4\left(a^2x^2+1\right)^2}}{4a}-\frac{\arctan(ax)}{4a^2\left(a^2x^2+1\right)^2}\right)}{c^3}+\frac{-\left(a^2\left(\frac{\int\frac{1}{\left(a^2x^2+1\right)^2}dx}{2a}-\frac{\arctan(ax)}{2a^2\left(a^2x^2+1\right)}\right)\right)+i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\,\text{arc}}{c^3}$$

$$\Big\downarrow \ 215$$

$$-\frac{a^2\left(\frac{\frac{3}{4}\left(\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}\right)+\frac{x}{4\left(a^2x^2+1\right)^2}}{4a}-\frac{\arctan(ax)}{4a^2\left(a^2x^2+1\right)^2}\right)}{c^3}+\frac{-\left(a^2\left(\frac{\frac{1}{2}\int\frac{1}{a^2x^2+1}dx+\frac{x}{2\left(a^2x^2+1\right)}}{2a}-\frac{\arctan(ax)}{2a^2\left(a^2x^2+1\right)}\right)\right)+i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)}{c^3}$$

$$\Big\downarrow \ 216$$

$$-\frac{a^2\left(\frac{\frac{3}{4}\left(\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}\right)+\frac{x}{4\left(a^2x^2+1\right)^2}}{4a}-\frac{\arctan(ax)}{4a^2(a^2x^2+1)^2}\right)}{c^3}+$$

$$\frac{-\left(a^2\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)\right)+i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-}{c^3}$$

input
```
Int[ArcTan[a*x]/(x*(c + a^2*c*x^2)^3),x]
```

output
```
-((a^2*(-1/4*ArcTan[a*x]/(a^2*(1 + a^2*x^2)^2) + (x/(4*(1 + a^2*x^2)^2) +
(3*(x/(2*(1 + a^2*x^2)) + ArcTan[a*x]/(2*a)))/4)/(4*a)))/c^3) + ((-1/2*I)*
ArcTan[a*x]^2 - a^2*(-1/2*ArcTan[a*x]/(a^2*(1 + a^2*x^2)) + (x/(2*(1 + a^2
*x^2)) + ArcTan[a*x]/(2*a))/(2*a)) + I*((-I)*ArcTan[a*x]*Log[2 - 2/(1 - I*
a*x)] - PolyLog[2, -1 + 2/(1 - I*a*x)]/2))/c^3
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

## Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.76 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.61

| method | result |
|---|---|
| parts | $\frac{\arctan(ax)}{4c^3(a^2x^2+1)^2}+\frac{\arctan(ax)}{2c^3(a^2x^2+1)}-\frac{\arctan(ax)\ln(a^2x^2+1)}{2c^3}+\frac{\arctan(ax)\ln(x)}{c^3}-\frac{a\left(-\frac{i\ln(x)(-\ln(-iax+1)+\ln(iax}{a}\right.}{}$ |
| derivativedivides | $-\frac{\arctan(ax)\ln(a^2x^2+1)}{2c^3}+\frac{\arctan(ax)}{4c^3(a^2x^2+1)^2}+\frac{\arctan(ax)}{2c^3(a^2x^2+1)}+\frac{\arctan(ax)\ln(ax)}{c^3}-\frac{\frac{\frac{11}{8}a^3x^3+\frac{13}{8}ax}{(a^2x^2+1)^2}+\frac{11\arctan(ax}{8}}{}$ |
| default | $-\frac{\arctan(ax)\ln(a^2x^2+1)}{2c^3}+\frac{\arctan(ax)}{4c^3(a^2x^2+1)^2}+\frac{\arctan(ax)}{2c^3(a^2x^2+1)}+\frac{\arctan(ax)\ln(ax)}{c^3}-\frac{\frac{\frac{11}{8}a^3x^3+\frac{13}{8}ax}{(a^2x^2+1)^2}+\frac{11\arctan(ax}{8}}{}$ |
| risch | $\frac{\ln(iax+1)ax}{64c^3(iax-1)^2}-\frac{5\ln(iax+1)ax}{64c^3(iax-1)}+\frac{\ln(-iax+1)ax}{64c^3(-iax-1)^2}-\frac{5\ln(-iax+1)ax}{64c^3(-iax-1)}+\frac{i\ln(-iax+1)a^2x^2}{128c^3(-iax-1)^2}-\frac{i\ln(iax+1)a^2x}{128c^3(iax-1)^2}$ |

input
```
int(arctan(a*x)/x/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*arctan(a*x)/c^3/(a^2*x^2+1)^2+1/2*arctan(a*x)/c^3/(a^2*x^2+1)-1/2/c^3*
arctan(a*x)*ln(a^2*x^2+1)+1/c^3*arctan(a*x)*ln(x)-1/2/c^3*a*(-I*ln(x)*(-ln
(1-I*a*x)+ln(1+I*a*x))/a-I*(dilog(1+I*a*x)-dilog(1-I*a*x))/a-1/4/a^2*sum(1
/_alpha*(2*ln(x-_alpha)*ln(a^2*x^2+1)-a^2*(1/a^2/_alpha*ln(x-_alpha)^2+2*_
alpha*ln(x-_alpha)*ln(1/2*(x+_alpha)/_alpha)+2*_alpha*dilog(1/2*(x+_alpha)
/_alpha))),_alpha=RootOf(_Z^2*a^2+1))+1/2*(11/8*a^2*x^3+13/8*x)/(a^2*x^2+1
)^2+11/16/a*arctan(a*x))
```

## Fricas [F]

$$\int \frac{\arctan(ax)}{x\left(c+a^2cx^2\right)^3}\,dx=\int \frac{\arctan\left(ax\right)}{\left(a^2cx^2+c\right)^3x}\,dx$$

input
```
integrate(arctan(a*x)/x/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
integral(arctan(a*x)/(a^6*c^3*x^7 + 3*a^4*c^3*x^5 + 3*a^2*c^3*x^3 + c^3*x)
, x)
```

## Sympy [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)}{x\left(c+a^2cx^2\right)^3}\,dx = \text{Exception raised: RecursionError}$$

input
```
integrate(atan(a*x)/x/(a**2*c*x**2+c)**3,x)
```

output
```
Exception raised: RecursionError >> maximum recursion depth exceeded
```

## Maxima [F]

$$\int \frac{\arctan(ax)}{x\left(c+a^2cx^2\right)^3}\,dx = \int \frac{\arctan\left(ax\right)}{\left(a^2cx^2+c\right)^3x}\,dx$$

input
```
integrate(arctan(a*x)/x/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)/((a^2*c*x^2 + c)^3*x), x)
```

## Giac [F]

$$\int \frac{\arctan(ax)}{x\left(c+a^2cx^2\right)^3}\,dx = \int \frac{\arctan\left(ax\right)}{\left(a^2cx^2+c\right)^3x}\,dx$$

input
```
integrate(arctan(a*x)/x/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
integrate(arctan(a*x)/((a^2*c*x^2 + c)^3*x), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)}{x\left(c+a^2cx^2\right)^3}\,dx=\int \frac{\mathrm{atan}(a\,x)}{x\,\left(c\,a^2\,x^2+c\right)^3}\,dx$$

input
```
int(atan(a*x)/(x*(c + a^2*c*x^2)^3),x)
```

output
```
int(atan(a*x)/(x*(c + a^2*c*x^2)^3), x)
```

## Reduce [F]

$$\int \frac{\arctan(ax)}{x\left(c+a^2cx^2\right)^3}\,dx=\frac{\int \frac{atan(ax)}{a^6x^7+3a^4x^5+3a^2x^3+x}dx}{c^3}$$

input
```
int(atan(a*x)/x/(a^2*c*x^2+c)^3,x)
```

output
```
int(atan(a*x)/(a**6*x**7 + 3*a**4*x**5 + 3*a**2*x**3 + x),x)/c**3
```

# 3.197 $\int \frac{\arctan(ax)}{x^2(c+a^2cx^2)^3}\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 142

$$\int \frac{\arctan(ax)}{x^2(c+a^2cx^2)^3}\,dx = -\frac{a}{16c^3(1+a^2x^2)^2} - \frac{7a}{16c^3(1+a^2x^2)} - \frac{\arctan(ax)}{c^3x} - \frac{a^2x\arctan(ax)}{4c^3(1+a^2x^2)^2} - \frac{7a^2x\arctan(ax)}{8c^3(1+a^2x^2)} - \frac{15a\arctan(ax)^2}{16c^3} + \frac{a\log(x)}{c^3} - \frac{a\log(1+a^2x^2)}{2c^3}$$

output

```
-1/16*a/c^3/(a^2*x^2+1)^2-7/16*a/c^3/(a^2*x^2+1)-arctan(a*x)/c^3/x-1/4*a^2
*x*arctan(a*x)/c^3/(a^2*x^2+1)^2-7/8*a^2*x*arctan(a*x)/c^3/(a^2*x^2+1)-15/
16*a*arctan(a*x)^2/c^3+a*ln(x)/c^3-1/2*a*ln(a^2*x^2+1)/c^3
```

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.83

$$\int \frac{\arctan(ax)}{x^2(c+a^2cx^2)^3}\,dx = \frac{-2(8+25a^2x^2+15a^4x^4)\arctan(ax) - 15ax(1+a^2x^2)^2\arctan(ax)^2 + ax\Big(-8-7a^2x^2+16(1+a^2x^2)^2}{16c^3x(1+a^2x^2)^2}$$

input
```
Integrate[ArcTan[a*x]/(x^2*(c + a^2*c*x^2)^3),x]
```

output
```
(-2*(8 + 25*a^2*x^2 + 15*a^4*x^4)*ArcTan[a*x] - 15*a*x*(1 + a^2*x^2)^2*Arc
Tan[a*x]^2 + a*x*(-8 - 7*a^2*x^2 + 16*(1 + a^2*x^2)^2*Log[x] - 8*(1 + a^2*
x^2)^2*Log[1 + a^2*x^2]))/(16*c^3*x*(1 + a^2*x^2)^2)
```

## Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.46, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {5501, 27, 5431, 5427, 241, 5501, 5427, 241, 5453, 5361, 243, 47, 14, 16, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)}{x^2 (a^2cx^2+c)^3} \, dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{\arctan(ax)}{c^2x^2(a^2x^2+1)^2}dx}{c} - a^2 \int \frac{\arctan(ax)}{c^3 (a^2x^2+1)^3}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx}{c^3} - \frac{a^2 \int \frac{\arctan(ax)}{(a^2x^2+1)^3}dx}{c^3}$$

$$\Big\downarrow \text{5431}$$

$$\frac{\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx}{c^3} - \frac{a^2 \left(\frac{3}{4}\int \frac{\arctan(ax)}{(a^2x^2+1)^2}dx + \frac{x\arctan(ax)}{4(a^2x^2+1)^2} + \frac{1}{16a(a^2x^2+1)^2}\right)}{c^3}$$

$$\Big\downarrow \text{5427}$$

$$\frac{\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx}{c^3} - \frac{a^2 \left(\frac{3}{4}\left(-\frac{1}{2}a\int \frac{x}{(a^2x^2+1)^2}dx + \frac{x\arctan(ax)}{2(a^2x^2+1)} + \frac{\arctan(ax)^2}{4a}\right) + \frac{x\arctan(ax)}{4(a^2x^2+1)^2} + \frac{1}{16a(a^2x^2+1)^2}\right)}{c^3}$$

$$\Big\downarrow \text{241}$$

$$\frac{\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx}{c^3}-\frac{a^2\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}\right)}{c^3}$$

$$\Big\downarrow \ 5501$$

$$\frac{\int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx-a^2\int \frac{\arctan(ax)}{(a^2x^2+1)^2}dx}{c^3}-\frac{a^2\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}\right)}{c^3}$$

$$\Big\downarrow \ 5427$$

$$\frac{\int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx-a^2\left(-\frac{1}{2}a\int \frac{x}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)}{c^3}-\frac{a^2\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}\right)}{c^3}$$

$$\Big\downarrow \ 241$$

$$\frac{\int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx-a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)}{c^3}-\frac{a^2\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}\right)}{c^3}$$

$$\Big\downarrow \ 5453$$

$$\frac{-a^2\int \frac{\arctan(ax)}{a^2x^2+1}dx+\int \frac{\arctan(ax)}{x^2}dx-\left(a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)\right)}{c^3}-\frac{a^2\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}\right)}{c^3}$$

$$\Big\downarrow \ 5361$$

$$\frac{-a^2\int \frac{\arctan(ax)}{a^2x^2+1}dx+a\int \frac{1}{x(a^2x^2+1)}dx-\left(a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)\right)-\frac{\arctan(ax)}{x}}{c^3}-\frac{a^2\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}\right)}{c^3}$$

$$\Big\downarrow \ 243$$

$$\frac{-a^2\int \frac{\arctan(ax)}{a^2x^2+1}dx+\frac{1}{2}a\int \frac{1}{x^2(a^2x^2+1)}dx^2-\left(a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)\right)-\frac{\arctan(ax)}{x}}{c^3}-\frac{a^2\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}\right)}{c^3}$$

$$\Big\downarrow 47$$

$$\frac{-a^2\int \frac{\arctan(ax)}{a^2x^2+1}dx+\frac{1}{2}a\left(\int \frac{1}{x^2}dx^2-a^2\int \frac{1}{a^2x^2+1}dx^2\right)-\left(a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)\right)-\frac{\arctan(ax)}{x}}{c^3}-\frac{a^2\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}\right)}{c^3}$$

$$\Big\downarrow 14$$

$$\frac{-a^2\int \frac{\arctan(ax)}{a^2x^2+1}dx+\frac{1}{2}a\left(\log\left(x^2\right)-a^2\int \frac{1}{a^2x^2+1}dx^2\right)-\left(a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)\right)-\frac{\arctan(ax)}{x}}{c^3}-\frac{a^2\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}\right)}{c^3}$$

$$\Big\downarrow 16$$

$$\frac{-a^2\int \frac{\arctan(ax)}{a^2x^2+1}dx-\left(a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)\right)+\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{\arctan(ax)}{x}}{c^3}-\frac{a^2\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}\right)}{c^3}$$

$$\Big\downarrow 5419$$

$$\frac{-\left(a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)\right)+\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{1}{2}a\arctan(ax)^2-\frac{\arctan(ax)}{x}}{c^3}-\frac{a^2\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}\right)}{c^3}$$

input
```
Int[ArcTan[a*x]/(x^2*(c + a^2*c*x^2)^3),x]
```

output
```
-((a^2*(1/(16*a*(1 + a^2*x^2)^2) + (x*ArcTan[a*x])/(4*(1 + a^2*x^2)^2) + (
3*(1/(4*a*(1 + a^2*x^2)) + (x*ArcTan[a*x])/(2*(1 + a^2*x^2)) + ArcTan[a*x]
^2/(4*a)))/4))/c^3) + (-(ArcTan[a*x]/x) - (a*ArcTan[a*x]^2)/2 - a^2*(1/(4*
a*(1 + a^2*x^2)) + (x*ArcTan[a*x])/(2*(1 + a^2*x^2)) + ArcTan[a*x]^2/(4*a)
) + (a*(Log[x^2] - Log[1 + a^2*x^2]))/2)/c^3
```

## Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5431
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbol
] :> Simp[b*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (-Simp[x*(d + e*x
^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*d*(q + 1))), x] + Simp[(2*q + 3)/(2*d*(
q + 1))   Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a,
 b, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -3/2]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

## Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.96

| method | result |
|---|---|
| derivativedivides | $a\left(-\frac{7\arctan(ax)a^3x^3}{8c^3(a^2x^2+1)^2}-\frac{9ax\arctan(ax)}{8c^3(a^2x^2+1)^2}-\frac{15\arctan(ax)^2}{8c^3}-\frac{\arctan(ax)}{c^3ax}-\frac{-\frac{15\arctan(ax)^2}{2}+\frac{7}{2(a^2x^2+1)}+4\,\mathrm{l}}{}\right.$ |
| default | $a\left(-\frac{7\arctan(ax)a^3x^3}{8c^3(a^2x^2+1)^2}-\frac{9ax\arctan(ax)}{8c^3(a^2x^2+1)^2}-\frac{15\arctan(ax)^2}{8c^3}-\frac{\arctan(ax)}{c^3ax}-\frac{-\frac{15\arctan(ax)^2}{2}+\frac{7}{2(a^2x^2+1)}+4\,\mathrm{l}}{}\right.$ |
| parts | $-\frac{7\arctan(ax)a^4x^3}{8c^3(a^2x^2+1)^2}-\frac{9a^2x\arctan(ax)}{8c^3(a^2x^2+1)^2}-\frac{15a\arctan(ax)^2}{8c^3}-\frac{\arctan(ax)}{c^3x}-\frac{-\frac{15a\arctan(ax)^2}{16}-\frac{a\left(-\frac{7}{2(a^2x^2+1)}\right.}{}}{}$ |
| parallelrisch | $\frac{-15a^5\arctan(ax)^2x^5+16\ln(x)a^5x^5-8\ln(a^2x^2+1)x^5a^5+8a^5x^5-30x^4\arctan(ax)a^4-30a^3\arctan(ax)^2x^3+32\ln(x)a}{16x\,c^3(}$ |
| risch | $\frac{7ia^2\ln(-iax+1)x}{64c^3(-iax-1)}-\frac{ia^2\ln(-iax+1)x}{64c^3(-iax-1)^2}+\frac{a^3\ln(iax+1)x^2}{128c^3(iax-1)^2}+\frac{a^3\ln(-iax+1)x^2}{128c^3(-iax-1)^2}-\frac{7a\ln(-iax+1)}{64c^3(-iax-1)}+\frac{3a\ln(-ia}{128c^3(-ia}$ |

input
```
int(arctan(a*x)/x^2/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
a*(-7/8/c^3*arctan(a*x)/(a^2*x^2+1)^2*a^3*x^3-9/8*a*x*arctan(a*x)/c^3/(a^2
*x^2+1)^2-15/8*arctan(a*x)^2/c^3-1/c^3*arctan(a*x)/a/x-1/8/c^3*(-15/2*arct
an(a*x)^2+7/2/(a^2*x^2+1)+4*ln(a^2*x^2+1)+1/2/(a^2*x^2+1)^2-8*ln(a*x)))
```

## Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.05

$$\int \frac{\arctan(ax)}{x^2(c+a^2cx^2)^3}\,dx = -\frac{7a^3x^3+15(a^5x^5+2a^3x^3+ax)\arctan(ax)^2+8ax+2(15a^4x^4+25a^2x^2+8)\arctan(ax)+8(a^5x^5}{16(a^4c^3x^5+2a^2c^3x^3+c^3x)}$$

input
```
integrate(arctan(a*x)/x^2/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
-1/16*(7*a^3*x^3 + 15*(a^5*x^5 + 2*a^3*x^3 + a*x)*arctan(a*x)^2 + 8*a*x +
2*(15*a^4*x^4 + 25*a^2*x^2 + 8)*arctan(a*x) + 8*(a^5*x^5 + 2*a^3*x^3 + a*x
)*log(a^2*x^2 + 1) - 16*(a^5*x^5 + 2*a^3*x^3 + a*x)*log(x))/(a^4*c^3*x^5 +
 2*a^2*c^3*x^3 + c^3*x)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 604 vs. $2(134) = 268$.

Time = 1.22 (sec) , antiderivative size = 604, normalized size of antiderivative = 4.25

$$\int \frac{\arctan(ax)}{x^2 (c + a^2cx^2)^3} dx = \begin{cases} \frac{16a^5x^5 \log(x)}{16a^4c^3x^5+32a^2c^3x^3+16c^3x} - \frac{8a^5x^5 \log\left(x^2+\frac{1}{a^2}\right)}{16a^4c^3x^5+32a^2c^3x^3+16c^3x} - \frac{15a^5x^5 \operatorname{atan}^2(ax)}{16a^4c^3x^5+32a^2c^3x^3+16c^3x} - \frac{30a^4x^4 \operatorname{atan}(ax)}{16a^4c^3x^5+32a^2c^3x^3+16c^3x} + \frac{3}{16a^4c^3x} \\ 0 \end{cases}$$

input
```
integrate(atan(a*x)/x**2/(a**2*c*x**2+c)**3,x)
```

output
```
Piecewise((16*a**5*x**5*log(x)/(16*a**4*c**3*x**5 + 32*a**2*c**3*x**3 + 16
*c**3*x) - 8*a**5*x**5*log(x**2 + a**(-2))/(16*a**4*c**3*x**5 + 32*a**2*c*
*3*x**3 + 16*c**3*x) - 15*a**5*x**5*atan(a*x)**2/(16*a**4*c**3*x**5 + 32*a
**2*c**3*x**3 + 16*c**3*x) - 30*a**4*x**4*atan(a*x)/(16*a**4*c**3*x**5 + 3
2*a**2*c**3*x**3 + 16*c**3*x) + 32*a**3*x**3*log(x)/(16*a**4*c**3*x**5 + 3
2*a**2*c**3*x**3 + 16*c**3*x) - 16*a**3*x**3*log(x**2 + a**(-2))/(16*a**4*
c**3*x**5 + 32*a**2*c**3*x**3 + 16*c**3*x) - 30*a**3*x**3*atan(a*x)**2/(16
*a**4*c**3*x**5 + 32*a**2*c**3*x**3 + 16*c**3*x) - 7*a**3*x**3/(16*a**4*c*
*3*x**5 + 32*a**2*c**3*x**3 + 16*c**3*x) - 50*a**2*x**2*atan(a*x)/(16*a**4
*c**3*x**5 + 32*a**2*c**3*x**3 + 16*c**3*x) + 16*a*x*log(x)/(16*a**4*c**3*
x**5 + 32*a**2*c**3*x**3 + 16*c**3*x) - 8*a*x*log(x**2 + a**(-2))/(16*a**4
*c**3*x**5 + 32*a**2*c**3*x**3 + 16*c**3*x) - 15*a*x*atan(a*x)**2/(16*a**4
*c**3*x**5 + 32*a**2*c**3*x**3 + 16*c**3*x) - 8*a*x/(16*a**4*c**3*x**5 + 3
2*a**2*c**3*x**3 + 16*c**3*x) - 16*atan(a*x)/(16*a**4*c**3*x**5 + 32*a**2*
c**3*x**3 + 16*c**3*x), Ne(a, 0)), (0, True))
```

## Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.27

$$\int \frac{\arctan(ax)}{x^2 (c + a^2cx^2)^3} \, dx = -\frac{1}{8} \left( \frac{15 \, a^4x^4 + 25 \, a^2x^2 + 8}{a^4c^3x^5 + 2 \, a^2c^3x^3 + c^3x} + \frac{15 \, a \arctan(ax)}{c^3} \right) \arctan(ax) - \frac{(7 \, a^2x^2 - 15 \, (a^4x^4 + 2 \, a^2x^2 + 1) \arctan(ax)^2 + 8 \, (a^4x^4 + 2 \, a^2x^2 + 1) \log(a^2x^2 + 1) - 16 \, (a^4x^4 + 2 \, a^2}{16 \, (a^4c^3x^4 + 2 \, a^2c^3x^2 + c^3)}$$

input
```
integrate(arctan(a*x)/x^2/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
-1/8*((15*a^4*x^4 + 25*a^2*x^2 + 8)/(a^4*c^3*x^5 + 2*a^2*c^3*x^3 + c^3*x)
+ 15*a*arctan(a*x)/c^3)*arctan(a*x) - 1/16*(7*a^2*x^2 - 15*(a^4*x^4 + 2*a^
2*x^2 + 1)*arctan(a*x)^2 + 8*(a^4*x^4 + 2*a^2*x^2 + 1)*log(a^2*x^2 + 1) -
16*(a^4*x^4 + 2*a^2*x^2 + 1)*log(x) + 8)*a/(a^4*c^3*x^4 + 2*a^2*c^3*x^2 +
c^3)
```

## Giac [F]

$$\int \frac{\arctan(ax)}{x^2 (c + a^2cx^2)^3} \, dx = \int \frac{\arctan(ax)}{(a^2cx^2 + c)^3 x^2} \, dx$$

input
```
integrate(arctan(a*x)/x^2/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
integrate(arctan(a*x)/((a^2*c*x^2 + c)^3*x^2), x)
```

## Mupad [B] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.94

$$\int \frac{\arctan(ax)}{x^2 (c+a^2cx^2)^3} \, dx = \frac{a \ln(x)}{c^3} - \frac{a \ln(a^2 x^2+1)}{2 c^3} - \frac{\frac{7 a^3 x^2}{2}+4 a}{8 a^4 c^3 x^4+16 a^2 c^3 x^2+8 c^3} - \frac{\mathrm{atan}(a x) \left(\frac{1}{a^2 c^3}+\frac{25 x^2}{8 c^3}+\frac{15 a^2 x^4}{8 c^3}\right)}{\frac{x}{a^2}+2 x^3+a^2 x^5} - \frac{15 a \, \mathrm{atan}(a x)^2}{16 c^3}$$

input
```
int(atan(a*x)/(x^2*(c + a^2*c*x^2)^3),x)
```

output
```
(a*log(x))/c^3 - (a*log(a^2*x^2 + 1))/(2*c^3) - (4*a + (7*a^3*x^2)/2)/(8*c
^3 + 16*a^2*c^3*x^2 + 8*a^4*c^3*x^4) - (atan(a*x)*(1/(a^2*c^3) + (25*x^2)/
(8*c^3) + (15*a^2*x^4)/(8*c^3)))/(x/a^2 + 2*x^3 + a^2*x^5) - (15*a*atan(a*
x)^2)/(16*c^3)
```

## Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.30

$$\int \frac{\arctan(ax)}{x^2 (c+a^2cx^2)^3} \, dx = \frac{-30 atan(ax)^2 a^5x^5 - 60 atan(ax)^2 a^3x^3 - 30 atan(ax)^2 ax - 60 atan(ax) a^4x^4 - 100 atan(ax) a^2x^2 - 32 a}{}$$

input
```
int(atan(a*x)/x^2/(a^2*c*x^2+c)^3,x)
```

output
```
( - 30*atan(a*x)**2*a**5*x**5 - 60*atan(a*x)**2*a**3*x**3 - 30*atan(a*x)**
2*a*x - 60*atan(a*x)*a**4*x**4 - 100*atan(a*x)*a**2*x**2 - 32*atan(a*x) -
16*log(a**2*x**2 + 1)*a**5*x**5 - 32*log(a**2*x**2 + 1)*a**3*x**3 - 16*log
(a**2*x**2 + 1)*a*x + 32*log(x)*a**5*x**5 + 64*log(x)*a**3*x**3 + 32*log(x
)*a*x + 7*a**5*x**5 - 9*a*x)/(32*c**3*x*(a**4*x**4 + 2*a**2*x**2 + 1))
```

# 3.198 $\int \frac{\arctan(ax)}{x^3(c+a^2cx^2)^3}\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 205

$$\int \frac{\arctan(ax)}{x^3(c+a^2cx^2)^3}\,dx = -\frac{a}{2c^3x}+\frac{a^3x}{16c^3(1+a^2x^2)^2}+\frac{19a^3x}{32c^3(1+a^2x^2)}+\frac{3a^2\arctan(ax)}{32c^3} -\frac{\arctan(ax)}{2c^3x^2}-\frac{a^2\arctan(ax)}{4c^3(1+a^2x^2)^2}-\frac{a^2\arctan(ax)}{c^3(1+a^2x^2)} +\frac{3ia^2\arctan(ax)^2}{2c^3}-\frac{3a^2\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{c^3} +\frac{3ia^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{2c^3}$$

output
```
-1/2*a/c^3/x+1/16*a^3*x/c^3/(a^2*x^2+1)^2+19/32*a^3*x/c^3/(a^2*x^2+1)+3/32
*a^2*arctan(a*x)/c^3-1/2*arctan(a*x)/c^3/x^2-1/4*a^2*arctan(a*x)/c^3/(a^2*
x^2+1)^2-a^2*arctan(a*x)/c^3/(a^2*x^2+1)+3/2*I*a^2*arctan(a*x)^2/c^3-3*a^2
*arctan(a*x)*ln(2-2/(1-I*a*x))/c^3+3/2*I*a^2*polylog(2,-1+2/(1-I*a*x))/c^3
```

## Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.54

$$\int \frac{\arctan(ax)}{x^3 (c+a^2cx^2)^3} dx$$
$$= \frac{a^2\left(-\frac{64}{ax}+192i\arctan(ax)^2+\arctan(ax)\left(-64-\frac{64}{a^2x^2}-80\cos(2\arctan(ax))-4\cos(4\arctan(ax))-38\right.\right.}{128c}$$

input
```
Integrate[ArcTan[a*x]/(x^3*(c + a^2*c*x^2)^3),x]
```

output
```
(a^2*(-64/(a*x) + (192*I)*ArcTan[a*x]^2 + ArcTan[a*x]*(-64 - 64/(a^2*x^2)
- 80*Cos[2*ArcTan[a*x]] - 4*Cos[4*ArcTan[a*x]] - 384*Log[1 - E^((2*I)*ArcT
an[a*x])]) + (192*I)*PolyLog[2, E^((2*I)*ArcTan[a*x])] + 40*Sin[2*ArcTan[a
*x]] + Sin[4*ArcTan[a*x]]))/(128*c^3)
```

## Rubi [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 446 vs. 2(205) = 410.

Time = 2.75 (sec) , antiderivative size = 446, normalized size of antiderivative = 2.18, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 1.050$, Rules used = {5501, 27, 5501, 5453, 5361, 264, 216, 5459, 5403, 2897, 5465, 215, 215, 216, 5501, 5459, 5403, 2897, 5465, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)}{x^3 (a^2cx^2+c)^3} dx$$
$$\Big\downarrow \; 5501$$
$$\frac{\int \frac{\arctan(ax)}{c^2x^3(a^2x^2+1)^2}dx}{c} - a^2\int \frac{\arctan(ax)}{c^3x(a^2x^2+1)^3}dx$$
$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\arctan(ax)}{x^3(a^2x^2+1)^2}dx}{c^3}-\frac{a^2\int \frac{\arctan(ax)}{x(a^2x^2+1)^3}dx}{c^3}$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{\arctan(ax)}{x^3(a^2x^2+1)}dx-a^2\int \frac{\arctan(ax)}{x(a^2x^2+1)^2}dx}{c^3}-\frac{a^2\left(\int \frac{\arctan(ax)}{x(a^2x^2+1)^2}dx-a^2\int \frac{x\arctan(ax)}{(a^2x^2+1)^3}dx\right)}{c^3}$$

$$\Big\downarrow \text{5453}$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)}{x(a^2x^2+1)^2}dx\right)-a^2\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx+\int \frac{\arctan(ax)}{x^3}dx}{c^3}-\frac{a^2\left(\int \frac{\arctan(ax)}{x(a^2x^2+1)^2}dx-a^2\int \frac{x\arctan(ax)}{(a^2x^2+1)^3}dx\right)}{c^3}$$

$$\Big\downarrow \text{5361}$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)}{x(a^2x^2+1)^2}dx\right)-a^2\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx+\frac{1}{2}a\int \frac{1}{x^2(a^2x^2+1)}dx-\frac{\arctan(ax)}{2x^2}}{c^3}-\frac{a^2\left(\int \frac{\arctan(ax)}{x(a^2x^2+1)^2}dx-a^2\int \frac{x\arctan(ax)}{(a^2x^2+1)^3}dx\right)}{c^3}$$

$$\Big\downarrow \text{264}$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)}{x(a^2x^2+1)^2}dx\right)-a^2\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx+\frac{1}{2}a\left(a^2\left(-\int \frac{1}{a^2x^2+1}dx\right)-\frac{1}{x}\right)-\frac{\arctan(ax)}{2x^2}}{c^3}-\frac{a^2\left(\int \frac{\arctan(ax)}{x(a^2x^2+1)^2}dx-a^2\int \frac{x\arctan(ax)}{(a^2x^2+1)^3}dx\right)}{c^3}$$

$$\Big\downarrow \text{216}$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)}{x(a^2x^2+1)^2}dx\right)-a^2\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx-\frac{\arctan(ax)}{2x^2}+\frac{1}{2}a\left(-a\arctan(ax)-\frac{1}{x}\right)}{c^3}-\frac{a^2\left(\int \frac{\arctan(ax)}{x(a^2x^2+1)^2}dx-a^2\int \frac{x\arctan(ax)}{(a^2x^2+1)^3}dx\right)}{c^3}$$

$$\Big\downarrow \text{5459}$$

$$-\frac{a^2\left(\int \frac{\arctan(ax)}{x(a^2x^2+1)^2}dx-a^2\int \frac{x\arctan(ax)}{(a^2x^2+1)^3}dx\right)}{c^3}+\frac{-a^2\int \frac{\arctan(ax)}{x(a^2x^2+1)^2}dx-\left(a^2\left(i\int \frac{\arctan(ax)}{x(ax+i)}dx-\frac{1}{2}i\arctan(ax)^2\right)\right)-\frac{\arctan(ax)}{2x^2}+\frac{1}{2}a\left(-a\arctan(ax)-\frac{1}{x}\right)}{c^3}$$

$$\Big\downarrow \text{5403}$$

$$-\frac{a^2\left(\int \frac{\arctan(ax)}{x(a^2x^2+1)^2}dx - a^2\int \frac{x\arctan(ax)}{(a^2x^2+1)^3}dx\right)}{c^3} + \frac{a^2\left(-\int \frac{\arctan(ax)}{x(a^2x^2+1)^2}dx\right) - a^2\left(i\left(ia\int \frac{\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx - i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)\right) - \frac{1}{2}i\arctan(ax)^2\right) - \frac{\arctan(}{2x^2}}{c^3}$$

$\downarrow$ 2897

$$-\frac{a^2\left(\int \frac{\arctan(ax)}{x(a^2x^2+1)^2}dx - a^2\int \frac{x\arctan(ax)}{(a^2x^2+1)^3}dx\right)}{c^3} + \frac{-a^2\int \frac{\arctan(ax)}{x(a^2x^2+1)^2}dx - \left(a^2\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right) - \frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right) - \frac{1}{2}i\arctan(ax)^2\right)\right) -}{c^3}$$

$\downarrow$ 5465

$$-\frac{a^2\left(\int \frac{\arctan(ax)}{x(a^2x^2+1)^2}dx - a^2\left(\frac{\int \frac{1}{(a^2x^2+1)^3}dx}{4a} - \frac{\arctan(ax)}{4a^2(a^2x^2+1)^2}\right)\right)}{c^3} + \frac{-a^2\int \frac{\arctan(ax)}{x(a^2x^2+1)^2}dx - \left(a^2\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right) - \frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right) - \frac{1}{2}i\arctan(ax)^2\right)\right) -}{c^3}$$

$\downarrow$ 215

$$-\frac{a^2\left(\int \frac{\arctan(ax)}{x(a^2x^2+1)^2}dx - a^2\left(\frac{\frac{3}{4}\int \frac{1}{(a^2x^2+1)^2}dx + \frac{x}{4(a^2x^2+1)^2}}{4a} - \frac{\arctan(ax)}{4a^2(a^2x^2+1)^2}\right)\right)}{c^3} + \frac{-a^2\int \frac{\arctan(ax)}{x(a^2x^2+1)^2}dx - \left(a^2\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right) - \frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right) - \frac{1}{2}i\arctan(ax)^2\right)\right) -}{c^3}$$

$\downarrow$ 215

$$-\frac{a^2\left(\int \frac{\arctan(ax)}{x(a^2x^2+1)^2}dx - a^2\left(\frac{\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{a^2x^2+1}dx + \frac{x}{2(a^2x^2+1)}\right) + \frac{x}{4(a^2x^2+1)^2}}{4a} - \frac{\arctan(ax)}{4a^2(a^2x^2+1)^2}\right)\right)}{c^3} + \frac{-a^2\int \frac{\arctan(ax)}{x(a^2x^2+1)^2}dx - \left(a^2\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right) - \frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right) - \frac{1}{2}i\arctan(ax)^2\right)\right) -}{c^3}$$

$\downarrow$ 216

$$-\frac{a^2\left(\int \frac{\arctan(ax)}{x(a^2x^2+1)^2}dx-a^2\left(\frac{\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}\right)+\frac{x}{4(a^2x^2+1)^2}}{4a}-\frac{\arctan(ax)}{4a^2(a^2x^2+1)^2}\right)\right)}{c^3}+\frac{-a^2\int \frac{\arctan(ax)}{x(a^2x^2+1)^2}dx-\left(a^2\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)\right)-}{c^3}$$

$$\Big\downarrow \; 5501$$

$$-\frac{a^2\left(-a^2\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx-\left(a^2\left(\frac{\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}\right)+\frac{x}{4(a^2x^2+1)^2}}{4a}-\frac{\arctan(ax)}{4a^2(a^2x^2+1)^2}\right)\right)\right)}{c^3}+\frac{-a^2\left(\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx-a^2\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx\right)-\left(a^2\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)\right.\right.}{c^3}$$

$$\Big\downarrow \; 5459$$

$$\frac{-a^2\left(a^2\left(-\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx\right)+i\int \frac{\arctan(ax)}{x(ax+i)}dx-\frac{1}{2}i\arctan(ax)^2\right)-\left(a^2\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\operatorname{Po}\right.\right.\right.}{c^3}$$
$$\frac{a^2\left(-a^2\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+i\int \frac{\arctan(ax)}{x(ax+i)}dx-\left(a^2\left(\frac{\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}\right)+\frac{x}{4(a^2x^2+1)^2}}{4a}-\frac{\arctan(ax)}{4a^2(a^2x^2+1)^2}\right)\right)-\frac{1}{2}i\operatorname{arc}\right.}{c^3}$$

$$\Big\downarrow \; 5403$$

$$\frac{-a^2\left(a^2\left(-\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx\right)+i\left(ia\int \frac{\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)-\left(a^2\right.}{}$$
$$\frac{a^2\left(-a^2\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+i\left(ia\int \frac{\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)\right)-\left(a^2\left(\frac{\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}\right.}{4a}\right.\right.\right.}{c^3}$$

$$\Big\downarrow \; 2897$$

$$\frac{-a^2\left(a^2\left(-\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx\right)+i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)}{a^2\left(-a^2\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx-\left(a^2\left(\frac{\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}\right)+\frac{x}{4(a^2x^2+1)^2}}{4a}-\frac{\arctan(ax)}{4a^2(a^2x^2+1)^2}\right)\right)+i\left(-i\arctan(ax)\log\left(2-\right.\right.\right.}$$
$$c^3$$

$\downarrow$ 5465

$$\frac{-a^2\left(-\left(a^2\left(\frac{\int\frac{1}{(a^2x^2+1)^2}dx}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)\right)+i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\right.}{a^2\left(-a^2\left(\frac{\int\frac{1}{(a^2x^2+1)^2}dx}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)-\left(a^2\left(\frac{\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}\right)+\frac{x}{4(a^2x^2+1)^2}}{4a}-\frac{\arctan(ax)}{4a^2(a^2x^2+1)^2}\right)\right)+i\left(-i\,\text{arc}\right.\right.}$$
$$c^3$$

$\downarrow$ 215

$$\frac{-a^2\left(-\left(a^2\left(\frac{\frac{1}{2}\int\frac{1}{a^2x^2+1}dx+\frac{x}{2(a^2x^2+1)}}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)\right)+i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}\right.\right.\right.}{a^2\left(-a^2\left(\frac{\frac{1}{2}\int\frac{1}{a^2x^2+1}dx+\frac{x}{2(a^2x^2+1)}}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)-\left(a^2\left(\frac{\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}\right)+\frac{x}{4(a^2x^2+1)^2}}{4a}-\frac{\arctan(ax)}{4a^2(a^2x^2+1)^2}\right)\right)+\right.}$$
$$c^3$$

$\downarrow$ 216

$$\frac{-a^2\left(-\left(a^2\left(\frac{\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)\right)+i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-\right.\right.\right.}{a^2\left(-\left(a^2\left(\frac{\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)\right)-a^2\left(\frac{\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}\right)+\frac{x}{4(a^2x^2+1)^2}}{4a}-\frac{\arctan(ax)}{4a^2(a^2x^2+1)^2}\right)+i\left(\right.\right.}$$
$$c^3$$

input
```
Int[ArcTan[a*x]/(x^3*(c + a^2*c*x^2)^3),x]
```

output
```
(-1/2*ArcTan[a*x]/x^2 + (a*(-x^(-1) - a*ArcTan[a*x]))/2 - a^2*((-1/2*I)*Ar
cTan[a*x]^2 + I*((-I)*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)] - PolyLog[2, -1 +
 2/(1 - I*a*x)]/2)) - a^2*((-1/2*I)*ArcTan[a*x]^2 - a^2*(-1/2*ArcTan[a*x]/
(a^2*(1 + a^2*x^2)) + (x/(2*(1 + a^2*x^2)) + ArcTan[a*x]/(2*a))/(2*a)) + I
*((-I)*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)] - PolyLog[2, -1 + 2/(1 - I*a*x)]
/2)))/c^3 - (a^2*((-1/2*I)*ArcTan[a*x]^2 - a^2*(-1/2*ArcTan[a*x]/(a^2*(1 +
 a^2*x^2)) + (x/(2*(1 + a^2*x^2)) + ArcTan[a*x]/(2*a))/(2*a)) - a^2*(-1/4*
ArcTan[a*x]/(a^2*(1 + a^2*x^2)^2) + (x/(4*(1 + a^2*x^2)^2) + (3*(x/(2*(1 +
 a^2*x^2)) + ArcTan[a*x]/(2*a)))/4)/(4*a)) + I*((-I)*ArcTan[a*x]*Log[2 - 2
/(1 - I*a*x)] - PolyLog[2, -1 + 2/(1 - I*a*x)]/2)))/c^3
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

## Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.32 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.42

| method | result |
|---|---|
| parts | $-\frac{a^2\arctan(ax)}{4c^3(a^2x^2+1)^2}-\frac{a^2\arctan(ax)}{c^3(a^2x^2+1)}+\frac{3\arctan(ax)a^2\ln(a^2x^2+1)}{2c^3}-\frac{\arctan(ax)}{2c^3x^2}-\frac{3\arctan(ax)a^2\ln(x)}{c^3}-\frac{a\left(-\cdots\right.}{\cdots}$ |
| derivativedivides | $a^2\left(\frac{3\arctan(ax)\ln(a^2x^2+1)}{2c^3}-\frac{\arctan(ax)}{c^3(a^2x^2+1)}-\frac{\arctan(ax)}{4c^3(a^2x^2+1)^2}-\frac{\arctan(ax)}{2c^3a^2x^2}-\frac{3\arctan(ax)\ln(ax)}{c^3}-\frac{6i\ln(a\cdots}{\cdots}\right.$ |
| default | $a^2\left(\frac{3\arctan(ax)\ln(a^2x^2+1)}{2c^3}-\frac{\arctan(ax)}{c^3(a^2x^2+1)}-\frac{\arctan(ax)}{4c^3(a^2x^2+1)^2}-\frac{\arctan(ax)}{2c^3a^2x^2}-\frac{3\arctan(ax)\ln(ax)}{c^3}-\frac{6i\ln(a\cdots}{\cdots}\right.$ |
| risch | $-\frac{a}{2c^3x}+\frac{19a^2\arctan(ax)}{64c^3}+\frac{9a^3\ln(-iax+1)x}{64c^3(-iax-1)}-\frac{ia^2\ln(-iax+1)}{32c^3(-iax+1)^2}-\frac{a^3\ln(iax+1)x}{64c^3(iax-1)^2}+\frac{9a^3\ln(iax+1)x}{64c^3(iax-1)}+\frac{ia^2\cdots}{32c\cdots}$ |

input
```
int(arctan(a*x)/x^3/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a^2*arctan(a*x)/c^3/(a^2*x^2+1)^2-a^2*arctan(a*x)/c^3/(a^2*x^2+1)+3/2
/c^3*arctan(a*x)*a^2*ln(a^2*x^2+1)-1/2*arctan(a*x)/c^3/x^2-3/c^3*arctan(a*
x)*a^2*ln(x)-1/2/c^3*a*(-6*a^2*(-1/2*I*ln(x)*(-ln(1-I*a*x)+ln(1+I*a*x))/a-
1/2*I*(dilog(1+I*a*x)-dilog(1-I*a*x))/a)+1/2*a^2*((-19/8*a^2*x^3-21/8*x)/(
a^2*x^2+1)^2-3/8/a*arctan(a*x))+1/x+3/4*sum(1/_alpha*(2*ln(x-_alpha)*ln(a^
2*x^2+1)-a^2*(1/a^2/_alpha*ln(x-_alpha)^2+2*_alpha*ln(x-_alpha)*ln(1/2*(x+
_alpha)/_alpha)+2*_alpha*dilog(1/2*(x+_alpha)/_alpha))),_alpha=RootOf(_Z^2
*a^2+1)))
```

## Fricas [F]

$$\int \frac{\arctan(ax)}{x^3 (c + a^2cx^2)^3} \, dx = \int \frac{\arctan(ax)}{(a^2cx^2 + c)^3 x^3} \, dx$$

input
```
integrate(arctan(a*x)/x^3/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
integral(arctan(a*x)/(a^6*c^3*x^9 + 3*a^4*c^3*x^7 + 3*a^2*c^3*x^5 + c^3*x^
3), x)
```

## Sympy [F]

$$\int \frac{\arctan(ax)}{x^3 (c + a^2cx^2)^3} \, dx = \frac{\int \frac{\operatorname{atan}(ax)}{a^6x^9+3a^4x^7+3a^2x^5+x^3} \, dx}{c^3}$$

input
```
integrate(atan(a*x)/x**3/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(atan(a*x)/(a**6*x**9 + 3*a**4*x**7 + 3*a**2*x**5 + x**3), x)/c**3
```

## Maxima [F]

$$\int \frac{\arctan(ax)}{x^3 (c + a^2cx^2)^3} \, dx = \int \frac{\arctan(ax)}{(a^2cx^2 + c)^3 x^3} \, dx$$

input
```
integrate(arctan(a*x)/x^3/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)/((a^2*c*x^2 + c)^3*x^3), x)
```

## Giac [F]

$$\int \frac{\arctan(ax)}{x^3 (c + a^2 c x^2)^3} \, dx = \int \frac{\arctan(ax)}{(a^2 c x^2 + c)^3 x^3} \, dx$$

input
```
integrate(arctan(a*x)/x^3/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
integrate(arctan(a*x)/((a^2*c*x^2 + c)^3*x^3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)}{x^3 (c + a^2 c x^2)^3} \, dx = \int \frac{\mathrm{atan}(a\,x)}{x^3\,(c\,a^2\,x^2 + c)^3} \, dx$$

input
```
int(atan(a*x)/(x^3*(c + a^2*c*x^2)^3),x)
```

output
```
int(atan(a*x)/(x^3*(c + a^2*c*x^2)^3), x)
```

## Reduce [F]

$$\int \frac{\arctan(ax)}{x^3 (c + a^2 c x^2)^3} \, dx = \frac{\int \frac{atan(ax)}{a^6 x^9 + 3a^4 x^7 + 3a^2 x^5 + x^3} dx}{c^3}$$

input
```
int(atan(a*x)/x^3/(a^2*c*x^2+c)^3,x)
```

output
```
int(atan(a*x)/(a**6*x**9 + 3*a**4*x**7 + 3*a**2*x**5 + x**3),x)/c**3
```

# 3.199 $\int \frac{\arctan(ax)}{x^4(c+a^2cx^2)^3}\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 183

$$\int \frac{\arctan(ax)}{x^4(c+a^2cx^2)^3}\,dx = -\frac{a}{6c^3x^2}+\frac{a^3}{16c^3(1+a^2x^2)^2}+\frac{11a^3}{16c^3(1+a^2x^2)}-\frac{\arctan(ax)}{3c^3x^3}+\frac{3a^2\arctan(ax)}{c^3x}+\frac{a^4x\arctan(ax)}{4c^3(1+a^2x^2)^2}+\frac{11a^4x\arctan(ax)}{8c^3(1+a^2x^2)}+\frac{35a^3\arctan(ax)^2}{16c^3}-\frac{10a^3\log(x)}{3c^3}+\frac{5a^3\log(1+a^2x^2)}{3c^3}$$

output
```
-1/6*a/c^3/x^2+1/16*a^3/c^3/(a^2*x^2+1)^2+11/16*a^3/c^3/(a^2*x^2+1)-1/3*ar
ctan(a*x)/c^3/x^3+3*a^2*arctan(a*x)/c^3/x+1/4*a^4*x*arctan(a*x)/c^3/(a^2*x
^2+1)^2+11/8*a^4*x*arctan(a*x)/c^3/(a^2*x^2+1)+35/16*a^3*arctan(a*x)^2/c^3
-10/3*a^3*ln(x)/c^3+5/3*a^3*ln(a^2*x^2+1)/c^3
```

## Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.78

$$\int \frac{\arctan(ax)}{x^4 (c + a^2 cx^2)^3} dx$$
$$= \frac{2(-8 + 56a^2x^2 + 175a^4x^4 + 105a^6x^6) \arctan(ax) + 105a^3x^3(1 + a^2x^2)^2 \arctan(ax)^2 + ax\Big(-8 + 20a^2x^2}{48c^3x^3 (1 + a^2x^2)^2}$$

input
```
Integrate[ArcTan[a*x]/(x^4*(c + a^2*c*x^2)^3),x]
```

output
```
(2*(-8 + 56*a^2*x^2 + 175*a^4*x^4 + 105*a^6*x^6)*ArcTan[a*x] + 105*a^3*x^3
*(1 + a^2*x^2)^2*ArcTan[a*x]^2 + a*x*(-8 + 20*a^2*x^2 + 25*a^4*x^4 - 160*(
a*x + a^3*x^3)^2*Log[x] + 80*(a*x + a^3*x^3)^2*Log[1 + a^2*x^2]))/(48*c^3*
x^3*(1 + a^2*x^2)^2)
```

## Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 412 vs. 2(183) = 366.

Time = 3.16 (sec) , antiderivative size = 412, normalized size of antiderivative = 2.25, number of steps used = 29, number of rules used = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.400, Rules used = {5501, 27, 5501, 5431, 5427, 241, 5453, 5361, 243, 54, 2009, 5453, 5361, 243, 47, 14, 16, 5419, 5501, 5427, 241, 5453, 5361, 243, 47, 14, 16, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)}{x^4 (a^2cx^2 + c)^3} dx$$
$$\Big\downarrow \text{5501}$$
$$\frac{\int \frac{\arctan(ax)}{c^2x^4(a^2x^2+1)^2}dx}{c} - a^2 \int \frac{\arctan(ax)}{c^3x^2 (a^2x^2 + 1)^3}dx$$
$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\arctan(ax)}{x^4(a^2x^2+1)^2}dx}{c^3}-\frac{a^2\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^3}dx}{c^3}$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{\arctan(ax)}{x^4(a^2x^2+1)}dx-a^2\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx}{c^3}-\frac{a^2\left(\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx-a^2\int \frac{\arctan(ax)}{(a^2x^2+1)^3}dx\right)}{c^3}$$

$$\Big\downarrow \text{5431}$$

$$\frac{\int \frac{\arctan(ax)}{x^4(a^2x^2+1)}dx-a^2\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx}{c^3}-\frac{a^2\left(\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx-a^2\left(\frac{3}{4}\int \frac{\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{1}{16a(a^2x^2+1)^2}\right)\right)}{c^3}$$

$$\Big\downarrow \text{5427}$$

$$\frac{\int \frac{\arctan(ax)}{x^4(a^2x^2+1)}dx-a^2\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx}{c^3}-\frac{a^2\left(\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx-a^2\left(\frac{3}{4}\left(-\frac{1}{2}a\int \frac{x}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{1}{16a(a^2x^2+1)^2}\right)\right)}{c^3}$$

$$\Big\downarrow \text{241}$$

$$\frac{\int \frac{\arctan(ax)}{x^4(a^2x^2+1)}dx-a^2\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx}{c^3}-\frac{a^2\left(\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx-a^2\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}\right)\right)}{c^3}$$

$$\Big\downarrow \text{5453}$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx\right)-a^2\int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx+\int \frac{\arctan(ax)}{x^4}dx}{c^3}-\frac{a^2\left(\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx-a^2\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}\right)\right)}{c^3}$$

$$\Big\downarrow \text{5361}$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx\right)-a^2\int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx+\frac{1}{3}a\int \frac{1}{x^3(a^2x^2+1)}dx-\frac{\arctan(ax)}{3x^3}}{c^3}-\frac{a^2\left(\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx-a^2\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}\right)\right)}{c^3}$$

$$\Big\downarrow \text{243}$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx\right)-a^2\int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx+\frac{1}{6}a\int \frac{1}{x^4(a^2x^2+1)}dx^2-\frac{\arctan(ax)}{3x^3}}{c^3}-\frac{a^2\left(\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx-a^2\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}\right)\right)}{c^3}$$

$$\Big\downarrow \text{ 54}$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx\right)-a^2\int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx+\frac{1}{6}a\int \left(\frac{a^4}{a^2x^2+1}-\frac{a^2}{x^2}+\frac{1}{x^4}\right)dx^2-\frac{\arctan(ax)}{3x^3}}{c^3}-\frac{a^2\left(\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx-a^2\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}\right)\right)}{c^3}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx\right)-a^2\int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx+\frac{1}{6}a\left(a^2\left(-\log\left(x^2\right)\right)+a^2\log\left(a^2x^2+1\right)-\frac{1}{x^2}\right)-\frac{\arctan(ax)}{3x^3}}{c^3}-\frac{a^2\left(\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx-a^2\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}\right)\right)}{c^3}$$

$$\Big\downarrow \text{ 5453}$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx\right)-a^2\left(\int \frac{\arctan(ax)}{x^2}dx-a^2\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{6}a\left(a^2\left(-\log\left(x^2\right)\right)+a^2\log\left(a^2x^2+1\right)-\frac{1}{x^2}\right)}{c^3}$$
$$\frac{a^2\left(\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx-a^2\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}\right)\right)}{c^3}$$

$$\Big\downarrow \text{ 5361}$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx\right)-a^2\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+a\int \frac{1}{x(a^2x^2+1)}dx-\frac{\arctan(ax)}{x}\right)+\frac{1}{6}a\left(a^2\left(-\log\left(x^2\right)\right)+a^2\log\right.}{c^3}$$
$$\frac{a^2\left(\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx-a^2\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}\right)\right)}{c^3}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx\right)-a^2\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{2}a\int \frac{1}{x^2(a^2x^2+1)}dx^2-\frac{\arctan(ax)}{x}\right)+\frac{1}{6}a\left(a^2\left(-\log\left(x^2\right)\right)+a^2\right.}{c^3}$$
$$\frac{a^2\left(\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx-a^2\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}\right)\right)}{c^3}$$

$$\Big\downarrow \text{ 47}$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx\right)-a^2\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{2}a\left(\int \frac{1}{x^2}dx^2-a^2\int \frac{1}{a^2x^2+1}dx^2\right)-\frac{\arctan(ax)}{x}\right)+\frac{1}{6}a\left(a^2\left(-\log\right.\right.}{c^3}$$

$$\frac{a^2\left(\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx-a^2\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}\right)\right)}{c^3}$$

$\big\downarrow$ 14

$$\frac{a^2\left(-\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx\right)-a^2\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{2}a\left(\log\left(x^2\right)-a^2\int \frac{1}{a^2x^2+1}dx^2\right)-\frac{\arctan(ax)}{x}\right)+\frac{1}{6}a\left(a^2\left(-\log\right.\right.}{c^3}$$

$$\frac{a^2\left(\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx-a^2\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}\right)\right)}{c^3}$$

$\big\downarrow$ 16

$$\frac{a^2\left(-\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx\right)-a^2\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{\arctan(ax)}{x}\right)+\frac{1}{6}a\left(a^2\left(-\log\right.\right.}{c^3}$$

$$\frac{a^2\left(\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx-a^2\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}\right)\right)}{c^3}$$

$\big\downarrow$ 5419

$$\frac{-a^2\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx-\left(a^2\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{1}{2}a\arctan(ax)^2-\frac{\arctan(ax)}{x}\right)\right)+\frac{1}{6}a\left(a^2\left(-\log\left(x^2\right)\right)\right.}{c^3}$$

$$\frac{a^2\left(\int \frac{\arctan(ax)}{x^2(a^2x^2+1)^2}dx-a^2\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}\right)\right)}{c^3}$$

$\big\downarrow$ 5501

$$\frac{-a^2\left(\int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx-a^2\int \frac{\arctan(ax)}{(a^2x^2+1)^2}dx\right)-\left(a^2\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{1}{2}a\arctan(ax)^2-\frac{\arctan(ax)}{x}\right)\right)}{c^3}$$

$$\frac{a^2\left(-a^2\int \frac{\arctan(ax)}{(a^2x^2+1)^2}dx+\int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx-\left(a^2\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+\right.\right.\right.}{c^3}$$

$\big\downarrow$ 5427

$$\frac{-a^2\left(\int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx-a^2\left(-\frac{1}{2}a\int \frac{x}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)\right)-\left(a^2\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)\right.\right.}{c^3}$$

$$\frac{a^2\left(-a^2\left(-\frac{1}{2}a\int \frac{x}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx-\left(a^2\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{}{4a}\right.\right.\right.\right.}{c^3}$$

$\Big\downarrow$ 241

$$\frac{-a^2\left(\int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx - a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)} + \frac{1}{4a(a^2x^2+1)} + \frac{\arctan(ax)^2}{4a}\right)\right) - \left(a^2\left(\frac{1}{2}a\left(\log\left(x^2\right) - \log\left(a^2x^2+1\right)\right) - \frac{1}{2}a \arc\right.\right.}{c^3}$$

$$\frac{a^2\left(\int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx - \left(a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)} + \frac{1}{4a(a^2x^2+1)} + \frac{\arctan(ax)^2}{4a}\right)\right) - a^2\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2} + \frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)} + \frac{1}{4a(a^2x^2+1)}\right.\right.\right.}{c^3}$$

$\Big\downarrow$ 5453

$$\frac{-a^2\left(-a^2\int \frac{\arctan(ax)}{a^2x^2+1}dx + \int \frac{\arctan(ax)}{x^2}dx - \left(a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)} + \frac{1}{4a(a^2x^2+1)} + \frac{\arctan(ax)^2}{4a}\right)\right)\right) - \left(a^2\left(\frac{1}{2}a\left(\log\left(x^2\right) - \text{lo}\right.\right.\right.}{c^3}$$

$$\frac{a^2\left(-a^2\int \frac{\arctan(ax)}{a^2x^2+1}dx + \int \frac{\arctan(ax)}{x^2}dx - \left(a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)} + \frac{1}{4a(a^2x^2+1)} + \frac{\arctan(ax)^2}{4a}\right)\right) - a^2\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2} + \frac{3}{4}\left(\frac{x\,\text{arct}}{2(a^2}\right.\right.\right.}{c^3}$$

$\Big\downarrow$ 5361

$$\frac{-a^2\left(-a^2\int \frac{\arctan(ax)}{a^2x^2+1}dx + a\int \frac{1}{x(a^2x^2+1)}dx - \left(a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)} + \frac{1}{4a(a^2x^2+1)} + \frac{\arctan(ax)^2}{4a}\right)\right) - \frac{\arctan(ax)}{x}\right) - \left(a^2\left(\frac{1}{2}a\right.\right.}{c^3}$$

$$\frac{a^2\left(-a^2\int \frac{\arctan(ax)}{a^2x^2+1}dx + a\int \frac{1}{x(a^2x^2+1)}dx - \left(a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)} + \frac{1}{4a(a^2x^2+1)} + \frac{\arctan(ax)^2}{4a}\right)\right) - a^2\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2} + \frac{3}{4}\left(\frac{x\,\text{ar}}{2(}\right.\right.\right.}{c^3}$$

$\Big\downarrow$ 243

$$\frac{-a^2\left(-a^2\int \frac{\arctan(ax)}{a^2x^2+1}dx + \frac{1}{2}a\int \frac{1}{x^2(a^2x^2+1)}dx^2 - \left(a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)} + \frac{1}{4a(a^2x^2+1)} + \frac{\arctan(ax)^2}{4a}\right)\right) - \frac{\arctan(ax)}{x}\right) - \left(a^2\right.}{c^3}$$

$$\frac{a^2\left(-a^2\int \frac{\arctan(ax)}{a^2x^2+1}dx + \frac{1}{2}a\int \frac{1}{x^2(a^2x^2+1)}dx^2 - \left(a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)} + \frac{1}{4a(a^2x^2+1)} + \frac{\arctan(ax)^2}{4a}\right)\right) - a^2\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2} + \frac{3}{4}\right.\right.}{c^3}$$

$\Big\downarrow$ 47

$$\frac{-a^2\left(-a^2\int \frac{\arctan(ax)}{a^2x^2+1}dx + \frac{1}{2}a\left(\int \frac{1}{x^2}dx^2 - a^2\int \frac{1}{a^2x^2+1}dx^2\right) - \left(a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)} + \frac{1}{4a(a^2x^2+1)} + \frac{\arctan(ax)^2}{4a}\right)\right) - \frac{\text{arctar}}{x}\right.}{c^3}$$

$$\frac{a^2\left(-a^2\int \frac{\arctan(ax)}{a^2x^2+1}dx + \frac{1}{2}a\left(\int \frac{1}{x^2}dx^2 - a^2\int \frac{1}{a^2x^2+1}dx^2\right) - \left(a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)} + \frac{1}{4a(a^2x^2+1)} + \frac{\arctan(ax)^2}{4a}\right)\right) - a^2\left(\frac{x\,\text{ar}}{4(a}\right.\right.}{c^3}$$

$\Big\downarrow$ 14

$$\frac{-a^2\left(-a^2\int\frac{\arctan(ax)}{a^2x^2+1}dx+\frac{1}{2}a\left(\log\left(x^2\right)-a^2\int\frac{1}{a^2x^2+1}dx^2\right)-\left(a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)\right)-\frac{\arctan}{x}\right.}{a^2\left(-a^2\int\frac{\arctan(ax)}{a^2x^2+1}dx+\frac{1}{2}a\left(\log\left(x^2\right)-a^2\int\frac{1}{a^2x^2+1}dx^2\right)-\left(a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)\right)-a^2\left(\frac{x\,\mathrm{ar}}{4(a}\right.\right.}$$

$$c^3$$

$$\Big\downarrow 16$$

$$\frac{-a^2\left(-a^2\int\frac{\arctan(ax)}{a^2x^2+1}dx-\left(a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)\right)+\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{\arctan(}{x}\right.}{a^2\left(-a^2\int\frac{\arctan(ax)}{a^2x^2+1}dx-\left(a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)\right)-a^2\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+}\right.\right.\right.}$$

$$c^3$$

$$\Big\downarrow 5419$$

$$\frac{-\left(a^2\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{1}{2}a\arctan(ax)^2-\frac{\arctan(ax)}{x}\right)\right)-a^2\left(-\left(a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\mathrm{arc}\right.\right.\right.}{a^2\left(-\left(a^2\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)\right)-a^2\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\right.\right.}$$

$$c^3$$

input
```
Int[ArcTan[a*x]/(x^4*(c + a^2*c*x^2)^3),x]
```

output
```
-((a^2*(-(ArcTan[a*x]/x) - (a*ArcTan[a*x]^2)/2 - a^2*(1/(4*a*(1 + a^2*x^2)
) + (x*ArcTan[a*x])/(2*(1 + a^2*x^2)) + ArcTan[a*x]^2/(4*a)) - a^2*(1/(16*
a*(1 + a^2*x^2)^2) + (x*ArcTan[a*x])/(4*(1 + a^2*x^2)^2) + (3*(1/(4*a*(1 +
 a^2*x^2)) + (x*ArcTan[a*x])/(2*(1 + a^2*x^2)) + ArcTan[a*x]^2/(4*a)))/4)
+ (a*(Log[x^2] - Log[1 + a^2*x^2]))/2))/c^3) + (-1/3*ArcTan[a*x]/x^3 - a^2
*(-(ArcTan[a*x]/x) - (a*ArcTan[a*x]^2)/2 + (a*(Log[x^2] - Log[1 + a^2*x^2]
))/2) - a^2*(-(ArcTan[a*x]/x) - (a*ArcTan[a*x]^2)/2 - a^2*(1/(4*a*(1 + a^2
*x^2)) + (x*ArcTan[a*x])/(2*(1 + a^2*x^2)) + ArcTan[a*x]^2/(4*a)) + (a*(Lo
g[x^2] - Log[1 + a^2*x^2]))/2) + (a*(-x^(-2) - a^2*Log[x^2] + a^2*Log[1 +
a^2*x^2]))/6)/c^3
```

### Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5431
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbol
] :> Simp[b*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (-Simp[x*(d + e*x
^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*d*(q + 1))), x] + Simp[(2*q + 3)/(2*d*(
q + 1))   Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a,
 b, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -3/2]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

## Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.88

| method | result |
|---|---|
| derivativedivides | $a^3\left(\frac{11\arctan(ax)a^3x^3}{8c^3(a^2x^2+1)^2}+\frac{13ax\arctan(ax)}{8c^3(a^2x^2+1)^2}+\frac{35\arctan(ax)^2}{8c^3}-\frac{\arctan(ax)}{3c^3a^3x^3}+\frac{3\arctan(ax)}{c^3ax}-\frac{-\frac{33}{2(a^2x^2+1)}-}{}\right.$ |
| default | $a^3\left(\frac{11\arctan(ax)a^3x^3}{8c^3(a^2x^2+1)^2}+\frac{13ax\arctan(ax)}{8c^3(a^2x^2+1)^2}+\frac{35\arctan(ax)^2}{8c^3}-\frac{\arctan(ax)}{3c^3a^3x^3}+\frac{3\arctan(ax)}{c^3ax}-\frac{-\frac{33}{2(a^2x^2+1)}-}{}\right.$ |
| parts | $\frac{11\arctan(ax)a^6x^3}{8c^3(a^2x^2+1)^2}+\frac{13a^4x\arctan(ax)}{8c^3(a^2x^2+1)^2}+\frac{35a^3\arctan(ax)^2}{8c^3}-\frac{\arctan(ax)}{3c^3x^3}+\frac{3a^2\arctan(ax)}{c^3x}-\frac{\frac{105a^3\arctan(a}{16}}{}$ |
| parallelrisch | $-\frac{-105a^7\arctan(ax)^2x^7+160\ln(x)a^7x^7-80\ln(a^2x^2+1)x^7a^7+4a^7x^7-210a^6\arctan(ax)x^6-210a^5\arctan(ax)^2x^5+}{}$ |
| risch | $-\frac{35a^3\ln(iax+1)^2}{64c^3}+\frac{(105a^7x^7\ln(-iax+1)-210ia^6x^6+210a^5x^5\ln(-iax+1)-350ia^4x^4+105a^3x^3\ln(-iax+1)-11}{96x^3c^3(a^2x^2+1)^2}$ |

input
```
int(arctan(a*x)/x^4/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
a^3*(11/8/c^3*arctan(a*x)/(a^2*x^2+1)^2*a^3*x^3+13/8*a*x*arctan(a*x)/c^3/(
a^2*x^2+1)^2+35/8*arctan(a*x)^2/c^3-1/3/c^3*arctan(a*x)/a^3/x^3+3/c^3*arct
an(a*x)/a/x-1/24/c^3*(-33/2/(a^2*x^2+1)-3/2/(a^2*x^2+1)^2-40*ln(a^2*x^2+1)
+4/a^2/x^2+80*ln(a*x)+105/2*arctan(a*x)^2))
```

## Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.98

$$\int \frac{\arctan(ax)}{x^4\left(c+a^2cx^2\right)^3}\,dx$$
$$=\frac{25\,a^5x^5+20\,a^3x^3+105\left(a^7x^7+2\,a^5x^5+a^3x^3\right)\arctan\left(ax\right)^2-8\,ax+2\left(105\,a^6x^6+175\,a^4x^4+56\,a^2x^2-\right.}{48\left(a^4c^3x^7+2\,a^2c^3\right.}$$

input
```
integrate(arctan(a*x)/x^4/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
1/48*(25*a^5*x^5 + 20*a^3*x^3 + 105*(a^7*x^7 + 2*a^5*x^5 + a^3*x^3)*arctan
(a*x)^2 - 8*a*x + 2*(105*a^6*x^6 + 175*a^4*x^4 + 56*a^2*x^2 - 8)*arctan(a*
x) + 80*(a^7*x^7 + 2*a^5*x^5 + a^3*x^3)*log(a^2*x^2 + 1) - 160*(a^7*x^7 +
2*a^5*x^5 + a^3*x^3)*log(x))/(a^4*c^3*x^7 + 2*a^2*c^3*x^5 + c^3*x^3)
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 724 vs. $2(177) = 354$.

Time = 1.76 (sec) , antiderivative size = 724, normalized size of antiderivative = 3.96

$$\int \frac{\arctan(ax)}{x^4 (c + a^2cx^2)^3} \, dx$$
$$= \begin{cases} -\frac{160a^7x^7 \log(x)}{48a^4c^3x^7+96a^2c^3x^5+48c^3x^3} + \frac{80a^7x^7 \log\left(x^2+\frac{1}{a^2}\right)}{48a^4c^3x^7+96a^2c^3x^5+48c^3x^3} + \frac{105a^7x^7 \operatorname{atan}^2(ax)}{48a^4c^3x^7+96a^2c^3x^5+48c^3x^3} + \frac{210a^6x^6 \operatorname{atan}(ax)}{48a^4c^3x^7+96a^2c^3x^5+48c^3x^3} - \\ 0 \end{cases}$$

input
```
integrate(atan(a*x)/x**4/(a**2*c*x**2+c)**3,x)
```

output
```
Piecewise((-160*a**7*x**7*log(x)/(48*a**4*c**3*x**7 + 96*a**2*c**3*x**5 +
48*c**3*x**3) + 80*a**7*x**7*log(x**2 + a**(-2))/(48*a**4*c**3*x**7 + 96*a
**2*c**3*x**5 + 48*c**3*x**3) + 105*a**7*x**7*atan(a*x)**2/(48*a**4*c**3*x
**7 + 96*a**2*c**3*x**5 + 48*c**3*x**3) + 210*a**6*x**6*atan(a*x)/(48*a**4
*c**3*x**7 + 96*a**2*c**3*x**5 + 48*c**3*x**3) - 320*a**5*x**5*log(x)/(48*
a**4*c**3*x**7 + 96*a**2*c**3*x**5 + 48*c**3*x**3) + 160*a**5*x**5*log(x**
2 + a**(-2))/(48*a**4*c**3*x**7 + 96*a**2*c**3*x**5 + 48*c**3*x**3) + 210*
a**5*x**5*atan(a*x)**2/(48*a**4*c**3*x**7 + 96*a**2*c**3*x**5 + 48*c**3*x*
*3) + 25*a**5*x**5/(48*a**4*c**3*x**7 + 96*a**2*c**3*x**5 + 48*c**3*x**3)
+ 350*a**4*x**4*atan(a*x)/(48*a**4*c**3*x**7 + 96*a**2*c**3*x**5 + 48*c**3
*x**3) - 160*a**3*x**3*log(x)/(48*a**4*c**3*x**7 + 96*a**2*c**3*x**5 + 48*
c**3*x**3) + 80*a**3*x**3*log(x**2 + a**(-2))/(48*a**4*c**3*x**7 + 96*a**2
*c**3*x**5 + 48*c**3*x**3) + 105*a**3*x**3*atan(a*x)**2/(48*a**4*c**3*x**7
 + 96*a**2*c**3*x**5 + 48*c**3*x**3) + 20*a**3*x**3/(48*a**4*c**3*x**7 + 9
6*a**2*c**3*x**5 + 48*c**3*x**3) + 112*a**2*x**2*atan(a*x)/(48*a**4*c**3*x
**7 + 96*a**2*c**3*x**5 + 48*c**3*x**3) - 8*a*x/(48*a**4*c**3*x**7 + 96*a*
*2*c**3*x**5 + 48*c**3*x**3) - 16*atan(a*x)/(48*a**4*c**3*x**7 + 96*a**2*c
**3*x**5 + 48*c**3*x**3), Ne(a, 0)), (0, True))
```

## Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.22

$$\int \frac{\arctan(ax)}{x^4 (c + a^2cx^2)^3} dx$$
$$= \frac{1}{24} \left( \frac{105 a^3 \arctan(ax)}{c^3} + \frac{105 a^6x^6 + 175 a^4x^4 + 56 a^2x^2 - 8}{a^4c^3x^7 + 2 a^2c^3x^5 + c^3x^3} \right) \arctan(ax)$$
$$+ \frac{(25 a^4x^4 + 20 a^2x^2 - 105 (a^6x^6 + 2 a^4x^4 + a^2x^2) \arctan(ax)^2 + 80 (a^6x^6 + 2 a^4x^4 + a^2x^2) \log(a^2x^2 +}{48 (a^4c^3x^6 + 2 a^2c^3x^4 + c^3x^2)}$$

input
```
integrate(arctan(a*x)/x^4/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
1/24*(105*a^3*arctan(a*x)/c^3 + (105*a^6*x^6 + 175*a^4*x^4 + 56*a^2*x^2 -
8)/(a^4*c^3*x^7 + 2*a^2*c^3*x^5 + c^3*x^3))*arctan(a*x) + 1/48*(25*a^4*x^4
 + 20*a^2*x^2 - 105*(a^6*x^6 + 2*a^4*x^4 + a^2*x^2)*arctan(a*x)^2 + 80*(a^
6*x^6 + 2*a^4*x^4 + a^2*x^2)*log(a^2*x^2 + 1) - 160*(a^6*x^6 + 2*a^4*x^4 +
 a^2*x^2)*log(x) - 8)*a/(a^4*c^3*x^6 + 2*a^2*c^3*x^4 + c^3*x^2)
```

## Giac [F]

$$\int \frac{\arctan(ax)}{x^4 (c + a^2cx^2)^3} dx = \int \frac{\arctan(ax)}{(a^2cx^2 + c)^3 x^4} dx$$

input
```
integrate(arctan(a*x)/x^4/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
integrate(arctan(a*x)/((a^2*c*x^2 + c)^3*x^4), x)
```

## Mupad [B] (verification not implemented)

Time = 0.85 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.89

$$\int \frac{\arctan(ax)}{x^4 (c+a^2cx^2)^3}\,dx = \frac{\frac{25\,a^5\,x^4}{2}+10\,a^3\,x^2-4\,a}{24\,a^4\,c^3\,x^6+48\,a^2\,c^3\,x^4+24\,c^3\,x^2} + \frac{\mathrm{atan}(a\,x)\,\left(\frac{7\,x^2}{3\,c^3}-\frac{1}{3\,a^2\,c^3}+\frac{175\,a^2\,x^4}{24\,c^3}+\frac{35\,a^4\,x^6}{8\,c^3}\right)}{2\,x^5+\frac{x^3}{a^2}+a^2\,x^7} + \frac{5\,a^3\,\ln\left(a^2\,x^2+1\right)}{3\,c^3} - \frac{10\,a^3\,\ln\left(x\right)}{3\,c^3} + \frac{35\,a^3\,\mathrm{atan}(a\,x)^2}{16\,c^3}$$

input
```
int(atan(a*x)/(x^4*(c + a^2*c*x^2)^3),x)
```

output
```
(10*a^3*x^2 - 4*a + (25*a^5*x^4)/2)/(24*c^3*x^2 + 48*a^2*c^3*x^4 + 24*a^4*
c^3*x^6) + (atan(a*x)*((7*x^2)/(3*c^3) - 1/(3*a^2*c^3) + (175*a^2*x^4)/(24
*c^3) + (35*a^4*x^6)/(8*c^3)))/(2*x^5 + x^3/a^2 + a^2*x^7) + (5*a^3*log(a^
2*x^2 + 1))/(3*c^3) - (10*a^3*log(x))/(3*c^3) + (35*a^3*atan(a*x)^2)/(16*c
^3)
```

## Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.18

$$\int \frac{\arctan(ax)}{x^4 (c+a^2cx^2)^3}\,dx = \frac{210atan(ax)^2\,a^7x^7+420atan(ax)^2\,a^5x^5+210atan(ax)^2\,a^3x^3+420atan(ax)\,a^6x^6+700atan(ax)\,a^4x^4+}{}$$

input
```
int(atan(a*x)/x^4/(a^2*c*x^2+c)^3,x)
```

output
```
(210*atan(a*x)**2*a**7*x**7 + 420*atan(a*x)**2*a**5*x**5 + 210*atan(a*x)**
2*a**3*x**3 + 420*atan(a*x)*a**6*x**6 + 700*atan(a*x)*a**4*x**4 + 224*atan
(a*x)*a**2*x**2 - 32*atan(a*x) + 160*log(a**2*x**2 + 1)*a**7*x**7 + 320*lo
g(a**2*x**2 + 1)*a**5*x**5 + 160*log(a**2*x**2 + 1)*a**3*x**3 - 320*log(x)
*a**7*x**7 - 640*log(x)*a**5*x**5 - 320*log(x)*a**3*x**3 - 25*a**7*x**7 +
15*a**3*x**3 - 16*a*x)/(96*c**3*x**3*(a**4*x**4 + 2*a**2*x**2 + 1))
```

# 3.200 $\int x^3\sqrt{c+a^2cx^2}\arctan(ax)\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 160

$$\int x^3\sqrt{c+a^2cx^2}\arctan(ax)\,dx=\frac{x\sqrt{c+a^2cx^2}}{24a^3}-\frac{x^3\sqrt{c+a^2cx^2}}{20a}-\frac{2\sqrt{c+a^2cx^2}\arctan(ax)}{15a^4}+\frac{x^2\sqrt{c+a^2cx^2}\arctan(ax)}{15a^2}+\frac{1}{5}x^4\sqrt{c+a^2cx^2}\arctan(ax)+\frac{11\sqrt{c}\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right)}{120a^4}$$

output
```
1/24*x*(a^2*c*x^2+c)^(1/2)/a^3-1/20*x^3*(a^2*c*x^2+c)^(1/2)/a-2/15*(a^2*c*
x^2+c)^(1/2)*arctan(a*x)/a^4+1/15*x^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)/a^2+
1/5*x^4*(a^2*c*x^2+c)^(1/2)*arctan(a*x)+11/120*c^(1/2)*arctanh(a*c^(1/2)*x
/(a^2*c*x^2+c)^(1/2))/a^4
```

## Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.66

$$\int x^3\sqrt{c+a^2cx^2}\arctan(ax)\,dx$$
$$=\frac{ax(5-6a^2x^2)\sqrt{c+a^2cx^2}+8\sqrt{c+a^2cx^2}(-2+a^2x^2+3a^4x^4)\arctan(ax)+11\sqrt{c}\log\left(acx+\sqrt{c}\sqrt{c+a\right.}{120a^4}$$

input
```
Integrate[x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x],x]
```

output
```
(a*x*(5 - 6*a^2*x^2)*Sqrt[c + a^2*c*x^2] + 8*Sqrt[c + a^2*c*x^2]*(-2 + a^2
*x^2 + 3*a^4*x^4)*ArcTan[a*x] + 11*Sqrt[c]*Log[a*c*x + Sqrt[c]*Sqrt[c + a^
2*c*x^2]])/(120*a^4)
```

## Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.87, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {5481, 262, 262, 224, 219, 5487, 262, 224, 219, 5465, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\arctan(ax)\sqrt{a^2cx^2+c}\,dx$$

$$\Big\downarrow \text{5481}$$

$$\frac{1}{5}c\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{5}ac\int\frac{x^4}{\sqrt{a^2cx^2+c}}dx+\frac{1}{5}x^4\arctan(ax)\sqrt{a^2cx^2+c}$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{5}c\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{5}ac\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\int\frac{x^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)+\frac{1}{5}x^4\arctan(ax)\sqrt{a^2cx^2+c}$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{5}c\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{5}ac\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int \frac{1}{\sqrt{a^2cx^2+c}}dx}{2a^2}\right)}{4a^2}\right)+\frac{1}{5}x^4\arctan(ax)\sqrt{a^2cx^2+c}$$

$\downarrow$ 224

$$\frac{1}{5}c\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{5}ac\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int \frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}}{2a^2}\right)}{4a^2}\right)+\frac{1}{5}x^4\arctan(ax)\sqrt{a^2cx^2+c}$$

$\downarrow$ 219

$$\frac{1}{5}c\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{5}x^4\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{5}ac\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}\right)$$

$\downarrow$ 5487

$$\frac{1}{5}c\left(-\frac{2\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}-\frac{\int \frac{x^2}{\sqrt{a^2cx^2+c}}dx}{3a}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}\right)+\frac{1}{5}x^4\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{5}ac\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}\right)$$

$\downarrow$ 262

$$\frac{1}{5}c\left(-\frac{2\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int \frac{1}{\sqrt{a^2cx^2+c}}dx}{2a^2}}{3a}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}\right)+$$
$$\frac{1}{5}x^4\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{5}ac\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}\right)$$

$$\Big\downarrow \ 224$$

$$\frac{1}{5}c\left(-\frac{2\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int \frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}}{2a^2}}{3a}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}\right)+$$
$$\frac{1}{5}x^4\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{5}ac\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}\right)$$

$$\Big\downarrow \ 219$$

$$\frac{1}{5}c\left(-\frac{2\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}\right)+$$
$$\frac{1}{5}x^4\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{5}ac\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}\right)$$

$$\Big\downarrow \ 5465$$

$$\frac{1}{5}c\left(-\frac{2\left(\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\int\frac{1}{\sqrt{a^2cx^2+c}}dx}{a}\right)}{3a^2}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}\right)+$$
$$\frac{1}{5}x^4\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{5}ac\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}\right)$$

$\downarrow$ 224

$$\frac{1}{5}c\left(-\frac{2\left(\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\int\frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}}{a}\right)}{3a^2}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}\right)$$
$$\frac{1}{5}x^4\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{5}ac\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}\right)$$

$\downarrow$ 219

$$\frac{1}{5}x^4\arctan(ax)\sqrt{a^2cx^2+c}+$$
$$\frac{1}{5}c\left(-\frac{2\left(\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a^2\sqrt{c}}\right)}{3a^2}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}\right)$$
$$\frac{1}{5}ac\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}\right)$$

input
```
Int[x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x],x]
```

output
```
(x^4*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/5 - (a*c*((x^3*Sqrt[c + a^2*c*x^2])/
(4*a^2*c) - (3*((x*Sqrt[c + a^2*c*x^2])/(2*a^2*c) - ArcTanh[(a*Sqrt[c]*x)/
Sqrt[c + a^2*c*x^2]]/(2*a^3*Sqrt[c])))/(4*a^2)))/5 + (c*((x^2*Sqrt[c + a^2
*c*x^2]*ArcTan[a*x])/(3*a^2*c) - ((x*Sqrt[c + a^2*c*x^2])/(2*a^2*c) - ArcT
anh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]]/(2*a^3*Sqrt[c]))/(3*a) - (2*((Sqrt[
c + a^2*c*x^2]*ArcTan[a*x])/(a^2*c) - ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c
*x^2]]/(a^2*Sqrt[c])))/(3*a^2)))/5
```

### Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5481
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTan[c*x
])/(f*(m + 2))), x] + (Simp[d/(m + 2)   Int[(f*x)^m*((a + b*ArcTan[c*x])/Sq
rt[d + e*x^2]), x], x] - Simp[b*c*(d/(f*(m + 2)))   Int[(f*x)^(m + 1)/Sqrt[
d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] &&
NeQ[m, -2]
```

rule 5487
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcTan[c*x])^p/(c^2*d*m)), x] + (-Simp[b*f*(p/(c*m))   Int[(f*x)^(m - 1)*((
a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[f^2*((m - 1)/(c^
2*m))   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2]), x], x])
/; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.35 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.10

| method | result |
|---|---|
| default | $\frac{\sqrt{c(ax-i)(ax+i)}\,(24x^4\arctan(ax)a^4-6a^3x^3+8x^2a^2\arctan(ax)+5ax-16\arctan(ax))}{120a^4}+\frac{11\sqrt{c(ax-i)(ax+i)}\,\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}+i\right)}{120a^4\sqrt{a^2x^2+1}}$ |

input
```
int(x^3*(a^2*c*x^2+c)^(1/2)*arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/120/a^4*(c*(a*x-I)*(a*x+I))^(1/2)*(24*x^4*arctan(a*x)*a^4-6*a^3*x^3+8*x^
2*a^2*arctan(a*x)+5*a*x-16*arctan(a*x))+11/120/a^4*(c*(a*x-I)*(a*x+I))^(1/
2)*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+I)/(a^2*x^2+1)^(1/2)-11/120/a^4*(c*(a*x-
I)*(a*x+I))^(1/2)*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-I)/(a^2*x^2+1)^(1/2)
```

## Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.59

$$\int x^3\sqrt{c+a^2cx^2}\arctan(ax)\,dx = -\frac{2\left(6\,a^3x^3-5\,ax-8\left(3\,a^4x^4+a^2x^2-2\right)\arctan\left(ax\right)\right)\sqrt{a^2cx^2+c}-11\,\sqrt{c}\log\left(-2\,a^2cx^2-2\,\sqrt{a^2cx^2+\right.}{240\,a^4}$$

input
```
integrate(x^3*(a^2*c*x^2+c)^(1/2)*arctan(a*x),x, algorithm="fricas")
```

output
```
-1/240*(2*(6*a^3*x^3 - 5*a*x - 8*(3*a^4*x^4 + a^2*x^2 - 2)*arctan(a*x))*sq
rt(a^2*c*x^2 + c) - 11*sqrt(c)*log(-2*a^2*c*x^2 - 2*sqrt(a^2*c*x^2 + c)*a*
sqrt(c)*x - c))/a^4
```

## Sympy [F]

$$\int x^3\sqrt{c+a^2cx^2}\arctan(ax)\,dx = \int x^3\sqrt{c\left(a^2x^2+1\right)}\,\mathrm{atan}\left(ax\right)dx$$

input
```
integrate(x**3*(a**2*c*x**2+c)**(1/2)*atan(a*x),x)
```

output
```
Integral(x**3*sqrt(c*(a**2*x**2 + 1))*atan(a*x), x)
```

## Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.79

$$\int x^3\sqrt{c+a^2cx^2}\arctan(ax)\,dx = -\frac{1}{120}\left(a\left(\frac{3\left(\frac{2\left(a^2x^2+1\right)^{\frac{3}{2}}x}{a^2}-\frac{\sqrt{a^2x^2+1}x}{a^2}-\frac{\operatorname{arsinh}(ax)}{a^3}\right)}{a^2}-\frac{8\left(\sqrt{a^2x^2+1}x+\frac{\operatorname{arsinh}(ax)}{a}\right)}{a^4}\right)-8\left(\frac{3\left(a^2x^2+1\right)}{a^2}\right.\right.$$

input
```
integrate(x^3*(a^2*c*x^2+c)^(1/2)*arctan(a*x),x, algorithm="maxima")
```

output
```
-1/120*(a*(3*(2*(a^2*x^2 + 1)^(3/2)*x/a^2 - sqrt(a^2*x^2 + 1)*x/a^2 - arcs
inh(a*x)/a^3)/a^2 - 8*(sqrt(a^2*x^2 + 1)*x + arcsinh(a*x)/a)/a^4) - 8*(3*(
a^2*x^2 + 1)^(3/2)*x^2/a^2 - 2*(a^2*x^2 + 1)^(3/2)/a^4)*arctan(a*x))*sqrt(
c)
```

## Giac [F(-2)]

Exception generated.

$$\int x^3\sqrt{c+a^2cx^2}\arctan(ax)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a^2*c*x^2+c)^(1/2)*arctan(a*x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{c+a^2cx^2}\arctan(ax)\,dx = \int x^3\,\mathrm{atan}(a\,x)\,\sqrt{c\,a^2\,x^2+c}\,dx$$

input
```
int(x^3*atan(a*x)*(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(x^3*atan(a*x)*(c + a^2*c*x^2)^(1/2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.73

$$\int x^3\sqrt{c+a^2cx^2}\arctan(ax)\,dx$$
$$=\frac{\sqrt{c}\,\big(24\sqrt{a^2x^2+1}\,atan(ax)\,a^4x^4+8\sqrt{a^2x^2+1}\,atan(ax)\,a^2x^2-16\sqrt{a^2x^2+1}\,atan(ax)-6\sqrt{a^2x^2+1}\,a^3}{120a^4}$$

input
```
int(x^3*(a^2*c*x^2+c)^(1/2)*atan(a*x),x)
```

output
```
(sqrt(c)*(24*sqrt(a**2*x**2 + 1)*atan(a*x)*a**4*x**4 + 8*sqrt(a**2*x**2 +
1)*atan(a*x)*a**2*x**2 - 16*sqrt(a**2*x**2 + 1)*atan(a*x) - 6*sqrt(a**2*x*
*2 + 1)*a**3*x**3 + 5*sqrt(a**2*x**2 + 1)*a*x + 11*log(sqrt(a**2*x**2 + 1)
 + a*x)))/(120*a**4)
```

# 3.201 $\int x^2\sqrt{c+a^2cx^2}\arctan(ax)\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 298

$$\int x^2\sqrt{c+a^2cx^2}\arctan(ax)\,dx=\frac{\sqrt{c+a^2cx^2}}{8a^3}-\frac{(c+a^2cx^2)^{3/2}}{12a^3c}+\frac{x\sqrt{c+a^2cx^2}\arctan(ax)}{8a^2}+\frac{1}{4}x^3\sqrt{c+a^2cx^2}\arctan(ax)+\frac{ic\sqrt{1+a^2x^2}\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{4a^3\sqrt{c+a^2cx^2}}-\frac{ic\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{8a^3\sqrt{c+a^2cx^2}}+\frac{ic\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{8a^3\sqrt{c+a^2cx^2}}$$

output
```
1/8*(a^2*c*x^2+c)^(1/2)/a^3-1/12*(a^2*c*x^2+c)^(3/2)/a^3/c+1/8*x*(a^2*c*x^
2+c)^(1/2)*arctan(a*x)/a^2+1/4*x^3*(a^2*c*x^2+c)^(1/2)*arctan(a*x)+1/4*I*c
*(a^2*x^2+1)^(1/2)*arctan(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/a^3
/(a^2*c*x^2+c)^(1/2)-1/8*I*c*(a^2*x^2+1)^(1/2)*polylog(2,-I*(1+I*a*x)^(1/2
)/(1-I*a*x)^(1/2))/a^3/(a^2*c*x^2+c)^(1/2)+1/8*I*c*(a^2*x^2+1)^(1/2)*polyl
og(2,I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/a^3/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (warning: unable to verify)

Time = 2.03 (sec) , antiderivative size = 278, normalized size of antiderivative = 0.93

$$\int x^2\sqrt{c+a^2cx^2}\arctan(ax)\,dx$$
$$=\frac{\sqrt{c\,(1+a^2x^2)}\Big(-6i\,\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)+6i\,\text{PolyLog}\left(2,ie^{i\arctan(ax)}\right)-\frac{1}{4}(1+a^2x^2)^2\left(-\frac{2}{\sqrt{1+a^2x^2}}\right.}{}$$

input
```
Integrate[x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x],x]
```

output
```
(Sqrt[c*(1 + a^2*x^2)]*((-6*I)*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + (6*I)*
PolyLog[2, I*E^(I*ArcTan[a*x])] - ((1 + a^2*x^2)^2*(-2/Sqrt[1 + a^2*x^2] -
 6*Cos[3*ArcTan[a*x]] + 3*ArcTan[a*x]*((-14*a*x)/Sqrt[1 + a^2*x^2] + 3*Log
[1 - I*E^(I*ArcTan[a*x])] + 4*Cos[2*ArcTan[a*x]]*(Log[1 - I*E^(I*ArcTan[a*
x])] - Log[1 + I*E^(I*ArcTan[a*x])]) + Cos[4*ArcTan[a*x]]*(Log[1 - I*E^(I*
ArcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])]) - 3*Log[1 + I*E^(I*ArcTan[a*
x])] + 2*Sin[3*ArcTan[a*x]])))/4))/(48*a^3*Sqrt[1 + a^2*x^2])
```

## Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 284, normalized size of antiderivative = 0.95, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {5481, 243, 53, 2009, 5487, 241, 5425, 5421}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\arctan(ax)\sqrt{a^2cx^2+c}\,dx$$

$$\Big\downarrow \text{5481}$$

$$\frac{1}{4}c\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{4}ac\int\frac{x^3}{\sqrt{a^2cx^2+c}}dx+\frac{1}{4}x^3\arctan(ax)\sqrt{a^2cx^2+c}$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{4}c\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{8}ac\int\frac{x^2}{\sqrt{a^2cx^2+c}}dx^2+\frac{1}{4}x^3\arctan(ax)\sqrt{a^2cx^2+c}$$

$$\Big\downarrow \text{53}$$

$$\frac{1}{4}c\int \frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{8}ac\int \left(\frac{\sqrt{a^2cx^2+c}}{a^2c}-\frac{1}{a^2\sqrt{a^2cx^2+c}}\right)dx^2+\frac{1}{4}x^3\arctan(ax)\sqrt{a^2cx^2+c}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}c\int \frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{4}x^3\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{8}ac\left(\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}\right)$$

$$\Big\downarrow \text{5487}$$

$$\frac{1}{4}c\left(-\frac{\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a^2}-\frac{\int \frac{x}{\sqrt{a^2cx^2+c}}dx}{2a}+\frac{x\arctan(ax)\sqrt{a^2cx^2+c}}{2a^2c}\right)+\frac{1}{4}x^3\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{8}ac\left(\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}\right)$$

$$\Big\downarrow \text{241}$$

$$\frac{1}{4}c\left(-\frac{\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a^2}+\frac{x\arctan(ax)\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\sqrt{a^2cx^2+c}}{2a^3c}\right)+\frac{1}{4}x^3\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{8}ac\left(\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}\right)$$

$$\Big\downarrow \text{5425}$$

$$\frac{1}{4}c\left(-\frac{\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{2a^2\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\sqrt{a^2cx^2+c}}{2a^3c}\right)+\frac{1}{4}x^3\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{8}ac\left(\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}\right)$$

$$\Big\downarrow \text{5421}$$

$$\frac{1}{4}x^3\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{8}ac\left(\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}\right)+\frac{1}{4}c\left(-\frac{\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2a^2\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)\sqrt{a^2}}{2a^2c}\right.$$

input
```
Int[x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x],x]
```

output
```
-1/8*(a*c*((-2*Sqrt[c + a^2*c*x^2])/(a^4*c) + (2*(c + a^2*c*x^2)^(3/2))/(3
*a^4*c^2))) + (x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/4 + (c*(-1/2*Sqrt[c +
a^2*c*x^2]/(a^3*c) + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(2*a^2*c) - (Sqrt
[1 + a^2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]]
)/a + (I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*PolyLo
g[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/(2*a^2*Sqrt[c + a^2*c*x^2])
))/4
```

### Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5481
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTan[c*x
])/(f*(m + 2))), x] + (Simp[d/(m + 2)   Int[(f*x)^m*((a + b*ArcTan[c*x])/Sq
rt[d + e*x^2]), x], x] - Simp[b*c*(d/(f*(m + 2)))   Int[(f*x)^(m + 1)/Sqrt[
d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] &&
NeQ[m, -2]
```

rule 5487
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcTan[c*x])^p/(c^2*d*m)), x] + (-Simp[b*f*(p/(c*m))   Int[(f*x)^(m - 1)*((
a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[f^2*((m - 1)/(c^
2*m))   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2]), x], x])
/; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

## Maple [A] (verified)

Time = 1.42 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.67

| method | result |
|---|---|
| default | $\frac{\sqrt{c(ax-i)(ax+i)}\,(6\arctan(ax)x^3a^3-2a^2x^2+3\arctan(ax)ax+1)}{24a^3}+\frac{\sqrt{c(ax-i)(ax+i)}\left(\arctan(ax)\ln\left(1+\frac{i(iax+1)}{\sqrt{a^2x^2+1}}\right)-\arctan(a\right.}{8a}$ |

input
```
int(x^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/24/a^3*(c*(a*x-I)*(a*x+I))^(1/2)*(6*arctan(a*x)*x^3*a^3-2*a^2*x^2+3*arct
an(a*x)*a*x+1)+1/8*(c*(a*x-I)*(a*x+I))^(1/2)*(arctan(a*x)*ln(1+I*(1+I*a*x)
/(a^2*x^2+1)^(1/2))-arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+I*dilo
g(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2)
))/a^3/(a^2*x^2+1)^(1/2)
```

## Fricas [F]

$$\int x^2\sqrt{c+a^2cx^2}\arctan(ax)\,dx=\int \sqrt{a^2cx^2+c}x^2\arctan\left(ax\right)\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^2*arctan(a*x), x)
```

## Sympy [F]

$$\int x^2\sqrt{c+a^2cx^2}\arctan(ax)\,dx=\int x^2\sqrt{c\left(a^2x^2+1\right)}\operatorname{atan}\left(ax\right)dx$$

input
```
integrate(x**2*(a**2*c*x**2+c)**(1/2)*atan(a*x),x)
```

output
```
Integral(x**2*sqrt(c*(a**2*x**2 + 1))*atan(a*x), x)
```

## Maxima [F]

$$\int x^2\sqrt{c+a^2cx^2}\arctan(ax)\,dx=\int \sqrt{a^2cx^2+c}x^2\arctan\left(ax\right)\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*x^2*arctan(a*x), x)
```

## Giac [F]

$$\int x^2\sqrt{c+a^2cx^2}\arctan(ax)\,dx = \int \sqrt{a^2cx^2+c}\,x^2\arctan\left(ax\right)\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x),x, algorithm="giac")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*x^2*arctan(a*x), x)
```

## Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{c+a^2cx^2}\arctan(ax)\,dx = \int x^2\,\mathrm{atan}(a\,x)\,\sqrt{c\,a^2\,x^2+c}\,dx$$

input
```
int(x^2*atan(a*x)*(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(x^2*atan(a*x)*(c + a^2*c*x^2)^(1/2), x)
```

## Reduce [F]

$$\int x^2\sqrt{c+a^2cx^2}\arctan(ax)\,dx = \sqrt{c}\left(\int\sqrt{a^2x^2+1}\,atan(ax)\,x^2dx\right)$$

input
```
int(x^2*(a^2*c*x^2+c)^(1/2)*atan(a*x),x)
```

output
```
sqrt(c)*int(sqrt(a**2*x**2 + 1)*atan(a*x)*x**2,x)
```

# 3.202 $\int x\sqrt{c+a^2cx^2}\arctan(ax)\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 86

$$\int x\sqrt{c+a^2cx^2}\arctan(ax)\,dx = -\frac{x\sqrt{c+a^2cx^2}}{6a}+\frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{3a^2c}-\frac{\sqrt{c}\,\mathrm{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right)}{6a^2}$$

output
```
-1/6*x*(a^2*c*x^2+c)^(1/2)/a+1/3*(a^2*c*x^2+c)^(3/2)*arctan(a*x)/a^2/c-1/6
*c^(1/2)*arctanh(a*c^(1/2)*x/(a^2*c*x^2+c)^(1/2))/a^2
```

## Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00

$$\int x\sqrt{c+a^2cx^2}\arctan(ax)\,dx = -\frac{ax\sqrt{c+a^2cx^2}-2(1+a^2x^2)\sqrt{c+a^2cx^2}\arctan(ax)+\sqrt{c}\log\left(acx+\sqrt{c}\sqrt{c+a^2cx^2}\right)}{6a^2}$$

input
```
Integrate[x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x],x]
```

output
```
-1/6*(a*x*Sqrt[c + a^2*c*x^2] - 2*(1 + a^2*x^2)*Sqrt[c + a^2*c*x^2]*ArcTan
[a*x] + Sqrt[c]*Log[a*c*x + Sqrt[c]*Sqrt[c + a^2*c*x^2]])/a^2
```

## Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5465, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\arctan(ax)\sqrt{a^2cx^2+c}\,dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\int\sqrt{a^2cx^2+c}dx}{3a}$$

$$\Big\downarrow \text{211}$$

$$\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{1}{2}c\int\frac{1}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\sqrt{a^2cx^2+c}}{3a}$$

$$\Big\downarrow \text{224}$$

$$\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{1}{2}c\int\frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}+\frac{1}{2}x\sqrt{a^2cx^2+c}}{3a}$$

$$\Big\downarrow \text{219}$$

$$\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}}{3a}$$

input
```
Int[x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x],x]
```

output
```
((c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/(3*a^2*c) - ((x*Sqrt[c + a^2*c*x^2])/2
 + (Sqrt[c]*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/(2*a))/(3*a)
```

### Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.11 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.81

| method | result |
|---|---|
| default | $\frac{\sqrt{c(ax-i)(ax+i)}\,(2x^2a^2\arctan(ax)-ax+2\arctan(ax))}{6a^2}+\frac{\sqrt{c(ax-i)(ax+i)}\,\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}-i\right)}{6a^2\sqrt{a^2x^2+1}}-\frac{\sqrt{c(ax-i)(ax+i)}\,\ln\left(\frac{iax+}{\sqrt{a^2x^2}}\right.}{6a^2\sqrt{a^2x^2+1}}$ |

input
```
int(x*(a^2*c*x^2+c)^(1/2)*arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/6/a^2*(c*(a*x-I)*(a*x+I))^(1/2)*(2*x^2*a^2*arctan(a*x)-a*x+2*arctan(a*x)
)+1/6/a^2*(c*(a*x-I)*(a*x+I))^(1/2)*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-I)/(a^2
*x^2+1)^(1/2)-1/6/a^2*(c*(a*x-I)*(a*x+I))^(1/2)*ln((1+I*a*x)/(a^2*x^2+1)^(
1/2)+I)/(a^2*x^2+1)^(1/2)
```

## Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.90

$$\int x\sqrt{c+a^2cx^2}\arctan(ax)\,dx = -\frac{2\sqrt{a^2cx^2+c}(ax-2\,(a^2x^2+1)\arctan{(ax)})-\sqrt{c}\log\left(-2\,a^2cx^2+2\sqrt{a^2cx^2+c}a\sqrt{c}x-c\right)}{12\,a^2}$$

input
```
integrate(x*(a^2*c*x^2+c)^(1/2)*arctan(a*x),x, algorithm="fricas")
```

output
```
-1/12*(2*sqrt(a^2*c*x^2 + c)*(a*x - 2*(a^2*x^2 + 1)*arctan(a*x)) - sqrt(c)
*log(-2*a^2*c*x^2 + 2*sqrt(a^2*c*x^2 + c)*a*sqrt(c)*x - c))/a^2
```

## Sympy [F]

$$\int x\sqrt{c+a^2cx^2}\arctan(ax)\,dx = \int x\sqrt{c\,(a^2x^2+1)}\operatorname{atan}{(ax)}\,dx$$

input
```
integrate(x*(a**2*c*x**2+c)**(1/2)*atan(a*x),x)
```

output
```
Integral(x*sqrt(c*(a**2*x**2 + 1))*atan(a*x), x)
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 260 vs. $2(70) = 140$.

Time = 0.20 (sec) , antiderivative size = 260, normalized size of antiderivative = 3.02

$$\int x\sqrt{c+a^2cx^2}\arctan(ax)\,dx$$
$$= \frac{4\,(a^2x^2+1)^{\frac{3}{2}}\sqrt{c}\arctan{(ax)}-2\,(a^4x^4+10\,a^2x^2+9)^{\frac{1}{4}}\left(ax\cos\left(\frac{1}{2}\arctan{(4\,ax,-a^2x^2+3)}\right)+2\sin\left(\frac{1}{2}\,a\right.\right.}{}$$

input
```
integrate(x*(a^2*c*x^2+c)^(1/2)*arctan(a*x),x, algorithm="maxima")
```

output
```
1/12*(4*(a^2*x^2 + 1)^(3/2)*sqrt(c)*arctan(a*x) - 2*(a^4*x^4 + 10*a^2*x^2
+ 9)^(1/4)*(a*x*cos(1/2*arctan2(4*a*x, -a^2*x^2 + 3)) + 2*sin(1/2*arctan2(
4*a*x, -a^2*x^2 + 3)))*sqrt(c) + sqrt(c)*(arctan2((a^4*x^4 + 10*a^2*x^2 +
9)^(1/4)*sin(1/2*arctan2(4*a*x, a^2*x^2 - 3)) + 2, a*x + (a^4*x^4 + 10*a^2
*x^2 + 9)^(1/4)*cos(1/2*arctan2(4*a*x, a^2*x^2 - 3))) + arctan2((a^4*x^4 +
 10*a^2*x^2 + 9)^(1/4)*sin(1/2*arctan2(4*a*x, a^2*x^2 - 3)) - 2, -a*x + (a
^4*x^4 + 10*a^2*x^2 + 9)^(1/4)*cos(1/2*arctan2(4*a*x, a^2*x^2 - 3)))))/a^2
```

## Giac [F(-2)]

Exception generated.

$$\int x\sqrt{c+a^2cx^2}\arctan(ax)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(1/2)*arctan(a*x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int x\sqrt{c+a^2cx^2}\arctan(ax)\,dx = \int x\,\mathrm{atan}(a\,x)\,\sqrt{c\,a^2\,x^2+c}\,dx$$

input
```
int(x*atan(a*x)*(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(x*atan(a*x)*(c + a^2*c*x^2)^(1/2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.90

$$\int x\sqrt{c+a^2cx^2}\arctan(ax)\,dx$$
$$=\frac{\sqrt{c}\left(2\sqrt{a^2x^2+1}\,atan(ax)\,a^2x^2+2\sqrt{a^2x^2+1}\,atan(ax)-\sqrt{a^2x^2+1}\,ax-\log\left(\sqrt{a^2x^2+1}+ax\right)\right)}{6a^2}$$

input

```
int(x*(a^2*c*x^2+c)^(1/2)*atan(a*x),x)
```

output

```
(sqrt(c)*(2*sqrt(a**2*x**2 + 1)*atan(a*x)*a**2*x**2 + 2*sqrt(a**2*x**2 + 1
)*atan(a*x) - sqrt(a**2*x**2 + 1)*a*x - log(sqrt(a**2*x**2 + 1) + a*x)))/(
6*a**2)
```

# 3.203 $\int \sqrt{c+a^2cx^2}\arctan(ax)\,dx$



## Optimal result

Integrand size = 19, antiderivative size = 244

$$\int \sqrt{c+a^2cx^2}\arctan(ax)\,dx = -\frac{\sqrt{c+a^2cx^2}}{2a}+\frac{1}{2}x\sqrt{c+a^2cx^2}\arctan(ax) - \frac{ic\sqrt{1+a^2x^2}\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a\sqrt{c+a^2cx^2}} + \frac{ic\sqrt{1+a^2x^2}\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2a\sqrt{c+a^2cx^2}} - \frac{ic\sqrt{1+a^2x^2}\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2a\sqrt{c+a^2cx^2}}$$

output
```
-1/2*(a^2*c*x^2+c)^(1/2)/a+1/2*x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)-I*c*(a^2*
x^2+1)^(1/2)*arctan(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/a/(a^2*c*
x^2+c)^(1/2)+1/2*I*c*(a^2*x^2+1)^(1/2)*polylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a
*x)^(1/2))/a/(a^2*c*x^2+c)^(1/2)-1/2*I*c*(a^2*x^2+1)^(1/2)*polylog(2,I*(1+
I*a*x)^(1/2)/(1-I*a*x)^(1/2))/a/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.58

$$\int \sqrt{c+a^2cx^2}\arctan(ax)\,dx$$
$$= \frac{\sqrt{c(1+a^2x^2)}\left(\sqrt{1+a^2x^2}(-1+ax\arctan(ax))+\arctan(ax)\left(\log\left(1-ie^{i\arctan(ax)}\right)-\log\left(1+ie^{i\arctan(ax)}\right)\right)\right.}{2a\sqrt{1+a^2x^2}}$$

input
```
Integrate[Sqrt[c + a^2*c*x^2]*ArcTan[a*x],x]
```

output
```
(Sqrt[c*(1 + a^2*x^2)]*(Sqrt[1 + a^2*x^2]*(-1 + a*x*ArcTan[a*x]) + ArcTan[
a*x]*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])]) + I*Pol
yLog[2, (-I)*E^(I*ArcTan[a*x])] - I*PolyLog[2, I*E^(I*ArcTan[a*x])]))/(2*a
*Sqrt[1 + a^2*x^2])
```

## Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.77, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {5413, 5425, 5421}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(ax)\sqrt{a^2cx^2+c}\,dx$$

$$\Big\downarrow \text{5413}$$

$$\frac{1}{2}c\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}$$

$$\Big\downarrow \text{5425}$$

$$\frac{c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}$$

$$\Big\downarrow \text{5421}$$

$$\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}$$

input
```
Int[Sqrt[c + a^2*c*x^2]*ArcTan[a*x],x]
```

output
```
-1/2*Sqrt[c + a^2*c*x^2]/a + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/2 + (c*Sq
rt[1 + a^2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x
]])/a + (I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*Poly
Log[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/(2*Sqrt[c + a^2*c*x^2])
```

## Defintions of rubi rules used

rule 5413
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Simp[x*(d + e*x^2)
^q*((a + b*ArcTan[c*x])/(2*q + 1)), x] + Simp[2*d*(q/(2*q + 1))   Int[(d +
e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0]
```

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

## Maple [A] (verified)

Time = 1.31 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.73

| method | result |
|---|---|
| default | $\frac{\sqrt{c(ax-i)(ax+i)}\,(\arctan(ax)ax-1)}{2a}-\frac{\sqrt{c(ax-i)(ax+i)}\left(\arctan(ax)\ln\left(1+\frac{i(iax+1)}{\sqrt{a^2x^2+1}}\right)-\arctan(ax)\ln\left(1-\frac{i(iax+1)}{\sqrt{a^2x^2+1}}\right)+i\,\mathrm{dilog}\right.}{2a\sqrt{a^2x^2+1}}$ |

input
```
int((a^2*c*x^2+c)^(1/2)*arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/2/a*(c*(a*x-I)*(a*x+I))^(1/2)*(arctan(a*x)*a*x-1)-1/2*(c*(a*x-I)*(a*x+I)
)^(1/2)*(arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-arctan(a*x)*ln(1-
I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+I*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*
dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/a/(a^2*x^2+1)^(1/2)
```

## Fricas [F]

$$\int \sqrt{c+a^2cx^2}\arctan(ax)\,dx=\int \sqrt{a^2cx^2+c}\arctan(ax)\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)*arctan(a*x),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arctan(a*x), x)
```

## Sympy [F]

$$\int \sqrt{c+a^2cx^2}\arctan(ax)\,dx=\int \sqrt{c(a^2x^2+1)}\operatorname{atan}(ax)\,dx$$

input
```
integrate((a**2*c*x**2+c)**(1/2)*atan(a*x),x)
```

output
```
Integral(sqrt(c*(a**2*x**2 + 1))*atan(a*x), x)
```

## Maxima [F]

$$\int \sqrt{c+a^2cx^2}\arctan(ax)\,dx = \int \sqrt{a^2cx^2+c}\arctan(ax)\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)*arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*arctan(a*x), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \sqrt{c+a^2cx^2}\arctan(ax)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(1/2)*arctan(a*x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+a^2cx^2}\arctan(ax)\,dx = \int \mathrm{atan}(a\,x)\,\sqrt{c\,a^2\,x^2+c}\,dx$$

input
```
int(atan(a*x)*(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(atan(a*x)*(c + a^2*c*x^2)^(1/2), x)
```

## Reduce [F]

$$\int \sqrt{c+a^2cx^2}\arctan(ax)\,dx=\sqrt{c}\left(\int \sqrt{a^2x^2+1}\,atan(ax)\,dx\right)$$

input
```
int((a^2*c*x^2+c)^(1/2)*atan(a*x),x)
```

output
```
sqrt(c)*int(sqrt(a**2*x**2 + 1)*atan(a*x),x)
```

# 3.204 $\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{x}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 229

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{x}\,dx = \sqrt{c+a^2cx^2}\arctan(ax) - \frac{2c\sqrt{1+a^2x^2}\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} - \sqrt{c}\,\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right) + \frac{ic\sqrt{1+a^2x^2}\,\text{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} - \frac{ic\sqrt{1+a^2x^2}\,\text{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}}$$

output
```
(a^2*c*x^2+c)^(1/2)*arctan(a*x)-2*c*(a^2*x^2+1)^(1/2)*arctan(a*x)*arctanh(
(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/(a^2*c*x^2+c)^(1/2)-c^(1/2)*arctanh(a*c^(
1/2)*x/(a^2*c*x^2+c)^(1/2))+I*c*(a^2*x^2+1)^(1/2)*polylog(2,-(1+I*a*x)^(1/
2)/(1-I*a*x)^(1/2))/(a^2*c*x^2+c)^(1/2)-I*c*(a^2*x^2+1)^(1/2)*polylog(2,(1
+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.72

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{x}\,dx$$
$$= \frac{\sqrt{c+a^2cx^2}\left(\sqrt{1+a^2x^2}\arctan(ax)+\arctan(ax)\log\left(1-e^{i\arctan(ax)}\right)-\arctan(ax)\log\left(1+e^{i\arctan(ax)}\right)\right.}{}$$

input
```
Integrate[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/x,x]
```

output
```
(Sqrt[c + a^2*c*x^2]*(Sqrt[1 + a^2*x^2]*ArcTan[a*x] + ArcTan[a*x]*Log[1 -
E^(I*ArcTan[a*x])] - ArcTan[a*x]*Log[1 + E^(I*ArcTan[a*x])] + Log[Cos[ArcT
an[a*x]/2] - Sin[ArcTan[a*x]/2]] - Log[Cos[ArcTan[a*x]/2] + Sin[ArcTan[a*x
]/2]] + I*PolyLog[2, -E^(I*ArcTan[a*x])] - I*PolyLog[2, E^(I*ArcTan[a*x])]
))/Sqrt[1 + a^2*x^2]
```

## Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.76, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {5481, 224, 219, 5493, 5489}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x}\,dx$$

$$\Big\downarrow \; 5481$$

$$c\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx - ac\int \frac{1}{\sqrt{a^2cx^2+c}}dx + \arctan(ax)\sqrt{a^2cx^2+c}$$

$$\Big\downarrow \; 224$$

$$c\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx - ac\int \frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}} + \arctan(ax)\sqrt{a^2cx^2+c}$$

$$\Big\downarrow \; 219$$

$$c\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx+\arctan(ax)\sqrt{a^2cx^2+c}-\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \text{5493}$$

$$\frac{c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+\arctan(ax)\sqrt{a^2cx^2+c}-\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \text{5489}$$

$$\frac{c\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{PolyLog}\left(2,\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)\right)}{\sqrt{a^2cx^2+c}}+\arctan(ax)\sqrt{a^2cx^2+c}-\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)$$

input
```
Int[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/x,x]
```

output
```
Sqrt[c + a^2*c*x^2]*ArcTan[a*x] - Sqrt[c]*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a
^2*c*x^2]] + (c*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/
Sqrt[1 - I*a*x]] + I*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])] - I*Po
lyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]]))/Sqrt[c + a^2*c*x^2]
```

## Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 5481
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTan[c*x
])/(f*(m + 2))), x] + (Simp[d/(m + 2)   Int[(f*x)^m*((a + b*ArcTan[c*x])/Sq
rt[d + e*x^2]), x], x] - Simp[b*c*(d/(f*(m + 2)))   Int[(f*x)^(m + 1)/Sqrt[
d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] &&
NeQ[m, -2]
```

rule 5489
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sq
rt[1 - I*c*x]], x] + (Simp[I*(b/Sqrt[d])*PolyLog[2, -Sqrt[1 + I*c*x]/Sqrt[1
 - I*c*x]], x] - Simp[I*(b/Sqrt[d])*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x]], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

## Maple [A] (verified)

Time = 1.49 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.66

| method | result |
|---|---|
| default | $\sqrt{c\left(ax-i\right)\left(ax+i\right)}\,\arctan\left(ax\right)-\frac{\sqrt{c(ax-i)(ax+i)}\left(\arctan(ax)\ln\left(1+\frac{iax+1}{\sqrt{a^2x^2+1}}\right)-i\,\mathrm{dilog}\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)-i\,\mathrm{dilog}\left(1\right.\right.}{\sqrt{a^2x^2+1}}$ |

input
```
int((a^2*c*x^2+c)^(1/2)*arctan(a*x)/x,x,method=_RETURNVERBOSE)
```

output
```
(c*(a*x-I)*(a*x+I))^(1/2)*arctan(a*x)-(c*(a*x-I)*(a*x+I))^(1/2)*(arctan(a*
x)*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*dilog((1+I*a*x)/(a^2*x^2+1)^(1/2))-
I*dilog(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*I*arctan((1+I*a*x)/(a^2*x^2+1)^(1
/2)))/(a^2*x^2+1)^(1/2)
```

## Fricas [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{x}\,dx=\int \frac{\sqrt{a^2cx^2+c}\arctan{(ax)}}{x}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)*arctan(a*x)/x,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)/x, x)
```

## Sympy [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{x}\,dx=\int \frac{\sqrt{c\,(a^2x^2+1)}\,\operatorname{atan}{(ax)}}{x}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(1/2)*atan(a*x)/x,x)
```

output
```
Integral(sqrt(c*(a**2*x**2 + 1))*atan(a*x)/x, x)
```

## Maxima [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{x}\,dx=\int \frac{\sqrt{a^2cx^2+c}\arctan{(ax)}}{x}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)*arctan(a*x)/x,x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*arctan(a*x)/x, x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(1/2)*arctan(a*x)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{x}\,dx = \int \frac{\operatorname{atan}(a\,x)\,\sqrt{c\,a^2\,x^2+c}}{x}\,dx$$

input
```
int((atan(a*x)*(c + a^2*c*x^2)^(1/2))/x,x)
```

output
```
int((atan(a*x)*(c + a^2*c*x^2)^(1/2))/x, x)
```

## Reduce [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{x}\,dx = \sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,atan(ax)}{x}dx\right)$$

input
```
int((a^2*c*x^2+c)^(1/2)*atan(a*x)/x,x)
```

output
```
sqrt(c)*int((sqrt(a**2*x**2 + 1)*atan(a*x))/x,x)
```

# 3.205 $\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{x^2}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 242

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{x^2}\,dx = -\frac{\sqrt{c+a^2cx^2}\arctan(ax)}{x} - \frac{2iac\sqrt{1+a^2x^2}\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} - a\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+a^2cx^2}}{\sqrt{c}}\right) + \frac{iac\sqrt{1+a^2x^2}\,\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} - \frac{iac\sqrt{1+a^2x^2}\,\text{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}}$$

output
```
-(a^2*c*x^2+c)^(1/2)*arctan(a*x)/x-2*I*a*c*(a^2*x^2+1)^(1/2)*arctan(a*x)*a
rctan((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/(a^2*c*x^2+c)^(1/2)-a*c^(1/2)*arcta
nh((a^2*c*x^2+c)^(1/2)/c^(1/2))+I*a*c*(a^2*x^2+1)^(1/2)*polylog(2,-I*(1+I*
a*x)^(1/2)/(1-I*a*x)^(1/2))/(a^2*c*x^2+c)^(1/2)-I*a*c*(a^2*x^2+1)^(1/2)*po
lylog(2,I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{x^2}\,dx = -\frac{a\sqrt{c(1+a^2x^2)}\left(\frac{\sqrt{1+a^2x^2}\arctan(ax)}{ax}-\arctan(ax)\log\left(1-ie^{i\arctan(ax)}\right)+\arctan(ax)\log\left(1+ie^{i\arctan(ax)}\right)\right.}{}$$

input
```
Integrate[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/x^2,x]
```

output
```
-((a*Sqrt[c*(1 + a^2*x^2)]*((Sqrt[1 + a^2*x^2]*ArcTan[a*x])/(a*x) - ArcTan
[a*x]*Log[1 - I*E^(I*ArcTan[a*x])] + ArcTan[a*x]*Log[1 + I*E^(I*ArcTan[a*x
])] + Log[Cos[ArcTan[a*x]/2]] - Log[Sin[ArcTan[a*x]/2]] - I*PolyLog[2, (-I
)*E^(I*ArcTan[a*x])] + I*PolyLog[2, I*E^(I*ArcTan[a*x])]))/Sqrt[1 + a^2*x^
2])
```

## Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.84, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {5485, 5425, 5421, 5479, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}\,dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5425}$$

$$\frac{a^2c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5421}$$

$$c\int \frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx+\frac{a^2c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5479}$$

$$c\left(a\int \frac{1}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right)+\frac{a^2c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{243}$$

$$c\left(\frac{1}{2}a\int \frac{1}{x^2\sqrt{a^2cx^2+c}}dx^2-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right)+\frac{a^2c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{73}$$

$$c\left(\frac{\int \frac{1}{\frac{x^4}{a^2c}-\frac{1}{a^2}}d\sqrt{a^2cx^2+c}}{ac}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right)+\frac{a^2c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{221}$$

$$c\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)+\frac{a^2c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{\sqrt{a^2cx^2+c}}$$

input
```
Int[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/x^2,x]
```

output
```
c*(-((Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(c*x)) - (a*ArcTanh[Sqrt[c + a^2*c*
x^2]/Sqrt[c]])/Sqrt[c]) + (a^2*c*Sqrt[1 + a^2*x^2]*(((-2*I)*ArcTan[a*x]*Ar
cTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/a + (I*PolyLog[2, ((-I)*Sqrt[1 + I*
a*x])/Sqrt[1 - I*a*x]])/a - (I*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a
*x]])/a))/Sqrt[c + a^2*c*x^2]
```

### Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

## Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 929 vs. 2(199) = 398.

Time = 1.54 (sec) , antiderivative size = 930, normalized size of antiderivative = 3.84

| method | result |
|---|---|
| default | $\frac{\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}-1\right)\left(ia^2x^2+2ax-i\right)\sqrt{c(ax-i)(ax+i)}}{4\sqrt{a^2x^2+1}\,x}-\frac{\ln\left(1+\frac{iax+1}{\sqrt{a^2x^2+1}}\right)\left(ia^2x^2+2ax-i\right)\sqrt{c(ax-i)(ax+i)}}{4\sqrt{a^2x^2+1}\,x}-\frac{\sqrt{c(ax-i)(ax+i)}}{}$ |

input
```
int((a^2*c*x^2+c)^(1/2)*arctan(a*x)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-1)*(I*a^2*x^2+2*a*x-I)/(a^2*x^2+1)^(1/2
)*(c*(a*x-I)*(a*x+I))^(1/2)/x-1/4*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))*(I*a^2
*x^2+2*a*x-I)/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(a*x+I))^(1/2)/x-1/4/(a^2*x^2+1
)^(1/2)*(c*(a*x-I)*(a*x+I))^(1/2)*(I*a^2*x^2-2*a*x-I)*ln((1+I*a*x)/(a^2*x^
2+1)^(1/2)-1)/x+1/4/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(a*x+I))^(1/2)*(I*a^2*x^2
-2*a*x-I)*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))/x+1/2*(c*(a*x-I)*(a*x+I))^(1/2
)*(I*a*x-1)*arctan(a*x)/x+1/4/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(a*x+I))^(1/2)*
(I*a^2*x^2-2*a*x-I)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)/x-1/4/
(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(a*x+I))^(1/2)*(I*a^2*x^2-2*a*x-I)*ln(1-I*(1+
I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)/x-1/4*arctan(a*x)*ln(1+I*(1+I*a*x)/(
a^2*x^2+1)^(1/2))*(I*a^2*x^2+2*a*x-I)/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(a*x+I)
)^(1/2)/x+1/4*arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(I*a^2*x^2+2
*a*x-I)/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(a*x+I))^(1/2)/x-1/4*dilog(1+I*(1+I*a
*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2-2*I*a*x-1)/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(a
*x+I))^(1/2)/x+1/4/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(a*x+I))^(1/2)*(a^2*x^2+2*
I*a*x-1)*dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/x-1/4/(a^2*x^2+1)^(1/2)*(c
*(a*x-I)*(a*x+I))^(1/2)*(a^2*x^2+2*I*a*x-1)*dilog(1-I*(1+I*a*x)/(a^2*x^2+1
)^(1/2))/x-1/2*arctan(a*x)*(1+I*a*x)*(c*(a*x-I)*(a*x+I))^(1/2)/x+1/4*dilog
(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2-2*I*a*x-1)/(a^2*x^2+1)^(1/2)*(c
*(a*x-I)*(a*x+I))^(1/2)/x
```

## Fricas [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{x^2}\,dx=\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)*arctan(a*x)/x^2,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)/x^2, x)
```

## Sympy [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{x^2}\,dx = \int \frac{\sqrt{c\,(a^2x^2+1)}\operatorname{atan}(ax)}{x^2}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(1/2)*atan(a*x)/x**2,x)
```

output
```
Integral(sqrt(c*(a**2*x**2 + 1))*atan(a*x)/x**2, x)
```

## Maxima [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{x^2}\,dx = \int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)*arctan(a*x)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*arctan(a*x)/x^2, x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(1/2)*arctan(a*x)/x^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{x^2}\,dx = \int \frac{\mathrm{atan}(a\,x)\,\sqrt{c\,a^2\,x^2+c}}{x^2}\,dx$$

input
```
int((atan(a*x)*(c + a^2*c*x^2)^(1/2))/x^2,x)
```

output
```
int((atan(a*x)*(c + a^2*c*x^2)^(1/2))/x^2, x)
```

## Reduce [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{x^2}\,dx = \sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,atan(ax)}{x^2}dx\right)$$

input
```
int((a^2*c*x^2+c)^(1/2)*atan(a*x)/x^2,x)
```

output
```
sqrt(c)*int((sqrt(a**2*x**2 + 1)*atan(a*x))/x**2,x)
```

# 3.206 $\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{x^3}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 240

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{x^3}\,dx = -\frac{a\sqrt{c+a^2cx^2}}{2x}-\frac{\sqrt{c+a^2cx^2}\arctan(ax)}{2x^2} - \frac{a^2c\sqrt{1+a^2x^2}\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} + \frac{ia^2c\sqrt{1+a^2x^2}\,\text{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2\sqrt{c+a^2cx^2}} - \frac{ia^2c\sqrt{1+a^2x^2}\,\text{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2\sqrt{c+a^2cx^2}}$$

output
```
-1/2*a*(a^2*c*x^2+c)^(1/2)/x-1/2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)/x^2-a^2*c
*(a^2*x^2+1)^(1/2)*arctan(a*x)*arctanh((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/(a
^2*c*x^2+c)^(1/2)+1/2*I*a^2*c*(a^2*x^2+1)^(1/2)*polylog(2,-(1+I*a*x)^(1/2)
/(1-I*a*x)^(1/2))/(a^2*c*x^2+c)^(1/2)-1/2*I*a^2*c*(a^2*x^2+1)^(1/2)*polylo
g(2,(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.69

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{x^3}\,dx$$
$$= \frac{a^2\sqrt{c(1+a^2x^2)}\left(-2\cot\left(\frac{1}{2}\arctan(ax)\right)-\arctan(ax)\csc^2\left(\frac{1}{2}\arctan(ax)\right)+4\arctan(ax)\log\left(1-e^{i\arctan(ax)}\right)\right.}{}$$

input
```
Integrate[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/x^3,x]
```

output
```
(a^2*Sqrt[c*(1 + a^2*x^2)]*(-2*Cot[ArcTan[a*x]/2] - ArcTan[a*x]*Csc[ArcTan
[a*x]/2]^2 + 4*ArcTan[a*x]*Log[1 - E^(I*ArcTan[a*x])] - 4*ArcTan[a*x]*Log[
1 + E^(I*ArcTan[a*x])] + (4*I)*PolyLog[2, -E^(I*ArcTan[a*x])] - (4*I)*Poly
Log[2, E^(I*ArcTan[a*x])] + ArcTan[a*x]*Sec[ArcTan[a*x]/2]^2 - 2*Tan[ArcTa
n[a*x]/2]))/(8*Sqrt[1 + a^2*x^2])
```

## Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {5481, 242, 5497, 242, 5493, 5489}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^3}\,dx$$

$$\Big\downarrow \text{5481}$$

$$-c\int \frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx+ac\int \frac{1}{x^2\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}$$

$$\Big\downarrow \text{242}$$

$$-c\int \frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}-\frac{a\sqrt{a^2cx^2+c}}{x}$$

$$\Big\downarrow \text{5497}$$

$$-c\left(-\frac{1}{2}a^2\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx+\frac{1}{2}a\int \frac{1}{x^2\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{2cx^2}\right)-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}-\frac{a\sqrt{a^2cx^2+c}}{x}$$

$\big\downarrow$ 242

$$-c\left(-\frac{1}{2}a^2\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{2cx^2}-\frac{a\sqrt{a^2cx^2+c}}{2cx}\right)-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}-\frac{a\sqrt{a^2cx^2+c}}{x}$$

$\big\downarrow$ 5493

$$-c\left(-\frac{a^2\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{x\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{2cx^2}-\frac{a\sqrt{a^2cx^2+c}}{2cx}\right)-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}-\frac{a\sqrt{a^2cx^2+c}}{x}$$

$\big\downarrow$ 5489

$$-c\left(-\frac{a^2\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{PolyLog}\left(2,\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)\right)}{2\sqrt{a^2cx^2+c}}-\text{ar}\right.$$
$$\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}-\frac{a\sqrt{a^2cx^2+c}}{x}$$

input
```
Int[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/x^3,x]
```

output
```
-((a*Sqrt[c + a^2*c*x^2])/x) - (Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/x^2 - c*(
-1/2*(a*Sqrt[c + a^2*c*x^2])/(c*x) - (Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(2*
c*x^2) - (a^2*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sq
rt[1 - I*a*x]] + I*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])] - I*Poly
Log[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]]))/(2*Sqrt[c + a^2*c*x^2]))
```

### Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 5481
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTan[c*x
])/(f*(m + 2))), x] + (Simp[d/(m + 2)   Int[(f*x)^m*((a + b*ArcTan[c*x])/Sq
rt[d + e*x^2]), x], x] - Simp[b*c*(d/(f*(m + 2)))   Int[(f*x)^(m + 1)/Sqrt[
d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] &&
NeQ[m, -2]
```

rule 5489
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sq
rt[1 - I*c*x]], x] + (Simp[I*(b/Sqrt[d])*PolyLog[2, -Sqrt[1 + I*c*x]/Sqrt[1
 - I*c*x]], x] - Simp[I*(b/Sqrt[d])*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x]], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5497
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)^(m
 + 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[c^2*((m
+ 2)/(f^2*(m + 1)))   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x
^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
 && LtQ[m, -1] && NeQ[m, -2]
```

## Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.70

| method | result |
|---|---|
| default | $-\frac{\sqrt{c(ax-i)(ax+i)}\,(ax+\arctan(ax))}{2x^2}+\frac{ia^2\sqrt{c(ax-i)(ax+i)}\left(i\arctan(ax)\ln\left(1+\frac{iax+1}{\sqrt{a^2x^2+1}}\right)-i\arctan(ax)\ln\left(1-\frac{iax+1}{\sqrt{a^2x^2+1}}\right)-\right.}{2\sqrt{a^2x^2+1}}$ |

input
```
int((a^2*c*x^2+c)^(1/2)*arctan(a*x)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(c*(a*x-I)*(a*x+I))^(1/2)*(a*x+arctan(a*x))/x^2+1/2*I*a^2*(c*(a*x-I)*
(a*x+I))^(1/2)*(I*arctan(a*x)*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*arctan(a
*x)*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2
))+polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)
```

## Fricas [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{x^3}\,dx=\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)*arctan(a*x)/x^3,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)/x^3, x)
```

## Sympy [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{x^3}\,dx=\int \frac{\sqrt{c\,(a^2x^2+1)}\operatorname{atan}(ax)}{x^3}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(1/2)*atan(a*x)/x**3,x)
```

output
```
Integral(sqrt(c*(a**2*x**2 + 1))*atan(a*x)/x**3, x)
```

## Maxima [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{x^3}\,dx = \int \frac{\sqrt{a^2cx^2+c}\arctan{(ax)}}{x^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)*arctan(a*x)/x^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*arctan(a*x)/x^3, x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(1/2)*arctan(a*x)/x^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{x^3}\,dx = \int \frac{\mathrm{atan}(a\,x)\,\sqrt{c\,a^2\,x^2+c}}{x^3}\,dx$$

input
```
int((atan(a*x)*(c + a^2*c*x^2)^(1/2))/x^3,x)
```

output
```
int((atan(a*x)*(c + a^2*c*x^2)^(1/2))/x^3, x)
```

## Reduce [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{x^3}\,dx = \sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,atan(ax)}{x^3}dx\right)$$

input
```
int((a^2*c*x^2+c)^(1/2)*atan(a*x)/x^3,x)
```

output
```
sqrt(c)*int((sqrt(a**2*x**2 + 1)*atan(a*x))/x**3,x)
```

# 3.207 $\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{x^4}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 84

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{x^4}\,dx = -\frac{a\sqrt{c+a^2cx^2}}{6x^2} - \frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{3cx^3} - \frac{1}{6}a^3\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c+a^2cx^2}}{\sqrt{c}}\right)$$

output
```
-1/6*a*(a^2*c*x^2+c)^(1/2)/x^2-1/3*(a^2*c*x^2+c)^(3/2)*arctan(a*x)/c/x^3-1
/6*a^3*c^(1/2)*arctanh((a^2*c*x^2+c)^(1/2)/c^(1/2))
```

## Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.25

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{x^4}\,dx$$
$$= \frac{-2(1+a^2x^2)\sqrt{c+a^2cx^2}\arctan(ax) + a^3\sqrt{c}x^3\log(x) - ax\left(\sqrt{c+a^2cx^2} + a^2\sqrt{c}x^2\log\left(c+\sqrt{c}\sqrt{c+a^2c\right.\right.}{6x^3}$$

input
```
Integrate[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/x^4,x]
```

output
```
(-2*(1 + a^2*x^2)*Sqrt[c + a^2*c*x^2]*ArcTan[a*x] + a^3*Sqrt[c]*x^3*Log[x]
 - a*x*(Sqrt[c + a^2*c*x^2] + a^2*Sqrt[c]*x^2*Log[c + Sqrt[c]*Sqrt[c + a^2
*c*x^2]]))/(6*x^3)
```

## Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {5479, 243, 51, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^4}dx$$

$$\Big\downarrow \text{5479}$$

$$\frac{1}{3}a\int \frac{\sqrt{a^2cx^2+c}}{x^3}dx-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3cx^3}$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{6}a\int \frac{\sqrt{a^2cx^2+c}}{x^4}dx^2-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3cx^3}$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{6}a\left(\frac{1}{2}a^2c\int \frac{1}{x^2\sqrt{a^2cx^2+c}}dx^2-\frac{\sqrt{a^2cx^2+c}}{x^2}\right)-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3cx^3}$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{6}a\left(\int \frac{1}{\frac{x^4}{a^2c}-\frac{1}{a^2}}d\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{x^2}\right)-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3cx^3}$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{6}a\left(a^2\left(-\sqrt{c}\right)\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)-\frac{\sqrt{a^2cx^2+c}}{x^2}\right)-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3cx^3}$$

input
```
Int[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/x^4,x]
```

output
```
-1/3*((c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/(c*x^3) + (a*(-(Sqrt[c + a^2*c*x^
2]/x^2) - a^2*Sqrt[c]*ArcTanh[Sqrt[c + a^2*c*x^2]/Sqrt[c]]))/6
```

## Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.27 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.82

| method | result |
|---|---|
| default | $-\frac{\sqrt{c(ax-i)(ax+i)}\,(2x^2a^2\arctan(ax)+ax+2\arctan(ax))}{6x^3}-\frac{a^3\sqrt{c(ax-i)(ax+i)}\,\ln\left(1+\frac{iax+1}{\sqrt{a^2x^2+1}}\right)}{6\sqrt{a^2x^2+1}}+\frac{a^3\sqrt{c(ax-i)(ax+i)}\,\ln}{6\sqrt{a^2x^2+}}$ |

input
```
int((a^2*c*x^2+c)^(1/2)*arctan(a*x)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(c*(a*x-I)*(a*x+I))^(1/2)*(2*x^2*a^2*arctan(a*x)+a*x+2*arctan(a*x))/x
^3-1/6*a^3*(c*(a*x-I)*(a*x+I))^(1/2)*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^
2*x^2+1)^(1/2)+1/6*a^3*(c*(a*x-I)*(a*x+I))^(1/2)*ln((1+I*a*x)/(a^2*x^2+1)^
(1/2)-1)/(a^2*x^2+1)^(1/2)
```

## Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{x^4}\,dx = \frac{a^3\sqrt{c}x^3\log\left(-\frac{a^2cx^2-2\sqrt{a^2cx^2+c}\sqrt{c}+2c}{x^2}\right)-2\sqrt{a^2cx^2+c}(ax+2(a^2x^2+1)\arctan(ax))}{12x^3}$$

input
```
integrate((a^2*c*x^2+c)^(1/2)*arctan(a*x)/x^4,x, algorithm="fricas")
```

output
```
1/12*(a^3*sqrt(c)*x^3*log(-(a^2*c*x^2 - 2*sqrt(a^2*c*x^2 + c)*sqrt(c) + 2*
c)/x^2) - 2*sqrt(a^2*c*x^2 + c)*(a*x + 2*(a^2*x^2 + 1)*arctan(a*x)))/x^3
```

## Sympy [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{x^4}\,dx = \int \frac{\sqrt{c\left(a^2x^2+1\right)}\operatorname{atan}(ax)}{x^4}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(1/2)*atan(a*x)/x**4,x)
```

output
```
Integral(sqrt(c*(a**2*x**2 + 1))*atan(a*x)/x**4, x)
```

## Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{x^4}\,dx = -\frac{1}{6}\left(\left(a^2\operatorname{arsinh}\left(\frac{1}{a|x|}\right)-\sqrt{a^2x^2+1}a^2+\frac{\left(a^2x^2+1\right)^{\frac{3}{2}}}{x^2}\right)a+\frac{2\left(a^2x^2+1\right)^{\frac{3}{2}}\arctan(ax)}{x^3}\right)\sqrt{c}$$

input
```
integrate((a^2*c*x^2+c)^(1/2)*arctan(a*x)/x^4,x, algorithm="maxima")
```

output
```
-1/6*((a^2*arcsinh(1/(a*abs(x))) - sqrt(a^2*x^2 + 1)*a^2 + (a^2*x^2 + 1)^(
3/2)/x^2)*a + 2*(a^2*x^2 + 1)^(3/2)*arctan(a*x)/x^3)*sqrt(c)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(1/2)*arctan(a*x)/x^4,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{x^4}\,dx = \int \frac{\mathrm{atan}(a\,x)\,\sqrt{c\,a^2\,x^2+c}}{x^4}\,dx$$

input
```
int((atan(a*x)*(c + a^2*c*x^2)^(1/2))/x^4,x)
```

output
```
int((atan(a*x)*(c + a^2*c*x^2)^(1/2))/x^4, x)
```

## Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.27

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{x^4}\,dx = \frac{\sqrt{c}\left(-2\sqrt{a^2x^2+1}\,atan(ax)\,a^2x^2-2\sqrt{a^2x^2+1}\,atan(ax)-\sqrt{a^2x^2+1}\,ax+\log\left(\sqrt{a^2x^2+1}+ax-1\right)a^3\right.}{6x^3}$$

input
```
int((a^2*c*x^2+c)^(1/2)*atan(a*x)/x^4,x)
```

output
```
(sqrt(c)*( - 2*sqrt(a**2*x**2 + 1)*atan(a*x)*a**2*x**2 - 2*sqrt(a**2*x**2
+ 1)*atan(a*x) - sqrt(a**2*x**2 + 1)*a*x + log(sqrt(a**2*x**2 + 1) + a*x -
 1)*a**3*x**3 - log(sqrt(a**2*x**2 + 1) + a*x + 1)*a**3*x**3))/(6*x**3)
```

# 3.208 $\int x^3(c+a^2cx^2)^{3/2}\arctan(ax)\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 217

$$\int x^3(c+a^2cx^2)^{3/2}\arctan(ax)\,dx=\frac{3cx\sqrt{c+a^2cx^2}}{112a^3}-\frac{23cx^3\sqrt{c+a^2cx^2}}{840a}-\frac{1}{42}acx^5\sqrt{c+a^2cx^2}-\frac{2c\sqrt{c+a^2cx^2}\arctan(ax)}{35a^4}+\frac{cx^2\sqrt{c+a^2cx^2}\arctan(ax)}{35a^2}+\frac{8}{35}cx^4\sqrt{c+a^2cx^2}\arctan(ax)+\frac{1}{7}a^2cx^6\sqrt{c+a^2cx^2}\arctan(ax)+\frac{17c^{3/2}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right)}{560a^4}$$

output
```
3/112*c*x*(a^2*c*x^2+c)^(1/2)/a^3-23/840*c*x^3*(a^2*c*x^2+c)^(1/2)/a-1/42*
a*c*x^5*(a^2*c*x^2+c)^(1/2)-2/35*c*(a^2*c*x^2+c)^(1/2)*arctan(a*x)/a^4+1/3
5*c*x^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)/a^2+8/35*c*x^4*(a^2*c*x^2+c)^(1/2)
*arctan(a*x)+1/7*a^2*c*x^6*(a^2*c*x^2+c)^(1/2)*arctan(a*x)+17/560*c^(3/2)*
arctanh(a*c^(1/2)*x/(a^2*c*x^2+c)^(1/2))/a^4
```

## Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.55

$$\int x^3(c+a^2cx^2)^{3/2}\arctan(ax)\,dx = \frac{acx\sqrt{c+a^2cx^2}(45-46a^2x^2-40a^4x^4)+48c(1+a^2x^2)^2(-2+5a^2x^2)\sqrt{c+a}}{1680a^4}$$

input
```
Integrate[x^3*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x],x]
```

output
```
(a*c*x*Sqrt[c + a^2*c*x^2]*(45 - 46*a^2*x^2 - 40*a^4*x^4) + 48*c*(1 + a^2*
x^2)^2*(-2 + 5*a^2*x^2)*Sqrt[c + a^2*c*x^2]*ArcTan[a*x] + 51*c^(3/2)*Log[a
*c*x + Sqrt[c]*Sqrt[c + a^2*c*x^2]])/(1680*a^4)
```

## Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 783 vs. 2(217) = 434.

Time = 2.48 (sec) , antiderivative size = 783, normalized size of antiderivative = 3.61, number of steps used = 27, number of rules used = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 1.182$, Rules used = {5485, 5481, 262, 262, 224, 219, 262, 224, 219, 5487, 262, 224, 219, 262, 224, 219, 5465, 224, 219, 5487, 262, 224, 219, 5465, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\arctan(ax)\left(a^2cx^2+c\right)^{3/2}dx$$

$$\Big\downarrow \ 5485$$

$$a^2c\int x^5\sqrt{a^2cx^2+c}\arctan(ax)dx+c\int x^3\sqrt{a^2cx^2+c}\arctan(ax)dx$$

$$\Big\downarrow \ 5481$$

$$a^2c\left(\frac{1}{7}c\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{7}ac\int\frac{x^6}{\sqrt{a^2cx^2+c}}dx+\frac{1}{7}x^6\arctan(ax)\sqrt{a^2cx^2+c}\right)+ c\left(\frac{1}{5}c\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{5}ac\int\frac{x^4}{\sqrt{a^2cx^2+c}}dx+\frac{1}{5}x^4\arctan(ax)\sqrt{a^2cx^2+c}\right)$$

$$\Big\downarrow \text{262}$$

$$c\left(\frac{1}{5}c\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{5}ac\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\int\frac{x^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)+\frac{1}{5}x^4\arctan(ax)\sqrt{a^2cx^2+c}\right)+$$
$$a^2c\left(\frac{1}{7}c\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{7}ac\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\int\frac{x^4}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)+\frac{1}{7}x^6\arctan(ax)\sqrt{a^2cx^2+c}\right)$$

$$\Big\downarrow \text{262}$$

$$c\left(\frac{1}{5}c\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{5}ac\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int\frac{1}{\sqrt{a^2cx^2+c}}dx}{2a^2}\right)}{4a^2}\right)+\frac{1}{5}x^4\arctan(ax)\sqrt{a^2cx^2+\right.$$
$$a^2c\left(\frac{1}{7}c\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{7}ac\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\int\frac{x^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{6a^2}\right)+\frac{1}{7}x^6\arctan(ax)\sqrt{a^2c\right.$$

$$\Big\downarrow \text{224}$$

$$c\left(\frac{1}{5}c\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{5}ac\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int\frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}}{2a^2}\right)}{4a^2}\right)+\frac{1}{5}x^4\arctan(ax)\right.$$
$$a^2c\left(\frac{1}{7}c\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{7}ac\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\int\frac{x^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{6a^2}\right)+\frac{1}{7}x^6\arctan(ax)\sqrt{a^2c\right.$$

$$\Big\downarrow \text{219}$$

$$a^2c\left(\frac{1}{7}c\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{7}ac\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\int\frac{x^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{6a^2}\right)+\frac{1}{7}x^6\arctan(ax)\sqrt{a^2c\ldots}\right.$$

$$c\left(\frac{1}{5}c\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{5}x^4\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{5}ac\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{\ldots}}{\sqrt{a^2c\ldots}}\right)}{2a^3\sqrt{c}}\right.}{4a^2}\ldots\right.\right.$$

$$\Big\downarrow \text{262}$$

$$a^2c\left(\frac{1}{7}c\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{7}ac\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int\frac{1}{\sqrt{a^2cx^2+c}}dx}{2a^2}\right)}{4a^2}\right)}{6a^2}\right)+\frac{1}{7}x^6\operatorname{arc}\ldots\right.$$

$$c\left(\frac{1}{5}c\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{5}x^4\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{5}ac\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{\ldots}}{\sqrt{a^2c\ldots}}\right)}{2a^3\sqrt{c}}\right.}{4a^2}\ldots\right.\right.$$

$$\Big\downarrow \text{224}$$

$$a^2c\left(\frac{1}{7}c\int \frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{7}ac\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int \frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}}{2a^2}\right)}{4a^2}\right)}{6a^2}\right)\right) -$$

$$c\left(\frac{1}{5}c\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{5}x^4\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{5}ac\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{}}{\sqrt{a^2c}}\right)}{2a^3\sqrt{c}}\right.}{4a^2}\right.\right.$$

$$\downarrow 219$$

$$c\left(\frac{1}{5}c\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{5}x^4\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{5}ac\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{}}{\sqrt{a^2c}}\right)}{2a^3\sqrt{c}}\right.}{4a^2}\right.\right.$$

$$a^2c\left(\frac{1}{7}c\int \frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{7}x^6\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{7}ac\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+}}{2a^2c}\right.}{}\right.}{6a^2}\right.\right.$$

$$\downarrow 5487$$

$$c\left(\frac{1}{5}c\left(-\frac{2\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}-\frac{\int\frac{x^2}{\sqrt{a^2cx^2+c}}dx}{3a}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}\right)+\frac{1}{5}x^4\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{5}ac\right.$$

$$a^2c\left(\frac{1}{7}c\left(-\frac{4\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a^2}-\frac{\int\frac{x^4}{\sqrt{a^2cx^2+c}}dx}{5a}+\frac{x^4\arctan(ax)\sqrt{a^2cx^2+c}}{5a^2c}\right)+\frac{1}{7}x^6\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{7}\right.$$

$$\Big\downarrow 262$$

$$c\left(\frac{1}{5}c\left(-\frac{2\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int\frac{1}{\sqrt{a^2cx^2+c}}dx}{2a^2}}{3a}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}\right)+\frac{1}{5}x^4\arctan(ax)\sqrt{a^2cx^2}\right.$$

$$a^2c\left(\frac{1}{7}c\left(-\frac{4\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a^2}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\int\frac{x^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}}{5a}+\frac{x^4\arctan(ax)\sqrt{a^2cx^2+c}}{5a^2c}\right)+\frac{1}{7}x^6\arctan(ax)\sqrt{\right.$$

$$\Big\downarrow 224$$

$$c\left(\frac{1}{5}c\left(-\frac{2\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int \frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}}{2a^2}}{3a}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}\right)+\frac{1}{5}x^4\arctan(ax)\right.$$

$$a^2c\left(\frac{1}{7}c\left(-\frac{4\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a^2}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\int \frac{x^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}}{5a}+\frac{x^4\arctan(ax)\sqrt{a^2cx^2+c}}{5a^2c}\right)+\frac{1}{7}x^6\arctan(ax)\sqrt{c}\right.$$

$$\downarrow 219$$

$$c\left(\frac{1}{5}c\left(-\frac{2\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}\right)+\frac{1}{5}x^4\arctan(ax)\sqrt{}\right.$$

$$a^2c\left(\frac{1}{7}c\left(-\frac{4\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a^2}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\int \frac{x^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}}{5a}+\frac{x^4\arctan(ax)\sqrt{a^2cx^2+c}}{5a^2c}\right)+\frac{1}{7}x^6\arctan(ax)\sqrt{c}\right.$$

$$\downarrow 262$$

$$c\left(\frac{1}{5}c\left(-\frac{2\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}\right)+\frac{1}{5}x^4\arctan(ax)\sqrt{}\right.$$

$$a^2c\left(\frac{1}{7}c\left(-\frac{4\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a^2}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int \frac{1}{\sqrt{a^2cx^2+c}}dx}{2a^2}\right)}{4a^2}}{5a}+\frac{x^4\arctan(ax)\sqrt{a^2cx^2+c}}{5a^2c}\right)+\frac{1}{7}x^6 a\right.$$

$$\Big\downarrow \; 224$$

$$c\left(\frac{1}{5}c\left(-\frac{2\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}\right)+\frac{1}{5}x^4\arctan(ax)\sqrt{}\right.$$

$$a^2c\left(\frac{1}{7}c\left(-\frac{4\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a^2}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int \frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}}{2a^2}\right)}{4a^2}}{5a}+\frac{x^4\arctan(ax)\sqrt{a^2cx^2+c}}{5a^2c}\right)\right.$$

$$\Big\downarrow \; 219$$

$$c\left(\frac{1}{5}c\left(-\frac{2\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}\right)+\frac{1}{5}x^4\arctan(ax)\sqrt{\right.$$

$$a^2c\left(\frac{1}{7}c\left(-\frac{4\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a^2}+\frac{x^4\arctan(ax)\sqrt{a^2cx^2+c}}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}}{5a}\right)\right.$$

$$\Big\downarrow \text{5465}$$

$$c\left(\frac{1}{5}c\left(-\frac{2\left(\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\int \frac{1}{\sqrt{a^2cx^2+c}}dx}{a}\right)}{3a^2}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}\right)\right.$$

$$a^2c\left(\frac{1}{7}c\left(-\frac{4\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a^2}+\frac{x^4\arctan(ax)\sqrt{a^2cx^2+c}}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}}{5a}\right)\right.$$

$$\Big\downarrow \text{224}$$

$$c\left(\frac{1}{5}c\left(-\frac{2\left(\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\int\frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}}{a}\right)}{3a^2}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{\cdots}{\sqrt{c}\cdots}\right)}{2a^3\sqrt{\cdots}}}{3a}\right.\right.$$

$$a^2c\left(\frac{1}{7}c\left(-\frac{4\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a^2}+\frac{x^4\arctan(ax)\sqrt{a^2cx^2+c}}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}}{5a}\right)\right.$$

$$\Big\downarrow \text{219}$$

$$a^2c\left(\frac{1}{7}c\left(-\frac{4\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a^2}+\frac{x^4\arctan(ax)\sqrt{a^2cx^2+c}}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}}{5a}\right)\right.$$

$$c\left(\frac{1}{5}x^4\arctan(ax)\sqrt{a^2cx^2+c}+\frac{1}{5}c\left(-\frac{2\left(\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a^2\sqrt{c}}\right)}{3a^2}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+\cdots}}{3a^2c}\right.\right.$$

$$\Big\downarrow \text{5487}$$

$$a^2c\left(\frac{1}{7}c\left(-\frac{4\left(-\frac{2\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}-\frac{\int\frac{x^2}{\sqrt{a^2cx^2+c}}dx}{3a}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}\right)}{5a^2}+\frac{x^4\arctan(ax)\sqrt{a^2cx^2+c}}{5a^2c}-\frac{x^3\sqrt{a^2cx^2+}}{4a^2c}\right.\right.$$

$$c\left(\frac{1}{5}x^4\arctan(ax)\sqrt{a^2cx^2+c}+\frac{1}{5}c\left(-\frac{2\left(\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a^2\sqrt{c}}\right)}{3a^2}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+}}{3a^2c}\right.\right.$$

$$\downarrow 262$$

$$a^2c\left(\frac{1}{7}c\left(-\frac{4\left(-\frac{2\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int\frac{1}{\sqrt{a^2cx^2+c}}dx}{2a^2}}{3a}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}\right)}{5a^2}+\frac{x^4\arctan(ax)\sqrt{a^2cx^2+c}}{5a^2c}-\right.\right.$$

$$c\left(\frac{1}{5}x^4\arctan(ax)\sqrt{a^2cx^2+c}+\frac{1}{5}c\left(-\frac{2\left(\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a^2\sqrt{c}}\right)}{3a^2}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+}}{3a^2c}\right.\right.$$

$$\downarrow 224$$

$$a^2c\left(\frac{1}{7}c\left(-\frac{4\left(-\frac{2\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int\frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}}{2a^2}}{3a}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}\right)}{5a^2}+\frac{x^4\arctan(ax)\sqrt{a^2c}}{5a^2c}\right.\right.$$

$$c\left(\frac{1}{5}x^4\arctan(ax)\sqrt{a^2cx^2+c}+\frac{1}{5}c\left(-\frac{2\left(\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a^2\sqrt{c}}\right)}{3a^2}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+}}{3a^2c}\right.\right.$$

$\downarrow$ 219

$$a^2c\left(\frac{1}{7}c\left(-\frac{4\left(-\frac{2\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}\right)}{5a^2}+\frac{x^4\arctan(ax)\sqrt{a^2cx}}{5a^2c}\right.\right.$$

$$c\left(\frac{1}{5}x^4\arctan(ax)\sqrt{a^2cx^2+c}+\frac{1}{5}c\left(-\frac{2\left(\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a^2\sqrt{c}}\right)}{3a^2}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+}}{3a^2c}\right.\right.$$

$\downarrow$ 5465

$$a^2c\left(\frac{1}{7}c\left(-\frac{4\left(-\frac{2\left(\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\int\frac{1}{\sqrt{a^2cx^2+c}}dx}{a}\right)}{3a^2}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}\right)}{5a^2}+x\right.\right.$$

$$c\left(\frac{1}{5}x^4\arctan(ax)\sqrt{a^2cx^2+c}+\frac{1}{5}c\left(-\frac{2\left(\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a^2\sqrt{c}}\right)}{3a^2}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+}}{3a^2c}\right.\right.$$

$$\downarrow 224$$

$$c\left(\frac{1}{7}\sqrt{a^2cx^2+c}\arctan(ax)x^6-\frac{1}{7}ac\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}\right)}{6a^2}\right)\right)$$

$$c\left(\frac{1}{5}\sqrt{a^2cx^2+c}\arctan(ax)x^4-\frac{1}{5}ac\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}\right)+\frac{1}{5}c\left(\sqrt{a^2cx^2}\right.\right.$$

$$\downarrow 219$$

$$c\left(\frac{1}{5}x^4\arctan(ax)\sqrt{a^2cx^2+c}+\frac{1}{5}c\left(-\frac{2\left(\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a^2\sqrt{c}}\right)}{3a^2}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+}}{3a^2c}\right.\right.$$

$$a^2c\left(\frac{1}{7}x^6\arctan(ax)\sqrt{a^2cx^2+c}+\frac{1}{7}c\left(\frac{x^4\arctan(ax)\sqrt{a^2cx^2+c}}{5a^2c}-\frac{4\left(-\frac{2\left(\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a^2\sqrt{c}}\right)}{3a^2}\right.}{}\right.\right.$$

input
```
Int[x^3*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x],x]
```

output
```
c*((x^4*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/5 - (a*c*((x^3*Sqrt[c + a^2*c*x^2
])/(4*a^2*c) - (3*((x*Sqrt[c + a^2*c*x^2])/(2*a^2*c) - ArcTanh[(a*Sqrt[c]*
x)/Sqrt[c + a^2*c*x^2]]/(2*a^3*Sqrt[c])))/(4*a^2)))/5 + (c*((x^2*Sqrt[c +
a^2*c*x^2]*ArcTan[a*x])/(3*a^2*c) - ((x*Sqrt[c + a^2*c*x^2])/(2*a^2*c) - A
rcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]]/(2*a^3*Sqrt[c]))/(3*a) - (2*((Sq
rt[c + a^2*c*x^2]*ArcTan[a*x])/(a^2*c) - ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^
2*c*x^2]]/(a^2*Sqrt[c])))/(3*a^2)))/5) + a^2*c*((x^6*Sqrt[c + a^2*c*x^2]*A
rcTan[a*x])/7 - (a*c*((x^5*Sqrt[c + a^2*c*x^2])/(6*a^2*c) - (5*((x^3*Sqrt[
c + a^2*c*x^2])/(4*a^2*c) - (3*((x*Sqrt[c + a^2*c*x^2])/(2*a^2*c) - ArcTan
h[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]]/(2*a^3*Sqrt[c])))/(4*a^2)))/(6*a^2)))
/7 + (c*((x^4*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(5*a^2*c) - ((x^3*Sqrt[c +
a^2*c*x^2])/(4*a^2*c) - (3*((x*Sqrt[c + a^2*c*x^2])/(2*a^2*c) - ArcTanh[(a
*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]]/(2*a^3*Sqrt[c])))/(4*a^2))/(5*a) - (4*((x
^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(3*a^2*c) - ((x*Sqrt[c + a^2*c*x^2])/(
2*a^2*c) - ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]]/(2*a^3*Sqrt[c]))/(3*
a) - (2*((Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(a^2*c) - ArcTanh[(a*Sqrt[c]*x)
/Sqrt[c + a^2*c*x^2]]/(a^2*Sqrt[c])))/(3*a^2)))/(5*a^2)))/7)
```

### Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5481
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTan[c*x
])/(f*(m + 2))), x] + (Simp[d/(m + 2)   Int[(f*x)^m*((a + b*ArcTan[c*x])/Sq
rt[d + e*x^2]), x], x] - Simp[b*c*(d/(f*(m + 2)))   Int[(f*x)^(m + 1)/Sqrt[
d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] &&
NeQ[m, -2]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5487
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcTan[c*x])^p/(c^2*d*m)), x] + (-Simp[b*f*(p/(c*m))   Int[(f*x)^(m - 1)*((
a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[f^2*((m - 1)/(c^
2*m))   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2]), x], x])
/; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.40 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.92

| method | result |
|---|---|
| default | $\frac{c\sqrt{c(ax-i)(ax+i)}\,(240a^6\arctan(ax)x^6-40a^5x^5+384x^4\arctan(ax)a^4-46a^3x^3+48x^2a^2\arctan(ax)+45ax-96\arctan(ax))}{1680a^4}+\frac{17}{}$ |

input
```
int(x^3*(a^2*c*x^2+c)^(3/2)*arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/1680*c/a^4*(c*(a*x-I)*(a*x+I))^(1/2)*(240*a^6*arctan(a*x)*x^6-40*a^5*x^5
+384*x^4*arctan(a*x)*a^4-46*a^3*x^3+48*x^2*a^2*arctan(a*x)+45*a*x-96*arcta
n(a*x))+17/560*c/a^4*(c*(a*x-I)*(a*x+I))^(1/2)*ln((1+I*a*x)/(a^2*x^2+1)^(1
/2)+I)/(a^2*x^2+1)^(1/2)-17/560*c/a^4*(c*(a*x-I)*(a*x+I))^(1/2)*ln((1+I*a*
x)/(a^2*x^2+1)^(1/2)-I)/(a^2*x^2+1)^(1/2)
```

## Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.54

$$\int x^3(c+a^2cx^2)^{3/2}\arctan(ax)\,dx = \frac{51\,c^{\frac{3}{2}}\log\left(-2\,a^2cx^2-2\,\sqrt{a^2cx^2+c}a\sqrt{c}x-c\right)-2\left(40\,a^5cx^5+46\,a^3cx^3-45\,ac\right.}{3360\,a^4}$$

input
```
integrate(x^3*(a^2*c*x^2+c)^(3/2)*arctan(a*x),x, algorithm="fricas")
```

output
```
1/3360*(51*c^(3/2)*log(-2*a^2*c*x^2 - 2*sqrt(a^2*c*x^2 + c)*a*sqrt(c)*x -
c) - 2*(40*a^5*c*x^5 + 46*a^3*c*x^3 - 45*a*c*x - 48*(5*a^6*c*x^6 + 8*a^4*c
*x^4 + a^2*c*x^2 - 2*c)*arctan(a*x))*sqrt(a^2*c*x^2 + c))/a^4
```

## Sympy [F]

$$\int x^3 \left(c + a^2 c x^2\right)^{3/2} \arctan(ax)\, dx = \int x^3 \left(c\left(a^2 x^2 + 1\right)\right)^{\frac{3}{2}} \operatorname{atan}(ax)\, dx$$

input
```
integrate(x**3*(a**2*c*x**2+c)**(3/2)*atan(a*x),x)
```

output
```
Integral(x**3*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x), x)
```

## Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.99

$$\int x^3 \left(c + a^2 c x^2\right)^{3/2} \arctan(ax)\, dx =$$
$$-\frac{1}{1680}\left(\left(\left(5\left(\frac{8\left(a^2x^2+1\right)^{\frac{3}{2}}x^3}{a^2} - \frac{6\left(a^2x^2+1\right)^{\frac{3}{2}}x}{a^4} + \frac{3\sqrt{a^2x^2+1}x}{a^4} + \frac{3\operatorname{arsinh}(ax)}{a^5}\right)c + \frac{18c\left(\frac{2\left(a^2x^2+1\right)^{\frac{3}{2}}x}{a^2}\right.}{}\right.\right.\right.$$

input
```
integrate(x^3*(a^2*c*x^2+c)^(3/2)*arctan(a*x),x, algorithm="maxima")
```

output
```
-1/1680*((5*(8*(a^2*x^2 + 1)^(3/2)*x^3/a^2 - 6*(a^2*x^2 + 1)^(3/2)*x/a^4 +
 3*sqrt(a^2*x^2 + 1)*x/a^4 + 3*arcsinh(a*x)/a^5)*c + 18*c*(2*(a^2*x^2 + 1)
^(3/2)*x/a^2 - sqrt(a^2*x^2 + 1)*x/a^2 - arcsinh(a*x)/a^3)/a^2 - 48*(sqrt(
a^2*x^2 + 1)*x + arcsinh(a*x)/a)*c/a^4)*a - 48*(5*(a^2*x^2 + 1)^(3/2)*c*x^
4 + 3*(a^2*x^2 + 1)^(3/2)*c*x^2/a^2 - 2*(a^2*x^2 + 1)^(3/2)*c/a^4)*arctan(
a*x))*sqrt(c)
```

## Giac [F(-2)]

Exception generated.

$$\int x^3 (c + a^2 c x^2)^{3/2} \arctan(ax)\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a^2*c*x^2+c)^(3/2)*arctan(a*x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int x^3 (c + a^2 c x^2)^{3/2} \arctan(ax)\, dx = \int x^3 \operatorname{atan}(a\,x)\, \left(c\,a^2\,x^2 + c\right)^{3/2} dx$$

input
```
int(x^3*atan(a*x)*(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(x^3*atan(a*x)*(c + a^2*c*x^2)^(3/2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.73

$$\int x^3 (c + a^2 c x^2)^{3/2} \arctan(ax)\, dx = \frac{\sqrt{c}\, c\big(240\sqrt{a^2x^2+1}\, atan(ax)\, a^6x^6 + 384\sqrt{a^2x^2+1}\, atan(ax)\, a^4x^4 + 48\sqrt{a^2x^2}}{}$$

input
```
int(x^3*(a^2*c*x^2+c)^(3/2)*atan(a*x),x)
```

output
```
(sqrt(c)*c*(240*sqrt(a**2*x**2 + 1)*atan(a*x)*a**6*x**6 + 384*sqrt(a**2*x*
*2 + 1)*atan(a*x)*a**4*x**4 + 48*sqrt(a**2*x**2 + 1)*atan(a*x)*a**2*x**2 -
 96*sqrt(a**2*x**2 + 1)*atan(a*x) - 40*sqrt(a**2*x**2 + 1)*a**5*x**5 - 46*
sqrt(a**2*x**2 + 1)*a**3*x**3 + 45*sqrt(a**2*x**2 + 1)*a*x + 51*log(sqrt(a
**2*x**2 + 1) + a*x)))/(1680*a**4)
```

# 3.209 $\int x^2(c+a^2cx^2)^{3/2}\arctan(ax)\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 357

$$\int x^2\left(c+a^2cx^2\right)^{3/2}\arctan(ax)\,dx=\frac{c\sqrt{c+a^2cx^2}}{16a^3}+\frac{\left(c+a^2cx^2\right)^{3/2}}{72a^3}-\frac{\left(c+a^2cx^2\right)^{5/2}}{30a^3c}+\frac{cx\sqrt{c+a^2cx^2}\arctan(ax)}{16a^2}+\frac{7}{24}cx^3\sqrt{c+a^2cx^2}\arctan(ax)+\frac{1}{6}a^2cx^5\sqrt{c+a^2cx^2}\arctan(ax)+\frac{ic^2\sqrt{1+a^2x^2}\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{8a^3\sqrt{c+a^2cx^2}}$$

output
```
1/16*c*(a^2*c*x^2+c)^(1/2)/a^3+1/72*(a^2*c*x^2+c)^(3/2)/a^3-1/30*(a^2*c*x^
2+c)^(5/2)/a^3/c+1/16*c*x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)/a^2+7/24*c*x^3*(
a^2*c*x^2+c)^(1/2)*arctan(a*x)+1/6*a^2*c*x^5*(a^2*c*x^2+c)^(1/2)*arctan(a*
x)+1/8*I*c^2*(a^2*x^2+1)^(1/2)*arctan(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*x
)^(1/2))/a^3/(a^2*c*x^2+c)^(1/2)-1/16*I*c^2*(a^2*x^2+1)^(1/2)*polylog(2,-I
*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/a^3/(a^2*c*x^2+c)^(1/2)+1/16*I*c^2*(a^2*
x^2+1)^(1/2)*polylog(2,I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/a^3/(a^2*c*x^2+c
)^(1/2)
```

## Mathematica [A] (warning: unable to verify)

Time = 4.36 (sec) , antiderivative size = 576, normalized size of antiderivative = 1.61

$$\int x^2(c+a^2cx^2)^{3/2}\arctan(ax)\,dx = \frac{c\sqrt{c+a^2cx^2}\left(\frac{3}{4}(1+a^2x^2)^{5/2}+\frac{55}{8}(1+a^2x^2)^3\cos(3\arctan(ax))-\frac{45}{8}(1+a^2x^2)\right.}{}$$

input
```
Integrate[x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x],x]
```

output
```
(c*Sqrt[c + a^2*c*x^2]*((3*(1 + a^2*x^2)^(5/2))/4 + (55*(1 + a^2*x^2)^3*Co
s[3*ArcTan[a*x]])/8 - (45*(1 + a^2*x^2)^3*Cos[5*ArcTan[a*x]])/8 - (90*I)*P
olyLog[2, (-I)*E^(I*ArcTan[a*x])] + (90*I)*PolyLog[2, I*E^(I*ArcTan[a*x])]
 - (15*(1 + a^2*x^2)^2*(-2/Sqrt[1 + a^2*x^2] - 6*Cos[3*ArcTan[a*x]] + 3*Ar
cTan[a*x]*((-14*a*x)/Sqrt[1 + a^2*x^2] + 3*Log[1 - I*E^(I*ArcTan[a*x])] +
4*Cos[2*ArcTan[a*x]]*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^(I*ArcTan
[a*x])]) + Cos[4*ArcTan[a*x]]*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^
(I*ArcTan[a*x])]) - 3*Log[1 + I*E^(I*ArcTan[a*x])] + 2*Sin[3*ArcTan[a*x]])
))/2 + (15*(1 + a^2*x^2)^3*ArcTan[a*x]*((156*a*x)/Sqrt[1 + a^2*x^2] + 30*L
og[1 - I*E^(I*ArcTan[a*x])] + 3*Cos[6*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a
*x])] + 45*Cos[2*ArcTan[a*x]]*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^
(I*ArcTan[a*x])]) + 18*Cos[4*ArcTan[a*x]]*(Log[1 - I*E^(I*ArcTan[a*x])] -
Log[1 + I*E^(I*ArcTan[a*x])]) - 30*Log[1 + I*E^(I*ArcTan[a*x])] - 3*Cos[6*
ArcTan[a*x]]*Log[1 + I*E^(I*ArcTan[a*x])] - 94*Sin[3*ArcTan[a*x]] + 6*Sin[
5*ArcTan[a*x]]))/16))/(1440*a^3*Sqrt[1 + a^2*x^2])
```

## Rubi [A] (verified)

Time = 2.53 (sec) , antiderivative size = 693, normalized size of antiderivative = 1.94, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {5485, 5481, 243, 53, 2009, 5487, 241, 243, 53, 2009, 5425, 5421, 5487, 241, 5425, 5421}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arctan(ax) \left(a^2cx^2+c\right)^{3/2} dx$$

$$\Big\downarrow \text{ 5485}$$

$$c\int x^2\sqrt{a^2cx^2+c}\arctan(ax)dx+a^2c\int x^4\sqrt{a^2cx^2+c}\arctan(ax)dx$$

$$\Big\downarrow \text{ 5481}$$

$$c\left(\frac{1}{4}c\int \frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{4}ac\int \frac{x^3}{\sqrt{a^2cx^2+c}}dx+\frac{1}{4}x^3\arctan(ax)\sqrt{a^2cx^2+c}\right)+$$
$$a^2c\left(\frac{1}{6}c\int \frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{6}ac\int \frac{x^5}{\sqrt{a^2cx^2+c}}dx+\frac{1}{6}x^5\arctan(ax)\sqrt{a^2cx^2+c}\right)$$

$$\Big\downarrow \text{ 243}$$

$$c\left(\frac{1}{4}c\int \frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{8}ac\int \frac{x^2}{\sqrt{a^2cx^2+c}}dx^2+\frac{1}{4}x^3\arctan(ax)\sqrt{a^2cx^2+c}\right)+$$
$$a^2c\left(\frac{1}{6}c\int \frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{12}ac\int \frac{x^4}{\sqrt{a^2cx^2+c}}dx^2+\frac{1}{6}x^5\arctan(ax)\sqrt{a^2cx^2+c}\right)$$

$$\Big\downarrow \text{ 53}$$

$$c\left(\frac{1}{4}c\int \frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{8}ac\int \left(\frac{\sqrt{a^2cx^2+c}}{a^2c}-\frac{1}{a^2\sqrt{a^2cx^2+c}}\right)dx^2+\frac{1}{4}x^3\arctan(ax)\sqrt{a^2cx^2+c}\right)+$$
$$a^2c\left(\frac{1}{6}c\int \frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{12}ac\int \left(\frac{\left(a^2cx^2+c\right)^{3/2}}{a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}+\frac{1}{a^4\sqrt{a^2cx^2+c}}\right)dx^2+\frac{1}{6}x^5\arctan(ax$$

$$\Big\downarrow \text{ 2009}$$

$$a^2c\left(\frac{1}{6}c\int \frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{6}x^5\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{12}ac\left(\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2}}{a}\right.\right.$$
$$c\left(\frac{1}{4}c\int \frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{4}x^3\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{8}ac\left(\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}\right)\right)$$

$$\Big\downarrow \text{ 5487}$$

$$a^2c\left(\frac{1}{6}c\left(-\frac{3\int \frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int \frac{x^3}{\sqrt{a^2cx^2+c}}dx}{4a}+\frac{x^3\arctan(ax)\sqrt{a^2cx^2+c}}{4a^2c}\right)+\frac{1}{6}x^5\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{12}\right.$$
$$c\left(\frac{1}{4}c\left(-\frac{\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a^2}-\frac{\int \frac{x}{\sqrt{a^2cx^2+c}}dx}{2a}+\frac{x\arctan(ax)\sqrt{a^2cx^2+c}}{2a^2c}\right)+\frac{1}{4}x^3\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{8}ac\left(\frac{2(}{}\right.\right.$$

$$\Big\downarrow \text{241}$$

$$a^2c\left(\frac{1}{6}c\left(-\frac{3\int \frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int \frac{x^3}{\sqrt{a^2cx^2+c}}dx}{4a}+\frac{x^3\arctan(ax)\sqrt{a^2cx^2+c}}{4a^2c}\right)+\frac{1}{6}x^5\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{12}\right.$$
$$c\left(\frac{1}{4}c\left(-\frac{\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a^2}+\frac{x\arctan(ax)\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\sqrt{a^2cx^2+c}}{2a^3c}\right)+\frac{1}{4}x^3\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{8}ac\left(\frac{2(a^2\right.\right.$$

$$\Big\downarrow \text{243}$$

$$a^2c\left(\frac{1}{6}c\left(-\frac{3\int \frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int \frac{x^2}{\sqrt{a^2cx^2+c}}dx^2}{8a}+\frac{x^3\arctan(ax)\sqrt{a^2cx^2+c}}{4a^2c}\right)+\frac{1}{6}x^5\arctan(ax)\sqrt{a^2cx^2+c}-\right.$$
$$c\left(\frac{1}{4}c\left(-\frac{\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a^2}+\frac{x\arctan(ax)\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\sqrt{a^2cx^2+c}}{2a^3c}\right)+\frac{1}{4}x^3\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{8}ac\left(\frac{2(a^2\right.\right.$$

$$\Big\downarrow \text{53}$$

$$a^2c\left(\frac{1}{6}c\left(-\frac{3\int \frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int \left(\frac{\sqrt{a^2cx^2+c}}{a^2c}-\frac{1}{a^2\sqrt{a^2cx^2+c}}\right)dx^2}{8a}+\frac{x^3\arctan(ax)\sqrt{a^2cx^2+c}}{4a^2c}\right)+\frac{1}{6}x^5\arctan(ax\right.$$
$$c\left(\frac{1}{4}c\left(-\frac{\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a^2}+\frac{x\arctan(ax)\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\sqrt{a^2cx^2+c}}{2a^3c}\right)+\frac{1}{4}x^3\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{8}ac\left(\frac{2(a^2\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$a^2c\left(\frac{1}{6}c\left(-\frac{3\int \frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a^2}+\frac{x^3\arctan(ax)\sqrt{a^2cx^2+c}}{4a^2c}-\frac{\frac{2(a^2cx^2+c)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}\right)+\frac{1}{6}x^5\arctan(ax)\sqrt{a^2c}\right.$$
$$c\left(\frac{1}{4}c\left(-\frac{\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a^2}+\frac{x\arctan(ax)\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\sqrt{a^2cx^2+c}}{2a^3c}\right)+\frac{1}{4}x^3\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{8}ac\left(\frac{2(a^2\right.\right.$$

$$\Big\downarrow \text{5425}$$

$$a^2c\left(\frac{1}{6}c\left(-\frac{3\int \frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a^2}+\frac{x^3\arctan(ax)\sqrt{a^2cx^2+c}}{4a^2c}-\frac{\frac{2(a^2cx^2+c)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}\right)+\frac{1}{6}x^5\arctan(ax)\sqrt{a^2c}\right.$$
$$c\left(\frac{1}{4}c\left(-\frac{\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{2a^2\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\sqrt{a^2cx^2+c}}{2a^3c}\right)+\frac{1}{4}x^3\arctan(ax)\sqrt{a^2cx^2+c}-\right.$$

$\downarrow$ 5421

$$a^2c\left(\frac{1}{6}c\left(-\frac{3\int \frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a^2}+\frac{x^3\arctan(ax)\sqrt{a^2cx^2+c}}{4a^2c}-\frac{\frac{2(a^2cx^2+c)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}\right)+\frac{1}{6}x^5\arctan(ax)\sqrt{a^2c\ldots}\right.$$

$$c\left(\frac{1}{4}x^3\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{8}ac\left(\frac{2(a^2cx^2+c)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}\right)+\frac{1}{4}c\left(-\frac{\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\,\mathrm{arcta}\ldots}{a}\right.}{\ldots}\right.\right.$$

$\downarrow$ 5487

$$a^2c\left(\frac{1}{6}c\left(-\frac{3\left(-\frac{\int\frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a^2}-\frac{\int\frac{x}{\sqrt{a^2cx^2+c}}dx}{2a}+\frac{x\arctan(ax)\sqrt{a^2cx^2+c}}{2a^2c}\right)}{4a^2}+\frac{x^3\arctan(ax)\sqrt{a^2cx^2+c}}{4a^2c}-\frac{\frac{2(a^2cx^2+c)^{3/2}}{3a^4c^2}\ldots}{8\ldots}\right.\right.$$

$$c\left(\frac{1}{4}x^3\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{8}ac\left(\frac{2(a^2cx^2+c)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}\right)+\frac{1}{4}c\left(-\frac{\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\,\mathrm{arcta}\ldots}{a}\right.}{\ldots}\right.\right.$$

$\downarrow$ 241

$$a^2c\left(\frac{1}{6}c\left(-\frac{3\left(-\frac{\int\frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a^2}+\frac{x\arctan(ax)\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\sqrt{a^2cx^2+c}}{2a^3c}\right)}{4a^2}+\frac{x^3\arctan(ax)\sqrt{a^2cx^2+c}}{4a^2c}-\frac{\frac{2(a^2cx^2+c)^{3/2}}{3a^4c^2}-\ldots}{8a}\right.\right.$$

$$c\left(\frac{1}{4}x^3\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{8}ac\left(\frac{2(a^2cx^2+c)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}\right)+\frac{1}{4}c\left(-\frac{\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\,\mathrm{arcta}\ldots}{a}\right.}{\ldots}\right.\right.$$

$\downarrow$ 5425

$$a^2c\left(\frac{1}{6}c\left(-\frac{3\left(-\frac{\sqrt{a^2x^2+1}\int\frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{2a^2\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\sqrt{a^2cx^2+c}}{2a^3c}\right)}{4a^2}+\frac{x^3\arctan(ax)\sqrt{a^2cx^2+c}}{4a^2c}-\frac{2(a^2cx^2+}{3a^4c^2}\right.\right.$$

$$c\left(\frac{1}{4}x^3\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{8}ac\left(\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}\right)+\frac{1}{4}c\left(-\frac{\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arcta}{a}\right.}{}\right.\right.$$

↓ 5421

$$c\left(\frac{1}{4}x^3\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{8}ac\left(\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}\right)+\frac{1}{4}c\left(-\frac{\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arcta}{a}\right.}{}\right.\right.$$

$$a^2c\left(\frac{1}{6}x^5\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{12}ac\left(\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}\right)+\frac{1}{6}c\left(\frac{x^3\arctan}{}\right.\right.$$

input
```
Int[x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x],x]
```

output
```
c*(-1/8*(a*c*((-2*Sqrt[c + a^2*c*x^2])/(a^4*c) + (2*(c + a^2*c*x^2)^(3/2))
/(3*a^4*c^2))) + (x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/4 + (c*(-1/2*Sqrt[c
 + a^2*c*x^2]/(a^3*c) + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(2*a^2*c) - (S
qrt[1 + a^2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*
x]])/a + (I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*Pol
yLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/(2*a^2*Sqrt[c + a^2*c*x^
2])))/4) + a^2*c*(-1/12*(a*c*((2*Sqrt[c + a^2*c*x^2])/(a^6*c) - (4*(c + a^
2*c*x^2)^(3/2))/(3*a^6*c^2) + (2*(c + a^2*c*x^2)^(5/2))/(5*a^6*c^3))) + (x
^5*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/6 + (c*(-1/8*((-2*Sqrt[c + a^2*c*x^2])
/(a^4*c) + (2*(c + a^2*c*x^2)^(3/2))/(3*a^4*c^2))/a + (x^3*Sqrt[c + a^2*c*
x^2]*ArcTan[a*x])/(4*a^2*c) - (3*(-1/2*Sqrt[c + a^2*c*x^2]/(a^3*c) + (x*Sq
rt[c + a^2*c*x^2]*ArcTan[a*x])/(2*a^2*c) - (Sqrt[1 + a^2*x^2]*(((-2*I)*Arc
Tan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/a + (I*PolyLog[2, ((-I)*
Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*PolyLog[2, (I*Sqrt[1 + I*a*x])/S
qrt[1 - I*a*x]])/a))/(2*a^2*Sqrt[c + a^2*c*x^2])))/(4*a^2)))/6)
```

## Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5481
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTan[c*x
])/(f*(m + 2))), x] + (Simp[d/(m + 2)   Int[(f*x)^m*((a + b*ArcTan[c*x])/Sq
rt[d + e*x^2]), x], x] - Simp[b*c*(d/(f*(m + 2)))   Int[(f*x)^(m + 1)/Sqrt[
d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] &&
NeQ[m, -2]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5487
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcTan[c*x])^p/(c^2*d*m)), x] + (-Simp[b*f*(p/(c*m))   Int[(f*x)^(m - 1)*((
a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[f^2*((m - 1)/(c^
2*m))   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2]), x], x])
/; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

## Maple [A] (verified)

Time = 1.36 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.62

| method | result |
|---|---|
| default | $\frac{c\sqrt{c(ax-i)(ax+i)}\,(120\arctan(ax)a^5x^5-24a^4x^4+210\arctan(ax)x^3a^3-38a^2x^2+45\arctan(ax)ax+31)}{720a^3}+\frac{c\sqrt{c(ax-i)(ax+i)}\,\left(\text{ar}\right.}{}$ |

input
```
int(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/720*c/a^3*(c*(a*x-I)*(a*x+I))^(1/2)*(120*arctan(a*x)*a^5*x^5-24*a^4*x^4+
210*arctan(a*x)*x^3*a^3-38*a^2*x^2+45*arctan(a*x)*a*x+31)+1/16*c*(c*(a*x-I
)*(a*x+I))^(1/2)*(arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-arctan(a
*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+I*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^
(1/2))-I*dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/a^3/(a^2*x^2+1)^(1/2)
```

## Fricas [F]

$$\int x^2\left(c+a^2cx^2\right)^{3/2}\arctan(ax)\,dx=\int\left(a^2cx^2+c\right)^{\frac{3}{2}}x^2\arctan\left(ax\right)\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x),x, algorithm="fricas")
```

output
```
integral((a^2*c*x^4 + c*x^2)*sqrt(a^2*c*x^2 + c)*arctan(a*x), x)
```

## Sympy [F]

$$\int x^2\left(c+a^2cx^2\right)^{3/2}\arctan(ax)\,dx=\int x^2\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}\left(ax\right)dx$$

input
```
integrate(x**2*(a**2*c*x**2+c)**(3/2)*atan(a*x),x)
```

output
```
Integral(x**2*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x), x)
```

## Maxima [F]

$$\int x^2 (c + a^2 c x^2)^{3/2} \arctan(ax)\, dx = \int (a^2 c x^2 + c)^{\frac{3}{2}} x^2 \arctan(ax)\, dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*x^2*arctan(a*x), x)
```

## Giac [F]

$$\int x^2 (c + a^2 c x^2)^{3/2} \arctan(ax)\, dx = \int (a^2 c x^2 + c)^{\frac{3}{2}} x^2 \arctan(ax)\, dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*x^2*arctan(a*x), x)
```

## Mupad [F(-1)]

Timed out.

$$\int x^2 (c + a^2 c x^2)^{3/2} \arctan(ax)\, dx = \int x^2 \operatorname{atan}(a\, x)\, \left(c\, a^2\, x^2 + c\right)^{3/2} dx$$

input
```
int(x^2*atan(a*x)*(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(x^2*atan(a*x)*(c + a^2*c*x^2)^(3/2), x)
```

## Reduce [F]

$$\int x^2(c+a^2cx^2)^{3/2}\arctan(ax)\,dx=\sqrt{c}\,c\left(\left(\int\sqrt{a^2x^2+1}\,atan(ax)\,x^4dx\right)a^2+\int\sqrt{a^2x^2+1}\,atan(ax)\,x^2dx\right)$$

input
```
int(x^2*(a^2*c*x^2+c)^(3/2)*atan(a*x),x)
```

output
```
sqrt(c)*c*(int(sqrt(a**2*x**2 + 1)*atan(a*x)*x**4,x)*a**2 + int(sqrt(a**2*
x**2 + 1)*atan(a*x)*x**2,x))
```

# 3.210 $\int x(c+a^2cx^2)^{3/2}\arctan(ax)\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 109

$$\int x(c+a^2cx^2)^{3/2}\arctan(ax)\,dx=-\frac{3cx\sqrt{c+a^2cx^2}}{40a}-\frac{x(c+a^2cx^2)^{3/2}}{20a}+\frac{(c+a^2cx^2)^{5/2}\arctan(ax)}{5a^2c}-\frac{3c^{3/2}\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right)}{40a^2}$$

output
```
-3/40*c*x*(a^2*c*x^2+c)^(1/2)/a-1/20*x*(a^2*c*x^2+c)^(3/2)/a+1/5*(a^2*c*x^
2+c)^(5/2)*arctan(a*x)/a^2/c-3/40*c^(3/2)*arctanh(a*c^(1/2)*x/(a^2*c*x^2+c
)^(1/2))/a^2
```

## Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.93

$$\int x(c+a^2cx^2)^{3/2}\arctan(ax)\,dx=-\frac{acx(5+2a^2x^2)\sqrt{c+a^2cx^2}-8c(1+a^2x^2)^2\sqrt{c+a^2cx^2}\arctan(ax)+3c^{3/2}\log\left(acx+\sqrt{c}\sqrt{c+a^2cx^2}\right)}{40a^2}$$

input
```
Integrate[x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x],x]
```

output
```
-1/40*(a*c*x*(5 + 2*a^2*x^2)*Sqrt[c + a^2*c*x^2] - 8*c*(1 + a^2*x^2)^2*Sqr
t[c + a^2*c*x^2]*ArcTan[a*x] + 3*c^(3/2)*Log[a*c*x + Sqrt[c]*Sqrt[c + a^2*
c*x^2]])/a^2
```

## Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5465, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \arctan(ax) \left(a^2cx^2+c\right)^{3/2} \, dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{\arctan(ax) \left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{\int \left(a^2cx^2+c\right)^{3/2}dx}{5a}$$

$$\Big\downarrow \text{211}$$

$$\frac{\arctan(ax) \left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{\frac{3}{4}c\int \sqrt{a^2cx^2+c}dx+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}}{5a}$$

$$\Big\downarrow \text{211}$$

$$\frac{\arctan(ax) \left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{\frac{3}{4}c\left(\frac{1}{2}c\int \frac{1}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}}{5a}$$

$$\Big\downarrow \text{224}$$

$$\frac{\arctan(ax) \left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{\frac{3}{4}c\left(\frac{1}{2}c\int \frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}}{5a}$$

$$\Big\downarrow \text{219}$$

$$\frac{\arctan(ax) \left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{\frac{3}{4}c\left(\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}}{5a}$$

input
```
Int[x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x],x]
```

output
```
((c + a^2*c*x^2)^(5/2)*ArcTan[a*x])/(5*a^2*c) - ((x*(c + a^2*c*x^2)^(3/2))
/4 + (3*c*((x*Sqrt[c + a^2*c*x^2])/2 + (Sqrt[c]*ArcTanh[(a*Sqrt[c]*x)/Sqrt
[c + a^2*c*x^2]])/(2*a)))/4)/(5*a)
```

**Defintions of rubi rules used**

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.18 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.64

| method | result |
|---|---|
| default | $\frac{c\sqrt{c(ax-i)(ax+i)}\,(8x^4\arctan(ax)a^4-2a^3x^3+16x^2a^2\arctan(ax)-5ax+8\arctan(ax))}{40a^2}-\frac{3c\sqrt{c(ax-i)(ax+i)}\,\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}+i\right)}{40a^2\sqrt{a^2x^2+1}}$ |

input
```
int(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/40*c/a^2*(c*(a*x-I)*(a*x+I))^(1/2)*(8*x^4*arctan(a*x)*a^4-2*a^3*x^3+16*x
^2*a^2*arctan(a*x)-5*a*x+8*arctan(a*x))-3/40*c/a^2*(c*(a*x-I)*(a*x+I))^(1/
2)*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+I)/(a^2*x^2+1)^(1/2)+3/40*c/a^2*(c*(a*x-
I)*(a*x+I))^(1/2)*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-I)/(a^2*x^2+1)^(1/2)
```

## Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.90

$$\int x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)\,dx = \frac{3\,c^{\frac{3}{2}}\log\left(-2\,a^2cx^2+2\,\sqrt{a^2cx^2+c}a\sqrt{c}x-c\right)-2\left(2\,a^3cx^3+5\,acx-8\left(a^4cx^4+\right.\right.}{80\,a^2}$$

input
```
integrate(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x),x, algorithm="fricas")
```

output
```
1/80*(3*c^(3/2)*log(-2*a^2*c*x^2 + 2*sqrt(a^2*c*x^2 + c)*a*sqrt(c)*x - c)
- 2*(2*a^3*c*x^3 + 5*a*c*x - 8*(a^4*c*x^4 + 2*a^2*c*x^2 + c)*arctan(a*x))*
sqrt(a^2*c*x^2 + c))/a^2
```

## Sympy [F]

$$\int x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)\,dx = \int x\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}(ax)\,dx$$

input
```
integrate(x*(a**2*c*x**2+c)**(3/2)*atan(a*x),x)
```

output
```
Integral(x*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x), x)
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 406 vs. $2(89) = 178$.

Time = 0.24 (sec) , antiderivative size = 406, normalized size of antiderivative = 3.72

$$\int x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)\,dx = \frac{40\left(a^2cx^2+c\right)\sqrt{a^2x^2+1}\sqrt{c}\arctan\left(ax\right)-20\left(a^4x^4+10\,a^2x^2+9\right)^{\frac{1}{4}}\left(acx\cos\left(\frac{1}{2}\right.\right.}{}$$

input
```
integrate(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x),x, algorithm="maxima")
```

output
```
1/120*(40*(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)*sqrt(c)*arctan(a*x) - 20*(a^4*
x^4 + 10*a^2*x^2 + 9)^(1/4)*(a*c*x*cos(1/2*arctan2(4*a*x, -a^2*x^2 + 3)) +
 2*c*sin(1/2*arctan2(4*a*x, -a^2*x^2 + 3)))*sqrt(c) - ((a*(3*(2*(a^2*x^2 +
 1)^(3/2)*x/a^2 - sqrt(a^2*x^2 + 1)*x/a^2 - arcsinh(a*x)/a^3)/a^2 - 8*(sqr
t(a^2*x^2 + 1)*x + arcsinh(a*x)/a)/a^4) - 8*(3*(a^2*x^2 + 1)^(3/2)*x^2/a^2
 - 2*(a^2*x^2 + 1)^(3/2)/a^4)*arctan(a*x))*a^4*c - 10*c*arctan2((a^4*x^4 +
 10*a^2*x^2 + 9)^(1/4)*sin(1/2*arctan2(4*a*x, a^2*x^2 - 3)) + 2, a*x + (a^
4*x^4 + 10*a^2*x^2 + 9)^(1/4)*cos(1/2*arctan2(4*a*x, a^2*x^2 - 3))) - 10*c
*arctan2((a^4*x^4 + 10*a^2*x^2 + 9)^(1/4)*sin(1/2*arctan2(4*a*x, a^2*x^2 -
 3)) - 2, -a*x + (a^4*x^4 + 10*a^2*x^2 + 9)^(1/4)*cos(1/2*arctan2(4*a*x, a
^2*x^2 - 3))))*sqrt(c))/a^2
```

## Giac [F(-2)]

Exception generated.

$$\int x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)\,dx=\int x\,\mathrm{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^{3/2}\,dx$$

input
```
int(x*atan(a*x)*(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(x*atan(a*x)*(c + a^2*c*x^2)^(3/2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.08

$$\int x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)\,dx=\frac{\sqrt{c}\,c\left(8\sqrt{a^2x^2+1}\,atan(ax)\,a^4x^4+16\sqrt{a^2x^2+1}\,atan(ax)\,a^2x^2+8\sqrt{a^2x^2+1}\,a\right.}{40a^2}$$

input
```
int(x*(a^2*c*x^2+c)^(3/2)*atan(a*x),x)
```

output
```
(sqrt(c)*c*(8*sqrt(a**2*x**2 + 1)*atan(a*x)*a**4*x**4 + 16*sqrt(a**2*x**2
+ 1)*atan(a*x)*a**2*x**2 + 8*sqrt(a**2*x**2 + 1)*atan(a*x) - 2*sqrt(a**2*x
**2 + 1)*a**3*x**3 - 5*sqrt(a**2*x**2 + 1)*a*x - 3*log(sqrt(a**2*x**2 + 1)
 + a*x)))/(40*a**2)
```

# 3.211 $\int (c+a^2cx^2)^{3/2}\arctan(ax)\,dx$



## Optimal result

Integrand size = 19, antiderivative size = 298

$$\int (c+a^2cx^2)^{3/2}\arctan(ax)\,dx = -\frac{3c\sqrt{c+a^2cx^2}}{8a} - \frac{(c+a^2cx^2)^{3/2}}{12a} + \frac{3}{8}cx\sqrt{c+a^2cx^2}\arctan(ax) + \frac{1}{4}x(c+a^2cx^2)^{3/2}\arctan(ax) - \frac{3ic^2\sqrt{1+a^2x^2}\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{4a\sqrt{c+a^2cx^2}} + \frac{3ic^2\sqrt{1+a^2x^2}\,\text{PolyLog}\left(2,-\right.}{8a\sqrt{c+a^2cx^2}}$$

output
```
-3/8*c*(a^2*c*x^2+c)^(1/2)/a-1/12*(a^2*c*x^2+c)^(3/2)/a+3/8*c*x*(a^2*c*x^2
+c)^(1/2)*arctan(a*x)+1/4*x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)-3/4*I*c^2*(a^2
*x^2+1)^(1/2)*arctan(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/a/(a^2*c
*x^2+c)^(1/2)+3/8*I*c^2*(a^2*x^2+1)^(1/2)*polylog(2,-I*(1+I*a*x)^(1/2)/(1-
I*a*x)^(1/2))/a/(a^2*c*x^2+c)^(1/2)-3/8*I*c^2*(a^2*x^2+1)^(1/2)*polylog(2,
I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/a/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (warning: unable to verify)

Time = 1.93 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.18

$$\int (c+a^2cx^2)^{3/2} \arctan(ax)\,dx = \frac{c\sqrt{c+a^2cx^2}\Big(2(1+a^2x^2)^{3/2}+96\sqrt{1+a^2x^2}(-1+ax\arctan(ax))+6(1+a^2x$$

input
```
Integrate[(c + a^2*c*x^2)^(3/2)*ArcTan[a*x],x]
```

output
```
(c*Sqrt[c + a^2*c*x^2]*(2*(1 + a^2*x^2)^(3/2) + 96*Sqrt[1 + a^2*x^2]*(-1 +
 a*x*ArcTan[a*x]) + 6*(1 + a^2*x^2)^2*Cos[3*ArcTan[a*x]] + 96*ArcTan[a*x]*
(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])]) + (72*I)*Pol
yLog[2, (-I)*E^(I*ArcTan[a*x])] - (72*I)*PolyLog[2, I*E^(I*ArcTan[a*x])] -
 3*(1 + a^2*x^2)^2*ArcTan[a*x]*((-14*a*x)/Sqrt[1 + a^2*x^2] + 3*Log[1 - I*
E^(I*ArcTan[a*x])] + 4*Cos[2*ArcTan[a*x]]*(Log[1 - I*E^(I*ArcTan[a*x])] -
Log[1 + I*E^(I*ArcTan[a*x])]) + Cos[4*ArcTan[a*x]]*(Log[1 - I*E^(I*ArcTan[
a*x])] - Log[1 + I*E^(I*ArcTan[a*x])]) - 3*Log[1 + I*E^(I*ArcTan[a*x])] +
2*Sin[3*ArcTan[a*x]])))/(192*a*Sqrt[1 + a^2*x^2])
```

## Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.80, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {5413, 5413, 5425, 5421}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(ax)\left(a^2cx^2+c\right)^{3/2}dx$$

$$\Big\downarrow \text{5413}$$

$$\frac{3}{4}c\int \sqrt{a^2cx^2+c}\arctan(ax)dx+\frac{1}{4}x\arctan(ax)\left(a^2cx^2+c\right)^{3/2}-\frac{\left(a^2cx^2+c\right)^{3/2}}{12a}$$

$$\Big\downarrow \text{5413}$$

$$\frac{3}{4}c\left(\frac{1}{2}c\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)+$$
$$\frac{1}{4}x\arctan(ax)\left(a^2cx^2+c\right)^{3/2}-\frac{\left(a^2cx^2+c\right)^{3/2}}{12a}$$

$$\Big\downarrow \text{5425}$$

$$\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)+$$
$$\frac{1}{4}x\arctan(ax)\left(a^2cx^2+c\right)^{3/2}-\frac{\left(a^2cx^2+c\right)^{3/2}}{12a}$$

$$\Big\downarrow \text{5421}$$

$$\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a^2\right.$$
$$\frac{1}{4}x\arctan(ax)\left(a^2cx^2+c\right)^{3/2}-\frac{\left(a^2cx^2+c\right)^{3/2}}{12a}$$

input
```
Int[(c + a^2*c*x^2)^(3/2)*ArcTan[a*x],x]
```

output
```
-1/12*(c + a^2*c*x^2)^(3/2)/a + (x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/4 +
(3*c*(-1/2*Sqrt[c + a^2*c*x^2]/a + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/2 +
 (c*Sqrt[1 + a^2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 -
 I*a*x]])/a + (I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (
I*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/(2*Sqrt[c + a^2*c*x
^2])))/4
```

### Defintions of rubi rules used

rule 5413
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Simp[x*(d + e*x^2)
^q*((a + b*ArcTan[c*x])/(2*q + 1)), x] + Simp[2*d*(q/(2*q + 1))   Int[(d +
e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0]
```

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

## Maple [A] (verified)

Time = 1.36 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.67

| method | result |
|---|---|
| default | $\frac{c\sqrt{c(ax-i)(ax+i)}\,(6\arctan(ax)x^3a^3-2a^2x^2+15\arctan(ax)ax-11)}{24a}-\frac{3c\sqrt{c(ax-i)(ax+i)}\left(\arctan(ax)\ln\left(1+\frac{i(iax+1)}{\sqrt{a^2x^2+1}}\right)-\mathrm{arc}\right.}{}$ |

input
```
int((a^2*c*x^2+c)^(3/2)*arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/24*c/a*(c*(a*x-I)*(a*x+I))^(1/2)*(6*arctan(a*x)*x^3*a^3-2*a^2*x^2+15*arc
tan(a*x)*a*x-11)-3/8*c*(c*(a*x-I)*(a*x+I))^(1/2)*(arctan(a*x)*ln(1+I*(1+I*
a*x)/(a^2*x^2+1)^(1/2))-arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+I*
dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(
1/2)))/a/(a^2*x^2+1)^(1/2)
```

## Fricas [F]

$$\int \left(c+a^2cx^2\right)^{3/2}\arctan(ax)\,dx=\int \left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x),x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^(3/2)*arctan(a*x), x)
```

## Sympy [F]

$$\int \left(c+a^2cx^2\right)^{3/2}\arctan(ax)\,dx=\int \left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}\left(ax\right)dx$$

input
```
integrate((a**2*c*x**2+c)**(3/2)*atan(a*x),x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(3/2)*atan(a*x), x)
```

## Maxima [F]

$$\int \left(c+a^2cx^2\right)^{3/2}\arctan(ax)\,dx=\int \left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*arctan(a*x), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^{3/2}\arctan(ax)\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \left(c+a^2cx^2\right)^{3/2}\arctan(ax)\,dx=\int \operatorname{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^{3/2}\,dx$$

input
```
int(atan(a*x)*(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(atan(a*x)*(c + a^2*c*x^2)^(3/2), x)
```

## Reduce [F]

$$\int \left(c+a^2cx^2\right)^{3/2}\arctan(ax)\,dx=\sqrt{c}\,c\left(\left(\int \sqrt{a^2x^2+1}\,atan(ax)\,x^2dx\right)a^2+\int \sqrt{a^2x^2+1}\,atan(ax)\,dx\right)$$

input
```
int((a^2*c*x^2+c)^(3/2)*atan(a*x),x)
```

output
```
sqrt(c)*c*(int(sqrt(a**2*x**2 + 1)*atan(a*x)*x**2,x)*a**2 + int(sqrt(a**2*
x**2 + 1)*atan(a*x),x))
```

# 3.212 $\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{x}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 281

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{x}\,dx = -\frac{1}{6}acx\sqrt{c+a^2cx^2}+c\sqrt{c+a^2cx^2}\arctan(ax) + \frac{1}{3}(c+a^2cx^2)^{3/2}\arctan(ax) - \frac{2c^2\sqrt{1+a^2x^2}\arctan(ax)\operatorname{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} - \frac{7}{6}c^{3/2}\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right) + \frac{ic^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} - \frac{ic^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}}$$

output
```
-1/6*a*c*x*(a^2*c*x^2+c)^(1/2)+c*(a^2*c*x^2+c)^(1/2)*arctan(a*x)+1/3*(a^2*
c*x^2+c)^(3/2)*arctan(a*x)-2*c^2*(a^2*x^2+1)^(1/2)*arctan(a*x)*arctanh((1+
I*a*x)^(1/2)/(1-I*a*x)^(1/2))/(a^2*c*x^2+c)^(1/2)-7/6*c^(3/2)*arctanh(a*c^
(1/2)*x/(a^2*c*x^2+c)^(1/2))+I*c^2*(a^2*x^2+1)^(1/2)*polylog(2,-(1+I*a*x)^
(1/2)/(1-I*a*x)^(1/2))/(a^2*c*x^2+c)^(1/2)-I*c^2*(a^2*x^2+1)^(1/2)*polylog
(2,(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.83

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{x}\,dx = \frac{c\sqrt{c+a^2cx^2}\left(-ax\sqrt{1+a^2x^2}+8\sqrt{1+a^2x^2}\arctan(ax)+2a^2x^2\sqrt{1+a^2\right.}}{}$$

input
```
Integrate[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/x,x]
```

output
```
(c*Sqrt[c + a^2*c*x^2]*(-(a*x*Sqrt[1 + a^2*x^2]) + 8*Sqrt[1 + a^2*x^2]*Arc
Tan[a*x] + 2*a^2*x^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x] + 6*ArcTan[a*x]*Log[1 -
 E^(I*ArcTan[a*x])] - 6*ArcTan[a*x]*Log[1 + E^(I*ArcTan[a*x])] + Log[-(a*x
) + Sqrt[1 + a^2*x^2]] + 6*Log[Cos[ArcTan[a*x]/2] - Sin[ArcTan[a*x]/2]] -
6*Log[Cos[ArcTan[a*x]/2] + Sin[ArcTan[a*x]/2]] + (6*I)*PolyLog[2, -E^(I*Ar
cTan[a*x])] - (6*I)*PolyLog[2, E^(I*ArcTan[a*x])]))/(6*Sqrt[1 + a^2*x^2])
```

## Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.98, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {5485, 5465, 211, 224, 219, 5481, 224, 219, 5493, 5489}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{x}\,dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\int x\sqrt{a^2cx^2+c}\arctan(ax)dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x}dx$$

$$\Big\downarrow \text{5465}$$

$$a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\int \sqrt{a^2cx^2+c}dx}{3a}\right)+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x}dx$$

$$\Big\downarrow \text{211}$$

$$a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{1}{2}c\int\frac{1}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\sqrt{a^2cx^2+c}}{3a}\right)+c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x}dx$$

$$\Big\downarrow \; 224$$

$$a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{1}{2}c\int\frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}+\frac{1}{2}x\sqrt{a^2cx^2+c}}{3a}\right)+c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x}dx$$

$$\Big\downarrow \; 219$$

$$c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x}dx+a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{\sqrt{c}\,\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}}{3a}\right)$$

$$\Big\downarrow \; 5481$$

$$c\left(c\int\frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx-ac\int\frac{1}{\sqrt{a^2cx^2+c}}dx+\arctan(ax)\sqrt{a^2cx^2+c}\right)+a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{\sqrt{c}\,\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}}{3a}\right)$$

$$\Big\downarrow \; 224$$

$$c\left(c\int\frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx-ac\int\frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}+\arctan(ax)\sqrt{a^2cx^2+c}\right)+a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{\sqrt{c}\,\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}}{3a}\right)$$

$$\Big\downarrow \; 219$$

$$c\left(c\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx+\arctan(ax)\sqrt{a^2cx^2+c}-\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)\right)+$$
$$a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}}{3a}\right)$$

$\Big\downarrow$ 5493

$$c\left(\frac{c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+\arctan(ax)\sqrt{a^2cx^2+c}-\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)\right)+$$
$$a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}}{3a}\right)$$

$\Big\downarrow$ 5489

$$a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}}{3a}\right)+$$
$$c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{PolyLog}\left(2,\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)\right)}{\sqrt{a^2cx^2+c}}+\arctan(\right.$$

input
```
Int[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/x,x]
```

output
```
a^2*c*(((c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/(3*a^2*c) - ((x*Sqrt[c + a^2*c*
x^2])/2 + (Sqrt[c]*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/(2*a))/(3*a
)) + c*(Sqrt[c + a^2*c*x^2]*ArcTan[a*x] - Sqrt[c]*ArcTanh[(a*Sqrt[c]*x)/Sq
rt[c + a^2*c*x^2]] + (c*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]*ArcTanh[Sqrt[1 +
 I*a*x]/Sqrt[1 - I*a*x]] + I*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])
] - I*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]]))/Sqrt[c + a^2*c*x^2])
```

## Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5481
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTan[c*x
])/(f*(m + 2))), x] + (Simp[d/(m + 2)   Int[(f*x)^m*((a + b*ArcTan[c*x])/Sq
rt[d + e*x^2]), x], x] - Simp[b*c*(d/(f*(m + 2)))   Int[(f*x)^(m + 1)/Sqrt[
d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] &&
NeQ[m, -2]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5489
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sq
rt[1 - I*c*x]], x] + (Simp[I*(b/Sqrt[d])*PolyLog[2, -Sqrt[1 + I*c*x]/Sqrt[1
 - I*c*x]], x] - Simp[I*(b/Sqrt[d])*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x]], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

## Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.66

| method | result |
|---|---|
| default | $-\frac{\sqrt{c(ax-i)(ax+i)}\left(-2\arctan(ax)\sqrt{a^2x^2+1}\,a^2x^2+\sqrt{a^2x^2+1}\,ax-8\arctan(ax)\sqrt{a^2x^2+1}+6\arctan(ax)\ln\left(1+\frac{iax+1}{\sqrt{a^2x^2+1}}\right)-6i\,\mathrm{dil}}{6\sqrt{a^2x^2+1}}$ |

input
```
int((a^2*c*x^2+c)^(3/2)*arctan(a*x)/x,x,method=_RETURNVERBOSE)
```

output
```
-1/6/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(a*x+I))^(1/2)*(-2*arctan(a*x)*(a^2*x^2+
1)^(1/2)*a^2*x^2+(a^2*x^2+1)^(1/2)*a*x-8*arctan(a*x)*(a^2*x^2+1)^(1/2)+6*a
rctan(a*x)*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I*dilog(1+(1+I*a*x)/(a^2*x^
2+1)^(1/2))-14*I*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I*dilog((1+I*a*x)/(
a^2*x^2+1)^(1/2)))*c
```

## Fricas [F]

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{x}\,dx=\int \frac{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)}{x}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)/x,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^(3/2)*arctan(a*x)/x, x)
```

## Sympy [F]

$$\int \frac{(c + a^2 c x^2)^{3/2} \arctan(ax)}{x} \, dx = \int \frac{(c(a^2x^2 + 1))^{\frac{3}{2}} \operatorname{atan}(ax)}{x} \, dx$$

input
```
integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)/x,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)/x, x)
```

## Maxima [F]

$$\int \frac{(c + a^2 c x^2)^{3/2} \arctan(ax)}{x} \, dx = \int \frac{(a^2cx^2 + c)^{\frac{3}{2}} \arctan(ax)}{x} \, dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)/x,x, algorithm="maxima")
```

output
```
1/3*(a^2*c*x^2 + c)*sqrt(a^2*x^2 + 1)*sqrt(c)*arctan(a*x) - 1/6*(a^4*x^4 +
 10*a^2*x^2 + 9)^(1/4)*(a*c*x*cos(1/2*arctan2(4*a*x, -a^2*x^2 + 3)) + 2*c*
sin(1/2*arctan2(4*a*x, -a^2*x^2 + 3)))*sqrt(c) + 1/12*(c*arctan2((a^4*x^4
+ 10*a^2*x^2 + 9)^(1/4)*sin(1/2*arctan2(4*a*x, a^2*x^2 - 3)) + 2, a*x + (a
^4*x^4 + 10*a^2*x^2 + 9)^(1/4)*cos(1/2*arctan2(4*a*x, a^2*x^2 - 3))) + c*a
rctan2((a^4*x^4 + 10*a^2*x^2 + 9)^(1/4)*sin(1/2*arctan2(4*a*x, a^2*x^2 - 3
)) - 2, -a*x + (a^4*x^4 + 10*a^2*x^2 + 9)^(1/4)*cos(1/2*arctan2(4*a*x, a^2
*x^2 - 3))) + 12*c*integrate(sqrt(a^2*x^2 + 1)*arctan(a*x)/x, x))*sqrt(c)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{x}\,dx = \int \frac{\mathrm{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^{3/2}}{x}\,dx$$

input
```
int((atan(a*x)*(c + a^2*c*x^2)^(3/2))/x,x)
```

output
```
int((atan(a*x)*(c + a^2*c*x^2)^(3/2))/x, x)
```

## Reduce [F]

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{x}\,dx = \frac{\sqrt{c}\,c\Big(2\sqrt{a^2x^2+1}\,atan(ax)\,a^2x^2+2\sqrt{a^2x^2+1}\,atan(ax)-\sqrt{a^2x^2+1}\,a}{6}$$

input
```
int((a^2*c*x^2+c)^(3/2)*atan(a*x)/x,x)
```

output
```
(sqrt(c)*c*(2*sqrt(a**2*x**2 + 1)*atan(a*x)*a**2*x**2 + 2*sqrt(a**2*x**2 +
 1)*atan(a*x) - sqrt(a**2*x**2 + 1)*a*x + 6*int((sqrt(a**2*x**2 + 1)*atan(
a*x))/x,x) - log(sqrt(a**2*x**2 + 1) + a*x)))/6
```

# 3.213 $\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{x^2}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 300

$$\begin{aligned}\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{x^2}\,dx &= -\frac{1}{2}ac\sqrt{c+a^2cx^2} - \frac{c\sqrt{c+a^2cx^2}\arctan(ax)}{x}\\ &+ \frac{1}{2}a^2cx\sqrt{c+a^2cx^2}\arctan(ax) - \frac{3iac^2\sqrt{1+a^2x^2}\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}}\\ &- ac^{3/2}\text{arctanh}\left(\frac{\sqrt{c+a^2cx^2}}{\sqrt{c}}\right) + \frac{3iac^2\sqrt{1+a^2x^2}\,\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2\sqrt{c+a^2cx^2}}\\ &- \frac{3iac^2\sqrt{1+a^2x^2}\,\text{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2\sqrt{c+a^2cx^2}}\end{aligned}$$

output

```
-1/2*a*c*(a^2*c*x^2+c)^(1/2)-c*(a^2*c*x^2+c)^(1/2)*arctan(a*x)/x+1/2*a^2*c
*x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)-3*I*a*c^2*(a^2*x^2+1)^(1/2)*arctan(a*x)
*arctan((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/(a^2*c*x^2+c)^(1/2)-a*c^(3/2)*arc
tanh((a^2*c*x^2+c)^(1/2)/c^(1/2))+3/2*I*a*c^2*(a^2*x^2+1)^(1/2)*polylog(2,
-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/(a^2*c*x^2+c)^(1/2)-3/2*I*a*c^2*(a^2*x
^2+1)^(1/2)*polylog(2,I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/(a^2*c*x^2+c)^(1/
2)
```

## Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.73

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{x^2}\,dx = \frac{c\sqrt{c+a^2cx^2}\left(-ax\sqrt{1+a^2x^2}-2\sqrt{1+a^2x^2}\arctan(ax)+a^2x^2\sqrt{1+a^2x\right.}}{}$$

input
```
Integrate[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/x^2,x]
```

output
```
(c*Sqrt[c + a^2*c*x^2]*(-(a*x*Sqrt[1 + a^2*x^2]) - 2*Sqrt[1 + a^2*x^2]*Arc
Tan[a*x] + a^2*x^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x] + 3*a*x*ArcTan[a*x]*Log[1
 - I*E^(I*ArcTan[a*x])] - 3*a*x*ArcTan[a*x]*Log[1 + I*E^(I*ArcTan[a*x])] -
 2*a*x*Log[Cos[ArcTan[a*x]/2]] + 2*a*x*Log[Sin[ArcTan[a*x]/2]] + (3*I)*a*x
*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - (3*I)*a*x*PolyLog[2, I*E^(I*ArcTan[a
*x])]))/(2*x*Sqrt[1 + a^2*x^2])
```

## Rubi [A] (verified)

Time = 1.64 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.33, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5485, 5413, 5425, 5421, 5485, 5425, 5421, 5479, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{x^2}dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\int \sqrt{a^2cx^2+c}\arctan(ax)dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^2}dx$$

$$\Big\downarrow \text{5413}$$

$$a^2c\left(\frac{1}{2}c\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)+ c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^2}dx$$

$$\Big\downarrow \text{5425}$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\int\frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)+$$
$$c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^2}dx$$

$$\Big\downarrow \text{5421}$$

$$c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^2}dx+$$
$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a\right.$$

$$\Big\downarrow \text{5485}$$

$$c\left(a^2c\int\frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+c\int\frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx\right)+$$
$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a\right.$$

$$\Big\downarrow \text{5425}$$

$$c\left(\frac{a^2c\sqrt{a^2x^2+1}\int\frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+c\int\frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx\right)+$$
$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a\right.$$

$$\Big\downarrow \text{5421}$$

$$c\left(c\int \frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx+\frac{a^2c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right.}{\sqrt{a^2cx^2+c}}\right.$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a\right.$$

$\downarrow$ 5479

$$c\left(c\left(a\int \frac{1}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right)+\frac{a^2c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2\right.}{a}\right.}{\sqrt{a^2cx^2+c}}\right.$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a\right.$$

$\downarrow$ 243

$$c\left(c\left(\frac{1}{2}a\int \frac{1}{x^2\sqrt{a^2cx^2+c}}dx^2-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right)+\frac{a^2c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyL}}{}\right.}{\sqrt{a^2cx^2+c}}\right.$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a\right.$$

$\downarrow$ 73

$$c\left(c\left(\frac{\int \frac{1}{\frac{x^4}{a^2c}-\frac{1}{a^2}}d\sqrt{a^2cx^2+c}}{ac}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right)+\frac{a^2c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{PolyLog}}{}\right.}{\sqrt{a^2cx^2+c}}\right.$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a}\right.$$

$$\Big\downarrow \ 221$$

$$c\left(c\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\,\mathrm{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)+\frac{a^2c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{PolyLo}}{}\right.}{\sqrt{a^2cx^2+c}}\right.$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a}\right.$$

input

```
Int[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/x^2,x]
```

output

```
a^2*c*(-1/2*Sqrt[c + a^2*c*x^2]/a + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/2
+ (c*Sqrt[1 + a^2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1
- I*a*x]])/a + (I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a -
(I*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/(2*Sqrt[c + a^2*c*
x^2])) + c*(c*(-((Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(c*x)) - (a*ArcTanh[Sqr
t[c + a^2*c*x^2]/Sqrt[c]])/Sqrt[c]) + (a^2*c*Sqrt[1 + a^2*x^2]*(((-2*I)*Ar
cTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/a + (I*PolyLog[2, ((-I)
*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*PolyLog[2, (I*Sqrt[1 + I*a*x])/
Sqrt[1 - I*a*x]])/a))/Sqrt[c + a^2*c*x^2])
```

### Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5413
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Simp[x*(d + e*x^2)
^q*((a + b*ArcTan[c*x])/(2*q + 1)), x] + Simp[2*d*(q/(2*q + 1))   Int[(d +
e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0]
```

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

## Maple [A] (verified)

Time = 1.59 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.88

| method | result |
|---|---|
| default | $-\frac{\sqrt{c(ax-i)(ax+i)}\left(-\arctan(ax)\sqrt{a^2x^2+1}\,a^2x^2+3\arctan(ax)\ln\left(1+\frac{i(iax+1)}{\sqrt{a^2x^2+1}}\right)ax-3\arctan(ax)\ln\left(1-\frac{i(iax+1)}{\sqrt{a^2x^2+1}}\right)ax-3i\,\text{dilog}\ldots\right)}{2\ldots}$ |

input
```
int((a^2*c*x^2+c)^(3/2)*arctan(a*x)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(a*x+I))^(1/2)*(-arctan(a*x)*(a^2*x^2+1)
^(1/2)*a^2*x^2+3*arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*a*x-3*arc
tan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*a*x-3*I*dilog(1+I*(1+I*a*x)/(
a^2*x^2+1)^(1/2))*a*x+3*I*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*a*x+(a^2*
x^2+1)^(1/2)*a*x+2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))*a*x-2*ln((1+I*a*x)/(a
^2*x^2+1)^(1/2)-1)*a*x+2*arctan(a*x)*(a^2*x^2+1)^(1/2))*c/x
```

## Fricas [F]

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{x^2}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)}{x^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)/x^2,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^(3/2)*arctan(a*x)/x^2, x)
```

## Sympy [F]

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{x^2}\,dx = \int \frac{(c(a^2x^2+1))^{\frac{3}{2}}\operatorname{atan}(ax)}{x^2}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)/x**2,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)/x**2, x)
```

## Maxima [F]

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{x^2}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)}{x^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)/x^2,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*arctan(a*x)/x^2, x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)/x^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{x^2}\,dx = \int \frac{\operatorname{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^{3/2}}{x^2}\,dx$$

input
```
int((atan(a*x)*(c + a^2*c*x^2)^(3/2))/x^2,x)
```

output
```
int((atan(a*x)*(c + a^2*c*x^2)^(3/2))/x^2, x)
```

## Reduce [F]

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{x^2}\,dx = \sqrt{c}\,c\left(\int \frac{\sqrt{a^2x^2+1}\,atan(ax)}{x^2}dx + \left(\int \sqrt{a^2x^2+1}\,atan(ax)\,dx\right)a^2\right)$$

input
```
int((a^2*c*x^2+c)^(3/2)*atan(a*x)/x^2,x)
```

output
```
sqrt(c)*c*(int((sqrt(a**2*x**2 + 1)*atan(a*x))/x**2,x) + int(sqrt(a**2*x**
2 + 1)*atan(a*x),x)*a**2)
```

# 3.214 $\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{x^3}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 304

$$\begin{aligned}\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{x^3}\,dx =& -\frac{ac\sqrt{c+a^2cx^2}}{2x}+a^2c\sqrt{c+a^2cx^2}\arctan(ax)\\ &-\frac{c\sqrt{c+a^2cx^2}\arctan(ax)}{2x^2}-\frac{3a^2c^2\sqrt{1+a^2x^2}\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}}\\ &-a^2c^{3/2}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right)+\frac{3ia^2c^2\sqrt{1+a^2x^2}\,\text{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2\sqrt{c+a^2cx^2}}\\ &-\frac{3ia^2c^2\sqrt{1+a^2x^2}\,\text{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2\sqrt{c+a^2cx^2}}\end{aligned}$$

output

```
-1/2*a*c*(a^2*c*x^2+c)^(1/2)/x+a^2*c*(a^2*c*x^2+c)^(1/2)*arctan(a*x)-1/2*c
*(a^2*c*x^2+c)^(1/2)*arctan(a*x)/x^2-3*a^2*c^2*(a^2*x^2+1)^(1/2)*arctan(a*
x)*arctanh((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/(a^2*c*x^2+c)^(1/2)-a^2*c^(3/2
)*arctanh(a*c^(1/2)*x/(a^2*c*x^2+c)^(1/2))+3/2*I*a^2*c^2*(a^2*x^2+1)^(1/2)
*polylog(2,-(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/(a^2*c*x^2+c)^(1/2)-3/2*I*a^2
*c^2*(a^2*x^2+1)^(1/2)*polylog(2,(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/(a^2*c*x
^2+c)^(1/2)
```

## Mathematica [A] (warning: unable to verify)

Time = 1.30 (sec) , antiderivative size = 301, normalized size of antiderivative = 0.99

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{x^3}\,dx = \frac{a^2c\sqrt{c+a^2cx^2}\left(-2-2\cot^2\left(\frac{1}{2}\arctan(ax)\right)+4ax\arctan(ax)\csc^2\left(\frac{1}{2}\arctan\right.\right.}{}$$

input
```
Integrate[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/x^3,x]
```

output
```
(a^2*c*Sqrt[c + a^2*c*x^2]*(-2 - 2*Cot[ArcTan[a*x]/2]^2 + 4*a*x*ArcTan[a*x
]*Csc[ArcTan[a*x]/2]^2 - ArcTan[a*x]*Cot[ArcTan[a*x]/2]*Csc[ArcTan[a*x]/2]
^2 + 12*ArcTan[a*x]*Cot[ArcTan[a*x]/2]*Log[1 - E^(I*ArcTan[a*x])] - 12*Arc
Tan[a*x]*Cot[ArcTan[a*x]/2]*Log[1 + E^(I*ArcTan[a*x])] + 8*Cot[ArcTan[a*x]
/2]*Log[Cos[ArcTan[a*x]/2] - Sin[ArcTan[a*x]/2]] - 8*Cot[ArcTan[a*x]/2]*Lo
g[Cos[ArcTan[a*x]/2] + Sin[ArcTan[a*x]/2]] + (12*I)*Cot[ArcTan[a*x]/2]*Pol
yLog[2, -E^(I*ArcTan[a*x])] - (12*I)*Cot[ArcTan[a*x]/2]*PolyLog[2, E^(I*Ar
cTan[a*x])] + ArcTan[a*x]*Csc[ArcTan[a*x]/2]*Sec[ArcTan[a*x]/2])*Tan[ArcTa
n[a*x]/2])/(8*Sqrt[1 + a^2*x^2])
```

## Rubi [A] (verified)

Time = 1.68 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.36, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5485, 5481, 224, 219, 242, 5493, 5489, 5497, 242, 5493, 5489}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{x^3}\,dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x}dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^3}dx$$

$$\Big\downarrow \text{5481}$$

$$a^2c\left(c\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx-ac\int \frac{1}{\sqrt{a^2cx^2+c}}dx+\arctan(ax)\sqrt{a^2cx^2+c}\right)+c\left(-c\int \frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx+ac\int \frac{1}{x^2\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}\right)$$

$$\Big\downarrow \text{224}$$

$$a^2c\left(c\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx-ac\int \frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}+\arctan(ax)\sqrt{a^2cx^2+c}\right)+c\left(-c\int \frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx+ac\int \frac{1}{x^2\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}\right)$$

$$\Big\downarrow \text{219}$$

$$a^2c\left(c\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx+\arctan(ax)\sqrt{a^2cx^2+c}-\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)\right)+c\left(-c\int \frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx+ac\int \frac{1}{x^2\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}\right)$$

$$\Big\downarrow \text{242}$$

$$a^2c\left(c\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx+\arctan(ax)\sqrt{a^2cx^2+c}-\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)\right)+c\left(-c\int \frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}-\frac{a\sqrt{a^2cx^2+c}}{x}\right)$$

$$\Big\downarrow \text{5493}$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+\arctan(ax)\sqrt{a^2cx^2+c}-\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)\right)+c\left(-c\int \frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}-\frac{a\sqrt{a^2cx^2+c}}{x}\right)$$

$$\Big\downarrow \text{5489}$$

$$c\left(-c\int \frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}-\frac{a\sqrt{a^2cx^2+c}}{x}\right)+a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{PolyLog}\left(2,\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)\right)}{\sqrt{a^2cx^2+c}}+\text{arcta}\right.$$

$$\Big\downarrow \text{5497}$$

$$c\left(-c\left(-\frac{1}{2}a^2\int\frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx+\frac{1}{2}a\int\frac{1}{x^2\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{2cx^2}\right)-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}\right.$$
$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{PolyLog}\left(2,\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)\right)}{\sqrt{a^2cx^2+c}}+\text{arcta}\right.$$

$\downarrow$ 242

$$c\left(-c\left(-\frac{1}{2}a^2\int\frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{2cx^2}-\frac{a\sqrt{a^2cx^2+c}}{2cx}\right)-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}-\frac{a\sqrt{a^2c}}{x}\right.$$
$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{PolyLog}\left(2,\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)\right)}{\sqrt{a^2cx^2+c}}+\text{arcta}\right.$$

$\downarrow$ 5493

$$c\left(-c\left(-\frac{a^2\sqrt{a^2x^2+1}\int\frac{\arctan(ax)}{x\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{2cx^2}-\frac{a\sqrt{a^2cx^2+c}}{2cx}\right)-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}-a\right.$$
$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{PolyLog}\left(2,\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)\right)}{\sqrt{a^2cx^2+c}}+\text{arcta}\right.$$

$\downarrow$ 5489

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{PolyLog}\left(2,\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)\right)}{\sqrt{a^2cx^2+c}}+\text{arcta}\right.$$
$$c\left(-c\left(-\frac{a^2\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{PolyLog}\left(2,\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)\right)}{2\sqrt{a^2cx^2+c}}-\right.\right.$$

input
```
Int[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/x^3,x]
```

output
```
c*(-((a*Sqrt[c + a^2*c*x^2])/x) - (Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/x^2 -
c*(-1/2*(a*Sqrt[c + a^2*c*x^2])/(c*x) - (Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/
(2*c*x^2) - (a^2*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]
/Sqrt[1 - I*a*x]] + I*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])] - I*P
olyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]]))/(2*Sqrt[c + a^2*c*x^2]))) + a
^2*c*(Sqrt[c + a^2*c*x^2]*ArcTan[a*x] - Sqrt[c]*ArcTanh[(a*Sqrt[c]*x)/Sqrt
[c + a^2*c*x^2]] + (c*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]*ArcTanh[Sqrt[1 + I
*a*x]/Sqrt[1 - I*a*x]] + I*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])]
- I*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]]))/Sqrt[c + a^2*c*x^2])
```

### Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 5481
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTan[c*x
])/(f*(m + 2))), x] + (Simp[d/(m + 2)   Int[(f*x)^m*((a + b*ArcTan[c*x])/Sq
rt[d + e*x^2]), x], x] - Simp[b*c*(d/(f*(m + 2)))   Int[(f*x)^(m + 1)/Sqrt[
d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] &&
NeQ[m, -2]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5489
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sq
rt[1 - I*c*x]], x] + (Simp[I*(b/Sqrt[d])*PolyLog[2, -Sqrt[1 + I*c*x]/Sqrt[1
 - I*c*x]], x] - Simp[I*(b/Sqrt[d])*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x]], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5497
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)^(m
 + 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[c^2*((m
+ 2)/(f^2*(m + 1)))   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x
^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
 && LtQ[m, -1] && NeQ[m, -2]
```

## Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.69

| method | result |
|---|---|
| default | $-\frac{\sqrt{c(ax-i)(ax+i)}\left(-2\arctan(ax)\sqrt{a^2x^2+1}\,a^2x^2+3\arctan(ax)\ln\left(1+\frac{iax+1}{\sqrt{a^2x^2+1}}\right)a^2x^2-3i\,\mathrm{dilog}\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)a^2x^2-3i\,\mathrm{dilog}\left(1+\right.\right.}{2\sqrt{a^2x^2+1}\,x^2}$ |

input
```
int((a^2*c*x^2+c)^(3/2)*arctan(a*x)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(a*x+I))^(1/2)*(-2*arctan(a*x)*(a^2*x^2+
1)^(1/2)*a^2*x^2+3*arctan(a*x)*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))*a^2*x^2-3
*I*dilog((1+I*a*x)/(a^2*x^2+1)^(1/2))*a^2*x^2-3*I*dilog(1+(1+I*a*x)/(a^2*x
^2+1)^(1/2))*a^2*x^2-4*I*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))*a^2*x^2+(a^2*
x^2+1)^(1/2)*a*x+arctan(a*x)*(a^2*x^2+1)^(1/2))*c/x^2
```

## Fricas [F]

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{x^3}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)}{x^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)/x^3,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^(3/2)*arctan(a*x)/x^3, x)
```

## Sympy [F]

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{x^3}\,dx = \int \frac{(c(a^2x^2+1))^{\frac{3}{2}}\operatorname{atan}(ax)}{x^3}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)/x**3,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)/x**3, x)
```

## Maxima [F]

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{x^3}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)}{x^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)/x^3,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*arctan(a*x)/x^3, x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)/x^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{x^3}\,dx = \int \frac{\mathrm{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^{3/2}}{x^3}\,dx$$

input
```
int((atan(a*x)*(c + a^2*c*x^2)^(3/2))/x^3,x)
```

output
```
int((atan(a*x)*(c + a^2*c*x^2)^(3/2))/x^3, x)
```

## Reduce [F]

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{x^3}\,dx = \sqrt{c}\,c\left(\int \frac{\sqrt{a^2x^2+1}\,atan(ax)}{x^3}dx + \left(\int \frac{\sqrt{a^2x^2+1}\,atan(ax)}{x}dx\right)a^2\right)$$

input
```
int((a^2*c*x^2+c)^(3/2)*atan(a*x)/x^3,x)
```

output
```
sqrt(c)*c*(int((sqrt(a**2*x**2 + 1)*atan(a*x))/x**3,x) + int((sqrt(a**2*x*
*2 + 1)*atan(a*x))/x,x)*a**2)
```

# 3.215 $\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{x^4}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 310

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{x^4}\,dx = -\frac{ac\sqrt{c+a^2cx^2}}{6x^2} - \frac{a^2c\sqrt{c+a^2cx^2}\arctan(ax)}{x} - \frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{3x^3} - \frac{2ia^3c^2\sqrt{1+a^2x^2}\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} - \frac{7}{6}a^3c^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c+a^2cx^2}}{\sqrt{c}}\right) + \frac{ia^3c^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} - \frac{ia^3c^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,\right.}{\sqrt{c+a^2cx^2}}$$

output
```
-1/6*a*c*(a^2*c*x^2+c)^(1/2)/x^2-a^2*c*(a^2*c*x^2+c)^(1/2)*arctan(a*x)/x-1
/3*(a^2*c*x^2+c)^(3/2)*arctan(a*x)/x^3-2*I*a^3*c^2*(a^2*x^2+1)^(1/2)*arcta
n(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/(a^2*c*x^2+c)^(1/2)-7/6*a^3
*c^(3/2)*arctanh((a^2*c*x^2+c)^(1/2)/c^(1/2))+I*a^3*c^2*(a^2*x^2+1)^(1/2)*
polylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/(a^2*c*x^2+c)^(1/2)-I*a^3*c^
2*(a^2*x^2+1)^(1/2)*polylog(2,I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/(a^2*c*x^
2+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.85

$$\int \frac{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)}{x^4}\,dx =$$

$$-\frac{c\sqrt{c+a^2cx^2}\left(ax\sqrt{1+a^2x^2}+2\sqrt{1+a^2x^2}\arctan(ax)+8a^2x^2\sqrt{1+a^2x^2}\arctan(ax)+a^3x^3\text{arctanh}\left(\sqrt{1\right.\right.}{}$$

input
```
Integrate[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/x^4,x]
```

output
```
-1/6*(c*Sqrt[c + a^2*c*x^2]*(a*x*Sqrt[1 + a^2*x^2] + 2*Sqrt[1 + a^2*x^2]*A
rcTan[a*x] + 8*a^2*x^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x] + a^3*x^3*ArcTanh[Sqr
t[1 + a^2*x^2]] - 6*a^3*x^3*ArcTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])] + 6*a
^3*x^3*ArcTan[a*x]*Log[1 + I*E^(I*ArcTan[a*x])] + 6*a^3*x^3*Log[Cos[ArcTan
[a*x]/2]] - 6*a^3*x^3*Log[Sin[ArcTan[a*x]/2]] - (6*I)*a^3*x^3*PolyLog[2, (
-I)*E^(I*ArcTan[a*x])] + (6*I)*a^3*x^3*PolyLog[2, I*E^(I*ArcTan[a*x])]))/(
x^3*Sqrt[1 + a^2*x^2])
```

## Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 296, normalized size of antiderivative = 0.95, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.591, Rules used = {5485, 5479, 243, 51, 73, 221, 5485, 5425, 5421, 5479, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{x^4}\,dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^2}dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^4}dx$$

$$\Big\downarrow \text{5479}$$

$$a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^2}dx+c\left(\frac{1}{3}a\int \frac{\sqrt{a^2cx^2+c}}{x^3}dx-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)$$

$\Big\downarrow$ 243

$$a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^2}dx+c\left(\frac{1}{6}a\int \frac{\sqrt{a^2cx^2+c}}{x^4}dx^2-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)$$

$\Big\downarrow$ 51

$$a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^2}dx+ c\left(\frac{1}{6}a\left(\frac{1}{2}a^2c\int \frac{1}{x^2\sqrt{a^2cx^2+c}}dx^2-\frac{\sqrt{a^2cx^2+c}}{x^2}\right)-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)$$

$\Big\downarrow$ 73

$$a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^2}dx+ c\left(\frac{1}{6}a\left(\int \frac{1}{\frac{x^4}{a^2c}-\frac{1}{a^2}}d\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{x^2}\right)-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)$$

$\Big\downarrow$ 221

$$a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^2}dx+ c\left(\frac{1}{6}a\left(a^2\left(-\sqrt{c}\right)\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)-\frac{\sqrt{a^2cx^2+c}}{x^2}\right)-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)$$

$\Big\downarrow$ 5485

$$a^2c\left(a^2c\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx\right)+ c\left(\frac{1}{6}a\left(a^2\left(-\sqrt{c}\right)\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)-\frac{\sqrt{a^2cx^2+c}}{x^2}\right)-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)$$

$\Big\downarrow$ 5425

$$a^2c\left(\frac{a^2c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx\right)+ c\left(\frac{1}{6}a\left(a^2\left(-\sqrt{c}\right)\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)-\frac{\sqrt{a^2cx^2+c}}{x^2}\right)-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)$$

$\Big\downarrow$ 5421

$$c\left(\frac{1}{6}a\left(a^2\left(-\sqrt{c}\right)\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)-\frac{\sqrt{a^2cx^2+c}}{x^2}\right)-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)+$$
$$a^2c\left(c\int\frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx+\frac{a^2c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax}}{\sqrt{1-i}}\right.}{a}\right.}{\sqrt{a^2cx^2+c}}\right.$$

$\downarrow$ 5479

$$c\left(\frac{1}{6}a\left(a^2\left(-\sqrt{c}\right)\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)-\frac{\sqrt{a^2cx^2+c}}{x^2}\right)-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)+$$
$$a^2c\left(c\left(a\int\frac{1}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right)+\frac{a^2c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}}{}\right.}{\sqrt{a^2cx^2+c}}\right.$$

$\downarrow$ 243

$$c\left(\frac{1}{6}a\left(a^2\left(-\sqrt{c}\right)\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)-\frac{\sqrt{a^2cx^2+c}}{x^2}\right)-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)+$$
$$a^2c\left(c\left(\frac{1}{2}a\int\frac{1}{x^2\sqrt{a^2cx^2+c}}dx^2-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right)+\frac{a^2c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{Pol}}{}\right.}{\sqrt{a^2cx^2+}}\right.$$

$\downarrow$ 73

$$c\left(\frac{1}{6}a\left(a^2\left(-\sqrt{c}\right)\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)-\frac{\sqrt{a^2cx^2+c}}{x^2}\right)-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)+$$
$$a^2c\left(c\left(\frac{\int\frac{1}{\frac{x^4}{a^2c}-\frac{1}{a^2}}d\sqrt{a^2cx^2+c}}{ac}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right)+\frac{a^2c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{Poly}}{}\right.}{\sqrt{a^2cx^2+c}}\right.$$

$\downarrow$ 221

$$c\left(\frac{1}{6}a\left(a^2(-\sqrt{c})\operatorname{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)-\frac{\sqrt{a^2cx^2+c}}{x^2}\right)-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)+$$

$$a^2c\left(c\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\operatorname{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)+\frac{a^2c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{Poly}}{}\right.}{\sqrt{a^2cx^2+c}}\right.$$

input
```
Int[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/x^4,x]
```

output
```
c*(-1/3*((c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/(c*x^3) + (a*(-(Sqrt[c + a^2*c
*x^2]/x^2) - a^2*Sqrt[c]*ArcTanh[Sqrt[c + a^2*c*x^2]/Sqrt[c]]))/6) + a^2*c
*(c*(-((Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(c*x)) - (a*ArcTanh[Sqrt[c + a^2*
c*x^2]/Sqrt[c]])/Sqrt[c]) + (a^2*c*Sqrt[1 + a^2*x^2]*(((-2*I)*ArcTan[a*x]*
ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/a + (I*PolyLog[2, ((-I)*Sqrt[1 +
I*a*x])/Sqrt[1 - I*a*x]])/a - (I*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I
*a*x]])/a))/Sqrt[c + a^2*c*x^2])
```

## Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

## Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2334 vs. $2(257) = 514$.

Time = 1.78 (sec) , antiderivative size = 2335, normalized size of antiderivative = 7.53

| method | result | size |
|---|---|---|
| default | Expression too large to display | 2335 |

input
```
int((a^2*c*x^2+c)^(3/2)*arctan(a*x)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/48*c*(a^3*x^3-I*a^2*x^2+a*x-I)*(c*(a*x-I)*(a*x+I))^(1/2)/x^3-7/96*c*ln(
1+(1+I*a*x)/(a^2*x^2+1)^(1/2))*(I*a^4*x^4+4*a^3*x^3-6*I*a^2*x^2-4*a*x+I)/(
a^2*x^2+1)^(1/2)*(c*(a*x-I)*(a*x+I))^(1/2)/x^3+1/8*c*arctan(a*x)*ln(1+I*(1
+I*a*x)/(a^2*x^2+1)^(1/2))*(I*a^2*x^2-2*a*x-I)*(a^2*x^2+1)^(1/2)*(c*(a*x-I
)*(a*x+I))^(1/2)/x^3-1/8*c*arctan(a*x)*(I*a^3*x^3+3*a^2*x^2-3*I*a*x-1)*(c*
(a*x-I)*(a*x+I))^(1/2)/x^3-1/16*c*dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(
a^4*x^4-4*I*a^3*x^3-6*a^2*x^2+4*I*a*x+1)/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(a*x
+I))^(1/2)/x^3-1/8*c*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+2*I*a
*x-1)*(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(a*x+I))^(1/2)/x^3+1/16/(a^2*x^2+1)^(1/
2)*(c*(a*x-I)*(a*x+I))^(1/2)*(a^4*x^4+4*I*a^3*x^3-6*a^2*x^2-4*I*a*x+1)*dil
og(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*c/x^3+1/48*(c*(a*x-I)*(a*x+I))^(1/2)*(
a^3*x^3+3*I*a^2*x^2-3*a*x-I)*c/x^3-1/16*c*arctan(a*x)*ln(1+I*(1+I*a*x)/(a^
2*x^2+1)^(1/2))*(I*a^4*x^4+4*a^3*x^3-6*I*a^2*x^2-4*a*x+I)/(a^2*x^2+1)^(1/2
)*(c*(a*x-I)*(a*x+I))^(1/2)/x^3-1/8*c*dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2
))*(a^2*x^2-2*I*a*x-1)*(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(a*x+I))^(1/2)/x^3+1/8
*c*dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+2*I*a*x-1)*(a^2*x^2+1)^
(1/2)*(c*(a*x-I)*(a*x+I))^(1/2)/x^3-7/48*c*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-
1)*(I*a^2*x^2-2*a*x-I)*(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(a*x+I))^(1/2)/x^3-7/9
6/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(a*x+I))^(1/2)*(I*a^4*x^4-4*a^3*x^3-6*I*a^2
*x^2+4*a*x+I)*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-1)*c/x^3-7/24*c*arctan(a*x...
```

## Fricas [F]

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{x^4}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)}{x^4}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)/x^4,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^(3/2)*arctan(a*x)/x^4, x)
```

## Sympy [F]

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{x^4}\,dx = \int \frac{(c(a^2x^2+1))^{\frac{3}{2}}\operatorname{atan}(ax)}{x^4}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)/x**4,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)/x**4, x)
```

## Maxima [F]

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{x^4}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)}{x^4}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)/x^4,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*arctan(a*x)/x^4, x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)/x^4,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{x^4}\,dx = \int \frac{\operatorname{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^{3/2}}{x^4}\,dx$$

input
```
int((atan(a*x)*(c + a^2*c*x^2)^(3/2))/x^4,x)
```

output
```
int((atan(a*x)*(c + a^2*c*x^2)^(3/2))/x^4, x)
```

## Reduce [F]

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{x^4}\,dx = \frac{\sqrt{c}\,c\Big(4atan\left(\sqrt{a^2x^2+1}+ax\right)a^3x^3-2\sqrt{a^2x^2+1}\,atan(ax)\,a^2x^2-2\sqrt{a^2}$$

input
```
int((a^2*c*x^2+c)^(3/2)*atan(a*x)/x^4,x)
```

output
```
(sqrt(c)*c*(4*atan(sqrt(a**2*x**2 + 1) + a*x)*a**3*x**3 - 2*sqrt(a**2*x**2
 + 1)*atan(a*x)*a**2*x**2 - 2*sqrt(a**2*x**2 + 1)*atan(a*x) - 2*atan(a*x)*
a**3*x**3 - sqrt(a**2*x**2 + 1)*a*x + 6*int((sqrt(a**2*x**2 + 1)*atan(a*x)
)/x**2,x)*a**2*x**3 + log(sqrt(a**2*x**2 + 1) + a*x - 1)*a**3*x**3 - log(s
qrt(a**2*x**2 + 1) + a*x + 1)*a**3*x**3))/(6*x**3)
```

# 3.216 $\int x^3(c+a^2cx^2)^{5/2}\arctan(ax)\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 289

$$\int x^3(c+a^2cx^2)^{5/2}\arctan(ax)\,dx = \frac{47c^2x\sqrt{c+a^2cx^2}}{2688a^3} - \frac{205c^2x^3\sqrt{c+a^2cx^2}}{12096a} - \frac{103ac^2x^5\sqrt{c+a^2cx^2}}{3024} - \frac{1}{72}a^3c^2x^7\sqrt{c+a^2cx^2} - \frac{2c^2\sqrt{c+a^2cx^2}\arctan(ax)}{63a^4} + \frac{c^2x^2\sqrt{c+a^2cx^2}\arctan(ax)}{63a^2} + \frac{5}{21}c^2x^4\sqrt{c+a^2cx^2}\arctan(ax) + \frac{19}{63}a^2c^2x^6\sqrt{c+a^2cx^2}\arctan(ax) + \frac{1}{9}a^4c^2x^8\sqrt{c+a^2cx^2}\arctan(ax) + \frac{115c^{5/2}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right)}{8064a^4}$$

output
```
47/2688*c^2*x*(a^2*c*x^2+c)^(1/2)/a^3-205/12096*c^2*x^3*(a^2*c*x^2+c)^(1/2
)/a-103/3024*a*c^2*x^5*(a^2*c*x^2+c)^(1/2)-1/72*a^3*c^2*x^7*(a^2*c*x^2+c)^
(1/2)-2/63*c^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)/a^4+1/63*c^2*x^2*(a^2*c*x^2
+c)^(1/2)*arctan(a*x)/a^2+5/21*c^2*x^4*(a^2*c*x^2+c)^(1/2)*arctan(a*x)+19/
63*a^2*c^2*x^6*(a^2*c*x^2+c)^(1/2)*arctan(a*x)+1/9*a^4*c^2*x^8*(a^2*c*x^2+
c)^(1/2)*arctan(a*x)+115/8064*c^(5/2)*arctanh(a*c^(1/2)*x/(a^2*c*x^2+c)^(1
/2))/a^4
```

## Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.45

$$\int x^3(c+a^2cx^2)^{5/2}\arctan(ax)\,dx = \frac{c^2\left(-ax\sqrt{c+a^2cx^2}(-423+410a^2x^2+824a^4x^4+336a^6x^6)+384(1+a^2x^2)^3\right.}{24192}$$

input
```
Integrate[x^3*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x],x]
```

output
```
(c^2*(-(a*x*Sqrt[c + a^2*c*x^2]*(-423 + 410*a^2*x^2 + 824*a^4*x^4 + 336*a^
6*x^6)) + 384*(1 + a^2*x^2)^3*(-2 + 7*a^2*x^2)*Sqrt[c + a^2*c*x^2]*ArcTan[
a*x] + 345*Sqrt[c]*Log[a*c*x + Sqrt[c]*Sqrt[c + a^2*c*x^2]]))/(24192*a^4)
```

## Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\arctan(ax)\left(a^2cx^2+c\right)^{5/2}dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\int x^5\left(a^2cx^2+c\right)^{3/2}\arctan(ax)dx+c\int x^3\left(a^2cx^2+c\right)^{3/2}\arctan(ax)dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\left(a^2c\int x^7\sqrt{a^2cx^2+c}\arctan(ax)dx+c\int x^5\sqrt{a^2cx^2+c}\arctan(ax)dx\right)+c\left(a^2c\int x^5\sqrt{a^2cx^2+c}\arctan(ax)dx+c\int x^3\sqrt{a^2cx^2+c}\arctan(ax)dx\right)$$

$$\Big\downarrow \text{5481}$$

$$a^2c\left(a^2c\left(\frac{1}{9}c\int\frac{x^7\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{9}ac\int\frac{x^8}{\sqrt{a^2cx^2+c}}dx+\frac{1}{9}x^8\arctan(ax)\sqrt{a^2cx^2+c}\right)+c\left(\frac{1}{7}c\int\frac{x^5\arctan(a}{\sqrt{a^2cx^2+}}\right.\right.$$
$$c\left(a^2c\left(\frac{1}{7}c\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{7}ac\int\frac{x^6}{\sqrt{a^2cx^2+c}}dx+\frac{1}{7}x^6\arctan(ax)\sqrt{a^2cx^2+c}\right)+c\left(\frac{1}{5}c\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}\right.\right.$$

$$\Big\downarrow \text{262}$$

$$c\left(c\left(\frac{1}{5}c\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{5}ac\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\int \frac{x^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)+\frac{1}{5}x^4\arctan(ax)\sqrt{a^2cx^2+c}\right)+a^2c\left(\right.\right.$$
$$a^2c\left(a^2c\left(\frac{1}{9}c\int \frac{x^7\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{9}ac\left(\frac{x^7\sqrt{a^2cx^2+c}}{8a^2c}-\frac{7\int \frac{x^6}{\sqrt{a^2cx^2+c}}dx}{8a^2}\right)+\frac{1}{9}x^8\arctan(ax)\sqrt{a^2cx^2+c}\right)+\right.$$

$$\Big\downarrow \text{262}$$

$$c\left(c\left(\frac{1}{5}c\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{5}ac\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int \frac{1}{\sqrt{a^2cx^2+c}}dx}{2a^2}\right)}{4a^2}\right)+\frac{1}{5}x^4\arctan(ax)\sqrt{a^2cx^2+c}\right.\right.$$
$$a^2c\left(c\left(\frac{1}{7}c\int \frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{7}ac\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\int \frac{x^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{6a^2}\right)+\frac{1}{7}x^6\arctan(ax)\sqrt{\right.\right.$$

$$\Big\downarrow \text{224}$$

$$c\left(c\left(\frac{1}{5}c\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{5}ac\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int \frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}}{2a^2}\right)}{4a^2}\right)+\frac{1}{5}x^4\arctan(a\right.\right.$$
$$a^2c\left(c\left(\frac{1}{7}c\int \frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{7}ac\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\int \frac{x^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{6a^2}\right)+\frac{1}{7}x^6\arctan(ax)\sqrt{\right.\right.$$

$$\Big\downarrow \text{219}$$

$$a^2c\left(c\left(\frac{1}{7}c\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{7}ac\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\int\frac{x^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{6a^2}\right)+\frac{1}{7}x^6\arctan(ax)\sqrt{\ }\right.\right.$$

$$c\left(a^2c\left(\frac{1}{7}c\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{7}ac\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\int\frac{x^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{6a^2}\right)+\frac{1}{7}x^6\arctan(ax)\sqrt{\ }\right.\right.$$

$$\Big\downarrow \text{262}$$

$$a^2c\left(c\left(\frac{1}{7}c\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{7}ac\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int\frac{1}{\sqrt{a^2cx^2+c}}dx}{2a^2}\right)}{4a^2}\right)}{6a^2}\right)+\frac{1}{7}x^6\right.\right.$$

$$c\left(a^2c\left(\frac{1}{7}c\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{7}ac\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int\frac{1}{\sqrt{a^2cx^2+c}}dx}{2a^2}\right)}{4a^2}\right)}{6a^2}\right)+\frac{1}{7}x^6\right.\right.$$

$$\Big\downarrow \text{224}$$

$$a^2c\left(c\left(\frac{1}{7}c\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{7}ac\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int\frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}}{2a^2}\right)}{4a^2}\right)}{6a^2}\right.\right.\right.$$

$$c\left(a^2c\left(\frac{1}{7}c\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{7}ac\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int\frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}}{2a^2}\right)}{4a^2}\right)}{6a^2}\right.\right.\right.$$

$$\downarrow 219$$

$$c\left(c\left(\frac{1}{5}c\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{5}x^4\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{5}ac\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{}{\sqrt{}}\right.}{2a^3\sqrt{}}\right.}{4a^2}\right.\right.\right.$$

$$a^2c\left(a^2c\left(\frac{1}{9}c\int \frac{x^7\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{9}ac\left(\frac{x^7\sqrt{a^2cx^2+c}}{8a^2c}-\frac{7\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\int \frac{x^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{6a^2}\right)}{8a^2}\right)+\right.\right.$$

$$\Big\downarrow \text{262}$$

$$c\left(c\left(\frac{1}{5}c\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{5}x^4\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{5}ac\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\ldots\right)}{2a^3\ldots}\right.}{4a^2}\right.\right.\right.$$

$$a^2c\left(a^2c\left(\frac{1}{9}c\int\frac{x^7\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{9}ac\left(\frac{x^7\sqrt{a^2cx^2+c}}{8a^2c}-\frac{7\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int\frac{1}{\sqrt{a^2cx^2+\ldots}}}{2a^2}\right.}{4a^2}\right.}{6a^2}\right.}{8a^2}\right.\right.\right.$$

$$\Big\downarrow 224$$

$$c\left(c\left(\frac{1}{9}\sqrt{a^2cx^2+c}\,\arctan(ax)x^8-\frac{1}{9}ac\left(\frac{x^7\sqrt{a^2cx^2+c}}{8a^2c}-\frac{7\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int\frac{1}{1-\frac{a^2cx}{a^2cx^2}}}{}\right)}{4a^2}\right)}{6a^2}\right)}{8a^2}\right.\right.\right.$$

$$c\left(c\left(\frac{1}{7}\sqrt{a^2cx^2+c}\,\arctan(ax)x^6-\frac{1}{7}ac\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}\right)}{6a^2}\right.\right.\right.$$

↓ 219

$$c\left(c\left(\frac{1}{9}\sqrt{a^2cx^2+c}\arctan(ax)x^8-\frac{1}{9}ac\left(\frac{x^7\sqrt{a^2cx^2+c}}{8a^2c}-\frac{7\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}(\ldots}{2a\ldots}\right.}{4a^2}\right.}{6a^2}\right.}{8a^2}\right.\right.\right.$$

$$c\left(c\left(\frac{1}{7}\sqrt{a^2cx^2+c}\arctan(ax)x^6-\frac{1}{7}ac\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}\right)}{6a^2}\right.\right.\right.$$

$$\Big\downarrow \ 5487$$

$$c\left(c\left(\frac{1}{9}\sqrt{a^2cx^2+c}\arctan(ax)x^8-\frac{1}{9}ac\left(\frac{x^7\sqrt{a^2cx^2+c}}{8a^2c}-\frac{7\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}(}{2a}\right.}{4a^2}\right.}{6a^2}\right.}{8a^2}\right.\right.\right.$$

$$c\left(c\left(\frac{1}{7}\sqrt{a^2cx^2+c}\arctan(ax)x^6-\frac{1}{7}ac\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}\right)}{6a^2}\right.\right.\right.$$

$$\Big\downarrow \; 262$$

$$c\left(c\left(\frac{1}{9}\sqrt{a^2cx^2+c}\arctan(ax)x^8-\frac{1}{9}ac\left(\frac{x^7\sqrt{a^2cx^2+c}}{8a^2c}-\frac{7\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}(}{2a}\right.}{4a^2}\right.}{6a^2}\right.}{8a^2}\right.\right.\right.$$

$$c\left(c\left(\frac{1}{7}\sqrt{a^2cx^2+c}\arctan(ax)x^6-\frac{1}{7}ac\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}\right)}{6a^2}\right.\right.\right.$$

$$\Big\downarrow \; 224$$

$$c\left(c\left(\frac{1}{9}\sqrt{a^2cx^2+c}\,\arctan(ax)x^8-\frac{1}{9}ac\left(\frac{x^7\sqrt{a^2cx^2+c}}{8a^2c}-\frac{7\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}(}{2a}\right.}{4a^2}\right.}{6a^2}\right.}{8a^2}\right.\right.\right.$$

$$c\left(c\left(\frac{1}{7}\sqrt{a^2cx^2+c}\,\arctan(ax)x^6-\frac{1}{7}ac\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}\right)}{6a^2}\right.\right.\right.$$

$\downarrow$ 219

$$c\left(c\left(\frac{1}{9}\sqrt{a^2cx^2+c}\arctan(ax)x^8-\frac{1}{9}ac\left(\frac{x^7\sqrt{a^2cx^2+c}}{8a^2c}-\frac{7\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}(}{2a}\right.}{4a^2}\right.}{6a^2}\right.}{8a^2}\right.\right.\right.$$

$$c\left(c\left(\frac{1}{7}\sqrt{a^2cx^2+c}\arctan(ax)x^6-\frac{1}{7}ac\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}\right)}{6a^2}\right.\right.\right.$$

$\downarrow$ 262

$$c\left(c\left(\frac{1}{9}\sqrt{a^2cx^2+c}\arctan(ax)x^8-\frac{1}{9}ac\left(\frac{x^7\sqrt{a^2cx^2+c}}{8a^2c}-\frac{7\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}(}{2a}\right.}{4a^2}\right.}{6a^2}\right.}{8a^2}\right.\right.\right.$$

$$c\left(c\left(\frac{1}{7}\sqrt{a^2cx^2+c}\arctan(ax)x^6-\frac{1}{7}ac\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}\right)}{6a^2}\right.\right.\right.$$

$\downarrow$ 224

$$c\left(c\left(\frac{1}{9}\sqrt{a^2cx^2+c}\arctan(ax)x^8-\frac{1}{9}ac\left(\frac{x^7\sqrt{a^2cx^2+c}}{8a^2c}-\frac{7\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}(}{2a}\right.}{4a^2}\right.}{6a^2}\right.}{8a^2}\right.\right.\right.$$

$$c\left(c\left(\frac{1}{7}\sqrt{a^2cx^2+c}\arctan(ax)x^6-\frac{1}{7}ac\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}\right)}{6a^2}\right.\right.\right.$$

$\downarrow$ 219

$$c\left(c\left(\frac{1}{9}\sqrt{a^2cx^2+c}\arctan(ax)x^8-\frac{1}{9}ac\left(\frac{x^7\sqrt{a^2cx^2+c}}{8a^2c}-\frac{7\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}(}{2a}\right.}{4a^2}\right.}{6a^2}\right.}{8a^2}\right.\right.\right.$$

$$c\left(c\left(\frac{1}{7}\sqrt{a^2cx^2+c}\arctan(ax)x^6-\frac{1}{7}ac\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}\right)}{6a^2}\right.\right.\right.$$

$$\Big\downarrow \text{262}$$

$$c\left(c\left(\frac{1}{9}\sqrt{a^2cx^2+c}\arctan(ax)x^8-\frac{1}{9}ac\left(\frac{x^7\sqrt{a^2cx^2+c}}{8a^2c}-\frac{7\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}(}{2a}\right.}{4a^2}\right.}{6a^2}\right.}{8a^2}\right.\right.\right.$$

$$c\left(c\left(\frac{1}{7}\sqrt{a^2cx^2+c}\arctan(ax)x^6-\frac{1}{7}ac\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}\right)}{6a^2}\right.\right.\right.$$

$\Big\downarrow$ 224

$$c\left(c\left(\frac{1}{9}\sqrt{a^2cx^2+c}\,\arctan(ax)x^8-\frac{1}{9}ac\left(\frac{x^7\sqrt{a^2cx^2+c}}{8a^2c}-\frac{7\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}(}{2a}\right.}{4a^2}\right.}{6a^2}\right.}{8a^2}\right.\right.\right.$$

$$c\left(c\left(\frac{1}{7}\sqrt{a^2cx^2+c}\,\arctan(ax)x^6-\frac{1}{7}ac\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}\right)}{6a^2}\right.\right.\right.$$

$\downarrow$ 219

$$c\left(c\left(\frac{1}{9}\sqrt{a^2cx^2+c}\arctan(ax)x^8-\frac{1}{9}ac\left(\frac{x^7\sqrt{a^2cx^2+c}}{8a^2c}-\frac{7\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}(}{2a}\right.}{4a^2}\right.}{6a^2}\right.}{8a^2}\right.\right.\right.$$

$$c\left(c\left(\frac{1}{7}\sqrt{a^2cx^2+c}\arctan(ax)x^6-\frac{1}{7}ac\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}\right)}{6a^2}\right.\right.\right.$$

$\downarrow$ 5465

$$c\left(c\left(\frac{1}{9}\sqrt{a^2cx^2+c}\,\arctan(ax)x^8-\frac{1}{9}ac\left(\frac{x^7\sqrt{a^2cx^2+c}}{8a^2c}-\frac{7\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}(}{2a}\right.}{4a^2}\right.}{6a^2}\right.}{8a^2}\right.\right.\right.$$

$$c\left(c\left(\frac{1}{7}\sqrt{a^2cx^2+c}\,\arctan(ax)x^6-\frac{1}{7}ac\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}\right)}{6a^2}\right.\right.\right.$$

$$\Big\downarrow \; 224$$

$$c\left(c\left(\frac{1}{9}\sqrt{a^2cx^2+c}\arctan(ax)x^8-\frac{1}{9}ac\left(\frac{x^7\sqrt{a^2cx^2+c}}{8a^2c}-\frac{7\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}(}{2a}\right.}{4a^2}\right.}{6a^2}\right.}{8a^2}\right.\right.\right.$$

$$c\left(c\left(\frac{1}{7}\sqrt{a^2cx^2+c}\arctan(ax)x^6-\frac{1}{7}ac\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}\right)}{6a^2}\right.\right.\right.$$

$\downarrow$ 219

$$c\left(c\left(\frac{1}{9}\sqrt{a^2cx^2+c}\arctan(ax)x^8-\frac{1}{9}ac\left(\frac{x^7\sqrt{a^2cx^2+c}}{8a^2c}-\frac{7\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}(}{2a}\right.}{4a^2}\right.}{6a^2}\right.}{8a^2}\right.\right.\right.$$

$$c\left(c\left(\frac{1}{7}\sqrt{a^2cx^2+c}\arctan(ax)x^6-\frac{1}{7}ac\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}\right)}{6a^2}\right.\right.\right.$$

$\downarrow$ 5487

$$c\left(c\left(\frac{1}{9}\sqrt{a^2cx^2+c}\arctan(ax)x^8-\frac{1}{9}ac\left(\frac{x^7\sqrt{a^2cx^2+c}}{8a^2c}-\frac{7\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}(}{2a}\right.}{4a^2}\right.}{6a^2}\right.}{8a^2}\right.\right.\right.$$

$$c\left(c\left(\frac{1}{7}\sqrt{a^2cx^2+c}\arctan(ax)x^6-\frac{1}{7}ac\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}\right)}{6a^2}\right.\right.\right.$$

$$\Big\downarrow \; 262$$

$$c\left(c\left(\frac{1}{9}\sqrt{a^2cx^2+c}\arctan(ax)x^8-\frac{1}{9}ac\left(\frac{x^7\sqrt{a^2cx^2+c}}{8a^2c}-\frac{7\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}(}{2a}\right.}{4a^2}\right.}{6a^2}\right.}{8a^2}\right.\right.\right.$$

$$c\left(c\left(\frac{1}{7}\sqrt{a^2cx^2+c}\arctan(ax)x^6-\frac{1}{7}ac\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}\right)}{6a^2}\right.\right.\right.$$

$$\Big\downarrow \text{224}$$

$$c\left(c\left(\frac{1}{9}\sqrt{a^2cx^2+c}\arctan(ax)x^8-\frac{1}{9}ac\left(\frac{x^7\sqrt{a^2cx^2+c}}{8a^2c}-\frac{7\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}(}{2a}\right.}{4a^2}\right.}{6a^2}\right.}{8a^2}\right.\right.\right.$$

$$c\left(c\left(\frac{1}{7}\sqrt{a^2cx^2+c}\arctan(ax)x^6-\frac{1}{7}ac\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}\right)}{6a^2}\right.\right.\right.$$

$\downarrow$ 219

$$c\left(c\left(\frac{1}{9}\sqrt{a^2cx^2+c}\arctan(ax)x^8-\frac{1}{9}ac\left(\frac{x^7\sqrt{a^2cx^2+c}}{8a^2c}-\frac{7\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}(}{2a}\right.}{4a^2}\right.}{6a^2}\right.}{8a^2}\right.\right.\right.$$

$$c\left(c\left(\frac{1}{7}\sqrt{a^2cx^2+c}\arctan(ax)x^6-\frac{1}{7}ac\left(\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}\right)}{6a^2}\right.\right.\right.$$

input
```
Int[x^3*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x],x]
```

output
```
$Aborted
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 6.51 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.78

| method | result |
|---|---|
| default | $\frac{c^2\sqrt{c(ax-i)(ax+i)}\,(2688\arctan(ax)a^8x^8-336a^7x^7+7296a^6\arctan(ax)x^6-824a^5x^5+5760x^4\arctan(ax)a^4-410a^3x^3+384x^2a^2 a}{24192a^4}$ |

input
```
int(x^3*(a^2*c*x^2+c)^(5/2)*arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/24192*c^2/a^4*(c*(a*x-I)*(a*x+I))^(1/2)*(2688*arctan(a*x)*a^8*x^8-336*a^
7*x^7+7296*a^6*arctan(a*x)*x^6-824*a^5*x^5+5760*x^4*arctan(a*x)*a^4-410*a^
3*x^3+384*x^2*a^2*arctan(a*x)+423*a*x-768*arctan(a*x))-115/8064*c^2/a^4*(c
*(a*x-I)*(a*x+I))^(1/2)*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-I)/(a^2*x^2+1)^(1/2
)+115/8064*c^2/a^4*(c*(a*x-I)*(a*x+I))^(1/2)*ln((1+I*a*x)/(a^2*x^2+1)^(1/2
)+I)/(a^2*x^2+1)^(1/2)
```

## Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.53

$$\int x^3(c+a^2cx^2)^{5/2}\arctan(ax)\,dx = \frac{345\,c^{\frac{5}{2}}\log\left(-2\,a^2cx^2-2\,\sqrt{a^2cx^2+c}a\sqrt{c}x-c\right)-2\left(336\,a^7c^2x^7+824\,a^5c^2x^5+\right.}{}$$

input
```
integrate(x^3*(a^2*c*x^2+c)^(5/2)*arctan(a*x),x, algorithm="fricas")
```

output
```
1/48384*(345*c^(5/2)*log(-2*a^2*c*x^2 - 2*sqrt(a^2*c*x^2 + c)*a*sqrt(c)*x
- c) - 2*(336*a^7*c^2*x^7 + 824*a^5*c^2*x^5 + 410*a^3*c^2*x^3 - 423*a*c^2*
x - 384*(7*a^8*c^2*x^8 + 19*a^6*c^2*x^6 + 15*a^4*c^2*x^4 + a^2*c^2*x^2 - 2
*c^2)*arctan(a*x))*sqrt(a^2*c*x^2 + c))/a^4
```

## Sympy [F]

$$\int x^3 \left(c+a^2 c x^2\right)^{5/2} \arctan(ax)\,dx = \int x^3 \left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}} \operatorname{atan}(ax)\,dx$$

input
```
integrate(x**3*(a**2*c*x**2+c)**(5/2)*atan(a*x),x)
```

output
```
Integral(x**3*(c*(a**2*x**2 + 1))**(5/2)*atan(a*x), x)
```

## Maxima [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.17

$$\int x^3 \left(c+a^2 c x^2\right)^{5/2} \arctan(ax)\,dx = -\frac{1}{24192}\left(\left(\left(7\left(\frac{48\left(a^2x^2+1\right)^{\frac{3}{2}}x^5}{a^2} - \frac{40\left(a^2x^2+1\right)^{\frac{3}{2}}x^3}{a^4} + \frac{30\left(a^2x^2+1\right)^{\frac{3}{2}}x}{a^6} - \frac{15\sqrt{a^2x^2+1}\,x}{a^6} - \frac{15\operatorname{arsinh}(a}{a^7}\right.\right.\right.\right.$$

input
```
integrate(x^3*(a^2*c*x^2+c)^(5/2)*arctan(a*x),x, algorithm="maxima")
```

output
```
-1/24192*((7*(48*(a^2*x^2 + 1)^(3/2)*x^5/a^2 - 40*(a^2*x^2 + 1)^(3/2)*x^3/
a^4 + 30*(a^2*x^2 + 1)^(3/2)*x/a^6 - 15*sqrt(a^2*x^2 + 1)*x/a^6 - 15*arcsi
nh(a*x)/a^7)*a^2*c^2 + 96*(8*(a^2*x^2 + 1)^(3/2)*x^3/a^2 - 6*(a^2*x^2 + 1)
^(3/2)*x/a^4 + 3*sqrt(a^2*x^2 + 1)*x/a^4 + 3*arcsinh(a*x)/a^5)*c^2 + 144*c
^2*(2*(a^2*x^2 + 1)^(3/2)*x/a^2 - sqrt(a^2*x^2 + 1)*x/a^2 - arcsinh(a*x)/a
^3)/a^2 - 384*(sqrt(a^2*x^2 + 1)*x + arcsinh(a*x)/a)*c^2/a^4)*a - 384*(7*(
a^2*x^2 + 1)^(3/2)*a^2*c^2*x^6 + 12*(a^2*x^2 + 1)^(3/2)*c^2*x^4 + 3*(a^2*x
^2 + 1)^(3/2)*c^2*x^2/a^2 - 2*(a^2*x^2 + 1)^(3/2)*c^2/a^4)*arctan(a*x))*sq
rt(c)
```

## Giac [F(-2)]

Exception generated.

$$\int x^3 \left(c+a^2 c x^2\right)^{5/2} \arctan(ax) \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a^2*c*x^2+c)^(5/2)*arctan(a*x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int x^3 \left(c+a^2 c x^2\right)^{5/2} \arctan(ax) \, dx = \int x^3 \operatorname{atan}(a\,x) \, \left(c\,a^2\,x^2+c\right)^{5/2} \, dx$$

input
```
int(x^3*atan(a*x)*(c + a^2*c*x^2)^(5/2),x)
```

output
```
int(x^3*atan(a*x)*(c + a^2*c*x^2)^(5/2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.69

$$\int x^3 (c + a^2 c x^2)^{5/2} \arctan(ax) \, dx = \frac{\sqrt{c}\, c^2 \left(2688\sqrt{a^2x^2+1}\, atan(ax)\, a^8 x^8 + 7296\sqrt{a^2x^2+1}\, atan(ax)\, a^6 x^6 + 5760\sqrt{}\right.}{}$$

input
```
int(x^3*(a^2*c*x^2+c)^(5/2)*atan(a*x),x)
```

output
```
(sqrt(c)*c**2*(2688*sqrt(a**2*x**2 + 1)*atan(a*x)*a**8*x**8 + 7296*sqrt(a*
*2*x**2 + 1)*atan(a*x)*a**6*x**6 + 5760*sqrt(a**2*x**2 + 1)*atan(a*x)*a**4
*x**4 + 384*sqrt(a**2*x**2 + 1)*atan(a*x)*a**2*x**2 - 768*sqrt(a**2*x**2 +
 1)*atan(a*x) - 336*sqrt(a**2*x**2 + 1)*a**7*x**7 - 824*sqrt(a**2*x**2 + 1
)*a**5*x**5 - 410*sqrt(a**2*x**2 + 1)*a**3*x**3 + 423*sqrt(a**2*x**2 + 1)*
a*x + 345*log(sqrt(a**2*x**2 + 1) + a*x)))/(24192*a**4)
```

# 3.217 $\int x^2(c+a^2cx^2)^{5/2}\arctan(ax)\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 418

$$\int x^2(c+a^2cx^2)^{5/2}\arctan(ax)\,dx = \frac{5c^2\sqrt{c+a^2cx^2}}{128a^3}+\frac{5c(c+a^2cx^2)^{3/2}}{576a^3}+\frac{(c+a^2cx^2)^{5/2}}{240a^3}-\frac{(c+a^2cx^2)^{7/2}}{56a^3c}+\frac{5c^2x\sqrt{c+a^2cx^2}\arctan(ax)}{128a^2}+\frac{59}{192}c^2x^3\sqrt{c+a^2cx^2}\arctan(ax)+\frac{17}{48}a^2c^2x^5\sqrt{c+a^2cx^2}\arctan(ax)+\frac{1}{8}a^4c^2x^7\sqrt{c+a^2cx^2}\arctan(ax)+\frac{5ic^3}{}$$

output
```
5/128*c^2*(a^2*c*x^2+c)^(1/2)/a^3+5/576*c*(a^2*c*x^2+c)^(3/2)/a^3+1/240*(a
^2*c*x^2+c)^(5/2)/a^3-1/56*(a^2*c*x^2+c)^(7/2)/a^3/c+5/128*c^2*x*(a^2*c*x^
2+c)^(1/2)*arctan(a*x)/a^2+59/192*c^2*x^3*(a^2*c*x^2+c)^(1/2)*arctan(a*x)+
17/48*a^2*c^2*x^5*(a^2*c*x^2+c)^(1/2)*arctan(a*x)+1/8*a^4*c^2*x^7*(a^2*c*x
^2+c)^(1/2)*arctan(a*x)+5/64*I*c^3*(a^2*x^2+1)^(1/2)*arctan(a*x)*arctan((1
+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/a^3/(a^2*c*x^2+c)^(1/2)-5/128*I*c^3*(a^2*x^
2+1)^(1/2)*polylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/a^3/(a^2*c*x^2+c)
^(1/2)+5/128*I*c^3*(a^2*x^2+1)^(1/2)*polylog(2,I*(1+I*a*x)^(1/2)/(1-I*a*x)
^(1/2))/a^3/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 907 vs. 2(418) = 836.

Time = 10.92 (sec) , antiderivative size = 907, normalized size of antiderivative = 2.17

$$\int x^2 (c + a^2 c x^2)^{5/2} \arctan(ax)\,dx = \text{Too large to display}$$

input
```
Integrate[x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x],x]
```

output
```
(c^2*Sqrt[c + a^2*c*x^2]*((-19067*(1 + a^2*x^2)^(7/2))/32 - (3829*(1 + a^2
*x^2)^4*Cos[3*ArcTan[a*x]])/32 - (3150*I)*PolyLog[2, (-I)*E^(I*ArcTan[a*x]
)] + (3150*I)*PolyLog[2, I*E^(I*ArcTan[a*x])] - 420*(1 + a^2*x^2)^2*(-2/Sq
rt[1 + a^2*x^2] - 6*Cos[3*ArcTan[a*x]] + 3*ArcTan[a*x]*((-14*a*x)/Sqrt[1 +
 a^2*x^2] + 3*Log[1 - I*E^(I*ArcTan[a*x])] + 4*Cos[2*ArcTan[a*x]]*(Log[1 -
 I*E^(I*ArcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])]) + Cos[4*ArcTan[a*x]]
*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])]) - 3*Log[1 +
 I*E^(I*ArcTan[a*x])] + 2*Sin[3*ArcTan[a*x]])) + 7*(1 + a^2*x^2)^3*(12/Sqr
t[1 + a^2*x^2] + 110*Cos[3*ArcTan[a*x]] - 90*Cos[5*ArcTan[a*x]] + 15*ArcTa
n[a*x]*((156*a*x)/Sqrt[1 + a^2*x^2] + 30*Log[1 - I*E^(I*ArcTan[a*x])] + 3*
Cos[6*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] + 45*Cos[2*ArcTan[a*x]]*(L
og[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])]) + 18*Cos[4*Arc
Tan[a*x]]*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])]) -
30*Log[1 + I*E^(I*ArcTan[a*x])] - 3*Cos[6*ArcTan[a*x]]*Log[1 + I*E^(I*ArcT
an[a*x])] - 94*Sin[3*ArcTan[a*x]] + 6*Sin[5*ArcTan[a*x]])) - (35*(1 + a^2*
x^2)^4*(314*Cos[5*ArcTan[a*x]] - 90*Cos[7*ArcTan[a*x]] + 3*ArcTan[a*x]*((-
3530*a*x)/Sqrt[1 + a^2*x^2] + 525*Log[1 - I*E^(I*ArcTan[a*x])] + 120*Cos[6
*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] + 15*Cos[8*ArcTan[a*x]]*Log[1 -
 I*E^(I*ArcTan[a*x])] + 840*Cos[2*ArcTan[a*x]]*(Log[1 - I*E^(I*ArcTan[a*x]
)] - Log[1 + I*E^(I*ArcTan[a*x])]) + 420*Cos[4*ArcTan[a*x]]*(Log[1 - I*...
```

## Rubi [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1650 vs. 2(418) = 836.

Time = 7.31 (sec) , antiderivative size = 1650, normalized size of antiderivative = 3.95, number of steps used = 25, number of rules used = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 1.091$, Rules used = {5485, 5485, 5481, 243, 53, 2009, 5487, 241, 243, 53, 2009, 5425, 5421, 5487, 241, 243, 53, 2009, 5425, 5421, 5487, 241, 5425, 5421}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arctan(ax) \left(a^2cx^2+c\right)^{5/2} dx$$

$$\Big\downarrow \text{ 5485}$$

$$c\int x^2\left(a^2cx^2+c\right)^{3/2}\arctan(ax)dx+a^2c\int x^4\left(a^2cx^2+c\right)^{3/2}\arctan(ax)dx$$

$$\Big\downarrow \text{ 5485}$$

$$c\left(c\int x^2\sqrt{a^2cx^2+c}\arctan(ax)dx+a^2c\int x^4\sqrt{a^2cx^2+c}\arctan(ax)dx\right)+ a^2c\left(a^2c\int x^6\sqrt{a^2cx^2+c}\arctan(ax)dx+c\int x^4\sqrt{a^2cx^2+c}\arctan(ax)dx\right)$$

$$\Big\downarrow \text{ 5481}$$

$$c\left(c\left(\frac{1}{4}c\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{4}ac\int\frac{x^3}{\sqrt{a^2cx^2+c}}dx+\frac{1}{4}x^3\arctan(ax)\sqrt{a^2cx^2+c}\right)+a^2c\left(\frac{1}{6}c\int\frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}\right.\right.$$
$$a^2c\left(a^2c\left(\frac{1}{8}c\int\frac{x^6\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{8}ac\int\frac{x^7}{\sqrt{a^2cx^2+c}}dx+\frac{1}{8}x^7\arctan(ax)\sqrt{a^2cx^2+c}\right)+c\left(\frac{1}{6}c\int\frac{x^4\arctan(a}{\sqrt{a^2cx^2+}}\right.\right.$$

$$\Big\downarrow \text{ 243}$$

$$c\left(c\left(\frac{1}{4}c\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{8}ac\int\frac{x^2}{\sqrt{a^2cx^2+c}}dx^2+\frac{1}{4}x^3\arctan(ax)\sqrt{a^2cx^2+c}\right)+a^2c\left(\frac{1}{6}c\int\frac{x^4\arctan(ax}{\sqrt{a^2cx^2+c}}\right.\right.$$
$$a^2c\left(a^2c\left(\frac{1}{8}c\int\frac{x^6\arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{16}ac\int\frac{x^6}{\sqrt{a^2cx^2+c}}dx^2+\frac{1}{8}x^7\arctan(ax)\sqrt{a^2cx^2+c}\right)+c\left(\frac{1}{6}c\int\frac{x^4\arctan}{\sqrt{a^2cx^2}}\right.\right.$$

$$\Big\downarrow \text{ 53}$$

$$c\left(c\left(\frac{1}{4}c\int \frac{x^2 \arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{8}ac\int \left(\frac{\sqrt{a^2cx^2+c}}{a^2c}-\frac{1}{a^2\sqrt{a^2cx^2+c}}\right)dx^2+\frac{1}{4}x^3\arctan(ax)\sqrt{a^2cx^2+c}\right)+a^2$$
$$a^2c\left(a^2c\left(\frac{1}{8}c\int \frac{x^6 \arctan(ax)}{\sqrt{a^2cx^2+c}}dx-\frac{1}{16}ac\int \left(\frac{(a^2cx^2+c)^{5/2}}{a^6c^3}-\frac{3(a^2cx^2+c)^{3/2}}{a^6c^2}+\frac{3\sqrt{a^2cx^2+c}}{a^6c}-\frac{1}{a^6\sqrt{a^2cx^2+}}\right.\right.\right.$$

$\Big\downarrow$ 2009

$$a^2c\left(a^2c\left(\frac{1}{8}c\int \frac{x^6 \arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{8}x^7\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{16}ac\left(\frac{2(a^2cx^2+c)^{7/2}}{7a^8c^4}-\frac{6(a^2cx^2+c)^{5/2}}{5a^8c^3}+\right.\right.\right.$$
$$c\left(a^2c\left(\frac{1}{6}c\int \frac{x^4 \arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{6}x^5\arctan(ax)\sqrt{a^2cx^2+c}-\frac{1}{12}ac\left(\frac{2(a^2cx^2+c)^{5/2}}{5a^6c^3}-\frac{4(a^2cx^2+c)^{3/2}}{3a^6c^2}+\frac{2\sqrt{}}{}\right.\right.\right.$$

$\Big\downarrow$ 5487

$$c\left(c\left(\frac{1}{8}\sqrt{a^2cx^2+c}\arctan(ax)x^7-\frac{1}{16}ac\left(\frac{2(a^2cx^2+c)^{7/2}}{7a^8c^4}-\frac{6(a^2cx^2+c)^{5/2}}{5a^8c^3}+\frac{2(a^2cx^2+c)^{3/2}}{a^8c^2}-\frac{2\sqrt{a^2cx^2+}}{a^8c}\right.\right.\right.$$
$$c\left(c\left(\frac{1}{6}\sqrt{a^2cx^2+c}\arctan(ax)x^5-\frac{1}{12}ac\left(\frac{2(a^2cx^2+c)^{5/2}}{5a^6c^3}-\frac{4(a^2cx^2+c)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}\right)+\frac{1}{6}c\left(\frac{\sqrt{a^2cx^2}}{}\right.\right.\right.$$

$\Big\downarrow$ 241

$$c\left(c\left(\frac{1}{8}\sqrt{a^2cx^2+c}\arctan(ax)x^7-\frac{1}{16}ac\left(\frac{2(a^2cx^2+c)^{7/2}}{7a^8c^4}-\frac{6(a^2cx^2+c)^{5/2}}{5a^8c^3}+\frac{2(a^2cx^2+c)^{3/2}}{a^8c^2}-\frac{2\sqrt{a^2cx^2+}}{a^8c}\right.\right.\right.$$
$$c\left(c\left(\frac{1}{6}\sqrt{a^2cx^2+c}\arctan(ax)x^5-\frac{1}{12}ac\left(\frac{2(a^2cx^2+c)^{5/2}}{5a^6c^3}-\frac{4(a^2cx^2+c)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}\right)+\frac{1}{6}c\left(\frac{\sqrt{a^2cx^2}}{}\right.\right.\right.$$

$\Big\downarrow$ 243

$$c\left(c\left(\frac{1}{8}\sqrt{a^2cx^2+c}\arctan(ax)x^7-\frac{1}{16}ac\left(\frac{2(a^2cx^2+c)^{7/2}}{7a^8c^4}-\frac{6(a^2cx^2+c)^{5/2}}{5a^8c^3}+\frac{2(a^2cx^2+c)^{3/2}}{a^8c^2}-\frac{2\sqrt{a^2cx^2+}}{a^8c}\right.\right.\right.$$
$$c\left(c\left(\frac{1}{6}\sqrt{a^2cx^2+c}\arctan(ax)x^5-\frac{1}{12}ac\left(\frac{2(a^2cx^2+c)^{5/2}}{5a^6c^3}-\frac{4(a^2cx^2+c)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}\right)+\frac{1}{6}c\left(\frac{\sqrt{a^2cx^2}}{}\right.\right.\right.$$

$\Big\downarrow$ 53

$$c\left(c\left(\frac{1}{8}\sqrt{a^2cx^2+c}\arctan(ax)x^7-\frac{1}{16}ac\left(\frac{2\left(a^2cx^2+c\right)^{7/2}}{7a^8c^4}-\frac{6\left(a^2cx^2+c\right)^{5/2}}{5a^8c^3}+\frac{2\left(a^2cx^2+c\right)^{3/2}}{a^8c^2}-\frac{2\sqrt{a^2cx^2+}}{a^8c}\right.\right.\right.$$

$$c\left(c\left(\frac{1}{6}\sqrt{a^2cx^2+c}\arctan(ax)x^5-\frac{1}{12}ac\left(\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}\right)+\frac{1}{6}c\left(\frac{\sqrt{a^2cx^2}}{}\right.\right.\right.$$

$\downarrow$ 2009

$$c\left(c\left(\frac{1}{8}\sqrt{a^2cx^2+c}\arctan(ax)x^7-\frac{1}{16}ac\left(\frac{2\left(a^2cx^2+c\right)^{7/2}}{7a^8c^4}-\frac{6\left(a^2cx^2+c\right)^{5/2}}{5a^8c^3}+\frac{2\left(a^2cx^2+c\right)^{3/2}}{a^8c^2}-\frac{2\sqrt{a^2cx^2+}}{a^8c}\right.\right.\right.$$

$$c\left(c\left(\frac{1}{6}\sqrt{a^2cx^2+c}\arctan(ax)x^5-\frac{1}{12}ac\left(\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}\right)+\frac{1}{6}c\left(\frac{\sqrt{a^2cx^2}}{}\right.\right.\right.$$

$\downarrow$ 5425

$$c\left(c\left(\frac{1}{8}\sqrt{a^2cx^2+c}\arctan(ax)x^7-\frac{1}{16}ac\left(\frac{2\left(a^2cx^2+c\right)^{7/2}}{7a^8c^4}-\frac{6\left(a^2cx^2+c\right)^{5/2}}{5a^8c^3}+\frac{2\left(a^2cx^2+c\right)^{3/2}}{a^8c^2}-\frac{2\sqrt{a^2cx^2+}}{a^8c}\right.\right.\right.$$

$$c\left(c\left(\frac{1}{6}\sqrt{a^2cx^2+c}\arctan(ax)x^5-\frac{1}{12}ac\left(\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}\right)+\frac{1}{6}c\left(\frac{\sqrt{a^2cx^2}}{}\right.\right.\right.$$

$\downarrow$ 5421

$$c\left(c\left(\frac{1}{8}\sqrt{a^2cx^2+c}\arctan(ax)x^7-\frac{1}{16}ac\left(\frac{2\left(a^2cx^2+c\right)^{7/2}}{7a^8c^4}-\frac{6\left(a^2cx^2+c\right)^{5/2}}{5a^8c^3}+\frac{2\left(a^2cx^2+c\right)^{3/2}}{a^8c^2}-\frac{2\sqrt{a^2cx^2+}}{a^8c}\right.\right.\right.$$

$$c\left(c\left(\frac{1}{6}\sqrt{a^2cx^2+c}\arctan(ax)x^5-\frac{1}{12}ac\left(\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}\right)+\frac{1}{6}c\left(\frac{\sqrt{a^2cx^2}}{}\right.\right.\right.$$

$\downarrow$ 5487

$$c\left(c\left(\frac{1}{8}\sqrt{a^2cx^2+c}\arctan(ax)x^7-\frac{1}{16}ac\left(\frac{2\left(a^2cx^2+c\right)^{7/2}}{7a^8c^4}-\frac{6\left(a^2cx^2+c\right)^{5/2}}{5a^8c^3}+\frac{2\left(a^2cx^2+c\right)^{3/2}}{a^8c^2}-\frac{2\sqrt{a^2cx^2+}}{a^8c}\right.\right.\right.$$

$$c\left(c\left(\frac{1}{6}\sqrt{a^2cx^2+c}\arctan(ax)x^5-\frac{1}{12}ac\left(\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}\right)+\frac{1}{6}c\left(\frac{\sqrt{a^2cx^2}}{}\right.\right.\right.$$

$$\Big\downarrow \text{241}$$

$$c\left(c\left(\frac{1}{8}\sqrt{a^2cx^2+c}\arctan(ax)x^7-\frac{1}{16}ac\left(\frac{2\left(a^2cx^2+c\right)^{7/2}}{7a^8c^4}-\frac{6\left(a^2cx^2+c\right)^{5/2}}{5a^8c^3}+\frac{2\left(a^2cx^2+c\right)^{3/2}}{a^8c^2}-\frac{2\sqrt{a^2cx^2+}}{a^8c}\right.\right.\right.$$

$$c\left(c\left(\frac{1}{6}\sqrt{a^2cx^2+c}\arctan(ax)x^5-\frac{1}{12}ac\left(\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}\right)+\frac{1}{6}c\left(\frac{\sqrt{a^2cx^2}}{}\right.\right.\right.$$

$$\Big\downarrow \text{243}$$

$$c\left(c\left(\frac{1}{8}\sqrt{a^2cx^2+c}\arctan(ax)x^7-\frac{1}{16}ac\left(\frac{2\left(a^2cx^2+c\right)^{7/2}}{7a^8c^4}-\frac{6\left(a^2cx^2+c\right)^{5/2}}{5a^8c^3}+\frac{2\left(a^2cx^2+c\right)^{3/2}}{a^8c^2}-\frac{2\sqrt{a^2cx^2+}}{a^8c}\right.\right.\right.$$

$$c\left(c\left(\frac{1}{6}\sqrt{a^2cx^2+c}\arctan(ax)x^5-\frac{1}{12}ac\left(\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}\right)+\frac{1}{6}c\left(\frac{\sqrt{a^2cx^2}}{}\right.\right.\right.$$

$$\Big\downarrow \text{53}$$

$$c\left(c\left(\frac{1}{8}\sqrt{a^2cx^2+c}\arctan(ax)x^7-\frac{1}{16}ac\left(\frac{2\left(a^2cx^2+c\right)^{7/2}}{7a^8c^4}-\frac{6\left(a^2cx^2+c\right)^{5/2}}{5a^8c^3}+\frac{2\left(a^2cx^2+c\right)^{3/2}}{a^8c^2}-\frac{2\sqrt{a^2cx^2+}}{a^8c}\right.\right.\right.$$

$$c\left(c\left(\frac{1}{6}\sqrt{a^2cx^2+c}\arctan(ax)x^5-\frac{1}{12}ac\left(\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}\right)+\frac{1}{6}c\left(\frac{\sqrt{a^2cx^2}}{}\right.\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$c\left(c\left(\frac{1}{8}\sqrt{a^2cx^2+c}\arctan(ax)x^7-\frac{1}{16}ac\left(\frac{2\left(a^2cx^2+c\right)^{7/2}}{7a^8c^4}-\frac{6\left(a^2cx^2+c\right)^{5/2}}{5a^8c^3}+\frac{2\left(a^2cx^2+c\right)^{3/2}}{a^8c^2}-\frac{2\sqrt{a^2cx^2+}}{a^8c}\right.\right.\right.$$

$$c\left(c\left(\frac{1}{6}\sqrt{a^2cx^2+c}\arctan(ax)x^5-\frac{1}{12}ac\left(\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}\right)+\frac{1}{6}c\left(\frac{\sqrt{a^2cx^2}}{}\right.\right.\right.$$

$$\Big\downarrow \text{5425}$$

$$c\left(c\left(\frac{1}{8}\sqrt{a^2cx^2+c}\arctan(ax)x^7-\frac{1}{16}ac\left(\frac{2\left(a^2cx^2+c\right)^{7/2}}{7a^8c^4}-\frac{6\left(a^2cx^2+c\right)^{5/2}}{5a^8c^3}+\frac{2\left(a^2cx^2+c\right)^{3/2}}{a^8c^2}-\frac{2\sqrt{a^2cx^2+}}{a^8c}\right.\right.\right.$$

$$c\left(c\left(\frac{1}{6}\sqrt{a^2cx^2+c}\arctan(ax)x^5-\frac{1}{12}ac\left(\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}\right)+\frac{1}{6}c\left(\frac{\sqrt{a^2cx^2}}{}\right.\right.\right.$$

$$\Big\downarrow \text{5421}$$

$$c\left(c\left(\frac{1}{8}\sqrt{a^2cx^2+c}\arctan(ax)x^7-\frac{1}{16}ac\left(\frac{2\left(a^2cx^2+c\right)^{7/2}}{7a^8c^4}-\frac{6\left(a^2cx^2+c\right)^{5/2}}{5a^8c^3}+\frac{2\left(a^2cx^2+c\right)^{3/2}}{a^8c^2}-\frac{2\sqrt{a^2cx^2+}}{a^8c}\right.\right.\right.$$

$$c\left(c\left(\frac{1}{6}\sqrt{a^2cx^2+c}\arctan(ax)x^5-\frac{1}{12}ac\left(\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}\right)+\frac{1}{6}c\left(\frac{\sqrt{a^2cx^2}}{}\right.\right.\right.$$

$\downarrow$ 5487

$$c\left(c\left(\frac{1}{8}\sqrt{a^2cx^2+c}\arctan(ax)x^7-\frac{1}{16}ac\left(\frac{2\left(a^2cx^2+c\right)^{7/2}}{7a^8c^4}-\frac{6\left(a^2cx^2+c\right)^{5/2}}{5a^8c^3}+\frac{2\left(a^2cx^2+c\right)^{3/2}}{a^8c^2}-\frac{2\sqrt{a^2cx^2+}}{a^8c}\right.\right.\right.$$

$$c\left(c\left(\frac{1}{6}\sqrt{a^2cx^2+c}\arctan(ax)x^5-\frac{1}{12}ac\left(\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}\right)+\frac{1}{6}c\left(\frac{\sqrt{a^2cx^2}}{}\right.\right.\right.$$

$\downarrow$ 241

$$c\left(c\left(\frac{1}{8}\sqrt{a^2cx^2+c}\arctan(ax)x^7-\frac{1}{16}ac\left(\frac{2\left(a^2cx^2+c\right)^{7/2}}{7a^8c^4}-\frac{6\left(a^2cx^2+c\right)^{5/2}}{5a^8c^3}+\frac{2\left(a^2cx^2+c\right)^{3/2}}{a^8c^2}-\frac{2\sqrt{a^2cx^2+}}{a^8c}\right.\right.\right.$$

$$c\left(c\left(\frac{1}{6}\sqrt{a^2cx^2+c}\arctan(ax)x^5-\frac{1}{12}ac\left(\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}\right)+\frac{1}{6}c\left(\frac{\sqrt{a^2cx^2}}{}\right.\right.\right.$$

$\downarrow$ 5425

$$c\left(c\left(\frac{1}{8}\sqrt{a^2cx^2+c}\arctan(ax)x^7-\frac{1}{16}ac\left(\frac{2\left(a^2cx^2+c\right)^{7/2}}{7a^8c^4}-\frac{6\left(a^2cx^2+c\right)^{5/2}}{5a^8c^3}+\frac{2\left(a^2cx^2+c\right)^{3/2}}{a^8c^2}-\frac{2\sqrt{a^2cx^2+}}{a^8c}\right.\right.\right.$$

$$c\left(c\left(\frac{1}{6}\sqrt{a^2cx^2+c}\arctan(ax)x^5-\frac{1}{12}ac\left(\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}\right)+\frac{1}{6}c\left(\frac{\sqrt{a^2cx^2}}{}\right.\right.\right.$$

$\downarrow$ 5421

$$c\left(c\left(\frac{1}{8}\sqrt{a^2cx^2+c}\arctan(ax)x^7-\frac{1}{16}ac\left(\frac{2\left(a^2cx^2+c\right)^{7/2}}{7a^8c^4}-\frac{6\left(a^2cx^2+c\right)^{5/2}}{5a^8c^3}+\frac{2\left(a^2cx^2+c\right)^{3/2}}{a^8c^2}-\frac{2\sqrt{a^2cx^2+}}{a^8c}\right.\right.\right.$$

$$c\left(c\left(\frac{1}{6}\sqrt{a^2cx^2+c}\arctan(ax)x^5-\frac{1}{12}ac\left(\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}\right)+\frac{1}{6}c\left(\frac{\sqrt{a^2cx^2}}{}\right.\right.\right.$$

input
```
Int[x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x],x]
```

output
```
c*(c*(-1/8*(a*c*((-2*Sqrt[c + a^2*c*x^2])/(a^4*c) + (2*(c + a^2*c*x^2)^(3/
2))/(3*a^4*c^2))) + (x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/4 + (c*(-1/2*Sqr
t[c + a^2*c*x^2]/(a^3*c) + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(2*a^2*c) -
 (Sqrt[1 + a^2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I
*a*x]])/a + (I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*
PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/(2*a^2*Sqrt[c + a^2*c
*x^2])))/4) + a^2*c*(-1/12*(a*c*((2*Sqrt[c + a^2*c*x^2])/(a^6*c) - (4*(c +
 a^2*c*x^2)^(3/2))/(3*a^6*c^2) + (2*(c + a^2*c*x^2)^(5/2))/(5*a^6*c^3))) +
 (x^5*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/6 + (c*(-1/8*((-2*Sqrt[c + a^2*c*x^
2])/(a^4*c) + (2*(c + a^2*c*x^2)^(3/2))/(3*a^4*c^2))/a + (x^3*Sqrt[c + a^2
*c*x^2]*ArcTan[a*x])/(4*a^2*c) - (3*(-1/2*Sqrt[c + a^2*c*x^2]/(a^3*c) + (x
*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(2*a^2*c) - (Sqrt[1 + a^2*x^2]*(((-2*I)*
ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/a + (I*PolyLog[2, ((-
I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*PolyLog[2, (I*Sqrt[1 + I*a*x]
)/Sqrt[1 - I*a*x]])/a))/(2*a^2*Sqrt[c + a^2*c*x^2])))/(4*a^2)))/6)) + a^2*
c*(c*(-1/12*(a*c*((2*Sqrt[c + a^2*c*x^2])/(a^6*c) - (4*(c + a^2*c*x^2)^(3/
2))/(3*a^6*c^2) + (2*(c + a^2*c*x^2)^(5/2))/(5*a^6*c^3))) + (x^5*Sqrt[c +
a^2*c*x^2]*ArcTan[a*x])/6 + (c*(-1/8*((-2*Sqrt[c + a^2*c*x^2])/(a^4*c) + (
2*(c + a^2*c*x^2)^(3/2))/(3*a^4*c^2))/a + (x^3*Sqrt[c + a^2*c*x^2]*ArcTan[
a*x])/(4*a^2*c) - (3*(-1/2*Sqrt[c + a^2*c*x^2]/(a^3*c) + (x*Sqrt[c + a^...
```

## Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5481
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTan[c*x
])/(f*(m + 2))), x] + (Simp[d/(m + 2)   Int[(f*x)^m*((a + b*ArcTan[c*x])/Sq
rt[d + e*x^2]), x], x] - Simp[b*c*(d/(f*(m + 2)))   Int[(f*x)^(m + 1)/Sqrt[
d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] &&
NeQ[m, -2]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5487
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcTan[c*x])^p/(c^2*d*m)), x] + (-Simp[b*f*(p/(c*m))   Int[(f*x)^(m - 1)*((
a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[f^2*((m - 1)/(c^
2*m))   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2]), x], x])
/; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

## Maple [A] (verified)

Time = 4.19 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.59

| method | result |
|---|---|
| default | $\frac{c^2\sqrt{c(ax-i)(ax+i)}\,(5040\arctan(ax)a^7x^7-720a^6x^6+14280\arctan(ax)a^5x^5-1992a^4x^4+12390\arctan(ax)x^3a^3-1474a^2x^2+1575}{40320a^3}$ |

input
```
int(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/40320*c^2/a^3*(c*(a*x-I)*(a*x+I))^(1/2)*(5040*arctan(a*x)*a^7*x^7-720*a^
6*x^6+14280*arctan(a*x)*a^5*x^5-1992*a^4*x^4+12390*arctan(a*x)*x^3*a^3-147
4*a^2*x^2+1575*arctan(a*x)*a*x+1373)+5/128*c^2*(c*(a*x-I)*(a*x+I))^(1/2)*(
arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-arctan(a*x)*ln(1-I*(1+I*a*
x)/(a^2*x^2+1)^(1/2))+I*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*dilog(1+I
*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/a^3/(a^2*x^2+1)^(1/2)
```

## Fricas [F]

$$\int x^2\left(c+a^2cx^2\right)^{5/2}\arctan(ax)\,dx=\int\left(a^2cx^2+c\right)^{\frac{5}{2}}x^2\arctan\left(ax\right)\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x),x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2)*sqrt(a^2*c*x^2 + c)*arcta
n(a*x), x)
```

## Sympy [F(-1)]

Timed out.

$$\int x^2 (c + a^2 c x^2)^{5/2} \arctan(ax) \, dx = \text{Timed out}$$

input
```
integrate(x**2*(a**2*c*x**2+c)**(5/2)*atan(a*x),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int x^2 (c + a^2 c x^2)^{5/2} \arctan(ax) \, dx = \int (a^2 c x^2 + c)^{\frac{5}{2}} x^2 \arctan(ax) \, dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)*x^2*arctan(a*x), x)
```

## Giac [F]

$$\int x^2 (c + a^2 c x^2)^{5/2} \arctan(ax) \, dx = \int (a^2 c x^2 + c)^{\frac{5}{2}} x^2 \arctan(ax) \, dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)*x^2*arctan(a*x), x)
```

## Mupad [F(-1)]

Timed out.

$$\int x^2(c+a^2cx^2)^{5/2}\arctan(ax)\,dx=\int x^2\,\mathrm{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^{5/2}\,dx$$

input
```
int(x^2*atan(a*x)*(c + a^2*c*x^2)^(5/2),x)
```

output
```
int(x^2*atan(a*x)*(c + a^2*c*x^2)^(5/2), x)
```

## Reduce [F]

$$\int x^2(c+a^2cx^2)^{5/2}\arctan(ax)\,dx=\sqrt{c}\,c^2\left(\left(\int\sqrt{a^2x^2+1}\,atan(ax)\,x^6dx\right)a^4+2\left(\int\sqrt{a^2x^2+1}\,atan(ax)\,x^4dx\right)a^2+\int\sqrt{a^2x^2+1}\,atan(ax)\,x^2dx\right)$$

input
```
int(x^2*(a^2*c*x^2+c)^(5/2)*atan(a*x),x)
```

output
```
sqrt(c)*c**2*(int(sqrt(a**2*x**2 + 1)*atan(a*x)*x**6,x)*a**4 + 2*int(sqrt(
a**2*x**2 + 1)*atan(a*x)*x**4,x)*a**2 + int(sqrt(a**2*x**2 + 1)*atan(a*x)*
x**2,x))
```

# 3.218 $\int x(c+a^2cx^2)^{5/2}\arctan(ax)\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 134

$$\int x(c+a^2cx^2)^{5/2}\arctan(ax)\,dx=-\frac{5c^2x\sqrt{c+a^2cx^2}}{112a}-\frac{5cx(c+a^2cx^2)^{3/2}}{168a}-\frac{x(c+a^2cx^2)^{5/2}}{42a}+\frac{(c+a^2cx^2)^{7/2}\arctan(ax)}{7a^2c}-\frac{5c^{5/2}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right)}{112a^2}$$

output
```
-5/112*c^2*x*(a^2*c*x^2+c)^(1/2)/a-5/168*c*x*(a^2*c*x^2+c)^(3/2)/a-1/42*x*
(a^2*c*x^2+c)^(5/2)/a+1/7*(a^2*c*x^2+c)^(7/2)*arctan(a*x)/a^2/c-5/112*c^(5
/2)*arctanh(a*c^(1/2)*x/(a^2*c*x^2+c)^(1/2))/a^2
```

## Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.83

$$\int x(c+a^2cx^2)^{5/2}\arctan(ax)\,dx=\frac{c^2\left(-ax\sqrt{c+a^2cx^2}(33+26a^2x^2+8a^4x^4)+48(1+a^2x^2)^3\sqrt{c+a^2cx^2}\arctan(\right.}{336a^2}$$

input
```
Integrate[x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x],x]
```

output
```
(c^2*(-(a*x*Sqrt[c + a^2*c*x^2]*(33 + 26*a^2*x^2 + 8*a^4*x^4)) + 48*(1 + a
^2*x^2)^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x] - 15*Sqrt[c]*Log[a*c*x + Sqrt[c]
*Sqrt[c + a^2*c*x^2]]))/(336*a^2)
```

## Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5465, 211, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \arctan(ax) \left(a^2cx^2+c\right)^{5/2} dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{\arctan(ax) \left(a^2cx^2+c\right)^{7/2}}{7a^2c}-\frac{\int \left(a^2cx^2+c\right)^{5/2}dx}{7a}$$

$$\Big\downarrow \text{211}$$

$$\frac{\arctan(ax) \left(a^2cx^2+c\right)^{7/2}}{7a^2c}-\frac{\frac{5}{6}c\int \left(a^2cx^2+c\right)^{3/2}dx+\frac{1}{6}x\left(a^2cx^2+c\right)^{5/2}}{7a}$$

$$\Big\downarrow \text{211}$$

$$\frac{\arctan(ax) \left(a^2cx^2+c\right)^{7/2}}{7a^2c}-\frac{\frac{5}{6}c\left(\frac{3}{4}c\int \sqrt{a^2cx^2+c}dx+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\right)+\frac{1}{6}x\left(a^2cx^2+c\right)^{5/2}}{7a}$$

$$\Big\downarrow \text{211}$$

$$\frac{\arctan(ax) \left(a^2cx^2+c\right)^{7/2}}{7a^2c}-\frac{\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{1}{2}c\int \frac{1}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\right)+\frac{1}{6}x\left(a^2cx^2+c\right)^{5/2}}{7a}$$

$$\Big\downarrow \text{224}$$

$$\frac{\arctan(ax)\left(a^2cx^2+c\right)^{7/2}}{7a^2c}-\frac{\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{1}{2}c\int\frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\right)+\frac{1}{6}x\left(a^2cx^2+c\right)^{5/2}}{7a}$$

$\downarrow$ 219

$$\frac{\arctan(ax)\left(a^2cx^2+c\right)^{7/2}}{7a^2c}-\frac{\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{\sqrt{c}\,\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\right)+\frac{1}{6}x\left(a^2cx^2+c\right)^{5/2}}{7a}$$

input
```
Int[x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x],x]
```

output
```
((c + a^2*c*x^2)^(7/2)*ArcTan[a*x])/(7*a^2*c) - ((x*(c + a^2*c*x^2)^(5/2))
/6 + (5*c*((x*(c + a^2*c*x^2)^(3/2))/4 + (3*c*((x*Sqrt[c + a^2*c*x^2])/2 +
 (Sqrt[c]*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/(2*a)))/4))/6)/(7*a)
```

## Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.67 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.53

| method | result |
|---|---|
| default | $\frac{c^2\sqrt{c(ax-i)(ax+i)}\,(48a^6\arctan(ax)x^6-8a^5x^5+144x^4\arctan(ax)a^4-26a^3x^3+144x^2a^2\arctan(ax)-33ax+48\arctan(ax))}{336a^2}+\frac{5c}{}$ |

input
```
int(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/336*c^2/a^2*(c*(a*x-I)*(a*x+I))^(1/2)*(48*a^6*arctan(a*x)*x^6-8*a^5*x^5+
144*x^4*arctan(a*x)*a^4-26*a^3*x^3+144*x^2*a^2*arctan(a*x)-33*a*x+48*arcta
n(a*x))+5/112*c^2/a^2*(c*(a*x-I)*(a*x+I))^(1/2)*ln((1+I*a*x)/(a^2*x^2+1)^(
1/2)-I)/(a^2*x^2+1)^(1/2)-5/112*c^2/a^2*(c*(a*x-I)*(a*x+I))^(1/2)*ln((1+I*
a*x)/(a^2*x^2+1)^(1/2)+I)/(a^2*x^2+1)^(1/2)
```

## Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.97

$$\int x(c+a^2cx^2)^{5/2}\arctan(ax)\,dx = \frac{15\,c^{\frac{5}{2}}\log\left(-2\,a^2cx^2+2\,\sqrt{a^2cx^2+c}a\sqrt{c}x-c\right)-2\,(8\,a^5c^2x^5+26\,a^3c^2x^3+33\,a}{672\,a^2}$$

input
```
integrate(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x),x, algorithm="fricas")
```

output
```
1/672*(15*c^(5/2)*log(-2*a^2*c*x^2 + 2*sqrt(a^2*c*x^2 + c)*a*sqrt(c)*x - c
) - 2*(8*a^5*c^2*x^5 + 26*a^3*c^2*x^3 + 33*a*c^2*x - 48*(a^6*c^2*x^6 + 3*a
^4*c^2*x^4 + 3*a^2*c^2*x^2 + c^2)*arctan(a*x))*sqrt(a^2*c*x^2 + c))/a^2
```

## Sympy [F]

$$\int x(c + a^2cx^2)^{5/2} \arctan(ax) \, dx = \int x\left(c\left(a^2x^2 + 1\right)\right)^{\frac{5}{2}} \operatorname{atan}(ax) \, dx$$

input
```
integrate(x*(a**2*c*x**2+c)**(5/2)*atan(a*x),x)
```

output
```
Integral(x*(c*(a**2*x**2 + 1))**(5/2)*atan(a*x), x)
```

## Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 637 vs. 2(110) = 220.

Time = 0.29 (sec) , antiderivative size = 637, normalized size of antiderivative = 4.75

$$\int x(c + a^2cx^2)^{5/2} \arctan(ax) \, dx = \text{Too large to display}$$

input
```
integrate(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x),x, algorithm="maxima")
```

output
```
1/1680*(560*(a^2*c^2*x^2 + c^2)*sqrt(a^2*x^2 + 1)*sqrt(c)*arctan(a*x) - 28
0*(a^4*x^4 + 10*a^2*x^2 + 9)^(1/4)*(a*c^2*x*cos(1/2*arctan2(4*a*x, -a^2*x^
2 + 3)) + 2*c^2*sin(1/2*arctan2(4*a*x, -a^2*x^2 + 3)))*sqrt(c) - ((a*(5*(8
*(a^2*x^2 + 1)^(3/2)*x^3/a^2 - 6*(a^2*x^2 + 1)^(3/2)*x/a^4 + 3*sqrt(a^2*x^
2 + 1)*x/a^4 + 3*arcsinh(a*x)/a^5)/a^2 - 24*(2*(a^2*x^2 + 1)^(3/2)*x/a^2 -
 sqrt(a^2*x^2 + 1)*x/a^2 - arcsinh(a*x)/a^3)/a^4 + 64*(sqrt(a^2*x^2 + 1)*x
 + arcsinh(a*x)/a)/a^6) - 16*(15*(a^2*x^2 + 1)^(3/2)*x^4/a^2 - 12*(a^2*x^2
 + 1)^(3/2)*x^2/a^4 + 8*(a^2*x^2 + 1)^(3/2)/a^6)*arctan(a*x))*a^6*c^2 + 28
*(a*(3*(2*(a^2*x^2 + 1)^(3/2)*x/a^2 - sqrt(a^2*x^2 + 1)*x/a^2 - arcsinh(a*
x)/a^3)/a^2 - 8*(sqrt(a^2*x^2 + 1)*x + arcsinh(a*x)/a)/a^4) - 8*(3*(a^2*x^
2 + 1)^(3/2)*x^2/a^2 - 2*(a^2*x^2 + 1)^(3/2)/a^4)*arctan(a*x))*a^4*c^2 - 1
40*c^2*arctan2((a^4*x^4 + 10*a^2*x^2 + 9)^(1/4)*sin(1/2*arctan2(4*a*x, a^2
*x^2 - 3)) + 2, a*x + (a^4*x^4 + 10*a^2*x^2 + 9)^(1/4)*cos(1/2*arctan2(4*a
*x, a^2*x^2 - 3))) - 140*c^2*arctan2((a^4*x^4 + 10*a^2*x^2 + 9)^(1/4)*sin(
1/2*arctan2(4*a*x, a^2*x^2 - 3)) - 2, -a*x + (a^4*x^4 + 10*a^2*x^2 + 9)^(1
/4)*cos(1/2*arctan2(4*a*x, a^2*x^2 - 3))))*sqrt(c))/a^2
```

## Giac [F(-2)]

Exception generated.

$$\int x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int x(c+a^2cx^2)^{5/2}\arctan(ax)\,dx=\int x\,\mathrm{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^{5/2}\,dx$$

input
```
int(x*atan(a*x)*(c + a^2*c*x^2)^(5/2),x)
```

output
```
int(x*atan(a*x)*(c + a^2*c*x^2)^(5/2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.19

$$\int x(c+a^2cx^2)^{5/2}\arctan(ax)\,dx=\frac{\sqrt{c}\,c^2\left(48\sqrt{a^2x^2+1}\,atan(ax)\,a^6x^6+144\sqrt{a^2x^2+1}\,atan(ax)\,a^4x^4+144\sqrt{a^2x^2}\right.}{}$$

input
```
int(x*(a^2*c*x^2+c)^(5/2)*atan(a*x),x)
```

output
```
(sqrt(c)*c**2*(48*sqrt(a**2*x**2 + 1)*atan(a*x)*a**6*x**6 + 144*sqrt(a**2*
x**2 + 1)*atan(a*x)*a**4*x**4 + 144*sqrt(a**2*x**2 + 1)*atan(a*x)*a**2*x**
2 + 48*sqrt(a**2*x**2 + 1)*atan(a*x) - 8*sqrt(a**2*x**2 + 1)*a**5*x**5 - 2
6*sqrt(a**2*x**2 + 1)*a**3*x**3 - 33*sqrt(a**2*x**2 + 1)*a*x - 15*log(sqrt
(a**2*x**2 + 1) + a*x)))/(336*a**2)
```

# 3.219 $\int (c+a^2cx^2)^{5/2}\arctan(ax)\,dx$



## Optimal result

Integrand size = 19, antiderivative size = 348

$$\int (c+a^2cx^2)^{5/2}\arctan(ax)\,dx = -\frac{5c^2\sqrt{c+a^2cx^2}}{16a} - \frac{5c(c+a^2cx^2)^{3/2}}{72a} - \frac{(c+a^2cx^2)^{5/2}}{30a}$$

$$+\frac{5}{16}c^2x\sqrt{c+a^2cx^2}\arctan(ax)+\frac{5}{24}cx(c+a^2cx^2)^{3/2}\arctan(ax)+\frac{1}{6}x(c+a^2cx^2)^{5/2}\arctan(ax)-\frac{5ic^3\sqrt{1+a}}{}$$

output
```
-5/16*c^2*(a^2*c*x^2+c)^(1/2)/a-5/72*c*(a^2*c*x^2+c)^(3/2)/a-1/30*(a^2*c*x
^2+c)^(5/2)/a+5/16*c^2*x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)+5/24*c*x*(a^2*c*x
^2+c)^(3/2)*arctan(a*x)+1/6*x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)-5/8*I*c^3*(a
^2*x^2+1)^(1/2)*arctan(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/a/(a^2
*c*x^2+c)^(1/2)+5/16*I*c^3*(a^2*x^2+1)^(1/2)*polylog(2,-I*(1+I*a*x)^(1/2)/
(1-I*a*x)^(1/2))/a/(a^2*c*x^2+c)^(1/2)-5/16*I*c^3*(a^2*x^2+1)^(1/2)*polylo
g(2,I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/a/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (warning: unable to verify)

Time = 4.30 (sec) , antiderivative size = 643, normalized size of antiderivative = 1.85

$$\int (c +a^2cx^2)^{5/2} \arctan(ax)\,dx = \frac{c^2\sqrt{c+a^2cx^2}\left(\frac{3}{4}(1+a^2x^2)^{5/2}+720\sqrt{1+a^2x^2}(-1+ax\arctan(ax))+\frac{55}{8}(1+\right.}{}$$

input
```
Integrate[(c + a^2*c*x^2)^(5/2)*ArcTan[a*x],x]
```

output
```
(c^2*Sqrt[c + a^2*c*x^2]*((3*(1 + a^2*x^2)^(5/2))/4 + 720*Sqrt[1 + a^2*x^2
]*(-1 + a*x*ArcTan[a*x]) + (55*(1 + a^2*x^2)^3*Cos[3*ArcTan[a*x]])/8 - (45
*(1 + a^2*x^2)^3*Cos[5*ArcTan[a*x]])/8 + 720*ArcTan[a*x]*(Log[1 - I*E^(I*A
rcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])]) + (450*I)*PolyLog[2, (-I)*E^(
I*ArcTan[a*x])] - (450*I)*PolyLog[2, I*E^(I*ArcTan[a*x])] - 15*(1 + a^2*x^
2)^2*(-2/Sqrt[1 + a^2*x^2] - 6*Cos[3*ArcTan[a*x]] + 3*ArcTan[a*x]*((-14*a*
x)/Sqrt[1 + a^2*x^2] + 3*Log[1 - I*E^(I*ArcTan[a*x])] + 4*Cos[2*ArcTan[a*x
]]*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])]) + Cos[4*A
rcTan[a*x]]*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])])
 - 3*Log[1 + I*E^(I*ArcTan[a*x])] + 2*Sin[3*ArcTan[a*x]])) + (15*(1 + a^2*x
^2)^3*ArcTan[a*x]*((156*a*x)/Sqrt[1 + a^2*x^2] + 30*Log[1 - I*E^(I*ArcTan[
a*x])] + 3*Cos[6*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] + 45*Cos[2*ArcT
an[a*x]]*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])]) + 1
8*Cos[4*ArcTan[a*x]]*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^(I*ArcTan
[a*x])]) - 30*Log[1 + I*E^(I*ArcTan[a*x])] - 3*Cos[6*ArcTan[a*x]]*Log[1 +
I*E^(I*ArcTan[a*x])] - 94*Sin[3*ArcTan[a*x]] + 6*Sin[5*ArcTan[a*x]]))/16))
/(1440*a*Sqrt[1 + a^2*x^2])
```

## Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.83, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {5413, 5413, 5413, 5425, 5421}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(ax)\left(a^2cx^2+c\right)^{5/2}dx$$

$\Big\downarrow$ 5413

$$\frac{5}{6}c\int \left(a^2cx^2+c\right)^{3/2}\arctan(ax)dx+\frac{1}{6}x\arctan(ax)\left(a^2cx^2+c\right)^{5/2}-\frac{\left(a^2cx^2+c\right)^{5/2}}{30a}$$

$\Big\downarrow$ 5413

$$\frac{5}{6}c\left(\frac{3}{4}c\int \sqrt{a^2cx^2+c}\arctan(ax)dx+\frac{1}{4}x\arctan(ax)\left(a^2cx^2+c\right)^{3/2}-\frac{\left(a^2cx^2+c\right)^{3/2}}{12a}\right)+\frac{1}{6}x\arctan(ax)\left(a^2cx^2+c\right)^{5/2}-\frac{\left(a^2cx^2+c\right)^{5/2}}{30a}$$

$\Big\downarrow$ 5413

$$\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{1}{2}c\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)+\frac{1}{4}x\arctan(ax)\left(a^2cx^2+c\right)^{3/2}-\frac{(\right.$$
$$\frac{1}{6}x\arctan(ax)\left(a^2cx^2+c\right)^{5/2}-\frac{\left(a^2cx^2+c\right)^{5/2}}{30a}$$

$\Big\downarrow$ 5425

$$\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)+\frac{1}{4}x\arctan(ax)\left(a^2cx^2+c\right)^{3}\right.$$
$$\frac{1}{6}x\arctan(ax)\left(a^2cx^2+c\right)^{5/2}-\frac{\left(a^2cx^2+c\right)^{5/2}}{30a}$$

$\Big\downarrow$ 5421

$$\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax\right.\right.$$
$$\frac{1}{6}x\arctan(ax)\left(a^2cx^2+c\right)^{5/2}-\frac{\left(a^2cx^2+c\right)^{5/2}}{30a}$$

input
```
Int[(c + a^2*c*x^2)^(5/2)*ArcTan[a*x],x]
```

output
```
-1/30*(c + a^2*c*x^2)^(5/2)/a + (x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x])/6 +
(5*c*(-1/12*(c + a^2*c*x^2)^(3/2)/a + (x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]
)/4 + (3*c*(-1/2*Sqrt[c + a^2*c*x^2]/a + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x
])/2 + (c*Sqrt[1 + a^2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sq
rt[1 - I*a*x]])/a + (I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])
/a - (I*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/(2*Sqrt[c + a
^2*c*x^2])))/4))/6
```

### Defintions of rubi rules used

rule 5413
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Simp[x*(d + e*x^2)
^q*((a + b*ArcTan[c*x])/(2*q + 1)), x] + Simp[2*d*(q/(2*q + 1))   Int[(d +
e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0]
```

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

## Maple [A] (verified)

Time = 2.01 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.65

| method | result |
|---|---|
| default | $\frac{c^2\sqrt{c(ax-i)(ax+i)}\,(120\arctan(ax)a^5x^5-24a^4x^4+390\arctan(ax)x^3a^3-98a^2x^2+495\arctan(ax)ax-299)}{720a}-\frac{5c^2\sqrt{c(ax-i)(ax+}}{}$ |

input
```
int((a^2*c*x^2+c)^(5/2)*arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/720*c^2/a*(c*(a*x-I)*(a*x+I))^(1/2)*(120*arctan(a*x)*a^5*x^5-24*a^4*x^4+
390*arctan(a*x)*x^3*a^3-98*a^2*x^2+495*arctan(a*x)*a*x-299)-5/16*c^2*(c*(a
*x-I)*(a*x+I))^(1/2)*(arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-arct
an(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+I*dilog(1-I*(1+I*a*x)/(a^2*x^2
+1)^(1/2))-I*dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/a/(a^2*x^2+1)^(1/2)
```

## Fricas [F]

$$\int (c+a^2cx^2)^{5/2}\arctan(ax)\,dx=\int (a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x),x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*arctan(a*
x), x)
```

## Sympy [F]

$$\int (c+a^2cx^2)^{5/2}\arctan(ax)\,dx=\int (c(a^2x^2+1))^{\frac{5}{2}}\operatorname{atan}(ax)\,dx$$

input
```
integrate((a**2*c*x**2+c)**(5/2)*atan(a*x),x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(5/2)*atan(a*x), x)
```

## Maxima [F]

$$\int \left(c+a^2cx^2\right)^{5/2}\arctan(ax)\,dx=\int \left(a^2cx^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)*arctan(a*x), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^{5/2}\arctan(ax)\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \left(c+a^2cx^2\right)^{5/2}\arctan(ax)\,dx=\int \mathrm{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^{5/2}\,dx$$

input
```
int(atan(a*x)*(c + a^2*c*x^2)^(5/2),x)
```

output
```
int(atan(a*x)*(c + a^2*c*x^2)^(5/2), x)
```

## Reduce [F]

$$\int \left(c+a^2cx^2\right)^{5/2}\arctan(ax)\,dx = \sqrt{c}\,c^2\left(\left(\int \sqrt{a^2x^2+1}\,atan(ax)\,x^4dx\right)a^4 + 2\left(\int \sqrt{a^2x^2+1}\,atan(ax)\,x^2dx\right)a^2 + \int \sqrt{a^2x^2+1}\,atan(ax)\,dx\right)$$

input
```
int((a^2*c*x^2+c)^(5/2)*atan(a*x),x)
```

output
```
sqrt(c)*c**2*(int(sqrt(a**2*x**2 + 1)*atan(a*x)*x**4,x)*a**4 + 2*int(sqrt(
a**2*x**2 + 1)*atan(a*x)*x**2,x)*a**2 + int(sqrt(a**2*x**2 + 1)*atan(a*x),
x))
```

# 3.220 $\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)}{x}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 329

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)}{x}\,dx = -\frac{29}{120}ac^2x\sqrt{c+a^2cx^2} - \frac{1}{20}acx(c+a^2cx^2)^{3/2} + c^2\sqrt{c+a^2cx^2}\arctan(ax) + \frac{1}{3}c(c+a^2cx^2)^{3/2}\arctan(ax) + \frac{1}{5}(c+a^2cx^2)^{5/2}\arctan(ax) - \frac{2c^3\sqrt{1+a^2x^2}\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} - \frac{1}{1}$$

output

```
-29/120*a*c^2*x*(a^2*c*x^2+c)^(1/2)-1/20*a*c*x*(a^2*c*x^2+c)^(3/2)+c^2*(a^
2*c*x^2+c)^(1/2)*arctan(a*x)+1/3*c*(a^2*c*x^2+c)^(3/2)*arctan(a*x)+1/5*(a^
2*c*x^2+c)^(5/2)*arctan(a*x)-2*c^3*(a^2*x^2+1)^(1/2)*arctan(a*x)*arctanh((
1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/(a^2*c*x^2+c)^(1/2)-149/120*c^(5/2)*arctan
h(a*c^(1/2)*x/(a^2*c*x^2+c)^(1/2))+I*c^3*(a^2*x^2+1)^(1/2)*polylog(2,-(1+I
*a*x)^(1/2)/(1-I*a*x)^(1/2))/(a^2*c*x^2+c)^(1/2)-I*c^3*(a^2*x^2+1)^(1/2)*p
olylog(2,(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.86

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)}{x}\,dx = \frac{c^2\sqrt{c+a^2cx^2}\left(-35ax\sqrt{1+a^2x^2}-6a^3x^3\sqrt{1+a^2x^2}+184\sqrt{1+a^2x^2}\,\mathrm{arc}\right.}{}$$

input
```
Integrate[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x])/x,x]
```

output
```
(c^2*Sqrt[c + a^2*c*x^2]*(-35*a*x*Sqrt[1 + a^2*x^2] - 6*a^3*x^3*Sqrt[1 + a
^2*x^2] + 184*Sqrt[1 + a^2*x^2]*ArcTan[a*x] + 88*a^2*x^2*Sqrt[1 + a^2*x^2]
*ArcTan[a*x] + 24*a^4*x^4*Sqrt[1 + a^2*x^2]*ArcTan[a*x] + 120*ArcTan[a*x]*
Log[1 - E^(I*ArcTan[a*x])] - 120*ArcTan[a*x]*Log[1 + E^(I*ArcTan[a*x])] +
29*Log[-(a*x) + Sqrt[1 + a^2*x^2]] + 120*Log[Cos[ArcTan[a*x]/2] - Sin[ArcT
an[a*x]/2]] - 120*Log[Cos[ArcTan[a*x]/2] + Sin[ArcTan[a*x]/2]] + (120*I)*P
olyLog[2, -E^(I*ArcTan[a*x])] - (120*I)*PolyLog[2, E^(I*ArcTan[a*x])]))/(1
20*Sqrt[1 + a^2*x^2])
```

## Rubi [A] (verified)

Time = 1.65 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.21, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {5485, 5465, 211, 211, 224, 219, 5485, 5465, 211, 224, 219, 5481, 224, 219, 5493, 5489}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{x}\,dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\int x\left(a^2cx^2+c\right)^{3/2}\arctan(ax)dx+c\int \frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}{x}dx$$

$$\Big\downarrow \text{5465}$$

$$a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{\int\left(a^2cx^2+c\right)^{3/2}dx}{5a}\right)+c\int\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}{x}dx$$

$$\Big\downarrow \ 211$$

$$a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{\frac{3}{4}c\int\sqrt{a^2cx^2+c}dx+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}}{5a}\right)+ c\int\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}{x}dx$$

$$\Big\downarrow \ 211$$

$$a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{\frac{3}{4}c\left(\frac{1}{2}c\int\frac{1}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}}{5a}\right)+ c\int\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}{x}dx$$

$$\Big\downarrow \ 224$$

$$a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{\frac{3}{4}c\left(\frac{1}{2}c\int\frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}}{5a}\right)+ c\int\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}{x}dx$$

$$\Big\downarrow \ 219$$

$$c\int\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}{x}dx+ a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{\frac{3}{4}c\left(\frac{\sqrt{c}\,\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}}{5a}\right)$$

$$\Big\downarrow \ 5485$$

$$c\left(a^2c\int x\sqrt{a^2cx^2+c}\arctan(ax)dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x}dx\right)+ a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{\frac{3}{4}c\left(\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}}{5a}\right)$$

$$\Big\downarrow \text{ 5465}$$

$$c\left(a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\int \sqrt{a^2cx^2+c}dx}{3a}\right)+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x}dx\right)+ a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{\frac{3}{4}c\left(\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}}{5a}\right)$$

$$\Big\downarrow \text{ 211}$$

$$c\left(a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{1}{2}c\int \frac{1}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\sqrt{a^2cx^2+c}}{3a}\right)+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x}dx\right)+ a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{\frac{3}{4}c\left(\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}}{5a}\right)$$

$$\Big\downarrow \text{ 224}$$

$$c\left(a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{1}{2}c\int \frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}+\frac{1}{2}x\sqrt{a^2cx^2+c}}{3a}\right)+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x}\right. a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{\frac{3}{4}c\left(\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}}{5a}\right)$$

$$\Big\downarrow \text{ 219}$$

$$c\left(c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x}dx+a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}}{3a}\right)\right.$$
$$a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{\frac{3}{4}c\left(\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}}{5a}\right)$$

$\Big\downarrow$ 5481

$$c\left(c\left(c\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx-ac\int \frac{1}{\sqrt{a^2cx^2+c}}dx+\arctan(ax)\sqrt{a^2cx^2+c}\right)+a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\right.\right.$$
$$a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{\frac{3}{4}c\left(\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}}{5a}\right)$$

$\Big\downarrow$ 224

$$c\left(c\left(c\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx-ac\int \frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}+\arctan(ax)\sqrt{a^2cx^2+c}\right)+a^2c\left(\frac{\arctan(ax)\left(a^2ca\right.}{3a^2c}\right.\right.$$
$$a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{\frac{3}{4}c\left(\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}}{5a}\right)$$

$\Big\downarrow$ 219

$$c\left(c\left(c\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx+\arctan(ax)\sqrt{a^2cx^2+c}-\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)\right)+a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+\right.}{3a^2c}\right.\right.$$

$$a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{\frac{3}{4}c\left(\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}}{5a}\right)$$

$\downarrow$ 5493

$$c\left(c\left(\frac{c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+\arctan(ax)\sqrt{a^2cx^2+c}-\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)\right)+a^2c\left(\frac{\arctan(ax)\left(a^2\right.}{3a^2}\right.\right.$$

$$a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{\frac{3}{4}c\left(\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}}{5a}\right)$$

$\downarrow$ 5489

$$a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{\frac{3}{4}c\left(\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}}{5a}\right)+$$

$$c\left(a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}}{3a}\right)+c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\arctan(ax)\right.}{}\right.\right.$$

input
```
Int[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x])/x,x]
```

output
```
a^2*c*(((c + a^2*c*x^2)^(5/2)*ArcTan[a*x])/(5*a^2*c) - ((x*(c + a^2*c*x^2)
^(3/2))/4 + (3*c*((x*Sqrt[c + a^2*c*x^2])/2 + (Sqrt[c]*ArcTanh[(a*Sqrt[c]*
x)/Sqrt[c + a^2*c*x^2]])/(2*a)))/4)/(5*a)) + c*(a^2*c*(((c + a^2*c*x^2)^(3
/2)*ArcTan[a*x])/(3*a^2*c) - ((x*Sqrt[c + a^2*c*x^2])/2 + (Sqrt[c]*ArcTanh
[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/(2*a))/(3*a)) + c*(Sqrt[c + a^2*c*x^2
]*ArcTan[a*x] - Sqrt[c]*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]] + (c*Sq
rt[1 + a^2*x^2]*(-2*ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]] +
 I*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])] - I*PolyLog[2, Sqrt[1 +
I*a*x]/Sqrt[1 - I*a*x]]))/Sqrt[c + a^2*c*x^2]))
```

### Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5481
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTan[c*x
])/(f*(m + 2))), x] + (Simp[d/(m + 2)   Int[(f*x)^m*((a + b*ArcTan[c*x])/Sq
rt[d + e*x^2]), x], x] - Simp[b*c*(d/(f*(m + 2)))   Int[(f*x)^(m + 1)/Sqrt[
d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] &&
NeQ[m, -2]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5489
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sq
rt[1 - I*c*x]], x] + (Simp[I*(b/Sqrt[d])*PolyLog[2, -Sqrt[1 + I*c*x]/Sqrt[1
 - I*c*x]], x] - Simp[I*(b/Sqrt[d])*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x]], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

## Maple [A] (verified)

Time = 2.30 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.60

| method | result |
|---|---|
| default | $\frac{c^2\sqrt{c(ax-i)(ax+i)}\,(24x^4\arctan(ax)a^4-6a^3x^3+88x^2a^2\arctan(ax)-35ax+184\arctan(ax))}{120}-\frac{c^2\sqrt{c(ax-i)(ax+i)}\left(60\arctan(a\right.}{}$ |

input
```
int((a^2*c*x^2+c)^(5/2)*arctan(a*x)/x,x,method=_RETURNVERBOSE)
```

output
```
1/120*c^2*(c*(a*x-I)*(a*x+I))^(1/2)*(24*x^4*arctan(a*x)*a^4-6*a^3*x^3+88*x
^2*a^2*arctan(a*x)-35*a*x+184*arctan(a*x))-1/60*c^2*(c*(a*x-I)*(a*x+I))^(1
/2)*(60*arctan(a*x)*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-149*I*arctan((1+I*a*
x)/(a^2*x^2+1)^(1/2))-60*I*dilog(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-60*I*dilog
((1+I*a*x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)
```

## Fricas [F]

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)}{x}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)}{x}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)/x,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*arctan(a*
x)/x, x)
```

## Sympy [F]

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)}{x}\,dx = \int \frac{(c(a^2x^2+1))^{\frac{5}{2}}\operatorname{atan}(ax)}{x}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)/x,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)/x, x)
```

## Maxima [F]

$$\int \frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)}{x}\,dx = \int \frac{\left(a^2cx^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)}{x}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)/x,x, algorithm="maxima")
```

output
```
2/3*(a^2*c^2*x^2 + c^2)*sqrt(a^2*x^2 + 1)*sqrt(c)*arctan(a*x) - 1/3*(a^4*x
^4 + 10*a^2*x^2 + 9)^(1/4)*(a*c^2*x*cos(1/2*arctan2(4*a*x, -a^2*x^2 + 3))
+ 2*c^2*sin(1/2*arctan2(4*a*x, -a^2*x^2 + 3)))*sqrt(c) - 1/120*((a*(3*(2*(
a^2*x^2 + 1)^(3/2)*x/a^2 - sqrt(a^2*x^2 + 1)*x/a^2 - arcsinh(a*x)/a^3)/a^2
 - 8*(sqrt(a^2*x^2 + 1)*x + arcsinh(a*x)/a)/a^4) - 8*(3*(a^2*x^2 + 1)^(3/2
)*x^2/a^2 - 2*(a^2*x^2 + 1)^(3/2)/a^4)*arctan(a*x))*a^4*c^2 - 20*c^2*arcta
n2((a^4*x^4 + 10*a^2*x^2 + 9)^(1/4)*sin(1/2*arctan2(4*a*x, a^2*x^2 - 3)) +
 2, a*x + (a^4*x^4 + 10*a^2*x^2 + 9)^(1/4)*cos(1/2*arctan2(4*a*x, a^2*x^2
- 3))) - 20*c^2*arctan2((a^4*x^4 + 10*a^2*x^2 + 9)^(1/4)*sin(1/2*arctan2(4
*a*x, a^2*x^2 - 3)) - 2, -a*x + (a^4*x^4 + 10*a^2*x^2 + 9)^(1/4)*cos(1/2*a
rctan2(4*a*x, a^2*x^2 - 3))) - 120*c^2*integrate(sqrt(a^2*x^2 + 1)*arctan(
a*x)/x, x))*sqrt(c)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)}{x}\,dx=\int \frac{\mathrm{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^{5/2}}{x}\,dx$$

input
```
int((atan(a*x)*(c + a^2*c*x^2)^(5/2))/x,x)
```

output
```
int((atan(a*x)*(c + a^2*c*x^2)^(5/2))/x, x)
```

## Reduce [F]

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)}{x}\,dx=\frac{\sqrt{c}\,c^2\Big(24\sqrt{a^2x^2+1}\,atan(ax)\,a^4x^4+88\sqrt{a^2x^2+1}\,atan(ax)\,a^2x^2+64\sqrt{}}{}$$

input
```
int((a^2*c*x^2+c)^(5/2)*atan(a*x)/x,x)
```

output
```
(sqrt(c)*c**2*(24*sqrt(a**2*x**2 + 1)*atan(a*x)*a**4*x**4 + 88*sqrt(a**2*x
**2 + 1)*atan(a*x)*a**2*x**2 + 64*sqrt(a**2*x**2 + 1)*atan(a*x) - 6*sqrt(a
**2*x**2 + 1)*a**3*x**3 - 35*sqrt(a**2*x**2 + 1)*a*x + 120*int((sqrt(a**2*
x**2 + 1)*atan(a*x))/x,x) - 29*log(sqrt(a**2*x**2 + 1) + a*x)))/120
```

# 3.221 $\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)}{x^2}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 355

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)}{x^2}\,dx = -\frac{7}{8}ac^2\sqrt{c+a^2cx^2} - \frac{1}{12}ac(c+a^2cx^2)^{3/2} - \frac{c^2\sqrt{c+a^2cx^2}\arctan(ax)}{x} + \frac{7}{8}a^2c^2x\sqrt{c+a^2cx^2}\arctan(ax) + \frac{1}{4}a^2cx(c+a^2cx^2)^{3/2}\arctan(ax) - \frac{15iac^3\sqrt{1+a^2x^2}\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{4\sqrt{c+a^2cx^2}} - ac^{5/2}\text{arctanh}\left(\frac{\sqrt{c+a^2c}}{\sqrt{c}}\right.$$

output
```
-7/8*a*c^2*(a^2*c*x^2+c)^(1/2)-1/12*a*c*(a^2*c*x^2+c)^(3/2)-c^2*(a^2*c*x^2
+c)^(1/2)*arctan(a*x)/x+7/8*a^2*c^2*x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)+1/4*
a^2*c*x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)-15/4*I*a*c^3*(a^2*x^2+1)^(1/2)*arc
tan(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/(a^2*c*x^2+c)^(1/2)-a*c^(
5/2)*arctanh((a^2*c*x^2+c)^(1/2)/c^(1/2))+15/8*I*a*c^3*(a^2*x^2+1)^(1/2)*p
olylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/(a^2*c*x^2+c)^(1/2)-15/8*I*a*
c^3*(a^2*x^2+1)^(1/2)*polylog(2,I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/(a^2*c*
x^2+c)^(1/2)
```

## Mathematica [A] (warning: unable to verify)

Time = 2.89 (sec) , antiderivative size = 491, normalized size of antiderivative = 1.38

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)}{x^2}\,dx = \frac{ac^2\sqrt{c+a^2cx^2}\left(\frac{1}{2}(1+a^2x^2)^{3/2}+48\sqrt{1+a^2x^2}(-1+ax\arctan(ax))+\right.}{}$$

input
```
Integrate[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x])/x^2,x]
```

output
```
(a*c^2*Sqrt[c + a^2*c*x^2]*((1 + a^2*x^2)^(3/2)/2 + 48*Sqrt[1 + a^2*x^2]*(
-1 + a*x*ArcTan[a*x]) + (3*(1 + a^2*x^2)^2*Cos[3*ArcTan[a*x]])/2 + 48*ArcT
an[a*x]*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])]) + (4
2*I)*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - 48*((Sqrt[1 + a^2*x^2]*ArcTan[a*
x])/(a*x) - ArcTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])] + ArcTan[a*x]*Log[1 +
 I*E^(I*ArcTan[a*x])] + Log[Cos[ArcTan[a*x]/2]] - Log[Sin[ArcTan[a*x]/2]]
- I*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + I*PolyLog[2, I*E^(I*ArcTan[a*x])]
) - (42*I)*PolyLog[2, I*E^(I*ArcTan[a*x])] - (3*(1 + a^2*x^2)^2*ArcTan[a*x
]*((-14*a*x)/Sqrt[1 + a^2*x^2] + 3*Log[1 - I*E^(I*ArcTan[a*x])] + 4*Cos[2*
ArcTan[a*x]]*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])])
 + Cos[4*ArcTan[a*x]]*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^(I*ArcTa
n[a*x])]) - 3*Log[1 + I*E^(I*ArcTan[a*x])] + 2*Sin[3*ArcTan[a*x]]))/4))/(4
8*Sqrt[1 + a^2*x^2])
```

## Rubi [A] (verified)

Time = 3.18 (sec) , antiderivative size = 647, normalized size of antiderivative = 1.82, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {5485, 5413, 5413, 5425, 5421, 5485, 5413, 5425, 5421, 5485, 5425, 5421, 5479, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{x^2}\,dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\int \left(a^2cx^2+c\right)^{3/2}\arctan(ax)dx+c\int \frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}{x^2}dx$$

$$\Big\downarrow \text{5413}$$

$$a^2c\left(\frac{3}{4}c\int \sqrt{a^2cx^2+c}\arctan(ax)dx+\frac{1}{4}x\arctan(ax)\left(a^2cx^2+c\right)^{3/2}-\frac{\left(a^2cx^2+c\right)^{3/2}}{12a}\right)+ c\int \frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}{x^2}dx$$

$$\Big\downarrow \text{5413}$$

$$a^2c\left(\frac{3}{4}c\left(\frac{1}{2}c\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)+\frac{1}{4}x\arctan(ax)\left(a^2cx^2+c\right)^{3/2}- \right. c\int \frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}{x^2}dx$$

$$\Big\downarrow \text{5425}$$

$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)+\frac{1}{4}x\arctan(ax)\left(a^2cx^2+c\right)^{3} \right. c\int \frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}{x^2}dx$$

$$\Big\downarrow \text{5421}$$

$$c\int \frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}{x^2}dx+ a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax\right.\right.$$

$$\Big\downarrow \text{5485}$$

$$c\left(a^2c\int \sqrt{a^2cx^2+c}\arctan(ax)dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^2}dx\right)+ a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax\right.\right.$$

$\Big\downarrow$ 5413

$$c\left(a^2c\left(\frac{1}{2}c\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^2}dx\right)+$$
$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(a\right.\right.$$

$\Big\downarrow$ 5425

$$c\left(a^2c\left(\frac{c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^2}d\right.$$
$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(a\right.\right.$$

$\Big\downarrow$ 5421

$$c\left(c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^2}dx+a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{Poly}}{}\right.}{2\sqrt{a^2cx^2+c}}\right.\right.$$
$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(a\right.\right.$$

$\Big\downarrow$ 5485

$$c\left(c\left(a^2c\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx\right)+a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}(}{}\right.}{2\sqrt{a^2cx^2+c}}\right.\right.$$

$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax\right.\right.$$

$$\Big\downarrow \text{5425}$$

$$c\left(c\left(\frac{a^2c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx\right)+a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\right.}{2\sqrt{a^2c}}\right.\right.$$

$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax\right.\right.$$

$$\Big\downarrow \text{5421}$$

$$c\left(c\left(c\int \frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx+\frac{a^2c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{iax}}{\sqrt{1-}}\right.}{a}\right.}{\sqrt{a^2cx^2+c}}\right.\right.$$

$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax\right.\right.$$

$$\Big\downarrow \text{5479}$$

$$c\left(c\left(c\left(a\int \frac{1}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right)+\frac{a^2c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLo}}{}\right.}{\sqrt{a^2cx^2+c}}\right.\right.$$

$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax\right.\right.$$

$$\Big\downarrow \text{243}$$

$$c\left(c\left(c\left(\frac{1}{2}a\int \frac{1}{x^2\sqrt{a^2cx^2+c}}dx^2-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right)+\frac{a^2c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{Po}}{}\right.}{\sqrt{a^2cx^2+}}\right.\right.$$

$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax\right.\right.$$

$$\Big\downarrow \text{73}$$

$$c\left(c\left(c\left(\frac{\int \frac{1}{\frac{x^4}{a^2c}-\frac{1}{a^2}}d\sqrt{a^2cx^2+c}}{ac}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right)+\frac{a^2c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{Poly}}{}\right.}{\sqrt{a^2cx^2+c}}\right.\right.$$

$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax\right.\right.$$

$$\Big\downarrow \text{221}$$

$$c\left(c\left(c\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)+\frac{a^2c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{Pol}}{}\right.}{\sqrt{a^2cx^2+}}\right.\right.$$

$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax\right.\right.$$

input
```
Int[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x])/x^2,x]
```

output
```
a^2*c*(-1/12*(c + a^2*c*x^2)^(3/2)/a + (x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x
])/4 + (3*c*(-1/2*Sqrt[c + a^2*c*x^2]/a + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*
x])/2 + (c*Sqrt[1 + a^2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/S
qrt[1 - I*a*x]])/a + (I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]]
)/a - (I*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/(2*Sqrt[c +
a^2*c*x^2])))/4) + c*(a^2*c*(-1/2*Sqrt[c + a^2*c*x^2]/a + (x*Sqrt[c + a^2*
c*x^2]*ArcTan[a*x])/2 + (c*Sqrt[1 + a^2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[S
qrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/a + (I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/
Sqrt[1 - I*a*x]])/a - (I*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/
a))/(2*Sqrt[c + a^2*c*x^2])) + c*(c*(-((Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(
c*x)) - (a*ArcTanh[Sqrt[c + a^2*c*x^2]/Sqrt[c]])/Sqrt[c]) + (a^2*c*Sqrt[1
+ a^2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/a
 + (I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*PolyLog[2
, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/Sqrt[c + a^2*c*x^2]))
```

## Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5413
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Simp[x*(d + e*x^2)
^q*((a + b*ArcTan[c*x])/(2*q + 1)), x] + Simp[2*d*(q/(2*q + 1))   Int[(d +
e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0]
```

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

## Maple [A] (verified)

Time = 2.34 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.75

| method | result |
|---|---|
| default | $\frac{c^2\sqrt{c(ax-i)(ax+i)}\,(6x^4\arctan(ax)a^4-2a^3x^3+27x^2a^2\arctan(ax)-23ax-24\arctan(ax))}{24x} - \frac{\sqrt{c(ax-i)(ax+i)}\,\Big(15\arctan(ax)\ln$ |

input
```
int((a^2*c*x^2+c)^(5/2)*arctan(a*x)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/24*c^2*(c*(a*x-I)*(a*x+I))^(1/2)*(6*x^4*arctan(a*x)*a^4-2*a^3*x^3+27*x^2
*a^2*arctan(a*x)-23*a*x-24*arctan(a*x))/x-1/8*(c*(a*x-I)*(a*x+I))^(1/2)/(a
^2*x^2+1)^(1/2)*(15*arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-15*arc
tan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-8*ln((1+I*a*x)/(a^2*x^2+1)^(1
/2)-1)+8*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-15*I*dilog(1+I*(1+I*a*x)/(a^2*x
^2+1)^(1/2))+15*I*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))*a*c^2
```

## Fricas [F]

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)}{x^2}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)}{x^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)/x^2,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*arctan(a*
x)/x^2, x)
```

## Sympy [F]

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)}{x^2}\,dx = \int \frac{(c(a^2x^2+1))^{\frac{5}{2}}\operatorname{atan}(ax)}{x^2}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)/x**2,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)/x**2, x)
```

## Maxima [F]

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)}{x^2}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)}{x^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)/x^2,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)*arctan(a*x)/x^2, x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)/x^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)}{x^2}\,dx = \int \frac{\operatorname{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^{5/2}}{x^2}\,dx$$

input
```
int((atan(a*x)*(c + a^2*c*x^2)^(5/2))/x^2,x)
```

output
```
int((atan(a*x)*(c + a^2*c*x^2)^(5/2))/x^2, x)
```

## Reduce [F]

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)}{x^2}\,dx = \sqrt{c}\,c^2\left(\int \frac{\sqrt{a^2x^2+1}\,atan(ax)}{x^2}dx + \left(\int \sqrt{a^2x^2+1}\,atan(ax)\,x^2dx\right)a^4 + 2\left(\int \sqrt{a^2x^2+1}\,atan(ax)\,dx\right)a^2\right)$$

input
```
int((a^2*c*x^2+c)^(5/2)*atan(a*x)/x^2,x)
```

output
```
sqrt(c)*c**2*(int((sqrt(a**2*x**2 + 1)*atan(a*x))/x**2,x) + int(sqrt(a**2*
x**2 + 1)*atan(a*x)*x**2,x)*a**4 + 2*int(sqrt(a**2*x**2 + 1)*atan(a*x),x)*
a**2)
```

# 3.222 $\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)}{x^3}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 364

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)}{x^3}\,dx = -\frac{ac^2\sqrt{c+a^2cx^2}}{2x} - \frac{1}{6}a^3c^2x\sqrt{c+a^2cx^2} + 2a^2c^2\sqrt{c+a^2cx^2}\arctan(ax) - \frac{c^2\sqrt{c+a^2cx^2}\arctan(ax)}{2x^2} + \frac{1}{3}a^2c(c+a^2cx^2)^{3/2}\arctan(ax) - \frac{5a^2c^3\sqrt{1+a^2x^2}\arctan(ax)\operatorname{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} - \frac{13}{6}a^2c^{5/2}\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right) + \frac{5ia^2c^3\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2\sqrt{c+a^2cx^2}} - \frac{5ia^2c^3\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(\right.}{2\sqrt{c+a^2cx^2}}$$

output
```
-1/2*a*c^2*(a^2*c*x^2+c)^(1/2)/x-1/6*a^3*c^2*x*(a^2*c*x^2+c)^(1/2)+2*a^2*c
^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)-1/2*c^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)
/x^2+1/3*a^2*c*(a^2*c*x^2+c)^(3/2)*arctan(a*x)-5*a^2*c^3*(a^2*x^2+1)^(1/2)
*arctan(a*x)*arctanh((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/(a^2*c*x^2+c)^(1/2)-
13/6*a^2*c^(5/2)*arctanh(a*c^(1/2)*x/(a^2*c*x^2+c)^(1/2))+5/2*I*a^2*c^3*(a
^2*x^2+1)^(1/2)*polylog(2,-(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/(a^2*c*x^2+c)^
(1/2)-5/2*I*a^2*c^3*(a^2*x^2+1)^(1/2)*polylog(2,(1+I*a*x)^(1/2)/(1-I*a*x)^
(1/2))/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (warning: unable to verify)

Time = 1.51 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.03

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)}{x^3}\,dx = \frac{a^2c^2\sqrt{c+a^2cx^2}\left(-6-6\cot^2\left(\frac{1}{2}\arctan(ax)\right)-2a^2x^2\csc^2\left(\frac{1}{2}\arctan(ax)\right)\right.}{}$$

input
```
Integrate[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x])/x^3,x]
```

output
```
(a^2*c^2*Sqrt[c + a^2*c*x^2]*(-6 - 6*Cot[ArcTan[a*x]/2]^2 - 2*a^2*x^2*Csc[
ArcTan[a*x]/2]^2 + 28*a*x*ArcTan[a*x]*Csc[ArcTan[a*x]/2]^2 + 4*a^3*x^3*Arc
Tan[a*x]*Csc[ArcTan[a*x]/2]^2 - 3*ArcTan[a*x]*Cot[ArcTan[a*x]/2]*Csc[ArcTa
n[a*x]/2]^2 + 60*ArcTan[a*x]*Cot[ArcTan[a*x]/2]*Log[1 - E^(I*ArcTan[a*x])]
 - 60*ArcTan[a*x]*Cot[ArcTan[a*x]/2]*Log[1 + E^(I*ArcTan[a*x])] + 4*Cot[Ar
cTan[a*x]/2]*Log[-(a*x) + Sqrt[1 + a^2*x^2]] + 48*Cot[ArcTan[a*x]/2]*Log[C
os[ArcTan[a*x]/2] - Sin[ArcTan[a*x]/2]] - 48*Cot[ArcTan[a*x]/2]*Log[Cos[Ar
cTan[a*x]/2] + Sin[ArcTan[a*x]/2]] + (60*I)*Cot[ArcTan[a*x]/2]*PolyLog[2,
-E^(I*ArcTan[a*x])] - (60*I)*Cot[ArcTan[a*x]/2]*PolyLog[2, E^(I*ArcTan[a*x
])] + 3*ArcTan[a*x]*Csc[ArcTan[a*x]/2]*Sec[ArcTan[a*x]/2])*Tan[ArcTan[a*x]
/2])/(24*Sqrt[1 + a^2*x^2])
```

## Rubi [A] (verified)

Time = 3.05 (sec) , antiderivative size = 696, normalized size of antiderivative = 1.91, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {5485, 5485, 5465, 211, 224, 219, 5481, 224, 219, 242, 5493, 5489, 5497, 242, 5493, 5489}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{x^3}\,dx$$

$$\Big\downarrow \; 5485$$

$$a^2c\int \frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}{x}dx+c\int \frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}{x^3}dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\left(a^2c\int x\sqrt{a^2cx^2+c}\arctan(ax)dx+c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x}dx\right)+$$
$$c\left(a^2c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x}dx+c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^3}dx\right)$$

$$\Big\downarrow \text{5465}$$

$$a^2c\left(a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\int\sqrt{a^2cx^2+c}dx}{3a}\right)+c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x}dx\right)+$$
$$c\left(a^2c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x}dx+c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^3}dx\right)$$

$$\Big\downarrow \text{211}$$

$$a^2c\left(a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{1}{2}c\int\frac{1}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\sqrt{a^2cx^2+c}}{3a}\right)+c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x}dx\right)+$$
$$c\left(a^2c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x}dx+c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^3}dx\right)$$

$$\Big\downarrow \text{224}$$

$$a^2c\left(a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{1}{2}c\int\frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}+\frac{1}{2}x\sqrt{a^2cx^2+c}}{3a}\right)+c\int\frac{\sqrt{a^2cx^2+c}\arctan(a}{x}\right.$$
$$c\left(a^2c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x}dx+c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^3}dx\right)$$

$$\Big\downarrow \text{219}$$

$$a^2c\left(c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x}dx+a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}}{3a}\right.\right.$$
$$c\left(a^2c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x}dx+c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^3}dx\right)$$

$$\Big\downarrow \text{5481}$$

$$a^2c\left(c\left(c\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx-ac\int \frac{1}{\sqrt{a^2cx^2+c}}dx+\arctan(ax)\sqrt{a^2cx^2+c}\right)+a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/}}{3a^2c}\right.\right.$$

$$c\left(a^2c\left(c\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx-ac\int \frac{1}{\sqrt{a^2cx^2+c}}dx+\arctan(ax)\sqrt{a^2cx^2+c}\right)+c\left(-c\int \frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx+ac\right.\right.$$

$\downarrow$ 224

$$a^2c\left(c\left(c\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx-ac\int \frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}+\arctan(ax)\sqrt{a^2cx^2+c}\right)+a^2c\left(\frac{\arctan(ax)\left(a^2\right.}{3a^2}\right.\right.$$

$$c\left(a^2c\left(c\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx-ac\int \frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}+\arctan(ax)\sqrt{a^2cx^2+c}\right)+c\left(-c\int \frac{\arctan(ax}{x^3\sqrt{a^2cx^2}}\right.\right.$$

$\downarrow$ 219

$$a^2c\left(c\left(c\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx+\arctan(ax)\sqrt{a^2cx^2+c}-\sqrt{c}\,\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)\right)+a^2c\left(\frac{\arctan(ax)\left(a^2cx^2\right.}{3a^2c}\right.\right.$$

$$c\left(a^2c\left(c\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx+\arctan(ax)\sqrt{a^2cx^2+c}-\sqrt{c}\,\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)\right)+c\left(-c\int \frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}d\right.\right.$$

$\downarrow$ 242

$$a^2c\left(c\left(c\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx+\arctan(ax)\sqrt{a^2cx^2+c}-\sqrt{c}\,\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)\right)+a^2c\left(\frac{\arctan(ax)\left(a^2cx^2\right.}{3a^2c}\right.\right.$$

$$c\left(a^2c\left(c\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx+\arctan(ax)\sqrt{a^2cx^2+c}-\sqrt{c}\,\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)\right)+c\left(-c\int \frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}d\right.\right.$$

$\downarrow$ 5493

$$a^2c\left(c\left(\frac{c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+\arctan(ax)\sqrt{a^2cx^2+c}-\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)\right)+a^2c\left(\frac{\arctan(ax)\ (}{3}\right.\right.$$

$$c\left(a^2c\left(\frac{c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+\arctan(ax)\sqrt{a^2cx^2+c}-\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)\right)+c\left(-c\int \frac{\arctan(}{x^3\sqrt{a^2cx}}\right.\right.$$

$$\Big\downarrow 5489$$

$$c\left(c\left(-c\int \frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}-\frac{a\sqrt{a^2cx^2+c}}{x}\right)+a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arc}\right.}{}\right.\right.$$

$$a^2c\left(a^2c\left(\frac{\arctan(ax)\ (a^2cx^2+c)^{3/2}}{3a^2c}-\frac{\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}}{3a}\right)+c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\arctan(a\right.}{}\right.\right.$$

$$\Big\downarrow 5497$$

$$c\left(c\left(-c\left(-\frac{1}{2}a^2\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx+\frac{1}{2}a\int \frac{1}{x^2\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{2cx^2}\right)-\frac{\arctan(ax)\sqrt{a^2cx^2-}}{x^2}\right.\right.$$

$$a^2c\left(a^2c\left(\frac{\arctan(ax)\ (a^2cx^2+c)^{3/2}}{3a^2c}-\frac{\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}}{3a}\right)+c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\arctan(a\right.}{}\right.\right.$$

$$\Big\downarrow 242$$

$$c\left(c\left(-c\left(-\frac{1}{2}a^2\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{2cx^2}-\frac{a\sqrt{a^2cx^2+c}}{2cx}\right)-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}-\frac{a\sqrt{}}{}\right.\right.$$

$$a^2c\left(a^2c\left(\frac{\arctan(ax)\ (a^2cx^2+c)^{3/2}}{3a^2c}-\frac{\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}}{3a}\right)+c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\arctan(a\right.}{}\right.\right.$$

$$\Big\downarrow 5493$$

$$c\left(c\left(-c\left(-\frac{a^2\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{x\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{2cx^2}-\frac{a\sqrt{a^2cx^2+c}}{2cx}\right)-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}-\right.\right.$$
$$a^2c\left(a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}}{3a}\right)+c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\arctan(a\right.}{}\right.\right.$$

$$\Big\downarrow \text{5489}$$

$$a^2c\left(a^2c\left(\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}}{3a}\right)+c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\arctan(a\right.}{}\right.\right.$$
$$c\left(a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{PolyLog}\left(2,\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)\right)}{\sqrt{a^2cx^2+c}}+\text{ar}\right.\right.$$

input
```
Int[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x])/x^3,x]
```

output
```
a^2*c*(a^2*c*(((c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/(3*a^2*c) - ((x*Sqrt[c +
 a^2*c*x^2])/2 + (Sqrt[c]*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/(2*a
))/(3*a)) + c*(Sqrt[c + a^2*c*x^2]*ArcTan[a*x] - Sqrt[c]*ArcTanh[(a*Sqrt[c
]*x)/Sqrt[c + a^2*c*x^2]] + (c*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]*ArcTanh[S
qrt[1 + I*a*x]/Sqrt[1 - I*a*x]] + I*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 -
I*a*x])] - I*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]]))/Sqrt[c + a^2*c*
x^2])) + c*(c*(-((a*Sqrt[c + a^2*c*x^2])/x) - (Sqrt[c + a^2*c*x^2]*ArcTan[
a*x])/x^2 - c*(-1/2*(a*Sqrt[c + a^2*c*x^2])/(c*x) - (Sqrt[c + a^2*c*x^2]*A
rcTan[a*x])/(2*c*x^2) - (a^2*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]*ArcTanh[Sqr
t[1 + I*a*x]/Sqrt[1 - I*a*x]] + I*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*
a*x])] - I*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]]))/(2*Sqrt[c + a^2*c
*x^2]))) + a^2*c*(Sqrt[c + a^2*c*x^2]*ArcTan[a*x] - Sqrt[c]*ArcTanh[(a*Sqr
t[c]*x)/Sqrt[c + a^2*c*x^2]] + (c*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]*ArcTan
h[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]] + I*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1
 - I*a*x])] - I*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]]))/Sqrt[c + a^2
*c*x^2]))
```

## Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5481
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTan[c*x
])/(f*(m + 2))), x] + (Simp[d/(m + 2)   Int[(f*x)^m*((a + b*ArcTan[c*x])/Sq
rt[d + e*x^2]), x], x] - Simp[b*c*(d/(f*(m + 2)))   Int[(f*x)^(m + 1)/Sqrt[
d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] &&
NeQ[m, -2]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5489
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sq
rt[1 - I*c*x]], x] + (Simp[I*(b/Sqrt[d])*PolyLog[2, -Sqrt[1 + I*c*x]/Sqrt[1
 - I*c*x]], x] - Simp[I*(b/Sqrt[d])*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x]], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5497
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)^(m
 + 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[c^2*((m
+ 2)/(f^2*(m + 1)))   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x
^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
 && LtQ[m, -1] && NeQ[m, -2]
```

## Maple [A] (verified)

Time = 3.09 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.70

| method | result |
|---|---|
| default | $-\frac{\sqrt{c(ax-i)(ax+i)}\left(-2\arctan(ax)\sqrt{a^2x^2+1}\,a^4x^4+\sqrt{a^2x^2+1}\,a^3x^3+15\arctan(ax)\ln\left(1+\frac{iax+1}{\sqrt{a^2x^2+1}}\right)a^2x^2-14\arctan(ax)\sqrt{a^2x^2}\right.}{}$ |

input
```
int((a^2*c*x^2+c)^(5/2)*arctan(a*x)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/6/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(a*x+I))^(1/2)*(-2*arctan(a*x)*(a^2*x^2+
1)^(1/2)*a^4*x^4+(a^2*x^2+1)^(1/2)*a^3*x^3+15*arctan(a*x)*ln(1+(1+I*a*x)/(
a^2*x^2+1)^(1/2))*a^2*x^2-14*arctan(a*x)*(a^2*x^2+1)^(1/2)*a^2*x^2-15*I*di
log((1+I*a*x)/(a^2*x^2+1)^(1/2))*a^2*x^2-26*I*arctan((1+I*a*x)/(a^2*x^2+1)
^(1/2))*a^2*x^2-15*I*dilog(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))*a^2*x^2+3*(a^2*x
^2+1)^(1/2)*a*x+3*arctan(a*x)*(a^2*x^2+1)^(1/2))*c^2/x^2
```

## Fricas [F]

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)}{x^3}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)}{x^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)/x^3,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*arctan(a*
x)/x^3, x)
```

## Sympy [F]

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)}{x^3}\,dx = \int \frac{(c(a^2x^2+1))^{\frac{5}{2}}\operatorname{atan}(ax)}{x^3}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)/x**3,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)/x**3, x)
```

## Maxima [F]

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)}{x^3}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)}{x^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)/x^3,x, algorithm="maxima")
```

output
```
1/3*(a^4*c^2*x^2 + a^2*c^2)*sqrt(a^2*x^2 + 1)*sqrt(c)*arctan(a*x) - 1/6*(a
^4*x^4 + 10*a^2*x^2 + 9)^(1/4)*(a^3*c^2*x*cos(1/2*arctan2(4*a*x, -a^2*x^2
+ 3)) + 2*a^2*c^2*sin(1/2*arctan2(4*a*x, -a^2*x^2 + 3)))*sqrt(c) + 1/12*(a
^2*c^2*arctan2((a^4*x^4 + 10*a^2*x^2 + 9)^(1/4)*sin(1/2*arctan2(4*a*x, a^2
*x^2 - 3)) + 2, a*x + (a^4*x^4 + 10*a^2*x^2 + 9)^(1/4)*cos(1/2*arctan2(4*a
*x, a^2*x^2 - 3))) + a^2*c^2*arctan2((a^4*x^4 + 10*a^2*x^2 + 9)^(1/4)*sin(
1/2*arctan2(4*a*x, a^2*x^2 - 3)) - 2, -a*x + (a^4*x^4 + 10*a^2*x^2 + 9)^(1
/4)*cos(1/2*arctan2(4*a*x, a^2*x^2 - 3))) + 24*a^2*c^2*integrate(sqrt(a^2*
x^2 + 1)*arctan(a*x)/x, x) + 12*c^2*integrate(sqrt(a^2*x^2 + 1)*arctan(a*x
)/x^3, x))*sqrt(c)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)/x^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)}{x^3}\,dx = \int \frac{\mathrm{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^{5/2}}{x^3}\,dx$$

input
```
int((atan(a*x)*(c + a^2*c*x^2)^(5/2))/x^3,x)
```

output
```
int((atan(a*x)*(c + a^2*c*x^2)^(5/2))/x^3, x)
```

## Reduce [F]

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)}{x^3}\,dx = \frac{\sqrt{c}\,c^2\Big(2\sqrt{a^2x^2+1}\,atan(ax)\,a^4x^2+2\sqrt{a^2x^2+1}\,atan(ax)\,a^2-\sqrt{a^2x^2+}$$

input
```
int((a^2*c*x^2+c)^(5/2)*atan(a*x)/x^3,x)
```

output
```
(sqrt(c)*c**2*(2*sqrt(a**2*x**2 + 1)*atan(a*x)*a**4*x**2 + 2*sqrt(a**2*x**
2 + 1)*atan(a*x)*a**2 - sqrt(a**2*x**2 + 1)*a**3*x + 6*int((sqrt(a**2*x**2
 + 1)*atan(a*x))/x**3,x) + 12*int((sqrt(a**2*x**2 + 1)*atan(a*x))/x,x)*a**
2 - log(sqrt(a**2*x**2 + 1) + a*x)*a**2))/6
```

# 3.223 $\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)}{x^4}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 372

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)}{x^4}\,dx = -\frac{1}{2}a^3c^2\sqrt{c+a^2cx^2} - \frac{ac^2\sqrt{c+a^2cx^2}}{6x^2} - \frac{2a^2c^2\sqrt{c+a^2cx^2}\arctan(ax)}{x} + \frac{1}{2}a^4c^2x\sqrt{c+a^2cx^2}\arctan(ax) - \frac{c(c+a^2cx^2)^{3/2}\arctan(ax)}{3x^3} - \frac{5ia^3c^3\sqrt{1+a^2x^2}\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} - \frac{13}{6}a^3c^{5/2}\text{arctanh}\left(\frac{\sqrt{c+a^2cx^2}}{\sqrt{c}}\right) + \frac{5ia^3c^3\sqrt{1+a^2x^2}\,\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2\sqrt{c+a^2cx^2}} - \frac{5ia^3c^3\sqrt{1+a^2x^2}\,\text{PolyLog}\left(\right.}{2\sqrt{c+a^2cx^2}}$$

output
```
-1/2*a^3*c^2*(a^2*c*x^2+c)^(1/2)-1/6*a*c^2*(a^2*c*x^2+c)^(1/2)/x^2-2*a^2*c
^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)/x+1/2*a^4*c^2*x*(a^2*c*x^2+c)^(1/2)*arc
tan(a*x)-1/3*c*(a^2*c*x^2+c)^(3/2)*arctan(a*x)/x^3-5*I*a^3*c^3*(a^2*x^2+1)
^(1/2)*arctan(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/(a^2*c*x^2+c)^(
1/2)-13/6*a^3*c^(5/2)*arctanh((a^2*c*x^2+c)^(1/2)/c^(1/2))+5/2*I*a^3*c^3*(
a^2*x^2+1)^(1/2)*polylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/(a^2*c*x^2+
c)^(1/2)-5/2*I*a^3*c^3*(a^2*x^2+1)^(1/2)*polylog(2,I*(1+I*a*x)^(1/2)/(1-I*
a*x)^(1/2))/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 313, normalized size of antiderivative = 0.84

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)}{x^4}\,dx = \frac{c^2\sqrt{c+a^2cx^2}\left(-ax\sqrt{1+a^2x^2}-3a^3x^3\sqrt{1+a^2x^2}-2\sqrt{1+a^2x^2}\arctan(\right.}{}$$

input
```
Integrate[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x])/x^4,x]
```

output
```
(c^2*Sqrt[c + a^2*c*x^2]*(-(a*x*Sqrt[1 + a^2*x^2]) - 3*a^3*x^3*Sqrt[1 + a^
2*x^2] - 2*Sqrt[1 + a^2*x^2]*ArcTan[a*x] - 14*a^2*x^2*Sqrt[1 + a^2*x^2]*Ar
cTan[a*x] + 3*a^4*x^4*Sqrt[1 + a^2*x^2]*ArcTan[a*x] - a^3*x^3*ArcTanh[Sqrt
[1 + a^2*x^2]] + 15*a^3*x^3*ArcTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])] - 15*
a^3*x^3*ArcTan[a*x]*Log[1 + I*E^(I*ArcTan[a*x])] - 12*a^3*x^3*Log[Cos[ArcT
an[a*x]/2]] + 12*a^3*x^3*Log[Sin[ArcTan[a*x]/2]] + (15*I)*a^3*x^3*PolyLog[
2, (-I)*E^(I*ArcTan[a*x])] - (15*I)*a^3*x^3*PolyLog[2, I*E^(I*ArcTan[a*x])
]))/(6*x^3*Sqrt[1 + a^2*x^2])
```

## Rubi [A] (verified)

Time = 3.30 (sec) , antiderivative size = 704, normalized size of antiderivative = 1.89, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.773$, Rules used = {5485, 5485, 5413, 5425, 5421, 5479, 243, 51, 73, 221, 5485, 5425, 5421, 5479, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{x^4}\,dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\int \frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}{x^2}dx+c\int \frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}{x^4}dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\left(a^2c\int \sqrt{a^2cx^2+c}\arctan(ax)dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^2}dx\right)+$$
$$c\left(a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^2}dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^4}dx\right)$$

$\downarrow$ 5413

$$a^2c\left(a^2c\left(\frac{1}{2}c\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^2}dx\right)$$
$$c\left(a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^2}dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^4}dx\right)$$

$\downarrow$ 5425

$$a^2c\left(a^2c\left(\frac{c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax}{x^2}\right.$$
$$c\left(a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^2}dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^4}dx\right)$$

$\downarrow$ 5421

$$c\left(a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^2}dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^4}dx\right)+$$
$$a^2c\left(c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^2}dx+a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{Po}}{}\right.}{2\sqrt{a^2cx^2+c}}\right.\right.$$

$\downarrow$ 5479

$$c\left(a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^2}dx+c\left(\frac{1}{3}a\int \frac{\sqrt{a^2cx^2+c}}{x^3}dx-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)\right)+$$
$$a^2c\left(c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^2}dx+a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{Po}}{}\right.}{2\sqrt{a^2cx^2+c}}\right.\right.$$

$\downarrow$ 243

$$c\left(a^2c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^2}dx+c\left(\frac{1}{6}a\int\frac{\sqrt{a^2cx^2+c}}{x^4}dx^2-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)\right)+$$
$$a^2c\left(c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^2}dx+a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{Po}}{}\right.}{2\sqrt{a^2cx^2+c}}\right.\right.$$

$\downarrow$ 51

$$c\left(a^2c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^2}dx+c\left(\frac{1}{6}a\left(\frac{1}{2}a^2c\int\frac{1}{x^2\sqrt{a^2cx^2+c}}dx^2-\frac{\sqrt{a^2cx^2+c}}{x^2}\right)-\frac{\arctan(ax)\left(a^2cx^2+\right.}{3cx^3}\right.\right.$$
$$a^2c\left(c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^2}dx+a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{Po}}{}\right.}{2\sqrt{a^2cx^2+c}}\right.\right.$$

$\downarrow$ 73

$$c\left(a^2c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^2}dx+c\left(\frac{1}{6}a\left(\int\frac{1}{\frac{x^4}{a^2c}-\frac{1}{a^2}}d\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{x^2}\right)-\frac{\arctan(ax)\left(a^2cx^2+c\right.}{3cx^3}\right.\right.$$
$$a^2c\left(c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^2}dx+a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{Po}}{}\right.}{2\sqrt{a^2cx^2+c}}\right.\right.$$

$\downarrow$ 221

$$c\left(a^2c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^2}dx+c\left(\frac{1}{6}a\left(a^2\left(-\sqrt{c}\right)\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)-\frac{\sqrt{a^2cx^2+c}}{x^2}\right)-\frac{\arctan(ax)\left(a\right.}{3c}\right.\right.$$
$$a^2c\left(c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^2}dx+a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{Po}}{}\right.}{2\sqrt{a^2cx^2+c}}\right.\right.$$

$\downarrow$ 5485

$$c\left(a^2c\left(a^2c\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx\right)+c\left(\frac{1}{6}a\left(a^2\left(-\sqrt{c}\right)\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)-\frac{\sqrt{a^2cx^2+}}{x^2}\right.\right.\right.$$

$$a^2c\left(c\left(a^2c\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx\right)+a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyL}}{}\right.}{2\sqrt{a^2cx^2+c}}\right.\right.$$

$\downarrow$ 5425

$$c\left(a^2c\left(\frac{a^2c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx\right)+c\left(\frac{1}{6}a\left(a^2\left(-\sqrt{c}\right)\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)-\frac{\sqrt{}}{}\right.\right.\right.$$

$$a^2c\left(c\left(\frac{a^2c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx\right)+a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}\right.}{2\sqrt{a}}\right.\right.$$

$\downarrow$ 5421

$$c\left(c\left(\frac{1}{6}a\left(a^2\left(-\sqrt{c}\right)\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)-\frac{\sqrt{a^2cx^2+c}}{x^2}\right)-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)+a^2c\left(c\int \frac{\text{arctan}}{x^2\sqrt{a^2c}}\right.\right.$$

$$a^2c\left(c\left(c\int \frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx+\frac{a^2c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i}{}\right.}{a}\right.}{\sqrt{a^2cx^2+c}}\right.\right.\right.$$

$\downarrow$ 5479

$$c\left(c\left(\frac{1}{6}a\left(a^2\left(-\sqrt{c}\right)\operatorname{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)-\frac{\sqrt{a^2cx^2+c}}{x^2}\right)-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)+a^2c\left(c\left(a\int\frac{}{x\sqrt{a}}\right.\right.\right.$$

$$a^2c\left(c\left(c\left(a\int\frac{1}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right)+\frac{a^2c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{Poly}}{}\right.}{\sqrt{a^2cx^2+c}}\right.\right.$$

$\downarrow$ 243

$$c\left(c\left(\frac{1}{6}a\left(a^2\left(-\sqrt{c}\right)\operatorname{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)-\frac{\sqrt{a^2cx^2+c}}{x^2}\right)-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)+a^2c\left(c\left(\frac{1}{2}a\int\frac{}{x^2}\right.\right.\right.$$

$$a^2c\left(c\left(c\left(\frac{1}{2}a\int\frac{1}{x^2\sqrt{a^2cx^2+c}}dx^2-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right)+\frac{a^2c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i}{}\right.}{\sqrt{a^2cx^2}}\right.\right.$$

$\downarrow$ 73

$$c\left(c\left(\frac{1}{6}a\left(a^2\left(-\sqrt{c}\right)\operatorname{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)-\frac{\sqrt{a^2cx^2+c}}{x^2}\right)-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)+a^2c\left(c\left(\frac{\int\frac{1}{\frac{x^4}{a^2c}-\frac{1}{a^2}}}{}\right.\right.\right.$$

$$a^2c\left(c\left(c\left(\frac{\int\frac{1}{\frac{x^4}{a^2c}-\frac{1}{a^2}}d\sqrt{a^2cx^2+c}}{ac}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right)+\frac{a^2c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{P}}{}\right.}{\sqrt{a^2cx^2}}\right.\right.$$

$\downarrow$ 221

$$a^2c\left(c\left(c\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)+\frac{a^2c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i}{\ldots}\right.}{\sqrt{a^2cx^2\ldots}}\right.\right.$$

$$c\left(c\left(\frac{1}{6}a\left(a^2(-\sqrt{c})\,\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)-\frac{\sqrt{a^2cx^2+c}}{x^2}\right)-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)+a^2c\left(c\left(-\frac{\arctan\ldots}{}\right.\right.\right.$$

input
```
Int[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x])/x^4,x]
```

output
```
a^2*c*(a^2*c*(-1/2*Sqrt[c + a^2*c*x^2]/a + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a
*x])/2 + (c*Sqrt[1 + a^2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/
Sqrt[1 - I*a*x]])/a + (I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]
])/a - (I*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/(2*Sqrt[c +
 a^2*c*x^2])) + c*(c*(-((Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(c*x)) - (a*ArcT
anh[Sqrt[c + a^2*c*x^2]/Sqrt[c]])/Sqrt[c]) + (a^2*c*Sqrt[1 + a^2*x^2]*(((-
2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/a + (I*PolyLog[2
, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*PolyLog[2, (I*Sqrt[1 + I
*a*x])/Sqrt[1 - I*a*x]])/a))/Sqrt[c + a^2*c*x^2])) + c*(c*(-1/3*((c + a^2*
c*x^2)^(3/2)*ArcTan[a*x])/(c*x^3) + (a*(-(Sqrt[c + a^2*c*x^2]/x^2) - a^2*S
qrt[c]*ArcTanh[Sqrt[c + a^2*c*x^2]/Sqrt[c]]))/6) + a^2*c*(c*(-((Sqrt[c + a
^2*c*x^2]*ArcTan[a*x])/(c*x)) - (a*ArcTanh[Sqrt[c + a^2*c*x^2]/Sqrt[c]])/S
qrt[c]) + (a^2*c*Sqrt[1 + a^2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*
a*x]/Sqrt[1 - I*a*x]])/a + (I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I
*a*x]])/a - (I*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/Sqrt[c
 + a^2*c*x^2]))
```

## Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5413
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Simp[x*(d + e*x^2)
^q*((a + b*ArcTan[c*x])/(2*q + 1)), x] + Simp[2*d*(q/(2*q + 1))   Int[(d +
e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0]
```

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

## Maple [A] (verified)

Time = 4.64 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.73

| method | result |
|---|---|
| default | $\frac{c^2\sqrt{c(ax-i)(ax+i)}\,(3x^4\arctan(ax)a^4-3a^3x^3-14x^2a^2\arctan(ax)-ax-2\arctan(ax))}{6x^3}+\frac{ic^2a^3\sqrt{c(ax-i)(ax+i)}\left(15i\arctan(ax\right.}{}$ |

input
```
int((a^2*c*x^2+c)^(5/2)*arctan(a*x)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/6*c^2*(c*(a*x-I)*(a*x+I))^(1/2)*(3*x^4*arctan(a*x)*a^4-3*a^3*x^3-14*x^2*
a^2*arctan(a*x)-a*x-2*arctan(a*x))/x^3+1/6*I*c^2*a^3*(c*(a*x-I)*(a*x+I))^(
1/2)*(15*I*arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-15*I*arctan(a*x
)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-13*I*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-
1)+13*I*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+15*dilog(1+I*(1+I*a*x)/(a^2*x^2+
1)^(1/2))-15*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)
```

## Fricas [F]

$$\int \frac{(c + a^2cx^2)^{5/2} \arctan(ax)}{x^4} \, dx = \int \frac{(a^2cx^2 + c)^{\frac{5}{2}} \arctan(ax)}{x^4} \, dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)/x^4,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*arctan(a*
x)/x^4, x)
```

## Sympy [F]

$$\int \frac{(c + a^2cx^2)^{5/2} \arctan(ax)}{x^4} \, dx = \int \frac{(c(a^2x^2 + 1))^{\frac{5}{2}} \operatorname{atan}(ax)}{x^4} \, dx$$

input
```
integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)/x**4,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)/x**4, x)
```

## Maxima [F]

$$\int \frac{(c + a^2cx^2)^{5/2} \arctan(ax)}{x^4} \, dx = \int \frac{(a^2cx^2 + c)^{\frac{5}{2}} \arctan(ax)}{x^4} \, dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)/x^4,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)*arctan(a*x)/x^4, x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)}{x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)/x^4,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)}{x^4}\,dx = \int \frac{\mathrm{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^{5/2}}{x^4}\,dx$$

input
```
int((atan(a*x)*(c + a^2*c*x^2)^(5/2))/x^4,x)
```

output
```
int((atan(a*x)*(c + a^2*c*x^2)^(5/2))/x^4, x)
```

## Reduce [F]

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)}{x^4}\,dx = \frac{\sqrt{c}\,c^2\left(4atan\left(\sqrt{a^2x^2+1}+ax\right)a^3x^3-2\sqrt{a^2x^2+1}\,atan(ax)\,a^2x^2-2\sqrt{c}\right.}{}$$

input
```
int((a^2*c*x^2+c)^(5/2)*atan(a*x)/x^4,x)
```

output
```
(sqrt(c)*c**2*(4*atan(sqrt(a**2*x**2 + 1) + a*x)*a**3*x**3 - 2*sqrt(a**2*x
**2 + 1)*atan(a*x)*a**2*x**2 - 2*sqrt(a**2*x**2 + 1)*atan(a*x) - 2*atan(a*
x)*a**3*x**3 - sqrt(a**2*x**2 + 1)*a*x + 12*int((sqrt(a**2*x**2 + 1)*atan(
a*x))/x**2,x)*a**2*x**3 + 6*int(sqrt(a**2*x**2 + 1)*atan(a*x),x)*a**4*x**3
 + log(sqrt(a**2*x**2 + 1) + a*x - 1)*a**3*x**3 - log(sqrt(a**2*x**2 + 1)
+ a*x + 1)*a**3*x**3))/(6*x**3)
```

# 3.224 $\int \frac{x^3 \arctan(ax)}{\sqrt{c+a^2cx^2}}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 120

$$\int \frac{x^3 \arctan(ax)}{\sqrt{c+a^2cx^2}}\,dx = -\frac{x\sqrt{c+a^2cx^2}}{6a^3c} - \frac{2\sqrt{c+a^2cx^2}\arctan(ax)}{3a^4c} + \frac{x^2\sqrt{c+a^2cx^2}\arctan(ax)}{3a^2c} + \frac{5\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right)}{6a^4\sqrt{c}}$$

output

```
-1/6*x*(a^2*c*x^2+c)^(1/2)/a^3/c-2/3*(a^2*c*x^2+c)^(1/2)*arctan(a*x)/a^4/c
+1/3*x^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)/a^2/c+5/6*arctanh(a*c^(1/2)*x/(a^
2*c*x^2+c)^(1/2))/a^4/c^(1/2)
```

## Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.76

$$\int \frac{x^3 \arctan(ax)}{\sqrt{c+a^2cx^2}}\,dx = \frac{-ax\sqrt{c+a^2cx^2} + 2(-2+a^2x^2)\sqrt{c+a^2cx^2}\arctan(ax) + 5\sqrt{c}\log\left(acx+\sqrt{c}\sqrt{c+a^2cx^2}\right)}{6a^4c}$$

input

```
Integrate[(x^3*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

output
```
(-(a*x*Sqrt[c + a^2*c*x^2]) + 2*(-2 + a^2*x^2)*Sqrt[c + a^2*c*x^2]*ArcTan[
a*x] + 5*Sqrt[c]*Log[a*c*x + Sqrt[c]*Sqrt[c + a^2*c*x^2]])/(6*a^4*c)
```

## Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.38, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {5487, 262, 224, 219, 5465, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \arctan(ax)}{\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5487}$$

$$-\frac{2\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}-\frac{\int \frac{x^2}{\sqrt{a^2cx^2+c}}dx}{3a}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}$$

$$\Big\downarrow \text{262}$$

$$-\frac{2\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int \frac{1}{\sqrt{a^2cx^2+c}}dx}{2a^2}}{3a}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}$$

$$\Big\downarrow \text{224}$$

$$-\frac{2\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int \frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}}{2a^2}}{3a}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}$$

$$\Big\downarrow \text{219}$$

$$-\frac{2\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}$$

$$\Big\downarrow \text{5465}$$

$$-\frac{2\left(\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\int \frac{1}{\sqrt{a^2cx^2+c}}dx}{a}\right)}{3a^2}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}$$

$$\Big\downarrow \text{224}$$

$$-\frac{2\left(\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\int \frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}}{a}\right)}{3a^2}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}$$

$$\Big\downarrow \text{219}$$

$$-\frac{2\left(\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a^2\sqrt{c}}\right)}{3a^2}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}$$

input
```
Int[(x^3*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

output
```
(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(3*a^2*c) - ((x*Sqrt[c + a^2*c*x^2])
/(2*a^2*c) - ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]]/(2*a^3*Sqrt[c]))/(
3*a) - (2*((Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(a^2*c) - ArcTanh[(a*Sqrt[c]*
x)/Sqrt[c + a^2*c*x^2]]/(a^2*Sqrt[c])))/(3*a^2)
```

## Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5487
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcTan[c*x])^p/(c^2*d*m)), x] + (-Simp[b*f*(p/(c*m))   Int[(f*x)^(m - 1)*((
a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[f^2*((m - 1)/(c^
2*m))   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2]), x], x])
/; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.29 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.38

| method | result |
|---|---|
| default | $\frac{(2x^2a^2\arctan(ax)-ax-4\arctan(ax))\sqrt{c(ax-i)(ax+i)}}{6c\,a^4}-\frac{5\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}-i\right)\sqrt{c(ax-i)(ax+i)}}{6\sqrt{a^2x^2+1}\,a^4c}+\frac{5\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}+i\right)\sqrt{c(ax-}}{6\sqrt{a^2x^2+1}\,a^4c}$ |

input
```
int(x^3*arctan(a*x)/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*x^2*a^2*arctan(a*x)-a*x-4*arctan(a*x))*(c*(a*x-I)*(a*x+I))^(1/2)/c/
a^4-5/6*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-I)*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x
^2+1)^(1/2)/a^4/c+5/6*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+I)*(c*(a*x-I)*(a*x+I)
)^(1/2)/(a^2*x^2+1)^(1/2)/a^4/c
```

## Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.67

$$\int \frac{x^3 \arctan(ax)}{\sqrt{c+a^2cx^2}}\,dx = -\frac{2\,\sqrt{a^2cx^2+c}(ax-2\,(a^2x^2-2)\arctan{(ax)})-5\,\sqrt{c}\log{\left(-2\,a^2cx^2-2\,\sqrt{a^2cx^2+c}a\sqrt{c}x-c\right)}}{12\,a^4c}$$

input
```
integrate(x^3*arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
-1/12*(2*sqrt(a^2*c*x^2 + c)*(a*x - 2*(a^2*x^2 - 2)*arctan(a*x)) - 5*sqrt(
c)*log(-2*a^2*c*x^2 - 2*sqrt(a^2*c*x^2 + c)*a*sqrt(c)*x - c))/(a^4*c)
```

## Sympy [F]

$$\int \frac{x^3 \arctan(ax)}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^3 \operatorname{atan}{(ax)}}{\sqrt{c\,(a^2x^2+1)}}\,dx$$

input
```
integrate(x**3*atan(a*x)/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x**3*atan(a*x)/sqrt(c*(a**2*x**2 + 1)), x)
```

## Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.74

$$\int \frac{x^3 \arctan(ax)}{\sqrt{c+a^2cx^2}}\,dx = -\frac{a\left(\frac{\frac{\sqrt{a^2x^2+1}x}{a^2}-\frac{\operatorname{arsinh}(ax)}{a^3}}{a^2}-\frac{4\operatorname{arsinh}(ax)}{a^5}\right)-2\left(\frac{\sqrt{a^2x^2+1}x^2}{a^2}-\frac{2\sqrt{a^2x^2+1}}{a^4}\right)\arctan{(ax)}}{6\sqrt{c}}$$

input
```
integrate(x^3*arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
-1/6*(a*((sqrt(a^2*x^2 + 1)*x/a^2 - arcsinh(a*x)/a^3)/a^2 - 4*arcsinh(a*x)
/a^5) - 2*(sqrt(a^2*x^2 + 1)*x^2/a^2 - 2*sqrt(a^2*x^2 + 1)/a^4)*arctan(a*x
))/sqrt(c)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x^3\arctan(ax)}{\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\arctan(ax)}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^3\,\mathrm{atan}(a\,x)}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int((x^3*atan(a*x))/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int((x^3*atan(a*x))/(c + a^2*c*x^2)^(1/2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.67

$$\int \frac{x^3 \arctan(ax)}{\sqrt{c+a^2cx^2}}\,dx$$
$$= \frac{\sqrt{c}\left(2\sqrt{a^2x^2+1}\,atan(ax)\,a^2x^2-4\sqrt{a^2x^2+1}\,atan(ax)-\sqrt{a^2x^2+1}\,ax+5\log\left(\sqrt{a^2x^2+1}+ax\right)\right)}{6a^4c}$$

input
```
int(x^3*atan(a*x)/(a^2*c*x^2+c)^(1/2),x)
```

output
```
(sqrt(c)*(2*sqrt(a**2*x**2 + 1)*atan(a*x)*a**2*x**2 - 4*sqrt(a**2*x**2 + 1
)*atan(a*x) - sqrt(a**2*x**2 + 1)*a*x + 5*log(sqrt(a**2*x**2 + 1) + a*x)))
/(6*a**4*c)
```

# 3.225 $\int \frac{x^2 \arctan(ax)}{\sqrt{c+a^2cx^2}}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 250

$$\int \frac{x^2 \arctan(ax)}{\sqrt{c+a^2cx^2}}\,dx = -\frac{\sqrt{c+a^2cx^2}}{2a^3c}+\frac{x\sqrt{c+a^2cx^2}\arctan(ax)}{2a^2c}+\frac{i\sqrt{1+a^2x^2}\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a^3\sqrt{c+a^2cx^2}}-\frac{i\sqrt{1+a^2x^2}\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2a^3\sqrt{c+a^2cx^2}}+\frac{i\sqrt{1+a^2x^2}\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2a^3\sqrt{c+a^2cx^2}}$$

output

```
-1/2*(a^2*c*x^2+c)^(1/2)/a^3/c+1/2*x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)/a^2/c
+I*(a^2*x^2+1)^(1/2)*arctan(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/a
^3/(a^2*c*x^2+c)^(1/2)-1/2*I*(a^2*x^2+1)^(1/2)*polylog(2,-I*(1+I*a*x)^(1/2
)/(1-I*a*x)^(1/2))/a^3/(a^2*c*x^2+c)^(1/2)+1/2*I*(a^2*x^2+1)^(1/2)*polylog
(2,I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/a^3/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.63

$$\int \frac{x^2 \arctan(ax)}{\sqrt{c+a^2cx^2}}\,dx = -\frac{\sqrt{c(1+a^2x^2)}\left(\sqrt{1+a^2x^2} - ax\sqrt{1+a^2x^2}\arctan(ax) + \arctan(ax)\log\left(1-ie^{i\arctan(ax)}\right) - \arctan(ax\right.}{2a^3c\sqrt{1+a^2x^2}}$$

input
```
Integrate[(x^2*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

output
```
-1/2*(Sqrt[c*(1 + a^2*x^2)]*(Sqrt[1 + a^2*x^2] - a*x*Sqrt[1 + a^2*x^2]*Arc
Tan[a*x] + ArcTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])] - ArcTan[a*x]*Log[1 +
I*E^(I*ArcTan[a*x])] + I*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - I*PolyLog[2,
 I*E^(I*ArcTan[a*x])]))/(a^3*c*Sqrt[1 + a^2*x^2])
```

## Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.80, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {5487, 241, 5425, 5421}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \arctan(ax)}{\sqrt{a^2cx^2+c}}\,dx$$

$$\Big\downarrow \text{5487}$$

$$-\frac{\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a^2} - \frac{\int \frac{x}{\sqrt{a^2cx^2+c}}dx}{2a} + \frac{x\arctan(ax)\sqrt{a^2cx^2+c}}{2a^2c}$$

$$\Big\downarrow \text{241}$$

$$-\frac{\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a^2} + \frac{x\arctan(ax)\sqrt{a^2cx^2+c}}{2a^2c} - \frac{\sqrt{a^2cx^2+c}}{2a^3c}$$

$$\Big\downarrow \text{5425}$$

$$-\frac{\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{2a^2\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\sqrt{a^2cx^2+c}}{2a^3c}$$

$$\Big\downarrow \text{5421}$$

$$-\frac{\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2a^2\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\sqrt{a^2cx^2+c}}{2a^3c}$$

input
```
Int[(x^2*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

output
```
-1/2*Sqrt[c + a^2*c*x^2]/(a^3*c) + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(2*
a^2*c) - (Sqrt[1 + a^2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sq
rt[1 - I*a*x]])/a + (I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])
/a - (I*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/(2*a^2*Sqrt[c
 + a^2*c*x^2])
```

## Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5487
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcTan[c*x])^p/(c^2*d*m)), x] + (-Simp[b*f*(p/(c*m))   Int[(f*x)^(m - 1)*((
a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[f^2*((m - 1)/(c^
2*m))   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2]), x], x])
/; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

## Maple [A] (verified)

Time = 1.40 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.74

| method | result |
|---|---|
| default | $\frac{(\arctan(ax)ax-1)\sqrt{c(ax-i)(ax+i)}}{2c\,a^3}+\frac{\left(\arctan(ax)\ln\left(1+\frac{i(iax+1)}{\sqrt{a^2x^2+1}}\right)-\arctan(ax)\ln\left(1-\frac{i(iax+1)}{\sqrt{a^2x^2+1}}\right)+i\,\mathrm{dilog}\left(1-\frac{i(iax+1)}{\sqrt{a^2x^2+1}}\right)-i\right.}{2\sqrt{a^2x^2+1}\,a^3c}$ |

input
```
int(x^2*arctan(a*x)/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(arctan(a*x)*a*x-1)*(c*(a*x-I)*(a*x+I))^(1/2)/c/a^3+1/2*(arctan(a*x)*l
n(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1
)^(1/2))+I*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*dilog(1+I*(1+I*a*x)/(a
^2*x^2+1)^(1/2)))*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/2)/a^3/c
```

## Fricas [F]

$$\int \frac{x^2 \arctan(ax)}{\sqrt{c+a^2cx^2}}\,dx=\int \frac{x^2 \arctan{(ax)}}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(x^2*arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(x^2*arctan(a*x)/sqrt(a^2*c*x^2 + c), x)
```

## Sympy [F]

$$\int \frac{x^2 \arctan(ax)}{\sqrt{c + a^2cx^2}}\,dx = \int \frac{x^2 \operatorname{atan}(ax)}{\sqrt{c(a^2x^2 + 1)}}\,dx$$

input
```
integrate(x**2*atan(a*x)/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x**2*atan(a*x)/sqrt(c*(a**2*x**2 + 1)), x)
```

## Maxima [F]

$$\int \frac{x^2 \arctan(ax)}{\sqrt{c + a^2cx^2}}\,dx = \int \frac{x^2 \arctan(ax)}{\sqrt{a^2cx^2 + c}}\,dx$$

input
```
integrate(x^2*arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2*arctan(a*x)/sqrt(a^2*c*x^2 + c), x)
```

## Giac [F]

$$\int \frac{x^2 \arctan(ax)}{\sqrt{c + a^2cx^2}}\,dx = \int \frac{x^2 \arctan(ax)}{\sqrt{a^2cx^2 + c}}\,dx$$

input
```
integrate(x^2*arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2*arctan(a*x)/sqrt(a^2*c*x^2 + c), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \arctan(ax)}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^2 \operatorname{atan}(a\,x)}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int((x^2*atan(a*x))/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int((x^2*atan(a*x))/(c + a^2*c*x^2)^(1/2), x)
```

## Reduce [F]

$$\int \frac{x^2 \arctan(ax)}{\sqrt{c+a^2cx^2}}\,dx = \frac{\int \frac{atan(ax)x^2}{\sqrt{a^2x^2+1}}dx}{\sqrt{c}}$$

input
```
int(x^2*atan(a*x)/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int((atan(a*x)*x**2)/sqrt(a**2*x**2 + 1),x)/sqrt(c)
```

# 3.226 $\int \frac{x \arctan(ax)}{\sqrt{c+a^2cx^2}} \, dx$



## Optimal result

Integrand size = 20, antiderivative size = 59

$$\int \frac{x \arctan(ax)}{\sqrt{c+a^2cx^2}} \, dx = \frac{\sqrt{c+a^2cx^2} \arctan(ax)}{a^2c} - \frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right)}{a^2\sqrt{c}}$$

output
```
(a^2*c*x^2+c)^(1/2)*arctan(a*x)/a^2/c-arctanh(a*c^(1/2)*x/(a^2*c*x^2+c)^(1
/2))/a^2/c^(1/2)
```

## Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.02

$$\int \frac{x \arctan(ax)}{\sqrt{c+a^2cx^2}} \, dx = \frac{\sqrt{c+a^2cx^2} \arctan(ax) - \sqrt{c} \log\left(acx + \sqrt{c}\sqrt{c+a^2cx^2}\right)}{a^2c}$$

input
```
Integrate[(x*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

output
```
(Sqrt[c + a^2*c*x^2]*ArcTan[a*x] - Sqrt[c]*Log[a*c*x + Sqrt[c]*Sqrt[c + a^
2*c*x^2]])/(a^2*c)
```

## Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {5465, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \arctan(ax)}{\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\int \frac{1}{\sqrt{a^2cx^2+c}}dx}{a}$$

$$\Big\downarrow \text{224}$$

$$\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\int \frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}}{a}$$

$$\Big\downarrow \text{219}$$

$$\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a^2\sqrt{c}}$$

input
```
Int[(x*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

output
```
(Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(a^2*c) - ArcTanh[(a*Sqrt[c]*x)/Sqrt[c +
 a^2*c*x^2]]/(a^2*Sqrt[c])
```

### Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.11 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.69

| method | result | size |
|---|---|---|
| default | $\frac{\left(\arctan(ax)\sqrt{a^2x^2+1}+\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}-i\right)-\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}+i\right)\right)\sqrt{c(ax-i)(ax+i)}}{\sqrt{a^2x^2+1}\,a^2c}$ | 100 |

input
```
int(x*arctan(a*x)/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(arctan(a*x)*(a^2*x^2+1)^(1/2)+ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-I)-ln((1+I*a
*x)/(a^2*x^2+1)^(1/2)+I))*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/2)/a^2/
c
```

## Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.08

$$\int \frac{x\arctan(ax)}{\sqrt{c+a^2cx^2}}\,dx = \frac{2\sqrt{a^2cx^2+c}\arctan(ax)+\sqrt{c}\log\left(-2\,a^2cx^2+2\sqrt{a^2cx^2+c}a\sqrt{c}x-c\right)}{2\,a^2c}$$

input
```
integrate(x*arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(2*sqrt(a^2*c*x^2 + c)*arctan(a*x) + sqrt(c)*log(-2*a^2*c*x^2 + 2*sqrt
(a^2*c*x^2 + c)*a*sqrt(c)*x - c))/(a^2*c)
```

## Sympy [F(-2)]

Exception generated.

$$\int \frac{x\arctan(ax)}{\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*atan(a*x)/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real zoo
```

## Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.03

$$\int \frac{x\arctan(ax)}{\sqrt{c+a^2cx^2}}\,dx = \frac{2\sqrt{a^2x^2+1}\arctan(ax) - \log\left(ax+\sqrt{a^2x^2+1}\right) + \log\left(-ax+\sqrt{a^2x^2+1}\right)}{2a^2\sqrt{c}}$$

input
```
integrate(x*arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
1/2*(2*sqrt(a^2*x^2 + 1)*arctan(a*x) - log(a*x + sqrt(a^2*x^2 + 1)) + log(
-a*x + sqrt(a^2*x^2 + 1)))/(a^2*sqrt(c))
```

## Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.02

$$\int \frac{x\arctan(ax)}{\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{a^2cx^2+c}\arctan{(ax)}}{a^2c} + \frac{\log\left(\left|-\sqrt{a^2c}x+\sqrt{a^2cx^2+c}\right|\right)}{a\sqrt{c}|a|}$$

input
```
integrate(x*arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
sqrt(a^2*c*x^2 + c)*arctan(a*x)/(a^2*c) + log(abs(-sqrt(a^2*c)*x + sqrt(a^
2*c*x^2 + c)))/(a*sqrt(c)*abs(a))
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x\arctan(ax)}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x\,\mathrm{atan}(a\,x)}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int((x*atan(a*x))/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int((x*atan(a*x))/(c + a^2*c*x^2)^(1/2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.71

$$\int \frac{x\arctan(ax)}{\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{c}\left(\sqrt{a^2x^2+1}\,atan(ax)-\log\left(\sqrt{a^2x^2+1}+ax\right)\right)}{a^2c}$$

input
```
int(x*atan(a*x)/(a^2*c*x^2+c)^(1/2),x)
```

output
```
(sqrt(c)*(sqrt(a**2*x**2 + 1)*atan(a*x) - log(sqrt(a**2*x**2 + 1) + a*x)))
/(a**2*c)
```

# 3.227 $\int \frac{\arctan(ax)}{\sqrt{c+a^2cx^2}}\,dx$



## Optimal result

Integrand size = 19, antiderivative size = 193

$$\int \frac{\arctan(ax)}{\sqrt{c+a^2cx^2}}\,dx = -\frac{2i\sqrt{1+a^2x^2}\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a\sqrt{c+a^2cx^2}} + \frac{i\sqrt{1+a^2x^2}\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a\sqrt{c+a^2cx^2}} - \frac{i\sqrt{1+a^2x^2}\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a\sqrt{c+a^2cx^2}}$$

output

```
-2*I*(a^2*x^2+1)^(1/2)*arctan(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))
/a/(a^2*c*x^2+c)^(1/2)+I*(a^2*x^2+1)^(1/2)*polylog(2,-I*(1+I*a*x)^(1/2)/(1
-I*a*x)^(1/2))/a/(a^2*c*x^2+c)^(1/2)-I*(a^2*x^2+1)^(1/2)*polylog(2,I*(1+I*
a*x)^(1/2)/(1-I*a*x)^(1/2))/a/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.47

$$\int \frac{\arctan(ax)}{\sqrt{c+a^2cx^2}}\,dx = -\frac{i\sqrt{c(1+a^2x^2)}\left(2\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)-\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)+\operatorname{PolyLog}\left(2,ie^{i\arctan(a}\right.\right.}{ac\sqrt{1+a^2x^2}}$$

input
```
Integrate[ArcTan[a*x]/Sqrt[c + a^2*c*x^2],x]
```

output
```
((-I)*Sqrt[c*(1 + a^2*x^2)]*(2*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x] - Pol
yLog[2, (-I)*E^(I*ArcTan[a*x])] + PolyLog[2, I*E^(I*ArcTan[a*x])]))/(a*c*S
qrt[1 + a^2*x^2])
```

## Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.73, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {5425, 5421}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}\,dx$$

$$\Big\downarrow \text{5425}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5421}$$

$$\frac{\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{\sqrt{a^2cx^2+c}}$$

input
```
Int[ArcTan[a*x]/Sqrt[c + a^2*c*x^2],x]
```

output
```
(Sqrt[1 + a^2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*
a*x]])/a + (I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*P
olyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/Sqrt[c + a^2*c*x^2]
```

### Defintions of rubi rules used

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

## Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.78

| method | result |
|---|---|
| default | $-\frac{\left(\arctan(ax)\ln\left(1+\frac{i(iax+1)}{\sqrt{a^2x^2+1}}\right)-\arctan(ax)\ln\left(1-\frac{i(iax+1)}{\sqrt{a^2x^2+1}}\right)+i\,\mathrm{dilog}\left(1-\frac{i(iax+1)}{\sqrt{a^2x^2+1}}\right)-i\,\mathrm{dilog}\left(1+\frac{i(iax+1)}{\sqrt{a^2x^2+1}}\right)\right)\sqrt{c(ax-i)(ax-}}{\sqrt{a^2x^2+1}\,ac}$ |

input
```
int(arctan(a*x)/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-arctan(a*x)*ln(1-I*(1+I*
a*x)/(a^2*x^2+1)^(1/2))+I*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*dilog(1
+I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/
2)/a/c
```

## Fricas [F]

$$\int \frac{\arctan(ax)}{\sqrt{c + a^2cx^2}} \, dx = \int \frac{\arctan(ax)}{\sqrt{a^2cx^2 + c}} \, dx$$

input
```
integrate(arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(arctan(a*x)/sqrt(a^2*c*x^2 + c), x)
```

## Sympy [F]

$$\int \frac{\arctan(ax)}{\sqrt{c + a^2cx^2}} \, dx = \int \frac{\operatorname{atan}(ax)}{\sqrt{c(a^2x^2 + 1)}} \, dx$$

input
```
integrate(atan(a*x)/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(atan(a*x)/sqrt(c*(a**2*x**2 + 1)), x)
```

## Maxima [F]

$$\int \frac{\arctan(ax)}{\sqrt{c + a^2cx^2}} \, dx = \int \frac{\arctan(ax)}{\sqrt{a^2cx^2 + c}} \, dx$$

input
```
integrate(arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)/sqrt(a^2*c*x^2 + c), x)
```

## Giac [F]

$$\int \frac{\arctan(ax)}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(arctan(a*x)/sqrt(a^2*c*x^2 + c), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\mathrm{atan}(a\,x)}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(atan(a*x)/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(atan(a*x)/(c + a^2*c*x^2)^(1/2), x)
```

## Reduce [F]

$$\int \frac{\arctan(ax)}{\sqrt{c+a^2cx^2}}\,dx = \frac{\int \frac{atan(ax)}{\sqrt{a^2x^2+1}}dx}{\sqrt{c}}$$

input
```
int(atan(a*x)/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(atan(a*x)/sqrt(a**2*x**2 + 1),x)/sqrt(c)
```

# 3.228 $\int \frac{\arctan(ax)}{x\sqrt{c+a^2cx^2}}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 177

$$\int \frac{\arctan(ax)}{x\sqrt{c+a^2cx^2}}\,dx = -\frac{2\sqrt{1+a^2x^2}\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} + \frac{i\sqrt{1+a^2x^2}\,\text{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} - \frac{i\sqrt{1+a^2x^2}\,\text{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}}$$

output
```
-2*(a^2*x^2+1)^(1/2)*arctan(a*x)*arctanh((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/
(a^2*c*x^2+c)^(1/2)+I*(a^2*x^2+1)^(1/2)*polylog(2,-(1+I*a*x)^(1/2)/(1-I*a*
x)^(1/2))/(a^2*c*x^2+c)^(1/2)-I*(a^2*x^2+1)^(1/2)*polylog(2,(1+I*a*x)^(1/2
)/(1-I*a*x)^(1/2))/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.56

$$\int \frac{\arctan(ax)}{x\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{1+a^2x^2}\left(\arctan(ax)\left(\log\left(1-e^{i\arctan(ax)}\right)-\log\left(1+e^{i\arctan(ax)}\right)\right)+i\,\mathrm{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-i\,\mathrm{Po}\right.}{\sqrt{c\left(1+a^2x^2\right)}}$$

input
```
Integrate[ArcTan[a*x]/(x*Sqrt[c + a^2*c*x^2]),x]
```

output
```
(Sqrt[1 + a^2*x^2]*(ArcTan[a*x]*(Log[1 - E^(I*ArcTan[a*x])] - Log[1 + E^(I
*ArcTan[a*x])]) + I*PolyLog[2, -E^(I*ArcTan[a*x])] - I*PolyLog[2, E^(I*Arc
Tan[a*x])]))/Sqrt[c*(1 + a^2*x^2)]
```

## Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.70, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5493, 5489}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}\,dx$$

$$\Big\downarrow \text{5493}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5489}$$

$$\frac{\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\mathrm{PolyLog}\left(2,\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)\right)}{\sqrt{a^2cx^2+c}}$$

input
```
Int[ArcTan[a*x]/(x*Sqrt[c + a^2*c*x^2]),x]
```

output
```
(Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]
] + I*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])] - I*PolyLog[2, Sqrt[1
 + I*a*x]/Sqrt[1 - I*a*x]]))/Sqrt[c + a^2*c*x^2]
```

### Defintions of rubi rules used

rule 5489
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sq
rt[1 - I*c*x]], x] + (Simp[I*(b/Sqrt[d])*PolyLog[2, -Sqrt[1 + I*c*x]/Sqrt[1
 - I*c*x]], x] - Simp[I*(b/Sqrt[d])*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x]], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

## Maple [A] (verified)

Time = 1.42 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.79

| method | result |
|---|---|
| default | $-\frac{i\left(i\arctan(ax)\ln\left(1-\frac{iax+1}{\sqrt{a^2x^2+1}}\right)-i\arctan(ax)\ln\left(1+\frac{iax+1}{\sqrt{a^2x^2+1}}\right)+\operatorname{polylog}\left(2,\frac{iax+1}{\sqrt{a^2x^2+1}}\right)-\operatorname{polylog}\left(2,-\frac{iax+1}{\sqrt{a^2x^2+1}}\right)\right)\sqrt{c(ax-i}}{\sqrt{a^2x^2+1}\,c}$ |

input
```
int(arctan(a*x)/x/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-I*(I*arctan(a*x)*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*arctan(a*x)*ln(1+(1+
I*a*x)/(a^2*x^2+1)^(1/2))+polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-polylog(2
,-(1+I*a*x)/(a^2*x^2+1)^(1/2)))*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/2
)/c
```

## Fricas [F]

$$\int \frac{\arctan(ax)}{x\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan(ax)}{\sqrt{a^2cx^2+cx}}\,dx$$

input
```
integrate(arctan(a*x)/x/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)/(a^2*c*x^3 + c*x), x)
```

## Sympy [F]

$$\int \frac{\arctan(ax)}{x\sqrt{c+a^2cx^2}}\,dx = \int \frac{\operatorname{atan}(ax)}{x\sqrt{c(a^2x^2+1)}}\,dx$$

input
```
integrate(atan(a*x)/x/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(atan(a*x)/(x*sqrt(c*(a**2*x**2 + 1))), x)
```

## Maxima [F]

$$\int \frac{\arctan(ax)}{x\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan(ax)}{\sqrt{a^2cx^2+cx}}\,dx$$

input
```
integrate(arctan(a*x)/x/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)/(sqrt(a^2*c*x^2 + c)*x), x)
```

## Giac [F]

$$\int \frac{\arctan(ax)}{x\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan{(ax)}}{\sqrt{a^2cx^2+cx}}\,dx$$

input
```
integrate(arctan(a*x)/x/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(arctan(a*x)/(sqrt(a^2*c*x^2 + c)*x), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)}{x\sqrt{c+a^2cx^2}}\,dx = \int \frac{\mathrm{atan}(a\,x)}{x\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(atan(a*x)/(x*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(atan(a*x)/(x*(c + a^2*c*x^2)^(1/2)), x)
```

## Reduce [F]

$$\int \frac{\arctan(ax)}{x\sqrt{c+a^2cx^2}}\,dx = \frac{\int \frac{atan(ax)}{\sqrt{a^2x^2+1}\,x}dx}{\sqrt{c}}$$

input
```
int(atan(a*x)/x/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(atan(a*x)/(sqrt(a**2*x**2 + 1)*x),x)/sqrt(c)
```

# 3.229 $\int \frac{\arctan(ax)}{x^2\sqrt{c+a^2cx^2}}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 56

$$\int \frac{\arctan(ax)}{x^2\sqrt{c+a^2cx^2}}\,dx = -\frac{\sqrt{c+a^2cx^2}\arctan(ax)}{cx} - \frac{a\operatorname{arctanh}\left(\frac{\sqrt{c+a^2cx^2}}{\sqrt{c}}\right)}{\sqrt{c}}$$

output
```
-(a^2*c*x^2+c)^(1/2)*arctan(a*x)/c/x-a*arctanh((a^2*c*x^2+c)^(1/2)/c^(1/2)
)/c^(1/2)
```

## Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.11

$$\int \frac{\arctan(ax)}{x^2\sqrt{c+a^2cx^2}}\,dx = -\frac{\sqrt{c+a^2cx^2}\arctan(ax)}{cx} + \frac{a\left(\log(x) - \log\left(c+\sqrt{c}\sqrt{c+a^2cx^2}\right)\right)}{\sqrt{c}}$$

input
```
Integrate[ArcTan[a*x]/(x^2*Sqrt[c + a^2*c*x^2]),x]
```

output
```
-((Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(c*x)) + (a*(Log[x] - Log[c + Sqrt[c]*
Sqrt[c + a^2*c*x^2]]))/Sqrt[c]
```

## Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {5479, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5479}$$

$$a\int \frac{1}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}a\int \frac{1}{x^2\sqrt{a^2cx^2+c}}dx^2-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}$$

$$\Big\downarrow \text{73}$$

$$\frac{\int \frac{1}{\frac{x^4}{a^2c}-\frac{1}{a^2}}d\sqrt{a^2cx^2+c}}{ac}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}$$

$$\Big\downarrow \text{221}$$

$$-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\,\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}$$

input
```
Int[ArcTan[a*x]/(x^2*Sqrt[c + a^2*c*x^2]),x]
```

output
```
-((Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(c*x)) - (a*ArcTanh[Sqrt[c + a^2*c*x^2
]/Sqrt[c]])/Sqrt[c]
```

### Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.38 (sec) , antiderivative size = 125, normalized size of antiderivative = 2.23

| method | result | size |
|---|---|---|
| default | $-\frac{\sqrt{c(ax-i)(ax+i)}\left(-\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}-1\right)\sqrt{a^2x^2+1}\,ax+\ln\left(1+\frac{iax+1}{\sqrt{a^2x^2+1}}\right)\sqrt{a^2x^2+1}\,ax+x^2a^2\arctan(ax)+\arctan(ax)\right)}{(a^2x^2+1)cx}$ | 125 |

input
```
int(arctan(a*x)/x^2/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(c*(a*x-I)*(a*x+I))^(1/2)*(-ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-1)*(a^2*x^2+1)
^(1/2)*a*x+ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)*a*x+x^2*a^2
*arctan(a*x)+arctan(a*x))/(a^2*x^2+1)/c/x
```

## Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.21

$$\int \frac{\arctan(ax)}{x^2\sqrt{c+a^2cx^2}}\,dx = \frac{a\sqrt{c}x\log\left(-\frac{a^2cx^2-2\sqrt{a^2cx^2+c}\sqrt{c}+2c}{x^2}\right)-2\sqrt{a^2cx^2+c}\arctan(ax)}{2\,cx}$$

input
```
integrate(arctan(a*x)/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(a*sqrt(c)*x*log(-(a^2*c*x^2 - 2*sqrt(a^2*c*x^2 + c)*sqrt(c) + 2*c)/x^
2) - 2*sqrt(a^2*c*x^2 + c)*arctan(a*x))/(c*x)
```

## Sympy [F]

$$\int \frac{\arctan(ax)}{x^2\sqrt{c+a^2cx^2}}\,dx = \int \frac{\operatorname{atan}(ax)}{x^2\sqrt{c(a^2x^2+1)}}\,dx$$

input
```
integrate(atan(a*x)/x**2/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(atan(a*x)/(x**2*sqrt(c*(a**2*x**2 + 1))), x)
```

## Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.64

$$\int \frac{\arctan(ax)}{x^2\sqrt{c+a^2cx^2}}\,dx = -\frac{a\,\text{arsinh}\left(\frac{1}{a|x|}\right)+\frac{\sqrt{a^2x^2+1}\arctan(ax)}{x}}{\sqrt{c}}$$

input
```
integrate(arctan(a*x)/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
-(a*arcsinh(1/(a*abs(x))) + sqrt(a^2*x^2 + 1)*arctan(a*x)/x)/sqrt(c)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)}{x^2\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Degree mismatch inside factorisatio
n over extensionsym2poly/r2sym(const gen & e,const index_m & i,const vecte
ur & l) E
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)}{x^2\sqrt{c+a^2cx^2}}\,dx = \int \frac{\mathrm{atan}(a\,x)}{x^2\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(atan(a*x)/(x^2*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(atan(a*x)/(x^2*(c + a^2*c*x^2)^(1/2)), x)
```

## Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.16

$$\int \frac{\arctan(ax)}{x^2\sqrt{c+a^2cx^2}}\,dx$$
$$= \frac{\sqrt{c}\left(-\sqrt{a^2x^2+1}\,atan(ax)+\log\left(\sqrt{a^2x^2+1}+ax-1\right)ax-\log\left(\sqrt{a^2x^2+1}+ax+1\right)ax\right)}{cx}$$

input
```
int(atan(a*x)/x^2/(a^2*c*x^2+c)^(1/2),x)
```

output
```
(sqrt(c)*( - sqrt(a**2*x**2 + 1)*atan(a*x) + log(sqrt(a**2*x**2 + 1) + a*x
 - 1)*a*x - log(sqrt(a**2*x**2 + 1) + a*x + 1)*a*x))/(c*x)
```

# 3.230 $\int \frac{\arctan(ax)}{x^3\sqrt{c+a^2cx^2}}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 242

$$\int \frac{\arctan(ax)}{x^3\sqrt{c+a^2cx^2}}\,dx = -\frac{a\sqrt{c+a^2cx^2}}{2cx} - \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{2cx^2} + \frac{a^2\sqrt{1+a^2x^2}\arctan(ax)\operatorname{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} - \frac{ia^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2\sqrt{c+a^2cx^2}} + \frac{ia^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2\sqrt{c+a^2cx^2}}$$

output
```
-1/2*a*(a^2*c*x^2+c)^(1/2)/c/x-1/2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)/c/x^2+a
^2*(a^2*x^2+1)^(1/2)*arctan(a*x)*arctanh((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/
(a^2*c*x^2+c)^(1/2)-1/2*I*a^2*(a^2*x^2+1)^(1/2)*polylog(2,-(1+I*a*x)^(1/2)
/(1-I*a*x)^(1/2))/(a^2*c*x^2+c)^(1/2)+1/2*I*a^2*(a^2*x^2+1)^(1/2)*polylog(
2,(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.68

$$\int \frac{\arctan(ax)}{x^3\sqrt{c+a^2cx^2}}\,dx = \frac{a^2\sqrt{1+a^2x^2}\left(-2\cot\left(\frac{1}{2}\arctan(ax)\right)-\arctan(ax)\csc^2\left(\frac{1}{2}\arctan(ax)\right)-4\arctan(ax)\log\left(1-e^{i\arctan(ax)}\right.\right.}{}$$

input
```
Integrate[ArcTan[a*x]/(x^3*Sqrt[c + a^2*c*x^2]),x]
```

output
```
(a^2*Sqrt[1 + a^2*x^2]*(-2*Cot[ArcTan[a*x]/2] - ArcTan[a*x]*Csc[ArcTan[a*x
]/2]^2 - 4*ArcTan[a*x]*Log[1 - E^(I*ArcTan[a*x])] + 4*ArcTan[a*x]*Log[1 +
E^(I*ArcTan[a*x])] - (4*I)*PolyLog[2, -E^(I*ArcTan[a*x])] + (4*I)*PolyLog[
2, E^(I*ArcTan[a*x])] + ArcTan[a*x]*Sec[ArcTan[a*x]/2]^2 - 2*Tan[ArcTan[a*
x]/2]))/(8*Sqrt[c*(1 + a^2*x^2)])
```

## Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.76, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {5497, 242, 5493, 5489}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}\,dx$$

$$\Big\downarrow \text{5497}$$

$$-\frac{1}{2}a^2\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx+\frac{1}{2}a\int \frac{1}{x^2\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{2cx^2}$$

$$\Big\downarrow \text{242}$$

$$-\frac{1}{2}a^2\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{2cx^2}-\frac{a\sqrt{a^2cx^2+c}}{2cx}$$

$$\Big\downarrow \text{5493}$$

$$-\frac{a^2\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{x\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{2cx^2}-\frac{a\sqrt{a^2cx^2+c}}{2cx}$$

$$\Big\downarrow \text{5489}$$

$$-\frac{a^2\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{PolyLog}\left(2,\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)\right)}{2\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{2cx^2}-\frac{a\sqrt{a^2cx^2+c}}{2cx}$$

input
```
Int[ArcTan[a*x]/(x^3*Sqrt[c + a^2*c*x^2]),x]
```

output
```
-1/2*(a*Sqrt[c + a^2*c*x^2])/(c*x) - (Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(2*
c*x^2) - (a^2*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sq
rt[1 - I*a*x]] + I*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])] - I*Poly
Log[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]]))/(2*Sqrt[c + a^2*c*x^2])
```

## Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 5489
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sq
rt[1 - I*c*x]], x] + (Simp[I*(b/Sqrt[d])*PolyLog[2, -Sqrt[1 + I*c*x]/Sqrt[1
 - I*c*x]], x] - Simp[I*(b/Sqrt[d])*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x]], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5497
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)^(m
 + 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[c^2*((m
+ 2)/(f^2*(m + 1)))   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x
^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
 && LtQ[m, -1] && NeQ[m, -2]
```

## Maple [A] (verified)

Time = 1.42 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.72

| method | result |
|---|---|
| default | $-\frac{(ax+\arctan(ax))\sqrt{c(ax-i)(ax+i)}}{2c\,x^2}+\frac{ia^2\left(i\arctan(ax)\ln\left(1-\frac{iax+1}{\sqrt{a^2x^2+1}}\right)-i\arctan(ax)\ln\left(1+\frac{iax+1}{\sqrt{a^2x^2+1}}\right)+\text{polylog}\left(2,\frac{iax+}{\sqrt{a^2x^2}}\right.\right.}{2\sqrt{a^2x^2+1}\,c}$ |

input
```
int(arctan(a*x)/x^3/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(a*x+arctan(a*x))*(c*(a*x-I)*(a*x+I))^(1/2)/c/x^2+1/2*I*a^2*(I*arctan
(a*x)*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*arctan(a*x)*ln(1+(1+I*a*x)/(a^2*
x^2+1)^(1/2))+polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-polylog(2,-(1+I*a*x)/
(a^2*x^2+1)^(1/2)))*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/2)/c
```

## Fricas [F]

$$\int \frac{\arctan(ax)}{x^3\sqrt{c+a^2cx^2}}\,dx=\int \frac{\arctan{(ax)}}{\sqrt{a^2cx^2+c}\,x^3}\,dx$$

input
```
integrate(arctan(a*x)/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)/(a^2*c*x^5 + c*x^3), x)
```

## Sympy [F]

$$\int \frac{\arctan(ax)}{x^3\sqrt{c+a^2cx^2}}\,dx = \int \frac{\operatorname{atan}(ax)}{x^3\sqrt{c(a^2x^2+1)}}\,dx$$

input
```
integrate(atan(a*x)/x**3/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(atan(a*x)/(x**3*sqrt(c*(a**2*x**2 + 1))), x)
```

## Maxima [F]

$$\int \frac{\arctan(ax)}{x^3\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}x^3}\,dx$$

input
```
integrate(arctan(a*x)/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)/(sqrt(a^2*c*x^2 + c)*x^3), x)
```

## Giac [F]

$$\int \frac{\arctan(ax)}{x^3\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}x^3}\,dx$$

input
```
integrate(arctan(a*x)/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(arctan(a*x)/(sqrt(a^2*c*x^2 + c)*x^3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)}{x^3\sqrt{c+a^2cx^2}}\,dx = \int \frac{\mathrm{atan}(a\,x)}{x^3\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(atan(a*x)/(x^3*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(atan(a*x)/(x^3*(c + a^2*c*x^2)^(1/2)), x)
```

## Reduce [F]

$$\int \frac{\arctan(ax)}{x^3\sqrt{c+a^2cx^2}}\,dx = \frac{\int \frac{atan(ax)}{\sqrt{a^2x^2+1}\,x^3}dx}{\sqrt{c}}$$

input
```
int(atan(a*x)/x^3/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(atan(a*x)/(sqrt(a**2*x**2 + 1)*x**3),x)/sqrt(c)
```

# 3.231 $\int \frac{\arctan(ax)}{x^4\sqrt{c+a^2cx^2}}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 118

$$\int \frac{\arctan(ax)}{x^4\sqrt{c+a^2cx^2}}\,dx = -\frac{a\sqrt{c+a^2cx^2}}{6cx^2} - \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{3cx^3} + \frac{2a^2\sqrt{c+a^2cx^2}\arctan(ax)}{3cx} + \frac{5a^3\text{arctanh}\left(\frac{\sqrt{c+a^2cx^2}}{\sqrt{c}}\right)}{6\sqrt{c}}$$

output
```
-1/6*a*(a^2*c*x^2+c)^(1/2)/c/x^2-1/3*(a^2*c*x^2+c)^(1/2)*arctan(a*x)/c/x^3
+2/3*a^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)/c/x+5/6*a^3*arctanh((a^2*c*x^2+c)
^(1/2)/c^(1/2))/c^(1/2)
```

## Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.93

$$\int \frac{\arctan(ax)}{x^4\sqrt{c+a^2cx^2}}\,dx = \frac{-ax\sqrt{c+a^2cx^2} + 2(-1+2a^2x^2)\sqrt{c+a^2cx^2}\arctan(ax) - 5a^3\sqrt{c}x^3\log(x) + 5a^3\sqrt{c}x^3\log\left(c+\sqrt{c}\sqrt{c+\right.}{6cx^3}$$

input
```
Integrate[ArcTan[a*x]/(x^4*Sqrt[c + a^2*c*x^2]),x]
```

output
```
(-(a*x*Sqrt[c + a^2*c*x^2]) + 2*(-1 + 2*a^2*x^2)*Sqrt[c + a^2*c*x^2]*ArcTa
n[a*x] - 5*a^3*Sqrt[c]*x^3*Log[x] + 5*a^3*Sqrt[c]*x^3*Log[c + Sqrt[c]*Sqrt
[c + a^2*c*x^2]])/(6*c*x^3)
```

## Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.27, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {5497, 243, 52, 73, 221, 5479, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)}{x^4\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{ 5497}$$

$$-\frac{2}{3}a^2\int \frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx+\frac{1}{3}a\int \frac{1}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{3cx^3}$$

$$\Big\downarrow \text{ 243}$$

$$-\frac{2}{3}a^2\int \frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx+\frac{1}{6}a\int \frac{1}{x^4\sqrt{a^2cx^2+c}}dx^2-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{3cx^3}$$

$$\Big\downarrow \text{ 52}$$

$$-\frac{2}{3}a^2\int \frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx+\frac{1}{6}a\left(-\frac{1}{2}a^2\int \frac{1}{x^2\sqrt{a^2cx^2+c}}dx^2-\frac{\sqrt{a^2cx^2+c}}{cx^2}\right)-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{3cx^3}$$

$$\Big\downarrow \text{ 73}$$

$$-\frac{2}{3}a^2\int \frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx+\frac{1}{6}a\left(-\frac{\int \frac{1}{\frac{x^4}{a^2c}-\frac{1}{a^2}}d\sqrt{a^2cx^2+c}}{c}-\frac{\sqrt{a^2cx^2+c}}{cx^2}\right)-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{3cx^3}$$

$$\Big\downarrow \text{ 221}$$

$$-\frac{2}{3}a^2\int \frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{3cx^3}+\frac{1}{6}a\left(\frac{a^2\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{\sqrt{a^2cx^2+c}}{cx^2}\right)$$

$$\Big\downarrow \text{5479}$$

$$-\frac{2}{3}a^2\left(a\int \frac{1}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right)-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{3cx^3}+\frac{1}{6}a\left(\frac{a^2\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{\sqrt{a^2cx^2+c}}{cx^2}\right)$$

$$\Big\downarrow \text{243}$$

$$-\frac{2}{3}a^2\left(\frac{1}{2}a\int \frac{1}{x^2\sqrt{a^2cx^2+c}}dx^2-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right)-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{3cx^3}+\frac{1}{6}a\left(\frac{a^2\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{\sqrt{a^2cx^2+c}}{cx^2}\right)$$

$$\Big\downarrow \text{73}$$

$$-\frac{2}{3}a^2\left(\frac{\int \frac{1}{\frac{x^4}{a^2c}-\frac{1}{a^2}}d\sqrt{a^2cx^2+c}}{ac}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right)-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{3cx^3}+\frac{1}{6}a\left(\frac{a^2\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{\sqrt{a^2cx^2+c}}{cx^2}\right)$$

$$\Big\downarrow \text{221}$$

$$-\frac{2}{3}a^2\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{3cx^3}+\frac{1}{6}a\left(\frac{a^2\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{\sqrt{a^2cx^2+c}}{cx^2}\right)$$

input
```
Int[ArcTan[a*x]/(x^4*Sqrt[c + a^2*c*x^2]),x]
```

output
```
-1/3*(Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(c*x^3) - (2*a^2*(-((Sqrt[c + a^2*c
*x^2]*ArcTan[a*x])/(c*x)) - (a*ArcTanh[Sqrt[c + a^2*c*x^2]/Sqrt[c]])/Sqrt[
c]))/3 + (a*(-(Sqrt[c + a^2*c*x^2]/(c*x^2)) + (a^2*ArcTanh[Sqrt[c + a^2*c*
x^2]/Sqrt[c]])/Sqrt[c]))/6
```

### Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5497
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)^(m
 + 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[c^2*((m
+ 2)/(f^2*(m + 1)))   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x
^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
 && LtQ[m, -1] && NeQ[m, -2]
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.41 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.38

| method | result |
|---|---|
| default | $\frac{(4x^2a^2\arctan(ax)-ax-2\arctan(ax))\sqrt{c(ax-i)(ax+i)}}{6c\,x^3}-\frac{5a^3\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}-1\right)\sqrt{c(ax-i)(ax+i)}}{6\sqrt{a^2x^2+1}\,c}+\frac{5a^3\ln\left(1+\frac{iax+1}{\sqrt{a^2x^2+1}}\right)\sqrt{c}}{6\sqrt{a^2x^2+1}}$ |

input
```
int(arctan(a*x)/x^4/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(4*x^2*a^2*arctan(a*x)-a*x-2*arctan(a*x))*(c*(a*x-I)*(a*x+I))^(1/2)/c/
x^3-5/6*a^3*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-1)*(c*(a*x-I)*(a*x+I))^(1/2)/(a
^2*x^2+1)^(1/2)/c+5/6*a^3*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))*(c*(a*x-I)*(a*
x+I))^(1/2)/(a^2*x^2+1)^(1/2)/c
```

## Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.75

$$\int \frac{\arctan(ax)}{x^4\sqrt{c+a^2cx^2}}\,dx = \frac{5\,a^3\sqrt{c}x^3\log\left(-\frac{a^2cx^2+2\,\sqrt{a^2cx^2+c}\sqrt{c}+2\,c}{x^2}\right)-2\,\sqrt{a^2cx^2+c}(ax-2\,(2\,a^2x^2-1)\arctan(ax))}{12\,cx^3}$$

input
```
integrate(arctan(a*x)/x^4/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
1/12*(5*a^3*sqrt(c)*x^3*log(-(a^2*c*x^2 + 2*sqrt(a^2*c*x^2 + c)*sqrt(c) +
2*c)/x^2) - 2*sqrt(a^2*c*x^2 + c)*(a*x - 2*(2*a^2*x^2 - 1)*arctan(a*x)))/(
c*x^3)
```

## Sympy [F]

$$\int \frac{\arctan(ax)}{x^4\sqrt{c+a^2cx^2}}\,dx = \int \frac{\operatorname{atan}(ax)}{x^4\sqrt{c\,(a^2x^2+1)}}\,dx$$

input
```
integrate(atan(a*x)/x**4/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(atan(a*x)/(x**4*sqrt(c*(a**2*x**2 + 1))), x)
```

## Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.69

$$\int \frac{\arctan(ax)}{x^4\sqrt{c+a^2cx^2}}\,dx = \frac{\left(5\,a^2\operatorname{arsinh}\left(\frac{1}{a|x|}\right) - \frac{\sqrt{a^2x^2+1}}{x^2}\right)a + 2\left(\frac{2\sqrt{a^2x^2+1}a^2}{x} - \frac{\sqrt{a^2x^2+1}}{x^3}\right)\arctan(ax)}{6\sqrt{c}}$$

input
```
integrate(arctan(a*x)/x^4/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
1/6*((5*a^2*arcsinh(1/(a*abs(x))) - sqrt(a^2*x^2 + 1)/x^2)*a + 2*(2*sqrt(a
^2*x^2 + 1)*a^2/x - sqrt(a^2*x^2 + 1)/x^3)*arctan(a*x))/sqrt(c)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)}{x^4\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)/x^4/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Degree mismatch inside factorisatio
n over extensionindex.cc index_m i_lex_is_greater Error: Bad Argument Valu
e
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)}{x^4\sqrt{c+a^2cx^2}}\,dx = \int \frac{\mathrm{atan}(a\,x)}{x^4\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(atan(a*x)/(x^4*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(atan(a*x)/(x^4*(c + a^2*c*x^2)^(1/2)), x)
```

## Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.94

$$\int \frac{\arctan(ax)}{x^4\sqrt{c+a^2cx^2}}\,dx$$
$$= \frac{\sqrt{c}\,\left(4\sqrt{a^2x^2+1}\,atan(ax)\,a^2x^2 - 2\sqrt{a^2x^2+1}\,atan(ax) - \sqrt{a^2x^2+1}\,ax - 5\log\left(\sqrt{a^2x^2+1}+ax-1\right)a^3}{6c\,x^3}$$

input
```
int(atan(a*x)/x^4/(a^2*c*x^2+c)^(1/2),x)
```

output
```
(sqrt(c)*(4*sqrt(a**2*x**2 + 1)*atan(a*x)*a**2*x**2 - 2*sqrt(a**2*x**2 + 1
)*atan(a*x) - sqrt(a**2*x**2 + 1)*a*x - 5*log(sqrt(a**2*x**2 + 1) + a*x -
1)*a**3*x**3 + 5*log(sqrt(a**2*x**2 + 1) + a*x + 1)*a**3*x**3))/(6*c*x**3)
```

# 3.232 $\int \frac{x^3 \arctan(ax)}{(c+a^2cx^2)^{3/2}} \, dx$



## Optimal result

Integrand size = 22, antiderivative size = 107

$$\int \frac{x^3 \arctan(ax)}{(c+a^2cx^2)^{3/2}} \, dx = -\frac{x}{a^3c\sqrt{c+a^2cx^2}} + \frac{\arctan(ax)}{a^4c\sqrt{c+a^2cx^2}} + \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{a^4c^2} - \frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right)}{a^4c^{3/2}}$$

output
```
-x/a^3/c/(a^2*c*x^2+c)^(1/2)+arctan(a*x)/a^4/c/(a^2*c*x^2+c)^(1/2)+(a^2*c*
x^2+c)^(1/2)*arctan(a*x)/a^4/c^2-arctanh(a*c^(1/2)*x/(a^2*c*x^2+c)^(1/2))/
a^4/c^(3/2)
```

## Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00

$$\int \frac{x^3 \arctan(ax)}{(c+a^2cx^2)^{3/2}} \, dx = \frac{-ax\sqrt{c+a^2cx^2} + (2+a^2x^2)\sqrt{c+a^2cx^2}\arctan(ax) - \sqrt{c}(1+a^2x^2)\log\left(acx+\sqrt{c}\right.}{a^4c^2(1+a^2x^2)}$$

input
```
Integrate[(x^3*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
```

output
```
(-(a*x*Sqrt[c + a^2*c*x^2]) + (2 + a^2*x^2)*Sqrt[c + a^2*c*x^2]*ArcTan[a*x
] - Sqrt[c]*(1 + a^2*x^2)*Log[a*c*x + Sqrt[c]*Sqrt[c + a^2*c*x^2]])/(a^4*c
^2*(1 + a^2*x^2))
```

## Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {5499, 5465, 208, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\arctan(ax)}{(a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \text{5499}$$

$$\frac{\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a^2c}-\frac{\int \frac{x\arctan(ax)}{(a^2cx^2+c)^{3/2}}dx}{a^2}$$

$$\Big\downarrow \text{5465}$$

$$\frac{\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\int \frac{1}{\sqrt{a^2cx^2+c}}dx}{a}}{a^2c}-\frac{\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}}dx}{a}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}}{a^2}$$

$$\Big\downarrow \text{208}$$

$$\frac{\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\int \frac{1}{\sqrt{a^2cx^2+c}}dx}{a}}{a^2c}-\frac{\frac{x}{ac\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}}{a^2}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\int \frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}}{a}}{a^2c}-\frac{\frac{x}{ac\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}}{a^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a^2\sqrt{c}}}{a^2c}-\frac{\frac{x}{ac\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}}{a^2}$$

input
```
Int[(x^3*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
```

output
```
-((x/(a*c*Sqrt[c + a^2*c*x^2]) - ArcTan[a*x]/(a^2*c*Sqrt[c + a^2*c*x^2]))/
a^2) + ((Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(a^2*c) - ArcTanh[(a*Sqrt[c]*x)/
Sqrt[c + a^2*c*x^2]]/(a^2*Sqrt[c]))/(a^2*c)
```

### Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5499
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ
[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.23 (sec) , antiderivative size = 218, normalized size of antiderivative = 2.04

| method | result |
|---|---|
| default | $\frac{\left(\arctan(ax)\sqrt{a^2x^2+1}\,a^2x^2-\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}+i\right)a^2x^2+\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}-i\right)a^2x^2-\sqrt{a^2x^2+1}\,ax+2\arctan(ax)\sqrt{a^2x^2+1}-\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right.\right.}{a^4c^2(a^4x^4+2a^2x^2+1)}$ |

input
```
int(x^3*arctan(a*x)/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(arctan(a*x)*(a^2*x^2+1)^(1/2)*a^2*x^2-ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+I)*a
^2*x^2+ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-I)*a^2*x^2-(a^2*x^2+1)^(1/2)*a*x+2*a
rctan(a*x)*(a^2*x^2+1)^(1/2)-ln((1+I*a*x)/(a^2*x^2+1)^(1/2)+I)+ln((1+I*a*x
)/(a^2*x^2+1)^(1/2)-I))*(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(a*x+I))^(1/2)/a^4/c^
2/(a^4*x^4+2*a^2*x^2+1)
```

## Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.95

$$\int \frac{x^3\arctan(ax)}{(c+a^2cx^2)^{3/2}}\,dx = \frac{(a^2x^2+1)\sqrt{c}\log\left(-2\,a^2cx^2+2\,\sqrt{a^2cx^2+c}a\sqrt{c}x-c\right)-2\,\sqrt{a^2cx^2+c}(ax-(a^2x^2\cdots}{2\,(a^6c^2x^2+a^4c^2)}$$

input
```
integrate(x^3*arctan(a*x)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
1/2*((a^2*x^2 + 1)*sqrt(c)*log(-2*a^2*c*x^2 + 2*sqrt(a^2*c*x^2 + c)*a*sqrt
(c)*x - c) - 2*sqrt(a^2*c*x^2 + c)*(a*x - (a^2*x^2 + 2)*arctan(a*x)))/(a^6
*c^2*x^2 + a^4*c^2)
```

## Sympy [F]

$$\int \frac{x^3 \arctan(ax)}{(c + a^2cx^2)^{3/2}}\,dx = \int \frac{x^3 \operatorname{atan}(ax)}{(c(a^2x^2 + 1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3*atan(a*x)/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(x**3*atan(a*x)/(c*(a**2*x**2 + 1))**(3/2), x)
```

## Maxima [F]

$$\int \frac{x^3 \arctan(ax)}{(c + a^2cx^2)^{3/2}}\,dx = \int \frac{x^3 \arctan(ax)}{(a^2cx^2 + c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3*arctan(a*x)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^3*arctan(a*x)/(a^2*c*x^2 + c)^(3/2), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x^3 \arctan(ax)}{(c + a^2cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctan(a*x)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \arctan(ax)}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^3\,\mathrm{atan}(a\,x)}{(c\,a^2\,x^2+c)^{3/2}}\,dx$$

input
```
int((x^3*atan(a*x))/(c + a^2*c*x^2)^(3/2),x)
```

output
```
int((x^3*atan(a*x))/(c + a^2*c*x^2)^(3/2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.13

$$\int \frac{x^3 \arctan(ax)}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{c}\,\left(\sqrt{a^2x^2+1}\,atan(ax)\,a^2x^2+2\sqrt{a^2x^2+1}\,atan(ax)-\sqrt{a^2x^2+1}\,ax-\log\left(\sqrt{a^2x^2}\right.\right.}{a^4c^2\,(a^2x^2+1)}$$

input
```
int(x^3*atan(a*x)/(a^2*c*x^2+c)^(3/2),x)
```

output
```
(sqrt(c)*(sqrt(a**2*x**2 + 1)*atan(a*x)*a**2*x**2 + 2*sqrt(a**2*x**2 + 1)*
atan(a*x) - sqrt(a**2*x**2 + 1)*a*x - log(sqrt(a**2*x**2 + 1) + a*x)*a**2*
x**2 - log(sqrt(a**2*x**2 + 1) + a*x) - a**2*x**2 - 1))/(a**4*c**2*(a**2*x
**2 + 1))
```

# 3.233 $\int \frac{x^2 \arctan(ax)}{(c+a^2cx^2)^{3/2}} \, dx$



## Optimal result

Integrand size = 22, antiderivative size = 251

$$\int \frac{x^2 \arctan(ax)}{(c+a^2cx^2)^{3/2}} \, dx = -\frac{1}{a^3c\sqrt{c+a^2cx^2}} - \frac{x\arctan(ax)}{a^2c\sqrt{c+a^2cx^2}} - \frac{2i\sqrt{1+a^2x^2}\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a^3c\sqrt{c+a^2cx^2}} + \frac{i\sqrt{1+a^2x^2}\,\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a^3c\sqrt{c+a^2cx^2}} - \frac{i\sqrt{1+a^2x^2}\,\text{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a^3c\sqrt{c+a^2cx^2}}$$

output
```
-1/a^3/c/(a^2*c*x^2+c)^(1/2)-x*arctan(a*x)/a^2/c/(a^2*c*x^2+c)^(1/2)-2*I*(
a^2*x^2+1)^(1/2)*arctan(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/a^3/c
/(a^2*c*x^2+c)^(1/2)+I*(a^2*x^2+1)^(1/2)*polylog(2,-I*(1+I*a*x)^(1/2)/(1-I
*a*x)^(1/2))/a^3/c/(a^2*c*x^2+c)^(1/2)-I*(a^2*x^2+1)^(1/2)*polylog(2,I*(1+
I*a*x)^(1/2)/(1-I*a*x)^(1/2))/a^3/c/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.62

$$\int \frac{x^2 \arctan(ax)}{(c+a^2cx^2)^{3/2}}\,dx =$$

$$-\frac{\sqrt{1+a^2x^2}\left(\frac{1}{\sqrt{1+a^2x^2}}+\frac{ax\arctan(ax)}{\sqrt{1+a^2x^2}}-\arctan(ax)\log\left(1-ie^{i\arctan(ax)}\right)+\arctan(ax)\log\left(1+ie^{i\arctan(ax)}\right)\cdots\right.}{a^3c\sqrt{c\left(1+a^2x^2\right)}}$$

input
```
Integrate[(x^2*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
```

output
```
-((Sqrt[1 + a^2*x^2]*(1/Sqrt[1 + a^2*x^2] + (a*x*ArcTan[a*x])/Sqrt[1 + a^2
*x^2] - ArcTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])] + ArcTan[a*x]*Log[1 + I*E
^(I*ArcTan[a*x])] - I*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + I*PolyLog[2, I*
E^(I*ArcTan[a*x])]))/(a^3*c*Sqrt[c*(1 + a^2*x^2)]))
```

## Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.78, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {5469, 5425, 5421}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\arctan(ax)}{(a^2cx^2+c)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 5469}$$

$$\frac{\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a^2c}-\frac{x\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{1}{a^3c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 5425}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{a^2c\sqrt{a^2cx^2+c}}-\frac{x\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{1}{a^3c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5421}$$

$$\frac{\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{a^2c\sqrt{a^2cx^2+c}}-\frac{x\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{1}{a^3c\sqrt{a^2cx^2+c}}$$

input
```
Int[(x^2*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
```

output
```
-(1/(a^3*c*Sqrt[c + a^2*c*x^2])) - (x*ArcTan[a*x])/(a^2*c*Sqrt[c + a^2*c*x
^2]) + (Sqrt[1 + a^2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt
[1 - I*a*x]])/a + (I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a
 - (I*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/(a^2*c*Sqrt[c +
 a^2*c*x^2])
```

### Defintions of rubi rules used

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5469
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^2*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c^3*d*(q + 1)^2)), x] + (Simp
[x*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*c^2*d*(q + 1))), x] - Simp[1
/(2*c^2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x]) /
; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -5/2]
```

## Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.98

| method | result |
|---|---|
| default | $-\frac{(\arctan(ax)+i)(ax-i)\sqrt{c(ax-i)(ax+i)}}{2(a^2x^2+1)a^3c^2}-\frac{\sqrt{c(ax-i)(ax+i)}\,(ax+i)(\arctan(ax)-i)}{2(a^2x^2+1)a^3c^2}-\frac{\left(\arctan(ax)\ln\left(1+\frac{i(iax+1)}{\sqrt{a^2x^2+1}}\right)-\mathrm{arcta}\right.}{}$ |

input
```
int(x^2*arctan(a*x)/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(arctan(a*x)+I)*(a*x-I)*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)/a^3/c^2
-1/2*(c*(a*x-I)*(a*x+I))^(1/2)*(a*x+I)*(arctan(a*x)-I)/(a^2*x^2+1)/a^3/c^2
-(arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-arctan(a*x)*ln(1-I*(1+I*
a*x)/(a^2*x^2+1)^(1/2))+I*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*dilog(1
+I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/
2)/a^3/c^2
```

## Fricas [F]

$$\int \frac{x^2\arctan(ax)}{(c+a^2cx^2)^{3/2}}\,dx=\int \frac{x^2\arctan{(ax)}}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*arctan(a*x)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^2*arctan(a*x)/(a^4*c^2*x^4 + 2*a^2*c^2*x^2
+ c^2), x)
```

## Sympy [F]

$$\int \frac{x^2 \arctan(ax)}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^2 \operatorname{atan}(ax)}{(c(a^2x^2+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2*atan(a*x)/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(x**2*atan(a*x)/(c*(a**2*x**2 + 1))**(3/2), x)
```

## Maxima [F]

$$\int \frac{x^2 \arctan(ax)}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^2 \arctan(ax)}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*arctan(a*x)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^2*arctan(a*x)/(a^2*c*x^2 + c)^(3/2), x)
```

## Giac [F]

$$\int \frac{x^2 \arctan(ax)}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^2 \arctan(ax)}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*arctan(a*x)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^2*arctan(a*x)/(a^2*c*x^2 + c)^(3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \arctan(ax)}{(c + a^2cx^2)^{3/2}} \, dx = \int \frac{x^2 \operatorname{atan}(a\,x)}{(c\,a^2\,x^2 + c)^{3/2}} \, dx$$

input
```
int((x^2*atan(a*x))/(c + a^2*c*x^2)^(3/2),x)
```

output
```
int((x^2*atan(a*x))/(c + a^2*c*x^2)^(3/2), x)
```

## Reduce [F]

$$\int \frac{x^2 \arctan(ax)}{(c + a^2cx^2)^{3/2}} \, dx = \frac{\int \frac{atan(ax)x^2}{\sqrt{a^2x^2+1}\,a^2x^2+\sqrt{a^2x^2+1}}dx}{\sqrt{c}\,c}$$

input
```
int(x^2*atan(a*x)/(a^2*c*x^2+c)^(3/2),x)
```

output
```
int((atan(a*x)*x**2)/(sqrt(a**2*x**2 + 1)*a**2*x**2 + sqrt(a**2*x**2 + 1))
,x)/(sqrt(c)*c)
```

# 3.234 $\int \frac{x \arctan(ax)}{(c+a^2cx^2)^{3/2}} \, dx$



## Optimal result

Integrand size = 20, antiderivative size = 49

$$\int \frac{x \arctan(ax)}{(c+a^2cx^2)^{3/2}} \, dx = \frac{x}{ac\sqrt{c+a^2cx^2}} - \frac{\arctan(ax)}{a^2c\sqrt{c+a^2cx^2}}$$

output
```
x/a/c/(a^2*c*x^2+c)^(1/2)-arctan(a*x)/a^2/c/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.86

$$\int \frac{x \arctan(ax)}{(c+a^2cx^2)^{3/2}} \, dx = \frac{\sqrt{c+a^2cx^2}(ax - \arctan(ax))}{a^2c^2(1+a^2x^2)}$$

input
```
Integrate[(x*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
```

output
```
(Sqrt[c + a^2*c*x^2]*(a*x - ArcTan[a*x]))/(a^2*c^2*(1 + a^2*x^2))
```

## Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5465, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \arctan(ax)}{(a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \text{ 5465}$$

$$\frac{\int \frac{1}{(a^2cx^2+c)^{3/2}}dx}{a} - \frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 208}$$

$$\frac{x}{ac\sqrt{a^2cx^2+c}} - \frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}$$

input
```
Int[(x*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
```

output
```
x/(a*c*Sqrt[c + a^2*c*x^2]) - ArcTan[a*x]/(a^2*c*Sqrt[c + a^2*c*x^2])
```

### Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.12 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.04

| method | result | size |
|---|---|---|
| default | $-\frac{(\arctan(ax)+i)(iax+1)\sqrt{c(ax-i)(ax+i)}}{2(a^2x^2+1)a^2c^2}+\frac{\sqrt{c(ax-i)(ax+i)}\,(iax-1)(\arctan(ax)-i)}{2(a^2x^2+1)a^2c^2}$ | 100 |
| orering | $\frac{2(a^2x^2+1)(a^2x^2-1)\arctan(ax)}{a^2(a^2c\,x^2+c)^{\frac{3}{2}}}+\frac{(a^2x^2+1)^2\left(\frac{\arctan(ax)}{(a^2c\,x^2+c)^{\frac{3}{2}}}+\frac{xa}{(a^2x^2+1)(a^2c\,x^2+c)^{\frac{3}{2}}}-\frac{3x^2\arctan(ax)c\,a^2}{(a^2c\,x^2+c)^{\frac{5}{2}}}\right)}{a^2}$ | 125 |

input
```
int(x*arctan(a*x)/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(arctan(a*x)+I)*(1+I*a*x)*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)/a^2/c
^2+1/2*(c*(a*x-I)*(a*x+I))^(1/2)*(I*a*x-1)*(arctan(a*x)-I)/(a^2*x^2+1)/a^2
/c^2
```

## Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int \frac{x\arctan(ax)}{(c+a^2cx^2)^{3/2}}\,dx=\frac{\sqrt{a^2cx^2+c}(ax-\arctan(ax))}{a^4c^2x^2+a^2c^2}$$

input
```
integrate(x*arctan(a*x)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
sqrt(a^2*c*x^2 + c)*(a*x - arctan(a*x))/(a^4*c^2*x^2 + a^2*c^2)
```

## Sympy [F(-2)]

Exception generated.

$$\int \frac{x\arctan(ax)}{(c+a^2cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*atan(a*x)/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real zoo
```

## Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.57

$$\int \frac{x\arctan(ax)}{(c+a^2cx^2)^{3/2}}\,dx = \frac{ax-\arctan(ax)}{\sqrt{a^2x^2+1}a^2c^{\frac{3}{2}}}$$

input
```
integrate(x*arctan(a*x)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
(a*x - arctan(a*x))/(sqrt(a^2*x^2 + 1)*a^2*c^(3/2))
```

## Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.92

$$\int \frac{x\arctan(ax)}{(c+a^2cx^2)^{3/2}}\,dx = \frac{x}{\sqrt{a^2cx^2+c}ac} - \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}a^2c}$$

input
```
integrate(x*arctan(a*x)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
x/(sqrt(a^2*c*x^2 + c)*a*c) - arctan(a*x)/(sqrt(a^2*c*x^2 + c)*a^2*c)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x\arctan(ax)}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x\,\mathrm{atan}(a\,x)}{(c\,a^2\,x^2+c)^{3/2}}\,dx$$

input
```
int((x*atan(a*x))/(c + a^2*c*x^2)^(3/2),x)
```

output
```
int((x*atan(a*x))/(c + a^2*c*x^2)^(3/2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.18

$$\int \frac{x\arctan(ax)}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{c}\left(-\sqrt{a^2x^2+1}\,atan(ax)+\sqrt{a^2x^2+1}\,ax+a^2x^2+1\right)}{a^2c^2\,(a^2x^2+1)}$$

input
```
int(x*atan(a*x)/(a^2*c*x^2+c)^(3/2),x)
```

output
```
(sqrt(c)*( - sqrt(a**2*x**2 + 1)*atan(a*x) + sqrt(a**2*x**2 + 1)*a*x + a**
2*x**2 + 1))/(a**2*c**2*(a**2*x**2 + 1))
```

# 3.235 $\int \frac{\arctan(ax)}{(c+a^2cx^2)^{3/2}}\,dx$



## Optimal result

Integrand size = 19, antiderivative size = 45

$$\int \frac{\arctan(ax)}{(c+a^2cx^2)^{3/2}}\,dx = \frac{1}{ac\sqrt{c+a^2cx^2}} + \frac{x\arctan(ax)}{c\sqrt{c+a^2cx^2}}$$

output
```
1/a/c/(a^2*c*x^2+c)^(1/2)+x*arctan(a*x)/c/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.84

$$\int \frac{\arctan(ax)}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{c+a^2cx^2}(1+ax\arctan(ax))}{c^2(a+a^3x^2)}$$

input
```
Integrate[ArcTan[a*x]/(c + a^2*c*x^2)^(3/2),x]
```

output
```
(Sqrt[c + a^2*c*x^2]*(1 + a*x*ArcTan[a*x]))/(c^2*(a + a^3*x^2))
```

## Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {5429}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)}{(a^2cx^2+c)^{3/2}}\,dx$$

$$\Big\downarrow \text{5429}$$

$$\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}$$

input
```
Int[ArcTan[a*x]/(c + a^2*c*x^2)^(3/2),x]
```

output
```
1/(a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x])/(c*Sqrt[c + a^2*c*x^2])
```

### Defintions of rubi rules used

rule 5429
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTan[c*x])/(d*Sqr
t[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. 2(41) = 82.

Time = 1.02 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.09

| method | result | size |
|---|---|---|
| orering | $\frac{4(a^2x^2+1)x\arctan(ax)}{(a^2c\,x^2+c)^{\frac{3}{2}}}+\frac{(a^2x^2+1)^2\left(\frac{a}{(a^2x^2+1)(a^2c\,x^2+c)^{\frac{3}{2}}}-\frac{3\arctan(ax)cx\,a^2}{(a^2c\,x^2+c)^{\frac{5}{2}}}\right)}{a^2}$ | 94 |
| default | $\frac{(\arctan(ax)+i)(ax-i)\sqrt{c(ax-i)(ax+i)}}{2a\,c^2(a^2x^2+1)}+\frac{\sqrt{c(ax-i)(ax+i)}\,(ax+i)(\arctan(ax)-i)}{2a\,c^2(a^2x^2+1)}$ | 98 |

input
```
int(arctan(a*x)/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
4*(a^2*x^2+1)*x*arctan(a*x)/(a^2*c*x^2+c)^(3/2)+1/a^2*(a^2*x^2+1)^2*(a/(a^
2*x^2+1)/(a^2*c*x^2+c)^(3/2)-3*arctan(a*x)/(a^2*c*x^2+c)^(5/2)*c*x*a^2)
```

## Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.89

$$\int \frac{\arctan(ax)}{(c+a^2cx^2)^{3/2}}\,dx=\frac{\sqrt{a^2cx^2+c}(ax\arctan(ax)+1)}{a^3c^2x^2+ac^2}$$

input
```
integrate(arctan(a*x)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
sqrt(a^2*c*x^2 + c)*(a*x*arctan(a*x) + 1)/(a^3*c^2*x^2 + a*c^2)
```

## Sympy [F]

$$\int \frac{\arctan(ax)}{(c+a^2cx^2)^{3/2}}\,dx=\int \frac{\operatorname{atan}(ax)}{(c(a^2x^2+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(atan(a*x)/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(atan(a*x)/(c*(a**2*x**2 + 1))**(3/2), x)
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int \frac{\arctan(ax)}{(c+a^2cx^2)^{3/2}}\,dx = \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}c} + \frac{1}{\sqrt{a^2cx^2+c}ac}$$

input
```
integrate(arctan(a*x)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
x*arctan(a*x)/(sqrt(a^2*c*x^2 + c)*c) + 1/(sqrt(a^2*c*x^2 + c)*a*c)
```

## Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int \frac{\arctan(ax)}{(c+a^2cx^2)^{3/2}}\,dx = \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}c} + \frac{1}{\sqrt{a^2cx^2+c}ac}$$

input
```
integrate(arctan(a*x)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
x*arctan(a*x)/(sqrt(a^2*c*x^2 + c)*c) + 1/(sqrt(a^2*c*x^2 + c)*a*c)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\mathrm{atan}(a\,x)}{(c\,a^2\,x^2+c)^{3/2}}\,dx$$

input
```
int(atan(a*x)/(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(atan(a*x)/(c + a^2*c*x^2)^(3/2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.29

$$\int \frac{\arctan(ax)}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{c}\left(-2atan\left(\sqrt{a^2x^2+1}+ax\right)a^2x^2 - 2atan\left(\sqrt{a^2x^2+1}+ax\right)+\sqrt{a^2x^2+1}\,atan(a\right.}{a\,c^2\left(a^2x^2+1\right)}$$

input
```
int(atan(a*x)/(a^2*c*x^2+c)^(3/2),x)
```

output
```
(sqrt(c)*( - 2*atan(sqrt(a**2*x**2 + 1) + a*x)*a**2*x**2 - 2*atan(sqrt(a**
2*x**2 + 1) + a*x) + sqrt(a**2*x**2 + 1)*atan(a*x)*a*x + atan(a*x)*a**2*x*
*2 + atan(a*x) + sqrt(a**2*x**2 + 1)))/(a*c**2*(a**2*x**2 + 1))
```

# 3.236 $\int \frac{\arctan(ax)}{x(c+a^2cx^2)^{3/2}} \, dx$



## Optimal result

Integrand size = 22, antiderivative size = 229

$$\int \frac{\arctan(ax)}{x(c+a^2cx^2)^{3/2}} \, dx = -\frac{ax}{c\sqrt{c+a^2cx^2}} + \frac{\arctan(ax)}{c\sqrt{c+a^2cx^2}} - \frac{2\sqrt{1+a^2x^2}\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{c\sqrt{c+a^2cx^2}} + \frac{i\sqrt{1+a^2x^2}\,\text{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{c\sqrt{c+a^2cx^2}} - \frac{i\sqrt{1+a^2x^2}\,\text{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{c\sqrt{c+a^2cx^2}}$$

output

```
-a*x/c/(a^2*c*x^2+c)^(1/2)+arctan(a*x)/c/(a^2*c*x^2+c)^(1/2)-2*(a^2*x^2+1)
^(1/2)*arctan(a*x)*arctanh((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/c/(a^2*c*x^2+c
)^(1/2)+I*(a^2*x^2+1)^(1/2)*polylog(2,-(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/c/
(a^2*c*x^2+c)^(1/2)-I*(a^2*x^2+1)^(1/2)*polylog(2,(1+I*a*x)^(1/2)/(1-I*a*x
)^(1/2))/c/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.22 (sec), antiderivative size = 141, normalized size of antiderivative = 0.62

$$\int \frac{\arctan(ax)}{x\left(c+a^2cx^2\right)^{3/2}}\,dx = \frac{\sqrt{1+a^2x^2}\left(-\frac{ax}{\sqrt{1+a^2x^2}}+\frac{\arctan(ax)}{\sqrt{1+a^2x^2}}+\arctan(ax)\log\left(1-e^{i\arctan(ax)}\right)-\arctan(ax)\right.}{c\sqrt{c\left(1+\right.}}$$

input
```
Integrate[ArcTan[a*x]/(x*(c + a^2*c*x^2)^(3/2)),x]
```

output
```
(Sqrt[1 + a^2*x^2]*(-((a*x)/Sqrt[1 + a^2*x^2]) + ArcTan[a*x]/Sqrt[1 + a^2*
x^2] + ArcTan[a*x]*Log[1 - E^(I*ArcTan[a*x])] - ArcTan[a*x]*Log[1 + E^(I*A
rcTan[a*x])] + I*PolyLog[2, -E^(I*ArcTan[a*x])] - I*PolyLog[2, E^(I*ArcTan
[a*x])]))/(c*Sqrt[c*(1 + a^2*x^2)])
```

## Rubi [A] (verified)

Time = 0.73 (sec), antiderivative size = 182, normalized size of antiderivative = 0.79, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {5501, 5465, 208, 5493, 5489}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)}{x\left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 5501}$$

$$\frac{\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx}{c}-a^2\int \frac{x\arctan(ax)}{\left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 5465}$$

$$\frac{\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx}{c}-a^2\left(\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}}dx}{a}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \text{ 208}$$

$$\frac{\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx}{c} - a^2\left(\frac{x}{ac\sqrt{a^2cx^2+c}} - \frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \text{ 5493}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{x\sqrt{a^2x^2+1}}dx}{c\sqrt{a^2cx^2+c}} - a^2\left(\frac{x}{ac\sqrt{a^2cx^2+c}} - \frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \text{ 5489}$$

$$-a^2\left(\frac{x}{ac\sqrt{a^2cx^2+c}} - \frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right) + \frac{\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right) + i\,\text{PolyLog}\left(2, -\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right) - i\,\text{PolyLog}\left(2, \frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)\right)}{c\sqrt{a^2cx^2+c}}$$

input
```
Int[ArcTan[a*x]/(x*(c + a^2*c*x^2)^(3/2)),x]
```

output
```
-(a^2*(x/(a*c*Sqrt[c + a^2*c*x^2]) - ArcTan[a*x]/(a^2*c*Sqrt[c + a^2*c*x^2
]))) + (Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 -
 I*a*x]] + I*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])] - I*PolyLog[2,
 Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]]))/(c*Sqrt[c + a^2*c*x^2])
```

## Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5489
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sq
rt[1 - I*c*x]], x] + (Simp[I*(b/Sqrt[d])*PolyLog[2, -Sqrt[1 + I*c*x]/Sqrt[1
 - I*c*x]], x] - Simp[I*(b/Sqrt[d])*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x]], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

## Maple [A] (verified)

Time = 1.33 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.01

| method | result |
|---|---|
| default | $\frac{(\arctan(ax)+i)(iax+1)\sqrt{c(ax-i)(ax+i)}}{2(a^2x^2+1)c^2}-\frac{\sqrt{c(ax-i)(ax+i)}\,(iax-1)(\arctan(ax)-i)}{2(a^2x^2+1)c^2}-\frac{i\left(i\arctan(ax)\ln\left(1-\frac{iax+1}{\sqrt{a^2x^2+1}}\right)-i\,\mathrm{ar}\right.}{}$ |

input
```
int(arctan(a*x)/x/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(arctan(a*x)+I)*(1+I*a*x)*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)/c^2-1/
2*(c*(a*x-I)*(a*x+I))^(1/2)*(I*a*x-1)*(arctan(a*x)-I)/(a^2*x^2+1)/c^2-I*(I
*arctan(a*x)*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*arctan(a*x)*ln(1+(1+I*a*x
)/(a^2*x^2+1)^(1/2))+polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-polylog(2,-(1+
I*a*x)/(a^2*x^2+1)^(1/2)))*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/2)/c^2
```

## Fricas [F]

$$\int \frac{\arctan(ax)}{x\left(c+a^2cx^2\right)^{3/2}}\,dx=\int \frac{\arctan\left(ax\right)}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(arctan(a*x)/x/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)/(a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^
2*x), x)
```

## Sympy [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)}{x\left(c+a^2cx^2\right)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(atan(a*x)/x/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real zoo
```

## Maxima [F]

$$\int \frac{\arctan(ax)}{x\left(c+a^2cx^2\right)^{3/2}}\,dx=\int \frac{\arctan\left(ax\right)}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(arctan(a*x)/x/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)/((a^2*c*x^2 + c)^(3/2)*x), x)
```

## Giac [F]

$$\int \frac{\arctan(ax)}{x\left(c+a^2cx^2\right)^{3/2}}\,dx=\int \frac{\arctan\left(ax\right)}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(arctan(a*x)/x/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(arctan(a*x)/((a^2*c*x^2 + c)^(3/2)*x), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)}{x\left(c+a^2cx^2\right)^{3/2}}\,dx=\int \frac{\mathrm{atan}(a\,x)}{x\,\left(c\,a^2\,x^2+c\right)^{3/2}}\,dx$$

input
```
int(atan(a*x)/(x*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(atan(a*x)/(x*(c + a^2*c*x^2)^(3/2)), x)
```

## Reduce [F]

$$\int \frac{\arctan(ax)}{x\left(c+a^2cx^2\right)^{3/2}}\,dx=\frac{\int \frac{atan(ax)}{\sqrt{a^2x^2+1}\,a^2x^3+\sqrt{a^2x^2+1}\,x}dx}{\sqrt{c}\,c}$$

input
```
int(atan(a*x)/x/(a^2*c*x^2+c)^(3/2),x)
```

output
```
int(atan(a*x)/(sqrt(a**2*x**2 + 1)*a**2*x**3 + sqrt(a**2*x**2 + 1)*x),x)/(
sqrt(c)*c)
```

# 3.237 $\int \frac{\arctan(ax)}{x^2(c+a^2cx^2)^{3/2}}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 103

$$\int \frac{\arctan(ax)}{x^2(c+a^2cx^2)^{3/2}}\,dx = -\frac{a}{c\sqrt{c+a^2cx^2}} - \frac{a^2x\arctan(ax)}{c\sqrt{c+a^2cx^2}} - \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{c^2x} - \frac{a\operatorname{arctanh}\left(\frac{\sqrt{c+a^2cx^2}}{\sqrt{c}}\right)}{c^{3/2}}$$

output
```
-a/c/(a^2*c*x^2+c)^(1/2)-a^2*x*arctan(a*x)/c/(a^2*c*x^2+c)^(1/2)-(a^2*c*x^
2+c)^(1/2)*arctan(a*x)/c^2/x-a*arctanh((a^2*c*x^2+c)^(1/2)/c^(1/2))/c^(3/2
)
```

## Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.18

$$\int \frac{\arctan(ax)}{x^2(c+a^2cx^2)^{3/2}}\,dx = -\frac{a\sqrt{c(1+a^2x^2)}}{c^2(1+a^2x^2)} - \frac{\sqrt{c(1+a^2x^2)}(1+2a^2x^2)\arctan(ax)}{c^2x(1+a^2x^2)} + \frac{a\log(x)}{c^{3/2}} - \frac{a\log\left(c+\sqrt{c}\sqrt{c(1+a^2x^2)}\right)}{c^{3/2}}$$

input
```
Integrate[ArcTan[a*x]/(x^2*(c + a^2*c*x^2)^(3/2)),x]
```

output
```
-((a*Sqrt[c*(1 + a^2*x^2)])/(c^2*(1 + a^2*x^2))) - (Sqrt[c*(1 + a^2*x^2)]*
(1 + 2*a^2*x^2)*ArcTan[a*x])/(c^2*x*(1 + a^2*x^2)) + (a*Log[x])/c^(3/2) -
(a*Log[c + Sqrt[c]*Sqrt[c*(1 + a^2*x^2)]])/c^(3/2)
```

## Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {5501, 5429, 5479, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)}{x^2 (a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx}{c}-a^2\int \frac{\arctan(ax)}{(a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \text{5429}$$

$$\frac{\int \frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx}{c}-a^2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \text{5479}$$

$$\frac{a\int \frac{1}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}}{c}-a^2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \text{243}$$

$$\frac{\frac{1}{2}a\int \frac{1}{x^2\sqrt{a^2cx^2+c}}dx^2-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}}{c}-a^2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{\int \frac{1}{\frac{x^4}{a^2c}-\frac{1}{a^2}}d\sqrt{a^2cx^2+c}}{ac}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}}{c}-a^2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}}{c}-a^2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)$$

input
```
Int[ArcTan[a*x]/(x^2*(c + a^2*c*x^2)^(3/2)),x]
```

output
```
-(a^2*(1/(a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x])/(c*Sqrt[c + a^2*c*x^2
]))) + (-((Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(c*x)) - (a*ArcTanh[Sqrt[c + a
^2*c*x^2]/Sqrt[c]])/Sqrt[c])/c
```

## Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5429
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTan[c*x])/(d*Sqr
t[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.31 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.13

| method | result |
|---|---|
| default | $-\frac{\left(\ln\left(1+\frac{iax+1}{\sqrt{a^2x^2+1}}\right)a^3x^3-\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}-1\right)a^3x^3+2\arctan(ax)\sqrt{a^2x^2+1}\,a^2x^2+\sqrt{a^2x^2+1}\,ax+\ln\left(1+\frac{iax+1}{\sqrt{a^2x^2+1}}\right)ax-\ln\left(\frac{iax\cdots}{\sqrt{a^2x\cdots}}\right.\cdots}{x\,c^2(a^4x^4+2a^2x^2+1)}$ |

input
```
int(arctan(a*x)/x^2/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))*a^3*x^3-ln((1+I*a*x)/(a^2*x^2+1)^(1/2)
-1)*a^3*x^3+2*arctan(a*x)*(a^2*x^2+1)^(1/2)*a^2*x^2+(a^2*x^2+1)^(1/2)*a*x+
ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))*a*x-ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-1)*a*
x+arctan(a*x)*(a^2*x^2+1)^(1/2))*(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(a*x+I))^(1/
2)/x/c^2/(a^4*x^4+2*a^2*x^2+1)
```

## Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.01

$$\int \frac{\arctan(ax)}{x^2 (c+a^2cx^2)^{3/2}}\,dx = \frac{(a^3x^3+ax)\sqrt{c}\log\left(-\frac{a^2cx^2-2\sqrt{a^2cx^2+c}\sqrt{c}+2c}{x^2}\right)-2\sqrt{a^2cx^2+c}(ax+(2a^2x^2+1)\,\mathrm{ar}}{2(a^2c^2x^3+c^2x)}$$

input
```
integrate(arctan(a*x)/x^2/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
1/2*((a^3*x^3 + a*x)*sqrt(c)*log(-(a^2*c*x^2 - 2*sqrt(a^2*c*x^2 + c)*sqrt(
c) + 2*c)/x^2) - 2*sqrt(a^2*c*x^2 + c)*(a*x + (2*a^2*x^2 + 1)*arctan(a*x))
)/(a^2*c^2*x^3 + c^2*x)
```

## Sympy [F]

$$\int \frac{\arctan(ax)}{x^2 (c+a^2cx^2)^{3/2}}\,dx = \int \frac{\operatorname{atan}(ax)}{x^2 (c(a^2x^2+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(atan(a*x)/x**2/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(atan(a*x)/(x**2*(c*(a**2*x**2 + 1))**(3/2)), x)
```

## Maxima [F]

$$\int \frac{\arctan(ax)}{x^2 (c+a^2cx^2)^{3/2}}\,dx = \int \frac{\arctan(ax)}{(a^2cx^2+c)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(arctan(a*x)/x^2/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)/((a^2*c*x^2 + c)^(3/2)*x^2), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)}{x^2 (c + a^2cx^2)^{3/2}} dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)/x^2/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)}{x^2 (c + a^2cx^2)^{3/2}} dx = \int \frac{\mathrm{atan}(a\,x)}{x^2\,(c\,a^2\,x^2 + c)^{3/2}} dx$$

input
```
int(atan(a*x)/(x^2*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(atan(a*x)/(x^2*(c + a^2*c*x^2)^(3/2)), x)
```

## Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.54

$$\int \frac{\arctan(ax)}{x^2 (c + a^2cx^2)^{3/2}} dx = \frac{\sqrt{c}\left(-2\sqrt{a^2x^2+1}\,atan(ax)\,a^2x^2 - \sqrt{a^2x^2+1}\,atan(ax) - \sqrt{a^2x^2+1}\,ax + \log(\sqrt{}\right.}{}$$

input
```
int(atan(a*x)/x^2/(a^2*c*x^2+c)^(3/2),x)
```

output

```
(sqrt(c)*( - 2*sqrt(a**2*x**2 + 1)*atan(a*x)*a**2*x**2 - sqrt(a**2*x**2 +
1)*atan(a*x) - sqrt(a**2*x**2 + 1)*a*x + log(sqrt(a**2*x**2 + 1) + a*x - 1
)*a**3*x**3 + log(sqrt(a**2*x**2 + 1) + a*x - 1)*a*x - log(sqrt(a**2*x**2
+ 1) + a*x + 1)*a**3*x**3 - log(sqrt(a**2*x**2 + 1) + a*x + 1)*a*x))/(c**2
*x*(a**2*x**2 + 1))
```

# 3.238 $\int \frac{\arctan(ax)}{x^3(c+a^2cx^2)^{3/2}}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 300

$$\int \frac{\arctan(ax)}{x^3\left(c+a^2cx^2\right)^{3/2}}\,dx = \frac{a^3x}{c\sqrt{c+a^2cx^2}} - \frac{a\sqrt{c+a^2cx^2}}{2c^2x} - \frac{a^2\arctan(ax)}{c\sqrt{c+a^2cx^2}} - \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{2c^2x^2} + \frac{3a^2\sqrt{1+a^2x^2}\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{c\sqrt{c+a^2cx^2}} - \frac{3ia^2\sqrt{1+a^2x^2}\,\text{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2c\sqrt{c+a^2cx^2}} + \frac{3ia^2\sqrt{1+a^2x^2}\,\text{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2c\sqrt{c+a^2cx^2}}$$

output
```
a^3*x/c/(a^2*c*x^2+c)^(1/2)-1/2*a*(a^2*c*x^2+c)^(1/2)/c^2/x-a^2*arctan(a*x
)/c/(a^2*c*x^2+c)^(1/2)-1/2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)/c^2/x^2+3*a^2*
(a^2*x^2+1)^(1/2)*arctan(a*x)*arctanh((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/c/(
a^2*c*x^2+c)^(1/2)-3/2*I*a^2*(a^2*x^2+1)^(1/2)*polylog(2,-(1+I*a*x)^(1/2)/
(1-I*a*x)^(1/2))/c/(a^2*c*x^2+c)^(1/2)+3/2*I*a^2*(a^2*x^2+1)^(1/2)*polylog
(2,(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/c/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (warning: unable to verify)

Time = 0.95 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.86

$$\int \frac{\arctan(ax)}{x^3 (c+a^2cx^2)^{3/2}} \, dx =$$
$$-\frac{a^2\left(-8ax+8\arctan(ax)+ax\csc^2\left(\frac{1}{2}\arctan(ax)\right)+\sqrt{1+a^2x^2}\arctan(ax)\csc^2\left(\frac{1}{2}\arctan(ax)\right)+12\sqrt{1-\right.}{}$$

input
```
Integrate[ArcTan[a*x]/(x^3*(c + a^2*c*x^2)^(3/2)),x]
```

output
```
-1/8*(a^2*(-8*a*x + 8*ArcTan[a*x] + a*x*Csc[ArcTan[a*x]/2]^2 + Sqrt[1 + a^
2*x^2]*ArcTan[a*x]*Csc[ArcTan[a*x]/2]^2 + 12*Sqrt[1 + a^2*x^2]*ArcTan[a*x]
*Log[1 - E^(I*ArcTan[a*x])] - 12*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Log[1 + E^(
I*ArcTan[a*x])] + (12*I)*Sqrt[1 + a^2*x^2]*PolyLog[2, -E^(I*ArcTan[a*x])]
- (12*I)*Sqrt[1 + a^2*x^2]*PolyLog[2, E^(I*ArcTan[a*x])] - Sqrt[1 + a^2*x^
2]*ArcTan[a*x]*Sec[ArcTan[a*x]/2]^2 + 2*Sqrt[1 + a^2*x^2]*Tan[ArcTan[a*x]/
2]))/(c*Sqrt[c + a^2*c*x^2])
```

## Rubi [A] (verified)

Time = 1.79 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.25, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {5501, 5497, 242, 5493, 5489, 5501, 5465, 208, 5493, 5489}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)}{x^3 (a^2cx^2+c)^{3/2}} \, dx$$

$$\Big\downarrow \text{ 5501}$$

$$\frac{\int \frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx}{c} - a^2\int \frac{\arctan(ax)}{x (a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \text{ 5497}$$

$$\frac{-\frac{1}{2}a^2\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx+\frac{1}{2}a\int \frac{1}{x^2\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{2cx^2}}{c}-a^2\int \frac{\arctan(ax)}{x\left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 242}$$

$$\frac{-\frac{1}{2}a^2\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{2cx^2}-\frac{a\sqrt{a^2cx^2+c}}{2cx}}{c}-a^2\int \frac{\arctan(ax)}{x\left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 5493}$$

$$\frac{-\frac{a^2\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{x\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{2cx^2}-\frac{a\sqrt{a^2cx^2+c}}{2cx}}{c}-a^2\int \frac{\arctan(ax)}{x\left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 5489}$$

$$-a^2\int \frac{\arctan(ax)}{x\left(a^2cx^2+c\right)^{3/2}}dx+$$
$$\frac{-\frac{a^2\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{PolyLog}\left(2,\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)\right)}{2\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{2cx^2}-\frac{a\sqrt{a^2cx^2+}}{2cx}}{c}$$

$$\Big\downarrow \text{ 5501}$$

$$-a^2\left(\frac{\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx}{c}-a^2\int \frac{x\arctan(ax)}{\left(a^2cx^2+c\right)^{3/2}}dx\right)+$$
$$\frac{-\frac{a^2\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{PolyLog}\left(2,\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)\right)}{2\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{2cx^2}-\frac{a\sqrt{a^2cx^2+}}{2cx}}{c}$$

$$\Big\downarrow \text{ 5465}$$

$$-a^2\left(\frac{\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx}{c}-a^2\left(\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}}dx}{a}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right)\right)+$$
$$\frac{-\frac{a^2\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{PolyLog}\left(2,\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)\right)}{2\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{2cx^2}-\frac{a\sqrt{a^2cx^2+}}{2cx}}{c}$$

$$\Big\downarrow \text{ 208}$$

$$-a^2\left(\frac{\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx}{c}-a^2\left(\frac{x}{ac\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right)\right)+$$
$$\frac{-\frac{a^2\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{PolyLog}\left(2,\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)\right)}{2\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{2cx^2}-\frac{a\sqrt{a^2cx^2+}}{2cx}}{c}$$

$$\Big\downarrow \text{5493}$$

$$-a^2\left(\frac{\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{x\sqrt{a^2x^2+1}}dx}{c\sqrt{a^2cx^2+c}}-a^2\left(\frac{x}{ac\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right)\right)+\frac{-\frac{a^2\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{PolyLog}\left(2,\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)\right)}{2\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{2cx^2}-\frac{a\sqrt{a^2cx^2+}}{2cx}}{c}$$

$$\Big\downarrow \text{5489}$$

$$\frac{-\frac{a^2\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{PolyLog}\left(2,\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)\right)}{2\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{2cx^2}-\frac{a\sqrt{a^2cx^2+}}{2cx}}{c}$$

$$a^2\left(-a^2\left(\frac{x}{ac\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right)+\frac{\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{}}{\sqrt{}}\right.\right.}{c\sqrt{a^2cx^2+c}}\right.$$

input
```
Int[ArcTan[a*x]/(x^3*(c + a^2*c*x^2)^(3/2)),x]
```

output
```
(-1/2*(a*Sqrt[c + a^2*c*x^2])/(c*x) - (Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(2
*c*x^2) - (a^2*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/S
qrt[1 - I*a*x]] + I*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])] - I*Pol
yLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]]))/(2*Sqrt[c + a^2*c*x^2]))/c - a^
2*(-(a^2*(x/(a*c*Sqrt[c + a^2*c*x^2]) - ArcTan[a*x]/(a^2*c*Sqrt[c + a^2*c*
x^2]))) + (Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[
1 - I*a*x]] + I*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])] - I*PolyLog
[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]]))/(c*Sqrt[c + a^2*c*x^2]))
```

## Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5489
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sq
rt[1 - I*c*x]], x] + (Simp[I*(b/Sqrt[d])*PolyLog[2, -Sqrt[1 + I*c*x]/Sqrt[1
 - I*c*x]], x] - Simp[I*(b/Sqrt[d])*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x]], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5497
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)^(m
 + 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[c^2*((m
+ 2)/(f^2*(m + 1)))   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x
^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
 && LtQ[m, -1] && NeQ[m, -2]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

## Maple [A] (verified)

Time = 1.39 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.91

| method | result |
|---|---|
| default | $-\frac{a^2(\arctan(ax)+i)(iax+1)\sqrt{c(ax-i)(ax+i)}}{2(a^2x^2+1)c^2}+\frac{\sqrt{c(ax-i)(ax+i)}\,(iax-1)(\arctan(ax)-i)a^2}{2(a^2x^2+1)c^2}-\frac{(ax+\arctan(ax))\sqrt{c(ax-i)(ax}}{2c^2x^2}$ |

input
```
int(arctan(a*x)/x^3/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^2*(arctan(a*x)+I)*(1+I*a*x)*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)/c
^2+1/2*(c*(a*x-I)*(a*x+I))^(1/2)*(I*a*x-1)*(arctan(a*x)-I)*a^2/(a^2*x^2+1)
/c^2-1/2*(a*x+arctan(a*x))*(c*(a*x-I)*(a*x+I))^(1/2)/c^2/x^2+3/2*I*a^2*(I*
arctan(a*x)*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*arctan(a*x)*ln(1+(1+I*a*x)
/(a^2*x^2+1)^(1/2))+polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-polylog(2,-(1+I
*a*x)/(a^2*x^2+1)^(1/2)))*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/2)/c^2
```

## Fricas [F]

$$\int \frac{\arctan(ax)}{x^3\,(c+a^2cx^2)^{3/2}}\,dx=\int \frac{\arctan{(ax)}}{(a^2cx^2+c)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate(arctan(a*x)/x^3/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)/(a^4*c^2*x^7 + 2*a^2*c^2*x^5 + c^
2*x^3), x)
```

## Sympy [F]

$$\int \frac{\arctan(ax)}{x^3 (c + a^2 c x^2)^{3/2}} \, dx = \int \frac{\operatorname{atan}(ax)}{x^3 (c (a^2 x^2 + 1))^{\frac{3}{2}}} \, dx$$

input
```
integrate(atan(a*x)/x**3/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(atan(a*x)/(x**3*(c*(a**2*x**2 + 1))**(3/2)), x)
```

## Maxima [F]

$$\int \frac{\arctan(ax)}{x^3 (c + a^2 c x^2)^{3/2}} \, dx = \int \frac{\arctan(ax)}{(a^2 c x^2 + c)^{\frac{3}{2}} x^3} \, dx$$

input
```
integrate(arctan(a*x)/x^3/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)/((a^2*c*x^2 + c)^(3/2)*x^3), x)
```

## Giac [F]

$$\int \frac{\arctan(ax)}{x^3 (c + a^2 c x^2)^{3/2}} \, dx = \int \frac{\arctan(ax)}{(a^2 c x^2 + c)^{\frac{3}{2}} x^3} \, dx$$

input
```
integrate(arctan(a*x)/x^3/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(arctan(a*x)/((a^2*c*x^2 + c)^(3/2)*x^3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)}{x^3 (c + a^2cx^2)^{3/2}} \, dx = \int \frac{\mathrm{atan}(a\,x)}{x^3 \left(c\,a^2\,x^2 + c\right)^{3/2}} \, dx$$

input
```
int(atan(a*x)/(x^3*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(atan(a*x)/(x^3*(c + a^2*c*x^2)^(3/2)), x)
```

## Reduce [F]

$$\int \frac{\arctan(ax)}{x^3 (c + a^2cx^2)^{3/2}} \, dx = \frac{\int \frac{atan(ax)}{\sqrt{a^2x^2+1}\,a^2x^5+\sqrt{a^2x^2+1}\,x^3}dx}{\sqrt{c}\,c}$$

input
```
int(atan(a*x)/x^3/(a^2*c*x^2+c)^(3/2),x)
```

output
```
int(atan(a*x)/(sqrt(a**2*x**2 + 1)*a**2*x**5 + sqrt(a**2*x**2 + 1)*x**3),x
)/(sqrt(c)*c)
```

# 3.239 $\int \frac{\arctan(ax)}{x^4(c+a^2cx^2)^{3/2}}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 165

$$\int \frac{\arctan(ax)}{x^4\left(c+a^2cx^2\right)^{3/2}}\,dx = \frac{a^3}{c\sqrt{c+a^2cx^2}} - \frac{a\sqrt{c+a^2cx^2}}{6c^2x^2} + \frac{a^4x\arctan(ax)}{c\sqrt{c+a^2cx^2}} - \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{3c^2x^3} + \frac{5a^2\sqrt{c+a^2cx^2}\arctan(ax)}{3c^2x} + \frac{11a^3\operatorname{arctanh}\left(\frac{\sqrt{c+a^2cx^2}}{\sqrt{c}}\right)}{6c^{3/2}}$$

output
```
a^3/c/(a^2*c*x^2+c)^(1/2)-1/6*a*(a^2*c*x^2+c)^(1/2)/c^2/x^2+a^4*x*arctan(a
*x)/c/(a^2*c*x^2+c)^(1/2)-1/3*(a^2*c*x^2+c)^(1/2)*arctan(a*x)/c^2/x^3+5/3*
a^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)/c^2/x+11/6*a^3*arctanh((a^2*c*x^2+c)^(
1/2)/c^(1/2))/c^(3/2)
```

## Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.87

$$\int \frac{\arctan(ax)}{x^4\left(c+a^2cx^2\right)^{3/2}}\,dx = \frac{\frac{a(-1+5a^2x^2)\sqrt{c+a^2cx^2}}{x^2+a^2x^4} + \frac{2\sqrt{c+a^2cx^2}(-1+4a^2x^2+8a^4x^4)\arctan(ax)}{x^3+a^2x^5} - 11a^3\sqrt{c}\log(x) + 11a^3\sqrt{\phantom{x}}}{6c^2}$$

input
```
Integrate[ArcTan[a*x]/(x^4*(c + a^2*c*x^2)^(3/2)),x]
```

output
```
((a*(-1 + 5*a^2*x^2)*Sqrt[c + a^2*c*x^2])/(x^2 + a^2*x^4) + (2*Sqrt[c + a^
2*c*x^2]*(-1 + 4*a^2*x^2 + 8*a^4*x^4)*ArcTan[a*x])/(x^3 + a^2*x^5) - 11*a^
3*Sqrt[c]*Log[x] + 11*a^3*Sqrt[c]*Log[c + Sqrt[c]*Sqrt[c + a^2*c*x^2]])/(6
*c^2)
```

## Rubi [A] (verified)

Time = 1.49 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.64, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {5501, 5497, 243, 52, 73, 221, 5479, 243, 73, 221, 5501, 5429, 5479, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)}{x^4 (a^2cx^2+c)^{3/2}} \, dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{\arctan(ax)}{x^4\sqrt{a^2cx^2+c}}dx}{c} - a^2 \int \frac{\arctan(ax)}{x^2 (a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \text{5497}$$

$$\frac{-\frac{2}{3}a^2 \int \frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx + \frac{1}{3}a \int \frac{1}{x^3\sqrt{a^2cx^2+c}}dx - \frac{\arctan(ax)\sqrt{a^2cx^2+c}}{3cx^3}}{c} - a^2 \int \frac{\arctan(ax)}{x^2 (a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{-\frac{2}{3}a^2 \int \frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx + \frac{1}{6}a \int \frac{1}{x^4\sqrt{a^2cx^2+c}}dx^2 - \frac{\arctan(ax)\sqrt{a^2cx^2+c}}{3cx^3}}{c} - a^2 \int \frac{\arctan(ax)}{x^2 (a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \text{52}$$

$$\frac{-\frac{2}{3}a^2 \int \frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx + \frac{1}{6}a\left(-\frac{1}{2}a^2 \int \frac{1}{x^2\sqrt{a^2cx^2+c}}dx^2 - \frac{\sqrt{a^2cx^2+c}}{cx^2}\right) - \frac{\arctan(ax)\sqrt{a^2cx^2+c}}{3cx^3}}{c} - a^2 \int \frac{\arctan(ax)}{x^2 (a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \text{73}$$

$$\frac{-\frac{2}{3}a^2\int \frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx+\frac{1}{6}a\left(-\frac{\int \frac{1}{\frac{x^4}{a^2c}-\frac{1}{a^2}}d\sqrt{a^2cx^2+c}}{c}-\frac{\sqrt{a^2cx^2+c}}{cx^2}\right)-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{3cx^3}}{c}-a^2\int \frac{\arctan(ax)}{x^2\left(a^2cx^2+c\right)^{3/2}}dx$$

$\big\downarrow$ 221

$$\frac{-\frac{2}{3}a^2\int \frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{3cx^3}+\frac{1}{6}a\left(\frac{a^2\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{\sqrt{a^2cx^2+c}}{cx^2}\right)}{c}-a^2\int \frac{\arctan(ax)}{x^2\left(a^2cx^2+c\right)^{3/2}}dx$$

$\big\downarrow$ 5479

$$\frac{-\frac{2}{3}a^2\left(a\int \frac{1}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right)-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{3cx^3}+\frac{1}{6}a\left(\frac{a^2\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{\sqrt{a^2cx^2+c}}{cx^2}\right)}{c}-a^2\int \frac{\arctan(ax)}{x^2\left(a^2cx^2+c\right)^{3/2}}dx$$

$\big\downarrow$ 243

$$\frac{-\frac{2}{3}a^2\left(\frac{1}{2}a\int \frac{1}{x^2\sqrt{a^2cx^2+c}}dx^2-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right)-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{3cx^3}+\frac{1}{6}a\left(\frac{a^2\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{\sqrt{a^2cx^2+c}}{cx^2}\right)}{c}-a^2\int \frac{\arctan(ax)}{x^2\left(a^2cx^2+c\right)^{3/2}}dx$$

$\big\downarrow$ 73

$$\frac{-\frac{2}{3}a^2\left(\frac{\int \frac{1}{\frac{x^4}{a^2c}-\frac{1}{a^2}}d\sqrt{a^2cx^2+c}}{ac}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right)-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{3cx^3}+\frac{1}{6}a\left(\frac{a^2\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{\sqrt{a^2cx^2+c}}{cx^2}\right)}{c}-a^2\int \frac{\arctan(ax)}{x^2\left(a^2cx^2+c\right)^{3/2}}dx$$

$\big\downarrow$ 221

$$\frac{-\frac{2}{3}a^2\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{3cx^3}+\frac{1}{6}a\left(\frac{a^2\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{\sqrt{a^2cx^2+c}}{cx^2}\right.}{c}$$

$$a^2\int\frac{\arctan(ax)}{x^2\left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 5501}$$

$$\frac{-\frac{2}{3}a^2\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{3cx^3}+\frac{1}{6}a\left(\frac{a^2\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{\sqrt{a^2cx^2+c}}{cx^2}\right.}{c}$$

$$a^2\left(\frac{\int\frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx}{c}-a^2\int\frac{\arctan(ax)}{\left(a^2cx^2+c\right)^{3/2}}dx\right)$$

$$\Big\downarrow \text{ 5429}$$

$$\frac{-\frac{2}{3}a^2\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{3cx^3}+\frac{1}{6}a\left(\frac{a^2\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{\sqrt{a^2cx^2+c}}{cx^2}\right.}{c}$$

$$a^2\left(\frac{\int\frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx}{c}-a^2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)\right)$$

$$\Big\downarrow \text{ 5479}$$

$$\frac{-\frac{2}{3}a^2\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{3cx^3}+\frac{1}{6}a\left(\frac{a^2\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{\sqrt{a^2cx^2+c}}{cx^2}\right.}{c}$$

$$a^2\left(\frac{a\int\frac{1}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}}{c}-a^2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)\right)$$

$$\Big\downarrow \text{ 243}$$

$$\frac{-\frac{2}{3}a^2\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{3cx^3}+\frac{1}{6}a\left(\frac{a^2\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{\sqrt{a^2cx^2+c}}{cx^2}\right.}{c}$$

$$a^2\left(\frac{\frac{1}{2}a\int\frac{1}{x^2\sqrt{a^2cx^2+c}}dx^2-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}}{c}-a^2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{-\frac{2}{3}a^2\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{3cx^3}+\frac{1}{6}a\left(\frac{a^2\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{\sqrt{a^2cx^2+c}}{cx^2}\right.}{c}$$

$$a^2\left(\frac{\frac{\int\frac{1}{\frac{x^4}{a^2c}-\frac{1}{a^2}}d\sqrt{a^2cx^2+c}}{ac}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}}{c}-a^2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{-\frac{2}{3}a^2\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{3cx^3}+\frac{1}{6}a\left(\frac{a^2\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{\sqrt{a^2cx^2+c}}{cx^2}\right.}{c}$$

$$a^2\left(\frac{-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}}{c}-a^2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)\right)$$

input
```
Int[ArcTan[a*x]/(x^4*(c + a^2*c*x^2)^(3/2)),x]
```

output
```
-(a^2*(-(a^2*(1/(a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x])/(c*Sqrt[c + a^
2*c*x^2]))) + (-((Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(c*x)) - (a*ArcTanh[Sqr
t[c + a^2*c*x^2]/Sqrt[c]])/Sqrt[c])/c)) + (-1/3*(Sqrt[c + a^2*c*x^2]*ArcTa
n[a*x])/(c*x^3) - (2*a^2*(-((Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(c*x)) - (a*
ArcTanh[Sqrt[c + a^2*c*x^2]/Sqrt[c]])/Sqrt[c]))/3 + (a*(-(Sqrt[c + a^2*c*x
^2]/(c*x^2)) + (a^2*ArcTanh[Sqrt[c + a^2*c*x^2]/Sqrt[c]])/Sqrt[c]))/6)/c
```

### Defintions of rubi rules used

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5429
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTan[c*x])/(d*Sqr
t[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5497
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)^(m
 + 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[c^2*((m
+ 2)/(f^2*(m + 1)))   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x
^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
 && LtQ[m, -1] && NeQ[m, -2]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.40 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.65

| method | result |
|---|---|
| default | $\frac{\sqrt{c(ax-i)(ax+i)}\,\sqrt{a^2x^2+1}\left(11\ln\left(1+\frac{iax+1}{\sqrt{a^2x^2+1}}\right)a^5x^5-11\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}-1\right)a^5x^5+16\arctan(ax)\sqrt{a^2x^2+1}\,a^4x^4+5\sqrt{a^2x^2+1}\,a^3\cdots\right.}{6x^3c^2(a^4x^4+\cdots}$ |

input
```
int(arctan(a*x)/x^4/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(c*(a*x-I)*(a*x+I))^(1/2)*(a^2*x^2+1)^(1/2)*(11*ln(1+(1+I*a*x)/(a^2*x^
2+1)^(1/2))*a^5*x^5-11*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-1)*a^5*x^5+16*arctan
(a*x)*(a^2*x^2+1)^(1/2)*a^4*x^4+5*(a^2*x^2+1)^(1/2)*a^3*x^3+11*ln(1+(1+I*a
*x)/(a^2*x^2+1)^(1/2))*a^3*x^3-11*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-1)*a^3*x^
3+8*arctan(a*x)*(a^2*x^2+1)^(1/2)*a^2*x^2-(a^2*x^2+1)^(1/2)*a*x-2*arctan(a
*x)*(a^2*x^2+1)^(1/2))/x^3/c^2/(a^4*x^4+2*a^2*x^2+1)
```

## Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.78

$$\int \frac{\arctan(ax)}{x^4 (c + a^2cx^2)^{3/2}}\,dx = \frac{11\,(a^5x^5 + a^3x^3)\sqrt{c}\log\left(-\frac{a^2cx^2+2\,\sqrt{a^2cx^2+c}\sqrt{c}+2\,c}{x^2}\right) + 2\,(5\,a^3x^3 - ax + 2\,(8\,a^4x^4 + 4}{12\,(a^2c^2x^5 + c^2x^3)}$$

input
```
integrate(arctan(a*x)/x^4/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
1/12*(11*(a^5*x^5 + a^3*x^3)*sqrt(c)*log(-(a^2*c*x^2 + 2*sqrt(a^2*c*x^2 +
c)*sqrt(c) + 2*c)/x^2) + 2*(5*a^3*x^3 - a*x + 2*(8*a^4*x^4 + 4*a^2*x^2 - 1
)*arctan(a*x))*sqrt(a^2*c*x^2 + c))/(a^2*c^2*x^5 + c^2*x^3)
```

## Sympy [F]

$$\int \frac{\arctan(ax)}{x^4 (c + a^2cx^2)^{3/2}}\,dx = \int \frac{\operatorname{atan}(ax)}{x^4 (c(a^2x^2 + 1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(atan(a*x)/x**4/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(atan(a*x)/(x**4*(c*(a**2*x**2 + 1))**(3/2)), x)
```

## Maxima [F]

$$\int \frac{\arctan(ax)}{x^4 (c + a^2cx^2)^{3/2}}\,dx = \int \frac{\arctan(ax)}{(a^2cx^2 + c)^{\frac{3}{2}}x^4}\,dx$$

input
```
integrate(arctan(a*x)/x^4/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)/((a^2*c*x^2 + c)^(3/2)*x^4), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)}{x^4 (c + a^2cx^2)^{3/2}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)/x^4/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Degree mismatch inside factorisatio
n over extensionDegree mismatch inside factorisation over extensionindex.c
c index_m
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)}{x^4 (c + a^2cx^2)^{3/2}} \, dx = \int \frac{\mathrm{atan}(a\,x)}{x^4\,(c\,a^2\,x^2 + c)^{3/2}} \, dx$$

input
```
int(atan(a*x)/(x^4*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(atan(a*x)/(x^4*(c + a^2*c*x^2)^(3/2)), x)
```

## Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.27

$$\int \frac{\arctan(ax)}{x^4 (c + a^2cx^2)^{3/2}} \, dx = \frac{\sqrt{c}\,(16\sqrt{a^2x^2+1}\,atan(ax)\,a^4x^4 + 8\sqrt{a^2x^2+1}\,atan(ax)\,a^2x^2 - 2\sqrt{a^2x^2+1}\,atan}{}$$

input
```
int(atan(a*x)/x^4/(a^2*c*x^2+c)^(3/2),x)
```

output

```
(sqrt(c)*(16*sqrt(a**2*x**2 + 1)*atan(a*x)*a**4*x**4 + 8*sqrt(a**2*x**2 +
1)*atan(a*x)*a**2*x**2 - 2*sqrt(a**2*x**2 + 1)*atan(a*x) + 5*sqrt(a**2*x**
2 + 1)*a**3*x**3 - sqrt(a**2*x**2 + 1)*a*x - 11*log(sqrt(a**2*x**2 + 1) +
a*x - 1)*a**5*x**5 - 11*log(sqrt(a**2*x**2 + 1) + a*x - 1)*a**3*x**3 + 11*
log(sqrt(a**2*x**2 + 1) + a*x + 1)*a**5*x**5 + 11*log(sqrt(a**2*x**2 + 1)
+ a*x + 1)*a**3*x**3))/(6*c**2*x**3*(a**2*x**2 + 1))
```

# 3.240 $\int \frac{x^5 \arctan(ax)}{(c+a^2cx^2)^{5/2}} dx$



## Optimal result

Integrand size = 22, antiderivative size = 170

$$\int \frac{x^5 \arctan(ax)}{(c+a^2cx^2)^{5/2}} dx = -\frac{x^3}{9a^3c(c+a^2cx^2)^{3/2}} - \frac{5x}{3a^5c^2\sqrt{c+a^2cx^2}} + \frac{x^2\arctan(ax)}{3a^4c(c+a^2cx^2)^{3/2}} + \frac{5\arctan(ax)}{3a^6c^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{a^6c^3} - \frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right)}{a^6c^{5/2}}$$

output

```
-1/9*x^3/a^3/c/(a^2*c*x^2+c)^(3/2)-5/3*x/a^5/c^2/(a^2*c*x^2+c)^(1/2)+1/3*x
^2*arctan(a*x)/a^4/c/(a^2*c*x^2+c)^(3/2)+5/3*arctan(a*x)/a^6/c^2/(a^2*c*x^
2+c)^(1/2)+(a^2*c*x^2+c)^(1/2)*arctan(a*x)/a^6/c^3-arctanh(a*c^(1/2)*x/(a^
2*c*x^2+c)^(1/2))/a^6/c^(5/2)
```

## Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.77

$$\int \frac{x^5 \arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx = -\frac{ax(15+16a^2x^2)\sqrt{c+a^2cx^2}-3\sqrt{c+a^2cx^2}(8+12a^2x^2+3a^4x^4)\arctan(ax)+9\sqrt{c}(1+a^2x^2)^2\log\left(acx\right.}{9a^6c^3\left(1+a^2x^2\right)^2}$$

input
```
Integrate[(x^5*ArcTan[a*x])/(c + a^2*c*x^2)^(5/2),x]
```

output
```
-1/9*(a*x*(15 + 16*a^2*x^2)*Sqrt[c + a^2*c*x^2] - 3*Sqrt[c + a^2*c*x^2]*(8
 + 12*a^2*x^2 + 3*a^4*x^4)*ArcTan[a*x] + 9*Sqrt[c]*(1 + a^2*x^2)^2*Log[a*c
*x + Sqrt[c]*Sqrt[c + a^2*c*x^2]])/(a^6*c^3*(1 + a^2*x^2)^2)
```

## Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.48, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {5499, 5473, 5465, 208, 5499, 5465, 208, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 \arctan(ax)}{(a^2cx^2+c)^{5/2}}\,dx$$

$$\Big\downarrow \text{5499}$$

$$\frac{\int \frac{x^3\arctan(ax)}{(a^2cx^2+c)^{3/2}}dx}{a^2c}-\frac{\int \frac{x^3\arctan(ax)}{(a^2cx^2+c)^{5/2}}dx}{a^2}$$

$$\Big\downarrow \text{5473}$$

$$\frac{\int \frac{x^3\arctan(ax)}{(a^2cx^2+c)^{3/2}}dx}{a^2c}-\frac{\frac{2\int \frac{x\arctan(ax)}{(a^2cx^2+c)^{3/2}}dx}{3a^2c}-\frac{x^2\arctan(ax)}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{x^3}{9ac(a^2cx^2+c)^{3/2}}}{a^2}$$

$$\Big\downarrow \text{5465}$$

$$\frac{\int \frac{x^3 \arctan(ax)}{(a^2cx^2+c)^{3/2}}dx}{a^2c}-\frac{\frac{2\left(\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}}dx}{a}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{x^2\arctan(ax)}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{x^3}{9ac(a^2cx^2+c)^{3/2}}}{a^2}$$

$$\Big\downarrow \; 208$$

$$\frac{\int \frac{x^3 \arctan(ax)}{(a^2cx^2+c)^{3/2}}dx}{a^2c}-\frac{-\frac{x^2\arctan(ax)}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x}{ac\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{x^3}{9ac(a^2cx^2+c)^{3/2}}}{a^2}$$

$$\Big\downarrow \; 5499$$

$$\frac{\frac{\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a^2c}-\frac{\int \frac{x\arctan(ax)}{\left(a^2cx^2+c\right)^{3/2}}dx}{a^2}}{a^2c}-\frac{-\frac{x^2\arctan(ax)}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x}{ac\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{x^3}{9ac(a^2cx^2+c)^{3/2}}}{a^2}$$

$$\Big\downarrow \; 5465$$

$$\frac{\frac{\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\int \frac{1}{\sqrt{a^2cx^2+c}}dx}{a}}{a^2c}-\frac{\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}}dx}{a}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-\frac{-\frac{x^2\arctan(ax)}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x}{ac\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{x^3}{9ac(a^2cx^2+c)^{3/2}}}{a^2}$$

$$\Big\downarrow \; 208$$

$$\frac{\frac{\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\int \frac{1}{\sqrt{a^2cx^2+c}}dx}{a}}{a^2c}-\frac{\frac{x}{ac\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-\frac{-\frac{x^2\arctan(ax)}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x}{ac\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{x^3}{9ac(a^2cx^2+c)^{3/2}}}{a^2}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\int \frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}}{a}}{a^2c}-\frac{\frac{x}{ac\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-\frac{-\frac{x^2\arctan(ax)}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x}{ac\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{x^3}{9ac(a^2cx^2+c)^{3/2}}}{a^2}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\frac{\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a^2\sqrt{c}}}{a^2c}-\frac{\frac{x}{ac\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-\frac{-\frac{x^2\arctan(ax)}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x}{ac\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{x^3}{9ac(a^2cx^2+c)^{3/2}}}{a^2}$$

input
```
Int[(x^5*ArcTan[a*x])/(c + a^2*c*x^2)^(5/2),x]
```

output
```
-((x^3/(9*a*c*(c + a^2*c*x^2)^(3/2)) - (x^2*ArcTan[a*x])/(3*a^2*c*(c + a^2
*c*x^2)^(3/2)) + (2*(x/(a*c*Sqrt[c + a^2*c*x^2]) - ArcTan[a*x]/(a^2*c*Sqrt
[c + a^2*c*x^2])))/(3*a^2*c))/a^2) + (-((x/(a*c*Sqrt[c + a^2*c*x^2]) - Arc
Tan[a*x]/(a^2*c*Sqrt[c + a^2*c*x^2]))/a^2) + ((Sqrt[c + a^2*c*x^2]*ArcTan[
a*x])/(a^2*c) - ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]]/(a^2*Sqrt[c]))/
(a^2*c))/(a^2*c)
```

## Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5473
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[b*(f*x)^m*((d + e*x^2)^(q + 1)/(c*d*m^2)), x] +
 (-Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(c^2*d*m))
, x] + Simp[f^2*((m - 1)/(c^2*d*m))   Int[(f*x)^(m - 2)*(d + e*x^2)^(q + 1)
*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2
*d] && EqQ[m + 2*q + 2, 0] && LtQ[q, -1]
```

rule 5499
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ
[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.36 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.92

| method | result |
|---|---|
| default | $\frac{\left(9 \arctan(ax)\sqrt{a^2x^2+1}\, a^4x^4 - 9\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}+i\right)a^4x^4 + 9\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}-i\right)a^4x^4 - 16\sqrt{a^2x^2+1}\, a^3x^3 + 36\arctan(ax)\sqrt{a^2x^2+1}\, a^2x}{9}$ |

input
```
int(x^5*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/9*(9*arctan(a*x)*(a^2*x^2+1)^(1/2)*a^4*x^4-9*ln((1+I*a*x)/(a^2*x^2+1)^(1
/2)+I)*a^4*x^4+9*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-I)*a^4*x^4-16*(a^2*x^2+1)^
(1/2)*a^3*x^3+36*arctan(a*x)*(a^2*x^2+1)^(1/2)*a^2*x^2-18*ln((1+I*a*x)/(a^
2*x^2+1)^(1/2)+I)*a^2*x^2+18*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-I)*a^2*x^2-15*
(a^2*x^2+1)^(1/2)*a*x+24*arctan(a*x)*(a^2*x^2+1)^(1/2)-9*ln((1+I*a*x)/(a^2
*x^2+1)^(1/2)+I)+9*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-I))*(c*(a*x-I)*(a*x+I))^
(1/2)/(a^2*x^2+1)^(1/2)/a^6/c^3/(a^4*x^4+2*a^2*x^2+1)
```

## Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.82

$$\int \frac{x^5 \arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx = \frac{9\left(a^4x^4+2\,a^2x^2+1\right)\sqrt{c}\log\left(-2\,a^2cx^2+2\,\sqrt{a^2cx^2+c}a\sqrt{c}x-c\right)-2\left(16\,a^3x^3+15\right.}{18\left(a^{10}c^3x^4+2\,a^8c^3x^2+a^6c^3\right)}$$

input
```
integrate(x^5*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
1/18*(9*(a^4*x^4 + 2*a^2*x^2 + 1)*sqrt(c)*log(-2*a^2*c*x^2 + 2*sqrt(a^2*c*
x^2 + c)*a*sqrt(c)*x - c) - 2*(16*a^3*x^3 + 15*a*x - 3*(3*a^4*x^4 + 12*a^2
*x^2 + 8)*arctan(a*x))*sqrt(a^2*c*x^2 + c))/(a^10*c^3*x^4 + 2*a^8*c^3*x^2
+ a^6*c^3)
```

## Sympy [F]

$$\int \frac{x^5 \arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^5 \operatorname{atan}(ax)}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**5*atan(a*x)/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(x**5*atan(a*x)/(c*(a**2*x**2 + 1))**(5/2), x)
```

## Maxima [F]

$$\int \frac{x^5 \arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^5 \arctan(ax)}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^5*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(x^5*arctan(a*x)/(a^2*c*x^2 + c)^(5/2), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x^5 \arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^5*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^5 \arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^5\,\mathrm{atan}(a\,x)}{(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int((x^5*atan(a*x))/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int((x^5*atan(a*x))/(c + a^2*c*x^2)^(5/2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.19

$$\int \frac{x^5 \arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx = \frac{\sqrt{c}\,\left(9\sqrt{a^2x^2+1}\,atan(ax)\,a^4x^4+36\sqrt{a^2x^2+1}\,atan(ax)\,a^2x^2+24\sqrt{a^2x^2+1}\,atan(a}{}$$

input
```
int(x^5*atan(a*x)/(a^2*c*x^2+c)^(5/2),x)
```

output
```
(sqrt(c)*(9*sqrt(a**2*x**2 + 1)*atan(a*x)*a**4*x**4 + 36*sqrt(a**2*x**2 +
1)*atan(a*x)*a**2*x**2 + 24*sqrt(a**2*x**2 + 1)*atan(a*x) - 16*sqrt(a**2*x
**2 + 1)*a**3*x**3 - 15*sqrt(a**2*x**2 + 1)*a*x - 9*log(sqrt(a**2*x**2 + 1
) + a*x)*a**4*x**4 - 18*log(sqrt(a**2*x**2 + 1) + a*x)*a**2*x**2 - 9*log(s
qrt(a**2*x**2 + 1) + a*x) + 4*a**4*x**4 + 8*a**2*x**2 + 4))/(9*a**6*c**3*(
a**4*x**4 + 2*a**2*x**2 + 1))
```

# 3.241 $\int \frac{x^4 \arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 308

$$\int \frac{x^4 \arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx = \frac{1}{9a^5c\,(c+a^2cx^2)^{3/2}} - \frac{4}{3a^5c^2\sqrt{c+a^2cx^2}} - \frac{x^3\arctan(ax)}{3a^2c\,(c+a^2cx^2)^{3/2}} - \frac{x\arctan(ax)}{a^4c^2\sqrt{c+a^2cx^2}} - \frac{2i\sqrt{1+a^2x^2}\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a^5c^2\sqrt{c+a^2cx^2}} + \frac{i\sqrt{1+a^2x^2}\,\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a^5c^2\sqrt{c+a^2cx^2}} - \frac{i\sqrt{1+a^2x^2}\,\text{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a^5c^2\sqrt{c+a^2cx^2}}$$

output
```
1/9/a^5/c/(a^2*c*x^2+c)^(3/2)-4/3/a^5/c^2/(a^2*c*x^2+c)^(1/2)-1/3*x^3*arct
an(a*x)/a^2/c/(a^2*c*x^2+c)^(3/2)-x*arctan(a*x)/a^4/c^2/(a^2*c*x^2+c)^(1/2
)-2*I*(a^2*x^2+1)^(1/2)*arctan(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2)
)/a^5/c^2/(a^2*c*x^2+c)^(1/2)+I*(a^2*x^2+1)^(1/2)*polylog(2,-I*(1+I*a*x)^(
1/2)/(1-I*a*x)^(1/2))/a^5/c^2/(a^2*c*x^2+c)^(1/2)-I*(a^2*x^2+1)^(1/2)*poly
log(2,I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/a^5/c^2/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.57

$$\int \frac{x^4 \arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx = \frac{\sqrt{c(1+a^2x^2)}\left(-\frac{45}{\sqrt{1+a^2x^2}}-\frac{45ax\arctan(ax)}{\sqrt{1+a^2x^2}}+\cos(3\arctan(ax))+36\arctan(ax)\left(\log\right.\right.}{}$$

input
```
Integrate[(x^4*ArcTan[a*x])/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(Sqrt[c*(1 + a^2*x^2)]*(-45/Sqrt[1 + a^2*x^2] - (45*a*x*ArcTan[a*x])/Sqrt[
1 + a^2*x^2] + Cos[3*ArcTan[a*x]] + 36*ArcTan[a*x]*(Log[1 - I*E^(I*ArcTan[
a*x])] - Log[1 + I*E^(I*ArcTan[a*x])]) + (36*I)*(PolyLog[2, (-I)*E^(I*ArcT
an[a*x])] - PolyLog[2, I*E^(I*ArcTan[a*x])]) + 3*ArcTan[a*x]*Sin[3*ArcTan[
a*x]]))/(36*a^5*c^3*Sqrt[1 + a^2*x^2])
```

## Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 290, normalized size of antiderivative = 0.94, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {5499, 5469, 5425, 5421, 5479, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \arctan(ax)}{(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{ 5499}$$

$$\frac{\int \frac{x^2\arctan(ax)}{(a^2cx^2+c)^{3/2}}dx}{a^2c}-\frac{\int \frac{x^2\arctan(ax)}{(a^2cx^2+c)^{5/2}}dx}{a^2}$$

$$\Big\downarrow \text{ 5469}$$

$$\frac{\frac{\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a^2c}-\frac{x\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{1}{a^3c\sqrt{a^2cx^2+c}}}{a^2c}-\frac{\int \frac{x^2\arctan(ax)}{(a^2cx^2+c)^{5/2}}dx}{a^2}$$

$$\Big\downarrow \text{ 5425}$$

$$\frac{\frac{\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{a^2c\sqrt{a^2cx^2+c}}-\frac{x\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{1}{a^3c\sqrt{a^2cx^2+c}}}{a^2c}-\frac{\int \frac{x^2\arctan(ax)}{(a^2cx^2+c)^{5/2}}dx}{a^2}$$

$$\Big\downarrow \text{5421}$$

$$-\frac{\int \frac{x^2\arctan(ax)}{(a^2cx^2+c)^{5/2}}dx}{a^2}+\frac{\frac{\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{a^2c\sqrt{a^2cx^2+c}}-\frac{x\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{1}{a^3c\sqrt{a^2cx^2+c}}}{a^2c}$$

$$\Big\downarrow \text{5479}$$

$$-\frac{\frac{x^3\arctan(ax)}{3c(a^2cx^2+c)^{3/2}}-\frac{1}{3}a\int \frac{x^3}{(a^2cx^2+c)^{5/2}}dx}{a^2}+\frac{\frac{\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{a^2c\sqrt{a^2cx^2+c}}-\frac{x\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{1}{a^3c\sqrt{a^2cx^2+c}}}{a^2c}$$

$$\Big\downarrow \text{243}$$

$$-\frac{\frac{x^3\arctan(ax)}{3c(a^2cx^2+c)^{3/2}}-\frac{1}{6}a\int \frac{x^2}{(a^2cx^2+c)^{5/2}}dx^2}{a^2}+\frac{\frac{\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{a^2c\sqrt{a^2cx^2+c}}-\frac{x\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{1}{a^3c\sqrt{a^2cx^2+c}}}{a^2c}$$

$$\Big\downarrow \text{53}$$

$$-\frac{\frac{x^3\arctan(ax)}{3c(a^2cx^2+c)^{3/2}}-\frac{1}{6}a\int \left(\frac{1}{a^2c(a^2cx^2+c)^{3/2}}-\frac{1}{a^2(a^2cx^2+c)^{5/2}}\right)dx^2}{a^2}+\frac{\frac{\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{a^2c\sqrt{a^2cx^2+c}}-\frac{x\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{1}{a^3c\sqrt{a^2cx^2+c}}}{a^2c}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{x^3\arctan(ax)}{3c(a^2cx^2+c)^{3/2}}-\frac{1}{6}a\left(\frac{2}{3a^4c(a^2cx^2+c)^{3/2}}-\frac{2}{a^4c^2\sqrt{a^2cx^2+c}}\right)}{a^2}+\frac{\frac{\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{a^2c\sqrt{a^2cx^2+c}}-\frac{x\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{1}{a^3c\sqrt{a^2cx^2+c}}}{a^2c}$$

input
```
Int[(x^4*ArcTan[a*x])/(c + a^2*c*x^2)^(5/2),x]
```

output
```
-((-1/6*(a*(2/(3*a^4*c*(c + a^2*c*x^2)^(3/2)) - 2/(a^4*c^2*Sqrt[c + a^2*c*
x^2]))) + (x^3*ArcTan[a*x])/(3*c*(c + a^2*c*x^2)^(3/2)))/a^2) + (-(1/(a^3*
c*Sqrt[c + a^2*c*x^2])) - (x*ArcTan[a*x])/(a^2*c*Sqrt[c + a^2*c*x^2]) + (S
qrt[1 + a^2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*
x]])/a + (I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*Pol
yLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/(a^2*c*Sqrt[c + a^2*c*x^
2]))/(a^2*c)
```

### Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5469
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^2*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c^3*d*(q + 1)^2)), x] + (Simp
[x*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*c^2*d*(q + 1))), x] - Simp[1
/(2*c^2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x]) /
; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -5/2]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5499
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ
[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

## Maple [A] (verified)

Time = 1.49 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.26

| method | result |
|---|---|
| default | $-\frac{(i+3\arctan(ax))(a^3x^3-3ia^2x^2-3ax+i)\sqrt{c(ax-i)(ax+i)}}{72(a^2x^2+1)^2a^5c^3}-\frac{5(\arctan(ax)+i)(ax-i)\sqrt{c(ax-i)(ax+i)}}{8c^3a^5(a^2x^2+1)}-\frac{5\sqrt{c(ax-i)(ax+i)}}{8c^3a^5}$ |

input
```
int(x^4*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/72*(I+3*arctan(a*x))*(a^3*x^3-3*I*a^2*x^2-3*a*x+I)*(c*(a*x-I)*(a*x+I))^
(1/2)/(a^2*x^2+1)^2/a^5/c^3-5/8*(arctan(a*x)+I)*(a*x-I)*(c*(a*x-I)*(a*x+I)
)^(1/2)/c^3/a^5/(a^2*x^2+1)-5/8*(c*(a*x-I)*(a*x+I))^(1/2)*(a*x+I)*(arctan(
a*x)-I)/c^3/a^5/(a^2*x^2+1)-1/72*(-I+3*arctan(a*x))*(c*(a*x-I)*(a*x+I))^(1
/2)*(a^3*x^3+3*I*a^2*x^2-3*a*x-I)/(a^4*x^4+2*a^2*x^2+1)/a^5/c^3-(arctan(a*
x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x
^2+1)^(1/2))+I*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*dilog(1+I*(1+I*a*x
)/(a^2*x^2+1)^(1/2)))*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/2)/c^3/a^5
```

## Fricas [F]

$$\int \frac{x^4 \arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^4 \arctan\left(ax\right)}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^4*arctan(a*x)/(a^6*c^3*x^6 + 3*a^4*c^3*x^4
+ 3*a^2*c^3*x^2 + c^3), x)
```

## Sympy [F]

$$\int \frac{x^4 \arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^4 \operatorname{atan}\left(ax\right)}{(c\left(a^2x^2+1\right))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**4*atan(a*x)/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(x**4*atan(a*x)/(c*(a**2*x**2 + 1))**(5/2), x)
```

## Maxima [F]

$$\int \frac{x^4 \arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^4 \arctan(ax)}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(x^4*arctan(a*x)/(a^2*c*x^2 + c)^(5/2), x)
```

## Giac [F]

$$\int \frac{x^4 \arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^4 \arctan(ax)}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
integrate(x^4*arctan(a*x)/(a^2*c*x^2 + c)^(5/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^4 \arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^4\,\mathrm{atan}(a\,x)}{(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int((x^4*atan(a*x))/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int((x^4*atan(a*x))/(c + a^2*c*x^2)^(5/2), x)
```

## Reduce [F]

$$\int \frac{x^4 \arctan(ax)}{(c + a^2cx^2)^{5/2}} \, dx = \frac{\int \frac{atan(ax)x^4}{\sqrt{a^2x^2+1}\,a^4x^4+2\sqrt{a^2x^2+1}\,a^2x^2+\sqrt{a^2x^2+1}}dx}{\sqrt{c}\,c^2}$$

input
```
int(x^4*atan(a*x)/(a^2*c*x^2+c)^(5/2),x)
```

output
```
int((atan(a*x)*x**4)/(sqrt(a**2*x**2 + 1)*a**4*x**4 + 2*sqrt(a**2*x**2 + 1
)*a**2*x**2 + sqrt(a**2*x**2 + 1)),x)/(sqrt(c)*c**2)
```

# 3.242 $\int \frac{x^3 \arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 112

$$\int \frac{x^3 \arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx = \frac{x^3}{9ac\,(c+a^2cx^2)^{3/2}} + \frac{2x}{3a^3c^2\sqrt{c+a^2cx^2}} - \frac{x^2\arctan(ax)}{3a^2c\,(c+a^2cx^2)^{3/2}} - \frac{2\arctan(ax)}{3a^4c^2\sqrt{c+a^2cx^2}}$$

output
```
1/9*x^3/a/c/(a^2*c*x^2+c)^(3/2)+2/3*x/a^3/c^2/(a^2*c*x^2+c)^(1/2)-1/3*x^2*
arctan(a*x)/a^2/c/(a^2*c*x^2+c)^(3/2)-2/3*arctan(a*x)/a^4/c^2/(a^2*c*x^2+c
)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.58

$$\int \frac{x^3 \arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx = \frac{\sqrt{c+a^2cx^2}(ax(6+7a^2x^2)-3(2+3a^2x^2)\arctan(ax))}{9a^4c^3\,(1+a^2x^2)^2}$$

input
```
Integrate[(x^3*ArcTan[a*x])/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(Sqrt[c + a^2*c*x^2]*(a*x*(6 + 7*a^2*x^2) - 3*(2 + 3*a^2*x^2)*ArcTan[a*x])
)/(9*a^4*c^3*(1 + a^2*x^2)^2)
```

## Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {5473, 5465, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \arctan(ax)}{(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{5473}$$

$$\frac{2\int \frac{x\arctan(ax)}{(a^2cx^2+c)^{3/2}}dx}{3a^2c}-\frac{x^2\arctan(ax)}{3a^2c\,(a^2cx^2+c)^{3/2}}+\frac{x^3}{9ac\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5465}$$

$$\frac{2\left(\frac{\int \frac{1}{(a^2cx^2+c)^{3/2}}dx}{a}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{x^2\arctan(ax)}{3a^2c\,(a^2cx^2+c)^{3/2}}+\frac{x^3}{9ac\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{208}$$

$$-\frac{x^2\arctan(ax)}{3a^2c\,(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x}{ac\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{x^3}{9ac\,(a^2cx^2+c)^{3/2}}$$

input
```
Int[(x^3*ArcTan[a*x])/(c + a^2*c*x^2)^(5/2),x]
```

output
```
x^3/(9*a*c*(c + a^2*c*x^2)^(3/2)) - (x^2*ArcTan[a*x])/(3*a^2*c*(c + a^2*c*
x^2)^(3/2)) + (2*(x/(a*c*Sqrt[c + a^2*c*x^2]) - ArcTan[a*x]/(a^2*c*Sqrt[c
+ a^2*c*x^2])))/(3*a^2*c)
```

### Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5473
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[b*(f*x)^m*((d + e*x^2)^(q + 1)/(c*d*m^2)), x] +
 (-Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(c^2*d*m))
, x] + Simp[f^2*((m - 1)/(c^2*d*m))   Int[(f*x)^(m - 2)*(d + e*x^2)^(q + 1)
*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2
*d] && EqQ[m + 2*q + 2, 0] && LtQ[q, -1]
```

## Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.38

| method | result |
|---|---|
| orering | $\frac{2(a^2x^2+1)(7a^4x^4-9a^2x^2-12)\arctan(ax)}{9a^4(a^2cx^2+c)^{\frac{5}{2}}}+\frac{(7a^2x^2+6)(a^2x^2+1)^2\Big(\frac{3x^2\arctan(ax)}{(a^2cx^2+c)^{\frac{5}{2}}}+\frac{x^3a}{(a^2x^2+1)(a^2cx^2+c)^{\frac{5}{2}}}-\frac{5x^4\arctan(ax)ca^2}{(a^2cx^2+c)^{\frac{7}{2}}}}{9x^2a^4}$ |
| default | $-\frac{(i+3\arctan(ax))(ia^3x^3+3a^2x^2-3iax-1)\sqrt{c(ax-i)(ax+i)}}{72(a^2x^2+1)^2a^4c^3}-\frac{3(\arctan(ax)+i)(iax+1)\sqrt{c(ax-i)(ax+i)}}{8c^3a^4(a^2x^2+1)}+\frac{3\sqrt{c(ax-i)(ax+}}{8c}$ |

input
```
int(x^3*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/9*(a^2*x^2+1)*(7*a^4*x^4-9*a^2*x^2-12)/a^4*arctan(a*x)/(a^2*c*x^2+c)^(5/
2)+1/9/x^2*(7*a^2*x^2+6)/a^4*(a^2*x^2+1)^2*(3*x^2*arctan(a*x)/(a^2*c*x^2+c
)^(5/2)+x^3*a/(a^2*x^2+1)/(a^2*c*x^2+c)^(5/2)-5*x^4*arctan(a*x)/(a^2*c*x^2
+c)^(7/2)*c*a^2)
```

## Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.66

$$\int \frac{x^3 \arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx = \frac{(7\,a^3x^3+6\,ax-3\,(3\,a^2x^2+2)\arctan{(ax)})\sqrt{a^2cx^2+c}}{9\,(a^8c^3x^4+2\,a^6c^3x^2+a^4c^3)}$$

input
```
integrate(x^3*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
1/9*(7*a^3*x^3 + 6*a*x - 3*(3*a^2*x^2 + 2)*arctan(a*x))*sqrt(a^2*c*x^2 + c
)/(a^8*c^3*x^4 + 2*a^6*c^3*x^2 + a^4*c^3)
```

## Sympy [F]

$$\int \frac{x^3 \arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^3\,\mathrm{atan}\,(ax)}{(c\,(a^2x^2+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**3*atan(a*x)/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(x**3*atan(a*x)/(c*(a**2*x**2 + 1))**(5/2), x)
```

## Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.58

$$\int \frac{x^3 \arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx = \frac{7\,a^3x^3+6\,ax-3\,(3\,a^2x^2+2)\arctan{(ax)}}{9\,(a^6c^2x^2+a^4c^2)\sqrt{a^2x^2+1}\sqrt{c}}$$

input
```
integrate(x^3*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
1/9*(7*a^3*x^3 + 6*a*x - 3*(3*a^2*x^2 + 2)*arctan(a*x))/((a^6*c^2*x^2 + a^
4*c^2)*sqrt(a^2*x^2 + 1)*sqrt(c))
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x^3 \arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^3\,\mathrm{atan}(a\,x)}{\left(c\,a^2\,x^2+c\right)^{5/2}}\,dx$$

input
```
int((x^3*atan(a*x))/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int((x^3*atan(a*x))/(c + a^2*c*x^2)^(5/2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.04

$$\int \frac{x^3 \arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx = \frac{\sqrt{c}\left(-9\sqrt{a^2x^2+1}\,atan(ax)\,a^2x^2-6\sqrt{a^2x^2+1}\,atan(ax)+7\sqrt{a^2x^2+1}\,a^3x^3+6\sqrt{c}\right.}{9a^4c^3\left(a^4x^4+2a^2x^2+1\right)}$$

input
```
int(x^3*atan(a*x)/(a^2*c*x^2+c)^(5/2),x)
```

output
```
(sqrt(c)*( - 9*sqrt(a**2*x**2 + 1)*atan(a*x)*a**2*x**2 - 6*sqrt(a**2*x**2
+ 1)*atan(a*x) + 7*sqrt(a**2*x**2 + 1)*a**3*x**3 + 6*sqrt(a**2*x**2 + 1)*a
*x - a**4*x**4 - 2*a**2*x**2 - 1))/(9*a**4*c**3*(a**4*x**4 + 2*a**2*x**2 +
 1))
```

# 3.243 $\int \frac{x^2 \arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 77

$$\int \frac{x^2 \arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx = -\frac{1}{9a^3c\,(c+a^2cx^2)^{3/2}} + \frac{1}{3a^3c^2\sqrt{c+a^2cx^2}} + \frac{x^3\arctan(ax)}{3c\,(c+a^2cx^2)^{3/2}}$$

output
```
-1/9/a^3/c/(a^2*c*x^2+c)^(3/2)+1/3/a^3/c^2/(a^2*c*x^2+c)^(1/2)+1/3*x^3*arc
tan(a*x)/c/(a^2*c*x^2+c)^(3/2)
```

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.74

$$\int \frac{x^2 \arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx = \frac{\sqrt{c+a^2cx^2}(2+3a^2x^2+3a^3x^3\arctan(ax))}{9a^3c^3\,(1+a^2x^2)^2}$$

input
```
Integrate[(x^2*ArcTan[a*x])/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(Sqrt[c + a^2*c*x^2]*(2 + 3*a^2*x^2 + 3*a^3*x^3*ArcTan[a*x]))/(9*a^3*c^3*(
1 + a^2*x^2)^2)
```

## Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {5479, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \arctan(ax)}{(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{5479}$$

$$\frac{x^3 \arctan(ax)}{3c(a^2cx^2+c)^{3/2}}-\frac{1}{3}a\int \frac{x^3}{(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{x^3 \arctan(ax)}{3c(a^2cx^2+c)^{3/2}}-\frac{1}{6}a\int \frac{x^2}{(a^2cx^2+c)^{5/2}}dx^2$$

$$\Big\downarrow \text{53}$$

$$\frac{x^3 \arctan(ax)}{3c(a^2cx^2+c)^{3/2}}-\frac{1}{6}a\int \left(\frac{1}{a^2c(a^2cx^2+c)^{3/2}}-\frac{1}{a^2(a^2cx^2+c)^{5/2}}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^3 \arctan(ax)}{3c(a^2cx^2+c)^{3/2}}-\frac{1}{6}a\left(\frac{2}{3a^4c(a^2cx^2+c)^{3/2}}-\frac{2}{a^4c^2\sqrt{a^2cx^2+c}}\right)$$

input
```
Int[(x^2*ArcTan[a*x])/(c + a^2*c*x^2)^(5/2),x]
```

output
```
-1/6*(a*(2/(3*a^4*c*(c + a^2*c*x^2)^(3/2)) - 2/(a^4*c^2*Sqrt[c + a^2*c*x^2
])) + (x^3*ArcTan[a*x])/(3*c*(c + a^2*c*x^2)^(3/2))
```

### Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(65) = 130$.

Time = 1.11 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.91

| method | result |
|---|---|
| orering | $\frac{4(a^2x^2+1)(3a^4x^4-1)\arctan(ax)}{9a^4x(a^2c\,x^2+c)^{\frac{5}{2}}}+\frac{(3a^2x^2+2)(a^2x^2+1)^2\left(\frac{2x\arctan(ax)}{(a^2c\,x^2+c)^{\frac{5}{2}}}+\frac{x^2a}{(a^2x^2+1)(a^2c\,x^2+c)^{\frac{5}{2}}}-\frac{5x^3\arctan(ax)c\,a^2}{(a^2c\,x^2+c)^{\frac{7}{2}}}\right)}{9a^4x^2}$ |
| default | $\frac{(i+3\arctan(ax))(a^3x^3-3ia^2x^2-3ax+i)\sqrt{c(ax-i)(ax+i)}}{72(a^2x^2+1)^2a^3c^3}+\frac{(\arctan(ax)+i)(ax-i)\sqrt{c(ax-i)(ax+i)}}{8c^3a^3(a^2x^2+1)}+\frac{\sqrt{c(ax-i)(ax+i)}\,(ax-}{8c^3a^3(a^2x}$ |

input
```
int(x^2*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
4/9*(a^2*x^2+1)*(3*a^4*x^4-1)/a^4/x*arctan(a*x)/(a^2*c*x^2+c)^(5/2)+1/9*(3
*a^2*x^2+2)/a^4*(a^2*x^2+1)^2/x^2*(2*x*arctan(a*x)/(a^2*c*x^2+c)^(5/2)+x^2
*a/(a^2*x^2+1)/(a^2*c*x^2+c)^(5/2)-5*x^3*arctan(a*x)/(a^2*c*x^2+c)^(7/2)*c
*a^2)
```

## Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.87

$$\int \frac{x^2 \arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx = \frac{(3\,a^3x^3\arctan{(ax)} + 3\,a^2x^2 + 2)\sqrt{a^2cx^2 + c}}{9\,(a^7c^3x^4 + 2\,a^5c^3x^2 + a^3c^3)}$$

input
```
integrate(x^2*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
1/9*(3*a^3*x^3*arctan(a*x) + 3*a^2*x^2 + 2)*sqrt(a^2*c*x^2 + c)/(a^7*c^3*x
^4 + 2*a^5*c^3*x^2 + a^3*c^3)
```

## Sympy [F]

$$\int \frac{x^2 \arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^2\operatorname{atan}{(ax)}}{(c\,(a^2x^2+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**2*atan(a*x)/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(x**2*atan(a*x)/(c*(a**2*x**2 + 1))**(5/2), x)
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.21

$$\int \frac{x^2 \arctan(ax)}{(c + a^2 c x^2)^{5/2}} \, dx = \frac{1}{9} a \left( \frac{3}{\sqrt{a^2 c x^2 + c} a^4 c^2} - \frac{1}{(a^2 c x^2 + c)^{\frac{3}{2}} a^4 c} \right) + \frac{1}{3} \left( \frac{x}{\sqrt{a^2 c x^2 + c} a^2 c^2} - \frac{x}{(a^2 c x^2 + c)^{\frac{3}{2}} a^2 c} \right) \arctan(ax)$$

input
```
integrate(x^2*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
1/9*a*(3/(sqrt(a^2*c*x^2 + c)*a^4*c^2) - 1/((a^2*c*x^2 + c)^(3/2)*a^4*c))
+ 1/3*(x/(sqrt(a^2*c*x^2 + c)*a^2*c^2) - x/((a^2*c*x^2 + c)^(3/2)*a^2*c))*
arctan(a*x)
```

## Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.75

$$\int \frac{x^2 \arctan(ax)}{(c + a^2 c x^2)^{5/2}} \, dx = \frac{x^3 \arctan(ax)}{3 (a^2 c x^2 + c)^{\frac{3}{2}} c} + \frac{3 a^2 c x^2 + 2 c}{9 (a^2 c x^2 + c)^{\frac{3}{2}} a^3 c^2}$$

input
```
integrate(x^2*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
1/3*x^3*arctan(a*x)/((a^2*c*x^2 + c)^(3/2)*c) + 1/9*(3*a^2*c*x^2 + 2*c)/((
a^2*c*x^2 + c)^(3/2)*a^3*c^2)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \arctan(ax)}{(c+a^2cx^2)^{5/2}}\, dx = \int \frac{x^2\, \mathrm{atan}(a\, x)}{(c\, a^2\, x^2+c)^{5/2}}\, dx$$

input
```
int((x^2*atan(a*x))/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int((x^2*atan(a*x))/(c + a^2*c*x^2)^(5/2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 175, normalized size of antiderivative = 2.27

$$\int \frac{x^2 \arctan(ax)}{(c+a^2cx^2)^{5/2}}\, dx = \frac{\sqrt{c}\left(-6atan\left(\sqrt{a^2x^2+1}+ax\right)a^4x^4-12atan\left(\sqrt{a^2x^2+1}+ax\right)a^2x^2-6atan\left(\sqrt{a^2x^2+1}+ax\right)\right.}{}$$

input
```
int(x^2*atan(a*x)/(a^2*c*x^2+c)^(5/2),x)
```

output
```
(sqrt(c)*( - 6*atan(sqrt(a**2*x**2 + 1) + a*x)*a**4*x**4 - 12*atan(sqrt(a*
*2*x**2 + 1) + a*x)*a**2*x**2 - 6*atan(sqrt(a**2*x**2 + 1) + a*x) + 3*sqrt
(a**2*x**2 + 1)*atan(a*x)*a**3*x**3 + 3*atan(a*x)*a**4*x**4 + 6*atan(a*x)*
a**2*x**2 + 3*atan(a*x) + 3*sqrt(a**2*x**2 + 1)*a**2*x**2 + 2*sqrt(a**2*x*
*2 + 1)))/(9*a**3*c**3*(a**4*x**4 + 2*a**2*x**2 + 1))
```

# 3.244 $\int \frac{x \arctan(ax)}{(c+a^2cx^2)^{5/2}} dx$



## Optimal result

Integrand size = 20, antiderivative size = 79

$$\int \frac{x \arctan(ax)}{(c+a^2cx^2)^{5/2}} dx = \frac{x}{9ac(c+a^2cx^2)^{3/2}} + \frac{2x}{9ac^2\sqrt{c+a^2cx^2}} - \frac{\arctan(ax)}{3a^2c(c+a^2cx^2)^{3/2}}$$

output
```
1/9*x/a/c/(a^2*c*x^2+c)^(3/2)+2/9*x/a/c^2/(a^2*c*x^2+c)^(1/2)-1/3*arctan(a
*x)/a^2/c/(a^2*c*x^2+c)^(3/2)
```

## Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.65

$$\int \frac{x \arctan(ax)}{(c+a^2cx^2)^{5/2}} dx = \frac{\sqrt{c+a^2cx^2}(3ax+2a^3x^3-3\arctan(ax))}{9c^3(a+a^3x^2)^2}$$

input
```
Integrate[(x*ArcTan[a*x])/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(Sqrt[c + a^2*c*x^2]*(3*a*x + 2*a^3*x^3 - 3*ArcTan[a*x]))/(9*c^3*(a + a^3*
x^2)^2)
```

## Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {5465, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \arctan(ax)}{(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{\int \frac{1}{(a^2cx^2+c)^{5/2}}dx}{3a}-\frac{\arctan(ax)}{3a^2c\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{209}$$

$$\frac{\frac{2\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}}dx}{3c}+\frac{x}{3c(a^2cx^2+c)^{3/2}}}{3a}-\frac{\arctan(ax)}{3a^2c\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{208}$$

$$\frac{\frac{2x}{3c^2\sqrt{a^2cx^2+c}}+\frac{x}{3c(a^2cx^2+c)^{3/2}}}{3a}-\frac{\arctan(ax)}{3a^2c\left(a^2cx^2+c\right)^{3/2}}$$

input
```
Int[(x*ArcTan[a*x])/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(x/(3*c*(c + a^2*c*x^2)^(3/2)) + (2*x)/(3*c^2*Sqrt[c + a^2*c*x^2]))/(3*a)
- ArcTan[a*x]/(3*a^2*c*(c + a^2*c*x^2)^(3/2))
```

### Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

## Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(67) = 134$.

Time = 1.36 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.82

| method | result |
|---|---|
| orering | $\frac{2\left(4a^{6}x^{6}+9a^{4}x^{4}+2a^{2}x^{2}-3\right)\arctan\left(ax\right)}{9a^{2}\left(a^{2}c\,x^{2}+c\right)^{\frac{5}{2}}}+\frac{\left(2a^{2}x^{2}+3\right)\left(a^{2}x^{2}+1\right)^{2}\left(\frac{\arctan\left(ax\right)}{\left(a^{2}c\,x^{2}+c\right)^{\frac{5}{2}}}+\frac{xa}{\left(a^{2}x^{2}+1\right)\left(a^{2}c\,x^{2}+c\right)^{\frac{5}{2}}}-\frac{5x^{2}\arctan\left(ax\right)c\,a^{2}}{\left(a^{2}c\,x^{2}+c\right)^{\frac{7}{2}}}\right)}{9a^{2}}$ |
| default | $\frac{\left(i+3\arctan\left(ax\right)\right)\left(ia^{3}x^{3}+3a^{2}x^{2}-3iax-1\right)\sqrt{c\left(ax-i\right)\left(ax+i\right)}}{72\left(a^{2}x^{2}+1\right)^{2}a^{2}c^{3}}-\frac{\left(\arctan\left(ax\right)+i\right)\left(iax+1\right)\sqrt{c\left(ax-i\right)\left(ax+i\right)}}{8c^{3}a^{2}\left(a^{2}x^{2}+1\right)}+\frac{\sqrt{c\left(ax-i\right)\left(ax+i\right)}\,(i}{8c^{3}a^{2}(}$ |

input
```
int(x*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/9*(4*a^6*x^6+9*a^4*x^4+2*a^2*x^2-3)/a^2*arctan(a*x)/(a^2*c*x^2+c)^(5/2)+
1/9*(2*a^2*x^2+3)/a^2*(a^2*x^2+1)^2*(arctan(a*x)/(a^2*c*x^2+c)^(5/2)+x*a/(
a^2*x^2+1)/(a^2*c*x^2+c)^(5/2)-5*x^2*arctan(a*x)/(a^2*c*x^2+c)^(7/2)*c*a^2
)
```

## Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.81

$$\int \frac{x \arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx = \frac{(2\,a^3x^3+3\,ax-3\,\arctan{(ax)})\sqrt{a^2cx^2+c}}{9\,(a^6c^3x^4+2\,a^4c^3x^2+a^2c^3)}$$

input
```
integrate(x*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
1/9*(2*a^3*x^3 + 3*a*x - 3*arctan(a*x))*sqrt(a^2*c*x^2 + c)/(a^6*c^3*x^4 +
 2*a^4*c^3*x^2 + a^2*c^3)
```

## Sympy [F(-2)]

Exception generated.

$$\int \frac{x \arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*atan(a*x)/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real zoo
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.84

$$\int \frac{x \arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx = \frac{(2\,a^3x^3+3\,ax-3\,\arctan{(ax)})\sqrt{a^2x^2+1}\sqrt{c}}{9\,(a^6c^3x^4+2\,a^4c^3x^2+a^2c^3)}$$

input
```
integrate(x*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
1/9*(2*a^3*x^3 + 3*a*x - 3*arctan(a*x))*sqrt(a^2*x^2 + 1)*sqrt(c)/(a^6*c^3
*x^4 + 2*a^4*c^3*x^2 + a^2*c^3)
```

## Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.73

$$\int \frac{x\arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx = \frac{\left(\frac{2\,ax^2}{c}+\frac{3}{ac}\right)x}{9\left(a^2cx^2+c\right)^{\frac{3}{2}}} - \frac{\arctan\left(ax\right)}{3\left(a^2cx^2+c\right)^{\frac{3}{2}}a^2c}$$

input
```
integrate(x*arctan(a*x)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
1/9*(2*a*x^2/c + 3/(a*c))*x/(a^2*c*x^2 + c)^(3/2) - 1/3*arctan(a*x)/((a^2*
c*x^2 + c)^(3/2)*a^2*c)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x\arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x\,\mathrm{atan}(a\,x)}{\left(c\,a^2\,x^2+c\right)^{5/2}}\,dx$$

input
```
int((x*atan(a*x))/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int((x*atan(a*x))/(c + a^2*c*x^2)^(5/2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.20

$$\int \frac{x\arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx = \frac{\sqrt{c}\left(-3\sqrt{a^2x^2+1}\,atan(ax)+2\sqrt{a^2x^2+1}\,a^3x^3+3\sqrt{a^2x^2+1}\,ax-2a^4x^4-4a^2x^2\right.}{9a^2c^3\left(a^4x^4+2a^2x^2+1\right)}$$

input
```
int(x*atan(a*x)/(a^2*c*x^2+c)^(5/2),x)
```

output
```
(sqrt(c)*( - 3*sqrt(a**2*x**2 + 1)*atan(a*x) + 2*sqrt(a**2*x**2 + 1)*a**3*
x**3 + 3*sqrt(a**2*x**2 + 1)*a*x - 2*a**4*x**4 - 4*a**2*x**2 - 2))/(9*a**2
*c**3*(a**4*x**4 + 2*a**2*x**2 + 1))
```

# 3.245 $\int \frac{\arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx$



## Optimal result

Integrand size = 19, antiderivative size = 101

$$\int \frac{\arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx = \frac{1}{9ac\,(c+a^2cx^2)^{3/2}} + \frac{2}{3ac^2\sqrt{c+a^2cx^2}} + \frac{x\arctan(ax)}{3c\,(c+a^2cx^2)^{3/2}} + \frac{2x\arctan(ax)}{3c^2\sqrt{c+a^2cx^2}}$$

output
```
1/9/a/c/(a^2*c*x^2+c)^(3/2)+2/3/a/c^2/(a^2*c*x^2+c)^(1/2)+1/3*x*arctan(a*x
)/c/(a^2*c*x^2+c)^(3/2)+2/3*x*arctan(a*x)/c^2/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.62

$$\int \frac{\arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx = \frac{\sqrt{c+a^2cx^2}(7+6a^2x^2+(9ax+6a^3x^3)\arctan(ax))}{9ac^3\,(1+a^2x^2)^2}$$

input
```
Integrate[ArcTan[a*x]/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(Sqrt[c + a^2*c*x^2]*(7 + 6*a^2*x^2 + (9*a*x + 6*a^3*x^3)*ArcTan[a*x]))/(9
*a*c^3*(1 + a^2*x^2)^2)
```

## Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = $\{5431, 5429\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)}{(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{5431}$$

$$\frac{2\int \frac{\arctan(ax)}{(a^2cx^2+c)^{3/2}}dx}{3c}+\frac{x\arctan(ax)}{3c\,(a^2cx^2+c)^{3/2}}+\frac{1}{9ac\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5429}$$

$$\frac{x\arctan(ax)}{3c\,(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{1}{9ac\,(a^2cx^2+c)^{3/2}}$$

input
```
Int[ArcTan[a*x]/(c + a^2*c*x^2)^(5/2),x]
```

output
```
1/(9*a*c*(c + a^2*c*x^2)^(3/2)) + (x*ArcTan[a*x])/(3*c*(c + a^2*c*x^2)^(3/
2)) + (2*(1/(a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x])/(c*Sqrt[c + a^2*c*
x^2])))/(3*c)
```

### Defintions of rubi rules used

rule 5429
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTan[c*x])/(d*Sqr
t[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

rule 5431
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbol
] :> Simp[b*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (-Simp[x*(d + e*x
^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*d*(q + 1))), x] + Simp[(2*q + 3)/(2*d*(
q + 1))   Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a,
 b, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -3/2]
```

## Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.13

| method | result |
|---|---|
| orering | $\frac{\left(4 a^{4} x^{5}+\frac{80}{9} a^{2} x^{3}+\frac{44}{9} x\right) \arctan \left(a x\right)}{\left(a^{2} c \,x^{2}+c\right)^{\frac{5}{2}}}+\frac{\left(6 a^{2} x^{2}+7\right) \left(a^{2} x^{2}+1\right)^{2} \left(\frac{a}{\left(a^{2} x^{2}+1\right) \left(a^{2} c \,x^{2}+c\right)^{\frac{5}{2}}}-\frac{5 \arctan \left(a x\right) c x \,a^{2}}{\left(a^{2} c \,x^{2}+c\right)^{\frac{7}{2}}}\right)}{9 a^{2}}$ |
| default | $-\frac{\left(i+3 \arctan \left(a x\right)\right) \left(a^{3} x^{3}-3 i a^{2} x^{2}-3 a x+i\right) \sqrt{c \left(a x-i\right) \left(a x+i\right)}}{72 \left(a^{2} x^{2}+1\right)^{2} a \,c^{3}}+\frac{3 \left(\arctan \left(a x\right)+i\right) \left(a x-i\right) \sqrt{c \left(a x-i\right) \left(a x+i\right)}}{8 c^{3} a \left(a^{2} x^{2}+1\right)}+\frac{3 \sqrt{c \left(a x-i\right) \left(a x+i\right)}}{8 c^{3} a}$ |

input
```
int(arctan(a*x)/(a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(4*a^4*x^5+80/9*a^2*x^3+44/9*x)*arctan(a*x)/(a^2*c*x^2+c)^(5/2)+1/9*(6*a^2
*x^2+7)/a^2*(a^2*x^2+1)^2*(a/(a^2*x^2+1)/(a^2*c*x^2+c)^(5/2)-5*arctan(a*x)
/(a^2*c*x^2+c)^(7/2)*c*x*a^2)
```

## Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.71

$$\int \frac{\arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx = \frac{\sqrt{a^2cx^2+c}(6\,a^2x^2+3\,(2\,a^3x^3+3\,ax)\arctan{(ax)}+7)}{9\,(a^5c^3x^4+2\,a^3c^3x^2+ac^3)}$$

input
```
integrate(arctan(a*x)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
1/9*sqrt(a^2*c*x^2 + c)*(6*a^2*x^2 + 3*(2*a^3*x^3 + 3*a*x)*arctan(a*x) + 7
)/(a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)
```

## Sympy [F]

$$\int \frac{\arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\operatorname{atan}{(ax)}}{(c\,(a^2x^2+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(atan(a*x)/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(atan(a*x)/(c*(a**2*x**2 + 1))**(5/2), x)
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.85

$$\int \frac{\arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx = \frac{1}{9}\,a\left(\frac{6}{\sqrt{a^2cx^2+c}a^2c^2}+\frac{1}{(a^2cx^2+c)^{\frac{3}{2}}a^2c}\right) + \frac{1}{3}\left(\frac{2\,x}{\sqrt{a^2cx^2+c}c^2}+\frac{x}{(a^2cx^2+c)^{\frac{3}{2}}c}\right)\arctan{(ax)}$$

input
```
integrate(arctan(a*x)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
1/9*a*(6/(sqrt(a^2*c*x^2 + c)*a^2*c^2) + 1/((a^2*c*x^2 + c)^(3/2)*a^2*c))
+ 1/3*(2*x/(sqrt(a^2*c*x^2 + c)*c^2) + x/((a^2*c*x^2 + c)^(3/2)*c))*arctan
(a*x)
```

## Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.69

$$\int \frac{\arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx = \frac{\left(\frac{2a^2x^2}{c}+\frac{3}{c}\right)x\arctan(ax)}{3\,(a^2cx^2+c)^{\frac{3}{2}}} + \frac{6\,a^2cx^2+7\,c}{9\,(a^2cx^2+c)^{\frac{3}{2}}ac^2}$$

input
```
integrate(arctan(a*x)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
1/3*(2*a^2*x^2/c + 3/c)*x*arctan(a*x)/(a^2*c*x^2 + c)^(3/2) + 1/9*(6*a^2*c
*x^2 + 7*c)/((a^2*c*x^2 + c)^(3/2)*a*c^2)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\mathrm{atan}(a\,x)}{(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int(atan(a*x)/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int(atan(a*x)/(c + a^2*c*x^2)^(5/2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.91

$$\int \frac{\arctan(ax)}{(c+a^2cx^2)^{5/2}}\,dx = \frac{\sqrt{c}\,\left(-12atan\left(\sqrt{a^2x^2+1}+ax\right)a^4x^4-24atan\left(\sqrt{a^2x^2+1}+ax\right)a^2x^2-12atan\left(\sqrt{\right.\right.}}{}$$

input
```
int(atan(a*x)/(a^2*c*x^2+c)^(5/2),x)
```

output
```
(sqrt(c)*( - 12*atan(sqrt(a**2*x**2 + 1) + a*x)*a**4*x**4 - 24*atan(sqrt(a
**2*x**2 + 1) + a*x)*a**2*x**2 - 12*atan(sqrt(a**2*x**2 + 1) + a*x) + 6*sq
rt(a**2*x**2 + 1)*atan(a*x)*a**3*x**3 + 9*sqrt(a**2*x**2 + 1)*atan(a*x)*a*
x + 6*atan(a*x)*a**4*x**4 + 12*atan(a*x)*a**2*x**2 + 6*atan(a*x) + 6*sqrt(
a**2*x**2 + 1)*a**2*x**2 + 7*sqrt(a**2*x**2 + 1)))/(9*a*c**3*(a**4*x**4 +
2*a**2*x**2 + 1))
```

# 3.246 $\int \frac{\arctan(ax)}{x(c+a^2cx^2)^{5/2}}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 279

$$\begin{aligned}\int \frac{\arctan(ax)}{x(c+a^2cx^2)^{5/2}}\,dx = & -\frac{ax}{9c(c+a^2cx^2)^{3/2}} - \frac{11ax}{9c^2\sqrt{c+a^2cx^2}} \\ & + \frac{\arctan(ax)}{3c(c+a^2cx^2)^{3/2}} + \frac{\arctan(ax)}{c^2\sqrt{c+a^2cx^2}} - \frac{2\sqrt{1+a^2x^2}\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{c^2\sqrt{c+a^2cx^2}} \\ & + \frac{i\sqrt{1+a^2x^2}\,\text{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{c^2\sqrt{c+a^2cx^2}} - \frac{i\sqrt{1+a^2x^2}\,\text{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{c^2\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
-1/9*a*x/c/(a^2*c*x^2+c)^(3/2)-11/9*a*x/c^2/(a^2*c*x^2+c)^(1/2)+1/3*arctan
(a*x)/c/(a^2*c*x^2+c)^(3/2)+arctan(a*x)/c^2/(a^2*c*x^2+c)^(1/2)-2*(a^2*x^2
+1)^(1/2)*arctan(a*x)*arctanh((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/c^2/(a^2*c*
x^2+c)^(1/2)+I*(a^2*x^2+1)^(1/2)*polylog(2,-(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2
))/c^2/(a^2*c*x^2+c)^(1/2)-I*(a^2*x^2+1)^(1/2)*polylog(2,(1+I*a*x)^(1/2)/(
1-I*a*x)^(1/2))/c^2/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.60

$$\int \frac{\arctan(ax)}{x\left(c+a^2cx^2\right)^{5/2}}\,dx = \frac{\left(1+a^2x^2\right)^{3/2}\left(-\frac{45ax}{\sqrt{1+a^2x^2}}+\frac{45\arctan(ax)}{\sqrt{1+a^2x^2}}+3\arctan(ax)\cos(3\arctan(ax))+36\,\mathrm{arct}\right.}{}$$

input
```
Integrate[ArcTan[a*x]/(x*(c + a^2*c*x^2)^(5/2)),x]
```

output
```
((1 + a^2*x^2)^(3/2)*((-45*a*x)/Sqrt[1 + a^2*x^2] + (45*ArcTan[a*x])/Sqrt[
1 + a^2*x^2] + 3*ArcTan[a*x]*Cos[3*ArcTan[a*x]] + 36*ArcTan[a*x]*Log[1 - E
^(I*ArcTan[a*x])] - 36*ArcTan[a*x]*Log[1 + E^(I*ArcTan[a*x])] + (36*I)*Pol
yLog[2, -E^(I*ArcTan[a*x])] - (36*I)*PolyLog[2, E^(I*ArcTan[a*x])] - Sin[3
*ArcTan[a*x]]))/(36*c*(c*(1 + a^2*x^2))^(3/2))
```

## Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {5501, 5465, 209, 208, 5501, 5465, 208, 5493, 5489}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)}{x\left(a^2cx^2+c\right)^{5/2}}dx$$

$$\Big\downarrow \; 5501$$

$$\frac{\int \frac{\arctan(ax)}{x\left(a^2cx^2+c\right)^{3/2}}dx}{c}-a^2\int \frac{x\arctan(ax)}{\left(a^2cx^2+c\right)^{5/2}}dx$$

$$\Big\downarrow \; 5465$$

$$\frac{\int \frac{\arctan(ax)}{x\left(a^2cx^2+c\right)^{3/2}}dx}{c}-a^2\left(\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{5/2}}dx}{3a}-\frac{\arctan(ax)}{3a^2c\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\Big\downarrow \; 209$$

$$\frac{\int \frac{\arctan(ax)}{x(a^2cx^2+c)^{3/2}}dx}{c} - a^2\left(\frac{\frac{2\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}}dx}{3c} + \frac{x}{3c(a^2cx^2+c)^{3/2}}}{3a} - \frac{\arctan(ax)}{3a^2c\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\Big\downarrow \text{208}$$

$$\frac{\int \frac{\arctan(ax)}{x(a^2cx^2+c)^{3/2}}dx}{c} - a^2\left(\frac{\frac{2x}{3c^2\sqrt{a^2cx^2+c}} + \frac{x}{3c(a^2cx^2+c)^{3/2}}}{3a} - \frac{\arctan(ax)}{3a^2c\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\Big\downarrow \text{5501}$$

$$\frac{\frac{\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx}{c} - a^2\int \frac{x\arctan(ax)}{(a^2cx^2+c)^{3/2}}dx}{c} - a^2\left(\frac{\frac{2x}{3c^2\sqrt{a^2cx^2+c}} + \frac{x}{3c(a^2cx^2+c)^{3/2}}}{3a} - \frac{\arctan(ax)}{3a^2c\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\Big\downarrow \text{5465}$$

$$\frac{\frac{\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx}{c} - a^2\left(\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}}dx}{a} - \frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right)}{c} - a^2\left(\frac{\frac{2x}{3c^2\sqrt{a^2cx^2+c}} + \frac{x}{3c(a^2cx^2+c)^{3/2}}}{3a} - \frac{\arctan(ax)}{3a^2c\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\Big\downarrow \text{208}$$

$$\frac{\frac{\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx}{c} - a^2\left(\frac{x}{ac\sqrt{a^2cx^2+c}} - \frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right)}{c} - a^2\left(\frac{\frac{2x}{3c^2\sqrt{a^2cx^2+c}} + \frac{x}{3c(a^2cx^2+c)^{3/2}}}{3a} - \frac{\arctan(ax)}{3a^2c\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\Big\downarrow \text{5493}$$

$$\frac{\frac{\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{x\sqrt{a^2x^2+1}}dx}{c\sqrt{a^2cx^2+c}} - a^2\left(\frac{x}{ac\sqrt{a^2cx^2+c}} - \frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right)}{c} - a^2\left(\frac{\frac{2x}{3c^2\sqrt{a^2cx^2+c}} + \frac{x}{3c(a^2cx^2+c)^{3/2}}}{3a} - \frac{\arctan(ax)}{3a^2c\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\Big\downarrow \text{5489}$$

$$\frac{-a^2\left(\frac{\frac{2x}{3c^2\sqrt{a^2cx^2+c}}+\frac{x}{3c(a^2cx^2+c)^{3/2}}}{3a}-\frac{\arctan(ax)}{3a^2c\,(a^2cx^2+c)^{3/2}}\right)+\frac{-a^2\left(\frac{x}{ac\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right)+\frac{\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{PolyLog}\left(2,\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)\right)}{c\sqrt{a^2cx^2+c}}}{c}}{c}$$

input
```
Int[ArcTan[a*x]/(x*(c + a^2*c*x^2)^(5/2)),x]
```

output
```
-(a^2*((x/(3*c*(c + a^2*c*x^2)^(3/2)) + (2*x)/(3*c^2*Sqrt[c + a^2*c*x^2]))
/(3*a) - ArcTan[a*x]/(3*a^2*c*(c + a^2*c*x^2)^(3/2)))) + (-(a^2*(x/(a*c*Sq
rt[c + a^2*c*x^2]) - ArcTan[a*x]/(a^2*c*Sqrt[c + a^2*c*x^2]))) + (Sqrt[1 +
 a^2*x^2]*(-2*ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]] + I*Pol
yLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])] - I*PolyLog[2, Sqrt[1 + I*a*x]
/Sqrt[1 - I*a*x]]))/(c*Sqrt[c + a^2*c*x^2]))/c
```

## Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5489
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sq
rt[1 - I*c*x]], x] + (Simp[I*(b/Sqrt[d])*PolyLog[2, -Sqrt[1 + I*c*x]/Sqrt[1
 - I*c*x]], x] - Simp[I*(b/Sqrt[d])*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x]], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

## Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.33

| method | result |
|---|---|
| default | $-\frac{(i+3\arctan(ax))(ia^3x^3+3a^2x^2-3iax-1)\sqrt{c(ax-i)(ax+i)}}{72(a^2x^2+1)^2c^3}+\frac{5(\arctan(ax)+i)(iax+1)\sqrt{c(ax-i)(ax+i)}}{8c^3(a^2x^2+1)}-\frac{5\sqrt{c(ax-i)(ax+}}{8}$ |

input
```
int(arctan(a*x)/x/(a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/72*(I+3*arctan(a*x))*(I*a^3*x^3+3*a^2*x^2-3*I*a*x-1)*(c*(a*x-I)*(a*x+I)
)^(1/2)/(a^2*x^2+1)^2/c^3+5/8*(arctan(a*x)+I)*(1+I*a*x)*(c*(a*x-I)*(a*x+I)
)^(1/2)/c^3/(a^2*x^2+1)-5/8*(c*(a*x-I)*(a*x+I))^(1/2)*(I*a*x-1)*(arctan(a*
x)-I)/c^3/(a^2*x^2+1)+1/72*(c*(a*x-I)*(a*x+I))^(1/2)*(I*a^3*x^3-3*a^2*x^2-
3*I*a*x+1)*(-I+3*arctan(a*x))/c^3/(a^4*x^4+2*a^2*x^2+1)-I*(I*arctan(a*x)*l
n(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*arctan(a*x)*ln(1+(1+I*a*x)/(a^2*x^2+1)^
(1/2))+polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-polylog(2,-(1+I*a*x)/(a^2*x^
2+1)^(1/2)))*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/2)/c^3
```

## Fricas [F]

$$\int \frac{\arctan(ax)}{x\left(c+a^2cx^2\right)^{5/2}}\,dx = \int \frac{\arctan\left(ax\right)}{\left(a^2cx^2+c\right)^{\frac{5}{2}}x}\,dx$$

input
```
integrate(arctan(a*x)/x/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)/(a^6*c^3*x^7 + 3*a^4*c^3*x^5 + 3*
a^2*c^3*x^3 + c^3*x), x)
```

## Sympy [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)}{x\left(c+a^2cx^2\right)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(atan(a*x)/x/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real zoo
```

## Maxima [F]

$$\int \frac{\arctan(ax)}{x\left(c+a^2cx^2\right)^{5/2}}\,dx = \int \frac{\arctan\left(ax\right)}{\left(a^2cx^2+c\right)^{\frac{5}{2}}x}\,dx$$

input
```
integrate(arctan(a*x)/x/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)/((a^2*c*x^2 + c)^(5/2)*x), x)
```

## Giac [F]

$$\int \frac{\arctan(ax)}{x\left(c+a^2cx^2\right)^{5/2}}\,dx=\int \frac{\arctan\left(ax\right)}{\left(a^2cx^2+c\right)^{\frac{5}{2}}x}\,dx$$

input
```
integrate(arctan(a*x)/x/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
integrate(arctan(a*x)/((a^2*c*x^2 + c)^(5/2)*x), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)}{x\left(c+a^2cx^2\right)^{5/2}}\,dx=\int \frac{\operatorname{atan}(a\,x)}{x\,\left(c\,a^2\,x^2+c\right)^{5/2}}\,dx$$

input
```
int(atan(a*x)/(x*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(atan(a*x)/(x*(c + a^2*c*x^2)^(5/2)), x)
```

## Reduce [F]

$$\int \frac{\arctan(ax)}{x\left(c+a^2cx^2\right)^{5/2}}\,dx=\frac{\int \frac{atan(ax)}{\sqrt{a^2x^2+1}\,a^4x^5+2\sqrt{a^2x^2+1}\,a^2x^3+\sqrt{a^2x^2+1}\,x}dx}{\sqrt{c}\,c^2}$$

input
```
int(atan(a*x)/x/(a^2*c*x^2+c)^(5/2),x)
```

output
```
int(atan(a*x)/(sqrt(a**2*x**2 + 1)*a**4*x**5 + 2*sqrt(a**2*x**2 + 1)*a**2*
x**3 + sqrt(a**2*x**2 + 1)*x),x)/(sqrt(c)*c**2)
```

# 3.247 $\int \frac{\arctan(ax)}{x^2(c+a^2cx^2)^{5/2}}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 158

$$\int \frac{\arctan(ax)}{x^2(c+a^2cx^2)^{5/2}}\,dx = -\frac{a}{9c(c+a^2cx^2)^{3/2}} - \frac{5a}{3c^2\sqrt{c+a^2cx^2}} - \frac{a^2x\arctan(ax)}{3c(c+a^2cx^2)^{3/2}} - \frac{5a^2x\arctan(ax)}{3c^2\sqrt{c+a^2cx^2}} - \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{c^3x} - \frac{a\,\text{arctanh}\left(\frac{\sqrt{c+a^2cx^2}}{\sqrt{c}}\right)}{c^{5/2}}$$

output
```
-1/9*a/c/(a^2*c*x^2+c)^(3/2)-5/3*a/c^2/(a^2*c*x^2+c)^(1/2)-1/3*a^2*x*arcta
n(a*x)/c/(a^2*c*x^2+c)^(3/2)-5/3*a^2*x*arctan(a*x)/c^2/(a^2*c*x^2+c)^(1/2)
-(a^2*c*x^2+c)^(1/2)*arctan(a*x)/c^3/x-a*arctanh((a^2*c*x^2+c)^(1/2)/c^(1/
2))/c^(5/2)
```

## Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.96

$$\int \frac{\arctan(ax)}{x^2(c+a^2cx^2)^{5/2}}\,dx = \frac{-3\sqrt{c+a^2cx^2}(3+12a^2x^2+8a^4x^4)\arctan(ax)+ax\Big(-((16+15a^2x^2)\sqrt{c+a^2c}}{9c^3x(1+a^2}$$

input
```
Integrate[ArcTan[a*x]/(x^2*(c + a^2*c*x^2)^(5/2)),x]
```

output
```
(-3*Sqrt[c + a^2*c*x^2]*(3 + 12*a^2*x^2 + 8*a^4*x^4)*ArcTan[a*x] + a*x*(-(
(16 + 15*a^2*x^2)*Sqrt[c + a^2*c*x^2]) + 9*Sqrt[c]*(1 + a^2*x^2)^2*Log[x]
- 9*Sqrt[c]*(1 + a^2*x^2)^2*Log[c + Sqrt[c]*Sqrt[c + a^2*c*x^2]]))/(9*c^3*
x*(1 + a^2*x^2)^2)
```

## Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.42, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {5501, 5431, 5429, 5501, 5429, 5479, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)}{x^2 (a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{\arctan(ax)}{x^2(a^2cx^2+c)^{3/2}}dx}{c}-a^2\int \frac{\arctan(ax)}{(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{5431}$$

$$\frac{\int \frac{\arctan(ax)}{x^2(a^2cx^2+c)^{3/2}}dx}{c}-a^2\left(\frac{2\int \frac{\arctan(ax)}{(a^2cx^2+c)^{3/2}}dx}{3c}+\frac{x\arctan(ax)}{3c(a^2cx^2+c)^{3/2}}+\frac{1}{9ac(a^2cx^2+c)^{3/2}}\right)$$

$$\Big\downarrow \text{5429}$$

$$\frac{\int \frac{\arctan(ax)}{x^2(a^2cx^2+c)^{3/2}}dx}{c}-a^2\left(\frac{x\arctan(ax)}{3c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{1}{9ac(a^2cx^2+c)^{3/2}}\right)$$

$$\Big\downarrow \text{5501}$$

$$\frac{\frac{\int \frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx}{c}-a^2\int \frac{\arctan(ax)}{(a^2cx^2+c)^{3/2}}dx}{c}-a^2\left(\frac{x\arctan(ax)}{3c\,(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{1}{9ac\,(a^2cx^2+c)^{3/2}}\right)$$

$$\Big\downarrow \; 5429$$

$$\frac{\frac{\int \frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx}{c}-a^2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{c}-a^2\left(\frac{x\arctan(ax)}{3c\,(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{1}{9ac\,(a^2cx^2+c)^{3/2}}\right)$$

$$\Big\downarrow \; 5479$$

$$\frac{\frac{a\int \frac{1}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}}{c}-a^2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{c}-a^2\left(\frac{x\arctan(ax)}{3c\,(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{1}{9ac\,(a^2cx^2+c)^{3/2}}\right)$$

$$\Big\downarrow \; 243$$

$$\frac{\frac{\frac{1}{2}a\int \frac{1}{x^2\sqrt{a^2cx^2+c}}dx^2-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}}{c}-a^2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{c}-a^2\left(\frac{x\arctan(ax)}{3c\,(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{1}{9ac\,(a^2cx^2+c)^{3/2}}\right)$$

$$\Big\downarrow \; 73$$

$$\frac{\frac{\frac{\int \frac{1}{\frac{x^4}{a^2c}-\frac{1}{a^2}}d\sqrt{a^2cx^2+c}}{ac}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}}{c}-a^2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{c}-a^2\left(\frac{x\arctan(ax)}{3c\,(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{1}{9ac\,(a^2cx^2+c)^{3/2}}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}}{c}-a^2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{c}-a^2\left(\frac{x\arctan(ax)}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{1}{9ac\left(a^2cx^2+c\right)^{3/2}}\right)$$

input
```
Int[ArcTan[a*x]/(x^2*(c + a^2*c*x^2)^(5/2)),x]
```

output
```
-(a^2*(1/(9*a*c*(c + a^2*c*x^2)^(3/2)) + (x*ArcTan[a*x])/(3*c*(c + a^2*c*x
^2)^(3/2)) + (2*(1/(a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x])/(c*Sqrt[c +
 a^2*c*x^2])))/(3*c))) + (-(a^2*(1/(a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a
*x])/(c*Sqrt[c + a^2*c*x^2]))) + (-((Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(c*x
)) - (a*ArcTanh[Sqrt[c + a^2*c*x^2]/Sqrt[c]])/Sqrt[c])/c)/c
```

## Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5429
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTan[c*x])/(d*Sqr
t[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

rule 5431
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbol
] :> Simp[b*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (-Simp[x*(d + e*x
^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*d*(q + 1))), x] + Simp[(2*q + 3)/(2*d*(
q + 1))   Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a,
 b, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -3/2]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.40 (sec) , antiderivative size = 325, normalized size of antiderivative = 2.06

| method | result |
|---|---|
| default | $-\frac{\left(9\ln\left(1+\frac{iax+1}{\sqrt{a^2x^2+1}}\right)a^5x^5-9\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}-1\right)a^5x^5+24\arctan(ax)\sqrt{a^2x^2+1}\,a^4x^4+15\sqrt{a^2x^2+1}\,a^3x^3+18\ln\left(1+\frac{iax+1}{\sqrt{a^2x^2+1}}\right)a^3}{}$ |

input
```
int(arctan(a*x)/x^2/(a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/9*(9*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))*a^5*x^5-9*ln((1+I*a*x)/(a^2*x^2+
1)^(1/2)-1)*a^5*x^5+24*arctan(a*x)*(a^2*x^2+1)^(1/2)*a^4*x^4+15*(a^2*x^2+1
)^(1/2)*a^3*x^3+18*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))*a^3*x^3-18*ln((1+I*a*
x)/(a^2*x^2+1)^(1/2)-1)*a^3*x^3+36*arctan(a*x)*(a^2*x^2+1)^(1/2)*a^2*x^2+1
6*(a^2*x^2+1)^(1/2)*a*x+9*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))*a*x-9*ln((1+I*
a*x)/(a^2*x^2+1)^(1/2)-1)*a*x+9*arctan(a*x)*(a^2*x^2+1)^(1/2))/(a^2*x^2+1)
^(1/2)*(c*(a*x-I)*(a*x+I))^(1/2)/x/c^3/(a^4*x^4+2*a^2*x^2+1)
```

## Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.90

$$\int \frac{\arctan(ax)}{x^2\left(c+a^2cx^2\right)^{5/2}}\,dx = \frac{9\left(a^5x^5+2\,a^3x^3+ax\right)\sqrt{c}\log\left(-\frac{a^2cx^2-2\,\sqrt{a^2cx^2+c}\sqrt{c}+2\,c}{x^2}\right)-2\left(15\,a^3x^3+16\,ax+3\left(\right.\right.}{18\left(a^4c^3x^5+2\,a^2c^3x^3+c^3x\right)}$$

input
```
integrate(arctan(a*x)/x^2/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
1/18*(9*(a^5*x^5 + 2*a^3*x^3 + a*x)*sqrt(c)*log(-(a^2*c*x^2 - 2*sqrt(a^2*c
*x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*(15*a^3*x^3 + 16*a*x + 3*(8*a^4*x^4 + 12
*a^2*x^2 + 3)*arctan(a*x))*sqrt(a^2*c*x^2 + c))/(a^4*c^3*x^5 + 2*a^2*c^3*x
^3 + c^3*x)
```

## Sympy [F]

$$\int \frac{\arctan(ax)}{x^2\left(c+a^2cx^2\right)^{5/2}}\,dx = \int \frac{\operatorname{atan}{\left(ax\right)}}{x^2\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(atan(a*x)/x**2/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(atan(a*x)/(x**2*(c*(a**2*x**2 + 1))**(5/2)), x)
```

## Maxima [F]

$$\int \frac{\arctan(ax)}{x^2 (c + a^2cx^2)^{5/2}}\,dx = \int \frac{\arctan(ax)}{(a^2cx^2 + c)^{\frac{5}{2}}x^2}\,dx$$

input
```
integrate(arctan(a*x)/x^2/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)/((a^2*c*x^2 + c)^(5/2)*x^2), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)}{x^2 (c + a^2cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)/x^2/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)}{x^2 (c + a^2cx^2)^{5/2}}\,dx = \int \frac{\mathrm{atan}(a\,x)}{x^2\,(c\,a^2\,x^2 + c)^{5/2}}\,dx$$

input
```
int(atan(a*x)/(x^2*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(atan(a*x)/(x^2*(c + a^2*c*x^2)^(5/2)), x)
```

## Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.63

$$\int \frac{\arctan(ax)}{x^2 (c + a^2cx^2)^{5/2}} \, dx = \frac{\sqrt{c} \left(-24\sqrt{a^2x^2+1}\, atan(ax)\, a^4x^4 - 36\sqrt{a^2x^2+1}\, atan(ax)\, a^2x^2 - 9\sqrt{a^2x^2+1}\, a\right.}{}$$

input
```
int(atan(a*x)/x^2/(a^2*c*x^2+c)^(5/2),x)
```

output
```
(sqrt(c)*( - 24*sqrt(a**2*x**2 + 1)*atan(a*x)*a**4*x**4 - 36*sqrt(a**2*x**
2 + 1)*atan(a*x)*a**2*x**2 - 9*sqrt(a**2*x**2 + 1)*atan(a*x) - 15*sqrt(a**
2*x**2 + 1)*a**3*x**3 - 16*sqrt(a**2*x**2 + 1)*a*x + 9*log(sqrt(a**2*x**2
+ 1) + a*x - 1)*a**5*x**5 + 18*log(sqrt(a**2*x**2 + 1) + a*x - 1)*a**3*x**
3 + 9*log(sqrt(a**2*x**2 + 1) + a*x - 1)*a*x - 9*log(sqrt(a**2*x**2 + 1) +
 a*x + 1)*a**5*x**5 - 18*log(sqrt(a**2*x**2 + 1) + a*x + 1)*a**3*x**3 - 9*
log(sqrt(a**2*x**2 + 1) + a*x + 1)*a*x))/(9*c**3*x*(a**4*x**4 + 2*a**2*x**
2 + 1))
```

# 3.248 $\int x^m (c+a^2cx^2)^3 \arctan(ax)\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 270

$$\begin{aligned}\int x^m(c+a^2cx^2)^3\arctan(ax)\,dx &= \frac{c^3x^{1+m}\arctan(ax)}{1+m}+\frac{3a^2c^3x^{3+m}\arctan(ax)}{3+m}+\frac{3a^4c^3x^{5+m}\arctan(ax)}{5+m}\\ &+\frac{a^6c^3x^{7+m}\arctan(ax)}{7+m}-\frac{ac^3x^{2+m}\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},-a^2x^2\right)}{2+3m+m^2}\\ &-\frac{3a^3c^3x^{4+m}\text{Hypergeometric2F1}\left(1,\frac{4+m}{2},\frac{6+m}{2},-a^2x^2\right)}{12+7m+m^2}\\ &-\frac{3a^5c^3x^{6+m}\text{Hypergeometric2F1}\left(1,\frac{6+m}{2},\frac{8+m}{2},-a^2x^2\right)}{(5+m)(6+m)}\\ &-\frac{a^7c^3x^{8+m}\text{Hypergeometric2F1}\left(1,\frac{8+m}{2},\frac{10+m}{2},-a^2x^2\right)}{(7+m)(8+m)}\end{aligned}$$

output

```
c^3*x^(1+m)*arctan(a*x)/(1+m)+3*a^2*c^3*x^(3+m)*arctan(a*x)/(3+m)+3*a^4*c^
3*x^(5+m)*arctan(a*x)/(5+m)+a^6*c^3*x^(7+m)*arctan(a*x)/(7+m)-a*c^3*x^(2+m
)*hypergeom([1, 1+1/2*m],[2+1/2*m],-a^2*x^2)/(m^2+3*m+2)-3*a^3*c^3*x^(4+m)
*hypergeom([1, 2+1/2*m],[3+1/2*m],-a^2*x^2)/(m^2+7*m+12)-3*a^5*c^3*x^(6+m)
*hypergeom([1, 3+1/2*m],[4+1/2*m],-a^2*x^2)/(5+m)/(6+m)-a^7*c^3*x^(8+m)*hy
pergeom([1, 4+1/2*m],[5+1/2*m],-a^2*x^2)/(7+m)/(8+m)
```

## Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.87

$$\int x^m \left(c+a^2cx^2\right)^3 \arctan(ax)\,dx = c^3x^{1+m}\left(\frac{\arctan(ax)}{1+m}+\frac{3a^2x^2\arctan(ax)}{3+m}+\frac{3a^4x^4\arctan(ax)}{5+m}+\frac{a^6x^6\arctan(ax)}{7+m}-\frac{a^7x^7\,\text{Hypergeometric2F1}\left(1,4+\frac{m}{2},5+\frac{m}{2},-a^2x^2\right)}{(7+m)(8+m)}-\frac{ax\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},-a^2x^2\right)}{2+3m+m^2}-\frac{3a^3x^3\,\text{Hypergeometric2F1}\left(1,\frac{4+m}{2},\frac{6+m}{2},-a^2x^2\right)}{12+7m+m^2}-\frac{3a^5x^5\,\text{Hypergeometric2F1}\left(1,\frac{6+m}{2},\frac{8+m}{2},-a^2x^2\right)}{(5+m)(6+m)}\right)$$

input
```
Integrate[x^m*(c + a^2*c*x^2)^3*ArcTan[a*x],x]
```

output
```
c^3*x^(1 + m)*(ArcTan[a*x]/(1 + m) + (3*a^2*x^2*ArcTan[a*x])/(3 + m) + (3*
a^4*x^4*ArcTan[a*x])/(5 + m) + (a^6*x^6*ArcTan[a*x])/(7 + m) - (a^7*x^7*Hy
pergeometric2F1[1, 4 + m/2, 5 + m/2, -(a^2*x^2)])/((7 + m)*(8 + m)) - (a*x
*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, -(a^2*x^2)])/(2 + 3*m + m^2) -
 (3*a^3*x^3*Hypergeometric2F1[1, (4 + m)/2, (6 + m)/2, -(a^2*x^2)])/(12 +
7*m + m^2) - (3*a^5*x^5*Hypergeometric2F1[1, (6 + m)/2, (8 + m)/2, -(a^2*x
^2)])/((5 + m)*(6 + m)))
```

## Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \arctan(ax)\left(a^2cx^2+c\right)^3\,dx$$

$$\Big\downarrow \text{5483}$$

$$\int \left(a^6c^3x^{m+6}\arctan(ax)+3a^4c^3x^{m+4}\arctan(ax)+3a^2c^3x^{m+2}\arctan(ax)+c^3x^m\arctan(ax)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^6c^3x^{m+7}\arctan(ax)}{m+7}+\frac{3a^4c^3x^{m+5}\arctan(ax)}{m+5}+\frac{3a^2c^3x^{m+3}\arctan(ax)}{m+3}-\frac{ac^3x^{m+2}\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},-a^2x^2\right)}{m^2+3m+2}-\frac{a^7c^3x^{m+8}\text{Hypergeometric2F1}\left(1,\frac{m+8}{2},\frac{m+10}{2},-a^2x^2\right)}{(m+7)(m+8)}-\frac{3a^5c^3x^{m+6}\text{Hypergeometric2F1}\left(1,\frac{m+6}{2},\frac{m+8}{2},-a^2x^2\right)}{(m+5)(m+6)}-\frac{3a^3c^3x^{m+4}\text{Hypergeometric2F1}\left(1,\frac{m+4}{2},\frac{m+6}{2},-a^2x^2\right)}{m^2+7m+12}+\frac{c^3x^{m+1}\arctan(ax)}{m+1}$$

input
```
Int[x^m*(c + a^2*c*x^2)^3*ArcTan[a*x],x]
```

output
```
(c^3*x^(1 + m)*ArcTan[a*x])/(1 + m) + (3*a^2*c^3*x^(3 + m)*ArcTan[a*x])/(3
 + m) + (3*a^4*c^3*x^(5 + m)*ArcTan[a*x])/(5 + m) + (a^6*c^3*x^(7 + m)*Arc
Tan[a*x])/(7 + m) - (a*c^3*x^(2 + m)*Hypergeometric2F1[1, (2 + m)/2, (4 +
m)/2, -(a^2*x^2)])/(2 + 3*m + m^2) - (3*a^3*c^3*x^(4 + m)*Hypergeometric2F
1[1, (4 + m)/2, (6 + m)/2, -(a^2*x^2)])/(12 + 7*m + m^2) - (3*a^5*c^3*x^(6
 + m)*Hypergeometric2F1[1, (6 + m)/2, (8 + m)/2, -(a^2*x^2)])/((5 + m)*(6
+ m)) - (a^7*c^3*x^(8 + m)*Hypergeometric2F1[1, (8 + m)/2, (10 + m)/2, -(a
^2*x^2)])/((7 + m)*(8 + m))
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

## Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5.

Time = 120.87 (sec) , antiderivative size = 600, normalized size of antiderivative = 2.22

| method | result |
|---|---|
| meijerg | $\frac{a^{-m-1} c^{3} \left(-\frac{4 x^{m} a^{m} \left(a^{6} m^{3} x^{6}+6 a^{6} m^{2} x^{6}+8 a^{6} m\, x^{6}-a^{4} m^{3} x^{4}-8 a^{4} m^{2} x^{4}-12 a^{4} m\, x^{4}+a^{2} m^{3} x^{2}+10 a^{2} m^{2} x^{2}+24 a^{2} m\, x^{2}-m^{3}-12 m^{2}-44 m-48\right)}{\left(7+m \right) m \left(2+m \right) \left(4+m \right) \left(6+m \right)}}{4}$ |

input
```
int(x^m*(a^2*c*x^2+c)^3*arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/4*a^(-m-1)*c^3*(-4*x^m*a^m*(a^6*m^3*x^6+6*a^6*m^2*x^6+8*a^6*m*x^6-a^4*m^
3*x^4-8*a^4*m^2*x^4-12*a^4*m*x^4+a^2*m^3*x^2+10*a^2*m^2*x^2+24*a^2*m*x^2-m
^3-12*m^2-44*m-48)/(7+m)/m/(2+m)/(4+m)/(6+m)+8*x^(8+m)*a^(8+m)/(14+2*m)/(a
^2*x^2)^(1/2)*arctan((a^2*x^2)^(1/2))+2/(8+m)*x^m*a^m*(-8-m)/(7+m)*LerchPh
i(-a^2*x^2,1,1/2*m))+3/4*a^(-m-1)*c^3*(-4*x^m*a^m*(a^4*m^2*x^4+2*a^4*m*x^4
-a^2*m^2*x^2-4*a^2*m*x^2+m^2+6*m+8)/(5+m)/m/(2+m)/(4+m)+8*x^(6+m)*a^(6+m)/
(10+2*m)/(a^2*x^2)^(1/2)*arctan((a^2*x^2)^(1/2))+2*x^m*a^m/(5+m)*LerchPhi(
-a^2*x^2,1,1/2*m))+3/4*a^(-m-1)*c^3*(-4*x^m*a^m*(a^2*m*x^2-m-2)/(3+m)/m/(2
+m)+8*x^(4+m)*a^(4+m)/(6+2*m)/(a^2*x^2)^(1/2)*arctan((a^2*x^2)^(1/2))+2/(4
+m)*x^m*a^m*(-m-4)/(3+m)*LerchPhi(-a^2*x^2,1,1/2*m))+1/4*a^(-m-1)*c^3*(4/(
2+m)*x^m*a^m*(-m-2)/(1+m)/m+8*x^(2+m)*a^(2+m)/(2+2*m)/(a^2*x^2)^(1/2)*arct
an((a^2*x^2)^(1/2))+2*x^m*a^m/(1+m)*LerchPhi(-a^2*x^2,1,1/2*m))
```

## Fricas [F]

$$\int x^m (c + a^2 c x^2)^3 \arctan(ax)\,dx = \int (a^2 c x^2 + c)^3 x^m \arctan(ax)\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^3*arctan(a*x),x, algorithm="fricas")
```

output
```
integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*x^m*arctan(a*
x), x)
```

## Sympy [F]

$$\int x^m (c + a^2 c x^2)^3 \arctan(ax)\,dx = c^3 \left( \int x^m \operatorname{atan}(ax)\,dx + \int 3a^2 x^2 x^m \operatorname{atan}(ax)\,dx + \int 3a^4 x^4 x^m \operatorname{atan}(ax)\,dx + \int a^6 x^6 x^m \operatorname{atan}(ax)\,dx \right)$$

input
```
integrate(x**m*(a**2*c*x**2+c)**3*atan(a*x),x)
```

output
```
c**3*(Integral(x**m*atan(a*x), x) + Integral(3*a**2*x**2*x**m*atan(a*x), x
) + Integral(3*a**4*x**4*x**m*atan(a*x), x) + Integral(a**6*x**6*x**m*atan
(a*x), x))
```

## Maxima [F]

$$\int x^m (c + a^2 c x^2)^3 \arctan(ax)\,dx = \int (a^2 c x^2 + c)^3 x^m \arctan(ax)\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^3*arctan(a*x),x, algorithm="maxima")
```

output
```
(((a^6*c^3*m^3 + 9*a^6*c^3*m^2 + 23*a^6*c^3*m + 15*a^6*c^3)*x^7 + 3*(a^4*c
^3*m^3 + 11*a^4*c^3*m^2 + 31*a^4*c^3*m + 21*a^4*c^3)*x^5 + 3*(a^2*c^3*m^3
+ 13*a^2*c^3*m^2 + 47*a^2*c^3*m + 35*a^2*c^3)*x^3 + (c^3*m^3 + 15*c^3*m^2
+ 71*c^3*m + 105*c^3)*x)*x^m*arctan(a*x) - (m^4 + 16*m^3 + 86*m^2 + 176*m
+ 105)*integrate(((a^7*c^3*m^3 + 9*a^7*c^3*m^2 + 23*a^7*c^3*m + 15*a^7*c^3
)*x^7 + 3*(a^5*c^3*m^3 + 11*a^5*c^3*m^2 + 31*a^5*c^3*m + 21*a^5*c^3)*x^5 +
 3*(a^3*c^3*m^3 + 13*a^3*c^3*m^2 + 47*a^3*c^3*m + 35*a^3*c^3)*x^3 + (a*c^3
*m^3 + 15*a*c^3*m^2 + 71*a*c^3*m + 105*a*c^3)*x)*x^m/(m^4 + 16*m^3 + (a^2*
m^4 + 16*a^2*m^3 + 86*a^2*m^2 + 176*a^2*m + 105*a^2)*x^2 + 86*m^2 + 176*m
+ 105), x))/(m^4 + 16*m^3 + 86*m^2 + 176*m + 105)
```

## Giac [F]

$$\int x^m\left(c+a^2cx^2\right)^3\arctan(ax)\,dx=\int\left(a^2cx^2+c\right)^3x^m\arctan\left(ax\right)\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^3*arctan(a*x),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^3*x^m*arctan(a*x), x)
```

## Mupad [F(-1)]

Timed out.

$$\int x^m\left(c+a^2cx^2\right)^3\arctan(ax)\,dx=\int x^m\,\mathrm{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^3\,dx$$

input
```
int(x^m*atan(a*x)*(c + a^2*c*x^2)^3,x)
```

output
```
int(x^m*atan(a*x)*(c + a^2*c*x^2)^3, x)
```

## Reduce [F]

$$\int x^m \left(c+a^2cx^2\right)^3 \arctan(ax)\,dx = \text{Too large to display}$$

input
```
int(x^m*(a^2*c*x^2+c)^3*atan(a*x),x)
```

output
```
(c**3*(x**m*atan(a*x)*a**7*m**7*x**7 + 21*x**m*atan(a*x)*a**7*m**6*x**7 +
175*x**m*atan(a*x)*a**7*m**5*x**7 + 735*x**m*atan(a*x)*a**7*m**4*x**7 + 16
24*x**m*atan(a*x)*a**7*m**3*x**7 + 1764*x**m*atan(a*x)*a**7*m**2*x**7 + 72
0*x**m*atan(a*x)*a**7*m*x**7 + 3*x**m*atan(a*x)*a**5*m**7*x**5 + 69*x**m*a
tan(a*x)*a**5*m**6*x**5 + 621*x**m*atan(a*x)*a**5*m**5*x**5 + 2775*x**m*at
an(a*x)*a**5*m**4*x**5 + 6432*x**m*atan(a*x)*a**5*m**3*x**5 + 7236*x**m*at
an(a*x)*a**5*m**2*x**5 + 3024*x**m*atan(a*x)*a**5*m*x**5 + 3*x**m*atan(a*x
)*a**3*m**7*x**3 + 75*x**m*atan(a*x)*a**3*m**6*x**3 + 741*x**m*atan(a*x)*a
**3*m**5*x**3 + 3657*x**m*atan(a*x)*a**3*m**4*x**3 + 9336*x**m*atan(a*x)*a
**3*m**3*x**3 + 11388*x**m*atan(a*x)*a**3*m**2*x**3 + 5040*x**m*atan(a*x)*
a**3*m*x**3 + x**m*atan(a*x)*a*m**7*x + 27*x**m*atan(a*x)*a*m**6*x + 295*x
**m*atan(a*x)*a*m**5*x + 1665*x**m*atan(a*x)*a*m**4*x + 5104*x**m*atan(a*x
)*a*m**3*x + 8028*x**m*atan(a*x)*a*m**2*x + 5040*x**m*atan(a*x)*a*m*x - x*
*m*a**6*m**6*x**6 - 15*x**m*a**6*m**5*x**6 - 85*x**m*a**6*m**4*x**6 - 225*
x**m*a**6*m**3*x**6 - 274*x**m*a**6*m**2*x**6 - 120*x**m*a**6*m*x**6 - 2*x
**m*a**4*m**6*x**4 - 40*x**m*a**4*m**5*x**4 - 286*x**m*a**4*m**4*x**4 - 89
6*x**m*a**4*m**3*x**4 - 1224*x**m*a**4*m**2*x**4 - 576*x**m*a**4*m*x**4 -
x**m*a**2*m**6*x**2 - 25*x**m*a**2*m**5*x**2 - 245*x**m*a**2*m**4*x**2 - 1
127*x**m*a**2*m**3*x**2 - 2274*x**m*a**2*m**2*x**2 - 1368*x**m*a**2*m*x**2
 - 48*x**m*m**3 - 576*x**m*m**2 - 2112*x**m*m - 2304*x**m + 48*int(x**m...
```

# 3.249 $\int x^m (c + a^2 c x^2)^2 \arctan(ax)\, dx$



## Optimal result

Integrand size = 20, antiderivative size = 201

$$\begin{aligned}
&\int x^m (c + a^2 c x^2)^2 \arctan(ax)\, dx \\
&= \frac{c^2 x^{1+m} \arctan(ax)}{1+m} + \frac{2a^2 c^2 x^{3+m} \arctan(ax)}{3+m} + \frac{a^4 c^2 x^{5+m} \arctan(ax)}{5+m} \\
&\quad - \frac{a c^2 x^{2+m} \text{Hypergeometric2F1}\left(1, \frac{2+m}{2}, \frac{4+m}{2}, -a^2 x^2\right)}{2+3m+m^2} \\
&\quad - \frac{2a^3 c^2 x^{4+m} \text{Hypergeometric2F1}\left(1, \frac{4+m}{2}, \frac{6+m}{2}, -a^2 x^2\right)}{12+7m+m^2} \\
&\quad - \frac{a^5 c^2 x^{6+m} \text{Hypergeometric2F1}\left(1, \frac{6+m}{2}, \frac{8+m}{2}, -a^2 x^2\right)}{(5+m)(6+m)}
\end{aligned}$$

output
```
c^2*x^(1+m)*arctan(a*x)/(1+m)+2*a^2*c^2*x^(3+m)*arctan(a*x)/(3+m)+a^4*c^2*
x^(5+m)*arctan(a*x)/(5+m)-a*c^2*x^(2+m)*hypergeom([1, 1+1/2*m],[2+1/2*m],-
a^2*x^2)/(m^2+3*m+2)-2*a^3*c^2*x^(4+m)*hypergeom([1, 2+1/2*m],[3+1/2*m],-a
^2*x^2)/(m^2+7*m+12)-a^5*c^2*x^(6+m)*hypergeom([1, 3+1/2*m],[4+1/2*m],-a^2
*x^2)/(5+m)/(6+m)
```

## Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.87

$$\int x^m (c+a^2cx^2)^2 \arctan(ax)\,dx = c^2x^{1+m}\left(\frac{\arctan(ax)}{1+m}+\frac{2a^2x^2\arctan(ax)}{3+m}+\frac{a^4x^4\arctan(ax)}{5+m}-\frac{ax\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},-a^2x^2\right)}{2+3m+m^2}-\frac{2a^3x^3\,\text{Hypergeometric2F1}\left(1,\frac{4+m}{2},\frac{6+m}{2},-a^2x^2\right)}{12+7m+m^2}-\frac{a^5x^5\,\text{Hypergeometric2F1}\left(1,\frac{6+m}{2},\frac{8+m}{2},-a^2x^2\right)}{(5+m)(6+m)}\right)$$

input
```
Integrate[x^m*(c + a^2*c*x^2)^2*ArcTan[a*x],x]
```

output
```
c^2*x^(1 + m)*(ArcTan[a*x]/(1 + m) + (2*a^2*x^2*ArcTan[a*x])/(3 + m) + (a^
4*x^4*ArcTan[a*x])/(5 + m) - (a*x*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/
2, -(a^2*x^2)])/(2 + 3*m + m^2) - (2*a^3*x^3*Hypergeometric2F1[1, (4 + m)/
2, (6 + m)/2, -(a^2*x^2)])/(12 + 7*m + m^2) - (a^5*x^5*Hypergeometric2F1[1
, (6 + m)/2, (8 + m)/2, -(a^2*x^2)])/((5 + m)*(6 + m)))
```

## Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \arctan(ax)\left(a^2cx^2+c\right)^2\,dx$$

$$\Big\downarrow \text{5483}$$

$$\int \left(a^4c^2x^{m+4}\arctan(ax)+2a^2c^2x^{m+2}\arctan(ax)+c^2x^m\arctan(ax)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^4c^2x^{m+5}\arctan(ax)}{m+5}+\frac{2a^2c^2x^{m+3}\arctan(ax)}{m+3}-\frac{ac^2x^{m+2}\,\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},-a^2x^2\right)}{m^2+3m+2}-\frac{a^5c^2x^{m+6}\,\text{Hypergeometric2F1}\left(1,\frac{m+6}{2},\frac{m+8}{2},-a^2x^2\right)}{(m+5)(m+6)}-\frac{2a^3c^2x^{m+4}\,\text{Hypergeometric2F1}\left(1,\frac{m+4}{2},\frac{m+6}{2},-a^2x^2\right)}{m^2+7m+12}+\frac{c^2x^{m+1}\arctan(ax)}{m+1}$$

input
```
Int[x^m*(c + a^2*c*x^2)^2*ArcTan[a*x],x]
```

output
```
(c^2*x^(1 + m)*ArcTan[a*x])/(1 + m) + (2*a^2*c^2*x^(3 + m)*ArcTan[a*x])/(3
 + m) + (a^4*c^2*x^(5 + m)*ArcTan[a*x])/(5 + m) - (a*c^2*x^(2 + m)*Hyperge
ometric2F1[1, (2 + m)/2, (4 + m)/2, -(a^2*x^2)])/(2 + 3*m + m^2) - (2*a^3*
c^2*x^(4 + m)*Hypergeometric2F1[1, (4 + m)/2, (6 + m)/2, -(a^2*x^2)])/(12
+ 7*m + m^2) - (a^5*c^2*x^(6 + m)*Hypergeometric2F1[1, (6 + m)/2, (8 + m)/
2, -(a^2*x^2)])/((5 + m)*(6 + m))
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

## Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5.

Time = 33.82 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.87

| method | result |
|---|---|
| meijerg | $\frac{a^{-m-1}c^{2}\left(-\frac{4x^{m}a^{m}\left(a^{4}m^{2}x^{4}+2a^{4}m\,x^{4}-a^{2}m^{2}x^{2}-4a^{2}m\,x^{2}+m^{2}+6m+8\right)}{(5+m)m(2+m)(4+m)}+\frac{8x^{6+m}a^{6+m}\arctan\left(\sqrt{a^{2}x^{2}}\right)}{(10+2m)\sqrt{a^{2}x^{2}}}+\frac{2x^{m}a^{m}\,\mathrm{LerchPhi}\left(-a^{2}x^{2},1,\frac{m}{2}\right.}{5+m}}{4}$ |

input
```
int(x^m*(a^2*c*x^2+c)^2*arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/4*a^(-m-1)*c^2*(-4*x^m*a^m*(a^4*m^2*x^4+2*a^4*m*x^4-a^2*m^2*x^2-4*a^2*m*
x^2+m^2+6*m+8)/(5+m)/m/(2+m)/(4+m)+8*x^(6+m)*a^(6+m)/(10+2*m)/(a^2*x^2)^(1
/2)*arctan((a^2*x^2)^(1/2))+2*x^m*a^m/(5+m)*LerchPhi(-a^2*x^2,1,1/2*m))+1/
2*a^(-m-1)*c^2*(-4*x^m*a^m*(a^2*m*x^2-m-2)/(3+m)/m/(2+m)+8*x^(4+m)*a^(4+m)
/(6+2*m)/(a^2*x^2)^(1/2)*arctan((a^2*x^2)^(1/2))+2/(4+m)*x^m*a^m*(-m-4)/(3
+m)*LerchPhi(-a^2*x^2,1,1/2*m))+1/4*a^(-m-1)*c^2*(4/(2+m)*x^m*a^m*(-m-2)/(
1+m)/m+8*x^(2+m)*a^(2+m)/(2+2*m)/(a^2*x^2)^(1/2)*arctan((a^2*x^2)^(1/2))+2
*x^m*a^m/(1+m)*LerchPhi(-a^2*x^2,1,1/2*m))
```

## Fricas [F]

$$\int x^m \left(c+a^2cx^2\right)^2 \arctan(ax)\,dx = \int \left(a^2cx^2+c\right)^2 x^m \arctan\left(ax\right)\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^2*arctan(a*x),x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*x^m*arctan(a*x), x)
```

## Sympy [F]

$$\int x^m \left(c+a^2cx^2\right)^2 \arctan(ax)\,dx = c^2\left(\int x^m \operatorname{atan}(ax)\,dx + \int 2a^2x^2x^m \operatorname{atan}(ax)\,dx + \int a^4x^4x^m \operatorname{atan}(ax)\,dx\right)$$

input
```
integrate(x**m*(a**2*c*x**2+c)**2*atan(a*x),x)
```

output
```
c**2*(Integral(x**m*atan(a*x), x) + Integral(2*a**2*x**2*x**m*atan(a*x), x
) + Integral(a**4*x**4*x**m*atan(a*x), x))
```

## Maxima [F]

$$\int x^m \left(c+a^2cx^2\right)^2 \arctan(ax)\,dx = \int \left(a^2cx^2+c\right)^2 x^m \arctan(ax)\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^2*arctan(a*x),x, algorithm="maxima")
```

output
```
(((a^4*c^2*m^2 + 4*a^4*c^2*m + 3*a^4*c^2)*x^5 + 2*(a^2*c^2*m^2 + 6*a^2*c^2
*m + 5*a^2*c^2)*x^3 + (c^2*m^2 + 8*c^2*m + 15*c^2)*x)*x^m*arctan(a*x) - (m
^3 + 9*m^2 + 23*m + 15)*integrate(((a^5*c^2*m^2 + 4*a^5*c^2*m + 3*a^5*c^2)
*x^5 + 2*(a^3*c^2*m^2 + 6*a^3*c^2*m + 5*a^3*c^2)*x^3 + (a*c^2*m^2 + 8*a*c^
2*m + 15*a*c^2)*x)*x^m/(m^3 + (a^2*m^3 + 9*a^2*m^2 + 23*a^2*m + 15*a^2)*x^
2 + 9*m^2 + 23*m + 15), x))/(m^3 + 9*m^2 + 23*m + 15)
```

## Giac [F]

$$\int x^m (c + a^2cx^2)^2 \arctan(ax) \, dx = \int (a^2cx^2 + c)^2 x^m \arctan(ax) \, dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^2*arctan(a*x),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^2*x^m*arctan(a*x), x)
```

## Mupad [F(-1)]

Timed out.

$$\int x^m (c + a^2cx^2)^2 \arctan(ax) \, dx = \int x^m \operatorname{atan}(a\,x)\, \left(c\,a^2\,x^2 + c\right)^2 \, dx$$

input
```
int(x^m*atan(a*x)*(c + a^2*c*x^2)^2,x)
```

output
```
int(x^m*atan(a*x)*(c + a^2*c*x^2)^2, x)
```

## Reduce [F]

$$\int x^m (c + a^2cx^2)^2 \arctan(ax) \, dx = \text{Too large to display}$$

input
```
int(x^m*(a^2*c*x^2+c)^2*atan(a*x),x)
```

output
```
(c**2*(x**m*atan(a*x)*a**5*m**5*x**5 + 10*x**m*atan(a*x)*a**5*m**4*x**5 +
35*x**m*atan(a*x)*a**5*m**3*x**5 + 50*x**m*atan(a*x)*a**5*m**2*x**5 + 24*x
**m*atan(a*x)*a**5*m*x**5 + 2*x**m*atan(a*x)*a**3*m**5*x**3 + 24*x**m*atan
(a*x)*a**3*m**4*x**3 + 98*x**m*atan(a*x)*a**3*m**3*x**3 + 156*x**m*atan(a*
x)*a**3*m**2*x**3 + 80*x**m*atan(a*x)*a**3*m*x**3 + x**m*atan(a*x)*a*m**5*
x + 14*x**m*atan(a*x)*a*m**4*x + 71*x**m*atan(a*x)*a*m**3*x + 154*x**m*ata
n(a*x)*a*m**2*x + 120*x**m*atan(a*x)*a*m*x - x**m*a**4*m**4*x**4 - 6*x**m*
a**4*m**3*x**4 - 11*x**m*a**4*m**2*x**4 - 6*x**m*a**4*m*x**4 - x**m*a**2*m
**4*x**2 - 12*x**m*a**2*m**3*x**2 - 39*x**m*a**2*m**2*x**2 - 28*x**m*a**2*
m*x**2 - 8*x**m*m**2 - 48*x**m*m - 64*x**m + 8*int(x**m/(a**2*m**3*x**3 +
9*a**2*m**2*x**3 + 23*a**2*m*x**3 + 15*a**2*x**3 + m**3*x + 9*m**2*x + 23*
m*x + 15*x),x)*m**6 + 120*int(x**m/(a**2*m**3*x**3 + 9*a**2*m**2*x**3 + 23
*a**2*m*x**3 + 15*a**2*x**3 + m**3*x + 9*m**2*x + 23*m*x + 15*x),x)*m**5 +
 680*int(x**m/(a**2*m**3*x**3 + 9*a**2*m**2*x**3 + 23*a**2*m*x**3 + 15*a**
2*x**3 + m**3*x + 9*m**2*x + 23*m*x + 15*x),x)*m**4 + 1800*int(x**m/(a**2*
m**3*x**3 + 9*a**2*m**2*x**3 + 23*a**2*m*x**3 + 15*a**2*x**3 + m**3*x + 9*
m**2*x + 23*m*x + 15*x),x)*m**3 + 2192*int(x**m/(a**2*m**3*x**3 + 9*a**2*m
**2*x**3 + 23*a**2*m*x**3 + 15*a**2*x**3 + m**3*x + 9*m**2*x + 23*m*x + 15
*x),x)*m**2 + 960*int(x**m/(a**2*m**3*x**3 + 9*a**2*m**2*x**3 + 23*a**2*m*
x**3 + 15*a**2*x**3 + m**3*x + 9*m**2*x + 23*m*x + 15*x),x)*m))/(a*m*(m...
```

# 3.250 $\int x^m(c+a^2cx^2)\arctan(ax)\,dx$



## Optimal result

Integrand size = 18, antiderivative size = 124

$$\int x^m\left(c+a^2cx^2\right)\arctan(ax)\,dx=\frac{cx^{1+m}\arctan(ax)}{1+m}+\frac{a^2cx^{3+m}\arctan(ax)}{3+m}-\frac{acx^{2+m}\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},-a^2x^2\right)}{2+3m+m^2}-\frac{a^3cx^{4+m}\,\text{Hypergeometric2F1}\left(1,\frac{4+m}{2},\frac{6+m}{2},-a^2x^2\right)}{12+7m+m^2}$$

output
```
c*x^(1+m)*arctan(a*x)/(1+m)+a^2*c*x^(3+m)*arctan(a*x)/(3+m)-a*c*x^(2+m)*hy
pergeom([1, 1+1/2*m],[2+1/2*m],-a^2*x^2)/(m^2+3*m+2)-a^3*c*x^(4+m)*hyperge
om([1, 2+1/2*m],[3+1/2*m],-a^2*x^2)/(m^2+7*m+12)
```

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.90

$$\int x^m\left(c+a^2cx^2\right)\arctan(ax)\,dx=cx^{1+m}\left(\left(\frac{1}{1+m}+\frac{a^2x^2}{3+m}\right)\arctan(ax)-\frac{ax\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},-a^2x^2\right)}{2+3m+m^2}-\frac{a^3x^3\,\text{Hypergeometric2F1}\left(1,\frac{4+m}{2},\frac{6+m}{2},-a^2x^2\right)}{12+7m+m^2}\right)$$

input
```
Integrate[x^m*(c + a^2*c*x^2)*ArcTan[a*x],x]
```

output
```
c*x^(1 + m)*(((1 + m)^(-1) + (a^2*x^2)/(3 + m))*ArcTan[a*x] - (a*x*Hyperge
ometric2F1[1, (2 + m)/2, (4 + m)/2, -(a^2*x^2)])/(2 + 3*m + m^2) - (a^3*x^
3*Hypergeometric2F1[1, (4 + m)/2, (6 + m)/2, -(a^2*x^2)])/(12 + 7*m + m^2)
)
```

## Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5485, 5361, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \arctan(ax)\left(a^2cx^2+c\right)dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\int x^{m+2}\arctan(ax)dx+c\int x^m\arctan(ax)dx$$

$$\Big\downarrow \text{5361}$$

$$a^2c\left(\frac{x^{m+3}\arctan(ax)}{m+3}-\frac{a\int\frac{x^{m+3}}{a^2x^2+1}dx}{m+3}\right)+c\left(\frac{x^{m+1}\arctan(ax)}{m+1}-\frac{a\int\frac{x^{m+1}}{a^2x^2+1}dx}{m+1}\right)$$

$$\Big\downarrow \text{278}$$

$$a^2c\left(\frac{x^{m+3}\arctan(ax)}{m+3}-\frac{ax^{m+4}\text{Hypergeometric2F1}\left(1,\frac{m+4}{2},\frac{m+6}{2},-a^2x^2\right)}{(m+3)(m+4)}\right)+c\left(\frac{x^{m+1}\arctan(ax)}{m+1}-\frac{ax^{m+2}\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},-a^2x^2\right)}{(m+1)(m+2)}\right)$$

input
```
Int[x^m*(c + a^2*c*x^2)*ArcTan[a*x],x]
```

output
```
c*((x^(1 + m)*ArcTan[a*x])/(1 + m) - (a*x^(2 + m)*Hypergeometric2F1[1, (2
+ m)/2, (4 + m)/2, -(a^2*x^2)])/((1 + m)*(2 + m))) + a^2*c*((x^(3 + m)*Arc
Tan[a*x])/(3 + m) - (a*x^(4 + m)*Hypergeometric2F1[1, (4 + m)/2, (6 + m)/2
, -(a^2*x^2)])/((3 + m)*(4 + m)))
```

### Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

## Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5.

Time = 8.49 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.79

| method | result |
|---|---|
| meijerg | $\frac{a^{-m-1} c \left(-\frac{4 x^{m} a^{m} \left(a^{2} m \,x^{2}-m-2\right)}{(3+m) m (2+m)}+\frac{8 x^{4+m} a^{4+m} \arctan\left(\sqrt{a^{2} x^{2}}\right)}{(6+2m) \sqrt{a^{2} x^{2}}}+\frac{2 x^{m} a^{m} (-m-4) \operatorname{LerchPhi}\left(-a^{2} x^{2}, 1, \frac{m}{2}\right)}{(4+m)(3+m)}\right)}{4}+\frac{a^{-m-1} c \left(\frac{4 x^{m} a^{m} (-}{(2+m)(1-}\right.}{}$ |

input
```
int(x^m*(a^2*c*x^2+c)*arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/4*a^(-m-1)*c*(-4*x^m*a^m*(a^2*m*x^2-m-2)/(3+m)/m/(2+m)+8*x^(4+m)*a^(4+m)
/(6+2*m)/(a^2*x^2)^(1/2)*arctan((a^2*x^2)^(1/2))+2/(4+m)*x^m*a^m*(-m-4)/(3
+m)*LerchPhi(-a^2*x^2,1,1/2*m))+1/4*a^(-m-1)*c*(4/(2+m)*x^m*a^m*(-m-2)/(1+
m)/m+8*x^(2+m)*a^(2+m)/(2+2*m)/(a^2*x^2)^(1/2)*arctan((a^2*x^2)^(1/2))+2*x
^m*a^m/(1+m)*LerchPhi(-a^2*x^2,1,1/2*m))
```

## Fricas [F]

$$\int x^m \left(c+a^2cx^2\right) \arctan(ax)\, dx = \int \left(a^2cx^2+c\right)x^m \arctan\left(ax\right)\, dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)*arctan(a*x),x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)*x^m*arctan(a*x), x)
```

## Sympy [F]

$$\int x^m \left(c+a^2cx^2\right) \arctan(ax)\, dx = c\left(\int x^m \operatorname{atan}\left(ax\right) dx + \int a^2x^2x^m \operatorname{atan}\left(ax\right) dx\right)$$

input
```
integrate(x**m*(a**2*c*x**2+c)*atan(a*x),x)
```

output
```
c*(Integral(x**m*atan(a*x), x) + Integral(a**2*x**2*x**m*atan(a*x), x))
```

## Maxima [F]

$$\int x^m (c + a^2cx^2) \arctan(ax) \, dx = \int (a^2cx^2 + c)x^m \arctan(ax) \, dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)*arctan(a*x),x, algorithm="maxima")
```

output
```
(((a^2*c*m + a^2*c)*x^3 + (c*m + 3*c)*x)*x^m*arctan(a*x) - (m^2 + 4*m + 3)
*integrate(((a^3*c*m + a^3*c)*x^3 + (a*c*m + 3*a*c)*x)*x^m/((a^2*m^2 + 4*a
^2*m + 3*a^2)*x^2 + m^2 + 4*m + 3), x))/(m^2 + 4*m + 3)
```

## Giac [F]

$$\int x^m (c + a^2cx^2) \arctan(ax) \, dx = \int (a^2cx^2 + c)x^m \arctan(ax) \, dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)*arctan(a*x),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)*x^m*arctan(a*x), x)
```

## Mupad [F(-1)]

Timed out.

$$\int x^m (c + a^2cx^2) \arctan(ax) \, dx = \int x^m \operatorname{atan}(a\,x) \, \left(c\,a^2\,x^2 + c\right) \, dx$$

input
```
int(x^m*atan(a*x)*(c + a^2*c*x^2),x)
```

output
```
int(x^m*atan(a*x)*(c + a^2*c*x^2), x)
```

## Reduce [F]

$$\int x^m(c+a^2cx^2)\arctan(ax)\,dx$$
$$= \frac{c\left(x^m atan(ax)\,a^3m^3x^3 + 3x^m atan(ax)\,a^3m^2x^3 + 2x^m atan(ax)\,a^3m\,x^3 + x^m atan(ax)\,a\,m^3x + 5x^m atan(\right.}{}$$

input
```
int(x^m*(a^2*c*x^2+c)*atan(a*x),x)
```

output
```
(c*(x**m*atan(a*x)*a**3*m**3*x**3 + 3*x**m*atan(a*x)*a**3*m**2*x**3 + 2*x*
*m*atan(a*x)*a**3*m*x**3 + x**m*atan(a*x)*a*m**3*x + 5*x**m*atan(a*x)*a*m*
*2*x + 6*x**m*atan(a*x)*a*m*x - x**m*a**2*m**2*x**2 - x**m*a**2*m*x**2 - 2
*x**m*m - 4*x**m + 2*int(x**m/(a**2*m**2*x**3 + 4*a**2*m*x**3 + 3*a**2*x**
3 + m**2*x + 4*m*x + 3*x),x)*m**4 + 12*int(x**m/(a**2*m**2*x**3 + 4*a**2*m
*x**3 + 3*a**2*x**3 + m**2*x + 4*m*x + 3*x),x)*m**3 + 22*int(x**m/(a**2*m*
*2*x**3 + 4*a**2*m*x**3 + 3*a**2*x**3 + m**2*x + 4*m*x + 3*x),x)*m**2 + 12
*int(x**m/(a**2*m**2*x**3 + 4*a**2*m*x**3 + 3*a**2*x**3 + m**2*x + 4*m*x +
 3*x),x)*m))/(a*m*(m**3 + 6*m**2 + 11*m + 6))
```

# 3.251 $\int \frac{x^m \arctan(ax)}{c+a^2cx^2} dx$



## Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{x^m \arctan(ax)}{c+a^2cx^2} dx = \text{Int}\left(\frac{x^m \arctan(ax)}{c+a^2cx^2}, x\right)$$

output
```
Defer(Int)(x^m*arctan(a*x)/(a^2*c*x^2+c),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x^m \arctan(ax)}{c+a^2cx^2} dx = \int \frac{x^m \arctan(ax)}{c+a^2cx^2} dx$$

input
```
Integrate[(x^m*ArcTan[a*x])/(c + a^2*c*x^2),x]
```

output
```
Integrate[(x^m*ArcTan[a*x])/(c + a^2*c*x^2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \arctan(ax)}{a^2cx^2+c} \, dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int \frac{x^m \arctan(ax)}{a^2cx^2+c}dx$$

input
```
Int[(x^m*ArcTan[a*x])/(c + a^2*c*x^2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.87 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan(ax)}{a^2c\,x^2+c}dx$$

input
```
int(x^m*arctan(a*x)/(a^2*c*x^2+c),x)
```

output
```
int(x^m*arctan(a*x)/(a^2*c*x^2+c),x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x^m \arctan(ax)}{c + a^2cx^2} \, dx = \int \frac{x^m \arctan{(ax)}}{a^2cx^2 + c} \, dx$$

input
```
integrate(x^m*arctan(a*x)/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
integral(x^m*arctan(a*x)/(a^2*c*x^2 + c), x)
```

## Sympy [N/A]

Not integrable

Time = 1.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{x^m \arctan(ax)}{c + a^2cx^2} \, dx = \frac{\int \frac{x^m \operatorname{atan}{(ax)}}{a^2x^2+1} \, dx}{c}$$

input
```
integrate(x**m*atan(a*x)/(a**2*c*x**2+c),x)
```

output
```
Integral(x**m*atan(a*x)/(a**2*x**2 + 1), x)/c
```

## Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x^m \arctan(ax)}{c + a^2cx^2} \, dx = \int \frac{x^m \arctan{(ax)}}{a^2cx^2 + c} \, dx$$

input
```
integrate(x^m*arctan(a*x)/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
integrate(x^m*arctan(a*x)/(a^2*c*x^2 + c), x)
```

## Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x^m \arctan(ax)}{c+a^2cx^2}\,dx = \int \frac{x^m \arctan{(ax)}}{a^2cx^2+c}\,dx$$

input
```
integrate(x^m*arctan(a*x)/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
integrate(x^m*arctan(a*x)/(a^2*c*x^2 + c), x)
```

## Mupad [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x^m \arctan(ax)}{c+a^2cx^2}\,dx = \int \frac{x^m \operatorname{atan}(a\,x)}{c\,a^2\,x^2+c}\,dx$$

input
```
int((x^m*atan(a*x))/(c + a^2*c*x^2),x)
```

output
```
int((x^m*atan(a*x))/(c + a^2*c*x^2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.25

$$\int \frac{x^m \arctan(ax)}{c + a^2cx^2} \, dx = \frac{\int \frac{x^m atan(ax)}{a^2x^2+1} dx}{c}$$

input
```
int(x^m*atan(a*x)/(a^2*c*x^2+c),x)
```

output
```
int((x**m*atan(a*x))/(a**2*x**2 + 1),x)/c
```

# 3.252 $\int \frac{x^m \arctan(ax)}{(c+a^2cx^2)^2} dx$



## Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{x^m \arctan(ax)}{(c+a^2cx^2)^2} dx = \text{Int}\left(\frac{x^m \arctan(ax)}{(c+a^2cx^2)^2}, x\right)$$

output
```
Defer(Int)(x^m*arctan(a*x)/(a^2*c*x^2+c)^2,x)
```

## Mathematica [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x^m \arctan(ax)}{(c+a^2cx^2)^2} dx = \int \frac{x^m \arctan(ax)}{(c+a^2cx^2)^2} dx$$

input
```
Integrate[(x^m*ArcTan[a*x])/(c + a^2*c*x^2)^2,x]
```

output
```
Integrate[(x^m*ArcTan[a*x])/(c + a^2*c*x^2)^2, x]
```

## Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \arctan(ax)}{(a^2cx^2+c)^2} \, dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int \frac{x^m \arctan(ax)}{(a^2cx^2+c)^2}dx$$

input
```
Int[(x^m*ArcTan[a*x])/(c + a^2*c*x^2)^2,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 1.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan(ax)}{(a^2c\,x^2+c)^2}dx$$

input
```
int(x^m*arctan(a*x)/(a^2*c*x^2+c)^2,x)
```

output
```
int(x^m*arctan(a*x)/(a^2*c*x^2+c)^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.85

$$\int \frac{x^m \arctan(ax)}{(c+a^2cx^2)^2}\,dx = \int \frac{x^m \arctan(ax)}{(a^2cx^2+c)^2}\,dx$$

input
```
integrate(x^m*arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
integral(x^m*arctan(a*x)/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2), x)
```

## Sympy [N/A]

Not integrable

Time = 4.62 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.45

$$\int \frac{x^m \arctan(ax)}{(c+a^2cx^2)^2}\,dx = \frac{\int \frac{x^m \operatorname{atan}(ax)}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

input
```
integrate(x**m*atan(a*x)/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(x**m*atan(a*x)/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

## Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x^m \arctan(ax)}{(c+a^2cx^2)^2}\,dx = \int \frac{x^m \arctan(ax)}{(a^2cx^2+c)^2}\,dx$$

input
```
integrate(x^m*arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
integrate(x^m*arctan(a*x)/(a^2*c*x^2 + c)^2, x)
```

## Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x^m \arctan(ax)}{(c+a^2cx^2)^2} \, dx = \int \frac{x^m \arctan{(ax)}}{(a^2cx^2+c)^2} \, dx$$

input
```
integrate(x^m*arctan(a*x)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate(x^m*arctan(a*x)/(a^2*c*x^2 + c)^2, x)
```

## Mupad [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x^m \arctan(ax)}{(c+a^2cx^2)^2} \, dx = \int \frac{x^m \operatorname{atan}(a\,x)}{(c\,a^2\,x^2+c)^2} \, dx$$

input
```
int((x^m*atan(a*x))/(c + a^2*c*x^2)^2,x)
```

output
```
int((x^m*atan(a*x))/(c + a^2*c*x^2)^2, x)
```

## Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.65

$$\int \frac{x^m \arctan(ax)}{(c + a^2cx^2)^2} \, dx = \frac{\int \frac{x^m atan(ax)}{a^4x^4+2a^2x^2+1} dx}{c^2}$$

input
```
int(x^m*atan(a*x)/(a^2*c*x^2+c)^2,x)
```

output
```
int((x**m*atan(a*x))/(a**4*x**4 + 2*a**2*x**2 + 1),x)/c**2
```

# 3.253 $\int x^m(c+a^2cx^2)^{5/2}\arctan(ax)\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int x^m\left(c+a^2cx^2\right)^{5/2}\arctan(ax)\,dx=\operatorname{Int}\left(x^m\left(c+a^2cx^2\right)^{5/2}\arctan(ax),x\right)$$

output
```
Defer(Int)(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.93 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int x^m\left(c+a^2cx^2\right)^{5/2}\arctan(ax)\,dx=\int x^m\left(c+a^2cx^2\right)^{5/2}\arctan(ax)\,dx$$

input
```
Integrate[x^m*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x],x]
```

output
```
Integrate[x^m*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x], x]
```

## Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \arctan(ax) \left(a^2cx^2+c\right)^{5/2} dx$$

$$\Big\downarrow \; 5560$$

$$\int x^m \arctan(ax) \left(a^2cx^2+c\right)^{5/2} dx$$

input
```
Int[x^m*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 1.68 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int x^m \left(a^2c\,x^2+c\right)^{\frac{5}{2}} \arctan\left(ax\right) dx$$

input
```
int(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x),x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.14

$$\int x^m \left(c+a^2 c x^2\right)^{5/2} \arctan(ax)\, dx = \int \left(a^2 c x^2+c\right)^{\frac{5}{2}} x^m \arctan\left(ax\right)\, dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x),x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*x^m*arcta
n(a*x), x)
```

## Sympy [F(-1)]

Timed out.

$$\int x^m \left(c+a^2 c x^2\right)^{5/2} \arctan(ax)\, dx = \text{Timed out}$$

input
```
integrate(x**m*(a**2*c*x**2+c)**(5/2)*atan(a*x),x)
```

output
```
Timed out
```

## Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x^m \left(c+a^2 c x^2\right)^{5/2} \arctan(ax)\, dx = \int \left(a^2 c x^2+c\right)^{\frac{5}{2}} x^m \arctan\left(ax\right)\, dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)*x^m*arctan(a*x), x)
```

## Giac [F(-2)]

Exception generated.

$$\int x^m (c + a^2 c x^2)^{5/2} \arctan(ax) \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x^m (c + a^2 c x^2)^{5/2} \arctan(ax) \, dx = \int x^m \operatorname{atan}(a\,x)\, \left(c\,a^2\,x^2 + c\right)^{5/2} \, dx$$

input
```
int(x^m*atan(a*x)*(c + a^2*c*x^2)^(5/2),x)
```

output
```
int(x^m*atan(a*x)*(c + a^2*c*x^2)^(5/2), x)
```

## Reduce [N/A]

Not integrable

Time = 8.19 (sec) , antiderivative size = 82, normalized size of antiderivative = 3.73

$$\int x^m (c + a^2 c x^2)^{5/2} \arctan(ax) \, dx = \sqrt{c}\, c^2 \left( \left( \int x^m \sqrt{a^2 x^2 + 1}\, atan(ax)\, x^4 dx \right) a^4 + 2 \left( \int x^m \sqrt{a^2 x^2 + 1}\, atan(ax)\, x^2 dx \right) a^2 + \int x^m \sqrt{a^2 x^2 + 1}\, atan(ax)\, dx \right)$$

input
```
int(x^m*(a^2*c*x^2+c)^(5/2)*atan(a*x),x)
```

output
```
sqrt(c)*c**2*(int(x**m*sqrt(a**2*x**2 + 1)*atan(a*x)*x**4,x)*a**4 + 2*int(
x**m*sqrt(a**2*x**2 + 1)*atan(a*x)*x**2,x)*a**2 + int(x**m*sqrt(a**2*x**2
+ 1)*atan(a*x),x))
```

# 3.254 $\int x^m(c+a^2cx^2)^{3/2}\arctan(ax)\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int x^m\left(c+a^2cx^2\right)^{3/2}\arctan(ax)\,dx=\text{Int}\left(x^m\left(c+a^2cx^2\right)^{3/2}\arctan(ax),x\right)$$

output
```
Defer(Int)(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int x^m\left(c+a^2cx^2\right)^{3/2}\arctan(ax)\,dx=\int x^m\left(c+a^2cx^2\right)^{3/2}\arctan(ax)\,dx$$

input
```
Integrate[x^m*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x],x]
```

output
```
Integrate[x^m*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x], x]
```

## Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \arctan(ax) \left(a^2cx^2+c\right)^{3/2} dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int x^m \arctan(ax) \left(a^2cx^2+c\right)^{3/2} dx$$

input
```
Int[x^m*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 1.53 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int x^{m} \left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} \arctan \left(a x \right)d x$$

input
```
int(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x),x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x^m \left(c+a^2cx^2\right)^{3/2} \arctan(ax)\,dx = \int \left(a^2cx^2+c\right)^{\frac{3}{2}} x^m \arctan\left(ax\right)\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x),x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^(3/2)*x^m*arctan(a*x), x)
```

## Sympy [F(-1)]

Timed out.

$$\int x^m \left(c+a^2cx^2\right)^{3/2} \arctan(ax)\,dx = \text{Timed out}$$

input
```
integrate(x**m*(a**2*c*x**2+c)**(3/2)*atan(a*x),x)
```

output
```
Timed out
```

## Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x^m \left(c+a^2cx^2\right)^{3/2} \arctan(ax)\,dx = \int \left(a^2cx^2+c\right)^{\frac{3}{2}} x^m \arctan\left(ax\right)\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*x^m*arctan(a*x), x)
```

## Giac [F(-2)]

Exception generated.

$$\int x^m \left(c+a^2cx^2\right)^{3/2} \arctan(ax)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x^m \left(c+a^2cx^2\right)^{3/2} \arctan(ax)\,dx = \int x^m\,\mathrm{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^{3/2}\,dx$$

input
```
int(x^m*atan(a*x)*(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(x^m*atan(a*x)*(c + a^2*c*x^2)^(3/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.36

$$\int x^m \left(c+a^2cx^2\right)^{3/2} \arctan(ax)\,dx = \sqrt{c}\,c\left(\left(\int x^m\sqrt{a^2x^2+1}\,atan(ax)\,x^2dx\right)a^2 + \int x^m\sqrt{a^2x^2+1}\,atan(ax)\,dx\right)$$

input
```
int(x^m*(a^2*c*x^2+c)^(3/2)*atan(a*x),x)
```

output
```
sqrt(c)*c*(int(x**m*sqrt(a**2*x**2 + 1)*atan(a*x)*x**2,x)*a**2 + int(x**m*
sqrt(a**2*x**2 + 1)*atan(a*x),x))
```

# 3.255 $\int x^m \sqrt{c+a^2cx^2} \arctan(ax)\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\begin{aligned}\int x^m \sqrt{c+a^2cx^2} \arctan(ax)\,dx\\ &= \frac{x^{1+m}\sqrt{c+a^2cx^2}\arctan(ax)}{2+m}\\ &\quad -\frac{ax^{2+m}\sqrt{c+a^2cx^2}\,\text{Hypergeometric2F1}\left(1,\frac{3+m}{2},\frac{4+m}{2},-a^2x^2\right)}{(2+m)^2}\\ &\quad +\frac{c\text{Int}\left(\frac{x^m\arctan(ax)}{\sqrt{c+a^2cx^2}},x\right)}{2+m}\end{aligned}$$

output
```
x^(1+m)*(a^2*c*x^2+c)^(1/2)*arctan(a*x)/(2+m)-a*x^(2+m)*(a^2*c*x^2+c)^(1/2
)*hypergeom([1, 3/2+1/2*m],[2+1/2*m],-a^2*x^2)/(2+m)^2+c*Defer(Int)(x^m*ar
ctan(a*x)/(a^2*c*x^2+c)^(1/2),x)/(2+m)
```

## Mathematica [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int x^m\sqrt{c+a^2cx^2}\arctan(ax)\,dx=\int x^m\sqrt{c+a^2cx^2}\arctan(ax)\,dx$$

input
```
Integrate[x^m*Sqrt[c + a^2*c*x^2]*ArcTan[a*x],x]
```

output
```
Integrate[x^m*Sqrt[c + a^2*c*x^2]*ArcTan[a*x], x]
```

## Rubi [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m\arctan(ax)\sqrt{a^2cx^2+c}\,dx$$

$$\Big\downarrow \text{5481}$$

$$\frac{c\int\frac{x^m\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{m+2}-\frac{ac\int\frac{x^{m+1}}{\sqrt{a^2cx^2+c}}dx}{m+2}+\frac{x^{m+1}\arctan(ax)\sqrt{a^2cx^2+c}}{m+2}$$

$$\Big\downarrow \text{279}$$

$$\frac{c\int\frac{x^m\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{m+2}-\frac{ac\sqrt{a^2x^2+1}\int\frac{x^{m+1}}{\sqrt{a^2x^2+1}}dx}{(m+2)\sqrt{a^2cx^2+c}}+\frac{x^{m+1}\arctan(ax)\sqrt{a^2cx^2+c}}{m+2}$$

$$\Big\downarrow \text{278}$$

$$\frac{c\int \frac{x^m \arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{m+2}+\frac{x^{m+1}\arctan(ax)\sqrt{a^2cx^2+c}}{m+2}-\frac{ac\sqrt{a^2x^2+1}x^{m+2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},-a^2x^2\right)}{(m+2)^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5560}$$

$$\frac{c\int \frac{x^m \arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{m+2}+\frac{x^{m+1}\arctan(ax)\sqrt{a^2cx^2+c}}{m+2}-\frac{ac\sqrt{a^2x^2+1}x^{m+2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},-a^2x^2\right)}{(m+2)^2\sqrt{a^2cx^2+c}}$$

input
```
Int[x^m*Sqrt[c + a^2*c*x^2]*ArcTan[a*x],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 1.46 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int x^m\sqrt{a^2c\,x^2+c}\,\arctan\left(ax\right)dx$$

input
```
int(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x),x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x^m\sqrt{c+a^2cx^2}\arctan(ax)\,dx=\int \sqrt{a^2cx^2+c}x^m\arctan\left(ax\right)\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^m*arctan(a*x), x)
```

## Sympy [N/A]

Not integrable

Time = 13.78 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x^m\sqrt{c+a^2cx^2}\arctan(ax)\,dx=\int x^m\sqrt{c\left(a^2x^2+1\right)}\operatorname{atan}\left(ax\right)dx$$

input
```
integrate(x**m*(a**2*c*x**2+c)**(1/2)*atan(a*x),x)
```

output
```
Integral(x**m*sqrt(c*(a**2*x**2 + 1))*atan(a*x), x)
```

## Maxima [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x^m\sqrt{c+a^2cx^2}\arctan(ax)\,dx=\int \sqrt{a^2cx^2+c}x^m\arctan\left(ax\right)\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*x^m*arctan(a*x), x)
```

## Giac [F(-2)]

Exception generated.

$$\int x^m\sqrt{c+a^2cx^2}\arctan(ax)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x^m\sqrt{c+a^2cx^2}\arctan(ax)\,dx = \int x^m\,\mathrm{atan}(a\,x)\,\sqrt{c\,a^2\,x^2+c}\,dx$$

input
```
int(x^m*atan(a*x)*(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(x^m*atan(a*x)*(c + a^2*c*x^2)^(1/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int x^m\sqrt{c+a^2cx^2}\arctan(ax)\,dx=\sqrt{c}\left(\int x^m\sqrt{a^2x^2+1}\,atan(ax)\,dx\right)$$

input
```
int(x^m*(a^2*c*x^2+c)^(1/2)*atan(a*x),x)
```

output
```
sqrt(c)*int(x**m*sqrt(a**2*x**2 + 1)*atan(a*x),x)
```

# 3.256 $\int \frac{x^m \arctan(ax)}{\sqrt{c+a^2cx^2}}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^m \arctan(ax)}{\sqrt{c+a^2cx^2}}\,dx = \text{Int}\left(\frac{x^m \arctan(ax)}{\sqrt{c+a^2cx^2}},x\right)$$

output
```
Defer(Int)(x^m*arctan(a*x)/(a^2*c*x^2+c)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m \arctan(ax)}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^m \arctan(ax)}{\sqrt{c+a^2cx^2}}\,dx$$

input
```
Integrate[(x^m*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

output
```
Integrate[(x^m*ArcTan[a*x])/Sqrt[c + a^2*c*x^2], x]
```

## Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \arctan(ax)}{\sqrt{a^2cx^2+c}}\,dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int \frac{x^m \arctan(ax)}{\sqrt{a^2cx^2+c}}dx$$

input
```
Int[(x^m*ArcTan[a*x])/Sqrt[c + a^2*c*x^2],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 1.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x^m \arctan\left(ax\right)}{\sqrt{a^2c\,x^2+c}}dx$$

input
```
int(x^m*arctan(a*x)/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(x^m*arctan(a*x)/(a^2*c*x^2+c)^(1/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan(ax)}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^m \arctan(ax)}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(x^m*arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(x^m*arctan(a*x)/sqrt(a^2*c*x^2 + c), x)
```

## Sympy [N/A]

Not integrable

Time = 8.71 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan(ax)}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^m \operatorname{atan}(ax)}{\sqrt{c\,(a^2x^2+1)}}\,dx$$

input
```
integrate(x**m*atan(a*x)/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x**m*atan(a*x)/sqrt(c*(a**2*x**2 + 1)), x)
```

## Maxima [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan(ax)}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^m \arctan(ax)}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(x^m*arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^m*arctan(a*x)/sqrt(a^2*c*x^2 + c), x)
```

## Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan(ax)}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^m \arctan(ax)}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(x^m*arctan(a*x)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^m*arctan(a*x)/sqrt(a^2*c*x^2 + c), x)
```

## Mupad [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan(ax)}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^m \operatorname{atan}(a\,x)}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int((x^m*atan(a*x))/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int((x^m*atan(a*x))/(c + a^2*c*x^2)^(1/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

$$\int \frac{x^m \arctan(ax)}{\sqrt{c+a^2cx^2}}\,dx = \frac{\int \frac{x^m \operatorname{atan}(ax)}{\sqrt{a^2x^2+1}}dx}{\sqrt{c}}$$

input
```
int(x^m*atan(a*x)/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int((x**m*atan(a*x))/sqrt(a**2*x**2 + 1),x)/sqrt(c)
```

# 3.257 $\int \frac{x^m \arctan(ax)}{(c+a^2cx^2)^{3/2}}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^m \arctan(ax)}{(c+a^2cx^2)^{3/2}}\,dx = \text{Int}\left(\frac{x^m \arctan(ax)}{(c+a^2cx^2)^{3/2}}, x\right)$$

output
```
Defer(Int)(x^m*arctan(a*x)/(a^2*c*x^2+c)^(3/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m \arctan(ax)}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^m \arctan(ax)}{(c+a^2cx^2)^{3/2}}\,dx$$

input
```
Integrate[(x^m*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
```

output
```
Integrate[(x^m*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \arctan(ax)}{(a^2cx^2+c)^{3/2}}\,dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x^m \arctan(ax)}{(a^2cx^2+c)^{3/2}}dx$$

input
```
Int[(x^m*ArcTan[a*x])/(c + a^2*c*x^2)^(3/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 1.57 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x^m \arctan(ax)}{(a^2c\,x^2+c)^{\frac{3}{2}}}dx$$

input
```
int(x^m*arctan(a*x)/(a^2*c*x^2+c)^(3/2),x)
```

output
```
int(x^m*arctan(a*x)/(a^2*c*x^2+c)^(3/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.23

$$\int \frac{x^m \arctan(ax)}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^m \arctan(ax)}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m*arctan(a*x)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^m*arctan(a*x)/(a^4*c^2*x^4 + 2*a^2*c^2*x^2
+ c^2), x)
```

## Sympy [N/A]

Not integrable

Time = 15.82 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan(ax)}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^m \operatorname{atan}(ax)}{(c(a^2x^2+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**m*atan(a*x)/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(x**m*atan(a*x)/(c*(a**2*x**2 + 1))**(3/2), x)
```

## Maxima [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan(ax)}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^m \arctan(ax)}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m*arctan(a*x)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^m*arctan(a*x)/(a^2*c*x^2 + c)^(3/2), x)
```

## Giac [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan(ax)}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^m \arctan(ax)}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m*arctan(a*x)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^m*arctan(a*x)/(a^2*c*x^2 + c)^(3/2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan(ax)}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^m \operatorname{atan}(a\,x)}{(c\,a^2\,x^2+c)^{3/2}}\,dx$$

input
```
int((x^m*atan(a*x))/(c + a^2*c*x^2)^(3/2),x)
```

output
```
int((x^m*atan(a*x))/(c + a^2*c*x^2)^(3/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.18

$$\int \frac{x^m \arctan(ax)}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\int \frac{x^m\, atan(ax)}{\sqrt{a^2x^2+1}\,a^2x^2+\sqrt{a^2x^2+1}}dx}{\sqrt{c}\,c}$$

input

```
int(x^m*atan(a*x)/(a^2*c*x^2+c)^(3/2),x)
```

output

```
int((x**m*atan(a*x))/(sqrt(a**2*x**2 + 1)*a**2*x**2 + sqrt(a**2*x**2 + 1))
,x)/(sqrt(c)*c)
```

# 3.258 $\int x^3(c+a^2cx^2)\arctan(ax)^2\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 124

$$\int x^3(c+a^2cx^2)\arctan(ax)^2\,dx = -\frac{cx^2}{180a^2}+\frac{cx^4}{60}+\frac{cx\arctan(ax)}{6a^3} - \frac{cx^3\arctan(ax)}{18a}-\frac{1}{15}acx^5\arctan(ax) - \frac{c\arctan(ax)^2}{12a^4}+\frac{1}{4}cx^4\arctan(ax)^2 + \frac{1}{6}a^2cx^6\arctan(ax)^2-\frac{7c\log(1+a^2x^2)}{90a^4}$$

output
```
-1/180*c*x^2/a^2+1/60*c*x^4+1/6*c*x*arctan(a*x)/a^3-1/18*c*x^3*arctan(a*x)
/a-1/15*a*c*x^5*arctan(a*x)-1/12*c*arctan(a*x)^2/a^4+1/4*c*x^4*arctan(a*x)
^2+1/6*a^2*c*x^6*arctan(a*x)^2-7/90*c*ln(a^2*x^2+1)/a^4
```

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.72

$$\int x^3 (c + a^2cx^2) \arctan(ax)^2 \, dx$$
$$= \frac{c(-a^2x^2 + 3a^4x^4 - 2ax(-15 + 5a^2x^2 + 6a^4x^4) \arctan(ax) + 15(-1 + 3a^4x^4 + 2a^6x^6) \arctan(ax)^2 - 14}{180a^4}$$

input
```
Integrate[x^3*(c + a^2*c*x^2)*ArcTan[a*x]^2,x]
```

output
```
(c*(-(a^2*x^2) + 3*a^4*x^4 - 2*a*x*(-15 + 5*a^2*x^2 + 6*a^4*x^4)*ArcTan[a*
x] + 15*(-1 + 3*a^4*x^4 + 2*a^6*x^6)*ArcTan[a*x]^2 - 14*Log[1 + a^2*x^2]))
/(180*a^4)
```

## Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 290 vs. 2(124) = 248.

Time = 2.14 (sec) , antiderivative size = 290, normalized size of antiderivative = 2.34, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.950$, Rules used = {5485, 5361, 5451, 5361, 243, 49, 2009, 5451, 5345, 240, 5361, 243, 49, 2009, 5419, 5451, 5345, 240, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \arctan(ax)^2 \left(a^2cx^2 + c\right) \, dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c \int x^5 \arctan(ax)^2 dx + c \int x^3 \arctan(ax)^2 dx$$

$$\Big\downarrow \text{5361}$$

$$a^2c\left(\frac{1}{6}x^6 \arctan(ax)^2 - \frac{1}{3}a \int \frac{x^6 \arctan(ax)}{a^2x^2+1}dx\right) + c\left(\frac{1}{4}x^4 \arctan(ax)^2 - \frac{1}{2}a \int \frac{x^4 \arctan(ax)}{a^2x^2+1}dx\right)$$

$$\Big\downarrow \ 5451$$

$$c\left(\frac{1}{4}x^4\arctan(ax)^2-\frac{1}{2}a\left(\frac{\int x^2\arctan(ax)dx}{a^2}-\frac{\int \frac{x^2\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)\right)+a^2c\left(\frac{1}{6}x^6\arctan(ax)^2-\frac{1}{3}a\left(\frac{\int x^4\arctan(ax)dx}{a^2}-\frac{\int \frac{x^4\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)\right)$$

$$\Big\downarrow \ 5361$$

$$c\left(\frac{1}{4}x^4\arctan(ax)^2-\frac{1}{2}a\left(\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{3}a\int\frac{x^3}{a^2x^2+1}dx}{a^2}-\frac{\int \frac{x^2\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)\right)+a^2c\left(\frac{1}{6}x^6\arctan(ax)^2-\frac{1}{3}a\left(\frac{\frac{1}{5}x^5\arctan(ax)-\frac{1}{5}a\int\frac{x^5}{a^2x^2+1}dx}{a^2}-\frac{\int \frac{x^4\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)\right)$$

$$\Big\downarrow \ 243$$

$$c\left(\frac{1}{4}x^4\arctan(ax)^2-\frac{1}{2}a\left(\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\int\frac{x^2}{a^2x^2+1}dx^2}{a^2}-\frac{\int \frac{x^2\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)\right)+a^2c\left(\frac{1}{6}x^6\arctan(ax)^2-\frac{1}{3}a\left(\frac{\frac{1}{5}x^5\arctan(ax)-\frac{1}{10}a\int\frac{x^4}{a^2x^2+1}dx^2}{a^2}-\frac{\int \frac{x^4\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)\right)$$

$$\Big\downarrow \ 49$$

$$c\left(\frac{1}{4}x^4\arctan(ax)^2-\frac{1}{2}a\left(\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\int\left(\frac{1}{a^2}-\frac{1}{a^2(a^2x^2+1)}\right)dx^2}{a^2}-\frac{\int \frac{x^2\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)\right)+a^2c\left(\frac{1}{6}x^6\arctan(ax)^2-\frac{1}{3}a\left(\frac{\frac{1}{5}x^5\arctan(ax)-\frac{1}{10}a\int\left(\frac{x^2}{a^2}+\frac{1}{a^4(a^2x^2+1)}-\frac{1}{a^4}\right)dx^2}{a^2}-\frac{\int \frac{x^4\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)\right)$$

$$\Big\downarrow \ 2009$$

$$c\left(\frac{1}{4}x^4\arctan(ax)^2-\frac{1}{2}a\left(\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log(a^2x^2+1)}{a^4}\right)}{a^2}-\frac{\int \frac{x^2\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)\right)+a^2c\left(\frac{1}{6}x^6\arctan(ax)^2-\frac{1}{3}a\left(\frac{\frac{1}{5}x^5\arctan(ax)-\frac{1}{10}a\left(-\frac{x^2}{a^4}+\frac{x^4}{2a^2}+\frac{\log(a^2x^2+1)}{a^6}\right)}{a^2}-\frac{\int \frac{x^4\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)\right)$$

$$\Big\downarrow \ 5451$$

$$c\left(\frac{1}{4}x^4\arctan(ax)^2-\frac{1}{2}a\left(\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log(a^2x^2+1)}{a^4}\right)}{a^2}-\frac{\frac{\int\arctan(ax)dx}{a^2}-\frac{\int\frac{\arctan(ax)}{a^2x^2+1}dx}{a^2}}{a^2}\right)\right)+$$

$$a^2c\left(\frac{1}{6}x^6\arctan(ax)^2-\frac{1}{3}a\left(\frac{\frac{1}{5}x^5\arctan(ax)-\frac{1}{10}a\left(-\frac{x^2}{a^4}+\frac{x^4}{2a^2}+\frac{\log(a^2x^2+1)}{a^6}\right)}{a^2}-\frac{\frac{\int x^2\arctan(ax)dx}{a^2}-\frac{\int\frac{x^2\arctan(ax)}{a^2x^2+1}d}{a^2}}{a^2}\right.\right.$$

$\downarrow$ 5345

$$c\left(\frac{1}{4}x^4\arctan(ax)^2-\frac{1}{2}a\left(\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log(a^2x^2+1)}{a^4}\right)}{a^2}-\frac{\frac{x\arctan(ax)-a\int\frac{x}{a^2x^2+1}dx}{a^2}-\frac{\int\frac{\arctan(ax)}{a^2x^2+1}dx}{a^2}}{a^2}\right)\right)$$

$$a^2c\left(\frac{1}{6}x^6\arctan(ax)^2-\frac{1}{3}a\left(\frac{\frac{1}{5}x^5\arctan(ax)-\frac{1}{10}a\left(-\frac{x^2}{a^4}+\frac{x^4}{2a^2}+\frac{\log(a^2x^2+1)}{a^6}\right)}{a^2}-\frac{\frac{\int x^2\arctan(ax)dx}{a^2}-\frac{\int\frac{x^2\arctan(ax)}{a^2x^2+1}d}{a^2}}{a^2}\right.\right.$$

$\downarrow$ 240

$$c\left(\frac{1}{4}x^4\arctan(ax)^2-\frac{1}{2}a\left(\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log(a^2x^2+1)}{a^4}\right)}{a^2}-\frac{\frac{x\arctan(ax)-\frac{\log(a^2x^2+1)}{2a}}{a^2}-\frac{\int\frac{\arctan(ax)}{a^2x^2+1}dx}{a^2}}{a^2}\right)\right)$$

$$a^2c\left(\frac{1}{6}x^6\arctan(ax)^2-\frac{1}{3}a\left(\frac{\frac{1}{5}x^5\arctan(ax)-\frac{1}{10}a\left(-\frac{x^2}{a^4}+\frac{x^4}{2a^2}+\frac{\log(a^2x^2+1)}{a^6}\right)}{a^2}-\frac{\frac{\int x^2\arctan(ax)dx}{a^2}-\frac{\int\frac{x^2\arctan(ax)}{a^2x^2+1}d}{a^2}}{a^2}\right.\right.$$

$\downarrow$ 5361

$$c\left(\frac{1}{4}x^4\arctan(ax)^2-\frac{1}{2}a\left(\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log(a^2x^2+1)}{a^4}\right)}{a^2}-\frac{\frac{x\arctan(ax)-\frac{\log(a^2x^2+1)}{2a}}{a^2}-\frac{\int\frac{\arctan(ax)}{a^2x^2+1}dx}{a^2}}{a^2}\right)\right)$$

$$a^2c\left(\frac{1}{6}x^6\arctan(ax)^2-\frac{1}{3}a\left(\frac{\frac{1}{5}x^5\arctan(ax)-\frac{1}{10}a\left(-\frac{x^2}{a^4}+\frac{x^4}{2a^2}+\frac{\log(a^2x^2+1)}{a^6}\right)}{a^2}-\frac{\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{3}a\int\frac{x^3}{a^2x^2+1}dx}{a^2}-\int}{a^2}\right.\right.$$

$\downarrow$ 243

$$c\left(\frac{1}{4}x^4\arctan(ax)^2-\frac{1}{2}a\left(\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log(a^2x^2+1)}{a^4}\right)}{a^2}-\frac{\frac{x\arctan(ax)-\frac{\log(a^2x^2+1)}{2a}}{a^2}-\frac{\int\frac{\arctan(ax)}{a^2x^2+1}dx}{a^2}}{a^2}\right)\right)-$$
$$a^2c\left(\frac{1}{6}x^6\arctan(ax)^2-\frac{1}{3}a\left(\frac{\frac{1}{5}x^5\arctan(ax)-\frac{1}{10}a\left(-\frac{x^2}{a^4}+\frac{x^4}{2a^2}+\frac{\log(a^2x^2+1)}{a^6}\right)}{a^2}-\frac{\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\int\frac{x^2}{a^2x^2+1}dx^2}{a^2}-}{a^2}\right.\right.$$

$\downarrow$ 49

$$c\left(\frac{1}{4}x^4\arctan(ax)^2-\frac{1}{2}a\left(\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log(a^2x^2+1)}{a^4}\right)}{a^2}-\frac{\frac{x\arctan(ax)-\frac{\log(a^2x^2+1)}{2a}}{a^2}-\frac{\int\frac{\arctan(ax)}{a^2x^2+1}dx}{a^2}}{a^2}\right)\right)-$$
$$a^2c\left(\frac{1}{6}x^6\arctan(ax)^2-\frac{1}{3}a\left(\frac{\frac{1}{5}x^5\arctan(ax)-\frac{1}{10}a\left(-\frac{x^2}{a^4}+\frac{x^4}{2a^2}+\frac{\log(a^2x^2+1)}{a^6}\right)}{a^2}-\frac{\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\int\left(\frac{1}{a^2}-\frac{1}{a^2(a^2x^2+}\right.}{a^2}}{a^2}\right.\right.$$

$\downarrow$ 2009

$$c\left(\frac{1}{4}x^4\arctan(ax)^2-\frac{1}{2}a\left(\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log(a^2x^2+1)}{a^4}\right)}{a^2}-\frac{\frac{x\arctan(ax)-\frac{\log(a^2x^2+1)}{2a}}{a^2}-\frac{\int\frac{\arctan(ax)}{a^2x^2+1}dx}{a^2}}{a^2}\right)\right)-$$
$$a^2c\left(\frac{1}{6}x^6\arctan(ax)^2-\frac{1}{3}a\left(\frac{\frac{1}{5}x^5\arctan(ax)-\frac{1}{10}a\left(-\frac{x^2}{a^4}+\frac{x^4}{2a^2}+\frac{\log(a^2x^2+1)}{a^6}\right)}{a^2}-\frac{\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log(a^2x^2+1)}{a^4}\right)}{a^2}}{a^2}\right.\right.$$

$\downarrow$ 5419

$$a^2c\left(\frac{1}{6}x^6\arctan(ax)^2-\frac{1}{3}a\left(\frac{\frac{1}{5}x^5\arctan(ax)-\frac{1}{10}a\left(-\frac{x^2}{a^4}+\frac{x^4}{2a^2}+\frac{\log(a^2x^2+1)}{a^6}\right)}{a^2}-\frac{\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log(a^2x^2+1)}{a^4}\right)}{a^2}}{a^2}\right.\right.$$
$$c\left(\frac{1}{4}x^4\arctan(ax)^2-\frac{1}{2}a\left(\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log(a^2x^2+1)}{a^4}\right)}{a^2}-\frac{\frac{x\arctan(ax)-\frac{\log(a^2x^2+1)}{2a}}{a^2}-\frac{\arctan(ax)^2}{2a^3}}{a^2}\right)\right)$$

$\downarrow$ 5451

$$a^2c\left(\frac{1}{6}x^6\arctan(ax)^2-\frac{1}{3}a\left(\frac{\frac{1}{5}x^5\arctan(ax)-\frac{1}{10}a\left(-\frac{x^2}{a^4}+\frac{x^4}{2a^2}+\frac{\log(a^2x^2+1)}{a^6}\right)}{a^2}-\frac{\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log(a^2x^2+1)}{a^4}\right.}{a^2}}{a^2}\right.\right.$$

$$c\left(\frac{1}{4}x^4\arctan(ax)^2-\frac{1}{2}a\left(\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log(a^2x^2+1)}{a^4}\right)}{a^2}-\frac{\frac{x\arctan(ax)-\frac{\log(a^2x^2+1)}{2a}}{a^2}-\frac{\arctan(ax)^2}{2a^3}}{a^2}\right)\right)$$

$\big\downarrow$ 5345

$$a^2c\left(\frac{1}{6}x^6\arctan(ax)^2-\frac{1}{3}a\left(\frac{\frac{1}{5}x^5\arctan(ax)-\frac{1}{10}a\left(-\frac{x^2}{a^4}+\frac{x^4}{2a^2}+\frac{\log(a^2x^2+1)}{a^6}\right)}{a^2}-\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log(a^2x^2+1)}{a^4}\right.}{a^2}\right.\right.$$

$$c\left(\frac{1}{4}x^4\arctan(ax)^2-\frac{1}{2}a\left(\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log(a^2x^2+1)}{a^4}\right)}{a^2}-\frac{\frac{x\arctan(ax)-\frac{\log(a^2x^2+1)}{2a}}{a^2}-\frac{\arctan(ax)^2}{2a^3}}{a^2}\right)\right)$$

$\big\downarrow$ 240

$$a^2c\left(\frac{1}{6}x^6\arctan(ax)^2-\frac{1}{3}a\left(\frac{\frac{1}{5}x^5\arctan(ax)-\frac{1}{10}a\left(-\frac{x^2}{a^4}+\frac{x^4}{2a^2}+\frac{\log(a^2x^2+1)}{a^6}\right)}{a^2}-\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log(a^2x^2+1)}{a^4}\right.}{a^2}\right.\right.$$

$$c\left(\frac{1}{4}x^4\arctan(ax)^2-\frac{1}{2}a\left(\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log(a^2x^2+1)}{a^4}\right)}{a^2}-\frac{\frac{x\arctan(ax)-\frac{\log(a^2x^2+1)}{2a}}{a^2}-\frac{\arctan(ax)^2}{2a^3}}{a^2}\right)\right)$$

$\big\downarrow$ 5419

$$c\left(\frac{1}{4}x^4\arctan(ax)^2-\frac{1}{2}a\left(\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log(a^2x^2+1)}{a^4}\right)}{a^2}-\frac{\frac{x\arctan(ax)-\frac{\log(a^2x^2+1)}{2a}}{a^2}-\frac{\arctan(ax)^2}{2a^3}}{a^2}\right)\right)+$$

$$a^2c\left(\frac{1}{6}x^6\arctan(ax)^2-\frac{1}{3}a\left(\frac{\frac{1}{5}x^5\arctan(ax)-\frac{1}{10}a\left(-\frac{x^2}{a^4}+\frac{x^4}{2a^2}+\frac{\log(a^2x^2+1)}{a^6}\right)}{a^2}-\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log(a^2x^2+1)}{a^4}\right.}{a^2}\right.\right.$$

input
```
Int[x^3*(c + a^2*c*x^2)*ArcTan[a*x]^2,x]
```

output
```
c*((x^4*ArcTan[a*x]^2)/4 - (a*(((x^3*ArcTan[a*x])/3 - (a*(x^2/a^2 - Log[1
+ a^2*x^2]/a^4))/6)/a^2 - (-1/2*ArcTan[a*x]^2/a^3 + (x*ArcTan[a*x] - Log[1
 + a^2*x^2]/(2*a))/a^2)/a^2))/2) + a^2*c*((x^6*ArcTan[a*x]^2)/6 - (a*(((x^
5*ArcTan[a*x])/5 - (a*(-(x^2/a^4) + x^4/(2*a^2) + Log[1 + a^2*x^2]/a^6))/1
0)/a^2 - (((x^3*ArcTan[a*x])/3 - (a*(x^2/a^2 - Log[1 + a^2*x^2]/a^4))/6)/a
^2 - (-1/2*ArcTan[a*x]^2/a^3 + (x*ArcTan[a*x] - Log[1 + a^2*x^2]/(2*a))/a^
2)/a^2)/a^2))/3)
```

## Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

## Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.87

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{c\arctan(ax)^2a^6x^6}{6}+\frac{c\arctan(ax)^2a^4x^4}{4}-\frac{c\left(\frac{2\arctan(ax)a^5x^5}{5}+\frac{\arctan(ax)x^3a^3}{3}-\arctan(ax)ax+\frac{\arctan(ax)^2}{2}-\frac{a^4x^4}{10}+\frac{a^2x^2}{30}+\frac{7}{}\right.}{6}}{a^4}$ |
| default | $\frac{\frac{c\arctan(ax)^2a^6x^6}{6}+\frac{c\arctan(ax)^2a^4x^4}{4}-\frac{c\left(\frac{2\arctan(ax)a^5x^5}{5}+\frac{\arctan(ax)x^3a^3}{3}-\arctan(ax)ax+\frac{\arctan(ax)^2}{2}-\frac{a^4x^4}{10}+\frac{a^2x^2}{30}+\frac{7}{}\right.}{6}}{a^4}$ |
| parallelrisch | $-\frac{-30c\arctan(ax)^2a^6x^6+12c\arctan(ax)a^5x^5-45c\arctan(ax)^2a^4x^4-3a^4cx^4+10c\arctan(ax)a^3x^3+a^2cx^2-30acx\,a}{180a^4}$ |
| parts | $\frac{a^2cx^6\arctan(ax)^2}{6}+\frac{cx^4\arctan(ax)^2}{4}-\frac{c\left(\frac{2a\arctan(ax)x^5}{5}+\frac{\arctan(ax)x^3}{3a}-\frac{\arctan(ax)x}{a^3}+\frac{\arctan(ax)^2}{a^4}-\frac{\frac{3a^4x^4}{2}-\frac{a^2x}{2}}{}\right.}{6}$ |
| risch | $-\frac{c(2a^6x^6+3a^4x^4-1)\ln(iax+1)^2}{48a^4}+\frac{c(30a^6x^6\ln(-iax+1)+12ia^5x^5+45x^4\ln(-iax+1)a^4+10ia^3x^3-30iax-15\ln(}{360a^4}$ |

input
```
int(x^3*(a^2*c*x^2+c)*arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^4*(1/6*c*arctan(a*x)^2*a^6*x^6+1/4*c*arctan(a*x)^2*a^4*x^4-1/6*c*(2/5*
arctan(a*x)*a^5*x^5+1/3*arctan(a*x)*x^3*a^3-arctan(a*x)*a*x+1/2*arctan(a*x
)^2-1/10*a^4*x^4+1/30*a^2*x^2+7/15*ln(a^2*x^2+1)))
```

## Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.78

$$\int x^3(c+a^2cx^2)\arctan(ax)^2\,dx$$
$$=\frac{3\,a^4cx^4-a^2cx^2+15\,(2\,a^6cx^6+3\,a^4cx^4-c)\arctan{(ax)}^2-2\,(6\,a^5cx^5+5\,a^3cx^3-15\,acx)\arctan{(ax)}-}{180\,a^4}$$

input
```
integrate(x^3*(a^2*c*x^2+c)*arctan(a*x)^2,x, algorithm="fricas")
```

output
```
1/180*(3*a^4*c*x^4 - a^2*c*x^2 + 15*(2*a^6*c*x^6 + 3*a^4*c*x^4 - c)*arctan
(a*x)^2 - 2*(6*a^5*c*x^5 + 5*a^3*c*x^3 - 15*a*c*x)*arctan(a*x) - 14*c*log(
a^2*x^2 + 1))/a^4
```

## Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.98

$$\int x^3(c+a^2cx^2)\arctan(ax)^2\,dx$$
$$=\begin{cases}\frac{a^2cx^6\operatorname{atan}^2(ax)}{6}-\frac{acx^5\operatorname{atan}(ax)}{15}+\frac{cx^4\operatorname{atan}^2(ax)}{4}+\frac{cx^4}{60}-\frac{cx^3\operatorname{atan}(ax)}{18a}-\frac{cx^2}{180a^2}+\frac{cx\operatorname{atan}(ax)}{6a^3}-\frac{7c\log\left(x^2+\frac{1}{a^2}\right)}{90a^4}-\frac{c\operatorname{atan}}{12} \\ 0\end{cases}$$

input
```
integrate(x**3*(a**2*c*x**2+c)*atan(a*x)**2,x)
```

output
```
Piecewise((a**2*c*x**6*atan(a*x)**2/6 - a*c*x**5*atan(a*x)/15 + c*x**4*ata
n(a*x)**2/4 + c*x**4/60 - c*x**3*atan(a*x)/(18*a) - c*x**2/(180*a**2) + c*
x*atan(a*x)/(6*a**3) - 7*c*log(x**2 + a**(-2))/(90*a**4) - c*atan(a*x)**2/
(12*a**4), Ne(a, 0)), (0, True))
```

## Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.94

$$\int x^3(c+a^2cx^2)\arctan(ax)^2\,dx$$
$$=-\frac{1}{90}a\left(\frac{6a^4cx^5+5a^2cx^3-15cx}{a^4}+\frac{15c\arctan(ax)}{a^5}\right)\arctan(ax)+\frac{1}{12}(2a^2cx^6+3cx^4)\arctan(ax)^2+\frac{3a^4cx^4-a^2cx^2+15c\arctan(ax)^2-14c\log(a^2x^2+1)}{180a^4}$$

input
```
integrate(x^3*(a^2*c*x^2+c)*arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-1/90*a*((6*a^4*c*x^5 + 5*a^2*c*x^3 - 15*c*x)/a^4 + 15*c*arctan(a*x)/a^5)*
arctan(a*x) + 1/12*(2*a^2*c*x^6 + 3*c*x^4)*arctan(a*x)^2 + 1/180*(3*a^4*c*
x^4 - a^2*c*x^2 + 15*c*arctan(a*x)^2 - 14*c*log(a^2*x^2 + 1))/a^4
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.85

$$\int x^3(c+a^2cx^2)\arctan(ax)^2\,dx=\frac{1}{12}\left(2\,a^2cx^6+3\,cx^4\right)\arctan\left(ax\right)^2$$
$$-\frac{12\,a^5cx^5\arctan\left(ax\right)-3\,a^4cx^4+10\,a^3cx^3\arctan\left(ax\right)+a^2cx^2-30\,acx\arctan\left(ax\right)+15\,c\arctan\left(ax\right)}{180\,a^4}$$

input
```
integrate(x^3*(a^2*c*x^2+c)*arctan(a*x)^2,x, algorithm="giac")
```

output
```
1/12*(2*a^2*c*x^6 + 3*c*x^4)*arctan(a*x)^2 - 1/180*(12*a^5*c*x^5*arctan(a*
x) - 3*a^4*c*x^4 + 10*a^3*c*x^3*arctan(a*x) + a^2*c*x^2 - 30*a*c*x*arctan(
a*x) + 15*c*arctan(a*x)^2 + 14*c*log(a^2*x^2 + 1))/a^4
```

## Mupad [B] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.82

$$\int x^3(c+a^2cx^2)\arctan(ax)^2\,dx=$$
$$-\frac{c\left(14\,\ln\left(a^2\,x^2+1\right)+a^2\,x^2-3\,a^4\,x^4+15\,\mathrm{atan}(a\,x)^2+10\,a^3\,x^3\,\mathrm{atan}(a\,x)+12\,a^5\,x^5\,\mathrm{atan}(a\,x)-30\,a\,x\right.}{180\,a^4}$$

input
```
int(x^3*atan(a*x)^2*(c + a^2*c*x^2),x)
```

output
```
-(c*(14*log(a^2*x^2 + 1) + a^2*x^2 - 3*a^4*x^4 + 15*atan(a*x)^2 + 10*a^3*x
^3*atan(a*x) + 12*a^5*x^5*atan(a*x) - 30*a*x*atan(a*x) - 45*a^4*x^4*atan(a
*x)^2 - 30*a^6*x^6*atan(a*x)^2))/(180*a^4)
```

## Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.83

$$\int x^3(c+a^2cx^2)\arctan(ax)^2\,dx$$
$$=\frac{c(30atan(ax)^2a^6x^6+45atan(ax)^2a^4x^4-15atan(ax)^2-12atan(ax)a^5x^5-10atan(ax)a^3x^3+30atan}{180a^4}$$

input
```
int(x^3*(a^2*c*x^2+c)*atan(a*x)^2,x)
```

output
```
(c*(30*atan(a*x)**2*a**6*x**6 + 45*atan(a*x)**2*a**4*x**4 - 15*atan(a*x)**
2 - 12*atan(a*x)*a**5*x**5 - 10*atan(a*x)*a**3*x**3 + 30*atan(a*x)*a*x - 1
4*log(a**2*x**2 + 1) + 3*a**4*x**4 - a**2*x**2))/(180*a**4)
```

# 3.259 $\int x^2(c+a^2cx^2)\arctan(ax)^2\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 156

$$\int x^2(c+a^2cx^2)\arctan(ax)^2\,dx = \frac{cx}{30a^2}+\frac{cx^3}{30}-\frac{c\arctan(ax)}{30a^3}-\frac{2cx^2\arctan(ax)}{15a} - \frac{1}{10}acx^4\arctan(ax)-\frac{2ic\arctan(ax)^2}{15a^3} + \frac{1}{3}cx^3\arctan(ax)^2+\frac{1}{5}a^2cx^5\arctan(ax)^2 - \frac{4c\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{15a^3} - \frac{2ic\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{15a^3}$$

output
```
1/30*c*x/a^2+1/30*c*x^3-1/30*c*arctan(a*x)/a^3-2/15*c*x^2*arctan(a*x)/a-1/
10*a*c*x^4*arctan(a*x)-2/15*I*c*arctan(a*x)^2/a^3+1/3*c*x^3*arctan(a*x)^2+
1/5*a^2*c*x^5*arctan(a*x)^2-4/15*c*arctan(a*x)*ln(2/(1+I*a*x))/a^3-2/15*I*
c*polylog(2,1-2/(1+I*a*x))/a^3
```

## Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.67

$$\int x^2(c+a^2cx^2)\arctan(ax)^2\,dx$$
$$=\frac{c\left(ax+a^3x^3+2(2i+5a^3x^3+3a^5x^5)\arctan(ax)^2-\arctan(ax)\left(1+4a^2x^2+3a^4x^4+8\log\left(1+e^{2i\arctan}\right.\right.\right.}{30a^3}$$

input
```
Integrate[x^2*(c + a^2*c*x^2)*ArcTan[a*x]^2,x]
```

output
```
(c*(a*x + a^3*x^3 + 2*(2*I + 5*a^3*x^3 + 3*a^5*x^5)*ArcTan[a*x]^2 - ArcTan
[a*x]*(1 + 4*a^2*x^2 + 3*a^4*x^4 + 8*Log[1 + E^((2*I)*ArcTan[a*x])]) + (4*
I)*PolyLog[2, -E^((2*I)*ArcTan[a*x])]))/(30*a^3)
```

## Rubi [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 318 vs. 2(156) = 312.

Time = 1.66 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.04, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {5485, 5361, 5451, 5361, 254, 262, 216, 2009, 5451, 5361, 262, 216, 5455, 5379, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\arctan(ax)^2\left(a^2cx^2+c\right)\,dx$$

$$\Big\downarrow \text{ 5485}$$

$$a^2c\int x^4\arctan(ax)^2dx+c\int x^2\arctan(ax)^2dx$$

$$\Big\downarrow \text{ 5361}$$

$$a^2c\left(\frac{1}{5}x^5\arctan(ax)^2-\frac{2}{5}a\int\frac{x^5\arctan(ax)}{a^2x^2+1}dx\right)+ c\left(\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\int\frac{x^3\arctan(ax)}{a^2x^2+1}dx\right)$$

$$\Big\downarrow \text{5451}$$

$$c\left(\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\int x\arctan(ax)dx}{a^2}-\frac{\int\frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)\right)+ a^2c\left(\frac{1}{5}x^5\arctan(ax)^2-\frac{2}{5}a\left(\frac{\int x^3\arctan(ax)dx}{a^2}-\frac{\int\frac{x^3\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)\right)$$

$$\Big\downarrow \text{5361}$$

$$c\left(\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\int\frac{x^2}{a^2x^2+1}dx}{a^2}-\frac{\int\frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)\right)+ a^2c\left(\frac{1}{5}x^5\arctan(ax)^2-\frac{2}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)-\frac{1}{4}a\int\frac{x^4}{a^2x^2+1}dx}{a^2}-\frac{\int\frac{x^3\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)\right)$$

$$\Big\downarrow \text{254}$$

$$c\left(\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\int\frac{x^2}{a^2x^2+1}dx}{a^2}-\frac{\int\frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)\right)+ a^2c\left(\frac{1}{5}x^5\arctan(ax)^2-\frac{2}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)-\frac{1}{4}a\int\left(\frac{x^2}{a^2}+\frac{1}{a^4(a^2x^2+1)}-\frac{1}{a^4}\right)dx}{a^2}-\frac{\int\frac{x^3\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)\right)$$

$$\Big\downarrow \text{262}$$

$$c\left(\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\int\frac{1}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\frac{\int\frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)\right)+ a^2c\left(\frac{1}{5}x^5\arctan(ax)^2-\frac{2}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)-\frac{1}{4}a\int\left(\frac{x^2}{a^2}+\frac{1}{a^4(a^2x^2+1)}-\frac{1}{a^4}\right)dx}{a^2}-\frac{\int\frac{x^3\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)\right)$$

$$\Big\downarrow \text{216}$$

$$a^2c\left(\frac{1}{5}x^5\arctan(ax)^2-\frac{2}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)-\frac{1}{4}a\int\left(\frac{x^2}{a^2}+\frac{1}{a^4(a^2x^2+1)}-\frac{1}{a^4}\right)dx}{a^2}-\frac{\int\frac{x^3\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)\right)+c\left(\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{\int\frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)\right)$$

$$\Big\downarrow \; 2009$$

$$c\left(\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{\int\frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)\right)+a^2c\left(\frac{1}{5}x^5\arctan(ax)^2-\frac{2}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)-\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)}{a^2}-\frac{\int\frac{x^3\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)\right)$$

$$\Big\downarrow \; 5451$$

$$c\left(\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{\int\frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)\right)+a^2c\left(\frac{1}{5}x^5\arctan(ax)^2-\frac{2}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)-\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)}{a^2}-\frac{\frac{\int x\arctan(ax)dx}{a^2}-\frac{\int\frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}}{a^2}\right)\right)$$

$$\Big\downarrow \; 5361$$

$$c\left(\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{\int\frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)\right)+a^2c\left(\frac{1}{5}x^5\arctan(ax)^2-\frac{2}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)-\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)}{a^2}-\frac{\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\int\frac{x^2}{a^2x^2+1}dx}{a^2}-\frac{\int\frac{x\arctan}{a^2 \cdots}}{\cdots}}{a^2}\right.\right.$$

$$\Big\downarrow \; 262$$

$$c\left(\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{\int\frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)\right)+a^2c\left(\frac{1}{5}x^5\arctan(ax)^2-\frac{2}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)-\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)}{a^2}-\frac{\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\int\frac{1}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\cdots}{a^2}\right.\right.$$

$$\Big\downarrow \; 216$$

$$c\left(\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{\int\frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)\right)+$$

$$a^2c\Bigg(\frac{1}{5}x^5\arctan(ax)^2-\frac{2}{5}a\Bigg(\frac{\frac{1}{4}x^4\arctan(ax)-\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)}{a^2}-\frac{\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\int}{a^2}$$

$\downarrow$ 5455

$$c\left(\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{-\frac{\int\frac{\arctan(ax)}{i-ax}dx}{a}-\frac{i\arctan(ax)^2}{2a^2}}{a^2}\right)\right)+$$

$$a^2c\Bigg(\frac{1}{5}x^5\arctan(ax)^2-\frac{2}{5}a\Bigg(\frac{\frac{1}{4}x^4\arctan(ax)-\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)}{a^2}-\frac{\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{-}{}}{a^2}$$

$\downarrow$ 5379

$$c\Bigg(\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\Bigg(\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}-\int\frac{\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(}{2a^2}}{a^2}$$

$$a^2c\Bigg(\frac{1}{5}x^5\arctan(ax)^2-\frac{2}{5}a\Bigg(\frac{\frac{1}{4}x^4\arctan(ax)-\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)}{a^2}-\frac{\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{-}{}}{}$$

$\downarrow$ 2849

$$c\Bigg(\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\Bigg(\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{-\frac{\frac{i\int\frac{\log\left(\frac{2}{iax+1}\right)}{1-\frac{2}{iax+1}}d\frac{1}{iax+1}}{a}+\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}}{a}-\frac{i\,\mathrm{a}}{}}{a^2}$$

$$a^2c\Bigg(\frac{1}{5}x^5\arctan(ax)^2-\frac{2}{5}a\Bigg(\frac{\frac{1}{4}x^4\arctan(ax)-\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)}{a^2}-\frac{\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{-}{}}{}$$

$\downarrow$ 2752

$$c\left(\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,\right.}{2a}}{a}}{a^2}\right.\right.$$

$$a^2c\left(\frac{1}{5}x^5\arctan(ax)^2-\frac{2}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)-\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)}{a^2}-\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\right.\right.$$

input
```
Int[x^2*(c + a^2*c*x^2)*ArcTan[a*x]^2,x]
```

output
```
c*((x^3*ArcTan[a*x]^2)/3 - (2*a*(((x^2*ArcTan[a*x])/2 - (a*(x/a^2 - ArcTan
[a*x]/a^3))/2)/a^2 - (((-1/2*I)*ArcTan[a*x]^2)/a^2 - ((ArcTan[a*x]*Log[2/(
1 + I*a*x)])/a + ((I/2)*PolyLog[2, 1 - 2/(1 + I*a*x)])/a)/a)/a^2))/3) + a^
2*c*((x^5*ArcTan[a*x]^2)/5 - (2*a*(((x^4*ArcTan[a*x])/4 - (a*(-(x/a^4) + x
^3/(3*a^2) + ArcTan[a*x]/a^5))/4)/a^2 - (((x^2*ArcTan[a*x])/2 - (a*(x/a^2
- ArcTan[a*x]/a^3))/2)/a^2 - (((-1/2*I)*ArcTan[a*x]^2)/a^2 - ((ArcTan[a*x]
*Log[2/(1 + I*a*x)])/a + ((I/2)*PolyLog[2, 1 - 2/(1 + I*a*x)])/a)/a)/a^2)/
a^2))/5)
```

## Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

## Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.43

| method | result |
|---|---|
| parts | $\frac{a^{2} c \,x^{5} \arctan \left(a x \right)^{2}}{5}+\frac{c \,x^{3} \arctan \left(a x \right)^{2}}{3}-\frac{2 c \left(\frac{3 a \arctan \left(a x \right) x^{4}}{4}+\frac{\arctan \left(a x \right) x^{2}}{a}-\frac{\arctan \left(a x \right) \ln \left(a^{2} x^{2}+1\right)}{a^{3}}-\frac{a^{3} x^{3}+a x -\mathrm{arc}}{}\right.}{}$ |
| derivativedivides | $\frac{\frac{c \arctan \left(a x \right)^{2} a^{5} x^{5}}{5}+\frac{c \arctan \left(a x \right)^{2} a^{3} x^{3}}{3}-\frac{2 c \left(\frac{3 x^{4} \arctan \left(a x \right) a^{4}}{4}+x^{2} a^{2} \arctan \left(a x \right)-\arctan \left(a x \right) \ln \left(a^{2} x^{2}+1\right)-\frac{a^{3} x^{3}}{4}-\frac{a x}{4}+\frac{\arctan (}{4}\right.}{}}{}$ |
| default | $\frac{\frac{c \arctan \left(a x \right)^{2} a^{5} x^{5}}{5}+\frac{c \arctan \left(a x \right)^{2} a^{3} x^{3}}{3}-\frac{2 c \left(\frac{3 x^{4} \arctan \left(a x \right) a^{4}}{4}+x^{2} a^{2} \arctan \left(a x \right)-\arctan \left(a x \right) \ln \left(a^{2} x^{2}+1\right)-\frac{a^{3} x^{3}}{4}-\frac{a x}{4}+\frac{\arctan (}{4}\right.}{}}{}$ |
| risch | $\frac{c \,x^{3}}{30}+\frac{c x}{30 a^{2}}-\frac{c \arctan \left(a x \right)}{30 a^{3}}-\frac{i c \ln \left(i a x +1\right) \ln \left(-i a x +1\right)}{15 a^{3}}+\frac{c \ln \left(i a x +1\right) \ln \left(-i a x +1\right) x^{3}}{6}-\frac{c \,a^{2} \ln \left(-i a x +1\right)^{2} x^{5}}{20}$ |

input
```
int(x^2*(a^2*c*x^2+c)*arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/5*a^2*c*x^5*arctan(a*x)^2+1/3*c*x^3*arctan(a*x)^2-2/15*c*(3/4*a*arctan(a
*x)*x^4+1/a*arctan(a*x)*x^2-1/a^3*arctan(a*x)*ln(a^2*x^2+1)-1/4/a^3*(a^3*x
^3+a*x-arctan(a*x)+2*I*(ln(a*x-I)*ln(a^2*x^2+1)-1/2*ln(a*x-I)^2-dilog(-1/2
*I*(a*x+I))-ln(a*x-I)*ln(-1/2*I*(a*x+I)))-2*I*(ln(a*x+I)*ln(a^2*x^2+1)-1/2
*ln(a*x+I)^2-dilog(1/2*I*(a*x-I))-ln(a*x+I)*ln(1/2*I*(a*x-I)))))
```

## Fricas [F]

$$\int x^2(c+a^2cx^2)\arctan(ax)^2\,dx=\int (a^2cx^2+c)x^2\arctan(ax)^2\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)*arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^4 + c*x^2)*arctan(a*x)^2, x)
```

## Sympy [F]

$$\int x^2(c+a^2cx^2)\arctan(ax)^2\,dx=c\left(\int x^2\,\mathrm{atan}^2(ax)\,dx+\int a^2x^4\,\mathrm{atan}^2(ax)\,dx\right)$$

input
```
integrate(x**2*(a**2*c*x**2+c)*atan(a*x)**2,x)
```

output
```
c*(Integral(x**2*atan(a*x)**2, x) + Integral(a**2*x**4*atan(a*x)**2, x))
```

## Maxima [F]

$$\int x^2(c+a^2cx^2)\arctan(ax)^2\,dx=\int (a^2cx^2+c)x^2\arctan(ax)^2\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)*arctan(a*x)^2,x, algorithm="maxima")
```

output
```
1/60*(3*a^2*c*x^5 + 5*c*x^3)*arctan(a*x)^2 - 1/240*(3*a^2*c*x^5 + 5*c*x^3)
*log(a^2*x^2 + 1)^2 + integrate(1/240*(180*(a^4*c*x^6 + 2*a^2*c*x^4 + c*x^
2)*arctan(a*x)^2 + 15*(a^4*c*x^6 + 2*a^2*c*x^4 + c*x^2)*log(a^2*x^2 + 1)^2
 - 8*(3*a^3*c*x^5 + 5*a*c*x^3)*arctan(a*x) + 4*(3*a^4*c*x^6 + 5*a^2*c*x^4)
*log(a^2*x^2 + 1))/(a^2*x^2 + 1), x)
```

## Giac [F]

$$\int x^2 (c+a^2cx^2) \arctan(ax)^2 \, dx = \int (a^2cx^2+c)x^2 \arctan{(ax)}^2 \, dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)*arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)*x^2*arctan(a*x)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int x^2 (c+a^2cx^2) \arctan(ax)^2 \, dx = \int x^2 \operatorname{atan}(a\,x)^2 \left(c\,a^2\,x^2+c\right) \, dx$$

input
```
int(x^2*atan(a*x)^2*(c + a^2*c*x^2),x)
```

output
```
int(x^2*atan(a*x)^2*(c + a^2*c*x^2), x)
```

## Reduce [F]

$$\int x^2 (c+a^2cx^2) \arctan(ax)^2 \, dx$$
$$= \frac{c\big(6atan(ax)^2\,a^5x^5+10atan(ax)^2\,a^3x^3+4atan(ax)^2\,ax-3atan(ax)\,a^4x^4-4atan(ax)\,a^2x^2-atan(ax)}{30a^3}$$

input
```
int(x^2*(a^2*c*x^2+c)*atan(a*x)^2,x)
```

output
```
(c*(6*atan(a*x)**2*a**5*x**5 + 10*atan(a*x)**2*a**3*x**3 + 4*atan(a*x)**2*
a*x - 3*atan(a*x)*a**4*x**4 - 4*atan(a*x)*a**2*x**2 - atan(a*x) - 4*int(at
an(a*x)**2,x)*a + a**3*x**3 + a*x))/(30*a**3)
```

# 3.260 $\int x(c+a^2cx^2)\arctan(ax)^2\,dx$



## Optimal result

Integrand size = 18, antiderivative size = 96

$$\int x\left(c+a^2cx^2\right)\arctan(ax)^2\,dx=\frac{c(1+a^2x^2)}{12a^2}-\frac{cx\arctan(ax)}{3a}-\frac{cx(1+a^2x^2)\arctan(ax)}{6a}+\frac{c(1+a^2x^2)^2\arctan(ax)^2}{4a^2}+\frac{c\log\left(1+a^2x^2\right)}{6a^2}$$

output
```
1/12*c*(a^2*x^2+1)/a^2-1/3*c*x*arctan(a*x)/a-1/6*c*x*(a^2*x^2+1)*arctan(a*
x)/a+1/4*c*(a^2*x^2+1)^2*arctan(a*x)^2/a^2+1/6*c*ln(a^2*x^2+1)/a^2
```

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.67

$$\int x\left(c+a^2cx^2\right)\arctan(ax)^2\,dx=\frac{c\left(a^2x^2-2ax(3+a^2x^2)\arctan(ax)+3(1+a^2x^2)^2\arctan(ax)^2+2\log\left(1+a^2x^2\right)\right)}{12a^2}$$

input
```
Integrate[x*(c + a^2*c*x^2)*ArcTan[a*x]^2,x]
```

output
```
(c*(a^2*x^2 - 2*a*x*(3 + a^2*x^2)*ArcTan[a*x] + 3*(1 + a^2*x^2)^2*ArcTan[a
*x]^2 + 2*Log[1 + a^2*x^2]))/(12*a^2)
```

## Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {5465, 27, 5413, 5345, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\arctan(ax)^2\left(a^2cx^2+c\right)\,dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{c\left(a^2x^2+1\right)^2\arctan(ax)^2}{4a^2}-\frac{\int c\left(a^2x^2+1\right)\arctan(ax)dx}{2a}$$

$$\Big\downarrow \text{27}$$

$$\frac{c\left(a^2x^2+1\right)^2\arctan(ax)^2}{4a^2}-\frac{c\int\left(a^2x^2+1\right)\arctan(ax)dx}{2a}$$

$$\Big\downarrow \text{5413}$$

$$\frac{c\left(a^2x^2+1\right)^2\arctan(ax)^2}{4a^2}-\frac{c\left(\frac{2}{3}\int\arctan(ax)dx+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)-\frac{a^2x^2+1}{6a}\right)}{2a}$$

$$\Big\downarrow \text{5345}$$

$$\frac{c\left(a^2x^2+1\right)^2\arctan(ax)^2}{4a^2}-\frac{c\left(\frac{2}{3}\left(x\arctan(ax)-a\int\frac{x}{a^2x^2+1}dx\right)+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)-\frac{a^2x^2+1}{6a}\right)}{2a}$$

$$\Big\downarrow \text{240}$$

$$\frac{c\left(a^2x^2+1\right)^2\arctan(ax)^2}{4a^2}-\frac{c\left(\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)+\frac{2}{3}\left(x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}\right)-\frac{a^2x^2+1}{6a}\right)}{2a}$$

input
```
Int[x*(c + a^2*c*x^2)*ArcTan[a*x]^2,x]
```

output
```
(c*(1 + a^2*x^2)^2*ArcTan[a*x]^2)/(4*a^2) - (c*(-1/6*(1 + a^2*x^2)/a + (x*
(1 + a^2*x^2)*ArcTan[a*x])/3 + (2*(x*ArcTan[a*x] - Log[1 + a^2*x^2]/(2*a))
)/3))/(2*a)
```

**Defintions of rubi rules used**

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5413
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Simp[x*(d + e*x^2)
^q*((a + b*ArcTan[c*x])/(2*q + 1)), x] + Simp[2*d*(q/(2*q + 1))   Int[(d +
e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

## Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.91

| method | result |
|---|---|
| parts | $\frac{x^4 c\, a^2 \arctan(ax)^2}{4}+\frac{c\, x^2 \arctan(ax)^2}{2}+\frac{c \arctan(ax)^2}{4a^2}-\frac{c\left(\frac{\arctan(ax) x^3 a^3}{3}+\arctan(ax) ax-\frac{a^2 x^2}{6}-\frac{\ln\left(a^2 x^2+1\right)}{3}\right)}{2a^2}$ |
| derivativedivides | $\frac{\frac{c \arctan(ax)^2 a^4 x^4}{4}+\frac{a^2 c\, x^2 \arctan(ax)^2}{2}+\frac{c \arctan(ax)^2}{4}-\frac{c\left(\frac{\arctan(ax) x^3 a^3}{3}+\arctan(ax) ax-\frac{a^2 x^2}{6}-\frac{\ln\left(a^2 x^2+1\right)}{3}\right)}{2}}{a^2}$ |
| default | $\frac{\frac{c \arctan(ax)^2 a^4 x^4}{4}+\frac{a^2 c\, x^2 \arctan(ax)^2}{2}+\frac{c \arctan(ax)^2}{4}-\frac{c\left(\frac{\arctan(ax) x^3 a^3}{3}+\arctan(ax) ax-\frac{a^2 x^2}{6}-\frac{\ln\left(a^2 x^2+1\right)}{3}\right)}{2}}{a^2}$ |
| parallelrisch | $\frac{3c \arctan(ax)^2 a^4 x^4-2c \arctan(ax) a^3 x^3+6a^2 c\, x^2 \arctan(ax)^2+a^2 c\, x^2-6acx \arctan(ax)+3c \arctan(ax)^2+2c \ln(a^2 x^2}{12a^2}$ |
| risch | $-\frac{c\left(a^2 x^2+1\right)^2 \ln(iax+1)^2}{16a^2}+\frac{c\left(3x^4 \ln(-iax+1) a^4+2i a^3 x^3+6a^2 x^2 \ln(-iax+1)+6iax+3\ln(-iax+1)\right) \ln(iax+1)}{24a^2}-$ |

input
```
int(x*(a^2*c*x^2+c)*arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4*c*a^2*arctan(a*x)^2+1/2*c*x^2*arctan(a*x)^2+1/4*c*arctan(a*x)^2/a^
2-1/2*c/a^2*(1/3*arctan(a*x)*x^3*a^3+arctan(a*x)*a*x-1/6*a^2*x^2-1/3*ln(a^
2*x^2+1))
```

## Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.77

$$\int x\left(c+a^2cx^2\right)\arctan(ax)^2\,dx$$
$$=\frac{a^2cx^2+3\left(a^4cx^4+2\,a^2cx^2+c\right)\arctan\left(ax\right)^2-2\left(a^3cx^3+3\,acx\right)\arctan\left(ax\right)+2\,c\log\left(a^2x^2+1\right)}{12\,a^2}$$

input
```
integrate(x*(a^2*c*x^2+c)*arctan(a*x)^2,x, algorithm="fricas")
```

output
```
1/12*(a^2*c*x^2 + 3*(a^4*c*x^4 + 2*a^2*c*x^2 + c)*arctan(a*x)^2 - 2*(a^3*c
*x^3 + 3*a*c*x)*arctan(a*x) + 2*c*log(a^2*x^2 + 1))/a^2
```

## Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.98

$$\int x\left(c+a^2cx^2\right)\arctan(ax)^2\,dx$$
$$=\begin{cases}\frac{a^2cx^4\operatorname{atan}^2(ax)}{4}-\frac{acx^3\operatorname{atan}(ax)}{6}+\frac{cx^2\operatorname{atan}^2(ax)}{2}+\frac{cx^2}{12}-\frac{cx\operatorname{atan}(ax)}{2a}+\frac{c\log\left(x^2+\frac{1}{a^2}\right)}{6a^2}+\frac{c\operatorname{atan}^2(ax)}{4a^2} & \text{for } a\neq 0\\ 0 & \text{otherwise}\end{cases}$$

input
```
integrate(x*(a**2*c*x**2+c)*atan(a*x)**2,x)
```

output
```
Piecewise((a**2*c*x**4*atan(a*x)**2/4 - a*c*x**3*atan(a*x)/6 + c*x**2*atan
(a*x)**2/2 + c*x**2/12 - c*x*atan(a*x)/(2*a) + c*log(x**2 + a**(-2))/(6*a*
*2) + c*atan(a*x)**2/(4*a**2), Ne(a, 0)), (0, True))
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.91

$$\int x\left(c+a^2cx^2\right)\arctan(ax)^2\,dx$$
$$=\frac{\left(a^2cx^2+c\right)^2\arctan(ax)^2}{4\,a^2c}+\frac{\left(c^2x^2+\frac{2\,c^2\log\left(a^2x^2+1\right)}{a^2}\right)a-2\,\left(a^2c^2x^3+3\,c^2x\right)\arctan(ax)}{12\,ac}$$

input
```
integrate(x*(a^2*c*x^2+c)*arctan(a*x)^2,x, algorithm="maxima")
```

output
```
1/4*(a^2*c*x^2 + c)^2*arctan(a*x)^2/(a^2*c) + 1/12*((c^2*x^2 + 2*c^2*log(a
^2*x^2 + 1)/a^2)*a - 2*(a^2*c^2*x^3 + 3*c^2*x)*arctan(a*x))/(a*c)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.86

$$\int x(c+a^2cx^2)\arctan(ax)^2\,dx=\frac{1}{4}\left(a^2cx^4+2\,cx^2\right)\arctan\left(ax\right)^2-\frac{2\,a^3cx^3\arctan\left(ax\right)-a^2cx^2+6\,acx\arctan\left(ax\right)-3\,c\arctan\left(ax\right)^2-2\,c\log\left(a^2x^2+1\right)}{12\,a^2}$$

input
```
integrate(x*(a^2*c*x^2+c)*arctan(a*x)^2,x, algorithm="giac")
```

output
```
1/4*(a^2*c*x^4 + 2*c*x^2)*arctan(a*x)^2 - 1/12*(2*a^3*c*x^3*arctan(a*x) -
a^2*c*x^2 + 6*a*c*x*arctan(a*x) - 3*c*arctan(a*x)^2 - 2*c*log(a^2*x^2 + 1)
)/a^2
```

## Mupad [B] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.86

$$\int x(c+a^2cx^2)\arctan(ax)^2\,dx=\frac{c\left(6\,x^2\,\mathrm{atan}(a\,x)^2+x^2\right)}{12}+\frac{\frac{c\left(3\,\mathrm{atan}(a\,x)^2+2\,\ln\left(a^2\,x^2+1\right)\right)}{12}-\frac{a\,c\,x\,\mathrm{atan}(a\,x)}{2}}{a^2}+\frac{a^2\,c\,x^4\,\mathrm{atan}(a\,x)^2}{4}-\frac{a\,c\,x^3\,\mathrm{atan}(a\,x)}{6}$$

input
```
int(x*atan(a*x)^2*(c + a^2*c*x^2),x)
```

output
```
(c*(6*x^2*atan(a*x)^2 + x^2))/12 + ((c*(2*log(a^2*x^2 + 1) + 3*atan(a*x)^2
))/12 - (a*c*x*atan(a*x))/2)/a^2 + (a^2*c*x^4*atan(a*x)^2)/4 - (a*c*x^3*at
an(a*x))/6
```

## Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.85

$$\int x\left(c+a^2cx^2\right)\arctan(ax)^2\,dx$$
$$=\frac{c\left(3atan(ax)^2\,a^4x^4+6atan(ax)^2\,a^2x^2+3atan(ax)^2-2atan(ax)\,a^3x^3-6atan(ax)\,ax+2\log(a^2x^2+1)\right.}{12a^2}$$

input
```
int(x*(a^2*c*x^2+c)*atan(a*x)^2,x)
```

output
```
(c*(3*atan(a*x)**2*a**4*x**4 + 6*atan(a*x)**2*a**2*x**2 + 3*atan(a*x)**2 -
 2*atan(a*x)*a**3*x**3 - 6*atan(a*x)*a*x + 2*log(a**2*x**2 + 1) + a**2*x**
2))/(12*a**2)
```

# 3.261 $\int (c+a^2cx^2)\arctan(ax)^2\,dx$



## Optimal result

Integrand size = 17, antiderivative size = 128

$$\int (c+a^2cx^2)\arctan(ax)^2\,dx = \frac{cx}{3}-\frac{c(1+a^2x^2)\arctan(ax)}{3a}+\frac{2ic\arctan(ax)^2}{3a}+\frac{2}{3}cx\arctan(ax)^2+\frac{1}{3}cx(1+a^2x^2)\arctan(ax)^2+\frac{4c\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{3a}+\frac{2ic\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{3a}$$

output

```
1/3*c*x-1/3*c*(a^2*x^2+1)*arctan(a*x)/a+2/3*I*c*arctan(a*x)^2/a+2/3*c*x*ar
ctan(a*x)^2+1/3*c*x*(a^2*x^2+1)*arctan(a*x)^2+4/3*c*arctan(a*x)*ln(2/(1+I*
a*x))/a+2/3*I*c*polylog(2,1-2/(1+I*a*x))/a
```

## Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.64

$$\int (c+a^2cx^2) \arctan(ax)^2 \, dx$$
$$= \frac{c\left(ax+(-2i+3ax+a^3x^3)\arctan(ax)^2-\arctan(ax)\left(1+a^2x^2-4\log\left(1+e^{2i\arctan(ax)}\right)\right)-2i\,\mathrm{PolyLog}\right.}{3a}$$

input
```
Integrate[(c + a^2*c*x^2)*ArcTan[a*x]^2,x]
```

output
```
(c*(a*x + (-2*I + 3*a*x + a^3*x^3)*ArcTan[a*x]^2 - ArcTan[a*x]*(1 + a^2*x^
2 - 4*Log[1 + E^((2*I)*ArcTan[a*x])]) - (2*I)*PolyLog[2, -E^((2*I)*ArcTan[
a*x])]))/(3*a)
```

## Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {5415, 24, 5345, 5455, 5379, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(ax)^2 \left(a^2cx^2+c\right) \, dx$$

$$\Big\downarrow \text{5415}$$

$$\frac{2}{3}c\int \arctan(ax)^2dx+\frac{c\int 1dx}{3}+\frac{1}{3}cx\left(a^2x^2+1\right)\arctan(ax)^2-\frac{c\left(a^2x^2+1\right)\arctan(ax)}{3a}$$

$$\Big\downarrow \text{24}$$

$$\frac{2}{3}c\int \arctan(ax)^2dx+\frac{1}{3}cx\left(a^2x^2+1\right)\arctan(ax)^2-\frac{c\left(a^2x^2+1\right)\arctan(ax)}{3a}+\frac{cx}{3}$$

$$\Big\downarrow \text{5345}$$

$$\frac{2}{3}c\left(x\arctan(ax)^2-2a\int\frac{x\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{3}cx\left(a^2x^2+1\right)\arctan(ax)^2-\frac{c\left(a^2x^2+1\right)\arctan(ax)}{3a}+\frac{cx}{3}$$

$\Big\downarrow$ 5455

$$\frac{2}{3}c\left(x\arctan(ax)^2-2a\left(-\frac{\int\frac{\arctan(ax)}{i-ax}dx}{a}-\frac{i\arctan(ax)^2}{2a^2}\right)\right)+\frac{1}{3}cx\left(a^2x^2+1\right)\arctan(ax)^2-\frac{c\left(a^2x^2+1\right)\arctan(ax)}{3a}+\frac{cx}{3}$$

$\Big\downarrow$ 5379

$$\frac{2}{3}c\left(x\arctan(ax)^2-2a\left(-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}-\int\frac{\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^2}{2a^2}\right)\right)+\frac{1}{3}cx\left(a^2x^2+1\right)\arctan(ax)^2-\frac{c\left(a^2x^2+1\right)\arctan(ax)}{3a}+\frac{cx}{3}$$

$\Big\downarrow$ 2849

$$\frac{2}{3}c\left(x\arctan(ax)^2-2a\left(-\frac{\frac{i\int\frac{\log\left(\frac{2}{iax+1}\right)}{1-\frac{2}{iax+1}}d\frac{1}{iax+1}}{a}+\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}}{a}-\frac{i\arctan(ax)^2}{2a^2}\right)\right)+\frac{1}{3}cx\left(a^2x^2+1\right)\arctan(ax)^2-\frac{c\left(a^2x^2+1\right)\arctan(ax)}{3a}+\frac{cx}{3}$$

$\Big\downarrow$ 2752

$$\frac{2}{3}c\left(x\arctan(ax)^2-2a\left(-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}\right)\right)+\frac{1}{3}cx\left(a^2x^2+1\right)\arctan(ax)^2-\frac{c\left(a^2x^2+1\right)\arctan(ax)}{3a}+\frac{cx}{3}$$

input
```
Int[(c + a^2*c*x^2)*ArcTan[a*x]^2,x]
```

output
```
(c*x)/3 - (c*(1 + a^2*x^2)*ArcTan[a*x])/(3*a) + (c*x*(1 + a^2*x^2)*ArcTan[
a*x]^2)/3 + (2*c*(x*ArcTan[a*x]^2 - 2*a*(((-1/2*I)*ArcTan[a*x]^2)/a^2 - ((
ArcTan[a*x]*Log[2/(1 + I*a*x)])/a + ((I/2)*PolyLog[2, 1 - 2/(1 + I*a*x)])/
a)/a)))/3
```

### Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5415
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTan[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x])^p,
x], x] + Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*(
a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[q, 0] && GtQ[p, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

## Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.56

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{c\arctan(ax)^2 a^3 x^3}{3}+acx\arctan(ax)^2-\frac{2c\left(\frac{x^2a^2\arctan(ax)}{2}+\arctan(ax)\ln\left(a^2x^2+1\right)-\frac{ax}{2}+\frac{\arctan(ax)}{2}+\frac{i\left(\ln(ax-i)\ln\left(a^2x^2+1\right.\right.}{}\right.}{}}{}$ |
| default | $\frac{\frac{c\arctan(ax)^2 a^3 x^3}{3}+acx\arctan(ax)^2-\frac{2c\left(\frac{x^2a^2\arctan(ax)}{2}+\arctan(ax)\ln\left(a^2x^2+1\right)-\frac{ax}{2}+\frac{\arctan(ax)}{2}+\frac{i\left(\ln(ax-i)\ln\left(a^2x^2+1\right.\right.}{}\right.}{}}{}$ |
| parts | $\frac{x^3ca^2\arctan(ax)^2}{3}+cx\arctan(ax)^2-\frac{2c\left(\frac{x^2\arctan(ax)a}{2}+\frac{\arctan(ax)\ln\left(a^2x^2+1\right)}{a}-\frac{ax-\arctan(ax)-i\left(\ln(ax-i\right.}{}\right.}{}$ |
| risch | $-\frac{2ic\ln\left(\frac{1}{2}+\frac{iax}{2}\right)\ln(-iax+1)}{3a}-\frac{c\arctan(ax)}{3a}-\frac{ic\ln(-iax+1)^2}{6a}+\frac{37ic}{54a}+\frac{ic\ln(iax+1)^2}{6a}+\frac{ic\ln(iax+1)\ln(-i}{3a}$ |

input
```
int((a^2*c*x^2+c)*arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a*(1/3*c*arctan(a*x)^2*a^3*x^3+a*c*x*arctan(a*x)^2-2/3*c*(1/2*x^2*a^2*ar
ctan(a*x)+arctan(a*x)*ln(a^2*x^2+1)-1/2*a*x+1/2*arctan(a*x)+1/2*I*(ln(a*x-
I)*ln(a^2*x^2+1)-1/2*ln(a*x-I)^2-dilog(-1/2*I*(a*x+I))-ln(a*x-I)*ln(-1/2*I
*(a*x+I)))-1/2*I*(ln(a*x+I)*ln(a^2*x^2+1)-1/2*ln(a*x+I)^2-dilog(1/2*I*(a*x
-I))-ln(a*x+I)*ln(1/2*I*(a*x-I)))))
```

## Fricas [F]

$$\int \left(c+a^2cx^2\right)\arctan(ax)^2\,dx=\int \left(a^2cx^2+c\right)\arctan\left(ax\right)^2\,dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)*arctan(a*x)^2, x)
```

## Sympy [F]

$$\int (c+a^2cx^2)\arctan(ax)^2\,dx = c\left(\int a^2x^2\operatorname{atan}^2(ax)\,dx + \int \operatorname{atan}^2(ax)\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)*atan(a*x)**2,x)
```

output
```
c*(Integral(a**2*x**2*atan(a*x)**2, x) + Integral(atan(a*x)**2, x))
```

## Maxima [F]

$$\int (c+a^2cx^2)\arctan(ax)^2\,dx = \int (a^2cx^2+c)\arctan(ax)^2\,dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^2,x, algorithm="maxima")
```

output
```
36*a^4*c*integrate(1/48*x^4*arctan(a*x)^2/(a^2*x^2 + 1), x) + 3*a^4*c*inte
grate(1/48*x^4*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) + 4*a^4*c*integrate(1/
48*x^4*log(a^2*x^2 + 1)/(a^2*x^2 + 1), x) - 8*a^3*c*integrate(1/48*x^3*arc
tan(a*x)/(a^2*x^2 + 1), x) + 72*a^2*c*integrate(1/48*x^2*arctan(a*x)^2/(a^
2*x^2 + 1), x) + 6*a^2*c*integrate(1/48*x^2*log(a^2*x^2 + 1)^2/(a^2*x^2 +
1), x) + 12*a^2*c*integrate(1/48*x^2*log(a^2*x^2 + 1)/(a^2*x^2 + 1), x) +
1/12*(a^2*c*x^3 + 3*c*x)*arctan(a*x)^2 + 1/4*c*arctan(a*x)^3/a - 24*a*c*in
tegrate(1/48*x*arctan(a*x)/(a^2*x^2 + 1), x) - 1/48*(a^2*c*x^3 + 3*c*x)*lo
g(a^2*x^2 + 1)^2 + 3*c*integrate(1/48*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x)
```

## Giac [F]

$$\int (c+a^2cx^2)\arctan(ax)^2\,dx=\int (a^2cx^2+c)\arctan(ax)^2\,dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)*arctan(a*x)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (c+a^2cx^2)\arctan(ax)^2\,dx=\int \mathrm{atan}(a\,x)^2\,\left(c\,a^2\,x^2+c\right)\,dx$$

input
```
int(atan(a*x)^2*(c + a^2*c*x^2),x)
```

output
```
int(atan(a*x)^2*(c + a^2*c*x^2), x)
```

## Reduce [F]

$$\int (c+a^2cx^2)\arctan(ax)^2\,dx$$
$$=\frac{c\left(atan(ax)^2a^3x^3+3atan(ax)^2ax-atan(ax)a^2x^2-atan(ax)-4\left(\int\frac{atan(ax)x}{a^2x^2+1}dx\right)a^2+ax\right)}{3a}$$

input
```
int((a^2*c*x^2+c)*atan(a*x)^2,x)
```

output
```
(c*(atan(a*x)**2*a**3*x**3 + 3*atan(a*x)**2*a*x - atan(a*x)*a**2*x**2 - at
an(a*x) - 4*int((atan(a*x)*x)/(a**2*x**2 + 1),x)*a**2 + a*x))/(3*a)
```

# 3.262 $\int \frac{(c+a^2cx^2)\arctan(ax)^2}{x}\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 169

$$\int \frac{(c+a^2cx^2)\arctan(ax)^2}{x}\,dx = -acx\arctan(ax)+\frac{1}{2}c\arctan(ax)^2+\frac{1}{2}a^2cx^2\arctan(ax)^2 + 2c\arctan(ax)^2\operatorname{arctanh}\left(1-\frac{2}{1+iax}\right) + \frac{1}{2}c\log\left(1+a^2x^2\right) - ic\arctan(ax)\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right) + ic\arctan(ax)\operatorname{PolyLog}\left(2,-1+\frac{2}{1+iax}\right) - \frac{1}{2}c\operatorname{PolyLog}\left(3,1-\frac{2}{1+iax}\right) + \frac{1}{2}c\operatorname{PolyLog}\left(3,-1+\frac{2}{1+iax}\right)$$

output

```
-a*c*x*arctan(a*x)+1/2*c*arctan(a*x)^2+1/2*a^2*c*x^2*arctan(a*x)^2-2*c*arc
tan(a*x)^2*arctanh(-1+2/(1+I*a*x))+1/2*c*ln(a^2*x^2+1)-I*c*arctan(a*x)*pol
ylog(2,1-2/(1+I*a*x))+I*c*arctan(a*x)*polylog(2,-1+2/(1+I*a*x))-1/2*c*poly
log(3,1-2/(1+I*a*x))+1/2*c*polylog(3,-1+2/(1+I*a*x))
```

## Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.08

$$\int \frac{(c+a^2cx^2)\arctan(ax)^2}{x}\,dx = -acx\arctan(ax)+\frac{1}{2}c(1+a^2x^2)\arctan(ax)^2 + \frac{2}{3}ic\arctan(ax)^3+c\arctan(ax)^2\log\left(1-e^{-2i\arctan(ax)}\right) - c\arctan(ax)^2\log\left(1+e^{2i\arctan(ax)}\right)+\frac{1}{2}c\log\left(1+a^2x^2\right) + ic\arctan(ax)\,\text{PolyLog}\left(2,e^{-2i\arctan(ax)}\right) + ic\arctan(ax)\,\text{PolyLog}\left(2,-e^{2i\arctan(ax)}\right) + \frac{1}{2}c\,\text{PolyLog}\left(3,e^{-2i\arctan(ax)}\right) - \frac{1}{2}c\,\text{PolyLog}\left(3,-e^{2i\arctan(ax)}\right)$$

input
```
Integrate[((c + a^2*c*x^2)*ArcTan[a*x]^2)/x,x]
```

output
```
-(a*c*x*ArcTan[a*x]) + (c*(1 + a^2*x^2)*ArcTan[a*x]^2)/2 + ((2*I)/3)*c*Arc
Tan[a*x]^3 + c*ArcTan[a*x]^2*Log[1 - E^((-2*I)*ArcTan[a*x])] - c*ArcTan[a*
x]^2*Log[1 + E^((2*I)*ArcTan[a*x])] + (c*Log[1 + a^2*x^2])/2 + I*c*ArcTan[
a*x]*PolyLog[2, E^((-2*I)*ArcTan[a*x])] + I*c*ArcTan[a*x]*PolyLog[2, -E^((
2*I)*ArcTan[a*x])] + (c*PolyLog[3, E^((-2*I)*ArcTan[a*x])])/2 - (c*PolyLog
[3, -E^((2*I)*ArcTan[a*x])])/2
```

## Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.24, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5485, 5357, 5361, 5451, 5345, 240, 5419, 5523, 5529, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2\left(a^2cx^2+c\right)}{x}dx$$

$$\Big\downarrow \text{ 5485}$$

$$a^2c\int x\arctan(ax)^2dx+c\int\frac{\arctan(ax)^2}{x}dx$$

$$\Big\downarrow \text{5357}$$

$$a^2c\int x\arctan(ax)^2dx+$$
$$c\left(2\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{1+iax}\right)-4a\int\frac{\arctan(ax)\text{arctanh}\left(1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx\right)$$

$$\Big\downarrow \text{5361}$$

$$a^2c\left(\frac{1}{2}x^2\arctan(ax)^2-a\int\frac{x^2\arctan(ax)}{a^2x^2+1}dx\right)+$$
$$c\left(2\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{1+iax}\right)-4a\int\frac{\arctan(ax)\text{arctanh}\left(1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx\right)$$

$$\Big\downarrow \text{5451}$$

$$a^2c\left(\frac{1}{2}x^2\arctan(ax)^2-a\left(\frac{\int\arctan(ax)dx}{a^2}-\frac{\int\frac{\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)\right)+$$
$$c\left(2\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{1+iax}\right)-4a\int\frac{\arctan(ax)\text{arctanh}\left(1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx\right)$$

$$\Big\downarrow \text{5345}$$

$$a^2c\left(\frac{1}{2}x^2\arctan(ax)^2-a\left(\frac{x\arctan(ax)-a\int\frac{x}{a^2x^2+1}dx}{a^2}-\frac{\int\frac{\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)\right)+$$
$$c\left(2\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{1+iax}\right)-4a\int\frac{\arctan(ax)\text{arctanh}\left(1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx\right)$$

$$\Big\downarrow \text{240}$$

$$a^2c\left(\frac{1}{2}x^2\arctan(ax)^2-a\left(\frac{x\arctan(ax)-\frac{\log(a^2x^2+1)}{2a}}{a^2}-\frac{\int\frac{\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)\right)+$$
$$c\left(2\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{1+iax}\right)-4a\int\frac{\arctan(ax)\text{arctanh}\left(1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx\right)$$

$$\Big\downarrow \text{5419}$$

$$a^2c\left(\frac{1}{2}x^2\arctan(ax)^2-a\left(\frac{x\arctan(ax)-\frac{\log(a^2x^2+1)}{2a}}{a^2}-\frac{\arctan(ax)^2}{2a^3}\right)\right)+$$
$$c\left(2\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{1+iax}\right)-4a\int\frac{\arctan(ax)\text{arctanh}\left(1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx\right)$$

$\Big\downarrow$ 5523

$$a^2c\left(\frac{1}{2}x^2\arctan(ax)^2-a\left(\frac{x\arctan(ax)-\frac{\log(a^2x^2+1)}{2a}}{a^2}-\frac{\arctan(ax)^2}{2a^3}\right)\right)+$$
$$c\left(2\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{1+iax}\right)-4a\left(\frac{1}{2}\int\frac{\arctan(ax)\log\left(2-\frac{2}{iax+1}\right)}{a^2x^2+1}dx-\frac{1}{2}\int\frac{\arctan(ax)\log\left(\frac{2}{iax+1}\right.}{a^2x^2+1}\right.\right.$$

$\Big\downarrow$ 5529

$$a^2c\left(\frac{1}{2}x^2\arctan(ax)^2-a\left(\frac{x\arctan(ax)-\frac{\log(a^2x^2+1)}{2a}}{a^2}-\frac{\arctan(ax)^2}{2a^3}\right)\right)+$$
$$c\left(2\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{1+iax}\right)-4a\left(\frac{1}{2}\left(\frac{i\arctan(ax)\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}-\frac{1}{2}i\int\frac{\text{PolyLog}\left(2,1-\right.}{a^2x^2+1}\right.\right.\right.$$

$\Big\downarrow$ 7164

$$a^2c\left(\frac{1}{2}x^2\arctan(ax)^2-a\left(\frac{x\arctan(ax)-\frac{\log(a^2x^2+1)}{2a}}{a^2}-\frac{\arctan(ax)^2}{2a^3}\right)\right)+$$
$$c\left(2\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{1+iax}\right)-4a\left(\frac{1}{2}\left(\frac{i\arctan(ax)\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}+\frac{\text{PolyLog}\left(3,1-\frac{2}{iax+1}\right.}{4a}\right.\right.\right.$$

input
```
Int[((c + a^2*c*x^2)*ArcTan[a*x]^2)/x,x]
```

output
```
a^2*c*((x^2*ArcTan[a*x]^2)/2 - a*(-1/2*ArcTan[a*x]^2/a^3 + (x*ArcTan[a*x]
- Log[1 + a^2*x^2]/(2*a))/a^2)) + c*(2*ArcTan[a*x]^2*ArcTanh[1 - 2/(1 + I*
a*x)] - 4*a*((((I/2)*ArcTan[a*x]*PolyLog[2, 1 - 2/(1 + I*a*x)])/a + PolyLo
g[3, 1 - 2/(1 + I*a*x)]/(4*a))/2 + (((-1/2*I)*ArcTan[a*x]*PolyLog[2, -1 +
2/(1 + I*a*x)])/a - PolyLog[3, -1 + 2/(1 + I*a*x)]/(4*a))/2))
```

## Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5357
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Simp[2*b*c*p   Int[(a + b
*ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
 FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5523
```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[1/2   Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e
*x^2)), x], x] - Simp[1/2   Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] &&
EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 20.72 (sec) , antiderivative size = 1055, normalized size of antiderivative = 6.24

| method | result | size |
|---|---|---|
| derivativedivides | Expression too large to display | 1055 |
| default | Expression too large to display | 1055 |
| parts | Expression too large to display | 1545 |

input
```
int((a^2*c*x^2+c)*arctan(a*x)^2/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*a^2*c*x^2*arctan(a*x)^2+c*arctan(a*x)^2*ln(a*x)-c*(arctan(a*x)^2*ln((1
+I*a*x)^2/(a^2*x^2+1)-1)-arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))+1
/2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*cs
gn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x)^
2-2*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))-arctan(a*x)^2*ln(1+(1+I*a*x)/(a
^2*x^2+1)^(1/2))-1/2*I*Pi*arctan(a*x)^2-2*polylog(3,-(1+I*a*x)/(a^2*x^2+1)
^(1/2))-1/2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I/((1+I*a*x)^2/(
a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)
+1))*arctan(a*x)^2+1/2*polylog(3,-(1+I*a*x)^2/(a^2*x^2+1))+1/2*I*Pi*csgn(I
/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x
)^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2-1/2*arctan(a*x)^2+ln((1+I*a*x)^2/(a^2*
x^2+1)+1)+2*I*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*I*arct
an(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-1/2*I*Pi*csgn(I*((1+I*a*x)^
2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^3*arctan(a*x)^2-I*arctan(a*x
)*polylog(2,-(1+I*a*x)^2/(a^2*x^2+1))-1/2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^
2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+
I*a*x)^2/(a^2*x^2+1)+1))*arctan(a*x)^2+1/2*I*Pi*csgn(((1+I*a*x)^2/(a^2*x^2
+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2+1/2*I*Pi*csgn(I*((1+I*
a*x)^2/(a^2*x^2+1)-1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^
2*x^2+1)+1))^2*arctan(a*x)^2-1/2*I*Pi*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/...
```

## Fricas [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)^2}{x}\,dx = \int \frac{(a^2cx^2+c)\arctan\left(ax\right)^2}{x}\,dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^2/x,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)*arctan(a*x)^2/x, x)
```

## Sympy [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)^2}{x}\,dx = c\left(\int \frac{\operatorname{atan}^2(ax)}{x}\,dx + \int a^2x\operatorname{atan}^2(ax)\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)*atan(a*x)**2/x,x)
```

output
```
c*(Integral(atan(a*x)**2/x, x) + Integral(a**2*x*atan(a*x)**2, x))
```

## Maxima [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)^2}{x}\,dx = \int \frac{(a^2cx^2+c)\arctan(ax)^2}{x}\,dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^2/x,x, algorithm="maxima")
```

output
```
1/8*a^2*c*x^2*arctan(a*x)^2 - 1/32*a^2*c*x^2*log(a^2*x^2 + 1)^2 + 12*a^4*c
*integrate(1/16*x^4*arctan(a*x)^2/(a^2*x^3 + x), x) + a^4*c*integrate(1/16
*x^4*log(a^2*x^2 + 1)^2/(a^2*x^3 + x), x) + 2*a^4*c*integrate(1/16*x^4*log
(a^2*x^2 + 1)/(a^2*x^3 + x), x) - 4*a^3*c*integrate(1/16*x^3*arctan(a*x)/(
a^2*x^3 + x), x) + 24*a^2*c*integrate(1/16*x^2*arctan(a*x)^2/(a^2*x^3 + x)
, x) + 1/48*c*log(a^2*x^2 + 1)^3 + 12*c*integrate(1/16*arctan(a*x)^2/(a^2*
x^3 + x), x) + c*integrate(1/16*log(a^2*x^2 + 1)^2/(a^2*x^3 + x), x)
```

## Giac [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)^2}{x}\,dx = \int \frac{(a^2cx^2+c)\arctan(ax)^2}{x}\,dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^2/x,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)*arctan(a*x)^2/x, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)\arctan(ax)^2}{x}\,dx = \int \frac{\mathrm{atan}(a\,x)^2\,(c\,a^2\,x^2+c)}{x}\,dx$$

input
```
int((atan(a*x)^2*(c + a^2*c*x^2))/x,x)
```

output
```
int((atan(a*x)^2*(c + a^2*c*x^2))/x, x)
```

## Reduce [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)^2}{x}\,dx = \frac{c\left(atan(ax)^2\,a^2x^2 + atan(ax)^2 - 2atan(ax)\,ax + 2\left(\int \frac{atan(ax)^2}{x}dx\right) + \log(a^2x^2+1)\right)}{2}$$

input
```
int((a^2*c*x^2+c)*atan(a*x)^2/x,x)
```

output
```
(c*(atan(a*x)**2*a**2*x**2 + atan(a*x)**2 - 2*atan(a*x)*a*x + 2*int(atan(a
*x)**2/x,x) + log(a**2*x**2 + 1)))/2
```

# 3.263 $\int \frac{(c+a^2cx^2)\arctan(ax)^2}{x^2}\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 113

$$\begin{aligned}\int \frac{(c+a^2cx^2)\arctan(ax)^2}{x^2}\,dx = &-\frac{c\arctan(ax)^2}{x}+a^2cx\arctan(ax)^2\\&+2ac\arctan(ax)\log\left(\frac{2}{1+iax}\right)\\&+2ac\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)\\&-iac\,\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)\\&+iac\,\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)\end{aligned}$$

output
```
-c*arctan(a*x)^2/x+a^2*c*x*arctan(a*x)^2+2*a*c*arctan(a*x)*ln(2/(1+I*a*x))
+2*a*c*arctan(a*x)*ln(2-2/(1-I*a*x))-I*a*c*polylog(2,-1+2/(1-I*a*x))+I*a*c
*polylog(2,1-2/(1+I*a*x))
```

## Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.83

$$\int \frac{(c+a^2cx^2)\arctan(ax)^2}{x^2}\,dx = c\left(\frac{(-i+ax)^2\arctan(ax)^2}{x} + 2a\arctan(ax)\left(\log\left(1-e^{2i\arctan(ax)}\right)+\log\left(1+e^{2i\arctan(ax)}\right)\right) - ia\,\text{PolyLog}\left(2,-e^{2i\arctan(ax)}\right) - ia\,\text{PolyLog}\left(2,e^{2i\arctan(ax)}\right)\right)$$

input
```
Integrate[((c + a^2*c*x^2)*ArcTan[a*x]^2)/x^2,x]
```

output
```
c*(((-I + a*x)^2*ArcTan[a*x]^2)/x + 2*a*ArcTan[a*x]*(Log[1 - E^((2*I)*ArcT
an[a*x])] + Log[1 + E^((2*I)*ArcTan[a*x])]) - I*a*PolyLog[2, -E^((2*I)*Arc
Tan[a*x])] - I*a*PolyLog[2, E^((2*I)*ArcTan[a*x])])
```

## Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.45, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5485, 5345, 5361, 5455, 5379, 2849, 2752, 5459, 5403, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2\,(a^2cx^2+c)}{x^2}\,dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\int \arctan(ax)^2dx + c\int \frac{\arctan(ax)^2}{x^2}dx$$

$$\Big\downarrow \text{5345}$$

$$a^2c\left(x\arctan(ax)^2 - 2a\int \frac{x\arctan(ax)}{a^2x^2+1}dx\right) + c\int \frac{\arctan(ax)^2}{x^2}dx$$

$$\Big\downarrow \text{5361}$$

$$a^2c\left(x\arctan(ax)^2-2a\int\frac{x\arctan(ax)}{a^2x^2+1}dx\right)+c\left(2a\int\frac{\arctan(ax)}{x\left(a^2x^2+1\right)}dx-\frac{\arctan(ax)^2}{x}\right)$$

$$\Big\downarrow \text{5455}$$

$$c\left(2a\int\frac{\arctan(ax)}{x\left(a^2x^2+1\right)}dx-\frac{\arctan(ax)^2}{x}\right)+a^2c\left(x\arctan(ax)^2-2a\left(-\frac{\int\frac{\arctan(ax)}{i-ax}dx}{a}-\frac{i\arctan(ax)^2}{2a^2}\right)\right)$$

$$\Big\downarrow \text{5379}$$

$$c\left(2a\int\frac{\arctan(ax)}{x\left(a^2x^2+1\right)}dx-\frac{\arctan(ax)^2}{x}\right)+a^2c\left(x\arctan(ax)^2-2a\left(-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}-\int\frac{\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^2}{2a^2}\right)\right)$$

$$\Big\downarrow \text{2849}$$

$$c\left(2a\int\frac{\arctan(ax)}{x\left(a^2x^2+1\right)}dx-\frac{\arctan(ax)^2}{x}\right)+a^2c\left(x\arctan(ax)^2-2a\left(-\frac{\frac{i\int\frac{\log\left(\frac{2}{iax+1}\right)}{1-\frac{2}{iax+1}}d\frac{1}{iax+1}}{a}+\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}}{a}-\frac{i\arctan(ax)^2}{2a^2}\right)\right)$$

$$\Big\downarrow \text{2752}$$

$$c\left(2a\int\frac{\arctan(ax)}{x\left(a^2x^2+1\right)}dx-\frac{\arctan(ax)^2}{x}\right)+a^2c\left(x\arctan(ax)^2-2a\left(-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}\right)\right)$$

$$\Big\downarrow \text{5459}$$

$$c\left(-\frac{\arctan(ax)^2}{x}+2a\left(i\int\frac{\arctan(ax)}{x(ax+i)}dx-\frac{1}{2}i\arctan(ax)^2\right)\right)+a^2c\left(x\arctan(ax)^2-2a\left(-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}\right)\right)$$

$$\Big\downarrow \text{5403}$$

$$c\left(-\frac{\arctan(ax)^2}{x}+2a\left(i\left(ia\int\frac{\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)\right)+$$
$$a^2c\left(x\arctan(ax)^2-2a\left(-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}\right)\right)$$

$$\Big\downarrow \text{2897}$$

$$a^2c\left(x\arctan(ax)^2-2a\left(-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}\right)\right)+$$
$$c\left(-\frac{\arctan(ax)^2}{x}+2a\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)\right)$$

input
```
Int[((c + a^2*c*x^2)*ArcTan[a*x]^2)/x^2,x]
```

output
```
c*(-(ArcTan[a*x]^2/x) + 2*a*((-1/2*I)*ArcTan[a*x]^2 + I*((-I)*ArcTan[a*x]*
Log[2 - 2/(1 - I*a*x)] - PolyLog[2, -1 + 2/(1 - I*a*x)]/2))) + a^2*c*(x*Ar
cTan[a*x]^2 - 2*a*(((-1/2*I)*ArcTan[a*x]^2)/a^2 - ((ArcTan[a*x]*Log[2/(1 +
 I*a*x)])/a + ((I/2)*PolyLog[2, 1 - 2/(1 + I*a*x)])/a)/a))
```

## Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

## Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 237 vs. 2(107) = 214.

Time = 0.48 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.11

| method | result |
|---|---|
| derivativedivides | $a\left(acx\arctan\left(ax\right)^{2}-\frac{c\arctan(ax)^{2}}{ax}-2c\left(-\arctan\left(ax\right)\ln\left(ax\right)+\arctan\left(ax\right)\ln\left(a^{2}x^{2}+\right.\right.\right.$ |
| default | $a\left(acx\arctan\left(ax\right)^{2}-\frac{c\arctan(ax)^{2}}{ax}-2c\left(-\arctan\left(ax\right)\ln\left(ax\right)+\arctan\left(ax\right)\ln\left(a^{2}x^{2}+\right.\right.\right.$ |
| parts | $a^{2}cx\arctan\left(ax\right)^{2}-\frac{c\arctan(ax)^{2}}{x}-2c\left(-a\arctan\left(ax\right)\ln\left(ax\right)+a\arctan\left(ax\right)\ln\left(a^{2}x^{2}\right.\right.$ |

input
```
int((a^2*c*x^2+c)*arctan(a*x)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
a*(a*c*x*arctan(a*x)^2-c*arctan(a*x)^2/a/x-2*c*(-arctan(a*x)*ln(a*x)+arcta
n(a*x)*ln(a^2*x^2+1)+1/2*I*(ln(a*x-I)*ln(a^2*x^2+1)-1/2*ln(a*x-I)^2-dilog(
-1/2*I*(a*x+I))-ln(a*x-I)*ln(-1/2*I*(a*x+I)))-1/2*I*(ln(a*x+I)*ln(a^2*x^2+
1)-1/2*ln(a*x+I)^2-dilog(1/2*I*(a*x-I))-ln(a*x+I)*ln(1/2*I*(a*x-I)))-1/2*I
*ln(a*x)*ln(1+I*a*x)+1/2*I*ln(a*x)*ln(1-I*a*x)-1/2*I*dilog(1+I*a*x)+1/2*I*
dilog(1-I*a*x)))
```

## Fricas [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)^2}{x^2}\,dx = \int \frac{(a^2cx^2+c)\arctan{(ax)}^2}{x^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^2/x^2,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)*arctan(a*x)^2/x^2, x)
```

## Sympy [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)^2}{x^2}\,dx = c\left(\int a^2\operatorname{atan}^2{(ax)}\,dx + \int \frac{\operatorname{atan}^2{(ax)}}{x^2}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)*atan(a*x)**2/x**2,x)
```

output
```
c*(Integral(a**2*atan(a*x)**2, x) + Integral(atan(a*x)**2/x**2, x))
```

## Maxima [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)^2}{x^2}\,dx = \int \frac{(a^2cx^2+c)\arctan{(ax)}^2}{x^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^2/x^2,x, algorithm="maxima")
```

output
```
1/16*(4*(a^2*c*x^2 - c)*arctan(a*x)^2 - (a^2*c*x^2 - c)*log(a^2*x^2 + 1)^2
 + 8*(24*a^4*c*integrate(1/16*x^4*arctan(a*x)^2/(a^2*x^4 + x^2), x) + 2*a^
4*c*integrate(1/16*x^4*log(a^2*x^2 + 1)^2/(a^2*x^4 + x^2), x) + 8*a^4*c*in
tegrate(1/16*x^4*log(a^2*x^2 + 1)/(a^2*x^4 + x^2), x) + a*c*arctan(a*x)^3
- 16*a^3*c*integrate(1/16*x^3*arctan(a*x)/(a^2*x^4 + x^2), x) + 4*a^2*c*in
tegrate(1/16*x^2*log(a^2*x^2 + 1)^2/(a^2*x^4 + x^2), x) - 8*a^2*c*integrat
e(1/16*x^2*log(a^2*x^2 + 1)/(a^2*x^4 + x^2), x) + 16*a*c*integrate(1/16*x*
arctan(a*x)/(a^2*x^4 + x^2), x) + 24*c*integrate(1/16*arctan(a*x)^2/(a^2*x
^4 + x^2), x) + 2*c*integrate(1/16*log(a^2*x^2 + 1)^2/(a^2*x^4 + x^2), x))
*x)/x
```

## Giac [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)^2}{x^2}\,dx=\int \frac{(a^2cx^2+c)\arctan{(ax)}^2}{x^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^2/x^2,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)*arctan(a*x)^2/x^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)\arctan(ax)^2}{x^2}\,dx=\int \frac{\mathrm{atan}(a\,x)^2\,(c\,a^2\,x^2+c)}{x^2}\,dx$$

input
```
int((atan(a*x)^2*(c + a^2*c*x^2))/x^2,x)
```

output
```
int((atan(a*x)^2*(c + a^2*c*x^2))/x^2, x)
```

## Reduce [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)^2}{x^2}\,dx$$
$$= \frac{c\left(-atan(ax)^2+\left(\int atan(ax)^2\,dx\right)a^2x+2\left(\int \frac{atan(ax)}{a^2x^3+x}dx\right)ax\right)}{x}$$

input
```
int((a^2*c*x^2+c)*atan(a*x)^2/x^2,x)
```

output
```
(c*( - atan(a*x)**2 + int(atan(a*x)**2,x)*a**2*x + 2*int(atan(a*x)/(a**2*x
**3 + x),x)*a*x))/x
```

# 3.264 $\int \frac{(c+a^2cx^2)\arctan(ax)^2}{x^3}\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 196

$$\begin{aligned}\int \frac{(c+a^2cx^2)\arctan(ax)^2}{x^3}\,dx = &-\frac{ac\arctan(ax)}{x}-\frac{1}{2}a^2c\arctan(ax)^2-\frac{c\arctan(ax)^2}{2x^2}\\ &+2a^2c\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{1+iax}\right)\\ &+a^2c\log(x)-\frac{1}{2}a^2c\log\left(1+a^2x^2\right)\\ &-ia^2c\arctan(ax)\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right)\\ &+ia^2c\arctan(ax)\operatorname{PolyLog}\left(2,-1+\frac{2}{1+iax}\right)\\ &-\frac{1}{2}a^2c\operatorname{PolyLog}\left(3,1-\frac{2}{1+iax}\right)\\ &+\frac{1}{2}a^2c\operatorname{PolyLog}\left(3,-1+\frac{2}{1+iax}\right)\end{aligned}$$

output
```
-a*c*arctan(a*x)/x-1/2*a^2*c*arctan(a*x)^2-1/2*c*arctan(a*x)^2/x^2-2*a^2*c
*arctan(a*x)^2*arctanh(-1+2/(1+I*a*x))+a^2*c*ln(x)-1/2*a^2*c*ln(a^2*x^2+1)
-I*a^2*c*arctan(a*x)*polylog(2,1-2/(1+I*a*x))+I*a^2*c*arctan(a*x)*polylog(
2,-1+2/(1+I*a*x))-1/2*a^2*c*polylog(3,1-2/(1+I*a*x))+1/2*a^2*c*polylog(3,-
1+2/(1+I*a*x))
```

## Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.12

$$\int \frac{(c+a^2cx^2)\arctan(ax)^2}{x^3}\,dx = -\frac{ac\arctan(ax)}{x}+\frac{c(-1-a^2x^2)\arctan(ax)^2}{2x^2} + \frac{2}{3}ia^2c\arctan(ax)^3 + a^2c\arctan(ax)^2\log\left(1-e^{-2i\arctan(ax)}\right) - a^2c\arctan(ax)^2\log\left(1+e^{2i\arctan(ax)}\right) + a^2c\log(x)-\frac{1}{2}a^2c\log\left(1+a^2x^2\right) + ia^2c\arctan(ax)\,\text{PolyLog}\left(2,e^{-2i\arctan(ax)}\right) + ia^2c\arctan(ax)\,\text{PolyLog}\left(2,-e^{2i\arctan(ax)}\right) + \frac{1}{2}a^2c\,\text{PolyLog}\left(3,e^{-2i\arctan(ax)}\right) - \frac{1}{2}a^2c\,\text{PolyLog}\left(3,-e^{2i\arctan(ax)}\right)$$

input
```
Integrate[((c + a^2*c*x^2)*ArcTan[a*x]^2)/x^3,x]
```

output
```
-((a*c*ArcTan[a*x])/x) + (c*(-1 - a^2*x^2)*ArcTan[a*x]^2)/(2*x^2) + ((2*I)
/3)*a^2*c*ArcTan[a*x]^3 + a^2*c*ArcTan[a*x]^2*Log[1 - E^((-2*I)*ArcTan[a*x
])] - a^2*c*ArcTan[a*x]^2*Log[1 + E^((2*I)*ArcTan[a*x])] + a^2*c*Log[x] -
(a^2*c*Log[1 + a^2*x^2])/2 + I*a^2*c*ArcTan[a*x]*PolyLog[2, E^((-2*I)*ArcT
an[a*x])] + I*a^2*c*ArcTan[a*x]*PolyLog[2, -E^((2*I)*ArcTan[a*x])] + (a^2*
c*PolyLog[3, E^((-2*I)*ArcTan[a*x])])/2 - (a^2*c*PolyLog[3, -E^((2*I)*ArcT
an[a*x])])/2
```

## Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.07, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.650$, Rules used = {5485, 5357, 5361, 5453, 5361, 243, 47, 14, 16, 5419, 5523, 5529, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2 \left(a^2cx^2+c\right)}{x^3}dx$$

$$\Big\downarrow \textbf{5485}$$

$$a^2c\int \frac{\arctan(ax)^2}{x}dx+c\int \frac{\arctan(ax)^2}{x^3}dx$$

$$\Big\downarrow \textbf{5357}$$

$$c\int \frac{\arctan(ax)^2}{x^3}dx+ a^2c\left(2\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{1+iax}\right)-4a\int \frac{\arctan(ax)\text{arctanh}\left(1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx\right)$$

$$\Big\downarrow \textbf{5361}$$

$$c\left(a\int \frac{\arctan(ax)}{x^2\left(a^2x^2+1\right)}dx-\frac{\arctan(ax)^2}{2x^2}\right)+ a^2c\left(2\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{1+iax}\right)-4a\int \frac{\arctan(ax)\text{arctanh}\left(1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx\right)$$

$$\Big\downarrow \textbf{5453}$$

$$c\left(a\left(\int \frac{\arctan(ax)}{x^2}dx-a^2\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)-\frac{\arctan(ax)^2}{2x^2}\right)+ a^2c\left(2\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{1+iax}\right)-4a\int \frac{\arctan(ax)\text{arctanh}\left(1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx\right)$$

$$\Big\downarrow \textbf{5361}$$

$$c\left(a\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+a\int \frac{1}{x\left(a^2x^2+1\right)}dx-\frac{\arctan(ax)}{x}\right)-\frac{\arctan(ax)^2}{2x^2}\right)+ a^2c\left(2\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{1+iax}\right)-4a\int \frac{\arctan(ax)\text{arctanh}\left(1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx\right)$$

$$\Big\downarrow \textbf{243}$$

$$c\left(a\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{2}a\int \frac{1}{x^2\left(a^2x^2+1\right)}dx^2-\frac{\arctan(ax)}{x}\right)-\frac{\arctan(ax)^2}{2x^2}\right)+ a^2c\left(2\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{1+iax}\right)-4a\int \frac{\arctan(ax)\text{arctanh}\left(1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx\right)$$

$$\Big\downarrow \textbf{47}$$

$$c\left(a\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{2}a\left(\int \frac{1}{x^2}dx^2-a^2\int \frac{1}{a^2x^2+1}dx^2\right)-\frac{\arctan(ax)}{x}\right)-\frac{\arctan(ax)^2}{2x^2}\right)+ a^2c\left(2\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{1+iax}\right)-4a\int \frac{\arctan(ax)\text{arctanh}\left(1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx\right)$$

$$\Big\downarrow \text{ 14}$$

$$c\left(a\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{2}a\left(\log\left(x^2\right)-a^2\int \frac{1}{a^2x^2+1}dx^2\right)-\frac{\arctan(ax)}{x}\right)-\frac{\arctan(ax)^2}{2x^2}\right)+ a^2c\left(2\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{1+iax}\right)-4a\int \frac{\arctan(ax)\text{arctanh}\left(1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx\right)$$

$$\Big\downarrow \text{ 16}$$

$$c\left(a\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{\arctan(ax)}{x}\right)-\frac{\arctan(ax)^2}{2x^2}\right)+ a^2c\left(2\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{1+iax}\right)-4a\int \frac{\arctan(ax)\text{arctanh}\left(1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx\right)$$

$$\Big\downarrow \text{ 5419}$$

$$c\left(a\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{1}{2}a\arctan(ax)^2-\frac{\arctan(ax)}{x}\right)-\frac{\arctan(ax)^2}{2x^2}\right)+ a^2c\left(2\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{1+iax}\right)-4a\int \frac{\arctan(ax)\text{arctanh}\left(1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx\right)$$

$$\Big\downarrow \text{ 5523}$$

$$c\left(a\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{1}{2}a\arctan(ax)^2-\frac{\arctan(ax)}{x}\right)-\frac{\arctan(ax)^2}{2x^2}\right)+ a^2c\left(2\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{1+iax}\right)-4a\left(\frac{1}{2}\int \frac{\arctan(ax)\log\left(2-\frac{2}{iax+1}\right)}{a^2x^2+1}dx-\frac{1}{2}\int \frac{\arctan(ax)\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx\right)\right)$$

$$\Big\downarrow \text{ 5529}$$

$$c\left(a\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{1}{2}a\arctan(ax)^2-\frac{\arctan(ax)}{x}\right)-\frac{\arctan(ax)^2}{2x^2}\right)+ a^2c\left(2\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{1+iax}\right)-4a\left(\frac{1}{2}\left(\frac{i\arctan(ax)\operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}-\frac{1}{2}i\int \frac{\operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx\right)\right.\right.$$

$\Big\downarrow$ 7164

$$c\left(a\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{1}{2}a\arctan(ax)^2-\frac{\arctan(ax)}{x}\right)-\frac{\arctan(ax)^2}{2x^2}\right)+ a^2c\left(2\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{1+iax}\right)-4a\left(\frac{1}{2}\left(\frac{i\arctan(ax)\operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}+\frac{\operatorname{PolyLog}\left(3,1-\frac{2}{iax}\right.}{4a}\right.\right.\right.$$

input
```
Int[((c + a^2*c*x^2)*ArcTan[a*x]^2)/x^3,x]
```

output
```
c*(-1/2*ArcTan[a*x]^2/x^2 + a*(-(ArcTan[a*x]/x) - (a*ArcTan[a*x]^2)/2 + (a
*(Log[x^2] - Log[1 + a^2*x^2]))/2)) + a^2*c*(2*ArcTan[a*x]^2*ArcTanh[1 - 2
/(1 + I*a*x)] - 4*a*((((I/2)*ArcTan[a*x]*PolyLog[2, 1 - 2/(1 + I*a*x)])/a
+ PolyLog[3, 1 - 2/(1 + I*a*x)]/(4*a))/2 + (((-1/2*I)*ArcTan[a*x]*PolyLog[
2, -1 + 2/(1 + I*a*x)])/a - PolyLog[3, -1 + 2/(1 + I*a*x)]/(4*a))/2))
```

## Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5357
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Simp[2*b*c*p   Int[(a + b
*ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
 FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5523
```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[1/2   Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e
*x^2)), x], x] - Simp[1/2   Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] &&
EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 23.74 (sec) , antiderivative size = 1134, normalized size of antiderivative = 5.79

| method | result | size |
|---|---|---|
| derivativedivides | Expression too large to display | 1134 |
| default | Expression too large to display | 1134 |
| parts | Expression too large to display | 1561 |

input
```
int((a^2*c*x^2+c)*arctan(a*x)^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
a^2*(-1/2*c*arctan(a*x)^2/a^2/x^2+c*arctan(a*x)^2*ln(a*x)-c*(arctan(a*x)^2
*ln((1+I*a*x)^2/(a^2*x^2+1)-1)-arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1
/2))+2*I*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*polylog(3,(1
+I*a*x)/(a^2*x^2+1)^(1/2))-arctan(a*x)^2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))
-1/2*I*Pi*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^3*
arctan(a*x)^2-2*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+1/2*I*Pi*csgn(((1+
I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2+1/2*p
olylog(3,-(1+I*a*x)^2/(a^2*x^2+1))+1/2*arctan(a*x)^2-ln(1+(1+I*a*x)/(a^2*x
^2+1)^(1/2))-ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-1)+2*I*arctan(a*x)*polylog(2,-
(1+I*a*x)/(a^2*x^2+1)^(1/2))+1/2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/(
(1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/
(a^2*x^2+1)+1))^2*arctan(a*x)^2+1/2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1
))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arcta
n(a*x)^2-1/2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2
+1)+1))^3*arctan(a*x)^2+1/2*arctan(a*x)*(I*a*x-(a^2*x^2+1)^(1/2)+1)/a/x-1/
2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*csg
n(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*arctan(a*x)^2-1
/2*I*Pi*arctan(a*x)^2+1/2*I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*
((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2-I
*arctan(a*x)*polylog(2,-(1+I*a*x)^2/(a^2*x^2+1))-1/2*I*Pi*csgn(I*((1+I*...
```

## Fricas [F]

$$\int \frac{(c + a^2cx^2)\arctan(ax)^2}{x^3}\,dx = \int \frac{(a^2cx^2 + c)\arctan\left(ax\right)^2}{x^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^2/x^3,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)*arctan(a*x)^2/x^3, x)
```

## Sympy [F]

$$\int \frac{(c + a^2cx^2) \arctan(ax)^2}{x^3} dx = c\left(\int \frac{\operatorname{atan}^2(ax)}{x^3} dx + \int \frac{a^2 \operatorname{atan}^2(ax)}{x} dx\right)$$

input
```
integrate((a**2*c*x**2+c)*atan(a*x)**2/x**3,x)
```

output
```
c*(Integral(atan(a*x)**2/x**3, x) + Integral(a**2*atan(a*x)**2/x, x))
```

## Maxima [F]

$$\int \frac{(c + a^2cx^2) \arctan(ax)^2}{x^3} dx = \int \frac{(a^2cx^2 + c) \arctan(ax)^2}{x^3} dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^2/x^3,x, algorithm="maxima")
```

output
```
1/96*((1152*a^4*c*integrate(1/16*x^4*arctan(a*x)^2/(a^2*x^5 + x^3), x) + a
^2*c*log(a^2*x^2 + 1)^3 + 2304*a^2*c*integrate(1/16*x^2*arctan(a*x)^2/(a^2
*x^5 + x^3), x) + 192*a^2*c*integrate(1/16*x^2*log(a^2*x^2 + 1)^2/(a^2*x^5
 + x^3), x) - 192*a^2*c*integrate(1/16*x^2*log(a^2*x^2 + 1)/(a^2*x^5 + x^3
), x) + 384*a*c*integrate(1/16*x*arctan(a*x)/(a^2*x^5 + x^3), x) + 1152*c*
integrate(1/16*arctan(a*x)^2/(a^2*x^5 + x^3), x) + 96*c*integrate(1/16*log
(a^2*x^2 + 1)^2/(a^2*x^5 + x^3), x))*x^2 - 12*c*arctan(a*x)^2 + 3*c*log(a^
2*x^2 + 1)^2)/x^2
```

## Giac [F]

$$\int \frac{(c + a^2cx^2) \arctan(ax)^2}{x^3} dx = \int \frac{(a^2cx^2 + c) \arctan(ax)^2}{x^3} dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^2/x^3,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)*arctan(a*x)^2/x^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)\arctan(ax)^2}{x^3}\,dx = \int \frac{\mathrm{atan}(a\,x)^2\,(c\,a^2\,x^2+c)}{x^3}\,dx$$

input
```
int((atan(a*x)^2*(c + a^2*c*x^2))/x^3,x)
```

output
```
int((atan(a*x)^2*(c + a^2*c*x^2))/x^3, x)
```

## Reduce [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)^2}{x^3}\,dx = \frac{c\left(-atan(ax)^2\,a^2x^2 - atan(ax)^2 - 2atan(ax)\,ax + 2\left(\int \frac{atan(ax)^2}{x}dx\right)a^2x^2 - \log(a^2x^2+1)\,a^2x^2 + 2\log(x$$

$$2x^2$$

input
```
int((a^2*c*x^2+c)*atan(a*x)^2/x^3,x)
```

output
```
(c*( - atan(a*x)**2*a**2*x**2 - atan(a*x)**2 - 2*atan(a*x)*a*x + 2*int(ata
n(a*x)**2/x,x)*a**2*x**2 - log(a**2*x**2 + 1)*a**2*x**2 + 2*log(x)*a**2*x*
*2))/(2*x**2)
```

# 3.265 $\int \frac{(c+a^2cx^2)\arctan(ax)^2}{x^4}\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 135

$$\begin{aligned}\int \frac{(c+a^2cx^2)\arctan(ax)^2}{x^4}\,dx = & -\frac{a^2c}{3x}-\frac{1}{3}a^3c\arctan(ax)-\frac{ac\arctan(ax)}{3x^2}\\ & -\frac{2}{3}ia^3c\arctan(ax)^2-\frac{c\arctan(ax)^2}{3x^3}-\frac{a^2c\arctan(ax)^2}{x}\\ & +\frac{4}{3}a^3c\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)\\ & -\frac{2}{3}ia^3c\,\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)\end{aligned}$$

output
```
-1/3*a^2*c/x-1/3*a^3*c*arctan(a*x)-1/3*a*c*arctan(a*x)/x^2-2/3*I*a^3*c*arc
tan(a*x)^2-1/3*c*arctan(a*x)^2/x^3-a^2*c*arctan(a*x)^2/x+4/3*a^3*c*arctan(
a*x)*ln(2-2/(1-I*a*x))-2/3*I*a^3*c*polylog(2,-1+2/(1-I*a*x))
```

## Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.76

$$\int \frac{(c+a^2cx^2)\arctan(ax)^2}{x^4}\,dx$$
$$=\frac{c\left(-a^2x^2+(1-2iax)(-i+ax)^2\arctan(ax)^2+ax\arctan(ax)\left(-1-a^2x^2+4a^2x^2\log\left(1-e^{2i\arctan(ax)}\right)\right.\right.}{3x^3}$$

input
```
Integrate[((c + a^2*c*x^2)*ArcTan[a*x]^2)/x^4,x]
```

output
```
(c*(-(a^2*x^2) + (1 - (2*I)*a*x)*(-I + a*x)^2*ArcTan[a*x]^2 + a*x*ArcTan[a
*x]*(-1 - a^2*x^2 + 4*a^2*x^2*Log[1 - E^((2*I)*ArcTan[a*x])]) - (2*I)*a^3*
x^3*PolyLog[2, E^((2*I)*ArcTan[a*x])]))/(3*x^3)
```

## Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.47, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {5485, 5361, 5453, 5361, 264, 216, 5459, 5403, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2\left(a^2cx^2+c\right)}{x^4}\,dx$$

$$\Big\downarrow \text{ 5485}$$

$$a^2c\int \frac{\arctan(ax)^2}{x^2}dx+c\int \frac{\arctan(ax)^2}{x^4}dx$$

$$\Big\downarrow \text{ 5361}$$

$$a^2c\left(2a\int \frac{\arctan(ax)}{x\left(a^2x^2+1\right)}dx-\frac{\arctan(ax)^2}{x}\right)+c\left(\frac{2}{3}a\int \frac{\arctan(ax)}{x^3\left(a^2x^2+1\right)}dx-\frac{\arctan(ax)^2}{3x^3}\right)$$

$$\Big\downarrow \text{ 5453}$$

$$a^2c\left(2a\int \frac{\arctan(ax)}{x\left(a^2x^2+1\right)}dx-\frac{\arctan(ax)^2}{x}\right)+$$
$$c\left(\frac{2}{3}a\left(\int \frac{\arctan(ax)}{x^3}dx-a^2\int \frac{\arctan(ax)}{x\left(a^2x^2+1\right)}dx\right)-\frac{\arctan(ax)^2}{3x^3}\right)$$

$$\Big\downarrow \text{ 5361}$$

$$a^2c\left(2a\int \frac{\arctan(ax)}{x\left(a^2x^2+1\right)}dx-\frac{\arctan(ax)^2}{x}\right)+$$
$$c\left(\frac{2}{3}a\left(a^2\left(-\int \frac{\arctan(ax)}{x\left(a^2x^2+1\right)}dx\right)+\frac{1}{2}a\int \frac{1}{x^2\left(a^2x^2+1\right)}dx-\frac{\arctan(ax)}{2x^2}\right)-\frac{\arctan(ax)^2}{3x^3}\right)$$

$$\Big\downarrow \text{ 264}$$

$$a^2c\left(2a\int \frac{\arctan(ax)}{x\left(a^2x^2+1\right)}dx-\frac{\arctan(ax)^2}{x}\right)+$$
$$c\left(\frac{2}{3}a\left(a^2\left(-\int \frac{\arctan(ax)}{x\left(a^2x^2+1\right)}dx\right)+\frac{1}{2}a\left(a^2\left(-\int \frac{1}{a^2x^2+1}dx\right)-\frac{1}{x}\right)-\frac{\arctan(ax)}{2x^2}\right)-\frac{\arctan(ax)^2}{3x^3}\right)$$

$$\Big\downarrow \text{ 216}$$

$$a^2c\left(2a\int \frac{\arctan(ax)}{x\left(a^2x^2+1\right)}dx-\frac{\arctan(ax)^2}{x}\right)+$$
$$c\left(\frac{2}{3}a\left(a^2\left(-\int \frac{\arctan(ax)}{x\left(a^2x^2+1\right)}dx\right)-\frac{\arctan(ax)}{2x^2}+\frac{1}{2}a\left(-a\arctan(ax)-\frac{1}{x}\right)\right)-\frac{\arctan(ax)^2}{3x^3}\right)$$

$$\Big\downarrow \text{ 5459}$$

$$c\left(-\frac{\arctan(ax)^2}{3x^3}+\frac{2}{3}a\left(-\left(a^2\left(i\int \frac{\arctan(ax)}{x(ax+i)}dx-\frac{1}{2}i\arctan(ax)^2\right)\right)-\frac{\arctan(ax)}{2x^2}+\frac{1}{2}a\left(-a\arctan(ax)-\frac{1}{x}\right.\right.\right.$$
$$a^2c\left(-\frac{\arctan(ax)^2}{x}+2a\left(i\int \frac{\arctan(ax)}{x(ax+i)}dx-\frac{1}{2}i\arctan(ax)^2\right)\right)$$

$$\Big\downarrow \text{ 5403}$$

$$a^2c\left(-\frac{\arctan(ax)^2}{x}+2a\left(i\left(ia\int \frac{\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)\right)$$
$$c\left(-\frac{\arctan(ax)^2}{3x^3}+\frac{2}{3}a\left(-\left(a^2\left(i\left(ia\int \frac{\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{2}i\arctan(ax\right.\right.\right.\right.$$

$$\Big\downarrow \text{ 2897}$$

$$c\left(-\frac{\arctan(ax)^2}{3x^3}+\frac{2}{3}a\left(-\left(a^2\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\operatorname{arcta}\right.\right.\right.\right.$$
$$a^2c\left(-\frac{\arctan(ax)^2}{x}+2a\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)\right.\right.$$

input
```
Int[((c + a^2*c*x^2)*ArcTan[a*x]^2)/x^4,x]
```

output
```
c*(-1/3*ArcTan[a*x]^2/x^3 + (2*a*(-1/2*ArcTan[a*x]/x^2 + (a*(-x^(-1) - a*A
rcTan[a*x]))/2 - a^2*((-1/2*I)*ArcTan[a*x]^2 + I*((-I)*ArcTan[a*x]*Log[2 -
 2/(1 - I*a*x)] - PolyLog[2, -1 + 2/(1 - I*a*x)]/2))))/3) + a^2*c*(-(ArcTa
n[a*x]^2/x) + 2*a*((-1/2*I)*ArcTan[a*x]^2 + I*((-I)*ArcTan[a*x]*Log[2 - 2/
(1 - I*a*x)] - PolyLog[2, -1 + 2/(1 - I*a*x)]/2)))
```

### Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

## Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 270 vs. $2(117) = 234$.

Time = 0.62 (sec) , antiderivative size = 271, normalized size of antiderivative = 2.01

| method | result |
|---|---|
| derivativedivides | $a^3\left(-\frac{c\arctan(ax)^2}{3a^3x^3}-\frac{c\arctan(ax)^2}{ax}-\frac{2c\left(\frac{\arctan(ax)}{2a^2x^2}-2\arctan(ax)\ln(ax)+\arctan(ax)\ln(a^2x^2+1)+\frac{1}{2ax}+\mathrm{arct}\right.}{}\right.$ |
| default | $a^3\left(-\frac{c\arctan(ax)^2}{3a^3x^3}-\frac{c\arctan(ax)^2}{ax}-\frac{2c\left(\frac{\arctan(ax)}{2a^2x^2}-2\arctan(ax)\ln(ax)+\arctan(ax)\ln(a^2x^2+1)+\frac{1}{2ax}+\mathrm{arct}\right.}{}\right.$ |
| parts | $-\frac{c\arctan(ax)^2}{3x^3}-\frac{a^2c\arctan(ax)^2}{x}-\frac{2c\left(\frac{a\arctan(ax)}{2x^2}-2a^3\arctan(ax)\ln(ax)+a^3\arctan(ax)\ln(a^2x^2+1)+\frac{a^3\left(\frac{1}{a}\right.}{}\right.}{}$ |

input

```
int((a^2*c*x^2+c)*arctan(a*x)^2/x^4,x,method=_RETURNVERBOSE)
```

output

```
a^3*(-1/3*c*arctan(a*x)^2/a^3/x^3-c*arctan(a*x)^2/a/x-2/3*c*(1/2*arctan(a*
x)/a^2/x^2-2*arctan(a*x)*ln(a*x)+arctan(a*x)*ln(a^2*x^2+1)+1/2/a/x+1/2*arc
tan(a*x)-I*ln(a*x)*ln(1+I*a*x)+I*ln(a*x)*ln(1-I*a*x)-I*dilog(1+I*a*x)+I*di
log(1-I*a*x)+1/2*I*(ln(a*x-I)*ln(a^2*x^2+1)-1/2*ln(a*x-I)^2-dilog(-1/2*I*(
a*x+I))-ln(a*x-I)*ln(-1/2*I*(a*x+I)))-1/2*I*(ln(a*x+I)*ln(a^2*x^2+1)-1/2*l
n(a*x+I)^2-dilog(1/2*I*(a*x-I))-ln(a*x+I)*ln(1/2*I*(a*x-I)))))
```

## Fricas [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)^2}{x^4}\,dx = \int \frac{(a^2cx^2+c)\arctan(ax)^2}{x^4}\,dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^2/x^4,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)*arctan(a*x)^2/x^4, x)
```

## Sympy [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)^2}{x^4}\,dx = c\left(\int \frac{\operatorname{atan}^2(ax)}{x^4}\,dx + \int \frac{a^2\operatorname{atan}^2(ax)}{x^2}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)*atan(a*x)**2/x**4,x)
```

output
```
c*(Integral(atan(a*x)**2/x**4, x) + Integral(a**2*atan(a*x)**2/x**2, x))
```

## Maxima [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)^2}{x^4}\,dx = \int \frac{(a^2cx^2+c)\arctan(ax)^2}{x^4}\,dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^2/x^4,x, algorithm="maxima")
```

output
```
1/48*(12*(a^3*c*arctan(a*x)^3 + 12*a^4*c*integrate(1/48*x^4*log(a^2*x^2 +
1)^2/(a^2*x^6 + x^4), x) - 48*a^4*c*integrate(1/48*x^4*log(a^2*x^2 + 1)/(a
^2*x^6 + x^4), x) + 96*a^3*c*integrate(1/48*x^3*arctan(a*x)/(a^2*x^6 + x^4
), x) + 288*a^2*c*integrate(1/48*x^2*arctan(a*x)^2/(a^2*x^6 + x^4), x) + 2
4*a^2*c*integrate(1/48*x^2*log(a^2*x^2 + 1)^2/(a^2*x^6 + x^4), x) - 16*a^2
*c*integrate(1/48*x^2*log(a^2*x^2 + 1)/(a^2*x^6 + x^4), x) + 32*a*c*integr
ate(1/48*x*arctan(a*x)/(a^2*x^6 + x^4), x) + 144*c*integrate(1/48*arctan(a
*x)^2/(a^2*x^6 + x^4), x) + 12*c*integrate(1/48*log(a^2*x^2 + 1)^2/(a^2*x^
6 + x^4), x))*x^3 - 4*(3*a^2*c*x^2 + c)*arctan(a*x)^2 + (3*a^2*c*x^2 + c)*
log(a^2*x^2 + 1)^2)/x^3
```

## Giac [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)^2}{x^4}\,dx=\int \frac{(a^2cx^2+c)\arctan{(ax)}^2}{x^4}\,dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^2/x^4,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)*arctan(a*x)^2/x^4, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)\arctan(ax)^2}{x^4}\,dx=\int \frac{\mathrm{atan}(a\,x)^2\,(c\,a^2\,x^2+c)}{x^4}\,dx$$

input
```
int((atan(a*x)^2*(c + a^2*c*x^2))/x^4,x)
```

output
```
int((atan(a*x)^2*(c + a^2*c*x^2))/x^4, x)
```

## Reduce [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)^2}{x^4}\,dx$$
$$=\frac{c\left(-3atan(ax)^2a^2x^2-atan(ax)^2-3atan(ax)\,a^3x^3-3atan(ax)\,ax-4\left(\int\frac{atan(ax)}{a^2x^5+x^3}dx\right)a\,x^3-3a^2x^2\right)}{3x^3}$$

input
```
int((a^2*c*x^2+c)*atan(a*x)^2/x^4,x)
```

output
```
(c*( - 3*atan(a*x)**2*a**2*x**2 - atan(a*x)**2 - 3*atan(a*x)*a**3*x**3 - 3
*atan(a*x)*a*x - 4*int(atan(a*x)/(a**2*x**5 + x**3),x)*a*x**3 - 3*a**2*x**
2))/(3*x**3)
```

# 3.266 $\int x^3(c+a^2cx^2)^2 \arctan(ax)^2 \, dx$



## Optimal result

Integrand size = 22, antiderivative size = 191

$$\begin{aligned}\int x^3(c+a^2cx^2)^2 \arctan(ax)^2 \, dx = &-\frac{5c^2x^2}{504a^2}+\frac{c^2x^4}{84}+\frac{1}{168}a^2c^2x^6+\frac{c^2x\arctan(ax)}{12a^3}\\ &-\frac{c^2x^3\arctan(ax)}{36a}-\frac{1}{12}ac^2x^5\arctan(ax)\\ &-\frac{1}{28}a^3c^2x^7\arctan(ax)-\frac{c^2\arctan(ax)^2}{24a^4}\\ &+\frac{1}{4}c^2x^4\arctan(ax)^2+\frac{1}{3}a^2c^2x^6\arctan(ax)^2\\ &+\frac{1}{8}a^4c^2x^8\arctan(ax)^2-\frac{2c^2\log(1+a^2x^2)}{63a^4}\end{aligned}$$

output
```
-5/504*c^2*x^2/a^2+1/84*c^2*x^4+1/168*a^2*c^2*x^6+1/12*c^2*x*arctan(a*x)/a
^3-1/36*c^2*x^3*arctan(a*x)/a-1/12*a*c^2*x^5*arctan(a*x)-1/28*a^3*c^2*x^7*
arctan(a*x)-1/24*c^2*arctan(a*x)^2/a^4+1/4*c^2*x^4*arctan(a*x)^2+1/3*a^2*c
^2*x^6*arctan(a*x)^2+1/8*a^4*c^2*x^8*arctan(a*x)^2-2/63*c^2*ln(a^2*x^2+1)/
a^4
```

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.58

$$\int x^3 \left(c+a^2cx^2\right)^2 \arctan(ax)^2\,dx$$
$$= \frac{c^2\left(-5a^2x^2+6a^4x^4+3a^6x^6-2ax(-21+7a^2x^2+21a^4x^4+9a^6x^6)\arctan(ax)+21(1+a^2x^2)^3(-1+3\right.}{504a^4}$$

input
```
Integrate[x^3*(c + a^2*c*x^2)^2*ArcTan[a*x]^2,x]
```

output
```
(c^2*(-5*a^2*x^2 + 6*a^4*x^4 + 3*a^6*x^6 - 2*a*x*(-21 + 7*a^2*x^2 + 21*a^4
*x^4 + 9*a^6*x^6)*ArcTan[a*x] + 21*(1 + a^2*x^2)^3*(-1 + 3*a^2*x^2)*ArcTan
[a*x]^2 - 16*Log[1 + a^2*x^2]))/(504*a^4)
```

## Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \arctan(ax)^2 \left(a^2cx^2+c\right)^2\,dx$$
$$\Big\downarrow \; 5483$$
$$\int \left(a^4c^2x^7\arctan(ax)^2+2a^2c^2x^5\arctan(ax)^2+c^2x^3\arctan(ax)^2\right)dx$$
$$\Big\downarrow \; 2009$$
$$\frac{1}{8}a^4c^2x^8\arctan(ax)^2-\frac{c^2\arctan(ax)^2}{24a^4}-\frac{1}{28}a^3c^2x^7\arctan(ax)+\frac{c^2x\arctan(ax)}{12a^3}+\frac{1}{3}a^2c^2x^6\arctan(ax)^2+\frac{1}{168}a^2c^2x^6-\frac{5c^2x^2}{504a^2}-\frac{2c^2\log\left(a^2x^2+1\right)}{63a^4}-\frac{1}{12}ac^2x^5\arctan(ax)+\frac{1}{4}c^2x^4\arctan(ax)^2-\frac{c^2x^3\arctan(ax)}{36a}+\frac{c^2x^4}{84}$$

input
```
Int[x^3*(c + a^2*c*x^2)^2*ArcTan[a*x]^2,x]
```

output
```
(-5*c^2*x^2)/(504*a^2) + (c^2*x^4)/84 + (a^2*c^2*x^6)/168 + (c^2*x*ArcTan[
a*x])/(12*a^3) - (c^2*x^3*ArcTan[a*x])/(36*a) - (a*c^2*x^5*ArcTan[a*x])/12
 - (a^3*c^2*x^7*ArcTan[a*x])/28 - (c^2*ArcTan[a*x]^2)/(24*a^4) + (c^2*x^4*
ArcTan[a*x]^2)/4 + (a^2*c^2*x^6*ArcTan[a*x]^2)/3 + (a^4*c^2*x^8*ArcTan[a*x
]^2)/8 - (2*c^2*Log[1 + a^2*x^2])/(63*a^4)
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

## Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.79

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{c^2 \arctan(ax)^2 a^8 x^8}{8}+\frac{c^2 \arctan(ax)^2 a^6 x^6}{3}+\frac{a^4 c^2 x^4 \arctan(ax)^2}{4}-\frac{c^2\left(\frac{3\arctan(ax) a^7 x^7}{7}+\arctan(ax) a^5 x^5+\frac{\arctan(ax) x^3 a^3}{3}-\text{arc}\right.}{}}{a^4}$ |
| default | $\frac{\frac{c^2 \arctan(ax)^2 a^8 x^8}{8}+\frac{c^2 \arctan(ax)^2 a^6 x^6}{3}+\frac{a^4 c^2 x^4 \arctan(ax)^2}{4}-\frac{c^2\left(\frac{3\arctan(ax) a^7 x^7}{7}+\arctan(ax) a^5 x^5+\frac{\arctan(ax) x^3 a^3}{3}-\text{arc}\right.}{}}{a^4}$ |
| parts | $\frac{a^4 c^2 x^8 \arctan(ax)^2}{8}+\frac{a^2 c^2 x^6 \arctan(ax)^2}{3}+\frac{c^2 x^4 \arctan(ax)^2}{4}-\frac{c^2\left(\frac{3a^3\arctan(ax) x^7}{7}+a\arctan(ax) x^5+\frac{\arctan(}{3a}\right.}{}$ |
| parallelrisch | $-\frac{-63c^2\arctan(ax)^2 a^8 x^8+18a^7 c^2\arctan(ax) x^7-168c^2\arctan(ax)^2 a^6 x^6-3a^6 c^2 x^6+42a^5 c^2\arctan(ax) x^5-126a^4 c^2}{}$ |
| risch | $-\frac{c^2\left(3a^8x^8+8a^6x^6+6a^4x^4-1\right)\ln(iax+1)^2}{96a^4}+\frac{c^2\left(63a^8x^8\ln(-iax+1)+18ia^7x^7+168a^6x^6\ln(-iax+1)+42ia^5x^5+12\right.}{1008a}$ |

input
```
int(x^3*(a^2*c*x^2+c)^2*arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^4*(1/8*c^2*arctan(a*x)^2*a^8*x^8+1/3*c^2*arctan(a*x)^2*a^6*x^6+1/4*a^4
*c^2*x^4*arctan(a*x)^2-1/12*c^2*(3/7*arctan(a*x)*a^7*x^7+arctan(a*x)*a^5*x
^5+1/3*arctan(a*x)*x^3*a^3-arctan(a*x)*a*x+1/2*arctan(a*x)^2-1/14*a^6*x^6-
1/7*a^4*x^4+5/42*a^2*x^2+8/21*ln(a^2*x^2+1)))
```

## Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.77

$$\int x^3 \left(c+a^2 c x^2\right)^2 \arctan(ax)^2 \, dx = \frac{3\,a^6 c^2 x^6 + 6\,a^4 c^2 x^4 - 5\,a^2 c^2 x^2 + 21\,(3\,a^8 c^2 x^8 + 8\,a^6 c^2 x^6 + 6\,a^4 c^2 x^4 - c^2)\arctan\left(ax\right)^2 - 16\,c^2 \log\left(a^2 x^2 +\right.}{504\,a^4}$$

input
```
integrate(x^3*(a^2*c*x^2+c)^2*arctan(a*x)^2,x, algorithm="fricas")
```

output
```
1/504*(3*a^6*c^2*x^6 + 6*a^4*c^2*x^4 - 5*a^2*c^2*x^2 + 21*(3*a^8*c^2*x^8 +
 8*a^6*c^2*x^6 + 6*a^4*c^2*x^4 - c^2)*arctan(a*x)^2 - 16*c^2*log(a^2*x^2 +
 1) - 2*(9*a^7*c^2*x^7 + 21*a^5*c^2*x^5 + 7*a^3*c^2*x^3 - 21*a*c^2*x)*arct
an(a*x))/a^4
```

## Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.97

$$\int x^3 \left(c+a^2 c x^2\right)^2 \arctan(ax)^2 \, dx = \begin{cases} \frac{a^4 c^2 x^8 \operatorname{atan}^2(ax)}{8} - \frac{a^3 c^2 x^7 \operatorname{atan}(ax)}{28} + \frac{a^2 c^2 x^6 \operatorname{atan}^2(ax)}{3} + \frac{a^2 c^2 x^6}{168} - \frac{a c^2 x^5 \operatorname{atan}(ax)}{12} + \frac{c^2 x^4 \operatorname{atan}^2(ax)}{4} + \frac{c^2 x^4}{84} - \frac{c^2 x^3 \operatorname{atan}(}{36a} \\ 0 \end{cases}$$

input
```
integrate(x**3*(a**2*c*x**2+c)**2*atan(a*x)**2,x)
```

output
```
Piecewise((a**4*c**2*x**8*atan(a*x)**2/8 - a**3*c**2*x**7*atan(a*x)/28 + a
**2*c**2*x**6*atan(a*x)**2/3 + a**2*c**2*x**6/168 - a*c**2*x**5*atan(a*x)/
12 + c**2*x**4*atan(a*x)**2/4 + c**2*x**4/84 - c**2*x**3*atan(a*x)/(36*a)
- 5*c**2*x**2/(504*a**2) + c**2*x*atan(a*x)/(12*a**3) - 2*c**2*log(x**2 +
a**(-2))/(63*a**4) - c**2*atan(a*x)**2/(24*a**4), Ne(a, 0)), (0, True))
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.88

$$\int x^3(c+a^2cx^2)^2\arctan(ax)^2\,dx = -\frac{1}{252}a\left(\frac{21\,c^2\arctan(ax)}{a^5}+\frac{9\,a^6c^2x^7+21\,a^4c^2x^5+7\,a^2c^2x^3-21\,c^2x}{a^4}\right)\arctan(ax) + \frac{1}{24}\left(3\,a^4c^2x^8+8\,a^2c^2x^6+6\,c^2x^4\right)\arctan(ax)^2 + \frac{3\,a^6c^2x^6+6\,a^4c^2x^4-5\,a^2c^2x^2+21\,c^2\arctan(ax)^2-16\,c^2\log(a^2x^2+1)}{504\,a^4}$$

input
```
integrate(x^3*(a^2*c*x^2+c)^2*arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-1/252*a*(21*c^2*arctan(a*x)/a^5 + (9*a^6*c^2*x^7 + 21*a^4*c^2*x^5 + 7*a^2
*c^2*x^3 - 21*c^2*x)/a^4)*arctan(a*x) + 1/24*(3*a^4*c^2*x^8 + 8*a^2*c^2*x^
6 + 6*c^2*x^4)*arctan(a*x)^2 + 1/504*(3*a^6*c^2*x^6 + 6*a^4*c^2*x^4 - 5*a^
2*c^2*x^2 + 21*c^2*arctan(a*x)^2 - 16*c^2*log(a^2*x^2 + 1))/a^4
```

## Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.84

$$\int x^3(c+a^2cx^2)^2\arctan(ax)^2\,dx = \frac{1}{24}\left(3\,a^4c^2x^8+8\,a^2c^2x^6+6\,c^2x^4\right)\arctan(ax)^2 - \frac{18\,a^7c^2x^7\arctan(ax)-3\,a^6c^2x^6+42\,a^5c^2x^5\arctan(ax)-6\,a^4c^2x^4+14\,a^3c^2x^3\arctan(ax)+5\,a^2c^2x^2}{504\,a^4}$$

input
```
integrate(x^3*(a^2*c*x^2+c)^2*arctan(a*x)^2,x, algorithm="giac")
```

output
```
1/24*(3*a^4*c^2*x^8 + 8*a^2*c^2*x^6 + 6*c^2*x^4)*arctan(a*x)^2 - 1/504*(18
*a^7*c^2*x^7*arctan(a*x) - 3*a^6*c^2*x^6 + 42*a^5*c^2*x^5*arctan(a*x) - 6*
a^4*c^2*x^4 + 14*a^3*c^2*x^3*arctan(a*x) + 5*a^2*c^2*x^2 - 42*a*c^2*x*arct
an(a*x) + 21*c^2*arctan(a*x)^2 + 16*c^2*log(a^2*x^2 + 1))/a^4
```

## Mupad [B] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.76

$$\int x^3\left(c+a^2cx^2\right)^2\arctan(ax)^2\,dx = \mathrm{atan}(a\,x)^2\left(\frac{c^2\,x^4}{4}-\frac{c^2}{24\,a^4}+\frac{a^2\,c^2\,x^6}{3}+\frac{a^4\,c^2\,x^8}{8}\right)+\frac{c^2\,x^4}{84}-a^2\,\mathrm{atan}(a\,x)\left(\frac{a\,c^2\,x^7}{28}-\frac{c^2\,x}{12\,a^5}+\frac{c^2\,x^5}{12\,a}+\frac{c^2\,x^3}{36\,a^3}\right)-\frac{2\,c^2\,\ln\left(a^2\,x^2+1\right)}{63\,a^4}-\frac{5\,c^2\,x^2}{504\,a^2}+\frac{a^2\,c^2\,x^6}{168}$$

input
```
int(x^3*atan(a*x)^2*(c + a^2*c*x^2)^2,x)
```

output
```
atan(a*x)^2*((c^2*x^4)/4 - c^2/(24*a^4) + (a^2*c^2*x^6)/3 + (a^4*c^2*x^8)/
8) + (c^2*x^4)/84 - a^2*atan(a*x)*((a*c^2*x^7)/28 - (c^2*x)/(12*a^5) + (c^
2*x^5)/(12*a) + (c^2*x^3)/(36*a^3)) - (2*c^2*log(a^2*x^2 + 1))/(63*a^4) -
(5*c^2*x^2)/(504*a^2) + (a^2*c^2*x^6)/168
```

## Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.73

$$\int x^3\left(c+a^2cx^2\right)^2\arctan(ax)^2\,dx = \frac{c^2\left(63atan(ax)^2a^8x^8+168atan(ax)^2a^6x^6+126atan(ax)^2a^4x^4-21atan(ax)^2-18atan(ax)a^7x^7-42a\right.}{504a^4}$$

input
```
int(x^3*(a^2*c*x^2+c)^2*atan(a*x)^2,x)
```

output
```
(c**2*(63*atan(a*x)**2*a**8*x**8 + 168*atan(a*x)**2*a**6*x**6 + 126*atan(a
*x)**2*a**4*x**4 - 21*atan(a*x)**2 - 18*atan(a*x)*a**7*x**7 - 42*atan(a*x)
*a**5*x**5 - 14*atan(a*x)*a**3*x**3 + 42*atan(a*x)*a*x - 16*log(a**2*x**2
+ 1) + 3*a**6*x**6 + 6*a**4*x**4 - 5*a**2*x**2))/(504*a**4)
```

# 3.267 $\int x^2(c+a^2cx^2)^2 \arctan(ax)^2\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 225

$$\begin{aligned}\int x^2(c+a^2cx^2)^2 \arctan(ax)^2\,dx = &-\frac{c^2x}{210a^2}+\frac{17c^2x^3}{630}+\frac{1}{105}a^2c^2x^5+\frac{c^2\arctan(ax)}{210a^3}\\ &-\frac{8c^2x^2\arctan(ax)}{105a}-\frac{9}{70}ac^2x^4\arctan(ax)\\ &-\frac{1}{21}a^3c^2x^6\arctan(ax)-\frac{8ic^2\arctan(ax)^2}{105a^3}\\ &+\frac{1}{3}c^2x^3\arctan(ax)^2+\frac{2}{5}a^2c^2x^5\arctan(ax)^2\\ &+\frac{1}{7}a^4c^2x^7\arctan(ax)^2-\frac{16c^2\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{105a^3}\\ &-\frac{8ic^2\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{105a^3}\end{aligned}$$

output

```
-1/210*c^2*x/a^2+17/630*c^2*x^3+1/105*a^2*c^2*x^5+1/210*c^2*arctan(a*x)/a^
3-8/105*c^2*x^2*arctan(a*x)/a-9/70*a*c^2*x^4*arctan(a*x)-1/21*a^3*c^2*x^6*
arctan(a*x)-8/105*I*c^2*arctan(a*x)^2/a^3+1/3*c^2*x^3*arctan(a*x)^2+2/5*a^
2*c^2*x^5*arctan(a*x)^2+1/7*a^4*c^2*x^7*arctan(a*x)^2-16/105*c^2*arctan(a*
x)*ln(2/(1+I*a*x))/a^3-8/105*I*c^2*polylog(2,1-2/(1+I*a*x))/a^3
```

## Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.59

$$\int x^2(c+a^2cx^2)^2\arctan(ax)^2\,dx$$
$$=\frac{c^2(ax(-3+17a^2x^2+6a^4x^4)+6(8i+35a^3x^3+42a^5x^5+15a^7x^7)\arctan(ax)^2-3\arctan(ax)\left(-1+16\right.}{630a^3}$$

input
```
Integrate[x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^2,x]
```

output
```
(c^2*(a*x*(-3 + 17*a^2*x^2 + 6*a^4*x^4) + 6*(8*I + 35*a^3*x^3 + 42*a^5*x^5
 + 15*a^7*x^7)*ArcTan[a*x]^2 - 3*ArcTan[a*x]*(-1 + 16*a^2*x^2 + 27*a^4*x^4
 + 10*a^6*x^6 + 32*Log[1 + E^((2*I)*ArcTan[a*x])]) + (48*I)*PolyLog[2, -E^
((2*I)*ArcTan[a*x])]))/(630*a^3)
```

## Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\arctan(ax)^2\left(a^2cx^2+c\right)^2dx$$
$$\Big\downarrow \text{5483}$$
$$\int \left(a^4c^2x^6\arctan(ax)^2+2a^2c^2x^4\arctan(ax)^2+c^2x^2\arctan(ax)^2\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{7}a^4c^2x^7\arctan(ax)^2-\frac{1}{21}a^3c^2x^6\arctan(ax)-\frac{8ic^2\arctan(ax)^2}{105a^3}+\frac{c^2\arctan(ax)}{210a^3}-\frac{16c^2\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{105a^3}-\frac{8ic^2\,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{105a^3}+\frac{2}{5}a^2c^2x^5\arctan(ax)^2+\frac{1}{105}a^2c^2x^5-\frac{c^2x}{210a^2}-\frac{9}{70}ac^2x^4\arctan(ax)+\frac{1}{3}c^2x^3\arctan(ax)^2-\frac{8c^2x^2\arctan(ax)}{105a}+\frac{17c^2x^3}{630}$$

input
```
Int[x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^2,x]
```

output
```
-1/210*(c^2*x)/a^2 + (17*c^2*x^3)/630 + (a^2*c^2*x^5)/105 + (c^2*ArcTan[a*
x])/(210*a^3) - (8*c^2*x^2*ArcTan[a*x])/(105*a) - (9*a*c^2*x^4*ArcTan[a*x]
)/70 - (a^3*c^2*x^6*ArcTan[a*x])/21 - (((8*I)/105)*c^2*ArcTan[a*x]^2)/a^3
+ (c^2*x^3*ArcTan[a*x]^2)/3 + (2*a^2*c^2*x^5*ArcTan[a*x]^2)/5 + (a^4*c^2*x
^7*ArcTan[a*x]^2)/7 - (16*c^2*ArcTan[a*x]*Log[2/(1 + I*a*x)])/(105*a^3) -
(((8*I)/105)*c^2*PolyLog[2, 1 - 2/(1 + I*a*x)])/a^3
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

## Maple [A] (verified)

Time = 2.07 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.19

| method | result |
|---|---|
| parts | $\frac{a^4c^2x^7\arctan(ax)^2}{7}+\frac{2a^2c^2x^5\arctan(ax)^2}{5}+\frac{c^2x^3\arctan(ax)^2}{3}-\frac{2c^2\left(\frac{5a^3\arctan(ax)x^6}{2}+\frac{27a\arctan(ax)x^4}{4}+\frac{4\operatorname{arc}}{}\right.}{}$ |
| derivativedivides | $\frac{\frac{c^2\arctan(ax)^2a^7x^7}{7}+\frac{2c^2\arctan(ax)^2a^5x^5}{5}+\frac{a^3c^2x^3\arctan(ax)^2}{3}-\frac{2c^2\left(\frac{5a^6\arctan(ax)x^6}{2}+\frac{27x^4\arctan(ax)a^4}{4}+4x^2a^2\arctan(a\right.}{}}{}$ |
| default | $\frac{\frac{c^2\arctan(ax)^2a^7x^7}{7}+\frac{2c^2\arctan(ax)^2a^5x^5}{5}+\frac{a^3c^2x^3\arctan(ax)^2}{3}-\frac{2c^2\left(\frac{5a^6\arctan(ax)x^6}{2}+\frac{27x^4\arctan(ax)a^4}{4}+4x^2a^2\arctan(a\right.}{}}{}$ |
| risch | $-\frac{177151ic^2}{2315250a^3}-\frac{c^2\ln(iax+1)^2x^3}{12}-\frac{c^2\ln(-iax+1)^2x^3}{12}+\frac{17x^3c^2}{630}-\frac{c^2x}{210a^2}+\frac{a^2c^2x^5}{105}+\frac{c^2\arctan(ax)}{210a^3}+\frac{c^2}{}$ |

input
```
int(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/7*a^4*c^2*x^7*arctan(a*x)^2+2/5*a^2*c^2*x^5*arctan(a*x)^2+1/3*c^2*x^3*ar
ctan(a*x)^2-2/105*c^2*(5/2*a^3*arctan(a*x)*x^6+27/4*a*arctan(a*x)*x^4+4/a*
arctan(a*x)*x^2-4/a^3*arctan(a*x)*ln(a^2*x^2+1)-1/4/a^3*(2*a^5*x^5+17/3*a^
3*x^3-a*x+arctan(a*x)+8*I*(ln(a*x-I)*ln(a^2*x^2+1)-1/2*ln(a*x-I)^2-dilog(-
1/2*I*(a*x+I))-ln(a*x-I)*ln(-1/2*I*(a*x+I)))-8*I*(ln(a*x+I)*ln(a^2*x^2+1)-
1/2*ln(a*x+I)^2-dilog(1/2*I*(a*x-I))-ln(a*x+I)*ln(1/2*I*(a*x-I)))))
```

## Fricas [F]

$$\int x^2(c+a^2cx^2)^2\arctan(ax)^2\,dx=\int\left(a^2cx^2+c\right)^2x^2\arctan\left(ax\right)^2\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2)*arctan(a*x)^2, x)
```

## Sympy [F]

$$\int x^2(c+a^2cx^2)^2\arctan(ax)^2\,dx=c^2\left(\int x^2\operatorname{atan}^2\left(ax\right)dx+\int 2a^2x^4\operatorname{atan}^2\left(ax\right)dx+\int a^4x^6\operatorname{atan}^2\left(ax\right)dx\right)$$

input
```
integrate(x**2*(a**2*c*x**2+c)**2*atan(a*x)**2,x)
```

output
```
c**2*(Integral(x**2*atan(a*x)**2, x) + Integral(2*a**2*x**4*atan(a*x)**2,
x) + Integral(a**4*x**6*atan(a*x)**2, x))
```

## Maxima [F]

$$\int x^2(c+a^2cx^2)^2\arctan(ax)^2\,dx=\int (a^2cx^2+c)^2x^2\arctan(ax)^2\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^2,x, algorithm="maxima")
```

output
```
1/420*(15*a^4*c^2*x^7 + 42*a^2*c^2*x^5 + 35*c^2*x^3)*arctan(a*x)^2 - 1/168
0*(15*a^4*c^2*x^7 + 42*a^2*c^2*x^5 + 35*c^2*x^3)*log(a^2*x^2 + 1)^2 + inte
grate(1/1680*(1260*(a^6*c^2*x^8 + 3*a^4*c^2*x^6 + 3*a^2*c^2*x^4 + c^2*x^2)
*arctan(a*x)^2 + 105*(a^6*c^2*x^8 + 3*a^4*c^2*x^6 + 3*a^2*c^2*x^4 + c^2*x^
2)*log(a^2*x^2 + 1)^2 - 8*(15*a^5*c^2*x^7 + 42*a^3*c^2*x^5 + 35*a*c^2*x^3)
*arctan(a*x) + 4*(15*a^6*c^2*x^8 + 42*a^4*c^2*x^6 + 35*a^2*c^2*x^4)*log(a^
2*x^2 + 1))/(a^2*x^2 + 1), x)
```

## Giac [F]

$$\int x^2(c+a^2cx^2)^2\arctan(ax)^2\,dx=\int (a^2cx^2+c)^2x^2\arctan(ax)^2\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^2*x^2*arctan(a*x)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int x^2(c+a^2cx^2)^2\arctan(ax)^2\,dx=\int x^2\,\mathrm{atan}(a\,x)^2\,\left(c\,a^2\,x^2+c\right)^2\,dx$$

input
```
int(x^2*atan(a*x)^2*(c + a^2*c*x^2)^2,x)
```

output
```
int(x^2*atan(a*x)^2*(c + a^2*c*x^2)^2, x)
```

## Reduce [F]

$$\int x^2(c+a^2cx^2)^2\arctan(ax)^2\,dx$$
$$=\frac{c^2\big(90atan(ax)^2\,a^7x^7+252atan(ax)^2\,a^5x^5+210atan(ax)^2\,a^3x^3+48atan(ax)^2\,ax-30atan(ax)\,a^6x^6-}{630a^3}$$

input
```
int(x^2*(a^2*c*x^2+c)^2*atan(a*x)^2,x)
```

output
```
(c**2*(90*atan(a*x)**2*a**7*x**7 + 252*atan(a*x)**2*a**5*x**5 + 210*atan(a
*x)**2*a**3*x**3 + 48*atan(a*x)**2*a*x - 30*atan(a*x)*a**6*x**6 - 81*atan(
a*x)*a**4*x**4 - 48*atan(a*x)*a**2*x**2 + 3*atan(a*x) - 48*int(atan(a*x)**
2,x)*a + 6*a**5*x**5 + 17*a**3*x**3 - 3*a*x))/(630*a**3)
```

# 3.268 $\int x(c+a^2cx^2)^2 \arctan(ax)^2 \, dx$



## Optimal result

Integrand size = 20, antiderivative size = 153

$$\int x(c+a^2cx^2)^2 \arctan(ax)^2 \, dx = \frac{2c^2(1+a^2x^2)}{45a^2} + \frac{c^2(1+a^2x^2)^2}{60a^2} - \frac{8c^2x\arctan(ax)}{45a} - \frac{4c^2x(1+a^2x^2)\arctan(ax)}{45a} - \frac{c^2x(1+a^2x^2)^2\arctan(ax)}{15a} + \frac{c^2(1+a^2x^2)^3\arctan(ax)^2}{6a^2} + \frac{4c^2\log(1+a^2x^2)}{45a^2}$$

output

```
2/45*c^2*(a^2*x^2+1)/a^2+1/60*c^2*(a^2*x^2+1)^2/a^2-8/45*c^2*x*arctan(a*x)
/a-4/45*c^2*x*(a^2*x^2+1)*arctan(a*x)/a-1/15*c^2*x*(a^2*x^2+1)^2*arctan(a*
x)/a+1/6*c^2*(a^2*x^2+1)^3*arctan(a*x)^2/a^2+4/45*c^2*ln(a^2*x^2+1)/a^2
```

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.55

$$\int x(c+a^2cx^2)^2\arctan(ax)^2\,dx$$
$$=\frac{c^2\Big(14a^2x^2+3a^4x^4-4ax(15+10a^2x^2+3a^4x^4)\arctan(ax)+30(1+a^2x^2)^3\arctan(ax)^2+16\log\left(1+a\right.}{180a^2}$$

input
```
Integrate[x*(c + a^2*c*x^2)^2*ArcTan[a*x]^2,x]
```

output
```
(c^2*(14*a^2*x^2 + 3*a^4*x^4 - 4*a*x*(15 + 10*a^2*x^2 + 3*a^4*x^4)*ArcTan[
a*x] + 30*(1 + a^2*x^2)^3*ArcTan[a*x]^2 + 16*Log[1 + a^2*x^2]))/(180*a^2)
```

## Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5465, 27, 5413, 5413, 5345, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\arctan(ax)^2\left(a^2cx^2+c\right)^2\,dx$$

$$\Big\downarrow \ 5465$$

$$\frac{c^2\left(a^2x^2+1\right)^3\arctan(ax)^2}{6a^2}-\frac{\int c^2\left(a^2x^2+1\right)^2\arctan(ax)dx}{3a}$$

$$\Big\downarrow \ 27$$

$$\frac{c^2\left(a^2x^2+1\right)^3\arctan(ax)^2}{6a^2}-\frac{c^2\int\left(a^2x^2+1\right)^2\arctan(ax)dx}{3a}$$

$$\Big\downarrow \ 5413$$

$$\frac{c^2\left(a^2x^2+1\right)^3\arctan(ax)^2}{6a^2}-\frac{c^2\left(\frac{4}{5}\int\left(a^2x^2+1\right)\arctan(ax)dx+\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan(ax)-\frac{\left(a^2x^2+1\right)^2}{20a}\right)}{3a}$$

$$\Big\downarrow \; 5413$$

$$\frac{c^2\left(a^2x^2+1\right)^3\arctan(ax)^2}{6a^2}-\frac{c^2\left(\frac{4}{5}\left(\frac{2}{3}\int\arctan(ax)dx+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)-\frac{a^2x^2+1}{6a}\right)+\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan(ax)-\frac{\left(a^2x^2+1\right)^2}{20a}\right)}{3a}$$

$$\Big\downarrow \; 5345$$

$$\frac{c^2\left(a^2x^2+1\right)^3\arctan(ax)^2}{6a^2}-\frac{c^2\left(\frac{4}{5}\left(\frac{2}{3}\left(x\arctan(ax)-a\int\frac{x}{a^2x^2+1}dx\right)+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)-\frac{a^2x^2+1}{6a}\right)+\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan(ax)-\frac{(a}{}\right.}{3a}$$

$$\Big\downarrow \; 240$$

$$\frac{c^2\left(a^2x^2+1\right)^3\arctan(ax)^2}{6a^2}-\frac{c^2\left(\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan(ax)+\frac{4}{5}\left(\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)+\frac{2}{3}\left(x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}\right)-\frac{a^2x^2+1}{6a}\right)-\frac{(a^2x^2}{2}\right.}{3a}$$

input
```
Int[x*(c + a^2*c*x^2)^2*ArcTan[a*x]^2,x]
```

output
```
(c^2*(1 + a^2*x^2)^3*ArcTan[a*x]^2)/(6*a^2) - (c^2*(-1/20*(1 + a^2*x^2)^2/
a + (x*(1 + a^2*x^2)^2*ArcTan[a*x])/5 + (4*(-1/6*(1 + a^2*x^2)/a + (x*(1 +
 a^2*x^2)*ArcTan[a*x])/3 + (2*(x*ArcTan[a*x] - Log[1 + a^2*x^2]/(2*a)))/3)
)/5))/(3*a)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5413
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Simp[x*(d + e*x^2)
^q*((a + b*ArcTan[c*x])/(2*q + 1)), x] + Simp[2*d*(q/(2*q + 1))   Int[(d +
e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

## Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.86

| method | result |
|---|---|
| parts | $\frac{a^4c^2x^6\arctan(ax)^2}{6}+\frac{a^2c^2x^4\arctan(ax)^2}{2}+\frac{c^2x^2\arctan(ax)^2}{2}+\frac{c^2\arctan(ax)^2}{6a^2}-\frac{c^2\left(\frac{\arctan(ax)a^5x^5}{5}+\frac{2\arctan\ldots}{}\right.}{}$ |
| derivativedivides | $\frac{\frac{c^2\arctan(ax)^2a^6x^6}{6}+\frac{a^4c^2x^4\arctan(ax)^2}{2}+\frac{a^2c^2x^2\arctan(ax)^2}{2}+\frac{c^2\arctan(ax)^2}{6}-\frac{c^2\left(\frac{\arctan(ax)a^5x^5}{5}+\frac{2\arctan(ax)x^3a^3}{3}+\text{arc}\ldots\right.}{3\ldots}}{a^2}$ |
| default | $\frac{\frac{c^2\arctan(ax)^2a^6x^6}{6}+\frac{a^4c^2x^4\arctan(ax)^2}{2}+\frac{a^2c^2x^2\arctan(ax)^2}{2}+\frac{c^2\arctan(ax)^2}{6}-\frac{c^2\left(\frac{\arctan(ax)a^5x^5}{5}+\frac{2\arctan(ax)x^3a^3}{3}+\text{arc}\ldots\right.}{3\ldots}}{a^2}$ |
| parallelrisch | $\frac{30c^2\arctan(ax)^2a^6x^6-12a^5c^2\arctan(ax)x^5+90a^4c^2x^4\arctan(ax)^2+3a^4c^2x^4-40a^3c^2x^3\arctan(ax)+90a^2c^2x^2\,\text{arcta}\ldots}{180a^2}$ |
| risch | $-\frac{c^2\left(a^2x^2+1\right)^3\ln(iax+1)^2}{24a^2}+\frac{c^2\left(15a^6x^6\ln(-iax+1)+6ia^5x^5+45x^4\ln(-iax+1)a^4+20ia^3x^3+45a^2x^2\ln(-iax+1)\ldots\right.}{180a^2}$ |

input
```
int(x*(a^2*c*x^2+c)^2*arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*a^4*c^2*x^6*arctan(a*x)^2+1/2*a^2*c^2*x^4*arctan(a*x)^2+1/2*c^2*x^2*ar
ctan(a*x)^2+1/6*c^2*arctan(a*x)^2/a^2-1/3*c^2/a^2*(1/5*arctan(a*x)*a^5*x^5
+2/3*arctan(a*x)*x^3*a^3+arctan(a*x)*a*x-1/20*a^4*x^4-7/30*a^2*x^2-4/15*ln
(a^2*x^2+1))
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.80

$$\int x\left(c+a^2cx^2\right)^2\arctan(ax)^2\,dx$$
$$=\frac{3\,a^4c^2x^4+14\,a^2c^2x^2+30\left(a^6c^2x^6+3\,a^4c^2x^4+3\,a^2c^2x^2+c^2\right)\arctan\left(ax\right)^2+16\,c^2\log\left(a^2x^2+1\right)-4\left(3\,a\ldots\right.}{180\,a^2}$$

input
```
integrate(x*(a^2*c*x^2+c)^2*arctan(a*x)^2,x, algorithm="fricas")
```

output
```
1/180*(3*a^4*c^2*x^4 + 14*a^2*c^2*x^2 + 30*(a^6*c^2*x^6 + 3*a^4*c^2*x^4 +
3*a^2*c^2*x^2 + c^2)*arctan(a*x)^2 + 16*c^2*log(a^2*x^2 + 1) - 4*(3*a^5*c^
2*x^5 + 10*a^3*c^2*x^3 + 15*a*c^2*x)*arctan(a*x))/a^2
```

## Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.03

$$\int x(c+a^2cx^2)^2 \arctan(ax)^2\,dx$$
$$= \begin{cases} \frac{a^4c^2x^6\operatorname{atan}^2(ax)}{6} - \frac{a^3c^2x^5\operatorname{atan}(ax)}{15} + \frac{a^2c^2x^4\operatorname{atan}^2(ax)}{2} + \frac{a^2c^2x^4}{60} - \frac{2ac^2x^3\operatorname{atan}(ax)}{9} + \frac{c^2x^2\operatorname{atan}^2(ax)}{2} + \frac{7c^2x^2}{90} - \frac{c^2x\operatorname{atan}}{3a} \\ 0 \end{cases}$$

input
```
integrate(x*(a**2*c*x**2+c)**2*atan(a*x)**2,x)
```

output
```
Piecewise((a**4*c**2*x**6*atan(a*x)**2/6 - a**3*c**2*x**5*atan(a*x)/15 + a
**2*c**2*x**4*atan(a*x)**2/2 + a**2*c**2*x**4/60 - 2*a*c**2*x**3*atan(a*x)
/9 + c**2*x**2*atan(a*x)**2/2 + 7*c**2*x**2/90 - c**2*x*atan(a*x)/(3*a) +
4*c**2*log(x**2 + a**(-2))/(45*a**2) + c**2*atan(a*x)**2/(6*a**2), Ne(a, 0
)), (0, True))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.73

$$\int x(c+a^2cx^2)^2 \arctan(ax)^2\,dx = \frac{(a^2cx^2+c)^3\arctan(ax)^2}{6\,a^2c}$$
$$+ \frac{\left(3\,a^2c^3x^4 + 14\,c^3x^2 + \frac{16\,c^3\log(a^2x^2+1)}{a^2}\right)a - 4\,(3\,a^4c^3x^5 + 10\,a^2c^3x^3 + 15\,c^3x)\arctan(ax)}{180\,ac}$$

input
```
integrate(x*(a^2*c*x^2+c)^2*arctan(a*x)^2,x, algorithm="maxima")
```

output
```
1/6*(a^2*c*x^2 + c)^3*arctan(a*x)^2/(a^2*c) + 1/180*((3*a^2*c^3*x^4 + 14*c
^3*x^2 + 16*c^3*log(a^2*x^2 + 1)/a^2)*a - 4*(3*a^4*c^3*x^5 + 10*a^2*c^3*x^
3 + 15*c^3*x)*arctan(a*x))/(a*c)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.05

$$\int x\left(c+a^2cx^2\right)^2\arctan(ax)^2\,dx=\frac{\left(a^2cx^2+c\right)^3\arctan\left(ax\right)^2}{6\,a^2c}$$
$$-\frac{3\left(4\,x^5\arctan\left(ax\right)-a\left(\frac{a^2x^4-2\,x^2}{a^4}+\frac{2\,\log\left(a^2x^2+1\right)}{a^6}\right)\right)a^4c^2+20\left(2\,x^3\arctan\left(ax\right)-a\left(\frac{x^2}{a^2}-\frac{\log\left(a^2x^2+1\right)}{a^4}\right)\right)c}{180\,a}$$

input
```
integrate(x*(a^2*c*x^2+c)^2*arctan(a*x)^2,x, algorithm="giac")
```

output
```
1/6*(a^2*c*x^2 + c)^3*arctan(a*x)^2/(a^2*c) - 1/180*(3*(4*x^5*arctan(a*x)
- a*((a^2*x^4 - 2*x^2)/a^4 + 2*log(a^2*x^2 + 1)/a^6))*a^4*c^2 + 20*(2*x^3*
arctan(a*x) - a*(x^2/a^2 - log(a^2*x^2 + 1)/a^4))*a^2*c^2 + 30*(2*a*x*arct
an(a*x) - log(a^2*x^2 + 1))*c^2/a)/a
```

## Mupad [B] (verification not implemented)

Time = 0.85 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.88

$$\int x\left(c+a^2cx^2\right)^2\arctan(ax)^2\,dx=\frac{\frac{c^2\left(30\,\mathrm{atan}(a\,x)^2+16\,\ln\left(a^2\,x^2+1\right)\right)}{180}-\frac{a\,c^2\,x\,\mathrm{atan}(a\,x)}{3}}{a^2}$$
$$+\frac{c^2\left(90\,x^2\,\mathrm{atan}(a\,x)^2+14\,x^2\right)}{180}$$
$$+\frac{a^2\,c^2\left(90\,x^4\,\mathrm{atan}(a\,x)^2+3\,x^4\right)}{180}-\frac{a^3\,c^2\,x^5\,\mathrm{atan}(a\,x)}{15}$$
$$+\frac{a^4\,c^2\,x^6\,\mathrm{atan}(a\,x)^2}{6}-\frac{2\,a\,c^2\,x^3\,\mathrm{atan}(a\,x)}{9}$$

input
```
int(x*atan(a*x)^2*(c + a^2*c*x^2)^2,x)
```

output
```
((c^2*(16*log(a^2*x^2 + 1) + 30*atan(a*x)^2))/180 - (a*c^2*x*atan(a*x))/3)
/a^2 + (c^2*(90*x^2*atan(a*x)^2 + 14*x^2))/180 + (a^2*c^2*(90*x^4*atan(a*x
)^2 + 3*x^4))/180 - (a^3*c^2*x^5*atan(a*x))/15 + (a^4*c^2*x^6*atan(a*x)^2)
/6 - (2*a*c^2*x^3*atan(a*x))/9
```

## Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.78

$$\int x\left(c+a^2cx^2\right)^2\arctan(ax)^2\,dx$$

$$=\frac{c^2\left(30atan(ax)^2\,a^6x^6+90atan(ax)^2\,a^4x^4+90atan(ax)^2\,a^2x^2+30atan(ax)^2-12atan(ax)\,a^5x^5-40ata\right.}{180a^2}$$

input
```
int(x*(a^2*c*x^2+c)^2*atan(a*x)^2,x)
```

output
```
(c**2*(30*atan(a*x)**2*a**6*x**6 + 90*atan(a*x)**2*a**4*x**4 + 90*atan(a*x
)**2*a**2*x**2 + 30*atan(a*x)**2 - 12*atan(a*x)*a**5*x**5 - 40*atan(a*x)*a
**3*x**3 - 60*atan(a*x)*a*x + 16*log(a**2*x**2 + 1) + 3*a**4*x**4 + 14*a**
2*x**2))/(180*a**2)
```

# 3.269 $\int (c+a^2cx^2)^2 \arctan(ax)^2\,dx$



## Optimal result

Integrand size = 19, antiderivative size = 205

$$\int (c+a^2cx^2)^2 \arctan(ax)^2\,dx = \frac{11c^2x}{30}+\frac{1}{30}a^2c^2x^3-\frac{4c^2(1+a^2x^2)\arctan(ax)}{15a} -\frac{c^2(1+a^2x^2)^2\arctan(ax)}{10a}+\frac{8ic^2\arctan(ax)^2}{15a} +\frac{8}{15}c^2x\arctan(ax)^2+\frac{4}{15}c^2x(1+a^2x^2)\arctan(ax)^2 +\frac{1}{5}c^2x(1+a^2x^2)^2\arctan(ax)^2 +\frac{16c^2\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{15a} +\frac{8ic^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{15a}$$

output
```
11/30*c^2*x+1/30*a^2*c^2*x^3-4/15*c^2*(a^2*x^2+1)*arctan(a*x)/a-1/10*c^2*(
a^2*x^2+1)^2*arctan(a*x)/a+8/15*I*c^2*arctan(a*x)^2/a+8/15*c^2*x*arctan(a*
x)^2+4/15*c^2*x*(a^2*x^2+1)*arctan(a*x)^2+1/5*c^2*x*(a^2*x^2+1)^2*arctan(a
*x)^2+16/15*c^2*arctan(a*x)*ln(2/(1+I*a*x))/a+8/15*I*c^2*polylog(2,1-2/(1+
I*a*x))/a
```

## Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.55

$$\int \left(c+a^2cx^2\right)^2\arctan(ax)^2\,dx$$
$$=\frac{c^2\left(ax(11+a^2x^2)+2(-8i+15ax+10a^3x^3+3a^5x^5)\arctan(ax)^2-\arctan(ax)\left(11+14a^2x^2+3a^4x^4-\right.\right.}{30a}$$

input
```
Integrate[(c + a^2*c*x^2)^2*ArcTan[a*x]^2,x]
```

output
```
(c^2*(a*x*(11 + a^2*x^2) + 2*(-8*I + 15*a*x + 10*a^3*x^3 + 3*a^5*x^5)*ArcT
an[a*x]^2 - ArcTan[a*x]*(11 + 14*a^2*x^2 + 3*a^4*x^4 - 32*Log[1 + E^((2*I)
*ArcTan[a*x])]) - (16*I)*PolyLog[2, -E^((2*I)*ArcTan[a*x])]))/(30*a)
```

## Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.526, Rules used = {5415, 27, 2009, 5415, 24, 5345, 5455, 5379, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(ax)^2\left(a^2cx^2+c\right)^2\,dx$$

$$\Big\downarrow \text{ 5415}$$

$$\frac{4}{5}c\int c\left(a^2x^2+1\right)\arctan(ax)^2dx+\frac{1}{10}c\int\left(a^2cx^2+c\right)dx+\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\arctan(ax)^2-\frac{c^2\left(a^2x^2+1\right)^2\arctan(ax)}{10a}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{4}{5}c^2\int\left(a^2x^2+1\right)\arctan(ax)^2dx+\frac{1}{10}c\int\left(a^2cx^2+c\right)dx+\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\arctan(ax)^2-\frac{c^2\left(a^2x^2+1\right)^2\arctan(ax)}{10a}$$

$$\Big\downarrow \text{2009}$$

$$\frac{4}{5}c^2\int \left(a^2x^2+1\right)\arctan(ax)^2dx+\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\arctan(ax)^2-\frac{c^2\left(a^2x^2+1\right)^2\arctan(ax)}{10a}+\frac{1}{10}c\left(\frac{1}{3}a^2cx^3+cx\right)$$

$$\Big\downarrow \text{5415}$$

$$\frac{4}{5}c^2\left(\frac{2}{3}\int \arctan(ax)^2dx+\frac{\int 1dx}{3}+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)^2-\frac{\left(a^2x^2+1\right)\arctan(ax)}{3a}\right)+\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\arctan(ax)^2-\frac{c^2\left(a^2x^2+1\right)^2\arctan(ax)}{10a}+\frac{1}{10}c\left(\frac{1}{3}a^2cx^3+cx\right)$$

$$\Big\downarrow \text{24}$$

$$\frac{4}{5}c^2\left(\frac{2}{3}\int \arctan(ax)^2dx+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)^2-\frac{\left(a^2x^2+1\right)\arctan(ax)}{3a}+\frac{x}{3}\right)+\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\arctan(ax)^2-\frac{c^2\left(a^2x^2+1\right)^2\arctan(ax)}{10a}+\frac{1}{10}c\left(\frac{1}{3}a^2cx^3+cx\right)$$

$$\Big\downarrow \text{5345}$$

$$\frac{4}{5}c^2\left(\frac{2}{3}\left(x\arctan(ax)^2-2a\int \frac{x\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)^2-\frac{\left(a^2x^2+1\right)\arctan(ax)}{3a}+\frac{x}{3}\right)+\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\arctan(ax)^2-\frac{c^2\left(a^2x^2+1\right)^2\arctan(ax)}{10a}+\frac{1}{10}c\left(\frac{1}{3}a^2cx^3+cx\right)$$

$$\Big\downarrow \text{5455}$$

$$\frac{4}{5}c^2\left(\frac{2}{3}\left(x\arctan(ax)^2-2a\left(-\frac{\int \frac{\arctan(ax)}{i-ax}dx}{a}-\frac{i\arctan(ax)^2}{2a^2}\right)\right)+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)^2-\frac{\left(a^2x^2+1\right)\text{ar}}{3a}\right.$$
$$\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\arctan(ax)^2-\frac{c^2\left(a^2x^2+1\right)^2\arctan(ax)}{10a}+\frac{1}{10}c\left(\frac{1}{3}a^2cx^3+cx\right)$$

$$\Big\downarrow \text{5379}$$

$$\frac{4}{5}c^2\left(\frac{2}{3}\left(x\arctan(ax)^2-2a\left(-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}-\int \frac{\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^2}{2a^2}\right)\right)+\frac{1}{3}x\left(a^2x^2+1\right)\text{arcta}\right.$$
$$\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\arctan(ax)^2-\frac{c^2\left(a^2x^2+1\right)^2\arctan(ax)}{10a}+\frac{1}{10}c\left(\frac{1}{3}a^2cx^3+cx\right)$$

$$\Big\downarrow \text{2849}$$

$$\frac{4}{5}c^2\left(\frac{2}{3}\left(x\arctan(ax)^2-2a\left(-\frac{\frac{i\int\frac{\log\left(\frac{2}{iax+1}\right)}{1-\frac{2}{iax+1}}d\frac{1}{iax+1}}{a}+\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}}{a}-\frac{i\arctan(ax)^2}{2a^2}\right)\right)+\frac{1}{3}x\left(a^2x^2+1\right)\text{ar}\right.$$
$$\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\arctan(ax)^2-\frac{c^2\left(a^2x^2+1\right)^2\arctan(ax)}{10a}+\frac{1}{10}c\left(\frac{1}{3}a^2cx^3+cx\right)$$

$$\Big\downarrow \text{2752}$$

$$\frac{4}{5}c^2\left(\frac{2}{3}\left(x\arctan(ax)^2-2a\left(-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}\right)\right)+\frac{1}{3}x\left(a^2x^2+1\right)\text{a}\right.$$
$$\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\arctan(ax)^2-\frac{c^2\left(a^2x^2+1\right)^2\arctan(ax)}{10a}+\frac{1}{10}c\left(\frac{1}{3}a^2cx^3+cx\right)$$

input
```
Int[(c + a^2*c*x^2)^2*ArcTan[a*x]^2,x]
```

output
```
(c*(c*x + (a^2*c*x^3)/3))/10 - (c^2*(1 + a^2*x^2)^2*ArcTan[a*x])/(10*a) +
(c^2*x*(1 + a^2*x^2)^2*ArcTan[a*x]^2)/5 + (4*c^2*(x/3 - ((1 + a^2*x^2)*Arc
Tan[a*x])/(3*a) + (x*(1 + a^2*x^2)*ArcTan[a*x]^2)/3 + (2*(x*ArcTan[a*x]^2
- 2*a*(((-1/2*I)*ArcTan[a*x]^2)/a^2 - ((ArcTan[a*x]*Log[2/(1 + I*a*x)])/a
+ ((I/2)*PolyLog[2, 1 - 2/(1 + I*a*x)])/a)/a)))/3))/5
```

## Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5415
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTan[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x])^p,
x], x] + Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*(
a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[q, 0] && GtQ[p, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

## Maple [A] (verified)

Time = 1.77 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.19

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{c^2 \arctan(ax)^2 a^5 x^5}{5}+\frac{2a^3c^2x^3\arctan(ax)^2}{3}+a\,c^2x\arctan(ax)^2-\frac{2c^2\left(\frac{3x^4\arctan(ax)a^4}{4}+\frac{7x^2a^2\arctan(ax)}{2}+4\arctan(ax)\ln\left(a\right.\right.}{}}{}$ |
| default | $\frac{\frac{c^2 \arctan(ax)^2 a^5 x^5}{5}+\frac{2a^3c^2x^3\arctan(ax)^2}{3}+a\,c^2x\arctan(ax)^2-\frac{2c^2\left(\frac{3x^4\arctan(ax)a^4}{4}+\frac{7x^2a^2\arctan(ax)}{2}+4\arctan(ax)\ln\left(a\right.\right.}{}}{}$ |
| parts | $\frac{a^4c^2x^5\arctan(ax)^2}{5}+\frac{2a^2c^2x^3\arctan(ax)^2}{3}+c^2x\arctan(ax)^2-\frac{2c^2\left(\frac{3a^3\arctan(ax)x^4}{4}+\frac{7x^2\arctan(ax)a}{2}+\right.}{}$ |
| risch | $\frac{7ic^2a\ln(iax+1)x^2}{30}+\frac{3739ic^2}{6750a}+\frac{c^2a^4\ln(iax+1)\ln(-iax+1)x^5}{10}+\frac{c^2a^2\ln(iax+1)\ln(-iax+1)x^3}{3}-\frac{2ic^2\ln(-iax}{15a}$ |

input
```
int((a^2*c*x^2+c)^2*arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a*(1/5*c^2*arctan(a*x)^2*a^5*x^5+2/3*a^3*c^2*x^3*arctan(a*x)^2+a*c^2*x*a
rctan(a*x)^2-2/15*c^2*(3/4*x^4*arctan(a*x)*a^4+7/2*x^2*a^2*arctan(a*x)+4*a
rctan(a*x)*ln(a^2*x^2+1)-1/4*a^3*x^3-11/4*a*x+11/4*arctan(a*x)+2*I*(ln(a*x
-I)*ln(a^2*x^2+1)-1/2*ln(a*x-I)^2-dilog(-1/2*I*(a*x+I))-ln(a*x-I)*ln(-1/2*
I*(a*x+I)))-2*I*(ln(a*x+I)*ln(a^2*x^2+1)-1/2*ln(a*x+I)^2-dilog(1/2*I*(a*x-
I))-ln(a*x+I)*ln(1/2*I*(a*x-I)))))
```

## Fricas [F]

$$\int \left(c+a^2cx^2\right)^2\arctan(ax)^2\,dx=\int \left(a^2cx^2+c\right)^2\arctan\left(ax\right)^2\,dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)^2, x)
```

## Sympy [F]

$$\int \left(c+a^2cx^2\right)^2 \arctan(ax)^2\,dx = c^2\left(\int 2a^2x^2\,\mathrm{atan}^2\,(ax)\,dx + \int a^4x^4\,\mathrm{atan}^2\,(ax)\,dx + \int \mathrm{atan}^2\,(ax)\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**2*atan(a*x)**2,x)
```

output
```
c**2*(Integral(2*a**2*x**2*atan(a*x)**2, x) + Integral(a**4*x**4*atan(a*x)
**2, x) + Integral(atan(a*x)**2, x))
```

## Maxima [F]

$$\int \left(c+a^2cx^2\right)^2 \arctan(ax)^2\,dx = \int \left(a^2cx^2+c\right)^2 \arctan\left(ax\right)^2\,dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^2,x, algorithm="maxima")
```

output
```
180*a^6*c^2*integrate(1/240*x^6*arctan(a*x)^2/(a^2*x^2 + 1), x) + 15*a^6*c
^2*integrate(1/240*x^6*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) + 12*a^6*c^2*i
ntegrate(1/240*x^6*log(a^2*x^2 + 1)/(a^2*x^2 + 1), x) - 24*a^5*c^2*integra
te(1/240*x^5*arctan(a*x)/(a^2*x^2 + 1), x) + 540*a^4*c^2*integrate(1/240*x
^4*arctan(a*x)^2/(a^2*x^2 + 1), x) + 45*a^4*c^2*integrate(1/240*x^4*log(a^
2*x^2 + 1)^2/(a^2*x^2 + 1), x) + 40*a^4*c^2*integrate(1/240*x^4*log(a^2*x^
2 + 1)/(a^2*x^2 + 1), x) - 80*a^3*c^2*integrate(1/240*x^3*arctan(a*x)/(a^2
*x^2 + 1), x) + 540*a^2*c^2*integrate(1/240*x^2*arctan(a*x)^2/(a^2*x^2 + 1
), x) + 45*a^2*c^2*integrate(1/240*x^2*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x
) + 60*a^2*c^2*integrate(1/240*x^2*log(a^2*x^2 + 1)/(a^2*x^2 + 1), x) + 1/
4*c^2*arctan(a*x)^3/a - 120*a*c^2*integrate(1/240*x*arctan(a*x)/(a^2*x^2 +
 1), x) + 1/60*(3*a^4*c^2*x^5 + 10*a^2*c^2*x^3 + 15*c^2*x)*arctan(a*x)^2 +
 15*c^2*integrate(1/240*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) - 1/240*(3*a^
4*c^2*x^5 + 10*a^2*c^2*x^3 + 15*c^2*x)*log(a^2*x^2 + 1)^2
```

## Giac [F]

$$\int \left(c+a^2cx^2\right)^2 \arctan(ax)^2\,dx = \int \left(a^2cx^2+c\right)^2 \arctan\left(ax\right)^2\,dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^2*arctan(a*x)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \left(c+a^2cx^2\right)^2 \arctan(ax)^2\,dx = \int \mathrm{atan}(a\,x)^2\,\left(c\,a^2\,x^2+c\right)^2\,dx$$

input
```
int(atan(a*x)^2*(c + a^2*c*x^2)^2,x)
```

output
```
int(atan(a*x)^2*(c + a^2*c*x^2)^2, x)
```

## Reduce [F]

$$\int \left(c+a^2cx^2\right)^2 \arctan(ax)^2\,dx$$
$$= \frac{c^2\Big(6atan(ax)^2\,a^5x^5+20atan(ax)^2\,a^3x^3+30atan(ax)^2\,ax-3atan(ax)\,a^4x^4-14atan(ax)\,a^2x^2-11ata}{30a}$$

input
```
int((a^2*c*x^2+c)^2*atan(a*x)^2,x)
```

output
```
(c**2*(6*atan(a*x)**2*a**5*x**5 + 20*atan(a*x)**2*a**3*x**3 + 30*atan(a*x)
**2*a*x - 3*atan(a*x)*a**4*x**4 - 14*atan(a*x)*a**2*x**2 - 11*atan(a*x) -
32*int((atan(a*x)*x)/(a**2*x**2 + 1),x)*a**2 + a**3*x**3 + 11*a*x))/(30*a)
```

# 3.270 $\int \frac{(c+a^2cx^2)^2 \arctan(ax)^2}{x} \, dx$



## Optimal result

Integrand size = 22, antiderivative size = 235

$$\begin{aligned}\int \frac{(c+a^2cx^2)^2 \arctan(ax)^2}{x} \, dx = {} & \frac{1}{12}a^2c^2x^2 - \frac{3}{2}ac^2x \arctan(ax) \\ & - \frac{1}{6}a^3c^2x^3 \arctan(ax) + \frac{3}{4}c^2 \arctan(ax)^2 \\ & + a^2c^2x^2 \arctan(ax)^2 + \frac{1}{4}a^4c^2x^4 \arctan(ax)^2 \\ & + 2c^2 \arctan(ax)^2 \operatorname{arctanh}\left(1 - \frac{2}{1+iax}\right) \\ & + \frac{2}{3}c^2 \log\left(1+a^2x^2\right) \\ & - ic^2 \arctan(ax) \operatorname{PolyLog}\left(2, 1 - \frac{2}{1+iax}\right) \\ & + ic^2 \arctan(ax) \operatorname{PolyLog}\left(2, -1 + \frac{2}{1+iax}\right) \\ & - \frac{1}{2}c^2 \operatorname{PolyLog}\left(3, 1 - \frac{2}{1+iax}\right) \\ & + \frac{1}{2}c^2 \operatorname{PolyLog}\left(3, -1 + \frac{2}{1+iax}\right)\end{aligned}$$

output
```
1/12*a^2*c^2*x^2-3/2*a*c^2*x*arctan(a*x)-1/6*a^3*c^2*x^3*arctan(a*x)+3/4*c
^2*arctan(a*x)^2+a^2*c^2*x^2*arctan(a*x)^2+1/4*a^4*c^2*x^4*arctan(a*x)^2-2
*c^2*arctan(a*x)^2*arctanh(-1+2/(1+I*a*x))+2/3*c^2*ln(a^2*x^2+1)-I*c^2*arc
tan(a*x)*polylog(2,1-2/(1+I*a*x))+I*c^2*arctan(a*x)*polylog(2,-1+2/(1+I*a*
x))-1/2*c^2*polylog(3,1-2/(1+I*a*x))+1/2*c^2*polylog(3,-1+2/(1+I*a*x))
```

## Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.93

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^2}{x}\,dx = \frac{1}{24}c^2\big(2 - i\pi^3 + 2a^2x^2 - 36ax\arctan(ax) - 4a^3x^3\arctan(ax) + 18\arctan(ax)^2 + 24a^2x^2\arctan(ax)^2 + 6a^4x^4\arctan(ax)^2 + 16i\arctan(ax)^3 + 24\arctan(ax)^2\log\left(1-e^{-2i\arctan(ax)}\right) - 24\arctan(ax)^2\log\left(1+e^{2i\arctan(ax)}\right) + 16\log\left(1+a^2x^2\right) + 24i\arctan(ax)\,\mathrm{PolyLog}\left(2,e^{-2i\arctan(ax)}\right) + 24i\arctan(ax)\,\mathrm{PolyLog}\left(2,-e^{2i\arctan(ax)}\right) + 12\,\mathrm{PolyLog}\left(3,e^{-2i\arctan(ax)}\right) - 12\,\mathrm{PolyLog}\left(3,-e^{2i\arctan(ax)}\right)\big)$$

input
```
Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x]^2)/x,x]
```

output
```
(c^2*(2 - I*Pi^3 + 2*a^2*x^2 - 36*a*x*ArcTan[a*x] - 4*a^3*x^3*ArcTan[a*x]
+ 18*ArcTan[a*x]^2 + 24*a^2*x^2*ArcTan[a*x]^2 + 6*a^4*x^4*ArcTan[a*x]^2 +
(16*I)*ArcTan[a*x]^3 + 24*ArcTan[a*x]^2*Log[1 - E^((-2*I)*ArcTan[a*x])] -
24*ArcTan[a*x]^2*Log[1 + E^((2*I)*ArcTan[a*x])] + 16*Log[1 + a^2*x^2] + (2
4*I)*ArcTan[a*x]*PolyLog[2, E^((-2*I)*ArcTan[a*x])] + (24*I)*ArcTan[a*x]*P
olyLog[2, -E^((2*I)*ArcTan[a*x])] + 12*PolyLog[3, E^((-2*I)*ArcTan[a*x])]
- 12*PolyLog[3, -E^((2*I)*ArcTan[a*x])]))/24
```

## Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2 \left(a^2cx^2+c\right)^2}{x} \, dx$$

$$\Big\downarrow \text{5483}$$

$$\int \left(a^4c^2x^3\arctan(ax)^2+2a^2c^2x\arctan(ax)^2+\frac{c^2\arctan(ax)^2}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}a^4c^2x^4\arctan(ax)^2-\frac{1}{6}a^3c^2x^3\arctan(ax)+a^2c^2x^2\arctan(ax)^2+\frac{1}{12}a^2c^2x^2+\frac{2}{3}c^2\log\left(a^2x^2+1\right)+2c^2\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{1+iax}\right)-ic^2\arctan(ax)\,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)+ic^2\arctan(ax)\,\text{PolyLog}\left(2,\frac{2}{iax+1}-1\right)-\frac{3}{2}ac^2x\arctan(ax)+\frac{3}{4}c^2\arctan(ax)^2-\frac{1}{2}c^2\,\text{PolyLog}\left(3,1-\frac{2}{iax+1}\right)+\frac{1}{2}c^2\,\text{PolyLog}\left(3,\frac{2}{iax+1}-1\right)$$

input
```
Int[((c + a^2*c*x^2)^2*ArcTan[a*x]^2)/x,x]
```

output
```
(a^2*c^2*x^2)/12 - (3*a*c^2*x*ArcTan[a*x])/2 - (a^3*c^2*x^3*ArcTan[a*x])/6
 + (3*c^2*ArcTan[a*x]^2)/4 + a^2*c^2*x^2*ArcTan[a*x]^2 + (a^4*c^2*x^4*ArcT
an[a*x]^2)/4 + 2*c^2*ArcTan[a*x]^2*ArcTanh[1 - 2/(1 + I*a*x)] + (2*c^2*Log
[1 + a^2*x^2])/3 - I*c^2*ArcTan[a*x]*PolyLog[2, 1 - 2/(1 + I*a*x)] + I*c^2
*ArcTan[a*x]*PolyLog[2, -1 + 2/(1 + I*a*x)] - (c^2*PolyLog[3, 1 - 2/(1 + I
*a*x)])/2 + (c^2*PolyLog[3, -1 + 2/(1 + I*a*x)])/2
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 25.81 (sec) , antiderivative size = 1185, normalized size of antiderivative = 5.04

| method | result | size |
|---|---|---|
| derivativedivides | Expression too large to display | 1185 |
| default | Expression too large to display | 1185 |
| parts | Expression too large to display | 1696 |

input
```
int((a^2*c*x^2+c)^2*arctan(a*x)^2/x,x,method=_RETURNVERBOSE)
```

output
```
1/4*a^4*c^2*x^4*arctan(a*x)^2+a^2*c^2*x^2*arctan(a*x)^2+c^2*arctan(a*x)^2*
ln(a*x)-1/2*c^2*(-I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I/((1+I*a*
x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*
x^2+1)+1))*arctan(a*x)^2-3/2*arctan(a*x)^2+arctan(a*x)*(a*x-I)*(a*x+I)^2-a
rctan(a*x)*(a*x-I)^2*(a*x+I)+I*arctan(a*x)*(a*x-I)^2+1/3*arctan(a*x)*(a*x-
I)^3-4*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))-4*polylog(3,-(1+I*a*x)/(a^2*
x^2+1)^(1/2))+I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^
2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*a
rctan(a*x)^2+2*arctan(a*x)*(a*x-I)+1/3*I*(a*x+I)-2*I*arctan(a*x)*(a*x-I)*(
a*x+I)+4*I*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*I*arctan(a
*x)*polylog(2,-(1+I*a*x)^2/(a^2*x^2+1))+polylog(3,-(1+I*a*x)^2/(a^2*x^2+1)
)+8/3*ln((1+I*a*x)^2/(a^2*x^2+1)+1)-1/6*(a*x+I)^2-I*Pi*arctan(a*x)^2+4*I*a
rctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+I*Pi*csgn(I/((1+I*a*x)^
2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2
+1)+1))^2*arctan(a*x)^2+I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I*((
1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2-I*P
i*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(((1
+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*arctan(a*x)^2+2*arct
an(a*x)^2*ln((1+I*a*x)^2/(a^2*x^2+1)-1)-2*arctan(a*x)^2*ln(1-(1+I*a*x)/(a^
2*x^2+1)^(1/2))-2*arctan(a*x)^2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*Pi*...
```

## Fricas [F]

$$\int \frac{(c+a^2cx^2)^2\arctan(ax)^2}{x}\,dx = \int \frac{(a^2cx^2+c)^2\arctan{(ax)}^2}{x}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^2/x,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)^2/x, x)
```

## Sympy [F]

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^2}{x}\,dx = c^2\left(\int \frac{\operatorname{atan}^2(ax)}{x}\,dx + \int 2a^2x \operatorname{atan}^2(ax)\,dx + \int a^4x^3 \operatorname{atan}^2(ax)\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**2*atan(a*x)**2/x,x)
```

output
```
c**2*(Integral(atan(a*x)**2/x, x) + Integral(2*a**2*x*atan(a*x)**2, x) + I
ntegral(a**4*x**3*atan(a*x)**2, x))
```

## Maxima [F]

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^2}{x}\,dx = \int \frac{(a^2cx^2+c)^2 \arctan(ax)^2}{x}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^2/x,x, algorithm="maxima")
```

output
```
12*a^6*c^2*integrate(1/16*x^6*arctan(a*x)^2/(a^2*x^3 + x), x) + a^6*c^2*in
tegrate(1/16*x^6*log(a^2*x^2 + 1)^2/(a^2*x^3 + x), x) + a^6*c^2*integrate(
1/16*x^6*log(a^2*x^2 + 1)/(a^2*x^3 + x), x) - 2*a^5*c^2*integrate(1/16*x^5
*arctan(a*x)/(a^2*x^3 + x), x) + 36*a^4*c^2*integrate(1/16*x^4*arctan(a*x)
^2/(a^2*x^3 + x), x) + 3*a^4*c^2*integrate(1/16*x^4*log(a^2*x^2 + 1)^2/(a^
2*x^3 + x), x) + 4*a^4*c^2*integrate(1/16*x^4*log(a^2*x^2 + 1)/(a^2*x^3 +
x), x) - 8*a^3*c^2*integrate(1/16*x^3*arctan(a*x)/(a^2*x^3 + x), x) + 36*a
^2*c^2*integrate(1/16*x^2*arctan(a*x)^2/(a^2*x^3 + x), x) + 1/32*c^2*log(a
^2*x^2 + 1)^3 + 1/16*(a^4*c^2*x^4 + 4*a^2*c^2*x^2)*arctan(a*x)^2 + 12*c^2*
integrate(1/16*arctan(a*x)^2/(a^2*x^3 + x), x) + c^2*integrate(1/16*log(a^
2*x^2 + 1)^2/(a^2*x^3 + x), x) - 1/64*(a^4*c^2*x^4 + 4*a^2*c^2*x^2)*log(a^
2*x^2 + 1)^2
```

## Giac [F]

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^2}{x} \, dx = \int \frac{(a^2cx^2+c)^2 \arctan(ax)^2}{x} \, dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^2/x,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^2*arctan(a*x)^2/x, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^2}{x} \, dx = \int \frac{\operatorname{atan}(a\,x)^2\,(c\,a^2\,x^2+c)^2}{x} \, dx$$

input
```
int((atan(a*x)^2*(c + a^2*c*x^2)^2)/x,x)
```

output
```
int((atan(a*x)^2*(c + a^2*c*x^2)^2)/x, x)
```

## Reduce [F]

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^2}{x} \, dx$$
$$= \frac{c^2\left(3atan(ax)^2 a^4x^4 + 12atan(ax)^2 a^2x^2 + 9atan(ax)^2 - 2atan(ax)\, a^3x^3 - 18atan(ax)\, ax + 12\left(\int \frac{atan(a}{x}\right.\right.}{12}$$

input
```
int((a^2*c*x^2+c)^2*atan(a*x)^2/x,x)
```

output
```
(c**2*(3*atan(a*x)**2*a**4*x**4 + 12*atan(a*x)**2*a**2*x**2 + 9*atan(a*x)*
*2 - 2*atan(a*x)*a**3*x**3 - 18*atan(a*x)*a*x + 12*int(atan(a*x)**2/x,x) +
 8*log(a**2*x**2 + 1) + a**2*x**2))/12
```

# 3.271 $\int \frac{(c+a^2cx^2)^2 \arctan(ax)^2}{x^2} \, dx$



## Optimal result

Integrand size = 22, antiderivative size = 205

$$\begin{aligned}\int \frac{(c+a^2cx^2)^2 \arctan(ax)^2}{x^2} \, dx &= \frac{1}{3}a^2c^2x - \frac{1}{3}ac^2\arctan(ax) - \frac{1}{3}a^3c^2x^2\arctan(ax) \\ &\quad + \frac{2}{3}iac^2\arctan(ax)^2 - \frac{c^2\arctan(ax)^2}{x} \\ &\quad + 2a^2c^2x\arctan(ax)^2 + \frac{1}{3}a^4c^2x^3\arctan(ax)^2 \\ &\quad + \frac{10}{3}ac^2\arctan(ax)\log\left(\frac{2}{1+iax}\right) \\ &\quad + 2ac^2\arctan(ax)\log\left(2-\frac{2}{1-iax}\right) \\ &\quad - iac^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right) \\ &\quad + \frac{5}{3}iac^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right)\end{aligned}$$

output
```
1/3*a^2*c^2*x-1/3*a*c^2*arctan(a*x)-1/3*a^3*c^2*x^2*arctan(a*x)+2/3*I*a*c^
2*arctan(a*x)^2-c^2*arctan(a*x)^2/x+2*a^2*c^2*x*arctan(a*x)^2+1/3*a^4*c^2*
x^3*arctan(a*x)^2+10/3*a*c^2*arctan(a*x)*ln(2/(1+I*a*x))+2*a*c^2*arctan(a*
x)*ln(2-2/(1-I*a*x))-I*a*c^2*polylog(2,-1+2/(1-I*a*x))+5/3*I*a*c^2*polylog
(2,1-2/(1+I*a*x))
```

## Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.81

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^2}{x^2} dx$$
$$= \frac{c^2 (a^2x^2 - ax\arctan(ax) - a^3x^3\arctan(ax) - 3\arctan(ax)^2 - 8iax\arctan(ax)^2 + 6a^2x^2\arctan(ax)^2 +}{}$$

input
```
Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x]^2)/x^2,x]
```

output
```
(c^2*(a^2*x^2 - a*x*ArcTan[a*x] - a^3*x^3*ArcTan[a*x] - 3*ArcTan[a*x]^2 -
(8*I)*a*x*ArcTan[a*x]^2 + 6*a^2*x^2*ArcTan[a*x]^2 + a^4*x^4*ArcTan[a*x]^2
+ 6*a*x*ArcTan[a*x]*Log[1 - E^((2*I)*ArcTan[a*x])] + 10*a*x*ArcTan[a*x]*Lo
g[1 + E^((2*I)*ArcTan[a*x])] - (5*I)*a*x*PolyLog[2, -E^((2*I)*ArcTan[a*x])
] - (3*I)*a*x*PolyLog[2, E^((2*I)*ArcTan[a*x])]))/(3*x)
```

## Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2 (a^2cx^2+c)^2}{x^2} dx$$
$$\Big\downarrow \text{ 5483}$$
$$\int \left(a^4c^2x^2\arctan(ax)^2 + 2a^2c^2\arctan(ax)^2 + \frac{c^2\arctan(ax)^2}{x^2}\right) dx$$
$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}a^4c^2x^3\arctan(ax)^2-\frac{1}{3}a^3c^2x^2\arctan(ax)+2a^2c^2x\arctan(ax)^2+\frac{1}{3}a^2c^2x+\frac{2}{3}iac^2\arctan(ax)^2-\frac{1}{3}ac^2\arctan(ax)-\frac{c^2\arctan(ax)^2}{x}+\frac{10}{3}ac^2\arctan(ax)\log\left(\frac{2}{1+iax}\right)+2ac^2\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-iac^2\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)+\frac{5}{3}iac^2\operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right)$$

input
```
Int[((c + a^2*c*x^2)^2*ArcTan[a*x]^2)/x^2,x]
```

output
```
(a^2*c^2*x)/3 - (a*c^2*ArcTan[a*x])/3 - (a^3*c^2*x^2*ArcTan[a*x])/3 + ((2*
I)/3)*a*c^2*ArcTan[a*x]^2 - (c^2*ArcTan[a*x]^2)/x + 2*a^2*c^2*x*ArcTan[a*x
]^2 + (a^4*c^2*x^3*ArcTan[a*x]^2)/3 + (10*a*c^2*ArcTan[a*x]*Log[2/(1 + I*a
*x)])/3 + 2*a*c^2*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)] - I*a*c^2*PolyLog[2,
-1 + 2/(1 - I*a*x)] + ((5*I)/3)*a*c^2*PolyLog[2, 1 - 2/(1 + I*a*x)]
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

## Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.39

| method | result |
|---|---|
| derivativedivides | $a\left(\frac{a^3c^2x^3\arctan(ax)^2}{3}+2a\,c^2x\arctan\left(ax\right)^2-\frac{c^2\arctan(ax)^2}{ax}-\frac{2c^2\left(\frac{x^2a^2\arctan(ax)}{2}-3\arctan(ax)\ln(a\right.}{}\right.$ |
| default | $a\left(\frac{a^3c^2x^3\arctan(ax)^2}{3}+2a\,c^2x\arctan\left(ax\right)^2-\frac{c^2\arctan(ax)^2}{ax}-\frac{2c^2\left(\frac{x^2a^2\arctan(ax)}{2}-3\arctan(ax)\ln(a\right.}{}\right.$ |
| parts | $\frac{a^4c^2x^3\arctan(ax)^2}{3}+2a^2c^2x\arctan\left(ax\right)^2-\frac{c^2\arctan(ax)^2}{x}-\frac{2c^2\left(\frac{\arctan(ax)x^2a^3}{2}-3a\arctan(ax)\ln(ax)\right.}{}$ |

input
```
int((a^2*c*x^2+c)^2*arctan(a*x)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
a*(1/3*a^3*c^2*x^3*arctan(a*x)^2+2*a*c^2*x*arctan(a*x)^2-c^2*arctan(a*x)^2
/a/x-2/3*c^2*(1/2*x^2*a^2*arctan(a*x)-3*arctan(a*x)*ln(a*x)+4*arctan(a*x)*
ln(a^2*x^2+1)-1/2*a*x+1/2*arctan(a*x)-3/2*I*ln(a*x)*ln(1+I*a*x)+3/2*I*ln(a
*x)*ln(1-I*a*x)-3/2*I*dilog(1+I*a*x)+3/2*I*dilog(1-I*a*x)+2*I*(ln(a*x-I)*l
n(a^2*x^2+1)-1/2*ln(a*x-I)^2-dilog(-1/2*I*(a*x+I))-ln(a*x-I)*ln(-1/2*I*(a*
x+I)))-2*I*(ln(a*x+I)*ln(a^2*x^2+1)-1/2*ln(a*x+I)^2-dilog(1/2*I*(a*x-I))-l
n(a*x+I)*ln(1/2*I*(a*x-I)))))
```

## Fricas [F]

$$\int \frac{(c+a^2cx^2)^2\arctan(ax)^2}{x^2}\,dx=\int \frac{\left(a^2cx^2+c\right)^2\arctan\left(ax\right)^2}{x^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^2/x^2,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)^2/x^2, x)
```

## Sympy [F]

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^2}{x^2}\,dx = c^2\left(\int 2a^2 \operatorname{atan}^2(ax)\,dx + \int \frac{\operatorname{atan}^2(ax)}{x^2}\,dx + \int a^4x^2 \operatorname{atan}^2(ax)\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**2*atan(a*x)**2/x**2,x)
```

output
```
c**2*(Integral(2*a**2*atan(a*x)**2, x) + Integral(atan(a*x)**2/x**2, x) +
Integral(a**4*x**2*atan(a*x)**2, x))
```

## Maxima [F]

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^2}{x^2}\,dx = \int \frac{(a^2cx^2+c)^2 \arctan(ax)^2}{x^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^2/x^2,x, algorithm="maxima")
```

output
```
1/48*(4*(a^4*c^2*x^4 + 6*a^2*c^2*x^2 - 3*c^2)*arctan(a*x)^2 - (a^4*c^2*x^4
 + 6*a^2*c^2*x^2 - 3*c^2)*log(a^2*x^2 + 1)^2 + 12*(144*a^6*c^2*integrate(1
/48*x^6*arctan(a*x)^2/(a^2*x^4 + x^2), x) + 12*a^6*c^2*integrate(1/48*x^6*
log(a^2*x^2 + 1)^2/(a^2*x^4 + x^2), x) + 16*a^6*c^2*integrate(1/48*x^6*log
(a^2*x^2 + 1)/(a^2*x^4 + x^2), x) - 32*a^5*c^2*integrate(1/48*x^5*arctan(a
*x)/(a^2*x^4 + x^2), x) + 432*a^4*c^2*integrate(1/48*x^4*arctan(a*x)^2/(a^
2*x^4 + x^2), x) + 36*a^4*c^2*integrate(1/48*x^4*log(a^2*x^2 + 1)^2/(a^2*x
^4 + x^2), x) + 96*a^4*c^2*integrate(1/48*x^4*log(a^2*x^2 + 1)/(a^2*x^4 +
x^2), x) + 3*a*c^2*arctan(a*x)^3 - 192*a^3*c^2*integrate(1/48*x^3*arctan(a
*x)/(a^2*x^4 + x^2), x) + 36*a^2*c^2*integrate(1/48*x^2*log(a^2*x^2 + 1)^2
/(a^2*x^4 + x^2), x) - 48*a^2*c^2*integrate(1/48*x^2*log(a^2*x^2 + 1)/(a^2
*x^4 + x^2), x) + 96*a*c^2*integrate(1/48*x*arctan(a*x)/(a^2*x^4 + x^2), x
) + 144*c^2*integrate(1/48*arctan(a*x)^2/(a^2*x^4 + x^2), x) + 12*c^2*inte
grate(1/48*log(a^2*x^2 + 1)^2/(a^2*x^4 + x^2), x))*x)/x
```

## Giac [F]

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^2}{x^2}\,dx = \int \frac{(a^2cx^2+c)^2 \arctan{(ax)}^2}{x^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^2/x^2,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^2*arctan(a*x)^2/x^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^2}{x^2}\,dx = \int \frac{\mathrm{atan}(a\,x)^2\,(c\,a^2\,x^2+c)^2}{x^2}\,dx$$

input
```
int((atan(a*x)^2*(c + a^2*c*x^2)^2)/x^2,x)
```

output
```
int((atan(a*x)^2*(c + a^2*c*x^2)^2)/x^2, x)
```

## Reduce [F]

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^2}{x^2}\,dx$$
$$= \frac{c^2\left(atan(ax)^2\,a^4x^4+6atan(ax)^2\,a^2x^2-3atan(ax)^2-atan(ax)\,a^3x^3-atan(ax)\,ax+6\left(\int \frac{atan(ax)}{a^2x^3+x}dx\right)a\right.}{3x}$$

input
```
int((a^2*c*x^2+c)^2*atan(a*x)^2/x^2,x)
```

output
```
(c**2*(atan(a*x)**2*a**4*x**4 + 6*atan(a*x)**2*a**2*x**2 - 3*atan(a*x)**2
- atan(a*x)*a**3*x**3 - atan(a*x)*a*x + 6*int(atan(a*x)/(a**2*x**3 + x),x)
*a*x - 10*int((atan(a*x)*x)/(a**2*x**2 + 1),x)*a**3*x + a**2*x**2))/(3*x)
```

# 3.272 $\int \frac{(c+a^2cx^2)^2 \arctan(ax)^2}{x^3} \, dx$



## Optimal result

Integrand size = 22, antiderivative size = 207

$$
\begin{aligned}
\int \frac{(c+a^2cx^2)^2 \arctan(ax)^2}{x^3} \, dx = & -\frac{ac^2 \arctan(ax)}{x} - a^3c^2x \arctan(ax) \\
& - \frac{c^2 \arctan(ax)^2}{2x^2} + \frac{1}{2}a^4c^2x^2 \arctan(ax)^2 \\
& + 4a^2c^2 \arctan(ax)^2 \operatorname{arctanh}\left(1 - \frac{2}{1+iax}\right) \\
& + a^2c^2 \log(x) \\
& - 2ia^2c^2 \arctan(ax) \operatorname{PolyLog}\left(2, 1 - \frac{2}{1+iax}\right) \\
& + 2ia^2c^2 \arctan(ax) \operatorname{PolyLog}\left(2, -1 + \frac{2}{1+iax}\right) \\
& - a^2c^2 \operatorname{PolyLog}\left(3, 1 - \frac{2}{1+iax}\right) \\
& + a^2c^2 \operatorname{PolyLog}\left(3, -1 + \frac{2}{1+iax}\right)
\end{aligned}
$$

output
```
-a*c^2*arctan(a*x)/x-a^3*c^2*x*arctan(a*x)-1/2*c^2*arctan(a*x)^2/x^2+1/2*a
^4*c^2*x^2*arctan(a*x)^2-4*a^2*c^2*arctan(a*x)^2*arctanh(-1+2/(1+I*a*x))+a
^2*c^2*ln(x)-2*I*a^2*c^2*arctan(a*x)*polylog(2,1-2/(1+I*a*x))+2*I*a^2*c^2*
arctan(a*x)*polylog(2,-1+2/(1+I*a*x))-a^2*c^2*polylog(3,1-2/(1+I*a*x))+a^2
*c^2*polylog(3,-1+2/(1+I*a*x))
```

## Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.09

$$\int \frac{(c+a^2cx^2)^2\arctan(ax)^2}{x^3}\,dx = a^2c^2\left(-\frac{i\pi^3}{12}-\frac{\arctan(ax)}{ax}-ax\arctan(ax)-\frac{\arctan(ax)^2}{2a^2x^2}+\frac{1}{2}a^2x^2\arctan(ax)^2+\frac{4}{3}i\arctan(ax)^3+2\arctan(ax)^2\log\left(1-e^{-2i\arctan(ax)}\right)-2\arctan(ax)^2\log\left(1+e^{2i\arctan(ax)}\right)+\log\left(\frac{ax}{\sqrt{1+a^2x^2}}\right)+\frac{1}{2}\log\left(1+a^2x^2\right)+2i\arctan(ax)\operatorname{PolyLog}\left(2,e^{-2i\arctan(ax)}\right)+2i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{2i\arctan(ax)}\right)+\operatorname{PolyLog}\left(3,e^{-2i\arctan(ax)}\right)-\operatorname{PolyLog}\left(3,-e^{2i\arctan(ax)}\right)\right)$$

input
```
Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x]^2)/x^3,x]
```

output
```
a^2*c^2*((-1/12*I)*Pi^3 - ArcTan[a*x]/(a*x) - a*x*ArcTan[a*x] - ArcTan[a*x
]^2/(2*a^2*x^2) + (a^2*x^2*ArcTan[a*x]^2)/2 + ((4*I)/3)*ArcTan[a*x]^3 + 2*
ArcTan[a*x]^2*Log[1 - E^((-2*I)*ArcTan[a*x])] - 2*ArcTan[a*x]^2*Log[1 + E^
((2*I)*ArcTan[a*x])] + Log[(a*x)/Sqrt[1 + a^2*x^2]] + Log[1 + a^2*x^2]/2 +
 (2*I)*ArcTan[a*x]*PolyLog[2, E^((-2*I)*ArcTan[a*x])] + (2*I)*ArcTan[a*x]*
PolyLog[2, -E^((2*I)*ArcTan[a*x])] + PolyLog[3, E^((-2*I)*ArcTan[a*x])] -
PolyLog[3, -E^((2*I)*ArcTan[a*x])])
```

## Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2 \left(a^2cx^2+c\right)^2}{x^3}dx$$

$$\Big\downarrow \text{5483}$$

$$\int \left(a^4c^2x\arctan(ax)^2+\frac{2a^2c^2\arctan(ax)^2}{x}+\frac{c^2\arctan(ax)^2}{x^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}a^4c^2x^2\arctan(ax)^2-a^3c^2x\arctan(ax)+4a^2c^2\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{1+iax}\right)-2ia^2c^2\arctan(ax)\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)+2ia^2c^2\arctan(ax)\text{PolyLog}\left(2,\frac{2}{iax+1}-1\right)-a^2c^2\text{PolyLog}\left(3,1-\frac{2}{iax+1}\right)+a^2c^2\text{PolyLog}\left(3,\frac{2}{iax+1}-1\right)+a^2c^2\log(x)-\frac{c^2\arctan(ax)^2}{2x^2}-\frac{ac^2\arctan(ax)}{x}$$

input
```
Int[((c + a^2*c*x^2)^2*ArcTan[a*x]^2)/x^3,x]
```

output
```
-((a*c^2*ArcTan[a*x])/x) - a^3*c^2*x*ArcTan[a*x] - (c^2*ArcTan[a*x]^2)/(2*
x^2) + (a^4*c^2*x^2*ArcTan[a*x]^2)/2 + 4*a^2*c^2*ArcTan[a*x]^2*ArcTanh[1 -
 2/(1 + I*a*x)] + a^2*c^2*Log[x] - (2*I)*a^2*c^2*ArcTan[a*x]*PolyLog[2, 1
- 2/(1 + I*a*x)] + (2*I)*a^2*c^2*ArcTan[a*x]*PolyLog[2, -1 + 2/(1 + I*a*x)
] - a^2*c^2*PolyLog[3, 1 - 2/(1 + I*a*x)] + a^2*c^2*PolyLog[3, -1 + 2/(1 +
 I*a*x)]
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 44.44 (sec) , antiderivative size = 1184, normalized size of antiderivative = 5.72

| method | result | size |
|---|---|---|
| derivativedivides | Expression too large to display | 1184 |
| default | Expression too large to display | 1184 |
| parts | Expression too large to display | 1614 |

input
```
int((a^2*c*x^2+c)^2*arctan(a*x)^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
a^2*(1/2*a^2*c^2*x^2*arctan(a*x)^2-1/2*c^2*arctan(a*x)^2/a^2/x^2+2*c^2*arc
tan(a*x)^2*ln(a*x)-c^2*(2*arctan(a*x)^2*ln((1+I*a*x)^2/(a^2*x^2+1)-1)-2*ar
ctan(a*x)^2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*Pi*arctan(a*x)^2-4*polylog
(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1
)^(1/2))+4*I*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-4*polylog(
3,(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1
+I*a*x)^2/(a^2*x^2+1)+1))^3*arctan(a*x)^2+polylog(3,-(1+I*a*x)^2/(a^2*x^2+
1))-2*I*arctan(a*x)*polylog(2,-(1+I*a*x)^2/(a^2*x^2+1))-ln(1+(1+I*a*x)/(a^
2*x^2+1)^(1/2))-ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-1)-I*Pi*csgn(((1+I*a*x)^2/(
a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^3*arctan(a*x)^2+4*I*arctan(a*x)
*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+ln((1+I*a*x)^2/(a^2*x^2+1)+1)+I*P
i*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a
*x)^2+arctan(a*x)*(a*x-I)+I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*
((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2+I
*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)
/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2-I*Pi*csgn(I*((1+I*a*x)^2/(a^
2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/
((1+I*a*x)^2/(a^2*x^2+1)+1))*arctan(a*x)^2+1/2*arctan(a*x)*(I*a*x-(a^2*x^2
+1)^(1/2)+1)/a/x+1/2*arctan(a*x)*(I*a*x+(a^2*x^2+1)^(1/2)+1)/a/x+I*Pi*csgn
(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(((1+I*...
```

## Fricas [F]

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^2}{x^3}\,dx = \int \frac{(a^2cx^2+c)^2 \arctan{(ax)}^2}{x^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^2/x^3,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)^2/x^3, x)
```

## Sympy [F]

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^2}{x^3}\,dx = c^2\left(\int \frac{\operatorname{atan}^2(ax)}{x^3}\,dx + \int \frac{2a^2\operatorname{atan}^2(ax)}{x}\,dx + \int a^4x\operatorname{atan}^2(ax)\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**2*atan(a*x)**2/x**3,x)
```

output
```
c**2*(Integral(atan(a*x)**2/x**3, x) + Integral(2*a**2*atan(a*x)**2/x, x)
+ Integral(a**4*x*atan(a*x)**2, x))
```

## Maxima [F]

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^2}{x^3}\,dx = \int \frac{(a^2cx^2+c)^2 \arctan(ax)^2}{x^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^2/x^3,x, algorithm="maxima")
```

output
```
-1/32*(2*a^4*c^2*x^4 - 4*a^4*c^2*x^2*integrate(4*x*arctan(a*x)^2 + x*log(a
^2*x^2 + 1)^2, x) - 8*a^3*c^2*x^2*integrate(-1/4*(12*(a^2*x^2 + 1)*a*x*arc
tan(a*x)^2 - 3*(a^2*x^2 + 1)*a*x*log(a^2*x^2 + 1)^2 + 12*(a^2*x^2 + 1)*arc
tan(a*x)*log(a^2*x^2 + 1) + (4*(a^2*x^2 + 1)^2*arctan(a*x)*cos(3*arctan(a*
x))*log(a^2*x^2 + 1) - 12*(a^2*x^2 + 1)^(3/2)*arctan(a*x)*cos(2*arctan(a*x
))*log(a^2*x^2 + 1) - 4*sqrt(a^2*x^2 + 1)*arctan(a*x)*log(a^2*x^2 + 1) + (
4*(a^2*x^2 + 1)^2*arctan(a*x)^2 - (a^2*x^2 + 1)^2*log(a^2*x^2 + 1)^2)*sin(
3*arctan(a*x)) - 3*(4*(a^2*x^2 + 1)^(3/2)*arctan(a*x)^2 - (a^2*x^2 + 1)^(3
/2)*log(a^2*x^2 + 1)^2)*sin(2*arctan(a*x)))*sqrt(a^2*x^2 + 1))/((a^2*x^2 +
 1)^4*cos(3*arctan(a*x))^2 + (a^2*x^2 + 1)^4*sin(3*arctan(a*x))^2 - 6*(a^2
*x^2 + 1)^(7/2)*sin(3*arctan(a*x))*sin(2*arctan(a*x)) + 9*(a^2*x^2 + 1)^3*
cos(2*arctan(a*x))^2 + 9*(a^2*x^2 + 1)^3*sin(2*arctan(a*x))^2 + a^2*x^2 +
6*(a^2*x^2 + 1)^2*cos(2*arctan(a*x)) + 9*(a^2*x^2 + 1)^2 - 2*(3*(a^2*x^2 +
 1)^(7/2)*cos(2*arctan(a*x)) + (a^2*x^2 + 1)^(5/2))*cos(3*arctan(a*x)) + 6
*((a^2*x^2 + 1)^2*a*x*sin(3*arctan(a*x)) - 3*(a^2*x^2 + 1)^(3/2)*a*x*sin(2
*arctan(a*x)) + (a^2*x^2 + 1)^2*cos(3*arctan(a*x)) - 3*(a^2*x^2 + 1)^(3/2)
*cos(2*arctan(a*x)) - sqrt(a^2*x^2 + 1))*sqrt(a^2*x^2 + 1) + 1), x) - 8*a^
3*c^2*x^2*integrate(1/4*(4*(a^2*x^2 + 1)*arctan(a*x)*log(a^2*x^2 + 1) - (4
*(a^2*x^2 + 1)*a*x*arctan(a*x)^2 - (a^2*x^2 + 1)*a*x*log(a^2*x^2 + 1)^2 +
4*(a^2*x^2 + 1)*arctan(a*x)*log(a^2*x^2 + 1))*cos(2*arctan(a*x)) - (4*(...
```

## Giac [F]

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^2}{x^3} \, dx = \int \frac{(a^2cx^2+c)^2 \arctan(ax)^2}{x^3} \, dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^2/x^3,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^2*arctan(a*x)^2/x^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^2}{x^3}\,dx = \int \frac{\mathrm{atan}(a\,x)^2\,(c\,a^2\,x^2+c)^2}{x^3}\,dx$$

input
```
int((atan(a*x)^2*(c + a^2*c*x^2)^2)/x^3,x)
```

output
```
int((atan(a*x)^2*(c + a^2*c*x^2)^2)/x^3, x)
```

## Reduce [F]

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^2}{x^3}\,dx = \frac{c^2\left(atan(ax)^2\,a^4x^4 - atan(ax)^2 - 2atan(ax)\,a^3x^3 - 2atan(ax)\,ax + 4\left(\int \frac{atan(ax)^2}{x}dx\right)a^2x^2 + 2\log(x)\,a^2 \right.}{2x^2}$$

input
```
int((a^2*c*x^2+c)^2*atan(a*x)^2/x^3,x)
```

output
```
(c**2*(atan(a*x)**2*a**4*x**4 - atan(a*x)**2 - 2*atan(a*x)*a**3*x**3 - 2*a
tan(a*x)*a*x + 4*int(atan(a*x)**2/x,x)*a**2*x**2 + 2*log(x)*a**2*x**2))/(2
*x**2)
```

# 3.273 $\int \frac{(c+a^2cx^2)^2 \arctan(ax)^2}{x^4} dx$



## Optimal result

Integrand size = 22, antiderivative size = 216

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^2}{x^4} dx = -\frac{a^2c^2}{3x} - \frac{1}{3}a^3c^2\arctan(ax) - \frac{ac^2\arctan(ax)}{3x^2} - \frac{2}{3}ia^3c^2\arctan(ax)^2 - \frac{c^2\arctan(ax)^2}{3x^3} - \frac{2a^2c^2\arctan(ax)^2}{x} + a^4c^2x\arctan(ax)^2 + 2a^3c^2\arctan(ax)\log\left(\frac{2}{1+iax}\right) + \frac{10}{3}a^3c^2\arctan(ax)\log\left(2-\frac{2}{1-iax}\right) - \frac{5}{3}ia^3c^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right) + ia^3c^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right)$$

output
```
-1/3*a^2*c^2/x-1/3*a^3*c^2*arctan(a*x)-1/3*a*c^2*arctan(a*x)/x^2-2/3*I*a^3
*c^2*arctan(a*x)^2-1/3*c^2*arctan(a*x)^2/x^3-2*a^2*c^2*arctan(a*x)^2/x+a^4
*c^2*x*arctan(a*x)^2+2*a^3*c^2*arctan(a*x)*ln(2/(1+I*a*x))+10/3*a^3*c^2*ar
ctan(a*x)*ln(2-2/(1-I*a*x))-5/3*I*a^3*c^2*polylog(2,-1+2/(1-I*a*x))+I*a^3*
c^2*polylog(2,1-2/(1+I*a*x))
```

## Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.88

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^2}{x^4} dx$$
$$= \frac{c^2(-a^2x^2 - ax\arctan(ax) - a^3x^3\arctan(ax) - \arctan(ax)^2 - 6a^2x^2\arctan(ax)^2 - 8ia^3x^3\arctan(ax)^2}{}$$

input
```
Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x]^2)/x^4,x]
```

output
```
(c^2*(-(a^2*x^2) - a*x*ArcTan[a*x] - a^3*x^3*ArcTan[a*x] - ArcTan[a*x]^2 -
 6*a^2*x^2*ArcTan[a*x]^2 - (8*I)*a^3*x^3*ArcTan[a*x]^2 + 3*a^4*x^4*ArcTan[
a*x]^2 + 10*a^3*x^3*ArcTan[a*x]*Log[1 - E^((2*I)*ArcTan[a*x])] + 6*a^3*x^3
*ArcTan[a*x]*Log[1 + E^((2*I)*ArcTan[a*x])] - (3*I)*a^3*x^3*PolyLog[2, -E^
((2*I)*ArcTan[a*x])] - (5*I)*a^3*x^3*PolyLog[2, E^((2*I)*ArcTan[a*x])]))/(
3*x^3)
```

## Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2 (a^2cx^2+c)^2}{x^4} dx$$
$$\Big\downarrow \text{ 5483}$$
$$\int \left(a^4c^2\arctan(ax)^2 + \frac{2a^2c^2\arctan(ax)^2}{x^2} + \frac{c^2\arctan(ax)^2}{x^4}\right) dx$$
$$\Big\downarrow \text{ 2009}$$

$$a^4c^2x\arctan(ax)^2 - \frac{2}{3}ia^3c^2\arctan(ax)^2 - \frac{1}{3}a^3c^2\arctan(ax) + 2a^3c^2\arctan(ax)\log\left(\frac{2}{1+iax}\right) + \frac{10}{3}a^3c^2\arctan(ax)\log\left(2 - \frac{2}{1-iax}\right) - \frac{5}{3}ia^3c^2\operatorname{PolyLog}\left(2, \frac{2}{1-iax} - 1\right) + ia^3c^2\operatorname{PolyLog}\left(2, 1 - \frac{2}{iax+1}\right) - \frac{2a^2c^2\arctan(ax)^2}{x} - \frac{a^2c^2}{3x} - \frac{c^2\arctan(ax)^2}{3x^3} - \frac{ac^2\arctan(ax)}{3x^2}$$

input
```
Int[((c + a^2*c*x^2)^2*ArcTan[a*x]^2)/x^4,x]
```

output
```
-1/3*(a^2*c^2)/x - (a^3*c^2*ArcTan[a*x])/3 - (a*c^2*ArcTan[a*x])/(3*x^2) -
 ((2*I)/3)*a^3*c^2*ArcTan[a*x]^2 - (c^2*ArcTan[a*x]^2)/(3*x^3) - (2*a^2*c^
2*ArcTan[a*x]^2)/x + a^4*c^2*x*ArcTan[a*x]^2 + 2*a^3*c^2*ArcTan[a*x]*Log[2
/(1 + I*a*x)] + (10*a^3*c^2*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)])/3 - ((5*I)
/3)*a^3*c^2*PolyLog[2, -1 + 2/(1 - I*a*x)] + I*a^3*c^2*PolyLog[2, 1 - 2/(1
 + I*a*x)]
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

## Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.34

| method | result |
|---|---|
| derivativedivides | $a^3\left(a\,c^2x\arctan\left(ax\right)^2-\frac{2c^2\arctan(ax)^2}{ax}-\frac{c^2\arctan(ax)^2}{3a^3x^3}-\frac{2c^2\left(4\arctan(ax)\ln(a^2x^2+1)+\frac{\arctan(ax)}{2a^2x^2}\right.}{}\right.$ |
| default | $a^3\left(a\,c^2x\arctan\left(ax\right)^2-\frac{2c^2\arctan(ax)^2}{ax}-\frac{c^2\arctan(ax)^2}{3a^3x^3}-\frac{2c^2\left(4\arctan(ax)\ln(a^2x^2+1)+\frac{\arctan(ax)}{2a^2x^2}\right.}{}\right.$ |
| parts | $a^4c^2x\arctan\left(ax\right)^2-\frac{c^2\arctan(ax)^2}{3x^3}-\frac{2a^2c^2\arctan(ax)^2}{x}-\frac{2c^2\left(4a^3\arctan(ax)\ln(a^2x^2+1)+\frac{a\arctan(ax}{2x^2}\right.}{}$ |

input
```
int((a^2*c*x^2+c)^2*arctan(a*x)^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
a^3*(a*c^2*x*arctan(a*x)^2-2*c^2*arctan(a*x)^2/a/x-1/3*c^2*arctan(a*x)^2/a
^3/x^3-2/3*c^2*(4*arctan(a*x)*ln(a^2*x^2+1)+1/2*arctan(a*x)/a^2/x^2-5*arct
an(a*x)*ln(a*x)+1/2/a/x+1/2*arctan(a*x)-5/2*I*ln(a*x)*ln(1+I*a*x)+5/2*I*ln
(a*x)*ln(1-I*a*x)-5/2*I*dilog(1+I*a*x)+5/2*I*dilog(1-I*a*x)+2*I*(ln(a*x-I)
*ln(a^2*x^2+1)-1/2*ln(a*x-I)^2-dilog(-1/2*I*(a*x+I))-ln(a*x-I)*ln(-1/2*I*(
a*x+I)))-2*I*(ln(a*x+I)*ln(a^2*x^2+1)-1/2*ln(a*x+I)^2-dilog(1/2*I*(a*x-I))
-ln(a*x+I)*ln(1/2*I*(a*x-I)))))
```

## Fricas [F]

$$\int\frac{\left(c+a^2cx^2\right)^2\arctan(ax)^2}{x^4}\,dx=\int\frac{\left(a^2cx^2+c\right)^2\arctan\left(ax\right)^2}{x^4}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^2/x^4,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)^2/x^4, x)
```

## Sympy [F]

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^2}{x^4}\,dx = c^2\left(\int a^4 \operatorname{atan}^2(ax)\,dx + \int \frac{\operatorname{atan}^2(ax)}{x^4}\,dx + \int \frac{2a^2 \operatorname{atan}^2(ax)}{x^2}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**2*atan(a*x)**2/x**4,x)
```

output
```
c**2*(Integral(a**4*atan(a*x)**2, x) + Integral(atan(a*x)**2/x**4, x) + In
tegral(2*a**2*atan(a*x)**2/x**2, x))
```

## Maxima [F]

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^2}{x^4}\,dx = \int \frac{(a^2cx^2+c)^2 \arctan(ax)^2}{x^4}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^2/x^4,x, algorithm="maxima")
```

output
```
1/48*(12*(144*a^6*c^2*integrate(1/48*x^6*arctan(a*x)^2/(a^2*x^6 + x^4), x)
 + 12*a^6*c^2*integrate(1/48*x^6*log(a^2*x^2 + 1)^2/(a^2*x^6 + x^4), x) +
48*a^6*c^2*integrate(1/48*x^6*log(a^2*x^2 + 1)/(a^2*x^6 + x^4), x) + 3*a^3
*c^2*arctan(a*x)^3 - 96*a^5*c^2*integrate(1/48*x^5*arctan(a*x)/(a^2*x^6 +
x^4), x) + 36*a^4*c^2*integrate(1/48*x^4*log(a^2*x^2 + 1)^2/(a^2*x^6 + x^4
), x) - 96*a^4*c^2*integrate(1/48*x^4*log(a^2*x^2 + 1)/(a^2*x^6 + x^4), x)
 + 192*a^3*c^2*integrate(1/48*x^3*arctan(a*x)/(a^2*x^6 + x^4), x) + 432*a^
2*c^2*integrate(1/48*x^2*arctan(a*x)^2/(a^2*x^6 + x^4), x) + 36*a^2*c^2*in
tegrate(1/48*x^2*log(a^2*x^2 + 1)^2/(a^2*x^6 + x^4), x) - 16*a^2*c^2*integ
rate(1/48*x^2*log(a^2*x^2 + 1)/(a^2*x^6 + x^4), x) + 32*a*c^2*integrate(1/
48*x*arctan(a*x)/(a^2*x^6 + x^4), x) + 144*c^2*integrate(1/48*arctan(a*x)^
2/(a^2*x^6 + x^4), x) + 12*c^2*integrate(1/48*log(a^2*x^2 + 1)^2/(a^2*x^6
+ x^4), x))*x^3 + 4*(3*a^4*c^2*x^4 - 6*a^2*c^2*x^2 - c^2)*arctan(a*x)^2 -
(3*a^4*c^2*x^4 - 6*a^2*c^2*x^2 - c^2)*log(a^2*x^2 + 1)^2)/x^3
```

## Giac [F]

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^2}{x^4}\,dx = \int \frac{(a^2cx^2+c)^2 \arctan(ax)^2}{x^4}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^2/x^4,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^2*arctan(a*x)^2/x^4, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^2}{x^4}\,dx = \int \frac{\mathrm{atan}(a\,x)^2\,(c\,a^2\,x^2+c)^2}{x^4}\,dx$$

input
```
int((atan(a*x)^2*(c + a^2*c*x^2)^2)/x^4,x)
```

output
```
int((atan(a*x)^2*(c + a^2*c*x^2)^2)/x^4, x)
```

## Reduce [F]

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^2}{x^4}\,dx = \frac{c^2\left(-6atan(ax)^2 a^2x^2 - atan(ax)^2 - atan(ax)\,a^3x^3 - atan(ax)\,ax + 3\left(\int atan(ax)^2\,dx\right)a^4x^3 + 10\left(\int \frac{ata}{a^2}\right.\right.}{3x^3}$$

input
```
int((a^2*c*x^2+c)^2*atan(a*x)^2/x^4,x)
```

output
```
(c**2*( - 6*atan(a*x)**2*a**2*x**2 - atan(a*x)**2 - atan(a*x)*a**3*x**3 -
atan(a*x)*a*x + 3*int(atan(a*x)**2,x)*a**4*x**3 + 10*int(atan(a*x)/(a**2*x
**3 + x),x)*a**3*x**3 - a**2*x**2))/(3*x**3)
```

# 3.274 $\int x^3(c+a^2cx^2)^3 \arctan(ax)^2 \, dx$



## Optimal result

Integrand size = 22, antiderivative size = 240

$$\begin{aligned}\int x^3(c+a^2cx^2)^3 \arctan(ax)^2 \, dx = & -\frac{107c^3x^2}{12600a^2}+\frac{53c^3x^4}{6300}+\frac{71a^2c^3x^6}{7560} \\ & +\frac{1}{360}a^4c^3x^8+\frac{c^3x\arctan(ax)}{20a^3} \\ & -\frac{c^3x^3\arctan(ax)}{60a}-\frac{9}{100}ac^3x^5\arctan(ax) \\ & -\frac{11}{140}a^3c^3x^7\arctan(ax)-\frac{1}{45}a^5c^3x^9\arctan(ax) \\ & -\frac{c^3\arctan(ax)^2}{40a^4}+\frac{1}{4}c^3x^4\arctan(ax)^2 \\ & +\frac{1}{2}a^2c^3x^6\arctan(ax)^2+\frac{3}{8}a^4c^3x^8\arctan(ax)^2 \\ & +\frac{1}{10}a^6c^3x^{10}\arctan(ax)^2-\frac{26c^3\log(1+a^2x^2)}{1575a^4}\end{aligned}$$

output
```
-107/12600*c^3*x^2/a^2+53/6300*c^3*x^4+71/7560*a^2*c^3*x^6+1/360*a^4*c^3*x
^8+1/20*c^3*x*arctan(a*x)/a^3-1/60*c^3*x^3*arctan(a*x)/a-9/100*a*c^3*x^5*a
rctan(a*x)-11/140*a^3*c^3*x^7*arctan(a*x)-1/45*a^5*c^3*x^9*arctan(a*x)-1/4
0*c^3*arctan(a*x)^2/a^4+1/4*c^3*x^4*arctan(a*x)^2+1/2*a^2*c^3*x^6*arctan(a
*x)^2+3/8*a^4*c^3*x^8*arctan(a*x)^2+1/10*a^6*c^3*x^10*arctan(a*x)^2-26/157
5*c^3*ln(a^2*x^2+1)/a^4
```

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.52

$$\int x^3 (c + a^2cx^2)^3 \arctan(ax)^2 \, dx$$
$$= \frac{c^3 \left(-321a^2x^2 + 318a^4x^4 + 355a^6x^6 + 105a^8x^8 - 6ax(-315 + 105a^2x^2 + 567a^4x^4 + 495a^6x^6 + 140a^8x^8)\right.}{37800a^4}$$

input
```
Integrate[x^3*(c + a^2*c*x^2)^3*ArcTan[a*x]^2,x]
```

output
```
(c^3*(-321*a^2*x^2 + 318*a^4*x^4 + 355*a^6*x^6 + 105*a^8*x^8 - 6*a*x*(-315
 + 105*a^2*x^2 + 567*a^4*x^4 + 495*a^6*x^6 + 140*a^8*x^8)*ArcTan[a*x] + 94
5*(1 + a^2*x^2)^4*(-1 + 4*a^2*x^2)*ArcTan[a*x]^2 - 624*Log[1 + a^2*x^2]))/
(37800*a^4)
```

## Rubi [A] (verified)

Time = 1.42 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \arctan(ax)^2 \left(a^2cx^2 + c\right)^3 \, dx$$
$$\Big\downarrow \; 5483$$
$$\int \left(a^6c^3x^9 \arctan(ax)^2 + 3a^4c^3x^7 \arctan(ax)^2 + 3a^2c^3x^5 \arctan(ax)^2 + c^3x^3 \arctan(ax)^2\right) dx$$
$$\Big\downarrow \; 2009$$

$$\frac{1}{10}a^6c^3x^{10}\arctan(ax)^2-\frac{1}{45}a^5c^3x^9\arctan(ax)+\frac{3}{8}a^4c^3x^8\arctan(ax)^2-\frac{c^3\arctan(ax)^2}{40a^4}+\frac{1}{360}a^4c^3x^8-\frac{11}{140}a^3c^3x^7\arctan(ax)+\frac{c^3x\arctan(ax)}{20a^3}+\frac{1}{2}a^2c^3x^6\arctan(ax)^2+\frac{71a^2c^3x^6}{7560}-\frac{107c^3x^2}{12600a^2}-\frac{26c^3\log\left(a^2x^2+1\right)}{1575a^4}-\frac{9}{100}ac^3x^5\arctan(ax)+\frac{1}{4}c^3x^4\arctan(ax)^2-\frac{c^3x^3\arctan(ax)}{60a}+\frac{53c^3x^4}{6300}$$

input
```
Int[x^3*(c + a^2*c*x^2)^3*ArcTan[a*x]^2,x]
```

output
```
(-107*c^3*x^2)/(12600*a^2) + (53*c^3*x^4)/6300 + (71*a^2*c^3*x^6)/7560 + (
a^4*c^3*x^8)/360 + (c^3*x*ArcTan[a*x])/(20*a^3) - (c^3*x^3*ArcTan[a*x])/(6
0*a) - (9*a*c^3*x^5*ArcTan[a*x])/100 - (11*a^3*c^3*x^7*ArcTan[a*x])/140 -
(a^5*c^3*x^9*ArcTan[a*x])/45 - (c^3*ArcTan[a*x]^2)/(40*a^4) + (c^3*x^4*Arc
Tan[a*x]^2)/4 + (a^2*c^3*x^6*ArcTan[a*x]^2)/2 + (3*a^4*c^3*x^8*ArcTan[a*x]
^2)/8 + (a^6*c^3*x^10*ArcTan[a*x]^2)/10 - (26*c^3*Log[1 + a^2*x^2])/(1575*
a^4)
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

## Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.78

| method | result |
|---|---|
| derivativedivides | $\frac{c^3 \arctan(ax)^2 a^{10} x^{10}}{10}+\frac{3c^3 \arctan(ax)^2 a^8 x^8}{8}+\frac{a^6 c^3 x^6 \arctan(ax)^2}{2}+\frac{a^4 c^3 x^4 \arctan(ax)^2}{4}-\frac{c^3\left(\frac{4 \arctan(ax) a^9 x^9}{9}+\frac{11 \arctan(ax}{7}\right.}{}$ |
| default | $\frac{c^3 \arctan(ax)^2 a^{10} x^{10}}{10}+\frac{3c^3 \arctan(ax)^2 a^8 x^8}{8}+\frac{a^6 c^3 x^6 \arctan(ax)^2}{2}+\frac{a^4 c^3 x^4 \arctan(ax)^2}{4}-\frac{c^3\left(\frac{4 \arctan(ax) a^9 x^9}{9}+\frac{11 \arctan(ax}{7}\right.}{}$ |
| parts | $\frac{a^6 c^3 x^{10} \arctan(ax)^2}{10}+\frac{3a^4 c^3 x^8 \arctan(ax)^2}{8}+\frac{a^2 c^3 x^6 \arctan(ax)^2}{2}+\frac{c^3 x^4 \arctan(ax)^2}{4}-\frac{c^3\left(\frac{4a^5 \arctan(ax) x^9}{9}\right.}{}$ |
| parallelrisch | $-\frac{-3780c^3 \arctan(ax)^2 a^{10} x^{10}+840a^9 c^3 \arctan(ax) x^9-14175c^3 \arctan(ax)^2 a^8 x^8-105a^8 c^3 x^8+2970c^3 \arctan(ax) a^7}{}$ |
| risch | $-\frac{c^3(4a^{10}x^{10}+15a^8x^8+20a^6x^6+10a^4x^4-1)\ln(iax+1)^2}{160a^4}+\frac{c^3(1260a^{10}x^{10}\ln(-iax+1)+280ia^9x^9+4725a^8x^8\ln(-}{}$ |

input
```
int(x^3*(a^2*c*x^2+c)^3*arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^4*(1/10*c^3*arctan(a*x)^2*a^10*x^10+3/8*c^3*arctan(a*x)^2*a^8*x^8+1/2*
a^6*c^3*x^6*arctan(a*x)^2+1/4*a^4*c^3*x^4*arctan(a*x)^2-1/20*c^3*(4/9*arct
an(a*x)*a^9*x^9+11/7*arctan(a*x)*a^7*x^7+9/5*arctan(a*x)*a^5*x^5+1/3*arcta
n(a*x)*x^3*a^3-arctan(a*x)*a*x+1/2*arctan(a*x)^2-1/18*a^8*x^8-71/378*a^6*x
^6-53/315*a^4*x^4+107/630*a^2*x^2+104/315*ln(a^2*x^2+1)))
```

## Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.75

$$\int x^3(c+a^2cx^2)^3 \arctan(ax)^2\,dx$$
$$=\frac{105\,a^8c^3x^8+355\,a^6c^3x^6+318\,a^4c^3x^4-321\,a^2c^3x^2-624\,c^3\log\left(a^2x^2+1\right)+945\left(4\,a^{10}c^3x^{10}+15\,a^8c^3x^8\right.}{}$$

input
```
integrate(x^3*(a^2*c*x^2+c)^3*arctan(a*x)^2,x, algorithm="fricas")
```

output
```
1/37800*(105*a^8*c^3*x^8 + 355*a^6*c^3*x^6 + 318*a^4*c^3*x^4 - 321*a^2*c^3
*x^2 - 624*c^3*log(a^2*x^2 + 1) + 945*(4*a^10*c^3*x^10 + 15*a^8*c^3*x^8 +
20*a^6*c^3*x^6 + 10*a^4*c^3*x^4 - c^3)*arctan(a*x)^2 - 6*(140*a^9*c^3*x^9
+ 495*a^7*c^3*x^7 + 567*a^5*c^3*x^5 + 105*a^3*c^3*x^3 - 315*a*c^3*x)*arcta
n(a*x))/a^4
```

## Sympy [A] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.00

$$\int x^3(c+a^2cx^2)^3\arctan(ax)^2\,dx$$
$$=\begin{cases}\frac{a^6c^3x^{10}\,\mathrm{atan}^2\,(ax)}{10}-\frac{a^5c^3x^9\,\mathrm{atan}\,(ax)}{45}+\frac{3a^4c^3x^8\,\mathrm{atan}^2\,(ax)}{8}+\frac{a^4c^3x^8}{360}-\frac{11a^3c^3x^7\,\mathrm{atan}\,(ax)}{140}+\frac{a^2c^3x^6\,\mathrm{atan}^2\,(ax)}{2}+\frac{71a^2c^3x^6}{7560}-\\0\end{cases}$$

input
```
integrate(x**3*(a**2*c*x**2+c)**3*atan(a*x)**2,x)
```

output
```
Piecewise((a**6*c**3*x**10*atan(a*x)**2/10 - a**5*c**3*x**9*atan(a*x)/45 +
 3*a**4*c**3*x**8*atan(a*x)**2/8 + a**4*c**3*x**8/360 - 11*a**3*c**3*x**7*
atan(a*x)/140 + a**2*c**3*x**6*atan(a*x)**2/2 + 71*a**2*c**3*x**6/7560 - 9
*a*c**3*x**5*atan(a*x)/100 + c**3*x**4*atan(a*x)**2/4 + 53*c**3*x**4/6300
- c**3*x**3*atan(a*x)/(60*a) - 107*c**3*x**2/(12600*a**2) + c**3*x*atan(a*
x)/(20*a**3) - 26*c**3*log(x**2 + a**(-2))/(1575*a**4) - c**3*atan(a*x)**2
/(40*a**4), Ne(a, 0)), (0, True))
```

## Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.84

$$\int x^3(c+a^2cx^2)^3\arctan(ax)^2\,dx=$$
$$-\frac{1}{6300}\,a\left(\frac{315\,c^3\arctan\left(ax\right)}{a^5}+\frac{140\,a^8c^3x^9+495\,a^6c^3x^7+567\,a^4c^3x^5+105\,a^2c^3x^3-315\,c^3x}{a^4}\right)\arctan\left(\right.$$
$$+\frac{1}{40}\left(4\,a^6c^3x^{10}+15\,a^4c^3x^8+20\,a^2c^3x^6+10\,c^3x^4\right)\arctan\left(ax\right)^2$$
$$+\frac{105\,a^8c^3x^8+355\,a^6c^3x^6+318\,a^4c^3x^4-321\,a^2c^3x^2+945\,c^3\arctan\left(ax\right)^2-624\,c^3\log\left(a^2x^2+1\right)}{37800\,a^4}$$

input
```
integrate(x^3*(a^2*c*x^2+c)^3*arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-1/6300*a*(315*c^3*arctan(a*x)/a^5 + (140*a^8*c^3*x^9 + 495*a^6*c^3*x^7 +
567*a^4*c^3*x^5 + 105*a^2*c^3*x^3 - 315*c^3*x)/a^4)*arctan(a*x) + 1/40*(4*
a^6*c^3*x^10 + 15*a^4*c^3*x^8 + 20*a^2*c^3*x^6 + 10*c^3*x^4)*arctan(a*x)^2
 + 1/37800*(105*a^8*c^3*x^8 + 355*a^6*c^3*x^6 + 318*a^4*c^3*x^4 - 321*a^2*
c^3*x^2 + 945*c^3*arctan(a*x)^2 - 624*c^3*log(a^2*x^2 + 1))/a^4
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.82

$$\int x^3 (c + a^2 c x^2)^3 \arctan(ax)^2 \, dx$$
$$= \frac{1}{40} \left(4 a^6 c^3 x^{10} + 15 a^4 c^3 x^8 + 20 a^2 c^3 x^6 + 10 c^3 x^4\right) \arctan(ax)^2$$
$$- \frac{840 a^9 c^3 x^9 \arctan(ax) - 105 a^8 c^3 x^8 + 2970 a^7 c^3 x^7 \arctan(ax) - 355 a^6 c^3 x^6 + 3402 a^5 c^3 x^5 \arctan(ax)}{}$$

input
```
integrate(x^3*(a^2*c*x^2+c)^3*arctan(a*x)^2,x, algorithm="giac")
```

output
```
1/40*(4*a^6*c^3*x^10 + 15*a^4*c^3*x^8 + 20*a^2*c^3*x^6 + 10*c^3*x^4)*arcta
n(a*x)^2 - 1/37800*(840*a^9*c^3*x^9*arctan(a*x) - 105*a^8*c^3*x^8 + 2970*a
^7*c^3*x^7*arctan(a*x) - 355*a^6*c^3*x^6 + 3402*a^5*c^3*x^5*arctan(a*x) -
318*a^4*c^3*x^4 + 630*a^3*c^3*x^3*arctan(a*x) + 321*a^2*c^3*x^2 - 1890*a*c
^3*x*arctan(a*x) + 945*c^3*arctan(a*x)^2 + 624*c^3*log(a^2*x^2 + 1))/a^4
```

## Mupad [B] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.74

$$\int x^3 (c+a^2cx^2)^3 \arctan(ax)^2 \, dx = \mathrm{atan}(a\,x)^2 \left(\frac{c^3\,x^4}{4} - \frac{c^3}{40\,a^4} + \frac{a^2\,c^3\,x^6}{2} + \frac{3\,a^4\,c^3\,x^8}{8} + \frac{a^6\,c^3\,x^{10}}{10}\right) + \frac{53\,c^3\,x^4}{6300} - \frac{26\,c^3\,\ln\left(a^2\,x^2+1\right)}{1575\,a^4} - \frac{107\,c^3\,x^2}{12600\,a^2} + \frac{71\,a^2\,c^3\,x^6}{7560} + \frac{a^4\,c^3\,x^8}{360} - a^2\,\mathrm{atan}(a\,x)\left(\frac{11\,a\,c^3\,x^7}{140} - \frac{c^3\,x}{20\,a^5} + \frac{9\,c^3\,x^5}{100\,a} + \frac{c^3\,x^3}{60\,a^3} + \frac{a^3\,c^3\,x^9}{45}\right)$$

input
```
int(x^3*atan(a*x)^2*(c + a^2*c*x^2)^3,x)
```

output
```
atan(a*x)^2*((c^3*x^4)/4 - c^3/(40*a^4) + (a^2*c^3*x^6)/2 + (3*a^4*c^3*x^8
)/8 + (a^6*c^3*x^10)/10) + (53*c^3*x^4)/6300 - (26*c^3*log(a^2*x^2 + 1))/(
1575*a^4) - (107*c^3*x^2)/(12600*a^2) + (71*a^2*c^3*x^6)/7560 + (a^4*c^3*x
^8)/360 - a^2*atan(a*x)*((11*a*c^3*x^7)/140 - (c^3*x)/(20*a^5) + (9*c^3*x^
5)/(100*a) + (c^3*x^3)/(60*a^3) + (a^3*c^3*x^9)/45)
```

## Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.72

$$\int x^3 (c+a^2cx^2)^3 \arctan(ax)^2 \, dx$$
$$= \frac{c^3 \left(3780 atan(ax)^2 a^{10}x^{10} + 14175 atan(ax)^2 a^8x^8 + 18900 atan(ax)^2 a^6x^6 + 9450 atan(ax)^2 a^4x^4 - 945 ata\right.}{}$$

input
```
int(x^3*(a^2*c*x^2+c)^3*atan(a*x)^2,x)
```

output
```
(c**3*(3780*atan(a*x)**2*a**10*x**10 + 14175*atan(a*x)**2*a**8*x**8 + 1890
0*atan(a*x)**2*a**6*x**6 + 9450*atan(a*x)**2*a**4*x**4 - 945*atan(a*x)**2
- 840*atan(a*x)*a**9*x**9 - 2970*atan(a*x)*a**7*x**7 - 3402*atan(a*x)*a**5
*x**5 - 630*atan(a*x)*a**3*x**3 + 1890*atan(a*x)*a*x - 624*log(a**2*x**2 +
 1) + 105*a**8*x**8 + 355*a**6*x**6 + 318*a**4*x**4 - 321*a**2*x**2))/(378
00*a**4)
```

# 3.275 $\int x^2(c+a^2cx^2)^3 \arctan(ax)^2\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 274

$$\begin{aligned}\int x^2(c+a^2cx^2)^3 \arctan(ax)^2\,dx =& -\frac{47c^3x}{3780a^2}+\frac{239c^3x^3}{11340}+\frac{59a^2c^3x^5}{3780}+\frac{1}{252}a^4c^3x^7 \\ &+\frac{47c^3\arctan(ax)}{3780a^3}-\frac{16c^3x^2\arctan(ax)}{315a} \\ &-\frac{89}{630}ac^3x^4\arctan(ax)-\frac{20}{189}a^3c^3x^6\arctan(ax) \\ &-\frac{1}{36}a^5c^3x^8\arctan(ax)-\frac{16ic^3\arctan(ax)^2}{315a^3} \\ &+\frac{1}{3}c^3x^3\arctan(ax)^2+\frac{3}{5}a^2c^3x^5\arctan(ax)^2 \\ &+\frac{3}{7}a^4c^3x^7\arctan(ax)^2+\frac{1}{9}a^6c^3x^9\arctan(ax)^2 \\ &-\frac{32c^3\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{315a^3} \\ &-\frac{16ic^3\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{315a^3}\end{aligned}$$

output
```
-47/3780*c^3*x/a^2+239/11340*c^3*x^3+59/3780*a^2*c^3*x^5+1/252*a^4*c^3*x^7
+47/3780*c^3*arctan(a*x)/a^3-16/315*c^3*x^2*arctan(a*x)/a-89/630*a*c^3*x^4
*arctan(a*x)-20/189*a^3*c^3*x^6*arctan(a*x)-1/36*a^5*c^3*x^8*arctan(a*x)-1
6/315*I*c^3*polylog(2,1-2/(1+I*a*x))/a^3+1/3*c^3*x^3*arctan(a*x)^2+3/5*a^2
*c^3*x^5*arctan(a*x)^2+3/7*a^4*c^3*x^7*arctan(a*x)^2+1/9*a^6*c^3*x^9*arcta
n(a*x)^2-32/315*c^3*arctan(a*x)*ln(2/(1+I*a*x))/a^3-16/315*I*c^3*arctan(a*
x)^2/a^3
```

## Mathematica [A] (verified)

Time = 1.58 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.57

$$\int x^2(c+a^2cx^2)^3 \arctan(ax)^2 \, dx$$
$$= \frac{c^3(ax(-141+239a^2x^2+177a^4x^4+45a^6x^6)+36(16i+105a^3x^3+189a^5x^5+135a^7x^7+35a^9x^9)\arctan}{}$$

input
```
Integrate[x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^2,x]
```

output
```
(c^3*(a*x*(-141 + 239*a^2*x^2 + 177*a^4*x^4 + 45*a^6*x^6) + 36*(16*I + 105
*a^3*x^3 + 189*a^5*x^5 + 135*a^7*x^7 + 35*a^9*x^9)*ArcTan[a*x]^2 - 3*ArcTa
n[a*x]*(-47 + 192*a^2*x^2 + 534*a^4*x^4 + 400*a^6*x^6 + 105*a^8*x^8 + 384*
Log[1 + E^((2*I)*ArcTan[a*x])]) + (576*I)*PolyLog[2, -E^((2*I)*ArcTan[a*x]
)]))/(11340*a^3)
```

## Rubi [A] (verified)

Time = 1.37 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arctan(ax)^2 \left(a^2cx^2+c\right)^3 \, dx$$

$$\Big\downarrow \text{5483}$$

$$\int \left(a^6c^3x^8\arctan(ax)^2+3a^4c^3x^6\arctan(ax)^2+3a^2c^3x^4\arctan(ax)^2+c^3x^2\arctan(ax)^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{9}a^6c^3x^9\arctan(ax)^2-\frac{1}{36}a^5c^3x^8\arctan(ax)+\frac{3}{7}a^4c^3x^7\arctan(ax)^2+\frac{1}{252}a^4c^3x^7-\frac{20}{189}a^3c^3x^6\arctan(ax)-\frac{16ic^3\arctan(ax)^2}{315a^3}+\frac{47c^3\arctan(ax)}{3780a^3}-\frac{32c^3\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{315a^3}-\frac{16ic^3\operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{315a^3}+\frac{3}{5}a^2c^3x^5\arctan(ax)^2+\frac{59a^2c^3x^5}{3780}-\frac{47c^3x}{3780a^2}-\frac{89}{630}ac^3x^4\arctan(ax)+\frac{1}{3}c^3x^3\arctan(ax)^2-\frac{16c^3x^2\arctan(ax)}{315a}+\frac{239c^3x^3}{11340}$$

input
```
Int[x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^2,x]
```

output
```
(-47*c^3*x)/(3780*a^2) + (239*c^3*x^3)/11340 + (59*a^2*c^3*x^5)/3780 + (a^
4*c^3*x^7)/252 + (47*c^3*ArcTan[a*x])/(3780*a^3) - (16*c^3*x^2*ArcTan[a*x]
)/(315*a) - (89*a*c^3*x^4*ArcTan[a*x])/630 - (20*a^3*c^3*x^6*ArcTan[a*x])/
189 - (a^5*c^3*x^8*ArcTan[a*x])/36 - (((16*I)/315)*c^3*ArcTan[a*x]^2)/a^3
+ (c^3*x^3*ArcTan[a*x]^2)/3 + (3*a^2*c^3*x^5*ArcTan[a*x]^2)/5 + (3*a^4*c^3
*x^7*ArcTan[a*x]^2)/7 + (a^6*c^3*x^9*ArcTan[a*x]^2)/9 - (32*c^3*ArcTan[a*x
]*Log[2/(1 + I*a*x)])/(315*a^3) - (((16*I)/315)*c^3*PolyLog[2, 1 - 2/(1 +
I*a*x)])/a^3
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

## Maple [A] (verified)

Time = 3.18 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.12

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{c^3 \arctan(ax)^2 a^9 x^9}{9}+\frac{3c^3 \arctan(ax)^2 a^7 x^7}{7}+\frac{3a^5c^3x^5 \arctan(ax)^2}{5}+\frac{a^3c^3x^3 \arctan(ax)^2}{3}-\frac{2c^3\left(\frac{35 \arctan(ax) a^8 x^8}{8}+\frac{50a^6 \arctan}{3}\right.}{}}{}$ |
| default | $\frac{\frac{c^3 \arctan(ax)^2 a^9 x^9}{9}+\frac{3c^3 \arctan(ax)^2 a^7 x^7}{7}+\frac{3a^5c^3x^5 \arctan(ax)^2}{5}+\frac{a^3c^3x^3 \arctan(ax)^2}{3}-\frac{2c^3\left(\frac{35 \arctan(ax) a^8 x^8}{8}+\frac{50a^6 \arctan}{3}\right.}{}}{}$ |
| parts | $\frac{a^6c^3x^9 \arctan(ax)^2}{9}+\frac{3a^4c^3x^7 \arctan(ax)^2}{7}+\frac{3a^2c^3x^5 \arctan(ax)^2}{5}+\frac{c^3x^3 \arctan(ax)^2}{3}-\frac{2c^3\left(\frac{35a^5 \arctan(ax)}{8}\right.}{}$ |
| risch | $\frac{239c^3x^3}{11340}-\frac{47c^3x}{3780a^2}+\frac{a^4c^3x^7}{252}+\frac{47c^3 \arctan(ax)}{3780a^3}+\frac{59a^2c^3x^5}{3780}-\frac{8ic^3 \ln(iax+1) \ln(-iax+1)}{315a^3}+\frac{c^3a^6 \ln(iax+1}{}$ |

input
```
int(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(1/9*c^3*arctan(a*x)^2*a^9*x^9+3/7*c^3*arctan(a*x)^2*a^7*x^7+3/5*a^5
*c^3*x^5*arctan(a*x)^2+1/3*a^3*c^3*x^3*arctan(a*x)^2-2/315*c^3*(35/8*arcta
n(a*x)*a^8*x^8+50/3*a^6*arctan(a*x)*x^6+89/4*x^4*arctan(a*x)*a^4+8*x^2*a^2
*arctan(a*x)-8*arctan(a*x)*ln(a^2*x^2+1)-5/8*a^7*x^7-59/24*a^5*x^5-239/72*
a^3*x^3+47/24*a*x-47/24*arctan(a*x)-4*I*(ln(a*x-I)*ln(a^2*x^2+1)-1/2*ln(a*
x-I)^2-dilog(-1/2*I*(a*x+I))-ln(a*x-I)*ln(-1/2*I*(a*x+I)))+4*I*(ln(a*x+I)*
ln(a^2*x^2+1)-1/2*ln(a*x+I)^2-dilog(1/2*I*(a*x-I))-ln(a*x+I)*ln(1/2*I*(a*x
-I)))))
```

## Fricas [F]

$$\int x^2(c+a^2cx^2)^3 \arctan(ax)^2\,dx=\int (a^2cx^2+c)^3x^2 \arctan(ax)^2\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^6*c^3*x^8 + 3*a^4*c^3*x^6 + 3*a^2*c^3*x^4 + c^3*x^2)*arctan(a*
x)^2, x)
```

## Sympy [F]

$$\int x^2 (c+a^2cx^2)^3 \arctan(ax)^2 \, dx = c^3 \left( \int x^2 \operatorname{atan}^2(ax) \, dx + \int 3a^2x^4 \operatorname{atan}^2(ax) \, dx + \int 3a^4x^6 \operatorname{atan}^2(ax) \, dx + \int a^6x^8 \operatorname{atan}^2(ax) \, dx \right)$$

input
```
integrate(x**2*(a**2*c*x**2+c)**3*atan(a*x)**2,x)
```

output
```
c**3*(Integral(x**2*atan(a*x)**2, x) + Integral(3*a**2*x**4*atan(a*x)**2,
x) + Integral(3*a**4*x**6*atan(a*x)**2, x) + Integral(a**6*x**8*atan(a*x)*
*2, x))
```

## Maxima [F]

$$\int x^2 (c+a^2cx^2)^3 \arctan(ax)^2 \, dx = \int (a^2cx^2+c)^3 x^2 \arctan(ax)^2 \, dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^2,x, algorithm="maxima")
```

output
```
1/1260*(35*a^6*c^3*x^9 + 135*a^4*c^3*x^7 + 189*a^2*c^3*x^5 + 105*c^3*x^3)*
arctan(a*x)^2 - 1/5040*(35*a^6*c^3*x^9 + 135*a^4*c^3*x^7 + 189*a^2*c^3*x^5
 + 105*c^3*x^3)*log(a^2*x^2 + 1)^2 + integrate(1/5040*(3780*(a^8*c^3*x^10
+ 4*a^6*c^3*x^8 + 6*a^4*c^3*x^6 + 4*a^2*c^3*x^4 + c^3*x^2)*arctan(a*x)^2 +
 315*(a^8*c^3*x^10 + 4*a^6*c^3*x^8 + 6*a^4*c^3*x^6 + 4*a^2*c^3*x^4 + c^3*x
^2)*log(a^2*x^2 + 1)^2 - 8*(35*a^7*c^3*x^9 + 135*a^5*c^3*x^7 + 189*a^3*c^3
*x^5 + 105*a*c^3*x^3)*arctan(a*x) + 4*(35*a^8*c^3*x^10 + 135*a^6*c^3*x^8 +
 189*a^4*c^3*x^6 + 105*a^2*c^3*x^4)*log(a^2*x^2 + 1))/(a^2*x^2 + 1), x)
```

## Giac [F]

$$\int x^2 \left(c+a^2 c x^2\right)^3 \arctan(ax)^2 \, dx = \int \left(a^2 c x^2 + c\right)^3 x^2 \arctan\left(ax\right)^2 \, dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^3*x^2*arctan(a*x)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int x^2 \left(c+a^2 c x^2\right)^3 \arctan(ax)^2 \, dx = \int x^2 \operatorname{atan}(a\,x)^2 \left(c\,a^2\,x^2 + c\right)^3 dx$$

input
```
int(x^2*atan(a*x)^2*(c + a^2*c*x^2)^3,x)
```

output
```
int(x^2*atan(a*x)^2*(c + a^2*c*x^2)^3, x)
```

## Reduce [F]

$$\int x^2 \left(c+a^2 c x^2\right)^3 \arctan(ax)^2 \, dx = \frac{c^3 \left(1260 atan(ax)^2 a^9 x^9 + 4860 atan(ax)^2 a^7 x^7 + 6804 atan(ax)^2 a^5 x^5 + 3780 atan(ax)^2 a^3 x^3 + 576 atan(a\right.}{}$$

input
```
int(x^2*(a^2*c*x^2+c)^3*atan(a*x)^2,x)
```

output
```
(c**3*(1260*atan(a*x)**2*a**9*x**9 + 4860*atan(a*x)**2*a**7*x**7 + 6804*at
an(a*x)**2*a**5*x**5 + 3780*atan(a*x)**2*a**3*x**3 + 576*atan(a*x)**2*a*x
- 315*atan(a*x)*a**8*x**8 - 1200*atan(a*x)*a**6*x**6 - 1602*atan(a*x)*a**4
*x**4 - 576*atan(a*x)*a**2*x**2 + 141*atan(a*x) - 576*int(atan(a*x)**2,x)*
a + 45*a**7*x**7 + 177*a**5*x**5 + 239*a**3*x**3 - 141*a*x))/(11340*a**3)
```

# 3.276 $\int x(c+a^2cx^2)^3 \arctan(ax)^2 \, dx$



## Optimal result

Integrand size = 20, antiderivative size = 200

$$\int x(c+a^2cx^2)^3 \arctan(ax)^2 \, dx = \frac{c^3(1+a^2x^2)}{35a^2} + \frac{3c^3(1+a^2x^2)^2}{280a^2} + \frac{c^3(1+a^2x^2)^3}{168a^2} - \frac{4c^3x\arctan(ax)}{35a} - \frac{2c^3x(1+a^2x^2)\arctan(ax)}{35a} - \frac{3c^3x(1+a^2x^2)^2\arctan(ax)}{70a} - \frac{c^3x(1+a^2x^2)^3\arctan(ax)}{28a} + \frac{c^3(1+a^2x^2)^4\arctan(ax)^2}{8a^2} + \frac{2c^3\log(1+a^2x^2)}{35a^2}$$

output
```
1/35*c^3*(a^2*x^2+1)/a^2+3/280*c^3*(a^2*x^2+1)^2/a^2+1/168*c^3*(a^2*x^2+1)
^3/a^2-4/35*c^3*x*arctan(a*x)/a-2/35*c^3*x*(a^2*x^2+1)*arctan(a*x)/a-3/70*
c^3*x*(a^2*x^2+1)^2*arctan(a*x)/a-1/28*c^3*x*(a^2*x^2+1)^3*arctan(a*x)/a+1
/8*c^3*(a^2*x^2+1)^4*arctan(a*x)^2/a^2+2/35*c^3*ln(a^2*x^2+1)/a^2
```

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.50

$$\int x\left(c+a^2cx^2\right)^3\arctan(ax)^2\,dx$$
$$=\frac{c^3\Big(57a^2x^2+24a^4x^4+5a^6x^6-6ax(35+35a^2x^2+21a^4x^4+5a^6x^6)\arctan(ax)+105(1+a^2x^2)^4\arctan}{840a^2}$$

input
```
Integrate[x*(c + a^2*c*x^2)^3*ArcTan[a*x]^2,x]
```

output
```
(c^3*(57*a^2*x^2 + 24*a^4*x^4 + 5*a^6*x^6 - 6*a*x*(35 + 35*a^2*x^2 + 21*a^
4*x^4 + 5*a^6*x^6)*ArcTan[a*x] + 105*(1 + a^2*x^2)^4*ArcTan[a*x]^2 + 48*Lo
g[1 + a^2*x^2]))/(840*a^2)
```

## Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {5465, 27, 5413, 5413, 5413, 5345, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\arctan(ax)^2\left(a^2cx^2+c\right)^3\,dx$$
$$\Big\downarrow \text{5465}$$
$$\frac{c^3\left(a^2x^2+1\right)^4\arctan(ax)^2}{8a^2}-\frac{\int c^3\left(a^2x^2+1\right)^3\arctan(ax)dx}{4a}$$
$$\Big\downarrow \text{27}$$
$$\frac{c^3\left(a^2x^2+1\right)^4\arctan(ax)^2}{8a^2}-\frac{c^3\int\left(a^2x^2+1\right)^3\arctan(ax)dx}{4a}$$
$$\Big\downarrow \text{5413}$$

$$\frac{c^3\left(a^2x^2+1\right)^4\arctan(ax)^2}{8a^2}-\frac{c^3\left(\frac{6}{7}\int\left(a^2x^2+1\right)^2\arctan(ax)dx+\frac{1}{7}x\left(a^2x^2+1\right)^3\arctan(ax)-\frac{\left(a^2x^2+1\right)^3}{42a}\right)}{4a}$$

$\big\downarrow$ 5413

$$\frac{c^3\left(a^2x^2+1\right)^4\arctan(ax)^2}{8a^2}-\frac{c^3\left(\frac{6}{7}\left(\frac{4}{5}\int\left(a^2x^2+1\right)\arctan(ax)dx+\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan(ax)-\frac{\left(a^2x^2+1\right)^2}{20a}\right)+\frac{1}{7}x\left(a^2x^2+1\right)^3\arctan(ax)-\frac{(a}{}\right.}{4a}$$

$\big\downarrow$ 5413

$$\frac{c^3\left(a^2x^2+1\right)^4\arctan(ax)^2}{8a^2}-\frac{c^3\left(\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\int\arctan(ax)dx+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)-\frac{a^2x^2+1}{6a}\right)+\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan(ax)-\frac{\left(a^2x^2+1\right)^2}{20a}\right)+\right.}{4a}$$

$\big\downarrow$ 5345

$$\frac{c^3\left(a^2x^2+1\right)^4\arctan(ax)^2}{8a^2}-\frac{c^3\left(\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\left(x\arctan(ax)-a\int\frac{x}{a^2x^2+1}dx\right)+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)-\frac{a^2x^2+1}{6a}\right)+\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan(ax)-\right.\right.\right.}{4a}$$

$\big\downarrow$ 240

$$\frac{c^3\left(a^2x^2+1\right)^4\arctan(ax)^2}{8a^2}-\frac{c^3\left(\frac{1}{7}x\left(a^2x^2+1\right)^3\arctan(ax)+\frac{6}{7}\left(\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan(ax)+\frac{4}{5}\left(\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)+\frac{2}{3}\left(x\arctan(ax)\right.\right.\right.\right.}{4a}$$

input
```
Int[x*(c + a^2*c*x^2)^3*ArcTan[a*x]^2,x]
```

output
```
(c^3*(1 + a^2*x^2)^4*ArcTan[a*x]^2)/(8*a^2) - (c^3*(-1/42*(1 + a^2*x^2)^3/
a + (x*(1 + a^2*x^2)^3*ArcTan[a*x])/7 + (6*(-1/20*(1 + a^2*x^2)^2/a + (x*(
1 + a^2*x^2)^2*ArcTan[a*x])/5 + (4*(-1/6*(1 + a^2*x^2)/a + (x*(1 + a^2*x^2
)*ArcTan[a*x])/3 + (2*(x*ArcTan[a*x] - Log[1 + a^2*x^2]/(2*a)))/3))/5))/7)
)/(4*a)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5413
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Simp[x*(d + e*x^2)
^q*((a + b*ArcTan[c*x])/(2*q + 1)), x] + Simp[2*d*(q/(2*q + 1))   Int[(d +
e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

## Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.84

| method | result |
|---|---|
| parts | $\frac{c^3 \arctan(ax)^2 a^6 x^8}{8} + \frac{c^3 \arctan(ax)^2 a^4 x^6}{2} + \frac{3c^3 \arctan(ax)^2 a^2 x^4}{4} + \frac{c^3 \arctan(ax)^2 x^2}{2} + \frac{c^3 \arctan(ax)^2}{8a^2} - \frac{c^3}{}$ |
| derivativedivides | $\frac{\frac{c^3 \arctan(ax)^2 a^8 x^8}{8} + \frac{a^6 c^3 x^6 \arctan(ax)^2}{2} + \frac{3a^4 c^3 x^4 \arctan(ax)^2}{4} + \frac{a^2 c^3 x^2 \arctan(ax)^2}{2} + \frac{c^3 \arctan(ax)^2}{8} - \frac{c^3\left(\frac{\arctan(ax) a^7 x^7}{7}\right.}{}}{a^2}$ |
| default | $\frac{\frac{c^3 \arctan(ax)^2 a^8 x^8}{8} + \frac{a^6 c^3 x^6 \arctan(ax)^2}{2} + \frac{3a^4 c^3 x^4 \arctan(ax)^2}{4} + \frac{a^2 c^3 x^2 \arctan(ax)^2}{2} + \frac{c^3 \arctan(ax)^2}{8} - \frac{c^3\left(\frac{\arctan(ax) a^7 x^7}{7}\right.}{}}{a^2}$ |
| parallelrisch | $105c^3 \arctan(ax)^2 a^8 x^8 - 30c^3 \arctan(ax) a^7 x^7 + 420a^6 c^3 x^6 \arctan(ax)^2 + 5a^6 c^3 x^6 - 126a^5 c^3 x^5 \arctan(ax) + 630a^4 c^3 x^4$ |
| risch | $-\frac{c^3 (a^2x^2+1)^4 \ln(iax+1)^2}{32a^2} + \frac{c^3 (35a^8x^8 \ln(-iax+1) + 10ia^7x^7 + 140a^6x^6 \ln(-iax+1) + 42ia^5x^5 + 210x^4 \ln(-iax+1}{560a^2}$ |

input
```
int(x*(a^2*c*x^2+c)^3*arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*c^3*arctan(a*x)^2*a^6*x^8+1/2*c^3*arctan(a*x)^2*a^4*x^6+3/4*c^3*arctan
(a*x)^2*a^2*x^4+1/2*c^3*arctan(a*x)^2*x^2+1/8*c^3*arctan(a*x)^2/a^2-1/4*c^
3/a^2*(1/7*arctan(a*x)*a^7*x^7+3/5*arctan(a*x)*a^5*x^5+arctan(a*x)*x^3*a^3
+arctan(a*x)*a*x-1/42*a^6*x^6-4/35*a^4*x^4-19/70*a^2*x^2-8/35*ln(a^2*x^2+1
))
```

## Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.78

$$\int x\left(c + a^2cx^2\right)^3 \arctan(ax)^2 \, dx$$
$$= \frac{5\,a^6c^3x^6 + 24\,a^4c^3x^4 + 57\,a^2c^3x^2 + 48\,c^3\log\left(a^2x^2+1\right) + 105\left(a^8c^3x^8 + 4\,a^6c^3x^6 + 6\,a^4c^3x^4 + 4\,a^2c^3x^2 + \right.}{840\,a^2}$$

input
```
integrate(x*(a^2*c*x^2+c)^3*arctan(a*x)^2,x, algorithm="fricas")
```

output
```
1/840*(5*a^6*c^3*x^6 + 24*a^4*c^3*x^4 + 57*a^2*c^3*x^2 + 48*c^3*log(a^2*x^
2 + 1) + 105*(a^8*c^3*x^8 + 4*a^6*c^3*x^6 + 6*a^4*c^3*x^4 + 4*a^2*c^3*x^2
+ c^3)*arctan(a*x)^2 - 6*(5*a^7*c^3*x^7 + 21*a^5*c^3*x^5 + 35*a^3*c^3*x^3
+ 35*a*c^3*x)*arctan(a*x))/a^2
```

## Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.04

$$\int x(c+a^2cx^2)^3 \arctan(ax)^2\,dx$$
$$=\begin{cases} \frac{a^6c^3x^8\operatorname{atan}^2(ax)}{8}-\frac{a^5c^3x^7\operatorname{atan}(ax)}{28}+\frac{a^4c^3x^6\operatorname{atan}^2(ax)}{2}+\frac{a^4c^3x^6}{168}-\frac{3a^3c^3x^5\operatorname{atan}(ax)}{20}+\frac{3a^2c^3x^4\operatorname{atan}^2(ax)}{4}+\frac{a^2c^3x^4}{35}-\frac{ac^3 \\ 0 \end{cases}$$

input
```
integrate(x*(a**2*c*x**2+c)**3*atan(a*x)**2,x)
```

output
```
Piecewise((a**6*c**3*x**8*atan(a*x)**2/8 - a**5*c**3*x**7*atan(a*x)/28 + a
**4*c**3*x**6*atan(a*x)**2/2 + a**4*c**3*x**6/168 - 3*a**3*c**3*x**5*atan(
a*x)/20 + 3*a**2*c**3*x**4*atan(a*x)**2/4 + a**2*c**3*x**4/35 - a*c**3*x**
3*atan(a*x)/4 + c**3*x**2*atan(a*x)**2/2 + 19*c**3*x**2/280 - c**3*x*atan(
a*x)/(4*a) + 2*c**3*log(x**2 + a**(-2))/(35*a**2) + c**3*atan(a*x)**2/(8*a
**2), Ne(a, 0)), (0, True))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.66

$$\int x(c+a^2cx^2)^3 \arctan(ax)^2\,dx=\frac{(a^2cx^2+c)^4\arctan(ax)^2}{8\,a^2c}$$
$$+\frac{\left(5\,a^4c^4x^6+24\,a^2c^4x^4+57\,c^4x^2+\frac{48\,c^4\log(a^2x^2+1)}{a^2}\right)a-6\left(5\,a^6c^4x^7+21\,a^4c^4x^5+35\,a^2c^4x^3+35\,c^4x\right)\text{ar}}{840\,ac}$$

input
```
integrate(x*(a^2*c*x^2+c)^3*arctan(a*x)^2,x, algorithm="maxima")
```

output
```
1/8*(a^2*c*x^2 + c)^4*arctan(a*x)^2/(a^2*c) + 1/840*((5*a^4*c^4*x^6 + 24*a
^2*c^4*x^4 + 57*c^4*x^2 + 48*c^4*log(a^2*x^2 + 1)/a^2)*a - 6*(5*a^6*c^4*x^
7 + 21*a^4*c^4*x^5 + 35*a^2*c^4*x^3 + 35*c^4*x)*arctan(a*x))/(a*c)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.12

$$\int x(c+a^2cx^2)^3\arctan(ax)^2\,dx = \frac{(a^2cx^2+c)^4\arctan(ax)^2}{8a^2c} - \frac{5\left(12x^7\arctan(ax) - a\left(\frac{2a^4x^6-3a^2x^4+6x^2}{a^6} - \frac{6\log(a^2x^2+1)}{a^8}\right)\right)a^6c^3 + 63\left(4x^5\arctan(ax) - a\left(\frac{a^2x^4-2x^2}{a^4} + \right.\right.}{168}$$

input
```
integrate(x*(a^2*c*x^2+c)^3*arctan(a*x)^2,x, algorithm="giac")
```

output
```
1/8*(a^2*c*x^2 + c)^4*arctan(a*x)^2/(a^2*c) - 1/1680*(5*(12*x^7*arctan(a*x
) - a*((2*a^4*x^6 - 3*a^2*x^4 + 6*x^2)/a^6 - 6*log(a^2*x^2 + 1)/a^8))*a^6*
c^3 + 63*(4*x^5*arctan(a*x) - a*((a^2*x^4 - 2*x^2)/a^4 + 2*log(a^2*x^2 + 1
)/a^6))*a^4*c^3 + 210*(2*x^3*arctan(a*x) - a*(x^2/a^2 - log(a^2*x^2 + 1)/a
^4))*a^2*c^3 + 210*(2*a*x*arctan(a*x) - log(a^2*x^2 + 1))*c^3/a)/a
```

## Mupad [B] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.78

$$\int x(c+a^2cx^2)^3\arctan(ax)^2\,dx = \mathrm{atan}(a\,x)^2\left(\frac{c^3}{8\,a^2} + \frac{c^3\,x^2}{2} + \frac{3\,a^2\,c^3\,x^4}{4} + \frac{a^4\,c^3\,x^6}{2} + \frac{a^6\,c^3\,x^8}{8}\right) + \frac{19\,c^3\,x^2}{280} - a^2\,\mathrm{atan}(a\,x)\,\left(\frac{c^3\,x}{4\,a^3} + \frac{3\,a\,c^3\,x^5}{20} + \frac{c^3\,x^3}{4\,a} + \frac{a^3\,c^3\,x^7}{28}\right) + \frac{2\,c^3\,\ln\left(a^2\,x^2+1\right)}{35\,a^2} + \frac{a^2\,c^3\,x^4}{35} + \frac{a^4\,c^3\,x^6}{168}$$

input
```
int(x*atan(a*x)^2*(c + a^2*c*x^2)^3,x)
```

output
```
atan(a*x)^2*(c^3/(8*a^2) + (c^3*x^2)/2 + (3*a^2*c^3*x^4)/4 + (a^4*c^3*x^6)
/2 + (a^6*c^3*x^8)/8) + (19*c^3*x^2)/280 - a^2*atan(a*x)*((c^3*x)/(4*a^3)
+ (3*a*c^3*x^5)/20 + (c^3*x^3)/(4*a) + (a^3*c^3*x^7)/28) + (2*c^3*log(a^2*
x^2 + 1))/(35*a^2) + (a^2*c^3*x^4)/35 + (a^4*c^3*x^6)/168
```

## Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.76

$$\int x(c+a^2cx^2)^3\arctan(ax)^2\,dx$$
$$=\frac{c^3\left(105atan(ax)^2a^8x^8+420atan(ax)^2a^6x^6+630atan(ax)^2a^4x^4+420atan(ax)^2a^2x^2+105atan(ax)^2-\right.}{}$$

input
```
int(x*(a^2*c*x^2+c)^3*atan(a*x)^2,x)
```

output
```
(c**3*(105*atan(a*x)**2*a**8*x**8 + 420*atan(a*x)**2*a**6*x**6 + 630*atan(
a*x)**2*a**4*x**4 + 420*atan(a*x)**2*a**2*x**2 + 105*atan(a*x)**2 - 30*ata
n(a*x)*a**7*x**7 - 126*atan(a*x)*a**5*x**5 - 210*atan(a*x)*a**3*x**3 - 210
*atan(a*x)*a*x + 48*log(a**2*x**2 + 1) + 5*a**6*x**6 + 24*a**4*x**4 + 57*a
**2*x**2))/(840*a**2)
```

# 3.277 $\int (c+a^2cx^2)^3 \arctan(ax)^2\,dx$



## Optimal result

Integrand size = 19, antiderivative size = 268

$$\begin{aligned}\int (c+a^2cx^2)^3 \arctan(ax)^2\,dx =& \frac{38c^3x}{105}+\frac{19}{315}a^2c^3x^3+\frac{1}{105}a^4c^3x^5 \\ &-\frac{8c^3(1+a^2x^2)\arctan(ax)}{35a} \\ &-\frac{3c^3(1+a^2x^2)^2\arctan(ax)}{35a} \\ &-\frac{c^3(1+a^2x^2)^3\arctan(ax)}{21a}+\frac{16ic^3\arctan(ax)^2}{35a} \\ &+\frac{16}{35}c^3x\arctan(ax)^2+\frac{8}{35}c^3x(1+a^2x^2)\arctan(ax)^2 \\ &+\frac{6}{35}c^3x(1+a^2x^2)^2\arctan(ax)^2 \\ &+\frac{1}{7}c^3x(1+a^2x^2)^3\arctan(ax)^2 \\ &+\frac{32c^3\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{35a} \\ &+\frac{16ic^3\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{35a}\end{aligned}$$

output
```
38/105*c^3*x+19/315*a^2*c^3*x^3+1/105*a^4*c^3*x^5-8/35*c^3*(a^2*x^2+1)*arc
tan(a*x)/a-3/35*c^3*(a^2*x^2+1)^2*arctan(a*x)/a-1/21*c^3*(a^2*x^2+1)^3*arc
tan(a*x)/a+16/35*I*c^3*arctan(a*x)^2/a+16/35*c^3*x*arctan(a*x)^2+8/35*c^3*
x*(a^2*x^2+1)*arctan(a*x)^2+6/35*c^3*x*(a^2*x^2+1)^2*arctan(a*x)^2+1/7*c^3
*x*(a^2*x^2+1)^3*arctan(a*x)^2+32/35*c^3*arctan(a*x)*ln(2/(1+I*a*x))/a+16/
35*I*c^3*polylog(2,1-2/(1+I*a*x))/a
```

## Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.51

$$\int (c+a^2cx^2)^3 \arctan(ax)^2 \, dx$$
$$= \frac{c^3(ax(114+19a^2x^2+3a^4x^4)+9(-16i+35ax+35a^3x^3+21a^5x^5+5a^7x^7)\arctan(ax)^2-3\arctan(ax}{315a}$$

input
```
Integrate[(c + a^2*c*x^2)^3*ArcTan[a*x]^2,x]
```

output
```
(c^3*(a*x*(114 + 19*a^2*x^2 + 3*a^4*x^4) + 9*(-16*I + 35*a*x + 35*a^3*x^3
+ 21*a^5*x^5 + 5*a^7*x^7)*ArcTan[a*x]^2 - 3*ArcTan[a*x]*(38 + 57*a^2*x^2 +
 24*a^4*x^4 + 5*a^6*x^6 - 96*Log[1 + E^((2*I)*ArcTan[a*x])]) - (144*I)*Pol
yLog[2, -E^((2*I)*ArcTan[a*x])]))/(315*a)
```

## Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.08, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.684$, Rules used = {5415, 27, 210, 2009, 5415, 2009, 5415, 24, 5345, 5455, 5379, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(ax)^2 \left(a^2cx^2+c\right)^3 \, dx$$
$$\Big\downarrow \text{ 5415}$$

$$\frac{6}{7}c\int c^2\left(a^2x^2+1\right)^2\arctan(ax)^2dx+\frac{1}{21}c\int\left(a^2cx^2+c\right)^2dx+\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)^2-\frac{c^3\left(a^2x^2+1\right)^3\arctan(ax)}{21a}$$

$\downarrow$ 27

$$\frac{6}{7}c^3\int\left(a^2x^2+1\right)^2\arctan(ax)^2dx+\frac{1}{21}c\int\left(a^2cx^2+c\right)^2dx+\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)^2-\frac{c^3\left(a^2x^2+1\right)^3\arctan(ax)}{21a}$$

$\downarrow$ 210

$$\frac{6}{7}c^3\int\left(a^2x^2+1\right)^2\arctan(ax)^2dx+\frac{1}{21}c\int\left(a^4c^2x^4+2a^2c^2x^2+c^2\right)dx+\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)^2-\frac{c^3\left(a^2x^2+1\right)^3\arctan(ax)}{21a}$$

$\downarrow$ 2009

$$\frac{6}{7}c^3\int\left(a^2x^2+1\right)^2\arctan(ax)^2dx+\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)^2-\frac{c^3\left(a^2x^2+1\right)^3\arctan(ax)}{21a}+\frac{1}{21}c\left(\frac{1}{5}a^4c^2x^5+\frac{2}{3}a^2c^2x^3+c^2x\right)$$

$\downarrow$ 5415

$$\frac{6}{7}c^3\left(\frac{4}{5}\int\left(a^2x^2+1\right)\arctan(ax)^2dx+\frac{1}{10}\int\left(a^2x^2+1\right)dx+\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan(ax)^2-\frac{\left(a^2x^2+1\right)^2\arctan(\right.}{10a}\right.$$
$$\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)^2-\frac{c^3\left(a^2x^2+1\right)^3\arctan(ax)}{21a}+\frac{1}{21}c\left(\frac{1}{5}a^4c^2x^5+\frac{2}{3}a^2c^2x^3+c^2x\right)$$

$\downarrow$ 2009

$$\frac{6}{7}c^3\left(\frac{4}{5}\int\left(a^2x^2+1\right)\arctan(ax)^2dx+\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan(ax)^2-\frac{\left(a^2x^2+1\right)^2\arctan(ax)}{10a}+\frac{1}{10}\left(\frac{a^2x^3}{3}+x\right)\right.$$
$$\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)^2-\frac{c^3\left(a^2x^2+1\right)^3\arctan(ax)}{21a}+\frac{1}{21}c\left(\frac{1}{5}a^4c^2x^5+\frac{2}{3}a^2c^2x^3+c^2x\right)$$

$\downarrow$ 5415

$$\frac{6}{7}c^3\left(\frac{4}{5}\left(\frac{2}{3}\int \arctan(ax)^2dx+\frac{\int 1dx}{3}+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)^2-\frac{\left(a^2x^2+1\right)\arctan(ax)}{3a}\right)+\frac{1}{5}x\left(a^2x^2+1\right)^2\right.$$
$$\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)^2-\frac{c^3\left(a^2x^2+1\right)^3\arctan(ax)}{21a}+\frac{1}{21}c\left(\frac{1}{5}a^4c^2x^5+\frac{2}{3}a^2c^2x^3+c^2x\right)$$

$\downarrow$ 24

$$\frac{6}{7}c^3\left(\frac{4}{5}\left(\frac{2}{3}\int \arctan(ax)^2dx+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)^2-\frac{\left(a^2x^2+1\right)\arctan(ax)}{3a}+\frac{x}{3}\right)+\frac{1}{5}x\left(a^2x^2+1\right)^2\mathrm{arct}\right.$$
$$\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)^2-\frac{c^3\left(a^2x^2+1\right)^3\arctan(ax)}{21a}+\frac{1}{21}c\left(\frac{1}{5}a^4c^2x^5+\frac{2}{3}a^2c^2x^3+c^2x\right)$$

$\downarrow$ 5345

$$\frac{6}{7}c^3\left(\frac{4}{5}\left(\frac{2}{3}\left(x\arctan(ax)^2-2a\int\frac{x\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)^2-\frac{\left(a^2x^2+1\right)\arctan(ax)}{3a}+\frac{x}{3}\right.\right.$$
$$\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)^2-\frac{c^3\left(a^2x^2+1\right)^3\arctan(ax)}{21a}+\frac{1}{21}c\left(\frac{1}{5}a^4c^2x^5+\frac{2}{3}a^2c^2x^3+c^2x\right)$$

$\downarrow$ 5455

$$\frac{6}{7}c^3\left(\frac{4}{5}\left(\frac{2}{3}\left(x\arctan(ax)^2-2a\left(-\frac{\int\frac{\arctan(ax)}{i-ax}dx}{a}-\frac{i\arctan(ax)^2}{2a^2}\right)\right)+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)^2-\frac{(a^2x^2+1}{}\right.\right.$$
$$\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)^2-\frac{c^3\left(a^2x^2+1\right)^3\arctan(ax)}{21a}+\frac{1}{21}c\left(\frac{1}{5}a^4c^2x^5+\frac{2}{3}a^2c^2x^3+c^2x\right)$$

$\downarrow$ 5379

$$\frac{6}{7}c^3\left(\frac{4}{5}\left(\frac{2}{3}\left(x\arctan(ax)^2-2a\left(-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}-\int\frac{\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^2}{2a^2}\right)\right)+\frac{1}{3}x\left(a^2x^2+1\right)\mathrm{ar}\right.\right.$$
$$\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)^2-\frac{c^3\left(a^2x^2+1\right)^3\arctan(ax)}{21a}+\frac{1}{21}c\left(\frac{1}{5}a^4c^2x^5+\frac{2}{3}a^2c^2x^3+c^2x\right)$$

$\downarrow$ 2849

$$\frac{6}{7}c^3\left(\frac{4}{5}\left(\frac{2}{3}\left(x\arctan(ax)^2-2a\left(-\frac{\frac{i\int\frac{\log\left(\frac{2}{iax+1}\right)}{1-\frac{2}{iax+1}}d\frac{1}{iax+1}}{a}+\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}}{a}-\frac{i\arctan(ax)^2}{2a^2}\right)\right)+\frac{1}{3}x(a^2x^2+1\right.\right.$$
$$\frac{1}{7}c^3x(a^2x^2+1)^3\arctan(ax)^2-\frac{c^3(a^2x^2+1)^3\arctan(ax)}{21a}+\frac{1}{21}c\left(\frac{1}{5}a^4c^2x^5+\frac{2}{3}a^2c^2x^3+c^2x\right)$$

$\Big\downarrow$ 2752

$$\frac{6}{7}c^3\left(\frac{4}{5}\left(\frac{2}{3}\left(x\arctan(ax)^2-2a\left(-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}\right)\right)+\frac{1}{3}x(a^2x^2+\right.\right.$$
$$\frac{1}{7}c^3x(a^2x^2+1)^3\arctan(ax)^2-\frac{c^3(a^2x^2+1)^3\arctan(ax)}{21a}+\frac{1}{21}c\left(\frac{1}{5}a^4c^2x^5+\frac{2}{3}a^2c^2x^3+c^2x\right)$$

input
```
Int[(c + a^2*c*x^2)^3*ArcTan[a*x]^2,x]
```

output
```
(c*(c^2*x + (2*a^2*c^2*x^3)/3 + (a^4*c^2*x^5)/5))/21 - (c^3*(1 + a^2*x^2)^
3*ArcTan[a*x])/(21*a) + (c^3*x*(1 + a^2*x^2)^3*ArcTan[a*x]^2)/7 + (6*c^3*(
(x + (a^2*x^3)/3)/10 - ((1 + a^2*x^2)^2*ArcTan[a*x])/(10*a) + (x*(1 + a^2*
x^2)^2*ArcTan[a*x]^2)/5 + (4*(x/3 - ((1 + a^2*x^2)*ArcTan[a*x])/(3*a) + (x
*(1 + a^2*x^2)*ArcTan[a*x]^2)/3 + (2*(x*ArcTan[a*x]^2 - 2*a*(((-1/2*I)*Arc
Tan[a*x]^2)/a^2 - ((ArcTan[a*x]*Log[2/(1 + I*a*x)])/a + ((I/2)*PolyLog[2,
1 - 2/(1 + I*a*x)])/a)/a)))/3))/5))/7
```

## Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5415
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTan[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x])^p,
x], x] + Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*(
a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[q, 0] && GtQ[p, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

## Maple [A] (verified)

Time = 2.62 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.04

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{c^3 \arctan(ax)^2 a^7 x^7}{7}+\frac{3a^5c^3x^5 \arctan(ax)^2}{5}+a^3c^3x^3 \arctan(ax)^2+a\,c^3x \arctan(ax)^2-\frac{2c^3\left(\frac{5a^6 \arctan(ax)x^6}{6}+4x^4 \arctan(a\cdots\right.}{\cdots}}{\cdots}$ |
| default | $\frac{\frac{c^3 \arctan(ax)^2 a^7 x^7}{7}+\frac{3a^5c^3x^5 \arctan(ax)^2}{5}+a^3c^3x^3 \arctan(ax)^2+a\,c^3x \arctan(ax)^2-\frac{2c^3\left(\frac{5a^6 \arctan(ax)x^6}{6}+4x^4 \arctan(a\cdots\right.}{\cdots}}{\cdots}$ |
| parts | $\frac{c^3 \arctan(ax)^2 a^6 x^7}{7}+\frac{3c^3 \arctan(ax)^2 a^4 x^5}{5}+c^3 \arctan(ax)^2 a^2x^3+c^3x \arctan(ax)^2-\frac{2c^3\left(\frac{5a^5 \cdots}{\cdots}\right.}{\cdots}$ |
| risch | $\frac{19c^3x^3a^2}{315}+\frac{a^4c^3x^5}{105}+\frac{38c^3x}{105}-\frac{38c^3 \arctan(ax)}{105a}-\frac{3c^3a^4 \ln(-iax+1)^2x^5}{20}-\frac{c^3a^6 \ln(-iax+1)^2x^7}{28}-\frac{c^3a^2 \ln(\cdots}{\cdots}$ |

input
```
int((a^2*c*x^2+c)^3*arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a*(1/7*c^3*arctan(a*x)^2*a^7*x^7+3/5*a^5*c^3*x^5*arctan(a*x)^2+a^3*c^3*x
^3*arctan(a*x)^2+a*c^3*x*arctan(a*x)^2-2/35*c^3*(5/6*a^6*arctan(a*x)*x^6+4
*x^4*arctan(a*x)*a^4+19/2*x^2*a^2*arctan(a*x)+8*arctan(a*x)*ln(a^2*x^2+1)-
1/6*a^5*x^5-19/18*a^3*x^3-19/3*a*x+19/3*arctan(a*x)+4*I*(ln(a*x-I)*ln(a^2*
x^2+1)-1/2*ln(a*x-I)^2-dilog(-1/2*I*(a*x+I))-ln(a*x-I)*ln(-1/2*I*(a*x+I)))
-4*I*(ln(a*x+I)*ln(a^2*x^2+1)-1/2*ln(a*x+I)^2-dilog(1/2*I*(a*x-I))-ln(a*x+
I)*ln(1/2*I*(a*x-I)))))
```

## Fricas [F]

$$\int (c+a^2cx^2)^3 \arctan(ax)^2\,dx=\int \left(a^2cx^2+c\right)^3 \arctan(ax)^2\,dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a*x)^2
, x)
```

## Sympy [F]

$$\int \left(c+a^2cx^2\right)^3 \arctan(ax)^2\,dx = c^3\left(\int 3a^2x^2\operatorname{atan}^2\left(ax\right)dx + \int 3a^4x^4\operatorname{atan}^2\left(ax\right)dx + \int a^6x^6\operatorname{atan}^2\left(ax\right)dx + \int \operatorname{atan}^2\left(ax\right)dx\right)$$

input
```
integrate((a**2*c*x**2+c)**3*atan(a*x)**2,x)
```

output
```
c**3*(Integral(3*a**2*x**2*atan(a*x)**2, x) + Integral(3*a**4*x**4*atan(a*
x)**2, x) + Integral(a**6*x**6*atan(a*x)**2, x) + Integral(atan(a*x)**2, x
))
```

## Maxima [F]

$$\int \left(c+a^2cx^2\right)^3 \arctan(ax)^2\,dx = \int \left(a^2cx^2+c\right)^3\arctan\left(ax\right)^2\,dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^2,x, algorithm="maxima")
```

output
```
420*a^8*c^3*integrate(1/560*x^8*arctan(a*x)^2/(a^2*x^2 + 1), x) + 35*a^8*c
^3*integrate(1/560*x^8*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) + 20*a^8*c^3*i
ntegrate(1/560*x^8*log(a^2*x^2 + 1)/(a^2*x^2 + 1), x) - 40*a^7*c^3*integra
te(1/560*x^7*arctan(a*x)/(a^2*x^2 + 1), x) + 1680*a^6*c^3*integrate(1/560*
x^6*arctan(a*x)^2/(a^2*x^2 + 1), x) + 140*a^6*c^3*integrate(1/560*x^6*log(
a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) + 84*a^6*c^3*integrate(1/560*x^6*log(a^2*
x^2 + 1)/(a^2*x^2 + 1), x) - 168*a^5*c^3*integrate(1/560*x^5*arctan(a*x)/(
a^2*x^2 + 1), x) + 2520*a^4*c^3*integrate(1/560*x^4*arctan(a*x)^2/(a^2*x^2
 + 1), x) + 210*a^4*c^3*integrate(1/560*x^4*log(a^2*x^2 + 1)^2/(a^2*x^2 +
1), x) + 140*a^4*c^3*integrate(1/560*x^4*log(a^2*x^2 + 1)/(a^2*x^2 + 1), x
) - 280*a^3*c^3*integrate(1/560*x^3*arctan(a*x)/(a^2*x^2 + 1), x) + 1680*a
^2*c^3*integrate(1/560*x^2*arctan(a*x)^2/(a^2*x^2 + 1), x) + 140*a^2*c^3*i
ntegrate(1/560*x^2*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) + 140*a^2*c^3*inte
grate(1/560*x^2*log(a^2*x^2 + 1)/(a^2*x^2 + 1), x) + 1/4*c^3*arctan(a*x)^3
/a - 280*a*c^3*integrate(1/560*x*arctan(a*x)/(a^2*x^2 + 1), x) + 35*c^3*in
tegrate(1/560*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) + 1/140*(5*a^6*c^3*x^7
+ 21*a^4*c^3*x^5 + 35*a^2*c^3*x^3 + 35*c^3*x)*arctan(a*x)^2 - 1/560*(5*a^6
*c^3*x^7 + 21*a^4*c^3*x^5 + 35*a^2*c^3*x^3 + 35*c^3*x)*log(a^2*x^2 + 1)^2
```

## Giac [F]

$$\int (c+a^2cx^2)^3 \arctan(ax)^2\,dx = \int (a^2cx^2+c)^3 \arctan(ax)^2\,dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^3*arctan(a*x)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \left(c+a^2cx^2\right)^3 \arctan(ax)^2\,dx = \int \operatorname{atan}(a\,x)^2\,\left(c\,a^2\,x^2+c\right)^3\,dx$$

input
```
int(atan(a*x)^2*(c + a^2*c*x^2)^3,x)
```

output
```
int(atan(a*x)^2*(c + a^2*c*x^2)^3, x)
```

## Reduce [F]

$$\int \left(c+a^2cx^2\right)^3 \arctan(ax)^2\,dx = \frac{c^3\left(45atan(ax)^2\,a^7x^7+189atan(ax)^2\,a^5x^5+315atan(ax)^2\,a^3x^3+315atan(ax)^2\,ax-15atan(ax)\,a^6x^6 -\right.}{3}$$

input
```
int((a^2*c*x^2+c)^3*atan(a*x)^2,x)
```

output
```
(c**3*(45*atan(a*x)**2*a**7*x**7 + 189*atan(a*x)**2*a**5*x**5 + 315*atan(a
*x)**2*a**3*x**3 + 315*atan(a*x)**2*a*x - 15*atan(a*x)*a**6*x**6 - 72*atan
(a*x)*a**4*x**4 - 171*atan(a*x)*a**2*x**2 - 114*atan(a*x) - 288*int((atan(
a*x)*x)/(a**2*x**2 + 1),x)*a**2 + 3*a**5*x**5 + 19*a**3*x**3 + 114*a*x))/(
315*a)
```

# 3.278 $\int \frac{(c+a^2cx^2)^3\arctan(ax)^2}{x}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 287

$$\begin{aligned}\int \frac{(c+a^2cx^2)^3\arctan(ax)^2}{x}\,dx &= \frac{29}{180}a^2c^3x^2+\frac{1}{60}a^4c^3x^4-\frac{11}{6}ac^3x\arctan(ax)\\ &-\frac{7}{18}a^3c^3x^3\arctan(ax)-\frac{1}{15}a^5c^3x^5\arctan(ax)\\ &+\frac{11}{12}c^3\arctan(ax)^2+\frac{3}{2}a^2c^3x^2\arctan(ax)^2\\ &+\frac{3}{4}a^4c^3x^4\arctan(ax)^2+\frac{1}{6}a^6c^3x^6\arctan(ax)^2\\ &+2c^3\arctan(ax)^2\operatorname{arctanh}\left(1-\frac{2}{1+iax}\right)\\ &+\frac{34}{45}c^3\log\left(1+a^2x^2\right)\\ &-ic^3\arctan(ax)\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right)\\ &+ic^3\arctan(ax)\operatorname{PolyLog}\left(2,-1+\frac{2}{1+iax}\right)\\ &-\frac{1}{2}c^3\operatorname{PolyLog}\left(3,1-\frac{2}{1+iax}\right)\\ &+\frac{1}{2}c^3\operatorname{PolyLog}\left(3,-1+\frac{2}{1+iax}\right)\end{aligned}$$

output
```
29/180*a^2*c^3*x^2+1/60*a^4*c^3*x^4-11/6*a*c^3*x*arctan(a*x)-7/18*a^3*c^3*
x^3*arctan(a*x)-1/15*a^5*c^3*x^5*arctan(a*x)+11/12*c^3*arctan(a*x)^2+3/2*a
^2*c^3*x^2*arctan(a*x)^2+3/4*a^4*c^3*x^4*arctan(a*x)^2+1/6*a^6*c^3*x^6*arc
tan(a*x)^2-2*c^3*arctan(a*x)^2*arctanh(-1+2/(1+I*a*x))+34/45*c^3*ln(a^2*x^
2+1)-I*c^3*arctan(a*x)*polylog(2,1-2/(1+I*a*x))+I*c^3*arctan(a*x)*polylog(
2,-1+2/(1+I*a*x))-1/2*c^3*polylog(3,1-2/(1+I*a*x))+1/2*c^3*polylog(3,-1+2/
(1+I*a*x))
```

## Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.88

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^2}{x} \, dx = \frac{1}{360}c^3\big(52 - 15i\pi^3 + 58a^2x^2 + 6a^4x^4 - 660ax\arctan(ax) - 140a^3x^3\arctan(ax) - 24a^5x^5\arctan(ax) + 330\arctan(ax)^2 + 540a^2x^2\arctan(ax)^2 + 270a^4x^4\arctan(ax)^2 + 60a^6x^6\arctan(ax)^2 + 240i\arctan(ax)^3 + 360\arctan(ax)^2\log\left(1-e^{-2i\arctan(ax)}\right) - 360\arctan(ax)^2\log\left(1+e^{2i\arctan(ax)}\right) + 272\log\left(1+a^2x^2\right) + 360i\arctan(ax)\operatorname{PolyLog}\left(2,e^{-2i\arctan(ax)}\right) + 360i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{2i\arctan(ax)}\right) + 180\operatorname{PolyLog}\left(3,e^{-2i\arctan(ax)}\right) - 180\operatorname{PolyLog}\left(3,-e^{2i\arctan(ax)}\right)\big)$$

input
```
Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x]^2)/x,x]
```

output
```
(c^3*(52 - (15*I)*Pi^3 + 58*a^2*x^2 + 6*a^4*x^4 - 660*a*x*ArcTan[a*x] - 14
0*a^3*x^3*ArcTan[a*x] - 24*a^5*x^5*ArcTan[a*x] + 330*ArcTan[a*x]^2 + 540*a
^2*x^2*ArcTan[a*x]^2 + 270*a^4*x^4*ArcTan[a*x]^2 + 60*a^6*x^6*ArcTan[a*x]^
2 + (240*I)*ArcTan[a*x]^3 + 360*ArcTan[a*x]^2*Log[1 - E^((-2*I)*ArcTan[a*x
])] - 360*ArcTan[a*x]^2*Log[1 + E^((2*I)*ArcTan[a*x])] + 272*Log[1 + a^2*x
^2] + (360*I)*ArcTan[a*x]*PolyLog[2, E^((-2*I)*ArcTan[a*x])] + (360*I)*Arc
Tan[a*x]*PolyLog[2, -E^((2*I)*ArcTan[a*x])] + 180*PolyLog[3, E^((-2*I)*Arc
Tan[a*x])] - 180*PolyLog[3, -E^((2*I)*ArcTan[a*x])]))/360
```

## Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2 (a^2cx^2+c)^3}{x} \, dx$$

$$\Big\downarrow \text{5483}$$

$$\int \left(a^6c^3x^5\arctan(ax)^2+3a^4c^3x^3\arctan(ax)^2+3a^2c^3x\arctan(ax)^2+\frac{c^3\arctan(ax)^2}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{6}a^6c^3x^6\arctan(ax)^2-\frac{1}{15}a^5c^3x^5\arctan(ax)+\frac{3}{4}a^4c^3x^4\arctan(ax)^2+\frac{1}{60}a^4c^3x^4-\frac{7}{18}a^3c^3x^3\arctan(ax)+\frac{3}{2}a^2c^3x^2\arctan(ax)^2+\frac{29}{180}a^2c^3x^2+\frac{34}{45}c^3\log(a^2x^2+1)+2c^3\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{1+iax}\right)-ic^3\arctan(ax)\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)+ic^3\arctan(ax)\text{PolyLog}\left(2,\frac{2}{iax+1}-1\right)-\frac{11}{6}ac^3x\arctan(ax)+\frac{11}{12}c^3\arctan(ax)^2-\frac{1}{2}c^3\text{PolyLog}\left(3,1-\frac{2}{iax+1}\right)+\frac{1}{2}c^3\text{PolyLog}\left(3,\frac{2}{iax+1}-1\right)$$

input

```
Int[((c + a^2*c*x^2)^3*ArcTan[a*x]^2)/x,x]
```

output

```
(29*a^2*c^3*x^2)/180 + (a^4*c^3*x^4)/60 - (11*a*c^3*x*ArcTan[a*x])/6 - (7*
a^3*c^3*x^3*ArcTan[a*x])/18 - (a^5*c^3*x^5*ArcTan[a*x])/15 + (11*c^3*ArcTa
n[a*x]^2)/12 + (3*a^2*c^3*x^2*ArcTan[a*x]^2)/2 + (3*a^4*c^3*x^4*ArcTan[a*x
]^2)/4 + (a^6*c^3*x^6*ArcTan[a*x]^2)/6 + 2*c^3*ArcTan[a*x]^2*ArcTanh[1 - 2
/(1 + I*a*x)] + (34*c^3*Log[1 + a^2*x^2])/45 - I*c^3*ArcTan[a*x]*PolyLog[2
, 1 - 2/(1 + I*a*x)] + I*c^3*ArcTan[a*x]*PolyLog[2, -1 + 2/(1 + I*a*x)] -
(c^3*PolyLog[3, 1 - 2/(1 + I*a*x)])/2 + (c^3*PolyLog[3, -1 + 2/(1 + I*a*x)
])/2
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 50.00 (sec) , antiderivative size = 1405, normalized size of antiderivative = 4.90

| method | result | size |
|---|---|---|
| derivativedivides | Expression too large to display | 1405 |
| default | Expression too large to display | 1405 |
| parts | Expression too large to display | 1948 |

input
```
int((a^2*c*x^2+c)^3*arctan(a*x)^2/x,x,method=_RETURNVERBOSE)
```

output
```
1/6*a^6*c^3*x^6*arctan(a*x)^2+3/4*a^4*c^3*x^4*arctan(a*x)^2+3/2*a^2*c^3*x^
2*arctan(a*x)^2+c^3*arctan(a*x)^2*ln(a*x)-1/6*c^3*(-3*I*Pi*csgn(I*((1+I*a*
x)^2/(a^2*x^2+1)-1))*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)
^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*arctan(a*x)^2+2*arctan(a*x)
*(a*x-I)*(a*x+I)^4-11/2*arctan(a*x)^2-8*I*arctan(a*x)*(a*x-I)*(a*x+I)^3+3*
I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1
)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2-3*I*Pi*csgn(I*((1+I*a*x)^2/
(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-
1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*arctan(a*x)^2+3*I*Pi*csgn(I*((1+I*a*x)^2/(
a^2*x^2+1)-1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)
+1))^2*arctan(a*x)^2-3*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^
2/(a^2*x^2+1)+1))^3*arctan(a*x)^2+3*I*Pi*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/
((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2-3*I*Pi*csgn(((1+I*a*x)^2/(a^2
*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^3*arctan(a*x)^2-8*I*arctan(a*x)*(a
*x-I)^3*(a*x+I)-6*I*arctan(a*x)*(a*x-I)*(a*x+I)+12*I*arctan(a*x)*(a*x-I)^2
*(a*x+I)^2+3*I*arctan(a*x)*(a*x-I)^2+12*I*arctan(a*x)*polylog(2,-(1+I*a*x)
/(a^2*x^2+1)^(1/2))+12*I*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2)
)-3*I*Pi*arctan(a*x)^2-6*I*arctan(a*x)*polylog(2,-(1+I*a*x)^2/(a^2*x^2+1))
-5*arctan(a*x)*(a*x-I)*(a*x+I)^2+5*arctan(a*x)*(a*x-I)^2*(a*x+I)+3*I*Pi*cs
gn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(((1+...
```

## Fricas [F]

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^2}{x} \, dx = \int \frac{(a^2cx^2+c)^3 \arctan(ax)^2}{x} \, dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^2/x,x, algorithm="fricas")
```

output
```
integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a*x)^2
/x, x)
```

## Sympy [F]

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^2}{x}\,dx = c^3\left(\int \frac{\operatorname{atan}^2(ax)}{x}\,dx + \int 3a^2x\operatorname{atan}^2(ax)\,dx + \int 3a^4x^3\operatorname{atan}^2(ax)\,dx + \int a^6x^5\operatorname{atan}^2(ax)\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**3*atan(a*x)**2/x,x)
```

output
```
c**3*(Integral(atan(a*x)**2/x, x) + Integral(3*a**2*x*atan(a*x)**2, x) + I
ntegral(3*a**4*x**3*atan(a*x)**2, x) + Integral(a**6*x**5*atan(a*x)**2, x)
)
```

## Maxima [F]

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^2}{x}\,dx = \int \frac{(a^2cx^2+c)^3 \arctan(ax)^2}{x}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^2/x,x, algorithm="maxima")
```

output
```
36*a^8*c^3*integrate(1/48*x^8*arctan(a*x)^2/(a^2*x^3 + x), x) + 3*a^8*c^3*
integrate(1/48*x^8*log(a^2*x^2 + 1)^2/(a^2*x^3 + x), x) + 2*a^8*c^3*integr
ate(1/48*x^8*log(a^2*x^2 + 1)/(a^2*x^3 + x), x) - 4*a^7*c^3*integrate(1/48
*x^7*arctan(a*x)/(a^2*x^3 + x), x) + 144*a^6*c^3*integrate(1/48*x^6*arctan
(a*x)^2/(a^2*x^3 + x), x) + 12*a^6*c^3*integrate(1/48*x^6*log(a^2*x^2 + 1)
^2/(a^2*x^3 + x), x) + 9*a^6*c^3*integrate(1/48*x^6*log(a^2*x^2 + 1)/(a^2*
x^3 + x), x) - 18*a^5*c^3*integrate(1/48*x^5*arctan(a*x)/(a^2*x^3 + x), x)
 + 216*a^4*c^3*integrate(1/48*x^4*arctan(a*x)^2/(a^2*x^3 + x), x) + 18*a^4
*c^3*integrate(1/48*x^4*log(a^2*x^2 + 1)^2/(a^2*x^3 + x), x) + 18*a^4*c^3*
integrate(1/48*x^4*log(a^2*x^2 + 1)/(a^2*x^3 + x), x) - 36*a^3*c^3*integra
te(1/48*x^3*arctan(a*x)/(a^2*x^3 + x), x) + 144*a^2*c^3*integrate(1/48*x^2
*arctan(a*x)^2/(a^2*x^3 + x), x) + 1/24*c^3*log(a^2*x^2 + 1)^3 + 36*c^3*in
tegrate(1/48*arctan(a*x)^2/(a^2*x^3 + x), x) + 3*c^3*integrate(1/48*log(a^
2*x^2 + 1)^2/(a^2*x^3 + x), x) + 1/48*(2*a^6*c^3*x^6 + 9*a^4*c^3*x^4 + 18*
a^2*c^3*x^2)*arctan(a*x)^2 - 1/192*(2*a^6*c^3*x^6 + 9*a^4*c^3*x^4 + 18*a^2
*c^3*x^2)*log(a^2*x^2 + 1)^2
```

## Giac [F]

$$\int \frac{(c+a^2cx^2)^3\arctan(ax)^2}{x}\,dx = \int \frac{(a^2cx^2+c)^3\arctan(ax)^2}{x}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^2/x,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^3*arctan(a*x)^2/x, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^3\arctan(ax)^2}{x}\,dx = \int \frac{\operatorname{atan}(a\,x)^2\,(c\,a^2\,x^2+c)^3}{x}\,dx$$

input
```
int((atan(a*x)^2*(c + a^2*c*x^2)^3)/x,x)
```

output
```
int((atan(a*x)^2*(c + a^2*c*x^2)^3)/x, x)
```

## Reduce [F]

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^2}{x}\,dx$$
$$= \frac{c^3\Big(30atan(ax)^2\,a^6x^6+135atan(ax)^2\,a^4x^4+270atan(ax)^2\,a^2x^2+165atan(ax)^2-12atan(ax)\,a^5x^5-70}{180}$$

input
```
int((a^2*c*x^2+c)^3*atan(a*x)^2/x,x)
```

output
```
(c**3*(30*atan(a*x)**2*a**6*x**6 + 135*atan(a*x)**2*a**4*x**4 + 270*atan(a
*x)**2*a**2*x**2 + 165*atan(a*x)**2 - 12*atan(a*x)*a**5*x**5 - 70*atan(a*x
)*a**3*x**3 - 330*atan(a*x)*a*x + 180*int(atan(a*x)**2/x,x) + 136*log(a**2
*x**2 + 1) + 3*a**4*x**4 + 29*a**2*x**2))/180
```

# 3.279 $\int \frac{\left(c+a^2cx^2\right)^3\arctan(ax)^2}{x^2}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 251

$$\begin{aligned}\int \frac{(c+a^2cx^2)^3\arctan(ax)^2}{x^2}\,dx &= \frac{7}{10}a^2c^3x+\frac{1}{30}a^4c^3x^3-\frac{7}{10}ac^3\arctan(ax)\\&\quad-\frac{4}{5}a^3c^3x^2\arctan(ax)-\frac{1}{10}a^5c^3x^4\arctan(ax)\\&\quad+\frac{6}{5}iac^3\arctan(ax)^2-\frac{c^3\arctan(ax)^2}{x}\\&\quad+3a^2c^3x\arctan(ax)^2+a^4c^3x^3\arctan(ax)^2\\&\quad+\frac{1}{5}a^6c^3x^5\arctan(ax)^2\\&\quad+\frac{22}{5}ac^3\arctan(ax)\log\left(\frac{2}{1+iax}\right)\\&\quad+2ac^3\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)\\&\quad-iac^3\,\mathrm{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)\\&\quad+\frac{11}{5}iac^3\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+iax}\right)\end{aligned}$$

output
```
7/10*a^2*c^3*x+1/30*a^4*c^3*x^3-7/10*a*c^3*arctan(a*x)-4/5*a^3*c^3*x^2*arc
tan(a*x)-1/10*a^5*c^3*x^4*arctan(a*x)+6/5*I*a*c^3*arctan(a*x)^2-c^3*arctan
(a*x)^2/x+3*a^2*c^3*x*arctan(a*x)^2+a^4*c^3*x^3*arctan(a*x)^2+1/5*a^6*c^3*
x^5*arctan(a*x)^2+22/5*a*c^3*arctan(a*x)*ln(2/(1+I*a*x))+2*a*c^3*arctan(a*
x)*ln(2-2/(1-I*a*x))-I*a*c^3*polylog(2,-1+2/(1-I*a*x))+11/5*I*a*c^3*polylo
g(2,1-2/(1+I*a*x))
```

## Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.80

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^2}{x^2} dx$$
$$= \frac{c^3(21a^2x^2 + a^4x^4 - 21ax\arctan(ax) - 24a^3x^3\arctan(ax) - 3a^5x^5\arctan(ax) - 30\arctan(ax)^2 - 96iax}{}$$

input
```
Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x]^2)/x^2,x]
```

output
```
(c^3*(21*a^2*x^2 + a^4*x^4 - 21*a*x*ArcTan[a*x] - 24*a^3*x^3*ArcTan[a*x] -
 3*a^5*x^5*ArcTan[a*x] - 30*ArcTan[a*x]^2 - (96*I)*a*x*ArcTan[a*x]^2 + 90*
a^2*x^2*ArcTan[a*x]^2 + 30*a^4*x^4*ArcTan[a*x]^2 + 6*a^6*x^6*ArcTan[a*x]^2
 + 60*a*x*ArcTan[a*x]*Log[1 - E^((2*I)*ArcTan[a*x])] + 132*a*x*ArcTan[a*x]
*Log[1 + E^((2*I)*ArcTan[a*x])] - (66*I)*a*x*PolyLog[2, -E^((2*I)*ArcTan[a
*x])] - (30*I)*a*x*PolyLog[2, E^((2*I)*ArcTan[a*x])]))/(30*x)
```

## Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2 (a^2cx^2+c)^3}{x^2} dx$$

$$\Big\downarrow \text{5483}$$

$$\int \left(a^6c^3x^4\arctan(ax)^2+3a^4c^3x^2\arctan(ax)^2+3a^2c^3\arctan(ax)^2+\frac{c^3\arctan(ax)^2}{x^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}a^6c^3x^5\arctan(ax)^2-\frac{1}{10}a^5c^3x^4\arctan(ax)+a^4c^3x^3\arctan(ax)^2+\frac{1}{30}a^4c^3x^3-\frac{4}{5}a^3c^3x^2\arctan(ax)+3a^2c^3x\arctan(ax)^2+\frac{7}{10}a^2c^3x+\frac{6}{5}iac^3\arctan(ax)^2-\frac{7}{10}ac^3\arctan(ax)-\frac{c^3\arctan(ax)^2}{x}+\frac{22}{5}ac^3\arctan(ax)\log\left(\frac{2}{1+iax}\right)+2ac^3\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-iac^3\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)+\frac{11}{5}iac^3\operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right)$$

input
```
Int[((c + a^2*c*x^2)^3*ArcTan[a*x]^2)/x^2,x]
```

output
```
(7*a^2*c^3*x)/10 + (a^4*c^3*x^3)/30 - (7*a*c^3*ArcTan[a*x])/10 - (4*a^3*c^
3*x^2*ArcTan[a*x])/5 - (a^5*c^3*x^4*ArcTan[a*x])/10 + ((6*I)/5)*a*c^3*ArcT
an[a*x]^2 - (c^3*ArcTan[a*x]^2)/x + 3*a^2*c^3*x*ArcTan[a*x]^2 + a^4*c^3*x^
3*ArcTan[a*x]^2 + (a^6*c^3*x^5*ArcTan[a*x]^2)/5 + (22*a*c^3*ArcTan[a*x]*Lo
g[2/(1 + I*a*x)])/5 + 2*a*c^3*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)] - I*a*c^3
*PolyLog[2, -1 + 2/(1 - I*a*x)] + ((11*I)/5)*a*c^3*PolyLog[2, 1 - 2/(1 + I
*a*x)]
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

## Maple [A] (verified)

Time = 1.32 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.28

| method | result |
|---|---|
| derivativedivides | $a\left(\frac{a^5c^3x^5\arctan(ax)^2}{5}+a^3c^3x^3\arctan(ax)^2+3a\,c^3x\arctan(ax)^2-\frac{c^3\arctan(ax)^2}{ax}-\frac{2c^3\left(\frac{x^4 ...}{}\right.}{}\right.$ |
| default | $a\left(\frac{a^5c^3x^5\arctan(ax)^2}{5}+a^3c^3x^3\arctan(ax)^2+3a\,c^3x\arctan(ax)^2-\frac{c^3\arctan(ax)^2}{ax}-\frac{2c^3\left(\frac{x^4 ...}{}\right.}{}\right.$ |
| parts | $\frac{a^6c^3x^5\arctan(ax)^2}{5}+a^4c^3x^3\arctan(ax)^2+3a^2c^3x\arctan(ax)^2-\frac{c^3\arctan(ax)^2}{x}-\frac{2c^3\left(\frac{\arctan( ...}{}\right.}{}$ |

input
```
int((a^2*c*x^2+c)^3*arctan(a*x)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
a*(1/5*a^5*c^3*x^5*arctan(a*x)^2+a^3*c^3*x^3*arctan(a*x)^2+3*a*c^3*x*arcta
n(a*x)^2-c^3*arctan(a*x)^2/a/x-2/5*c^3*(1/4*x^4*arctan(a*x)*a^4+2*x^2*a^2*
arctan(a*x)-5*arctan(a*x)*ln(a*x)+8*arctan(a*x)*ln(a^2*x^2+1)-1/12*a^3*x^3
-7/4*a*x+7/4*arctan(a*x)-5/2*I*ln(a*x)*ln(1+I*a*x)+5/2*I*ln(a*x)*ln(1-I*a*
x)-5/2*I*dilog(1+I*a*x)+5/2*I*dilog(1-I*a*x)+4*I*(ln(a*x-I)*ln(a^2*x^2+1)-
1/2*ln(a*x-I)^2-dilog(-1/2*I*(a*x+I))-ln(a*x-I)*ln(-1/2*I*(a*x+I)))-4*I*(l
n(a*x+I)*ln(a^2*x^2+1)-1/2*ln(a*x+I)^2-dilog(1/2*I*(a*x-I))-ln(a*x+I)*ln(1
/2*I*(a*x-I)))))
```

## Fricas [F]

$$\int \frac{(c+a^2cx^2)^3\arctan(ax)^2}{x^2}\,dx=\int \frac{(a^2cx^2+c)^3\arctan(ax)^2}{x^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^2/x^2,x, algorithm="fricas")
```

output
```
integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a*x)^2
/x^2, x)
```

## Sympy [F]

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^2}{x^2}\,dx = c^3\left(\int 3a^2 \operatorname{atan}^2(ax)\,dx + \int \frac{\operatorname{atan}^2(ax)}{x^2}\,dx + \int 3a^4x^2 \operatorname{atan}^2(ax)\,dx + \int a^6x^4 \operatorname{atan}^2(ax)\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**3*atan(a*x)**2/x**2,x)
```

output
```
c**3*(Integral(3*a**2*atan(a*x)**2, x) + Integral(atan(a*x)**2/x**2, x) +
Integral(3*a**4*x**2*atan(a*x)**2, x) + Integral(a**6*x**4*atan(a*x)**2, x
))
```

## Maxima [F]

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^2}{x^2}\,dx = \int \frac{(a^2cx^2+c)^3 \arctan(ax)^2}{x^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^2/x^2,x, algorithm="maxima")
```

output
```
1/80*(4*(a^6*c^3*x^6 + 5*a^4*c^3*x^4 + 15*a^2*c^3*x^2 - 5*c^3)*arctan(a*x)
^2 - (a^6*c^3*x^6 + 5*a^4*c^3*x^4 + 15*a^2*c^3*x^2 - 5*c^3)*log(a^2*x^2 +
1)^2 + 80*(60*a^8*c^3*integrate(1/80*x^8*arctan(a*x)^2/(a^2*x^4 + x^2), x)
 + 5*a^8*c^3*integrate(1/80*x^8*log(a^2*x^2 + 1)^2/(a^2*x^4 + x^2), x) + 4
*a^8*c^3*integrate(1/80*x^8*log(a^2*x^2 + 1)/(a^2*x^4 + x^2), x) - 8*a^7*c
^3*integrate(1/80*x^7*arctan(a*x)/(a^2*x^4 + x^2), x) + 240*a^6*c^3*integr
ate(1/80*x^6*arctan(a*x)^2/(a^2*x^4 + x^2), x) + 20*a^6*c^3*integrate(1/80
*x^6*log(a^2*x^2 + 1)^2/(a^2*x^4 + x^2), x) + 20*a^6*c^3*integrate(1/80*x^
6*log(a^2*x^2 + 1)/(a^2*x^4 + x^2), x) - 40*a^5*c^3*integrate(1/80*x^5*arc
tan(a*x)/(a^2*x^4 + x^2), x) + 360*a^4*c^3*integrate(1/80*x^4*arctan(a*x)^
2/(a^2*x^4 + x^2), x) + 30*a^4*c^3*integrate(1/80*x^4*log(a^2*x^2 + 1)^2/(
a^2*x^4 + x^2), x) + 60*a^4*c^3*integrate(1/80*x^4*log(a^2*x^2 + 1)/(a^2*x
^4 + x^2), x) + a*c^3*arctan(a*x)^3 - 120*a^3*c^3*integrate(1/80*x^3*arcta
n(a*x)/(a^2*x^4 + x^2), x) + 20*a^2*c^3*integrate(1/80*x^2*log(a^2*x^2 + 1
)^2/(a^2*x^4 + x^2), x) - 20*a^2*c^3*integrate(1/80*x^2*log(a^2*x^2 + 1)/(
a^2*x^4 + x^2), x) + 40*a*c^3*integrate(1/80*x*arctan(a*x)/(a^2*x^4 + x^2)
, x) + 60*c^3*integrate(1/80*arctan(a*x)^2/(a^2*x^4 + x^2), x) + 5*c^3*int
egrate(1/80*log(a^2*x^2 + 1)^2/(a^2*x^4 + x^2), x))*x)/x
```

## Giac [F]

$$\int \frac{(c+a^2cx^2)^3\arctan(ax)^2}{x^2}\,dx=\int \frac{(a^2cx^2+c)^3\arctan{(ax)}^2}{x^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^2/x^2,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^3*arctan(a*x)^2/x^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^3\arctan(ax)^2}{x^2}\,dx=\int \frac{\mathrm{atan}(a\,x)^2\,{(c\,a^2\,x^2+c)}^3}{x^2}\,dx$$

input
```
int((atan(a*x)^2*(c + a^2*c*x^2)^3)/x^2,x)
```

output
```
int((atan(a*x)^2*(c + a^2*c*x^2)^3)/x^2, x)
```

## Reduce [F]

$$\int \frac{(c+a^2cx^2)^3\arctan(ax)^2}{x^2}\,dx = \frac{c^3\left(6atan(ax)^2a^6x^6+30atan(ax)^2a^4x^4+90atan(ax)^2a^2x^2-30atan(ax)^2-3atan(ax)a^5x^5-24atan(\right.}{30x}$$

input
```
int((a^2*c*x^2+c)^3*atan(a*x)^2/x^2,x)
```

output
```
(c**3*(6*atan(a*x)**2*a**6*x**6 + 30*atan(a*x)**2*a**4*x**4 + 90*atan(a*x)
**2*a**2*x**2 - 30*atan(a*x)**2 - 3*atan(a*x)*a**5*x**5 - 24*atan(a*x)*a**
3*x**3 - 21*atan(a*x)*a*x + 60*int(atan(a*x)/(a**2*x**3 + x),x)*a*x - 132*
int((atan(a*x)*x)/(a**2*x**2 + 1),x)*a**3*x + a**4*x**4 + 21*a**2*x**2))/(
30*x)
```

# 3.280 $\int \frac{(c+a^2cx^2)^3 \arctan(ax)^2}{x^3} dx$



## Optimal result

Integrand size = 22, antiderivative size = 299

$$\begin{aligned}\int \frac{(c+a^2cx^2)^3 \arctan(ax)^2}{x^3} dx &= \frac{1}{12}a^4c^3x^2 - \frac{ac^3\arctan(ax)}{x} \\ &- \frac{5}{2}a^3c^3x\arctan(ax) - \frac{1}{6}a^5c^3x^3\arctan(ax) \\ &+ \frac{3}{4}a^2c^3\arctan(ax)^2 - \frac{c^3\arctan(ax)^2}{2x^2} \\ &+ \frac{3}{2}a^4c^3x^2\arctan(ax)^2 + \frac{1}{4}a^6c^3x^4\arctan(ax)^2 \\ &+ 6a^2c^3\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{1+iax}\right) \\ &+ a^2c^3\log(x) + \frac{2}{3}a^2c^3\log\left(1+a^2x^2\right) \\ &- 3ia^2c^3\arctan(ax)\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right) \\ &+ 3ia^2c^3\arctan(ax)\operatorname{PolyLog}\left(2,-1+\frac{2}{1+iax}\right) \\ &- \frac{3}{2}a^2c^3\operatorname{PolyLog}\left(3,1-\frac{2}{1+iax}\right) \\ &+ \frac{3}{2}a^2c^3\operatorname{PolyLog}\left(3,-1+\frac{2}{1+iax}\right)\end{aligned}$$

output
```
1/12*a^4*c^3*x^2-a*c^3*arctan(a*x)/x-5/2*a^3*c^3*x*arctan(a*x)-1/6*a^5*c^3
*x^3*arctan(a*x)+3/4*a^2*c^3*arctan(a*x)^2-1/2*c^3*arctan(a*x)^2/x^2+3/2*a
^4*c^3*x^2*arctan(a*x)^2+1/4*a^6*c^3*x^4*arctan(a*x)^2-6*a^2*c^3*arctan(a*
x)^2*arctanh(-1+2/(1+I*a*x))+a^2*c^3*ln(x)+2/3*a^2*c^3*ln(a^2*x^2+1)-3*I*a
^2*c^3*arctan(a*x)*polylog(2,1-2/(1+I*a*x))+3*I*a^2*c^3*arctan(a*x)*polylo
g(2,-1+2/(1+I*a*x))-3/2*a^2*c^3*polylog(3,1-2/(1+I*a*x))+3/2*a^2*c^3*polyl
og(3,-1+2/(1+I*a*x))
```

## Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.11

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^2}{x^3}\,dx$$
$$= \frac{c^3\Big(2a^2x^2 - 3ia^2\pi^3x^2 + 2a^4x^4 - 24ax\arctan(ax) - 60a^3x^3\arctan(ax) - 4a^5x^5\arctan(ax) - 12\arctan(\ldots}{}$$

input
```
Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x]^2)/x^3,x]
```

output
```
(c^3*(2*a^2*x^2 - (3*I)*a^2*Pi^3*x^2 + 2*a^4*x^4 - 24*a*x*ArcTan[a*x] - 60
*a^3*x^3*ArcTan[a*x] - 4*a^5*x^5*ArcTan[a*x] - 12*ArcTan[a*x]^2 + 18*a^2*x
^2*ArcTan[a*x]^2 + 36*a^4*x^4*ArcTan[a*x]^2 + 6*a^6*x^6*ArcTan[a*x]^2 + (4
8*I)*a^2*x^2*ArcTan[a*x]^3 + 72*a^2*x^2*ArcTan[a*x]^2*Log[1 - E^((-2*I)*Ar
cTan[a*x])] - 72*a^2*x^2*ArcTan[a*x]^2*Log[1 + E^((2*I)*ArcTan[a*x])] + 24
*a^2*x^2*Log[(a*x)/Sqrt[1 + a^2*x^2]] + 28*a^2*x^2*Log[1 + a^2*x^2] + (72*
I)*a^2*x^2*ArcTan[a*x]*PolyLog[2, E^((-2*I)*ArcTan[a*x])] + (72*I)*a^2*x^2
*ArcTan[a*x]*PolyLog[2, -E^((2*I)*ArcTan[a*x])] + 36*a^2*x^2*PolyLog[3, E^
((-2*I)*ArcTan[a*x])] - 36*a^2*x^2*PolyLog[3, -E^((2*I)*ArcTan[a*x])]))/(2
4*x^2)
```

## Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2 \left(a^2cx^2+c\right)^3}{x^3} \, dx$$

$$\Big\downarrow \text{5483}$$

$$\int \left(a^6c^3x^3\arctan(ax)^2+3a^4c^3x\arctan(ax)^2+\frac{3a^2c^3\arctan(ax)^2}{x}+\frac{c^3\arctan(ax)^2}{x^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}a^6c^3x^4\arctan(ax)^2-\frac{1}{6}a^5c^3x^3\arctan(ax)+\frac{3}{2}a^4c^3x^2\arctan(ax)^2+\frac{1}{12}a^4c^3x^2-\frac{5}{2}a^3c^3x\arctan(ax)+6a^2c^3\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{1+iax}\right)-3ia^2c^3\arctan(ax)\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)+3ia^2c^3\arctan(ax)\text{PolyLog}\left(2,\frac{2}{iax+1}-1\right)+\frac{3}{4}a^2c^3\arctan(ax)^2-\frac{3}{2}a^2c^3\text{PolyLog}\left(3,1-\frac{2}{iax+1}\right)+\frac{3}{2}a^2c^3\text{PolyLog}\left(3,\frac{2}{iax+1}-1\right)+\frac{2}{3}a^2c^3\log\left(a^2x^2+1\right)+a^2c^3\log(x)-\frac{c^3\arctan(ax)^2}{2x^2}-\frac{ac^3\arctan(ax)}{x}$$

input
```
Int[((c + a^2*c*x^2)^3*ArcTan[a*x]^2)/x^3,x]
```

output
```
(a^4*c^3*x^2)/12 - (a*c^3*ArcTan[a*x])/x - (5*a^3*c^3*x*ArcTan[a*x])/2 - (
a^5*c^3*x^3*ArcTan[a*x])/6 + (3*a^2*c^3*ArcTan[a*x]^2)/4 - (c^3*ArcTan[a*x
]^2)/(2*x^2) + (3*a^4*c^3*x^2*ArcTan[a*x]^2)/2 + (a^6*c^3*x^4*ArcTan[a*x]^
2)/4 + 6*a^2*c^3*ArcTan[a*x]^2*ArcTanh[1 - 2/(1 + I*a*x)] + a^2*c^3*Log[x]
 + (2*a^2*c^3*Log[1 + a^2*x^2])/3 - (3*I)*a^2*c^3*ArcTan[a*x]*PolyLog[2, 1
 - 2/(1 + I*a*x)] + (3*I)*a^2*c^3*ArcTan[a*x]*PolyLog[2, -1 + 2/(1 + I*a*x
)] - (3*a^2*c^3*PolyLog[3, 1 - 2/(1 + I*a*x)])/2 + (3*a^2*c^3*PolyLog[3, -
1 + 2/(1 + I*a*x)])/2
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 60.52 (sec) , antiderivative size = 1318, normalized size of antiderivative = 4.41

| method | result | size |
|---|---|---|
| derivativedivides | Expression too large to display | 1318 |
| default | Expression too large to display | 1318 |
| parts | Expression too large to display | 1754 |

input
```
int((a^2*c*x^2+c)^3*arctan(a*x)^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
a^2*(1/4*a^4*c^3*x^4*arctan(a*x)^2+3/2*a^2*c^3*x^2*arctan(a*x)^2+3*c^3*arc
tan(a*x)^2*ln(a*x)-1/2*c^3*arctan(a*x)^2/a^2/x^2-1/2*c^3*(-3*I*Pi*csgn(I*(
(1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1
+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*arctan(a*x)^2-3/2*ar
ctan(a*x)^2-2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-1)+3*I*Pi*csgn(I/((1+I*a*x)^2
/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+
1)+1))^2*arctan(a*x)^2-3*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x
)^2/(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2
+1)+1))*arctan(a*x)^2+3*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I*((
1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2-2*l
n(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)
/((1+I*a*x)^2/(a^2*x^2+1)+1))^3*arctan(a*x)^2+3*I*Pi*csgn(((1+I*a*x)^2/(a^
2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2-3*I*Pi*csgn(((1+I
*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^3*arctan(a*x)^2-2*I*ar
ctan(a*x)*(a*x-I)*(a*x+I)+arctan(a*x)*(I*a*x+(a^2*x^2+1)^(1/2)+1)/a/x+arct
an(a*x)*(I*a*x-(a^2*x^2+1)^(1/2)+1)/a/x+12*I*arctan(a*x)*polylog(2,-(1+I*a
*x)/(a^2*x^2+1)^(1/2))+12*I*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1
/2))-3*I*Pi*arctan(a*x)^2-6*I*arctan(a*x)*polylog(2,-(1+I*a*x)^2/(a^2*x^2+
1))+arctan(a*x)*(a*x-I)*(a*x+I)^2-arctan(a*x)*(a*x-I)^2*(a*x+I)+3*I*Pi*csg
n(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(((1+I...
```

## Fricas [F]

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^2}{x^3}\,dx = \int \frac{(a^2cx^2+c)^3 \arctan(ax)^2}{x^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^2/x^3,x, algorithm="fricas")
```

output
```
integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a*x)^2
/x^3, x)
```

## Sympy [F]

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^2}{x^3}\,dx = c^3\left(\int \frac{\operatorname{atan}^2(ax)}{x^3}\,dx + \int \frac{3a^2\operatorname{atan}^2(ax)}{x}\,dx + \int 3a^4x\operatorname{atan}^2(ax)\,dx + \int a^6x^3\operatorname{atan}^2(ax)\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**3*atan(a*x)**2/x**3,x)
```

output
```
c**3*(Integral(atan(a*x)**2/x**3, x) + Integral(3*a**2*atan(a*x)**2/x, x)
+ Integral(3*a**4*x*atan(a*x)**2, x) + Integral(a**6*x**3*atan(a*x)**2, x)
)
```

## Maxima [F]

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^2}{x^3}\,dx = \int \frac{(a^2cx^2+c)^3\arctan(ax)^2}{x^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^2/x^3,x, algorithm="maxima")
```

output
```
1/64*(4*(192*a^8*c^3*integrate(1/16*x^8*arctan(a*x)^2/(a^2*x^5 + x^3), x)
+ 16*a^8*c^3*integrate(1/16*x^8*log(a^2*x^2 + 1)^2/(a^2*x^5 + x^3), x) + 1
6*a^8*c^3*integrate(1/16*x^8*log(a^2*x^2 + 1)/(a^2*x^5 + x^3), x) - 32*a^7
*c^3*integrate(1/16*x^7*arctan(a*x)/(a^2*x^5 + x^3), x) + 768*a^6*c^3*inte
grate(1/16*x^6*arctan(a*x)^2/(a^2*x^5 + x^3), x) + 64*a^6*c^3*integrate(1/
16*x^6*log(a^2*x^2 + 1)^2/(a^2*x^5 + x^3), x) + 96*a^6*c^3*integrate(1/16*
x^6*log(a^2*x^2 + 1)/(a^2*x^5 + x^3), x) - 192*a^5*c^3*integrate(1/16*x^5*
arctan(a*x)/(a^2*x^5 + x^3), x) + 1152*a^4*c^3*integrate(1/16*x^4*arctan(a
*x)^2/(a^2*x^5 + x^3), x) + a^2*c^3*log(a^2*x^2 + 1)^3 + 768*a^2*c^3*integ
rate(1/16*x^2*arctan(a*x)^2/(a^2*x^5 + x^3), x) + 64*a^2*c^3*integrate(1/1
6*x^2*log(a^2*x^2 + 1)^2/(a^2*x^5 + x^3), x) - 32*a^2*c^3*integrate(1/16*x
^2*log(a^2*x^2 + 1)/(a^2*x^5 + x^3), x) + 64*a*c^3*integrate(1/16*x*arctan
(a*x)/(a^2*x^5 + x^3), x) + 192*c^3*integrate(1/16*arctan(a*x)^2/(a^2*x^5
+ x^3), x) + 16*c^3*integrate(1/16*log(a^2*x^2 + 1)^2/(a^2*x^5 + x^3), x))
*x^2 + 4*(a^6*c^3*x^6 + 6*a^4*c^3*x^4 - 2*c^3)*arctan(a*x)^2 - (a^6*c^3*x^
6 + 6*a^4*c^3*x^4 - 2*c^3)*log(a^2*x^2 + 1)^2)/x^2
```

## Giac [F]

$$\int \frac{(c+a^2cx^2)^3\arctan(ax)^2}{x^3}\,dx=\int \frac{(a^2cx^2+c)^3\arctan(ax)^2}{x^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^2/x^3,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^3*arctan(a*x)^2/x^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^3\arctan(ax)^2}{x^3}\,dx=\int \frac{\mathrm{atan}(a\,x)^2\,(c\,a^2\,x^2+c)^3}{x^3}\,dx$$

input
```
int((atan(a*x)^2*(c + a^2*c*x^2)^3)/x^3,x)
```

output
```
int((atan(a*x)^2*(c + a^2*c*x^2)^3)/x^3, x)
```

## Reduce [F]

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^2}{x^3}\,dx$$
$$= \frac{c^3\Big(3atan(ax)^2 a^6x^6 + 18atan(ax)^2 a^4x^4 + 9atan(ax)^2 a^2x^2 - 6atan(ax)^2 - 2atan(ax)\,a^5x^5 - 30atan(ax}{12x^2}$$

input
```
int((a^2*c*x^2+c)^3*atan(a*x)^2/x^3,x)
```

output
```
(c**3*(3*atan(a*x)**2*a**6*x**6 + 18*atan(a*x)**2*a**4*x**4 + 9*atan(a*x)*
*2*a**2*x**2 - 6*atan(a*x)**2 - 2*atan(a*x)*a**5*x**5 - 30*atan(a*x)*a**3*
x**3 - 12*atan(a*x)*a*x + 36*int(atan(a*x)**2/x,x)*a**2*x**2 + 8*log(a**2*
x**2 + 1)*a**2*x**2 + 12*log(x)*a**2*x**2 + a**4*x**4))/(12*x**2)
```

# 3.281 $\int \frac{(c+a^2cx^2)^3 \arctan(ax)^2}{x^4} \, dx$



## Optimal result

Integrand size = 22, antiderivative size = 250

$$\begin{aligned}\int \frac{(c+a^2cx^2)^3 \arctan(ax)^2}{x^4} \, dx = & -\frac{a^2c^3}{3x} + \frac{1}{3}a^4c^3x - \frac{2}{3}a^3c^3\arctan(ax) \\ & -\frac{ac^3\arctan(ax)}{3x^2} - \frac{1}{3}a^5c^3x^2\arctan(ax) \\ & -\frac{c^3\arctan(ax)^2}{3x^3} - \frac{3a^2c^3\arctan(ax)^2}{x} \\ & +3a^4c^3x\arctan(ax)^2 + \frac{1}{3}a^6c^3x^3\arctan(ax)^2 \\ & +\frac{16}{3}a^3c^3\arctan(ax)\log\left(\frac{2}{1+iax}\right) \\ & +\frac{16}{3}a^3c^3\arctan(ax)\log\left(2-\frac{2}{1-iax}\right) \\ & -\frac{8}{3}ia^3c^3\,\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right) \\ & +\frac{8}{3}ia^3c^3\,\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)\end{aligned}$$

output
```
-1/3*a^2*c^3/x+1/3*a^4*c^3*x-2/3*a^3*c^3*arctan(a*x)-1/3*a*c^3*arctan(a*x)
/x^2-1/3*a^5*c^3*x^2*arctan(a*x)-1/3*c^3*arctan(a*x)^2/x^3-3*a^2*c^3*arcta
n(a*x)^2/x+3*a^4*c^3*x*arctan(a*x)^2+1/3*a^6*c^3*x^3*arctan(a*x)^2+16/3*a^
3*c^3*arctan(a*x)*ln(2/(1+I*a*x))+16/3*a^3*c^3*arctan(a*x)*ln(2-2/(1-I*a*x
))-8/3*I*a^3*c^3*polylog(2,-1+2/(1-I*a*x))+8/3*I*a^3*c^3*polylog(2,1-2/(1+
I*a*x))
```

## Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.88

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^2}{x^4} dx$$
$$= \frac{c^3(-a^2x^2 + a^4x^4 - ax\arctan(ax) - 2a^3x^3\arctan(ax) - a^5x^5\arctan(ax) - \arctan(ax)^2 - 9a^2x^2\arctan($$

input
```
Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x]^2)/x^4,x]
```

output
```
(c^3*(-(a^2*x^2) + a^4*x^4 - a*x*ArcTan[a*x] - 2*a^3*x^3*ArcTan[a*x] - a^5
*x^5*ArcTan[a*x] - ArcTan[a*x]^2 - 9*a^2*x^2*ArcTan[a*x]^2 - (16*I)*a^3*x^
3*ArcTan[a*x]^2 + 9*a^4*x^4*ArcTan[a*x]^2 + a^6*x^6*ArcTan[a*x]^2 + 16*a^3
*x^3*ArcTan[a*x]*Log[1 - E^((2*I)*ArcTan[a*x])] + 16*a^3*x^3*ArcTan[a*x]*L
og[1 + E^((2*I)*ArcTan[a*x])] - (8*I)*a^3*x^3*PolyLog[2, -E^((2*I)*ArcTan[
a*x])] - (8*I)*a^3*x^3*PolyLog[2, E^((2*I)*ArcTan[a*x])]))/(3*x^3)
```

## Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2 (a^2cx^2+c)^3}{x^4} dx$$

$$\Big\downarrow \text{5483}$$

$$\int \left(a^6c^3x^2\arctan(ax)^2+3a^4c^3\arctan(ax)^2+\frac{3a^2c^3\arctan(ax)^2}{x^2}+\frac{c^3\arctan(ax)^2}{x^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}a^6c^3x^3\arctan(ax)^2-\frac{1}{3}a^5c^3x^2\arctan(ax)+3a^4c^3x\arctan(ax)^2+\frac{1}{3}a^4c^3x-\frac{2}{3}a^3c^3\arctan(ax)+\frac{16}{3}a^3c^3\arctan(ax)\log\left(\frac{2}{1+iax}\right)+\frac{16}{3}a^3c^3\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{8}{3}ia^3c^3\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)+\frac{8}{3}ia^3c^3\operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right)-\frac{3a^2c^3\arctan(ax)^2}{x}-\frac{a^2c^3}{3x}-\frac{c^3\arctan(ax)^2}{3x^3}-\frac{ac^3\arctan(ax)}{3x^2}$$

input
```
Int[((c + a^2*c*x^2)^3*ArcTan[a*x]^2)/x^4,x]
```

output
```
-1/3*(a^2*c^3)/x + (a^4*c^3*x)/3 - (2*a^3*c^3*ArcTan[a*x])/3 - (a*c^3*ArcT
an[a*x])/(3*x^2) - (a^5*c^3*x^2*ArcTan[a*x])/3 - (c^3*ArcTan[a*x]^2)/(3*x^
3) - (3*a^2*c^3*ArcTan[a*x]^2)/x + 3*a^4*c^3*x*ArcTan[a*x]^2 + (a^6*c^3*x^
3*ArcTan[a*x]^2)/3 + (16*a^3*c^3*ArcTan[a*x]*Log[2/(1 + I*a*x)])/3 + (16*a
^3*c^3*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)])/3 - ((8*I)/3)*a^3*c^3*PolyLog[2
, -1 + 2/(1 - I*a*x)] + ((8*I)/3)*a^3*c^3*PolyLog[2, 1 - 2/(1 + I*a*x)]
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

## Maple [A] (verified)

Time = 1.77 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.29

| method | result |
|---|---|
| derivativedivides | $a^3\left(\frac{a^3c^3x^3\arctan(ax)^2}{3}+3a\,c^3x\arctan\left(ax\right)^2-\frac{3c^3\arctan(ax)^2}{ax}-\frac{c^3\arctan(ax)^2}{3a^3x^3}-\frac{2c^3\left(\frac{x^2a^2\arctan(}{2}\right.}{}\right.$ |
| default | $a^3\left(\frac{a^3c^3x^3\arctan(ax)^2}{3}+3a\,c^3x\arctan\left(ax\right)^2-\frac{3c^3\arctan(ax)^2}{ax}-\frac{c^3\arctan(ax)^2}{3a^3x^3}-\frac{2c^3\left(\frac{x^2a^2\arctan(}{2}\right.}{}\right.$ |
| parts | $\frac{a^6c^3x^3\arctan(ax)^2}{3}+3a^4c^3x\arctan\left(ax\right)^2-\frac{c^3\arctan(ax)^2}{3x^3}-\frac{3a^2c^3\arctan(ax)^2}{x}-\frac{2c^3\left(\frac{a^5\arctan(ax)x^}{2}\right.}{}$ |

input
```
int((a^2*c*x^2+c)^3*arctan(a*x)^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
a^3*(1/3*a^3*c^3*x^3*arctan(a*x)^2+3*a*c^3*x*arctan(a*x)^2-3*c^3*arctan(a*
x)^2/a/x-1/3*c^3*arctan(a*x)^2/a^3/x^3-2/3*c^3*(1/2*x^2*a^2*arctan(a*x)+1/
2*arctan(a*x)/a^2/x^2-8*arctan(a*x)*ln(a*x)+8*arctan(a*x)*ln(a^2*x^2+1)-1/
2*a*x+1/2/a/x+arctan(a*x)-4*I*ln(a*x)*ln(1+I*a*x)+4*I*ln(a*x)*ln(1-I*a*x)-
4*I*dilog(1+I*a*x)+4*I*dilog(1-I*a*x)+4*I*(ln(a*x-I)*ln(a^2*x^2+1)-1/2*ln(
a*x-I)^2-dilog(-1/2*I*(a*x+I))-ln(a*x-I)*ln(-1/2*I*(a*x+I)))-4*I*(ln(a*x+I
)*ln(a^2*x^2+1)-1/2*ln(a*x+I)^2-dilog(1/2*I*(a*x-I))-ln(a*x+I)*ln(1/2*I*(a
*x-I)))))
```

## Fricas [F]

$$\int\frac{(c+a^2cx^2)^3\arctan(ax)^2}{x^4}\,dx=\int\frac{\left(a^2cx^2+c\right)^3\arctan\left(ax\right)^2}{x^4}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^2/x^4,x, algorithm="fricas")
```

output
```
integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a*x)^2
/x^4, x)
```

## Sympy [F]

$$\int \frac{(c+a^2cx^2)^3\arctan(ax)^2}{x^4}\,dx = c^3\left(\int 3a^4\operatorname{atan}^2(ax)\,dx + \int \frac{\operatorname{atan}^2(ax)}{x^4}\,dx + \int \frac{3a^2\operatorname{atan}^2(ax)}{x^2}\,dx + \int a^6x^2\operatorname{atan}^2(ax)\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**3*atan(a*x)**2/x**4,x)
```

output
```
c**3*(Integral(3*a**4*atan(a*x)**2, x) + Integral(atan(a*x)**2/x**4, x) +
Integral(3*a**2*atan(a*x)**2/x**2, x) + Integral(a**6*x**2*atan(a*x)**2, x
))
```

## Maxima [F]

$$\int \frac{(c+a^2cx^2)^3\arctan(ax)^2}{x^4}\,dx = \int \frac{(a^2cx^2+c)^3\arctan(ax)^2}{x^4}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^2/x^4,x, algorithm="maxima")
```

output
```
1/48*(24*(72*a^8*c^3*integrate(1/48*x^8*arctan(a*x)^2/(a^2*x^6 + x^4), x)
+ 6*a^8*c^3*integrate(1/48*x^8*log(a^2*x^2 + 1)^2/(a^2*x^6 + x^4), x) + 8*
a^8*c^3*integrate(1/48*x^8*log(a^2*x^2 + 1)/(a^2*x^6 + x^4), x) - 16*a^7*c
^3*integrate(1/48*x^7*arctan(a*x)/(a^2*x^6 + x^4), x) + 288*a^6*c^3*integr
ate(1/48*x^6*arctan(a*x)^2/(a^2*x^6 + x^4), x) + 24*a^6*c^3*integrate(1/48
*x^6*log(a^2*x^2 + 1)^2/(a^2*x^6 + x^4), x) + 72*a^6*c^3*integrate(1/48*x^
6*log(a^2*x^2 + 1)/(a^2*x^6 + x^4), x) + 3*a^3*c^3*arctan(a*x)^3 - 144*a^5
*c^3*integrate(1/48*x^5*arctan(a*x)/(a^2*x^6 + x^4), x) + 36*a^4*c^3*integ
rate(1/48*x^4*log(a^2*x^2 + 1)^2/(a^2*x^6 + x^4), x) - 72*a^4*c^3*integrat
e(1/48*x^4*log(a^2*x^2 + 1)/(a^2*x^6 + x^4), x) + 144*a^3*c^3*integrate(1/
48*x^3*arctan(a*x)/(a^2*x^6 + x^4), x) + 288*a^2*c^3*integrate(1/48*x^2*ar
ctan(a*x)^2/(a^2*x^6 + x^4), x) + 24*a^2*c^3*integrate(1/48*x^2*log(a^2*x^
2 + 1)^2/(a^2*x^6 + x^4), x) - 8*a^2*c^3*integrate(1/48*x^2*log(a^2*x^2 +
1)/(a^2*x^6 + x^4), x) + 16*a*c^3*integrate(1/48*x*arctan(a*x)/(a^2*x^6 +
x^4), x) + 72*c^3*integrate(1/48*arctan(a*x)^2/(a^2*x^6 + x^4), x) + 6*c^3
*integrate(1/48*log(a^2*x^2 + 1)^2/(a^2*x^6 + x^4), x))*x^3 + 4*(a^6*c^3*x
^6 + 9*a^4*c^3*x^4 - 9*a^2*c^3*x^2 - c^3)*arctan(a*x)^2 - (a^6*c^3*x^6 + 9
*a^4*c^3*x^4 - 9*a^2*c^3*x^2 - c^3)*log(a^2*x^2 + 1)^2)/x^3
```

## Giac [F]

$$\int \frac{(c + a^2cx^2)^3 \arctan(ax)^2}{x^4} \, dx = \int \frac{(a^2cx^2 + c)^3 \arctan(ax)^2}{x^4} \, dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^2/x^4,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^3*arctan(a*x)^2/x^4, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^2}{x^4}\,dx = \int \frac{\operatorname{atan}(a\,x)^2\,\left(c\,a^2\,x^2+c\right)^3}{x^4}\,dx$$

input
```
int((atan(a*x)^2*(c + a^2*c*x^2)^3)/x^4,x)
```

output
```
int((atan(a*x)^2*(c + a^2*c*x^2)^3)/x^4, x)
```

## Reduce [F]

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^2}{x^4}\,dx$$
$$= \frac{c^3\Big(atan(ax)^2\,a^6x^6 + 9atan(ax)^2\,a^4x^4 - 9atan(ax)^2\,a^2x^2 - atan(ax)^2 - atan(ax)\,a^5x^5 - 10atan(ax)\,a^3}{3x^3}$$

input
```
int((a^2*c*x^2+c)^3*atan(a*x)^2/x^4,x)
```

output
```
(c**3*(atan(a*x)**2*a**6*x**6 + 9*atan(a*x)**2*a**4*x**4 - 9*atan(a*x)**2*
a**2*x**2 - atan(a*x)**2 - atan(a*x)*a**5*x**5 - 10*atan(a*x)*a**3*x**3 -
9*atan(a*x)*a*x - 16*int(atan(a*x)/(a**2*x**5 + x**3),x)*a*x**3 - 16*int((
atan(a*x)*x)/(a**2*x**2 + 1),x)*a**5*x**3 + a**4*x**4 - 9*a**2*x**2))/(3*x
**3)
```

# 3.282 $\int \frac{x^4 \arctan(ax)^2}{c+a^2cx^2} dx$



## Optimal result

Integrand size = 22, antiderivative size = 166

$$\int \frac{x^4 \arctan(ax)^2}{c+a^2cx^2} dx = \frac{x}{3a^4c} - \frac{\arctan(ax)}{3a^5c} - \frac{x^2\arctan(ax)}{3a^3c} - \frac{4i\arctan(ax)^2}{3a^5c} - \frac{x\arctan(ax)^2}{a^4c} + \frac{x^3\arctan(ax)^2}{3a^2c} + \frac{\arctan(ax)^3}{3a^5c} - \frac{8\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{3a^5c} - \frac{4i\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{3a^5c}$$

output
```
1/3*x/a^4/c-1/3*arctan(a*x)/a^5/c-1/3*x^2*arctan(a*x)/a^3/c-4/3*I*arctan(a
*x)^2/a^5/c-x*arctan(a*x)^2/a^4/c+1/3*x^3*arctan(a*x)^2/a^2/c+1/3*arctan(a
*x)^3/a^5/c-8/3*arctan(a*x)*ln(2/(1+I*a*x))/a^5/c-4/3*I*polylog(2,1-2/(1+I
*a*x))/a^5/c
```

## Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.54

$$\int \frac{x^4\arctan(ax)^2}{c+a^2cx^2} dx = \frac{ax + (4i - 3ax + a^3x^3)\arctan(ax)^2 + \arctan(ax)^3 - \arctan(ax)\left(1 + a^2x^2 + 8\log\left(1 + e^{2i\arctan(ax)}\right)\right) +}{3a^5c}$$

input
```
Integrate[(x^4*ArcTan[a*x]^2)/(c + a^2*c*x^2),x]
```

output
```
(a*x + (4*I - 3*a*x + a^3*x^3)*ArcTan[a*x]^2 + ArcTan[a*x]^3 - ArcTan[a*x]
*(1 + a^2*x^2 + 8*Log[1 + E^((2*I)*ArcTan[a*x])]) + (4*I)*PolyLog[2, -E^((
2*I)*ArcTan[a*x])])/(3*a^5*c)
```

## Rubi [A] (verified)

Time = 1.50 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.46, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {5451, 27, 5361, 5451, 5345, 5361, 262, 216, 5419, 5455, 5379, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \arctan(ax)^2}{a^2cx^2+c}dx$$

$$\Big\downarrow \text{ 5451}$$

$$\frac{\int x^2\arctan(ax)^2dx}{a^2c}-\frac{\int \frac{x^2\arctan(ax)^2}{c(a^2x^2+1)}dx}{a^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int x^2\arctan(ax)^2dx}{a^2c}-\frac{\int \frac{x^2\arctan(ax)^2}{a^2x^2+1}dx}{a^2c}$$

$$\Big\downarrow \text{ 5361}$$

$$\frac{\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\int \frac{x^3\arctan(ax)}{a^2x^2+1}dx}{a^2c}-\frac{\int \frac{x^2\arctan(ax)^2}{a^2x^2+1}dx}{a^2c}$$

$$\Big\downarrow \text{ 5451}$$

$$\frac{\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\int x\arctan(ax)dx}{a^2}-\frac{\int \frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2c}-\frac{\frac{\int \arctan(ax)^2dx}{a^2}-\frac{\int \frac{\arctan(ax)^2}{a^2x^2+1}dx}{a^2}}{a^2c}$$

$$\Big\downarrow \text{ 5345}$$

$$\frac{\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\int x\arctan(ax)dx}{a^2}-\frac{\int\frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2c}-\frac{\frac{x\arctan(ax)^2-2a\int\frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}-\frac{\int\frac{\arctan(ax)^2}{a^2x^2+1}dx}{a^2}}{a^2c}$$

$$\Big\downarrow \text{ 5361}$$

$$\frac{\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\int\frac{x^2}{a^2x^2+1}dx}{a^2}-\frac{\int\frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2c}-\frac{\frac{x\arctan(ax)^2-2a\int\frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}-\frac{\int\frac{\arctan(ax)^2}{a^2x^2+1}dx}{a^2}}{a^2c}$$

$$\Big\downarrow \text{ 262}$$

$$\frac{\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\int\frac{1}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\frac{\int\frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2c}-\frac{\frac{x\arctan(ax)^2-2a\int\frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}-\frac{\int\frac{\arctan(ax)^2}{a^2x^2+1}dx}{a^2}}{a^2c}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{\int\frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2c}-\frac{\frac{x\arctan(ax)^2-2a\int\frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}-\frac{\int\frac{\arctan(ax)^2}{a^2x^2+1}dx}{a^2}}{a^2c}$$

$$\Big\downarrow \text{ 5419}$$

$$\frac{\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{\int\frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2c}-\frac{\frac{x\arctan(ax)^2-2a\int\frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}-\frac{\arctan(ax)^3}{3a^3}}{a^2c}$$

$$\Big\downarrow \text{ 5455}$$

$$\frac{\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{-\frac{\int\frac{\arctan(ax)}{i-ax}dx}{a}-\frac{i\arctan(ax)^2}{2a^2}}{a^2}\right)}{a^2c}-\frac{-\frac{\arctan(ax)^3}{3a^3}+\frac{x\arctan(ax)^2-2a\left(-\frac{\int\frac{\arctan(ax)}{i-ax}dx}{a}-\frac{i\arctan(ax)^2}{2a^2}\right)}{a^2}}{a^2c}$$

$$\Big\downarrow \; 5379$$

$$\frac{\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}-\int\frac{\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^2}{2a^2}}{a^2}\right)}{a^2c}-\frac{-\frac{\arctan(ax)^3}{3a^3}+\frac{x\arctan(ax)^2-2a\left(-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}-\int\frac{\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^2}{2a^2}\right)}{a^2}}{a^2c}$$

$$\Big\downarrow \; 2849$$

$$\frac{\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{-\frac{\frac{i\int\frac{\log\left(\frac{2}{iax+1}\right)}{1-\frac{2}{iax+1}}d\frac{1}{iax+1}}{a}+\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}}{a}-\frac{i\arctan(ax)^2}{2a^2}}{a^2}\right)}{a^2c}-\frac{-\frac{\arctan(ax)^3}{3a^3}+\frac{x\arctan(ax)^2-2a\left(-\frac{\frac{i\int\frac{\log\left(\frac{2}{iax+1}\right)}{1-\frac{2}{iax+1}}d\frac{1}{iax+1}}{a}+\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}}{a}-\frac{i\arctan(ax)^2}{2a^2}\right)}{a^2}}{a^2c}$$

$$\Big\downarrow \; 2752$$

$$\frac{\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}}{a^2}\right)}{a^2c}-\frac{-\frac{\arctan(ax)^3}{3a^3}+\frac{x\arctan(ax)^2-2a\left(-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}\right)}{a^2}}{a^2c}$$

input
```
Int[(x^4*ArcTan[a*x]^2)/(c + a^2*c*x^2),x]
```

output
```
((x^3*ArcTan[a*x]^2)/3 - (2*a*(((x^2*ArcTan[a*x])/2 - (a*(x/a^2 - ArcTan[a
*x]/a^3))/2)/a^2 - (((-1/2*I)*ArcTan[a*x]^2)/a^2 - ((ArcTan[a*x]*Log[2/(1
+ I*a*x)])/a + ((I/2)*PolyLog[2, 1 - 2/(1 + I*a*x)])/a)/a)/a^2))/3)/(a^2*c
) - (-1/3*ArcTan[a*x]^3/a^3 + (x*ArcTan[a*x]^2 - 2*a*(((-1/2*I)*ArcTan[a*x
]^2)/a^2 - ((ArcTan[a*x]*Log[2/(1 + I*a*x)])/a + ((I/2)*PolyLog[2, 1 - 2/(
1 + I*a*x)])/a)/a))/a^2)/(a^2*c)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

## Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.35

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{\arctan(ax)^2 a^3 x^3}{3c} - \frac{\arctan(ax)^2 ax}{c} + \frac{\arctan(ax)^3}{c} - \frac{2\left(\frac{x^2a^2\arctan(ax)}{2} - 2\arctan(ax)\ln\left(a^2x^2+1\right) - \frac{ax}{2} + \frac{\arctan(ax)}{2} - i\left(\ln(ax-i)\right.\right.}{}}{}$ |
| default | $\frac{\frac{\arctan(ax)^2 a^3 x^3}{3c} - \frac{\arctan(ax)^2 ax}{c} + \frac{\arctan(ax)^3}{c} - \frac{2\left(\frac{x^2a^2\arctan(ax)}{2} - 2\arctan(ax)\ln\left(a^2x^2+1\right) - \frac{ax}{2} + \frac{\arctan(ax)}{2} - i\left(\ln(ax-i)\right.\right.}{}}{}$ |
| parts | $\frac{x^3\arctan(ax)^2}{3a^2c} - \frac{x\arctan(ax)^2}{a^4c} + \frac{\arctan(ax)^3}{a^5c} - \frac{2\left(\frac{\arctan(ax)^3}{3a^5} + \frac{\frac{x^2a^2\arctan(ax)}{2} - 2\arctan(ax)\ln\left(a^2x^2+1\right) - \frac{ax}{2} +}{}\right.}{}$ |

input
```
int(x^4*arctan(a*x)^2/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
1/a^5*(1/3/c*arctan(a*x)^2*a^3*x^3-1/c*arctan(a*x)^2*a*x+1/c*arctan(a*x)^3
-2/3/c*(1/2*x^2*a^2*arctan(a*x)-2*arctan(a*x)*ln(a^2*x^2+1)-1/2*a*x+1/2*ar
ctan(a*x)-I*(ln(a*x-I)*ln(a^2*x^2+1)-1/2*ln(a*x-I)^2-dilog(-1/2*I*(a*x+I))
-ln(a*x-I)*ln(-1/2*I*(a*x+I)))+I*(ln(a*x+I)*ln(a^2*x^2+1)-1/2*ln(a*x+I)^2-
dilog(1/2*I*(a*x-I))-ln(a*x+I)*ln(1/2*I*(a*x-I)))+arctan(a*x)^3))
```

## Fricas [F]

$$\int \frac{x^4\arctan(ax)^2}{c+a^2cx^2}\,dx = \int \frac{x^4\arctan(ax)^2}{a^2cx^2+c}\,dx$$

input
```
integrate(x^4*arctan(a*x)^2/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
integral(x^4*arctan(a*x)^2/(a^2*c*x^2 + c), x)
```

## Sympy [F]

$$\int \frac{x^4 \arctan(ax)^2}{c+a^2cx^2} \, dx = \frac{\int \frac{x^4 \operatorname{atan}^2(ax)}{a^2x^2+1} \, dx}{c}$$

input
```
integrate(x**4*atan(a*x)**2/(a**2*c*x**2+c),x)
```

output
```
Integral(x**4*atan(a*x)**2/(a**2*x**2 + 1), x)/c
```

## Maxima [F]

$$\int \frac{x^4 \arctan(ax)^2}{c+a^2cx^2} \, dx = \int \frac{x^4 \arctan{(ax)}^2}{a^2cx^2+c} \, dx$$

input
```
integrate(x^4*arctan(a*x)^2/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
1/48*(4*(432*a^4*integrate(1/48*x^4*arctan(a*x)^2/(a^6*c*x^2 + a^4*c), x)
+ 36*a^4*integrate(1/48*x^4*log(a^2*x^2 + 1)^2/(a^6*c*x^2 + a^4*c), x) + 4
8*a^4*integrate(1/48*x^4*log(a^2*x^2 + 1)/(a^6*c*x^2 + a^4*c), x) - 96*a^3
*integrate(1/48*x^3*arctan(a*x)/(a^6*c*x^2 + a^4*c), x) - 144*a^2*integrat
e(1/48*x^2*log(a^2*x^2 + 1)/(a^6*c*x^2 + a^4*c), x) + 288*a*integrate(1/48
*x*arctan(a*x)/(a^6*c*x^2 + a^4*c), x) - arctan(a*x)^3/(a^5*c) - 36*integr
ate(1/48*log(a^2*x^2 + 1)^2/(a^6*c*x^2 + a^4*c), x))*a^5*c + 4*(a^3*x^3 -
3*a*x)*arctan(a*x)^2 + 8*arctan(a*x)^3 - (a^3*x^3 - 3*a*x)*log(a^2*x^2 + 1
)^2)/(a^5*c)
```

## Giac [F]

$$\int \frac{x^4 \arctan(ax)^2}{c+a^2cx^2}\,dx = \int \frac{x^4 \arctan{(ax)}^2}{a^2cx^2+c}\,dx$$

input
```
integrate(x^4*arctan(a*x)^2/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
integrate(x^4*arctan(a*x)^2/(a^2*c*x^2 + c), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^4 \arctan(ax)^2}{c+a^2cx^2}\,dx = \int \frac{x^4 \operatorname{atan}(a\,x)^2}{c\,a^2\,x^2+c}\,dx$$

input
```
int((x^4*atan(a*x)^2)/(c + a^2*c*x^2),x)
```

output
```
int((x^4*atan(a*x)^2)/(c + a^2*c*x^2), x)
```

## Reduce [F]

$$\int \frac{x^4 \arctan(ax)^2}{c+a^2cx^2}\,dx = \frac{atan(ax)^3 + atan(ax)^2\,a^3x^3 - 3atan(ax)^2\,ax - atan(ax)\,a^2x^2 - atan(ax) + 8\left(\int \frac{atan(ax)x}{a^2x^2+1}dx\right)a^2 + ax}{3a^5c}$$

input
```
int(x^4*atan(a*x)^2/(a^2*c*x^2+c),x)
```

output
```
(atan(a*x)**3 + atan(a*x)**2*a**3*x**3 - 3*atan(a*x)**2*a*x - atan(a*x)*a*
*2*x**2 - atan(a*x) + 8*int((atan(a*x)*x)/(a**2*x**2 + 1),x)*a**2 + a*x)/(
3*a**5*c)
```

# 3.283 $\int \frac{x^3 \arctan(ax)^2}{c+a^2cx^2} dx$



## Optimal result

Integrand size = 22, antiderivative size = 169

$$\int \frac{x^3 \arctan(ax)^2}{c+a^2cx^2} dx = -\frac{x \arctan(ax)}{a^3c} + \frac{\arctan(ax)^2}{2a^4c} + \frac{x^2 \arctan(ax)^2}{2a^2c} + \frac{i \arctan(ax)^3}{3a^4c} + \frac{\arctan(ax)^2 \log\left(\frac{2}{1+iax}\right)}{a^4c} + \frac{\log(1+a^2x^2)}{2a^4c} + \frac{i \arctan(ax) \operatorname{PolyLog}\left(2, 1-\frac{2}{1+iax}\right)}{a^4c} + \frac{\operatorname{PolyLog}\left(3, 1-\frac{2}{1+iax}\right)}{2a^4c}$$

output

```
-x*arctan(a*x)/a^3/c+1/2*arctan(a*x)^2/a^4/c+1/2*x^2*arctan(a*x)^2/a^2/c+1
/3*I*arctan(a*x)^3/a^4/c+arctan(a*x)^2*ln(2/(1+I*a*x))/a^4/c+1/2*ln(a^2*x^
2+1)/a^4/c+I*arctan(a*x)*polylog(2,1-2/(1+I*a*x))/a^4/c+1/2*polylog(3,1-2/
(1+I*a*x))/a^4/c
```

## Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.73

$$\int \frac{x^3 \arctan(ax)^2}{c+a^2cx^2}\,dx$$
$$= \frac{-ax\arctan(ax)+\frac{1}{2}(1+a^2x^2)\arctan(ax)^2-\frac{1}{3}i\arctan(ax)^3+\arctan(ax)^2\log\left(1+e^{2i\arctan(ax)}\right)-\log\left(\right.}{a^4c}$$

input
```
Integrate[(x^3*ArcTan[a*x]^2)/(c + a^2*c*x^2),x]
```

output
```
(-(a*x*ArcTan[a*x]) + ((1 + a^2*x^2)*ArcTan[a*x]^2)/2 - (I/3)*ArcTan[a*x]^
3 + ArcTan[a*x]^2*Log[1 + E^((2*I)*ArcTan[a*x])] - Log[1/Sqrt[1 + a^2*x^2]
] - I*ArcTan[a*x]*PolyLog[2, -E^((2*I)*ArcTan[a*x])] + PolyLog[3, -E^((2*I
)*ArcTan[a*x])]/2)/(a^4*c)
```

## Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5451, 27, 5361, 5451, 5345, 240, 5419, 5455, 5379, 5529, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \arctan(ax)^2}{a^2cx^2+c}\,dx$$
$$\Big\downarrow \text{5451}$$
$$\frac{\int x\arctan(ax)^2dx}{a^2c}-\frac{\int \frac{x\arctan(ax)^2}{c(a^2x^2+1)}dx}{a^2}$$
$$\Big\downarrow \text{27}$$
$$\frac{\int x\arctan(ax)^2dx}{a^2c}-\frac{\int \frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2c}$$
$$\Big\downarrow \text{5361}$$

$$\frac{\frac{1}{2}x^2\arctan(ax)^2-a\int\frac{x^2\arctan(ax)}{a^2x^2+1}dx}{a^2c}-\frac{\int\frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2c}$$

$\Big\downarrow$ 5451

$$\frac{\frac{1}{2}x^2\arctan(ax)^2-a\left(\frac{\int\arctan(ax)dx}{a^2}-\frac{\int\frac{\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2c}-\frac{\int\frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2c}$$

$\Big\downarrow$ 5345

$$\frac{\frac{1}{2}x^2\arctan(ax)^2-a\left(\frac{x\arctan(ax)-a\int\frac{x}{a^2x^2+1}dx}{a^2}-\frac{\int\frac{\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2c}-\frac{\int\frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2c}$$

$\Big\downarrow$ 240

$$\frac{\frac{1}{2}x^2\arctan(ax)^2-a\left(\frac{x\arctan(ax)-\frac{\log(a^2x^2+1)}{2a}}{a^2}-\frac{\int\frac{\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2c}-\frac{\int\frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2c}$$

$\Big\downarrow$ 5419

$$\frac{\frac{1}{2}x^2\arctan(ax)^2-a\left(\frac{x\arctan(ax)-\frac{\log(a^2x^2+1)}{2a}}{a^2}-\frac{\arctan(ax)^2}{2a^3}\right)}{a^2c}-\frac{\int\frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2c}$$

$\Big\downarrow$ 5455

$$\frac{\frac{1}{2}x^2\arctan(ax)^2-a\left(\frac{x\arctan(ax)-\frac{\log(a^2x^2+1)}{2a}}{a^2}-\frac{\arctan(ax)^2}{2a^3}\right)}{a^2c}-\frac{-\frac{\int\frac{\arctan(ax)^2}{i-ax}dx}{a}-\frac{i\arctan(ax)^3}{3a^2}}{a^2c}$$

$\Big\downarrow$ 5379

$$\frac{\frac{1}{2}x^2\arctan(ax)^2-a\left(\frac{x\arctan(ax)-\frac{\log(a^2x^2+1)}{2a}}{a^2}-\frac{\arctan(ax)^2}{2a^3}\right)}{a^2c}-\frac{-\frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}-2\int\frac{\arctan(ax)\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^3}{3a^2}}{a^2c}$$

$\Big\downarrow$ 5529

$$\frac{\frac{1}{2}x^2\arctan(ax)^2 - a\left(\frac{x\arctan(ax)-\frac{\log(a^2x^2+1)}{2a}}{a^2} - \frac{\arctan(ax)^2}{2a^3}\right)}{a^2c} - \frac{-\frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a} - 2\left(\frac{1}{2}i\int\frac{\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx - \frac{i\arctan(ax)\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}\right)}{a} - \frac{i\arctan(ax)^3}{3a^2}}{a^2c}$$

$$\Big\downarrow \textbf{7164}$$

$$\frac{\frac{1}{2}x^2\arctan(ax)^2 - a\left(\frac{x\arctan(ax)-\frac{\log(a^2x^2+1)}{2a}}{a^2} - \frac{\arctan(ax)^2}{2a^3}\right)}{a^2c} - \frac{-\frac{i\arctan(ax)^3}{3a^2} - \frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a} - 2\left(-\frac{i\arctan(ax)\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a} - \frac{\text{PolyLog}\left(3,1-\frac{2}{iax+1}\right)}{4a}\right)}{a}}{a^2c}$$

input
```
Int[(x^3*ArcTan[a*x]^2)/(c + a^2*c*x^2),x]
```

output
```
((x^2*ArcTan[a*x]^2)/2 - a*(-1/2*ArcTan[a*x]^2/a^3 + (x*ArcTan[a*x] - Log[
1 + a^2*x^2]/(2*a))/a^2))/(a^2*c) - (((-1/3*I)*ArcTan[a*x]^3)/a^2 - ((ArcT
an[a*x]^2*Log[2/(1 + I*a*x)])/a - 2*(((-1/2*I)*ArcTan[a*x]*PolyLog[2, 1 -
2/(1 + I*a*x)])/a - PolyLog[3, 1 - 2/(1 + I*a*x)]/(4*a)))/a)/(a^2*c)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 19.53 (sec) , antiderivative size = 874, normalized size of antiderivative = 5.17

| method | result | size |
|---|---|---|
| derivativedivides | Expression too large to display | 874 |
| default | Expression too large to display | 874 |
| parts | Expression too large to display | 929 |

input
```
int(x^3*arctan(a*x)^2/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
1/a^4*(1/2/c*arctan(a*x)^2*a^2*x^2-1/2/c*arctan(a*x)^2*ln(a^2*x^2+1)-1/c*(
I*arctan(a*x)*polylog(2,-(1+I*a*x)^2/(a^2*x^2+1))-1/2*polylog(3,-(1+I*a*x)
^2/(a^2*x^2+1))-arctan(a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2))+1/4*I*Pi*csg
n(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*
(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*arctan(a*x)^2+1/2*I
*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)
^2)^2*arctan(a*x)^2+ln((1+I*a*x)^2/(a^2*x^2+1)+1)+1/4*I*Pi*csgn(I*(1+I*a*x
)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3*arctan(a*x)^2-1/2*arctan(
a*x)^2-1/4*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))^2*csgn(I*((1+I*a*x)^2/
(a^2*x^2+1)+1)^2)*arctan(a*x)^2-1/2*I*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2
))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^2*arctan(a*x)^2-1/4*I*Pi*csgn(I*(1+I*a*
x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+
1)^2)^2*arctan(a*x)^2-1/4*I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(
I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2*arctan(a*x)^2+1
/4*I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^3*arctan(a*x)^2+1/3*I*arctan(a*x)^
3-1/4*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3*arctan(a*x)^2+arctan(a*
x)*(a*x-I)+1/4*I*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))^2*csgn(I*(1+I*a*x)
^2/(a^2*x^2+1))*arctan(a*x)^2-arctan(a*x)^2*ln(2)))
```

## Fricas [F]

$$\int \frac{x^3 \arctan(ax)^2}{c + a^2cx^2} dx = \int \frac{x^3 \arctan{(ax)}^2}{a^2cx^2 + c} dx$$

input
```
integrate(x^3*arctan(a*x)^2/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
integral(x^3*arctan(a*x)^2/(a^2*c*x^2 + c), x)
```

## Sympy [F]

$$\int \frac{x^3 \arctan(ax)^2}{c + a^2cx^2} dx = \frac{\int \frac{x^3 \operatorname{atan}^2{(ax)}}{a^2x^2+1} dx}{c}$$

input
```
integrate(x**3*atan(a*x)**2/(a**2*c*x**2+c),x)
```

output
```
Integral(x**3*atan(a*x)**2/(a**2*x**2 + 1), x)/c
```

## Maxima [F]

$$\int \frac{x^3 \arctan(ax)^2}{c + a^2cx^2} dx = \int \frac{x^3 \arctan{(ax)}^2}{a^2cx^2 + c} dx$$

input
```
integrate(x^3*arctan(a*x)^2/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
integrate(x^3*arctan(a*x)^2/(a^2*c*x^2 + c), x)
```

## Giac [F]

$$\int \frac{x^3 \arctan(ax)^2}{c+a^2cx^2}\,dx = \int \frac{x^3 \arctan{(ax)}^2}{a^2cx^2+c}\,dx$$

input
```
integrate(x^3*arctan(a*x)^2/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
integrate(x^3*arctan(a*x)^2/(a^2*c*x^2 + c), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \arctan(ax)^2}{c+a^2cx^2}\,dx = \int \frac{x^3 \operatorname{atan}(a\,x)^2}{c\,a^2\,x^2+c}\,dx$$

input
```
int((x^3*atan(a*x)^2)/(c + a^2*c*x^2),x)
```

output
```
int((x^3*atan(a*x)^2)/(c + a^2*c*x^2), x)
```

## Reduce [F]

$$\int \frac{x^3 \arctan(ax)^2}{c+a^2cx^2}\,dx = \frac{\int \frac{atan(ax)^2 x^3}{a^2x^2+1}dx}{c}$$

input
```
int(x^3*atan(a*x)^2/(a^2*c*x^2+c),x)
```

output
```
int((atan(a*x)**2*x**3)/(a**2*x**2 + 1),x)/c
```

# 3.284 $\int \frac{x^2 \arctan(ax)^2}{c+a^2cx^2} \, dx$



## Optimal result

Integrand size = 22, antiderivative size = 98

$$\int \frac{x^2 \arctan(ax)^2}{c+a^2cx^2} \, dx = \frac{i \arctan(ax)^2}{a^3c} + \frac{x \arctan(ax)^2}{a^2c} - \frac{\arctan(ax)^3}{3a^3c} + \frac{2 \arctan(ax) \log\left(\frac{2}{1+iax}\right)}{a^3c} + \frac{i \operatorname{PolyLog}\left(2, 1-\frac{2}{1+iax}\right)}{a^3c}$$

output
```
I*arctan(a*x)^2/a^3/c+x*arctan(a*x)^2/a^2/c-1/3*arctan(a*x)^3/a^3/c+2*arct
an(a*x)*ln(2/(1+I*a*x))/a^3/c+I*polylog(2,1-2/(1+I*a*x))/a^3/c
```

## Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.70

$$\int \frac{x^2 \arctan(ax)^2}{c+a^2cx^2} \, dx = \frac{-\frac{1}{3}\arctan(ax)\left((3i-3ax)\arctan(ax)+\arctan(ax)^2-6\log\left(1+e^{2i\arctan(ax)}\right)\right)-i \operatorname{PolyLog}\left(2,-e^{2i\,\mathrm{arcta}}\right.}{a^3c}$$

input
```
Integrate[(x^2*ArcTan[a*x]^2)/(c + a^2*c*x^2),x]
```

output
```
(-1/3*(ArcTan[a*x]*((3*I - 3*a*x)*ArcTan[a*x] + ArcTan[a*x]^2 - 6*Log[1 +
E^((2*I)*ArcTan[a*x])])) - I*PolyLog[2, -E^((2*I)*ArcTan[a*x])])/(a^3*c)
```

## Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {5451, 27, 5345, 5419, 5455, 5379, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \arctan(ax)^2}{a^2cx^2+c} \, dx$$

$$\Big\downarrow \text{5451}$$

$$\frac{\int \arctan(ax)^2dx}{a^2c} - \frac{\int \frac{\arctan(ax)^2}{c(a^2x^2+1)}dx}{a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \arctan(ax)^2dx}{a^2c} - \frac{\int \frac{\arctan(ax)^2}{a^2x^2+1}dx}{a^2c}$$

$$\Big\downarrow \text{5345}$$

$$\frac{x\arctan(ax)^2 - 2a\int \frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2c} - \frac{\int \frac{\arctan(ax)^2}{a^2x^2+1}dx}{a^2c}$$

$$\Big\downarrow \text{5419}$$

$$\frac{x\arctan(ax)^2 - 2a\int \frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2c} - \frac{\arctan(ax)^3}{3a^3c}$$

$$\Big\downarrow \text{5455}$$

$$-\frac{\arctan(ax)^3}{3a^3c} + \frac{x\arctan(ax)^2 - 2a\left(-\frac{\int \frac{\arctan(ax)}{i-ax}dx}{a} - \frac{i\arctan(ax)^2}{2a^2}\right)}{a^2c}$$

$$\Big\downarrow \text{5379}$$

$$-\frac{\arctan(ax)^3}{3a^3c}+\frac{x\arctan(ax)^2-2a\left(-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}-\int\frac{\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^2}{2a^2}\right)}{a^2c}$$

$\Big\downarrow$ 2849

$$-\frac{\arctan(ax)^3}{3a^3c}+\frac{x\arctan(ax)^2-2a\left(-\frac{\frac{i\int\frac{\log\left(\frac{2}{iax+1}\right)}{1-\frac{2}{iax+1}}d\frac{1}{iax+1}}{a}+\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}}{a}-\frac{i\arctan(ax)^2}{2a^2}\right)}{a^2c}$$

$\Big\downarrow$ 2752

$$-\frac{\arctan(ax)^3}{3a^3c}+\frac{x\arctan(ax)^2-2a\left(-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}\right)}{a^2c}$$

input
```
Int[(x^2*ArcTan[a*x]^2)/(c + a^2*c*x^2),x]
```

output
```
-1/3*ArcTan[a*x]^3/(a^3*c) + (x*ArcTan[a*x]^2 - 2*a*(((-1/2*I)*ArcTan[a*x]
^2)/a^2 - ((ArcTan[a*x]*Log[2/(1 + I*a*x)])/a + ((I/2)*PolyLog[2, 1 - 2/(1
 + I*a*x)])/a)/a))/(a^2*c)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

## Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(92) = 184$.

Time = 0.91 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.91

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{\arctan(ax)^2 ax}{c}-\frac{\arctan(ax)^3}{c}-\frac{2\left(\frac{\arctan(ax)\ln\left(a^2x^2+1\right)}{2}+\frac{i\left(\ln(ax-i)\ln\left(a^2x^2+1\right)-\frac{\ln(ax-i)^2}{2}-\mathrm{dilog}\left(-\frac{i(ax+i)}{2}\right)-\ln(ax-i)\,\mathrm{l}\cdots\right.}{4}\cdots\right.}{\cdots}}{a^3}$ |
| default | $\frac{\frac{\arctan(ax)^2 ax}{c}-\frac{\arctan(ax)^3}{c}-\frac{2\left(\frac{\arctan(ax)\ln\left(a^2x^2+1\right)}{2}+\frac{i\left(\ln(ax-i)\ln\left(a^2x^2+1\right)-\frac{\ln(ax-i)^2}{2}-\mathrm{dilog}\left(-\frac{i(ax+i)}{2}\right)-\ln(ax-i)\,\mathrm{l}\cdots\right.}{4}\cdots\right.}{\cdots}}{a^3}$ |
| parts | $\frac{x\arctan(ax)^2}{a^2c}-\frac{\arctan(ax)^3}{a^3c}-\frac{2\left(\frac{\arctan(ax)\ln\left(a^2x^2+1\right)}{2}+\frac{i\left(\ln(ax-i)\ln\left(a^2x^2+1\right)-\frac{\ln(ax-i)^2}{2}-\mathrm{dilog}\left(-\frac{i(ax+i)}{2}\right)-\mathrm{ln}\cdots\right.}{4}\cdots\right.}{\cdots}$ |

input
```
int(x^2*arctan(a*x)^2/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(1/c*arctan(a*x)^2*a*x-1/c*arctan(a*x)^3-2/c*(1/2*arctan(a*x)*ln(a^2
*x^2+1)+1/4*I*(ln(a*x-I)*ln(a^2*x^2+1)-1/2*ln(a*x-I)^2-dilog(-1/2*I*(a*x+I
))-ln(a*x-I)*ln(-1/2*I*(a*x+I)))-1/4*I*(ln(a*x+I)*ln(a^2*x^2+1)-1/2*ln(a*x
+I)^2-dilog(1/2*I*(a*x-I))-ln(a*x+I)*ln(1/2*I*(a*x-I)))-1/3*arctan(a*x)^3)
)
```

## Fricas [F]

$$\int \frac{x^2\arctan(ax)^2}{c+a^2cx^2}\,dx = \int \frac{x^2\arctan\left(ax\right)^2}{a^2cx^2+c}\,dx$$

input
```
integrate(x^2*arctan(a*x)^2/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
integral(x^2*arctan(a*x)^2/(a^2*c*x^2 + c), x)
```

## Sympy [F]

$$\int \frac{x^2 \arctan(ax)^2}{c + a^2cx^2}\,dx = \frac{\int \frac{x^2 \operatorname{atan}^2(ax)}{a^2x^2+1}\,dx}{c}$$

input
```
integrate(x**2*atan(a*x)**2/(a**2*c*x**2+c),x)
```

output
```
Integral(x**2*atan(a*x)**2/(a**2*x**2 + 1), x)/c
```

## Maxima [F]

$$\int \frac{x^2 \arctan(ax)^2}{c + a^2cx^2}\,dx = \int \frac{x^2 \arctan(ax)^2}{a^2cx^2 + c}\,dx$$

input
```
integrate(x^2*arctan(a*x)^2/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
1/48*(4*(144*a^2*integrate(1/16*x^2*arctan(a*x)^2/(a^4*c*x^2 + a^2*c), x)
+ 12*a^2*integrate(1/16*x^2*log(a^2*x^2 + 1)^2/(a^4*c*x^2 + a^2*c), x) + 4
8*a^2*integrate(1/16*x^2*log(a^2*x^2 + 1)/(a^4*c*x^2 + a^2*c), x) - 96*a*i
ntegrate(1/16*x*arctan(a*x)/(a^4*c*x^2 + a^2*c), x) + arctan(a*x)^3/(a^3*c
) + 12*integrate(1/16*log(a^2*x^2 + 1)^2/(a^4*c*x^2 + a^2*c), x))*a^3*c +
12*a*x*arctan(a*x)^2 - 3*a*x*log(a^2*x^2 + 1)^2 - 8*arctan(a*x)^3)/(a^3*c)
```

## Giac [F]

$$\int \frac{x^2 \arctan(ax)^2}{c + a^2cx^2}\,dx = \int \frac{x^2 \arctan(ax)^2}{a^2cx^2 + c}\,dx$$

input
```
integrate(x^2*arctan(a*x)^2/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
integrate(x^2*arctan(a*x)^2/(a^2*c*x^2 + c), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \arctan(ax)^2}{c+a^2cx^2}\,dx = \int \frac{x^2\,\mathrm{atan}(a\,x)^2}{c\,a^2\,x^2+c}\,dx$$

input
```
int((x^2*atan(a*x)^2)/(c + a^2*c*x^2),x)
```

output
```
int((x^2*atan(a*x)^2)/(c + a^2*c*x^2), x)
```

## Reduce [F]

$$\int \frac{x^2 \arctan(ax)^2}{c+a^2cx^2}\,dx = \frac{\int \frac{atan(ax)^2x^2}{a^2x^2+1}dx}{c}$$

input
```
int(x^2*atan(a*x)^2/(a^2*c*x^2+c),x)
```

output
```
int((atan(a*x)**2*x**2)/(a**2*x**2 + 1),x)/c
```

# 3.285 $\int \frac{x \arctan(ax)^2}{c+a^2cx^2} dx$



## Optimal result

Integrand size = 20, antiderivative size = 102

$$\int \frac{x \arctan(ax)^2}{c+a^2cx^2} dx = -\frac{i \arctan(ax)^3}{3a^2c} - \frac{\arctan(ax)^2 \log\left(\frac{2}{1+iax}\right)}{a^2c} - \frac{i \arctan(ax) \operatorname{PolyLog}\left(2, 1-\frac{2}{1+iax}\right)}{a^2c} - \frac{\operatorname{PolyLog}\left(3, 1-\frac{2}{1+iax}\right)}{2a^2c}$$

output
```
-1/3*I*arctan(a*x)^3/a^2/c-arctan(a*x)^2*ln(2/(1+I*a*x))/a^2/c-I*arctan(a*
x)*polylog(2,1-2/(1+I*a*x))/a^2/c-1/2*polylog(3,1-2/(1+I*a*x))/a^2/c
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.90

$$\int \frac{x \arctan(ax)^2}{c+a^2cx^2} dx = -\frac{2i \arctan(ax)^3 + 6 \arctan(ax)^2 \log\left(\frac{2i}{i-ax}\right) + 6i \arctan(ax) \operatorname{PolyLog}\left(2, \frac{i+ax}{-i+ax}\right) + 3 \operatorname{PolyLog}\left(3, \frac{i+ax}{-i+ax}\right)}{6a^2c}$$

input
```
Integrate[(x*ArcTan[a*x]^2)/(c + a^2*c*x^2),x]
```

output
```
-1/6*((2*I)*ArcTan[a*x]^3 + 6*ArcTan[a*x]^2*Log[(2*I)/(I - a*x)] + (6*I)*A
rcTan[a*x]*PolyLog[2, (I + a*x)/(-I + a*x)] + 3*PolyLog[3, (I + a*x)/(-I +
 a*x)])/(a^2*c)
```

## Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5455, 5379, 5529, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \arctan(ax)^2}{a^2cx^2+c}dx$$

$$\Big\downarrow \text{5455}$$

$$-\frac{\int \frac{\arctan(ax)^2}{i-ax}dx}{ac}-\frac{i\arctan(ax)^3}{3a^2c}$$

$$\Big\downarrow \text{5379}$$

$$-\frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}-2\int \frac{\arctan(ax)\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{ac}-\frac{i\arctan(ax)^3}{3a^2c}$$

$$\Big\downarrow \text{5529}$$

$$-\frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}-2\left(\frac{1}{2}i\int \frac{\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx-\frac{i\arctan(ax)\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}\right)}{ac}-\frac{i\arctan(ax)^3}{3a^2c}$$

$$\Big\downarrow \text{7164}$$

$$-\frac{i\arctan(ax)^3}{3a^2c}-\frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}-2\left(-\frac{i\arctan(ax)\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}-\frac{\text{PolyLog}\left(3,1-\frac{2}{iax+1}\right)}{4a}\right)}{ac}$$

input
```
Int[(x*ArcTan[a*x]^2)/(c + a^2*c*x^2),x]
```

output
```
((-1/3*I)*ArcTan[a*x]^3)/(a^2*c) - ((ArcTan[a*x]^2*Log[2/(1 + I*a*x)])/a -
 2*(((-1/2*I)*ArcTan[a*x]*PolyLog[2, 1 - 2/(1 + I*a*x)])/a - PolyLog[3, 1
- 2/(1 + I*a*x)]/(4*a)))/(a*c)
```

### Defintions of rubi rules used

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 13.18 (sec) , antiderivative size = 756, normalized size of antiderivative = 7.41

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{\arctan(ax)^2\ln\left(a^2x^2+1\right)}{2c} - \frac{\arctan(ax)^2\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right) - \frac{i\arctan(ax)^3}{3} + \frac{\left(-i\pi\,\mathrm{csgn}\left(\frac{i}{\left(\frac{(iax+1)^2}{a^2x^2+1}+1\right)^2}\right)\mathrm{csgn}\left(\frac{i(iax+1)^2}{a^2x^2+1}\right)\mathrm{csgn}\cdots\right.}{\cdots}}{\cdots}}{\cdots}$ |
| default | $\frac{\frac{\arctan(ax)^2\ln\left(a^2x^2+1\right)}{2c} - \frac{\arctan(ax)^2\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right) - \frac{i\arctan(ax)^3}{3} + \frac{\left(-i\pi\,\mathrm{csgn}\left(\frac{i}{\left(\frac{(iax+1)^2}{a^2x^2+1}+1\right)^2}\right)\mathrm{csgn}\left(\frac{i(iax+1)^2}{a^2x^2+1}\right)\mathrm{csgn}\cdots\right.}{\cdots}}{\cdots}}{\cdots}$ |
| parts | $\frac{\ln\left(a^2x^2+1\right)\arctan(ax)^2}{2a^2c} - \frac{\frac{\arctan(ax)^2\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)}{a} - \frac{i\arctan(ax)^3}{3a} + \frac{\left(-i\pi\,\mathrm{csgn}\left(\frac{i}{\left(\frac{(iax+1)^2}{a^2x^2+1}+1\right)^2}\right)\mathrm{csgn}\left(\frac{i(iax+1)^2}{a^2x^2+1}\right)\cdots\right.}{\cdots}}{\cdots}$ |

input
```
int(x*arctan(a*x)^2/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
1/a^2*(1/2/c*arctan(a*x)^2*ln(a^2*x^2+1)-1/c*(arctan(a*x)^2*ln((1+I*a*x)/(
a^2*x^2+1)^(1/2))-1/3*I*arctan(a*x)^3+1/4*(-I*Pi*csgn(I/((1+I*a*x)^2/(a^2*
x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1
)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)+I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2
)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2+I*Pi*csg
n(I*((1+I*a*x)^2/(a^2*x^2+1)+1))^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)-2
*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+
1)^2)^2+I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3-I*Pi*csgn(I*(1+I*a*x)
/(a^2*x^2+1)^(1/2))^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))+2*I*Pi*csgn(I*(1+I*a
*x)/(a^2*x^2+1)^(1/2))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^2-I*Pi*csgn(I*(1+I*
a*x)^2/(a^2*x^2+1))^3+I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x
)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2-I*Pi*csgn(I*(1+I*a*x)^2/(
a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3+4*ln(2))*arctan(a*x)^2-I*arcta
n(a*x)*polylog(2,-(1+I*a*x)^2/(a^2*x^2+1))+1/2*polylog(3,-(1+I*a*x)^2/(a^2
*x^2+1))))
```

## Fricas [F]

$$\int \frac{x \arctan(ax)^2}{c + a^2 c x^2} \, dx = \int \frac{x \arctan(ax)^2}{a^2 c x^2 + c} \, dx$$

input
```
integrate(x*arctan(a*x)^2/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
integral(x*arctan(a*x)^2/(a^2*c*x^2 + c), x)
```

## Sympy [F]

$$\int \frac{x \arctan(ax)^2}{c + a^2 c x^2} \, dx = \frac{\int \frac{x \operatorname{atan}^2(ax)}{a^2 x^2 + 1} \, dx}{c}$$

input
```
integrate(x*atan(a*x)**2/(a**2*c*x**2+c),x)
```

output
```
Integral(x*atan(a*x)**2/(a**2*x**2 + 1), x)/c
```

## Maxima [F]

$$\int \frac{x \arctan(ax)^2}{c + a^2 c x^2} \, dx = \int \frac{x \arctan(ax)^2}{a^2 c x^2 + c} \, dx$$

input
```
integrate(x*arctan(a*x)^2/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
integrate(x*arctan(a*x)^2/(a^2*c*x^2 + c), x)
```

## Giac [F]

$$\int \frac{x \arctan(ax)^2}{c+a^2cx^2}\,dx = \int \frac{x \arctan(ax)^2}{a^2cx^2+c}\,dx$$

input
```
integrate(x*arctan(a*x)^2/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
integrate(x*arctan(a*x)^2/(a^2*c*x^2 + c), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x \arctan(ax)^2}{c+a^2cx^2}\,dx = \int \frac{x\,\mathrm{atan}(a\,x)^2}{c\,a^2\,x^2+c}\,dx$$

input
```
int((x*atan(a*x)^2)/(c + a^2*c*x^2),x)
```

output
```
int((x*atan(a*x)^2)/(c + a^2*c*x^2), x)
```

## Reduce [F]

$$\int \frac{x \arctan(ax)^2}{c+a^2cx^2}\,dx = \frac{\int \frac{atan(ax)^2x}{a^2x^2+1}dx}{c}$$

input
```
int(x*atan(a*x)^2/(a^2*c*x^2+c),x)
```

output
```
int((atan(a*x)**2*x)/(a**2*x**2 + 1),x)/c
```

# 3.286 $\int \frac{\arctan(ax)^2}{c+a^2cx^2} dx$



## Optimal result

Integrand size = 19, antiderivative size = 16

$$\int \frac{\arctan(ax)^2}{c+a^2cx^2} dx = \frac{\arctan(ax)^3}{3ac}$$

output
```
1/3*arctan(a*x)^3/a/c
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax)^2}{c+a^2cx^2} dx = \frac{\arctan(ax)^3}{3ac}$$

input
```
Integrate[ArcTan[a*x]^2/(c + a^2*c*x^2),x]
```

output
```
ArcTan[a*x]^3/(3*a*c)
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{5419\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2}{a^2cx^2+c}\,dx$$

$$\Big\downarrow \text{5419}$$

$$\frac{\arctan(ax)^3}{3ac}$$

input
```
Int[ArcTan[a*x]^2/(c + a^2*c*x^2),x]
```

output
```
ArcTan[a*x]^3/(3*a*c)
```

### Defintions of rubi rules used

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

## Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\arctan(ax)^3}{3ac}$ | 15 |
| default | $\frac{\arctan(ax)^3}{3ac}$ | 15 |
| parallelrisch | $\frac{\arctan(ax)^3}{3ac}$ | 15 |
| parts | $\frac{\arctan(ax)^3}{3ac}$ | 15 |
| risch | $\frac{i\ln(iax+1)^3}{24ca}-\frac{i\ln(-iax+1)\ln(iax+1)^2}{8ca}+\frac{i\ln(-iax+1)^2\ln(iax+1)}{8ca}-\frac{i\ln(-iax+1)^3}{24ca}$ | 94 |

input
```
int(arctan(a*x)^2/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
1/3*arctan(a*x)^3/a/c
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\arctan(ax)^2}{c+a^2cx^2}\,dx=\frac{\arctan{(ax)}^3}{3\,ac}$$

input
```
integrate(arctan(a*x)^2/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
1/3*arctan(a*x)^3/(a*c)
```

## Sympy [F]

$$\int \frac{\arctan(ax)^2}{c + a^2cx^2} \, dx = \frac{\int \frac{\operatorname{atan}^2(ax)}{a^2x^2+1} \, dx}{c}$$

input
```
integrate(atan(a*x)**2/(a**2*c*x**2+c),x)
```

output
```
Integral(atan(a*x)**2/(a**2*x**2 + 1), x)/c
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\arctan(ax)^2}{c + a^2cx^2} \, dx = \frac{\arctan(ax)^3}{3\,ac}$$

input
```
integrate(arctan(a*x)^2/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
1/3*arctan(a*x)^3/(a*c)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\arctan(ax)^2}{c + a^2cx^2} \, dx = \frac{\arctan(ax)^3}{3\,ac}$$

input
```
integrate(arctan(a*x)^2/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
1/3*arctan(a*x)^3/(a*c)
```

## Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\arctan(ax)^2}{c + a^2cx^2}\,dx = \frac{\operatorname{atan}(a\,x)^3}{3\,a\,c}$$

input
```
int(atan(a*x)^2/(c + a^2*c*x^2),x)
```

output
```
atan(a*x)^3/(3*a*c)
```

## Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\arctan(ax)^2}{c + a^2cx^2}\,dx = \frac{atan(ax)^3}{3ac}$$

input
```
int(atan(a*x)^2/(a^2*c*x^2+c),x)
```

output
```
atan(a*x)**3/(3*a*c)
```

# 3.287 $\int \frac{\arctan(ax)^2}{x(c+a^2cx^2)}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 91

$$\int \frac{\arctan(ax)^2}{x(c+a^2cx^2)}\,dx = -\frac{i\arctan(ax)^3}{3c}+\frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{c} - \frac{i\arctan(ax)\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{c} + \frac{\operatorname{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)}{2c}$$

output
```
-1/3*I*arctan(a*x)^3/c+arctan(a*x)^2*ln(2-2/(1-I*a*x))/c-I*arctan(a*x)*pol
ylog(2,-1+2/(1-I*a*x))/c+1/2*polylog(3,-1+2/(1-I*a*x))/c
```

## Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.91

$$\int \frac{\arctan(ax)^2}{x(c+a^2cx^2)}\,dx = \frac{i\arctan(ax)^3}{3c}+\frac{\arctan(ax)^2\log\left(1-e^{-2i\arctan(ax)}\right)}{c} + \frac{i\arctan(ax)\operatorname{PolyLog}\left(2,e^{-2i\arctan(ax)}\right)}{c} + \frac{\operatorname{PolyLog}\left(3,e^{-2i\arctan(ax)}\right)}{2c}$$

input
```
Integrate[ArcTan[a*x]^2/(x*(c + a^2*c*x^2)),x]
```

output
```
((I/3)*ArcTan[a*x]^3)/c + (ArcTan[a*x]^2*Log[1 - E^((-2*I)*ArcTan[a*x])])/
c + (I*ArcTan[a*x]*PolyLog[2, E^((-2*I)*ArcTan[a*x])])/c + PolyLog[3, E^((
-2*I)*ArcTan[a*x])]/(2*c)
```

## Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.18, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {5459, 5403, 5527, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2}{x(a^2cx^2+c)}dx$$

$$\Big\downarrow \text{5459}$$

$$\frac{i\int \frac{\arctan(ax)^2}{x(ax+i)}dx}{c}-\frac{i\arctan(ax)^3}{3c}$$

$$\Big\downarrow \text{5403}$$

$$\frac{i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)}{c}-\frac{i\arctan(ax)^3}{3c}$$

$$\Big\downarrow \text{5527}$$

$$\frac{i\left(2ia\left(\frac{i\arctan(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-\frac{1}{2}i\int \frac{\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{a^2x^2+1}dx\right)-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)}{c}-\frac{i\arctan(ax)^3}{3c}$$

$$\Big\downarrow \text{7164}$$

$$\frac{i\left(2ia\left(\frac{i\arctan(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-\frac{\operatorname{PolyLog}\left(3,\frac{2}{1-iax}-1\right)}{4a}\right)-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)}{c}-\frac{i\arctan(ax)^3}{3c}$$

input
```
Int[ArcTan[a*x]^2/(x*(c + a^2*c*x^2)),x]
```

output
```
((-1/3*I)*ArcTan[a*x]^3)/c + (I*((-I)*ArcTan[a*x]^2*Log[2 - 2/(1 - I*a*x)]
 + (2*I)*a*(((I/2)*ArcTan[a*x]*PolyLog[2, -1 + 2/(1 - I*a*x)])/a - PolyLog
[3, -1 + 2/(1 - I*a*x)]/(4*a))))/c
```

## Defintions of rubi rules used

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5527
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 60.60 (sec) , antiderivative size = 1578, normalized size of antiderivative = 17.34

| method | result | size |
|---|---|---|
| derivativedivides | Expression too large to display | 1578 |
| default | Expression too large to display | 1578 |
| parts | Expression too large to display | 1989 |

input
```
int(arctan(a*x)^2/x/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
1/c*arctan(a*x)^2*ln(a*x)-1/2/c*arctan(a*x)^2*ln(a^2*x^2+1)-1/c*(-arctan(a
*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2))+arctan(a*x)^2*ln((1+I*a*x)^2/(a^2*x^
2+1)-1)+1/3*I*arctan(a*x)^3-1/4*(-I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+
I*a*x)^2/(a^2*x^2+1)+1)^2)^3+2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1
+I*a*x)^2/(a^2*x^2+1)+1))^3+2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csg
n(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*
a*x)^2/(a^2*x^2+1)+1))+2*I*Pi*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^
2/(a^2*x^2+1)+1))^3-2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2
/(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)
+1))^2-I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*
x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)-2*I*
Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^
2)^2-2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I*((1+I*a*x)^2/(a^2*x
^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2+I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+
1)+1)^2)^3+2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2
+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))-2*I*
Pi*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2+2*I*Pi*
csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^2-2*I*
Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/
((1+I*a*x)^2/(a^2*x^2+1)+1))^2+I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))...
```

## Fricas [F]

$$\int \frac{\arctan(ax)^2}{x\,(c+a^2cx^2)}\,dx = \int \frac{\arctan\left(ax\right)^2}{(a^2cx^2+c)x}\,dx$$

input
```
integrate(arctan(a*x)^2/x/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
integral(arctan(a*x)^2/(a^2*c*x^3 + c*x), x)
```

## Sympy [F]

$$\int \frac{\arctan(ax)^2}{x\,(c+a^2cx^2)}\,dx = \frac{\int \frac{\operatorname{atan}^2\left(ax\right)}{a^2x^3+x}\,dx}{c}$$

input
```
integrate(atan(a*x)**2/x/(a**2*c*x**2+c),x)
```

output
```
Integral(atan(a*x)**2/(a**2*x**3 + x), x)/c
```

## Maxima [F]

$$\int \frac{\arctan(ax)^2}{x\,(c+a^2cx^2)}\,dx = \int \frac{\arctan\left(ax\right)^2}{(a^2cx^2+c)x}\,dx$$

input
```
integrate(arctan(a*x)^2/x/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)^2/((a^2*c*x^2 + c)*x), x)
```

## Giac [F]

$$\int \frac{\arctan(ax)^2}{x(c+a^2cx^2)}\,dx = \int \frac{\arctan(ax)^2}{(a^2cx^2+c)x}\,dx$$

input
```
integrate(arctan(a*x)^2/x/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^2/((a^2*c*x^2 + c)*x), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^2}{x(c+a^2cx^2)}\,dx = \int \frac{\operatorname{atan}(a\,x)^2}{x\,(c\,a^2\,x^2+c)}\,dx$$

input
```
int(atan(a*x)^2/(x*(c + a^2*c*x^2)),x)
```

output
```
int(atan(a*x)^2/(x*(c + a^2*c*x^2)), x)
```

## Reduce [F]

$$\int \frac{\arctan(ax)^2}{x(c+a^2cx^2)}\,dx = \frac{\int \frac{atan(ax)^2}{a^2x^3+x}dx}{c}$$

input
```
int(atan(a*x)^2/x/(a^2*c*x^2+c),x)
```

output
```
int(atan(a*x)**2/(a**2*x**3 + x),x)/c
```

# 3.288 $\int \frac{\arctan(ax)^2}{x^2(c+a^2cx^2)}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 92

$$\int \frac{\arctan(ax)^2}{x^2(c+a^2cx^2)}\,dx = -\frac{ia\arctan(ax)^2}{c} - \frac{\arctan(ax)^2}{cx} - \frac{a\arctan(ax)^3}{3c} + \frac{2a\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{c} - \frac{ia\,\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{c}$$

output
```
-I*a*arctan(a*x)^2/c-arctan(a*x)^2/c/x-1/3*a*arctan(a*x)^3/c+2*a*arctan(a*
x)*ln(2-2/(1-I*a*x))/c-I*a*polylog(2,-1+2/(1-I*a*x))/c
```

## Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.79

$$\int \frac{\arctan(ax)^2}{x^2(c+a^2cx^2)}\,dx$$
$$= \frac{a\left(-\frac{1}{3}\arctan(ax)\left(\frac{3\arctan(ax)}{ax}+\arctan(ax)(3i+\arctan(ax))-6\log\left(1-e^{2i\arctan(ax)}\right)\right)-i\,\text{PolyLog}\left(2,\right.}{c}$$

input
```
Integrate[ArcTan[a*x]^2/(x^2*(c + a^2*c*x^2)),x]
```

output
```
(a*(-1/3*(ArcTan[a*x]*((3*ArcTan[a*x])/(a*x) + ArcTan[a*x]*(3*I + ArcTan[a
*x]) - 6*Log[1 - E^((2*I)*ArcTan[a*x])])) - I*PolyLog[2, E^((2*I)*ArcTan[a
*x])]))/c
```

## Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {5453, 27, 5361, 5419, 5459, 5403, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2}{x^2(a^2cx^2+c)}dx$$

$$\Big\downarrow \text{ 5453}$$

$$\frac{\int \frac{\arctan(ax)^2}{x^2}dx}{c}-a^2\int \frac{\arctan(ax)^2}{c(a^2x^2+1)}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{\arctan(ax)^2}{x^2}dx}{c}-\frac{a^2\int \frac{\arctan(ax)^2}{a^2x^2+1}dx}{c}$$

$$\Big\downarrow \text{ 5361}$$

$$\frac{2a\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx-\frac{\arctan(ax)^2}{x}}{c}-\frac{a^2\int \frac{\arctan(ax)^2}{a^2x^2+1}dx}{c}$$

$$\Big\downarrow \text{ 5419}$$

$$\frac{2a\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx-\frac{\arctan(ax)^2}{x}}{c}-\frac{a\arctan(ax)^3}{3c}$$

$$\Big\downarrow \text{ 5459}$$

$$-\frac{a\arctan(ax)^3}{3c}+\frac{-\frac{\arctan(ax)^2}{x}+2a\left(i\int \frac{\arctan(ax)}{x(ax+i)}dx-\frac{1}{2}i\arctan(ax)^2\right)}{c}$$

$$\Big\downarrow \text{ 5403}$$

$$-\frac{a\arctan(ax)^3}{3c}+\frac{-\frac{\arctan(ax)^2}{x}+2a\left(i\left(ia\int\frac{\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)}{c}$$

$$\Big\downarrow \text{ 2897}$$

$$-\frac{a\arctan(ax)^3}{3c}+\frac{-\frac{\arctan(ax)^2}{x}+2a\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)}{c}$$

input
```
Int[ArcTan[a*x]^2/(x^2*(c + a^2*c*x^2)),x]
```

output
```
-1/3*(a*ArcTan[a*x]^3)/c + (-(ArcTan[a*x]^2/x) + 2*a*((-1/2*I)*ArcTan[a*x]
^2 + I*((-I)*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)] - PolyLog[2, -1 + 2/(1 - I
*a*x)]/2)))/c
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

## Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 251 vs. $2(86) = 172$.

Time = 0.71 (sec) , antiderivative size = 252, normalized size of antiderivative = 2.74

| method | result |
|---|---|
| derivativedivides | $a\left(-\frac{\arctan(ax)^2}{cax}-\frac{\arctan(ax)^3}{c}-\frac{2\left(-\frac{\arctan(ax)^3}{3}+\frac{\arctan(ax)\ln\left(a^2x^2+1\right)}{2}-\arctan(ax)\ln(ax)+\frac{i\left(\ln(ax-i)\ln\left(a\right.\right.}{}\right.}{}\right.$ |
| default | $a\left(-\frac{\arctan(ax)^2}{cax}-\frac{\arctan(ax)^3}{c}-\frac{2\left(-\frac{\arctan(ax)^3}{3}+\frac{\arctan(ax)\ln\left(a^2x^2+1\right)}{2}-\arctan(ax)\ln(ax)+\frac{i\left(\ln(ax-i)\ln\left(a\right.\right.}{}\right.}{}\right.$ |
| parts | $-\frac{a\arctan(ax)^3}{c}-\frac{\arctan(ax)^2}{cx}-\frac{2\left(-\frac{a\arctan(ax)^3}{3}-a\left(\arctan(ax)\ln(ax)-\frac{\arctan(ax)\ln\left(a^2x^2+1\right)}{2}+\frac{i\ln(ax)\ln(iax}{2}\right.\right.}{}$ |

input
```
int(arctan(a*x)^2/x^2/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
a*(-1/c*arctan(a*x)^2/a/x-1/c*arctan(a*x)^3-2/c*(-1/3*arctan(a*x)^3+1/2*ar
ctan(a*x)*ln(a^2*x^2+1)-arctan(a*x)*ln(a*x)+1/4*I*(ln(a*x-I)*ln(a^2*x^2+1)
-1/2*ln(a*x-I)^2-dilog(-1/2*I*(a*x+I))-ln(a*x-I)*ln(-1/2*I*(a*x+I)))-1/4*I
*(ln(a*x+I)*ln(a^2*x^2+1)-1/2*ln(a*x+I)^2-dilog(1/2*I*(a*x-I))-ln(a*x+I)*l
n(1/2*I*(a*x-I)))-1/2*I*ln(a*x)*ln(1+I*a*x)+1/2*I*ln(a*x)*ln(1-I*a*x)-1/2*
I*dilog(1+I*a*x)+1/2*I*dilog(1-I*a*x)))
```

## Fricas [F]

$$\int \frac{\arctan(ax)^2}{x^2\left(c+a^2cx^2\right)}\,dx=\int \frac{\arctan\left(ax\right)^2}{(a^2cx^2+c)x^2}\,dx$$

input
```
integrate(arctan(a*x)^2/x^2/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
integral(arctan(a*x)^2/(a^2*c*x^4 + c*x^2), x)
```

## Sympy [F]

$$\int \frac{\arctan(ax)^2}{x^2\left(c+a^2cx^2\right)}\,dx=\frac{\int \frac{\operatorname{atan}^2(ax)}{a^2x^4+x^2}\,dx}{c}$$

input
```
integrate(atan(a*x)**2/x**2/(a**2*c*x**2+c),x)
```

output
```
Integral(atan(a*x)**2/(a**2*x**4 + x**2), x)/c
```

## Maxima [F]

$$\int \frac{\arctan(ax)^2}{x^2\left(c+a^2cx^2\right)}\,dx=\int \frac{\arctan\left(ax\right)^2}{(a^2cx^2+c)x^2}\,dx$$

input
```
integrate(arctan(a*x)^2/x^2/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
-1/48*(8*a*x*arctan(a*x)^3 - 4*(a*arctan(a*x)^3/c + 12*a^2*integrate(1/16*
x^2*log(a^2*x^2 + 1)^2/(a^2*c*x^4 + c*x^2), x) - 48*a^2*integrate(1/16*x^2
*log(a^2*x^2 + 1)/(a^2*c*x^4 + c*x^2), x) + 96*a*integrate(1/16*x*arctan(a
*x)/(a^2*c*x^4 + c*x^2), x) + 144*integrate(1/16*arctan(a*x)^2/(a^2*c*x^4
+ c*x^2), x) + 12*integrate(1/16*log(a^2*x^2 + 1)^2/(a^2*c*x^4 + c*x^2), x
))*c*x + 12*arctan(a*x)^2 - 3*log(a^2*x^2 + 1)^2)/(c*x)
```

## Giac [F]

$$\int \frac{\arctan(ax)^2}{x^2\left(c+a^2cx^2\right)}\,dx=\int \frac{\arctan\left(ax\right)^2}{(a^2cx^2+c)x^2}\,dx$$

input
```
integrate(arctan(a*x)^2/x^2/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^2/((a^2*c*x^2 + c)*x^2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^2}{x^2 (c + a^2 c x^2)} \, dx = \int \frac{\mathrm{atan}(a\,x)^2}{x^2\,\left(c\,a^2\,x^2 + c\right)} \, dx$$

input
```
int(atan(a*x)^2/(x^2*(c + a^2*c*x^2)),x)
```

output
```
int(atan(a*x)^2/(x^2*(c + a^2*c*x^2)), x)
```

## Reduce [F]

$$\int \frac{\arctan(ax)^2}{x^2 (c + a^2 c x^2)} \, dx = \frac{-atan(ax)^3 ax - 3atan(ax)^2 + 6\left(\int \frac{atan(ax)}{a^2 x^3 + x} dx\right) ax}{3cx}$$

input
```
int(atan(a*x)^2/x^2/(a^2*c*x^2+c),x)
```

output
```
( - atan(a*x)**3*a*x - 3*atan(a*x)**2 + 6*int(atan(a*x)/(a**2*x**3 + x),x)
*a*x)/(3*c*x)
```

# 3.289 $\int \frac{\arctan(ax)^2}{x^3(c+a^2cx^2)} dx$



## Optimal result

Integrand size = 22, antiderivative size = 178

$$\int \frac{\arctan(ax)^2}{x^3(c+a^2cx^2)} dx = -\frac{a\arctan(ax)}{cx} - \frac{a^2\arctan(ax)^2}{2c} - \frac{\arctan(ax)^2}{2cx^2} + \frac{ia^2\arctan(ax)^3}{3c} + \frac{a^2\log(x)}{c} - \frac{a^2\log(1+a^2x^2)}{2c} - \frac{a^2\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{c} + \frac{ia^2\arctan(ax)\,\mathrm{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{c} - \frac{a^2\,\mathrm{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)}{2c}$$

output
```
-a*arctan(a*x)/c/x-1/2*a^2*arctan(a*x)^2/c-1/2*arctan(a*x)^2/c/x^2+1/3*I*a
^2*arctan(a*x)^3/c+a^2*ln(x)/c-1/2*a^2*ln(a^2*x^2+1)/c-a^2*arctan(a*x)^2*l
n(2-2/(1-I*a*x))/c+I*a^2*arctan(a*x)*polylog(2,-1+2/(1-I*a*x))/c-1/2*a^2*p
olylog(3,-1+2/(1-I*a*x))/c
```

## Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.80

$$\int \frac{\arctan(ax)^2}{x^3\left(c+a^2cx^2\right)}\,dx = \frac{a^2\left(\frac{i\pi^3}{24}-\frac{\arctan(ax)}{ax}-\frac{(1+a^2x^2)\arctan(ax)^2}{2a^2x^2}-\frac{1}{3}i\arctan(ax)^3-\arctan(ax)^2\log\left(1-e^{-2i\arctan(ax)}\right)+\log\left(\frac{a}{\sqrt{1+}}\right.\right.}{c}$$

input
```
Integrate[ArcTan[a*x]^2/(x^3*(c + a^2*c*x^2)),x]
```

output
```
(a^2*((I/24)*Pi^3 - ArcTan[a*x]/(a*x) - ((1 + a^2*x^2)*ArcTan[a*x]^2)/(2*a
^2*x^2) - (I/3)*ArcTan[a*x]^3 - ArcTan[a*x]^2*Log[1 - E^((-2*I)*ArcTan[a*x
])] + Log[(a*x)/Sqrt[1 + a^2*x^2]] - I*ArcTan[a*x]*PolyLog[2, E^((-2*I)*Ar
cTan[a*x])] - PolyLog[3, E^((-2*I)*ArcTan[a*x])]/2))/c
```

## Rubi [A] (verified)

Time = 1.43 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.97, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {5453, 27, 5361, 5453, 5361, 243, 47, 14, 16, 5419, 5459, 5403, 5527, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2}{x^3\left(a^2cx^2+c\right)}dx$$

$$\Big\downarrow \; 5453$$

$$\frac{\int \frac{\arctan(ax)^2}{x^3}dx}{c}-a^2\int \frac{\arctan(ax)^2}{cx\left(a^2x^2+1\right)}dx$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\arctan(ax)^2}{x^3}dx}{c}-\frac{a^2\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \; 5361$$

$$\frac{a\int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx-\frac{\arctan(ax)^2}{2x^2}}{c}-\frac{a^2\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \text{5453}$$

$$\frac{a\left(\int \frac{\arctan(ax)}{x^2}dx-a^2\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)-\frac{\arctan(ax)^2}{2x^2}}{c}-\frac{a^2\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \text{5361}$$

$$\frac{a\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+a\int \frac{1}{x(a^2x^2+1)}dx-\frac{\arctan(ax)}{x}\right)-\frac{\arctan(ax)^2}{2x^2}}{c}-\frac{a^2\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \text{243}$$

$$\frac{a\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{2}a\int \frac{1}{x^2(a^2x^2+1)}dx^2-\frac{\arctan(ax)}{x}\right)-\frac{\arctan(ax)^2}{2x^2}}{c}-\frac{a^2\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \text{47}$$

$$\frac{a\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{2}a\left(\int \frac{1}{x^2}dx^2-a^2\int \frac{1}{a^2x^2+1}dx^2\right)-\frac{\arctan(ax)}{x}\right)-\frac{\arctan(ax)^2}{2x^2}}{c}-\frac{a^2\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \text{14}$$

$$\frac{a\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{2}a\left(\log\left(x^2\right)-a^2\int \frac{1}{a^2x^2+1}dx^2\right)-\frac{\arctan(ax)}{x}\right)-\frac{\arctan(ax)^2}{2x^2}}{c}-\frac{a^2\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \text{16}$$

$$\frac{a\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{\arctan(ax)}{x}\right)-\frac{\arctan(ax)^2}{2x^2}}{c}-\frac{a^2\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \text{5419}$$

$$\frac{a\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{1}{2}a\arctan(ax)^2-\frac{\arctan(ax)}{x}\right)-\frac{\arctan(ax)^2}{2x^2}}{c}-\frac{a^2\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx}{c}$$

$$\downarrow \text{5459}$$

$$\frac{a\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{1}{2}a\arctan(ax)^2-\frac{\arctan(ax)}{x}\right)-\frac{\arctan(ax)^2}{2x^2}}{c}-\frac{a^2\left(i\int\frac{\arctan(ax)^2}{x(ax+i)}dx-\frac{1}{3}i\arctan(ax)^3\right)}{c}$$

$$\downarrow \text{5403}$$

$$\frac{a\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{1}{2}a\arctan(ax)^2-\frac{\arctan(ax)}{x}\right)-\frac{\arctan(ax)^2}{2x^2}}{c}-\frac{a^2\left(i\left(2ia\int\frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3\right)}{c}$$

$$\downarrow \text{5527}$$

$$\frac{a\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{1}{2}a\arctan(ax)^2-\frac{\arctan(ax)}{x}\right)-\frac{\arctan(ax)^2}{2x^2}}{c}-\frac{a^2\left(i\left(2ia\left(\frac{i\arctan(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-\frac{1}{2}i\int\frac{\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{a^2x^2+1}dx\right)-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3\right)}{c}$$

$$\downarrow \text{7164}$$

$$\frac{a\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{1}{2}a\arctan(ax)^2-\frac{\arctan(ax)}{x}\right)-\frac{\arctan(ax)^2}{2x^2}}{c}-\frac{a^2\left(i\left(2ia\left(\frac{i\arctan(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-\frac{\operatorname{PolyLog}\left(3,\frac{2}{1-iax}-1\right)}{4a}\right)-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3\right)}{c}$$

input
```
Int[ArcTan[a*x]^2/(x^3*(c + a^2*c*x^2)),x]
```

output
```
(-1/2*ArcTan[a*x]^2/x^2 + a*(-(ArcTan[a*x]/x) - (a*ArcTan[a*x]^2)/2 + (a*(
Log[x^2] - Log[1 + a^2*x^2]))/2))/c - (a^2*((-1/3*I)*ArcTan[a*x]^3 + I*((-
I)*ArcTan[a*x]^2*Log[2 - 2/(1 - I*a*x)] + (2*I)*a*(((I/2)*ArcTan[a*x]*Poly
Log[2, -1 + 2/(1 - I*a*x)])/a - PolyLog[3, -1 + 2/(1 - I*a*x)]/(4*a)))))/c
```

### Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5527
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 67.41 (sec) , antiderivative size = 1826, normalized size of antiderivative = 10.26

| method | result | size |
|---|---|---|
| derivativedivides | Expression too large to display | 1826 |
| default | Expression too large to display | 1826 |
| parts | Expression too large to display | 2266 |

input
```
int(arctan(a*x)^2/x^3/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
a^2*(-1/2/c*arctan(a*x)^2/a^2/x^2-1/c*arctan(a*x)^2*ln(a*x)+1/2/c*arctan(a
*x)^2*ln(a^2*x^2+1)-1/c*(1/2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn
(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a
*x)^2/(a^2*x^2+1)+1))*arctan(a*x)^2-1/4*I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+
1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((
1+I*a*x)^2/(a^2*x^2+1)+1)^2)*arctan(a*x)^2+1/2*arctan(a*x)^2-ln((1+I*a*x)/
(a^2*x^2+1)^(1/2)-1)-ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-1/3*I*arctan(a*x)^3
+1/2*arctan(a*x)*(I*a*x+(a^2*x^2+1)^(1/2)+1)/a/x+1/2*arctan(a*x)*(I*a*x-(a
^2*x^2+1)^(1/2)+1)/a/x+2*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*polylog(
3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+1/2*I*Pi*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/
((1+I*a*x)^2/(a^2*x^2+1)+1))^3*arctan(a*x)^2+arctan(a*x)^2*ln((1+I*a*x)/(a
^2*x^2+1)^(1/2))+1/2*I*Pi*arctan(a*x)^2-2*I*arctan(a*x)*polylog(2,(1+I*a*x
)/(a^2*x^2+1)^(1/2))-2*I*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2
))+1/2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1)
)*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*arctan(a*x
)^2-1/2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*
x^2+1)+1)^2)^2*arctan(a*x)^2-1/4*I*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))^
2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*arctan(a*x)^2-1/2*I*Pi*csgn(I*((1+I*a*x)
^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+
1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2-1/2*I*Pi*csgn(I*((1+...
```

## Fricas [F]

$$\int \frac{\arctan(ax)^2}{x^3(c+a^2cx^2)}\,dx = \int \frac{\arctan(ax)^2}{(a^2cx^2+c)x^3}\,dx$$

input
```
integrate(arctan(a*x)^2/x^3/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
integral(arctan(a*x)^2/(a^2*c*x^5 + c*x^3), x)
```

## Sympy [F]

$$\int \frac{\arctan(ax)^2}{x^3 (c + a^2cx^2)}\, dx = \frac{\int \frac{\operatorname{atan}^2(ax)}{a^2x^5+x^3}\, dx}{c}$$

input
```
integrate(atan(a*x)**2/x**3/(a**2*c*x**2+c),x)
```

output
```
Integral(atan(a*x)**2/(a**2*x**5 + x**3), x)/c
```

## Maxima [F]

$$\int \frac{\arctan(ax)^2}{x^3 (c + a^2cx^2)}\, dx = \int \frac{\arctan(ax)^2}{(a^2cx^2 + c)x^3}\, dx$$

input
```
integrate(arctan(a*x)^2/x^3/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)^2/((a^2*c*x^2 + c)*x^3), x)
```

## Giac [F]

$$\int \frac{\arctan(ax)^2}{x^3 (c + a^2cx^2)}\, dx = \int \frac{\arctan(ax)^2}{(a^2cx^2 + c)x^3}\, dx$$

input
```
integrate(arctan(a*x)^2/x^3/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^2/((a^2*c*x^2 + c)*x^3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^2}{x^3(c+a^2cx^2)}\,dx = \int \frac{\operatorname{atan}(a\,x)^2}{x^3\,\left(c\,a^2\,x^2+c\right)}\,dx$$

input
```
int(atan(a*x)^2/(x^3*(c + a^2*c*x^2)),x)
```

output
```
int(atan(a*x)^2/(x^3*(c + a^2*c*x^2)), x)
```

## Reduce [F]

$$\int \frac{\arctan(ax)^2}{x^3(c+a^2cx^2)}\,dx = \frac{\int \frac{atan(ax)^2}{a^2x^5+x^3}dx}{c}$$

input
```
int(atan(a*x)^2/x^3/(a^2*c*x^2+c),x)
```

output
```
int(atan(a*x)**2/(a**2*x**5 + x**3),x)/c
```

# 3.290 $\int \frac{\arctan(ax)^2}{x^4(c+a^2cx^2)}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 166

$$\int \frac{\arctan(ax)^2}{x^4(c+a^2cx^2)}\,dx = -\frac{a^2}{3cx}-\frac{a^3\arctan(ax)}{3c}-\frac{a\arctan(ax)}{3cx^2}+\frac{4ia^3\arctan(ax)^2}{3c}-\frac{\arctan(ax)^2}{3cx^3}+\frac{a^2\arctan(ax)^2}{cx}+\frac{a^3\arctan(ax)^3}{3c}-\frac{8a^3\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{3c}+\frac{4ia^3\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{3c}$$

output
```
-1/3*a^2/c/x-1/3*a^3*arctan(a*x)/c-1/3*a*arctan(a*x)/c/x^2+4/3*I*a^3*arcta
n(a*x)^2/c-1/3*arctan(a*x)^2/c/x^3+a^2*arctan(a*x)^2/c/x+1/3*a^3*arctan(a*
x)^3/c-8/3*a^3*arctan(a*x)*ln(2-2/(1-I*a*x))/c+4/3*I*a^3*polylog(2,-1+2/(1
-I*a*x))/c
```

## Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.72

$$\int \frac{\arctan(ax)^2}{x^4\left(c+a^2cx^2\right)}\,dx = \frac{a^3\left(-\frac{1-4\arctan(ax)^2+\frac{\left(1+a^2x^2\right)\arctan(ax)^2}{a^2x^2}}{ax}+\arctan(ax)\left(-\frac{1+a^2x^2}{a^2x^2}+\arctan(ax)(4i+\arctan(ax))-8\log\left(1-\right.\right.\right.}{3c}$$

input
```
Integrate[ArcTan[a*x]^2/(x^4*(c + a^2*c*x^2)),x]
```

output
```
(a^3*(-((1 - 4*ArcTan[a*x]^2 + ((1 + a^2*x^2)*ArcTan[a*x]^2)/(a^2*x^2))/(a
*x)) + ArcTan[a*x]*(-((1 + a^2*x^2)/(a^2*x^2)) + ArcTan[a*x]*(4*I + ArcTan
[a*x]) - 8*Log[1 - E^((2*I)*ArcTan[a*x])]) + (4*I)*PolyLog[2, E^((2*I)*Arc
Tan[a*x])]))/(3*c)
```

## Rubi [A] (verified)

Time = 1.40 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.30, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5453, 27, 5361, 5453, 5361, 264, 216, 5419, 5459, 5403, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2}{x^4\left(a^2cx^2+c\right)}\,dx$$

$$\Big\downarrow \text{ 5453}$$

$$\frac{\int \frac{\arctan(ax)^2}{x^4}dx}{c}-a^2\int \frac{\arctan(ax)^2}{cx^2\left(a^2x^2+1\right)}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{\arctan(ax)^2}{x^4}dx}{c}-\frac{a^2\int \frac{\arctan(ax)^2}{x^2\left(a^2x^2+1\right)}dx}{c}$$

$$\Big\downarrow \text{5361}$$

$$\frac{\frac{2}{3}a\int \frac{\arctan(ax)}{x^3(a^2x^2+1)}dx-\frac{\arctan(ax)^2}{3x^3}}{c}-\frac{a^2\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \text{5453}$$

$$\frac{\frac{2}{3}a\left(\int \frac{\arctan(ax)}{x^3}dx-a^2\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx\right)-\frac{\arctan(ax)^2}{3x^3}}{c}-\frac{a^2\left(\int \frac{\arctan(ax)^2}{x^2}dx-a^2\int \frac{\arctan(ax)^2}{a^2x^2+1}dx\right)}{c}$$

$$\Big\downarrow \text{5361}$$

$$\frac{\frac{2}{3}a\left(a^2\left(-\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx\right)+\frac{1}{2}a\int \frac{1}{x^2(a^2x^2+1)}dx-\frac{\arctan(ax)}{2x^2}\right)-\frac{\arctan(ax)^2}{3x^3}}{c}-\frac{a^2\left(a^2\left(-\int \frac{\arctan(ax)^2}{a^2x^2+1}dx\right)+2a\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx-\frac{\arctan(ax)^2}{x}\right)}{c}$$

$$\Big\downarrow \text{264}$$

$$\frac{\frac{2}{3}a\left(a^2\left(-\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx\right)+\frac{1}{2}a\left(a^2\left(-\int \frac{1}{a^2x^2+1}dx\right)-\frac{1}{x}\right)-\frac{\arctan(ax)}{2x^2}\right)-\frac{\arctan(ax)^2}{3x^3}}{c}-\frac{a^2\left(a^2\left(-\int \frac{\arctan(ax)^2}{a^2x^2+1}dx\right)+2a\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx-\frac{\arctan(ax)^2}{x}\right)}{c}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{2}{3}a\left(a^2\left(-\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx\right)-\frac{\arctan(ax)}{2x^2}+\frac{1}{2}a\left(-a\arctan(ax)-\frac{1}{x}\right)\right)-\frac{\arctan(ax)^2}{3x^3}}{c}-\frac{a^2\left(a^2\left(-\int \frac{\arctan(ax)^2}{a^2x^2+1}dx\right)+2a\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx-\frac{\arctan(ax)^2}{x}\right)}{c}$$

$$\Big\downarrow \text{5419}$$

$$\frac{\frac{2}{3}a\left(a^2\left(-\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx\right)-\frac{\arctan(ax)}{2x^2}+\frac{1}{2}a\left(-a\arctan(ax)-\frac{1}{x}\right)\right)-\frac{\arctan(ax)^2}{3x^3}}{c}-\frac{a^2\left(2a\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx-\frac{1}{3}a\arctan(ax)^3-\frac{\arctan(ax)^2}{x}\right)}{c}$$

$$\Big\downarrow \text{5459}$$

$$\frac{-\frac{\arctan(ax)^2}{3x^3}+\frac{2}{3}a\left(-\left(a^2\left(i\int \frac{\arctan(ax)}{x(ax+i)}dx-\frac{1}{2}i\arctan(ax)^2\right)\right)-\frac{\arctan(ax)}{2x^2}+\frac{1}{2}a\left(-a\arctan(ax)-\frac{1}{x}\right)\right)}{c}-\frac{a^2\left(2a\left(i\int \frac{\arctan(ax)}{x(ax+i)}dx-\frac{1}{2}i\arctan(ax)^2\right)-\frac{1}{3}a\arctan(ax)^3-\frac{\arctan(ax)^2}{x}\right)}{c}$$

$\downarrow$ 5403

$$\frac{-\frac{\arctan(ax)^2}{3x^3}+\frac{2}{3}a\left(-\left(a^2\left(i\left(ia\int\frac{\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)\right)-\frac{\arctan(}{2x^2}\right.}{c}$$

$$\frac{a^2\left(2a\left(i\left(ia\int\frac{\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)-\frac{1}{3}a\arctan(ax)^3-\frac{\arctan(ax}{x}\right.}{c}$$

$\downarrow$ 2897

$$\frac{-\frac{\arctan(ax)^2}{3x^3}+\frac{2}{3}a\left(-\left(a^2\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)\right)-\frac{a}{}\right.}{c}$$

$$\frac{a^2\left(2a\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)-\frac{1}{3}a\arctan(ax)^3-\frac{\text{arc}}{}\right.}{c}$$

input
```
Int[ArcTan[a*x]^2/(x^4*(c + a^2*c*x^2)),x]
```

output
```
(-1/3*ArcTan[a*x]^2/x^3 + (2*a*(-1/2*ArcTan[a*x]/x^2 + (a*(-x^(-1) - a*Arc
Tan[a*x]))/2 - a^2*((-1/2*I)*ArcTan[a*x]^2 + I*((-I)*ArcTan[a*x]*Log[2 - 2
/(1 - I*a*x)] - PolyLog[2, -1 + 2/(1 - I*a*x)]/2))))/3)/c - (a^2*(-(ArcTan
[a*x]^2/x) - (a*ArcTan[a*x]^3)/3 + 2*a*((-1/2*I)*ArcTan[a*x]^2 + I*((-I)*A
rcTan[a*x]*Log[2 - 2/(1 - I*a*x)] - PolyLog[2, -1 + 2/(1 - I*a*x)]/2))))/c
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

## Maple [A] (verified)

Time = 1.32 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.77

| method | result |
|---|---|
| derivativedivides | $a^3\left(-\frac{\arctan(ax)^2}{3c\,a^3x^3}+\frac{\arctan(ax)^2}{cax}+\frac{\arctan(ax)^3}{c}-\frac{2\left(-2\arctan(ax)\ln(a^2x^2+1)+\frac{\arctan(ax)}{2a^2x^2}+4\arctan(ax)\ln\right.}{}\right.$ |
| default | $a^3\left(-\frac{\arctan(ax)^2}{3c\,a^3x^3}+\frac{\arctan(ax)^2}{cax}+\frac{\arctan(ax)^3}{c}-\frac{2\left(-2\arctan(ax)\ln(a^2x^2+1)+\frac{\arctan(ax)}{2a^2x^2}+4\arctan(ax)\ln\right.}{}\right.$ |
| parts | $\frac{a^3\arctan(ax)^3}{c}-\frac{\arctan(ax)^2}{3c\,x^3}+\frac{a^2\arctan(ax)^2}{cx}-\frac{2\left(a^3\left(-2\arctan(ax)\ln(a^2x^2+1)+\frac{\arctan(ax)}{2a^2x^2}+4\arctan(ax)\right.\right.}{}$ |

input
```
int(arctan(a*x)^2/x^4/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
a^3*(-1/3/c*arctan(a*x)^2/a^3/x^3+1/c*arctan(a*x)^2/a/x+1/c*arctan(a*x)^3-
2/3/c*(-2*arctan(a*x)*ln(a^2*x^2+1)+1/2*arctan(a*x)/a^2/x^2+4*arctan(a*x)*
ln(a*x)+1/2/a/x+1/2*arctan(a*x)-I*(ln(a*x-I)*ln(a^2*x^2+1)-1/2*ln(a*x-I)^2
-dilog(-1/2*I*(a*x+I))-ln(a*x-I)*ln(-1/2*I*(a*x+I)))+I*(ln(a*x+I)*ln(a^2*x
^2+1)-1/2*ln(a*x+I)^2-dilog(1/2*I*(a*x-I))-ln(a*x+I)*ln(1/2*I*(a*x-I)))+2*
I*ln(a*x)*ln(1+I*a*x)-2*I*ln(a*x)*ln(1-I*a*x)+2*I*dilog(1+I*a*x)-2*I*dilog
(1-I*a*x)+arctan(a*x)^3))
```

## Fricas [F]

$$\int \frac{\arctan(ax)^2}{x^4 (c + a^2cx^2)} dx = \int \frac{\arctan(ax)^2}{(a^2cx^2 + c)x^4} dx$$

input
```
integrate(arctan(a*x)^2/x^4/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
integral(arctan(a*x)^2/(a^2*c*x^6 + c*x^4), x)
```

## Sympy [F]

$$\int \frac{\arctan(ax)^2}{x^4 (c + a^2cx^2)} dx = \frac{\int \frac{\operatorname{atan}^2(ax)}{a^2x^6+x^4} dx}{c}$$

input
```
integrate(atan(a*x)**2/x**4/(a**2*c*x**2+c),x)
```

output
```
Integral(atan(a*x)**2/(a**2*x**6 + x**4), x)/c
```

## Maxima [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^2}{x^4 (c + a^2cx^2)} dx = \text{Timed out}$$

input
```
integrate(arctan(a*x)^2/x^4/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
Timed out
```

## Giac [F]

$$\int \frac{\arctan(ax)^2}{x^4\left(c+a^2cx^2\right)}\,dx=\int \frac{\arctan\left(ax\right)^2}{(a^2cx^2+c)x^4}\,dx$$

input
```
integrate(arctan(a*x)^2/x^4/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^2/((a^2*c*x^2 + c)*x^4), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^2}{x^4\left(c+a^2cx^2\right)}\,dx=\int \frac{\mathrm{atan}(a\,x)^2}{x^4\,\left(c\,a^2\,x^2+c\right)}\,dx$$

input
```
int(atan(a*x)^2/(x^4*(c + a^2*c*x^2)),x)
```

output
```
int(atan(a*x)^2/(x^4*(c + a^2*c*x^2)), x)
```

## Reduce [F]

$$\int \frac{\arctan(ax)^2}{x^4\left(c+a^2cx^2\right)}\,dx$$
$$=\frac{atan(ax)^3\,a^3x^3+3atan(ax)^2\,a^2x^2-atan(ax)^2+3atan(ax)\,a^3x^3+3atan(ax)\,ax+8\left(\int \frac{atan(ax)}{a^2x^5+x^3}dx\right)a\,x^{3}}{3c\,x^3}$$

input
```
int(atan(a*x)^2/x^4/(a^2*c*x^2+c),x)
```

output
```
(atan(a*x)**3*a**3*x**3 + 3*atan(a*x)**2*a**2*x**2 - atan(a*x)**2 + 3*atan
(a*x)*a**3*x**3 + 3*atan(a*x)*a*x + 8*int(atan(a*x)/(a**2*x**5 + x**3),x)*
a*x**3 + 3*a**2*x**2)/(3*c*x**3)
```

# 3.291 $\int \frac{x^3 \arctan(ax)^2}{(c+a^2cx^2)^2}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 192

$$\begin{aligned}\int \frac{x^3 \arctan(ax)^2}{(c+a^2cx^2)^2}\,dx = &-\frac{1}{4a^4c^2(1+a^2x^2)}-\frac{x\arctan(ax)}{2a^3c^2(1+a^2x^2)}-\frac{\arctan(ax)^2}{4a^4c^2}\\ &+\frac{\arctan(ax)^2}{2a^4c^2(1+a^2x^2)}-\frac{i\arctan(ax)^3}{3a^4c^2}-\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a^4c^2}\\ &-\frac{i\arctan(ax)\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{a^4c^2}-\frac{\mathrm{PolyLog}\left(3,1-\frac{2}{1+iax}\right)}{2a^4c^2}\end{aligned}$$

output
```
-1/4/a^4/c^2/(a^2*x^2+1)-1/2*x*arctan(a*x)/a^3/c^2/(a^2*x^2+1)-1/4*arctan(
a*x)^2/a^4/c^2+1/2*arctan(a*x)^2/a^4/c^2/(a^2*x^2+1)-1/3*I*arctan(a*x)^3/a
^4/c^2-arctan(a*x)^2*ln(2/(1+I*a*x))/a^4/c^2-I*arctan(a*x)*polylog(2,1-2/(
1+I*a*x))/a^4/c^2-1/2*polylog(3,1-2/(1+I*a*x))/a^4/c^2
```

## Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.61

$$\int \frac{x^3 \arctan(ax)^2}{(c+a^2cx^2)^2}\,dx$$
$$= \frac{\frac{1}{3}i\arctan(ax)^3+\frac{1}{8}(-1+2\arctan(ax)^2)\cos(2\arctan(ax))-\arctan(ax)^2\log\left(1+e^{2i\arctan(ax)}\right)+i\arctan}{a^4c^2}$$

input
```
Integrate[(x^3*ArcTan[a*x]^2)/(c + a^2*c*x^2)^2,x]
```

output
```
((I/3)*ArcTan[a*x]^3 + ((-1 + 2*ArcTan[a*x]^2)*Cos[2*ArcTan[a*x]])/8 - Arc
Tan[a*x]^2*Log[1 + E^((2*I)*ArcTan[a*x])] + I*ArcTan[a*x]*PolyLog[2, -E^((
2*I)*ArcTan[a*x])] - PolyLog[3, -E^((2*I)*ArcTan[a*x])]/2 - (ArcTan[a*x]*S
in[2*ArcTan[a*x]])/4)/(a^4*c^2)
```

## Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {5499, 27, 5455, 5379, 5465, 5427, 241, 5529, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \arctan(ax)^2}{(a^2cx^2+c)^2}\,dx$$

$$\Big\downarrow \; 5499$$

$$\frac{\int \frac{x\arctan(ax)^2}{c(a^2x^2+1)}dx}{a^2c}-\frac{\int \frac{x\arctan(ax)^2}{c^2(a^2x^2+1)^2}dx}{a^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2c^2}-\frac{\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx}{a^2c^2}$$

$$\Big\downarrow \; 5455$$

$$-\frac{\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx}{a^2c^2}+\frac{-\frac{\int \frac{\arctan(ax)^2}{i-ax}dx}{a}-\frac{i\arctan(ax)^3}{3a^2}}{a^2c^2}$$

$$\Big\downarrow \ 5379$$

$$-\frac{\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx}{a^2c^2}+\frac{-\frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}-2\int \frac{\arctan(ax)\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^3}{3a^2}}{a^2c^2}$$

$$\Big\downarrow \ 5465$$

$$-\frac{\frac{\int \frac{\arctan(ax)}{\left(a^2x^2+1\right)^2}dx}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}}{a^2c^2}+\frac{-\frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}-2\int \frac{\arctan(ax)\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^3}{3a^2}}{a^2c^2}$$

$$\Big\downarrow \ 5427$$

$$-\frac{\frac{-\frac{1}{2}a\int \frac{x}{\left(a^2x^2+1\right)^2}dx+\frac{x\arctan(ax)}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^2}{4a}}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}}{a^2c^2}+\frac{-\frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}-2\int \frac{\arctan(ax)\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^3}{3a^2}}{a^2c^2}$$

$$\Big\downarrow \ 241$$

$$-\frac{\frac{\frac{x\arctan(ax)}{2\left(a^2x^2+1\right)}+\frac{1}{4a\left(a^2x^2+1\right)}+\frac{\arctan(ax)^2}{4a}}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}}{a^2c^2}+\frac{-\frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}-2\int \frac{\arctan(ax)\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^3}{3a^2}}{a^2c^2}$$

$$\Big\downarrow \ 5529$$

$$-\frac{\frac{\frac{x\arctan(ax)}{2\left(a^2x^2+1\right)}+\frac{1}{4a\left(a^2x^2+1\right)}+\frac{\arctan(ax)^2}{4a}}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}}{a^2c^2}+\frac{-\frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}-2\left(\frac{1}{2}i\int \frac{\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx-\frac{i\arctan(ax)\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}\right)}{a}-\frac{i\arctan(ax)^3}{3a^2}}{a^2c^2}$$

$$\Big\downarrow \ 7164$$

$$-\frac{\frac{\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}}{a^2c^2}+\frac{-\frac{i\arctan(ax)^3}{3a^2}-\frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}-2\left(-\frac{i\arctan(ax)\operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}-\frac{\operatorname{PolyLog}\left(3,1-\frac{2}{iax+1}\right)}{4a}\right)}{a}}{a^2c^2}$$

input
```
Int[(x^3*ArcTan[a*x]^2)/(c + a^2*c*x^2)^2,x]
```

output
```
-((-1/2*ArcTan[a*x]^2/(a^2*(1 + a^2*x^2)) + (1/(4*a*(1 + a^2*x^2)) + (x*Ar
cTan[a*x])/(2*(1 + a^2*x^2)) + ArcTan[a*x]^2/(4*a))/a)/(a^2*c^2)) + (((-1/
3*I)*ArcTan[a*x]^3)/a^2 - ((ArcTan[a*x]^2*Log[2/(1 + I*a*x)])/a - 2*(((-1/
2*I)*ArcTan[a*x]*PolyLog[2, 1 - 2/(1 + I*a*x)])/a - PolyLog[3, 1 - 2/(1 +
I*a*x)]/(4*a)))/a)/(a^2*c^2)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5499
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ
[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 31.13 (sec) , antiderivative size = 855, normalized size of antiderivative = 4.45

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{\arctan(ax)^2 \ln\left(a^2x^2+1\right)}{2c^2}+\frac{\arctan(ax)^2}{2c^2\left(a^2x^2+1\right)}-\frac{\arctan(ax)^2 \ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)-\frac{i\arctan(ax)^3}{3}-\frac{i\arctan(ax)(ax+i)}{8ax-8i}-\frac{ax+i}{16(ax-i)}+\frac{i\arctan}{8}}{}}{}$ |
| default | $\frac{\frac{\arctan(ax)^2}{2c^2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^2 \ln\left(a^2x^2+1\right)}{2c^2}-\frac{\arctan(ax)^2 \ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)-\frac{i\arctan(ax)^3}{3}-\frac{i\arctan(ax)(ax+i)}{8ax-8i}-\frac{ax+i}{16(ax-i)}+\frac{i\arctan}{8}}{}}{}$ |
| parts | $\frac{\arctan(ax)^2 \ln(a^2x^2+1)}{2c^2a^4}+\frac{\arctan(ax)^2}{2a^4c^2(a^2x^2+1)}-\frac{a\left(\frac{\arctan(ax)^2 \ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)}{a^5}-\frac{i\arctan(ax)^3}{3a^5}-\frac{i\arctan(ax)(ax+i)}{8a^5(ax-i)}-\frac{}{16a}\right.}{}$ |

input
```
int(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^4*(1/2/c^2*arctan(a*x)^2*ln(a^2*x^2+1)+1/2*arctan(a*x)^2/c^2/(a^2*x^2+
1)-1/c^2*(arctan(a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2))-1/3*I*arctan(a*x)^
3-I*arctan(a*x)*(a*x+I)/(8*a*x-8*I)-1/16*(a*x+I)/(a*x-I)+I*arctan(a*x)*(a*
x-I)/(8*a*x+8*I)-1/16*(a*x-I)/(a*x+I)-I*arctan(a*x)*polylog(2,-(1+I*a*x)^2
/(a^2*x^2+1))+1/2*polylog(3,-(1+I*a*x)^2/(a^2*x^2+1))+1/4*(I*Pi*csgn(I*((1
+I*a*x)^2/(a^2*x^2+1)+1))^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)-2*I*Pi*c
sgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2
+I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3-I*Pi*csgn(I*(1+I*a*x)^2/(a^2
*x^2+1))^3+2*I*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*csgn(I*(1+I*a*x)^2/(
a^2*x^2+1))^2+I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2
*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2-I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2
+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/(
(1+I*a*x)^2/(a^2*x^2+1)+1)^2)-I*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))^2*c
sgn(I*(1+I*a*x)^2/(a^2*x^2+1))-I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a
*x)^2/(a^2*x^2+1)+1)^2)^3+I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(
I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2+4*ln(2)+1)*arct
an(a*x)^2))
```

## Fricas [F]

$$\int \frac{x^3 \arctan(ax)^2}{(c+a^2cx^2)^2}\,dx = \int \frac{x^3 \arctan{(ax)}^2}{(a^2cx^2+c)^2}\,dx$$

input
```
integrate(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
integral(x^3*arctan(a*x)^2/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2), x)
```

## Sympy [F]

$$\int \frac{x^3 \arctan(ax)^2}{(c+a^2cx^2)^2}\,dx = \frac{\int \frac{x^3 \operatorname{atan}^2{(ax)}}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

input
```
integrate(x**3*atan(a*x)**2/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(x**3*atan(a*x)**2/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

## Maxima [F]

$$\int \frac{x^3 \arctan(ax)^2}{(c+a^2cx^2)^2}\,dx = \int \frac{x^3 \arctan(ax)^2}{(a^2cx^2+c)^2}\,dx$$

input
```
integrate(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
integrate(x^3*arctan(a*x)^2/(a^2*c*x^2 + c)^2, x)
```

## Giac [F]

$$\int \frac{x^3 \arctan(ax)^2}{(c+a^2cx^2)^2}\,dx = \int \frac{x^3 \arctan(ax)^2}{(a^2cx^2+c)^2}\,dx$$

input
```
integrate(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate(x^3*arctan(a*x)^2/(a^2*c*x^2 + c)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \arctan(ax)^2}{(c+a^2cx^2)^2}\,dx = \int \frac{x^3 \operatorname{atan}(a\,x)^2}{(c\,a^2\,x^2+c)^2}\,dx$$

input
```
int((x^3*atan(a*x)^2)/(c + a^2*c*x^2)^2,x)
```

output
```
int((x^3*atan(a*x)^2)/(c + a^2*c*x^2)^2, x)
```

## Reduce [F]

$$\int \frac{x^3 \arctan(ax)^2}{\left(c+a^2cx^2\right)^2}\,dx = \frac{\int \frac{atan(ax)^2x^3}{a^4x^4+2a^2x^2+1}dx}{c^2}$$

input
```
int(x^3*atan(a*x)^2/(a^2*c*x^2+c)^2,x)
```

output
```
int((atan(a*x)**2*x**3)/(a**4*x**4 + 2*a**2*x**2 + 1),x)/c**2
```

# 3.292 $\int \frac{x^2 \arctan(ax)^2}{(c+a^2cx^2)^2} dx$



## Optimal result

Integrand size = 22, antiderivative size = 106

$$\int \frac{x^2 \arctan(ax)^2}{(c+a^2cx^2)^2} dx = \frac{x}{4a^2c^2(1+a^2x^2)} + \frac{\arctan(ax)}{4a^3c^2} - \frac{\arctan(ax)}{2a^3c^2(1+a^2x^2)} - \frac{x\arctan(ax)^2}{2a^2c^2(1+a^2x^2)} + \frac{\arctan(ax)^3}{6a^3c^2}$$

output
```
1/4*x/a^2/c^2/(a^2*x^2+1)+1/4*arctan(a*x)/a^3/c^2-1/2*arctan(a*x)/a^3/c^2/
(a^2*x^2+1)-1/2*x*arctan(a*x)^2/a^2/c^2/(a^2*x^2+1)+1/6*arctan(a*x)^3/a^3/
c^2
```

## Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.64

$$\int \frac{x^2 \arctan(ax)^2}{(c+a^2cx^2)^2} dx = \frac{3ax + 3(-1+a^2x^2)\arctan(ax) - 6ax\arctan(ax)^2 + 2(1+a^2x^2)\arctan(ax)^3}{12a^3c^2(1+a^2x^2)}$$

input
```
Integrate[(x^2*ArcTan[a*x]^2)/(c + a^2*c*x^2)^2,x]
```

output
```
(3*a*x + 3*(-1 + a^2*x^2)*ArcTan[a*x] - 6*a*x*ArcTan[a*x]^2 + 2*(1 + a^2*x
^2)*ArcTan[a*x]^3)/(12*a^3*c^2*(1 + a^2*x^2))
```

## Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {5471, 27, 5465, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \arctan(ax)^2}{(a^2cx^2+c)^2}dx$$

$$\Big\downarrow \text{ 5471}$$

$$\frac{\int \frac{x\arctan(ax)}{c^2(a^2x^2+1)^2}dx}{a}+\frac{\arctan(ax)^3}{6a^3c^2}-\frac{x\arctan(ax)^2}{2a^2c^2(a^2x^2+1)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx}{ac^2}+\frac{\arctan(ax)^3}{6a^3c^2}-\frac{x\arctan(ax)^2}{2a^2c^2(a^2x^2+1)}$$

$$\Big\downarrow \text{ 5465}$$

$$\frac{\frac{\int \frac{1}{(a^2x^2+1)^2}dx}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}}{ac^2}+\frac{\arctan(ax)^3}{6a^3c^2}-\frac{x\arctan(ax)^2}{2a^2c^2(a^2x^2+1)}$$

$$\Big\downarrow \text{ 215}$$

$$\frac{\frac{\frac{1}{2}\int \frac{1}{a^2x^2+1}dx+\frac{x}{2(a^2x^2+1)}}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}}{ac^2}+\frac{\arctan(ax)^3}{6a^3c^2}-\frac{x\arctan(ax)^2}{2a^2c^2(a^2x^2+1)}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{\arctan(ax)^3}{6a^3c^2}-\frac{x\arctan(ax)^2}{2a^2c^2(a^2x^2+1)}+\frac{\frac{\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}}{ac^2}$$

input
```
Int[(x^2*ArcTan[a*x]^2)/(c + a^2*c*x^2)^2,x]
```

output
```
-1/2*(x*ArcTan[a*x]^2)/(a^2*c^2*(1 + a^2*x^2)) + ArcTan[a*x]^3/(6*a^3*c^2)
 + (-1/2*ArcTan[a*x]/(a^2*(1 + a^2*x^2)) + (x/(2*(1 + a^2*x^2)) + ArcTan[a
*x]/(2*a))/(2*a))/(a*c^2)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5471
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^2)/((d_) + (e_.)*(x_)^2)
^2, x_Symbol] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(2*b*c^3*d^2*(p + 1)), x]
 + (-Simp[x*((a + b*ArcTan[c*x])^p/(2*c^2*d*(d + e*x^2))), x] + Simp[b*(p/(
2*c))   Int[x*((a + b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

## Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.71

| method | result |
|---|---|
| parallelrisch | $\frac{2\arctan(ax)^3a^2x^2+3x^2a^2\arctan(ax)-6a\arctan(ax)^2x+2\arctan(ax)^3+3ax-3\arctan(ax)}{12c^2(a^2x^2+1)a^3}$ |
| derivativedivides | $\frac{-\frac{ax\arctan(ax)^2}{2c^2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^3}{2c^2}-\frac{\frac{\arctan(ax)^3}{3}+\frac{\arctan(ax)}{2a^2x^2+2}-\frac{ax}{4\left(a^2x^2+1\right)}-\frac{\arctan(ax)}{4}}{c^2}}{a^3}$ |
| default | $\frac{-\frac{ax\arctan(ax)^2}{2c^2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^3}{2c^2}-\frac{\frac{\arctan(ax)^3}{3}+\frac{\arctan(ax)}{2a^2x^2+2}-\frac{ax}{4\left(a^2x^2+1\right)}-\frac{\arctan(ax)}{4}}{c^2}}{a^3}$ |
| parts | $-\frac{x\arctan(ax)^2}{2a^2c^2(a^2x^2+1)}+\frac{\arctan(ax)^3}{2a^3c^2}-\frac{\frac{\arctan(ax)^3}{3a^3}-\frac{-\frac{\arctan(ax)}{2\left(a^2x^2+1\right)}+\frac{ax}{4a^2x^2+4}+\frac{\arctan(ax)}{4}}{a^3}}{c^2}$ |
| risch | $\frac{i\ln(iax+1)^3}{48c^2a^3}-\frac{i(a^2x^2\ln(-iax+1)+\ln(-iax+1)+2iax)\ln(iax+1)^2}{16a^3c^2(a^2x^2+1)}+\frac{i\left(a^2x^2\ln(-iax+1)^2+\ln(-iax+1)^2+4iax\right.}{16a^3c^2(ax+i)(ax-}$ |

input
```
int(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/12*(2*arctan(a*x)^3*a^2*x^2+3*x^2*a^2*arctan(a*x)-6*a*arctan(a*x)^2*x+2*
arctan(a*x)^3+3*a*x-3*arctan(a*x))/c^2/(a^2*x^2+1)/a^3
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.65

$$\int \frac{x^2\arctan(ax)^2}{(c+a^2cx^2)^2}\,dx$$
$$=-\frac{6\,ax\arctan\left(ax\right)^2-2\left(a^2x^2+1\right)\arctan\left(ax\right)^3-3\,ax-3\left(a^2x^2-1\right)\arctan\left(ax\right)}{12\left(a^5c^2x^2+a^3c^2\right)}$$

input
```
integrate(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
-1/12*(6*a*x*arctan(a*x)^2 - 2*(a^2*x^2 + 1)*arctan(a*x)^3 - 3*a*x - 3*(a^
2*x^2 - 1)*arctan(a*x))/(a^5*c^2*x^2 + a^3*c^2)
```

## Sympy [F]

$$\int \frac{x^2 \arctan(ax)^2}{(c+a^2cx^2)^2}\,dx = \frac{\int \frac{x^2 \operatorname{atan}^2(ax)}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

input
```
integrate(x**2*atan(a*x)**2/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(x**2*atan(a*x)**2/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

## Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.42

$$\int \frac{x^2 \arctan(ax)^2}{(c+a^2cx^2)^2}\,dx = -\frac{1}{2}\left(\frac{x}{a^4c^2x^2+a^2c^2} - \frac{\arctan(ax)}{a^3c^2}\right)\arctan(ax)^2 + \frac{(2(a^2x^2+1)\arctan(ax)^3 + 3\,ax + 3(a^2x^2+1)\arctan(ax))a^2}{12(a^7c^2x^2+a^5c^2)} - \frac{((a^2x^2+1)\arctan(ax)^2+1)a\arctan(ax)}{2(a^6c^2x^2+a^4c^2)}$$

input
```
integrate(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
-1/2*(x/(a^4*c^2*x^2 + a^2*c^2) - arctan(a*x)/(a^3*c^2))*arctan(a*x)^2 + 1
/12*(2*(a^2*x^2 + 1)*arctan(a*x)^3 + 3*a*x + 3*(a^2*x^2 + 1)*arctan(a*x))*
a^2/(a^7*c^2*x^2 + a^5*c^2) - 1/2*((a^2*x^2 + 1)*arctan(a*x)^2 + 1)*a*arct
an(a*x)/(a^6*c^2*x^2 + a^4*c^2)
```

## Giac [F]

$$\int \frac{x^2 \arctan(ax)^2}{(c+a^2cx^2)^2}\,dx = \int \frac{x^2 \arctan(ax)^2}{(a^2cx^2+c)^2}\,dx$$

input
```
integrate(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate(x^2*arctan(a*x)^2/(a^2*c*x^2 + c)^2, x)
```

## Mupad [B] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.91

$$\int \frac{x^2 \arctan(ax)^2}{(c+a^2cx^2)^2}\,dx = \frac{x}{2\,(2\,a^4\,c^2\,x^2+2\,a^2\,c^2)} + \frac{\mathrm{atan}(a\,x)}{4\,a^3\,c^2} + \frac{\mathrm{atan}(a\,x)^3}{6\,a^3\,c^2} - \frac{\mathrm{atan}(a\,x)}{2\,a^5\,c^2\,\left(\frac{1}{a^2}+x^2\right)} - \frac{x\,\mathrm{atan}(a\,x)^2}{2\,a^4\,c^2\,\left(\frac{1}{a^2}+x^2\right)}$$

input
```
int((x^2*atan(a*x)^2)/(c + a^2*c*x^2)^2,x)
```

output
```
x/(2*(2*a^2*c^2 + 2*a^4*c^2*x^2)) + atan(a*x)/(4*a^3*c^2) + atan(a*x)^3/(6
*a^3*c^2) - atan(a*x)/(2*a^5*c^2*(1/a^2 + x^2)) - (x*atan(a*x)^2)/(2*a^4*c
^2*(1/a^2 + x^2))
```

## Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.70

$$\int \frac{x^2 \arctan(ax)^2}{(c+a^2cx^2)^2}\,dx = \frac{2atan(ax)^3 a^2x^2 + 2atan(ax)^3 - 6atan(ax)^2 ax + 3atan(ax)\,a^2x^2 - 3atan(ax) + 3ax}{12a^3c^2\,(a^2x^2+1)}$$

input
```
int(x^2*atan(a*x)^2/(a^2*c*x^2+c)^2,x)
```

output
```
(2*atan(a*x)**3*a**2*x**2 + 2*atan(a*x)**3 - 6*atan(a*x)**2*a*x + 3*atan(a
*x)*a**2*x**2 - 3*atan(a*x) + 3*a*x)/(12*a**3*c**2*(a**2*x**2 + 1))
```

# 3.293 $\int \frac{x \arctan(ax)^2}{(c+a^2cx^2)^2} dx$



## Optimal result

Integrand size = 20, antiderivative size = 91

$$\int \frac{x \arctan(ax)^2}{(c+a^2cx^2)^2} dx = \frac{1}{4a^2c^2(1+a^2x^2)} + \frac{x \arctan(ax)}{2ac^2(1+a^2x^2)} + \frac{\arctan(ax)^2}{4a^2c^2} - \frac{\arctan(ax)^2}{2a^2c^2(1+a^2x^2)}$$

output
```
1/4/a^2/c^2/(a^2*x^2+1)+1/2*x*arctan(a*x)/a/c^2/(a^2*x^2+1)+1/4*arctan(a*x
)^2/a^2/c^2-1/2*arctan(a*x)^2/a^2/c^2/(a^2*x^2+1)
```

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.52

$$\int \frac{x \arctan(ax)^2}{(c+a^2cx^2)^2} dx = \frac{1 + 2ax \arctan(ax) + (-1 + a^2x^2) \arctan(ax)^2}{4a^2c^2(1+a^2x^2)}$$

input
```
Integrate[(x*ArcTan[a*x]^2)/(c + a^2*c*x^2)^2,x]
```

output
```
(1 + 2*a*x*ArcTan[a*x] + (-1 + a^2*x^2)*ArcTan[a*x]^2)/(4*a^2*c^2*(1 + a^2
*x^2))
```

## Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5465, 27, 5427, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \arctan(ax)^2}{(a^2cx^2+c)^2} \, dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{\int \frac{\arctan(ax)}{c^2(a^2x^2+1)^2}dx}{a} - \frac{\arctan(ax)^2}{2a^2c^2(a^2x^2+1)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\arctan(ax)}{(a^2x^2+1)^2}dx}{ac^2} - \frac{\arctan(ax)^2}{2a^2c^2(a^2x^2+1)}$$

$$\Big\downarrow \text{5427}$$

$$\frac{-\frac{1}{2}a\int \frac{x}{(a^2x^2+1)^2}dx + \frac{x\arctan(ax)}{2(a^2x^2+1)} + \frac{\arctan(ax)^2}{4a}}{ac^2} - \frac{\arctan(ax)^2}{2a^2c^2(a^2x^2+1)}$$

$$\Big\downarrow \text{241}$$

$$\frac{\frac{x\arctan(ax)}{2(a^2x^2+1)} + \frac{1}{4a(a^2x^2+1)} + \frac{\arctan(ax)^2}{4a}}{ac^2} - \frac{\arctan(ax)^2}{2a^2c^2(a^2x^2+1)}$$

input
```
Int[(x*ArcTan[a*x]^2)/(c + a^2*c*x^2)^2,x]
```

output
```
-1/2*ArcTan[a*x]^2/(a^2*c^2*(1 + a^2*x^2)) + (1/(4*a*(1 + a^2*x^2)) + (x*A
rcTan[a*x])/(2*(1 + a^2*x^2)) + ArcTan[a*x]^2/(4*a))/(a*c^2)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

## Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.64

| method | result |
|---|---|
| parallelrisch | $\frac{\arctan(ax)^2x^2a^2-a^2x^2+2\arctan(ax)ax-\arctan(ax)^2}{4c^2(a^2x^2+1)a^2}$ |
| derivativedivides | $\frac{-\frac{\arctan(ax)^2}{2c^2\left(a^2x^2+1\right)}+\frac{\frac{\arctan(ax)ax}{2a^2x^2+2}+\frac{\arctan(ax)^2}{4}+\frac{1}{4a^2x^2+4}}{c^2}}{a^2}$ |
| default | $\frac{-\frac{\arctan(ax)^2}{2c^2\left(a^2x^2+1\right)}+\frac{\frac{\arctan(ax)ax}{2a^2x^2+2}+\frac{\arctan(ax)^2}{4}+\frac{1}{4a^2x^2+4}}{c^2}}{a^2}$ |
| parts | $-\frac{\arctan(ax)^2}{2a^2c^2(a^2x^2+1)}+\frac{\frac{\arctan(ax)ax}{2a^2x^2+2}+\frac{\arctan(ax)^2}{4}+\frac{1}{4a^2x^2+4}}{a^2c^2}$ |
| risch | $-\frac{(a^2x^2-1)\ln(iax+1)^2}{16a^2c^2(a^2x^2+1)}+\frac{(-\ln(-iax+1)+a^2x^2\ln(-iax+1)-2iax)\ln(iax+1)}{8(ax+i)a^2c^2(ax-i)}-\frac{-4+a^2x^2\ln(-iax+1)^2-\ln(-i}{16(ax+i)a^2c}$ |
| orering | $\frac{(a^2x^2+1)(10a^4x^4-3a^2x^2+1)\arctan(ax)^2}{4a^4x^2(a^2c\,x^2+c)^2}+\frac{(a^2x^2+1)^2(5a^2x^2-1)\left(\frac{\arctan(ax)^2}{\left(a^2c\,x^2+c\right)^2}+\frac{2x\arctan(ax)a}{\left(a^2c\,x^2+c\right)^2\left(a^2x^2+1\right)}-\frac{4x^2\,\mathrm{arc}}{\left(a^2\right.}\right.}{4a^4x^2}$ |

input
```
int(x*arctan(a*x)^2/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*(arctan(a*x)^2*x^2*a^2-a^2*x^2+2*arctan(a*x)*a*x-arctan(a*x)^2)/c^2/(a
^2*x^2+1)/a^2
```

## Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.53

$$\int \frac{x\arctan(ax)^2}{(c+a^2cx^2)^2}\,dx=\frac{2\,ax\arctan\left(ax\right)+\left(a^2x^2-1\right)\arctan\left(ax\right)^2+1}{4\left(a^4c^2x^2+a^2c^2\right)}$$

input
```
integrate(x*arctan(a*x)^2/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
1/4*(2*a*x*arctan(a*x) + (a^2*x^2 - 1)*arctan(a*x)^2 + 1)/(a^4*c^2*x^2 + a
^2*c^2)
```

## Sympy [F]

$$\int \frac{x\arctan(ax)^2}{(c+a^2cx^2)^2}\,dx = \frac{\int \frac{x\,\mathrm{atan}^2\,(ax)}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

input
```
integrate(x*atan(a*x)**2/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(x*atan(a*x)**2/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.14

$$\int \frac{x\arctan(ax)^2}{(c+a^2cx^2)^2}\,dx = \frac{\left(\frac{x}{a^2cx^2+c}+\frac{\arctan(ax)}{ac}\right)\arctan(ax)}{2\,ac} - \frac{(a^2x^2+1)\arctan(ax)^2-1}{4\,(a^4cx^2+a^2c)c} - \frac{\arctan(ax)^2}{2\,(a^2cx^2+c)a^2c}$$

input
```
integrate(x*arctan(a*x)^2/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
1/2*(x/(a^2*c*x^2 + c) + arctan(a*x)/(a*c))*arctan(a*x)/(a*c) - 1/4*((a^2*
x^2 + 1)*arctan(a*x)^2 - 1)/((a^4*c*x^2 + a^2*c)*c) - 1/2*arctan(a*x)^2/((
a^2*c*x^2 + c)*a^2*c)
```

## Giac [F]

$$\int \frac{x\arctan(ax)^2}{(c+a^2cx^2)^2}\,dx = \int \frac{x\arctan(ax)^2}{(a^2cx^2+c)^2}\,dx$$

input
```
integrate(x*arctan(a*x)^2/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate(x*arctan(a*x)^2/(a^2*c*x^2 + c)^2, x)
```

## Mupad [B] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.55

$$\int \frac{x \arctan(ax)^2}{(c+a^2cx^2)^2}\, dx = \frac{a^2\, x^2\, \mathrm{atan}(a\, x)^2 + 2\, a\, x\, \mathrm{atan}(a\, x) - \mathrm{atan}(a\, x)^2 + 1}{4\, a^2\, c^2\, (a^2\, x^2 + 1)}$$

input
```
int((x*atan(a*x)^2)/(c + a^2*c*x^2)^2,x)
```

output
```
(2*a*x*atan(a*x) - atan(a*x)^2 + a^2*x^2*atan(a*x)^2 + 1)/(4*a^2*c^2*(a^2*
x^2 + 1))
```

## Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.63

$$\int \frac{x \arctan(ax)^2}{(c+a^2cx^2)^2}\, dx = \frac{atan(ax)^2\, a^2x^2 - atan(ax)^2 + 2atan(ax)\, ax - a^2x^2}{4a^2c^2\, (a^2x^2 + 1)}$$

input
```
int(x*atan(a*x)^2/(a^2*c*x^2+c)^2,x)
```

output
```
(atan(a*x)**2*a**2*x**2 - atan(a*x)**2 + 2*atan(a*x)*a*x - a**2*x**2)/(4*a
**2*c**2*(a**2*x**2 + 1))
```

# 3.294 $\int \frac{\arctan(ax)^2}{(c+a^2cx^2)^2} dx$



## Optimal result

Integrand size = 19, antiderivative size = 100

$$\int \frac{\arctan(ax)^2}{(c+a^2cx^2)^2} dx = -\frac{x}{4c^2(1+a^2x^2)} - \frac{\arctan(ax)}{4ac^2} + \frac{\arctan(ax)}{2ac^2(1+a^2x^2)} + \frac{x\arctan(ax)^2}{2c^2(1+a^2x^2)} + \frac{\arctan(ax)^3}{6ac^2}$$

output
```
-1/4*x/c^2/(a^2*x^2+1)-1/4*arctan(a*x)/a/c^2+1/2*arctan(a*x)/a/c^2/(a^2*x^
2+1)+1/2*x*arctan(a*x)^2/c^2/(a^2*x^2+1)+1/6*arctan(a*x)^3/a/c^2
```

## Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.65

$$\int \frac{\arctan(ax)^2}{(c+a^2cx^2)^2} dx = \frac{-3ax+(3-3a^2x^2)\arctan(ax)+6ax\arctan(ax)^2+2(1+a^2x^2)\arctan(ax)^3}{12c^2(a+a^3x^2)}$$

input
```
Integrate[ArcTan[a*x]^2/(c + a^2*c*x^2)^2,x]
```

output
```
(-3*a*x + (3 - 3*a^2*x^2)*ArcTan[a*x] + 6*a*x*ArcTan[a*x]^2 + 2*(1 + a^2*x
^2)*ArcTan[a*x]^3)/(12*c^2*(a + a^3*x^2))
```

## Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {5427, 27, 5465, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2}{(a^2cx^2+c)^2}dx$$

$$\Big\downarrow \text{5427}$$

$$-a\int \frac{x\arctan(ax)}{c^2(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2c^2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6ac^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx}{c^2}+\frac{x\arctan(ax)^2}{2c^2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6ac^2}$$

$$\Big\downarrow \text{5465}$$

$$-\frac{a\left(\frac{\int \frac{1}{(a^2x^2+1)^2}dx}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)}{c^2}+\frac{x\arctan(ax)^2}{2c^2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6ac^2}$$

$$\Big\downarrow \text{215}$$

$$-\frac{a\left(\frac{\frac{1}{2}\int \frac{1}{a^2x^2+1}dx+\frac{x}{2(a^2x^2+1)}}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)}{c^2}+\frac{x\arctan(ax)^2}{2c^2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6ac^2}$$

$$\Big\downarrow \text{216}$$

$$\frac{x\arctan(ax)^2}{2c^2(a^2x^2+1)}-\frac{a\left(\frac{\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)}{c^2}+\frac{\arctan(ax)^3}{6ac^2}$$

input
```
Int[ArcTan[a*x]^2/(c + a^2*c*x^2)^2,x]
```

output
```
(x*ArcTan[a*x]^2)/(2*c^2*(1 + a^2*x^2)) + ArcTan[a*x]^3/(6*a*c^2) - (a*(-1
/2*ArcTan[a*x]/(a^2*(1 + a^2*x^2)) + (x/(2*(1 + a^2*x^2)) + ArcTan[a*x]/(2
*a))/(2*a)))/c^2
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

## Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.75

| method | result |
|---|---|
| parallelrisch | $\frac{2\arctan(ax)^3 a^2x^2-3x^2a^2\arctan(ax)+6a\arctan(ax)^2x+2\arctan(ax)^3-3ax+3\arctan(ax)}{12c^2(a^2x^2+1)a}$ |
| derivativedivides | $\frac{\frac{ax\arctan(ax)^2}{2c^2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^3}{2c^2}-\frac{\frac{\arctan(ax)^3}{3}-\frac{\arctan(ax)}{2\left(a^2x^2+1\right)}+\frac{ax}{4a^2x^2+4}+\frac{\arctan(ax)}{4}}{c^2}}{a}$ |
| default | $\frac{\frac{ax\arctan(ax)^2}{2c^2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^3}{2c^2}-\frac{\frac{\arctan(ax)^3}{3}-\frac{\arctan(ax)}{2\left(a^2x^2+1\right)}+\frac{ax}{4a^2x^2+4}+\frac{\arctan(ax)}{4}}{c^2}}{a}$ |
| parts | $\frac{x\arctan(ax)^2}{2c^2(a^2x^2+1)}+\frac{\arctan(ax)^3}{2a\,c^2}-\frac{\frac{\arctan(ax)^3}{3a}+\frac{-\frac{\arctan(ax)}{2\left(a^2x^2+1\right)}+\frac{ax}{4a^2x^2+4}+\frac{\arctan(ax)}{4}}{a}}{c^2}$ |
| risch | $\frac{i\ln(iax+1)^3}{48a\,c^2}-\frac{i(a^2x^2\ln(-iax+1)+\ln(-iax+1)-2iax)\ln(iax+1)^2}{16c^2(a^2x^2+1)a}+\frac{i\left(a^2x^2\ln(-iax+1)^2+\ln(-iax+1)^2-4iax\ln\right.}{16c^2(ax+i)(ax-i)}$ |

input
```
int(arctan(a*x)^2/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/12*(2*arctan(a*x)^3*a^2*x^2-3*x^2*a^2*arctan(a*x)+6*a*arctan(a*x)^2*x+2*
arctan(a*x)^3-3*a*x+3*arctan(a*x))/c^2/(a^2*x^2+1)/a
```

## Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.67

$$\int \frac{\arctan(ax)^2}{(c+a^2cx^2)^2}\,dx = \frac{6\,ax\arctan{(ax)}^2+2\,(a^2x^2+1)\arctan{(ax)}^3-3\,ax-3\,(a^2x^2-1)\arctan{(ax)}}{12\,(a^3c^2x^2+ac^2)}$$

input
```
integrate(arctan(a*x)^2/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
1/12*(6*a*x*arctan(a*x)^2 + 2*(a^2*x^2 + 1)*arctan(a*x)^3 - 3*a*x - 3*(a^2
*x^2 - 1)*arctan(a*x))/(a^3*c^2*x^2 + a*c^2)
```

## Sympy [F]

$$\int \frac{\arctan(ax)^2}{(c+a^2cx^2)^2}\,dx = \frac{\int \frac{\operatorname{atan}^2(ax)}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

input
```
integrate(atan(a*x)**2/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(atan(a*x)**2/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

## Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.46

$$\int \frac{\arctan(ax)^2}{(c+a^2cx^2)^2}\,dx = \frac{1}{2}\left(\frac{x}{a^2c^2x^2+c^2}+\frac{\arctan(ax)}{ac^2}\right)\arctan(ax)^2 + \frac{\left(2\,(a^2x^2+1)\arctan(ax)^3-3\,ax-3\,(a^2x^2+1)\arctan(ax)\right)a^2}{12\,(a^5c^2x^2+a^3c^2)} - \frac{\left((a^2x^2+1)\arctan(ax)^2-1\right)a\arctan(ax)}{2\,(a^4c^2x^2+a^2c^2)}$$

input
```
integrate(arctan(a*x)^2/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
1/2*(x/(a^2*c^2*x^2 + c^2) + arctan(a*x)/(a*c^2))*arctan(a*x)^2 + 1/12*(2*
(a^2*x^2 + 1)*arctan(a*x)^3 - 3*a*x - 3*(a^2*x^2 + 1)*arctan(a*x))*a^2/(a^
5*c^2*x^2 + a^3*c^2) - 1/2*((a^2*x^2 + 1)*arctan(a*x)^2 - 1)*a*arctan(a*x)
/(a^4*c^2*x^2 + a^2*c^2)
```

## Giac [F]

$$\int \frac{\arctan(ax)^2}{(c+a^2cx^2)^2}\,dx = \int \frac{\arctan(ax)^2}{(a^2cx^2+c)^2}\,dx$$

input
```
integrate(arctan(a*x)^2/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^2/(a^2*c*x^2 + c)^2, x)
```

## Mupad [B] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.01

$$\int \frac{\arctan(ax)^2}{(c+a^2cx^2)^2}\,dx = \frac{\operatorname{atan}(a\,x)}{2\,(a^3\,c^2\,x^2+a\,c^2)} - \frac{x}{2\,(2\,a^2\,c^2\,x^2+2\,c^2)} + \frac{x\,\operatorname{atan}(a\,x)^2}{2\,(a^2\,c^2\,x^2+c^2)} - \frac{\operatorname{atan}(a\,x)}{4\,a\,c^2} + \frac{\operatorname{atan}(a\,x)^3}{6\,a\,c^2}$$

input
```
int(atan(a*x)^2/(c + a^2*c*x^2)^2,x)
```

output
```
atan(a*x)/(2*(a*c^2 + a^3*c^2*x^2)) - x/(2*(2*c^2 + 2*a^2*c^2*x^2)) + (x*a
tan(a*x)^2)/(2*(c^2 + a^2*c^2*x^2)) - atan(a*x)/(4*a*c^2) + atan(a*x)^3/(6
*a*c^2)
```

## Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.74

$$\int \frac{\arctan(ax)^2}{(c+a^2cx^2)^2}\,dx = \frac{2atan(ax)^3 a^2x^2 + 2atan(ax)^3 + 6atan(ax)^2 ax - 3atan(ax)\,a^2x^2 + 3atan(ax) - 3ax}{12a\,c^2\,(a^2x^2+1)}$$

input
```
int(atan(a*x)^2/(a^2*c*x^2+c)^2,x)
```

output
```
(2*atan(a*x)**3*a**2*x**2 + 2*atan(a*x)**3 + 6*atan(a*x)**2*a*x - 3*atan(a
*x)*a**2*x**2 + 3*atan(a*x) - 3*a*x)/(12*a*c**2*(a**2*x**2 + 1))
```

# 3.295 $\int \frac{\arctan(ax)^2}{x(c+a^2cx^2)^2}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 170

$$\int \frac{\arctan(ax)^2}{x(c+a^2cx^2)^2}\,dx = -\frac{1}{4c^2(1+a^2x^2)} - \frac{ax\arctan(ax)}{2c^2(1+a^2x^2)} - \frac{\arctan(ax)^2}{4c^2} + \frac{\arctan(ax)^2}{2c^2(1+a^2x^2)} - \frac{i\arctan(ax)^3}{3c^2} + \frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{c^2} - \frac{i\arctan(ax)\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{c^2} + \frac{\operatorname{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)}{2c^2}$$

output
```
-1/4/c^2/(a^2*x^2+1)-1/2*a*x*arctan(a*x)/c^2/(a^2*x^2+1)-1/4*arctan(a*x)^2
/c^2+1/2*arctan(a*x)^2/c^2/(a^2*x^2+1)-1/3*I*arctan(a*x)^3/c^2+arctan(a*x)
^2*ln(2-2/(1-I*a*x))/c^2-I*arctan(a*x)*polylog(2,-1+2/(1-I*a*x))/c^2+1/2*p
olylog(3,-1+2/(1-I*a*x))/c^2
```

## Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.70

$$\int \frac{\arctan(ax)^2}{x\left(c+a^2cx^2\right)^2}\,dx$$
$$= \frac{-i\pi^3+8i\arctan(ax)^3-3\cos(2\arctan(ax))+6\arctan(ax)^2\cos(2\arctan(ax))+24\arctan(ax)^2\log\left(1-\right.}{}$$

input
```
Integrate[ArcTan[a*x]^2/(x*(c + a^2*c*x^2)^2),x]
```

output
```
((-I)*Pi^3 + (8*I)*ArcTan[a*x]^3 - 3*Cos[2*ArcTan[a*x]] + 6*ArcTan[a*x]^2*
Cos[2*ArcTan[a*x]] + 24*ArcTan[a*x]^2*Log[1 - E^((-2*I)*ArcTan[a*x])] + (2
4*I)*ArcTan[a*x]*PolyLog[2, E^((-2*I)*ArcTan[a*x])] + 12*PolyLog[3, E^((-2
*I)*ArcTan[a*x])] - 6*ArcTan[a*x]*Sin[2*ArcTan[a*x]])/(24*c^2)
```

## Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.15, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {5501, 27, 5459, 5403, 5465, 5427, 241, 5527, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2}{x\left(a^2cx^2+c\right)^2}\,dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{\arctan(ax)^2}{cx(a^2x^2+1)}dx}{c}-a^2\int \frac{x\arctan(ax)^2}{c^2\left(a^2x^2+1\right)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx}{c^2}-\frac{a^2\int \frac{x\arctan(ax)^2}{\left(a^2x^2+1\right)^2}dx}{c^2}$$

$$\Big\downarrow \text{5459}$$

$$-\frac{a^2\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx}{c^2}+\frac{i\int \frac{\arctan(ax)^2}{x(ax+i)}dx-\frac{1}{3}i\arctan(ax)^3}{c^2}$$

$\Big\downarrow$ 5403

$$-\frac{a^2\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx}{c^2}+\frac{i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3}{c^2}$$

$\Big\downarrow$ 5465

$$-\frac{a^2\left(\frac{\int \frac{\arctan(ax)}{\left(a^2x^2+1\right)^2}dx}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)}{c^2}+\frac{i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3}{c^2}$$

$\Big\downarrow$ 5427

$$-\frac{a^2\left(\frac{-\frac{1}{2}a\int \frac{x}{\left(a^2x^2+1\right)^2}dx+\frac{x\arctan(ax)}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^2}{4a}}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)}{c^2}+\frac{i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3}{c^2}$$

$\Big\downarrow$ 241

$$-\frac{a^2\left(\frac{\frac{x\arctan(ax)}{2\left(a^2x^2+1\right)}+\frac{1}{4a\left(a^2x^2+1\right)}+\frac{\arctan(ax)^2}{4a}}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)}{c^2}+\frac{i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3}{c^2}$$

$\Big\downarrow$ 5527

$$-\frac{a^2\left(\frac{\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)}{c^2}+\frac{i\left(2ia\left(\frac{i\arctan(ax)\,\mathrm{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-\frac{1}{2}i\int\frac{\mathrm{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{a^2x^2+1}dx\right)-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(}{c^2}$$

$$\Big\downarrow \text{7164}$$

$$-\frac{a^2\left(\frac{\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)}{c^2}+\frac{i\left(2ia\left(\frac{i\arctan(ax)\,\mathrm{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-\frac{\mathrm{PolyLog}\left(3,\frac{2}{1-iax}-1\right)}{4a}\right)-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3}{c^2}$$

input
```
Int[ArcTan[a*x]^2/(x*(c + a^2*c*x^2)^2),x]
```

output
```
-((a^2*(-1/2*ArcTan[a*x]^2/(a^2*(1 + a^2*x^2)) + (1/(4*a*(1 + a^2*x^2)) +
(x*ArcTan[a*x])/(2*(1 + a^2*x^2)) + ArcTan[a*x]^2/(4*a))/a))/c^2) + ((-1/3
*I)*ArcTan[a*x]^3 + I*((-I)*ArcTan[a*x]^2*Log[2 - 2/(1 - I*a*x)] + (2*I)*a
*(((I/2)*ArcTan[a*x]*PolyLog[2, -1 + 2/(1 - I*a*x)])/a - PolyLog[3, -1 + 2
/(1 - I*a*x)]/(4*a))))/c^2
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

rule 5527
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 63.29 (sec) , antiderivative size = 1677, normalized size of antiderivative = 9.86

| method | result | size |
|---|---|---|
| derivativedivides | Expression too large to display | 1677 |
| default | Expression too large to display | 1677 |
| parts | Expression too large to display | 2098 |

input
```
int(arctan(a*x)^2/x/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c^2*arctan(a*x)^2*ln(a*x)-1/2/c^2*arctan(a*x)^2*ln(a^2*x^2+1)+1/2*arctan
(a*x)^2/c^2/(a^2*x^2+1)-1/c^2*(-arctan(a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/
2))+1/3*I*arctan(a*x)^3-I*arctan(a*x)*(a*x+I)/(8*a*x-8*I)-1/16*(a*x+I)/(a*
x-I)+I*arctan(a*x)*(a*x-I)/(8*a*x+8*I)-1/16*(a*x-I)/(a*x+I)+arctan(a*x)^2*
ln((1+I*a*x)^2/(a^2*x^2+1)-1)-arctan(a*x)^2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/
2))+2*I*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*polylog(3,-(
1+I*a*x)/(a^2*x^2+1)^(1/2))-arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2)
)+2*I*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*polylog(3,(1+I*
a*x)/(a^2*x^2+1)^(1/2))+1/4*(2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))*cs
gn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2+I*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1
/2))^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))-2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2
+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^3+2*I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+
1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2-I
*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+
1)^2)-I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3-2*I*Pi*csgn(I*(1+I*a*x)
/(a^2*x^2+1)^(1/2))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^2-2*I*Pi*csgn(((1+I*a*
x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^3-I*Pi*csgn(I*(1+I*a*x)^2
/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2
)^2+I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2
+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)-2*I*...
```

## Fricas [F]

$$\int \frac{\arctan(ax)^2}{x\left(c+a^2cx^2\right)^2}\,dx = \int \frac{\arctan\left(ax\right)^2}{\left(a^2cx^2+c\right)^2x}\,dx$$

input
```
integrate(arctan(a*x)^2/x/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
integral(arctan(a*x)^2/(a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^2*x), x)
```

## Sympy [F]

$$\int \frac{\arctan(ax)^2}{x\left(c+a^2cx^2\right)^2}\,dx = \frac{\int \frac{\operatorname{atan}^2\left(ax\right)}{a^4x^5+2a^2x^3+x}\,dx}{c^2}$$

input
```
integrate(atan(a*x)**2/x/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(atan(a*x)**2/(a**4*x**5 + 2*a**2*x**3 + x), x)/c**2
```

## Maxima [F]

$$\int \frac{\arctan(ax)^2}{x\left(c+a^2cx^2\right)^2}\,dx = \int \frac{\arctan\left(ax\right)^2}{\left(a^2cx^2+c\right)^2x}\,dx$$

input
```
integrate(arctan(a*x)^2/x/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)^2/((a^2*c*x^2 + c)^2*x), x)
```

## Giac [F]

$$\int \frac{\arctan(ax)^2}{x(c+a^2cx^2)^2}\,dx = \int \frac{\arctan(ax)^2}{(a^2cx^2+c)^2x}\,dx$$

input
```
integrate(arctan(a*x)^2/x/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^2/((a^2*c*x^2 + c)^2*x), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^2}{x(c+a^2cx^2)^2}\,dx = \int \frac{\mathrm{atan}(a\,x)^2}{x\,(c\,a^2\,x^2+c)^2}\,dx$$

input
```
int(atan(a*x)^2/(x*(c + a^2*c*x^2)^2),x)
```

output
```
int(atan(a*x)^2/(x*(c + a^2*c*x^2)^2), x)
```

## Reduce [F]

$$\int \frac{\arctan(ax)^2}{x(c+a^2cx^2)^2}\,dx = \frac{\int \frac{atan(ax)^2}{a^4x^5+2a^2x^3+x}dx}{c^2}$$

input
```
int(atan(a*x)^2/x/(a^2*c*x^2+c)^2,x)
```

output
```
int(atan(a*x)**2/(a**4*x**5 + 2*a**2*x**3 + x),x)/c**2
```

# 3.296 $\int \frac{\arctan(ax)^2}{x^2(c+a^2cx^2)^2}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 177

$$\int \frac{\arctan(ax)^2}{x^2(c+a^2cx^2)^2}\,dx = \frac{a^2x}{4c^2(1+a^2x^2)} + \frac{a\arctan(ax)}{4c^2} - \frac{a\arctan(ax)}{2c^2(1+a^2x^2)} - \frac{ia\arctan(ax)^2}{c^2} - \frac{\arctan(ax)^2}{c^2x} - \frac{a^2x\arctan(ax)^2}{2c^2(1+a^2x^2)} - \frac{a\arctan(ax)^3}{2c^2} + \frac{2a\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{c^2} - \frac{ia\,\mathrm{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{c^2}$$

output
```
1/4*a^2*x/c^2/(a^2*x^2+1)+1/4*a*arctan(a*x)/c^2-1/2*a*arctan(a*x)/c^2/(a^2
*x^2+1)-I*a*arctan(a*x)^2/c^2-arctan(a*x)^2/c^2/x-1/2*a^2*x*arctan(a*x)^2/
c^2/(a^2*x^2+1)-1/2*a*arctan(a*x)^3/c^2+2*a*arctan(a*x)*ln(2-2/(1-I*a*x))/
c^2-I*a*polylog(2,-1+2/(1-I*a*x))/c^2
```

## Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.62

$$\int \frac{\arctan(ax)^2}{x^2 (c+a^2cx^2)^2} dx = -\frac{4ax\arctan(ax)^3 + 2ax\arctan(ax)\left(\cos(2\arctan(ax)) - 8\log\left(1-e^{2i\arctan(ax)}\right)\right) + 8iax\,\mathrm{PolyLog}\left(2, e^{2i\arctan(ax)}\right) - ax\sin(2\arctan(ax)) + 2\arctan(ax)^2(4 + 4iax + ax\sin(2\arctan(ax)))}{8c^2x}$$

input
```
Integrate[ArcTan[a*x]^2/(x^2*(c + a^2*c*x^2)^2),x]
```

output
```
-1/8*(4*a*x*ArcTan[a*x]^3 + 2*a*x*ArcTan[a*x]*(Cos[2*ArcTan[a*x]] - 8*Log[
1 - E^((2*I)*ArcTan[a*x])]) + (8*I)*a*x*PolyLog[2, E^((2*I)*ArcTan[a*x])]
- a*x*Sin[2*ArcTan[a*x]] + 2*ArcTan[a*x]^2*(4 + (4*I)*a*x + a*x*Sin[2*ArcT
an[a*x]]))/(c^2*x)
```

## Rubi [A] (verified)

Time = 1.35 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.11, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {5501, 27, 5427, 5453, 5361, 5419, 5459, 5403, 2897, 5465, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2}{x^2 (a^2cx^2+c)^2} dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{\arctan(ax)^2}{cx^2(a^2x^2+1)}dx}{c} - a^2\int \frac{\arctan(ax)^2}{c^2(a^2x^2+1)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)}dx}{c^2} - \frac{a^2\int \frac{\arctan(ax)^2}{(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \text{5427}$$

$$\frac{\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)}dx}{c^2}-\frac{a^2\left(-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)}{c^2}$$

$$\Big\downarrow \text{ 5453}$$

$$\frac{\int \frac{\arctan(ax)^2}{x^2}dx-a^2\int \frac{\arctan(ax)^2}{a^2x^2+1}dx}{c^2}-\frac{a^2\left(-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)}{c^2}$$

$$\Big\downarrow \text{ 5361}$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)^2}{a^2x^2+1}dx\right)+2a\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx-\frac{\arctan(ax)^2}{x}}{c^2}-\frac{a^2\left(-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)}{c^2}$$

$$\Big\downarrow \text{ 5419}$$

$$\frac{2a\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx-\frac{1}{3}a\arctan(ax)^3-\frac{\arctan(ax)^2}{x}}{c^2}-\frac{a^2\left(-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)}{c^2}$$

$$\Big\downarrow \text{ 5459}$$

$$-\frac{a^2\left(-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)}{c^2}+\frac{2a\left(i\int \frac{\arctan(ax)}{x(ax+i)}dx-\frac{1}{2}i\arctan(ax)^2\right)-\frac{1}{3}a\arctan(ax)^3-\frac{\arctan(ax)^2}{x}}{c^2}$$

$$\Big\downarrow \text{ 5403}$$

$$-\frac{a^2\left(-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)}{c^2}+\frac{2a\left(i\left(ia\int \frac{\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)-\frac{1}{3}a\arctan(ax)^3-\frac{\arctan(ax)^2}{x}}{c^2}$$

$$\Big\downarrow \text{ 2897}$$

$$-\frac{a^2\left(-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)}{c^2}+\frac{2a\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)-\frac{1}{3}a\arctan(ax)^3-\frac{\arctan(}{x}}{c^2}$$

$$\Big\downarrow \text{ 5465}$$

$$-\frac{a^2\left(-a\left(\frac{\int \frac{1}{\left(a^2x^2+1\right)^2}dx}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)}{c^2}+\frac{2a\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)-\frac{1}{3}a\arctan(ax)^3-\frac{\arctan(}{x}}{c^2}$$

$\downarrow$ 215

$$-\frac{a^2\left(-a\left(\frac{\frac{1}{2}\int \frac{1}{a^2x^2+1}dx+\frac{x}{2\left(a^2x^2+1\right)}}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)}{c^2}+\frac{2a\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)-\frac{1}{3}a\arctan(ax)^3-\frac{\arctan(}{x}}{c^2}$$

$\downarrow$ 216

$$-\frac{a^2\left(\frac{x\arctan(ax)^2}{2(a^2x^2+1)}-a\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)+\frac{\arctan(ax)^3}{6a}\right)}{c^2}+\frac{2a\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)-\frac{1}{3}a\arctan(ax)^3-\frac{\arctan(}{x}}{c^2}$$

input
```
Int[ArcTan[a*x]^2/(x^2*(c + a^2*c*x^2)^2),x]
```

output
```
-((a^2*((x*ArcTan[a*x]^2)/(2*(1 + a^2*x^2)) + ArcTan[a*x]^3/(6*a) - a*(-1/
2*ArcTan[a*x]/(a^2*(1 + a^2*x^2)) + (x/(2*(1 + a^2*x^2)) + ArcTan[a*x]/(2*
a))/(2*a))))/c^2) + (-(ArcTan[a*x]^2/x) - (a*ArcTan[a*x]^3)/3 + 2*a*((-1/2
*I)*ArcTan[a*x]^2 + I*((-I)*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)] - PolyLog[2
, -1 + 2/(1 - I*a*x)]/2)))/c^2
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

## Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.77

| method | result |
|---|---|
| derivativedivides | $a\left(-\frac{ax\arctan(ax)^2}{2c^2(a^2x^2+1)}-\frac{3\arctan(ax)^3}{2c^2}-\frac{\arctan(ax)^2}{c^2ax}-\frac{-\arctan(ax)^3+\arctan(ax)\ln(a^2x^2+1)+\frac{\arctan(ax)}{2a^2x^2+2}-2\right.$ |
| default | $a\left(-\frac{ax\arctan(ax)^2}{2c^2(a^2x^2+1)}-\frac{3\arctan(ax)^3}{2c^2}-\frac{\arctan(ax)^2}{c^2ax}-\frac{-\arctan(ax)^3+\arctan(ax)\ln(a^2x^2+1)+\frac{\arctan(ax)}{2a^2x^2+2}-2\right.$ |
| parts | $-\frac{a^2x\arctan(ax)^2}{2c^2(a^2x^2+1)}-\frac{3a\arctan(ax)^3}{2c^2}-\frac{\arctan(ax)^2}{c^2x}-\frac{2\left(-\frac{a\arctan(ax)^3}{2}-\frac{a\left(-\arctan(ax)\ln\left(a^2x^2+1\right)-\frac{\arctan(ax}{2\left(a^2x^2+1\right.}\right.}{}\right.}{}$ |

input
```
int(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
a*(-1/2*a*x*arctan(a*x)^2/c^2/(a^2*x^2+1)-3/2*arctan(a*x)^3/c^2-1/c^2*arct
an(a*x)^2/a/x-1/c^2*(-arctan(a*x)^3+arctan(a*x)*ln(a^2*x^2+1)+1/2*arctan(a
*x)/(a^2*x^2+1)-2*arctan(a*x)*ln(a*x)-1/4*a*x/(a^2*x^2+1)-1/4*arctan(a*x)-
I*ln(a*x)*ln(1+I*a*x)+I*ln(a*x)*ln(1-I*a*x)-I*dilog(1+I*a*x)+I*dilog(1-I*a
*x)+1/2*I*(ln(a*x-I)*ln(a^2*x^2+1)-1/2*ln(a*x-I)^2-dilog(-1/2*I*(a*x+I))-l
n(a*x-I)*ln(-1/2*I*(a*x+I)))-1/2*I*(ln(a*x+I)*ln(a^2*x^2+1)-1/2*ln(a*x+I)^
2-dilog(1/2*I*(a*x-I))-ln(a*x+I)*ln(1/2*I*(a*x-I)))))
```

## Fricas [F]

$$\int \frac{\arctan(ax)^2}{x^2\left(c+a^2cx^2\right)^2}\,dx=\int \frac{\arctan\left(ax\right)^2}{\left(a^2cx^2+c\right)^2x^2}\,dx$$

input
```
integrate(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
integral(arctan(a*x)^2/(a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2), x)
```

## Sympy [F]

$$\int \frac{\arctan(ax)^2}{x^2 (c + a^2cx^2)^2} \, dx = \frac{\int \frac{\operatorname{atan}^2(ax)}{a^4x^6+2a^2x^4+x^2} \, dx}{c^2}$$

input
```
integrate(atan(a*x)**2/x**2/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(atan(a*x)**2/(a**4*x**6 + 2*a**2*x**4 + x**2), x)/c**2
```

## Maxima [F]

$$\int \frac{\arctan(ax)^2}{x^2 (c + a^2cx^2)^2} \, dx = \int \frac{\arctan(ax)^2}{(a^2cx^2 + c)^2 x^2} \, dx$$

input
```
integrate(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
-1/32*(6*a^3*x^3*arctan2(1, a*x) - 6*a^2*x^2 + 8*(a^3*x^3 + a*x)*arctan(a*
x)^3 + 12*a*x*arctan(a*x) + 4*(3*a^2*x^2 + 2)*arctan(a*x)^2 + 6*a*x*arctan
2(1, a*x) - (3*a^2*x^2 + 2)*log(a^2*x^2 + 1)^2 + 192*(a^6*c^2*x^3 + a^4*c^
2*x)*integrate(1/16*x^2*log(a^2*x^2 + 1)/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^
2), x) - 128*(a^2*c^2*x^3 + c^2*x)*integrate(1/64*(4*(a^2*x^2 + 1)^(7/2)*a
^2*arctan(a*x)*log(a^2*x^2 + 1)*sin(6*arctan(a*x)) - 24*(a^2*x^2 + 1)^3*a^
2*arctan(a*x)*log(a^2*x^2 + 1)*sin(5*arctan(a*x)) + 52*(a^2*x^2 + 1)^(5/2)
*a^2*arctan(a*x)*log(a^2*x^2 + 1)*sin(4*arctan(a*x)) - 48*(a^2*x^2 + 1)^2*
a^2*arctan(a*x)*log(a^2*x^2 + 1)*sin(3*arctan(a*x)) + 16*(a^2*x^2 + 1)^(3/
2)*a^2*arctan(a*x)*log(a^2*x^2 + 1)*sin(2*arctan(a*x)) - (4*(a^2*x^2 + 1)^
(7/2)*a^2*arctan(a*x)^2 - (a^2*x^2 + 1)^(7/2)*a^2*log(a^2*x^2 + 1)^2)*cos(
6*arctan(a*x)) + 6*(4*(a^2*x^2 + 1)^3*a^2*arctan(a*x)^2 - (a^2*x^2 + 1)^3*
a^2*log(a^2*x^2 + 1)^2)*cos(5*arctan(a*x)) - 13*(4*(a^2*x^2 + 1)^(5/2)*a^2
*arctan(a*x)^2 - (a^2*x^2 + 1)^(5/2)*a^2*log(a^2*x^2 + 1)^2)*cos(4*arctan(
a*x)) + 12*(4*(a^2*x^2 + 1)^2*a^2*arctan(a*x)^2 - (a^2*x^2 + 1)^2*a^2*log(
a^2*x^2 + 1)^2)*cos(3*arctan(a*x)) - 4*(4*(a^2*x^2 + 1)^(3/2)*a^2*arctan(a
*x)^2 - (a^2*x^2 + 1)^(3/2)*a^2*log(a^2*x^2 + 1)^2)*cos(2*arctan(a*x)))*sq
rt(a^2*x^2 + 1)/((a^2*c^2*x^2 + c^2)*(a^2*x^2 + 1)^6*cos(6*arctan(a*x))^2
+ (a^2*c^2*x^2 + c^2)*(a^2*x^2 + 1)^6*sin(6*arctan(a*x))^2 + 36*(a^2*c^2*x
^2 + c^2)*(a^2*x^2 + 1)^5*cos(5*arctan(a*x))^2 + 36*(a^2*c^2*x^2 + c^2)...
```

## Giac [F]

$$\int \frac{\arctan(ax)^2}{x^2 (c + a^2cx^2)^2}\, dx = \int \frac{\arctan\left(ax\right)^2}{\left(a^2cx^2 + c\right)^2 x^2}\, dx$$

input
```
integrate(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^2/((a^2*c*x^2 + c)^2*x^2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^2}{x^2 (c+a^2cx^2)^2} \, dx = \int \frac{\mathrm{atan}(a\,x)^2}{x^2\,(c\,a^2\,x^2+c)^2} \, dx$$

input
```
int(atan(a*x)^2/(x^2*(c + a^2*c*x^2)^2),x)
```

output
```
int(atan(a*x)^2/(x^2*(c + a^2*c*x^2)^2), x)
```

## Reduce [F]

$$\int \frac{\arctan(ax)^2}{x^2 (c+a^2cx^2)^2} \, dx = \frac{-2atan(ax)^3 a^3x^3 - 2atan(ax)^3 ax - 6atan(ax)^2 a^2x^2 - 4atan(ax)^2 + 3atan(ax) a^3x^3 - 3atan(ax) ax}{4c^2x (a^2x^2+1)}$$

input
```
int(atan(a*x)^2/x^2/(a^2*c*x^2+c)^2,x)
```

output
```
( - 2*atan(a*x)**3*a**3*x**3 - 2*atan(a*x)**3*a*x - 6*atan(a*x)**2*a**2*x**
*2 - 4*atan(a*x)**2 + 3*atan(a*x)*a**3*x**3 - 3*atan(a*x)*a*x + 8*int(atan
(a*x)/(a**4*x**5 + 2*a**2*x**3 + x),x)*a**3*x**3 + 8*int(atan(a*x)/(a**4*x
**5 + 2*a**2*x**3 + x),x)*a*x + 3*a**2*x**2)/(4*c**2*x*(a**2*x**2 + 1))
```

# 3.297 $\int \frac{\arctan(ax)^2}{x^3(c+a^2cx^2)^2}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 250

$$\begin{aligned}\int \frac{\arctan(ax)^2}{x^3(c+a^2cx^2)^2}\,dx =& \frac{a^2}{4c^2(1+a^2x^2)} - \frac{a\arctan(ax)}{c^2x} + \frac{a^3x\arctan(ax)}{2c^2(1+a^2x^2)} - \frac{a^2\arctan(ax)^2}{4c^2} \\ &- \frac{\arctan(ax)^2}{2c^2x^2} - \frac{a^2\arctan(ax)^2}{2c^2(1+a^2x^2)} + \frac{2ia^2\arctan(ax)^3}{3c^2} + \frac{a^2\log(x)}{c^2} \\ &- \frac{a^2\log(1+a^2x^2)}{2c^2} - \frac{2a^2\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{c^2} \\ &+ \frac{2ia^2\arctan(ax)\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{c^2} \\ &- \frac{a^2\operatorname{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)}{c^2}\end{aligned}$$

output
```
1/4*a^2/c^2/(a^2*x^2+1)-a*arctan(a*x)/c^2/x+1/2*a^3*x*arctan(a*x)/c^2/(a^2
*x^2+1)-1/4*a^2*arctan(a*x)^2/c^2-1/2*arctan(a*x)^2/c^2/x^2-1/2*a^2*arctan
(a*x)^2/c^2/(a^2*x^2+1)+2/3*I*a^2*arctan(a*x)^3/c^2+a^2*ln(x)/c^2-1/2*a^2*
ln(a^2*x^2+1)/c^2-2*a^2*arctan(a*x)^2*ln(2-2/(1-I*a*x))/c^2+2*I*a^2*arctan
(a*x)*polylog(2,-1+2/(1-I*a*x))/c^2-a^2*polylog(3,-1+2/(1-I*a*x))/c^2
```

## Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.73

$$\int \frac{\arctan(ax)^2}{x^3\left(c+a^2cx^2\right)^2}\,dx$$
$$= \frac{a^2\left(\frac{i\pi^3}{12}-\frac{\arctan(ax)}{ax}-\frac{(1+a^2x^2)\arctan(ax)^2}{2a^2x^2}-\frac{2}{3}i\arctan(ax)^3+\frac{1}{8}\cos(2\arctan(ax))-\frac{1}{4}\arctan(ax)^2\cos(2\arctan$$

input
```
Integrate[ArcTan[a*x]^2/(x^3*(c + a^2*c*x^2)^2),x]
```

output
```
(a^2*((I/12)*Pi^3 - ArcTan[a*x]/(a*x) - ((1 + a^2*x^2)*ArcTan[a*x]^2)/(2*a
^2*x^2) - ((2*I)/3)*ArcTan[a*x]^3 + Cos[2*ArcTan[a*x]]/8 - (ArcTan[a*x]^2*
Cos[2*ArcTan[a*x]])/4 - 2*ArcTan[a*x]^2*Log[1 - E^((-2*I)*ArcTan[a*x])] +
Log[(a*x)/Sqrt[1 + a^2*x^2]] - (2*I)*ArcTan[a*x]*PolyLog[2, E^((-2*I)*ArcT
an[a*x])] - PolyLog[3, E^((-2*I)*ArcTan[a*x])] + (ArcTan[a*x]*Sin[2*ArcTan
[a*x]])/4))/c^2
```

## Rubi [A] (verified)

Time = 3.24 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.46, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.955$, Rules used = {5501, 27, 5453, 5361, 5453, 5361, 243, 47, 14, 16, 5419, 5459, 5403, 5501, 5459, 5403, 5465, 5427, 241, 5527, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2}{x^3\left(a^2cx^2+c\right)^2}dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{\arctan(ax)^2}{cx^3(a^2x^2+1)}dx}{c}-a^2\int \frac{\arctan(ax)^2}{c^2x\left(a^2x^2+1\right)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\arctan(ax)^2}{x^3(a^2x^2+1)}dx}{c^2}-\frac{a^2\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \text{5453}$$

$$\frac{\int \frac{\arctan(ax)^2}{x^3}dx-a^2\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx}{c^2}-\frac{a^2\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \text{5361}$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right)+a\int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx-\frac{\arctan(ax)^2}{2x^2}}{c^2}-\frac{a^2\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \text{5453}$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right)+a\left(\int \frac{\arctan(ax)}{x^2}dx-a^2\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)-\frac{\arctan(ax)^2}{2x^2}}{c^2}-\frac{a^2\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \text{5361}$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right)+a\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+a\int \frac{1}{x(a^2x^2+1)}dx-\frac{\arctan(ax)}{x}\right)-\frac{\arctan(ax)^2}{2x^2}}{c^2}-\frac{a^2\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \text{243}$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right)+a\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{2}a\int \frac{1}{x^2(a^2x^2+1)}dx^2-\frac{\arctan(ax)}{x}\right)-\frac{\arctan(ax)^2}{2x^2}}{c^2}-\frac{a^2\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \text{47}$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right)+a\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{2}a\left(\int \frac{1}{x^2}dx^2-a^2\int \frac{1}{a^2x^2+1}dx^2\right)-\frac{\arctan(ax)}{x}\right)-\frac{\arctan(ax)^2}{2x^2}}{c^2}-\frac{a^2\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \text{14}$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right)+a\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{2}a\left(\log\left(x^2\right)-a^2\int \frac{1}{a^2x^2+1}dx^2\right)-\frac{\arctan(ax)}{x}\right)-\frac{\arctan(ax)^2}{2x^2}}{c^2}-\frac{a^2\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \ 16$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right)+a\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{\arctan(ax)}{x}\right)-\frac{\arctan(ax)^2}{2x^2}}{c^2}-\frac{a^2\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \ 5419$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right)+a\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{1}{2}a\arctan(ax)^2-\frac{\arctan(ax)}{x}\right)-\frac{\arctan(ax)^2}{2x^2}}{c^2}-\frac{a^2\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \ 5459$$

$$-\frac{a^2\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx}{c^2}+\frac{-\left(a^2\left(i\int \frac{\arctan(ax)^2}{x(ax+i)}dx-\frac{1}{3}i\arctan(ax)^3\right)\right)+a\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{1}{2}a\arctan(ax)^2-\frac{\arctan(ax)}{x}\right)-}{c^2}$$

$$\Big\downarrow \ 5403$$

$$-\frac{a^2\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx}{c^2}+\frac{-\left(a^2\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3\right)\right)+a\left(\frac{1}{2}a\left(\log\left(x^2\right)\right.\right.}{c^2}$$

$$\Big\downarrow \ 5501$$

$$-\frac{a^2\left(\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx-a^2\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx\right)}{c^2}+\frac{-\left(a^2\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3\right)\right)+a\left(\frac{1}{2}a\left(\log\left(x^2\right)\right.\right.}{c^2}$$

$$\Big\downarrow \; 5459$$

$$\frac{-\left(a^2\left(i\left(2ia\int\frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3\right)\right)+a\left(\frac{1}{2}a\left(\log\left(x^2\right)\right.\right.}{c^2}$$

$$\frac{a^2\left(-a^2\int\frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+i\int\frac{\arctan(ax)^2}{x(ax+i)}dx-\frac{1}{3}i\arctan(ax)^3\right)}{c^2}$$

$$\Big\downarrow \; 5403$$

$$\frac{-\left(a^2\left(i\left(2ia\int\frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3\right)\right)+a\left(\frac{1}{2}a\left(\log\left(x^2\right)\right.\right.}{c^2}$$

$$\frac{a^2\left(-a^2\int\frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+i\left(2ia\int\frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3\right)}{c^2}$$

$$\Big\downarrow \; 5465$$

$$\frac{-\left(a^2\left(i\left(2ia\int\frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3\right)\right)+a\left(\frac{1}{2}a\left(\log\left(x^2\right)\right.\right.}{c^2}$$

$$\frac{a^2\left(-a^2\left(\frac{\int\frac{\arctan(ax)}{\left(a^2x^2+1\right)^2}dx}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)+i\left(2ia\int\frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}\right.}{c^2}$$

$$\Big\downarrow \; 5427$$

$$\frac{-\left(a^2\left(i\left(2ia\int\frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3\right)\right)+a\left(\frac{1}{2}a\left(\log\left(x^2\right)\right.\right.}{c^2}$$

$$\frac{a^2\left(-a^2\left(\frac{-\frac{1}{2}a\int\frac{x}{\left(a^2x^2+1\right)^2}dx+\frac{x\arctan(ax)}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^2}{4a}}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)+i\left(2ia\int\frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax\right.\right.}{c^2}$$

$$\Big\downarrow \; 241$$

$$-\frac{\left(a^2\left(i\left(2ia\int\frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3\right)\right)+a\left(\frac{1}{2}a(\log(x^2)\right.}{c^2}$$

$$\frac{a^2\left(i\left(2ia\int\frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-a^2\left(\frac{\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}}{a}-\right.\right.}{c^2}$$

$\Big\downarrow$ 5527

$$-\frac{\left(a^2\left(i\left(2ia\left(\frac{i\arctan(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-\frac{1}{2}i\int\frac{\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{a^2x^2+1}dx\right)-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\right.\right.}{c^2}$$

$$\frac{a^2\left(i\left(2ia\left(\frac{i\arctan(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-\frac{1}{2}i\int\frac{\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{a^2x^2+1}dx\right)-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-a^2\left(\right.\right.}{c^2}$$

$\Big\downarrow$ 7164

$$-\frac{\left(a^2\left(i\left(2ia\left(\frac{i\arctan(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-\frac{\operatorname{PolyLog}\left(3,\frac{2}{1-iax}-1\right)}{4a}\right)-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan\right.\right.}{c^2}$$

$$\frac{a^2\left(-a^2\left(\frac{\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)+i\left(2ia\left(\frac{i\arctan(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-\frac{\operatorname{PolyLog}\left(3,\frac{2}{1-iax}\right.}{4a}\right.\right.\right.}{c^2}$$

input

```
Int[ArcTan[a*x]^2/(x^3*(c + a^2*c*x^2)^2),x]
```

output

```
-((a^2*((-1/3*I)*ArcTan[a*x]^3 - a^2*(-1/2*ArcTan[a*x]^2/(a^2*(1 + a^2*x^2
)) + (1/(4*a*(1 + a^2*x^2)) + (x*ArcTan[a*x])/(2*(1 + a^2*x^2)) + ArcTan[a
*x]^2/(4*a))/a) + I*((-I)*ArcTan[a*x]^2*Log[2 - 2/(1 - I*a*x)] + (2*I)*a*(
((I/2)*ArcTan[a*x]*PolyLog[2, -1 + 2/(1 - I*a*x)])/a - PolyLog[3, -1 + 2/(
1 - I*a*x)]/(4*a)))))/c^2) + (-1/2*ArcTan[a*x]^2/x^2 + a*(-(ArcTan[a*x]/x)
 - (a*ArcTan[a*x]^2)/2 + (a*(Log[x^2] - Log[1 + a^2*x^2]))/2) - a^2*((-1/3
*I)*ArcTan[a*x]^3 + I*((-I)*ArcTan[a*x]^2*Log[2 - 2/(1 - I*a*x)] + (2*I)*a
*(((I/2)*ArcTan[a*x]*PolyLog[2, -1 + 2/(1 - I*a*x)])/a - PolyLog[3, -1 + 2
/(1 - I*a*x)]/(4*a)))))/c^2
```

## Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

rule 5527
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 71.10 (sec) , antiderivative size = 1927, normalized size of antiderivative = 7.71

| method | result | size |
|---|---|---|
| derivativedivides | Expression too large to display | 1927 |
| default | Expression too large to display | 1927 |
| parts | Expression too large to display | 2375 |

input
```
int(arctan(a*x)^2/x^3/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*(-1/2/c^2*arctan(a*x)^2/a^2/x^2-2/c^2*arctan(a*x)^2*ln(a*x)+1/c^2*arct
an(a*x)^2*ln(a^2*x^2+1)-1/2*arctan(a*x)^2/c^2/(a^2*x^2+1)-1/c^2*(-1/2*I*Pi
*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csg
n(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*arctan(a*x)^2+I
*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)
)*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*arctan(a
*x)^2+1/4*arctan(a*x)^2-ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-1)-ln(1+(1+I*a*x)/(
a^2*x^2+1)^(1/2))+1/2*arctan(a*x)*(I*a*x+(a^2*x^2+1)^(1/2)+1)/a/x+1/2*arct
an(a*x)*(I*a*x-(a^2*x^2+1)^(1/2)+1)/a/x+1/2*I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x
^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2*arc
tan(a*x)^2+I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1
)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*arctan
(a*x)^2-I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*
x^2+1)+1)^2)^2*arctan(a*x)^2-I*arctan(a*x)*(a*x-I)/(8*a*x+8*I)-1/2*I*Pi*cs
gn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3*arctan(a*x)^
2-I*Pi*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arc
tan(a*x)^2-1/2*I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^3*arctan(a*x)^2+1/2*I*
Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3*arctan(a*x)^2+I*Pi*csgn(I*((1+I
*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^3*arctan(a*x)^2+I*Pi*c
sgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^3*arctan(a...
```

## Fricas [F]

$$\int \frac{\arctan(ax)^2}{x^3 (c + a^2cx^2)^2} \, dx = \int \frac{\arctan{(ax)}^2}{(a^2cx^2 + c)^2 x^3} \, dx$$

input
```
integrate(arctan(a*x)^2/x^3/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
integral(arctan(a*x)^2/(a^4*c^2*x^7 + 2*a^2*c^2*x^5 + c^2*x^3), x)
```

## Sympy [F]

$$\int \frac{\arctan(ax)^2}{x^3 (c + a^2cx^2)^2} \, dx = \frac{\int \frac{\operatorname{atan}^2(ax)}{a^4x^7 + 2a^2x^5 + x^3} \, dx}{c^2}$$

input
```
integrate(atan(a*x)**2/x**3/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(atan(a*x)**2/(a**4*x**7 + 2*a**2*x**5 + x**3), x)/c**2
```

## Maxima [F]

$$\int \frac{\arctan(ax)^2}{x^3 (c + a^2cx^2)^2} \, dx = \int \frac{\arctan(ax)^2}{(a^2cx^2 + c)^2 x^3} \, dx$$

input
```
integrate(arctan(a*x)^2/x^3/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)^2/((a^2*c*x^2 + c)^2*x^3), x)
```

## Giac [F]

$$\int \frac{\arctan(ax)^2}{x^3 (c + a^2cx^2)^2} \, dx = \int \frac{\arctan(ax)^2}{(a^2cx^2 + c)^2 x^3} \, dx$$

input
```
integrate(arctan(a*x)^2/x^3/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^2/((a^2*c*x^2 + c)^2*x^3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^2}{x^3 \left(c+a^2cx^2\right)^2}\,dx = \int \frac{\mathrm{atan}(a\,x)^2}{x^3\,\left(c\,a^2\,x^2+c\right)^2}\,dx$$

input
```
int(atan(a*x)^2/(x^3*(c + a^2*c*x^2)^2),x)
```

output
```
int(atan(a*x)^2/(x^3*(c + a^2*c*x^2)^2), x)
```

## Reduce [F]

$$\int \frac{\arctan(ax)^2}{x^3 \left(c+a^2cx^2\right)^2}\,dx = \frac{\int \frac{atan(ax)^2}{a^4x^7+2a^2x^5+x^3}dx}{c^2}$$

input
```
int(atan(a*x)^2/x^3/(a^2*c*x^2+c)^2,x)
```

output
```
int(atan(a*x)**2/(a**4*x**7 + 2*a**2*x**5 + x**3),x)/c**2
```

# 3.298 $\int \frac{\arctan(ax)^2}{x^4(c+a^2cx^2)^2}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 242

$$\int \frac{\arctan(ax)^2}{x^4(c+a^2cx^2)^2}\,dx = -\frac{a^2}{3c^2x}-\frac{a^4x}{4c^2(1+a^2x^2)}-\frac{7a^3\arctan(ax)}{12c^2}-\frac{a\arctan(ax)}{3c^2x^2}+\frac{a^3\arctan(ax)}{2c^2(1+a^2x^2)}+\frac{7ia^3\arctan(ax)^2}{3c^2}-\frac{\arctan(ax)^2}{3c^2x^3}+\frac{2a^2\arctan(ax)^2}{c^2x}+\frac{a^4x\arctan(ax)^2}{2c^2(1+a^2x^2)}+\frac{5a^3\arctan(ax)^3}{6c^2}-\frac{14a^3\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{3c^2}+\frac{7ia^3\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{3c^2}$$

output
```
-1/3*a^2/c^2/x-1/4*a^4*x/c^2/(a^2*x^2+1)-7/12*a^3*arctan(a*x)/c^2-1/3*a*ar
ctan(a*x)/c^2/x^2+1/2*a^3*arctan(a*x)/c^2/(a^2*x^2+1)+7/3*I*a^3*arctan(a*x
)^2/c^2-1/3*arctan(a*x)^2/c^2/x^3+2*a^2*arctan(a*x)^2/c^2/x+1/2*a^4*x*arct
an(a*x)^2/c^2/(a^2*x^2+1)+5/6*a^3*arctan(a*x)^3/c^2-14/3*a^3*arctan(a*x)*l
n(2-2/(1-I*a*x))/c^2+7/3*I*a^3*polylog(2,-1+2/(1-I*a*x))/c^2
```

## Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.69

$$\int \frac{\arctan(ax)^2}{x^4(c+a^2cx^2)^2}\,dx$$
$$= \frac{20a^3x^3\arctan(ax)^3 + 2ax\arctan(ax)\left(-4-4a^2x^2+3a^2x^2\cos(2\arctan(ax)) - 56a^2x^2\log\left(1-e^{2i\arctan}\right.\right.}{}$$

input
```
Integrate[ArcTan[a*x]^2/(x^4*(c + a^2*c*x^2)^2),x]
```

output
```
(20*a^3*x^3*ArcTan[a*x]^3 + 2*a*x*ArcTan[a*x]*(-4 - 4*a^2*x^2 + 3*a^2*x^2*
Cos[2*ArcTan[a*x]] - 56*a^2*x^2*Log[1 - E^((2*I)*ArcTan[a*x])]) + (56*I)*a
^3*x^3*PolyLog[2, E^((2*I)*ArcTan[a*x])] - a^2*x^2*(8 + 3*a*x*Sin[2*ArcTan
[a*x]]) + ArcTan[a*x]^2*(-8 + 48*a^2*x^2 + (56*I)*a^3*x^3 + 6*a^3*x^3*Sin[
2*ArcTan[a*x]]))/(24*c^2*x^3)
```

## Rubi [A] (verified)

Time = 3.72 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.69, number of steps used = 23, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 1.045$, Rules used = {5501, 27, 5453, 5361, 5453, 5361, 264, 216, 5419, 5459, 5403, 2897, 5501, 5427, 5453, 5361, 5419, 5459, 5403, 2897, 5465, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2}{x^4(a^2cx^2+c)^2}\,dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{\arctan(ax)^2}{cx^4(a^2x^2+1)}dx}{c} - a^2\int \frac{\arctan(ax)^2}{c^2x^2(a^2x^2+1)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\arctan(ax)^2}{x^4(a^2x^2+1)}dx}{c^2} - \frac{a^2\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \ 5453$$

$$\frac{\int \frac{\arctan(ax)^2}{x^4}dx - a^2\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)}dx}{c^2} - \frac{a^2\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \ 5361$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)}dx\right) + \frac{2}{3}a\int \frac{\arctan(ax)}{x^3(a^2x^2+1)}dx - \frac{\arctan(ax)^2}{3x^3}}{c^2} - \frac{a^2\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \ 5453$$

$$\frac{-\left(a^2\left(\int \frac{\arctan(ax)^2}{x^2}dx - a^2\int \frac{\arctan(ax)^2}{a^2x^2+1}dx\right)\right) + \frac{2}{3}a\left(\int \frac{\arctan(ax)}{x^3}dx - a^2\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx\right) - \frac{\arctan(ax)^2}{3x^3}}{c^2} - \frac{a^2\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \ 5361$$

$$\frac{-\left(a^2\left(a^2\left(-\int \frac{\arctan(ax)^2}{a^2x^2+1}dx\right) + 2a\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx - \frac{\arctan(ax)^2}{x}\right)\right) + \frac{2}{3}a\left(a^2\left(-\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx\right) + \frac{1}{2}a\int \frac{1}{x^2(a^2x^2+1)}dx}{c^2}$$

$$\frac{a^2\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \ 264$$

$$\frac{-\left(a^2\left(a^2\left(-\int \frac{\arctan(ax)^2}{a^2x^2+1}dx\right) + 2a\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx - \frac{\arctan(ax)^2}{x}\right)\right) + \frac{2}{3}a\left(a^2\left(-\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx\right) + \frac{1}{2}a\left(a^2\left(-\int \frac{1}{a^2x^2+}\right.\right.\right.}{c^2}$$

$$\frac{a^2\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \ 216$$

$$\frac{-\left(a^2\left(a^2\left(-\int \frac{\arctan(ax)^2}{a^2x^2+1}dx\right) + 2a\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx - \frac{\arctan(ax)^2}{x}\right)\right) + \frac{2}{3}a\left(a^2\left(-\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx\right) - \frac{\arctan(ax)}{2x^2} + \frac{1}{2}a(-\right.}{c^2}$$

$$\frac{a^2\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \ 5419$$

$$\frac{-\left(a^2\left(2a\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx-\frac{1}{3}a\arctan(ax)^3-\frac{\arctan(ax)^2}{x}\right)\right)+\frac{2}{3}a\left(a^2\left(-\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx\right)-\frac{\arctan(ax)}{2x^2}+\frac{1}{2}a(-a\arcta}{c^2}$$

$$\frac{a^2\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx}{c^2}$$

$\downarrow$ 5459

$$-\frac{a^2\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx}{c^2}+$$

$$\frac{\frac{2}{3}a\left(-\left(a^2\left(i\int \frac{\arctan(ax)}{x(ax+i)}dx-\frac{1}{2}i\arctan(ax)^2\right)\right)-\frac{\arctan(ax)}{2x^2}+\frac{1}{2}a\left(-a\arctan(ax)-\frac{1}{x}\right)\right)-\left(a^2\left(2a\left(i\int \frac{\arctan(ax)}{x(ax+i)}dx\right.\right.\right.}{c^2}$$

$\downarrow$ 5403

$$-\frac{a^2\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx}{c^2}+$$

$$-\left(a^2\left(2a\left(i\left(ia\int \frac{\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)-\frac{1}{3}a\arctan(ax)^3-\frac{\arctan}{}\right.\right.$$

$\downarrow$ 2897

$$-\frac{a^2\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx}{c^2}+$$

$$\frac{2}{3}a\left(-\left(a^2\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)\right)-\frac{\arctan(ax)}{2x^2}+\frac{1}{2}a(\right.$$

$\downarrow$ 5501

$$-\frac{a^2\left(\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)}dx-a^2\int \frac{\arctan(ax)^2}{(a^2x^2+1)^2}dx\right)}{c^2}+$$

$$\frac{2}{3}a\left(-\left(a^2\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)\right)-\frac{\arctan(ax)}{2x^2}+\frac{1}{2}a(\right.$$

$\downarrow$ 5427

$$-\frac{a^2\left(\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)}dx-a^2\left(-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)\right)}{c^2}+$$

$$\frac{2}{3}a\left(-\left(a^2\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)\right)-\frac{\arctan(ax)}{2x^2}+\frac{1}{2}a(\right.$$

$\downarrow$ 5453

$$-\frac{a^2\left(-\left(a^2\left(-a\int\frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)\right)-a^2\int\frac{\arctan(ax)^2}{a^2x^2+1}dx+\int\frac{\arctan(ax)^2}{x^2}dx\right)}{c^2}+$$

$$\frac{2}{3}a\left(-\left(a^2\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)\right)-\frac{\arctan(ax)}{2x^2}+\frac{1}{2}a(\right.$$

$$\Big\downarrow \; 5361$$

$$-\frac{a^2\left(-\left(a^2\left(-a\int\frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)\right)-a^2\int\frac{\arctan(ax)^2}{a^2x^2+1}dx+2a\int\frac{\arctan(ax)}{x(a^2x^2+1)}dx-\frac{\arctan(ax)^2}{x}\right.}{c^2}$$

$$\frac{2}{3}a\left(-\left(a^2\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)\right)-\frac{\arctan(ax)}{2x^2}+\frac{1}{2}a(\right.$$

$$\Big\downarrow \; 5419$$

$$-\frac{a^2\left(-a^2\left(-a\int\frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)+2a\int\frac{\arctan(ax)}{x(a^2x^2+1)}dx-\frac{1}{3}a\arctan(ax)^3-\frac{\arctan(ax)^2}{x}\right)}{c^2}+$$

$$\frac{2}{3}a\left(-\left(a^2\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)\right)-\frac{\arctan(ax)}{2x^2}+\frac{1}{2}a(\right.$$

$$\Big\downarrow \; 5459$$

$$\frac{2}{3}a\left(-\left(a^2\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)\right)-\frac{\arctan(ax)}{2x^2}+\frac{1}{2}a(\right.$$

$$\frac{a^2\left(-a^2\left(-a\int\frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)+2a\left(i\int\frac{\arctan(ax)}{x(ax+i)}dx-\frac{1}{2}i\arctan(ax)^2\right)-\frac{1}{3}a\arctan(ax\right.}{c^2}$$

$$\Big\downarrow \; 5403$$

$$\frac{2}{3}a\left(-\left(a^2\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)\right)-\frac{\arctan(ax)}{2x^2}+\frac{1}{2}a(\right.$$

$$\frac{a^2\left(-a^2\left(-a\int\frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)+2a\left(i\left(ia\int\frac{\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)\log\left(2-\frac{}{1-}\right.\right.\right.\right.}{c^2}$$

$$\Big\downarrow \; 2897$$

$$\frac{\frac{2}{3}a\left(-\left(a^2\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)\right)-\frac{\arctan(ax)}{2x^2}+\frac{1}{2}a\left(\right.}{\frac{a^2\left(-a^2\left(-a\int\frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)+2a\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\right.\right.\right.\right.}{c^2}}$$

$\Big\downarrow$ 5465

$$\frac{\frac{2}{3}a\left(-\left(a^2\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)\right)-\frac{\arctan(ax)}{2x^2}+\frac{1}{2}a\left(\right.}{\frac{a^2\left(-a^2\left(-a\left(\frac{\int\frac{1}{(a^2x^2+1)^2}dx}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)+2a\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)\right.\right.\right.}{c^2}}$$

$\Big\downarrow$ 215

$$\frac{\frac{2}{3}a\left(-\left(a^2\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)\right)-\frac{\arctan(ax)}{2x^2}+\frac{1}{2}a\left(\right.}{\frac{a^2\left(-a^2\left(-a\left(\frac{\frac{1}{2}\int\frac{1}{a^2x^2+1}dx+\frac{x}{2(a^2x^2+1)}}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)+2a\left(i\left(-i\arctan(ax)\log\left(2-\right.\right.\right.\right.}{c^2}}$$

$\Big\downarrow$ 216

$$\frac{\frac{2}{3}a\left(-\left(a^2\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)\right)-\frac{\arctan(ax)}{2x^2}+\frac{1}{2}a\left(\right.}{\frac{a^2\left(-a^2\left(\frac{x\arctan(ax)^2}{2(a^2x^2+1)}-a\left(\frac{\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)+\frac{\arctan(ax)^3}{6a}\right)+2a\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-\right.\right.\right.\right.}{c^2}}$$

input
```
Int[ArcTan[a*x]^2/(x^4*(c + a^2*c*x^2)^2),x]
```

output
```
(-1/3*ArcTan[a*x]^2/x^3 - a^2*(-(ArcTan[a*x]^2/x) - (a*ArcTan[a*x]^3)/3 +
2*a*((-1/2*I)*ArcTan[a*x]^2 + I*((-I)*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)] -
 PolyLog[2, -1 + 2/(1 - I*a*x)]/2))) + (2*a*(-1/2*ArcTan[a*x]/x^2 + (a*(-x
^(-1) - a*ArcTan[a*x]))/2 - a^2*((-1/2*I)*ArcTan[a*x]^2 + I*((-I)*ArcTan[a
*x]*Log[2 - 2/(1 - I*a*x)] - PolyLog[2, -1 + 2/(1 - I*a*x)]/2))))/3)/c^2 -
 (a^2*(-(ArcTan[a*x]^2/x) - (a*ArcTan[a*x]^3)/3 - a^2*((x*ArcTan[a*x]^2)/(
2*(1 + a^2*x^2)) + ArcTan[a*x]^3/(6*a) - a*(-1/2*ArcTan[a*x]/(a^2*(1 + a^2
*x^2)) + (x/(2*(1 + a^2*x^2)) + ArcTan[a*x]/(2*a))/(2*a))) + 2*a*((-1/2*I)
*ArcTan[a*x]^2 + I*((-I)*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)] - PolyLog[2, -
1 + 2/(1 - I*a*x)]/2))))/c^2
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

## Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.45

| method | result |
|---|---|
| derivativedivides | $a^3\left(-\frac{\arctan(ax)^2}{3c^2a^3x^3}+\frac{2\arctan(ax)^2}{c^2ax}+\frac{ax\arctan(ax)^2}{2c^2(a^2x^2+1)}+\frac{5\arctan(ax)^3}{2c^2}-\frac{\frac{\arctan(ax)}{a^2x^2}+14\arctan(ax)\ln(ax)-7}{}\right.$ |
| default | $a^3\left(-\frac{\arctan(ax)^2}{3c^2a^3x^3}+\frac{2\arctan(ax)^2}{c^2ax}+\frac{ax\arctan(ax)^2}{2c^2(a^2x^2+1)}+\frac{5\arctan(ax)^3}{2c^2}-\frac{\frac{\arctan(ax)}{a^2x^2}+14\arctan(ax)\ln(ax)-7}{}\right.$ |
| parts | $\frac{a^4x\arctan(ax)^2}{2c^2(a^2x^2+1)}+\frac{5a^3\arctan(ax)^3}{2c^2}-\frac{\arctan(ax)^2}{3c^2x^3}+\frac{2a^2\arctan(ax)^2}{c^2x}-\frac{2\left(\frac{5a^3\arctan(ax)^3}{2}+\frac{a^3\left(\frac{\arctan(ax)}{a^2x^2}+14\right.}{}\right.}{}$ |

input
```
int(arctan(a*x)^2/x^4/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
a^3*(-1/3/c^2*arctan(a*x)^2/a^3/x^3+2/c^2*arctan(a*x)^2/a/x+1/2*a*x*arctan
(a*x)^2/c^2/(a^2*x^2+1)+5/2*arctan(a*x)^3/c^2-1/3/c^2*(arctan(a*x)/a^2/x^2
+14*arctan(a*x)*ln(a*x)-7*arctan(a*x)*ln(a^2*x^2+1)-3/2*arctan(a*x)/(a^2*x
^2+1)+7*I*ln(a*x)*ln(1+I*a*x)-7*I*ln(a*x)*ln(1-I*a*x)+7*I*dilog(1+I*a*x)-7
*I*dilog(1-I*a*x)-7/2*I*(ln(a*x-I)*ln(a^2*x^2+1)-1/2*ln(a*x-I)^2-dilog(-1/
2*I*(a*x+I))-ln(a*x-I)*ln(-1/2*I*(a*x+I)))+7/2*I*(ln(a*x+I)*ln(a^2*x^2+1)-
1/2*ln(a*x+I)^2-dilog(1/2*I*(a*x-I))-ln(a*x+I)*ln(1/2*I*(a*x-I)))+1/a/x+3/
4*a*x/(a^2*x^2+1)+7/4*arctan(a*x)+5*arctan(a*x)^3))
```

## Fricas [F]

$$\int \frac{\arctan(ax)^2}{x^4\left(c+a^2cx^2\right)^2}\,dx = \int \frac{\arctan\left(ax\right)^2}{\left(a^2cx^2+c\right)^2x^4}\,dx$$

input
```
integrate(arctan(a*x)^2/x^4/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
integral(arctan(a*x)^2/(a^4*c^2*x^8 + 2*a^2*c^2*x^6 + c^2*x^4), x)
```

## Sympy [F]

$$\int \frac{\arctan(ax)^2}{x^4\left(c+a^2cx^2\right)^2}\,dx = \frac{\int \frac{\operatorname{atan}^2\left(ax\right)}{a^4x^8+2a^2x^6+x^4}\,dx}{c^2}$$

input
```
integrate(atan(a*x)**2/x**4/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(atan(a*x)**2/(a**4*x**8 + 2*a**2*x**6 + x**4), x)/c**2
```

## Maxima [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^2}{x^4(c+a^2cx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(arctan(a*x)^2/x^4/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
Timed out
```

## Giac [F]

$$\int \frac{\arctan(ax)^2}{x^4(c+a^2cx^2)^2}\,dx = \int \frac{\arctan(ax)^2}{(a^2cx^2+c)^2x^4}\,dx$$

input
```
integrate(arctan(a*x)^2/x^4/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^2/((a^2*c*x^2 + c)^2*x^4), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^2}{x^4(c+a^2cx^2)^2}\,dx = \int \frac{\operatorname{atan}(a\,x)^2}{x^4\,(c\,a^2\,x^2+c)^2}\,dx$$

input
```
int(atan(a*x)^2/(x^4*(c + a^2*c*x^2)^2),x)
```

output
```
int(atan(a*x)^2/(x^4*(c + a^2*c*x^2)^2), x)
```

## Reduce [F]

$$\int \frac{\arctan(ax)^2}{x^4 (c+a^2cx^2)^2} dx$$
$$= \frac{5atan(ax)^3 a^5x^5 + 5atan(ax)^3 a^3x^3 + 15atan(ax)^2 a^4x^4 + 10atan(ax)^2 a^2x^2 - 2atan(ax)^2 + 15atan(ax)}{6c^2x^3 (a^2x^2 + 1}$$

input
```
int(atan(a*x)^2/x^4/(a^2*c*x^2+c)^2,x)
```

output
```
(5*atan(a*x)**3*a**5*x**5 + 5*atan(a*x)**3*a**3*x**3 + 15*atan(a*x)**2*a**
4*x**4 + 10*atan(a*x)**2*a**2*x**2 - 2*atan(a*x)**2 + 15*atan(a*x)*a**3*x*
*3 + 5*atan(a*x)*a*x + 14*int(atan(a*x)/(a**4*x**7 + 2*a**2*x**5 + x**3),x
)*a**3*x**5 + 14*int(atan(a*x)/(a**4*x**7 + 2*a**2*x**5 + x**3),x)*a*x**3
+ 5*a**2*x**2)/(6*c**2*x**3*(a**2*x**2 + 1))
```

# 3.299 $\int \frac{x^3 \arctan(ax)^2}{(c+a^2cx^2)^3} dx$



## Optimal result

Integrand size = 22, antiderivative size = 140

$$\int \frac{x^3 \arctan(ax)^2}{(c+a^2cx^2)^3} dx = -\frac{x^4}{32c^3(1+a^2x^2)^2} + \frac{3}{32a^4c^3(1+a^2x^2)} + \frac{x^3\arctan(ax)}{8ac^3(1+a^2x^2)^2} + \frac{3x\arctan(ax)}{16a^3c^3(1+a^2x^2)} - \frac{3\arctan(ax)^2}{32a^4c^3} + \frac{x^4\arctan(ax)^2}{4c^3(1+a^2x^2)^2}$$

output
```
-1/32*x^4/c^3/(a^2*x^2+1)^2+3/32/a^4/c^3/(a^2*x^2+1)+1/8*x^3*arctan(a*x)/a
/c^3/(a^2*x^2+1)^2+3/16*x*arctan(a*x)/a^3/c^3/(a^2*x^2+1)-3/32*arctan(a*x)
^2/a^4/c^3+1/4*x^4*arctan(a*x)^2/c^3/(a^2*x^2+1)^2
```

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.53

$$\int \frac{x^3 \arctan(ax)^2}{(c+a^2cx^2)^3} dx = \frac{4+5a^2x^2+2ax(3+5a^2x^2)\arctan(ax)+(-3-6a^2x^2+5a^4x^4)\arctan(ax)^2}{32a^4c^3(1+a^2x^2)^2}$$

input
```
Integrate[(x^3*ArcTan[a*x]^2)/(c + a^2*c*x^2)^3,x]
```

output
```
(4 + 5*a^2*x^2 + 2*a*x*(3 + 5*a^2*x^2)*ArcTan[a*x] + (-3 - 6*a^2*x^2 + 5*a
^4*x^4)*ArcTan[a*x]^2)/(32*a^4*c^3*(1 + a^2*x^2)^2)
```

## Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {5479, 27, 5473, 5469, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \arctan(ax)^2}{(a^2cx^2+c)^3}dx$$

$$\Big\downarrow \text{5479}$$

$$\frac{x^4 \arctan(ax)^2}{4c^3(a^2x^2+1)^2}-\frac{1}{2}a\int \frac{x^4 \arctan(ax)}{c^3(a^2x^2+1)^3}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{x^4 \arctan(ax)^2}{4c^3(a^2x^2+1)^2}-\frac{a\int \frac{x^4 \arctan(ax)}{(a^2x^2+1)^3}dx}{2c^3}$$

$$\Big\downarrow \text{5473}$$

$$\frac{x^4 \arctan(ax)^2}{4c^3(a^2x^2+1)^2}-\frac{a\left(\frac{3\int \frac{x^2 \arctan(ax)}{(a^2x^2+1)^2}dx}{4a^2}-\frac{x^3 \arctan(ax)}{4a^2(a^2x^2+1)^2}+\frac{x^4}{16a(a^2x^2+1)^2}\right)}{2c^3}$$

$$\Big\downarrow \text{5469}$$

$$\frac{x^4 \arctan(ax)^2}{4c^3(a^2x^2+1)^2}-\frac{a\left(\frac{3\left(\frac{\int \frac{\arctan(ax)}{a^2x^2+1}dx}{2a^2}-\frac{x\arctan(ax)}{2a^2(a^2x^2+1)}-\frac{1}{4a^3(a^2x^2+1)}\right)}{4a^2}-\frac{x^3 \arctan(ax)}{4a^2(a^2x^2+1)^2}+\frac{x^4}{16a(a^2x^2+1)^2}\right)}{2c^3}$$

$$\Big\downarrow \text{5419}$$

$$\frac{x^4 \arctan(ax)^2}{4c^3 (a^2x^2+1)^2} - \frac{a \left( -\frac{x^3 \arctan(ax)}{4a^2(a^2x^2+1)^2} + \frac{x^4}{16a(a^2x^2+1)^2} + \frac{3\left( \frac{\arctan(ax)^2}{4a^3} - \frac{x \arctan(ax)}{2a^2(a^2x^2+1)} - \frac{1}{4a^3(a^2x^2+1)} \right)}{4a^2} \right)}{2c^3}$$

input
```
Int[(x^3*ArcTan[a*x]^2)/(c + a^2*c*x^2)^3,x]
```

output
```
(x^4*ArcTan[a*x]^2)/(4*c^3*(1 + a^2*x^2)^2) - (a*(x^4/(16*a*(1 + a^2*x^2)^
2) - (x^3*ArcTan[a*x])/(4*a^2*(1 + a^2*x^2)^2) + (3*(-1/4*1/(a^3*(1 + a^2*
x^2)) - (x*ArcTan[a*x])/(2*a^2*(1 + a^2*x^2)) + ArcTan[a*x]^2/(4*a^3)))/(4
*a^2)))/(2*c^3)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5469
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^2*((d_) + (e_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[(-b)*((d + e*x^2)^(q + 1)/(4*c^3*d*(q + 1)^2)), x] + (Simp
[x*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*c^2*d*(q + 1))), x] - Simp[1
/(2*c^2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x]) /
; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -5/2]
```

rule 5473
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[b*(f*x)^m*((d + e*x^2)^(q + 1)/(c*d*m^2)), x] +
 (-Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(c^2*d*m))
, x] + Simp[f^2*((m - 1)/(c^2*d*m))   Int[(f*x)^(m - 2)*(d + e*x^2)^(q + 1)
*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2
*d] && EqQ[m + 2*q + 2, 0] && LtQ[q, -1]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

## Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.66

| method | result |
|---|---|
| parallelrisch | $\frac{5a^4\arctan(ax)^2x^4-4a^4x^4+10\arctan(ax)x^3a^3-6\arctan(ax)^2x^2a^2-3a^2x^2+6\arctan(ax)ax-3\arctan(ax)^2}{32c^3(a^2x^2+1)^2a^4}$ |
| derivativedivides | $\frac{-\frac{\arctan(ax)^2}{2c^3(a^2x^2+1)}+\frac{\arctan(ax)^2}{4c^3(a^2x^2+1)^2}-\frac{-\frac{5\arctan(ax)a^3x^3}{8(a^2x^2+1)^2}-\frac{3\arctan(ax)ax}{8(a^2x^2+1)^2}-\frac{5\arctan(ax)^2}{16}-\frac{5}{16(a^2x^2+1)}+\frac{1}{16(a^2x^2+1)^2}}{2c^3}}{a^4}$ |
| default | $\frac{-\frac{\arctan(ax)^2}{2c^3(a^2x^2+1)}+\frac{\arctan(ax)^2}{4c^3(a^2x^2+1)^2}-\frac{-\frac{5\arctan(ax)a^3x^3}{8(a^2x^2+1)^2}-\frac{3\arctan(ax)ax}{8(a^2x^2+1)^2}-\frac{5\arctan(ax)^2}{16}-\frac{5}{16(a^2x^2+1)}+\frac{1}{16(a^2x^2+1)^2}}{2c^3}}{a^4}$ |
| parts | $-\frac{\arctan(ax)^2}{2c^3a^4(a^2x^2+1)}+\frac{\arctan(ax)^2}{4c^3a^4(a^2x^2+1)^2}-\frac{-\frac{5\arctan(ax)x^3}{8a(a^2x^2+1)^2}-\frac{3\arctan(ax)x}{8a^3(a^2x^2+1)^2}-\frac{5\arctan(ax)^2}{8a^4}+\frac{-\frac{5}{2(a^2x^2+1)}+\frac{1}{2(a^2x^2+1}}{8a^4}}{2c^3}$ |
| risch | $-\frac{(5a^4x^4-6a^2x^2-3)\ln(iax+1)^2}{128a^4c^3(a^2x^2+1)^2}+\frac{(-6a^2x^2\ln(-iax+1)-3\ln(-iax+1)+5x^4\ln(-iax+1)a^4-10ia^3x^3-6iax)\ln(i}{64a^4(ax+i)^2(ax-i)^2c^3}$ |
| orering | $-\frac{(a^2x^2+1)(12a^6x^6-13a^4x^4+9a^2x^2+15)\arctan(ax)^2}{16a^4(a^2cx^2+c)^3}-\frac{(a^2x^2+1)^2(16a^4x^4-5a^2x^2-12)\left(\frac{3x^2\arctan(ax)^2}{(a^2cx^2+c)^3}+\frac{2}{(a^2c}\right.}{32x^2a^4}$ |

input
```
int(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/32*(5*a^4*arctan(a*x)^2*x^4-4*a^4*x^4+10*arctan(a*x)*x^3*a^3-6*arctan(a*
x)^2*x^2*a^2-3*a^2*x^2+6*arctan(a*x)*a*x-3*arctan(a*x)^2)/c^3/(a^2*x^2+1)^
2/a^4
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.62

$$\int \frac{x^3 \arctan(ax)^2}{(c+a^2cx^2)^3}\,dx = \frac{5\,a^2x^2 + (5\,a^4x^4 - 6\,a^2x^2 - 3)\arctan(ax)^2 + 2\,(5\,a^3x^3 + 3\,ax)\arctan(ax) + 4}{32\,(a^8c^3x^4 + 2\,a^6c^3x^2 + a^4c^3)}$$

input
```
integrate(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
1/32*(5*a^2*x^2 + (5*a^4*x^4 - 6*a^2*x^2 - 3)*arctan(a*x)^2 + 2*(5*a^3*x^3
 + 3*a*x)*arctan(a*x) + 4)/(a^8*c^3*x^4 + 2*a^6*c^3*x^2 + a^4*c^3)
```

## Sympy [F]

$$\int \frac{x^3 \arctan(ax)^2}{(c+a^2cx^2)^3}\,dx = \frac{\int \frac{x^3 \operatorname{atan}^2(ax)}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

input
```
integrate(x**3*atan(a*x)**2/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(x**3*atan(a*x)**2/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x)
/c**3
```

## Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.32

$$\int \frac{x^3 \arctan(ax)^2}{(c+a^2cx^2)^3}\,dx = \frac{1}{16}\,a\left(\frac{5\,a^2x^3 + 3\,x}{a^8c^3x^4 + 2\,a^6c^3x^2 + a^4c^3} + \frac{5\,\arctan(ax)}{a^5c^3}\right)\arctan(ax) + \frac{(5\,a^2x^2 - 5\,(a^4x^4 + 2\,a^2x^2 + 1)\arctan(ax)^2 + 4)a^2}{32\,(a^{10}c^3x^4 + 2\,a^8c^3x^2 + a^6c^3)} - \frac{(2\,a^2x^2 + 1)\arctan(ax)^2}{4\,(a^8c^3x^4 + 2\,a^6c^3x^2 + a^4c^3)}$$

input
```
integrate(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
1/16*a*((5*a^2*x^3 + 3*x)/(a^8*c^3*x^4 + 2*a^6*c^3*x^2 + a^4*c^3) + 5*arct
an(a*x)/(a^5*c^3))*arctan(a*x) + 1/32*(5*a^2*x^2 - 5*(a^4*x^4 + 2*a^2*x^2
+ 1)*arctan(a*x)^2 + 4)*a^2/(a^10*c^3*x^4 + 2*a^8*c^3*x^2 + a^6*c^3) - 1/4
*(2*a^2*x^2 + 1)*arctan(a*x)^2/(a^8*c^3*x^4 + 2*a^6*c^3*x^2 + a^4*c^3)
```

## Giac [F]

$$\int \frac{x^3 \arctan(ax)^2}{(c+a^2cx^2)^3}\,dx = \int \frac{x^3 \arctan{(ax)}^2}{(a^2cx^2+c)^3}\,dx$$

input
```
integrate(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
integrate(x^3*arctan(a*x)^2/(a^2*c*x^2 + c)^3, x)
```

## Mupad [B] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.61

$$\int \frac{x^3 \arctan(ax)^2}{(c+a^2cx^2)^3}\,dx = \frac{5\,a^4\,x^4\,\mathrm{atan}(a\,x)^2+10\,a^3\,x^3\,\mathrm{atan}(a\,x)-6\,a^2\,x^2\,\mathrm{atan}(a\,x)^2+5\,a^2\,x^2+6\,a\,x\,\mathrm{atan}(a\,x)-3\,\mathrm{atan}(a\,x)^2+4}{32\,a^4\,c^3\,{\left(a^2\,x^2+1\right)}^2}$$

input
```
int((x^3*atan(a*x)^2)/(c + a^2*c*x^2)^3,x)
```

output
```
(5*a^2*x^2 - 3*atan(a*x)^2 + 10*a^3*x^3*atan(a*x) + 6*a*x*atan(a*x) - 6*a^
2*x^2*atan(a*x)^2 + 5*a^4*x^4*atan(a*x)^2 + 4)/(32*a^4*c^3*(a^2*x^2 + 1)^2
)
```

## Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.66

$$\int \frac{x^3 \arctan(ax)^2}{(c+a^2cx^2)^3} \, dx$$
$$= \frac{10atan(ax)^2 a^4x^4 - 12atan(ax)^2 a^2x^2 - 6atan(ax)^2 + 20atan(ax) a^3x^3 + 12atan(ax) ax - 5a^4x^4 + 3}{64a^4c^3 (a^4x^4 + 2a^2x^2 + 1)}$$

input
```
int(x^3*atan(a*x)^2/(a^2*c*x^2+c)^3,x)
```

output
```
(10*atan(a*x)**2*a**4*x**4 - 12*atan(a*x)**2*a**2*x**2 - 6*atan(a*x)**2 +
20*atan(a*x)*a**3*x**3 + 12*atan(a*x)*a*x - 5*a**4*x**4 + 3)/(64*a**4*c**3
*(a**4*x**4 + 2*a**2*x**2 + 1))
```

# 3.300 $\int \frac{x^2 \arctan(ax)^2}{(c+a^2cx^2)^3} dx$



## Optimal result

Integrand size = 22, antiderivative size = 181

$$\int \frac{x^2 \arctan(ax)^2}{(c+a^2cx^2)^3} dx = \frac{x}{32a^2c^3(1+a^2x^2)^2} - \frac{x}{64a^2c^3(1+a^2x^2)} - \frac{\arctan(ax)}{64a^3c^3} - \frac{\arctan(ax)}{8a^3c^3(1+a^2x^2)^2} + \frac{\arctan(ax)}{8a^3c^3(1+a^2x^2)} - \frac{x\arctan(ax)^2}{4a^2c^3(1+a^2x^2)^2} + \frac{x\arctan(ax)^2}{8a^2c^3(1+a^2x^2)} + \frac{\arctan(ax)^3}{24a^3c^3}$$

output
```
1/32*x/a^2/c^3/(a^2*x^2+1)^2-1/64*x/a^2/c^3/(a^2*x^2+1)-1/64*arctan(a*x)/a
^3/c^3-1/8*arctan(a*x)/a^3/c^3/(a^2*x^2+1)^2+1/8*arctan(a*x)/a^3/c^3/(a^2*
x^2+1)-1/4*x*arctan(a*x)^2/a^2/c^3/(a^2*x^2+1)^2+1/8*x*arctan(a*x)^2/a^2/c
^3/(a^2*x^2+1)+1/24*arctan(a*x)^3/a^3/c^3
```

## Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.52

$$\int \frac{x^2 \arctan(ax)^2}{(c+a^2cx^2)^3} dx$$
$$= \frac{3ax - 3a^3x^3 - 3(1-6a^2x^2+a^4x^4)\arctan(ax) + 24ax(-1+a^2x^2)\arctan(ax)^2 + 8(1+a^2x^2)^2 \arctan(a}{192a^3c^3(1+a^2x^2)^2}$$

input
```
Integrate[(x^2*ArcTan[a*x]^2)/(c + a^2*c*x^2)^3,x]
```

output
```
(3*a*x - 3*a^3*x^3 - 3*(1 - 6*a^2*x^2 + a^4*x^4)*ArcTan[a*x] + 24*a*x*(-1
+ a^2*x^2)*ArcTan[a*x]^2 + 8*(1 + a^2*x^2)^2*ArcTan[a*x]^3)/(192*a^3*c^3*(
1 + a^2*x^2)^2)
```

## Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.72, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5499, 27, 5427, 5435, 215, 215, 216, 5427, 5465, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \arctan(ax)^2}{(a^2cx^2+c)^3} dx$$

$$\Big\downarrow \text{5499}$$

$$\frac{\int \frac{\arctan(ax)^2}{c^2(a^2x^2+1)^2}dx}{a^2c} - \frac{\int \frac{\arctan(ax)^2}{c^3(a^2x^2+1)^3}dx}{a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\arctan(ax)^2}{(a^2x^2+1)^2}dx}{a^2c^3} - \frac{\int \frac{\arctan(ax)^2}{(a^2x^2+1)^3}dx}{a^2c^3}$$

$$\Big\downarrow \text{5427}$$

$$\frac{-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}}{a^2c^3}-\frac{\int \frac{\arctan(ax)^2}{(a^2x^2+1)^3}dx}{a^2c^3}$$

$$\Big\downarrow \; 5435$$

$$\frac{-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}}{a^2c^3}-\frac{\frac{3}{4}\int \frac{\arctan(ax)^2}{(a^2x^2+1)^2}dx-\frac{1}{8}\int \frac{1}{(a^2x^2+1)^3}dx+\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}}{a^2c^3}$$

$$\Big\downarrow \; 215$$

$$\frac{-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}}{a^2c^3}-\frac{\frac{3}{4}\int \frac{\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{1}{8}\left(-\frac{3}{4}\int \frac{1}{(a^2x^2+1)^2}dx-\frac{x}{4(a^2x^2+1)^2}\right)+\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}}{a^2c^3}$$

$$\Big\downarrow \; 215$$

$$\frac{-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}}{a^2c^3}-\frac{\frac{3}{4}\int \frac{\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{a^2x^2+1}dx+\frac{x}{2(a^2x^2+1)}\right)-\frac{x}{4(a^2x^2+1)^2}\right)+\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}}{a^2c^3}$$

$$\Big\downarrow \; 216$$

$$\frac{-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}}{a^2c^3}-\frac{\frac{3}{4}\int \frac{\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}\right)-\frac{x}{4(a^2x^2+1)^2}\right)}{a^2c^3}$$

$$\Big\downarrow \; 5427$$

$$\frac{-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}}{a^2c^3}-\frac{\frac{3}{4}\left(-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)+\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}\right)-\right.}{a^2c^3}$$

$$\Big\downarrow \; 5465$$

$$\frac{-a\left(\frac{\int \frac{1}{\left(a^2x^2+1\right)^2}dx}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}}{a^2c^3}-$$

$$\frac{\frac{3}{4}\left(-a\left(\frac{\int \frac{1}{\left(a^2x^2+1\right)^2}dx}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)+\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2(a^2x^2+1}\right.\right.}{a^2c^3}$$

$$\Big\downarrow \textbf{215}$$

$$\frac{-a\left(\frac{\frac{1}{2}\int \frac{1}{a^2x^2+1}dx+\frac{x}{2\left(a^2x^2+1\right)}}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}}{a^2c^3}-$$

$$\frac{\frac{3}{4}\left(-a\left(\frac{\frac{1}{2}\int \frac{1}{a^2x^2+1}dx+\frac{x}{2\left(a^2x^2+1\right)}}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)+\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\right.\right.}{a^2c^3}$$

$$\Big\downarrow \textbf{216}$$

$$\frac{\frac{x\arctan(ax)^2}{2(a^2x^2+1)}-a\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)+\frac{\arctan(ax)^3}{6a}}{a^2c^3}-$$

$$\frac{\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}\right)-\frac{x}{4(a^2x^2+1)^2}\right)+\frac{3}{4}\left(\frac{x\arctan(ax)^2}{2(a^2x^2+1)}-a\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(a}{2a}}{2a}\right.\right.}{a^2c^3}$$

input
```
Int[(x^2*ArcTan[a*x]^2)/(c + a^2*c*x^2)^3,x]
```

output
```
((x*ArcTan[a*x]^2)/(2*(1 + a^2*x^2)) + ArcTan[a*x]^3/(6*a) - a*(-1/2*ArcTa
n[a*x]/(a^2*(1 + a^2*x^2)) + (x/(2*(1 + a^2*x^2)) + ArcTan[a*x]/(2*a))/(2*
a)))/(a^2*c^3) - (ArcTan[a*x]/(8*a*(1 + a^2*x^2)^2) + (x*ArcTan[a*x]^2)/(4
*(1 + a^2*x^2)^2) + (-1/4*x/(1 + a^2*x^2)^2 - (3*(x/(2*(1 + a^2*x^2)) + Ar
cTan[a*x]/(2*a)))/4)/8 + (3*((x*ArcTan[a*x]^2)/(2*(1 + a^2*x^2)) + ArcTan[
a*x]^3/(6*a) - a*(-1/2*ArcTan[a*x]/(a^2*(1 + a^2*x^2)) + (x/(2*(1 + a^2*x^
2)) + ArcTan[a*x]/(2*a))/(2*a))))/4)/(a^2*c^3)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5435
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[b*p*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p - 1)/(4*c*d*
(q + 1)^2)), x] + (-Simp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*d*
(q + 1))), x] + Simp[(2*q + 3)/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p, x], x] - Simp[b^2*p*((p - 1)/(4*(q + 1)^2))   Int[(d + e
*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5499
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ
[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

## Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.68

| method | result |
|---|---|
| parallelrisch | $\frac{8\arctan(ax)^3 a^4x^4-3x^4\arctan(ax)a^4+24a^3\arctan(ax)^2x^3+16\arctan(ax)^3a^2x^2-3a^3x^3+18x^2a^2\arctan(ax)-24a\,\mathrm{ar}}{192c^3(a^2x^2+1)^2a^3}$ |
| derivativedivides | $\frac{\frac{\arctan(ax)^2a^3x^3}{8c^3(a^2x^2+1)^2}-\frac{ax\arctan(ax)^2}{8c^3(a^2x^2+1)^2}+\frac{\arctan(ax)^3}{8c^3}-\frac{\frac{\arctan(ax)^3}{3}-\frac{\arctan(ax)}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2(a^2x^2+1)^2}+\frac{\frac{1}{8}a^3x^3-\frac{1}{8}ax}{2(a^2x^2+1)^2}+\frac{\arctan(ax)}{16}}{4c^3}}{a^3}$ |
| default | $\frac{\frac{\arctan(ax)^2a^3x^3}{8c^3(a^2x^2+1)^2}-\frac{ax\arctan(ax)^2}{8c^3(a^2x^2+1)^2}+\frac{\arctan(ax)^3}{8c^3}-\frac{\frac{\arctan(ax)^3}{3}-\frac{\arctan(ax)}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2(a^2x^2+1)^2}+\frac{\frac{1}{8}a^3x^3-\frac{1}{8}ax}{2(a^2x^2+1)^2}+\frac{\arctan(ax)}{16}}{4c^3}}{a^3}$ |
| parts | $\frac{\arctan(ax)^2x^3}{8c^3(a^2x^2+1)^2}-\frac{x\arctan(ax)^2}{8a^2c^3(a^2x^2+1)^2}+\frac{\arctan(ax)^3}{8a^3c^3}-\frac{\frac{\arctan(ax)^3}{3a^3}+\frac{-\frac{\arctan(ax)}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2(a^2x^2+1)^2}+\frac{\frac{1}{8}a^3x^3-\frac{1}{8}ax}{2(a^2x^2+1)^2}+\mathrm{arct}}{a^3}}{4c^3}$ |
| risch | $\frac{i\ln(iax+1)^3}{192a^3c^3}-\frac{i(x^4\ln(-iax+1)a^4+2a^2x^2\ln(-iax+1)-2ia^3x^3+\ln(-iax+1)+2iax)\ln(iax+1)^2}{64a^3c^3(a^2x^2+1)^2}+\frac{i\left(a^4x^4\ln(-ia\right.}{}$ |

input
```
int(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/192*(8*arctan(a*x)^3*a^4*x^4-3*x^4*arctan(a*x)*a^4+24*a^3*arctan(a*x)^2*
x^3+16*arctan(a*x)^3*a^2*x^2-3*a^3*x^3+18*x^2*a^2*arctan(a*x)-24*a*arctan(
a*x)^2*x+8*arctan(a*x)^3+3*a*x-3*arctan(a*x))/c^3/(a^2*x^2+1)^2/a^3
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.63

$$\int \frac{x^2 \arctan(ax)^2}{(c+a^2cx^2)^3}\,dx = -\frac{3\,a^3x^3 - 8\,(a^4x^4 + 2\,a^2x^2 + 1)\arctan(ax)^3 - 24\,(a^3x^3 - ax)\arctan(ax)^2 - 3\,ax + 3\,(a^4x^4 - 6\,a^2x^2 + }{192\,(a^7c^3x^4 + 2\,a^5c^3x^2 + a^3c^3)}$$

input
```
integrate(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
-1/192*(3*a^3*x^3 - 8*(a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x)^3 - 24*(a^3*x^
3 - a*x)*arctan(a*x)^2 - 3*a*x + 3*(a^4*x^4 - 6*a^2*x^2 + 1)*arctan(a*x))/
(a^7*c^3*x^4 + 2*a^5*c^3*x^2 + a^3*c^3)
```

## Sympy [F]

$$\int \frac{x^2 \arctan(ax)^2}{(c+a^2cx^2)^3}\,dx = \frac{\int \frac{x^2 \operatorname{atan}^2(ax)}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

input
```
integrate(x**2*atan(a*x)**2/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(x**2*atan(a*x)**2/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x)
/c**3
```

## Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.28

$$\int \frac{x^2 \arctan(ax)^2}{(c+a^2cx^2)^3}\,dx = \frac{1}{8}\left(\frac{a^2x^3 - x}{a^6c^3x^4 + 2\,a^4c^3x^2 + a^2c^3} + \frac{\arctan(ax)}{a^3c^3}\right)\arctan(ax)^2 - \frac{\left(3\,a^3x^3 - 8\,(a^4x^4 + 2\,a^2x^2 + 1)\arctan(ax)^3 - 3\,ax + 3\,(a^4x^4 + 2\,a^2x^2 + 1)\arctan(ax)\right)a^2}{192\,(a^9c^3x^4 + 2\,a^7c^3x^2 + a^5c^3)} + \frac{\left(a^2x^2 - (a^4x^4 + 2\,a^2x^2 + 1)\arctan(ax)^2\right)a\arctan(ax)}{8\,(a^8c^3x^4 + 2\,a^6c^3x^2 + a^4c^3)}$$

input
```
integrate(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
1/8*((a^2*x^3 - x)/(a^6*c^3*x^4 + 2*a^4*c^3*x^2 + a^2*c^3) + arctan(a*x)/(
a^3*c^3))*arctan(a*x)^2 - 1/192*(3*a^3*x^3 - 8*(a^4*x^4 + 2*a^2*x^2 + 1)*a
rctan(a*x)^3 - 3*a*x + 3*(a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x))*a^2/(a^9*c
^3*x^4 + 2*a^7*c^3*x^2 + a^5*c^3) + 1/8*(a^2*x^2 - (a^4*x^4 + 2*a^2*x^2 +
1)*arctan(a*x)^2)*a*arctan(a*x)/(a^8*c^3*x^4 + 2*a^6*c^3*x^2 + a^4*c^3)
```

## Giac [F]

$$\int \frac{x^2 \arctan(ax)^2}{(c+a^2cx^2)^3}\,dx = \int \frac{x^2 \arctan(ax)^2}{(a^2cx^2+c)^3}\,dx$$

input
```
integrate(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
integrate(x^2*arctan(a*x)^2/(a^2*c*x^2 + c)^3, x)
```

## Mupad [B] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.83

$$\int \frac{x^2 \arctan(ax)^2}{(c+a^2cx^2)^3}\,dx = \frac{\frac{x}{8a^2}-\frac{x^3}{8}}{8\,a^4\,c^3\,x^4+16\,a^2\,c^3\,x^2+8\,c^3} - \frac{\mathrm{atan}(a\,x)^2\left(\frac{x}{8\,a^4\,c^3}-\frac{x^3}{8\,a^2\,c^3}\right)}{\frac{1}{a^2}+2\,x^2+a^2\,x^4} - \frac{\mathrm{atan}(a\,x)}{64\,a^3\,c^3} + \frac{\mathrm{atan}(a\,x)^3}{24\,a^3\,c^3} + \frac{x^2\,\mathrm{atan}(a\,x)}{8\,a^3\,c^3\,\left(\frac{1}{a^2}+2\,x^2+a^2\,x^4\right)}$$

input
```
int((x^2*atan(a*x)^2)/(c + a^2*c*x^2)^3,x)
```

output
```
(x/(8*a^2) - x^3/8)/(8*c^3 + 16*a^2*c^3*x^2 + 8*a^4*c^3*x^4) - (atan(a*x)^
2*(x/(8*a^4*c^3) - x^3/(8*a^2*c^3)))/(1/a^2 + 2*x^2 + a^2*x^4) - atan(a*x)
/(64*a^3*c^3) + atan(a*x)^3/(24*a^3*c^3) + (x^2*atan(a*x))/(8*a^3*c^3*(1/a
^2 + 2*x^2 + a^2*x^4))
```

## Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.72

$$\int \frac{x^2 \arctan(ax)^2}{(c+a^2cx^2)^3}\,dx = \frac{8atan(ax)^3 a^4x^4 + 16atan(ax)^3 a^2x^2 + 8atan(ax)^3 + 24atan(ax)^2 a^3x^3 - 24atan(ax)^2 ax - 3atan(ax) a}{192a^3c^3\left(a^4x^4+2a^2x^2+1\right)}$$

input
```
int(x^2*atan(a*x)^2/(a^2*c*x^2+c)^3,x)
```

output
```
(8*atan(a*x)**3*a**4*x**4 + 16*atan(a*x)**3*a**2*x**2 + 8*atan(a*x)**3 + 2
4*atan(a*x)**2*a**3*x**3 - 24*atan(a*x)**2*a*x - 3*atan(a*x)*a**4*x**4 + 1
8*atan(a*x)*a**2*x**2 - 3*atan(a*x) - 3*a**3*x**3 + 3*a*x)/(192*a**3*c**3*
(a**4*x**4 + 2*a**2*x**2 + 1))
```

# 3.301 $\int \frac{x \arctan(ax)^2}{(c+a^2cx^2)^3} \, dx$



## Optimal result

Integrand size = 20, antiderivative size = 138

$$\int \frac{x \arctan(ax)^2}{(c+a^2cx^2)^3} \, dx = \frac{1}{32a^2c^3(1+a^2x^2)^2} + \frac{3}{32a^2c^3(1+a^2x^2)} + \frac{x\arctan(ax)}{8ac^3(1+a^2x^2)^2} + \frac{3x\arctan(ax)}{16ac^3(1+a^2x^2)} + \frac{3\arctan(ax)^2}{32a^2c^3} - \frac{\arctan(ax)^2}{4a^2c^3(1+a^2x^2)^2}$$

output
```
1/32/a^2/c^3/(a^2*x^2+1)^2+3/32/a^2/c^3/(a^2*x^2+1)+1/8*x*arctan(a*x)/a/c^
3/(a^2*x^2+1)^2+3/16*x*arctan(a*x)/a/c^3/(a^2*x^2+1)+3/32*arctan(a*x)^2/a^
2/c^3-1/4*arctan(a*x)^2/a^2/c^3/(a^2*x^2+1)^2
```

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.51

$$\int \frac{x \arctan(ax)^2}{(c+a^2cx^2)^3} \, dx = \frac{4+3a^2x^2+2ax(5+3a^2x^2)\arctan(ax)+(-5+6a^2x^2+3a^4x^4)\arctan(ax)^2}{32c^3(a+a^3x^2)^2}$$

input
```
Integrate[(x*ArcTan[a*x]^2)/(c + a^2*c*x^2)^3,x]
```

output
```
(4 + 3*a^2*x^2 + 2*a*x*(5 + 3*a^2*x^2)*ArcTan[a*x] + (-5 + 6*a^2*x^2 + 3*a
^4*x^4)*ArcTan[a*x]^2)/(32*c^3*(a + a^3*x^2)^2)
```

## Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5465, 27, 5431, 5427, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \arctan(ax)^2}{(a^2cx^2+c)^3} \, dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{\int \frac{\arctan(ax)}{c^3(a^2x^2+1)^3}dx}{2a} - \frac{\arctan(ax)^2}{4a^2c^3(a^2x^2+1)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\arctan(ax)}{(a^2x^2+1)^3}dx}{2ac^3} - \frac{\arctan(ax)^2}{4a^2c^3(a^2x^2+1)^2}$$

$$\Big\downarrow \text{5431}$$

$$\frac{\frac{3}{4}\int \frac{\arctan(ax)}{(a^2x^2+1)^2}dx + \frac{x\arctan(ax)}{4(a^2x^2+1)^2} + \frac{1}{16a(a^2x^2+1)^2}}{2ac^3} - \frac{\arctan(ax)^2}{4a^2c^3(a^2x^2+1)^2}$$

$$\Big\downarrow \text{5427}$$

$$\frac{\frac{3}{4}\left(-\frac{1}{2}a\int \frac{x}{(a^2x^2+1)^2}dx + \frac{x\arctan(ax)}{2(a^2x^2+1)} + \frac{\arctan(ax)^2}{4a}\right) + \frac{x\arctan(ax)}{4(a^2x^2+1)^2} + \frac{1}{16a(a^2x^2+1)^2}}{2ac^3} - \frac{\arctan(ax)^2}{4a^2c^3(a^2x^2+1)^2}$$

$$\Big\downarrow \text{241}$$

$$\frac{\frac{x\arctan(ax)}{4(a^2x^2+1)^2} + \frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)} + \frac{1}{4a(a^2x^2+1)} + \frac{\arctan(ax)^2}{4a}\right) + \frac{1}{16a(a^2x^2+1)^2}}{2ac^3} - \frac{\arctan(ax)^2}{4a^2c^3(a^2x^2+1)^2}$$

input
```
Int[(x*ArcTan[a*x]^2)/(c + a^2*c*x^2)^3,x]
```

output
```
-1/4*ArcTan[a*x]^2/(a^2*c^3*(1 + a^2*x^2)^2) + (1/(16*a*(1 + a^2*x^2)^2) +
 (x*ArcTan[a*x])/(4*(1 + a^2*x^2)^2) + (3*(1/(4*a*(1 + a^2*x^2)) + (x*ArcT
an[a*x])/(2*(1 + a^2*x^2)) + ArcTan[a*x]^2/(4*a)))/4)/(2*a*c^3)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5431
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbol
] :> Simp[b*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (-Simp[x*(d + e*x
^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*d*(q + 1))), x] + Simp[(2*q + 3)/(2*d*(
q + 1))   Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a,
 b, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -3/2]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

## Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.67

| method | result |
|---|---|
| parallelrisch | $\frac{3a^4\arctan(ax)^2x^4-4a^4x^4+6\arctan(ax)x^3a^3+6\arctan(ax)^2x^2a^2-5a^2x^2+10\arctan(ax)ax-5\arctan(ax)^2}{32c^3(a^2x^2+1)^2a^2}$ |
| derivativedivides | $\frac{-\frac{\arctan(ax)^2}{4c^3(a^2x^2+1)^2}+\frac{\frac{\arctan(ax)ax}{4(a^2x^2+1)^2}+\frac{3\arctan(ax)ax}{8(a^2x^2+1)}+\frac{3\arctan(ax)^2}{16}+\frac{3}{16(a^2x^2+1)}+\frac{1}{16(a^2x^2+1)^2}}{2c^3}}{a^2}$ |
| default | $\frac{-\frac{\arctan(ax)^2}{4c^3(a^2x^2+1)^2}+\frac{\frac{\arctan(ax)ax}{4(a^2x^2+1)^2}+\frac{3\arctan(ax)ax}{8(a^2x^2+1)}+\frac{3\arctan(ax)^2}{16}+\frac{3}{16(a^2x^2+1)}+\frac{1}{16(a^2x^2+1)^2}}{2c^3}}{a^2}$ |
| parts | $-\frac{\arctan(ax)^2}{4a^2c^3(a^2x^2+1)^2}+\frac{\frac{\arctan(ax)ax}{4(a^2x^2+1)^2}+\frac{3\arctan(ax)ax}{8(a^2x^2+1)}+\frac{3\arctan(ax)^2}{16}+\frac{3}{16(a^2x^2+1)}+\frac{1}{16(a^2x^2+1)^2}}{2a^2c^3}$ |
| risch | $-\frac{(3a^4x^4+6a^2x^2-5)\ln(iax+1)^2}{128a^2c^3(a^2x^2+1)^2}+\frac{(-5\ln(-iax+1)+3x^4\ln(-iax+1)a^4+6a^2x^2\ln(-iax+1)-6ia^3x^3-10iax)\ln(i}{64(ax+i)^2c^3(ax-i)^2a^2}$ |
| orering | $-\frac{(a^2x^2+1)(60a^6x^6+49a^4x^4-34a^2x^2+15)\arctan(ax)^2}{32a^2(a^2cx^2+c)^3}-\frac{(a^2x^2+1)^2(24a^4x^4+23a^2x^2-10)\left(\frac{\arctan(ax)^2}{(a^2cx^2+c)^3}+\frac{2}{(a^2c}\right.}{32a^2}$ |

input
```
int(x*arctan(a*x)^2/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/32*(3*a^4*arctan(a*x)^2*x^4-4*a^4*x^4+6*arctan(a*x)*x^3*a^3+6*arctan(a*x
)^2*x^2*a^2-5*a^2*x^2+10*arctan(a*x)*a*x-5*arctan(a*x)^2)/c^3/(a^2*x^2+1)^
2/a^2
```

## Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.63

$$\int \frac{x\arctan(ax)^2}{(c+a^2cx^2)^3}\,dx = \frac{3\,a^2x^2+(3\,a^4x^4+6\,a^2x^2-5)\arctan(ax)^2+2\,(3\,a^3x^3+5\,ax)\arctan(ax)+4}{32\,(a^6c^3x^4+2\,a^4c^3x^2+a^2c^3)}$$

input
```
integrate(x*arctan(a*x)^2/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
1/32*(3*a^2*x^2 + (3*a^4*x^4 + 6*a^2*x^2 - 5)*arctan(a*x)^2 + 2*(3*a^3*x^3
 + 5*a*x)*arctan(a*x) + 4)/(a^6*c^3*x^4 + 2*a^4*c^3*x^2 + a^2*c^3)
```

## Sympy [F]

$$\int \frac{x \arctan(ax)^2}{(c+a^2cx^2)^3}\,dx = \frac{\int \frac{x \operatorname{atan}^2(ax)}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

input
```
integrate(x*atan(a*x)**2/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(x*atan(a*x)**2/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x)/c*
*3
```

## Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.18

$$\int \frac{x \arctan(ax)^2}{(c+a^2cx^2)^3}\,dx = \frac{\left(\frac{3a^2x^3+5x}{a^4c^2x^4+2a^2c^2x^2+c^2} + \frac{3\arctan(ax)}{ac^2}\right)\arctan(ax)}{16\,ac} + \frac{3a^2x^2 - 3(a^4x^4+2a^2x^2+1)\arctan(ax)^2 + 4}{32(a^6c^2x^4+2a^4c^2x^2+a^2c^2)c} - \frac{\arctan(ax)^2}{4(a^2cx^2+c)^2a^2c}$$

input
```
integrate(x*arctan(a*x)^2/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
1/16*((3*a^2*x^3 + 5*x)/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2) + 3*arctan(a*x
)/(a*c^2))*arctan(a*x)/(a*c) + 1/32*(3*a^2*x^2 - 3*(a^4*x^4 + 2*a^2*x^2 +
1)*arctan(a*x)^2 + 4)/((a^6*c^2*x^4 + 2*a^4*c^2*x^2 + a^2*c^2)*c) - 1/4*ar
ctan(a*x)^2/((a^2*c*x^2 + c)^2*a^2*c)
```

## Giac [F]

$$\int \frac{x\arctan(ax)^2}{(c+a^2cx^2)^3}\,dx = \int \frac{x\arctan(ax)^2}{(a^2cx^2+c)^3}\,dx$$

input
```
integrate(x*arctan(a*x)^2/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
integrate(x*arctan(a*x)^2/(a^2*c*x^2 + c)^3, x)
```

## Mupad [B] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.62

$$\int \frac{x\arctan(ax)^2}{(c+a^2cx^2)^3}\,dx = \frac{3\,a^4\,x^4\,\mathrm{atan}(a\,x)^2+6\,a^3\,x^3\,\mathrm{atan}(a\,x)+6\,a^2\,x^2\,\mathrm{atan}(a\,x)^2+3\,a^2\,x^2+10\,a\,x\,\mathrm{atan}(a\,x)-5\,\mathrm{atan}(a\,x)^2+4}{32\,a^2\,c^3\,(a^2\,x^2+1)^2}$$

input
```
int((x*atan(a*x)^2)/(c + a^2*c*x^2)^3,x)
```

output
```
(3*a^2*x^2 - 5*atan(a*x)^2 + 6*a^3*x^3*atan(a*x) + 10*a*x*atan(a*x) + 6*a^
2*x^2*atan(a*x)^2 + 3*a^4*x^4*atan(a*x)^2 + 4)/(32*a^2*c^3*(a^2*x^2 + 1)^2
)
```

## Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.67

$$\int \frac{x\arctan(ax)^2}{(c+a^2cx^2)^3}\,dx = \frac{6atan(ax)^2\,a^4x^4+12atan(ax)^2\,a^2x^2-10atan(ax)^2+12atan(ax)\,a^3x^3+20atan(ax)\,ax-3a^4x^4+5}{64a^2c^3\,(a^4x^4+2a^2x^2+1)}$$

input
```
int(x*atan(a*x)^2/(a^2*c*x^2+c)^3,x)
```

output
```
(6*atan(a*x)**2*a**4*x**4 + 12*atan(a*x)**2*a**2*x**2 - 10*atan(a*x)**2 +
12*atan(a*x)*a**3*x**3 + 20*atan(a*x)*a*x - 3*a**4*x**4 + 5)/(64*a**2*c**3
*(a**4*x**4 + 2*a**2*x**2 + 1))
```

# 3.302 $\int \frac{\arctan(ax)^2}{(c+a^2cx^2)^3} \, dx$



## Optimal result

Integrand size = 19, antiderivative size = 169

$$\int \frac{\arctan(ax)^2}{(c+a^2cx^2)^3} \, dx = -\frac{x}{32c^3(1+a^2x^2)^2} - \frac{15x}{64c^3(1+a^2x^2)} - \frac{15\arctan(ax)}{64ac^3} + \frac{\arctan(ax)}{8ac^3(1+a^2x^2)^2} + \frac{3\arctan(ax)}{8ac^3(1+a^2x^2)} + \frac{x\arctan(ax)^2}{4c^3(1+a^2x^2)^2} + \frac{3x\arctan(ax)^2}{8c^3(1+a^2x^2)} + \frac{\arctan(ax)^3}{8ac^3}$$

output
```
-1/32*x/c^3/(a^2*x^2+1)^2-15/64*x/c^3/(a^2*x^2+1)-15/64*arctan(a*x)/a/c^3+
1/8*arctan(a*x)/a/c^3/(a^2*x^2+1)^2+3/8*arctan(a*x)/a/c^3/(a^2*x^2+1)+1/4*
x*arctan(a*x)^2/c^3/(a^2*x^2+1)^2+3/8*x*arctan(a*x)^2/c^3/(a^2*x^2+1)+1/8*
arctan(a*x)^3/a/c^3
```

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.58

$$\int \frac{\arctan(ax)^2}{(c+a^2cx^2)^3}\,dx$$
$$= \frac{-ax(17+15a^2x^2)+(17-6a^2x^2-15a^4x^4)\arctan(ax)+8ax(5+3a^2x^2)\arctan(ax)^2+8(1+a^2x^2)^2 \text{ar}}{64ac^3(1+a^2x^2)^2}$$

input
```
Integrate[ArcTan[a*x]^2/(c + a^2*c*x^2)^3,x]
```

output
```
(-(a*x*(17 + 15*a^2*x^2)) + (17 - 6*a^2*x^2 - 15*a^4*x^4)*ArcTan[a*x] + 8*
a*x*(5 + 3*a^2*x^2)*ArcTan[a*x]^2 + 8*(1 + a^2*x^2)^2*ArcTan[a*x]^3)/(64*a
*c^3*(1 + a^2*x^2)^2)
```

## Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.30, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {5435, 27, 215, 215, 218, 5427, 5465, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2}{(a^2cx^2+c)^3}\,dx$$
$$\Big\downarrow \; 5435$$
$$\frac{3\int \frac{\arctan(ax)^2}{c^2(a^2x^2+1)^2}dx}{4c}-\frac{1}{8}\int \frac{1}{(a^2cx^2+c)^3}dx+\frac{x\arctan(ax)^2}{4c^3(a^2x^2+1)^2}+\frac{\arctan(ax)}{8ac^3(a^2x^2+1)^2}$$
$$\Big\downarrow \; 27$$
$$\frac{3\int \frac{\arctan(ax)^2}{(a^2x^2+1)^2}dx}{4c^3}-\frac{1}{8}\int \frac{1}{(a^2cx^2+c)^3}dx+\frac{x\arctan(ax)^2}{4c^3(a^2x^2+1)^2}+\frac{\arctan(ax)}{8ac^3(a^2x^2+1)^2}$$
$$\Big\downarrow \; 215$$

$$\frac{3\int \frac{\arctan(ax)^2}{(a^2x^2+1)^2}dx}{4c^3}+\frac{1}{8}\left(-\frac{3\int \frac{1}{(a^2cx^2+c)^2}dx}{4c}-\frac{x}{4c^3\,(a^2x^2+1)^2}\right)+\frac{x\arctan(ax)^2}{4c^3\,(a^2x^2+1)^2}+\frac{\arctan(ax)}{8ac^3\,(a^2x^2+1)^2}$$

$$\Big\downarrow \text{215}$$

$$\frac{3\int \frac{\arctan(ax)^2}{(a^2x^2+1)^2}dx}{4c^3}+\frac{1}{8}\left(-\frac{3\left(\frac{\int \frac{1}{a^2cx^2+c}dx}{2c}+\frac{x}{2c^2(a^2x^2+1)}\right)}{4c}-\frac{x}{4c^3\,(a^2x^2+1)^2}\right)+\frac{x\arctan(ax)^2}{4c^3\,(a^2x^2+1)^2}+\frac{\arctan(ax)}{8ac^3\,(a^2x^2+1)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{3\int \frac{\arctan(ax)^2}{(a^2x^2+1)^2}dx}{4c^3}+\frac{x\arctan(ax)^2}{4c^3\,(a^2x^2+1)^2}+\frac{\arctan(ax)}{8ac^3\,(a^2x^2+1)^2}+\frac{1}{8}\left(-\frac{3\left(\frac{x}{2c^2(a^2x^2+1)}+\frac{\arctan(ax)}{2ac^2}\right)}{4c}-\frac{x}{4c^3\,(a^2x^2+1)^2}\right)$$

$$\Big\downarrow \text{5427}$$

$$\frac{3\left(-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)}{4c^3}+\frac{x\arctan(ax)^2}{4c^3\,(a^2x^2+1)^2}+\frac{\arctan(ax)}{8ac^3\,(a^2x^2+1)^2}+\frac{1}{8}\left(-\frac{3\left(\frac{x}{2c^2(a^2x^2+1)}+\frac{\arctan(ax)}{2ac^2}\right)}{4c}-\frac{x}{4c^3\,(a^2x^2+1)^2}\right)$$

$$\Big\downarrow \text{5465}$$

$$\frac{3\left(-a\left(\frac{\int \frac{1}{(a^2x^2+1)^2}dx}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)}{4c^3}+\frac{x\arctan(ax)^2}{4c^3\,(a^2x^2+1)^2}+\frac{\arctan(ax)}{8ac^3\,(a^2x^2+1)^2}+\frac{1}{8}\left(-\frac{3\left(\frac{x}{2c^2(a^2x^2+1)}+\frac{\arctan(ax)}{2ac^2}\right)}{4c}-\frac{x}{4c^3\,(a^2x^2+1)^2}\right)$$

$$\Big\downarrow \text{215}$$

$$\frac{3\left(-a\left(\frac{\frac{1}{2}\int\frac{1}{a^2x^2+1}dx+\frac{x}{2\left(a^2x^2+1\right)}}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)}{4c^3}+\frac{x\arctan(ax)^2}{4c^3\left(a^2x^2+1\right)^2}+\frac{\arctan(ax)}{8ac^3\left(a^2x^2+1\right)^2}+\frac{1}{8}\left(-\frac{3\left(\frac{x}{2c^2(a^2x^2+1)}+\frac{\arctan(ax)}{2ac^2}\right)}{4c}-\frac{x}{4c^3\left(a^2x^2+1\right)^2}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{x\arctan(ax)^2}{4c^3\left(a^2x^2+1\right)^2}+\frac{\arctan(ax)}{8ac^3\left(a^2x^2+1\right)^2}+\frac{3\left(\frac{x\arctan(ax)^2}{2(a^2x^2+1)}-a\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)+\frac{\arctan(ax)^3}{6a}\right)}{4c^3}+\frac{1}{8}\left(-\frac{3\left(\frac{x}{2c^2(a^2x^2+1)}+\frac{\arctan(ax)}{2ac^2}\right)}{4c}-\frac{x}{4c^3\left(a^2x^2+1\right)^2}\right)$$

input
```
Int[ArcTan[a*x]^2/(c + a^2*c*x^2)^3,x]
```

output
```
ArcTan[a*x]/(8*a*c^3*(1 + a^2*x^2)^2) + (x*ArcTan[a*x]^2)/(4*c^3*(1 + a^2*
x^2)^2) + (-1/4*x/(c^3*(1 + a^2*x^2)^2) - (3*(x/(2*c^2*(1 + a^2*x^2)) + Ar
cTan[a*x]/(2*a*c^2)))/(4*c))/8 + (3*((x*ArcTan[a*x]^2)/(2*(1 + a^2*x^2)) +
 ArcTan[a*x]^3/(6*a) - a*(-1/2*ArcTan[a*x]/(a^2*(1 + a^2*x^2)) + (x/(2*(1
+ a^2*x^2)) + ArcTan[a*x]/(2*a))/(2*a))))/(4*c^3)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5435
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[b*p*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p - 1)/(4*c*d*
(q + 1)^2)), x] + (-Simp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*d*
(q + 1))), x] + Simp[(2*q + 3)/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p, x], x] - Simp[b^2*p*((p - 1)/(4*(q + 1)^2))   Int[(d + e
*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

## Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.73

| method | result |
|---|---|
| parallelrisch | $\frac{8\arctan(ax)^3a^4x^4-15x^4\arctan(ax)a^4+24a^3\arctan(ax)^2x^3+16\arctan(ax)^3a^2x^2-15a^3x^3-6x^2a^2\arctan(ax)+40a\,a}{64c^3(a^2x^2+1)^2a}$ |
| derivativedivides | $\frac{\frac{ax\arctan(ax)^2}{4c^3(a^2x^2+1)^2}+\frac{3ax\arctan(ax)^2}{8c^3(a^2x^2+1)}+\frac{3\arctan(ax)^3}{8c^3}-\frac{-\frac{3\arctan(ax)}{2(a^2x^2+1)}-\frac{\arctan(ax)}{2(a^2x^2+1)^2}+\frac{\frac{15}{8}a^3x^3+\frac{17}{8}ax}{2(a^2x^2+1)^2}+\frac{15\arctan(ax)}{16}+\arctan(ax)^3}{4c^3}}{a}$ |
| default | $\frac{\frac{ax\arctan(ax)^2}{4c^3(a^2x^2+1)^2}+\frac{3ax\arctan(ax)^2}{8c^3(a^2x^2+1)}+\frac{3\arctan(ax)^3}{8c^3}-\frac{-\frac{3\arctan(ax)}{2(a^2x^2+1)}-\frac{\arctan(ax)}{2(a^2x^2+1)^2}+\frac{\frac{15}{8}a^3x^3+\frac{17}{8}ax}{2(a^2x^2+1)^2}+\frac{15\arctan(ax)}{16}+\arctan(ax)^3}{4c^3}}{a}$ |
| parts | $\frac{x\arctan(ax)^2}{4c^3(a^2x^2+1)^2}+\frac{3x\arctan(ax)^2}{8c^3(a^2x^2+1)}+\frac{3\arctan(ax)^3}{8a\,c^3}-\frac{\frac{-\frac{3\arctan(ax)}{2(a^2x^2+1)}-\frac{\arctan(ax)}{2(a^2x^2+1)^2}+\frac{\frac{15}{8}a^3x^3+\frac{17}{8}ax}{2(a^2x^2+1)^2}+\frac{15\arctan(ax)}{16}}{a}+}{4c^3}$ |
| risch | $\frac{i\ln(iax+1)^3}{64a\,c^3}-\frac{i(3x^4\ln(-iax+1)a^4+6a^2x^2\ln(-iax+1)-6ia^3x^3+3\ln(-iax+1)-10iax)\ln(iax+1)^2}{64c^3(a^2x^2+1)^2a}+\frac{i\left(3a^4x^4\ln\right.}{}$ |

input
```
int(arctan(a*x)^2/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/64*(8*arctan(a*x)^3*a^4*x^4-15*x^4*arctan(a*x)*a^4+24*a^3*arctan(a*x)^2*
x^3+16*arctan(a*x)^3*a^2*x^2-15*a^3*x^3-6*x^2*a^2*arctan(a*x)+40*a*arctan(
a*x)^2*x+8*arctan(a*x)^3-17*a*x+17*arctan(a*x))/c^3/(a^2*x^2+1)^2/a
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.67

$$\int \frac{\arctan(ax)^2}{(c+a^2cx^2)^3}\,dx =$$
$$-\frac{15\,a^3x^3-8\,(a^4x^4+2\,a^2x^2+1)\arctan{(ax)}^3-8\,(3\,a^3x^3+5\,ax)\arctan{(ax)}^2+17\,ax+(15\,a^4x^4+6\,a}{64\,(a^5c^3x^4+2\,a^3c^3x^2+ac^3)}$$

input
```
integrate(arctan(a*x)^2/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
-1/64*(15*a^3*x^3 - 8*(a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x)^3 - 8*(3*a^3*x
^3 + 5*a*x)*arctan(a*x)^2 + 17*a*x + (15*a^4*x^4 + 6*a^2*x^2 - 17)*arctan(
a*x))/(a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)
```

## Sympy [F]

$$\int \frac{\arctan(ax)^2}{(c+a^2cx^2)^3}\,dx = \frac{\int \frac{\operatorname{atan}^2(ax)}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

input
```
integrate(atan(a*x)**2/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(atan(a*x)**2/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x)/c**3
```

## Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.37

$$\begin{aligned}\int \frac{\arctan(ax)^2}{(c+a^2cx^2)^3}\,dx =& \frac{1}{8}\left(\frac{3a^2x^3+5x}{a^4c^3x^4+2a^2c^3x^2+c^3}+\frac{3\arctan(ax)}{ac^3}\right)\arctan(ax)^2\\&-\frac{(15a^3x^3-8(a^4x^4+2a^2x^2+1)\arctan(ax)^3+17ax+15(a^4x^4+2a^2x^2+1)\arctan(ax))a^2}{64(a^7c^3x^4+2a^5c^3x^2+a^3c^3)}\\&+\frac{(3a^2x^2-3(a^4x^4+2a^2x^2+1)\arctan(ax)^2+4)a\arctan(ax)}{8(a^6c^3x^4+2a^4c^3x^2+a^2c^3)}\end{aligned}$$

input
```
integrate(arctan(a*x)^2/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
1/8*((3*a^2*x^3 + 5*x)/(a^4*c^3*x^4 + 2*a^2*c^3*x^2 + c^3) + 3*arctan(a*x)
/(a*c^3))*arctan(a*x)^2 - 1/64*(15*a^3*x^3 - 8*(a^4*x^4 + 2*a^2*x^2 + 1)*a
rctan(a*x)^3 + 17*a*x + 15*(a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x))*a^2/(a^7
*c^3*x^4 + 2*a^5*c^3*x^2 + a^3*c^3) + 1/8*(3*a^2*x^2 - 3*(a^4*x^4 + 2*a^2*
x^2 + 1)*arctan(a*x)^2 + 4)*a*arctan(a*x)/(a^6*c^3*x^4 + 2*a^4*c^3*x^2 + a
^2*c^3)
```

## Giac [F]

$$\int \frac{\arctan(ax)^2}{(c+a^2cx^2)^3}\,dx = \int \frac{\arctan(ax)^2}{(a^2cx^2+c)^3}\,dx$$

input
```
integrate(arctan(a*x)^2/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^2/(a^2*c*x^2 + c)^3, x)
```

## Mupad [B] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.93

$$\int \frac{\arctan(ax)^2}{(c+a^2cx^2)^3}\,dx = \frac{\mathrm{atan}(a\,x)\,\left(\frac{1}{2\,a^3\,c^3}+\frac{3\,x^2}{8\,a\,c^3}\right)}{\frac{1}{a^2}+2\,x^2+a^2\,x^4}-\frac{15\,\mathrm{atan}(a\,x)}{64\,a\,c^3}-\frac{\frac{15\,a^2\,x^3}{8}+\frac{17\,x}{8}}{8\,a^4\,c^3\,x^4+16\,a^2\,c^3\,x^2+8\,c^3}+\frac{\mathrm{atan}(a\,x)^2\,\left(\frac{3\,x^3}{8\,c^3}+\frac{5\,x}{8\,a^2\,c^3}\right)}{\frac{1}{a^2}+2\,x^2+a^2\,x^4}+\frac{\mathrm{atan}(a\,x)^3}{8\,a\,c^3}$$

input
```
int(atan(a*x)^2/(c + a^2*c*x^2)^3,x)
```

output
```
(atan(a*x)*(1/(2*a^3*c^3) + (3*x^2)/(8*a*c^3)))/(1/a^2 + 2*x^2 + a^2*x^4)
- (15*atan(a*x))/(64*a*c^3) - ((17*x)/8 + (15*a^2*x^3)/8)/(8*c^3 + 16*a^2*
c^3*x^2 + 8*a^4*c^3*x^4) + (atan(a*x)^2*((3*x^3)/(8*c^3) + (5*x)/(8*a^2*c^
3)))/(1/a^2 + 2*x^2 + a^2*x^4) + atan(a*x)^3/(8*a*c^3)
```

## Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.77

$$\int \frac{\arctan(ax)^2}{(c+a^2cx^2)^3}\,dx$$
$$= \frac{8atan(ax)^3 a^4x^4 + 16atan(ax)^3 a^2x^2 + 8atan(ax)^3 + 24atan(ax)^2 a^3x^3 + 40atan(ax)^2 ax - 15atan(ax)}{64a\,c^3\left(a^4x^4 + 2a^2x^2 + 1\right)}$$

input
```
int(atan(a*x)^2/(a^2*c*x^2+c)^3,x)
```

output
```
(8*atan(a*x)**3*a**4*x**4 + 16*atan(a*x)**3*a**2*x**2 + 8*atan(a*x)**3 + 2
4*atan(a*x)**2*a**3*x**3 + 40*atan(a*x)**2*a*x - 15*atan(a*x)*a**4*x**4 -
6*atan(a*x)*a**2*x**2 + 17*atan(a*x) - 15*a**3*x**3 - 17*a*x)/(64*a*c**3*(
a**4*x**4 + 2*a**2*x**2 + 1))
```

# 3.303 $\int \frac{\arctan(ax)^2}{x(c+a^2cx^2)^3}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 236

$$\begin{aligned}\int \frac{\arctan(ax)^2}{x(c+a^2cx^2)^3}\,dx = &-\frac{1}{32c^3(1+a^2x^2)^2}-\frac{11}{32c^3(1+a^2x^2)}-\frac{ax\arctan(ax)}{8c^3(1+a^2x^2)^2}\\&-\frac{11ax\arctan(ax)}{16c^3(1+a^2x^2)}-\frac{11\arctan(ax)^2}{32c^3}+\frac{\arctan(ax)^2}{4c^3(1+a^2x^2)^2}\\&+\frac{\arctan(ax)^2}{2c^3(1+a^2x^2)}-\frac{i\arctan(ax)^3}{3c^3}+\frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{c^3}\\&-\frac{i\arctan(ax)\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{c^3}\\&+\frac{\operatorname{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)}{2c^3}\end{aligned}$$

output
```
-1/32/c^3/(a^2*x^2+1)^2-11/32/c^3/(a^2*x^2+1)-1/8*a*x*arctan(a*x)/c^3/(a^2
*x^2+1)^2-11/16*a*x*arctan(a*x)/c^3/(a^2*x^2+1)-11/32*arctan(a*x)^2/c^3+1/
4*arctan(a*x)^2/c^3/(a^2*x^2+1)^2+1/2*arctan(a*x)^2/c^3/(a^2*x^2+1)-1/3*I*
arctan(a*x)^3/c^3+arctan(a*x)^2*ln(2-2/(1-I*a*x))/c^3-I*arctan(a*x)*polylo
g(2,-1+2/(1-I*a*x))/c^3+1/2*polylog(3,-1+2/(1-I*a*x))/c^3
```

## Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.66

$$\int \frac{\arctan(ax)^2}{x\left(c+a^2cx^2\right)^3}\,dx = \frac{-32i\pi^3+256i\arctan(ax)^3-144\cos(2\arctan(ax))+288\arctan(ax)^2\cos(2\arctan(ax))-3\cos(4\arcta}{}$$

input
```
Integrate[ArcTan[a*x]^2/(x*(c + a^2*c*x^2)^3),x]
```

output
```
((-32*I)*Pi^3 + (256*I)*ArcTan[a*x]^3 - 144*Cos[2*ArcTan[a*x]] + 288*ArcTa
n[a*x]^2*Cos[2*ArcTan[a*x]] - 3*Cos[4*ArcTan[a*x]] + 24*ArcTan[a*x]^2*Cos[
4*ArcTan[a*x]] + 768*ArcTan[a*x]^2*Log[1 - E^((-2*I)*ArcTan[a*x])] + (768*
I)*ArcTan[a*x]*PolyLog[2, E^((-2*I)*ArcTan[a*x])] + 384*PolyLog[3, E^((-2*
I)*ArcTan[a*x])] - 288*ArcTan[a*x]*Sin[2*ArcTan[a*x]] - 12*ArcTan[a*x]*Sin
[4*ArcTan[a*x]])/(768*c^3)
```

## Rubi [A] (verified)

Time = 2.12 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.39, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {5501, 27, 5465, 5431, 5427, 241, 5501, 5459, 5403, 5465, 5427, 241, 5527, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2}{x\left(a^2cx^2+c\right)^3}\,dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{\arctan(ax)^2}{c^2x\left(a^2x^2+1\right)^2}dx}{c}-a^2\int \frac{x\arctan(ax)^2}{c^3\left(a^2x^2+1\right)^3}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\arctan(ax)^2}{x\left(a^2x^2+1\right)^2}dx}{c^3}-\frac{a^2\int \frac{x\arctan(ax)^2}{\left(a^2x^2+1\right)^3}dx}{c^3}$$

$$\Big\downarrow \text{5465}$$

$$\frac{\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx}{c^3} - \frac{a^2\left(\frac{\int \frac{\arctan(ax)}{(a^2x^2+1)^3}dx}{2a} - \frac{\arctan(ax)^2}{4a^2(a^2x^2+1)^2}\right)}{c^3}$$

$$\Big\downarrow \text{5431}$$

$$\frac{\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx}{c^3} - \frac{a^2\left(\frac{\frac{3}{4}\int \frac{\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{1}{16a(a^2x^2+1)^2}}{2a} - \frac{\arctan(ax)^2}{4a^2(a^2x^2+1)^2}\right)}{c^3}$$

$$\Big\downarrow \text{5427}$$

$$\frac{\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx}{c^3} - \frac{a^2\left(\frac{\frac{3}{4}\left(-\frac{1}{2}a\int \frac{x}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{1}{16a(a^2x^2+1)^2}}{2a} - \frac{\arctan(ax)^2}{4a^2(a^2x^2+1)^2}\right)}{c^3}$$

$$\Big\downarrow \text{241}$$

$$\frac{\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx}{c^3} - \frac{a^2\left(\frac{\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}}{2a} - \frac{\arctan(ax)^2}{4a^2(a^2x^2+1)^2}\right)}{c^3}$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx - a^2\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx}{c^3} - \frac{a^2\left(\frac{\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}}{2a} - \frac{\arctan(ax)^2}{4a^2(a^2x^2+1)^2}\right)}{c^3}$$

$$\Big\downarrow \text{5459}$$

$$-\frac{a^2\left(\frac{\frac{x\arctan(ax)}{4\left(a^2x^2+1\right)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2\left(a^2x^2+1\right)}+\frac{1}{4a\left(a^2x^2+1\right)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a\left(a^2x^2+1\right)^2}}{2a}-\frac{\arctan(ax)^2}{4a^2(a^2x^2+1)^2}\right)}{c^3}+\frac{-a^2\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+i\int \frac{\arctan(ax)^2}{x(ax+i)}dx-\frac{1}{3}i\arctan(ax)^3}{c^3}$$

$$\Big\downarrow \; 5403$$

$$-\frac{a^2\left(\frac{\frac{x\arctan(ax)}{4\left(a^2x^2+1\right)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2\left(a^2x^2+1\right)}+\frac{1}{4a\left(a^2x^2+1\right)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a\left(a^2x^2+1\right)^2}}{2a}-\frac{\arctan(ax)^2}{4a^2(a^2x^2+1)^2}\right)}{c^3}+\frac{-a^2\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3}{c^3}$$

$$\Big\downarrow \; 5465$$

$$-\frac{a^2\left(\frac{\frac{x\arctan(ax)}{4\left(a^2x^2+1\right)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2\left(a^2x^2+1\right)}+\frac{1}{4a\left(a^2x^2+1\right)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a\left(a^2x^2+1\right)^2}}{2a}-\frac{\arctan(ax)^2}{4a^2(a^2x^2+1)^2}\right)}{c^3}+\frac{-a^2\left(\frac{\int\frac{\arctan(ax)}{\left(a^2x^2+1\right)^2}dx}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)+i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\,\mathrm{arc}}{c^3}$$

$$\Big\downarrow \; 5427$$

$$-\frac{a^2\left(\frac{\frac{x\arctan(ax)}{4\left(a^2x^2+1\right)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2\left(a^2x^2+1\right)}+\frac{1}{4a\left(a^2x^2+1\right)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a\left(a^2x^2+1\right)^2}}{2a}-\frac{\arctan(ax)^2}{4a^2(a^2x^2+1)^2}\right)}{c^3}+\frac{-a^2\left(\frac{-\frac{1}{2}a\int\frac{x}{\left(a^2x^2+1\right)^2}dx+\frac{x\arctan(ax)}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^2}{4a}}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)+i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\,\mathrm{lo}\right.}{c^3}$$

$$\Big\downarrow \; 241$$

$$-\frac{a^2\left(\frac{\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}}{2a}-\frac{\arctan(ax)^2}{4a^2(a^2x^2+1)^2}\right)}{c^3}+$$

$$\frac{i\left(2ia\int\frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-a^2\left(\frac{\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}}{a}-\frac{\arctan}{2a^2(a}\right.}{c^3}$$

$$\Big\downarrow \text{5527}$$

$$-\frac{a^2\left(\frac{\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}}{2a}-\frac{\arctan(ax)^2}{4a^2(a^2x^2+1)^2}\right)}{c^3}+$$

$$\frac{i\left(2ia\left(\frac{i\arctan(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-\frac{1}{2}i\int\frac{\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{a^2x^2+1}dx\right)-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-a^2\left(\frac{\frac{x\arctan}{2(a^2x}}{}\right.}{c^3}$$

$$\Big\downarrow \text{7164}$$

$$-\frac{a^2\left(\frac{\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}}{2a}-\frac{\arctan(ax)^2}{4a^2(a^2x^2+1)^2}\right)}{c^3}+$$

$$\frac{-a^2\left(\frac{\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)+i\left(2ia\left(\frac{i\arctan(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-\frac{\operatorname{PolyLog}\left(3,\frac{2}{1-iax}-1\right)}{4a}\right.\right.}{c^3}$$

input
```
Int[ArcTan[a*x]^2/(x*(c + a^2*c*x^2)^3),x]
```

output
```
-((a^2*(-1/4*ArcTan[a*x]^2/(a^2*(1 + a^2*x^2)^2) + (1/(16*a*(1 + a^2*x^2)^
2) + (x*ArcTan[a*x])/(4*(1 + a^2*x^2)^2) + (3*(1/(4*a*(1 + a^2*x^2)) + (x*
ArcTan[a*x])/(2*(1 + a^2*x^2)) + ArcTan[a*x]^2/(4*a)))/4)/(2*a)))/c^3) + (
(-1/3*I)*ArcTan[a*x]^3 - a^2*(-1/2*ArcTan[a*x]^2/(a^2*(1 + a^2*x^2)) + (1/
(4*a*(1 + a^2*x^2)) + (x*ArcTan[a*x])/(2*(1 + a^2*x^2)) + ArcTan[a*x]^2/(4
*a))/a) + I*((-I)*ArcTan[a*x]^2*Log[2 - 2/(1 - I*a*x)] + (2*I)*a*(((I/2)*A
rcTan[a*x]*PolyLog[2, -1 + 2/(1 - I*a*x)])/a - PolyLog[3, -1 + 2/(1 - I*a*
x)]/(4*a))))/c^3
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5431
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbol
] :> Simp[b*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (-Simp[x*(d + e*x
^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*d*(q + 1))), x] + Simp[(2*q + 3)/(2*d*(
q + 1))   Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a,
 b, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -3/2]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

rule 5527
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 66.50 (sec) , antiderivative size = 1722, normalized size of antiderivative = 7.30

| method | result | size |
|---|---|---|
| derivativedivides | Expression too large to display | 1722 |
| default | Expression too large to display | 1722 |
| parts | Expression too large to display | 2148 |

input
```
int(arctan(a*x)^2/x/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*arctan(a*x)^2/c^3/(a^2*x^2+1)+1/4*arctan(a*x)^2/c^3/(a^2*x^2+1)^2-1/2/
c^3*arctan(a*x)^2*ln(a^2*x^2+1)+1/c^3*arctan(a*x)^2*ln(a*x)-1/2/c^3*(-2*ar
ctan(a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2))+4*I*arctan(a*x)*polylog(2,-(1+
I*a*x)/(a^2*x^2+1)^(1/2))+2/3*I*arctan(a*x)^3-3/16*(a*x+I)/(a*x-I)+3*I*arc
tan(a*x)*(a*x-I)/(8*a*x+8*I)-3/16*(a*x-I)/(a*x+I)+2*arctan(a*x)^2*ln((1+I*
a*x)^2/(a^2*x^2+1)-1)-2*arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*
I*arctan(a*x)*(a*x+I)/(8*a*x-8*I)-4*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))
-2*arctan(a*x)^2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+4*I*arctan(a*x)*polylog
(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-4*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-
1/16*(16*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+
1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))+16*I*Pi*
csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^3+16*I*Pi*cs
gn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn
(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))+8*I*Pi*csgn(I*
((1+I*a*x)^2/(a^2*x^2+1)+1))^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)+8*I*P
i*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x
)^2/(a^2*x^2+1)+1)^2)^2+16*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a
*x)^2/(a^2*x^2+1)+1))^3-8*I*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))^2*csgn(
I*(1+I*a*x)^2/(a^2*x^2+1))-16*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+
I*a*x)^2/(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/...
```

## Fricas [F]

$$\int \frac{\arctan(ax)^2}{x\left(c+a^2cx^2\right)^3}\,dx=\int \frac{\arctan\left(ax\right)^2}{\left(a^2cx^2+c\right)^3x}\,dx$$

input
```
integrate(arctan(a*x)^2/x/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
integral(arctan(a*x)^2/(a^6*c^3*x^7 + 3*a^4*c^3*x^5 + 3*a^2*c^3*x^3 + c^3*
x), x)
```

## Sympy [F]

$$\int \frac{\arctan(ax)^2}{x\left(c+a^2cx^2\right)^3}\,dx = \frac{\int \frac{\operatorname{atan}^2(ax)}{a^6x^7+3a^4x^5+3a^2x^3+x}\,dx}{c^3}$$

input
```
integrate(atan(a*x)**2/x/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(atan(a*x)**2/(a**6*x**7 + 3*a**4*x**5 + 3*a**2*x**3 + x), x)/c**3
```

## Maxima [F]

$$\int \frac{\arctan(ax)^2}{x\left(c+a^2cx^2\right)^3}\,dx = \int \frac{\arctan(ax)^2}{\left(a^2cx^2+c\right)^3x}\,dx$$

input
```
integrate(arctan(a*x)^2/x/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)^2/((a^2*c*x^2 + c)^3*x), x)
```

## Giac [F]

$$\int \frac{\arctan(ax)^2}{x\left(c+a^2cx^2\right)^3}\,dx = \int \frac{\arctan(ax)^2}{\left(a^2cx^2+c\right)^3x}\,dx$$

input
```
integrate(arctan(a*x)^2/x/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^2/((a^2*c*x^2 + c)^3*x), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^2}{x\left(c+a^2cx^2\right)^3}\,dx = \int \frac{\mathrm{atan}(a\,x)^2}{x\,\left(c\,a^2\,x^2+c\right)^3}\,dx$$

input
```
int(atan(a*x)^2/(x*(c + a^2*c*x^2)^3),x)
```

output
```
int(atan(a*x)^2/(x*(c + a^2*c*x^2)^3), x)
```

## Reduce [F]

$$\int \frac{\arctan(ax)^2}{x\left(c+a^2cx^2\right)^3}\,dx = \frac{\int \frac{atan(ax)^2}{a^6x^7+3a^4x^5+3a^2x^3+x}dx}{c^3}$$

input
```
int(atan(a*x)^2/x/(a^2*c*x^2+c)^3,x)
```

output
```
int(atan(a*x)**2/(a**6*x**7 + 3*a**4*x**5 + 3*a**2*x**3 + x),x)/c**3
```

# 3.304 $\int \frac{\arctan(ax)^2}{x^2(c+a^2cx^2)^3}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 250

$$\int \frac{\arctan(ax)^2}{x^2(c+a^2cx^2)^3}\,dx = \frac{a^2x}{32c^3(1+a^2x^2)^2}+\frac{31a^2x}{64c^3(1+a^2x^2)}+\frac{31a\arctan(ax)}{64c^3} - \frac{a\arctan(ax)}{8c^3(1+a^2x^2)^2}-\frac{7a\arctan(ax)}{8c^3(1+a^2x^2)}-\frac{ia\arctan(ax)^2}{c^3} - \frac{\arctan(ax)^2}{c^3x}-\frac{a^2x\arctan(ax)^2}{4c^3(1+a^2x^2)^2} - \frac{7a^2x\arctan(ax)^2}{8c^3(1+a^2x^2)}-\frac{5a\arctan(ax)^3}{8c^3} + \frac{2a\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{c^3}-\frac{ia\,\text{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{c^3}$$

output
```
1/32*a^2*x/c^3/(a^2*x^2+1)^2+31/64*a^2*x/c^3/(a^2*x^2+1)+31/64*a*arctan(a*
x)/c^3-1/8*a*arctan(a*x)/c^3/(a^2*x^2+1)^2-7/8*a*arctan(a*x)/c^3/(a^2*x^2+
1)-I*a*arctan(a*x)^2/c^3-arctan(a*x)^2/c^3/x-1/4*a^2*x*arctan(a*x)^2/c^3/(
a^2*x^2+1)^2-7/8*a^2*x*arctan(a*x)^2/c^3/(a^2*x^2+1)-5/8*a*arctan(a*x)^3/c
^3+2*a*arctan(a*x)*ln(2-2/(1-I*a*x))/c^3-I*a*polylog(2,-1+2/(1-I*a*x))/c^3
```

## Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.56

$$\int \frac{\arctan(ax)^2}{x^2(c+a^2cx^2)^3}\,dx = -\frac{160ax\arctan(ax)^3 + 4ax\arctan(ax)\left(32\cos(2\arctan(ax)) + \cos(4\arctan(ax)) - 128\log\left(1-e^{2i\arctan(ax)}\right.\right.}{}$$

input
```
Integrate[ArcTan[a*x]^2/(x^2*(c + a^2*c*x^2)^3),x]
```

output
```
-1/256*(160*a*x*ArcTan[a*x]^3 + 4*a*x*ArcTan[a*x]*(32*Cos[2*ArcTan[a*x]] +
 Cos[4*ArcTan[a*x]] - 128*Log[1 - E^((2*I)*ArcTan[a*x])]) + (256*I)*a*x*Po
lyLog[2, E^((2*I)*ArcTan[a*x])] - a*x*(64*Sin[2*ArcTan[a*x]] + Sin[4*ArcTa
n[a*x]]) + 8*ArcTan[a*x]^2*(32 + (32*I)*a*x + 16*a*x*Sin[2*ArcTan[a*x]] +
a*x*Sin[4*ArcTan[a*x]]))/(c^3*x)
```

## Rubi [A] (verified)

Time = 2.74 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.60, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.955$, Rules used = {5501, 27, 5435, 215, 215, 216, 5427, 5465, 215, 216, 5501, 5427, 5453, 5361, 5419, 5459, 5403, 2897, 5465, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2}{x^2(a^2cx^2+c)^3}\,dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{\arctan(ax)^2}{c^2x^2(a^2x^2+1)^2}dx}{c} - a^2\int \frac{\arctan(ax)^2}{c^3(a^2x^2+1)^3}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx}{c^3} - \frac{a^2\int \frac{\arctan(ax)^2}{(a^2x^2+1)^3}dx}{c^3}$$

$$\Big\downarrow \text{5435}$$

$$\frac{\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx}{c^3}-\frac{a^2\left(\frac{3}{4}\int \frac{\arctan(ax)^2}{(a^2x^2+1)^2}dx-\frac{1}{8}\int \frac{1}{(a^2x^2+1)^3}dx+\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}\right)}{c^3}$$

$$\Big\downarrow \text{215}$$

$$\frac{\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx}{c^3}-\frac{a^2\left(\frac{3}{4}\int \frac{\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{1}{8}\left(-\frac{3}{4}\int \frac{1}{(a^2x^2+1)^2}dx-\frac{x}{4(a^2x^2+1)^2}\right)+\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}\right)}{c^3}$$

$$\Big\downarrow \text{215}$$

$$\frac{\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx}{c^3}-\frac{a^2\left(\frac{3}{4}\int \frac{\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{a^2x^2+1}dx+\frac{x}{2(a^2x^2+1)}\right)-\frac{x}{4(a^2x^2+1)^2}\right)+\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}\right)}{c^3}$$

$$\Big\downarrow \text{216}$$

$$\frac{\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx}{c^3}-\frac{a^2\left(\frac{3}{4}\int \frac{\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}\right)-\frac{x}{4(a^2x^2+1)^2}\right)\right)}{c^3}$$

$$\Big\downarrow \text{5427}$$

$$\frac{\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx}{c^3}-\frac{a^2\left(\frac{3}{4}\left(-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)+\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}\right.\right.\right.}{c^3}$$

$$\Big\downarrow \text{5465}$$

$$\frac{\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx}{c^3}-\frac{a^2\left(\frac{3}{4}\left(-a\left(\frac{\int \frac{1}{(a^2x^2+1)^2}dx}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)+\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{}{2(a\right.\right.\right.}{c^3}$$

$$\Big\downarrow \text{215}$$

$$\frac{\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx}{c^3} - \frac{a^2\left(\frac{3}{4}\left(-a\left(\frac{\frac{1}{2}\int \frac{1}{a^2x^2+1}dx+\frac{x}{2\left(a^2x^2+1\right)}}{2a} - \frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right) + \frac{x\arctan(ax)^2}{2(a^2x^2+1)} + \frac{\arctan(ax)^3}{6a}\right) + \frac{x\arctan(ax)^2}{4(a^2x^2+1)^2} + \frac{\arctan(ax)}{8a(a^2x^2+1)^2} + \frac{1}{8}\left(\right.}{c^3}$$

$$\Big\downarrow \textbf{216}$$

$$\frac{\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx}{c^3} - \frac{a^2\left(\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2} + \frac{\arctan(ax)}{8a(a^2x^2+1)^2} + \frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)} + \frac{\arctan(ax)}{2a}\right) - \frac{x}{4(a^2x^2+1)^2}\right) + \frac{3}{4}\left(\frac{x\arctan(ax)^2}{2(a^2x^2+1)} - a\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+\text{ar}}{2a}\right.\right.\right.}{c^3}$$

$$\Big\downarrow \textbf{5501}$$

$$\frac{\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)}dx - a^2\int \frac{\arctan(ax)^2}{(a^2x^2+1)^2}dx}{c^3} - \frac{a^2\left(\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2} + \frac{\arctan(ax)}{8a(a^2x^2+1)^2} + \frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)} + \frac{\arctan(ax)}{2a}\right) - \frac{x}{4(a^2x^2+1)^2}\right) + \frac{3}{4}\left(\frac{x\arctan(ax)^2}{2(a^2x^2+1)} - a\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+\text{ar}}{2a}\right.\right.\right.}{c^3}$$

$$\Big\downarrow \textbf{5427}$$

$$\frac{\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)}dx - a^2\left(-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx + \frac{x\arctan(ax)^2}{2(a^2x^2+1)} + \frac{\arctan(ax)^3}{6a}\right)}{c^3} - \frac{a^2\left(\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2} + \frac{\arctan(ax)}{8a(a^2x^2+1)^2} + \frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)} + \frac{\arctan(ax)}{2a}\right) - \frac{x}{4(a^2x^2+1)^2}\right) + \frac{3}{4}\left(\frac{x\arctan(ax)^2}{2(a^2x^2+1)} - a\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+\text{ar}}{2a}\right.\right.\right.}{c^3}$$

$$\Big\downarrow \textbf{5453}$$

$$\frac{-\left(a^2\left(-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx + \frac{x\arctan(ax)^2}{2(a^2x^2+1)} + \frac{\arctan(ax)^3}{6a}\right)\right) - a^2\int \frac{\arctan(ax)^2}{a^2x^2+1}dx + \int \frac{\arctan(ax)^2}{x^2}dx}{c^3} - \frac{a^2\left(\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2} + \frac{\arctan(ax)}{8a(a^2x^2+1)^2} + \frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)} + \frac{\arctan(ax)}{2a}\right) - \frac{x}{4(a^2x^2+1)^2}\right) + \frac{3}{4}\left(\frac{x\arctan(ax)^2}{2(a^2x^2+1)} - a\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+\text{ar}}{2a}\right.\right.\right.}{c^3}$$

$$\Big\downarrow \textbf{5361}$$

$$\frac{-\left(a^2\left(-a\int\frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)\right)-a^2\int\frac{\arctan(ax)^2}{a^2x^2+1}dx+2a\int\frac{\arctan(ax)}{x(a^2x^2+1)}dx-\frac{\arctan(ax)^2}{x}}{c^3}-$$

$$\frac{a^2\left(\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}\right)-\frac{x}{4(a^2x^2+1)^2}\right)+\frac{3}{4}\left(\frac{x\arctan(ax)^2}{2(a^2x^2+1)}-a\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+\text{ar}}{2a}\right.\right.\right.}{c^3}$$

$\downarrow$ 5419

$$\frac{-a^2\left(-a\int\frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)+2a\int\frac{\arctan(ax)}{x(a^2x^2+1)}dx-\frac{1}{3}a\arctan(ax)^3-\frac{\arctan(ax)^2}{x}}{c^3}-$$

$$\frac{a^2\left(\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}\right)-\frac{x}{4(a^2x^2+1)^2}\right)+\frac{3}{4}\left(\frac{x\arctan(ax)^2}{2(a^2x^2+1)}-a\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+\text{ar}}{2a}\right.\right.\right.}{c^3}$$

$\downarrow$ 5459

$$-\frac{a^2\left(\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}\right)-\frac{x}{4(a^2x^2+1)^2}\right)+\frac{3}{4}\left(\frac{x\arctan(ax)^2}{2(a^2x^2+1)}-a\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+}{2a}\right.\right.\right.}{c^3}$$

$$\frac{-a^2\left(-a\int\frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)+2a\left(i\int\frac{\arctan(ax)}{x(ax+i)}dx-\frac{1}{2}i\arctan(ax)^2\right)-\frac{1}{3}a\arctan(ax)^3-}{c^3}$$

$\downarrow$ 5403

$$-\frac{a^2\left(\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}\right)-\frac{x}{4(a^2x^2+1)^2}\right)+\frac{3}{4}\left(\frac{x\arctan(ax)^2}{2(a^2x^2+1)}-a\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+}{2a}\right.\right.\right.}{c^3}$$

$$\frac{-a^2\left(-a\int\frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)+2a\left(i\left(ia\int\frac{\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)\right.\right.}{c^3}$$

$\downarrow$ 2897

$$-\frac{a^2\left(\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}\right)-\frac{x}{4(a^2x^2+1)^2}\right)+\frac{3}{4}\left(\frac{x\arctan(ax)^2}{2(a^2x^2+1)}-a\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+}{2a}\right.\right.\right.}{c^3}$$

$$\frac{-a^2\left(-a\int\frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)+2a\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-i}\right.\right.\right.}{c^3}$$

$\downarrow$ 5465

$$-\frac{a^2\left(\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}\right)-\frac{x}{4(a^2x^2+1)^2}\right)+\frac{3}{4}\left(\frac{x\arctan(ax)^2}{2(a^2x^2+1)}-a\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+}{2a}\right.\right.\right.}{c^3}$$

$$\frac{-a^2\left(-a\left(\frac{\int\frac{1}{\left(a^2x^2+1\right)^2}dx}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)+2a\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\right.\right.}{c^3}$$

$\downarrow$ 215

$$-\frac{a^2\left(\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}\right)-\frac{x}{4(a^2x^2+1)^2}\right)+\frac{3}{4}\left(\frac{x\arctan(ax)^2}{2(a^2x^2+1)}-a\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+}{2a}\right.\right.\right.}{c^3}$$

$$\frac{-a^2\left(-a\left(\frac{\frac{1}{2}\int\frac{1}{a^2x^2+1}dx+\frac{x}{2\left(a^2x^2+1\right)}}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)+2a\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-}\right.\right.\right.}{c^3}$$

$\downarrow$ 216

$$-\frac{a^2\left(\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}\right)-\frac{x}{4(a^2x^2+1)^2}\right)+\frac{3}{4}\left(\frac{x\arctan(ax)^2}{2(a^2x^2+1)}-a\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+}{2a}\right.\right.\right.}{c^3}$$

$$\frac{-a^2\left(\frac{x\arctan(ax)^2}{2(a^2x^2+1)}-a\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)+\frac{\arctan(ax)^3}{6a}\right)+2a\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)\right.\right.}{c^3}$$

input
```
Int[ArcTan[a*x]^2/(x^2*(c + a^2*c*x^2)^3),x]
```

output
```
-((a^2*(ArcTan[a*x]/(8*a*(1 + a^2*x^2)^2) + (x*ArcTan[a*x]^2)/(4*(1 + a^2*
x^2)^2) + (-1/4*x/(1 + a^2*x^2)^2 - (3*(x/(2*(1 + a^2*x^2)) + ArcTan[a*x]/
(2*a)))/4)/8 + (3*((x*ArcTan[a*x]^2)/(2*(1 + a^2*x^2)) + ArcTan[a*x]^3/(6*
a) - a*(-1/2*ArcTan[a*x]/(a^2*(1 + a^2*x^2)) + (x/(2*(1 + a^2*x^2)) + ArcT
an[a*x]/(2*a))/(2*a))))/4))/c^3) + (-(ArcTan[a*x]^2/x) - (a*ArcTan[a*x]^3)
/3 - a^2*((x*ArcTan[a*x]^2)/(2*(1 + a^2*x^2)) + ArcTan[a*x]^3/(6*a) - a*(-
1/2*ArcTan[a*x]/(a^2*(1 + a^2*x^2)) + (x/(2*(1 + a^2*x^2)) + ArcTan[a*x]/(
2*a))/(2*a))) + 2*a*((-1/2*I)*ArcTan[a*x]^2 + I*((-I)*ArcTan[a*x]*Log[2 -
2/(1 - I*a*x)] - PolyLog[2, -1 + 2/(1 - I*a*x)]/2)))/c^3
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5435
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[b*p*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p - 1)/(4*c*d*
(q + 1)^2)), x] + (-Simp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*d*
(q + 1))), x] + Simp[(2*q + 3)/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p, x], x] - Simp[b^2*p*((p - 1)/(4*(q + 1)^2))   Int[(d + e
*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

## Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.48

| method | result |
|---|---|
| derivativedivides | $a\left(-\frac{\arctan(ax)^2}{c^3ax}-\frac{7\arctan(ax)^2a^3x^3}{8c^3(a^2x^2+1)^2}-\frac{9ax\arctan(ax)^2}{8c^3(a^2x^2+1)^2}-\frac{15\arctan(ax)^3}{8c^3}-\frac{-5\arctan(ax)^3-8\arctan(ax}{}\right.$ |
| default | $a\left(-\frac{\arctan(ax)^2}{c^3ax}-\frac{7\arctan(ax)^2a^3x^3}{8c^3(a^2x^2+1)^2}-\frac{9ax\arctan(ax)^2}{8c^3(a^2x^2+1)^2}-\frac{15\arctan(ax)^3}{8c^3}-\frac{-5\arctan(ax)^3-8\arctan(ax}{}\right.$ |
| parts | $-\frac{7\arctan(ax)^2a^4x^3}{8c^3(a^2x^2+1)^2}-\frac{9a^2x\arctan(ax)^2}{8c^3(a^2x^2+1)^2}-\frac{15a\arctan(ax)^3}{8c^3}-\frac{\arctan(ax)^2}{c^3x}-\frac{2\left(-\frac{5a\arctan(ax)^3}{8}-\frac{a\left(8\arctan(\right.}{}\right.}{}$ |

input
```
int(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
a*(-1/c^3*arctan(a*x)^2/a/x-7/8/c^3*arctan(a*x)^2/(a^2*x^2+1)^2*a^3*x^3-9/
8*a*x*arctan(a*x)^2/c^3/(a^2*x^2+1)^2-15/8*arctan(a*x)^3/c^3-1/4/c^3*(-5*a
rctan(a*x)^3-8*arctan(a*x)*ln(a*x)+1/2*arctan(a*x)/(a^2*x^2+1)^2+7/2*arcta
n(a*x)/(a^2*x^2+1)+4*arctan(a*x)*ln(a^2*x^2+1)-4*I*ln(a*x)*ln(1+I*a*x)+4*I
*ln(a*x)*ln(1-I*a*x)-4*I*dilog(1+I*a*x)+4*I*dilog(1-I*a*x)+2*I*(ln(a*x-I)*
ln(a^2*x^2+1)-1/2*ln(a*x-I)^2-dilog(-1/2*I*(a*x+I))-ln(a*x-I)*ln(-1/2*I*(a
*x+I)))-2*I*(ln(a*x+I)*ln(a^2*x^2+1)-1/2*ln(a*x+I)^2-dilog(1/2*I*(a*x-I))-
ln(a*x+I)*ln(1/2*I*(a*x-I)))-1/2*(31/8*a^3*x^3+33/8*a*x)/(a^2*x^2+1)^2-31/
16*arctan(a*x)))
```

## Fricas [F]

$$\int \frac{\arctan(ax)^2}{x^2 \left(c+a^2cx^2\right)^3}\,dx = \int \frac{\arctan\left(ax\right)^2}{\left(a^2cx^2+c\right)^3x^2}\,dx$$

input
```
integrate(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
integral(arctan(a*x)^2/(a^6*c^3*x^8 + 3*a^4*c^3*x^6 + 3*a^2*c^3*x^4 + c^3*
x^2), x)
```

## Sympy [F]

$$\int \frac{\arctan(ax)^2}{x^2 \left(c+a^2cx^2\right)^3}\,dx = \frac{\int \frac{\operatorname{atan}^2\left(ax\right)}{a^6x^8+3a^4x^6+3a^2x^4+x^2}\,dx}{c^3}$$

input
```
integrate(atan(a*x)**2/x**2/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(atan(a*x)**2/(a**6*x**8 + 3*a**4*x**6 + 3*a**2*x**4 + x**2), x)/c
**3
```

## Maxima [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^2}{x^2 (c+a^2cx^2)^3} \, dx = \text{Timed out}$$

input
```
integrate(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
Timed out
```

## Giac [F]

$$\int \frac{\arctan(ax)^2}{x^2 (c+a^2cx^2)^3} \, dx = \int \frac{\arctan(ax)^2}{(a^2cx^2+c)^3x^2} \, dx$$

input
```
integrate(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^2/((a^2*c*x^2 + c)^3*x^2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^2}{x^2 (c+a^2cx^2)^3} \, dx = \int \frac{\mathrm{atan}(a\,x)^2}{x^2\,(c\,a^2\,x^2+c)^3} \, dx$$

input
```
int(atan(a*x)^2/(x^2*(c + a^2*c*x^2)^3),x)
```

output
```
int(atan(a*x)^2/(x^2*(c + a^2*c*x^2)^3), x)
```

## Reduce [F]

$$\int \frac{\arctan(ax)^2}{x^2 (c + a^2cx^2)^3} dx$$
$$= \frac{-40atan(ax)^3 a^5x^5 - 80atan(ax)^3 a^3x^3 - 40atan(ax)^3 ax - 120atan(ax)^2 a^4x^4 - 200atan(ax)^2 a^2x^2 - }{}$$

input
```
int(atan(a*x)^2/x^2/(a^2*c*x^2+c)^3,x)
```

output
```
( - 40*atan(a*x)**3*a**5*x**5 - 80*atan(a*x)**3*a**3*x**3 - 40*atan(a*x)**
3*a*x - 120*atan(a*x)**2*a**4*x**4 - 200*atan(a*x)**2*a**2*x**2 - 64*atan(
a*x)**2 + 75*atan(a*x)*a**5*x**5 + 30*atan(a*x)*a**3*x**3 - 85*atan(a*x)*a
*x + 128*int(atan(a*x)/(a**6*x**7 + 3*a**4*x**5 + 3*a**2*x**3 + x),x)*a**5
*x**5 + 256*int(atan(a*x)/(a**6*x**7 + 3*a**4*x**5 + 3*a**2*x**3 + x),x)*a
**3*x**3 + 128*int(atan(a*x)/(a**6*x**7 + 3*a**4*x**5 + 3*a**2*x**3 + x),x
)*a*x + 75*a**4*x**4 + 85*a**2*x**2)/(64*c**3*x*(a**4*x**4 + 2*a**2*x**2 +
 1))
```

# 3.305 $\int \frac{\arctan(ax)^2}{x^3(c+a^2cx^2)^3}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 322

$$\begin{aligned}\int \frac{\arctan(ax)^2}{x^3(c+a^2cx^2)^3}\,dx &= \frac{a^2}{32c^3(1+a^2x^2)^2}+\frac{19a^2}{32c^3(1+a^2x^2)}-\frac{a\arctan(ax)}{c^3x}\\&+\frac{a^3x\arctan(ax)}{8c^3(1+a^2x^2)^2}+\frac{19a^3x\arctan(ax)}{16c^3(1+a^2x^2)}\\&+\frac{3a^2\arctan(ax)^2}{32c^3}-\frac{\arctan(ax)^2}{2c^3x^2}-\frac{a^2\arctan(ax)^2}{4c^3(1+a^2x^2)^2}\\&-\frac{a^2\arctan(ax)^2}{c^3(1+a^2x^2)}+\frac{ia^2\arctan(ax)^3}{c^3}+\frac{a^2\log(x)}{c^3}\\&-\frac{a^2\log(1+a^2x^2)}{2c^3}-\frac{3a^2\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{c^3}\\&+\frac{3ia^2\arctan(ax)\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{c^3}\\&-\frac{3a^2\operatorname{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)}{2c^3}\end{aligned}$$

output
```
1/32*a^2/c^3/(a^2*x^2+1)^2+19/32*a^2/c^3/(a^2*x^2+1)-a*arctan(a*x)/c^3/x+1
/8*a^3*x*arctan(a*x)/c^3/(a^2*x^2+1)^2+19/16*a^3*x*arctan(a*x)/c^3/(a^2*x^
2+1)+3/32*a^2*arctan(a*x)^2/c^3-1/2*arctan(a*x)^2/c^3/x^2-1/4*a^2*arctan(a
*x)^2/c^3/(a^2*x^2+1)^2-a^2*arctan(a*x)^2/c^3/(a^2*x^2+1)+I*a^2*arctan(a*x
)^3/c^3+a^2*ln(x)/c^3-1/2*a^2*ln(a^2*x^2+1)/c^3-3*a^2*arctan(a*x)^2*ln(2-2
/(1-I*a*x))/c^3+3*I*a^2*arctan(a*x)*polylog(2,-1+2/(1-I*a*x))/c^3-3/2*a^2*
polylog(3,-1+2/(1-I*a*x))/c^3
```

## Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.70

$$\int \frac{\arctan(ax)^2}{x^3(c+a^2cx^2)^3}\,dx = \frac{a^2\left(\frac{i\pi^3}{8}-\frac{\arctan(ax)}{ax}-\frac{(1+a^2x^2)\arctan(ax)^2}{2a^2x^2}-i\arctan(ax)^3+\frac{5}{16}\cos(2\arctan(ax))-\frac{5}{8}\arctan(ax)^2\cos(2\arct\right.}{}$$

input
```
Integrate[ArcTan[a*x]^2/(x^3*(c + a^2*c*x^2)^3),x]
```

output
```
(a^2*((I/8)*Pi^3 - ArcTan[a*x]/(a*x) - ((1 + a^2*x^2)*ArcTan[a*x]^2)/(2*a^
2*x^2) - I*ArcTan[a*x]^3 + (5*Cos[2*ArcTan[a*x]])/16 - (5*ArcTan[a*x]^2*Co
s[2*ArcTan[a*x]])/8 + Cos[4*ArcTan[a*x]]/256 - (ArcTan[a*x]^2*Cos[4*ArcTan
[a*x]])/32 - 3*ArcTan[a*x]^2*Log[1 - E^((-2*I)*ArcTan[a*x])] + Log[(a*x)/S
qrt[1 + a^2*x^2]] - (3*I)*ArcTan[a*x]*PolyLog[2, E^((-2*I)*ArcTan[a*x])] -
 (3*PolyLog[3, E^((-2*I)*ArcTan[a*x])])/2 + (5*ArcTan[a*x]*Sin[2*ArcTan[a*
x]])/8 + (ArcTan[a*x]*Sin[4*ArcTan[a*x]])/64))/c^3
```

## Rubi [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 689 vs. 2(322) = 644.

Time = 5.99 (sec) , antiderivative size = 689, normalized size of antiderivative = 2.14, number of steps used = 27, number of rules used = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 1.182$, Rules used = {5501, 27, 5501, 5453, 5361, 5453, 5361, 243, 47, 14, 16, 5419, 5459, 5403, 5465, 5431, 5427, 241, 5501, 5459, 5403, 5465, 5427, 241, 5527, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2}{x^3\left(a^2cx^2+c\right)^3}dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{\arctan(ax)^2}{c^2x^3(a^2x^2+1)^2}dx}{c}-a^2\int \frac{\arctan(ax)^2}{c^3x\left(a^2x^2+1\right)^3}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\arctan(ax)^2}{x^3(a^2x^2+1)^2}dx}{c^3}-\frac{a^2\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^3}dx}{c^3}$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{\arctan(ax)^2}{x^3(a^2x^2+1)}dx-a^2\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx}{c^3}-\frac{a^2\left(\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx-a^2\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^3}dx\right)}{c^3}$$

$$\Big\downarrow \text{5453}$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx\right)-a^2\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx+\int \frac{\arctan(ax)^2}{x^3}dx}{c^3}-\frac{a^2\left(\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx-a^2\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^3}dx\right)}{c^3}$$

$$\Big\downarrow \text{5361}$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx\right)-a^2\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx+a\int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx-\frac{\arctan(ax)^2}{2x^2}}{c^3}-\frac{a^2\left(\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx-a^2\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^3}dx\right)}{c^3}$$

$$\Big\downarrow \text{5453}$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx\right)-a^2\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx+a\left(\int \frac{\arctan(ax)}{x^2}dx-a^2\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)-\frac{\arctan(ax)^2}{2x^2}}{c^3}-\frac{a^2\left(\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx-a^2\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^3}dx\right)}{c^3}$$

$$\Big\downarrow \text{5361}$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx\right) - a^2\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx + a\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right) + a\int \frac{1}{x(a^2x^2+1)}dx - \frac{\arctan(ax)}{x}\right) - \frac{\arctan(ax)^2}{2x^2}}{c^3}$$

$$\frac{a^2\left(\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx - a^2\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^3}dx\right)}{c^3}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx\right) - a^2\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx + a\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right) + \frac{1}{2}a\int \frac{1}{x^2(a^2x^2+1)}dx^2 - \frac{\arctan(ax)}{x}\right) - \frac{\arctan(}{2x^2}}{c^3}$$

$$\frac{a^2\left(\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx - a^2\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^3}dx\right)}{c^3}$$

$$\Big\downarrow \text{ 47}$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx\right) - a^2\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx + a\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right) + \frac{1}{2}a\left(\int \frac{1}{x^2}dx^2 - a^2\int \frac{1}{a^2x^2+1}dx^2\right) - \frac{\arctan(a}{x}}{c^3}$$

$$\frac{a^2\left(\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx - a^2\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^3}dx\right)}{c^3}$$

$$\Big\downarrow \text{ 14}$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx\right) - a^2\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx + a\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right) + \frac{1}{2}a\left(\log\left(x^2\right) - a^2\int \frac{1}{a^2x^2+1}dx^2\right) - \frac{\arctan(a}{x}}{c^3}$$

$$\frac{a^2\left(\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx - a^2\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^3}dx\right)}{c^3}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx\right) - a^2\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx + a\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right) + \frac{1}{2}a\left(\log\left(x^2\right) - \log\left(a^2x^2+1\right)\right) - \frac{\arctan(ax}{x}}{c^3}$$

$$\frac{a^2\left(\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx - a^2\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^3}dx\right)}{c^3}$$

$$\Big\downarrow \text{ 5419}$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx\right) - a^2\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx + a\left(\frac{1}{2}a\left(\log\left(x^2\right) - \log\left(a^2x^2+1\right)\right) - \frac{1}{2}a\arctan(ax)^2 - \frac{\arctan(ax)}{x}\right) -}{c^3}$$

$$\frac{a^2\left(\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx - a^2\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^3}dx\right)}{c^3}$$

$$\Big\downarrow \text{5459}$$

$$-\frac{a^2\left(\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx-a^2\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^3}dx\right)}{c^3}+\frac{-a^2\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx-\left(a^2\left(i\int \frac{\arctan(ax)^2}{x(ax+i)}dx-\frac{1}{3}i\arctan(ax)^3\right)\right)+a\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{1}{2}a\arctan(\right.}{c^3}$$

$$\Big\downarrow \text{5403}$$

$$-\frac{a^2\left(\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx-a^2\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^3}dx\right)}{c^3}+\frac{a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx\right)-a^2\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3\right.}{c^3}$$

$$\Big\downarrow \text{5465}$$

$$-\frac{a^2\left(\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx-a^2\left(\frac{\int \frac{\arctan(ax)}{(a^2x^2+1)^3}dx}{2a}-\frac{\arctan(ax)^2}{4a^2(a^2x^2+1)^2}\right)\right)}{c^3}+\frac{a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx\right)-a^2\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3\right.}{c^3}$$

$$\Big\downarrow \text{5431}$$

$$-\frac{a^2\left(\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx-a^2\left(\frac{\frac{3}{4}\int \frac{\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{1}{16a(a^2x^2+1)^2}}{2a}-\frac{\arctan(ax)^2}{4a^2(a^2x^2+1)^2}\right)\right)}{c^3}+\frac{a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx\right)-a^2\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3\right.}{c^3}$$

$$\Big\downarrow \text{5427}$$

$$-\frac{a^2\left(\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx-a^2\left(\frac{\frac{3}{4}\left(-\frac{1}{2}a\int \frac{x}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{1}{16a(a^2x^2+1)^2}}{2a}-\frac{\arctan(ax)^2}{4a^2(a^2x^2+1)^2}\right)\right)}{c^3}$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx\right)-a^2\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3\right.}{c^3}$$

$\downarrow$ 241

$$-\frac{a^2\left(\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx-a^2\left(\frac{\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}}{2a}-\frac{\arctan(ax)^2}{4a^2(a^2x^2+1)^2}\right)\right)}{c^3}+$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)^2}dx\right)-a^2\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3\right.}{c^3}$$

$\downarrow$ 5501

$$-\frac{a^2\left(-a^2\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx-\left(a^2\left(\frac{\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}}{2a}\right.\right.\right.}{c^3}$$

$$\frac{-\left(a^2\left(\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx-a^2\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx\right)\right)-a^2\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-\ldots}\right.\right.\right.}{c^3}$$

$\downarrow$ 5459

$$-\left(a^2\left(-a^2\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+i\int \frac{\arctan(ax)^2}{x(ax+i)}dx-\frac{1}{3}i\arctan(ax)^3\right)\right)-a^2\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\,\mathrm{arc}\ldots\right.\right.$$

$$\frac{a^2\left(-a^2\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+i\int \frac{\arctan(ax)^2}{x(ax+i)}dx-a^2\left(\frac{\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}}{2a}-\right.\right.}{c^3}$$

$\downarrow$ 5403

$$-\left(a^2\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3\right)\right)-a^2\left(-a^2\int \frac{x\,\mathrm{ar}\ldots}{(a\ldots}\right.$$

$$\frac{a^2\left(-a^2\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-a^2\left(\frac{\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\ldots}{\ldots}\right.\right.}{c^3}$$

$\downarrow$ 5465

$$-\left(a^2\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3\right)\right)-a^2\left(-a^2\left(\frac{\int \frac{\mathrm{ar}}{(\mathrm{a}}}{}\right.\right.$$

$$\frac{}{a^2\left(-a^2\left(\frac{\int \frac{\arctan(ax)}{(a^2x^2+1)^2}dx}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)+i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-a\right.}$$

$c^3$

$\downarrow$ 5427

$$-\left(a^2\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3\right)\right)-a^2\left(-a^2\left(\frac{-\frac{1}{2}a}{}\right.\right.$$

$$\frac{}{a^2\left(-a^2\left(\frac{-\frac{1}{2}a\int \frac{x}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)+i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(a\right.\right.}$$

$\downarrow$ 241

$$-\left(a^2\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3\right)\right)-a^2\left(i\left(2ia\int \frac{\mathrm{ar}}{}\right.\right.$$

$$\frac{}{a^2\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-a^2\left(\frac{\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}}{a}-\right.\right.}$$

$\downarrow$ 5527

$$-\left(a^2\left(i\left(2ia\left(\frac{i\arctan(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-\frac{1}{2}i\int \frac{\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{a^2x^2+1}dx\right)-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\right.\right.$$

$$\frac{}{a^2\left(i\left(2ia\left(\frac{i\arctan(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-\frac{1}{2}i\int \frac{\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{a^2x^2+1}dx\right)-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-a^2\left(\right.\right.}$$

$\downarrow$ 7164

$$-a^2\left(-a^2\left(\frac{\frac{x\arctan(ax)}{2\left(a^2x^2+1\right)}+\frac{1}{4a\left(a^2x^2+1\right)}+\frac{\arctan(ax)^2}{4a}}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)+i\left(2ia\left(\frac{i\arctan(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-\frac{\operatorname{PolyLog}\left(3,\frac{2}{1-i}\right.}{4a}\right.\right.\right.$$

$$a^2\left(-a^2\left(\frac{\frac{x\arctan(ax)}{2\left(a^2x^2+1\right)}+\frac{1}{4a\left(a^2x^2+1\right)}+\frac{\arctan(ax)^2}{4a}}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)-a^2\left(\frac{\frac{x\arctan(ax)}{4\left(a^2x^2+1\right)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2\left(a^2x^2+1\right)}+\frac{1}{4a\left(a^2x^2+1\right)}+\frac{\arctan(ax)^2}{4a}\right)+}{2a}\right.\right.$$

input
```
Int[ArcTan[a*x]^2/(x^3*(c + a^2*c*x^2)^3),x]
```

output
```
-((a^2*((-1/3*I)*ArcTan[a*x]^3 - a^2*(-1/2*ArcTan[a*x]^2/(a^2*(1 + a^2*x^2
)) + (1/(4*a*(1 + a^2*x^2)) + (x*ArcTan[a*x])/(2*(1 + a^2*x^2)) + ArcTan[a
*x]^2/(4*a))/a) - a^2*(-1/4*ArcTan[a*x]^2/(a^2*(1 + a^2*x^2)^2) + (1/(16*a
*(1 + a^2*x^2)^2) + (x*ArcTan[a*x])/(4*(1 + a^2*x^2)^2) + (3*(1/(4*a*(1 +
a^2*x^2)) + (x*ArcTan[a*x])/(2*(1 + a^2*x^2)) + ArcTan[a*x]^2/(4*a)))/4)/(
2*a)) + I*((-I)*ArcTan[a*x]^2*Log[2 - 2/(1 - I*a*x)] + (2*I)*a*(((I/2)*Arc
Tan[a*x]*PolyLog[2, -1 + 2/(1 - I*a*x)])/a - PolyLog[3, -1 + 2/(1 - I*a*x)
]/(4*a)))))/c^3) + (-1/2*ArcTan[a*x]^2/x^2 + a*(-(ArcTan[a*x]/x) - (a*ArcT
an[a*x]^2)/2 + (a*(Log[x^2] - Log[1 + a^2*x^2]))/2) - a^2*((-1/3*I)*ArcTan
[a*x]^3 + I*((-I)*ArcTan[a*x]^2*Log[2 - 2/(1 - I*a*x)] + (2*I)*a*(((I/2)*A
rcTan[a*x]*PolyLog[2, -1 + 2/(1 - I*a*x)])/a - PolyLog[3, -1 + 2/(1 - I*a*
x)]/(4*a)))) - a^2*((-1/3*I)*ArcTan[a*x]^3 - a^2*(-1/2*ArcTan[a*x]^2/(a^2*
(1 + a^2*x^2)) + (1/(4*a*(1 + a^2*x^2)) + (x*ArcTan[a*x])/(2*(1 + a^2*x^2)
) + ArcTan[a*x]^2/(4*a))/a) + I*((-I)*ArcTan[a*x]^2*Log[2 - 2/(1 - I*a*x)]
 + (2*I)*a*(((I/2)*ArcTan[a*x]*PolyLog[2, -1 + 2/(1 - I*a*x)])/a - PolyLog
[3, -1 + 2/(1 - I*a*x)]/(4*a)))))/c^3
```

## Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5431
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbol
] :> Simp[b*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (-Simp[x*(d + e*x
^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*d*(q + 1))), x] + Simp[(2*q + 3)/(2*d*(
q + 1))   Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a,
 b, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -3/2]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

rule 5527
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 94.58 (sec) , antiderivative size = 1970, normalized size of antiderivative = 6.12

| method | result | size |
|---|---|---|
| derivativedivides | Expression too large to display | 1970 |
| default | Expression too large to display | 1970 |
| parts | Expression too large to display | 2425 |

input
```
int(arctan(a*x)^2/x^3/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
a^2*(-1/2/c^3*arctan(a*x)^2/a^2/x^2-3/c^3*arctan(a*x)^2*ln(a*x)+3/2/c^3*ar
ctan(a*x)^2*ln(a^2*x^2+1)-arctan(a*x)^2/c^3/(a^2*x^2+1)-1/4*arctan(a*x)^2/
c^3/(a^2*x^2+1)^2-1/2/c^3*(-3/16*arctan(a*x)^2-2*ln((1+I*a*x)/(a^2*x^2+1)^
(1/2)-1)-1/128*cos(4*arctan(a*x))-2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*I*
Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^
2)^2*arctan(a*x)^2+12*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))+12*polylog(3,
-(1+I*a*x)/(a^2*x^2+1)^(1/2))-3/2*I*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))
^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*arctan(a*x)^2-3*I*Pi*csgn(((1+I*a*x)^2/
(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2+3*I*Pi*csgn(((
1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^3*arctan(a*x)^2+3*I
*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^3*arct
an(a*x)^2-3/2*I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)
+1)^2)^3*arctan(a*x)^2-3/2*I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^3*arctan(a
*x)^2+3/2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3*arctan(a*x)^2-3/2*I
*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*
csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*arctan(a*x)^
2+3*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I/((1+I*a*x)^2/(a^2*x^2+
1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*arc
tan(a*x)^2+3*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2
+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*a...
```

## Fricas [F]

$$\int \frac{\arctan(ax)^2}{x^3 (c+a^2cx^2)^3} \, dx = \int \frac{\arctan{(ax)}^2}{(a^2cx^2+c)^3x^3} \, dx$$

input
```
integrate(arctan(a*x)^2/x^3/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
integral(arctan(a*x)^2/(a^6*c^3*x^9 + 3*a^4*c^3*x^7 + 3*a^2*c^3*x^5 + c^3*
x^3), x)
```

## Sympy [F]

$$\int \frac{\arctan(ax)^2}{x^3 (c + a^2cx^2)^3} dx = \frac{\int \frac{\operatorname{atan}^2(ax)}{a^6x^9+3a^4x^7+3a^2x^5+x^3} dx}{c^3}$$

input
```
integrate(atan(a*x)**2/x**3/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(atan(a*x)**2/(a**6*x**9 + 3*a**4*x**7 + 3*a**2*x**5 + x**3), x)/c
**3
```

## Maxima [F]

$$\int \frac{\arctan(ax)^2}{x^3 (c + a^2cx^2)^3} dx = \int \frac{\arctan(ax)^2}{(a^2cx^2 + c)^3 x^3} dx$$

input
```
integrate(arctan(a*x)^2/x^3/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)^2/((a^2*c*x^2 + c)^3*x^3), x)
```

## Giac [F]

$$\int \frac{\arctan(ax)^2}{x^3 (c + a^2cx^2)^3} dx = \int \frac{\arctan(ax)^2}{(a^2cx^2 + c)^3 x^3} dx$$

input
```
integrate(arctan(a*x)^2/x^3/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^2/((a^2*c*x^2 + c)^3*x^3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^2}{x^3\left(c+a^2cx^2\right)^3}\,dx = \int \frac{\operatorname{atan}(a\,x)^2}{x^3\left(c\,a^2\,x^2+c\right)^3}\,dx$$

input
```
int(atan(a*x)^2/(x^3*(c + a^2*c*x^2)^3),x)
```

output
```
int(atan(a*x)^2/(x^3*(c + a^2*c*x^2)^3), x)
```

## Reduce [F]

$$\int \frac{\arctan(ax)^2}{x^3\left(c+a^2cx^2\right)^3}\,dx = \frac{\int \frac{atan(ax)^2}{a^6x^9+3a^4x^7+3a^2x^5+x^3}dx}{c^3}$$

input
```
int(atan(a*x)^2/x^3/(a^2*c*x^2+c)^3,x)
```

output
```
int(atan(a*x)**2/(a**6*x**9 + 3*a**4*x**7 + 3*a**2*x**5 + x**3),x)/c**3
```

# 3.306 $\int \frac{\arctan(ax)^2}{x^4(c+a^2cx^2)^3}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 317

$$\int \frac{\arctan(ax)^2}{x^4(c+a^2cx^2)^3}\,dx = -\frac{a^2}{3c^3x}-\frac{a^4x}{32c^3(1+a^2x^2)^2}-\frac{47a^4x}{64c^3(1+a^2x^2)} -\frac{205a^3\arctan(ax)}{192c^3}-\frac{a\arctan(ax)}{3c^3x^2}+\frac{a^3\arctan(ax)}{8c^3(1+a^2x^2)^2} +\frac{11a^3\arctan(ax)}{8c^3(1+a^2x^2)}+\frac{10ia^3\arctan(ax)^2}{3c^3}-\frac{\arctan(ax)^2}{3c^3x^3} +\frac{3a^2\arctan(ax)^2}{c^3x}+\frac{a^4x\arctan(ax)^2}{4c^3(1+a^2x^2)^2}+\frac{11a^4x\arctan(ax)^2}{8c^3(1+a^2x^2)} +\frac{35a^3\arctan(ax)^3}{24c^3}-\frac{20a^3\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{3c^3} +\frac{10ia^3\,\mathrm{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{3c^3}$$

output

```
-1/3*a^2/c^3/x-1/32*a^4*x/c^3/(a^2*x^2+1)^2-47/64*a^4*x/c^3/(a^2*x^2+1)-20
5/192*a^3*arctan(a*x)/c^3-1/3*a*arctan(a*x)/c^3/x^2+1/8*a^3*arctan(a*x)/c^
3/(a^2*x^2+1)^2+11/8*a^3*arctan(a*x)/c^3/(a^2*x^2+1)+10/3*I*a^3*arctan(a*x
)^2/c^3-1/3*arctan(a*x)^2/c^3/x^3+3*a^2*arctan(a*x)^2/c^3/x+1/4*a^4*x*arct
an(a*x)^2/c^3/(a^2*x^2+1)^2+11/8*a^4*x*arctan(a*x)^2/c^3/(a^2*x^2+1)+35/24
*a^3*arctan(a*x)^3/c^3-20/3*a^3*arctan(a*x)*ln(2-2/(1-I*a*x))/c^3+10/3*I*a
^3*polylog(2,-1+2/(1-I*a*x))/c^3
```

## Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.60

$$\int \frac{\arctan(ax)^2}{x^4(c+a^2cx^2)^3}\,dx$$
$$= \frac{a^3\left(-\frac{256(1+a^2x^2)\arctan(ax)}{a^2x^2} - \frac{256(1+a^2x^2)\arctan(ax)^2}{a^3x^3} + 1120\arctan(ax)^3 + \frac{256(-1+10\arctan(ax)^2)}{ax} + 576\arctan(a\right.}{}$$

input
```
Integrate[ArcTan[a*x]^2/(x^4*(c + a^2*c*x^2)^3),x]
```

output
```
(a^3*((-256*(1 + a^2*x^2)*ArcTan[a*x])/(a^2*x^2) - (256*(1 + a^2*x^2)*ArcT
an[a*x]^2)/(a^3*x^3) + 1120*ArcTan[a*x]^3 + (256*(-1 + 10*ArcTan[a*x]^2))/
(a*x) + 576*ArcTan[a*x]*Cos[2*ArcTan[a*x]] + 12*ArcTan[a*x]*Cos[4*ArcTan[a
*x]] - 5120*ArcTan[a*x]*Log[1 - E^((2*I)*ArcTan[a*x])] + (2560*I)*(ArcTan[
a*x]^2 + PolyLog[2, E^((2*I)*ArcTan[a*x])]) + 288*(-1 + 2*ArcTan[a*x]^2)*S
in[2*ArcTan[a*x]] + 3*(-1 + 8*ArcTan[a*x]^2)*Sin[4*ArcTan[a*x]]))/(768*c^3
)
```

## Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2}{x^4(a^2cx^2+c)^3}\,dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{\arctan(ax)^2}{c^2x^4(a^2x^2+1)^2}dx}{c} - a^2\int \frac{\arctan(ax)^2}{c^3x^2(a^2x^2+1)^3}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\arctan(ax)^2}{x^4(a^2x^2+1)^2}dx}{c^3} - \frac{a^2\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^3}dx}{c^3}$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{\arctan(ax)^2}{x^4(a^2x^2+1)}dx - a^2\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx}{c^3} - \frac{a^2\left(\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx - a^2\int \frac{\arctan(ax)^2}{(a^2x^2+1)^3}dx\right)}{c^3}$$

$$\Big\downarrow \text{5435}$$

$$\frac{\int \frac{\arctan(ax)^2}{x^4(a^2x^2+1)}dx - a^2\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx}{c^3} - \frac{a^2\left(\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx - a^2\left(\frac{3}{4}\int \frac{\arctan(ax)^2}{(a^2x^2+1)^2}dx - \frac{1}{8}\int \frac{1}{(a^2x^2+1)^3}dx + \frac{x\arctan(ax)^2}{4(a^2x^2+1)^2} + \frac{\arctan(ax)}{8a(a^2x^2+1)^2}\right)\right)}{c^3}$$

$$\Big\downarrow \text{215}$$

$$\frac{\int \frac{\arctan(ax)^2}{x^4(a^2x^2+1)}dx - a^2\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx}{c^3} - \frac{a^2\left(\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx - a^2\left(\frac{3}{4}\int \frac{\arctan(ax)^2}{(a^2x^2+1)^2}dx + \frac{1}{8}\left(-\frac{3}{4}\int \frac{1}{(a^2x^2+1)^2}dx - \frac{x}{4(a^2x^2+1)^2}\right) + \frac{x\arctan(ax)^2}{4(a^2x^2+1)^2} + \frac{\arctan(ax)}{8a(a^2x^2+1)^2}\right)\right)}{c^3}$$

$$\Big\downarrow \text{215}$$

$$\frac{\int \frac{\arctan(ax)^2}{x^4(a^2x^2+1)}dx - a^2\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx}{c^3} - \frac{a^2\left(\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx - a^2\left(\frac{3}{4}\int \frac{\arctan(ax)^2}{(a^2x^2+1)^2}dx + \frac{1}{8}\left(-\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{a^2x^2+1}dx + \frac{x}{2(a^2x^2+1)}\right) - \frac{x}{4(a^2x^2+1)^2}\right) + \frac{x\arctan(ax)^2}{4(a^2x^2+1)^2} + \frac{a}{8a}\right.}{c^3}$$

$$\Big\downarrow \text{216}$$

$$\frac{\int \frac{\arctan(ax)^2}{x^4(a^2x^2+1)}dx - a^2\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx}{c^3} - \frac{a^2\left(\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx - a^2\left(\frac{3}{4}\int \frac{\arctan(ax)^2}{(a^2x^2+1)^2}dx + \frac{x\arctan(ax)^2}{4(a^2x^2+1)^2} + \frac{\arctan(ax)}{8a(a^2x^2+1)^2} + \frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)} + \frac{\arctan(ax)}{2a}\right) - \frac{x}{4(a^2x^2+}\right.\right.\right.}{c^3}$$

$$\Big\downarrow \text{5427}$$

$$\frac{\int \frac{\arctan(ax)^2}{x^4(a^2x^2+1)}dx - a^2\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx}{c^3} - \frac{a^2\left(\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx - a^2\left(\frac{3}{4}\left(-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx + \frac{x\arctan(ax)^2}{2(a^2x^2+1)} + \frac{\arctan(ax)^3}{6a}\right) + \frac{x\arctan(ax)^2}{4(a^2x^2+1)^2} + \frac{\arctan(ax)}{8a(a^2x^2+1)^2} + \frac{1}{8}\left(-\frac{3}{4}\right.\right.\right.}{c^3}$$

$$\Big\downarrow \text{5453}$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx\right) - a^2\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)}dx + \int \frac{\arctan(ax)^2}{x^4}dx}{c^3} - \frac{a^2\left(\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx - a^2\left(\frac{3}{4}\left(-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx + \frac{x\arctan(ax)^2}{2(a^2x^2+1)} + \frac{\arctan(ax)^3}{6a}\right) + \frac{x\arctan(ax)^2}{4(a^2x^2+1)^2} + \frac{\arctan(ax)}{8a(a^2x^2+1)^2} + \frac{1}{8}\left(-\frac{3}{4}\right.\right.\right.}{c^3}$$

$$\Big\downarrow \text{5361}$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx\right)-a^2\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)}dx+\frac{2}{3}a\int \frac{\arctan(ax)}{x^3(a^2x^2+1)}dx-\frac{\arctan(ax)^2}{3x^3}}{c^3}-\frac{a^2\left(\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx-a^2\left(\frac{3}{4}\left(-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)+\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\left(-\frac{3}{4}\right.\right.\right.}{c^3}$$

$$\Big\downarrow \text{5453}$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx\right)-a^2\left(\int \frac{\arctan(ax)^2}{x^2}dx-a^2\int \frac{\arctan(ax)^2}{a^2x^2+1}dx\right)+\frac{2}{3}a\left(\int \frac{\arctan(ax)}{x^3}dx-a^2\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx\right)-\text{arc}}{c^3}$$
$$\frac{a^2\left(\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx-a^2\left(\frac{3}{4}\left(-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)+\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\left(-\frac{3}{4}\right.\right.\right.}{c^3}$$

$$\Big\downarrow \text{5361}$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx\right)-a^2\left(a^2\left(-\int \frac{\arctan(ax)^2}{a^2x^2+1}dx\right)+2a\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx-\frac{\arctan(ax)^2}{x}\right)+\frac{2}{3}a\left(a^2\left(-\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx\right)\right.}{c^3}$$
$$\frac{a^2\left(\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx-a^2\left(\frac{3}{4}\left(-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)+\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\left(-\frac{3}{4}\right.\right.\right.}{c^3}$$

$$\Big\downarrow \text{264}$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx\right)-a^2\left(a^2\left(-\int \frac{\arctan(ax)^2}{a^2x^2+1}dx\right)+2a\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx-\frac{\arctan(ax)^2}{x}\right)+\frac{2}{3}a\left(a^2\left(-\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx\right)\right.}{c^3}$$
$$\frac{a^2\left(\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx-a^2\left(\frac{3}{4}\left(-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)+\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\left(-\frac{3}{4}\right.\right.\right.}{c^3}$$

$$\Big\downarrow \text{216}$$

$$\frac{a^2\left(-\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx\right)-a^2\left(a^2\left(-\int \frac{\arctan(ax)^2}{a^2x^2+1}dx\right)+2a\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx-\frac{\arctan(ax)^2}{x}\right)+\frac{2}{3}a\left(a^2\left(-\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx\right)\right.}{c^3}$$
$$\frac{a^2\left(\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx-a^2\left(\frac{3}{4}\left(-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)+\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\left(-\frac{3}{4}\right.\right.\right.}{c^3}$$

$$\Big\downarrow \text{5419}$$

$$\frac{-\left(a^2\left(2a\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx-\frac{1}{3}a\arctan(ax)^3-\frac{\arctan(ax)^2}{x}\right)\right)-a^2\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx+\frac{2}{3}a\left(a^2\left(-\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx\right)-\frac{\text{arct}}{}\right.}{c^3}$$

$$\frac{a^2\left(\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx-a^2\left(\frac{3}{4}\left(-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)+\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\left(-\frac{3}{4}\right.\right.\right.}{c^3}$$

$\downarrow$ 5459

$$-\frac{a^2\left(\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx-a^2\left(\frac{3}{4}\left(-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)+\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\left(-\right.\right.\right.}{c^3}$$

$$\frac{-a^2\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx+\frac{2}{3}a\left(-\left(a^2\left(i\int \frac{\arctan(ax)}{x(ax+i)}dx-\frac{1}{2}i\arctan(ax)^2\right)\right)-\frac{\arctan(ax)}{2x^2}+\frac{1}{2}a\left(-a\arctan(ax)-\frac{1}{x}\right)\right)-}{c^3}$$

$\downarrow$ 5403

$$-\frac{a^2\left(\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx-a^2\left(\frac{3}{4}\left(-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)+\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\left(-\right.\right.\right.}{c^3}$$

$$a^2\left(-\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx\right)-a^2\left(2a\left(i\left(ia\int \frac{\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)-\right.$$

$\downarrow$ 2897

$$-\frac{a^2\left(\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx-a^2\left(\frac{3}{4}\left(-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)+\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\left(-\right.\right.\right.}{c^3}$$

$$-a^2\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx+\frac{2}{3}a\left(-\left(a^2\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)\right)-\frac{1}{2}i\arctan(ax)\right.\right.$$

$\downarrow$ 5465

$$-\frac{a^2\left(\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx-a^2\left(\frac{3}{4}\left(-a\left(\frac{\int \frac{1}{(a^2x^2+1)^2}dx}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)+\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\right.\right.}{c^3}$$

$$-a^2\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx+\frac{2}{3}a\left(-\left(a^2\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)\right)-\frac{1}{2}i\arctan(ax)\right.\right.$$

$\downarrow$ 215

$$-\frac{a^2\left(\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx - a^2\left(\frac{3}{4}\left(-a\left(\frac{\frac{1}{2}\int \frac{1}{a^2x^2+1}dx+\frac{x}{2(a^2x^2+1)}}{2a} - \frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right) + \frac{x\arctan(ax)^2}{2(a^2x^2+1)} + \frac{\arctan(ax)^3}{6a}\right) + \frac{x\arctan}{4(a^2x^2}\right.\right.}{-a^2\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx + \frac{2}{3}a\left(-\left(a^2\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right) - \frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right) - \frac{1}{2}i\arctan(ax)\right.\right.\right.}$$

$$c^3$$

$$\Big\downarrow 216$$

$$-\frac{a^2\left(\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx - a^2\left(\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2} + \frac{\arctan(ax)}{8a(a^2x^2+1)^2} + \frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)} + \frac{\arctan(ax)}{2a}\right) - \frac{x}{4(a^2x^2+1)^2}\right) + \frac{3}{4}\left(\frac{x\arctan}{2(a^2}\right.\right.\right.}{-a^2\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)^2}dx + \frac{2}{3}a\left(-\left(a^2\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right) - \frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right) - \frac{1}{2}i\arctan(ax)\right.\right.\right.}$$

$$c^3$$

$$\Big\downarrow 5501$$

$$-\frac{a^2\left(-a^2\int \frac{\arctan(ax)^2}{(a^2x^2+1)^2}dx + \int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)}dx - \left(a^2\left(\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2} + \frac{\arctan(ax)}{8a(a^2x^2+1)^2} + \frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)} + \frac{\arctan(ax)}{2a}\right) - \right.\right.\right.\right.}{-a^2\left(\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)}dx - a^2\int \frac{\arctan(ax)^2}{(a^2x^2+1)^2}dx\right) + \frac{2}{3}a\left(-\left(a^2\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right) - \frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax} -\right.\right.\right.\right.\right.}$$

$$c^3$$

$$\Big\downarrow 5427$$

$$-\frac{a^2\left(-a^2\left(-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx + \frac{x\arctan(ax)^2}{2(a^2x^2+1)} + \frac{\arctan(ax)^3}{6a}\right) + \int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)}dx - \left(a^2\left(\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2} + \frac{\arctan(ax)}{8a(a^2x^2+1)^2} + \right.\right.\right.}{-a^2\left(\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)}dx - a^2\left(-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx + \frac{x\arctan(ax)^2}{2(a^2x^2+1)} + \frac{\arctan(ax)^3}{6a}\right)\right) + \frac{2}{3}a\left(-\left(a^2\left(i\left(-i\arctan(ax)\log\left(2\right.\right.\right.\right.\right.\right.}$$

$$\Big\downarrow 5453$$

$$-\frac{a^2\left(-a^2\left(-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx + \frac{x\arctan(ax)^2}{2(a^2x^2+1)} + \frac{\arctan(ax)^3}{6a}\right) - a^2\int \frac{\arctan(ax)^2}{a^2x^2+1}dx + \int \frac{\arctan(ax)^2}{x^2}dx - \left(a^2\left(\frac{x\arctan(}{4(a^2x^2-}\right.\right.\right.}{-a^2\left(-\left(a^2\left(-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx + \frac{x\arctan(ax)^2}{2(a^2x^2+1)} + \frac{\arctan(ax)^3}{6a}\right)\right) - a^2\int \frac{\arctan(ax)^2}{a^2x^2+1}dx + \int \frac{\arctan(ax)^2}{x^2}dx\right) + \frac{2}{3}a\left(-\left(a^2\right.\right.}$$

$$\Big\downarrow \text{5361}$$

$$-\frac{a^2\left(-a^2\left(-a\int\frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)-a^2\int\frac{\arctan(ax)^2}{a^2x^2+1}dx+2a\int\frac{\arctan(ax)}{x(a^2x^2+1)}dx-\left(a^2\left(\frac{x\arctan}{4(a^2x}\right.\right.\right.}{-a^2\left(-\left(a^2\left(-a\int\frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)\right)-a^2\int\frac{\arctan(ax)^2}{a^2x^2+1}dx+2a\int\frac{\arctan(ax)}{x(a^2x^2+1)}dx-\frac{\arctan(ax)^2}{x}\right.}$$

$$\Big\downarrow \text{5419}$$

$$-\frac{a^2\left(-a^2\left(-a\int\frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)+2a\int\frac{\arctan(ax)}{x(a^2x^2+1)}dx-a^2\left(\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\right.\right.}{-a^2\left(-a^2\left(-a\int\frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)+2a\int\frac{\arctan(ax)}{x(a^2x^2+1)}dx-\frac{1}{3}a\arctan(ax)^3-\frac{\arctan(ax)^2}{x}\right)+}$$

$$\Big\downarrow \text{5459}$$

$$\frac{-a^2\left(-a^2\left(-a\int\frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)+2a\left(i\int\frac{\arctan(ax)}{x(ax+i)}dx-\frac{1}{2}i\arctan(ax)^2\right)-\frac{1}{3}a\arctan(\right.}{a^2\left(-a^2\left(-a\int\frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)+2a\left(i\int\frac{\arctan(ax)}{x(ax+i)}dx-\frac{1}{2}i\arctan(ax)^2\right)-a^2\left(\frac{x\arctan(}{4(a^2x^2+}\right.\right.}$$

$$\Big\downarrow \text{5403}$$

$$\frac{-a^2\left(-a^2\left(-a\int\frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)+2a\left(i\left(ia\int\frac{\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)\log\left(2-\right.\right.\right.\right.}{a^2\left(-a^2\left(-a\int\frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)+2a\left(i\left(ia\int\frac{\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)\log\left(2-\frac{}{1-}\right.\right.\right.\right.}$$

$$\Big\downarrow \text{2897}$$

$$\frac{-a^2\left(-a^2\left(-a\int\frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)+2a\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(\right.\right.\right.\right.}{a^2\left(-a^2\left(-a\int\frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)-a^2\left(\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)}+\right.\right.\right.\right.}$$

$$\Big\downarrow \ 5465$$

$$\frac{-a^2\left(-a^2\left(-a\left(\frac{\int\frac{1}{(a^2x^2+1)^2}dx}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)+2a\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right.\right.\right.\right.}{a^2\left(-a^2\left(-a\left(\frac{\int\frac{1}{(a^2x^2+1)^2}dx}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)-a^2\left(\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\left(-\right.\right.\right.}$$

input
```
Int[ArcTan[a*x]^2/(x^4*(c + a^2*c*x^2)^3),x]
```

output
```
$Aborted
```

## Maple [A] (verified)

Time = 2.08 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.29

| method | result |
|---|---|
| derivativedivides | $a^3\left(\frac{11\arctan(ax)^2a^3x^3}{8c^3(a^2x^2+1)^2}+\frac{13ax\arctan(ax)^2}{8c^3(a^2x^2+1)^2}+\frac{35\arctan(ax)^3}{8c^3}-\frac{\arctan(ax)^2}{3c^3a^3x^3}+\frac{3\arctan(ax)^2}{c^3ax}-\frac{-40\arctan}{}\right.$ |
| default | $a^3\left(\frac{11\arctan(ax)^2a^3x^3}{8c^3(a^2x^2+1)^2}+\frac{13ax\arctan(ax)^2}{8c^3(a^2x^2+1)^2}+\frac{35\arctan(ax)^3}{8c^3}-\frac{\arctan(ax)^2}{3c^3a^3x^3}+\frac{3\arctan(ax)^2}{c^3ax}-\frac{-40\arctan}{}\right.$ |
| parts | $\frac{11\arctan(ax)^2a^6x^3}{8c^3(a^2x^2+1)^2}+\frac{13a^4x\arctan(ax)^2}{8c^3(a^2x^2+1)^2}+\frac{35a^3\arctan(ax)^3}{8c^3}-\frac{\arctan(ax)^2}{3c^3x^3}+\frac{3a^2\arctan(ax)^2}{c^3x}-\frac{2\left(\frac{35a^3\,\mathrm{ar}}{}\right.}{}$ |

input
```
int(arctan(a*x)^2/x^4/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
a^3*(11/8/c^3*arctan(a*x)^2/(a^2*x^2+1)^2*a^3*x^3+13/8*a*x*arctan(a*x)^2/c
^3/(a^2*x^2+1)^2+35/8*arctan(a*x)^3/c^3-1/3/c^3*arctan(a*x)^2/a^3/x^3+3/c^
3*arctan(a*x)^2/a/x-1/12/c^3*(-40*arctan(a*x)*ln(a^2*x^2+1)-3/2*arctan(a*x
)/(a^2*x^2+1)^2-33/2*arctan(a*x)/(a^2*x^2+1)+4*arctan(a*x)/a^2/x^2+80*arct
an(a*x)*ln(a*x)+40*I*ln(a*x)*ln(1+I*a*x)-40*I*ln(a*x)*ln(1-I*a*x)+40*I*dil
og(1+I*a*x)-40*I*dilog(1-I*a*x)-20*I*(ln(a*x-I)*ln(a^2*x^2+1)-1/2*ln(a*x-I
)^2-dilog(-1/2*I*(a*x+I))-ln(a*x-I)*ln(-1/2*I*(a*x+I)))+20*I*(ln(a*x+I)*ln
(a^2*x^2+1)-1/2*ln(a*x+I)^2-dilog(1/2*I*(a*x-I))-ln(a*x+I)*ln(1/2*I*(a*x-I
)))+1/2*(141/8*a^3*x^3+147/8*a*x)/(a^2*x^2+1)^2+205/16*arctan(a*x)+4/a/x+3
5*arctan(a*x)^3))
```

## Fricas [F]

$$\int \frac{\arctan(ax)^2}{x^4\left(c+a^2cx^2\right)^3}\,dx=\int \frac{\arctan\left(ax\right)^2}{\left(a^2cx^2+c\right)^3x^4}\,dx$$

input
```
integrate(arctan(a*x)^2/x^4/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
integral(arctan(a*x)^2/(a^6*c^3*x^10 + 3*a^4*c^3*x^8 + 3*a^2*c^3*x^6 + c^3
*x^4), x)
```

## Sympy [F]

$$\int \frac{\arctan(ax)^2}{x^4\left(c+a^2cx^2\right)^3}\,dx=\frac{\int \frac{\operatorname{atan}^2\left(ax\right)}{a^6x^{10}+3a^4x^8+3a^2x^6+x^4}\,dx}{c^3}$$

input
```
integrate(atan(a*x)**2/x**4/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(atan(a*x)**2/(a**6*x**10 + 3*a**4*x**8 + 3*a**2*x**6 + x**4), x)/
c**3
```

## Maxima [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^2}{x^4\left(c+a^2cx^2\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(arctan(a*x)^2/x^4/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
Timed out
```

## Giac [F]

$$\int \frac{\arctan(ax)^2}{x^4\left(c+a^2cx^2\right)^3}\,dx = \int \frac{\arctan\left(ax\right)^2}{\left(a^2cx^2+c\right)^3x^4}\,dx$$

input
```
integrate(arctan(a*x)^2/x^4/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^2/((a^2*c*x^2 + c)^3*x^4), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^2}{x^4\left(c+a^2cx^2\right)^3}\,dx = \int \frac{\mathrm{atan}(a\,x)^2}{x^4\,\left(c\,a^2\,x^2+c\right)^3}\,dx$$

input
```
int(atan(a*x)^2/(x^4*(c + a^2*c*x^2)^3),x)
```

output
```
int(atan(a*x)^2/(x^4*(c + a^2*c*x^2)^3), x)
```

## Reduce [F]

$$\int \frac{\arctan(ax)^2}{x^4 (c + a^2 c x^2)^3} \, dx$$
$$= \frac{840 atan(ax)^3 a^7 x^7 + 1680 atan(ax)^3 a^5 x^5 + 840 atan(ax)^3 a^3 x^3 + 2520 atan(ax)^2 a^6 x^6 + 4200 atan(ax)^2 a}{}$$

input
```
int(atan(a*x)^2/x^4/(a^2*c*x^2+c)^3,x)
```

output
```
(840*atan(a*x)**3*a**7*x**7 + 1680*atan(a*x)**3*a**5*x**5 + 840*atan(a*x)*
*3*a**3*x**3 + 2520*atan(a*x)**2*a**6*x**6 + 4200*atan(a*x)**2*a**4*x**4 +
 1344*atan(a*x)**2*a**2*x**2 - 192*atan(a*x)**2 - 735*atan(a*x)*a**7*x**7
+ 1050*atan(a*x)*a**5*x**5 + 2625*atan(a*x)*a**3*x**3 + 448*atan(a*x)*a*x
+ 1280*int(atan(a*x)/(a**6*x**9 + 3*a**4*x**7 + 3*a**2*x**5 + x**3),x)*a**
5*x**7 + 2560*int(atan(a*x)/(a**6*x**9 + 3*a**4*x**7 + 3*a**2*x**5 + x**3)
,x)*a**3*x**5 + 1280*int(atan(a*x)/(a**6*x**9 + 3*a**4*x**7 + 3*a**2*x**5
+ x**3),x)*a*x**3 - 735*a**6*x**6 - 385*a**4*x**4 + 448*a**2*x**2)/(576*c*
*3*x**3*(a**4*x**4 + 2*a**2*x**2 + 1))
```

# 3.307 $\int x^3\sqrt{c+a^2cx^2}\arctan(ax)^2\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 385

$$\begin{aligned}\int x^3\sqrt{c+a^2cx^2}\arctan(ax)^2\,dx &= -\frac{11\sqrt{c+a^2cx^2}}{60a^4}+\frac{(c+a^2cx^2)^{3/2}}{30a^4c}\\&+\frac{x\sqrt{c+a^2cx^2}\arctan(ax)}{12a^3}\\&-\frac{x^3\sqrt{c+a^2cx^2}\arctan(ax)}{10a}\\&-\frac{2\sqrt{c+a^2cx^2}\arctan(ax)^2}{15a^4}\\&+\frac{x^2\sqrt{c+a^2cx^2}\arctan(ax)^2}{15a^2}\\&+\frac{1}{5}x^4\sqrt{c+a^2cx^2}\arctan(ax)^2\\&-\frac{11ic\sqrt{1+a^2x^2}\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{30a^4\sqrt{c+a^2cx^2}}\\&+\frac{11ic\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{60a^4\sqrt{c+a^2cx^2}}\\&-\frac{11ic\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{60a^4\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
-11/60*(a^2*c*x^2+c)^(1/2)/a^4+1/30*(a^2*c*x^2+c)^(3/2)/a^4/c+1/12*x*(a^2*
c*x^2+c)^(1/2)*arctan(a*x)/a^3-1/10*x^3*(a^2*c*x^2+c)^(1/2)*arctan(a*x)/a-
2/15*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2/a^4+1/15*x^2*(a^2*c*x^2+c)^(1/2)*ar
ctan(a*x)^2/a^2+1/5*x^4*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2-11/30*I*c*(a^2*x
^2+1)^(1/2)*arctan(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/a^4/(a^2*c
*x^2+c)^(1/2)+11/60*I*c*(a^2*x^2+1)^(1/2)*polylog(2,-I*(1+I*a*x)^(1/2)/(1-
I*a*x)^(1/2))/a^4/(a^2*c*x^2+c)^(1/2)-11/60*I*c*(a^2*x^2+1)^(1/2)*polylog(
2,I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/a^4/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (warning: unable to verify)

Time = 0.91 (sec) , antiderivative size = 360, normalized size of antiderivative = 0.94

$$\int x^3\sqrt{c+a^2cx^2}\arctan(ax)^2\,dx = -\frac{(1+a^2x^2)^2\sqrt{c(1+a^2x^2)}\left(50-32\arctan(ax)^2+72\cos(2\arctan(ax))+160\arctan(ax)^2\cos(2\arctan\right.}{}$$

input
```
Integrate[x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2,x]
```

output
```
-1/960*((1 + a^2*x^2)^2*Sqrt[c*(1 + a^2*x^2)]*(50 - 32*ArcTan[a*x]^2 + 72*
Cos[2*ArcTan[a*x]] + 160*ArcTan[a*x]^2*Cos[2*ArcTan[a*x]] + 22*Cos[4*ArcTa
n[a*x]] - (110*ArcTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2]
 - 55*ArcTan[a*x]*Cos[3*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] - 11*Arc
Tan[a*x]*Cos[5*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] + (110*ArcTan[a*x
]*Log[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + 55*ArcTan[a*x]*Cos[3*A
rcTan[a*x]]*Log[1 + I*E^(I*ArcTan[a*x])] + 11*ArcTan[a*x]*Cos[5*ArcTan[a*x
]]*Log[1 + I*E^(I*ArcTan[a*x])] - ((176*I)*PolyLog[2, (-I)*E^(I*ArcTan[a*x
])])/(1 + a^2*x^2)^(5/2) + ((176*I)*PolyLog[2, I*E^(I*ArcTan[a*x])])/(1 +
a^2*x^2)^(5/2) + 4*ArcTan[a*x]*Sin[2*ArcTan[a*x]] - 22*ArcTan[a*x]*Sin[4*A
rcTan[a*x]]))/a^4
```

## Rubi [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1189 vs. 2(385) = 770.

Time = 5.97 (sec) , antiderivative size = 1189, normalized size of antiderivative = 3.09, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.792$, Rules used = {5485, 5487, 5465, 5425, 5421, 5487, 241, 243, 53, 2009, 5425, 5421, 5465, 5425, 5421, 5487, 241, 5425, 5421}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \arctan(ax)^2 \sqrt{a^2cx^2+c}\,dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\int \frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int \frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5487}$$

$$c\left(-\frac{2\int \frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{2\int \frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)^2\sqrt{a^2cx^2+c}}{3a^2c}\right)+a^2c\left(-\frac{2\int \frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int \frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a^2}+\frac{x^4\arctan(ax)^2\sqrt{a^2cx^2+c}}{5a^2c}\right)$$

$$\Big\downarrow \text{5465}$$

$$c\left(-\frac{2\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}\right)}{3a^2}-\frac{2\int \frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}+\frac{x^2\arctan(ax)^2\sqrt{a^2cx^2+c}}{3a^2c}\right)+a^2c\left(-\frac{2\int \frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int \frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a^2}+\frac{x^4\arctan(ax)^2\sqrt{a^2cx^2+c}}{5a^2c}\right)$$

$$\Big\downarrow \text{5425}$$

$$c\left(-\frac{2\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\int\frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{a\sqrt{a^2cx^2+c}}\right)}{3a^2}-\frac{2\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}+\frac{x^2\arctan(ax)^2\sqrt{a^2cx^2+c}}{3a^2c}\right)+$$

$$a^2c\left(-\frac{2\int\frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int\frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a^2}+\frac{x^4\arctan(ax)^2\sqrt{a^2cx^2+c}}{5a^2c}\right)$$

$$\Big\downarrow \; 5421$$

$$a^2c\left(-\frac{2\int\frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int\frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a^2}+\frac{x^4\arctan(ax)^2\sqrt{a^2cx^2+c}}{5a^2c}\right)+$$

$$c\left(-\frac{2\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{2\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}}{}\right.}{a\sqrt{a^2cx^2+c}}\right.}{3a^2}\right.$$

$$\Big\downarrow \; 5487$$

$$a^2c\left(-\frac{4\left(-\frac{2\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{2\int\frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)^2\sqrt{a^2cx^2+c}}{3a^2c}\right)}{5a^2}-\frac{2\left(-\frac{3\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int\frac{x^3}{\sqrt{a^2cx^2+c}}dx}{4a}+\right.}{5a}\right.$$

$$c\left(-\frac{2\left(-\frac{\int\frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a^2}-\frac{\int\frac{x}{\sqrt{a^2cx^2+c}}dx}{2a}+\frac{x\arctan(ax)\sqrt{a^2cx^2+c}}{2a^2c}\right)}{3a}-\frac{2\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\,\text{arc}}{a}\right.}{}\right.}{}\right.$$

$$\Big\downarrow \; 241$$

$$a^2c\left(-\frac{4\left(-\frac{2\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{2\int\frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)^2\sqrt{a^2cx^2+c}}{3a^2c}\right)}{5a^2}-\frac{2\left(-\frac{3\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int\frac{x^3}{\sqrt{a^2cx^2+c}}dx}{4a}+\right.}{5a}\right.$$

$$c\left(-\frac{2\left(-\frac{\int\frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a^2}+\frac{x\arctan(ax)\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\sqrt{a^2cx^2+c}}{2a^3c}\right)}{3a}-\frac{2\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\right.}{a}\right.}{}\right.}{3a}\right.$$

$\downarrow$ 243

$$a^2c\left(-\frac{4\left(-\frac{2\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{2\int\frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)^2\sqrt{a^2cx^2+c}}{3a^2c}\right)}{5a^2}-\frac{2\left(-\frac{3\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int\frac{x^2}{\sqrt{a^2cx^2+c}}dx^2}{8a}+\right.}{5a}\right.$$

$$c\left(-\frac{2\left(-\frac{\int\frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a^2}+\frac{x\arctan(ax)\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\sqrt{a^2cx^2+c}}{2a^3c}\right)}{3a}-\frac{2\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\right.}{a}\right.}{}\right.}{3a}\right.$$

$\downarrow$ 53

$$a^2c\left(-\frac{4\left(-\frac{2\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{2\int\frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)^2\sqrt{a^2cx^2+c}}{3a^2c}\right)}{5a^2}-\frac{2\left(-\frac{3\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int\left(\frac{\sqrt{a^2cx^2+c}}{a^2c}-\frac{}{a^2\cdots}\right.}{8a}\cdots\right.}{5a\cdots}\right.$$

$$c\left(-\frac{2\left(-\frac{\int\frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a^2}+\frac{x\arctan(ax)\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\sqrt{a^2cx^2+c}}{2a^3c}\right)}{3a}-\frac{2\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\cdots\right.}{a}\cdots\right.}{\cdots}\cdots\right.}{3a\cdots}\right.$$

$\Big\downarrow$ 2009

$$a^2c\left(-\frac{4\left(-\frac{2\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{2\int\frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)^2\sqrt{a^2cx^2+c}}{3a^2c}\right)}{5a^2}-\frac{2\left(-\frac{3\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a^2}+\frac{x^3\arctan(ax)\sqrt{a^2c\cdots}}{4a^2c}\cdots\right.}{5a}\right.$$

$$c\left(-\frac{2\left(-\frac{\int\frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a^2}+\frac{x\arctan(ax)\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\sqrt{a^2cx^2+c}}{2a^3c}\right)}{3a}-\frac{2\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\cdots\right.}{a}\cdots\right.}{\cdots}\cdots\right.}{3a\cdots}\right.$$

$\Big\downarrow$ 5425

$$a^2c\left(-\frac{4\left(-\frac{2\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{2\int\frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)^2\sqrt{a^2cx^2+c}}{3a^2c}\right)}{5a^2}-\frac{2\left(-\frac{3\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a^2}+\frac{x^3\arctan(ax)\sqrt{a^2c}}{4a^2c}\right.}{5a}\right.$$

$$c\left(-\frac{2\left(-\frac{\sqrt{a^2x^2+1}\int\frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{2a^2\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\sqrt{a^2cx^2+c}}{2a^3c}\right)}{3a}-\frac{2\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(a}{}\right.}{}\right.}{}\right.$$

$\downarrow$ 5421

$$a^2c\left(-\frac{4\left(-\frac{2\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{2\int\frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)^2\sqrt{a^2cx^2+c}}{3a^2c}\right)}{5a^2}-\frac{2\left(-\frac{3\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a^2}+\frac{x^3\arctan(ax)\sqrt{a^2c}}{4a^2c}\right.}{5a}\right.$$

$$c\left(-\frac{2\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{a\sqrt{a^2cx^2+c}}\right)}{3a^2}+\frac{x^2}{}\right.$$

$\downarrow$ 5465

$$a^2c\left(-\frac{4\left(-\frac{2\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\int\frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}\right)}{3a^2}-\frac{2\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}+\frac{x^2\arctan(ax)^2\sqrt{a^2cx^2+c}}{3a^2c}\right)}{5a^2}-\frac{2\left(-\frac{3\int\frac{x^2\arctan(ax)\cdots}{\sqrt{a^2cx^2+c}}}{4a^2}\cdots\right.}{\cdots}\right.$$

$$c\left(-\frac{2\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{a\sqrt{a^2cx^2+c}}\right)}{3a^2}+\frac{x^2\cdots}{\cdots}\right.$$

$$\Big\downarrow \; 5425$$

$$a^2c\left(-\frac{4\left(-\frac{2\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\int\frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{a\sqrt{a^2cx^2+c}}\right)}{3a^2}-\frac{2\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}+\frac{x^2\arctan(ax)^2\sqrt{a^2cx^2+c}}{3a^2c}\right)}{5a^2}-\frac{2\left(-\frac{3\int\frac{x^2\cdots}{\sqrt{\cdots}}}{\cdots}\cdots\right.}{\cdots}\right.$$

$$c\left(-\frac{2\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{a\sqrt{a^2cx^2+c}}\right)}{3a^2}+\frac{x^2\cdots}{\cdots}\right.$$

$$\Big\downarrow \; 5421$$

$$c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}-\frac{3\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-\frac{4\left(\frac{x^2\sqrt{\cdots}}{\cdots}\right.}{\cdots}\right.$$

$$c\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)}{2a^2c}-\frac{\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\cdots\right.}{2a^2\sqrt{a^2cx^2+c}}\right.}{3a}\right.$$

$\downarrow$ 5487

$$c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)}{2a^2c}-\frac{\int\frac{x}{\sqrt{a^2cx^2}}}{2a}\right.}{4a^2}\right.}{5a}\right.$$

$$c\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)}{2a^2c}-\frac{\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\right.}{2a^2\sqrt{a^2cx^2+c}}\right.}{3a}\right.$$

$$\downarrow 241$$

$$c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)}{2a^2c}-\frac{\int\frac{\arctan(}{\sqrt{a^2cx^2}}}{2a^2}\right.}{4a^2}\right.}{5a}\right.$$

$$c\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)}{2a^2c}-\frac{\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\right.}{2a^2\sqrt{a^2cx^2+c}}\right.}{3a}\right.$$

$\downarrow$ 5425

$$c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)}{2a^2c}-\frac{\sqrt{a^2x^2+1}}{2a^2}\right.}{4a^2}\right.}{5a}\right.$$

$$c\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)}{2a^2c}-\frac{\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\right.}{2a^2\sqrt{a^2cx^2+c}}\right.}{3a}\right.$$

$\downarrow$ 5421

$$c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)}{2a^2c}-\frac{\sqrt{a^2x^2+1}\cdots}{\cdots}\right.}{\cdots}\right.}{5\cdots}\right.$$

$$c\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)}{2a^2c}-\frac{\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\cdots\right.}{2a^2\sqrt{a^2cx^2+c}}\right.}{3a}\right.$$

input
```
Int[x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2,x]
```

output
```
c*((x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(3*a^2*c) - (2*(-1/2*Sqrt[c + a
^2*c*x^2]/(a^3*c) + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(2*a^2*c) - (Sqrt[
1 + a^2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])
/a + (I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*PolyLog
[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/(2*a^2*Sqrt[c + a^2*c*x^2]))
)/(3*a) - (2*((Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(a^2*c) - (2*Sqrt[1 + a^
2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/a + (
I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*PolyLog[2, (I
*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/(a*Sqrt[c + a^2*c*x^2])))/(3*a^2))
 + a^2*c*((x^4*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(5*a^2*c) - (2*(-1/8*((-
2*Sqrt[c + a^2*c*x^2])/(a^4*c) + (2*(c + a^2*c*x^2)^(3/2))/(3*a^4*c^2))/a
+ (x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(4*a^2*c) - (3*(-1/2*Sqrt[c + a^2*
c*x^2]/(a^3*c) + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(2*a^2*c) - (Sqrt[1 +
 a^2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/a
+ (I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*PolyLog[2,
 (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/(2*a^2*Sqrt[c + a^2*c*x^2])))/(
4*a^2)))/(5*a) - (4*((x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(3*a^2*c) - (
2*(-1/2*Sqrt[c + a^2*c*x^2]/(a^3*c) + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/
(2*a^2*c) - (Sqrt[1 + a^2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]
/Sqrt[1 - I*a*x]])/a + (I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*...
```

### Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5487
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcTan[c*x])^p/(c^2*d*m)), x] + (-Simp[b*f*(p/(c*m))   Int[(f*x)^(m - 1)*((
a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[f^2*((m - 1)/(c^
2*m))   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2]), x], x])
/; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

## Maple [A] (verified)

Time = 2.02 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.61

| method | result |
|---|---|
| default | $\frac{\sqrt{c(ax-i)(ax+i)}\,\left(12a^4\arctan(ax)^2x^4-6\arctan(ax)x^3a^3+4\arctan(ax)^2x^2a^2+2a^2x^2+5\arctan(ax)ax-8\arctan(ax)^2-9\right)}{60a^4} - \frac{11}{}$ |

input
```
int(x^3*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/60/a^4*(c*(a*x-I)*(a*x+I))^(1/2)*(12*a^4*arctan(a*x)^2*x^4-6*arctan(a*x)
*x^3*a^3+4*arctan(a*x)^2*x^2*a^2+2*a^2*x^2+5*arctan(a*x)*a*x-8*arctan(a*x)
^2-9)-11/60*(c*(a*x-I)*(a*x+I))^(1/2)*(arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x
^2+1)^(1/2))-arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+I*dilog(1-I*(
1+I*a*x)/(a^2*x^2+1)^(1/2))-I*dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/a^4/
(a^2*x^2+1)^(1/2)
```

## Fricas [F]

$$\int x^3\sqrt{c+a^2cx^2}\arctan(ax)^2\,dx = \int \sqrt{a^2cx^2+c}\,x^3\arctan{(ax)}^2\,dx$$

input
```
integrate(x^3*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^3*arctan(a*x)^2, x)
```

## Sympy [F]

$$\int x^3\sqrt{c+a^2cx^2}\arctan(ax)^2\,dx = \int x^3\sqrt{c\,(a^2x^2+1)}\,\mathrm{atan}^2\,(ax)\,dx$$

input
```
integrate(x**3*(a**2*c*x**2+c)**(1/2)*atan(a*x)**2,x)
```

output
```
Integral(x**3*sqrt(c*(a**2*x**2 + 1))*atan(a*x)**2, x)
```

## Maxima [F]

$$\int x^3\sqrt{c+a^2cx^2}\arctan(ax)^2\,dx=\int\sqrt{a^2cx^2+c}x^3\arctan{(ax)}^2\;dx$$

input
```
integrate(x^3*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*x^3*arctan(a*x)^2, x)
```

## Giac [F(-2)]

Exception generated.

$$\int x^3\sqrt{c+a^2cx^2}\arctan(ax)^2\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{c+a^2cx^2}\arctan(ax)^2\,dx=\int x^3\,\mathrm{atan}(a\,x)^2\,\sqrt{c\,a^2\,x^2+c}\,dx$$

input
```
int(x^3*atan(a*x)^2*(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(x^3*atan(a*x)^2*(c + a^2*c*x^2)^(1/2), x)
```

## Reduce [F]

$$\int x^3\sqrt{c+a^2cx^2}\arctan(ax)^2\,dx=\sqrt{c}\left(\int\sqrt{a^2x^2+1}\,atan(ax)^2\,x^3dx\right)$$

input
```
int(x^3*(a^2*c*x^2+c)^(1/2)*atan(a*x)^2,x)
```

output
```
sqrt(c)*int(sqrt(a**2*x**2 + 1)*atan(a*x)**2*x**3,x)
```

# 3.308 $\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^2\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 436

$$\begin{aligned}\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^2\,dx &= \frac{x\sqrt{c+a^2cx^2}}{12a^2}+\frac{\sqrt{c+a^2cx^2}\arctan(ax)}{12a^3}\\
&-\frac{x^2\sqrt{c+a^2cx^2}\arctan(ax)}{6a}\\
&+\frac{x\sqrt{c+a^2cx^2}\arctan(ax)^2}{8a^2}\\
&+\frac{1}{4}x^3\sqrt{c+a^2cx^2}\arctan(ax)^2\\
&+\frac{ic\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2}{4a^3\sqrt{c+a^2cx^2}}\\
&-\frac{\sqrt{c}\,\mathrm{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right)}{6a^3}\\
&-\frac{ic\sqrt{1+a^2x^2}\arctan(ax)\,\mathrm{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)}{4a^3\sqrt{c+a^2cx^2}}\\
&+\frac{ic\sqrt{1+a^2x^2}\arctan(ax)\,\mathrm{PolyLog}\left(2,ie^{i\arctan(ax)}\right)}{4a^3\sqrt{c+a^2cx^2}}\\
&+\frac{c\sqrt{1+a^2x^2}\,\mathrm{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)}{4a^3\sqrt{c+a^2cx^2}}\\
&-\frac{c\sqrt{1+a^2x^2}\,\mathrm{PolyLog}\left(3,ie^{i\arctan(ax)}\right)}{4a^3\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
1/12*x*(a^2*c*x^2+c)^(1/2)/a^2+1/12*(a^2*c*x^2+c)^(1/2)*arctan(a*x)/a^3-1/
6*x^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)/a+1/8*x*(a^2*c*x^2+c)^(1/2)*arctan(a
*x)^2/a^2+1/4*x^3*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2+1/4*I*c*(a^2*x^2+1)^(1
/2)*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^2/a^3/(a^2*c*x^2+c)^(1
/2)-1/6*c^(1/2)*arctanh(a*c^(1/2)*x/(a^2*c*x^2+c)^(1/2))/a^3-1/4*I*c*(a^2*
x^2+1)^(1/2)*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a^3/(a^
2*c*x^2+c)^(1/2)+1/4*I*c*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(2,I*(1+I*a*
x)/(a^2*x^2+1)^(1/2))/a^3/(a^2*c*x^2+c)^(1/2)+1/4*c*(a^2*x^2+1)^(1/2)*poly
log(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a^3/(a^2*c*x^2+c)^(1/2)-1/4*c*(a^2*x
^2+1)^(1/2)*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a^3/(a^2*c*x^2+c)^(1/
2)
```

## Mathematica [A] (warning: unable to verify)

Time = 0.91 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.61

$$\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^2\,dx$$
$$= \frac{\sqrt{c+a^2cx^2}\left(8\left(-2\coth^{-1}\left(\frac{ax}{\sqrt{1+a^2x^2}}\right)+3i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2-3i\arctan(ax)\operatorname{PolyLog}\left(2,-\right.\right.\right.}{}$$

input
```
Integrate[x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2,x]
```

output
```
(Sqrt[c + a^2*c*x^2]*(8*(-2*ArcCoth[(a*x)/Sqrt[1 + a^2*x^2]] + (3*I)*ArcTa
n[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 - (3*I)*ArcTan[a*x]*PolyLog[2, (-I)*E^(
I*ArcTan[a*x])] + (3*I)*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] + 3*Po
lyLog[3, (-I)*E^(I*ArcTan[a*x])] - 3*PolyLog[3, I*E^(I*ArcTan[a*x])]) + (1
 + a^2*x^2)^(3/2)*(ArcTan[a*x]*(2 + 6*Sqrt[1 + a^2*x^2]*Cos[3*ArcTan[a*x]]
) - 3*ArcTan[a*x]^2*(-7*a*x + Sqrt[1 + a^2*x^2]*Sin[3*ArcTan[a*x]]) + 2*(a
*x + Sqrt[1 + a^2*x^2]*Sin[3*ArcTan[a*x]]))))/(96*a^3*Sqrt[1 + a^2*x^2])
```

## Rubi [A] (verified)

Time = 5.94 (sec) , antiderivative size = 706, normalized size of antiderivative = 1.62, number of steps used = 26, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.042, Rules used = {5485, 5487, 5425, 5423, 3042, 4669, 3011, 2720, 5465, 224, 219, 5487, 262, 224, 219, 5425, 5423, 3042, 4669, 3011, 2720, 5465, 224, 219, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arctan(ax)^2\sqrt{a^2cx^2+c}\,dx$$

$$\Big\downarrow \text{5485}$$

$$c\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+a^2c\int \frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5487}$$

$$c\left(-\frac{\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a^2}+\frac{x\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a^2c}\right)+a^2c\left(-\frac{3\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}+\frac{x^3\arctan(ax)^2\sqrt{a^2cx^2+c}}{4a^2c}\right)$$

$$\Big\downarrow \text{5425}$$

$$c\left(-\frac{\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{2a^2\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a^2c}\right)+a^2c\left(-\frac{3\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}+\frac{x^3\arctan(ax)^2\sqrt{a^2cx^2+c}}{4a^2c}\right)$$

$$\Big\downarrow \text{5423}$$

$$a^2c\left(-\frac{3\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}+\frac{x^3\arctan(ax)^2\sqrt{a^2cx^2+c}}{4a^2c}\right)+c\left(-\frac{\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{\sqrt{a^2x^2+1}\int \sqrt{a^2x^2+1}\arctan(ax)^2d\arctan(ax)}{2a^3\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a^2c}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^2c\left(-\frac{3\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}+\frac{x^3\arctan(ax)^2\sqrt{a^2cx^2+c}}{4a^2c}\right)+$$
$$c\left(-\frac{\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{2a^3\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a^2c}\right.$$

$\downarrow$ 4669

$$a^2c\left(-\frac{3\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}+\frac{x^3\arctan(ax)^2\sqrt{a^2cx^2+c}}{4a^2c}\right)+$$
$$c\left(-\frac{\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{\sqrt{a^2x^2+1}\left(-2\int \arctan(ax)\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+2\int \arctan(ax)\log\left(1+\right.\right.}{2a^3\sqrt{a^2cx^2+c}}\right.$$

$\downarrow$ 3011

$$a^2c\left(-\frac{3\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}+\frac{x^3\arctan(ax)^2\sqrt{a^2cx^2+c}}{4a^2c}\right)+$$
$$c\left(-\frac{\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-i\int \operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\,\mathrm{a}\right.\right.}{}\right.$$

$\downarrow$ 2720

$$a^2c\left(-\frac{3\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}+\frac{x^3\arctan(ax)^2\sqrt{a^2cx^2+c}}{4a^2c}\right)+$$
$$c\left(-\frac{\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\,\mathrm{ar}}\right.\right.\right.}{}\right.$$

$\downarrow$ 5465

$$a^2c\left(-\frac{3\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}+\frac{x^3\arctan(ax)^2\sqrt{a^2cx^2+c}}{4a^2c}\right)+$$
$$c\left(-\frac{\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\int \frac{1}{\sqrt{a^2cx^2+c}}dx}{a}}{a}-\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\,\mathrm{F}\right.\right.}{}\right.$$

$\downarrow$ 224

$$a^2c\left(-\frac{3\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}+\frac{x^3\arctan(ax)^2\sqrt{a^2cx^2+c}}{4a^2c}\right)+$$

$$c\left(-\frac{\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\int \frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}}{a}}{a}-\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\,\mathrm{ar}}\right.\right.}{}\right.$$

$\downarrow$ 219

$$a^2c\left(-\frac{3\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}+\frac{x^3\arctan(ax)^2\sqrt{a^2cx^2+c}}{4a^2c}\right)+$$

$$c\left(-\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(a}\right.\right.}{}\right.$$

$\downarrow$ 5487

$$a^2c\left(-\frac{-\frac{2\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}-\frac{\int \frac{x^2}{\sqrt{a^2cx^2+c}}dx}{3a}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}}{2a}-\frac{3\left(-\frac{\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a^2}+\frac{x\arctan(ax)^2}{2a^2c}\right.}{4a^2}\right.$$

$$c\left(-\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(a}\right.\right.}{}\right.$$

$\downarrow$ 262

$$a^2c\left(-\frac{-\frac{2\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int \frac{1}{\sqrt{a^2cx^2+c}}dx}{2a^2}}{3a}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}}{2a}-\frac{3\left(-\frac{\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a^2}+\right.}{4a^2}\right.$$

$$c\left(-\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(a}\right.\right.}{}\right.$$

$\downarrow$ 224

$$a^2c\left(-\frac{-\frac{2\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int\frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}}{2a^2}}{3a}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}}{2a}-\frac{3\left(-\frac{\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{\int\frac{\arctan(a}{\sqrt{a^2cx^2}}}{2a^2}\right.}{4a}\right.$$

$$c\left(-\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(a}\right.\right.}{}\right.$$

$\downarrow$ 219

$$a^2c\left(-\frac{3\left(-\frac{\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{\int\frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a^2}+\frac{x\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a^2c}\right)}{4a^2}-\frac{-\frac{2\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}-\frac{x}{}}{2a}\right.$$

$$c\left(-\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(a}\right.\right.}{}\right.$$

$\downarrow$ 5425

$$a^2c\left(-\frac{3\left(-\frac{\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{\sqrt{a^2x^2+1}\int\frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{2a^2\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a^2c}\right)}{4a^2}-\frac{-\frac{2\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)\sqrt{a^2cx}}{3a^2c}}{2}\right.$$

$$c\left(-\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(a}\right.\right.}{}\right.$$

$\downarrow$ 5423

$$a^2c\left(-\frac{-\frac{2\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}}{2a}-\frac{3\left(-\frac{\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{\sqrt{a^2x^2+1}\int\right.}{}\right.$$

$$c\left(-\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(a}\right.\right.}{}\right.$$

$\downarrow$ 3042

$$a^2c\left(-\frac{-\frac{2\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}}{2a}-\frac{3\left(-\frac{\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{\sqrt{a^2x^2+1}\int\right.}{}\right.$$

$$c\left(-\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(a}\right.\right.}{}\right.$$

$\downarrow$ 4669

$$c\left(-\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(a}\right.\right.}{}\right.$$

$$a^2c\left(-\frac{-\frac{2\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}}{2a}-\frac{3\left(-\frac{\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{\sqrt{a^2x^2+1}(-}{}\right.}{}\right.$$

$\downarrow$ 3011

$$c\left(-\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\,\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\,\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(a\cdots}\right.\right.}{\cdots}\right.$$

$$a^2c\left(-\frac{-\frac{2\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}}{2a}-\frac{3\left(-\frac{\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{\sqrt{a^2x^2+1}(2\cdots}{\cdots}\right.}{\cdots}\right.$$

$\downarrow$ 2720

$$c\left(-\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\,\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\,\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(a\cdots}\right.\right.}{\cdots}\right.$$

$$a^2c\left(-\frac{-\frac{2\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}}{2a}-\frac{3\left(-\frac{\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{\sqrt{a^2x^2+1}(2\cdots}{\cdots}\right.}{\cdots}\right.$$

$\downarrow$ 5465

$$c\left(-\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\,\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\,\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(a\cdots}\right.\right.}{\cdots}\right.$$

$$a^2c\left(-\frac{-\frac{2\left(\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\int\frac{1}{\sqrt{a^2cx^2+c}}dx}{a}\right)}{3a^2}+\frac{x^2\arctan(ax)\sqrt{a^2cx^2+c}}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}}{2a}-\frac{3\left(-\frac{\arctan(a\cdots}{\cdots}\right.}{\cdots}\right.$$

$\downarrow$ 224

$$c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^2}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a^2c}-\frac{\int \frac{}{1-}}{}\right.}{3a^2}}{2a}\right.$$

$$c\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a^2\sqrt{c}}}{a}-\frac{\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)a\right.}{}\right.$$

$\downarrow$ 219

$$c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^2}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a^2c}-\frac{\operatorname{arct}}{}\right.}{3a^2}}{2a}\right.$$

$$c\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a^2\sqrt{c}}}{a}-\frac{\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)a\right.}{}\right.$$

$\downarrow$ 7143

$$c\left(-\frac{\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a^2\sqrt{c}}}{a}+\frac{x\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\right.\right.}{}\right.$$

$$a^2c\left(\frac{x^3\arctan(ax)^2\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(-\frac{\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a^2\sqrt{c}}}{a}+\frac{x\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\sqrt{a^2x^2+1}\left(2\left(\right.\right.}{}\right.}{}\right.$$

input
```
Int[x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2,x]
```

output
```
a^2*c*((x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(4*a^2*c) - ((x^2*Sqrt[c +
a^2*c*x^2]*ArcTan[a*x])/(3*a^2*c) - ((x*Sqrt[c + a^2*c*x^2])/(2*a^2*c) - A
rcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]]/(2*a^3*Sqrt[c]))/(3*a) - (2*((Sq
rt[c + a^2*c*x^2]*ArcTan[a*x])/(a^2*c) - ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^
2*c*x^2]]/(a^2*Sqrt[c])))/(3*a^2))/(2*a) - (3*((x*Sqrt[c + a^2*c*x^2]*ArcT
an[a*x]^2)/(2*a^2*c) - ((Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(a^2*c) - ArcTan
h[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]]/(a^2*Sqrt[c]))/a - (Sqrt[1 + a^2*x^2]
*((-2*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 + 2*(I*ArcTan[a*x]*PolyLo
g[2, (-I)*E^(I*ArcTan[a*x])] - PolyLog[3, (-I)*E^(I*ArcTan[a*x])]) - 2*(I*
ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] - PolyLog[3, I*E^(I*ArcTan[a*x
])])))/(2*a^3*Sqrt[c + a^2*c*x^2])))/(4*a^2)) + c*((x*Sqrt[c + a^2*c*x^2]*
ArcTan[a*x]^2)/(2*a^2*c) - ((Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(a^2*c) - Ar
cTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]]/(a^2*Sqrt[c]))/a - (Sqrt[1 + a^2*
x^2]*((-2*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 + 2*(I*ArcTan[a*x]*Po
lyLog[2, (-I)*E^(I*ArcTan[a*x])] - PolyLog[3, (-I)*E^(I*ArcTan[a*x])]) - 2
*(I*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] - PolyLog[3, I*E^(I*ArcTan
[a*x])])))/(2*a^3*Sqrt[c + a^2*c*x^2]))
```

## Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5487
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcTan[c*x])^p/(c^2*d*m)), x] + (-Simp[b*f*(p/(c*m))   Int[(f*x)^(m - 1)*((
a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[f^2*((m - 1)/(c^
2*m))   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2]), x], x])
/; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

## Maple [A] (verified)

Time = 2.12 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.69

| method | result |
|---|---|
| default | $\frac{\sqrt{c(ax-i)(ax+i)}\,\left(6a^3\arctan(ax)^2x^3-4x^2a^2\arctan(ax)+3a\arctan(ax)^2x+2ax+2\arctan(ax)\right)}{24a^3}+\frac{i\sqrt{c(ax-i)(ax+i)}\,\left(3i\arctan\right.}{}$ |

input
```
int(x^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/24/a^3*(c*(a*x-I)*(a*x+I))^(1/2)*(6*a^3*arctan(a*x)^2*x^3-4*x^2*a^2*arct
an(a*x)+3*a*arctan(a*x)^2*x+2*a*x+2*arctan(a*x))+1/24*I*(c*(a*x-I)*(a*x+I)
)^(1/2)*(3*I*arctan(a*x)^2*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*I*arctan(
a*x)^2*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*arctan(a*x)*polylog(2,I*(1+I*
a*x)/(a^2*x^2+1)^(1/2))-6*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(
1/2))+6*I*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I*polylog(3,-I*(1+I*a
*x)/(a^2*x^2+1)^(1/2))+8*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2)))/a^3/(a^2*x^2
+1)^(1/2)
```

## Fricas [F]

$$\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^2\,dx=\int\sqrt{a^2cx^2+c}x^2\arctan\left(ax\right)^2\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^2*arctan(a*x)^2, x)
```

## Sympy [F]

$$\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^2\,dx=\int x^2\sqrt{c\left(a^2x^2+1\right)}\operatorname{atan}^2\left(ax\right)dx$$

input
```
integrate(x**2*(a**2*c*x**2+c)**(1/2)*atan(a*x)**2,x)
```

output
```
Integral(x**2*sqrt(c*(a**2*x**2 + 1))*atan(a*x)**2, x)
```

## Maxima [F]

$$\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^2\,dx=\int \sqrt{a^2cx^2+c}x^2\arctan\left(ax\right)^2\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*x^2*arctan(a*x)^2, x)
```

## Giac [F]

$$\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^2\,dx=\int \sqrt{a^2cx^2+c}x^2\arctan\left(ax\right)^2\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*x^2*arctan(a*x)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^2\,dx=\int x^2\,\mathrm{atan}(a\,x)^2\,\sqrt{c\,a^2\,x^2+c}\,dx$$

input
```
int(x^2*atan(a*x)^2*(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(x^2*atan(a*x)^2*(c + a^2*c*x^2)^(1/2), x)
```

## Reduce [F]

$$\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^2\,dx=\sqrt{c}\left(\int\sqrt{a^2x^2+1}\,atan(ax)^2\,x^2dx\right)$$

input
```
int(x^2*(a^2*c*x^2+c)^(1/2)*atan(a*x)^2,x)
```

output
```
sqrt(c)*int(sqrt(a**2*x**2 + 1)*atan(a*x)**2*x**2,x)
```

# 3.309 $\int x\sqrt{c+a^2cx^2}\arctan(ax)^2\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 279

$$\int x\sqrt{c+a^2cx^2}\arctan(ax)^2\,dx = \frac{\sqrt{c+a^2cx^2}}{3a^2} - \frac{x\sqrt{c+a^2cx^2}\arctan(ax)}{3a} + \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^2}{3a^2c} + \frac{2ic\sqrt{1+a^2x^2}\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3a^2\sqrt{c+a^2cx^2}} - \frac{ic\sqrt{1+a^2x^2}\,\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3a^2\sqrt{c+a^2cx^2}} + \frac{ic\sqrt{1+a^2x^2}\,\text{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3a^2\sqrt{c+a^2cx^2}}$$

output
```
1/3*(a^2*c*x^2+c)^(1/2)/a^2-1/3*x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)/a+1/3*(a
^2*c*x^2+c)^(3/2)*arctan(a*x)^2/a^2/c+2/3*I*c*(a^2*x^2+1)^(1/2)*arctan(a*x
)*arctan((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/a^2/(a^2*c*x^2+c)^(1/2)-1/3*I*c*
(a^2*x^2+1)^(1/2)*polylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/a^2/(a^2*c
*x^2+c)^(1/2)+1/3*I*c*(a^2*x^2+1)^(1/2)*polylog(2,I*(1+I*a*x)^(1/2)/(1-I*a
*x)^(1/2))/a^2/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (warning: unable to verify)

Time = 0.45 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.93

$$\int x\sqrt{c+a^2cx^2}\arctan(ax)^2\,dx$$
$$= \frac{(1+a^2x^2)\sqrt{c(1+a^2x^2)}\left(2+4\arctan(ax)^2+2\cos(2\arctan(ax))-\frac{3\arctan(ax)\log\left(1-ie^{i\arctan(ax)}\right)}{\sqrt{1+a^2x^2}}-\arctan\right.}{}$$

input
```
Integrate[x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2,x]
```

output
```
((1 + a^2*x^2)*Sqrt[c*(1 + a^2*x^2)]*(2 + 4*ArcTan[a*x]^2 + 2*Cos[2*ArcTan
[a*x]] - (3*ArcTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] -
ArcTan[a*x]*Cos[3*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] + (3*ArcTan[a*
x]*Log[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + ArcTan[a*x]*Cos[3*Arc
Tan[a*x]]*Log[1 + I*E^(I*ArcTan[a*x])] - ((4*I)*PolyLog[2, (-I)*E^(I*ArcTa
n[a*x])])/(1 + a^2*x^2)^(3/2) + ((4*I)*PolyLog[2, I*E^(I*ArcTan[a*x])])/(1
 + a^2*x^2)^(3/2) - 2*ArcTan[a*x]*Sin[2*ArcTan[a*x]]))/(12*a^2)
```

## Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.81, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {5465, 5413, 5425, 5421}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\arctan(ax)^2\sqrt{a^2cx^2+c}\,dx$$
$$\Big\downarrow \text{5465}$$
$$\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{2\int\sqrt{a^2cx^2+c}\arctan(ax)dx}{3a}$$
$$\Big\downarrow \text{5413}$$

$$\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{2\left(\frac{1}{2}c\int\frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)}{3a}$$

$\downarrow$ 5425

$$\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{2\left(\frac{c\sqrt{a^2x^2+1}\int\frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)}{3a}$$

$\downarrow$ 5421

$$\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{2\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a}}{}\right.}{3a}$$

input
```
Int[x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2,x]
```

output
```
((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/(3*a^2*c) - (2*(-1/2*Sqrt[c + a^2*c*
x^2]/a + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/2 + (c*Sqrt[1 + a^2*x^2]*(((-
2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/a + (I*PolyLog[2
, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*PolyLog[2, (I*Sqrt[1 + I
*a*x])/Sqrt[1 - I*a*x]])/a))/(2*Sqrt[c + a^2*c*x^2])))/(3*a)
```

## Defintions of rubi rules used

rule 5413
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Simp[x*(d + e*x^2)
^q*((a + b*ArcTan[c*x])/(2*q + 1)), x] + Simp[2*d*(q/(2*q + 1))   Int[(d +
e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0]
```

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

## Maple [A] (verified)

Time = 1.80 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.71

| method | result |
|---|---|
| default | $\frac{\sqrt{c(ax-i)(ax+i)}\left(\arctan(ax)^2x^2a^2-\arctan(ax)ax+\arctan(ax)^2+1\right)}{3a^2}+\frac{\sqrt{c(ax-i)(ax+i)}\left(\arctan(ax)\ln\left(1+\frac{i(iax+1)}{\sqrt{a^2x^2+1}}\right)-\operatorname{arcta}\right.}{}$ |

input
```
int(x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3/a^2*(c*(a*x-I)*(a*x+I))^(1/2)*(arctan(a*x)^2*x^2*a^2-arctan(a*x)*a*x+a
rctan(a*x)^2+1)+1/3*(c*(a*x-I)*(a*x+I))^(1/2)*(arctan(a*x)*ln(1+I*(1+I*a*x
)/(a^2*x^2+1)^(1/2))-arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+I*dil
og(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2
)))/a^2/(a^2*x^2+1)^(1/2)
```

## Fricas [F]

$$\int x\sqrt{c+a^2cx^2}\arctan(ax)^2\,dx=\int \sqrt{a^2cx^2+c}x\arctan\left(ax\right)^2\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x*arctan(a*x)^2, x)
```

## Sympy [F]

$$\int x\sqrt{c+a^2cx^2}\arctan(ax)^2\,dx=\int x\sqrt{c\left(a^2x^2+1\right)}\operatorname{atan}^2\left(ax\right)dx$$

input
```
integrate(x*(a**2*c*x**2+c)**(1/2)*atan(a*x)**2,x)
```

output
```
Integral(x*sqrt(c*(a**2*x**2 + 1))*atan(a*x)**2, x)
```

## Maxima [F]

$$\int x\sqrt{c+a^2cx^2}\arctan(ax)^2\,dx=\int \sqrt{a^2cx^2+c}x\arctan\left(ax\right)^2\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*x*arctan(a*x)^2, x)
```

## Giac [F(-2)]

Exception generated.

$$\int x\sqrt{c+a^2cx^2}\arctan(ax)^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int x\sqrt{c+a^2cx^2}\arctan(ax)^2\,dx = \int x\,\mathrm{atan}(a\,x)^2\,\sqrt{c\,a^2\,x^2+c}\,dx$$

input
```
int(x*atan(a*x)^2*(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(x*atan(a*x)^2*(c + a^2*c*x^2)^(1/2), x)
```

## Reduce [F]

$$\int x\sqrt{c+a^2cx^2}\arctan(ax)^2\,dx = \sqrt{c}\left(\int\sqrt{a^2x^2+1}\,atan(ax)^2\,xdx\right)$$

input
```
int(x*(a^2*c*x^2+c)^(1/2)*atan(a*x)^2,x)
```

output
```
sqrt(c)*int(sqrt(a**2*x**2 + 1)*atan(a*x)**2*x,x)
```

# 3.310 $\int \sqrt{c+a^2cx^2}\arctan(ax)^2\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 340

$$\begin{aligned}\int \sqrt{c+a^2cx^2}\arctan(ax)^2\,dx = &-\frac{\sqrt{c+a^2cx^2}\arctan(ax)}{a}+\frac{1}{2}x\sqrt{c+a^2cx^2}\arctan(ax)^2\\ &-\frac{ic\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2}{a\sqrt{c+a^2cx^2}}\\ &+\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right)}{a}\\ &+\frac{ic\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)}{a\sqrt{c+a^2cx^2}}\\ &-\frac{ic\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,ie^{i\arctan(ax)}\right)}{a\sqrt{c+a^2cx^2}}\\ &-\frac{c\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)}{a\sqrt{c+a^2cx^2}}\\ &+\frac{c\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,ie^{i\arctan(ax)}\right)}{a\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
-(a^2*c*x^2+c)^(1/2)*arctan(a*x)/a+1/2*x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2
-I*c*(a^2*x^2+1)^(1/2)*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^2/a
/(a^2*c*x^2+c)^(1/2)+c^(1/2)*arctanh(a*c^(1/2)*x/(a^2*c*x^2+c)^(1/2))/a+I*
c*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/
a/(a^2*c*x^2+c)^(1/2)-I*c*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(2,I*(1+I*a
*x)/(a^2*x^2+1)^(1/2))/a/(a^2*c*x^2+c)^(1/2)-c*(a^2*x^2+1)^(1/2)*polylog(3
,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a/(a^2*c*x^2+c)^(1/2)+c*(a^2*x^2+1)^(1/2)
*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.59

$$\int \sqrt{c+a^2cx^2}\arctan(ax)^2\,dx$$
$$=\frac{\sqrt{c\left(1+a^2x^2\right)}\left(2\coth^{-1}\left(\frac{ax}{\sqrt{1+a^2x^2}}\right)-2\sqrt{1+a^2x^2}\arctan(ax)+ax\sqrt{1+a^2x^2}\arctan(ax)^2-2i\arctan\left(e\right.\right.}{}$$

input
```
Integrate[Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2,x]
```

output
```
(Sqrt[c*(1 + a^2*x^2)]*(2*ArcCoth[(a*x)/Sqrt[1 + a^2*x^2]] - 2*Sqrt[1 + a^
2*x^2]*ArcTan[a*x] + a*x*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2 - (2*I)*ArcTan[E^
(I*ArcTan[a*x])]*ArcTan[a*x]^2 + (2*I)*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*Ar
cTan[a*x])] - (2*I)*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] - 2*PolyLo
g[3, (-I)*E^(I*ArcTan[a*x])] + 2*PolyLog[3, I*E^(I*ArcTan[a*x])]))/(2*a*Sq
rt[1 + a^2*x^2])
```

## Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.67, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {5415, 224, 219, 5425, 5423, 3042, 4669, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(ax)^2\sqrt{a^2cx^2+c}\,dx$$

$$\Big\downarrow \text{5415}$$

$$\frac{1}{2}c\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int \frac{1}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{2}c\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int \frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}c\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}$$

$$\Big\downarrow \text{5425}$$

$$\frac{c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}$$

$$\Big\downarrow \text{5423}$$

$$\frac{c\sqrt{a^2x^2+1}\int \sqrt{a^2x^2+1}\arctan(ax)^2d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{c\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}$$

$$\Big\downarrow \text{4669}$$

$$\frac{c\sqrt{a^2x^2+1}\left(-2\int \arctan(ax)\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+2\int \arctan(ax)\log\left(1+ie^{i\arctan(ax)}\right)d\arctan(\right.}{2a\sqrt{a^2cx^2+c}}$$
$$\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}$$

$\Big\downarrow$ 3011

$$\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-i\int \operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(ax)\right)-2\left(i\arcta\right.\right.}{2a\sqrt{a}}$$
$$\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}$$

$\Big\downarrow$ 2720

$$\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)\right.}{}$$
$$\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}$$

$\Big\downarrow$ 7143

$$\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)\right)-2\left(i\arctan(ax)\operatorname{PolyLog}\left(\right.\right.\right.}{2a\sqrt{a^2cx^2+c}}$$
$$\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}$$

input
```
Int[Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2,x]
```

output
```
-((Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/a) + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x
]^2)/2 + (Sqrt[c]*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/a + (c*Sqrt[
1 + a^2*x^2]*((-2*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 + 2*(I*ArcTan
[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - PolyLog[3, (-I)*E^(I*ArcTan[a*x
])]) - 2*(I*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] - PolyLog[3, I*E^(
I*ArcTan[a*x])])))/(2*a*Sqrt[c + a^2*c*x^2])
```

### Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5415
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTan[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x])^p,
x], x] + Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*(
a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[q, 0] && GtQ[p, 1]
```

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

## Maple [A] (verified)

Time = 1.97 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.79

| method | result |
|---|---|
| default | $\frac{\sqrt{c(ax-i)(ax+i)}\, \arctan(ax)(\arctan(ax)ax-2)}{2a} - \frac{i\sqrt{c(ax-i)(ax+i)}\left(i\arctan(ax)^2\ln\left(1-\frac{i(iax+1)}{\sqrt{a^2x^2+1}}\right)-i\arctan(ax)^2\ln\left(1+\frac{i(i}{\sqrt{a}}\right.\right.}{}$ |

input
```
int((a^2*c*x^2+c)^(1/2)*arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/a*(c*(a*x-I)*(a*x+I))^(1/2)*arctan(a*x)*(arctan(a*x)*a*x-2)-1/2*I*(c*(
a*x-I)*(a*x+I))^(1/2)*(I*arctan(a*x)^2*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))
-I*arctan(a*x)^2*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*arctan(a*x)*polylog
(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^
2*x^2+1)^(1/2))+2*I*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*I*polylog(3
,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+4*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2)))/a/
(a^2*x^2+1)^(1/2)
```

## Fricas [F]

$$\int \sqrt{c+a^2cx^2}\arctan(ax)^2\,dx = \int \sqrt{a^2cx^2+c}\arctan\left(ax\right)^2\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)*arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2, x)
```

## Sympy [F]

$$\int \sqrt{c+a^2cx^2}\arctan(ax)^2\,dx = \int \sqrt{c\left(a^2x^2+1\right)}\operatorname{atan}^2\left(ax\right)dx$$

input
```
integrate((a**2*c*x**2+c)**(1/2)*atan(a*x)**2,x)
```

output
```
Integral(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**2, x)
```

## Maxima [F]

$$\int \sqrt{c+a^2cx^2}\arctan(ax)^2\,dx = \int \sqrt{a^2cx^2+c}\arctan{(ax)}^2\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)*arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2, x)
```

## Giac [F(-2)]

Exception generated.

$$\int \sqrt{c+a^2cx^2}\arctan(ax)^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(1/2)*arctan(a*x)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+a^2cx^2}\arctan(ax)^2\,dx = \int \mathrm{atan}(a\,x)^2\,\sqrt{c\,a^2\,x^2+c}\,dx$$

input
```
int(atan(a*x)^2*(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(atan(a*x)^2*(c + a^2*c*x^2)^(1/2), x)
```

## Reduce [F]

$$\int \sqrt{c+a^2cx^2}\arctan(ax)^2\,dx = \sqrt{c}\left(\int \sqrt{a^2x^2+1}\,atan(ax)^2\,dx\right)$$

input
```
int((a^2*c*x^2+c)^(1/2)*atan(a*x)^2,x)
```

output
```
sqrt(c)*int(sqrt(a**2*x**2 + 1)*atan(a*x)**2,x)
```

# 3.311 $\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 439

$$\begin{aligned}\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x}\,dx &= \sqrt{c+a^2cx^2}\arctan(ax)^2 \\ &+ \frac{4ic\sqrt{1+a^2x^2}\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} \\ &- \frac{2c\sqrt{1+a^2x^2}\arctan(ax)^2\text{arctanh}\left(e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} \\ &+ \frac{2ic\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} \\ &- \frac{2ic\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} \\ &- \frac{2ic\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} \\ &+ \frac{2ic\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} \\ &- \frac{2c\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,-e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} \\ &+ \frac{2c\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2+4*I*c*(a^2*x^2+1)^(1/2)*arctan(a*x)*arct
an((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/(a^2*c*x^2+c)^(1/2)-2*c*(a^2*x^2+1)^(1
/2)*arctan(a*x)^2*arctanh((1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)
+2*I*c*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2
))/(a^2*c*x^2+c)^(1/2)-2*I*c*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(2,(1+I*
a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)-2*I*c*(a^2*x^2+1)^(1/2)*polylo
g(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/(a^2*c*x^2+c)^(1/2)+2*I*c*(a^2*x^2
+1)^(1/2)*polylog(2,I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/(a^2*c*x^2+c)^(1/2)
-2*c*(a^2*x^2+1)^(1/2)*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+
c)^(1/2)+2*c*(a^2*x^2+1)^(1/2)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2
*c*x^2+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.57

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x}\,dx$$
$$= \frac{\sqrt{c+a^2cx^2}\left(\sqrt{1+a^2x^2}\arctan(ax)^2+\arctan(ax)^2\log\left(1-e^{i\arctan(ax)}\right)-2\arctan(ax)\log\left(1-ie^{i\arctan(ax)}\right)\right.}{}$$

input
```
Integrate[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/x,x]
```

output
```
(Sqrt[c + a^2*c*x^2]*(Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2 + ArcTan[a*x]^2*Log[
1 - E^(I*ArcTan[a*x])] - 2*ArcTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])] + 2*Ar
cTan[a*x]*Log[1 + I*E^(I*ArcTan[a*x])] - ArcTan[a*x]^2*Log[1 + E^(I*ArcTan
[a*x])] + (2*I)*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])] - (2*I)*PolyLog
[2, (-I)*E^(I*ArcTan[a*x])] + (2*I)*PolyLog[2, I*E^(I*ArcTan[a*x])] - (2*I
)*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])] - 2*PolyLog[3, -E^(I*ArcTan[a*
x])] + 2*PolyLog[3, E^(I*ArcTan[a*x])]))/Sqrt[1 + a^2*x^2]
```

## Rubi [A] (verified)

Time = 1.79 (sec) , antiderivative size = 305, normalized size of antiderivative = 0.69, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {5485, 5465, 5425, 5421, 5493, 5491, 3042, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{x}dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\int \frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5465}$$

$$a^2c\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}\right)+c\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5425}$$

$$a^2c\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{a\sqrt{a^2cx^2+c}}\right)+c\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5421}$$

$$c\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+$$
$$a^2c\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{ia\cdots}}{\sqrt{1-\cdots}}\right)}{a}\right.}{a\sqrt{a^2cx^2+c}}\right.$$

$$\Big\downarrow \text{5493}$$

$$\frac{c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+$$
$$a^2c\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{ia}}{\sqrt{1-}}\right.}{a}\right.}{a\sqrt{a^2cx^2+c}}\right.$$

$$\Big\downarrow \text{5491}$$

$$\frac{c\sqrt{a^2x^2+1}\int \frac{\sqrt{a^2x^2+1}\arctan(ax)^2}{ax}d\arctan(ax)}{\sqrt{a^2cx^2+c}}+$$
$$a^2c\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{ia}}{\sqrt{1-}}\right.}{a}\right.}{a\sqrt{a^2cx^2+c}}\right.$$

$$\Big\downarrow \text{3042}$$

$$\frac{c\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc(\arctan(ax))d\arctan(ax)}{\sqrt{a^2cx^2+c}}+$$
$$a^2c\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{ia}}{\sqrt{1-}}\right.}{a}\right.}{a\sqrt{a^2cx^2+c}}\right.$$

$$\Big\downarrow \text{4671}$$

$$\frac{c\sqrt{a^2x^2+1}\left(-2\int \arctan(ax)\log\left(1-e^{i\arctan(ax)}\right)d\arctan(ax)+2\int \arctan(ax)\log\left(1+e^{i\arctan(ax)}\right)d\arctan(a\right.}{\sqrt{a^2cx^2+c}}$$
$$a^2c\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{ia}}{\sqrt{1-}}\right.}{a}\right.}{a\sqrt{a^2cx^2+c}}\right.$$

$$\Big\downarrow \text{3011}$$

$$\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-i\int\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)d\arctan(ax)\right)-2\left(i\arctan\right.\right.}{\sqrt{a^2cx}}$$

$$a^2c\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{ia}}{\sqrt{1-}}\right.}{a}\right.}{a\sqrt{a^2cx^2+c}}\right.$$

$\big\downarrow$ 2720

$$\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)-\right.}{}$$

$$a^2c\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{ia}}{\sqrt{1-}}\right.}{a}\right.}{a\sqrt{a^2cx^2+c}}\right.$$

$\big\downarrow$ 7143

$$\frac{c\sqrt{a^2x^2+1}\left(-2\arctan(ax)^2\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)+2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\operatorname{PolyLog}\left(3,-e^{i a}\right.\right.\right.}{\sqrt{a^2cx^2+c}}$$

$$a^2c\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{ia}}{\sqrt{1-}}\right.}{a}\right.}{a\sqrt{a^2cx^2+c}}\right.$$

input

```
Int[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/x,x]
```

output

```
a^2*c*((Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(a^2*c) - (2*Sqrt[1 + a^2*x^2]*
(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/a + (I*PolyL
og[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*PolyLog[2, (I*Sqrt[1
 + I*a*x])/Sqrt[1 - I*a*x]])/a))/(a*Sqrt[c + a^2*c*x^2])) + (c*Sqrt[1 + a^
2*x^2]*(-2*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])] + 2*(I*ArcTan[a*x]*Pol
yLog[2, -E^(I*ArcTan[a*x])] - PolyLog[3, -E^(I*ArcTan[a*x])]) - 2*(I*ArcTa
n[a*x]*PolyLog[2, E^(I*ArcTan[a*x])] - PolyLog[3, E^(I*ArcTan[a*x])])))/Sq
rt[c + a^2*c*x^2]
```

### Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5491
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Simp[1/Sqrt[d]   Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTa
n[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] &&
GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

## Maple [A] (verified)

Time = 2.39 (sec) , antiderivative size = 337, normalized size of antiderivative = 0.77

| method | result |
|---|---|
| default | $\sqrt{c\left(ax-i\right)\left(ax+i\right)}\,\arctan\left(ax\right)^{2}+\frac{i\sqrt{c(ax-i)(ax+i)}\left(i\arctan(ax)^{2}\ln\left(1+\frac{iax+1}{\sqrt{a^{2}x^{2}+1}}\right)-i\arctan(ax)^{2}\ln\left(1-\frac{iax}{\sqrt{a^{2}x}}\right.\right.}{}$ |

input
```
int((a^2*c*x^2+c)^(1/2)*arctan(a*x)^2/x,x,method=_RETURNVERBOSE)
```

output
```
(c*(a*x-I)*(a*x+I))^(1/2)*arctan(a*x)^2+I*(c*(a*x-I)*(a*x+I))^(1/2)*(I*arc
tan(a*x)^2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*arctan(a*x)^2*ln(1-(1+I*a*x
)/(a^2*x^2+1)^(1/2))-2*I*arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+2
*I*arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*arctan(a*x)*polylog(2
,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+
1)^(1/2))+2*I*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*I*polylog(3,(1+I*a
*x)/(a^2*x^2+1)^(1/2))-2*dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*dilog(1-
I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)
```

## Fricas [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x}\,dx = \int \frac{\sqrt{a^2cx^2+c}\arctan\left(ax\right)^2}{x}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)*arctan(a*x)^2/x,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2/x, x)
```

## Sympy [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x}\,dx = \int \frac{\sqrt{c\left(a^2x^2+1\right)}\operatorname{atan}^2\left(ax\right)}{x}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(1/2)*atan(a*x)**2/x,x)
```

output
```
Integral(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**2/x, x)
```

## Maxima [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x}\,dx = \int \frac{\sqrt{a^2cx^2+c}\arctan\left(ax\right)^2}{x}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)*arctan(a*x)^2/x,x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2/x, x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(1/2)*arctan(a*x)^2/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x}\,dx = \int \frac{\mathrm{atan}(a\,x)^2\,\sqrt{c\,a^2\,x^2+c}}{x}\,dx$$

input
```
int((atan(a*x)^2*(c + a^2*c*x^2)^(1/2))/x,x)
```

output
```
int((atan(a*x)^2*(c + a^2*c*x^2)^(1/2))/x, x)
```

## Reduce [F]

$$\int \frac{\sqrt{c+a^2cx^2}\,\arctan(ax)^2}{x}\,dx = \sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,atan(ax)^2}{x}dx\right)$$

input
```
int((a^2*c*x^2+c)^(1/2)*atan(a*x)^2/x,x)
```

output
```
sqrt(c)*int((sqrt(a**2*x**2 + 1)*atan(a*x)**2)/x,x)
```

# 3.312 $\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x^2}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 458

$$\begin{aligned}\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x^2}\,dx &= -\frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x} \\ &- \frac{2iac\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2}{\sqrt{c+a^2cx^2}} \\ &- \frac{4ac\sqrt{1+a^2x^2}\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} \\ &+ \frac{2iac\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} \\ &- \frac{2iac\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,ie^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} \\ &+ \frac{2iac\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} \\ &- \frac{2iac\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} \\ &- \frac{2ac\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} \\ &+ \frac{2ac\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,ie^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
-(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2/x-2*I*a*c*(a^2*x^2+1)^(1/2)*arctan((1+I
*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2)-4*a*c*(a^2*x^2+
1)^(1/2)*arctan(a*x)*arctanh((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/(a^2*c*x^2+c
)^(1/2)+2*I*a*c*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*
x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)-2*I*a*c*(a^2*x^2+1)^(1/2)*arctan(a*x)*po
lylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)+2*I*a*c*(a^2*x^
2+1)^(1/2)*polylog(2,-(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/(a^2*c*x^2+c)^(1/2)
-2*I*a*c*(a^2*x^2+1)^(1/2)*polylog(2,(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/(a^2
*c*x^2+c)^(1/2)-2*a*c*(a^2*x^2+1)^(1/2)*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)
^(1/2))/(a^2*c*x^2+c)^(1/2)+2*a*c*(a^2*x^2+1)^(1/2)*polylog(3,I*(1+I*a*x)/
(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.58

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x^2}\,dx =$$
$$-\frac{a\sqrt{c\left(1+a^2x^2\right)}\left(\frac{\sqrt{1+a^2x^2}\arctan(ax)^2}{ax}-2\arctan(ax)\log\left(1-e^{i\arctan(ax)}\right)-\arctan(ax)^2\log\left(1-ie^{i\arctan(}\right.\right.}{}$$

input
```
Integrate[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/x^2,x]
```

output
```
-((a*Sqrt[c*(1 + a^2*x^2)]*((Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2)/(a*x) - 2*Ar
cTan[a*x]*Log[1 - E^(I*ArcTan[a*x])] - ArcTan[a*x]^2*Log[1 - I*E^(I*ArcTan
[a*x])] + ArcTan[a*x]^2*Log[1 + I*E^(I*ArcTan[a*x])] + 2*ArcTan[a*x]*Log[1
 + E^(I*ArcTan[a*x])] - (2*I)*PolyLog[2, -E^(I*ArcTan[a*x])] - (2*I)*ArcTa
n[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + (2*I)*ArcTan[a*x]*PolyLog[2, I
*E^(I*ArcTan[a*x])] + (2*I)*PolyLog[2, E^(I*ArcTan[a*x])] + 2*PolyLog[3, (
-I)*E^(I*ArcTan[a*x])] - 2*PolyLog[3, I*E^(I*ArcTan[a*x])]))/Sqrt[1 + a^2*
x^2])
```

## Rubi [A] (verified)

Time = 2.06 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.66, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {5485, 5425, 5423, 3042, 4669, 3011, 2720, 5479, 5493, 5489, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{x^2}dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5425}$$

$$\frac{a^2c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5423}$$

$$\frac{ac\sqrt{a^2x^2+1}\int \sqrt{a^2x^2+1}\arctan(ax)^2d\arctan(ax)}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{3042}$$

$$c\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx+\frac{ac\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{4669}$$

$$c\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx+$$
$$\frac{ac\sqrt{a^2x^2+1}\left(-2\int \arctan(ax)\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+2\int \arctan(ax)\log\left(1+ie^{i\arctan(ax)}\right)d\arctan\right.}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3011}$$

$$c\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx+$$
$$\frac{ac\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-i\int \operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(ax)\right)-2\left(i\arctan\right.\right.}{\sqrt{a^2}}$$

$$\Big\downarrow \text{2720}$$

$$c\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx+$$
$$\underline{ac\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right.}$$

$$\Big\downarrow \text{5479}$$

$$c\left(2a\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}\right)+$$
$$\underline{ac\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right.}$$

$$\Big\downarrow \text{5493}$$

$$c\left(\frac{2a\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}\right)+$$
$$\underline{ac\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right.}$$

$$\Big\downarrow \text{5489}$$

$$\underline{ac\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right.}$$

$$c\left(-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}+\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\operatorname{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\operatorname{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\operatorname{Po}\right.}{\sqrt{a^2cx^2+c}}\right.$$

$$\Big\downarrow \text{7143}$$

$$c\left(-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}+\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\operatorname{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\operatorname{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\operatorname{Po}\right.}{\sqrt{a^2cx^2+c}}\right.$$
$$\frac{ac\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)\right)-2\left(i\arctan(ax)\operatorname{PolyLog}\right.\right.}{\sqrt{a^2cx^2+c}}$$

input
```
Int[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/x^2,x]
```

output
```
c*(-((Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(c*x)) + (2*a*Sqrt[1 + a^2*x^2]*(
-2*ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]] + I*PolyLog[2, -(S
qrt[1 + I*a*x]/Sqrt[1 - I*a*x])] - I*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I
*a*x]]))/Sqrt[c + a^2*c*x^2]) + (a*c*Sqrt[1 + a^2*x^2]*((-2*I)*ArcTan[E^(I
*ArcTan[a*x])]*ArcTan[a*x]^2 + 2*(I*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTa
n[a*x])] - PolyLog[3, (-I)*E^(I*ArcTan[a*x])]) - 2*(I*ArcTan[a*x]*PolyLog[
2, I*E^(I*ArcTan[a*x])] - PolyLog[3, I*E^(I*ArcTan[a*x])])))/Sqrt[c + a^2*
c*x^2]
```

### Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5489
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sq
rt[1 - I*c*x]], x] + (Simp[I*(b/Sqrt[d])*PolyLog[2, -Sqrt[1 + I*c*x]/Sqrt[1
 - I*c*x]], x] - Simp[I*(b/Sqrt[d])*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x]], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

## Maple [A] (verified)

Time = 2.55 (sec) , antiderivative size = 308, normalized size of antiderivative = 0.67

| method | result |
|---|---|
| default | $-\frac{\sqrt{c(ax-i)(ax+i)}\,\arctan(ax)^2}{x}+\frac{a\sqrt{c(ax-i)(ax+i)}\left(\arctan(ax)^2\ln\left(1-\frac{i(iax+1)}{\sqrt{a^2x^2+1}}\right)-\arctan(ax)^2\ln\left(1+\frac{i(iax+1)}{\sqrt{a^2x^2+1}}\right)-2i\,\mathrm{arct}\right.}{}$ |

input
```
int((a^2*c*x^2+c)^(1/2)*arctan(a*x)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
-(c*(a*x-I)*(a*x+I))^(1/2)*arctan(a*x)^2/x+a*(c*(a*x-I)*(a*x+I))^(1/2)*(ar
ctan(a*x)^2*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-arctan(a*x)^2*ln(1+I*(1+I*
a*x)/(a^2*x^2+1)^(1/2))-2*I*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^
(1/2))+2*I*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*arctan(
a*x)*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*I*dilog((1+I*a*x)/(a^2*x^2+1)^(1/
2))+2*I*dilog(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*polylog(3,I*(1+I*a*x)/(a^2*
x^2+1)^(1/2))-2*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/
2)
```

## Fricas [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x^2}\,dx=\int \frac{\sqrt{a^2cx^2+c}\arctan{(ax)}^2}{x^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)*arctan(a*x)^2/x^2,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2/x^2, x)
```

## Sympy [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x^2}\,dx = \int \frac{\sqrt{c\left(a^2x^2+1\right)}\operatorname{atan}^2\left(ax\right)}{x^2}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(1/2)*atan(a*x)**2/x**2,x)
```

output
```
Integral(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**2/x**2, x)
```

## Maxima [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x^2}\,dx = \int \frac{\sqrt{a^2cx^2+c}\arctan\left(ax\right)^2}{x^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)*arctan(a*x)^2/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2/x^2, x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(1/2)*arctan(a*x)^2/x^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x^2}\,dx = \int \frac{\operatorname{atan}(a\,x)^2\,\sqrt{c\,a^2\,x^2+c}}{x^2}\,dx$$

input
```
int((atan(a*x)^2*(c + a^2*c*x^2)^(1/2))/x^2,x)
```

output
```
int((atan(a*x)^2*(c + a^2*c*x^2)^(1/2))/x^2, x)
```

## Reduce [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x^2}\,dx = \sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,atan(ax)^2}{x^2}dx\right)$$

input
```
int((a^2*c*x^2+c)^(1/2)*atan(a*x)^2/x^2,x)
```

output
```
sqrt(c)*int((sqrt(a**2*x**2 + 1)*atan(a*x)**2)/x**2,x)
```

# 3.313 $\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x^3}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 328

$$\begin{aligned}\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x^3}\,dx &= -\frac{a\sqrt{c+a^2cx^2}\arctan(ax)}{x}-\frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{2x^2}\\ &\quad-\frac{a^2c\sqrt{1+a^2x^2}\arctan(ax)^2\text{arctanh}\left(e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}\\ &\quad-a^2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c+a^2cx^2}}{\sqrt{c}}\right)\\ &\quad+\frac{ia^2c\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}\\ &\quad-\frac{ia^2c\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}\\ &\quad-\frac{a^2c\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,-e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}\\ &\quad+\frac{a^2c\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
-a*(a^2*c*x^2+c)^(1/2)*arctan(a*x)/x-1/2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2
/x^2-a^2*c*(a^2*x^2+1)^(1/2)*arctan(a*x)^2*arctanh((1+I*a*x)/(a^2*x^2+1)^(
1/2))/(a^2*c*x^2+c)^(1/2)-a^2*c^(1/2)*arctanh((a^2*c*x^2+c)^(1/2)/c^(1/2))
+I*a^2*c*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1
/2))/(a^2*c*x^2+c)^(1/2)-I*a^2*c*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(2,(
1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)-a^2*c*(a^2*x^2+1)^(1/2)*po
lylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)+a^2*c*(a^2*x^2+1
)^(1/2)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 1.34 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x^3}\,dx = \frac{a^2\sqrt{c\,(1+a^2x^2)}\left(-4\arctan(ax)\cot\left(\frac{1}{2}\arctan(ax)\right)-\arctan(ax)^2\csc^2\left(\frac{1}{2}\arctan(ax)\right)+4\arctan(ax)^2\left(\right.\right.}{}$$

input
```
Integrate[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/x^3,x]
```

output
```
(a^2*Sqrt[c*(1 + a^2*x^2)]*(-4*ArcTan[a*x]*Cot[ArcTan[a*x]/2] - ArcTan[a*x
]^2*Csc[ArcTan[a*x]/2]^2 + 4*ArcTan[a*x]^2*(Log[1 - E^(I*ArcTan[a*x])] - L
og[1 + E^(I*ArcTan[a*x])]) + 8*Log[Tan[ArcTan[a*x]/2]] + (8*I)*ArcTan[a*x]
*(PolyLog[2, -E^(I*ArcTan[a*x])] - PolyLog[2, E^(I*ArcTan[a*x])]) + 8*(-Po
lyLog[3, -E^(I*ArcTan[a*x])] + PolyLog[3, E^(I*ArcTan[a*x])]) + ArcTan[a*x
]^2*Sec[ArcTan[a*x]/2]^2 - 4*ArcTan[a*x]*Tan[ArcTan[a*x]/2]))/(8*Sqrt[1 +
a^2*x^2])
```

## Rubi [A] (verified)

Time = 3.05 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.08, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.792$, Rules used = {5485, 5493, 5491, 3042, 4671, 3011, 2720, 5497, 5479, 243, 73, 221, 5493, 5491, 3042, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed

below.

$$\int \frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{x^3}dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5493}$$

$$\frac{a^2c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5491}$$

$$\frac{a^2c\sqrt{a^2x^2+1}\int \frac{\sqrt{a^2x^2+1}\arctan(ax)^2}{ax}d\arctan(ax)}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2c\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc(\arctan(ax))d\arctan(ax)}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{4671}$$

$$c\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx+ \frac{a^2c\sqrt{a^2x^2+1}\left(-2\int \arctan(ax)\log\left(1-e^{i\arctan(ax)}\right)d\arctan(ax)+2\int \arctan(ax)\log\left(1+e^{i\arctan(ax)}\right)d\arctan\right.}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3011}$$

$$c\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx+ \frac{a^2c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-i\int \text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)d\arctan(ax)\right)-2\left(i\arct\right.\right.}{\sqrt{a^2c}}$$

$$\Big\downarrow \text{2720}$$

$$c\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx+ \frac{a^2c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)\right.}{}$$

$$\Big\downarrow \text{5497}$$

$$\frac{c\left(-\frac{1}{2}a^2\int\frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+a\int\frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{2cx^2}\right)+}{a^2c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)\right.}$$

$$\Big\downarrow \text{5479}$$

$$\frac{c\left(-\frac{1}{2}a^2\int\frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+a\left(a\int\frac{1}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right)-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{2cx^2}\right)+}{a^2c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)\right.}$$

$$\Big\downarrow \text{243}$$

$$\frac{c\left(-\frac{1}{2}a^2\int\frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+a\left(\frac{1}{2}a\int\frac{1}{x^2\sqrt{a^2cx^2+c}}dx^2-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right)-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{2cx^2}\right.}{a^2c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)\right.}$$

$$\Big\downarrow \text{73}$$

$$\frac{c\left(-\frac{1}{2}a^2\int\frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+a\left(\frac{\int\frac{1}{\frac{x^4}{a^2c}-\frac{1}{a^2}}d\sqrt{a^2cx^2+c}}{ac}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right)-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{2cx^2}\right)}{a^2c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)\right.}$$

$$\Big\downarrow \text{221}$$

$$\frac{c\left(-\frac{1}{2}a^2\int\frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+a\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\operatorname{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{2cx^2}\right.}{a^2c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)\right.}$$

$$\Big\downarrow \text{5493}$$

$$\frac{c\left(-\frac{a^2\sqrt{a^2x^2+1}\int\frac{\arctan(ax)^2}{x\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+a\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\operatorname{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{\arctan(ax)^2\sqrt{a^2cx}}{2cx^2}\right.}{a^2c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)\right.}$$

$\Big\downarrow$ 5491

$$c\left(-\frac{a^2\sqrt{a^2x^2+1}\int\frac{\sqrt{a^2x^2+1}\arctan(ax)^2}{ax}d\arctan(ax)}{2\sqrt{a^2cx^2+c}}+a\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\right.$$
$$a^2c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\,\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\,\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)\right.$$

$\Big\downarrow$ 3042

$$c\left(-\frac{a^2\sqrt{a^2x^2+1}\int\arctan(ax)^2\csc(\arctan(ax))d\arctan(ax)}{2\sqrt{a^2cx^2+c}}+a\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\text{arctanh}\left(\frac{\sqrt{a^2cx^2}}{\sqrt{c}}\right.}{\sqrt{c}}\right.\right.$$
$$a^2c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\,\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\,\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)\right.$$

$\Big\downarrow$ 4671

$$a^2c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\,\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\,\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)\right.$$

$$c\left(-\frac{a^2\sqrt{a^2x^2+1}\left(-2\int\arctan(ax)\log\left(1-e^{i\arctan(ax)}\right)d\arctan(ax)+2\int\arctan(ax)\log\left(1+e^{i\arctan(ax)}\right)d\,\text{arc}\right.}{2\sqrt{a^2cx^2+c}}\right.$$

$\Big\downarrow$ 3011

$$a^2c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\,\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\,\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)\right.$$

$$c\left(-\frac{a^2\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\,\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-i\int\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)d\arctan(ax)\right)-2(i\right.}{2}\right.$$

$\Big\downarrow$ 2720

$$a^2c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\,\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\,\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)\right.$$

$$c\left(-\frac{a^2\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\,\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\,\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(\right.\right.}{}\right.$$

$\Big\downarrow$ 7143

$$\frac{a^2c\sqrt{a^2x^2+1}\left(-2\arctan(ax)^2\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)+2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\operatorname{PolyLog}\left(3,-e\right.\right.\right.}{\sqrt{a^2cx^2+c}}$$

$$c\left(-\frac{a^2\sqrt{a^2x^2+1}\left(-2\arctan(ax)^2\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)+2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\operatorname{PolyLog}\left(3\right.\right.\right.}{2\sqrt{a^2cx^2+c}}\right.$$

input
```
Int[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/x^3,x]
```

output
```
c*(-1/2*(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(c*x^2) + a*(-((Sqrt[c + a^2*c
*x^2]*ArcTan[a*x])/(c*x)) - (a*ArcTanh[Sqrt[c + a^2*c*x^2]/Sqrt[c]])/Sqrt[
c]) - (a^2*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])]
+ 2*(I*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])] - PolyLog[3, -E^(I*ArcTa
n[a*x])]) - 2*(I*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])] - PolyLog[3, E^
(I*ArcTan[a*x])])))/(2*Sqrt[c + a^2*c*x^2])) + (a^2*c*Sqrt[1 + a^2*x^2]*(-
2*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])] + 2*(I*ArcTan[a*x]*PolyLog[2, -
E^(I*ArcTan[a*x])] - PolyLog[3, -E^(I*ArcTan[a*x])]) - 2*(I*ArcTan[a*x]*Po
lyLog[2, E^(I*ArcTan[a*x])] - PolyLog[3, E^(I*ArcTan[a*x])])))/Sqrt[c + a^
2*c*x^2]
```

## Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5491
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Simp[1/Sqrt[d]   Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTa
n[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] &&
GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5497
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)^(m
 + 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[c^2*((m
+ 2)/(f^2*(m + 1)))   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x
^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
 && LtQ[m, -1] && NeQ[m, -2]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

## Maple [A] (verified)

Time = 2.32 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.78

| method | result |
|---|---|
| default | $-\frac{\sqrt{c(ax-i)(ax+i)}\,\arctan(ax)(2ax+\arctan(ax))}{2x^2}-\frac{a^2\sqrt{c(ax-i)(ax+i)}\,\left(\arctan(ax)^2\ln\left(1+\frac{iax+1}{\sqrt{a^2x^2+1}}\right)-\arctan(ax)^2\ln\left(1-\right.\right.}{}$ |

input
```
int((a^2*c*x^2+c)^(1/2)*arctan(a*x)^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(c*(a*x-I)*(a*x+I))^(1/2)*arctan(a*x)*(2*a*x+arctan(a*x))/x^2-1/2*a^2
*(c*(a*x-I)*(a*x+I))^(1/2)*(arctan(a*x)^2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2)
)-arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*I*arctan(a*x)*polylog(
2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*I*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x
^2+1)^(1/2))+2*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*polylog(3,(1+I*a*
x)/(a^2*x^2+1)^(1/2))+4*arctanh((1+I*a*x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^
(1/2)
```

## Fricas [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x^3}\,dx = \int \frac{\sqrt{a^2cx^2+c}\arctan{(ax)}^2}{x^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)*arctan(a*x)^2/x^3,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2/x^3, x)
```

## Sympy [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x^3}\,dx = \int \frac{\sqrt{c\,(a^2x^2+1)}\operatorname{atan}^2{(ax)}}{x^3}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(1/2)*atan(a*x)**2/x**3,x)
```

output
```
Integral(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**2/x**3, x)
```

## Maxima [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x^3}\,dx = \int \frac{\sqrt{a^2cx^2+c}\arctan\left(ax\right)^2}{x^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)*arctan(a*x)^2/x^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2/x^3, x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(1/2)*arctan(a*x)^2/x^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x^3}\,dx = \int \frac{\mathrm{atan}\left(a\,x\right)^2\,\sqrt{c\,a^2\,x^2+c}}{x^3}\,dx$$

input
```
int((atan(a*x)^2*(c + a^2*c*x^2)^(1/2))/x^3,x)
```

output
```
int((atan(a*x)^2*(c + a^2*c*x^2)^(1/2))/x^3, x)
```

## Reduce [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x^3}\,dx = \sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,atan(ax)^2}{x^3}dx\right)$$

input

```
int((a^2*c*x^2+c)^(1/2)*atan(a*x)^2/x^3,x)
```

output

```
sqrt(c)*int((sqrt(a**2*x**2 + 1)*atan(a*x)**2)/x**3,x)
```

# 3.314 $\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x^4}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 275

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x^4}\,dx = -\frac{a^2\sqrt{c+a^2cx^2}}{3x} - \frac{a\sqrt{c+a^2cx^2}\arctan(ax)}{3x^2} - \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^2}{3cx^3} - \frac{2a^3c\sqrt{1+a^2x^2}\arctan(ax)\operatorname{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3\sqrt{c+a^2cx^2}} + \frac{ia^3c\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3\sqrt{c+a^2cx^2}} - \frac{ia^3c\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3\sqrt{c+a^2cx^2}}$$

output
```
-1/3*a^2*(a^2*c*x^2+c)^(1/2)/x-1/3*a*(a^2*c*x^2+c)^(1/2)*arctan(a*x)/x^2-1
/3*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/c/x^3-2/3*a^3*c*(a^2*x^2+1)^(1/2)*arc
tan(a*x)*arctanh((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/(a^2*c*x^2+c)^(1/2)+1/3*
I*a^3*c*(a^2*x^2+1)^(1/2)*polylog(2,-(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/(a^2
*c*x^2+c)^(1/2)-1/3*I*a^3*c*(a^2*x^2+1)^(1/2)*polylog(2,(1+I*a*x)^(1/2)/(1
-I*a*x)^(1/2))/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (warning: unable to verify)

Time = 1.22 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x^4}\,dx =$$
$$-\frac{c\sqrt{1+a^2x^2}\left(-4ia^3x^3\,\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)+4ia^3x^3\,\text{PolyLog}\left(2,e^{i\arctan(ax)}\right)+\sqrt{1+a^2x^2}\left(4a^2x^2\right.\right.}{}$$

input
```
Integrate[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/x^4,x]
```

output
```
-1/12*(c*Sqrt[1 + a^2*x^2]*((-4*I)*a^3*x^3*PolyLog[2, -E^(I*ArcTan[a*x])]
+ (4*I)*a^3*x^3*PolyLog[2, E^(I*ArcTan[a*x])] + Sqrt[1 + a^2*x^2]*(4*a^2*x
^2 + 4*(1 + a^2*x^2)*ArcTan[a*x]^2 + ArcTan[a*x]*(a*x*(4 - 3*Sqrt[1 + a^2*
x^2]*Log[1 - E^(I*ArcTan[a*x])] + 3*Sqrt[1 + a^2*x^2]*Log[1 + E^(I*ArcTan[
a*x])]) + (1 + a^2*x^2)*(Log[1 - E^(I*ArcTan[a*x])] - Log[1 + E^(I*ArcTan[
a*x])])*Sin[3*ArcTan[a*x]]))))/(x^3*Sqrt[c + a^2*c*x^2])
```

## Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {5479, 5481, 242, 5497, 242, 5493, 5489}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{x^4}dx$$

$$\Big\downarrow \text{ 5479}$$

$$\frac{2}{3}a\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^3}dx-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3cx^3}$$

$$\Big\downarrow \text{ 5481}$$

$$\frac{2}{3}a\left(-c\int \frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx+ac\int \frac{1}{x^2\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}\right)-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3cx^3}$$

$$\Big\downarrow \text{242}$$

$$\frac{2}{3}a\left(-c\int \frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}-\frac{a\sqrt{a^2cx^2+c}}{x}\right)-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3cx^3}$$

$$\Big\downarrow \text{5497}$$

$$\frac{2}{3}a\left(-c\left(-\frac{1}{2}a^2\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx+\frac{1}{2}a\int \frac{1}{x^2\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{2cx^2}\right)-\frac{\arctan(ax)\sqrt{a^2cx^2+}}{x^2}\right.$$
$$\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3cx^3}$$

$$\Big\downarrow \text{242}$$

$$\frac{2}{3}a\left(-c\left(-\frac{1}{2}a^2\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{2cx^2}-\frac{a\sqrt{a^2cx^2+c}}{2cx}\right)-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}-\frac{a\sqrt{a^2}}{}\right.$$
$$\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3cx^3}$$

$$\Big\downarrow \text{5493}$$

$$\frac{2}{3}a\left(-c\left(-\frac{a^2\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{x\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{2cx^2}-\frac{a\sqrt{a^2cx^2+c}}{2cx}\right)-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}-\right.$$
$$\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3cx^3}$$

$$\Big\downarrow \text{5489}$$

$$-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3cx^3}+$$
$$\frac{2}{3}a\left(-c\left(-\frac{a^2\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{PolyLog}\left(2,\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)\right)}{2\sqrt{a^2cx^2+c}}\right.\right.$$

input
```
Int[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/x^4,x]
```

output
```
-1/3*((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/(c*x^3) + (2*a*(-((a*Sqrt[c + a
^2*c*x^2])/x) - (Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/x^2 - c*(-1/2*(a*Sqrt[c
+ a^2*c*x^2])/(c*x) - (Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(2*c*x^2) - (a^2*S
qrt[1 + a^2*x^2]*(-2*ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]]
+ I*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])] - I*PolyLog[2, Sqrt[1 +
 I*a*x]/Sqrt[1 - I*a*x]]))/(2*Sqrt[c + a^2*c*x^2]))))/3
```

## Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5481
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTan[c*x
])/(f*(m + 2))), x] + (Simp[d/(m + 2)   Int[(f*x)^m*((a + b*ArcTan[c*x])/Sq
rt[d + e*x^2]), x], x] - Simp[b*c*(d/(f*(m + 2)))   Int[(f*x)^(m + 1)/Sqrt[
d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] &&
NeQ[m, -2]
```

rule 5489
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sq
rt[1 - I*c*x]], x] + (Simp[I*(b/Sqrt[d])*PolyLog[2, -Sqrt[1 + I*c*x]/Sqrt[1
 - I*c*x]], x] - Simp[I*(b/Sqrt[d])*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x]], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5497
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)^(m
 + 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[c^2*((m
+ 2)/(f^2*(m + 1)))   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x
^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
 && LtQ[m, -1] && NeQ[m, -2]
```

## Maple [A] (verified)

Time = 2.22 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.71

| method | result |
|---|---|
| default | $-\frac{\sqrt{c(ax-i)(ax+i)}\,\left(\arctan(ax)^2x^2a^2+a^2x^2+\arctan(ax)ax+\arctan(ax)^2\right)}{3x^3}+\frac{ia^3\sqrt{c(ax-i)(ax+i)}\,\left(i\arctan(ax)\ln\left(1+\frac{iax+}{\sqrt{a^2x^2}}\right.\right.}{}$ |

input
```
int((a^2*c*x^2+c)^(1/2)*arctan(a*x)^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(c*(a*x-I)*(a*x+I))^(1/2)*(arctan(a*x)^2*x^2*a^2+a^2*x^2+arctan(a*x)*
a*x+arctan(a*x)^2)/x^3+1/3*I*a^3*(c*(a*x-I)*(a*x+I))^(1/2)*(I*arctan(a*x)*
ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*arctan(a*x)*ln(1-(1+I*a*x)/(a^2*x^2+1)
^(1/2))-polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+polylog(2,-(1+I*a*x)/(a^2*x
^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)
```

## Fricas [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x^4}\,dx = \int \frac{\sqrt{a^2cx^2+c}\arctan\left(ax\right)^2}{x^4}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)*arctan(a*x)^2/x^4,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2/x^4, x)
```

## Sympy [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x^4}\,dx = \int \frac{\sqrt{c\left(a^2x^2+1\right)}\operatorname{atan}^2\left(ax\right)}{x^4}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(1/2)*atan(a*x)**2/x**4,x)
```

output
```
Integral(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**2/x**4, x)
```

## Maxima [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x^4}\,dx = \int \frac{\sqrt{a^2cx^2+c}\arctan\left(ax\right)^2}{x^4}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)*arctan(a*x)^2/x^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2/x^4, x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(1/2)*arctan(a*x)^2/x^4,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x^4}\,dx = \int \frac{\mathrm{atan}(a\,x)^2\,\sqrt{c\,a^2\,x^2+c}}{x^4}\,dx$$

input
```
int((atan(a*x)^2*(c + a^2*c*x^2)^(1/2))/x^4,x)
```

output
```
int((atan(a*x)^2*(c + a^2*c*x^2)^(1/2))/x^4, x)
```

## Reduce [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{x^4}\,dx = \sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,atan(ax)^2}{x^4}dx\right)$$

input
```
int((a^2*c*x^2+c)^(1/2)*atan(a*x)^2/x^4,x)
```

output
```
sqrt(c)*int((sqrt(a**2*x**2 + 1)*atan(a*x)**2)/x**4,x)
```

# 3.315 $\int x^3(c+a^2cx^2)^{3/2}\arctan(ax)^2\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 476

$$\int x^3(c+a^2cx^2)^{3/2}\arctan(ax)^2\,dx = -\frac{17c\sqrt{c+a^2cx^2}}{280a^4}-\frac{17(c+a^2cx^2)^{3/2}}{1260a^4}+\frac{(c+a^2cx^2)^{5/2}}{105a^4c}+\frac{3cx\sqrt{c+a^2cx^2}\arctan(ax)}{56a^3}-\frac{23cx^3\sqrt{c+a^2cx^2}\arctan(ax)}{420a}-\frac{1}{21}acx^5\sqrt{c+a^2cx^2}\arctan(ax)-\frac{2c\sqrt{c+a^2cx^2}\arctan(ax)^2}{35a^4}+\frac{cx^2\sqrt{c+a^2cx^2}\arctan(ax)^2}{35a^2}+\frac{8}{35}cx^4\sqrt{c+a$$

output
```
-17/280*c*(a^2*c*x^2+c)^(1/2)/a^4-17/1260*(a^2*c*x^2+c)^(3/2)/a^4+1/105*(a
^2*c*x^2+c)^(5/2)/a^4/c+3/56*c*x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)/a^3-23/42
0*c*x^3*(a^2*c*x^2+c)^(1/2)*arctan(a*x)/a-1/21*a*c*x^5*(a^2*c*x^2+c)^(1/2)
*arctan(a*x)-2/35*c*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2/a^4+1/35*c*x^2*(a^2*
c*x^2+c)^(1/2)*arctan(a*x)^2/a^2+8/35*c*x^4*(a^2*c*x^2+c)^(1/2)*arctan(a*x
)^2+1/7*a^2*c*x^6*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2-17/140*I*c^2*(a^2*x^2+
1)^(1/2)*arctan(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/a^4/(a^2*c*x^
2+c)^(1/2)+17/280*I*c^2*(a^2*x^2+1)^(1/2)*polylog(2,-I*(1+I*a*x)^(1/2)/(1-
I*a*x)^(1/2))/a^4/(a^2*c*x^2+c)^(1/2)-17/280*I*c^2*(a^2*x^2+1)^(1/2)*polyl
og(2,I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/a^4/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (warning: unable to verify)

Time = 3.27 (sec) , antiderivative size = 797, normalized size of antiderivative = 1.67

$$\int x^3 (c + a^2 c x^2)^{3/2} \arctan(ax)^2 \, dx = \text{Too large to display}$$

input
```
Integrate[x^3*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2,x]
```

output
```
(c*(1 + a^2*x^2)^2*Sqrt[c + a^2*c*x^2]*(-168*(50 - 32*ArcTan[a*x]^2 + 72*C
os[2*ArcTan[a*x]] + 160*ArcTan[a*x]^2*Cos[2*ArcTan[a*x]] + 22*Cos[4*ArcTan
[a*x]] - (110*ArcTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2]
- 55*ArcTan[a*x]*Cos[3*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] - 11*ArcT
an[a*x]*Cos[5*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] + (110*ArcTan[a*x]
*Log[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + 55*ArcTan[a*x]*Cos[3*Ar
cTan[a*x]]*Log[1 + I*E^(I*ArcTan[a*x])] + 11*ArcTan[a*x]*Cos[5*ArcTan[a*x]
]*Log[1 + I*E^(I*ArcTan[a*x])] - ((176*I)*PolyLog[2, (-I)*E^(I*ArcTan[a*x]
)])/(1 + a^2*x^2)^(5/2) + ((176*I)*PolyLog[2, I*E^(I*ArcTan[a*x])])/(1 + a
^2*x^2)^(5/2) + 4*ArcTan[a*x]*Sin[2*ArcTan[a*x]] - 22*ArcTan[a*x]*Sin[4*Ar
cTan[a*x]]) + (1 + a^2*x^2)*(4116 + 10944*ArcTan[a*x]^2 + 6262*Cos[2*ArcTa
n[a*x]] - 5376*ArcTan[a*x]^2*Cos[2*ArcTan[a*x]] + 2764*Cos[4*ArcTan[a*x]]
+ 6720*ArcTan[a*x]^2*Cos[4*ArcTan[a*x]] + 618*Cos[6*ArcTan[a*x]] - (10815*
ArcTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - 6489*ArcTan[
a*x]*Cos[3*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] - 2163*ArcTan[a*x]*Co
s[5*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] - 309*ArcTan[a*x]*Cos[7*ArcT
an[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] + (10815*ArcTan[a*x]*Log[1 + I*E^(I*
ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + 6489*ArcTan[a*x]*Cos[3*ArcTan[a*x]]*Log
[1 + I*E^(I*ArcTan[a*x])] + 2163*ArcTan[a*x]*Cos[5*ArcTan[a*x]]*Log[1 + I*
E^(I*ArcTan[a*x])] + 309*ArcTan[a*x]*Cos[7*ArcTan[a*x]]*Log[1 + I*E^(I*...
```

## Rubi [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 3185 vs. $2(476) = 952$.

Time = 19.45 (sec) , antiderivative size = 3185, normalized size of antiderivative = 6.69, number of steps used = 31, number of rules used = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 1.250$, Rules used = {5485, 5485, 5487, 5465, 5425, 5421, 5487, 241, 243, 53, 2009, 5425, 5421, 5465, 5425, 5421, 5487, 241, 243, 53, 2009, 5425, 5421, 5465, 5425, 5421, 5487, 241, 5425, 5421}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \arctan(ax)^2 \left(a^2cx^2+c\right)^{3/2} dx$$

$$\Big\downarrow \text{ 5485}$$

$$a^2c\int x^5\sqrt{a^2cx^2+c}\arctan(ax)^2dx+c\int x^3\sqrt{a^2cx^2+c}\arctan(ax)^2dx$$

$$\Big\downarrow \text{ 5485}$$

$$a^2c\left(a^2c\int \frac{x^7\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int \frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx\right)+ c\left(a^2c\int \frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int \frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx\right)$$

$$\Big\downarrow \text{ 5487}$$

$$c\left(c\left(-\frac{2\int \frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{2\int \frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)^2\sqrt{a^2cx^2+c}}{3a^2c}\right)+a^2c\left(-\frac{2\int \frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int\right.\right.$$
$$a^2c\left(a^2c\left(-\frac{2\int \frac{x^6\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{7a}-\frac{6\int \frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{7a^2}+\frac{x^6\arctan(ax)^2\sqrt{a^2cx^2+c}}{7a^2c}\right)+c\left(-\frac{2\int \frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a}-\right.\right.$$

$$\Big\downarrow \text{ 5465}$$

$$c\left(c\left(-\frac{2\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}\right)}{3a^2}-\frac{2\int \frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}+\frac{x^2\arctan(ax)^2\sqrt{a^2cx^2+c}}{3a^2c}\right)+a^2c\left(-\frac{2}{}\right.\right.$$
$$a^2c\left(a^2c\left(-\frac{2\int \frac{x^6\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{7a}-\frac{6\int \frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{7a^2}+\frac{x^6\arctan(ax)^2\sqrt{a^2cx^2+c}}{7a^2c}\right)+c\left(-\frac{2\int \frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a}-\right.\right.$$

$$\Big\downarrow \text{ 5425}$$

$$c\left(c\left(-\frac{2\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\int\frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{a\sqrt{a^2cx^2+c}}\right)}{3a^2}-\frac{2\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}+\frac{x^2\arctan(ax)^2\sqrt{a^2cx^2+c}}{3a^2c}\right)+a\right.$$

$$a^2c\left(a^2c\left(-\frac{2\int\frac{x^6\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{7a}-\frac{6\int\frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{7a^2}+\frac{x^6\arctan(ax)^2\sqrt{a^2cx^2+c}}{7a^2c}\right)+c\left(-\frac{2\int\frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a}-\right.\right.$$

$\downarrow$ 5421

$$a^2c\left(a^2c\left(-\frac{2\int\frac{x^6\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{7a}-\frac{6\int\frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{7a^2}+\frac{x^6\arctan(ax)^2\sqrt{a^2cx^2+c}}{7a^2c}\right)+c\left(-\frac{2\int\frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a}-\right.\right.$$

$$c\left(a^2c\left(-\frac{2\int\frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int\frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a^2}+\frac{x^4\arctan(ax)^2\sqrt{a^2cx^2+c}}{5a^2c}\right)+c\left(-\frac{2\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{2\,\cdots}{\cdots}\right.\right.$$

$\downarrow$ 5487

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\int\frac{x^5}{\sqrt{a^2cx^2+c}}dx}{6a}-\frac{5\int\frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)}{7a}-\frac{6\left(\frac{\sqrt{a^2cx^2+c}\,\text{arct}\cdots}{5a^2c}\right.}{}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\int\frac{x^3}{\sqrt{a^2cx^2+c}}dx}{4a}-\frac{3\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-\frac{4\left(\frac{x^2\sqrt{a^2cx^2+c}\,\text{ar}\cdots}{3a^2c}\right.}{}\right.\right.$$

$\downarrow$ 241

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\int\frac{x^5}{\sqrt{a^2cx^2+c}}dx}{6a}-\frac{5\int\frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)}{7a}-\frac{6\left(\frac{\sqrt{a^2cx^2+c}\arct}{5a^2c}\right.}{}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\int\frac{x^3}{\sqrt{a^2cx^2+c}}dx}{4a}-\frac{3\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-\frac{4\left(\frac{x^2\sqrt{a^2cx^2+c}\,a}{3a^2c}\right.}{}\right.\right.$$

$\downarrow$ 243

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\int\frac{x^4}{\sqrt{a^2cx^2+c}}dx^2}{12a}-\frac{5\int\frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)}{7a}-\frac{6\left(\frac{\sqrt{a^2cx^2+c}\arc}{5a^2}\right.}{}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\int\frac{x^2}{\sqrt{a^2cx^2+c}}dx^2}{8a}-\frac{3\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-\frac{4\left(\frac{x^2\sqrt{a^2cx^2+c}\,a}{3a^2}\right.}{}\right.\right.$$

$\downarrow$ 53

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\int\left(\frac{\left(a^2cx^2+c\right)^{3/2}}{a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}+\frac{1}{a^4\sqrt{a^2cx^2+c}}\right)dx^2}{12a}-\frac{5\int\frac{x^4\text{arc}}{\sqrt{a^2}}}{6}\right.}{7a}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\int\left(\frac{\sqrt{a^2cx^2+c}}{a^2c}-\frac{1}{a^2\sqrt{a^2cx^2+c}}\right)dx^2}{8a}-\frac{3\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-\right.\right.$$

$\Big\downarrow$ 2009

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\int\frac{x^4\arctan(ax}{\sqrt{a^2cx^2+c}}}{6a^2}\right.}{7a}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}-\frac{3\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-\frac{4\left(\frac{x}{}\right.}{}\right.\right.$$

$\Big\downarrow$ 5425

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\int\frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}}{6a^2}\right.}{7a}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}-\frac{3\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-\frac{4\left(\right.}{}\right.\right.$$

$\downarrow$ 5421

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\int\frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}}{6a^2}\right.}{7a}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}-\frac{3\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-\frac{4\left(\right.}{}\right.\right.$$

$\downarrow$ 5465

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\int\frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right.}{7a}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}-\frac{3\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-\frac{4\left(\cdots\right.}{\cdots}\right.\right.$$

$\downarrow$ 5425

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\int\frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right.}{7a}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}-\frac{3\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-\frac{4\left(\cdots\right.}{\cdots}\right.\right.$$

$\downarrow$ 5421

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\int\frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}}{6a^2}\right.}{7a}\right.\right.$$

$$\left(\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}-\frac{3\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-\frac{4\left(\right.}{}\right.\right.\right.$$

↓ 5487

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\left(\frac{\sqrt{a^2cx^2+c}\,a\ldots}{4a\ldots}\ldots\right.}{\ldots}\ldots\right.}{7a}\ldots\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)}{2a^2c}-\frac{\int\ldots}{\sqrt{a\ldots}}\ldots\right.}{4a^2\ldots}\ldots\right.}{5a}\ldots\right.\right.$$

↓ 241

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\,\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\,\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\left(\frac{\sqrt{a^2cx^2+c}\,a}{4a}\right.}{}\right.}{7a}\right.\right.$$

$$\left(\left(\frac{\sqrt{a^2cx^2+c}\,\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\,\arctan(ax)x^3}{4a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}\,\arctan(ax)}{2a^2c}-\frac{\int\frac{\mathrm{arc}}{\sqrt{a}}}{}\right.}{4a^2}\right.}{5a}\right.\right.$$

↓ 243

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\left(\frac{\sqrt{a^2cx^2+c}\,\mathrm{a}}{4a}\right.}{}\right.}{7a}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)}{2a^2c}-\frac{\int\frac{\mathrm{arc}}{\sqrt{a}}}{}\right.}{4a^2}\right.}{5a}\right.\right.$$

↓ 53

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\,\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\,\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\left(\cdots\right.}{\cdots}\right.}{7a}\cdots\right.\right.$$

$$\left(\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\,\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\,\arctan(ax)x^3}{4a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}\,\arctan(ax)}{2a^2c}-\cdots\right.}{4a^2}\cdots\right.}{5a}\cdots\right.\right.\right.\right.$$

↓ 2009

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\left(\frac{\sqrt{a^2cx^2+c}\,a}{4a}\right.}{}\right.}{7a}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)}{2a^2c}-\frac{\int\frac{\text{arc}}{\sqrt{a}}}{}\right.}{4a^2}\right.}{5a}\right.\right.$$

↓ 5425

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\left(\frac{\sqrt{a^2cx^2+c}\,a}{4a}\right.}{}\right.}{7a}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)}{2a^2c}-\frac{\sqrt{a^2}}{}\right.}{4}\right.}{5a}\right.\right.$$

↓ 5421

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\,\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\,\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\left(\frac{\sqrt{a^2cx^2+c}\,a}{4a}\right.}{}\right.}{7a}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\,\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\,\arctan(ax)x^3}{4a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}\,\arctan(ax)}{2a^2c}-\frac{\sqrt{a^2}}{}\right.}{}\right.}{}\right.\right.$$

↓ 5465

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\left(\frac{\sqrt{a^2cx^2+c}\,\mathrm{a}}{4a}\right.}{}\right.}{7a}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)}{2a^2c}-\frac{\sqrt{a^2}}{}\right.}{}\right.}{}\right.\right.$$

↓ 5425

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\left(\frac{\sqrt{a^2cx^2+c}\,a}{4a}\right.}{}\right.}{7a}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)}{2a^2c}-\frac{\sqrt{a^2}}{}\right.}{}\right.}{}\right.\right.$$

$\downarrow$ 5421

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\left(\frac{\sqrt{a^2cx^2+c}\,a}{4a}\right.}{}\right.}{7a}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)}{2a^2c}-\frac{\sqrt{a^2c}}{}\right.}{}\right.}{}\right.\right.$$

↓ 5487

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\left(\frac{\sqrt{a^2cx^2+c}\,a\cdots}{4a\cdots}\right.}{\cdots}\right.}{\cdots}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)}{2a^2c}-\frac{\sqrt{a^2\cdots}}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.\right.$$

↓ 241

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\left(\frac{\sqrt{a^2cx^2+c}\,\mathrm{a}}{4a}\right.}{}\right.}{}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)}{2a^2c}-\frac{\sqrt{a^2}}{}\right.}{}\right.}{}\right.\right.$$

↓ 5425

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\left(\frac{\sqrt{a^2cx^2+c}\,\mathrm{a}\ldots}{4a\ldots}\right.}{\ldots}\right.}{\ldots}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)}{2a^2c}-\frac{\sqrt{a^2\ldots}}{\ldots}\right.}{\ldots}\right.}{\ldots}\right.\right.$$

↓ 5421

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\left(\frac{\sqrt{a^2cx^2+c}\,a\cdots}{4a\cdots}\right.}{\cdots}\right.}{\cdots}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)}{2a^2c}-\frac{\sqrt{a^2\cdots}}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.\right.$$

input
```
Int[x^3*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2,x]
```

output
```
c*(c*((x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(3*a^2*c) - (2*(-1/2*Sqrt[c
+ a^2*c*x^2]/(a^3*c) + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(2*a^2*c) - (Sq
rt[1 + a^2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x
]])/a + (I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*Poly
Log[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/(2*a^2*Sqrt[c + a^2*c*x^2
])))/(3*a) - (2*((Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(a^2*c) - (2*Sqrt[1 +
 a^2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/a
+ (I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*PolyLog[2,
 (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/(a*Sqrt[c + a^2*c*x^2])))/(3*a^
2)) + a^2*c*((x^4*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(5*a^2*c) - (2*(-1/8*
((-2*Sqrt[c + a^2*c*x^2])/(a^4*c) + (2*(c + a^2*c*x^2)^(3/2))/(3*a^4*c^2))
/a + (x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(4*a^2*c) - (3*(-1/2*Sqrt[c + a
^2*c*x^2]/(a^3*c) + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(2*a^2*c) - (Sqrt[
1 + a^2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])
/a + (I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*PolyLog
[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/(2*a^2*Sqrt[c + a^2*c*x^2]))
)/(4*a^2)))/(5*a) - (4*((x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(3*a^2*c)
- (2*(-1/2*Sqrt[c + a^2*c*x^2]/(a^3*c) + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x
])/(2*a^2*c) - (Sqrt[1 + a^2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a
*x]/Sqrt[1 - I*a*x]])/a + (I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 -...
```

## Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5487
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcTan[c*x])^p/(c^2*d*m)), x] + (-Simp[b*f*(p/(c*m))   Int[(f*x)^(m - 1)*((
a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[f^2*((m - 1)/(c^
2*m))   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2]), x], x])
/; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

## Maple [A] (verified)

Time = 2.21 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.57

| method | result |
|---|---|
| default | $\frac{c\sqrt{c(ax-i)(ax+i)}\,\left(360a^6x^6\arctan(ax)^2-120\arctan(ax)a^5x^5+576a^4\arctan(ax)^2x^4+24a^4x^4-138\arctan(ax)x^3a^3+72\arctan(a\right.}{2520a^4}$ |

input
```
int(x^3*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2520*c/a^4*(c*(a*x-I)*(a*x+I))^(1/2)*(360*a^6*x^6*arctan(a*x)^2-120*arct
an(a*x)*a^5*x^5+576*a^4*arctan(a*x)^2*x^4+24*a^4*x^4-138*arctan(a*x)*x^3*a
^3+72*arctan(a*x)^2*x^2*a^2+14*a^2*x^2+135*arctan(a*x)*a*x-144*arctan(a*x)
^2-163)-17/280*c*(c*(a*x-I)*(a*x+I))^(1/2)*(arctan(a*x)*ln(1+I*(1+I*a*x)/(
a^2*x^2+1)^(1/2))-arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+I*dilog(
1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))
/a^4/(a^2*x^2+1)^(1/2)
```

## Fricas [F]

$$\int x^3\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2\,dx=\int\left(a^2cx^2+c\right)^{\frac{3}{2}}x^3\arctan\left(ax\right)^2\,dx$$

input
```
integrate(x^3*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^5 + c*x^3)*sqrt(a^2*c*x^2 + c)*arctan(a*x)^2, x)
```

## Sympy [F]

$$\int x^3 \left(c+a^2 c x^2\right)^{3/2} \arctan(ax)^2 \, dx = \int x^3 \left(c\left(a^2 x^2+1\right)\right)^{\frac{3}{2}} \operatorname{atan}^2(ax) \, dx$$

input
```
integrate(x**3*(a**2*c*x**2+c)**(3/2)*atan(a*x)**2,x)
```

output
```
Integral(x**3*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**2, x)
```

## Maxima [F]

$$\int x^3 \left(c+a^2 c x^2\right)^{3/2} \arctan(ax)^2 \, dx = \int \left(a^2 c x^2+c\right)^{\frac{3}{2}} x^3 \arctan(ax)^2 \, dx$$

input
```
integrate(x^3*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*x^3*arctan(a*x)^2, x)
```

## Giac [F(-2)]

Exception generated.

$$\int x^3 \left(c+a^2 c x^2\right)^{3/2} \arctan(ax)^2 \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int x^3 (c + a^2 c x^2)^{3/2} \arctan(ax)^2 \, dx = \int x^3 \operatorname{atan}(a\,x)^2 \left(c\,a^2\,x^2 + c\right)^{3/2} dx$$

input
```
int(x^3*atan(a*x)^2*(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(x^3*atan(a*x)^2*(c + a^2*c*x^2)^(3/2), x)
```

## Reduce [F]

$$\int x^3 (c + a^2 c x^2)^{3/2} \arctan(ax)^2 \, dx = \sqrt{c}\,c\left(\left(\int \sqrt{a^2x^2+1}\,atan(ax)^2\,x^5dx\right)a^2 + \int \sqrt{a^2x^2+1}\,atan(ax)^2\,x^3dx\right)$$

input
```
int(x^3*(a^2*c*x^2+c)^(3/2)*atan(a*x)^2,x)
```

output
```
sqrt(c)*c*(int(sqrt(a**2*x**2 + 1)*atan(a*x)**2*x**5,x)*a**2 + int(sqrt(a*
*2*x**2 + 1)*atan(a*x)**2*x**3,x))
```

# 3.316 $\int x^2(c+a^2cx^2)^{3/2}\arctan(ax)^2\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 531

$$\begin{aligned}
\int x^2(c+a^2cx^2)^{3/2}\arctan(ax)^2\,dx &= \frac{cx\sqrt{c+a^2cx^2}}{36a^2}+\frac{1}{60}cx^3\sqrt{c+a^2cx^2} \\
&+\frac{31c\sqrt{c+a^2cx^2}\arctan(ax)}{360a^3}-\frac{19cx^2\sqrt{c+a^2cx^2}\arctan(ax)}{180a} \\
&-\frac{1}{15}acx^4\sqrt{c+a^2cx^2}\arctan(ax)+\frac{cx\sqrt{c+a^2cx^2}\arctan(ax)^2}{16a^2} \\
&+\frac{7}{24}cx^3\sqrt{c+a^2cx^2}\arctan(ax)^2+\frac{1}{6}a^2cx^5\sqrt{c+a^2cx^2}\arctan(ax)^2 \\
&+\frac{ic^2\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2}{8a^3\sqrt{c+a^2cx^2}}-\frac{41c^{3/2}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right)}{360a^3} \\
&-\frac{ic^2\sqrt{1+a^2x^2}\arctan(ax)\,\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)}{8a^3\sqrt{c+a^2cx^2}} \\
&+\frac{ic^2\sqrt{1+a^2x^2}\arctan(ax)\,\text{PolyLog}\left(2,ie^{i\arctan(ax)}\right)}{8a^3\sqrt{c+a^2cx^2}} \\
&+\frac{c^2\sqrt{1+a^2x^2}\,\text{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)}{8a^3\sqrt{c+a^2cx^2}} \\
&-\frac{c^2\sqrt{1+a^2x^2}\,\text{PolyLog}\left(3,ie^{i\arctan(ax)}\right)}{8a^3\sqrt{c+a^2cx^2}}
\end{aligned}$$

output
```
1/36*c*x*(a^2*c*x^2+c)^(1/2)/a^2+1/60*c*x^3*(a^2*c*x^2+c)^(1/2)+31/360*c*(
a^2*c*x^2+c)^(1/2)*arctan(a*x)/a^3-19/180*c*x^2*(a^2*c*x^2+c)^(1/2)*arctan
(a*x)/a-1/15*a*c*x^4*(a^2*c*x^2+c)^(1/2)*arctan(a*x)+1/16*c*x*(a^2*c*x^2+c
)^(1/2)*arctan(a*x)^2/a^2+7/24*c*x^3*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2+1/6
*a^2*c*x^5*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2+1/8*I*c^2*(a^2*x^2+1)^(1/2)*a
rctan((1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^2/a^3/(a^2*c*x^2+c)^(1/2)-4
1/360*c^(3/2)*arctanh(a*c^(1/2)*x/(a^2*c*x^2+c)^(1/2))/a^3-1/8*I*c^2*(a^2*
x^2+1)^(1/2)*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a^3/(a^
2*c*x^2+c)^(1/2)+1/8*I*c^2*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(2,I*(1+I*
a*x)/(a^2*x^2+1)^(1/2))/a^3/(a^2*c*x^2+c)^(1/2)+1/8*c^2*(a^2*x^2+1)^(1/2)*
polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a^3/(a^2*c*x^2+c)^(1/2)-1/8*c^2*
(a^2*x^2+1)^(1/2)*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a^3/(a^2*c*x^2+
c)^(1/2)
```

## Mathematica [A] (warning: unable to verify)

Time = 2.47 (sec) , antiderivative size = 527, normalized size of antiderivative = 0.99

$$\int x^2(c+a^2cx^2)^{3/2}\arctan(ax)^2\,dx = \frac{c\sqrt{c+a^2cx^2}\left(960\left(-2\coth^{-1}\left(\frac{ax}{\sqrt{1+a^2x^2}}\right)+3i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2\right.\right.}{}$$

input
```
Integrate[x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2,x]
```

output
```
(c*Sqrt[c + a^2*c*x^2]*(960*(-2*ArcCoth[(a*x)/Sqrt[1 + a^2*x^2]] + (3*I)*A
rcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 - (3*I)*ArcTan[a*x]*PolyLog[2, (-I)
*E^(I*ArcTan[a*x])] + (3*I)*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] +
3*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] - 3*PolyLog[3, I*E^(I*ArcTan[a*x])])
+ 32*(19*ArcCoth[(a*x)/Sqrt[1 + a^2*x^2]] - (45*I)*ArcTan[E^(I*ArcTan[a*x]
)]*ArcTan[a*x]^2 + (45*I)*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] -
 (45*I)*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] - 45*PolyLog[3, (-I)*E
^(I*ArcTan[a*x])] + 45*PolyLog[3, I*E^(I*ArcTan[a*x])]) + 120*(1 + a^2*x^2
)^(3/2)*(ArcTan[a*x]*(2 + 6*Sqrt[1 + a^2*x^2]*Cos[3*ArcTan[a*x]]) - 3*ArcT
an[a*x]^2*(-7*a*x + Sqrt[1 + a^2*x^2]*Sin[3*ArcTan[a*x]]) + 2*(a*x + Sqrt[
1 + a^2*x^2]*Sin[3*ArcTan[a*x]])) + (1 + a^2*x^2)^3*((-56*a*x)/Sqrt[1 + a^
2*x^2] + ArcTan[a*x]*(12/Sqrt[1 + a^2*x^2] + 110*Cos[3*ArcTan[a*x]] - 90*C
os[5*ArcTan[a*x]]) - 108*Sin[3*ArcTan[a*x]] - 52*Sin[5*ArcTan[a*x]] + 15*A
rcTan[a*x]^2*((78*a*x)/Sqrt[1 + a^2*x^2] - 47*Sin[3*ArcTan[a*x]] + 3*Sin[5
*ArcTan[a*x]]))))/(11520*a^3*Sqrt[1 + a^2*x^2])
```

## Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arctan(ax)^2 \left(a^2cx^2+c\right)^{3/2} dx$$

$$\Big\downarrow \text{ 5485}$$

$$c\int x^2\sqrt{a^2cx^2+c}\arctan(ax)^2dx+a^2c\int x^4\sqrt{a^2cx^2+c}\arctan(ax)^2dx$$

$$\Big\downarrow \text{ 5485}$$

$$c\left(c\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+a^2c\int \frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx\right)+ a^2c\left(a^2c\int \frac{x^6\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int \frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx\right)$$

$$\Big\downarrow \text{ 5487}$$

$$c\left(c\left(-\frac{\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a^2}+\frac{x\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a^2c}\right)+a^2c\left(-\frac{3\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\right.\right.$$

$$a^2c\left(c\left(-\frac{3\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}+\frac{x^3\arctan(ax)^2\sqrt{a^2cx^2+c}}{4a^2c}\right)+a^2c\left(-\frac{\int \frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\right.\right.$$

$\downarrow$ 5425

$$c\left(c\left(-\frac{\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{2a^2\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a^2c}\right)+a^2c\left(-\frac{3\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right.\right.$$

$$a^2c\left(c\left(-\frac{3\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}+\frac{x^3\arctan(ax)^2\sqrt{a^2cx^2+c}}{4a^2c}\right)+a^2c\left(-\frac{\int \frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\right.\right.$$

$\downarrow$ 5423

$$a^2c\left(c\left(-\frac{3\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}+\frac{x^3\arctan(ax)^2\sqrt{a^2cx^2+c}}{4a^2c}\right)+a^2c\left(-\frac{\int \frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\right.\right.$$

$$c\left(a^2c\left(-\frac{3\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}+\frac{x^3\arctan(ax)^2\sqrt{a^2cx^2+c}}{4a^2c}\right)+c\left(-\frac{\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}-\right.\right.$$

$\downarrow$ 3042

$$a^2c\left(c\left(-\frac{3\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}+\frac{x^3\arctan(ax)^2\sqrt{a^2cx^2+c}}{4a^2c}\right)+a^2c\left(-\frac{\int \frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\right.\right.$$

$$c\left(a^2c\left(-\frac{3\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}+\frac{x^3\arctan(ax)^2\sqrt{a^2cx^2+c}}{4a^2c}\right)+c\left(-\frac{\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}-\right.\right.$$

$\downarrow$ 4669

$$a^2c\left(c\left(-\frac{3\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}+\frac{x^3\arctan(ax)^2\sqrt{a^2cx^2+c}}{4a^2c}\right)+a^2c\left(-\frac{\int \frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\right.\right.$$

$$c\left(a^2c\left(-\frac{3\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}+\frac{x^3\arctan(ax)^2\sqrt{a^2cx^2+c}}{4a^2c}\right)+c\left(-\frac{\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}-\right.\right.$$

$\downarrow$ 3011

$$a^2c\left(c\left(-\frac{3\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}+\frac{x^3\arctan(ax)^2\sqrt{a^2cx^2+c}}{4a^2c}\right)+a^2c\left(-\frac{\int \frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{5\int \cdots}{\cdots}\right.\right.$$
$$c\left(a^2c\left(-\frac{3\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}+\frac{x^3\arctan(ax)^2\sqrt{a^2cx^2+c}}{4a^2c}\right)+c\left(-\frac{\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{\sqrt{a^2x^2 \cdots}}{\cdots}\right.\right.$$

$\downarrow$ 2720

$$a^2c\left(c\left(-\frac{3\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}+\frac{x^3\arctan(ax)^2\sqrt{a^2cx^2+c}}{4a^2c}\right)+a^2c\left(-\frac{\int \frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{5\int \cdots}{\cdots}\right.\right.$$
$$c\left(a^2c\left(-\frac{3\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}+\frac{x^3\arctan(ax)^2\sqrt{a^2cx^2+c}}{4a^2c}\right)+c\left(-\frac{\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{\sqrt{a^2x^2 \cdots}}{\cdots}\right.\right.$$

$\downarrow$ 5465

$$a^2c\left(c\left(-\frac{3\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}+\frac{x^3\arctan(ax)^2\sqrt{a^2cx^2+c}}{4a^2c}\right)+a^2c\left(-\frac{\int \frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{5\int \cdots}{\cdots}\right.\right.$$
$$c\left(a^2c\left(-\frac{3\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}+\frac{x^3\arctan(ax)^2\sqrt{a^2cx^2+c}}{4a^2c}\right)+c\left(-\frac{\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\int \frac{\cdots}{\sqrt{\cdots}}}{\cdots}}{a}\cdots\right.\right.$$

$\downarrow$ 224

$$a^2c\left(c\left(-\frac{3\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}+\frac{x^3\arctan(ax)^2\sqrt{a^2cx^2+c}}{4a^2c}\right)+a^2c\left(-\frac{\int \frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{5\int \cdots}{\cdots}\right.\right.$$
$$c\left(a^2c\left(-\frac{3\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}+\frac{x^3\arctan(ax)^2\sqrt{a^2cx^2+c}}{4a^2c}\right)+c\left(-\frac{\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\int \frac{\cdots}{1-\cdots}}{\cdots}}{a}\cdots\right.\right.$$

$\downarrow$ 219

$$a^2c\left(c\left(-\frac{3\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}+\frac{x^3\arctan(ax)^2\sqrt{a^2cx^2+c}}{4a^2c}\right)+a^2c\left(-\frac{\int \frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{5\int\right.\right.$$

$$c\left(a^2c\left(-\frac{3\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}+\frac{x^3\arctan(ax)^2\sqrt{a^2cx^2+c}}{4a^2c}\right)+c\left(-\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(a\right.\right.}{}\right.\right.$$

$\downarrow$ 5487

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\int\frac{x^4}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a^2}}{3a}-\frac{5\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{4a^2c}\right.}{}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^2}{3a^2c}-\frac{\int\frac{x^2}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{2\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}}{2a}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax}{2a^2c}\right.}{}\right.\right.$$

$\downarrow$ 262

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\int\frac{x^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}}{5a}-\frac{4\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a^2}}{3a}-\frac{5\left(\sqrt{a^2c}\right.}{}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^2}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int\frac{1}{\sqrt{a^2cx^2+c}}dx}{2a^2}}{3a}-\frac{2\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}}{2a}-\frac{3\left(x\sqrt{a^2cx^2}\right.}{}\right.\right.$$

$\downarrow$ 224

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\int\frac{x^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}}{5a}-\frac{4\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a^2}}{3a}-\frac{5\left(\sqrt{a^2c\cdots\right.}{\cdots}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^2}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int\frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}}{2a^2}}{3a}-\frac{2\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}}{2a}-\frac{3\left(\cdots\right.}{\cdots}\right.\right.$$

$\downarrow$ 219

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\int\frac{x^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}}{5a}-\frac{4\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a^2}}{3a}-\frac{5\left(\sqrt{a^2c\cdots\right.}{\cdots}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^2}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}-\frac{2\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}}{2a}-\frac{3\left(\cdots\right.}{\cdots}\right.\right.$$

$\downarrow$ 262

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int\frac{1}{\sqrt{a^2cx^2+c}}dx}{2a^2}\right)}{4a^2}}{5a}-\frac{4\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}\cdots}{5a^2}\cdots}{3a}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^2}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}-\frac{2\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}}{2a}-\frac{3\left(\cdots\right.}{\cdots}\right.\right.$$

$\downarrow$ 224

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int\frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}}{2a^2}\right)}{4a^2}}{5a}-\frac{4\int\frac{x^3 \ldots}{\sqrt{\ldots}}}{\ldots}}{3a}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^2}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}-\frac{2\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}}{2a}-\frac{3\left(\ldots\right.}{\ldots}\right.\right.$$

$\downarrow$ 219

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}}{5a}-\frac{4\int\frac{x^3 \ldots}{\sqrt{\ldots}}}{\ldots}}{3a}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^2}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}-\frac{2\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}}{2a}-\frac{3\left(\ldots\right.}{\ldots}\right.\right.$$

$\downarrow$ 5425

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}}{5a}-\frac{4\int\frac{x^3 \ldots}{\sqrt{\ldots}}}{\ldots}}{3a}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^2}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}-\frac{2\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}}{2a}-\frac{3\left(\ldots\right.}{\ldots}\right.\right.$$

$\downarrow$ 5423

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}}{5a}-\frac{4\int\frac{x^3 a\cdots}{\sqrt{\cdots}}\cdots}{\cdots}}{3a}\cdots\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^2}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}-\frac{2\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}}{2a}-\frac{3\left(\cdots\right.}{\cdots}\right.\right.$$

$\downarrow$ 3042

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}}{5a}-\frac{4\int\frac{x^3 a\cdots}{\sqrt{\cdots}}\cdots}{\cdots}}{3a}\cdots\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^2}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}-\frac{2\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}}{2a}-\frac{3\left(\cdots\right.}{\cdots}\right.\right.$$

$\downarrow$ 4669

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}}{5a}-\frac{4\int\frac{x^3 a\cdots}{\sqrt{\cdots}}\cdots}{\cdots}}{3a}\cdots\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^2}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}-\frac{2\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}}{2a}-\frac{3\left(\cdots\right.}{\cdots}\right.\right.$$

$\downarrow$ 3011

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}}{5a}-\frac{4\int\frac{x^3 a\cdots}{\sqrt{\cdots}}\cdots}{\cdots}}{3a}\cdots\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^2}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}-\frac{2\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}-\frac{3\left(\cdots\right.}{\cdots}}{2a}\cdots\right.\right.$$

$\downarrow$ 2720

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}}{5a}-\frac{4\int\frac{x^3 a\cdots}{\sqrt{\cdots}}\cdots}{\cdots}}{3a}\cdots\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^2}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}-\frac{2\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}-\frac{3\left(\cdots\right.}{\cdots}}{2a}\cdots\right.\right.$$

$\downarrow$ 5465

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}}{5a}-\frac{4\int\frac{x^3 a\cdots}{\sqrt{\cdots}}\cdots}{\cdots}}{3a}\cdots\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^2}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a^2c}-\frac{\int\cdots}{\cdots}\cdots\right.}{3a^2}\cdots}{2a}\cdots\right.\right.$$

$\downarrow$ 224

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}}{5a}-\frac{4\int\frac{x^3 a}{\sqrt{}}}{}}{3a}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^2}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a^2c}-\int\right.}{3a}}{2a}\right.\right.$$

$$\Big\downarrow \; 219$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}}{5a}-\frac{4\int\frac{x^3 a}{\sqrt{}}}{}}{3a}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^2}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a^2c}-\right.}{3a}}{2a}\right.\right.$$

$$\Big\downarrow \; 5487$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}}{5a}}{3a}-\frac{4\left(\sqrt{a^2\right.}{}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^2}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}}{2a}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a^2c}-\right.}{3a}\right.\right.$$

$\downarrow$ 262

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}}{5a}}{3a}-\frac{4\left(\sqrt{a^2\right.}{}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^2}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}}{2a}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a^2c}-\right.}{3a}\right.\right.$$

input
```
Int[x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2,x]
```

output
```
$Aborted
```

## Maple [A] (verified)

Time = 2.20 (sec) , antiderivative size = 338, normalized size of antiderivative = 0.64

| method | result |
|---|---|
| default | $\frac{c\sqrt{c(ax-i)(ax+i)}\,\left(120a^5\arctan(ax)^2x^5-48x^4\arctan(ax)a^4+210a^3\arctan(ax)^2x^3+12a^3x^3-76x^2a^2\arctan(ax)+45a\arctan(ax\right.}{720a^3}$ |

input
```
int(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/720*c/a^3*(c*(a*x-I)*(a*x+I))^(1/2)*(120*a^5*arctan(a*x)^2*x^5-48*x^4*ar
ctan(a*x)*a^4+210*a^3*arctan(a*x)^2*x^3+12*a^3*x^3-76*x^2*a^2*arctan(a*x)+
45*a*arctan(a*x)^2*x+20*a*x+62*arctan(a*x))+1/720*I*c*(c*(a*x-I)*(a*x+I))^
(1/2)*(45*I*arctan(a*x)^2*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-45*I*arctan(
a*x)^2*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+90*arctan(a*x)*polylog(2,I*(1+I
*a*x)/(a^2*x^2+1)^(1/2))-90*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)
^(1/2))+90*I*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-90*I*polylog(3,-I*(1
+I*a*x)/(a^2*x^2+1)^(1/2))+164*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2)))/a^3/(a
^2*x^2+1)^(1/2)
```

## Fricas [F]

$$\int x^2\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2\,dx=\int\left(a^2cx^2+c\right)^{\frac{3}{2}}x^2\arctan\left(ax\right)^2\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^4 + c*x^2)*sqrt(a^2*c*x^2 + c)*arctan(a*x)^2, x)
```

## Sympy [F]

$$\int x^2\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2\,dx=\int x^2\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^2(ax)\,dx$$

input
```
integrate(x**2*(a**2*c*x**2+c)**(3/2)*atan(a*x)**2,x)
```

output
```
Integral(x**2*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**2, x)
```

## Maxima [F]

$$\int x^2\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2\,dx=\int\left(a^2cx^2+c\right)^{\frac{3}{2}}x^2\arctan(ax)^2\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*x^2*arctan(a*x)^2, x)
```

## Giac [F]

$$\int x^2\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2\,dx=\int\left(a^2cx^2+c\right)^{\frac{3}{2}}x^2\arctan(ax)^2\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*x^2*arctan(a*x)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int x^2(c+a^2cx^2)^{3/2}\arctan(ax)^2\,dx=\int x^2\,\mathrm{atan}(a\,x)^2\,\left(c\,a^2\,x^2+c\right)^{3/2}dx$$

input
```
int(x^2*atan(a*x)^2*(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(x^2*atan(a*x)^2*(c + a^2*c*x^2)^(3/2), x)
```

## Reduce [F]

$$\int x^2(c+a^2cx^2)^{3/2}\arctan(ax)^2\,dx=\sqrt{c}\,c\left(\left(\int\sqrt{a^2x^2+1}\,atan(ax)^2\,x^4dx\right)a^2+\int\sqrt{a^2x^2+1}\,atan(ax)^2\,x^2dx\right)$$

input
```
int(x^2*(a^2*c*x^2+c)^(3/2)*atan(a*x)^2,x)
```

output
```
sqrt(c)*c*(int(sqrt(a**2*x**2 + 1)*atan(a*x)**2*x**4,x)*a**2 + int(sqrt(a*
*2*x**2 + 1)*atan(a*x)**2*x**2,x))
```

# 3.317 $\int x(c+a^2cx^2)^{3/2}\arctan(ax)^2\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 334

$$\begin{aligned}\int x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2\,dx &= \frac{3c\sqrt{c+a^2cx^2}}{20a^2}+\frac{\left(c+a^2cx^2\right)^{3/2}}{30a^2}\\ &-\frac{3cx\sqrt{c+a^2cx^2}\arctan(ax)}{20a}-\frac{x(c+a^2cx^2)^{3/2}\arctan(ax)}{10a}\\ &+\frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2}{5a^2c}+\frac{3ic^2\sqrt{1+a^2x^2}\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{10a^2\sqrt{c+a^2cx^2}}\\ &-\frac{3ic^2\sqrt{1+a^2x^2}\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{20a^2\sqrt{c+a^2cx^2}}+\frac{3ic^2\sqrt{1+a^2x^2}\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{20a^2\sqrt{c+a^2cx^2}}\end{aligned}$$

output

```
3/20*c*(a^2*c*x^2+c)^(1/2)/a^2+1/30*(a^2*c*x^2+c)^(3/2)/a^2-3/20*c*x*(a^2*
c*x^2+c)^(1/2)*arctan(a*x)/a-1/10*x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)/a+1/5*
(a^2*c*x^2+c)^(5/2)*arctan(a*x)^2/a^2/c+3/10*I*c^2*(a^2*x^2+1)^(1/2)*arcta
n(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/a^2/(a^2*c*x^2+c)^(1/2)-3/2
0*I*c^2*(a^2*x^2+1)^(1/2)*polylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/a^
2/(a^2*c*x^2+c)^(1/2)+3/20*I*c^2*(a^2*x^2+1)^(1/2)*polylog(2,I*(1+I*a*x)^(
1/2)/(1-I*a*x)^(1/2))/a^2/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (warning: unable to verify)

Time = 2.85 (sec) , antiderivative size = 601, normalized size of antiderivative = 1.80

$$\int x(c+a^2cx^2)^{3/2}\arctan(ax)^2\,dx = \frac{c(1+a^2x^2)\sqrt{c+a^2cx^2}\left(80\left(2+4\arctan(ax)^2+2\cos(2\arctan(ax))-\frac{3\arctan}{}\right.\right.}{}$$

input
```
Integrate[x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2,x]
```

output
```
(c*(1 + a^2*x^2)*Sqrt[c + a^2*c*x^2]*(80*(2 + 4*ArcTan[a*x]^2 + 2*Cos[2*Ar
cTan[a*x]] - (3*ArcTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2
] - ArcTan[a*x]*Cos[3*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] + (3*ArcTa
n[a*x]*Log[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + ArcTan[a*x]*Cos[3
*ArcTan[a*x]]*Log[1 + I*E^(I*ArcTan[a*x])] - ((4*I)*PolyLog[2, (-I)*E^(I*A
rcTan[a*x])])/(1 + a^2*x^2)^(3/2) + ((4*I)*PolyLog[2, I*E^(I*ArcTan[a*x])]
)/(1 + a^2*x^2)^(3/2) - 2*ArcTan[a*x]*Sin[2*ArcTan[a*x]]) - (1 + a^2*x^2)*
(50 - 32*ArcTan[a*x]^2 + 72*Cos[2*ArcTan[a*x]] + 160*ArcTan[a*x]^2*Cos[2*A
rcTan[a*x]] + 22*Cos[4*ArcTan[a*x]] - (110*ArcTan[a*x]*Log[1 - I*E^(I*ArcT
an[a*x])])/Sqrt[1 + a^2*x^2] - 55*ArcTan[a*x]*Cos[3*ArcTan[a*x]]*Log[1 - I
*E^(I*ArcTan[a*x])] - 11*ArcTan[a*x]*Cos[5*ArcTan[a*x]]*Log[1 - I*E^(I*Arc
Tan[a*x])] + (110*ArcTan[a*x]*Log[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x
^2] + 55*ArcTan[a*x]*Cos[3*ArcTan[a*x]]*Log[1 + I*E^(I*ArcTan[a*x])] + 11*
ArcTan[a*x]*Cos[5*ArcTan[a*x]]*Log[1 + I*E^(I*ArcTan[a*x])] - ((176*I)*Pol
yLog[2, (-I)*E^(I*ArcTan[a*x])])/(1 + a^2*x^2)^(5/2) + ((176*I)*PolyLog[2,
 I*E^(I*ArcTan[a*x])])/(1 + a^2*x^2)^(5/2) + 4*ArcTan[a*x]*Sin[2*ArcTan[a*
x]] - 22*ArcTan[a*x]*Sin[4*ArcTan[a*x]])))/(960*a^2)
```

## Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.83, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {5465, 5413, 5413, 5425, 5421}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \arctan(ax)^2 \left(a^2cx^2+c\right)^{3/2} dx$$

$\big\downarrow$ 5465

$$\frac{\arctan(ax)^2 \left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{2\int \left(a^2cx^2+c\right)^{3/2}\arctan(ax)dx}{5a}$$

$\big\downarrow$ 5413

$$\frac{\arctan(ax)^2 \left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{2\left(\frac{3}{4}c\int \sqrt{a^2cx^2+c}\arctan(ax)dx+\frac{1}{4}x\arctan(ax)\left(a^2cx^2+c\right)^{3/2}-\frac{\left(a^2cx^2+c\right)^{3/2}}{12a}\right)}{5a}$$

$\big\downarrow$ 5413

$$\frac{\arctan(ax)^2 \left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{2\left(\frac{3}{4}c\left(\frac{1}{2}c\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)+\frac{1}{4}x\arctan(ax)\left(a^2cx^2+c\right)^{3/2}-\frac{\left(a^2cx^2+c\right)^{3/2}}{12a}\right)}{5a}$$

$\big\downarrow$ 5425

$$\frac{\arctan(ax)^2 \left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{2\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)+\frac{1}{4}x\arctan(ax)\left(a^2cx^2+c\right)^{3/2}-\frac{\left(a^2cx^2+ \right.}{12a}\right.}{5a}$$

$\big\downarrow$ 5421

$$\frac{\arctan(ax)^2 \left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{2\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}\right.\right.}{5a}$$

input
```
Int[x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2,x]
```

output
```
((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2)/(5*a^2*c) - (2*(-1/12*(c + a^2*c*x^2
)^(3/2)/a + (x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/4 + (3*c*(-1/2*Sqrt[c +
a^2*c*x^2]/a + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/2 + (c*Sqrt[1 + a^2*x^2
]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/a + (I*Pol
yLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*PolyLog[2, (I*Sqrt
[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/(2*Sqrt[c + a^2*c*x^2])))/4))/(5*a)
```

## Defintions of rubi rules used

rule 5413
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Simp[x*(d + e*x^2)
^q*((a + b*ArcTan[c*x])/(2*q + 1)), x] + Simp[2*d*(q/(2*q + 1))   Int[(d +
e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0]
```

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

## Maple [A] (verified)

Time = 2.05 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.71

| method | result |
|---|---|
| default | $\frac{c\sqrt{c(ax-i)(ax+i)}\left(12a^4\arctan(ax)^2x^4-6\arctan(ax)x^3a^3+24\arctan(ax)^2x^2a^2+2a^2x^2-15\arctan(ax)ax+12\arctan(ax)^2+11\right)}{60a^2}$ |

input
```
int(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/60*c/a^2*(c*(a*x-I)*(a*x+I))^(1/2)*(12*a^4*arctan(a*x)^2*x^4-6*arctan(a*
x)*x^3*a^3+24*arctan(a*x)^2*x^2*a^2+2*a^2*x^2-15*arctan(a*x)*a*x+12*arctan
(a*x)^2+11)+3/20*c*(c*(a*x-I)*(a*x+I))^(1/2)*(arctan(a*x)*ln(1+I*(1+I*a*x)
/(a^2*x^2+1)^(1/2))-arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+I*dilo
g(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2)
))/a^2/(a^2*x^2+1)^(1/2)
```

## Fricas [F]

$$\int x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2\,dx=\int\left(a^2cx^2+c\right)^{\frac{3}{2}}x\arctan\left(ax\right)^2\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^3 + c*x)*sqrt(a^2*c*x^2 + c)*arctan(a*x)^2, x)
```

## Sympy [F]

$$\int x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2\,dx=\int x\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^2\left(ax\right)dx$$

input
```
integrate(x*(a**2*c*x**2+c)**(3/2)*atan(a*x)**2,x)
```

output
```
Integral(x*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**2, x)
```

## Maxima [F]

$$\int x(c+a^2cx^2)^{3/2}\arctan(ax)^2\,dx=\int(a^2cx^2+c)^{\frac{3}{2}}x\arctan(ax)^2\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*x*arctan(a*x)^2, x)
```

## Giac [F(-2)]

Exception generated.

$$\int x(c+a^2cx^2)^{3/2}\arctan(ax)^2\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int x(c+a^2cx^2)^{3/2}\arctan(ax)^2\,dx=\int x\,\mathrm{atan}(a\,x)^2\,\left(c\,a^2\,x^2+c\right)^{3/2}\,dx$$

input
```
int(x*atan(a*x)^2*(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(x*atan(a*x)^2*(c + a^2*c*x^2)^(3/2), x)
```

## Reduce [F]

$$\int x(c+a^2cx^2)^{3/2}\arctan(ax)^2\,dx=\sqrt{c}\,c\left(\left(\int\sqrt{a^2x^2+1}\,atan(ax)^2\,x^3dx\right)a^2+\int\sqrt{a^2x^2+1}\,atan(ax)^2\,xdx\right)$$

input
```
int(x*(a^2*c*x^2+c)^(3/2)*atan(a*x)^2,x)
```

output
```
sqrt(c)*c*(int(sqrt(a**2*x**2 + 1)*atan(a*x)**2*x**3,x)*a**2 + int(sqrt(a*
*2*x**2 + 1)*atan(a*x)**2*x,x))
```

# 3.318 $\int (c+a^2cx^2)^{3/2}\arctan(ax)^2\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 438

$$\int (c+a^2cx^2)^{3/2}\arctan(ax)^2\,dx = \frac{1}{12}cx\sqrt{c+a^2cx^2}-\frac{3c\sqrt{c+a^2cx^2}\arctan(ax)}{4a} -\frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{6a}+\frac{3}{8}cx\sqrt{c+a^2cx^2}\arctan(ax)^2 +\frac{1}{4}x(c+a^2cx^2)^{3/2}\arctan(ax)^2-\frac{3ic^2\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2}{4a\sqrt{c+a^2cx^2}}+\frac{5c^{3/2}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right)}{6a}$$

output

```
1/12*c*x*(a^2*c*x^2+c)^(1/2)-3/4*c*(a^2*c*x^2+c)^(1/2)*arctan(a*x)/a-1/6*(
a^2*c*x^2+c)^(3/2)*arctan(a*x)/a+3/8*c*x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2
+1/4*x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2-3/4*I*c^2*(a^2*x^2+1)^(1/2)*arcta
n((1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^2/a/(a^2*c*x^2+c)^(1/2)+5/6*c^(
3/2)*arctanh(a*c^(1/2)*x/(a^2*c*x^2+c)^(1/2))/a+3/4*I*c^2*(a^2*x^2+1)^(1/2
)*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a/(a^2*c*x^2+c)^(1
/2)-3/4*I*c^2*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2
+1)^(1/2))/a/(a^2*c*x^2+c)^(1/2)-3/4*c^2*(a^2*x^2+1)^(1/2)*polylog(3,-I*(1
+I*a*x)/(a^2*x^2+1)^(1/2))/a/(a^2*c*x^2+c)^(1/2)+3/4*c^2*(a^2*x^2+1)^(1/2)
*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 439, normalized size of antiderivative = 1.00

$$\int (c+a^2cx^2)^{3/2} \arctan(ax)^2 \, dx = \frac{c\sqrt{c+a^2cx^2}\left(2ax\sqrt{1+a^2x^2}+2a^3x^3\sqrt{1+a^2x^2}+80\coth^{-1}\left(\frac{ax}{\sqrt{1+a^2x^2}}\right)-94\sqrt{\ldots}\right.}{}$$

input
```
Integrate[(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2,x]
```

output
```
(c*Sqrt[c + a^2*c*x^2]*(2*a*x*Sqrt[1 + a^2*x^2] + 2*a^3*x^3*Sqrt[1 + a^2*x
^2] + 80*ArcCoth[(a*x)/Sqrt[1 + a^2*x^2]] - 94*Sqrt[1 + a^2*x^2]*ArcTan[a*
x] + 2*a^2*x^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x] + 69*a*x*Sqrt[1 + a^2*x^2]*Ar
cTan[a*x]^2 + 21*a^3*x^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2 - (72*I)*ArcTan[E
^(I*ArcTan[a*x])]*ArcTan[a*x]^2 + 6*ArcTan[a*x]*Cos[3*ArcTan[a*x]] + 12*a^
2*x^2*ArcTan[a*x]*Cos[3*ArcTan[a*x]] + 6*a^4*x^4*ArcTan[a*x]*Cos[3*ArcTan[
a*x]] + (72*I)*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - (72*I)*Arc
Tan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] - 72*PolyLog[3, (-I)*E^(I*ArcTan[
a*x])] + 72*PolyLog[3, I*E^(I*ArcTan[a*x])] + 2*Sin[3*ArcTan[a*x]] + 4*a^2
*x^2*Sin[3*ArcTan[a*x]] + 2*a^4*x^4*Sin[3*ArcTan[a*x]] - 3*ArcTan[a*x]^2*S
in[3*ArcTan[a*x]] - 6*a^2*x^2*ArcTan[a*x]^2*Sin[3*ArcTan[a*x]] - 3*a^4*x^4
*ArcTan[a*x]^2*Sin[3*ArcTan[a*x]]))/(96*a*Sqrt[1 + a^2*x^2])
```

## Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 344, normalized size of antiderivative = 0.79, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {5415, 211, 224, 219, 5415, 224, 219, 5425, 5423, 3042, 4669, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(ax)^2 \left(a^2cx^2+c\right)^{3/2} \, dx$$

$$\Big\downarrow \text{5415}$$

$$\frac{3}{4}c\int \sqrt{a^2cx^2+c}\arctan(ax)^2dx+\frac{1}{6}c\int \sqrt{a^2cx^2+c}dx+\frac{1}{4}x\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{6a}$$

$$\Big\downarrow \text{211}$$

$$\frac{3}{4}c\int \sqrt{a^2cx^2+c}\arctan(ax)^2dx+\frac{1}{6}c\left(\frac{1}{2}c\int \frac{1}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{6a}$$

$$\Big\downarrow \text{224}$$

$$\frac{3}{4}c\int \sqrt{a^2cx^2+c}\arctan(ax)^2dx+\frac{1}{6}c\left(\frac{1}{2}c\int \frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{6a}$$

$$\Big\downarrow \text{219}$$

$$\frac{3}{4}c\int \sqrt{a^2cx^2+c}\arctan(ax)^2dx+\frac{1}{4}x\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{6a}+\frac{1}{6}c\left(\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)$$

$$\Big\downarrow \text{5415}$$

$$\frac{3}{4}c\left(\frac{1}{2}c\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int \frac{1}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}\right)+\frac{1}{4}x\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{6a}+\frac{1}{6}c\left(\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)$$

$$\Big\downarrow \text{224}$$

$$\frac{3}{4}c\left(\frac{1}{2}c\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int \frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+}}{a}\right.$$
$$\frac{1}{4}x\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{6a}+$$
$$\frac{1}{6}c\left(\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{3}{4}c\left(\frac{1}{2}c\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}\right)+$$
$$\frac{1}{4}x\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{6a}+$$
$$\frac{1}{6}c\left(\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)$$

$$\Big\downarrow \text{5425}$$

$$\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right.}{a}\right.$$
$$\frac{1}{4}x\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{6a}+$$
$$\frac{1}{6}c\left(\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)$$

$$\Big\downarrow \text{5423}$$

$$\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\int \sqrt{a^2x^2+1}\arctan(ax)^2d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}\right.$$
$$\frac{1}{4}x\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{6a}+$$
$$\frac{1}{6}c\left(\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{}}{a}\right.$$

$$\frac{1}{4}x\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{6a}+$$

$$\frac{1}{6}c\left(\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)$$

$$\Big\downarrow \text{4669}$$

$$\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\int \arctan(ax)\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+2\int \arctan(ax)\log\left(1+ie^{i\arctan(ax)}\right)d\,\text{arc}\right.}{2a\sqrt{a^2cx^2+c}}\right.$$

$$\frac{1}{4}x\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{6a}+$$

$$\frac{1}{6}c\left(\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)$$

$$\Big\downarrow \text{3011}$$

$$\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-i\int \text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(ax)\right)-2(i\right.}{2}\right.$$

$$\frac{1}{4}x\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{6a}+$$

$$\frac{1}{6}c\left(\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)$$

$$\Big\downarrow \text{2720}$$

$$\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(}\right.\right.}{}\right.$$

$$\frac{1}{4}x\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{6a}+$$

$$\frac{1}{6}c\left(\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)$$

$$\Big\downarrow \text{7143}$$

$$\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)\right)-2\left(i\arctan(ax)\operatorname{PolyL}\right.\right.}{2a\sqrt{a^2cx^2+c}}\right.$$

$$\frac{1}{4}x\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{6a}+$$

$$\frac{1}{6}c\left(\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)$$

input
```
Int[(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2,x]
```

output
```
-1/6*((c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/a + (x*(c + a^2*c*x^2)^(3/2)*ArcT
an[a*x]^2)/4 + (c*((x*Sqrt[c + a^2*c*x^2])/2 + (Sqrt[c]*ArcTanh[(a*Sqrt[c]
*x)/Sqrt[c + a^2*c*x^2]])/(2*a)))/6 + (3*c*(-((Sqrt[c + a^2*c*x^2]*ArcTan[
a*x])/a) + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/2 + (Sqrt[c]*ArcTanh[(a*S
qrt[c]*x)/Sqrt[c + a^2*c*x^2]])/a + (c*Sqrt[1 + a^2*x^2]*((-2*I)*ArcTan[E^
(I*ArcTan[a*x])]*ArcTan[a*x]^2 + 2*(I*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*Arc
Tan[a*x])] - PolyLog[3, (-I)*E^(I*ArcTan[a*x])]) - 2*(I*ArcTan[a*x]*PolyLo
g[2, I*E^(I*ArcTan[a*x])] - PolyLog[3, I*E^(I*ArcTan[a*x])])))/(2*a*Sqrt[c
 + a^2*c*x^2])))/4
```

## Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5415
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTan[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x])^p,
x], x] + Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*(
a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[q, 0] && GtQ[p, 1]
```

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

## Maple [A] (verified)

Time = 2.12 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.69

| method | result |
|---|---|
| default | $\frac{c\sqrt{c(ax-i)(ax+i)}\left(6a^3\arctan(ax)^2x^3-4x^2a^2\arctan(ax)+15a\arctan(ax)^2x+2ax-22\arctan(ax)\right)}{24a} - \frac{ic\sqrt{c(ax-i)(ax+i)}\left(9i\,\mathrm{ar}\right.}{}$ |

input
```
int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/24*c/a*(c*(a*x-I)*(a*x+I))^(1/2)*(6*a^3*arctan(a*x)^2*x^3-4*x^2*a^2*arct
an(a*x)+15*a*arctan(a*x)^2*x+2*a*x-22*arctan(a*x))-1/24*I*c*(c*(a*x-I)*(a*
x+I))^(1/2)*(9*I*arctan(a*x)^2*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-9*I*arc
tan(a*x)^2*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+18*arctan(a*x)*polylog(2,I*
(1+I*a*x)/(a^2*x^2+1)^(1/2))-18*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^
2+1)^(1/2))+18*I*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-18*I*polylog(3,-
I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+40*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2)))/a/(
a^2*x^2+1)^(1/2)
```

## Fricas [F]

$$\int \left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2\,dx=\int \left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^2\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^2, x)
```

## Sympy [F]

$$\int \left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2\,dx=\int \left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^2\left(ax\right)dx$$

input
```
integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)**2,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**2, x)
```

## Maxima [F]

$$\int \left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2\,dx=\int \left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^2\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^2, x)
```

## Giac [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2\,dx = \int \mathrm{atan}(a\,x)^2\,\left(c\,a^2\,x^2+c\right)^{3/2}\,dx$$

input
```
int(atan(a*x)^2*(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(atan(a*x)^2*(c + a^2*c*x^2)^(3/2), x)
```

## Reduce [F]

$$\int \left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2\,dx = \sqrt{c}\,c\left(\left(\int \sqrt{a^2x^2+1}\,atan(ax)^2\,x^2dx\right)a^2 + \int \sqrt{a^2x^2+1}\,atan(ax)^2\,dx\right)$$

input
```
int((a^2*c*x^2+c)^(3/2)*atan(a*x)^2,x)
```

output
```
sqrt(c)*c*(int(sqrt(a**2*x**2 + 1)*atan(a*x)**2*x**2,x)*a**2 + int(sqrt(a*
*2*x**2 + 1)*atan(a*x)**2,x))
```

# 3.319 $\int \frac{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2}{x}\,dx$

## Optimal result

Integrand size = 24, antiderivative size = 530

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^2}{x}\,dx = \frac{1}{3}c\sqrt{c+a^2cx^2} - \frac{1}{3}acx\sqrt{c+a^2cx^2}\arctan(ax) + c\sqrt{c+a^2cx^2}\arctan(ax)^2 + \frac{1}{3}\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2 + \frac{14ic^2\sqrt{1+a^2x^2}\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3\sqrt{c+a^2cx^2}} - \frac{2c^2\sqrt{1+a^2x^2}\arctan(ax)^2\text{arctanh}\left(e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} + \frac{2ic^2\sqrt{1+a^2x^2}\arctan(ax)\,\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} - \frac{2ic^2\sqrt{1+a^2x^2}\arctan(ax)\,\text{PolyLog}\left(2,e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} - \frac{7ic^2\sqrt{1+a^2x^2}\,\text{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3\sqrt{c+a^2cx^2}} + \frac{7ic^2\sqrt{1+a^2x^2}\,\text{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3\sqrt{c+a^2cx^2}} - \frac{2c^2\sqrt{1+a^2x^2}\,\text{PolyLog}\left(3,-e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} + \frac{2c^2\sqrt{1+a^2x^2}\,\text{PolyLog}\left(3,e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}$$

output

```
1/3*c*(a^2*c*x^2+c)^(1/2)-1/3*a*c*x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)+c*(a^2
*c*x^2+c)^(1/2)*arctan(a*x)^2+1/3*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2+14/3*I
*c^2*(a^2*x^2+1)^(1/2)*arctan(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))
/(a^2*c*x^2+c)^(1/2)-2*c^2*(a^2*x^2+1)^(1/2)*arctan(a*x)^2*arctanh((1+I*a*
x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)+2*I*c^2*(a^2*x^2+1)^(1/2)*arctan
(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)-2*I*c^2*
(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*
c*x^2+c)^(1/2)-7/3*I*c^2*(a^2*x^2+1)^(1/2)*polylog(2,-I*(1+I*a*x)^(1/2)/(1
-I*a*x)^(1/2))/(a^2*c*x^2+c)^(1/2)+7/3*I*c^2*(a^2*x^2+1)^(1/2)*polylog(2,I
*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/(a^2*c*x^2+c)^(1/2)-2*c^2*(a^2*x^2+1)^(1
/2)*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)+2*c^2*(a^2
*x^2+1)^(1/2)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (warning: unable to verify)

Time = 2.05 (sec) , antiderivative size = 496, normalized size of antiderivative = 0.94

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^2}{x}\,dx = \frac{1}{12}c\sqrt{c+a^2cx^2}\left(\frac{12\left(\sqrt{1+a^2x^2}\arctan(ax)^2+\arctan(ax)^2\log\left(1-e^{i\arctan(ax)}\right)\right.}{}\right.$$

$$+(1+a^2x^2)\left(2+4\arctan(ax)^2+2\cos(2\arctan(ax))-\frac{3\arctan(ax)\log\left(1-ie^{i\arctan(ax)}\right)}{\sqrt{1+a^2x^2}}-\arctan(ax)\cos(3\right.$$

input

```
Integrate[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/x,x]
```

output

```
(c*Sqrt[c + a^2*c*x^2]*((12*(Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2 + ArcTan[a*x]
^2*Log[1 - E^(I*ArcTan[a*x])] - 2*ArcTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])]
 + 2*ArcTan[a*x]*Log[1 + I*E^(I*ArcTan[a*x])] - ArcTan[a*x]^2*Log[1 + E^(I
*ArcTan[a*x])] + (2*I)*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])] - (2*I)*
PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + (2*I)*PolyLog[2, I*E^(I*ArcTan[a*x])]
 - (2*I)*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])] - 2*PolyLog[3, -E^(I*Ar
cTan[a*x])] + 2*PolyLog[3, E^(I*ArcTan[a*x])]))/Sqrt[1 + a^2*x^2] + (1 + a
^2*x^2)*(2 + 4*ArcTan[a*x]^2 + 2*Cos[2*ArcTan[a*x]] - (3*ArcTan[a*x]*Log[1
 - I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - ArcTan[a*x]*Cos[3*ArcTan[a*x]
]*Log[1 - I*E^(I*ArcTan[a*x])] + (3*ArcTan[a*x]*Log[1 + I*E^(I*ArcTan[a*x]
)])/Sqrt[1 + a^2*x^2] + ArcTan[a*x]*Cos[3*ArcTan[a*x]]*Log[1 + I*E^(I*ArcT
an[a*x])] - ((4*I)*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/(1 + a^2*x^2)^(3/2)
 + ((4*I)*PolyLog[2, I*E^(I*ArcTan[a*x])])/(1 + a^2*x^2)^(3/2) - 2*ArcTan[
a*x]*Sin[2*ArcTan[a*x]])))/12
```

## Rubi [A] (verified)

Time = 3.45 (sec) , antiderivative size = 540, normalized size of antiderivative = 1.02, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {5485, 5465, 5413, 5425, 5421, 5485, 5465, 5425, 5421, 5493, 5491, 3042, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2 \left(a^2cx^2+c\right)^{3/2}}{x}dx$$

$$\Big\downarrow \text{ 5485}$$

$$a^2c\int x\sqrt{a^2cx^2+c}\arctan(ax)^2dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x}dx$$

$$\Big\downarrow \text{ 5465}$$

$$a^2c\left(\frac{\arctan(ax)^2 \left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{2\int \sqrt{a^2cx^2+c}\arctan(ax)dx}{3a}\right)+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x}dx$$

$$\Big\downarrow \text{ 5413}$$

$$a^2c\left(\frac{\arctan(ax)^2 \left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{2\left(\frac{1}{2}c\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)}{3a}\right)+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x}dx$$

$$\Big\downarrow \text{ 5425}$$

$$a^2c\left(\frac{\arctan(ax)^2 \left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{2\left(\frac{c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)}{3a}\right)+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x}dx$$

$$\Big\downarrow \text{ 5421}$$

$$c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x}dx+a^2c\left(\frac{\arctan(ax)^2 \left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{2\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right.}{2\sqrt{a^2cx^2+c}}\right.}{3a}\right.$$

$\downarrow$ 5485

$$c\left(a^2c\int \frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx\right)+$$

$$a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{2\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right.}{2\sqrt{a^2cx^2+c}}\right.}{3a}\right.$$

$\downarrow$ 5465

$$c\left(a^2c\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\int\frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}\right)+c\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx\right)+$$

$$a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{2\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right.}{2\sqrt{a^2cx^2+c}}\right.}{3a}\right.$$

$\downarrow$ 5425

$$c\left(a^2c\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\int\frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{a\sqrt{a^2cx^2+c}}\right)+c\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx\right)+$$

$$a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{2\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right.}{2\sqrt{a^2cx^2+c}}\right.}{3a}\right.$$

$\downarrow$ 5421

$$c\left(c\int\frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+a^2c\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\right.}{a}\right.}{a\sqrt{a^2cx^2+c}}\right.\right.$$

$$a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{2\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right.}{2\sqrt{a^2cx^2+c}}\right.}{3a}\right.$$

$\downarrow$ 5493

$$c\left(\frac{c\sqrt{a^2x^2+1}\int\frac{\arctan(ax)^2}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+a^2c\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{Pol}}{}\right.}{a\sqrt{a^2cx^2-}}\right.\right.$$

$$a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{2\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right.}{2\sqrt{a^2cx^2+c}}\right.}{3a}\right.$$

$\downarrow$ 5491

$$c\left(\frac{c\sqrt{a^2x^2+1}\int \frac{\sqrt{a^2x^2+1}\arctan(ax)^2}{ax}d\arctan(ax)}{\sqrt{a^2cx^2+c}}+a^2c\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\text{arc}}{a}\right.}{}\right.\right.$$

$$a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{2\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right.}{2\sqrt{a^2cx^2+c}}\right.}{3a}\right.$$

$\downarrow$ 3042

$$c\left(\frac{c\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc(\arctan(ax))d\arctan(ax)}{\sqrt{a^2cx^2+c}}+a^2c\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\,\text{a}}{}\right.}{}\right.\right.$$

$$a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{2\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right.}{2\sqrt{a^2cx^2+c}}\right.}{3a}\right.$$

$\downarrow$ 4671

$$c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\int \arctan(ax)\log\left(1-e^{i\arctan(ax)}\right)d\arctan(ax)+2\int \arctan(ax)\log\left(1+e^{i\arctan(ax)}\right)d\arctan\right.}{\sqrt{a^2cx^2+c}}\right.$$

$$a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{2\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right.}{2\sqrt{a^2cx^2+c}}\right.}{3a}\right.$$

$\Big\downarrow$ 3011

$$c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\,\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-i\int \text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)d\arctan(ax)\right)-2\left(i\arc\right.\right.}{\sqrt{a}}\right.$$

$$a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{2\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right.}{2\sqrt{a^2cx^2+c}}\right.}{3a}\right.$$

$\Big\downarrow$ 2720

$$c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)\right.}{}\right.$$

$$a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{2\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right.}{2\sqrt{a^2cx^2+c}}\right.}{3a}\right.$$

$\downarrow$ 7143

$$c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\arctan(ax)^2\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)+2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\operatorname{PolyLog}\left(3,-\right.\right.\right.}{\sqrt{a^2cx^2+c}}\right.$$

$$a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{2\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right.}{2\sqrt{a^2cx^2+c}}\right.}{3a}\right.$$

input
```
Int[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/x,x]
```

output
```
a^2*c*(((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/(3*a^2*c) - (2*(-1/2*Sqrt[c +
 a^2*c*x^2]/a + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/2 + (c*Sqrt[1 + a^2*x^
2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/a + (I*Po
lyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*PolyLog[2, (I*Sqr
t[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/(2*Sqrt[c + a^2*c*x^2])))/(3*a)) + c*(
a^2*c*((Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(a^2*c) - (2*Sqrt[1 + a^2*x^2]*
(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/a + (I*PolyL
og[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*PolyLog[2, (I*Sqrt[1
 + I*a*x])/Sqrt[1 - I*a*x]])/a))/(a*Sqrt[c + a^2*c*x^2])) + (c*Sqrt[1 + a^
2*x^2]*(-2*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])] + 2*(I*ArcTan[a*x]*Pol
yLog[2, -E^(I*ArcTan[a*x])] - PolyLog[3, -E^(I*ArcTan[a*x])]) - 2*(I*ArcTa
n[a*x]*PolyLog[2, E^(I*ArcTan[a*x])] - PolyLog[3, E^(I*ArcTan[a*x])])))/Sq
rt[c + a^2*c*x^2])
```

## Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5413
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Simp[x*(d + e*x^2)
^q*((a + b*ArcTan[c*x])/(2*q + 1)), x] + Simp[2*d*(q/(2*q + 1))   Int[(d +
e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0]
```

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5491
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Simp[1/Sqrt[d]   Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTa
n[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] &&
GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

## Maple [A] (verified)

Time = 2.57 (sec) , antiderivative size = 365, normalized size of antiderivative = 0.69

| method | result |
|---|---|
| default | $\frac{c\sqrt{c(ax-i)(ax+i)}\left(\arctan(ax)^2x^2a^2-\arctan(ax)ax+4\arctan(ax)^2+1\right)}{3}+\frac{ic\sqrt{c(ax-i)(ax+i)}\left(3i\arctan(ax)^2\ln\left(1+\frac{iax+1}{\sqrt{a^2x^2+1}}\right.\right.}{}$ |

input
```
int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/x,x,method=_RETURNVERBOSE)
```

output
```
1/3*c*(c*(a*x-I)*(a*x+I))^(1/2)*(arctan(a*x)^2*x^2*a^2-arctan(a*x)*a*x+4*a
rctan(a*x)^2+1)+1/3*I*c*(c*(a*x-I)*(a*x+I))^(1/2)*(3*I*arctan(a*x)^2*ln(1+
(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*I*arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^
(1/2))-7*I*arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+7*I*arctan(a*x)
*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*arctan(a*x)*polylog(2,-(1+I*a*x)/(a
^2*x^2+1)^(1/2))-6*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*
polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I*polylog(3,(1+I*a*x)/(a^2*x^2+1
)^(1/2))-7*dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+7*dilog(1-I*(1+I*a*x)/(a
^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)
```

## Fricas [F]

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^2}{x}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)^2}{x}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/x,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^2/x, x)
```

## Sympy [F]

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^2}{x}\,dx = \int \frac{(c(a^2x^2+1))^{\frac{3}{2}}\operatorname{atan}^2(ax)}{x}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)**2/x,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**2/x, x)
```

## Maxima [F]

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^2}{x}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)^2}{x}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/x,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^2/x, x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^2}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^2}{x}\,dx = \int \frac{\mathrm{atan}(a\,x)^2\,(c\,a^2\,x^2+c)^{3/2}}{x}\,dx$$

input
```
int((atan(a*x)^2*(c + a^2*c*x^2)^(3/2))/x,x)
```

output
```
int((atan(a*x)^2*(c + a^2*c*x^2)^(3/2))/x, x)
```

## Reduce [F]

$$\int \frac{(c + a^2cx^2)^{3/2} \arctan(ax)^2}{x} \, dx = \sqrt{c}\, c \left( \int \frac{\sqrt{a^2x^2 + 1}\, atan(ax)^2}{x} dx + \left( \int \sqrt{a^2x^2 + 1}\, atan(ax)^2 \, x dx \right) a^2 \right)$$

input
```
int((a^2*c*x^2+c)^(3/2)*atan(a*x)^2/x,x)
```

output
```
sqrt(c)*c*(int((sqrt(a**2*x**2 + 1)*atan(a*x)**2)/x,x) + int(sqrt(a**2*x**
2 + 1)*atan(a*x)**2*x,x)*a**2)
```

# 3.320 $\int \frac{(c+a^2cx^2)^{3/2} \arctan(ax)^2}{x^2} \, dx$

## Optimal result

Integrand size = 24, antiderivative size = 556

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^2}{x^2}\,dx = -ac\sqrt{c+a^2cx^2}\arctan(ax) - \frac{c\sqrt{c+a^2cx^2}\arctan(ax)^2}{x} + \frac{1}{2}a^2cx\sqrt{c+a^2cx^2}\arctan(ax)^2 - \frac{3iac^2\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2}{\sqrt{c+a^2cx^2}} - \frac{4ac^2\sqrt{1+a^2x^2}\arctan(ax)\operatorname{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} + ac^{3/2}\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right) + \frac{3iac^2\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} - \frac{3iac^2\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,ie^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} + \frac{2iac^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} - \frac{2iac^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} - \frac{3ac^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} + \frac{3ac^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,ie^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}$$

output
```
-a*c*(a^2*c*x^2+c)^(1/2)*arctan(a*x)-c*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2/x
+1/2*a^2*c*x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2-3*I*a*c^2*(a^2*x^2+1)^(1/2)
*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2)-4*a
*c^2*(a^2*x^2+1)^(1/2)*arctan(a*x)*arctanh((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2)
)/(a^2*c*x^2+c)^(1/2)+a*c^(3/2)*arctanh(a*c^(1/2)*x/(a^2*c*x^2+c)^(1/2))+3
*I*a*c^2*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^
(1/2))/(a^2*c*x^2+c)^(1/2)-3*I*a*c^2*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog
(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)+2*I*a*c^2*(a^2*x^2+1
)^(1/2)*polylog(2,-(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/(a^2*c*x^2+c)^(1/2)-2*
I*a*c^2*(a^2*x^2+1)^(1/2)*polylog(2,(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/(a^2*
c*x^2+c)^(1/2)-3*a*c^2*(a^2*x^2+1)^(1/2)*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1
)^(1/2))/(a^2*c*x^2+c)^(1/2)+3*a*c^2*(a^2*x^2+1)^(1/2)*polylog(3,I*(1+I*a*
x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 376, normalized size of antiderivative = 0.68

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^2}{x^2}\,dx = \frac{c\sqrt{c+a^2cx^2}\left(2ax\coth^{-1}\left(\frac{ax}{\sqrt{1+a^2x^2}}\right)-2ax\sqrt{1+a^2x^2}\arctan(ax)-2\sqrt{}\right.}{}$$

input
```
Integrate[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/x^2,x]
```

output
```
(c*Sqrt[c + a^2*c*x^2]*(2*a*x*ArcCoth[(a*x)/Sqrt[1 + a^2*x^2]] - 2*a*x*Sqr
t[1 + a^2*x^2]*ArcTan[a*x] - 2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2 + a^2*x^2*S
qrt[1 + a^2*x^2]*ArcTan[a*x]^2 - (2*I)*a*x*ArcTan[E^(I*ArcTan[a*x])]*ArcTa
n[a*x]^2 + 4*a*x*ArcTan[a*x]*Log[1 - E^(I*ArcTan[a*x])] + 2*a*x*ArcTan[a*x
]^2*Log[1 - I*E^(I*ArcTan[a*x])] - 2*a*x*ArcTan[a*x]^2*Log[1 + I*E^(I*ArcT
an[a*x])] - 4*a*x*ArcTan[a*x]*Log[1 + E^(I*ArcTan[a*x])] + (4*I)*a*x*PolyL
og[2, -E^(I*ArcTan[a*x])] + (6*I)*a*x*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*Arc
Tan[a*x])] - (6*I)*a*x*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] - (4*I)
*a*x*PolyLog[2, E^(I*ArcTan[a*x])] - 6*a*x*PolyLog[3, (-I)*E^(I*ArcTan[a*x
])] + 6*a*x*PolyLog[3, I*E^(I*ArcTan[a*x])]))/(2*x*Sqrt[1 + a^2*x^2])
```

## Rubi [A] (verified)

Time = 4.49 (sec) , antiderivative size = 536, normalized size of antiderivative = 0.96, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.875$, Rules used = {5485, 5415, 224, 219, 5425, 5423, 3042, 4669, 3011, 2720, 5485, 5425, 5423, 3042, 4669, 3011, 2720, 5479, 5493, 5489, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2 \left(a^2cx^2+c\right)^{3/2}}{x^2}dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\int \sqrt{a^2cx^2+c}\arctan(ax)^2dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx$$

$$\Big\downarrow \text{5415}$$

$$a^2c\left(\frac{1}{2}c\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int \frac{1}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}\right)+ c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx$$

$$\Big\downarrow \text{224}$$

$$a^2c\left(\frac{1}{2}c\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int \frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+}}{a}\right. c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx$$

$$\Big\downarrow \text{219}$$

$$a^2c\left(\frac{1}{2}c\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}\right)+ c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx$$

$$\Big\downarrow \text{5425}$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\int\frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}\right.$$
$$c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx$$

$\downarrow$ 5423

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\int\sqrt{a^2x^2+1}\arctan(ax)^2d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}\right.$$
$$c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx$$

$\downarrow$ 3042

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\int\arctan(ax)^2\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{}}{a}\right.$$
$$c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx$$

$\downarrow$ 4669

$$c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx+$$
$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\int\arctan(ax)\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+2\int\arctan(ax)\log\left(1+ie^{i\arctan(ax)}\right)d\text{ar}\right.}{2a\sqrt{a^2cx^2+c}}\right.$$

$\downarrow$ 3011

$$c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx+$$
$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-i\int\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(ax)\right)-2(i\right.}{2}\right.$$

$\downarrow$ 2720

$$c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx+$$
$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan}\right.\right.}{}\right.$$

$$\Big\downarrow \text{5485}$$

$$c\left(a^2c\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx\right)+$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan}\right.\right.}{}\right.$$

$$\Big\downarrow \text{5425}$$

$$c\left(\frac{a^2c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx\right)+$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan}\right.\right.}{}\right.$$

$$\Big\downarrow \text{5423}$$

$$c\left(\frac{ac\sqrt{a^2x^2+1}\int \sqrt{a^2x^2+1}\arctan(ax)^2d\arctan(ax)}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx\right)+$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan}\right.\right.}{}\right.$$

$$\Big\downarrow \text{3042}$$

$$c\left(c\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx+\frac{ac\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{\sqrt{a^2cx^2+c}}\right)+$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan}\right.\right.}{}\right.$$

$$\Big\downarrow \text{4669}$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan}\right.\right.}{}\right.$$

$$c\left(c\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx+\frac{ac\sqrt{a^2x^2+1}\left(-2\int \arctan(ax)\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+2\int \arctan(ax)\log\right.}{\sqrt{a^2cx^2+c}}\right.$$

$$\Big\downarrow \text{3011}$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan}\right.\right.}{}\right.$$

$$c\left(c\int\frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx+\frac{ac\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-i\int\operatorname{PolyLog}\left(2,-ie^{i\arctan(a}\right.\right.\right.}{}\right.$$

$\downarrow$ 2720

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan}\right.\right.}{}\right.$$

$$c\left(c\int\frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx+\frac{ac\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,\right.\right.\right.}{}\right.$$

$\downarrow$ 5479

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan}\right.\right.}{}\right.$$

$$c\left(c\left(2a\int\frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}\right)+\frac{ac\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax}\right.\right.\right.}{}\right.$$

$\downarrow$ 5493

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan}\right.\right.}{}\right.$$

$$c\left(c\left(\frac{2a\sqrt{a^2x^2+1}\int\frac{\arctan(ax)}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}\right)+\frac{ac\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\,a}\right.\right.\right.}{}\right.$$

$\downarrow$ 5489

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan}\right.\right.}{}\right.$$

$$c\left(\frac{ac\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(}\right.\right.}{}\right.$$

$\downarrow$ 7143

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)\right)-2\left(i\arctan(ax)\operatorname{Poly}\right.\right.}{2a\sqrt{a^2cx^2+c}}\right.$$

$$c\left(c\left(-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}+\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\operatorname{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\operatorname{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\right.}{\sqrt{a^2cx^2+c}}\right.\right.$$

input
```
Int[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/x^2,x]
```

output
```
a^2*c*(-((Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/a) + (x*Sqrt[c + a^2*c*x^2]*Arc
Tan[a*x]^2)/2 + (Sqrt[c]*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/a + (
c*Sqrt[1 + a^2*x^2]*((-2*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 + 2*(I
*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - PolyLog[3, (-I)*E^(I*Arc
Tan[a*x])]) - 2*(I*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] - PolyLog[3
, I*E^(I*ArcTan[a*x])])))/(2*a*Sqrt[c + a^2*c*x^2])) + c*(c*(-((Sqrt[c + a
^2*c*x^2]*ArcTan[a*x]^2)/(c*x)) + (2*a*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]*A
rcTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]] + I*PolyLog[2, -(Sqrt[1 + I*a*x]/S
qrt[1 - I*a*x])] - I*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]]))/Sqrt[c
+ a^2*c*x^2]) + (a*c*Sqrt[1 + a^2*x^2]*((-2*I)*ArcTan[E^(I*ArcTan[a*x])]*A
rcTan[a*x]^2 + 2*(I*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - PolyL
og[3, (-I)*E^(I*ArcTan[a*x])]) - 2*(I*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan
[a*x])] - PolyLog[3, I*E^(I*ArcTan[a*x])])))/Sqrt[c + a^2*c*x^2])
```

### Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5415
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTan[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x])^p,
x], x] + Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*(
a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[q, 0] && GtQ[p, 1]
```

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5489
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sq
rt[1 - I*c*x]], x] + (Simp[I*(b/Sqrt[d])*PolyLog[2, -Sqrt[1 + I*c*x]/Sqrt[1
 - I*c*x]], x] - Simp[I*(b/Sqrt[d])*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x]], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

## Maple [A] (verified)

Time = 2.54 (sec) , antiderivative size = 356, normalized size of antiderivative = 0.64

| method | result |
|---|---|
| default | $\frac{c\sqrt{c(ax-i)(ax+i)}\,\arctan(ax)(x^2a^2\arctan(ax)-2ax-2\arctan(ax))}{2x}+\frac{ica\sqrt{c(ax-i)(ax+i)}\left(3i\arctan(ax)^2\ln\left(1+\frac{i(iax+1)}{\sqrt{a^2x^2+1}}\right)-\right.}{}$ |

input

```
int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/x^2,x,method=_RETURNVERBOSE)
```

output

```
1/2*c*(c*(a*x-I)*(a*x+I))^(1/2)*arctan(a*x)*(x^2*a^2*arctan(a*x)-2*a*x-2*a
rctan(a*x))/x+1/2*I*c*a*(c*(a*x-I)*(a*x+I))^(1/2)*(3*I*arctan(a*x)^2*ln(1+
I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*I*arctan(a*x)^2*ln(1-I*(1+I*a*x)/(a^2*x^2
+1)^(1/2))+4*I*arctan(a*x)*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*arctan(a*x)
*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*arctan(a*x)*polylog(2,I*(1+I*
a*x)/(a^2*x^2+1)^(1/2))+6*I*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I*
polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+4*dilog((1+I*a*x)/(a^2*x^2+1)^(1/
2))-4*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))+4*dilog(1+(1+I*a*x)/(a^2*x^2+1)^
(1/2)))/(a^2*x^2+1)^(1/2)
```

## Fricas [F]

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^2}{x^2}\,dx=\int \frac{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)^2}{x^2}\,dx$$

input

```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/x^2,x, algorithm="fricas")
```

output

```
integral((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^2/x^2, x)
```

## Sympy [F]

$$\int \frac{(c + a^2cx^2)^{3/2}\arctan(ax)^2}{x^2}\,dx = \int \frac{(c(a^2x^2 + 1))^{\frac{3}{2}}\operatorname{atan}^2(ax)}{x^2}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)**2/x**2,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**2/x**2, x)
```

## Maxima [F]

$$\int \frac{(c + a^2cx^2)^{3/2}\arctan(ax)^2}{x^2}\,dx = \int \frac{(a^2cx^2 + c)^{\frac{3}{2}}\arctan(ax)^2}{x^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/x^2,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^2/x^2, x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{(c + a^2cx^2)^{3/2}\arctan(ax)^2}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/x^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^2}{x^2}\,dx = \int \frac{\operatorname{atan}(a\,x)^2\,(c\,a^2\,x^2+c)^{3/2}}{x^2}\,dx$$

input
```
int((atan(a*x)^2*(c + a^2*c*x^2)^(3/2))/x^2,x)
```

output
```
int((atan(a*x)^2*(c + a^2*c*x^2)^(3/2))/x^2, x)
```

## Reduce [F]

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^2}{x^2}\,dx = \sqrt{c}\,c\left(\int \frac{\sqrt{a^2x^2+1}\,atan(ax)^2}{x^2}dx + \left(\int \sqrt{a^2x^2+1}\,atan(ax)^2\,dx\right)a^2\right)$$

input
```
int((a^2*c*x^2+c)^(3/2)*atan(a*x)^2/x^2,x)
```

output
```
sqrt(c)*c*(int((sqrt(a**2*x**2 + 1)*atan(a*x)**2)/x**2,x) + int(sqrt(a**2*
x**2 + 1)*atan(a*x)**2,x)*a**2)
```

# 3.321 $\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^2}{x^3}\,dx$

## Optimal result

Integrand size = 24, antiderivative size = 567

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^2}{x^3}\,dx = -\frac{ac\sqrt{c+a^2cx^2}\arctan(ax)}{x} + a^2c\sqrt{c+a^2cx^2}\arctan(ax)^2 - \frac{c\sqrt{c+a^2cx^2}\arctan(ax)^2}{2x^2} + \frac{4ia^2c^2\sqrt{1+a^2x^2}\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} - \frac{3a^2c^2\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} - a^2c^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c+a^2cx^2}}{\sqrt{c}}\right) + \frac{3ia^2c^2\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} - \frac{3ia^2c^2\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} - \frac{2ia^2c^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} + \frac{2ia^2c^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} - \frac{3a^2c^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,-e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} + \frac{3a^2c^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}$$

output
```
-a*c*(a^2*c*x^2+c)^(1/2)*arctan(a*x)/x+a^2*c*(a^2*c*x^2+c)^(1/2)*arctan(a*
x)^2-1/2*c*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2/x^2+4*I*a^2*c^2*(a^2*x^2+1)^(
1/2)*arctan(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/(a^2*c*x^2+c)^(1/
2)-3*a^2*c^2*(a^2*x^2+1)^(1/2)*arctan(a*x)^2*arctanh((1+I*a*x)/(a^2*x^2+1)
^(1/2))/(a^2*c*x^2+c)^(1/2)-a^2*c^(3/2)*arctanh((a^2*c*x^2+c)^(1/2)/c^(1/2
))+3*I*a^2*c^2*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2
+1)^(1/2))/(a^2*c*x^2+c)^(1/2)-3*I*a^2*c^2*(a^2*x^2+1)^(1/2)*arctan(a*x)*p
olylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)-2*I*a^2*c^2*(a^2
*x^2+1)^(1/2)*polylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/(a^2*c*x^2+c)^
(1/2)+2*I*a^2*c^2*(a^2*x^2+1)^(1/2)*polylog(2,I*(1+I*a*x)^(1/2)/(1-I*a*x)^
(1/2))/(a^2*c*x^2+c)^(1/2)-3*a^2*c^2*(a^2*x^2+1)^(1/2)*polylog(3,-(1+I*a*x
)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)+3*a^2*c^2*(a^2*x^2+1)^(1/2)*polyl
og(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (warning: unable to verify)

Time = 2.17 (sec) , antiderivative size = 455, normalized size of antiderivative = 0.80

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^2}{x^3}\,dx = \frac{a^2c\sqrt{c+a^2cx^2}\left(-4\arctan(ax)-4\arctan(ax)\cot^2\left(\frac{1}{2}\arctan(ax)\right)+4a\right.}{}$$

input
```
Integrate[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/x^3,x]
```

output
```
(a^2*c*Sqrt[c + a^2*c*x^2]*(-4*ArcTan[a*x] - 4*ArcTan[a*x]*Cot[ArcTan[a*x]
/2]^2 + 4*a*x*ArcTan[a*x]^2*Csc[ArcTan[a*x]/2]^2 - ArcTan[a*x]^2*Cot[ArcTa
n[a*x]/2]*Csc[ArcTan[a*x]/2]^2 + 12*ArcTan[a*x]^2*Cot[ArcTan[a*x]/2]*Log[1
 - E^(I*ArcTan[a*x])] - 16*ArcTan[a*x]*Cot[ArcTan[a*x]/2]*Log[1 - I*E^(I*A
rcTan[a*x])] + 16*ArcTan[a*x]*Cot[ArcTan[a*x]/2]*Log[1 + I*E^(I*ArcTan[a*x
])] - 12*ArcTan[a*x]^2*Cot[ArcTan[a*x]/2]*Log[1 + E^(I*ArcTan[a*x])] + 8*C
ot[ArcTan[a*x]/2]*Log[Tan[ArcTan[a*x]/2]] + (24*I)*ArcTan[a*x]*Cot[ArcTan[
a*x]/2]*PolyLog[2, -E^(I*ArcTan[a*x])] - (16*I)*Cot[ArcTan[a*x]/2]*PolyLog
[2, (-I)*E^(I*ArcTan[a*x])] + (16*I)*Cot[ArcTan[a*x]/2]*PolyLog[2, I*E^(I*
ArcTan[a*x])] - (24*I)*ArcTan[a*x]*Cot[ArcTan[a*x]/2]*PolyLog[2, E^(I*ArcT
an[a*x])] - 24*Cot[ArcTan[a*x]/2]*PolyLog[3, -E^(I*ArcTan[a*x])] + 24*Cot[
ArcTan[a*x]/2]*PolyLog[3, E^(I*ArcTan[a*x])] + ArcTan[a*x]^2*Csc[ArcTan[a*
x]/2]*Sec[ArcTan[a*x]/2])*Tan[ArcTan[a*x]/2])/(8*Sqrt[1 + a^2*x^2])
```

## Rubi [A] (verified)

Time = 6.21 (sec) , antiderivative size = 667, normalized size of antiderivative = 1.18, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.958, Rules used = {5485, 5485, 5465, 5425, 5421, 5493, 5491, 3042, 4671, 3011, 2720, 5497, 5479, 243, 73, 221, 5493, 5491, 3042, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2 \left(a^2cx^2+c\right)^{3/2}}{x^3} \, dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x}dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^3}dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\left(a^2c\int \frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx\right)+c\left(a^2c\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx\right)$$

$$\Big\downarrow \text{5465}$$

$$a^2c\left(a^2c\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}\right)+c\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx\right)+c\left(a^2c\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx\right)$$

$$\Big\downarrow \text{5425}$$

$$a^2c\left(a^2c\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{a\sqrt{a^2cx^2+c}}\right)+c\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx\right)+c\left(a^2c\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx\right)$$

$$\Big\downarrow \text{5421}$$

$$c\left(a^2c\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx + c\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx\right) +$$
$$a^2c\left(c\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx + a^2c\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c} - \frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a} + \frac{i\,\mathrm{PolyLog}(}{}\right.}{a\sqrt{a^2cx^2+c}}\right.\right.$$

$\downarrow$ 5493

$$c\left(\frac{a^2c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}} + c\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx\right) +$$
$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}} + a^2c\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c} - \frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a} + \right.}{a\sqrt{a^2cx}}\right.\right.$$

$\downarrow$ 5491

$$c\left(\frac{a^2c\sqrt{a^2x^2+1}\int \frac{\sqrt{a^2x^2+1}\arctan(ax)^2}{ax}d\arctan(ax)}{\sqrt{a^2cx^2+c}} + c\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx\right) +$$
$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\int \frac{\sqrt{a^2x^2+1}\arctan(ax)^2}{ax}d\arctan(ax)}{\sqrt{a^2cx^2+c}} + a^2c\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c} - \frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)}{}\right.}{}\right.\right.$$

$\downarrow$ 3042

$$c\left(\frac{a^2c\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc(\arctan(ax))d\arctan(ax)}{\sqrt{a^2cx^2+c}} + c\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx\right) +$$
$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc(\arctan(ax))d\arctan(ax)}{\sqrt{a^2cx^2+c}} + a^2c\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c} - \frac{2\sqrt{a^2x^2+1}\left(-\right.}{}\right.\right.$$

$\downarrow$ 4671

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\int \arctan(ax)\log\left(1-e^{i\arctan(ax)}\right)d\arctan(ax)+2\int \arctan(ax)\log\left(1+e^{i\arctan(ax)}\right)d\arctan\right.}{\sqrt{a^2cx^2+c}}\right.$$

$$c\left(c\int\frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx+\frac{a^2c\sqrt{a^2x^2+1}\left(-2\int \arctan(ax)\log\left(1-e^{i\arctan(ax)}\right)d\arctan(ax)+2\int \arctan(ax)\log\right.}{\sqrt{a^2cx^2+c}}\right.$$

$\big\downarrow$ 3011

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-i\int \operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)d\arctan(ax)\right)-2\left(i\,a\right.\right.}{\sqrt{}}\right.$$

$$c\left(c\int\frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx+\frac{a^2c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-i\int \operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right.\right.\right.}{}\right.$$

$\big\downarrow$ 2720

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(a}\right.\right.}{}\right.$$

$$c\left(c\int\frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx+\frac{a^2c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,\right.\right.\right.}{}\right.$$

$\big\downarrow$ 5497

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(a}\right.\right.}{}\right.$$

$$c\left(c\left(-\frac{1}{2}a^2\int\frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+a\int\frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{2cx^2}\right)+\frac{a^2c\sqrt{a^2x^2+1}\left(2\left(i\arctan\right.\right.}{}\right.$$

$\big\downarrow$ 5479

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(a}\right.\right.}{c\left(c\left(-\frac{1}{2}a^2\int\frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+a\left(a\int\frac{1}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right)-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{2cx^2}\right)\right.}\right.$$

$\Big\downarrow$ 243

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(a}\right.\right.}{c\left(c\left(-\frac{1}{2}a^2\int\frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+a\left(\frac{1}{2}a\int\frac{1}{x^2\sqrt{a^2cx^2+c}}dx^2-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right)-\frac{\arctan(ax)^2\sqrt{a^2cx^2}}{2cx^2}\right.\right.}\right.$$

$\Big\downarrow$ 73

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(a}\right.\right.}{c\left(c\left(-\frac{1}{2}a^2\int\frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+a\left(\frac{\int\frac{1}{\frac{x^4}{a^2c}-\frac{1}{a^2}}d\sqrt{a^2cx^2+c}}{ac}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right)-\frac{\arctan(ax)^2\sqrt{a^2cx^2+}}{2cx^2}\right.\right.}\right.$$

$\Big\downarrow$ 221

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(a}\right.\right.}{c\left(c\left(-\frac{1}{2}a^2\int\frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+a\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\operatorname{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{\arctan(ax)^2\sqrt{a^2cx^2}}{2cx^2}\right.\right.}\right.$$

$\Big\downarrow$ 5493

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(a\right.\right.}{}\right.$$

$$c\left(c\left(-\frac{a^2\sqrt{a^2x^2+1}\int\frac{\arctan(ax)^2}{x\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+a\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\operatorname{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{\arctan(ax)^2\sqrt{a^2}}{2cx^2}\right.\right.$$

$$\Big\downarrow \text{5491}$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(a\right.\right.}{}\right.$$

$$c\left(c\left(-\frac{a^2\sqrt{a^2x^2+1}\int\frac{\sqrt{a^2x^2+1}\arctan(ax)^2}{ax}d\arctan(ax)}{2\sqrt{a^2cx^2+c}}+a\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\operatorname{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(a\right.\right.}{}\right.$$

$$c\left(c\left(-\frac{a^2\sqrt{a^2x^2+1}\int\arctan(ax)^2\csc(\arctan(ax))d\arctan(ax)}{2\sqrt{a^2cx^2+c}}+a\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\operatorname{arctanh}\left(\frac{\sqrt{a}}{}\right.}{\sqrt{c}}\right.\right.\right.$$

$$\Big\downarrow \text{4671}$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(a\right.\right.}{}\right.$$

$$c\left(\frac{a^2c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(a\right.\right.}{}\right.$$

$$\Big\downarrow \text{3011}$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(a\right.\right.}{}\right.$$

$$c\left(\frac{a^2c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(a\right.\right.}{}\right.$$

$\downarrow$ 2720

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{i}}{\sqrt{1}}\right.}{a}\right.}{a\sqrt{a^2cx^2+c}}\right.\right.$$

$$c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)\arctan(ax)^2+2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\right.\right.}{}\right.$$

$\downarrow$ 7143

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\arctan(ax)^2\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)+2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\operatorname{PolyLog}\left(3,\right.\right.\right.}{\sqrt{a^2cx^2+c}}\right.$$

$$c\left(\frac{a^2c\sqrt{a^2x^2+1}\left(-2\arctan(ax)^2\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)+2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\operatorname{PolyLog}\left(3,\right.\right.\right.}{\sqrt{a^2cx^2+c}}\right.$$

input
```
Int[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/x^3,x]
```

output
```
a^2*c*(a^2*c*((Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(a^2*c) - (2*Sqrt[1 + a^
2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/a + (
I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*PolyLog[2, (I
*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/(a*Sqrt[c + a^2*c*x^2])) + (c*Sqrt
[1 + a^2*x^2]*(-2*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])] + 2*(I*ArcTan[a
*x]*PolyLog[2, -E^(I*ArcTan[a*x])] - PolyLog[3, -E^(I*ArcTan[a*x])]) - 2*(
I*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])] - PolyLog[3, E^(I*ArcTan[a*x])
])))/Sqrt[c + a^2*c*x^2]) + c*(c*(-1/2*(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)
/(c*x^2) + a*(-((Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(c*x)) - (a*ArcTanh[Sqrt
[c + a^2*c*x^2]/Sqrt[c]])/Sqrt[c]) - (a^2*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x
]^2*ArcTanh[E^(I*ArcTan[a*x])] + 2*(I*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[
a*x])] - PolyLog[3, -E^(I*ArcTan[a*x])]) - 2*(I*ArcTan[a*x]*PolyLog[2, E^(
I*ArcTan[a*x])] - PolyLog[3, E^(I*ArcTan[a*x])])))/(2*Sqrt[c + a^2*c*x^2])
) + (a^2*c*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])]
+ 2*(I*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])] - PolyLog[3, -E^(I*ArcTa
n[a*x])]) - 2*(I*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])] - PolyLog[3, E^
(I*ArcTan[a*x])])))/Sqrt[c + a^2*c*x^2])
```

## Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5491
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Simp[1/Sqrt[d]   Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTa
n[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] &&
GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5497
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)^(m
 + 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[c^2*((m
+ 2)/(f^2*(m + 1)))   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x
^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
 && LtQ[m, -1] && NeQ[m, -2]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

## Maple [A] (verified)

Time = 2.56 (sec) , antiderivative size = 412, normalized size of antiderivative = 0.73

| method | result |
|---|---|
| default | $\frac{c\sqrt{c(ax-i)(ax+i)}\,\arctan(ax)(2x^2a^2\arctan(ax)-2ax-\arctan(ax))}{2x^2}-\frac{c\,a^2\sqrt{c(ax-i)(ax+i)}\left(3\arctan(ax)^2\ln\left(1+\frac{iax+1}{\sqrt{a^2x^2+1}}\right)-\right.}{}$ |

input
```
int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*c*(c*(a*x-I)*(a*x+I))^(1/2)*arctan(a*x)*(2*x^2*a^2*arctan(a*x)-2*a*x-a
rctan(a*x))/x^2-1/2*c*a^2*(c*(a*x-I)*(a*x+I))^(1/2)*(3*arctan(a*x)^2*ln(1+
(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1
/2))-6*I*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*arctan(a*
x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-4*arctan(a*x)*ln(1+I*(1+I*a*x)/(
a^2*x^2+1)^(1/2))+4*arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+4*I*di
log(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-4*I*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(
1/2))-2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-1)+2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/
2))+6*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*polylog(3,(1+I*a*x)/(a^2*x
^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)
```

## Fricas [F]

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^2}{x^3}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)^2}{x^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/x^3,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^2/x^3, x)
```

## Sympy [F]

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^2}{x^3}\,dx = \int \frac{(c(a^2x^2+1))^{\frac{3}{2}}\operatorname{atan}^2(ax)}{x^3}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)**2/x**3,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**2/x**3, x)
```

## Maxima [F]

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^2}{x^3}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)^2}{x^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/x^3,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^2/x^3, x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^2}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/x^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^2}{x^3}\,dx = \int \frac{\mathrm{atan}(a\,x)^2\,(c\,a^2\,x^2+c)^{3/2}}{x^3}\,dx$$

input
```
int((atan(a*x)^2*(c + a^2*c*x^2)^(3/2))/x^3,x)
```

output
```
int((atan(a*x)^2*(c + a^2*c*x^2)^(3/2))/x^3, x)
```

## Reduce [F]

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^2}{x^3}\,dx = \sqrt{c}\,c\left(\int \frac{\sqrt{a^2x^2+1}\,atan(ax)^2}{x^3}dx + \left(\int \frac{\sqrt{a^2x^2+1}\,atan(ax)^2}{x}dx\right)a^2\right)$$

input
```
int((a^2*c*x^2+c)^(3/2)*atan(a*x)^2/x^3,x)
```

output
```
sqrt(c)*c*(int((sqrt(a**2*x**2 + 1)*atan(a*x)**2)/x**3,x) + int((sqrt(a**2
*x**2 + 1)*atan(a*x)**2)/x,x)*a**2)
```

## 3.322 $\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^2}{x^4}\,dx$

## Optimal result

Integrand size = 24, antiderivative size = 579

$$
\begin{aligned}
&\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^2}{x^4}\,dx = \\
&-\frac{a^2c\sqrt{c+a^2cx^2}}{3x}-\frac{ac\sqrt{c+a^2cx^2}\arctan(ax)}{3x^2} \\
&-\frac{a^2c\sqrt{c+a^2cx^2}\arctan(ax)^2}{x}-\frac{(c+a^2cx^2)^{3/2}\arctan(ax)^2}{3x^3} \\
&-\frac{2ia^3c^2\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2}{\sqrt{c+a^2cx^2}} \\
&-\frac{14a^3c^2\sqrt{1+a^2x^2}\arctan(ax)\operatorname{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3\sqrt{c+a^2cx^2}} \\
&+\frac{2ia^3c^2\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} \\
&-\frac{2ia^3c^2\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,ie^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} \\
&+\frac{7ia^3c^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3\sqrt{c+a^2cx^2}} \\
&-\frac{7ia^3c^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3\sqrt{c+a^2cx^2}} \\
&-\frac{2a^3c^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} \\
&+\frac{2a^3c^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,ie^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}
\end{aligned}
$$

output
```
-1/3*a^2*c*(a^2*c*x^2+c)^(1/2)/x-1/3*a*c*(a^2*c*x^2+c)^(1/2)*arctan(a*x)/x
^2-a^2*c*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2/x-1/3*(a^2*c*x^2+c)^(3/2)*arcta
n(a*x)^2/x^3-2*I*a^3*c^2*(a^2*x^2+1)^(1/2)*arctan((1+I*a*x)/(a^2*x^2+1)^(1
/2))*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2)-14/3*a^3*c^2*(a^2*x^2+1)^(1/2)*arct
an(a*x)*arctanh((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/(a^2*c*x^2+c)^(1/2)+2*I*a
^3*c^2*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1
/2))/(a^2*c*x^2+c)^(1/2)-2*I*a^3*c^2*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog
(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)+7/3*I*a^3*c^2*(a^2*x
^2+1)^(1/2)*polylog(2,-(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/(a^2*c*x^2+c)^(1/2
)-7/3*I*a^3*c^2*(a^2*x^2+1)^(1/2)*polylog(2,(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2
))/(a^2*c*x^2+c)^(1/2)-2*a^3*c^2*(a^2*x^2+1)^(1/2)*polylog(3,-I*(1+I*a*x)/
(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)+2*a^3*c^2*(a^2*x^2+1)^(1/2)*polylog
(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (warning: unable to verify)

Time = 4.32 (sec) , antiderivative size = 453, normalized size of antiderivative = 0.78

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^2}{x^4}\,dx = \frac{a^3c^2\sqrt{1+a^2x^2}\left(8i\,\mathrm{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-24\left(\frac{\sqrt{1+a^2x^2}\arctan(ax)^2}{ax}-\right.\right.}{}$$

input
```
Integrate[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/x^4,x]
```

output
```
(a^3*c^2*Sqrt[1 + a^2*x^2]*((8*I)*PolyLog[2, -E^(I*ArcTan[a*x])] - 24*((Sq
rt[1 + a^2*x^2]*ArcTan[a*x]^2)/(a*x) - 2*ArcTan[a*x]*Log[1 - E^(I*ArcTan[a
*x])] - ArcTan[a*x]^2*Log[1 - I*E^(I*ArcTan[a*x])] + ArcTan[a*x]^2*Log[1 +
 I*E^(I*ArcTan[a*x])] + 2*ArcTan[a*x]*Log[1 + E^(I*ArcTan[a*x])] - (2*I)*P
olyLog[2, -E^(I*ArcTan[a*x])] - (2*I)*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*Arc
Tan[a*x])] + (2*I)*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] + (2*I)*Pol
yLog[2, E^(I*ArcTan[a*x])] + 2*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] - 2*Poly
Log[3, I*E^(I*ArcTan[a*x])]) - (2*(1 + a^2*x^2)^(3/2)*(2 + 4*ArcTan[a*x]^2
 - 2*Cos[2*ArcTan[a*x]] + ((4*I)*a^3*x^3*PolyLog[2, E^(I*ArcTan[a*x])])/(1
 + a^2*x^2)^(3/2) + ArcTan[a*x]*(2*Sin[2*ArcTan[a*x]] + ((Log[1 - E^(I*Arc
Tan[a*x])] - Log[1 + E^(I*ArcTan[a*x])])*(-3*a*x + Sqrt[1 + a^2*x^2]*Sin[3
*ArcTan[a*x]]))/Sqrt[1 + a^2*x^2])))/(a^3*x^3)))/(24*Sqrt[c + a^2*c*x^2])
```

## Rubi [A] (verified)

Time = 4.65 (sec) , antiderivative size = 575, normalized size of antiderivative = 0.99, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.792$, Rules used = {5485, 5479, 5481, 242, 5485, 5425, 5423, 3042, 4669, 3011, 2720, 5479, 5493, 5489, 5497, 242, 5493, 5489, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2 \left(a^2cx^2+c\right)^{3/2}}{x^4}dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^4}dx$$

$$\Big\downarrow \text{5479}$$

$$a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx+ c\left(\frac{2}{3}a\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^3}dx-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)$$

$$\Big\downarrow \text{5481}$$

$$a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx+ c\left(\frac{2}{3}a\left(-c\int \frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx+ac\int \frac{1}{x^2\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}\right)-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)$$

$$\Big\downarrow \text{242}$$

$$a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx+ c\left(\frac{2}{3}a\left(-c\int \frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}-\frac{a\sqrt{a^2cx^2+c}}{x}\right)-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)$$

$$\Big\downarrow \text{5485}$$

$$a^2c\left(a^2c\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx\right)+ c\left(\frac{2}{3}a\left(-c\int \frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}-\frac{a\sqrt{a^2cx^2+c}}{x}\right)-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)$$

$$\Big\downarrow \text{5425}$$

$$a^2c\left(\frac{a^2c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx\right)+$$
$$c\left(\frac{2}{3}a\left(-c\int \frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}-\frac{a\sqrt{a^2cx^2+c}}{x}\right)-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)$$

$$\Big\downarrow \text{5423}$$

$$a^2c\left(\frac{ac\sqrt{a^2x^2+1}\int \sqrt{a^2x^2+1}\arctan(ax)^2d\arctan(ax)}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx\right)+$$
$$c\left(\frac{2}{3}a\left(-c\int \frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}-\frac{a\sqrt{a^2cx^2+c}}{x}\right)-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^2c\left(c\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx+\frac{ac\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{\sqrt{a^2cx^2+c}}\right)+$$
$$c\left(\frac{2}{3}a\left(-c\int \frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}-\frac{a\sqrt{a^2cx^2+c}}{x}\right)-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)$$

$$\Big\downarrow \text{4669}$$

$$c\left(\frac{2}{3}a\left(-c\int \frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}-\frac{a\sqrt{a^2cx^2+c}}{x}\right)-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)+$$
$$a^2c\left(c\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx+\frac{ac\sqrt{a^2x^2+1}\left(-2\int \arctan(ax)\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+2\int \arctan(ax)\right.}{\sqrt{a^2cx^2+c}}\right.$$

$$\Big\downarrow \text{3011}$$

$$c\left(\frac{2}{3}a\left(-c\int \frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}-\frac{a\sqrt{a^2cx^2+c}}{x}\right)-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)+$$
$$a^2c\left(c\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx+\frac{ac\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-i\int \text{PolyLog}\left(2,-ie^{i\arctan}\right.\right.\right.}{}\right.$$

$$\Big\downarrow \text{2720}$$

$$c\left(\frac{2}{3}a\left(-c\int\frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}-\frac{a\sqrt{a^2cx^2+c}}{x}\right)-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)+$$
$$a^2c\left(c\int\frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx+\frac{ac\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(\right.\right.\right.}{}\right.$$

$\downarrow$ 5479

$$c\left(\frac{2}{3}a\left(-c\int\frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}-\frac{a\sqrt{a^2cx^2+c}}{x}\right)-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)+$$
$$a^2c\left(c\left(2a\int\frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}\right)+\frac{ac\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan}\right.\right.\right.}{}\right.$$

$\downarrow$ 5493

$$c\left(\frac{2}{3}a\left(-c\int\frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}-\frac{a\sqrt{a^2cx^2+c}}{x}\right)-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)+$$
$$a^2c\left(c\left(\frac{2a\sqrt{a^2x^2+1}\int\frac{\arctan(ax)}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}\right)+\frac{ac\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie\right.\right.\right.}{}\right.$$

$\downarrow$ 5489

$$c\left(\frac{2}{3}a\left(-c\int\frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}-\frac{a\sqrt{a^2cx^2+c}}{x}\right)-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)+$$
$$a^2c\left(\frac{ac\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan}\right.\right.}{}\right.$$

$\downarrow$ 5497

$$c\left(\frac{2}{3}a\left(-c\left(-\frac{1}{2}a^2\int\frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx+\frac{1}{2}a\int\frac{1}{x^2\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{2cx^2}\right)-\frac{\arctan(ax)\sqrt{a^2cx^2}}{x^2}\right.\right.$$
$$a^2c\left(\frac{ac\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan}\right.\right.}{}\right.$$

$\downarrow$ 242

$$c\left(\frac{2}{3}a\left(-c\left(-\frac{1}{2}a^2\int\frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{2cx^2}-\frac{a\sqrt{a^2cx^2+c}}{2cx}\right)-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}-a\right.\right.$$

$$a^2c\left(\frac{ac\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan}\right.\right.}{}\right.$$

$$\Big\downarrow \text{5493}$$

$$c\left(\frac{2}{3}a\left(-c\left(-\frac{a^2\sqrt{a^2x^2+1}\int\frac{\arctan(ax)}{x\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{2cx^2}-\frac{a\sqrt{a^2cx^2+c}}{2cx}\right)-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}\right.\right.$$

$$a^2c\left(\frac{ac\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan}\right.\right.}{}\right.$$

$$\Big\downarrow \text{5489}$$

$$a^2c\left(\frac{ac\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan}\right.\right.}{}\right.$$

$$c\left(-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3cx^3}+\frac{2}{3}a\left(-c\left(-\frac{a^2\sqrt{a^2x^2+1}\left(-2\arctan(ax)\operatorname{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\operatorname{PolyLog}\left(2,-\right.\right.}{2\sqrt{a^2cx^2+c}}\right.\right.\right.$$

$$\Big\downarrow \text{7143}$$

$$a^2c\left(c\left(-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}+\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\operatorname{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\operatorname{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-\right.}{\sqrt{a^2cx^2+c}}\right.\right.$$

$$c\left(-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3cx^3}+\frac{2}{3}a\left(-c\left(-\frac{a^2\sqrt{a^2x^2+1}\left(-2\arctan(ax)\operatorname{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\operatorname{PolyLog}\left(2,-\right.\right.}{2\sqrt{a^2cx^2+c}}\right.\right.\right.$$

input
```
Int[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/x^4,x]
```

output
```
c*(-1/3*((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/(c*x^3) + (2*a*(-((a*Sqrt[c
+ a^2*c*x^2])/x) - (Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/x^2 - c*(-1/2*(a*Sqrt
[c + a^2*c*x^2])/(c*x) - (Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(2*c*x^2) - (a^
2*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x
]] + I*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])] - I*PolyLog[2, Sqrt[
1 + I*a*x]/Sqrt[1 - I*a*x]]))/(2*Sqrt[c + a^2*c*x^2]))))/3) + a^2*c*(c*(-(
(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(c*x)) + (2*a*Sqrt[1 + a^2*x^2]*(-2*Ar
cTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]] + I*PolyLog[2, -(Sqrt[1
 + I*a*x]/Sqrt[1 - I*a*x])] - I*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]
]))/Sqrt[c + a^2*c*x^2]) + (a*c*Sqrt[1 + a^2*x^2]*((-2*I)*ArcTan[E^(I*ArcT
an[a*x])]*ArcTan[a*x]^2 + 2*(I*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x
])] - PolyLog[3, (-I)*E^(I*ArcTan[a*x])]) - 2*(I*ArcTan[a*x]*PolyLog[2, I*
E^(I*ArcTan[a*x])] - PolyLog[3, I*E^(I*ArcTan[a*x])])))/Sqrt[c + a^2*c*x^2
])
```

## Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5481
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTan[c*x
])/(f*(m + 2))), x] + (Simp[d/(m + 2)   Int[(f*x)^m*((a + b*ArcTan[c*x])/Sq
rt[d + e*x^2]), x], x] - Simp[b*c*(d/(f*(m + 2)))   Int[(f*x)^(m + 1)/Sqrt[
d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] &&
NeQ[m, -2]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5489
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sq
rt[1 - I*c*x]], x] + (Simp[I*(b/Sqrt[d])*PolyLog[2, -Sqrt[1 + I*c*x]/Sqrt[1
 - I*c*x]], x] - Simp[I*(b/Sqrt[d])*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x]], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5497
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)^(m
 + 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[c^2*((m
+ 2)/(f^2*(m + 1)))   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x
^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
 && LtQ[m, -1] && NeQ[m, -2]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

## Maple [A] (verified)

Time = 2.67 (sec) , antiderivative size = 343, normalized size of antiderivative = 0.59

| method | result |
|---|---|
| default | $-\frac{c\sqrt{c(ax-i)(ax+i)}\,\left(4\arctan(ax)^{2}x^{2}a^{2}+a^{2}x^{2}+\arctan(ax)ax+\arctan(ax)^{2}\right)}{3x^{3}}-\frac{c\,a^{3}\sqrt{c(ax-i)(ax+i)}\,\left(3\arctan(ax)^{2}\ln\left(1+\frac{i(}{\sqrt{}}\right.\right.}{}$ |

input
```
int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*c*(c*(a*x-I)*(a*x+I))^(1/2)*(4*arctan(a*x)^2*x^2*a^2+a^2*x^2+arctan(a
*x)*a*x+arctan(a*x)^2)/x^3-1/3*c*a^3*(c*(a*x-I)*(a*x+I))^(1/2)*(3*arctan(a
*x)^2*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*arctan(a*x)^2*ln(1-I*(1+I*a*x)
/(a^2*x^2+1)^(1/2))-6*I*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/
2))+6*I*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+7*arctan(a*x)
*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-7*I*dilog(1+(1+I*a*x)/(a^2*x^2+1)^(1/2)
)-7*I*dilog((1+I*a*x)/(a^2*x^2+1)^(1/2))+6*polylog(3,-I*(1+I*a*x)/(a^2*x^2
+1)^(1/2))-6*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)
```

## Fricas [F]

$$\int \frac{\left(c+a^{2}cx^{2}\right)^{3/2}\arctan(ax)^{2}}{x^{4}}\,dx=\int \frac{\left(a^{2}cx^{2}+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^{2}}{x^{4}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/x^4,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^2/x^4, x)
```

## Sympy [F]

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^2}{x^4}\,dx = \int \frac{(c(a^2x^2+1))^{\frac{3}{2}}\operatorname{atan}^2(ax)}{x^4}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)**2/x**4,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**2/x**4, x)
```

## Maxima [F]

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^2}{x^4}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)^2}{x^4}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/x^4,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^2/x^4, x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^2}{x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/x^4,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^2}{x^4}\,dx=\int \frac{\operatorname{atan}(a\,x)^2\,(c\,a^2\,x^2+c)^{3/2}}{x^4}\,dx$$

input
```
int((atan(a*x)^2*(c + a^2*c*x^2)^(3/2))/x^4,x)
```

output
```
int((atan(a*x)^2*(c + a^2*c*x^2)^(3/2))/x^4, x)
```

## Reduce [F]

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^2}{x^4}\,dx=\sqrt{c}\,c\left(\int \frac{\sqrt{a^2x^2+1}\,atan(ax)^2}{x^4}dx+\left(\int \frac{\sqrt{a^2x^2+1}\,atan(ax)^2}{x^2}dx\right)a^2\right)$$

input
```
int((a^2*c*x^2+c)^(3/2)*atan(a*x)^2/x^4,x)
```

output
```
sqrt(c)*c*(int((sqrt(a**2*x**2 + 1)*atan(a*x)**2)/x**4,x) + int((sqrt(a**2
*x**2 + 1)*atan(a*x)**2)/x**2,x)*a**2)
```

# 3.323 $\int x^3(c+a^2cx^2)^{5/2}\arctan(ax)^2\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 578

$$\int x^3\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2\,dx=-\frac{115c^2\sqrt{c+a^2cx^2}}{4032a^4}-\frac{115c(c+a^2cx^2)^{3/2}}{18144a^4}$$
$$-\frac{23(c+a^2cx^2)^{5/2}}{7560a^4}+\frac{(c+a^2cx^2)^{7/2}}{252a^4c}+\frac{47c^2x\sqrt{c+a^2cx^2}\arctan(ax)}{1344a^3}$$
$$-\frac{205c^2x^3\sqrt{c+a^2cx^2}\arctan(ax)}{6048a}-\frac{103ac^2x^5\sqrt{c+a^2cx^2}\arctan(ax)}{1512}$$
$$-\frac{1}{36}a^3c^2x^7\sqrt{c+a^2cx^2}\arctan(ax)-\frac{2c^2\sqrt{c+a^2cx^2}\arctan(ax)^2}{63a^4}+\frac{c^2x^2\sqrt{c+a^2cx^2}\arctan(ax)^2}{63a^2}+\frac{5}{21}c^2x^4\sqrt{c$$

output
```
-115/4032*c^2*(a^2*c*x^2+c)^(1/2)/a^4-115/18144*c*(a^2*c*x^2+c)^(3/2)/a^4-
23/7560*(a^2*c*x^2+c)^(5/2)/a^4+1/252*(a^2*c*x^2+c)^(7/2)/a^4/c+47/1344*c^
2*x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)/a^3-205/6048*c^2*x^3*(a^2*c*x^2+c)^(1/
2)*arctan(a*x)/a-103/1512*a*c^2*x^5*(a^2*c*x^2+c)^(1/2)*arctan(a*x)-1/36*a
^3*c^2*x^7*(a^2*c*x^2+c)^(1/2)*arctan(a*x)-2/63*c^2*(a^2*c*x^2+c)^(1/2)*ar
ctan(a*x)^2/a^4+1/63*c^2*x^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2/a^2+5/21*c^
2*x^4*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2+19/63*a^2*c^2*x^6*(a^2*c*x^2+c)^(1
/2)*arctan(a*x)^2+1/9*a^4*c^2*x^8*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2-115/20
16*I*c^3*(a^2*x^2+1)^(1/2)*arctan(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*x)^(1
/2))/a^4/(a^2*c*x^2+c)^(1/2)+115/4032*I*c^3*(a^2*x^2+1)^(1/2)*polylog(2,-I
*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/a^4/(a^2*c*x^2+c)^(1/2)-115/4032*I*c^3*(
a^2*x^2+1)^(1/2)*polylog(2,I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/a^4/(a^2*c*x
^2+c)^(1/2)
```

## Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1320 vs. 2(578) = 1156.

Time = 5.93 (sec) , antiderivative size = 1320, normalized size of antiderivative = 2.28

$$\int x^3\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2\,dx=\text{Too large to display}$$

input
```
Integrate[x^3*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2,x]
```

output
```
((c + a^2*c*x^2)^(5/2)*(-48384*(50 - 32*ArcTan[a*x]^2 + 72*Cos[2*ArcTan[a*
x]] + 160*ArcTan[a*x]^2*Cos[2*ArcTan[a*x]] + 22*Cos[4*ArcTan[a*x]] - (110*
ArcTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - 55*ArcTan[a*
x]*Cos[3*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] - 11*ArcTan[a*x]*Cos[5*
ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] + (110*ArcTan[a*x]*Log[1 + I*E^(
I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + 55*ArcTan[a*x]*Cos[3*ArcTan[a*x]]*Log
[1 + I*E^(I*ArcTan[a*x])] + 11*ArcTan[a*x]*Cos[5*ArcTan[a*x]]*Log[1 + I*E^
(I*ArcTan[a*x])] - ((176*I)*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/(1 + a^2*x
^2)^(5/2) + ((176*I)*PolyLog[2, I*E^(I*ArcTan[a*x])])/(1 + a^2*x^2)^(5/2)
+ 4*ArcTan[a*x]*Sin[2*ArcTan[a*x]] - 22*ArcTan[a*x]*Sin[4*ArcTan[a*x]]) +
576*(1 + a^2*x^2)*(4116 + 10944*ArcTan[a*x]^2 + 6262*Cos[2*ArcTan[a*x]] -
5376*ArcTan[a*x]^2*Cos[2*ArcTan[a*x]] + 2764*Cos[4*ArcTan[a*x]] + 6720*Arc
Tan[a*x]^2*Cos[4*ArcTan[a*x]] + 618*Cos[6*ArcTan[a*x]] - (10815*ArcTan[a*x
]*Log[1 - I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - 6489*ArcTan[a*x]*Cos[3
*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] - 2163*ArcTan[a*x]*Cos[5*ArcTan
[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] - 309*ArcTan[a*x]*Cos[7*ArcTan[a*x]]*L
og[1 - I*E^(I*ArcTan[a*x])] + (10815*ArcTan[a*x]*Log[1 + I*E^(I*ArcTan[a*x
])])/Sqrt[1 + a^2*x^2] + 6489*ArcTan[a*x]*Cos[3*ArcTan[a*x]]*Log[1 + I*E^(
I*ArcTan[a*x])] + 2163*ArcTan[a*x]*Cos[5*ArcTan[a*x]]*Log[1 + I*E^(I*ArcTa
n[a*x])] + 309*ArcTan[a*x]*Cos[7*ArcTan[a*x]]*Log[1 + I*E^(I*ArcTan[a*x...
```

## Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \arctan(ax)^2 \left(a^2cx^2+c\right)^{5/2} dx$$

$$\Big\downarrow \text{ 5485}$$

$$a^2c\int x^5\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2dx+c\int x^3\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2dx$$

$$\Big\downarrow \text{ 5485}$$

$$a^2c\left(a^2c\int x^7\sqrt{a^2cx^2+c}\arctan(ax)^2dx+c\int x^5\sqrt{a^2cx^2+c}\arctan(ax)^2dx\right)+ c\left(a^2c\int x^5\sqrt{a^2cx^2+c}\arctan(ax)^2dx+c\int x^3\sqrt{a^2cx^2+c}\arctan(ax)^2dx\right)$$

$$\Big\downarrow \text{ 5485}$$

$$a^2c\left(a^2c\left(a^2c\int\frac{x^9\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int\frac{x^7\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx\right)+c\left(a^2c\int\frac{x^7\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int\frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}\right.\right.$$
$$c\left(a^2c\left(a^2c\int\frac{x^7\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int\frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx\right)+c\left(a^2c\int\frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int\frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}d\right.\right.$$

$$\Big\downarrow \text{5487}$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^8}{9a^2c}-\frac{2\int\frac{x^8\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{9a}-\frac{8\int\frac{x^7\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{9a^2}\right)a^2+c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{7a^2c}\right.\right.\right.$$
$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\int\frac{x^6\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{7a}-\frac{6\int\frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{7a^2}\right)a^2+c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{5a^2c}\right.\right.\right.$$

$$\Big\downarrow \text{5465}$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^8}{9a^2c}-\frac{2\int\frac{x^8\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{9a}-\frac{8\int\frac{x^7\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{9a^2}\right)a^2+c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{7a^2c}\right.\right.\right.$$
$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\int\frac{x^6\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{7a}-\frac{6\int\frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{7a^2}\right)a^2+c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{5a^2c}\right.\right.\right.$$

$$\Big\downarrow \text{5425}$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^8}{9a^2c}-\frac{2\int\frac{x^8\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{9a}-\frac{8\int\frac{x^7\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{9a^2}\right)a^2+c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{7a^2c}\right.\right.\right.$$
$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\int\frac{x^6\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{7a}-\frac{6\int\frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{7a^2}\right)a^2+c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{5a^2c}\right.\right.\right.$$

$$\Big\downarrow \text{5421}$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^8}{9a^2c}-\frac{2\int\frac{x^8\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{9a}-\frac{8\int\frac{x^7\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{9a^2}\right)a^2+c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{7a^2c}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\int\frac{x^6\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{7a}-\frac{6\int\frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{7a^2}\right)a^2+c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{5a^2c}\right.\right.\right.$$

$\downarrow$ 5487

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^8}{9a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^7}{8a^2c}-\frac{\int\frac{x^7}{\sqrt{a^2cx^2+c}}dx}{8a}-\frac{7\int\frac{x^6\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{8a^2}\right)}{9a}-\frac{8\left(\frac{\sqrt{a^2cx^2+c}}{7}\right.}{}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\int\frac{x^5}{\sqrt{a^2cx^2+c}}dx}{6a}-\frac{5\int\frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)}{7a}-\frac{6\left(\frac{\sqrt{a^2cx^2+c}}{5}\right.}{}\right.\right.\right.$$

$\downarrow$ 241

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^8}{9a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^7}{8a^2c}-\frac{\int\frac{x^7}{\sqrt{a^2cx^2+c}}dx}{8a}-\frac{7\int\frac{x^6\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{8a^2}\right)}{9a}-\frac{8\left(\frac{\sqrt{a^2cx^2+c}}{7}\right.}{}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\int\frac{x^5}{\sqrt{a^2cx^2+c}}dx}{6a}-\frac{5\int\frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)}{7a}-\frac{6\left(\frac{\sqrt{a^2cx^2+c}}{5}\right.}{}\right.\right.\right.$$

$\downarrow$ 243

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^8}{9a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^7}{8a^2c}-\frac{\int\frac{x^6}{\sqrt{a^2cx^2+c}}dx^2}{16a}-\frac{7\int\frac{x^6\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{8a^2}\right)}{9a}-\frac{8\left(\sqrt{a^2cx^2+}\right.}{}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\int\frac{x^4}{\sqrt{a^2cx^2+c}}dx^2}{12a}-\frac{5\int\frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)}{7a}-\frac{6\left(\sqrt{a^2cx^2+}\right.}{}\right.\right.\right.$$

$\downarrow$ 53

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^8}{9a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^7}{8a^2c}-\frac{\int\left(\frac{\left(a^2cx^2+c\right)^{5/2}}{a^6c^3}-\frac{3\left(a^2cx^2+c\right)^{3/2}}{a^6c^2}+\frac{3\sqrt{a^2cx^2+c}}{a^6c}-\frac{1}{a^6\sqrt{a^2cx^2+c}}\right.}{16a}\right.}{9a}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\int\left(\frac{\left(a^2cx^2+c\right)^{3/2}}{a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}+\frac{1}{a^4\sqrt{a^2cx^2+c}}\right)dx^2}{12a}-\frac{5\int\right.}{7a}\right.\right.\right.$$

$\downarrow$ 2009

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^8}{9a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^7}{8a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{7/2}}{7a^8c^4}-\frac{6\left(a^2cx^2+c\right)^{5/2}}{5a^8c^3}+\frac{2\left(a^2cx^2+c\right)^{3/2}}{a^8c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^8c}}{16a}\right.}{9a}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\int\frac{x^4\arctan}{\sqrt{a^2cx^2}}}{6a^2}\right.}{7a}\right.\right.\right.$$

$\downarrow$ 5425

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^8}{9a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^7}{8a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{7/2}}{7a^8c^4}-\frac{6\left(a^2cx^2+c\right)^{5/2}}{5a^8c^3}+\frac{2\left(a^2cx^2+c\right)^{3/2}}{a^8c^2}-\frac{2\sqrt{a^2cx^2}}{a^8c}}{16a}\right.}{9a}\right.\right.\right.$$

$$\left.c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\int\frac{x^4\arctan}{\sqrt{a^2c}}}{6a^2}\right.}{7a}\right.\right.$$

$\Big\downarrow$ 5421

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^8}{9a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^7}{8a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{7/2}}{7a^8c^4}-\frac{6\left(a^2cx^2+c\right)^{5/2}}{5a^8c^3}+\frac{2\left(a^2cx^2+c\right)^{3/2}}{a^8c^2}-\frac{2\sqrt{a^2cx^2}}{a^8c}}{16a}\right.}{9a}\right.\right.\right.$$

$$\left.c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\int\frac{x^4\arctan}{\sqrt{a^2c}}}{6a^2}\right.}{7a}\right.\right.$$

$\Big\downarrow$ 5465

$$c\Bigg(c\bigg(c\bigg(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^8}{9a^2c}-\frac{2\Big(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^7}{8a^2c}-\frac{\frac{2(a^2cx^2+c)^{7/2}}{7a^8c^4}-\frac{6(a^2cx^2+c)^{5/2}}{5a^8c^3}+\frac{2(a^2cx^2+c)^{3/2}}{a^8c^2}-\frac{2\sqrt{a^2cx^2}}{a^8c}}{16a}}{9a}$$

$$c\Bigg(c\bigg(c\bigg(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\Big(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\frac{2(a^2cx^2+c)^{5/2}}{5a^6c^3}-\frac{4(a^2cx^2+c)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\int\frac{x^4\,\mathrm{arcta}}{\sqrt{a^2c}}}{6a^2}}{7a}$$

$\downarrow$ 5425

$$c\Bigg(c\bigg(c\bigg(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^8}{9a^2c}-\frac{2\Big(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^7}{8a^2c}-\frac{\frac{2(a^2cx^2+c)^{7/2}}{7a^8c^4}-\frac{6(a^2cx^2+c)^{5/2}}{5a^8c^3}+\frac{2(a^2cx^2+c)^{3/2}}{a^8c^2}-\frac{2\sqrt{a^2cx^2}}{a^8c}}{16a}}{9a}$$

$$c\Bigg(c\bigg(c\bigg(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\Big(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\frac{2(a^2cx^2+c)^{5/2}}{5a^6c^3}-\frac{4(a^2cx^2+c)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\int\frac{x^4\,\mathrm{arcta}}{\sqrt{a^2c}}}{6a^2}}{7a}$$

$\downarrow$ 5421

$$c\left(c\left(c\left(\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^8}{9a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^7}{8a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{7/2}}{7a^8c^4}-\frac{6\left(a^2cx^2+c\right)^{5/2}}{5a^8c^3}+\frac{2\left(a^2cx^2+c\right)^{3/2}}{a^8c^2}-\frac{2\sqrt{a^2cx^2+\cdots}}{a^8c}\cdots}{16a}\cdots\right.}{9a}\cdots\right.\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\int\frac{x^4\arctan\cdots}{\sqrt{a^2c\cdots}}}{6a^2\cdots}\cdots\right.}{7a}\cdots\right.\right.\right.$$

↓ 5487

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^8}{9a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^7}{8a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{7/2}}{7a^8c^4}-\frac{6\left(a^2cx^2+c\right)^{5/2}}{5a^8c^3}+\frac{2\left(a^2cx^2+c\right)^{3/2}}{a^8c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^8c}}{16a}\right.}{9a}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\left(\frac{\sqrt{a^2cx^2}\cdots}{}\right.}{}\right.}{7a}\right.\right.\right.$$

↓ 241

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^8}{9a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^7}{8a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{7/2}}{7a^8c^4}-\frac{6\left(a^2cx^2+c\right)^{5/2}}{5a^8c^3}+\frac{2\left(a^2cx^2+c\right)^{3/2}}{a^8c^2}-\frac{2\sqrt{a^2cx^2+\cdots}}{a^8c}}{16a}\right.}{9a}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\left(\frac{\sqrt{a^2cx^2\cdots}}{\cdots}\right.}{\cdots}\right.}{7a}\right.\right.\right.$$

↓ 243

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^8}{9a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^7}{8a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{7/2}}{7a^8c^4}-\frac{6\left(a^2cx^2+c\right)^{5/2}}{5a^8c^3}+\frac{2\left(a^2cx^2+c\right)^{3/2}}{a^8c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^8c}}{16a}\right.}{9a}\right.\right.\right.$$

$$\left(\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\left(\frac{\sqrt{a^2cx^2}}{}\right.}{}\right.}{7a}\right.\right.\right.\right.$$

↓ 53

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^8}{9a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^7}{8a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{7/2}}{7a^8c^4}-\frac{6\left(a^2cx^2+c\right)^{5/2}}{5a^8c^3}+\frac{2\left(a^2cx^2+c\right)^{3/2}}{a^8c^2}-\frac{2\sqrt{a^2cx^2+\cdots}}{a^8c}}{16a}\right.}{}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\left(\frac{\sqrt{a^2cx^2\cdots}}{}\right.}{}\right.}{7a}\right.\right.\right.$$

↓ 2009

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^8}{9a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^7}{8a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{7/2}}{7a^8c^4}-\frac{6\left(a^2cx^2+c\right)^{5/2}}{5a^8c^3}+\frac{2\left(a^2cx^2+c\right)^{3/2}}{a^8c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^8c}}{16a}\right.}{}\right.\right.\right.$$

$$\left(\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\left(\frac{\sqrt{a^2cx^2}}{}\right.}{}\right.}{7a}\right.\right.\right.$$

↓ 5425

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^8}{9a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^7}{8a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{7/2}}{7a^8c^4}-\frac{6\left(a^2cx^2+c\right)^{5/2}}{5a^8c^3}+\frac{2\left(a^2cx^2+c\right)^{3/2}}{a^8c^2}-\frac{2\sqrt{a^2cx^2+}}{a^8c}}{16a}\right.}{}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\left(\frac{\sqrt{a^2cx^2}}{}\right.}{}\right.}{7a}\right.\right.\right.$$

↓ 5421

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^8}{9a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^7}{8a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{7/2}}{7a^8c^4}-\frac{6\left(a^2cx^2+c\right)^{5/2}}{5a^8c^3}+\frac{2\left(a^2cx^2+c\right)^{3/2}}{a^8c^2}-\frac{2\sqrt{a^2cx^2+\cdots}}{a^8c}}{16a}\right.}{}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\left(\frac{\sqrt{a^2cx^2\cdots}}{}\right.}{}\right.}{7a}\right.\right.\right.$$

↓ 5465

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\,\arctan(ax)^2x^8}{9a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\,\arctan(ax)x^7}{8a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{7/2}}{7a^8c^4}-\frac{6\left(a^2cx^2+c\right)^{5/2}}{5a^8c^3}+\frac{2\left(a^2cx^2+c\right)^{3/2}}{a^8c^2}-\frac{2\sqrt{a^2cx^2+\cdots}}{a^8c}}{16a}\right.}{\cdots}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\,\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\,\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\left(\frac{\sqrt{a^2cx^2\cdots}}{\cdots}\right.}{\cdots}\right.}{7a}\right.\right.\right.$$

↓ 5425

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^8}{9a^2c} - \frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^7}{8a^2c} - \frac{\frac{2\left(a^2cx^2+c\right)^{7/2}}{7a^8c^4} - \frac{6\left(a^2cx^2+c\right)^{5/2}}{5a^8c^3} + \frac{2\left(a^2cx^2+c\right)^{3/2}}{a^8c^2} - \frac{2\sqrt{a^2cx^2+c}}{a^8c}}{16a}\right.}{}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c} - \frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c} - \frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3} - \frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2} + \frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a} - \frac{5\left(\frac{\sqrt{a^2cx^2}}{}\right.}{}\right.}{7a}\right.\right.\right.$$

$\downarrow$ 5421

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^8}{9a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^7}{8a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{7/2}}{7a^8c^4}-\frac{6\left(a^2cx^2+c\right)^{5/2}}{5a^8c^3}+\frac{2\left(a^2cx^2+c\right)^{3/2}}{a^8c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^8c}}{16a}\right.}{}\right.\right.\right.$$

$$\left(\left(\left(\quad 2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\left(\frac{\sqrt{a^2cx^2}}{}\right.}{}\right.\right.\right.\right.$$

↓ 5487

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^8}{9a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^7}{8a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{7/2}}{7a^8c^4}-\frac{6\left(a^2cx^2+c\right)^{5/2}}{5a^8c^3}+\frac{2\left(a^2cx^2+c\right)^{3/2}}{a^8c^2}-\frac{2\sqrt{a^2cx^2+\cdots}}{a^8c}}{16a}\right.}{}\right.\right.\right.$$

$$\left(\left(\left(\cdots 2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\left(\frac{\sqrt{a^2cx^2\cdots}}{}\right.}{}\right.\right.\right.\right.$$

$$\sqrt{a^2cx^2+c}\arctan(ax)^2x^6 \cdots$$

↓ 241

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\,\arctan(ax)^2x^8}{9a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\,\arctan(ax)x^7}{8a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{7/2}}{7a^8c^4}-\frac{6\left(a^2cx^2+c\right)^{5/2}}{5a^8c^3}+\frac{2\left(a^2cx^2+c\right)^{3/2}}{a^8c^2}-\frac{2\sqrt{a^2cx^2+\cdots}}{a^8c}}{16a}\cdots\right)}{\cdots}\right.\right.\right.$$

$$\left(\left(\left(\frac{\sqrt{a^2cx^2+\cdots}\,\arctan(ax)^2x^6}{\cdots}\cdots 2\left(\frac{\sqrt{a^2cx^2+c}\,\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\left(\frac{\sqrt{a^2cx^2\cdots}}{\cdots}\right)}{\cdots}\right.\right.\right.\right.$$

↓ 243

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^8}{9a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^7}{8a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{7/2}}{7a^8c^4}-\frac{6\left(a^2cx^2+c\right)^{5/2}}{5a^8c^3}+\frac{2\left(a^2cx^2+c\right)^{3/2}}{a^8c^2}-\frac{2\sqrt{a^2cx^2+}}{a^8c}}{16a}\right.}{}\right.\right.\right.$$

$$\left(\left(\left(\frac{\sqrt{a^2cx^2+}\arctan(ax)^2x^6}{}\right.\right.\right.\quad 2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}+\frac{2\sqrt{a^2cx^2+c}}{a^6c}}{12a}-\frac{5\left(\frac{\sqrt{a^2cx^2}}{}\right.}{}\right.$$

input
```
Int[x^3*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2,x]
```

output
```
$Aborted
```

## Maple [A] (verified)

Time = 11.80 (sec) , antiderivative size = 309, normalized size of antiderivative = 0.53

| method | result |
|---|---|
| default | $\frac{c^2\sqrt{c(ax-i)(ax+i)}\,\left(20160\arctan(ax)^2a^8x^8-5040\arctan(ax)a^7x^7+54720a^6x^6\arctan(ax)^2+720a^6x^6-12360\arctan(ax)a^5x^5+4\right.}{1}$ |

input
```
int(x^3*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/181440*c^2/a^4*(c*(a*x-I)*(a*x+I))^(1/2)*(20160*arctan(a*x)^2*a^8*x^8-50
40*arctan(a*x)*a^7*x^7+54720*a^6*x^6*arctan(a*x)^2+720*a^6*x^6-12360*arcta
n(a*x)*a^5*x^5+43200*a^4*arctan(a*x)^2*x^4+1608*a^4*x^4-6150*arctan(a*x)*x
^3*a^3+2880*arctan(a*x)^2*x^2*a^2-94*a^2*x^2+6345*arctan(a*x)*a*x-5760*arc
tan(a*x)^2-6157)-115/4032*c^2*(c*(a*x-I)*(a*x+I))^(1/2)*(arctan(a*x)*ln(1+
I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1
/2))+I*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*dilog(1+I*(1+I*a*x)/(a^2*x
^2+1)^(1/2)))/a^4/(a^2*x^2+1)^(1/2)
```

## Fricas [F]

$$\int x^3\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2\,dx=\int \left(a^2cx^2+c\right)^{\frac{5}{2}}x^3\arctan\left(ax\right)^2\,dx$$

input
```
integrate(x^3*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^7 + 2*a^2*c^2*x^5 + c^2*x^3)*sqrt(a^2*c*x^2 + c)*arcta
n(a*x)^2, x)
```

## Sympy [F]

$$\int x^3 \left(c+a^2 c x^2\right)^{5/2} \arctan(ax)^2 \, dx = \int x^3 \left(c\left(a^2 x^2+1\right)\right)^{\frac{5}{2}} \operatorname{atan}^2(ax) \, dx$$

input
```
integrate(x**3*(a**2*c*x**2+c)**(5/2)*atan(a*x)**2,x)
```

output
```
Integral(x**3*(c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**2, x)
```

## Maxima [F]

$$\int x^3 \left(c+a^2 c x^2\right)^{5/2} \arctan(ax)^2 \, dx = \int \left(a^2 c x^2+c\right)^{\frac{5}{2}} x^3 \arctan(ax)^2 \, dx$$

input
```
integrate(x^3*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)*x^3*arctan(a*x)^2, x)
```

## Giac [F(-2)]

Exception generated.

$$\int x^3 \left(c+a^2 c x^2\right)^{5/2} \arctan(ax)^2 \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int x^3 \left(c+a^2 c x^2\right)^{5/2} \arctan(ax)^2 \, dx = \int x^3 \operatorname{atan}(a\,x)^2 \left(c\,a^2\,x^2+c\right)^{5/2} dx$$

input
```
int(x^3*atan(a*x)^2*(c + a^2*c*x^2)^(5/2),x)
```

output
```
int(x^3*atan(a*x)^2*(c + a^2*c*x^2)^(5/2), x)
```

## Reduce [F]

$$\int x^3 \left(c+a^2 c x^2\right)^{5/2} \arctan(ax)^2 \, dx = \sqrt{c}\,c^2 \left(\left(\int \sqrt{a^2x^2+1}\,atan(ax)^2\,x^7dx\right) a^4 + 2\left(\int \sqrt{a^2x^2+1}\,atan(ax)^2\,x^5dx\right) a^2 + \int \sqrt{a^2x^2+1}\,atan(ax)^2\,x^3dx\right)$$

input
```
int(x^3*(a^2*c*x^2+c)^(5/2)*atan(a*x)^2,x)
```

output
```
sqrt(c)*c**2*(int(sqrt(a**2*x**2 + 1)*atan(a*x)**2*x**7,x)*a**4 + 2*int(sq
rt(a**2*x**2 + 1)*atan(a*x)**2*x**5,x)*a**2 + int(sqrt(a**2*x**2 + 1)*atan
(a*x)**2*x**3,x))
```

# 3.324 $\int x^2(c+a^2cx^2)^{5/2}\arctan(ax)^2\,dx$

## Optimal result

Integrand size = 24, antiderivative size = 638

$$\int x^2\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2\,dx=\frac{43c^2x\sqrt{c+a^2cx^2}}{4032a^2}+\frac{29c^2x^3\sqrt{c+a^2cx^2}}{1680}+\frac{1}{168}a^2c^2x^5\sqrt{c+a^2cx^2}+\frac{1373c^2\sqrt{c+a^2cx^2}\arctan(ax)}{20160a^3}-\frac{737c^2x^2\sqrt{c+a^2cx^2}\arctan(ax)}{10080a}-\frac{83}{840}ac^2x^4\sqrt{c+a^2cx^2}\arctan(ax)-\frac{1}{28}a^3c^2x^6\sqrt{c+a^2cx^2}\arctan(ax)+\frac{5c^2x\sqrt{c+a^2cx^2}\arctan(ax)^2}{128a^2}+\frac{59}{192}c^2x^3\sqrt{c+a^2cx^2}\arctan(ax)^2+\frac{17}{48}a^2c^2x^5\sqrt{c+a^2cx^2}\arctan(ax)^2+\frac{1}{8}a^4c^2x^7\sqrt{c+a^2cx^2}\arctan(ax)^2+\frac{5ic^3\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2}{64a^3\sqrt{c+a^2cx^2}}-\frac{397c^{5/2}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right)}{5040a^3}-\frac{5ic^3\sqrt{1+a^2x^2}\arctan(ax)\,\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)}{64a^3\sqrt{c+a^2cx^2}}+\frac{5ic^3\sqrt{1+a^2x^2}\arctan(ax)\,\text{PolyLog}\left(2,ie^{i\arctan(ax)}\right)}{64a^3\sqrt{c+a^2cx^2}}+\frac{5c^3\sqrt{1+a^2x^2}\,\text{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)}{64a^3\sqrt{c+a^2cx^2}}-\frac{5c^3\sqrt{1+a^2x^2}\,\text{PolyLog}\left(3,ie^{i\arctan(ax)}\right)}{64a^3\sqrt{c+a^2cx^2}}$$

output
```
43/4032*c^2*x*(a^2*c*x^2+c)^(1/2)/a^2+29/1680*c^2*x^3*(a^2*c*x^2+c)^(1/2)+
1/168*a^2*c^2*x^5*(a^2*c*x^2+c)^(1/2)+1373/20160*c^2*(a^2*c*x^2+c)^(1/2)*a
rctan(a*x)/a^3-737/10080*c^2*x^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)/a-83/840*
a*c^2*x^4*(a^2*c*x^2+c)^(1/2)*arctan(a*x)-1/28*a^3*c^2*x^6*(a^2*c*x^2+c)^(
1/2)*arctan(a*x)+5/128*c^2*x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2/a^2+59/192*
c^2*x^3*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2+17/48*a^2*c^2*x^5*(a^2*c*x^2+c)^
(1/2)*arctan(a*x)^2+1/8*a^4*c^2*x^7*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2+5/64
*I*c^3*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/
2))/a^3/(a^2*c*x^2+c)^(1/2)-397/5040*c^(5/2)*arctanh(a*c^(1/2)*x/(a^2*c*x^
2+c)^(1/2))/a^3+5/64*I*c^3*(a^2*x^2+1)^(1/2)*arctan((1+I*a*x)/(a^2*x^2+1)^
(1/2))*arctan(a*x)^2/a^3/(a^2*c*x^2+c)^(1/2)-5/64*I*c^3*(a^2*x^2+1)^(1/2)*
arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a^3/(a^2*c*x^2+c)^(1
/2)+5/64*c^3*(a^2*x^2+1)^(1/2)*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a
^3/(a^2*c*x^2+c)^(1/2)-5/64*c^3*(a^2*x^2+1)^(1/2)*polylog(3,I*(1+I*a*x)/(a
^2*x^2+1)^(1/2))/a^3/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (warning: unable to verify)

Time = 3.40 (sec) , antiderivative size = 759, normalized size of antiderivative = 1.19

$$\int x^2 \left(c+a^2 c x^2\right)^{5/2} \arctan(ax)^2 \, dx = \text{Too large to display}$$

input
```
Integrate[x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2,x]
```

output
```
(c^2*Sqrt[c + a^2*c*x^2]*(53760*a*x*(1 + a^2*x^2)^(3/2) - 25088*a*x*(1 + a
^2*x^2)^(5/2) + 7006*a*x*(1 + a^2*x^2)^(7/2) - 203264*ArcCoth[(a*x)/Sqrt[1
 + a^2*x^2]] + 53760*(1 + a^2*x^2)^(3/2)*ArcTan[a*x] + 5376*(1 + a^2*x^2)^
(5/2)*ArcTan[a*x] - 38134*(1 + a^2*x^2)^(7/2)*ArcTan[a*x] + 564480*a*x*(1
+ a^2*x^2)^(3/2)*ArcTan[a*x]^2 + 524160*a*x*(1 + a^2*x^2)^(5/2)*ArcTan[a*x
]^2 + 185325*a*x*(1 + a^2*x^2)^(7/2)*ArcTan[a*x]^2 + (201600*I)*ArcTan[E^(
I*ArcTan[a*x])]*ArcTan[a*x]^2 + 161280*(1 + a^2*x^2)^2*ArcTan[a*x]*Cos[3*A
rcTan[a*x]] + 49280*(1 + a^2*x^2)^3*ArcTan[a*x]*Cos[3*ArcTan[a*x]] - 7658*
(1 + a^2*x^2)^4*ArcTan[a*x]*Cos[3*ArcTan[a*x]] - 40320*(1 + a^2*x^2)^3*Arc
Tan[a*x]*Cos[5*ArcTan[a*x]] - 10990*(1 + a^2*x^2)^4*ArcTan[a*x]*Cos[5*ArcT
an[a*x]] + 3150*(1 + a^2*x^2)^4*ArcTan[a*x]*Cos[7*ArcTan[a*x]] - (201600*I
)*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + (201600*I)*ArcTan[a*x]*
PolyLog[2, I*E^(I*ArcTan[a*x])] + 201600*PolyLog[3, (-I)*E^(I*ArcTan[a*x])
] - 201600*PolyLog[3, I*E^(I*ArcTan[a*x])] + 53760*(1 + a^2*x^2)^2*Sin[3*A
rcTan[a*x]] - 48384*(1 + a^2*x^2)^3*Sin[3*ArcTan[a*x]] + 12246*(1 + a^2*x^
2)^4*Sin[3*ArcTan[a*x]] - 80640*(1 + a^2*x^2)^2*ArcTan[a*x]^2*Sin[3*ArcTan
[a*x]] - 315840*(1 + a^2*x^2)^3*ArcTan[a*x]^2*Sin[3*ArcTan[a*x]] - 93975*(
1 + a^2*x^2)^4*ArcTan[a*x]^2*Sin[3*ArcTan[a*x]] - 23296*(1 + a^2*x^2)^3*Si
n[5*ArcTan[a*x]] + 7678*(1 + a^2*x^2)^4*Sin[5*ArcTan[a*x]] + 20160*(1 + a^
2*x^2)^3*ArcTan[a*x]^2*Sin[5*ArcTan[a*x]] + 41685*(1 + a^2*x^2)^4*ArcTa...
```

## Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arctan(ax)^2 \left(a^2cx^2+c\right)^{5/2} dx$$

$$\Big\downarrow \text{ 5485}$$

$$c\int x^2\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2dx+a^2c\int x^4\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2dx$$

$$\Big\downarrow \text{ 5485}$$

$$c\left(c\int x^2\sqrt{a^2cx^2+c}\arctan(ax)^2dx+a^2c\int x^4\sqrt{a^2cx^2+c}\arctan(ax)^2dx\right)+ a^2c\left(a^2c\int x^6\sqrt{a^2cx^2+c}\arctan(ax)^2dx+c\int x^4\sqrt{a^2cx^2+c}\arctan(ax)^2dx\right)$$

$$\Big\downarrow \text{ 5485}$$

$$c\left(c\left(c\int \frac{x^2 \arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx + a^2c\int \frac{x^4 \arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx\right) + a^2c\left(a^2c\int \frac{x^6 \arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx + c\int \frac{x^4 \arctan(ax)^2}{\sqrt{a^2cx^2+c}}d\right.\right.$$
$$a^2c\left(a^2c\left(a^2c\int \frac{x^8 \arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx + c\int \frac{x^6 \arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx\right) + c\left(a^2c\int \frac{x^6 \arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx + c\int \frac{x^4 \arctan(ax)^2}{\sqrt{a^2cx^2+c}}\right.\right.$$

$\downarrow$ 5487

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^7}{8a^2c} - \frac{\int \frac{x^7\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a} - \frac{7\int \frac{x^6\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{8a^2}\right)a^2 + c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{6a^2c}\right.\right.\right.$$
$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c} - \frac{\int \frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a} - \frac{5\int \frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)a^2 + c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{4a^2c}\right.\right.\right.$$

$\downarrow$ 5425

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^7}{8a^2c} - \frac{\int \frac{x^7\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a} - \frac{7\int \frac{x^6\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{8a^2}\right)a^2 + c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{6a^2c}\right.\right.\right.$$
$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c} - \frac{\int \frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a} - \frac{5\int \frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)a^2 + c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{4a^2c}\right.\right.\right.$$

$\downarrow$ 5423

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^7}{8a^2c} - \frac{\int \frac{x^7\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a} - \frac{7\int \frac{x^6\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{8a^2}\right)a^2 + c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{6a^2c}\right.\right.\right.$$
$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c} - \frac{\int \frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a} - \frac{5\int \frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)a^2 + c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{4a^2c}\right.\right.\right.$$

$\downarrow$ 3042

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^7}{8a^2c} - \frac{\int \frac{x^7\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a} - \frac{7\int \frac{x^6\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{8a^2}\right)a^2 + c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{6a^2c}\right.\right.\right.$$
$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c} - \frac{\int \frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a} - \frac{5\int \frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)a^2 + c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{4a^2c}\right.\right.\right.$$

$\downarrow$ 4669

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^7}{8a^2c}-\frac{\int\frac{x^7\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a}-\frac{7\int\frac{x^6\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{8a^2}\right)a^2+c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x}{6a^2c}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{5\int\frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)a^2+c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x}{4a^2c}\right.\right.\right.$$

$\downarrow$ 3011

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^7}{8a^2c}-\frac{\int\frac{x^7\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a}-\frac{7\int\frac{x^6\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{8a^2}\right)a^2+c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x}{6a^2c}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{5\int\frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)a^2+c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x}{4a^2c}\right.\right.\right.$$

$\downarrow$ 2720

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^7}{8a^2c}-\frac{\int\frac{x^7\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a}-\frac{7\int\frac{x^6\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{8a^2}\right)a^2+c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x}{6a^2c}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{5\int\frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)a^2+c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x}{4a^2c}\right.\right.\right.$$

$\downarrow$ 5465

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^7}{8a^2c}-\frac{\int\frac{x^7\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a}-\frac{7\int\frac{x^6\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{8a^2}\right)a^2+c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x}{6a^2c}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{5\int\frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)a^2+c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x}{4a^2c}\right.\right.\right.$$

$\downarrow$ 224

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^7}{8a^2c}-\frac{\int\frac{x^7\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a}-\frac{7\int\frac{x^6\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{8a^2}\right)a^2+c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x}{6a^2c}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{5\int\frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)a^2+c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x}{4a^2c}\right.\right.\right.$$

$\downarrow$ 219

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^7}{8a^2c}-\frac{\int\frac{x^7\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a}-\frac{7\int\frac{x^6\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{8a^2}\right)a^2+c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x}{6a^2c}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{5\int\frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)a^2+c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x}{4a^2c}\right.\right.\right.$$

$\downarrow$ 5487

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^7}{8a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^6}{7a^2c}-\frac{\int\frac{x^6}{\sqrt{a^2cx^2+c}}dx}{7a}-\frac{6\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{7a^2}}{4a}-\frac{7\left(\frac{\sqrt{a^2cx^2+c}\arctan}{6a^2c}\right.}{}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\int\frac{x^4}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a^2}}{3a}-\frac{5\left(\frac{\sqrt{a^2cx^2+c}\arctan}{4a^2c}\right.}{}\right.\right.\right.$$

$\downarrow$ 262

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^7}{8a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^6}{7a^2c}-\frac{\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\int\frac{x^4}{\sqrt{a^2cx^2+c}}dx}{6a^2}}{7a}-\frac{6\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{7a^2}}{4a}-\frac{7\left(\sqrt{\right.}{}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\int\frac{x^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}}{5a}-\frac{4\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a^2}}{3a}-\frac{5\left(\sqrt{\right.}{}\right.\right.\right.$$

$\downarrow$ 224

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^7}{8a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^6}{7a^2c}-\frac{\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\int\frac{x^4}{\sqrt{a^2cx^2+c}}dx}{6a^2}}{7a}-\frac{6\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{7a^2}}{4a}-\frac{7\left(\sqrt{}\right.}{}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\int\frac{x^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}}{5a}-\frac{4\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a^2}}{3a}-\frac{5\left(\sqrt{}\right.}{}\right.\right.\right.$$

$\downarrow$ 219

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^7}{8a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^6}{7a^2c}-\frac{\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\int\frac{x^4}{\sqrt{a^2cx^2+c}}dx}{6a^2}}{7a}-\frac{6\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{7a^2}}{4a}-\frac{7\left(\sqrt{}\right.}{}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\int\frac{x^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}}{5a}-\frac{4\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a^2}}{3a}-\frac{5\left(\sqrt{}\right.}{}\right.\right.\right.$$

$\downarrow$ 262

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^7}{8a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^6}{7a^2c}-\frac{\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\int\frac{x^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{6a^2}}{7a}-\frac{6\int\frac{x^5\,\mathrm{arct}}{\sqrt{a^2c}}}{7a}}{4a}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int\frac{1}{\sqrt{a^2cx^2+c}}dx}{2a^2}\right)}{4a^2}}{5a}-\frac{4\int\frac{x^3\arctan}{\sqrt{a^2cx^2}}}{5a^2}}{3a}\right.\right.\right.$$

$\downarrow$ 224

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^7}{8a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^6}{7a^2c}-\frac{\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\int\frac{x^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{6a^2}}{7a}-\frac{6\int\frac{x^5\operatorname{arct}}{\sqrt{a^2c}}}{7a}}{4a}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int\frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}}{2a^2}\right)}{4a^2}}{5a}-\frac{4}{}}{3a}\right.\right.\right.$$

$\big\downarrow$ 219

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^7}{8a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^6}{7a^2c}-\frac{\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\int\frac{x^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{6a^2}}{7a}-\frac{6\int\frac{x^5\operatorname{arct}}{\sqrt{a^2c}}}{7a}}{4a}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}}{5a}-\frac{4\int}{}}{3a}\right.\right.\right.$$

$\big\downarrow$ 262

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^7}{8a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^6}{7a^2c}-\frac{\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int\frac{1}{\sqrt{a^2cx^2+c}}dx}{2a^2}\right)}{4a^2}\right.}{6a^2}}{7a}}{4a}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}}{5a}}{3a}-\frac{4\int\right.\right.\right.$$

$\downarrow$ 224

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^7}{8a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^6}{7a^2c}-\frac{\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int\frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{}{\sqrt{}}}{2a^2}\right.}{4a^2}\right.}{6a^2}}{7a}}{4a}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}}{5a}}{3a}-\frac{4\int\right.\right.\right.$$

$\downarrow$ 219

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^7}{8a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^6}{7a^2c}-\frac{\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\cdots}{\sqrt{a^2\cdots}}\right)}{2a^3\sqrt{c}}\cdots\right.}{4a^2}\cdots\right.}{6a^2}\cdots}{7a}\cdots}{4a}\cdots\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}}{5a}}{3a}-\frac{4\int\cdots}{\cdots}\right.\right.\right.$$

$\downarrow$ 5425

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^7}{8a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^6}{7a^2c}-\frac{\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\cdots}{\sqrt{a^2\cdots}}\right)}{2a^3\sqrt{c}}\cdots\right.}{4a^2}\cdots\right.}{6a^2}\cdots}{7a}\cdots}{4a}\cdots\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}}{5a}}{3a}-\frac{4\int\cdots}{\cdots}\right.\right.\right.$$

$\downarrow$ 5423

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^7}{8a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^6}{7a^2c}-\frac{\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a}{\sqrt{a^2}}\right.}{2a^3\sqrt{c}}\right.}{4a^2}\right.}{6a^2}}{7a}}{4a}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}}{5a}}{3a}-\frac{4\int\right.\right.\right.$$

$\downarrow$ 3042

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^7}{8a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^6}{7a^2c}-\frac{\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a}{\sqrt{a^2}}\right.}{2a^3\sqrt{c}}\right.}{4a^2}\right.}{6a^2}}{7a}}{4a}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}}{5a}}{3a}-\frac{4\int\right.\right.\right.$$

$\downarrow$ 4669

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^7}{8a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^6}{7a^2c}-\frac{\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}\right)}{6a^2}}{7a}}{4a}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}}{5a}}{3a}-\frac{4\int}{}\right.\right.\right.$$

$\downarrow$ 3011

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^7}{8a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^6}{7a^2c}-\frac{\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}\right)}{6a^2}}{7a}}{4a}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}}{5a}}{3a}-\frac{4\int}{}\right.\right.\right.$$

$\downarrow$ 2720

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^7}{8a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^6}{7a^2c}-\frac{\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\cdots}{\sqrt{a^2\cdots}}\right)}{2a^3\sqrt{c}}\cdots\right.}{4a^2}\cdots\right.}{6a^2}\cdots}{7a}\cdots}{4a}\cdots\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}}{5a}}{3a}-\frac{4\int\cdots}{\cdots}\right.\right.\right.$$

$\downarrow$ 5465

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^7}{8a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^6}{7a^2c}-\frac{\frac{x^5\sqrt{a^2cx^2+c}}{6a^2c}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\cdots}{\sqrt{a^2\cdots}}\right)}{2a^3\sqrt{c}}\cdots\right.}{4a^2}\cdots\right.}{6a^2}\cdots}{7a}\cdots}{4a}\cdots\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\left(\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}\right)}{4a^2}}{5a}}{3a}-\frac{4\int\cdots}{\cdots}\right.\right.\right.$$

input
```
Int[x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2,x]
```

output
```
$Aborted
```

## Maple [A] (verified)

Time = 7.59 (sec) , antiderivative size = 376, normalized size of antiderivative = 0.59

| method | result |
|---|---|
| default | $\frac{c^2\sqrt{c(ax-i)(ax+i)}\,\left(5040\arctan(ax)^2a^7x^7-1440a^6\arctan(ax)x^6+14280a^5\arctan(ax)^2x^5+240a^5x^5-3984x^4\arctan(ax)a^4+123}{40320a^3}$ |

input
```
int(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/40320*c^2/a^3*(c*(a*x-I)*(a*x+I))^(1/2)*(5040*arctan(a*x)^2*a^7*x^7-1440
*a^6*arctan(a*x)*x^6+14280*a^5*arctan(a*x)^2*x^5+240*a^5*x^5-3984*x^4*arct
an(a*x)*a^4+12390*a^3*arctan(a*x)^2*x^3+696*a^3*x^3-2948*x^2*a^2*arctan(a*
x)+1575*a*arctan(a*x)^2*x+430*a*x+2746*arctan(a*x))-1/40320*I*c^2*(c*(a*x-
I)*(a*x+I))^(1/2)*(1575*I*arctan(a*x)^2*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2)
)-1575*I*arctan(a*x)^2*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+3150*arctan(a*x
)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-3150*arctan(a*x)*polylog(2,I*(
1+I*a*x)/(a^2*x^2+1)^(1/2))+3150*I*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2
))-3150*I*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6352*arctan((1+I*a*x)/(
a^2*x^2+1)^(1/2)))/a^3/(a^2*x^2+1)^(1/2)
```

## Fricas [F]

$$\int x^2\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2\,dx=\int \left(a^2cx^2+c\right)^{\frac{5}{2}}x^2\arctan\left(ax\right)^2\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2)*sqrt(a^2*c*x^2 + c)*arcta
n(a*x)^2, x)
```

## Sympy [F]

$$\int x^2\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2\,dx=\int x^2\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}\operatorname{atan}^2(ax)\,dx$$

input
```
integrate(x**2*(a**2*c*x**2+c)**(5/2)*atan(a*x)**2,x)
```

output
```
Integral(x**2*(c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**2, x)
```

## Maxima [F]

$$\int x^2\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2\,dx=\int\left(a^2cx^2+c\right)^{\frac{5}{2}}x^2\arctan(ax)^2\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)*x^2*arctan(a*x)^2, x)
```

## Giac [F]

$$\int x^2\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2\,dx=\int\left(a^2cx^2+c\right)^{\frac{5}{2}}x^2\arctan(ax)^2\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)*x^2*arctan(a*x)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int x^2 (c+a^2cx^2)^{5/2} \arctan(ax)^2 \, dx = \int x^2 \operatorname{atan}(a\,x)^2 \left(c\,a^2\,x^2+c\right)^{5/2} dx$$

input
```
int(x^2*atan(a*x)^2*(c + a^2*c*x^2)^(5/2),x)
```

output
```
int(x^2*atan(a*x)^2*(c + a^2*c*x^2)^(5/2), x)
```

## Reduce [F]

$$\int x^2 (c+a^2cx^2)^{5/2} \arctan(ax)^2 \, dx = \sqrt{c}\,c^2\left(\left(\int \sqrt{a^2x^2+1}\,atan(ax)^2\,x^6dx\right)a^4 + 2\left(\int \sqrt{a^2x^2+1}\,atan(ax)^2\,x^4dx\right)a^2 + \int \sqrt{a^2x^2+1}\,atan(ax)^2\,x^2dx\right)$$

input
```
int(x^2*(a^2*c*x^2+c)^(5/2)*atan(a*x)^2,x)
```

output
```
sqrt(c)*c**2*(int(sqrt(a**2*x**2 + 1)*atan(a*x)**2*x**6,x)*a**4 + 2*int(sq
rt(a**2*x**2 + 1)*atan(a*x)**2*x**4,x)*a**2 + int(sqrt(a**2*x**2 + 1)*atan
(a*x)**2*x**2,x))
```

# 3.325 $\int x(c + a^2cx^2)^{5/2} \arctan(ax)^2 \, dx$



## Optimal result

Integrand size = 22, antiderivative size = 387

$$\int x(c + a^2cx^2)^{5/2} \arctan(ax)^2 \, dx = \frac{5c^2\sqrt{c + a^2cx^2}}{56a^2} + \frac{5c(c + a^2cx^2)^{3/2}}{252a^2} + \frac{(c + a^2cx^2)^{5/2}}{105a^2} - \frac{5c^2x\sqrt{c + a^2cx^2}\arctan(ax)}{56a} - \frac{5cx(c + a^2cx^2)^{3/2}\arctan(ax)}{84a} - \frac{x(c + a^2cx^2)^{5/2}\arctan(ax)}{21a} + \frac{(c + a^2cx^2)^{7/2}\arctan(ax)^2}{7a^2c} + \frac{5ic^3\sqrt{1 + a^2x^2}\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{28a^2\sqrt{c + a^2cx^2}} - \frac{5ic^3\sqrt{1 + a^2x^2}\,\text{PolyLog}\left(2, -\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{56a^2\sqrt{c + a^2cx^2}} + \frac{5ic^3\sqrt{1 + a^2x^2}\,\text{PolyLog}\left(2, \frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{56a^2\sqrt{c + a^2cx^2}}$$

output
```
5/56*c^2*(a^2*c*x^2+c)^(1/2)/a^2+5/252*c*(a^2*c*x^2+c)^(3/2)/a^2+1/105*(a^
2*c*x^2+c)^(5/2)/a^2-5/56*c^2*x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)/a-5/84*c*x
*(a^2*c*x^2+c)^(3/2)*arctan(a*x)/a-1/21*x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)/
a+1/7*(a^2*c*x^2+c)^(7/2)*arctan(a*x)^2/a^2/c+5/28*I*c^3*(a^2*x^2+1)^(1/2)
*arctan(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/a^2/(a^2*c*x^2+c)^(1/
2)-5/56*I*c^3*(a^2*x^2+1)^(1/2)*polylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/
2))/a^2/(a^2*c*x^2+c)^(1/2)+5/56*I*c^3*(a^2*x^2+1)^(1/2)*polylog(2,I*(1+I*
a*x)^(1/2)/(1-I*a*x)^(1/2))/a^2/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1039 vs. $2(387) = 774$.

Time = 6.77 (sec) , antiderivative size = 1039, normalized size of antiderivative = 2.68

$$\int x(c + a^2cx^2)^{5/2} \arctan(ax)^2 \, dx = \text{Too large to display}$$

input
```
Integrate[x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2,x]
```

output
```
(c^2*(1 + a^2*x^2)*Sqrt[c + a^2*c*x^2]*(13440*(2 + 4*ArcTan[a*x]^2 + 2*Cos
[2*ArcTan[a*x]] - (3*ArcTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^
2*x^2] - ArcTan[a*x]*Cos[3*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] + (3*
ArcTan[a*x]*Log[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + ArcTan[a*x]*
Cos[3*ArcTan[a*x]]*Log[1 + I*E^(I*ArcTan[a*x])] - ((4*I)*PolyLog[2, (-I)*E
^(I*ArcTan[a*x])])/(1 + a^2*x^2)^(3/2) + ((4*I)*PolyLog[2, I*E^(I*ArcTan[a
*x])])/(1 + a^2*x^2)^(3/2) - 2*ArcTan[a*x]*Sin[2*ArcTan[a*x]]) - 336*(1 +
a^2*x^2)*(50 - 32*ArcTan[a*x]^2 + 72*Cos[2*ArcTan[a*x]] + 160*ArcTan[a*x]^
2*Cos[2*ArcTan[a*x]] + 22*Cos[4*ArcTan[a*x]] - (110*ArcTan[a*x]*Log[1 - I*
E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - 55*ArcTan[a*x]*Cos[3*ArcTan[a*x]]*
Log[1 - I*E^(I*ArcTan[a*x])] - 11*ArcTan[a*x]*Cos[5*ArcTan[a*x]]*Log[1 - I
*E^(I*ArcTan[a*x])] + (110*ArcTan[a*x]*Log[1 + I*E^(I*ArcTan[a*x])])/Sqrt[
1 + a^2*x^2] + 55*ArcTan[a*x]*Cos[3*ArcTan[a*x]]*Log[1 + I*E^(I*ArcTan[a*x
])] + 11*ArcTan[a*x]*Cos[5*ArcTan[a*x]]*Log[1 + I*E^(I*ArcTan[a*x])] - ((1
76*I)*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/(1 + a^2*x^2)^(5/2) + ((176*I)*P
olyLog[2, I*E^(I*ArcTan[a*x])])/(1 + a^2*x^2)^(5/2) + 4*ArcTan[a*x]*Sin[2*
ArcTan[a*x]] - 22*ArcTan[a*x]*Sin[4*ArcTan[a*x]]) + (1 + a^2*x^2)^2*(4116
+ 10944*ArcTan[a*x]^2 + 6262*Cos[2*ArcTan[a*x]] - 5376*ArcTan[a*x]^2*Cos[2
*ArcTan[a*x]] + 2764*Cos[4*ArcTan[a*x]] + 6720*ArcTan[a*x]^2*Cos[4*ArcTan[
a*x]] + 618*Cos[6*ArcTan[a*x]] - (10815*ArcTan[a*x]*Log[1 - I*E^(I*ArcT...
```

## Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 327, normalized size of antiderivative = 0.84, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {5465, 5413, 5413, 5413, 5425, 5421}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \arctan(ax)^2 \left(a^2cx^2+c\right)^{5/2} dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{\arctan(ax)^2 \left(a^2cx^2+c\right)^{7/2}}{7a^2c}-\frac{2\int \left(a^2cx^2+c\right)^{5/2}\arctan(ax)dx}{7a}$$

$$\Big\downarrow \text{5413}$$

$$\frac{\arctan(ax)^2 \left(a^2cx^2+c\right)^{7/2}}{7a^2c}-\frac{2\left(\frac{5}{6}c\int \left(a^2cx^2+c\right)^{3/2}\arctan(ax)dx+\frac{1}{6}x\arctan(ax)\left(a^2cx^2+c\right)^{5/2}-\frac{\left(a^2cx^2+c\right)^{5/2}}{30a}\right)}{7a}$$

$$\Big\downarrow \text{5413}$$

$$\frac{\arctan(ax)^2 \left(a^2cx^2+c\right)^{7/2}}{7a^2c}-\frac{2\left(\frac{5}{6}c\left(\frac{3}{4}c\int \sqrt{a^2cx^2+c}\arctan(ax)dx+\frac{1}{4}x\arctan(ax)\left(a^2cx^2+c\right)^{3/2}-\frac{\left(a^2cx^2+c\right)^{3/2}}{12a}\right)+\frac{1}{6}x\arctan(ax)\left(a^2cx^2+\right.}{7a}$$

$$\Big\downarrow \text{5413}$$

$$\frac{\arctan(ax)^2 \left(a^2cx^2+c\right)^{7/2}}{7a^2c}-\frac{2\left(\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{1}{2}c\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)+\frac{1}{4}x\arctan(ax)\left(a^2cx^2+c\right)^{3/2}-\frac{\left(a^2cx^2+c\right.}{12a}\right.}{7a}$$

$$\Big\downarrow \text{5425}$$

$$\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{7/2}}{7a^2c}-$$

$$\frac{2\left(\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\int\frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)+\frac{1}{4}x\arctan(ax)\left(a^2cx^2+c\right)^{3/2}-\frac{(a^2}{}\right.\right.}{7a}$$

$\downarrow$ 5421

$$\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{7/2}}{7a^2c}-$$

$$\frac{2\left(\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2}\right.\right.\right.}{}$$

input
```
Int[x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2,x]
```

output
```
((c + a^2*c*x^2)^(7/2)*ArcTan[a*x]^2)/(7*a^2*c) - (2*(-1/30*(c + a^2*c*x^2
)^(5/2)/a + (x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x])/6 + (5*c*(-1/12*(c + a^2
*c*x^2)^(3/2)/a + (x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/4 + (3*c*(-1/2*Sqr
t[c + a^2*c*x^2]/a + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/2 + (c*Sqrt[1 + a
^2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/a +
(I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*PolyLog[2, (
I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/(2*Sqrt[c + a^2*c*x^2])))/4))/6))
/(7*a)
```

## Defintions of rubi rules used

rule 5413
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Simp[x*(d + e*x^2)
^q*((a + b*ArcTan[c*x])/(2*q + 1)), x] + Simp[2*d*(q/(2*q + 1))   Int[(d +
e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0]
```

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

## Maple [A] (verified)

Time = 4.86 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.71

| method | result |
|---|---|
| default | $\frac{c^2\sqrt{c(ax-i)(ax+i)}\left(360a^6x^6\arctan(ax)^2-120\arctan(ax)a^5x^5+1080a^4\arctan(ax)^2x^4+24a^4x^4-390\arctan(ax)x^3a^3+1080\arctan\right.}{2520a^2}$ |

input
```
int(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2520*c^2/a^2*(c*(a*x-I)*(a*x+I))^(1/2)*(360*a^6*x^6*arctan(a*x)^2-120*ar
ctan(a*x)*a^5*x^5+1080*a^4*arctan(a*x)^2*x^4+24*a^4*x^4-390*arctan(a*x)*x^
3*a^3+1080*arctan(a*x)^2*x^2*a^2+98*a^2*x^2-495*arctan(a*x)*a*x+360*arctan
(a*x)^2+299)+5/56*c^2*(c*(a*x-I)*(a*x+I))^(1/2)*(arctan(a*x)*ln(1+I*(1+I*a
*x)/(a^2*x^2+1)^(1/2))-arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+I*d
ilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1
/2)))/a^2/(a^2*x^2+1)^(1/2)
```

## Fricas [F]

$$\int x(c+a^2cx^2)^{5/2}\arctan(ax)^2\,dx=\int (a^2cx^2+c)^{\frac{5}{2}}x\arctan(ax)^2\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^2*x)*sqrt(a^2*c*x^2 + c)*arctan(
a*x)^2, x)
```

## Sympy [F]

$$\int x(c+a^2cx^2)^{5/2}\arctan(ax)^2\,dx=\int x(c(a^2x^2+1))^{\frac{5}{2}}\operatorname{atan}^2(ax)\,dx$$

input
```
integrate(x*(a**2*c*x**2+c)**(5/2)*atan(a*x)**2,x)
```

output
```
Integral(x*(c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**2, x)
```

## Maxima [F]

$$\int x(c+a^2cx^2)^{5/2}\arctan(ax)^2\,dx=\int (a^2cx^2+c)^{\frac{5}{2}}x\arctan(ax)^2\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)*x*arctan(a*x)^2, x)
```

## Giac [F(-2)]

Exception generated.

$$\int x(c+a^2cx^2)^{5/2}\arctan(ax)^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int x(c+a^2cx^2)^{5/2}\arctan(ax)^2\,dx = \int x\,\mathrm{atan}(a\,x)^2\,\left(c\,a^2\,x^2+c\right)^{5/2}\,dx$$

input
```
int(x*atan(a*x)^2*(c + a^2*c*x^2)^(5/2),x)
```

output
```
int(x*atan(a*x)^2*(c + a^2*c*x^2)^(5/2), x)
```

## Reduce [F]

$$\int x(c+a^2cx^2)^{5/2}\arctan(ax)^2\,dx = \sqrt{c}\,c^2\left(\left(\int \sqrt{a^2x^2+1}\,atan(ax)^2\,x^5dx\right)a^4 + 2\left(\int \sqrt{a^2x^2+1}\,atan(ax)^2\,x^3dx\right)a^2 + \int \sqrt{a^2x^2+1}\,atan(ax)^2\,xdx\right)$$

input
```
int(x*(a^2*c*x^2+c)^(5/2)*atan(a*x)^2,x)
```

output
```
sqrt(c)*c**2*(int(sqrt(a**2*x**2 + 1)*atan(a*x)**2*x**5,x)*a**4 + 2*int(sq
rt(a**2*x**2 + 1)*atan(a*x)**2*x**3,x)*a**2 + int(sqrt(a**2*x**2 + 1)*atan
(a*x)**2*x,x))
```

# 3.326 $\int (c + a^2cx^2)^{5/2} \arctan(ax)^2 \, dx$



## Optimal result

Integrand size = 21, antiderivative size = 516

$$\int (c + a^2cx^2)^{5/2} \arctan(ax)^2 \, dx = \frac{17}{180}c^2x\sqrt{c+a^2cx^2} + \frac{1}{60}cx(c+a^2cx^2)^{3/2} - \frac{5c^2\sqrt{c+a^2cx^2}\arctan(ax)}{8a} - \frac{5c(c+a^2cx^2)^{3/2}\arctan(ax)}{36a} - \frac{(c+a^2cx^2)^{5/2}\arctan(ax)}{15a} + \frac{5}{16}c^2x\sqrt{c+a^2cx^2}\arctan(ax)^2 + \frac{5}{24}cx(c+a^2cx^2)^{3/2}\arctan(ax)^2 + \frac{1}{6}x(c+a^2cx^2)^{5/2}\arctan(ax)^2 - \frac{5ic^3\sqrt{1-}}{}$$

output
```
17/180*c^2*x*(a^2*c*x^2+c)^(1/2)+1/60*c*x*(a^2*c*x^2+c)^(3/2)-5/8*c^2*(a^2
*c*x^2+c)^(1/2)*arctan(a*x)/a-5/36*c*(a^2*c*x^2+c)^(3/2)*arctan(a*x)/a-1/1
5*(a^2*c*x^2+c)^(5/2)*arctan(a*x)/a+5/16*c^2*x*(a^2*c*x^2+c)^(1/2)*arctan(
a*x)^2+5/24*c*x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2+1/6*x*(a^2*c*x^2+c)^(5/2
)*arctan(a*x)^2-5/8*I*c^3*(a^2*x^2+1)^(1/2)*arctan((1+I*a*x)/(a^2*x^2+1)^(
1/2))*arctan(a*x)^2/a/(a^2*c*x^2+c)^(1/2)+259/360*c^(5/2)*arctanh(a*c^(1/2
)*x/(a^2*c*x^2+c)^(1/2))/a+5/8*I*c^3*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog
(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a/(a^2*c*x^2+c)^(1/2)-5/8*I*c^3*(a^2*x^
2+1)^(1/2)*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a/(a^2*c*x
^2+c)^(1/2)-5/8*c^3*(a^2*x^2+1)^(1/2)*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(
1/2))/a/(a^2*c*x^2+c)^(1/2)+5/8*c^3*(a^2*x^2+1)^(1/2)*polylog(3,I*(1+I*a*x
)/(a^2*x^2+1)^(1/2))/a/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (warning: unable to verify)

Time = 1.24 (sec) , antiderivative size = 771, normalized size of antiderivative = 1.49

$$\int (c+a^2cx^2)^{5/2} \arctan(ax)^2 \, dx = \frac{c^2\sqrt{c+a^2cx^2}\Big(424ax\sqrt{1+a^2x^2}+368a^3x^3\sqrt{1+a^2x^2}-56a^5x^5\sqrt{1+a^2x^2}+8$$

input
```
Integrate[(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2,x]
```

output
```
(c^2*Sqrt[c + a^2*c*x^2]*(424*a*x*Sqrt[1 + a^2*x^2] + 368*a^3*x^3*Sqrt[1 +
 a^2*x^2] - 56*a^5*x^5*Sqrt[1 + a^2*x^2] + 8288*ArcCoth[(a*x)/Sqrt[1 + a^2
*x^2]] - 11028*Sqrt[1 + a^2*x^2]*ArcTan[a*x] + 504*a^2*x^2*Sqrt[1 + a^2*x^
2]*ArcTan[a*x] + 12*a^4*x^4*Sqrt[1 + a^2*x^2]*ArcTan[a*x] + 11970*a*x*Sqrt
[1 + a^2*x^2]*ArcTan[a*x]^2 + 7380*a^3*x^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2
 + 1170*a^5*x^5*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2 - (7200*I)*ArcTan[E^(I*Arc
Tan[a*x])]*ArcTan[a*x]^2 + 1550*ArcTan[a*x]*Cos[3*ArcTan[a*x]] + 3210*a^2*
x^2*ArcTan[a*x]*Cos[3*ArcTan[a*x]] + 1770*a^4*x^4*ArcTan[a*x]*Cos[3*ArcTan
[a*x]] + 110*a^6*x^6*ArcTan[a*x]*Cos[3*ArcTan[a*x]] - 90*ArcTan[a*x]*Cos[5
*ArcTan[a*x]] - 270*a^2*x^2*ArcTan[a*x]*Cos[5*ArcTan[a*x]] - 270*a^4*x^4*A
rcTan[a*x]*Cos[5*ArcTan[a*x]] - 90*a^6*x^6*ArcTan[a*x]*Cos[5*ArcTan[a*x]]
 + (7200*I)*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - (7200*I)*ArcTa
n[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] - 7200*PolyLog[3, (-I)*E^(I*ArcTan[
a*x])] + 7200*PolyLog[3, I*E^(I*ArcTan[a*x])] + 372*Sin[3*ArcTan[a*x]] + 6
36*a^2*x^2*Sin[3*ArcTan[a*x]] + 156*a^4*x^4*Sin[3*ArcTan[a*x]] - 108*a^6*x
^6*Sin[3*ArcTan[a*x]] - 1425*ArcTan[a*x]^2*Sin[3*ArcTan[a*x]] - 3555*a^2*x
^2*ArcTan[a*x]^2*Sin[3*ArcTan[a*x]] - 2835*a^4*x^4*ArcTan[a*x]^2*Sin[3*Arc
Tan[a*x]] - 705*a^6*x^6*ArcTan[a*x]^2*Sin[3*ArcTan[a*x]] - 52*Sin[5*ArcTan
[a*x]] - 156*a^2*x^2*Sin[5*ArcTan[a*x]] - 156*a^4*x^4*Sin[5*ArcTan[a*x]] -
 52*a^6*x^6*Sin[5*ArcTan[a*x]] + 45*ArcTan[a*x]^2*Sin[5*ArcTan[a*x]] + ...
```

## Rubi [A] (verified)

Time = 1.87 (sec) , antiderivative size = 485, normalized size of antiderivative = 0.94, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.905, Rules used = {5415, 211, 211, 224, 219, 5415, 211, 224, 219, 5415, 224, 219, 5425, 5423, 3042, 4669, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(ax)^2 \left(a^2cx^2+c\right)^{5/2} dx$$

$$\Big\downarrow \ 5415$$

$$\frac{5}{6}c\int \left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2dx+\frac{1}{15}c\int \left(a^2cx^2+c\right)^{3/2}dx+\frac{1}{6}x\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{15a}$$

$$\Big\downarrow \ 211$$

$$\frac{5}{6}c\int \left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2dx+\frac{1}{15}c\left(\frac{3}{4}c\int \sqrt{a^2cx^2+c}dx+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\right)+\frac{1}{6}x\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{15a}$$

$$\Big\downarrow \ 211$$

$$\frac{5}{6}c\int \left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2dx+\frac{1}{15}c\left(\frac{3}{4}c\left(\frac{1}{2}c\int \frac{1}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\right)+\frac{1}{6}x\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{15a}$$

$$\Big\downarrow \ 224$$

$$\frac{5}{6}c\int \left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2dx+\frac{1}{15}c\left(\frac{3}{4}c\left(\frac{1}{2}c\int \frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\right)+\frac{1}{6}x\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{15a}$$

$$\Big\downarrow \ 219$$

$$\frac{5}{6}c\int \left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2dx+\frac{1}{6}x\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{15a}+\frac{1}{15}c\left(\frac{3}{4}c\left(\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\right)$$

$\downarrow$ 5415

$$\frac{5}{6}c\left(\frac{3}{4}c\int \sqrt{a^2cx^2+c}\arctan(ax)^2dx+\frac{1}{6}c\int \sqrt{a^2cx^2+c}dx+\frac{1}{4}x\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}-\frac{\arctan(ax)\left(a^2c\right.}{6a}\right.$$
$$\frac{1}{6}x\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{15a}+\frac{1}{15}c\left(\frac{3}{4}c\left(\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\right)$$

$\downarrow$ 211

$$\frac{5}{6}c\left(\frac{3}{4}c\int \sqrt{a^2cx^2+c}\arctan(ax)^2dx+\frac{1}{6}c\left(\frac{1}{2}c\int \frac{1}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\arctan(ax)^2\left(a^2cx^2+\right.\right.$$
$$\frac{1}{6}x\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{15a}+\frac{1}{15}c\left(\frac{3}{4}c\left(\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\right)$$

$\downarrow$ 224

$$\frac{5}{6}c\left(\frac{3}{4}c\int \sqrt{a^2cx^2+c}\arctan(ax)^2dx+\frac{1}{6}c\left(\frac{1}{2}c\int \frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\arctan(ax\right.$$
$$\frac{1}{6}x\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{15a}+\frac{1}{15}c\left(\frac{3}{4}c\left(\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\right)$$

$\downarrow$ 219

$$\frac{5}{6}c\left(\frac{3}{4}c\int \sqrt{a^2cx^2+c}\arctan(ax)^2dx+\frac{1}{4}x\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{6a}+\frac{1}{6}c\left(\frac{\sqrt{c}a\right.\right.$$
$$\frac{1}{6}x\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{15a}+$$
$$\frac{1}{15}c\left(\frac{3}{4}c\left(\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\right)$$

$\downarrow$ 5415

$$\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{1}{2}c\int\frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int\frac{1}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}\right)+\right.$$
$$\frac{1}{6}x\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{15a}+$$
$$\frac{1}{15}c\left(\frac{3}{4}c\left(\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\right)$$

$\downarrow$ 224

$$\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{1}{2}c\int\frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int\frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2c}}{a}\right.\right.$$
$$\frac{1}{6}x\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{15a}+$$
$$\frac{1}{15}c\left(\frac{3}{4}c\left(\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\right)$$

$\downarrow$ 219

$$\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{1}{2}c\int\frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}\right.\right.$$
$$\frac{1}{6}x\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{15a}+$$
$$\frac{1}{15}c\left(\frac{3}{4}c\left(\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\right)$$

$\downarrow$ 5425

$$\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{}}{\sqrt{a^2c}}\right.}{a}\right.\right.$$

$$\frac{1}{6}x\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{15a}+$$

$$\left.\frac{1}{15}c\left(\frac{3}{4}c\left(\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\right)\right)$$

$\downarrow$ 5423

$$\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\int \sqrt{a^2x^2+1}\arctan(ax)^2d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2}}{a}\right.\right.$$

$$\frac{1}{6}x\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{15a}+$$

$$\left.\frac{1}{15}c\left(\frac{3}{4}c\left(\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\right)\right)$$

$\downarrow$ 3042

$$\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\arctan(a\right.\right.$$

$$\frac{1}{6}x\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{15a}+$$

$$\left.\frac{1}{15}c\left(\frac{3}{4}c\left(\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\right)\right)$$

$\downarrow$ 4669

$$\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\int \arctan(ax)\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+2\int \arctan(ax)\log\left(1+ie^{i\arctan(ax)}\right)\right.}{2a\sqrt{a^2cx^2+c}}\right.\right.$$

$$\frac{1}{6}x\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{15a}+$$

$$\left.\frac{1}{15}c\left(\frac{3}{4}c\left(\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\right)\right)$$

$\downarrow$ 3011

$$\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-i\int\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(ax)\right)-\right.}{\phantom{x}}\right.\right.$$

$$\frac{1}{6}x\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{15a}+$$

$$\left.\frac{1}{15}c\left(\frac{3}{4}c\left(\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\right)\right)$$

$\downarrow$ 2720

$$\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right.\right.}{\phantom{x}}\right.\right.$$

$$\frac{1}{6}x\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{15a}+$$

$$\left.\frac{1}{15}c\left(\frac{3}{4}c\left(\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\right)\right)$$

$\downarrow$ 7143

$$\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)\right)-2\left(i\arctan(ax)\right.\right.}{2a\sqrt{a^2cx^2+}}\right.\right.$$

$$\frac{1}{6}x\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{15a}+$$

$$\left.\frac{1}{15}c\left(\frac{3}{4}c\left(\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\right)\right)$$

input
```
Int[(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2,x]
```

output
```
-1/15*((c + a^2*c*x^2)^(5/2)*ArcTan[a*x])/a + (x*(c + a^2*c*x^2)^(5/2)*Arc
Tan[a*x]^2)/6 + (c*((x*(c + a^2*c*x^2)^(3/2))/4 + (3*c*((x*Sqrt[c + a^2*c*
x^2])/2 + (Sqrt[c]*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/(2*a)))/4))
/15 + (5*c*(-1/6*((c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/a + (x*(c + a^2*c*x^2
)^(3/2)*ArcTan[a*x]^2)/4 + (c*((x*Sqrt[c + a^2*c*x^2])/2 + (Sqrt[c]*ArcTan
h[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/(2*a)))/6 + (3*c*(-((Sqrt[c + a^2*c*
x^2]*ArcTan[a*x])/a) + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/2 + (Sqrt[c]*
ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/a + (c*Sqrt[1 + a^2*x^2]*((-2*
I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 + 2*(I*ArcTan[a*x]*PolyLog[2, (
-I)*E^(I*ArcTan[a*x])] - PolyLog[3, (-I)*E^(I*ArcTan[a*x])]) - 2*(I*ArcTan
[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] - PolyLog[3, I*E^(I*ArcTan[a*x])])))
/(2*a*Sqrt[c + a^2*c*x^2])))/4))/6
```

## Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5415
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTan[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x])^p,
x], x] + Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*(
a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[q, 0] && GtQ[p, 1]
```

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

## Maple [A] (verified)

Time = 3.53 (sec) , antiderivative size = 342, normalized size of antiderivative = 0.66

| method | result |
|---|---|
| default | $\frac{c^{2} \sqrt{c(ax-i)(ax+i)}\,\left(120 a^{5} \arctan(ax)^{2} x^{5}-48 x^{4} \arctan(ax) a^{4}+390 a^{3} \arctan(ax)^{2} x^{3}+12 a^{3} x^{3}-196 x^{2} a^{2} \arctan(ax)+495 a \arctan\right.}{720 a}$ |

input
```
int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/720*c^2/a*(c*(a*x-I)*(a*x+I))^(1/2)*(120*a^5*arctan(a*x)^2*x^5-48*x^4*ar
ctan(a*x)*a^4+390*a^3*arctan(a*x)^2*x^3+12*a^3*x^3-196*x^2*a^2*arctan(a*x)
+495*a*arctan(a*x)^2*x+80*a*x-598*arctan(a*x))-1/720*I*c^2*(c*(a*x-I)*(a*x
+I))^(1/2)*(225*I*arctan(a*x)^2*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-225*I*
arctan(a*x)^2*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+450*arctan(a*x)*polylog(
2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-450*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a
^2*x^2+1)^(1/2))+450*I*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-450*I*poly
log(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+1036*arctan((1+I*a*x)/(a^2*x^2+1)^(1
/2)))/a/(a^2*x^2+1)^(1/2)
```

## Fricas [F]

$$\int \left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2\,dx=\int \left(a^2cx^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^2\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*arctan(a*
x)^2, x)
```

## Sympy [F]

$$\int \left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2\,dx=\int \left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}\operatorname{atan}^2\left(ax\right)dx$$

input
```
integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)**2,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**2, x)
```

## Maxima [F]

$$\int \left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2\,dx=\int \left(a^2cx^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^2\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^2, x)
```

## Giac [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2\,dx=\int \operatorname{atan}(a\,x)^2\,\left(c\,a^2\,x^2+c\right)^{5/2}\,dx$$

input
```
int(atan(a*x)^2*(c + a^2*c*x^2)^(5/2),x)
```

output
```
int(atan(a*x)^2*(c + a^2*c*x^2)^(5/2), x)
```

## Reduce [F]

$$\int \left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2\,dx=\sqrt{c}\,c^2\left(\left(\int \sqrt{a^2x^2+1}\,atan(ax)^2\,x^4dx\right)a^4+2\left(\int \sqrt{a^2x^2+1}\,atan(ax)^2\,x^2dx\right)a^2+\int \sqrt{a^2x^2+1}\,atan(ax)^2\,dx\right)$$

input
```
int((a^2*c*x^2+c)^(5/2)*atan(a*x)^2,x)
```

output
```
sqrt(c)*c**2*(int(sqrt(a**2*x**2 + 1)*atan(a*x)**2*x**4,x)*a**4 + 2*int(sq
rt(a**2*x**2 + 1)*atan(a*x)**2*x**2,x)*a**2 + int(sqrt(a**2*x**2 + 1)*atan
(a*x)**2,x))
```

# 3.327 $\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^2}{x}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 605

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^2}{x}\,dx = \frac{29}{60}c^2\sqrt{c+a^2cx^2} + \frac{1}{30}c\left(c+a^2cx^2\right)^{3/2} - \frac{29}{60}ac^2x\sqrt{c+a^2cx^2}\arctan(ax) - \frac{1}{10}acx\left(c+a^2cx^2\right)^{3/2}\arctan(ax)+c^2\sqrt{c+a^2cx^2}\arctan(ax)^2+\frac{1}{3}c\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2+\frac{1}{5}\left(c+a^2cx^2\right)$$

output
```
29/60*c^2*(a^2*c*x^2+c)^(1/2)+1/30*c*(a^2*c*x^2+c)^(3/2)-29/60*a*c^2*x*(a^
2*c*x^2+c)^(1/2)*arctan(a*x)-1/10*a*c*x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)+c^
2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2+1/3*c*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^
2+1/5*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^2+149/30*I*c^3*(a^2*x^2+1)^(1/2)*arc
tan(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/(a^2*c*x^2+c)^(1/2)-2*c^3
*(a^2*x^2+1)^(1/2)*arctan(a*x)^2*arctanh((1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2
*c*x^2+c)^(1/2)+2*I*c^3*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(2,-(1+I*a*x)
/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)-2*I*c^3*(a^2*x^2+1)^(1/2)*arctan(a
*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)-149/60*I*c^
3*(a^2*x^2+1)^(1/2)*polylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/(a^2*c*x
^2+c)^(1/2)+149/60*I*c^3*(a^2*x^2+1)^(1/2)*polylog(2,I*(1+I*a*x)^(1/2)/(1-
I*a*x)^(1/2))/(a^2*c*x^2+c)^(1/2)-2*c^3*(a^2*x^2+1)^(1/2)*polylog(3,-(1+I*
a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)+2*c^3*(a^2*x^2+1)^(1/2)*polylo
g(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (warning: unable to verify)

Time = 6.96 (sec) , antiderivative size = 889, normalized size of antiderivative = 1.47

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^2}{x}\,dx = \text{Too large to display}$$

input
```
Integrate[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2)/x,x]
```

output
```
c^2*Sqrt[c*(1 + a^2*x^2)]*(ArcTan[a*x]^2 + (ArcTan[a*x]^2*(Log[1 - E^(I*Ar
cTan[a*x])] - Log[1 + E^(I*ArcTan[a*x])]))/Sqrt[1 + a^2*x^2] - (2*(ArcTan[
a*x]*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])]) + I*(Po
lyLog[2, (-I)*E^(I*ArcTan[a*x])] - PolyLog[2, I*E^(I*ArcTan[a*x])])))/Sqrt
[1 + a^2*x^2] + ((2*I)*ArcTan[a*x]*(PolyLog[2, -E^(I*ArcTan[a*x])] - PolyL
og[2, E^(I*ArcTan[a*x])]))/Sqrt[1 + a^2*x^2] + (2*(-PolyLog[3, -E^(I*ArcTa
n[a*x])] + PolyLog[3, E^(I*ArcTan[a*x])]))/Sqrt[1 + a^2*x^2]) + (c^2*(1 +
a^2*x^2)*Sqrt[c*(1 + a^2*x^2)]*(2 + 4*ArcTan[a*x]^2 + 2*Cos[2*ArcTan[a*x]]
 - (3*ArcTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - ArcTan
[a*x]*Cos[3*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] + (3*ArcTan[a*x]*Log
[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + ArcTan[a*x]*Cos[3*ArcTan[a*
x]]*Log[1 + I*E^(I*ArcTan[a*x])] - ((4*I)*PolyLog[2, (-I)*E^(I*ArcTan[a*x]
)])/(1 + a^2*x^2)^(3/2) + ((4*I)*PolyLog[2, I*E^(I*ArcTan[a*x])])/(1 + a^2
*x^2)^(3/2) - 2*ArcTan[a*x]*Sin[2*ArcTan[a*x]]))/6 - (c^2*(1 + a^2*x^2)^2*
Sqrt[c*(1 + a^2*x^2)]*(50 - 32*ArcTan[a*x]^2 + 72*Cos[2*ArcTan[a*x]] + 160
*ArcTan[a*x]^2*Cos[2*ArcTan[a*x]] + 22*Cos[4*ArcTan[a*x]] - (110*ArcTan[a*
x]*Log[1 - I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - 55*ArcTan[a*x]*Cos[3*
ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] - 11*ArcTan[a*x]*Cos[5*ArcTan[a*
x]]*Log[1 - I*E^(I*ArcTan[a*x])] + (110*ArcTan[a*x]*Log[1 + I*E^(I*ArcTan[
a*x])])/Sqrt[1 + a^2*x^2] + 55*ArcTan[a*x]*Cos[3*ArcTan[a*x]]*Log[1 + I...
```

## Rubi [A] (verified)

Time = 6.17 (sec) , antiderivative size = 825, normalized size of antiderivative = 1.36, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.917$, Rules used = {5485, 5465, 5413, 5413, 5425, 5421, 5485, 5465, 5413, 5425, 5421, 5485, 5465, 5425, 5421, 5493, 5491, 3042, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2 \left(a^2cx^2+c\right)^{5/2}}{x} \, dx$$

$$\Big\downarrow \text{ 5485}$$

$$a^2c\int x\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2dx+c\int \frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{x}dx$$

$$\Big\downarrow \text{ 5465}$$

$$a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{2\int\left(a^2cx^2+c\right)^{3/2}\arctan(ax)dx}{5a}\right)+$$
$$c\int\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{x}dx$$

$$\Big\downarrow \text{5413}$$

$$a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{2\left(\frac{3}{4}c\int\sqrt{a^2cx^2+c}\arctan(ax)dx+\frac{1}{4}x\arctan(ax)\left(a^2cx^2+c\right)^{3/2}-\frac{(a^2cx^2+c)}{12a}\right.}{5a}\right.$$
$$c\int\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{x}dx$$

$$\Big\downarrow \text{5413}$$

$$a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{2\left(\frac{3}{4}c\left(\frac{1}{2}c\int\frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)+\frac{1}{4}x\,\text{arcta}\right.}{5a}\right.$$
$$c\int\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{x}dx$$

$$\Big\downarrow \text{5425}$$

$$a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{2\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\int\frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)+\frac{1}{4}x\right.}{5a}\right.$$
$$c\int\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{x}dx$$

$$\Big\downarrow \text{5421}$$

$$c\int \frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{x}dx+$$

$$a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{2\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{ia}}{\sqrt{1-}}\right.}{a}\right.}{2\sqrt{a^2cx^2+c}}\right.\right.}{}\right.$$

↓ 5485

$$c\left(a^2c\int x\sqrt{a^2cx^2+c}\arctan(ax)^2dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x}dx\right)+$$

$$a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{2\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{ia}}{\sqrt{1-}}\right.}{a}\right.}{2\sqrt{a^2cx^2+c}}\right.\right.}{}\right.$$

↓ 5465

$$c\left(a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{2\int \sqrt{a^2cx^2+c}\arctan(ax)dx}{3a}\right)+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x}dx\right)+$$

$$a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{2\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{ia}}{\sqrt{1-}}\right.}{a}\right.}{2\sqrt{a^2cx^2+c}}\right.\right.}{}\right.$$

↓ 5413

$$c\left(a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{2\left(\frac{1}{2}c\int\frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)}{3a}\right)+c\int\sqrt{a\right.$$

$$a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{2\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{iax}}{\sqrt{1-}}\right.}{a}\right.}{2\sqrt{a^2cx^2+c}}\right.\right.}{}\right.$$

$\downarrow$ 5425

$$c\left(a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{2\left(\frac{c\sqrt{a^2x^2+1}\int\frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)}{3a}\right)+c\right.$$

$$a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{2\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{iax}}{\sqrt{1-}}\right.}{a}\right.}{2\sqrt{a^2cx^2+c}}\right.\right.}{}\right.$$

$\downarrow$ 5421

$$c\left(c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x}dx+a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{2\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\right.}{2\sqrt{a}}\right.}{}\right.\right.$$

$$a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{2\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{ia}}{\sqrt{1-}}\right.}{a}\right.}{2\sqrt{a^2cx^2+c}}\right.\right.}{}\right.$$

$\downarrow$ 5485

$$c\left(c\left(a^2c\int \frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx\right)+a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{2\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan}{}\right.}{}\right.}{}\right.\right.$$

$$a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{2\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{ia}}{\sqrt{1-}}\right.}{a}\right.}{2\sqrt{a^2cx^2+c}}\right.\right.}{}\right.$$

$\downarrow$ 5465

$$c\left(c\left(a^2c\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\int\frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}\right)+c\int\frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx\right)+a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right.}{3a^2c}\right.\right.$$

$$a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{2\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{ia}}{\sqrt{1-}}\right.}{a}\right.}{2\sqrt{a^2cx^2+c}}\right.\right.}{}\right.$$

$\downarrow$ 5425

$$c\left(c\left(a^2c\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\int\frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{a\sqrt{a^2cx^2+c}}\right)+c\int\frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx\right)+a^2c\left(\frac{\arctan(ax)^2}{3}\right.\right.$$

$$a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{2\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{ia}}{\sqrt{1-}}\right.}{a}\right.}{2\sqrt{a^2cx^2+c}}\right.\right.}{}\right.$$

$\downarrow$ 5421

$$c\left(c\left(c\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+a^2c\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}}{}\right.}{a\sqrt{a^2cx^2+c}}\right.\right.\right.$$

$$a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{2\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax}}{\sqrt{1-}}\right.}{a}\right.}{2\sqrt{a^2cx^2+c}}\right.\right.}{}\right.$$

$\downarrow$ 5493

$$c\left(c\left(\frac{c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+a^2c\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\right.}{a\sqrt{a^2cx}}\right.\right.\right.$$

$$a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{2\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax}}{\sqrt{1-}}\right.}{a}\right.}{2\sqrt{a^2cx^2+c}}\right.\right.}{}\right.$$

$\downarrow$ 5491

$$c\left(c\left(\frac{c\sqrt{a^2x^2+1}\int\frac{\sqrt{a^2x^2+1}\arctan(ax)^2}{ax}d\arctan(ax)}{\sqrt{a^2cx^2+c}}+a^2c\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)}{}\right.}{}\right.\right.\right.$$

$$a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{2\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{ia}}{\sqrt{1-}}\right.}{a}\right.}{2\sqrt{a^2cx^2+c}}\right.\right.}{}\right.$$

$\downarrow$ 3042

$$c\left(c\left(\frac{c\sqrt{a^2x^2+1}\int\arctan(ax)^2\csc(\arctan(ax))d\arctan(ax)}{\sqrt{a^2cx^2+c}}+a^2c\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\right.}{}\right.\right.\right.$$

$$a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{2\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{ia}}{\sqrt{1-}}\right.}{a}\right.}{2\sqrt{a^2cx^2+c}}\right.\right.}{}\right.$$

$\downarrow$ 4671

$$c\left(\frac{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^2}{5a^2c}-\frac{2\left(\frac{1}{4}x\arctan(ax)\left(a^2cx^2+c\right)^{3/2}-\frac{\left(a^2cx^2+c\right)^{3/2}}{12a}+\frac{3}{4}c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax\right.\right.}{}\right.$$

$$c\left(c\left(\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}(}{}\right.}{2\sqrt{a^2cx^2+c}}\right.}{3a}\right.\right.$$

$\big\downarrow$ 3011

$$c\left(\frac{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^2}{5a^2c}-\frac{2\left(\frac{1}{4}x\arctan(ax)\left(a^2cx^2+c\right)^{3/2}-\frac{\left(a^2cx^2+c\right)^{3/2}}{12a}+\frac{3}{4}c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax\right.\right.}{}\right.$$

$$c\left(c\left(\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}(}{}\right.}{2\sqrt{a^2cx^2+c}}\right.}{3a}\right.\right.$$

$\big\downarrow$ 2720

$$c\left(\frac{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^2}{5a^2c}-\frac{2\left(\frac{1}{4}x\arctan(ax)\left(a^2cx^2+c\right)^{3/2}-\frac{\left(a^2cx^2+c\right)^{3/2}}{12a}+\frac{3}{4}c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax\right.\right.}{}\right.$$

$$c\left(c\left(\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{PolyLog}(}{}\right.}{2\sqrt{a^2cx^2+c}}\right.}{3a}\right.\right.$$

$\downarrow$ 7143

$$c\left(\frac{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^2}{5a^2c}-\frac{2\left(\frac{1}{4}x\arctan(ax)\left(a^2cx^2+c\right)^{3/2}-\frac{\left(a^2cx^2+c\right)^{3/2}}{12a}+\frac{3}{4}c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax\right.\right.}{}\right.$$

$$c\left(c\left(\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{PolyLog}(}{}\right.}{2\sqrt{a^2cx^2+c}}\right.}{3a}\right.\right.$$

input
```
Int[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2)/x,x]
```

output
```
a^2*c*(((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2)/(5*a^2*c) - (2*(-1/12*(c + a^
2*c*x^2)^(3/2)/a + (x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x])/4 + (3*c*(-1/2*Sq
rt[c + a^2*c*x^2]/a + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/2 + (c*Sqrt[1 +
a^2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/a +
 (I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*PolyLog[2,
(I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/(2*Sqrt[c + a^2*c*x^2])))/4))/(5
*a)) + c*(a^2*c*(((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/(3*a^2*c) - (2*(-1/
2*Sqrt[c + a^2*c*x^2]/a + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/2 + (c*Sqrt[
1 + a^2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])
/a + (I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*PolyLog
[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/(2*Sqrt[c + a^2*c*x^2])))/(3
*a)) + c*(a^2*c*((Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(a^2*c) - (2*Sqrt[1 +
 a^2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/a
+ (I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*PolyLog[2,
 (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/(a*Sqrt[c + a^2*c*x^2])) + (c*S
qrt[1 + a^2*x^2]*(-2*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])] + 2*(I*ArcTa
n[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])] - PolyLog[3, -E^(I*ArcTan[a*x])]) -
2*(I*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])] - PolyLog[3, E^(I*ArcTan[a*
x])])))/Sqrt[c + a^2*c*x^2]))
```

### Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5413
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Simp[x*(d + e*x^2)
^q*((a + b*ArcTan[c*x])/(2*q + 1)), x] + Simp[2*d*(q/(2*q + 1))   Int[(d +
e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0]
```

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5491
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Simp[1/Sqrt[d]   Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTa
n[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] &&
GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

## Maple [A] (verified)

Time = 4.21 (sec) , antiderivative size = 404, normalized size of antiderivative = 0.67

| method | result |
|---|---|
| default | $\frac{c^2\sqrt{c(ax-i)(ax+i)}\left(12a^4\arctan(ax)^2x^4-6\arctan(ax)x^3a^3+44\arctan(ax)^2x^2a^2+2a^2x^2-35\arctan(ax)ax+92\arctan(ax)^2+31\right)}{60}$ |

input
```
int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2/x,x,method=_RETURNVERBOSE)
```

output
```
1/60*c^2*(c*(a*x-I)*(a*x+I))^(1/2)*(12*a^4*arctan(a*x)^2*x^4-6*arctan(a*x)
*x^3*a^3+44*arctan(a*x)^2*x^2*a^2+2*a^2*x^2-35*arctan(a*x)*a*x+92*arctan(a
*x)^2+31)+1/60*I*c^2*(c*(a*x-I)*(a*x+I))^(1/2)*(60*I*arctan(a*x)^2*ln(1+(1
+I*a*x)/(a^2*x^2+1)^(1/2))-60*I*arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(
1/2))-149*I*arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+149*I*arctan(a
*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+120*arctan(a*x)*polylog(2,-(1+I*a*
x)/(a^2*x^2+1)^(1/2))-120*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2
))+120*I*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-120*I*polylog(3,(1+I*a*x)
/(a^2*x^2+1)^(1/2))-149*dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+149*dilog(1
-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)
```

## Fricas [F]

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^2}{x}\,dx=\int \frac{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)^2}{x}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2/x,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*arctan(a*
x)^2/x, x)
```

## Sympy [F]

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^2}{x}\,dx=\int \frac{(c(a^2x^2+1))^{\frac{5}{2}}\operatorname{atan}^2(ax)}{x}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)**2/x,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**2/x, x)
```

## Maxima [F]

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^2}{x}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}\arctan{(ax)}^2}{x}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2/x,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^2/x, x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^2}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^2}{x}\,dx = \int \frac{\mathrm{atan}(a\,x)^2\,{(c\,a^2\,x^2+c)}^{5/2}}{x}\,dx$$

input
```
int((atan(a*x)^2*(c + a^2*c*x^2)^(5/2))/x,x)
```

output
```
int((atan(a*x)^2*(c + a^2*c*x^2)^(5/2))/x, x)
```

## Reduce [F]

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^2}{x}\,dx = \sqrt{c}\,c^2\left(\int \frac{\sqrt{a^2x^2+1}\,atan(ax)^2}{x}dx + \left(\int \sqrt{a^2x^2+1}\,atan(ax)^2\,x^3dx\right)a^4 + 2\left(\int \sqrt{a^2x^2+1}\,atan(ax)^2\,xdx\right)a^2\right)$$

input

```
int((a^2*c*x^2+c)^(5/2)*atan(a*x)^2/x,x)
```

output

```
sqrt(c)*c**2*(int((sqrt(a**2*x**2 + 1)*atan(a*x)**2)/x,x) + int(sqrt(a**2*
x**2 + 1)*atan(a*x)**2*x**3,x)*a**4 + 2*int(sqrt(a**2*x**2 + 1)*atan(a*x)*
*2*x,x)*a**2)
```

# 3.328 $\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^2}{x^2}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 655

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^2}{x^2}\,dx = \frac{1}{12}a^2c^2x\sqrt{c+a^2cx^2}$$

$$-\frac{7}{4}ac^2\sqrt{c+a^2cx^2}\arctan(ax)-\frac{1}{6}ac(c+a^2cx^2)^{3/2}\arctan(ax)$$

$$-\frac{c^2\sqrt{c+a^2cx^2}\arctan(ax)^2}{x}+\frac{7}{8}a^2c^2x\sqrt{c+a^2cx^2}\arctan(ax)^2$$

$$+\frac{1}{4}a^2cx(c+a^2cx^2)^{3/2}\arctan(ax)^2-\frac{15iac^3\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2}{4\sqrt{c+a^2cx^2}}-\frac{4ac^3\sqrt{1+a^2x^2}\,\mathrm{arc}}{\sqrt{c}}$$

output
```
1/12*a^2*c^2*x*(a^2*c*x^2+c)^(1/2)-7/4*a*c^2*(a^2*c*x^2+c)^(1/2)*arctan(a*
x)-1/6*a*c*(a^2*c*x^2+c)^(3/2)*arctan(a*x)-c^2*(a^2*c*x^2+c)^(1/2)*arctan(
a*x)^2/x+7/8*a^2*c^2*x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2+1/4*a^2*c*x*(a^2*
c*x^2+c)^(3/2)*arctan(a*x)^2-15/4*I*a*c^3*(a^2*x^2+1)^(1/2)*arctan((1+I*a*
x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2)-4*a*c^3*(a^2*x^2+1
)^(1/2)*arctan(a*x)*arctanh((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/(a^2*c*x^2+c)
^(1/2)+11/6*a*c^(5/2)*arctanh(a*c^(1/2)*x/(a^2*c*x^2+c)^(1/2))+15/4*I*a*c^
3*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/
(a^2*c*x^2+c)^(1/2)-15/4*I*a*c^3*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(2,I
*(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)+2*I*a*c^3*(a^2*x^2+1)^(1
/2)*polylog(2,-(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/(a^2*c*x^2+c)^(1/2)-2*I*a*
c^3*(a^2*x^2+1)^(1/2)*polylog(2,(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/(a^2*c*x^
2+c)^(1/2)-15/4*a*c^3*(a^2*x^2+1)^(1/2)*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)
^(1/2))/(a^2*c*x^2+c)^(1/2)+15/4*a*c^3*(a^2*x^2+1)^(1/2)*polylog(3,I*(1+I*
a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (warning: unable to verify)

Time = 1.35 (sec) , antiderivative size = 626, normalized size of antiderivative = 0.96

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^2}{x^2}\,dx = \frac{c^2\sqrt{c+a^2cx^2}\left(2a^2x^2\sqrt{1+a^2x^2}+2a^4x^4\sqrt{1+a^2x^2}+176ax\coth^{-1}\left(\frac{}{\sqrt{1}}\right.\right.}{}$$

input
```
Integrate[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2)/x^2,x]
```

output
```
(c^2*Sqrt[c + a^2*c*x^2]*(2*a^2*x^2*Sqrt[1 + a^2*x^2] + 2*a^4*x^4*Sqrt[1 +
 a^2*x^2] + 176*a*x*ArcCoth[(a*x)/Sqrt[1 + a^2*x^2]] - 190*a*x*Sqrt[1 + a^
2*x^2]*ArcTan[a*x] + 2*a^3*x^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x] - 96*Sqrt[1 +
 a^2*x^2]*ArcTan[a*x]^2 + 117*a^2*x^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2 + 21
*a^4*x^4*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2 - (168*I)*a*x*ArcTan[E^(I*ArcTan[
a*x])]*ArcTan[a*x]^2 + 6*a*x*ArcTan[a*x]*Cos[3*ArcTan[a*x]] + 12*a^3*x^3*A
rcTan[a*x]*Cos[3*ArcTan[a*x]] + 6*a^5*x^5*ArcTan[a*x]*Cos[3*ArcTan[a*x]] +
 192*a*x*ArcTan[a*x]*Log[1 - E^(I*ArcTan[a*x])] + 96*a*x*ArcTan[a*x]^2*Log
[1 - I*E^(I*ArcTan[a*x])] - 96*a*x*ArcTan[a*x]^2*Log[1 + I*E^(I*ArcTan[a*x
])] - 192*a*x*ArcTan[a*x]*Log[1 + E^(I*ArcTan[a*x])] + (192*I)*a*x*PolyLog
[2, -E^(I*ArcTan[a*x])] + (360*I)*a*x*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*Arc
Tan[a*x])] - (360*I)*a*x*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] - (19
2*I)*a*x*PolyLog[2, E^(I*ArcTan[a*x])] - 360*a*x*PolyLog[3, (-I)*E^(I*ArcT
an[a*x])] + 360*a*x*PolyLog[3, I*E^(I*ArcTan[a*x])] + 2*a*x*Sin[3*ArcTan[a
*x]] + 4*a^3*x^3*Sin[3*ArcTan[a*x]] + 2*a^5*x^5*Sin[3*ArcTan[a*x]] - 3*a*x
*ArcTan[a*x]^2*Sin[3*ArcTan[a*x]] - 6*a^3*x^3*ArcTan[a*x]^2*Sin[3*ArcTan[a
*x]] - 3*a^5*x^5*ArcTan[a*x]^2*Sin[3*ArcTan[a*x]]))/(96*x*Sqrt[1 + a^2*x^2
])
```

## Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2 \left(a^2cx^2+c\right)^{5/2}}{x^2}dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\int \left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2dx+c\int \frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{x^2}dx$$

$$\Big\downarrow \text{5415}$$

$$a^2c\left(\frac{3}{4}c\int \sqrt{a^2cx^2+c}\arctan(ax)^2dx+\frac{1}{6}c\int \sqrt{a^2cx^2+c}dx+\frac{1}{4}x\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}-\frac{\arctan(ax)\left(a^2\right.}{6a}\right.$$
$$c\int \frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{x^2}dx$$

$$\Big\downarrow \text{211}$$

$$a^2c\left(\frac{3}{4}c\int \sqrt{a^2cx^2+c}\arctan(ax)^2dx+\frac{1}{6}c\left(\frac{1}{2}c\int \frac{1}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\arctan(ax)^2\left(a^2cx^2+\right.\right.$$
$$c\int \frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{x^2}dx$$

$\big\downarrow$ 224

$$a^2c\left(\frac{3}{4}c\int \sqrt{a^2cx^2+c}\arctan(ax)^2dx+\frac{1}{6}c\left(\frac{1}{2}c\int \frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\arctan(ax\right.$$
$$c\int \frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{x^2}dx$$

$\big\downarrow$ 219

$$a^2c\left(\frac{3}{4}c\int \sqrt{a^2cx^2+c}\arctan(ax)^2dx+\frac{1}{4}x\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{6a}+\frac{1}{6}c\left(\frac{\sqrt{ca}}{}\right.\right.$$
$$c\int \frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{x^2}dx$$

$\big\downarrow$ 5415

$$a^2c\left(\frac{3}{4}c\left(\frac{1}{2}c\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int \frac{1}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}\right)+\right.$$
$$c\int \frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{x^2}dx$$

$\big\downarrow$ 224

$$a^2c\left(\frac{3}{4}c\left(\frac{1}{2}c\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int \frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2c}}{a}\right.\right.$$
$$c\int \frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{x^2}dx$$

$\big\downarrow$ 219

$$a^2c\left(\frac{3}{4}c\left(\frac{1}{2}c\int\frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}\right.\right.$$

$$c\int\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{x^2}dx$$

$$\Big\downarrow \text{5425}$$

$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\int\frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\frac{\sqrt{c}\text{arctanh}\left(\frac{a}{\sqrt{a^2}}\right.}{a}\right.\right.$$

$$c\int\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{x^2}dx$$

$$\Big\downarrow \text{5423}$$

$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\int\sqrt{a^2x^2+1}\arctan(ax)^2d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2}}{a}\right.\right.$$

$$c\int\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{x^2}dx$$

$$\Big\downarrow \text{3042}$$

$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\int\arctan(ax)^2\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(}{}\right.\right.$$

$$c\int\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{x^2}dx$$

$$\Big\downarrow \text{4669}$$

$$c\int\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{x^2}dx+$$

$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\int\arctan(ax)\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+2\int\arctan(ax)\log\left(1+ie^{i\arctan(ax)}\right)\right.}{2a\sqrt{a^2cx^2+c}}\right.\right.$$

$$\Big\downarrow \text{3011}$$

$$c\int \frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{x^2}dx+$$
$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\,\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-i\int \text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(ax)\right)\right.\right.\right.$$

$$\Big\downarrow \; 2720$$

$$c\int \frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{x^2}dx+$$
$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\,\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\,\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\right.\right.\right.$$

$$\Big\downarrow \; 5485$$

$$c\left(a^2c\int \sqrt{a^2cx^2+c}\arctan(ax)^2dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx\right)+$$
$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\,\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\,\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\right.\right.\right.$$

$$\Big\downarrow \; 5415$$

$$c\left(a^2c\left(\frac{1}{2}c\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int \frac{1}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}\right)+c\right.$$
$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\,\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\,\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\right.\right.\right.$$

$$\Big\downarrow \; 224$$

$$c\left(a^2c\left(\frac{1}{2}c\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int \frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx}}{a}\right.\right.$$
$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\,\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\,\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\right.\right.\right.$$

$$\Big\downarrow \; 219$$

$$c\left(a^2c\left(\frac{1}{2}c\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}\right.\right.$$
$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\ldots}\right.\right.}{}\right.\right.$$

$\downarrow$ 5425

$$c\left(a^2c\left(\frac{c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c\ldots}}{\sqrt{a^2cx\ldots}}\right.}{a}\right.\right.$$
$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\ldots}\right.\right.}{}\right.\right.$$

$\downarrow$ 5423

$$c\left(a^2c\left(\frac{c\sqrt{a^2x^2+1}\int \sqrt{a^2x^2+1}\arctan(ax)^2d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+\ldots}}{a}\right.\right.$$
$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\ldots}\right.\right.}{}\right.\right.$$

$\downarrow$ 3042

$$c\left(a^2c\left(\frac{c\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(a\ldots}{}\right.\right.$$
$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\ldots}\right.\right.}{}\right.\right.$$

$\downarrow$ 4669

$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\ldots}\right.\right.}{}\right.\right.$$
$$c\left(c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx+a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\int \arctan(ax)\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+2\int\ldots\right.}{}\right.\right.$$

$$\Big\downarrow \text{3011}$$

$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\,\cdots}\right.\right.}{\cdots}\right.\right.$$

$$c\left(c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx+a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-i\int\operatorname{PolyLog}\left(\cdots\right.\right.\right.}{\cdots}\right.\right.$$

$$\Big\downarrow \text{2720}$$

$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\,\cdots}\right.\right.}{\cdots}\right.\right.$$

$$c\left(c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx+a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(a\cdots}\right.\right.}{\cdots}\right.\right.$$

$$\Big\downarrow \text{5485}$$

$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\,\cdots}\right.\right.}{\cdots}\right.\right.$$

$$c\left(c\left(a^2c\int\frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int\frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx\right)+a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(\cdots}\right.\right.\right.}{\cdots}\right.\right.$$

$$\Big\downarrow \text{5425}$$

$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\,\cdots}\right.\right.}{\cdots}\right.\right.$$

$$c\left(c\left(\frac{a^2c\sqrt{a^2x^2+1}\int\frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+c\int\frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx\right)+a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-i\cdots\right.\right.\right.}{\cdots}\right.\right.$$

$$\Big\downarrow \text{5423}$$

$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\ldots}\right.\right.}{\ldots}\right.\right.$$

$$c\left(c\left(\frac{ac\sqrt{a^2x^2+1}\int\sqrt{a^2x^2+1}\arctan(ax)^2d\arctan(ax)}{\sqrt{a^2cx^2+c}}+c\int\frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx\right)+a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan\ldots\right.\right.}{\ldots}\right.\right.$$

$\downarrow$ 3042

$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\ldots}\right.\right.}{\ldots}\right.\right.$$

$$c\left(c\left(c\int\frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx+\frac{ac\sqrt{a^2x^2+1}\int\arctan(ax)^2\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{\sqrt{a^2cx^2+c}}\right)+a^2c\left(\frac{c\sqrt{a^2x^2+\ldots}}{\ldots}\right.\right.$$

$\downarrow$ 4669

$$c\left(\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2-\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}{6a}+\frac{1}{6}c\left(\frac{1}{2}\sqrt{a^2cx^2+c}x+\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}\right)\ldots\right.$$

$$c\left(c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^2-\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a}+\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}+\frac{c\sqrt{a^2x^2+1}\left(-2i\arctan\ldots\right.}{\ldots}\right.\right.$$

$\downarrow$ 3011

$$c\left(\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2-\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}{6a}+\frac{1}{6}c\left(\frac{1}{2}\sqrt{a^2cx^2+c}x+\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a}\right)\ldots\right.$$

$$c\left(c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^2-\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a}+\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}+\frac{c\sqrt{a^2x^2+1}\left(-2i\arctan\ldots\right.}{\ldots}\right.\right.$$

input
```
Int[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2)/x^2,x]
```

output
```
$Aborted
```

## Maple [A] (verified)

Time = 4.09 (sec) , antiderivative size = 399, normalized size of antiderivative = 0.61

| method | result |
|---|---|
| default | $\frac{c^{2}\sqrt{c(ax-i)(ax+i)}\,\left(6a^{4}\arctan(ax)^{2}x^{4}-4\arctan(ax)x^{3}a^{3}+27\arctan(ax)^{2}x^{2}a^{2}+2a^{2}x^{2}-46\arctan(ax)ax-24\arctan(ax)^{2}\right)}{24x}-$ |

input
```
int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/24*c^2*(c*(a*x-I)*(a*x+I))^(1/2)*(6*a^4*arctan(a*x)^2*x^4-4*arctan(a*x)*
x^3*a^3+27*arctan(a*x)^2*x^2*a^2+2*a^2*x^2-46*arctan(a*x)*a*x-24*arctan(a*
x)^2)/x-1/24*I*c^2*a*(c*(a*x-I)*(a*x+I))^(1/2)*(45*I*arctan(a*x)^2*ln(1-I*
(1+I*a*x)/(a^2*x^2+1)^(1/2))-45*I*arctan(a*x)^2*ln(1+I*(1+I*a*x)/(a^2*x^2+
1)^(1/2))-48*I*arctan(a*x)*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+90*arctan(a*x
)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-90*arctan(a*x)*polylog(2,-I*(1+
I*a*x)/(a^2*x^2+1)^(1/2))+90*I*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-90
*I*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-48*dilog(1+(1+I*a*x)/(a^2*x^2
+1)^(1/2))-48*dilog((1+I*a*x)/(a^2*x^2+1)^(1/2))+88*arctan((1+I*a*x)/(a^2*
x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)
```

## Fricas [F]

$$\int \frac{\left(c+a^{2}cx^{2}\right)^{5/2}\arctan(ax)^{2}}{x^{2}}\,dx=\int \frac{\left(a^{2}cx^{2}+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^{2}}{x^{2}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2/x^2,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*arctan(a*
x)^2/x^2, x)
```

## Sympy [F]

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^2}{x^2}\,dx = \int \frac{(c(a^2x^2+1))^{\frac{5}{2}}\operatorname{atan}^2(ax)}{x^2}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)**2/x**2,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**2/x**2, x)
```

## Maxima [F]

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^2}{x^2}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)^2}{x^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2/x^2,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^2/x^2, x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^2}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2/x^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^2}{x^2}\,dx=\int \frac{\mathrm{atan}(a\,x)^2\,(c\,a^2\,x^2+c)^{5/2}}{x^2}\,dx$$

input

```
int((atan(a*x)^2*(c + a^2*c*x^2)^(5/2))/x^2,x)
```

output

```
int((atan(a*x)^2*(c + a^2*c*x^2)^(5/2))/x^2, x)
```

## Reduce [F]

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^2}{x^2}\,dx=\sqrt{c}\,c^2\left(\int \frac{\sqrt{a^2x^2+1}\,atan(ax)^2}{x^2}dx+\left(\int \sqrt{a^2x^2+1}\,atan(ax)^2\,x^2dx\right)a^4+2\left(\int \sqrt{a^2x^2+1}\,atan(ax)^2\,dx\right)a^2\right)$$

input

```
int((a^2*c*x^2+c)^(5/2)*atan(a*x)^2/x^2,x)
```

output

```
sqrt(c)*c**2*(int((sqrt(a**2*x**2 + 1)*atan(a*x)**2)/x**2,x) + int(sqrt(a*
*2*x**2 + 1)*atan(a*x)**2*x**2,x)*a**4 + 2*int(sqrt(a**2*x**2 + 1)*atan(a*
x)**2,x)*a**2)
```

# 3.329 $\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^2}{x^3}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 661

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^2}{x^3}\,dx = \frac{1}{3}a^2c^2\sqrt{c+a^2cx^2} - \frac{ac^2\sqrt{c+a^2cx^2}\arctan(ax)}{x} - \frac{1}{3}a^3c^2x\sqrt{c+a^2cx^2}\arctan(ax) + 2a^2c^2\sqrt{c+a^2cx^2}\arctan(ax)^2 - \frac{c^2\sqrt{c+a^2cx^2}\arctan(ax)^2}{2x^2} + \frac{1}{3}a^2c(c+a^2cx^2)^{3/2}\arctan(ax)^2 + \frac{26ia^2c^3\sqrt{1+a^2x^2}\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3\sqrt{c+a^2cx^2}} - \frac{5a^2c^3\sqrt{1+a^2x^2}\arctan(ax)^2\text{arctanh}\left(e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} - a^2c^{5/2}\text{arctanh}\left(\frac{\sqrt{c+a^2cx^2}}{\sqrt{c}}\right) + \frac{5ia^2c^3\sqrt{1+a^2x^2}\arctan(ax)\,\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} - \frac{5ia^2c^3\sqrt{1+a^2x}}{}$$

output
```
1/3*a^2*c^2*(a^2*c*x^2+c)^(1/2)-a*c^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)/x-1/
3*a^3*c^2*x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)+2*a^2*c^2*(a^2*c*x^2+c)^(1/2)*
arctan(a*x)^2-1/2*c^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2/x^2+1/3*a^2*c*(a^2
*c*x^2+c)^(3/2)*arctan(a*x)^2+26/3*I*a^2*c^3*(a^2*x^2+1)^(1/2)*arctan(a*x)
*arctan((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/(a^2*c*x^2+c)^(1/2)-5*a^2*c^3*(a^
2*x^2+1)^(1/2)*arctan(a*x)^2*arctanh((1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x
^2+c)^(1/2)-a^2*c^(5/2)*arctanh((a^2*c*x^2+c)^(1/2)/c^(1/2))+5*I*a^2*c^3*(
a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*
c*x^2+c)^(1/2)-5*I*a^2*c^3*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(2,(1+I*a*
x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)-13/3*I*a^2*c^3*(a^2*x^2+1)^(1/2)
*polylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/(a^2*c*x^2+c)^(1/2)+13/3*I*
a^2*c^3*(a^2*x^2+1)^(1/2)*polylog(2,I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/(a^
2*c*x^2+c)^(1/2)-5*a^2*c^3*(a^2*x^2+1)^(1/2)*polylog(3,-(1+I*a*x)/(a^2*x^2
+1)^(1/2))/(a^2*c*x^2+c)^(1/2)+5*a^2*c^3*(a^2*x^2+1)^(1/2)*polylog(3,(1+I*
a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (warning: unable to verify)

Time = 7.28 (sec) , antiderivative size = 761, normalized size of antiderivative = 1.15

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^2}{x^3}\,dx = \text{Too large to display}$$

input
```
Integrate[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2)/x^3,x]
```

output
```
2*a^2*c^2*Sqrt[c*(1 + a^2*x^2)]*(ArcTan[a*x]^2 + (ArcTan[a*x]^2*(Log[1 - E
^(I*ArcTan[a*x])] - Log[1 + E^(I*ArcTan[a*x])]))/Sqrt[1 + a^2*x^2] - (2*(A
rcTan[a*x]*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])]) +
 I*(PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - PolyLog[2, I*E^(I*ArcTan[a*x])]))
)/Sqrt[1 + a^2*x^2] + ((2*I)*ArcTan[a*x]*(PolyLog[2, -E^(I*ArcTan[a*x])] -
 PolyLog[2, E^(I*ArcTan[a*x])]))/Sqrt[1 + a^2*x^2] + (2*(-PolyLog[3, -E^(I
*ArcTan[a*x])] + PolyLog[3, E^(I*ArcTan[a*x])]))/Sqrt[1 + a^2*x^2]) + (a^2
*c^2*(1 + a^2*x^2)*Sqrt[c*(1 + a^2*x^2)]*(2 + 4*ArcTan[a*x]^2 + 2*Cos[2*Ar
cTan[a*x]] - (3*ArcTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2
] - ArcTan[a*x]*Cos[3*ArcTan[a*x]]*Log[1 - I*E^(I*ArcTan[a*x])] + (3*ArcTa
n[a*x]*Log[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + ArcTan[a*x]*Cos[3
*ArcTan[a*x]]*Log[1 + I*E^(I*ArcTan[a*x])] - ((4*I)*PolyLog[2, (-I)*E^(I*A
rcTan[a*x])])/(1 + a^2*x^2)^(3/2) + ((4*I)*PolyLog[2, I*E^(I*ArcTan[a*x])]
)/(1 + a^2*x^2)^(3/2) - 2*ArcTan[a*x]*Sin[2*ArcTan[a*x]]))/12 + (a^2*c^2*S
qrt[c*(1 + a^2*x^2)]*(-4*ArcTan[a*x]*Cot[ArcTan[a*x]/2] - ArcTan[a*x]^2*Cs
c[ArcTan[a*x]/2]^2 + 4*ArcTan[a*x]^2*(Log[1 - E^(I*ArcTan[a*x])] - Log[1 +
 E^(I*ArcTan[a*x])]) + 8*Log[Tan[ArcTan[a*x]/2]] + (8*I)*ArcTan[a*x]*(Poly
Log[2, -E^(I*ArcTan[a*x])] - PolyLog[2, E^(I*ArcTan[a*x])]) + 8*(-PolyLog[
3, -E^(I*ArcTan[a*x])] + PolyLog[3, E^(I*ArcTan[a*x])]) + ArcTan[a*x]^2*Se
c[ArcTan[a*x]/2]^2 - 4*ArcTan[a*x]*Tan[ArcTan[a*x]/2]))/(8*Sqrt[1 + a^2...
```

## Rubi [A] (verified)

Time = 11.33 (sec) , antiderivative size = 1215, normalized size of antiderivative = 1.84, number of steps used = 29, number of rules used = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 1.167$, Rules used = {5485, 5485, 5465, 5413, 5425, 5421, 5485, 5465, 5425, 5421, 5493, 5491, 3042, 4671, 3011, 2720, 5497, 5479, 243, 73, 221, 5493, 5491, 3042, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2 \left(a^2cx^2+c\right)^{5/2}}{x^3} \, dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\int \frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{x}dx+c\int \frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{x^3}dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\left(a^2c\int x\sqrt{a^2cx^2+c}\arctan(ax)^2dx+c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x}dx\right)+ c\left(a^2c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x}dx+c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^3}dx\right)$$

$$\Big\downarrow \text{5465}$$

$$a^2c\left(a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{2\int\sqrt{a^2cx^2+c}\arctan(ax)dx}{3a}\right)+c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x}dx\right)+ c\left(a^2c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x}dx+c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^3}dx\right)$$

$$\Big\downarrow \text{5413}$$

$$a^2c\left(a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{2\left(\frac{1}{2}c\int\frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)}{3a}\right)+c\int\frac{\sqrt{}}{}\right. c\left(a^2c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x}dx+c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^3}dx\right)$$

$$\Big\downarrow \text{5425}$$

$$a^2c\left(a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{2\left(\frac{c\sqrt{a^2x^2+1}\int\frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)}{3a}\right)\right)+ c\left(a^2c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x}dx+c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^3}dx\right)$$

$$\Big\downarrow \text{5421}$$

$$c\left(a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x}dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^3}dx\right)+$$

$$a^2c\left(c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x}dx+a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{2\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}\right.}{2}\right.}{}\right.\right.$$

$\downarrow$ 5485

$$c\left(a^2c\left(a^2c\int \frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx\right)+c\left(a^2c\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx\right)\right)+$$

$$a^2c\left(c\left(a^2c\int \frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx\right)+a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{2\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\,a}{}\right.}{}\right.}{}\right.\right.$$

$\downarrow$ 5465

$$c\left(a^2c\left(a^2c\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}\right)+c\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx\right)+c\left(a^2c\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx\right.\right.$$

$$a^2c\left(c\left(a^2c\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}\right)+c\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx\right)+a^2c\left(\frac{\arctan(ax)^2\left(a^2cx^2\right.}{3a^2c}\right.\right.$$

$\downarrow$ 5425

$$c\left(a^2c\left(a^2c\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\int\frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{a\sqrt{a^2cx^2+c}}\right)+c\int\frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx\right)+c\left(a^2c\int\frac{\text{arcta}}{x\sqrt{a^2}}\right.\right.$$

$$a^2c\left(c\left(a^2c\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\int\frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{a\sqrt{a^2cx^2+c}}\right)+c\int\frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx\right)+a^2c\left(\frac{\arctan(ax}{}\right.\right.$$

$\downarrow$ 5421

$$a^2c\left(c\left(c\int\frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+a^2c\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyL}}{}\right.}{a\sqrt{a^2cx^2+}}\right.\right.\right.$$

$$c\left(c\left(a^2c\int\frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+c\int\frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx\right)+a^2c\left(c\int\frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+a^2c\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+}}{a^2c}\right.\right.\right.$$

$\downarrow$ 5493

$$c\left(c\left(\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(\right.}{}\right.}{2\sqrt{a^2cx^2+c}}\right.}{3a}\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2\right.}{a}\right.}{a\sqrt{a^2cx^2+c}}\right.\right.\right.$$

$\downarrow$ 5491

$$c\left(c\left(\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(\right.}{}\right.}{2\sqrt{a^2cx^2+c}}\right.}{3a}\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2\right.}{a}\right.}{a\sqrt{a^2cx^2+c}}\right.\right.\right.$$

$\downarrow$ 3042

$$a^2c\left(c\left(\frac{c\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc(\arctan(ax))d\arctan(ax)}{\sqrt{a^2cx^2+c}}+a^2c\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(\right.}{}\right.\right.\right.$$

$$c\left(c\left(\frac{a^2c\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc(\arctan(ax))d\arctan(ax)}{\sqrt{a^2cx^2+c}}+c\int\frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx\right)+a^2c\left(\frac{c\sqrt{a^2x^2+1}\int}{}\right.\right.$$

$\downarrow$ 4671

$$c\left(c\left(\frac{(a^2cx^2+c)^{3/2}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(\right.}{}\right.}{2\sqrt{a^2cx^2+c}}\right.}{3a}\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2\right.}{a}\right.}{a\sqrt{a^2cx^2+c}}\right.\right.\right.$$

$\downarrow$ 3011

$$c\left(c\left(\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(\right.}{}\right.}{2\sqrt{a^2cx^2+c}}\right.}{3a}\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2\right.}{a}\right.}{a\sqrt{a^2cx^2+c}}\right.\right.\right.$$

$$\Big\downarrow \text{ 2720}$$

$$c\left(c\left(\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(\right.}{}\right.}{2\sqrt{a^2cx^2+c}}\right.}{3a}\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2\right.}{a}\right.}{a\sqrt{a^2cx^2+c}}\right.\right.\right.$$

$$\Big\downarrow \text{ 5497}$$

$$c\left(c\left(\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(\right.}{}\right.}{2\sqrt{a^2cx^2+c}}\right.}{3a}\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\mathrm{PolyLog}\left(2\right.}{a}\right.}{a\sqrt{a^2cx^2+c}}\right.\right.\right.$$

$\downarrow$ 5479

$$c\left(c\left(\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(\right.}{}\right.}{2\sqrt{a^2cx^2+c}}\right.}{3a}\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\mathrm{PolyLog}\left(2\right.}{a}\right.}{a\sqrt{a^2cx^2+c}}\right.\right.\right.$$

$\downarrow$ 243

$$c\left(c\left(\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(\right.}{}\right.}{2\sqrt{a^2cx^2+c}}\right.}{3a}\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2\right.}{a}\right.}{a\sqrt{a^2cx^2+c}}\right.\right.\right.$$

$\downarrow$ 73

$$c\left(c\left(\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(\right.}{}\right.}{2\sqrt{a^2cx^2+c}}\right.}{3a}\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2\right.}{a}\right.}{a\sqrt{a^2cx^2+c}}\right.\right.\right.$$

$\downarrow$ 221

$$c\left(c\left(\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(\right.}{}\right.}{2\sqrt{a^2cx^2+c}}\right.}{3a}\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2\right.}{a}\right.}{a\sqrt{a^2cx^2+c}}\right.\right.\right.$$

$\downarrow$ 5493

$$c\left(c\left(\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(\right.}{}\right.}{2\sqrt{a^2cx^2+c}}\right.}{3a}\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2\right.}{a}\right.}{a\sqrt{a^2cx^2+c}}\right.\right.\right.$$

$\downarrow$ 5491

$$c\left(c\left(\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(\cdots\right.}{\cdots}\right.}{2\sqrt{a^2cx^2+c}}\right.}{3a}\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2\cdots\right.}{a}\right.}{a\sqrt{a^2cx^2+c}}\right.\right.\right.$$

$\downarrow$ 3042

$$c\left(c\left(\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(\cdots\right.}{\cdots}\right.}{2\sqrt{a^2cx^2+c}}\right.}{3a}\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2\cdots\right.}{a}\right.}{a\sqrt{a^2cx^2+c}}\right.\right.\right.$$

$\downarrow$ 4671

$$c\left(c\left(\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(\right.}{}\right.}{2\sqrt{a^2cx^2+c}}\right.}{3a}\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2\right.}{a}\right.}{a\sqrt{a^2cx^2+c}}\right.\right.\right.$$

$$\Big\downarrow \; 3011$$

$$c\left(c\left(\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(\right.}{}\right.}{2\sqrt{a^2cx^2+c}}\right.}{3a}\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2\right.}{a}\right.}{a\sqrt{a^2cx^2+c}}\right.\right.\right.$$

$$\Big\downarrow \; 2720$$

$$c\left(c\left(\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(\right.}{}\right.}{2\sqrt{a^2cx^2+c}}\right.}{3a}\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2\right.}{a}\right.}{a\sqrt{a^2cx^2+c}}\right.\right.\right.$$

$\downarrow$ 7143

$$c\left(c\left(\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(\right.}{}\right.}{2\sqrt{a^2cx^2+c}}\right.}{3a}\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2\right.}{a}\right.}{a\sqrt{a^2cx^2+c}}\right.\right.\right.$$

input
```
Int[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2)/x^3,x]
```

output
```
a^2*c*(a^2*c*(((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/(3*a^2*c) - (2*(-1/2*S
qrt[c + a^2*c*x^2]/a + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/2 + (c*Sqrt[1 +
 a^2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/a
+ (I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*PolyLog[2,
 (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/(2*Sqrt[c + a^2*c*x^2])))/(3*a)
) + c*(a^2*c*((Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(a^2*c) - (2*Sqrt[1 + a^
2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/a + (
I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*PolyLog[2, (I
*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/(a*Sqrt[c + a^2*c*x^2])) + (c*Sqrt
[1 + a^2*x^2]*(-2*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])] + 2*(I*ArcTan[a
*x]*PolyLog[2, -E^(I*ArcTan[a*x])] - PolyLog[3, -E^(I*ArcTan[a*x])]) - 2*(
I*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])] - PolyLog[3, E^(I*ArcTan[a*x])
])))/Sqrt[c + a^2*c*x^2])) + c*(a^2*c*(a^2*c*((Sqrt[c + a^2*c*x^2]*ArcTan[
a*x]^2)/(a^2*c) - (2*Sqrt[1 + a^2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1
+ I*a*x]/Sqrt[1 - I*a*x]])/a + (I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1
 - I*a*x]])/a - (I*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/(a
*Sqrt[c + a^2*c*x^2])) + (c*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]^2*ArcTanh[E^
(I*ArcTan[a*x])] + 2*(I*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])] - PolyL
og[3, -E^(I*ArcTan[a*x])]) - 2*(I*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])
] - PolyLog[3, E^(I*ArcTan[a*x])])))/Sqrt[c + a^2*c*x^2]) + c*(c*(-1/2*...
```

## Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5413
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Simp[x*(d + e*x^2)
^q*((a + b*ArcTan[c*x])/(2*q + 1)), x] + Simp[2*d*(q/(2*q + 1))   Int[(d +
e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0]
```

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5491
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Simp[1/Sqrt[d]   Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTa
n[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] &&
GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5497
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)^(m
 + 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[c^2*((m
+ 2)/(f^2*(m + 1)))   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x
^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
 && LtQ[m, -1] && NeQ[m, -2]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

## Maple [A] (verified)

Time = 4.10 (sec) , antiderivative size = 454, normalized size of antiderivative = 0.69

| method | result |
|---|---|
| default | $\frac{c^2\sqrt{c(ax-i)(ax+i)}\left(2a^4\arctan(ax)^2x^4-2\arctan(ax)x^3a^3+14\arctan(ax)^2x^2a^2+2a^2x^2-6\arctan(ax)ax-3\arctan(ax)^2\right)}{6x^2}-\frac{c^2 \cdots}{}$ |

input
```
int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/6*c^2*(c*(a*x-I)*(a*x+I))^(1/2)*(2*a^4*arctan(a*x)^2*x^4-2*arctan(a*x)*x
^3*a^3+14*arctan(a*x)^2*x^2*a^2+2*a^2*x^2-6*arctan(a*x)*a*x-3*arctan(a*x)^
2)/x^2-1/6*c^2*a^2*(c*(a*x-I)*(a*x+I))^(1/2)*(15*arctan(a*x)^2*ln(1+(1+I*a
*x)/(a^2*x^2+1)^(1/2))-15*arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-
30*I*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+30*I*arctan(a*x)*
polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-26*arctan(a*x)*ln(1+I*(1+I*a*x)/(a^
2*x^2+1)^(1/2))+26*arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+26*I*di
log(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-26*I*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^
(1/2))-6*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-1)+6*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1
/2))+30*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-30*polylog(3,(1+I*a*x)/(a^
2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)
```

## Fricas [F]

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^2}{x^3}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)^2}{x^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2/x^3,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*arctan(a*
x)^2/x^3, x)
```

## Sympy [F]

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^2}{x^3}\,dx = \int \frac{(c(a^2x^2+1))^{\frac{5}{2}}\operatorname{atan}^2(ax)}{x^3}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)**2/x**3,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**2/x**3, x)
```

## Maxima [F]

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^2}{x^3}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)^2}{x^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2/x^3,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^2/x^3, x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^2}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2/x^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^2}{x^3}\,dx = \int \frac{\operatorname{atan}(a\,x)^2\,(c\,a^2\,x^2+c)^{5/2}}{x^3}\,dx$$

input
```
int((atan(a*x)^2*(c + a^2*c*x^2)^(5/2))/x^3,x)
```

output
```
int((atan(a*x)^2*(c + a^2*c*x^2)^(5/2))/x^3, x)
```

## Reduce [F]

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^2}{x^3}\,dx = \sqrt{c}\,c^2\left(\int \frac{\sqrt{a^2x^2+1}\,atan(ax)^2}{x^3}dx + 2\left(\int \frac{\sqrt{a^2x^2+1}\,atan(ax)^2}{x}dx\right)a^2 + \left(\int \sqrt{a^2x^2+1}\,atan(ax)^2\,xdx\right)a^4\right)$$

input
```
int((a^2*c*x^2+c)^(5/2)*atan(a*x)^2/x^3,x)
```

output
```
sqrt(c)*c**2*(int((sqrt(a**2*x**2 + 1)*atan(a*x)**2)/x**3,x) + 2*int((sqrt
(a**2*x**2 + 1)*atan(a*x)**2)/x,x)*a**2 + int(sqrt(a**2*x**2 + 1)*atan(a*x
)**2*x,x)*a**4)
```

# 3.330 $\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^2}{x^4}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 675

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^2}{x^4}\,dx = -\frac{a^2c^2\sqrt{c+a^2cx^2}}{3x}$$
$$-a^3c^2\sqrt{c+a^2cx^2}\arctan(ax)-\frac{ac^2\sqrt{c+a^2cx^2}\arctan(ax)}{3x^2}$$
$$-\frac{2a^2c^2\sqrt{c+a^2cx^2}\arctan(ax)^2}{x}+\frac{1}{2}a^4c^2x\sqrt{c+a^2cx^2}\arctan(ax)^2$$
$$-\frac{c(c+a^2cx^2)^{3/2}\arctan(ax)^2}{3x^3}-\frac{5ia^3c^3\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2}{\sqrt{c+a^2cx^2}}$$
$$-\frac{26a^3c^3\sqrt{1+a^2x^2}\arctan(ax)\mathrm{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3\sqrt{c+a^2cx^2}}$$
$$+a^3c^{5/2}\mathrm{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right)+\frac{5ia^3c^3\sqrt{1+a^2x^2}\arctan(ax)\,\mathrm{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}-\frac{5ia^3c^3\sqrt{1+a^2x}}{}$$

output
```
-1/3*a^2*c^2*(a^2*c*x^2+c)^(1/2)/x-a^3*c^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)
-1/3*a*c^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)/x^2-2*a^2*c^2*(a^2*c*x^2+c)^(1/
2)*arctan(a*x)^2/x+1/2*a^4*c^2*x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2-1/3*c*(
a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/x^3-5*I*a^3*c^3*(a^2*x^2+1)^(1/2)*arctan(
(1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2)-26/3*a^3*c^
3*(a^2*x^2+1)^(1/2)*arctan(a*x)*arctanh((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/(
a^2*c*x^2+c)^(1/2)+a^3*c^(5/2)*arctanh(a*c^(1/2)*x/(a^2*c*x^2+c)^(1/2))+5*
I*a^3*c^3*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)
^(1/2))/(a^2*c*x^2+c)^(1/2)-5*I*a^3*c^3*(a^2*x^2+1)^(1/2)*arctan(a*x)*poly
log(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)+13/3*I*a^3*c^3*(a
^2*x^2+1)^(1/2)*polylog(2,-(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/(a^2*c*x^2+c)^
(1/2)-13/3*I*a^3*c^3*(a^2*x^2+1)^(1/2)*polylog(2,(1+I*a*x)^(1/2)/(1-I*a*x)
^(1/2))/(a^2*c*x^2+c)^(1/2)-5*a^3*c^3*(a^2*x^2+1)^(1/2)*polylog(3,-I*(1+I*
a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)+5*a^3*c^3*(a^2*x^2+1)^(1/2)*po
lylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (warning: unable to verify)

Time = 2.52 (sec) , antiderivative size = 644, normalized size of antiderivative = 0.95

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^2}{x^4}\,dx =$$
$$-\frac{c^3\sqrt{1+a^2x^2}\left(2(1+a^2x^2)^{3/2}-12a^3x^3\coth^{-1}\left(\frac{ax}{\sqrt{1+a^2x^2}}\right)+12a^3x^3\sqrt{1+a^2x^2}\arctan(ax)+24a^2x^2\sqrt{1+\cdots}\right.}{\cdots}$$

input
```
Integrate[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2)/x^4,x]
```

output
```
-1/12*(c^3*Sqrt[1 + a^2*x^2]*(2*(1 + a^2*x^2)^(3/2) - 12*a^3*x^3*ArcCoth[(
a*x)/Sqrt[1 + a^2*x^2]] + 12*a^3*x^3*Sqrt[1 + a^2*x^2]*ArcTan[a*x] + 24*a^
2*x^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2 - 6*a^4*x^4*Sqrt[1 + a^2*x^2]*ArcTan
[a*x]^2 + 4*(1 + a^2*x^2)^(3/2)*ArcTan[a*x]^2 + (12*I)*a^3*x^3*ArcTan[E^(I
*ArcTan[a*x])]*ArcTan[a*x]^2 - 2*(1 + a^2*x^2)^(3/2)*Cos[2*ArcTan[a*x]] -
3*a*x*ArcTan[a*x]*Log[1 - E^(I*ArcTan[a*x])] - 51*a^3*x^3*ArcTan[a*x]*Log[
1 - E^(I*ArcTan[a*x])] - 24*a^3*x^3*ArcTan[a*x]^2*Log[1 - I*E^(I*ArcTan[a*
x])] + 24*a^3*x^3*ArcTan[a*x]^2*Log[1 + I*E^(I*ArcTan[a*x])] + 3*a*x*ArcTa
n[a*x]*Log[1 + E^(I*ArcTan[a*x])] + 51*a^3*x^3*ArcTan[a*x]*Log[1 + E^(I*Ar
cTan[a*x])] - (52*I)*a^3*x^3*PolyLog[2, -E^(I*ArcTan[a*x])] - (60*I)*a^3*x
^3*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + (60*I)*a^3*x^3*ArcTan[
a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] + (52*I)*a^3*x^3*PolyLog[2, E^(I*ArcT
an[a*x])] + 60*a^3*x^3*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] - 60*a^3*x^3*Pol
yLog[3, I*E^(I*ArcTan[a*x])] + 2*(1 + a^2*x^2)^(3/2)*ArcTan[a*x]*Sin[2*Arc
Tan[a*x]] + (1 + a^2*x^2)^(3/2)*ArcTan[a*x]*Log[1 - E^(I*ArcTan[a*x])]*Sin
[3*ArcTan[a*x]] - (1 + a^2*x^2)^(3/2)*ArcTan[a*x]*Log[1 + E^(I*ArcTan[a*x]
)]*Sin[3*ArcTan[a*x]]))/(x^3*Sqrt[c + a^2*c*x^2])
```

## Rubi [A] (verified)

Time = 11.38 (sec) , antiderivative size = 1119, normalized size of antiderivative = 1.66, number of steps used = 30, number of rules used = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 1.208$, Rules used = {5485, 5485, 5415, 224, 219, 5425, 5423, 3042, 4669, 3011, 2720, 5479, 5481, 242, 5485, 5425, 5423, 3042, 4669, 3011, 2720, 5479, 5493, 5489, 5497, 242, 5493, 5489, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2 \left(a^2cx^2+c\right)^{5/2}}{x^4}dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\int \frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{x^2}dx+c\int \frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{x^4}dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\left(a^2c\int \sqrt{a^2cx^2+c}\arctan(ax)^2dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx\right)+ c\left(a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^4}dx\right)$$

$\downarrow$ 5415

$$a^2c\left(a^2c\left(\frac{1}{2}c\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int \frac{1}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}\right)\right. c\left(a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^4}dx\right)$$

$\downarrow$ 224

$$a^2c\left(a^2c\left(\frac{1}{2}c\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int \frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2}}{a}\right.\right. c\left(a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^4}dx\right)$$

$\downarrow$ 219

$$a^2c\left(a^2c\left(\frac{1}{2}c\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right.}{a}\right.\right. c\left(a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^4}dx\right)$$

$\downarrow$ 5425

$$a^2c\left(a^2c\left(\frac{c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\frac{\sqrt{c}\text{arctanh}\left(\frac{a}{\sqrt{a}}\right.}{a}\right.\right. c\left(a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^4}dx\right)$$

$\downarrow$ 5423

$$a^2c\left(a^2c\left(\frac{c\sqrt{a^2x^2+1}\int\sqrt{a^2x^2+1}\arctan(ax)^2d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2}}{a}\right.\right.$$
$$c\left(a^2c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx+c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^4}dx\right)$$

$\downarrow$ 3042

$$a^2c\left(a^2c\left(\frac{c\sqrt{a^2x^2+1}\int\arctan(ax)^2\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\arctan(\right.\right.$$
$$c\left(a^2c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx+c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^4}dx\right)$$

$\downarrow$ 4669

$$c\left(a^2c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx+c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^4}dx\right)+$$
$$a^2c\left(c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx+a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\int\arctan(ax)\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+2\right.}{}\right.\right.$$

$\downarrow$ 3011

$$c\left(a^2c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx+c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^4}dx\right)+$$
$$a^2c\left(c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx+a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-i\int\operatorname{PolyLo}\right.\right.}{}\right.\right.$$

$\downarrow$ 2720

$$c\left(a^2c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx+c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^4}dx\right)+$$
$$a^2c\left(c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx+a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan}\right.\right.}{}\right.\right.$$

$\downarrow$ 5479

$$c\left(a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx+c\left(\frac{2}{3}a\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)}{x^3}dx-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)\right)+$$
$$a^2c\left(c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx+a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan}\right.\right.}{}\right.\right.$$

$\downarrow$ 5481

$$c\left(a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx+c\left(\frac{2}{3}a\left(-c\int \frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx+ac\int \frac{1}{x^2\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a}}{x^2}\right.\right.\right.$$
$$a^2c\left(c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx+a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan}\right.\right.}{}\right.\right.$$

$\downarrow$ 242

$$c\left(a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx+c\left(\frac{2}{3}a\left(-c\int \frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}-\frac{a\sqrt{a^2cx^2+c}}{x}\right)\right.\right.$$
$$a^2c\left(c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^2}dx+a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan}\right.\right.}{}\right.\right.$$

$\downarrow$ 5485

$$c\left(a^2c\left(a^2c\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx\right)+c\left(\frac{2}{3}a\left(-c\int \frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+}}{x^2}\right.\right.\right.$$
$$a^2c\left(c\left(a^2c\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx\right)+a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan}\right.\right.\right.}{}\right.\right.$$

$\downarrow$ 5425

$$c\left(a^2c\left(\frac{a^2c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx\right)+c\left(\frac{2}{3}a\left(-c\int \frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{}}{x^2}\right.\right.\right.$$
$$a^2c\left(c\left(\frac{a^2c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx\right)+a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,\right.\right.\right.}{}\right.\right.$$

$\downarrow$ 5423

$$c\left(a^2c\left(\frac{ac\sqrt{a^2x^2+1}\int \sqrt{a^2x^2+1}\arctan(ax)^2d\arctan(ax)}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx\right)+c\left(\frac{2}{3}a\left(-c\int \frac{\arctan(a}{x^3\sqrt{a^2cx^2}}\right.\right.\right.$$

$$a^2c\left(c\left(\frac{ac\sqrt{a^2x^2+1}\int \sqrt{a^2x^2+1}\arctan(ax)^2d\arctan(ax)}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx\right)+a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\,\mathrm{ar}\right.\right.}{}\right.\right.$$

$\downarrow$ 3042

$$c\left(a^2c\left(c\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx+\frac{ac\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{\sqrt{a^2cx^2+c}}\right)+c\left(\frac{2}{3}a\left(-c\int\right.\right.\right.$$

$$a^2c\left(c\left(c\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx+\frac{ac\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{\sqrt{a^2cx^2+c}}\right)+a^2c\left(\frac{c\sqrt{a^2x^2}}{}\right.\right.$$

$\downarrow$ 4669

$$a^2c\left(a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\,\mathrm{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\,\mathrm{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i}\right.\right.}{}\right.\right.$$

$$c\left(c\left(\frac{2}{3}a\left(-c\int \frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}-\frac{a\sqrt{a^2cx^2+c}}{x}\right)-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)+\right.$$

$\downarrow$ 3011

$$a^2c\left(a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\,\mathrm{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\,\mathrm{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i}\right.\right.}{}\right.\right.$$

$$c\left(c\left(\frac{2}{3}a\left(-c\int \frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{x^2}-\frac{a\sqrt{a^2cx^2+c}}{x}\right)-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)+\right.$$

$\downarrow$ 2720

$$c\left(c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^2-\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a}+\frac{\sqrt{c}\,\mathrm{arctanh}\left(\frac{a\sqrt{cx}}{\sqrt{a^2cx^2+c}}\right)}{a}+\frac{c\sqrt{a^2x^2+1}\left(-2i\arctan\right.}{}\right.\right.$$

$$c\left(c\left(c\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx+\frac{ac\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2+2\left(i\arctan(ax)\,\mathrm{PolyLog}\left(2,-\right.\right.\right.}{}\right.\right.$$

$\downarrow$ 5479

$$c\left(c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^2-\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a}+\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{cx}}{\sqrt{a^2cx^2+c}}\right)}{a}+\frac{c\sqrt{a^2x^2+1}(-2i\arctan\right.\right.$$

$$c\left(c\left(c\left(2a\int\frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx-\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{cx}\right)+\frac{ac\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2\right.}{}\right.\right.$$

$\downarrow$ 5493

$$c\left(c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^2-\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a}+\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{cx}}{\sqrt{a^2cx^2+c}}\right)}{a}+\frac{c\sqrt{a^2x^2+1}(-2i\arctan\right.\right.$$

$$c\left(c\left(c\left(\frac{2a\sqrt{a^2x^2+1}\int\frac{\arctan(ax)}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}-\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{cx}\right)+\frac{ac\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\text{arcta}\right.}{}\right.\right.$$

$\downarrow$ 5489

$$c\left(c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^2-\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a}+\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{cx}}{\sqrt{a^2cx^2+c}}\right)}{a}+\frac{c\sqrt{a^2x^2+1}(-2i\arctan\right.\right.$$

$$c\left(c\left(c\left(\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{PolyLog}\left(2,\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)\right)}{\sqrt{a^2cx^2+c}}-\right.\right.\right.$$

$\downarrow$ 5497

$$c\left(c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^2-\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a}+\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{cx}}{\sqrt{a^2cx^2+c}}\right)}{a}+\frac{c\sqrt{a^2x^2+1}(-2i\arctan\right.\right.$$

$$c\left(c\left(c\left(\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{PolyLog}\left(2,\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)\right)}{\sqrt{a^2cx^2+c}}-\right.\right.\right.$$

$\downarrow$ 242

$$c\left(c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^2-\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a}+\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{cx}}{\sqrt{a^2cx^2+c}}\right)}{a}+\frac{c\sqrt{a^2x^2+1}(-2i\arctan\right.\right.$$

$$c\left(c\left(c\left(\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{PolyLog}\left(2,\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)\right)}{\sqrt{a^2cx^2+c}}-\right.\right.\right.$$

$\downarrow$ 5493

$$c\left(c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^2-\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a}+\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}+\frac{c\sqrt{a^2x^2+1}(-2i\arctan\right.\right.$$

$$c\left(c\left(c\left(\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{PolyLog}\left(2,\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)\right)}{\sqrt{a^2cx^2+c}}-\right.\right.\right.$$

$\downarrow$ 5489

$$c\left(c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^2-\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a}+\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}+\frac{c\sqrt{a^2x^2+1}(-2i\arctan\right.\right.$$

$$c\left(c\left(c\left(\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{PolyLog}\left(2,\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)\right)}{\sqrt{a^2cx^2+c}}-\right.\right.\right.$$

$\downarrow$ 7143

$$c\left(c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^2-\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a}+\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}+\frac{c\sqrt{a^2x^2+1}(-2i\arctan\right.\right.$$

$$c\left(c\left(c\left(\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{PolyLog}\left(2,\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)\right)}{\sqrt{a^2cx^2+c}}-\right.\right.\right.$$

input
```
Int[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2)/x^4,x]
```

output
```
a^2*c*(a^2*c*(-((Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/a) + (x*Sqrt[c + a^2*c*x
^2]*ArcTan[a*x]^2)/2 + (Sqrt[c]*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]]
)/a + (c*Sqrt[1 + a^2*x^2]*((-2*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2
 + 2*(I*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - PolyLog[3, (-I)*E
^(I*ArcTan[a*x])]) - 2*(I*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] - Po
lyLog[3, I*E^(I*ArcTan[a*x])])))/(2*a*Sqrt[c + a^2*c*x^2])) + c*(c*(-((Sqr
t[c + a^2*c*x^2]*ArcTan[a*x]^2)/(c*x)) + (2*a*Sqrt[1 + a^2*x^2]*(-2*ArcTan
[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]] + I*PolyLog[2, -(Sqrt[1 + I
*a*x]/Sqrt[1 - I*a*x])] - I*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]]))/
Sqrt[c + a^2*c*x^2]) + (a*c*Sqrt[1 + a^2*x^2]*((-2*I)*ArcTan[E^(I*ArcTan[a
*x])]*ArcTan[a*x]^2 + 2*(I*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])]
 - PolyLog[3, (-I)*E^(I*ArcTan[a*x])]) - 2*(I*ArcTan[a*x]*PolyLog[2, I*E^(I
*ArcTan[a*x])] - PolyLog[3, I*E^(I*ArcTan[a*x])])))/Sqrt[c + a^2*c*x^2]))
 + c*(c*(-1/3*((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/(c*x^3) + (2*a*(-((a*Sq
rt[c + a^2*c*x^2])/x) - (Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/x^2 - c*(-1/2*(a
*Sqrt[c + a^2*c*x^2])/(c*x) - (Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(2*c*x^2)
 - (a^2*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 -
I*a*x]] + I*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])] - I*PolyLog[2,
Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]]))/(2*Sqrt[c + a^2*c*x^2]))))/3) + a^2*c*(
c*(-((Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(c*x)) + (2*a*Sqrt[1 + a^2*x^2...
```

## Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5415
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTan[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x])^p,
x], x] + Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*(
a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[q, 0] && GtQ[p, 1]
```

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5481
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*ArcTan[c*x
])/(f*(m + 2))), x] + (Simp[d/(m + 2)   Int[(f*x)^m*((a + b*ArcTan[c*x])/Sq
rt[d + e*x^2]), x], x] - Simp[b*c*(d/(f*(m + 2)))   Int[(f*x)^(m + 1)/Sqrt[
d + e*x^2], x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] &&
NeQ[m, -2]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5489
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sq
rt[1 - I*c*x]], x] + (Simp[I*(b/Sqrt[d])*PolyLog[2, -Sqrt[1 + I*c*x]/Sqrt[1
 - I*c*x]], x] - Simp[I*(b/Sqrt[d])*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x]], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5497
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)^(m
 + 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[c^2*((m
+ 2)/(f^2*(m + 1)))   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x
^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
 && LtQ[m, -1] && NeQ[m, -2]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

## Maple [A] (verified)

Time = 5.63 (sec) , antiderivative size = 401, normalized size of antiderivative = 0.59

| method | result |
|---|---|
| default | $\frac{c^2\sqrt{c(ax-i)(ax+i)}\,\left(3a^4\arctan(ax)^2x^4-6\arctan(ax)x^3a^3-14\arctan(ax)^2x^2a^2-2a^2x^2-2\arctan(ax)ax-2\arctan(ax)^2\right)}{6x^3}+\frac{ic^2}{\cdots}$ |

input
```
int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/6*c^2*(c*(a*x-I)*(a*x+I))^(1/2)*(3*a^4*arctan(a*x)^2*x^4-6*arctan(a*x)*x
^3*a^3-14*arctan(a*x)^2*x^2*a^2-2*a^2*x^2-2*arctan(a*x)*a*x-2*arctan(a*x)^
2)/x^3+1/6*I*c^2*a^3*(c*(a*x-I)*(a*x+I))^(1/2)*(15*I*arctan(a*x)^2*ln(1+I*
(1+I*a*x)/(a^2*x^2+1)^(1/2))-15*I*arctan(a*x)^2*ln(1-I*(1+I*a*x)/(a^2*x^2+
1)^(1/2))+26*I*arctan(a*x)*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+30*arctan(a*x
)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-30*arctan(a*x)*polylog(2,I*(1+
I*a*x)/(a^2*x^2+1)^(1/2))+30*I*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-3
0*I*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+26*dilog((1+I*a*x)/(a^2*x^2+1
)^(1/2))+26*dilog(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-12*arctan((1+I*a*x)/(a^2*
x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)
```

## Fricas [F]

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^2}{x^4}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}\arctan{(ax)}^2}{x^4}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2/x^4,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*arctan(a*
x)^2/x^4, x)
```

## Sympy [F]

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^2}{x^4}\,dx = \int \frac{(c(a^2x^2+1))^{\frac{5}{2}}\operatorname{atan}^2{(ax)}}{x^4}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)**2/x**4,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**2/x**4, x)
```

## Maxima [F]

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^2}{x^4}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)^2}{x^4}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2/x^4,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^2/x^4, x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^2}{x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^2/x^4,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^2}{x^4}\,dx = \int \frac{\mathrm{atan}(a\,x)^2\,(c\,a^2\,x^2+c)^{5/2}}{x^4}\,dx$$

input
```
int((atan(a*x)^2*(c + a^2*c*x^2)^(5/2))/x^4,x)
```

output
```
int((atan(a*x)^2*(c + a^2*c*x^2)^(5/2))/x^4, x)
```

## Reduce [F]

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^2}{x^4}\,dx = \sqrt{c}\,c^2\left(\int \frac{\sqrt{a^2x^2+1}\,atan(ax)^2}{x^4}dx + 2\left(\int \frac{\sqrt{a^2x^2+1}\,atan(ax)^2}{x^2}dx\right)a^2 + \left(\int \sqrt{a^2x^2+1}\,atan(ax)^2\,dx\right)a^4\right)$$

input
```
int((a^2*c*x^2+c)^(5/2)*atan(a*x)^2/x^4,x)
```

output
```
sqrt(c)*c**2*(int((sqrt(a**2*x**2 + 1)*atan(a*x)**2)/x**4,x) + 2*int((sqrt
(a**2*x**2 + 1)*atan(a*x)**2)/x**2,x)*a**2 + int(sqrt(a**2*x**2 + 1)*atan(
a*x)**2,x)*a**4)
```

# 3.331 $\int \frac{x^3 \arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 315

$$\begin{aligned}\int \frac{x^3 \arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx &= \frac{\sqrt{c+a^2cx^2}}{3a^4c} - \frac{x\sqrt{c+a^2cx^2}\arctan(ax)}{3a^3c} \\ &\quad - \frac{2\sqrt{c+a^2cx^2}\arctan(ax)^2}{3a^4c} + \frac{x^2\sqrt{c+a^2cx^2}\arctan(ax)^2}{3a^2c} \\ &\quad - \frac{10i\sqrt{1+a^2x^2}\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3a^4\sqrt{c+a^2cx^2}} \\ &\quad + \frac{5i\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3a^4\sqrt{c+a^2cx^2}} \\ &\quad - \frac{5i\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3a^4\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
1/3*(a^2*c*x^2+c)^(1/2)/a^4/c-1/3*x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)/a^3/c-
2/3*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2/a^4/c+1/3*x^2*(a^2*c*x^2+c)^(1/2)*ar
ctan(a*x)^2/a^2/c-10/3*I*(a^2*x^2+1)^(1/2)*arctan(a*x)*arctan((1+I*a*x)^(1
/2)/(1-I*a*x)^(1/2))/a^4/(a^2*c*x^2+c)^(1/2)+5/3*I*(a^2*x^2+1)^(1/2)*polyl
og(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/a^4/(a^2*c*x^2+c)^(1/2)-5/3*I*(a^
2*x^2+1)^(1/2)*polylog(2,I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/a^4/(a^2*c*x^2
+c)^(1/2)
```

## Mathematica [A] (warning: unable to verify)

Time = 0.55 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.89

$$\int \frac{x^3 \arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx = \frac{(1+a^2x^2)\sqrt{c(1+a^2x^2)}\Big(2-2\arctan(ax)^2+2\cos(2\arctan(ax))-6\arctan(ax)^2\cos(2\arctan(ax))+}{}$$

input
```
Integrate[(x^3*ArcTan[a*x]^2)/Sqrt[c + a^2*c*x^2],x]
```

output
```
((1 + a^2*x^2)*Sqrt[c*(1 + a^2*x^2)]*(2 - 2*ArcTan[a*x]^2 + 2*Cos[2*ArcTan
[a*x]] - 6*ArcTan[a*x]^2*Cos[2*ArcTan[a*x]] + (15*ArcTan[a*x]*Log[1 - I*E^
(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + 5*ArcTan[a*x]*Cos[3*ArcTan[a*x]]*Log
[1 - I*E^(I*ArcTan[a*x])] - (15*ArcTan[a*x]*Log[1 + I*E^(I*ArcTan[a*x])])/
Sqrt[1 + a^2*x^2] - 5*ArcTan[a*x]*Cos[3*ArcTan[a*x]]*Log[1 + I*E^(I*ArcTan
[a*x])] + ((20*I)*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/(1 + a^2*x^2)^(3/2)
- ((20*I)*PolyLog[2, I*E^(I*ArcTan[a*x])])/(1 + a^2*x^2)^(3/2) - 2*ArcTan[
a*x]*Sin[2*ArcTan[a*x]]))/(12*a^4*c)
```

## Rubi [A] (verified)

Time = 1.51 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.33, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5487, 5465, 5425, 5421, 5487, 241, 5425, 5421}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \arctan(ax)^2}{\sqrt{a^2cx^2+c}}\,dx$$

$$\Big\downarrow \text{5487}$$

$$-\frac{2\int \frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{2\int \frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)^2\sqrt{a^2cx^2+c}}{3a^2c}$$

$$\Big\downarrow \text{5465}$$

$$-\frac{2\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\int\frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}\right)}{3a^2}-\frac{2\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}+\frac{x^2\arctan(ax)^2\sqrt{a^2cx^2+c}}{3a^2c}$$

$$\Big\downarrow \text{5425}$$

$$-\frac{2\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\int\frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{a\sqrt{a^2cx^2+c}}\right)}{3a^2}-\frac{2\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}+\frac{x^2\arctan(ax)^2\sqrt{a^2cx^2+c}}{3a^2c}$$

$$\Big\downarrow \text{5421}$$

$$-\frac{2\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{2\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{a\sqrt{a^2cx^2+c}}\right)}{3a^2}+\frac{x^2\arctan(ax)^2\sqrt{a^2cx^2+c}}{3a^2c}$$

$$\Big\downarrow \text{5487}$$

$$-\frac{2\left(-\frac{\int\frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a^2}-\frac{\int\frac{x}{\sqrt{a^2cx^2+c}}dx}{2a}+\frac{x\arctan(ax)\sqrt{a^2cx^2+c}}{2a^2c}\right)}{3a}-\frac{2\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{a\sqrt{a^2cx^2+c}}\right)}{3a^2}+\frac{x^2\arctan(ax)^2\sqrt{a^2cx^2+c}}{3a^2c}$$

$$\Big\downarrow \text{241}$$

$$-\frac{2\left(-\frac{\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a^2}+\frac{x\arctan(ax)\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\sqrt{a^2cx^2+c}}{2a^3c}\right)}{3a}-\frac{2\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{a\sqrt{a^2cx^2+c}}\right)}{3a^2}+\frac{x^2\arctan(ax)^2\sqrt{a^2cx^2+c}}{3a^2c}$$

$$\Big\downarrow \textbf{5425}$$

$$-\frac{2\left(-\frac{\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{2a^2\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\sqrt{a^2cx^2+c}}{2a^3c}\right)}{3a}-\frac{2\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{a\sqrt{a^2cx^2+c}}\right)}{3a^2}+\frac{x^2\arctan(ax)^2\sqrt{a^2cx^2+c}}{3a^2c}$$

$$\Big\downarrow \textbf{5421}$$

$$-\frac{2\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{a\sqrt{a^2cx^2+c}}\right)}{3a^2}+\frac{x^2\arctan(ax)^2\sqrt{a^2cx^2+c}}{3a^2c}-\frac{2\left(-\frac{\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2a^2\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\sqrt{a^2cx^2+c}}{2a^3c}\right)}{3a}$$

input
```
Int[(x^3*ArcTan[a*x]^2)/Sqrt[c + a^2*c*x^2],x]
```

output
```
(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(3*a^2*c) - (2*(-1/2*Sqrt[c + a^2*
c*x^2]/(a^3*c) + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(2*a^2*c) - (Sqrt[1 +
 a^2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/a
+ (I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*PolyLog[2,
 (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/(2*a^2*Sqrt[c + a^2*c*x^2])))/(
3*a) - (2*((Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(a^2*c) - (2*Sqrt[1 + a^2*x
^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/a + (I*P
olyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*PolyLog[2, (I*Sq
rt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/(a*Sqrt[c + a^2*c*x^2])))/(3*a^2)
```

## Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5487
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcTan[c*x])^p/(c^2*d*m)), x] + (-Simp[b*f*(p/(c*m))   Int[(f*x)^(m - 1)*((
a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[f^2*((m - 1)/(c^
2*m))   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2]), x], x])
/; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

## Maple [A] (verified)

Time = 2.22 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.65

| method | result |
|---|---|
| default | $\frac{\left(\arctan(ax)^2 x^2 a^2-\arctan(ax)ax-2\arctan(ax)^2+1\right)\sqrt{c(ax-i)(ax+i)}}{3c\,a^4}-\frac{5\left(\arctan(ax)\ln\left(1+\frac{i(iax+1)}{\sqrt{a^2x^2+1}}\right)-\arctan(ax)\ln\left(1-\frac{i}{\sqrt{}}\right.\right.}{}$ |

input
```
int(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(arctan(a*x)^2*x^2*a^2-arctan(a*x)*a*x-2*arctan(a*x)^2+1)*(c*(a*x-I)*(
a*x+I))^(1/2)/c/a^4-5/3*(arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-a
rctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+I*dilog(1-I*(1+I*a*x)/(a^2*
x^2+1)^(1/2))-I*dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))*(c*(a*x-I)*(a*x+I)
)^(1/2)/(a^2*x^2+1)^(1/2)/a^4/c
```

## Fricas [F]

$$\int \frac{x^3\arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^3\arctan\left(ax\right)^2}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(x^3*arctan(a*x)^2/sqrt(a^2*c*x^2 + c), x)
```

## Sympy [F]

$$\int \frac{x^3 \arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^3 \operatorname{atan}^2(ax)}{\sqrt{c\,(a^2x^2+1)}}\,dx$$

input
```
integrate(x**3*atan(a*x)**2/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x**3*atan(a*x)**2/sqrt(c*(a**2*x**2 + 1)), x)
```

## Maxima [F]

$$\int \frac{x^3 \arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^3 \arctan(ax)^2}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3*arctan(a*x)^2/sqrt(a^2*c*x^2 + c), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x^3 \arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^3\,\mathrm{atan}(a\,x)^2}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int((x^3*atan(a*x)^2)/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int((x^3*atan(a*x)^2)/(c + a^2*c*x^2)^(1/2), x)
```

## Reduce [F]

$$\int \frac{x^3 \arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx = \frac{\int \frac{atan(ax)^2x^3}{\sqrt{a^2x^2+1}}dx}{\sqrt{c}}$$

input
```
int(x^3*atan(a*x)^2/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int((atan(a*x)**2*x**3)/sqrt(a**2*x**2 + 1),x)/sqrt(c)
```

# 3.332 $\int \frac{x^2 \arctan(ax)^2}{\sqrt{c+a^2cx^2}} \, dx$



## Optimal result

Integrand size = 24, antiderivative size = 344

$$\begin{aligned}
\int \frac{x^2 \arctan(ax)^2}{\sqrt{c+a^2cx^2}} \, dx = & -\frac{\sqrt{c+a^2cx^2}\arctan(ax)}{a^3c} + \frac{x\sqrt{c+a^2cx^2}\arctan(ax)^2}{2a^2c} \\
& + \frac{i\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2}{a^3\sqrt{c+a^2cx^2}} \\
& + \frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right)}{a^3\sqrt{c}} \\
& - \frac{i\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)}{a^3\sqrt{c+a^2cx^2}} \\
& + \frac{i\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,ie^{i\arctan(ax)}\right)}{a^3\sqrt{c+a^2cx^2}} \\
& + \frac{\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)}{a^3\sqrt{c+a^2cx^2}} \\
& - \frac{\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,ie^{i\arctan(ax)}\right)}{a^3\sqrt{c+a^2cx^2}}
\end{aligned}$$

output
```
-(a^2*c*x^2+c)^(1/2)*arctan(a*x)/a^3/c+1/2*x*(a^2*c*x^2+c)^(1/2)*arctan(a*
x)^2/a^2/c+I*(a^2*x^2+1)^(1/2)*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(
a*x)^2/a^3/(a^2*c*x^2+c)^(1/2)+arctanh(a*c^(1/2)*x/(a^2*c*x^2+c)^(1/2))/a^
3/c^(1/2)-I*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+
1)^(1/2))/a^3/(a^2*c*x^2+c)^(1/2)+I*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(
2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a^3/(a^2*c*x^2+c)^(1/2)+(a^2*x^2+1)^(1/2)
*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a^3/(a^2*c*x^2+c)^(1/2)-(a^2*x^
2+1)^(1/2)*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a^3/(a^2*c*x^2+c)^(1/2
)
```

## Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.51

$$\int \frac{x^2 \arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{c+a^2cx^2}\left(\arctan(ax)(-2+ax\arctan(ax)) + \frac{2\left(\coth^{-1}\left(\frac{ax}{\sqrt{1+a^2x^2}}\right)+i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2-i\arctan(ax)\,\mathrm{P}\cdots\right)}{\cdots}\right)}{2a^3c}$$

input
```
Integrate[(x^2*ArcTan[a*x]^2)/Sqrt[c + a^2*c*x^2],x]
```

output
```
(Sqrt[c + a^2*c*x^2]*(ArcTan[a*x]*(-2 + a*x*ArcTan[a*x]) + (2*(ArcCoth[(a*
x)/Sqrt[1 + a^2*x^2]] + I*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 - I*ArcT
an[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + I*ArcTan[a*x]*PolyLog[2, I*E^
(I*ArcTan[a*x])] + PolyLog[3, (-I)*E^(I*ArcTan[a*x])] - PolyLog[3, I*E^(I*
ArcTan[a*x])]))/Sqrt[1 + a^2*x^2]))/(2*a^3*c)
```

## Rubi [A] (verified)

Time = 1.38 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.70, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {5487, 5425, 5423, 3042, 4669, 3011, 2720, 5465, 224, 219, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx$$

$\Big\downarrow$ 5487

$$-\frac{\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a^2}+\frac{x\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a^2c}$$

$\Big\downarrow$ 5425

$$-\frac{\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{2a^2\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a^2c}$$

$\Big\downarrow$ 5423

$$-\frac{\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{\sqrt{a^2x^2+1}\int \sqrt{a^2x^2+1}\arctan(ax)^2d\arctan(ax)}{2a^3\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a^2c}$$

$\Big\downarrow$ 3042

$$-\frac{\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{2a^3\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a^2c}$$

$\Big\downarrow$ 4669

$$-\frac{\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{\sqrt{a^2x^2+1}\left(-2\int \arctan(ax)\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+2\int \arctan(ax)\log\left(1+ie^{i\arctan(ax)}\right)d\arctan(a\right.}{2a^3\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a^2c}$$

$\Big\downarrow$ 3011

$$-\frac{\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-i\int \operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(ax)\right)-2\left(i\arctan\right.\right.}{2a^3\sqrt{a}}+\frac{x\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a^2c}$$

$\Big\downarrow$ 2720

$$-\frac{\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)-\right.}{}$$

$$\frac{x\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a^2c}$$

$\downarrow$ 5465

$$-\frac{\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\int \frac{1}{\sqrt{a^2cx^2+c}}dx}{a}}{a}-\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)-\right.}{}$$

$$\frac{x\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a^2c}$$

$\downarrow$ 224

$$-\frac{\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\int \frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}}{a}}{a}-\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)-\right.}{}$$

$$\frac{x\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a^2c}$$

$\downarrow$ 219

$$-\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)\right.}{}$$

$$\frac{\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a^2\sqrt{c}}}{a}+\frac{x\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a^2c}$$

$\downarrow$ 7143

$$-\frac{\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a^2\sqrt{c}}}{a}+\frac{x\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)\right)-2\left(i\arctan(ax)\operatorname{PolyLog}\left(2\right.\right.\right.}{2a^3\sqrt{a^2cx^2+c}}$$

input
```
Int[(x^2*ArcTan[a*x]^2)/Sqrt[c + a^2*c*x^2],x]
```

output
```
(x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(2*a^2*c) - ((Sqrt[c + a^2*c*x^2]*Ar
cTan[a*x])/(a^2*c) - ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]]/(a^2*Sqrt[
c]))/a - (Sqrt[1 + a^2*x^2]*((-2*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^
2 + 2*(I*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - PolyLog[3, (-I)*
E^(I*ArcTan[a*x])]) - 2*(I*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] - P
olyLog[3, I*E^(I*ArcTan[a*x])])))/(2*a^3*Sqrt[c + a^2*c*x^2])
```

## Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5487
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcTan[c*x])^p/(c^2*d*m)), x] + (-Simp[b*f*(p/(c*m))   Int[(f*x)^(m - 1)*((
a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[f^2*((m - 1)/(c^
2*m))   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2]), x], x])
/; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

## Maple [A] (verified)

Time = 2.27 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.80

| method | result |
|---|---|
| default | $\frac{(\arctan(ax)ax-2)\arctan(ax)\sqrt{c(ax-i)(ax+i)}}{2c\,a^3}+\frac{i\left(i\arctan(ax)^2\ln\left(1-\frac{i(iax+1)}{\sqrt{a^2x^2+1}}\right)-i\arctan(ax)^2\ln\left(1+\frac{i(iax+1)}{\sqrt{a^2x^2+1}}\right)+2\arctan\right.}{}$ |

input
```
int(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(arctan(a*x)*a*x-2)*arctan(a*x)*(c*(a*x-I)*(a*x+I))^(1/2)/c/a^3+1/2*I*
(I*arctan(a*x)^2*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*arctan(a*x)^2*ln(1+
I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^
2+1)^(1/2))-2*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*I*po
lylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*I*polylog(3,-I*(1+I*a*x)/(a^2*x^2
+1)^(1/2))-4*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2)))*(c*(a*x-I)*(a*x+I))^(1/2
)/(a^2*x^2+1)^(1/2)/a^3/c
```

## Fricas [F]

$$\int \frac{x^2\arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx=\int \frac{x^2\arctan\left(ax\right)^2}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(x^2*arctan(a*x)^2/sqrt(a^2*c*x^2 + c), x)
```

## Sympy [F]

$$\int \frac{x^2 \arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^2 \operatorname{atan}^2(ax)}{\sqrt{c\,(a^2x^2+1)}}\,dx$$

input
```
integrate(x**2*atan(a*x)**2/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x**2*atan(a*x)**2/sqrt(c*(a**2*x**2 + 1)), x)
```

## Maxima [F]

$$\int \frac{x^2 \arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^2 \arctan(ax)^2}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2*arctan(a*x)^2/sqrt(a^2*c*x^2 + c), x)
```

## Giac [F]

$$\int \frac{x^2 \arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^2 \arctan(ax)^2}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2*arctan(a*x)^2/sqrt(a^2*c*x^2 + c), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \arctan(ax)^2}{\sqrt{c + a^2cx^2}} \, dx = \int \frac{x^2 \operatorname{atan}(a\,x)^2}{\sqrt{c\,a^2\,x^2 + c}} \, dx$$

input
```
int((x^2*atan(a*x)^2)/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int((x^2*atan(a*x)^2)/(c + a^2*c*x^2)^(1/2), x)
```

## Reduce [F]

$$\int \frac{x^2 \arctan(ax)^2}{\sqrt{c + a^2cx^2}} \, dx = \frac{\int \frac{atan(ax)^2 x^2}{\sqrt{a^2x^2+1}} dx}{\sqrt{c}}$$

input
```
int(x^2*atan(a*x)^2/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int((atan(a*x)**2*x**2)/sqrt(a**2*x**2 + 1),x)/sqrt(c)
```

# 3.333 $\int \frac{x\arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 220

$$\int \frac{x\arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{a^2c} + \frac{4i\sqrt{1+a^2x^2}\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a^2\sqrt{c+a^2cx^2}} - \frac{2i\sqrt{1+a^2x^2}\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a^2\sqrt{c+a^2cx^2}} + \frac{2i\sqrt{1+a^2x^2}\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a^2\sqrt{c+a^2cx^2}}$$

output
```
(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2/a^2/c+4*I*(a^2*x^2+1)^(1/2)*arctan(a*x)*
arctan((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/a^2/(a^2*c*x^2+c)^(1/2)-2*I*(a^2*x
^2+1)^(1/2)*polylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/a^2/(a^2*c*x^2+c
)^(1/2)+2*I*(a^2*x^2+1)^(1/2)*polylog(2,I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))
/a^2/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.57

$$\int \frac{x\arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{c(1+a^2x^2)}\left(\arctan(ax)^2 - \frac{2(\arctan(ax)(\log(1-ie^{i\arctan(ax)})-\log(1+ie^{i\arctan(ax)}))+i(\text{PolyLog}(2,-ie^{i\arctan(ax)})-\text{PolyLog}}{\sqrt{1+a^2x^2}}\right.}{a^2c}$$

input
```
Integrate[(x*ArcTan[a*x]^2)/Sqrt[c + a^2*c*x^2],x]
```

output
```
(Sqrt[c*(1 + a^2*x^2)]*(ArcTan[a*x]^2 - (2*(ArcTan[a*x]*(Log[1 - I*E^(I*Ar
cTan[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])]) + I*(PolyLog[2, (-I)*E^(I*ArcT
an[a*x])] - PolyLog[2, I*E^(I*ArcTan[a*x])])))/Sqrt[1 + a^2*x^2]))/(a^2*c)
```

## Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.78, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {5465, 5425, 5421}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}\,dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c} - \frac{2\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}$$

$$\Big\downarrow \text{5425}$$

$$\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c} - \frac{2\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{a\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5421}$$

$$\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c} - \frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a} + \frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a} - \frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{a\sqrt{a^2cx^2+c}}$$

input
```
Int[(x*ArcTan[a*x]^2)/Sqrt[c + a^2*c*x^2],x]
```

output
```
(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(a^2*c) - (2*Sqrt[1 + a^2*x^2]*(((-2*I
)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/a + (I*PolyLog[2, (
(-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*PolyLog[2, (I*Sqrt[1 + I*a*
x])/Sqrt[1 - I*a*x]])/a))/(a*Sqrt[c + a^2*c*x^2])
```

## Defintions of rubi rules used

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

## Maple [A] (verified)

Time = 1.93 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.82

| method | result |
|---|---|
| default | $\frac{\arctan(ax)^2\sqrt{c(ax-i)(ax+i)}}{a^2c}+\frac{2\left(\arctan(ax)\ln\left(1+\frac{i(iax+1)}{\sqrt{a^2x^2+1}}\right)-\arctan(ax)\ln\left(1-\frac{i(iax+1)}{\sqrt{a^2x^2+1}}\right)+i\,\mathrm{dilog}\left(1-\frac{i(iax+1)}{\sqrt{a^2x^2+1}}\right)-i\,\mathrm{dilog}}{\sqrt{a^2x^2+1}\,a^2c}$ |

input
```
int(x*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arctan(a*x)^2*(c*(a*x-I)*(a*x+I))^(1/2)/a^2/c+2*(arctan(a*x)*ln(1+I*(1+I*a
*x)/(a^2*x^2+1)^(1/2))-arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+I*d
ilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1
/2)))*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/2)/a^2/c
```

## Fricas [F]

$$\int \frac{x\arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx=\int \frac{x\arctan\left(ax\right)^2}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(x*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(x*arctan(a*x)^2/sqrt(a^2*c*x^2 + c), x)
```

## Sympy [F]

$$\int \frac{x\arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx=\int \frac{x\,\mathrm{atan}^2\left(ax\right)}{\sqrt{c\left(a^2x^2+1\right)}}\,dx$$

input
```
integrate(x*atan(a*x)**2/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x*atan(a*x)**2/sqrt(c*(a**2*x**2 + 1)), x)
```

## Maxima [F]

$$\int \frac{x \arctan(ax)^2}{\sqrt{c + a^2cx^2}} \, dx = \int \frac{x \arctan{(ax)}^2}{\sqrt{a^2cx^2 + c}} \, dx$$

input
```
integrate(x*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x*arctan(a*x)^2/sqrt(a^2*c*x^2 + c), x)
```

## Giac [F]

$$\int \frac{x \arctan(ax)^2}{\sqrt{c + a^2cx^2}} \, dx = \int \frac{x \arctan{(ax)}^2}{\sqrt{a^2cx^2 + c}} \, dx$$

input
```
integrate(x*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x*arctan(a*x)^2/sqrt(a^2*c*x^2 + c), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x \arctan(ax)^2}{\sqrt{c + a^2cx^2}} \, dx = \int \frac{x \operatorname{atan}(a\,x)^2}{\sqrt{c\,a^2\,x^2 + c}} \, dx$$

input
```
int((x*atan(a*x)^2)/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int((x*atan(a*x)^2)/(c + a^2*c*x^2)^(1/2), x)
```

## Reduce [F]

$$\int \frac{x \arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx = \frac{\int \frac{atan(ax)^2 x}{\sqrt{a^2x^2+1}}dx}{\sqrt{c}}$$

input
```
int(x*atan(a*x)^2/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int((atan(a*x)**2*x)/sqrt(a**2*x**2 + 1),x)/sqrt(c)
```

# 3.334 $\int \frac{\arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 256

$$\begin{aligned}\int \frac{\arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx = &-\frac{2i\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2}{a\sqrt{c+a^2cx^2}}\\ &+\frac{2i\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)}{a\sqrt{c+a^2cx^2}}\\ &-\frac{2i\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,ie^{i\arctan(ax)}\right)}{a\sqrt{c+a^2cx^2}}\\ &-\frac{2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)}{a\sqrt{c+a^2cx^2}}\\ &+\frac{2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,ie^{i\arctan(ax)}\right)}{a\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
-2*I*(a^2*x^2+1)^(1/2)*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^2/a
/(a^2*c*x^2+c)^(1/2)+2*I*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(2,-I*(1+I*a
*x)/(a^2*x^2+1)^(1/2))/a/(a^2*c*x^2+c)^(1/2)-2*I*(a^2*x^2+1)^(1/2)*arctan(
a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a/(a^2*c*x^2+c)^(1/2)-2*(a^2
*x^2+1)^(1/2)*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a/(a^2*c*x^2+c)^(1
/2)+2*(a^2*x^2+1)^(1/2)*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a/(a^2*c*
x^2+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.55

$$\int \frac{\arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx = \frac{2\sqrt{c(1+a^2x^2)}\left(-i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2+i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-i\arctan\right.}{ac\sqrt{1+a^2x^2}}$$

input
```
Integrate[ArcTan[a*x]^2/Sqrt[c + a^2*c*x^2],x]
```

output
```
(2*Sqrt[c*(1 + a^2*x^2)]*((-I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 + I
*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - I*ArcTan[a*x]*PolyLog[2,
 I*E^(I*ArcTan[a*x])] - PolyLog[3, (-I)*E^(I*ArcTan[a*x])] + PolyLog[3, I*
E^(I*ArcTan[a*x])]))/(a*c*Sqrt[1 + a^2*x^2])
```

## Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.56, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5425, 5423, 3042, 4669, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}\,dx$$

$$\Big\downarrow \text{5425}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5423}$$

$$\frac{\sqrt{a^2x^2+1}\int \sqrt{a^2x^2+1}\arctan(ax)^2d\arctan(ax)}{a\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{a\sqrt{a^2cx^2+c}}$$

$\Big\downarrow$ 4669

$$\frac{\sqrt{a^2x^2+1}\left(-2\int \arctan(ax)\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+2\int \arctan(ax)\log\left(1+ie^{i\arctan(ax)}\right)d\arctan(a\right.}{a\sqrt{a^2cx^2+c}}$$

$\Big\downarrow$ 3011

$$\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-i\int \text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(ax)\right)-2\left(i\arctan\right.\right.}{a\sqrt{a^2}}$$

$\Big\downarrow$ 2720

$$\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)-\right.$$

$\Big\downarrow$ 7143

$$\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\text{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)\right)-2\left(i\arctan(ax)\text{PolyLog}\left(2\right.\right.\right.}{a\sqrt{a^2cx^2+c}}$$

input
```
Int[ArcTan[a*x]^2/Sqrt[c + a^2*c*x^2],x]
```

output
```
(Sqrt[1 + a^2*x^2]*((-2*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 + 2*(I*
ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - PolyLog[3, (-I)*E^(I*ArcT
an[a*x])]) - 2*(I*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] - PolyLog[3,
 I*E^(I*ArcTan[a*x])])))/(a*Sqrt[c + a^2*c*x^2])
```

### Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

## Maple [F]

$$\int \frac{\arctan(ax)^2}{\sqrt{a^2c\,x^2+c}}dx$$

input
```
int(arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x)
```

## Fricas [F]

$$\int \frac{\arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(arctan(a*x)^2/sqrt(a^2*c*x^2 + c), x)
```

## Sympy [F]

$$\int \frac{\arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\operatorname{atan}^2(ax)}{\sqrt{c(a^2x^2+1)}}\,dx$$

input
```
integrate(atan(a*x)**2/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(atan(a*x)**2/sqrt(c*(a**2*x**2 + 1)), x)
```

## Maxima [F]

$$\int \frac{\arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)^2/sqrt(a^2*c*x^2 + c), x)
```

## Giac [F]

$$\int \frac{\arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^2/sqrt(a^2*c*x^2 + c), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\mathrm{atan}(a\,x)^2}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(atan(a*x)^2/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(atan(a*x)^2/(c + a^2*c*x^2)^(1/2), x)
```

## Reduce [F]

$$\int \frac{\arctan(ax)^2}{\sqrt{c + a^2cx^2}}\,dx = \frac{\int \frac{atan(ax)^2}{\sqrt{a^2x^2+1}}dx}{\sqrt{c}}$$

input
```
int(atan(a*x)^2/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(atan(a*x)**2/sqrt(a**2*x**2 + 1),x)/sqrt(c)
```

# 3.335 $\int \frac{\arctan(ax)^2}{x\sqrt{c+a^2cx^2}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 227

$$\int \frac{\arctan(ax)^2}{x\sqrt{c+a^2cx^2}}\,dx = -\frac{2\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} + \frac{2i\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} - \frac{2i\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} - \frac{2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,-e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} + \frac{2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}$$

output

```
-2*(a^2*x^2+1)^(1/2)*arctan(a*x)^2*arctanh((1+I*a*x)/(a^2*x^2+1)^(1/2))/(a
^2*c*x^2+c)^(1/2)+2*I*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(2,-(1+I*a*x)/(
a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)-2*I*(a^2*x^2+1)^(1/2)*arctan(a*x)*po
lylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)-2*(a^2*x^2+1)^(1/
2)*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)+2*(a^2*x^2+
1)^(1/2)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.64

$$\int \frac{\arctan(ax)^2}{x\sqrt{c+a^2cx^2}}\,dx$$
$$= \frac{\sqrt{1+a^2x^2}\left(\arctan(ax)^2\log\left(1-e^{i\arctan(ax)}\right)-\arctan(ax)^2\log\left(1+e^{i\arctan(ax)}\right)+2i\arctan(ax)\operatorname{PolyLog}\right.}{\sqrt{}}$$

input
```
Integrate[ArcTan[a*x]^2/(x*Sqrt[c + a^2*c*x^2]),x]
```

output
```
(Sqrt[1 + a^2*x^2]*(ArcTan[a*x]^2*Log[1 - E^(I*ArcTan[a*x])] - ArcTan[a*x]
^2*Log[1 + E^(I*ArcTan[a*x])] + (2*I)*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[
a*x])] - (2*I)*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])] - 2*PolyLog[3, -E
^(I*ArcTan[a*x])] + 2*PolyLog[3, E^(I*ArcTan[a*x])]))/Sqrt[c*(1 + a^2*x^2)
]
```

## Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.56, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {5493, 5491, 3042, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}\,dx$$
$$\Big\downarrow \; 5493$$
$$\frac{\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}$$
$$\Big\downarrow \; 5491$$
$$\frac{\sqrt{a^2x^2+1}\int \frac{\sqrt{a^2x^2+1}\arctan(ax)^2}{ax}d\arctan(ax)}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc(\arctan(ax))d\arctan(ax)}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{4671}$$

$$\frac{\sqrt{a^2x^2+1}\left(-2\int \arctan(ax)\log\left(1-e^{i\arctan(ax)}\right)d\arctan(ax)+2\int \arctan(ax)\log\left(1+e^{i\arctan(ax)}\right)d\arctan(ax\right.}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3011}$$

$$\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-i\int \operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)d\arctan(ax)\right)-2\left(i\arctan(\right.\right.}{\sqrt{a^2cx}}$$

$$\Big\downarrow \text{2720}$$

$$\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)-2\right.}{}$$

$$\Big\downarrow \text{7143}$$

$$\frac{\sqrt{a^2x^2+1}\left(-2\arctan(ax)^2\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)+2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\operatorname{PolyLog}\left(3,-e^{i\operatorname{ar}}\right.\right.\right.}{\sqrt{a^2cx^2+c}}$$

input

```
Int[ArcTan[a*x]^2/(x*Sqrt[c + a^2*c*x^2]),x]
```

output

```
(Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])] + 2*(I*Arc
Tan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])] - PolyLog[3, -E^(I*ArcTan[a*x])])
- 2*(I*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])] - PolyLog[3, E^(I*ArcTan[
a*x])])))/Sqrt[c + a^2*c*x^2]
```

### Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5491
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Simp[1/Sqrt[d]   Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTa
n[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] &&
GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

## Maple [A] (verified)

Time = 2.17 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.87

| method | result |
|---|---|
| default | $\frac{\left(\arctan(ax)^2\ln\left(1-\frac{iax+1}{\sqrt{a^2x^2+1}}\right)-\arctan(ax)^2\ln\left(1+\frac{iax+1}{\sqrt{a^2x^2+1}}\right)-2i\arctan(ax)\operatorname{polylog}\left(2,\frac{iax+1}{\sqrt{a^2x^2+1}}\right)+2i\arctan(ax)\operatorname{polylog}\left(2,\right.\right.}{\sqrt{a^2x^2+1}\,c}$ |

input
```
int(arctan(a*x)^2/x/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-arctan(a*x)^2*ln(1+(1+I*a
*x)/(a^2*x^2+1)^(1/2))-2*I*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/
2))+2*I*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*polylog(3,(1
+I*a*x)/(a^2*x^2+1)^(1/2))-2*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2)))*(c*(
a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/2)/c
```

## Fricas [F]

$$\int \frac{\arctan(ax)^2}{x\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan\left(ax\right)^2}{\sqrt{a^2cx^2+c}x}\,dx$$

input
```
integrate(arctan(a*x)^2/x/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2/(a^2*c*x^3 + c*x), x)
```

## Sympy [F]

$$\int \frac{\arctan(ax)^2}{x\sqrt{c+a^2cx^2}}\,dx = \int \frac{\operatorname{atan}^2(ax)}{x\sqrt{c(a^2x^2+1)}}\,dx$$

input
```
integrate(atan(a*x)**2/x/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(atan(a*x)**2/(x*sqrt(c*(a**2*x**2 + 1))), x)
```

## Maxima [F]

$$\int \frac{\arctan(ax)^2}{x\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}\,x}\,dx$$

input
```
integrate(arctan(a*x)^2/x/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)^2/(sqrt(a^2*c*x^2 + c)*x), x)
```

## Giac [F]

$$\int \frac{\arctan(ax)^2}{x\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}\,x}\,dx$$

input
```
integrate(arctan(a*x)^2/x/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^2/(sqrt(a^2*c*x^2 + c)*x), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^2}{x\sqrt{c+a^2cx^2}}\,dx = \int \frac{\mathrm{atan}(a\,x)^2}{x\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(atan(a*x)^2/(x*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(atan(a*x)^2/(x*(c + a^2*c*x^2)^(1/2)), x)
```

## Reduce [F]

$$\int \frac{\arctan(ax)^2}{x\sqrt{c+a^2cx^2}}\,dx = \frac{\int \frac{atan(ax)^2}{\sqrt{a^2x^2+1}\,x}dx}{\sqrt{c}}$$

input
```
int(atan(a*x)^2/x/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(atan(a*x)**2/(sqrt(a**2*x**2 + 1)*x),x)/sqrt(c)
```

# 3.336 $\int \frac{\arctan(ax)^2}{x^2\sqrt{c+a^2cx^2}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 208

$$\int \frac{\arctan(ax)^2}{x^2\sqrt{c+a^2cx^2}}\,dx = -\frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{cx} - \frac{4a\sqrt{1+a^2x^2}\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} + \frac{2ia\sqrt{1+a^2x^2}\,\text{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} - \frac{2ia\sqrt{1+a^2x^2}\,\text{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}}$$

output

```
-(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2/c/x-4*a*(a^2*x^2+1)^(1/2)*arctan(a*x)*a
rctanh((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/(a^2*c*x^2+c)^(1/2)+2*I*a*(a^2*x^2
+1)^(1/2)*polylog(2,-(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/(a^2*c*x^2+c)^(1/2)-
2*I*a*(a^2*x^2+1)^(1/2)*polylog(2,(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/(a^2*c*
x^2+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.62

$$\int \frac{\arctan(ax)^2}{x^2\sqrt{c+a^2cx^2}}\,dx =$$
$$-\frac{a\sqrt{1+a^2x^2}\left(\arctan(ax)\left(\frac{\sqrt{1+a^2x^2}\arctan(ax)}{ax}-2\log\left(1-e^{i\arctan(ax)}\right)+2\log\left(1+e^{i\arctan(ax)}\right)\right)-2i\,\mathrm{Poly}\right.}{\sqrt{c\left(1+a^2x^2\right)}}$$

input
```
Integrate[ArcTan[a*x]^2/(x^2*Sqrt[c + a^2*c*x^2]),x]
```

output
```
-((a*Sqrt[1 + a^2*x^2]*(ArcTan[a*x]*((Sqrt[1 + a^2*x^2]*ArcTan[a*x])/(a*x)
 - 2*Log[1 - E^(I*ArcTan[a*x])] + 2*Log[1 + E^(I*ArcTan[a*x])]) - (2*I)*Po
lyLog[2, -E^(I*ArcTan[a*x])] + (2*I)*PolyLog[2, E^(I*ArcTan[a*x])]))/Sqrt[
c*(1 + a^2*x^2)])
```

## Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.75, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {5479, 5493, 5489}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}\,dx$$
$$\Big\downarrow \text{ 5479}$$
$$2a\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}$$
$$\Big\downarrow \text{ 5493}$$
$$\frac{2a\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}$$
$$\Big\downarrow \text{ 5489}$$

$$-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}+\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{PolyLog}\left(2,\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)\right)}{\sqrt{a^2cx^2+c}}$$

input
```
Int[ArcTan[a*x]^2/(x^2*Sqrt[c + a^2*c*x^2]),x]
```

output
```
-((Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(c*x)) + (2*a*Sqrt[1 + a^2*x^2]*(-2*
ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]] + I*PolyLog[2, -(Sqrt
[1 + I*a*x]/Sqrt[1 - I*a*x])] - I*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*
x]]))/Sqrt[c + a^2*c*x^2]
```

### Defintions of rubi rules used

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5489
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sq
rt[1 - I*c*x]], x] + (Simp[I*(b/Sqrt[d])*PolyLog[2, -Sqrt[1 + I*c*x]/Sqrt[1
 - I*c*x]], x] - Simp[I*(b/Sqrt[d])*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x]], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

## Maple [A] (verified)

Time = 2.19 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.82

| method | result |
|---|---|
| default | $-\frac{\arctan(ax)^2\sqrt{c(ax-i)(ax+i)}}{cx}-\frac{2ia\left(i\arctan(ax)\ln\left(1-\frac{iax+1}{\sqrt{a^2x^2+1}}\right)-i\arctan(ax)\ln\left(1+\frac{iax+1}{\sqrt{a^2x^2+1}}\right)+\text{polylog}\left(2,\frac{iax+1}{\sqrt{a^2x^2+1}}\right)\right.}{\sqrt{a^2x^2+1}\,c}$ |

input
```
int(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-arctan(a*x)^2*(c*(a*x-I)*(a*x+I))^(1/2)/c/x-2*I*a*(I*arctan(a*x)*ln(1-(1+
I*a*x)/(a^2*x^2+1)^(1/2))-I*arctan(a*x)*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+
polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1
/2)))*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/2)/c
```

## Fricas [F]

$$\int \frac{\arctan(ax)^2}{x^2\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan\left(ax\right)^2}{\sqrt{a^2cx^2+c}x^2}\,dx$$

input
```
integrate(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2/(a^2*c*x^4 + c*x^2), x)
```

## Sympy [F]

$$\int \frac{\arctan(ax)^2}{x^2\sqrt{c+a^2cx^2}}\,dx = \int \frac{\operatorname{atan}^2\left(ax\right)}{x^2\sqrt{c\left(a^2x^2+1\right)}}\,dx$$

input
```
integrate(atan(a*x)**2/x**2/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(atan(a*x)**2/(x**2*sqrt(c*(a**2*x**2 + 1))), x)
```

## Maxima [F]

$$\int \frac{\arctan(ax)^2}{x^2\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}x^2}\,dx$$

input
```
integrate(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)^2/(sqrt(a^2*c*x^2 + c)*x^2), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^2}{x^2\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^2}{x^2\sqrt{c+a^2cx^2}}\,dx = \int \frac{\mathrm{atan}(a\,x)^2}{x^2\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(atan(a*x)^2/(x^2*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(atan(a*x)^2/(x^2*(c + a^2*c*x^2)^(1/2)), x)
```

## Reduce [F]

$$\int \frac{\arctan(ax)^2}{x^2\sqrt{c+a^2cx^2}}\,dx = \frac{\int \frac{atan(ax)^2}{\sqrt{a^2x^2+1}\,x^2}dx}{\sqrt{c}}$$

input
```
int(atan(a*x)^2/x^2/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(atan(a*x)**2/(sqrt(a**2*x**2 + 1)*x**2),x)/sqrt(c)
```

# 3.337 $\int \frac{\arctan(ax)^2}{x^3\sqrt{c+a^2cx^2}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 328

$$\begin{aligned}\int \frac{\arctan(ax)^2}{x^3\sqrt{c+a^2cx^2}}\,dx = &-\frac{a\sqrt{c+a^2cx^2}\arctan(ax)}{cx}-\frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{2cx^2}\\&+\frac{a^2\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}\\&-\frac{a^2\operatorname{arctanh}\left(\frac{\sqrt{c+a^2cx^2}}{\sqrt{c}}\right)}{\sqrt{c}}\\&-\frac{ia^2\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}\\&+\frac{ia^2\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}\\&+\frac{a^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,-e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}\\&-\frac{a^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
-a*(a^2*c*x^2+c)^(1/2)*arctan(a*x)/c/x-1/2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)
^2/c/x^2+a^2*(a^2*x^2+1)^(1/2)*arctan(a*x)^2*arctanh((1+I*a*x)/(a^2*x^2+1)
^(1/2))/(a^2*c*x^2+c)^(1/2)-a^2*arctanh((a^2*c*x^2+c)^(1/2)/c^(1/2))/c^(1/
2)-I*a^2*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1
/2))/(a^2*c*x^2+c)^(1/2)+I*a^2*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(2,(1+
I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)+a^2*(a^2*x^2+1)^(1/2)*polylo
g(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)-a^2*(a^2*x^2+1)^(1/2
)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.70

$$\int \frac{\arctan(ax)^2}{x^3\sqrt{c+a^2cx^2}}\,dx = \frac{a^2\sqrt{1+a^2x^2}\left(-4\arctan(ax)\cot\left(\frac{1}{2}\arctan(ax)\right)-\arctan(ax)^2\csc^2\left(\frac{1}{2}\arctan(ax)\right)-4\arctan(ax)^2\log\left(\right.\right.}{}$$

input
```
Integrate[ArcTan[a*x]^2/(x^3*Sqrt[c + a^2*c*x^2]),x]
```

output
```
(a^2*Sqrt[1 + a^2*x^2]*(-4*ArcTan[a*x]*Cot[ArcTan[a*x]/2] - ArcTan[a*x]^2*
Csc[ArcTan[a*x]/2]^2 - 4*ArcTan[a*x]^2*Log[1 - E^(I*ArcTan[a*x])] + 4*ArcT
an[a*x]^2*Log[1 + E^(I*ArcTan[a*x])] + 8*Log[Tan[ArcTan[a*x]/2]] - (8*I)*A
rcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])] + (8*I)*ArcTan[a*x]*PolyLog[2, E
^(I*ArcTan[a*x])] + 8*PolyLog[3, -E^(I*ArcTan[a*x])] - 8*PolyLog[3, E^(I*A
rcTan[a*x])] + ArcTan[a*x]^2*Sec[ArcTan[a*x]/2]^2 - 4*ArcTan[a*x]*Tan[ArcT
an[a*x]/2]))/(8*Sqrt[c*(1 + a^2*x^2)])
```

## Rubi [A] (verified)

Time = 1.39 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.67, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5497, 5479, 243, 73, 221, 5493, 5491, 3042, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{ 5497}$$

$$-\frac{1}{2}a^2\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+a\int \frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{2cx^2}$$

$$\Big\downarrow \text{ 5479}$$

$$-\frac{1}{2}a^2\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+a\left(a\int \frac{1}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right)-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{2cx^2}$$

$$\Big\downarrow \text{ 243}$$

$$-\frac{1}{2}a^2\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+a\left(\frac{1}{2}a\int \frac{1}{x^2\sqrt{a^2cx^2+c}}dx^2-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right)-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{2cx^2}$$

$$\Big\downarrow \text{ 73}$$

$$-\frac{1}{2}a^2\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+a\left(\frac{\int \frac{1}{\frac{x^4}{a^2c}-\frac{1}{a^2}}d\sqrt{a^2cx^2+c}}{ac}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right)-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{2cx^2}$$

$$\Big\downarrow \text{ 221}$$

$$-\frac{1}{2}a^2\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+a\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{2cx^2}$$

$$\Big\downarrow \text{ 5493}$$

$$-\frac{a^2\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{x\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+a\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{2cx^2}$$

$$\Big\downarrow \text{ 5491}$$

$$-\frac{a^2\sqrt{a^2x^2+1}\int \frac{\sqrt{a^2x^2+1}\arctan(ax)^2}{ax}d\arctan(ax)}{2\sqrt{a^2cx^2+c}}+$$
$$a\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{2cx^2}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{a^2\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc(\arctan(ax))d\arctan(ax)}{2\sqrt{a^2cx^2+c}}+$$
$$a\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{2cx^2}$$

$$\Big\downarrow \text{ 4671}$$

$$-\frac{a^2\sqrt{a^2x^2+1}\left(-2\int \arctan(ax)\log\left(1-e^{i\arctan(ax)}\right)d\arctan(ax)+2\int \arctan(ax)\log\left(1+e^{i\arctan(ax)}\right)d\arctan\right.}{2\sqrt{a^2cx^2+c}}$$
$$a\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{2cx^2}$$

$$\Big\downarrow \text{ 3011}$$

$$-\frac{a^2\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-i\int \operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)d\arctan(ax)\right)-2\left(i\,\text{arc}\right.\right.}{2\sqrt{a}}$$
$$a\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{2cx^2}$$

$$\Big\downarrow \text{ 2720}$$

$$-\frac{a^2\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right.\right.}{}$$
$$a\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{2cx^2}$$

$$\Big\downarrow \text{ 7143}$$

$$-\frac{a^2\sqrt{a^2x^2+1}\left(-2\arctan(ax)^2\text{arctanh}\left(e^{i\arctan(ax)}\right)+2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\operatorname{PolyLog}\left(3,-\right.\right.\right.}{2\sqrt{a^2cx^2+c}}$$

$$a\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{2cx^2}$$

input
```
Int[ArcTan[a*x]^2/(x^3*Sqrt[c + a^2*c*x^2]),x]
```

output
```
-1/2*(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(c*x^2) + a*(-((Sqrt[c + a^2*c*x^
2]*ArcTan[a*x])/(c*x)) - (a*ArcTanh[Sqrt[c + a^2*c*x^2]/Sqrt[c]])/Sqrt[c])
 - (a^2*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])] + 2
*(I*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])] - PolyLog[3, -E^(I*ArcTan[a
*x])]) - 2*(I*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])] - PolyLog[3, E^(I*
ArcTan[a*x])])))/(2*Sqrt[c + a^2*c*x^2])
```

## Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5491
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Simp[1/Sqrt[d]   Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTa
n[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] &&
GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5497
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)^(m
 + 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[c^2*((m
+ 2)/(f^2*(m + 1)))   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x
^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
 && LtQ[m, -1] && NeQ[m, -2]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

## Maple [A] (verified)

Time = 2.40 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.80

| method | result |
|---|---|
| default | $-\frac{(2ax+\arctan(ax))\arctan(ax)\sqrt{c(ax-i)(ax+i)}}{2c\,x^2}+\frac{a^2\left(\arctan(ax)^2\ln\left(1+\frac{iax+1}{\sqrt{a^2x^2+1}}\right)-\arctan(ax)^2\ln\left(1-\frac{iax+1}{\sqrt{a^2x^2+1}}\right)-2i\,\mathrm{arct}\right.}{}$ |

input
```
int(arctan(a*x)^2/x^3/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(2*a*x+arctan(a*x))*arctan(a*x)*(c*(a*x-I)*(a*x+I))^(1/2)/c/x^2+1/2*a
^2*(arctan(a*x)^2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-arctan(a*x)^2*ln(1-(1+
I*a*x)/(a^2*x^2+1)^(1/2))-2*I*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)
^(1/2))+2*I*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*polylog(3
,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))-4*
arctanh((1+I*a*x)/(a^2*x^2+1)^(1/2)))*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1
)^(1/2)/c
```

## Fricas [F]

$$\int \frac{\arctan(ax)^2}{x^3\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan\left(ax\right)^2}{\sqrt{a^2cx^2+c}x^3}\,dx$$

input
```
integrate(arctan(a*x)^2/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2/(a^2*c*x^5 + c*x^3), x)
```

## Sympy [F]

$$\int \frac{\arctan(ax)^2}{x^3\sqrt{c+a^2cx^2}}\,dx = \int \frac{\operatorname{atan}^2\left(ax\right)}{x^3\sqrt{c\left(a^2x^2+1\right)}}\,dx$$

input
```
integrate(atan(a*x)**2/x**3/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(atan(a*x)**2/(x**3*sqrt(c*(a**2*x**2 + 1))), x)
```

## Maxima [F]

$$\int \frac{\arctan(ax)^2}{x^3\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan\left(ax\right)^2}{\sqrt{a^2cx^2+c}x^3}\,dx$$

input
```
integrate(arctan(a*x)^2/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)^2/(sqrt(a^2*c*x^2 + c)*x^3), x)
```

## Giac [F]

$$\int \frac{\arctan(ax)^2}{x^3\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}\,x^3}\,dx$$

input
```
integrate(arctan(a*x)^2/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^2/(sqrt(a^2*c*x^2 + c)*x^3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^2}{x^3\sqrt{c+a^2cx^2}}\,dx = \int \frac{\mathrm{atan}(a\,x)^2}{x^3\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(atan(a*x)^2/(x^3*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(atan(a*x)^2/(x^3*(c + a^2*c*x^2)^(1/2)), x)
```

## Reduce [F]

$$\int \frac{\arctan(ax)^2}{x^3\sqrt{c+a^2cx^2}}\,dx = \frac{\int \frac{atan(ax)^2}{\sqrt{a^2x^2+1}\,x^3}dx}{\sqrt{c}}$$

input
```
int(atan(a*x)^2/x^3/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(atan(a*x)**2/(sqrt(a**2*x**2 + 1)*x**3),x)/sqrt(c)
```

# 3.338 $\int \frac{\arctan(ax)^2}{x^4\sqrt{c+a^2cx^2}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 311

$$\int \frac{\arctan(ax)^2}{x^4\sqrt{c+a^2cx^2}}\,dx = -\frac{a^2\sqrt{c+a^2cx^2}}{3cx}-\frac{a\sqrt{c+a^2cx^2}\arctan(ax)}{3cx^2}-\frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{3cx^3}+\frac{2a^2\sqrt{c+a^2cx^2}\arctan(ax)^2}{3cx}+\frac{10a^3\sqrt{1+a^2x^2}\arctan(ax)\operatorname{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3\sqrt{c+a^2cx^2}}-\frac{5ia^3\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3\sqrt{c+a^2cx^2}}+\frac{5ia^3\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3\sqrt{c+a^2cx^2}}$$

output
```
-1/3*a^2*(a^2*c*x^2+c)^(1/2)/c/x-1/3*a*(a^2*c*x^2+c)^(1/2)*arctan(a*x)/c/x
^2-1/3*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2/c/x^3+2/3*a^2*(a^2*c*x^2+c)^(1/2)
*arctan(a*x)^2/c/x+10/3*a^3*(a^2*x^2+1)^(1/2)*arctan(a*x)*arctanh((1+I*a*x
)^(1/2)/(1-I*a*x)^(1/2))/(a^2*c*x^2+c)^(1/2)-5/3*I*a^3*(a^2*x^2+1)^(1/2)*p
olylog(2,-(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/(a^2*c*x^2+c)^(1/2)+5/3*I*a^3*(
a^2*x^2+1)^(1/2)*polylog(2,(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/(a^2*c*x^2+c)^
(1/2)
```

## Mathematica [A] (warning: unable to verify)

Time = 1.91 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.73

$$\int \frac{\arctan(ax)^2}{x^4\sqrt{c+a^2cx^2}}\,dx$$
$$= \frac{a^3\sqrt{c+a^2cx^2}\left(-20i\,\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)+\frac{(1+a^2x^2)^{3/2}\left(\arctan(ax)^2(2-6\cos(2\arctan(ax)))+2(-1+\cos(2\arctan(a\right.}{}\right.}{1}$$

input
```
Integrate[ArcTan[a*x]^2/(x^4*Sqrt[c + a^2*c*x^2]),x]
```

output
```
(a^3*Sqrt[c + a^2*c*x^2]*((-20*I)*PolyLog[2, -E^(I*ArcTan[a*x])] + ((1 + a
^2*x^2)^(3/2)*(ArcTan[a*x]^2*(2 - 6*Cos[2*ArcTan[a*x]]) + 2*(-1 + Cos[2*Ar
cTan[a*x]]) + ((20*I)*a^3*x^3*PolyLog[2, E^(I*ArcTan[a*x])])/(1 + a^2*x^2)
^(3/2) + ArcTan[a*x]*(-2*Sin[2*ArcTan[a*x]] + (5*(Log[1 - E^(I*ArcTan[a*x]
)] - Log[1 + E^(I*ArcTan[a*x])])*(-3*a*x + Sqrt[1 + a^2*x^2]*Sin[3*ArcTan[
a*x]]))/Sqrt[1 + a^2*x^2])))/(a^3*x^3)))/(12*c*Sqrt[1 + a^2*x^2])
```

## Rubi [A] (verified)

Time = 1.62 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.23, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5497, 5479, 5493, 5489, 5497, 242, 5493, 5489}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2}{x^4\sqrt{a^2cx^2+c}}\,dx$$

$$\Big\downarrow \text{5497}$$

$$-\frac{2}{3}a^2\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx+\frac{2}{3}a\int \frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{3cx^3}$$

$$\Big\downarrow \text{5479}$$

$$-\frac{2}{3}a^2\left(2a\int\frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}\right)+\frac{2}{3}a\int\frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{3cx^3}$$

$\downarrow$ 5493

$$-\frac{2}{3}a^2\left(\frac{2a\sqrt{a^2x^2+1}\int\frac{\arctan(ax)}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}\right)+\frac{2}{3}a\int\frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{3cx^3}$$

$\downarrow$ 5489

$$\frac{2}{3}a\int\frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx-\frac{2}{3}a^2\left(-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}+\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\right.}{\sqrt{a^2cx^2+c}}\right.$$
$$-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{3cx^3}$$

$\downarrow$ 5497

$$\frac{2}{3}a\left(-\frac{1}{2}a^2\int\frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx+\frac{1}{2}a\int\frac{1}{x^2\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{2cx^2}\right)-$$
$$\frac{2}{3}a^2\left(-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}+\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\right.}{\sqrt{a^2cx^2+c}}\right.$$
$$-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{3cx^3}$$

$\downarrow$ 242

$$\frac{2}{3}a\left(-\frac{1}{2}a^2\int\frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{2cx^2}-\frac{a\sqrt{a^2cx^2+c}}{2cx}\right)-$$
$$\frac{2}{3}a^2\left(-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}+\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\right.}{\sqrt{a^2cx^2+c}}\right.$$
$$-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{3cx^3}$$

$\downarrow$ 5493

$$\frac{2}{3}a\left(-\frac{a^2\sqrt{a^2x^2+1}\int\frac{\arctan(ax)}{x\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{2cx^2}-\frac{a\sqrt{a^2cx^2+c}}{2cx}\right)-$$
$$\frac{2}{3}a^2\left(-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}+\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\right.}{\sqrt{a^2cx^2+c}}\right.$$
$$\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{3cx^3}$$

$$\Big\downarrow \text{5489}$$

$$-\frac{2}{3}a^2\left(-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}+\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-\right.}{\sqrt{a^2cx^2+c}}\right.$$
$$\frac{2}{3}a\left(-\frac{a^2\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{PolyLog}\left(2,\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)\right)}{2\sqrt{a^2cx^2+c}}-\text{ar}\right.$$
$$\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{3cx^3}$$

input
```
Int[ArcTan[a*x]^2/(x^4*Sqrt[c + a^2*c*x^2]),x]
```

output
```
-1/3*(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(c*x^3) - (2*a^2*(-((Sqrt[c + a^2
*c*x^2]*ArcTan[a*x]^2)/(c*x)) + (2*a*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]*Arc
Tanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]] + I*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqr
t[1 - I*a*x])] - I*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]]))/Sqrt[c +
a^2*c*x^2]))/3 + (2*a*(-1/2*(a*Sqrt[c + a^2*c*x^2])/(c*x) - (Sqrt[c + a^2*
c*x^2]*ArcTan[a*x])/(2*c*x^2) - (a^2*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]*Arc
Tanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]] + I*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqr
t[1 - I*a*x])] - I*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]]))/(2*Sqrt[c
 + a^2*c*x^2])))/3
```

### Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5489
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sq
rt[1 - I*c*x]], x] + (Simp[I*(b/Sqrt[d])*PolyLog[2, -Sqrt[1 + I*c*x]/Sqrt[1
 - I*c*x]], x] - Simp[I*(b/Sqrt[d])*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x]], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5497
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)^(m
 + 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[c^2*((m
+ 2)/(f^2*(m + 1)))   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x
^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
 && LtQ[m, -1] && NeQ[m, -2]
```

## Maple [A] (verified)

Time = 2.26 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.66

| method | result |
|---|---|
| default | $\frac{\left(2 \arctan(ax)^2 x^2 a^2 - a^2 x^2 - \arctan(ax) ax - \arctan(ax)^2\right)\sqrt{c(ax-i)(ax+i)}}{3c\,x^3} + \frac{5ia^3\left(i \arctan(ax) \ln\left(1-\frac{iax+1}{\sqrt{a^2x^2+1}}\right) - i \arctan(ax)\right.}{}$ |

input
```
int(arctan(a*x)^2/x^4/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(2*arctan(a*x)^2*x^2*a^2-a^2*x^2-arctan(a*x)*a*x-arctan(a*x)^2)*(c*(a*
x-I)*(a*x+I))^(1/2)/c/x^3+5/3*I*a^3*(I*arctan(a*x)*ln(1-(1+I*a*x)/(a^2*x^2
+1)^(1/2))-I*arctan(a*x)*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+polylog(2,(1+I*
a*x)/(a^2*x^2+1)^(1/2))-polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2)))*(c*(a*x-I
)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/2)/c
```

## Fricas [F]

$$\int \frac{\arctan(ax)^2}{x^4\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+cx^4}}\,dx$$

input
```
integrate(arctan(a*x)^2/x^4/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2/(a^2*c*x^6 + c*x^4), x)
```

## Sympy [F]

$$\int \frac{\arctan(ax)^2}{x^4\sqrt{c+a^2cx^2}}\,dx = \int \frac{\operatorname{atan}^2(ax)}{x^4\sqrt{c(a^2x^2+1)}}\,dx$$

input
```
integrate(atan(a*x)**2/x**4/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(atan(a*x)**2/(x**4*sqrt(c*(a**2*x**2 + 1))), x)
```

## Maxima [F]

$$\int \frac{\arctan(ax)^2}{x^4\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+cx^4}}\,dx$$

input
```
integrate(arctan(a*x)^2/x^4/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)^2/(sqrt(a^2*c*x^2 + c)*x^4), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^2}{x^4\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^2/x^4/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^2}{x^4\sqrt{c+a^2cx^2}}\,dx = \int \frac{\mathrm{atan}(a\,x)^2}{x^4\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(atan(a*x)^2/(x^4*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(atan(a*x)^2/(x^4*(c + a^2*c*x^2)^(1/2)), x)
```

## Reduce [F]

$$\int \frac{\arctan(ax)^2}{x^4\sqrt{c+a^2cx^2}}\,dx = \frac{\int \frac{atan(ax)^2}{\sqrt{a^2x^2+1}\,x^4}dx}{\sqrt{c}}$$

input
```
int(atan(a*x)^2/x^4/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(atan(a*x)**2/(sqrt(a**2*x**2 + 1)*x**4),x)/sqrt(c)
```

# 3.339 $\int \frac{x^3 \arctan(ax)^2}{(c+a^2cx^2)^{3/2}} \, dx$



## Optimal result

Integrand size = 24, antiderivative size = 305

$$\begin{aligned}\int \frac{x^3 \arctan(ax)^2}{(c+a^2cx^2)^{3/2}} \, dx = & -\frac{2}{a^4c\sqrt{c+a^2cx^2}} - \frac{2x\arctan(ax)}{a^3c\sqrt{c+a^2cx^2}} + \frac{\arctan(ax)^2}{a^4c\sqrt{c+a^2cx^2}} \\ & + \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{a^4c^2} + \frac{4i\sqrt{1+a^2x^2}\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a^4c\sqrt{c+a^2cx^2}} \\ & - \frac{2i\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a^4c\sqrt{c+a^2cx^2}} + \frac{2i\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a^4c\sqrt{c+a^2cx^2}}\end{aligned}$$

output

```
-2/a^4/c/(a^2*c*x^2+c)^(1/2)-2*x*arctan(a*x)/a^3/c/(a^2*c*x^2+c)^(1/2)+arc
tan(a*x)^2/a^4/c/(a^2*c*x^2+c)^(1/2)+(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2/a^4
/c^2+4*I*(a^2*x^2+1)^(1/2)*arctan(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*x)^(1
/2))/a^4/c/(a^2*c*x^2+c)^(1/2)-2*I*(a^2*x^2+1)^(1/2)*polylog(2,-I*(1+I*a*x
)^(1/2)/(1-I*a*x)^(1/2))/a^4/c/(a^2*c*x^2+c)^(1/2)+2*I*(a^2*x^2+1)^(1/2)*p
olylog(2,I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/a^4/c/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.69

$$\int \frac{x^3 \arctan(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{c(1+a^2x^2)}\left(-2+3\arctan(ax)^2-2\cos(2\arctan(ax))+\arctan(ax)^2\cos(2\arct\right.}{}$$

input
```
Integrate[(x^3*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(3/2),x]
```

output
```
(Sqrt[c*(1 + a^2*x^2)]*(-2 + 3*ArcTan[a*x]^2 - 2*Cos[2*ArcTan[a*x]] + ArcT
an[a*x]^2*Cos[2*ArcTan[a*x]] - (4*ArcTan[a*x]*Log[1 - I*E^(I*ArcTan[a*x])]
)/Sqrt[1 + a^2*x^2] + (4*ArcTan[a*x]*Log[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1
+ a^2*x^2] - ((4*I)*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2]
+ ((4*I)*PolyLog[2, I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - 2*ArcTan[a*x
]*Sin[2*ArcTan[a*x]]))/(2*a^4*c^2)
```

## Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {5499, 5465, 5425, 5421, 5429}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \arctan(ax)^2}{(a^2cx^2+c)^{3/2}}\,dx$$

$$\Big\downarrow \text{5499}$$

$$\frac{\int \frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a^2c}-\frac{\int \frac{x\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{a^2}$$

$$\Big\downarrow \text{5465}$$

$$\frac{\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}}{a^2c}-\frac{\frac{2\int \frac{\arctan(ax)}{(a^2cx^2+c)^{3/2}}dx}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}}{a^2}$$

$$\Big\downarrow \text{5425}$$

$$\frac{\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\int\frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{a\sqrt{a^2cx^2+c}}}{a^2c}-\frac{\frac{2\int\frac{\arctan(ax)}{\left(a^2cx^2+c\right)^{3/2}}dx}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}}{a^2}$$

$$\Big\downarrow \text{5421}$$

$$-\frac{\frac{2\int\frac{\arctan(ax)}{\left(a^2cx^2+c\right)^{3/2}}dx}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}}{a^2}+\frac{\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{a\sqrt{a^2cx^2+c}}}{a^2c}$$

$$\Big\downarrow \text{5429}$$

$$-\frac{\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}}{a^2}+\frac{\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{a\sqrt{a^2cx^2+c}}}{a^2c}$$

input
```
Int[(x^3*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(3/2),x]
```

output
```
-((-(ArcTan[a*x]^2/(a^2*c*Sqrt[c + a^2*c*x^2])) + (2*(1/(a*c*Sqrt[c + a^2*
c*x^2]) + (x*ArcTan[a*x])/(c*Sqrt[c + a^2*c*x^2])))/a)/a^2) + ((Sqrt[c + a
^2*c*x^2]*ArcTan[a*x]^2)/(a^2*c) - (2*Sqrt[1 + a^2*x^2]*(((-2*I)*ArcTan[a*
x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/a + (I*PolyLog[2, ((-I)*Sqrt[1
 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1
- I*a*x]])/a))/(a*Sqrt[c + a^2*c*x^2]))/(a^2*c)
```

### Defintions of rubi rules used

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5429
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTan[c*x])/(d*Sqr
t[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5499
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ
[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

## Maple [A] (verified)

Time = 2.13 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.96

| method | result |
|---|---|
| default | $\frac{\left(\arctan(ax)^2-2+2i\arctan(ax)\right)(iax+1)\sqrt{c(ax-i)(ax+i)}}{2(a^2x^2+1)a^4c^2}-\frac{\sqrt{c(ax-i)(ax+i)}\,(iax-1)\left(\arctan(ax)^2-2-2i\arctan(ax)\right)}{2(a^2x^2+1)a^4c^2}+$ arc |

input
```
int(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(arctan(a*x)^2-2+2*I*arctan(a*x))*(1+I*a*x)*(c*(a*x-I)*(a*x+I))^(1/2)/
(a^2*x^2+1)/a^4/c^2-1/2*(c*(a*x-I)*(a*x+I))^(1/2)*(I*a*x-1)*(arctan(a*x)^2
-2-2*I*arctan(a*x))/(a^2*x^2+1)/a^4/c^2+arctan(a*x)^2*(c*(a*x-I)*(a*x+I))^
(1/2)/a^4/c^2+2*(arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-arctan(a*
x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+I*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(
1/2))-I*dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))*(c*(a*x-I)*(a*x+I))^(1/2)/
(a^2*x^2+1)^(1/2)/a^4/c^2
```

## Fricas [F]

$$\int \frac{x^3\arctan(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx=\int \frac{x^3\arctan\left(ax\right)^2}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^3*arctan(a*x)^2/(a^4*c^2*x^4 + 2*a^2*c^2*x^
2 + c^2), x)
```

## Sympy [F]

$$\int \frac{x^3 \arctan(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^3 \operatorname{atan}^2(ax)}{(c(a^2x^2+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3*atan(a*x)**2/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(x**3*atan(a*x)**2/(c*(a**2*x**2 + 1))**(3/2), x)
```

## Maxima [F]

$$\int \frac{x^3 \arctan(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^3 \arctan(ax)^2}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^3*arctan(a*x)^2/(a^2*c*x^2 + c)^(3/2), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x^3 \arctan(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \arctan(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^3\,\mathrm{atan}(a\,x)^2}{(c\,a^2\,x^2+c)^{3/2}}\,dx$$

input
```
int((x^3*atan(a*x)^2)/(c + a^2*c*x^2)^(3/2),x)
```

output
```
int((x^3*atan(a*x)^2)/(c + a^2*c*x^2)^(3/2), x)
```

## Reduce [F]

$$\int \frac{x^3 \arctan(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\int \frac{atan(ax)^2x^3}{\sqrt{a^2x^2+1}\,a^2x^2+\sqrt{a^2x^2+1}}dx}{\sqrt{c}\,c}$$

input
```
int(x^3*atan(a*x)^2/(a^2*c*x^2+c)^(3/2),x)
```

output
```
int((atan(a*x)**2*x**3)/(sqrt(a**2*x**2 + 1)*a**2*x**2 + sqrt(a**2*x**2 +
1)),x)/(sqrt(c)*c)
```

# 3.340 $\int \frac{x^2\arctan(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 349

$$\begin{aligned}\int \frac{x^2\arctan(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx &= \frac{2x}{a^2c\sqrt{c+a^2cx^2}}-\frac{2\arctan(ax)}{a^3c\sqrt{c+a^2cx^2}}\\&-\frac{x\arctan(ax)^2}{a^2c\sqrt{c+a^2cx^2}}-\frac{2i\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2}{a^3c\sqrt{c+a^2cx^2}}\\&+\frac{2i\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)}{a^3c\sqrt{c+a^2cx^2}}\\&-\frac{2i\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,ie^{i\arctan(ax)}\right)}{a^3c\sqrt{c+a^2cx^2}}\\&-\frac{2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)}{a^3c\sqrt{c+a^2cx^2}}+\frac{2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,ie^{i\arctan(ax)}\right)}{a^3c\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
2*x/a^2/c/(a^2*c*x^2+c)^(1/2)-2*arctan(a*x)/a^3/c/(a^2*c*x^2+c)^(1/2)-x*ar
ctan(a*x)^2/a^2/c/(a^2*c*x^2+c)^(1/2)-2*I*(a^2*x^2+1)^(1/2)*arctan((1+I*a*
x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^2/a^3/c/(a^2*c*x^2+c)^(1/2)+2*I*(a^2*x^2
+1)^(1/2)*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a^3/c/(a^2
*c*x^2+c)^(1/2)-2*I*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a
^2*x^2+1)^(1/2))/a^3/c/(a^2*c*x^2+c)^(1/2)-2*(a^2*x^2+1)^(1/2)*polylog(3,-
I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a^3/c/(a^2*c*x^2+c)^(1/2)+2*(a^2*x^2+1)^(1/
2)*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a^3/c/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.65

$$\int \frac{x^2 \arctan(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx =$$

$$-\frac{\sqrt{1+a^2x^2}\left(-\frac{2ax}{\sqrt{1+a^2x^2}}+\frac{2\arctan(ax)}{\sqrt{1+a^2x^2}}+\frac{ax\arctan(ax)^2}{\sqrt{1+a^2x^2}}-\arctan(ax)^2\log\left(1-ie^{i\arctan(ax)}\right)+\arctan(ax)^2\log\left(\right.\right.}{}$$

input
```
Integrate[(x^2*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(3/2),x]
```

output
```
-((Sqrt[1 + a^2*x^2]*((-2*a*x)/Sqrt[1 + a^2*x^2] + (2*ArcTan[a*x])/Sqrt[1
+ a^2*x^2] + (a*x*ArcTan[a*x]^2)/Sqrt[1 + a^2*x^2] - ArcTan[a*x]^2*Log[1 -
 I*E^(I*ArcTan[a*x])] + ArcTan[a*x]^2*Log[1 + I*E^(I*ArcTan[a*x])] - (2*I)
*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + (2*I)*ArcTan[a*x]*PolyLo
g[2, I*E^(I*ArcTan[a*x])] + 2*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] - 2*PolyL
og[3, I*E^(I*ArcTan[a*x])]))/(a^3*c*Sqrt[c*(1 + a^2*x^2)]))
```

## Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.64, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5499, 5425, 5423, 3042, 4669, 3011, 2720, 5433, 208, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \arctan(ax)^2}{(a^2cx^2+c)^{3/2}}\,dx$$

$$\Big\downarrow \text{5499}$$

$$\frac{\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a^2c}-\frac{\int \frac{\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{a^2}$$

$$\Big\downarrow \text{5425}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{a^2c\sqrt{a^2cx^2+c}}-\frac{\int \frac{\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{a^2}$$

$\downarrow$ 5423

$$\frac{\sqrt{a^2x^2+1}\int \sqrt{a^2x^2+1}\arctan(ax)^2d\arctan(ax)}{a^3c\sqrt{a^2cx^2+c}}-\frac{\int \frac{\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{a^2}$$

$\downarrow$ 3042

$$\frac{\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{a^3c\sqrt{a^2cx^2+c}}-\frac{\int \frac{\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{a^2}$$

$\downarrow$ 4669

$$-\frac{\int \frac{\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{a^2}+\frac{\sqrt{a^2x^2+1}\left(-2\int \arctan(ax)\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+2\int \arctan(ax)\log\left(1+ie^{i\arctan(ax)}\right)d\arctan(a\right.}{a^3c\sqrt{a^2cx^2+c}}$$

$\downarrow$ 3011

$$-\frac{\int \frac{\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{a^2}+\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-i\int \operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(ax)\right)-2\left(i\arctan\right.\right.}{a^3c\sqrt{a}}$$

$\downarrow$ 2720

$$-\frac{\int \frac{\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{a^2}+\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)-\right.}{}$$

$\downarrow$ 5433

$$-\frac{-2\int \frac{1}{(a^2cx^2+c)^{3/2}}dx+\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}}{a^2}+\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)-\right.}{}$$

$\downarrow$ 208

$$-\frac{\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}}{a^2}+\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)-\right.}{}$$

$\big\downarrow$ 7143

$$-\frac{\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}}{a^2}+\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)\right)-2\left(i\arctan(ax)\operatorname{PolyLog}\left(2\right.\right.\right.}{a^3c\sqrt{a^2cx^2+c}}$$

input
```
Int[(x^2*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(3/2),x]
```

output
```
-(((-2*x)/(c*Sqrt[c + a^2*c*x^2]) + (2*ArcTan[a*x])/(a*c*Sqrt[c + a^2*c*x^
2]) + (x*ArcTan[a*x]^2)/(c*Sqrt[c + a^2*c*x^2]))/a^2) + (Sqrt[1 + a^2*x^2]
*((-2*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 + 2*(I*ArcTan[a*x]*PolyLo
g[2, (-I)*E^(I*ArcTan[a*x])] - PolyLog[3, (-I)*E^(I*ArcTan[a*x])]) - 2*(I*
ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] - PolyLog[3, I*E^(I*ArcTan[a*x
])])))/(a^3*c*Sqrt[c + a^2*c*x^2])
```

## Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5433
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x] - Simp[b^2*p*(p
- 1)   Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2), x], x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

rule 5499
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ
[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

## Maple [F]

$$\int \frac{x^2 \arctan(ax)^2}{(a^2 c\, x^2 + c)^{\frac{3}{2}}}dx$$

input
```
int(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x)
```

output
```
int(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x)
```

## Fricas [F]

$$\int \frac{x^2 \arctan(ax)^2}{(c + a^2cx^2)^{3/2}}\, dx = \int \frac{x^2 \arctan(ax)^2}{(a^2cx^2 + c)^{\frac{3}{2}}}\, dx$$

input
```
integrate(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^2*arctan(a*x)^2/(a^4*c^2*x^4 + 2*a^2*c^2*x^
2 + c^2), x)
```

## Sympy [F]

$$\int \frac{x^2 \arctan(ax)^2}{(c + a^2cx^2)^{3/2}} \, dx = \int \frac{x^2 \operatorname{atan}^2(ax)}{(c(a^2x^2 + 1))^{\frac{3}{2}}} \, dx$$

input
```
integrate(x**2*atan(a*x)**2/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(x**2*atan(a*x)**2/(c*(a**2*x**2 + 1))**(3/2), x)
```

## Maxima [F]

$$\int \frac{x^2 \arctan(ax)^2}{(c + a^2cx^2)^{3/2}} \, dx = \int \frac{x^2 \arctan(ax)^2}{(a^2cx^2 + c)^{\frac{3}{2}}} \, dx$$

input
```
integrate(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^2*arctan(a*x)^2/(a^2*c*x^2 + c)^(3/2), x)
```

## Giac [F]

$$\int \frac{x^2 \arctan(ax)^2}{(c + a^2cx^2)^{3/2}} \, dx = \int \frac{x^2 \arctan(ax)^2}{(a^2cx^2 + c)^{\frac{3}{2}}} \, dx$$

input
```
integrate(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^2*arctan(a*x)^2/(a^2*c*x^2 + c)^(3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \arctan(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^2\,\mathrm{atan}(a\,x)^2}{(c\,a^2\,x^2+c)^{3/2}}\,dx$$

input
```
int((x^2*atan(a*x)^2)/(c + a^2*c*x^2)^(3/2),x)
```

output
```
int((x^2*atan(a*x)^2)/(c + a^2*c*x^2)^(3/2), x)
```

## Reduce [F]

$$\int \frac{x^2 \arctan(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\int \frac{atan(ax)^2x^2}{\sqrt{a^2x^2+1}\,a^2x^2+\sqrt{a^2x^2+1}}dx}{\sqrt{c}\,c}$$

input
```
int(x^2*atan(a*x)^2/(a^2*c*x^2+c)^(3/2),x)
```

output
```
int((atan(a*x)**2*x**2)/(sqrt(a**2*x**2 + 1)*a**2*x**2 + sqrt(a**2*x**2 +
1)),x)/(sqrt(c)*c)
```

# 3.341 $\int \frac{x \arctan(ax)^2}{(c+a^2cx^2)^{3/2}} \, dx$



## Optimal result

Integrand size = 22, antiderivative size = 78

$$\int \frac{x \arctan(ax)^2}{(c+a^2cx^2)^{3/2}} \, dx = \frac{2}{a^2c\sqrt{c+a^2cx^2}} + \frac{2x\arctan(ax)}{ac\sqrt{c+a^2cx^2}} - \frac{\arctan(ax)^2}{a^2c\sqrt{c+a^2cx^2}}$$

output
```
2/a^2/c/(a^2*c*x^2+c)^(1/2)+2*x*arctan(a*x)/a/c/(a^2*c*x^2+c)^(1/2)-arctan
(a*x)^2/a^2/c/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.64

$$\int \frac{x \arctan(ax)^2}{(c+a^2cx^2)^{3/2}} \, dx = \frac{\sqrt{c+a^2cx^2}(2+2ax\arctan(ax)-\arctan(ax)^2)}{a^2c^2(1+a^2x^2)}$$

input
```
Integrate[(x*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(3/2),x]
```

output
```
(Sqrt[c + a^2*c*x^2]*(2 + 2*a*x*ArcTan[a*x] - ArcTan[a*x]^2))/(a^2*c^2*(1
 + a^2*x^2))
```

## Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.01, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {5465, 5429}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \arctan(ax)^2}{(a^2cx^2+c)^{3/2}} dx$$

$$\Big\downarrow \ 5465$$

$$\frac{2\int \frac{\arctan(ax)}{(a^2cx^2+c)^{3/2}}dx}{a} - \frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \ 5429$$

$$\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}} + \frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a} - \frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}$$

input
```
Int[(x*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(3/2),x]
```

output
```
-(ArcTan[a*x]^2/(a^2*c*Sqrt[c + a^2*c*x^2])) + (2*(1/(a*c*Sqrt[c + a^2*c*x
^2]) + (x*ArcTan[a*x])/(c*Sqrt[c + a^2*c*x^2])))/a
```

### Defintions of rubi rules used

rule 5429
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTan[c*x])/(d*Sqr
t[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.72 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.49

| method | result |
|---|---|
| default | $-\frac{\left(\arctan(ax)^2-2+2i\arctan(ax)\right)(iax+1)\sqrt{c(ax-i)(ax+i)}}{2(a^2x^2+1)a^2c^2}+\frac{\sqrt{c(ax-i)(ax+i)}\,(iax-1)\left(\arctan(ax)^2-2-2i\arctan(ax)\right)}{2(a^2x^2+1)a^2c^2}$ |
| orering | $\frac{(a^2x^2+1)(8a^4x^4-3a^2x^2+2)\arctan(ax)^2}{a^4x^2(a^2cx^2+c)^{\frac{3}{2}}}+\frac{(a^2x^2+1)^2(7a^2x^2-2)\left(\frac{\arctan(ax)^2}{(a^2cx^2+c)^{\frac{3}{2}}}+\frac{2x\arctan(ax)a}{(a^2cx^2+c)^{\frac{3}{2}}(a^2x^2+1)}-\frac{3x^2\arctan(ax)^2ca^2}{(a^2cx^2+c)^{\frac{5}{2}}}\right)}{x^2a^4}$ |

input
```
int(x*arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(arctan(a*x)^2-2+2*I*arctan(a*x))*(1+I*a*x)*(c*(a*x-I)*(a*x+I))^(1/2)
/(a^2*x^2+1)/a^2/c^2+1/2*(c*(a*x-I)*(a*x+I))^(1/2)*(I*a*x-1)*(arctan(a*x)^
2-2-2*I*arctan(a*x))/(a^2*x^2+1)/a^2/c^2
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.65

$$\int \frac{x\arctan(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx=\frac{\sqrt{a^2cx^2+c}\left(2\,ax\arctan\left(ax\right)-\arctan\left(ax\right)^2+2\right)}{a^4c^2x^2+a^2c^2}$$

input
```
integrate(x*arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
sqrt(a^2*c*x^2 + c)*(2*a*x*arctan(a*x) - arctan(a*x)^2 + 2)/(a^4*c^2*x^2 +
 a^2*c^2)
```

## Sympy [F]

$$\int \frac{x\arctan(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x\,\operatorname{atan}^2(ax)}{\left(c\,(a^2x^2+1)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*atan(a*x)**2/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(x*atan(a*x)**2/(c*(a**2*x**2 + 1))**(3/2), x)
```

## Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.94

$$\int \frac{x\arctan(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx = \sqrt{c}\left(\frac{2\,x\arctan(ax)}{\sqrt{a^2x^2+1}ac^2} - \frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}a^2c^2} + \frac{2}{\sqrt{a^2x^2+1}a^2c^2}\right)$$

input
```
integrate(x*arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
sqrt(c)*(2*x*arctan(a*x)/(sqrt(a^2*x^2 + 1)*a*c^2) - arctan(a*x)^2/(sqrt(a
^2*x^2 + 1)*a^2*c^2) + 2/(sqrt(a^2*x^2 + 1)*a^2*c^2))
```

## Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.92

$$\int \frac{x \arctan(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx = \frac{2\,x \arctan{(ax)}}{\sqrt{a^2cx^2+c}ac} - \frac{\arctan{(ax)}^2}{\sqrt{a^2cx^2+c}a^2c} + \frac{2}{\sqrt{a^2cx^2+c}a^2c}$$

input
```
integrate(x*arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
2*x*arctan(a*x)/(sqrt(a^2*c*x^2 + c)*a*c) - arctan(a*x)^2/(sqrt(a^2*c*x^2
+ c)*a^2*c) + 2/(sqrt(a^2*c*x^2 + c)*a^2*c)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x \arctan(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x\,\mathrm{atan}(a\,x)^2}{(c\,a^2\,x^2+c)^{3/2}}\,dx$$

input
```
int((x*atan(a*x)^2)/(c + a^2*c*x^2)^(3/2),x)
```

output
```
int((x*atan(a*x)^2)/(c + a^2*c*x^2)^(3/2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.63

$$\int \frac{x \arctan(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{c}\left(-4atan\left(\sqrt{a^2x^2+1}+ax\right)a^2x^2 - 4atan\left(\sqrt{a^2x^2+1}+ax\right) - \sqrt{a^2x^2+1}\,atan(a}{a^2c^2\left(a^2x^2\right.}$$

input
```
int(x*atan(a*x)^2/(a^2*c*x^2+c)^(3/2),x)
```

output
```
(sqrt(c)*( - 4*atan(sqrt(a**2*x**2 + 1) + a*x)*a**2*x**2 - 4*atan(sqrt(a**
2*x**2 + 1) + a*x) - sqrt(a**2*x**2 + 1)*atan(a*x)**2 + 2*sqrt(a**2*x**2 +
 1)*atan(a*x)*a*x + 2*atan(a*x)*a**2*x**2 + 2*atan(a*x) + 2*sqrt(a**2*x**2
 + 1)))/(a**2*c**2*(a**2*x**2 + 1))
```

# 3.342 $\int \frac{\arctan(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 72

$$\int \frac{\arctan(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx=-\frac{2x}{c\sqrt{c+a^2cx^2}}+\frac{2\arctan(ax)}{ac\sqrt{c+a^2cx^2}}+\frac{x\arctan(ax)^2}{c\sqrt{c+a^2cx^2}}$$

output
```
-2*x/c/(a^2*c*x^2+c)^(1/2)+2*arctan(a*x)/a/c/(a^2*c*x^2+c)^(1/2)+x*arctan(
a*x)^2/c/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.68

$$\int \frac{\arctan(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx=\frac{\sqrt{c+a^2cx^2}(-2ax+2\arctan(ax)+ax\arctan(ax)^2)}{c^2\,(a+a^3x^2)}$$

input
```
Integrate[ArcTan[a*x]^2/(c + a^2*c*x^2)^(3/2),x]
```

output
```
(Sqrt[c + a^2*c*x^2]*(-2*a*x + 2*ArcTan[a*x] + a*x*ArcTan[a*x]^2))/(c^2*(a
 + a^3*x^2))
```

## Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5433, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2}{(a^2cx^2+c)^{3/2}} \, dx$$

$$\Big\downarrow \text{ 5433}$$

$$-2\int \frac{1}{(a^2cx^2+c)^{3/2}}dx + \frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}} + \frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 208}$$

$$\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}} + \frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}} - \frac{2x}{c\sqrt{a^2cx^2+c}}$$

input
```
Int[ArcTan[a*x]^2/(c + a^2*c*x^2)^(3/2),x]
```

output
```
(-2*x)/(c*Sqrt[c + a^2*c*x^2]) + (2*ArcTan[a*x])/(a*c*Sqrt[c + a^2*c*x^2])
 + (x*ArcTan[a*x]^2)/(c*Sqrt[c + a^2*c*x^2])
```

### Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 5433
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x] - Simp[b^2*p*(p
- 1)   Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2), x], x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.64 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.58

| method | result |
|---|---|
| default | $\frac{\left(\arctan(ax)^{2}-2+2i\arctan(ax)\right)(ax-i)\sqrt{c(ax-i)(ax+i)}}{2(a^{2}x^{2}+1)a\,c^{2}}+\frac{\sqrt{c(ax-i)(ax+i)}\,(ax+i)\left(\arctan(ax)^{2}-2-2i\arctan(ax)\right)}{2(a^{2}x^{2}+1)a\,c^{2}}$ |
| orering | $\frac{(-12a^{4}x^{5}-11a^{2}x^{3}+x)\arctan(ax)^{2}}{(a^{2}c\,x^{2}+c)^{\frac{3}{2}}}-\frac{(a^{2}x^{2}+1)^{2}(8a^{2}x^{2}-1)\left(\frac{2\arctan(ax)a}{(a^{2}c\,x^{2}+c)^{\frac{3}{2}}(a^{2}x^{2}+1)}-\frac{3\arctan(ax)^{2}cx\,a^{2}}{(a^{2}c\,x^{2}+c)^{\frac{5}{2}}}\right)}{a^{2}}-\frac{x(a^{2}x^{2}+1)^{3}\Big(}{}$ |

input
```
int(arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(arctan(a*x)^2-2+2*I*arctan(a*x))*(a*x-I)*(c*(a*x-I)*(a*x+I))^(1/2)/(a
^2*x^2+1)/a/c^2+1/2*(c*(a*x-I)*(a*x+I))^(1/2)*(a*x+I)*(arctan(a*x)^2-2-2*I
*arctan(a*x))/(a^2*x^2+1)/a/c^2
```

## Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.71

$$\int \frac{\arctan(ax)^{2}}{(c+a^{2}cx^{2})^{3/2}}\,dx=\frac{\sqrt{a^{2}cx^{2}+c}\left(ax\arctan(ax)^{2}-2\,ax+2\,\arctan(ax)\right)}{a^{3}c^{2}x^{2}+ac^{2}}$$

input
```
integrate(arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
sqrt(a^2*c*x^2 + c)*(a*x*arctan(a*x)^2 - 2*a*x + 2*arctan(a*x))/(a^3*c^2*x
^2 + a*c^2)
```

## Sympy [F]

$$\int \frac{\arctan(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\operatorname{atan}^2(ax)}{(c(a^2x^2+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(atan(a*x)**2/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(atan(a*x)**2/(c*(a**2*x**2 + 1))**(3/2), x)
```

## Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.74

$$\int \frac{\arctan(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx = \frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}c} - \frac{2(ax-\arctan(ax))}{\sqrt{a^2x^2+1}ac^{\frac{3}{2}}}$$

input
```
integrate(arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
x*arctan(a*x)^2/(sqrt(a^2*c*x^2 + c)*c) - 2*(a*x - arctan(a*x))/(sqrt(a^2*
x^2 + 1)*a*c^(3/2))
```

## Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx = -2a\left(\frac{x}{\sqrt{a^2cx^2+c}ac} - \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}a^2c}\right) + \frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}c}$$

input
```
integrate(arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
-2*a*(x/(sqrt(a^2*c*x^2 + c)*a*c) - arctan(a*x)/(sqrt(a^2*c*x^2 + c)*a^2*c
)) + x*arctan(a*x)^2/(sqrt(a^2*c*x^2 + c)*c)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\mathrm{atan}(a\,x)^2}{(c\,a^2\,x^2+c)^{3/2}}\,dx$$

input
```
int(atan(a*x)^2/(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(atan(a*x)^2/(c + a^2*c*x^2)^(3/2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.10

$$\int \frac{\arctan(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{c}\left(\sqrt{a^2x^2+1}\,atan(ax)^2\,ax+2\sqrt{a^2x^2+1}\,atan(ax)-2\sqrt{a^2x^2+1}\,ax-2a^2x^2-2\right)}{a\,c^2\,(a^2x^2+1)}$$

input
```
int(atan(a*x)^2/(a^2*c*x^2+c)^(3/2),x)
```

output
```
(sqrt(c)*(sqrt(a**2*x**2 + 1)*atan(a*x)**2*a*x + 2*sqrt(a**2*x**2 + 1)*ata
n(a*x) - 2*sqrt(a**2*x**2 + 1)*a*x - 2*a**2*x**2 - 2))/(a*c**2*(a**2*x**2
+ 1))
```

# 3.343 $\int \frac{\arctan(ax)^2}{x(c+a^2cx^2)^{3/2}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 310

$$\begin{aligned}\int \frac{\arctan(ax)^2}{x(c+a^2cx^2)^{3/2}}\,dx &= -\frac{2}{c\sqrt{c+a^2cx^2}}-\frac{2ax\arctan(ax)}{c\sqrt{c+a^2cx^2}} \\ &+\frac{\arctan(ax)^2}{c\sqrt{c+a^2cx^2}}-\frac{2\sqrt{1+a^2x^2}\arctan(ax)^2\text{arctanh}\left(e^{i\arctan(ax)}\right)}{c\sqrt{c+a^2cx^2}} \\ &+\frac{2i\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)}{c\sqrt{c+a^2cx^2}} \\ &-\frac{2i\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,e^{i\arctan(ax)}\right)}{c\sqrt{c+a^2cx^2}} \\ &-\frac{2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,-e^{i\arctan(ax)}\right)}{c\sqrt{c+a^2cx^2}}+\frac{2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,e^{i\arctan(ax)}\right)}{c\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
-2/c/(a^2*c*x^2+c)^(1/2)-2*a*x*arctan(a*x)/c/(a^2*c*x^2+c)^(1/2)+arctan(a*
x)^2/c/(a^2*c*x^2+c)^(1/2)-2*(a^2*x^2+1)^(1/2)*arctan(a*x)^2*arctanh((1+I*
a*x)/(a^2*x^2+1)^(1/2))/c/(a^2*c*x^2+c)^(1/2)+2*I*(a^2*x^2+1)^(1/2)*arctan
(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))/c/(a^2*c*x^2+c)^(1/2)-2*I*(a
^2*x^2+1)^(1/2)*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))/c/(a^2*
c*x^2+c)^(1/2)-2*(a^2*x^2+1)^(1/2)*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))
/c/(a^2*c*x^2+c)^(1/2)+2*(a^2*x^2+1)^(1/2)*polylog(3,(1+I*a*x)/(a^2*x^2+1)
^(1/2))/c/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.66

$$\int \frac{\arctan(ax)^2}{x\,(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{1+a^2x^2}\left(-\frac{2}{\sqrt{1+a^2x^2}}-\frac{2ax\arctan(ax)}{\sqrt{1+a^2x^2}}+\frac{\arctan(ax)^2}{\sqrt{1+a^2x^2}}+\arctan(ax)^2\log\left(1-e^{i\arctan(ax}\right.\right.}{}$$

input
```
Integrate[ArcTan[a*x]^2/(x*(c + a^2*c*x^2)^(3/2)),x]
```

output
```
(Sqrt[1 + a^2*x^2]*(-2/Sqrt[1 + a^2*x^2] - (2*a*x*ArcTan[a*x])/Sqrt[1 + a^
2*x^2] + ArcTan[a*x]^2/Sqrt[1 + a^2*x^2] + ArcTan[a*x]^2*Log[1 - E^(I*ArcT
an[a*x])] - ArcTan[a*x]^2*Log[1 + E^(I*ArcTan[a*x])] + (2*I)*ArcTan[a*x]*P
olyLog[2, -E^(I*ArcTan[a*x])] - (2*I)*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a
*x])] - 2*PolyLog[3, -E^(I*ArcTan[a*x])] + 2*PolyLog[3, E^(I*ArcTan[a*x])]
))/(c*Sqrt[c*(1 + a^2*x^2)])
```

## Rubi [A] (verified)

Time = 1.45 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.69, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5501, 5465, 5429, 5493, 5491, 3042, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2}{x\,(a^2cx^2+c)^{3/2}}\,dx$$

$$\Big\downarrow \; 5501$$

$$\frac{\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx}{c} - a^2\int \frac{x\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \; 5465$$

$$\frac{\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx}{c} - a^2\left(\frac{2\int \frac{\arctan(ax)}{(a^2cx^2+c)^{3/2}}dx}{a} - \frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \text{5429}$$

$$\frac{\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx}{c}-a^2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \text{5493}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{x\sqrt{a^2x^2+1}}dx}{c\sqrt{a^2cx^2+c}}-a^2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \text{5491}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{\sqrt{a^2x^2+1}\arctan(ax)^2}{ax}d\arctan(ax)}{c\sqrt{a^2cx^2+c}}- a^2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc(\arctan(ax))d\arctan(ax)}{c\sqrt{a^2cx^2+c}}- a^2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \text{4671}$$

$$-a^2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)+ \frac{\sqrt{a^2x^2+1}\left(-2\int \arctan(ax)\log\left(1-e^{i\arctan(ax)}\right)d\arctan(ax)+2\int \arctan(ax)\log\left(1+e^{i\arctan(ax)}\right)d\arctan(ax\right.}{c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3011}$$

$$-a^2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)+ \frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-i\int \operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)d\arctan(ax)\right)-2\left(i\arctan(\right.\right.}{c\sqrt{a^2cx}}$$

$$\Big\downarrow \text{2720}$$

$$-a^2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)+$$
$$\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)-2\right.$$

$\Big\downarrow$ 7143

$$-a^2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)+$$
$$\frac{\sqrt{a^2x^2+1}\left(-2\arctan(ax)^2\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)+2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\operatorname{PolyLog}\left(3,-e^{i\arctan(ax)}\right)\right)\right.}{c\sqrt{a^2cx^2+c}}$$

input
```
Int[ArcTan[a*x]^2/(x*(c + a^2*c*x^2)^(3/2)),x]
```

output
```
-(a^2*(-(ArcTan[a*x]^2/(a^2*c*Sqrt[c + a^2*c*x^2])) + (2*(1/(a*c*Sqrt[c +
a^2*c*x^2]) + (x*ArcTan[a*x])/(c*Sqrt[c + a^2*c*x^2])))/a)) + (Sqrt[1 + a^
2*x^2]*(-2*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])] + 2*(I*ArcTan[a*x]*Pol
yLog[2, -E^(I*ArcTan[a*x])] - PolyLog[3, -E^(I*ArcTan[a*x])]) - 2*(I*ArcTa
n[a*x]*PolyLog[2, E^(I*ArcTan[a*x])] - PolyLog[3, E^(I*ArcTan[a*x])])))/(c
*Sqrt[c + a^2*c*x^2])
```

## Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5429
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTan[c*x])/(d*Sqr
t[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5491
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Simp[1/Sqrt[d]   Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTa
n[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] &&
GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

## Maple [A] (verified)

Time = 2.10 (sec) , antiderivative size = 306, normalized size of antiderivative = 0.99

| method | result |
|---|---|
| default | $\frac{\left(\arctan(ax)^2-2+2i\arctan(ax)\right)(iax+1)\sqrt{c(ax-i)(ax+i)}}{2(a^2x^2+1)c^2}-\frac{\sqrt{c(ax-i)(ax+i)}\,(iax-1)\left(\arctan(ax)^2-2-2i\arctan(ax)\right)}{2(a^2x^2+1)c^2}+\frac{(}{}$ |

input
```
int(arctan(a*x)^2/x/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(arctan(a*x)^2-2+2*I*arctan(a*x))*(1+I*a*x)*(c*(a*x-I)*(a*x+I))^(1/2)/
(a^2*x^2+1)/c^2-1/2*(c*(a*x-I)*(a*x+I))^(1/2)*(I*a*x-1)*(arctan(a*x)^2-2-2
*I*arctan(a*x))/(a^2*x^2+1)/c^2+(arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^
(1/2))-arctan(a*x)^2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*I*arctan(a*x)*pol
ylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*I*arctan(a*x)*polylog(2,-(1+I*a*x)/(
a^2*x^2+1)^(1/2))+2*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*polylog(3,-(1
+I*a*x)/(a^2*x^2+1)^(1/2)))*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/2)/c^
2
```

## Fricas [F]

$$\int \frac{\arctan(ax)^2}{x\left(c+a^2cx^2\right)^{3/2}}\,dx=\int \frac{\arctan\left(ax\right)^2}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(arctan(a*x)^2/x/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2/(a^4*c^2*x^5 + 2*a^2*c^2*x^3 +
c^2*x), x)
```

## Sympy [F]

$$\int \frac{\arctan(ax)^2}{x\left(c+a^2cx^2\right)^{3/2}}\,dx = \int \frac{\operatorname{atan}^2(ax)}{x\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(atan(a*x)**2/x/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(atan(a*x)**2/(x*(c*(a**2*x**2 + 1))**(3/2)), x)
```

## Maxima [F]

$$\int \frac{\arctan(ax)^2}{x\left(c+a^2cx^2\right)^{3/2}}\,dx = \int \frac{\arctan(ax)^2}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(arctan(a*x)^2/x/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)^2/((a^2*c*x^2 + c)^(3/2)*x), x)
```

## Giac [F]

$$\int \frac{\arctan(ax)^2}{x\left(c+a^2cx^2\right)^{3/2}}\,dx = \int \frac{\arctan(ax)^2}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(arctan(a*x)^2/x/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^2/((a^2*c*x^2 + c)^(3/2)*x), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^2}{x\,(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\mathrm{atan}(a\,x)^2}{x\,(c\,a^2\,x^2+c)^{3/2}}\,dx$$

input
```
int(atan(a*x)^2/(x*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(atan(a*x)^2/(x*(c + a^2*c*x^2)^(3/2)), x)
```

## Reduce [F]

$$\int \frac{\arctan(ax)^2}{x\,(c+a^2cx^2)^{3/2}}\,dx = \frac{\int \frac{atan(ax)^2}{\sqrt{a^2x^2+1}\,a^2x^3+\sqrt{a^2x^2+1}\,x}dx}{\sqrt{c}\,c}$$

input
```
int(atan(a*x)^2/x/(a^2*c*x^2+c)^(3/2),x)
```

output
```
int(atan(a*x)**2/(sqrt(a**2*x**2 + 1)*a**2*x**3 + sqrt(a**2*x**2 + 1)*x),x
)/(sqrt(c)*c)
```

# 3.344 $\int \frac{\arctan(ax)^2}{x^2(c+a^2cx^2)^{3/2}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 293

$$\int \frac{\arctan(ax)^2}{x^2\left(c+a^2cx^2\right)^{3/2}}\,dx = \frac{2a^2x}{c\sqrt{c+a^2cx^2}} - \frac{2a\arctan(ax)}{c\sqrt{c+a^2cx^2}} - \frac{a^2x\arctan(ax)^2}{c\sqrt{c+a^2cx^2}} - \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{c^2x} - \frac{4a\sqrt{1+a^2x^2}\arctan(ax)\operatorname{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{c\sqrt{c+a^2cx^2}} + \frac{2ia\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{c\sqrt{c+a^2cx^2}} - \frac{2ia\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{c\sqrt{c+a^2cx^2}}$$

output
```
2*a^2*x/c/(a^2*c*x^2+c)^(1/2)-2*a*arctan(a*x)/c/(a^2*c*x^2+c)^(1/2)-a^2*x*
arctan(a*x)^2/c/(a^2*c*x^2+c)^(1/2)-(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2/c^2/
x-4*a*(a^2*x^2+1)^(1/2)*arctan(a*x)*arctanh((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2
))/c/(a^2*c*x^2+c)^(1/2)+2*I*a*(a^2*x^2+1)^(1/2)*polylog(2,-(1+I*a*x)^(1/2
)/(1-I*a*x)^(1/2))/c/(a^2*c*x^2+c)^(1/2)-2*I*a*(a^2*x^2+1)^(1/2)*polylog(2
,(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/c/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.77

$$\int \frac{\arctan(ax)^2}{x^2 (c + a^2cx^2)^{3/2}}\,dx = \frac{a\Big(4ax - 4\arctan(ax) - 2ax\arctan(ax)^2 - \frac{1}{2}ax\arctan(ax)^2\csc^2\left(\frac{1}{2}\arctan(ax)\right)}{}$$

input
```
Integrate[ArcTan[a*x]^2/(x^2*(c + a^2*c*x^2)^(3/2)),x]
```

output
```
(a*(4*a*x - 4*ArcTan[a*x] - 2*a*x*ArcTan[a*x]^2 - (a*x*ArcTan[a*x]^2*Csc[A
rcTan[a*x]/2]^2)/2 + 4*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Log[1 - E^(I*ArcTan[a
*x])] - 4*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Log[1 + E^(I*ArcTan[a*x])] + (4*I)
*Sqrt[1 + a^2*x^2]*PolyLog[2, -E^(I*ArcTan[a*x])] - (4*I)*Sqrt[1 + a^2*x^2
]*PolyLog[2, E^(I*ArcTan[a*x])] - (2*(1 + a^2*x^2)*ArcTan[a*x]^2*Sin[ArcTa
n[a*x]/2]^2)/(a*x)))/(2*c*Sqrt[c + a^2*c*x^2])
```

## Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.81, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5501, 5433, 208, 5479, 5493, 5489}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2}{x^2 (a^2cx^2 + c)^{3/2}}\,dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx}{c} - a^2\int \frac{\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \text{5433}$$

$$\frac{\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx}{c} - a^2\left(-2\int \frac{1}{(a^2cx^2+c)^{3/2}}dx + \frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}} + \frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \text{208}$$

$$\frac{\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx}{c}-a^2\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \text{5479}$$

$$\frac{2a\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}}{c}-a^2\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \text{5493}$$

$$\frac{\frac{2a\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}}{c}-a^2\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \text{5489}$$

$$-a^2\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)+\frac{-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}+\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{PolyLog}\left(2,\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)\right)}{\sqrt{a^2cx^2+c}}}{c}$$

input
```
Int[ArcTan[a*x]^2/(x^2*(c + a^2*c*x^2)^(3/2)),x]
```

output
```
-(a^2*((-2*x)/(c*Sqrt[c + a^2*c*x^2]) + (2*ArcTan[a*x])/(a*c*Sqrt[c + a^2*
c*x^2]) + (x*ArcTan[a*x]^2)/(c*Sqrt[c + a^2*c*x^2]))) + (-((Sqrt[c + a^2*c
*x^2]*ArcTan[a*x]^2)/(c*x)) + (2*a*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]*ArcTa
nh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]] + I*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[
1 - I*a*x])] - I*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]]))/Sqrt[c + a^
2*c*x^2])/c
```

## Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 5433
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x] - Simp[b^2*p*(p
- 1)   Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2), x], x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5489
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sq
rt[1 - I*c*x]], x] + (Simp[I*(b/Sqrt[d])*PolyLog[2, -Sqrt[1 + I*c*x]/Sqrt[1
 - I*c*x]], x] - Simp[I*(b/Sqrt[d])*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x]], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

## Maple [A] (verified)

Time = 2.18 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.95

| method | result |
|---|---|
| default | $-\frac{a\left(\arctan(ax)^2-2+2i\arctan(ax)\right)(ax-i)\sqrt{c(ax-i)(ax+i)}}{2(a^2x^2+1)c^2}-\frac{\sqrt{c(ax-i)(ax+i)}\,(ax+i)\left(\arctan(ax)^2-2-2i\arctan(ax)\right)a}{2(a^2x^2+1)c^2}-$ |

input
```
int(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*a*(arctan(a*x)^2-2+2*I*arctan(a*x))*(a*x-I)*(c*(a*x-I)*(a*x+I))^(1/2)
/(a^2*x^2+1)/c^2-1/2*(c*(a*x-I)*(a*x+I))^(1/2)*(a*x+I)*(arctan(a*x)^2-2-2*
I*arctan(a*x))*a/(a^2*x^2+1)/c^2-arctan(a*x)^2*(c*(a*x-I)*(a*x+I))^(1/2)/c
^2/x-2*I*a*(I*arctan(a*x)*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*arctan(a*x)*
ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-p
olylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2)))*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2
+1)^(1/2)/c^2
```

## Fricas [F]

$$\int \frac{\arctan(ax)^2}{x^2\left(c+a^2cx^2\right)^{3/2}}\,dx=\int \frac{\arctan\left(ax\right)^2}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2/(a^4*c^2*x^6 + 2*a^2*c^2*x^4 +
c^2*x^2), x)
```

## Sympy [F]

$$\int \frac{\arctan(ax)^2}{x^2 (c + a^2cx^2)^{3/2}} \, dx = \int \frac{\operatorname{atan}^2(ax)}{x^2 (c (a^2x^2 + 1))^{\frac{3}{2}}} \, dx$$

input
```
integrate(atan(a*x)**2/x**2/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(atan(a*x)**2/(x**2*(c*(a**2*x**2 + 1))**(3/2)), x)
```

## Maxima [F]

$$\int \frac{\arctan(ax)^2}{x^2 (c + a^2cx^2)^{3/2}} \, dx = \int \frac{\arctan(ax)^2}{(a^2cx^2 + c)^{\frac{3}{2}} x^2} \, dx$$

input
```
integrate(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)^2/((a^2*c*x^2 + c)^(3/2)*x^2), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^2}{x^2 (c + a^2cx^2)^{3/2}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^2}{x^2 (c + a^2cx^2)^{3/2}} \, dx = \int \frac{\mathrm{atan}(a\,x)^2}{x^2\,(c\,a^2\,x^2 + c)^{3/2}} \, dx$$

input
```
int(atan(a*x)^2/(x^2*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(atan(a*x)^2/(x^2*(c + a^2*c*x^2)^(3/2)), x)
```

## Reduce [F]

$$\int \frac{\arctan(ax)^2}{x^2 (c + a^2cx^2)^{3/2}} \, dx = \frac{\int \frac{atan(ax)^2}{\sqrt{a^2x^2+1}\,a^2x^4+\sqrt{a^2x^2+1}\,x^2}dx}{\sqrt{c}\,c}$$

input
```
int(atan(a*x)^2/x^2/(a^2*c*x^2+c)^(3/2),x)
```

output
```
int(atan(a*x)**2/(sqrt(a**2*x**2 + 1)*a**2*x**4 + sqrt(a**2*x**2 + 1)*x**2
),x)/(sqrt(c)*c)
```

# 3.345 $\int \frac{\arctan(ax)^2}{x^3(c+a^2cx^2)^{3/2}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 422

$$\begin{aligned}\int \frac{\arctan(ax)^2}{x^3\left(c+a^2cx^2\right)^{3/2}}\,dx &= \frac{2a^2}{c\sqrt{c+a^2cx^2}}+\frac{2a^3x\arctan(ax)}{c\sqrt{c+a^2cx^2}} \\ &- \frac{a\sqrt{c+a^2cx^2}\arctan(ax)}{c^2x}-\frac{a^2\arctan(ax)^2}{c\sqrt{c+a^2cx^2}}-\frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{2c^2x^2} \\ &+ \frac{3a^2\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)}{c\sqrt{c+a^2cx^2}}-\frac{a^2\operatorname{arctanh}\left(\frac{\sqrt{c+a^2cx^2}}{\sqrt{c}}\right)}{c^{3/2}} \\ &- \frac{3ia^2\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)}{c\sqrt{c+a^2cx^2}} \\ &+ \frac{3ia^2\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,e^{i\arctan(ax)}\right)}{c\sqrt{c+a^2cx^2}} \\ &+ \frac{3a^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,-e^{i\arctan(ax)}\right)}{c\sqrt{c+a^2cx^2}} \\ &- \frac{3a^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,e^{i\arctan(ax)}\right)}{c\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
2*a^2/c/(a^2*c*x^2+c)^(1/2)+2*a^3*x*arctan(a*x)/c/(a^2*c*x^2+c)^(1/2)-a*(a
^2*c*x^2+c)^(1/2)*arctan(a*x)/c^2/x-a^2*arctan(a*x)^2/c/(a^2*c*x^2+c)^(1/2
)-1/2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2/c^2/x^2+3*a^2*(a^2*x^2+1)^(1/2)*ar
ctan(a*x)^2*arctanh((1+I*a*x)/(a^2*x^2+1)^(1/2))/c/(a^2*c*x^2+c)^(1/2)-a^2
*arctanh((a^2*c*x^2+c)^(1/2)/c^(1/2))/c^(3/2)-3*I*a^2*(a^2*x^2+1)^(1/2)*ar
ctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))/c/(a^2*c*x^2+c)^(1/2)+3*
I*a^2*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))
/c/(a^2*c*x^2+c)^(1/2)+3*a^2*(a^2*x^2+1)^(1/2)*polylog(3,-(1+I*a*x)/(a^2*x
^2+1)^(1/2))/c/(a^2*c*x^2+c)^(1/2)-3*a^2*(a^2*x^2+1)^(1/2)*polylog(3,(1+I*
a*x)/(a^2*x^2+1)^(1/2))/c/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (warning: unable to verify)

Time = 1.64 (sec) , antiderivative size = 371, normalized size of antiderivative = 0.88

$$\int \frac{\arctan(ax)^2}{x^3\left(c+a^2cx^2\right)^{3/2}}\,dx = \frac{a^2\left(16+16ax\arctan(ax)-8\arctan(ax)^2-2ax\arctan(ax)\csc^2\left(\frac{1}{2}\arctan(ax)\right)\right.}{}$$

input
```
Integrate[ArcTan[a*x]^2/(x^3*(c + a^2*c*x^2)^(3/2)),x]
```

output
```
(a^2*(16 + 16*a*x*ArcTan[a*x] - 8*ArcTan[a*x]^2 - 2*a*x*ArcTan[a*x]*Csc[Ar
cTan[a*x]/2]^2 - Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*Csc[ArcTan[a*x]/2]^2 - 12
*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*Log[1 - E^(I*ArcTan[a*x])] + 12*Sqrt[1 +
a^2*x^2]*ArcTan[a*x]^2*Log[1 + E^(I*ArcTan[a*x])] + 8*Sqrt[1 + a^2*x^2]*Lo
g[Tan[ArcTan[a*x]/2]] - (24*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, -E
^(I*ArcTan[a*x])] + (24*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, E^(I*A
rcTan[a*x])] + 24*Sqrt[1 + a^2*x^2]*PolyLog[3, -E^(I*ArcTan[a*x])] - 24*Sq
rt[1 + a^2*x^2]*PolyLog[3, E^(I*ArcTan[a*x])] + Sqrt[1 + a^2*x^2]*ArcTan[a
*x]^2*Sec[ArcTan[a*x]/2]^2 - 4*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Tan[ArcTan[a*
x]/2]))/(8*c*Sqrt[c + a^2*c*x^2])
```

## Rubi [A] (verified)

Time = 4.49 (sec) , antiderivative size = 445, normalized size of antiderivative = 1.05, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {5501, 5497, 5479, 243, 73, 221, 5493, 5491, 3042, 4671, 3011, 2720, 5501, 5465, 5429, 5493, 5491, 3042, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2}{x^3\left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx}{c}-a^2\int \frac{\arctan(ax)^2}{x\left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{5497}$$

$$\frac{-\frac{1}{2}a^2\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+a\int \frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{2cx^2}}{c}-a^2\int \frac{\arctan(ax)^2}{x\left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{5479}$$

$$\frac{-\frac{1}{2}a^2\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+a\left(a\int \frac{1}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right)-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{2cx^2}}{c}-a^2\int \frac{\arctan(ax)^2}{x\left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{-\frac{1}{2}a^2\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+a\left(\frac{1}{2}a\int \frac{1}{x^2\sqrt{a^2cx^2+c}}dx^2-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right)-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{2cx^2}}{c}-a^2\int \frac{\arctan(ax)^2}{x\left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{73}$$

$$\frac{-\frac{1}{2}a^2\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+a\left(\frac{\int \frac{1}{\frac{x^4}{a^2c}-\frac{1}{a^2}}d\sqrt{a^2cx^2+c}}{ac}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right)-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{2cx^2}}{c}-a^2\int \frac{\arctan(ax)^2}{x\left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{221}$$

$$\frac{-\frac{1}{2}a^2\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+a\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{2cx^2}}{c}-a^2\int \frac{\arctan(ax)^2}{x\left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{5493}$$

$$\frac{-\frac{a^2\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{x\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+a\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{2cx^2}}{c}-a^2\int \frac{\arctan(ax)^2}{x\left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{5491}$$

$$\frac{-\frac{a^2\sqrt{a^2x^2+1}\int \frac{\sqrt{a^2x^2+1}\arctan(ax)^2}{ax}d\arctan(ax)}{2\sqrt{a^2cx^2+c}}+a\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{2cx^2}}{c}-a^2\int \frac{\arctan(ax)^2}{x\left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{a^2\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc(\arctan(ax))d\arctan(ax)}{2\sqrt{a^2cx^2+c}}+a\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{2cx^2}}{c}-a^2\int \frac{\arctan(ax)^2}{x\left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{4671}$$

$$-a^2\int \frac{\arctan(ax)^2}{x\left(a^2cx^2+c\right)^{3/2}}dx+\frac{-\frac{a^2\sqrt{a^2x^2+1}\left(-2\int \arctan(ax)\log\left(1-e^{i\arctan(ax)}\right)d\arctan(ax)+2\int \arctan(ax)\log\left(1+e^{i\arctan(ax)}\right)d\arctan(ax)-2\arctan(ax)^2\text{arctanh}(e^{i}\right.}{2\sqrt{a^2cx^2+c}}}{c}$$

$$\Big\downarrow \text{3011}$$

$$-a^2\int \frac{\arctan(ax)^2}{x\left(a^2cx^2+c\right)^{3/2}}dx+$$

$$-\frac{a^2\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-i\int \operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)d\arctan(ax)\right)-2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,e^{i\arctan(ax)}\right)-\right.\right.}{2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 2720}$$

$$-a^2\int \frac{\arctan(ax)^2}{x\left(a^2cx^2+c\right)^{3/2}}dx+$$

$$-\frac{a^2\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)-2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,e\right.\right.\right.}{2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 5501}$$

$$-a^2\left(\frac{\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx}{c}-a^2\int \frac{x\arctan(ax)^2}{\left(a^2cx^2+c\right)^{3/2}}dx\right)+$$

$$-\frac{a^2\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)-2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,e\right.\right.\right.}{2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 5465}$$

$$-a^2\left(\frac{\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx}{c}-a^2\left(\frac{2\int \frac{\arctan(ax)}{\left(a^2cx^2+c\right)^{3/2}}dx}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)\right)+$$

$$-\frac{a^2\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)-2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,e\right.\right.\right.}{2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 5429}$$

$$-a^2\left(\frac{\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx}{c}-a^2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)\right)+$$

$$-\frac{a^2\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)-2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,e\right.\right.\right.}{2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 5493}$$

$$-a^2\left(\frac{\sqrt{a^2x^2+1}\int\frac{\arctan(ax)^2}{x\sqrt{a^2x^2+1}}dx}{c\sqrt{a^2cx^2+c}}-a^2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)\right)+$$

$$-\frac{a^2\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)-2\left(i\arctan(ax)\operatorname{PolyLog}(2,e\right.}{2\sqrt{a^2cx^2+c}}$$

$\downarrow$ 5491

$$-a^2\left(\frac{\sqrt{a^2x^2+1}\int\frac{\sqrt{a^2x^2+1}\arctan(ax)^2}{ax}d\arctan(ax)}{c\sqrt{a^2cx^2+c}}-a^2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)\right)+$$

$$-\frac{a^2\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)-2\left(i\arctan(ax)\operatorname{PolyLog}(2,e\right.}{2\sqrt{a^2cx^2+c}}$$

$\downarrow$ 3042

$$-a^2\left(\frac{\sqrt{a^2x^2+1}\int\arctan(ax)^2\csc(\arctan(ax))d\arctan(ax)}{c\sqrt{a^2cx^2+c}}-a^2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+\right.}$$

$$-\frac{a^2\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)-2\left(i\arctan(ax)\operatorname{PolyLog}(2,e\right.}{2\sqrt{a^2cx^2+c}}$$

$\downarrow$ 4671

$$-\frac{a^2\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)-2\left(i\arctan(ax)\operatorname{PolyLog}(2,e\right.}{2\sqrt{a^2cx^2+c}}$$

$$a^2\left(-a^2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)+\frac{\sqrt{a^2x^2+1}\left(-2\int\arctan(ax)\log\left(1-e^{i\arctan(ax)}\right)a\right.}{}\right.$$

$\downarrow$ 3011

$$-\frac{\frac{a^2\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)-2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,e\right.\right.}{2\sqrt{a^2cx^2+c}}}{a^2\left(-a^2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)+\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax}\right.\right.\right.}{}}$$

$$\Big\downarrow \text{2720}$$

$$-\frac{\frac{a^2\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)-2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,e\right.\right.}{2\sqrt{a^2cx^2+c}}}{a^2\left(-a^2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)+\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax}\right.\right.\right.}{}}$$

$$\Big\downarrow \text{7143}$$

$$-\frac{\frac{a^2\sqrt{a^2x^2+1}\left(-2\arctan(ax)^2\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)+2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\operatorname{PolyLog}\left(3,-e^{i\arctan(ax)}\right)\right)-2\left(i\arctan(ax)\right.\right.}{2\sqrt{a^2cx^2+c}}}{a^2\left(-a^2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)+\frac{\sqrt{a^2x^2+1}\left(-2\arctan(ax)^2\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)+\right.}{}}$$

input
```
Int[ArcTan[a*x]^2/(x^3*(c + a^2*c*x^2)^(3/2)),x]
```

output
```
(-1/2*(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(c*x^2) + a*(-((Sqrt[c + a^2*c*x
^2]*ArcTan[a*x])/(c*x)) - (a*ArcTanh[Sqrt[c + a^2*c*x^2]/Sqrt[c]])/Sqrt[c]
) - (a^2*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])] +
2*(I*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])] - PolyLog[3, -E^(I*ArcTan[
a*x])]) - 2*(I*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])] - PolyLog[3, E^(I
*ArcTan[a*x])])))/(2*Sqrt[c + a^2*c*x^2]))/c - a^2*(-(a^2*(-(ArcTan[a*x]^2
/(a^2*c*Sqrt[c + a^2*c*x^2])) + (2*(1/(a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTa
n[a*x])/(c*Sqrt[c + a^2*c*x^2])))/a)) + (Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]
^2*ArcTanh[E^(I*ArcTan[a*x])] + 2*(I*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a
*x])] - PolyLog[3, -E^(I*ArcTan[a*x])]) - 2*(I*ArcTan[a*x]*PolyLog[2, E^(I
*ArcTan[a*x])] - PolyLog[3, E^(I*ArcTan[a*x])])))/(c*Sqrt[c + a^2*c*x^2]))
```

### Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5429
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTan[c*x])/(d*Sqr
t[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5491
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Simp[1/Sqrt[d]   Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTa
n[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] &&
GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5497
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)^(m
 + 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[c^2*((m
+ 2)/(f^2*(m + 1)))   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x
^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
 && LtQ[m, -1] && NeQ[m, -2]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

## Maple [A] (verified)

Time = 2.32 (sec) , antiderivative size = 376, normalized size of antiderivative = 0.89

| method | result |
|---|---|
| default | $-\frac{a^2\left(\arctan(ax)^2-2+2i\arctan(ax)\right)(iax+1)\sqrt{c(ax-i)(ax+i)}}{2(a^2x^2+1)c^2}+\frac{\sqrt{c(ax-i)(ax+i)}\,(iax-1)\left(\arctan(ax)^2-2-2i\arctan(ax)\right)a^2}{2(a^2x^2+1)c^2}$ |

input
```
int(arctan(a*x)^2/x^3/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^2*(arctan(a*x)^2-2+2*I*arctan(a*x))*(1+I*a*x)*(c*(a*x-I)*(a*x+I))^(
1/2)/(a^2*x^2+1)/c^2+1/2*(c*(a*x-I)*(a*x+I))^(1/2)*(I*a*x-1)*(arctan(a*x)^
2-2-2*I*arctan(a*x))*a^2/(a^2*x^2+1)/c^2-1/2*(2*a*x+arctan(a*x))*arctan(a*
x)*(c*(a*x-I)*(a*x+I))^(1/2)/c^2/x^2-1/2*a^2*(3*arctan(a*x)^2*ln(1-(1+I*a*
x)/(a^2*x^2+1)^(1/2))-3*arctan(a*x)^2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*
I*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*arctan(a*x)*polyl
og(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+4*arctanh((1+I*a*x)/(a^2*x^2+1)^(1/2))+
6*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*polylog(3,-(1+I*a*x)/(a^2*x^2+1
)^(1/2)))*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/2)/c^2
```

## Fricas [F]

$$\int \frac{\arctan(ax)^2}{x^3\,(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\arctan\left(ax\right)^2}{(a^2cx^2+c)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate(arctan(a*x)^2/x^3/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2/(a^4*c^2*x^7 + 2*a^2*c^2*x^5 +
c^2*x^3), x)
```

## Sympy [F]

$$\int \frac{\arctan(ax)^2}{x^3 (c + a^2 c x^2)^{3/2}}\,dx = \int \frac{\operatorname{atan}^2(ax)}{x^3 \left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(atan(a*x)**2/x**3/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(atan(a*x)**2/(x**3*(c*(a**2*x**2 + 1))**(3/2)), x)
```

## Maxima [F]

$$\int \frac{\arctan(ax)^2}{x^3 (c + a^2 c x^2)^{3/2}}\,dx = \int \frac{\arctan(ax)^2}{(a^2cx^2+c)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate(arctan(a*x)^2/x^3/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)^2/((a^2*c*x^2 + c)^(3/2)*x^3), x)
```

## Giac [F]

$$\int \frac{\arctan(ax)^2}{x^3 (c + a^2 c x^2)^{3/2}}\,dx = \int \frac{\arctan(ax)^2}{(a^2cx^2+c)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate(arctan(a*x)^2/x^3/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^2/((a^2*c*x^2 + c)^(3/2)*x^3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^2}{x^3 (c + a^2cx^2)^{3/2}} \, dx = \int \frac{\operatorname{atan}(a\,x)^2}{x^3 \left(c\,a^2\,x^2 + c\right)^{3/2}} \, dx$$

input
```
int(atan(a*x)^2/(x^3*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(atan(a*x)^2/(x^3*(c + a^2*c*x^2)^(3/2)), x)
```

## Reduce [F]

$$\int \frac{\arctan(ax)^2}{x^3 (c + a^2cx^2)^{3/2}} \, dx = \frac{\int \frac{atan(ax)^2}{\sqrt{a^2x^2+1}\,a^2x^5+\sqrt{a^2x^2+1}\,x^3}dx}{\sqrt{c}\,c}$$

input
```
int(atan(a*x)^2/x^3/(a^2*c*x^2+c)^(3/2),x)
```

output
```
int(atan(a*x)**2/(sqrt(a**2*x**2 + 1)*a**2*x**5 + sqrt(a**2*x**2 + 1)*x**3
),x)/(sqrt(c)*c)
```

# 3.346 $\int \frac{\arctan(ax)^2}{x^4(c+a^2cx^2)^{3/2}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 397

$$\begin{aligned}\int \frac{\arctan(ax)^2}{x^4(c+a^2cx^2)^{3/2}}\,dx &= -\frac{2a^4x}{c\sqrt{c+a^2cx^2}}-\frac{a^2\sqrt{c+a^2cx^2}}{3c^2x}+\frac{2a^3\arctan(ax)}{c\sqrt{c+a^2cx^2}}\\ &\quad-\frac{a\sqrt{c+a^2cx^2}\arctan(ax)}{3c^2x^2}+\frac{a^4x\arctan(ax)^2}{c\sqrt{c+a^2cx^2}}-\frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{3c^2x^3}\\ &\quad+\frac{5a^2\sqrt{c+a^2cx^2}\arctan(ax)^2}{3c^2x}+\frac{22a^3\sqrt{1+a^2x^2}\arctan(ax)\operatorname{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3c\sqrt{c+a^2cx^2}}\\ &\quad-\frac{11ia^3\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3c\sqrt{c+a^2cx^2}}\\ &\quad+\frac{11ia^3\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{3c\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
-2*a^4*x/c/(a^2*c*x^2+c)^(1/2)-1/3*a^2*(a^2*c*x^2+c)^(1/2)/c^2/x+2*a^3*arc
tan(a*x)/c/(a^2*c*x^2+c)^(1/2)-1/3*a*(a^2*c*x^2+c)^(1/2)*arctan(a*x)/c^2/x
^2+a^4*x*arctan(a*x)^2/c/(a^2*c*x^2+c)^(1/2)-1/3*(a^2*c*x^2+c)^(1/2)*arcta
n(a*x)^2/c^2/x^3+5/3*a^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2/c^2/x+22/3*a^3*
(a^2*x^2+1)^(1/2)*arctan(a*x)*arctanh((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/c/(
a^2*c*x^2+c)^(1/2)-11/3*I*a^3*(a^2*x^2+1)^(1/2)*polylog(2,-(1+I*a*x)^(1/2)
/(1-I*a*x)^(1/2))/c/(a^2*c*x^2+c)^(1/2)+11/3*I*a^3*(a^2*x^2+1)^(1/2)*polyl
og(2,(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/c/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (warning: unable to verify)

Time = 2.36 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.68

$$\int \frac{\arctan(ax)^2}{x^4 (c+a^2cx^2)^{3/2}}\,dx = \frac{a^3\sqrt{1+a^2x^2}\left(-88i\,\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)+\frac{(1+a^2x^2)^{3/2}\left(-22+28\cos(2\arctan(ax))-\right.}{}\right.}{}$$

input
```
Integrate[ArcTan[a*x]^2/(x^4*(c + a^2*c*x^2)^(3/2)),x]
```

output
```
(a^3*Sqrt[1 + a^2*x^2]*((-88*I)*PolyLog[2, -E^(I*ArcTan[a*x])] + ((1 + a^2
*x^2)^(3/2)*(-22 + 28*Cos[2*ArcTan[a*x]] - 6*Cos[4*ArcTan[a*x]] + ArcTan[a
*x]^2*(25 - 36*Cos[2*ArcTan[a*x]] + 3*Cos[4*ArcTan[a*x]]) + ((88*I)*a^3*x^
3*PolyLog[2, E^(I*ArcTan[a*x])])/(1 + a^2*x^2)^(3/2) + ArcTan[a*x]*((66*a*
x*(-Log[1 - E^(I*ArcTan[a*x])] + Log[1 + E^(I*ArcTan[a*x])]))/Sqrt[1 + a^2
*x^2] + 8*Sin[2*ArcTan[a*x]] + 22*(Log[1 - E^(I*ArcTan[a*x])] - Log[1 + E^
(I*ArcTan[a*x])])*Sin[3*ArcTan[a*x]] - 6*Sin[4*ArcTan[a*x]])))/(a^3*x^3)))
/(24*c*Sqrt[c + a^2*c*x^2])
```

## Rubi [A] (verified)

Time = 4.09 (sec) , antiderivative size = 629, normalized size of antiderivative = 1.58, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5501, 5497, 5479, 5493, 5489, 5497, 242, 5493, 5489, 5501, 5433, 208, 5479, 5493, 5489}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2}{x^4\left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{\arctan(ax)^2}{x^4\sqrt{a^2cx^2+c}}dx}{c}-a^2\int \frac{\arctan(ax)^2}{x^2\left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{5497}$$

$$\frac{-\frac{2}{3}a^2\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx+\frac{2}{3}a\int \frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{3cx^3}}{c}-a^2\int \frac{\arctan(ax)^2}{x^2\left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{5479}$$

$$\frac{-\frac{2}{3}a^2\left(2a\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}\right)+\frac{2}{3}a\int \frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{3cx^3}}{c}-a^2\int \frac{\arctan(ax)^2}{x^2\left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{5493}$$

$$\frac{-\frac{2}{3}a^2\left(\frac{2a\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}\right)+\frac{2}{3}a\int \frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{3cx^3}}{c}-a^2\int \frac{\arctan(ax)^2}{x^2\left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{5489}$$

$$-a^2\int\frac{\arctan(ax)^2}{x^2\left(a^2cx^2+c\right)^{3/2}}dx+$$
$$\frac{\frac{2}{3}a\int\frac{\arctan(ax)}{x^3\sqrt{a^2cx^2+c}}dx-\frac{2}{3}a^2\left(-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}+\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{Po}\right.}{\sqrt{a^2cx^2+c}}\right.}{c}$$

$\big\downarrow$ 5497

$$-a^2\int\frac{\arctan(ax)^2}{x^2\left(a^2cx^2+c\right)^{3/2}}dx+$$
$$\frac{\frac{2}{3}a\left(-\frac{1}{2}a^2\int\frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx+\frac{1}{2}a\int\frac{1}{x^2\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{2cx^2}\right)-\frac{2}{3}a^2\left(-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}+\frac{2a\sqrt{a^2x^2+1}\left(-2\right.}{}\right.}{c}$$

$\big\downarrow$ 242

$$-a^2\int\frac{\arctan(ax)^2}{x^2\left(a^2cx^2+c\right)^{3/2}}dx+$$
$$\frac{\frac{2}{3}a\left(-\frac{1}{2}a^2\int\frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{2cx^2}-\frac{a\sqrt{a^2cx^2+c}}{2cx}\right)-\frac{2}{3}a^2\left(-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}+\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\right.}{}\right.}{c}$$

$\big\downarrow$ 5493

$$-a^2\int\frac{\arctan(ax)^2}{x^2\left(a^2cx^2+c\right)^{3/2}}dx+$$
$$\frac{\frac{2}{3}a\left(-\frac{a^2\sqrt{a^2x^2+1}\int\frac{\arctan(ax)}{x\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{2cx^2}-\frac{a\sqrt{a^2cx^2+c}}{2cx}\right)-\frac{2}{3}a^2\left(-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}+\frac{2a\sqrt{a^2x^2+1}\left(-2\,\text{arcta}\right.}{}\right.}{c}$$

$\big\downarrow$ 5489

$$-a^2\int\frac{\arctan(ax)^2}{x^2\left(a^2cx^2+c\right)^{3/2}}dx+$$
$$\frac{-\frac{2}{3}a^2\left(-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}+\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{PolyLog}\left(2,\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)\right)}{\sqrt{a^2cx^2+c}}\right)-}{}$$

$\big\downarrow$ 5501

$$-a^2\left(\frac{\int\frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx}{c}-a^2\int\frac{\arctan(ax)^2}{\left(a^2cx^2+c\right)^{3/2}}dx\right)+$$
$$\frac{-\frac{2}{3}a^2\left(-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}+\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{PolyLog}\left(2,\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)\right)}{\sqrt{a^2cx^2+c}}\right)-}{}$$

$$\Big\downarrow \text{5433}$$

$$-a^2\left(\frac{\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx}{c}-a^2\left(-2\int \frac{1}{(a^2cx^2+c)^{3/2}}dx+\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}\right)\right)+$$
$$-\frac{2}{3}a^2\left(-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}+\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{PolyLog}\left(2,\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)\right)}{\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \text{208}$$

$$-a^2\left(\frac{\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx}{c}-a^2\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)\right)+$$
$$-\frac{2}{3}a^2\left(-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}+\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{PolyLog}\left(2,\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)\right)}{\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \text{5479}$$

$$-a^2\left(\frac{2a\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}}{c}-a^2\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)\right)+$$
$$-\frac{2}{3}a^2\left(-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}+\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{PolyLog}\left(2,\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)\right)}{\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \text{5493}$$

$$-a^2\left(\frac{\frac{2a\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}}{c}-a^2\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)\right)+$$
$$-\frac{2}{3}a^2\left(-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}+\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{PolyLog}\left(2,\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)\right)}{\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \text{5489}$$

$$-\frac{2}{3}a^2\left(-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}+\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{PolyLog}\left(2,\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)\right)}{\sqrt{a^2cx^2+c}}\right) -$$

$$a^2\left(-a^2\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)+\frac{-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}+\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arcta}\right.}{}}{}\right.$$

input
```
Int[ArcTan[a*x]^2/(x^4*(c + a^2*c*x^2)^(3/2)),x]
```

output
```
-(a^2*(-(a^2*((-2*x)/(c*Sqrt[c + a^2*c*x^2]) + (2*ArcTan[a*x])/(a*c*Sqrt[c
 + a^2*c*x^2]) + (x*ArcTan[a*x]^2)/(c*Sqrt[c + a^2*c*x^2]))) + (-((Sqrt[c
+ a^2*c*x^2]*ArcTan[a*x]^2)/(c*x)) + (2*a*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x
]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]] + I*PolyLog[2, -(Sqrt[1 + I*a*x
]/Sqrt[1 - I*a*x])] - I*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]]))/Sqrt
[c + a^2*c*x^2])/c)) + (-1/3*(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(c*x^3) -
 (2*a^2*(-((Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(c*x)) + (2*a*Sqrt[1 + a^2*
x^2]*(-2*ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]] + I*PolyLog[
2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])] - I*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt
[1 - I*a*x]]))/Sqrt[c + a^2*c*x^2]))/3 + (2*a*(-1/2*(a*Sqrt[c + a^2*c*x^2]
)/(c*x) - (Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(2*c*x^2) - (a^2*Sqrt[1 + a^2*
x^2]*(-2*ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]] + I*PolyLog[
2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])] - I*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt
[1 - I*a*x]]))/(2*Sqrt[c + a^2*c*x^2])))/3)/c
```

## Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 5433
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x] - Simp[b^2*p*(p
- 1)   Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2), x], x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5489
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sq
rt[1 - I*c*x]], x] + (Simp[I*(b/Sqrt[d])*PolyLog[2, -Sqrt[1 + I*c*x]/Sqrt[1
 - I*c*x]], x] - Simp[I*(b/Sqrt[d])*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x]], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5497
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)^(m
 + 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[c^2*((m
+ 2)/(f^2*(m + 1)))   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x
^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
 && LtQ[m, -1] && NeQ[m, -2]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

## Maple [A] (verified)

Time = 2.18 (sec) , antiderivative size = 318, normalized size of antiderivative = 0.80

| method | result |
|---|---|
| default | $\frac{a^3\left(\arctan(ax)^2-2+2i\arctan(ax)\right)(ax-i)\sqrt{c(ax-i)(ax+i)}}{2(a^2x^2+1)c^2}+\frac{\sqrt{c(ax-i)(ax+i)}\,(ax+i)\left(\arctan(ax)^2-2-2i\arctan(ax)\right)a^3}{2(a^2x^2+1)c^2}+$ |

input
```
int(arctan(a*x)^2/x^4/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*a^3*(arctan(a*x)^2-2+2*I*arctan(a*x))*(a*x-I)*(c*(a*x-I)*(a*x+I))^(1/2
)/(a^2*x^2+1)/c^2+1/2*(c*(a*x-I)*(a*x+I))^(1/2)*(a*x+I)*(arctan(a*x)^2-2-2
*I*arctan(a*x))*a^3/(a^2*x^2+1)/c^2+1/3*(5*arctan(a*x)^2*x^2*a^2-a^2*x^2-a
rctan(a*x)*a*x-arctan(a*x)^2)*(c*(a*x-I)*(a*x+I))^(1/2)/x^3/c^2+11/3*I*a^3
*(I*arctan(a*x)*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*arctan(a*x)*ln(1+(1+I*
a*x)/(a^2*x^2+1)^(1/2))+polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-polylog(2,-
(1+I*a*x)/(a^2*x^2+1)^(1/2)))*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/2)/
c^2
```

## Fricas [F]

$$\int \frac{\arctan(ax)^2}{x^4\left(c+a^2cx^2\right)^{3/2}}\,dx=\int \frac{\arctan\left(ax\right)^2}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x^4}\,dx$$

input
```
integrate(arctan(a*x)^2/x^4/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2/(a^4*c^2*x^8 + 2*a^2*c^2*x^6 +
c^2*x^4), x)
```

## Sympy [F]

$$\int \frac{\arctan(ax)^2}{x^4 (c + a^2cx^2)^{3/2}} \, dx = \int \frac{\operatorname{atan}^2{(ax)}}{x^4 \left(c \left(a^2x^2 + 1\right)\right)^{\frac{3}{2}}} \, dx$$

input
```
integrate(atan(a*x)**2/x**4/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(atan(a*x)**2/(x**4*(c*(a**2*x**2 + 1))**(3/2)), x)
```

## Maxima [F]

$$\int \frac{\arctan(ax)^2}{x^4 (c + a^2cx^2)^{3/2}} \, dx = \int \frac{\arctan\left(ax\right)^2}{\left(a^2cx^2 + c\right)^{\frac{3}{2}} x^4} \, dx$$

input
```
integrate(arctan(a*x)^2/x^4/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)^2/((a^2*c*x^2 + c)^(3/2)*x^4), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^2}{x^4 (c + a^2cx^2)^{3/2}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^2/x^4/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^2}{x^4 (c + a^2cx^2)^{3/2}}\, dx = \int \frac{\mathrm{atan}(a\,x)^2}{x^4\,(c\,a^2\,x^2 + c)^{3/2}}\, dx$$

input
```
int(atan(a*x)^2/(x^4*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(atan(a*x)^2/(x^4*(c + a^2*c*x^2)^(3/2)), x)
```

## Reduce [F]

$$\int \frac{\arctan(ax)^2}{x^4 (c + a^2cx^2)^{3/2}}\, dx = \frac{\int \frac{atan(ax)^2}{\sqrt{a^2x^2+1}\,a^2x^6+\sqrt{a^2x^2+1}\,x^4}dx}{\sqrt{c}\,c}$$

input
```
int(atan(a*x)^2/x^4/(a^2*c*x^2+c)^(3/2),x)
```

output
```
int(atan(a*x)**2/(sqrt(a**2*x**2 + 1)*a**2*x**6 + sqrt(a**2*x**2 + 1)*x**4
),x)/(sqrt(c)*c)
```

# 3.347 $\int \frac{x^5 \arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 400

$$\begin{aligned}\int \frac{x^5 \arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx &= \frac{2}{27a^6c(c+a^2cx^2)^{3/2}} - \frac{32}{9a^6c^2\sqrt{c+a^2cx^2}} \\ &- \frac{2x^3\arctan(ax)}{9a^3c(c+a^2cx^2)^{3/2}} - \frac{10x\arctan(ax)}{3a^5c^2\sqrt{c+a^2cx^2}} + \frac{x^2\arctan(ax)^2}{3a^4c(c+a^2cx^2)^{3/2}} \\ &+ \frac{5\arctan(ax)^2}{3a^6c^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{a^6c^3} \\ &+ \frac{4i\sqrt{1+a^2x^2}\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a^6c^2\sqrt{c+a^2cx^2}} \\ &- \frac{2i\sqrt{1+a^2x^2}\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a^6c^2\sqrt{c+a^2cx^2}} + \frac{2i\sqrt{1+a^2x^2}\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a^6c^2\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
2/27/a^6/c/(a^2*c*x^2+c)^(3/2)-32/9/a^6/c^2/(a^2*c*x^2+c)^(1/2)-2/9*x^3*ar
ctan(a*x)/a^3/c/(a^2*c*x^2+c)^(3/2)-10/3*x*arctan(a*x)/a^5/c^2/(a^2*c*x^2+
c)^(1/2)+1/3*x^2*arctan(a*x)^2/a^4/c/(a^2*c*x^2+c)^(3/2)+5/3*arctan(a*x)^2
/a^6/c^2/(a^2*c*x^2+c)^(1/2)+(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2/a^6/c^3+4*I
*(a^2*x^2+1)^(1/2)*arctan(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/a^6
/c^2/(a^2*c*x^2+c)^(1/2)-2*I*(a^2*x^2+1)^(1/2)*polylog(2,-I*(1+I*a*x)^(1/2
)/(1-I*a*x)^(1/2))/a^6/c^2/(a^2*c*x^2+c)^(1/2)+2*I*(a^2*x^2+1)^(1/2)*polyl
og(2,I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/a^6/c^2/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (warning: unable to verify)

Time = 1.05 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.57

$$\int \frac{x^5 \arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = \frac{8(-95+\cos(2\arctan(ax))) - 9(1+a^2x^2)\arctan(ax)^2(-45-20\cos(2\arctan(ax)}{}$$

input
```
Integrate[(x^5*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(8*(-95 + Cos[2*ArcTan[a*x]]) - 9*(1 + a^2*x^2)*ArcTan[a*x]^2*(-45 - 20*Co
s[2*ArcTan[a*x]] + Cos[4*ArcTan[a*x]]) - (432*I)*Sqrt[1 + a^2*x^2]*PolyLog
[2, (-I)*E^(I*ArcTan[a*x])] + (432*I)*Sqrt[1 + a^2*x^2]*PolyLog[2, I*E^(I*
ArcTan[a*x])] + 6*ArcTan[a*x]*(-124*a*x - 72*Sqrt[1 + a^2*x^2]*Log[1 - I*E
^(I*ArcTan[a*x])] + 72*Sqrt[1 + a^2*x^2]*Log[1 + I*E^(I*ArcTan[a*x])] + (1
 + a^2*x^2)*Sin[4*ArcTan[a*x]]))/(216*a^6*c^2*Sqrt[c + a^2*c*x^2])
```

## Rubi [A] (verified)

Time = 2.54 (sec) , antiderivative size = 482, normalized size of antiderivative = 1.20, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5499, 5475, 243, 53, 2009, 5465, 5429, 5499, 5465, 5425, 5421, 5429}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 \arctan(ax)^2}{(a^2cx^2+c)^{5/2}}\,dx$$

$$\Big\downarrow \ 5499$$

$$\frac{\int \frac{x^3\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{a^2c} - \frac{\int \frac{x^3\arctan(ax)^2}{(a^2cx^2+c)^{5/2}}dx}{a^2}$$

$$\Big\downarrow \ 5475$$

$$\frac{\int \frac{x^3\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{a^2c} - \frac{\frac{2\int \frac{x\arctan(ax)^2}{\left(a^2cx^2+c\right)^{3/2}}dx}{3a^2c} - \frac{2}{9}\int \frac{x^3}{(a^2cx^2+c)^{5/2}}dx - \frac{x^2\arctan(ax)^2}{3a^2c(a^2cx^2+c)^{3/2}} + \frac{2x^3\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}}{a^2}$$

$$\Big\downarrow \text{243}$$

$$\frac{\int \frac{x^3\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{a^2c}-\frac{\frac{2\int \frac{x\arctan(ax)^2}{\left(a^2cx^2+c\right)^{3/2}}dx}{3a^2c}-\frac{1}{9}\int \frac{x^2}{(a^2cx^2+c)^{5/2}}dx^2-\frac{x^2\arctan(ax)^2}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2x^3\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}}{a^2}$$

$$\Big\downarrow \text{53}$$

$$\frac{\int \frac{x^3\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{a^2c}-\frac{\frac{2\int \frac{x\arctan(ax)^2}{\left(a^2cx^2+c\right)^{3/2}}dx}{3a^2c}-\frac{1}{9}\int \left(\frac{1}{a^2c(a^2cx^2+c)^{3/2}}-\frac{1}{a^2(a^2cx^2+c)^{5/2}}\right)dx^2-\frac{x^2\arctan(ax)^2}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2x^3\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}}{a^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\int \frac{x^3\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{a^2c}-\frac{\frac{2\int \frac{x\arctan(ax)^2}{\left(a^2cx^2+c\right)^{3/2}}dx}{3a^2c}-\frac{x^2\arctan(ax)^2}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2x^3\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}+\frac{1}{9}\left(\frac{2}{a^4c^2\sqrt{a^2cx^2+c}}-\frac{2}{3a^4c(a^2cx^2+c)^{3/2}}\right)}{a^2}$$

$$\Big\downarrow \text{5465}$$

$$\frac{\int \frac{x^3\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{a^2c}-\frac{\frac{2\left(\frac{2\int \frac{\arctan(ax)}{\left(a^2cx^2+c\right)^{3/2}}dx}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{x^2\arctan(ax)^2}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2x^3\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}+\frac{1}{9}\left(\frac{2}{a^4c^2\sqrt{a^2cx^2+c}}-\frac{2}{3a^4c(a^2cx^2+c)^{3/2}}\right)}{a^2}$$

$$\Big\downarrow \text{5429}$$

$$\frac{\int \frac{x^3\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{a^2c}-\frac{-\frac{x^2\arctan(ax)^2}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{2x^3\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}+\frac{1}{9}\left(\frac{2}{a^4c^2\sqrt{a^2cx^2+c}}-\frac{2}{3a^4c(a^2cx^2+c)^{3/2}}\right)}{a^2}$$

$$\Big\downarrow \text{5499}$$

$$\frac{\frac{\int \frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a^2c}-\frac{\int \frac{x\arctan(ax)^2}{\left(a^2cx^2+c\right)^{3/2}}dx}{a^2}}{a^2c}-\frac{-\frac{x^2\arctan(ax)^2}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{2x^3\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}+\frac{1}{9}\left(\frac{2}{a^4c^2\sqrt{a^2cx^2+c}}-\frac{2}{3a^4c(a^2cx^2+c)^{3/2}}\right.}{a^2}$$

$\big\downarrow$ 5465

$$\frac{\frac{\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}}{a^2c}-\frac{\frac{2\int \frac{\arctan(ax)}{\left(a^2cx^2+c\right)^{3/2}}dx}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-\frac{-\frac{x^2\arctan(ax)^2}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{2x^3\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}+\frac{1}{9}\left(\frac{2}{a^4c^2\sqrt{a^2cx^2+c}}-\frac{2}{3a^4c(a^2cx^2+c)^{3/2}}\right.}{a^2}$$

$\big\downarrow$ 5425

$$\frac{\frac{\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{a\sqrt{a^2cx^2+c}}}{a^2c}-\frac{\frac{2\int \frac{\arctan(ax)}{\left(a^2cx^2+c\right)^{3/2}}dx}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-\frac{-\frac{x^2\arctan(ax)^2}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{2x^3\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}+\frac{1}{9}\left(\frac{2}{a^4c^2\sqrt{a^2cx^2+c}}-\frac{2}{3a^4c(a^2cx^2+c)^{3/2}}\right.}{a^2}$$

$\big\downarrow$ 5421

$$-\frac{-\frac{x^2\arctan(ax)^2}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{2x^3\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}+\frac{1}{9}\left(\frac{2}{a^4c^2\sqrt{a^2cx^2+c}}-\frac{2}{3a^4c(a^2cx^2+c)^{3}}\right.}{a^2}$$
$$\frac{-\frac{\frac{2\int \frac{\arctan(ax)}{\left(a^2cx^2+c\right)^{3/2}}dx}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}}{a^2}+\frac{\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\mathrm{PolyLog}}{}\right.}{a\sqrt{a^2cx^2+c}}}{a^2c}}{a^2c}$$

$\big\downarrow$ 5429

$$-\frac{-\frac{x^2\arctan(ax)^2}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{2x^3\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}+\frac{1}{9}\left(\frac{2}{a^4c^2\sqrt{a^2cx^2+c}}-\frac{2}{3a^4c(a^2cx^2+c)^{3/2}}\right)}{a^2}$$

$$\frac{-\frac{\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}}{a^2}+\frac{\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right.}{a\sqrt{a^2cx^2+c}}}{a^2c}}{a^2c}$$

input
```
Int[(x^5*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(5/2),x]
```

output
```
-(((-2/(3*a^4*c*(c + a^2*c*x^2)^(3/2)) + 2/(a^4*c^2*Sqrt[c + a^2*c*x^2]))/
9 + (2*x^3*ArcTan[a*x])/(9*a*c*(c + a^2*c*x^2)^(3/2)) - (x^2*ArcTan[a*x]^2
)/(3*a^2*c*(c + a^2*c*x^2)^(3/2)) + (2*(-(ArcTan[a*x]^2/(a^2*c*Sqrt[c + a^
2*c*x^2])) + (2*(1/(a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x])/(c*Sqrt[c +
 a^2*c*x^2])))/a))/(3*a^2*c))/a^2) + (-((-(ArcTan[a*x]^2/(a^2*c*Sqrt[c + a
^2*c*x^2])) + (2*(1/(a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x])/(c*Sqrt[c
+ a^2*c*x^2])))/a)/a^2) + ((Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(a^2*c) - (
2*Sqrt[1 + a^2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I
*a*x]])/a + (I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*
PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/(a*Sqrt[c + a^2*c*x^2
]))/(a^2*c))/(a^2*c)
```

## Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5429
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTan[c*x])/(d*Sqr
t[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5475
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_)*((d_) + (e_.)
*(x_)^2)^(q_), x_Symbol] :> Simp[b*p*(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*Ar
cTan[c*x])^(p - 1)/(c*d*m^2)), x] + (-Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(q +
 1)*((a + b*ArcTan[c*x])^p/(c^2*d*m)), x] + Simp[f^2*((m - 1)/(c^2*d*m))
Int[(f*x)^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p, x], x] - Simp[
b^2*p*((p - 1)/m^2)   Int[(f*x)^m*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2)
, x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && EqQ[m + 2*
q + 2, 0] && LtQ[q, -1] && GtQ[p, 1]
```

rule 5499
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ
[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

## Maple [A] (verified)

Time = 2.33 (sec) , antiderivative size = 454, normalized size of antiderivative = 1.14

| method | result |
|---|---|
| default | $\frac{\left(6i\arctan(ax)+9\arctan(ax)^2-2\right)\left(i a^3x^3+3a^2x^2-3iax-1\right)\sqrt{c(ax-i)(ax+i)}}{216(a^2x^2+1)^2a^6c^3}+\frac{7\left(\arctan(ax)^2-2+2i\arctan(ax)\right)(iax+1)\sqrt{c(a}{8c^3a^6(a^2x^2+1)}$ |

input
```
int(x^5*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/216*(6*I*arctan(a*x)+9*arctan(a*x)^2-2)*(I*a^3*x^3+3*a^2*x^2-3*I*a*x-1)*
(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^2/a^6/c^3+7/8*(arctan(a*x)^2-2+2*I*a
rctan(a*x))*(1+I*a*x)*(c*(a*x-I)*(a*x+I))^(1/2)/c^3/a^6/(a^2*x^2+1)-7/8*(c
*(a*x-I)*(a*x+I))^(1/2)*(I*a*x-1)*(arctan(a*x)^2-2-2*I*arctan(a*x))/c^3/a^
6/(a^2*x^2+1)-1/216*(c*(a*x-I)*(a*x+I))^(1/2)*(I*a^3*x^3-3*a^2*x^2-3*I*a*x
+1)*(-6*I*arctan(a*x)+9*arctan(a*x)^2-2)/c^3/a^6/(a^4*x^4+2*a^2*x^2+1)+arc
tan(a*x)^2*(c*(a*x-I)*(a*x+I))^(1/2)/a^6/c^3+2*(arctan(a*x)*ln(1+I*(1+I*a*
x)/(a^2*x^2+1)^(1/2))-arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+I*di
log(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/
2)))*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/2)/c^3/a^6
```

## Fricas [F]

$$\int \frac{x^5\arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx=\int \frac{x^5\arctan\left(ax\right)^2}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^5*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^5*arctan(a*x)^2/(a^6*c^3*x^6 + 3*a^4*c^3*x^
4 + 3*a^2*c^3*x^2 + c^3), x)
```

## Sympy [F]

$$\int \frac{x^5 \arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^5 \operatorname{atan}^2(ax)}{(c(a^2x^2+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**5*atan(a*x)**2/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(x**5*atan(a*x)**2/(c*(a**2*x**2 + 1))**(5/2), x)
```

## Maxima [F]

$$\int \frac{x^5 \arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^5 \arctan(ax)^2}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^5*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(x^5*arctan(a*x)^2/(a^2*c*x^2 + c)^(5/2), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x^5 \arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^5*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^5 \arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^5\,\mathrm{atan}(a\,x)^2}{(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int((x^5*atan(a*x)^2)/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int((x^5*atan(a*x)^2)/(c + a^2*c*x^2)^(5/2), x)
```

## Reduce [F]

$$\int \frac{x^5 \arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = \frac{\int \frac{atan(ax)^2x^5}{\sqrt{a^2x^2+1}\,a^4x^4+2\sqrt{a^2x^2+1}\,a^2x^2+\sqrt{a^2x^2+1}}dx}{\sqrt{c}\,c^2}$$

input
```
int(x^5*atan(a*x)^2/(a^2*c*x^2+c)^(5/2),x)
```

output
```
int((atan(a*x)**2*x**5)/(sqrt(a**2*x**2 + 1)*a**4*x**4 + 2*sqrt(a**2*x**2
+ 1)*a**2*x**2 + sqrt(a**2*x**2 + 1)),x)/(sqrt(c)*c**2)
```

# 3.348 $\int \frac{x^4 \arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 444

$$
\begin{aligned}
\int \frac{x^4 \arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx &= \frac{2x^3}{27a^2c(c+a^2cx^2)^{3/2}} + \frac{22x}{9a^4c^2\sqrt{c+a^2cx^2}} \\
&- \frac{2x^2\arctan(ax)}{9a^3c(c+a^2cx^2)^{3/2}} - \frac{22\arctan(ax)}{9a^5c^2\sqrt{c+a^2cx^2}} - \frac{x^3\arctan(ax)^2}{3a^2c(c+a^2cx^2)^{3/2}} \\
&- \frac{x\arctan(ax)^2}{a^4c^2\sqrt{c+a^2cx^2}} - \frac{2i\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2}{a^5c^2\sqrt{c+a^2cx^2}} \\
&+ \frac{2i\sqrt{1+a^2x^2}\arctan(ax)\,\mathrm{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)}{a^5c^2\sqrt{c+a^2cx^2}} \\
&- \frac{2i\sqrt{1+a^2x^2}\arctan(ax)\,\mathrm{PolyLog}\left(2,ie^{i\arctan(ax)}\right)}{a^5c^2\sqrt{c+a^2cx^2}} \\
&- \frac{2\sqrt{1+a^2x^2}\,\mathrm{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)}{a^5c^2\sqrt{c+a^2cx^2}} \\
&+ \frac{2\sqrt{1+a^2x^2}\,\mathrm{PolyLog}\left(3,ie^{i\arctan(ax)}\right)}{a^5c^2\sqrt{c+a^2cx^2}}
\end{aligned}
$$

output
```
2/27*x^3/a^2/c/(a^2*c*x^2+c)^(3/2)+22/9*x/a^4/c^2/(a^2*c*x^2+c)^(1/2)-2/9*
x^2*arctan(a*x)/a^3/c/(a^2*c*x^2+c)^(3/2)-22/9*arctan(a*x)/a^5/c^2/(a^2*c*
x^2+c)^(1/2)-1/3*x^3*arctan(a*x)^2/a^2/c/(a^2*c*x^2+c)^(3/2)-x*arctan(a*x)
^2/a^4/c^2/(a^2*c*x^2+c)^(1/2)-2*I*(a^2*x^2+1)^(1/2)*arctan((1+I*a*x)/(a^2
*x^2+1)^(1/2))*arctan(a*x)^2/a^5/c^2/(a^2*c*x^2+c)^(1/2)+2*I*(a^2*x^2+1)^(
1/2)*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a^5/c^2/(a^2*c*
x^2+c)^(1/2)-2*I*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*
x^2+1)^(1/2))/a^5/c^2/(a^2*c*x^2+c)^(1/2)-2*(a^2*x^2+1)^(1/2)*polylog(3,-I
*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a^5/c^2/(a^2*c*x^2+c)^(1/2)+2*(a^2*x^2+1)^(1
/2)*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a^5/c^2/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.54

$$\int \frac{x^4 \arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = \frac{\sqrt{c(1+a^2x^2)}\left(-\frac{270\arctan(ax)}{\sqrt{1+a^2x^2}} - \frac{135ax(-2+\arctan(ax)^2)}{\sqrt{1+a^2x^2}} + 6\arctan(ax)\cos(3\arctan(ax$$

input
```
Integrate[(x^4*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(Sqrt[c*(1 + a^2*x^2)]*((-270*ArcTan[a*x])/Sqrt[1 + a^2*x^2] - (135*a*x*(-
2 + ArcTan[a*x]^2))/Sqrt[1 + a^2*x^2] + 6*ArcTan[a*x]*Cos[3*ArcTan[a*x]] +
 108*ArcTan[a*x]^2*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a
*x])]) + (216*I)*ArcTan[a*x]*(PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - PolyLog
[2, I*E^(I*ArcTan[a*x])]) - 216*(PolyLog[3, (-I)*E^(I*ArcTan[a*x])] - Poly
Log[3, I*E^(I*ArcTan[a*x])]) + (-2 + 9*ArcTan[a*x]^2)*Sin[3*ArcTan[a*x]]))
/(108*a^5*c^3*Sqrt[1 + a^2*x^2])
```

## Rubi [A] (verified)

Time = 2.66 (sec) , antiderivative size = 391, normalized size of antiderivative = 0.88, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {5499, 5479, 5473, 5465, 208, 5499, 5425, 5423, 3042, 4669, 3011, 2720, 5433, 208, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\arctan(ax)^2}{(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{ 5499}$$

$$\frac{\int \frac{x^2\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{a^2c}-\frac{\int \frac{x^2\arctan(ax)^2}{(a^2cx^2+c)^{5/2}}dx}{a^2}$$

$$\Big\downarrow \text{ 5479}$$

$$\frac{\int \frac{x^2\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{a^2c}-\frac{\frac{x^3\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}}-\frac{2}{3}a\int \frac{x^3\arctan(ax)}{(a^2cx^2+c)^{5/2}}dx}{a^2}$$

$$\Big\downarrow \text{ 5473}$$

$$\frac{\int \frac{x^2\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{a^2c}-\frac{\frac{x^3\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}}-\frac{2}{3}a\left(\frac{2\int \frac{x\arctan(ax)}{(a^2cx^2+c)^{3/2}}dx}{3a^2c}-\frac{x^2\arctan(ax)}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{x^3}{9ac(a^2cx^2+c)^{3/2}}\right)}{a^2}$$

$$\Big\downarrow \text{ 5465}$$

$$\frac{\int \frac{x^2\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{a^2c}-\frac{\frac{x^3\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}}-\frac{2}{3}a\left(\frac{2\left(\frac{\int \frac{1}{(a^2cx^2+c)^{3/2}}dx}{a}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{x^2\arctan(ax)}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{x^3}{9ac(a^2cx^2+c)^{3/2}}\right)}{a^2}$$

$$\Big\downarrow \text{ 208}$$

$$\frac{\int \frac{x^2\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{a^2c}-\frac{\frac{x^3\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}}-\frac{2}{3}a\left(-\frac{x^2\arctan(ax)}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x}{ac\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{x^3}{9ac(a^2cx^2+c)^{3/2}}\right)}{a^2}$$

$$\Big\downarrow \text{ 5499}$$

$$\frac{\frac{\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a^2c}-\frac{\int \frac{\arctan(ax)^2}{\left(a^2cx^2+c\right)^{3/2}}dx}{a^2}}{a^2c}-\frac{\frac{x^3\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}}-\frac{2}{3}a\left(-\frac{x^2\arctan(ax)}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x}{ac\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{x^3}{9ac(a^2cx^2+c)^{3/2}}\right)}{a^2}$$

$$\Big\downarrow \; 5425$$

$$\frac{\frac{\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{a^2c\sqrt{a^2cx^2+c}}-\frac{\int \frac{\arctan(ax)^2}{\left(a^2cx^2+c\right)^{3/2}}dx}{a^2}}{a^2c}-\frac{\frac{x^3\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}}-\frac{2}{3}a\left(-\frac{x^2\arctan(ax)}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x}{ac\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{x^3}{9ac(a^2cx^2+c)^{3/2}}\right)}{a^2}$$

$$\Big\downarrow \; 5423$$

$$\frac{\frac{\sqrt{a^2x^2+1}\int \sqrt{a^2x^2+1}\arctan(ax)^2d\arctan(ax)}{a^3c\sqrt{a^2cx^2+c}}-\frac{\int \frac{\arctan(ax)^2}{\left(a^2cx^2+c\right)^{3/2}}dx}{a^2}}{a^2c}-\frac{\frac{x^3\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}}-\frac{2}{3}a\left(-\frac{x^2\arctan(ax)}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x}{ac\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{x^3}{9ac(a^2cx^2+c)^{3/2}}\right)}{a^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{a^3c\sqrt{a^2cx^2+c}}-\frac{\int \frac{\arctan(ax)^2}{\left(a^2cx^2+c\right)^{3/2}}dx}{a^2}}{a^2c}-\frac{\frac{x^3\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}}-\frac{2}{3}a\left(-\frac{x^2\arctan(ax)}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x}{ac\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{x^3}{9ac(a^2cx^2+c)^{3/2}}\right)}{a^2}$$

$$\Big\downarrow \; 4669$$

$$-\frac{\frac{x^3\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}}-\frac{2}{3}a\left(-\frac{x^2\arctan(ax)}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x}{ac\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{x^3}{9ac(a^2cx^2+c)^{3/2}}\right)}{a^2}+\frac{-\frac{\int \frac{\arctan(ax)^2}{\left(a^2cx^2+c\right)^{3/2}}dx}{a^2}+\frac{\sqrt{a^2x^2+1}\left(-2\int \arctan(ax)\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+2\int \arctan(ax)\log\left(1+ie^{i\arctan(ax)}\right)d\arctan(ax)-2i\,\mathrm{arc}\right.}{a^3c\sqrt{a^2cx^2+c}}}{a^2c}$$

$\Big\downarrow$ 3011

$$-\frac{\frac{x^3\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}}-\frac{2}{3}a\left(-\frac{x^2\arctan(ax)}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x}{ac\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{x^3}{9ac(a^2cx^2+c)^{3/2}}\right)}{a^2}+\frac{-\frac{\int\frac{\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{a^2}+\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-i\int\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(ax)\right)-2\left(i\arctan(ax)\operatorname{Pol}\right.}{a^3c\sqrt{a^2cx^2+c}}}{a^2c}$$

$\Big\downarrow$ 2720

$$-\frac{\frac{x^3\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}}-\frac{2}{3}a\left(-\frac{x^2\arctan(ax)}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x}{ac\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{x^3}{9ac(a^2cx^2+c)^{3/2}}\right)}{a^2}+\frac{-\frac{\int\frac{\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{a^2}+\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)-2\left(i\,\mathrm{ar}\right.}{a}}{a^2c}$$

$\Big\downarrow$ 5433

$$-\frac{\frac{x^3\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}}-\frac{2}{3}a\left(-\frac{x^2\arctan(ax)}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x}{ac\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{x^3}{9ac(a^2cx^2+c)^{3/2}}\right)}{a^2}+\frac{-\frac{-2\int\frac{1}{(a^2cx^2+c)^{3/2}}dx+\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}}{a^2}+\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i}\right.\right.\right.}{}$$

$\Big\downarrow$ 208

$$-\frac{\frac{x^3\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}}-\frac{2}{3}a\left(-\frac{x^2\arctan(ax)}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x}{ac\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{x^3}{9ac(a^2cx^2+c)^{3/2}}\right)}{a^2}+\frac{-\frac{\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}}{a^2}+\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)\right.\right.}{}$$

$\Big\downarrow$ 7143

$$-\frac{\frac{x^3\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}}-\frac{2}{3}a\left(-\frac{x^2\arctan(ax)}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x}{ac\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{x^3}{9ac(a^2cx^2+c)^{3/2}}\right)}{a^2}+\frac{-\frac{\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}}{a^2}+\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)\right)-2\left(i\arctan(a\right.\right.}{a^3c\sqrt{a^2c}}}{a^2c}$$

input
```
Int[(x^4*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(5/2),x]
```

output
```
-(((x^3*ArcTan[a*x]^2)/(3*c*(c + a^2*c*x^2)^(3/2)) - (2*a*(x^3/(9*a*c*(c +
 a^2*c*x^2)^(3/2)) - (x^2*ArcTan[a*x])/(3*a^2*c*(c + a^2*c*x^2)^(3/2)) + (
2*(x/(a*c*Sqrt[c + a^2*c*x^2]) - ArcTan[a*x]/(a^2*c*Sqrt[c + a^2*c*x^2])))
/(3*a^2*c)))/3)/a^2) + (-(((-2*x)/(c*Sqrt[c + a^2*c*x^2]) + (2*ArcTan[a*x]
)/(a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x]^2)/(c*Sqrt[c + a^2*c*x^2]))/a
^2) + (Sqrt[1 + a^2*x^2]*((-2*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 +
 2*(I*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - PolyLog[3, (-I)*E^(
I*ArcTan[a*x])]) - 2*(I*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] - Poly
Log[3, I*E^(I*ArcTan[a*x])])))/(a^3*c*Sqrt[c + a^2*c*x^2]))/(a^2*c)
```

### Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5433
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x] - Simp[b^2*p*(p
- 1)   Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2), x], x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5473
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[b*(f*x)^m*((d + e*x^2)^(q + 1)/(c*d*m^2)), x] +
 (-Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(c^2*d*m))
, x] + Simp[f^2*((m - 1)/(c^2*d*m))   Int[(f*x)^(m - 2)*(d + e*x^2)^(q + 1)
*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2
*d] && EqQ[m + 2*q + 2, 0] && LtQ[q, -1]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5499
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ
[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

## Maple [F]

$$\int \frac{x^4 \arctan (ax)^2}{(a^2c\,x^2+c)^{\frac{5}{2}}}dx$$

input
```
int(x^4*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x)
```

output
```
int(x^4*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x)
```

## Fricas [F]

$$\int \frac{x^4 \arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^4 \arctan (ax)^2}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^4*arctan(a*x)^2/(a^6*c^3*x^6 + 3*a^4*c^3*x^
4 + 3*a^2*c^3*x^2 + c^3), x)
```

## Sympy [F]

$$\int \frac{x^4 \arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^4 \operatorname{atan}^2(ax)}{(c(a^2x^2+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**4*atan(a*x)**2/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(x**4*atan(a*x)**2/(c*(a**2*x**2 + 1))**(5/2), x)
```

## Maxima [F]

$$\int \frac{x^4 \arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^4 \arctan(ax)^2}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(x^4*arctan(a*x)^2/(a^2*c*x^2 + c)^(5/2), x)
```

## Giac [F]

$$\int \frac{x^4 \arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^4 \arctan(ax)^2}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
integrate(x^4*arctan(a*x)^2/(a^2*c*x^2 + c)^(5/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^4 \arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^4\,\mathrm{atan}(a\,x)^2}{(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int((x^4*atan(a*x)^2)/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int((x^4*atan(a*x)^2)/(c + a^2*c*x^2)^(5/2), x)
```

## Reduce [F]

$$\int \frac{x^4 \arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = \frac{\int \frac{atan(ax)^2 x^4}{\sqrt{a^2x^2+1}\,a^4x^4+2\sqrt{a^2x^2+1}\,a^2x^2+\sqrt{a^2x^2+1}}dx}{\sqrt{c}\,c^2}$$

input
```
int(x^4*atan(a*x)^2/(a^2*c*x^2+c)^(5/2),x)
```

output
```
int((atan(a*x)**2*x**4)/(sqrt(a**2*x**2 + 1)*a**4*x**4 + 2*sqrt(a**2*x**2
+ 1)*a**2*x**2 + sqrt(a**2*x**2 + 1)),x)/(sqrt(c)*c**2)
```

# 3.349 $\int \frac{x^3 \arctan(ax)^2}{(c+a^2cx^2)^{5/2}} \, dx$



## Optimal result

Integrand size = 24, antiderivative size = 172

$$\int \frac{x^3 \arctan(ax)^2}{(c+a^2cx^2)^{5/2}} \, dx = -\frac{2}{27a^4c(c+a^2cx^2)^{3/2}} + \frac{14}{9a^4c^2\sqrt{c+a^2cx^2}} + \frac{2x^3\arctan(ax)}{9ac(c+a^2cx^2)^{3/2}} + \frac{4x\arctan(ax)}{3a^3c^2\sqrt{c+a^2cx^2}} - \frac{x^2\arctan(ax)^2}{3a^2c(c+a^2cx^2)^{3/2}} - \frac{2\arctan(ax)^2}{3a^4c^2\sqrt{c+a^2cx^2}}$$

output
```
-2/27/a^4/c/(a^2*c*x^2+c)^(3/2)+14/9/a^4/c^2/(a^2*c*x^2+c)^(1/2)+2/9*x^3*a
rctan(a*x)/a/c/(a^2*c*x^2+c)^(3/2)+4/3*x*arctan(a*x)/a^3/c^2/(a^2*c*x^2+c)
^(1/2)-1/3*x^2*arctan(a*x)^2/a^2/c/(a^2*c*x^2+c)^(3/2)-2/3*arctan(a*x)^2/a
^4/c^2/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.47

$$\int \frac{x^3 \arctan(ax)^2}{(c+a^2cx^2)^{5/2}} \, dx = \frac{\sqrt{c+a^2cx^2}(40+42a^2x^2+6ax(6+7a^2x^2)\arctan(ax)-9(2+3a^2x^2)\arctan(ax)^2}{27a^4c^3(1+a^2x^2)^2}$$

input
```
Integrate[(x^3*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(Sqrt[c + a^2*c*x^2]*(40 + 42*a^2*x^2 + 6*a*x*(6 + 7*a^2*x^2)*ArcTan[a*x]
- 9*(2 + 3*a^2*x^2)*ArcTan[a*x]^2))/(27*a^4*c^3*(1 + a^2*x^2)^2)
```

## Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.19, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5475, 243, 53, 2009, 5465, 5429}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \arctan(ax)^2}{(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{5475}$$

$$\frac{2\int \frac{x\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{3a^2c} - \frac{2}{9}\int \frac{x^3}{(a^2cx^2+c)^{5/2}}dx - \frac{x^2\arctan(ax)^2}{3a^2c\,(a^2cx^2+c)^{3/2}} + \frac{2x^3\arctan(ax)}{9ac\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{243}$$

$$\frac{2\int \frac{x\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{3a^2c} - \frac{1}{9}\int \frac{x^2}{(a^2cx^2+c)^{5/2}}dx^2 - \frac{x^2\arctan(ax)^2}{3a^2c\,(a^2cx^2+c)^{3/2}} + \frac{2x^3\arctan(ax)}{9ac\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{53}$$

$$\frac{2\int \frac{x\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{3a^2c} - \frac{1}{9}\int \left(\frac{1}{a^2c\,(a^2cx^2+c)^{3/2}} - \frac{1}{a^2\,(a^2cx^2+c)^{5/2}}\right)dx^2 - \frac{x^2\arctan(ax)^2}{3a^2c\,(a^2cx^2+c)^{3/2}} + \frac{2x^3\arctan(ax)}{9ac\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\int \frac{x\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{3a^2c} - \frac{x^2\arctan(ax)^2}{3a^2c\,(a^2cx^2+c)^{3/2}} + \frac{2x^3\arctan(ax)}{9ac\,(a^2cx^2+c)^{3/2}} + \frac{1}{9}\left(\frac{2}{a^4c^2\sqrt{a^2cx^2+c}} - \frac{2}{3a^4c\,(a^2cx^2+c)^{3/2}}\right)$$

$$\Big\downarrow \text{5465}$$

$$\frac{2\left(\frac{2\int\frac{\arctan(ax)}{(a^2cx^2+c)^{3/2}}dx}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{x^2\arctan(ax)^2}{3a^2c\,(a^2cx^2+c)^{3/2}}+\frac{2x^3\arctan(ax)}{9ac\,(a^2cx^2+c)^{3/2}}+\frac{1}{9}\left(\frac{2}{a^4c^2\sqrt{a^2cx^2+c}}-\frac{2}{3a^4c\,(a^2cx^2+c)^{3/2}}\right)$$

$$\Big\downarrow \text{5429}$$

$$-\frac{x^2\arctan(ax)^2}{3a^2c\,(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{2x^3\arctan(ax)}{9ac\,(a^2cx^2+c)^{3/2}}+\frac{1}{9}\left(\frac{2}{a^4c^2\sqrt{a^2cx^2+c}}-\frac{2}{3a^4c\,(a^2cx^2+c)^{3/2}}\right)$$

input
```
Int[(x^3*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(-2/(3*a^4*c*(c + a^2*c*x^2)^(3/2)) + 2/(a^4*c^2*Sqrt[c + a^2*c*x^2]))/9 +
 (2*x^3*ArcTan[a*x])/(9*a*c*(c + a^2*c*x^2)^(3/2)) - (x^2*ArcTan[a*x]^2)/(
3*a^2*c*(c + a^2*c*x^2)^(3/2)) + (2*(-(ArcTan[a*x]^2/(a^2*c*Sqrt[c + a^2*c
*x^2])) + (2*(1/(a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x])/(c*Sqrt[c + a^
2*c*x^2])))/a))/(3*a^2*c)
```

## Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5429
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTan[c*x])/(d*Sqr
t[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5475
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_)*((d_) + (e_.)
*(x_)^2)^(q_), x_Symbol] :> Simp[b*p*(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*Ar
cTan[c*x])^(p - 1)/(c*d*m^2)), x] + (-Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(q +
 1)*((a + b*ArcTan[c*x])^p/(c^2*d*m)), x] + Simp[f^2*((m - 1)/(c^2*d*m))
Int[(f*x)^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p, x], x] - Simp[
b^2*p*((p - 1)/m^2)   Int[(f*x)^m*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2)
, x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && EqQ[m + 2*
q + 2, 0] && LtQ[q, -1] && GtQ[p, 1]
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.90 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.60

| method | result |
|---|---|
| default | $-\frac{\left(6i\arctan(ax)+9\arctan(ax)^2-2\right)\left(ia^3x^3+3a^2x^2-3iax-1\right)\sqrt{c(ax-i)(ax+i)}}{216(a^2x^2+1)^2a^4c^3}-\frac{3\left(\arctan(ax)^2-2+2i\arctan(ax)\right)(iax+1)\sqrt{c}}{8c^3a^4(a^2x^2+1)}$ |
| orering | $\frac{(a^2x^2+1)\left(168a^6x^6-81a^4x^4+40a^2x^2+240\right)\arctan(ax)^2}{27a^6x^2(a^2c\,x^2+c)^{\frac{5}{2}}}+\frac{(a^2x^2+1)^2\left(49a^4x^4+4a^2x^2-40\right)\left(\frac{3x^2\arctan(ax)^2}{(a^2c\,x^2+c)^{\frac{5}{2}}}+\frac{2x^3\arctan(ax)}{(a^2c\,x^2+c)^{\frac{5}{2}}(a^2x}\right.}{9a^6x^4}$ |

input
```
int(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/216*(6*I*arctan(a*x)+9*arctan(a*x)^2-2)*(I*a^3*x^3+3*a^2*x^2-3*I*a*x-1)
*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^2/a^4/c^3-3/8*(arctan(a*x)^2-2+2*I*
arctan(a*x))*(1+I*a*x)*(c*(a*x-I)*(a*x+I))^(1/2)/c^3/a^4/(a^2*x^2+1)+3/8*(
c*(a*x-I)*(a*x+I))^(1/2)*(I*a*x-1)*(arctan(a*x)^2-2-2*I*arctan(a*x))/c^3/a
^4/(a^2*x^2+1)+1/216*(c*(a*x-I)*(a*x+I))^(1/2)*(I*a^3*x^3-3*a^2*x^2-3*I*a*
x+1)*(-6*I*arctan(a*x)+9*arctan(a*x)^2-2)/c^3/a^4/(a^4*x^4+2*a^2*x^2+1)
```

## Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.53

$$\int \frac{x^3 \arctan(ax)^2}{\left(c+a^2cx^2\right)^{5/2}}\,dx = \frac{\sqrt{a^2cx^2+c}\left(42\,a^2x^2 - 9\left(3\,a^2x^2+2\right)\arctan\left(ax\right)^2 + 6\left(7\,a^3x^3+6\,ax\right)\arctan\left(ax\right) + \right.}{27\left(a^8c^3x^4+2\,a^6c^3x^2+a^4c^3\right)}$$

input
```
integrate(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
1/27*sqrt(a^2*c*x^2 + c)*(42*a^2*x^2 - 9*(3*a^2*x^2 + 2)*arctan(a*x)^2 + 6
*(7*a^3*x^3 + 6*a*x)*arctan(a*x) + 40)/(a^8*c^3*x^4 + 2*a^6*c^3*x^2 + a^4*
c^3)
```

## Sympy [F]

$$\int \frac{x^3 \arctan(ax)^2}{\left(c+a^2cx^2\right)^{5/2}}\,dx = \int \frac{x^3 \operatorname{atan}^2\left(ax\right)}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**3*atan(a*x)**2/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(x**3*atan(a*x)**2/(c*(a**2*x**2 + 1))**(5/2), x)
```

## Maxima [F]

$$\int \frac{x^3 \arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^3 \arctan(ax)^2}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(x^3*arctan(a*x)^2/(a^2*c*x^2 + c)^(5/2), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x^3 \arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^3 \operatorname{atan}(a\,x)^2}{(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int((x^3*atan(a*x)^2)/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int((x^3*atan(a*x)^2)/(c + a^2*c*x^2)^(5/2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.37

$$\int \frac{x^3 \arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = \frac{\sqrt{c}\left(-84atan\left(\sqrt{a^2x^2+1}+ax\right)a^4x^4-168atan\left(\sqrt{a^2x^2+1}+ax\right)a^2x^2-84atan\left(\right.\right.}{}$$

input
```
int(x^3*atan(a*x)^2/(a^2*c*x^2+c)^(5/2),x)
```

output
```
(sqrt(c)*( - 84*atan(sqrt(a**2*x**2 + 1) + a*x)*a**4*x**4 - 168*atan(sqrt(
a**2*x**2 + 1) + a*x)*a**2*x**2 - 84*atan(sqrt(a**2*x**2 + 1) + a*x) - 27*
sqrt(a**2*x**2 + 1)*atan(a*x)**2*a**2*x**2 - 18*sqrt(a**2*x**2 + 1)*atan(a
*x)**2 + 42*sqrt(a**2*x**2 + 1)*atan(a*x)*a**3*x**3 + 36*sqrt(a**2*x**2 +
1)*atan(a*x)*a*x + 42*atan(a*x)*a**4*x**4 + 84*atan(a*x)*a**2*x**2 + 42*at
an(a*x) + 42*sqrt(a**2*x**2 + 1)*a**2*x**2 + 40*sqrt(a**2*x**2 + 1)))/(27*
a**4*c**3*(a**4*x**4 + 2*a**2*x**2 + 1))
```

# 3.350 $\int \frac{x^2 \arctan(ax)^2}{(c+a^2cx^2)^{5/2}} dx$



## Optimal result

Integrand size = 24, antiderivative size = 139

$$\int \frac{x^2 \arctan(ax)^2}{(c+a^2cx^2)^{5/2}} dx = -\frac{2x^3}{27c(c+a^2cx^2)^{3/2}} - \frac{4x}{9a^2c^2\sqrt{c+a^2cx^2}} + \frac{2x^2\arctan(ax)}{9ac(c+a^2cx^2)^{3/2}} + \frac{4\arctan(ax)}{9a^3c^2\sqrt{c+a^2cx^2}} + \frac{x^3\arctan(ax)^2}{3c(c+a^2cx^2)^{3/2}}$$

output
```
-2/27*x^3/c/(a^2*c*x^2+c)^(3/2)-4/9*x/a^2/c^2/(a^2*c*x^2+c)^(1/2)+2/9*x^2*
arctan(a*x)/a/c/(a^2*c*x^2+c)^(3/2)+4/9*arctan(a*x)/a^3/c^2/(a^2*c*x^2+c)^
(1/2)+1/3*x^3*arctan(a*x)^2/c/(a^2*c*x^2+c)^(3/2)
```

## Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.58

$$\int \frac{x^2 \arctan(ax)^2}{(c+a^2cx^2)^{5/2}} dx = \frac{\sqrt{c+a^2cx^2}(-2ax(6+7a^2x^2)+6(2+3a^2x^2)\arctan(ax)+9a^3x^3\arctan(ax)^2)}{27a^3c^3(1+a^2x^2)^2}$$

input
```
Integrate[(x^2*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(Sqrt[c + a^2*c*x^2]*(-2*a*x*(6 + 7*a^2*x^2) + 6*(2 + 3*a^2*x^2)*ArcTan[a*
x] + 9*a^3*x^3*ArcTan[a*x]^2))/(27*a^3*c^3*(1 + a^2*x^2)^2)
```

## Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5479, 5473, 5465, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \arctan(ax)^2}{\left(a^2cx^2+c\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 5479}$$

$$\frac{x^3 \arctan(ax)^2}{3c\left(a^2cx^2+c\right)^{3/2}}-\frac{2}{3}a\int \frac{x^3 \arctan(ax)}{\left(a^2cx^2+c\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 5473}$$

$$\frac{x^3 \arctan(ax)^2}{3c\left(a^2cx^2+c\right)^{3/2}}-\frac{2}{3}a\left(\frac{2\int \frac{x\arctan(ax)}{\left(a^2cx^2+c\right)^{3/2}}dx}{3a^2c}-\frac{x^2\arctan(ax)}{3a^2c\left(a^2cx^2+c\right)^{3/2}}+\frac{x^3}{9ac\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\Big\downarrow \text{ 5465}$$

$$\frac{x^3 \arctan(ax)^2}{3c\left(a^2cx^2+c\right)^{3/2}}-\frac{2}{3}a\left(\frac{2\left(\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}}dx}{a}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{x^2\arctan(ax)}{3a^2c\left(a^2cx^2+c\right)^{3/2}}+\frac{x^3}{9ac\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\Big\downarrow \text{ 208}$$

$$\frac{x^3 \arctan(ax)^2}{3c\left(a^2cx^2+c\right)^{3/2}}-\frac{2}{3}a\left(-\frac{x^2\arctan(ax)}{3a^2c\left(a^2cx^2+c\right)^{3/2}}+\frac{2\left(\frac{x}{ac\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{x^3}{9ac\left(a^2cx^2+c\right)^{3/2}}\right)$$

input
```
Int[(x^2*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(x^3*ArcTan[a*x]^2)/(3*c*(c + a^2*c*x^2)^(3/2)) - (2*a*(x^3/(9*a*c*(c + a^
2*c*x^2)^(3/2)) - (x^2*ArcTan[a*x])/(3*a^2*c*(c + a^2*c*x^2)^(3/2)) + (2*(
x/(a*c*Sqrt[c + a^2*c*x^2]) - ArcTan[a*x]/(a^2*c*Sqrt[c + a^2*c*x^2])))/(3
*a^2*c)))/3
```

### Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5473
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[b*(f*x)^m*((d + e*x^2)^(q + 1)/(c*d*m^2)), x] +
 (-Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(c^2*d*m))
, x] + Simp[f^2*((m - 1)/(c^2*d*m))   Int[(f*x)^(m - 2)*(d + e*x^2)^(q + 1)
*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2
*d] && EqQ[m + 2*q + 2, 0] && LtQ[q, -1]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.70 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.96

| method | result |
|---|---|
| default | $\frac{\left(6 i \arctan(ax)+9 \arctan(ax)^{2}-2\right) \left(a^{3} x^{3}-3 i a^{2} x^{2}-3 a x+i\right) \sqrt{c (a x-i) (a x+i)}}{216 \left(a^{2} x^{2}+1\right)^{2} a^{3} c^{3}}+\frac{\left(\arctan(ax)^{2}-2+2 i \arctan(ax)\right) (a x-i) \sqrt{c (a x-}}{8 c^{3} a^{3} \left(a^{2} x^{2}+1\right)}$ |
| orering | $-\frac{\left(a^{2} x^{2}+1\right) \left(84 a^{6} x^{6}-13 a^{4} x^{4}+48\right) \arctan(ax)^{2}}{27 a^{4} x \left(a^{2} c\, x^{2}+c\right)^{\frac{5}{2}}}-\frac{\left(a^{2} x^{2}+1\right)^{2} \left(56 a^{4} x^{4}+11 a^{2} x^{2}-30\right) \left(\frac{2 x \arctan(ax)^{2}}{\left(a^{2} c\, x^{2}+c\right)^{\frac{5}{2}}}+\frac{2 x^{2} \arctan(ax) a}{\left(a^{2} c\, x^{2}+c\right)^{\frac{5}{2}} \left(a^{2} x^{2}+1\right)}-5\right.}{27 x^{2} a^{4}}$ |

input
```
int(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/216*(6*I*arctan(a*x)+9*arctan(a*x)^2-2)*(a^3*x^3-3*I*a^2*x^2-3*a*x+I)*(c
*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^2/a^3/c^3+1/8*(arctan(a*x)^2-2+2*I*arc
tan(a*x))*(a*x-I)*(c*(a*x-I)*(a*x+I))^(1/2)/c^3/a^3/(a^2*x^2+1)+1/8*(c*(a*
x-I)*(a*x+I))^(1/2)*(a*x+I)*(arctan(a*x)^2-2-2*I*arctan(a*x))/c^3/a^3/(a^2
*x^2+1)+1/216*(-6*I*arctan(a*x)+9*arctan(a*x)^2-2)*(c*(a*x-I)*(a*x+I))^(1/
2)*(a^3*x^3+3*I*a^2*x^2-3*a*x-I)/(a^4*x^4+2*a^2*x^2+1)/a^3/c^3
```

## Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.63

$$\int \frac{x^2 \arctan(ax)^2}{\left(c+a^2cx^2\right)^{5/2}}\,dx = \frac{\left(9\,a^3x^3\arctan\left(ax\right)^2-14\,a^3x^3-12\,ax+6\,\left(3\,a^2x^2+2\right)\arctan\left(ax\right)\right)\sqrt{a^2cx^2+c}}{27\,\left(a^7c^3x^4+2\,a^5c^3x^2+a^3c^3\right)}$$

input
```
integrate(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
1/27*(9*a^3*x^3*arctan(a*x)^2 - 14*a^3*x^3 - 12*a*x + 6*(3*a^2*x^2 + 2)*ar
ctan(a*x))*sqrt(a^2*c*x^2 + c)/(a^7*c^3*x^4 + 2*a^5*c^3*x^2 + a^3*c^3)
```

## Sympy [F]

$$\int \frac{x^2 \arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^2 \operatorname{atan}^2(ax)}{(c(a^2x^2+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**2*atan(a*x)**2/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(x**2*atan(a*x)**2/(c*(a**2*x**2 + 1))**(5/2), x)
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.84

$$\int \frac{x^2 \arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = \frac{1}{3}\left(\frac{x}{\sqrt{a^2cx^2+c}a^2c^2} - \frac{x}{(a^2cx^2+c)^{\frac{3}{2}}a^2c}\right)\arctan(ax)^2 - \frac{2\,(7\,a^3x^3+6\,ax-3\,(3\,a^2x^2+2)\arctan(ax))a}{27\,(a^6c^2x^2+a^4c^2)\sqrt{a^2x^2+1}\sqrt{c}}$$

input
```
integrate(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
1/3*(x/(sqrt(a^2*c*x^2 + c)*a^2*c^2) - x/((a^2*c*x^2 + c)^(3/2)*a^2*c))*ar
ctan(a*x)^2 - 2/27*(7*a^3*x^3 + 6*a*x - 3*(3*a^2*x^2 + 2)*arctan(a*x))*a/(
(a^6*c^2*x^2 + a^4*c^2)*sqrt(a^2*x^2 + 1)*sqrt(c))
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x^2 \arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\, dx = \int \frac{x^2\, \mathrm{atan}(a\, x)^2}{(c\, a^2\, x^2 + c)^{5/2}}\, dx$$

input
```
int((x^2*atan(a*x)^2)/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int((x^2*atan(a*x)^2)/(c + a^2*c*x^2)^(5/2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.01

$$\int \frac{x^2 \arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\, dx = \frac{\sqrt{c}\left(9\sqrt{a^2x^2+1}\, atan(ax)^2\, a^3x^3 + 18\sqrt{a^2x^2+1}\, atan(ax)\, a^2x^2 + 12\sqrt{a^2x^2+1}\, atan\right.}{27a^3c^3\left(a^4x^4+2a^2x^2\right.}$$

input
```
int(x^2*atan(a*x)^2/(a^2*c*x^2+c)^(5/2),x)
```

output
```
(sqrt(c)*(9*sqrt(a**2*x**2 + 1)*atan(a*x)**2*a**3*x**3 + 18*sqrt(a**2*x**2
 + 1)*atan(a*x)*a**2*x**2 + 12*sqrt(a**2*x**2 + 1)*atan(a*x) - 14*sqrt(a**
2*x**2 + 1)*a**3*x**3 - 12*sqrt(a**2*x**2 + 1)*a*x + 2*a**4*x**4 + 4*a**2*
x**2 + 2))/(27*a**3*c**3*(a**4*x**4 + 2*a**2*x**2 + 1))
```

# 3.351 $\int \frac{x \arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\, dx$



## Optimal result

Integrand size = 22, antiderivative size = 137

$$\int \frac{x \arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\, dx = \frac{2}{27a^2c(c+a^2cx^2)^{3/2}} + \frac{4}{9a^2c^2\sqrt{c+a^2cx^2}} + \frac{2x\arctan(ax)}{9ac(c+a^2cx^2)^{3/2}} + \frac{4x\arctan(ax)}{9ac^2\sqrt{c+a^2cx^2}} - \frac{\arctan(ax)^2}{3a^2c(c+a^2cx^2)^{3/2}}$$

output
```
2/27/a^2/c/(a^2*c*x^2+c)^(3/2)+4/9/a^2/c^2/(a^2*c*x^2+c)^(1/2)+2/9*x*arcta
n(a*x)/a/c/(a^2*c*x^2+c)^(3/2)+4/9*x*arctan(a*x)/a/c^2/(a^2*c*x^2+c)^(1/2)
-1/3*arctan(a*x)^2/a^2/c/(a^2*c*x^2+c)^(3/2)
```

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.52

$$\int \frac{x \arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\, dx = \frac{\sqrt{c+a^2cx^2}(2(7+6a^2x^2)+6ax(3+2a^2x^2)\arctan(ax)-9\arctan(ax)^2)}{27c^3(a+a^3x^2)^2}$$

input
```
Integrate[(x*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(Sqrt[c + a^2*c*x^2]*(2*(7 + 6*a^2*x^2) + 6*a*x*(3 + 2*a^2*x^2)*ArcTan[a*x
] - 9*ArcTan[a*x]^2))/(27*c^3*(a + a^3*x^2)^2)
```

## Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {5465, 5431, 5429}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \arctan(ax)^2}{(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{ 5465}$$

$$\frac{2\int \frac{\arctan(ax)}{(a^2cx^2+c)^{5/2}}dx}{3a}-\frac{\arctan(ax)^2}{3a^2c\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{ 5431}$$

$$\frac{2\left(\frac{2\int \frac{\arctan(ax)}{\left(a^2cx^2+c\right)^{3/2}}dx}{3c}+\frac{x\arctan(ax)}{3c(a^2cx^2+c)^{3/2}}+\frac{1}{9ac(a^2cx^2+c)^{3/2}}\right)}{3a}-\frac{\arctan(ax)^2}{3a^2c\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{ 5429}$$

$$\frac{2\left(\frac{x\arctan(ax)}{3c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{1}{9ac(a^2cx^2+c)^{3/2}}\right)}{3a}-\frac{\arctan(ax)^2}{3a^2c\,(a^2cx^2+c)^{3/2}}$$

input
```
Int[(x*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(5/2),x]
```

output
```
-1/3*ArcTan[a*x]^2/(a^2*c*(c + a^2*c*x^2)^(3/2)) + (2*(1/(9*a*c*(c + a^2*c
*x^2)^(3/2)) + (x*ArcTan[a*x])/(3*c*(c + a^2*c*x^2)^(3/2)) + (2*(1/(a*c*Sq
rt[c + a^2*c*x^2]) + (x*ArcTan[a*x])/(c*Sqrt[c + a^2*c*x^2])))/(3*c)))/(3*
a)
```

### Defintions of rubi rules used

rule 5429
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTan[c*x])/(d*Sqr
t[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

rule 5431
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbol
] :> Simp[b*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (-Simp[x*(d + e*x
^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*d*(q + 1))), x] + Simp[(2*q + 3)/(2*d*(
q + 1))   Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a,
 b, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -3/2]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.10 (sec) , antiderivative size = 276, normalized size of antiderivative = 2.01

| method | result |
|---|---|
| default | $\frac{\left(6i\arctan(ax)+9\arctan(ax)^2-2\right)\left(ia^3x^3+3a^2x^2-3iax-1\right)\sqrt{c(ax-i)(ax+i)}}{216(a^2x^2+1)^2a^2c^3}-\frac{\left(\arctan(ax)^2-2+2i\arctan(ax)\right)(iax+1)\sqrt{c(ax}}{8c^3a^2(a^2x^2+1)}$ |
| orering | $\frac{(a^2x^2+1)(144a^6x^6+152a^4x^4-27a^2x^2+14)\arctan(ax)^2}{27a^4x^2(a^2cx^2+c)^{\frac{5}{2}}}+\frac{(a^2x^2+1)^2(66a^4x^4+67a^2x^2-14)\left(\frac{\arctan(ax)^2}{(a^2cx^2+c)^{\frac{5}{2}}}+\frac{2x\arctan(ax)a}{(a^2cx^2+c)^{\frac{5}{2}}(a^2x}\right.}{27x^2a^4}$ |

input
```
int(x*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/216*(6*I*arctan(a*x)+9*arctan(a*x)^2-2)*(I*a^3*x^3+3*a^2*x^2-3*I*a*x-1)*
(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^2/a^2/c^3-1/8*(arctan(a*x)^2-2+2*I*a
rctan(a*x))*(1+I*a*x)*(c*(a*x-I)*(a*x+I))^(1/2)/c^3/a^2/(a^2*x^2+1)+1/8*(c
*(a*x-I)*(a*x+I))^(1/2)*(I*a*x-1)*(arctan(a*x)^2-2-2*I*arctan(a*x))/c^3/a^
2/(a^2*x^2+1)-1/216*(c*(a*x-I)*(a*x+I))^(1/2)*(I*a^3*x^3-3*a^2*x^2-3*I*a*x
+1)*(-6*I*arctan(a*x)+9*arctan(a*x)^2-2)/a^2/c^3/(a^4*x^4+2*a^2*x^2+1)
```

## Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.60

$$\int \frac{x\arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = \frac{\sqrt{a^2cx^2+c}\left(12\,a^2x^2+6\,(2\,a^3x^3+3\,ax)\arctan\,(ax)-9\,\arctan\,(ax)^2+14\right)}{27\,(a^6c^3x^4+2\,a^4c^3x^2+a^2c^3)}$$

input
```
integrate(x*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
1/27*sqrt(a^2*c*x^2 + c)*(12*a^2*x^2 + 6*(2*a^3*x^3 + 3*a*x)*arctan(a*x) -
 9*arctan(a*x)^2 + 14)/(a^6*c^3*x^4 + 2*a^4*c^3*x^2 + a^2*c^3)
```

## Sympy [F]

$$\int \frac{x\arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x\,\mathrm{atan}^2\,(ax)}{(c\,(a^2x^2+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*atan(a*x)**2/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(x*atan(a*x)**2/(c*(a**2*x**2 + 1))**(5/2), x)
```

## Maxima [F]

$$\int \frac{x \arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x \arctan(ax)^2}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(x*arctan(a*x)^2/(a^2*c*x^2 + c)^(5/2), x)
```

## Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.71

$$\int \frac{x \arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = \frac{2\left(\frac{2\,ax^2}{c}+\frac{3}{ac}\right)x\arctan(ax)}{9\,(a^2cx^2+c)^{\frac{3}{2}}} - \frac{\arctan(ax)^2}{3\,(a^2cx^2+c)^{\frac{3}{2}}a^2c} + \frac{2\,(6\,a^2cx^2+7\,c)}{27\,(a^2cx^2+c)^{\frac{3}{2}}a^2c^2}$$

input
```
integrate(x*arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
2/9*(2*a*x^2/c + 3/(a*c))*x*arctan(a*x)/(a^2*c*x^2 + c)^(3/2) - 1/3*arctan
(a*x)^2/((a^2*c*x^2 + c)^(3/2)*a^2*c) + 2/27*(6*a^2*c*x^2 + 7*c)/((a^2*c*x
^2 + c)^(3/2)*a^2*c^2)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x \arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x\,\mathrm{atan}(a\,x)^2}{(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int((x*atan(a*x)^2)/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int((x*atan(a*x)^2)/(c + a^2*c*x^2)^(5/2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.54

$$\int \frac{x \arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = \frac{\sqrt{c}\left(-24atan\left(\sqrt{a^2x^2+1}+ax\right)a^4x^4-48atan\left(\sqrt{a^2x^2+1}+ax\right)a^2x^2-24atan\left(\sqrt{\right.\right.}{}$$

input
```
int(x*atan(a*x)^2/(a^2*c*x^2+c)^(5/2),x)
```

output
```
(sqrt(c)*( - 24*atan(sqrt(a**2*x**2 + 1) + a*x)*a**4*x**4 - 48*atan(sqrt(a
**2*x**2 + 1) + a*x)*a**2*x**2 - 24*atan(sqrt(a**2*x**2 + 1) + a*x) - 9*sq
rt(a**2*x**2 + 1)*atan(a*x)**2 + 12*sqrt(a**2*x**2 + 1)*atan(a*x)*a**3*x**
3 + 18*sqrt(a**2*x**2 + 1)*atan(a*x)*a*x + 12*atan(a*x)*a**4*x**4 + 24*ata
n(a*x)*a**2*x**2 + 12*atan(a*x) + 12*sqrt(a**2*x**2 + 1)*a**2*x**2 + 14*sq
rt(a**2*x**2 + 1)))/(27*a**2*c**3*(a**4*x**4 + 2*a**2*x**2 + 1))
```

# 3.352 $\int \frac{\arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 157

$$\int \frac{\arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = -\frac{2x}{27c\,(c+a^2cx^2)^{3/2}} - \frac{40x}{27c^2\sqrt{c+a^2cx^2}} + \frac{2\arctan(ax)}{9ac\,(c+a^2cx^2)^{3/2}} + \frac{4\arctan(ax)}{3ac^2\sqrt{c+a^2cx^2}} + \frac{x\arctan(ax)^2}{3c\,(c+a^2cx^2)^{3/2}} + \frac{2x\arctan(ax)^2}{3c^2\sqrt{c+a^2cx^2}}$$

output
```
-2/27*x/c/(a^2*c*x^2+c)^(3/2)-40/27*x/c^2/(a^2*c*x^2+c)^(1/2)+2/9*arctan(a
*x)/a/c/(a^2*c*x^2+c)^(3/2)+4/3*arctan(a*x)/a/c^2/(a^2*c*x^2+c)^(1/2)+1/3*
x*arctan(a*x)^2/c/(a^2*c*x^2+c)^(3/2)+2/3*x*arctan(a*x)^2/c^2/(a^2*c*x^2+c
)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.55

$$\int \frac{\arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = \frac{\sqrt{c+a^2cx^2}(-2ax(21+20a^2x^2)+6(7+6a^2x^2)\arctan(ax)+9ax(3+2a^2x^2)\,\text{arcta}}{27ac^3\,(1+a^2x^2)^2}$$

input
```
Integrate[ArcTan[a*x]^2/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(Sqrt[c + a^2*c*x^2]*(-2*a*x*(21 + 20*a^2*x^2) + 6*(7 + 6*a^2*x^2)*ArcTan[
a*x] + 9*a*x*(3 + 2*a^2*x^2)*ArcTan[a*x]^2))/(27*a*c^3*(1 + a^2*x^2)^2)
```

## Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.18, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {5435, 209, 208, 5433, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2}{(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{5435}$$

$$\frac{2\int \frac{\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{3c}-\frac{2}{9}\int \frac{1}{(a^2cx^2+c)^{5/2}}dx+\frac{x\arctan(ax)^2}{3c\,(a^2cx^2+c)^{3/2}}+\frac{2\arctan(ax)}{9ac\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{209}$$

$$\frac{2\int \frac{\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{3c}-\frac{2}{9}\left(\frac{2\int \frac{1}{(a^2cx^2+c)^{3/2}}dx}{3c}+\frac{x}{3c\,(a^2cx^2+c)^{3/2}}\right)+\frac{x\arctan(ax)^2}{3c\,(a^2cx^2+c)^{3/2}}+\frac{2\arctan(ax)}{9ac\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{208}$$

$$\frac{2\int \frac{\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{3c}+\frac{x\arctan(ax)^2}{3c\,(a^2cx^2+c)^{3/2}}+\frac{2\arctan(ax)}{9ac\,(a^2cx^2+c)^{3/2}}-\frac{2}{9}\left(\frac{2x}{3c^2\sqrt{a^2cx^2+c}}+\frac{x}{3c\,(a^2cx^2+c)^{3/2}}\right)$$

$$\Big\downarrow \text{5433}$$

$$\frac{2\left(-2\int \frac{1}{(a^2cx^2+c)^{3/2}}dx+\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{x\arctan(ax)^2}{3c\,(a^2cx^2+c)^{3/2}}+\frac{2\arctan(ax)}{9ac\,(a^2cx^2+c)^{3/2}}-\frac{2}{9}\left(\frac{2x}{3c^2\sqrt{a^2cx^2+c}}+\frac{x}{3c\,(a^2cx^2+c)^{3/2}}\right)$$

$$\Big\downarrow \text{208}$$

$$\frac{x\arctan(ax)^2}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{2\arctan(ax)}{9ac\left(a^2cx^2+c\right)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{3c}-\frac{2}{9}\left(\frac{2x}{3c^2\sqrt{a^2cx^2+c}}+\frac{x}{3c\left(a^2cx^2+c\right)^{3/2}}\right)$$

input
```
Int[ArcTan[a*x]^2/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(-2*(x/(3*c*(c + a^2*c*x^2)^(3/2)) + (2*x)/(3*c^2*Sqrt[c + a^2*c*x^2])))/9
 + (2*ArcTan[a*x])/(9*a*c*(c + a^2*c*x^2)^(3/2)) + (x*ArcTan[a*x]^2)/(3*c*
(c + a^2*c*x^2)^(3/2)) + (2*((-2*x)/(c*Sqrt[c + a^2*c*x^2]) + (2*ArcTan[a*
x])/(a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x]^2)/(c*Sqrt[c + a^2*c*x^2]))
)/(3*c)
```

## Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 5433
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x] - Simp[b^2*p*(p
- 1)   Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2), x], x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

rule 5435
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[b*p*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p - 1)/(4*c*d*
(q + 1)^2)), x] + (-Simp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*d*
(q + 1))), x] + Simp[(2*q + 3)/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p, x], x] - Simp[b^2*p*((p - 1)/(4*(q + 1)^2))   Int[(d + e
*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.82 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.73

| method | result |
|---|---|
| default | $-\frac{\left(6i\arctan(ax)+9\arctan(ax)^2-2\right)\left(a^3x^3-3ia^2x^2-3ax+i\right)\sqrt{c(ax-i)(ax+i)}}{216(a^2x^2+1)^2 a c^3}+\frac{3\left(\arctan(ax)^2-2+2i\arctan(ax)\right)(ax-i)\sqrt{c(a}}{8c^3a(a^2x^2+1)}$ |
| orering | $\frac{\left(-\frac{200}{9}a^6x^7-\frac{1222}{27}a^4x^5-\frac{595}{27}a^2x^3+x\right)\arctan(ax)^2}{(a^2cx^2+c)^{\frac{5}{2}}}-\frac{(a^2x^2+1)^2(80a^4x^4+78a^2x^2-7)\left(\frac{2\arctan(ax)a}{(a^2cx^2+c)^{\frac{5}{2}}(a^2x^2+1)}-\frac{5\arctan(ax)^2cx\,a}{(a^2cx^2+c)^{\frac{7}{2}}}\right.}{9a^2}$ |

input
```
int(arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/216*(6*I*arctan(a*x)+9*arctan(a*x)^2-2)*(a^3*x^3-3*I*a^2*x^2-3*a*x+I)*(
c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^2/a/c^3+3/8*(arctan(a*x)^2-2+2*I*arct
an(a*x))*(a*x-I)*(c*(a*x-I)*(a*x+I))^(1/2)/c^3/a/(a^2*x^2+1)+3/8*(c*(a*x-I
)*(a*x+I))^(1/2)*(a*x+I)*(arctan(a*x)^2-2-2*I*arctan(a*x))/c^3/a/(a^2*x^2+
1)-1/216*(-6*I*arctan(a*x)+9*arctan(a*x)^2-2)*(c*(a*x-I)*(a*x+I))^(1/2)*(a
^3*x^3+3*I*a^2*x^2-3*a*x-I)/(a^4*x^4+2*a^2*x^2+1)/a/c^3
```

## Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.59

$$\int \frac{\arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = -\frac{\left(40\,a^3x^3-9\,(2\,a^3x^3+3\,ax)\arctan{(ax)}^2+42\,ax-6\,(6\,a^2x^2+7)\arctan{(ax)}\right)\sqrt{a^2cx^2+c}}{27\,(a^5c^3x^4+2\,a^3c^3x^2+ac^3)}$$

input
```
integrate(arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
-1/27*(40*a^3*x^3 - 9*(2*a^3*x^3 + 3*a*x)*arctan(a*x)^2 + 42*a*x - 6*(6*a^
2*x^2 + 7)*arctan(a*x))*sqrt(a^2*c*x^2 + c)/(a^5*c^3*x^4 + 2*a^3*c^3*x^2 +
 a*c^3)
```

## Sympy [F]

$$\int \frac{\arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\operatorname{atan}^2{(ax)}}{(c\,(a^2x^2+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(atan(a*x)**2/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(atan(a*x)**2/(c*(a**2*x**2 + 1))**(5/2), x)
```

## Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.71

$$\int \frac{\arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = \frac{1}{3}\left(\frac{2\,x}{\sqrt{a^2cx^2+c}c^2}+\frac{x}{(a^2cx^2+c)^{\frac{3}{2}}c}\right)\arctan{(ax)}^2 - \frac{2\,(20\,a^3x^3+21\,ax-3\,(6\,a^2x^2+7)\arctan{(ax)})a}{27\,(a^4c^2x^2+a^2c^2)\sqrt{a^2x^2+1}\sqrt{c}}$$

input
```
integrate(arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
1/3*(2*x/(sqrt(a^2*c*x^2 + c)*c^2) + x/((a^2*c*x^2 + c)^(3/2)*c))*arctan(a
*x)^2 - 2/27*(20*a^3*x^3 + 21*a*x - 3*(6*a^2*x^2 + 7)*arctan(a*x))*a/((a^4
*c^2*x^2 + a^2*c^2)*sqrt(a^2*x^2 + 1)*sqrt(c))
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^2/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\mathrm{atan}(a\,x)^2}{(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int(atan(a*x)^2/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int(atan(a*x)^2/(c + a^2*c*x^2)^(5/2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.03

$$\int \frac{\arctan(ax)^2}{(c+a^2cx^2)^{5/2}}\,dx = \frac{\sqrt{c}\,\left(18\sqrt{a^2x^2+1}\,atan(ax)^2\,a^3x^3+27\sqrt{a^2x^2+1}\,atan(ax)^2\,ax+36\sqrt{a^2x^2+1}\,atan}{27a}$$

input
```
int(atan(a*x)^2/(a^2*c*x^2+c)^(5/2),x)
```

output
```
(sqrt(c)*(18*sqrt(a**2*x**2 + 1)*atan(a*x)**2*a**3*x**3 + 27*sqrt(a**2*x**
2 + 1)*atan(a*x)**2*a*x + 36*sqrt(a**2*x**2 + 1)*atan(a*x)*a**2*x**2 + 42*
sqrt(a**2*x**2 + 1)*atan(a*x) - 40*sqrt(a**2*x**2 + 1)*a**3*x**3 - 42*sqrt
(a**2*x**2 + 1)*a*x + 16*a**4*x**4 + 32*a**2*x**2 + 16))/(27*a*c**3*(a**4*
x**4 + 2*a**2*x**2 + 1))
```

# 3.353 $\int \frac{\arctan(ax)^2}{x(c+a^2cx^2)^{5/2}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 389

$$\begin{aligned}\int \frac{\arctan(ax)^2}{x(c+a^2cx^2)^{5/2}}\,dx &= -\frac{2}{27c(c+a^2cx^2)^{3/2}}-\frac{22}{9c^2\sqrt{c+a^2cx^2}}\\
&-\frac{2ax\arctan(ax)}{9c(c+a^2cx^2)^{3/2}}-\frac{22ax\arctan(ax)}{9c^2\sqrt{c+a^2cx^2}}+\frac{\arctan(ax)^2}{3c(c+a^2cx^2)^{3/2}}\\
&+\frac{\arctan(ax)^2}{c^2\sqrt{c+a^2cx^2}}-\frac{2\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)}{c^2\sqrt{c+a^2cx^2}}\\
&+\frac{2i\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)}{c^2\sqrt{c+a^2cx^2}}\\
&-\frac{2i\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,e^{i\arctan(ax)}\right)}{c^2\sqrt{c+a^2cx^2}}\\
&-\frac{2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,-e^{i\arctan(ax)}\right)}{c^2\sqrt{c+a^2cx^2}}+\frac{2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,e^{i\arctan(ax)}\right)}{c^2\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
-2/27/c/(a^2*c*x^2+c)^(3/2)-22/9/c^2/(a^2*c*x^2+c)^(1/2)-2/9*a*x*arctan(a*
x)/c/(a^2*c*x^2+c)^(3/2)-22/9*a*x*arctan(a*x)/c^2/(a^2*c*x^2+c)^(1/2)+1/3*
arctan(a*x)^2/c/(a^2*c*x^2+c)^(3/2)+arctan(a*x)^2/c^2/(a^2*c*x^2+c)^(1/2)-
2*(a^2*x^2+1)^(1/2)*arctan(a*x)^2*arctanh((1+I*a*x)/(a^2*x^2+1)^(1/2))/c^2
/(a^2*c*x^2+c)^(1/2)+2*I*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(2,-(1+I*a*x
)/(a^2*x^2+1)^(1/2))/c^2/(a^2*c*x^2+c)^(1/2)-2*I*(a^2*x^2+1)^(1/2)*arctan(
a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))/c^2/(a^2*c*x^2+c)^(1/2)-2*(a^2
*x^2+1)^(1/2)*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))/c^2/(a^2*c*x^2+c)^(1
/2)+2*(a^2*x^2+1)^(1/2)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))/c^2/(a^2*c*
x^2+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.63

$$\int \frac{\arctan(ax)^2}{x\left(c+a^2cx^2\right)^{5/2}}\,dx = \frac{\left(1+a^2x^2\right)^{3/2}\left(-\frac{270}{\sqrt{1+a^2x^2}}-\frac{270ax\arctan(ax)}{\sqrt{1+a^2x^2}}+\frac{135\arctan(ax)^2}{\sqrt{1+a^2x^2}}-2\cos(3\arctan(ax))+9\right.}{}$$

input
```
Integrate[ArcTan[a*x]^2/(x*(c + a^2*c*x^2)^(5/2)),x]
```

output
```
((1 + a^2*x^2)^(3/2)*(-270/Sqrt[1 + a^2*x^2] - (270*a*x*ArcTan[a*x])/Sqrt[
1 + a^2*x^2] + (135*ArcTan[a*x]^2)/Sqrt[1 + a^2*x^2] - 2*Cos[3*ArcTan[a*x]
] + 9*ArcTan[a*x]^2*Cos[3*ArcTan[a*x]] + 108*ArcTan[a*x]^2*Log[1 - E^(I*Ar
cTan[a*x])] - 108*ArcTan[a*x]^2*Log[1 + E^(I*ArcTan[a*x])] + (216*I)*ArcTa
n[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])] - (216*I)*ArcTan[a*x]*PolyLog[2, E^(
I*ArcTan[a*x])] - 216*PolyLog[3, -E^(I*ArcTan[a*x])] + 216*PolyLog[3, E^(I
*ArcTan[a*x])] - 6*ArcTan[a*x]*Sin[3*ArcTan[a*x]]))/(108*c*(c*(1 + a^2*x^2
))^(3/2))
```

## Rubi [A] (verified)

Time = 2.84 (sec) , antiderivative size = 365, normalized size of antiderivative = 0.94, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {5501, 5465, 5431, 5429, 5501, 5465, 5429, 5493, 5491, 3042, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2}{x\left(a^2cx^2+c\right)^{5/2}}dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{\arctan(ax)^2}{x(a^2cx^2+c)^{3/2}}dx}{c}-a^2\int \frac{x\arctan(ax)^2}{\left(a^2cx^2+c\right)^{5/2}}dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{\int \frac{\arctan(ax)^2}{x(a^2cx^2+c)^{3/2}}dx}{c}-a^2\left(\frac{2\int \frac{\arctan(ax)}{(a^2cx^2+c)^{5/2}}dx}{3a}-\frac{\arctan(ax)^2}{3a^2c\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\Big\downarrow \text{5431}$$

$$\frac{\int \frac{\arctan(ax)^2}{x(a^2cx^2+c)^{3/2}}dx}{c}-a^2\left(\frac{2\left(\frac{2\int \frac{\arctan(ax)}{\left(a^2cx^2+c\right)^{3/2}}dx}{3c}+\frac{x\arctan(ax)}{3c(a^2cx^2+c)^{3/2}}+\frac{1}{9ac(a^2cx^2+c)^{3/2}}\right)}{3a}-\frac{\arctan(ax)^2}{3a^2c\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\Big\downarrow \text{5429}$$

$$\frac{\int \frac{\arctan(ax)^2}{x(a^2cx^2+c)^{3/2}}dx}{c}-a^2\left(\frac{2\left(\frac{x\arctan(ax)}{3c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{1}{9ac(a^2cx^2+c)^{3/2}}\right)}{3a}-\frac{\arctan(ax)^2}{3a^2c\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\Big\downarrow \text{5501}$$

$$\frac{\frac{\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx}{c} - a^2\int \frac{x\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{c} - a^2\left(\frac{2\left(\frac{x\arctan(ax)}{3c(a^2cx^2+c)^{3/2}} + \frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{3c} + \frac{1}{9ac(a^2cx^2+c)^{3/2}}\right)}{3a} - \frac{\arctan(ax)^2}{3a^2c\,(a^2cx^2+c)^{3/2}}\right)$$

$$\Big\downarrow \text{5465}$$

$$\frac{\frac{\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx}{c} - a^2\left(\frac{2\int \frac{\arctan(ax)}{\left(a^2cx^2+c\right)^{3/2}}dx}{a} - \frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)}{c} - a^2\left(\frac{2\left(\frac{x\arctan(ax)}{3c(a^2cx^2+c)^{3/2}} + \frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{3c} + \frac{1}{9ac(a^2cx^2+c)^{3/2}}\right)}{3a} - \frac{\arctan(ax)^2}{3a^2c\,(a^2cx^2+c)^{3/2}}\right)$$

$$\Big\downarrow \text{5429}$$

$$\frac{\frac{\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx}{c} - a^2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a} - \frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)}{c} - a^2\left(\frac{2\left(\frac{x\arctan(ax)}{3c(a^2cx^2+c)^{3/2}} + \frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{3c} + \frac{1}{9ac(a^2cx^2+c)^{3/2}}\right)}{3a} - \frac{\arctan(ax)^2}{3a^2c\,(a^2cx^2+c)^{3/2}}\right)$$

$$\Big\downarrow \text{5493}$$

$$\frac{\frac{\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{x\sqrt{a^2x^2+1}}dx}{c\sqrt{a^2cx^2+c}}-a^2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)}{c}-a^2\left(\frac{2\left(\frac{x\arctan(ax)}{3c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{1}{9ac(a^2cx^2+c)^{3/2}}\right)}{3a}-\frac{\arctan(ax)^2}{3a^2c\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\Big\downarrow \text{ 5491}$$

$$\frac{\frac{\sqrt{a^2x^2+1}\int \frac{\sqrt{a^2x^2+1}\arctan(ax)^2}{ax}d\arctan(ax)}{c\sqrt{a^2cx^2+c}}-a^2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)}{c}-a^2\left(\frac{2\left(\frac{x\arctan(ax)}{3c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{1}{9ac(a^2cx^2+c)^{3/2}}\right)}{3a}-\frac{\arctan(ax)^2}{3a^2c\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc(\arctan(ax))d\arctan(ax)}{c\sqrt{a^2cx^2+c}}-a^2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)}{c}-a^2\left(\frac{2\left(\frac{x\arctan(ax)}{3c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{1}{9ac(a^2cx^2+c)^{3/2}}\right)}{3a}-\frac{\arctan(ax)^2}{3a^2c\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\Big\downarrow \text{ 4671}$$

$$\frac{-a^2\left(\frac{2\left(\frac{x\arctan(ax)}{3c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{1}{9ac(a^2cx^2+c)^{3/2}}\right)}{3a}-\frac{\arctan(ax)^2}{3a^2c\left(a^2cx^2+c\right)^{3/2}}\right)+ -a^2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)+\frac{\sqrt{a^2x^2+1}\left(-2\int\arctan(ax)\log\left(1-e^{i\arctan(ax)}\right)d\arctan(ax)+2\int\arctan(ax)\log}{c\sqrt{a^2cx^2+}}}{c}$$

$\downarrow$ 3011

$$\frac{-a^2\left(\frac{2\left(\frac{x\arctan(ax)}{3c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{1}{9ac(a^2cx^2+c)^{3/2}}\right)}{3a}-\frac{\arctan(ax)^2}{3a^2c\left(a^2cx^2+c\right)^{3/2}}\right)+ -a^2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)+\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-i\int\operatorname{PolyLog}\left(2,-e^{i\arctan(}\right.\right.\right.}{}}{c}$$

$\downarrow$ 2720

$$-a^2\left(\frac{2\left(\frac{x\arctan(ax)}{3c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{1}{9ac(a^2cx^2+c)^{3/2}}\right)}{3a}-\frac{\arctan(ax)^2}{3a^2c\left(a^2cx^2+c\right)^{3/2}}\right)+ -a^2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)+\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}(\right.\right.}{}$$

$\downarrow$ 7143

$$-a^2\left(\frac{2\left(\frac{x\arctan(ax)}{3c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{1}{9ac(a^2cx^2+c)^{3/2}}\right)}{3a}-\frac{\arctan(ax)^2}{3a^2c\,(a^2cx^2+c)^{3/2}}\right)+$$

$$\frac{-a^2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)+\frac{\sqrt{a^2x^2+1}\left(-2\arctan(ax)^2\text{arctanh}(e^{i\arctan(ax)})+2(i\arctan(ax)\,\text{PolyLog}(2,-\right.}{}}{c}$$

input
```
Int[ArcTan[a*x]^2/(x*(c + a^2*c*x^2)^(5/2)),x]
```

output
```
-(a^2*(-1/3*ArcTan[a*x]^2/(a^2*c*(c + a^2*c*x^2)^(3/2)) + (2*(1/(9*a*c*(c
+ a^2*c*x^2)^(3/2)) + (x*ArcTan[a*x])/(3*c*(c + a^2*c*x^2)^(3/2)) + (2*(1/
(a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x])/(c*Sqrt[c + a^2*c*x^2])))/(3*c
)))/(3*a)) + (-(a^2*(-(ArcTan[a*x]^2/(a^2*c*Sqrt[c + a^2*c*x^2])) + (2*(1
/(a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x])/(c*Sqrt[c + a^2*c*x^2])))/a))
 + (Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])] + 2*(I*
ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])] - PolyLog[3, -E^(I*ArcTan[a*x])
]) - 2*(I*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])] - PolyLog[3, E^(I*ArcT
an[a*x])])))/(c*Sqrt[c + a^2*c*x^2]))/c
```

## Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5429
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTan[c*x])/(d*Sqr
t[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

rule 5431
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbol
] :> Simp[b*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (-Simp[x*(d + e*x
^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*d*(q + 1))), x] + Simp[(2*q + 3)/(2*d*(
q + 1))   Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a,
 b, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -3/2]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5491
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Simp[1/Sqrt[d]   Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTa
n[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] &&
GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

## Maple [A] (verified)

Time = 2.38 (sec) , antiderivative size = 460, normalized size of antiderivative = 1.18

| method | result |
|---|---|
| default | $-\frac{\left(6i\arctan(ax)+9\arctan(ax)^2-2\right)\left(ia^3x^3+3a^2x^2-3iax-1\right)\sqrt{c(ax-i)(ax+i)}}{216c^3(a^2x^2+1)^2}+\frac{5\left(\arctan(ax)^2-2+2i\arctan(ax)\right)(iax+1)\sqrt{c}}{8(a^2x^2+1)c^3}$ |

input
```
int(arctan(a*x)^2/x/(a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/216*(6*I*arctan(a*x)+9*arctan(a*x)^2-2)*(I*a^3*x^3+3*a^2*x^2-3*I*a*x-1)
*(c*(a*x-I)*(a*x+I))^(1/2)/c^3/(a^2*x^2+1)^2+5/8*(arctan(a*x)^2-2+2*I*arct
an(a*x))*(1+I*a*x)*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)/c^3-5/8*(c*(a*x-I
)*(a*x+I))^(1/2)*(I*a*x-1)*(arctan(a*x)^2-2-2*I*arctan(a*x))/(a^2*x^2+1)/c
^3+1/216*(c*(a*x-I)*(a*x+I))^(1/2)*(I*a^3*x^3-3*a^2*x^2-3*I*a*x+1)*(-6*I*a
rctan(a*x)+9*arctan(a*x)^2-2)/c^3/(a^4*x^4+2*a^2*x^2+1)+(arctan(a*x)^2*ln(
1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-arctan(a*x)^2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1
/2))-2*I*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*I*arctan(a*x
)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*polylog(3,(1+I*a*x)/(a^2*x^2+1
)^(1/2))-2*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2)))*(c*(a*x-I)*(a*x+I))^(1
/2)/(a^2*x^2+1)^(1/2)/c^3
```

## Fricas [F]

$$\int \frac{\arctan(ax)^2}{x(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\arctan(ax)^2}{(a^2cx^2+c)^{\frac{5}{2}}x}\,dx$$

input
```
integrate(arctan(a*x)^2/x/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2/(a^6*c^3*x^7 + 3*a^4*c^3*x^5 +
3*a^2*c^3*x^3 + c^3*x), x)
```

## Sympy [F]

$$\int \frac{\arctan(ax)^2}{x(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\operatorname{atan}^2(ax)}{x\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(atan(a*x)**2/x/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(atan(a*x)**2/(x*(c*(a**2*x**2 + 1))**(5/2)), x)
```

## Maxima [F]

$$\int \frac{\arctan(ax)^2}{x(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\arctan(ax)^2}{(a^2cx^2+c)^{\frac{5}{2}}x}\,dx$$

input
```
integrate(arctan(a*x)^2/x/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)^2/((a^2*c*x^2 + c)^(5/2)*x), x)
```

## Giac [F]

$$\int \frac{\arctan(ax)^2}{x\,(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\arctan\left(ax\right)^2}{(a^2cx^2+c)^{\frac{5}{2}}x}\,dx$$

input
```
integrate(arctan(a*x)^2/x/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^2/((a^2*c*x^2 + c)^(5/2)*x), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^2}{x\,(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\mathrm{atan}(a\,x)^2}{x\,(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int(atan(a*x)^2/(x*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(atan(a*x)^2/(x*(c + a^2*c*x^2)^(5/2)), x)
```

## Reduce [F]

$$\int \frac{\arctan(ax)^2}{x\,(c+a^2cx^2)^{5/2}}\,dx = \frac{\int \frac{atan(ax)^2}{\sqrt{a^2x^2+1}\,a^4x^5+2\sqrt{a^2x^2+1}\,a^2x^3+\sqrt{a^2x^2+1}\,x}dx}{\sqrt{c}\,c^2}$$

input
```
int(atan(a*x)^2/x/(a^2*c*x^2+c)^(5/2),x)
```

output
```
int(atan(a*x)**2/(sqrt(a**2*x**2 + 1)*a**4*x**5 + 2*sqrt(a**2*x**2 + 1)*a*
*2*x**3 + sqrt(a**2*x**2 + 1)*x),x)/(sqrt(c)*c**2)
```

# 3.354 $\int \frac{\arctan(ax)^2}{x^2(c+a^2cx^2)^{5/2}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 381

$$\begin{aligned}\int \frac{\arctan(ax)^2}{x^2(c+a^2cx^2)^{5/2}}\,dx &= \frac{2a^2x}{27c(c+a^2cx^2)^{3/2}}+\frac{94a^2x}{27c^2\sqrt{c+a^2cx^2}}\\ &\quad-\frac{2a\arctan(ax)}{9c(c+a^2cx^2)^{3/2}}-\frac{10a\arctan(ax)}{3c^2\sqrt{c+a^2cx^2}}-\frac{a^2x\arctan(ax)^2}{3c(c+a^2cx^2)^{3/2}}-\frac{5a^2x\arctan(ax)^2}{3c^2\sqrt{c+a^2cx^2}}\\ &\quad-\frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{c^3x}-\frac{4a\sqrt{1+a^2x^2}\arctan(ax)\operatorname{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{c^2\sqrt{c+a^2cx^2}}\\ &\quad+\frac{2ia\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{c^2\sqrt{c+a^2cx^2}}-\frac{2ia\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{c^2\sqrt{c+a^2cx^2}}\end{aligned}$$

output

```
2/27*a^2*x/c/(a^2*c*x^2+c)^(3/2)+94/27*a^2*x/c^2/(a^2*c*x^2+c)^(1/2)-2/9*a
*arctan(a*x)/c/(a^2*c*x^2+c)^(3/2)-10/3*a*arctan(a*x)/c^2/(a^2*c*x^2+c)^(1
/2)-1/3*a^2*x*arctan(a*x)^2/c/(a^2*c*x^2+c)^(3/2)-5/3*a^2*x*arctan(a*x)^2/
c^2/(a^2*c*x^2+c)^(1/2)-(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2/c^3/x-4*a*(a^2*x
^2+1)^(1/2)*arctan(a*x)*arctanh((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/c^2/(a^2*
c*x^2+c)^(1/2)+2*I*a*(a^2*x^2+1)^(1/2)*polylog(2,-(1+I*a*x)^(1/2)/(1-I*a*x
)^(1/2))/c^2/(a^2*c*x^2+c)^(1/2)-2*I*a*(a^2*x^2+1)^(1/2)*polylog(2,(1+I*a*
x)^(1/2)/(1-I*a*x)^(1/2))/c^2/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (warning: unable to verify)

Time = 1.28 (sec) , antiderivative size = 296, normalized size of antiderivative = 0.78

$$\int \frac{\arctan(ax)^2}{x^2 (c + a^2cx^2)^{5/2}} \, dx =$$
$$-\frac{a\left(-378ax + 378\arctan(ax) + 189ax\arctan(ax)^2 + 6\sqrt{1+a^2x^2}\arctan(ax)\cos(3\arctan(ax)) + 27ax \operatorname{ar}\right.}{}$$

input
```
Integrate[ArcTan[a*x]^2/(x^2*(c + a^2*c*x^2)^(5/2)),x]
```

output
```
-1/108*(a*(-378*a*x + 378*ArcTan[a*x] + 189*a*x*ArcTan[a*x]^2 + 6*Sqrt[1 +
 a^2*x^2]*ArcTan[a*x]*Cos[3*ArcTan[a*x]] + 27*a*x*ArcTan[a*x]^2*Csc[ArcTan
[a*x]/2]^2 - 216*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Log[1 - E^(I*ArcTan[a*x])]
+ 216*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Log[1 + E^(I*ArcTan[a*x])] - (216*I)*S
qrt[1 + a^2*x^2]*PolyLog[2, -E^(I*ArcTan[a*x])] + (216*I)*Sqrt[1 + a^2*x^2
]*PolyLog[2, E^(I*ArcTan[a*x])] - 2*Sqrt[1 + a^2*x^2]*Sin[3*ArcTan[a*x]] +
 9*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*Sin[3*ArcTan[a*x]] + 54*Sqrt[1 + a^2*x^
2]*ArcTan[a*x]^2*Tan[ArcTan[a*x]/2]))/(c^2*Sqrt[c + a^2*c*x^2])
```

## Rubi [A] (verified)

Time = 2.12 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.13, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5501, 5435, 209, 208, 5433, 208, 5501, 5433, 208, 5479, 5493, 5489}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^2}{x^2 (a^2cx^2+c)^{5/2}} \, dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{\arctan(ax)^2}{x^2(a^2cx^2+c)^{3/2}}dx}{c} - a^2 \int \frac{\arctan(ax)^2}{(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{5435}$$

$$\frac{\int \frac{\arctan(ax)^2}{x^2(a^2cx^2+c)^{3/2}}dx}{c} - a^2\left(\frac{2\int \frac{\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{3c} - \frac{2}{9}\int \frac{1}{(a^2cx^2+c)^{5/2}}dx + \frac{x\arctan(ax)^2}{3c\,(a^2cx^2+c)^{3/2}} + \frac{2\arctan(ax)}{9ac\,(a^2cx^2+c)^{3/2}}\right)$$

$$\Big\downarrow \ 209$$

$$\frac{\int \frac{\arctan(ax)^2}{x^2(a^2cx^2+c)^{3/2}}dx}{c} - a^2\left(\frac{2\int \frac{\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{3c} - \frac{2}{9}\left(\frac{2\int \frac{1}{(a^2cx^2+c)^{3/2}}dx}{3c} + \frac{x}{3c\,(a^2cx^2+c)^{3/2}}\right) + \frac{x\arctan(ax)^2}{3c\,(a^2cx^2+c)^{3/2}} + \frac{2\arctan(ax)}{9ac\,(a^2cx^2+c)^{3/2}}\right)$$

$$\Big\downarrow \ 208$$

$$\frac{\int \frac{\arctan(ax)^2}{x^2(a^2cx^2+c)^{3/2}}dx}{c} - a^2\left(\frac{2\int \frac{\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{3c} + \frac{x\arctan(ax)^2}{3c\,(a^2cx^2+c)^{3/2}} + \frac{2\arctan(ax)}{9ac\,(a^2cx^2+c)^{3/2}} - \frac{2}{9}\left(\frac{2x}{3c^2\sqrt{a^2cx^2+c}} + \frac{x}{3c\,(a^2cx^2+c)^{3/2}}\right)\right)$$

$$\Big\downarrow \ 5433$$

$$\frac{\int \frac{\arctan(ax)^2}{x^2(a^2cx^2+c)^{3/2}}dx}{c} - a^2\left(\frac{2\left(-2\int \frac{1}{(a^2cx^2+c)^{3/2}}dx + \frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}} + \frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}\right)}{3c} + \frac{x\arctan(ax)^2}{3c\,(a^2cx^2+c)^{3/2}} + \frac{2\arctan(ax)}{9ac\,(a^2cx^2+c)^{3/2}} - \frac{2}{9}\left(\frac{2x}{3c^2\sqrt{a^2c\ldots}}\right.\right.$$

$$\Big\downarrow \ 208$$

$$\frac{\int \frac{\arctan(ax)^2}{x^2(a^2cx^2+c)^{3/2}}dx}{c} - a^2\left(\frac{x\arctan(ax)^2}{3c\,(a^2cx^2+c)^{3/2}} + \frac{2\arctan(ax)}{9ac\,(a^2cx^2+c)^{3/2}} + \frac{2\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}} + \frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}} - \frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{3c} - \frac{2}{9}\left(\frac{2x}{3c^2\sqrt{a^2cx^2+c}} + \ldots\right.\right.$$

$$\Big\downarrow \ 5501$$

$$\frac{\frac{\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx}{c} - a^2\int \frac{\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{c} - a^2\left(\frac{x\arctan(ax)^2}{3c\,(a^2cx^2+c)^{3/2}} + \frac{2\arctan(ax)}{9ac\,(a^2cx^2+c)^{3/2}} + \frac{2\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}} + \frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}} - \frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{3c} - \frac{2}{9}\left(\frac{2x}{3c^2\sqrt{a^2cx^2+c}} + \ldots\right.\right.$$

$\downarrow$ 5433

$$\frac{\frac{\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx}{c} - a^2\left(-2\int \frac{1}{(a^2cx^2+c)^{3/2}}dx + \frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}} + \frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}\right)}{c} - a^2\left(\frac{x\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}} + \frac{2\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}} + \frac{2\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}} + \frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}} - \frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{3c} - \frac{2}{9}\left(\frac{2x}{3c^2\sqrt{a^2cx^2+c}} + \right.\right.$$

$\downarrow$ 208

$$\frac{\frac{\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx}{c} - a^2\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}} + \frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}} - \frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{c} - a^2\left(\frac{x\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}} + \frac{2\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}} + \frac{2\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}} + \frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}} - \frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{3c} - \frac{2}{9}\left(\frac{2x}{3c^2\sqrt{a^2cx^2+c}} + \right.\right.$$

$\downarrow$ 5479

$$\frac{\frac{2a\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx - \frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}}{c} - a^2\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}} + \frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}} - \frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{c} - a^2\left(\frac{x\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}} + \frac{2\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}} + \frac{2\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}} + \frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}} - \frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{3c} - \frac{2}{9}\left(\frac{2x}{3c^2\sqrt{a^2cx^2+c}} + \right.\right.$$

$\downarrow$ 5493

$$\frac{\frac{\frac{2a\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}} - \frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}}{c} - a^2\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}} + \frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}} - \frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{c} - a^2\left(\frac{x\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}} + \frac{2\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}} + \frac{2\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}} + \frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}} - \frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{3c} - \frac{2}{9}\left(\frac{2x}{3c^2\sqrt{a^2cx^2+c}} + \right.\right.$$

$\downarrow$ 5489

$$-a^2\left(\frac{x\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}} + \frac{2\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}} + \frac{2\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}} + \frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}} - \frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{3c} - \frac{2}{9}\left(\frac{2x}{3c^2\sqrt{a^2cx^2+c}} \right.\right.$$

$$\frac{-a^2\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}} + \frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}} - \frac{2x}{c\sqrt{a^2cx^2+c}}\right) + \frac{-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx} + \frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2\right.\right.}{\sqrt{a^2cx^2+c}}}{c}}{c}$$

input
```
Int[ArcTan[a*x]^2/(x^2*(c + a^2*c*x^2)^(5/2)),x]
```

output
```
-(a^2*((-2*(x/(3*c*(c + a^2*c*x^2)^(3/2)) + (2*x)/(3*c^2*Sqrt[c + a^2*c*x^
2])))/9 + (2*ArcTan[a*x])/(9*a*c*(c + a^2*c*x^2)^(3/2)) + (x*ArcTan[a*x]^2
)/(3*c*(c + a^2*c*x^2)^(3/2)) + (2*((-2*x)/(c*Sqrt[c + a^2*c*x^2]) + (2*Ar
cTan[a*x])/(a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x]^2)/(c*Sqrt[c + a^2*c
*x^2])))/(3*c))) + (-(a^2*((-2*x)/(c*Sqrt[c + a^2*c*x^2]) + (2*ArcTan[a*x]
)/(a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x]^2)/(c*Sqrt[c + a^2*c*x^2])))
 + (-((Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(c*x)) + (2*a*Sqrt[1 + a^2*x^2]*(
-2*ArcTan[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]] + I*PolyLog[2, -(S
qrt[1 + I*a*x]/Sqrt[1 - I*a*x])] - I*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I
*a*x]]))/Sqrt[c + a^2*c*x^2])/c)/c
```

### Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 5433
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x] - Simp[b^2*p*(p
- 1)   Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2), x], x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

rule 5435
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[b*p*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p - 1)/(4*c*d*
(q + 1)^2)), x] + (-Simp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*d*
(q + 1))), x] + Simp[(2*q + 3)/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p, x], x] - Simp[b^2*p*((p - 1)/(4*(q + 1)^2))   Int[(d + e
*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5489
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sq
rt[1 - I*c*x]], x] + (Simp[I*(b/Sqrt[d])*PolyLog[2, -Sqrt[1 + I*c*x]/Sqrt[1
 - I*c*x]], x] - Simp[I*(b/Sqrt[d])*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x]], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

## Maple [A] (verified)

Time = 2.37 (sec) , antiderivative size = 579, normalized size of antiderivative = 1.52

| method | result |
|---|---|
| default | $-\frac{\left(-54\arctan(ax)\ln\left(1-\frac{iax+1}{\sqrt{a^2x^2+1}}\right)a^5x^5+54\arctan(ax)\ln\left(1+\frac{iax+1}{\sqrt{a^2x^2+1}}\right)a^5x^5+108i\,\mathrm{polylog}\left(2,\frac{iax+1}{\sqrt{a^2x^2+1}}\right)a^3x^3+54i\,\mathrm{polylog}\left(\right.\right.}{}$ |

input
```
int(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/27*(-54*arctan(a*x)*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))*a^5*x^5+54*arctan
(a*x)*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))*a^5*x^5+108*I*polylog(2,(1+I*a*x)/
(a^2*x^2+1)^(1/2))*a^3*x^3+54*I*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))*a*x
+72*arctan(a*x)^2*(a^2*x^2+1)^(1/2)*a^4*x^4-94*a^4*x^4*(a^2*x^2+1)^(1/2)-1
08*arctan(a*x)*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))*a^3*x^3+108*arctan(a*x)*l
n(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))*a^3*x^3+90*arctan(a*x)*(a^2*x^2+1)^(1/2)*
a^3*x^3-54*I*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))*a*x-108*I*polylog(2,-
(1+I*a*x)/(a^2*x^2+1)^(1/2))*a^3*x^3+108*arctan(a*x)^2*(a^2*x^2+1)^(1/2)*a
^2*x^2-96*a^2*x^2*(a^2*x^2+1)^(1/2)-54*arctan(a*x)*ln(1-(1+I*a*x)/(a^2*x^2
+1)^(1/2))*a*x+54*arctan(a*x)*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))*a*x+96*arc
tan(a*x)*(a^2*x^2+1)^(1/2)*a*x-54*I*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2)
)*a^5*x^5+54*I*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))*a^5*x^5+27*arctan(a*
x)^2*(a^2*x^2+1)^(1/2))*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/2)/x/c^3/
(a^4*x^4+2*a^2*x^2+1)
```

## Fricas [F]

$$\int \frac{\arctan(ax)^2}{x^2\left(c+a^2cx^2\right)^{5/2}}\,dx=\int \frac{\arctan\left(ax\right)^2}{\left(a^2cx^2+c\right)^{\frac{5}{2}}x^2}\,dx$$

input
```
integrate(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2/(a^6*c^3*x^8 + 3*a^4*c^3*x^6 +
3*a^2*c^3*x^4 + c^3*x^2), x)
```

## Sympy [F]

$$\int \frac{\arctan(ax)^2}{x^2(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\operatorname{atan}^2(ax)}{x^2\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(atan(a*x)**2/x**2/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(atan(a*x)**2/(x**2*(c*(a**2*x**2 + 1))**(5/2)), x)
```

## Maxima [F]

$$\int \frac{\arctan(ax)^2}{x^2(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\arctan(ax)^2}{(a^2cx^2+c)^{\frac{5}{2}}x^2}\,dx$$

input
```
integrate(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)^2/((a^2*c*x^2 + c)^(5/2)*x^2), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^2}{x^2(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^2/x^2/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^2}{x^2(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\operatorname{atan}(a\,x)^2}{x^2\,(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int(atan(a*x)^2/(x^2*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(atan(a*x)^2/(x^2*(c + a^2*c*x^2)^(5/2)), x)
```

## Reduce [F]

$$\int \frac{\arctan(ax)^2}{x^2(c+a^2cx^2)^{5/2}}\,dx = \frac{\int \frac{atan(ax)^2}{\sqrt{a^2x^2+1}\,a^4x^6+2\sqrt{a^2x^2+1}\,a^2x^4+\sqrt{a^2x^2+1}\,x^2}dx}{\sqrt{c}\,c^2}$$

input
```
int(atan(a*x)^2/x^2/(a^2*c*x^2+c)^(5/2),x)
```

output
```
int(atan(a*x)**2/(sqrt(a**2*x**2 + 1)*a**4*x**6 + 2*sqrt(a**2*x**2 + 1)*a*
*2*x**4 + sqrt(a**2*x**2 + 1)*x**2),x)/(sqrt(c)*c**2)
```

# 3.355 $\int x^m(c+a^2cx^2)^2\arctan(ax)^2\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int x^m(c+a^2cx^2)^2\arctan(ax)^2\,dx=\text{Int}\left(x^m(c+a^2cx^2)^2\arctan(ax)^2,x\right)$$

output
```
Defer(Int)(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^2,x)
```

## Mathematica [N/A]

Not integrable

Time = 1.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int x^m(c+a^2cx^2)^2\arctan(ax)^2\,dx=\int x^m(c+a^2cx^2)^2\arctan(ax)^2\,dx$$

input
```
Integrate[x^m*(c + a^2*c*x^2)^2*ArcTan[a*x]^2,x]
```

output
```
Integrate[x^m*(c + a^2*c*x^2)^2*ArcTan[a*x]^2, x]
```

## Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \arctan(ax)^2 \left(a^2cx^2+c\right)^2 dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int x^m \arctan(ax)^2 \left(a^2cx^2+c\right)^2 dx$$

input
```
Int[x^m*(c + a^2*c*x^2)^2*ArcTan[a*x]^2,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 2.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x^m \left(a^2c\,x^2+c\right)^2 \arctan\left(ax\right)^2 dx$$

input
```
int(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^2,x)
```

output
```
int(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

$$\int x^m \left(c+a^2cx^2\right)^2 \arctan(ax)^2\,dx = \int \left(a^2cx^2+c\right)^2 x^m \arctan\left(ax\right)^2\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*x^m*arctan(a*x)^2, x)
```

## Sympy [N/A]

Not integrable

Time = 11.98 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.41

$$\int x^m \left(c+a^2cx^2\right)^2 \arctan(ax)^2\,dx = c^2\left(\int x^m \operatorname{atan}^2\left(ax\right)dx + \int 2a^2x^2x^m \operatorname{atan}^2\left(ax\right)dx + \int a^4x^4x^m \operatorname{atan}^2\left(ax\right)dx\right)$$

input
```
integrate(x**m*(a**2*c*x**2+c)**2*atan(a*x)**2,x)
```

output
```
c**2*(Integral(x**m*atan(a*x)**2, x) + Integral(2*a**2*x**2*x**m*atan(a*x)
**2, x) + Integral(a**4*x**4*x**m*atan(a*x)**2, x))
```

## Maxima [N/A]

Not integrable

Time = 10.39 (sec) , antiderivative size = 841, normalized size of antiderivative = 38.23

$$\int x^m (c + a^2 c x^2)^2 \arctan(ax)^2 \, dx = \int (a^2 c x^2 + c)^2 x^m \arctan(ax)^2 \, dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^2,x, algorithm="maxima")
```

output
```
1/16*(4*((a^4*c^2*m^2 + 4*a^4*c^2*m + 3*a^4*c^2)*x^5 + 2*(a^2*c^2*m^2 + 6*
a^2*c^2*m + 5*a^2*c^2)*x^3 + (c^2*m^2 + 8*c^2*m + 15*c^2)*x)*x^m*arctan(a*
x)^2 - ((a^4*c^2*m^2 + 4*a^4*c^2*m + 3*a^4*c^2)*x^5 + 2*(a^2*c^2*m^2 + 6*a
^2*c^2*m + 5*a^2*c^2)*x^3 + (c^2*m^2 + 8*c^2*m + 15*c^2)*x)*x^m*log(a^2*x^
2 + 1)^2 + 16*(m^3 + 9*m^2 + 23*m + 15)*integrate(1/16*(12*((a^6*c^2*m^3 +
 9*a^6*c^2*m^2 + 23*a^6*c^2*m + 15*a^6*c^2)*x^6 + c^2*m^3 + 3*(a^4*c^2*m^3
 + 9*a^4*c^2*m^2 + 23*a^4*c^2*m + 15*a^4*c^2)*x^4 + 9*c^2*m^2 + 23*c^2*m +
 3*(a^2*c^2*m^3 + 9*a^2*c^2*m^2 + 23*a^2*c^2*m + 15*a^2*c^2)*x^2 + 15*c^2)
*x^m*arctan(a*x)^2 + ((a^6*c^2*m^3 + 9*a^6*c^2*m^2 + 23*a^6*c^2*m + 15*a^6
*c^2)*x^6 + c^2*m^3 + 3*(a^4*c^2*m^3 + 9*a^4*c^2*m^2 + 23*a^4*c^2*m + 15*a
^4*c^2)*x^4 + 9*c^2*m^2 + 23*c^2*m + 3*(a^2*c^2*m^3 + 9*a^2*c^2*m^2 + 23*a
^2*c^2*m + 15*a^2*c^2)*x^2 + 15*c^2)*x^m*log(a^2*x^2 + 1)^2 - 8*((a^5*c^2*
m^2 + 4*a^5*c^2*m + 3*a^5*c^2)*x^5 + 2*(a^3*c^2*m^2 + 6*a^3*c^2*m + 5*a^3*
c^2)*x^3 + (a*c^2*m^2 + 8*a*c^2*m + 15*a*c^2)*x)*x^m*arctan(a*x) + 4*((a^6
*c^2*m^2 + 4*a^6*c^2*m + 3*a^6*c^2)*x^6 + 2*(a^4*c^2*m^2 + 6*a^4*c^2*m + 5
*a^4*c^2)*x^4 + (a^2*c^2*m^2 + 8*a^2*c^2*m + 15*a^2*c^2)*x^2)*x^m*log(a^2*
x^2 + 1))/(m^3 + (a^2*m^3 + 9*a^2*m^2 + 23*a^2*m + 15*a^2)*x^2 + 9*m^2 + 2
3*m + 15), x))/(m^3 + 9*m^2 + 23*m + 15)
```

## Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int x^m (c + a^2 c x^2)^2 \arctan(ax)^2 \, dx = \int (a^2 c x^2 + c)^2 x^m \arctan(ax)^2 \, dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^2*x^m*arctan(a*x)^2, x)
```

## Mupad [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int x^m (c + a^2cx^2)^2 \arctan(ax)^2 \, dx = \int x^m \operatorname{atan}(a\,x)^2 \left(c\,a^2\,x^2 + c\right)^2 dx$$

input
```
int(x^m*atan(a*x)^2*(c + a^2*c*x^2)^2,x)
```

output
```
int(x^m*atan(a*x)^2*(c + a^2*c*x^2)^2, x)
```

## Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 2267, normalized size of antiderivative = 103.05

$$\int x^m (c + a^2cx^2)^2 \arctan(ax)^2 \, dx = \text{Too large to display}$$

input
```
int(x^m*(a^2*c*x^2+c)^2*atan(a*x)^2,x)
```

output

```
(c**2*(x**m*atan(a*x)**2*a**5*m**5*x**5 + 10*x**m*atan(a*x)**2*a**5*m**4*x
**5 + 35*x**m*atan(a*x)**2*a**5*m**3*x**5 + 50*x**m*atan(a*x)**2*a**5*m**2
*x**5 + 24*x**m*atan(a*x)**2*a**5*m*x**5 + 2*x**m*atan(a*x)**2*a**3*m**5*x
**3 + 24*x**m*atan(a*x)**2*a**3*m**4*x**3 + 98*x**m*atan(a*x)**2*a**3*m**3
*x**3 + 156*x**m*atan(a*x)**2*a**3*m**2*x**3 + 80*x**m*atan(a*x)**2*a**3*m
*x**3 + x**m*atan(a*x)**2*a*m**5*x + 14*x**m*atan(a*x)**2*a*m**4*x + 71*x*
*m*atan(a*x)**2*a*m**3*x + 154*x**m*atan(a*x)**2*a*m**2*x + 120*x**m*atan(
a*x)**2*a*m*x - 2*x**m*atan(a*x)*a**4*m**4*x**4 - 12*x**m*atan(a*x)*a**4*m
**3*x**4 - 22*x**m*atan(a*x)*a**4*m**2*x**4 - 12*x**m*atan(a*x)*a**4*m*x**
4 - 2*x**m*atan(a*x)*a**2*m**4*x**2 - 24*x**m*atan(a*x)*a**2*m**3*x**2 - 7
8*x**m*atan(a*x)*a**2*m**2*x**2 - 56*x**m*atan(a*x)*a**2*m*x**2 - 16*x**m*
atan(a*x)*m**2 - 96*x**m*atan(a*x)*m - 128*x**m*atan(a*x) + 2*x**m*a**3*m*
*3*x**3 + 6*x**m*a**3*m**2*x**3 + 4*x**m*a**3*m*x**3 + 12*x**m*a*m**2*x +
44*x**m*a*m*x - 12*int(x**m/(a**2*m**5*x**2 + 15*a**2*m**4*x**2 + 85*a**2*
m**3*x**2 + 225*a**2*m**2*x**2 + 274*a**2*m*x**2 + 120*a**2*x**2 + m**5 +
15*m**4 + 85*m**3 + 225*m**2 + 274*m + 120),x)*a*m**8 - 220*int(x**m/(a**2
*m**5*x**2 + 15*a**2*m**4*x**2 + 85*a**2*m**3*x**2 + 225*a**2*m**2*x**2 +
274*a**2*m*x**2 + 120*a**2*x**2 + m**5 + 15*m**4 + 85*m**3 + 225*m**2 + 27
4*m + 120),x)*a*m**7 - 1568*int(x**m/(a**2*m**5*x**2 + 15*a**2*m**4*x**2 +
 85*a**2*m**3*x**2 + 225*a**2*m**2*x**2 + 274*a**2*m*x**2 + 120*a**2*x*...
```

# 3.356 $\int x^m(c+a^2cx^2)\arctan(ax)^2\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 20

$$\int x^m(c+a^2cx^2)\arctan(ax)^2\,dx = \text{Int}\left(x^m(c+a^2cx^2)\arctan(ax)^2,x\right)$$

output
```
Defer(Int)(x^m*(a^2*c*x^2+c)*arctan(a*x)^2,x)
```

## Mathematica [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x^m(c+a^2cx^2)\arctan(ax)^2\,dx = \int x^m(c+a^2cx^2)\arctan(ax)^2\,dx$$

input
```
Integrate[x^m*(c + a^2*c*x^2)*ArcTan[a*x]^2,x]
```

output
```
Integrate[x^m*(c + a^2*c*x^2)*ArcTan[a*x]^2, x]
```

## Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \arctan(ax)^2 \left(a^2cx^2+c\right) \, dx$$

$$\Big\downarrow \text{5560}$$

$$\int x^m \arctan(ax)^2 \left(a^2cx^2+c\right) dx$$

input
```
Int[x^m*(c + a^2*c*x^2)*ArcTan[a*x]^2,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 1.87 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int x^m \left(a^2 c\,x^2+c\right) \arctan\left(a x\right)^2 dx$$

input
```
int(x^m*(a^2*c*x^2+c)*arctan(a*x)^2,x)
```

output
```
int(x^m*(a^2*c*x^2+c)*arctan(a*x)^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x^m (c + a^2cx^2) \arctan(ax)^2 \, dx = \int (a^2cx^2 + c)x^m \arctan(ax)^2 \, dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)*arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)*x^m*arctan(a*x)^2, x)
```

## Sympy [N/A]

Not integrable

Time = 5.78 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.55

$$\int x^m (c + a^2cx^2) \arctan(ax)^2 \, dx = c\left(\int x^m \operatorname{atan}^2(ax) \, dx + \int a^2x^2x^m \operatorname{atan}^2(ax) \, dx\right)$$

input
```
integrate(x**m*(a**2*c*x**2+c)*atan(a*x)**2,x)
```

output
```
c*(Integral(x**m*atan(a*x)**2, x) + Integral(a**2*x**2*x**m*atan(a*x)**2,
x))
```

## Maxima [N/A]

Not integrable

Time = 4.62 (sec) , antiderivative size = 386, normalized size of antiderivative = 19.30

$$\int x^m (c + a^2cx^2) \arctan(ax)^2 \, dx = \int (a^2cx^2 + c)x^m \arctan(ax)^2 \, dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)*arctan(a*x)^2,x, algorithm="maxima")
```

output
```
1/16*(4*((a^2*c*m + a^2*c)*x^3 + (c*m + 3*c)*x)*x^m*arctan(a*x)^2 - ((a^2*
c*m + a^2*c)*x^3 + (c*m + 3*c)*x)*x^m*log(a^2*x^2 + 1)^2 + 16*(m^2 + 4*m +
 3)*integrate(1/16*(12*((a^4*c*m^2 + 4*a^4*c*m + 3*a^4*c)*x^4 + c*m^2 + 2*
(a^2*c*m^2 + 4*a^2*c*m + 3*a^2*c)*x^2 + 4*c*m + 3*c)*x^m*arctan(a*x)^2 + (
(a^4*c*m^2 + 4*a^4*c*m + 3*a^4*c)*x^4 + c*m^2 + 2*(a^2*c*m^2 + 4*a^2*c*m +
 3*a^2*c)*x^2 + 4*c*m + 3*c)*x^m*log(a^2*x^2 + 1)^2 - 8*((a^3*c*m + a^3*c)
*x^3 + (a*c*m + 3*a*c)*x)*x^m*arctan(a*x) + 4*((a^4*c*m + a^4*c)*x^4 + (a^
2*c*m + 3*a^2*c)*x^2)*x^m*log(a^2*x^2 + 1))/((a^2*m^2 + 4*a^2*m + 3*a^2)*x
^2 + m^2 + 4*m + 3), x))/(m^2 + 4*m + 3)
```

## Giac [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x^m (c + a^2cx^2) \arctan(ax)^2 \, dx = \int (a^2cx^2 + c) x^m \arctan(ax)^2 \, dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)*arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)*x^m*arctan(a*x)^2, x)
```

## Mupad [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x^m (c + a^2cx^2) \arctan(ax)^2 \, dx = \int x^m \operatorname{atan}(a\,x)^2 \left(c\,a^2\,x^2 + c\right) \, dx$$

input
```
int(x^m*atan(a*x)^2*(c + a^2*c*x^2),x)
```

output
```
int(x^m*atan(a*x)^2*(c + a^2*c*x^2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 940, normalized size of antiderivative = 47.00

$$\int x^m(c+a^2cx^2)\arctan(ax)^2\,dx = \text{Too large to display}$$

input
```
int(x^m*(a^2*c*x^2+c)*atan(a*x)^2,x)
```

output
```
(c*(x**m*atan(a*x)**2*a**3*m**3*x**3 + 3*x**m*atan(a*x)**2*a**3*m**2*x**3
+ 2*x**m*atan(a*x)**2*a**3*m*x**3 + x**m*atan(a*x)**2*a*m**3*x + 5*x**m*at
an(a*x)**2*a*m**2*x + 6*x**m*atan(a*x)**2*a*m*x - 2*x**m*atan(a*x)*a**2*m*
*2*x**2 - 2*x**m*atan(a*x)*a**2*m*x**2 - 4*x**m*atan(a*x)*m - 8*x**m*atan(
a*x) + 2*x**m*a*m*x - 2*int(x**m/(a**2*m**3*x**2 + 6*a**2*m**2*x**2 + 11*a
**2*m*x**2 + 6*a**2*x**2 + m**3 + 6*m**2 + 11*m + 6),x)*a*m**5 - 10*int(x*
*m/(a**2*m**3*x**2 + 6*a**2*m**2*x**2 + 11*a**2*m*x**2 + 6*a**2*x**2 + m**
3 + 6*m**2 + 11*m + 6),x)*a*m**4 - 2*int(x**m/(a**2*m**3*x**2 + 6*a**2*m**
2*x**2 + 11*a**2*m*x**2 + 6*a**2*x**2 + m**3 + 6*m**2 + 11*m + 6),x)*a*m**
3 + 58*int(x**m/(a**2*m**3*x**2 + 6*a**2*m**2*x**2 + 11*a**2*m*x**2 + 6*a*
*2*x**2 + m**3 + 6*m**2 + 11*m + 6),x)*a*m**2 + 100*int(x**m/(a**2*m**3*x*
*2 + 6*a**2*m**2*x**2 + 11*a**2*m*x**2 + 6*a**2*x**2 + m**3 + 6*m**2 + 11*
m + 6),x)*a*m + 48*int(x**m/(a**2*m**3*x**2 + 6*a**2*m**2*x**2 + 11*a**2*m
*x**2 + 6*a**2*x**2 + m**3 + 6*m**2 + 11*m + 6),x)*a + 4*int((x**m*atan(a*
x))/(a**2*m**3*x**3 + 6*a**2*m**2*x**3 + 11*a**2*m*x**3 + 6*a**2*x**3 + m*
*3*x + 6*m**2*x + 11*m*x + 6*x),x)*m**5 + 32*int((x**m*atan(a*x))/(a**2*m*
*3*x**3 + 6*a**2*m**2*x**3 + 11*a**2*m*x**3 + 6*a**2*x**3 + m**3*x + 6*m**
2*x + 11*m*x + 6*x),x)*m**4 + 92*int((x**m*atan(a*x))/(a**2*m**3*x**3 + 6*
a**2*m**2*x**3 + 11*a**2*m*x**3 + 6*a**2*x**3 + m**3*x + 6*m**2*x + 11*m*x
 + 6*x),x)*m**3 + 112*int((x**m*atan(a*x))/(a**2*m**3*x**3 + 6*a**2*m**...
```

# 3.357 $\int \frac{x^m \arctan(ax)^2}{c+a^2cx^2}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^m \arctan(ax)^2}{c+a^2cx^2}\,dx = \text{Int}\left(\frac{x^m \arctan(ax)^2}{c+a^2cx^2},x\right)$$

output
```
Defer(Int)(x^m*arctan(a*x)^2/(a^2*c*x^2+c),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m \arctan(ax)^2}{c+a^2cx^2}\,dx = \int \frac{x^m \arctan(ax)^2}{c+a^2cx^2}\,dx$$

input
```
Integrate[(x^m*ArcTan[a*x]^2)/(c + a^2*c*x^2),x]
```

output
```
Integrate[(x^m*ArcTan[a*x]^2)/(c + a^2*c*x^2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \arctan(ax)^2}{a^2cx^2+c} \, dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x^m \arctan(ax)^2}{a^2cx^2+c}dx$$

input
```
Int[(x^m*ArcTan[a*x]^2)/(c + a^2*c*x^2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 1.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan \left(a x \right)^2}{a^2 c \,x^2+c}d x$$

input
```
int(x^m*arctan(a*x)^2/(a^2*c*x^2+c),x)
```

output
```
int(x^m*arctan(a*x)^2/(a^2*c*x^2+c),x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m \arctan(ax)^2}{c + a^2cx^2} \, dx = \int \frac{x^m \arctan(ax)^2}{a^2cx^2 + c} \, dx$$

input
```
integrate(x^m*arctan(a*x)^2/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
integral(x^m*arctan(a*x)^2/(a^2*c*x^2 + c), x)
```

## Sympy [N/A]

Not integrable

Time = 1.50 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x^m \arctan(ax)^2}{c + a^2cx^2} \, dx = \frac{\int \frac{x^m \operatorname{atan}^2(ax)}{a^2x^2+1} \, dx}{c}$$

input
```
integrate(x**m*atan(a*x)**2/(a**2*c*x**2+c),x)
```

output
```
Integral(x**m*atan(a*x)**2/(a**2*x**2 + 1), x)/c
```

## Maxima [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m \arctan(ax)^2}{c + a^2cx^2} \, dx = \int \frac{x^m \arctan(ax)^2}{a^2cx^2 + c} \, dx$$

input
```
integrate(x^m*arctan(a*x)^2/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
integrate(x^m*arctan(a*x)^2/(a^2*c*x^2 + c), x)
```

## Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m \arctan(ax)^2}{c + a^2cx^2} \, dx = \int \frac{x^m \arctan{(ax)}^2}{a^2cx^2 + c} \, dx$$

input
```
integrate(x^m*arctan(a*x)^2/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
integrate(x^m*arctan(a*x)^2/(a^2*c*x^2 + c), x)
```

## Mupad [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m \arctan(ax)^2}{c + a^2cx^2} \, dx = \int \frac{x^m \operatorname{atan}(a\,x)^2}{c\,a^2\,x^2 + c} \, dx$$

input
```
int((x^m*atan(a*x)^2)/(c + a^2*c*x^2),x)
```

output
```
int((x^m*atan(a*x)^2)/(c + a^2*c*x^2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

$$\int \frac{x^m \arctan(ax)^2}{c + a^2cx^2} \, dx = \frac{\int \frac{x^m atan(ax)^2}{a^2x^2+1} dx}{c}$$

input
```
int(x^m*atan(a*x)^2/(a^2*c*x^2+c),x)
```

output
```
int((x**m*atan(a*x)**2)/(a**2*x**2 + 1),x)/c
```

# 3.358 $\int \frac{x^m \arctan(ax)^2}{(c+a^2cx^2)^2} dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^m \arctan(ax)^2}{(c+a^2cx^2)^2} dx = \operatorname{Int}\left(\frac{x^m \arctan(ax)^2}{(c+a^2cx^2)^2}, x\right)$$

output
```
Defer(Int)(x^m*arctan(a*x)^2/(a^2*c*x^2+c)^2,x)
```

## Mathematica [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m \arctan(ax)^2}{(c+a^2cx^2)^2} dx = \int \frac{x^m \arctan(ax)^2}{(c+a^2cx^2)^2} dx$$

input
```
Integrate[(x^m*ArcTan[a*x]^2)/(c + a^2*c*x^2)^2,x]
```

output
```
Integrate[(x^m*ArcTan[a*x]^2)/(c + a^2*c*x^2)^2, x]
```

## Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \arctan(ax)^2}{(a^2cx^2+c)^2}dx$$

$$\Big\downarrow \text{5560}$$

$$\int \frac{x^m \arctan(ax)^2}{(a^2cx^2+c)^2}dx$$

input
```
Int[(x^m*ArcTan[a*x]^2)/(c + a^2*c*x^2)^2,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 1.81 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan(ax)^2}{(a^2c\,x^2+c)^2}dx$$

input
```
int(x^m*arctan(a*x)^2/(a^2*c*x^2+c)^2,x)
```

output
```
int(x^m*arctan(a*x)^2/(a^2*c*x^2+c)^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.77

$$\int \frac{x^m \arctan(ax)^2}{(c+a^2cx^2)^2} \, dx = \int \frac{x^m \arctan{(ax)}^2}{(a^2cx^2+c)^2} \, dx$$

input
```
integrate(x^m*arctan(a*x)^2/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
integral(x^m*arctan(a*x)^2/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2), x)
```

## Sympy [N/A]

Not integrable

Time = 2.77 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.41

$$\int \frac{x^m \arctan(ax)^2}{(c+a^2cx^2)^2} \, dx = \frac{\int \frac{x^m \operatorname{atan}^2{(ax)}}{a^4x^4+2a^2x^2+1} \, dx}{c^2}$$

input
```
integrate(x**m*atan(a*x)**2/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(x**m*atan(a*x)**2/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

## Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m \arctan(ax)^2}{(c+a^2cx^2)^2} \, dx = \int \frac{x^m \arctan{(ax)}^2}{(a^2cx^2+c)^2} \, dx$$

input
```
integrate(x^m*arctan(a*x)^2/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
integrate(x^m*arctan(a*x)^2/(a^2*c*x^2 + c)^2, x)
```

## Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m \arctan(ax)^2}{\left(c+a^2cx^2\right)^2} \, dx = \int \frac{x^m \arctan\left(ax\right)^2}{\left(a^2cx^2+c\right)^2} \, dx$$

input
```
integrate(x^m*arctan(a*x)^2/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate(x^m*arctan(a*x)^2/(a^2*c*x^2 + c)^2, x)
```

## Mupad [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m \arctan(ax)^2}{\left(c+a^2cx^2\right)^2} \, dx = \int \frac{x^m \operatorname{atan}(a\,x)^2}{\left(c\,a^2\,x^2+c\right)^2} \, dx$$

input
```
int((x^m*atan(a*x)^2)/(c + a^2*c*x^2)^2,x)
```

output
```
int((x^m*atan(a*x)^2)/(c + a^2*c*x^2)^2, x)
```

## Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.59

$$\int \frac{x^m \arctan(ax)^2}{(c + a^2cx^2)^2} dx = \frac{\int \frac{x^m atan(ax)^2}{a^4x^4 + 2a^2x^2 + 1} dx}{c^2}$$

input
```
int(x^m*atan(a*x)^2/(a^2*c*x^2+c)^2,x)
```

output
```
int((x**m*atan(a*x)**2)/(a**4*x**4 + 2*a**2*x**2 + 1),x)/c**2
```

# 3.359 $\int x^m(c+a^2cx^2)^{3/2}\arctan(ax)^2\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int x^m\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2\,dx=\text{Int}\left(x^m\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2,x\right)$$

output
```
Defer(Int)(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x)
```

## Mathematica [N/A]

Not integrable

Time = 0.98 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int x^m\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2\,dx=\int x^m\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2\,dx$$

input
```
Integrate[x^m*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2,x]
```

output
```
Integrate[x^m*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2, x]
```

## Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \arctan(ax)^2 \left(a^2cx^2+c\right)^{3/2} dx$$

$$\big\downarrow \text{5560}$$

$$\int x^m \arctan(ax)^2 \left(a^2cx^2+c\right)^{3/2} dx$$

input
```
Int[x^m*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 2.43 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int x^m \left(a^2 c\, x^2+c\right)^{\frac{3}{2}} \arctan\left(a x\right)^2 dx$$

input
```
int(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x)
```

output
```
int(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^m \left(c+a^2 c x^2\right)^{3/2} \arctan(ax)^2 \, dx = \int \left(a^2 c x^2+c\right)^{\frac{3}{2}} x^m \arctan\left(ax\right)^2 \, dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^(3/2)*x^m*arctan(a*x)^2, x)
```

## Sympy [F(-1)]

Timed out.

$$\int x^m \left(c+a^2 c x^2\right)^{3/2} \arctan(ax)^2 \, dx = \text{Timed out}$$

input
```
integrate(x**m*(a**2*c*x**2+c)**(3/2)*atan(a*x)**2,x)
```

output
```
Timed out
```

## Maxima [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^m \left(c+a^2 c x^2\right)^{3/2} \arctan(ax)^2 \, dx = \int \left(a^2 c x^2+c\right)^{\frac{3}{2}} x^m \arctan\left(ax\right)^2 \, dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*x^m*arctan(a*x)^2, x)
```

## Giac [F(-2)]

Exception generated.

$$\int x^m \left(c+a^2cx^2\right)^{3/2} \arctan(ax)^2 \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^m \left(c+a^2cx^2\right)^{3/2} \arctan(ax)^2 \, dx = \int x^m \operatorname{atan}(a\,x)^2 \left(c\,a^2\,x^2+c\right)^{3/2} dx$$

input
```
int(x^m*atan(a*x)^2*(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(x^m*atan(a*x)^2*(c + a^2*c*x^2)^(3/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.33

$$\int x^m \left(c+a^2cx^2\right)^{3/2} \arctan(ax)^2 \, dx = \sqrt{c}\,c\left(\left(\int x^m\sqrt{a^2x^2+1}\,atan(ax)^2\,x^2dx\right)a^2 + \int x^m\sqrt{a^2x^2+1}\,atan(ax)^2\,dx\right)$$

input
```
int(x^m*(a^2*c*x^2+c)^(3/2)*atan(a*x)^2,x)
```

output
```
sqrt(c)*c*(int(x**m*sqrt(a**2*x**2 + 1)*atan(a*x)**2*x**2,x)*a**2 + int(x*
*m*sqrt(a**2*x**2 + 1)*atan(a*x)**2,x))
```

# 3.360 $\int x^m \sqrt{c+a^2 c x^2} \arctan(ax)^2 \, dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int x^m \sqrt{c+a^2 c x^2} \arctan(ax)^2 \, dx = \text{Int}\left(x^m \sqrt{c+a^2 c x^2} \arctan(ax)^2, x\right)$$

output
```
Defer(Int)(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2,x)
```

## Mathematica [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int x^m \sqrt{c+a^2 c x^2} \arctan(ax)^2 \, dx = \int x^m \sqrt{c+a^2 c x^2} \arctan(ax)^2 \, dx$$

input
```
Integrate[x^m*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2,x]
```

output
```
Integrate[x^m*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2, x]
```

## Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \arctan(ax)^2 \sqrt{a^2cx^2+c}\,dx$$

$$\Big\downarrow \ 5560$$

$$\int x^m \arctan(ax)^2 \sqrt{a^2cx^2+c}dx$$

input
```
Int[x^m*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 2.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int x^m \sqrt{a^2c\,x^2+c}\,\arctan\left(ax\right)^2 dx$$

input
```
int(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2,x)
```

output
```
int(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^m \sqrt{c + a^2 c x^2} \arctan(ax)^2 \, dx = \int \sqrt{a^2 c x^2 + c} x^m \arctan(ax)^2 \, dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^m*arctan(a*x)^2, x)
```

## Sympy [N/A]

Not integrable

Time = 29.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^m \sqrt{c + a^2 c x^2} \arctan(ax)^2 \, dx = \int x^m \sqrt{c (a^2 x^2 + 1)} \operatorname{atan}^2(ax) \, dx$$

input
```
integrate(x**m*(a**2*c*x**2+c)**(1/2)*atan(a*x)**2,x)
```

output
```
Integral(x**m*sqrt(c*(a**2*x**2 + 1))*atan(a*x)**2, x)
```

## Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^m \sqrt{c + a^2 c x^2} \arctan(ax)^2 \, dx = \int \sqrt{a^2 c x^2 + c} x^m \arctan(ax)^2 \, dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*x^m*arctan(a*x)^2, x)
```

## Giac [F(-2)]

Exception generated.

$$\int x^m\sqrt{c+a^2cx^2}\arctan(ax)^2\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^m\sqrt{c+a^2cx^2}\arctan(ax)^2\,dx=\int x^m\,\mathrm{atan}(a\,x)^2\,\sqrt{c\,a^2\,x^2+c}\,dx$$

input
```
int(x^m*atan(a*x)^2*(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(x^m*atan(a*x)^2*(c + a^2*c*x^2)^(1/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int x^m \sqrt{c+a^2cx^2} \arctan(ax)^2 \, dx = \sqrt{c} \left( \int x^m \sqrt{a^2x^2+1} \, atan(ax)^2 \, dx \right)$$

input
```
int(x^m*(a^2*c*x^2+c)^(1/2)*atan(a*x)^2,x)
```

output
```
sqrt(c)*int(x**m*sqrt(a**2*x**2 + 1)*atan(a*x)**2,x)
```

# 3.361 $\int \frac{x^m \arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m \arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx = \mathrm{Int}\left(\frac{x^m \arctan(ax)^2}{\sqrt{c+a^2cx^2}},x\right)$$

output
```
Defer(Int)(x^m*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^m \arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx$$

input
```
Integrate[(x^m*ArcTan[a*x]^2)/Sqrt[c + a^2*c*x^2],x]
```

output
```
Integrate[(x^m*ArcTan[a*x]^2)/Sqrt[c + a^2*c*x^2], x]
```

## Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \arctan(ax)^2}{\sqrt{a^2cx^2+c}} \, dx$$

$$\Big\downarrow \text{5560}$$

$$\int \frac{x^m \arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx$$

input
```
Int[(x^m*ArcTan[a*x]^2)/Sqrt[c + a^2*c*x^2],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 2.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m \arctan\left(ax\right)^2}{\sqrt{a^2c\,x^2+c}}dx$$

input
```
int(x^m*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(x^m*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^m \arctan(ax)^2}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(x^m*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(x^m*arctan(a*x)^2/sqrt(a^2*c*x^2 + c), x)
```

## Sympy [N/A]

Not integrable

Time = 17.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^m \operatorname{atan}^2(ax)}{\sqrt{c(a^2x^2+1)}}\,dx$$

input
```
integrate(x**m*atan(a*x)**2/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x**m*atan(a*x)**2/sqrt(c*(a**2*x**2 + 1)), x)
```

## Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^m \arctan(ax)^2}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(x^m*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^m*arctan(a*x)^2/sqrt(a^2*c*x^2 + c), x)
```

## Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^m \arctan{(ax)}^2}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(x^m*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^m*arctan(a*x)^2/sqrt(a^2*c*x^2 + c), x)
```

## Mupad [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan(ax)^2}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^m\,\mathrm{atan}{(a\,x)}^2}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int((x^m*atan(a*x)^2)/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int((x^m*atan(a*x)^2)/(c + a^2*c*x^2)^(1/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int \frac{x^m \arctan(ax)^2}{\sqrt{c + a^2cx^2}}\, dx = \frac{\int \frac{x^m atan(ax)^2}{\sqrt{a^2x^2+1}}dx}{\sqrt{c}}$$

input
```
int(x^m*atan(a*x)^2/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int((x**m*atan(a*x)**2)/sqrt(a**2*x**2 + 1),x)/sqrt(c)
```

# 3.362 $\int \frac{x^m \arctan(ax)^2}{(c+a^2cx^2)^{3/2}} \, dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m \arctan(ax)^2}{(c+a^2cx^2)^{3/2}} \, dx = \text{Int}\left(\frac{x^m \arctan(ax)^2}{(c+a^2cx^2)^{3/2}}, x\right)$$

output
```
Defer(Int)(x^m*arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \arctan(ax)^2}{(c+a^2cx^2)^{3/2}} \, dx = \int \frac{x^m \arctan(ax)^2}{(c+a^2cx^2)^{3/2}} \, dx$$

input
```
Integrate[(x^m*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(3/2),x]
```

output
```
Integrate[(x^m*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(3/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int \frac{x^m \arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx$$

input
```
Int[(x^m*ArcTan[a*x]^2)/(c + a^2*c*x^2)^(3/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 2.44 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m \arctan\left(ax\right)^2}{\left(a^2c\,x^2+c\right)^{\frac{3}{2}}}dx$$

input
```
int(x^m*arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x)
```

output
```
int(x^m*arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.12

$$\int \frac{x^m \arctan(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^m \arctan{(ax)}^2}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m*arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^m*arctan(a*x)^2/(a^4*c^2*x^4 + 2*a^2*c^2*x^
2 + c^2), x)
```

## Sympy [N/A]

Not integrable

Time = 14.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^m \operatorname{atan}^2{(ax)}}{(c\,(a^2x^2+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**m*atan(a*x)**2/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(x**m*atan(a*x)**2/(c*(a**2*x**2 + 1))**(3/2), x)
```

## Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^m \arctan{(ax)}^2}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m*arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^m*arctan(a*x)^2/(a^2*c*x^2 + c)^(3/2), x)
```

## Giac [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^m \arctan{(ax)}^2}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m*arctan(a*x)^2/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^m*arctan(a*x)^2/(a^2*c*x^2 + c)^(3/2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^m\,\mathrm{atan}(a\,x)^2}{(c\,a^2\,x^2+c)^{3/2}}\,dx$$

input
```
int((x^m*atan(a*x)^2)/(c + a^2*c*x^2)^(3/2),x)
```

output
```
int((x^m*atan(a*x)^2)/(c + a^2*c*x^2)^(3/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.08

$$\int \frac{x^m \arctan(ax)^2}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\int \frac{x^m atan(ax)^2}{\sqrt{a^2x^2+1}\,a^2x^2+\sqrt{a^2x^2+1}}dx}{\sqrt{c}\,c}$$

input
```
int(x^m*atan(a*x)^2/(a^2*c*x^2+c)^(3/2),x)
```

output
```
int((x**m*atan(a*x)**2)/(sqrt(a**2*x**2 + 1)*a**2*x**2 + sqrt(a**2*x**2 +
1)),x)/(sqrt(c)*c)
```

# 3.363 $\int x^3(c+a^2cx^2)\arctan(ax)^3\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 219

$$\begin{aligned}\int x^3(c+a^2cx^2)\arctan(ax)^3\,dx &= \frac{cx}{15a^3}-\frac{cx^3}{60a}-\frac{c\arctan(ax)}{15a^4}-\frac{cx^2\arctan(ax)}{60a^2}\\&+\frac{1}{20}cx^4\arctan(ax)+\frac{7ic\arctan(ax)^2}{30a^4}\\&+\frac{cx\arctan(ax)^2}{4a^3}-\frac{cx^3\arctan(ax)^2}{12a}\\&-\frac{1}{10}acx^5\arctan(ax)^2-\frac{c\arctan(ax)^3}{12a^4}\\&+\frac{1}{4}cx^4\arctan(ax)^3+\frac{1}{6}a^2cx^6\arctan(ax)^3\\&+\frac{7c\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{15a^4}\\&+\frac{7ic\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{30a^4}\end{aligned}$$

output
```
1/15*c*x/a^3-1/60*c*x^3/a-1/15*c*arctan(a*x)/a^4-1/60*c*x^2*arctan(a*x)/a^
2+1/20*c*x^4*arctan(a*x)+7/30*I*c*arctan(a*x)^2/a^4+1/4*c*x*arctan(a*x)^2/
a^3-1/12*c*x^3*arctan(a*x)^2/a-1/10*a*c*x^5*arctan(a*x)^2-1/12*c*arctan(a*
x)^3/a^4+1/4*c*x^4*arctan(a*x)^3+1/6*a^2*c*x^6*arctan(a*x)^3+7/15*c*arctan
(a*x)*ln(2/(1+I*a*x))/a^4+7/30*I*c*polylog(2,1-2/(1+I*a*x))/a^4
```

## Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.62

$$\int x^3(c+a^2cx^2)\arctan(ax)^3\,dx$$
$$=\frac{c(4ax-a^3x^3-(14i-15ax+5a^3x^3+6a^5x^5)\arctan(ax)^2+5(-1+3a^4x^4+2a^6x^6)\arctan(ax)^3+\text{arct}}{60a^4}$$

input
```
Integrate[x^3*(c + a^2*c*x^2)*ArcTan[a*x]^3,x]
```

output
```
(c*(4*a*x - a^3*x^3 - (14*I - 15*a*x + 5*a^3*x^3 + 6*a^5*x^5)*ArcTan[a*x]^
2 + 5*(-1 + 3*a^4*x^4 + 2*a^6*x^6)*ArcTan[a*x]^3 + ArcTan[a*x]*(-4 - a^2*x
^2 + 3*a^4*x^4 + 28*Log[1 + E^((2*I)*ArcTan[a*x])]) - (14*I)*PolyLog[2, -E
^((2*I)*ArcTan[a*x])]))/(60*a^4)
```

## Rubi [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 711 vs. 2(219) = 438.

Time = 4.75 (sec) , antiderivative size = 711, normalized size of antiderivative = 3.25, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.100, Rules used = {5485, 5361, 5451, 5361, 5451, 5345, 5361, 254, 262, 216, 2009, 5419, 5451, 5345, 5361, 262, 216, 5419, 5455, 5379, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\arctan(ax)^3\left(a^2cx^2+c\right)\,dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\int x^5\arctan(ax)^3dx+c\int x^3\arctan(ax)^3dx$$

$$\Big\downarrow \text{5361}$$

$$a^2c\left(\frac{1}{6}x^6\arctan(ax)^3-\frac{1}{2}a\int\frac{x^6\arctan(ax)^2}{a^2x^2+1}dx\right)+ c\left(\frac{1}{4}x^4\arctan(ax)^3-\frac{3}{4}a\int\frac{x^4\arctan(ax)^2}{a^2x^2+1}dx\right)$$

$$\Big\downarrow \; 5451$$

$$c\left(\frac{1}{4}x^4\arctan(ax)^3-\frac{3}{4}a\left(\frac{\int x^2\arctan(ax)^2dx}{a^2}-\frac{\int\frac{x^2\arctan(ax)^2}{a^2x^2+1}dx}{a^2}\right)\right)+ a^2c\left(\frac{1}{6}x^6\arctan(ax)^3-\frac{1}{2}a\left(\frac{\int x^4\arctan(ax)^2dx}{a^2}-\frac{\int\frac{x^4\arctan(ax)^2}{a^2x^2+1}dx}{a^2}\right)\right)$$

$$\Big\downarrow \; 5361$$

$$c\left(\frac{1}{4}x^4\arctan(ax)^3-\frac{3}{4}a\left(\frac{\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\int\frac{x^3\arctan(ax)}{a^2x^2+1}dx}{a^2}-\frac{\int\frac{x^2\arctan(ax)^2}{a^2x^2+1}dx}{a^2}\right)\right)+ a^2c\left(\frac{1}{6}x^6\arctan(ax)^3-\frac{1}{2}a\left(\frac{\frac{1}{5}x^5\arctan(ax)^2-\frac{2}{5}a\int\frac{x^5\arctan(ax)}{a^2x^2+1}dx}{a^2}-\frac{\int\frac{x^4\arctan(ax)^2}{a^2x^2+1}dx}{a^2}\right)\right)$$

$$\Big\downarrow \; 5451$$

$$c\left(\frac{1}{4}x^4\arctan(ax)^3-\frac{3}{4}a\left(\frac{\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\int x\arctan(ax)dx}{a^2}-\frac{\int\frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\frac{\frac{\int\arctan(ax)^2dx}{a^2}-\frac{\int\frac{\arctan(ax)^2}{a^2x^2+1}\cdots}{a^2}}{a^2}\cdots\right.\right.$$

$$a^2c\left(\frac{1}{6}x^6\arctan(ax)^3-\frac{1}{2}a\left(\frac{\frac{1}{5}x^5\arctan(ax)^2-\frac{2}{5}a\left(\frac{\int x^3\arctan(ax)dx}{a^2}-\frac{\int\frac{x^3\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\frac{\frac{\int x^2\arctan(ax)^2dx}{a^2}-\cdots}{a^2}\cdots\right.\right.$$

$$\Big\downarrow \; 5345$$

$$c\left(\frac{1}{4}x^4\arctan(ax)^3-\frac{3}{4}a\left(\frac{\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\int x\arctan(ax)dx}{a^2}-\frac{\int\frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\frac{\frac{x\arctan(ax)^2-2a\int\frac{x\arctan(ax)}{a^2x^2+1}d\cdots}{a^2}\cdots}{a^2}\cdots\right.\right.$$

$$a^2c\left(\frac{1}{6}x^6\arctan(ax)^3-\frac{1}{2}a\left(\frac{\frac{1}{5}x^5\arctan(ax)^2-\frac{2}{5}a\left(\frac{\int x^3\arctan(ax)dx}{a^2}-\frac{\int\frac{x^3\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\frac{\frac{\int x^2\arctan(ax)^2dx}{a^2}-\cdots}{a^2}\cdots\right.\right.$$

$\Big\downarrow$ 5361

$$c\left(\frac{1}{4}x^4\arctan(ax)^3-\frac{3}{4}a\left(\frac{\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\int\frac{x^2}{a^2x^2+1}dx}{a^2}-\frac{\int\frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\frac{x\arctan(ax)^2-2a}{a}\right.\right.$$

$$a^2c\left(\frac{1}{6}x^6\arctan(ax)^3-\frac{1}{2}a\left(\frac{\frac{1}{5}x^5\arctan(ax)^2-\frac{2}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)-\frac{1}{4}a\int\frac{x^4}{a^2x^2+1}dx}{a^2}-\frac{\int\frac{x^3\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\frac{\frac{1}{3}x^3\arctan(a}{}\right.\right.$$

$\Big\downarrow$ 254

$$c\left(\frac{1}{4}x^4\arctan(ax)^3-\frac{3}{4}a\left(\frac{\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\int\frac{x^2}{a^2x^2+1}dx}{a^2}-\frac{\int\frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\frac{x\arctan(ax)^2-2a}{a}\right.\right.$$

$$a^2c\left(\frac{1}{6}x^6\arctan(ax)^3-\frac{1}{2}a\left(\frac{\frac{1}{5}x^5\arctan(ax)^2-\frac{2}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)-\frac{1}{4}a\int\left(\frac{x^2}{a^2}+\frac{1}{a^4(a^2x^2+1)}-\frac{1}{a^4}\right)dx}{a^2}-\frac{\int\frac{x^3\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2}\right.\right.$$

$\Big\downarrow$ 262

$$c\left(\frac{1}{4}x^4\arctan(ax)^3-\frac{3}{4}a\left(\frac{\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\int\frac{1}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\frac{\int\frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\frac{x\arctan\ldots}{}\right.\right.$$

$$a^2c\left(\frac{1}{6}x^6\arctan(ax)^3-\frac{1}{2}a\left(\frac{\frac{1}{5}x^5\arctan(ax)^2-\frac{2}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)-\frac{1}{4}a\int\left(\frac{x^2}{a^2}+\frac{1}{a^4(a^2x^2+1)}-\frac{1}{a^4}\right)dx}{a^2}-\frac{\int\frac{x^3\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2}\ldots\right.\right.$$

$$\Big\downarrow \text{ 216}$$

$$a^2c\left(\frac{1}{6}x^6\arctan(ax)^3-\frac{1}{2}a\left(\frac{\frac{1}{5}x^5\arctan(ax)^2-\frac{2}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)-\frac{1}{4}a\int\left(\frac{x^2}{a^2}+\frac{1}{a^4(a^2x^2+1)}-\frac{1}{a^4}\right)dx}{a^2}-\frac{\int\frac{x^3\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2}\ldots\right.\right.$$

$$c\left(\frac{1}{4}x^4\arctan(ax)^3-\frac{3}{4}a\left(\frac{\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{\int\frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\frac{x\arctan(ax)\ldots}{}\right.\right.$$

$$\Big\downarrow \text{ 2009}$$

$$c\left(\frac{1}{4}x^4\arctan(ax)^3-\frac{3}{4}a\left(\frac{\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{\int\frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\frac{x\arctan(ax)\ldots}{}\right.\right.$$

$$a^2c\left(\frac{1}{6}x^6\arctan(ax)^3-\frac{1}{2}a\left(\frac{\frac{1}{5}x^5\arctan(ax)^2-\frac{2}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)-\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)}{a^2}-\frac{\int\frac{x^3\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\frac{\frac{1}{3}\ldots}{}\right.\right.$$

$\downarrow$ 5419

$$c\left(\frac{1}{4}x^4\arctan(ax)^3-\frac{3}{4}a\left(\frac{\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{\int\frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\frac{x\arctan(ax)\cdots}{\cdots}\right.\right.$$
$$a^2c\left(\frac{1}{6}x^6\arctan(ax)^3-\frac{1}{2}a\left(\frac{\frac{1}{5}x^5\arctan(ax)^2-\frac{2}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)-\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)}{a^2}-\frac{\int\frac{x^3\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\frac{\frac{1}{3}\cdots}{\cdots}\right.\right.$$

$\downarrow$ 5451

$$c\left(\frac{1}{4}x^4\arctan(ax)^3-\frac{3}{4}a\left(\frac{\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{\int\frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\frac{x\arctan(ax)\cdots}{\cdots}\right.\right.$$
$$a^2c\left(\frac{1}{6}x^6\arctan(ax)^3-\frac{1}{2}a\left(\frac{\frac{1}{5}x^5\arctan(ax)^2-\frac{2}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)-\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)}{a^2}-\frac{\frac{\int x\arctan(ax)dx}{a^2}-\frac{\int\frac{x\arctan\cdots}{a^2x^2\cdots}}{a^2}}{a^2}\cdots\right.}{a^2}\cdots\right.\right.$$

$\downarrow$ 5345

$$c\left(\frac{1}{4}x^4\arctan(ax)^3-\frac{3}{4}a\left(\frac{\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{\int\frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\frac{x\arctan(ax)\cdots}{\cdots}\right.\right.$$
$$a^2c\left(\frac{1}{6}x^6\arctan(ax)^3-\frac{1}{2}a\left(\frac{\frac{1}{5}x^5\arctan(ax)^2-\frac{2}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)-\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)}{a^2}-\frac{\frac{\int x\arctan(ax)dx}{a^2}-\frac{\int\frac{x\arctan\cdots}{a^2x^2\cdots}}{a^2}}{a^2}\cdots\right.}{a^2}\cdots\right.\right.$$

$\downarrow$ 5361

$$c\left(\frac{1}{4}x^4\arctan(ax)^3-\frac{3}{4}a\left(\frac{\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{\int\frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\frac{x\arctan(ax)\cdots}{\cdots}\right.\right.$$

$$a^2c\left(\frac{1}{6}x^6\arctan(ax)^3-\frac{1}{2}a\left(\frac{\frac{1}{5}x^5\arctan(ax)^2-\frac{2}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)-\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)}{a^2}-\frac{\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\int\frac{x^2}{a^2x^2+\cdots}}{a^2}\cdots}{a^2}\right.}{a^2}\right.\right.$$

$\downarrow$ 262

$$c\left(\frac{1}{4}x^4\arctan(ax)^3-\frac{3}{4}a\left(\frac{\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{\int\frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\frac{x\arctan(ax)\cdots}{\cdots}\right.\right.$$

$$a^2c\left(\frac{1}{6}x^6\arctan(ax)^3-\frac{1}{2}a\left(\frac{\frac{1}{5}x^5\arctan(ax)^2-\frac{2}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)-\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)}{a^2}-\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\int\cdots}{\cdots}\right.}{a^2}\right.}{a^2}\right.\right.$$

$\downarrow$ 216

$$c\left(\frac{1}{4}x^4\arctan(ax)^3-\frac{3}{4}a\left(\frac{\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{\int\frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\frac{x\arctan(ax)\cdots}{\cdots}\right.\right.$$

$$a^2c\left(\frac{1}{6}x^6\arctan(ax)^3-\frac{1}{2}a\left(\frac{\frac{1}{5}x^5\arctan(ax)^2-\frac{2}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)-\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)}{a^2}-\frac{\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\cdots\right.}{a^2}}{a\cdots}\right.}{a^2}\right.\right.$$

$\downarrow$ 5419

$$c\left(\frac{1}{4}x^4\arctan(ax)^3-\frac{3}{4}a\left(\frac{\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{\int\frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\frac{x\arctan(ax)}{}\right.\right.$$

$$a^2c\left(\frac{1}{6}x^6\arctan(ax)^3-\frac{1}{2}a\left(\frac{\frac{1}{5}x^5\arctan(ax)^2-\frac{2}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)-\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)}{a^2}-\frac{\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\text{ar}\right.}{a^2}}{}\right.}{a^2}\right.\right.$$

$\downarrow$ 5455

$$c\left(\frac{1}{4}x^4\arctan(ax)^3-\frac{3}{4}a\left(\frac{\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{-\frac{\int\frac{\arctan(ax)}{i-ax}dx}{a}-\frac{i\arctan(ax)^2}{2a^2}}{a^2}\right)}{a^2}\right.\right.$$

$$a^2c\left(\frac{1}{6}x^6\arctan(ax)^3-\frac{1}{2}a\left(\frac{\frac{1}{5}x^5\arctan(ax)^2-\frac{2}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)-\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)}{a^2}-\frac{\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\text{ar}\right.}{a^2}}{}\right.}{a^2}\right.\right.$$

$\downarrow$ 5379

$$c\left(\frac{1}{4}x^4\arctan(ax)^3-\frac{3}{4}a\left(\frac{\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}-\int\frac{\log\left(\frac{2}{iax+\cdots}\right)}{a^2x^2+\cdots}}{a}}{a^2}\cdots\right.}{a^2}\cdots\right.\right.$$

$$a^2c\left(\frac{1}{6}x^6\arctan(ax)^3-\frac{1}{2}a\left(\frac{\frac{1}{5}x^5\arctan(ax)^2-\frac{2}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)-\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)}{a^2}-\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\cdots\right)}{a^2}\cdots\right.}{a^2}\cdots\right.\right.$$

$\Big\downarrow$ 2849

$$c\left(\frac{1}{6}x^6\arctan(ax)^3-\frac{1}{2}a\left(\frac{\frac{1}{5}x^5\arctan(ax)^2-\frac{2}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)-\frac{1}{4}a\left(\frac{x^3}{3a^2}-\frac{x}{a^4}+\frac{\arctan(ax)}{a^5}\right)}{a^2}-\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan\cdots}{a\cdots}\right)}{a^2}\cdots\right.}{a^2}\cdots\right.\right.$$

$$c\left(\frac{1}{4}x^4\arctan(ax)^3-\frac{3}{4}a\left(\frac{\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{-\frac{i\arctan(ax)^2}{2a^2}-\frac{\arctan(ax)\log\left(\frac{2}{iax+\cdots}\right)}{a}}{a^2}\cdots\right.}{a^2}\cdots\right.\right.$$

$\Big\downarrow$ 2752

$$c\left(\frac{1}{4}x^4\arctan(ax)^3-\frac{3}{4}a\left(\frac{\frac{1}{3}x^3\arctan(ax)^2-\frac{2}{3}a\left(\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\frac{\arctan(ax)}{a^3}\right)}{a^2}-\frac{-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}\cdots}{a}}{a^2}\cdots\right.}{a^2}\cdots\right.\right.$$

$$a^2c\left(\frac{1}{6}x^6\arctan(ax)^3-\frac{1}{2}a\left(\frac{\frac{1}{5}x^5\arctan(ax)^2-\frac{2}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)-\frac{1}{4}a\left(\frac{\arctan(ax)}{a^5}-\frac{x}{a^4}+\frac{x^3}{3a^2}\right)}{a^2}-\frac{\frac{1}{2}x^2\arctan(ax)-\frac{1}{2}a\left(\frac{x}{a^2}-\cdots\right.}{a^2}\cdots\right.}{a^2}\cdots\right.\right.$$

input

```
Int[x^3*(c + a^2*c*x^2)*ArcTan[a*x]^3,x]
```

output

```
c*((x^4*ArcTan[a*x]^3)/4 - (3*a*(((x^3*ArcTan[a*x]^2)/3 - (2*a*(((x^2*ArcT
an[a*x])/2 - (a*(x/a^2 - ArcTan[a*x]/a^3))/2)/a^2 - (((-1/2*I)*ArcTan[a*x]
^2)/a^2 - ((ArcTan[a*x]*Log[2/(1 + I*a*x)])/a + ((I/2)*PolyLog[2, 1 - 2/(1
 + I*a*x)])/a)/a)/a^2))/3)/a^2 - (-1/3*ArcTan[a*x]^3/a^3 + (x*ArcTan[a*x]^
2 - 2*a*(((-1/2*I)*ArcTan[a*x]^2)/a^2 - ((ArcTan[a*x]*Log[2/(1 + I*a*x)])/
a + ((I/2)*PolyLog[2, 1 - 2/(1 + I*a*x)])/a)/a))/a^2)/a^2))/4) + a^2*c*((x
^6*ArcTan[a*x]^3)/6 - (a*(((x^5*ArcTan[a*x]^2)/5 - (2*a*(((x^4*ArcTan[a*x]
)/4 - (a*(-(x/a^4) + x^3/(3*a^2) + ArcTan[a*x]/a^5))/4)/a^2 - (((x^2*ArcTa
n[a*x])/2 - (a*(x/a^2 - ArcTan[a*x]/a^3))/2)/a^2 - (((-1/2*I)*ArcTan[a*x]^
2)/a^2 - ((ArcTan[a*x]*Log[2/(1 + I*a*x)])/a + ((I/2)*PolyLog[2, 1 - 2/(1
+ I*a*x)])/a)/a)/a^2)/a^2))/5)/a^2 - (((x^3*ArcTan[a*x]^2)/3 - (2*a*(((x^2
*ArcTan[a*x])/2 - (a*(x/a^2 - ArcTan[a*x]/a^3))/2)/a^2 - (((-1/2*I)*ArcTan
[a*x]^2)/a^2 - ((ArcTan[a*x]*Log[2/(1 + I*a*x)])/a + ((I/2)*PolyLog[2, 1 -
 2/(1 + I*a*x)])/a)/a)/a^2))/3)/a^2 - (-1/3*ArcTan[a*x]^3/a^3 + (x*ArcTan[
a*x]^2 - 2*a*(((-1/2*I)*ArcTan[a*x]^2)/a^2 - ((ArcTan[a*x]*Log[2/(1 + I*a*
x)])/a + ((I/2)*PolyLog[2, 1 - 2/(1 + I*a*x)])/a)/a))/a^2)/a^2)/a^2))/2)
```

### Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

## Maple [A] (verified)

Time = 3.29 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.25

| method | result |
|---|---|
| derivativedivides | $\frac{c\arctan(ax)^3a^6x^6}{6}+\frac{c\arctan(ax)^3a^4x^4}{4}-\frac{c\arctan(ax)^3}{12}-\frac{c\left(\frac{2a^5\arctan(ax)^2x^5}{5}+\frac{a^3\arctan(ax)^2x^3}{3}-a\arctan(ax)^2x-\frac{x^4\arctan(}{5}\right.}{}$ |
| default | $\frac{c\arctan(ax)^3a^6x^6}{6}+\frac{c\arctan(ax)^3a^4x^4}{4}-\frac{c\arctan(ax)^3}{12}-\frac{c\left(\frac{2a^5\arctan(ax)^2x^5}{5}+\frac{a^3\arctan(ax)^2x^3}{3}-a\arctan(ax)^2x-\frac{x^4\arctan(}{5}\right.}{}$ |
| parts | $\frac{a^2cx^6\arctan(ax)^3}{6}+\frac{cx^4\arctan(ax)^3}{4}-\frac{c\left(\frac{2a\arctan(ax)^2x^5}{5}+\frac{\arctan(ax)^2x^3}{3a}-\frac{\arctan(ax)^2x}{a^3}+\frac{\arctan(ax)^3}{a^4}-\frac{2\left(\frac{3x^4a}{}\right.}{}\right.}{}$ |

input
```
int(x^3*(a^2*c*x^2+c)*arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a^4*(1/6*c*arctan(a*x)^3*a^6*x^6+1/4*c*arctan(a*x)^3*a^4*x^4-1/12*c*arct
an(a*x)^3-1/4*c*(2/5*a^5*arctan(a*x)^2*x^5+1/3*a^3*arctan(a*x)^2*x^3-a*arc
tan(a*x)^2*x-1/5*x^4*arctan(a*x)*a^4+1/15*x^2*a^2*arctan(a*x)+14/15*arctan
(a*x)*ln(a^2*x^2+1)+1/15*a^3*x^3-4/15*a*x+4/15*arctan(a*x)+7/15*I*(ln(a*x-
I)*ln(a^2*x^2+1)-1/2*ln(a*x-I)^2-dilog(-1/2*I*(a*x+I))-ln(a*x-I)*ln(-1/2*I
*(a*x+I)))-7/15*I*(ln(a*x+I)*ln(a^2*x^2+1)-1/2*ln(a*x+I)^2-dilog(1/2*I*(a*
x-I))-ln(a*x+I)*ln(1/2*I*(a*x-I)))))
```

## Fricas [F]

$$\int x^3\left(c+a^2cx^2\right)\arctan(ax)^3\,dx=\int\left(a^2cx^2+c\right)x^3\arctan\left(ax\right)^3\,dx$$

input
```
integrate(x^3*(a^2*c*x^2+c)*arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^5 + c*x^3)*arctan(a*x)^3, x)
```

## Sympy [F]

$$\int x^3(c+a^2cx^2)\arctan(ax)^3\,dx=c\left(\int x^3\operatorname{atan}^3(ax)\,dx+\int a^2x^5\operatorname{atan}^3(ax)\,dx\right)$$

input
```
integrate(x**3*(a**2*c*x**2+c)*atan(a*x)**3,x)
```

output
```
c*(Integral(x**3*atan(a*x)**3, x) + Integral(a**2*x**5*atan(a*x)**3, x))
```

## Maxima [F]

$$\int x^3(c+a^2cx^2)\arctan(ax)^3\,dx=\int(a^2cx^2+c)x^3\arctan(ax)^3\,dx$$

input
```
integrate(x^3*(a^2*c*x^2+c)*arctan(a*x)^3,x, algorithm="maxima")
```

output
```
1/960*(20*(23040*a^7*c*integrate(1/960*x^7*arctan(a*x)^3/(a^5*x^2 + a^3),
x) - 5760*a^6*c*integrate(1/960*x^6*arctan(a*x)^2/(a^5*x^2 + a^3), x) - 14
40*a^6*c*integrate(1/960*x^6*log(a^2*x^2 + 1)^2/(a^5*x^2 + a^3), x) - 1152
*a^6*c*integrate(1/960*x^6*log(a^2*x^2 + 1)/(a^5*x^2 + a^3), x) + 46080*a^
5*c*integrate(1/960*x^5*arctan(a*x)^3/(a^5*x^2 + a^3), x) + 2304*a^5*c*int
egrate(1/960*x^5*arctan(a*x)/(a^5*x^2 + a^3), x) - 8640*a^4*c*integrate(1/
960*x^4*arctan(a*x)^2/(a^5*x^2 + a^3), x) - 2160*a^4*c*integrate(1/960*x^4
*log(a^2*x^2 + 1)^2/(a^5*x^2 + a^3), x) - 960*a^4*c*integrate(1/960*x^4*lo
g(a^2*x^2 + 1)/(a^5*x^2 + a^3), x) + 23040*a^3*c*integrate(1/960*x^3*arcta
n(a*x)^3/(a^5*x^2 + a^3), x) + 1920*a^3*c*integrate(1/960*x^3*arctan(a*x)/
(a^5*x^2 + a^3), x) + 2880*a^2*c*integrate(1/960*x^2*log(a^2*x^2 + 1)/(a^5
*x^2 + a^3), x) - 5760*a*c*integrate(1/960*x*arctan(a*x)/(a^5*x^2 + a^3),
x) + 720*c*integrate(1/960*log(a^2*x^2 + 1)^2/(a^5*x^2 + a^3), x) + c*arct
an(a*x)^3/a^4)*a^4 + 40*(2*a^6*c*x^6 + 3*a^4*c*x^4 - c)*arctan(a*x)^3 - 4*
(6*a^5*c*x^5 + 5*a^3*c*x^3 - 15*a*c*x)*arctan(a*x)^2 + (6*a^5*c*x^5 + 5*a^
3*c*x^3 - 15*a*c*x)*log(a^2*x^2 + 1)^2)/a^4
```

## Giac [F]

$$\int x^3(c+a^2cx^2)\arctan(ax)^3\,dx = \int (a^2cx^2+c)x^3\arctan(ax)^3\,dx$$

input
```
integrate(x^3*(a^2*c*x^2+c)*arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)*x^3*arctan(a*x)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int x^3(c+a^2cx^2)\arctan(ax)^3\,dx = \int x^3\operatorname{atan}(a\,x)^3\,(c\,a^2\,x^2+c)\,dx$$

input
```
int(x^3*atan(a*x)^3*(c + a^2*c*x^2),x)
```

output
```
int(x^3*atan(a*x)^3*(c + a^2*c*x^2), x)
```

## Reduce [F]

$$\int x^3(c+a^2cx^2)\arctan(ax)^3\,dx$$
$$= \frac{c\Big(10atan(ax)^3a^6x^6+15atan(ax)^3a^4x^4-5atan(ax)^3-6atan(ax)^2a^5x^5-5atan(ax)^2a^3x^3+15atan(a}{60a^4}$$

input
```
int(x^3*(a^2*c*x^2+c)*atan(a*x)^3,x)
```

output
```
(c*(10*atan(a*x)**3*a**6*x**6 + 15*atan(a*x)**3*a**4*x**4 - 5*atan(a*x)**3
 - 6*atan(a*x)**2*a**5*x**5 - 5*atan(a*x)**2*a**3*x**3 + 15*atan(a*x)**2*a
*x + 3*atan(a*x)*a**4*x**4 - atan(a*x)*a**2*x**2 - 4*atan(a*x) - 28*int((a
tan(a*x)*x)/(a**2*x**2 + 1),x)*a**2 - a**3*x**3 + 4*a*x))/(60*a**4)
```

# 3.364 $\int x^2(c+a^2cx^2)\arctan(ax)^3\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 211

$$\begin{aligned}\int x^2(c+a^2cx^2)\arctan(ax)^3\,dx = &-\frac{cx^2}{20a}+\frac{cx\arctan(ax)}{10a^2}+\frac{1}{10}cx^3\arctan(ax)\\&-\frac{c\arctan(ax)^2}{20a^3}-\frac{cx^2\arctan(ax)^2}{5a}\\&-\frac{3}{20}acx^4\arctan(ax)^2-\frac{2ic\arctan(ax)^3}{15a^3}\\&+\frac{1}{3}cx^3\arctan(ax)^3+\frac{1}{5}a^2cx^5\arctan(ax)^3\\&-\frac{2c\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{5a^3}\\&-\frac{2ic\arctan(ax)\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{5a^3}\\&-\frac{c\operatorname{PolyLog}\left(3,1-\frac{2}{1+iax}\right)}{5a^3}\end{aligned}$$

output
```
-1/20*c*x^2/a+1/10*c*x*arctan(a*x)/a^2+1/10*c*x^3*arctan(a*x)-1/20*c*arcta
n(a*x)^2/a^3-1/5*c*x^2*arctan(a*x)^2/a-3/20*a*c*x^4*arctan(a*x)^2-2/15*I*c
*arctan(a*x)^3/a^3+1/3*c*x^3*arctan(a*x)^3+1/5*a^2*c*x^5*arctan(a*x)^3-2/5
*c*arctan(a*x)^2*ln(2/(1+I*a*x))/a^3-2/5*I*c*arctan(a*x)*polylog(2,1-2/(1+
I*a*x))/a^3-1/5*c*polylog(3,1-2/(1+I*a*x))/a^3
```

## Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.81

$$\int x^2(c+a^2cx^2)\arctan(ax)^3\,dx$$
$$= \frac{c(-3-3a^2x^2+6ax\arctan(ax)+6a^3x^3\arctan(ax)-3\arctan(ax)^2-12a^2x^2\arctan(ax)^2-9a^4x^4\arctan}{}$$

input
```
Integrate[x^2*(c + a^2*c*x^2)*ArcTan[a*x]^3,x]
```

output
```
(c*(-3 - 3*a^2*x^2 + 6*a*x*ArcTan[a*x] + 6*a^3*x^3*ArcTan[a*x] - 3*ArcTan[
a*x]^2 - 12*a^2*x^2*ArcTan[a*x]^2 - 9*a^4*x^4*ArcTan[a*x]^2 + (8*I)*ArcTan
[a*x]^3 + 20*a^3*x^3*ArcTan[a*x]^3 + 12*a^5*x^5*ArcTan[a*x]^3 - 24*ArcTan[
a*x]^2*Log[1 + E^((2*I)*ArcTan[a*x])] + (24*I)*ArcTan[a*x]*PolyLog[2, -E^(
(2*I)*ArcTan[a*x])] - 12*PolyLog[3, -E^((2*I)*ArcTan[a*x])]))/(60*a^3)
```

## Rubi [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 503 vs. 2(211) = 422.

Time = 4.13 (sec) , antiderivative size = 503, normalized size of antiderivative = 2.38, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {5485, 5361, 5451, 5361, 5451, 5345, 240, 5361, 243, 49, 2009, 5419, 5451, 5345, 240, 5419, 5455, 5379, 5529, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\arctan(ax)^3\left(a^2cx^2+c\right)\,dx$$

$$\Big\downarrow \text{ 5485}$$

$$a^2c\int x^4\arctan(ax)^3dx+c\int x^2\arctan(ax)^3dx$$

$$\Big\downarrow \text{ 5361}$$

$$a^2c\left(\frac{1}{5}x^5\arctan(ax)^3-\frac{3}{5}a\int\frac{x^5\arctan(ax)^2}{a^2x^2+1}dx\right)+c\left(\frac{1}{3}x^3\arctan(ax)^3-a\int\frac{x^3\arctan(ax)^2}{a^2x^2+1}dx\right)$$

$\downarrow$ 5451

$$c\left(\frac{1}{3}x^3\arctan(ax)^3-a\left(\frac{\int x\arctan(ax)^2dx}{a^2}-\frac{\int\frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2}\right)\right)+a^2c\left(\frac{1}{5}x^5\arctan(ax)^3-\frac{3}{5}a\left(\frac{\int x^3\arctan(ax)^2dx}{a^2}-\frac{\int\frac{x^3\arctan(ax)^2}{a^2x^2+1}dx}{a^2}\right)\right)$$

$\downarrow$ 5361

$$c\left(\frac{1}{3}x^3\arctan(ax)^3-a\left(\frac{\frac{1}{2}x^2\arctan(ax)^2-a\int\frac{x^2\arctan(ax)}{a^2x^2+1}dx}{a^2}-\frac{\int\frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2}\right)\right)+a^2c\left(\frac{1}{5}x^5\arctan(ax)^3-\frac{3}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)^2-\frac{1}{2}a\int\frac{x^4\arctan(ax)}{a^2x^2+1}dx}{a^2}-\frac{\int\frac{x^3\arctan(ax)^2}{a^2x^2+1}dx}{a^2}\right)\right)$$

$\downarrow$ 5451

$$c\left(\frac{1}{3}x^3\arctan(ax)^3-a\left(\frac{\frac{1}{2}x^2\arctan(ax)^2-a\left(\frac{\int\arctan(ax)dx}{a^2}-\frac{\int\frac{\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\frac{\int\frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2}\right)\right)+a^2c\left(\frac{1}{5}x^5\arctan(ax)^3-\frac{3}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)^2-\frac{1}{2}a\left(\frac{\int x^2\arctan(ax)dx}{a^2}-\frac{\int\frac{x^2\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\frac{\frac{\int x\arctan(ax)^2dx}{a^2}-\int\cdots}{a^2}\cdots$$

$\downarrow$ 5345

$$c\left(\frac{1}{3}x^3\arctan(ax)^3-a\left(\frac{\frac{1}{2}x^2\arctan(ax)^2-a\left(\frac{x\arctan(ax)-a\int\frac{x}{a^2x^2+1}dx}{a^2}-\frac{\int\frac{\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\frac{\int\frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2}\right)\right)+a^2c\left(\frac{1}{5}x^5\arctan(ax)^3-\frac{3}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)^2-\frac{1}{2}a\left(\frac{\int x^2\arctan(ax)dx}{a^2}-\frac{\int\frac{x^2\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\frac{\frac{\int x\arctan(ax)^2dx}{a^2}-\int\cdots}{a^2}\cdots$$

$\Big\downarrow$ 240

$$c\left(\frac{1}{3}x^3\arctan(ax)^3-a\left(\frac{\frac{1}{2}x^2\arctan(ax)^2-a\left(\frac{x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}}{a^2}-\frac{\int\frac{\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\frac{\int\frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2}\right)\right)+$$
$$a^2c\left(\frac{1}{5}x^5\arctan(ax)^3-\frac{3}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)^2-\frac{1}{2}a\left(\frac{\int x^2\arctan(ax)dx}{a^2}-\frac{\int\frac{x^2\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\frac{\frac{\int x\arctan(ax)^2dx}{a^2}-\int\cdots}{a^2}\cdots\right.\right.$$

$\Big\downarrow$ 5361

$$c\left(\frac{1}{3}x^3\arctan(ax)^3-a\left(\frac{\frac{1}{2}x^2\arctan(ax)^2-a\left(\frac{x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}}{a^2}-\frac{\int\frac{\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\frac{\int\frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2}\right)\right)+$$
$$a^2c\left(\frac{1}{5}x^5\arctan(ax)^3-\frac{3}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)^2-\frac{1}{2}a\left(\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{3}a\int\frac{x^3}{a^2x^2+1}dx}{a^2}-\frac{\int\frac{x^2\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\frac{\frac{1}{2}x^2\arctan(a\cdots}{\cdots}\right.\right.$$

$\Big\downarrow$ 243

$$c\left(\frac{1}{3}x^3\arctan(ax)^3-a\left(\frac{\frac{1}{2}x^2\arctan(ax)^2-a\left(\frac{x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}}{a^2}-\frac{\int\frac{\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\frac{\int\frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2}\right)\right)+$$
$$a^2c\left(\frac{1}{5}x^5\arctan(ax)^3-\frac{3}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)^2-\frac{1}{2}a\left(\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\int\frac{x^2}{a^2x^2+1}dx^2}{a^2}-\frac{\int\frac{x^2\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\frac{\frac{1}{2}x^2\arctan(\cdots}{\cdots}\right.\right.$$

$\Big\downarrow$ 49

$$c\left(\frac{1}{3}x^3\arctan(ax)^3-a\left(\frac{\frac{1}{2}x^2\arctan(ax)^2-a\left(\frac{x\arctan(ax)-\frac{\log(a^2x^2+1)}{2a}}{a^2}-\frac{\int\frac{\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\frac{\int\frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2}\right)\right)+$$

$$a^2c\left(\frac{1}{5}x^5\arctan(ax)^3-\frac{3}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)^2-\frac{1}{2}a\left(\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\int\left(\frac{1}{a^2}-\frac{1}{a^2(a^2x^2+1)}\right)dx^2}{a^2}-\frac{\int\frac{x^2\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\right.\right.$$

$\big\downarrow$ 2009

$$c\left(\frac{1}{3}x^3\arctan(ax)^3-a\left(\frac{\frac{1}{2}x^2\arctan(ax)^2-a\left(\frac{x\arctan(ax)-\frac{\log(a^2x^2+1)}{2a}}{a^2}-\frac{\int\frac{\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\frac{\int\frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2}\right)\right)+$$

$$a^2c\left(\frac{1}{5}x^5\arctan(ax)^3-\frac{3}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)^2-\frac{1}{2}a\left(\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log(a^2x^2+1)}{a^4}\right)}{a^2}-\frac{\int\frac{x^2\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\frac{\frac{1}{2}x^2}{\ldots}\right.\right.$$

$\big\downarrow$ 5419

$$a^2c\left(\frac{1}{5}x^5\arctan(ax)^3-\frac{3}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)^2-\frac{1}{2}a\left(\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log(a^2x^2+1)}{a^4}\right)}{a^2}-\frac{\int\frac{x^2\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\frac{\frac{1}{2}x^2}{\ldots}\right.\right.$$

$$c\left(\frac{1}{3}x^3\arctan(ax)^3-a\left(\frac{\frac{1}{2}x^2\arctan(ax)^2-a\left(\frac{x\arctan(ax)-\frac{\log(a^2x^2+1)}{2a}}{a^2}-\frac{\arctan(ax)^2}{2a^3}\right)}{a^2}-\frac{\int\frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2}\right)\right)$$

$\downarrow$ 5451

$$a^2c\left(\frac{1}{5}x^5\arctan(ax)^3-\frac{3}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)^2-\frac{1}{2}a\left(\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log\left(a^2x^2+1\right)}{a^4}\right)}{a^2}-\frac{\frac{\int\arctan(ax)dx}{a^2}-\frac{\int\frac{\arctan(ax)}{a^2x^2+1}d\cdots}{a^2}}{a^2}\cdots\right.}{a^2}\cdots\right.\right.$$

$$c\left(\frac{1}{3}x^3\arctan(ax)^3-a\left(\frac{\frac{1}{2}x^2\arctan(ax)^2-a\left(\frac{x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}}{a^2}-\frac{\arctan(ax)^2}{2a^3}\right)}{a^2}-\frac{\int\frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2}\right)\right)\Bigg)$$

$\downarrow$ 5345

$$a^2c\left(\frac{1}{5}x^5\arctan(ax)^3-\frac{3}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)^2-\frac{1}{2}a\left(\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log\left(a^2x^2+1\right)}{a^4}\right)}{a^2}-\frac{\frac{x\arctan(ax)-a\int\frac{x}{a^2x^2+1}dx}{a^2}-\cdots}{a^2}\cdots\right.}{a^2}\cdots\right.\right.$$

$$c\left(\frac{1}{3}x^3\arctan(ax)^3-a\left(\frac{\frac{1}{2}x^2\arctan(ax)^2-a\left(\frac{x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}}{a^2}-\frac{\arctan(ax)^2}{2a^3}\right)}{a^2}-\frac{\int\frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2}\right)\right)\Bigg)$$

$\downarrow$ 240

$$a^2c\left(\frac{1}{5}x^5\arctan(ax)^3-\frac{3}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)^2-\frac{1}{2}a\left(\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log(a^2x^2+1)}{a^4}\right)}{a^2}-\frac{\frac{x\arctan(ax)-\frac{\log(a^2x^2+1)}{2a}}{a^2}-\int}{a^2}\right.}{a^2}\right.\right.$$

$$c\left(\frac{1}{3}x^3\arctan(ax)^3-a\left(\frac{\frac{1}{2}x^2\arctan(ax)^2-a\left(\frac{x\arctan(ax)-\frac{\log(a^2x^2+1)}{2a}}{a^2}-\frac{\arctan(ax)^2}{2a^3}\right)}{a^2}-\frac{\int\frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2}\right)\right)$$

$\downarrow$ 5419

$$c\left(\frac{1}{3}x^3\arctan(ax)^3-a\left(\frac{\frac{1}{2}x^2\arctan(ax)^2-a\left(\frac{x\arctan(ax)-\frac{\log(a^2x^2+1)}{2a}}{a^2}-\frac{\arctan(ax)^2}{2a^3}\right)}{a^2}-\frac{\int\frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2}\right)\right)+$$

$$a^2c\left(\frac{1}{5}x^5\arctan(ax)^3-\frac{3}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)^2-\frac{1}{2}a\left(\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log(a^2x^2+1)}{a^4}\right)}{a^2}-\frac{\frac{x\arctan(ax)-\frac{\log(a^2x^2+1)}{2a}}{a^2}-a}{a^2}\right.}{a^2}\right.\right.$$

$\downarrow$ 5455

$$c\left(\frac{1}{3}x^3\arctan(ax)^3-a\left(\frac{\frac{1}{2}x^2\arctan(ax)^2-a\left(\frac{x\arctan(ax)-\frac{\log(a^2x^2+1)}{2a}}{a^2}-\frac{\arctan(ax)^2}{2a^3}\right)}{a^2}-\frac{-\frac{\int\frac{\arctan(ax)^2}{i-ax}dx}{a}-\frac{i\arctan(}{3a^2}}{a^2}\right.\right.$$

$$a^2c\left(\frac{1}{5}x^5\arctan(ax)^3-\frac{3}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)^2-\frac{1}{2}a\left(\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log(a^2x^2+1)}{a^4}\right)}{a^2}-\frac{\frac{x\arctan(ax)-\frac{\log(a^2x^2+1)}{2a}}{a^2}-a}{a^2}\right.}{a^2}\right.\right.$$

$\downarrow$ 5379

$$c\left(\frac{1}{3}x^3\arctan(ax)^3-a\left(\frac{\frac{1}{2}x^2\arctan(ax)^2-a\left(\frac{x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}}{a^2}-\frac{\arctan(ax)^2}{2a^3}\right)}{a^2}-\frac{-\frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}-2\int}{a}}{}\right.\right.$$

$$a^2c\left(\frac{1}{5}x^5\arctan(ax)^3-\frac{3}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)^2-\frac{1}{2}a\left(\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log\left(a^2x^2+1\right)}{a^4}\right)}{a^2}-\frac{\frac{x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}}{a^2}-}{a^2}\right.}{a^2}\right.\right.$$

$\downarrow$ 5529

$$c\left(\frac{1}{3}x^3\arctan(ax)^3-a\left(\frac{\frac{1}{2}x^2\arctan(ax)^2-a\left(\frac{x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}}{a^2}-\frac{\arctan(ax)^2}{2a^3}\right)}{a^2}-\frac{-\frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}-2\left(\right.}{}}{}\right.\right.$$

$$a^2c\left(\frac{1}{5}x^5\arctan(ax)^3-\frac{3}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)^2-\frac{1}{2}a\left(\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log\left(a^2x^2+1\right)}{a^4}\right)}{a^2}-\frac{\frac{x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}}{a^2}-}{a^2}\right.}{a^2}\right.\right.$$

$\downarrow$ 7164

$$c\left(\frac{1}{3}x^3\arctan(ax)^3-a\left(\frac{\frac{1}{2}x^2\arctan(ax)^2-a\left(\frac{x\arctan(ax)-\frac{\log(a^2x^2+1)}{2a}}{a^2}-\frac{\arctan(ax)^2}{2a^3}\right)}{a^2}-\frac{-\frac{i\arctan(ax)^3}{3a^2}-\frac{\arctan(ax)^2}{}\cdots}{}\right.\right.$$

$$a^2c\left(\frac{1}{5}x^5\arctan(ax)^3-\frac{3}{5}a\left(\frac{\frac{1}{4}x^4\arctan(ax)^2-\frac{1}{2}a\left(\frac{\frac{1}{3}x^3\arctan(ax)-\frac{1}{6}a\left(\frac{x^2}{a^2}-\frac{\log(a^2x^2+1)}{a^4}\right)}{a^2}-\frac{\frac{x\arctan(ax)-\frac{\log(a^2x^2+1)}{2a}}{a^2}-\cdots}{a^2}\right.}{a^2}\right.\right.$$

input
```
Int[x^2*(c + a^2*c*x^2)*ArcTan[a*x]^3,x]
```

output
```
c*((x^3*ArcTan[a*x]^3)/3 - a*(((x^2*ArcTan[a*x]^2)/2 - a*(-1/2*ArcTan[a*x]
^2/a^3 + (x*ArcTan[a*x] - Log[1 + a^2*x^2]/(2*a))/a^2))/a^2 - (((-1/3*I)*A
rcTan[a*x]^3)/a^2 - ((ArcTan[a*x]^2*Log[2/(1 + I*a*x)])/a - 2*(((-1/2*I)*A
rcTan[a*x]*PolyLog[2, 1 - 2/(1 + I*a*x)])/a - PolyLog[3, 1 - 2/(1 + I*a*x)
]/(4*a)))/a)/a^2)) + a^2*c*((x^5*ArcTan[a*x]^3)/5 - (3*a*(((x^4*ArcTan[a*x
]^2)/4 - (a*(((x^3*ArcTan[a*x])/3 - (a*(x^2/a^2 - Log[1 + a^2*x^2]/a^4))/6
)/a^2 - (-1/2*ArcTan[a*x]^2/a^3 + (x*ArcTan[a*x] - Log[1 + a^2*x^2]/(2*a))
/a^2)/a^2))/2)/a^2 - (((x^2*ArcTan[a*x]^2)/2 - a*(-1/2*ArcTan[a*x]^2/a^3 +
 (x*ArcTan[a*x] - Log[1 + a^2*x^2]/(2*a))/a^2))/a^2 - (((-1/3*I)*ArcTan[a*
x]^3)/a^2 - ((ArcTan[a*x]^2*Log[2/(1 + I*a*x)])/a - 2*(((-1/2*I)*ArcTan[a*
x]*PolyLog[2, 1 - 2/(1 + I*a*x)])/a - PolyLog[3, 1 - 2/(1 + I*a*x)]/(4*a))
)/a)/a^2)/a^2))/5)
```

## Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 71.03 (sec) , antiderivative size = 990, normalized size of antiderivative = 4.69

| method | result | size |
|---|---|---|
| derivativedivides | Expression too large to display | 990 |
| default | Expression too large to display | 990 |
| parts | Expression too large to display | 992 |

input
```
int(x^2*(a^2*c*x^2+c)*arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(1/5*c*arctan(a*x)^3*a^5*x^5+1/3*c*arctan(a*x)^3*a^3*x^3-1/5*c*(-1/2
*I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)
)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*arctan(a*x
)^2+1/4*arctan(a*x)^2-1/2*I*(a*x+I)+arctan(a*x)^2*x^2*a^2+1/2*I*Pi*csgn(I*
((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3*arctan(a*x)^2-1/2*I*Pi*csgn(I*(1+I*a*x)^2
/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3*arctan(a*x)^2+3*I*arctan(a*x
)*(a*x-I)*(a*x+I)-1/2*I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^3*arctan(a*x)^2
+3/2*arctan(a*x)*(a*x-I)^2*(a*x+I)-2*I*arctan(a*x)*polylog(2,-(1+I*a*x)^2/
(a^2*x^2+1))-3/2*arctan(a*x)*(a*x-I)*(a*x+I)^2+3/4*a^4*arctan(a*x)^2*x^4-1
/2*arctan(a*x)*(a*x-I)^3+arctan(a*x)*(a*x-I)-2/3*I*arctan(a*x)^3+2*arctan(
a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2))+polylog(3,-(1+I*a*x)^2/(a^2*x^2+1))
+1/4*(a*x+I)^2-3/2*I*arctan(a*x)*(a*x-I)^2-arctan(a*x)^2*ln(a^2*x^2+1)+2*a
rctan(a*x)^2*ln(2)+1/2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))^2*csgn(I*(
(1+I*a*x)^2/(a^2*x^2+1)+1)^2)*arctan(a*x)^2-I*Pi*csgn(I*((1+I*a*x)^2/(a^2*
x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2*arctan(a*x)^2-1/2*I*Pi*
csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*arct
an(a*x)^2+I*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*csgn(I*(1+I*a*x)^2/(a^2
*x^2+1))^2*arctan(a*x)^2+1/2*I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(
1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2*arctan(a*x)^2+1/2*
I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+...
```

## Fricas [F]

$$\int x^2(c+a^2cx^2)\arctan(ax)^3\,dx=\int (a^2cx^2+c)x^2\arctan(ax)^3\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)*arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^4 + c*x^2)*arctan(a*x)^3, x)
```

## Sympy [F]

$$\int x^2(c+a^2cx^2)\arctan(ax)^3\,dx = c\left(\int x^2\,\mathrm{atan}^3\,(ax)\,dx+\int a^2x^4\,\mathrm{atan}^3\,(ax)\,dx\right)$$

input
```
integrate(x**2*(a**2*c*x**2+c)*atan(a*x)**3,x)
```

output
```
c*(Integral(x**2*atan(a*x)**3, x) + Integral(a**2*x**4*atan(a*x)**3, x))
```

## Maxima [F]

$$\int x^2(c+a^2cx^2)\arctan(ax)^3\,dx = \int (a^2cx^2+c)x^2\arctan(ax)^3\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)*arctan(a*x)^3,x, algorithm="maxima")
```

output
```
1/120*(3*a^2*c*x^5 + 5*c*x^3)*arctan(a*x)^3 - 1/160*(3*a^2*c*x^5 + 5*c*x^3
)*arctan(a*x)*log(a^2*x^2 + 1)^2 + integrate(1/160*(140*(a^4*c*x^6 + 2*a^2
*c*x^4 + c*x^2)*arctan(a*x)^3 - 4*(3*a^3*c*x^5 + 5*a*c*x^3)*arctan(a*x)^2
+ 4*(3*a^4*c*x^6 + 5*a^2*c*x^4)*arctan(a*x)*log(a^2*x^2 + 1) + (3*a^3*c*x^
5 + 5*a*c*x^3 + 15*(a^4*c*x^6 + 2*a^2*c*x^4 + c*x^2)*arctan(a*x))*log(a^2*
x^2 + 1)^2)/(a^2*x^2 + 1), x)
```

## Giac [F]

$$\int x^2(c+a^2cx^2)\arctan(ax)^3\,dx = \int (a^2cx^2+c)x^2\arctan(ax)^3\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)*arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)*x^2*arctan(a*x)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int x^2(c+a^2cx^2)\arctan(ax)^3\,dx=\int x^2\,\mathrm{atan}(a\,x)^3\,\left(c\,a^2\,x^2+c\right)\,dx$$

input
```
int(x^2*atan(a*x)^3*(c + a^2*c*x^2),x)
```

output
```
int(x^2*atan(a*x)^3*(c + a^2*c*x^2), x)
```

## Reduce [F]

$$\int x^2(c+a^2cx^2)\arctan(ax)^3\,dx$$
$$=\frac{c\left(12atan(ax)^3\,a^5x^5+20atan(ax)^3\,a^3x^3+8atan(ax)^3\,ax-9atan(ax)^2\,a^4x^4-12atan(ax)^2\,a^2x^2-3ata\right.}{60a^3}$$

input
```
int(x^2*(a^2*c*x^2+c)*atan(a*x)^3,x)
```

output
```
(c*(12*atan(a*x)**3*a**5*x**5 + 20*atan(a*x)**3*a**3*x**3 + 8*atan(a*x)**3
*a*x - 9*atan(a*x)**2*a**4*x**4 - 12*atan(a*x)**2*a**2*x**2 - 3*atan(a*x)*
*2 + 6*atan(a*x)*a**3*x**3 + 6*atan(a*x)*a*x - 8*int(atan(a*x)**3,x)*a - 3
*a**2*x**2))/(60*a**3)
```

# 3.365 $\int x(c + a^2cx^2) \arctan(ax)^3 dx$



## Optimal result

Integrand size = 18, antiderivative size = 160

$$\int x(c + a^2cx^2) \arctan(ax)^3 dx = -\frac{cx}{4a} + \frac{c(1 + a^2x^2) \arctan(ax)}{4a^2} - \frac{ic \arctan(ax)^2}{2a^2} - \frac{cx \arctan(ax)^2}{2a} - \frac{cx(1 + a^2x^2) \arctan(ax)^2}{4a} + \frac{c(1 + a^2x^2)^2 \arctan(ax)^3}{4a^2} - \frac{c \arctan(ax) \log\left(\frac{2}{1+iax}\right)}{a^2} - \frac{ic \operatorname{PolyLog}\left(2, 1 - \frac{2}{1+iax}\right)}{2a^2}$$

output
```
-1/4*c*x/a+1/4*c*(a^2*x^2+1)*arctan(a*x)/a^2-1/2*I*c*arctan(a*x)^2/a^2-1/2
*c*x*arctan(a*x)^2/a-1/4*c*x*(a^2*x^2+1)*arctan(a*x)^2/a+1/4*c*(a^2*x^2+1)
^2*arctan(a*x)^3/a^2-c*arctan(a*x)*ln(2/(1+I*a*x))/a^2-1/2*I*c*polylog(2,1
-2/(1+I*a*x))/a^2
```

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.63

$$\int x(c+a^2cx^2)\arctan(ax)^3\,dx$$
$$= \frac{c\Big(-ax-(-2i+3ax+a^3x^3)\arctan(ax)^2+(1+a^2x^2)^2\arctan(ax)^3+\arctan(ax)\left(1+a^2x^2-4\log\left(1\right.\right.}{4a^2}$$

input
```
Integrate[x*(c + a^2*c*x^2)*ArcTan[a*x]^3,x]
```

output
```
(c*(-(a*x) - (-2*I + 3*a*x + a^3*x^3)*ArcTan[a*x]^2 + (1 + a^2*x^2)^2*ArcT
an[a*x]^3 + ArcTan[a*x]*(1 + a^2*x^2 - 4*Log[1 + E^((2*I)*ArcTan[a*x])]) +
 (2*I)*PolyLog[2, -E^((2*I)*ArcTan[a*x])]))/(4*a^2)
```

## Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5465, 27, 5415, 24, 5345, 5455, 5379, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\arctan(ax)^3\left(a^2cx^2+c\right)\,dx$$

$$\Big\downarrow \; 5465$$

$$\frac{c(a^2x^2+1)^2\arctan(ax)^3}{4a^2}-\frac{3\int c(a^2x^2+1)\arctan(ax)^2dx}{4a}$$

$$\Big\downarrow \; 27$$

$$\frac{c(a^2x^2+1)^2\arctan(ax)^3}{4a^2}-\frac{3c\int (a^2x^2+1)\arctan(ax)^2dx}{4a}$$

$$\Big\downarrow \; 5415$$

$$\frac{c(a^2x^2+1)^2\arctan(ax)^3}{4a^2}-\frac{3c\left(\frac{2}{3}\int\arctan(ax)^2dx+\frac{\int 1dx}{3}+\frac{1}{3}x(a^2x^2+1)\arctan(ax)^2-\frac{(a^2x^2+1)\arctan(ax)}{3a}\right)}{4a}$$

$$\Big\downarrow \text{24}$$

$$\frac{c(a^2x^2+1)^2\arctan(ax)^3}{4a^2}-\frac{3c\left(\frac{2}{3}\int\arctan(ax)^2dx+\frac{1}{3}x(a^2x^2+1)\arctan(ax)^2-\frac{(a^2x^2+1)\arctan(ax)}{3a}+\frac{x}{3}\right)}{4a}$$

$$\Big\downarrow \text{5345}$$

$$\frac{c(a^2x^2+1)^2\arctan(ax)^3}{4a^2}-\frac{3c\left(\frac{2}{3}\left(x\arctan(ax)^2-2a\int\frac{x\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{3}x(a^2x^2+1)\arctan(ax)^2-\frac{(a^2x^2+1)\arctan(ax)}{3a}+\frac{x}{3}\right)}{4a}$$

$$\Big\downarrow \text{5455}$$

$$\frac{c(a^2x^2+1)^2\arctan(ax)^3}{4a^2}-\frac{3c\left(\frac{2}{3}\left(x\arctan(ax)^2-2a\left(-\frac{\int\frac{\arctan(ax)}{i-ax}dx}{a}-\frac{i\arctan(ax)^2}{2a^2}\right)\right)+\frac{1}{3}x(a^2x^2+1)\arctan(ax)^2-\frac{(a^2x^2+1)\arctan(ax)}{3a}+\frac{x}{3}\right)}{4a}$$

$$\Big\downarrow \text{5379}$$

$$\frac{c(a^2x^2+1)^2\arctan(ax)^3}{4a^2}-\frac{3c\left(\frac{2}{3}\left(x\arctan(ax)^2-2a\left(-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}-\int\frac{\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^2}{2a^2}\right)\right)+\frac{1}{3}x(a^2x^2+1)\arctan(ax)^2-\right.}{4a}$$

$$\Big\downarrow \text{2849}$$

$$\frac{c(a^2x^2+1)^2\arctan(ax)^3}{4a^2}-\frac{3c\left(\frac{2}{3}\left(x\arctan(ax)^2-2a\left(-\frac{\frac{i\int\frac{\log\left(\frac{2}{iax+1}\right)}{1-\frac{2}{iax+1}}d\frac{1}{iax+1}}{a}+\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}}{a}-\frac{i\arctan(ax)^2}{2a^2}\right)\right)+\frac{1}{3}x(a^2x^2+1)\arctan(ax\right.}{4a}$$

$$\Big\downarrow \text{2752}$$

$$\frac{c(a^2x^2+1)^2\arctan(ax)^3}{4a^2} - \frac{3c\left(\frac{2}{3}\left(x\arctan(ax)^2-2a\left(-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}\right)\right)+\frac{1}{3}x(a^2x^2+1)\arctan(ax)^2}{4a}$$

input
```
Int[x*(c + a^2*c*x^2)*ArcTan[a*x]^3,x]
```

output
```
(c*(1 + a^2*x^2)^2*ArcTan[a*x]^3)/(4*a^2) - (3*c*(x/3 - ((1 + a^2*x^2)*Arc
Tan[a*x])/(3*a) + (x*(1 + a^2*x^2)*ArcTan[a*x]^2)/3 + (2*(x*ArcTan[a*x]^2
- 2*a*(((-1/2*I)*ArcTan[a*x]^2)/a^2 - ((ArcTan[a*x]*Log[2/(1 + I*a*x)])/a
+ ((I/2)*PolyLog[2, 1 - 2/(1 + I*a*x)])/a)/a)))/3))/(4*a)
```

## Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5415
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTan[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x])^p,
x], x] + Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*(
a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[q, 0] && GtQ[p, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

## Maple [A] (verified)

Time = 4.04 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.48

| method | result |
|---|---|
| parts | $\frac{c\arctan(ax)^3a^2x^4}{4}+\frac{c\arctan(ax)^3x^2}{2}+\frac{c\arctan(ax)^3}{4a^2}-\frac{3c\left(\frac{a^3\arctan(ax)^2x^3}{3}+a\arctan(ax)^2x-\frac{x^2a^2\arctan(ax)}{3}\right.}{}$ |
| derivativedivides | $\frac{\frac{c\arctan(ax)^3a^4x^4}{4}+\frac{a^2cx^2\arctan(ax)^3}{2}+\frac{c\arctan(ax)^3}{4}-\frac{3c\left(\frac{a^3\arctan(ax)^2x^3}{3}+a\arctan(ax)^2x-\frac{x^2a^2\arctan(ax)}{3}-\frac{2\arctan(ax}{}\right.}{}}{}$ |
| default | $\frac{\frac{c\arctan(ax)^3a^4x^4}{4}+\frac{a^2cx^2\arctan(ax)^3}{2}+\frac{c\arctan(ax)^3}{4}-\frac{3c\left(\frac{a^3\arctan(ax)^2x^3}{3}+a\arctan(ax)^2x-\frac{x^2a^2\arctan(ax)}{3}-\frac{2\arctan(ax}{}\right.}{}}{}$ |

input
```
int(x*(a^2*c*x^2+c)*arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*c*arctan(a*x)^3*a^2*x^4+1/2*c*arctan(a*x)^3*x^2+1/4*c*arctan(a*x)^3/a^
2-3/4/a^2*c*(1/3*a^3*arctan(a*x)^2*x^3+a*arctan(a*x)^2*x-1/3*x^2*a^2*arcta
n(a*x)-2/3*arctan(a*x)*ln(a^2*x^2+1)+1/3*a*x-1/3*arctan(a*x)-1/3*I*(ln(a*x
-I)*ln(a^2*x^2+1)-1/2*ln(a*x-I)^2-dilog(-1/2*I*(a*x+I))-ln(a*x-I)*ln(-1/2*
I*(a*x+I)))+1/3*I*(ln(a*x+I)*ln(a^2*x^2+1)-1/2*ln(a*x+I)^2-dilog(1/2*I*(a*
x-I))-ln(a*x+I)*ln(1/2*I*(a*x-I))))
```

## Fricas [F]

$$\int x\left(c+a^2cx^2\right)\arctan(ax)^3\,dx=\int\left(a^2cx^2+c\right)x\arctan\left(ax\right)^3\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)*arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^3 + c*x)*arctan(a*x)^3, x)
```

## Sympy [F]

$$\int x(c+a^2cx^2)\arctan(ax)^3\,dx = c\left(\int x\,\mathrm{atan}^3\,(ax)\,dx + \int a^2x^3\,\mathrm{atan}^3\,(ax)\,dx\right)$$

input
```
integrate(x*(a**2*c*x**2+c)*atan(a*x)**3,x)
```

output
```
c*(Integral(x*atan(a*x)**3, x) + Integral(a**2*x**3*atan(a*x)**3, x))
```

## Maxima [F]

$$\int x(c+a^2cx^2)\arctan(ax)^3\,dx = \int (a^2cx^2+c)x\arctan(ax)^3\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)*arctan(a*x)^3,x, algorithm="maxima")
```

output
```
1/64*(8*(a^4*c*x^4 + 2*a^2*c*x^2 + c)*arctan(a*x)^3 + 4*(512*a^5*c*integra
te(1/64*x^5*arctan(a*x)^3/(a^3*x^2 + a), x) - 192*a^4*c*integrate(1/64*x^4
*arctan(a*x)^2/(a^3*x^2 + a), x) - 48*a^4*c*integrate(1/64*x^4*log(a^2*x^2
 + 1)^2/(a^3*x^2 + a), x) - 64*a^4*c*integrate(1/64*x^4*log(a^2*x^2 + 1)/(
a^3*x^2 + a), x) + 1024*a^3*c*integrate(1/64*x^3*arctan(a*x)^3/(a^3*x^2 +
a), x) + 128*a^3*c*integrate(1/64*x^3*arctan(a*x)/(a^3*x^2 + a), x) - 384*
a^2*c*integrate(1/64*x^2*arctan(a*x)^2/(a^3*x^2 + a), x) - 96*a^2*c*integr
ate(1/64*x^2*log(a^2*x^2 + 1)^2/(a^3*x^2 + a), x) - 192*a^2*c*integrate(1/
64*x^2*log(a^2*x^2 + 1)/(a^3*x^2 + a), x) + 512*a*c*integrate(1/64*x*arcta
n(a*x)^3/(a^3*x^2 + a), x) + 384*a*c*integrate(1/64*x*arctan(a*x)/(a^3*x^2
 + a), x) - c*arctan(a*x)^3/a^2 - 48*c*integrate(1/64*log(a^2*x^2 + 1)^2/(
a^3*x^2 + a), x))*a^2 - 4*(a^3*c*x^3 + 3*a*c*x)*arctan(a*x)^2 + (a^3*c*x^3
 + 3*a*c*x)*log(a^2*x^2 + 1)^2)/a^2
```

## Giac [F]

$$\int x(c+a^2cx^2)\arctan(ax)^3\,dx=\int (a^2cx^2+c)x\arctan(ax)^3\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)*arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)*x*arctan(a*x)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int x(c+a^2cx^2)\arctan(ax)^3\,dx=\int x\,\mathrm{atan}(a\,x)^3\,(c\,a^2\,x^2+c)\,dx$$

input
```
int(x*atan(a*x)^3*(c + a^2*c*x^2),x)
```

output
```
int(x*atan(a*x)^3*(c + a^2*c*x^2), x)
```

## Reduce [F]

$$\int x(c+a^2cx^2)\arctan(ax)^3\,dx$$
$$=\frac{c\Big(atan(ax)^3\,a^4x^4+2atan(ax)^3\,a^2x^2+atan(ax)^3-atan(ax)^2\,a^3x^3-3atan(ax)^2\,ax+atan(ax)\,a^2x^2+}{4a^2}$$

input
```
int(x*(a^2*c*x^2+c)*atan(a*x)^3,x)
```

output
```
(c*(atan(a*x)**3*a**4*x**4 + 2*atan(a*x)**3*a**2*x**2 + atan(a*x)**3 - ata
n(a*x)**2*a**3*x**3 - 3*atan(a*x)**2*a*x + atan(a*x)*a**2*x**2 + atan(a*x)
 + 4*int((atan(a*x)*x)/(a**2*x**2 + 1),x)*a**2 - a*x))/(4*a**2)
```

# 3.366 $\int (c+a^2cx^2) \arctan(ax)^3 \, dx$



## Optimal result

Integrand size = 17, antiderivative size = 172

$$\int (c+a^2cx^2) \arctan(ax)^3 \, dx = cx\arctan(ax) - \frac{c(1+a^2x^2)\arctan(ax)^2}{2a} + \frac{2ic\arctan(ax)^3}{3a} + \frac{2}{3}cx\arctan(ax)^3 + \frac{1}{3}cx(1+a^2x^2)\arctan(ax)^3 + \frac{2c\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a} - \frac{c\log(1+a^2x^2)}{2a} + \frac{2ic\arctan(ax)\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{a} + \frac{c\operatorname{PolyLog}\left(3,1-\frac{2}{1+iax}\right)}{a}$$

output
```
c*x*arctan(a*x)-1/2*c*(a^2*x^2+1)*arctan(a*x)^2/a+2/3*I*c*arctan(a*x)^3/a+
2/3*c*x*arctan(a*x)^3+1/3*c*x*(a^2*x^2+1)*arctan(a*x)^3+2*c*arctan(a*x)^2*
ln(2/(1+I*a*x))/a-1/2*c*ln(a^2*x^2+1)/a+2*I*c*arctan(a*x)*polylog(2,1-2/(1
+I*a*x))/a+c*polylog(3,1-2/(1+I*a*x))/a
```

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.84

$$\int \left(c+a^2cx^2\right) \arctan(ax)^3 \, dx$$
$$= \frac{c\left(6ax \arctan(ax) - 3\arctan(ax)^2 - 3a^2x^2 \arctan(ax)^2 - 4i \arctan(ax)^3 + 6ax \arctan(ax)^3 + 2a^3x^3 \operatorname{arct}\right.}{}$$

input
```
Integrate[(c + a^2*c*x^2)*ArcTan[a*x]^3,x]
```

output
```
(c*(6*a*x*ArcTan[a*x] - 3*ArcTan[a*x]^2 - 3*a^2*x^2*ArcTan[a*x]^2 - (4*I)*
ArcTan[a*x]^3 + 6*a*x*ArcTan[a*x]^3 + 2*a^3*x^3*ArcTan[a*x]^3 + 12*ArcTan[
a*x]^2*Log[1 + E^((2*I)*ArcTan[a*x])] - 3*Log[1 + a^2*x^2] - (12*I)*ArcTan
[a*x]*PolyLog[2, -E^((2*I)*ArcTan[a*x])] + 6*PolyLog[3, -E^((2*I)*ArcTan[a
*x])]))/(6*a)
```

## Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {5415, 5345, 240, 5455, 5379, 5529, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(ax)^3 \left(a^2cx^2+c\right) \, dx$$

$$\Big\downarrow \text{ 5415}$$

$$c\int \arctan(ax)dx + \frac{2}{3}c\int \arctan(ax)^3dx + \frac{1}{3}cx\left(a^2x^2+1\right)\arctan(ax)^3 - \frac{c\left(a^2x^2+1\right)\arctan(ax)^2}{2a}$$

$$\Big\downarrow \text{ 5345}$$

$$c\left(x\arctan(ax)-a\int\frac{x}{a^2x^2+1}dx\right)+\frac{2}{3}c\left(x\arctan(ax)^3-3a\int\frac{x\arctan(ax)^2}{a^2x^2+1}dx\right)+\frac{1}{3}cx\left(a^2x^2+1\right)\arctan(ax)^3-\frac{c\left(a^2x^2+1\right)\arctan(ax)^2}{2a}$$

$\big\downarrow$ 240

$$\frac{2}{3}c\left(x\arctan(ax)^3-3a\int\frac{x\arctan(ax)^2}{a^2x^2+1}dx\right)+\frac{1}{3}cx\left(a^2x^2+1\right)\arctan(ax)^3-\frac{c\left(a^2x^2+1\right)\arctan(ax)^2}{2a}+c\left(x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}\right)$$

$\big\downarrow$ 5455

$$\frac{2}{3}c\left(x\arctan(ax)^3-3a\left(-\frac{\int\frac{\arctan(ax)^2}{i-ax}dx}{a}-\frac{i\arctan(ax)^3}{3a^2}\right)\right)+\frac{1}{3}cx\left(a^2x^2+1\right)\arctan(ax)^3-\frac{c\left(a^2x^2+1\right)\arctan(ax)^2}{2a}+c\left(x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}\right)$$

$\big\downarrow$ 5379

$$\frac{2}{3}c\left(x\arctan(ax)^3-3a\left(-\frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}-2\int\frac{\arctan(ax)\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^3}{3a^2}\right)\right)+\frac{1}{3}cx\left(a^2x^2+1\right)\arctan(ax)^3-\frac{c\left(a^2x^2+1\right)\arctan(ax)^2}{2a}+c\left(x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}\right)$$

$\big\downarrow$ 5529

$$\frac{2}{3}c\left(x\arctan(ax)^3-3a\left(-\frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}-2\left(\frac{1}{2}i\int\frac{\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx-\frac{i\arctan(ax)\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}\right)}{a}\right.\right.$$
$$\frac{1}{3}cx\left(a^2x^2+1\right)\arctan(ax)^3-\frac{c\left(a^2x^2+1\right)\arctan(ax)^2}{2a}+c\left(x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}\right)$$

$\big\downarrow$ 7164

$$\frac{2}{3}c\left(x\arctan(ax)^3-3a\left(-\frac{i\arctan(ax)^3}{3a^2}-\frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}-2\left(-\frac{i\arctan(ax)\operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}-\frac{\operatorname{PolyLog}\left(3,1\right.}{4a}\right.}{a}\right.\right.$$

$$\frac{1}{3}cx\left(a^2x^2+1\right)\arctan(ax)^3-\frac{c\left(a^2x^2+1\right)\arctan(ax)^2}{2a}+c\left(x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}\right)$$

input
```
Int[(c + a^2*c*x^2)*ArcTan[a*x]^3,x]
```

output
```
-1/2*(c*(1 + a^2*x^2)*ArcTan[a*x]^2)/a + (c*x*(1 + a^2*x^2)*ArcTan[a*x]^3)
/3 + c*(x*ArcTan[a*x] - Log[1 + a^2*x^2]/(2*a)) + (2*c*(x*ArcTan[a*x]^3 -
3*a*(((-1/3*I)*ArcTan[a*x]^3)/a^2 - ((ArcTan[a*x]^2*Log[2/(1 + I*a*x)])/a
- 2*(((-1/2*I)*ArcTan[a*x]*PolyLog[2, 1 - 2/(1 + I*a*x)])/a - PolyLog[3, 1
 - 2/(1 + I*a*x)]/(4*a)))/a)))/3
```

## Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5415
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTan[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x])^p,
x], x] + Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*(
a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[q, 0] && GtQ[p, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 71.57 (sec) , antiderivative size = 890, normalized size of antiderivative = 5.17

| method | result | size |
|---|---|---|
| parts | Expression too large to display | 890 |
| derivativedivides | Expression too large to display | 893 |
| default | Expression too large to display | 893 |

input
```
int((a^2*c*x^2+c)*arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/3*c*arctan(a*x)^3*a^2*x^3+c*x*arctan(a*x)^3-c*(1/2*x^2*arctan(a*x)^2*a+1
/a*arctan(a*x)^2*ln(a^2*x^2+1)-1/a*(-1/2*I*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)
^(1/2))^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*arctan(a*x)^2+polylog(3,-(1+I*a*
x)^2/(a^2*x^2+1))+2*arctan(a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2))-1/2*I*Pi
*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3*arctan(a*
x)^2+1/2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3*arctan(a*x)^2-I*Pi*c
sgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2
*arctan(a*x)^2-1/2*I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^3*arctan(a*x)^2+ar
ctan(a*x)*(a*x-I)-2*I*arctan(a*x)*polylog(2,-(1+I*a*x)^2/(a^2*x^2+1))+1/2*
I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*
a*x)^2/(a^2*x^2+1)+1)^2)^2*arctan(a*x)^2-2/3*I*arctan(a*x)^3+1/2*I*Pi*csgn
(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x
)^2/(a^2*x^2+1)+1)^2)^2*arctan(a*x)^2-1/2*I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^
2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/
((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*arctan(a*x)^2+I*Pi*csgn(I*(1+I*a*x)/(a^2*x^
2+1)^(1/2))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^2*arctan(a*x)^2+1/2*I*Pi*csgn(
I*((1+I*a*x)^2/(a^2*x^2+1)+1))^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*arc
tan(a*x)^2+ln((1+I*a*x)^2/(a^2*x^2+1)+1)-1/2*arctan(a*x)^2+2*arctan(a*x)^2
*ln(2)))
```

## Fricas [F]

$$\int (c + a^2cx^2) \arctan(ax)^3 \, dx = \int (a^2cx^2 + c) \arctan(ax)^3 \, dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)*arctan(a*x)^3, x)
```

## Sympy [F]

$$\int (c+a^2cx^2)\arctan(ax)^3\,dx = c\left(\int a^2x^2\,\mathrm{atan}^3\,(ax)\,dx+\int \mathrm{atan}^3\,(ax)\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)*atan(a*x)**3,x)
```

output
```
c*(Integral(a**2*x**2*atan(a*x)**3, x) + Integral(atan(a*x)**3, x))
```

## Maxima [F]

$$\int (c+a^2cx^2)\arctan(ax)^3\,dx = \int (a^2cx^2+c)\arctan(ax)^3\,dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^3,x, algorithm="maxima")
```

output
```
28*a^4*c*integrate(1/32*x^4*arctan(a*x)^3/(a^2*x^2 + 1), x) + 3*a^4*c*inte
grate(1/32*x^4*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) + 4*a^4*c*
integrate(1/32*x^4*arctan(a*x)*log(a^2*x^2 + 1)/(a^2*x^2 + 1), x) - 4*a^3*
c*integrate(1/32*x^3*arctan(a*x)^2/(a^2*x^2 + 1), x) + a^3*c*integrate(1/3
2*x^3*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) + 1/24*(a^2*c*x^3 + 3*c*x)*arct
an(a*x)^3 + 7/32*c*arctan(a*x)^4/a + 56*a^2*c*integrate(1/32*x^2*arctan(a*
x)^3/(a^2*x^2 + 1), x) + 6*a^2*c*integrate(1/32*x^2*arctan(a*x)*log(a^2*x^
2 + 1)^2/(a^2*x^2 + 1), x) + 12*a^2*c*integrate(1/32*x^2*arctan(a*x)*log(a
^2*x^2 + 1)/(a^2*x^2 + 1), x) - 1/32*(a^2*c*x^3 + 3*c*x)*arctan(a*x)*log(a
^2*x^2 + 1)^2 - 12*a*c*integrate(1/32*x*arctan(a*x)^2/(a^2*x^2 + 1), x) +
3*a*c*integrate(1/32*x*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) + 3*c*integrat
e(1/32*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x)
```

## Giac [F]

$$\int \left(c+a^2cx^2\right)\arctan(ax)^3\,dx=\int \left(a^2cx^2+c\right)\arctan\left(ax\right)^3\,dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)*arctan(a*x)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \left(c+a^2cx^2\right)\arctan(ax)^3\,dx=\int \mathrm{atan}(a\,x)^3\,\left(c\,a^2\,x^2+c\right)\,dx$$

input
```
int(atan(a*x)^3*(c + a^2*c*x^2),x)
```

output
```
int(atan(a*x)^3*(c + a^2*c*x^2), x)
```

## Reduce [F]

$$\int \left(c+a^2cx^2\right)\arctan(ax)^3\,dx$$
$$=\frac{c\left(2atan(ax)^3\,a^3x^3+6atan(ax)^3\,ax-3atan(ax)^2\,a^2x^2-3atan(ax)^2+6atan(ax)\,ax-12\left(\int\frac{atan(ax)^2x}{a^2x^2+1}d\right.\right.}{6a}$$

input
```
int((a^2*c*x^2+c)*atan(a*x)^3,x)
```

output
```
(c*(2*atan(a*x)**3*a**3*x**3 + 6*atan(a*x)**3*a*x - 3*atan(a*x)**2*a**2*x**
*2 - 3*atan(a*x)**2 + 6*atan(a*x)*a*x - 12*int((atan(a*x)**2*x)/(a**2*x**2
 + 1),x)*a**2 - 3*log(a**2*x**2 + 1)))/(6*a)
```

# 3.367 $\int \frac{(c+a^2cx^2)\arctan(ax)^3}{x}\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 276

$$\begin{aligned}\int \frac{(c+a^2cx^2)\arctan(ax)^3}{x}\,dx = &-\frac{3}{2}ic\arctan(ax)^2-\frac{3}{2}acx\arctan(ax)^2\\ &+\frac{1}{2}c\arctan(ax)^3+\frac{1}{2}a^2cx^2\arctan(ax)^3\\ &+2c\arctan(ax)^3\text{arctanh}\left(1-\frac{2}{1+iax}\right)\\ &-3c\arctan(ax)\log\left(\frac{2}{1+iax}\right)\\ &-\frac{3}{2}ic\,\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)\\ &-\frac{3}{2}ic\arctan(ax)^2\,\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right)\\ &+\frac{3}{2}ic\arctan(ax)^2\,\text{PolyLog}\left(2,-1+\frac{2}{1+iax}\right)\\ &-\frac{3}{2}c\arctan(ax)\,\text{PolyLog}\left(3,1-\frac{2}{1+iax}\right)\\ &+\frac{3}{2}c\arctan(ax)\,\text{PolyLog}\left(3,-1+\frac{2}{1+iax}\right)\\ &+\frac{3}{4}ic\,\text{PolyLog}\left(4,1-\frac{2}{1+iax}\right)\\ &-\frac{3}{4}ic\,\text{PolyLog}\left(4,-1+\frac{2}{1+iax}\right)\end{aligned}$$

output
```
-3/2*I*c*arctan(a*x)^2-3/2*a*c*x*arctan(a*x)^2+1/2*c*arctan(a*x)^3+1/2*a^2
*c*x^2*arctan(a*x)^3-2*c*arctan(a*x)^3*arctanh(-1+2/(1+I*a*x))-3*c*arctan(
a*x)*ln(2/(1+I*a*x))-3/2*I*c*polylog(2,1-2/(1+I*a*x))-3/2*I*c*arctan(a*x)^
2*polylog(2,1-2/(1+I*a*x))+3/2*I*c*arctan(a*x)^2*polylog(2,-1+2/(1+I*a*x))
-3/2*c*arctan(a*x)*polylog(3,1-2/(1+I*a*x))+3/2*c*arctan(a*x)*polylog(3,-1
+2/(1+I*a*x))+3/4*I*c*polylog(4,1-2/(1+I*a*x))-3/4*I*c*polylog(4,-1+2/(1+I
*a*x))
```

## Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.96

$$\int \frac{(c+a^2cx^2)\arctan(ax)^3}{x}\,dx = \frac{1}{2}c(1+a^2x^2)\arctan(ax)^3 - \frac{3}{2}c\big(-i\arctan(ax)^2 + ax\arctan(ax)^2 + 2\arctan(ax)\log\left(1+e^{2i\arctan(ax)}\right) - i\,\text{PolyLog}\left(2,-e^{2i\arctan(ax)}\right)\big) - \frac{1}{64}ic\big(\pi^4 - 32\arctan(ax)^4 + 64i\arctan(ax)^3\log\left(1-e^{-2i\arctan(ax)}\right) - 64i\arctan(ax)^3\log\left(1+e^{2i\arctan(ax)}\right) - 96\arctan(ax)^2\,\text{PolyLog}\left(2,e^{-2i\arctan(ax)}\right) - 96\arctan(ax)^2\,\text{PolyLog}\left(2,-e^{2i\arctan(ax)}\right) + 96i\arctan(ax)\,\text{PolyLog}\left(3,e^{-2i\arctan(ax)}\right) - 96i\arctan(ax)\,\text{PolyLog}\left(3,-e^{2i\arctan(ax)}\right) + 48\,\text{PolyLog}\left(4,e^{-2i\arctan(ax)}\right) + 48\,\text{PolyLog}\left(4,-e^{2i\arctan(ax)}\right)\big)$$

input
```
Integrate[((c + a^2*c*x^2)*ArcTan[a*x]^3)/x,x]
```

output
```
(c*(1 + a^2*x^2)*ArcTan[a*x]^3)/2 - (3*c*((-I)*ArcTan[a*x]^2 + a*x*ArcTan[
a*x]^2 + 2*ArcTan[a*x]*Log[1 + E^((2*I)*ArcTan[a*x])] - I*PolyLog[2, -E^((
2*I)*ArcTan[a*x])]))/2 - (I/64)*c*(Pi^4 - 32*ArcTan[a*x]^4 + (64*I)*ArcTan
[a*x]^3*Log[1 - E^((-2*I)*ArcTan[a*x])] - (64*I)*ArcTan[a*x]^3*Log[1 + E^(
(2*I)*ArcTan[a*x])] - 96*ArcTan[a*x]^2*PolyLog[2, E^((-2*I)*ArcTan[a*x])]
- 96*ArcTan[a*x]^2*PolyLog[2, -E^((2*I)*ArcTan[a*x])] + (96*I)*ArcTan[a*x]
*PolyLog[3, E^((-2*I)*ArcTan[a*x])] - (96*I)*ArcTan[a*x]*PolyLog[3, -E^((2
*I)*ArcTan[a*x])] + 48*PolyLog[4, E^((-2*I)*ArcTan[a*x])] + 48*PolyLog[4,
-E^((2*I)*ArcTan[a*x])])
```

## Rubi [A] (verified)

Time = 2.33 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.23, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {5485, 5357, 5361, 5451, 5345, 5419, 5455, 5379, 2849, 2752, 5523, 5529, 5533, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3 \left(a^2cx^2+c\right)}{x} \, dx$$

$$\Big\downarrow \text{ 5485}$$

$$a^2c\int x\arctan(ax)^3dx+c\int \frac{\arctan(ax)^3}{x}dx$$

$$\Big\downarrow \text{ 5357}$$

$$a^2c\int x\arctan(ax)^3dx+ c\left(2\arctan(ax)^3\text{arctanh}\left(1-\frac{2}{1+iax}\right)-6a\int \frac{\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx\right)$$

$$\Big\downarrow \text{ 5361}$$

$$a^2c\left(\frac{1}{2}x^2\arctan(ax)^3-\frac{3}{2}a\int \frac{x^2\arctan(ax)^2}{a^2x^2+1}dx\right)+ c\left(2\arctan(ax)^3\text{arctanh}\left(1-\frac{2}{1+iax}\right)-6a\int \frac{\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx\right)$$

$$\Big\downarrow \text{5451}$$

$$a^2c\left(\frac{1}{2}x^2\arctan(ax)^3-\frac{3}{2}a\left(\frac{\int \arctan(ax)^2dx}{a^2}-\frac{\int \frac{\arctan(ax)^2}{a^2x^2+1}dx}{a^2}\right)\right)+c\left(2\arctan(ax)^3\text{arctanh}\left(1-\frac{2}{1+iax}\right)-6a\int \frac{\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx\right)$$

$$\Big\downarrow \text{5345}$$

$$a^2c\left(\frac{1}{2}x^2\arctan(ax)^3-\frac{3}{2}a\left(\frac{x\arctan(ax)^2-2a\int \frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}-\frac{\int \frac{\arctan(ax)^2}{a^2x^2+1}dx}{a^2}\right)\right)+c\left(2\arctan(ax)^3\text{arctanh}\left(1-\frac{2}{1+iax}\right)-6a\int \frac{\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx\right)$$

$$\Big\downarrow \text{5419}$$

$$a^2c\left(\frac{1}{2}x^2\arctan(ax)^3-\frac{3}{2}a\left(\frac{x\arctan(ax)^2-2a\int \frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}-\frac{\arctan(ax)^3}{3a^3}\right)\right)+c\left(2\arctan(ax)^3\text{arctanh}\left(1-\frac{2}{1+iax}\right)-6a\int \frac{\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx\right)$$

$$\Big\downarrow \text{5455}$$

$$c\left(2\arctan(ax)^3\text{arctanh}\left(1-\frac{2}{1+iax}\right)-6a\int \frac{\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx\right)+a^2c\left(\frac{1}{2}x^2\arctan(ax)^3-\frac{3}{2}a\left(-\frac{\arctan(ax)^3}{3a^3}+\frac{x\arctan(ax)^2-2a\left(-\frac{\int \frac{\arctan(ax)}{i-ax}dx}{a}-\frac{i\arctan(ax)^2}{2a^2}\right)}{a^2}\right)\right)$$

$$\Big\downarrow \text{5379}$$

$$c\left(2\arctan(ax)^3\text{arctanh}\left(1-\frac{2}{1+iax}\right)-6a\int \frac{\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx\right)+a^2c\left(\frac{1}{2}x^2\arctan(ax)^3-\frac{3}{2}a\left(-\frac{\arctan(ax)^3}{3a^3}+\frac{x\arctan(ax)^2-2a\left(-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}-\int \frac{\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(a}{2a^2}\right.}{a^2}\right.\right.$$

$\downarrow$ 2849

$$c\left(2\arctan(ax)^3\text{arctanh}\left(1-\frac{2}{1+iax}\right)-6a\int\frac{\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx\right)+$$

$$a^2c\left(\frac{1}{2}x^2\arctan(ax)^3-\frac{3}{2}a\left(-\frac{\arctan(ax)^3}{3a^3}+\frac{x\arctan(ax)^2-2a\left(-\frac{\frac{i\int\frac{\log\left(\frac{2}{iax+1}\right)}{1-\frac{2}{iax+1}}d\frac{1}{iax+1}}{a}+\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}}{a}-\frac{i\,\text{ar}}{}\right.}{a^2}\right.\right.$$

$\downarrow$ 2752

$$c\left(2\arctan(ax)^3\text{arctanh}\left(1-\frac{2}{1+iax}\right)-6a\int\frac{\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx\right)+$$

$$a^2c\left(\frac{1}{2}x^2\arctan(ax)^3-\frac{3}{2}a\left(-\frac{\arctan(ax)^3}{3a^3}+\frac{x\arctan(ax)^2-2a\left(-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,1-\right.}{2a}}{a}\right.}{a^2}\right.\right.$$

$\downarrow$ 5523

$$c\left(2\arctan(ax)^3\text{arctanh}\left(1-\frac{2}{1+iax}\right)-6a\left(\frac{1}{2}\int\frac{\arctan(ax)^2\log\left(2-\frac{2}{iax+1}\right)}{a^2x^2+1}dx-\frac{1}{2}\int\frac{\arctan(ax)^2\log\left(\frac{2}{iax-}\right.}{a^2x^2+1}\right.\right.$$

$$a^2c\left(\frac{1}{2}x^2\arctan(ax)^3-\frac{3}{2}a\left(-\frac{\arctan(ax)^3}{3a^3}+\frac{x\arctan(ax)^2-2a\left(-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,1-\right.}{2a}}{a}\right.}{a^2}\right.\right.$$

$\downarrow$ 5529

$$c\left(2\arctan(ax)^3\text{arctanh}\left(1-\frac{2}{1+iax}\right)-6a\left(\frac{1}{2}\left(\frac{i\arctan(ax)^2\,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}-i\int\frac{\arctan(ax)\,\text{Poly}}{a^2x}\right.\right.\right.$$

$$a^2c\left(\frac{1}{2}x^2\arctan(ax)^3-\frac{3}{2}a\left(-\frac{\arctan(ax)^3}{3a^3}+\frac{x\arctan(ax)^2-2a\left(-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\,\text{PolyLog}(2,1}{2a}}{a}\right.}{a^2}\right.\right.$$

$\downarrow$ 5533

$$c\left(2\arctan(ax)^3\text{arctanh}\left(1-\frac{2}{1+iax}\right)-6a\left(\frac{1}{2}\left(\frac{i\arctan(ax)^2\,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}-i\left(\frac{i\arctan(ax)\,\text{Poly}}{2}\right.\right.\right.\right.$$

$$a^2c\left(\frac{1}{2}x^2\arctan(ax)^3-\frac{3}{2}a\left(-\frac{\arctan(ax)^3}{3a^3}+\frac{x\arctan(ax)^2-2a\left(-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\,\text{PolyLog}(2,1}{2a}}{a}\right.}{a^2}\right.\right.$$

$\downarrow$ 7164

$$a^2c\left(\frac{1}{2}x^2\arctan(ax)^3-\frac{3}{2}a\left(-\frac{\arctan(ax)^3}{3a^3}+\frac{x\arctan(ax)^2-2a\left(-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\,\text{PolyLog}(2,1}{2a}}{a}\right.}{a^2}\right.\right.$$

$$c\left(2\arctan(ax)^3\text{arctanh}\left(1-\frac{2}{1+iax}\right)-6a\left(\frac{1}{2}\left(\frac{i\arctan(ax)^2\,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}-i\left(\frac{i\arctan(ax)\,\text{Poly}}{2}\right.\right.\right.\right.$$

input
```
Int[((c + a^2*c*x^2)*ArcTan[a*x]^3)/x,x]
```

output
```
a^2*c*((x^2*ArcTan[a*x]^3)/2 - (3*a*(-1/3*ArcTan[a*x]^3/a^3 + (x*ArcTan[a*
x]^2 - 2*a*(((-1/2*I)*ArcTan[a*x]^2)/a^2 - ((ArcTan[a*x]*Log[2/(1 + I*a*x)
])/a + ((I/2)*PolyLog[2, 1 - 2/(1 + I*a*x)])/a)/a))/a^2))/2) + c*(2*ArcTan
[a*x]^3*ArcTanh[1 - 2/(1 + I*a*x)] - 6*a*((((I/2)*ArcTan[a*x]^2*PolyLog[2,
 1 - 2/(1 + I*a*x)])/a - I*(((I/2)*ArcTan[a*x]*PolyLog[3, 1 - 2/(1 + I*a*x
)])/a + PolyLog[4, 1 - 2/(1 + I*a*x)]/(4*a)))/2 + (((-1/2*I)*ArcTan[a*x]^2
*PolyLog[2, -1 + 2/(1 + I*a*x)])/a + I*(((I/2)*ArcTan[a*x]*PolyLog[3, -1 +
 2/(1 + I*a*x)])/a + PolyLog[4, -1 + 2/(1 + I*a*x)]/(4*a)))/2))
```

## Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5357
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Simp[2*b*c*p   Int[(a + b
*ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
 FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5523
```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[1/2   Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e
*x^2)), x], x] - Simp[1/2   Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] &&
EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 5533
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k + 1
, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &&
EqQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

## Maple [A] (verified)

Time = 21.72 (sec) , antiderivative size = 460, normalized size of antiderivative = 1.67

| method | result |
|---|---|
| derivativedivides | $\frac{c\arctan(ax)^2(-3-i\arctan(ax)+\arctan(ax)ax)(ax+i)}{2}+6ic\,\mathrm{polylog}\left(4,-\frac{iax+1}{\sqrt{a^2x^2+1}}\right)-3c\arctan\left(ax\right.$ |
| default | $\frac{c\arctan(ax)^2(-3-i\arctan(ax)+\arctan(ax)ax)(ax+i)}{2}+6ic\,\mathrm{polylog}\left(4,-\frac{iax+1}{\sqrt{a^2x^2+1}}\right)-3c\arctan\left(ax\right.$ |

input
```
int((a^2*c*x^2+c)*arctan(a*x)^3/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*c*arctan(a*x)^2*(-3-I*arctan(a*x)+arctan(a*x)*a*x)*(a*x+I)+6*I*c*polyl
og(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*c*arctan(a*x)*ln((1+I*a*x)^2/(a^2*x^2
+1)+1)+3/2*I*c*arctan(a*x)^2*polylog(2,-(1+I*a*x)^2/(a^2*x^2+1))-c*arctan(
a*x)^3*ln((1+I*a*x)^2/(a^2*x^2+1)+1)-3*I*c*arctan(a*x)^2*polylog(2,-(1+I*a
*x)/(a^2*x^2+1)^(1/2))-3/2*c*arctan(a*x)*polylog(3,-(1+I*a*x)^2/(a^2*x^2+1
))-3/4*I*c*polylog(4,-(1+I*a*x)^2/(a^2*x^2+1))+c*arctan(a*x)^3*ln(1-(1+I*a
*x)/(a^2*x^2+1)^(1/2))+6*I*c*polylog(4,(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*c*ar
ctan(a*x)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))+3/2*I*c*polylog(2,-(1+I*a
*x)^2/(a^2*x^2+1))+c*arctan(a*x)^3*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*I*c
*arctan(a*x)^2*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*c*arctan(a*x)*poly
log(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+3*I*c*arctan(a*x)^2
```

## Fricas [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)^3}{x}\,dx = \int \frac{(a^2cx^2+c)\arctan{(ax)}^3}{x}\,dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^3/x,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)*arctan(a*x)^3/x, x)
```

## Sympy [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)^3}{x}\,dx = c\left(\int \frac{\operatorname{atan}^3{(ax)}}{x}\,dx + \int a^2x\operatorname{atan}^3{(ax)}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)*atan(a*x)**3/x,x)
```

output
```
c*(Integral(atan(a*x)**3/x, x) + Integral(a**2*x*atan(a*x)**3, x))
```

## Maxima [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)^3}{x}\,dx=\int \frac{(a^2cx^2+c)\arctan{(ax)}^3}{x}\,dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^3/x,x, algorithm="maxima")
```

output
```
1/16*a^2*c*x^2*arctan(a*x)^3 - 3/64*a^2*c*x^2*arctan(a*x)*log(a^2*x^2 + 1)
^2 + integrate(1/64*(12*a^4*c*x^4*arctan(a*x)*log(a^2*x^2 + 1) - 12*a^3*c*
x^3*arctan(a*x)^2 + 56*(a^4*c*x^4 + 2*a^2*c*x^2 + c)*arctan(a*x)^3 + 3*(a^
3*c*x^3 + 2*(a^4*c*x^4 + 2*a^2*c*x^2 + c)*arctan(a*x))*log(a^2*x^2 + 1)^2)
/(a^2*x^3 + x), x)
```

## Giac [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)^3}{x}\,dx=\int \frac{(a^2cx^2+c)\arctan{(ax)}^3}{x}\,dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^3/x,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)*arctan(a*x)^3/x, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)\arctan(ax)^3}{x}\,dx=\int \frac{\mathrm{atan}(a\,x)^3\,(c\,a^2\,x^2+c)}{x}\,dx$$

input
```
int((atan(a*x)^3*(c + a^2*c*x^2))/x,x)
```

output
```
int((atan(a*x)^3*(c + a^2*c*x^2))/x, x)
```

## Reduce [F]

$$\int \frac{(c + a^2cx^2)\arctan(ax)^3}{x}\,dx$$
$$= \frac{c\left(atan(ax)^3 a^2x^2 + atan(ax)^3 - 3atan(ax)^2 ax + 3atan(ax)\, a^2x^2 + 3atan(ax) + 2\left(\int \frac{atan(ax)^3}{x}dx\right) - 6\left(\right.\right.}{2}$$

input
```
int((a^2*c*x^2+c)*atan(a*x)^3/x,x)
```

output
```
(c*(atan(a*x)**3*a**2*x**2 + atan(a*x)**3 - 3*atan(a*x)**2*a*x + 3*atan(a*
x)*a**2*x**2 + 3*atan(a*x) + 2*int(atan(a*x)**3/x,x) - 6*int((atan(a*x)*x*
*3)/(a**2*x**2 + 1),x)*a**4 - 3*a*x))/2
```

# 3.368 $\int \frac{(c+a^2cx^2)\arctan(ax)^3}{x^2}\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 169

$$\int \frac{(c+a^2cx^2)\arctan(ax)^3}{x^2}\,dx = -\frac{c\arctan(ax)^3}{x}+a^2cx\arctan(ax)^3 + 3ac\arctan(ax)^2\log\left(\frac{2}{1+iax}\right) + 3ac\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right) - 3iac\arctan(ax)\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right) + 3iac\arctan(ax)\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right) + \frac{3}{2}ac\operatorname{PolyLog}\left(3,-1+\frac{2}{1-iax}\right) + \frac{3}{2}ac\operatorname{PolyLog}\left(3,1-\frac{2}{1+iax}\right)$$

output
```
-c*arctan(a*x)^3/x+a^2*c*x*arctan(a*x)^3+3*a*c*arctan(a*x)^2*ln(2/(1+I*a*x
))+3*a*c*arctan(a*x)^2*ln(2-2/(1-I*a*x))-3*I*a*c*arctan(a*x)*polylog(2,-1+
2/(1-I*a*x))+3*I*a*c*arctan(a*x)*polylog(2,1-2/(1+I*a*x))+3/2*a*c*polylog(
3,-1+2/(1-I*a*x))+3/2*a*c*polylog(3,1-2/(1+I*a*x))
```

## Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.11

$$\int \frac{(c+a^2cx^2)\arctan(ax)^3}{x^2}\,dx = -iac\arctan(ax)^3+a^2cx\arctan(ax)^3 + 3ac\arctan(ax)^2\log\left(1+e^{2i\arctan(ax)}\right) - 3iac\arctan(ax)\operatorname{PolyLog}\left(2,-e^{2i\arctan(ax)}\right) + ac\left(-\frac{i\pi^3}{8}+i\arctan(ax)^3-\frac{\arctan(ax)^3}{ax} + 3\arctan(ax)^2\log\left(1-e^{-2i\arctan(ax)}\right) + 3i\arctan(ax)\operatorname{PolyLog}\left(2,e^{-2i\arctan(ax)}\right) + \frac{3}{2}\operatorname{PolyLog}\left(3,e^{-2i\arctan(ax)}\right)\right) + \frac{3}{2}ac\operatorname{PolyLog}\left(3,-e^{2i\arctan(ax)}\right)$$

input
```
Integrate[((c + a^2*c*x^2)*ArcTan[a*x]^3)/x^2,x]
```

output
```
(-I)*a*c*ArcTan[a*x]^3 + a^2*c*x*ArcTan[a*x]^3 + 3*a*c*ArcTan[a*x]^2*Log[1
 + E^((2*I)*ArcTan[a*x])] - (3*I)*a*c*ArcTan[a*x]*PolyLog[2, -E^((2*I)*Arc
Tan[a*x])] + a*c*((-1/8*I)*Pi^3 + I*ArcTan[a*x]^3 - ArcTan[a*x]^3/(a*x) +
3*ArcTan[a*x]^2*Log[1 - E^((-2*I)*ArcTan[a*x])] + (3*I)*ArcTan[a*x]*PolyLo
g[2, E^((-2*I)*ArcTan[a*x])] + (3*PolyLog[3, E^((-2*I)*ArcTan[a*x])])/2) +
 (3*a*c*PolyLog[3, -E^((2*I)*ArcTan[a*x])])/2
```

## Rubi [A] (verified)

Time = 1.71 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.40, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5485, 5345, 5361, 5455, 5379, 5459, 5403, 5527, 5529, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3\,(a^2cx^2+c)}{x^2}\,dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\int \arctan(ax)^3dx+c\int \frac{\arctan(ax)^3}{x^2}dx$$

$$\Big\downarrow \text{5345}$$

$$a^2c\left(x\arctan(ax)^3-3a\int \frac{x\arctan(ax)^2}{a^2x^2+1}dx\right)+c\int \frac{\arctan(ax)^3}{x^2}dx$$

$$\Big\downarrow \text{5361}$$

$$a^2c\left(x\arctan(ax)^3-3a\int \frac{x\arctan(ax)^2}{a^2x^2+1}dx\right)+c\left(3a\int \frac{\arctan(ax)^2}{x\left(a^2x^2+1\right)}dx-\frac{\arctan(ax)^3}{x}\right)$$

$$\Big\downarrow \text{5455}$$

$$c\left(3a\int \frac{\arctan(ax)^2}{x\left(a^2x^2+1\right)}dx-\frac{\arctan(ax)^3}{x}\right)+ a^2c\left(x\arctan(ax)^3-3a\left(-\frac{\int \frac{\arctan(ax)^2}{i-ax}dx}{a}-\frac{i\arctan(ax)^3}{3a^2}\right)\right)$$

$$\Big\downarrow \text{5379}$$

$$c\left(3a\int \frac{\arctan(ax)^2}{x\left(a^2x^2+1\right)}dx-\frac{\arctan(ax)^3}{x}\right)+ a^2c\left(x\arctan(ax)^3-3a\left(-\frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}-2\int \frac{\arctan(ax)\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^3}{3a^2}\right)\right)$$

$$\Big\downarrow \text{5459}$$

$$a^2c\left(x\arctan(ax)^3-3a\left(-\frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}-2\int \frac{\arctan(ax)\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^3}{3a^2}\right)\right)+ c\left(-\frac{\arctan(ax)^3}{x}+3a\left(i\int \frac{\arctan(ax)^2}{x(ax+i)}dx-\frac{1}{3}i\arctan(ax)^3\right)\right)$$

$$\Big\downarrow \text{5403}$$

$$a^2c\left(x\arctan(ax)^3-3a\left(-\frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}-2\int \frac{\arctan(ax)\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^3}{3a^2}\right)\right)+ c\left(-\frac{\arctan(ax)^3}{x}+3a\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\,\text{arcta}\right.\right.$$

$$\Big\downarrow \ 5527$$

$$c\left(-\frac{\arctan(ax)^3}{x}+3a\left(i\left(2ia\left(\frac{i\arctan(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-\frac{1}{2}i\int\frac{\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{a^2x^2+1}dx\right)-i\operatorname{ar}\right.\right.\right.$$
$$a^2c\left(x\arctan(ax)^3-3a\left(-\frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}-2\int\frac{\arctan(ax)\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^3}{3a^2}\right)\right)$$

$$\Big\downarrow \ 5529$$

$$a^2c\left(x\arctan(ax)^3-3a\left(-\frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}-2\left(\frac{1}{2}i\int\frac{\operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx-\frac{i\arctan(ax)\operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}\right)}{a}\right.\right.$$
$$c\left(-\frac{\arctan(ax)^3}{x}+3a\left(i\left(2ia\left(\frac{i\arctan(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-\frac{1}{2}i\int\frac{\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{a^2x^2+1}dx\right)-i\operatorname{ar}\right.\right.\right.$$

$$\Big\downarrow \ 7164$$

$$a^2c\left(x\arctan(ax)^3-3a\left(-\frac{i\arctan(ax)^3}{3a^2}-\frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}-2\left(-\frac{i\arctan(ax)\operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}-\frac{\operatorname{PolyLog}\left(3,\right.}{4a}\right.}{a}\right.\right.$$
$$c\left(-\frac{\arctan(ax)^3}{x}+3a\left(i\left(2ia\left(\frac{i\arctan(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-\frac{\operatorname{PolyLog}\left(3,\frac{2}{1-iax}-1\right)}{4a}\right)-i\arctan(ax\right.\right.\right.$$

input
```
Int[((c + a^2*c*x^2)*ArcTan[a*x]^3)/x^2,x]
```

output
```
c*(-(ArcTan[a*x]^3/x) + 3*a*((-1/3*I)*ArcTan[a*x]^3 + I*((-I)*ArcTan[a*x]^
2*Log[2 - 2/(1 - I*a*x)] + (2*I)*a*(((I/2)*ArcTan[a*x]*PolyLog[2, -1 + 2/(
1 - I*a*x)])/a - PolyLog[3, -1 + 2/(1 - I*a*x)]/(4*a))))) + a^2*c*(x*ArcTa
n[a*x]^3 - 3*a*(((-1/3*I)*ArcTan[a*x]^3)/a^2 - ((ArcTan[a*x]^2*Log[2/(1 +
I*a*x)])/a - 2*(((-1/2*I)*ArcTan[a*x]*PolyLog[2, 1 - 2/(1 + I*a*x)])/a - P
olyLog[3, 1 - 2/(1 + I*a*x)]/(4*a)))/a))
```

### Defintions of rubi rules used

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5527
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 152.19 (sec) , antiderivative size = 1653, normalized size of antiderivative = 9.78

| method | result | size |
|---|---|---|
| parts | Expression too large to display | 1653 |
| derivativedivides | Expression too large to display | 1654 |
| default | Expression too large to display | 1654 |

input
```
int((a^2*c*x^2+c)*arctan(a*x)^3/x^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*c*x*arctan(a*x)^3-c*arctan(a*x)^3/x-3*c*(-a*arctan(a*x)^2*ln(a*x)+a*ar
ctan(a*x)^2*ln(a^2*x^2+1)-2*a*(arctan(a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2
))-1/3*I*arctan(a*x)^3-1/2*arctan(a*x)^2*ln((1+I*a*x)^2/(a^2*x^2+1)-1)+1/2
*arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*arctan(a*x)*polylog(2,(
1+I*a*x)/(a^2*x^2+1)^(1/2))+polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))+1/2*arc
tan(a*x)^2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*arctan(a*x)*polylog(2,-(1+I
*a*x)/(a^2*x^2+1)^(1/2))+polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+1/4*(-I*P
i*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))-I*
Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(((
1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2-I*Pi*csgn(I*(1+I*
a*x)^2/(a^2*x^2+1))^3-I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^
2*x^2+1)+1)^2)^3-I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x
)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2+I*Pi*csgn(I*((1+I*a*x)^2
/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)
-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))-2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)
)*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2+I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^
2+1)-1))*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+
1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))-I*Pi*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/(
(1+I*a*x)^2/(a^2*x^2+1)+1))^2+I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))^2*c
sgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)+I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+...
```

## Fricas [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)^3}{x^2}\,dx = \int \frac{(a^2cx^2+c)\arctan{(ax)}^3}{x^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^3/x^2,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)*arctan(a*x)^3/x^2, x)
```

## Sympy [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)^3}{x^2}\,dx = c\left(\int a^2\operatorname{atan}^3(ax)\,dx + \int \frac{\operatorname{atan}^3(ax)}{x^2}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)*atan(a*x)**3/x**2,x)
```

output
```
c*(Integral(a**2*atan(a*x)**3, x) + Integral(atan(a*x)**3/x**2, x))
```

## Maxima [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)^3}{x^2}\,dx = \int \frac{(a^2cx^2+c)\arctan(ax)^3}{x^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^3/x^2,x, algorithm="maxima")
```

output
```
1/64*(8*(a^2*c*x^2 - c)*arctan(a*x)^3 - 6*(a^2*c*x^2 - c)*arctan(a*x)*log(
a^2*x^2 + 1)^2 + (28*a*c*arctan(a*x)^4 + 1792*a^4*c*integrate(1/32*x^4*arc
tan(a*x)^3/(a^2*x^4 + x^2), x) + 192*a^4*c*integrate(1/32*x^4*arctan(a*x)*
log(a^2*x^2 + 1)^2/(a^2*x^4 + x^2), x) + 768*a^4*c*integrate(1/32*x^4*arct
an(a*x)*log(a^2*x^2 + 1)/(a^2*x^4 + x^2), x) - 768*a^3*c*integrate(1/32*x^
3*arctan(a*x)^2/(a^2*x^4 + x^2), x) + a*c*log(a^2*x^2 + 1)^3 + 384*a^2*c*i
ntegrate(1/32*x^2*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*x^4 + x^2), x) - 768
*a^2*c*integrate(1/32*x^2*arctan(a*x)*log(a^2*x^2 + 1)/(a^2*x^4 + x^2), x)
 + 768*a*c*integrate(1/32*x*arctan(a*x)^2/(a^2*x^4 + x^2), x) - 192*a*c*in
tegrate(1/32*x*log(a^2*x^2 + 1)^2/(a^2*x^4 + x^2), x) + 1792*c*integrate(1
/32*arctan(a*x)^3/(a^2*x^4 + x^2), x) + 192*c*integrate(1/32*arctan(a*x)*l
og(a^2*x^2 + 1)^2/(a^2*x^4 + x^2), x))*x)/x
```

## Giac [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)^3}{x^2}\,dx = \int \frac{(a^2cx^2+c)\arctan(ax)^3}{x^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^3/x^2,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)*arctan(a*x)^3/x^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)\arctan(ax)^3}{x^2}\,dx = \int \frac{\operatorname{atan}(a\,x)^3\,(c\,a^2\,x^2+c)}{x^2}\,dx$$

input
```
int((atan(a*x)^3*(c + a^2*c*x^2))/x^2,x)
```

output
```
int((atan(a*x)^3*(c + a^2*c*x^2))/x^2, x)
```

## Reduce [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)^3}{x^2}\,dx = \frac{c\left(-atan(ax)^3+\left(\int atan(ax)^3\,dx\right)a^2x+3\left(\int \frac{atan(ax)^2}{a^2x^3+x}dx\right)ax\right)}{x}$$

input
```
int((a^2*c*x^2+c)*atan(a*x)^3/x^2,x)
```

output
```
(c*( - atan(a*x)**3 + int(atan(a*x)**3,x)*a**2*x + 3*int(atan(a*x)**2/(a**
2*x**3 + x),x)*a*x))/x
```

# 3.369 $\int \frac{(c+a^2cx^2)\arctan(ax)^3}{x^3}\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 310

$$\begin{aligned}\int \frac{(c+a^2cx^2)\arctan(ax)^3}{x^3}\,dx = &-\frac{3}{2}ia^2c\arctan(ax)^2-\frac{3ac\arctan(ax)^2}{2x}\\
&-\frac{1}{2}a^2c\arctan(ax)^3-\frac{c\arctan(ax)^3}{2x^2}\\
&+2a^2c\arctan(ax)^3\operatorname{arctanh}\left(1-\frac{2}{1+iax}\right)\\
&+3a^2c\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)\\
&-\frac{3}{2}ia^2c\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)\\
&-\frac{3}{2}ia^2c\arctan(ax)^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right)\\
&+\frac{3}{2}ia^2c\arctan(ax)^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1+iax}\right)\\
&-\frac{3}{2}a^2c\arctan(ax)\operatorname{PolyLog}\left(3,1-\frac{2}{1+iax}\right)\\
&+\frac{3}{2}a^2c\arctan(ax)\operatorname{PolyLog}\left(3,-1+\frac{2}{1+iax}\right)\\
&+\frac{3}{4}ia^2c\operatorname{PolyLog}\left(4,1-\frac{2}{1+iax}\right)\\
&-\frac{3}{4}ia^2c\operatorname{PolyLog}\left(4,-1+\frac{2}{1+iax}\right)\end{aligned}$$

output
```
-3/2*I*a^2*c*arctan(a*x)^2-3/2*a*c*arctan(a*x)^2/x-1/2*a^2*c*arctan(a*x)^3
-1/2*c*arctan(a*x)^3/x^2-2*a^2*c*arctan(a*x)^3*arctanh(-1+2/(1+I*a*x))+3*a
^2*c*arctan(a*x)*ln(2-2/(1-I*a*x))-3/2*I*a^2*c*polylog(2,-1+2/(1-I*a*x))-3
/2*I*a^2*c*arctan(a*x)^2*polylog(2,1-2/(1+I*a*x))+3/2*I*a^2*c*arctan(a*x)^
2*polylog(2,-1+2/(1+I*a*x))-3/2*a^2*c*arctan(a*x)*polylog(3,1-2/(1+I*a*x))
+3/2*a^2*c*arctan(a*x)*polylog(3,-1+2/(1+I*a*x))+3/4*I*a^2*c*polylog(4,1-2
/(1+I*a*x))-3/4*I*a^2*c*polylog(4,-1+2/(1+I*a*x))
```

## Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.96

$$\int \frac{(c+a^2cx^2)\arctan(ax)^3}{x^3}\,dx = \frac{1}{2}a^2c\arctan(ax)^3 + \frac{c(-1-a^2x^2)\arctan(ax)^3}{2x^2} + \frac{3}{2}a^2c\left(-\frac{1}{3}\arctan(ax)\left(\frac{3\arctan(ax)}{ax} + \arctan(ax)(3i+\arctan(ax)) - 6\log\left(1-e^{2i\arctan(ax)}\right)\right) - i\,\text{PolyLog}\left(2,e^{2i\arctan(ax)}\right)\right) - \frac{1}{64}ia^2c\left(\pi^4 - 32\arctan(ax)^4 + 64i\arctan(ax)^3\log\left(1-e^{-2i\arctan(ax)}\right) - 64i\arctan(ax)^3\log\left(1+e^{2i\arctan(ax)}\right) - 96\arctan(ax)^2\,\text{PolyLog}\left(2,e^{-2i\arctan(ax)}\right) - 96\arctan(ax)^2\,\text{PolyLog}\left(2,-e^{2i\arctan(ax)}\right) + 96i\arctan(ax)\,\text{PolyLog}\left(3,e^{-2i\arctan(ax)}\right) - 96i\arctan(ax)\,\text{PolyLog}\left(3,-e^{2i\arctan(ax)}\right) + 48\,\text{PolyLog}\left(4,e^{-2i\arctan(ax)}\right) + 48\,\text{PolyLog}\left(4,-e^{2i\arctan(ax)}\right)\right)$$

input
```
Integrate[((c + a^2*c*x^2)*ArcTan[a*x]^3)/x^3,x]
```

output
```
(a^2*c*ArcTan[a*x]^3)/2 + (c*(-1 - a^2*x^2)*ArcTan[a*x]^3)/(2*x^2) + (3*a^
2*c*(-1/3*(ArcTan[a*x]*((3*ArcTan[a*x])/(a*x) + ArcTan[a*x]*(3*I + ArcTan[
a*x]) - 6*Log[1 - E^((2*I)*ArcTan[a*x])])) - I*PolyLog[2, E^((2*I)*ArcTan[
a*x])]))/2 - (I/64)*a^2*c*(Pi^4 - 32*ArcTan[a*x]^4 + (64*I)*ArcTan[a*x]^3*
Log[1 - E^((-2*I)*ArcTan[a*x])] - (64*I)*ArcTan[a*x]^3*Log[1 + E^((2*I)*Ar
cTan[a*x])] - 96*ArcTan[a*x]^2*PolyLog[2, E^((-2*I)*ArcTan[a*x])] - 96*Arc
Tan[a*x]^2*PolyLog[2, -E^((2*I)*ArcTan[a*x])] + (96*I)*ArcTan[a*x]*PolyLog
[3, E^((-2*I)*ArcTan[a*x])] - (96*I)*ArcTan[a*x]*PolyLog[3, -E^((2*I)*ArcT
an[a*x])] + 48*PolyLog[4, E^((-2*I)*ArcTan[a*x])] + 48*PolyLog[4, -E^((2*I
)*ArcTan[a*x])])
```

## Rubi [A] (verified)

Time = 2.25 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.06, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.650$, Rules used = {5485, 5357, 5361, 5453, 5361, 5419, 5459, 5403, 2897, 5523, 5529, 5533, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3\left(a^2cx^2+c\right)}{x^3}dx$$

$$\Big\downarrow \ 5485$$

$$a^2c\int \frac{\arctan(ax)^3}{x}dx+c\int \frac{\arctan(ax)^3}{x^3}dx$$

$$\Big\downarrow \ 5357$$

$$c\int \frac{\arctan(ax)^3}{x^3}dx+ a^2c\left(2\arctan(ax)^3\text{arctanh}\left(1-\frac{2}{1+iax}\right)-6a\int \frac{\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx\right)$$

$$\Big\downarrow \ 5361$$

$$c\left(\frac{3}{2}a\int \frac{\arctan(ax)^2}{x^2\left(a^2x^2+1\right)}dx-\frac{\arctan(ax)^3}{2x^2}\right)+ a^2c\left(2\arctan(ax)^3\text{arctanh}\left(1-\frac{2}{1+iax}\right)-6a\int \frac{\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx\right)$$

$$\Big\downarrow \text{5453}$$

$$c\left(\frac{3}{2}a\left(\int \frac{\arctan(ax)^2}{x^2}dx - a^2\int \frac{\arctan(ax)^2}{a^2x^2+1}dx\right) - \frac{\arctan(ax)^3}{2x^2}\right) + a^2c\left(2\arctan(ax)^3\text{arctanh}\left(1-\frac{2}{1+iax}\right) - 6a\int \frac{\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx\right)$$

$$\Big\downarrow \text{5361}$$

$$c\left(\frac{3}{2}a\left(a^2\left(-\int \frac{\arctan(ax)^2}{a^2x^2+1}dx\right) + 2a\int \frac{\arctan(ax)}{x\,(a^2x^2+1)}dx - \frac{\arctan(ax)^2}{x}\right) - \frac{\arctan(ax)^3}{2x^2}\right) + a^2c\left(2\arctan(ax)^3\text{arctanh}\left(1-\frac{2}{1+iax}\right) - 6a\int \frac{\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx\right)$$

$$\Big\downarrow \text{5419}$$

$$c\left(\frac{3}{2}a\left(2a\int \frac{\arctan(ax)}{x\,(a^2x^2+1)}dx - \frac{1}{3}a\arctan(ax)^3 - \frac{\arctan(ax)^2}{x}\right) - \frac{\arctan(ax)^3}{2x^2}\right) + a^2c\left(2\arctan(ax)^3\text{arctanh}\left(1-\frac{2}{1+iax}\right) - 6a\int \frac{\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx\right)$$

$$\Big\downarrow \text{5459}$$

$$a^2c\left(2\arctan(ax)^3\text{arctanh}\left(1-\frac{2}{1+iax}\right) - 6a\int \frac{\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx\right) + c\left(-\frac{\arctan(ax)^3}{2x^2} + \frac{3}{2}a\left(2a\left(i\int \frac{\arctan(ax)}{x(ax+i)}dx - \frac{1}{2}i\arctan(ax)^2\right) - \frac{1}{3}a\arctan(ax)^3 - \frac{\arctan(ax)^2}{x}\right)\right)$$

$$\Big\downarrow \text{5403}$$

$$a^2c\left(2\arctan(ax)^3\text{arctanh}\left(1-\frac{2}{1+iax}\right) - 6a\int \frac{\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx\right) + c\left(-\frac{\arctan(ax)^3}{2x^2} + \frac{3}{2}a\left(2a\left(i\left(ia\int \frac{\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx - i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)\right) - \frac{1}{2}i\arctan(ax)^2\right)\right.\right.$$

$$\Big\downarrow \text{2897}$$

$$a^2c\left(2\arctan(ax)^3\text{arctanh}\left(1-\frac{2}{1+iax}\right) - 6a\int \frac{\arctan(ax)^2\text{arctanh}\left(1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx\right) + c\left(-\frac{\arctan(ax)^3}{2x^2} + \frac{3}{2}a\left(2a\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right) - \frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right) - \frac{1}{2}i\arctan(a\right.\right.\right.$$

$\downarrow$ 5523

$$a^2c\left(2\arctan(ax)^3\text{arctanh}\left(1-\frac{2}{1+iax}\right)-6a\left(\frac{1}{2}\int\frac{\arctan(ax)^2\log\left(2-\frac{2}{iax+1}\right)}{a^2x^2+1}dx-\frac{1}{2}\int\frac{\arctan(ax)^2\log\left(\frac{}{i}\right.}{a^2x^2+1}\right.\right.$$
$$c\left(-\frac{\arctan(ax)^3}{2x^2}+\frac{3}{2}a\left(2a\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(a\right.\right.\right.$$

$\downarrow$ 5529

$$a^2c\left(2\arctan(ax)^3\text{arctanh}\left(1-\frac{2}{1+iax}\right)-6a\left(\frac{1}{2}\left(\frac{i\arctan(ax)^2\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}-i\int\frac{\arctan(ax)\text{Po}}{a}\right.\right.\right.$$
$$c\left(-\frac{\arctan(ax)^3}{2x^2}+\frac{3}{2}a\left(2a\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(a\right.\right.\right.$$

$\downarrow$ 5533

$$a^2c\left(2\arctan(ax)^3\text{arctanh}\left(1-\frac{2}{1+iax}\right)-6a\left(\frac{1}{2}\left(\frac{i\arctan(ax)^2\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}-i\left(\frac{i\arctan(ax)\text{Po}}{}\right.\right.\right.\right.$$
$$c\left(-\frac{\arctan(ax)^3}{2x^2}+\frac{3}{2}a\left(2a\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(a\right.\right.\right.$$

$\downarrow$ 7164

$$a^2c\left(2\arctan(ax)^3\text{arctanh}\left(1-\frac{2}{1+iax}\right)-6a\left(\frac{1}{2}\left(\frac{i\arctan(ax)^2\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}-i\left(\frac{i\arctan(ax)\text{Po}}{}\right.\right.\right.\right.$$
$$c\left(-\frac{\arctan(ax)^3}{2x^2}+\frac{3}{2}a\left(2a\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(a\right.\right.\right.$$

input
```
Int[((c + a^2*c*x^2)*ArcTan[a*x]^3)/x^3,x]
```

output
```
c*(-1/2*ArcTan[a*x]^3/x^2 + (3*a*(-(ArcTan[a*x]^2/x) - (a*ArcTan[a*x]^3)/3
 + 2*a*((-1/2*I)*ArcTan[a*x]^2 + I*((-I)*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)
] - PolyLog[2, -1 + 2/(1 - I*a*x)]/2))))/2) + a^2*c*(2*ArcTan[a*x]^3*ArcTa
nh[1 - 2/(1 + I*a*x)] - 6*a*((((I/2)*ArcTan[a*x]^2*PolyLog[2, 1 - 2/(1 + I
*a*x)])/a - I*(((I/2)*ArcTan[a*x]*PolyLog[3, 1 - 2/(1 + I*a*x)])/a + PolyL
og[4, 1 - 2/(1 + I*a*x)]/(4*a)))/2 + (((-1/2*I)*ArcTan[a*x]^2*PolyLog[2, -
1 + 2/(1 + I*a*x)])/a + I*(((I/2)*ArcTan[a*x]*PolyLog[3, -1 + 2/(1 + I*a*x
)])/a + PolyLog[4, -1 + 2/(1 + I*a*x)]/(4*a)))/2))
```

## Defintions of rubi rules used

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5357
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
b*ArcTan[c*x])^p*ArcTanh[1 - 2/(1 + I*c*x)], x] - Simp[2*b*c*p   Int[(a + b
*ArcTan[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 + I*c*x)]/(1 + c^2*x^2)), x], x] /;
 FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5523
```
Int[(ArcTanh[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[1/2   Int[Log[1 + u]*((a + b*ArcTan[c*x])^p/(d + e
*x^2)), x], x] - Simp[1/2   Int[Log[1 - u]*((a + b*ArcTan[c*x])^p/(d + e*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*d] &&
EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 5533
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k + 1
, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &&
EqQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

## Maple [A] (verified)

Time = 29.46 (sec) , antiderivative size = 525, normalized size of antiderivative = 1.69

| method | result |
|---|---|
| derivativedivides | $a^2\left(-\frac{c\arctan(ax)^2(-i\arctan(ax)+\arctan(ax)ax-3iax)(ax+i)}{2a^2x^2}-3ic\,\mathrm{polylog}\left(2,\frac{iax+1}{\sqrt{a^2x^2+1}}\right)+c\,\mathrm{arcta}\right.$ |
| default | $a^2\left(-\frac{c\arctan(ax)^2(-i\arctan(ax)+\arctan(ax)ax-3iax)(ax+i)}{2a^2x^2}-3ic\,\mathrm{polylog}\left(2,\frac{iax+1}{\sqrt{a^2x^2+1}}\right)+c\,\mathrm{arcta}\right.$ |

input
```
int((a^2*c*x^2+c)*arctan(a*x)^3/x^3,x,method=_RETURNVERBOSE)
```

output
```
a^2*(-1/2*c*arctan(a*x)^2*(-I*arctan(a*x)+arctan(a*x)*a*x-3*I*a*x)*(a*x+I)
/a^2/x^2-3*I*c*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+c*arctan(a*x)^3*ln(1
-(1+I*a*x)/(a^2*x^2+1)^(1/2))+3/2*I*c*arctan(a*x)^2*polylog(2,-(1+I*a*x)^2
/(a^2*x^2+1))-3/2*c*arctan(a*x)*polylog(3,-(1+I*a*x)^2/(a^2*x^2+1))-3/4*I*
c*polylog(4,-(1+I*a*x)^2/(a^2*x^2+1))-c*arctan(a*x)^3*ln((1+I*a*x)^2/(a^2*
x^2+1)+1)-3*I*c*arctan(a*x)^2*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+3*c*a
rctan(a*x)*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*I*c*arctan(a*x)^2+3*c*arcta
n(a*x)*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*I*c*arctan(a*x)^2*polylog(2,-(1
+I*a*x)/(a^2*x^2+1)^(1/2))+c*arctan(a*x)^3*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2
))+6*I*c*polylog(4,(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*c*arctan(a*x)*polylog(3,
-(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*I*c*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2)
)+6*c*arctan(a*x)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*c*polylog(4,-
(1+I*a*x)/(a^2*x^2+1)^(1/2)))
```

## Fricas [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)^3}{x^3}\,dx=\int \frac{(a^2cx^2+c)\arctan(ax)^3}{x^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^3/x^3,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)*arctan(a*x)^3/x^3, x)
```

## Sympy [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)^3}{x^3}\,dx=c\left(\int \frac{\operatorname{atan}^3(ax)}{x^3}\,dx+\int \frac{a^2\operatorname{atan}^3(ax)}{x}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)*atan(a*x)**3/x**3,x)
```

output
```
c*(Integral(atan(a*x)**3/x**3, x) + Integral(a**2*atan(a*x)**3/x, x))
```

## Maxima [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)^3}{x^3}\,dx=\int \frac{(a^2cx^2+c)\arctan(ax)^3}{x^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^3/x^3,x, algorithm="maxima")
```

output
```
-1/64*(4*c*arctan(a*x)^3 - 3*c*arctan(a*x)*log(a^2*x^2 + 1)^2 - 64*x^2*int
egrate(-1/64*(12*a^2*c*x^2*arctan(a*x)*log(a^2*x^2 + 1) - 12*a*c*x*arctan(
a*x)^2 - 56*(a^4*c*x^4 + 2*a^2*c*x^2 + c)*arctan(a*x)^3 + 3*(a*c*x - 2*(a^
4*c*x^4 + 2*a^2*c*x^2 + c)*arctan(a*x))*log(a^2*x^2 + 1)^2)/(a^2*x^5 + x^3
), x))/x^2
```

## Giac [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)^3}{x^3}\,dx = \int \frac{(a^2cx^2+c)\arctan{(ax)}^3}{x^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^3/x^3,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)*arctan(a*x)^3/x^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)\arctan(ax)^3}{x^3}\,dx = \int \frac{\operatorname{atan}(a\,x)^3\,(c\,a^2\,x^2+c)}{x^3}\,dx$$

input
```
int((atan(a*x)^3*(c + a^2*c*x^2))/x^3,x)
```

output
```
int((atan(a*x)^3*(c + a^2*c*x^2))/x^3, x)
```

## Reduce [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)^3}{x^3}\,dx$$
$$= \frac{c\left(-atan(ax)^3\,a^2x^2 - atan(ax)^3 - 3atan(ax)^2\,ax - 3atan(ax)\,a^2x^2 - 3atan(ax) - 6\left(\int \frac{atan(ax)}{a^2x^5+x^3}dx\right)x^2 +}{2x^2}$$

input
```
int((a^2*c*x^2+c)*atan(a*x)^3/x^3,x)
```

output
```
(c*( - atan(a*x)**3*a**2*x**2 - atan(a*x)**3 - 3*atan(a*x)**2*a*x - 3*atan
(a*x)*a**2*x**2 - 3*atan(a*x) - 6*int(atan(a*x)/(a**2*x**5 + x**3),x)*x**2
 + 2*int(atan(a*x)**3/x,x)*a**2*x**2 - 3*a*x))/(2*x**2)
```

# 3.370 $\int \frac{(c+a^2cx^2)\arctan(ax)^3}{x^4}\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 189

$$\begin{aligned}\int \frac{(c+a^2cx^2)\arctan(ax)^3}{x^4}\,dx =& -\frac{a^2c\arctan(ax)}{x}-\frac{1}{2}a^3c\arctan(ax)^2-\frac{ac\arctan(ax)^2}{2x^2}\\&-\frac{2}{3}ia^3c\arctan(ax)^3-\frac{c\arctan(ax)^3}{3x^3}\\&-\frac{a^2c\arctan(ax)^3}{x}+a^3c\log(x)-\frac{1}{2}a^3c\log\left(1+a^2x^2\right)\\&+2a^3c\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\\&-2ia^3c\arctan(ax)\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)\\&+a^3c\operatorname{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)\end{aligned}$$

output
```
-a^2*c*arctan(a*x)/x-1/2*a^3*c*arctan(a*x)^2-1/2*a*c*arctan(a*x)^2/x^2-2/3
*I*a^3*c*arctan(a*x)^3-1/3*c*arctan(a*x)^3/x^3-a^2*c*arctan(a*x)^3/x+a^3*c
*ln(x)-1/2*a^3*c*ln(a^2*x^2+1)+2*a^3*c*arctan(a*x)^2*ln(2-2/(1-I*a*x))-2*I
*a^3*c*arctan(a*x)*polylog(2,-1+2/(1-I*a*x))+a^3*c*polylog(3,-1+2/(1-I*a*x
))
```

## Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.94

$$\int \frac{(c+a^2cx^2)\arctan(ax)^3}{x^4}\,dx = \frac{1}{12}c\left(-ia^3\pi^3 - \frac{12a^2\arctan(ax)}{x} - 6a^3\arctan(ax)^2 - \frac{6a\arctan(ax)^2}{x^2} + 8ia^3\arctan(ax)^3 - \frac{4\arctan(ax)^3}{x^3} - \frac{12a^2\arctan(ax)^3}{x} + 24a^3\arctan(ax)^2\log\left(1-e^{-2i\arctan(ax)}\right) + 12a^3\log\left(\frac{ax}{\sqrt{1+a^2x^2}}\right) + 24ia^3\arctan(ax)\operatorname{PolyLog}\left(2,e^{-2i\arctan(ax)}\right) + 12a^3\operatorname{PolyLog}\left(3,e^{-2i\arctan(ax)}\right)\right)$$

input
```
Integrate[((c + a^2*c*x^2)*ArcTan[a*x]^3)/x^4,x]
```

output
```
(c*((-I)*a^3*Pi^3 - (12*a^2*ArcTan[a*x])/x - 6*a^3*ArcTan[a*x]^2 - (6*a*Ar
cTan[a*x]^2)/x^2 + (8*I)*a^3*ArcTan[a*x]^3 - (4*ArcTan[a*x]^3)/x^3 - (12*a
^2*ArcTan[a*x]^3)/x + 24*a^3*ArcTan[a*x]^2*Log[1 - E^((-2*I)*ArcTan[a*x])]
 + 12*a^3*Log[(a*x)/Sqrt[1 + a^2*x^2]] + (24*I)*a^3*ArcTan[a*x]*PolyLog[2,
 E^((-2*I)*ArcTan[a*x])] + 12*a^3*PolyLog[3, E^((-2*I)*ArcTan[a*x])]))/12
```

## Rubi [A] (verified)

Time = 2.19 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.61, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {5485, 5361, 5453, 5361, 5453, 5361, 243, 47, 14, 16, 5419, 5459, 5403, 5527, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3\left(a^2cx^2+c\right)}{x^4}\,dx$$

$$\Big\downarrow \text{ 5485}$$

$$a^2c\int \frac{\arctan(ax)^3}{x^2}dx+c\int \frac{\arctan(ax)^3}{x^4}dx$$

$$\Big\downarrow \text{ 5361}$$

$$a^2c\left(3a\int \frac{\arctan(ax)^2}{x\left(a^2x^2+1\right)}dx-\frac{\arctan(ax)^3}{x}\right)+c\left(a\int \frac{\arctan(ax)^2}{x^3\left(a^2x^2+1\right)}dx-\frac{\arctan(ax)^3}{3x^3}\right)$$

$$\Big\downarrow \text{ 5453}$$

$$a^2c\left(3a\int \frac{\arctan(ax)^2}{x\left(a^2x^2+1\right)}dx-\frac{\arctan(ax)^3}{x}\right)+ c\left(a\left(\int \frac{\arctan(ax)^2}{x^3}dx-a^2\int \frac{\arctan(ax)^2}{x\left(a^2x^2+1\right)}dx\right)-\frac{\arctan(ax)^3}{3x^3}\right)$$

$$\Big\downarrow \text{ 5361}$$

$$a^2c\left(3a\int \frac{\arctan(ax)^2}{x\left(a^2x^2+1\right)}dx-\frac{\arctan(ax)^3}{x}\right)+ c\left(a\left(a^2\left(-\int \frac{\arctan(ax)^2}{x\left(a^2x^2+1\right)}dx\right)+a\int \frac{\arctan(ax)}{x^2\left(a^2x^2+1\right)}dx-\frac{\arctan(ax)^2}{2x^2}\right)-\frac{\arctan(ax)^3}{3x^3}\right)$$

$$\Big\downarrow \text{ 5453}$$

$$a^2c\left(3a\int \frac{\arctan(ax)^2}{x\left(a^2x^2+1\right)}dx-\frac{\arctan(ax)^3}{x}\right)+ c\left(a\left(a^2\left(-\int \frac{\arctan(ax)^2}{x\left(a^2x^2+1\right)}dx\right)+a\left(\int \frac{\arctan(ax)}{x^2}dx-a^2\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)-\frac{\arctan(ax)^2}{2x^2}\right)-\frac{\arctan(ax)^3}{3x^3}\right)$$

$$\Big\downarrow \text{ 5361}$$

$$a^2c\left(3a\int \frac{\arctan(ax)^2}{x\left(a^2x^2+1\right)}dx-\frac{\arctan(ax)^3}{x}\right)+ c\left(a\left(a^2\left(-\int \frac{\arctan(ax)^2}{x\left(a^2x^2+1\right)}dx\right)+a\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+a\int \frac{1}{x\left(a^2x^2+1\right)}dx-\frac{\arctan(ax)}{x}\right)-\frac{\arctan(ax)^2}{2x^2}\right)-\frac{\arctan(ax)^3}{3x^3}\right)$$

$$\Big\downarrow \text{ 243}$$

$$a^2c\left(3a\int \frac{\arctan(ax)^2}{x\left(a^2x^2+1\right)}dx-\frac{\arctan(ax)^3}{x}\right)+ c\left(a\left(a^2\left(-\int \frac{\arctan(ax)^2}{x\left(a^2x^2+1\right)}dx\right)+a\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{2}a\int \frac{1}{x^2\left(a^2x^2+1\right)}dx^2-\frac{\arctan(ax)}{x}\right)-\frac{\arctan(ax)^2}{2x^2}\right)-\frac{\arctan(ax)^3}{3x^3}\right)$$

$$\Big\downarrow \text{ 47}$$

$$a^2c\left(3a\int \frac{\arctan(ax)^2}{x\left(a^2x^2+1\right)}dx-\frac{\arctan(ax)^3}{x}\right)+$$
$$c\left(a\left(a^2\left(-\int \frac{\arctan(ax)^2}{x\left(a^2x^2+1\right)}dx\right)+a\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{2}a\left(\int \frac{1}{x^2}dx^2-a^2\int \frac{1}{a^2x^2+1}dx^2\right)-\frac{\arctan(}{x}\right.\right.\right.$$

$\downarrow$ 14

$$a^2c\left(3a\int \frac{\arctan(ax)^2}{x\left(a^2x^2+1\right)}dx-\frac{\arctan(ax)^3}{x}\right)+$$
$$c\left(a\left(a^2\left(-\int \frac{\arctan(ax)^2}{x\left(a^2x^2+1\right)}dx\right)+a\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{2}a\left(\log\left(x^2\right)-a^2\int \frac{1}{a^2x^2+1}dx^2\right)-\frac{\arctan(a}{x}\right.\right.\right.$$

$\downarrow$ 16

$$a^2c\left(3a\int \frac{\arctan(ax)^2}{x\left(a^2x^2+1\right)}dx-\frac{\arctan(ax)^3}{x}\right)+$$
$$c\left(a\left(a^2\left(-\int \frac{\arctan(ax)^2}{x\left(a^2x^2+1\right)}dx\right)+a\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{\arctan(ax)}{x}\right)\right.\right.$$

$\downarrow$ 5419

$$a^2c\left(3a\int \frac{\arctan(ax)^2}{x\left(a^2x^2+1\right)}dx-\frac{\arctan(ax)^3}{x}\right)+$$
$$c\left(a\left(a^2\left(-\int \frac{\arctan(ax)^2}{x\left(a^2x^2+1\right)}dx\right)+a\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{1}{2}a\arctan(ax)^2-\frac{\arctan(ax)}{x}\right)-\frac{\arctan(}{2x}\right.\right.$$

$\downarrow$ 5459

$$c\left(-\frac{\arctan(ax)^3}{3x^3}+a\left(-\left(a^2\left(i\int \frac{\arctan(ax)^2}{x(ax+i)}dx-\frac{1}{3}i\arctan(ax)^3\right)\right)+a\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{1}{2}a\right.\right.\right.$$
$$a^2c\left(-\frac{\arctan(ax)^3}{x}+3a\left(i\int \frac{\arctan(ax)^2}{x(ax+i)}dx-\frac{1}{3}i\arctan(ax)^3\right)\right)$$

$\downarrow$ 5403

$$a^2c\left(-\frac{\arctan(ax)^3}{x}+3a\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\,\mathrm{arc}\right.\right.$$
$$c\left(-\frac{\arctan(ax)^3}{3x^3}+a\left(-\left(a^2\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\right.\right.\right.\right.$$

$\downarrow$ 5527

$$a^2c\left(-\frac{\arctan(ax)^3}{x}+3a\left(i\left(2ia\left(\frac{i\arctan(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-\frac{1}{2}i\int\frac{\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{a^2x^2+1}dx\right)-i\right.\right.\right.$$
$$c\left(-\frac{\arctan(ax)^3}{3x^3}+a\left(-\left(a^2\left(i\left(2ia\left(\frac{i\arctan(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-\frac{1}{2}i\int\frac{\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{a^2x^2+1}dx\right.\right.\right.\right.\right.\right.$$

$\downarrow$ 7164

$$c\left(-\frac{\arctan(ax)^3}{3x^3}+a\left(-\left(a^2\left(i\left(2ia\left(\frac{i\arctan(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-\frac{\operatorname{PolyLog}\left(3,\frac{2}{1-iax}-1\right)}{4a}\right)-i\arg\right.\right.\right.\right.\right.$$
$$a^2c\left(-\frac{\arctan(ax)^3}{x}+3a\left(i\left(2ia\left(\frac{i\arctan(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-\frac{\operatorname{PolyLog}\left(3,\frac{2}{1-iax}-1\right)}{4a}\right)-i\arctan(\right.\right.\right.$$

input
```
Int[((c + a^2*c*x^2)*ArcTan[a*x]^3)/x^4,x]
```

output
```
a^2*c*(-(ArcTan[a*x]^3/x) + 3*a*((-1/3*I)*ArcTan[a*x]^3 + I*((-I)*ArcTan[a
*x]^2*Log[2 - 2/(1 - I*a*x)] + (2*I)*a*(((I/2)*ArcTan[a*x]*PolyLog[2, -1 +
 2/(1 - I*a*x)])/a - PolyLog[3, -1 + 2/(1 - I*a*x)]/(4*a))))) + c*(-1/3*Ar
cTan[a*x]^3/x^3 + a*(-1/2*ArcTan[a*x]^2/x^2 + a*(-(ArcTan[a*x]/x) - (a*Arc
Tan[a*x]^2)/2 + (a*(Log[x^2] - Log[1 + a^2*x^2]))/2) - a^2*((-1/3*I)*ArcTa
n[a*x]^3 + I*((-I)*ArcTan[a*x]^2*Log[2 - 2/(1 - I*a*x)] + (2*I)*a*(((I/2)*
ArcTan[a*x]*PolyLog[2, -1 + 2/(1 - I*a*x)])/a - PolyLog[3, -1 + 2/(1 - I*a
*x)]/(4*a))))))
```

## Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5527
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 188.51 (sec) , antiderivative size = 1848, normalized size of antiderivative = 9.78

| method | result | size |
|---|---|---|
| derivativedivides | Expression too large to display | 1848 |
| default | Expression too large to display | 1848 |
| parts | Expression too large to display | 1850 |

input
```
int((a^2*c*x^2+c)*arctan(a*x)^3/x^4,x,method=_RETURNVERBOSE)
```

output
```
a^3*(-c*arctan(a*x)^3/a/x-1/3*c*arctan(a*x)^3/a^3/x^3-c*(1/2*I*Pi*csgn(I/(
(1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*
a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*arctan(a*x)^2-I*Pi*csgn(
I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*
((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*arctan(a*x)^2+1/2
*arctan(a*x)^2-ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-1)-ln(1+(1+I*a*x)/(a^2*x^2+1
)^(1/2))-4*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))-4*polylog(3,-(1+I*a*x)/(
a^2*x^2+1)^(1/2))+I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*
x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2-2*arctan(
a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2))+1/2*I*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+
1)^(1/2))^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*arctan(a*x)^2-1/2*I*Pi*csgn(I/
((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2
/(a^2*x^2+1)+1)^2)^2*arctan(a*x)^2-1/2*I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)
)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2*arctan(a
*x)^2-I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))
*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*arctan(a*x)
^2+I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1
)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2-1/2*I*Pi*csgn(I*((1+I*a*
x)^2/(a^2*x^2+1)+1))^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*arctan(a*x)^2
-I*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1...
```

## Fricas [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)^3}{x^4}\,dx = \int \frac{(a^2cx^2+c)\arctan(ax)^3}{x^4}\,dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^3/x^4,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)*arctan(a*x)^3/x^4, x)
```

## Sympy [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)^3}{x^4}\,dx = c\left(\int \frac{\operatorname{atan}^3(ax)}{x^4}\,dx + \int \frac{a^2\operatorname{atan}^3(ax)}{x^2}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)*atan(a*x)**3/x**4,x)
```

output
```
c*(Integral(atan(a*x)**3/x**4, x) + Integral(a**2*atan(a*x)**3/x**2, x))
```

## Maxima [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)^3}{x^4}\,dx = \int \frac{(a^2cx^2+c)\arctan(ax)^3}{x^4}\,dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^3/x^4,x, algorithm="maxima")
```

output
```
1/96*(3*(7*a^3*c*arctan(a*x)^4 + 96*a^4*c*integrate(1/32*x^4*arctan(a*x)*l
og(a^2*x^2 + 1)^2/(a^2*x^6 + x^4), x) - 384*a^4*c*integrate(1/32*x^4*arcta
n(a*x)*log(a^2*x^2 + 1)/(a^2*x^6 + x^4), x) + 384*a^3*c*integrate(1/32*x^3
*arctan(a*x)^2/(a^2*x^6 + x^4), x) - 96*a^3*c*integrate(1/32*x^3*log(a^2*x
^2 + 1)^2/(a^2*x^6 + x^4), x) + 1792*a^2*c*integrate(1/32*x^2*arctan(a*x)^
3/(a^2*x^6 + x^4), x) + 192*a^2*c*integrate(1/32*x^2*arctan(a*x)*log(a^2*x
^2 + 1)^2/(a^2*x^6 + x^4), x) - 128*a^2*c*integrate(1/32*x^2*arctan(a*x)*l
og(a^2*x^2 + 1)/(a^2*x^6 + x^4), x) + 128*a*c*integrate(1/32*x*arctan(a*x)
^2/(a^2*x^6 + x^4), x) - 32*a*c*integrate(1/32*x*log(a^2*x^2 + 1)^2/(a^2*x
^6 + x^4), x) + 896*c*integrate(1/32*arctan(a*x)^3/(a^2*x^6 + x^4), x) + 9
6*c*integrate(1/32*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*x^6 + x^4), x))*x^3
 - 4*(3*a^2*c*x^2 + c)*arctan(a*x)^3 + 3*(3*a^2*c*x^2 + c)*arctan(a*x)*log
(a^2*x^2 + 1)^2)/x^3
```

## Giac [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)^3}{x^4}\,dx=\int \frac{(a^2cx^2+c)\arctan{(ax)}^3}{x^4}\,dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^3/x^4,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)*arctan(a*x)^3/x^4, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)\arctan(ax)^3}{x^4}\,dx=\int \frac{\operatorname{atan}(a\,x)^3\,(c\,a^2\,x^2+c)}{x^4}\,dx$$

input
```
int((atan(a*x)^3*(c + a^2*c*x^2))/x^4,x)
```

output
```
int((atan(a*x)^3*(c + a^2*c*x^2))/x^4, x)
```

## Reduce [F]

$$\int \frac{(c+a^2cx^2)\arctan(ax)^3}{x^4}\,dx$$
$$=\frac{c\left(-6atan(ax)^3a^2x^2-2atan(ax)^3-9atan(ax)^2a^3x^3-9atan(ax)^2ax-18atan(ax)a^2x^2-12\left(\int \frac{atan(c}{a^2x^5+}\right.\right.}{6x^3}$$

input
```
int((a^2*c*x^2+c)*atan(a*x)^3/x^4,x)
```

output
```
(c*( - 6*atan(a*x)**3*a**2*x**2 - 2*atan(a*x)**3 - 9*atan(a*x)**2*a**3*x**
3 - 9*atan(a*x)**2*a*x - 18*atan(a*x)*a**2*x**2 - 12*int(atan(a*x)**2/(a**
2*x**5 + x**3),x)*a*x**3 - 9*log(a**2*x**2 + 1)*a**3*x**3 + 18*log(x)*a**3
*x**3))/(6*x**3)
```

# 3.371 $\int x^3(c+a^2cx^2)^2 \arctan(ax)^3\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 313

$$\begin{aligned}\int x^3\left(c+a^2cx^2\right)^2\arctan(ax)^3\,dx &= \frac{c^2x}{21a^3}-\frac{c^2x^3}{168a}-\frac{1}{280}ac^2x^5-\frac{c^2\arctan(ax)}{21a^4}\\&\quad-\frac{5c^2x^2\arctan(ax)}{168a^2}+\frac{1}{28}c^2x^4\arctan(ax)\\&\quad+\frac{1}{56}a^2c^2x^6\arctan(ax)+\frac{2ic^2\arctan(ax)^2}{21a^4}\\&\quad+\frac{c^2x\arctan(ax)^2}{8a^3}-\frac{c^2x^3\arctan(ax)^2}{24a}\\&\quad-\frac{1}{8}ac^2x^5\arctan(ax)^2-\frac{3}{56}a^3c^2x^7\arctan(ax)^2\\&\quad-\frac{c^2\arctan(ax)^3}{24a^4}+\frac{1}{4}c^2x^4\arctan(ax)^3\\&\quad+\frac{1}{3}a^2c^2x^6\arctan(ax)^3+\frac{1}{8}a^4c^2x^8\arctan(ax)^3\\&\quad+\frac{4c^2\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{21a^4}\\&\quad+\frac{2ic^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{21a^4}\end{aligned}$$

output
```
1/21*c^2*x/a^3-1/168*c^2*x^3/a-1/280*a*c^2*x^5-1/21*c^2*arctan(a*x)/a^4-5/
168*c^2*x^2*arctan(a*x)/a^2+1/28*c^2*x^4*arctan(a*x)+1/56*a^2*c^2*x^6*arct
an(a*x)+2/21*I*c^2*polylog(2,1-2/(1+I*a*x))/a^4+1/8*c^2*x*arctan(a*x)^2/a^
3-1/24*c^2*x^3*arctan(a*x)^2/a-1/8*a*c^2*x^5*arctan(a*x)^2-3/56*a^3*c^2*x^
7*arctan(a*x)^2-1/24*c^2*arctan(a*x)^3/a^4+1/4*c^2*x^4*arctan(a*x)^3+1/3*a
^2*c^2*x^6*arctan(a*x)^3+1/8*a^4*c^2*x^8*arctan(a*x)^3+4/21*c^2*arctan(a*x
)*ln(2/(1+I*a*x))/a^4+2/21*I*c^2*arctan(a*x)^2/a^4
```

## Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.53

$$\int x^3 \left(c+a^2cx^2\right)^2 \arctan(ax)^3 \, dx$$

$$= \frac{c^2\Big(40ax - 5a^3x^3 - 3a^5x^5 - 5(16i - 21ax + 7a^3x^3 + 21a^5x^5 + 9a^7x^7)\arctan(ax)^2 + 35(1+a^2x^2)^3 (-1}{}$$

input
```
Integrate[x^3*(c + a^2*c*x^2)^2*ArcTan[a*x]^3,x]
```

output
```
(c^2*(40*a*x - 5*a^3*x^3 - 3*a^5*x^5 - 5*(16*I - 21*a*x + 7*a^3*x^3 + 21*a
^5*x^5 + 9*a^7*x^7)*ArcTan[a*x]^2 + 35*(1 + a^2*x^2)^3*(-1 + 3*a^2*x^2)*Ar
cTan[a*x]^3 + 5*ArcTan[a*x]*(-8 - 5*a^2*x^2 + 6*a^4*x^4 + 3*a^6*x^6 + 32*L
og[1 + E^((2*I)*ArcTan[a*x])]) - (80*I)*PolyLog[2, -E^((2*I)*ArcTan[a*x])]
))/(840*a^4)
```

## Rubi [A] (verified)

Time = 2.49 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \arctan(ax)^3 \left(a^2cx^2+c\right)^2 \, dx$$

$$\Big\downarrow \text{5483}$$

$$\int \left(a^4c^2x^7 \arctan(ax)^3 + 2a^2c^2x^5 \arctan(ax)^3 + c^2x^3 \arctan(ax)^3\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{8}a^4c^2x^8 \arctan(ax)^3 - \frac{c^2 \arctan(ax)^3}{24a^4} + \frac{2ic^2 \arctan(ax)^2}{21a^4} - \frac{c^2 \arctan(ax)}{21a^4} + \frac{4c^2 \arctan(ax) \log\left(\frac{2}{1+iax}\right)}{21a^4} + \frac{2ic^2 \operatorname{PolyLog}\left(2, 1 - \frac{2}{iax+1}\right)}{21a^4} - \frac{3}{56}a^3c^2x^7 \arctan(ax)^2 + \frac{c^2x \arctan(ax)^2}{8a^3} + \frac{c^2x}{21a^3} + \frac{1}{3}a^2c^2x^6 \arctan(ax)^3 + \frac{1}{56}a^2c^2x^6 \arctan(ax) - \frac{5c^2x^2 \arctan(ax)}{168a^2} - \frac{1}{8}ac^2x^5 \arctan(ax)^2 + \frac{1}{4}c^2x^4 \arctan(ax)^3 + \frac{1}{28}c^2x^4 \arctan(ax) - \frac{c^2x^3 \arctan(ax)^2}{24a} - \frac{1}{280}ac^2x^5 - \frac{c^2x^3}{168a}$$

input
```
Int[x^3*(c + a^2*c*x^2)^2*ArcTan[a*x]^3,x]
```

output
```
(c^2*x)/(21*a^3) - (c^2*x^3)/(168*a) - (a*c^2*x^5)/280 - (c^2*ArcTan[a*x])
/(21*a^4) - (5*c^2*x^2*ArcTan[a*x])/(168*a^2) + (c^2*x^4*ArcTan[a*x])/28 +
 (a^2*c^2*x^6*ArcTan[a*x])/56 + (((2*I)/21)*c^2*ArcTan[a*x]^2)/a^4 + (c^2*
x*ArcTan[a*x]^2)/(8*a^3) - (c^2*x^3*ArcTan[a*x]^2)/(24*a) - (a*c^2*x^5*Arc
Tan[a*x]^2)/8 - (3*a^3*c^2*x^7*ArcTan[a*x]^2)/56 - (c^2*ArcTan[a*x]^3)/(24
*a^4) + (c^2*x^4*ArcTan[a*x]^3)/4 + (a^2*c^2*x^6*ArcTan[a*x]^3)/3 + (a^4*c
^2*x^8*ArcTan[a*x]^3)/8 + (4*c^2*ArcTan[a*x]*Log[2/(1 + I*a*x)])/(21*a^4)
+ (((2*I)/21)*c^2*PolyLog[2, 1 - 2/(1 + I*a*x)])/a^4
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

## Maple [A] (verified)

Time = 5.51 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.06

| method | result |
|---|---|
| derivativedivides | $\frac{c^2\arctan(ax)^3a^8x^8}{8}+\frac{c^2\arctan(ax)^3a^6x^6}{3}+\frac{a^4c^2x^4\arctan(ax)^3}{4}-\frac{c^2\arctan(ax)^3}{24}-\frac{c^2\left(\frac{3\arctan(ax)^2a^7x^7}{7}+a^5\arctan(ax)^2x^5+\right.}{}$ |
| default | $\frac{c^2\arctan(ax)^3a^8x^8}{8}+\frac{c^2\arctan(ax)^3a^6x^6}{3}+\frac{a^4c^2x^4\arctan(ax)^3}{4}-\frac{c^2\arctan(ax)^3}{24}-\frac{c^2\left(\frac{3\arctan(ax)^2a^7x^7}{7}+a^5\arctan(ax)^2x^5+\right.}{}$ |
| parts | $\frac{a^4c^2x^8\arctan(ax)^3}{8}+\frac{a^2c^2x^6\arctan(ax)^3}{3}+\frac{c^2x^4\arctan(ax)^3}{4}-\frac{c^2\left(\frac{3a^3\arctan(ax)^2x^7}{7}+a\arctan(ax)^2x^5+\right.}{}$ |

input
```
int(x^3*(a^2*c*x^2+c)^2*arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a^4*(1/8*c^2*arctan(a*x)^3*a^8*x^8+1/3*c^2*arctan(a*x)^3*a^6*x^6+1/4*a^4
*c^2*x^4*arctan(a*x)^3-1/24*c^2*arctan(a*x)^3-1/8*c^2*(3/7*arctan(a*x)^2*a
^7*x^7+a^5*arctan(a*x)^2*x^5+1/3*a^3*arctan(a*x)^2*x^3-a*arctan(a*x)^2*x-1
/7*a^6*arctan(a*x)*x^6-2/7*x^4*arctan(a*x)*a^4+5/21*x^2*a^2*arctan(a*x)+16
/21*arctan(a*x)*ln(a^2*x^2+1)+1/35*a^5*x^5+1/21*a^3*x^3-8/21*a*x+8/21*arct
an(a*x)+8/21*I*(ln(a*x-I)*ln(a^2*x^2+1)-1/2*ln(a*x-I)^2-dilog(-1/2*I*(a*x+
I))-ln(a*x-I)*ln(-1/2*I*(a*x+I)))-8/21*I*(ln(a*x+I)*ln(a^2*x^2+1)-1/2*ln(a
*x+I)^2-dilog(1/2*I*(a*x-I))-ln(a*x+I)*ln(1/2*I*(a*x-I)))))
```

## Fricas [F]

$$\int x^3\left(c+a^2cx^2\right)^2\arctan(ax)^3\,dx=\int\left(a^2cx^2+c\right)^2x^3\arctan\left(ax\right)^3\,dx$$

input
```
integrate(x^3*(a^2*c*x^2+c)^2*arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^7 + 2*a^2*c^2*x^5 + c^2*x^3)*arctan(a*x)^3, x)
```

## Sympy [F]

$$\int x^3 (c + a^2 c x^2)^2 \arctan(ax)^3 \, dx = c^2 \left( \int x^3 \operatorname{atan}^3(ax) \, dx + \int 2a^2 x^5 \operatorname{atan}^3(ax) \, dx + \int a^4 x^7 \operatorname{atan}^3(ax) \, dx \right)$$

input
```
integrate(x**3*(a**2*c*x**2+c)**2*atan(a*x)**3,x)
```

output
```
c**2*(Integral(x**3*atan(a*x)**3, x) + Integral(2*a**2*x**5*atan(a*x)**3,
x) + Integral(a**4*x**7*atan(a*x)**3, x))
```

## Maxima [F]

$$\int x^3 (c + a^2 c x^2)^2 \arctan(ax)^3 \, dx = \int (a^2 c x^2 + c)^2 x^3 \arctan(ax)^3 \, dx$$

input
```
integrate(x^3*(a^2*c*x^2+c)^2*arctan(a*x)^3,x, algorithm="maxima")
```

output
```
1/2688*(28*(129024*a^9*c^2*integrate(1/2688*x^9*arctan(a*x)^3/(a^5*x^2 + a
^3), x) - 24192*a^8*c^2*integrate(1/2688*x^8*arctan(a*x)^2/(a^5*x^2 + a^3)
, x) - 6048*a^8*c^2*integrate(1/2688*x^8*log(a^2*x^2 + 1)^2/(a^5*x^2 + a^3
), x) - 3456*a^8*c^2*integrate(1/2688*x^8*log(a^2*x^2 + 1)/(a^5*x^2 + a^3)
, x) + 387072*a^7*c^2*integrate(1/2688*x^7*arctan(a*x)^3/(a^5*x^2 + a^3),
x) + 6912*a^7*c^2*integrate(1/2688*x^7*arctan(a*x)/(a^5*x^2 + a^3), x) - 6
4512*a^6*c^2*integrate(1/2688*x^6*arctan(a*x)^2/(a^5*x^2 + a^3), x) - 1612
8*a^6*c^2*integrate(1/2688*x^6*log(a^2*x^2 + 1)^2/(a^5*x^2 + a^3), x) - 80
64*a^6*c^2*integrate(1/2688*x^6*log(a^2*x^2 + 1)/(a^5*x^2 + a^3), x) + 387
072*a^5*c^2*integrate(1/2688*x^5*arctan(a*x)^3/(a^5*x^2 + a^3), x) + 16128
*a^5*c^2*integrate(1/2688*x^5*arctan(a*x)/(a^5*x^2 + a^3), x) - 48384*a^4*
c^2*integrate(1/2688*x^4*arctan(a*x)^2/(a^5*x^2 + a^3), x) - 12096*a^4*c^2
*integrate(1/2688*x^4*log(a^2*x^2 + 1)^2/(a^5*x^2 + a^3), x) - 2688*a^4*c^
2*integrate(1/2688*x^4*log(a^2*x^2 + 1)/(a^5*x^2 + a^3), x) + 129024*a^3*c
^2*integrate(1/2688*x^3*arctan(a*x)^3/(a^5*x^2 + a^3), x) + 5376*a^3*c^2*i
ntegrate(1/2688*x^3*arctan(a*x)/(a^5*x^2 + a^3), x) + 8064*a^2*c^2*integra
te(1/2688*x^2*log(a^2*x^2 + 1)/(a^5*x^2 + a^3), x) - 16128*a*c^2*integrate
(1/2688*x*arctan(a*x)/(a^5*x^2 + a^3), x) + 2016*c^2*integrate(1/2688*log(
a^2*x^2 + 1)^2/(a^5*x^2 + a^3), x) + c^2*arctan(a*x)^3/a^4)*a^4 + 56*(3*a^
8*c^2*x^8 + 8*a^6*c^2*x^6 + 6*a^4*c^2*x^4 - c^2)*arctan(a*x)^3 - 4*(9*a...
```

## Giac [F]

$$\int x^3 \left(c+a^2cx^2\right)^2 \arctan(ax)^3 \, dx = \int \left(a^2cx^2+c\right)^2 x^3 \arctan\left(ax\right)^3 \, dx$$

input
```
integrate(x^3*(a^2*c*x^2+c)^2*arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^2*x^3*arctan(a*x)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int x^3(c+a^2cx^2)^2 \arctan(ax)^3\,dx = \int x^3\,\mathrm{atan}(a\,x)^3\,\left(c\,a^2\,x^2+c\right)^2\,dx$$

input
```
int(x^3*atan(a*x)^3*(c + a^2*c*x^2)^2,x)
```

output
```
int(x^3*atan(a*x)^3*(c + a^2*c*x^2)^2, x)
```

## Reduce [F]

$$\int x^3(c+a^2cx^2)^2 \arctan(ax)^3\,dx = \frac{c^2\Big(105atan(ax)^3\,a^8x^8+280atan(ax)^3\,a^6x^6+210atan(ax)^3\,a^4x^4-35atan(ax)^3-45atan(ax)^2\,a^7x^7-1$$

input
```
int(x^3*(a^2*c*x^2+c)^2*atan(a*x)^3,x)
```

output
```
(c**2*(105*atan(a*x)**3*a**8*x**8 + 280*atan(a*x)**3*a**6*x**6 + 210*atan(
a*x)**3*a**4*x**4 - 35*atan(a*x)**3 - 45*atan(a*x)**2*a**7*x**7 - 105*atan
(a*x)**2*a**5*x**5 - 35*atan(a*x)**2*a**3*x**3 + 105*atan(a*x)**2*a*x + 15
*atan(a*x)*a**6*x**6 + 30*atan(a*x)*a**4*x**4 - 25*atan(a*x)*a**2*x**2 - 4
0*atan(a*x) - 160*int((atan(a*x)*x)/(a**2*x**2 + 1),x)*a**2 - 3*a**5*x**5
- 5*a**3*x**3 + 40*a*x))/(840*a**4)
```

# 3.372 $\int x^2(c+a^2cx^2)^2 \arctan(ax)^3\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 321

$$\begin{aligned}\int x^2(c+a^2cx^2)^2\arctan(ax)^3\,dx =& -\frac{11c^2x^2}{420a}-\frac{1}{140}ac^2x^4-\frac{c^2x\arctan(ax)}{70a^2}\\&+\frac{17}{210}c^2x^3\arctan(ax)+\frac{1}{35}a^2c^2x^5\arctan(ax)\\&+\frac{c^2\arctan(ax)^2}{140a^3}-\frac{4c^2x^2\arctan(ax)^2}{35a}\\&-\frac{27}{140}ac^2x^4\arctan(ax)^2-\frac{1}{14}a^3c^2x^6\arctan(ax)^2\\&-\frac{8ic^2\arctan(ax)^3}{105a^3}+\frac{1}{3}c^2x^3\arctan(ax)^3\\&+\frac{2}{5}a^2c^2x^5\arctan(ax)^3+\frac{1}{7}a^4c^2x^7\arctan(ax)^3\\&-\frac{8c^2\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{35a^3}+\frac{c^2\log(1+a^2x^2)}{30a^3}\\&-\frac{8ic^2\arctan(ax)\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{35a^3}\\&-\frac{4c^2\operatorname{PolyLog}\left(3,1-\frac{2}{1+iax}\right)}{35a^3}\end{aligned}$$

output
```
-11/420*c^2*x^2/a-1/140*a*c^2*x^4-1/70*c^2*x*arctan(a*x)/a^2+17/210*c^2*x^
3*arctan(a*x)+1/35*a^2*c^2*x^5*arctan(a*x)+1/140*c^2*arctan(a*x)^2/a^3-4/3
5*c^2*x^2*arctan(a*x)^2/a-27/140*a*c^2*x^4*arctan(a*x)^2-1/14*a^3*c^2*x^6*
arctan(a*x)^2-8/105*I*c^2*arctan(a*x)^3/a^3+1/3*c^2*x^3*arctan(a*x)^3+2/5*
a^2*c^2*x^5*arctan(a*x)^3+1/7*a^4*c^2*x^7*arctan(a*x)^3-8/35*c^2*arctan(a*
x)^2*ln(2/(1+I*a*x))/a^3+1/30*c^2*ln(a^2*x^2+1)/a^3-8/35*I*c^2*arctan(a*x)
*polylog(2,1-2/(1+I*a*x))/a^3-4/35*c^2*polylog(3,1-2/(1+I*a*x))/a^3
```

## Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.73

$$\int x^2\left(c+a^2cx^2\right)^2\arctan(ax)^3\,dx$$
$$=\frac{c^2\left(-8-11a^2x^2-3a^4x^4-6ax\arctan(ax)+34a^3x^3\arctan(ax)+12a^5x^5\arctan(ax)+3\arctan(ax)^2-\right.}{}$$

input
```
Integrate[x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^3,x]
```

output
```
(c^2*(-8 - 11*a^2*x^2 - 3*a^4*x^4 - 6*a*x*ArcTan[a*x] + 34*a^3*x^3*ArcTan[
a*x] + 12*a^5*x^5*ArcTan[a*x] + 3*ArcTan[a*x]^2 - 48*a^2*x^2*ArcTan[a*x]^2
 - 81*a^4*x^4*ArcTan[a*x]^2 - 30*a^6*x^6*ArcTan[a*x]^2 + (32*I)*ArcTan[a*x
]^3 + 140*a^3*x^3*ArcTan[a*x]^3 + 168*a^5*x^5*ArcTan[a*x]^3 + 60*a^7*x^7*A
rcTan[a*x]^3 - 96*ArcTan[a*x]^2*Log[1 + E^((2*I)*ArcTan[a*x])] + 14*Log[1
+ a^2*x^2] + (96*I)*ArcTan[a*x]*PolyLog[2, -E^((2*I)*ArcTan[a*x])] - 48*Po
lyLog[3, -E^((2*I)*ArcTan[a*x])]))/(420*a^3)
```

## Rubi [A] (verified)

Time = 2.01 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arctan(ax)^3 \left(a^2cx^2+c\right)^2 \, dx$$

$$\Big\downarrow \; 5483$$

$$\int \left(a^4c^2x^6 \arctan(ax)^3 + 2a^2c^2x^4 \arctan(ax)^3 + c^2x^2 \arctan(ax)^3\right) dx$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{7}a^4c^2x^7 \arctan(ax)^3 - \frac{8ic^2 \arctan(ax) \operatorname{PolyLog}\left(2, 1-\frac{2}{iax+1}\right)}{35a^3} - \frac{1}{14}a^3c^2x^6 \arctan(ax)^2 - \frac{8ic^2 \arctan(ax)^3}{105a^3} + \frac{c^2 \arctan(ax)^2}{140a^3} - \frac{8c^2 \arctan(ax)^2 \log\left(\frac{2}{1+iax}\right)}{35a^3} - \frac{4c^2 \operatorname{PolyLog}\left(3, 1-\frac{2}{iax+1}\right)}{35a^3} + \frac{2}{5}a^2c^2x^5 \arctan(ax)^3 + \frac{1}{35}a^2c^2x^5 \arctan(ax) - \frac{c^2x \arctan(ax)}{70a^2} + \frac{c^2 \log\left(a^2x^2+1\right)}{30a^3} - \frac{27}{140}ac^2x^4 \arctan(ax)^2 + \frac{1}{3}c^2x^3 \arctan(ax)^3 + \frac{17}{210}c^2x^3 \arctan(ax) - \frac{4c^2x^2 \arctan(ax)^2}{35a} - \frac{1}{140}ac^2x^4 - \frac{11c^2x^2}{420a}$$

input
```
Int[x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^3,x]
```

output
```
(-11*c^2*x^2)/(420*a) - (a*c^2*x^4)/140 - (c^2*x*ArcTan[a*x])/(70*a^2) + (
17*c^2*x^3*ArcTan[a*x])/210 + (a^2*c^2*x^5*ArcTan[a*x])/35 + (c^2*ArcTan[a
*x]^2)/(140*a^3) - (4*c^2*x^2*ArcTan[a*x]^2)/(35*a) - (27*a*c^2*x^4*ArcTan
[a*x]^2)/140 - (a^3*c^2*x^6*ArcTan[a*x]^2)/14 - (((8*I)/105)*c^2*ArcTan[a*
x]^3)/a^3 + (c^2*x^3*ArcTan[a*x]^3)/3 + (2*a^2*c^2*x^5*ArcTan[a*x]^3)/5 +
(a^4*c^2*x^7*ArcTan[a*x]^3)/7 - (8*c^2*ArcTan[a*x]^2*Log[2/(1 + I*a*x)])/(
35*a^3) + (c^2*Log[1 + a^2*x^2])/(30*a^3) - (((8*I)/35)*c^2*ArcTan[a*x]*Po
lyLog[2, 1 - 2/(1 + I*a*x)])/a^3 - (4*c^2*PolyLog[3, 1 - 2/(1 + I*a*x)])/(
35*a^3)
```

**Defintions of rubi rules used**

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 90.18 (sec) , antiderivative size = 1256, normalized size of antiderivative = 3.91

| method | result | size |
|---|---|---|
| derivativedivides | Expression too large to display | 1256 |
| default | Expression too large to display | 1256 |
| parts | Expression too large to display | 1256 |

input
```
int(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(1/7*c^2*arctan(a*x)^3*a^7*x^7+2/5*c^2*arctan(a*x)^3*a^5*x^5+1/3*c^2
*arctan(a*x)^3*a^3*x^3-1/35*c^2*(-2*I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1
)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*
a*x)^2/(a^2*x^2+1)+1)^2)*arctan(a*x)^2-2*I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+
1))^3*arctan(a*x)^2+2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3*arctan(
a*x)^2-1/4*arctan(a*x)^2+4*arctan(a*x)^2*x^2*a^2+2*I*Pi*csgn(I/((1+I*a*x)^
2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1
)+1)^2)^2*arctan(a*x)^2+4*I*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*csgn(I*
(1+I*a*x)^2/(a^2*x^2+1))^2*arctan(a*x)^2-2*I*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+
1)^(1/2))^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*arctan(a*x)^2+1/4*(a*x+I)^4+2*
I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*
a*x)^2/(a^2*x^2+1)+1)^2)^2*arctan(a*x)^2+2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x
^2+1)+1))^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*arctan(a*x)^2-2*I*Pi*csg
n(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3*arctan(a*x)^2
+20*I*arctan(a*x)*(a*x-I)^3*(a*x+I)-30*I*arctan(a*x)*(a*x-I)^2*(a*x+I)^2+2
0*I*arctan(a*x)*(a*x-I)*(a*x+I)^3-3*I*arctan(a*x)*(a*x-I)*(a*x+I)-4*I*Pi*c
sgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2
*arctan(a*x)^2-8*I*arctan(a*x)*polylog(2,-(1+I*a*x)^2/(a^2*x^2+1))+27/4*a^
4*arctan(a*x)^2*x^4+5/2*a^6*x^6*arctan(a*x)^2-arctan(a*x)*(a*x-I)^5-I*(a*x
+I)^3-8/3*I*arctan(a*x)^3-5/6*I*(a*x+I)+4*arctan(a*x)*(a*x-I)+43/6*arct...
```

## Fricas [F]

$$\int x^2 \left(c+a^2cx^2\right)^2 \arctan(ax)^3\,dx=\int \left(a^2cx^2+c\right)^2 x^2 \arctan\left(ax\right)^3\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2)*arctan(a*x)^3, x)
```

## Sympy [F]

$$\int x^2 \left(c+a^2 c x^2\right)^2 \arctan(ax)^3 \, dx = c^2 \left(\int x^2 \operatorname{atan}^3(ax)\, dx + \int 2a^2 x^4 \operatorname{atan}^3(ax)\, dx + \int a^4 x^6 \operatorname{atan}^3(ax)\, dx\right)$$

input
```
integrate(x**2*(a**2*c*x**2+c)**2*atan(a*x)**3,x)
```

output
```
c**2*(Integral(x**2*atan(a*x)**3, x) + Integral(2*a**2*x**4*atan(a*x)**3,
x) + Integral(a**4*x**6*atan(a*x)**3, x))
```

## Maxima [F]

$$\int x^2 \left(c+a^2 c x^2\right)^2 \arctan(ax)^3 \, dx = \int \left(a^2 c x^2 + c\right)^2 x^2 \arctan(ax)^3 \, dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^3,x, algorithm="maxima")
```

output
```
1/840*(15*a^4*c^2*x^7 + 42*a^2*c^2*x^5 + 35*c^2*x^3)*arctan(a*x)^3 - 1/112
0*(15*a^4*c^2*x^7 + 42*a^2*c^2*x^5 + 35*c^2*x^3)*arctan(a*x)*log(a^2*x^2 +
 1)^2 + integrate(1/1120*(980*(a^6*c^2*x^8 + 3*a^4*c^2*x^6 + 3*a^2*c^2*x^4
 + c^2*x^2)*arctan(a*x)^3 - 4*(15*a^5*c^2*x^7 + 42*a^3*c^2*x^5 + 35*a*c^2*
x^3)*arctan(a*x)^2 + 4*(15*a^6*c^2*x^8 + 42*a^4*c^2*x^6 + 35*a^2*c^2*x^4)*
arctan(a*x)*log(a^2*x^2 + 1) + (15*a^5*c^2*x^7 + 42*a^3*c^2*x^5 + 35*a*c^2
*x^3 + 105*(a^6*c^2*x^8 + 3*a^4*c^2*x^6 + 3*a^2*c^2*x^4 + c^2*x^2)*arctan(
a*x))*log(a^2*x^2 + 1)^2)/(a^2*x^2 + 1), x)
```

## Giac [F]

$$\int x^2 \left(c+a^2 c x^2\right)^2 \arctan(ax)^3 \, dx = \int \left(a^2 c x^2 + c\right)^2 x^2 \arctan\left(ax\right)^3 \, dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^2*x^2*arctan(a*x)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int x^2 \left(c+a^2 c x^2\right)^2 \arctan(ax)^3 \, dx = \int x^2 \operatorname{atan}(a\,x)^3 \left(c\,a^2\,x^2+c\right)^2 dx$$

input
```
int(x^2*atan(a*x)^3*(c + a^2*c*x^2)^2,x)
```

output
```
int(x^2*atan(a*x)^3*(c + a^2*c*x^2)^2, x)
```

## Reduce [F]

$$\int x^2 \left(c+a^2 c x^2\right)^2 \arctan(ax)^3 \, dx = \frac{c^2 \left(60 atan(ax)^3 a^7 x^7 + 168 atan(ax)^3 a^5 x^5 + 140 atan(ax)^3 a^3 x^3 + 32 atan(ax)^3 ax - 30 atan(ax)^2 a^6 x^6 - \right.}{}$$

input
```
int(x^2*(a^2*c*x^2+c)^2*atan(a*x)^3,x)
```

output
```
(c**2*(60*atan(a*x)**3*a**7*x**7 + 168*atan(a*x)**3*a**5*x**5 + 140*atan(a
*x)**3*a**3*x**3 + 32*atan(a*x)**3*a*x - 30*atan(a*x)**2*a**6*x**6 - 81*at
an(a*x)**2*a**4*x**4 - 48*atan(a*x)**2*a**2*x**2 + 3*atan(a*x)**2 + 12*ata
n(a*x)*a**5*x**5 + 34*atan(a*x)*a**3*x**3 - 6*atan(a*x)*a*x - 32*int(atan(
a*x)**3,x)*a + 14*log(a**2*x**2 + 1) - 3*a**4*x**4 - 11*a**2*x**2))/(420*a
**3)
```

# 3.373 $\int x(c+a^2cx^2)^2 \arctan(ax)^3 \, dx$



## Optimal result

Integrand size = 20, antiderivative size = 242

$$\begin{aligned}\int x(c+a^2cx^2)^2 \arctan(ax)^3 \, dx = & -\frac{11c^2x}{60a} - \frac{1}{60}ac^2x^3 + \frac{2c^2(1+a^2x^2)\arctan(ax)}{15a^2} \\ & + \frac{c^2(1+a^2x^2)^2\arctan(ax)}{20a^2} - \frac{4ic^2\arctan(ax)^2}{15a^2} \\ & - \frac{4c^2x\arctan(ax)^2}{15a} - \frac{2c^2x(1+a^2x^2)\arctan(ax)^2}{15a} \\ & - \frac{c^2x(1+a^2x^2)^2\arctan(ax)^2}{10a} \\ & + \frac{c^2(1+a^2x^2)^3\arctan(ax)^3}{6a^2} \\ & - \frac{8c^2\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{15a^2} \\ & - \frac{4ic^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{15a^2}\end{aligned}$$

output

```
-11/60*c^2*x/a-1/60*a*c^2*x^3+2/15*c^2*(a^2*x^2+1)*arctan(a*x)/a^2+1/20*c^
2*(a^2*x^2+1)^2*arctan(a*x)/a^2-4/15*I*c^2*arctan(a*x)^2/a^2-4/15*c^2*x*ar
ctan(a*x)^2/a-2/15*c^2*x*(a^2*x^2+1)*arctan(a*x)^2/a-1/10*c^2*x*(a^2*x^2+1
)^2*arctan(a*x)^2/a+1/6*c^2*(a^2*x^2+1)^3*arctan(a*x)^3/a^2-8/15*c^2*arcta
n(a*x)*ln(2/(1+I*a*x))/a^2-4/15*I*c^2*polylog(2,1-2/(1+I*a*x))/a^2
```

## Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.54

$$\int x(c+a^2cx^2)^2\arctan(ax)^3\,dx$$
$$=\frac{c^2\left(-ax(11+a^2x^2)-2(-8i+15ax+10a^3x^3+3a^5x^5)\arctan(ax)^2+10(1+a^2x^2)^3\arctan(ax)^3+\operatorname{arct}\right.}{60a^2}$$

input
```
Integrate[x*(c + a^2*c*x^2)^2*ArcTan[a*x]^3,x]
```

output
```
(c^2*(-(a*x*(11 + a^2*x^2)) - 2*(-8*I + 15*a*x + 10*a^3*x^3 + 3*a^5*x^5)*A
rcTan[a*x]^2 + 10*(1 + a^2*x^2)^3*ArcTan[a*x]^3 + ArcTan[a*x]*(11 + 14*a^2
*x^2 + 3*a^4*x^4 - 32*Log[1 + E^((2*I)*ArcTan[a*x])]) + (16*I)*PolyLog[2,
-E^((2*I)*ArcTan[a*x])]))/(60*a^2)
```

## Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.96, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.550, Rules used = {5465, 27, 5415, 2009, 5415, 24, 5345, 5455, 5379, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\arctan(ax)^3\left(a^2cx^2+c\right)^2\,dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{c^2\left(a^2x^2+1\right)^3\arctan(ax)^3}{6a^2}-\frac{\int c^2\left(a^2x^2+1\right)^2\arctan(ax)^2dx}{2a}$$

$$\Big\downarrow \text{27}$$

$$\frac{c^2\left(a^2x^2+1\right)^3\arctan(ax)^3}{6a^2}-\frac{c^2\int\left(a^2x^2+1\right)^2\arctan(ax)^2dx}{2a}$$

$$\Big\downarrow \text{5415}$$

$$\frac{c^2\left(a^2x^2+1\right)^3\arctan(ax)^3}{6a^2}-\frac{c^2\left(\frac{4}{5}\int\left(a^2x^2+1\right)\arctan(ax)^2dx+\frac{1}{10}\int\left(a^2x^2+1\right)dx+\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan(ax)^2-\frac{\left(a^2x^2+1\right)^2\arctan(ax)}{10a}\right)}{2a}$$

$\big\downarrow$ 2009

$$\frac{c^2\left(a^2x^2+1\right)^3\arctan(ax)^3}{6a^2}-\frac{c^2\left(\frac{4}{5}\int\left(a^2x^2+1\right)\arctan(ax)^2dx+\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan(ax)^2-\frac{\left(a^2x^2+1\right)^2\arctan(ax)}{10a}+\frac{1}{10}\left(\frac{a^2x^3}{3}+x\right)\right)}{2a}$$

$\big\downarrow$ 5415

$$\frac{c^2\left(a^2x^2+1\right)^3\arctan(ax)^3}{6a^2}-\frac{c^2\left(\frac{4}{5}\left(\frac{2}{3}\int\arctan(ax)^2dx+\frac{\int 1dx}{3}+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)^2-\frac{\left(a^2x^2+1\right)\arctan(ax)}{3a}\right)+\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan(ax)\right.}{2a}$$

$\big\downarrow$ 24

$$\frac{c^2\left(a^2x^2+1\right)^3\arctan(ax)^3}{6a^2}-\frac{c^2\left(\frac{4}{5}\left(\frac{2}{3}\int\arctan(ax)^2dx+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)^2-\frac{\left(a^2x^2+1\right)\arctan(ax)}{3a}+\frac{x}{3}\right)+\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan(ax)^2-\right.}{2a}$$

$\big\downarrow$ 5345

$$\frac{c^2\left(a^2x^2+1\right)^3\arctan(ax)^3}{6a^2}-\frac{c^2\left(\frac{4}{5}\left(\frac{2}{3}\left(x\arctan(ax)^2-2a\int\frac{x\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)^2-\frac{\left(a^2x^2+1\right)\arctan(ax)}{3a}+\frac{x}{3}\right)+\frac{1}{5}x\left(a^2x^2\right.\right.}{2a}$$

$\big\downarrow$ 5455

$$\frac{c^2\left(a^2x^2+1\right)^3\arctan(ax)^3}{6a^2}-\frac{c^2\left(\frac{4}{5}\left(\frac{2}{3}\left(x\arctan(ax)^2-2a\left(-\frac{\int\frac{\arctan(ax)}{i-ax}dx}{a}-\frac{i\arctan(ax)^2}{2a^2}\right)\right)+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)^2-\frac{\left(a^2x^2+1\right)\arctan(ax)}{3a}\right.\right.}{2a}$$

$\big\downarrow$ 5379

$$\frac{c^2\left(a^2x^2+1\right)^3\arctan(ax)^3}{6a^2}-\frac{c^2\left(\frac{4}{5}\left(\frac{2}{3}\left(x\arctan(ax)^2-2a\left(-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}-\int\frac{\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^2}{2a^2}\right)\right)\right)+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)^2\right.}{2a}$$

$\downarrow$ 2849

$$\frac{c^2\left(a^2x^2+1\right)^3\arctan(ax)^3}{6a^2}-\frac{c^2\left(\frac{4}{5}\left(\frac{2}{3}\left(x\arctan(ax)^2-2a\left(-\frac{\frac{i\int\frac{\log\left(\frac{2}{iax+1}\right)}{1-\frac{2}{iax+1}}d\frac{1}{iax+1}}{a}+\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}}{a}-\frac{i\arctan(ax)^2}{2a^2}\right)\right)\right)+\frac{1}{3}x\left(a^2x^2+1\right)\arctan\right.}{2a}$$

$\downarrow$ 2752

$$\frac{c^2\left(a^2x^2+1\right)^3\arctan(ax)^3}{6a^2}-\frac{c^2\left(\frac{4}{5}\left(\frac{2}{3}\left(x\arctan(ax)^2-2a\left(-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}\right)\right)\right)+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(a\right.}{2a}$$

input
```
Int[x*(c + a^2*c*x^2)^2*ArcTan[a*x]^3,x]
```

output
```
(c^2*(1 + a^2*x^2)^3*ArcTan[a*x]^3)/(6*a^2) - (c^2*((x + (a^2*x^3)/3)/10 -
 ((1 + a^2*x^2)^2*ArcTan[a*x])/(10*a) + (x*(1 + a^2*x^2)^2*ArcTan[a*x]^2)/
5 + (4*(x/3 - ((1 + a^2*x^2)*ArcTan[a*x])/(3*a) + (x*(1 + a^2*x^2)*ArcTan[
a*x]^2)/3 + (2*(x*ArcTan[a*x]^2 - 2*a*(((-1/2*I)*ArcTan[a*x]^2)/a^2 - ((Ar
cTan[a*x]*Log[2/(1 + I*a*x)])/a + ((I/2)*PolyLog[2, 1 - 2/(1 + I*a*x)])/a)
/a)))/3))/5))/(2*a)
```

### Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5415
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTan[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x])^p,
x], x] + Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*(
a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[q, 0] && GtQ[p, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

## Maple [A] (verified)

Time = 5.71 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.22

| method | result |
|---|---|
| parts | $\frac{c^2\arctan(ax)^3a^4x^6}{6}+\frac{c^2\arctan(ax)^3a^2x^4}{2}+\frac{c^2\arctan(ax)^3x^2}{2}+\frac{c^2\arctan(ax)^3}{6a^2}-\frac{c^2\left(\frac{a^5\arctan(ax)^2x^5}{5}+\frac{2a^3 a}{}\right.}{}$ |
| derivativedivides | $\frac{\frac{c^2\arctan(ax)^3a^6x^6}{6}+\frac{a^4c^2x^4\arctan(ax)^3}{2}+\frac{a^2c^2x^2\arctan(ax)^3}{2}+\frac{c^2\arctan(ax)^3}{6}-\frac{c^2\left(\frac{a^5\arctan(ax)^2x^5}{5}+\frac{2a^3\arctan(ax)^2x^3}{3}+\right.}{}}{}$ |
| default | $\frac{\frac{c^2\arctan(ax)^3a^6x^6}{6}+\frac{a^4c^2x^4\arctan(ax)^3}{2}+\frac{a^2c^2x^2\arctan(ax)^3}{2}+\frac{c^2\arctan(ax)^3}{6}-\frac{c^2\left(\frac{a^5\arctan(ax)^2x^5}{5}+\frac{2a^3\arctan(ax)^2x^3}{3}+\right.}{}}{}$ |

input
```
int(x*(a^2*c*x^2+c)^2*arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/6*c^2*arctan(a*x)^3*a^4*x^6+1/2*c^2*arctan(a*x)^3*a^2*x^4+1/2*c^2*arctan
(a*x)^3*x^2+1/6*c^2*arctan(a*x)^3/a^2-1/2/a^2*c^2*(1/5*a^5*arctan(a*x)^2*x
^5+2/3*a^3*arctan(a*x)^2*x^3+a*arctan(a*x)^2*x-1/10*x^4*arctan(a*x)*a^4-7/
15*x^2*a^2*arctan(a*x)-8/15*arctan(a*x)*ln(a^2*x^2+1)+1/30*a^3*x^3+11/30*a
*x-11/30*arctan(a*x)-4/15*I*(ln(a*x-I)*ln(a^2*x^2+1)-1/2*ln(a*x-I)^2-dilog
(-1/2*I*(a*x+I))-ln(a*x-I)*ln(-1/2*I*(a*x+I)))+4/15*I*(ln(a*x+I)*ln(a^2*x^
2+1)-1/2*ln(a*x+I)^2-dilog(1/2*I*(a*x-I))-ln(a*x+I)*ln(1/2*I*(a*x-I))))
```

## Fricas [F]

$$\int x(c+a^2cx^2)^2\arctan(ax)^3\,dx=\int\left(a^2cx^2+c\right)^2x\arctan\left(ax\right)^3\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^2*arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^2*x)*arctan(a*x)^3, x)
```

## Sympy [F]

$$\int x(c+a^2cx^2)^2\arctan(ax)^3\,dx=c^2\left(\int x\operatorname{atan}^3\left(ax\right)dx+\int 2a^2x^3\operatorname{atan}^3\left(ax\right)dx+\int a^4x^5\operatorname{atan}^3\left(ax\right)dx\right)$$

input
```
integrate(x*(a**2*c*x**2+c)**2*atan(a*x)**3,x)
```

output
```
c**2*(Integral(x*atan(a*x)**3, x) + Integral(2*a**2*x**3*atan(a*x)**3, x)
+ Integral(a**4*x**5*atan(a*x)**3, x))
```

## Maxima [F]

$$\int x(c+a^2cx^2)^2 \arctan(ax)^3\,dx = \int (a^2cx^2+c)^2 x \arctan(ax)^3\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^2*arctan(a*x)^3,x, algorithm="maxima")
```

output
```
1/480*(40*(a^6*c^2*x^6 + 3*a^4*c^2*x^4 + 3*a^2*c^2*x^2 + c^2)*arctan(a*x)^
3 + 20*(5760*a^7*c^2*integrate(1/480*x^7*arctan(a*x)^3/(a^3*x^2 + a), x) -
 1440*a^6*c^2*integrate(1/480*x^6*arctan(a*x)^2/(a^3*x^2 + a), x) - 360*a^
6*c^2*integrate(1/480*x^6*log(a^2*x^2 + 1)^2/(a^3*x^2 + a), x) - 288*a^6*c
^2*integrate(1/480*x^6*log(a^2*x^2 + 1)/(a^3*x^2 + a), x) + 17280*a^5*c^2*
integrate(1/480*x^5*arctan(a*x)^3/(a^3*x^2 + a), x) + 576*a^5*c^2*integrat
e(1/480*x^5*arctan(a*x)/(a^3*x^2 + a), x) - 4320*a^4*c^2*integrate(1/480*x
^4*arctan(a*x)^2/(a^3*x^2 + a), x) - 1080*a^4*c^2*integrate(1/480*x^4*log(
a^2*x^2 + 1)^2/(a^3*x^2 + a), x) - 960*a^4*c^2*integrate(1/480*x^4*log(a^2
*x^2 + 1)/(a^3*x^2 + a), x) + 17280*a^3*c^2*integrate(1/480*x^3*arctan(a*x
)^3/(a^3*x^2 + a), x) + 1920*a^3*c^2*integrate(1/480*x^3*arctan(a*x)/(a^3*
x^2 + a), x) - 4320*a^2*c^2*integrate(1/480*x^2*arctan(a*x)^2/(a^3*x^2 + a
), x) - 1080*a^2*c^2*integrate(1/480*x^2*log(a^2*x^2 + 1)^2/(a^3*x^2 + a),
 x) - 1440*a^2*c^2*integrate(1/480*x^2*log(a^2*x^2 + 1)/(a^3*x^2 + a), x)
+ 5760*a*c^2*integrate(1/480*x*arctan(a*x)^3/(a^3*x^2 + a), x) + 2880*a*c^
2*integrate(1/480*x*arctan(a*x)/(a^3*x^2 + a), x) - c^2*arctan(a*x)^3/a^2
- 360*c^2*integrate(1/480*log(a^2*x^2 + 1)^2/(a^3*x^2 + a), x))*a^2 - 4*(3
*a^5*c^2*x^5 + 10*a^3*c^2*x^3 + 15*a*c^2*x)*arctan(a*x)^2 + (3*a^5*c^2*x^5
 + 10*a^3*c^2*x^3 + 15*a*c^2*x)*log(a^2*x^2 + 1)^2)/a^2
```

## Giac [F]

$$\int x(c+a^2cx^2)^2 \arctan(ax)^3\,dx = \int (a^2cx^2+c)^2 x \arctan(ax)^3\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^2*arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^2*x*arctan(a*x)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int x(c+a^2cx^2)^2 \arctan(ax)^3 \, dx = \int x \operatorname{atan}(a\,x)^3 \left(c\,a^2\,x^2+c\right)^2 dx$$

input
```
int(x*atan(a*x)^3*(c + a^2*c*x^2)^2,x)
```

output
```
int(x*atan(a*x)^3*(c + a^2*c*x^2)^2, x)
```

## Reduce [F]

$$\int x(c+a^2cx^2)^2 \arctan(ax)^3 \, dx$$
$$= \frac{c^2\Big(10atan(ax)^3\,a^6x^6+30atan(ax)^3\,a^4x^4+30atan(ax)^3\,a^2x^2+10atan(ax)^3-6atan(ax)^2\,a^5x^5-20ata}{}$$

input
```
int(x*(a^2*c*x^2+c)^2*atan(a*x)^3,x)
```

output
```
(c**2*(10*atan(a*x)**3*a**6*x**6 + 30*atan(a*x)**3*a**4*x**4 + 30*atan(a*x
)**3*a**2*x**2 + 10*atan(a*x)**3 - 6*atan(a*x)**2*a**5*x**5 - 20*atan(a*x)
**2*a**3*x**3 - 30*atan(a*x)**2*a*x + 3*atan(a*x)*a**4*x**4 + 14*atan(a*x)
*a**2*x**2 + 11*atan(a*x) + 32*int((atan(a*x)*x)/(a**2*x**2 + 1),x)*a**2 -
 a**3*x**3 - 11*a*x))/(60*a**2)
```

# 3.374 $\int \left(c+a^2cx^2\right)^2 \arctan(ax)^3 \, dx$



## Optimal result

Integrand size = 19, antiderivative size = 289

$$\begin{aligned}\int \left(c+a^2cx^2\right)^2 \arctan(ax)^3 \, dx =& -\frac{c^2(1+a^2x^2)}{20a}+c^2x\arctan(ax) \\ &+\frac{1}{10}c^2x\left(1+a^2x^2\right)\arctan(ax) \\ &-\frac{2c^2(1+a^2x^2)\arctan(ax)^2}{5a} \\ &-\frac{3c^2(1+a^2x^2)^2\arctan(ax)^2}{20a}+\frac{8ic^2\arctan(ax)^3}{15a} \\ &+\frac{8}{15}c^2x\arctan(ax)^3+\frac{4}{15}c^2x\left(1+a^2x^2\right)\arctan(ax)^3 \\ &+\frac{1}{5}c^2x\left(1+a^2x^2\right)^2\arctan(ax)^3 \\ &+\frac{8c^2\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{5a}-\frac{c^2\log\left(1+a^2x^2\right)}{2a} \\ &+\frac{8ic^2\arctan(ax)\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{5a} \\ &+\frac{4c^2\operatorname{PolyLog}\left(3,1-\frac{2}{1+iax}\right)}{5a}\end{aligned}$$

output
```
-1/20*c^2*(a^2*x^2+1)/a+c^2*x*arctan(a*x)+1/10*c^2*x*(a^2*x^2+1)*arctan(a*
x)-2/5*c^2*(a^2*x^2+1)*arctan(a*x)^2/a-3/20*c^2*(a^2*x^2+1)^2*arctan(a*x)^
2/a+8/15*I*c^2*arctan(a*x)^3/a+8/15*c^2*x*arctan(a*x)^3+4/15*c^2*x*(a^2*x^
2+1)*arctan(a*x)^3+1/5*c^2*x*(a^2*x^2+1)^2*arctan(a*x)^3+8/5*c^2*arctan(a*
x)^2*ln(2/(1+I*a*x))/a-1/2*c^2*ln(a^2*x^2+1)/a+8/5*I*c^2*arctan(a*x)*polyl
og(2,1-2/(1+I*a*x))/a+4/5*c^2*polylog(3,1-2/(1+I*a*x))/a
```

## Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.67

$$\int (c+a^2cx^2)^2 \arctan(ax)^3 \, dx = \frac{c^2(-3-3a^2x^2+66ax\arctan(ax)+6a^3x^3\arctan(ax)-33\arctan(ax)^2-42a^2x^2\arctan(ax)^2-9a^4x^4 a}{}$$

input
```
Integrate[(c + a^2*c*x^2)^2*ArcTan[a*x]^3,x]
```

output
```
(c^2*(-3 - 3*a^2*x^2 + 66*a*x*ArcTan[a*x] + 6*a^3*x^3*ArcTan[a*x] - 33*Arc
Tan[a*x]^2 - 42*a^2*x^2*ArcTan[a*x]^2 - 9*a^4*x^4*ArcTan[a*x]^2 - (32*I)*A
rcTan[a*x]^3 + 60*a*x*ArcTan[a*x]^3 + 40*a^3*x^3*ArcTan[a*x]^3 + 12*a^5*x^
5*ArcTan[a*x]^3 + 96*ArcTan[a*x]^2*Log[1 + E^((2*I)*ArcTan[a*x])] - 30*Log
[1 + a^2*x^2] - (96*I)*ArcTan[a*x]*PolyLog[2, -E^((2*I)*ArcTan[a*x])] + 48
*PolyLog[3, -E^((2*I)*ArcTan[a*x])]))/(60*a)
```

## Rubi [A] (verified)

Time = 1.42 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.08, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.632$, Rules used = {5415, 27, 5413, 5345, 240, 5415, 5345, 240, 5455, 5379, 5529, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(ax)^3 \left(a^2cx^2+c\right)^2 \, dx$$

$$\Big\downarrow \text{5415}$$

$$\frac{3}{10}c\int c\left(a^2x^2+1\right)\arctan(ax)dx+\frac{4}{5}c\int c\left(a^2x^2+1\right)\arctan(ax)^3dx+\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\arctan(ax)^3-\frac{3c^2\left(a^2x^2+1\right)^2\arctan(ax)^2}{20a}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{10}c^2\int\left(a^2x^2+1\right)\arctan(ax)dx+\frac{4}{5}c^2\int\left(a^2x^2+1\right)\arctan(ax)^3dx+\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\arctan(ax)^3-\frac{3c^2\left(a^2x^2+1\right)^2\arctan(ax)^2}{20a}$$

$$\Big\downarrow \text{5413}$$

$$\frac{3}{10}c^2\left(\frac{2}{3}\int\arctan(ax)dx+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)-\frac{a^2x^2+1}{6a}\right)+\frac{4}{5}c^2\int\left(a^2x^2+1\right)\arctan(ax)^3dx+\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\arctan(ax)^3-\frac{3c^2\left(a^2x^2+1\right)^2\arctan(ax)^2}{20a}$$

$$\Big\downarrow \text{5345}$$

$$\frac{3}{10}c^2\left(\frac{2}{3}\left(x\arctan(ax)-a\int\frac{x}{a^2x^2+1}dx\right)+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)-\frac{a^2x^2+1}{6a}\right)+\frac{4}{5}c^2\int\left(a^2x^2+1\right)\arctan(ax)^3dx+\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\arctan(ax)^3-\frac{3c^2\left(a^2x^2+1\right)^2\arctan(ax)^2}{20a}$$

$$\Big\downarrow \text{240}$$

$$\frac{4}{5}c^2\int\left(a^2x^2+1\right)\arctan(ax)^3dx+\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\arctan(ax)^3-\frac{3c^2\left(a^2x^2+1\right)^2\arctan(ax)^2}{20a}+\frac{3}{10}c^2\left(\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)+\frac{2}{3}\left(x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}\right)-\frac{a^2x^2+1}{6a}\right)$$

$$\Big\downarrow \text{5415}$$

$$\frac{4}{5}c^2\left(\int\arctan(ax)dx+\frac{2}{3}\int\arctan(ax)^3dx+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)^3-\frac{\left(a^2x^2+1\right)\arctan(ax)^2}{2a}\right)+\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\arctan(ax)^3-\frac{3c^2\left(a^2x^2+1\right)^2\arctan(ax)^2}{20a}+\frac{3}{10}c^2\left(\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)+\frac{2}{3}\left(x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}\right)-\frac{a^2x^2+1}{6a}\right)$$

$$\Big\downarrow \text{5345}$$

$$\frac{4}{5}c^2\left(\frac{2}{3}\left(x\arctan(ax)^3-3a\int\frac{x\arctan(ax)^2}{a^2x^2+1}dx\right)-a\int\frac{x}{a^2x^2+1}dx+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)^3-\frac{\left(a^2x^2+1\right)}{2}\right.$$
$$\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\arctan(ax)^3-\frac{3c^2\left(a^2x^2+1\right)^2\arctan(ax)^2}{20a}+$$
$$\frac{3}{10}c^2\left(\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)+\frac{2}{3}\left(x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}\right)-\frac{a^2x^2+1}{6a}\right)$$

$$\Big\downarrow \; 240$$

$$\frac{4}{5}c^2\left(\frac{2}{3}\left(x\arctan(ax)^3-3a\int\frac{x\arctan(ax)^2}{a^2x^2+1}dx\right)+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)^3-\frac{\left(a^2x^2+1\right)\arctan(ax)^2}{2a}-\log\right.$$
$$\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\arctan(ax)^3-\frac{3c^2\left(a^2x^2+1\right)^2\arctan(ax)^2}{20a}+$$
$$\frac{3}{10}c^2\left(\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)+\frac{2}{3}\left(x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}\right)-\frac{a^2x^2+1}{6a}\right)$$

$$\Big\downarrow \; 5455$$

$$\frac{4}{5}c^2\left(\frac{2}{3}\left(x\arctan(ax)^3-3a\left(-\frac{\int\frac{\arctan(ax)^2}{i-ax}dx}{a}-\frac{i\arctan(ax)^3}{3a^2}\right)\right)+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)^3-\frac{\left(a^2x^2+1\right)\text{a}}{2a}\right.$$
$$\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\arctan(ax)^3-\frac{3c^2\left(a^2x^2+1\right)^2\arctan(ax)^2}{20a}+$$
$$\frac{3}{10}c^2\left(\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)+\frac{2}{3}\left(x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}\right)-\frac{a^2x^2+1}{6a}\right)$$

$$\Big\downarrow \; 5379$$

$$\frac{4}{5}c^2\left(\frac{2}{3}\left(x\arctan(ax)^3-3a\left(-\frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}-2\int\frac{\arctan(ax)\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^3}{3a^2}\right)\right)+\frac{1}{3}x\left(a^2x^2\right.\right.$$
$$\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\arctan(ax)^3-\frac{3c^2\left(a^2x^2+1\right)^2\arctan(ax)^2}{20a}+$$
$$\frac{3}{10}c^2\left(\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)+\frac{2}{3}\left(x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}\right)-\frac{a^2x^2+1}{6a}\right)$$

$$\Big\downarrow \; 5529$$

$$\frac{4}{5}c^2\left(\frac{2}{3}\left(x\arctan(ax)^3-3a\left(-\frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}-2\left(\frac{1}{2}i\int\frac{\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx-\frac{i\arctan(ax)\,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}\right.}{a}\right.\right.\right.$$

$$\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\arctan(ax)^3-\frac{3c^2\left(a^2x^2+1\right)^2\arctan(ax)^2}{20a}+$$

$$\frac{3}{10}c^2\left(\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)+\frac{2}{3}\left(x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}\right)-\frac{a^2x^2+1}{6a}\right)$$

$$\Big\downarrow \text{ 7164}$$

$$\frac{4}{5}c^2\left(\frac{2}{3}\left(x\arctan(ax)^3-3a\left(-\frac{i\arctan(ax)^3}{3a^2}-\frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}-2\left(-\frac{i\arctan(ax)\,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}-\text{PolyL}\right.}{a}\right.\right.\right.$$

$$\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\arctan(ax)^3-\frac{3c^2\left(a^2x^2+1\right)^2\arctan(ax)^2}{20a}+$$

$$\frac{3}{10}c^2\left(\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)+\frac{2}{3}\left(x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}\right)-\frac{a^2x^2+1}{6a}\right)$$

input
```
Int[(c + a^2*c*x^2)^2*ArcTan[a*x]^3,x]
```

output
```
(-3*c^2*(1 + a^2*x^2)^2*ArcTan[a*x]^2)/(20*a) + (c^2*x*(1 + a^2*x^2)^2*Arc
Tan[a*x]^3)/5 + (3*c^2*(-1/6*(1 + a^2*x^2)/a + (x*(1 + a^2*x^2)*ArcTan[a*x
])/3 + (2*(x*ArcTan[a*x] - Log[1 + a^2*x^2]/(2*a)))/3))/10 + (4*c^2*(x*Arc
Tan[a*x] - ((1 + a^2*x^2)*ArcTan[a*x]^2)/(2*a) + (x*(1 + a^2*x^2)*ArcTan[a
*x]^3)/3 - Log[1 + a^2*x^2]/(2*a) + (2*(x*ArcTan[a*x]^3 - 3*a*(((-1/3*I)*A
rcTan[a*x]^3)/a^2 - ((ArcTan[a*x]^2*Log[2/(1 + I*a*x)])/a - 2*(((-1/2*I)*A
rcTan[a*x]*PolyLog[2, 1 - 2/(1 + I*a*x)])/a - PolyLog[3, 1 - 2/(1 + I*a*x)
]/(4*a)))/a))/3))/5
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5413
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Simp[x*(d + e*x^2)
^q*((a + b*ArcTan[c*x])/(2*q + 1)), x] + Simp[2*d*(q/(2*q + 1))   Int[(d +
e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0]
```

rule 5415
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTan[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x])^p,
x], x] + Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*(
a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[q, 0] && GtQ[p, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 94.36 (sec) , antiderivative size = 1038, normalized size of antiderivative = 3.59

| method | result | size |
|---|---|---|
| derivativedivides | Expression too large to display | 1038 |
| default | Expression too large to display | 1038 |
| parts | Expression too large to display | 1039 |

input
```
int((a^2*c*x^2+c)^2*arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a*(1/5*c^2*arctan(a*x)^3*a^5*x^5+2/3*c^2*arctan(a*x)^3*a^3*x^3+c^2*arcta
n(a*x)^3*a*x-1/5*c^2*(2*I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*
(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*
x^2+1)+1)^2)*arctan(a*x)^2+11/4*arctan(a*x)^2-1/2*I*(a*x+I)+7/2*arctan(a*x
)^2*x^2*a^2+3/4*a^4*arctan(a*x)^2*x^4-4*arctan(a*x)*(a*x-I)-1/2*arctan(a*x
)*(a*x-I)^3+3*I*arctan(a*x)*(a*x-I)*(a*x+I)+2*I*Pi*csgn(I*(1+I*a*x)^2/(a^2
*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3*arctan(a*x)^2+2*I*Pi*csgn(I*(1+I*
a*x)^2/(a^2*x^2+1))^3*arctan(a*x)^2-2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)
+1)^2)^3*arctan(a*x)^2+8/3*I*arctan(a*x)^3-8*arctan(a*x)^2*ln((1+I*a*x)/(a
^2*x^2+1)^(1/2))-4*polylog(3,-(1+I*a*x)^2/(a^2*x^2+1))+3/2*arctan(a*x)*(a*
x-I)^2*(a*x+I)-3/2*arctan(a*x)*(a*x-I)*(a*x+I)^2-5*ln((1+I*a*x)^2/(a^2*x^2
+1)+1)+1/4*(a*x+I)^2-2*I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(
1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2*arctan(a*x)^2+2*I*
Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*a
rctan(a*x)^2+4*arctan(a*x)^2*ln(a^2*x^2+1)-3/2*I*arctan(a*x)*(a*x-I)^2+8*I
*arctan(a*x)*polylog(2,-(1+I*a*x)^2/(a^2*x^2+1))-8*arctan(a*x)^2*ln(2)-2*I
*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+
1)^2)*arctan(a*x)^2+4*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+
I*a*x)^2/(a^2*x^2+1)+1)^2)^2*arctan(a*x)^2-4*I*Pi*csgn(I*(1+I*a*x)/(a^2*x^
2+1)^(1/2))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^2*arctan(a*x)^2-2*I*Pi*csgn...
```

## Fricas [F]

$$\int \left(c+a^2cx^2\right)^2 \arctan(ax)^3\,dx = \int \left(a^2cx^2+c\right)^2 \arctan\left(ax\right)^3\,dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)^3, x)
```

## Sympy [F]

$$\int \left(c+a^2cx^2\right)^2 \arctan(ax)^3 \, dx = c^2 \left( \int 2a^2x^2 \operatorname{atan}^3(ax) \, dx + \int a^4x^4 \operatorname{atan}^3(ax) \, dx + \int \operatorname{atan}^3(ax) \, dx \right)$$

input
```
integrate((a**2*c*x**2+c)**2*atan(a*x)**3,x)
```

output
```
c**2*(Integral(2*a**2*x**2*atan(a*x)**3, x) + Integral(a**4*x**4*atan(a*x)
**3, x) + Integral(atan(a*x)**3, x))
```

## Maxima [F]

$$\int \left(c+a^2cx^2\right)^2 \arctan(ax)^3 \, dx = \int \left(a^2cx^2+c\right)^2 \arctan(ax)^3 \, dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^3,x, algorithm="maxima")
```

output
```
140*a^6*c^2*integrate(1/160*x^6*arctan(a*x)^3/(a^2*x^2 + 1), x) + 15*a^6*c
^2*integrate(1/160*x^6*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) +
12*a^6*c^2*integrate(1/160*x^6*arctan(a*x)*log(a^2*x^2 + 1)/(a^2*x^2 + 1),
 x) - 12*a^5*c^2*integrate(1/160*x^5*arctan(a*x)^2/(a^2*x^2 + 1), x) + 3*a
^5*c^2*integrate(1/160*x^5*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) + 420*a^4*
c^2*integrate(1/160*x^4*arctan(a*x)^3/(a^2*x^2 + 1), x) + 45*a^4*c^2*integ
rate(1/160*x^4*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) + 40*a^4*c
^2*integrate(1/160*x^4*arctan(a*x)*log(a^2*x^2 + 1)/(a^2*x^2 + 1), x) - 40
*a^3*c^2*integrate(1/160*x^3*arctan(a*x)^2/(a^2*x^2 + 1), x) + 10*a^3*c^2*
integrate(1/160*x^3*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) + 7/32*c^2*arctan
(a*x)^4/a + 420*a^2*c^2*integrate(1/160*x^2*arctan(a*x)^3/(a^2*x^2 + 1), x
) + 45*a^2*c^2*integrate(1/160*x^2*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*x^2
 + 1), x) + 60*a^2*c^2*integrate(1/160*x^2*arctan(a*x)*log(a^2*x^2 + 1)/(a
^2*x^2 + 1), x) + 1/120*(3*a^4*c^2*x^5 + 10*a^2*c^2*x^3 + 15*c^2*x)*arctan
(a*x)^3 - 60*a*c^2*integrate(1/160*x*arctan(a*x)^2/(a^2*x^2 + 1), x) + 15*
a*c^2*integrate(1/160*x*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) - 1/160*(3*a^
4*c^2*x^5 + 10*a^2*c^2*x^3 + 15*c^2*x)*arctan(a*x)*log(a^2*x^2 + 1)^2 + 15
*c^2*integrate(1/160*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x)
```

## Giac [F]

$$\int \left(c+a^2cx^2\right)^2 \arctan(ax)^3\,dx = \int \left(a^2cx^2+c\right)^2 \arctan\left(ax\right)^3\,dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^2*arctan(a*x)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \left(c+a^2cx^2\right)^2 \arctan(ax)^3 \, dx = \int \mathrm{atan}(a\,x)^3 \left(c\,a^2\,x^2+c\right)^2 dx$$

input
```
int(atan(a*x)^3*(c + a^2*c*x^2)^2,x)
```

output
```
int(atan(a*x)^3*(c + a^2*c*x^2)^2, x)
```

## Reduce [F]

$$\int \left(c+a^2cx^2\right)^2 \arctan(ax)^3 \, dx = \frac{c^2 \left(12atan(ax)^3 a^5x^5 + 40atan(ax)^3 a^3x^3 + 60atan(ax)^3 ax - 9atan(ax)^2 a^4x^4 - 42atan(ax)^2 a^2x^2 - 33\right.}{60a}$$

input
```
int((a^2*c*x^2+c)^2*atan(a*x)^3,x)
```

output
```
(c**2*(12*atan(a*x)**3*a**5*x**5 + 40*atan(a*x)**3*a**3*x**3 + 60*atan(a*x
)**3*a*x - 9*atan(a*x)**2*a**4*x**4 - 42*atan(a*x)**2*a**2*x**2 - 33*atan(
a*x)**2 + 6*atan(a*x)*a**3*x**3 + 66*atan(a*x)*a*x - 96*int((atan(a*x)**2*
x)/(a**2*x**2 + 1),x)*a**2 - 30*log(a**2*x**2 + 1) - 3*a**2*x**2))/(60*a)
```

# 3.375 $\int \frac{(c+a^2cx^2)^2 \arctan(ax)^3}{x} \, dx$

## Optimal result

Integrand size = 22, antiderivative size = 370

$$\int \frac{(c+a^2cx^2)^2\arctan(ax)^3}{x}\,dx = -\frac{1}{4}ac^2x+\frac{1}{4}c^2\arctan(ax)+\frac{1}{4}a^2c^2x^2\arctan(ax) - 2ic^2\arctan(ax)^2-\frac{9}{4}ac^2x\arctan(ax)^2 - \frac{1}{4}a^3c^2x^3\arctan(ax)^2+\frac{3}{4}c^2\arctan(ax)^3 + a^2c^2x^2\arctan(ax)^3+\frac{1}{4}a^4c^2x^4\arctan(ax)^3 + 2c^2\arctan(ax)^3\text{arctanh}\left(1-\frac{2}{1+iax}\right) - 4c^2\arctan(ax)\log\left(\frac{2}{1+iax}\right) - 2ic^2\,\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right) - \frac{3}{2}ic^2\arctan(ax)^2\,\text{PolyLog}\left(2,1-\frac{2}{1+iax}\right) + \frac{3}{2}ic^2\arctan(ax)^2\,\text{PolyLog}\left(2,-1+\frac{2}{1+iax}\right) - \frac{3}{2}c^2\arctan(ax)\,\text{PolyLog}\left(3,1-\frac{2}{1+iax}\right) + \frac{3}{2}c^2\arctan(ax)\,\text{PolyLog}\left(3,-1+\frac{2}{1+iax}\right) + \frac{3}{4}ic^2\,\text{PolyLog}\left(4,1-\frac{2}{1+iax}\right) - \frac{3}{4}ic^2\,\text{PolyLog}\left(4,-1+\frac{2}{1+iax}\right)$$

output

```
-1/4*a*c^2*x+1/4*c^2*arctan(a*x)+1/4*a^2*c^2*x^2*arctan(a*x)-3/4*I*c^2*pol
ylog(4,-1+2/(1+I*a*x))-9/4*a*c^2*x*arctan(a*x)^2-1/4*a^3*c^2*x^3*arctan(a*
x)^2+3/4*c^2*arctan(a*x)^3+a^2*c^2*x^2*arctan(a*x)^3+1/4*a^4*c^2*x^4*arcta
n(a*x)^3-2*c^2*arctan(a*x)^3*arctanh(-1+2/(1+I*a*x))-4*c^2*arctan(a*x)*ln(
2/(1+I*a*x))-2*I*c^2*polylog(2,1-2/(1+I*a*x))+3/4*I*c^2*polylog(4,1-2/(1+I
*a*x))-3/2*I*c^2*arctan(a*x)^2*polylog(2,1-2/(1+I*a*x))-3/2*c^2*arctan(a*x
)*polylog(3,1-2/(1+I*a*x))+3/2*c^2*arctan(a*x)*polylog(3,-1+2/(1+I*a*x))+3
/2*I*c^2*arctan(a*x)^2*polylog(2,-1+2/(1+I*a*x))-2*I*c^2*arctan(a*x)^2
```

## Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.82

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^3}{x} \, dx = \frac{1}{64}c^2\big(-i\pi^4 - 16ax + 16\arctan(ax) + 16a^2x^2\arctan(ax) + 128i\arctan(ax)^2 - 144ax\arctan(ax)^2 - 16a^3x^3\arctan(ax)^2 + 48\arctan(ax)^3 + 64a^2x^2\arctan(ax)^3 + 16a^4x^4\arctan(ax)^3 + 32i\arctan(ax)^4 + 64\arctan(ax)^3\log\left(1-e^{-2i\arctan(ax)}\right) - 256\arctan(ax)\log\left(1+e^{2i\arctan(ax)}\right) - 64\arctan(ax)^3\log\left(1+e^{2i\arctan(ax)}\right) + 96i\arctan(ax)^2\operatorname{PolyLog}\left(2,e^{-2i\arctan(ax)}\right) + 32i\left(4+3\arctan(ax)^2\right)\operatorname{PolyLog}\left(2,-e^{2i\arctan(ax)}\right) + 96\arctan(ax)\operatorname{PolyLog}\left(3,e^{-2i\arctan(ax)}\right) - 96\arctan(ax)\operatorname{PolyLog}\left(3,-e^{2i\arctan(ax)}\right) - 48i\operatorname{PolyLog}\left(4,e^{-2i\arctan(ax)}\right) - 48i\operatorname{PolyLog}\left(4,-e^{2i\arctan(ax)}\right)\big)$$

input
```
Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x]^3)/x,x]
```

output
```
(c^2*((-I)*Pi^4 - 16*a*x + 16*ArcTan[a*x] + 16*a^2*x^2*ArcTan[a*x] + (128*
I)*ArcTan[a*x]^2 - 144*a*x*ArcTan[a*x]^2 - 16*a^3*x^3*ArcTan[a*x]^2 + 48*A
rcTan[a*x]^3 + 64*a^2*x^2*ArcTan[a*x]^3 + 16*a^4*x^4*ArcTan[a*x]^3 + (32*I
)*ArcTan[a*x]^4 + 64*ArcTan[a*x]^3*Log[1 - E^((-2*I)*ArcTan[a*x])] - 256*A
rcTan[a*x]*Log[1 + E^((2*I)*ArcTan[a*x])] - 64*ArcTan[a*x]^3*Log[1 + E^((2
*I)*ArcTan[a*x])] + (96*I)*ArcTan[a*x]^2*PolyLog[2, E^((-2*I)*ArcTan[a*x])
] + (32*I)*(4 + 3*ArcTan[a*x]^2)*PolyLog[2, -E^((2*I)*ArcTan[a*x])] + 96*A
rcTan[a*x]*PolyLog[3, E^((-2*I)*ArcTan[a*x])] - 96*ArcTan[a*x]*PolyLog[3,
-E^((2*I)*ArcTan[a*x])] - (48*I)*PolyLog[4, E^((-2*I)*ArcTan[a*x])] - (48*
I)*PolyLog[4, -E^((2*I)*ArcTan[a*x])]))/64
```

## Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan (ax)^3 \left(a^2 c x^2+c\right)^2}{x} \, dx$$

$$\Big\downarrow \text{ 5483}$$

$$\int \left(a^4 c^2 x^3 \arctan (ax)^3+2 a^2 c^2 x \arctan (ax)^3+\frac{c^2 \arctan (ax)^3}{x}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4} a^4 c^2 x^4 \arctan (ax)^3-\frac{1}{4} a^3 c^2 x^3 \arctan (ax)^2+a^2 c^2 x^2 \arctan (ax)^3+\frac{1}{4} a^2 c^2 x^2 \arctan (ax)+2 c^2 \arctan (ax)^3 \text{arctanh}\left(1-\frac{2}{1+iax}\right)-\frac{3}{2} i c^2 \arctan (ax)^2 \text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)+\frac{3}{2} i c^2 \arctan (ax)^2 \text{PolyLog}\left(2,\frac{2}{iax+1}-1\right)-\frac{3}{2} c^2 \arctan (ax) \text{PolyLog}\left(3,1-\frac{2}{iax+1}\right)+\frac{3}{2} c^2 \arctan (ax) \text{PolyLog}\left(3,\frac{2}{iax+1}-1\right)-\frac{9}{4} a c^2 x \arctan (ax)^2+\frac{3}{4} c^2 \arctan (ax)^3-2 i c^2 \arctan (ax)^2+\frac{1}{4} c^2 \arctan (ax)-4 c^2 \arctan (ax) \log \left(\frac{2}{1+iax}\right)-2 i c^2 \text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)+\frac{3}{4} i c^2 \text{PolyLog}\left(4,1-\frac{2}{iax+1}\right)-\frac{3}{4} i c^2 \text{PolyLog}\left(4,\frac{2}{iax+1}-1\right)-\frac{1}{4} a c^2 x$$

input
```
Int[((c + a^2*c*x^2)^2*ArcTan[a*x]^3)/x,x]
```

output
```
-1/4*(a*c^2*x) + (c^2*ArcTan[a*x])/4 + (a^2*c^2*x^2*ArcTan[a*x])/4 - (2*I)
*c^2*ArcTan[a*x]^2 - (9*a*c^2*x*ArcTan[a*x]^2)/4 - (a^3*c^2*x^3*ArcTan[a*x
]^2)/4 + (3*c^2*ArcTan[a*x]^3)/4 + a^2*c^2*x^2*ArcTan[a*x]^3 + (a^4*c^2*x^
4*ArcTan[a*x]^3)/4 + 2*c^2*ArcTan[a*x]^3*ArcTanh[1 - 2/(1 + I*a*x)] - 4*c^
2*ArcTan[a*x]*Log[2/(1 + I*a*x)] - (2*I)*c^2*PolyLog[2, 1 - 2/(1 + I*a*x)]
 - ((3*I)/2)*c^2*ArcTan[a*x]^2*PolyLog[2, 1 - 2/(1 + I*a*x)] + ((3*I)/2)*c
^2*ArcTan[a*x]^2*PolyLog[2, -1 + 2/(1 + I*a*x)] - (3*c^2*ArcTan[a*x]*PolyL
og[3, 1 - 2/(1 + I*a*x)])/2 + (3*c^2*ArcTan[a*x]*PolyLog[3, -1 + 2/(1 + I*
a*x)])/2 + ((3*I)/4)*c^2*PolyLog[4, 1 - 2/(1 + I*a*x)] - ((3*I)/4)*c^2*Pol
yLog[4, -1 + 2/(1 + I*a*x)]
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

## Maple [A] (verified)

Time = 31.67 (sec) , antiderivative size = 566, normalized size of antiderivative = 1.53

| method | result |
|---|---|
| derivativedivides | $\frac{c^2\left(-3i\arctan(ax)^3+3x\arctan(ax)^3a-i\arctan(ax)^3a^2x^2+\arctan(ax)^3a^3x^3-8\arctan(ax)^2+i\arctan(ax)^2ax-\text{arcta}\right.}{4}$ |
| default | $\frac{c^2\left(-3i\arctan(ax)^3+3x\arctan(ax)^3a-i\arctan(ax)^3a^2x^2+\arctan(ax)^3a^3x^3-8\arctan(ax)^2+i\arctan(ax)^2ax-\text{arcta}\right.}{4}$ |

input
```
int((a^2*c*x^2+c)^2*arctan(a*x)^3/x,x,method=_RETURNVERBOSE)
```

output
```
1/4*c^2*(-3*I*arctan(a*x)^3+3*x*arctan(a*x)^3*a-I*arctan(a*x)^3*a^2*x^2+ar
ctan(a*x)^3*a^3*x^3-8*arctan(a*x)^2+I*arctan(a*x)^2*a*x-arctan(a*x)^2*x^2*
a^2-I*arctan(a*x)+arctan(a*x)*a*x-1)*(a*x+I)+c^2*arctan(a*x)^3*ln(1+(1+I*a
*x)/(a^2*x^2+1)^(1/2))-3/4*I*c^2*polylog(4,-(1+I*a*x)^2/(a^2*x^2+1))+6*c^2
*arctan(a*x)*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*c^2*polylog(4,-(1
+I*a*x)/(a^2*x^2+1)^(1/2))-c^2*arctan(a*x)^3*ln((1+I*a*x)^2/(a^2*x^2+1)+1)
+4*I*c^2*arctan(a*x)^2-3/2*c^2*arctan(a*x)*polylog(3,-(1+I*a*x)^2/(a^2*x^2
+1))-3*I*c^2*arctan(a*x)^2*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+c^2*arc
tan(a*x)^3*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*c^2*polylog(4,(1+I*a*x)/(
a^2*x^2+1)^(1/2))+6*c^2*arctan(a*x)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))
-3*I*c^2*arctan(a*x)^2*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*I*c^2*poly
log(2,-(1+I*a*x)^2/(a^2*x^2+1))-4*c^2*arctan(a*x)*ln((1+I*a*x)^2/(a^2*x^2+
1)+1)+3/2*I*c^2*arctan(a*x)^2*polylog(2,-(1+I*a*x)^2/(a^2*x^2+1))
```

## Fricas [F]

$$\int \frac{(c+a^2cx^2)^2\arctan(ax)^3}{x}\,dx = \int \frac{(a^2cx^2+c)^2\arctan(ax)^3}{x}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^3/x,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)^3/x, x)
```

## Sympy [F]

$$\int \frac{(c+a^2cx^2)^2\arctan(ax)^3}{x}\,dx = c^2\left(\int \frac{\operatorname{atan}^3(ax)}{x}\,dx + \int 2a^2x\operatorname{atan}^3(ax)\,dx + \int a^4x^3\operatorname{atan}^3(ax)\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**2*atan(a*x)**3/x,x)
```

output
```
c**2*(Integral(atan(a*x)**3/x, x) + Integral(2*a**2*x*atan(a*x)**3, x) + I
ntegral(a**4*x**3*atan(a*x)**3, x))
```

## Maxima [F]

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^3}{x}\,dx = \int \frac{(a^2cx^2+c)^2 \arctan(ax)^3}{x}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^3/x,x, algorithm="maxima")
```

output
```
1/32*(a^4*c^2*x^4 + 4*a^2*c^2*x^2)*arctan(a*x)^3 - 3/128*(a^4*c^2*x^4 + 4*
a^2*c^2*x^2)*arctan(a*x)*log(a^2*x^2 + 1)^2 + integrate(1/128*(112*(a^6*c^
2*x^6 + 3*a^4*c^2*x^4 + 3*a^2*c^2*x^2 + c^2)*arctan(a*x)^3 - 12*(a^5*c^2*x
^5 + 4*a^3*c^2*x^3)*arctan(a*x)^2 + 12*(a^6*c^2*x^6 + 4*a^4*c^2*x^4)*arcta
n(a*x)*log(a^2*x^2 + 1) + 3*(a^5*c^2*x^5 + 4*a^3*c^2*x^3 + 4*(a^6*c^2*x^6
+ 3*a^4*c^2*x^4 + 3*a^2*c^2*x^2 + c^2)*arctan(a*x))*log(a^2*x^2 + 1)^2)/(a
^2*x^3 + x), x)
```

## Giac [F]

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^3}{x}\,dx = \int \frac{(a^2cx^2+c)^2 \arctan(ax)^3}{x}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^3/x,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^2*arctan(a*x)^3/x, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^3}{x}\,dx = \int \frac{\mathrm{atan}(a\,x)^3\,(c\,a^2\,x^2+c)^2}{x}\,dx$$

input
```
int((atan(a*x)^3*(c + a^2*c*x^2)^2)/x,x)
```

output
```
int((atan(a*x)^3*(c + a^2*c*x^2)^2)/x, x)
```

## Reduce [F]

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^3}{x}\,dx$$
$$= \frac{c^2\Big(atan(ax)^3\,a^4x^4 + 4atan(ax)^3\,a^2x^2 + 3atan(ax)^3 - atan(ax)^2\,a^3x^3 - 9atan(ax)^2\,ax + 13atan(ax)\,a^2}{4}$$

input
```
int((a^2*c*x^2+c)^2*atan(a*x)^3/x,x)
```

output
```
(c**2*(atan(a*x)**3*a**4*x**4 + 4*atan(a*x)**3*a**2*x**2 + 3*atan(a*x)**3
- atan(a*x)**2*a**3*x**3 - 9*atan(a*x)**2*a*x + 13*atan(a*x)*a**2*x**2 + 1
3*atan(a*x) + 4*int(atan(a*x)**3/x,x) - 24*int((atan(a*x)*x**3)/(a**2*x**2
 + 1),x)*a**4 - 8*int((atan(a*x)*x)/(a**2*x**2 + 1),x)*a**2 - 13*a*x))/4
```

# 3.376 $\int \frac{(c+a^2cx^2)^2 \arctan(ax)^3}{x^2} \, dx$



## Optimal result

Integrand size = 22, antiderivative size = 284

$$\begin{aligned}\int \frac{(c+a^2cx^2)^2 \arctan(ax)^3}{x^2} \, dx &= a^2c^2x\arctan(ax) - \frac{1}{2}ac^2\arctan(ax)^2 \\ &\quad - \frac{1}{2}a^3c^2x^2\arctan(ax)^2 \\ &\quad + \frac{2}{3}iac^2\arctan(ax)^3 - \frac{c^2\arctan(ax)^3}{x} \\ &\quad + 2a^2c^2x\arctan(ax)^3 + \frac{1}{3}a^4c^2x^3\arctan(ax)^3 \\ &\quad + 5ac^2\arctan(ax)^2\log\left(\frac{2}{1+iax}\right) \\ &\quad - \frac{1}{2}ac^2\log\left(1+a^2x^2\right) \\ &\quad + 3ac^2\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right) \\ &\quad - 3iac^2\arctan(ax)\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right) \\ &\quad + 5iac^2\arctan(ax)\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right) \\ &\quad + \frac{3}{2}ac^2\operatorname{PolyLog}\left(3,-1+\frac{2}{1-iax}\right) \\ &\quad + \frac{5}{2}ac^2\operatorname{PolyLog}\left(3,1-\frac{2}{1+iax}\right)\end{aligned}$$

output
```
a^2*c^2*x*arctan(a*x)-1/2*a*c^2*arctan(a*x)^2-1/2*a^3*c^2*x^2*arctan(a*x)^
2+2/3*I*a*c^2*arctan(a*x)^3-c^2*arctan(a*x)^3/x+2*a^2*c^2*x*arctan(a*x)^3+
1/3*a^4*c^2*x^3*arctan(a*x)^3+5*a*c^2*arctan(a*x)^2*ln(2/(1+I*a*x))-1/2*a*
c^2*ln(a^2*x^2+1)+3*a*c^2*arctan(a*x)^2*ln(2-2/(1-I*a*x))-3*I*a*c^2*arctan
(a*x)*polylog(2,-1+2/(1-I*a*x))+5*I*a*c^2*arctan(a*x)*polylog(2,1-2/(1+I*a
*x))+3/2*a*c^2*polylog(3,-1+2/(1-I*a*x))+5/2*a*c^2*polylog(3,1-2/(1+I*a*x)
)
```

## Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.87

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^3}{x^2} dx$$
$$= \frac{c^2(-3ia\pi^3x + 24a^2x^2\arctan(ax) - 12ax\arctan(ax)^2 - 12a^3x^3\arctan(ax)^2 - 24\arctan(ax)^3 - 16iax\,\mathrm{a}}{}$$

input
```
Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x]^3)/x^2,x]
```

output
```
(c^2*((-3*I)*a*Pi^3*x + 24*a^2*x^2*ArcTan[a*x] - 12*a*x*ArcTan[a*x]^2 - 12
*a^3*x^3*ArcTan[a*x]^2 - 24*ArcTan[a*x]^3 - (16*I)*a*x*ArcTan[a*x]^3 + 48*
a^2*x^2*ArcTan[a*x]^3 + 8*a^4*x^4*ArcTan[a*x]^3 + 72*a*x*ArcTan[a*x]^2*Log
[1 - E^((-2*I)*ArcTan[a*x])] + 120*a*x*ArcTan[a*x]^2*Log[1 + E^((2*I)*ArcT
an[a*x])] - 12*a*x*Log[1 + a^2*x^2] + (72*I)*a*x*ArcTan[a*x]*PolyLog[2, E^
((-2*I)*ArcTan[a*x])] - (120*I)*a*x*ArcTan[a*x]*PolyLog[2, -E^((2*I)*ArcTa
n[a*x])] + 36*a*x*PolyLog[3, E^((-2*I)*ArcTan[a*x])] + 60*a*x*PolyLog[3, -
E^((2*I)*ArcTan[a*x])]))/(24*x)
```

## Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3 (a^2cx^2+c)^2}{x^2} dx$$

$$\Big\downarrow \text{5483}$$

$$\int \left(a^4c^2x^2\arctan(ax)^3+2a^2c^2\arctan(ax)^3+\frac{c^2\arctan(ax)^3}{x^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}a^4c^2x^3\arctan(ax)^3-\frac{1}{2}a^3c^2x^2\arctan(ax)^2+2a^2c^2x\arctan(ax)^3+a^2c^2x\arctan(ax)-\frac{1}{2}ac^2\log\left(a^2x^2+1\right)-3iac^2\arctan(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)+5iac^2\arctan(ax)\operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right)+\frac{2}{3}iac^2\arctan(ax)^3-\frac{1}{2}ac^2\arctan(ax)^2-\frac{c^2\arctan(ax)^3}{x}+5ac^2\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)+3ac^2\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)+\frac{3}{2}ac^2\operatorname{PolyLog}\left(3,\frac{2}{1-iax}-1\right)+\frac{5}{2}ac^2\operatorname{PolyLog}\left(3,1-\frac{2}{iax+1}\right)$$

input
```
Int[((c + a^2*c*x^2)^2*ArcTan[a*x]^3)/x^2,x]
```

output
```
a^2*c^2*x*ArcTan[a*x] - (a*c^2*ArcTan[a*x]^2)/2 - (a^3*c^2*x^2*ArcTan[a*x]
^2)/2 + ((2*I)/3)*a*c^2*ArcTan[a*x]^3 - (c^2*ArcTan[a*x]^3)/x + 2*a^2*c^2*
x*ArcTan[a*x]^3 + (a^4*c^2*x^3*ArcTan[a*x]^3)/3 + 5*a*c^2*ArcTan[a*x]^2*Lo
g[2/(1 + I*a*x)] - (a*c^2*Log[1 + a^2*x^2])/2 + 3*a*c^2*ArcTan[a*x]^2*Log[
2 - 2/(1 - I*a*x)] - (3*I)*a*c^2*ArcTan[a*x]*PolyLog[2, -1 + 2/(1 - I*a*x)
] + (5*I)*a*c^2*ArcTan[a*x]*PolyLog[2, 1 - 2/(1 + I*a*x)] + (3*a*c^2*PolyL
og[3, -1 + 2/(1 - I*a*x)])/2 + (5*a*c^2*PolyLog[3, 1 - 2/(1 + I*a*x)])/2
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 307.72 (sec) , antiderivative size = 1851, normalized size of antiderivative = 6.52

| method | result | size |
|---|---|---|
| derivativedivides | Expression too large to display | 1851 |
| default | Expression too large to display | 1851 |
| parts | Expression too large to display | 1851 |

input
```
int((a^2*c*x^2+c)^2*arctan(a*x)^3/x^2,x,method=_RETURNVERBOSE)
```

output
```
a*(1/3*c^2*arctan(a*x)^3*a^3*x^3+2*c^2*arctan(a*x)^3*a*x-c^2*arctan(a*x)^3
/a/x-c^2*(-3/2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I/((1+I*a*x)^
2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2
+1)+1))*arctan(a*x)^2+2*I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*
(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*
x^2+1)+1)^2)*arctan(a*x)^2+1/2*arctan(a*x)^2+1/2*arctan(a*x)^2*x^2*a^2-3/2
*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^3*ar
ctan(a*x)^2-3/2*I*Pi*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^
2+1)+1))^3*arctan(a*x)^2+3/2*I*Pi*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a
*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2+6*I*arctan(a*x)*polylog(2,(1+I*a*x)/
(a^2*x^2+1)^(1/2))-6*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*polylog(3,-(
1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+
1)^(1/2))-arctan(a*x)*(a*x-I)+5*I*arctan(a*x)*polylog(2,-(1+I*a*x)^2/(a^2*
x^2+1))-3/2*I*Pi*arctan(a*x)^2+2*I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I
*a*x)^2/(a^2*x^2+1)+1)^2)^3*arctan(a*x)^2+2*I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x
^2+1))^3*arctan(a*x)^2-2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3*arct
an(a*x)^2+8/3*I*arctan(a*x)^3-8*arctan(a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/
2))-5/2*polylog(3,-(1+I*a*x)^2/(a^2*x^2+1))-ln((1+I*a*x)^2/(a^2*x^2+1)+1)+
3/2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*c
sgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a...
```

## Fricas [F]

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^3}{x^2}\,dx = \int \frac{(a^2cx^2+c)^2 \arctan(ax)^3}{x^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^3/x^2,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)^3/x^2, x)
```

## Sympy [F]

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^3}{x^2}\,dx = c^2\left(\int 2a^2 \operatorname{atan}^3(ax)\,dx + \int \frac{\operatorname{atan}^3(ax)}{x^2}\,dx + \int a^4x^2 \operatorname{atan}^3(ax)\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**2*atan(a*x)**3/x**2,x)
```

output
```
c**2*(Integral(2*a**2*atan(a*x)**3, x) + Integral(atan(a*x)**3/x**2, x) +
Integral(a**4*x**2*atan(a*x)**3, x))
```

## Maxima [F]

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^3}{x^2}\,dx = \int \frac{(a^2cx^2+c)^2 \arctan(ax)^3}{x^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^3/x^2,x, algorithm="maxima")
```

output
```
1/96*(4*(a^4*c^2*x^4 + 6*a^2*c^2*x^2 - 3*c^2)*arctan(a*x)^3 - 3*(a^4*c^2*x
^4 + 6*a^2*c^2*x^2 - 3*c^2)*arctan(a*x)*log(a^2*x^2 + 1)^2 + 3*(896*a^6*c^
2*integrate(1/32*x^6*arctan(a*x)^3/(a^2*x^4 + x^2), x) + 96*a^6*c^2*integr
ate(1/32*x^6*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*x^4 + x^2), x) + 128*a^6*
c^2*integrate(1/32*x^6*arctan(a*x)*log(a^2*x^2 + 1)/(a^2*x^4 + x^2), x) -
128*a^5*c^2*integrate(1/32*x^5*arctan(a*x)^2/(a^2*x^4 + x^2), x) + 32*a^5*
c^2*integrate(1/32*x^5*log(a^2*x^2 + 1)^2/(a^2*x^4 + x^2), x) + 21*a*c^2*a
rctan(a*x)^4 + 2688*a^4*c^2*integrate(1/32*x^4*arctan(a*x)^3/(a^2*x^4 + x^
2), x) + 288*a^4*c^2*integrate(1/32*x^4*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^
2*x^4 + x^2), x) + 768*a^4*c^2*integrate(1/32*x^4*arctan(a*x)*log(a^2*x^2
+ 1)/(a^2*x^4 + x^2), x) - 768*a^3*c^2*integrate(1/32*x^3*arctan(a*x)^2/(a
^2*x^4 + x^2), x) + a*c^2*log(a^2*x^2 + 1)^3 + 288*a^2*c^2*integrate(1/32*
x^2*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*x^4 + x^2), x) - 384*a^2*c^2*integ
rate(1/32*x^2*arctan(a*x)*log(a^2*x^2 + 1)/(a^2*x^4 + x^2), x) + 384*a*c^2
*integrate(1/32*x*arctan(a*x)^2/(a^2*x^4 + x^2), x) - 96*a*c^2*integrate(1
/32*x*log(a^2*x^2 + 1)^2/(a^2*x^4 + x^2), x) + 896*c^2*integrate(1/32*arct
an(a*x)^3/(a^2*x^4 + x^2), x) + 96*c^2*integrate(1/32*arctan(a*x)*log(a^2*
x^2 + 1)^2/(a^2*x^4 + x^2), x))*x)/x
```

## Giac [F]

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^3}{x^2} \, dx = \int \frac{(a^2cx^2+c)^2 \arctan(ax)^3}{x^2} \, dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^3/x^2,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^2*arctan(a*x)^3/x^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^3}{x^2} \, dx = \int \frac{\operatorname{atan}(a\,x)^3\,(c\,a^2\,x^2+c)^2}{x^2} \, dx$$

input
```
int((atan(a*x)^3*(c + a^2*c*x^2)^2)/x^2,x)
```

output
```
int((atan(a*x)^3*(c + a^2*c*x^2)^2)/x^2, x)
```

## Reduce [F]

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^3}{x^2} \, dx = \frac{c^2\Big(2atan(ax)^3\,a^4x^4+12atan(ax)^3\,a^2x^2-6atan(ax)^3-3atan(ax)^2\,a^3x^3-3atan(ax)^2\,ax+6atan(ax)}{6x}$$

input
```
int((a^2*c*x^2+c)^2*atan(a*x)^3/x^2,x)
```

output
```
(c**2*(2*atan(a*x)**3*a**4*x**4 + 12*atan(a*x)**3*a**2*x**2 - 6*atan(a*x)*
*3 - 3*atan(a*x)**2*a**3*x**3 - 3*atan(a*x)**2*a*x + 6*atan(a*x)*a**2*x**2
 + 18*int(atan(a*x)**2/(a**2*x**3 + x),x)*a*x - 30*int((atan(a*x)**2*x)/(a
**2*x**2 + 1),x)*a**3*x - 3*log(a**2*x**2 + 1)*a*x))/(6*x)
```

## 3.377 $\int \frac{(c+a^2cx^2)^2\arctan(ax)^3}{x^3}\,dx$

## Optimal result

Integrand size = 22, antiderivative size = 399

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^3}{x^3} \, dx = -3ia^2c^2 \arctan(ax)^2 - \frac{3ac^2 \arctan(ax)^2}{2x} - \frac{3}{2}a^3c^2x \arctan(ax)^2 - \frac{c^2 \arctan(ax)^3}{2x^2} + \frac{1}{2}a^4c^2x^2 \arctan(ax)^3 + 4a^2c^2 \arctan(ax)^3 \operatorname{arctanh}\left(1 - \frac{2}{1+iax}\right) - 3a^2c^2 \arctan(ax) \log\left(\frac{2}{1+iax}\right) + 3a^2c^2 \arctan(ax) \log\left(2 - \frac{2}{1-iax}\right) - \frac{3}{2}ia^2c^2 \operatorname{PolyLog}\left(2, -1 + \frac{2}{1-iax}\right) - \frac{3}{2}ia^2c^2 \operatorname{PolyLog}\left(2, 1 - \frac{2}{1+iax}\right) - 3ia^2c^2 \arctan(ax)^2 \operatorname{PolyLog}\left(2, 1 - \frac{2}{1+iax}\right) + 3ia^2c^2 \arctan(ax)^2 \operatorname{PolyLog}\left(2, -1 + \frac{2}{1+iax}\right) - 3a^2c^2 \arctan(ax) \operatorname{PolyLog}\left(3, 1 - \frac{2}{1+iax}\right) + 3a^2c^2 \arctan(ax) \operatorname{PolyLog}\left(3, -1 + \frac{2}{1+iax}\right) + \frac{3}{2}ia^2c^2 \operatorname{PolyLog}\left(4, 1 - \frac{2}{1+iax}\right) - \frac{3}{2}ia^2c^2 \operatorname{PolyLog}\left(4, -1 + \frac{2}{1+iax}\right)$$

output
```
3/2*I*a^2*c^2*polylog(4,1-2/(1+I*a*x))-3/2*a*c^2*arctan(a*x)^2/x-3/2*a^3*c
^2*x*arctan(a*x)^2-1/2*c^2*arctan(a*x)^3/x^2+1/2*a^4*c^2*x^2*arctan(a*x)^3
-4*a^2*c^2*arctan(a*x)^3*arctanh(-1+2/(1+I*a*x))-3*a^2*c^2*arctan(a*x)*ln(
2/(1+I*a*x))+3*a^2*c^2*arctan(a*x)*ln(2-2/(1-I*a*x))-3*I*a^2*c^2*arctan(a*
x)^2-3*I*a^2*c^2*arctan(a*x)^2*polylog(2,1-2/(1+I*a*x))-3/2*I*a^2*c^2*poly
log(2,1-2/(1+I*a*x))-3/2*I*a^2*c^2*polylog(4,-1+2/(1+I*a*x))-3*a^2*c^2*arc
tan(a*x)*polylog(3,1-2/(1+I*a*x))+3*a^2*c^2*arctan(a*x)*polylog(3,-1+2/(1+
I*a*x))-3/2*I*a^2*c^2*polylog(2,-1+2/(1-I*a*x))+3*I*a^2*c^2*arctan(a*x)^2*
polylog(2,-1+2/(1+I*a*x))
```

## Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.76

$$\int \frac{(c+a^2cx^2)^2\arctan(ax)^3}{x^3}\,dx = \frac{1}{32}a^2c^2\left(-i\pi^4-\frac{48\arctan(ax)^2}{ax}-48ax\arctan(ax)^2-\frac{16\arctan(ax)^3}{a^2x^2}+16a^2x^2\arctan(ax)^3+32i\arctan(ax)^4+64\arctan(ax)^3\log\left(1-e^{-2i\arctan(ax)}\right)+96\arctan(ax)\log\left(1-e^{2i\arctan(ax)}\right)-96\arctan(ax)\log\left(1+e^{2i\arctan(ax)}\right)-64\arctan(ax)^3\log\left(1+e^{2i\arctan(ax)}\right)+96i\arctan(ax)^2\operatorname{PolyLog}\left(2,e^{-2i\arctan(ax)}\right)+48i\left(1+2\arctan(ax)^2\right)\operatorname{PolyLog}\left(2,-e^{2i\arctan(ax)}\right)-48i\operatorname{PolyLog}\left(2,e^{2i\arctan(ax)}\right)+96\arctan(ax)\operatorname{PolyLog}\left(3,e^{-2i\arctan(ax)}\right)-96\arctan(ax)\operatorname{PolyLog}\left(3,-e^{2i\arctan(ax)}\right)-48i\operatorname{PolyLog}\left(4,e^{-2i\arctan(ax)}\right)-48i\operatorname{PolyLog}\left(4,-e^{2i\arctan(ax)}\right)\right)$$

input
```
Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x]^3)/x^3,x]
```

output
```
(a^2*c^2*((-I)*Pi^4 - (48*ArcTan[a*x]^2)/(a*x) - 48*a*x*ArcTan[a*x]^2 - (1
6*ArcTan[a*x]^3)/(a^2*x^2) + 16*a^2*x^2*ArcTan[a*x]^3 + (32*I)*ArcTan[a*x]
^4 + 64*ArcTan[a*x]^3*Log[1 - E^((-2*I)*ArcTan[a*x])] + 96*ArcTan[a*x]*Log
[1 - E^((2*I)*ArcTan[a*x])] - 96*ArcTan[a*x]*Log[1 + E^((2*I)*ArcTan[a*x])
] - 64*ArcTan[a*x]^3*Log[1 + E^((2*I)*ArcTan[a*x])] + (96*I)*ArcTan[a*x]^2
*PolyLog[2, E^((-2*I)*ArcTan[a*x])] + (48*I)*(1 + 2*ArcTan[a*x]^2)*PolyLog
[2, -E^((2*I)*ArcTan[a*x])] - (48*I)*PolyLog[2, E^((2*I)*ArcTan[a*x])] + 9
6*ArcTan[a*x]*PolyLog[3, E^((-2*I)*ArcTan[a*x])] - 96*ArcTan[a*x]*PolyLog[
3, -E^((2*I)*ArcTan[a*x])] - (48*I)*PolyLog[4, E^((-2*I)*ArcTan[a*x])] - (
48*I)*PolyLog[4, -E^((2*I)*ArcTan[a*x])]))/32
```

## Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3 \left(a^2cx^2+c\right)^2}{x^3} \, dx$$

$$\Big\downarrow \text{ 5483}$$

$$\int \left(a^4c^2x\arctan(ax)^3+\frac{2a^2c^2\arctan(ax)^3}{x}+\frac{c^2\arctan(ax)^3}{x^3}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}a^4c^2x^2\arctan(ax)^3-\frac{3}{2}a^3c^2x\arctan(ax)^2+4a^2c^2\arctan(ax)^3\text{arctanh}\left(1-\frac{2}{1+iax}\right)-3ia^2c^2\arctan(ax)^2\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)+3ia^2c^2\arctan(ax)^2\text{PolyLog}\left(2,\frac{2}{iax+1}-1\right)-3a^2c^2\arctan(ax)\text{PolyLog}\left(3,1-\frac{2}{iax+1}\right)+3a^2c^2\arctan(ax)\text{PolyLog}\left(3,\frac{2}{iax+1}-1\right)-3ia^2c^2\arctan(ax)^2-3a^2c^2\arctan(ax)\log\left(\frac{2}{1+iax}\right)+3a^2c^2\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{3}{2}ia^2c^2\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)-\frac{3}{2}ia^2c^2\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)+\frac{3}{2}ia^2c^2\text{PolyLog}\left(4,1-\frac{2}{iax+1}\right)-\frac{3}{2}ia^2c^2\text{PolyLog}\left(4,\frac{2}{iax+1}-1\right)-\frac{c^2\arctan(ax)^3}{2x^2}-\frac{3ac^2\arctan(ax)^2}{2x}$$

input
```
Int[((c + a^2*c*x^2)^2*ArcTan[a*x]^3)/x^3,x]
```

output
```
(-3*I)*a^2*c^2*ArcTan[a*x]^2 - (3*a*c^2*ArcTan[a*x]^2)/(2*x) - (3*a^3*c^2*
x*ArcTan[a*x]^2)/2 - (c^2*ArcTan[a*x]^3)/(2*x^2) + (a^4*c^2*x^2*ArcTan[a*x
]^3)/2 + 4*a^2*c^2*ArcTan[a*x]^3*ArcTanh[1 - 2/(1 + I*a*x)] - 3*a^2*c^2*Ar
cTan[a*x]*Log[2/(1 + I*a*x)] + 3*a^2*c^2*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)
] - ((3*I)/2)*a^2*c^2*PolyLog[2, -1 + 2/(1 - I*a*x)] - ((3*I)/2)*a^2*c^2*P
olyLog[2, 1 - 2/(1 + I*a*x)] - (3*I)*a^2*c^2*ArcTan[a*x]^2*PolyLog[2, 1 -
2/(1 + I*a*x)] + (3*I)*a^2*c^2*ArcTan[a*x]^2*PolyLog[2, -1 + 2/(1 + I*a*x)
] - 3*a^2*c^2*ArcTan[a*x]*PolyLog[3, 1 - 2/(1 + I*a*x)] + 3*a^2*c^2*ArcTan
[a*x]*PolyLog[3, -1 + 2/(1 + I*a*x)] + ((3*I)/2)*a^2*c^2*PolyLog[4, 1 - 2/
(1 + I*a*x)] - ((3*I)/2)*a^2*c^2*PolyLog[4, -1 + 2/(1 + I*a*x)]
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

## Maple [A] (verified)

Time = 71.29 (sec) , antiderivative size = 622, normalized size of antiderivative = 1.56

| method | result |
|---|---|
| derivativedivides | $a^2\left(\frac{c^2\arctan(ax)^2(x^2a^2\arctan(ax)-\arctan(ax)-3ax)(ax-i)(ax+i)}{2a^2x^2}+2c^2\arctan(ax)^3\ln\left(1-\frac{iax+1}{\sqrt{a^2x^2-}}\right.\right.$ |
| default | $a^2\left(\frac{c^2\arctan(ax)^2(x^2a^2\arctan(ax)-\arctan(ax)-3ax)(ax-i)(ax+i)}{2a^2x^2}+2c^2\arctan(ax)^3\ln\left(1-\frac{iax+1}{\sqrt{a^2x^2-}}\right.\right.$ |

input
```
int((a^2*c*x^2+c)^2*arctan(a*x)^3/x^3,x,method=_RETURNVERBOSE)
```

output
```
a^2*(1/2*c^2*arctan(a*x)^2*(x^2*a^2*arctan(a*x)-arctan(a*x)-3*a*x)*(a*x-I)
*(a*x+I)/a^2/x^2+2*c^2*arctan(a*x)^3*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))+3/2
*I*c^2*polylog(2,-(1+I*a*x)^2/(a^2*x^2+1))-3*c^2*arctan(a*x)*polylog(3,-(1
+I*a*x)^2/(a^2*x^2+1))+12*I*c^2*polylog(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+12
*c^2*arctan(a*x)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I*c^2*arctan(a*x
)^2*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+3*c^2*arctan(a*x)*ln(1+(1+I*a*x
)/(a^2*x^2+1)^(1/2))-3*I*c^2*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+12*c^2
*arctan(a*x)*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*I*c^2*polylog(2,-(1
+I*a*x)/(a^2*x^2+1)^(1/2))+2*c^2*arctan(a*x)^3*ln(1+(1+I*a*x)/(a^2*x^2+1)^
(1/2))-3/2*I*c^2*polylog(4,-(1+I*a*x)^2/(a^2*x^2+1))+3*c^2*arctan(a*x)*ln(
1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I*c^2*arctan(a*x)^2*polylog(2,-(1+I*a*x)/
(a^2*x^2+1)^(1/2))-2*c^2*arctan(a*x)^3*ln((1+I*a*x)^2/(a^2*x^2+1)+1)+3*I*c
^2*arctan(a*x)^2*polylog(2,-(1+I*a*x)^2/(a^2*x^2+1))-3*c^2*arctan(a*x)*ln(
(1+I*a*x)^2/(a^2*x^2+1)+1)+12*I*c^2*polylog(4,(1+I*a*x)/(a^2*x^2+1)^(1/2))
)
```

## Fricas [F]

$$\int \frac{(c+a^2cx^2)^2\arctan(ax)^3}{x^3}\,dx=\int \frac{(a^2cx^2+c)^2\arctan(ax)^3}{x^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^3/x^3,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)^3/x^3, x)
```

## Sympy [F]

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^3}{x^3} \, dx = c^2\left(\int \frac{\operatorname{atan}^3(ax)}{x^3} \, dx + \int \frac{2a^2 \operatorname{atan}^3(ax)}{x} \, dx + \int a^4x \operatorname{atan}^3(ax) \, dx\right)$$

input
```
integrate((a**2*c*x**2+c)**2*atan(a*x)**3/x**3,x)
```

output
```
c**2*(Integral(atan(a*x)**3/x**3, x) + Integral(2*a**2*atan(a*x)**3/x, x)
+ Integral(a**4*x*atan(a*x)**3, x))
```

## Maxima [F]

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^3}{x^3} \, dx = \int \frac{(a^2cx^2+c)^2 \arctan(ax)^3}{x^3} \, dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^3/x^3,x, algorithm="maxima")
```

output
```
1/64*(12*a^4*c^2*x^2*integrate(4*x*arctan(a*x)^3 + x*arctan(a*x)*log(a^2*x
^2 + 1)^2, x) + 8*a^3*c^2*x^2*integrate(-1/8*(24*(a^2*x^2 + 1)*a*x*arctan(
a*x)^3 - 18*(a^2*x^2 + 1)*a*x*arctan(a*x)*log(a^2*x^2 + 1)^2 + 36*(a^2*x^2
 + 1)*arctan(a*x)^2*log(a^2*x^2 + 1) - 3*(a^2*x^2 + 1)*log(a^2*x^2 + 1)^3
- sqrt(a^2*x^2 + 1)*(12*sqrt(a^2*x^2 + 1)*arctan(a*x)^2*log(a^2*x^2 + 1) -
 sqrt(a^2*x^2 + 1)*log(a^2*x^2 + 1)^3 - (12*(a^2*x^2 + 1)^2*arctan(a*x)^2*
log(a^2*x^2 + 1) - (a^2*x^2 + 1)^2*log(a^2*x^2 + 1)^3)*cos(3*arctan(a*x))
+ 3*(12*(a^2*x^2 + 1)^(3/2)*arctan(a*x)^2*log(a^2*x^2 + 1) - (a^2*x^2 + 1)
^(3/2)*log(a^2*x^2 + 1)^3)*cos(2*arctan(a*x)) - 2*(4*(a^2*x^2 + 1)^2*arcta
n(a*x)^3 - 3*(a^2*x^2 + 1)^2*arctan(a*x)*log(a^2*x^2 + 1)^2)*sin(3*arctan(
a*x)) + 6*(4*(a^2*x^2 + 1)^(3/2)*arctan(a*x)^3 - 3*(a^2*x^2 + 1)^(3/2)*arc
tan(a*x)*log(a^2*x^2 + 1)^2)*sin(2*arctan(a*x))))/((a^2*x^2 + 1)^4*cos(3*a
rctan(a*x))^2 + (a^2*x^2 + 1)^4*sin(3*arctan(a*x))^2 - 6*(a^2*x^2 + 1)^(7/
2)*sin(3*arctan(a*x))*sin(2*arctan(a*x)) + 9*(a^2*x^2 + 1)^3*cos(2*arctan(
a*x))^2 + 9*(a^2*x^2 + 1)^3*sin(2*arctan(a*x))^2 + a^2*x^2 + 6*(a^2*x^2 +
1)^2*cos(2*arctan(a*x)) + 9*(a^2*x^2 + 1)^2 - 2*(3*(a^2*x^2 + 1)^(7/2)*cos
(2*arctan(a*x)) + (a^2*x^2 + 1)^(5/2))*cos(3*arctan(a*x)) + 6*((a^2*x^2 +
1)^2*a*x*sin(3*arctan(a*x)) - 3*(a^2*x^2 + 1)^(3/2)*a*x*sin(2*arctan(a*x))
 + (a^2*x^2 + 1)^2*cos(3*arctan(a*x)) - 3*(a^2*x^2 + 1)^(3/2)*cos(2*arctan
(a*x)) - sqrt(a^2*x^2 + 1))*sqrt(a^2*x^2 + 1) + 1), x) - 12*a^3*c^2*x^2...
```

## Giac [F]

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^3}{x^3}\,dx = \int \frac{(a^2cx^2+c)^2 \arctan(ax)^3}{x^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^3/x^3,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^2*arctan(a*x)^3/x^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^2\arctan(ax)^3}{x^3}\,dx=\int \frac{\operatorname{atan}(a\,x)^3\,(c\,a^2\,x^2+c)^2}{x^3}\,dx$$

input
```
int((atan(a*x)^3*(c + a^2*c*x^2)^2)/x^3,x)
```

output
```
int((atan(a*x)^3*(c + a^2*c*x^2)^2)/x^3, x)
```

## Reduce [F]

$$\int \frac{(c+a^2cx^2)^2\arctan(ax)^3}{x^3}\,dx$$
$$=\frac{c^2\Big(atan(ax)^3\,a^4x^4-atan(ax)^3-3atan(ax)^2\,a^3x^3-3atan(ax)^2\,ax+3atan(ax)\,a^4x^4-3atan(ax)-6\Big(}{2x^2}$$

input
```
int((a^2*c*x^2+c)^2*atan(a*x)^3/x^3,x)
```

output
```
(c**2*(atan(a*x)**3*a**4*x**4 - atan(a*x)**3 - 3*atan(a*x)**2*a**3*x**3 -
3*atan(a*x)**2*a*x + 3*atan(a*x)*a**4*x**4 - 3*atan(a*x) - 6*int(atan(a*x)
/(a**2*x**5 + x**3),x)*x**2 + 4*int(atan(a*x)**3/x,x)*a**2*x**2 - 6*int((a
tan(a*x)*x**3)/(a**2*x**2 + 1),x)*a**6*x**2 - 3*a**3*x**3 - 3*a*x))/(2*x**
2)
```

# 3.378 $\int \frac{(c+a^2cx^2)^2 \arctan(ax)^3}{x^4} \, dx$



## Optimal result

Integrand size = 22, antiderivative size = 311

$$
\begin{aligned}
\int \frac{(c+a^2cx^2)^2 \arctan(ax)^3}{x^4} \, dx = & -\frac{a^2c^2\arctan(ax)}{x} - \frac{1}{2}a^3c^2\arctan(ax)^2 \\
& - \frac{ac^2\arctan(ax)^2}{2x^2} - \frac{2}{3}ia^3c^2\arctan(ax)^3 \\
& - \frac{c^2\arctan(ax)^3}{3x^3} - \frac{2a^2c^2\arctan(ax)^3}{x} \\
& + a^4c^2x\arctan(ax)^3 + a^3c^2\log(x) \\
& + 3a^3c^2\arctan(ax)^2\log\left(\frac{2}{1+iax}\right) \\
& - \frac{1}{2}a^3c^2\log\left(1+a^2x^2\right) \\
& + 5a^3c^2\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right) \\
& - 5ia^3c^2\arctan(ax)\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right) \\
& + 3ia^3c^2\arctan(ax)\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right) \\
& + \frac{5}{2}a^3c^2\operatorname{PolyLog}\left(3,-1+\frac{2}{1-iax}\right) \\
& + \frac{3}{2}a^3c^2\operatorname{PolyLog}\left(3,1-\frac{2}{1+iax}\right)
\end{aligned}
$$

output
```
-a^2*c^2*arctan(a*x)/x-1/2*a^3*c^2*arctan(a*x)^2-1/2*a*c^2*arctan(a*x)^2/x
^2-2/3*I*a^3*c^2*arctan(a*x)^3-1/3*c^2*arctan(a*x)^3/x^3-2*a^2*c^2*arctan(
a*x)^3/x+a^4*c^2*x*arctan(a*x)^3+a^3*c^2*ln(x)+3*a^3*c^2*arctan(a*x)^2*ln(
2/(1+I*a*x))-1/2*a^3*c^2*ln(a^2*x^2+1)+5*a^3*c^2*arctan(a*x)^2*ln(2-2/(1-I
*a*x))-5*I*a^3*c^2*arctan(a*x)*polylog(2,-1+2/(1-I*a*x))+3*I*a^3*c^2*arcta
n(a*x)*polylog(2,1-2/(1+I*a*x))+5/2*a^3*c^2*polylog(3,-1+2/(1-I*a*x))+3/2*
a^3*c^2*polylog(3,1-2/(1+I*a*x))
```

## Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.93

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^3}{x^4} dx$$
$$= \frac{c^2\left(-5ia^3\pi^3x^3 - 24a^2x^2\arctan(ax) - 12ax\arctan(ax)^2 - 12a^3x^3\arctan(ax)^2 - 8\arctan(ax)^3 - 48a^2x^2\right.}{}$$

input
```
Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x]^3)/x^4,x]
```

output
```
(c^2*((-5*I)*a^3*Pi^3*x^3 - 24*a^2*x^2*ArcTan[a*x] - 12*a*x*ArcTan[a*x]^2
- 12*a^3*x^3*ArcTan[a*x]^2 - 8*ArcTan[a*x]^3 - 48*a^2*x^2*ArcTan[a*x]^3 +
(16*I)*a^3*x^3*ArcTan[a*x]^3 + 24*a^4*x^4*ArcTan[a*x]^3 + 120*a^3*x^3*ArcT
an[a*x]^2*Log[1 - E^((-2*I)*ArcTan[a*x])] + 72*a^3*x^3*ArcTan[a*x]^2*Log[1
 + E^((2*I)*ArcTan[a*x])] + 24*a^3*x^3*Log[(a*x)/Sqrt[1 + a^2*x^2]] + (120
*I)*a^3*x^3*ArcTan[a*x]*PolyLog[2, E^((-2*I)*ArcTan[a*x])] - (72*I)*a^3*x^
3*ArcTan[a*x]*PolyLog[2, -E^((2*I)*ArcTan[a*x])] + 60*a^3*x^3*PolyLog[3, E
^((-2*I)*ArcTan[a*x])] + 36*a^3*x^3*PolyLog[3, -E^((2*I)*ArcTan[a*x])]))/(
24*x^3)
```

## Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3 \left(a^2cx^2+c\right)^2}{x^4} \, dx$$

$$\Big\downarrow \text{ 5483}$$

$$\int \left(a^4c^2\arctan(ax)^3+\frac{2a^2c^2\arctan(ax)^3}{x^2}+\frac{c^2\arctan(ax)^3}{x^4}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$a^4c^2x\arctan(ax)^3-5ia^3c^2\arctan(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)+3ia^3c^2\arctan(ax)\operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right)-\frac{2}{3}ia^3c^2\arctan(ax)^3-\frac{1}{2}a^3c^2\arctan(ax)^2+3a^3c^2\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)+5a^3c^2\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)+\frac{5}{2}a^3c^2\operatorname{PolyLog}\left(3,\frac{2}{1-iax}-1\right)+\frac{3}{2}a^3c^2\operatorname{PolyLog}\left(3,1-\frac{2}{iax+1}\right)+a^3c^2\log(x)-\frac{2a^2c^2\arctan(ax)^3}{x}-\frac{a^2c^2\arctan(ax)}{x}-\frac{1}{2}a^3c^2\log\left(a^2x^2+1\right)-\frac{c^2\arctan(ax)^3}{3x^3}-\frac{ac^2\arctan(ax)^2}{2x^2}$$

input
```
Int[((c + a^2*c*x^2)^2*ArcTan[a*x]^3)/x^4,x]
```

output
```
-((a^2*c^2*ArcTan[a*x])/x) - (a^3*c^2*ArcTan[a*x]^2)/2 - (a*c^2*ArcTan[a*x
]^2)/(2*x^2) - ((2*I)/3)*a^3*c^2*ArcTan[a*x]^3 - (c^2*ArcTan[a*x]^3)/(3*x^
3) - (2*a^2*c^2*ArcTan[a*x]^3)/x + a^4*c^2*x*ArcTan[a*x]^3 + a^3*c^2*Log[x
] + 3*a^3*c^2*ArcTan[a*x]^2*Log[2/(1 + I*a*x)] - (a^3*c^2*Log[1 + a^2*x^2]
)/2 + 5*a^3*c^2*ArcTan[a*x]^2*Log[2 - 2/(1 - I*a*x)] - (5*I)*a^3*c^2*ArcTa
n[a*x]*PolyLog[2, -1 + 2/(1 - I*a*x)] + (3*I)*a^3*c^2*ArcTan[a*x]*PolyLog[
2, 1 - 2/(1 + I*a*x)] + (5*a^3*c^2*PolyLog[3, -1 + 2/(1 - I*a*x)])/2 + (3*
a^3*c^2*PolyLog[3, 1 - 2/(1 + I*a*x)])/2
```

**Defintions of rubi rules used**

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 12.31 (sec) , antiderivative size = 1924, normalized size of antiderivative = 6.19

Expression too large to display

input
```
int((a^2*c*x^2+c)^2*arctan(a*x)^3/x^4,x)
```

output
```
a^3*(c^2*arctan(a*x)^3*a*x-1/3*c^2*arctan(a*x)^3/a^3/x^3-2*c^2*arctan(a*x)
^3/a/x-c^2*(2*I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^
2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^
2)*arctan(a*x)^2-5/2*I*Pi*arctan(a*x)^2+10*I*arctan(a*x)*polylog(2,-(1+I*a
*x)/(a^2*x^2+1)^(1/2))+1/2*arctan(a*x)^2-ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-1)
-ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-5/2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1
)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a
*x)^2/(a^2*x^2+1)+1))*arctan(a*x)^2+1/2*arctan(a*x)^2/a^2/x^2+5/2*I*Pi*csg
n(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*
a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2+5/2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x
^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1
+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2-10*polylog(3,(1+I*a*x)/(a^2*x^2+
1)^(1/2))-10*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+10*I*arctan(a*x)*poly
log(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+3*I*arctan(a*x)*polylog(2,-(1+I*a*x)^2/
(a^2*x^2+1))+5/2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I*((1+I*a*x
)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2+2*I*Pi*csg
n(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3*arctan(a*x)^2
+2*I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^3*arctan(a*x)^2-2*I*Pi*csgn(I*((1+
I*a*x)^2/(a^2*x^2+1)+1)^2)^3*arctan(a*x)^2-5/2*I*Pi*csgn(I*((1+I*a*x)^2/(a
^2*x^2+1)-1))*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(...
```

## Fricas [F]

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^3}{x^4}\,dx = \int \frac{(a^2cx^2+c)^2 \arctan(ax)^3}{x^4}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^3/x^4,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)^3/x^4, x)
```

## Sympy [F]

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^3}{x^4}\,dx = c^2\left(\int a^4 \operatorname{atan}^3(ax)\,dx + \int \frac{\operatorname{atan}^3(ax)}{x^4}\,dx + \int \frac{2a^2\operatorname{atan}^3(ax)}{x^2}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**2*atan(a*x)**3/x**4,x)
```

output
```
c**2*(Integral(a**4*atan(a*x)**3, x) + Integral(atan(a*x)**3/x**4, x) + In
tegral(2*a**2*atan(a*x)**3/x**2, x))
```

## Maxima [F]

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^3}{x^4}\,dx = \int \frac{(a^2cx^2+c)^2 \arctan(ax)^3}{x^4}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^3/x^4,x, algorithm="maxima")
```

output
```
1/192*(3*(42*a^3*c^2*arctan(a*x)^4 + 1792*a^6*c^2*integrate(1/32*x^6*arcta
n(a*x)^3/(a^2*x^6 + x^4), x) + 192*a^6*c^2*integrate(1/32*x^6*arctan(a*x)*
log(a^2*x^2 + 1)^2/(a^2*x^6 + x^4), x) + 768*a^6*c^2*integrate(1/32*x^6*ar
ctan(a*x)*log(a^2*x^2 + 1)/(a^2*x^6 + x^4), x) - 768*a^5*c^2*integrate(1/3
2*x^5*arctan(a*x)^2/(a^2*x^6 + x^4), x) + a^3*c^2*log(a^2*x^2 + 1)^3 + 576
*a^4*c^2*integrate(1/32*x^4*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*x^6 + x^4)
, x) - 1536*a^4*c^2*integrate(1/32*x^4*arctan(a*x)*log(a^2*x^2 + 1)/(a^2*x
^6 + x^4), x) + 1536*a^3*c^2*integrate(1/32*x^3*arctan(a*x)^2/(a^2*x^6 + x
^4), x) - 384*a^3*c^2*integrate(1/32*x^3*log(a^2*x^2 + 1)^2/(a^2*x^6 + x^4
), x) + 5376*a^2*c^2*integrate(1/32*x^2*arctan(a*x)^3/(a^2*x^6 + x^4), x)
+ 576*a^2*c^2*integrate(1/32*x^2*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*x^6 +
 x^4), x) - 256*a^2*c^2*integrate(1/32*x^2*arctan(a*x)*log(a^2*x^2 + 1)/(a
^2*x^6 + x^4), x) + 256*a*c^2*integrate(1/32*x*arctan(a*x)^2/(a^2*x^6 + x^
4), x) - 64*a*c^2*integrate(1/32*x*log(a^2*x^2 + 1)^2/(a^2*x^6 + x^4), x)
+ 1792*c^2*integrate(1/32*arctan(a*x)^3/(a^2*x^6 + x^4), x) + 192*c^2*inte
grate(1/32*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*x^6 + x^4), x))*x^3 + 8*(3*
a^4*c^2*x^4 - 6*a^2*c^2*x^2 - c^2)*arctan(a*x)^3 - 6*(3*a^4*c^2*x^4 - 6*a^
2*c^2*x^2 - c^2)*arctan(a*x)*log(a^2*x^2 + 1)^2)/x^3
```

## Giac [F]

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^3}{x^4} \, dx = \int \frac{(a^2cx^2+c)^2 \arctan(ax)^3}{x^4} \, dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^3/x^4,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^2*arctan(a*x)^3/x^4, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^3}{x^4}\,dx = \int \frac{\operatorname{atan}(a\,x)^3\,(c\,a^2\,x^2+c)^2}{x^4}\,dx$$

input
```
int((atan(a*x)^3*(c + a^2*c*x^2)^2)/x^4,x)
```

output
```
int((atan(a*x)^3*(c + a^2*c*x^2)^2)/x^4, x)
```

## Reduce [F]

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^3}{x^4}\,dx$$
$$= \frac{c^2\Big(-12atan(ax)^3\,a^2x^2 - 2atan(ax)^3 - 3atan(ax)^2\,a^3x^3 - 3atan(ax)^2\,ax - 6atan(ax)\,a^2x^2 + 6\big(\int atan(}{6x^3}$$

input
```
int((a^2*c*x^2+c)^2*atan(a*x)^3/x^4,x)
```

output
```
(c**2*( - 12*atan(a*x)**3*a**2*x**2 - 2*atan(a*x)**3 - 3*atan(a*x)**2*a**3
*x**3 - 3*atan(a*x)**2*a*x - 6*atan(a*x)*a**2*x**2 + 6*int(atan(a*x)**3,x)
*a**4*x**3 + 30*int(atan(a*x)**2/(a**2*x**3 + x),x)*a**3*x**3 - 3*log(a**2
*x**2 + 1)*a**3*x**3 + 6*log(x)*a**3*x**3))/(6*x**3)
```

# 3.379 $\int x^3(c+a^2cx^2)^3\arctan(ax)^3\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 381

$$\begin{aligned}\int x^3(c+a^2cx^2)^3\arctan(ax)^3\,dx=&\ \frac{389c^3x}{12600a^3}-\frac{17c^3x^3}{9450a}-\frac{1}{252}ac^3x^5-\frac{1}{840}a^3c^3x^7\\&-\frac{389c^3\arctan(ax)}{12600a^4}-\frac{107c^3x^2\arctan(ax)}{4200a^2}\\&+\frac{53c^3x^4\arctan(ax)}{2100}+\frac{71a^2c^3x^6\arctan(ax)}{2520}\\&+\frac{1}{120}a^4c^3x^8\arctan(ax)+\frac{26ic^3\arctan(ax)^2}{525a^4}\\&+\frac{3c^3x\arctan(ax)^2}{40a^3}-\frac{c^3x^3\arctan(ax)^2}{40a}\\&-\frac{27}{200}ac^3x^5\arctan(ax)^2-\frac{33}{280}a^3c^3x^7\arctan(ax)^2\\&-\frac{1}{30}a^5c^3x^9\arctan(ax)^2-\frac{c^3\arctan(ax)^3}{40a^4}\\&+\frac{1}{4}c^3x^4\arctan(ax)^3+\frac{1}{2}a^2c^3x^6\arctan(ax)^3\\&+\frac{3}{8}a^4c^3x^8\arctan(ax)^3+\frac{1}{10}a^6c^3x^{10}\arctan(ax)^3\\&+\frac{52c^3\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{525a^4}\\&+\frac{26ic^3\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{525a^4}\end{aligned}$$

output
```
389/12600*c^3*x/a^3-17/9450*c^3*x^3/a-1/252*a*c^3*x^5-1/840*a^3*c^3*x^7-38
9/12600*c^3*arctan(a*x)/a^4-107/4200*c^3*x^2*arctan(a*x)/a^2+53/2100*c^3*x
^4*arctan(a*x)+71/2520*a^2*c^3*x^6*arctan(a*x)+1/120*a^4*c^3*x^8*arctan(a*
x)+26/525*I*c^3*polylog(2,1-2/(1+I*a*x))/a^4+3/40*c^3*x*arctan(a*x)^2/a^3-
1/40*c^3*x^3*arctan(a*x)^2/a-27/200*a*c^3*x^5*arctan(a*x)^2-33/280*a^3*c^3
*x^7*arctan(a*x)^2-1/30*a^5*c^3*x^9*arctan(a*x)^2-1/40*c^3*arctan(a*x)^3/a
^4+1/4*c^3*x^4*arctan(a*x)^3+1/2*a^2*c^3*x^6*arctan(a*x)^3+3/8*a^4*c^3*x^8
*arctan(a*x)^3+1/10*a^6*c^3*x^10*arctan(a*x)^3+52/525*c^3*arctan(a*x)*ln(2
/(1+I*a*x))/a^4+26/525*I*c^3*arctan(a*x)^2/a^4
```

## Mathematica [A] (verified)

Time = 1.45 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.50

$$\int x^3 \left(c+a^2cx^2\right)^3 \arctan(ax)^3 \, dx$$
$$= \frac{c^3\Big(-ax(-1167+68a^2x^2+150a^4x^4+45a^6x^6)-9(208i-315ax+105a^3x^3+567a^5x^5+495a^7x^7+14}{}$$

input
```
Integrate[x^3*(c + a^2*c*x^2)^3*ArcTan[a*x]^3,x]
```

output
```
(c^3*(-(a*x*(-1167 + 68*a^2*x^2 + 150*a^4*x^4 + 45*a^6*x^6)) - 9*(208*I -
315*a*x + 105*a^3*x^3 + 567*a^5*x^5 + 495*a^7*x^7 + 140*a^9*x^9)*ArcTan[a*
x]^2 + 945*(1 + a^2*x^2)^4*(-1 + 4*a^2*x^2)*ArcTan[a*x]^3 + 3*ArcTan[a*x]*
(-389 - 321*a^2*x^2 + 318*a^4*x^4 + 355*a^6*x^6 + 105*a^8*x^8 + 1248*Log[1
 + E^((2*I)*ArcTan[a*x])]) - (1872*I)*PolyLog[2, -E^((2*I)*ArcTan[a*x])]))
/(37800*a^4)
```

## Rubi [A] (verified)

Time = 3.86 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \arctan(ax)^3 \left(a^2cx^2+c\right)^3 \, dx$$

$$\Big\downarrow \text{5483}$$

$$\int \left(a^6c^3x^9 \arctan(ax)^3 + 3a^4c^3x^7 \arctan(ax)^3 + 3a^2c^3x^5 \arctan(ax)^3 + c^3x^3 \arctan(ax)^3\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{10}a^6c^3x^{10}\arctan(ax)^3 - \frac{1}{30}a^5c^3x^9\arctan(ax)^2 + \frac{3}{8}a^4c^3x^8\arctan(ax)^3 + \frac{1}{120}a^4c^3x^8\arctan(ax) - \frac{c^3\arctan(ax)^3}{40a^4} + \frac{26ic^3\arctan(ax)^2}{525a^4} - \frac{389c^3\arctan(ax)}{12600a^4} + \frac{52c^3\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{525a^4} + \frac{26ic^3\operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{525a^4} - \frac{33}{280}a^3c^3x^7\arctan(ax)^2 + \frac{3c^3x\arctan(ax)^2}{40a^3} - \frac{1}{840}a^3c^3x^7 + \frac{389c^3x}{12600a^3} + \frac{1}{2}a^2c^3x^6\arctan(ax)^3 + \frac{71a^2c^3x^6\arctan(ax)}{2520} - \frac{107c^3x^2\arctan(ax)}{4200a^2} - \frac{27}{200}ac^3x^5\arctan(ax)^2 + \frac{1}{4}c^3x^4\arctan(ax)^3 + \frac{53c^3x^4\arctan(ax)}{2100} - \frac{c^3x^3\arctan(ax)^2}{40a} - \frac{1}{252}ac^3x^5 - \frac{17c^3x^3}{9450a}$$

input
```
Int[x^3*(c + a^2*c*x^2)^3*ArcTan[a*x]^3,x]
```

output
```
(389*c^3*x)/(12600*a^3) - (17*c^3*x^3)/(9450*a) - (a*c^3*x^5)/252 - (a^3*c
^3*x^7)/840 - (389*c^3*ArcTan[a*x])/(12600*a^4) - (107*c^3*x^2*ArcTan[a*x]
)/(4200*a^2) + (53*c^3*x^4*ArcTan[a*x])/2100 + (71*a^2*c^3*x^6*ArcTan[a*x]
)/2520 + (a^4*c^3*x^8*ArcTan[a*x])/120 + (((26*I)/525)*c^3*ArcTan[a*x]^2)/
a^4 + (3*c^3*x*ArcTan[a*x]^2)/(40*a^3) - (c^3*x^3*ArcTan[a*x]^2)/(40*a) -
(27*a*c^3*x^5*ArcTan[a*x]^2)/200 - (33*a^3*c^3*x^7*ArcTan[a*x]^2)/280 - (a
^5*c^3*x^9*ArcTan[a*x]^2)/30 - (c^3*ArcTan[a*x]^3)/(40*a^4) + (c^3*x^4*Arc
Tan[a*x]^3)/4 + (a^2*c^3*x^6*ArcTan[a*x]^3)/2 + (3*a^4*c^3*x^8*ArcTan[a*x]
^3)/8 + (a^6*c^3*x^10*ArcTan[a*x]^3)/10 + (52*c^3*ArcTan[a*x]*Log[2/(1 + I
*a*x)])/(525*a^4) + (((26*I)/525)*c^3*PolyLog[2, 1 - 2/(1 + I*a*x)])/a^4
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

## Maple [A] (verified)

Time = 6.90 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.01

| method | result |
|---|---|
| derivativedivides | $\frac{c^3 \arctan(ax)^3 a^{10} x^{10}}{10} + \frac{3c^3 \arctan(ax)^3 a^8 x^8}{8} + \frac{a^6 c^3 x^6 \arctan(ax)^3}{2} + \frac{a^4 c^3 x^4 \arctan(ax)^3}{4} - \frac{c^3 \arctan(ax)^3}{40} - \frac{3c^3\left(\frac{4\arctan(ax)^2}{9}\right.}{}$ |
| default | $\frac{c^3 \arctan(ax)^3 a^{10} x^{10}}{10} + \frac{3c^3 \arctan(ax)^3 a^8 x^8}{8} + \frac{a^6 c^3 x^6 \arctan(ax)^3}{2} + \frac{a^4 c^3 x^4 \arctan(ax)^3}{4} - \frac{c^3 \arctan(ax)^3}{40} - \frac{3c^3\left(\frac{4\arctan(ax)^2}{9}\right.}{}$ |
| parts | $\frac{a^6 c^3 x^{10} \arctan(ax)^3}{10} + \frac{3a^4 c^3 x^8 \arctan(ax)^3}{8} + \frac{a^2 c^3 x^6 \arctan(ax)^3}{2} + \frac{c^3 x^4 \arctan(ax)^3}{4} - \frac{3c^3\left(\frac{4a^5\arctan(ax)^2}{9}\right.}{}$ |

input
```
int(x^3*(a^2*c*x^2+c)^3*arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a^4*(1/10*c^3*arctan(a*x)^3*a^10*x^10+3/8*c^3*arctan(a*x)^3*a^8*x^8+1/2*
a^6*c^3*x^6*arctan(a*x)^3+1/4*a^4*c^3*x^4*arctan(a*x)^3-1/40*c^3*arctan(a*
x)^3-3/40*c^3*(4/9*arctan(a*x)^2*a^9*x^9+11/7*arctan(a*x)^2*a^7*x^7+9/5*a^
5*arctan(a*x)^2*x^5+1/3*a^3*arctan(a*x)^2*x^3-a*arctan(a*x)^2*x-1/9*arctan
(a*x)*a^8*x^8-71/189*a^6*arctan(a*x)*x^6-106/315*x^4*arctan(a*x)*a^4+107/3
15*x^2*a^2*arctan(a*x)+208/315*arctan(a*x)*ln(a^2*x^2+1)+1/63*a^7*x^7+10/1
89*a^5*x^5+68/2835*a^3*x^3-389/945*a*x+389/945*arctan(a*x)+104/315*I*(ln(a
*x-I)*ln(a^2*x^2+1)-1/2*ln(a*x-I)^2-dilog(-1/2*I*(a*x+I))-ln(a*x-I)*ln(-1/
2*I*(a*x+I)))-104/315*I*(ln(a*x+I)*ln(a^2*x^2+1)-1/2*ln(a*x+I)^2-dilog(1/2
*I*(a*x-I))-ln(a*x+I)*ln(1/2*I*(a*x-I)))))
```

## Fricas [F]

$$\int x^3 \left(c + a^2 c x^2\right)^3 \arctan(ax)^3 \, dx = \int \left(a^2 c x^2 + c\right)^3 x^3 \arctan\left(ax\right)^3 \, dx$$

input
```
integrate(x^3*(a^2*c*x^2+c)^3*arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^6*c^3*x^9 + 3*a^4*c^3*x^7 + 3*a^2*c^3*x^5 + c^3*x^3)*arctan(a*
x)^3, x)
```

## Sympy [F]

$$\int x^3 \left(c + a^2 c x^2\right)^3 \arctan(ax)^3 \, dx = c^3 \left(\int x^3 \operatorname{atan}^3(ax)\, dx + \int 3a^2 x^5 \operatorname{atan}^3(ax)\, dx + \int 3a^4 x^7 \operatorname{atan}^3(ax)\, dx + \int a^6 x^9 \operatorname{atan}^3(ax)\, dx\right)$$

input
```
integrate(x**3*(a**2*c*x**2+c)**3*atan(a*x)**3,x)
```

output
```
c**3*(Integral(x**3*atan(a*x)**3, x) + Integral(3*a**2*x**5*atan(a*x)**3,
x) + Integral(3*a**4*x**7*atan(a*x)**3, x) + Integral(a**6*x**9*atan(a*x)*
*3, x))
```

## Maxima [F]

$$\int x^3 (c + a^2 c x^2)^3 \arctan(ax)^3 \, dx = \int (a^2 c x^2 + c)^3 x^3 \arctan(ax)^3 \, dx$$

input
```
integrate(x^3*(a^2*c*x^2+c)^3*arctan(a*x)^3,x, algorithm="maxima")
```

output
```
1/67200*(420*(5376000*a^11*c^3*integrate(1/67200*x^11*arctan(a*x)^3/(a^5*x
^2 + a^3), x) - 806400*a^10*c^3*integrate(1/67200*x^10*arctan(a*x)^2/(a^5*
x^2 + a^3), x) - 201600*a^10*c^3*integrate(1/67200*x^10*log(a^2*x^2 + 1)^2
/(a^5*x^2 + a^3), x) - 89600*a^10*c^3*integrate(1/67200*x^10*log(a^2*x^2 +
 1)/(a^5*x^2 + a^3), x) + 21504000*a^9*c^3*integrate(1/67200*x^9*arctan(a*
x)^3/(a^5*x^2 + a^3), x) + 179200*a^9*c^3*integrate(1/67200*x^9*arctan(a*x
)/(a^5*x^2 + a^3), x) - 3024000*a^8*c^3*integrate(1/67200*x^8*arctan(a*x)^
2/(a^5*x^2 + a^3), x) - 756000*a^8*c^3*integrate(1/67200*x^8*log(a^2*x^2 +
 1)^2/(a^5*x^2 + a^3), x) - 316800*a^8*c^3*integrate(1/67200*x^8*log(a^2*x
^2 + 1)/(a^5*x^2 + a^3), x) + 32256000*a^7*c^3*integrate(1/67200*x^7*arcta
n(a*x)^3/(a^5*x^2 + a^3), x) + 633600*a^7*c^3*integrate(1/67200*x^7*arctan
(a*x)/(a^5*x^2 + a^3), x) - 4032000*a^6*c^3*integrate(1/67200*x^6*arctan(a
*x)^2/(a^5*x^2 + a^3), x) - 1008000*a^6*c^3*integrate(1/67200*x^6*log(a^2*
x^2 + 1)^2/(a^5*x^2 + a^3), x) - 362880*a^6*c^3*integrate(1/67200*x^6*log(
a^2*x^2 + 1)/(a^5*x^2 + a^3), x) + 21504000*a^5*c^3*integrate(1/67200*x^5*
arctan(a*x)^3/(a^5*x^2 + a^3), x) + 725760*a^5*c^3*integrate(1/67200*x^5*a
rctan(a*x)/(a^5*x^2 + a^3), x) - 2016000*a^4*c^3*integrate(1/67200*x^4*arc
tan(a*x)^2/(a^5*x^2 + a^3), x) - 504000*a^4*c^3*integrate(1/67200*x^4*log(
a^2*x^2 + 1)^2/(a^5*x^2 + a^3), x) - 67200*a^4*c^3*integrate(1/67200*x^4*l
og(a^2*x^2 + 1)/(a^5*x^2 + a^3), x) + 5376000*a^3*c^3*integrate(1/67200...
```

## Giac [F]

$$\int x^3 (c + a^2 c x^2)^3 \arctan(ax)^3 \, dx = \int (a^2 c x^2 + c)^3 x^3 \arctan(ax)^3 \, dx$$

input
```
integrate(x^3*(a^2*c*x^2+c)^3*arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^3*x^3*arctan(a*x)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int x^3 \left(c+a^2cx^2\right)^3 \arctan(ax)^3 \, dx = \int x^3 \operatorname{atan}(a\,x)^3 \left(c\,a^2\,x^2+c\right)^3 dx$$

input
```
int(x^3*atan(a*x)^3*(c + a^2*c*x^2)^3,x)
```

output
```
int(x^3*atan(a*x)^3*(c + a^2*c*x^2)^3, x)
```

## Reduce [F]

$$\int x^3 \left(c+a^2cx^2\right)^3 \arctan(ax)^3 \, dx$$
$$= \frac{c^3 \Big(3780 atan(ax)^3 a^{10}x^{10} + 14175 atan(ax)^3 a^8x^8 + 18900 atan(ax)^3 a^6x^6 + 9450 atan(ax)^3 a^4x^4 - 945 at}{}$$

input
```
int(x^3*(a^2*c*x^2+c)^3*atan(a*x)^3,x)
```

output
```
(c**3*(3780*atan(a*x)**3*a**10*x**10 + 14175*atan(a*x)**3*a**8*x**8 + 1890
0*atan(a*x)**3*a**6*x**6 + 9450*atan(a*x)**3*a**4*x**4 - 945*atan(a*x)**3
- 1260*atan(a*x)**2*a**9*x**9 - 4455*atan(a*x)**2*a**7*x**7 - 5103*atan(a*
x)**2*a**5*x**5 - 945*atan(a*x)**2*a**3*x**3 + 2835*atan(a*x)**2*a*x + 315
*atan(a*x)*a**8*x**8 + 1065*atan(a*x)*a**6*x**6 + 954*atan(a*x)*a**4*x**4
- 963*atan(a*x)*a**2*x**2 - 1167*atan(a*x) - 3744*int((atan(a*x)*x)/(a**2*
x**2 + 1),x)*a**2 - 45*a**7*x**7 - 150*a**5*x**5 - 68*a**3*x**3 + 1167*a*x
))/(37800*a**4)
```

# 3.380 $\int x^2 (c + a^2 c x^2)^3 \arctan(ax)^3 \, dx$



## Optimal result

Integrand size = 22, antiderivative size = 389

$$\begin{aligned}\int x^2 (c + a^2 c x^2)^3 \arctan(ax)^3 \, dx = & -\frac{107c^3x^2}{7560a} - \frac{11ac^3x^4}{1260} - \frac{1}{504}a^3c^3x^6 \\ & - \frac{47c^3x\arctan(ax)}{1260a^2} + \frac{239c^3x^3\arctan(ax)}{3780} \\ & + \frac{59a^2c^3x^5\arctan(ax)}{1260} + \frac{1}{84}a^4c^3x^7\arctan(ax) \\ & + \frac{47c^3\arctan(ax)^2}{2520a^3} - \frac{8c^3x^2\arctan(ax)^2}{105a} \\ & - \frac{89}{420}ac^3x^4\arctan(ax)^2 - \frac{10}{63}a^3c^3x^6\arctan(ax)^2 \\ & - \frac{1}{24}a^5c^3x^8\arctan(ax)^2 - \frac{16ic^3\arctan(ax)^3}{315a^3} \\ & + \frac{1}{3}c^3x^3\arctan(ax)^3 + \frac{3}{5}a^2c^3x^5\arctan(ax)^3 \\ & + \frac{3}{7}a^4c^3x^7\arctan(ax)^3 + \frac{1}{9}a^6c^3x^9\arctan(ax)^3 \\ & - \frac{16c^3\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{105a^3} + \frac{31c^3\log(1+a^2x^2)}{945a^3} \\ & - \frac{16ic^3\arctan(ax)\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{105a^3} \\ & - \frac{8c^3\operatorname{PolyLog}\left(3,1-\frac{2}{1+iax}\right)}{105a^3}\end{aligned}$$

output
```
-107/7560*c^3*x^2/a-11/1260*a*c^3*x^4-1/504*a^3*c^3*x^6-47/1260*c^3*x*arct
an(a*x)/a^2+239/3780*c^3*x^3*arctan(a*x)+59/1260*a^2*c^3*x^5*arctan(a*x)+1
/84*a^4*c^3*x^7*arctan(a*x)+47/2520*c^3*arctan(a*x)^2/a^3-8/105*c^3*x^2*ar
ctan(a*x)^2/a-89/420*a*c^3*x^4*arctan(a*x)^2-10/63*a^3*c^3*x^6*arctan(a*x)
^2-1/24*a^5*c^3*x^8*arctan(a*x)^2-16/315*I*c^3*arctan(a*x)^3/a^3+1/3*c^3*x
^3*arctan(a*x)^3+3/5*a^2*c^3*x^5*arctan(a*x)^3+3/7*a^4*c^3*x^7*arctan(a*x)
^3+1/9*a^6*c^3*x^9*arctan(a*x)^3-16/105*c^3*arctan(a*x)^2*ln(2/(1+I*a*x))/
a^3+31/945*c^3*ln(a^2*x^2+1)/a^3-16/105*I*c^3*arctan(a*x)*polylog(2,1-2/(1
+I*a*x))/a^3-8/105*c^3*polylog(3,1-2/(1+I*a*x))/a^3
```

## Mathematica [A] (verified)

Time = 1.39 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.72

$$\int x^2 (c + a^2 c x^2)^3 \arctan(ax)^3 \, dx$$
$$= \frac{c^3 (-56 - 107a^2x^2 - 66a^4x^4 - 15a^6x^6 - 282ax \arctan(ax) + 478a^3x^3 \arctan(ax) + 354a^5x^5 \arctan(ax)}{}$$

input
```
Integrate[x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^3,x]
```

output
```
(c^3*(-56 - 107*a^2*x^2 - 66*a^4*x^4 - 15*a^6*x^6 - 282*a*x*ArcTan[a*x] +
478*a^3*x^3*ArcTan[a*x] + 354*a^5*x^5*ArcTan[a*x] + 90*a^7*x^7*ArcTan[a*x]
 + 141*ArcTan[a*x]^2 - 576*a^2*x^2*ArcTan[a*x]^2 - 1602*a^4*x^4*ArcTan[a*x
]^2 - 1200*a^6*x^6*ArcTan[a*x]^2 - 315*a^8*x^8*ArcTan[a*x]^2 + (384*I)*Arc
Tan[a*x]^3 + 2520*a^3*x^3*ArcTan[a*x]^3 + 4536*a^5*x^5*ArcTan[a*x]^3 + 324
0*a^7*x^7*ArcTan[a*x]^3 + 840*a^9*x^9*ArcTan[a*x]^3 - 1152*ArcTan[a*x]^2*L
og[1 + E^((2*I)*ArcTan[a*x])] + 248*Log[1 + a^2*x^2] + (1152*I)*ArcTan[a*x
]*PolyLog[2, -E^((2*I)*ArcTan[a*x])] - 576*PolyLog[3, -E^((2*I)*ArcTan[a*x
])]))/(7560*a^3)
```

## Rubi [A] (verified)

Time = 3.19 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arctan(ax)^3 \left(a^2cx^2+c\right)^3 dx$$

$$\Big\downarrow \text{ 5483}$$

$$\int \left(a^6c^3x^8\arctan(ax)^3+3a^4c^3x^6\arctan(ax)^3+3a^2c^3x^4\arctan(ax)^3+c^3x^2\arctan(ax)^3\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{9}a^6c^3x^9\arctan(ax)^3-\frac{1}{24}a^5c^3x^8\arctan(ax)^2+\frac{3}{7}a^4c^3x^7\arctan(ax)^3+\frac{1}{84}a^4c^3x^7\arctan(ax)-\frac{16ic^3\arctan(ax)\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{105a^3}-\frac{10}{63}a^3c^3x^6\arctan(ax)^2-\frac{16ic^3\arctan(ax)^3}{315a^3}+\frac{47c^3\arctan(ax)^2}{2520a^3}-\frac{16c^3\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{105a^3}-\frac{8c^3\text{PolyLog}\left(3,1-\frac{2}{iax+1}\right)}{105a^3}-\frac{1}{504}a^3c^3x^6+\frac{3}{5}a^2c^3x^5\arctan(ax)^3+\frac{59a^2c^3x^5\arctan(ax)}{1260}-\frac{47c^3x\arctan(ax)}{1260a^2}+\frac{31c^3\log\left(a^2x^2+1\right)}{945a^3}-\frac{89}{420}ac^3x^4\arctan(ax)^2+\frac{1}{3}c^3x^3\arctan(ax)^3+\frac{239c^3x^3\arctan(ax)}{3780}-\frac{8c^3x^2\arctan(ax)^2}{105a}-\frac{11ac^3x^4}{1260}-\frac{107c^3x^2}{7560a}$$

input
```
Int[x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^3,x]
```

output
```
(-107*c^3*x^2)/(7560*a) - (11*a*c^3*x^4)/1260 - (a^3*c^3*x^6)/504 - (47*c^
3*x*ArcTan[a*x])/(1260*a^2) + (239*c^3*x^3*ArcTan[a*x])/3780 + (59*a^2*c^3
*x^5*ArcTan[a*x])/1260 + (a^4*c^3*x^7*ArcTan[a*x])/84 + (47*c^3*ArcTan[a*x
]^2)/(2520*a^3) - (8*c^3*x^2*ArcTan[a*x]^2)/(105*a) - (89*a*c^3*x^4*ArcTan
[a*x]^2)/420 - (10*a^3*c^3*x^6*ArcTan[a*x]^2)/63 - (a^5*c^3*x^8*ArcTan[a*x
]^2)/24 - (((16*I)/315)*c^3*ArcTan[a*x]^3)/a^3 + (c^3*x^3*ArcTan[a*x]^3)/3
 + (3*a^2*c^3*x^5*ArcTan[a*x]^3)/5 + (3*a^4*c^3*x^7*ArcTan[a*x]^3)/7 + (a^
6*c^3*x^9*ArcTan[a*x]^3)/9 - (16*c^3*ArcTan[a*x]^2*Log[2/(1 + I*a*x)])/(10
5*a^3) + (31*c^3*Log[1 + a^2*x^2])/(945*a^3) - (((16*I)/105)*c^3*ArcTan[a*
x]*PolyLog[2, 1 - 2/(1 + I*a*x)])/a^3 - (8*c^3*PolyLog[3, 1 - 2/(1 + I*a*x
)])/(105*a^3)
```

**Defintions of rubi rules used**

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 136.07 (sec) , antiderivative size = 1576, normalized size of antiderivative = 4.05

| method | result | size |
|---|---|---|
| derivativedivides | Expression too large to display | 1576 |
| default | Expression too large to display | 1576 |
| parts | Expression too large to display | 1576 |

input
```
int(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(1/9*c^3*arctan(a*x)^3*a^9*x^9+3/7*c^3*arctan(a*x)^3*a^7*x^7+3/5*c^3
*arctan(a*x)^3*a^5*x^5+1/3*c^3*arctan(a*x)^3*a^3*x^3-1/105*c^3*(-4*I*Pi*cs
gn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I
*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*arctan(a*x)^2-47/2
4*arctan(a*x)^2+8*arctan(a*x)^2*x^2*a^2-53/24*(a*x+I)^4-4*I*Pi*csgn(I*(1+I
*a*x)/(a^2*x^2+1)^(1/2))^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*arctan(a*x)^2+4
*I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I
*a*x)^2/(a^2*x^2+1)+1)^2)^2*arctan(a*x)^2+4*I*Pi*csgn(I/((1+I*a*x)^2/(a^2*
x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)
^2*arctan(a*x)^2-8*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a
*x)^2/(a^2*x^2+1)+1)^2)^2*arctan(a*x)^2+89/4*a^4*arctan(a*x)^2*x^4+5/24*(a
*x+I)^6+50/3*a^6*x^6*arctan(a*x)^2+35/8*arctan(a*x)^2*a^8*x^8+8*arctan(a*x
)*(a*x-I)+16*arctan(a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2))+8*polylog(3,-(1
+I*a*x)^2/(a^2*x^2+1))-320/3*arctan(a*x)*(a*x-I)^4*(a*x+I)+62/9*ln((1+I*a*
x)^2/(a^2*x^2+1)+1)+8*I*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*csgn(I*(1+I
*a*x)^2/(a^2*x^2+1))^2*arctan(a*x)^2+4*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1
)+1))^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*arctan(a*x)^2-8/9*(a*x+I)^2-
16*I*arctan(a*x)*polylog(2,-(1+I*a*x)^2/(a^2*x^2+1))-640/3*arctan(a*x)*(a*
x-I)^2*(a*x+I)^3-35/4*arctan(a*x)*(a*x-I)*(a*x+I)^6-175/4*arctan(a*x)*(a*x
-I)^3*(a*x+I)^4-8*arctan(a*x)^2*ln(a^2*x^2+1)+115*I*arctan(a*x)*(a*x-I)...
```

## Fricas [F]

$$\int x^2 \left(c+a^2 c x^2\right)^3 \arctan(ax)^3 \, dx = \int \left(a^2 c x^2 + c\right)^3 x^2 \arctan\left(ax\right)^3 \, dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^6*c^3*x^8 + 3*a^4*c^3*x^6 + 3*a^2*c^3*x^4 + c^3*x^2)*arctan(a*
x)^3, x)
```

## Sympy [F]

$$\int x^2(c+a^2cx^2)^3\arctan(ax)^3\,dx = c^3\left(\int x^2\operatorname{atan}^3(ax)\,dx+\int 3a^2x^4\operatorname{atan}^3(ax)\,dx + \int 3a^4x^6\operatorname{atan}^3(ax)\,dx+\int a^6x^8\operatorname{atan}^3(ax)\,dx\right)$$

input
```
integrate(x**2*(a**2*c*x**2+c)**3*atan(a*x)**3,x)
```

output
```
c**3*(Integral(x**2*atan(a*x)**3, x) + Integral(3*a**2*x**4*atan(a*x)**3,
x) + Integral(3*a**4*x**6*atan(a*x)**3, x) + Integral(a**6*x**8*atan(a*x)*
*3, x))
```

## Maxima [F]

$$\int x^2(c+a^2cx^2)^3\arctan(ax)^3\,dx = \int (a^2cx^2+c)^3x^2\arctan(ax)^3\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^3,x, algorithm="maxima")
```

output
```
1/2520*(35*a^6*c^3*x^9 + 135*a^4*c^3*x^7 + 189*a^2*c^3*x^5 + 105*c^3*x^3)*
arctan(a*x)^3 - 1/3360*(35*a^6*c^3*x^9 + 135*a^4*c^3*x^7 + 189*a^2*c^3*x^5
 + 105*c^3*x^3)*arctan(a*x)*log(a^2*x^2 + 1)^2 + integrate(1/3360*(2940*(a
^8*c^3*x^10 + 4*a^6*c^3*x^8 + 6*a^4*c^3*x^6 + 4*a^2*c^3*x^4 + c^3*x^2)*arc
tan(a*x)^3 - 4*(35*a^7*c^3*x^9 + 135*a^5*c^3*x^7 + 189*a^3*c^3*x^5 + 105*a
*c^3*x^3)*arctan(a*x)^2 + 4*(35*a^8*c^3*x^10 + 135*a^6*c^3*x^8 + 189*a^4*c
^3*x^6 + 105*a^2*c^3*x^4)*arctan(a*x)*log(a^2*x^2 + 1) + (35*a^7*c^3*x^9 +
 135*a^5*c^3*x^7 + 189*a^3*c^3*x^5 + 105*a*c^3*x^3 + 315*(a^8*c^3*x^10 + 4
*a^6*c^3*x^8 + 6*a^4*c^3*x^6 + 4*a^2*c^3*x^4 + c^3*x^2)*arctan(a*x))*log(a
^2*x^2 + 1)^2)/(a^2*x^2 + 1), x)
```

## Giac [F]

$$\int x^2\left(c+a^2cx^2\right)^3\arctan(ax)^3\,dx=\int \left(a^2cx^2+c\right)^3x^2\arctan\left(ax\right)^3\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^3*x^2*arctan(a*x)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int x^2\left(c+a^2cx^2\right)^3\arctan(ax)^3\,dx=\int x^2\,\mathrm{atan}(a\,x)^3\,\left(c\,a^2\,x^2+c\right)^3\,dx$$

input
```
int(x^2*atan(a*x)^3*(c + a^2*c*x^2)^3,x)
```

output
```
int(x^2*atan(a*x)^3*(c + a^2*c*x^2)^3, x)
```

## Reduce [F]

$$\int x^2\left(c+a^2cx^2\right)^3\arctan(ax)^3\,dx$$
$$=\frac{c^3\left(840atan(ax)^3\,a^9x^9+3240atan(ax)^3\,a^7x^7+4536atan(ax)^3\,a^5x^5+2520atan(ax)^3\,a^3x^3+384atan(ax\right.}{}$$

input
```
int(x^2*(a^2*c*x^2+c)^3*atan(a*x)^3,x)
```

output

```
(c**3*(840*atan(a*x)**3*a**9*x***9 + 3240*atan(a*x)**3*a**7*x**7 + 4536*ata
n(a*x)**3*a**5*x**5 + 2520*atan(a*x)**3*a**3*x**3 + 384*atan(a*x)**3*a*x -
 315*atan(a*x)**2*a**8*x**8 - 1200*atan(a*x)**2*a**6*x**6 - 1602*atan(a*x)
**2*a**4*x**4 - 576*atan(a*x)**2*a**2*x**2 + 141*atan(a*x)**2 + 90*atan(a*
x)*a**7*x**7 + 354*atan(a*x)*a**5*x**5 + 478*atan(a*x)*a**3*x**3 - 282*ata
n(a*x)*a*x - 384*int(atan(a*x)**3,x)*a + 248*log(a**2*x**2 + 1) - 15*a**6*
x**6 - 66*a**4*x**4 - 107*a**2*x**2))/(7560*a**3)
```

# 3.381 $\int x(c+a^2cx^2)^3 \arctan(ax)^3 \, dx$



## Optimal result

Integrand size = 20, antiderivative size = 308

$$\begin{aligned}\int x(c+a^2cx^2)^3 \arctan(ax)^3 \, dx = & -\frac{19c^3x}{140a}-\frac{19}{840}ac^3x^3-\frac{1}{280}a^3c^3x^5 \\ & +\frac{3c^3(1+a^2x^2)\arctan(ax)}{35a^2} \\ & +\frac{9c^3(1+a^2x^2)^2\arctan(ax)}{280a^2} \\ & +\frac{c^3(1+a^2x^2)^3\arctan(ax)}{56a^2}-\frac{6ic^3\arctan(ax)^2}{35a^2} \\ & -\frac{6c^3x\arctan(ax)^2}{35a}-\frac{3c^3x(1+a^2x^2)\arctan(ax)^2}{35a} \\ & -\frac{9c^3x(1+a^2x^2)^2\arctan(ax)^2}{140a} \\ & -\frac{3c^3x(1+a^2x^2)^3\arctan(ax)^2}{56a} \\ & +\frac{c^3(1+a^2x^2)^4\arctan(ax)^3}{8a^2} \\ & -\frac{12c^3\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{35a^2} \\ & -\frac{6ic^3\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{35a^2}\end{aligned}$$

output
```
-19/140*c^3*x/a-19/840*a*c^3*x^3-1/280*a^3*c^3*x^5+3/35*c^3*(a^2*x^2+1)*ar
ctan(a*x)/a^2+9/280*c^3*(a^2*x^2+1)^2*arctan(a*x)/a^2+1/56*c^3*(a^2*x^2+1)
^3*arctan(a*x)/a^2-6/35*I*c^3*arctan(a*x)^2/a^2-6/35*c^3*x*arctan(a*x)^2/a
-3/35*c^3*x*(a^2*x^2+1)*arctan(a*x)^2/a-9/140*c^3*x*(a^2*x^2+1)^2*arctan(a
*x)^2/a-3/56*c^3*x*(a^2*x^2+1)^3*arctan(a*x)^2/a+1/8*c^3*(a^2*x^2+1)^4*arc
tan(a*x)^3/a^2-12/35*c^3*arctan(a*x)*ln(2/(1+I*a*x))/a^2-6/35*I*c^3*polylo
g(2,1-2/(1+I*a*x))/a^2
```

## Mathematica [A] (verified)

Time = 0.99 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.51

$$\int x(c + a^2cx^2)^3 \arctan(ax)^3 \, dx$$
$$= \frac{c^3\left(-ax(114 + 19a^2x^2 + 3a^4x^4) - 9(-16i + 35ax + 35a^3x^3 + 21a^5x^5 + 5a^7x^7) \arctan(ax)^2 + 105(1 + a\right.}{}$$

input
```
Integrate[x*(c + a^2*c*x^2)^3*ArcTan[a*x]^3,x]
```

output
```
(c^3*(-(a*x*(114 + 19*a^2*x^2 + 3*a^4*x^4)) - 9*(-16*I + 35*a*x + 35*a^3*x
^3 + 21*a^5*x^5 + 5*a^7*x^7)*ArcTan[a*x]^2 + 105*(1 + a^2*x^2)^4*ArcTan[a*
x]^3 + 3*ArcTan[a*x]*(38 + 57*a^2*x^2 + 24*a^4*x^4 + 5*a^6*x^6 - 96*Log[1
+ E^((2*I)*ArcTan[a*x])]) + (144*I)*PolyLog[2, -E^((2*I)*ArcTan[a*x])]))/(
840*a^2)
```

## Rubi [A] (verified)

Time = 1.31 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {5465, 27, 5415, 210, 2009, 5415, 2009, 5415, 24, 5345, 5455, 5379, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \arctan(ax)^3 \left(a^2cx^2 + c\right)^3 \, dx$$

$$\Big\downarrow \; 5465$$

$$\frac{c^3\left(a^2x^2+1\right)^4\arctan(ax)^3}{8a^2}-\frac{3\int c^3\left(a^2x^2+1\right)^3\arctan(ax)^2dx}{8a}$$

$$\Big\downarrow \; 27$$

$$\frac{c^3\left(a^2x^2+1\right)^4\arctan(ax)^3}{8a^2}-\frac{3c^3\int\left(a^2x^2+1\right)^3\arctan(ax)^2dx}{8a}$$

$$\Big\downarrow \; 5415$$

$$\frac{c^3\left(a^2x^2+1\right)^4\arctan(ax)^3}{8a^2}-\frac{3c^3\left(\frac{6}{7}\int\left(a^2x^2+1\right)^2\arctan(ax)^2dx+\frac{1}{21}\int\left(a^2x^2+1\right)^2dx+\frac{1}{7}x\left(a^2x^2+1\right)^3\arctan(ax)^2-\frac{\left(a^2x^2+1\right)^3\arctan(ax)}{21a}\right)}{8a}$$

$$\Big\downarrow \; 210$$

$$\frac{c^3\left(a^2x^2+1\right)^4\arctan(ax)^3}{8a^2}-\frac{3c^3\left(\frac{6}{7}\int\left(a^2x^2+1\right)^2\arctan(ax)^2dx+\frac{1}{21}\int\left(a^4x^4+2a^2x^2+1\right)dx+\frac{1}{7}x\left(a^2x^2+1\right)^3\arctan(ax)^2-\frac{\left(a^2x^2+1\right)^3\text{arcta}}{21a}\right.}{8a}$$

$$\Big\downarrow \; 2009$$

$$\frac{c^3\left(a^2x^2+1\right)^4\arctan(ax)^3}{8a^2}-\frac{3c^3\left(\frac{6}{7}\int\left(a^2x^2+1\right)^2\arctan(ax)^2dx+\frac{1}{7}x\left(a^2x^2+1\right)^3\arctan(ax)^2-\frac{\left(a^2x^2+1\right)^3\arctan(ax)}{21a}+\frac{1}{21}\left(\frac{a^4x^5}{5}+\frac{2a^2x^3}{3}+x\right)\right.}{8a}$$

$$\Big\downarrow \; 5415$$

$$\frac{c^3\left(a^2x^2+1\right)^4\arctan(ax)^3}{8a^2}-\frac{3c^3\left(\frac{6}{7}\left(\frac{4}{5}\int\left(a^2x^2+1\right)\arctan(ax)^2dx+\frac{1}{10}\int\left(a^2x^2+1\right)dx+\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan(ax)^2-\frac{\left(a^2x^2+1\right)^2\arctan(ax)}{10a}\right)\right.}{8a}$$

$$\Big\downarrow \; 2009$$

$$\frac{c^3\left(a^2x^2+1\right)^4\arctan(ax)^3}{8a^2}-\frac{3c^3\left(\frac{6}{7}\left(\frac{4}{5}\int\left(a^2x^2+1\right)\arctan(ax)^2dx+\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan(ax)^2-\frac{\left(a^2x^2+1\right)^2\arctan(ax)}{10a}+\frac{1}{10}\left(\frac{a^2x^3}{3}+x\right)\right)+\frac{1}{7}x\right.}{8a}$$

$$\Big\downarrow \text{5415}$$

$$\frac{c^3(a^2x^2+1)^4\arctan(ax)^3}{8a^2}-$$
$$3c^3\left(\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\int\arctan(ax)^2dx+\frac{\int 1dx}{3}+\frac{1}{3}x(a^2x^2+1)\arctan(ax)^2-\frac{(a^2x^2+1)\arctan(ax)}{3a}\right)+\frac{1}{5}x(a^2x^2+1)^2\arctan\right.\right.$$

$$\Big\downarrow \text{24}$$

$$\frac{c^3(a^2x^2+1)^4\arctan(ax)^3}{8a^2}-$$
$$3c^3\left(\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\int\arctan(ax)^2dx+\frac{1}{3}x(a^2x^2+1)\arctan(ax)^2-\frac{(a^2x^2+1)\arctan(ax)}{3a}+\frac{x}{3}\right)+\frac{1}{5}x(a^2x^2+1)^2\arctan(ax\right.\right.$$

$$\Big\downarrow \text{5345}$$

$$\frac{c^3(a^2x^2+1)^4\arctan(ax)^3}{8a^2}-$$
$$3c^3\left(\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\left(x\arctan(ax)^2-2a\int\frac{x\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{3}x(a^2x^2+1)\arctan(ax)^2-\frac{(a^2x^2+1)\arctan(ax)}{3a}+\frac{x}{3}\right)+\frac{1}{5}x(a\right.\right.$$

$$\Big\downarrow \text{5455}$$

$$\frac{c^3(a^2x^2+1)^4\arctan(ax)^3}{8a^2}-$$
$$3c^3\left(\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\left(x\arctan(ax)^2-2a\left(-\frac{\int\frac{\arctan(ax)}{i-ax}dx}{a}-\frac{i\arctan(ax)^2}{2a^2}\right)\right)+\frac{1}{3}x(a^2x^2+1)\arctan(ax)^2-\frac{(a^2x^2+1)\arctan(}{3a}\right.\right.\right.$$

$$\Big\downarrow \text{5379}$$

$$\frac{c^3(a^2x^2+1)^4\arctan(ax)^3}{8a^2}-$$
$$3c^3\left(\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\left(x\arctan(ax)^2-2a\left(-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}-\int\frac{\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^2}{2a^2}\right)\right)+\frac{1}{3}x(a^2x^2+1)\arctan(\right.\right.\right.$$

$$\Big\downarrow \text{2849}$$

$$\frac{c^3(a^2x^2+1)^4\arctan(ax)^3}{8a^2}-$$
$$3c^3\left(\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\left(x\arctan(ax)^2-2a\left(-\frac{\frac{i\int\frac{\log\left(\frac{2}{iax+1}\right)}{1-\frac{2}{iax+1}}d\frac{1}{iax+1}}{a}+\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}}{a}-\frac{i\arctan(ax)^2}{2a^2}\right)\right)+\frac{1}{3}x(a^2x^2+1)\text{ar}\right.\right.\right.$$

$$\downarrow \text{2752}$$

$$\frac{c^3(a^2x^2+1)^4\arctan(ax)^3}{8a^2}-$$

$$\frac{3c^3\left(\frac{6}{7}\left(\frac{4}{5}\left(\frac{2}{3}\left(x\arctan(ax)^2-2a\left(-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}\right)\right)\right)+\frac{1}{3}x(a^2x^2+1)\,\text{arct}\right.\right.\right.}{}$$

input
```
Int[x*(c + a^2*c*x^2)^3*ArcTan[a*x]^3,x]
```

output
```
(c^3*(1 + a^2*x^2)^4*ArcTan[a*x]^3)/(8*a^2) - (3*c^3*((x + (2*a^2*x^3)/3 +
 (a^4*x^5)/5)/21 - ((1 + a^2*x^2)^3*ArcTan[a*x])/(21*a) + (x*(1 + a^2*x^2)
^3*ArcTan[a*x]^2)/7 + (6*((x + (a^2*x^3)/3)/10 - ((1 + a^2*x^2)^2*ArcTan[a
*x])/(10*a) + (x*(1 + a^2*x^2)^2*ArcTan[a*x]^2)/5 + (4*(x/3 - ((1 + a^2*x^
2)*ArcTan[a*x])/(3*a) + (x*(1 + a^2*x^2)*ArcTan[a*x]^2)/3 + (2*(x*ArcTan[a
*x]^2 - 2*a*(((-1/2*I)*ArcTan[a*x]^2)/a^2 - ((ArcTan[a*x]*Log[2/(1 + I*a*x
)])/a + ((I/2)*PolyLog[2, 1 - 2/(1 + I*a*x)])/a)/a)))/3))/5))/7))/(8*a)
```

## Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5415
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTan[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x])^p,
x], x] + Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*(
a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[q, 0] && GtQ[p, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

## Maple [A] (verified)

Time = 7.21 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.12

| method | result |
|---|---|
| parts | $\frac{c^3\arctan(ax)^3a^6x^8}{8}+\frac{c^3\arctan(ax)^3a^4x^6}{2}+\frac{3c^3\arctan(ax)^3a^2x^4}{4}+\frac{c^3\arctan(ax)^3x^2}{2}+\frac{c^3\arctan(ax)^3}{8a^2}-\frac{3c\cdots}{\cdots}$ |
| derivativedivides | $\frac{\frac{c^3\arctan(ax)^3a^8x^8}{8}+\frac{a^6c^3x^6\arctan(ax)^3}{2}+\frac{3a^4c^3x^4\arctan(ax)^3}{4}+\frac{a^2c^3x^2\arctan(ax)^3}{2}+\frac{c^3\arctan(ax)^3}{8}-\frac{3c^3\left(\frac{\arctan(ax)^2a^7x\cdots}{7}\cdots\right.}{\cdots}}{\cdots}$ |
| default | $\frac{\frac{c^3\arctan(ax)^3a^8x^8}{8}+\frac{a^6c^3x^6\arctan(ax)^3}{2}+\frac{3a^4c^3x^4\arctan(ax)^3}{4}+\frac{a^2c^3x^2\arctan(ax)^3}{2}+\frac{c^3\arctan(ax)^3}{8}-\frac{3c^3\left(\frac{\arctan(ax)^2a^7x\cdots}{7}\cdots\right.}{\cdots}}{\cdots}$ |

input
```
int(x*(a^2*c*x^2+c)^3*arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*c^3*arctan(a*x)^3*a^6*x^8+1/2*c^3*arctan(a*x)^3*a^4*x^6+3/4*c^3*arctan
(a*x)^3*a^2*x^4+1/2*c^3*arctan(a*x)^3*x^2+1/8*c^3*arctan(a*x)^3/a^2-3/8/a^
2*c^3*(1/7*arctan(a*x)^2*a^7*x^7+3/5*a^5*arctan(a*x)^2*x^5+a^3*arctan(a*x)
^2*x^3+a*arctan(a*x)^2*x-1/21*a^6*arctan(a*x)*x^6-8/35*x^4*arctan(a*x)*a^4
-19/35*x^2*a^2*arctan(a*x)-16/35*arctan(a*x)*ln(a^2*x^2+1)+1/105*a^5*x^5+1
9/315*a^3*x^3+38/105*a*x-38/105*arctan(a*x)-8/35*I*(ln(a*x-I)*ln(a^2*x^2+1
)-1/2*ln(a*x-I)^2-dilog(-1/2*I*(a*x+I))-ln(a*x-I)*ln(-1/2*I*(a*x+I)))+8/35
*I*(ln(a*x+I)*ln(a^2*x^2+1)-1/2*ln(a*x+I)^2-dilog(1/2*I*(a*x-I))-ln(a*x+I)
*ln(1/2*I*(a*x-I))))
```

## Fricas [F]

$$\int x(c+a^2cx^2)^3\arctan(ax)^3\,dx=\int\left(a^2cx^2+c\right)^3x\arctan\left(ax\right)^3\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^3*arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^6*c^3*x^7 + 3*a^4*c^3*x^5 + 3*a^2*c^3*x^3 + c^3*x)*arctan(a*x)
^3, x)
```

## Sympy [F]

$$\int x\left(c+a^2cx^2\right)^3\arctan(ax)^3\,dx=c^3\left(\int x\,\mathrm{atan}^3\,(ax)\,dx+\int 3a^2x^3\,\mathrm{atan}^3\,(ax)\,dx+\int 3a^4x^5\,\mathrm{atan}^3\,(ax)\,dx+\int a^6x^7\,\mathrm{atan}^3\,(ax)\,dx\right)$$

input
```
integrate(x*(a**2*c*x**2+c)**3*atan(a*x)**3,x)
```

output
```
c**3*(Integral(x*atan(a*x)**3, x) + Integral(3*a**2*x**3*atan(a*x)**3, x)
+ Integral(3*a**4*x**5*atan(a*x)**3, x) + Integral(a**6*x**7*atan(a*x)**3,
 x))
```

## Maxima [F]

$$\int x\left(c+a^2cx^2\right)^3\arctan(ax)^3\,dx=\int\left(a^2cx^2+c\right)^3x\arctan\left(ax\right)^3\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^3*arctan(a*x)^3,x, algorithm="maxima")
```

output
```
1/4480*(280*(a^8*c^3*x^8 + 4*a^6*c^3*x^6 + 6*a^4*c^3*x^4 + 4*a^2*c^3*x^2 +
 c^3)*arctan(a*x)^3 + 140*(71680*a^9*c^3*integrate(1/4480*x^9*arctan(a*x)^
3/(a^3*x^2 + a), x) - 13440*a^8*c^3*integrate(1/4480*x^8*arctan(a*x)^2/(a^
3*x^2 + a), x) - 3360*a^8*c^3*integrate(1/4480*x^8*log(a^2*x^2 + 1)^2/(a^3
*x^2 + a), x) - 1920*a^8*c^3*integrate(1/4480*x^8*log(a^2*x^2 + 1)/(a^3*x^
2 + a), x) + 286720*a^7*c^3*integrate(1/4480*x^7*arctan(a*x)^3/(a^3*x^2 +
a), x) + 3840*a^7*c^3*integrate(1/4480*x^7*arctan(a*x)/(a^3*x^2 + a), x) -
 53760*a^6*c^3*integrate(1/4480*x^6*arctan(a*x)^2/(a^3*x^2 + a), x) - 1344
0*a^6*c^3*integrate(1/4480*x^6*log(a^2*x^2 + 1)^2/(a^3*x^2 + a), x) - 8064
*a^6*c^3*integrate(1/4480*x^6*log(a^2*x^2 + 1)/(a^3*x^2 + a), x) + 430080*
a^5*c^3*integrate(1/4480*x^5*arctan(a*x)^3/(a^3*x^2 + a), x) + 16128*a^5*c
^3*integrate(1/4480*x^5*arctan(a*x)/(a^3*x^2 + a), x) - 80640*a^4*c^3*inte
grate(1/4480*x^4*arctan(a*x)^2/(a^3*x^2 + a), x) - 20160*a^4*c^3*integrate
(1/4480*x^4*log(a^2*x^2 + 1)^2/(a^3*x^2 + a), x) - 13440*a^4*c^3*integrate
(1/4480*x^4*log(a^2*x^2 + 1)/(a^3*x^2 + a), x) + 286720*a^3*c^3*integrate(
1/4480*x^3*arctan(a*x)^3/(a^3*x^2 + a), x) + 26880*a^3*c^3*integrate(1/448
0*x^3*arctan(a*x)/(a^3*x^2 + a), x) - 53760*a^2*c^3*integrate(1/4480*x^2*a
rctan(a*x)^2/(a^3*x^2 + a), x) - 13440*a^2*c^3*integrate(1/4480*x^2*log(a^
2*x^2 + 1)^2/(a^3*x^2 + a), x) - 13440*a^2*c^3*integrate(1/4480*x^2*log(a^
2*x^2 + 1)/(a^3*x^2 + a), x) + 71680*a*c^3*integrate(1/4480*x*arctan(a*...
```

## Giac [F]

$$\int x(c+a^2cx^2)^3\arctan(ax)^3\,dx=\int \left(a^2cx^2+c\right)^3x\arctan\left(ax\right)^3\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^3*arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^3*x*arctan(a*x)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int x(c+a^2cx^2)^3 \arctan(ax)^3\,dx = \int x\,\mathrm{atan}(a\,x)^3\,\left(c\,a^2\,x^2+c\right)^3\,dx$$

input
```
int(x*atan(a*x)^3*(c + a^2*c*x^2)^3,x)
```

output
```
int(x*atan(a*x)^3*(c + a^2*c*x^2)^3, x)
```

## Reduce [F]

$$\int x(c+a^2cx^2)^3 \arctan(ax)^3\,dx$$
$$= \frac{c^3\Big(105atan(ax)^3\,a^8x^8 + 420atan(ax)^3\,a^6x^6 + 630atan(ax)^3\,a^4x^4 + 420atan(ax)^3\,a^2x^2 + 105atan(ax)^3 -}{}$$

input
```
int(x*(a^2*c*x^2+c)^3*atan(a*x)^3,x)
```

output
```
(c**3*(105*atan(a*x)**3*a**8*x**8 + 420*atan(a*x)**3*a**6*x**6 + 630*atan(
a*x)**3*a**4*x**4 + 420*atan(a*x)**3*a**2*x**2 + 105*atan(a*x)**3 - 45*ata
n(a*x)**2*a**7*x**7 - 189*atan(a*x)**2*a**5*x**5 - 315*atan(a*x)**2*a**3*x
**3 - 315*atan(a*x)**2*a*x + 15*atan(a*x)*a**6*x**6 + 72*atan(a*x)*a**4*x*
*4 + 171*atan(a*x)*a**2*x**2 + 114*atan(a*x) + 288*int((atan(a*x)*x)/(a**2
*x**2 + 1),x)*a**2 - 3*a**5*x**5 - 19*a**3*x**3 - 114*a*x))/(840*a**2)
```

# 3.382 $\int \left(c+a^2 c x^2\right)^3 \arctan(ax)^3\,dx$

## Optimal result

Integrand size = 19, antiderivative size = 388

$$\int (c+a^2cx^2)^3 \arctan(ax)^3 \, dx = -\frac{13c^3(1+a^2x^2)}{210a} - \frac{c^3(1+a^2x^2)^2}{140a} + \frac{14}{15}c^3x\arctan(ax) + \frac{13}{105}c^3x(1+a^2x^2)\arctan(ax) + \frac{1}{35}c^3x(1+a^2x^2)^2\arctan(ax) - \frac{12c^3(1+a^2x^2)\arctan(ax)^2}{35a} - \frac{9c^3(1+a^2x^2)^2\arctan(ax)^2}{70a} - \frac{c^3(1+a^2x^2)^3\arctan(ax)^2}{14a} + \frac{16ic^3\arctan(ax)^3}{35a} + \frac{16}{35}c^3x\arctan(ax)^3 + \frac{8}{35}c^3x(1+a^2x^2)\arctan(ax)^3 + \frac{6}{35}c^3x(1+a^2x^2)^2\arctan(ax)^3 + \frac{1}{7}c^3x(1+a^2x^2)^3\arctan(ax)^3 + \frac{48c^3\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{35a} - \frac{7c^3\log(1+a^2x^2)}{15a} + \frac{48ic^3\arctan(ax)\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{35a} + \frac{24c^3\operatorname{PolyLog}\left(3,1-\frac{2}{1+iax}\right)}{35a}$$

output
```
-13/210*c^3*(a^2*x^2+1)/a-1/140*c^3*(a^2*x^2+1)^2/a+14/15*c^3*x*arctan(a*x
)+13/105*c^3*x*(a^2*x^2+1)*arctan(a*x)+1/35*c^3*x*(a^2*x^2+1)^2*arctan(a*x
)-12/35*c^3*(a^2*x^2+1)*arctan(a*x)^2/a-9/70*c^3*(a^2*x^2+1)^2*arctan(a*x)
^2/a-1/14*c^3*(a^2*x^2+1)^3*arctan(a*x)^2/a+48/35*I*c^3*arctan(a*x)*polylo
g(2,1-2/(1+I*a*x))/a+16/35*c^3*x*arctan(a*x)^3+8/35*c^3*x*(a^2*x^2+1)*arct
an(a*x)^3+6/35*c^3*x*(a^2*x^2+1)^2*arctan(a*x)^3+1/7*c^3*x*(a^2*x^2+1)^3*a
rctan(a*x)^3+48/35*c^3*arctan(a*x)^2*ln(2/(1+I*a*x))/a-7/15*c^3*ln(a^2*x^2
+1)/a+16/35*I*c^3*arctan(a*x)^3/a+24/35*c^3*polylog(3,1-2/(1+I*a*x))/a
```

## Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.63

$$\int \left(c+a^2cx^2\right)^3 \arctan(ax)^3 \, dx$$
$$= \frac{c^3\left(-29-32a^2x^2-3a^4x^4+456ax\arctan(ax)+76a^3x^3\arctan(ax)+12a^5x^5\arctan(ax)-228\arctan(a\right.}{}$$

input
```
Integrate[(c + a^2*c*x^2)^3*ArcTan[a*x]^3,x]
```

output
```
(c^3*(-29 - 32*a^2*x^2 - 3*a^4*x^4 + 456*a*x*ArcTan[a*x] + 76*a^3*x^3*ArcT
an[a*x] + 12*a^5*x^5*ArcTan[a*x] - 228*ArcTan[a*x]^2 - 342*a^2*x^2*ArcTan[
a*x]^2 - 144*a^4*x^4*ArcTan[a*x]^2 - 30*a^6*x^6*ArcTan[a*x]^2 - (192*I)*Ar
cTan[a*x]^3 + 420*a*x*ArcTan[a*x]^3 + 420*a^3*x^3*ArcTan[a*x]^3 + 252*a^5*
x^5*ArcTan[a*x]^3 + 60*a^7*x^7*ArcTan[a*x]^3 + 576*ArcTan[a*x]^2*Log[1 + E
^((2*I)*ArcTan[a*x])] - 196*Log[1 + a^2*x^2] - (576*I)*ArcTan[a*x]*PolyLog
[2, -E^((2*I)*ArcTan[a*x])] + 288*PolyLog[3, -E^((2*I)*ArcTan[a*x])]))/(42
0*a)
```

## Rubi [A] (verified)

Time = 2.37 (sec) , antiderivative size = 473, normalized size of antiderivative = 1.22, number of steps used = 17, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.895$, Rules used = {5415, 27, 5413, 5413, 5345, 240, 5415, 5413, 5345, 240, 5415, 5345, 240, 5455, 5379, 5529, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(ax)^3 \left(a^2cx^2+c\right)^3 \, dx$$

$$\Big\downarrow \; 5415$$

$$\frac{1}{7}c\int c^2\left(a^2x^2+1\right)^2\arctan(ax)dx+\frac{6}{7}c\int c^2\left(a^2x^2+1\right)^2\arctan(ax)^3dx+\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)^3-\frac{c^3\left(a^2x^2+1\right)^3\arctan(ax)^2}{14a}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{7}c^3\int \left(a^2x^2+1\right)^2 \arctan(ax)dx+\frac{6}{7}c^3\int \left(a^2x^2+1\right)^2 \arctan(ax)^3dx+\frac{1}{7}c^3x\left(a^2x^2+1\right)^3 \arctan(ax)^3-\frac{c^3\left(a^2x^2+1\right)^3 \arctan(ax)^2}{14a}$$

$$\Big\downarrow \text{5413}$$

$$\frac{1}{7}c^3\left(\frac{4}{5}\int \left(a^2x^2+1\right)\arctan(ax)dx+\frac{1}{5}x\left(a^2x^2+1\right)^2 \arctan(ax)-\frac{\left(a^2x^2+1\right)^2}{20a}\right)+\frac{6}{7}c^3\int \left(a^2x^2+1\right)^2 \arctan(ax)^3dx+\frac{1}{7}c^3x\left(a^2x^2+1\right)^3 \arctan(ax)^3-\frac{c^3\left(a^2x^2+1\right)^3 \arctan(ax)^2}{14a}$$

$$\Big\downarrow \text{5413}$$

$$\frac{1}{7}c^3\left(\frac{4}{5}\left(\frac{2}{3}\int \arctan(ax)dx+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)-\frac{a^2x^2+1}{6a}\right)+\frac{1}{5}x\left(a^2x^2+1\right)^2 \arctan(ax)-\frac{\left(a^2x^2+1\right)}{20a}\right. \\ \frac{6}{7}c^3\int \left(a^2x^2+1\right)^2 \arctan(ax)^3dx+\frac{1}{7}c^3x\left(a^2x^2+1\right)^3 \arctan(ax)^3-\frac{c^3\left(a^2x^2+1\right)^3 \arctan(ax)^2}{14a}$$

$$\Big\downarrow \text{5345}$$

$$\frac{1}{7}c^3\left(\frac{4}{5}\left(\frac{2}{3}\left(x\arctan(ax)-a\int \frac{x}{a^2x^2+1}dx\right)+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)-\frac{a^2x^2+1}{6a}\right)+\frac{1}{5}x\left(a^2x^2+1\right)^2 \text{arctan}\right. \\ \frac{6}{7}c^3\int \left(a^2x^2+1\right)^2 \arctan(ax)^3dx+\frac{1}{7}c^3x\left(a^2x^2+1\right)^3 \arctan(ax)^3-\frac{c^3\left(a^2x^2+1\right)^3 \arctan(ax)^2}{14a}$$

$$\Big\downarrow \text{240}$$

$$\frac{6}{7}c^3\int \left(a^2x^2+1\right)^2 \arctan(ax)^3dx+\frac{1}{7}c^3x\left(a^2x^2+1\right)^3 \arctan(ax)^3-\frac{c^3\left(a^2x^2+1\right)^3 \arctan(ax)^2}{14a}+\frac{1}{7}c^3\left(\frac{1}{5}x\left(a^2x^2+1\right)^2 \arctan(ax)+\frac{4}{5}\left(\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)+\frac{2}{3}\left(x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}\right)-\frac{a^2x^2+}{6a}\right.\right.$$

$$\Big\downarrow \text{5415}$$

$$\frac{6}{7}c^3\left(\frac{3}{10}\int\left(a^2x^2+1\right)\arctan(ax)dx+\frac{4}{5}\int\left(a^2x^2+1\right)\arctan(ax)^3dx+\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan(ax)^3-\frac{3\left(a^2x^2+\right.}{}\right.$$
$$\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)^3-\frac{c^3\left(a^2x^2+1\right)^3\arctan(ax)^2}{14a}+$$
$$\frac{1}{7}c^3\left(\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan(ax)+\frac{4}{5}\left(\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)+\frac{2}{3}\left(x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}\right)-\frac{a^2x^2+}{6a}\right.\right.$$

$\downarrow$ 5413

$$\frac{6}{7}c^3\left(\frac{3}{10}\left(\frac{2}{3}\int\arctan(ax)dx+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)-\frac{a^2x^2+1}{6a}\right)+\frac{4}{5}\int\left(a^2x^2+1\right)\arctan(ax)^3dx+\frac{1}{5}x(a\right.$$
$$\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)^3-\frac{c^3\left(a^2x^2+1\right)^3\arctan(ax)^2}{14a}+$$
$$\frac{1}{7}c^3\left(\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan(ax)+\frac{4}{5}\left(\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)+\frac{2}{3}\left(x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}\right)-\frac{a^2x^2+}{6a}\right.\right.$$

$\downarrow$ 5345

$$\frac{6}{7}c^3\left(\frac{3}{10}\left(\frac{2}{3}\left(x\arctan(ax)-a\int\frac{x}{a^2x^2+1}dx\right)+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)-\frac{a^2x^2+1}{6a}\right)+\frac{4}{5}\int\left(a^2x^2+1\right)\text{arct}\right.$$
$$\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)^3-\frac{c^3\left(a^2x^2+1\right)^3\arctan(ax)^2}{14a}+$$
$$\frac{1}{7}c^3\left(\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan(ax)+\frac{4}{5}\left(\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)+\frac{2}{3}\left(x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}\right)-\frac{a^2x^2+}{6a}\right.\right.$$

$\downarrow$ 240

$$\frac{6}{7}c^3\left(\frac{4}{5}\int\left(a^2x^2+1\right)\arctan(ax)^3dx+\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan(ax)^3-\frac{3\left(a^2x^2+1\right)^2\arctan(ax)^2}{20a}+\frac{3}{10}\left(\frac{1}{3}x\left(a^2x^2\right.\right.\right.$$
$$\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)^3-\frac{c^3\left(a^2x^2+1\right)^3\arctan(ax)^2}{14a}+$$
$$\frac{1}{7}c^3\left(\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan(ax)+\frac{4}{5}\left(\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)+\frac{2}{3}\left(x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}\right)-\frac{a^2x^2+}{6a}\right.\right.$$

$\downarrow$ 5415

$$\frac{6}{7}c^3\left(\frac{4}{5}\left(\int \arctan(ax)dx+\frac{2}{3}\int \arctan(ax)^3dx+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)^3-\frac{\left(a^2x^2+1\right)\arctan(ax)^2}{2a}\right)+\frac{1}{5}x(\right.$$
$$\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)^3-\frac{c^3\left(a^2x^2+1\right)^3\arctan(ax)^2}{14a}+$$
$$\frac{1}{7}c^3\left(\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan(ax)+\frac{4}{5}\left(\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)+\frac{2}{3}\left(x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}\right)-\frac{a^2x^2+}{6a}\right.\right.$$

$\downarrow$ 5345

$$\frac{6}{7}c^3\left(\frac{4}{5}\left(\frac{2}{3}\left(x\arctan(ax)^3-3a\int \frac{x\arctan(ax)^2}{a^2x^2+1}dx\right)-a\int \frac{x}{a^2x^2+1}dx+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)^3-\frac{(a^2x^2+}{}\right.\right.$$
$$\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)^3-\frac{c^3\left(a^2x^2+1\right)^3\arctan(ax)^2}{14a}+$$
$$\frac{1}{7}c^3\left(\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan(ax)+\frac{4}{5}\left(\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)+\frac{2}{3}\left(x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}\right)-\frac{a^2x^2+}{6a}\right.\right.$$

$\downarrow$ 240

$$\frac{6}{7}c^3\left(\frac{4}{5}\left(\frac{2}{3}\left(x\arctan(ax)^3-3a\int \frac{x\arctan(ax)^2}{a^2x^2+1}dx\right)+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)^3-\frac{\left(a^2x^2+1\right)\arctan(ax)^2}{2a}-\right.\right.$$
$$\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)^3-\frac{c^3\left(a^2x^2+1\right)^3\arctan(ax)^2}{14a}+$$
$$\frac{1}{7}c^3\left(\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan(ax)+\frac{4}{5}\left(\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)+\frac{2}{3}\left(x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}\right)-\frac{a^2x^2+}{6a}\right.\right.$$

$\downarrow$ 5455

$$\frac{6}{7}c^3\left(\frac{4}{5}\left(\frac{2}{3}\left(x\arctan(ax)^3-3a\left(-\frac{\int \frac{\arctan(ax)^2}{i-ax}dx}{a}-\frac{i\arctan(ax)^3}{3a^2}\right)\right)+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)^3-\frac{(a^2x^2+1}{}\right.\right.$$
$$\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)^3-\frac{c^3\left(a^2x^2+1\right)^3\arctan(ax)^2}{14a}+$$
$$\frac{1}{7}c^3\left(\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan(ax)+\frac{4}{5}\left(\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)+\frac{2}{3}\left(x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}\right)-\frac{a^2x^2+}{6a}\right.\right.$$

$\downarrow$ 5379

$$\frac{6}{7}c^3\left(\frac{4}{5}\left(\frac{2}{3}\left(x\arctan(ax)^3-3a\left(-\frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}-2\int\frac{\arctan(ax)\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^3}{3a^2}\right)\right)+\frac{1}{3}x(\right.\right.$$
$$\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)^3-\frac{c^3\left(a^2x^2+1\right)^3\arctan(ax)^2}{14a}+$$
$$\frac{1}{7}c^3\left(\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan(ax)+\frac{4}{5}\left(\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)+\frac{2}{3}\left(x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}\right)-\frac{a^2x^2+}{6a}\right.\right.$$

$$\Big\downarrow \text{5529}$$

$$\frac{6}{7}c^3\left(\frac{4}{5}\left(\frac{2}{3}\left(x\arctan(ax)^3-3a\left(-\frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}-2\left(\frac{1}{2}i\int\frac{\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx-\frac{i\arctan(ax)\text{PolyLog}\left(2,1-\frac{}{i}\right.}{2a}\right.}{a}\right.\right.\right.\right.$$
$$\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)^3-\frac{c^3\left(a^2x^2+1\right)^3\arctan(ax)^2}{14a}+$$
$$\frac{1}{7}c^3\left(\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan(ax)+\frac{4}{5}\left(\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)+\frac{2}{3}\left(x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}\right)-\frac{a^2x^2+}{6a}\right.\right.$$

$$\Big\downarrow \text{7164}$$

$$\frac{6}{7}c^3\left(\frac{4}{5}\left(\frac{2}{3}\left(x\arctan(ax)^3-3a\left(-\frac{i\arctan(ax)^3}{3a^2}-\frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}-2\left(-\frac{i\arctan(ax)\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}-\frac{\text{P}}{}\right.}{a}\right.\right.\right.\right.$$
$$\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)^3-\frac{c^3\left(a^2x^2+1\right)^3\arctan(ax)^2}{14a}+$$
$$\frac{1}{7}c^3\left(\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan(ax)+\frac{4}{5}\left(\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)+\frac{2}{3}\left(x\arctan(ax)-\frac{\log\left(a^2x^2+1\right)}{2a}\right)-\frac{a^2x^2+}{6a}\right.\right.$$

input
```
Int[(c + a^2*c*x^2)^3*ArcTan[a*x]^3,x]
```

output
```
-1/14*(c^3*(1 + a^2*x^2)^3*ArcTan[a*x]^2)/a + (c^3*x*(1 + a^2*x^2)^3*ArcTa
n[a*x]^3)/7 + (c^3*(-1/20*(1 + a^2*x^2)^2/a + (x*(1 + a^2*x^2)^2*ArcTan[a*
x])/5 + (4*(-1/6*(1 + a^2*x^2)/a + (x*(1 + a^2*x^2)*ArcTan[a*x])/3 + (2*(x
*ArcTan[a*x] - Log[1 + a^2*x^2]/(2*a)))/3))/5))/7 + (6*c^3*((-3*(1 + a^2*x
^2)^2*ArcTan[a*x]^2)/(20*a) + (x*(1 + a^2*x^2)^2*ArcTan[a*x]^3)/5 + (3*(-1
/6*(1 + a^2*x^2)/a + (x*(1 + a^2*x^2)*ArcTan[a*x])/3 + (2*(x*ArcTan[a*x] -
 Log[1 + a^2*x^2]/(2*a)))/3))/10 + (4*(x*ArcTan[a*x] - ((1 + a^2*x^2)*ArcT
an[a*x]^2)/(2*a) + (x*(1 + a^2*x^2)*ArcTan[a*x]^3)/3 - Log[1 + a^2*x^2]/(2
*a) + (2*(x*ArcTan[a*x]^3 - 3*a*(((-1/3*I)*ArcTan[a*x]^3)/a^2 - ((ArcTan[a
*x]^2*Log[2/(1 + I*a*x)])/a - 2*(((-1/2*I)*ArcTan[a*x]*PolyLog[2, 1 - 2/(1
 + I*a*x)])/a - PolyLog[3, 1 - 2/(1 + I*a*x)]/(4*a)))/a)))/3))/5))/7
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5413
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Simp[x*(d + e*x^2)
^q*((a + b*ArcTan[c*x])/(2*q + 1)), x] + Simp[2*d*(q/(2*q + 1))   Int[(d +
e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0]
```

rule 5415
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTan[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x])^p,
x], x] + Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*(
a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[q, 0] && GtQ[p, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 111.60 (sec) , antiderivative size = 1267, normalized size of antiderivative = 3.27

| method | result | size |
|---|---|---|
| derivativedivides | Expression too large to display | 1267 |
| default | Expression too large to display | 1267 |
| parts | Expression too large to display | 1268 |

input

```
int((a^2*c*x^2+c)^3*arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

output

```
1/a*(1/7*c^3*arctan(a*x)^3*a^7*x^7+3/5*c^3*arctan(a*x)^3*a^5*x^5+c^3*arcta
n(a*x)^3*a^3*x^3+c^3*arctan(a*x)^3*a*x-3/35*c^3*(4*I*Pi*csgn(I/((1+I*a*x)^
2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^
2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*arctan(a*x)^2+4*I*Pi*csgn(I*(1+I*a
*x)/(a^2*x^2+1)^(1/2))^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*arctan(a*x)^2+8*I
*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)
^2)^2*arctan(a*x)^2+19/3*arctan(a*x)^2+19/2*arctan(a*x)^2*x^2*a^2+1/12*(a*
x+I)^4-8*I*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*csgn(I*(1+I*a*x)^2/(a^2*
x^2+1))^2*arctan(a*x)^2-4*I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(
I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2*arctan(a*x)^2-4
*I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I
*a*x)^2/(a^2*x^2+1)+1)^2)^2*arctan(a*x)^2-4*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*
x^2+1)+1))^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*arctan(a*x)^2+16/3*I*ar
ctan(a*x)^3-13/9*I*(a*x+I)-1/3*I*(a*x+I)^3+4*a^4*arctan(a*x)^2*x^4+5/6*a^6
*x^6*arctan(a*x)^2-8*arctan(a*x)*(a*x-I)+16*I*arctan(a*x)*polylog(2,-(1+I*
a*x)^2/(a^2*x^2+1))-5/3*I*arctan(a*x)*(a*x-I)^4-16*arctan(a*x)^2*ln((1+I*a
*x)/(a^2*x^2+1)^(1/2))-8*polylog(3,-(1+I*a*x)^2/(a^2*x^2+1))+5/3*arctan(a*
x)*(a*x-I)^4*(a*x+I)-98/9*ln((1+I*a*x)^2/(a^2*x^2+1)+1)+7/18*(a*x+I)^2-10*
I*arctan(a*x)*(a*x-I)^2*(a*x+I)^2+6*I*arctan(a*x)*(a*x-I)*(a*x+I)+20/3*I*a
rctan(a*x)*(a*x-I)^3*(a*x+I)+20/3*I*arctan(a*x)*(a*x-I)*(a*x+I)^3-4*I*P...
```

## Fricas [F]

$$\int \left(c+a^2cx^2\right)^3 \arctan(ax)^3 \, dx = \int \left(a^2cx^2+c\right)^3 \arctan\left(ax\right)^3 \, dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a*x)^3
, x)
```

## Sympy [F]

$$\int \left(c+a^2cx^2\right)^3 \arctan(ax)^3 \, dx = c^3 \left(\int 3a^2x^2 \operatorname{atan}^3\left(ax\right) dx + \int 3a^4x^4 \operatorname{atan}^3\left(ax\right) dx + \int a^6x^6 \operatorname{atan}^3\left(ax\right) dx + \int \operatorname{atan}^3\left(ax\right) dx\right)$$

input
```
integrate((a**2*c*x**2+c)**3*atan(a*x)**3,x)
```

output
```
c**3*(Integral(3*a**2*x**2*atan(a*x)**3, x) + Integral(3*a**4*x**4*atan(a*
x)**3, x) + Integral(a**6*x**6*atan(a*x)**3, x) + Integral(atan(a*x)**3, x
))
```

## Maxima [F]

$$\int \left(c+a^2cx^2\right)^3 \arctan(ax)^3 \, dx = \int \left(a^2cx^2+c\right)^3 \arctan\left(ax\right)^3 \, dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^3,x, algorithm="maxima")
```

output
```
980*a^8*c^3*integrate(1/1120*x^8*arctan(a*x)^3/(a^2*x^2 + 1), x) + 105*a^8
*c^3*integrate(1/1120*x^8*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x)
 + 60*a^8*c^3*integrate(1/1120*x^8*arctan(a*x)*log(a^2*x^2 + 1)/(a^2*x^2 +
 1), x) - 60*a^7*c^3*integrate(1/1120*x^7*arctan(a*x)^2/(a^2*x^2 + 1), x)
+ 15*a^7*c^3*integrate(1/1120*x^7*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) + 3
920*a^6*c^3*integrate(1/1120*x^6*arctan(a*x)^3/(a^2*x^2 + 1), x) + 420*a^6
*c^3*integrate(1/1120*x^6*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x)
 + 252*a^6*c^3*integrate(1/1120*x^6*arctan(a*x)*log(a^2*x^2 + 1)/(a^2*x^2
+ 1), x) - 252*a^5*c^3*integrate(1/1120*x^5*arctan(a*x)^2/(a^2*x^2 + 1), x
) + 63*a^5*c^3*integrate(1/1120*x^5*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) +
 5880*a^4*c^3*integrate(1/1120*x^4*arctan(a*x)^3/(a^2*x^2 + 1), x) + 630*a
^4*c^3*integrate(1/1120*x^4*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1),
x) + 420*a^4*c^3*integrate(1/1120*x^4*arctan(a*x)*log(a^2*x^2 + 1)/(a^2*x^
2 + 1), x) - 420*a^3*c^3*integrate(1/1120*x^3*arctan(a*x)^2/(a^2*x^2 + 1),
 x) + 105*a^3*c^3*integrate(1/1120*x^3*log(a^2*x^2 + 1)^2/(a^2*x^2 + 1), x
) + 7/32*c^3*arctan(a*x)^4/a + 3920*a^2*c^3*integrate(1/1120*x^2*arctan(a*
x)^3/(a^2*x^2 + 1), x) + 420*a^2*c^3*integrate(1/1120*x^2*arctan(a*x)*log(
a^2*x^2 + 1)^2/(a^2*x^2 + 1), x) + 420*a^2*c^3*integrate(1/1120*x^2*arctan
(a*x)*log(a^2*x^2 + 1)/(a^2*x^2 + 1), x) - 420*a*c^3*integrate(1/1120*x*ar
ctan(a*x)^2/(a^2*x^2 + 1), x) + 105*a*c^3*integrate(1/1120*x*log(a^2*x^...
```

## Giac [F]

$$\int (c+a^2cx^2)^3 \arctan(ax)^3\,dx = \int (a^2cx^2+c)^3 \arctan(ax)^3\,dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^3*arctan(a*x)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \left(c+a^2cx^2\right)^3 \arctan(ax)^3\,dx = \int \operatorname{atan}(a\,x)^3\,\left(c\,a^2\,x^2+c\right)^3\,dx$$

input
```
int(atan(a*x)^3*(c + a^2*c*x^2)^3,x)
```

output
```
int(atan(a*x)^3*(c + a^2*c*x^2)^3, x)
```

## Reduce [F]

$$\int \left(c+a^2cx^2\right)^3 \arctan(ax)^3\,dx = \frac{c^3\left(60atan(ax)^3\,a^7x^7+252atan(ax)^3\,a^5x^5+420atan(ax)^3\,a^3x^3+420atan(ax)^3\,ax-30atan(ax)^2\,a^6x^6\right.}{}$$

input
```
int((a^2*c*x^2+c)^3*atan(a*x)^3,x)
```

output
```
(c**3*(60*atan(a*x)**3*a**7*x**7 + 252*atan(a*x)**3*a**5*x**5 + 420*atan(a
*x)**3*a**3*x**3 + 420*atan(a*x)**3*a*x - 30*atan(a*x)**2*a**6*x**6 - 144*
atan(a*x)**2*a**4*x**4 - 342*atan(a*x)**2*a**2*x**2 - 228*atan(a*x)**2 + 1
2*atan(a*x)*a**5*x**5 + 76*atan(a*x)*a**3*x**3 + 456*atan(a*x)*a*x - 576*i
nt((atan(a*x)**2*x)/(a**2*x**2 + 1),x)*a**2 - 196*log(a**2*x**2 + 1) - 3*a
**4*x**4 - 32*a**2*x**2))/(420*a)
```

## 3.383 $\int \frac{(c+a^2cx^2)^3 \arctan(ax)^3}{x} \, dx$

## Optimal result

Integrand size = 22, antiderivative size = 447

$$\int \frac{(c+a^2cx^2)^3\arctan(ax)^3}{x}\,dx = -\frac{13}{30}ac^3x - \frac{1}{60}a^3c^3x^3 + \frac{13}{30}c^3\arctan(ax) + \frac{29}{60}a^2c^3x^2\arctan(ax) + \frac{1}{20}a^4c^3x^4\arctan(ax) - \frac{34}{15}ic^3\arctan(ax)^2 - \frac{11}{4}ac^3x\arctan(ax)^2 - \frac{7}{12}a^3c^3x^3\arctan(ax)^2 - \frac{1}{10}a^5c^3x^5\arctan(ax)^2 + \frac{11}{12}c^3\arctan(ax)^3 + \frac{3}{2}a^2c^3x^2\arctan(ax)^3 + \frac{3}{4}a^4c^3x^4\arctan(ax)^3 + \frac{1}{6}a^6c^3x^6\arctan(ax)^3 + 2c^3\arctan(ax)^3\operatorname{arctanh}\left(1-\frac{2}{1+iax}\right) - \frac{68}{15}c^3\arctan(ax)\log\left(\frac{2}{1+iax}\right) - \frac{34}{15}ic^3\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right) - \frac{3}{2}ic^3\arctan(ax)^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right) + \frac{3}{2}ic^3\arctan(ax)^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1+iax}\right) - \frac{3}{2}c^3\arctan(ax)\operatorname{PolyLog}\left(3,1-\frac{2}{1+iax}\right) + \frac{3}{2}c^3\arctan(ax)\operatorname{PolyLog}\left(3,-1+\frac{2}{1+iax}\right) + \frac{3}{4}ic^3\operatorname{PolyLog}\left(4,1-\frac{2}{1+iax}\right) - \frac{3}{4}ic^3\operatorname{PolyLog}\left(4,-1+\frac{2}{1+iax}\right)$$

output
```
-13/30*a*c^3*x-1/60*a^3*c^3*x^3+13/30*c^3*arctan(a*x)+29/60*a^2*c^3*x^2*ar
ctan(a*x)+1/20*a^4*c^3*x^4*arctan(a*x)+3/4*I*c^3*polylog(4,1-2/(1+I*a*x))-
11/4*a*c^3*x*arctan(a*x)^2-7/12*a^3*c^3*x^3*arctan(a*x)^2-1/10*a^5*c^3*x^5
*arctan(a*x)^2+11/12*c^3*arctan(a*x)^3+3/2*a^2*c^3*x^2*arctan(a*x)^3+3/4*a
^4*c^3*x^4*arctan(a*x)^3+1/6*a^6*c^3*x^6*arctan(a*x)^3-2*c^3*arctan(a*x)^3
*arctanh(-1+2/(1+I*a*x))-68/15*c^3*arctan(a*x)*ln(2/(1+I*a*x))-34/15*I*c^3
*polylog(2,1-2/(1+I*a*x))-3/2*I*c^3*arctan(a*x)^2*polylog(2,1-2/(1+I*a*x))
-34/15*I*c^3*arctan(a*x)^2-3/2*c^3*arctan(a*x)*polylog(3,1-2/(1+I*a*x))+3/
2*c^3*arctan(a*x)*polylog(3,-1+2/(1+I*a*x))+3/2*I*c^3*arctan(a*x)^2*polylo
g(2,-1+2/(1+I*a*x))-3/4*I*c^3*polylog(4,-1+2/(1+I*a*x))
```

## Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 350, normalized size of antiderivative = 0.78

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^3}{x}\,dx = \frac{1}{960}c^3\big(-15i\pi^4 - 416ax - 16a^3x^3 + 416\arctan(ax) + 464a^2x^2\arctan(ax) + 48a^4x^4\arctan(ax) + 2176i\arctan(ax)^2 - 2640ax\arctan(ax)^2 - 560a^3x^3\arctan(ax)^2 - 96a^5x^5\arctan(ax)^2 + 880\arctan(ax)^3 + 1440a^2x^2\arctan(ax)^3 + 720a^4x^4\arctan(ax)^3 + 160a^6x^6\arctan(ax)^3 + 480i\arctan(ax)^4 + 960\arctan(ax)^3\log\left(1-e^{-2i\arctan(ax)}\right) - 4352\arctan(ax)\log\left(1+e^{2i\arctan(ax)}\right) - 960\arctan(ax)^3\log\left(1+e^{2i\arctan(ax)}\right) + 1440i\arctan(ax)^2\operatorname{PolyLog}\left(2,e^{-2i\arctan(ax)}\right) + 32i\left(68+45\arctan(ax)^2\right)\operatorname{PolyLog}\left(2,-e^{2i\arctan(ax)}\right) + 1440\arctan(ax)\operatorname{PolyLog}\left(3,e^{-2i\arctan(ax)}\right) - 1440\arctan(ax)\operatorname{PolyLog}\left(3,-e^{2i\arctan(ax)}\right) - 720i\operatorname{PolyLog}\left(4,e^{-2i\arctan(ax)}\right) - 720i\operatorname{PolyLog}\left(4,-e^{2i\arctan(ax)}\right)\big)$$

input
```
Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x]^3)/x,x]
```

output
```
(c^3*((-15*I)*Pi^4 - 416*a*x - 16*a^3*x^3 + 416*ArcTan[a*x] + 464*a^2*x^2*
ArcTan[a*x] + 48*a^4*x^4*ArcTan[a*x] + (2176*I)*ArcTan[a*x]^2 - 2640*a*x*A
rcTan[a*x]^2 - 560*a^3*x^3*ArcTan[a*x]^2 - 96*a^5*x^5*ArcTan[a*x]^2 + 880*
ArcTan[a*x]^3 + 1440*a^2*x^2*ArcTan[a*x]^3 + 720*a^4*x^4*ArcTan[a*x]^3 + 1
60*a^6*x^6*ArcTan[a*x]^3 + (480*I)*ArcTan[a*x]^4 + 960*ArcTan[a*x]^3*Log[1
 - E^((-2*I)*ArcTan[a*x])] - 4352*ArcTan[a*x]*Log[1 + E^((2*I)*ArcTan[a*x]
)] - 960*ArcTan[a*x]^3*Log[1 + E^((2*I)*ArcTan[a*x])] + (1440*I)*ArcTan[a*
x]^2*PolyLog[2, E^((-2*I)*ArcTan[a*x])] + (32*I)*(68 + 45*ArcTan[a*x]^2)*P
olyLog[2, -E^((2*I)*ArcTan[a*x])] + 1440*ArcTan[a*x]*PolyLog[3, E^((-2*I)*
ArcTan[a*x])] - 1440*ArcTan[a*x]*PolyLog[3, -E^((2*I)*ArcTan[a*x])] - (720
*I)*PolyLog[4, E^((-2*I)*ArcTan[a*x])] - (720*I)*PolyLog[4, -E^((2*I)*ArcT
an[a*x])]))/960
```

## Rubi [A] (verified)

Time = 2.00 (sec) , antiderivative size = 447, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3 \left(a^2cx^2+c\right)^3}{x} \, dx$$

$$\Big\downarrow \text{5483}$$

$$\int \left(a^6c^3x^5 \arctan(ax)^3 + 3a^4c^3x^3 \arctan(ax)^3 + 3a^2c^3x \arctan(ax)^3 + \frac{c^3 \arctan(ax)^3}{x}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{6}a^6c^3x^6\arctan(ax)^3-\frac{1}{10}a^5c^3x^5\arctan(ax)^2+\frac{3}{4}a^4c^3x^4\arctan(ax)^3+\frac{1}{20}a^4c^3x^4\arctan(ax)-\frac{7}{12}a^3c^3x^3\arctan(ax)^2-\frac{1}{60}a^3c^3x^3+\frac{3}{2}a^2c^3x^2\arctan(ax)^3+\frac{29}{60}a^2c^3x^2\arctan(ax)+2c^3\arctan(ax)^3\operatorname{arctanh}\left(1-\frac{2}{1+iax}\right)-\frac{3}{2}ic^3\arctan(ax)^2\operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right)+\frac{3}{2}ic^3\arctan(ax)^2\operatorname{PolyLog}\left(2,\frac{2}{iax+1}-1\right)-\frac{3}{2}c^3\arctan(ax)\operatorname{PolyLog}\left(3,1-\frac{2}{iax+1}\right)+\frac{3}{2}c^3\arctan(ax)\operatorname{PolyLog}\left(3,\frac{2}{iax+1}-1\right)-\frac{11}{4}ac^3x\arctan(ax)^2+\frac{11}{12}c^3\arctan(ax)^3-\frac{34}{15}ic^3\arctan(ax)^2+\frac{13}{30}c^3\arctan(ax)-\frac{68}{15}c^3\arctan(ax)\log\left(\frac{2}{1+iax}\right)-\frac{34}{15}ic^3\operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right)+\frac{3}{4}ic^3\operatorname{PolyLog}\left(4,1-\frac{2}{iax+1}\right)-\frac{3}{4}ic^3\operatorname{PolyLog}\left(4,\frac{2}{iax+1}-1\right)-\frac{13}{30}ac^3x$$

input

```
Int[((c + a^2*c*x^2)^3*ArcTan[a*x]^3)/x,x]
```

output

```
(-13*a*c^3*x)/30 - (a^3*c^3*x^3)/60 + (13*c^3*ArcTan[a*x])/30 + (29*a^2*c^
3*x^2*ArcTan[a*x])/60 + (a^4*c^3*x^4*ArcTan[a*x])/20 - ((34*I)/15)*c^3*Arc
Tan[a*x]^2 - (11*a*c^3*x*ArcTan[a*x]^2)/4 - (7*a^3*c^3*x^3*ArcTan[a*x]^2)/
12 - (a^5*c^3*x^5*ArcTan[a*x]^2)/10 + (11*c^3*ArcTan[a*x]^3)/12 + (3*a^2*c
^3*x^2*ArcTan[a*x]^3)/2 + (3*a^4*c^3*x^4*ArcTan[a*x]^3)/4 + (a^6*c^3*x^6*A
rcTan[a*x]^3)/6 + 2*c^3*ArcTan[a*x]^3*ArcTanh[1 - 2/(1 + I*a*x)] - (68*c^3
*ArcTan[a*x]*Log[2/(1 + I*a*x)])/15 - ((34*I)/15)*c^3*PolyLog[2, 1 - 2/(1
+ I*a*x)] - ((3*I)/2)*c^3*ArcTan[a*x]^2*PolyLog[2, 1 - 2/(1 + I*a*x)] + ((
3*I)/2)*c^3*ArcTan[a*x]^2*PolyLog[2, -1 + 2/(1 + I*a*x)] - (3*c^3*ArcTan[a
*x]*PolyLog[3, 1 - 2/(1 + I*a*x)])/2 + (3*c^3*ArcTan[a*x]*PolyLog[3, -1 +
2/(1 + I*a*x)])/2 + ((3*I)/4)*c^3*PolyLog[4, 1 - 2/(1 + I*a*x)] - ((3*I)/4
)*c^3*PolyLog[4, -1 + 2/(1 + I*a*x)]
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

## Maple [A] (verified)

Time = 39.12 (sec) , antiderivative size = 664, normalized size of antiderivative = 1.49

| method | result |
|---|---|
| derivativedivides | $\frac{c^3\left(iax+55x\arctan(ax)^3a+29i\arctan(ax)^2ax+35\arctan(ax)^3a^3x^3-10i\arctan(ax)^3a^4x^4+10\arctan(ax)^3a^5x^5-136\right.}{}$ |
| default | $\frac{c^3\left(iax+55x\arctan(ax)^3a+29i\arctan(ax)^2ax+35\arctan(ax)^3a^3x^3-10i\arctan(ax)^3a^4x^4+10\arctan(ax)^3a^5x^5-136\right.}{}$ |

input
```
int((a^2*c*x^2+c)^3*arctan(a*x)^3/x,x,method=_RETURNVERBOSE)
```

output
```
1/60*c^3*(I*a*x+55*x*arctan(a*x)^3*a+29*I*arctan(a*x)^2*a*x+35*arctan(a*x)
^3*a^3*x^3-10*I*arctan(a*x)^3*a^4*x^4+10*arctan(a*x)^3*a^5*x^5-136*arctan(
a*x)^2-35*I*arctan(a*x)^3*a^2*x^2-29*arctan(a*x)^2*x^2*a^2-3*I*arctan(a*x)
*a^2*x^2-6*a^4*arctan(a*x)^2*x^4+6*I*arctan(a*x)^2*a^3*x^3+26*arctan(a*x)*
a*x-55*I*arctan(a*x)^3+3*arctan(a*x)*x^3*a^3-25-26*I*arctan(a*x)-a^2*x^2)*
(a*x+I)+3/2*I*c^3*arctan(a*x)^2*polylog(2,-(1+I*a*x)^2/(a^2*x^2+1))-68/15*
c^3*arctan(a*x)*ln((1+I*a*x)^2/(a^2*x^2+1)+1)-3/4*I*c^3*polylog(4,-(1+I*a*
x)^2/(a^2*x^2+1))+c^3*arctan(a*x)^3*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*I*
c^3*arctan(a*x)^2*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*c^3*arctan(a*x)
*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*c^3*polylog(4,(1+I*a*x)/(a^2*
x^2+1)^(1/2))-c^3*arctan(a*x)^3*ln((1+I*a*x)^2/(a^2*x^2+1)+1)-3*I*c^3*arct
an(a*x)^2*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-3/2*c^3*arctan(a*x)*poly
log(3,-(1+I*a*x)^2/(a^2*x^2+1))+34/15*I*c^3*polylog(2,-(1+I*a*x)^2/(a^2*x^
2+1))+c^3*arctan(a*x)^3*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*c^3*polylog(
4,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*c^3*arctan(a*x)*polylog(3,(1+I*a*x)/(a^2
*x^2+1)^(1/2))+68/15*I*c^3*arctan(a*x)^2
```

## Fricas [F]

$$\int \frac{(c+a^2cx^2)^3\arctan(ax)^3}{x}\,dx=\int \frac{(a^2cx^2+c)^3\arctan(ax)^3}{x}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^3/x,x, algorithm="fricas")
```

output
```
integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a*x)^3
/x, x)
```

## Sympy [F]

$$\int \frac{(c+a^2cx^2)^3\arctan(ax)^3}{x}\,dx=c^3\left(\int \frac{\operatorname{atan}^3(ax)}{x}\,dx+\int 3a^2x\operatorname{atan}^3(ax)\,dx+\int 3a^4x^3\operatorname{atan}^3(ax)\,dx+\int a^6x^5\operatorname{atan}^3(ax)\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**3*atan(a*x)**3/x,x)
```

output
```
c**3*(Integral(atan(a*x)**3/x, x) + Integral(3*a**2*x*atan(a*x)**3, x) + I
ntegral(3*a**4*x**3*atan(a*x)**3, x) + Integral(a**6*x**5*atan(a*x)**3, x)
)
```

## Maxima [F]

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^3}{x}\,dx = \int \frac{(a^2cx^2+c)^3 \arctan(ax)^3}{x}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^3/x,x, algorithm="maxima")
```

output
```
1/96*(2*a^6*c^3*x^6 + 9*a^4*c^3*x^4 + 18*a^2*c^3*x^2)*arctan(a*x)^3 - 1/12
8*(2*a^6*c^3*x^6 + 9*a^4*c^3*x^4 + 18*a^2*c^3*x^2)*arctan(a*x)*log(a^2*x^2
 + 1)^2 + integrate(1/128*(112*(a^8*c^3*x^8 + 4*a^6*c^3*x^6 + 6*a^4*c^3*x^
4 + 4*a^2*c^3*x^2 + c^3)*arctan(a*x)^3 - 4*(2*a^7*c^3*x^7 + 9*a^5*c^3*x^5
+ 18*a^3*c^3*x^3)*arctan(a*x)^2 + 4*(2*a^8*c^3*x^8 + 9*a^6*c^3*x^6 + 18*a^
4*c^3*x^4)*arctan(a*x)*log(a^2*x^2 + 1) + (2*a^7*c^3*x^7 + 9*a^5*c^3*x^5 +
 18*a^3*c^3*x^3 + 12*(a^8*c^3*x^8 + 4*a^6*c^3*x^6 + 6*a^4*c^3*x^4 + 4*a^2*
c^3*x^2 + c^3)*arctan(a*x))*log(a^2*x^2 + 1)^2)/(a^2*x^3 + x), x)
```

## Giac [F]

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^3}{x}\,dx = \int \frac{(a^2cx^2+c)^3 \arctan(ax)^3}{x}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^3/x,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^3*arctan(a*x)^3/x, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^3}{x}\,dx = \int \frac{\operatorname{atan}(a\,x)^3\,(c\,a^2\,x^2+c)^3}{x}\,dx$$

input
```
int((atan(a*x)^3*(c + a^2*c*x^2)^3)/x,x)
```

output
```
int((atan(a*x)^3*(c + a^2*c*x^2)^3)/x, x)
```

## Reduce [F]

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^3}{x}\,dx$$
$$= \frac{c^3\Big(10atan(ax)^3\,a^6x^6+45atan(ax)^3\,a^4x^4+90atan(ax)^3\,a^2x^2+55atan(ax)^3-6atan(ax)^2\,a^5x^5-35ata}{}$$

input
```
int((a^2*c*x^2+c)^3*atan(a*x)^3/x,x)
```

output
```
(c**3*(10*atan(a*x)**3*a**6*x**6 + 45*atan(a*x)**3*a**4*x**4 + 90*atan(a*x
)**3*a**2*x**2 + 55*atan(a*x)**3 - 6*atan(a*x)**2*a**5*x**5 - 35*atan(a*x)
**2*a**3*x**3 - 165*atan(a*x)**2*a*x + 3*atan(a*x)*a**4*x**4 + 299*atan(a*
x)*a**2*x**2 + 296*atan(a*x) + 60*int(atan(a*x)**3/x,x) - 540*int((atan(a*
x)*x**3)/(a**2*x**2 + 1),x)*a**4 - 268*int((atan(a*x)*x)/(a**2*x**2 + 1),x
)*a**2 - a**3*x**3 - 296*a*x))/60
```

# 3.384 $\int \frac{(c+a^2cx^2)^3 \arctan(ax)^3}{x^2} \, dx$

## Optimal result

Integrand size = 22, antiderivative size = 354

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^3}{x^2} dx = -\frac{1}{20}a^3c^3x^2 + \frac{21}{10}a^2c^3x\arctan(ax) + \frac{1}{10}a^4c^3x^3\arctan(ax) - \frac{21}{20}ac^3\arctan(ax)^2 - \frac{6}{5}a^3c^3x^2\arctan(ax)^2 - \frac{3}{20}a^5c^3x^4\arctan(ax)^2 + \frac{6}{5}iac^3\arctan(ax)^3 - \frac{c^3\arctan(ax)^3}{x} + 3a^2c^3x\arctan(ax)^3 + a^4c^3x^3\arctan(ax)^3 + \frac{1}{5}a^6c^3x^5\arctan(ax)^3 + \frac{33}{5}ac^3\arctan(ax)^2\log\left(\frac{2}{1+iax}\right) - ac^3\log(1+a^2x^2) + 3ac^3\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right) - 3iac^3\arctan(ax)\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right) + \frac{33}{5}iac^3\arctan(ax)\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right) + \frac{3}{2}ac^3\operatorname{PolyLog}\left(3,-1+\frac{2}{1-iax}\right) + \frac{33}{10}ac^3\operatorname{PolyLog}\left(3,1-\frac{2}{1+iax}\right)$$

output
```
-1/20*a^3*c^3*x^2+21/10*a^2*c^3*x*arctan(a*x)+1/10*a^4*c^3*x^3*arctan(a*x)
-21/20*a*c^3*arctan(a*x)^2-6/5*a^3*c^3*x^2*arctan(a*x)^2-3/20*a^5*c^3*x^4*
arctan(a*x)^2+6/5*I*a*c^3*arctan(a*x)^3-c^3*arctan(a*x)^3/x+3*a^2*c^3*x*ar
ctan(a*x)^3+a^4*c^3*x^3*arctan(a*x)^3+1/5*a^6*c^3*x^5*arctan(a*x)^3+33/5*a
*c^3*arctan(a*x)^2*ln(2/(1+I*a*x))-a*c^3*ln(a^2*x^2+1)+3*a*c^3*arctan(a*x)
^2*ln(2-2/(1-I*a*x))+33/5*I*a*c^3*arctan(a*x)*polylog(2,1-2/(1+I*a*x))-3*I
*a*c^3*arctan(a*x)*polylog(2,-1+2/(1-I*a*x))+3/2*a*c^3*polylog(3,-1+2/(1-I
*a*x))+33/10*a*c^3*polylog(3,1-2/(1+I*a*x))
```

## Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 298, normalized size of antiderivative = 0.84

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^3}{x^2} dx$$
$$= \frac{c^3(-2ax - 5ia\pi^3x - 2a^3x^3 + 84a^2x^2 \arctan(ax) + 4a^4x^4 \arctan(ax) - 42ax \arctan(ax)^2 - 48a^3x^3 \text{arcta}}{}$$

input
```
Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x]^3)/x^2,x]
```

output
```
(c^3*(-2*a*x - (5*I)*a*Pi^3*x - 2*a^3*x^3 + 84*a^2*x^2*ArcTan[a*x] + 4*a^4
*x^4*ArcTan[a*x] - 42*a*x*ArcTan[a*x]^2 - 48*a^3*x^3*ArcTan[a*x]^2 - 6*a^5
*x^5*ArcTan[a*x]^2 - 40*ArcTan[a*x]^3 - (48*I)*a*x*ArcTan[a*x]^3 + 120*a^2
*x^2*ArcTan[a*x]^3 + 40*a^4*x^4*ArcTan[a*x]^3 + 8*a^6*x^6*ArcTan[a*x]^3 +
120*a*x*ArcTan[a*x]^2*Log[1 - E^((-2*I)*ArcTan[a*x])] + 264*a*x*ArcTan[a*x
]^2*Log[1 + E^((2*I)*ArcTan[a*x])] - 40*a*x*Log[1 + a^2*x^2] + (120*I)*a*x
*ArcTan[a*x]*PolyLog[2, E^((-2*I)*ArcTan[a*x])] - (264*I)*a*x*ArcTan[a*x]*
PolyLog[2, -E^((2*I)*ArcTan[a*x])] + 60*a*x*PolyLog[3, E^((-2*I)*ArcTan[a*
x])] + 132*a*x*PolyLog[3, -E^((2*I)*ArcTan[a*x])]))/(40*x)
```

## Rubi [A] (verified)

Time = 1.58 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3 (a^2cx^2+c)^3}{x^2} dx$$
$$\Big\downarrow \text{ 5483}$$
$$\int \left(a^6c^3x^4 \arctan(ax)^3 + 3a^4c^3x^2 \arctan(ax)^3 + 3a^2c^3 \arctan(ax)^3 + \frac{c^3 \arctan(ax)^3}{x^2}\right) dx$$
$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{5}a^6c^3x^5\arctan(ax)^3-\frac{3}{20}a^5c^3x^4\arctan(ax)^2+a^4c^3x^3\arctan(ax)^3+\frac{1}{10}a^4c^3x^3\arctan(ax)-\frac{6}{5}a^3c^3x^2\arctan(ax)^2-\frac{1}{20}a^3c^3x^2+3a^2c^3x\arctan(ax)^3+\frac{21}{10}a^2c^3x\arctan(ax)-ac^3\log\left(a^2x^2+1\right)-3iac^3\arctan(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)+\frac{33}{5}iac^3\arctan(ax)\operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right)+\frac{6}{5}iac^3\arctan(ax)^3-\frac{21}{20}ac^3\arctan(ax)^2-\frac{c^3\arctan(ax)^3}{x}+\frac{33}{5}ac^3\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)+3ac^3\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)+\frac{3}{2}ac^3\operatorname{PolyLog}\left(3,\frac{2}{1-iax}-1\right)+\frac{33}{10}ac^3\operatorname{PolyLog}\left(3,1-\frac{2}{iax+1}\right)$$

input
```
Int[((c + a^2*c*x^2)^3*ArcTan[a*x]^3)/x^2,x]
```

output
```
-1/20*(a^3*c^3*x^2) + (21*a^2*c^3*x*ArcTan[a*x])/10 + (a^4*c^3*x^3*ArcTan[
a*x])/10 - (21*a*c^3*ArcTan[a*x]^2)/20 - (6*a^3*c^3*x^2*ArcTan[a*x]^2)/5 -
 (3*a^5*c^3*x^4*ArcTan[a*x]^2)/20 + ((6*I)/5)*a*c^3*ArcTan[a*x]^3 - (c^3*A
rcTan[a*x]^3)/x + 3*a^2*c^3*x*ArcTan[a*x]^3 + a^4*c^3*x^3*ArcTan[a*x]^3 +
(a^6*c^3*x^5*ArcTan[a*x]^3)/5 + (33*a*c^3*ArcTan[a*x]^2*Log[2/(1 + I*a*x)]
)/5 - a*c^3*Log[1 + a^2*x^2] + 3*a*c^3*ArcTan[a*x]^2*Log[2 - 2/(1 - I*a*x)
] - (3*I)*a*c^3*ArcTan[a*x]*PolyLog[2, -1 + 2/(1 - I*a*x)] + ((33*I)/5)*a*
c^3*ArcTan[a*x]*PolyLog[2, 1 - 2/(1 + I*a*x)] + (3*a*c^3*PolyLog[3, -1 + 2
/(1 - I*a*x)])/2 + (33*a*c^3*PolyLog[3, 1 - 2/(1 + I*a*x)])/10
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 10.59 (sec) , antiderivative size = 1988, normalized size of antiderivative = 5.62

Expression too large to display

input
```
int((a^2*c*x^2+c)^3*arctan(a*x)^3/x^2,x)
```

output
```
a*(1/5*c^3*arctan(a*x)^3*a^5*x^5+c^3*arctan(a*x)^3*a^3*x^3+3*c^3*arctan(a*
x)^3*a*x-c^3*arctan(a*x)^3/a/x-3/5*c^3*(4*I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^
2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/
((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*arctan(a*x)^2+4*I*Pi*csgn(I*(1+I*a*x)/(a^2*
x^2+1)^(1/2))^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*arctan(a*x)^2+8*I*Pi*csgn(
I*((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2*arc
tan(a*x)^2+7/4*arctan(a*x)^2+2*arctan(a*x)^2*x^2*a^2-8*I*Pi*csgn(I*(1+I*a*
x)/(a^2*x^2+1)^(1/2))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^2*arctan(a*x)^2-4*I*
Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((
1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2*arctan(a*x)^2-4*I*Pi*csgn(I*(1+I*a*x)^2/(a^
2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2*
arctan(a*x)^2-4*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))^2*csgn(I*((1+I*a*
x)^2/(a^2*x^2+1)+1)^2)*arctan(a*x)^2-10*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1
/2))-10*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+16/3*I*arctan(a*x)^3+1/4*a
^4*arctan(a*x)^2*x^4-3*arctan(a*x)*(a*x-I)-5/2*I*Pi*csgn(I*((1+I*a*x)^2/(a
^2*x^2+1)-1))*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2
*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*arctan(a*x)^2-16*arctan(a*x)^2*ln(
(1+I*a*x)/(a^2*x^2+1)^(1/2))-11/2*polylog(3,-(1+I*a*x)^2/(a^2*x^2+1))+5*ar
ctan(a*x)^2*ln((1+I*a*x)^2/(a^2*x^2+1)-1)-10/3*ln((1+I*a*x)^2/(a^2*x^2+1)+
1)+1/12*(a*x+I)^2-1/6*I*(a*x+I)-5*arctan(a*x)^2*ln(a*x)-5*arctan(a*x)^2...
```

## Fricas [F]

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^3}{x^2} \, dx = \int \frac{(a^2cx^2+c)^3 \arctan(ax)^3}{x^2} \, dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^3/x^2,x, algorithm="fricas")
```

output
```
integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a*x)^3
/x^2, x)
```

## Sympy [F]

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^3}{x^2} \, dx = c^3 \left( \int 3a^2 \operatorname{atan}^3(ax) \, dx + \int \frac{\operatorname{atan}^3(ax)}{x^2} \, dx + \int 3a^4x^2 \operatorname{atan}^3(ax) \, dx + \int a^6x^4 \operatorname{atan}^3(ax) \, dx \right)$$

input
```
integrate((a**2*c*x**2+c)**3*atan(a*x)**3/x**2,x)
```

output
```
c**3*(Integral(3*a**2*atan(a*x)**3, x) + Integral(atan(a*x)**3/x**2, x) +
Integral(3*a**4*x**2*atan(a*x)**3, x) + Integral(a**6*x**4*atan(a*x)**3, x
))
```

## Maxima [F]

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^3}{x^2} \, dx = \int \frac{(a^2cx^2+c)^3 \arctan(ax)^3}{x^2} \, dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^3/x^2,x, algorithm="maxima")
```

output
```
1/320*(8*(a^6*c^3*x^6 + 5*a^4*c^3*x^4 + 15*a^2*c^3*x^2 - 5*c^3)*arctan(a*x
)^3 - 6*(a^6*c^3*x^6 + 5*a^4*c^3*x^4 + 15*a^2*c^3*x^2 - 5*c^3)*arctan(a*x)
*log(a^2*x^2 + 1)^2 + 5*(8960*a^8*c^3*integrate(1/160*x^8*arctan(a*x)^3/(a
^2*x^4 + x^2), x) + 960*a^8*c^3*integrate(1/160*x^8*arctan(a*x)*log(a^2*x^
2 + 1)^2/(a^2*x^4 + x^2), x) + 768*a^8*c^3*integrate(1/160*x^8*arctan(a*x)
*log(a^2*x^2 + 1)/(a^2*x^4 + x^2), x) - 768*a^7*c^3*integrate(1/160*x^7*ar
ctan(a*x)^2/(a^2*x^4 + x^2), x) + 192*a^7*c^3*integrate(1/160*x^7*log(a^2*
x^2 + 1)^2/(a^2*x^4 + x^2), x) + 35840*a^6*c^3*integrate(1/160*x^6*arctan(
a*x)^3/(a^2*x^4 + x^2), x) + 3840*a^6*c^3*integrate(1/160*x^6*arctan(a*x)*
log(a^2*x^2 + 1)^2/(a^2*x^4 + x^2), x) + 3840*a^6*c^3*integrate(1/160*x^6*
arctan(a*x)*log(a^2*x^2 + 1)/(a^2*x^4 + x^2), x) - 3840*a^5*c^3*integrate(
1/160*x^5*arctan(a*x)^2/(a^2*x^4 + x^2), x) + 960*a^5*c^3*integrate(1/160*
x^5*log(a^2*x^2 + 1)^2/(a^2*x^4 + x^2), x) + 56*a*c^3*arctan(a*x)^4 + 5376
0*a^4*c^3*integrate(1/160*x^4*arctan(a*x)^3/(a^2*x^4 + x^2), x) + 5760*a^4
*c^3*integrate(1/160*x^4*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*x^4 + x^2), x
) + 11520*a^4*c^3*integrate(1/160*x^4*arctan(a*x)*log(a^2*x^2 + 1)/(a^2*x^
4 + x^2), x) - 11520*a^3*c^3*integrate(1/160*x^3*arctan(a*x)^2/(a^2*x^4 +
x^2), x) + 3*a*c^3*log(a^2*x^2 + 1)^3 + 3840*a^2*c^3*integrate(1/160*x^2*a
rctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*x^4 + x^2), x) - 3840*a^2*c^3*integrate
(1/160*x^2*arctan(a*x)*log(a^2*x^2 + 1)/(a^2*x^4 + x^2), x) + 3840*a*c^...
```

## Giac [F]

$$\int \frac{(c+a^2cx^2)^3\arctan(ax)^3}{x^2}\,dx=\int \frac{(a^2cx^2+c)^3\arctan(ax)^3}{x^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^3/x^2,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^3*arctan(a*x)^3/x^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^3}{x^2}\,dx = \int \frac{\mathrm{atan}(a\,x)^3\,(c\,a^2\,x^2+c)^3}{x^2}\,dx$$

input
```
int((atan(a*x)^3*(c + a^2*c*x^2)^3)/x^2,x)
```

output
```
int((atan(a*x)^3*(c + a^2*c*x^2)^3)/x^2, x)
```

## Reduce [F]

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^3}{x^2}\,dx$$
$$= \frac{c^3\Big(4atan(ax)^3\,a^6x^6 + 20atan(ax)^3\,a^4x^4 + 60atan(ax)^3\,a^2x^2 - 20atan(ax)^3 - 3atan(ax)^2\,a^5x^5 - 24atan}{}$$

input
```
int((a^2*c*x^2+c)^3*atan(a*x)^3/x^2,x)
```

output
```
(c**3*(4*atan(a*x)**3*a**6*x**6 + 20*atan(a*x)**3*a**4*x**4 + 60*atan(a*x)
**3*a**2*x**2 - 20*atan(a*x)**3 - 3*atan(a*x)**2*a**5*x**5 - 24*atan(a*x)*
*2*a**3*x**3 - 21*atan(a*x)**2*a*x + 2*atan(a*x)*a**4*x**4 + 42*atan(a*x)*
a**2*x**2 + 60*int(atan(a*x)**2/(a**2*x**3 + x),x)*a*x - 132*int((atan(a*x
)**2*x)/(a**2*x**2 + 1),x)*a**3*x - 20*log(a**2*x**2 + 1)*a*x - a**3*x**3)
)/(20*x)
```

# 3.385 $\int \frac{(c+a^2cx^2)^3\arctan(ax)^3}{x^3}\,dx$

## Optimal result

Integrand size = 22, antiderivative size = 503

$$\int \frac{(c+a^2cx^2)^3\arctan(ax)^3}{x^3}\,dx = -\frac{1}{4}a^3c^3x+\frac{1}{4}a^2c^3\arctan(ax)+\frac{1}{4}a^4c^3x^2\arctan(ax) - 5ia^2c^3\arctan(ax)^2-\frac{3ac^3\arctan(ax)^2}{2x} - \frac{15}{4}a^3c^3x\arctan(ax)^2-\frac{1}{4}a^5c^3x^3\arctan(ax)^2 + \frac{3}{4}a^2c^3\arctan(ax)^3-\frac{c^3\arctan(ax)^3}{2x^2} + \frac{3}{2}a^4c^3x^2\arctan(ax)^3+\frac{1}{4}a^6c^3x^4\arctan(ax)^3 + 6a^2c^3\arctan(ax)^3\operatorname{arctanh}\left(1-\frac{2}{1+iax}\right) - 7a^2c^3\arctan(ax)\log\left(\frac{2}{1+iax}\right) + 3a^2c^3\arctan(ax)\log\left(2-\frac{2}{1-iax}\right) - \frac{3}{2}ia^2c^3\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right) - \frac{7}{2}ia^2c^3\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right) - \frac{9}{2}ia^2c^3\arctan(ax)^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right) + \frac{9}{2}ia^2c^3\arctan(ax)^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1+iax}\right) - \frac{9}{2}a^2c^3\arctan(ax)\operatorname{PolyLog}\left(3,1-\frac{2}{1+iax}\right) + \frac{9}{2}a^2c^3\arctan(ax)\operatorname{PolyLog}\left(3,-1+\frac{2}{1+iax}\right) + \frac{9}{4}ia^2c^3\operatorname{PolyLog}\left(4,1-\frac{2}{1+iax}\right) - \frac{9}{4}ia^2c^3\operatorname{PolyLog}\left(4,-1+\frac{2}{1+iax}\right)$$

output
```
-1/4*a^3*c^3*x+1/4*a^2*c^3*arctan(a*x)+1/4*a^4*c^3*x^2*arctan(a*x)-9/4*I*a
^2*c^3*polylog(4,-1+2/(1+I*a*x))-3/2*a*c^3*arctan(a*x)^2/x-15/4*a^3*c^3*x*
arctan(a*x)^2-1/4*a^5*c^3*x^3*arctan(a*x)^2+3/4*a^2*c^3*arctan(a*x)^3-1/2*
c^3*arctan(a*x)^3/x^2+3/2*a^4*c^3*x^2*arctan(a*x)^3+1/4*a^6*c^3*x^4*arctan
(a*x)^3-6*a^2*c^3*arctan(a*x)^3*arctanh(-1+2/(1+I*a*x))-7*a^2*c^3*arctan(a
*x)*ln(2/(1+I*a*x))+3*a^2*c^3*arctan(a*x)*ln(2-2/(1-I*a*x))+9/2*I*a^2*c^3*
arctan(a*x)^2*polylog(2,-1+2/(1+I*a*x))-9/2*I*a^2*c^3*arctan(a*x)^2*polylo
g(2,1-2/(1+I*a*x))+9/4*I*a^2*c^3*polylog(4,1-2/(1+I*a*x))-5*I*a^2*c^3*arct
an(a*x)^2-9/2*a^2*c^3*arctan(a*x)*polylog(3,1-2/(1+I*a*x))+9/2*a^2*c^3*arc
tan(a*x)*polylog(3,-1+2/(1+I*a*x))-7/2*I*a^2*c^3*polylog(2,1-2/(1+I*a*x))-
3/2*I*a^2*c^3*polylog(2,-1+2/(1-I*a*x))
```

## Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 464, normalized size of antiderivative = 0.92

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^3}{x^3} dx$$
$$= \frac{c^3(-3ia^2\pi^4x^2 - 16a^3x^3 + 16a^2x^2\arctan(ax) + 16a^4x^4\arctan(ax) - 96ax\arctan(ax)^2 + 128ia^2x^2 \text{arcta}}{}$$

input
```
Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x]^3)/x^3,x]
```

output
```
(c^3*((-3*I)*a^2*Pi^4*x^2 - 16*a^3*x^3 + 16*a^2*x^2*ArcTan[a*x] + 16*a^4*x
^4*ArcTan[a*x] - 96*a*x*ArcTan[a*x]^2 + (128*I)*a^2*x^2*ArcTan[a*x]^2 - 24
0*a^3*x^3*ArcTan[a*x]^2 - 16*a^5*x^5*ArcTan[a*x]^2 - 32*ArcTan[a*x]^3 + 48
*a^2*x^2*ArcTan[a*x]^3 + 96*a^4*x^4*ArcTan[a*x]^3 + 16*a^6*x^6*ArcTan[a*x]
^3 + (96*I)*a^2*x^2*ArcTan[a*x]^4 + 192*a^2*x^2*ArcTan[a*x]^3*Log[1 - E^((
-2*I)*ArcTan[a*x])] + 192*a^2*x^2*ArcTan[a*x]*Log[1 - E^((2*I)*ArcTan[a*x]
)] - 448*a^2*x^2*ArcTan[a*x]*Log[1 + E^((2*I)*ArcTan[a*x])] - 192*a^2*x^2*
ArcTan[a*x]^3*Log[1 + E^((2*I)*ArcTan[a*x])] + (288*I)*a^2*x^2*ArcTan[a*x]
^2*PolyLog[2, E^((-2*I)*ArcTan[a*x])] + (32*I)*a^2*x^2*(7 + 9*ArcTan[a*x]^
2)*PolyLog[2, -E^((2*I)*ArcTan[a*x])] - (96*I)*a^2*x^2*PolyLog[2, E^((2*I)
*ArcTan[a*x])] + 288*a^2*x^2*ArcTan[a*x]*PolyLog[3, E^((-2*I)*ArcTan[a*x])
] - 288*a^2*x^2*ArcTan[a*x]*PolyLog[3, -E^((2*I)*ArcTan[a*x])] - (144*I)*a
^2*x^2*PolyLog[4, E^((-2*I)*ArcTan[a*x])] - (144*I)*a^2*x^2*PolyLog[4, -E^
((2*I)*ArcTan[a*x])]))/(64*x^2)
```

## Rubi [A] (verified)

Time = 1.53 (sec) , antiderivative size = 503, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3 \left(a^2cx^2+c\right)^3}{x^3}dx$$

$$\Big\downarrow \text{5483}$$

$$\int \left(a^6c^3x^3\arctan(ax)^3+3a^4c^3x\arctan(ax)^3+\frac{3a^2c^3\arctan(ax)^3}{x}+\frac{c^3\arctan(ax)^3}{x^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}a^6c^3x^4\arctan(ax)^3-\frac{1}{4}a^5c^3x^3\arctan(ax)^2+\frac{3}{2}a^4c^3x^2\arctan(ax)^3+\frac{1}{4}a^4c^3x^2\arctan(ax)-\frac{15}{4}a^3c^3x\arctan(ax)^2-\frac{1}{4}a^3c^3x+6a^2c^3\arctan(ax)^3\text{arctanh}\left(1-\frac{2}{1+iax}\right)-\frac{9}{2}ia^2c^3\arctan(ax)^2\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)+\frac{9}{2}ia^2c^3\arctan(ax)^2\text{PolyLog}\left(2,\frac{2}{iax+1}-1\right)-\frac{9}{2}a^2c^3\arctan(ax)\text{PolyLog}\left(3,1-\frac{2}{iax+1}\right)+\frac{9}{2}a^2c^3\arctan(ax)\text{PolyLog}\left(3,\frac{2}{iax+1}-1\right)+\frac{3}{4}a^2c^3\arctan(ax)^3-5ia^2c^3\arctan(ax)^2+\frac{1}{4}a^2c^3\arctan(ax)-7a^2c^3\arctan(ax)\log\left(\frac{2}{1+iax}\right)+3a^2c^3\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{3}{2}ia^2c^3\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)-\frac{7}{2}ia^2c^3\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)+\frac{9}{4}ia^2c^3\text{PolyLog}\left(4,1-\frac{2}{iax+1}\right)-\frac{9}{4}ia^2c^3\text{PolyLog}\left(4,\frac{2}{iax+1}-1\right)-\frac{c^3\arctan(ax)^3}{2x^2}-\frac{3ac^3\arctan(ax)^2}{2x}$$

input
```
Int[((c + a^2*c*x^2)^3*ArcTan[a*x]^3)/x^3,x]
```

output
```
-1/4*(a^3*c^3*x) + (a^2*c^3*ArcTan[a*x])/4 + (a^4*c^3*x^2*ArcTan[a*x])/4 -
 (5*I)*a^2*c^3*ArcTan[a*x]^2 - (3*a*c^3*ArcTan[a*x]^2)/(2*x) - (15*a^3*c^3
*x*ArcTan[a*x]^2)/4 - (a^5*c^3*x^3*ArcTan[a*x]^2)/4 + (3*a^2*c^3*ArcTan[a*
x]^3)/4 - (c^3*ArcTan[a*x]^3)/(2*x^2) + (3*a^4*c^3*x^2*ArcTan[a*x]^3)/2 +
(a^6*c^3*x^4*ArcTan[a*x]^3)/4 + 6*a^2*c^3*ArcTan[a*x]^3*ArcTanh[1 - 2/(1 +
 I*a*x)] - 7*a^2*c^3*ArcTan[a*x]*Log[2/(1 + I*a*x)] + 3*a^2*c^3*ArcTan[a*x
]*Log[2 - 2/(1 - I*a*x)] - ((3*I)/2)*a^2*c^3*PolyLog[2, -1 + 2/(1 - I*a*x)
] - ((7*I)/2)*a^2*c^3*PolyLog[2, 1 - 2/(1 + I*a*x)] - ((9*I)/2)*a^2*c^3*Ar
cTan[a*x]^2*PolyLog[2, 1 - 2/(1 + I*a*x)] + ((9*I)/2)*a^2*c^3*ArcTan[a*x]^
2*PolyLog[2, -1 + 2/(1 + I*a*x)] - (9*a^2*c^3*ArcTan[a*x]*PolyLog[3, 1 - 2
/(1 + I*a*x)])/2 + (9*a^2*c^3*ArcTan[a*x]*PolyLog[3, -1 + 2/(1 + I*a*x)])/
2 + ((9*I)/4)*a^2*c^3*PolyLog[4, 1 - 2/(1 + I*a*x)] - ((9*I)/4)*a^2*c^3*Po
lyLog[4, -1 + 2/(1 + I*a*x)]
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

## Maple [A] (verified)

Time = 80.68 (sec) , antiderivative size = 763, normalized size of antiderivative = 1.52

| method | result |
|---|---|
| derivativedivides | $a^2 \left( \frac{c^3 \left(2i \arctan(ax)^3+6i \arctan(ax)^2 ax-2x \arctan(ax)^3 a-14 \arctan(ax)^2 x^2 a^2-5i \arctan(ax)^3 a^2 x^2+i \arctan(ax)\right.}{} \right.$ |
| default | $a^2 \left( \frac{c^3 \left(2i \arctan(ax)^3+6i \arctan(ax)^2 ax-2x \arctan(ax)^3 a-14 \arctan(ax)^2 x^2 a^2-5i \arctan(ax)^3 a^2 x^2+i \arctan(ax)\right.}{} \right.$ |

input
```
int((a^2*c*x^2+c)^3*arctan(a*x)^3/x^3,x,method=_RETURNVERBOSE)
```

output
```
a^2*(1/4*c^3*(2*I*arctan(a*x)^3+6*I*arctan(a*x)^2*a*x-2*x*arctan(a*x)^3*a-
14*arctan(a*x)^2*x^2*a^2-5*I*arctan(a*x)^3*a^2*x^2+I*arctan(a*x)^2*a^3*x^3
+5*arctan(a*x)^3*a^3*x^3-a^4*arctan(a*x)^2*x^4-I*arctan(a*x)^3*a^4*x^4+arc
tan(a*x)^3*a^5*x^5-a^2*x^2-I*arctan(a*x)*a^2*x^2+arctan(a*x)*x^3*a^3)*(a*x
+I)/a^2/x^2-9/4*I*c^3*polylog(4,-(1+I*a*x)^2/(a^2*x^2+1))-3*c^3*arctan(a*x
)^3*ln((1+I*a*x)^2/(a^2*x^2+1)+1)-9*I*c^3*arctan(a*x)^2*polylog(2,(1+I*a*x
)/(a^2*x^2+1)^(1/2))+18*c^3*arctan(a*x)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1
/2))-9*I*c^3*arctan(a*x)^2*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+3*c^3*a
rctan(a*x)^3*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+7/2*I*c^3*polylog(2,-(1+I*a
*x)^2/(a^2*x^2+1))+3*c^3*arctan(a*x)^3*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))+1
8*I*c^3*polylog(4,(1+I*a*x)/(a^2*x^2+1)^(1/2))-7*c^3*arctan(a*x)*ln((1+I*a
*x)^2/(a^2*x^2+1)+1)+9/2*I*c^3*arctan(a*x)^2*polylog(2,-(1+I*a*x)^2/(a^2*x
^2+1))-9/2*c^3*arctan(a*x)*polylog(3,-(1+I*a*x)^2/(a^2*x^2+1))+18*I*c^3*po
lylog(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+3*c^3*arctan(a*x)*ln(1+(1+I*a*x)/(a^
2*x^2+1)^(1/2))-3*I*c^3*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+3*c^3*arcta
n(a*x)*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))+4*I*c^3*arctan(a*x)^2+18*c^3*arct
an(a*x)*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*I*c^3*polylog(2,-(1+I*a*
x)/(a^2*x^2+1)^(1/2)))
```

## Fricas [F]

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^3}{x^3} \, dx = \int \frac{(a^2cx^2+c)^3 \arctan(ax)^3}{x^3} \, dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^3/x^3,x, algorithm="fricas")
```

output
```
integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a*x)^3
/x^3, x)
```

## Sympy [F]

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^3}{x^3}\,dx = c^3\left(\int \frac{\operatorname{atan}^3(ax)}{x^3}\,dx + \int \frac{3a^2\operatorname{atan}^3(ax)}{x}\,dx + \int 3a^4x\operatorname{atan}^3(ax)\,dx + \int a^6x^3\operatorname{atan}^3(ax)\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**3*atan(a*x)**3/x**3,x)
```

output
```
c**3*(Integral(atan(a*x)**3/x**3, x) + Integral(3*a**2*atan(a*x)**3/x, x)
+ Integral(3*a**4*x*atan(a*x)**3, x) + Integral(a**6*x**3*atan(a*x)**3, x)
)
```

## Maxima [F]

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^3}{x^3}\,dx = \int \frac{(a^2cx^2+c)^3 \arctan(ax)^3}{x^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^3/x^3,x, algorithm="maxima")
```

output
```
1/128*(4*(a^6*c^3*x^6 + 6*a^4*c^3*x^4 - 2*c^3)*arctan(a*x)^3 - 3*(a^6*c^3*
x^6 + 6*a^4*c^3*x^4 - 2*c^3)*arctan(a*x)*log(a^2*x^2 + 1)^2 + 128*x^2*inte
grate(1/128*(112*(a^8*c^3*x^8 + 4*a^6*c^3*x^6 + 6*a^4*c^3*x^4 + 4*a^2*c^3*
x^2 + c^3)*arctan(a*x)^3 - 12*(a^7*c^3*x^7 + 6*a^5*c^3*x^5 - 2*a*c^3*x)*ar
ctan(a*x)^2 + 12*(a^8*c^3*x^8 + 6*a^6*c^3*x^6 - 2*a^2*c^3*x^2)*arctan(a*x)
*log(a^2*x^2 + 1) + 3*(a^7*c^3*x^7 + 6*a^5*c^3*x^5 - 2*a*c^3*x + 4*(a^8*c^
3*x^8 + 4*a^6*c^3*x^6 + 6*a^4*c^3*x^4 + 4*a^2*c^3*x^2 + c^3)*arctan(a*x))*
log(a^2*x^2 + 1)^2)/(a^2*x^5 + x^3), x))/x^2
```

## Giac [F]

$$\int \frac{(c+a^2cx^2)^3\arctan(ax)^3}{x^3}\,dx = \int \frac{(a^2cx^2+c)^3\arctan(ax)^3}{x^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^3/x^3,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^3*arctan(a*x)^3/x^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^3\arctan(ax)^3}{x^3}\,dx = \int \frac{\mathrm{atan}(a\,x)^3\,(c\,a^2\,x^2+c)^3}{x^3}\,dx$$

input
```
int((atan(a*x)^3*(c + a^2*c*x^2)^3)/x^3,x)
```

output
```
int((atan(a*x)^3*(c + a^2*c*x^2)^3)/x^3, x)
```

## Reduce [F]

$$\int \frac{(c+a^2cx^2)^3\arctan(ax)^3}{x^3}\,dx$$
$$= \frac{c^3\Big(atan(ax)^3\,a^6x^6+6atan(ax)^3\,a^4x^4+3atan(ax)^3\,a^2x^2-2atan(ax)^3-atan(ax)^2\,a^5x^5-15atan(ax)^2}{}$$

input
```
int((a^2*c*x^2+c)^3*atan(a*x)^3/x^3,x)
```

output
```
(c**3*(atan(a*x)**3*a**6*x**6 + 6*atan(a*x)**3*a**4*x**4 + 3*atan(a*x)**3*
a**2*x**2 - 2*atan(a*x)**3 - atan(a*x)**2*a**5*x**5 - 15*atan(a*x)**2*a**3
*x**3 - 6*atan(a*x)**2*a*x + 19*atan(a*x)*a**4*x**4 + 13*atan(a*x)*a**2*x*
*2 - 6*atan(a*x) - 12*int(atan(a*x)/(a**2*x**5 + x**3),x)*x**2 + 12*int(at
an(a*x)**3/x,x)*a**2*x**2 - 36*int((atan(a*x)*x**3)/(a**2*x**2 + 1),x)*a**
6*x**2 - 8*int((atan(a*x)*x)/(a**2*x**2 + 1),x)*a**4*x**2 - 19*a**3*x**3 -
 6*a*x))/(4*x**2)
```

# 3.386 $\int \frac{(c+a^2cx^2)^3 \arctan(ax)^3}{x^4} \, dx$

## Optimal result

Integrand size = 22, antiderivative size = 336

$$\int \frac{(c+a^2cx^2)^3\arctan(ax)^3}{x^4}\,dx = -\frac{a^2c^3\arctan(ax)}{x}+a^4c^3x\arctan(ax) - a^3c^3\arctan(ax)^2-\frac{ac^3\arctan(ax)^2}{2x^2} - \frac{1}{2}a^5c^3x^2\arctan(ax)^2-\frac{c^3\arctan(ax)^3}{3x^3} - \frac{3a^2c^3\arctan(ax)^3}{x}+3a^4c^3x\arctan(ax)^3 + \frac{1}{3}a^6c^3x^3\arctan(ax)^3+a^3c^3\log(x) + 8a^3c^3\arctan(ax)^2\log\left(\frac{2}{1+iax}\right) - a^3c^3\log\left(1+a^2x^2\right) + 8a^3c^3\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right) - 8ia^3c^3\arctan(ax)\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right) + 8ia^3c^3\arctan(ax)\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right) + 4a^3c^3\operatorname{PolyLog}\left(3,-1+\frac{2}{1-iax}\right) + 4a^3c^3\operatorname{PolyLog}\left(3,1-\frac{2}{1+iax}\right)$$

output
```
-a^2*c^3*arctan(a*x)/x+a^4*c^3*x*arctan(a*x)-a^3*c^3*arctan(a*x)^2-1/2*a*c
^3*arctan(a*x)^2/x^2-1/2*a^5*c^3*x^2*arctan(a*x)^2-1/3*c^3*arctan(a*x)^3/x
^3-3*a^2*c^3*arctan(a*x)^3/x+3*a^4*c^3*x*arctan(a*x)^3+1/3*a^6*c^3*x^3*arc
tan(a*x)^3+a^3*c^3*ln(x)+8*a^3*c^3*arctan(a*x)^2*ln(2/(1+I*a*x))-a^3*c^3*l
n(a^2*x^2+1)+8*a^3*c^3*arctan(a*x)^2*ln(2-2/(1-I*a*x))-8*I*a^3*c^3*arctan(
a*x)*polylog(2,-1+2/(1-I*a*x))+8*I*a^3*c^3*arctan(a*x)*polylog(2,1-2/(1+I*
a*x))+4*a^3*c^3*polylog(3,-1+2/(1-I*a*x))+4*a^3*c^3*polylog(3,1-2/(1+I*a*x
))
```

## Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 331, normalized size of antiderivative = 0.99

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^3}{x^4} dx$$
$$= \frac{c^3\left(-2ia^3\pi^3x^3 - 6a^2x^2\arctan(ax) + 6a^4x^4\arctan(ax) - 3ax\arctan(ax)^2 - 6a^3x^3\arctan(ax)^2 - 3a^5x^5 \right.}{}$$

input
```
Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x]^3)/x^4,x]
```

output
```
(c^3*((-2*I)*a^3*Pi^3*x^3 - 6*a^2*x^2*ArcTan[a*x] + 6*a^4*x^4*ArcTan[a*x]
- 3*a*x*ArcTan[a*x]^2 - 6*a^3*x^3*ArcTan[a*x]^2 - 3*a^5*x^5*ArcTan[a*x]^2
- 2*ArcTan[a*x]^3 - 18*a^2*x^2*ArcTan[a*x]^3 + 18*a^4*x^4*ArcTan[a*x]^3 +
2*a^6*x^6*ArcTan[a*x]^3 + 48*a^3*x^3*ArcTan[a*x]^2*Log[1 - E^((-2*I)*ArcTa
n[a*x])] + 48*a^3*x^3*ArcTan[a*x]^2*Log[1 + E^((2*I)*ArcTan[a*x])] + 6*a^3
*x^3*Log[(a*x)/Sqrt[1 + a^2*x^2]] - 3*a^3*x^3*Log[1 + a^2*x^2] + (48*I)*a^
3*x^3*ArcTan[a*x]*PolyLog[2, E^((-2*I)*ArcTan[a*x])] - (48*I)*a^3*x^3*ArcT
an[a*x]*PolyLog[2, -E^((2*I)*ArcTan[a*x])] + 24*a^3*x^3*PolyLog[3, E^((-2*
I)*ArcTan[a*x])] + 24*a^3*x^3*PolyLog[3, -E^((2*I)*ArcTan[a*x])]))/(6*x^3)
```

## Rubi [A] (verified)

Time = 1.39 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {5483, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3 \left(a^2cx^2+c\right)^3}{x^4} dx$$
$$\Big\downarrow \text{ 5483}$$
$$\int \left(a^6c^3x^2\arctan(ax)^3 + 3a^4c^3\arctan(ax)^3 + \frac{3a^2c^3\arctan(ax)^3}{x^2} + \frac{c^3\arctan(ax)^3}{x^4}\right) dx$$

$\downarrow$ 2009

$$\frac{1}{3}a^6c^3x^3\arctan(ax)^3-\frac{1}{2}a^5c^3x^2\arctan(ax)^2+3a^4c^3x\arctan(ax)^3+a^4c^3x\arctan(ax)-8ia^3c^3\arctan(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)+8ia^3c^3\arctan(ax)\operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right)-a^3c^3\arctan(ax)^2+8a^3c^3\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)+8a^3c^3\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)+4a^3c^3\operatorname{PolyLog}\left(3,\frac{2}{1-iax}-1\right)+4a^3c^3\operatorname{PolyLog}\left(3,1-\frac{2}{iax+1}\right)+a^3c^3\log(x)-\frac{3a^2c^3\arctan(ax)^3}{x}-\frac{a^2c^3\arctan(ax)}{x}-a^3c^3\log\left(a^2x^2+1\right)-\frac{c^3\arctan(ax)^3}{3x^3}-\frac{ac^3\arctan(ax)^2}{2x^2}$$

input
```
Int[((c + a^2*c*x^2)^3*ArcTan[a*x]^3)/x^4,x]
```

output
```
-((a^2*c^3*ArcTan[a*x])/x) + a^4*c^3*x*ArcTan[a*x] - a^3*c^3*ArcTan[a*x]^2
 - (a*c^3*ArcTan[a*x]^2)/(2*x^2) - (a^5*c^3*x^2*ArcTan[a*x]^2)/2 - (c^3*Ar
cTan[a*x]^3)/(3*x^3) - (3*a^2*c^3*ArcTan[a*x]^3)/x + 3*a^4*c^3*x*ArcTan[a*
x]^3 + (a^6*c^3*x^3*ArcTan[a*x]^3)/3 + a^3*c^3*Log[x] + 8*a^3*c^3*ArcTan[a
*x]^2*Log[2/(1 + I*a*x)] - a^3*c^3*Log[1 + a^2*x^2] + 8*a^3*c^3*ArcTan[a*x
]^2*Log[2 - 2/(1 - I*a*x)] - (8*I)*a^3*c^3*ArcTan[a*x]*PolyLog[2, -1 + 2/(
1 - I*a*x)] + (8*I)*a^3*c^3*ArcTan[a*x]*PolyLog[2, 1 - 2/(1 + I*a*x)] + 4*
a^3*c^3*PolyLog[3, -1 + 2/(1 - I*a*x)] + 4*a^3*c^3*PolyLog[3, 1 - 2/(1 + I
*a*x)]
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5483
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a +
 b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*
d] && IGtQ[p, 0] && IGtQ[q, 1] && (EqQ[p, 1] || IntegerQ[m])
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 13.51 (sec) , antiderivative size = 1992, normalized size of antiderivative = 5.93

Expression too large to display

input
```
int((a^2*c*x^2+c)^3*arctan(a*x)^3/x^4,x)
```

output
```
a^3*(1/3*c^3*arctan(a*x)^3*a^3*x^3+3*c^3*arctan(a*x)^3*a*x-1/3*c^3*arctan(
a*x)^3/a^3/x^3-3*c^3*arctan(a*x)^3/a/x-c^3*(4*I*Pi*csgn(I/((1+I*a*x)^2/(a^
2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2
+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*arctan(a*x)^2+4*I*Pi*csgn(I*(1+I*a*x)/(
a^2*x^2+1)^(1/2))^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*arctan(a*x)^2+8*I*Pi*c
sgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2
*arctan(a*x)^2+arctan(a*x)^2-ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-1)-ln(1+(1+I*a
*x)/(a^2*x^2+1)^(1/2))+1/2*arctan(a*x)^2*x^2*a^2+4*I*Pi*csgn(I*((1+I*a*x)^
2/(a^2*x^2+1)-1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2
+1)+1))^2*arctan(a*x)^2-8*I*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*csgn(I*
(1+I*a*x)^2/(a^2*x^2+1))^2*arctan(a*x)^2-4*I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x
^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^
2*arctan(a*x)^2-4*I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/
(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2*arctan(a*x)^2-4*I*Pi*csgn(I*(
(1+I*a*x)^2/(a^2*x^2+1)+1))^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*arctan
(a*x)^2-16*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))-16*polylog(3,-(1+I*a*x)/
(a^2*x^2+1)^(1/2))+16/3*I*arctan(a*x)^3-4*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^
2+1)-1))*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+
1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*arctan(a*x)^2-arctan(a*x)*(a*x-I)-16*ar
ctan(a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2))-4*polylog(3,-(1+I*a*x)^2/(a...
```

## Fricas [F]

$$\int \frac{(c+a^2cx^2)^3\arctan(ax)^3}{x^4}\,dx=\int \frac{(a^2cx^2+c)^3\arctan(ax)^3}{x^4}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^3/x^4,x, algorithm="fricas")
```

output
```
integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a*x)^3
/x^4, x)
```

## Sympy [F]

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^3}{x^4}\,dx = c^3\left(\int 3a^4 \operatorname{atan}^3(ax)\,dx + \int \frac{\operatorname{atan}^3(ax)}{x^4}\,dx + \int \frac{3a^2 \operatorname{atan}^3(ax)}{x^2}\,dx + \int a^6x^2 \operatorname{atan}^3(ax)\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**3*atan(a*x)**3/x**4,x)
```

output
```
c**3*(Integral(3*a**4*atan(a*x)**3, x) + Integral(atan(a*x)**3/x**4, x) +
Integral(3*a**2*atan(a*x)**3/x**2, x) + Integral(a**6*x**2*atan(a*x)**3, x
))
```

## Maxima [F]

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^3}{x^4}\,dx = \int \frac{(a^2cx^2+c)^3 \arctan(ax)^3}{x^4}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^3/x^4,x, algorithm="maxima")
```

output
```
1/192*(3*(1792*a^8*c^3*integrate(1/32*x^8*arctan(a*x)^3/(a^2*x^6 + x^4), x
) + 192*a^8*c^3*integrate(1/32*x^8*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*x^6
 + x^4), x) + 256*a^8*c^3*integrate(1/32*x^8*arctan(a*x)*log(a^2*x^2 + 1)/
(a^2*x^6 + x^4), x) - 256*a^7*c^3*integrate(1/32*x^7*arctan(a*x)^2/(a^2*x^
6 + x^4), x) + 64*a^7*c^3*integrate(1/32*x^7*log(a^2*x^2 + 1)^2/(a^2*x^6 +
 x^4), x) + 84*a^3*c^3*arctan(a*x)^4 + 7168*a^6*c^3*integrate(1/32*x^6*arc
tan(a*x)^3/(a^2*x^6 + x^4), x) + 768*a^6*c^3*integrate(1/32*x^6*arctan(a*x
)*log(a^2*x^2 + 1)^2/(a^2*x^6 + x^4), x) + 2304*a^6*c^3*integrate(1/32*x^6
*arctan(a*x)*log(a^2*x^2 + 1)/(a^2*x^6 + x^4), x) - 2304*a^5*c^3*integrate
(1/32*x^5*arctan(a*x)^2/(a^2*x^6 + x^4), x) + 3*a^3*c^3*log(a^2*x^2 + 1)^3
 + 1152*a^4*c^3*integrate(1/32*x^4*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*x^6
 + x^4), x) - 2304*a^4*c^3*integrate(1/32*x^4*arctan(a*x)*log(a^2*x^2 + 1)
/(a^2*x^6 + x^4), x) + 2304*a^3*c^3*integrate(1/32*x^3*arctan(a*x)^2/(a^2*
x^6 + x^4), x) - 576*a^3*c^3*integrate(1/32*x^3*log(a^2*x^2 + 1)^2/(a^2*x^
6 + x^4), x) + 7168*a^2*c^3*integrate(1/32*x^2*arctan(a*x)^3/(a^2*x^6 + x^
4), x) + 768*a^2*c^3*integrate(1/32*x^2*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^
2*x^6 + x^4), x) - 256*a^2*c^3*integrate(1/32*x^2*arctan(a*x)*log(a^2*x^2
+ 1)/(a^2*x^6 + x^4), x) + 256*a*c^3*integrate(1/32*x*arctan(a*x)^2/(a^2*x
^6 + x^4), x) - 64*a*c^3*integrate(1/32*x*log(a^2*x^2 + 1)^2/(a^2*x^6 + x^
4), x) + 1792*c^3*integrate(1/32*arctan(a*x)^3/(a^2*x^6 + x^4), x) + 19...
```

## Giac [F]

$$\int \frac{(c+a^2cx^2)^3\arctan(ax)^3}{x^4}\,dx=\int \frac{(a^2cx^2+c)^3\arctan{(ax)}^3}{x^4}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^3/x^4,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^3*arctan(a*x)^3/x^4, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^3}{x^4} \, dx = \int \frac{\operatorname{atan}(a\,x)^3\,(c\,a^2\,x^2+c)^3}{x^4} \, dx$$

input
```
int((atan(a*x)^3*(c + a^2*c*x^2)^3)/x^4,x)
```

output
```
int((atan(a*x)^3*(c + a^2*c*x^2)^3)/x^4, x)
```

## Reduce [F]

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^3}{x^4} \, dx = \frac{c^3\Big(2atan(ax)^3\,a^6x^6+18atan(ax)^3\,a^4x^4-18atan(ax)^3\,a^2x^2-2atan(ax)^3-3atan(ax)^2\,a^5x^5-30atan(}{}$$

input
```
int((a^2*c*x^2+c)^3*atan(a*x)^3/x^4,x)
```

output
```
(c**3*(2*atan(a*x)**3*a**6*x**6 + 18*atan(a*x)**3*a**4*x**4 - 18*atan(a*x)
**3*a**2*x**2 - 2*atan(a*x)**3 - 3*atan(a*x)**2*a**5*x**5 - 30*atan(a*x)**
2*a**3*x**3 - 27*atan(a*x)**2*a*x + 6*atan(a*x)*a**4*x**4 - 54*atan(a*x)*a
**2*x**2 - 48*int(atan(a*x)**2/(a**2*x**5 + x**3),x)*a*x**3 - 48*int((atan
(a*x)**2*x)/(a**2*x**2 + 1),x)*a**5*x**3 - 30*log(a**2*x**2 + 1)*a**3*x**3
 + 54*log(x)*a**3*x**3))/(6*x**3)
```

# 3.387 $\int \frac{x^4 \arctan(ax)^3}{c+a^2cx^2} dx$



## Optimal result

Integrand size = 22, antiderivative size = 217

$$\begin{aligned}\int \frac{x^4 \arctan(ax)^3}{c+a^2cx^2} dx = {} & \frac{x \arctan(ax)}{a^4c} - \frac{\arctan(ax)^2}{2a^5c} - \frac{x^2 \arctan(ax)^2}{2a^3c} \\ & - \frac{4i \arctan(ax)^3}{3a^5c} - \frac{x \arctan(ax)^3}{a^4c} + \frac{x^3 \arctan(ax)^3}{3a^2c} \\ & + \frac{\arctan(ax)^4}{4a^5c} - \frac{4 \arctan(ax)^2 \log\left(\frac{2}{1+iax}\right)}{a^5c} - \frac{\log(1+a^2x^2)}{2a^5c} \\ & - \frac{4i \arctan(ax) \operatorname{PolyLog}\left(2, 1-\frac{2}{1+iax}\right)}{a^5c} \\ & - \frac{2 \operatorname{PolyLog}\left(3, 1-\frac{2}{1+iax}\right)}{a^5c}\end{aligned}$$

output

```
x*arctan(a*x)/a^4/c-1/2*arctan(a*x)^2/a^5/c-1/2*x^2*arctan(a*x)^2/a^3/c-4/
3*I*arctan(a*x)^3/a^5/c-x*arctan(a*x)^3/a^4/c+1/3*x^3*arctan(a*x)^3/a^2/c+
1/4*arctan(a*x)^4/a^5/c-4*arctan(a*x)^2*ln(2/(1+I*a*x))/a^5/c-1/2*ln(a^2*x
^2+1)/a^5/c-4*I*arctan(a*x)*polylog(2,1-2/(1+I*a*x))/a^5/c-2*polylog(3,1-2
/(1+I*a*x))/a^5/c
```

## Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.71

$$\int \frac{x^4 \arctan(ax)^3}{c+a^2cx^2}\,dx$$
$$= \frac{12ax\arctan(ax) - 6\arctan(ax)^2 - 6a^2x^2\arctan(ax)^2 + 16i\arctan(ax)^3 - 12ax\arctan(ax)^3 + 4a^3x^3\operatorname{ar}}{}$$

input
```
Integrate[(x^4*ArcTan[a*x]^3)/(c + a^2*c*x^2),x]
```

output
```
(12*a*x*ArcTan[a*x] - 6*ArcTan[a*x]^2 - 6*a^2*x^2*ArcTan[a*x]^2 + (16*I)*A
rcTan[a*x]^3 - 12*a*x*ArcTan[a*x]^3 + 4*a^3*x^3*ArcTan[a*x]^3 + 3*ArcTan[a
*x]^4 - 48*ArcTan[a*x]^2*Log[1 + E^((2*I)*ArcTan[a*x])] - 6*Log[1 + a^2*x^
2] + (48*I)*ArcTan[a*x]*PolyLog[2, -E^((2*I)*ArcTan[a*x])] - 24*PolyLog[3,
 -E^((2*I)*ArcTan[a*x])])/(12*a^5*c)
```

## Rubi [A] (verified)

Time = 2.63 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.53, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.682$, Rules used = {5451, 27, 5361, 5451, 5345, 5361, 5419, 5451, 5345, 240, 5419, 5455, 5379, 5529, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \arctan(ax)^3}{a^2cx^2+c}dx$$
$$\Big\downarrow \text{ 5451}$$
$$\frac{\int x^2\arctan(ax)^3dx}{a^2c} - \frac{\int \frac{x^2\arctan(ax)^3}{c(a^2x^2+1)}dx}{a^2}$$
$$\Big\downarrow \text{ 27}$$
$$\frac{\int x^2\arctan(ax)^3dx}{a^2c} - \frac{\int \frac{x^2\arctan(ax)^3}{a^2x^2+1}dx}{a^2c}$$

$$\Big\downarrow \text{5361}$$

$$\frac{\frac{1}{3}x^3\arctan(ax)^3-a\int\frac{x^3\arctan(ax)^2}{a^2x^2+1}dx}{a^2c}-\frac{\int\frac{x^2\arctan(ax)^3}{a^2x^2+1}dx}{a^2c}$$

$$\Big\downarrow \text{5451}$$

$$\frac{\frac{1}{3}x^3\arctan(ax)^3-a\left(\frac{\int x\arctan(ax)^2dx}{a^2}-\frac{\int\frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2}\right)}{a^2c}-\frac{\frac{\int\arctan(ax)^3dx}{a^2}-\frac{\int\frac{\arctan(ax)^3}{a^2x^2+1}dx}{a^2}}{a^2c}$$

$$\Big\downarrow \text{5345}$$

$$\frac{\frac{1}{3}x^3\arctan(ax)^3-a\left(\frac{\int x\arctan(ax)^2dx}{a^2}-\frac{\int\frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2}\right)}{a^2c}-\frac{\frac{x\arctan(ax)^3-3a\int\frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2}-\frac{\int\frac{\arctan(ax)^3}{a^2x^2+1}dx}{a^2}}{a^2c}$$

$$\Big\downarrow \text{5361}$$

$$\frac{\frac{1}{3}x^3\arctan(ax)^3-a\left(\frac{\frac{1}{2}x^2\arctan(ax)^2-a\int\frac{x^2\arctan(ax)}{a^2x^2+1}dx}{a^2}-\frac{\int\frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2}\right)}{a^2c}-\frac{\frac{x\arctan(ax)^3-3a\int\frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2}-\frac{\int\frac{\arctan(ax)^3}{a^2x^2+1}dx}{a^2}}{a^2c}$$

$$\Big\downarrow \text{5419}$$

$$\frac{\frac{1}{3}x^3\arctan(ax)^3-a\left(\frac{\frac{1}{2}x^2\arctan(ax)^2-a\int\frac{x^2\arctan(ax)}{a^2x^2+1}dx}{a^2}-\frac{\int\frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2}\right)}{a^2c}-\frac{\frac{x\arctan(ax)^3-3a\int\frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2}-\frac{\arctan(ax)^4}{4a^3}}{a^2c}$$

$$\Big\downarrow \text{5451}$$

$$\frac{\frac{1}{3}x^3\arctan(ax)^3-a\left(\frac{\frac{1}{2}x^2\arctan(ax)^2-a\left(\frac{\int\arctan(ax)dx}{a^2}-\frac{\int\frac{\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2}-\frac{\int\frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2}\right)}{a^2c}-\frac{\frac{x\arctan(ax)^3-3a\int\frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2}-\frac{\arctan(ax)^4}{4a^3}}{a^2c}$$

$$\Big\downarrow \text{5345}$$

$$\frac{\frac{1}{3}x^3\arctan(ax)^3 - a\left(\frac{\frac{1}{2}x^2\arctan(ax)^2 - a\left(\frac{x\arctan(ax)-a\int\frac{x}{a^2x^2+1}dx}{a^2} - \frac{\int\frac{\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2} - \frac{\int\frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2}\right)}{a^2c} - \frac{\frac{x\arctan(ax)^3-3a\int\frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2} - \frac{\arctan(ax)^4}{4a^3}}{a^2c}$$

$$\Big\downarrow \text{240}$$

$$\frac{\frac{1}{3}x^3\arctan(ax)^3 - a\left(\frac{\frac{1}{2}x^2\arctan(ax)^2 - a\left(\frac{x\arctan(ax)-\frac{\log(a^2x^2+1)}{2a}}{a^2} - \frac{\int\frac{\arctan(ax)}{a^2x^2+1}dx}{a^2}\right)}{a^2} - \frac{\int\frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2}\right)}{a^2c} - \frac{\frac{x\arctan(ax)^3-3a\int\frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2} - \frac{\arctan(ax)^4}{4a^3}}{a^2c}$$

$$\Big\downarrow \text{5419}$$

$$\frac{\frac{1}{3}x^3\arctan(ax)^3 - a\left(\frac{\frac{1}{2}x^2\arctan(ax)^2 - a\left(\frac{x\arctan(ax)-\frac{\log(a^2x^2+1)}{2a}}{a^2} - \frac{\arctan(ax)^2}{2a^3}\right)}{a^2} - \frac{\int\frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2}\right)}{a^2c} - \frac{\frac{x\arctan(ax)^3-3a\int\frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2} - \frac{\arctan(ax)^4}{4a^3}}{a^2c}$$

$$\Big\downarrow \text{5455}$$

$$\frac{\frac{1}{3}x^3\arctan(ax)^3 - a\left(\frac{\frac{1}{2}x^2\arctan(ax)^2 - a\left(\frac{x\arctan(ax)-\frac{\log(a^2x^2+1)}{2a}}{a^2} - \frac{\arctan(ax)^2}{2a^3}\right)}{a^2} - \frac{-\frac{\int\frac{\arctan(ax)^2}{i-ax}dx}{a} - \frac{i\arctan(ax)^3}{3a^2}}{a^2}\right)}{a^2c} - \frac{-\frac{\arctan(ax)^4}{4a^3} + \frac{x\arctan(ax)^3-3a\left(-\frac{\int\frac{\arctan(ax)^2}{i-ax}dx}{a} - \frac{i\arctan(ax)^3}{3a^2}\right)}{a^2}}{a^2c}$$

$\downarrow$ 5379

$$\frac{\frac{1}{3}x^3\arctan(ax)^3-a\left(\frac{\frac{1}{2}x^2\arctan(ax)^2-a\left(\frac{x\arctan(ax)-\frac{\log(a^2x^2+1)}{2a}}{a^2}-\frac{\arctan(ax)^2}{2a^3}\right)}{a^2}-\frac{-\frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}-2\int\frac{\arctan(ax)\log\left(\frac{2}{iax\cdots}\right)}{a^2x^2+1}}{a}}{a^2}\cdots\right.}{a^2c}$$

$$\frac{-\frac{\arctan(ax)^4}{4a^3}+\frac{x\arctan(ax)^3-3a\left(-\frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}-2\int\frac{\arctan(ax)\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^3}{3a^2}\right)}{a^2}}{a^2c}$$

$\downarrow$ 5529

$$\frac{\frac{1}{3}x^3\arctan(ax)^3-a\left(\frac{\frac{1}{2}x^2\arctan(ax)^2-a\left(\frac{x\arctan(ax)-\frac{\log(a^2x^2+1)}{2a}}{a^2}-\frac{\arctan(ax)^2}{2a^3}\right)}{a^2}-\frac{-\frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}-2\left(\frac{1}{2}i\int\frac{\text{PolyLog}\left(2,1-\cdots\right)}{a^2x^2+1}\cdots\right.}{a\cdots}}{\cdots}\cdots\right.}{a^2c}$$

$$\frac{-\frac{\arctan(ax)^4}{4a^3}+\frac{x\arctan(ax)^3-3a\left(-\frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}-2\left(\frac{1}{2}i\int\frac{\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx-\frac{i\arctan(ax)\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}\right)}{a}-\frac{i\arctan(ax)\cdots}{3a^2}\cdots\right.}{a^2}}{a^2c}$$

$\downarrow$ 7164

$$\frac{\frac{1}{3}x^3\arctan(ax)^3-a\left(\frac{\frac{1}{2}x^2\arctan(ax)^2-a\left(\frac{x\arctan(ax)-\frac{\log(a^2x^2+1)}{2a}}{a^2}-\frac{\arctan(ax)^2}{2a^3}\right)}{a^2}-\frac{-\frac{i\arctan(ax)^3}{3a^2}-\frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}-2\left(-\frac{i\,a\cdots}{\cdots}\right.}{\cdots}}{\cdots}\cdots\right.}{a^2c}$$

$$\frac{-\frac{\arctan(ax)^4}{4a^3}+\frac{x\arctan(ax)^3-3a\left(-\frac{i\arctan(ax)^3}{3a^2}-\frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}-2\left(-\frac{i\arctan(ax)\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}-\frac{\text{PolyLog}\left(3,1-\frac{2}{iax+1}\right)}{4a}\right)}{a}\right)}{a^2}}{a^2c}$$

input

```
Int[(x^4*ArcTan[a*x]^3)/(c + a^2*c*x^2),x]
```

output
```
((x^3*ArcTan[a*x]^3)/3 - a*(((x^2*ArcTan[a*x]^2)/2 - a*(-1/2*ArcTan[a*x]^2
/a^3 + (x*ArcTan[a*x] - Log[1 + a^2*x^2]/(2*a))/a^2))/a^2 - (((-1/3*I)*Arc
Tan[a*x]^3)/a^2 - ((ArcTan[a*x]^2*Log[2/(1 + I*a*x)])/a - 2*(((-1/2*I)*Arc
Tan[a*x]*PolyLog[2, 1 - 2/(1 + I*a*x)])/a - PolyLog[3, 1 - 2/(1 + I*a*x)]/
(4*a)))/a)/a^2))/(a^2*c) - (-1/4*ArcTan[a*x]^4/a^3 + (x*ArcTan[a*x]^3 - 3*
a*(((-1/3*I)*ArcTan[a*x]^3)/a^2 - ((ArcTan[a*x]^2*Log[2/(1 + I*a*x)])/a -
2*(((-1/2*I)*ArcTan[a*x]*PolyLog[2, 1 - 2/(1 + I*a*x)])/a - PolyLog[3, 1 -
 2/(1 + I*a*x)]/(4*a)))/a))/a^2)/(a^2*c)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 65.30 (sec) , antiderivative size = 919, normalized size of antiderivative = 4.24

| method | result | size |
|---|---|---|
| derivativedivides | Expression too large to display | 919 |
| default | Expression too large to display | 919 |
| parts | Expression too large to display | 929 |

input
```
int(x^4*arctan(a*x)^3/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
1/a^5*(1/3/c*arctan(a*x)^3*a^3*x^3-1/c*arctan(a*x)^3*a*x+1/c*arctan(a*x)^4
-1/c*(1/2*arctan(a*x)^2*x^2*a^2-2*arctan(a*x)^2*ln(a^2*x^2+1)-ln((1+I*a*x)
^2/(a^2*x^2+1)+1)+I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))^2*csgn(I*((1+I*
a*x)^2/(a^2*x^2+1)+1)^2)*arctan(a*x)^2+I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)
)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2*arctan(a
*x)^2-I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x
^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*arcta
n(a*x)^2-I*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))^2*csgn(I*(1+I*a*x)^2/(a^
2*x^2+1))*arctan(a*x)^2-I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(
a^2*x^2+1)+1)^2)^3*arctan(a*x)^2-arctan(a*x)*(a*x-I)-2*I*Pi*csgn(I*((1+I*a
*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2*arctan(a*x)^
2+I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1
)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2*arctan(a*x)^2-4/3*I*arctan(a*x)^3+2*I*P
i*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^2*ar
ctan(a*x)^2-4*I*arctan(a*x)*polylog(2,-(1+I*a*x)^2/(a^2*x^2+1))+1/2*arctan
(a*x)^2+4*arctan(a*x)^2*ln(2)-I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^3*arcta
n(a*x)^2+I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3*arctan(a*x)^2+2*poly
log(3,-(1+I*a*x)^2/(a^2*x^2+1))+4*arctan(a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(
1/2))+3/4*arctan(a*x)^4))
```

## Fricas [F]

$$\int \frac{x^4 \arctan(ax)^3}{c+a^2cx^2}\,dx = \int \frac{x^4 \arctan\left(ax\right)^3}{a^2cx^2+c}\,dx$$

input
```
integrate(x^4*arctan(a*x)^3/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
integral(x^4*arctan(a*x)^3/(a^2*c*x^2 + c), x)
```

## Sympy [F]

$$\int \frac{x^4 \arctan(ax)^3}{c + a^2cx^2}\,dx = \frac{\int \frac{x^4 \operatorname{atan}^3(ax)}{a^2x^2+1}\,dx}{c}$$

input
```
integrate(x**4*atan(a*x)**3/(a**2*c*x**2+c),x)
```

output
```
Integral(x**4*atan(a*x)**3/(a**2*x**2 + 1), x)/c
```

## Maxima [F]

$$\int \frac{x^4 \arctan(ax)^3}{c + a^2cx^2}\,dx = \int \frac{x^4 \arctan(ax)^3}{a^2cx^2 + c}\,dx$$

input
```
integrate(x^4*arctan(a*x)^3/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
1/3072*(48*(7168*a^4*integrate(1/128*x^4*arctan(a*x)^3/(a^6*c*x^2 + a^4*c)
, x) + 768*a^4*integrate(1/128*x^4*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^6*c*x
^2 + a^4*c), x) + 1024*a^4*integrate(1/128*x^4*arctan(a*x)*log(a^2*x^2 + 1
)/(a^6*c*x^2 + a^4*c), x) - 1024*a^3*integrate(1/128*x^3*arctan(a*x)^2/(a^
6*c*x^2 + a^4*c), x) + 256*a^3*integrate(1/128*x^3*log(a^2*x^2 + 1)^2/(a^6
*c*x^2 + a^4*c), x) - 3072*a^2*integrate(1/128*x^2*arctan(a*x)*log(a^2*x^2
 + 1)/(a^6*c*x^2 + a^4*c), x) + 768*a*integrate(1/128*x*arctan(a*x)^2*log(
a^2*x^2 + 1)/(a^6*c*x^2 + a^4*c), x) + 192*a*integrate(1/128*x*log(a^2*x^2
 + 1)^3/(a^6*c*x^2 + a^4*c), x) + 3072*a*integrate(1/128*x*arctan(a*x)^2/(
a^6*c*x^2 + a^4*c), x) - 768*a*integrate(1/128*x*log(a^2*x^2 + 1)^2/(a^6*c
*x^2 + a^4*c), x) - 3*arctan(a*x)^4/(a^5*c) - 384*integrate(1/128*arctan(a
*x)*log(a^2*x^2 + 1)^2/(a^6*c*x^2 + a^4*c), x))*a^5*c + 128*(a^3*x^3 - 3*a
*x)*arctan(a*x)^3 + 240*arctan(a*x)^4 - 9*log(a^2*x^2 + 1)^4 - 24*(4*(a^3*
x^3 - 3*a*x)*arctan(a*x) + 3*arctan(a*x)^2)*log(a^2*x^2 + 1)^2)/(a^5*c)
```

## Giac [F]

$$\int \frac{x^4 \arctan(ax)^3}{c+a^2cx^2}\,dx = \int \frac{x^4 \arctan\left(ax\right)^3}{a^2cx^2+c}\,dx$$

input
```
integrate(x^4*arctan(a*x)^3/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
integrate(x^4*arctan(a*x)^3/(a^2*c*x^2 + c), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^4 \arctan(ax)^3}{c+a^2cx^2}\,dx = \int \frac{x^4\,\mathrm{atan}(a\,x)^3}{c\,a^2\,x^2+c}\,dx$$

input
```
int((x^4*atan(a*x)^3)/(c + a^2*c*x^2),x)
```

output
```
int((x^4*atan(a*x)^3)/(c + a^2*c*x^2), x)
```

## Reduce [F]

$$\int \frac{x^4 \arctan(ax)^3}{c+a^2cx^2}\,dx$$
$$= \frac{3atan(ax)^4 + 4atan(ax)^3 a^3x^3 - 12atan(ax)^3 ax - 6atan(ax)^2 a^2x^2 - 6atan(ax)^2 + 12atan(ax)\,ax + 48}{12a^5c}$$

input
```
int(x^4*atan(a*x)^3/(a^2*c*x^2+c),x)
```

output
```
(3*atan(a*x)**4 + 4*atan(a*x)**3*a**3*x**3 - 12*atan(a*x)**3*a*x - 6*atan(
a*x)**2*a**2*x**2 - 6*atan(a*x)**2 + 12*atan(a*x)*a*x + 48*int((atan(a*x)*
*2*x)/(a**2*x**2 + 1),x)*a**2 - 6*log(a**2*x**2 + 1))/(12*a**5*c)
```

# 3.388 $\int \frac{x^3 \arctan(ax)^3}{c+a^2cx^2} dx$



## Optimal result

Integrand size = 22, antiderivative size = 260

$$\int \frac{x^3 \arctan(ax)^3}{c+a^2cx^2} dx = -\frac{3i \arctan(ax)^2}{2a^4c} - \frac{3x \arctan(ax)^2}{2a^3c} + \frac{\arctan(ax)^3}{2a^4c} + \frac{x^2 \arctan(ax)^3}{2a^2c} + \frac{i \arctan(ax)^4}{4a^4c} - \frac{3 \arctan(ax) \log\left(\frac{2}{1+iax}\right)}{a^4c} + \frac{\arctan(ax)^3 \log\left(\frac{2}{1+iax}\right)}{a^4c} - \frac{3i \operatorname{PolyLog}\left(2, 1-\frac{2}{1+iax}\right)}{2a^4c} + \frac{3i \arctan(ax)^2 \operatorname{PolyLog}\left(2, 1-\frac{2}{1+iax}\right)}{2a^4c} + \frac{3 \arctan(ax) \operatorname{PolyLog}\left(3, 1-\frac{2}{1+iax}\right)}{2a^4c} - \frac{3i \operatorname{PolyLog}\left(4, 1-\frac{2}{1+iax}\right)}{4a^4c}$$

output
```
-3/2*I*arctan(a*x)^2/a^4/c-3/2*x*arctan(a*x)^2/a^3/c+1/2*arctan(a*x)^3/a^4
/c+1/2*x^2*arctan(a*x)^3/a^2/c+1/4*I*arctan(a*x)^4/a^4/c-3*arctan(a*x)*ln(
2/(1+I*a*x))/a^4/c+arctan(a*x)^3*ln(2/(1+I*a*x))/a^4/c-3/2*I*polylog(2,1-2
/(1+I*a*x))/a^4/c+3/2*I*arctan(a*x)^2*polylog(2,1-2/(1+I*a*x))/a^4/c+3/2*a
rctan(a*x)*polylog(3,1-2/(1+I*a*x))/a^4/c-3/4*I*polylog(4,1-2/(1+I*a*x))/a
^4/c
```

## Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.62

$$\int \frac{x^3 \arctan(ax)^3}{c + a^2cx^2} \, dx$$
$$= \frac{6i \arctan(ax)^2 - 6ax \arctan(ax)^2 + 2(1 + a^2x^2) \arctan(ax)^3 - i \arctan(ax)^4 - 12 \arctan(ax) \log\left(1 + e^{2i \arctan(ax)}\right) \cdots}{\cdots}$$

input
```
Integrate[(x^3*ArcTan[a*x]^3)/(c + a^2*c*x^2),x]
```

output
```
((6*I)*ArcTan[a*x]^2 - 6*a*x*ArcTan[a*x]^2 + 2*(1 + a^2*x^2)*ArcTan[a*x]^3
 - I*ArcTan[a*x]^4 - 12*ArcTan[a*x]*Log[1 + E^((2*I)*ArcTan[a*x])] + 4*Arc
Tan[a*x]^3*Log[1 + E^((2*I)*ArcTan[a*x])] - (6*I)*(-1 + ArcTan[a*x]^2)*Pol
yLog[2, -E^((2*I)*ArcTan[a*x])] + 6*ArcTan[a*x]*PolyLog[3, -E^((2*I)*ArcTa
n[a*x])] + (3*I)*PolyLog[4, -E^((2*I)*ArcTan[a*x])])/(4*a^4*c)
```

## Rubi [A] (verified)

Time = 1.84 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.03, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {5451, 27, 5361, 5451, 5345, 5419, 5455, 5379, 2849, 2752, 5529, 5533, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \arctan(ax)^3}{a^2cx^2 + c} \, dx$$

$$\Big\downarrow \text{5451}$$

$$\frac{\int x \arctan(ax)^3 dx}{a^2c} - \frac{\int \frac{x \arctan(ax)^3}{c(a^2x^2+1)} dx}{a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int x \arctan(ax)^3 dx}{a^2c} - \frac{\int \frac{x \arctan(ax)^3}{a^2x^2+1} dx}{a^2c}$$

$$\Big\downarrow \text{5361}$$

$$\frac{\frac{1}{2}x^2\arctan(ax)^3-\frac{3}{2}a\int\frac{x^2\arctan(ax)^2}{a^2x^2+1}dx}{a^2c}-\frac{\int\frac{x\arctan(ax)^3}{a^2x^2+1}dx}{a^2c}$$

$$\Big\downarrow \text{5451}$$

$$\frac{\frac{1}{2}x^2\arctan(ax)^3-\frac{3}{2}a\left(\frac{\int\arctan(ax)^2dx}{a^2}-\frac{\int\frac{\arctan(ax)^2}{a^2x^2+1}dx}{a^2}\right)}{a^2c}-\frac{\int\frac{x\arctan(ax)^3}{a^2x^2+1}dx}{a^2c}$$

$$\Big\downarrow \text{5345}$$

$$\frac{\frac{1}{2}x^2\arctan(ax)^3-\frac{3}{2}a\left(\frac{x\arctan(ax)^2-2a\int\frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}-\frac{\int\frac{\arctan(ax)^2}{a^2x^2+1}dx}{a^2}\right)}{a^2c}-\frac{\int\frac{x\arctan(ax)^3}{a^2x^2+1}dx}{a^2c}$$

$$\Big\downarrow \text{5419}$$

$$\frac{\frac{1}{2}x^2\arctan(ax)^3-\frac{3}{2}a\left(\frac{x\arctan(ax)^2-2a\int\frac{x\arctan(ax)}{a^2x^2+1}dx}{a^2}-\frac{\arctan(ax)^3}{3a^3}\right)}{a^2c}-\frac{\int\frac{x\arctan(ax)^3}{a^2x^2+1}dx}{a^2c}$$

$$\Big\downarrow \text{5455}$$

$$\frac{\frac{1}{2}x^2\arctan(ax)^3-\frac{3}{2}a\left(-\frac{\arctan(ax)^3}{3a^3}+\frac{x\arctan(ax)^2-2a\left(-\frac{\int\frac{\arctan(ax)}{i-ax}dx}{a}-\frac{i\arctan(ax)^2}{2a^2}\right)}{a^2}\right)}{a^2c}-\frac{-\frac{\int\frac{\arctan(ax)^3}{i-ax}dx}{a}-\frac{i\arctan(ax)^4}{4a^2}}{a^2c}$$

$$\Big\downarrow \text{5379}$$

$$\frac{\frac{1}{2}x^2\arctan(ax)^3-\frac{3}{2}a\left(-\frac{\arctan(ax)^3}{3a^3}+\frac{x\arctan(ax)^2-2a\left(-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}-\int\frac{\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^2}{2a^2}\right)}{a^2}\right)}{a^2c}-\frac{-\frac{\frac{\arctan(ax)^3\log\left(\frac{2}{1+iax}\right)}{a}-3\int\frac{\arctan(ax)^2\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^4}{4a^2}}{a^2c}$$

$$\Big\downarrow \text{2849}$$

$$\frac{\frac{1}{2}x^2\arctan(ax)^3-\frac{3}{2}a\left(-\frac{\arctan(ax)^3}{3a^3}+\frac{x\arctan(ax)^2-2a\left(-\frac{\frac{i\int\frac{\log\left(\frac{2}{iax+1}\right)}{1-\frac{2}{iax+1}}d\frac{1}{iax+1}}{a}+\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}}{a}-\frac{i\arctan(ax)^2}{2a^2}\right)}{a^2}\right)}{a^2c}-\frac{-\frac{\frac{\arctan(ax)^3\log\left(\frac{2}{1+iax}\right)}{a}-3\int\frac{\arctan(ax)^2\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^4}{4a^2}}{a^2c}$$

$\downarrow$ 2752

$$\frac{\frac{1}{2}x^2\arctan(ax)^3-\frac{3}{2}a\left(-\frac{\arctan(ax)^3}{3a^3}+\frac{x\arctan(ax)^2-2a\left(-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}\right)}{a^2}\right)}{a^2c}-\frac{-\frac{\frac{\arctan(ax)^3\log\left(\frac{2}{1+iax}\right)}{a}-3\int\frac{\arctan(ax)^2\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^4}{4a^2}}{a^2c}$$

$\downarrow$ 5529

$$\frac{\frac{1}{2}x^2\arctan(ax)^3-\frac{3}{2}a\left(-\frac{\arctan(ax)^3}{3a^3}+\frac{x\arctan(ax)^2-2a\left(-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}\right)}{a^2}\right)}{a^2c}-\frac{-\frac{\frac{\arctan(ax)^3\log\left(\frac{2}{1+iax}\right)}{a}-3\left(i\int\frac{\arctan(ax)\,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx-\frac{i\arctan(ax)^2\,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}\right)}{a}-\frac{i\arctan(ax)^4}{4a^2}}{a^2c}$$

$\downarrow$ 5533

$$\frac{\frac{1}{2}x^2\arctan(ax)^3-\frac{3}{2}a\left(-\frac{\arctan(ax)^3}{3a^3}+\frac{x\arctan(ax)^2-2a\left(-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}\right)}{a^2}\right)}{a^2c}-$$

$$\frac{-\frac{\frac{\arctan(ax)^3\log\left(\frac{2}{1+iax}\right)}{a}-3\left(i\left(\frac{i\arctan(ax)\,\mathrm{PolyLog}\left(3,1-\frac{2}{iax+1}\right)}{2a}-\frac{1}{2}i\int\frac{\mathrm{PolyLog}\left(3,1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx\right)-\frac{i\arctan(ax)^2\,\mathrm{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}\right)}{a}-\frac{i\,\mathrm{arcta}}{4a}}{a^2c}$$

$$\Big\downarrow \text{ 7164}$$

$$\frac{\frac{1}{2}x^2\arctan(ax)^3-\frac{3}{2}a\left(-\frac{\arctan(ax)^3}{3a^3}+\frac{x\arctan(ax)^2-2a\left(-\frac{i\arctan(ax)^2}{2a^2}-\frac{\frac{\arctan(ax)\log\left(\frac{2}{1+iax}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}}{a}\right)}{a^2}\right)}{a^2c}-$$

$$\frac{-\frac{i\arctan(ax)^4}{4a^2}-\frac{\frac{\arctan(ax)^3\log\left(\frac{2}{1+iax}\right)}{a}-3\left(i\left(\frac{i\arctan(ax)\,\mathrm{PolyLog}\left(3,1-\frac{2}{iax+1}\right)}{2a}+\frac{\mathrm{PolyLog}\left(4,1-\frac{2}{iax+1}\right)}{4a}\right)-\frac{i\arctan(ax)^2\,\mathrm{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}\right)}{a}}{a^2c}$$

input

```
Int[(x^3*ArcTan[a*x]^3)/(c + a^2*c*x^2),x]
```

output

```
((x^2*ArcTan[a*x]^3)/2 - (3*a*(-1/3*ArcTan[a*x]^3/a^3 + (x*ArcTan[a*x]^2 -
 2*a*(((-1/2*I)*ArcTan[a*x]^2)/a^2 - ((ArcTan[a*x]*Log[2/(1 + I*a*x)])/a +
 ((I/2)*PolyLog[2, 1 - 2/(1 + I*a*x)])/a)/a))/a^2))/2)/(a^2*c) - (((-1/4*I
)*ArcTan[a*x]^4)/a^2 - ((ArcTan[a*x]^3*Log[2/(1 + I*a*x)])/a - 3*(((-1/2*I
)*ArcTan[a*x]^2*PolyLog[2, 1 - 2/(1 + I*a*x)])/a + I*(((I/2)*ArcTan[a*x]*P
olyLog[3, 1 - 2/(1 + I*a*x)])/a + PolyLog[4, 1 - 2/(1 + I*a*x)]/(4*a))))/a
)/(a^2*c)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 5533
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k + 1
, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &&
EqQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

## Maple [A] (verified)

Time = 70.46 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.00

| method | result |
|---|---|
| derivativedivides | $-\frac{i \arctan(ax)^4}{4c}+\frac{\arctan(ax)^2(-3-i\arctan(ax)+\arctan(ax)ax)(ax+i)}{2c}+\frac{\arctan(ax)^3 \ln\left(\frac{(iax+1)^2}{a^2x^2+1}+1\right)}{c}-\frac{3i\arctan(ax)^2 \operatorname{polylog}\left(\right.}{2c}$ |
| default | $-\frac{i \arctan(ax)^4}{4c}+\frac{\arctan(ax)^2(-3-i\arctan(ax)+\arctan(ax)ax)(ax+i)}{2c}+\frac{\arctan(ax)^3 \ln\left(\frac{(iax+1)^2}{a^2x^2+1}+1\right)}{c}-\frac{3i\arctan(ax)^2 \operatorname{polylog}\left(\right.}{2c}$ |

input
```
int(x^3*arctan(a*x)^3/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
1/a^4*(-1/4*I*arctan(a*x)^4/c+1/2/c*arctan(a*x)^2*(-3-I*arctan(a*x)+arctan
(a*x)*a*x)*(a*x+I)+1/c*arctan(a*x)^3*ln((1+I*a*x)^2/(a^2*x^2+1)+1)-3/2*I/c
*arctan(a*x)^2*polylog(2,-(1+I*a*x)^2/(a^2*x^2+1))+3/2/c*arctan(a*x)*polyl
og(3,-(1+I*a*x)^2/(a^2*x^2+1))+3/4*I/c*polylog(4,-(1+I*a*x)^2/(a^2*x^2+1))
+3*I/c*arctan(a*x)^2-3/c*arctan(a*x)*ln((1+I*a*x)^2/(a^2*x^2+1)+1)+3/2*I/c
*polylog(2,-(1+I*a*x)^2/(a^2*x^2+1)))
```

## Fricas [F]

$$\int \frac{x^3 \arctan(ax)^3}{c+a^2cx^2}\,dx = \int \frac{x^3 \arctan\left(ax\right)^3}{a^2cx^2+c}\,dx$$

input
```
integrate(x^3*arctan(a*x)^3/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
integral(x^3*arctan(a*x)^3/(a^2*c*x^2 + c), x)
```

## Sympy [F]

$$\int \frac{x^3 \arctan(ax)^3}{c+a^2cx^2}\,dx = \frac{\int \frac{x^3 \operatorname{atan}^3(ax)}{a^2x^2+1}\,dx}{c}$$

input
```
integrate(x**3*atan(a*x)**3/(a**2*c*x**2+c),x)
```

output
```
Integral(x**3*atan(a*x)**3/(a**2*x**2 + 1), x)/c
```

## Maxima [F]

$$\int \frac{x^3 \arctan(ax)^3}{c+a^2cx^2} \, dx = \int \frac{x^3 \arctan{(ax)}^3}{a^2cx^2+c} \, dx$$

input
```
integrate(x^3*arctan(a*x)^3/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
integrate(x^3*arctan(a*x)^3/(a^2*c*x^2 + c), x)
```

## Giac [F]

$$\int \frac{x^3 \arctan(ax)^3}{c+a^2cx^2} \, dx = \int \frac{x^3 \arctan{(ax)}^3}{a^2cx^2+c} \, dx$$

input
```
integrate(x^3*arctan(a*x)^3/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
integrate(x^3*arctan(a*x)^3/(a^2*c*x^2 + c), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \arctan(ax)^3}{c+a^2cx^2} \, dx = \int \frac{x^3 \operatorname{atan}(a\,x)^3}{c\,a^2\,x^2+c} \, dx$$

input
```
int((x^3*atan(a*x)^3)/(c + a^2*c*x^2),x)
```

output
```
int((x^3*atan(a*x)^3)/(c + a^2*c*x^2), x)
```

## Reduce [F]

$$\int \frac{x^3 \arctan(ax)^3}{c + a^2cx^2} \, dx = \frac{\int \frac{atan(ax)^3 x^3}{a^2x^2+1} dx}{c}$$

input

```
int(x^3*atan(a*x)^3/(a^2*c*x^2+c),x)
```

output

```
int((atan(a*x)**3*x**3)/(a**2*x**2 + 1),x)/c
```

# 3.389 $\int \frac{x^2 \arctan(ax)^3}{c+a^2cx^2} \, dx$



## Optimal result

Integrand size = 22, antiderivative size = 130

$$\int \frac{x^2 \arctan(ax)^3}{c+a^2cx^2} \, dx = \frac{i \arctan(ax)^3}{a^3c} + \frac{x \arctan(ax)^3}{a^2c} - \frac{\arctan(ax)^4}{4a^3c} + \frac{3 \arctan(ax)^2 \log\left(\frac{2}{1+iax}\right)}{a^3c} + \frac{3i \arctan(ax) \operatorname{PolyLog}\left(2, 1-\frac{2}{1+iax}\right)}{a^3c} + \frac{3 \operatorname{PolyLog}\left(3, 1-\frac{2}{1+iax}\right)}{2a^3c}$$

output
```
I*arctan(a*x)^3/a^3/c+x*arctan(a*x)^3/a^2/c-1/4*arctan(a*x)^4/a^3/c+3*arct
an(a*x)^2*ln(2/(1+I*a*x))/a^3/c+3*I*arctan(a*x)*polylog(2,1-2/(1+I*a*x))/a
^3/c+3/2*polylog(3,1-2/(1+I*a*x))/a^3/c
```

## Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.72

$$\int \frac{x^2 \arctan(ax)^3}{c+a^2cx^2} dx = \frac{-\frac{1}{4}\arctan(ax)^2\left((4i-4ax)\arctan(ax)+\arctan(ax)^2-12\log\left(1+e^{2i\arctan(ax)}\right)\right)-3i\arctan(ax)\,\mathrm{PolyL}}{a^3c}$$

input
```
Integrate[(x^2*ArcTan[a*x]^3)/(c + a^2*c*x^2),x]
```

output
```
(-1/4*(ArcTan[a*x]^2*((4*I - 4*a*x)*ArcTan[a*x] + ArcTan[a*x]^2 - 12*Log[1
 + E^((2*I)*ArcTan[a*x])])) - (3*I)*ArcTan[a*x]*PolyLog[2, -E^((2*I)*ArcTa
n[a*x])] + (3*PolyLog[3, -E^((2*I)*ArcTan[a*x])])/2)/(a^3*c)
```

## Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {5451, 27, 5345, 5419, 5455, 5379, 5529, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\arctan(ax)^3}{a^2cx^2+c}dx$$

$$\Big\downarrow \; 5451$$

$$\frac{\int \arctan(ax)^3dx}{a^2c}-\frac{\int \frac{\arctan(ax)^3}{c(a^2x^2+1)}dx}{a^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \arctan(ax)^3dx}{a^2c}-\frac{\int \frac{\arctan(ax)^3}{a^2x^2+1}dx}{a^2c}$$

$$\Big\downarrow \; 5345$$

$$\frac{x\arctan(ax)^3-3a\int \frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2c}-\frac{\int \frac{\arctan(ax)^3}{a^2x^2+1}dx}{a^2c}$$

$$\Big\downarrow \text{5419}$$

$$\frac{x\arctan(ax)^3-3a\int\frac{x\arctan(ax)^2}{a^2x^2+1}dx}{a^2c}-\frac{\arctan(ax)^4}{4a^3c}$$

$$\Big\downarrow \text{5455}$$

$$-\frac{\arctan(ax)^4}{4a^3c}+\frac{x\arctan(ax)^3-3a\left(-\frac{\int\frac{\arctan(ax)^2}{i-ax}dx}{a}-\frac{i\arctan(ax)^3}{3a^2}\right)}{a^2c}$$

$$\Big\downarrow \text{5379}$$

$$-\frac{\arctan(ax)^4}{4a^3c}+\frac{x\arctan(ax)^3-3a\left(-\frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}-2\int\frac{\arctan(ax)\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^3}{3a^2}\right)}{a^2c}$$

$$\Big\downarrow \text{5529}$$

$$-\frac{\arctan(ax)^4}{4a^3c}+\frac{x\arctan(ax)^3-3a\left(-\frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}-2\left(\frac{1}{2}i\int\frac{\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx-\frac{i\arctan(ax)\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}\right)}{a}-\frac{i\arctan(ax)^3}{3a^2}\right)}{a^2c}$$

$$\Big\downarrow \text{7164}$$

$$-\frac{\arctan(ax)^4}{4a^3c}+\frac{x\arctan(ax)^3-3a\left(-\frac{i\arctan(ax)^3}{3a^2}-\frac{\frac{\arctan(ax)^2\log\left(\frac{2}{1+iax}\right)}{a}-2\left(-\frac{i\arctan(ax)\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}-\frac{\text{PolyLog}\left(3,1-\frac{2}{iax+1}\right)}{4a}\right)}{a}\right)}{a^2c}$$

input
```
Int[(x^2*ArcTan[a*x]^3)/(c + a^2*c*x^2),x]
```

output
```
-1/4*ArcTan[a*x]^4/(a^3*c) + (x*ArcTan[a*x]^3 - 3*a*(((-1/3*I)*ArcTan[a*x]
^3)/a^2 - ((ArcTan[a*x]^2*Log[2/(1 + I*a*x)])/a - 2*(((-1/2*I)*ArcTan[a*x]
*PolyLog[2, 1 - 2/(1 + I*a*x)])/a - PolyLog[3, 1 - 2/(1 + I*a*x)]/(4*a)))/
a))/(a^2*c)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 58.12 (sec) , antiderivative size = 785, normalized size of antiderivative = 6.04

| method | result |
|---|---|
| derivativedivides | $\frac{\frac{\arctan(ax)^3 ax}{c}-\frac{\arctan(ax)^4}{c}-\frac{3\left(\frac{\arctan(ax)^2\ln\left(a^2x^2+1\right)}{2}-\arctan(ax)^2\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)+\frac{i\arctan(ax)^3}{3}-\frac{\left(-i\pi\ \mathrm{csgn}\left(\frac{}{\left(\frac{(iax+1}{a^2x^2+}\right.}\right.\right.}{}\right.}{}}{}$ |
| default | $\frac{\frac{\arctan(ax)^3 ax}{c}-\frac{\arctan(ax)^4}{c}-\frac{3\left(\frac{\arctan(ax)^2\ln\left(a^2x^2+1\right)}{2}-\arctan(ax)^2\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)+\frac{i\arctan(ax)^3}{3}-\frac{\left(-i\pi\ \mathrm{csgn}\left(\frac{}{\left(\frac{(iax+1}{a^2x^2+}\right.}\right.\right.}{}\right.}{}}{}$ |
| parts | $\frac{x\arctan(ax)^3}{a^2c}-\frac{\arctan(ax)^4}{a^3c}-\frac{3\left(\frac{\arctan(ax)^2\ln\left(a^2x^2+1\right)}{2}-\arctan(ax)^2\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)+\frac{i\arctan(ax)^3}{3}-\frac{\left(-i\pi\ \mathrm{csgn}\left(\right.\right.}{}\right.}{}$ |

input
```
int(x^2*arctan(a*x)^3/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(1/c*arctan(a*x)^3*a*x-1/c*arctan(a*x)^4-3/c*(1/2*arctan(a*x)^2*ln(a
^2*x^2+1)-arctan(a*x)^2*ln((1+I*a*x)/(a^2*x^2+1)^(1/2))+1/3*I*arctan(a*x)^
3-1/4*(-I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2
*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)+I*P
i*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1
+I*a*x)^2/(a^2*x^2+1)+1)^2)^2+I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))^2*c
sgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)-2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1
)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2+I*Pi*csgn(I*((1+I*a*x)^2/(a^
2*x^2+1)+1)^2)^3-I*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))^2*csgn(I*(1+I*a*
x)^2/(a^2*x^2+1))+2*I*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*csgn(I*(1+I*a
*x)^2/(a^2*x^2+1))^2-I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^3+I*Pi*csgn(I*(1
+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^
2+1)+1)^2)^2-I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+
1)^2)^3+4*ln(2))*arctan(a*x)^2+I*arctan(a*x)*polylog(2,-(1+I*a*x)^2/(a^2*x
^2+1))-1/2*polylog(3,-(1+I*a*x)^2/(a^2*x^2+1))-1/4*arctan(a*x)^4))
```

## Fricas [F]

$$\int \frac{x^2 \arctan(ax)^3}{c+a^2cx^2}\,dx = \int \frac{x^2 \arctan\left(ax\right)^3}{a^2cx^2+c}\,dx$$

input
```
integrate(x^2*arctan(a*x)^3/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
integral(x^2*arctan(a*x)^3/(a^2*c*x^2 + c), x)
```

## Sympy [F]

$$\int \frac{x^2 \arctan(ax)^3}{c+a^2cx^2}\,dx = \frac{\int \frac{x^2 \operatorname{atan}^3(ax)}{a^2x^2+1}\,dx}{c}$$

input
```
integrate(x**2*atan(a*x)**3/(a**2*c*x**2+c),x)
```

output
```
Integral(x**2*atan(a*x)**3/(a**2*x**2 + 1), x)/c
```

## Maxima [F]

$$\int \frac{x^2 \arctan(ax)^3}{c + a^2cx^2} \, dx = \int \frac{x^2 \arctan{(ax)}^3}{a^2cx^2 + c} \, dx$$

input
```
integrate(x^2*arctan(a*x)^3/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
1/1024*(16*(7168*a^2*integrate(1/128*x^2*arctan(a*x)^3/(a^4*c*x^2 + a^2*c)
, x) + 768*a^2*integrate(1/128*x^2*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^4*c*x
^2 + a^2*c), x) + 3072*a^2*integrate(1/128*x^2*arctan(a*x)*log(a^2*x^2 + 1
)/(a^4*c*x^2 + a^2*c), x) - 768*a*integrate(1/128*x*arctan(a*x)^2*log(a^2*
x^2 + 1)/(a^4*c*x^2 + a^2*c), x) - 192*a*integrate(1/128*x*log(a^2*x^2 + 1
)^3/(a^4*c*x^2 + a^2*c), x) - 3072*a*integrate(1/128*x*arctan(a*x)^2/(a^4*
c*x^2 + a^2*c), x) + 768*a*integrate(1/128*x*log(a^2*x^2 + 1)^2/(a^4*c*x^2
 + a^2*c), x) + 3*arctan(a*x)^4/(a^3*c) + 384*integrate(1/128*arctan(a*x)*
log(a^2*x^2 + 1)^2/(a^4*c*x^2 + a^2*c), x))*a^3*c + 128*a*x*arctan(a*x)^3
- 80*arctan(a*x)^4 + 3*log(a^2*x^2 + 1)^4 - 24*(4*a*x*arctan(a*x) - arctan
(a*x)^2)*log(a^2*x^2 + 1)^2)/(a^3*c)
```

## Giac [F]

$$\int \frac{x^2 \arctan(ax)^3}{c + a^2cx^2} \, dx = \int \frac{x^2 \arctan{(ax)}^3}{a^2cx^2 + c} \, dx$$

input
```
integrate(x^2*arctan(a*x)^3/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
integrate(x^2*arctan(a*x)^3/(a^2*c*x^2 + c), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \arctan(ax)^3}{c+a^2cx^2} \, dx = \int \frac{x^2 \operatorname{atan}(a\,x)^3}{c\,a^2\,x^2+c} \, dx$$

input
```
int((x^2*atan(a*x)^3)/(c + a^2*c*x^2),x)
```

output
```
int((x^2*atan(a*x)^3)/(c + a^2*c*x^2), x)
```

## Reduce [F]

$$\int \frac{x^2 \arctan(ax)^3}{c+a^2cx^2} \, dx = \frac{\int \frac{atan(ax)^3 x^2}{a^2x^2+1}dx}{c}$$

input
```
int(x^2*atan(a*x)^3/(a^2*c*x^2+c),x)
```

output
```
int((atan(a*x)**3*x**2)/(a**2*x**2 + 1),x)/c
```

# 3.390 $\int \frac{x \arctan(ax)^3}{c+a^2cx^2} dx$



## Optimal result

Integrand size = 20, antiderivative size = 138

$$\int \frac{x \arctan(ax)^3}{c+a^2cx^2} dx = -\frac{i \arctan(ax)^4}{4a^2c} - \frac{\arctan(ax)^3 \log\left(\frac{2}{1+iax}\right)}{a^2c} - \frac{3i \arctan(ax)^2 \operatorname{PolyLog}\left(2, 1-\frac{2}{1+iax}\right)}{2a^2c} - \frac{3 \arctan(ax) \operatorname{PolyLog}\left(3, 1-\frac{2}{1+iax}\right)}{2a^2c} + \frac{3i \operatorname{PolyLog}\left(4, 1-\frac{2}{1+iax}\right)}{4a^2c}$$

output
```
-1/4*I*arctan(a*x)^4/a^2/c-arctan(a*x)^3*ln(2/(1+I*a*x))/a^2/c-3/2*I*arcta
n(a*x)^2*polylog(2,1-2/(1+I*a*x))/a^2/c-3/2*arctan(a*x)*polylog(3,1-2/(1+I
*a*x))/a^2/c+3/4*I*polylog(4,1-2/(1+I*a*x))/a^2/c
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.86

$$\int \frac{x \arctan(ax)^3}{c + a^2cx^2} \, dx = -\frac{i\left(\arctan(ax)^4 - 4i\arctan(ax)^3 \log\left(\frac{2i}{i-ax}\right) + 6\arctan(ax)^2 \operatorname{PolyLog}\left(2, \frac{i+ax}{-i+ax}\right) - 6i\arctan(ax) \operatorname{PolyLo}\right.}{4a^2c}$$

input
```
Integrate[(x*ArcTan[a*x]^3)/(c + a^2*c*x^2),x]
```

output
```
((-1/4*I)*(ArcTan[a*x]^4 - (4*I)*ArcTan[a*x]^3*Log[(2*I)/(I - a*x)] + 6*Ar
cTan[a*x]^2*PolyLog[2, (I + a*x)/(-I + a*x)] - (6*I)*ArcTan[a*x]*PolyLog[3
, (I + a*x)/(-I + a*x)] - 3*PolyLog[4, (I + a*x)/(-I + a*x)]))/(a^2*c)
```

## Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5455, 5379, 5529, 5533, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \arctan(ax)^3}{a^2cx^2 + c} \, dx$$

$$\Big\downarrow \; 5455$$

$$-\frac{\int \frac{\arctan(ax)^3}{i-ax}dx}{ac} - \frac{i\arctan(ax)^4}{4a^2c}$$

$$\Big\downarrow \; 5379$$

$$-\frac{\frac{\arctan(ax)^3 \log\left(\frac{2}{1+iax}\right)}{a} - 3\int \frac{\arctan(ax)^2 \log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{ac} - \frac{i\arctan(ax)^4}{4a^2c}$$

$$\Big\downarrow \; 5529$$

$$-\frac{\frac{\arctan(ax)^3\log\left(\frac{2}{1+iax}\right)}{a}-3\left(i\int\frac{\arctan(ax)\operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx-\frac{i\arctan(ax)^2\operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}\right)}{ac}-\frac{i\arctan(ax)^4}{4a^2c}$$

$\Big\downarrow$ 5533

$$-\frac{\frac{\arctan(ax)^3\log\left(\frac{2}{1+iax}\right)}{a}-3\left(i\left(\frac{i\arctan(ax)\operatorname{PolyLog}\left(3,1-\frac{2}{iax+1}\right)}{2a}-\frac{1}{2}i\int\frac{\operatorname{PolyLog}\left(3,1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx\right)-\frac{i\arctan(ax)^2\operatorname{PolyLog}\left(2,1-\right.}{2a}\right.}{ac}$$

$$\frac{i\arctan(ax)^4}{4a^2c}$$

$\Big\downarrow$ 7164

$$-\frac{i\arctan(ax)^4}{4a^2c}-$$

$$\frac{\frac{\arctan(ax)^3\log\left(\frac{2}{1+iax}\right)}{a}-3\left(i\left(\frac{i\arctan(ax)\operatorname{PolyLog}\left(3,1-\frac{2}{iax+1}\right)}{2a}+\frac{\operatorname{PolyLog}\left(4,1-\frac{2}{iax+1}\right)}{4a}\right)-\frac{i\arctan(ax)^2\operatorname{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}\right)}{ac}$$

input
```
Int[(x*ArcTan[a*x]^3)/(c + a^2*c*x^2),x]
```

output
```
((-1/4*I)*ArcTan[a*x]^4)/(a^2*c) - ((ArcTan[a*x]^3*Log[2/(1 + I*a*x)])/a -
 3*(((-1/2*I)*ArcTan[a*x]^2*PolyLog[2, 1 - 2/(1 + I*a*x)])/a + I*(((I/2)*A
rcTan[a*x]*PolyLog[3, 1 - 2/(1 + I*a*x)])/a + PolyLog[4, 1 - 2/(1 + I*a*x)
]/(4*a))))/(a*c)
```

## Defintions of rubi rules used

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 5533
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k + 1
, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &&
EqQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 69.65 (sec) , antiderivative size = 789, normalized size of antiderivative = 5.72

| method | result |
|---|---|
| derivativedivides | $\frac{\ln(a^2x^2+1)\arctan(ax)^3}{2c} - \frac{3\left(\frac{2\arctan(ax)^3\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)}{3} - \frac{i\arctan(ax)^4}{6} + \frac{\left(-i\pi\,\mathrm{csgn}\left(\frac{i}{\left(\frac{(iax+1)^2}{a^2x^2+1}+1\right)^2}\right)\mathrm{csgn}\left(\frac{i(iax+1)^2}{a^2x^2+1}\right) \cdots\right.}{\cdots}\right.}{\cdots}$ |
| default | $\frac{\ln(a^2x^2+1)\arctan(ax)^3}{2c} - \frac{3\left(\frac{2\arctan(ax)^3\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)}{3} - \frac{i\arctan(ax)^4}{6} + \frac{\left(-i\pi\,\mathrm{csgn}\left(\frac{i}{\left(\frac{(iax+1)^2}{a^2x^2+1}+1\right)^2}\right)\mathrm{csgn}\left(\frac{i(iax+1)^2}{a^2x^2+1}\right) \cdots\right.}{\cdots}\right.}{\cdots}$ |
| parts | $\frac{\ln(a^2x^2+1)\arctan(ax)^3}{2c\,a^2} - \frac{3\left(\frac{2\arctan(ax)^3\ln\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)}{3a} - \frac{i\arctan(ax)^4}{6a} + \frac{\left(-i\pi\,\mathrm{csgn}\left(\frac{i}{\left(\frac{(iax+1)^2}{a^2x^2+1}+1\right)^2}\right)\mathrm{csgn}\left(\frac{i(iax \cdots}{a^2x^2 \cdots}\right) \cdots\right.}{\cdots}\right.}{\cdots}$ |

input

```
int(x*arctan(a*x)^3/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output

```
1/a^2*(1/2/c*ln(a^2*x^2+1)*arctan(a*x)^3-3/2/c*(2/3*arctan(a*x)^3*ln((1+I*
a*x)/(a^2*x^2+1)^(1/2))-1/6*I*arctan(a*x)^4+1/6*(-I*Pi*csgn(I/((1+I*a*x)^2
/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2
*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)+I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1
)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2+I*
Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1
)^2)-2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x
^2+1)+1)^2)^2+I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3-I*Pi*csgn(I*(1+
I*a*x)/(a^2*x^2+1)^(1/2))^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))+2*I*Pi*csgn(I*
(1+I*a*x)/(a^2*x^2+1)^(1/2))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^2-I*Pi*csgn(I
*(1+I*a*x)^2/(a^2*x^2+1))^3+I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1
+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2-I*Pi*csgn(I*(1+I*a*
x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3+4*ln(2))*arctan(a*x)^3-I
*arctan(a*x)^2*polylog(2,-(1+I*a*x)^2/(a^2*x^2+1))+arctan(a*x)*polylog(3,-
(1+I*a*x)^2/(a^2*x^2+1))+1/2*I*polylog(4,-(1+I*a*x)^2/(a^2*x^2+1))))
```

## Fricas [F]

$$\int \frac{x\arctan(ax)^3}{c+a^2cx^2}\,dx = \int \frac{x\arctan{(ax)}^3}{a^2cx^2+c}\,dx$$

input
```
integrate(x*arctan(a*x)^3/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
integral(x*arctan(a*x)^3/(a^2*c*x^2 + c), x)
```

## Sympy [F]

$$\int \frac{x\arctan(ax)^3}{c+a^2cx^2}\,dx = \frac{\int \frac{x\,\mathrm{atan}^3{(ax)}}{a^2x^2+1}\,dx}{c}$$

input
```
integrate(x*atan(a*x)**3/(a**2*c*x**2+c),x)
```

output
```
Integral(x*atan(a*x)**3/(a**2*x**2 + 1), x)/c
```

## Maxima [F]

$$\int \frac{x\arctan(ax)^3}{c+a^2cx^2}\,dx = \int \frac{x\arctan{(ax)}^3}{a^2cx^2+c}\,dx$$

input
```
integrate(x*arctan(a*x)^3/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
integrate(x*arctan(a*x)^3/(a^2*c*x^2 + c), x)
```

## Giac [F]

$$\int \frac{x\arctan(ax)^3}{c+a^2cx^2}\,dx=\int \frac{x\arctan\left(ax\right)^3}{a^2cx^2+c}\,dx$$

input
```
integrate(x*arctan(a*x)^3/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
integrate(x*arctan(a*x)^3/(a^2*c*x^2 + c), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x\arctan(ax)^3}{c+a^2cx^2}\,dx=\int \frac{x\,\mathrm{atan}(a\,x)^3}{c\,a^2\,x^2+c}\,dx$$

input
```
int((x*atan(a*x)^3)/(c + a^2*c*x^2),x)
```

output
```
int((x*atan(a*x)^3)/(c + a^2*c*x^2), x)
```

## Reduce [F]

$$\int \frac{x\arctan(ax)^3}{c+a^2cx^2}\,dx=\frac{\int \frac{atan(ax)^3x}{a^2x^2+1}dx}{c}$$

input
```
int(x*atan(a*x)^3/(a^2*c*x^2+c),x)
```

output
```
int((atan(a*x)**3*x)/(a**2*x**2 + 1),x)/c
```

# 3.391 $\int \frac{\arctan(ax)^3}{c+a^2cx^2}\,dx$



## Optimal result

Integrand size = 19, antiderivative size = 16

$$\int \frac{\arctan(ax)^3}{c+a^2cx^2}\,dx = \frac{\arctan(ax)^4}{4ac}$$

output
```
1/4*arctan(a*x)^4/a/c
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax)^3}{c+a^2cx^2}\,dx = \frac{\arctan(ax)^4}{4ac}$$

input
```
Integrate[ArcTan[a*x]^3/(c + a^2*c*x^2),x]
```

output
```
ArcTan[a*x]^4/(4*a*c)
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3}{a^2cx^2+c} \, dx$$

$$\Big\downarrow \; 5419$$

$$\frac{\arctan(ax)^4}{4ac}$$

input
```
Int[ArcTan[a*x]^3/(c + a^2*c*x^2),x]
```

output
```
ArcTan[a*x]^4/(4*a*c)
```

### Defintions of rubi rules used

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

## Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

| method | result |
|---|---|
| derivativedivides | $\frac{\arctan(ax)^4}{4ac}$ |
| default | $\frac{\arctan(ax)^4}{4ac}$ |
| parallelrisch | $\frac{\arctan(ax)^4}{4ac}$ |
| parts | $\frac{\arctan(ax)^4}{4ac}$ |
| risch | $\frac{\ln(iax+1)^4}{64ca}-\frac{\ln(-iax+1)\ln(iax+1)^3}{16ca}+\frac{3\ln(-iax+1)^2\ln(iax+1)^2}{32ca}-\frac{\ln(-iax+1)^3\ln(iax+1)}{16ca}+\frac{\ln(-iax+1)^4}{64ca}$ |

input
```
int(arctan(a*x)^3/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
1/4*arctan(a*x)^4/a/c
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\arctan(ax)^3}{c+a^2cx^2}\,dx=\frac{\arctan{(ax)}^4}{4\,ac}$$

input
```
integrate(arctan(a*x)^3/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
1/4*arctan(a*x)^4/(a*c)
```

## Sympy [F]

$$\int \frac{\arctan(ax)^3}{c+a^2cx^2}\,dx=\frac{\int \frac{\operatorname{atan}^3(ax)}{a^2x^2+1}\,dx}{c}$$

input
```
integrate(atan(a*x)**3/(a**2*c*x**2+c),x)
```

output
```
Integral(atan(a*x)**3/(a**2*x**2 + 1), x)/c
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\arctan(ax)^3}{c+a^2cx^2}\,dx=\frac{\arctan(ax)^4}{4\,ac}$$

input
```
integrate(arctan(a*x)^3/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
1/4*arctan(a*x)^4/(a*c)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\arctan(ax)^3}{c+a^2cx^2}\,dx=\frac{\arctan(ax)^4}{4\,ac}$$

input
```
integrate(arctan(a*x)^3/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
1/4*arctan(a*x)^4/(a*c)
```

## Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\arctan(ax)^3}{c+a^2cx^2}\,dx=\frac{\mathrm{atan}(a\,x)^4}{4\,a\,c}$$

input
```
int(atan(a*x)^3/(c + a^2*c*x^2),x)
```

output
```
atan(a*x)^4/(4*a*c)
```

## Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\arctan(ax)^3}{c+a^2cx^2}\,dx=\frac{atan(ax)^4}{4ac}$$

input
```
int(atan(a*x)^3/(a^2*c*x^2+c),x)
```

output
```
atan(a*x)**4/(4*a*c)
```

# 3.392 $\int \frac{\arctan(ax)^3}{x(c+a^2cx^2)}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 124

$$\int \frac{\arctan(ax)^3}{x(c+a^2cx^2)}\,dx = -\frac{i\arctan(ax)^4}{4c}+\frac{\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)}{c} - \frac{3i\arctan(ax)^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{2c} + \frac{3\arctan(ax)\operatorname{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)}{2c} + \frac{3i\operatorname{PolyLog}\left(4,-1+\frac{2}{1-iax}\right)}{4c}$$

output
```
-1/4*I*arctan(a*x)^4/c+arctan(a*x)^3*ln(2-2/(1-I*a*x))/c-3/2*I*arctan(a*x)
^2*polylog(2,-1+2/(1-I*a*x))/c+3/2*arctan(a*x)*polylog(3,-1+2/(1-I*a*x))/c
+3/4*I*polylog(4,-1+2/(1-I*a*x))/c
```

## Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.81

$$\int \frac{\arctan(ax)^3}{x\left(c+a^2cx^2\right)}\,dx = -\frac{i\left(\pi^4-16\arctan(ax)^4+64i\arctan(ax)^3\log\left(1-e^{-2i\arctan(ax)}\right)-96\arctan(ax)^2\operatorname{PolyLog}\left(2,e^{-2i\arctan(ax)}\right)\right.}{64c}$$

input
```
Integrate[ArcTan[a*x]^3/(x*(c + a^2*c*x^2)),x]
```

output
```
((-1/64*I)*(Pi^4 - 16*ArcTan[a*x]^4 + (64*I)*ArcTan[a*x]^3*Log[1 - E^((-2*
I)*ArcTan[a*x])] - 96*ArcTan[a*x]^2*PolyLog[2, E^((-2*I)*ArcTan[a*x])] + (
96*I)*ArcTan[a*x]*PolyLog[3, E^((-2*I)*ArcTan[a*x])] + 48*PolyLog[4, E^((-
2*I)*ArcTan[a*x])]))/c
```

## Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {5459, 5403, 5527, 5531, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3}{x\left(a^2cx^2+c\right)}dx$$

$$\Big\downarrow \; 5459$$

$$\frac{i\int \frac{\arctan(ax)^3}{x(ax+i)}dx}{c}-\frac{i\arctan(ax)^4}{4c}$$

$$\Big\downarrow \; 5403$$

$$\frac{i\left(3ia\int \frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right)}{c}-\frac{i\arctan(ax)^4}{4c}$$

$$\Big\downarrow \; 5527$$

$$\frac{i\left(3ia\left(\frac{i\arctan(ax)^2\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-i\int\frac{\arctan(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{a^2x^2+1}dx\right)-i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right)}{c}-\frac{i\arctan(ax)^4}{4c}$$

$\downarrow$ 5531

$$\frac{i\left(3ia\left(\frac{i\arctan(ax)^2\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-i\left(\frac{1}{2}i\int\frac{\operatorname{PolyLog}\left(3,\frac{2}{1-iax}-1\right)}{a^2x^2+1}dx-\frac{i\arctan(ax)\operatorname{PolyLog}\left(3,\frac{2}{1-iax}-1\right)}{2a}\right)\right)-i\arctan}{c}$$
$$\frac{i\arctan(ax)^4}{4c}$$

$\downarrow$ 7164

$$\frac{i\left(3ia\left(\frac{i\arctan(ax)^2\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-i\left(\frac{\operatorname{PolyLog}\left(4,\frac{2}{1-iax}-1\right)}{4a}-\frac{i\arctan(ax)\operatorname{PolyLog}\left(3,\frac{2}{1-iax}-1\right)}{2a}\right)\right)-i\arctan(ax)^3\operatorname{lo}}{c}$$
$$\frac{i\arctan(ax)^4}{4c}$$

input
```
Int[ArcTan[a*x]^3/(x*(c + a^2*c*x^2)),x]
```

output
```
((-1/4*I)*ArcTan[a*x]^4)/c + (I*((-I)*ArcTan[a*x]^3*Log[2 - 2/(1 - I*a*x)]
 + (3*I)*a*(((I/2)*ArcTan[a*x]^2*PolyLog[2, -1 + 2/(1 - I*a*x)])/a - I*(((
-1/2*I)*ArcTan[a*x]*PolyLog[3, -1 + 2/(1 - I*a*x)])/a + PolyLog[4, -1 + 2/
(1 - I*a*x)]/(4*a)))))/c
```

## Defintions of rubi rules used

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5527
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

rule 5531
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/
(2*c*d)), x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k
+ 1, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0]
&& EqQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.94 (sec) , antiderivative size = 1640, normalized size of antiderivative = 13.23

Expression too large to display

input
```
int(arctan(a*x)^3/x/(a^2*c*x^2+c),x)
```

output
```
1/c*arctan(a*x)^3*ln(a*x)-1/2/c*ln(a^2*x^2+1)*arctan(a*x)^3-3/2/c*(-2/3*ar
ctan(a*x)^3*ln((1+I*a*x)/(a^2*x^2+1)^(1/2))+1/6*I*arctan(a*x)^4+2/3*arctan
(a*x)^3*ln((1+I*a*x)^2/(a^2*x^2+1)-1)-2/3*arctan(a*x)^3*ln(1-(1+I*a*x)/(a^
2*x^2+1)^(1/2))+2*I*arctan(a*x)^2*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-4
*arctan(a*x)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))-4*I*polylog(4,(1+I*a*x
)/(a^2*x^2+1)^(1/2))-2/3*arctan(a*x)^3*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+2
*I*arctan(a*x)^2*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-4*arctan(a*x)*pol
ylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-4*I*polylog(4,-(1+I*a*x)/(a^2*x^2+1)^
(1/2))-1/6*(-I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+
1)^2)^3+2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)
+1))^3+2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I/((1+I*a*x)^2/(a^2
*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1)
)+2*I*Pi*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^3-2
*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn
(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2-I*Pi*csgn(I/((
1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a
*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)-2*I*Pi*csgn(I*((1+I*a*x)^
2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2-2*I*Pi*csgn(I*((
1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2
/(a^2*x^2+1)+1))^2+I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3+2*I*Pi*...
```

## Fricas [F]

$$\int \frac{\arctan(ax)^3}{x\,(c+a^2cx^2)}\,dx=\int \frac{\arctan\left(ax\right)^3}{(a^2cx^2+c)x}\,dx$$

input
```
integrate(arctan(a*x)^3/x/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
integral(arctan(a*x)^3/(a^2*c*x^3 + c*x), x)
```

## Sympy [F]

$$\int \frac{\arctan(ax)^3}{x\left(c+a^2cx^2\right)}\,dx = \frac{\int \frac{\operatorname{atan}^3(ax)}{a^2x^3+x}\,dx}{c}$$

input
```
integrate(atan(a*x)**3/x/(a**2*c*x**2+c),x)
```

output
```
Integral(atan(a*x)**3/(a**2*x**3 + x), x)/c
```

## Maxima [F]

$$\int \frac{\arctan(ax)^3}{x\left(c+a^2cx^2\right)}\,dx = \int \frac{\arctan\left(ax\right)^3}{(a^2cx^2+c)x}\,dx$$

input
```
integrate(arctan(a*x)^3/x/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)^3/((a^2*c*x^2 + c)*x), x)
```

## Giac [F]

$$\int \frac{\arctan(ax)^3}{x\left(c+a^2cx^2\right)}\,dx = \int \frac{\arctan\left(ax\right)^3}{(a^2cx^2+c)x}\,dx$$

input
```
integrate(arctan(a*x)^3/x/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^3/((a^2*c*x^2 + c)*x), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^3}{x\,(c+a^2cx^2)}\,dx=\int \frac{\mathrm{atan}(a\,x)^3}{x\,\left(c\,a^2\,x^2+c\right)}\,dx$$

input
```
int(atan(a*x)^3/(x*(c + a^2*c*x^2)),x)
```

output
```
int(atan(a*x)^3/(x*(c + a^2*c*x^2)), x)
```

## Reduce [F]

$$\int \frac{\arctan(ax)^3}{x\,(c+a^2cx^2)}\,dx=\frac{\int \frac{atan(ax)^3}{a^2x^3+x}dx}{c}$$

input
```
int(atan(a*x)^3/x/(a^2*c*x^2+c),x)
```

output
```
int(atan(a*x)**3/(a**2*x**3 + x),x)/c
```

# 3.393 $\int \frac{\arctan(ax)^3}{x^2(c+a^2cx^2)}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 122

$$\int \frac{\arctan(ax)^3}{x^2(c+a^2cx^2)}\,dx = -\frac{ia\arctan(ax)^3}{c}-\frac{\arctan(ax)^3}{cx}-\frac{a\arctan(ax)^4}{4c}+\frac{3a\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{c}-\frac{3ia\arctan(ax)\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{c}+\frac{3a\operatorname{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)}{2c}$$

output
```
-I*a*arctan(a*x)^3/c-arctan(a*x)^3/c/x-1/4*a*arctan(a*x)^4/c+3*a*arctan(a*
x)^2*ln(2-2/(1-I*a*x))/c-3*I*a*arctan(a*x)*polylog(2,-1+2/(1-I*a*x))/c+3/2
*a*polylog(3,-1+2/(1-I*a*x))/c
```

## Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.89

$$\int \frac{\arctan(ax)^3}{x^2\,(c+a^2cx^2)}\,dx = \frac{a\left(-\frac{i\pi^3}{8}+i\arctan(ax)^3-\frac{\arctan(ax)^3}{ax}-\frac{1}{4}\arctan(ax)^4+3\arctan(ax)^2\log\left(1-e^{-2i\arctan(ax)}\right)+3i\arctan(\right.}{c}$$

input
```
Integrate[ArcTan[a*x]^3/(x^2*(c + a^2*c*x^2)),x]
```

output
```
(a*((-1/8*I)*Pi^3 + I*ArcTan[a*x]^3 - ArcTan[a*x]^3/(a*x) - ArcTan[a*x]^4/
4 + 3*ArcTan[a*x]^2*Log[1 - E^((-2*I)*ArcTan[a*x])] + (3*I)*ArcTan[a*x]*Po
lyLog[2, E^((-2*I)*ArcTan[a*x])] + (3*PolyLog[3, E^((-2*I)*ArcTan[a*x])])/
2))/c
```

## Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {5453, 27, 5361, 5419, 5459, 5403, 5527, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3}{x^2\,(a^2cx^2+c)}\,dx$$

$$\Big\downarrow \text{5453}$$

$$\frac{\int \frac{\arctan(ax)^3}{x^2}dx}{c}-a^2\int \frac{\arctan(ax)^3}{c\,(a^2x^2+1)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\arctan(ax)^3}{x^2}dx}{c}-\frac{a^2\int \frac{\arctan(ax)^3}{a^2x^2+1}dx}{c}$$

$$\Big\downarrow \text{5361}$$

$$\frac{3a\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx - \frac{\arctan(ax)^3}{x}}{c} - \frac{a^2\int \frac{\arctan(ax)^3}{a^2x^2+1}dx}{c}$$

$$\Big\downarrow \text{5419}$$

$$\frac{3a\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx - \frac{\arctan(ax)^3}{x}}{c} - \frac{a\arctan(ax)^4}{4c}$$

$$\Big\downarrow \text{5459}$$

$$-\frac{a\arctan(ax)^4}{4c} + \frac{-\frac{\arctan(ax)^3}{x} + 3a\left(i\int \frac{\arctan(ax)^2}{x(ax+i)}dx - \frac{1}{3}i\arctan(ax)^3\right)}{c}$$

$$\Big\downarrow \text{5403}$$

$$-\frac{a\arctan(ax)^4}{4c} + \frac{-\frac{\arctan(ax)^3}{x} + 3a\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx - i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right) - \frac{1}{3}i\arctan(ax)^3\right)}{c}$$

$$\Big\downarrow \text{5527}$$

$$-\frac{a\arctan(ax)^4}{4c} + \frac{-\frac{\arctan(ax)^3}{x} + 3a\left(i\left(2ia\left(\frac{i\arctan(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a} - \frac{1}{2}i\int \frac{\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{a^2x^2+1}dx\right) - i\arctan(ax)^2\log\left(2-\frac{2}{1-\ldots}\right.\right.\right.}{c}$$

$$\Big\downarrow \text{7164}$$

$$-\frac{a\arctan(ax)^4}{4c} + \frac{-\frac{\arctan(ax)^3}{x} + 3a\left(i\left(2ia\left(\frac{i\arctan(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a} - \frac{\operatorname{PolyLog}\left(3,\frac{2}{1-iax}-1\right)}{4a}\right) - i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right) \ldots\right.}{c}$$

input
```
Int[ArcTan[a*x]^3/(x^2*(c + a^2*c*x^2)),x]
```

output
```
-1/4*(a*ArcTan[a*x]^4)/c + (-(ArcTan[a*x]^3/x) + 3*a*((-1/3*I)*ArcTan[a*x]
^3 + I*((-I)*ArcTan[a*x]^2*Log[2 - 2/(1 - I*a*x)] + (2*I)*a*(((I/2)*ArcTan
[a*x]*PolyLog[2, -1 + 2/(1 - I*a*x)])/a - PolyLog[3, -1 + 2/(1 - I*a*x)]/(
4*a)))))/c
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5527
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.70 (sec) , antiderivative size = 1611, normalized size of antiderivative = 13.20

Expression too large to display

input
```
int(arctan(a*x)^3/x^2/(a^2*c*x^2+c),x)
```

output
```
a*(-1/c*arctan(a*x)^3/a/x-1/c*arctan(a*x)^4-3/c*(-1/4*arctan(a*x)^4+1/2*ar
ctan(a*x)^2*ln(a^2*x^2+1)-arctan(a*x)^2*ln(a*x)-arctan(a*x)^2*ln((1+I*a*x)
/(a^2*x^2+1)^(1/2))+arctan(a*x)^2*ln((1+I*a*x)^2/(a^2*x^2+1)-1)+1/3*I*arct
an(a*x)^3-1/4*(-I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+
1)+1)^2)^3+2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2
+1)+1))^3+2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I/((1+I*a*x)^2/(
a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)
+1))+2*I*Pi*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^
3-2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*c
sgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2-I*Pi*csgn(I
/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+
I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)-2*I*Pi*csgn(I*((1+I*a*
x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2-2*I*Pi*csgn(I
*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x
)^2/(a^2*x^2+1)+1))^2+I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3+2*I*Pi*
csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(((1+I
*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))-2*I*Pi*csgn(((1+I*a*x)
^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2+2*I*Pi*csgn(I*(1+I*a*x)/(
a^2*x^2+1)^(1/2))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^2-2*I*Pi*csgn(I/((1+I*a*
x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a...
```

## Fricas [F]

$$\int \frac{\arctan(ax)^3}{x^2\left(c+a^2cx^2\right)}\,dx=\int \frac{\arctan\left(ax\right)^3}{(a^2cx^2+c)x^2}\,dx$$

input
```
integrate(arctan(a*x)^3/x^2/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
integral(arctan(a*x)^3/(a^2*c*x^4 + c*x^2), x)
```

## Sympy [F]

$$\int \frac{\arctan(ax)^3}{x^2(c+a^2cx^2)}\,dx = \frac{\int \frac{\operatorname{atan}^3(ax)}{a^2x^4+x^2}\,dx}{c}$$

input
```
integrate(atan(a*x)**3/x**2/(a**2*c*x**2+c),x)
```

output
```
Integral(atan(a*x)**3/(a**2*x**4 + x**2), x)/c
```

## Maxima [F]

$$\int \frac{\arctan(ax)^3}{x^2(c+a^2cx^2)}\,dx = \int \frac{\arctan(ax)^3}{(a^2cx^2+c)x^2}\,dx$$

input
```
integrate(arctan(a*x)^3/x^2/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
-1/1024*(80*a*x*arctan(a*x)^4 - 3*a*x*log(a^2*x^2 + 1)^4 - (48*a*arctan(a*
x)^4/c - 12288*a^3*integrate(1/128*x^3*arctan(a*x)^2*log(a^2*x^2 + 1)/(a^2
*c*x^4 + c*x^2), x) - 3*a*log(a^2*x^2 + 1)^4/c + 6144*a^2*integrate(1/128*
x^2*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*c*x^4 + c*x^2), x) - 49152*a^2*int
egrate(1/128*x^2*arctan(a*x)*log(a^2*x^2 + 1)/(a^2*c*x^4 + c*x^2), x) + 49
152*a*integrate(1/128*x*arctan(a*x)^2/(a^2*c*x^4 + c*x^2), x) - 12288*a*in
tegrate(1/128*x*log(a^2*x^2 + 1)^2/(a^2*c*x^4 + c*x^2), x) + 114688*integr
ate(1/128*arctan(a*x)^3/(a^2*c*x^4 + c*x^2), x) + 12288*integrate(1/128*ar
ctan(a*x)*log(a^2*x^2 + 1)^2/(a^2*c*x^4 + c*x^2), x))*c*x + 128*arctan(a*x
)^3 - 24*(a*x*arctan(a*x)^2 + 4*arctan(a*x))*log(a^2*x^2 + 1)^2)/(c*x)
```

## Giac [F]

$$\int \frac{\arctan(ax)^3}{x^2(c+a^2cx^2)}\,dx = \int \frac{\arctan(ax)^3}{(a^2cx^2+c)x^2}\,dx$$

input
```
integrate(arctan(a*x)^3/x^2/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^3/((a^2*c*x^2 + c)*x^2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^3}{x^2(c+a^2cx^2)}\,dx = \int \frac{\mathrm{atan}(a\,x)^3}{x^2\,(c\,a^2\,x^2+c)}\,dx$$

input
```
int(atan(a*x)^3/(x^2*(c + a^2*c*x^2)),x)
```

output
```
int(atan(a*x)^3/(x^2*(c + a^2*c*x^2)), x)
```

## Reduce [F]

$$\int \frac{\arctan(ax)^3}{x^2(c+a^2cx^2)}\,dx = \frac{-atan(ax)^4\,ax-4atan(ax)^3+12\left(\int \frac{atan(ax)^2}{a^2x^3+x}dx\right)ax}{4cx}$$

input
```
int(atan(a*x)^3/x^2/(a^2*c*x^2+c),x)
```

output
```
( - atan(a*x)**4*a*x - 4*atan(a*x)**3 + 12*int(atan(a*x)**2/(a**2*x**3 + x
),x)*a*x)/(4*c*x)
```

# 3.394 $\int \frac{\arctan(ax)^3}{x^3(c+a^2cx^2)}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 262

$$\int \frac{\arctan(ax)^3}{x^3(c+a^2cx^2)}\,dx = -\frac{3ia^2\arctan(ax)^2}{2c}-\frac{3a\arctan(ax)^2}{2cx}-\frac{a^2\arctan(ax)^3}{2c}-\frac{\arctan(ax)^3}{2cx^2}+\frac{ia^2\arctan(ax)^4}{4c}+\frac{3a^2\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{c}-\frac{a^2\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)}{c}-\frac{3ia^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{2c}+\frac{3ia^2\arctan(ax)^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{2c}-\frac{3a^2\arctan(ax)\operatorname{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)}{2c}-\frac{3ia^2\operatorname{PolyLog}\left(4,-1+\frac{2}{1-iax}\right)}{4c}$$

output

```
-3/2*I*a^2*arctan(a*x)^2/c-3/2*a*arctan(a*x)^2/c/x-1/2*a^2*arctan(a*x)^3/c
-1/2*arctan(a*x)^3/c/x^2+1/4*I*a^2*arctan(a*x)^4/c+3*a^2*arctan(a*x)*ln(2-
2/(1-I*a*x))/c-a^2*arctan(a*x)^3*ln(2-2/(1-I*a*x))/c-3/2*I*a^2*polylog(2,-
1+2/(1-I*a*x))/c+3/2*I*a^2*arctan(a*x)^2*polylog(2,-1+2/(1-I*a*x))/c-3/2*a
^2*arctan(a*x)*polylog(3,-1+2/(1-I*a*x))/c-3/4*I*a^2*polylog(4,-1+2/(1-I*a
*x))/c
```

## Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.72

$$\int \frac{\arctan(ax)^3}{x^3 (c+a^2cx^2)} dx = \frac{ia^2 \left(\pi^4 - 96\arctan(ax)^2 + \frac{96i\arctan(ax)^2}{ax} + \frac{32i(1+a^2x^2)\arctan(ax)^3}{a^2x^2} - 16\arctan(ax)^4 + 64i\arctan(ax)^3\log(1$$

input
```
Integrate[ArcTan[a*x]^3/(x^3*(c + a^2*c*x^2)),x]
```

output
```
((I/64)*a^2*(Pi^4 - 96*ArcTan[a*x]^2 + ((96*I)*ArcTan[a*x]^2)/(a*x) + ((32
*I)*(1 + a^2*x^2)*ArcTan[a*x]^3)/(a^2*x^2) - 16*ArcTan[a*x]^4 + (64*I)*Arc
Tan[a*x]^3*Log[1 - E^((-2*I)*ArcTan[a*x])] - (192*I)*ArcTan[a*x]*Log[1 - E
^((2*I)*ArcTan[a*x])] - 96*ArcTan[a*x]^2*PolyLog[2, E^((-2*I)*ArcTan[a*x])
] - 96*PolyLog[2, E^((2*I)*ArcTan[a*x])] + (96*I)*ArcTan[a*x]*PolyLog[3, E
^((-2*I)*ArcTan[a*x])] + 48*PolyLog[4, E^((-2*I)*ArcTan[a*x])]))/c
```

## Rubi [A] (verified)

Time = 1.86 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.98, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {5453, 27, 5361, 5453, 5361, 5419, 5459, 5403, 2897, 5527, 5531, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3}{x^3 (a^2cx^2+c)} dx$$

$$\Big\downarrow \text{5453}$$

$$\frac{\int \frac{\arctan(ax)^3}{x^3}dx}{c} - a^2 \int \frac{\arctan(ax)^3}{cx(a^2x^2+1)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\arctan(ax)^3}{x^3}dx}{c} - \frac{a^2 \int \frac{\arctan(ax)^3}{x(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \text{5361}$$

$$\frac{\frac{3}{2}a\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)}dx-\frac{\arctan(ax)^3}{2x^2}}{c}-\frac{a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \text{5453}$$

$$\frac{\frac{3}{2}a\left(\int \frac{\arctan(ax)^2}{x^2}dx-a^2\int \frac{\arctan(ax)^2}{a^2x^2+1}dx\right)-\frac{\arctan(ax)^3}{2x^2}}{c}-\frac{a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \text{5361}$$

$$\frac{\frac{3}{2}a\left(a^2\left(-\int \frac{\arctan(ax)^2}{a^2x^2+1}dx\right)+2a\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx-\frac{\arctan(ax)^2}{x}\right)-\frac{\arctan(ax)^3}{2x^2}}{c}-\frac{a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \text{5419}$$

$$\frac{\frac{3}{2}a\left(2a\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx-\frac{1}{3}a\arctan(ax)^3-\frac{\arctan(ax)^2}{x}\right)-\frac{\arctan(ax)^3}{2x^2}}{c}-\frac{a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \text{5459}$$

$$\frac{-\frac{\arctan(ax)^3}{2x^2}+\frac{3}{2}a\left(2a\left(i\int \frac{\arctan(ax)}{x(ax+i)}dx-\frac{1}{2}i\arctan(ax)^2\right)-\frac{1}{3}a\arctan(ax)^3-\frac{\arctan(ax)^2}{x}\right)}{c}-\frac{a^2\left(i\int \frac{\arctan(ax)^3}{x(ax+i)}dx-\frac{1}{4}i\arctan(ax)^4\right)}{c}$$

$$\Big\downarrow \text{5403}$$

$$\frac{-\frac{\arctan(ax)^3}{2x^2}+\frac{3}{2}a\left(2a\left(i\left(ia\int \frac{\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)-\frac{1}{3}a\arctan(a\right.}{c}$$

$$\frac{a^2\left(i\left(3ia\int \frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{4}i\arctan(ax)^4\right)}{c}$$

$$\Big\downarrow \text{2897}$$

$$\frac{-\frac{\arctan(ax)^3}{2x^2}+\frac{3}{2}a\left(2a\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)-\frac{1}{3}a\,\text{arc}\right.}{c}$$

$$\frac{a^2\left(i\left(3ia\int \frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{4}i\arctan(ax)^4\right)}{c}$$

$$\Big\downarrow \text{5527}$$

$$\frac{-\frac{\arctan(ax)^3}{2x^2}+\frac{3}{2}a\left(2a\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)-\frac{1}{3}a\text{ arc}}{c}$$

$$\frac{a^2\left(i\left(3ia\left(\frac{i\arctan(ax)^2\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-i\int\frac{\arctan(ax)\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{a^2x^2+1}dx\right)-i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right)}{c}$$

$$\Big\downarrow \text{5531}$$

$$\frac{-\frac{\arctan(ax)^3}{2x^2}+\frac{3}{2}a\left(2a\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)-\frac{1}{3}a\text{ arc}}{c}$$

$$\frac{a^2\left(i\left(3ia\left(\frac{i\arctan(ax)^2\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-i\left(\frac{1}{2}i\int\frac{\text{PolyLog}\left(3,\frac{2}{1-iax}-1\right)}{a^2x^2+1}dx-\frac{i\arctan(ax)\text{PolyLog}\left(3,\frac{2}{1-iax}-1\right)}{2a}\right)\right)\right)-i\text{ arc}}{c}$$

$$\Big\downarrow \text{7164}$$

$$\frac{-\frac{\arctan(ax)^3}{2x^2}+\frac{3}{2}a\left(2a\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)-\frac{1}{3}a\text{ arc}}{c}$$

$$\frac{a^2\left(i\left(3ia\left(\frac{i\arctan(ax)^2\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-i\left(\frac{\text{PolyLog}\left(4,\frac{2}{1-iax}-1\right)}{4a}-\frac{i\arctan(ax)\text{PolyLog}\left(3,\frac{2}{1-iax}-1\right)}{2a}\right)\right)\right)-i\arctan(ax}{c}$$

input
```
Int[ArcTan[a*x]^3/(x^3*(c + a^2*c*x^2)),x]
```

output
```
(-1/2*ArcTan[a*x]^3/x^2 + (3*a*(-(ArcTan[a*x]^2/x) - (a*ArcTan[a*x]^3)/3 +
 2*a*((-1/2*I)*ArcTan[a*x]^2 + I*((-I)*ArcTan[a*x]*Log[2 - 2/(1 - I*a*x)]
- PolyLog[2, -1 + 2/(1 - I*a*x)]/2))))/2)/c - (a^2*((-1/4*I)*ArcTan[a*x]^4
 + I*((-I)*ArcTan[a*x]^3*Log[2 - 2/(1 - I*a*x)] + (3*I)*a*(((I/2)*ArcTan[a
*x]^2*PolyLog[2, -1 + 2/(1 - I*a*x)])/a - I*(((-1/2*I)*ArcTan[a*x]*PolyLog
[3, -1 + 2/(1 - I*a*x)])/a + PolyLog[4, -1 + 2/(1 - I*a*x)]/(4*a))))))/c
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5527
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

rule 5531
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/
(2*c*d)), x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k
+ 1, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0]
&& EqQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

## Maple [A] (verified)

Time = 94.10 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.68

| method | result |
|---|---|
| derivativedivides | $a^2\left(-\frac{3i\arctan(ax)^2}{c}-\frac{\arctan(ax)^2(-i\arctan(ax)+\arctan(ax)ax-3iax)(ax+i)}{2c\,a^2x^2}-\frac{6i\,\mathrm{polylog}\left(4,-\frac{iax+1}{\sqrt{a^2x^2+1}}\right)}{c}\right.$ |
| default | $a^2\left(-\frac{3i\arctan(ax)^2}{c}-\frac{\arctan(ax)^2(-i\arctan(ax)+\arctan(ax)ax-3iax)(ax+i)}{2c\,a^2x^2}-\frac{6i\,\mathrm{polylog}\left(4,-\frac{iax+1}{\sqrt{a^2x^2+1}}\right)}{c}\right.$ |

input
```
int(arctan(a*x)^3/x^3/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
a^2*(-3*I/c*arctan(a*x)^2-1/2/c*arctan(a*x)^2*(-I*arctan(a*x)+arctan(a*x)*
a*x-3*I*a*x)*(a*x+I)/a^2/x^2-6*I/c*polylog(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2))
-1/c*arctan(a*x)^3*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I/c*polylog(4,(1+I*
a*x)/(a^2*x^2+1)^(1/2))-6/c*arctan(a*x)*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(
1/2))-3*I/c*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-1/c*arctan(a*x)^3*ln(1-
(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*I/c*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))
-6/c*arctan(a*x)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))+3*I/c*arctan(a*x)^
2*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+1/4*I/c*arctan(a*x)^4+3/c*arctan
(a*x)*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+3*I/c*arctan(a*x)^2*polylog(2,(1+I
*a*x)/(a^2*x^2+1)^(1/2))+3/c*arctan(a*x)*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))
)
```

## Fricas [F]

$$\int \frac{\arctan(ax)^3}{x^3\left(c+a^2cx^2\right)}\,dx = \int \frac{\arctan\left(ax\right)^3}{(a^2cx^2+c)x^3}\,dx$$

input
```
integrate(arctan(a*x)^3/x^3/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
integral(arctan(a*x)^3/(a^2*c*x^5 + c*x^3), x)
```

## Sympy [F]

$$\int \frac{\arctan(ax)^3}{x^3\left(c+a^2cx^2\right)}\,dx = \frac{\int \frac{\operatorname{atan}^3(ax)}{a^2x^5+x^3}\,dx}{c}$$

input
```
integrate(atan(a*x)**3/x**3/(a**2*c*x**2+c),x)
```

output
```
Integral(atan(a*x)**3/(a**2*x**5 + x**3), x)/c
```

## Maxima [F]

$$\int \frac{\arctan(ax)^3}{x^3(c+a^2cx^2)}\,dx = \int \frac{\arctan(ax)^3}{(a^2cx^2+c)x^3}\,dx$$

input
```
integrate(arctan(a*x)^3/x^3/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)^3/((a^2*c*x^2 + c)*x^3), x)
```

## Giac [F]

$$\int \frac{\arctan(ax)^3}{x^3(c+a^2cx^2)}\,dx = \int \frac{\arctan(ax)^3}{(a^2cx^2+c)x^3}\,dx$$

input
```
integrate(arctan(a*x)^3/x^3/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^3/((a^2*c*x^2 + c)*x^3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^3}{x^3(c+a^2cx^2)}\,dx = \int \frac{\operatorname{atan}(a\,x)^3}{x^3\,(c\,a^2\,x^2+c)}\,dx$$

input
```
int(atan(a*x)^3/(x^3*(c + a^2*c*x^2)),x)
```

output
```
int(atan(a*x)^3/(x^3*(c + a^2*c*x^2)), x)
```

## Reduce [F]

$$\int \frac{\arctan(ax)^3}{x^3 (c + a^2 c x^2)} \, dx = \frac{\int \frac{atan(ax)^3}{a^2 x^5 + x^3} dx}{c}$$

input
```
int(atan(a*x)^3/x^3/(a^2*c*x^2+c),x)
```

output
```
int(atan(a*x)**3/(a**2*x**5 + x**3),x)/c
```

# 3.395 $\int \frac{\arctan(ax)^3}{x^4(c+a^2cx^2)}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 227

$$\begin{aligned}\int \frac{\arctan(ax)^3}{x^4(c+a^2cx^2)}\,dx = & -\frac{a^2\arctan(ax)}{cx}-\frac{a^3\arctan(ax)^2}{2c}-\frac{a\arctan(ax)^2}{2cx^2}\\ & +\frac{4ia^3\arctan(ax)^3}{3c}-\frac{\arctan(ax)^3}{3cx^3}\\ & +\frac{a^2\arctan(ax)^3}{cx}+\frac{a^3\arctan(ax)^4}{4c}+\frac{a^3\log(x)}{c}\\ & -\frac{a^3\log(1+a^2x^2)}{2c}-\frac{4a^3\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{c}\\ & +\frac{4ia^3\arctan(ax)\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{c}\\ & -\frac{2a^3\operatorname{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)}{c}\end{aligned}$$

output

```
-a^2*arctan(a*x)/c/x-1/2*a^3*arctan(a*x)^2/c-1/2*a*arctan(a*x)^2/c/x^2+4/3
*I*a^3*arctan(a*x)^3/c-1/3*arctan(a*x)^3/c/x^3+a^2*arctan(a*x)^3/c/x+1/4*a
^3*arctan(a*x)^4/c+a^3*ln(x)/c-1/2*a^3*ln(a^2*x^2+1)/c-4*a^3*arctan(a*x)^2
*ln(2-2/(1-I*a*x))/c+4*I*a^3*arctan(a*x)*polylog(2,-1+2/(1-I*a*x))/c-2*a^3
*polylog(3,-1+2/(1-I*a*x))/c
```

## Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.79

$$\int \frac{\arctan(ax)^3}{x^4 (c + a^2cx^2)} dx$$
$$= \frac{a^3 \left(\frac{i\pi^3}{6} - \frac{\arctan(ax)}{ax} - \frac{1}{2}\arctan(ax)^2 - \frac{\arctan(ax)^2}{2a^2x^2} - \frac{4}{3}i\arctan(ax)^3 - \frac{\arctan(ax)^3}{3a^3x^3} + \frac{\arctan(ax)^3}{ax} + \frac{1}{4}\arctan(ax)\right.}{}$$

input
```
Integrate[ArcTan[a*x]^3/(x^4*(c + a^2*c*x^2)),x]
```

output
```
(a^3*((I/6)*Pi^3 - ArcTan[a*x]/(a*x) - ArcTan[a*x]^2/2 - ArcTan[a*x]^2/(2*
a^2*x^2) - ((4*I)/3)*ArcTan[a*x]^3 - ArcTan[a*x]^3/(3*a^3*x^3) + ArcTan[a*
x]^3/(a*x) + ArcTan[a*x]^4/4 - 4*ArcTan[a*x]^2*Log[1 - E^((-2*I)*ArcTan[a*
x])] + Log[(a*x)/Sqrt[1 + a^2*x^2]] - (4*I)*ArcTan[a*x]*PolyLog[2, E^((-2*
I)*ArcTan[a*x])] - 2*PolyLog[3, E^((-2*I)*ArcTan[a*x])]))/c
```

## Rubi [A] (verified)

Time = 2.48 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.41, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.773$, Rules used = {5453, 27, 5361, 5453, 5361, 5419, 5453, 5361, 243, 47, 14, 16, 5419, 5459, 5403, 5527, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3}{x^4 (a^2cx^2 + c)} dx$$
$$\Big\downarrow \text{5453}$$
$$\frac{\int \frac{\arctan(ax)^3}{x^4}dx}{c} - a^2 \int \frac{\arctan(ax)^3}{cx^2 (a^2x^2 + 1)}dx$$
$$\Big\downarrow \text{27}$$
$$\frac{\int \frac{\arctan(ax)^3}{x^4}dx}{c} - \frac{a^2 \int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \text{5361}$$

$$\frac{a\int \frac{\arctan(ax)^2}{x^3(a^2x^2+1)}dx-\frac{\arctan(ax)^3}{3x^3}}{c}-\frac{a^2\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \text{5453}$$

$$\frac{a\left(\int \frac{\arctan(ax)^2}{x^3}dx-a^2\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right)-\frac{\arctan(ax)^3}{3x^3}}{c}-\frac{a^2\left(\int \frac{\arctan(ax)^3}{x^2}dx-a^2\int \frac{\arctan(ax)^3}{a^2x^2+1}dx\right)}{c}$$

$$\Big\downarrow \text{5361}$$

$$\frac{a\left(a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right)+a\int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx-\frac{\arctan(ax)^2}{2x^2}\right)-\frac{\arctan(ax)^3}{3x^3}}{c}-\frac{a^2\left(3a\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx-a^2\int \frac{\arctan(ax)^3}{a^2x^2+1}dx-\frac{\arctan(ax)^3}{x}\right)}{c}$$

$$\Big\downarrow \text{5419}$$

$$\frac{a\left(a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right)+a\int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx-\frac{\arctan(ax)^2}{2x^2}\right)-\frac{\arctan(ax)^3}{3x^3}}{c}-\frac{a^2\left(3a\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx-\frac{1}{4}a\arctan(ax)^4-\frac{\arctan(ax)^3}{x}\right)}{c}$$

$$\Big\downarrow \text{5453}$$

$$\frac{a\left(a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right)+a\left(\int \frac{\arctan(ax)}{x^2}dx-a^2\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)-\frac{\arctan(ax)^2}{2x^2}\right)-\frac{\arctan(ax)^3}{3x^3}}{c}-\frac{a^2\left(3a\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx-\frac{1}{4}a\arctan(ax)^4-\frac{\arctan(ax)^3}{x}\right)}{c}$$

$$\Big\downarrow \text{5361}$$

$$\frac{a\left(a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right)+a\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+a\int \frac{1}{x(a^2x^2+1)}dx-\frac{\arctan(ax)}{x}\right)-\frac{\arctan(ax)^2}{2x^2}\right)-\frac{\arctan(ax)^3}{3x^3}}{c}-\frac{a^2\left(3a\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx-\frac{1}{4}a\arctan(ax)^4-\frac{\arctan(ax)^3}{x}\right)}{c}$$

$$\Big\downarrow \text{243}$$

$$\frac{a\left(a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right)+a\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{2}a\int \frac{1}{x^2(a^2x^2+1)}dx^2-\frac{\arctan(ax)}{x}\right)-\frac{\arctan(ax)^2}{2x^2}\right)-\frac{\arctan(ax)^3}{3x^3}}{c}-\frac{a^2\left(3a\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx-\frac{1}{4}a\arctan(ax)^4-\frac{\arctan(ax)^3}{x}\right)}{c}$$

$\big\downarrow$ 47

$$\frac{a\left(a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right)+a\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{2}a\left(\int \frac{1}{x^2}dx^2-a^2\int \frac{1}{a^2x^2+1}dx^2\right)-\frac{\arctan(ax)}{x}\right)-\frac{\arctan(ax)^2}{2x^2}\right)}{c}$$

$$\frac{a^2\left(3a\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx-\frac{1}{4}a\arctan(ax)^4-\frac{\arctan(ax)^3}{x}\right)}{c}$$

$\big\downarrow$ 14

$$\frac{a\left(a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right)+a\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{2}a\left(\log\left(x^2\right)-a^2\int \frac{1}{a^2x^2+1}dx^2\right)-\frac{\arctan(ax)}{x}\right)-\frac{\arctan(ax)^2}{2x^2}\right)}{c}$$

$$\frac{a^2\left(3a\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx-\frac{1}{4}a\arctan(ax)^4-\frac{\arctan(ax)^3}{x}\right)}{c}$$

$\big\downarrow$ 16

$$\frac{a\left(a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right)+a\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{\arctan(ax)}{x}\right)-\frac{\arctan(ax)^2}{2x^2}\right)}{c}$$

$$\frac{a^2\left(3a\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx-\frac{1}{4}a\arctan(ax)^4-\frac{\arctan(ax)^3}{x}\right)}{c}$$

$\big\downarrow$ 5419

$$\frac{a\left(a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right)+a\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{1}{2}a\arctan(ax)^2-\frac{\arctan(ax)}{x}\right)-\frac{\arctan(ax)^2}{2x^2}\right)-\frac{\text{arcta}}{}}{c}$$

$$\frac{a^2\left(3a\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx-\frac{1}{4}a\arctan(ax)^4-\frac{\arctan(ax)^3}{x}\right)}{c}$$

$\big\downarrow$ 5459

$$\frac{-\frac{\arctan(ax)^3}{3x^3}+a\left(-\left(a^2\left(i\int \frac{\arctan(ax)^2}{x(ax+i)}dx-\frac{1}{3}i\arctan(ax)^3\right)\right)+a\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{1}{2}a\arctan(ax\right.\right.}{c}$$

$$\frac{a^2\left(3a\left(i\int \frac{\arctan(ax)^2}{x(ax+i)}dx-\frac{1}{3}i\arctan(ax)^3\right)-\frac{1}{4}a\arctan(ax)^4-\frac{\arctan(ax)^3}{x}\right)}{c}$$

$\big\downarrow$ 5403

$$-\frac{\arctan(ax)^3}{3x^3}+a\left(-\left(a^2\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3\right)\right.\right.$$
$$\overline{\phantom{xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx}c}$$

$$\frac{a^2\left(3a\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3\right)-\frac{1}{4}a\arctan(ax)^4\right.}{c}$$

$$\Big\downarrow 5527$$

$$-\frac{\arctan(ax)^3}{3x^3}+a\left(-\left(a^2\left(i\left(2ia\left(\frac{i\arctan(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-\frac{1}{2}i\int \frac{\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{a^2x^2+1}dx\right)-i\arctan(ax)^2\log\left(\right.\right.\right.\right.\right.$$

$$\frac{a^2\left(3a\left(i\left(2ia\left(\frac{i\arctan(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-\frac{1}{2}i\int \frac{\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{a^2x^2+1}dx\right)-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}\right.\right.}{c}$$

$$\Big\downarrow 7164$$

$$-\frac{\arctan(ax)^3}{3x^3}+a\left(-\left(a^2\left(i\left(2ia\left(\frac{i\arctan(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-\frac{\operatorname{PolyLog}\left(3,\frac{2}{1-iax}-1\right)}{4a}\right)-i\arctan(ax)^2\log\left(2-\frac{2}{1-\right.\right.\right.\right.\right.$$
$$\overline{\phantom{xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx}c}$$

$$\frac{a^2\left(3a\left(i\left(2ia\left(\frac{i\arctan(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-\frac{\operatorname{PolyLog}\left(3,\frac{2}{1-iax}-1\right)}{4a}\right)-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan\right.\right.}{c}$$

input
```
Int[ArcTan[a*x]^3/(x^4*(c + a^2*c*x^2)),x]
```

output
```
-((a^2*(-(ArcTan[a*x]^3/x) - (a*ArcTan[a*x]^4)/4 + 3*a*((-1/3*I)*ArcTan[a*
x]^3 + I*((-I)*ArcTan[a*x]^2*Log[2 - 2/(1 - I*a*x)] + (2*I)*a*(((I/2)*ArcT
an[a*x]*PolyLog[2, -1 + 2/(1 - I*a*x)])/a - PolyLog[3, -1 + 2/(1 - I*a*x)]
/(4*a))))))/c) + (-1/3*ArcTan[a*x]^3/x^3 + a*(-1/2*ArcTan[a*x]^2/x^2 + a*(
-(ArcTan[a*x]/x) - (a*ArcTan[a*x]^2)/2 + (a*(Log[x^2] - Log[1 + a^2*x^2]))
/2) - a^2*((-1/3*I)*ArcTan[a*x]^3 + I*((-I)*ArcTan[a*x]^2*Log[2 - 2/(1 - I
*a*x)] + (2*I)*a*(((I/2)*ArcTan[a*x]*PolyLog[2, -1 + 2/(1 - I*a*x)])/a - P
olyLog[3, -1 + 2/(1 - I*a*x)]/(4*a))))))/c
```

### Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5527
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.87 (sec) , antiderivative size = 1872, normalized size of antiderivative = 8.25

Expression too large to display

input
```
int(arctan(a*x)^3/x^4/(a^2*c*x^2+c),x)
```

output
```
a^3*(1/c*arctan(a*x)^4-1/3/c*arctan(a*x)^3/a^3/x^3+1/c*arctan(a*x)^3/a/x-1
/c*(1/2*arctan(a*x)^2-ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-1)-ln(1+(1+I*a*x)/(a^
2*x^2+1)^(1/2))+8*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))+8*polylog(3,-(1+I
*a*x)/(a^2*x^2+1)^(1/2))-I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/
(a^2*x^2+1)+1)^2)^3*arctan(a*x)^2-I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^3*a
rctan(a*x)^2+2*I*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*csgn(I*(1+I*a*x)^2
/(a^2*x^2+1))^2*arctan(a*x)^2+I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*c
sgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2*arctan(a*x)
^2+I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1
+I*a*x)^2/(a^2*x^2+1)+1)^2)^2*arctan(a*x)^2-2*I*Pi*csgn(I*((1+I*a*x)^2/(a^
2*x^2+1)-1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1
))^2*arctan(a*x)^2+I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))^2*csgn(I*((1+I
*a*x)^2/(a^2*x^2+1)+1)^2)*arctan(a*x)^2-I*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^
(1/2))^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*arctan(a*x)^2-2*I*Pi*csgn(I*((1+I
*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(((1+I*a*x)^2/(a^2
*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x)^2+4*arctan(a*x)^2*ln
((1+I*a*x)/(a^2*x^2+1)^(1/2))-4*arctan(a*x)^2*ln((1+I*a*x)^2/(a^2*x^2+1)-1
)+1/2*arctan(a*x)*(I*a*x-(a^2*x^2+1)^(1/2)+1)/a/x+1/2*arctan(a*x)*(I*a*x+(
a^2*x^2+1)^(1/2)+1)/a/x-2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*
((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2*arctan(a*x)^2-2*I*Pi*csgn(I/((1+I*a*x)...
```

## Fricas [F]

$$\int \frac{\arctan(ax)^3}{x^4\left(c+a^2cx^2\right)}\,dx=\int \frac{\arctan\left(ax\right)^3}{(a^2cx^2+c)x^4}\,dx$$

input
```
integrate(arctan(a*x)^3/x^4/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
integral(arctan(a*x)^3/(a^2*c*x^6 + c*x^4), x)
```

## Sympy [F]

$$\int \frac{\arctan(ax)^3}{x^4\left(c+a^2cx^2\right)}\,dx=\frac{\int \frac{\operatorname{atan}^3(ax)}{a^2x^6+x^4}\,dx}{c}$$

input
```
integrate(atan(a*x)**3/x**4/(a**2*c*x**2+c),x)
```

output
```
Integral(atan(a*x)**3/(a**2*x**6 + x**4), x)/c
```

## Maxima [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^3}{x^4\left(c+a^2cx^2\right)}\,dx=\text{Timed out}$$

input
```
integrate(arctan(a*x)^3/x^4/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
Timed out
```

## Giac [F]

$$\int \frac{\arctan(ax)^3}{x^4\left(c+a^2cx^2\right)}\,dx=\int \frac{\arctan(ax)^3}{(a^2cx^2+c)x^4}\,dx$$

input
```
integrate(arctan(a*x)^3/x^4/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^3/((a^2*c*x^2 + c)*x^4), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^3}{x^4(c+a^2cx^2)}\,dx = \int \frac{\mathrm{atan}(a\,x)^3}{x^4\,\left(c\,a^2\,x^2+c\right)}\,dx$$

input
```
int(atan(a*x)^3/(x^4*(c + a^2*c*x^2)),x)
```

output
```
int(atan(a*x)^3/(x^4*(c + a^2*c*x^2)), x)
```

## Reduce [F]

$$\int \frac{\arctan(ax)^3}{x^4(c+a^2cx^2)}\,dx = \frac{3atan(ax)^4\,a^3x^3+12atan(ax)^3\,a^2x^2-4atan(ax)^3+18atan(ax)^2\,a^3x^3+18atan(ax)^2\,ax+36atan(ax)}{12c\,x^3}$$

input
```
int(atan(a*x)^3/x^4/(a^2*c*x^2+c),x)
```

output
```
(3*atan(a*x)**4*a**3*x**3 + 12*atan(a*x)**3*a**2*x**2 - 4*atan(a*x)**3 + 1
8*atan(a*x)**2*a**3*x**3 + 18*atan(a*x)**2*a*x + 36*atan(a*x)*a**2*x**2 +
48*int(atan(a*x)**2/(a**2*x**5 + x**3),x)*a*x**3 + 18*log(a**2*x**2 + 1)*a
**3*x**3 - 36*log(x)*a**3*x**3)/(12*c*x**3)
```

# 3.396 $\int \frac{x^3 \arctan(ax)^3}{(c+a^2cx^2)^2}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 270

$$\begin{aligned}\int \frac{x^3 \arctan(ax)^3}{(c+a^2cx^2)^2}\,dx &= \frac{3x}{8a^3c^2(1+a^2x^2)}+\frac{3\arctan(ax)}{8a^4c^2}-\frac{3\arctan(ax)}{4a^4c^2(1+a^2x^2)}\\ &\quad-\frac{3x\arctan(ax)^2}{4a^3c^2(1+a^2x^2)}-\frac{\arctan(ax)^3}{4a^4c^2}+\frac{\arctan(ax)^3}{2a^4c^2(1+a^2x^2)}\\ &\quad-\frac{i\arctan(ax)^4}{4a^4c^2}-\frac{\arctan(ax)^3\log\left(\frac{2}{1+iax}\right)}{a^4c^2}\\ &\quad-\frac{3i\arctan(ax)^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+iax}\right)}{2a^4c^2}\\ &\quad-\frac{3\arctan(ax)\operatorname{PolyLog}\left(3,1-\frac{2}{1+iax}\right)}{2a^4c^2}\\ &\quad+\frac{3i\operatorname{PolyLog}\left(4,1-\frac{2}{1+iax}\right)}{4a^4c^2}\end{aligned}$$

output
```
3/8*x/a^3/c^2/(a^2*x^2+1)+3/8*arctan(a*x)/a^4/c^2-3/4*arctan(a*x)/a^4/c^2/
(a^2*x^2+1)-3/4*x*arctan(a*x)^2/a^3/c^2/(a^2*x^2+1)-1/4*arctan(a*x)^3/a^4/
c^2+1/2*arctan(a*x)^3/a^4/c^2/(a^2*x^2+1)-1/4*I*arctan(a*x)^4/a^4/c^2-arct
an(a*x)^3*ln(2/(1+I*a*x))/a^4/c^2-3/2*I*arctan(a*x)^2*polylog(2,1-2/(1+I*a
*x))/a^4/c^2-3/2*arctan(a*x)*polylog(3,1-2/(1+I*a*x))/a^4/c^2+3/4*I*polylo
g(4,1-2/(1+I*a*x))/a^4/c^2
```

## Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.58

$$\int \frac{x^3 \arctan(ax)^3}{(c+a^2cx^2)^2}\,dx$$
$$= \frac{4i\arctan(ax)^4 - 6\arctan(ax)\cos(2\arctan(ax)) + 4\arctan(ax)^3\cos(2\arctan(ax)) - 16\arctan(ax)^3\log}{}$$

input
```
Integrate[(x^3*ArcTan[a*x]^3)/(c + a^2*c*x^2)^2,x]
```

output
```
((4*I)*ArcTan[a*x]^4 - 6*ArcTan[a*x]*Cos[2*ArcTan[a*x]] + 4*ArcTan[a*x]^3*
Cos[2*ArcTan[a*x]] - 16*ArcTan[a*x]^3*Log[1 + E^((2*I)*ArcTan[a*x])] + (24
*I)*ArcTan[a*x]^2*PolyLog[2, -E^((2*I)*ArcTan[a*x])] - 24*ArcTan[a*x]*Poly
Log[3, -E^((2*I)*ArcTan[a*x])] - (12*I)*PolyLog[4, -E^((2*I)*ArcTan[a*x])]
 + 3*Sin[2*ArcTan[a*x]] - 6*ArcTan[a*x]^2*Sin[2*ArcTan[a*x]])/(16*a^4*c^2)
```

## Rubi [A] (verified)

Time = 1.72 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {5499, 27, 5455, 5379, 5465, 5427, 5465, 215, 216, 5529, 5533, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \arctan(ax)^3}{(a^2cx^2+c)^2}\,dx$$
$$\Big\downarrow \text{5499}$$
$$\frac{\int \frac{x\arctan(ax)^3}{c(a^2x^2+1)}dx}{a^2c} - \frac{\int \frac{x\arctan(ax)^3}{c^2(a^2x^2+1)^2}dx}{a^2}$$
$$\Big\downarrow \text{27}$$
$$\frac{\int \frac{x\arctan(ax)^3}{a^2x^2+1}dx}{a^2c^2} - \frac{\int \frac{x\arctan(ax)^3}{(a^2x^2+1)^2}dx}{a^2c^2}$$

$$\Big\downarrow \text{5455}$$

$$-\frac{\int \frac{x\arctan(ax)^3}{(a^2x^2+1)^2}dx}{a^2c^2}+\frac{-\frac{\int \frac{\arctan(ax)^3}{i-ax}dx}{a}-\frac{i\arctan(ax)^4}{4a^2}}{a^2c^2}$$

$$\Big\downarrow \text{5379}$$

$$-\frac{\int \frac{x\arctan(ax)^3}{(a^2x^2+1)^2}dx}{a^2c^2}+\frac{-\frac{\frac{\arctan(ax)^3\log\left(\frac{2}{1+iax}\right)}{a}-3\int \frac{\arctan(ax)^2\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^4}{4a^2}}{a^2c^2}$$

$$\Big\downarrow \text{5465}$$

$$-\frac{\frac{3\int \frac{\arctan(ax)^2}{\left(a^2x^2+1\right)^2}dx}{2a}-\frac{\arctan(ax)^3}{2a^2(a^2x^2+1)}}{a^2c^2}+\frac{-\frac{\frac{\arctan(ax)^3\log\left(\frac{2}{1+iax}\right)}{a}-3\int \frac{\arctan(ax)^2\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^4}{4a^2}}{a^2c^2}$$

$$\Big\downarrow \text{5427}$$

$$-\frac{\frac{3\left(-a\int \frac{x\arctan(ax)}{\left(a^2x^2+1\right)^2}dx+\frac{x\arctan(ax)^2}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^3}{6a}\right)}{2a}-\frac{\arctan(ax)^3}{2a^2(a^2x^2+1)}}{a^2c^2}+\frac{-\frac{\frac{\arctan(ax)^3\log\left(\frac{2}{1+iax}\right)}{a}-3\int \frac{\arctan(ax)^2\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^4}{4a^2}}{a^2c^2}$$

$$\Big\downarrow \text{5465}$$

$$-\frac{\frac{3\left(-a\left(\frac{\int \frac{1}{\left(a^2x^2+1\right)^2}dx}{2a}-\frac{\arctan(ax)}{2a^2\left(a^2x^2+1\right)}\right)+\frac{x\arctan(ax)^2}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^3}{6a}\right)}{2a}-\frac{\arctan(ax)^3}{2a^2(a^2x^2+1)}}{a^2c^2}+\frac{-\frac{\frac{\arctan(ax)^3\log\left(\frac{2}{1+iax}\right)}{a}-3\int \frac{\arctan(ax)^2\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^4}{4a^2}}{a^2c^2}$$

$$\Big\downarrow \text{215}$$

$$-\frac{\frac{3\left(-a\left(\frac{\frac{1}{2}\int \frac{1}{a^2x^2+1}dx+\frac{x}{2\left(a^2x^2+1\right)}}{2a}-\frac{\arctan(ax)}{2a^2\left(a^2x^2+1\right)}\right)+\frac{x\arctan(ax)^2}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^3}{6a}\right)}{2a}-\frac{\arctan(ax)^3}{2a^2(a^2x^2+1)}}{a^2c^2}+\frac{-\frac{\frac{\arctan(ax)^3\log\left(\frac{2}{1+iax}\right)}{a}-3\int \frac{\arctan(ax)^2\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^4}{4a^2}}{a^2c^2}$$

$$\Big\downarrow \text{216}$$

$$-\frac{\frac{3\left(\frac{x\arctan(ax)^2}{2\left(a^2x^2+1\right)}-a\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\arctan(ax)}{2a^2\left(a^2x^2+1\right)}\right)+\frac{\arctan(ax)^3}{6a}\right)}{2a}-\frac{\arctan(ax)^3}{2a^2(a^2x^2+1)}}{a^2c^2}+\frac{-\frac{\frac{\arctan(ax)^3\log\left(\frac{2}{1+iax}\right)}{a}-3\int\frac{\arctan(ax)^2\log\left(\frac{2}{iax+1}\right)}{a^2x^2+1}dx}{a}-\frac{i\arctan(ax)^4}{4a^2}}{a^2c^2}$$

$$\Big\downarrow \; 5529$$

$$-\frac{\frac{3\left(\frac{x\arctan(ax)^2}{2\left(a^2x^2+1\right)}-a\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\arctan(ax)}{2a^2\left(a^2x^2+1\right)}\right)+\frac{\arctan(ax)^3}{6a}\right)}{2a}-\frac{\arctan(ax)^3}{2a^2(a^2x^2+1)}}{a^2c^2}+\frac{-\frac{\frac{\arctan(ax)^3\log\left(\frac{2}{1+iax}\right)}{a}-3\left(i\int\frac{\arctan(ax)\,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx-\frac{i\arctan(ax)^2\,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}\right)}{a}-\frac{i\arctan(ax)^4}{4a^2}}{a^2c^2}$$

$$\Big\downarrow \; 5533$$

$$-\frac{\frac{3\left(\frac{x\arctan(ax)^2}{2\left(a^2x^2+1\right)}-a\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\arctan(ax)}{2a^2\left(a^2x^2+1\right)}\right)+\frac{\arctan(ax)^3}{6a}\right)}{2a}-\frac{\arctan(ax)^3}{2a^2(a^2x^2+1)}}{a^2c^2}+\frac{-\frac{\frac{\arctan(ax)^3\log\left(\frac{2}{1+iax}\right)}{a}-3\left(i\left(\frac{i\arctan(ax)\,\text{PolyLog}\left(3,1-\frac{2}{iax+1}\right)}{2a}-\frac{1}{2}i\int\frac{\text{PolyLog}\left(3,1-\frac{2}{iax+1}\right)}{a^2x^2+1}dx\right)-\frac{i\arctan(ax)^2\,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}\right)}{a}-\frac{i\arctan(ax)^4}{4a^2}}{a^2c^2}$$

$$\Big\downarrow \; 7164$$

$$-\frac{\frac{3\left(\frac{x\arctan(ax)^2}{2\left(a^2x^2+1\right)}-a\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\arctan(ax)}{2a^2\left(a^2x^2+1\right)}\right)+\frac{\arctan(ax)^3}{6a}\right)}{2a}-\frac{\arctan(ax)^3}{2a^2(a^2x^2+1)}}{a^2c^2}+\frac{-\frac{i\arctan(ax)^4}{4a^2}-\frac{\frac{\arctan(ax)^3\log\left(\frac{2}{1+iax}\right)}{a}-3\left(i\left(\frac{i\arctan(ax)\,\text{PolyLog}\left(3,1-\frac{2}{iax+1}\right)}{2a}+\frac{\text{PolyLog}\left(4,1-\frac{2}{iax+1}\right)}{4a}\right)-\frac{i\arctan(ax)^2\,\text{PolyLog}\left(2,1-\frac{2}{iax+1}\right)}{2a}\right)}{a}}{a^2c^2}$$

input
```
Int[(x^3*ArcTan[a*x]^3)/(c + a^2*c*x^2)^2,x]
```

output
```
-((-1/2*ArcTan[a*x]^3/(a^2*(1 + a^2*x^2)) + (3*((x*ArcTan[a*x]^2)/(2*(1 +
a^2*x^2)) + ArcTan[a*x]^3/(6*a) - a*(-1/2*ArcTan[a*x]/(a^2*(1 + a^2*x^2))
+ (x/(2*(1 + a^2*x^2)) + ArcTan[a*x]/(2*a))/(2*a))))/(2*a))/(a^2*c^2)) + (
((-1/4*I)*ArcTan[a*x]^4)/a^2 - ((ArcTan[a*x]^3*Log[2/(1 + I*a*x)])/a - 3*(
((-1/2*I)*ArcTan[a*x]^2*PolyLog[2, 1 - 2/(1 + I*a*x)])/a + I*(((I/2)*ArcTa
n[a*x]*PolyLog[3, 1 - 2/(1 + I*a*x)])/a + PolyLog[4, 1 - 2/(1 + I*a*x)]/(4
*a))))/a)/(a^2*c^2)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5499
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ
[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

rule 5529
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c
^2*d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 5533
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k + 1
, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] &&
EqQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I - c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 78.96 (sec) , antiderivative size = 936, normalized size of antiderivative = 3.47

| method | result | size |
|---|---|---|
| derivativedivides | Expression too large to display | 936 |
| default | Expression too large to display | 936 |
| parts | Expression too large to display | 971 |

input
```
int(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/a^4*(1/2*arctan(a*x)^3/c^2/(a^2*x^2+1)+1/2/c^2*arctan(a*x)^3*ln(a^2*x^2+
1)-3/2/c^2*(2/3*arctan(a*x)^3*ln((1+I*a*x)/(a^2*x^2+1)^(1/2))-1/6*I*arctan
(a*x)^4-I*arctan(a*x)^2*(a*x+I)/(8*a*x-8*I)-1/8*arctan(a*x)*(a*x+I)/(a*x-I
)+I*(a*x+I)/(16*a*x-16*I)+I*arctan(a*x)^2*(a*x-I)/(8*a*x+8*I)-1/8*arctan(a
*x)*(a*x-I)/(a*x+I)-I*(a*x-I)/(16*a*x+16*I)-I*arctan(a*x)^2*polylog(2,-(1+
I*a*x)^2/(a^2*x^2+1))+arctan(a*x)*polylog(3,-(1+I*a*x)^2/(a^2*x^2+1))+1/2*
I*polylog(4,-(1+I*a*x)^2/(a^2*x^2+1))+1/6*(I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x
^2+1)+1))^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)-2*I*Pi*csgn(I*((1+I*a*x)
^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2+I*Pi*csgn(I*((1
+I*a*x)^2/(a^2*x^2+1)+1)^2)^3-I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^3+2*I*P
i*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^2+I*
Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*
x)^2/(a^2*x^2+1)+1)^2)^2-I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I
*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2
*x^2+1)+1)^2)-I*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))^2*csgn(I*(1+I*a*x)^
2/(a^2*x^2+1))-I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1
)+1)^2)^3+I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a
^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2+4*ln(2)+1)*arctan(a*x)^3))
```

## Fricas [F]

$$\int \frac{x^3 \arctan(ax)^3}{(c + a^2cx^2)^2} \, dx = \int \frac{x^3 \arctan{(ax)}^3}{(a^2cx^2 + c)^2} \, dx$$

input
```
integrate(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
integral(x^3*arctan(a*x)^3/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2), x)
```

## Sympy [F]

$$\int \frac{x^3 \arctan(ax)^3}{(c + a^2cx^2)^2} \, dx = \frac{\int \frac{x^3 \operatorname{atan}^3{(ax)}}{a^4x^4+2a^2x^2+1} \, dx}{c^2}$$

input
```
integrate(x**3*atan(a*x)**3/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(x**3*atan(a*x)**3/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

## Maxima [F]

$$\int \frac{x^3 \arctan(ax)^3}{(c + a^2cx^2)^2} \, dx = \int \frac{x^3 \arctan{(ax)}^3}{(a^2cx^2 + c)^2} \, dx$$

input
```
integrate(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
integrate(x^3*arctan(a*x)^3/(a^2*c*x^2 + c)^2, x)
```

## Giac [F]

$$\int \frac{x^3 \arctan(ax)^3}{(c+a^2cx^2)^2} \, dx = \int \frac{x^3 \arctan{(ax)}^3}{(a^2cx^2+c)^2} \, dx$$

input
```
integrate(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate(x^3*arctan(a*x)^3/(a^2*c*x^2 + c)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \arctan(ax)^3}{(c+a^2cx^2)^2} \, dx = \int \frac{x^3 \operatorname{atan}(a\,x)^3}{(c\,a^2\,x^2+c)^2} \, dx$$

input
```
int((x^3*atan(a*x)^3)/(c + a^2*c*x^2)^2,x)
```

output
```
int((x^3*atan(a*x)^3)/(c + a^2*c*x^2)^2, x)
```

## Reduce [F]

$$\int \frac{x^3 \arctan(ax)^3}{(c+a^2cx^2)^2} \, dx = \frac{\int \frac{atan(ax)^3x^3}{a^4x^4+2a^2x^2+1}dx}{c^2}$$

input
```
int(x^3*atan(a*x)^3/(a^2*c*x^2+c)^2,x)
```

output
```
int((atan(a*x)**3*x**3)/(a**4*x**4 + 2*a**2*x**2 + 1),x)/c**2
```

# 3.397 $\int \frac{x^2 \arctan(ax)^3}{(c+a^2cx^2)^2} dx$



## Optimal result

Integrand size = 22, antiderivative size = 135

$$\int \frac{x^2 \arctan(ax)^3}{(c+a^2cx^2)^2} dx = \frac{3}{8a^3c^2(1+a^2x^2)} + \frac{3x\arctan(ax)}{4a^2c^2(1+a^2x^2)} + \frac{3\arctan(ax)^2}{8a^3c^2} - \frac{3\arctan(ax)^2}{4a^3c^2(1+a^2x^2)} - \frac{x\arctan(ax)^3}{2a^2c^2(1+a^2x^2)} + \frac{\arctan(ax)^4}{8a^3c^2}$$

output
```
3/8/a^3/c^2/(a^2*x^2+1)+3/4*x*arctan(a*x)/a^2/c^2/(a^2*x^2+1)+3/8*arctan(a
*x)^2/a^3/c^2-3/4*arctan(a*x)^2/a^3/c^2/(a^2*x^2+1)-1/2*x*arctan(a*x)^3/a^
2/c^2/(a^2*x^2+1)+1/8*arctan(a*x)^4/a^3/c^2
```

## Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.55

$$\int \frac{x^2 \arctan(ax)^3}{(c+a^2cx^2)^2} dx = \frac{3+6ax\arctan(ax)+3(-1+a^2x^2)\arctan(ax)^2-4ax\arctan(ax)^3+(1+a^2x^2)\arctan(ax)^4}{8a^3c^2(1+a^2x^2)}$$

input
```
Integrate[(x^2*ArcTan[a*x]^3)/(c + a^2*c*x^2)^2,x]
```

output
```
(3 + 6*a*x*ArcTan[a*x] + 3*(-1 + a^2*x^2)*ArcTan[a*x]^2 - 4*a*x*ArcTan[a*x
]^3 + (1 + a^2*x^2)*ArcTan[a*x]^4)/(8*a^3*c^2*(1 + a^2*x^2))
```

## Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {5471, 27, 5465, 5427, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \arctan(ax)^3}{(a^2cx^2+c)^2}dx$$

$$\Big\downarrow \text{5471}$$

$$\frac{3\int \frac{x\arctan(ax)^2}{c^2(a^2x^2+1)^2}dx}{2a}+\frac{\arctan(ax)^4}{8a^3c^2}-\frac{x\arctan(ax)^3}{2a^2c^2(a^2x^2+1)}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx}{2ac^2}+\frac{\arctan(ax)^4}{8a^3c^2}-\frac{x\arctan(ax)^3}{2a^2c^2(a^2x^2+1)}$$

$$\Big\downarrow \text{5465}$$

$$\frac{3\left(\frac{\int \frac{\arctan(ax)}{(a^2x^2+1)^2}dx}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)}{2ac^2}+\frac{\arctan(ax)^4}{8a^3c^2}-\frac{x\arctan(ax)^3}{2a^2c^2(a^2x^2+1)}$$

$$\Big\downarrow \text{5427}$$

$$\frac{3\left(\frac{-\frac{1}{2}a\int \frac{x}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)}{2ac^2}+\frac{\arctan(ax)^4}{8a^3c^2}-\frac{x\arctan(ax)^3}{2a^2c^2(a^2x^2+1)}$$

$$\Big\downarrow \text{241}$$

$$\frac{\arctan(ax)^4}{8a^3c^2} - \frac{x\arctan(ax)^3}{2a^2c^2(a^2x^2+1)} + \frac{3\left(\frac{\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}}{a} - \frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)}{2ac^2}$$

input
```
Int[(x^2*ArcTan[a*x]^3)/(c + a^2*c*x^2)^2,x]
```

output
```
-1/2*(x*ArcTan[a*x]^3)/(a^2*c^2*(1 + a^2*x^2)) + ArcTan[a*x]^4/(8*a^3*c^2)
 + (3*(-1/2*ArcTan[a*x]^2/(a^2*(1 + a^2*x^2)) + (1/(4*a*(1 + a^2*x^2)) + (
x*ArcTan[a*x])/(2*(1 + a^2*x^2)) + ArcTan[a*x]^2/(4*a))/a))/(2*a*c^2)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5471
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^2)/((d_) + (e_.)*(x_)^2)
^2, x_Symbol] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(2*b*c^3*d^2*(p + 1)), x]
 + (-Simp[x*((a + b*ArcTan[c*x])^p/(2*c^2*d*(d + e*x^2))), x] + Simp[b*(p/(
2*c))   Int[x*((a + b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

## Maple [A] (verified)

Time = 1.79 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.65

| method | result |
|---|---|
| parallelrisch | $\frac{\arctan(ax)^4 a^2x^2+3\arctan(ax)^2x^2a^2-4x\arctan(ax)^3a-3a^2x^2+\arctan(ax)^4+6\arctan(ax)ax-3\arctan(ax)^2}{8c^2(a^2x^2+1)a^3}$ |
| derivativedivides | $\frac{-\frac{\arctan(ax)^3ax}{2c^2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^4}{2c^2}-\frac{3\left(\frac{\arctan(ax)^4}{4}+\frac{\arctan(ax)^2}{2a^2x^2+2}-\frac{\arctan(ax)ax}{2\left(a^2x^2+1\right)}-\frac{\arctan(ax)^2}{4}-\frac{1}{4\left(a^2x^2+1\right)}\right)}{2c^2}}{a^3}$ |
| default | $\frac{-\frac{\arctan(ax)^3ax}{2c^2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^4}{2c^2}-\frac{3\left(\frac{\arctan(ax)^4}{4}+\frac{\arctan(ax)^2}{2a^2x^2+2}-\frac{\arctan(ax)ax}{2\left(a^2x^2+1\right)}-\frac{\arctan(ax)^2}{4}-\frac{1}{4\left(a^2x^2+1\right)}\right)}{2c^2}}{a^3}$ |
| parts | $-\frac{x\arctan(ax)^3}{2a^2c^2(a^2x^2+1)}+\frac{\arctan(ax)^4}{2a^3c^2}-\frac{3\left(\frac{\arctan(ax)^4}{4a^3}-\frac{-\frac{\arctan(ax)^2}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)ax}{2a^2x^2+2}+\frac{\arctan(ax)^2}{4}+\frac{1}{4a^2x^2+4}}{a^3}\right)}{2c^2}$ |
| risch | $\frac{\ln(iax+1)^4}{128c^2a^3}-\frac{\left(a^2x^2\ln(-iax+1)+\ln(-iax+1)+2iax\right)\ln(iax+1)^3}{32a^3c^2(a^2x^2+1)}+\frac{3\left(a^2x^2\ln(-iax+1)^2+4iax\ln(-iax+1)-2a^2\right.}{64a^3c^2(ax+i)(a}$ |

input
```
int(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*(arctan(a*x)^4*a^2*x^2+3*arctan(a*x)^2*x^2*a^2-4*x*arctan(a*x)^3*a-3*a
^2*x^2+arctan(a*x)^4+6*arctan(a*x)*a*x-3*arctan(a*x)^2)/c^2/(a^2*x^2+1)/a^
3
```

## Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.56

$$\int \frac{x^2 \arctan(ax)^3}{(c+a^2cx^2)^2} dx = -\frac{4\,ax\arctan(ax)^3 - (a^2x^2+1)\arctan(ax)^4 - 6\,ax\arctan(ax) - 3\,(a^2x^2-1)\arctan(ax)^2 - 3}{8\,(a^5c^2x^2 + a^3c^2)}$$

input
```
integrate(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
-1/8*(4*a*x*arctan(a*x)^3 - (a^2*x^2 + 1)*arctan(a*x)^4 - 6*a*x*arctan(a*x
) - 3*(a^2*x^2 - 1)*arctan(a*x)^2 - 3)/(a^5*c^2*x^2 + a^3*c^2)
```

## Sympy [F]

$$\int \frac{x^2 \arctan(ax)^3}{(c+a^2cx^2)^2} dx = \frac{\int \frac{x^2 \operatorname{atan}^3(ax)}{a^4x^4+2a^2x^2+1}\, dx}{c^2}$$

input
```
integrate(x**2*atan(a*x)**3/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(x**2*atan(a*x)**3/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

## Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.61

$$\int \frac{x^2 \arctan(ax)^3}{(c+a^2cx^2)^2} dx = -\frac{1}{2}\left(\frac{x}{a^4c^2x^2+a^2c^2} - \frac{\arctan(ax)}{a^3c^2}\right)\arctan(ax)^3 - \frac{3\left((a^2x^2+1)\arctan(ax)^2+1\right)a\arctan(ax)^2}{4\,(a^6c^2x^2+a^4c^2)} - \frac{1}{8}\left(\frac{\left((a^2x^2+1)\arctan(ax)^4 + 3\,(a^2x^2+1)\arctan(ax)^2 - 3\right)a^2}{a^8c^2x^2+a^6c^2} - \frac{2\left(2\,(a^2x^2+1)\arctan(ax)^3 + 3\,ax\right.}{a^7c^2}\right.$$

input
```
integrate(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
-1/2*(x/(a^4*c^2*x^2 + a^2*c^2) - arctan(a*x)/(a^3*c^2))*arctan(a*x)^3 - 3
/4*((a^2*x^2 + 1)*arctan(a*x)^2 + 1)*a*arctan(a*x)^2/(a^6*c^2*x^2 + a^4*c^
2) - 1/8*(((a^2*x^2 + 1)*arctan(a*x)^4 + 3*(a^2*x^2 + 1)*arctan(a*x)^2 - 3
)*a^2/(a^8*c^2*x^2 + a^6*c^2) - 2*(2*(a^2*x^2 + 1)*arctan(a*x)^3 + 3*a*x +
 3*(a^2*x^2 + 1)*arctan(a*x))*a*arctan(a*x)/(a^7*c^2*x^2 + a^5*c^2))*a
```

## Giac [F]

$$\int \frac{x^2 \arctan(ax)^3}{(c+a^2cx^2)^2}\,dx = \int \frac{x^2 \arctan{(ax)}^3}{(a^2cx^2+c)^2}\,dx$$

input
```
integrate(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate(x^2*arctan(a*x)^3/(a^2*c*x^2 + c)^2, x)
```

## Mupad [B] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.88

$$\int \frac{x^2 \arctan(ax)^3}{(c+a^2cx^2)^2}\,dx = \frac{3}{2\,a^2\,\left(4\,a^3\,c^2\,x^2+4\,a\,c^2\right)} + \mathrm{atan}(a\,x)^2\left(\frac{3}{8\,a^3\,c^2}-\frac{3}{4\,a^5\,c^2\,\left(\frac{1}{a^2}+x^2\right)}\right) + \frac{\mathrm{atan}(a\,x)^4}{8\,a^3\,c^2} + \frac{3\,x\,\mathrm{atan}(a\,x)}{4\,a^4\,c^2\,\left(\frac{1}{a^2}+x^2\right)} - \frac{x\,\mathrm{atan}(a\,x)^3}{2\,a^4\,c^2\,\left(\frac{1}{a^2}+x^2\right)}$$

input
```
int((x^2*atan(a*x)^3)/(c + a^2*c*x^2)^2,x)
```

output
```
3/(2*a^2*(4*a*c^2 + 4*a^3*c^2*x^2)) + atan(a*x)^2*(3/(8*a^3*c^2) - 3/(4*a^
5*c^2*(1/a^2 + x^2))) + atan(a*x)^4/(8*a^3*c^2) + (3*x*atan(a*x))/(4*a^4*c
^2*(1/a^2 + x^2)) - (x*atan(a*x)^3)/(2*a^4*c^2*(1/a^2 + x^2))
```

## Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.64

$$\int \frac{x^2 \arctan(ax)^3}{(c + a^2cx^2)^2} \, dx$$
$$= \frac{atan(ax)^4 a^2x^2 + atan(ax)^4 - 4atan(ax)^3 ax + 3atan(ax)^2 a^2x^2 - 3atan(ax)^2 + 6atan(ax) ax - 3a^2x^2}{8a^3c^2 (a^2x^2 + 1)}$$

input
```
int(x^2*atan(a*x)^3/(a^2*c*x^2+c)^2,x)
```

output
```
(atan(a*x)**4*a**2*x**2 + atan(a*x)**4 - 4*atan(a*x)**3*a*x + 3*atan(a*x)*
*2*a**2*x**2 - 3*atan(a*x)**2 + 6*atan(a*x)*a*x - 3*a**2*x**2)/(8*a**3*c**
2*(a**2*x**2 + 1))
```

# 3.398 $\int \frac{x\arctan(ax)^3}{(c+a^2cx^2)^2}\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 133

$$\int \frac{x\arctan(ax)^3}{(c+a^2cx^2)^2}\,dx = -\frac{3x}{8ac^2(1+a^2x^2)} - \frac{3\arctan(ax)}{8a^2c^2} + \frac{3\arctan(ax)}{4a^2c^2(1+a^2x^2)} + \frac{3x\arctan(ax)^2}{4ac^2(1+a^2x^2)} + \frac{\arctan(ax)^3}{4a^2c^2} - \frac{\arctan(ax)^3}{2a^2c^2(1+a^2x^2)}$$

output
```
-3/8*x/a/c^2/(a^2*x^2+1)-3/8*arctan(a*x)/a^2/c^2+3/4*arctan(a*x)/a^2/c^2/(
a^2*x^2+1)+3/4*x*arctan(a*x)^2/a/c^2/(a^2*x^2+1)+1/4*arctan(a*x)^3/a^2/c^2
-1/2*arctan(a*x)^3/a^2/c^2/(a^2*x^2+1)
```

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.51

$$\int \frac{x\arctan(ax)^3}{(c+a^2cx^2)^2}\,dx = \frac{-3ax+(3-3a^2x^2)\arctan(ax)+6ax\arctan(ax)^2+2(-1+a^2x^2)\arctan(ax)^3}{8a^2c^2(1+a^2x^2)}$$

input
```
Integrate[(x*ArcTan[a*x]^3)/(c + a^2*c*x^2)^2,x]
```

output
```
(-3*a*x + (3 - 3*a^2*x^2)*ArcTan[a*x] + 6*a*x*ArcTan[a*x]^2 + 2*(-1 + a^2*
x^2)*ArcTan[a*x]^3)/(8*a^2*c^2*(1 + a^2*x^2))
```

## Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5465, 27, 5427, 5465, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\arctan(ax)^3}{(a^2cx^2+c)^2}dx$$

$$\Big\downarrow \text{ 5465}$$

$$\frac{3\int \frac{\arctan(ax)^2}{c^2(a^2x^2+1)^2}dx}{2a}-\frac{\arctan(ax)^3}{2a^2c^2(a^2x^2+1)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{3\int \frac{\arctan(ax)^2}{(a^2x^2+1)^2}dx}{2ac^2}-\frac{\arctan(ax)^3}{2a^2c^2(a^2x^2+1)}$$

$$\Big\downarrow \text{ 5427}$$

$$\frac{3\left(-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)}{2ac^2}-\frac{\arctan(ax)^3}{2a^2c^2(a^2x^2+1)}$$

$$\Big\downarrow \text{ 5465}$$

$$\frac{3\left(-a\left(\frac{\int \frac{1}{(a^2x^2+1)^2}dx}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)}{2ac^2}-\frac{\arctan(ax)^3}{2a^2c^2(a^2x^2+1)}$$

$$\Big\downarrow \text{ 215}$$

$$\frac{3\left(-a\left(\frac{\frac{1}{2}\int \frac{1}{a^2x^2+1}dx+\frac{x}{2(a^2x^2+1)}}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)}{2ac^2}-\frac{\arctan(ax)^3}{2a^2c^2(a^2x^2+1)}$$

$$\Big\downarrow \; 216$$

$$\frac{3\left(\frac{x\arctan(ax)^2}{2(a^2x^2+1)}-a\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)+\frac{\arctan(ax)^3}{6a}\right)}{2ac^2}-\frac{\arctan(ax)^3}{2a^2c^2\left(a^2x^2+1\right)}$$

input
```
Int[(x*ArcTan[a*x]^3)/(c + a^2*c*x^2)^2,x]
```

output
```
-1/2*ArcTan[a*x]^3/(a^2*c^2*(1 + a^2*x^2)) + (3*((x*ArcTan[a*x]^2)/(2*(1 +
 a^2*x^2)) + ArcTan[a*x]^3/(6*a) - a*(-1/2*ArcTan[a*x]/(a^2*(1 + a^2*x^2))
 + (x/(2*(1 + a^2*x^2)) + ArcTan[a*x]/(2*a))/(2*a))))/(2*a*c^2)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

## Maple [A] (verified)

Time = 1.59 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.56

| method | result |
|---|---|
| parallelrisch | $\frac{2\arctan(ax)^3a^2x^2-3x^2a^2\arctan(ax)+6a\arctan(ax)^2x-2\arctan(ax)^3-3ax+3\arctan(ax)}{8c^2(a^2x^2+1)a^2}$ |
| derivativedivides | $\frac{-\frac{\arctan(ax)^3}{2c^2\left(a^2x^2+1\right)}+\frac{\frac{3a\arctan(ax)^2x}{2\left(2a^2x^2+2\right)}+\frac{\arctan(ax)^3}{4}+\frac{3\arctan(ax)}{2\left(2a^2x^2+2\right)}-\frac{3ax}{8\left(a^2x^2+1\right)}-\frac{3\arctan(ax)}{8}}{c^2}}{a^2}$ |
| default | $\frac{-\frac{\arctan(ax)^3}{2c^2\left(a^2x^2+1\right)}+\frac{\frac{3a\arctan(ax)^2x}{2\left(2a^2x^2+2\right)}+\frac{\arctan(ax)^3}{4}+\frac{3\arctan(ax)}{2\left(2a^2x^2+2\right)}-\frac{3ax}{8\left(a^2x^2+1\right)}-\frac{3\arctan(ax)}{8}}{c^2}}{a^2}$ |
| parts | $-\frac{\arctan(ax)^3}{2a^2c^2(a^2x^2+1)}+\frac{\frac{3a\arctan(ax)^2x}{2\left(2a^2x^2+2\right)}+\frac{\arctan(ax)^3}{4}+\frac{3\arctan(ax)}{2\left(2a^2x^2+2\right)}-\frac{3ax}{8\left(a^2x^2+1\right)}-\frac{3\arctan(ax)}{8}}{a^2c^2}$ |
| risch | $\frac{i(a^2x^2-1)\ln(iax+1)^3}{32a^2c^2(a^2x^2+1)}-\frac{3i(-\ln(-iax+1)+a^2x^2\ln(-iax+1)-2iax)\ln(iax+1)^2}{32(ax+i)a^2c^2(ax-i)}+\frac{3i\left(-4+a^2x^2\ln(-iax+1)^2-\cdots\right.}{32(\cdots}$ |
| orering | $-\frac{(a^2x^2+1)(9a^6x^6-3a^4x^4+2a^2x^2-1)\arctan(ax)^3}{2a^4x^2(a^2cx^2+c)^2}-\frac{(a^2x^2+1)^2(17a^4x^4-6a^2x^2+2)\left(\frac{\arctan(ax)^3}{(a^2cx^2+c)^2}+\frac{3x\arctan\cdots}{(a^2cx^2+c)^2}\cdots\right.}{4a^4x^2}$ |

input
```
int(x*arctan(a*x)^3/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*(2*arctan(a*x)^3*a^2*x^2-3*x^2*a^2*arctan(a*x)+6*a*arctan(a*x)^2*x-2*a
rctan(a*x)^3-3*a*x+3*arctan(a*x))/c^2/(a^2*x^2+1)/a^2
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.52

$$\int \frac{x\arctan(ax)^3}{(c+a^2cx^2)^2}\,dx = \frac{6\,ax\arctan{(ax)}^2+2\,(a^2x^2-1)\arctan{(ax)}^3-3\,ax-3\,(a^2x^2-1)\arctan{(ax)}}{8\,(a^4c^2x^2+a^2c^2)}$$

input
```
integrate(x*arctan(a*x)^3/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
1/8*(6*a*x*arctan(a*x)^2 + 2*(a^2*x^2 - 1)*arctan(a*x)^3 - 3*a*x - 3*(a^2*
x^2 - 1)*arctan(a*x))/(a^4*c^2*x^2 + a^2*c^2)
```

## Sympy [F]

$$\int \frac{x\arctan(ax)^3}{(c+a^2cx^2)^2}\,dx = \frac{\int \frac{x\,\mathrm{atan}^3\,(ax)}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

input
```
integrate(x*atan(a*x)**3/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(x*atan(a*x)**3/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

## Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.31

$$\int \frac{x\arctan(ax)^3}{(c+a^2cx^2)^2}\,dx = \frac{3\left(\frac{x}{a^2cx^2+c}+\frac{\arctan(ax)}{ac}\right)\arctan{(ax)}^2}{4\,ac} + \frac{\frac{\left(2\,(a^2x^2+1)\arctan(ax)^3-3\,ax-3\,(a^2x^2+1)\arctan(ax)\right)a^2}{a^5cx^2+a^3c}-\frac{6\left((a^2x^2+1)\arctan(ax)^2-1\right)a\arctan(ax)}{a^4cx^2+a^2c}}{8\,ac} - \frac{\arctan{(ax)}^3}{2\,(a^2cx^2+c)a^2c}$$

input
```
integrate(x*arctan(a*x)^3/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
3/4*(x/(a^2*c*x^2 + c) + arctan(a*x)/(a*c))*arctan(a*x)^2/(a*c) + 1/8*((2*
(a^2*x^2 + 1)*arctan(a*x)^3 - 3*a*x - 3*(a^2*x^2 + 1)*arctan(a*x))*a^2/(a^
5*c*x^2 + a^3*c) - 6*((a^2*x^2 + 1)*arctan(a*x)^2 - 1)*a*arctan(a*x)/(a^4*
c*x^2 + a^2*c))/(a*c) - 1/2*arctan(a*x)^3/((a^2*c*x^2 + c)*a^2*c)
```

## Giac [F]

$$\int \frac{x\arctan(ax)^3}{(c+a^2cx^2)^2}\,dx = \int \frac{x\arctan{(ax)}^3}{(a^2cx^2+c)^2}\,dx$$

input
```
integrate(x*arctan(a*x)^3/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate(x*arctan(a*x)^3/(a^2*c*x^2 + c)^2, x)
```

## Mupad [B] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.86

$$\int \frac{x\arctan(ax)^3}{(c+a^2cx^2)^2}\,dx = \mathrm{atan}(a\,x)^3\left(\frac{1}{4\,a^2\,c^2}-\frac{1}{2\,a^4\,c^2\,\left(\frac{1}{a^2}+x^2\right)}\right)-\frac{3\,x}{2\,\left(4\,a^3\,c^2\,x^2+4\,a\,c^2\right)} -\frac{3\,\mathrm{atan}(a\,x)}{8\,a^2\,c^2}+\frac{3\,\mathrm{atan}(a\,x)}{4\,a^4\,c^2\,\left(\frac{1}{a^2}+x^2\right)}+\frac{3\,x\,\mathrm{atan}(a\,x)^2}{4\,a^3\,c^2\,\left(\frac{1}{a^2}+x^2\right)}$$

input
```
int((x*atan(a*x)^3)/(c + a^2*c*x^2)^2,x)
```

output
```
atan(a*x)^3*(1/(4*a^2*c^2) - 1/(2*a^4*c^2*(1/a^2 + x^2))) - (3*x)/(2*(4*a*
c^2 + 4*a^3*c^2*x^2)) - (3*atan(a*x))/(8*a^2*c^2) + (3*atan(a*x))/(4*a^4*c
^2*(1/a^2 + x^2)) + (3*x*atan(a*x)^2)/(4*a^3*c^2*(1/a^2 + x^2))
```

## Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.56

$$\int \frac{x\arctan(ax)^3}{(c+a^2cx^2)^2}\,dx = \frac{2atan(ax)^3\,a^2x^2-2atan(ax)^3+6atan(ax)^2\,ax-3atan(ax)\,a^2x^2+3atan(ax)-3ax}{8a^2c^2\,(a^2x^2+1)}$$

input
```
int(x*atan(a*x)^3/(a^2*c*x^2+c)^2,x)
```

output
```
(2*atan(a*x)**3*a**2*x**2 - 2*atan(a*x)**3 + 6*atan(a*x)**2*a*x - 3*atan(a
*x)*a**2*x**2 + 3*atan(a*x) - 3*a*x)/(8*a**2*c**2*(a**2*x**2 + 1))
```

# 3.399 $\int \frac{\arctan(ax)^3}{(c+a^2cx^2)^2} dx$



## Optimal result

Integrand size = 19, antiderivative size = 129

$$\int \frac{\arctan(ax)^3}{(c+a^2cx^2)^2} dx = -\frac{3}{8ac^2(1+a^2x^2)} - \frac{3x\arctan(ax)}{4c^2(1+a^2x^2)} - \frac{3\arctan(ax)^2}{8ac^2} + \frac{3\arctan(ax)^2}{4ac^2(1+a^2x^2)} + \frac{x\arctan(ax)^3}{2c^2(1+a^2x^2)} + \frac{\arctan(ax)^4}{8ac^2}$$

output
```
-3/8/a/c^2/(a^2*x^2+1)-3/4*x*arctan(a*x)/c^2/(a^2*x^2+1)-3/8*arctan(a*x)^2
/a/c^2+3/4*arctan(a*x)^2/a/c^2/(a^2*x^2+1)+1/2*x*arctan(a*x)^3/c^2/(a^2*x^
2+1)+1/8*arctan(a*x)^4/a/c^2
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.55

$$\int \frac{\arctan(ax)^3}{(c+a^2cx^2)^2} dx = \frac{-3-6ax\arctan(ax)+(3-3a^2x^2)\arctan(ax)^2+4ax\arctan(ax)^3+(1+a^2x^2)\arctan(ax)^4}{8c^2(a+a^3x^2)}$$

input
```
Integrate[ArcTan[a*x]^3/(c + a^2*c*x^2)^2,x]
```

output
```
(-3 - 6*a*x*ArcTan[a*x] + (3 - 3*a^2*x^2)*ArcTan[a*x]^2 + 4*a*x*ArcTan[a*x
]^3 + (1 + a^2*x^2)*ArcTan[a*x]^4)/(8*c^2*(a + a^3*x^2))
```

## Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {5427, 27, 5465, 5427, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3}{(a^2cx^2+c)^2}dx$$

$$\Big\downarrow \text{ 5427}$$

$$-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{c^2(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2c^2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8ac^2}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{3a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx}{2c^2}+\frac{x\arctan(ax)^3}{2c^2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8ac^2}$$

$$\Big\downarrow \text{ 5465}$$

$$-\frac{3a\left(\frac{\int \frac{\arctan(ax)}{(a^2x^2+1)^2}dx}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)}{2c^2}+\frac{x\arctan(ax)^3}{2c^2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8ac^2}$$

$$\Big\downarrow \text{ 5427}$$

$$-\frac{3a\left(\frac{-\frac{1}{2}a\int \frac{x}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)}{2c^2}+\frac{x\arctan(ax)^3}{2c^2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8ac^2}$$

$$\Big\downarrow \text{ 241}$$

$$\frac{x\arctan(ax)^3}{2c^2(a^2x^2+1)}-\frac{3a\left(\frac{\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)}{2c^2}+\frac{\arctan(ax)^4}{8ac^2}$$

input
```
Int[ArcTan[a*x]^3/(c + a^2*c*x^2)^2,x]
```

output
```
(x*ArcTan[a*x]^3)/(2*c^2*(1 + a^2*x^2)) + ArcTan[a*x]^4/(8*a*c^2) - (3*a*(
-1/2*ArcTan[a*x]^2/(a^2*(1 + a^2*x^2)) + (1/(4*a*(1 + a^2*x^2)) + (x*ArcTa
n[a*x])/(2*(1 + a^2*x^2)) + ArcTan[a*x]^2/(4*a))/a))/(2*c^2)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

## Maple [A] (verified)

Time = 1.68 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.68

| method | result |
|---|---|
| parallelrisch | $\frac{\arctan(ax)^4 a^2 x^2 - 3\arctan(ax)^2 x^2 a^2 + 4x\arctan(ax)^3 a + 3a^2x^2 + \arctan(ax)^4 - 6\arctan(ax)ax + 3\arctan(ax)^2}{8c^2(a^2x^2+1)a}$ |
| derivativedivides | $\frac{\frac{\arctan(ax)^3 ax}{2c^2(a^2x^2+1)} + \frac{\arctan(ax)^4}{2c^2} - \frac{3\left(\frac{\arctan(ax)^4}{4} - \frac{\arctan(ax)^2}{2(a^2x^2+1)} + \frac{\arctan(ax)ax}{2a^2x^2+2} + \frac{\arctan(ax)^2}{4} + \frac{1}{4a^2x^2+4}\right)}{2c^2}}{a}$ |
| default | $\frac{\frac{\arctan(ax)^3 ax}{2c^2(a^2x^2+1)} + \frac{\arctan(ax)^4}{2c^2} - \frac{3\left(\frac{\arctan(ax)^4}{4} - \frac{\arctan(ax)^2}{2(a^2x^2+1)} + \frac{\arctan(ax)ax}{2a^2x^2+2} + \frac{\arctan(ax)^2}{4} + \frac{1}{4a^2x^2+4}\right)}{2c^2}}{a}$ |
| parts | $\frac{x\arctan(ax)^3}{2c^2(a^2x^2+1)} + \frac{\arctan(ax)^4}{2a\,c^2} - \frac{3\left(\frac{\arctan(ax)^4}{4a} + \frac{-\frac{\arctan(ax)^2}{2(a^2x^2+1)} + \frac{\arctan(ax)ax}{2a^2x^2+2} + \frac{\arctan(ax)^2}{4} + \frac{1}{4a^2x^2+4}}{a}\right)}{2c^2}$ |
| risch | $\frac{\ln(iax+1)^4}{128a\,c^2} - \frac{(a^2x^2\ln(-iax+1)+\ln(-iax+1)-2iax)\ln(iax+1)^3}{32c^2(a^2x^2+1)a} + \frac{3\left(a^2x^2\ln(-iax+1)^2+2a^2x^2+\ln(-iax+1)^2-\right.}{64c^2(ax+i)(ax}$ |

input
```
int(arctan(a*x)^3/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*(arctan(a*x)^4*a^2*x^2-3*arctan(a*x)^2*x^2*a^2+4*x*arctan(a*x)^3*a+3*a
^2*x^2+arctan(a*x)^4-6*arctan(a*x)*a*x+3*arctan(a*x)^2)/c^2/(a^2*x^2+1)/a
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.57

$$\int \frac{\arctan(ax)^3}{(c+a^2cx^2)^2}\,dx = \frac{4\,ax\arctan(ax)^3 + (a^2x^2+1)\arctan(ax)^4 - 6\,ax\arctan(ax) - 3\,(a^2x^2-1)\arctan(ax)^2 - 3}{8\,(a^3c^2x^2+ac^2)}$$

input
```
integrate(arctan(a*x)^3/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
1/8*(4*a*x*arctan(a*x)^3 + (a^2*x^2 + 1)*arctan(a*x)^4 - 6*a*x*arctan(a*x)
 - 3*(a^2*x^2 - 1)*arctan(a*x)^2 - 3)/(a^3*c^2*x^2 + a*c^2)
```

## Sympy [F]

$$\int \frac{\arctan(ax)^3}{(c+a^2cx^2)^2}\,dx = \frac{\int \frac{\operatorname{atan}^3(ax)}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

input
```
integrate(atan(a*x)**3/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(atan(a*x)**3/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

## Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.65

$$\int \frac{\arctan(ax)^3}{(c+a^2cx^2)^2}\,dx = \frac{1}{2}\left(\frac{x}{a^2c^2x^2+c^2}+\frac{\arctan(ax)}{ac^2}\right)\arctan(ax)^3 - \frac{3\left((a^2x^2+1)\arctan(ax)^2-1\right)a\arctan(ax)^2}{4\left(a^4c^2x^2+a^2c^2\right)} - \frac{1}{8}\left(\frac{\left((a^2x^2+1)\arctan(ax)^4-3(a^2x^2+1)\arctan(ax)^2+3\right)a^2}{a^6c^2x^2+a^4c^2} - \frac{2\left(2(a^2x^2+1)\arctan(ax)^3-3ax\right.}{a^5c^2\ldots}\right.$$

input
```
integrate(arctan(a*x)^3/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
1/2*(x/(a^2*c^2*x^2 + c^2) + arctan(a*x)/(a*c^2))*arctan(a*x)^3 - 3/4*((a^
2*x^2 + 1)*arctan(a*x)^2 - 1)*a*arctan(a*x)^2/(a^4*c^2*x^2 + a^2*c^2) - 1/
8*(((a^2*x^2 + 1)*arctan(a*x)^4 - 3*(a^2*x^2 + 1)*arctan(a*x)^2 + 3)*a^2/(
a^6*c^2*x^2 + a^4*c^2) - 2*(2*(a^2*x^2 + 1)*arctan(a*x)^3 - 3*a*x - 3*(a^2
*x^2 + 1)*arctan(a*x))*a*arctan(a*x)/(a^5*c^2*x^2 + a^3*c^2))*a
```

## Giac [F]

$$\int \frac{\arctan(ax)^3}{(c+a^2cx^2)^2}\,dx = \int \frac{\arctan(ax)^3}{(a^2cx^2+c)^2}\,dx$$

input
```
integrate(arctan(a*x)^3/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^3/(a^2*c*x^2 + c)^2, x)
```

## Mupad [B] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.92

$$\int \frac{\arctan(ax)^3}{(c+a^2cx^2)^2}\,dx = \frac{\operatorname{atan}(a\,x)^4}{8\,a\,c^2} - \operatorname{atan}(a\,x)^2\left(\frac{3}{8\,a\,c^2} - \frac{3}{4\,a^3\,c^2\,\left(\frac{1}{a^2}+x^2\right)}\right) - \frac{3}{2\,a\,\left(4\,a^2\,c^2\,x^2+4\,c^2\right)} - \frac{3\,x\,\operatorname{atan}(a\,x)}{4\,a^2\,c^2\,\left(\frac{1}{a^2}+x^2\right)} + \frac{x\,\operatorname{atan}(a\,x)^3}{2\,a^2\,c^2\,\left(\frac{1}{a^2}+x^2\right)}$$

input
```
int(atan(a*x)^3/(c + a^2*c*x^2)^2,x)
```

output
```
atan(a*x)^4/(8*a*c^2) - atan(a*x)^2*(3/(8*a*c^2) - 3/(4*a^3*c^2*(1/a^2 + x
^2))) - 3/(2*a*(4*c^2 + 4*a^2*c^2*x^2)) - (3*x*atan(a*x))/(4*a^2*c^2*(1/a^
2 + x^2)) + (x*atan(a*x)^3)/(2*a^2*c^2*(1/a^2 + x^2))
```

## Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.67

$$\int \frac{\arctan(ax)^3}{(c+a^2cx^2)^2}\,dx = \frac{atan(ax)^4\,a^2x^2 + atan(ax)^4 + 4atan(ax)^3\,ax - 3atan(ax)^2\,a^2x^2 + 3atan(ax)^2 - 6atan(ax)\,ax + 3a^2x^2}{8a\,c^2\,(a^2x^2+1)}$$

input
```
int(atan(a*x)^3/(a^2*c*x^2+c)^2,x)
```

output
```
(atan(a*x)**4*a**2*x**2 + atan(a*x)**4 + 4*atan(a*x)**3*a*x - 3*atan(a*x)*
*2*a**2*x**2 + 3*atan(a*x)**2 - 6*atan(a*x)*a*x + 3*a**2*x**2)/(8*a*c**2*(
a**2*x**2 + 1))
```

# 3.400 $\int \frac{\arctan(ax)^3}{x(c+a^2cx^2)^2}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 240

$$\begin{aligned}\int \frac{\arctan(ax)^3}{x(c+a^2cx^2)^2}\,dx = {} & \frac{3ax}{8c^2(1+a^2x^2)} + \frac{3\arctan(ax)}{8c^2} - \frac{3\arctan(ax)}{4c^2(1+a^2x^2)} \\ & - \frac{3ax\arctan(ax)^2}{4c^2(1+a^2x^2)} - \frac{\arctan(ax)^3}{4c^2} + \frac{\arctan(ax)^3}{2c^2(1+a^2x^2)} \\ & - \frac{i\arctan(ax)^4}{4c^2} + \frac{\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)}{c^2} \\ & - \frac{3i\arctan(ax)^2\,\mathrm{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{2c^2} \\ & + \frac{3\arctan(ax)\,\mathrm{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)}{2c^2} \\ & + \frac{3i\,\mathrm{PolyLog}\left(4,-1+\frac{2}{1-iax}\right)}{4c^2}\end{aligned}$$

output

```
3/8*a*x/c^2/(a^2*x^2+1)+3/8*arctan(a*x)/c^2-3/4*arctan(a*x)/c^2/(a^2*x^2+1
)-3/4*a*x*arctan(a*x)^2/c^2/(a^2*x^2+1)-1/4*arctan(a*x)^3/c^2+1/2*arctan(a
*x)^3/c^2/(a^2*x^2+1)-1/4*I*arctan(a*x)^4/c^2+arctan(a*x)^3*ln(2-2/(1-I*a*
x))/c^2-3/2*I*arctan(a*x)^2*polylog(2,-1+2/(1-I*a*x))/c^2+3/2*arctan(a*x)*
polylog(3,-1+2/(1-I*a*x))/c^2+3/4*I*polylog(4,-1+2/(1-I*a*x))/c^2
```

## Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.65

$$\int \frac{\arctan(ax)^3}{x\left(c+a^2cx^2\right)^2}\,dx$$
$$= \frac{-i\pi^4+16i\arctan(ax)^4-24\arctan(ax)\cos(2\arctan(ax))+16\arctan(ax)^3\cos(2\arctan(ax))+64\,\mathrm{arct}}{}$$

input
```
Integrate[ArcTan[a*x]^3/(x*(c + a^2*c*x^2)^2),x]
```

output
```
((-I)*Pi^4 + (16*I)*ArcTan[a*x]^4 - 24*ArcTan[a*x]*Cos[2*ArcTan[a*x]] + 16
*ArcTan[a*x]^3*Cos[2*ArcTan[a*x]] + 64*ArcTan[a*x]^3*Log[1 - E^((-2*I)*Arc
Tan[a*x])] + (96*I)*ArcTan[a*x]^2*PolyLog[2, E^((-2*I)*ArcTan[a*x])] + 96*
ArcTan[a*x]*PolyLog[3, E^((-2*I)*ArcTan[a*x])] - (48*I)*PolyLog[4, E^((-2*
I)*ArcTan[a*x])] + 12*Sin[2*ArcTan[a*x]] - 24*ArcTan[a*x]^2*Sin[2*ArcTan[a
*x]])/(64*c^2)
```

## Rubi [A] (verified)

Time = 1.70 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.16, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {5501, 27, 5459, 5403, 5465, 5427, 5465, 215, 216, 5527, 5531, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3}{x\left(a^2cx^2+c\right)^2}\,dx$$
$$\Big\downarrow \text{ 5501}$$
$$\frac{\int \frac{\arctan(ax)^3}{cx(a^2x^2+1)}dx}{c}-a^2\int \frac{x\arctan(ax)^3}{c^2\left(a^2x^2+1\right)^2}dx$$
$$\Big\downarrow \text{ 27}$$
$$\frac{\int \frac{\arctan(ax)^3}{x(a^2x^2+1)}dx}{c^2}-\frac{a^2\int \frac{x\arctan(ax)^3}{(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \text{5459}$$

$$-\frac{a^2\int \frac{x\arctan(ax)^3}{(a^2x^2+1)^2}dx}{c^2}+\frac{i\int \frac{\arctan(ax)^3}{x(ax+i)}dx-\frac{1}{4}i\arctan(ax)^4}{c^2}$$

$$\Big\downarrow \text{5403}$$

$$-\frac{a^2\int \frac{x\arctan(ax)^3}{(a^2x^2+1)^2}dx}{c^2}+\frac{i\left(3ia\int \frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{4}i\arctan(ax)^4}{c^2}$$

$$\Big\downarrow \text{5465}$$

$$-\frac{a^2\left(\frac{3\int \frac{\arctan(ax)^2}{(a^2x^2+1)^2}dx}{2a}-\frac{\arctan(ax)^3}{2a^2(a^2x^2+1)}\right)}{c^2}+\frac{i\left(3ia\int \frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{4}i\arctan(ax)^4}{c^2}$$

$$\Big\downarrow \text{5427}$$

$$-\frac{a^2\left(\frac{3\left(-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)}{2a}-\frac{\arctan(ax)^3}{2a^2(a^2x^2+1)}\right)}{c^2}+\frac{i\left(3ia\int \frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{4}i\arctan(ax)^4}{c^2}$$

$$\Big\downarrow \text{5465}$$

$$-\frac{a^2\left(\frac{3\left(-a\left(\frac{\int \frac{1}{(a^2x^2+1)^2}dx}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)}{2a}-\frac{\arctan(ax)^3}{2a^2(a^2x^2+1)}\right)}{c^2}+\frac{i\left(3ia\int \frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{4}i\arctan(ax)^4}{c^2}$$

$$\Big\downarrow \text{215}$$

$$-\frac{a^2\left(\frac{3\left(-a\left(\frac{\frac{1}{2}\int\frac{1}{a^2x^2+1}dx+\frac{x}{2\left(a^2x^2+1\right)}}{2a}-\frac{\arctan(ax)}{2a^2\left(a^2x^2+1\right)}\right)+\frac{x\arctan(ax)^2}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^3}{6a}\right)}{2a}-\frac{\arctan(ax)^3}{2a^2(a^2x^2+1)}\right)}{c^2}+\frac{i\left(3ia\int\frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{4}i\arctan(ax)^4}{c^2}$$

$$\Big\downarrow \; 216$$

$$-\frac{a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{2\left(a^2x^2+1\right)}-a\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\arctan(ax)}{2a^2\left(a^2x^2+1\right)}\right)+\frac{\arctan(ax)^3}{6a}\right)}{2a}-\frac{\arctan(ax)^3}{2a^2(a^2x^2+1)}\right)}{c^2}+\frac{i\left(3ia\int\frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{4}i\arctan(ax)^4}{c^2}$$

$$\Big\downarrow \; 5527$$

$$-\frac{a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{2\left(a^2x^2+1\right)}-a\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\arctan(ax)}{2a^2\left(a^2x^2+1\right)}\right)+\frac{\arctan(ax)^3}{6a}\right)}{2a}-\frac{\arctan(ax)^3}{2a^2(a^2x^2+1)}\right)}{c^2}+\frac{i\left(3ia\left(\frac{i\arctan(ax)^2\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-i\int\frac{\arctan(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{a^2x^2+1}dx\right)-i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{4}}{c^2}$$

$$\Big\downarrow \; 5531$$

$$-\frac{a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{2\left(a^2x^2+1\right)}-a\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\arctan(ax)}{2a^2\left(a^2x^2+1\right)}\right)+\frac{\arctan(ax)^3}{6a}\right)}{2a}-\frac{\arctan(ax)^3}{2a^2(a^2x^2+1)}\right)}{c^2}+\frac{i\left(3ia\left(\frac{i\arctan(ax)^2\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-i\left(\frac{1}{2}i\int\frac{\operatorname{PolyLog}\left(3,\frac{2}{1-iax}-1\right)}{a^2x^2+1}dx-\frac{i\arctan(ax)\operatorname{PolyLog}\left(3,\frac{2}{1-iax}-1\right)}{2a}\right)\right)-i\arctan}{c^2}$$

$$\Big\downarrow \text{7164}$$

$$-\frac{a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{2\left(a^2x^2+1\right)}-a\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\arctan(ax)}{2a^2\left(a^2x^2+1\right)}\right)+\frac{\arctan(ax)^3}{6a}\right)}{2a}-\frac{\arctan(ax)^3}{2a^2(a^2x^2+1)}\right)}{c^2}+$$

$$\frac{i\left(3ia\left(\frac{i\arctan(ax)^2\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-i\left(\frac{\operatorname{PolyLog}\left(4,\frac{2}{1-iax}-1\right)}{4a}-\frac{i\arctan(ax)\operatorname{PolyLog}\left(3,\frac{2}{1-iax}-1\right)}{2a}\right)\right)-i\arctan(ax)^3\,\mathrm{lo}}{c^2}$$

input
```
Int[ArcTan[a*x]^3/(x*(c + a^2*c*x^2)^2),x]
```

output
```
-((a^2*(-1/2*ArcTan[a*x]^3/(a^2*(1 + a^2*x^2)) + (3*((x*ArcTan[a*x]^2)/(2*
(1 + a^2*x^2)) + ArcTan[a*x]^3/(6*a) - a*(-1/2*ArcTan[a*x]/(a^2*(1 + a^2*x
^2)) + (x/(2*(1 + a^2*x^2)) + ArcTan[a*x]/(2*a))/(2*a))))/(2*a)))/c^2) + (
(-1/4*I)*ArcTan[a*x]^4 + I*((-I)*ArcTan[a*x]^3*Log[2 - 2/(1 - I*a*x)] + (3
*I)*a*(((I/2)*ArcTan[a*x]^2*PolyLog[2, -1 + 2/(1 - I*a*x)])/a - I*(((-1/2*
I)*ArcTan[a*x]*PolyLog[3, -1 + 2/(1 - I*a*x)])/a + PolyLog[4, -1 + 2/(1 -
I*a*x)]/(4*a)))))/c^2
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

rule 5527
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

rule 5531
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/
(2*c*d)), x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k
+ 1, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0]
&& EqQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 7.50 (sec) , antiderivative size = 1787, normalized size of antiderivative = 7.45

Expression too large to display

input
```
int(arctan(a*x)^3/x/(a^2*c*x^2+c)^2,x)
```

output
```
-1/2/c^2*arctan(a*x)^3*ln(a^2*x^2+1)+1/2*arctan(a*x)^3/c^2/(a^2*x^2+1)+1/c
^2*arctan(a*x)^3*ln(a*x)-3/2/c^2*(-2/3*arctan(a*x)^3*ln((1+I*a*x)/(a^2*x^2
+1)^(1/2))-4*I*polylog(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-4*I*polylog(4,(1+I*
a*x)/(a^2*x^2+1)^(1/2))-1/8*arctan(a*x)*(a*x+I)/(a*x-I)-I*(a*x-I)/(16*a*x+
16*I)+1/6*I*arctan(a*x)^4-1/8*arctan(a*x)*(a*x-I)/(a*x+I)+I*arctan(a*x)^2*
(a*x-I)/(8*a*x+8*I)+2/3*arctan(a*x)^3*ln((1+I*a*x)^2/(a^2*x^2+1)-1)-2/3*ar
ctan(a*x)^3*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+I*(a*x+I)/(16*a*x-16*I)-4*ar
ctan(a*x)*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*I*arctan(a*x)^2*polylo
g(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-2/3*arctan(a*x)^3*ln(1-(1+I*a*x)/(a^2*x^2
+1)^(1/2))-I*arctan(a*x)^2*(a*x+I)/(8*a*x-8*I)-4*arctan(a*x)*polylog(3,(1+
I*a*x)/(a^2*x^2+1)^(1/2))+2*I*arctan(a*x)^2*polylog(2,-(1+I*a*x)/(a^2*x^2+
1)^(1/2))-1/6*(I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^
2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2-2*I*Pi*csgn(I/((1+I*a*x)^2/(a^2*
x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))
^2+2*I*Pi*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^3-
I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))^3+I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1
)+1))^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)-I*Pi*csgn(I*(1+I*a*x)^2/(a^2
*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3-I*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)
^(1/2))^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))+I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^
2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2...
```

## Fricas [F]

$$\int \frac{\arctan(ax)^3}{x\,(c+a^2cx^2)^2}\,dx=\int \frac{\arctan\left(ax\right)^3}{\left(a^2cx^2+c\right)^2x}\,dx$$

input
```
integrate(arctan(a*x)^3/x/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
integral(arctan(a*x)^3/(a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^2*x), x)
```

## Sympy [F]

$$\int \frac{\arctan(ax)^3}{x\left(c+a^2cx^2\right)^2}\,dx = \frac{\int \frac{\operatorname{atan}^3(ax)}{a^4x^5+2a^2x^3+x}\,dx}{c^2}$$

input
```
integrate(atan(a*x)**3/x/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(atan(a*x)**3/(a**4*x**5 + 2*a**2*x**3 + x), x)/c**2
```

## Maxima [F]

$$\int \frac{\arctan(ax)^3}{x\left(c+a^2cx^2\right)^2}\,dx = \int \frac{\arctan\left(ax\right)^3}{\left(a^2cx^2+c\right)^2x}\,dx$$

input
```
integrate(arctan(a*x)^3/x/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)^3/((a^2*c*x^2 + c)^2*x), x)
```

## Giac [F]

$$\int \frac{\arctan(ax)^3}{x\left(c+a^2cx^2\right)^2}\,dx = \int \frac{\arctan\left(ax\right)^3}{\left(a^2cx^2+c\right)^2x}\,dx$$

input
```
integrate(arctan(a*x)^3/x/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^3/((a^2*c*x^2 + c)^2*x), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^3}{x(c+a^2cx^2)^2}\,dx = \int \frac{\mathrm{atan}(a\,x)^3}{x\,(c\,a^2\,x^2+c)^2}\,dx$$

input
```
int(atan(a*x)^3/(x*(c + a^2*c*x^2)^2),x)
```

output
```
int(atan(a*x)^3/(x*(c + a^2*c*x^2)^2), x)
```

## Reduce [F]

$$\int \frac{\arctan(ax)^3}{x(c+a^2cx^2)^2}\,dx = \frac{\int \frac{atan(ax)^3}{a^4x^5+2a^2x^3+x}dx}{c^2}$$

input
```
int(atan(a*x)^3/x/(a^2*c*x^2+c)^2,x)
```

output
```
int(atan(a*x)**3/(a**4*x**5 + 2*a**2*x**3 + x),x)/c**2
```

# 3.401 $\int \frac{\arctan(ax)^3}{x^2(c+a^2cx^2)^2}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 234

$$\begin{aligned}\int \frac{\arctan(ax)^3}{x^2(c+a^2cx^2)^2}\,dx &= \frac{3a}{8c^2(1+a^2x^2)}+\frac{3a^2x\arctan(ax)}{4c^2(1+a^2x^2)}\\&\quad+\frac{3a\arctan(ax)^2}{8c^2}-\frac{3a\arctan(ax)^2}{4c^2(1+a^2x^2)}\\&\quad-\frac{ia\arctan(ax)^3}{c^2}-\frac{\arctan(ax)^3}{c^2x}-\frac{a^2x\arctan(ax)^3}{2c^2(1+a^2x^2)}\\&\quad-\frac{3a\arctan(ax)^4}{8c^2}+\frac{3a\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{c^2}\\&\quad-\frac{3ia\arctan(ax)\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{c^2}\\&\quad+\frac{3a\operatorname{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)}{2c^2}\end{aligned}$$

output
```
3/8*a/c^2/(a^2*x^2+1)+3/4*a^2*x*arctan(a*x)/c^2/(a^2*x^2+1)+3/8*a*arctan(a
*x)^2/c^2-3/4*a*arctan(a*x)^2/c^2/(a^2*x^2+1)-I*a*arctan(a*x)^3/c^2-arctan
(a*x)^3/c^2/x-1/2*a^2*x*arctan(a*x)^3/c^2/(a^2*x^2+1)-3/8*a*arctan(a*x)^4/
c^2+3*a*arctan(a*x)^2*ln(2-2/(1-I*a*x))/c^2-3*I*a*arctan(a*x)*polylog(2,-1
+2/(1-I*a*x))/c^2+3/2*a*polylog(3,-1+2/(1-I*a*x))/c^2
```

## Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.67

$$\int \frac{\arctan(ax)^3}{x^2 (c+a^2cx^2)^2} dx$$
$$= \frac{a\left(-2i\pi^3 + 16i\arctan(ax)^3 - \frac{16\arctan(ax)^3}{ax} - 6\arctan(ax)^4 + 3\cos(2\arctan(ax)) - 6\arctan(ax)^2\cos(2\text{a}\right.}{}$$

input
```
Integrate[ArcTan[a*x]^3/(x^2*(c + a^2*c*x^2)^2),x]
```

output
```
(a*((-2*I)*Pi^3 + (16*I)*ArcTan[a*x]^3 - (16*ArcTan[a*x]^3)/(a*x) - 6*ArcT
an[a*x]^4 + 3*Cos[2*ArcTan[a*x]] - 6*ArcTan[a*x]^2*Cos[2*ArcTan[a*x]] + 48
*ArcTan[a*x]^2*Log[1 - E^((-2*I)*ArcTan[a*x])] + (48*I)*ArcTan[a*x]*PolyLo
g[2, E^((-2*I)*ArcTan[a*x])] + 24*PolyLog[3, E^((-2*I)*ArcTan[a*x])] + 6*A
rcTan[a*x]*Sin[2*ArcTan[a*x]] - 4*ArcTan[a*x]^3*Sin[2*ArcTan[a*x]]))/(16*c
^2)
```

## Rubi [A] (verified)

Time = 1.95 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.12, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {5501, 27, 5427, 5453, 5361, 5419, 5459, 5403, 5465, 5427, 241, 5527, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3}{x^2 (a^2cx^2+c)^2} dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{\arctan(ax)^3}{cx^2(a^2x^2+1)}dx}{c} - a^2\int \frac{\arctan(ax)^3}{c^2 (a^2x^2+1)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)}dx}{c^2} - \frac{a^2\int \frac{\arctan(ax)^3}{(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \text{5427}$$

$$\frac{\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)}dx}{c^2}-\frac{a^2\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)}{c^2}$$

$$\Big\downarrow \text{5453}$$

$$\frac{\int \frac{\arctan(ax)^3}{x^2}dx-a^2\int \frac{\arctan(ax)^3}{a^2x^2+1}dx}{c^2}-\frac{a^2\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)}{c^2}$$

$$\Big\downarrow \text{5361}$$

$$\frac{3a\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx-a^2\int \frac{\arctan(ax)^3}{a^2x^2+1}dx-\frac{\arctan(ax)^3}{x}}{c^2}-\frac{a^2\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)}{c^2}$$

$$\Big\downarrow \text{5419}$$

$$\frac{3a\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx-\frac{1}{4}a\arctan(ax)^4-\frac{\arctan(ax)^3}{x}}{c^2}-\frac{a^2\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)}{c^2}$$

$$\Big\downarrow \text{5459}$$

$$-\frac{a^2\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)}{c^2}+\frac{3a\left(i\int \frac{\arctan(ax)^2}{x(ax+i)}dx-\frac{1}{3}i\arctan(ax)^3\right)-\frac{1}{4}a\arctan(ax)^4-\frac{\arctan(ax)^3}{x}}{c^2}$$

$$\Big\downarrow \text{5403}$$

$$-\frac{a^2\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)}{c^2}+\frac{3a\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3\right)-\frac{1}{4}a\arctan(ax)^4-\text{a}}{c^2}$$

$$\Big\downarrow \text{5465}$$

$$-\frac{a^2\left(-\frac{3}{2}a\left(\frac{\int \frac{\arctan(ax)}{(a^2x^2+1)^2}dx}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)}{c^2}+$$

$$\frac{3a\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3\right)-\frac{1}{4}a\arctan(ax)^4-\text{a}}{c^2}$$

$\downarrow$ 5427

$$-\frac{a^2\left(-\frac{3}{2}a\left(\frac{-\frac{1}{2}a\int \frac{x}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)}{c^2}+$$

$$\frac{3a\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3\right)-\frac{1}{4}a\arctan(ax)^4-\text{a}}{c^2}$$

$\downarrow$ 241

$$-\frac{a^2\left(\frac{x\arctan(ax)^3}{2(a^2x^2+1)}-\frac{3}{2}a\left(\frac{\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)+\frac{\arctan(ax)^4}{8a}\right)}{c^2}+$$

$$\frac{3a\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3\right)-\frac{1}{4}a\arctan(ax)^4-\text{a}}{c^2}$$

$\downarrow$ 5527

$$-\frac{a^2\left(\frac{x\arctan(ax)^3}{2(a^2x^2+1)}-\frac{3}{2}a\left(\frac{\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)+\frac{\arctan(ax)^4}{8a}\right)}{c^2}+$$

$$\frac{3a\left(i\left(2ia\left(\frac{i\arctan(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-\frac{1}{2}i\int \frac{\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{a^2x^2+1}dx\right)-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\,\text{arc}}{c^2}$$

$\downarrow$ 7164

$$-\frac{a^2\left(\frac{x\arctan(ax)^3}{2(a^2x^2+1)}-\frac{3}{2}a\left(\frac{\frac{x\arctan(ax)}{2\left(a^2x^2+1\right)}+\frac{1}{4a\left(a^2x^2+1\right)}+\frac{\arctan(ax)^2}{4a}}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)+\frac{\arctan(ax)^4}{8a}\right)}{c^2}+$$

$$\frac{3a\left(i\left(2ia\left(\frac{i\arctan(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-\frac{\operatorname{PolyLog}\left(3,\frac{2}{1-iax}-1\right)}{4a}\right)-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax\right.}{c^2}$$

input
```
Int[ArcTan[a*x]^3/(x^2*(c + a^2*c*x^2)^2),x]
```

output
```
-((a^2*((x*ArcTan[a*x]^3)/(2*(1 + a^2*x^2)) + ArcTan[a*x]^4/(8*a) - (3*a*(
-1/2*ArcTan[a*x]^2/(a^2*(1 + a^2*x^2)) + (1/(4*a*(1 + a^2*x^2)) + (x*ArcTa
n[a*x])/(2*(1 + a^2*x^2)) + ArcTan[a*x]^2/(4*a))/a))/2))/c^2) + (-(ArcTan[
a*x]^3/x) - (a*ArcTan[a*x]^4)/4 + 3*a*((-1/3*I)*ArcTan[a*x]^3 + I*((-I)*Ar
cTan[a*x]^2*Log[2 - 2/(1 - I*a*x)] + (2*I)*a*(((I/2)*ArcTan[a*x]*PolyLog[2
, -1 + 2/(1 - I*a*x)])/a - PolyLog[3, -1 + 2/(1 - I*a*x)]/(4*a)))))/c^2
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

rule 5527
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 7.39 (sec) , antiderivative size = 1731, normalized size of antiderivative = 7.40

Expression too large to display

input
```
int(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^2,x)
```

output
```
a*(-1/2/c^2*arctan(a*x)^3*a*x/(a^2*x^2+1)-3/2/c^2*arctan(a*x)^4-1/c^2*arct
an(a*x)^3/a/x-3/2/c^2*(-3/4*arctan(a*x)^4+arctan(a*x)^2*ln(a^2*x^2+1)+1/2*
arctan(a*x)^2/(a^2*x^2+1)-2*arctan(a*x)^2*ln(a*x)-2*arctan(a*x)^2*ln((1+I*
a*x)/(a^2*x^2+1)^(1/2))-I*arctan(a*x)*(a*x-I)/(8*a*x+8*I)+I*arctan(a*x)*(a
*x+I)/(8*a*x-8*I)+1/16*(a*x+I)/(a*x-I)+4*I*arctan(a*x)*polylog(2,(1+I*a*x)
/(a^2*x^2+1)^(1/2))+1/16*(a*x-I)/(a*x+I)+2*arctan(a*x)^2*ln((1+I*a*x)^2/(a
^2*x^2+1)-1)-2*arctan(a*x)^2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+4*I*arctan(
a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-4*polylog(3,-(1+I*a*x)/(a^2*x
^2+1)^(1/2))-2*arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))+2/3*I*arcta
n(a*x)^3-4*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))-1/4*(2*I*Pi*csgn(I/((1+I
*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2
*x^2+1)+1)^2)^2-2*I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a
*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)
+1)^2)+2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))^2*csgn(I*((1+I*a*x)^2/(a
^2*x^2+1)+1)^2)-4*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*
x)^2/(a^2*x^2+1)+1)^2)^2+2*I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+
I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2-2*I*Pi*csgn(I*(1+I*a
*x)^2/(a^2*x^2+1))^3+2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^3-2*I*Pi
*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))-4*I
*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)...
```

## Fricas [F]

$$\int \frac{\arctan(ax)^3}{x^2 (c + a^2 c x^2)^2} \, dx = \int \frac{\arctan(ax)^3}{(a^2 c x^2 + c)^2 x^2} \, dx$$

input
```
integrate(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
integral(arctan(a*x)^3/(a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2), x)
```

## Sympy [F]

$$\int \frac{\arctan(ax)^3}{x^2\left(c+a^2cx^2\right)^2}\,dx = \frac{\int \frac{\operatorname{atan}^3(ax)}{a^4x^6+2a^2x^4+x^2}\,dx}{c^2}$$

input
```
integrate(atan(a*x)**3/x**2/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(atan(a*x)**3/(a**4*x**6 + 2*a**2*x**4 + x**2), x)/c**2
```

## Maxima [F]

$$\int \frac{\arctan(ax)^3}{x^2\left(c+a^2cx^2\right)^2}\,dx = \int \frac{\arctan\left(ax\right)^3}{\left(a^2cx^2+c\right)^2x^2}\,dx$$

input
```
integrate(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
-1/2048*(240*(a^3*x^3 + a*x)*arctan(a*x)^4 - 9*(a^3*x^3 + a*x)*log(a^2*x^2
 + 1)^4 + 128*(3*a^2*x^2 + 2)*arctan(a*x)^3 - 24*(3*(a^3*x^3 + a*x)*arctan
(a*x)^2 + 4*(3*a^2*x^2 + 2)*arctan(a*x))*log(a^2*x^2 + 1)^2 - 4*(a^2*c^2*x
^3 + c^2*x)*(72*a^5*(a^2/(a^8*c^2*x^2 + a^6*c^2) + log(a^2*x^2 + 1)/(a^6*c
^2*x^2 + a^4*c^2)) - 18432*a^5*integrate(1/256*x^5*arctan(a*x)^2*log(a^2*x
^2 + 1)/(a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2), x) - 4608*a^5*integrate(1
/256*x^5*log(a^2*x^2 + 1)^3/(a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2), x) +
36864*a^4*integrate(1/256*x^4*arctan(a*x)^3/(a^4*c^2*x^6 + 2*a^2*c^2*x^4 +
 c^2*x^2), x) + 9216*a^4*integrate(1/256*x^4*arctan(a*x)*log(a^2*x^2 + 1)^
2/(a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2), x) - 73728*a^4*integrate(1/256*
x^4*arctan(a*x)*log(a^2*x^2 + 1)/(a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2),
x) + 9*a^3*log(a^2*x^2 + 1)^3/(a^4*c^2*x^2 + a^2*c^2) + 27*(2*a^4*(a^2/(a^
10*c^2*x^2 + a^8*c^2) + log(a^2*x^2 + 1)/(a^8*c^2*x^2 + a^6*c^2)) + a^2*lo
g(a^2*x^2 + 1)^2/(a^6*c^2*x^2 + a^4*c^2))*a^3 - 18432*a^3*integrate(1/256*
x^3*arctan(a*x)^2*log(a^2*x^2 + 1)/(a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2)
, x) + 73728*a^3*integrate(1/256*x^3*arctan(a*x)^2/(a^4*c^2*x^6 + 2*a^2*c^
2*x^4 + c^2*x^2), x) + 36*a^3*log(a^2*x^2 + 1)^2/(a^4*c^2*x^2 + a^2*c^2) +
 36864*a^2*integrate(1/256*x^2*arctan(a*x)^3/(a^4*c^2*x^6 + 2*a^2*c^2*x^4
+ c^2*x^2), x) + 9216*a^2*integrate(1/256*x^2*arctan(a*x)*log(a^2*x^2 + 1)
^2/(a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2), x) - 49152*a^2*integrate(1/...
```

## Giac [F]

$$\int \frac{\arctan(ax)^3}{x^2 (c + a^2 c x^2)^2} \, dx = \int \frac{\arctan(ax)^3}{(a^2 c x^2 + c)^2 x^2} \, dx$$

input
```
integrate(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^3/((a^2*c*x^2 + c)^2*x^2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^3}{x^2 (c+a^2cx^2)^2} \, dx = \int \frac{\mathrm{atan}(a\,x)^3}{x^2\,(c\,a^2\,x^2+c)^2} \, dx$$

input
```
int(atan(a*x)^3/(x^2*(c + a^2*c*x^2)^2),x)
```

output
```
int(atan(a*x)^3/(x^2*(c + a^2*c*x^2)^2), x)
```

## Reduce [F]

$$\int \frac{\arctan(ax)^3}{x^2 (c+a^2cx^2)^2} \, dx = \frac{-3atan(ax)^4\,a^3x^3 - 3atan(ax)^4\,ax - 12atan(ax)^3\,a^2x^2 - 8atan(ax)^3 + 9atan(ax)^2\,a^3x^3 - 9atan(ax)^2\,a}{8c^2x\,(a^2x^2+1)}$$

input
```
int(atan(a*x)^3/x^2/(a^2*c*x^2+c)^2,x)
```

output
```
( - 3*atan(a*x)**4*a**3*x**3 - 3*atan(a*x)**4*a*x - 12*atan(a*x)**3*a**2*x
**2 - 8*atan(a*x)**3 + 9*atan(a*x)**2*a**3*x**3 - 9*atan(a*x)**2*a*x + 18*
atan(a*x)*a**2*x**2 + 24*int(atan(a*x)**2/(a**4*x**5 + 2*a**2*x**3 + x),x)
*a**3*x**3 + 24*int(atan(a*x)**2/(a**4*x**5 + 2*a**2*x**3 + x),x)*a*x - 9*
a**3*x**3)/(8*c**2*x*(a**2*x**2 + 1))
```

## 3.402 $\int \frac{\arctan(ax)^3}{x^3(c+a^2cx^2)^2}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 374

$$\begin{aligned}\int \frac{\arctan(ax)^3}{x^3(c+a^2cx^2)^2}\,dx &= -\frac{3a^3x}{8c^2(1+a^2x^2)} - \frac{3a^2\arctan(ax)}{8c^2} + \frac{3a^2\arctan(ax)}{4c^2(1+a^2x^2)} \\ &\quad - \frac{3ia^2\arctan(ax)^2}{2c^2} - \frac{3a\arctan(ax)^2}{2c^2x} + \frac{3a^3x\arctan(ax)^2}{4c^2(1+a^2x^2)} \\ &\quad - \frac{a^2\arctan(ax)^3}{4c^2} - \frac{\arctan(ax)^3}{2c^2x^2} - \frac{a^2\arctan(ax)^3}{2c^2(1+a^2x^2)} \\ &\quad + \frac{ia^2\arctan(ax)^4}{2c^2} + \frac{3a^2\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{c^2} \\ &\quad - \frac{2a^2\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)}{c^2} \\ &\quad - \frac{3ia^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{2c^2} \\ &\quad + \frac{3ia^2\arctan(ax)^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{c^2} \\ &\quad - \frac{3a^2\arctan(ax)\operatorname{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)}{c^2} \\ &\quad - \frac{3ia^2\operatorname{PolyLog}\left(4,-1+\frac{2}{1-iax}\right)}{2c^2}\end{aligned}$$

output
```
-3/8*a^3*x/c^2/(a^2*x^2+1)-3/8*a^2*arctan(a*x)/c^2+3/4*a^2*arctan(a*x)/c^2
/(a^2*x^2+1)-3/2*I*a^2*polylog(4,-1+2/(1-I*a*x))/c^2-3/2*a*arctan(a*x)^2/c
^2/x+3/4*a^3*x*arctan(a*x)^2/c^2/(a^2*x^2+1)-1/4*a^2*arctan(a*x)^3/c^2-1/2
*arctan(a*x)^3/c^2/x^2-1/2*a^2*arctan(a*x)^3/c^2/(a^2*x^2+1)+1/2*I*a^2*arc
tan(a*x)^4/c^2+3*a^2*arctan(a*x)*ln(2-2/(1-I*a*x))/c^2-2*a^2*arctan(a*x)^3
*ln(2-2/(1-I*a*x))/c^2-3/2*I*a^2*polylog(2,-1+2/(1-I*a*x))/c^2+3*I*a^2*arc
tan(a*x)^2*polylog(2,-1+2/(1-I*a*x))/c^2-3*a^2*arctan(a*x)*polylog(3,-1+2/
(1-I*a*x))/c^2-3/2*I*a^2*arctan(a*x)^2/c^2
```

## Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.65

$$\int \frac{\arctan(ax)^3}{x^3\left(c+a^2cx^2\right)^2}\,dx = \frac{a^2\left(i\pi^4-48i\arctan(ax)^2-\frac{48\arctan(ax)^2}{ax}-\frac{16(1+a^2x^2)\arctan(ax)^3}{a^2x^2}-16i\arctan(ax)^4+12\arctan(ax)\cos(2a\right.}{}$$

input
```
Integrate[ArcTan[a*x]^3/(x^3*(c + a^2*c*x^2)^2),x]
```

output
```
(a^2*(I*Pi^4 - (48*I)*ArcTan[a*x]^2 - (48*ArcTan[a*x]^2)/(a*x) - (16*(1 +
a^2*x^2)*ArcTan[a*x]^3)/(a^2*x^2) - (16*I)*ArcTan[a*x]^4 + 12*ArcTan[a*x]*
Cos[2*ArcTan[a*x]] - 8*ArcTan[a*x]^3*Cos[2*ArcTan[a*x]] - 64*ArcTan[a*x]^3
*Log[1 - E^((-2*I)*ArcTan[a*x])] + 96*ArcTan[a*x]*Log[1 - E^((2*I)*ArcTan[
a*x])] - (96*I)*ArcTan[a*x]^2*PolyLog[2, E^((-2*I)*ArcTan[a*x])] - (48*I)*
PolyLog[2, E^((2*I)*ArcTan[a*x])] - 96*ArcTan[a*x]*PolyLog[3, E^((-2*I)*Ar
cTan[a*x])] + (48*I)*PolyLog[4, E^((-2*I)*ArcTan[a*x])] - 6*Sin[2*ArcTan[a
*x]] + 12*ArcTan[a*x]^2*Sin[2*ArcTan[a*x]]))/(32*c^2)
```

## Rubi [A] (verified)

Time = 4.53 (sec) , antiderivative size = 532, normalized size of antiderivative = 1.42, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.955, Rules used = {5501, 27, 5453, 5361, 5453, 5361, 5419, 5459, 5403, 2897, 5501, 5459, 5403, 5465, 5427, 5465, 215, 216, 5527, 5531, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3}{x^3 (a^2cx^2+c)^2}dx$$

$$\Big\downarrow \text{ 5501}$$

$$\frac{\int \frac{\arctan(ax)^3}{cx^3(a^2x^2+1)}dx}{c} - a^2\int \frac{\arctan(ax)^3}{c^2x (a^2x^2+1)^2}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{\arctan(ax)^3}{x^3(a^2x^2+1)}dx}{c^2} - \frac{a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \text{ 5453}$$

$$\frac{\int \frac{\arctan(ax)^3}{x^3}dx - a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)}dx}{c^2} - \frac{a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \text{ 5361}$$

$$\frac{\frac{3}{2}a\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)}dx - a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)}dx - \frac{\arctan(ax)^3}{2x^2}}{c^2} - \frac{a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \text{ 5453}$$

$$\frac{\frac{3}{2}a\left(\int \frac{\arctan(ax)^2}{x^2}dx - a^2\int \frac{\arctan(ax)^2}{a^2x^2+1}dx\right) - a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)}dx - \frac{\arctan(ax)^3}{2x^2}}{c^2} - \frac{a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \text{ 5361}$$

$$\frac{\frac{3}{2}a\left(a^2\left(-\int \frac{\arctan(ax)^2}{a^2x^2+1}dx\right) + 2a\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx - \frac{\arctan(ax)^2}{x}\right) - a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)}dx - \frac{\arctan(ax)^3}{2x^2}}{c^2} - \frac{a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \text{5419}$$

$$\frac{\frac{3}{2}a\left(2a\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx-\frac{1}{3}a\arctan(ax)^3-\frac{\arctan(ax)^2}{x}\right)-a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)}dx-\frac{\arctan(ax)^3}{2x^2}}{c^2}-\frac{a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \text{5459}$$

$$-\frac{a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx}{c^2}+\frac{-a^2\left(i\int \frac{\arctan(ax)^3}{x(ax+i)}dx-\frac{1}{4}i\arctan(ax)^4\right)+\frac{3}{2}a\left(2a\left(i\int \frac{\arctan(ax)}{x(ax+i)}dx-\frac{1}{2}i\arctan(ax)^2\right)-\frac{1}{3}a\arctan(ax)^3-\frac{\arctan(a}{x}\right.}{c^2}$$

$$\Big\downarrow \text{5403}$$

$$-\frac{a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx}{c^2}+\frac{\frac{3}{2}a\left(2a\left(i\left(ia\int \frac{\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{2}i\arctan(ax)^2\right)-\frac{1}{3}a\arctan(ax)^3-\frac{\arctan(ax}{x}\right.}{c^2}$$

$$\Big\downarrow \text{2897}$$

$$-\frac{a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx}{c^2}+\frac{-a^2\left(i\left(3ia\int \frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{4}i\arctan(ax)^4\right)+\frac{3}{2}a\left(2a\left(i\left(-i\arct\right.\right.\right.}{}$$

$$\Big\downarrow \text{5501}$$

$$-\frac{a^2\left(\int \frac{\arctan(ax)^3}{x(a^2x^2+1)}dx-a^2\int \frac{x\arctan(ax)^3}{(a^2x^2+1)^2}dx\right)}{c^2}+\frac{-a^2\left(i\left(3ia\int \frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{4}i\arctan(ax)^4\right)+\frac{3}{2}a\left(2a\left(i\left(-i\arct\right.\right.\right.}{}$$

$$\Big\downarrow \text{5459}$$

$$\frac{-a^2\left(i\left(3ia\int\frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{4}i\arctan(ax)^4\right)+\frac{3}{2}a\left(2a\left(i\left(-i\arctan\right.\right.\right.}{\frac{a^2\left(-a^2\int\frac{x\arctan(ax)^3}{(a^2x^2+1)^2}dx+i\int\frac{\arctan(ax)^3}{x(ax+i)}dx-\frac{1}{4}i\arctan(ax)^4\right)}{c^2}}$$

$\downarrow$ 5403

$$\frac{-a^2\left(i\left(3ia\int\frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{4}i\arctan(ax)^4\right)+\frac{3}{2}a\left(2a\left(i\left(-i\arctan\right.\right.\right.}{\frac{a^2\left(-a^2\int\frac{x\arctan(ax)^3}{(a^2x^2+1)^2}dx+i\left(3ia\int\frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{4}i\arctan(ax)^4\right.}{c^2}}$$

$\downarrow$ 5465

$$\frac{-a^2\left(i\left(3ia\int\frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{4}i\arctan(ax)^4\right)+\frac{3}{2}a\left(2a\left(i\left(-i\arctan\right.\right.\right.}{\frac{a^2\left(-a^2\left(\frac{3\int\frac{\arctan(ax)^2}{\left(a^2x^2+1\right)^2}dx}{2a}-\frac{\arctan(ax)^3}{2a^2(a^2x^2+1)}\right)+i\left(3ia\int\frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right)-\right.}{c^2}}$$

$\downarrow$ 5427

$$\frac{-a^2\left(i\left(3ia\int\frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{4}i\arctan(ax)^4\right)+\frac{3}{2}a\left(2a\left(i\left(-i\arctan\right.\right.\right.}{\frac{a^2\left(-a^2\left(\frac{3\left(-a\int\frac{x\arctan(ax)}{\left(a^2x^2+1\right)^2}dx+\frac{x\arctan(ax)^2}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^3}{6a}\right)}{2a}-\frac{\arctan(ax)^3}{2a^2(a^2x^2+1)}\right)+i\left(3ia\int\frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan\right.\right.}{c^2}}$$

$\downarrow$ 5465

$$\frac{-a^2\left(i\left(3ia\int\frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{4}i\arctan(ax)^4\right)+\frac{3}{2}a\left(2a\left(i\left(-i\arctan\right.\right.\right.}{}$$

$$\frac{a^2\left(-a^2\left(\frac{3\left(-a\left(\frac{\int\frac{1}{\left(a^2x^2+1\right)^2}dx}{2a}-\frac{\arctan(ax)}{2a^2\left(a^2x^2+1\right)}\right)+\frac{x\arctan(ax)^2}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^3}{6a}\right)}{2a}-\frac{\arctan(ax)^3}{2a^2(a^2x^2+1)}\right)+i\left(3ia\int\frac{\arctan(ax)^2\log\left(2-\frac{2}{1-i}\right.}{a^2x^2+1}\right.\right.}{c^2}$$

$\downarrow$ 215

$$\frac{-a^2\left(i\left(3ia\int\frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{4}i\arctan(ax)^4\right)+\frac{3}{2}a\left(2a\left(i\left(-i\arctan\right.\right.\right.}{}$$

$$\frac{a^2\left(-a^2\left(\frac{3\left(-a\left(\frac{\frac{1}{2}\int\frac{1}{a^2x^2+1}dx+\frac{x}{2\left(a^2x^2+1\right)}}{2a}-\frac{\arctan(ax)}{2a^2\left(a^2x^2+1\right)}\right)+\frac{x\arctan(ax)^2}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^3}{6a}\right)}{2a}-\frac{\arctan(ax)^3}{2a^2(a^2x^2+1)}\right)+i\left(3ia\int\frac{\arctan(ax)^2}{a^2}\right.\right.}{c^2}$$

$\downarrow$ 216

$$\frac{-a^2\left(i\left(3ia\int\frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{4}i\arctan(ax)^4\right)+\frac{3}{2}a\left(2a\left(i\left(-i\arctan\right.\right.\right.}{}$$

$$\frac{a^2\left(i\left(3ia\int\frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right)-a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{2\left(a^2x^2+1\right)}-a\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}}{2a}\right.\right.}{2a}\right.\right.}{c^2}$$

$\downarrow$ 5527

$$\frac{-a^2\left(i\left(3ia\left(\frac{i\arctan(ax)^2\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-i\int\frac{\arctan(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{a^2x^2+1}dx\right)-i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right.\right.}{a^2\left(i\left(3ia\left(\frac{i\arctan(ax)^2\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-i\int\frac{\arctan(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{a^2x^2+1}dx\right)-i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right)\right.}$$

$c^2$

$\downarrow$ 5531

$$\frac{-a^2\left(i\left(3ia\left(\frac{i\arctan(ax)^2\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-i\left(\frac{1}{2}i\int\frac{\operatorname{PolyLog}\left(3,\frac{2}{1-iax}-1\right)}{a^2x^2+1}dx-\frac{i\arctan(ax)\operatorname{PolyLog}\left(3,\frac{2}{1-iax}-1\right)}{2a}\right)\right)\right)-i\,a}{a^2\left(i\left(3ia\left(\frac{i\arctan(ax)^2\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-i\left(\frac{1}{2}i\int\frac{\operatorname{PolyLog}\left(3,\frac{2}{1-iax}-1\right)}{a^2x^2+1}dx-\frac{i\arctan(ax)\operatorname{PolyLog}\left(3,\frac{2}{1-iax}-1\right)}{2a}\right)\right)\right)-i\,\mathrm{ar}\right.}$$

$\downarrow$ 7164

$$\frac{-a^2\left(i\left(3ia\left(\frac{i\arctan(ax)^2\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-i\left(\frac{\operatorname{PolyLog}\left(4,\frac{2}{1-iax}-1\right)}{4a}-\frac{i\arctan(ax)\operatorname{PolyLog}\left(3,\frac{2}{1-iax}-1\right)}{2a}\right)\right)\right)-i\arctan(}{a^2\left(-a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{2\left(a^2x^2+1\right)}-a\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\arctan(ax)}{2a^2\left(a^2x^2+1\right)}\right)+\frac{\arctan(ax)^3}{6a}\right)}{2a}-\frac{\arctan(ax)^3}{2a^2(a^2x^2+1)}\right)+i\left(3ia\left(\frac{i\arctan(ax)^2\,\mathrm{Poly}}{2\cdots}\right.\right.\right.}$$

input
```
Int[ArcTan[a*x]^3/(x^3*(c + a^2*c*x^2)^2),x]
```

output
```
-((a^2*((-1/4*I)*ArcTan[a*x]^4 - a^2*(-1/2*ArcTan[a*x]^3/(a^2*(1 + a^2*x^2
)) + (3*((x*ArcTan[a*x]^2)/(2*(1 + a^2*x^2)) + ArcTan[a*x]^3/(6*a) - a*(-1
/2*ArcTan[a*x]/(a^2*(1 + a^2*x^2)) + (x/(2*(1 + a^2*x^2)) + ArcTan[a*x]/(2
*a))/(2*a))))/(2*a)) + I*((-I)*ArcTan[a*x]^3*Log[2 - 2/(1 - I*a*x)] + (3*I
)*a*(((I/2)*ArcTan[a*x]^2*PolyLog[2, -1 + 2/(1 - I*a*x)])/a - I*(((-1/2*I)
*ArcTan[a*x]*PolyLog[3, -1 + 2/(1 - I*a*x)])/a + PolyLog[4, -1 + 2/(1 - I*
a*x)]/(4*a))))))/c^2) + (-1/2*ArcTan[a*x]^3/x^2 + (3*a*(-(ArcTan[a*x]^2/x)
 - (a*ArcTan[a*x]^3)/3 + 2*a*((-1/2*I)*ArcTan[a*x]^2 + I*((-I)*ArcTan[a*x]
*Log[2 - 2/(1 - I*a*x)] - PolyLog[2, -1 + 2/(1 - I*a*x)]/2))))/2 - a^2*((-
1/4*I)*ArcTan[a*x]^4 + I*((-I)*ArcTan[a*x]^3*Log[2 - 2/(1 - I*a*x)] + (3*I
)*a*(((I/2)*ArcTan[a*x]^2*PolyLog[2, -1 + 2/(1 - I*a*x)])/a - I*(((-1/2*I)
*ArcTan[a*x]*PolyLog[3, -1 + 2/(1 - I*a*x)])/a + PolyLog[4, -1 + 2/(1 - I*
a*x)]/(4*a))))))/c^2
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

rule 5527
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

rule 5531
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/
(2*c*d)), x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k
+ 1, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0]
&& EqQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

## Maple [A] (verified)

Time = 107.02 (sec) , antiderivative size = 531, normalized size of antiderivative = 1.42

| method | result |
|---|---|
| derivativedivides | $a^2\left(-\frac{3i\,\mathrm{polylog}\left(2,\frac{iax+1}{\sqrt{a^2x^2+1}}\right)}{c^2}+\frac{\left(6i\arctan(ax)^2+4\arctan(ax)^3-3i-6\arctan(ax)\right)(ax-i)}{32c^2(ax+i)}+\frac{\left(-6i\arctan(ax)\right.}{}\right.$ |
| default | $a^2\left(-\frac{3i\,\mathrm{polylog}\left(2,\frac{iax+1}{\sqrt{a^2x^2+1}}\right)}{c^2}+\frac{\left(6i\arctan(ax)^2+4\arctan(ax)^3-3i-6\arctan(ax)\right)(ax-i)}{32c^2(ax+i)}+\frac{\left(-6i\arctan(ax)\right.}{}\right.$ |

input
```
int(arctan(a*x)^3/x^3/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*(-3*I/c^2*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+1/32*(6*I*arctan(a*x)
^2+4*arctan(a*x)^3-3*I-6*arctan(a*x))*(a*x-I)/c^2/(a*x+I)+1/32*(-6*I*arcta
n(a*x)^2+4*arctan(a*x)^3+3*I-6*arctan(a*x))*(a*x+I)/c^2/(a*x-I)-1/2/c^2*ar
ctan(a*x)^2*(-I*arctan(a*x)+arctan(a*x)*a*x-3*I*a*x)*(a*x+I)/a^2/x^2-3*I/c
^2*arctan(a*x)^2+3/c^2*arctan(a*x)*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+1/2*I
/c^2*arctan(a*x)^4+3/c^2*arctan(a*x)*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-12*
I/c^2*polylog(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I/c^2*arctan(a*x)^2*polylo
g(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-2/c^2*arctan(a*x)^3*ln(1-(1+I*a*x)/(a^2*
x^2+1)^(1/2))-12*I/c^2*polylog(4,(1+I*a*x)/(a^2*x^2+1)^(1/2))-12/c^2*arcta
n(a*x)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*I/c^2*polylog(2,-(1+I*a*x)
/(a^2*x^2+1)^(1/2))-2/c^2*arctan(a*x)^3*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+
6*I/c^2*arctan(a*x)^2*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-12/c^2*arctan
(a*x)*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2)))
```

## Fricas [F]

$$\int \frac{\arctan(ax)^3}{x^3\,(c+a^2cx^2)^2}\,dx=\int \frac{\arctan\left(ax\right)^3}{\left(a^2cx^2+c\right)^2x^3}\,dx$$

input
```
integrate(arctan(a*x)^3/x^3/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
integral(arctan(a*x)^3/(a^4*c^2*x^7 + 2*a^2*c^2*x^5 + c^2*x^3), x)
```

## Sympy [F]

$$\int \frac{\arctan(ax)^3}{x^3 (c + a^2cx^2)^2} dx = \frac{\int \frac{\operatorname{atan}^3(ax)}{a^4x^7+2a^2x^5+x^3} dx}{c^2}$$

input
```
integrate(atan(a*x)**3/x**3/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(atan(a*x)**3/(a**4*x**7 + 2*a**2*x**5 + x**3), x)/c**2
```

## Maxima [F]

$$\int \frac{\arctan(ax)^3}{x^3 (c + a^2cx^2)^2} dx = \int \frac{\arctan(ax)^3}{(a^2cx^2 + c)^2 x^3} dx$$

input
```
integrate(arctan(a*x)^3/x^3/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)^3/((a^2*c*x^2 + c)^2*x^3), x)
```

## Giac [F]

$$\int \frac{\arctan(ax)^3}{x^3 (c + a^2cx^2)^2} dx = \int \frac{\arctan(ax)^3}{(a^2cx^2 + c)^2 x^3} dx$$

input
```
integrate(arctan(a*x)^3/x^3/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^3/((a^2*c*x^2 + c)^2*x^3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^3}{x^3 (c+a^2cx^2)^2}\,dx = \int \frac{\mathrm{atan}(a\,x)^3}{x^3\,(c\,a^2\,x^2+c)^2}\,dx$$

input
```
int(atan(a*x)^3/(x^3*(c + a^2*c*x^2)^2),x)
```

output
```
int(atan(a*x)^3/(x^3*(c + a^2*c*x^2)^2), x)
```

## Reduce [F]

$$\int \frac{\arctan(ax)^3}{x^3 (c+a^2cx^2)^2}\,dx = \frac{\int \frac{atan(ax)^3}{a^4x^7+2a^2x^5+x^3}dx}{c^2}$$

input
```
int(atan(a*x)^3/x^3/(a^2*c*x^2+c)^2,x)
```

output
```
int(atan(a*x)**3/(a**4*x**7 + 2*a**2*x**5 + x**3),x)/c**2
```

# 3.403 $\int \frac{\arctan(ax)^3}{x^4(c+a^2cx^2)^2}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 332

$$\begin{aligned}\int \frac{\arctan(ax)^3}{x^4(c+a^2cx^2)^2}\,dx = &-\frac{3a^3}{8c^2(1+a^2x^2)}-\frac{a^2\arctan(ax)}{c^2x}-\frac{3a^4x\arctan(ax)}{4c^2(1+a^2x^2)}\\&-\frac{7a^3\arctan(ax)^2}{8c^2}-\frac{a\arctan(ax)^2}{2c^2x^2}+\frac{3a^3\arctan(ax)^2}{4c^2(1+a^2x^2)}\\&+\frac{7ia^3\arctan(ax)^3}{3c^2}-\frac{\arctan(ax)^3}{3c^2x^3}+\frac{2a^2\arctan(ax)^3}{c^2x}\\&+\frac{a^4x\arctan(ax)^3}{2c^2(1+a^2x^2)}+\frac{5a^3\arctan(ax)^4}{8c^2}+\frac{a^3\log(x)}{c^2}\\&-\frac{a^3\log(1+a^2x^2)}{2c^2}-\frac{7a^3\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{c^2}\\&+\frac{7ia^3\arctan(ax)\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{c^2}\\&-\frac{7a^3\operatorname{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)}{2c^2}\end{aligned}$$

output
```
-3/8*a^3/c^2/(a^2*x^2+1)-a^2*arctan(a*x)/c^2/x-3/4*a^4*x*arctan(a*x)/c^2/(
a^2*x^2+1)-7/8*a^3*arctan(a*x)^2/c^2-1/2*a*arctan(a*x)^2/c^2/x^2+3/4*a^3*a
rctan(a*x)^2/c^2/(a^2*x^2+1)+7*I*a^3*arctan(a*x)*polylog(2,-1+2/(1-I*a*x))
/c^2-1/3*arctan(a*x)^3/c^2/x^3+2*a^2*arctan(a*x)^3/c^2/x+1/2*a^4*x*arctan(
a*x)^3/c^2/(a^2*x^2+1)+5/8*a^3*arctan(a*x)^4/c^2+a^3*ln(x)/c^2-1/2*a^3*ln(
a^2*x^2+1)/c^2-7*a^3*arctan(a*x)^2*ln(2-2/(1-I*a*x))/c^2+7/3*I*a^3*arctan(
a*x)^3/c^2-7/2*a^3*polylog(3,-1+2/(1-I*a*x))/c^2
```

## Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.73

$$\int \frac{\arctan(ax)^3}{x^4\left(c+a^2cx^2\right)^2}\,dx = \frac{a^3\left(\frac{7i\pi^3}{24}-\frac{\arctan(ax)}{ax}-\frac{1}{2}\arctan(ax)^2-\frac{\arctan(ax)^2}{2a^2x^2}-\frac{7}{3}i\arctan(ax)^3-\frac{\arctan(ax)^3}{3a^3x^3}+\frac{2\arctan(ax)^3}{ax}+\frac{5}{8}\arctan(a\right.}{}$$

input
```
Integrate[ArcTan[a*x]^3/(x^4*(c + a^2*c*x^2)^2),x]
```

output
```
(a^3*(((7*I)/24)*Pi^3 - ArcTan[a*x]/(a*x) - ArcTan[a*x]^2/2 - ArcTan[a*x]^
2/(2*a^2*x^2) - ((7*I)/3)*ArcTan[a*x]^3 - ArcTan[a*x]^3/(3*a^3*x^3) + (2*A
rcTan[a*x]^3)/(a*x) + (5*ArcTan[a*x]^4)/8 - (3*Cos[2*ArcTan[a*x]])/16 + (3
*ArcTan[a*x]^2*Cos[2*ArcTan[a*x]])/8 - 7*ArcTan[a*x]^2*Log[1 - E^((-2*I)*A
rcTan[a*x])] + Log[(a*x)/Sqrt[1 + a^2*x^2]] - (7*I)*ArcTan[a*x]*PolyLog[2,
 E^((-2*I)*ArcTan[a*x])] - (7*PolyLog[3, E^((-2*I)*ArcTan[a*x])])/2 - (3*A
rcTan[a*x]*Sin[2*ArcTan[a*x]])/8 + (ArcTan[a*x]^3*Sin[2*ArcTan[a*x]])/4))/
c^2
```

## Rubi [A] (verified)

Time = 6.46 (sec) , antiderivative size = 580, normalized size of antiderivative = 1.75, number of steps used = 29, number of rules used = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 1.273$, Rules used = {5501, 27, 5453, 5361, 5453, 5361, 5419, 5453, 5361, 243, 47, 14, 16, 5419, 5459, 5403, 5501, 5427, 5453, 5361, 5419, 5459, 5403, 5465, 5427, 241, 5527, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed

below.

$$\int \frac{\arctan(ax)^3}{x^4\left(a^2cx^2+c\right)^2}dx$$

$$\Big\downarrow \text{ 5501}$$

$$\frac{\int \frac{\arctan(ax)^3}{cx^4(a^2x^2+1)}dx}{c}-a^2\int \frac{\arctan(ax)^3}{c^2x^2\left(a^2x^2+1\right)^2}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{\arctan(ax)^3}{x^4(a^2x^2+1)}dx}{c^2}-\frac{a^2\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \text{ 5453}$$

$$\frac{\int \frac{\arctan(ax)^3}{x^4}dx-a^2\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)}dx}{c^2}-\frac{a^2\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \text{ 5361}$$

$$\frac{-a^2\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)}dx+a\int \frac{\arctan(ax)^2}{x^3(a^2x^2+1)}dx-\frac{\arctan(ax)^3}{3x^3}}{c^2}-\frac{a^2\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \text{ 5453}$$

$$\frac{-a^2\left(\int \frac{\arctan(ax)^3}{x^2}dx-a^2\int \frac{\arctan(ax)^3}{a^2x^2+1}dx\right)+a\left(\int \frac{\arctan(ax)^2}{x^3}dx-a^2\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right)-\frac{\arctan(ax)^3}{3x^3}}{c^2}-\frac{a^2\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \text{ 5361}$$

$$\frac{a\left(a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right)+a\int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx-\frac{\arctan(ax)^2}{2x^2}\right)-a^2\left(3a\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx-a^2\int \frac{\arctan(ax)^3}{a^2x^2+1}dx-\frac{\arctan(ax)^3}{x}\right.}{c^2}-\frac{a^2\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx}{c^2}$$

$$\Big\downarrow \text{ 5419}$$

$$\frac{-a^2\left(3a\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx-\frac{1}{4}a\arctan(ax)^4-\frac{\arctan(ax)^3}{x}\right)+a\left(a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right)+a\int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx-\frac{\arctan(ax}{2x^2}\right.}{c^2}$$

$$\frac{a^2\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx}{c^2}$$

$\downarrow$ 5453

$$\frac{-a^2\left(3a\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx-\frac{1}{4}a\arctan(ax)^4-\frac{\arctan(ax)^3}{x}\right)+a\left(a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right)+a\left(\int \frac{\arctan(ax)}{x^2}dx-a^2\int \frac{\arct}{a^2}\right.\right.}{c^2}$$

$$\frac{a^2\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx}{c^2}$$

$\downarrow$ 5361

$$\frac{-a^2\left(3a\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx-\frac{1}{4}a\arctan(ax)^4-\frac{\arctan(ax)^3}{x}\right)+a\left(a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right)+a\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+\right.\right.}{c^2}$$

$$\frac{a^2\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx}{c^2}$$

$\downarrow$ 243

$$\frac{-a^2\left(3a\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx-\frac{1}{4}a\arctan(ax)^4-\frac{\arctan(ax)^3}{x}\right)+a\left(a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right)+a\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+\right.\right.}{c^2}$$

$$\frac{a^2\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx}{c^2}$$

$\downarrow$ 47

$$\frac{-a^2\left(3a\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx-\frac{1}{4}a\arctan(ax)^4-\frac{\arctan(ax)^3}{x}\right)+a\left(a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right)+a\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+\right.\right.}{c^2}$$

$$\frac{a^2\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx}{c^2}$$

$\downarrow$ 14

$$\frac{-a^2\left(3a\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx-\frac{1}{4}a\arctan(ax)^4-\frac{\arctan(ax)^3}{x}\right)+a\left(a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right)+a\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+\right.\right.}{c^2}$$

$$\frac{a^2\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx}{c^2}$$

$$\downarrow 16$$

$$\frac{-a^2\left(3a\int\frac{\arctan(ax)^2}{x(a^2x^2+1)}dx-\frac{1}{4}a\arctan(ax)^4-\frac{\arctan(ax)^3}{x}\right)+a\left(a^2\left(-\int\frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right)+a\left(a^2\left(-\int\frac{\arctan(ax)}{a^2x^2+1}dx\right)+\right.\right.}{c^2}$$

$$\frac{a^2\int\frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx}{c^2}$$

$$\downarrow 5419$$

$$\frac{-a^2\left(3a\int\frac{\arctan(ax)^2}{x(a^2x^2+1)}dx-\frac{1}{4}a\arctan(ax)^4-\frac{\arctan(ax)^3}{x}\right)+a\left(a^2\left(-\int\frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right)+a\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2\right.\right.\right.\right.}{c^2}$$

$$\frac{a^2\int\frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx}{c^2}$$

$$\downarrow 5459$$

$$-\frac{a^2\int\frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx}{c^2}+$$

$$\frac{a\left(-\left(a^2\left(i\int\frac{\arctan(ax)^2}{x(ax+i)}dx-\frac{1}{3}i\arctan(ax)^3\right)\right)+a\left(\frac{1}{2}a\left(\log\left(x^2\right)-\log\left(a^2x^2+1\right)\right)-\frac{1}{2}a\arctan(ax)^2-\frac{\arctan(ax)}{x}\right)\right.}{c^2}$$

$$\downarrow 5403$$

$$-\frac{a^2\int\frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx}{c^2}+$$

$$-a^2\left(3a\left(i\left(2ia\int\frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3\right)-\frac{1}{4}a\arctan(ax\right.$$

$$\downarrow 5501$$

$$-\frac{a^2\left(\int\frac{\arctan(ax)^3}{x^2(a^2x^2+1)}dx-a^2\int\frac{\arctan(ax)^3}{(a^2x^2+1)^2}dx\right)}{c^2}+$$

$$-a^2\left(3a\left(i\left(2ia\int\frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3\right)-\frac{1}{4}a\arctan(ax\right.$$

$$\downarrow 5427$$

$$-\frac{a^2\left(\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)}dx - a^2\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx + \frac{x\arctan(ax)^3}{2(a^2x^2+1)} + \frac{\arctan(ax)^4}{8a}\right)\right)}{c^2} +$$
$$\frac{-a^2\left(3a\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx - i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right) - \frac{1}{3}i\arctan(ax)^3\right) - \frac{1}{4}a\arctan(ax\right.}{}$$

$\downarrow$ 5453

$$-\frac{a^2\left(-\left(a^2\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx + \frac{x\arctan(ax)^3}{2(a^2x^2+1)} + \frac{\arctan(ax)^4}{8a}\right)\right) - a^2\int \frac{\arctan(ax)^3}{a^2x^2+1}dx + \int \frac{\arctan(ax)^3}{x^2}dx\right)}{c^2} +$$
$$\frac{-a^2\left(3a\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx - i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right) - \frac{1}{3}i\arctan(ax)^3\right) - \frac{1}{4}a\arctan(ax\right.}{}$$

$\downarrow$ 5361

$$-\frac{a^2\left(-a^2\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx + \frac{x\arctan(ax)^3}{2(a^2x^2+1)} + \frac{\arctan(ax)^4}{8a}\right) + 3a\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx - a^2\int \frac{\arctan(ax)^3}{a^2x^2+1}dx - \frac{\arctan(ax)^3}{x}\right.}{c^2}$$
$$\frac{-a^2\left(3a\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx - i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right) - \frac{1}{3}i\arctan(ax)^3\right) - \frac{1}{4}a\arctan(ax\right.}{}$$

$\downarrow$ 5419

$$-\frac{a^2\left(-a^2\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx + \frac{x\arctan(ax)^3}{2(a^2x^2+1)} + \frac{\arctan(ax)^4}{8a}\right) + 3a\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx - \frac{1}{4}a\arctan(ax)^4 - \frac{\arctan(ax)^3}{x}\right)}{c^2}$$
$$\frac{-a^2\left(3a\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx - i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right) - \frac{1}{3}i\arctan(ax)^3\right) - \frac{1}{4}a\arctan(ax\right.}{}$$

$\downarrow$ 5459

$$\frac{-a^2\left(3a\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx - i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right) - \frac{1}{3}i\arctan(ax)^3\right) - \frac{1}{4}a\arctan(ax\right.}{}$$
$$\frac{a^2\left(-a^2\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx + \frac{x\arctan(ax)^3}{2(a^2x^2+1)} + \frac{\arctan(ax)^4}{8a}\right) + 3a\left(i\int \frac{\arctan(ax)^2}{x(ax+i)}dx - \frac{1}{3}i\arctan(ax)^3\right) - \frac{1}{4}a\arctan\right.}{c^2}$$

$\downarrow$ 5403

$$-a^2\left(3a\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3\right)-\frac{1}{4}a\arctan(ax\right.$$

$$a^2\left(-a^2\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)+3a\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax\right.\right.\right.$$

$$c^2$$

$\downarrow$ 5465

$$-a^2\left(3a\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3\right)-\frac{1}{4}a\arctan(ax\right.$$

$$a^2\left(-a^2\left(-\frac{3}{2}a\left(\frac{\int \frac{\arctan(ax)}{(a^2x^2+1)^2}dx}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)+3a\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}a\right.\right.\right.$$

$$c^2$$

$\downarrow$ 5427

$$-a^2\left(3a\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3\right)-\frac{1}{4}a\arctan(ax\right.$$

$$a^2\left(-a^2\left(-\frac{3}{2}a\left(\frac{-\frac{1}{2}a\int \frac{x}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)+3a\left(i\left(2ic\right.\right.\right.$$

$$c$$

$\downarrow$ 241

$$-a^2\left(3a\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3\right)-\frac{1}{4}a\arctan(ax\right.$$

$$a^2\left(3a\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3\right)-a^2\left(\frac{x\arctan(ax)^3}{2(a^2x^2+1)}\right.\right.$$

$$c^2$$

$\downarrow$ 5527

$$-a^2\left(3a\left(i\left(2ia\left(\frac{i\arctan(ax)\,\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-\frac{1}{2}i\int\frac{\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{a^2x^2+1}dx\right)-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\right.\right.$$

$$a^2\left(3a\left(i\left(2ia\left(\frac{i\arctan(ax)\,\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-\frac{1}{2}i\int\frac{\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{a^2x^2+1}dx\right)-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}\right.\right.$$

$$\Big\downarrow \; 7164$$

$$a\left(-\left(a^2\left(i\left(2ia\left(\frac{i\arctan(ax)\,\text{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-\frac{\text{PolyLog}\left(3,\frac{2}{1-iax}-1\right)}{4a}\right)-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\,\text{arc}\right.\right.\right.$$

$$a^2\left(-a^2\left(\frac{x\arctan(ax)^3}{2(a^2x^2+1)}-\frac{3}{2}a\left(\frac{\frac{x\arctan(ax)}{2\left(a^2x^2+1\right)}+\frac{1}{4a\left(a^2x^2+1\right)}+\frac{\arctan(ax)^2}{4a}}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)+\frac{\arctan(ax)^4}{8a}\right)+3a\left(i\left(2ia\left(\frac{i\arctan}{}\right.\right.\right.\right.$$

input
```
Int[ArcTan[a*x]^3/(x^4*(c + a^2*c*x^2)^2),x]
```

output
```
-((a^2*(-(ArcTan[a*x]^3/x) - (a*ArcTan[a*x]^4)/4 - a^2*((x*ArcTan[a*x]^3)/
(2*(1 + a^2*x^2)) + ArcTan[a*x]^4/(8*a) - (3*a*(-1/2*ArcTan[a*x]^2/(a^2*(1
 + a^2*x^2)) + (1/(4*a*(1 + a^2*x^2)) + (x*ArcTan[a*x])/(2*(1 + a^2*x^2))
 + ArcTan[a*x]^2/(4*a))/a))/2) + 3*a*((-1/3*I)*ArcTan[a*x]^3 + I*((-I)*ArcT
an[a*x]^2*Log[2 - 2/(1 - I*a*x)] + (2*I)*a*(((I/2)*ArcTan[a*x]*PolyLog[2,
-1 + 2/(1 - I*a*x)])/a - PolyLog[3, -1 + 2/(1 - I*a*x)]/(4*a))))))/c^2) +
(-1/3*ArcTan[a*x]^3/x^3 - a^2*(-(ArcTan[a*x]^3/x) - (a*ArcTan[a*x]^4)/4 +
3*a*((-1/3*I)*ArcTan[a*x]^3 + I*((-I)*ArcTan[a*x]^2*Log[2 - 2/(1 - I*a*x)]
 + (2*I)*a*(((I/2)*ArcTan[a*x]*PolyLog[2, -1 + 2/(1 - I*a*x)])/a - PolyLog
[3, -1 + 2/(1 - I*a*x)]/(4*a))))) + a*(-1/2*ArcTan[a*x]^2/x^2 + a*(-(ArcTa
n[a*x]/x) - (a*ArcTan[a*x]^2)/2 + (a*(Log[x^2] - Log[1 + a^2*x^2]))/2) - a
^2*((-1/3*I)*ArcTan[a*x]^3 + I*((-I)*ArcTan[a*x]^2*Log[2 - 2/(1 - I*a*x)]
+ (2*I)*a*(((I/2)*ArcTan[a*x]*PolyLog[2, -1 + 2/(1 - I*a*x)])/a - PolyLog[
3, -1 + 2/(1 - I*a*x)]/(4*a))))))/c^2
```

## Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

rule 5527
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 9.99 (sec) , antiderivative size = 1990, normalized size of antiderivative = 5.99

Expression too large to display

input
```
int(arctan(a*x)^3/x^4/(a^2*c*x^2+c)^2,x)
```

output
```
a^3*(1/2/c^2*arctan(a*x)^3*a*x/(a^2*x^2+1)+5/2/c^2*arctan(a*x)^4-1/3/c^2*a
rctan(a*x)^3/a^3/x^3+2/c^2*arctan(a*x)^3/a/x-1/2/c^2*(7/4*arctan(a*x)^2-2*
ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-1)-3/2*arctan(a*x)^2/(a^2*x^2+1)-2*ln(1+(1+
I*a*x)/(a^2*x^2+1)^(1/2))+28*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))+28*pol
ylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+14*arctan(a*x)^2*ln((1+I*a*x)/(a^2*x^
2+1)^(1/2))-3*I*arctan(a*x)*(a*x+I)/(8*a*x-8*I)+3*I*arctan(a*x)*(a*x-I)/(8
*a*x+8*I)-7*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+
1)+1))*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arc
tan(a*x)^2+7*I*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*csgn(I*(1+I*a*x)^2/(
a^2*x^2+1))^2*arctan(a*x)^2-7*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csg
n(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2*arctan(a*x)
^2+7/2*I*Pi*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)
/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^2*arctan(a*x)^2-7/2*I*Pi*csgn(I*(1+I*a*x)/
(a^2*x^2+1)^(1/2))^2*csgn(I*(1+I*a*x)^2/(a^2*x^2+1))*arctan(a*x)^2+7/2*I*P
i*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1
+I*a*x)^2/(a^2*x^2+1)+1)^2)^2*arctan(a*x)^2-7*I*Pi*csgn(I/((1+I*a*x)^2/(a^
2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1
))^2*arctan(a*x)^2+7/2*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))^2*csgn(I*(
(1+I*a*x)^2/(a^2*x^2+1)+1)^2)*arctan(a*x)^2-14*arctan(a*x)^2*ln((1+I*a*x)^
2/(a^2*x^2+1)-1)+7*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2...
```

## Fricas [F]

$$\int \frac{\arctan(ax)^3}{x^4\left(c+a^2cx^2\right)^2}\,dx=\int \frac{\arctan\left(ax\right)^3}{\left(a^2cx^2+c\right)^2x^4}\,dx$$

input
```
integrate(arctan(a*x)^3/x^4/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
integral(arctan(a*x)^3/(a^4*c^2*x^8 + 2*a^2*c^2*x^6 + c^2*x^4), x)
```

## Sympy [F]

$$\int \frac{\arctan(ax)^3}{x^4 (c + a^2cx^2)^2} \, dx = \frac{\int \frac{\operatorname{atan}^3(ax)}{a^4x^8+2a^2x^6+x^4} \, dx}{c^2}$$

input
```
integrate(atan(a*x)**3/x**4/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(atan(a*x)**3/(a**4*x**8 + 2*a**2*x**6 + x**4), x)/c**2
```

## Maxima [F]

$$\int \frac{\arctan(ax)^3}{x^4 (c + a^2cx^2)^2} \, dx = \int \frac{\arctan(ax)^3}{(a^2cx^2 + c)^2 x^4} \, dx$$

input
```
integrate(arctan(a*x)^3/x^4/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
1/6144*(1200*(a^5*x^5 + a^3*x^3)*arctan(a*x)^4 - 45*(a^5*x^5 + a^3*x^3)*lo
g(a^2*x^2 + 1)^4 + 128*(15*a^4*x^4 + 10*a^2*x^2 - 2)*arctan(a*x)^3 - 24*(1
5*(a^5*x^5 + a^3*x^3)*arctan(a*x)^2 + 4*(15*a^4*x^4 + 10*a^2*x^2 - 2)*arct
an(a*x))*log(a^2*x^2 + 1)^2 - 12*(a^2*c^2*x^5 + c^2*x^3)*(120*a^7*(a^2/(a^
8*c^2*x^2 + a^6*c^2) + log(a^2*x^2 + 1)/(a^6*c^2*x^2 + a^4*c^2)) - 30720*a
^7*integrate(1/256*x^7*arctan(a*x)^2*log(a^2*x^2 + 1)/(a^4*c^2*x^8 + 2*a^2
*c^2*x^6 + c^2*x^4), x) - 7680*a^7*integrate(1/256*x^7*log(a^2*x^2 + 1)^3/
(a^4*c^2*x^8 + 2*a^2*c^2*x^6 + c^2*x^4), x) + 61440*a^6*integrate(1/256*x^
6*arctan(a*x)^3/(a^4*c^2*x^8 + 2*a^2*c^2*x^6 + c^2*x^4), x) + 15360*a^6*in
tegrate(1/256*x^6*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^4*c^2*x^8 + 2*a^2*c^2*
x^6 + c^2*x^4), x) - 122880*a^6*integrate(1/256*x^6*arctan(a*x)*log(a^2*x^
2 + 1)/(a^4*c^2*x^8 + 2*a^2*c^2*x^6 + c^2*x^4), x) + 15*a^5*log(a^2*x^2 +
1)^3/(a^4*c^2*x^2 + a^2*c^2) + 45*(2*a^4*(a^2/(a^10*c^2*x^2 + a^8*c^2) + l
og(a^2*x^2 + 1)/(a^8*c^2*x^2 + a^6*c^2)) + a^2*log(a^2*x^2 + 1)^2/(a^6*c^2
*x^2 + a^4*c^2))*a^5 - 30720*a^5*integrate(1/256*x^5*arctan(a*x)^2*log(a^2
*x^2 + 1)/(a^4*c^2*x^8 + 2*a^2*c^2*x^6 + c^2*x^4), x) + 122880*a^5*integra
te(1/256*x^5*arctan(a*x)^2/(a^4*c^2*x^8 + 2*a^2*c^2*x^6 + c^2*x^4), x) + 6
0*a^5*log(a^2*x^2 + 1)^2/(a^4*c^2*x^2 + a^2*c^2) + 61440*a^4*integrate(1/2
56*x^4*arctan(a*x)^3/(a^4*c^2*x^8 + 2*a^2*c^2*x^6 + c^2*x^4), x) + 15360*a
^4*integrate(1/256*x^4*arctan(a*x)*log(a^2*x^2 + 1)^2/(a^4*c^2*x^8 + 2*...
```

## Giac [F]

$$\int \frac{\arctan(ax)^3}{x^4 (c + a^2 c x^2)^2} \, dx = \int \frac{\arctan{(ax)}^3}{(a^2 c x^2 + c)^2 x^4} \, dx$$

input
```
integrate(arctan(a*x)^3/x^4/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^3/((a^2*c*x^2 + c)^2*x^4), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^3}{x^4(c+a^2cx^2)^2}\,dx = \int \frac{\mathrm{atan}(a\,x)^3}{x^4\,(c\,a^2\,x^2+c)^2}\,dx$$

input
```
int(atan(a*x)^3/(x^4*(c + a^2*c*x^2)^2),x)
```

output
```
int(atan(a*x)^3/(x^4*(c + a^2*c*x^2)^2), x)
```

## Reduce [F]

$$\int \frac{\arctan(ax)^3}{x^4(c+a^2cx^2)^2}\,dx = \frac{15atan(ax)^4\,a^5x^5+15atan(ax)^4\,a^3x^3+60atan(ax)^3\,a^4x^4+40atan(ax)^3\,a^2x^2-8atan(ax)^3+90atan(a}{}$$

input
```
int(atan(a*x)^3/x^4/(a^2*c*x^2+c)^2,x)
```

output
```
(15*atan(a*x)**4*a**5*x**5 + 15*atan(a*x)**4*a**3*x**3 + 60*atan(a*x)**3*a
**4*x**4 + 40*atan(a*x)**3*a**2*x**2 - 8*atan(a*x)**3 + 90*atan(a*x)**2*a*
*3*x**3 + 30*atan(a*x)**2*a*x + 60*atan(a*x)*a**2*x**2 + 84*int(atan(a*x)*
*2/(a**4*x**7 + 2*a**2*x**5 + x**3),x)*a**3*x**5 + 84*int(atan(a*x)**2/(a*
*4*x**7 + 2*a**2*x**5 + x**3),x)*a*x**3 + 30*log(a**2*x**2 + 1)*a**5*x**5
+ 30*log(a**2*x**2 + 1)*a**3*x**3 - 60*log(x)*a**5*x**5 - 60*log(x)*a**3*x
**3 + 30*a**5*x**5)/(24*c**2*x**3*(a**2*x**2 + 1))
```

# 3.404 $\int \frac{x^3 \arctan(ax)^3}{(c+a^2cx^2)^3} \, dx$



## Optimal result

Integrand size = 22, antiderivative size = 212

$$\int \frac{x^3 \arctan(ax)^3}{(c+a^2cx^2)^3} \, dx = -\frac{3x^3}{128ac^3(1+a^2x^2)^2} - \frac{45x}{256a^3c^3(1+a^2x^2)} - \frac{27\arctan(ax)}{256a^4c^3} - \frac{3x^4\arctan(ax)}{32c^3(1+a^2x^2)^2} + \frac{9\arctan(ax)}{32a^4c^3(1+a^2x^2)} + \frac{3x^3\arctan(ax)^2}{16ac^3(1+a^2x^2)^2} + \frac{9x\arctan(ax)^2}{32a^3c^3(1+a^2x^2)} - \frac{3\arctan(ax)^3}{32a^4c^3} + \frac{x^4\arctan(ax)^3}{4c^3(1+a^2x^2)^2}$$

output
```
-3/128*x^3/a/c^3/(a^2*x^2+1)^2-45/256*x/a^3/c^3/(a^2*x^2+1)-27/256*arctan(
a*x)/a^4/c^3-3/32*x^4*arctan(a*x)/c^3/(a^2*x^2+1)^2+9/32*arctan(a*x)/a^4/c
^3/(a^2*x^2+1)+3/16*x^3*arctan(a*x)^2/a/c^3/(a^2*x^2+1)^2+9/32*x*arctan(a*
x)^2/a^3/c^3/(a^2*x^2+1)-3/32*arctan(a*x)^3/a^4/c^3+1/4*x^4*arctan(a*x)^3/
c^3/(a^2*x^2+1)^2
```

## Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.50

$$\int \frac{x^3 \arctan(ax)^3}{(c+a^2cx^2)^3} \, dx = \frac{-3ax(15+17a^2x^2)+(45+18a^2x^2-51a^4x^4)\arctan(ax)+24ax(3+5a^2x^2)\arctan(ax)^2+8(-3-6a^2}{256a^4c^3(1+a^2x^2)^2}$$

input
```
Integrate[(x^3*ArcTan[a*x]^3)/(c + a^2*c*x^2)^3,x]
```

output
```
(-3*a*x*(15 + 17*a^2*x^2) + (45 + 18*a^2*x^2 - 51*a^4*x^4)*ArcTan[a*x] + 2
4*a*x*(3 + 5*a^2*x^2)*ArcTan[a*x]^2 + 8*(-3 - 6*a^2*x^2 + 5*a^4*x^4)*ArcTa
n[a*x]^3)/(256*a^4*c^3*(1 + a^2*x^2)^2)
```

## Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.23, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {5479, 27, 5475, 252, 252, 216, 5471, 5465, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \arctan(ax)^3}{(a^2cx^2+c)^3} \, dx$$

$$\Big\downarrow \text{5479}$$

$$\frac{x^4 \arctan(ax)^3}{4c^3(a^2x^2+1)^2} - \frac{3}{4}a \int \frac{x^4 \arctan(ax)^2}{c^3(a^2x^2+1)^3} dx$$

$$\Big\downarrow \text{27}$$

$$\frac{x^4 \arctan(ax)^3}{4c^3(a^2x^2+1)^2} - \frac{3a \int \frac{x^4 \arctan(ax)^2}{(a^2x^2+1)^3} dx}{4c^3}$$

$$\Big\downarrow \text{5475}$$

$$\frac{x^4\arctan(ax)^3}{4c^3\left(a^2x^2+1\right)^2}-\frac{3a\left(\frac{3\int\frac{x^2\arctan(ax)^2}{\left(a^2x^2+1\right)^2}dx}{4a^2}-\frac{1}{8}\int\frac{x^4}{(a^2x^2+1)^3}dx+\frac{x^4\arctan(ax)}{8a(a^2x^2+1)^2}-\frac{x^3\arctan(ax)^2}{4a^2(a^2x^2+1)^2}\right)}{4c^3}$$

$\Big\downarrow$ 252

$$\frac{x^4\arctan(ax)^3}{4c^3\left(a^2x^2+1\right)^2}-\frac{3a\left(\frac{3\int\frac{x^2\arctan(ax)^2}{\left(a^2x^2+1\right)^2}dx}{4a^2}+\frac{1}{8}\left(\frac{x^3}{4a^2(a^2x^2+1)^2}-\frac{3\int\frac{x^2}{\left(a^2x^2+1\right)^2}dx}{4a^2}\right)+\frac{x^4\arctan(ax)}{8a(a^2x^2+1)^2}-\frac{x^3\arctan(ax)^2}{4a^2(a^2x^2+1)^2}\right)}{4c^3}$$

$\Big\downarrow$ 252

$$\frac{x^4\arctan(ax)^3}{4c^3\left(a^2x^2+1\right)^2}-\frac{3a\left(\frac{3\int\frac{x^2\arctan(ax)^2}{\left(a^2x^2+1\right)^2}dx}{4a^2}+\frac{1}{8}\left(\frac{x^3}{4a^2(a^2x^2+1)^2}-\frac{3\left(\frac{\int\frac{1}{a^2x^2+1}dx}{2a^2}-\frac{x}{2a^2\left(a^2x^2+1\right)}\right)}{4a^2}\right)+\frac{x^4\arctan(ax)}{8a(a^2x^2+1)^2}-\frac{x^3\arctan(ax)^2}{4a^2(a^2x^2+1)^2}\right)}{4c^3}$$

$\Big\downarrow$ 216

$$\frac{x^4\arctan(ax)^3}{4c^3\left(a^2x^2+1\right)^2}-\frac{3a\left(\frac{3\int\frac{x^2\arctan(ax)^2}{\left(a^2x^2+1\right)^2}dx}{4a^2}+\frac{x^4\arctan(ax)}{8a(a^2x^2+1)^2}-\frac{x^3\arctan(ax)^2}{4a^2(a^2x^2+1)^2}+\frac{1}{8}\left(\frac{x^3}{4a^2(a^2x^2+1)^2}-\frac{3\left(\frac{\arctan(ax)}{2a^3}-\frac{x}{2a^2\left(a^2x^2+1\right)}\right)}{4a^2}\right)\right)}{4c^3}$$

$\Big\downarrow$ 5471

$$\frac{x^4\arctan(ax)^3}{4c^3\left(a^2x^2+1\right)^2}-\frac{3a\left(\frac{3\left(\frac{\int\frac{x\arctan(ax)}{\left(a^2x^2+1\right)^2}dx}{a}+\frac{\arctan(ax)^3}{6a^3}-\frac{x\arctan(ax)^2}{2a^2\left(a^2x^2+1\right)}\right)}{4a^2}+\frac{x^4\arctan(ax)}{8a(a^2x^2+1)^2}-\frac{x^3\arctan(ax)^2}{4a^2(a^2x^2+1)^2}+\frac{1}{8}\left(\frac{x^3}{4a^2(a^2x^2+1)^2}-\frac{3\left(\frac{\arctan(ax)}{2a^3}-\frac{x}{2a^2\left(a^2x^2+1\right)}\right)}{4a^2}\right)\right)}{4c^3}$$

$\Big\downarrow$ 5465

$$\frac{x^4\arctan(ax)^3}{4c^3\left(a^2x^2+1\right)^2}-\frac{3a\left(\frac{3\left(\frac{\frac{\int\frac{1}{\left(a^2x^2+1\right)^2}dx}{2a}-\frac{\arctan(ax)}{2a^2\left(a^2x^2+1\right)}}{a}+\frac{\arctan(ax)^3}{6a^3}-\frac{x\arctan(ax)^2}{2a^2\left(a^2x^2+1\right)}\right)}{4a^2}+\frac{x^4\arctan(ax)}{8a(a^2x^2+1)^2}-\frac{x^3\arctan(ax)^2}{4a^2(a^2x^2+1)^2}+\frac{1}{8}\left(\frac{x^3}{4a^2(a^2x^2+1)^2}-\frac{3\left(\frac{\text{arcta}\cdots}{2\cdots}\cdots\right)}{\cdots}\cdots\right)\cdots\right)}{4c^3}$$

$$\Big\downarrow \; 215$$

$$\frac{x^4\arctan(ax)^3}{4c^3\left(a^2x^2+1\right)^2}-\frac{3a\left(\frac{3\left(\frac{\frac{\frac{1}{2}\int\frac{1}{a^2x^2+1}dx+\frac{x}{2\left(a^2x^2+1\right)}}{2a}-\frac{\arctan(ax)}{2a^2\left(a^2x^2+1\right)}}{a}+\frac{\arctan(ax)^3}{6a^3}-\frac{x\arctan(ax)^2}{2a^2\left(a^2x^2+1\right)}\right)}{4a^2}+\frac{x^4\arctan(ax)}{8a(a^2x^2+1)^2}-\frac{x^3\arctan(ax)^2}{4a^2(a^2x^2+1)^2}+\frac{1}{8}\left(\frac{x^3}{4a^2(a^2x^2+1)^2}\cdots\right)\cdots\right)}{4c^3}$$

$$\Big\downarrow \; 216$$

$$\frac{x^4\arctan(ax)^3}{4c^3\left(a^2x^2+1\right)^2}-\frac{3a\left(\frac{x^4\arctan(ax)}{8a(a^2x^2+1)^2}-\frac{x^3\arctan(ax)^2}{4a^2(a^2x^2+1)^2}+\frac{3\left(\frac{\arctan(ax)^3}{6a^3}-\frac{x\arctan(ax)^2}{2a^2\left(a^2x^2+1\right)}+\frac{\frac{\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\arctan(ax)}{2a^2\left(a^2x^2+1\right)}}{a}\right)}{4a^2}+\frac{1}{8}\left(\frac{x^3}{4a^2(a^2x^2+1)^2}-\cdots\right)\cdots\right)}{4c^3}$$

input

```
Int[(x^3*ArcTan[a*x]^3)/(c + a^2*c*x^2)^3,x]
```

output
```
(x^4*ArcTan[a*x]^3)/(4*c^3*(1 + a^2*x^2)^2) - (3*a*((x^4*ArcTan[a*x])/(8*a
*(1 + a^2*x^2)^2) - (x^3*ArcTan[a*x]^2)/(4*a^2*(1 + a^2*x^2)^2) + (x^3/(4*
a^2*(1 + a^2*x^2)^2) - (3*(-1/2*x/(a^2*(1 + a^2*x^2)) + ArcTan[a*x]/(2*a^3
)))/(4*a^2))/8 + (3*(-1/2*(x*ArcTan[a*x]^2)/(a^2*(1 + a^2*x^2)) + ArcTan[a
*x]^3/(6*a^3) + (-1/2*ArcTan[a*x]/(a^2*(1 + a^2*x^2)) + (x/(2*(1 + a^2*x^2
)) + ArcTan[a*x]/(2*a))/(2*a))/a))/(4*a^2)))/(4*c^3)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5471
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^2)/((d_) + (e_.)*(x_)^2)
^2, x_Symbol] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(2*b*c^3*d^2*(p + 1)), x]
 + (-Simp[x*((a + b*ArcTan[c*x])^p/(2*c^2*d*(d + e*x^2))), x] + Simp[b*(p/(
2*c))   Int[x*((a + b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5475
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_)*((d_) + (e_.)
*(x_)^2)^(q_), x_Symbol] :> Simp[b*p*(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*Ar
cTan[c*x])^(p - 1)/(c*d*m^2)), x] + (-Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(q +
 1)*((a + b*ArcTan[c*x])^p/(c^2*d*m)), x] + Simp[f^2*((m - 1)/(c^2*d*m))
Int[(f*x)^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p, x], x] - Simp[
b^2*p*((p - 1)/m^2)   Int[(f*x)^m*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2)
, x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && EqQ[m + 2*
q + 2, 0] && LtQ[q, -1] && GtQ[p, 1]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

## Maple [A] (verified)

Time = 1.86 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.58

| method | result |
|---|---|
| parallelrisch | $\frac{40\arctan(ax)^3a^4x^4-51x^4\arctan(ax)a^4+120a^3\arctan(ax)^2x^3-48\arctan(ax)^3a^2x^2-51a^3x^3+18x^2a^2\arctan(ax)+7}{256c^3(a^2x^2+1)^2a^4}$ |
| derivativedivides | $\frac{-\frac{\arctan(ax)^3}{2c^3(a^2x^2+1)}+\frac{\arctan(ax)^3}{4c^3(a^2x^2+1)^2}-\frac{3\left(-\frac{5\arctan(ax)^2a^3x^3}{8(a^2x^2+1)^2}-\frac{3\arctan(ax)^2ax}{8(a^2x^2+1)^2}-\frac{5\arctan(ax)^3}{24}-\frac{5\arctan(ax)}{8(a^2x^2+1)}+\frac{\arctan(ax)}{8(a^2x^2+1)^2}+\frac{17}{8}\right.}{4c^3}}{a^4}$ |
| default | $\frac{-\frac{\arctan(ax)^3}{2c^3(a^2x^2+1)}+\frac{\arctan(ax)^3}{4c^3(a^2x^2+1)^2}-\frac{3\left(-\frac{5\arctan(ax)^2a^3x^3}{8(a^2x^2+1)^2}-\frac{3\arctan(ax)^2ax}{8(a^2x^2+1)^2}-\frac{5\arctan(ax)^3}{24}-\frac{5\arctan(ax)}{8(a^2x^2+1)}+\frac{\arctan(ax)}{8(a^2x^2+1)^2}+\frac{17}{8}\right.}{4c^3}}{a^4}$ |
| parts | $-\frac{\arctan(ax)^3}{2c^3a^4(a^2x^2+1)}+\frac{\arctan(ax)^3}{4c^3a^4(a^2x^2+1)^2}-\frac{3\left(-\frac{5\arctan(ax)^2x^3}{8a(a^2x^2+1)^2}-\frac{3\arctan(ax)^2x}{8a^3(a^2x^2+1)^2}-\frac{5\arctan(ax)^3}{8a^4}+\frac{-\frac{5\arctan(ax)}{2(a^2x^2+1)}+\frac{\arctan}{2(a^2}}{}\right.}{4c^3}$ |
| risch | $\frac{i(5a^4x^4-6a^2x^2-3)\ln(iax+1)^3}{256a^4c^3(a^2x^2+1)^2}-\frac{3i(-6a^2x^2\ln(-iax+1)-3\ln(-iax+1)+5x^4\ln(-iax+1)a^4-10ia^3x^3-6iax)\ln(i}{256a^4(ax+i)^2(ax-i)^2c^3}$ |
| orering | Expression too large to display |

input
```
int(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/256*(40*arctan(a*x)^3*a^4*x^4-51*x^4*arctan(a*x)*a^4+120*a^3*arctan(a*x)
^2*x^3-48*arctan(a*x)^3*a^2*x^2-51*a^3*x^3+18*x^2*a^2*arctan(a*x)+72*a*arc
tan(a*x)^2*x-24*arctan(a*x)^3-45*a*x+45*arctan(a*x))/c^3/(a^2*x^2+1)^2/a^4
```

## Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.55

$$\int \frac{x^3\arctan(ax)^3}{(c+a^2cx^2)^3}\,dx = -\frac{51\,a^3x^3-8\left(5\,a^4x^4-6\,a^2x^2-3\right)\arctan\left(ax\right)^3-24\left(5\,a^3x^3+3\,ax\right)\arctan\left(ax\right)^2+45\,ax+3\left(17\,a^4x^4\right.}{256\left(a^8c^3x^4+2\,a^6c^3x^2+a^4c^3\right)}$$

input
```
integrate(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
-1/256*(51*a^3*x^3 - 8*(5*a^4*x^4 - 6*a^2*x^2 - 3)*arctan(a*x)^3 - 24*(5*a
^3*x^3 + 3*a*x)*arctan(a*x)^2 + 45*a*x + 3*(17*a^4*x^4 - 6*a^2*x^2 - 15)*a
rctan(a*x))/(a^8*c^3*x^4 + 2*a^6*c^3*x^2 + a^4*c^3)
```

## Sympy [F]

$$\int \frac{x^3 \arctan(ax)^3}{(c+a^2cx^2)^3}\,dx = \frac{\int \frac{x^3 \operatorname{atan}^3(ax)}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

input
```
integrate(x**3*atan(a*x)**3/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(x**3*atan(a*x)**3/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x)
/c**3
```

## Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.36

$$\int \frac{x^3 \arctan(ax)^3}{(c+a^2cx^2)^3}\,dx = \frac{3}{32}\,a\left(\frac{5\,a^2x^3+3\,x}{a^8c^3x^4+2\,a^6c^3x^2+a^4c^3}+\frac{5\,\arctan(ax)}{a^5c^3}\right)\arctan(ax)^2 - \frac{(2\,a^2x^2+1)\arctan(ax)^3}{4\,(a^8c^3x^4+2\,a^6c^3x^2+a^4c^3)} - \frac{1}{256}\left(\frac{(51\,a^3x^3-40\,(a^4x^4+2\,a^2x^2+1)\arctan(ax)^3+45\,ax+51\,(a^4x^4+2\,a^2x^2+1)\arctan(ax))a^2}{a^{11}c^3x^4+2\,a^9c^3x^2+a^7c^3}\right.$$

input
```
integrate(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
3/32*a*((5*a^2*x^3 + 3*x)/(a^8*c^3*x^4 + 2*a^6*c^3*x^2 + a^4*c^3) + 5*arct
an(a*x)/(a^5*c^3))*arctan(a*x)^2 - 1/4*(2*a^2*x^2 + 1)*arctan(a*x)^3/(a^8*
c^3*x^4 + 2*a^6*c^3*x^2 + a^4*c^3) - 1/256*((51*a^3*x^3 - 40*(a^4*x^4 + 2*
a^2*x^2 + 1)*arctan(a*x)^3 + 45*a*x + 51*(a^4*x^4 + 2*a^2*x^2 + 1)*arctan(
a*x))*a^2/(a^11*c^3*x^4 + 2*a^9*c^3*x^2 + a^7*c^3) - 24*(5*a^2*x^2 - 5*(a^
4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x)^2 + 4)*a*arctan(a*x)/(a^10*c^3*x^4 + 2*
a^8*c^3*x^2 + a^6*c^3))*a
```

## Giac [F]

$$\int \frac{x^3 \arctan(ax)^3}{(c+a^2cx^2)^3}\,dx = \int \frac{x^3 \arctan{(ax)}^3}{(a^2cx^2+c)^3}\,dx$$

input
```
integrate(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
integrate(x^3*arctan(a*x)^3/(a^2*c*x^2 + c)^3, x)
```

## Mupad [B] (verification not implemented)

Time = 0.85 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.97

$$\int \frac{x^3 \arctan(ax)^3}{(c+a^2cx^2)^3}\,dx = \frac{\mathrm{atan}(a\,x)^2\,\left(\frac{9\,x}{32\,a^5\,c^3}+\frac{15\,x^3}{32\,a^3\,c^3}\right)}{\frac{1}{a^2}+2\,x^2+a^2\,x^4} - \mathrm{atan}(a\,x)^3\,\left(\frac{\frac{1}{4\,a^6\,c^3}+\frac{x^2}{2\,a^4\,c^3}}{\frac{1}{a^2}+2\,x^2+a^2\,x^4}-\frac{5}{32\,a^4\,c^3}\right) - \frac{\frac{51\,a^2\,x^3}{8}+\frac{45\,x}{8}}{32\,a^7\,c^3\,x^4+64\,a^5\,c^3\,x^2+32\,a^3\,c^3} - \frac{51\,\mathrm{atan}(a\,x)}{256\,a^4\,c^3} + \frac{\mathrm{atan}(a\,x)\,\left(\frac{3}{8\,a^6\,c^3}+\frac{15\,x^2}{32\,a^4\,c^3}\right)}{\frac{1}{a^2}+2\,x^2+a^2\,x^4}$$

input
```
int((x^3*atan(a*x)^3)/(c + a^2*c*x^2)^3,x)
```

output
```
(atan(a*x)^2*((9*x)/(32*a^5*c^3) + (15*x^3)/(32*a^3*c^3)))/(1/a^2 + 2*x^2
+ a^2*x^4) - atan(a*x)^3*((1/(4*a^6*c^3) + x^2/(2*a^4*c^3))/(1/a^2 + 2*x^2
 + a^2*x^4) - 5/(32*a^4*c^3)) - ((45*x)/8 + (51*a^2*x^3)/8)/(32*a^3*c^3 +
64*a^5*c^3*x^2 + 32*a^7*c^3*x^4) - (51*atan(a*x))/(256*a^4*c^3) + (atan(a*
x)*(3/(8*a^6*c^3) + (15*x^2)/(32*a^4*c^3)))/(1/a^2 + 2*x^2 + a^2*x^4)
```

## Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.61

$$\int \frac{x^3 \arctan(ax)^3}{(c+a^2cx^2)^3}\,dx$$
$$= \frac{40atan(ax)^3 a^4x^4 - 48atan(ax)^3 a^2x^2 - 24atan(ax)^3 + 120atan(ax)^2 a^3x^3 + 72atan(ax)^2 ax - 51atan(\ldots}{256a^4c^3(a^4x^4+2a^2x^2+1)}$$

input
```
int(x^3*atan(a*x)^3/(a^2*c*x^2+c)^3,x)
```

output
```
(40*atan(a*x)**3*a**4*x**4 - 48*atan(a*x)**3*a**2*x**2 - 24*atan(a*x)**3 +
 120*atan(a*x)**2*a**3*x**3 + 72*atan(a*x)**2*a*x - 51*atan(a*x)*a**4*x**4
 + 18*atan(a*x)*a**2*x**2 + 45*atan(a*x) - 51*a**3*x**3 - 45*a*x)/(256*a**
4*c**3*(a**4*x**4 + 2*a**2*x**2 + 1))
```

# 3.405 $\int \frac{x^2 \arctan(ax)^3}{(c+a^2cx^2)^3}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 237

$$\begin{aligned}\int \frac{x^2 \arctan(ax)^3}{(c+a^2cx^2)^3}\,dx = {} & \frac{3}{128a^3c^3(1+a^2x^2)^2} - \frac{3}{128a^3c^3(1+a^2x^2)} \\ & + \frac{3x\arctan(ax)}{32a^2c^3(1+a^2x^2)^2} - \frac{3x\arctan(ax)}{64a^2c^3(1+a^2x^2)} \\ & - \frac{3\arctan(ax)^2}{128a^3c^3} - \frac{3\arctan(ax)^2}{16a^3c^3(1+a^2x^2)^2} + \frac{3\arctan(ax)^2}{16a^3c^3(1+a^2x^2)} \\ & - \frac{x\arctan(ax)^3}{4a^2c^3(1+a^2x^2)^2} + \frac{x\arctan(ax)^3}{8a^2c^3(1+a^2x^2)} + \frac{\arctan(ax)^4}{32a^3c^3}\end{aligned}$$

output
```
3/128/a^3/c^3/(a^2*x^2+1)^2-3/128/a^3/c^3/(a^2*x^2+1)+3/32*x*arctan(a*x)/a
^2/c^3/(a^2*x^2+1)^2-3/64*x*arctan(a*x)/a^2/c^3/(a^2*x^2+1)-3/128*arctan(a
*x)^2/a^3/c^3-3/16*arctan(a*x)^2/a^3/c^3/(a^2*x^2+1)^2+3/16*arctan(a*x)^2/
a^3/c^3/(a^2*x^2+1)-1/4*x*arctan(a*x)^3/a^2/c^3/(a^2*x^2+1)^2+1/8*x*arctan
(a*x)^3/a^2/c^3/(a^2*x^2+1)+1/32*arctan(a*x)^4/a^3/c^3
```

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.47

$$\int \frac{x^2 \arctan(ax)^3}{(c+a^2cx^2)^3}\,dx = \frac{-3a^2x^2+(6ax-6a^3x^3)\arctan(ax)-3(1-6a^2x^2+a^4x^4)\arctan(ax)^2+16ax(-1+a^2x^2)\arctan(ax)^3}{128a^3c^3\left(1+a^2x^2\right)^2}$$

input
```
Integrate[(x^2*ArcTan[a*x]^3)/(c + a^2*c*x^2)^3,x]
```

output
```
(-3*a^2*x^2 + (6*a*x - 6*a^3*x^3)*ArcTan[a*x] - 3*(1 - 6*a^2*x^2 + a^4*x^4
)*ArcTan[a*x]^2 + 16*a*x*(-1 + a^2*x^2)*ArcTan[a*x]^3 + 4*(1 + a^2*x^2)^2*
ArcTan[a*x]^4)/(128*a^3*c^3*(1 + a^2*x^2)^2)
```

## Rubi [A] (verified)

Time = 1.64 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.73, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5499, 27, 5427, 5435, 5427, 5431, 5427, 241, 5465, 5427, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \arctan(ax)^3}{(a^2cx^2+c)^3}\,dx$$

$$\Big\downarrow \; 5499$$

$$\frac{\int \frac{\arctan(ax)^3}{c^2(a^2x^2+1)^2}dx}{a^2c}-\frac{\int \frac{\arctan(ax)^3}{c^3(a^2x^2+1)^3}dx}{a^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\arctan(ax)^3}{(a^2x^2+1)^2}dx}{a^2c^3}-\frac{\int \frac{\arctan(ax)^3}{(a^2x^2+1)^3}dx}{a^2c^3}$$

$$\Big\downarrow \; 5427$$

$$\frac{-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}}{a^2c^3}-\frac{\int \frac{\arctan(ax)^3}{(a^2x^2+1)^3}dx}{a^2c^3}$$

$\downarrow$ 5435

$$\frac{-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}}{a^2c^3}-\frac{-\frac{3}{8}\int \frac{\arctan(ax)}{(a^2x^2+1)^3}dx+\frac{3}{4}\int \frac{\arctan(ax)^3}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{4(a^2x^2+1)^2}+\frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2}}{a^2c^3}$$

$\downarrow$ 5427

$$\frac{-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}}{a^2c^3}-\frac{-\frac{3}{8}\int \frac{\arctan(ax)}{(a^2x^2+1)^3}dx+\frac{3}{4}\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)+\frac{x\arctan(ax)^3}{4(a^2x^2+1)^2}+\frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2}}{a^2c^3}$$

$\downarrow$ 5431

$$\frac{-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}}{a^2c^3}-\frac{-\frac{3}{8}\left(\frac{3}{4}\int \frac{\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{1}{16a(a^2x^2+1)^2}\right)+\frac{3}{4}\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)+\frac{x\arcta}{4(a^2x}}{a^2c^3}$$

$\downarrow$ 5427

$$\frac{-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}}{a^2c^3}-\frac{-\frac{3}{8}\left(\frac{3}{4}\left(-\frac{1}{2}a\int \frac{x}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{1}{16a(a^2x^2+1)^2}\right)+\frac{3}{4}\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\right.}{a^2c^3}$$

$\downarrow$ 241

$$\frac{-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}}{a^2c^3}-\frac{\frac{3}{4}\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)+\frac{x\arctan(ax)^3}{4(a^2x^2+1)^2}+\frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2}-\frac{3}{8}\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax}{2(a^2x^2+1)}\right.\right.}{a^2c^3}$$

$\downarrow$ 5465

$$\frac{-\frac{3}{2}a\left(\frac{\int \frac{\arctan(ax)}{\left(a^2x^2+1\right)^2}dx}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}}{a^2c^3}-$$

$$\frac{\frac{3}{4}\left(-\frac{3}{2}a\left(\frac{\int \frac{\arctan(ax)}{\left(a^2x^2+1\right)^2}dx}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)+\frac{x\arctan(ax)^3}{4(a^2x^2+1)^2}+\frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2}-\frac{3}{8}\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}\right.}{a^2c^3}$$

$\Big\downarrow$ 5427

$$\frac{-\frac{3}{2}a\left(\frac{-\frac{1}{2}a\int \frac{x}{\left(a^2x^2+1\right)^2}dx+\frac{x\arctan(ax)}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^2}{4a}}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}}{a^2c^3}-$$

$$\frac{\frac{3}{4}\left(-\frac{3}{2}a\left(\frac{-\frac{1}{2}a\int \frac{x}{\left(a^2x^2+1\right)^2}dx+\frac{x\arctan(ax)}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^2}{4a}}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)+\frac{x\arctan(ax)^3}{4(a^2x^2+1)^2}+\frac{3a}{16a}}{a^2c^3}$$

$\Big\downarrow$ 241

$$\frac{\frac{x\arctan(ax)^3}{2(a^2x^2+1)}-\frac{3}{2}a\left(\frac{\frac{x\arctan(ax)}{2\left(a^2x^2+1\right)}+\frac{1}{4a\left(a^2x^2+1\right)}+\frac{\arctan(ax)^2}{4a}}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)+\frac{\arctan(ax)^4}{8a}}{a^2c^3}-$$

$$\frac{\frac{x\arctan(ax)^3}{4(a^2x^2+1)^2}+\frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2}-\frac{3}{8}\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}\right)+\frac{3}{4}\left(\frac{x\arctan}{2(a^2}\right.}{a^2c^3}$$

input
```
Int[(x^2*ArcTan[a*x]^3)/(c + a^2*c*x^2)^3,x]
```

output
```
((x*ArcTan[a*x]^3)/(2*(1 + a^2*x^2)) + ArcTan[a*x]^4/(8*a) - (3*a*(-1/2*Ar
cTan[a*x]^2/(a^2*(1 + a^2*x^2)) + (1/(4*a*(1 + a^2*x^2)) + (x*ArcTan[a*x])
/(2*(1 + a^2*x^2)) + ArcTan[a*x]^2/(4*a))/a))/2)/(a^2*c^3) - ((3*ArcTan[a*
x]^2)/(16*a*(1 + a^2*x^2)^2) + (x*ArcTan[a*x]^3)/(4*(1 + a^2*x^2)^2) - (3*
(1/(16*a*(1 + a^2*x^2)^2) + (x*ArcTan[a*x])/(4*(1 + a^2*x^2)^2) + (3*(1/(4
*a*(1 + a^2*x^2)) + (x*ArcTan[a*x])/(2*(1 + a^2*x^2)) + ArcTan[a*x]^2/(4*a
)))/4))/8 + (3*((x*ArcTan[a*x]^3)/(2*(1 + a^2*x^2)) + ArcTan[a*x]^4/(8*a)
- (3*a*(-1/2*ArcTan[a*x]^2/(a^2*(1 + a^2*x^2)) + (1/(4*a*(1 + a^2*x^2)) +
(x*ArcTan[a*x])/(2*(1 + a^2*x^2)) + ArcTan[a*x]^2/(4*a))/a))/2))/4)/(a^2*c
^3)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5431
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbol
] :> Simp[b*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (-Simp[x*(d + e*x
^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*d*(q + 1))), x] + Simp[(2*q + 3)/(2*d*(
q + 1))   Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a,
 b, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -3/2]
```

rule 5435
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[b*p*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p - 1)/(4*c*d*
(q + 1)^2)), x] + (-Simp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*d*
(q + 1))), x] + Simp[(2*q + 3)/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p, x], x] - Simp[b^2*p*((p - 1)/(4*(q + 1)^2))   Int[(d + e
*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5499
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ
[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

## Maple [A] (verified)

Time = 1.68 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.61

| method | result |
|---|---|
| parallelrisch | $\frac{4a^4\arctan(ax)^4x^4-3a^4\arctan(ax)^2x^4+16\arctan(ax)^3a^3x^3+8\arctan(ax)^4a^2x^2-6\arctan(ax)x^3a^3+18\arctan(ax)^2}{128c^3(a^2x^2+1)^2a^3}$ |
| derivativedivides | $\frac{\frac{\arctan(ax)^3a^3x^3}{8c^3(a^2x^2+1)^2}-\frac{\arctan(ax)^3ax}{8c^3(a^2x^2+1)^2}+\frac{\arctan(ax)^4}{8c^3}-\frac{3\left(\frac{\arctan(ax)^4}{4}-\frac{\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^2}{2(a^2x^2+1)^2}+\frac{\arctan(ax)a^3x^3}{8(a^2x^2+1)^2}-\frac{\arctan(ax)a}{8(a^2x^2+1)}\right.}{8c^3}}{a^3}$ |
| default | $\frac{\frac{\arctan(ax)^3a^3x^3}{8c^3(a^2x^2+1)^2}-\frac{\arctan(ax)^3ax}{8c^3(a^2x^2+1)^2}+\frac{\arctan(ax)^4}{8c^3}-\frac{3\left(\frac{\arctan(ax)^4}{4}-\frac{\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^2}{2(a^2x^2+1)^2}+\frac{\arctan(ax)a^3x^3}{8(a^2x^2+1)^2}-\frac{\arctan(ax)a}{8(a^2x^2+1)}\right.}{8c^3}}{a^3}$ |
| parts | $\frac{\arctan(ax)^3x^3}{8c^3(a^2x^2+1)^2}-\frac{x\arctan(ax)^3}{8a^2c^3(a^2x^2+1)^2}+\frac{\arctan(ax)^4}{8a^3c^3}-\frac{3\left(\frac{\arctan(ax)^4}{4a^3}+\frac{-\frac{\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^2}{2(a^2x^2+1)^2}+\frac{\arctan(ax)a^3x^3}{8(a^2x^2+1)^2}}{}\right.}{8c^3}$ |
| risch | $\frac{\ln(iax+1)^4}{512a^3c^3}-\frac{(x^4\ln(-iax+1)a^4+2a^2x^2\ln(-iax+1)-2ia^3x^3+\ln(-iax+1)+2iax)\ln(iax+1)^3}{128a^3c^3(a^2x^2+1)^2}+\frac{3(2a^4x^4\ln(-ia}{}$ |

input
```
int(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/128*(4*a^4*arctan(a*x)^4*x^4-3*a^4*arctan(a*x)^2*x^4+16*arctan(a*x)^3*a^
3*x^3+8*arctan(a*x)^4*a^2*x^2-6*arctan(a*x)*x^3*a^3+18*arctan(a*x)^2*x^2*a
^2-16*x*arctan(a*x)^3*a-3*a^2*x^2+4*arctan(a*x)^4+6*arctan(a*x)*a*x-3*arct
an(a*x)^2)/c^3/(a^2*x^2+1)^2/a^3
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.55

$$\int \frac{x^2 \arctan(ax)^3}{(c+a^2cx^2)^3}\,dx = \frac{4\,(a^4x^4+2\,a^2x^2+1)\arctan\,(ax)^4-3\,a^2x^2+16\,(a^3x^3-ax)\arctan\,(ax)^3-3\,(a^4x^4-6\,a^2x^2+1)\arctan}{128\,(a^7c^3x^4+2\,a^5c^3x^2+a^3c^3)}$$

input
```
integrate(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
1/128*(4*(a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x)^4 - 3*a^2*x^2 + 16*(a^3*x^3
 - a*x)*arctan(a*x)^3 - 3*(a^4*x^4 - 6*a^2*x^2 + 1)*arctan(a*x)^2 - 6*(a^3
*x^3 - a*x)*arctan(a*x))/(a^7*c^3*x^4 + 2*a^5*c^3*x^2 + a^3*c^3)
```

## Sympy [F]

$$\int \frac{x^2 \arctan(ax)^3}{(c+a^2cx^2)^3}\,dx = \frac{\int \frac{x^2 \operatorname{atan}^3(ax)}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

input
```
integrate(x**2*atan(a*x)**3/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(x**2*atan(a*x)**3/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x)
/c**3
```

## Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.41

$$\int \frac{x^2 \arctan(ax)^3}{(c+a^2cx^2)^3}\,dx = \frac{1}{8}\left(\frac{a^2x^3-x}{a^6c^3x^4+2\,a^4c^3x^2+a^2c^3}+\frac{\arctan\,(ax)}{a^3c^3}\right)\arctan\,(ax)^3$$
$$+\frac{3\,\left(a^2x^2-(a^4x^4+2\,a^2x^2+1)\arctan\,(ax)^2\right)a\arctan\,(ax)^2}{16\,(a^8c^3x^4+2\,a^6c^3x^2+a^4c^3)}$$
$$-\frac{1}{128}\left(\frac{\left(4\,(a^4x^4+2\,a^2x^2+1)\arctan\,(ax)^4+3\,a^2x^2-3\,(a^4x^4+2\,a^2x^2+1)\arctan\,(ax)^2\right)a^2}{a^{10}c^3x^4+2\,a^8c^3x^2+a^6c^3}+\frac{2\,(3\,a^3x}{}\right.$$

input
```
integrate(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
1/8*((a^2*x^3 - x)/(a^6*c^3*x^4 + 2*a^4*c^3*x^2 + a^2*c^3) + arctan(a*x)/(
a^3*c^3))*arctan(a*x)^3 + 3/16*(a^2*x^2 - (a^4*x^4 + 2*a^2*x^2 + 1)*arctan
(a*x)^2)*a*arctan(a*x)^2/(a^8*c^3*x^4 + 2*a^6*c^3*x^2 + a^4*c^3) - 1/128*(
(4*(a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x)^4 + 3*a^2*x^2 - 3*(a^4*x^4 + 2*a^
2*x^2 + 1)*arctan(a*x)^2)*a^2/(a^10*c^3*x^4 + 2*a^8*c^3*x^2 + a^6*c^3) + 2
*(3*a^3*x^3 - 8*(a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x)^3 - 3*a*x + 3*(a^4*x
^4 + 2*a^2*x^2 + 1)*arctan(a*x))*a*arctan(a*x)/(a^9*c^3*x^4 + 2*a^7*c^3*x^
2 + a^5*c^3))*a
```

## Giac [F]

$$\int \frac{x^2 \arctan(ax)^3}{(c+a^2cx^2)^3}\,dx = \int \frac{x^2 \arctan(ax)^3}{(a^2cx^2+c)^3}\,dx$$

input
```
integrate(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
integrate(x^2*arctan(a*x)^3/(a^2*c*x^2 + c)^3, x)
```

## Mupad [B] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.79

$$\begin{aligned}\int \frac{x^2 \arctan(ax)^3}{(c+a^2cx^2)^3}\,dx &= \frac{\mathrm{atan}(a\,x)\,\left(\frac{3\,x}{64\,a^4\,c^3}-\frac{3\,x^3}{64\,a^2\,c^3}\right)}{\frac{1}{a^2}+2\,x^2+a^2\,x^4} \\ &\quad - \frac{3\,x^2}{2\,\left(64\,a^5\,c^3\,x^4+128\,a^3\,c^3\,x^2+64\,a\,c^3\right)} \\ &\quad - \mathrm{atan}(a\,x)^2\left(\frac{3}{128\,a^3\,c^3}-\frac{3\,x^2}{16\,a^3\,c^3\,\left(\frac{1}{a^2}+2\,x^2+a^2\,x^4\right)}\right) \\ &\quad - \frac{\mathrm{atan}(a\,x)^3\,\left(\frac{x}{8\,a^4\,c^3}-\frac{x^3}{8\,a^2\,c^3}\right)}{\frac{1}{a^2}+2\,x^2+a^2\,x^4}+\frac{\mathrm{atan}(a\,x)^4}{32\,a^3\,c^3}\end{aligned}$$

input
```
int((x^2*atan(a*x)^3)/(c + a^2*c*x^2)^3,x)
```

output
```
(atan(a*x)*((3*x)/(64*a^4*c^3) - (3*x^3)/(64*a^2*c^3)))/(1/a^2 + 2*x^2 + a
^2*x^4) - (3*x^2)/(2*(64*a*c^3 + 128*a^3*c^3*x^2 + 64*a^5*c^3*x^4)) - atan
(a*x)^2*(3/(128*a^3*c^3) - (3*x^2)/(16*a^3*c^3*(1/a^2 + 2*x^2 + a^2*x^4)))
 - (atan(a*x)^3*(x/(8*a^4*c^3) - x^3/(8*a^2*c^3)))/(1/a^2 + 2*x^2 + a^2*x^
4) + atan(a*x)^4/(32*a^3*c^3)
```

## Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.65

$$\int \frac{x^2 \arctan(ax)^3}{(c + a^2cx^2)^3} \, dx = \frac{8atan(ax)^4 a^4x^4 + 16atan(ax)^4 a^2x^2 + 8atan(ax)^4 + 32atan(ax)^3 a^3x^3 - 32atan(ax)^3 ax - 6atan(ax)^2 \cdots}{256a^3c^3 (a^4x^4 + 2a^2x^2 \cdots}$$

input
```
int(x^2*atan(a*x)^3/(a^2*c*x^2+c)^3,x)
```

output
```
(8*atan(a*x)**4*a**4*x**4 + 16*atan(a*x)**4*a**2*x**2 + 8*atan(a*x)**4 + 3
2*atan(a*x)**3*a**3*x**3 - 32*atan(a*x)**3*a*x - 6*atan(a*x)**2*a**4*x**4
+ 36*atan(a*x)**2*a**2*x**2 - 6*atan(a*x)**2 - 12*atan(a*x)*a**3*x**3 + 12
*atan(a*x)*a*x + 3*a**4*x**4 + 3)/(256*a**3*c**3*(a**4*x**4 + 2*a**2*x**2
+ 1))
```

# 3.406 $\int \frac{x \arctan(ax)^3}{(c+a^2cx^2)^3} dx$



## Optimal result

Integrand size = 20, antiderivative size = 208

$$\int \frac{x \arctan(ax)^3}{(c+a^2cx^2)^3} dx = -\frac{3x}{128ac^3(1+a^2x^2)^2} - \frac{45x}{256ac^3(1+a^2x^2)} - \frac{45\arctan(ax)}{256a^2c^3} + \frac{3\arctan(ax)}{32a^2c^3(1+a^2x^2)^2} + \frac{9\arctan(ax)}{32a^2c^3(1+a^2x^2)} + \frac{3x\arctan(ax)^2}{16ac^3(1+a^2x^2)^2} + \frac{9x\arctan(ax)^2}{32ac^3(1+a^2x^2)} + \frac{3\arctan(ax)^3}{32a^2c^3} - \frac{\arctan(ax)^3}{4a^2c^3(1+a^2x^2)^2}$$

output
```
-3/128*x/a/c^3/(a^2*x^2+1)^2-45/256*x/a/c^3/(a^2*x^2+1)-45/256*arctan(a*x)
/a^2/c^3+3/32*arctan(a*x)/a^2/c^3/(a^2*x^2+1)^2+9/32*arctan(a*x)/a^2/c^3/(
a^2*x^2+1)+3/16*x*arctan(a*x)^2/a/c^3/(a^2*x^2+1)^2+9/32*x*arctan(a*x)^2/a
/c^3/(a^2*x^2+1)+3/32*arctan(a*x)^3/a^2/c^3-1/4*arctan(a*x)^3/a^2/c^3/(a^2
*x^2+1)^2
```

## Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.50

$$\int \frac{x\arctan(ax)^3}{(c+a^2cx^2)^3}\,dx = \frac{-3ax(17+15a^2x^2)-3(-17+6a^2x^2+15a^4x^4)\arctan(ax)+24ax(5+3a^2x^2)\arctan(ax)^2+8(-5+6$$
$$256c^3\,(a+a^3x^2)^2$$

input
```
Integrate[(x*ArcTan[a*x]^3)/(c + a^2*c*x^2)^3,x]
```

output
```
(-3*a*x*(17 + 15*a^2*x^2) - 3*(-17 + 6*a^2*x^2 + 15*a^4*x^4)*ArcTan[a*x] +
 24*a*x*(5 + 3*a^2*x^2)*ArcTan[a*x]^2 + 8*(-5 + 6*a^2*x^2 + 3*a^4*x^4)*Arc
Tan[a*x]^3)/(256*c^3*(a + a^3*x^2)^2)
```

## Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.14, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5465, 27, 5435, 215, 215, 216, 5427, 5465, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\arctan(ax)^3}{(a^2cx^2+c)^3}\,dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{3\int \frac{\arctan(ax)^2}{c^3(a^2x^2+1)^3}dx}{4a}-\frac{\arctan(ax)^3}{4a^2c^3\,(a^2x^2+1)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\int \frac{\arctan(ax)^2}{(a^2x^2+1)^3}dx}{4ac^3}-\frac{\arctan(ax)^3}{4a^2c^3\,(a^2x^2+1)^2}$$

$$\Big\downarrow \text{5435}$$

$$\frac{3\left(\frac{3}{4}\int \frac{\arctan(ax)^2}{(a^2x^2+1)^2}dx-\frac{1}{8}\int \frac{1}{(a^2x^2+1)^3}dx+\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}\right)}{4ac^3}-\frac{\arctan(ax)^3}{4a^2c^3\left(a^2x^2+1\right)^2}$$

$\Big\downarrow$ 215

$$\frac{3\left(\frac{3}{4}\int \frac{\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{1}{8}\left(-\frac{3}{4}\int \frac{1}{(a^2x^2+1)^2}dx-\frac{x}{4(a^2x^2+1)^2}\right)+\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}\right)}{4ac^3}-\frac{\arctan(ax)^3}{4a^2c^3\left(a^2x^2+1\right)^2}$$

$\Big\downarrow$ 215

$$\frac{3\left(\frac{3}{4}\int \frac{\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{a^2x^2+1}dx+\frac{x}{2(a^2x^2+1)}\right)-\frac{x}{4(a^2x^2+1)^2}\right)+\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}\right)}{4ac^3}-\frac{\arctan(ax)^3}{4a^2c^3\left(a^2x^2+1\right)^2}$$

$\Big\downarrow$ 216

$$\frac{3\left(\frac{3}{4}\int \frac{\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}\right)-\frac{x}{4(a^2x^2+1)^2}\right)\right)}{4ac^3}-\frac{\arctan(ax)^3}{4a^2c^3\left(a^2x^2+1\right)^2}$$

$\Big\downarrow$ 5427

$$\frac{3\left(\frac{3}{4}\left(-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)+\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}\right.\right.\right.}{4ac^3}$$

$$\frac{\arctan(ax)^3}{4a^2c^3\left(a^2x^2+1\right)^2}$$

$\Big\downarrow$ 5465

$$\frac{3\left(\frac{3}{4}\left(-a\left(\frac{\int \frac{1}{(a^2x^2+1)^2}dx}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)+\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{3}{2(a^2x\right.\right.\right.}{4ac^3}$$

$$\frac{\arctan(ax)^3}{4a^2c^3\left(a^2x^2+1\right)^2}$$

$\Big\downarrow$ 215

$$\frac{3\left(\frac{3}{4}\left(-a\left(\frac{\frac{1}{2}\int\frac{1}{a^2x^2+1}dx+\frac{x}{2(a^2x^2+1)}}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)+\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\left(-\right.}{4ac^3}$$

$$\frac{\arctan(ax)^3}{4a^2c^3\left(a^2x^2+1\right)^2}$$

$\downarrow$ 216

$$\frac{3\left(\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}\right)-\frac{x}{4(a^2x^2+1)^2}\right)+\frac{3}{4}\left(\frac{x\arctan(ax)^2}{2(a^2x^2+1)}-a\left(\frac{\frac{x}{2(a^2x^2+1)}+\text{arct}}{2a}\right.\right.\right.}{4ac^3}$$

$$\frac{\arctan(ax)^3}{4a^2c^3\left(a^2x^2+1\right)^2}$$

input
```
Int[(x*ArcTan[a*x]^3)/(c + a^2*c*x^2)^3,x]
```

output
```
-1/4*ArcTan[a*x]^3/(a^2*c^3*(1 + a^2*x^2)^2) + (3*(ArcTan[a*x]/(8*a*(1 + a
^2*x^2)^2) + (x*ArcTan[a*x]^2)/(4*(1 + a^2*x^2)^2) + (-1/4*x/(1 + a^2*x^2)
^2 - (3*(x/(2*(1 + a^2*x^2)) + ArcTan[a*x]/(2*a)))/4)/8 + (3*((x*ArcTan[a*
x]^2)/(2*(1 + a^2*x^2)) + ArcTan[a*x]^3/(6*a) - a*(-1/2*ArcTan[a*x]/(a^2*(
1 + a^2*x^2)) + (x/(2*(1 + a^2*x^2)) + ArcTan[a*x]/(2*a))/(2*a))))/4))/(4*
a*c^3)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5435
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[b*p*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p - 1)/(4*c*d*
(q + 1)^2)), x] + (-Simp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*d*
(q + 1))), x] + Simp[(2*q + 3)/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p, x], x] - Simp[b^2*p*((p - 1)/(4*(q + 1)^2))   Int[(d + e
*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

## Maple [A] (verified)

Time = 1.67 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.59

| method | result |
|---|---|
| parallelrisch | $\frac{24\arctan(ax)^3a^4x^4-45x^4\arctan(ax)a^4+72a^3\arctan(ax)^2x^3+48\arctan(ax)^3a^2x^2-45a^3x^3-18x^2a^2\arctan(ax)+12}{256c^3(a^2x^2+1)^2a^2}$ |
| derivativedivides | $\frac{-\frac{\arctan(ax)^3}{4c^3(a^2x^2+1)^2}+\frac{\frac{3\arctan(ax)^2ax}{16(a^2x^2+1)^2}+\frac{9a\arctan(ax)^2x}{32(a^2x^2+1)}+\frac{3\arctan(ax)^3}{32}+\frac{9\arctan(ax)}{32(a^2x^2+1)}+\frac{3\arctan(ax)}{32(a^2x^2+1)^2}-\frac{3(\frac{15}{8}a^3x^3+\frac{17}{8}ax)}{32(a^2x^2+1)^2}-45}{c^3}}{a^2}$ |
| default | $\frac{-\frac{\arctan(ax)^3}{4c^3(a^2x^2+1)^2}+\frac{\frac{3\arctan(ax)^2ax}{16(a^2x^2+1)^2}+\frac{9a\arctan(ax)^2x}{32(a^2x^2+1)}+\frac{3\arctan(ax)^3}{32}+\frac{9\arctan(ax)}{32(a^2x^2+1)}+\frac{3\arctan(ax)}{32(a^2x^2+1)^2}-\frac{3(\frac{15}{8}a^3x^3+\frac{17}{8}ax)}{32(a^2x^2+1)^2}-45}{c^3}}{a^2}$ |
| parts | $-\frac{\arctan(ax)^3}{4a^2c^3(a^2x^2+1)^2}+\frac{\frac{3\arctan(ax)^2ax}{16(a^2x^2+1)^2}+\frac{9a\arctan(ax)^2x}{32(a^2x^2+1)}+\frac{3\arctan(ax)^3}{32}+\frac{9\arctan(ax)}{32(a^2x^2+1)}+\frac{3\arctan(ax)}{32(a^2x^2+1)^2}-\frac{3(\frac{15}{8}a^3x^3+\frac{17}{8}a}{32(a^2x^2+1)^2}}{a^2c^3}$ |
| risch | $\frac{i(3a^4x^4+6a^2x^2-5)\ln(iax+1)^3}{256a^2c^3(a^2x^2+1)^2}-\frac{3i(-5\ln(-iax+1)+3x^4\ln(-iax+1)a^4+6a^2x^2\ln(-iax+1)-6ia^3x^3-10iax)\ln(i}{256(ax+i)^2c^3(ax-i)^2a^2}$ |
| orering | $-\frac{(a^2x^2+1)(450a^8x^8+444a^6x^6-79a^4x^4+40a^2x^2-17)\arctan(ax)^3}{64a^4x^2(a^2cx^2+c)^3}-\frac{(a^2x^2+1)^2(1035a^6x^6+948a^4x^4-239a^2x^2+}{}$ |

input
```
int(x*arctan(a*x)^3/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/256*(24*arctan(a*x)^3*a^4*x^4-45*x^4*arctan(a*x)*a^4+72*a^3*arctan(a*x)^
2*x^3+48*arctan(a*x)^3*a^2*x^2-45*a^3*x^3-18*x^2*a^2*arctan(a*x)+120*a*arc
tan(a*x)^2*x-40*arctan(a*x)^3-51*a*x+51*arctan(a*x))/c^3/(a^2*x^2+1)^2/a^2
```

## Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.56

$$\int \frac{x\arctan(ax)^3}{(c+a^2cx^2)^3}\,dx = -\frac{45\,a^3x^3-8\,(3\,a^4x^4+6\,a^2x^2-5)\arctan(ax)^3-24\,(3\,a^3x^3+5\,ax)\arctan(ax)^2+51\,ax+3\,(15\,a^4x^4}{256\,(a^6c^3x^4+2\,a^4c^3x^2+a^2c^3)}$$

input
```
integrate(x*arctan(a*x)^3/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
-1/256*(45*a^3*x^3 - 8*(3*a^4*x^4 + 6*a^2*x^2 - 5)*arctan(a*x)^3 - 24*(3*a
^3*x^3 + 5*a*x)*arctan(a*x)^2 + 51*a*x + 3*(15*a^4*x^4 + 6*a^2*x^2 - 17)*a
rctan(a*x))/(a^6*c^3*x^4 + 2*a^4*c^3*x^2 + a^2*c^3)
```

## Sympy [F]

$$\int \frac{x \arctan(ax)^3}{(c + a^2cx^2)^3} \, dx = \frac{\int \frac{x \operatorname{atan}^3(ax)}{a^6x^6+3a^4x^4+3a^2x^2+1} \, dx}{c^3}$$

input
```
integrate(x*atan(a*x)**3/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(x*atan(a*x)**3/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x)/c*
*3
```

## Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.31

$$\int \frac{x \arctan(ax)^3}{(c + a^2cx^2)^3} \, dx = \frac{3 \left(\frac{3\,a^2x^3+5\,x}{a^4c^2x^4+2\,a^2c^2x^2+c^2} + \frac{3\,\arctan(ax)}{ac^2}\right) \arctan(ax)^2}{32\,ac} - \frac{3 \left(\frac{\left(15\,a^3x^3-8\,(a^4x^4+2\,a^2x^2+1)\arctan(ax)^3+17\,ax+15\,(a^4x^4+2\,a^2x^2+1)\arctan(ax)\right)a^2}{a^7c^2x^4+2\,a^5c^2x^2+a^3c^2} - \frac{8\left(3\,a^2x^2-3\,(a^4x^4+2\,a^2x^2+1)\arctan(ax\right.}{a^6c^2x^4+2\,a^4c^2x^2+a^2c}\right.}{256\,ac} - \frac{\arctan(ax)^3}{4\,(a^2cx^2+c)^2a^2c}$$

input
```
integrate(x*arctan(a*x)^3/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
3/32*((3*a^2*x^3 + 5*x)/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2) + 3*arctan(a*x
)/(a*c^2))*arctan(a*x)^2/(a*c) - 3/256*((15*a^3*x^3 - 8*(a^4*x^4 + 2*a^2*x
^2 + 1)*arctan(a*x)^3 + 17*a*x + 15*(a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x))
*a^2/(a^7*c^2*x^4 + 2*a^5*c^2*x^2 + a^3*c^2) - 8*(3*a^2*x^2 - 3*(a^4*x^4 +
 2*a^2*x^2 + 1)*arctan(a*x)^2 + 4)*a*arctan(a*x)/(a^6*c^2*x^4 + 2*a^4*c^2*
x^2 + a^2*c^2))/(a*c) - 1/4*arctan(a*x)^3/((a^2*c*x^2 + c)^2*a^2*c)
```

## Giac [F]

$$\int \frac{x \arctan(ax)^3}{(c+a^2cx^2)^3}\,dx = \int \frac{x \arctan(ax)^3}{(a^2cx^2+c)^3}\,dx$$

input
```
integrate(x*arctan(a*x)^3/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
integrate(x*arctan(a*x)^3/(a^2*c*x^2 + c)^3, x)
```

## Mupad [B] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.91

$$\int \frac{x \arctan(ax)^3}{(c+a^2cx^2)^3}\,dx = \mathrm{atan}(a\,x)^3\left(\frac{3}{32\,a^2\,c^3}-\frac{1}{4\,a^4\,c^3\,\left(\frac{1}{a^2}+2\,x^2+a^2\,x^4\right)}\right) - \frac{\frac{45\,a^2\,x^3}{8}+\frac{51\,x}{8}}{32\,a^5\,c^3\,x^4+64\,a^3\,c^3\,x^2+32\,a\,c^3} + \frac{\mathrm{atan}(a\,x)^2\,\left(\frac{15\,x}{32\,a^3\,c^3}+\frac{9\,x^3}{32\,a\,c^3}\right)}{\frac{1}{a^2}+2\,x^2+a^2\,x^4} - \frac{45\,\mathrm{atan}(a\,x)}{256\,a^2\,c^3} + \frac{\mathrm{atan}(a\,x)\,\left(\frac{3}{8\,a^4\,c^3}+\frac{9\,x^2}{32\,a^2\,c^3}\right)}{\frac{1}{a^2}+2\,x^2+a^2\,x^4}$$

input
```
int((x*atan(a*x)^3)/(c + a^2*c*x^2)^3,x)
```

output
```
atan(a*x)^3*(3/(32*a^2*c^3) - 1/(4*a^4*c^3*(1/a^2 + 2*x^2 + a^2*x^4))) - (
(51*x)/8 + (45*a^2*x^3)/8)/(32*a*c^3 + 64*a^3*c^3*x^2 + 32*a^5*c^3*x^4) +
(atan(a*x)^2*((15*x)/(32*a^3*c^3) + (9*x^3)/(32*a*c^3)))/(1/a^2 + 2*x^2 +
a^2*x^4) - (45*atan(a*x))/(256*a^2*c^3) + (atan(a*x)*(3/(8*a^4*c^3) + (9*x
^2)/(32*a^2*c^3)))/(1/a^2 + 2*x^2 + a^2*x^4)
```

## Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.62

$$\int \frac{x \arctan(ax)^3}{(c + a^2cx^2)^3} \, dx$$
$$= \frac{24atan(ax)^3 a^4x^4 + 48atan(ax)^3 a^2x^2 - 40atan(ax)^3 + 72atan(ax)^2 a^3x^3 + 120atan(ax)^2 ax - 45atan(}{256a^2c^3 (a^4x^4 + 2a^2x^2 + 1)}$$

input
```
int(x*atan(a*x)^3/(a^2*c*x^2+c)^3,x)
```

output
```
(24*atan(a*x)**3*a**4*x**4 + 48*atan(a*x)**3*a**2*x**2 - 40*atan(a*x)**3 +
 72*atan(a*x)**2*a**3*x**3 + 120*atan(a*x)**2*a*x - 45*atan(a*x)*a**4*x**4
 - 18*atan(a*x)*a**2*x**2 + 51*atan(a*x) - 45*a**3*x**3 - 51*a*x)/(256*a**
2*c**3*(a**4*x**4 + 2*a**2*x**2 + 1))
```

# 3.407 $\int \frac{\arctan(ax)^3}{(c+a^2cx^2)^3} dx$



## Optimal result

Integrand size = 19, antiderivative size = 225

$$\int \frac{\arctan(ax)^3}{(c+a^2cx^2)^3} dx = -\frac{3}{128ac^3(1+a^2x^2)^2} - \frac{45}{128ac^3(1+a^2x^2)} - \frac{3x\arctan(ax)}{32c^3(1+a^2x^2)^2} - \frac{45x\arctan(ax)}{64c^3(1+a^2x^2)} - \frac{45\arctan(ax)^2}{128ac^3} + \frac{3\arctan(ax)^2}{16ac^3(1+a^2x^2)^2} + \frac{9\arctan(ax)^2}{16ac^3(1+a^2x^2)} + \frac{x\arctan(ax)^3}{4c^3(1+a^2x^2)^2} + \frac{3x\arctan(ax)^3}{8c^3(1+a^2x^2)} + \frac{3\arctan(ax)^4}{32ac^3}$$

output
```
-3/128/a/c^3/(a^2*x^2+1)^2-45/128/a/c^3/(a^2*x^2+1)-3/32*x*arctan(a*x)/c^3
/(a^2*x^2+1)^2-45/64*x*arctan(a*x)/c^3/(a^2*x^2+1)-45/128*arctan(a*x)^2/a/
c^3+3/16*arctan(a*x)^2/a/c^3/(a^2*x^2+1)^2+9/16*arctan(a*x)^2/a/c^3/(a^2*x
^2+1)+1/4*x*arctan(a*x)^3/c^3/(a^2*x^2+1)^2+3/8*x*arctan(a*x)^3/c^3/(a^2*x
^2+1)+3/32*arctan(a*x)^4/a/c^3
```

## Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.51

$$\int \frac{\arctan(ax)^3}{(c+a^2cx^2)^3}\,dx = -\frac{48+45a^2x^2+6ax(17+15a^2x^2)\arctan(ax)+3(-17+6a^2x^2+15a^4x^4)\arctan(ax)^2-16ax(5+3a^2}{128ac^3(1+a^2x^2)^2}$$

input
```
Integrate[ArcTan[a*x]^3/(c + a^2*c*x^2)^3,x]
```

output
```
-1/128*(48 + 45*a^2*x^2 + 6*a*x*(17 + 15*a^2*x^2)*ArcTan[a*x] + 3*(-17 + 6
*a^2*x^2 + 15*a^4*x^4)*ArcTan[a*x]^2 - 16*a*x*(5 + 3*a^2*x^2)*ArcTan[a*x]^
3 - 12*(1 + a^2*x^2)^2*ArcTan[a*x]^4)/(a*c^3*(1 + a^2*x^2)^2)
```

## Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.26, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {5435, 27, 5427, 5431, 5427, 241, 5465, 5427, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3}{(a^2cx^2+c)^3}\,dx$$

$$\Big\downarrow \text{5435}$$

$$-\frac{3}{8}\int \frac{\arctan(ax)}{c^3(a^2x^2+1)^3}dx+\frac{3\int \frac{\arctan(ax)^3}{c^2(a^2x^2+1)^2}dx}{4c}+\frac{x\arctan(ax)^3}{4c^3(a^2x^2+1)^2}+\frac{3\arctan(ax)^2}{16ac^3(a^2x^2+1)^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3\int \frac{\arctan(ax)}{(a^2x^2+1)^3}dx}{8c^3}+\frac{3\int \frac{\arctan(ax)^3}{(a^2x^2+1)^2}dx}{4c^3}+\frac{x\arctan(ax)^3}{4c^3(a^2x^2+1)^2}+\frac{3\arctan(ax)^2}{16ac^3(a^2x^2+1)^2}$$

$$\Big\downarrow \text{5427}$$

$$-\frac{3\int \frac{\arctan(ax)}{(a^2x^2+1)^3}dx}{8c^3}+\frac{3\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)}{4c^3}+\frac{x\arctan(ax)^3}{4c^3\left(a^2x^2+1\right)^2}+\frac{3\arctan(ax)^2}{16ac^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \text{5431}$$

$$-\frac{3\left(\frac{3}{4}\int \frac{\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{1}{16a(a^2x^2+1)^2}\right)}{8c^3}+\frac{3\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)}{4c^3}+\frac{x\arctan(ax)^3}{4c^3\left(a^2x^2+1\right)^2}+\frac{3\arctan(ax)^2}{16ac^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \text{5427}$$

$$-\frac{3\left(\frac{3}{4}\left(-\frac{1}{2}a\int \frac{x}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{1}{16a(a^2x^2+1)^2}\right)}{8c^3}+\frac{3\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)}{4c^3}+\frac{x\arctan(ax)^3}{4c^3\left(a^2x^2+1\right)^2}+\frac{3\arctan(ax)^2}{16ac^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \text{241}$$

$$\frac{3\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)}{4c^3}+\frac{x\arctan(ax)^3}{4c^3\left(a^2x^2+1\right)^2}+\frac{3\arctan(ax)^2}{16ac^3\left(a^2x^2+1\right)^2}-\frac{3\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}\right)}{8c^3}$$

$$\Big\downarrow \text{5465}$$

$$\frac{3\left(-\frac{3}{2}a\left(\frac{\int \frac{\arctan(ax)}{\left(a^2x^2+1\right)^2}dx}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)}{4c^3}+\frac{x\arctan(ax)^3}{4c^3\left(a^2x^2+1\right)^2}+\frac{3\arctan(ax)^2}{16ac^3\left(a^2x^2+1\right)^2}-\frac{3\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}\right)}{8c^3}$$

$$\Big\downarrow \text{5427}$$

$$\frac{3\left(-\frac{3}{2}a\left(\frac{-\frac{1}{2}a\int\frac{x}{\left(a^2x^2+1\right)^2}dx+\frac{x\arctan(ax)}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^2}{4a}}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)}{4c^3}+$$
$$\frac{x\arctan(ax)^3}{4c^3\left(a^2x^2+1\right)^2}+\frac{3\arctan(ax)^2}{16ac^3\left(a^2x^2+1\right)^2}-$$
$$\frac{3\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}\right)}{8c^3}$$

$$\Big\downarrow \ 241$$

$$\frac{x\arctan(ax)^3}{4c^3\left(a^2x^2+1\right)^2}+\frac{3\arctan(ax)^2}{16ac^3\left(a^2x^2+1\right)^2}-$$
$$\frac{3\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}\right)}{8c^3}+$$
$$\frac{3\left(\frac{x\arctan(ax)^3}{2(a^2x^2+1)}-\frac{3}{2}a\left(\frac{\frac{x\arctan(ax)}{2\left(a^2x^2+1\right)}+\frac{1}{4a\left(a^2x^2+1\right)}+\frac{\arctan(ax)^2}{4a}}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)+\frac{\arctan(ax)^4}{8a}\right)}{4c^3}$$

input
```
Int[ArcTan[a*x]^3/(c + a^2*c*x^2)^3,x]
```

output
```
(3*ArcTan[a*x]^2)/(16*a*c^3*(1 + a^2*x^2)^2) + (x*ArcTan[a*x]^3)/(4*c^3*(1
 + a^2*x^2)^2) - (3*(1/(16*a*(1 + a^2*x^2)^2) + (x*ArcTan[a*x])/(4*(1 + a^
2*x^2)^2) + (3*(1/(4*a*(1 + a^2*x^2)) + (x*ArcTan[a*x])/(2*(1 + a^2*x^2))
+ ArcTan[a*x]^2/(4*a)))/4))/(8*c^3) + (3*((x*ArcTan[a*x]^3)/(2*(1 + a^2*x^
2)) + ArcTan[a*x]^4/(8*a) - (3*a*(-1/2*ArcTan[a*x]^2/(a^2*(1 + a^2*x^2)) +
 (1/(4*a*(1 + a^2*x^2)) + (x*ArcTan[a*x])/(2*(1 + a^2*x^2)) + ArcTan[a*x]^
2/(4*a))/a))/2))/(4*c^3)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5431
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbol
] :> Simp[b*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (-Simp[x*(d + e*x
^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*d*(q + 1))), x] + Simp[(2*q + 3)/(2*d*(
q + 1))   Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a,
 b, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -3/2]
```

rule 5435
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[b*p*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p - 1)/(4*c*d*
(q + 1)^2)), x] + (-Simp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*d*
(q + 1))), x] + Simp[(2*q + 3)/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p, x], x] - Simp[b^2*p*((p - 1)/(4*(q + 1)^2))   Int[(d + e
*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

## Maple [A] (verified)

Time = 1.79 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.68

| method | result |
|---|---|
| parallelrisch | $\frac{12a^4\arctan(ax)^4x^4-45a^4\arctan(ax)^2x^4+48\arctan(ax)^3a^3x^3+48a^4x^4+24\arctan(ax)^4a^2x^2-90\arctan(ax)x^3a^3-1}{128c^3(a^2x^2+1)^2a}$ |
| derivativedivides | $\frac{\frac{\arctan(ax)^3ax}{4c^3(a^2x^2+1)^2}+\frac{3\arctan(ax)^3ax}{8c^3(a^2x^2+1)}+\frac{3\arctan(ax)^4}{8c^3}-\frac{3\left(-\frac{\arctan(ax)^2}{2(a^2x^2+1)^2}-\frac{3\arctan(ax)^2}{2(a^2x^2+1)}+\frac{15\arctan(ax)a^3x^3}{8(a^2x^2+1)^2}+\frac{17\arctan(ax)ax}{8(a^2x^2+1)^2}+1\right.}{8c^3}}{a}$ |
| default | $\frac{\frac{\arctan(ax)^3ax}{4c^3(a^2x^2+1)^2}+\frac{3\arctan(ax)^3ax}{8c^3(a^2x^2+1)}+\frac{3\arctan(ax)^4}{8c^3}-\frac{3\left(-\frac{\arctan(ax)^2}{2(a^2x^2+1)^2}-\frac{3\arctan(ax)^2}{2(a^2x^2+1)}+\frac{15\arctan(ax)a^3x^3}{8(a^2x^2+1)^2}+\frac{17\arctan(ax)ax}{8(a^2x^2+1)^2}+1\right.}{8c^3}}{a}$ |
| parts | $\frac{x\arctan(ax)^3}{4c^3(a^2x^2+1)^2}+\frac{3x\arctan(ax)^3}{8c^3(a^2x^2+1)}+\frac{3\arctan(ax)^4}{8a\,c^3}-\frac{3\left(\frac{-\frac{\arctan(ax)^2}{2(a^2x^2+1)^2}-\frac{3\arctan(ax)^2}{2(a^2x^2+1)}+\frac{15\arctan(ax)a^3x^3}{8(a^2x^2+1)^2}+\frac{17\arctan}{8(a^2x^2}}{a}\right.}{}$ |
| risch | $\frac{3\ln(iax+1)^4}{512a\,c^3}-\frac{(3x^4\ln(-iax+1)a^4+6a^2x^2\ln(-iax+1)-6ia^3x^3+3\ln(-iax+1)-10iax)\ln(iax+1)^3}{128c^3(a^2x^2+1)^2a}+\frac{3\left(6a^4x^4\ln\right.}{}$ |

input
```
int(arctan(a*x)^3/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/128*(12*a^4*arctan(a*x)^4*x^4-45*a^4*arctan(a*x)^2*x^4+48*arctan(a*x)^3*
a^3*x^3+48*a^4*x^4+24*arctan(a*x)^4*a^2*x^2-90*arctan(a*x)*x^3*a^3-18*arct
an(a*x)^2*x^2*a^2+80*x*arctan(a*x)^3*a+51*a^2*x^2+12*arctan(a*x)^4-102*arc
tan(a*x)*a*x+51*arctan(a*x)^2)/c^3/(a^2*x^2+1)^2/a
```

## Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.59

$$\int\frac{\arctan(ax)^3}{(c+a^2cx^2)^3}\,dx$$
$$=\frac{12\left(a^4x^4+2\,a^2x^2+1\right)\arctan\left(ax\right)^4-45\,a^2x^2+16\left(3\,a^3x^3+5\,ax\right)\arctan\left(ax\right)^3-3\left(15\,a^4x^4+6\,a^2x^2-\right.}{128\left(a^5c^3x^4+2\,a^3c^3x^2+ac^3\right)}$$

input
```
integrate(arctan(a*x)^3/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
1/128*(12*(a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x)^4 - 45*a^2*x^2 + 16*(3*a^3
*x^3 + 5*a*x)*arctan(a*x)^3 - 3*(15*a^4*x^4 + 6*a^2*x^2 - 17)*arctan(a*x)^
2 - 6*(15*a^3*x^3 + 17*a*x)*arctan(a*x) - 48)/(a^5*c^3*x^4 + 2*a^3*c^3*x^2
 + a*c^3)
```

## Sympy [F]

$$\int \frac{\arctan(ax)^3}{(c+a^2cx^2)^3}\,dx = \frac{\int \frac{\operatorname{atan}^3(ax)}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

input
```
integrate(atan(a*x)**3/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(atan(a*x)**3/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x)/c**3
```

## Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.49

$$\int \frac{\arctan(ax)^3}{(c+a^2cx^2)^3}\,dx = \frac{1}{8}\left(\frac{3\,a^2x^3+5\,x}{a^4c^3x^4+2\,a^2c^3x^2+c^3}+\frac{3\,\arctan(ax)}{ac^3}\right)\arctan(ax)^3 + \frac{3\left(3\,a^2x^2-3\left(a^4x^4+2\,a^2x^2+1\right)\arctan(ax)^2+4\right)a\arctan(ax)^2}{16\left(a^6c^3x^4+2\,a^4c^3x^2+a^2c^3\right)} - \frac{3}{128}\left(\frac{\left(4\left(a^4x^4+2\,a^2x^2+1\right)\arctan(ax)^4+15\,a^2x^2-15\left(a^4x^4+2\,a^2x^2+1\right)\arctan(ax)^2+16\right)a^2}{a^8c^3x^4+2\,a^6c^3x^2+a^4c^3}+\right.$$

input
```
integrate(arctan(a*x)^3/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
1/8*((3*a^2*x^3 + 5*x)/(a^4*c^3*x^4 + 2*a^2*c^3*x^2 + c^3) + 3*arctan(a*x)
/(a*c^3))*arctan(a*x)^3 + 3/16*(3*a^2*x^2 - 3*(a^4*x^4 + 2*a^2*x^2 + 1)*ar
ctan(a*x)^2 + 4)*a*arctan(a*x)^2/(a^6*c^3*x^4 + 2*a^4*c^3*x^2 + a^2*c^3) -
 3/128*((4*(a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x)^4 + 15*a^2*x^2 - 15*(a^4*
x^4 + 2*a^2*x^2 + 1)*arctan(a*x)^2 + 16)*a^2/(a^8*c^3*x^4 + 2*a^6*c^3*x^2
+ a^4*c^3) + 2*(15*a^3*x^3 - 8*(a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x)^3 + 1
7*a*x + 15*(a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x))*a*arctan(a*x)/(a^7*c^3*x
^4 + 2*a^5*c^3*x^2 + a^3*c^3))*a
```

## Giac [F]

$$\int \frac{\arctan(ax)^3}{(c+a^2cx^2)^3}\,dx = \int \frac{\arctan(ax)^3}{(a^2cx^2+c)^3}\,dx$$

input
```
integrate(arctan(a*x)^3/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^3/(a^2*c*x^2 + c)^3, x)
```

## Mupad [B] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.88

$$\begin{aligned}\int \frac{\arctan(ax)^3}{(c+a^2cx^2)^3}\,dx &= \operatorname{atan}(a\,x)^2\left(\frac{\frac{3}{4\,a^3\,c^3}+\frac{9\,x^2}{16\,a\,c^3}}{\frac{1}{a^2}+2\,x^2+a^2\,x^4}-\frac{45}{128\,a\,c^3}\right)\\&\quad-\frac{\frac{45\,a\,x^2}{2}+\frac{24}{a}}{64\,a^4\,c^3\,x^4+128\,a^2\,c^3\,x^2+64\,c^3}-\frac{\operatorname{atan}(a\,x)\,\left(\frac{45\,x^3}{64\,c^3}+\frac{51\,x}{64\,a^2\,c^3}\right)}{\frac{1}{a^2}+2\,x^2+a^2\,x^4}\\&\quad+\frac{\operatorname{atan}(a\,x)^3\,\left(\frac{3\,x^3}{8\,c^3}+\frac{5\,x}{8\,a^2\,c^3}\right)}{\frac{1}{a^2}+2\,x^2+a^2\,x^4}+\frac{3\,\operatorname{atan}(a\,x)^4}{32\,a\,c^3}\end{aligned}$$

input
```
int(atan(a*x)^3/(c + a^2*c*x^2)^3,x)
```

output
```
atan(a*x)^2*((3/(4*a^3*c^3) + (9*x^2)/(16*a*c^3))/(1/a^2 + 2*x^2 + a^2*x^4
) - 45/(128*a*c^3)) - ((45*a*x^2)/2 + 24/a)/(64*c^3 + 128*a^2*c^3*x^2 + 64
*a^4*c^3*x^4) - (atan(a*x)*((45*x^3)/(64*c^3) + (51*x)/(64*a^2*c^3)))/(1/a
^2 + 2*x^2 + a^2*x^4) + (atan(a*x)^3*((3*x^3)/(8*c^3) + (5*x)/(8*a^2*c^3))
)/(1/a^2 + 2*x^2 + a^2*x^4) + (3*atan(a*x)^4)/(32*a*c^3)
```

## Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.68

$$\int \frac{\arctan(ax)^3}{(c+a^2cx^2)^3}\,dx = \frac{24atan(ax)^4 a^4x^4 + 48atan(ax)^4 a^2x^2 + 24atan(ax)^4 + 96atan(ax)^3 a^3x^3 + 160atan(ax)^3 ax - 90atan(\cdots}{256a\,c^3\,(a^4x^4 + \cdots}$$

input
```
int(atan(a*x)^3/(a^2*c*x^2+c)^3,x)
```

output
```
(24*atan(a*x)**4*a**4*x**4 + 48*atan(a*x)**4*a**2*x**2 + 24*atan(a*x)**4 +
 96*atan(a*x)**3*a**3*x**3 + 160*atan(a*x)**3*a*x - 90*atan(a*x)**2*a**4*x
**4 - 36*atan(a*x)**2*a**2*x**2 + 102*atan(a*x)**2 - 180*atan(a*x)*a**3*x*
*3 - 204*atan(a*x)*a*x + 45*a**4*x**4 - 51)/(256*a*c**3*(a**4*x**4 + 2*a**
2*x**2 + 1))
```

# 3.408 $\int \frac{\arctan(ax)^3}{x(c+a^2cx^2)^3}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 332

$$\begin{aligned}\int \frac{\arctan(ax)^3}{x(c+a^2cx^2)^3}\,dx &= \frac{3ax}{128c^3(1+a^2x^2)^2}+\frac{141ax}{256c^3(1+a^2x^2)}+\frac{141\arctan(ax)}{256c^3}\\ &-\frac{3\arctan(ax)}{32c^3(1+a^2x^2)^2}-\frac{33\arctan(ax)}{32c^3(1+a^2x^2)}-\frac{3ax\arctan(ax)^2}{16c^3(1+a^2x^2)^2}\\ &-\frac{33ax\arctan(ax)^2}{32c^3(1+a^2x^2)}-\frac{11\arctan(ax)^3}{32c^3}+\frac{\arctan(ax)^3}{4c^3(1+a^2x^2)^2}\\ &+\frac{\arctan(ax)^3}{2c^3(1+a^2x^2)}-\frac{i\arctan(ax)^4}{4c^3}+\frac{\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)}{c^3}\\ &-\frac{3i\arctan(ax)^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{2c^3}\\ &+\frac{3\arctan(ax)\operatorname{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)}{2c^3}\\ &+\frac{3i\operatorname{PolyLog}\left(4,-1+\frac{2}{1-iax}\right)}{4c^3}\end{aligned}$$

output
```
3/128*a*x/c^3/(a^2*x^2+1)^2+141/256*a*x/c^3/(a^2*x^2+1)+141/256*arctan(a*x
)/c^3-3/32*arctan(a*x)/c^3/(a^2*x^2+1)^2-33/32*arctan(a*x)/c^3/(a^2*x^2+1)
-3/16*a*x*arctan(a*x)^2/c^3/(a^2*x^2+1)^2-33/32*a*x*arctan(a*x)^2/c^3/(a^2
*x^2+1)-11/32*arctan(a*x)^3/c^3+1/4*arctan(a*x)^3/c^3/(a^2*x^2+1)^2+1/2*ar
ctan(a*x)^3/c^3/(a^2*x^2+1)-1/4*I*arctan(a*x)^4/c^3+arctan(a*x)^3*ln(2-2/(
1-I*a*x))/c^3-3/2*I*arctan(a*x)^2*polylog(2,-1+2/(1-I*a*x))/c^3+3/2*arctan
(a*x)*polylog(3,-1+2/(1-I*a*x))/c^3+3/4*I*polylog(4,-1+2/(1-I*a*x))/c^3
```

## Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.63

$$\int \frac{\arctan(ax)^3}{x\,(c+a^2cx^2)^3}\,dx = \frac{-16i\pi^4 + 256i\arctan(ax)^4 - 576\arctan(ax)\cos(2\arctan(ax)) + 384\arctan(ax)^3\cos(2\arctan(ax)) - 1}{}$$

input
```
Integrate[ArcTan[a*x]^3/(x*(c + a^2*c*x^2)^3),x]
```

output
```
((-16*I)*Pi^4 + (256*I)*ArcTan[a*x]^4 - 576*ArcTan[a*x]*Cos[2*ArcTan[a*x]]
 + 384*ArcTan[a*x]^3*Cos[2*ArcTan[a*x]] - 12*ArcTan[a*x]*Cos[4*ArcTan[a*x]
] + 32*ArcTan[a*x]^3*Cos[4*ArcTan[a*x]] + 1024*ArcTan[a*x]^3*Log[1 - E^((-
2*I)*ArcTan[a*x])] + (1536*I)*ArcTan[a*x]^2*PolyLog[2, E^((-2*I)*ArcTan[a*
x])] + 1536*ArcTan[a*x]*PolyLog[3, E^((-2*I)*ArcTan[a*x])] - (768*I)*PolyL
og[4, E^((-2*I)*ArcTan[a*x])] + 288*Sin[2*ArcTan[a*x]] - 576*ArcTan[a*x]^2
*Sin[2*ArcTan[a*x]] + 3*Sin[4*ArcTan[a*x]] - 24*ArcTan[a*x]^2*Sin[4*ArcTan
[a*x]])/(1024*c^3)
```

## Rubi [A] (verified)

Time = 3.52 (sec) , antiderivative size = 515, normalized size of antiderivative = 1.55, number of steps used = 22, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {5501, 27, 5465, 5435, 215, 215, 216, 5427, 5465, 215, 216, 5501, 5459, 5403, 5465, 5427, 5465, 215, 216, 5527, 5531, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3}{x(a^2cx^2+c)^3}dx$$

$$\Big\downarrow \textbf{5501}$$

$$\frac{\int \frac{\arctan(ax)^3}{c^2x(a^2x^2+1)^2}dx}{c} - a^2\int \frac{x\arctan(ax)^3}{c^3(a^2x^2+1)^3}dx$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx}{c^3} - \frac{a^2\int \frac{x\arctan(ax)^3}{(a^2x^2+1)^3}dx}{c^3}$$

$$\Big\downarrow \textbf{5465}$$

$$\frac{\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx}{c^3} - \frac{a^2\left(\frac{3\int \frac{\arctan(ax)^2}{(a^2x^2+1)^3}dx}{4a} - \frac{\arctan(ax)^3}{4a^2(a^2x^2+1)^2}\right)}{c^3}$$

$$\Big\downarrow \textbf{5435}$$

$$\frac{\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx}{c^3} - \frac{a^2\left(\frac{3\left(\frac{3}{4}\int \frac{\arctan(ax)^2}{(a^2x^2+1)^2}dx - \frac{1}{8}\int \frac{1}{(a^2x^2+1)^3}dx + \frac{x\arctan(ax)^2}{4(a^2x^2+1)^2} + \frac{\arctan(ax)}{8a(a^2x^2+1)^2}\right)}{4a} - \frac{\arctan(ax)^3}{4a^2(a^2x^2+1)^2}\right)}{c^3}$$

$$\Big\downarrow \textbf{215}$$

$$\frac{\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx}{c^3} - \frac{a^2\left(\frac{3\left(\frac{3}{4}\int \frac{\arctan(ax)^2}{(a^2x^2+1)^2}dx + \frac{1}{8}\left(-\frac{3}{4}\int \frac{1}{(a^2x^2+1)^2}dx - \frac{x}{4(a^2x^2+1)^2}\right) + \frac{x\arctan(ax)^2}{4(a^2x^2+1)^2} + \frac{\arctan(ax)}{8a(a^2x^2+1)^2}\right)}{4a} - \frac{\arctan(ax)^3}{4a^2(a^2x^2+1)^2}\right)}{c^3}$$

$$\Big\downarrow \textbf{215}$$

$$\frac{\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx}{c^3} - \frac{a^2\left(\frac{3\left(\frac{3}{4}\int \frac{\arctan(ax)^2}{(a^2x^2+1)^2}dx + \frac{1}{8}\left(-\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{a^2x^2+1}dx + \frac{x}{2(a^2x^2+1)}\right) - \frac{x}{4(a^2x^2+1)^2}\right) + \frac{x\arctan(ax)^2}{4(a^2x^2+1)^2} + \frac{\arctan(ax)}{8a(a^2x^2+1)^2}\right)}{4a} - \frac{\arctan(ax)^3}{4a^2(a^2x^2+1)^2}\right)}{c^3}$$

$\downarrow$ 216

$$\frac{\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx}{c^3} - \frac{a^2\left(\frac{3\left(\frac{3}{4}\int \frac{\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}\right)-\frac{x}{4(a^2x^2+1)^2}\right)\right)}{4a}-\frac{\arctan(ax)^3}{4a^2(a^2x^2+1)^2}\right)}{c^3}$$

$\downarrow$ 5427

$$\frac{\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx}{c^3} - \frac{a^2\Bigg(\frac{3\Big(\frac{3}{4}\left(-a\int \frac{x\arctan(ax)}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)+\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\Big(-\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}\right)-\frac{x}{4(a^2x^2+1)^2}\Big)}{4a}}{c^3}$$

$\downarrow$ 5465

$$\frac{\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx}{c^3} - \frac{a^2\Bigg(\frac{3\Big(\frac{3}{4}\left(-a\left(\frac{\int \frac{1}{(a^2x^2+1)^2}dx}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)+\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\Big(-\frac{3}{4}\Big(\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}}{4a}}{c^3}$$

$\downarrow$ 215

$$\frac{\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx}{c^3} - \frac{a^2\Bigg(\frac{3\Big(\frac{3}{4}\left(-a\left(\frac{\frac{1}{2}\int \frac{1}{a^2x^2+1}dx+\frac{x}{2(a^2x^2+1)}}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}+\frac{\arctan(ax)^3}{6a}\right)+\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\Big(-\frac{3}{4}\Big(\frac{x}{2(a^2x^2+1)}}{4a}}{c^3}$$

$\downarrow$ 216

$$\frac{\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx}{c^3} - \frac{a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}\right)-\frac{x}{4(a^2x^2+1)^2}\right)+\frac{3}{4}\left(\frac{x\arctan(ax)^2}{2(a^2x^2+1)}-a\left(\frac{\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\text{arcta}}{2a^2(a}\right.\right.\right.}{4a}\right.}{c^3}$$

$\downarrow$ 5501

$$\frac{\int \frac{\arctan(ax)^3}{x(a^2x^2+1)}dx - a^2\int \frac{x\arctan(ax)^3}{(a^2x^2+1)^2}dx}{c^3} - \frac{a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}\right)-\frac{x}{4(a^2x^2+1)^2}\right)+\frac{3}{4}\left(\frac{x\arctan(ax)^2}{2(a^2x^2+1)}-a\left(\frac{\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\text{arcta}}{2a^2(a}\right.\right.\right.}{4a}\right.}{c^3}$$

$\downarrow$ 5459

$$-\frac{a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}\right)-\frac{x}{4(a^2x^2+1)^2}\right)+\frac{3}{4}\left(\frac{x\arctan(ax)^2}{2(a^2x^2+1)}-a\left(\frac{\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\text{ar}}{2a^2}\right.\right.\right.}{4a}\right.}{c^3}$$

$$\frac{-a^2\int \frac{x\arctan(ax)^3}{(a^2x^2+1)^2}dx + i\int \frac{\arctan(ax)^3}{x(ax+i)}dx - \frac{1}{4}i\arctan(ax)^4}{c^3}$$

$\downarrow$ 5403

$$-\frac{a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}\right)-\frac{x}{4(a^2x^2+1)^2}\right)+\frac{3}{4}\left(\frac{x\arctan(ax)^2}{2(a^2x^2+1)}-a\left(\frac{\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\text{ar}}{2a^2}\right.\right.\right.}{4a}\right.}{c^3}$$

$$\frac{-a^2\int \frac{x\arctan(ax)^3}{(a^2x^2+1)^2}dx + i\left(3ia\int \frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx - i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right) - \frac{1}{4}i\arctan(ax)^4}{c^3}$$

$\Big\downarrow$ 5465

$$-\frac{a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{4\left(a^2x^2+1\right)^2}+\frac{\arctan(ax)}{8a\left(a^2x^2+1\right)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}\right)-\frac{x}{4\left(a^2x^2+1\right)^2}\right)+\frac{3}{4}\left(\frac{x\arctan(ax)^2}{2\left(a^2x^2+1\right)}-a\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\text{ar}}{2a^2}\cdots\right.\right.\right.}{4a}\cdots}{c^3}\cdots$$

$$\frac{-a^2\left(\frac{3\int\frac{\arctan(ax)^2}{\left(a^2x^2+1\right)^2}dx}{2a}-\frac{\arctan(ax)^3}{2a^2(a^2x^2+1)}\right)+i\left(3ia\int\frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{4}i\,a\cdots}{c^3}$$

$\Big\downarrow$ 5427

$$-\frac{a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{4\left(a^2x^2+1\right)^2}+\frac{\arctan(ax)}{8a\left(a^2x^2+1\right)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}\right)-\frac{x}{4\left(a^2x^2+1\right)^2}\right)+\frac{3}{4}\left(\frac{x\arctan(ax)^2}{2\left(a^2x^2+1\right)}-a\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\text{ar}}{2a^2}\cdots\right.\right.\right.}{4a}\cdots}{c^3}\cdots$$

$$\frac{-a^2\left(\frac{3\left(-a\int\frac{x\arctan(ax)}{\left(a^2x^2+1\right)^2}dx+\frac{x\arctan(ax)^2}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^3}{6a}\right)}{2a}-\frac{\arctan(ax)^3}{2a^2(a^2x^2+1)}\right)+i\left(3ia\int\frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(a\cdots\right.}{c^3}$$

$\Big\downarrow$ 5465

$$-\frac{a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{4\left(a^2x^2+1\right)^2}+\frac{\arctan(ax)}{8a\left(a^2x^2+1\right)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}\right)-\frac{x}{4\left(a^2x^2+1\right)^2}\right)+\frac{3}{4}\left(\frac{x\arctan(ax)^2}{2\left(a^2x^2+1\right)}-a\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\text{ar}}{2a^2}\cdots\right.\right.\right.}{4a}\cdots}{c^3}\cdots$$

$$\frac{-a^2\left(\frac{3\left(-a\left(\frac{\int\frac{1}{\left(a^2x^2+1\right)^2}dx}{2a}-\frac{\arctan(ax)}{2a^2\left(a^2x^2+1\right)}\right)+\frac{x\arctan(ax)^2}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^3}{6a}\right)}{2a}-\frac{\arctan(ax)^3}{2a^2(a^2x^2+1)}\right)+i\left(3ia\int\frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}a\cdots\right.}{c^3}$$

$\Big\downarrow$ 215

$$-\frac{a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{4\left(a^2x^2+1\right)^2}+\frac{\arctan(ax)}{8a\left(a^2x^2+1\right)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}\right)-\frac{x}{4\left(a^2x^2+1\right)^2}\right)+\frac{3}{4}\left(\frac{x\arctan(ax)^2}{2\left(a^2x^2+1\right)}-a\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\text{ar}}{2a^2}\right.\right.\right.}{4a}\right.}{c^3}$$

$$\frac{-a^2\left(\frac{3\left(-a\left(\frac{\frac{1}{2}\int\frac{1}{a^2x^2+1}dx+\frac{x}{2\left(a^2x^2+1\right)}}{2a}-\frac{\arctan(ax)}{2a^2\left(a^2x^2+1\right)}\right)+\frac{x\arctan(ax)^2}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^3}{6a}\right)}{2a}-\frac{\arctan(ax)^3}{2a^2(a^2x^2+1)}\right)+i\left(3ia\int\frac{\arctan(ax)^2\log\left(2\right.}{a^2x^2+1}\right.}{c^3}$$

$$\Big\downarrow \; 216$$

$$-\frac{a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{4\left(a^2x^2+1\right)^2}+\frac{\arctan(ax)}{8a\left(a^2x^2+1\right)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}\right)-\frac{x}{4\left(a^2x^2+1\right)^2}\right)+\frac{3}{4}\left(\frac{x\arctan(ax)^2}{2\left(a^2x^2+1\right)}-a\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\text{ar}}{2a^2}\right.\right.\right.}{4a}\right.}{c^3}$$

$$\frac{i\left(3ia\int\frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right)-a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{2\left(a^2x^2+1\right)}-a\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{}{2a}\right.\right.}{2a}\right.}{c^3}$$

$$\Big\downarrow \; 5527$$

$$-\frac{a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{4\left(a^2x^2+1\right)^2}+\frac{\arctan(ax)}{8a\left(a^2x^2+1\right)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}\right)-\frac{x}{4\left(a^2x^2+1\right)^2}\right)+\frac{3}{4}\left(\frac{x\arctan(ax)^2}{2\left(a^2x^2+1\right)}-a\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\text{ar}}{2a^2}\right.\right.\right.}{4a}\right.}{c^3}$$

$$\frac{i\left(3ia\left(\frac{i\arctan(ax)^2\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-i\int\frac{\arctan(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{a^2x^2+1}dx\right)-i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right)-a^{\cdots}}{c^3}$$

$\downarrow$ 5531

$$-\frac{a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}\right)-\frac{x}{4(a^2x^2+1)^2}\right)+\frac{3}{4}\left(\frac{x\arctan(ax)^2}{2(a^2x^2+1)}-a\left(\frac{\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\text{ar}}{2a^2}\right.\right.\right.}{4a}\right.}{c^3}$$

$$i\left(3ia\left(\frac{i\arctan(ax)^2\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-i\left(\frac{1}{2}i\int\frac{\operatorname{PolyLog}\left(3,\frac{2}{1-iax}-1\right)}{a^2x^2+1}dx-\frac{i\arctan(ax)\operatorname{PolyLog}\left(3,\frac{2}{1-iax}-1\right)}{2a}\right)\right)\right)-i\arctan$$

$\downarrow$ 7164

$$-\frac{a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2}+\frac{\arctan(ax)}{8a(a^2x^2+1)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}\right)-\frac{x}{4(a^2x^2+1)^2}\right)+\frac{3}{4}\left(\frac{x\arctan(ax)^2}{2(a^2x^2+1)}-a\left(\frac{\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\text{ar}}{2a^2}\right.\right.\right.}{4a}\right.}{c^3}$$

$$-a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{2(a^2x^2+1)}-a\left(\frac{\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)+\frac{\arctan(ax)^3}{6a}\right)}{2a}-\frac{\arctan(ax)^3}{2a^2(a^2x^2+1)}\right)+i\left(3ia\left(\frac{i\arctan(ax)^2\operatorname{PolyLog}(}{2a}\right.\right.$$

input
```
Int[ArcTan[a*x]^3/(x*(c + a^2*c*x^2)^3),x]
```

output
```
-((a^2*(-1/4*ArcTan[a*x]^3/(a^2*(1 + a^2*x^2)^2) + (3*(ArcTan[a*x]/(8*a*(1
 + a^2*x^2)^2) + (x*ArcTan[a*x]^2)/(4*(1 + a^2*x^2)^2) + (-1/4*x/(1 + a^2*
x^2)^2 - (3*(x/(2*(1 + a^2*x^2)) + ArcTan[a*x]/(2*a)))/4)/8 + (3*((x*ArcTa
n[a*x]^2)/(2*(1 + a^2*x^2)) + ArcTan[a*x]^3/(6*a) - a*(-1/2*ArcTan[a*x]/(a
^2*(1 + a^2*x^2)) + (x/(2*(1 + a^2*x^2)) + ArcTan[a*x]/(2*a))/(2*a))))/4))
/(4*a)))/c^3) + ((-1/4*I)*ArcTan[a*x]^4 - a^2*(-1/2*ArcTan[a*x]^3/(a^2*(1
+ a^2*x^2)) + (3*((x*ArcTan[a*x]^2)/(2*(1 + a^2*x^2)) + ArcTan[a*x]^3/(6*a
) - a*(-1/2*ArcTan[a*x]/(a^2*(1 + a^2*x^2)) + (x/(2*(1 + a^2*x^2)) + ArcTa
n[a*x]/(2*a))/(2*a))))/(2*a)) + I*((-I)*ArcTan[a*x]^3*Log[2 - 2/(1 - I*a*x
)] + (3*I)*a*(((I/2)*ArcTan[a*x]^2*PolyLog[2, -1 + 2/(1 - I*a*x)])/a - I*(
((-1/2*I)*ArcTan[a*x]*PolyLog[3, -1 + 2/(1 - I*a*x)])/a + PolyLog[4, -1 +
2/(1 - I*a*x)]/(4*a)))))/c^3
```

**Defintions of rubi rules used**

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5435
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[b*p*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p - 1)/(4*c*d*
(q + 1)^2)), x] + (-Simp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*d*
(q + 1))), x] + Simp[(2*q + 3)/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p, x], x] - Simp[b^2*p*((p - 1)/(4*(q + 1)^2))   Int[(d + e
*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

rule 5527
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

rule 5531
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_.
)*(x_)^2), x_Symbol] :> Simp[(-I)*(a + b*ArcTan[c*x])^p*(PolyLog[k + 1, u]/
(2*c*d)), x] + Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[k
+ 1, u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0]
&& EqQ[e, c^2*d] && EqQ[u^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.09 (sec) , antiderivative size = 1841, normalized size of antiderivative = 5.55

Expression too large to display

input
```
int(arctan(a*x)^3/x/(a^2*c*x^2+c)^3,x)
```

output
```
-1/2/c^3*arctan(a*x)^3*ln(a^2*x^2+1)+1/4*arctan(a*x)^3/c^3/(a^2*x^2+1)^2+1
/2*arctan(a*x)^3/c^3/(a^2*x^2+1)+1/c^3*arctan(a*x)^3*ln(a*x)-3/4/c^3*(-4/3
*arctan(a*x)^3*ln((1+I*a*x)/(a^2*x^2+1)^(1/2))-8*I*polylog(4,-(1+I*a*x)/(a
^2*x^2+1)^(1/2))-8*I*polylog(4,(1+I*a*x)/(a^2*x^2+1)^(1/2))+3*I*arctan(a*x
)^2*(a*x-I)/(8*a*x+8*I)-3/8*arctan(a*x)*(a*x-I)/(a*x+I)+3*I*(a*x+I)/(16*a*
x-16*I)+4/3*arctan(a*x)^3*ln((1+I*a*x)^2/(a^2*x^2+1)-1)-4/3*arctan(a*x)^3*
ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+1/3*I*arctan(a*x)^4-8*arctan(a*x)*polylo
g(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*I*(a*x-I)/(16*a*x+16*I)-3/8*arctan(a*x
)*(a*x+I)/(a*x-I)+4*I*arctan(a*x)^2*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2)
)-4/3*arctan(a*x)^3*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*I*arctan(a*x)^2*(a
*x+I)/(8*a*x-8*I)-8*arctan(a*x)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))+4*I
*arctan(a*x)^2*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-1/24*(16*I*Pi*csgn(I
*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(((1+I*a*x)^
2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))+16*I*Pi*csgn(((1+I*a*x)^2/(a
^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^3+16*I*Pi*csgn(I*((1+I*a*x)^2/(a
^2*x^2+1)-1))*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I*((1+I*a*x)^2/(a^2
*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))+8*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^
2+1)+1))^2*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)+8*I*Pi*csgn(I*(1+I*a*x)^2
/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2
)^2+16*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)...
```

## Fricas [F]

$$\int \frac{\arctan(ax)^3}{x\,(c+a^2cx^2)^3}\,dx = \int \frac{\arctan\left(ax\right)^3}{\left(a^2cx^2+c\right)^3x}\,dx$$

input
```
integrate(arctan(a*x)^3/x/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
integral(arctan(a*x)^3/(a^6*c^3*x^7 + 3*a^4*c^3*x^5 + 3*a^2*c^3*x^3 + c^3*
x), x)
```

## Sympy [F]

$$\int \frac{\arctan(ax)^3}{x\left(c+a^2cx^2\right)^3}\,dx = \frac{\int \frac{\operatorname{atan}^3(ax)}{a^6x^7+3a^4x^5+3a^2x^3+x}\,dx}{c^3}$$

input
```
integrate(atan(a*x)**3/x/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(atan(a*x)**3/(a**6*x**7 + 3*a**4*x**5 + 3*a**2*x**3 + x), x)/c**3
```

## Maxima [F]

$$\int \frac{\arctan(ax)^3}{x\left(c+a^2cx^2\right)^3}\,dx = \int \frac{\arctan(ax)^3}{\left(a^2cx^2+c\right)^3x}\,dx$$

input
```
integrate(arctan(a*x)^3/x/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)^3/((a^2*c*x^2 + c)^3*x), x)
```

## Giac [F]

$$\int \frac{\arctan(ax)^3}{x\left(c+a^2cx^2\right)^3}\,dx = \int \frac{\arctan(ax)^3}{\left(a^2cx^2+c\right)^3x}\,dx$$

input
```
integrate(arctan(a*x)^3/x/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^3/((a^2*c*x^2 + c)^3*x), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^3}{x\left(c+a^2cx^2\right)^3}\,dx=\int \frac{\mathrm{atan}(a\,x)^3}{x\,\left(c\,a^2\,x^2+c\right)^3}\,dx$$

input
```
int(atan(a*x)^3/(x*(c + a^2*c*x^2)^3),x)
```

output
```
int(atan(a*x)^3/(x*(c + a^2*c*x^2)^3), x)
```

## Reduce [F]

$$\int \frac{\arctan(ax)^3}{x\left(c+a^2cx^2\right)^3}\,dx=\frac{\int \frac{atan(ax)^3}{a^6x^7+3a^4x^5+3a^2x^3+x}dx}{c^3}$$

input
```
int(atan(a*x)^3/x/(a^2*c*x^2+c)^3,x)
```

output
```
int(atan(a*x)**3/(a**6*x**7 + 3*a**4*x**5 + 3*a**2*x**3 + x),x)/c**3
```

# 3.409 $\int \frac{\arctan(ax)^3}{x^2(c+a^2cx^2)^3}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 332

$$\begin{aligned}\int \frac{\arctan(ax)^3}{x^2(c+a^2cx^2)^3}\,dx &= \frac{3a}{128c^3(1+a^2x^2)^2}+\frac{93a}{128c^3(1+a^2x^2)}+\frac{3a^2x\arctan(ax)}{32c^3(1+a^2x^2)^2}\\
&\quad+\frac{93a^2x\arctan(ax)}{64c^3(1+a^2x^2)}+\frac{93a\arctan(ax)^2}{128c^3}\\
&\quad-\frac{3a\arctan(ax)^2}{16c^3(1+a^2x^2)^2}-\frac{21a\arctan(ax)^2}{16c^3(1+a^2x^2)}-\frac{ia\arctan(ax)^3}{c^3}\\
&\quad-\frac{\arctan(ax)^3}{c^3x}-\frac{a^2x\arctan(ax)^3}{4c^3(1+a^2x^2)^2}-\frac{7a^2x\arctan(ax)^3}{8c^3(1+a^2x^2)}\\
&\quad-\frac{15a\arctan(ax)^4}{32c^3}+\frac{3a\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{c^3}\\
&\quad-\frac{3ia\arctan(ax)\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{c^3}\\
&\quad+\frac{3a\operatorname{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)}{2c^3}\end{aligned}$$

output
```
3/128*a/c^3/(a^2*x^2+1)^2+93/128*a/c^3/(a^2*x^2+1)+3/32*a^2*x*arctan(a*x)/
c^3/(a^2*x^2+1)^2+93/64*a^2*x*arctan(a*x)/c^3/(a^2*x^2+1)+93/128*a*arctan(
a*x)^2/c^3-3/16*a*arctan(a*x)^2/c^3/(a^2*x^2+1)^2-21/16*a*arctan(a*x)^2/c^
3/(a^2*x^2+1)-I*a*arctan(a*x)^3/c^3-arctan(a*x)^3/c^3/x-1/4*a^2*x*arctan(a
*x)^3/c^3/(a^2*x^2+1)^2-7/8*a^2*x*arctan(a*x)^3/c^3/(a^2*x^2+1)-15/32*a*ar
ctan(a*x)^4/c^3+3*a*arctan(a*x)^2*ln(2-2/(1-I*a*x))/c^3-3*I*a*arctan(a*x)*
polylog(2,-1+2/(1-I*a*x))/c^3+3/2*a*polylog(3,-1+2/(1-I*a*x))/c^3
```

## Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.70

$$\int \frac{\arctan(ax)^3}{x^2 (c + a^2cx^2)^3} \, dx = \frac{a\left(-\frac{i\pi^3}{8} + i\arctan(ax)^3 - \frac{\arctan(ax)^3}{ax} - \frac{ax\arctan(ax)^3}{1+a^2x^2} - \frac{15}{32}\arctan(ax)^4 + \frac{3}{8}\cos(2\arctan(ax)) - \frac{3}{4}\arctan(a$$

input
```
Integrate[ArcTan[a*x]^3/(x^2*(c + a^2*c*x^2)^3),x]
```

output
```
(a*((-1/8*I)*Pi^3 + I*ArcTan[a*x]^3 - ArcTan[a*x]^3/(a*x) - (a*x*ArcTan[a*
x]^3)/(1 + a^2*x^2) - (15*ArcTan[a*x]^4)/32 + (3*Cos[2*ArcTan[a*x]])/8 - (
3*ArcTan[a*x]^2*Cos[2*ArcTan[a*x]])/4 + (3*Cos[4*ArcTan[a*x]])/1024 - (3*A
rcTan[a*x]^2*Cos[4*ArcTan[a*x]])/128 + 3*ArcTan[a*x]^2*Log[1 - E^((-2*I)*A
rcTan[a*x])] + (3*I)*ArcTan[a*x]*PolyLog[2, E^((-2*I)*ArcTan[a*x])] + (3*P
olyLog[3, E^((-2*I)*ArcTan[a*x])])/2 + (3*ArcTan[a*x]*Sin[2*ArcTan[a*x]])/
4 + (3*ArcTan[a*x]*Sin[4*ArcTan[a*x]])/256 - (ArcTan[a*x]^3*Sin[4*ArcTan[a
*x]])/32))/c^3
```

## Rubi [A] (verified)

Time = 4.49 (sec) , antiderivative size = 539, normalized size of antiderivative = 1.62, number of steps used = 22, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {5501, 27, 5435, 5427, 5431, 5427, 241, 5465, 5427, 241, 5501, 5427, 5453, 5361, 5419, 5459, 5403, 5465, 5427, 241, 5527, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed

below.

$$\int \frac{\arctan(ax)^3}{x^2\left(a^2cx^2+c\right)^3}dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{\arctan(ax)^3}{c^2x^2(a^2x^2+1)^2}dx}{c} - a^2\int \frac{\arctan(ax)^3}{c^3\left(a^2x^2+1\right)^3}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx}{c^3} - \frac{a^2\int \frac{\arctan(ax)^3}{(a^2x^2+1)^3}dx}{c^3}$$

$$\Big\downarrow \text{5435}$$

$$\frac{\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx}{c^3} - \frac{a^2\left(-\frac{3}{8}\int \frac{\arctan(ax)}{(a^2x^2+1)^3}dx + \frac{3}{4}\int \frac{\arctan(ax)^3}{(a^2x^2+1)^2}dx + \frac{x\arctan(ax)^3}{4(a^2x^2+1)^2} + \frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2}\right)}{c^3}$$

$$\Big\downarrow \text{5427}$$

$$\frac{\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx}{c^3} - \frac{a^2\left(-\frac{3}{8}\int \frac{\arctan(ax)}{(a^2x^2+1)^3}dx + \frac{3}{4}\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx + \frac{x\arctan(ax)^3}{2(a^2x^2+1)} + \frac{\arctan(ax)^4}{8a}\right) + \frac{x\arctan(ax)^3}{4(a^2x^2+1)^2} + \frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2}\right)}{c^3}$$

$$\Big\downarrow \text{5431}$$

$$\frac{\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx}{c^3} - \frac{a^2\left(-\frac{3}{8}\left(\frac{3}{4}\int \frac{\arctan(ax)}{(a^2x^2+1)^2}dx + \frac{x\arctan(ax)}{4(a^2x^2+1)^2} + \frac{1}{16a(a^2x^2+1)^2}\right) + \frac{3}{4}\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx + \frac{x\arctan(ax)^3}{2(a^2x^2+1)} + \frac{\arctan(ax)^4}{8a}\right) + \frac{x}{4}\cdots}{c^3}$$

$$\Big\downarrow \text{5427}$$

$$\frac{\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx}{c^3} - \frac{a^2\left(-\frac{3}{8}\left(\frac{3}{4}\left(-\frac{1}{2}a\int \frac{x}{(a^2x^2+1)^2}dx + \frac{x\arctan(ax)}{2(a^2x^2+1)} + \frac{\arctan(ax)^2}{4a}\right) + \frac{x\arctan(ax)}{4(a^2x^2+1)^2} + \frac{1}{16a(a^2x^2+1)^2}\right) + \frac{3}{4}\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx\cdots}{c^3}$$

$$\Big\downarrow \text{241}$$

$$\frac{\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx}{c^3} - \frac{a^2\left(\frac{3}{4}\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx + \frac{x\arctan(ax)^3}{2(a^2x^2+1)} + \frac{\arctan(ax)^4}{8a}\right) + \frac{x\arctan(ax)^3}{4(a^2x^2+1)^2} + \frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2} - \frac{3}{8}\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2} + \frac{3}{4}\left(\frac{x\arcta}{2(a^2x^2}\right.\right.\right.}{c^3}$$

$\downarrow$ 5465

$$\frac{\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx}{c^3} - \frac{a^2\left(\frac{3}{4}\left(-\frac{3}{2}a\left(\frac{\int \frac{\arctan(ax)}{(a^2x^2+1)^2}dx}{a} - \frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right) + \frac{x\arctan(ax)^3}{2(a^2x^2+1)} + \frac{\arctan(ax)^4}{8a}\right) + \frac{x\arctan(ax)^3}{4(a^2x^2+1)^2} + \frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2} - \frac{3}{8}\left(\frac{x\arcta}{4(a^2x^2}\right.\right.}{c^3}$$

$\downarrow$ 5427

$$\frac{\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx}{c^3} - \frac{a^2\left(\frac{3}{4}\left(-\frac{3}{2}a\left(\frac{-\frac{1}{2}a\int \frac{x}{(a^2x^2+1)^2}dx + \frac{x\arctan(ax)}{2(a^2x^2+1)} + \frac{\arctan(ax)^2}{4a}}{a} - \frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right) + \frac{x\arctan(ax)^3}{2(a^2x^2+1)} + \frac{\arctan(ax)^4}{8a}\right) + \frac{x\arctan(ax)^3}{4(a^2x^2+1)^2}\right.}{c^3}$$

$\downarrow$ 241

$$\frac{\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx}{c^3} - \frac{a^2\left(\frac{x\arctan(ax)^3}{4(a^2x^2+1)^2} + \frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2} - \frac{3}{8}\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2} + \frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)} + \frac{1}{4a(a^2x^2+1)} + \frac{\arctan(ax)^2}{4a}\right) + \frac{1}{16a(a^2x^2+1)^2}\right) + \frac{3}{4}\left(\right.\right.}{c^3}$$

$\downarrow$ 5501

$$\frac{\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)}dx - a^2\int \frac{\arctan(ax)^3}{(a^2x^2+1)^2}dx}{c^3} - \frac{a^2\left(\frac{x\arctan(ax)^3}{4(a^2x^2+1)^2} + \frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2} - \frac{3}{8}\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2} + \frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)} + \frac{1}{4a(a^2x^2+1)} + \frac{\arctan(ax)^2}{4a}\right) + \frac{1}{16a(a^2x^2+1)^2}\right) + \frac{3}{4}\left(\right.\right.}{c^3}$$

$\downarrow$ 5427

$$\frac{\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)}dx - a^2\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx + \frac{x\arctan(ax)^3}{2(a^2x^2+1)} + \frac{\arctan(ax)^4}{8a}\right)}{c^3} -$$

$$\frac{a^2\left(\frac{x\arctan(ax)^3}{4(a^2x^2+1)^2} + \frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2} - \frac{3}{8}\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2} + \frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)} + \frac{1}{4a(a^2x^2+1)} + \frac{\arctan(ax)^2}{4a}\right) + \frac{1}{16a(a^2x^2+1)^2}\right) + \frac{3}{4}\left(\right.\right.}{c^3}$$

$\downarrow$ 5453

$$\frac{-\left(a^2\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx + \frac{x\arctan(ax)^3}{2(a^2x^2+1)} + \frac{\arctan(ax)^4}{8a}\right)\right) - a^2\int \frac{\arctan(ax)^3}{a^2x^2+1}dx + \int \frac{\arctan(ax)^3}{x^2}dx}{c^3} -$$

$$\frac{a^2\left(\frac{x\arctan(ax)^3}{4(a^2x^2+1)^2} + \frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2} - \frac{3}{8}\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2} + \frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)} + \frac{1}{4a(a^2x^2+1)} + \frac{\arctan(ax)^2}{4a}\right) + \frac{1}{16a(a^2x^2+1)^2}\right) + \frac{3}{4}\left(\right.\right.}{c^3}$$

$\downarrow$ 5361

$$\frac{-a^2\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx + \frac{x\arctan(ax)^3}{2(a^2x^2+1)} + \frac{\arctan(ax)^4}{8a}\right) + 3a\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx - a^2\int \frac{\arctan(ax)^3}{a^2x^2+1}dx - \frac{\arctan(ax)^3}{x}}{c^3} -$$

$$\frac{a^2\left(\frac{x\arctan(ax)^3}{4(a^2x^2+1)^2} + \frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2} - \frac{3}{8}\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2} + \frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)} + \frac{1}{4a(a^2x^2+1)} + \frac{\arctan(ax)^2}{4a}\right) + \frac{1}{16a(a^2x^2+1)^2}\right) + \frac{3}{4}\left(\right.\right.}{c^3}$$

$\downarrow$ 5419

$$\frac{-a^2\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx + \frac{x\arctan(ax)^3}{2(a^2x^2+1)} + \frac{\arctan(ax)^4}{8a}\right) + 3a\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx - \frac{1}{4}a\arctan(ax)^4 - \frac{\arctan(ax)^3}{x}}{c^3} -$$

$$\frac{a^2\left(\frac{x\arctan(ax)^3}{4(a^2x^2+1)^2} + \frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2} - \frac{3}{8}\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2} + \frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)} + \frac{1}{4a(a^2x^2+1)} + \frac{\arctan(ax)^2}{4a}\right) + \frac{1}{16a(a^2x^2+1)^2}\right) + \frac{3}{4}\left(\right.\right.}{c^3}$$

$\downarrow$ 5459

$$-\frac{a^2\left(\frac{x\arctan(ax)^3}{4(a^2x^2+1)^2} + \frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2} - \frac{3}{8}\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2} + \frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)} + \frac{1}{4a(a^2x^2+1)} + \frac{\arctan(ax)^2}{4a}\right) + \frac{1}{16a(a^2x^2+1)^2}\right) + \frac{3}{4}\right.}{c^3}$$

$$\frac{-a^2\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx + \frac{x\arctan(ax)^3}{2(a^2x^2+1)} + \frac{\arctan(ax)^4}{8a}\right) + 3a\left(i\int \frac{\arctan(ax)^2}{x(ax+i)}dx - \frac{1}{3}i\arctan(ax)^3\right) - \frac{1}{4}a\arctan(ax}{c^3}$$

$\downarrow$ 5403

$$-\frac{a^2\left(\frac{x\arctan(ax)^3}{4(a^2x^2+1)^2}+\frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2}-\frac{3}{8}\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}\right)+\frac{3}{4}\right.}{c^3}$$

$$\frac{-a^2\left(-\frac{3}{2}a\int\frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)+3a\left(i\left(2ia\int\frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\right.\right.}{c^3}$$

$\downarrow$ 5465

$$-\frac{a^2\left(\frac{x\arctan(ax)^3}{4(a^2x^2+1)^2}+\frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2}-\frac{3}{8}\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}\right)+\frac{3}{4}\right.}{c^3}$$

$$\frac{-a^2\left(-\frac{3}{2}a\left(\frac{\int\frac{\arctan(ax)}{(a^2x^2+1)^2}dx}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)+3a\left(i\left(2ia\int\frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-\right.\right.}{c^3}$$

$\downarrow$ 5427

$$-\frac{a^2\left(\frac{x\arctan(ax)^3}{4(a^2x^2+1)^2}+\frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2}-\frac{3}{8}\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}\right)+\frac{3}{4}\right.}{c^3}$$

$$\frac{-a^2\left(-\frac{3}{2}a\left(\frac{-\frac{1}{2}a\int\frac{x}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)+3a\left(i\left(2ia\int\right.\right.}{c^3}$$

$\downarrow$ 241

$$-\frac{a^2\left(\frac{x\arctan(ax)^3}{4(a^2x^2+1)^2}+\frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2}-\frac{3}{8}\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}\right)+\frac{3}{4}\right.}{c^3}$$

$$\frac{3a\left(i\left(2ia\int\frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\arctan(ax)^3\right)-a^2\left(\frac{x\arctan(ax)^3}{2(a^2x^2+1)}-\right.}{c^3}$$

$\downarrow$ 5527

$$-\frac{a^2\left(\frac{x\arctan(ax)^3}{4(a^2x^2+1)^2}+\frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2}-\frac{3}{8}\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}\right)+\frac{3}{4}\right.}{c^3}$$

$$3a\left(i\left(2ia\left(\frac{i\arctan(ax)\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{2a}-\frac{1}{2}i\int\frac{\operatorname{PolyLog}\left(2,\frac{2}{1-iax}-1\right)}{a^2x^2+1}dx\right)-i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{3}i\operatorname{arc}\right.$$

$\downarrow$ 7164

$$-\frac{a^2\left(\frac{x\arctan(ax)^3}{4(a^2x^2+1)^2}+\frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2}-\frac{3}{8}\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}\right)+\frac{1}{16a(a^2x^2+1)^2}\right)+\frac{3}{4}\right.}{c^3}$$

$$-a^2\left(\frac{x\arctan(ax)^3}{2(a^2x^2+1)}-\frac{3}{2}a\left(\frac{\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}}{a}-\frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right)+\frac{\arctan(ax)^4}{8a}\right)+3a\left(i\left(2ia\left(\frac{i\arctan(ax)\,\mathrm{P}}{}\right.\right.\right.$$

input
```
Int[ArcTan[a*x]^3/(x^2*(c + a^2*c*x^2)^3),x]
```

output
```
-((a^2*((3*ArcTan[a*x]^2)/(16*a*(1 + a^2*x^2)^2) + (x*ArcTan[a*x]^3)/(4*(1
 + a^2*x^2)^2) - (3*(1/(16*a*(1 + a^2*x^2)^2) + (x*ArcTan[a*x])/(4*(1 + a^
2*x^2)^2) + (3*(1/(4*a*(1 + a^2*x^2)) + (x*ArcTan[a*x])/(2*(1 + a^2*x^2))
+ ArcTan[a*x]^2/(4*a)))/4))/8 + (3*((x*ArcTan[a*x]^3)/(2*(1 + a^2*x^2)) +
ArcTan[a*x]^4/(8*a) - (3*a*(-1/2*ArcTan[a*x]^2/(a^2*(1 + a^2*x^2)) + (1/(4
*a*(1 + a^2*x^2)) + (x*ArcTan[a*x])/(2*(1 + a^2*x^2)) + ArcTan[a*x]^2/(4*a
))/a))/2))/4))/c^3) + (-(ArcTan[a*x]^3/x) - (a*ArcTan[a*x]^4)/4 - a^2*((x*
ArcTan[a*x]^3)/(2*(1 + a^2*x^2)) + ArcTan[a*x]^4/(8*a) - (3*a*(-1/2*ArcTan
[a*x]^2/(a^2*(1 + a^2*x^2)) + (1/(4*a*(1 + a^2*x^2)) + (x*ArcTan[a*x])/(2*
(1 + a^2*x^2)) + ArcTan[a*x]^2/(4*a))/a))/2) + 3*a*((-1/3*I)*ArcTan[a*x]^3
 + I*((-I)*ArcTan[a*x]^2*Log[2 - 2/(1 - I*a*x)] + (2*I)*a*(((I/2)*ArcTan[a
*x]*PolyLog[2, -1 + 2/(1 - I*a*x)])/a - PolyLog[3, -1 + 2/(1 - I*a*x)]/(4*
a)))))/c^3
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5431
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbol
] :> Simp[b*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (-Simp[x*(d + e*x
^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*d*(q + 1))), x] + Simp[(2*q + 3)/(2*d*(
q + 1))   Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a,
 b, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -3/2]
```

rule 5435
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[b*p*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p - 1)/(4*c*d*
(q + 1)^2)), x] + (-Simp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*d*
(q + 1))), x] + Simp[(2*q + 3)/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p, x], x] - Simp[b^2*p*((p - 1)/(4*(q + 1)^2))   Int[(d + e
*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

rule 5527
```
Int[(Log[u_]*((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^2
), x_Symbol] :> Simp[I*(a + b*ArcTan[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*p*(I/2)   Int[(a + b*ArcTan[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[e, c^2*
d] && EqQ[(1 - u)^2 - (1 - 2*(I/(I + c*x)))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.66 (sec) , antiderivative size = 1799, normalized size of antiderivative = 5.42

Expression too large to display

input
```
int(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^3,x)
```

output
```
a*(-7/8/c^3*arctan(a*x)^3/(a^2*x^2+1)^2*a^3*x^3-9/8/c^3*arctan(a*x)^3/(a^2
*x^2+1)^2*a*x-15/8/c^3*arctan(a*x)^4-1/c^3*arctan(a*x)^3/a/x-3/8/c^3*(-15/
4*arctan(a*x)^4+7/2*arctan(a*x)^2/(a^2*x^2+1)+4*arctan(a*x)^2*ln(a^2*x^2+1
)+1/2*arctan(a*x)^2/(a^2*x^2+1)^2-8*arctan(a*x)^2*ln(a*x)-8*arctan(a*x)^2*
ln((1+I*a*x)/(a^2*x^2+1)^(1/2))-I*arctan(a*x)*(a*x-I)/(a*x+I)+16*I*arctan(
a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+16*I*arctan(a*x)*polylog(2,-(1
+I*a*x)/(a^2*x^2+1)^(1/2))+1/2*(a*x+I)/(a*x-I)+I*(a*x+I)*arctan(a*x)/(a*x-
I)+1/2*(a*x-I)/(a*x+I)+8*arctan(a*x)^2*ln((1+I*a*x)^2/(a^2*x^2+1)-1)-8*arc
tan(a*x)^2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-16*polylog(3,-(1+I*a*x)/(a^2*
x^2+1)^(1/2))-8*arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))+8/3*I*arct
an(a*x)^3-16*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))-1/16*(32*I*Pi*csgn(I*(
1+I*a*x)^2/(a^2*x^2+1))*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x
^2+1)+1)^2)^2+32*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1))^2*csgn(I*((1+I*a
*x)^2/(a^2*x^2+1)+1)^2)+64*I*Pi*csgn(I*(1+I*a*x)/(a^2*x^2+1)^(1/2))*csgn(I
*(1+I*a*x)^2/(a^2*x^2+1))^2+32*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)+1)^2)^
3+64*I*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I/((1+I*a*x)^2/(a^2*x^2
+1)+1))*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))-64
*I*Pi*csgn(((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))^2+64*I
*Pi*csgn(I*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn((
(1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))-32*I*Pi*csgn(I*...
```

## Fricas [F]

$$\int \frac{\arctan(ax)^3}{x^2 (c + a^2cx^2)^3}\, dx = \int \frac{\arctan{(ax)}^3}{(a^2cx^2 + c)^3 x^2}\, dx$$

input
```
integrate(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
integral(arctan(a*x)^3/(a^6*c^3*x^8 + 3*a^4*c^3*x^6 + 3*a^2*c^3*x^4 + c^3*
x^2), x)
```

## Sympy [F]

$$\int \frac{\arctan(ax)^3}{x^2 (c + a^2 c x^2)^3}\, dx = \frac{\int \frac{\operatorname{atan}^3(ax)}{a^6x^8+3a^4x^6+3a^2x^4+x^2}\, dx}{c^3}$$

input
```
integrate(atan(a*x)**3/x**2/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(atan(a*x)**3/(a**6*x**8 + 3*a**4*x**6 + 3*a**2*x**4 + x**2), x)/c
**3
```

## Maxima [F]

$$\int \frac{\arctan(ax)^3}{x^2 (c + a^2 c x^2)^3}\, dx = \int \frac{\arctan(ax)^3}{(a^2cx^2 + c)^3 x^2}\, dx$$

input
```
integrate(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
-1/16384*(2400*(a^5*x^5 + 2*a^3*x^3 + a*x)*arctan(a*x)^4 - 90*(a^5*x^5 + 2
*a^3*x^3 + a*x)*log(a^2*x^2 + 1)^4 + 256*(15*a^4*x^4 + 25*a^2*x^2 + 8)*arc
tan(a*x)^3 - 48*(15*(a^5*x^5 + 2*a^3*x^3 + a*x)*arctan(a*x)^2 + 4*(15*a^4*
x^4 + 25*a^2*x^2 + 8)*arctan(a*x))*log(a^2*x^2 + 1)^2 - (a^4*c^3*x^5 + 2*a
^2*c^3*x^3 + c^3*x)*(360*((8*a^2*x^2 + 7)*a^2/(a^12*c^3*x^4 + 2*a^10*c^3*x
^2 + a^8*c^3) + 2*(4*a^2*x^2 + 3)*log(a^2*x^2 + 1)/(a^10*c^3*x^4 + 2*a^8*c
^3*x^2 + a^6*c^3))*a^7 - 2949120*a^7*integrate(1/1024*x^7*arctan(a*x)^2*lo
g(a^2*x^2 + 1)/(a^6*c^3*x^8 + 3*a^4*c^3*x^6 + 3*a^2*c^3*x^4 + c^3*x^2), x)
 - 737280*a^7*integrate(1/1024*x^7*log(a^2*x^2 + 1)^3/(a^6*c^3*x^8 + 3*a^4
*c^3*x^6 + 3*a^2*c^3*x^4 + c^3*x^2), x) + 360*(2*a^2*x^2 + 1)*a^5*log(a^2*
x^2 + 1)^3/(a^8*c^3*x^4 + 2*a^6*c^3*x^2 + a^4*c^3) + 5898240*a^6*integrate
(1/1024*x^6*arctan(a*x)^3/(a^6*c^3*x^8 + 3*a^4*c^3*x^6 + 3*a^2*c^3*x^4 + c
^3*x^2), x) + 1474560*a^6*integrate(1/1024*x^6*arctan(a*x)*log(a^2*x^2 + 1
)^2/(a^6*c^3*x^8 + 3*a^4*c^3*x^6 + 3*a^2*c^3*x^4 + c^3*x^2), x) - 11796480
*a^6*integrate(1/1024*x^6*arctan(a*x)*log(a^2*x^2 + 1)/(a^6*c^3*x^8 + 3*a^
4*c^3*x^6 + 3*a^2*c^3*x^4 + c^3*x^2), x) + 720*(2*a^2*x^2 + 1)*a^5*log(a^2
*x^2 + 1)^2/(a^8*c^3*x^4 + 2*a^6*c^3*x^2 + a^4*c^3) + 270*(((16*a^2*x^2 +
15)*a^2/(a^14*c^3*x^4 + 2*a^12*c^3*x^2 + a^10*c^3) + 2*(8*a^2*x^2 + 7)*log
(a^2*x^2 + 1)/(a^12*c^3*x^4 + 2*a^10*c^3*x^2 + a^8*c^3))*a^4 + 2*(4*a^2*x^
2 + 3)*a^2*log(a^2*x^2 + 1)^2/(a^10*c^3*x^4 + 2*a^8*c^3*x^2 + a^6*c^3))...
```

## Giac [F]

$$\int \frac{\arctan(ax)^3}{x^2\left(c+a^2cx^2\right)^3}\,dx=\int \frac{\arctan\left(ax\right)^3}{\left(a^2cx^2+c\right)^3x^2}\,dx$$

input
```
integrate(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^3/((a^2*c*x^2 + c)^3*x^2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^3}{x^2 (c+a^2cx^2)^3} dx = \int \frac{\operatorname{atan}(a\,x)^3}{x^2 \left(c\,a^2\,x^2+c\right)^3} dx$$

input
```
int(atan(a*x)^3/(x^2*(c + a^2*c*x^2)^3),x)
```

output
```
int(atan(a*x)^3/(x^2*(c + a^2*c*x^2)^3), x)
```

## Reduce [F]

$$\int \frac{\arctan(ax)^3}{x^2 (c+a^2cx^2)^3} dx$$
$$= \frac{-120atan(ax)^4 a^5x^5 - 240atan(ax)^4 a^3x^3 - 120atan(ax)^4 ax - 480atan(ax)^3 a^4x^4 - 800atan(ax)^3 a^2x^2}{}$$

input
```
int(atan(a*x)^3/x^2/(a^2*c*x^2+c)^3,x)
```

output
```
( - 120*atan(a*x)**4*a**5*x**5 - 240*atan(a*x)**4*a**3*x**3 - 120*atan(a*x
)**4*a*x - 480*atan(a*x)**3*a**4*x**4 - 800*atan(a*x)**3*a**2*x**2 - 256*a
tan(a*x)**3 + 450*atan(a*x)**2*a**5*x**5 + 180*atan(a*x)**2*a**3*x**3 - 51
0*atan(a*x)**2*a*x + 900*atan(a*x)*a**4*x**4 + 1020*atan(a*x)*a**2*x**2 +
768*int(atan(a*x)**2/(a**6*x**7 + 3*a**4*x**5 + 3*a**2*x**3 + x),x)*a**5*x
**5 + 1536*int(atan(a*x)**2/(a**6*x**7 + 3*a**4*x**5 + 3*a**2*x**3 + x),x)
*a**3*x**3 + 768*int(atan(a*x)**2/(a**6*x**7 + 3*a**4*x**5 + 3*a**2*x**3 +
 x),x)*a*x - 225*a**5*x**5 + 255*a*x)/(256*c**3*x*(a**4*x**4 + 2*a**2*x**2
 + 1))
```

# 3.410 $\int \frac{\arctan(ax)^3}{x^3 (c+a^2cx^2)^3} \, dx$

## Optimal result

Integrand size = 22, antiderivative size = 478

$$\int \frac{\arctan(ax)^3}{x^3(c+a^2cx^2)^3}\,dx = -\frac{3a^3x}{128c^3(1+a^2x^2)^2}-\frac{237a^3x}{256c^3(1+a^2x^2)}-\frac{237a^2\arctan(ax)}{256c^3}+\frac{3a^2\arctan(ax)}{32c^3(1+a^2x^2)^2}+\frac{57a^2\arctan(ax)}{32c^3(1+a^2x^2)}-\frac{3ia^2\arctan(ax)^2}{2c^3}-\frac{3a\arctan(ax)^2}{2c^3x}+\frac{3a^3x\arctan(ax)^2}{16c^3(1+a^2x^2)^2}+\frac{57a^3x\arctan(ax)^2}{32c^3(1+a^2x^2)}+\frac{3a^2\arctan(ax)^3}{32c^3}-\frac{\arctan(ax)^3}{2c^3x^2}-\frac{a^2\arctan(ax)^3}{4c^3(1+a^2x^2)^2}-\frac{a^2\arctan(ax)^3}{c^3(1+a^2x^2)}+\frac{3ia^2\arctan(ax)^4}{4c^3}+\frac{3a^2\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{c^3}-\frac{3a^2\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)}{c^3}-\frac{3ia^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{2c^3}+\frac{9ia^2\arctan(ax)^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{2c^3}-\frac{9a^2\arctan(ax)\operatorname{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)}{2c^3}-\frac{9ia^2\operatorname{PolyLog}\left(4,-1+\frac{2}{1-iax}\right)}{4c^3}$$

output

```
-3/128*a^3*x/c^3/(a^2*x^2+1)^2-237/256*a^3*x/c^3/(a^2*x^2+1)-237/256*a^2*a
rctan(a*x)/c^3+3/32*a^2*arctan(a*x)/c^3/(a^2*x^2+1)^2+57/32*a^2*arctan(a*x
)/c^3/(a^2*x^2+1)-3/2*I*a^2*polylog(2,-1+2/(1-I*a*x))/c^3-3/2*a*arctan(a*x
)^2/c^3/x+3/16*a^3*x*arctan(a*x)^2/c^3/(a^2*x^2+1)^2+57/32*a^3*x*arctan(a*
x)^2/c^3/(a^2*x^2+1)+3/32*a^2*arctan(a*x)^3/c^3-1/2*arctan(a*x)^3/c^3/x^2-
1/4*a^2*arctan(a*x)^3/c^3/(a^2*x^2+1)^2-a^2*arctan(a*x)^3/c^3/(a^2*x^2+1)-
9/4*I*a^2*polylog(4,-1+2/(1-I*a*x))/c^3+3*a^2*arctan(a*x)*ln(2-2/(1-I*a*x)
)/c^3-3*a^2*arctan(a*x)^3*ln(2-2/(1-I*a*x))/c^3+3/4*I*a^2*arctan(a*x)^4/c^
3-3/2*I*a^2*arctan(a*x)^2/c^3-9/2*a^2*arctan(a*x)*polylog(3,-1+2/(1-I*a*x)
)/c^3+9/2*I*a^2*arctan(a*x)^2*polylog(2,-1+2/(1-I*a*x))/c^3
```

## Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 295, normalized size of antiderivative = 0.62

$$\int \frac{\arctan(ax)^3}{x^3 (c+a^2cx^2)^3} dx$$
$$= \frac{a^2\left(48i\pi^4 - 1536i\arctan(ax)^2 - \frac{1536\arctan(ax)^2}{ax} - \frac{512(1+a^2x^2)\arctan(ax)^3}{a^2x^2} - 768i\arctan(ax)^4 + 960\arctan(a\right.}{}$$

input
```
Integrate[ArcTan[a*x]^3/(x^3*(c + a^2*c*x^2)^3),x]
```

output
```
(a^2*((48*I)*Pi^4 - (1536*I)*ArcTan[a*x]^2 - (1536*ArcTan[a*x]^2)/(a*x) -
(512*(1 + a^2*x^2)*ArcTan[a*x]^3)/(a^2*x^2) - (768*I)*ArcTan[a*x]^4 + 960*
ArcTan[a*x]*Cos[2*ArcTan[a*x]] - 640*ArcTan[a*x]^3*Cos[2*ArcTan[a*x]] + 12
*ArcTan[a*x]*Cos[4*ArcTan[a*x]] - 32*ArcTan[a*x]^3*Cos[4*ArcTan[a*x]] - 30
72*ArcTan[a*x]^3*Log[1 - E^((-2*I)*ArcTan[a*x])] + 3072*ArcTan[a*x]*Log[1
- E^((2*I)*ArcTan[a*x])] - (4608*I)*ArcTan[a*x]^2*PolyLog[2, E^((-2*I)*Arc
Tan[a*x])] - (1536*I)*PolyLog[2, E^((2*I)*ArcTan[a*x])] - 4608*ArcTan[a*x]
*PolyLog[3, E^((-2*I)*ArcTan[a*x])] + (2304*I)*PolyLog[4, E^((-2*I)*ArcTan
[a*x])] - 480*Sin[2*ArcTan[a*x]] + 960*ArcTan[a*x]^2*Sin[2*ArcTan[a*x]] -
3*Sin[4*ArcTan[a*x]] + 24*ArcTan[a*x]^2*Sin[4*ArcTan[a*x]]))/(1024*c^3)
```

## Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3}{x^3 (a^2cx^2+c)^3} dx$$

$$\Big\downarrow \text{ 5501}$$

$$\frac{\int \frac{\arctan(ax)^3}{c^2x^3(a^2x^2+1)^2}dx}{c} - a^2\int \frac{\arctan(ax)^3}{c^3x\,(a^2x^2+1)^3}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{\arctan(ax)^3}{x^3(a^2x^2+1)^2}dx}{c^3}-\frac{a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^3}dx}{c^3}$$

$\Big\downarrow$ 5501

$$\frac{\int \frac{\arctan(ax)^3}{x^3(a^2x^2+1)}dx-a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx}{c^3}-\frac{a^2\left(\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx-a^2\int \frac{x\arctan(ax)^3}{(a^2x^2+1)^3}dx\right)}{c^3}$$

$\Big\downarrow$ 5453

$$\frac{a^2\left(-\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx\right)-a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)}dx+\int \frac{\arctan(ax)^3}{x^3}dx}{c^3}-\frac{a^2\left(\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx-a^2\int \frac{x\arctan(ax)^3}{(a^2x^2+1)^3}dx\right)}{c^3}$$

$\Big\downarrow$ 5361

$$\frac{\frac{3}{2}a\int \frac{\arctan(ax)^2}{x^2(a^2x^2+1)}dx-a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx-a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)}dx-\frac{\arctan(ax)^3}{2x^2}}{c^3}-\frac{a^2\left(\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx-a^2\int \frac{x\arctan(ax)^3}{(a^2x^2+1)^3}dx\right)}{c^3}$$

$\Big\downarrow$ 5453

$$\frac{\frac{3}{2}a\left(\int \frac{\arctan(ax)^2}{x^2}dx-a^2\int \frac{\arctan(ax)^2}{a^2x^2+1}dx\right)-a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx-a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)}dx-\frac{\arctan(ax)^3}{2x^2}}{c^3}-\frac{a^2\left(\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx-a^2\int \frac{x\arctan(ax)^3}{(a^2x^2+1)^3}dx\right)}{c^3}$$

$\Big\downarrow$ 5361

$$\frac{\frac{3}{2}a\left(a^2\left(-\int \frac{\arctan(ax)^2}{a^2x^2+1}dx\right)+2a\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx-\frac{\arctan(ax)^2}{x}\right)-a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx-a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)}dx-\frac{\arctan(ax)^3}{2x^2}}{c^3}-\frac{a^2\left(\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx-a^2\int \frac{x\arctan(ax)^3}{(a^2x^2+1)^3}dx\right)}{c^3}$$

$\Big\downarrow$ 5419

$$\frac{\frac{3}{2}a\left(2a\int \frac{\arctan(ax)}{x(a^2x^2+1)}dx-\frac{1}{3}a\arctan(ax)^3-\frac{\arctan(ax)^2}{x}\right)-a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx-a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)}dx-\frac{\arctan(ax)^3}{2x^2}}{c^3}-\frac{a^2\left(\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx-a^2\int \frac{x\arctan(ax)^3}{(a^2x^2+1)^3}dx\right)}{c^3}$$

$$\Big\downarrow \text{5459}$$

$$-\frac{a^2\left(\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx - a^2\int \frac{x\arctan(ax)^3}{(a^2x^2+1)^3}dx\right)}{c^3} + \frac{-a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx - a^2\left(i\int \frac{\arctan(ax)^3}{x(ax+i)}dx - \frac{1}{4}i\arctan(ax)^4\right) + \frac{3}{2}a\left(2a\left(i\int \frac{\arctan(ax)}{x(ax+i)}dx - \frac{1}{2}i\arctan(ax)^2\right) - \frac{1}{3}a\, a}{c^3}$$

$$\Big\downarrow \text{5403}$$

$$-\frac{a^2\left(\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx - a^2\int \frac{x\arctan(ax)^3}{(a^2x^2+1)^3}dx\right)}{c^3} + \frac{-a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx + \frac{3}{2}a\left(2a\left(i\left(ia\int \frac{\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx - i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)\right) - \frac{1}{2}i\arctan(ax)^2\right) - \frac{1}{3}a\, a}{}$$

$$\Big\downarrow \text{2897}$$

$$-\frac{a^2\left(\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx - a^2\int \frac{x\arctan(ax)^3}{(a^2x^2+1)^3}dx\right)}{c^3} + \frac{-a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx - a^2\left(i\left(3ia\int \frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx - i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right) - \frac{1}{4}i\arctan(ax)^4\right)}{}$$

$$\Big\downarrow \text{5465}$$

$$-\frac{a^2\left(\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx - a^2\left(\frac{3\int \frac{\arctan(ax)^2}{(a^2x^2+1)^3}dx}{4a} - \frac{\arctan(ax)^3}{4a^2(a^2x^2+1)^2}\right)\right)}{c^3} + \frac{-a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx - a^2\left(i\left(3ia\int \frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx - i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right) - \frac{1}{4}i\arctan(ax)^4\right)}{}$$

$$\Big\downarrow \text{5435}$$

$$-\frac{a^2\left(\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx - a^2\left(\frac{3\left(\frac{3}{4}\int \frac{\arctan(ax)^2}{(a^2x^2+1)^2}dx - \frac{1}{8}\int \frac{1}{(a^2x^2+1)^3}dx + \frac{x\arctan(ax)^2}{4(a^2x^2+1)^2} + \frac{\arctan(ax)}{8a(a^2x^2+1)^2}\right)}{4a} - \frac{\arctan(ax)^3}{4a^2(a^2x^2+1)^2}\right)\right)}{c^3} + \frac{-a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx - a^2\left(i\left(3ia\int \frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx - i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right) - \frac{1}{4}i\arctan(ax)^4\right)}{}$$

$$\Big\downarrow \text{215}$$

$$-\frac{a^2\left(\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx - a^2\left(\frac{3\left(\frac{3}{4}\int \frac{\arctan(ax)^2}{\left(a^2x^2+1\right)^2}dx+\frac{1}{8}\left(-\frac{3}{4}\int \frac{1}{\left(a^2x^2+1\right)^2}dx-\frac{x}{4\left(a^2x^2+1\right)^2}\right)+\frac{x\arctan(ax)^2}{4\left(a^2x^2+1\right)^2}+\frac{\arctan(ax)}{8a\left(a^2x^2+1\right)^2}\right)}{4a} - \frac{\arctan(a}{4a^2(a^2x^2}\right.\right.}{c^3}$$

$$-a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx - a^2\left(i\left(3ia\int \frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx - i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right) - \frac{1}{4}i\arctan(ax)^4\right)$$

$\downarrow$ 215

$$-\frac{a^2\left(\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx - a^2\left(\frac{3\left(\frac{3}{4}\int \frac{\arctan(ax)^2}{\left(a^2x^2+1\right)^2}dx+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{1}{2}\int \frac{1}{a^2x^2+1}dx+\frac{x}{2\left(a^2x^2+1\right)}\right)-\frac{x}{4\left(a^2x^2+1\right)^2}\right)+\frac{x\arctan(ax)^2}{4\left(a^2x^2+1\right)^2}+\frac{\arctan(ax)}{8a\left(a^2x^2+1\right)^2}\right)}{4a}\right.\right.}{c^3}$$

$$-a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx - a^2\left(i\left(3ia\int \frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx - i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right) - \frac{1}{4}i\arctan(ax)^4\right)$$

$\downarrow$ 216

$$-\frac{a^2\left(\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx - a^2\left(\frac{3\left(\frac{3}{4}\int \frac{\arctan(ax)^2}{\left(a^2x^2+1\right)^2}dx+\frac{x\arctan(ax)^2}{4\left(a^2x^2+1\right)^2}+\frac{\arctan(ax)}{8a\left(a^2x^2+1\right)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}\right)-\frac{x}{4\left(a^2x^2+1\right)^2}\right)\right)}{4a} - \right.\right.}{c^3}$$

$$-a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx - a^2\left(i\left(3ia\int \frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx - i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right) - \frac{1}{4}i\arctan(ax)^4\right)$$

$\downarrow$ 5427

$$-\frac{a^2\left(\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx - a^2\left(\frac{3\left(\frac{3}{4}\left(-a\int \frac{x\arctan(ax)}{\left(a^2x^2+1\right)^2}dx+\frac{x\arctan(ax)^2}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^3}{6a}\right)+\frac{x\arctan(ax)^2}{4\left(a^2x^2+1\right)^2}+\frac{\arctan(ax)}{8a\left(a^2x^2+1\right)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2\left(a^2x^2+1\right)}\right.\right.\right.}{4a}\right.\right.}{c^3}$$

$$-a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx - a^2\left(i\left(3ia\int \frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx - i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right) - \frac{1}{4}i\arctan(ax)^4\right)$$

$\downarrow$ 5465

$$-\frac{a^2\left(\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx - a^2\left(\frac{3\left(\frac{3}{4}\left(-a\left(\frac{\int \frac{1}{(a^2x^2+1)^2}dx}{2a} - \frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right) + \frac{x\arctan(ax)^2}{2(a^2x^2+1)} + \frac{\arctan(ax)^3}{6a}\right) + \frac{x\arctan(ax)^2}{4(a^2x^2+1)^2} + \frac{\arctan(ax)}{8a(a^2x^2+1)^2} + \frac{1}{8}\right.}{4a}\right.\right.}{c^3}$$

$$-a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx - a^2\left(i\left(3ia\int \frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx - i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right) - \frac{1}{4}i\arctan(ax)^4\right)$$

$\downarrow$ 215

$$-\frac{a^2\left(\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx - a^2\left(\frac{3\left(\frac{3}{4}\left(-a\left(\frac{\frac{1}{2}\int \frac{1}{a^2x^2+1}dx + \frac{x}{2(a^2x^2+1)}}{2a} - \frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right) + \frac{x\arctan(ax)^2}{2(a^2x^2+1)} + \frac{\arctan(ax)^3}{6a}\right) + \frac{x\arctan(ax)^2}{4(a^2x^2+1)^2} + \frac{\mathrm{arc}}{8a(a}\right.}{4a}\right.\right.}{c^3}$$

$$-a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx - a^2\left(i\left(3ia\int \frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx - i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right) - \frac{1}{4}i\arctan(ax)^4\right)$$

$\downarrow$ 216

$$-\frac{a^2\left(\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx - a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{4(a^2x^2+1)^2} + \frac{\arctan(ax)}{8a(a^2x^2+1)^2} + \frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2(a^2x^2+1)} + \frac{\arctan(ax)}{2a}\right) - \frac{x}{4(a^2x^2+1)^2}\right) + \frac{3}{4}\left(\frac{x\arctan(ax)^2}{2(a^2x^2+1)} - a\left(\right.\right.\right.}{4a}\right.\right.}{c^3}$$

$$-a^2\int \frac{\arctan(ax)^3}{x(a^2x^2+1)^2}dx - a^2\left(i\left(3ia\int \frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx - i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right) - \frac{1}{4}i\arctan(ax)^4\right)$$

$\downarrow$ 5501

$$-\frac{a^2\left(-a^2\int \frac{x\arctan(ax)^3}{(a^2x^2+1)^2}dx+\int \frac{\arctan(ax)^3}{x(a^2x^2+1)}dx-\left(a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{4\left(a^2x^2+1\right)^2}+\frac{\arctan(ax)}{8a\left(a^2x^2+1\right)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}\right)-\frac{x}{4\left(a^2x^2\right.}\right.\right.}{}\right.\right.\right.}{c^3}$$

$$-a^2\left(\int \frac{\arctan(ax)^3}{x(a^2x^2+1)}dx-a^2\int \frac{x\arctan(ax)^3}{(a^2x^2+1)^2}dx\right)-a^2\left(i\left(3ia\int \frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^3\log\left(2-\frac{2}{1-ia\ldots}\right.\right.\right.$$

$\downarrow$ 5459

$$-a^2\left(-a^2\int \frac{x\arctan(ax)^3}{(a^2x^2+1)^2}dx+i\int \frac{\arctan(ax)^3}{x(ax+i)}dx-\frac{1}{4}i\arctan(ax)^4\right)-a^2\left(i\left(3ia\int \frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\,\mathrm{arcta}\ldots\right.\right.$$

$$\frac{a^2\left(-a^2\int \frac{x\arctan(ax)^3}{(a^2x^2+1)^2}dx+i\int \frac{\arctan(ax)^3}{x(ax+i)}dx-a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{4\left(a^2x^2+1\right)^2}+\frac{\arctan(ax)}{8a\left(a^2x^2+1\right)^2}+\frac{1}{8}\left(-\frac{3}{4}\left(\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}\right)-\frac{x}{4\left(a^2x^2+1\right.}\right.\right.}{}\right.\right.}{c^3}$$

$\downarrow$ 5403

$$-a^2\left(i\left(3ia\int \frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{4}i\arctan(ax)^4\right)-a^2\left(-a^2\int \frac{x\,\mathrm{arctan}\ldots}{(a^2x^2\ldots}\right.$$

$$a^2\left(-a^2\int \frac{x\arctan(ax)^3}{(a^2x^2+1)^2}dx+i\left(3ia\int \frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right)-a^2\left(\frac{3\left(\frac{x\,\mathrm{arctan}(a\ldots}{4\left(a^2x^2+1\ldots\right.}\right.}{}\right.\right.$$

$\downarrow$ 5465

$$-a^2\left(i\left(3ia\int \frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right)-\frac{1}{4}i\arctan(ax)^4\right)-a^2\left(-a^2\left(\frac{3\int \frac{\arcta}{(a^2}}{}\right.\right.$$

$$a^2\left(-a^2\left(\frac{3\int \frac{\arctan(ax)^2}{\left(a^2x^2+1\right)^2}dx}{2a}-\frac{\arctan(ax)^3}{2a^2(a^2x^2+1)}\right)+i\left(3ia\int \frac{\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx-i\arctan(ax)^3\log\left(2-\frac{2}{1-iax}\right)\right)-\right.$$

$\downarrow$ 5427

$$-\frac{\arctan(ax)^3}{2x^2}+\frac{3}{2}a\left(-\frac{1}{3}a\arctan(ax)^3-\frac{\arctan(ax)^2}{x}+2a\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-\right.\right.\right.\right.$$

$$a^2\left(-\frac{1}{4}i\arctan(ax)^4-a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{4\left(a^2x^2+1\right)^2}+\frac{\arctan(ax)}{8a\left(a^2x^2+1\right)^2}+\frac{1}{8}\left(-\frac{x}{4\left(a^2x^2+1\right)^2}-\frac{3}{4}\left(\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}\right)\right)+\frac{3}{4}\left(\frac{\arctan(ax)^3}{6a}+\frac{x\arcta}{2(a^2}\right.\right.}{4a}\right.\right.$$

$\downarrow$ 5465

$$-\frac{\arctan(ax)^3}{2x^2}+\frac{3}{2}a\left(-\frac{1}{3}a\arctan(ax)^3-\frac{\arctan(ax)^2}{x}+2a\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-\right.\right.\right.\right.$$

$$a^2\left(-\frac{1}{4}i\arctan(ax)^4-a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{4\left(a^2x^2+1\right)^2}+\frac{\arctan(ax)}{8a\left(a^2x^2+1\right)^2}+\frac{1}{8}\left(-\frac{x}{4\left(a^2x^2+1\right)^2}-\frac{3}{4}\left(\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}\right)\right)+\frac{3}{4}\left(\frac{\arctan(ax)^3}{6a}+\frac{x\arcta}{2(a^2}\right.\right.}{4a}\right.\right.$$

$\downarrow$ 215

$$-\frac{\arctan(ax)^3}{2x^2}+\frac{3}{2}a\left(-\frac{1}{3}a\arctan(ax)^3-\frac{\arctan(ax)^2}{x}+2a\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\operatorname{PolyLog}\left(2,\frac{2}{1-iax}\right.\right.\right.\right.$$

$$a^2\left(-\frac{1}{4}i\arctan(ax)^4-a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{4\left(a^2x^2+1\right)^2}+\frac{\arctan(ax)}{8a\left(a^2x^2+1\right)^2}+\frac{1}{8}\left(-\frac{x}{4\left(a^2x^2+1\right)^2}-\frac{3}{4}\left(\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}\right)\right)+\frac{3}{4}\left(\frac{\arctan(ax)^3}{6a}+\frac{x\arctan}{2\left(a^2\right.}\right.}{4a}\right.\right.$$

$\downarrow$ 216

$$-\frac{\arctan(ax)^3}{2x^2}+\frac{3}{2}a\left(-\frac{1}{3}a\arctan(ax)^3-\frac{\arctan(ax)^2}{x}+2a\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\operatorname{PolyLog}\left(2,\frac{2}{1-iax}\right.\right.\right.\right.$$

$$a^2\left(-\frac{1}{4}i\arctan(ax)^4-a^2\left(\frac{3\left(\frac{\arctan(ax)^3}{6a}+\frac{x\arctan(ax)^2}{2\left(a^2x^2+1\right)}-a\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\arctan(ax)}{2a^2\left(a^2x^2+1\right)}\right)\right)}{2a}-\frac{\arctan(ax)^3}{2a^2(a^2x^2+1)}\right)-a^2\left(\right.\right.$$

$\downarrow$ 5527

$$-\frac{\arctan(ax)^3}{2x^2}+\frac{3}{2}a\left(-\frac{1}{3}a\arctan(ax)^3-\frac{\arctan(ax)^2}{x}+2a\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\operatorname{PolyLog}\left(2,\frac{2}{1-iax}\right.\right.\right.\right.$$

$$a^2\left(-\frac{1}{4}i\arctan(ax)^4-a^2\left(\frac{3\left(\frac{\arctan(ax)^3}{6a}+\frac{x\arctan(ax)^2}{2\left(a^2x^2+1\right)}-a\left(\frac{\frac{x}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\arctan(ax)}{2a^2\left(a^2x^2+1\right)}\right)\right)}{2a}-\frac{\arctan(ax)^3}{2a^2(a^2x^2+1)}\right)-a^2\left(\right.\right.$$

$$\Big\downarrow \text{5531}$$

$$-\frac{\arctan(ax)^3}{2x^2}+\frac{3}{2}a\left(-\frac{1}{3}a\arctan(ax)^3-\frac{\arctan(ax)^2}{x}+2a\left(i\left(-i\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)-\frac{1}{2}\text{PolyLog}\left(2,\frac{2}{1-iax}-\right.\right.\right.\right.$$

$$a^2\left(-\frac{1}{4}i\arctan(ax)^4-a^2\left(\frac{3\left(\frac{\arctan(ax)^3}{6a}+\frac{x\arctan(ax)^2}{2(a^2x^2+1)}-a\left(\frac{\frac{x}{2(a^2x^2+1)}+\frac{\arctan(ax)}{2a}}{2a}-\frac{\arctan(ax)}{2a^2(a^2x^2+1)}\right)\right)}{2a}-\frac{\arctan(ax)^3}{2a^2(a^2x^2+1)}\right)-a^2\left(\right.\right.$$

input
```
Int[ArcTan[a*x]^3/(x^3*(c + a^2*c*x^2)^3),x]
```

output
```
$Aborted
```

## Maple [A] (verified)

Time = 92.94 (sec) , antiderivative size = 579, normalized size of antiderivative = 1.21

| method | result |
|---|---|
| derivativedivides | $a^2\left(-\frac{3i\arctan(ax)^2}{c^3}+\frac{5\left(6i\arctan(ax)^2+4\arctan(ax)^3-3i-6\arctan(ax)\right)(ax-i)}{64c^3(ax+i)}+\frac{5\left(-6i\arctan(ax)^2+4\arctan\right.}{6}\right.$ |
| default | $a^2\left(-\frac{3i\arctan(ax)^2}{c^3}+\frac{5\left(6i\arctan(ax)^2+4\arctan(ax)^3-3i-6\arctan(ax)\right)(ax-i)}{64c^3(ax+i)}+\frac{5\left(-6i\arctan(ax)^2+4\arctan\right.}{6}\right.$ |

input
```
int(arctan(a*x)^3/x^3/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
a^2*(-3*I/c^3*arctan(a*x)^2+5/64*(6*I*arctan(a*x)^2+4*arctan(a*x)^3-3*I-6*
arctan(a*x))*(a*x-I)/c^3/(a*x+I)+5/64*(-6*I*arctan(a*x)^2+4*arctan(a*x)^3+
3*I-6*arctan(a*x))*(a*x+I)/c^3/(a*x-I)-1/2/c^3*arctan(a*x)^2*(-I*arctan(a*
x)+arctan(a*x)*a*x-3*I*a*x)*(a*x+I)/a^2/x^2-3*I/c^3*polylog(2,(1+I*a*x)/(a
^2*x^2+1)^(1/2))-3/c^3*arctan(a*x)^3*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*I
/c^3*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-18/c^3*arctan(a*x)*polylog(3,
-(1+I*a*x)/(a^2*x^2+1)^(1/2))-18*I/c^3*polylog(4,(1+I*a*x)/(a^2*x^2+1)^(1/
2))-3/c^3*arctan(a*x)^3*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-18*I/c^3*polylog
(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-18/c^3*arctan(a*x)*polylog(3,(1+I*a*x)/(a
^2*x^2+1)^(1/2))+3/4*I/c^3*arctan(a*x)^4+9*I/c^3*arctan(a*x)^2*polylog(2,-
(1+I*a*x)/(a^2*x^2+1)^(1/2))+3/c^3*arctan(a*x)*ln(1+(1+I*a*x)/(a^2*x^2+1)^
(1/2))+9*I/c^3*arctan(a*x)^2*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+3/c^3*
arctan(a*x)*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-1/256*arctan(a*x)*(8*arctan(
a*x)^2-3)/c^3*cos(4*arctan(a*x))+3/1024*(8*arctan(a*x)^2-1)/c^3*sin(4*arct
an(a*x)))
```

## Fricas [F]

$$\int \frac{\arctan(ax)^3}{x^3\left(c+a^2cx^2\right)^3}\,dx=\int \frac{\arctan\left(ax\right)^3}{\left(a^2cx^2+c\right)^3x^3}\,dx$$

input
```
integrate(arctan(a*x)^3/x^3/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
integral(arctan(a*x)^3/(a^6*c^3*x^9 + 3*a^4*c^3*x^7 + 3*a^2*c^3*x^5 + c^3*
x^3), x)
```

## Sympy [F]

$$\int \frac{\arctan(ax)^3}{x^3\left(c+a^2cx^2\right)^3}\,dx=\frac{\int \frac{\operatorname{atan}^3\left(ax\right)}{a^6x^9+3a^4x^7+3a^2x^5+x^3}\,dx}{c^3}$$

input
```
integrate(atan(a*x)**3/x**3/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(atan(a*x)**3/(a**6*x**9 + 3*a**4*x**7 + 3*a**2*x**5 + x**3), x)/c
**3
```

## Maxima [F]

$$\int \frac{\arctan(ax)^3}{x^3\left(c+a^2cx^2\right)^3}\,dx=\int \frac{\arctan\left(ax\right)^3}{\left(a^2cx^2+c\right)^3x^3}\,dx$$

input
```
integrate(arctan(a*x)^3/x^3/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)^3/((a^2*c*x^2 + c)^3*x^3), x)
```

## Giac [F]

$$\int \frac{\arctan(ax)^3}{x^3\left(c+a^2cx^2\right)^3}\,dx=\int \frac{\arctan\left(ax\right)^3}{\left(a^2cx^2+c\right)^3x^3}\,dx$$

input
```
integrate(arctan(a*x)^3/x^3/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^3/((a^2*c*x^2 + c)^3*x^3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^3}{x^3\left(c+a^2cx^2\right)^3}\,dx=\int \frac{\mathrm{atan}(a\,x)^3}{x^3\left(c\,a^2\,x^2+c\right)^3}\,dx$$

input
```
int(atan(a*x)^3/(x^3*(c + a^2*c*x^2)^3),x)
```

output
```
int(atan(a*x)^3/(x^3*(c + a^2*c*x^2)^3), x)
```

## Reduce [F]

$$\int \frac{\arctan(ax)^3}{x^3 (c + a^2 c x^2)^3} \, dx = \frac{\int \frac{atan(ax)^3}{a^6 x^9 + 3a^4 x^7 + 3a^2 x^5 + x^3} dx}{c^3}$$

input
```
int(atan(a*x)^3/x^3/(a^2*c*x^2+c)^3,x)
```

output
```
int(atan(a*x)**3/(a**6*x**9 + 3*a**4*x**7 + 3*a**2*x**5 + x**3),x)/c**3
```

# 3.411 $\int \frac{\arctan(ax)^3}{x^4(c+a^2cx^2)^3}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 432

$$\begin{aligned}\int \frac{\arctan(ax)^3}{x^4\left(c+a^2cx^2\right)^3}\,dx = &-\frac{3a^3}{128c^3\left(1+a^2x^2\right)^2}-\frac{141a^3}{128c^3\left(1+a^2x^2\right)}-\frac{a^2\arctan(ax)}{c^3x}\\&-\frac{3a^4x\arctan(ax)}{32c^3\left(1+a^2x^2\right)^2}-\frac{141a^4x\arctan(ax)}{64c^3\left(1+a^2x^2\right)}\\&-\frac{205a^3\arctan(ax)^2}{128c^3}-\frac{a\arctan(ax)^2}{2c^3x^2}+\frac{3a^3\arctan(ax)^2}{16c^3\left(1+a^2x^2\right)^2}\\&+\frac{33a^3\arctan(ax)^2}{16c^3\left(1+a^2x^2\right)}+\frac{10ia^3\arctan(ax)^3}{3c^3}\\&-\frac{\arctan(ax)^3}{3c^3x^3}+\frac{3a^2\arctan(ax)^3}{c^3x}+\frac{a^4x\arctan(ax)^3}{4c^3\left(1+a^2x^2\right)^2}\\&+\frac{11a^4x\arctan(ax)^3}{8c^3\left(1+a^2x^2\right)}+\frac{35a^3\arctan(ax)^4}{32c^3}+\frac{a^3\log(x)}{c^3}\\&-\frac{a^3\log\left(1+a^2x^2\right)}{2c^3}-\frac{10a^3\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)}{c^3}\\&+\frac{10ia^3\arctan(ax)\operatorname{PolyLog}\left(2,-1+\frac{2}{1-iax}\right)}{c^3}\\&-\frac{5a^3\operatorname{PolyLog}\left(3,-1+\frac{2}{1-iax}\right)}{c^3}\end{aligned}$$

output
```
-3/128*a^3/c^3/(a^2*x^2+1)^2-141/128*a^3/c^3/(a^2*x^2+1)-a^2*arctan(a*x)/c
^3/x-3/32*a^4*x*arctan(a*x)/c^3/(a^2*x^2+1)^2-141/64*a^4*x*arctan(a*x)/c^3
/(a^2*x^2+1)-205/128*a^3*arctan(a*x)^2/c^3-1/2*a*arctan(a*x)^2/c^3/x^2+3/1
6*a^3*arctan(a*x)^2/c^3/(a^2*x^2+1)^2+33/16*a^3*arctan(a*x)^2/c^3/(a^2*x^2
+1)+10/3*I*a^3*arctan(a*x)^3/c^3-1/3*arctan(a*x)^3/c^3/x^3+3*a^2*arctan(a*
x)^3/c^3/x+1/4*a^4*x*arctan(a*x)^3/c^3/(a^2*x^2+1)^2+11/8*a^4*x*arctan(a*x
)^3/c^3/(a^2*x^2+1)+35/32*a^3*arctan(a*x)^4/c^3+a^3*ln(x)/c^3-1/2*a^3*ln(a
^2*x^2+1)/c^3-10*a^3*arctan(a*x)^2*ln(2-2/(1-I*a*x))/c^3+10*I*a^3*arctan(a
*x)*polylog(2,-1+2/(1-I*a*x))/c^3-5*a^3*polylog(3,-1+2/(1-I*a*x))/c^3
```

## Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 301, normalized size of antiderivative = 0.70

$$\int \frac{\arctan(ax)^3}{x^4(c+a^2cx^2)^3}\,dx = \frac{a^3\left(\frac{5i\pi^3}{12}-\frac{\arctan(ax)}{ax}-\frac{1}{2}\arctan(ax)^2-\frac{\arctan(ax)^2}{2a^2x^2}-\frac{10}{3}i\arctan(ax)^3-\frac{\arctan(ax)^3}{3a^3x^3}+\frac{3\arctan(ax)^3}{ax}+\frac{35}{32}\arctan\right.}{}$$

input
```
Integrate[ArcTan[a*x]^3/(x^4*(c + a^2*c*x^2)^3),x]
```

output
```
(a^3*(((5*I)/12)*Pi^3 - ArcTan[a*x]/(a*x) - ArcTan[a*x]^2/2 - ArcTan[a*x]^
2/(2*a^2*x^2) - ((10*I)/3)*ArcTan[a*x]^3 - ArcTan[a*x]^3/(3*a^3*x^3) + (3*
ArcTan[a*x]^3)/(a*x) + (35*ArcTan[a*x]^4)/32 - (9*Cos[2*ArcTan[a*x]])/16 +
 (9*ArcTan[a*x]^2*Cos[2*ArcTan[a*x]])/8 - (3*Cos[4*ArcTan[a*x]])/1024 + (3
*ArcTan[a*x]^2*Cos[4*ArcTan[a*x]])/128 - 10*ArcTan[a*x]^2*Log[1 - E^((-2*I
)*ArcTan[a*x])] + Log[(a*x)/Sqrt[1 + a^2*x^2]] - (10*I)*ArcTan[a*x]*PolyLo
g[2, E^((-2*I)*ArcTan[a*x])] - 5*PolyLog[3, E^((-2*I)*ArcTan[a*x])] - (9*A
rcTan[a*x]*Sin[2*ArcTan[a*x]])/8 + (3*ArcTan[a*x]^3*Sin[2*ArcTan[a*x]])/4
- (3*ArcTan[a*x]*Sin[4*ArcTan[a*x]])/256 + (ArcTan[a*x]^3*Sin[4*ArcTan[a*x
]])/32))/c^3
```

## Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3}{x^4 (a^2cx^2+c)^3} \, dx$$

$$\Big\downarrow \text{ 5501}$$

$$\frac{\int \frac{\arctan(ax)^3}{c^2x^4(a^2x^2+1)^2}dx}{c} - a^2 \int \frac{\arctan(ax)^3}{c^3x^2(a^2x^2+1)^3}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{\arctan(ax)^3}{x^4(a^2x^2+1)^2}dx}{c^3} - \frac{a^2\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^3}dx}{c^3}$$

$$\Big\downarrow \text{ 5501}$$

$$\frac{\int \frac{\arctan(ax)^3}{x^4(a^2x^2+1)}dx - a^2\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx}{c^3} - \frac{a^2\left(\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx - a^2\int \frac{\arctan(ax)^3}{(a^2x^2+1)^3}dx\right)}{c^3}$$

$$\Big\downarrow \text{ 5435}$$

$$\frac{\int \frac{\arctan(ax)^3}{x^4(a^2x^2+1)}dx - a^2\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx}{c^3} - \frac{a^2\left(\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx - a^2\left(-\frac{3}{8}\int \frac{\arctan(ax)}{(a^2x^2+1)^3}dx + \frac{3}{4}\int \frac{\arctan(ax)^3}{(a^2x^2+1)^2}dx + \frac{x\arctan(ax)^3}{4(a^2x^2+1)^2} + \frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2}\right)\right)}{c^3}$$

$$\Big\downarrow \text{ 5427}$$

$$\frac{\int \frac{\arctan(ax)^3}{x^4(a^2x^2+1)}dx - a^2\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx}{c^3} - \frac{a^2\left(\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx - a^2\left(-\frac{3}{8}\int \frac{\arctan(ax)}{(a^2x^2+1)^3}dx + \frac{3}{4}\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx + \frac{x\arctan(ax)^3}{2(a^2x^2+1)} + \frac{\arctan(ax)^4}{8a}\right) + \frac{x\arctan(ax)^3}{4(a^2x^2+1)^2}\right.\right.}{c^3}$$

$$\Big\downarrow \text{ 5431}$$

$$\frac{\int \frac{\arctan(ax)^3}{x^4(a^2x^2+1)}dx - a^2\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx}{c^3} - \frac{a^2\left(\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx - a^2\left(-\frac{3}{8}\left(\frac{3}{4}\int \frac{\arctan(ax)}{(a^2x^2+1)^2}dx + \frac{x\arctan(ax)}{4(a^2x^2+1)^2} + \frac{1}{16a(a^2x^2+1)^2}\right) + \frac{3}{4}\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx + \frac{x\arctan(\right.\right.\right.}{c^3}$$

$$\Big\downarrow \text{ 5427}$$

$$\frac{\int \frac{\arctan(ax)^3}{x^4(a^2x^2+1)}dx - a^2\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx}{c^3} - \frac{a^2\left(\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx - a^2\left(-\frac{3}{8}\left(\frac{3}{4}\left(-\frac{1}{2}a\int \frac{x}{(a^2x^2+1)^2}dx + \frac{x\arctan(ax)}{2(a^2x^2+1)} + \frac{\arctan(ax)^2}{4a}\right) + \frac{x\arctan(ax)}{4(a^2x^2+1)^2} + \frac{1}{16a(a^2x^2+1)^2}\right) + \frac{3}{4}\right.}{c^3}$$

$\Big\downarrow$ 241

$$\frac{\int \frac{\arctan(ax)^3}{x^4(a^2x^2+1)}dx - a^2\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx}{c^3} - \frac{a^2\left(\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx - a^2\left(\frac{3}{4}\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx + \frac{x\arctan(ax)^3}{2(a^2x^2+1)} + \frac{\arctan(ax)^4}{8a}\right) + \frac{x\arctan(ax)^3}{4(a^2x^2+1)^2} + \frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2} - \frac{3}{8}\left(\right.\right.}{c^3}$$

$\Big\downarrow$ 5453

$$\frac{a^2\left(-\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx\right) - a^2\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)}dx + \int \frac{\arctan(ax)^3}{x^4}dx}{c^3} - \frac{a^2\left(\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx - a^2\left(\frac{3}{4}\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx + \frac{x\arctan(ax)^3}{2(a^2x^2+1)} + \frac{\arctan(ax)^4}{8a}\right) + \frac{x\arctan(ax)^3}{4(a^2x^2+1)^2} + \frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2} - \frac{3}{8}\left(\right.\right.}{c^3}$$

$\Big\downarrow$ 5361

$$\frac{-a^2\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx - a^2\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)}dx + a\int \frac{\arctan(ax)^2}{x^3(a^2x^2+1)}dx - \frac{\arctan(ax)^3}{3x^3}}{c^3} - \frac{a^2\left(\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx - a^2\left(\frac{3}{4}\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx + \frac{x\arctan(ax)^3}{2(a^2x^2+1)} + \frac{\arctan(ax)^4}{8a}\right) + \frac{x\arctan(ax)^3}{4(a^2x^2+1)^2} + \frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2} - \frac{3}{8}\left(\right.\right.}{c^3}$$

$\Big\downarrow$ 5453

$$\frac{-a^2\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx - a^2\left(\int \frac{\arctan(ax)^3}{x^2}dx - a^2\int \frac{\arctan(ax)^3}{a^2x^2+1}dx\right) + a\left(\int \frac{\arctan(ax)^2}{x^3}dx - a^2\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right) - \frac{\arctan}{3x}}{c^3}$$
$$\frac{a^2\left(\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx - a^2\left(\frac{3}{4}\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx + \frac{x\arctan(ax)^3}{2(a^2x^2+1)} + \frac{\arctan(ax)^4}{8a}\right) + \frac{x\arctan(ax)^3}{4(a^2x^2+1)^2} + \frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2} - \frac{3}{8}\left(\right.\right.}{c^3}$$

$\Big\downarrow$ 5361

$$\frac{a\left(a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right) + a\int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx - \frac{\arctan(ax)^2}{2x^2}\right) - a^2\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx - a^2\left(3a\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx - a^2\int \right.}{c^3}$$
$$\frac{a^2\left(\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx - a^2\left(\frac{3}{4}\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx + \frac{x\arctan(ax)^3}{2(a^2x^2+1)} + \frac{\arctan(ax)^4}{8a}\right) + \frac{x\arctan(ax)^3}{4(a^2x^2+1)^2} + \frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2} - \frac{3}{8}\left(\right.\right.}{c^3}$$

$\Big\downarrow$ 5419

$$\frac{-a^2\left(3a\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx-\frac{1}{4}a\arctan(ax)^4-\frac{\arctan(ax)^3}{x}\right)+a\left(a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right)+a\int \frac{\arctan(ax)}{x^2(a^2x^2+1)}dx-\frac{\arctan(ax}{2x^2}\right.}{c^3}$$

$$\frac{a^2\left(\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx-a^2\left(\frac{3}{4}\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)+\frac{x\arctan(ax)^3}{4(a^2x^2+1)^2}+\frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2}-\frac{3}{8}\left(\right.\right.\right.}{c^3}$$

$\downarrow$ 5453

$$\frac{-a^2\left(3a\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx-\frac{1}{4}a\arctan(ax)^4-\frac{\arctan(ax)^3}{x}\right)+a\left(a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right)+a\left(\int \frac{\arctan(ax)}{x^2}dx-a^2\int \frac{\text{arct}}{a^2}\right.\right.}{c^3}$$

$$\frac{a^2\left(\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx-a^2\left(\frac{3}{4}\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)+\frac{x\arctan(ax)^3}{4(a^2x^2+1)^2}+\frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2}-\frac{3}{8}\left(\right.\right.\right.}{c^3}$$

$\downarrow$ 5361

$$\frac{-a^2\left(3a\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx-\frac{1}{4}a\arctan(ax)^4-\frac{\arctan(ax)^3}{x}\right)+a\left(a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right)+a\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+\right.\right.}{c^3}$$

$$\frac{a^2\left(\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx-a^2\left(\frac{3}{4}\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)+\frac{x\arctan(ax)^3}{4(a^2x^2+1)^2}+\frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2}-\frac{3}{8}\left(\right.\right.\right.}{c^3}$$

$\downarrow$ 243

$$\frac{-a^2\left(3a\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx-\frac{1}{4}a\arctan(ax)^4-\frac{\arctan(ax)^3}{x}\right)+a\left(a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right)+a\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+\right.\right.}{c^3}$$

$$\frac{a^2\left(\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx-a^2\left(\frac{3}{4}\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)+\frac{x\arctan(ax)^3}{4(a^2x^2+1)^2}+\frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2}-\frac{3}{8}\left(\right.\right.\right.}{c^3}$$

$\downarrow$ 47

$$\frac{-a^2\left(3a\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx-\frac{1}{4}a\arctan(ax)^4-\frac{\arctan(ax)^3}{x}\right)+a\left(a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right)+a\left(a^2\left(-\int \frac{\arctan(ax)}{a^2x^2+1}dx\right)+\right.\right.}{c^3}$$

$$\frac{a^2\left(\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx-a^2\left(\frac{3}{4}\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)+\frac{x\arctan(ax)^3}{4(a^2x^2+1)^2}+\frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2}-\frac{3}{8}\left(\right.\right.\right.}{c^3}$$

$\downarrow$ 14

$$\frac{-a^2\left(3a\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx-\frac{1}{4}a\arctan(ax)^4-\frac{\arctan(ax)^3}{x}\right)-a^2\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx+a\left(a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right)+a\left(a^2\left(\right.\right.\right.}{c^3}$$

$$\frac{a^2\left(\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx-a^2\left(\frac{3}{4}\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)+\frac{x\arctan(ax)^3}{4(a^2x^2+1)^2}+\frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2}-\frac{3}{8}\left(\right.\right.\right.}{c^3}$$

$\downarrow$ 16

$$\frac{-a^2\left(3a\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx - \frac{1}{4}a\arctan(ax)^4 - \frac{\arctan(ax)^3}{x}\right) - a^2\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx + a\left(a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right) + a\left(a^2\left(\right.\right.\right.}{c^3}$$

$$\frac{a^2\left(\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx - a^2\left(\frac{3}{4}\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx + \frac{x\arctan(ax)^3}{2(a^2x^2+1)} + \frac{\arctan(ax)^4}{8a}\right) + \frac{x\arctan(ax)^3}{4(a^2x^2+1)^2} + \frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2} - \frac{3}{8}\left(\right.\right.\right.}{c^3}$$

$\downarrow$ 5419

$$\frac{-a^2\left(3a\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx - \frac{1}{4}a\arctan(ax)^4 - \frac{\arctan(ax)^3}{x}\right) - a^2\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx + a\left(a^2\left(-\int \frac{\arctan(ax)^2}{x(a^2x^2+1)}dx\right) + a\left(\frac{1}{2}a\left(\right.\right.\right.}{c^3}$$

$$\frac{a^2\left(\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx - a^2\left(\frac{3}{4}\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx + \frac{x\arctan(ax)^3}{2(a^2x^2+1)} + \frac{\arctan(ax)^4}{8a}\right) + \frac{x\arctan(ax)^3}{4(a^2x^2+1)^2} + \frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2} - \frac{3}{8}\left(\right.\right.\right.}{c^3}$$

$\downarrow$ 5459

$$-\frac{a^2\left(\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx - a^2\left(\frac{3}{4}\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx + \frac{x\arctan(ax)^3}{2(a^2x^2+1)} + \frac{\arctan(ax)^4}{8a}\right) + \frac{x\arctan(ax)^3}{4(a^2x^2+1)^2} + \frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2} - \frac{3}{8}\right.\right.}{c^3}$$

$$\frac{-a^2\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx + a\left(-\left(a^2\left(i\int \frac{\arctan(ax)^2}{x(ax+i)}dx - \frac{1}{3}i\arctan(ax)^3\right)\right) + a\left(\frac{1}{2}a\left(\log\left(x^2\right) - \log\left(a^2x^2+1\right)\right) - \frac{1}{2}a\,\mathrm{ar}\right.\right.}{}$$

$\downarrow$ 5403

$$-\frac{a^2\left(\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx - a^2\left(\frac{3}{4}\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx + \frac{x\arctan(ax)^3}{2(a^2x^2+1)} + \frac{\arctan(ax)^4}{8a}\right) + \frac{x\arctan(ax)^3}{4(a^2x^2+1)^2} + \frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2} - \frac{3}{8}\right.\right.}{c^3}$$

$$\frac{-a^2\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx - a^2\left(3a\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx - i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right) - \frac{1}{3}i\arctan(a\right.\right.}{}$$

$\downarrow$ 5465

$$-\frac{a^2\left(\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx - a^2\left(\frac{3}{4}\left(-\frac{3}{2}a\left(\frac{\int \frac{\arctan(ax)}{(a^2x^2+1)^2}dx}{a} - \frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right) + \frac{x\arctan(ax)^3}{2(a^2x^2+1)} + \frac{\arctan(ax)^4}{8a}\right) + \frac{x\arctan(ax)^3}{4(a^2x^2+1)^2} +\right.\right.}{c^3}$$

$$\frac{-a^2\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx - a^2\left(3a\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx - i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right) - \frac{1}{3}i\arctan(a\right.\right.}{}$$

$\downarrow$ 5427

$$-\frac{a^2\left(\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx - a^2\left(\frac{3}{4}\left(-\frac{3}{2}a\left(\frac{-\frac{1}{2}a\int \frac{x}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{\arctan(ax)^2}{4a}}{a} - \frac{\arctan(ax)^2}{2a^2(a^2x^2+1)}\right) + \frac{x\arctan(ax)^3}{2(a^2x^2+1)} + \text{arc}\cdots}{-a^2\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx - a^2\left(3a\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx - i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right) - \frac{1}{3}i\arctan(a\cdots}$$

$\downarrow$ 241

$$-\frac{a^2\left(\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx - a^2\left(\frac{x\arctan(ax)^3}{4(a^2x^2+1)^2} + \frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2} - \frac{3}{8}\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2} + \frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)} + \frac{1}{4a(a^2x^2+1)} + \frac{\arctan(ax)^2}{4a}\cdots}{-a^2\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)^2}dx - a^2\left(3a\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx - i\arctan(ax)^2\log\left(2-\frac{2}{1-iax}\right)\right) - \frac{1}{3}i\arctan(a\cdots}$$

$c^3$

$\downarrow$ 5501

$$-\frac{a^2\left(-a^2\int \frac{\arctan(ax)^3}{(a^2x^2+1)^2}dx + \int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)}dx - \left(a^2\left(\frac{x\arctan(ax)^3}{4(a^2x^2+1)^2} + \frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2} - \frac{3}{8}\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2} + \frac{3}{4}\left(\frac{x\arctan(ax)}{2(a^2x^2+1)} + \cdots}{-a^2\left(\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)}dx - a^2\int \frac{\arctan(ax)^3}{(a^2x^2+1)^2}dx\right) - a^2\left(3a\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\frac{2}{1-iax}\right)}{a^2x^2+1}dx - i\arctan(ax)^2\log\left(2-\frac{\cdots}{1-\cdots}\right.\right.\right.\right.}$$

$\downarrow$ 5427

$$-\frac{a^2\left(-a^2\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx + \frac{x\arctan(ax)^3}{2(a^2x^2+1)} + \frac{\arctan(ax)^4}{8a}\right) + \int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)}dx - \left(a^2\left(\frac{x\arctan(ax)^3}{4(a^2x^2+1)^2} + \frac{3\arctan(ax)^2}{16a(a^2x^2+1)\cdots}{-a^2\left(\int \frac{\arctan(ax)^3}{x^2(a^2x^2+1)}dx - a^2\left(-\frac{3}{2}a\int \frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx + \frac{x\arctan(ax)^3}{2(a^2x^2+1)} + \frac{\arctan(ax)^4}{8a}\right)\right) - a^2\left(3a\left(i\left(2ia\int \frac{\arctan(ax)\log\left(2-\cdots\right.}{a^2x^2+1}\cdots}$$

$\downarrow$ 5453

$$-\frac{a^2\left(-a^2\left(-\frac{3}{2}a\int\frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)-a^2\int\frac{\arctan(ax)^3}{a^2x^2+1}dx+\int\frac{\arctan(ax)^3}{x^2}dx-\left(a^2\left(\frac{x\arctan}{4(a^2\cdots}\right.\right.\right.}{-a^2\left(-\left(a^2\left(-\frac{3}{2}a\int\frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}+\frac{\arctan(ax)^4}{8a}\right)\right)-a^2\int\frac{\arctan(ax)^3}{a^2x^2+1}dx+\int\frac{\arctan(ax)^3}{x^2}dx\right)-a^2\left(3a\left(\cdots\right.\right.}$$

$$\Big\downarrow \text{5361}$$

$$-\frac{\arctan(ax)^3}{3x^3}-a^2\left(-\frac{\arctan(ax)^3}{x}-a^2\left(\frac{\arctan(ax)^4}{8a}+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}-\frac{3}{2}a\int\frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx\right)+3a\int\frac{\arctan(ax)^2}{x(a^2x^2+1)}dx-a^2\int\cdots\right.$$

$$a^2\left(-\frac{\arctan(ax)^3}{x}-a^2\left(\frac{x\arctan(ax)^3}{4(a^2x^2+1)^2}+\frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2}-\frac{3}{8}\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{\arctan(ax)^2}{4a}+\frac{x\arctan(ax)}{2(a^2x^2+1)}+\frac{1}{4a(a^2x^2+1)}\right)+\cdots\right.\right.\right.$$

$$\Big\downarrow \text{5419}$$

$$-\frac{\arctan(ax)^3}{3x^3}-a^2\left(-\frac{1}{4}a\arctan(ax)^4-\frac{\arctan(ax)^3}{x}-a^2\left(\frac{\arctan(ax)^4}{8a}+\frac{x\arctan(ax)^3}{2(a^2x^2+1)}-\frac{3}{2}a\int\frac{x\arctan(ax)^2}{(a^2x^2+1)^2}dx\right)+3a\int\frac{\text{arc}\cdots}{x(\cdots}\right.$$

$$a^2\left(-\frac{1}{4}a\arctan(ax)^4-\frac{\arctan(ax)^3}{x}-a^2\left(\frac{x\arctan(ax)^3}{4(a^2x^2+1)^2}+\frac{3\arctan(ax)^2}{16a(a^2x^2+1)^2}-\frac{3}{8}\left(\frac{x\arctan(ax)}{4(a^2x^2+1)^2}+\frac{3}{4}\left(\frac{\arctan(ax)^2}{4a}+\frac{x\arctan(ax)}{2(a^2x^2+1)}\cdots\right.\right.\right.\right.$$

input
```
Int[ArcTan[a*x]^3/(x^4*(c + a^2*c*x^2)^3),x]
```

output
```
$Aborted
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 9.28 (sec) , antiderivative size = 2062, normalized size of antiderivative = 4.77

Expression too large to display

input
```
int(arctan(a*x)^3/x^4/(a^2*c*x^2+c)^3,x)
```

output
```
a^3*(11/8/c^3*arctan(a*x)^3/(a^2*x^2+1)^2*a^3*x^3+13/8/c^3*arctan(a*x)^3/(
a^2*x^2+1)^2*a*x+35/8/c^3*arctan(a*x)^4-1/3/c^3*arctan(a*x)^3/a^3/x^3+3/c^
3*arctan(a*x)^3/a/x-1/8/c^3*(205/16*arctan(a*x)^2-8*ln((1+I*a*x)/(a^2*x^2+
1)^(1/2)-1)+3/128*cos(4*arctan(a*x))-33/2*arctan(a*x)^2/(a^2*x^2+1)-8*ln(1
+(1+I*a*x)/(a^2*x^2+1)^(1/2))-3/2*arctan(a*x)^2/(a^2*x^2+1)^2+40*I*Pi*csgn
(I*((1+I*a*x)^2/(a^2*x^2+1)-1))*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1))*csgn(I
*((1+I*a*x)^2/(a^2*x^2+1)-1)/((1+I*a*x)^2/(a^2*x^2+1)+1))*arctan(a*x)^2-20
*I*Pi*csgn(I/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)
)*csgn(I*(1+I*a*x)^2/(a^2*x^2+1)/((1+I*a*x)^2/(a^2*x^2+1)+1)^2)*arctan(a*x
)^2+160*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))+160*polylog(3,-(1+I*a*x)/(a
^2*x^2+1)^(1/2))-80/3*I*arctan(a*x)^3+80*arctan(a*x)^2*ln((1+I*a*x)/(a^2*x
^2+1)^(1/2))-160*I*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+40*
I*Pi*arctan(a*x)^2-80*arctan(a*x)^2*ln((1+I*a*x)^2/(a^2*x^2+1)-1)+4*arctan
(a*x)*(I*a*x-(a^2*x^2+1)^(1/2)+1)/a/x+4*arctan(a*x)*(I*a*x+(a^2*x^2+1)^(1/
2)+1)/a/x+80*arctan(a*x)^2*ln(a*x)+80*arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^
2+1)^(1/2))+80*arctan(a*x)^2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-40*arctan(a
*x)^2*ln(a^2*x^2+1)+105/4*arctan(a*x)^4+4*arctan(a*x)^2/a^2/x^2-160*I*arct
an(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+3/32*arctan(a*x)*sin(4*arct
an(a*x))+80*arctan(a*x)^2*ln(2)-9*I*arctan(a*x)*(a*x+I)/(2*a*x-2*I)+9*I*ar
ctan(a*x)*(a*x-I)/(2*a*x+2*I)-9/4*(a*x+I)/(a*x-I)-9/4*(a*x-I)/(a*x+I)+4...
```

## Fricas [F]

$$\int \frac{\arctan(ax)^3}{x^4\left(c+a^2cx^2\right)^3}\,dx=\int \frac{\arctan\left(ax\right)^3}{\left(a^2cx^2+c\right)^3x^4}\,dx$$

input
```
integrate(arctan(a*x)^3/x^4/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
integral(arctan(a*x)^3/(a^6*c^3*x^10 + 3*a^4*c^3*x^8 + 3*a^2*c^3*x^6 + c^3
*x^4), x)
```

## Sympy [F]

$$\int \frac{\arctan(ax)^3}{x^4 (c + a^2cx^2)^3}\,dx = \frac{\int \frac{\operatorname{atan}^3(ax)}{a^6x^{10}+3a^4x^8+3a^2x^6+x^4}\,dx}{c^3}$$

input
```
integrate(atan(a*x)**3/x**4/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(atan(a*x)**3/(a**6*x**10 + 3*a**4*x**8 + 3*a**2*x**6 + x**4), x)/
c**3
```

## Maxima [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^3}{x^4 (c + a^2cx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(arctan(a*x)^3/x^4/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
Timed out
```

## Giac [F]

$$\int \frac{\arctan(ax)^3}{x^4 (c + a^2cx^2)^3}\,dx = \int \frac{\arctan(ax)^3}{(a^2cx^2 + c)^3x^4}\,dx$$

input
```
integrate(arctan(a*x)^3/x^4/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^3/((a^2*c*x^2 + c)^3*x^4), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^3}{x^4 (c + a^2cx^2)^3} \, dx = \int \frac{\mathrm{atan}(a\,x)^3}{x^4 \left(c\,a^2\,x^2 + c\right)^3} \, dx$$

input
```
int(atan(a*x)^3/(x^4*(c + a^2*c*x^2)^3),x)
```

output
```
int(atan(a*x)^3/(x^4*(c + a^2*c*x^2)^3), x)
```

## Reduce [F]

$$\int \frac{\arctan(ax)^3}{x^4 (c + a^2cx^2)^3} \, dx$$
$$= \frac{1680atan(ax)^4 a^5x^5 + 5600atan(ax)^3 a^4x^4 + 2100atan(ax)^2 a^5x^5 + 1792\log(a^2x^2 + 1)\,a^5x^5 - 3584\log(}{}$$

input
```
int(atan(a*x)^3/x^4/(a^2*c*x^2+c)^3,x)
```

output
```
(840*atan(a*x)**4*a**7*x**7 + 1680*atan(a*x)**4*a**5*x**5 + 840*atan(a*x)*
*4*a**3*x**3 + 3360*atan(a*x)**3*a**6*x**6 + 5600*atan(a*x)**3*a**4*x**4 +
 1792*atan(a*x)**3*a**2*x**2 - 256*atan(a*x)**3 - 1470*atan(a*x)**2*a**7*x
**7 + 2100*atan(a*x)**2*a**5*x**5 + 5250*atan(a*x)**2*a**3*x**3 + 896*atan
(a*x)**2*a*x - 2940*atan(a*x)*a**6*x**6 - 1540*atan(a*x)*a**4*x**4 + 1792*
atan(a*x)*a**2*x**2 + 2560*int(atan(a*x)**2/(a**6*x**9 + 3*a**4*x**7 + 3*a
**2*x**5 + x**3),x)*a**5*x**7 + 5120*int(atan(a*x)**2/(a**6*x**9 + 3*a**4*
x**7 + 3*a**2*x**5 + x**3),x)*a**3*x**5 + 2560*int(atan(a*x)**2/(a**6*x**9
 + 3*a**4*x**7 + 3*a**2*x**5 + x**3),x)*a*x**3 + 896*log(a**2*x**2 + 1)*a*
*7*x**7 + 1792*log(a**2*x**2 + 1)*a**5*x**5 + 896*log(a**2*x**2 + 1)*a**3*
x**3 - 1792*log(x)*a**7*x**7 - 3584*log(x)*a**5*x**5 - 1792*log(x)*a**3*x*
*3 + 1183*a**7*x**7 - 1281*a**3*x**3)/(768*c**3*x**3*(a**4*x**4 + 2*a**2*x
**2 + 1))
```

# 3.412 $\int x^3\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 523

$$\begin{aligned}\int x^3\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx &= -\frac{x\sqrt{c+a^2cx^2}}{20a^3}-\frac{9\sqrt{c+a^2cx^2}\arctan(ax)}{20a^4}+\frac{x^2\sqrt{c+a^2cx^2}\arctan(ax)}{10a^2}\\
&\quad+\frac{x\sqrt{c+a^2cx^2}\arctan(ax)^2}{8a^3}-\frac{3x^3\sqrt{c+a^2cx^2}\arctan(ax)^2}{20a}\\
&\quad-\frac{11ic\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2}{20a^4\sqrt{c+a^2cx^2}}-\frac{2\sqrt{c+a^2cx^2}\arctan(ax)^3}{15a^4}\\
&\quad+\frac{x^2\sqrt{c+a^2cx^2}\arctan(ax)^3}{15a^2}+\frac{1}{5}x^4\sqrt{c+a^2cx^2}\arctan(ax)^3\\
&\quad+\frac{\sqrt{c}\,\mathrm{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right)}{2a^4}+\frac{11ic\sqrt{1+a^2x^2}\arctan(ax)\,\mathrm{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)}{20a^4\sqrt{c+a^2cx^2}}\\
&\quad-\frac{11ic\sqrt{1+a^2x^2}\arctan(ax)\,\mathrm{PolyLog}\left(2,ie^{i\arctan(ax)}\right)}{20a^4\sqrt{c+a^2cx^2}}\\
&\quad-\frac{11c\sqrt{1+a^2x^2}\,\mathrm{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)}{20a^4\sqrt{c+a^2cx^2}}+\frac{11c\sqrt{1+a^2x^2}\,\mathrm{PolyLog}\left(3,ie^{i\arctan(ax)}\right)}{20a^4\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
-1/20*x*(a^2*c*x^2+c)^(1/2)/a^3-9/20*(a^2*c*x^2+c)^(1/2)*arctan(a*x)/a^4+1
/10*x^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)/a^2+1/8*x*(a^2*c*x^2+c)^(1/2)*arct
an(a*x)^2/a^3-3/20*x^3*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2/a-11/20*I*c*(a^2*
x^2+1)^(1/2)*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^2/a^4/(a^2*c*
x^2+c)^(1/2)-2/15*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^3/a^4+1/15*x^2*(a^2*c*x^
2+c)^(1/2)*arctan(a*x)^3/a^2+1/5*x^4*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^3+1/2
*c^(1/2)*arctanh(a*c^(1/2)*x/(a^2*c*x^2+c)^(1/2))/a^4+11/20*I*c*(a^2*x^2+1
)^(1/2)*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a^4/(a^2*c*x
^2+c)^(1/2)-11/20*I*c*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(2,I*(1+I*a*x)/
(a^2*x^2+1)^(1/2))/a^4/(a^2*c*x^2+c)^(1/2)-11/20*c*(a^2*x^2+1)^(1/2)*polyl
og(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a^4/(a^2*c*x^2+c)^(1/2)+11/20*c*(a^2*
x^2+1)^(1/2)*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a^4/(a^2*c*x^2+c)^(1
/2)
```

## Mathematica [A] (warning: unable to verify)

Time = 0.90 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.50

$$\int x^3\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx$$
$$= \frac{\sqrt{c+a^2cx^2}\left(\frac{48\left(10\coth^{-1}\left(\frac{ax}{\sqrt{1+a^2x^2}}\right)-11i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2+11i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-11i\arctan(a\right.}{\sqrt{1+a^2x^2}}\right.}{}$$

input
```
Integrate[x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3,x]
```

output
```
(Sqrt[c + a^2*c*x^2]*((48*(10*ArcCoth[(a*x)/Sqrt[1 + a^2*x^2]] - (11*I)*Ar
cTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 + (11*I)*ArcTan[a*x]*PolyLog[2, (-I)
*E^(I*ArcTan[a*x])] - (11*I)*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] -
 11*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] + 11*PolyLog[3, I*E^(I*ArcTan[a*x])
]))/Sqrt[1 + a^2*x^2] - (1 + a^2*x^2)^2*((48*a*x)/(1 + a^2*x^2)^2 + 32*Arc
Tan[a*x]^3*(-1 + 5*Cos[2*ArcTan[a*x]]) + 6*ArcTan[a*x]*(25 + 36*Cos[2*ArcT
an[a*x]] + 11*Cos[4*ArcTan[a*x]]) + ArcTan[a*x]^2*(6*Sin[2*ArcTan[a*x]] -
33*Sin[4*ArcTan[a*x]]))))/(960*a^4)
```

## Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \arctan(ax)^3 \sqrt{a^2cx^2+c}\,dx$$

$$\Big\downarrow \text{ 5485}$$

$$a^2c\int \frac{x^5 \arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx+c\int \frac{x^3 \arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{ 5487}$$

$$c\left(-\frac{\int \frac{x^2 \arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{2\int \frac{x \arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2 \arctan(ax)^3\sqrt{a^2cx^2+c}}{3a^2c}\right)+a^2c\left(-\frac{3\int \frac{x^4 \arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int \frac{x^3 \arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{5a^2}+\frac{x^4 \arctan(ax)^3\sqrt{a^2cx^2+c}}{5a^2c}\right)$$

$$\Big\downarrow \text{ 5465}$$

$$c\left(-\frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a}\right)}{3a^2}-\frac{\int \frac{x^2 \arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a}+\frac{x^2 \arctan(ax)^3\sqrt{a^2cx^2+c}}{3a^2c}\right)+a^2c\left(-\frac{3\int \frac{x^4 \arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int \frac{x^3 \arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{5a^2}+\frac{x^4 \arctan(ax)^3\sqrt{a^2cx^2+c}}{5a^2c}\right)$$

$$\Big\downarrow \text{ 5425}$$

$$c\left(-\frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{a\sqrt{a^2cx^2+c}}\right)}{3a^2}-\frac{\int \frac{x^2 \arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a}+\frac{x^2 \arctan(ax)^3\sqrt{a^2cx^2+c}}{3a^2c}\right)+a^2c\left(-\frac{3\int \frac{x^4 \arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int \frac{x^3 \arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{5a^2}+\frac{x^4 \arctan(ax)^3\sqrt{a^2cx^2+c}}{5a^2c}\right)$$

$$\Big\downarrow \text{ 5423}$$

$$c\left(-\frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\int\sqrt{a^2x^2+1}\arctan(ax)^2d\arctan(ax)}{a^2\sqrt{a^2cx^2+c}}\right)}{3a^2}-\frac{\int\frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a}+\frac{x^2\arctan(ax)^3\sqrt{a^2c}}{3a^2c}\right.$$

$$a^2c\left(-\frac{3\int\frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int\frac{x^3\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{5a^2}+\frac{x^4\arctan(ax)^3\sqrt{a^2cx^2+c}}{5a^2c}\right)$$

$\downarrow$ 3042

$$c\left(-\frac{\int\frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\int\arctan(ax)^2\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{a^2\sqrt{a^2cx^2+c}}\right)}{3a^2}+\frac{x^2\arctan(a}{3}\right.$$

$$a^2c\left(-\frac{3\int\frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int\frac{x^3\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{5a^2}+\frac{x^4\arctan(ax)^3\sqrt{a^2cx^2+c}}{5a^2c}\right)$$

$\downarrow$ 4669

$$a^2c\left(-\frac{3\int\frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int\frac{x^3\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{5a^2}+\frac{x^4\arctan(ax)^3\sqrt{a^2cx^2+c}}{5a^2c}\right)+$$

$$c\left(-\frac{\int\frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(-2\int\arctan(ax)\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+2\int\arctan(ax)\log\right.}{a^2\sqrt{a^2cx^2+}}\right.}{3a^2}\right.$$

$\downarrow$ 3011

$$a^2c\left(-\frac{3\int\frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int\frac{x^3\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{5a^2}+\frac{x^4\arctan(ax)^3\sqrt{a^2cx^2+c}}{5a^2c}\right)+$$

$$c\left(-\frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-i\int\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(ax)\right)-2\left(i\arctan\right.\right.}{a^2}\right.}{3a^2}\right.$$

$\downarrow$ 2720

$$a^2c\left(-\frac{3\int\frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int\frac{x^3\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{5a^2}+\frac{x^4\arctan(ax)^3\sqrt{a^2cx^2+c}}{5a^2c}\right)+$$

$$c\left(-\frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan}\right.\right.}{}\right.}{}\right.$$

$\downarrow$ 5487

$$a^2c\left(-\frac{4\left(-\frac{\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{2\int \frac{x\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)^3\sqrt{a^2cx^2+c}}{3a^2c}\right)}{5a^2}-\frac{3\left(-\frac{3\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}\right.}{5a}\right.$$

$$c\left(-\frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan}\right.\right.}{}\right.}{}\right.$$

$\downarrow$ 5425

$$a^2c\left(-\frac{4\left(-\frac{\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{2\int \frac{x\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)^3\sqrt{a^2cx^2+c}}{3a^2c}\right)}{5a^2}-\frac{3\left(-\frac{3\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}\right.}{5a}\right.$$

$$c\left(-\frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan}\right.\right.}{}\right.}{}\right.$$

$\downarrow$ 5423

$$a^2c\left(-\frac{4\left(-\frac{\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{2\int \frac{x\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)^3\sqrt{a^2cx^2+c}}{3a^2c}\right)}{5a^2}-\frac{3\left(-\frac{3\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}\right.}{5a}\right.$$

$$c\left(-\frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan}\right.\right.}{}\right.}{}\right.$$

$\downarrow$ 3042

$$a^2c\left(-\frac{4\left(-\frac{\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{2\int \frac{x\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)^3\sqrt{a^2cx^2+c}}{3a^2c}\right)}{5a^2}-\frac{3\left(-\frac{3\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}\right.}{5a}\right.$$

$$c\left(-\frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right.\right.}{}\right.}{}\right.$$

$\downarrow$ 4669

$$a^2c\left(-\frac{4\left(-\frac{\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{2\int \frac{x\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)^3\sqrt{a^2cx^2+c}}{3a^2c}\right)}{5a^2}-\frac{3\left(-\frac{3\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}\right.}{5a}\right.$$

$$c\left(-\frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right.\right.}{}\right.}{}\right.$$

$\downarrow$ 3011

$$a^2c\left(-\frac{4\left(-\frac{\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{2\int \frac{x\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)^3\sqrt{a^2cx^2+c}}{3a^2c}\right)}{5a^2}-\frac{3\left(-\frac{3\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int \frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}\right.}{5a}\right.$$

$$c\left(-\frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right.\right.}{}\right.}{}\right.$$

$\downarrow$ 2720

$$a^2c\left(-\frac{4\left(-\frac{\int\frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{2\int\frac{x\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)^3\sqrt{a^2cx^2+c}}{3a^2c}\right)}{5a^2}-\frac{3\left(-\frac{3\int\frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}\right.}{5a}\right.$$

$$c\left(-\frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}(2,-ie^{i\arctan(ax)})-\int e^{-i\arctan(ax)}\operatorname{PolyLog}(2,-ie^{i\arctan(ax)})de^{i\arctan\right.\right.}{}\right.}{}\right.$$

↓ 5465

$$c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^4}{5a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{3\int\frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-\frac{4\left(\frac{x^2\sqrt{a^2cx^2+c}\,a}{3a^2}\right.}{}\right.$$

$$c\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a^2c}-\frac{\int\frac{1}{\sqrt{a^2cx^2+c}}dx}{a}}{a}-\frac{\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(\right.\right.}{}}{}\right.$$

↓ 224

$$c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^4}{5a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{3\int\frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-\frac{4\left(\frac{x^2\sqrt{a^2cx^2+c}\,a}{3a^2}\right.}{}\right.$$

$$c\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a^2c}-\frac{\int\frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}}{a}}{a}-\frac{\sqrt{a^2x^2+1}(-2i\arcta}{}}{}\right.$$

$$\Big\downarrow \text{219}$$

$$c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^4}{5a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{3\int\frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-\frac{4\left(\frac{x^2\sqrt{a^2cx^2+c}\,a}{3a^2}\right.}{}\right.$$

$$c\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a^2\sqrt{c}}}{a}-\frac{\sqrt{a^2x^2+1}(-2i\arctan}{}}{}\right.$$

$$\Big\downarrow \text{5425}$$

$$c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^4}{5a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{3\int\frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-\frac{4\left(\frac{x^2\sqrt{a^2cx^2+c}\,a}{3a^2}\right.}{}\right.$$

$$c\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a^2\sqrt{c}}}{a}-\frac{\sqrt{a^2x^2+1}(-2i\arctan}{}}{}\right.$$

$\downarrow$ 5423

$$c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^4}{5a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{3\int\frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-\frac{4\left(\frac{x^2\sqrt{a^2cx^2+c}\,a}{3a^2}\right.}{}\right.$$

$$c\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a^2\sqrt{c}}}{a}-\frac{\sqrt{a^2x^2+1}(-2i\arctan}{}}{}\right.$$

$\downarrow$ 3042

$$c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^4}{5a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{3\int\frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-\frac{4\left(\frac{x^2\sqrt{a^2cx^2+c}\,a}{3a^2}\right.}{}\right.$$

$$c\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a^2\sqrt{c}}}{a}-\frac{\sqrt{a^2x^2+1}(-2i\arctan}{}}{}\right.$$

$$\Big\downarrow \text{4669}$$

$$c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^4}{5a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{3\int\frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-\frac{4\left(\frac{x^2\sqrt{a^2cx^2+c}\,a}{3a^2}\right.}{}\right.$$

$$c\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a^2\sqrt{c}}}{a}-\frac{\sqrt{a^2x^2+1}(-2i\arctan}{}}{}\right.$$

$$\Big\downarrow \text{3011}$$

$$c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^4}{5a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{3\int\frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-\frac{4\left(\frac{x^2\sqrt{a^2cx^2+c}\,a}{3a^2}\right.}{}\right.$$

$$c\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a^2\sqrt{c}}}{a}-\frac{\sqrt{a^2x^2+1}(-2i\arctan}{}}{}\right.$$

$$\Big\downarrow \text{2720}$$

$$c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^4}{5a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{3\int\frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-\frac{4\left(\frac{x^2\sqrt{a^2cx^2+c}\,a}{3a^2}\right.}{}\right.$$

$$c\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a^2\sqrt{c}}}{a}-\frac{\sqrt{a^2x^2+1}(-2i\arctan}{}}{}\right.$$

$\downarrow$ 5487

$$c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^4}{5a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^2}{3a^2c}-\frac{\int\frac{x^2}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{2\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}}{2a}-\frac{3\left(\right.}{}\right.}{5a}\right.$$

$$c\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a^2\sqrt{c}}}{a}-\frac{\sqrt{a^2x^2+1}(-2i\arctan}{}}{}\right.$$

$\downarrow$ 262

$$c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^4}{5a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^2}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int\frac{1}{\sqrt{a^2cx^2+c}}dx}{2a^2}}{3a}-\frac{2\int\frac{x\arctan(\cdots}{\sqrt{a^2cx^2\cdots}}}{3a^2}}{2a}\right.}{5a}\right.$$

$$c\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a^2\sqrt{c}}}{a}-\frac{\sqrt{a^2x^2+1}(-2i\arctan\cdots}{\cdots}}{\cdots}\right.$$

$\downarrow$ 224

$$c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^4}{5a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^2}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int\frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}}{2a^2}}{3a}-\cdots}{2a}\right.}{5a}\right.$$

$$c\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a^2\sqrt{c}}}{a}-\frac{\sqrt{a^2x^2+1}(-2i\arctan\cdots}{\cdots}}{\cdots}\right.$$

$\downarrow$ 219

$$c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^4}{5a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^2}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}-\frac{2\int}{}}{2a}\right.}{5a}\right.$$

$$c\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a^2\sqrt{c}}}{a}-\frac{\sqrt{a^2x^2+1}(-2i\arctan}{}}{}\right.$$

$$\Big\downarrow \; 5425$$

$$c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^4}{5a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^2}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}-\frac{2\int}{}}{2a}\right.}{5a}\right.$$

$$c\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a^2c}-\frac{\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a^2\sqrt{c}}}{a}-\frac{\sqrt{a^2x^2+1}(-2i\arctan}{}}{}\right.$$

input
```
Int[x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3,x]
```

output
```
$Aborted
```

## Maple [A] (verified)

Time = 4.68 (sec) , antiderivative size = 417, normalized size of antiderivative = 0.80

| method | result |
|---|---|
| default | $\frac{\sqrt{c(ax-i)(ax+i)}\,\left(24\arctan(ax)^3a^4x^4-18a^3\arctan(ax)^2x^3+8\arctan(ax)^3a^2x^2+12x^2a^2\arctan(ax)+15a\arctan(ax)^2x-16\arcta\right.}{120a^4}$ |

input
```
int(x^3*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/120/a^4*(c*(a*x-I)*(a*x+I))^(1/2)*(24*arctan(a*x)^3*a^4*x^4-18*a^3*arcta
n(a*x)^2*x^3+8*arctan(a*x)^3*a^2*x^2+12*x^2*a^2*arctan(a*x)+15*a*arctan(a*
x)^2*x-16*arctan(a*x)^3-6*a*x-54*arctan(a*x))+11/120*(c*(a*x-I)*(a*x+I))^(
1/2)*(I*arctan(a*x)^3-3*arctan(a*x)^2*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+
6*I*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*polylog(3,-I*(
1+I*a*x)/(a^2*x^2+1)^(1/2)))/a^4/(a^2*x^2+1)^(1/2)+11/120*(c*(a*x-I)*(a*x+
I))^(1/2)*(-I*arctan(a*x)^3+3*arctan(a*x)^2*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(
1/2))-6*I*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*polylog(3
,I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/a^4/(a^2*x^2+1)^(1/2)-I/a^4*(c*(a*x-I)*(a
*x+I))^(1/2)*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^(1/2)
```

## Fricas [F]

$$\int x^3\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx=\int\sqrt{a^2cx^2+cx^3}\arctan\left(ax\right)^3\,dx$$

input
```
integrate(x^3*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^3*arctan(a*x)^3, x)
```

## Sympy [F]

$$\int x^3\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx=\int x^3\sqrt{c\left(a^2x^2+1\right)}\,\mathrm{atan}^3\left(ax\right)dx$$

input
```
integrate(x**3*(a**2*c*x**2+c)**(1/2)*atan(a*x)**3,x)
```

output
```
Integral(x**3*sqrt(c*(a**2*x**2 + 1))*atan(a*x)**3, x)
```

## Maxima [F]

$$\int x^3\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx=\int \sqrt{a^2cx^2+c}x^3\arctan\left(ax\right)^3\,dx$$

input
```
integrate(x^3*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*x^3*arctan(a*x)^3, x)
```

## Giac [F(-2)]

Exception generated.

$$\int x^3\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx = \int x^3\,\mathrm{atan}(a\,x)^3\,\sqrt{c\,a^2\,x^2+c}\,dx$$

input
```
int(x^3*atan(a*x)^3*(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(x^3*atan(a*x)^3*(c + a^2*c*x^2)^(1/2), x)
```

## Reduce [F]

$$\int x^3\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx = \sqrt{c}\left(\int \sqrt{a^2x^2+1}\,atan(ax)^3\,x^3dx\right)$$

input
```
int(x^3*(a^2*c*x^2+c)^(1/2)*atan(a*x)^3,x)
```

output
```
sqrt(c)*int(sqrt(a**2*x**2 + 1)*atan(a*x)**3*x**3,x)
```

# 3.413 $\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx$

## Optimal result

Integrand size = 24, antiderivative size = 747

$$\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx = -\frac{\sqrt{c+a^2cx^2}}{4a^3}+\frac{x\sqrt{c+a^2cx^2}\arctan(ax)}{4a^2}+\frac{\sqrt{c+a^2cx^2}\arctan(ax)^2}{8a^3} - \frac{x^2\sqrt{c+a^2cx^2}\arctan(ax)^2}{4a}+\frac{x\sqrt{c+a^2cx^2}\arctan(ax)^3}{8a^2} + \frac{1}{4}x^3\sqrt{c+a^2cx^2}\arctan(ax)^3+\frac{ic\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^3}{4a^3\sqrt{c+a^2cx^2}} + \frac{ic\sqrt{1+a^2x^2}\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a^3\sqrt{c+a^2cx^2}} - \frac{3ic\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)}{8a^3\sqrt{c+a^2cx^2}} + \frac{3ic\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{PolyLog}\left(2,ie^{i\arctan(ax)}\right)}{8a^3\sqrt{c+a^2cx^2}} - \frac{ic\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2a^3\sqrt{c+a^2cx^2}}+\frac{ic\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{2a^3\sqrt{c+a^2cx^2}} + \frac{3c\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)}{4a^3\sqrt{c+a^2cx^2}} - \frac{3c\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(3,ie^{i\arctan(ax)}\right)}{4a^3\sqrt{c+a^2cx^2}} + \frac{3ic\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(4,-ie^{i\arctan(ax)}\right)}{4a^3\sqrt{c+a^2cx^2}}-\frac{3ic\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(4,ie^{i\arctan(ax)}\right)}{4a^3\sqrt{c+a^2cx^2}}$$

output
```
-1/4*(a^2*c*x^2+c)^(1/2)/a^3+1/4*x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)/a^2+1/8
*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2/a^3-1/4*x^2*(a^2*c*x^2+c)^(1/2)*arctan(
a*x)^2/a+1/8*x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^3/a^2+1/4*x^3*(a^2*c*x^2+c)
^(1/2)*arctan(a*x)^3+I*c*(a^2*x^2+1)^(1/2)*arctan(a*x)*arctan((1+I*a*x)^(1
/2)/(1-I*a*x)^(1/2))/a^3/(a^2*c*x^2+c)^(1/2)+3/8*I*c*(a^2*x^2+1)^(1/2)*arc
tan(a*x)^2*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a^3/(a^2*c*x^2+c)^(1/2
)-1/2*I*c*(a^2*x^2+1)^(1/2)*polylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/
a^3/(a^2*c*x^2+c)^(1/2)+1/2*I*c*(a^2*x^2+1)^(1/2)*polylog(2,I*(1+I*a*x)^(1
/2)/(1-I*a*x)^(1/2))/a^3/(a^2*c*x^2+c)^(1/2)+3/4*I*c*(a^2*x^2+1)^(1/2)*pol
ylog(4,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a^3/(a^2*c*x^2+c)^(1/2)-3/8*I*c*(a^
2*x^2+1)^(1/2)*arctan(a*x)^2*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a^3
/(a^2*c*x^2+c)^(1/2)+3/4*c*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(3,-I*(1+I
*a*x)/(a^2*x^2+1)^(1/2))/a^3/(a^2*c*x^2+c)^(1/2)-3/4*c*(a^2*x^2+1)^(1/2)*a
rctan(a*x)*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a^3/(a^2*c*x^2+c)^(1/2
)-3/4*I*c*(a^2*x^2+1)^(1/2)*polylog(4,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a^3/(
a^2*c*x^2+c)^(1/2)+1/4*I*c*(a^2*x^2+1)^(1/2)*arctan((1+I*a*x)/(a^2*x^2+1)^
(1/2))*arctan(a*x)^3/a^3/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1844 vs. $2(747) = 1494$.

Time = 12.20 (sec) , antiderivative size = 1844, normalized size of antiderivative = 2.47

$$\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx = \text{Too large to display}$$

input
```
Integrate[x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3,x]
```

output
```
((Sqrt[c*(1 + a^2*x^2)]*(-1 + ArcTan[a*x]^2))/(4*Sqrt[1 + a^2*x^2]) + (Sqr
t[c*(1 + a^2*x^2)]*(-(ArcTan[a*x]*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1 +
I*E^(I*ArcTan[a*x])])) - I*(PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - PolyLog[2
, I*E^(I*ArcTan[a*x])])))/(2*Sqrt[1 + a^2*x^2]) + (Sqrt[c*(1 + a^2*x^2)]*(
-1/8*(Pi^3*Log[Cot[(Pi/2 - ArcTan[a*x])/2]]) - (3*Pi^2*((Pi/2 - ArcTan[a*x
])*(Log[1 - E^(I*(Pi/2 - ArcTan[a*x]))] - Log[1 + E^(I*(Pi/2 - ArcTan[a*x]
))]) + I*(PolyLog[2, -E^(I*(Pi/2 - ArcTan[a*x]))] - PolyLog[2, E^(I*(Pi/2
- ArcTan[a*x]))])))/4 + (3*Pi*((Pi/2 - ArcTan[a*x])^2*(Log[1 - E^(I*(Pi/2
- ArcTan[a*x]))] - Log[1 + E^(I*(Pi/2 - ArcTan[a*x]))]) + (2*I)*(Pi/2 - Ar
cTan[a*x])*(PolyLog[2, -E^(I*(Pi/2 - ArcTan[a*x]))] - PolyLog[2, E^(I*(Pi/
2 - ArcTan[a*x]))]) + 2*(-PolyLog[3, -E^(I*(Pi/2 - ArcTan[a*x]))] + PolyLo
g[3, E^(I*(Pi/2 - ArcTan[a*x]))])))/2 - 8*((I/64)*(Pi/2 - ArcTan[a*x])^4 +
 (I/4)*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2)^4 - ((Pi/2 - ArcTan[a*x])^3*Log[
1 + E^(I*(Pi/2 - ArcTan[a*x]))])/8 - (Pi^3*(I*(Pi/2 + (-1/2*Pi + ArcTan[a*
x])/2) - Log[1 + E^((2*I)*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2))]))/8 - (Pi/2
 + (-1/2*Pi + ArcTan[a*x])/2)^3*Log[1 + E^((2*I)*(Pi/2 + (-1/2*Pi + ArcTan
[a*x])/2))] + ((3*I)/8)*(Pi/2 - ArcTan[a*x])^2*PolyLog[2, -E^(I*(Pi/2 - Ar
cTan[a*x]))] + (3*Pi^2*((I/2)*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2)^2 - (Pi/2
 + (-1/2*Pi + ArcTan[a*x])/2)*Log[1 + E^((2*I)*(Pi/2 + (-1/2*Pi + ArcTan[a
*x])/2))] + (I/2)*PolyLog[2, -E^((2*I)*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/...
```

## Rubi [A] (verified)

Time = 10.22 (sec) , antiderivative size = 1306, normalized size of antiderivative = 1.75, number of steps used = 27, number of rules used = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 1.083$, Rules used = {5485, 5487, 5425, 5423, 3042, 4669, 3011, 5465, 5425, 5421, 5487, 5425, 5423, 3042, 4669, 3011, 5465, 5425, 5421, 5487, 241, 5425, 5421, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arctan(ax)^3 \sqrt{a^2cx^2+c}\,dx$$

$$\Big\downarrow \text{ 5485}$$

$$c\int \frac{x^2 \arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx + a^2c\int \frac{x^4 \arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{ 5487}$$

$$c\left(-\frac{3\int \frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{\int \frac{\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{2a^2}+\frac{x\arctan(ax)^3\sqrt{a^2cx^2+c}}{2a^2c}\right)+a^2c\left(-\frac{3\int \frac{x^2\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{3\int \frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a}+\frac{x^3\arctan(ax)^3\sqrt{a^2cx^2+c}}{4a^2c}\right)$$

$\downarrow$ 5425

$$c\left(-\frac{3\int \frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^3}{\sqrt{a^2x^2+1}}dx}{2a^2\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^3\sqrt{a^2cx^2+c}}{2a^2c}\right)+a^2c\left(-\frac{3\int \frac{x^2\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{3\int \frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a}+\frac{x^3\arctan(ax)^3\sqrt{a^2cx^2+c}}{4a^2c}\right)$$

$\downarrow$ 5423

$$a^2c\left(-\frac{3\int \frac{x^2\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{3\int \frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a}+\frac{x^3\arctan(ax)^3\sqrt{a^2cx^2+c}}{4a^2c}\right)+c\left(-\frac{3\int \frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{\sqrt{a^2x^2+1}\int \sqrt{a^2x^2+1}\arctan(ax)^3d\arctan(ax)}{2a^3\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^3\sqrt{a^2cx^2+c}}{2a^2c}\right)$$

$\downarrow$ 3042

$$a^2c\left(-\frac{3\int \frac{x^2\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{3\int \frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a}+\frac{x^3\arctan(ax)^3\sqrt{a^2cx^2+c}}{4a^2c}\right)+c\left(-\frac{3\int \frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{\sqrt{a^2x^2+1}\int \arctan(ax)^3\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{2a^3\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^3\sqrt{a^2cx^2+}}{2a^2c}\right.$$

$\downarrow$ 4669

$$a^2c\left(-\frac{3\int \frac{x^2\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{3\int \frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a}+\frac{x^3\arctan(ax)^3\sqrt{a^2cx^2+c}}{4a^2c}\right)+c\left(-\frac{3\int \frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{\sqrt{a^2x^2+1}\left(-3\int \arctan(ax)^2\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+3\int \arctan(ax)^2\log\right.}{2a^3\sqrt{a^2cx^2+c}}\right.$$

$\downarrow$ 3011

$$a^2c\left(-\frac{3\int \frac{x^2\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{3\int \frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a}+\frac{x^3\arctan(ax)^3\sqrt{a^2cx^2+c}}{4a^2c}\right)+$$
$$c\left(-\frac{3\int \frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int \arctan(ax)\operatorname{PolyLog}\left(2,-\right.\right.\right.}{}\right.$$

$$\Big\downarrow \text{5465}$$

$$a^2c\left(-\frac{3\int \frac{x^2\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{3\int \frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a}+\frac{x^3\arctan(ax)^3\sqrt{a^2cx^2+c}}{4a^2c}\right)+$$
$$c\left(-\frac{3\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}\right)}{2a}-\frac{\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int \operatorname{arc}\right.\right.}{}\right.$$

$$\Big\downarrow \text{5425}$$

$$a^2c\left(-\frac{3\int \frac{x^2\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{3\int \frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a}+\frac{x^3\arctan(ax)^3\sqrt{a^2cx^2+c}}{4a^2c}\right)+$$
$$c\left(-\frac{3\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{a\sqrt{a^2cx^2+c}}\right)}{2a}-\frac{\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\right.\right.}{}\right.$$

$$\Big\downarrow \text{5421}$$

$$a^2c\left(-\frac{3\int \frac{x^2\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{3\int \frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a}+\frac{x^3\arctan(ax)^3\sqrt{a^2cx^2+c}}{4a^2c}\right)+$$
$$c\left(-\frac{\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int \arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\operatorname{arctan}\right.\right.}{}\right.$$

$$\Big\downarrow \text{5487}$$

$$a^2c\left(-\frac{3\left(-\frac{2\int \frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{2\int \frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)^2\sqrt{a^2cx^2+c}}{3a^2c}\right)}{4a}-\frac{3\left(-\frac{3\int \frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{\int \frac{\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{2a^2}+\right.}{4a^2}\right.$$

$$c\left(-\frac{\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int \arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan\right.\right.}{}\right.$$

$\downarrow$ 5425

$$a^2c\left(-\frac{3\left(-\frac{2\int \frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{2\int \frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)^2\sqrt{a^2cx^2+c}}{3a^2c}\right)}{4a}-\frac{3\left(-\frac{3\int \frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{\sqrt{a^2x^2+1}\int \frac{\arctan(}{\sqrt{a^2x}}}{2a^2\sqrt{a^2cx^2+}}\right.}{4a^2}\right.$$

$$c\left(-\frac{\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int \arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan\right.\right.}{}\right.$$

$\downarrow$ 5423

$$a^2c\left(-\frac{3\left(-\frac{2\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{2\int\frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)^2\sqrt{a^2cx^2+c}}{3a^2c}\right)}{4a}-\frac{3\left(-\frac{3\int\frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{\sqrt{a^2x^2+1}\int\sqrt{a^2x^2}}{2a}\right.}{}\right.$$

$$c\left(-\frac{\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan\right.\right.}{}\right.$$

$\downarrow$ 3042

$$a^2c\left(-\frac{3\left(-\frac{2\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{2\int\frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)^2\sqrt{a^2cx^2+c}}{3a^2c}\right)}{4a}-\frac{3\left(-\frac{3\int\frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{\sqrt{a^2x^2+1}\int\arctan}{}\right.}{}\right.$$

$$c\left(-\frac{\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan\right.\right.}{}\right.$$

$\downarrow$ 4669

$$c\left(-\frac{\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan\right.\right.}{}\right.$$

$$a^2c\left(-\frac{3\left(-\frac{2\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{2\int\frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)^2\sqrt{a^2cx^2+c}}{3a^2c}\right)}{4a}-\frac{3\left(-\frac{3\int\frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{\sqrt{a^2x^2+1}(-3\int\text{arc}}{}\right.}{}\right.$$

$\downarrow$ 3011

$$c\left(-\frac{\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan\right.\right.}{}\right.$$

$$a^2c\left(-\frac{3\left(-\frac{2\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{2\int\frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)^2\sqrt{a^2cx^2+c}}{3a^2c}\right)}{4a}-\frac{3\left(-\frac{3\int\frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{\sqrt{a^2x^2+1}(3(i\,\text{arcta}}{}\right.}{}\right.$$

$\downarrow$ 5465

$$c\left(\frac{x^3\sqrt{a^2cx^2+c}\arctan(ax)^3}{4a^2c}-\frac{3\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\int\frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}\right)}{3a^2}-\frac{2\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}\right.}{4a}\right.$$

$$c\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)^3}{2a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\right.}{a\sqrt{a^2cx^2+c}}\right.}{2a}\right.$$

$\downarrow$ 5425

$$c\left(\frac{x^3\sqrt{a^2cx^2+c}\arctan(ax)^3}{4a^2c}-\frac{3\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\int\frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{a\sqrt{a^2cx^2+c}}\right)}{3a^2}-\frac{2\int\frac{x^2\text{arc}}{\sqrt{a^2}}}{3}\right.}{4a}\right.$$

$$c\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)^3}{2a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\right.}{a\sqrt{a^2cx^2+c}}\right.}{2a}\right.$$

$\downarrow$ 5421

$$c\left(\frac{x^3\sqrt{a^2cx^2+c}\arctan(ax)^3}{4a^2c}-\frac{3\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right.}{}\right.}{3a^2}\right.}{4a}\right.$$

$$c\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)^3}{2a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\right.}{a\sqrt{a^2cx^2+c}}\right.}{2a}\right.$$

↓ 5487

$$c\left(\frac{x^3\sqrt{a^2cx^2+c}\arctan(ax)^3}{4a^2c}-\frac{3\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right.}{}\right.}{3a^2}\right.}{}\right.$$

$$c\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)^3}{2a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\right.}{a\sqrt{a^2cx^2+c}}\right.}{2a}\right.$$

$\Big\downarrow$ 241

$$c\left(\frac{x^3\sqrt{a^2cx^2+c}\arctan(ax)^3}{4a^2c}-\frac{3\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right.}{}\right.}{3a^2}\right.}{}\right.$$

$$c\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)^3}{2a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\right.}{a\sqrt{a^2cx^2+c}}\right.}{2a}\right.$$

$\downarrow$ 5425

$$c\left(\frac{x^3\sqrt{a^2cx^2+c}\,\arctan(ax)^3}{4a^2c}-\frac{3\left(\frac{x^2\sqrt{a^2cx^2+c}\,\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\,\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right.}{}\right.}{3a^2}\right.}{}\right.$$

$$c\left(\frac{x\sqrt{a^2cx^2+c}\,\arctan(ax)^3}{2a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\,\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\right.}{a\sqrt{a^2cx^2+c}}\right.}{2a}\right.$$

$\downarrow$ 5421

$$c\left(\frac{x^3\sqrt{a^2cx^2+c}\arctan(ax)^3}{4a^2c}-\frac{3\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)}{2a^2c}-\frac{\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right.}{2\cdots}\right.}{3\cdots}\right.}{\cdots}\right.$$

$$c\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)^3}{2a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\cdots\right.}{a\sqrt{a^2cx^2+c}}\right.}{2a}\right.$$

$\downarrow$ 7163

$$c\left(\frac{x^3\sqrt{a^2cx^2+c}\arctan(ax)^3}{4a^2c}-\frac{3\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)}{2a^2c}-\frac{\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\cdots\right.}{2\cdots}\right.}{3a\cdots}\right.}{\cdots}\right.$$

$$c\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)^3}{2a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\cdots\right.}{a\sqrt{a^2cx^2+c}}\cdots\right.}{2a}\right.$$

$\downarrow$ 2720

$$c\left(\frac{x^3\sqrt{a^2cx^2+c}\arctan(ax)^3}{4a^2c}-\frac{3\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)}{2a^2c}-\frac{\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right.}{2\cdots}\right.}{3\cdots}\right.}{\cdots}\right.$$

$$c\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)^3}{2a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\cdots\right.}{a\sqrt{a^2cx^2+c}}\right.}{2a}\right.$$

↓ 7143

$$c\left(\frac{x^3\sqrt{a^2cx^2+c}\arctan(ax)^3}{4a^2c}-\frac{3\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)}{2a^2c}-\frac{\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right.}{2}\right.}{3}\right.}{}\right.$$

$$c\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)^3}{2a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\right.}{a\sqrt{a^2cx^2+c}}\right.}{2a}\right.$$

input
```
Int[x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3,x]
```

output
```
c*((x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/(2*a^2*c) - (3*((Sqrt[c + a^2*c*x
^2]*ArcTan[a*x]^2)/(a^2*c) - (2*Sqrt[1 + a^2*x^2]*(((-2*I)*ArcTan[a*x]*Arc
Tan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/a + (I*PolyLog[2, ((-I)*Sqrt[1 + I*a
*x])/Sqrt[1 - I*a*x]])/a - (I*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*
x]])/a))/(a*Sqrt[c + a^2*c*x^2])))/(2*a) - (Sqrt[1 + a^2*x^2]*((-2*I)*ArcT
an[E^(I*ArcTan[a*x])]*ArcTan[a*x]^3 + 3*(I*ArcTan[a*x]^2*PolyLog[2, (-I)*E
^(I*ArcTan[a*x])] - (2*I)*((-I)*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a*
x])] + PolyLog[4, (-I)*E^(I*ArcTan[a*x])])) - 3*(I*ArcTan[a*x]^2*PolyLog[2
, I*E^(I*ArcTan[a*x])] - (2*I)*((-I)*ArcTan[a*x]*PolyLog[3, I*E^(I*ArcTan[
a*x])] + PolyLog[4, I*E^(I*ArcTan[a*x])]))))/(2*a^3*Sqrt[c + a^2*c*x^2]))
+ a^2*c*((x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/(4*a^2*c) - (3*((x^2*Sqrt
[c + a^2*c*x^2]*ArcTan[a*x]^2)/(3*a^2*c) - (2*(-1/2*Sqrt[c + a^2*c*x^2]/(a
^3*c) + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(2*a^2*c) - (Sqrt[1 + a^2*x^2]
*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/a + (I*Poly
Log[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*PolyLog[2, (I*Sqrt[
1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/(2*a^2*Sqrt[c + a^2*c*x^2])))/(3*a) - (2
*((Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(a^2*c) - (2*Sqrt[1 + a^2*x^2]*(((-2
*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/a + (I*PolyLog[2,
 ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*PolyLog[2, (I*Sqrt[1 + I*
a*x])/Sqrt[1 - I*a*x]])/a))/(a*Sqrt[c + a^2*c*x^2])))/(3*a^2)))/(4*a) -...
```

### Defintions of rubi rules used

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5487
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcTan[c*x])^p/(c^2*d*m)), x] + (-Simp[b*f*(p/(c*m))   Int[(f*x)^(m - 1)*((
a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[f^2*((m - 1)/(c^
2*m))   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2]), x], x])
/; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

## Maple [A] (verified)

Time = 5.00 (sec) , antiderivative size = 460, normalized size of antiderivative = 0.62

| method | result |
|---|---|
| default | $\frac{\sqrt{c(ax-i)(ax+i)}\,\left(2\arctan(ax)^3a^3x^3-2\arctan(ax)^2x^2a^2+x\arctan(ax)^3a+2\arctan(ax)ax+\arctan(ax)^2-2\right)}{8a^3}-\frac{\sqrt{c(ax-i)(ax-}}{}$ |

input
```
int(x^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/8/a^3*(c*(a*x-I)*(a*x+I))^(1/2)*(2*arctan(a*x)^3*a^3*x^3-2*arctan(a*x)^2
*x^2*a^2+x*arctan(a*x)^3*a+2*arctan(a*x)*a*x+arctan(a*x)^2-2)-1/8*(c*(a*x-
I)*(a*x+I))^(1/2)*(arctan(a*x)^3*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-arcta
n(a*x)^3*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*I*arctan(a*x)^2*polylog(2,I
*(1+I*a*x)/(a^2*x^2+1)^(1/2))+3*I*arctan(a*x)^2*polylog(2,-I*(1+I*a*x)/(a^
2*x^2+1)^(1/2))+4*arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*arctan
(a*x)*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-4*arctan(a*x)*ln(1+I*(1+I*a
*x)/(a^2*x^2+1)^(1/2))-6*arctan(a*x)*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1
/2))+6*I*polylog(4,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+4*I*dilog(1+I*(1+I*a*x)/
(a^2*x^2+1)^(1/2))-4*I*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I*polylog(
4,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/a^3/(a^2*x^2+1)^(1/2)
```

## Fricas [F]

$$\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx=\int \sqrt{a^2cx^2+c}x^2\arctan\left(ax\right)^3\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^2*arctan(a*x)^3, x)
```

## Sympy [F]

$$\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx=\int x^2\sqrt{c\left(a^2x^2+1\right)}\operatorname{atan}^3\left(ax\right)dx$$

input
```
integrate(x**2*(a**2*c*x**2+c)**(1/2)*atan(a*x)**3,x)
```

output
```
Integral(x**2*sqrt(c*(a**2*x**2 + 1))*atan(a*x)**3, x)
```

## Maxima [F]

$$\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx=\int \sqrt{a^2cx^2+c}x^2\arctan(ax)^3\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*x^2*arctan(a*x)^3, x)
```

## Giac [F]

$$\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx=\int \sqrt{a^2cx^2+c}x^2\arctan(ax)^3\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*x^2*arctan(a*x)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx=\int x^2\,\mathrm{atan}(a\,x)^3\,\sqrt{c\,a^2\,x^2+c}\,dx$$

input
```
int(x^2*atan(a*x)^3*(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(x^2*atan(a*x)^3*(c + a^2*c*x^2)^(1/2), x)
```

## Reduce [F]

$$\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx=\sqrt{c}\left(\int\sqrt{a^2x^2+1}\,atan(ax)^3\,x^2dx\right)$$

input
```
int(x^2*(a^2*c*x^2+c)^(1/2)*atan(a*x)^3,x)
```

output
```
sqrt(c)*int(sqrt(a**2*x**2 + 1)*atan(a*x)**3*x**2,x)
```

# 3.414 $\int x\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 373

$$\begin{aligned}\int x\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx &= \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{a^2}-\frac{x\sqrt{c+a^2cx^2}\arctan(ax)^2}{2a}\\&\quad+\frac{ic\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2}{a^2\sqrt{c+a^2cx^2}}\\&\quad+\frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{3a^2c}-\frac{\sqrt{c}\,\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right)}{a^2}\\&\quad-\frac{ic\sqrt{1+a^2x^2}\arctan(ax)\,\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)}{a^2\sqrt{c+a^2cx^2}}\\&\quad+\frac{ic\sqrt{1+a^2x^2}\arctan(ax)\,\text{PolyLog}\left(2,ie^{i\arctan(ax)}\right)}{a^2\sqrt{c+a^2cx^2}}\\&\quad+\frac{c\sqrt{1+a^2x^2}\,\text{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)}{a^2\sqrt{c+a^2cx^2}}\\&\quad-\frac{c\sqrt{1+a^2x^2}\,\text{PolyLog}\left(3,ie^{i\arctan(ax)}\right)}{a^2\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
(a^2*c*x^2+c)^(1/2)*arctan(a*x)/a^2-1/2*x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^
2/a+I*c*(a^2*x^2+1)^(1/2)*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^
2/a^2/(a^2*c*x^2+c)^(1/2)+1/3*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3/a^2/c-c^(1
/2)*arctanh(a*c^(1/2)*x/(a^2*c*x^2+c)^(1/2))/a^2-I*c*(a^2*x^2+1)^(1/2)*arc
tan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a^2/(a^2*c*x^2+c)^(1/2)
+I*c*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2)
)/a^2/(a^2*c*x^2+c)^(1/2)+c*(a^2*x^2+1)^(1/2)*polylog(3,-I*(1+I*a*x)/(a^2*
x^2+1)^(1/2))/a^2/(a^2*c*x^2+c)^(1/2)-c*(a^2*x^2+1)^(1/2)*polylog(3,I*(1+I
*a*x)/(a^2*x^2+1)^(1/2))/a^2/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (warning: unable to verify)

Time = 0.51 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.55

$$\int x\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx = \frac{\sqrt{c+a^2cx^2}\left(-\frac{12\left(\coth^{-1}\left(\frac{ax}{\sqrt{1+a^2x^2}}\right)-i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2+i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-i\arctan(ax)\operatorname{PolyI}\right.}{\sqrt{1+a^2x^2}}\right.}{}$$

input
```
Integrate[x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3,x]
```

output
```
(Sqrt[c + a^2*c*x^2]*((-12*(ArcCoth[(a*x)/Sqrt[1 + a^2*x^2]] - I*ArcTan[E^
(I*ArcTan[a*x])]*ArcTan[a*x]^2 + I*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan
[a*x])] - I*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] - PolyLog[3, (-I)*
E^(I*ArcTan[a*x])] + PolyLog[3, I*E^(I*ArcTan[a*x])]))/Sqrt[1 + a^2*x^2] +
 (1 + a^2*x^2)*ArcTan[a*x]*(6 + 4*ArcTan[a*x]^2 + 6*Cos[2*ArcTan[a*x]] - 3
*ArcTan[a*x]*Sin[2*ArcTan[a*x]])))/(12*a^2)
```

## Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.71, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5465, 5415, 224, 219, 5425, 5423, 3042, 4669, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \arctan(ax)^3\sqrt{a^2cx^2+c}\,dx$$

$$\Big\downarrow \text{ 5465}$$

$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\int \sqrt{a^2cx^2+c}\arctan(ax)^2dx}{a}$$

$$\Big\downarrow \text{ 5415}$$

$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{1}{2}c\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int \frac{1}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}}{a}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{1}{2}c\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int \frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}}{a}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{1}{2}c\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\frac{\sqrt{c}\,\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}}{a}$$

$$\Big\downarrow \text{ 5425}$$

$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\frac{\sqrt{c}\,\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}}{a}$$

$$\Big\downarrow \text{5423}$$

$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{c\sqrt{a^2x^2+1}\int\sqrt{a^2x^2+1}\arctan(ax)^2d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{c\sqrt{a^2x^2+1}\int\arctan(ax)^2\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\frac{\sqrt{c}\text{arctanh}\left(\frac{\ldots}{\sqrt{\ldots}}\right)}{a}}{a}$$

$$\Big\downarrow \text{4669}$$

$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{c\sqrt{a^2x^2+1}\left(-2\int\arctan(ax)\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+2\int\arctan(ax)\log\left(1+ie^{i\arctan(ax)}\right)d\arctan(ax)-2i\arctan\left(e^{i\arctan(ax)}\right)\text{arctan}\ldots\right)}{2a\sqrt{a^2cx^2+c}}\ldots}{a}$$

$$\Big\downarrow \text{3011}$$

$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-i\int\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(ax)\right)-2\left(i\arctan(ax)\text{PolyLog}\left(2,ie^{i\arctan(ax)}\right)-i\ldots\right.\right.}{2a\sqrt{a^2cx^2+c}}\ldots}{\ldots}$$

$$\Big\downarrow \text{2720}$$

$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)-2\left(i\arctan(ax)\text{PolyLog}\left(2,ie^{i\ldots}\right.\right.\right.}{2a\sqrt{a^2cx^2+c}}\ldots}{\ldots}$$

$$\Big\downarrow \text{7143}$$

$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\text{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)\right)-2\left(i\arctan(ax)\text{PolyLog}\left(2,ie^{i\arctan(ax)}\right)-\text{PolyLog}\left(3,ie^{i\,\text{arc}\ldots}\right.\right.\right.}{2a\sqrt{a^2cx^2+c}}\ldots}{a}$$

input
```
Int[x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3,x]
```

output
```
((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3)/(3*a^2*c) - (-((Sqrt[c + a^2*c*x^2]*
ArcTan[a*x])/a) + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/2 + (Sqrt[c]*ArcTa
nh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/a + (c*Sqrt[1 + a^2*x^2]*((-2*I)*Ar
cTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 + 2*(I*ArcTan[a*x]*PolyLog[2, (-I)*E
^(I*ArcTan[a*x])] - PolyLog[3, (-I)*E^(I*ArcTan[a*x])]) - 2*(I*ArcTan[a*x]
*PolyLog[2, I*E^(I*ArcTan[a*x])] - PolyLog[3, I*E^(I*ArcTan[a*x])])))/(2*a
*Sqrt[c + a^2*c*x^2]))/a
```

### Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5415
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTan[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x])^p,
x], x] + Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*(
a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[q, 0] && GtQ[p, 1]
```

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

## Maple [A] (verified)

Time = 4.22 (sec) , antiderivative size = 370, normalized size of antiderivative = 0.99

| method | result |
|---|---|
| default | $\frac{\sqrt{c(ax-i)(ax+i)}\,\arctan(ax)\left(2\arctan(ax)^2x^2a^2-3\arctan(ax)ax+2\arctan(ax)^2+6\right)}{6a^2}+\frac{\sqrt{c(ax-i)(ax+i)}\,\left(i\arctan(ax)^3-3\,\mathrm{arc}\right.}{}$ |

input
```
int(x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/6/a^2*(c*(a*x-I)*(a*x+I))^(1/2)*arctan(a*x)*(2*arctan(a*x)^2*x^2*a^2-3*a
rctan(a*x)*a*x+2*arctan(a*x)^2+6)+1/6*(c*(a*x-I)*(a*x+I))^(1/2)*(I*arctan(
a*x)^3-3*arctan(a*x)^2*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*arctan(a*x)
*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*polylog(3,I*(1+I*a*x)/(a^2*x^2
+1)^(1/2)))/a^2/(a^2*x^2+1)^(1/2)+1/6*(c*(a*x-I)*(a*x+I))^(1/2)*(-I*arctan
(a*x)^3+3*arctan(a*x)^2*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I*arctan(a*x
)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*polylog(3,-I*(1+I*a*x)/(a^2*
x^2+1)^(1/2)))/a^2/(a^2*x^2+1)^(1/2)+2*I/a^2*(c*(a*x-I)*(a*x+I))^(1/2)*arc
tan((1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^(1/2)
```

## Fricas [F]

$$\int x\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx=\int\sqrt{a^2cx^2+c}\,x\arctan\left(ax\right)^3\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x*arctan(a*x)^3, x)
```

## Sympy [F]

$$\int x\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx=\int x\sqrt{c\left(a^2x^2+1\right)}\operatorname{atan}^3\left(ax\right)dx$$

input
```
integrate(x*(a**2*c*x**2+c)**(1/2)*atan(a*x)**3,x)
```

output
```
Integral(x*sqrt(c*(a**2*x**2 + 1))*atan(a*x)**3, x)
```

## Maxima [F]

$$\int x\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx=\int\sqrt{a^2cx^2+c}\,x\arctan\left(ax\right)^3\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*x*arctan(a*x)^3, x)
```

## Giac [F(-2)]

Exception generated.

$$\int x\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int x\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx=\int x\,\mathrm{atan}(a\,x)^3\,\sqrt{c\,a^2\,x^2+c}\,dx$$

input
```
int(x*atan(a*x)^3*(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(x*atan(a*x)^3*(c + a^2*c*x^2)^(1/2), x)
```

## Reduce [F]

$$\int x\sqrt{c+a^2cx^2}\arctan(ax)^3\,dx=\sqrt{c}\left(\int \sqrt{a^2x^2+1}\,atan(ax)^3\,xdx\right)$$

input
```
int(x*(a^2*c*x^2+c)^(1/2)*atan(a*x)^3,x)
```

output
```
sqrt(c)*int(sqrt(a**2*x**2 + 1)*atan(a*x)**3*x,x)
```

# 3.415 $\int \sqrt{c+a^2cx^2}\arctan(ax)^3\,dx$

## Optimal result

Integrand size = 21, antiderivative size = 626

$$\int \sqrt{c+a^2cx^2}\arctan(ax)^3\,dx = -\frac{3\sqrt{c+a^2cx^2}\arctan(ax)^2}{2a}+\frac{1}{2}x\sqrt{c+a^2cx^2}\arctan(ax)^3 - \frac{ic\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^3}{a\sqrt{c+a^2cx^2}} - \frac{6ic\sqrt{1+a^2x^2}\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a\sqrt{c+a^2cx^2}} + \frac{3ic\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)}{2a\sqrt{c+a^2cx^2}} - \frac{3ic\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{PolyLog}\left(2,ie^{i\arctan(ax)}\right)}{2a\sqrt{c+a^2cx^2}} + \frac{3ic\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a\sqrt{c+a^2cx^2}} - \frac{3ic\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a\sqrt{c+a^2cx^2}} - \frac{3c\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)}{a\sqrt{c+a^2cx^2}} + \frac{3c\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(3,ie^{i\arctan(ax)}\right)}{a\sqrt{c+a^2cx^2}} - \frac{3ic\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(4,-ie^{i\arctan(ax)}\right)}{a\sqrt{c+a^2cx^2}} + \frac{3ic\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(4,ie^{i\arctan(ax)}\right)}{a\sqrt{c+a^2cx^2}}$$

output
```
-3/2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2/a+1/2*x*(a^2*c*x^2+c)^(1/2)*arctan(
a*x)^3-I*c*(a^2*x^2+1)^(1/2)*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*
x)^3/a/(a^2*c*x^2+c)^(1/2)-6*I*c*(a^2*x^2+1)^(1/2)*arctan(a*x)*arctan((1+I
*a*x)^(1/2)/(1-I*a*x)^(1/2))/a/(a^2*c*x^2+c)^(1/2)+3/2*I*c*(a^2*x^2+1)^(1/
2)*arctan(a*x)^2*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a/(a^2*c*x^2+c)
^(1/2)-3/2*I*c*(a^2*x^2+1)^(1/2)*arctan(a*x)^2*polylog(2,I*(1+I*a*x)/(a^2*
x^2+1)^(1/2))/a/(a^2*c*x^2+c)^(1/2)+3*I*c*(a^2*x^2+1)^(1/2)*polylog(2,-I*(
1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/a/(a^2*c*x^2+c)^(1/2)-3*I*c*(a^2*x^2+1)^(1
/2)*polylog(2,I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/a/(a^2*c*x^2+c)^(1/2)-3*c
*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a
/(a^2*c*x^2+c)^(1/2)+3*c*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(3,I*(1+I*a*
x)/(a^2*x^2+1)^(1/2))/a/(a^2*c*x^2+c)^(1/2)-3*I*c*(a^2*x^2+1)^(1/2)*polylo
g(4,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a/(a^2*c*x^2+c)^(1/2)+3*I*c*(a^2*x^2+1
)^(1/2)*polylog(4,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.41

$$\int \sqrt{c+a^2cx^2}\arctan(ax)^3\,dx = -\frac{i\sqrt{c\left(1+a^2x^2\right)}\left(12\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)-3i\sqrt{1+a^2x^2}\arctan(ax)^2+iax\sqrt{1+a^2x^2}\arctan\right.}{}$$

input
```
Integrate[Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3,x]
```

output
```
((-1/2*I)*Sqrt[c*(1 + a^2*x^2)]*(12*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]
- (3*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2 + I*a*x*Sqrt[1 + a^2*x^2]*ArcTan[a
*x]^3 + 2*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^3 - 3*(2 + ArcTan[a*x]^2)*
PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + 3*(2 + ArcTan[a*x]^2)*PolyLog[2, I*E^
(I*ArcTan[a*x])] - (6*I)*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] +
(6*I)*ArcTan[a*x]*PolyLog[3, I*E^(I*ArcTan[a*x])] + 6*PolyLog[4, (-I)*E^(I
*ArcTan[a*x])] - 6*PolyLog[4, I*E^(I*ArcTan[a*x])]))/(a*Sqrt[1 + a^2*x^2])
```

## Rubi [A] (verified)

Time = 1.61 (sec) , antiderivative size = 400, normalized size of antiderivative = 0.64, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {5415, 5425, 5421, 5423, 3042, 4669, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(ax)^3\sqrt{a^2cx^2+c}dx$$

$$\Big\downarrow \text{5415}$$

$$3c\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}c\int \frac{\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^3\sqrt{a^2cx^2+c}-\frac{3\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a}$$

$$\Big\downarrow \text{5425}$$

$$\frac{3c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+\frac{c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^3}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^3\sqrt{a^2cx^2+c}-\frac{3\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a}$$

$$\Big\downarrow \text{5421}$$

$$\frac{c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^3}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^3\sqrt{a^2cx^2+c}-\frac{3\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a}$$

$$\Big\downarrow \text{5423}$$

$$\frac{c\sqrt{a^2x^2+1}\int \sqrt{a^2x^2+1}\arctan(ax)^3d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^3\sqrt{a^2cx^2+c}-\frac{3\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{c\sqrt{a^2x^2+1}\int \arctan(ax)^3\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^3\sqrt{a^2cx^2+c}-\frac{3\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a}$$

$$\Big\downarrow \text{4669}$$

$$\frac{c\sqrt{a^2x^2+1}\left(-3\int \arctan(ax)^2\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+3\int \arctan(ax)^2\log\left(1+ie^{i\arctan(ax)}\right)d\arcta\right.}{2a\sqrt{a^2cx^2+c}}$$

$$\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^3\sqrt{a^2cx^2+c}-\frac{3\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a}$$

$$\Big\downarrow \text{3011}$$

$$c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int \arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(ax\right.\right.$$

$$\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^3\sqrt{a^2cx^2+c}-\frac{3\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a}$$

$$\Big\downarrow \text{7163}$$

$$c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\left(i\int \operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)d\arctan(ax)-i\arct\right.\right.\right.$$

$$\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^3\sqrt{a^2cx^2+c}-\frac{3\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a}$$

$$\Big\downarrow \text{2720}$$

$$\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\left(\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right.\right.\right.}{}$$

$$\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{\sqrt{a^2cx^2+c}}+$$

$$\frac{1}{2}x\arctan(ax)^3\sqrt{a^2cx^2+c}-\frac{3\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a}$$

$$\Big\downarrow 7143$$

$$\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{\sqrt{a^2cx^2+c}}+$$

$$\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\left(\operatorname{PolyLog}\left(4,-ie^{i\arctan(ax)}\right)-i\arctan(ax)\operatorname{PolyLog}\right.\right.\right.}{}$$

$$\frac{1}{2}x\arctan(ax)^3\sqrt{a^2cx^2+c}-\frac{3\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a}$$

input
```
Int[Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3,x]
```

output
```
(-3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(2*a) + (x*Sqrt[c + a^2*c*x^2]*ArcT
an[a*x]^3)/2 + (3*c*Sqrt[1 + a^2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 +
 I*a*x]/Sqrt[1 - I*a*x]])/a + (I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1
- I*a*x]])/a - (I*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/Sqr
t[c + a^2*c*x^2] + (c*Sqrt[1 + a^2*x^2]*((-2*I)*ArcTan[E^(I*ArcTan[a*x])]*
ArcTan[a*x]^3 + 3*(I*ArcTan[a*x]^2*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - (2
*I)*((-I)*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] + PolyLog[4, (-I)
*E^(I*ArcTan[a*x])])) - 3*(I*ArcTan[a*x]^2*PolyLog[2, I*E^(I*ArcTan[a*x])]
 - (2*I)*((-I)*ArcTan[a*x]*PolyLog[3, I*E^(I*ArcTan[a*x])] + PolyLog[4, I*
E^(I*ArcTan[a*x])]))))/(2*a*Sqrt[c + a^2*c*x^2])
```

### Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5415
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTan[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x])^p,
x], x] + Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*(
a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[q, 0] && GtQ[p, 1]
```

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

## Maple [A] (verified)

Time = 4.77 (sec) , antiderivative size = 422, normalized size of antiderivative = 0.67

| method | result |
|---|---|
| default | $\frac{\sqrt{c(ax-i)(ax+i)}\,\arctan(ax)^2(\arctan(ax)ax-3)}{2a}+\frac{\sqrt{c(ax-i)(ax+i)}\left(\arctan(ax)^3\ln\left(1-\frac{i(iax+1)}{\sqrt{a^2x^2+1}}\right)-3i\arctan(ax)^2\operatorname{polylog}\left(\right.\right.}{}$ |

input
```
int((a^2*c*x^2+c)^(1/2)*arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2/a*(c*(a*x-I)*(a*x+I))^(1/2)*arctan(a*x)^2*(arctan(a*x)*a*x-3)+1/2*(c*(
a*x-I)*(a*x+I))^(1/2)*(arctan(a*x)^3*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-3
*I*arctan(a*x)^2*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-arctan(a*x)^3*ln
(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+3*I*arctan(a*x)^2*polylog(2,-I*(1+I*a*x)
/(a^2*x^2+1)^(1/2))+6*arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*ar
ctan(a*x)*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*polylog(4,I*(1+I*a*
x)/(a^2*x^2+1)^(1/2))-6*arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*
arctan(a*x)*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I*polylog(4,-I*(1+
I*a*x)/(a^2*x^2+1)^(1/2))+6*I*dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I*d
ilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/a/(a^2*x^2+1)^(1/2)
```

## Fricas [F]

$$\int \sqrt{c+a^2cx^2}\arctan(ax)^3\,dx=\int \sqrt{a^2cx^2+c}\arctan\left(ax\right)^3\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)*arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3, x)
```

## Sympy [F]

$$\int \sqrt{c+a^2cx^2}\arctan(ax)^3\,dx=\int \sqrt{c\left(a^2x^2+1\right)}\operatorname{atan}^3\left(ax\right)dx$$

input
```
integrate((a**2*c*x**2+c)**(1/2)*atan(a*x)**3,x)
```

output
```
Integral(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**3, x)
```

## Maxima [F]

$$\int \sqrt{c+a^2cx^2}\arctan(ax)^3\,dx = \int \sqrt{a^2cx^2+c}\arctan\left(ax\right)^3\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)*arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3, x)
```

## Giac [F(-2)]

Exception generated.

$$\int \sqrt{c+a^2cx^2}\arctan(ax)^3\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(1/2)*arctan(a*x)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+a^2cx^2}\arctan(ax)^3\,dx = \int \mathrm{atan}(a\,x)^3\,\sqrt{c\,a^2\,x^2+c}\,dx$$

input
```
int(atan(a*x)^3*(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(atan(a*x)^3*(c + a^2*c*x^2)^(1/2), x)
```

## Reduce [F]

$$\int \sqrt{c+a^2cx^2}\arctan(ax)^3\,dx=\sqrt{c}\left(\int \sqrt{a^2x^2+1}\,atan(ax)^3\,dx\right)$$

input
```
int((a^2*c*x^2+c)^(1/2)*atan(a*x)^3,x)
```

output
```
sqrt(c)*int(sqrt(a**2*x**2 + 1)*atan(a*x)**3,x)
```

# 3.416 $\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x}\,dx$

## Optimal result

Integrand size = 24, antiderivative size = 600

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x}\,dx = \frac{6ic\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2}{\sqrt{c+a^2cx^2}} + \sqrt{c+a^2cx^2}\arctan(ax)^3 - \frac{2c\sqrt{1+a^2x^2}\arctan(ax)^3\text{arctanh}\left(e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} + \frac{3ic\sqrt{1+a^2x^2}\arctan(ax)^2\,\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} - \frac{6ic\sqrt{1+a^2x^2}\arctan(ax)\,\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} + \frac{6ic\sqrt{1+a^2x^2}\arctan(ax)\,\text{PolyLog}\left(2,ie^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} - \frac{3ic\sqrt{1+a^2x^2}\arctan(ax)^2\,\text{PolyLog}\left(2,e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} - \frac{6c\sqrt{1+a^2x^2}\arctan(ax)\,\text{PolyLog}\left(3,-e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} + \frac{6c\sqrt{1+a^2x^2}\,\text{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} - \frac{6c\sqrt{1+a^2x^2}\,\text{PolyLog}\left(3,ie^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} + \frac{6c\sqrt{1+a^2x^2}\arctan(ax)\,\text{PolyLog}\left(3,e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} - \frac{6ic\sqrt{1+a^2x^2}\,\text{PolyLog}\left(4,-e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} + \frac{6ic\sqrt{1+a^2x^2}\,\text{PolyLog}\left(4,e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}$$

output
```
6*I*c*(a^2*x^2+1)^(1/2)*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^2/
(a^2*c*x^2+c)^(1/2)+(a^2*c*x^2+c)^(1/2)*arctan(a*x)^3-2*c*(a^2*x^2+1)^(1/2
)*arctan(a*x)^3*arctanh((1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)+3
*I*c*(a^2*x^2+1)^(1/2)*arctan(a*x)^2*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2
))/(a^2*c*x^2+c)^(1/2)-6*I*c*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(2,-I*(1
+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)+6*I*c*(a^2*x^2+1)^(1/2)*arc
tan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)-3*I*
c*(a^2*x^2+1)^(1/2)*arctan(a*x)^2*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))/(
a^2*c*x^2+c)^(1/2)-6*c*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(3,-(1+I*a*x)/
(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)+6*c*(a^2*x^2+1)^(1/2)*polylog(3,-I*
(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)-6*c*(a^2*x^2+1)^(1/2)*pol
ylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)+6*c*(a^2*x^2+1)^
(1/2)*arctan(a*x)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/
2)-6*I*c*(a^2*x^2+1)^(1/2)*polylog(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*
x^2+c)^(1/2)+6*I*c*(a^2*x^2+1)^(1/2)*polylog(4,(1+I*a*x)/(a^2*x^2+1)^(1/2)
)/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 366, normalized size of antiderivative = 0.61

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x}\,dx$$
$$= \frac{\sqrt{c+a^2cx^2}\left(-i\pi^4+8\sqrt{1+a^2x^2}\arctan(ax)^3+2i\arctan(ax)^4+8\arctan(ax)^3\log\left(1-e^{-i\arctan(ax)}\right)-2\right.}{}$$

input
```
Integrate[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/x,x]
```

output
```
(Sqrt[c + a^2*c*x^2]*((-I)*Pi^4 + 8*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^3 + (2*I
)*ArcTan[a*x]^4 + 8*ArcTan[a*x]^3*Log[1 - E^((-I)*ArcTan[a*x])] - 24*ArcTa
n[a*x]^2*Log[1 - I*E^(I*ArcTan[a*x])] + 24*ArcTan[a*x]^2*Log[1 + I*E^(I*Ar
cTan[a*x])] - 8*ArcTan[a*x]^3*Log[1 + E^(I*ArcTan[a*x])] + (24*I)*ArcTan[a
*x]^2*PolyLog[2, E^((-I)*ArcTan[a*x])] + (24*I)*ArcTan[a*x]^2*PolyLog[2, -
E^(I*ArcTan[a*x])] - (48*I)*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])]
 + (48*I)*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] + 48*ArcTan[a*x]*Pol
yLog[3, E^((-I)*ArcTan[a*x])] - 48*ArcTan[a*x]*PolyLog[3, -E^(I*ArcTan[a*x
])] + 48*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] - 48*PolyLog[3, I*E^(I*ArcTan[
a*x])] - (48*I)*PolyLog[4, E^((-I)*ArcTan[a*x])] - (48*I)*PolyLog[4, -E^(I
*ArcTan[a*x])]))/(8*Sqrt[1 + a^2*x^2])
```

## Rubi [A] (verified)

Time = 2.99 (sec) , antiderivative size = 357, normalized size of antiderivative = 0.60, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.708$, Rules used = {5485, 5465, 5425, 5423, 3042, 4669, 3011, 2720, 5493, 5491, 3042, 4671, 3011, 7143, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{x}dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\int \frac{x\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5465}$$

$$a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a}\right)+c\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5425}$$

$$a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{a\sqrt{a^2cx^2+c}}\right)+c\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5423}$$

$$a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\int\sqrt{a^2x^2+1}\arctan(ax)^2d\arctan(ax)}{a^2\sqrt{a^2cx^2+c}}\right)+c\int\frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx$$

$\downarrow$ 3042

$$c\int\frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx+a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\int\arctan(ax)^2\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{a^2\sqrt{a^2cx^2+c}}\right)$$

$\downarrow$ 4669

$$c\int\frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx+a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(-2\int\arctan(ax)\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+2\int\arctan(ax\right.}{a^2\sqrt{a^2cx^2}}\right.$$

$\downarrow$ 3011

$$c\int\frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx+a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-i\int\operatorname{PolyLog}\left(2,-ie^{i\arct}\right.\right.\right.}{}\right.$$

$\downarrow$ 2720

$$c\int\frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx+a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\right.\right.}{}\right.$$

$\downarrow$ 5493

$$\frac{c\sqrt{a^2x^2+1}\int\frac{\arctan(ax)^3}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\right.\right.}{}\right.$$

$\downarrow$ 5491

$$\frac{c\sqrt{a^2x^2+1}\int \frac{\sqrt{a^2x^2+1}\arctan(ax)^3}{ax}d\arctan(ax)}{\sqrt{a^2cx^2+c}}+$$
$$a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\right.\right.}{}\right.$$

$\downarrow$ 3042

$$\frac{c\sqrt{a^2x^2+1}\int \arctan(ax)^3\csc(\arctan(ax))d\arctan(ax)}{\sqrt{a^2cx^2+c}}+$$
$$a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\right.\right.}{}\right.$$

$\downarrow$ 4671

$$\frac{c\sqrt{a^2x^2+1}\left(-3\int \arctan(ax)^2\log\left(1-e^{i\arctan(ax)}\right)d\arctan(ax)+3\int \arctan(ax)^2\log\left(1+e^{i\arctan(ax)}\right)d\arctan(\right.}{\sqrt{a^2cx^2+c}}$$
$$a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\right.\right.}{}\right.$$

$\downarrow$ 3011

$$\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\int \arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)d\arctan(ax)\right)\right.}{}$$
$$a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\right.\right.}{}\right.$$

$\downarrow$ 7143

$$\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\int \arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)d\arctan(ax)\right)\right.}{}$$
$$a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\operatorname{PolyLog}\left(3,-ie^{i\arctan(}\right.\right.\right.}{}\right.$$

$\downarrow$ 7163

$$\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\left(i\int \operatorname{PolyLog}\left(3,-e^{i\arctan(ax)}\right)d\arctan(ax)-i\arctan\right.\right.\right.}{}$$
$$a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\operatorname{PolyLog}\left(3,-ie^{i\arctan(}\right.\right.\right.}{}\right.$$

$\downarrow$ 2720

$$\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\left(\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(3,-e^{i\arctan(ax)}\right)de^{i\arctan(ax}\right.\right.\right.}{}$$

$$a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\operatorname{PolyLog}\left(3,-ie^{i\arctan(\right.\right.\right.}{}\right.$$

$\downarrow$ 7143

$$\frac{c\sqrt{a^2x^2+1}\left(-2\arctan(ax)^3\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)+3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\left(\operatorname{PolyLog}\left(4,-\right.\right.\right.\right.}{}$$

$$a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\operatorname{PolyLog}\left(3,-ie^{i\arctan(\right.\right.\right.}{}\right.$$

input
```
Int[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/x,x]
```

output
```
a^2*c*((Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/(a^2*c) - (3*Sqrt[1 + a^2*x^2]*
((-2*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 + 2*(I*ArcTan[a*x]*PolyLog
[2, (-I)*E^(I*ArcTan[a*x])] - PolyLog[3, (-I)*E^(I*ArcTan[a*x])]) - 2*(I*A
rcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] - PolyLog[3, I*E^(I*ArcTan[a*x]
)])))/(a^2*Sqrt[c + a^2*c*x^2])) + (c*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]^3*
ArcTanh[E^(I*ArcTan[a*x])] + 3*(I*ArcTan[a*x]^2*PolyLog[2, -E^(I*ArcTan[a*
x])] - (2*I)*((-I)*ArcTan[a*x]*PolyLog[3, -E^(I*ArcTan[a*x])] + PolyLog[4,
 -E^(I*ArcTan[a*x])])) - 3*(I*ArcTan[a*x]^2*PolyLog[2, E^(I*ArcTan[a*x])]
- (2*I)*((-I)*ArcTan[a*x]*PolyLog[3, E^(I*ArcTan[a*x])] + PolyLog[4, E^(I*
ArcTan[a*x])]))))/Sqrt[c + a^2*c*x^2]
```

## Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5491
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Simp[1/Sqrt[d]   Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTa
n[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] &&
GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

## Maple [A] (verified)

Time = 6.24 (sec) , antiderivative size = 454, normalized size of antiderivative = 0.76

| method | result |
|---|---|
| default | $\sqrt{c\,(ax-i)\,(ax+i)}\,\arctan\left(ax\right)^{3}-\frac{\sqrt{c(ax-i)(ax+i)}\left(\arctan(ax)^{3}\ln\left(1+\frac{iax+1}{\sqrt{a^{2}x^{2}+1}}\right)-\arctan(ax)^{3}\ln\left(1-\frac{iax+1}{\sqrt{a^{2}x^{2}+}}\right.\right.}{}$ |

input
```
int((a^2*c*x^2+c)^(1/2)*arctan(a*x)^3/x,x,method=_RETURNVERBOSE)
```

output
```
(c*(a*x-I)*(a*x+I))^(1/2)*arctan(a*x)^3-(c*(a*x-I)*(a*x+I))^(1/2)*(arctan(
a*x)^3*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-arctan(a*x)^3*ln(1-(1+I*a*x)/(a^2
*x^2+1)^(1/2))-3*I*arctan(a*x)^2*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+3
*I*arctan(a*x)^2*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+3*arctan(a*x)^2*ln
(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*arctan(a*x)^2*ln(1+I*(1+I*a*x)/(a^2*x^
2+1)^(1/2))-6*I*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*a
rctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*arctan(a*x)*polylog
(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*arctan(a*x)*polylog(3,(1+I*a*x)/(a^2*x^
2+1)^(1/2))+6*I*polylog(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I*polylog(4,(1+I
*a*x)/(a^2*x^2+1)^(1/2))+6*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*poly
log(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)
```

## Fricas [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x}\,dx=\int \frac{\sqrt{a^2cx^2+c}\arctan\left(ax\right)^3}{x}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)*arctan(a*x)^3/x,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3/x, x)
```

## Sympy [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x}\,dx = \int \frac{\sqrt{c\left(a^2x^2+1\right)}\operatorname{atan}^3{(ax)}}{x}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(1/2)*atan(a*x)**3/x,x)
```

output
```
Integral(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**3/x, x)
```

## Maxima [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x}\,dx = \int \frac{\sqrt{a^2cx^2+c}\arctan\left(ax\right)^3}{x}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)*arctan(a*x)^3/x,x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3/x, x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(1/2)*arctan(a*x)^3/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x}\,dx = \int \frac{\mathrm{atan}(a\,x)^3\,\sqrt{c\,a^2\,x^2+c}}{x}\,dx$$

input
```
int((atan(a*x)^3*(c + a^2*c*x^2)^(1/2))/x,x)
```

output
```
int((atan(a*x)^3*(c + a^2*c*x^2)^(1/2))/x, x)
```

## Reduce [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x}\,dx = \sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,atan(ax)^3}{x}dx\right)$$

input
```
int((a^2*c*x^2+c)^(1/2)*atan(a*x)^3/x,x)
```

output
```
sqrt(c)*int((sqrt(a**2*x**2 + 1)*atan(a*x)**3)/x,x)
```

## 3.417 $\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x^2}\,dx$

## Optimal result

Integrand size = 24, antiderivative size = 622

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x^2}\,dx = -\frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x} - \frac{2iac\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^3}{\sqrt{c+a^2cx^2}} - \frac{6ac\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} + \frac{6iac\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} + \frac{3iac\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} - \frac{3iac\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{PolyLog}\left(2,ie^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} - \frac{6iac\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} - \frac{6ac\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,-e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} - \frac{6ac\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} + \frac{6ac\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(3,ie^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} + \frac{6ac\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} - \frac{6iac\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(4,-ie^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} + \frac{6iac\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(4,ie^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}$$

output
```
-(a^2*c*x^2+c)^(1/2)*arctan(a*x)^3/x-2*I*a*c*(a^2*x^2+1)^(1/2)*arctan((1+I
*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2)-6*a*c*(a^2*x^2+
1)^(1/2)*arctan(a*x)^2*arctanh((1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^
(1/2)+6*I*a*c*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+
1)^(1/2))/(a^2*c*x^2+c)^(1/2)+3*I*a*c*(a^2*x^2+1)^(1/2)*arctan(a*x)^2*poly
log(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)-3*I*a*c*(a^2*x^2
+1)^(1/2)*arctan(a*x)^2*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^
2+c)^(1/2)-6*I*a*c*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*
x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)-6*a*c*(a^2*x^2+1)^(1/2)*polylog(3,-(1+I*
a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)-6*a*c*(a^2*x^2+1)^(1/2)*arctan
(a*x)*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)+6*a*c*
(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^
2*c*x^2+c)^(1/2)+6*a*c*(a^2*x^2+1)^(1/2)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(
1/2))/(a^2*c*x^2+c)^(1/2)-6*I*a*c*(a^2*x^2+1)^(1/2)*polylog(4,-I*(1+I*a*x)
/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)+6*I*a*c*(a^2*x^2+1)^(1/2)*polylog(
4,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (warning: unable to verify)

Time = 2.21 (sec) , antiderivative size = 768, normalized size of antiderivative = 1.23

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/x^2,x]
```

output
```
(a*Sqrt[c + a^2*c*x^2]*Csc[ArcTan[a*x]/2]*((-7*I)*a*Pi^4*x - (8*I)*a*Pi^3*
x*ArcTan[a*x] + (24*I)*a*Pi^2*x*ArcTan[a*x]^2 - (32*I)*a*Pi*x*ArcTan[a*x]^
3 - 64*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^3 + (16*I)*a*x*ArcTan[a*x]^4 + 48*a*P
i^2*x*ArcTan[a*x]*Log[1 - I/E^(I*ArcTan[a*x])] - 96*a*Pi*x*ArcTan[a*x]^2*L
og[1 - I/E^(I*ArcTan[a*x])] - 8*a*Pi^3*x*Log[1 + I/E^(I*ArcTan[a*x])] + 64
*a*x*ArcTan[a*x]^3*Log[1 + I/E^(I*ArcTan[a*x])] + 192*a*x*ArcTan[a*x]^2*Lo
g[1 - E^(I*ArcTan[a*x])] + 8*a*Pi^3*x*Log[1 + I*E^(I*ArcTan[a*x])] - 48*a*
Pi^2*x*ArcTan[a*x]*Log[1 + I*E^(I*ArcTan[a*x])] + 96*a*Pi*x*ArcTan[a*x]^2*
Log[1 + I*E^(I*ArcTan[a*x])] - 64*a*x*ArcTan[a*x]^3*Log[1 + I*E^(I*ArcTan[
a*x])] - 192*a*x*ArcTan[a*x]^2*Log[1 + E^(I*ArcTan[a*x])] + 8*a*Pi^3*x*Log
[Tan[(Pi + 2*ArcTan[a*x])/4]] + (192*I)*a*x*ArcTan[a*x]^2*PolyLog[2, (-I)/
E^(I*ArcTan[a*x])] + (48*I)*a*Pi*x*(Pi - 4*ArcTan[a*x])*PolyLog[2, I/E^(I*
ArcTan[a*x])] + (384*I)*a*x*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])] + (
48*I)*a*Pi^2*x*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - (192*I)*a*Pi*x*ArcTan[
a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + (192*I)*a*x*ArcTan[a*x]^2*PolyLo
g[2, (-I)*E^(I*ArcTan[a*x])] - (384*I)*a*x*ArcTan[a*x]*PolyLog[2, E^(I*Arc
Tan[a*x])] + 384*a*x*ArcTan[a*x]*PolyLog[3, (-I)/E^(I*ArcTan[a*x])] - 192*
a*Pi*x*PolyLog[3, I/E^(I*ArcTan[a*x])] - 384*a*x*PolyLog[3, -E^(I*ArcTan[a
*x])] + 192*a*Pi*x*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] - 384*a*x*ArcTan[a*x
]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] + 384*a*x*PolyLog[3, E^(I*ArcTan[a...
```

## Rubi [A] (verified)

Time = 2.76 (sec) , antiderivative size = 360, normalized size of antiderivative = 0.58, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.708$, Rules used = {5485, 5425, 5423, 3042, 4669, 3011, 5479, 5493, 5491, 3042, 4671, 3011, 2720, 7143, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{x^2}dx$$

$$\Big\downarrow \text{ 5485}$$

$$a^2c\int \frac{\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^3}{x^2\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{ 5425}$$

$$\frac{a^2c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^3}{\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)^3}{x^2\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{ 5423}$$

$$\frac{ac\sqrt{a^2x^2+1}\int \sqrt{a^2x^2+1}\arctan(ax)^3d\arctan(ax)}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)^3}{x^2\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$c\int \frac{\arctan(ax)^3}{x^2\sqrt{a^2cx^2+c}}dx+\frac{ac\sqrt{a^2x^2+1}\int \arctan(ax)^3\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 4669}$$

$$c\int \frac{\arctan(ax)^3}{x^2\sqrt{a^2cx^2+c}}dx+$$
$$\frac{ac\sqrt{a^2x^2+1}\left(-3\int \arctan(ax)^2\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+3\int \arctan(ax)^2\log\left(1+ie^{i\arctan(ax)}\right)d\,\text{arct}\right.}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 3011}$$

$$c\int \frac{\arctan(ax)^3}{x^2\sqrt{a^2cx^2+c}}dx+$$
$$ac\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int \arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(a\right.\right.$$

$$\Big\downarrow \text{ 5479}$$

$$c\left(3a\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}\right)+$$
$$ac\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int \arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(a\right.\right.$$

$$\Big\downarrow \text{ 5493}$$

$$c\left(\frac{3a\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}\right)+$$
$$ac\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int \arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(a\right.\right.$$

$$\Big\downarrow \text{ 5491}$$

$$c\left(\frac{3a\sqrt{a^2x^2+1}\int \frac{\sqrt{a^2x^2+1}\arctan(ax)^2}{ax}d\arctan(ax)}{\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}\right)+$$
$$ac\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int \arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(a\right.\right.$$

$$\Big\downarrow \; 3042$$

$$c\left(\frac{3a\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc(\arctan(ax))d\arctan(ax)}{\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}\right)+$$
$$\frac{ac\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(a\right.\right.}{}$$

$$\Big\downarrow \; 4671$$

$$c\left(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(-2\int\arctan(ax)\log\left(1-e^{i\arctan(ax)}\right)d\arctan(ax)+2\int\arctan(ax\right.}{\sqrt{a^2cx^2+}}\right.$$
$$\frac{ac\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(a\right.\right.}{}$$

$$\Big\downarrow \; 3011$$

$$c\left(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-i\int\operatorname{PolyLog}\left(2,-e^{i\,\mathrm{arcta}}\right.\right.\right.}{}\right.$$
$$\frac{ac\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(a\right.\right.}{}$$

$$\Big\downarrow \; 2720$$

$$c\left(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\right.\right.}{}\right.$$
$$\frac{ac\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(a\right.\right.}{}$$

$$\Big\downarrow \; 7143$$

$$\frac{ac\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(a\right.\right.}{}$$
$$c\left(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(-2\arctan(ax)^2\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)+2\left(i\arctan(ax)\operatorname{PolyLog}(2,\right.\right.}{}\right.$$

$$\Big\downarrow \; 7163$$

$$\frac{ac\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\left(i\int\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)d\arctan(ax)-i\,\mathrm{arc}\right.\right.\right.}{}$$
$$c\left(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(-2\arctan(ax)^2\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)+2\left(i\arctan(ax)\operatorname{PolyLog}(2,\right.\right.}{}\right.$$

$\downarrow$ 2720

$$c\left(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(-2\arctan(ax)^2\text{arctanh}\left(e^{i\arctan(ax)}\right)+2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,\right.\right.\right.}{}\right.$$

$$\frac{ac\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\left(\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)de^{i\arctan}\right.\right.\right.}{}$$

$\downarrow$ 7143

$$c\left(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(-2\arctan(ax)^2\text{arctanh}\left(e^{i\arctan(ax)}\right)+2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,\right.\right.\right.}{}\right.$$

$$\frac{ac\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\left(\operatorname{PolyLog}\left(4,-ie^{i\arctan(ax)}\right)-i\arctan(ax)\operatorname{PolyLog}\right.\right.\right.}{}$$

input
```
Int[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/x^2,x]
```

output
```
c*(-((Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/(c*x)) + (3*a*Sqrt[1 + a^2*x^2]*(
-2*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])] + 2*(I*ArcTan[a*x]*PolyLog[2,
-E^(I*ArcTan[a*x])] - PolyLog[3, -E^(I*ArcTan[a*x])]) - 2*(I*ArcTan[a*x]*P
olyLog[2, E^(I*ArcTan[a*x])] - PolyLog[3, E^(I*ArcTan[a*x])])))/Sqrt[c + a
^2*c*x^2]) + (a*c*Sqrt[1 + a^2*x^2]*((-2*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcT
an[a*x]^3 + 3*(I*ArcTan[a*x]^2*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - (2*I)*
((-I)*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] + PolyLog[4, (-I)*E^(
I*ArcTan[a*x])])) - 3*(I*ArcTan[a*x]^2*PolyLog[2, I*E^(I*ArcTan[a*x])] - (
2*I)*((-I)*ArcTan[a*x]*PolyLog[3, I*E^(I*ArcTan[a*x])] + PolyLog[4, I*E^(I
*ArcTan[a*x])]))))/Sqrt[c + a^2*c*x^2]
```

## Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5491
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Simp[1/Sqrt[d]   Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTa
n[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] &&
GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

## Maple [A] (verified)

Time = 6.16 (sec) , antiderivative size = 466, normalized size of antiderivative = 0.75

| method | result |
|---|---|
| default | $-\frac{\sqrt{c(ax-i)(ax+i)}\,\arctan(ax)^3}{x}-\frac{ia\sqrt{c(ax-i)(ax+i)}\left(i\arctan(ax)^3\ln\left(1-\frac{i(iax+1)}{\sqrt{a^2x^2+1}}\right)-i\arctan(ax)^3\ln\left(1+\frac{i(iax+1)}{\sqrt{a^2x^2+1}}\right)-3i\,a$ |

input
```
int((a^2*c*x^2+c)^(1/2)*arctan(a*x)^3/x^2,x,method=_RETURNVERBOSE)
```

output
```
-(c*(a*x-I)*(a*x+I))^(1/2)*arctan(a*x)^3/x-I*a*(c*(a*x-I)*(a*x+I))^(1/2)*(
I*arctan(a*x)^3*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*arctan(a*x)^3*ln(1+I
*(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*I*arctan(a*x)^2*ln(1+(1+I*a*x)/(a^2*x^2+1)
^(1/2))+3*I*arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))+3*arctan(a*x)^
2*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*arctan(a*x)^2*polylog(2,-I*(1
+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*arctan(a*x)*polylog(3,I*(1+I*a*x)/(a^2*x^2+
1)^(1/2))-6*I*arctan(a*x)*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*arct
an(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*arctan(a*x)*polylog(2,(1
+I*a*x)/(a^2*x^2+1)^(1/2))-6*I*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I
*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*polylog(4,I*(1+I*a*x)/(a^2*x^2+1
)^(1/2))+6*polylog(4,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)
```

## Fricas [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x^2}\,dx=\int \frac{\sqrt{a^2cx^2+c}\arctan\left(ax\right)^3}{x^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)*arctan(a*x)^3/x^2,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3/x^2, x)
```

## Sympy [F]

$$\int \frac{\sqrt{c + a^2cx^2}\arctan(ax)^3}{x^2}\,dx = \int \frac{\sqrt{c\,(a^2x^2 + 1)}\,\mathrm{atan}^3\,(ax)}{x^2}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(1/2)*atan(a*x)**3/x**2,x)
```

output
```
Integral(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**3/x**2, x)
```

## Maxima [F]

$$\int \frac{\sqrt{c + a^2cx^2}\arctan(ax)^3}{x^2}\,dx = \int \frac{\sqrt{a^2cx^2 + c}\arctan\,(ax)^3}{x^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)*arctan(a*x)^3/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3/x^2, x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c + a^2cx^2}\arctan(ax)^3}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(1/2)*arctan(a*x)^3/x^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x^2}\,dx = \int \frac{\mathrm{atan}(a\,x)^3\,\sqrt{c\,a^2\,x^2+c}}{x^2}\,dx$$

input
```
int((atan(a*x)^3*(c + a^2*c*x^2)^(1/2))/x^2,x)
```

output
```
int((atan(a*x)^3*(c + a^2*c*x^2)^(1/2))/x^2, x)
```

## Reduce [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x^2}\,dx = \sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,atan(ax)^3}{x^2}dx\right)$$

input
```
int((a^2*c*x^2+c)^(1/2)*atan(a*x)^3/x^2,x)
```

output
```
sqrt(c)*int((sqrt(a**2*x**2 + 1)*atan(a*x)**3)/x**2,x)
```

# 3.418 $\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x^3}\,dx$

## Optimal result

Integrand size = 24, antiderivative size = 602

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x^3}\,dx = -\frac{3a\sqrt{c+a^2cx^2}\arctan(ax)^2}{2x} - \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{2x^2} - \frac{a^2c\sqrt{1+a^2x^2}\arctan(ax)^3\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} - \frac{6a^2c\sqrt{1+a^2x^2}\arctan(ax)\operatorname{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} + \frac{3ia^2c\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)}{2\sqrt{c+a^2cx^2}} - \frac{3ia^2c\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{PolyLog}\left(2,e^{i\arctan(ax)}\right)}{2\sqrt{c+a^2cx^2}} + \frac{3ia^2c\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} - \frac{3ia^2c\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} - \frac{3a^2c\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(3,-e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} + \frac{3a^2c\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(3,e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} - \frac{3ia^2c\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(4,-e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} + \frac{3ia^2c\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(4,e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}$$

output
```
-3/2*a*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2/x-1/2*(a^2*c*x^2+c)^(1/2)*arctan(
a*x)^3/x^2-a^2*c*(a^2*x^2+1)^(1/2)*arctan(a*x)^3*arctanh((1+I*a*x)/(a^2*x^
2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)-6*a^2*c*(a^2*x^2+1)^(1/2)*arctan(a*x)*arct
anh((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/(a^2*c*x^2+c)^(1/2)+3/2*I*a^2*c*(a^2*
x^2+1)^(1/2)*arctan(a*x)^2*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*
x^2+c)^(1/2)-3/2*I*a^2*c*(a^2*x^2+1)^(1/2)*arctan(a*x)^2*polylog(2,(1+I*a*
x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)+3*I*a^2*c*(a^2*x^2+1)^(1/2)*poly
log(2,-(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/(a^2*c*x^2+c)^(1/2)-3*I*a^2*c*(a^2
*x^2+1)^(1/2)*polylog(2,(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/(a^2*c*x^2+c)^(1/
2)-3*a^2*c*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^
(1/2))/(a^2*c*x^2+c)^(1/2)+3*a^2*c*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(3
,(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)-3*I*a^2*c*(a^2*x^2+1)^(1
/2)*polylog(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)+3*I*a^2*c*
(a^2*x^2+1)^(1/2)*polylog(4,(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/
2)
```

## Mathematica [A] (warning: unable to verify)

Time = 3.60 (sec) , antiderivative size = 345, normalized size of antiderivative = 0.57

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x^3}\,dx = \frac{a^2\sqrt{c(1+a^2x^2)}\left(-i\pi^4+2i\arctan(ax)^4-12\arctan(ax)^2\cot\left(\frac{1}{2}\arctan(ax)\right)-2\arctan(ax)^3\csc^2\left(\frac{1}{2}\operatorname{arc}\right.\right.}{}$$

input
```
Integrate[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/x^3,x]
```

output
```
(a^2*Sqrt[c*(1 + a^2*x^2)]*((-I)*Pi^4 + (2*I)*ArcTan[a*x]^4 - 12*ArcTan[a*
x]^2*Cot[ArcTan[a*x]/2] - 2*ArcTan[a*x]^3*Csc[ArcTan[a*x]/2]^2 + 8*ArcTan[
a*x]^3*Log[1 - E^((-I)*ArcTan[a*x])] + 48*ArcTan[a*x]*Log[1 - E^(I*ArcTan[
a*x])] - 48*ArcTan[a*x]*Log[1 + E^(I*ArcTan[a*x])] - 8*ArcTan[a*x]^3*Log[1
 + E^(I*ArcTan[a*x])] + (24*I)*ArcTan[a*x]^2*PolyLog[2, E^((-I)*ArcTan[a*x
])] + (24*I)*(2 + ArcTan[a*x]^2)*PolyLog[2, -E^(I*ArcTan[a*x])] - (48*I)*P
olyLog[2, E^(I*ArcTan[a*x])] + 48*ArcTan[a*x]*PolyLog[3, E^((-I)*ArcTan[a*
x])] - 48*ArcTan[a*x]*PolyLog[3, -E^(I*ArcTan[a*x])] - (48*I)*PolyLog[4, E
^((-I)*ArcTan[a*x])] - (48*I)*PolyLog[4, -E^(I*ArcTan[a*x])] + 2*ArcTan[a*
x]^3*Sec[ArcTan[a*x]/2]^2 - 12*ArcTan[a*x]^2*Tan[ArcTan[a*x]/2]))/(16*Sqrt
[1 + a^2*x^2])
```

## Rubi [A] (verified)

Time = 3.82 (sec) , antiderivative size = 560, normalized size of antiderivative = 0.93, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.708$, Rules used = {5485, 5493, 5491, 3042, 4671, 3011, 5497, 5479, 5493, 5489, 5491, 3042, 4671, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{x^3}dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^3}{x^3\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5493}$$

$$\frac{a^2c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^3}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)^3}{x^3\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5491}$$

$$\frac{a^2c\sqrt{a^2x^2+1}\int \frac{\sqrt{a^2x^2+1}\arctan(ax)^3}{ax}d\arctan(ax)}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)^3}{x^3\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2c\sqrt{a^2x^2+1}\int \arctan(ax)^3\csc(\arctan(ax))d\arctan(ax)}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)^3}{x^3\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{4671}$$

$$c\int \frac{\arctan(ax)^3}{x^3\sqrt{a^2cx^2+c}}dx+\frac{a^2c\sqrt{a^2x^2+1}\left(-3\int \arctan(ax)^2\log\left(1-e^{i\arctan(ax)}\right)d\arctan(ax)+3\int \arctan(ax)^2\log\left(1+e^{i\arctan(ax)}\right)d\arcta\right.}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3011}$$

$$c\int \frac{\arctan(ax)^3}{x^3\sqrt{a^2cx^2+c}}dx+\frac{a^2c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\int \arctan(ax)\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)d\arctan(a\right.\right.}{}$$

$$\Big\downarrow \text{5497}$$

$$c\left(\frac{3}{2}a\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx-\frac{1}{2}a^2\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{2cx^2}\right)+$$
$$\frac{a^2c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\int\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)d\arctan(a\right.\right.}{}$$

$$\Big\downarrow \text{5479}$$

$$c\left(\frac{3}{2}a\left(2a\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}\right)-\frac{1}{2}a^2\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{2cx^2}\right)$$
$$\frac{a^2c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\int\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)d\arctan(a\right.\right.}{}$$

$$\Big\downarrow \text{5493}$$

$$c\left(\frac{3}{2}a\left(\frac{2a\sqrt{a^2x^2+1}\int\frac{\arctan(ax)}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}\right)-\frac{a^2\sqrt{a^2x^2+1}\int\frac{\arctan(ax)^3}{x\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^3\sqrt{}}{2cx}\right.$$
$$\frac{a^2c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\int\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)d\arctan(a\right.\right.}{}$$

$$\Big\downarrow \text{5489}$$

$$\frac{a^2c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\int\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)d\arctan(a\right.\right.}{}$$
$$c\left(-\frac{a^2\sqrt{a^2x^2+1}\int\frac{\arctan(ax)^3}{x\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{3}{2}a\left(-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}+\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\operatorname{arctanh}\left(\frac{\sqrt{1+}}{\sqrt{1-}}\right.\right.}{}\right.\right.$$

$$\Big\downarrow \text{5491}$$

$$\frac{a^2c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\int\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)d\arctan(a\right.\right.}{}$$
$$c\left(-\frac{a^2\sqrt{a^2x^2+1}\int\frac{\sqrt{a^2x^2+1}\arctan(ax)^3}{ax}d\arctan(ax)}{2\sqrt{a^2cx^2+c}}+\frac{3}{2}a\left(-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}+\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan\right.}{}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\,\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\int\arctan(ax)\,\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)d\arctan(a\right.\right.}{}$$

$$c\left(-\frac{a^2\sqrt{a^2x^2+1}\int\arctan(ax)^3\csc(\arctan(ax))d\arctan(ax)}{2\sqrt{a^2cx^2+c}}+\frac{3}{2}a\left(-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}+\frac{2a\sqrt{a^2x^2+1}(}{}\right.\right.$$

$$\Big\downarrow \text{4671}$$

$$\frac{a^2c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\,\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\int\arctan(ax)\,\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)d\arctan(a\right.\right.}{}$$

$$c\left(-\frac{a^2\sqrt{a^2x^2+1}\left(-3\int\arctan(ax)^2\log\left(1-e^{i\arctan(ax)}\right)d\arctan(ax)+3\int\arctan(ax)^2\log\left(1+e^{i\arctan(ax)}\right)d\,a\right.}{2\sqrt{a^2cx^2+c}}\right.$$

$$\Big\downarrow \text{3011}$$

$$\frac{a^2c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\,\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\int\arctan(ax)\,\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)d\arctan(a\right.\right.}{}$$

$$c\left(-\frac{a^2\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\,\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\int\arctan(ax)\,\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)d\arcta\right.\right.}{}\right.$$

$$\Big\downarrow \text{7163}$$

$$\frac{a^2c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\,\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\left(i\int\text{PolyLog}\left(3,-e^{i\arctan(ax)}\right)d\arctan(ax)-i\,\text{arct}\right.\right.\right.}{}$$

$$c\left(-\frac{a^2\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\,\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\left(i\int\text{PolyLog}\left(3,-e^{i\arctan(ax)}\right)d\arctan(ax)-i\right.\right.\right.}{}\right.$$

$$\Big\downarrow \text{2720}$$

$$\frac{a^2c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\,\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\left(\int e^{-i\arctan(ax)}\,\text{PolyLog}\left(3,-e^{i\arctan(ax)}\right)de^{i\arctan(}\right.\right.\right.}{}$$

$$c\left(-\frac{a^2\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\,\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\left(\int e^{-i\arctan(ax)}\,\text{PolyLog}\left(3,-e^{i\arctan(ax)}\right)de^{i\,\text{ar}}\right.\right.\right.}{}\right.$$

$$\Big\downarrow \text{7143}$$

$$\frac{a^2c\sqrt{a^2x^2+1}\left(-2\arctan(ax)^3\text{arctanh}\left(e^{i\arctan(ax)}\right)+3\left(i\arctan(ax)^2\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\left(\text{PolyLog}\left(4\right.\right.\right.\right.}{}$$

$$c\left(\frac{3}{2}a\left(-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}+\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-\right.}{\sqrt{a^2cx^2+c}}\right.\right.$$

input
```
Int[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/x^3,x]
```

output
```
(a^2*c*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]^3*ArcTanh[E^(I*ArcTan[a*x])] + 3*
(I*ArcTan[a*x]^2*PolyLog[2, -E^(I*ArcTan[a*x])] - (2*I)*((-I)*ArcTan[a*x]*
PolyLog[3, -E^(I*ArcTan[a*x])] + PolyLog[4, -E^(I*ArcTan[a*x])])) - 3*(I*A
rcTan[a*x]^2*PolyLog[2, E^(I*ArcTan[a*x])] - (2*I)*((-I)*ArcTan[a*x]*PolyL
og[3, E^(I*ArcTan[a*x])] + PolyLog[4, E^(I*ArcTan[a*x])]))))/Sqrt[c + a^2*
c*x^2] + c*(-1/2*(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/(c*x^2) + (3*a*(-((Sq
rt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(c*x)) + (2*a*Sqrt[1 + a^2*x^2]*(-2*ArcTa
n[a*x]*ArcTanh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]] + I*PolyLog[2, -(Sqrt[1 +
I*a*x]/Sqrt[1 - I*a*x])] - I*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]]))
/Sqrt[c + a^2*c*x^2]))/2 - (a^2*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]^3*ArcTan
h[E^(I*ArcTan[a*x])] + 3*(I*ArcTan[a*x]^2*PolyLog[2, -E^(I*ArcTan[a*x])] -
 (2*I)*((-I)*ArcTan[a*x]*PolyLog[3, -E^(I*ArcTan[a*x])] + PolyLog[4, -E^(I
*ArcTan[a*x])])) - 3*(I*ArcTan[a*x]^2*PolyLog[2, E^(I*ArcTan[a*x])] - (2*I
)*((-I)*ArcTan[a*x]*PolyLog[3, E^(I*ArcTan[a*x])] + PolyLog[4, E^(I*ArcTan
[a*x])]))))/(2*Sqrt[c + a^2*c*x^2]))
```

## Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5489
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sq
rt[1 - I*c*x]], x] + (Simp[I*(b/Sqrt[d])*PolyLog[2, -Sqrt[1 + I*c*x]/Sqrt[1
 - I*c*x]], x] - Simp[I*(b/Sqrt[d])*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x]], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

rule 5491
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Simp[1/Sqrt[d]   Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTa
n[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] &&
GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5497
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)^(m
 + 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[c^2*((m
+ 2)/(f^2*(m + 1)))   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x
^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
 && LtQ[m, -1] && NeQ[m, -2]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

## Maple [A] (verified)

Time = 4.76 (sec) , antiderivative size = 404, normalized size of antiderivative = 0.67

| method | result |
|---|---|
| default | $-\frac{\sqrt{c(ax-i)(ax+i)}\,\arctan(ax)^2(3ax+\arctan(ax))}{2x^2}+\frac{ia^2\sqrt{c(ax-i)(ax+i)}\left(i\arctan(ax)^3\ln\left(1+\frac{iax+1}{\sqrt{a^2x^2+1}}\right)-i\arctan(ax)^3\ln\left(\right.\right.}{}$ |

input
```
int((a^2*c*x^2+c)^(1/2)*arctan(a*x)^3/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(c*(a*x-I)*(a*x+I))^(1/2)*arctan(a*x)^2*(3*a*x+arctan(a*x))/x^2+1/2*I
*a^2*(c*(a*x-I)*(a*x+I))^(1/2)*(I*arctan(a*x)^3*ln(1+(1+I*a*x)/(a^2*x^2+1)
^(1/2))-I*arctan(a*x)^3*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*arctan(a*x)*
ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+3*arctan(a*x)^2*polylog(2,-(1+I*a*x)/(a^
2*x^2+1)^(1/2))+6*I*arctan(a*x)*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*
I*arctan(a*x)*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*arctan(a*x)^2*polylog(2,
(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I*arctan(a*x)*polylog(3,(1+I*a*x)/(a^2*x^2+
1)^(1/2))+6*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*polylog(4,-(1+I*a*x)
/(a^2*x^2+1)^(1/2))-6*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*polylog(4,(
1+I*a*x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)
```

## Fricas [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x^3}\,dx=\int \frac{\sqrt{a^2cx^2+c}\arctan\left(ax\right)^3}{x^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)*arctan(a*x)^3/x^3,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3/x^3, x)
```

## Sympy [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x^3}\,dx = \int \frac{\sqrt{c\,(a^2x^2+1)}\,\mathrm{atan}^3\,(ax)}{x^3}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(1/2)*atan(a*x)**3/x**3,x)
```

output
```
Integral(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**3/x**3, x)
```

## Maxima [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x^3}\,dx = \int \frac{\sqrt{a^2cx^2+c}\arctan\,(ax)^3}{x^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)*arctan(a*x)^3/x^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3/x^3, x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(1/2)*arctan(a*x)^3/x^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x^3}\,dx = \int \frac{\mathrm{atan}(a\,x)^3\,\sqrt{c\,a^2\,x^2+c}}{x^3}\,dx$$

input
```
int((atan(a*x)^3*(c + a^2*c*x^2)^(1/2))/x^3,x)
```

output
```
int((atan(a*x)^3*(c + a^2*c*x^2)^(1/2))/x^3, x)
```

## Reduce [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x^3}\,dx = \sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,atan(ax)^3}{x^3}dx\right)$$

input
```
int((a^2*c*x^2+c)^(1/2)*atan(a*x)^3/x^3,x)
```

output
```
sqrt(c)*int((sqrt(a**2*x**2 + 1)*atan(a*x)**3)/x**3,x)
```

# 3.419 $\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x^4}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 361

$$\begin{aligned}\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x^4}\,dx = & -\frac{a^2\sqrt{c+a^2cx^2}\arctan(ax)}{x}-\frac{a\sqrt{c+a^2cx^2}\arctan(ax)^2}{2x^2}\\ & -\frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{3cx^3}\\ & -\frac{a^3c\sqrt{1+a^2x^2}\arctan(ax)^2\text{arctanh}\left(e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}\\ & -a^3\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c+a^2cx^2}}{\sqrt{c}}\right)\\ & +\frac{ia^3c\sqrt{1+a^2x^2}\arctan(ax)\,\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}\\ & -\frac{ia^3c\sqrt{1+a^2x^2}\arctan(ax)\,\text{PolyLog}\left(2,e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}\\ & -\frac{a^3c\sqrt{1+a^2x^2}\,\text{PolyLog}\left(3,-e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}\\ & +\frac{a^3c\sqrt{1+a^2x^2}\,\text{PolyLog}\left(3,e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
-a^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)/x-1/2*a*(a^2*c*x^2+c)^(1/2)*arctan(a*
x)^2/x^2-1/3*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3/c/x^3-a^3*c*(a^2*x^2+1)^(1/
2)*arctan(a*x)^2*arctanh((1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)-
a^3*c^(1/2)*arctanh((a^2*c*x^2+c)^(1/2)/c^(1/2))+I*a^3*c*(a^2*x^2+1)^(1/2)
*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)-I
*a^3*c*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2)
)/(a^2*c*x^2+c)^(1/2)-a^3*c*(a^2*x^2+1)^(1/2)*polylog(3,-(1+I*a*x)/(a^2*x^
2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)+a^3*c*(a^2*x^2+1)^(1/2)*polylog(3,(1+I*a*x
)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (warning: unable to verify)

Time = 2.52 (sec) , antiderivative size = 341, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x^4}\,dx = \frac{a^3c\sqrt{1+a^2x^2}\Big(-12\arctan(ax)\cot\left(\frac{1}{2}\arctan(ax)\right)-2\arctan(ax)^3\cot\left(\frac{1}{2}\arctan(ax)\right)-3\arctan(ax)^2\,\mathrm{cs}}{}$$

input
```
Integrate[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/x^4,x]
```

output
```
(a^3*c*Sqrt[1 + a^2*x^2]*(-12*ArcTan[a*x]*Cot[ArcTan[a*x]/2] - 2*ArcTan[a*
x]^3*Cot[ArcTan[a*x]/2] - 3*ArcTan[a*x]^2*Csc[ArcTan[a*x]/2]^2 - (a*x*ArcT
an[a*x]^3*Csc[ArcTan[a*x]/2]^4)/(2*Sqrt[1 + a^2*x^2]) + 12*ArcTan[a*x]^2*L
og[1 - E^(I*ArcTan[a*x])] - 12*ArcTan[a*x]^2*Log[1 + E^(I*ArcTan[a*x])] +
24*Log[Tan[ArcTan[a*x]/2]] + (24*I)*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*
x])] - (24*I)*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])] - 24*PolyLog[3, -E
^(I*ArcTan[a*x])] + 24*PolyLog[3, E^(I*ArcTan[a*x])] + 3*ArcTan[a*x]^2*Sec
[ArcTan[a*x]/2]^2 - (8*(1 + a^2*x^2)^(3/2)*ArcTan[a*x]^3*Sin[ArcTan[a*x]/2
]^4)/(a^3*x^3) - 12*ArcTan[a*x]*Tan[ArcTan[a*x]/2] - 2*ArcTan[a*x]^3*Tan[A
rcTan[a*x]/2]))/(24*Sqrt[c*(1 + a^2*x^2)])
```

## Rubi [A] (verified)

Time = 3.48 (sec) , antiderivative size = 387, normalized size of antiderivative = 1.07, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {5479, 5485, 5493, 5491, 3042, 4671, 3011, 2720, 5497, 5479, 243, 73, 221, 5493, 5491, 3042, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{x^4}dx$$

$$\Big\downarrow \text{5479}$$

$$a\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^3}dx-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}$$

$$\Big\downarrow \text{5485}$$

$$a\left(a^2c\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx\right)-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}$$

$$\Big\downarrow \text{5493}$$

$$a\left(\frac{a^2c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx\right)-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}$$

$$\Big\downarrow \text{5491}$$

$$a\left(\frac{a^2c\sqrt{a^2x^2+1}\int \frac{\sqrt{a^2x^2+1}\arctan(ax)^2}{ax}d\arctan(ax)}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx\right)-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}$$

$$\Big\downarrow \text{3042}$$

$$a\left(\frac{a^2c\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc(\arctan(ax))d\arctan(ax)}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx\right)-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}$$

$$\Big\downarrow \text{4671}$$

$$-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}+$$
$$a\left(c\int\frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx+\frac{a^2c\sqrt{a^2x^2+1}\left(-2\int\arctan(ax)\log\left(1-e^{i\arctan(ax)}\right)d\arctan(ax)+2\int\arctan(ax)\log\right.}{\sqrt{a^2cx^2+c}}\right.$$

$\big\downarrow$ 3011

$$-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}+$$
$$a\left(c\int\frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx+\frac{a^2c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-i\int\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)\right.\right.}{}\right.$$

$\big\downarrow$ 2720

$$-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}+$$
$$a\left(c\int\frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx+\frac{a^2c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,\right.\right.\right.}{}\right.$$

$\big\downarrow$ 5497

$$-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}+$$
$$a\left(c\left(-\frac{1}{2}a^2\int\frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+a\int\frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{2cx^2}\right)+\frac{a^2c\sqrt{a^2x^2+1}\left(2\left(i\arctan\right.\right.}{}\right.$$

$\big\downarrow$ 5479

$$-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}+$$
$$a\left(c\left(-\frac{1}{2}a^2\int\frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+a\left(a\int\frac{1}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right)-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{2cx^2}\right)\right.$$

$\big\downarrow$ 243

$$-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}+$$
$$a\left(c\left(-\frac{1}{2}a^2\int\frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+a\left(\frac{1}{2}a\int\frac{1}{x^2\sqrt{a^2cx^2+c}}dx^2-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right)-\frac{\arctan(ax)^2\sqrt{a^2cx^2}}{2cx^2}\right.\right.$$

$\big\downarrow$ 73

$$-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}+$$

$$a\left(c\left(-\frac{1}{2}a^2\int\frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+a\left(\frac{\int\frac{1}{\frac{x^4}{a^2c}-\frac{1}{a^2}}d\sqrt{a^2cx^2+c}}{ac}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right)-\frac{\arctan(ax)^2\sqrt{a^2cx^2+}}{2cx^2}\right.\right.$$

$\downarrow$ 221

$$-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}+$$

$$a\left(c\left(-\frac{1}{2}a^2\int\frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+a\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{\arctan(ax)^2\sqrt{a^2cx^2}}{2cx^2}\right.\right.$$

$\downarrow$ 5493

$$-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}+$$

$$a\left(c\left(-\frac{a^2\sqrt{a^2x^2+1}\int\frac{\arctan(ax)^2}{x\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+a\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{\arctan(ax)^2\sqrt{a}}{2cx^2}\right.\right.$$

$\downarrow$ 5491

$$-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}+$$

$$a\left(c\left(-\frac{a^2\sqrt{a^2x^2+1}\int\frac{\sqrt{a^2x^2+1}\arctan(ax)^2}{ax}d\arctan(ax)}{2\sqrt{a^2cx^2+c}}+a\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\text{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)\right.\right.$$

$\downarrow$ 3042

$$-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}+$$

$$a\left(c\left(-\frac{a^2\sqrt{a^2x^2+1}\int\arctan(ax)^2\csc(\arctan(ax))d\arctan(ax)}{2\sqrt{a^2cx^2+c}}+a\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\text{arctanh}\left(\frac{\sqrt{a}}{}\right.}{\sqrt{c}}\right.\right.\right.$$

$\downarrow$ 4671

$$-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}+$$

$$a\left(\frac{a^2c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\,\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\,\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(}\right.\right.}{}\right.$$

$\downarrow$ 3011

$$-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}+$$
$$a\left(\frac{a^2c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(a\right.\right.}{}\right.$$

$$\Big\downarrow \text{2720}$$

$$-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}+$$
$$a\left(\frac{a^2c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(a\right.\right.}{}\right.$$

$$\Big\downarrow \text{7143}$$

$$-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}+$$
$$a\left(\frac{a^2c\sqrt{a^2x^2+1}\left(-2\arctan(ax)^2\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)+2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\operatorname{PolyLog}(3\right.\right.}{\sqrt{a^2cx^2+c}}\right.$$

input
```
Int[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/x^4,x]
```

output
```
-1/3*((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3)/(c*x^3) + a*(c*(-1/2*(Sqrt[c +
a^2*c*x^2]*ArcTan[a*x]^2)/(c*x^2) + a*(-((Sqrt[c + a^2*c*x^2]*ArcTan[a*x])
/(c*x)) - (a*ArcTanh[Sqrt[c + a^2*c*x^2]/Sqrt[c]])/Sqrt[c]) - (a^2*Sqrt[1
+ a^2*x^2]*(-2*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])] + 2*(I*ArcTan[a*x]
*PolyLog[2, -E^(I*ArcTan[a*x])] - PolyLog[3, -E^(I*ArcTan[a*x])]) - 2*(I*A
rcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])] - PolyLog[3, E^(I*ArcTan[a*x])]))
)/(2*Sqrt[c + a^2*c*x^2])) + (a^2*c*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]^2*Ar
cTanh[E^(I*ArcTan[a*x])] + 2*(I*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])]
 - PolyLog[3, -E^(I*ArcTan[a*x])]) - 2*(I*ArcTan[a*x]*PolyLog[2, E^(I*ArcT
an[a*x])] - PolyLog[3, E^(I*ArcTan[a*x])])))/Sqrt[c + a^2*c*x^2])
```

## Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5491
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Simp[1/Sqrt[d]   Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTa
n[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] &&
GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5497
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)^(m
 + 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[c^2*((m
+ 2)/(f^2*(m + 1)))   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x
^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
 && LtQ[m, -1] && NeQ[m, -2]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

## Maple [A] (verified)

Time = 4.83 (sec) , antiderivative size = 315, normalized size of antiderivative = 0.87

| method | result |
|---|---|
| default | $-\frac{\sqrt{c(ax-i)(ax+i)}\,\arctan(ax)\left(2\arctan(ax)^2x^2a^2+6a^2x^2+3\arctan(ax)ax+2\arctan(ax)^2\right)}{6x^3}-\frac{a^3\sqrt{c(ax-i)(ax+i)}\,\left(\arctan(ax\right.}{}$ |

input
```
int((a^2*c*x^2+c)^(1/2)*arctan(a*x)^3/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(c*(a*x-I)*(a*x+I))^(1/2)*arctan(a*x)*(2*arctan(a*x)^2*x^2*a^2+6*a^2*
x^2+3*arctan(a*x)*a*x+2*arctan(a*x)^2)/x^3-1/2*a^3*(c*(a*x-I)*(a*x+I))^(1/
2)*(arctan(a*x)^2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*I*arctan(a*x)*polylo
g(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)
^(1/2))+2*I*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*polylog(3
,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2)))/(
a^2*x^2+1)^(1/2)-2*a^3*(c*(a*x-I)*(a*x+I))^(1/2)*arctanh((1+I*a*x)/(a^2*x^
2+1)^(1/2))/(a^2*x^2+1)^(1/2)
```

## Fricas [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x^4}\,dx = \int \frac{\sqrt{a^2cx^2+c}\arctan{(ax)}^3}{x^4}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)*arctan(a*x)^3/x^4,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3/x^4, x)
```

## Sympy [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x^4}\,dx = \int \frac{\sqrt{c\,(a^2x^2+1)}\,\mathrm{atan}^3{(ax)}}{x^4}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(1/2)*atan(a*x)**3/x**4,x)
```

output
```
Integral(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**3/x**4, x)
```

## Maxima [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x^4}\,dx = \int \frac{\sqrt{a^2cx^2+c}\arctan{(ax)}^3}{x^4}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)*arctan(a*x)^3/x^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3/x^4, x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(1/2)*arctan(a*x)^3/x^4,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x^4}\,dx = \int \frac{\mathrm{atan}(a\,x)^3\,\sqrt{c\,a^2\,x^2+c}}{x^4}\,dx$$

input
```
int((atan(a*x)^3*(c + a^2*c*x^2)^(1/2))/x^4,x)
```

output
```
int((atan(a*x)^3*(c + a^2*c*x^2)^(1/2))/x^4, x)
```

## Reduce [F]

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{x^4}\,dx = \sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,atan(ax)^3}{x^4}dx\right)$$

input
```
int((a^2*c*x^2+c)^(1/2)*atan(a*x)^3/x^4,x)
```

output
```
sqrt(c)*int((sqrt(a**2*x**2 + 1)*atan(a*x)**3)/x**4,x)
```

# 3.420 $\int x^3(c+a^2cx^2)^{3/2}\arctan(ax)^3\,dx$

## Optimal result

Integrand size = 24, antiderivative size = 652

$$\int x^3\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3\,dx=\frac{cx\sqrt{c+a^2cx^2}}{420a^3}-\frac{cx^3\sqrt{c+a^2cx^2}}{140a}-\frac{163c\sqrt{c+a^2cx^2}\arctan(ax)}{840a^4}+\frac{cx^2\sqrt{c+a^2cx^2}\arctan(ax)}{60a^2}+\frac{1}{35}cx^4\sqrt{c+a^2cx^2}\arctan(ax)+\frac{9cx\sqrt{c+a^2cx^2}\arctan(ax)^2}{112a^3}-\frac{23cx^3\sqrt{c+a^2cx^2}\arctan(ax)^2}{280a}-\frac{1}{14}acx^5\sqrt{c+a^2cx^2}\arctan(ax)^2-\frac{51ic^2\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2}{280a^4\sqrt{c+a^2cx^2}}-\frac{2c\sqrt{c+a^2cx^2}\arctan(ax)^3}{35a^4}+\frac{cx^2\sqrt{c+a^2cx^2}\arctan(ax)^3}{35a^2}+\frac{8}{35}cx^4\sqrt{c+a^2cx^2}\arctan(ax)^3+\frac{1}{7}a^2cx^6\sqrt{c+a^2cx^2}\arctan(ax)^3+\frac{23c^{3/2}\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right)}{120a^4}+\frac{51ic^2\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)}{280a^4\sqrt{c+a^2cx^2}}-\frac{51ic^2\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,ie^{i\arctan(ax)}\right)}{280a^4\sqrt{c+a^2cx^2}}-\frac{51c^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)}{280a^4\sqrt{c+a^2cx^2}}+\frac{51c^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,ie^{i\arctan(ax)}\right)}{280a^4\sqrt{c+a^2cx^2}}$$

output
```
1/420*c*x*(a^2*c*x^2+c)^(1/2)/a^3-1/140*c*x^3*(a^2*c*x^2+c)^(1/2)/a-163/84
0*c*(a^2*c*x^2+c)^(1/2)*arctan(a*x)/a^4+1/60*c*x^2*(a^2*c*x^2+c)^(1/2)*arc
tan(a*x)/a^2+1/35*c*x^4*(a^2*c*x^2+c)^(1/2)*arctan(a*x)+9/112*c*x*(a^2*c*x
^2+c)^(1/2)*arctan(a*x)^2/a^3-23/280*c*x^3*(a^2*c*x^2+c)^(1/2)*arctan(a*x)
^2/a-1/14*a*c*x^5*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2-51/280*I*c^2*(a^2*x^2+
1)^(1/2)*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a^4/(a^2*c*x
^2+c)^(1/2)-2/35*c*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^3/a^4+1/35*c*x^2*(a^2*c
*x^2+c)^(1/2)*arctan(a*x)^3/a^2+8/35*c*x^4*(a^2*c*x^2+c)^(1/2)*arctan(a*x)
^3+1/7*a^2*c*x^6*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^3+23/120*c^(3/2)*arctanh(
a*c^(1/2)*x/(a^2*c*x^2+c)^(1/2))/a^4+51/280*I*c^2*(a^2*x^2+1)^(1/2)*arctan
(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a^4/(a^2*c*x^2+c)^(1/2)-51
/280*I*c^2*(a^2*x^2+1)^(1/2)*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*
x)^2/a^4/(a^2*c*x^2+c)^(1/2)-51/280*c^2*(a^2*x^2+1)^(1/2)*polylog(3,-I*(1+
I*a*x)/(a^2*x^2+1)^(1/2))/a^4/(a^2*c*x^2+c)^(1/2)+51/280*c^2*(a^2*x^2+1)^(
1/2)*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a^4/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (warning: unable to verify)

Time = 2.29 (sec) , antiderivative size = 538, normalized size of antiderivative = 0.83

$$\int x^3(c+a^2cx^2)^{3/2}\arctan(ax)^3\,dx = \frac{c\sqrt{c+a^2cx^2}\Big(2688\Big(10\coth^{-1}\Big(\frac{ax}{\sqrt{1+a^2x^2}}\Big)-11i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)}{}$$

input
```
Integrate[x^3*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3,x]
```

output
```
(c*Sqrt[c + a^2*c*x^2]*(2688*(10*ArcCoth[(a*x)/Sqrt[1 + a^2*x^2]] - (11*I)
*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 + (11*I)*ArcTan[a*x]*PolyLog[2, (
-I)*E^(I*ArcTan[a*x])] - (11*I)*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])
] - 11*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] + 11*PolyLog[3, I*E^(I*ArcTan[a*
x])]) - 64*(259*ArcCoth[(a*x)/Sqrt[1 + a^2*x^2]] - (309*I)*ArcTan[E^(I*Arc
Tan[a*x])]*ArcTan[a*x]^2 + (309*I)*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan
[a*x])] - (309*I)*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] - 309*PolyLo
g[3, (-I)*E^(I*ArcTan[a*x])] + 309*PolyLog[3, I*E^(I*ArcTan[a*x])]) - 56*(
1 + a^2*x^2)^(5/2)*((48*a*x)/(1 + a^2*x^2)^2 + 32*ArcTan[a*x]^3*(-1 + 5*Co
s[2*ArcTan[a*x]]) + 6*ArcTan[a*x]*(25 + 36*Cos[2*ArcTan[a*x]] + 11*Cos[4*A
rcTan[a*x]]) + ArcTan[a*x]^2*(6*Sin[2*ArcTan[a*x]] - 33*Sin[4*ArcTan[a*x]]
)) + (1 + a^2*x^2)^(7/2)*(64*ArcTan[a*x]^3*(57 - 28*Cos[2*ArcTan[a*x]] + 3
5*Cos[4*ArcTan[a*x]]) + (8*ArcTan[a*x]*(647 + 764*Cos[2*ArcTan[a*x]] + 309
*Cos[4*ArcTan[a*x]]))/(1 + a^2*x^2) + 4*(101*Sin[2*ArcTan[a*x]] + 88*Sin[4
*ArcTan[a*x]] + 25*Sin[6*ArcTan[a*x]]) - 3*ArcTan[a*x]^2*(211*Sin[2*ArcTan
[a*x]] - 60*Sin[4*ArcTan[a*x]] + 103*Sin[6*ArcTan[a*x]]))))/(53760*a^4*Sqr
t[1 + a^2*x^2])
```

## Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \arctan(ax)^3 \left(a^2cx^2+c\right)^{3/2} dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\int x^5\sqrt{a^2cx^2+c}\arctan(ax)^3dx+c\int x^3\sqrt{a^2cx^2+c}\arctan(ax)^3dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\left(a^2c\int \frac{x^7\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx+c\int \frac{x^5\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx\right)+ c\left(a^2c\int \frac{x^5\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx+c\int \frac{x^3\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx\right)$$

$$\Big\downarrow \text{5487}$$

$$c\left(c\left(-\frac{\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{2\int \frac{x\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)^3\sqrt{a^2cx^2+c}}{3a^2c}\right)+a^2c\left(-\frac{3\int \frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int\right.\right.$$
$$a^2c\left(a^2c\left(-\frac{3\int \frac{x^6\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{7a}-\frac{6\int \frac{x^5\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{7a^2}+\frac{x^6\arctan(ax)^3\sqrt{a^2cx^2+c}}{7a^2c}\right)+c\left(-\frac{3\int \frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a}\right.\right.$$

$\downarrow$ 5465

$$c\left(c\left(-\frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a}\right)}{3a^2}-\frac{\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a}+\frac{x^2\arctan(ax)^3\sqrt{a^2cx^2+c}}{3a^2c}\right)+a^2c\left(-\frac{3}{}\right.\right.$$
$$a^2c\left(a^2c\left(-\frac{3\int \frac{x^6\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{7a}-\frac{6\int \frac{x^5\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{7a^2}+\frac{x^6\arctan(ax)^3\sqrt{a^2cx^2+c}}{7a^2c}\right)+c\left(-\frac{3\int \frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a}\right.\right.$$

$\downarrow$ 5425

$$c\left(c\left(-\frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{a\sqrt{a^2cx^2+c}}\right)}{3a^2}-\frac{\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a}+\frac{x^2\arctan(ax)^3\sqrt{a^2cx^2+c}}{3a^2c}\right)+a\right.$$
$$a^2c\left(a^2c\left(-\frac{3\int \frac{x^6\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{7a}-\frac{6\int \frac{x^5\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{7a^2}+\frac{x^6\arctan(ax)^3\sqrt{a^2cx^2+c}}{7a^2c}\right)+c\left(-\frac{3\int \frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a}\right.\right.$$

$\downarrow$ 5423

$$c\left(c\left(-\frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\int \sqrt{a^2x^2+1}\arctan(ax)^2d\arctan(ax)}{a^2\sqrt{a^2cx^2+c}}\right)}{3a^2}-\frac{\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a}+\frac{x^2\arctan(ax)^3\sqrt{a}}{3a^2c}\right.\right.$$
$$a^2c\left(a^2c\left(-\frac{3\int \frac{x^6\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{7a}-\frac{6\int \frac{x^5\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{7a^2}+\frac{x^6\arctan(ax)^3\sqrt{a^2cx^2+c}}{7a^2c}\right)+c\left(-\frac{3\int \frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a}\right.\right.$$

$\downarrow$ 3042

$$c\left(c\left(-\frac{\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{a^2\sqrt{a^2cx^2+c}}\right)}{3a^2}+\frac{x^2\arctan\right.\right.$$

$$a^2c\left(a^2c\left(-\frac{3\int \frac{x^6\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{7a}-\frac{6\int \frac{x^5\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{7a^2}+\frac{x^6\arctan(ax)^3\sqrt{a^2cx^2+c}}{7a^2c}\right)+c\left(-\frac{3\int \frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a}\right.\right.$$

$\downarrow$ 4669

$$a^2c\left(a^2c\left(-\frac{3\int \frac{x^6\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{7a}-\frac{6\int \frac{x^5\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{7a^2}+\frac{x^6\arctan(ax)^3\sqrt{a^2cx^2+c}}{7a^2c}\right)+c\left(-\frac{3\int \frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a}\right.\right.$$

$$c\left(a^2c\left(-\frac{3\int \frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int \frac{x^3\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{5a^2}+\frac{x^4\arctan(ax)^3\sqrt{a^2cx^2+c}}{5a^2c}\right)+c\left(-\frac{\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{2(}{}\right.\right.$$

$\downarrow$ 3011

$$a^2c\left(a^2c\left(-\frac{3\int \frac{x^6\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{7a}-\frac{6\int \frac{x^5\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{7a^2}+\frac{x^6\arctan(ax)^3\sqrt{a^2cx^2+c}}{7a^2c}\right)+c\left(-\frac{3\int \frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a}\right.\right.$$

$$c\left(a^2c\left(-\frac{3\int \frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int \frac{x^3\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{5a^2}+\frac{x^4\arctan(ax)^3\sqrt{a^2cx^2+c}}{5a^2c}\right)+c\left(-\frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}\right.}{}\right.\right.$$

$\downarrow$ 2720

$$a^2c\left(a^2c\left(-\frac{3\int \frac{x^6\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{7a}-\frac{6\int \frac{x^5\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{7a^2}+\frac{x^6\arctan(ax)^3\sqrt{a^2cx^2+c}}{7a^2c}\right)+c\left(-\frac{3\int \frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a}\right.\right.$$

$$c\left(a^2c\left(-\frac{3\int \frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int \frac{x^3\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{5a^2}+\frac{x^4\arctan(ax)^3\sqrt{a^2cx^2+c}}{5a^2c}\right)+c\left(-\frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}\right.}{}\right.\right.$$

$\downarrow$ 5487

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^6}{7a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{5\int\frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)}{7a}-\frac{6\left(\frac{\sqrt{a^2cx^2+c}}{}\right.}{}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^4}{5a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{3\int\frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-\frac{4\left(\frac{x^2\sqrt{a^2cx^2}}{}\right.}{}\right.\right.$$

$$\Big\downarrow \text{ 5425}$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^6}{7a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{5\int\frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)}{7a}-\frac{6\left(\frac{\sqrt{a^2cx^2+c}}{}\right.}{}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^4}{5a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{3\int\frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-\frac{4\left(\frac{x^2\sqrt{a^2cx^2}}{}\right.}{}\right.\right.$$

$$\Big\downarrow \text{ 5423}$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^6}{7a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{5\int\frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)}{7a}-\frac{6\left(\frac{\sqrt{a^2cx^2+c}}{}\right.}{}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^4}{5a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{3\int\frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-\frac{4\left(\frac{x^2\sqrt{a^2cx^2}}{}\right.}{}\right.\right.$$

$$\Big\downarrow \text{ 3042}$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^6}{7a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{5\int\frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)}{7a}-\frac{6\left(\frac{\sqrt{a^2cx^2+c}\cdots}{\cdots}\right.}{\cdots}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^4}{5a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{3\int\frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-\frac{4\left(\frac{x^2\sqrt{a^2cx^2}\cdots}{\cdots}\right.}{\cdots}\right.\right.$$

$\downarrow$ 4669

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^6}{7a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{5\int\frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)}{7a}-\frac{6\left(\frac{\sqrt{a^2cx^2+c}\cdots}{\cdots}\right.}{\cdots}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^4}{5a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{3\int\frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-\frac{4\left(\frac{x^2\sqrt{a^2cx^2}\cdots}{\cdots}\right.}{\cdots}\right.\right.$$

$\downarrow$ 3011

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^6}{7a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{5\int\frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)}{7a}-\frac{6\left(\frac{\sqrt{a^2cx^2+c}\cdots}{\cdots}\right.}{\cdots}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^4}{5a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{3\int\frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-\frac{4\left(\frac{x^2\sqrt{a^2cx^2}\cdots}{\cdots}\right.}{\cdots}\right.\right.$$

$\downarrow$ 2720

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^6}{7a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{5\int\frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)}{7a}-\frac{6\left(\frac{\sqrt{a^2cx^2+c}\right.}{}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^4}{5a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{3\int\frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-\frac{4\left(\frac{x^2\sqrt{a^2cx^2}\right.}{}\right.\right.$$

$\downarrow$ 5465

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^6}{7a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{5\int\frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)}{7a}-\frac{6\left(\frac{\sqrt{a^2cx^2+c}\right.}{}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^4}{5a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{3\int\frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-\frac{4\left(\frac{x^2\sqrt{a^2cx^2}\right.}{}\right.\right.$$

$\downarrow$ 224

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^6}{7a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{5\int\frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)}{7a}-\frac{6\left(\sqrt{a^2cx^2+c}\cdots\right.}{\cdots}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^4}{5a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{3\int\frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-\frac{4\left(\frac{x^2\sqrt{a^2cx^2}\cdots}{\cdots}\right.}{\cdots}\right.\right.$$

$$\Big\downarrow \text{219}$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^6}{7a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{5\int\frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)}{7a}-\frac{6\left(\sqrt{a^2cx^2+c}\cdots\right.}{\cdots}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^4}{5a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{3\int\frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-\frac{4\left(\frac{x^2\sqrt{a^2cx^2}\cdots}{\cdots}\right.}{\cdots}\right.\right.$$

$$\downarrow \text{5425}$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^6}{7a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{5\int\frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)}{7a}-\frac{6\left(\sqrt{a^2cx^2+c}\right.}{}\right.\right.$$

$$\left(\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^4}{5a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{3\int\frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-\frac{4\left(\frac{x^2\sqrt{a^2cx^2}}{}\right.}{}\right.\right.$$

$$\downarrow \text{5423}$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^6}{7a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{5\int\frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)}{7a}-\frac{6\left(\sqrt{a^2cx^2+c}\right.}{}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^4}{5a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{3\int\frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-\frac{4\left(\frac{x^2\sqrt{a^2cx^2}}{}\right.}{}\right.\right.$$

$$\downarrow \text{3042}$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^6}{7a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{5\int\frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)}{7a}-\frac{6\left(\frac{\sqrt{a^2cx^2+c}\right.}{}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^4}{5a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{3\int\frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-\frac{4\left(\frac{x^2\sqrt{a^2cx^2\right.}{}\right.\right.$$

$\downarrow$ 4669

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^6}{7a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{5\int\frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)}{7a}-\frac{6\left(\frac{\sqrt{a^2cx^2+c}\right.}{}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^4}{5a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{3\int\frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-\frac{4\left(\frac{x^2\sqrt{a^2cx^2\right.}{}\right.\right.$$

$\downarrow$ 3011

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^6}{7a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{5\int\frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)}{7a}-\frac{6\left(\sqrt{a^2cx^2+c}\right.}{}\right.\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^4}{5a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{3\int\frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-\frac{4\left(x^2\sqrt{a^2cx^2}\right.}{}\right.\right.$$

$\downarrow$ 2720

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^6}{7a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\int\frac{x^5\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{5\int\frac{x^4\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)}{7a}-\frac{6\left(\sqrt{a^2cx^2+c}\right.}{}\right.\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^4}{5a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{3\int\frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-\frac{4\left(x^2\sqrt{a^2cx^2}\right.}{}\right.\right.$$

$\downarrow$ 5487

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^6}{7a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\int\frac{x^4}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int\frac{x^3\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a^2}}{3a}-\right.}{7a}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^4}{5a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^2}{3a^2c}-\frac{\int\frac{x^2}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{2\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a^2}}{2a}-\right.}{5a}\right.\right.$$

$$\Big\downarrow \; 262$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^6}{7a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\int\frac{x^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}}{5a}-\frac{4\int\frac{x^3}{\sqrt{}}}{}}{3a}\right.}{7a}\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^4}{5a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^2}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int\frac{1}{\sqrt{a^2cx^2+c}}dx}{2a^2}}{3a}-\frac{2\int\frac{x\,\mathrm{arc}}{\sqrt{a^2}}}{3}}{2a}\right.}{5a}\right.\right.$$

$$\Big\downarrow \; 224$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^6}{7a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\int\frac{x^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}}{5a}-\frac{4\int\frac{x^3\cdots}{\sqrt{\cdots}}\cdots}{\cdots}}{3a}\cdots\right)}{7a}\cdots\right)\cdots\right.$$

$$\left.c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^4}{5a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^2}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\int\frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}\cdots}{2a^2}}{3a}\cdots}{2a}\cdots\right)}{5a}\cdots\right)\cdots\right)$$

$$\Big\downarrow \; 219$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^6}{7a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^4}{5a^2c}-\frac{\frac{x^3\sqrt{a^2cx^2+c}}{4a^2c}-\frac{3\int\frac{x^2}{\sqrt{a^2cx^2+c}}dx}{4a^2}}{5a}-\frac{4\int\frac{x^3}{\cdots}}{\cdots}}{3a}\cdots\right.}{7a}\cdots\right.\right.$$

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^4}{5a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^3}{4a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^2}{3a^2c}-\frac{\frac{x\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{2a^3\sqrt{c}}}{3a}\cdots}{2a}\cdots\right.}{5a}\cdots\right.\right.$$

input
```
Int[x^3*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3,x]
```

output
```
$Aborted
```

## Maple [A] (verified)

Time = 4.70 (sec) , antiderivative size = 469, normalized size of antiderivative = 0.72

| method | result |
|---|---|
| default | $\frac{c\sqrt{c(ax-i)(ax+i)}\left(240\arctan(ax)^3a^6x^6-120a^5\arctan(ax)^2x^5+384\arctan(ax)^3a^4x^4+48x^4\arctan(ax)a^4-138a^3\arctan(ax)^2x^3\cdots\right.}{1680a^4}$ |

input
```
int(x^3*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/1680*c/a^4*(c*(a*x-I)*(a*x+I))^(1/2)*(240*arctan(a*x)^3*a^6*x^6-120*a^5*
arctan(a*x)^2*x^5+384*arctan(a*x)^3*a^4*x^4+48*x^4*arctan(a*x)*a^4-138*a^3
*arctan(a*x)^2*x^3+48*arctan(a*x)^3*a^2*x^2-12*a^3*x^3+28*x^2*a^2*arctan(a
*x)+135*a*arctan(a*x)^2*x-96*arctan(a*x)^3+4*a*x-326*arctan(a*x))-17/560*c
*(c*(a*x-I)*(a*x+I))^(1/2)*(I*arctan(a*x)^3-3*arctan(a*x)^2*ln(1-I*(1+I*a*
x)/(a^2*x^2+1)^(1/2))+6*I*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1
/2))-6*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/a^4/(a^2*x^2+1)^(1/2)+17/
560*c*(c*(a*x-I)*(a*x+I))^(1/2)*(I*arctan(a*x)^3-3*arctan(a*x)^2*ln(1+I*(1
+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2
+1)^(1/2))-6*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/a^4/(a^2*x^2+1)^(1
/2)-23/60*I*c/a^4*(c*(a*x-I)*(a*x+I))^(1/2)*arctan((1+I*a*x)/(a^2*x^2+1)^(
1/2))/(a^2*x^2+1)^(1/2)
```

## Fricas [F]

$$\int x^3 (c + a^2 c x^2)^{3/2} \arctan(ax)^3 \, dx = \int (a^2 c x^2 + c)^{\frac{3}{2}} x^3 \arctan(ax)^3 \, dx$$

input
```
integrate(x^3*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^5 + c*x^3)*sqrt(a^2*c*x^2 + c)*arctan(a*x)^3, x)
```

## Sympy [F]

$$\int x^3 (c + a^2 c x^2)^{3/2} \arctan(ax)^3 \, dx = \int x^3 (c(a^2 x^2 + 1))^{\frac{3}{2}} \operatorname{atan}^3(ax) \, dx$$

input
```
integrate(x**3*(a**2*c*x**2+c)**(3/2)*atan(a*x)**3,x)
```

output
```
Integral(x**3*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**3, x)
```

## Maxima [F]

$$\int x^3 \left(c+a^2 c x^2\right)^{3/2} \arctan(ax)^3 \, dx = \int \left(a^2 c x^2 + c\right)^{\frac{3}{2}} x^3 \arctan\left(ax\right)^3 \, dx$$

input
```
integrate(x^3*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*x^3*arctan(a*x)^3, x)
```

## Giac [F(-2)]

Exception generated.

$$\int x^3 \left(c+a^2 c x^2\right)^{3/2} \arctan(ax)^3 \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int x^3 \left(c+a^2 c x^2\right)^{3/2} \arctan(ax)^3 \, dx = \int x^3 \operatorname{atan}(a\,x)^3 \left(c\,a^2\,x^2 + c\right)^{3/2} dx$$

input
```
int(x^3*atan(a*x)^3*(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(x^3*atan(a*x)^3*(c + a^2*c*x^2)^(3/2), x)
```

## Reduce [F]

$$\int x^3 (c + a^2cx^2)^{3/2} \arctan(ax)^3 \, dx = \sqrt{c}\, c\left(\left(\int \sqrt{a^2x^2+1}\, atan(ax)^3 \, x^5 dx\right) a^2 + \int \sqrt{a^2x^2+1}\, atan(ax)^3 \, x^3 dx\right)$$

input
```
int(x^3*(a^2*c*x^2+c)^(3/2)*atan(a*x)^3,x)
```

output
```
sqrt(c)*c*(int(sqrt(a**2*x**2 + 1)*atan(a*x)**3*x**5,x)*a**2 + int(sqrt(a*
*2*x**2 + 1)*atan(a*x)**3*x**3,x))
```

# 3.421 $\int x^2(c+a^2cx^2)^{3/2}\arctan(ax)^3\,dx$

## Optimal result

Integrand size = 24, antiderivative size = 882

$$\int x^2\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3\,dx=-\frac{c\sqrt{c+a^2cx^2}}{30a^3}-\frac{\left(c+a^2cx^2\right)^{3/2}}{60a^3}+\frac{cx\sqrt{c+a^2cx^2}\arctan(ax)}{12a^2}+\frac{1}{20}cx^3\sqrt{c+a^2cx^2}\arctan(ax)+\frac{31c\sqrt{c+a^2cx^2}\arctan(ax)^2}{240a^3}-\frac{19cx^2\sqrt{c+a^2cx^2}\arctan(ax)^2}{120a}-\frac{1}{10}acx^4\sqrt{c+a^2cx^2}\arctan(ax)^2+\frac{cx\sqrt{c+a^2cx^2}\arctan(ax)^3}{16a^2}+\frac{7}{24}cx^3\sqrt{c+a^2cx^2}\arctan(ax)^3+\frac{1}{6}a^2cx^5\sqrt{c+a^2cx^2}\arctan(ax)^3+\frac{ic^2\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^3}{8a^3\sqrt{c+a^2cx^2}}+\frac{41ic^2\sqrt{1+a^2x^2}\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{60a^3\sqrt{c+a^2cx^2}}-\frac{3ic^2\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)}{16a^3\sqrt{c+a^2cx^2}}+\frac{3ic^2\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{PolyLog}\left(2,ie^{i\arctan(ax)}\right)}{16a^3\sqrt{c+a^2cx^2}}-\frac{41ic^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{120a^3\sqrt{c+a^2cx^2}}+\frac{41ic^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{120a^3\sqrt{c+a^2cx^2}}+\frac{3c^2\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)}{8a^3\sqrt{c+a^2cx^2}}-\frac{3c^2\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(3,ie^{i\arctan(ax)}\right)}{8a^3\sqrt{c+a^2cx^2}}+\frac{3ic^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(4,-ie^{i\arctan(ax)}\right)}{8a^3\sqrt{c+a^2cx^2}}-\frac{3ic^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(4,ie^{i\arctan(ax)}\right)}{8a^3\sqrt{c+a^2cx^2}}$$

output
```
-1/30*c*(a^2*c*x^2+c)^(1/2)/a^3-1/60*(a^2*c*x^2+c)^(3/2)/a^3+1/12*c*x*(a^2
*c*x^2+c)^(1/2)*arctan(a*x)/a^2+1/20*c*x^3*(a^2*c*x^2+c)^(1/2)*arctan(a*x)
+31/240*c*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2/a^3-19/120*c*x^2*(a^2*c*x^2+c)
^(1/2)*arctan(a*x)^2/a-1/10*a*c*x^4*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2+1/16
*c*x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^3/a^2+7/24*c*x^3*(a^2*c*x^2+c)^(1/2)*
arctan(a*x)^3+1/6*a^2*c*x^5*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^3+41/120*I*c^2
*(a^2*x^2+1)^(1/2)*polylog(2,I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/a^3/(a^2*c
*x^2+c)^(1/2)-3/16*I*c^2*(a^2*x^2+1)^(1/2)*arctan(a*x)^2*polylog(2,-I*(1+I
*a*x)/(a^2*x^2+1)^(1/2))/a^3/(a^2*c*x^2+c)^(1/2)+1/8*I*c^2*(a^2*x^2+1)^(1/
2)*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^3/a^3/(a^2*c*x^2+c)^(1/
2)-41/120*I*c^2*(a^2*x^2+1)^(1/2)*polylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(
1/2))/a^3/(a^2*c*x^2+c)^(1/2)-3/8*I*c^2*(a^2*x^2+1)^(1/2)*polylog(4,I*(1+I
*a*x)/(a^2*x^2+1)^(1/2))/a^3/(a^2*c*x^2+c)^(1/2)+41/60*I*c^2*(a^2*x^2+1)^(
1/2)*arctan(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/a^3/(a^2*c*x^2+c)
^(1/2)+3/8*c^2*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(3,-I*(1+I*a*x)/(a^2*x
^2+1)^(1/2))/a^3/(a^2*c*x^2+c)^(1/2)-3/8*c^2*(a^2*x^2+1)^(1/2)*arctan(a*x)
*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a^3/(a^2*c*x^2+c)^(1/2)+3/8*I*c^
2*(a^2*x^2+1)^(1/2)*polylog(4,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a^3/(a^2*c*x
^2+c)^(1/2)+3/16*I*c^2*(a^2*x^2+1)^(1/2)*arctan(a*x)^2*polylog(2,I*(1+I*a*
x)/(a^2*x^2+1)^(1/2))/a^3/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 4015 vs. $2(882) = 1764$.

Time = 18.38 (sec) , antiderivative size = 4015, normalized size of antiderivative = 4.55

$$\int x^2 (c + a^2 c x^2)^{3/2} \arctan(ax)^3 \, dx = \text{Result too large to show}$$

input
```
Integrate[x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3,x]
```

output
```
(c*((Sqrt[c*(1 + a^2*x^2)]*(-1 + ArcTan[a*x]^2))/(4*Sqrt[1 + a^2*x^2]) + (
Sqrt[c*(1 + a^2*x^2)]*(-(ArcTan[a*x]*(Log[1 - I*E^(I*ArcTan[a*x])] - Log[1
 + I*E^(I*ArcTan[a*x])])) - I*(PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - PolyLo
g[2, I*E^(I*ArcTan[a*x])])))/(2*Sqrt[1 + a^2*x^2]) + (Sqrt[c*(1 + a^2*x^2)
]*(-1/8*(Pi^3*Log[Cot[(Pi/2 - ArcTan[a*x])/2]]) - (3*Pi^2*((Pi/2 - ArcTan[
a*x])*(Log[1 - E^(I*(Pi/2 - ArcTan[a*x]))] - Log[1 + E^(I*(Pi/2 - ArcTan[a
*x]))]) + I*(PolyLog[2, -E^(I*(Pi/2 - ArcTan[a*x]))] - PolyLog[2, E^(I*(Pi
/2 - ArcTan[a*x]))])))/4 + (3*Pi*((Pi/2 - ArcTan[a*x])^2*(Log[1 - E^(I*(Pi
/2 - ArcTan[a*x]))] - Log[1 + E^(I*(Pi/2 - ArcTan[a*x]))]) + (2*I)*(Pi/2 -
 ArcTan[a*x])*(PolyLog[2, -E^(I*(Pi/2 - ArcTan[a*x]))] - PolyLog[2, E^(I*(
Pi/2 - ArcTan[a*x]))]) + 2*(-PolyLog[3, -E^(I*(Pi/2 - ArcTan[a*x]))] + Pol
yLog[3, E^(I*(Pi/2 - ArcTan[a*x]))])))/2 - 8*((I/64)*(Pi/2 - ArcTan[a*x])^
4 + (I/4)*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2)^4 - ((Pi/2 - ArcTan[a*x])^3*L
og[1 + E^(I*(Pi/2 - ArcTan[a*x]))])/8 - (Pi^3*(I*(Pi/2 + (-1/2*Pi + ArcTan
[a*x])/2) - Log[1 + E^((2*I)*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2))]))/8 - (P
i/2 + (-1/2*Pi + ArcTan[a*x])/2)^3*Log[1 + E^((2*I)*(Pi/2 + (-1/2*Pi + Arc
Tan[a*x])/2))] + ((3*I)/8)*(Pi/2 - ArcTan[a*x])^2*PolyLog[2, -E^(I*(Pi/2 -
 ArcTan[a*x]))] + (3*Pi^2*((I/2)*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2)^2 - (P
i/2 + (-1/2*Pi + ArcTan[a*x])/2)*Log[1 + E^((2*I)*(Pi/2 + (-1/2*Pi + ArcTa
n[a*x])/2))] + (I/2)*PolyLog[2, -E^((2*I)*(Pi/2 + (-1/2*Pi + ArcTan[a*x...
```

## Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arctan(ax)^3 \left(a^2cx^2+c\right)^{3/2} dx$$

$$\Big\downarrow \text{ 5485}$$

$$c\int x^2\sqrt{a^2cx^2+c}\arctan(ax)^3dx+a^2c\int x^4\sqrt{a^2cx^2+c}\arctan(ax)^3dx$$

$$\Big\downarrow \text{ 5485}$$

$$c\left(c\int \frac{x^2\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx+a^2c\int \frac{x^4\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx\right)+ a^2c\left(a^2c\int \frac{x^6\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx+c\int \frac{x^4\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx\right)$$

$$\Big\downarrow \text{ 5487}$$

$$c\left(c\left(-\frac{3\int \frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{\int \frac{\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{2a^2}+\frac{x\arctan(ax)^3\sqrt{a^2cx^2+c}}{2a^2c}\right)+a^2c\left(-\frac{3\int \frac{x^2\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{3\int \frac{x^3}{}}{}\right.\right.$$
$$a^2c\left(c\left(-\frac{3\int \frac{x^2\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{3\int \frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a}+\frac{x^3\arctan(ax)^3\sqrt{a^2cx^2+c}}{4a^2c}\right)+a^2c\left(-\frac{\int \frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\right.\right.$$

$\downarrow$ 5425

$$c\left(c\left(-\frac{3\int \frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^3}{\sqrt{a^2x^2+1}}dx}{2a^2\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^3\sqrt{a^2cx^2+c}}{2a^2c}\right)+a^2c\left(-\frac{3\int \frac{x^2\arctan(ax)^3}{\sqrt{a^2cx^2+c}}}{4a^2}\right.\right.$$
$$a^2c\left(c\left(-\frac{3\int \frac{x^2\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{3\int \frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a}+\frac{x^3\arctan(ax)^3\sqrt{a^2cx^2+c}}{4a^2c}\right)+a^2c\left(-\frac{\int \frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\right.\right.$$

$\downarrow$ 5423

$$a^2c\left(c\left(-\frac{3\int \frac{x^2\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{3\int \frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a}+\frac{x^3\arctan(ax)^3\sqrt{a^2cx^2+c}}{4a^2c}\right)+a^2c\left(-\frac{\int \frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\right.\right.$$
$$c\left(a^2c\left(-\frac{3\int \frac{x^2\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{3\int \frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a}+\frac{x^3\arctan(ax)^3\sqrt{a^2cx^2+c}}{4a^2c}\right)+c\left(-\frac{3\int \frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\right.\right.$$

$\downarrow$ 3042

$$a^2c\left(c\left(-\frac{3\int \frac{x^2\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{3\int \frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a}+\frac{x^3\arctan(ax)^3\sqrt{a^2cx^2+c}}{4a^2c}\right)+a^2c\left(-\frac{\int \frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\right.\right.$$
$$c\left(a^2c\left(-\frac{3\int \frac{x^2\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{3\int \frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a}+\frac{x^3\arctan(ax)^3\sqrt{a^2cx^2+c}}{4a^2c}\right)+c\left(-\frac{3\int \frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\right.\right.$$

$\downarrow$ 4669

$$a^2c\left(c\left(-\frac{3\int \frac{x^2\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{3\int \frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a}+\frac{x^3\arctan(ax)^3\sqrt{a^2cx^2+c}}{4a^2c}\right)+a^2c\left(-\frac{\int \frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\right.\right.$$
$$c\left(a^2c\left(-\frac{3\int \frac{x^2\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{3\int \frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a}+\frac{x^3\arctan(ax)^3\sqrt{a^2cx^2+c}}{4a^2c}\right)+c\left(-\frac{3\int \frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\right.\right.$$

$\downarrow$ 3011

$$a^2c\left(c\left(-\frac{3\int\frac{x^2\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{3\int\frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a}+\frac{x^3\arctan(ax)^3\sqrt{a^2cx^2+c}}{4a^2c}\right)+a^2c\left(-\frac{\int\frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\right.\right.$$
$$c\left(a^2c\left(-\frac{3\int\frac{x^2\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{3\int\frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a}+\frac{x^3\arctan(ax)^3\sqrt{a^2cx^2+c}}{4a^2c}\right)+c\left(-\frac{3\int\frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\right.\right.$$

$\downarrow$ 5465

$$a^2c\left(c\left(-\frac{3\int\frac{x^2\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{3\int\frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a}+\frac{x^3\arctan(ax)^3\sqrt{a^2cx^2+c}}{4a^2c}\right)+a^2c\left(-\frac{\int\frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\right.\right.$$
$$c\left(a^2c\left(-\frac{3\int\frac{x^2\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{3\int\frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a}+\frac{x^3\arctan(ax)^3\sqrt{a^2cx^2+c}}{4a^2c}\right)+c\left(-\frac{3\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}\right.}{2a}\right.\right.$$

$\downarrow$ 5425

$$a^2c\left(c\left(-\frac{3\int\frac{x^2\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{3\int\frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a}+\frac{x^3\arctan(ax)^3\sqrt{a^2cx^2+c}}{4a^2c}\right)+a^2c\left(-\frac{\int\frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\right.\right.$$
$$c\left(a^2c\left(-\frac{3\int\frac{x^2\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{3\int\frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a}+\frac{x^3\arctan(ax)^3\sqrt{a^2cx^2+c}}{4a^2c}\right)+c\left(-\frac{3\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}\right.}{}\right.\right.$$

$\downarrow$ 5421

$$a^2c\left(c\left(-\frac{3\int\frac{x^2\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{3\int\frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a}+\frac{x^3\arctan(ax)^3\sqrt{a^2cx^2+c}}{4a^2c}\right)+a^2c\left(-\frac{\int\frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\right.\right.$$
$$c\left(a^2c\left(-\frac{3\int\frac{x^2\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{4a^2}-\frac{3\int\frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{4a}+\frac{x^3\arctan(ax)^3\sqrt{a^2cx^2+c}}{4a^2c}\right)+c\left(-\frac{\sqrt{a^2x^2+1}\left(3\left(i\arctan\right.\right.}{}\right.\right.$$

$\downarrow$ 5487

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\int\frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int\frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a^2}}{2a}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}\,\mathrm{ar}}{4a^2c}\right.}{}\right.\right.$$

$$c\left(c\left(\frac{x^3\sqrt{a^2cx^2+c}\arctan(ax)^3}{4a^2c}-\frac{3\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}{3a^2c}-\frac{2\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{2\int\frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{3a^2}\right)}{4a}-\frac{3\left(\frac{x\sqrt{a^2cx^2}}{}\right.}{}\right.\right.$$

$\downarrow$ 5425

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\int\frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int\frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a^2}}{2a}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}\,\mathrm{ar}}{4a^2c}\right.}{}\right.\right.$$

$$c\left(c\left(\frac{x^3\sqrt{a^2cx^2+c}\arctan(ax)^3}{4a^2c}-\frac{3\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}{3a^2c}-\frac{2\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{2\int\frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{3a^2}\right)}{4a}-\frac{3\left(\frac{x\sqrt{a^2cx^2}}{}\right.}{}\right.\right.$$

$\downarrow$ 5423

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\int\frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int\frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a^2}}{2a}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}\,\mathrm{ar}}{4a^2c}\right.}{}\right.\right.$$

$$c\left(c\left(\frac{x^3\sqrt{a^2cx^2+c}\arctan(ax)^3}{4a^2c}-\frac{3\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}{3a^2c}-\frac{2\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{2\int\frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{3a^2}\right)}{4a}-\frac{3\left(\frac{x\sqrt{a^2cx^2+}}{}\right.}{}\right.\right.$$

$\downarrow$ 3042

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\int\frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int\frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a^2}}{2a}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}\,\mathrm{ar}}{4a^2c}\right.}{}\right.\right.$$

$$c\left(c\left(\frac{x^3\sqrt{a^2cx^2+c}\arctan(ax)^3}{4a^2c}-\frac{3\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}{3a^2c}-\frac{2\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{2\int\frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{3a^2}\right)}{4a}-\frac{3\left(\frac{x\sqrt{a^2cx^2+}}{}\right.}{}\right.\right.$$

$\downarrow$ 4669

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\int\frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int\frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a^2}}{2a}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}\,\mathrm{ar}}{4a^2c}\right.}{}\right.\right.$$

$$c\left(c\left(\frac{x^3\sqrt{a^2cx^2+c}\arctan(ax)^3}{4a^2c}-\frac{3\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}{3a^2c}-\frac{2\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{2\int\frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{3a^2}\right)}{4a}-\frac{3\left(\frac{x\sqrt{a^2cx^2+}}{}\right.}{}\right.\right.$$

$\downarrow$ 3011

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\int\frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int\frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a^2}}{2a}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}\,\mathrm{ar}}{4a^2c}\right.}{}\right.\right.$$

$$c\left(c\left(\frac{x^3\sqrt{a^2cx^2+c}\arctan(ax)^3}{4a^2c}-\frac{3\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}{3a^2c}-\frac{2\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{3a}-\frac{2\int\frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{3a^2}\right)}{4a}-\frac{3\left(\frac{x\sqrt{a^2cx^2+}}{}\right.}{}\right.\right.$$

$\downarrow$ 5465

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\int\frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int\frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a^2}}{2a}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}\,\mathrm{ar}}{4a^2c}\right.}{}\right.\right.$$

$$c\left(c\left(\frac{x^3\sqrt{a^2cx^2+c}\arctan(ax)^3}{4a^2c}-\frac{3\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\int\frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}\right)}{3a^2}-\frac{2\int\frac{x^2\arctan(a}{\sqrt{a^2cx^2+}}}{3a}\right.}{4a}\right.\right.$$

$\downarrow$ 5425

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\int\frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int\frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a^2}}{2a}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}\,\mathrm{ar}}{4a^2c}\right.}{}\right.\right.$$

$$c\left(c\left(\frac{x^3\sqrt{a^2cx^2+c}\arctan(ax)^3}{4a^2c}-\frac{3\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\int\frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{a\sqrt{a^2cx^2+c}}\right)}{3a^2}-2\int\frac{x}{}\right.}{4a}\right.\right.$$

$\downarrow$ 5421

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c} - \frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c} - \frac{2\int\frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a} - \frac{4\int\frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a^2}}{2a} - \frac{5\left(\frac{x^3\sqrt{a^2cx^2+c}\,\text{ar}}{4a^2c}\right.}{}\right.\right.$$

$$c\left(c\left(\frac{x^3\sqrt{a^2cx^2+c}\arctan(ax)^3}{4a^2c} - \frac{3\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}{3a^2c} - \frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c} - \frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{ia}}{\sqrt{1-}}\right.}{a}\right.}{}\right.}{3a^2}\right.}{4a}\right.\right.$$

↓ 5487

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\int\frac{x^3}{\sqrt{a^2cx^2+c}}dx}{4a}-\frac{3\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-}{2a}\right.\right.$$

$$c\left(c\left(\frac{x^3\sqrt{a^2cx^2+c}\arctan(ax)^3}{4a^2c}-\frac{3\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{ia}}{\sqrt{1-}}\right.}{a}\right.}{}\right.}{3a^2}\right.}{}\right.\right.$$

↓ 241

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\int\frac{x^3}{\sqrt{a^2cx^2+c}}dx}{4a}-\frac{3\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}-}{2a}\right.\right.$$

$$c\left(c\left(\frac{x^3\sqrt{a^2cx^2+c}\arctan(ax)^3}{4a^2c}-\frac{3\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{ia}}{\sqrt{1-}}\right.}{a}\right.}{}\right)}{3a^2}\right.}{}\right.\right.$$

↓ 243

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\int\frac{x^2}{\sqrt{a^2cx^2+c}}dx^2}{8a}-\frac{3\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right)}{5a}}{2a}\right.\right.$$

$$c\left(c\left(\frac{x^3\sqrt{a^2cx^2+c}\arctan(ax)^3}{4a^2c}-\frac{3\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{ia}}{\sqrt{1-}}\right.}{a}\right.}{}\right)}{3a^2}\right)}{}\right.\right.$$

↓ 53

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\int\left(\frac{\sqrt{a^2cx^2+c}}{a^2c}-\frac{1}{a^2\sqrt{a^2cx^2+c}}\right)dx^2}{8a}-\frac{3\int x}{\cdots}\right.}{5a}}{2a}\right.\right.$$

$$c\left(c\left(\frac{x^3\sqrt{a^2cx^2+c}\arctan(ax)^3}{4a^2c}-\frac{3\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{ia}}{\sqrt{1-}}\right.}{a}\right.}{\cdots}\right.}{3a^2}\right.}{\cdots}\right.\right.$$

↓ 2009

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}-\frac{3\int\frac{x^2\arctan}{\sqrt{a^2}}}{4}\right.}{5a}}{2a}\right.\right.$$

$$c\left(c\left(\frac{x^3\sqrt{a^2cx^2+c}\arctan(ax)^3}{4a^2c}-\frac{3\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{ia}}{\sqrt{1-}}\right.}{a}\right.}{}\right.}{3a^2}\right.}{}\right.\right.$$

↓ 5425

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}-\frac{3\int\frac{x^2\operatorname{arc}}{\sqrt{a^2}}}{4}\right.}{5a}}{2a}\right.\right.$$

$$c\left(c\left(\frac{x^3\sqrt{a^2cx^2+c}\arctan(ax)^3}{4a^2c}-\frac{3\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{ia}}{\sqrt{1-}}\right.}{a}\right.}{}\right.}{3a^2}\right.}{}\right.\right.$$

↓ 5421

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}-\frac{3\int\frac{x^2\operatorname{arc}}{\sqrt{a^2}}}{4}\right.}{5a}}{2a}\right.\right.$$

$$c\left(c\left(\frac{x^3\sqrt{a^2cx^2+c}\arctan(ax)^3}{4a^2c}-\frac{3\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)}{2a^2c}-\frac{\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax}}{\sqrt{1-}}\right.}{a}\right.}{}\right.}{}\right.}{}\right.\right.$$

↓ 5423

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\,\arctan(ax)^3x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\,\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\,\arctan(ax)x^3}{4a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}-\frac{3\int\frac{x^2\,\mathrm{arc}}{\sqrt{a^2}}}{4}\right.}{5a}}{2a}\right.\right.$$

$$c\left(c\left(\frac{x^3\sqrt{a^2cx^2+c}\,\arctan(ax)^3}{4a^2c}-\frac{3\left(\frac{x^2\sqrt{a^2cx^2+c}\,\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{x\sqrt{a^2cx^2+c}\,\arctan(ax)}{2a^2c}-\frac{\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax}}{\sqrt{1-}}\right.}{a}\right.}{}\right.}{}\right.}{}\right.\right.$$

↓ 3042

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}-\frac{3\int\frac{x^2\operatorname{arc}}{\sqrt{a^2}}}{4}\right.}{5a}}{2a}\right.\right.$$

$$c\left(c\left(\frac{x^3\sqrt{a^2cx^2+c}\arctan(ax)^3}{4a^2c}-\frac{3\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)}{2a^2c}-\frac{\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax}}{\sqrt{1-}}\right.}{a}\right.}{}\right.}{}\right.}{}\right.\right.$$

↓ 4669

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}-\frac{3\int \frac{x^2\operatorname{arc}}{\sqrt{a^2}}}{4}\right.}{5a}}{2a}\right.\right.$$

$$c\left(c\left(\frac{x^3\sqrt{a^2cx^2+c}\arctan(ax)^3}{4a^2c}-\frac{3\left(\frac{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}{3a^2c}-\frac{2\left(\frac{x\sqrt{a^2cx^2+c}\arctan(ax)}{2a^2c}-\frac{\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax}}{\sqrt{1-}}\right.}{a}\right.}{}\right.}{}\right.}{}\right.\right.$$

input
```
Int[x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3,x]
```

output
```
$Aborted
```

## Maple [A] (verified)

Time = 4.96 (sec) , antiderivative size = 514, normalized size of antiderivative = 0.58

| method | result |
|---|---|
| default | $\frac{c\sqrt{c(ax-i)(ax+i)}\,\left(40\arctan(ax)^3a^5x^5-24a^4\arctan(ax)^2x^4+70\arctan(ax)^3a^3x^3+12\arctan(ax)x^3a^3-38\arctan(ax)^2x^2a^2+15x\right.}{240a^3}$ |

input
```
int(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/240*c/a^3*(c*(a*x-I)*(a*x+I))^(1/2)*(40*arctan(a*x)^3*a^5*x^5-24*a^4*arc
tan(a*x)^2*x^4+70*arctan(a*x)^3*a^3*x^3+12*arctan(a*x)*x^3*a^3-38*arctan(a
*x)^2*x^2*a^2+15*x*arctan(a*x)^3*a-4*a^2*x^2+20*arctan(a*x)*a*x+31*arctan(
a*x)^2-12)+1/240*c*(c*(a*x-I)*(a*x+I))^(1/2)*(15*arctan(a*x)^3*ln(1+I*(1+I
*a*x)/(a^2*x^2+1)^(1/2))-15*arctan(a*x)^3*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/
2))-45*I*arctan(a*x)^2*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+45*I*arct
an(a*x)^2*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+82*arctan(a*x)*ln(1+I*(
1+I*a*x)/(a^2*x^2+1)^(1/2))+90*arctan(a*x)*polylog(3,-I*(1+I*a*x)/(a^2*x^2
+1)^(1/2))-82*arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-90*arctan(a*
x)*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+90*I*polylog(4,-I*(1+I*a*x)/(a
^2*x^2+1)^(1/2))-90*I*polylog(4,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-82*I*dilog(
1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+82*I*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2
)))/a^3/(a^2*x^2+1)^(1/2)
```

## Fricas [F]

$$\int x^2\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3\,dx=\int\left(a^2cx^2+c\right)^{\frac{3}{2}}x^2\arctan\left(ax\right)^3\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^4 + c*x^2)*sqrt(a^2*c*x^2 + c)*arctan(a*x)^3, x)
```

## Sympy [F]

$$\int x^2 (c + a^2cx^2)^{3/2} \arctan(ax)^3 \, dx = \int x^2 \left(c(a^2x^2 + 1)\right)^{\frac{3}{2}} \operatorname{atan}^3(ax) \, dx$$

input
```
integrate(x**2*(a**2*c*x**2+c)**(3/2)*atan(a*x)**3,x)
```

output
```
Integral(x**2*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**3, x)
```

## Maxima [F]

$$\int x^2 (c + a^2cx^2)^{3/2} \arctan(ax)^3 \, dx = \int (a^2cx^2 + c)^{\frac{3}{2}} x^2 \arctan(ax)^3 \, dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*x^2*arctan(a*x)^3, x)
```

## Giac [F]

$$\int x^2 (c + a^2cx^2)^{3/2} \arctan(ax)^3 \, dx = \int (a^2cx^2 + c)^{\frac{3}{2}} x^2 \arctan(ax)^3 \, dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*x^2*arctan(a*x)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int x^2 \left(c + a^2 c x^2\right)^{3/2} \arctan(ax)^3 \, dx = \int x^2 \operatorname{atan}(a\,x)^3 \, \left(c\,a^2\,x^2 + c\right)^{3/2} \, dx$$

input
```
int(x^2*atan(a*x)^3*(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(x^2*atan(a*x)^3*(c + a^2*c*x^2)^(3/2), x)
```

## Reduce [F]

$$\int x^2 \left(c + a^2 c x^2\right)^{3/2} \arctan(ax)^3 \, dx = \sqrt{c}\, c \left( \left( \int \sqrt{a^2 x^2 + 1}\, atan(ax)^3 \, x^4 dx \right) a^2 + \int \sqrt{a^2 x^2 + 1}\, atan(ax)^3 \, x^2 dx \right)$$

input
```
int(x^2*(a^2*c*x^2+c)^(3/2)*atan(a*x)^3,x)
```

output
```
sqrt(c)*c*(int(sqrt(a**2*x**2 + 1)*atan(a*x)**3*x**4,x)*a**2 + int(sqrt(a*
*2*x**2 + 1)*atan(a*x)**3*x**2,x))
```

# 3.422 $\int x(c+a^2cx^2)^{3/2}\arctan(ax)^3\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 477

$$\begin{aligned}\int x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3\,dx &= -\frac{cx\sqrt{c+a^2cx^2}}{20a}\\
&+\frac{9c\sqrt{c+a^2cx^2}\arctan(ax)}{20a^2}+\frac{(c+a^2cx^2)^{3/2}\arctan(ax)}{10a^2}\\
&-\frac{9cx\sqrt{c+a^2cx^2}\arctan(ax)^2}{40a}-\frac{3x(c+a^2cx^2)^{3/2}\arctan(ax)^2}{20a}\\
&+\frac{9ic^2\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2}{20a^2\sqrt{c+a^2cx^2}}\\
&+\frac{(c+a^2cx^2)^{5/2}\arctan(ax)^3}{5a^2c}-\frac{c^{3/2}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right)}{2a^2}\\
&-\frac{9ic^2\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)}{20a^2\sqrt{c+a^2cx^2}}\\
&+\frac{9ic^2\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,ie^{i\arctan(ax)}\right)}{20a^2\sqrt{c+a^2cx^2}}\\
&+\frac{9c^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)}{20a^2\sqrt{c+a^2cx^2}}\\
&-\frac{9c^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,ie^{i\arctan(ax)}\right)}{20a^2\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
-1/20*c*x*(a^2*c*x^2+c)^(1/2)/a+9/20*c*(a^2*c*x^2+c)^(1/2)*arctan(a*x)/a^2
+1/10*(a^2*c*x^2+c)^(3/2)*arctan(a*x)/a^2-9/40*c*x*(a^2*c*x^2+c)^(1/2)*arc
tan(a*x)^2/a-3/20*x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/a+9/20*I*c^2*(a^2*x^
2+1)^(1/2)*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^2/a^2/(a^2*c*x^
2+c)^(1/2)+1/5*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^3/a^2/c-1/2*c^(3/2)*arctanh
(a*c^(1/2)*x/(a^2*c*x^2+c)^(1/2))/a^2-9/20*I*c^2*(a^2*x^2+1)^(1/2)*arctan(
a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a^2/(a^2*c*x^2+c)^(1/2)+9/2
0*I*c^2*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1
/2))/a^2/(a^2*c*x^2+c)^(1/2)+9/20*c^2*(a^2*x^2+1)^(1/2)*polylog(3,-I*(1+I*
a*x)/(a^2*x^2+1)^(1/2))/a^2/(a^2*c*x^2+c)^(1/2)-9/20*c^2*(a^2*x^2+1)^(1/2)
*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a^2/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (warning: unable to verify)

Time = 2.30 (sec) , antiderivative size = 439, normalized size of antiderivative = 0.92

$$\int x(c+a^2cx^2)^{3/2}\arctan(ax)^3\,dx = \frac{c\sqrt{c+a^2cx^2}\left(-960\left(\coth^{-1}\left(\frac{ax}{\sqrt{1+a^2x^2}}\right)-i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2+\right.\right.}{}$$

input
```
Integrate[x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3,x]
```

output
```
(c*Sqrt[c + a^2*c*x^2]*(-960*(ArcCoth[(a*x)/Sqrt[1 + a^2*x^2]] - I*ArcTan[
E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 + I*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcT
an[a*x])] - I*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] - PolyLog[3, (-I
)*E^(I*ArcTan[a*x])] + PolyLog[3, I*E^(I*ArcTan[a*x])]) + 48*(10*ArcCoth[(
a*x)/Sqrt[1 + a^2*x^2]] - (11*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 +
 (11*I)*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - (11*I)*ArcTan[a*x
]*PolyLog[2, I*E^(I*ArcTan[a*x])] - 11*PolyLog[3, (-I)*E^(I*ArcTan[a*x])]
+ 11*PolyLog[3, I*E^(I*ArcTan[a*x])]) + 80*(1 + a^2*x^2)^(3/2)*ArcTan[a*x]
*(6 + 4*ArcTan[a*x]^2 + 6*Cos[2*ArcTan[a*x]] - 3*ArcTan[a*x]*Sin[2*ArcTan[
a*x]]) - (1 + a^2*x^2)^(5/2)*((48*a*x)/(1 + a^2*x^2)^2 + 32*ArcTan[a*x]^3*
(-1 + 5*Cos[2*ArcTan[a*x]]) + 6*ArcTan[a*x]*(25 + 36*Cos[2*ArcTan[a*x]] +
11*Cos[4*ArcTan[a*x]]) + ArcTan[a*x]^2*(6*Sin[2*ArcTan[a*x]] - 33*Sin[4*Ar
cTan[a*x]]))))/(960*a^2*Sqrt[1 + a^2*x^2])
```

## Rubi [A] (verified)

Time = 1.59 (sec) , antiderivative size = 382, normalized size of antiderivative = 0.80, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.682, Rules used = {5465, 5415, 211, 224, 219, 5415, 224, 219, 5425, 5423, 3042, 4669, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \arctan(ax)^3 \left(a^2cx^2+c\right)^{3/2} dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{\arctan(ax)^3 \left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{3\int \left(a^2cx^2+c\right)^{3/2} \arctan(ax)^2dx}{5a}$$

$$\Big\downarrow \text{5415}$$

$$\frac{\arctan(ax)^3 \left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{3\left(\frac{3}{4}c\int \sqrt{a^2cx^2+c}\arctan(ax)^2dx+\frac{1}{6}c\int \sqrt{a^2cx^2+c}dx+\frac{1}{4}x\arctan(ax)^2 \left(a^2cx^2+c\right)^{3/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{6a}\right.}{5a}$$

$$\Big\downarrow \text{211}$$

$$\frac{\arctan(ax)^3 \left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{3\left(\frac{3}{4}c\int \sqrt{a^2cx^2+c}\arctan(ax)^2dx+\frac{1}{6}c\left(\frac{1}{2}c\int \frac{1}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\arctan(ax)^2 \left(a^2cx^2+c\right)^{3/2}-\right.}{5a}$$

$$\Big\downarrow \text{224}$$

$$\frac{\arctan(ax)^3 \left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{3\left(\frac{3}{4}c\int \sqrt{a^2cx^2+c}\arctan(ax)^2dx+\frac{1}{6}c\left(\frac{1}{2}c\int \frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\arctan(ax)^2 \left(a^2cx^2+\right.\right.}{5a}$$

$$\Big\downarrow \text{219}$$

$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-$$
$$\frac{3\left(\frac{3}{4}c\int\sqrt{a^2cx^2+c}\arctan(ax)^2dx+\frac{1}{4}x\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{6a}+\frac{1}{6}c\left(\frac{\sqrt{c}\text{arctanh}\left(\frac{}{\sqrt{a}}\right.}{2a}\right.\right.}{5a}$$

$\downarrow$ 5415

$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-$$
$$\frac{3\left(\frac{3}{4}c\left(\frac{1}{2}c\int\frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int\frac{1}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}\right)+\frac{1}{4}x\arctan(ax)^2\right.}{5a}$$

$\downarrow$ 224

$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-$$
$$\frac{3\left(\frac{3}{4}c\left(\frac{1}{2}c\int\frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int\frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}\right)+\frac{1}{4}x\,\text{arct}\right.}{5a}$$

$\downarrow$ 219

$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-$$
$$\frac{3\left(\frac{3}{4}c\left(\frac{1}{2}c\int\frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}\right)+\frac{1}{4}x\,\text{arcta}\right.}{5a}$$

$\downarrow$ 5425

$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-$$
$$\frac{3\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\int\frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}\right)+\frac{1}{4}x\right.}{5a}$$

$\downarrow$ 5423

$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-$$
$$3\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\int\sqrt{a^2x^2+1}\arctan(ax)^2d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\frac{\sqrt{c}\text{arctanh}\Big(\frac{}{\sqrt{}}\right.\right.$$

↓ 3042

$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-$$
$$3\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\int\arctan(ax)^2\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\frac{\sqrt{c}\text{arc}}{}\right.\right.$$

↓ 4669

$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-$$
$$3\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\int\arctan(ax)\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+2\int\arctan(ax)\log\left(1+ie^{i\arctan(ax)}\right)d\arctan(ax)-2i\arctan\left(e^{i\arctan(a}\right.\right.}{2a\sqrt{a^2cx^2+c}}\right.\right.$$

↓ 3011

$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-$$
$$3\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-i\int\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(ax)\right)-2\left(i\arctan(ax)\text{PolyLog}\left(2,ie^{i\arctan}\right.\right.\right.}{2a\sqrt{a^2cx^2+c}}\right.\right.$$

↓ 2720

$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-$$
$$3\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)-2\left(i\arctan(ax)\text{Poly}\right.\right.\right.}{2a\sqrt{a^2cx^2+c}}\right.\right.$$

↓ 7143

$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-$$

$$3\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)\right)-2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,ie^{i\arctan(ax)}\right)-\operatorname{PolyLo}\right.\right.}{2a\sqrt{a^2cx^2+c}}\right.\right.$$

input
```
Int[x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3,x]
```

output
```
((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3)/(5*a^2*c) - (3*(-1/6*((c + a^2*c*x^2
)^(3/2)*ArcTan[a*x])/a + (x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/4 + (c*((
x*Sqrt[c + a^2*c*x^2])/2 + (Sqrt[c]*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x
^2]])/(2*a)))/6 + (3*c*(-((Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/a) + (x*Sqrt[c
 + a^2*c*x^2]*ArcTan[a*x]^2)/2 + (Sqrt[c]*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a
^2*c*x^2]])/a + (c*Sqrt[1 + a^2*x^2]*((-2*I)*ArcTan[E^(I*ArcTan[a*x])]*Arc
Tan[a*x]^2 + 2*(I*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - PolyLog
[3, (-I)*E^(I*ArcTan[a*x])]) - 2*(I*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a
*x])] - PolyLog[3, I*E^(I*ArcTan[a*x])])))/(2*a*Sqrt[c + a^2*c*x^2])))/4))
/(5*a)
```

## Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5415
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTan[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x])^p,
x], x] + Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*(
a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[q, 0] && GtQ[p, 1]
```

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

## Maple [A] (verified)

Time = 4.44 (sec) , antiderivative size = 421, normalized size of antiderivative = 0.88

| method | result |
|---|---|
| default | $\frac{c\sqrt{c(ax-i)(ax+i)}\left(8\arctan(ax)^3a^4x^4-6a^3\arctan(ax)^2x^3+16\arctan(ax)^3a^2x^2+4x^2a^2\arctan(ax)-15a\arctan(ax)^2x+8\arctan(\right.}{40a^2}$ |

input
```
int(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/40*c/a^2*(c*(a*x-I)*(a*x+I))^(1/2)*(8*arctan(a*x)^3*a^4*x^4-6*a^3*arctan
(a*x)^2*x^3+16*arctan(a*x)^3*a^2*x^2+4*x^2*a^2*arctan(a*x)-15*a*arctan(a*x
)^2*x+8*arctan(a*x)^3-2*a*x+22*arctan(a*x))-3/40*c*(c*(a*x-I)*(a*x+I))^(1/
2)*(I*arctan(a*x)^3-3*arctan(a*x)^2*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*
I*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*polylog(3,-I*(1+
I*a*x)/(a^2*x^2+1)^(1/2)))/a^2/(a^2*x^2+1)^(1/2)+3/40*c*(c*(a*x-I)*(a*x+I)
)^(1/2)*(I*arctan(a*x)^3-3*arctan(a*x)^2*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2
))+6*I*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*polylog(3,I*
(1+I*a*x)/(a^2*x^2+1)^(1/2)))/a^2/(a^2*x^2+1)^(1/2)+I*c/a^2*(c*(a*x-I)*(a*
x+I))^(1/2)*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*x^2+1)^(1/2)
```

## Fricas [F]

$$\int x(c+a^2cx^2)^{3/2}\arctan(ax)^3\,dx=\int (a^2cx^2+c)^{\frac{3}{2}}x\arctan(ax)^3\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^3 + c*x)*sqrt(a^2*c*x^2 + c)*arctan(a*x)^3, x)
```

## Sympy [F]

$$\int x(c+a^2cx^2)^{3/2}\arctan(ax)^3\,dx=\int x(c(a^2x^2+1))^{\frac{3}{2}}\operatorname{atan}^3(ax)\,dx$$

input
```
integrate(x*(a**2*c*x**2+c)**(3/2)*atan(a*x)**3,x)
```

output
```
Integral(x*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**3, x)
```

## Maxima [F]

$$\int x(c+a^2cx^2)^{3/2}\arctan(ax)^3\,dx=\int (a^2cx^2+c)^{\frac{3}{2}}x\arctan(ax)^3\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*x*arctan(a*x)^3, x)
```

## Giac [F(-2)]

Exception generated.

$$\int x(c+a^2cx^2)^{3/2}\arctan(ax)^3\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int x(c+a^2cx^2)^{3/2}\arctan(ax)^3\,dx = \int x\,\mathrm{atan}(a\,x)^3\,\left(c\,a^2\,x^2+c\right)^{3/2}\,dx$$

input
```
int(x*atan(a*x)^3*(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(x*atan(a*x)^3*(c + a^2*c*x^2)^(3/2), x)
```

## Reduce [F]

$$\int x(c+a^2cx^2)^{3/2}\arctan(ax)^3\,dx = \sqrt{c}\,c\left(\left(\int \sqrt{a^2x^2+1}\,atan(ax)^3\,x^3dx\right)a^2 + \int \sqrt{a^2x^2+1}\,atan(ax)^3\,xdx\right)$$

input
```
int(x*(a^2*c*x^2+c)^(3/2)*atan(a*x)^3,x)
```

output
```
sqrt(c)*c*(int(sqrt(a**2*x**2 + 1)*atan(a*x)**3*x**3,x)*a**2 + int(sqrt(a*
*2*x**2 + 1)*atan(a*x)**3*x,x))
```

# 3.423 $\int (c+a^2cx^2)^{3/2}\arctan(ax)^3\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 760

$$\int (c+a^2cx^2)^{3/2}\arctan(ax)^3\,dx = -\frac{c\sqrt{c+a^2cx^2}}{4a}+\frac{1}{4}cx\sqrt{c+a^2cx^2}\arctan(ax)$$
$$-\frac{9c\sqrt{c+a^2cx^2}\arctan(ax)^2}{8a}-\frac{(c+a^2cx^2)^{3/2}\arctan(ax)^2}{4a}+\frac{3}{8}cx\sqrt{c+a^2cx^2}\arctan(ax)^3$$
$$+\frac{1}{4}x(c+a^2cx^2)^{3/2}\arctan(ax)^3-\frac{3ic^2\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^3}{4a\sqrt{c+a^2cx^2}}-\frac{5ic^2\sqrt{1+a^2x^2}\arctan(a}{a\sqrt{c+a^2}}$$

output
```
-1/4*c*(a^2*c*x^2+c)^(1/2)/a+1/4*c*x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)-9/8*c
*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2/a-1/4*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2
/a+3/8*c*x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^3+1/4*x*(a^2*c*x^2+c)^(3/2)*arc
tan(a*x)^3-9/8*I*c^2*(a^2*x^2+1)^(1/2)*arctan(a*x)^2*polylog(2,I*(1+I*a*x)
/(a^2*x^2+1)^(1/2))/a/(a^2*c*x^2+c)^(1/2)-9/4*I*c^2*(a^2*x^2+1)^(1/2)*poly
log(4,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a/(a^2*c*x^2+c)^(1/2)-5/2*I*c^2*(a^2
*x^2+1)^(1/2)*polylog(2,I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/a/(a^2*c*x^2+c)
^(1/2)+9/4*I*c^2*(a^2*x^2+1)^(1/2)*polylog(4,I*(1+I*a*x)/(a^2*x^2+1)^(1/2)
)/a/(a^2*c*x^2+c)^(1/2)+9/8*I*c^2*(a^2*x^2+1)^(1/2)*arctan(a*x)^2*polylog(
2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a/(a^2*c*x^2+c)^(1/2)+5/2*I*c^2*(a^2*x^2
+1)^(1/2)*polylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/a/(a^2*c*x^2+c)^(1
/2)-9/4*c^2*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(3,-I*(1+I*a*x)/(a^2*x^2+
1)^(1/2))/a/(a^2*c*x^2+c)^(1/2)+9/4*c^2*(a^2*x^2+1)^(1/2)*arctan(a*x)*poly
log(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a/(a^2*c*x^2+c)^(1/2)-5*I*c^2*(a^2*x^
2+1)^(1/2)*arctan(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/a/(a^2*c*x^
2+c)^(1/2)-3/4*I*c^2*(a^2*x^2+1)^(1/2)*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))
*arctan(a*x)^3/a/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 2105 vs. $2(760) = 1520$.

Time = 12.60 (sec) , antiderivative size = 2105, normalized size of antiderivative = 2.77

$$\int \left(c + a^2cx^2\right)^{3/2} \arctan(ax)^3 \, dx = \text{Result too large to show}$$

input
```
Integrate[(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3,x]
```

output
```
((-1/2*I)*c*Sqrt[c*(1 + a^2*x^2)]*(12*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x
] - (3*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2 + I*a*x*Sqrt[1 + a^2*x^2]*ArcTan
[a*x]^3 + 2*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^3 - 3*(2 + ArcTan[a*x]^2
)*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + 3*(2 + ArcTan[a*x]^2)*PolyLog[2, I*
E^(I*ArcTan[a*x])] - (6*I)*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])]
+ (6*I)*ArcTan[a*x]*PolyLog[3, I*E^(I*ArcTan[a*x])] + 6*PolyLog[4, (-I)*E^
(I*ArcTan[a*x])] - 6*PolyLog[4, I*E^(I*ArcTan[a*x])]))/(a*Sqrt[1 + a^2*x^2
]) + (c*((Sqrt[c*(1 + a^2*x^2)]*(-1 + ArcTan[a*x]^2))/(4*Sqrt[1 + a^2*x^2]
) + (Sqrt[c*(1 + a^2*x^2)]*(-(ArcTan[a*x]*(Log[1 - I*E^(I*ArcTan[a*x])] -
Log[1 + I*E^(I*ArcTan[a*x])])) - I*(PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - P
olyLog[2, I*E^(I*ArcTan[a*x])])))/(2*Sqrt[1 + a^2*x^2]) + (Sqrt[c*(1 + a^2
*x^2)]*(-1/8*(Pi^3*Log[Cot[(Pi/2 - ArcTan[a*x])/2]]) - (3*Pi^2*((Pi/2 - Ar
cTan[a*x])*(Log[1 - E^(I*(Pi/2 - ArcTan[a*x]))] - Log[1 + E^(I*(Pi/2 - Arc
Tan[a*x]))]) + I*(PolyLog[2, -E^(I*(Pi/2 - ArcTan[a*x]))] - PolyLog[2, E^(
I*(Pi/2 - ArcTan[a*x]))])))/4 + (3*Pi*((Pi/2 - ArcTan[a*x])^2*(Log[1 - E^(
I*(Pi/2 - ArcTan[a*x]))] - Log[1 + E^(I*(Pi/2 - ArcTan[a*x]))]) + (2*I)*(P
i/2 - ArcTan[a*x])*(PolyLog[2, -E^(I*(Pi/2 - ArcTan[a*x]))] - PolyLog[2, E
^(I*(Pi/2 - ArcTan[a*x]))]) + 2*(-PolyLog[3, -E^(I*(Pi/2 - ArcTan[a*x]))]
+ PolyLog[3, E^(I*(Pi/2 - ArcTan[a*x]))])))/2 - 8*((I/64)*(Pi/2 - ArcTan[a
*x])^4 + (I/4)*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2)^4 - ((Pi/2 - ArcTan[a...
```

## Rubi [A] (verified)

Time = 2.46 (sec) , antiderivative size = 652, normalized size of antiderivative = 0.86, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5415, 5413, 5415, 5425, 5421, 5423, 3042, 4669, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(ax)^3 \left(a^2cx^2+c\right)^{3/2} dx$$

$$\Big\downarrow \text{5415}$$

$$\frac{1}{2}c\int \sqrt{a^2cx^2+c}\arctan(ax)dx+\frac{3}{4}c\int \sqrt{a^2cx^2+c}\arctan(ax)^3dx+\frac{1}{4}x\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{4a}$$

$$\Big\downarrow \text{5413}$$

$$\frac{1}{2}c\left(\frac{1}{2}c\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)+$$
$$\frac{3}{4}c\int \sqrt{a^2cx^2+c}\arctan(ax)^3dx+\frac{1}{4}x\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{4a}$$

$\downarrow$ 5415

$$\frac{1}{2}c\left(\frac{1}{2}c\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)+$$
$$\frac{3}{4}c\left(3c\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}c\int \frac{\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^3\sqrt{a^2cx^2+c}-\frac{3\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a}\right)+$$
$$\frac{1}{4}x\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{4a}$$

$\downarrow$ 5425

$$\frac{1}{2}c\left(\frac{c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)+$$
$$\frac{3}{4}c\left(\frac{3c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+\frac{c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^3}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^3\sqrt{a^2cx^2+c}-\frac{3\arctan(ax)^2\sqrt{a}}{2a}\right.$$
$$\frac{1}{4}x\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{4a}$$

$\downarrow$ 5421

$$\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^3}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\mathrm{PolyLog}\left(2,\right.}{a}\right.}{\sqrt{a^2cx^2+c}}\right.$$
$$\frac{1}{2}c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\mathrm{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a^2}\right.$$
$$\frac{1}{4}x\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{4a}$$

$\downarrow$ 5423

$$\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\int\sqrt{a^2x^2+1}\arctan(ax)^3d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,\right.}{a}\right.}{\sqrt{a^2cx^2+c}}\right.$$

$$\frac{1}{2}c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a^2}\right.$$

$$\frac{1}{4}x\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{4a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\int\arctan(ax)^3\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\right.}{\sqrt{a^2c}}\right.$$

$$\frac{1}{2}c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a^2}\right.$$

$$\frac{1}{4}x\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{4a}$$

$$\Big\downarrow \text{4669}$$

$$\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(-3\int\arctan(ax)^2\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+3\int\arctan(ax)^2\log\left(1+ie^{i\arctan(ax)}\right)d\right.}{2a\sqrt{a^2cx^2+c}}\right.$$

$$\frac{1}{2}c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a^2}\right.$$

$$\frac{1}{4}x\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{4a}$$

$$\Big\downarrow \text{3011}$$

$$\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\operatorname{arcta}\right.\right.}{}\right.$$

$$\frac{1}{2}c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a^2}\right.$$

$$\frac{1}{4}x\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{4a}$$

$$\Big\downarrow \text{7163}$$

$$\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\left(i\int\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)d\arctan(ax)-i\right.\right.\right.}{}\right.$$

$$\frac{1}{2}c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a^2}\right.$$

$$\frac{1}{4}x\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{4a}$$

$$\Big\downarrow \text{2720}$$

$$\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\left(\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)de^{i\operatorname{ar}}\right.\right.\right.}{}\right.$$

$$\frac{1}{2}c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a^2}\right.$$

$$\frac{1}{4}x\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{4a}$$

$$\Big\downarrow \text{7143}$$

$$\frac{1}{2}c\left(\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a^2\right.$$

$$\frac{3}{4}c\left(\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{\sqrt{a^2cx^2+c}}+\frac{c\sqrt{a^2x^2+1}(3(i\right.$$

$$\frac{1}{4}x\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}{4a}$$

input
```
Int[(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3,x]
```

output
```
-1/4*((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2)/a + (x*(c + a^2*c*x^2)^(3/2)*Ar
cTan[a*x]^3)/4 + (c*(-1/2*Sqrt[c + a^2*c*x^2]/a + (x*Sqrt[c + a^2*c*x^2]*A
rcTan[a*x])/2 + (c*Sqrt[1 + a^2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 +
I*a*x]/Sqrt[1 - I*a*x]])/a + (I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 -
 I*a*x]])/a - (I*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/(2*S
qrt[c + a^2*c*x^2])))/2 + (3*c*((-3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(2*
a) + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/2 + (3*c*Sqrt[1 + a^2*x^2]*(((-
2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/a + (I*PolyLog[2
, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*PolyLog[2, (I*Sqrt[1 + I
*a*x])/Sqrt[1 - I*a*x]])/a))/Sqrt[c + a^2*c*x^2] + (c*Sqrt[1 + a^2*x^2]*((
-2*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^3 + 3*(I*ArcTan[a*x]^2*PolyLog
[2, (-I)*E^(I*ArcTan[a*x])] - (2*I)*((-I)*ArcTan[a*x]*PolyLog[3, (-I)*E^(I
*ArcTan[a*x])] + PolyLog[4, (-I)*E^(I*ArcTan[a*x])])) - 3*(I*ArcTan[a*x]^2
*PolyLog[2, I*E^(I*ArcTan[a*x])] - (2*I)*((-I)*ArcTan[a*x]*PolyLog[3, I*E^
(I*ArcTan[a*x])] + PolyLog[4, I*E^(I*ArcTan[a*x])]))))/(2*a*Sqrt[c + a^2*c
*x^2])))/4
```

### Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5413
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Simp[x*(d + e*x^2)
^q*((a + b*ArcTan[c*x])/(2*q + 1)), x] + Simp[2*d*(q/(2*q + 1))   Int[(d +
e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0]
```

rule 5415
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTan[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x])^p,
x], x] + Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*(
a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[q, 0] && GtQ[p, 1]
```

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

## Maple [A] (verified)

Time = 4.75 (sec) , antiderivative size = 466, normalized size of antiderivative = 0.61

| method | result |
|---|---|
| default | $\frac{c\sqrt{c(ax-i)(ax+i)}\left(2\arctan(ax)^3a^3x^3-2\arctan(ax)^2x^2a^2+5x\arctan(ax)^3a+2\arctan(ax)ax-11\arctan(ax)^2-2\right)}{8a} - \frac{c\sqrt{c(ax-}}{}$ |

input
```
int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*c/a*(c*(a*x-I)*(a*x+I))^(1/2)*(2*arctan(a*x)^3*a^3*x^3-2*arctan(a*x)^2
*x^2*a^2+5*x*arctan(a*x)^3*a+2*arctan(a*x)*a*x-11*arctan(a*x)^2-2)-1/8*c*(
c*(a*x-I)*(a*x+I))^(1/2)*(3*arctan(a*x)^3*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/
2))-9*I*arctan(a*x)^2*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*arctan(a
*x)^3*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+9*I*arctan(a*x)^2*polylog(2,I*(1
+I*a*x)/(a^2*x^2+1)^(1/2))+20*arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/
2))+18*arctan(a*x)*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+18*I*polylog(
4,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-20*arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2
+1)^(1/2))-18*arctan(a*x)*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-18*I*po
lylog(4,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-20*I*dilog(1+I*(1+I*a*x)/(a^2*x^2+1
)^(1/2))+20*I*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/a/(a^2*x^2+1)^(1/2)
```

## Fricas [F]

$$\int \left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3\,dx=\int \left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^3\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^3, x)
```

## Sympy [F]

$$\int \left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3\,dx=\int \left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^3(ax)\,dx$$

input
```
integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)**3,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**3, x)
```

## Maxima [F]

$$\int \left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3\,dx=\int \left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan(ax)^3\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^3, x)
```

## Giac [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3\,dx=\int \mathrm{atan}(a\,x)^3\,\left(c\,a^2\,x^2+c\right)^{3/2}\,dx$$

input
```
int(atan(a*x)^3*(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(atan(a*x)^3*(c + a^2*c*x^2)^(3/2), x)
```

## Reduce [F]

$$\int \left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3\,dx=\sqrt{c}\,c\left(\left(\int \sqrt{a^2x^2+1}\,atan(ax)^3\,x^2dx\right)a^2+\int \sqrt{a^2x^2+1}\,atan(ax)^3\,dx\right)$$

input
```
int((a^2*c*x^2+c)^(3/2)*atan(a*x)^3,x)
```

output
```
sqrt(c)*c*(int(sqrt(a**2*x**2 + 1)*atan(a*x)**3*x**2,x)*a**2 + int(sqrt(a*
*2*x**2 + 1)*atan(a*x)**3,x))
```

# 3.424 $\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 726

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x}\,dx = c\sqrt{c+a^2cx^2}\arctan(ax)$$
$$-\frac{1}{2}acx\sqrt{c+a^2cx^2}\arctan(ax)^2$$
$$+\frac{7ic^2\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2}{\sqrt{c+a^2cx^2}}+c\sqrt{c+a^2cx^2}\arctan(ax)^3$$
$$+\frac{1}{3}\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3-\frac{2c^2\sqrt{1+a^2x^2}\arctan(ax)^3\text{arctanh}\left(e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}$$
$$-c^{3/2}\text{arctanh}\left(\frac{a\sqrt{cx}}{\sqrt{c+a^2cx^2}}\right)+\frac{3ic^2\sqrt{1+a^2x^2}\arctan(ax)^2\,\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}-\frac{7ic^2\sqrt{1+a^2x^2}\,\text{arcta}}{}$$

output
```
c*(a^2*c*x^2+c)^(1/2)*arctan(a*x)-1/2*a*c*x*(a^2*c*x^2+c)^(1/2)*arctan(a*x
)^2+3*I*c^2*(a^2*x^2+1)^(1/2)*arctan(a*x)^2*polylog(2,-(1+I*a*x)/(a^2*x^2+
1)^(1/2))/(a^2*c*x^2+c)^(1/2)+c*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^3+1/3*(a^2
*c*x^2+c)^(3/2)*arctan(a*x)^3-2*c^2*(a^2*x^2+1)^(1/2)*arctan(a*x)^3*arctan
h((1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)-c^(3/2)*arctanh(a*c^(1/
2)*x/(a^2*c*x^2+c)^(1/2))+7*I*c^2*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(2,
I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)-7*I*c^2*(a^2*x^2+1)^(1/
2)*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/
2)-3*I*c^2*(a^2*x^2+1)^(1/2)*arctan(a*x)^2*polylog(2,(1+I*a*x)/(a^2*x^2+1)
^(1/2))/(a^2*c*x^2+c)^(1/2)+7*I*c^2*(a^2*x^2+1)^(1/2)*arctan((1+I*a*x)/(a^
2*x^2+1)^(1/2))*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2)-6*c^2*(a^2*x^2+1)^(1/2)*
arctan(a*x)*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)+7*
c^2*(a^2*x^2+1)^(1/2)*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2
+c)^(1/2)-7*c^2*(a^2*x^2+1)^(1/2)*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))
/(a^2*c*x^2+c)^(1/2)+6*c^2*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(3,(1+I*a*
x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)+6*I*c^2*(a^2*x^2+1)^(1/2)*polylo
g(4,(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)-6*I*c^2*(a^2*x^2+1)^(
1/2)*polylog(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 555, normalized size of antiderivative = 0.76

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x}\,dx = \frac{c\sqrt{c+a^2cx^2}\left(-3i\pi^4-24\coth^{-1}\left(\frac{ax}{\sqrt{1+a^2x^2}}\right)+\frac{24\arctan(ax)}{\sqrt{1+a^2x^2}}+\frac{24a^2x^2\arctan(\cdots}{\sqrt{1+a^2x^2}}\right.}{}$$

input
```
Integrate[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3)/x,x]
```

output
```
(c*Sqrt[c + a^2*c*x^2]*((-3*I)*Pi^4 - 24*ArcCoth[(a*x)/Sqrt[1 + a^2*x^2]]
+ (24*ArcTan[a*x])/Sqrt[1 + a^2*x^2] + (24*a^2*x^2*ArcTan[a*x])/Sqrt[1 + a
^2*x^2] - (12*a*x*ArcTan[a*x]^2)/Sqrt[1 + a^2*x^2] - (12*a^3*x^3*ArcTan[a*
x]^2)/Sqrt[1 + a^2*x^2] + (24*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 +
 (32*ArcTan[a*x]^3)/Sqrt[1 + a^2*x^2] + (40*a^2*x^2*ArcTan[a*x]^3)/Sqrt[1
+ a^2*x^2] + (8*a^4*x^4*ArcTan[a*x]^3)/Sqrt[1 + a^2*x^2] + (6*I)*ArcTan[a*
x]^4 + 24*ArcTan[a*x]^3*Log[1 - E^((-I)*ArcTan[a*x])] - 72*ArcTan[a*x]^2*L
og[1 - I*E^(I*ArcTan[a*x])] + 72*ArcTan[a*x]^2*Log[1 + I*E^(I*ArcTan[a*x])
] - 24*ArcTan[a*x]^3*Log[1 + E^(I*ArcTan[a*x])] + (72*I)*ArcTan[a*x]^2*Pol
yLog[2, E^((-I)*ArcTan[a*x])] + (72*I)*ArcTan[a*x]^2*PolyLog[2, -E^(I*ArcT
an[a*x])] - (168*I)*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + (168*
I)*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] + 144*ArcTan[a*x]*PolyLog[3
, E^((-I)*ArcTan[a*x])] - 144*ArcTan[a*x]*PolyLog[3, -E^(I*ArcTan[a*x])] +
 168*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] - 168*PolyLog[3, I*E^(I*ArcTan[a*x
])] - (144*I)*PolyLog[4, E^((-I)*ArcTan[a*x])] - (144*I)*PolyLog[4, -E^(I*
ArcTan[a*x])]))/(24*Sqrt[1 + a^2*x^2])
```

## Rubi [A] (verified)

Time = 6.27 (sec) , antiderivative size = 629, normalized size of antiderivative = 0.87, number of steps used = 29, number of rules used = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 1.167$, Rules used = {5485, 5465, 5415, 224, 219, 5425, 5423, 3042, 4669, 3011, 2720, 5485, 5465, 5425, 5423, 3042, 4669, 3011, 2720, 5493, 5491, 3042, 4671, 3011, 7143, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3 \left(a^2cx^2+c\right)^{3/2}}{x} \, dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c \int x\sqrt{a^2cx^2+c}\arctan(ax)^3dx + c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x}dx$$

$$\Big\downarrow \text{5465}$$

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\int \sqrt{a^2cx^2+c}\arctan(ax)^2dx}{a}\right)+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x}dx$$

$\Big\downarrow$ 5415

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{1}{2}c\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int \frac{1}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)}{a}\right.$$
$$c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x}dx$$

$\Big\downarrow$ 224

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{1}{2}c\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int \frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-}{a}\right.$$
$$c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x}dx$$

$\Big\downarrow$ 219

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{1}{2}c\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\frac{\sqrt{c}\text{arct}}{}}{a}\right.$$
$$c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x}dx$$

$\Big\downarrow$ 5425

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\frac{\sqrt{}}{}}{a}\right.$$
$$c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x}dx$$

$\Big\downarrow$ 5423

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{c\sqrt{a^2x^2+1}\int\sqrt{a^2x^2+1}\arctan(ax)^2d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(}{}}{a}\right.$$

$$c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x}dx$$

$$\Big\downarrow \text{3042}$$

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{c\sqrt{a^2x^2+1}\int\arctan(ax)^2\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}}{a}\right.$$

$$c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x}dx$$

$$\Big\downarrow \text{4669}$$

$$c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x}dx+$$

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{c\sqrt{a^2x^2+1}\left(-2\int\arctan(ax)\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+2\int\arctan(ax)\log\left(1+ie^{i\arctan(ax)}\right.\right.}{2a\sqrt{a^2cx^2+c}}}{}\right.$$

$$\Big\downarrow \text{3011}$$

$$c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x}dx+$$

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-i\int\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(ax)\right.\right.}{}\right.$$

$$\Big\downarrow \text{2720}$$

$$c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x}dx+$$

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)\right.\right.}{}\right.$$

$$\Big\downarrow \text{5485}$$

$$c\left(a^2c\int \frac{x\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx\right)+$$
$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)\right.\right.}{}\right.$$

$\downarrow$ 5465

$$c\left(a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a}\right)+c\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx\right)+$$
$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)\right.\right.}{}\right.$$

$\downarrow$ 5425

$$c\left(a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{a\sqrt{a^2cx^2+c}}\right)+c\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx\right)+$$
$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)\right.\right.}{}\right.$$

$\downarrow$ 5423

$$c\left(a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\int \sqrt{a^2x^2+1}\arctan(ax)^2d\arctan(ax)}{a^2\sqrt{a^2cx^2+c}}\right)+c\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx\right)$$
$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)\right.\right.}{}\right.$$

$\downarrow$ 3042

$$c\left(c\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx+a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\,\text{arcta}}{a^2\sqrt{a^2cx^2+c}}\right.\right.$$

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\,\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\,\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)\right.\right.}{}\right.$$

$$\Big\downarrow \; 4669$$

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\,\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\,\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)\right.\right.}{}\right.$$

$$c\left(c\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx+a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(-2\int \arctan(ax)\log\left(1-ie^{i\arctan(ax)}\right)d\,\text{a}\right.}{}\right.\right.$$

$$\Big\downarrow \; 3011$$

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\,\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\,\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)\right.\right.}{}\right.$$

$$c\left(c\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx+a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\,\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)\right.\right.}{}\right.\right.$$

$$\Big\downarrow \; 2720$$

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\,\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\,\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)\right.\right.}{}\right.$$

$$c\left(c\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx+a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\,\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)\right.\right.}{}\right.\right.$$

$$\Big\downarrow \; 5493$$

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)\right.\right.}{}\right.$$

$$c\left(\frac{c\sqrt{a^2x^2+1}\int\frac{\arctan(ax)^3}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arc}\right.\right.\right.}{}\right.\right.$$

$\downarrow$ 5491

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)\right.\right.}{}\right.$$

$$c\left(\frac{c\sqrt{a^2x^2+1}\int\frac{\sqrt{a^2x^2+1}\arctan(ax)^3}{ax}d\arctan(ax)}{\sqrt{a^2cx^2+c}}+a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{P}\right.\right.}{}\right.\right.$$

$\downarrow$ 3042

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)\right.\right.}{}\right.$$

$$c\left(\frac{c\sqrt{a^2x^2+1}\int\arctan(ax)^3\csc(\arctan(ax))d\arctan(ax)}{\sqrt{a^2cx^2+c}}+a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\operatorname{ar}\right.\right.}{}\right.\right.$$

$\downarrow$ 4671

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)\right.\right.}{}\right.$$

$$c\left(\frac{c\sqrt{a^2x^2+1}\left(-3\int\arctan(ax)^2\log\left(1-e^{i\arctan(ax)}\right)d\arctan(ax)+3\int\arctan(ax)^2\log\left(1+e^{i\arctan(ax)}\right)d\operatorname{arct}\right.}{\sqrt{a^2cx^2+c}}\right.$$

$\downarrow$ 3011

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)\right.\right.}{}\right.$$

$$c\left(\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\int\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)d\arctan(\right.\right.}{}\right.$$

$\downarrow$ 7143

$$c\left(\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\int\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)d\arctan(a\right.\right.}{}\right.$$

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)\right)-2\left(i\arctan(a\right.\right.}{2a\sqrt{a^2cx}}\right.$$

$\downarrow$ 7163

$$c\left(\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\left(i\int\operatorname{PolyLog}\left(3,-e^{i\arctan(ax)}\right)d\arctan(ax)-i\operatorname{arc}\right.\right.\right.}{}\right.$$

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)\right)-2\left(i\arctan(a\right.\right.}{2a\sqrt{a^2cx}}\right.$$

$\downarrow$ 2720

$$c\left(\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\left(\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(3,-e^{i\arctan(ax)}\right)de^{i\arctan}\right.\right.\right.}{}\right.$$

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)\right)-2\left(i\arctan(a\right.\right.}{2a\sqrt{a^2cx}}\right.$$

$\downarrow$ 7143

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)\right)-2\left(i\arctan(a\right.\right.}{2a\sqrt{a^2cx}}\right.$$

$$c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\arctan(ax)^3\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)+3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\left(\operatorname{PolyLog}\left(\right.\right.\right.\right.}{}\right.$$

input
```
Int[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3)/x,x]
```

output
```
a^2*c*(((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3)/(3*a^2*c) - (-((Sqrt[c + a^2*
c*x^2]*ArcTan[a*x])/a) + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/2 + (Sqrt[c
]*ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/a + (c*Sqrt[1 + a^2*x^2]*((-
2*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 + 2*(I*ArcTan[a*x]*PolyLog[2,
 (-I)*E^(I*ArcTan[a*x])] - PolyLog[3, (-I)*E^(I*ArcTan[a*x])]) - 2*(I*ArcT
an[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] - PolyLog[3, I*E^(I*ArcTan[a*x])])
))/(2*a*Sqrt[c + a^2*c*x^2]))/a) + c*(a^2*c*((Sqrt[c + a^2*c*x^2]*ArcTan[a
*x]^3)/(a^2*c) - (3*Sqrt[1 + a^2*x^2]*((-2*I)*ArcTan[E^(I*ArcTan[a*x])]*Ar
cTan[a*x]^2 + 2*(I*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - PolyLo
g[3, (-I)*E^(I*ArcTan[a*x])]) - 2*(I*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[
a*x])] - PolyLog[3, I*E^(I*ArcTan[a*x])])))/(a^2*Sqrt[c + a^2*c*x^2])) + (
c*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]^3*ArcTanh[E^(I*ArcTan[a*x])] + 3*(I*Ar
cTan[a*x]^2*PolyLog[2, -E^(I*ArcTan[a*x])] - (2*I)*((-I)*ArcTan[a*x]*PolyL
og[3, -E^(I*ArcTan[a*x])] + PolyLog[4, -E^(I*ArcTan[a*x])])) - 3*(I*ArcTan
[a*x]^2*PolyLog[2, E^(I*ArcTan[a*x])] - (2*I)*((-I)*ArcTan[a*x]*PolyLog[3,
 E^(I*ArcTan[a*x])] + PolyLog[4, E^(I*ArcTan[a*x])]))))/Sqrt[c + a^2*c*x^2
])
```

## Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5415
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTan[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x])^p,
x], x] + Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*(
a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[q, 0] && GtQ[p, 1]
```

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5491
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Simp[1/Sqrt[d]   Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTa
n[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] &&
GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

## Maple [A] (verified)

Time = 5.58 (sec) , antiderivative size = 511, normalized size of antiderivative = 0.70

| method | result |
|---|---|
| default | $\frac{c\sqrt{c(ax-i)(ax+i)}\,\arctan(ax)\left(2\arctan(ax)^2x^2a^2-3\arctan(ax)ax+8\arctan(ax)^2+6\right)}{6}-\frac{c\sqrt{c(ax-i)(ax+i)}\,\left(2\arctan(ax)^3\ln\left(\right.\right.}{}$ |

input
```
int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3/x,x,method=_RETURNVERBOSE)
```

output
```
1/6*c*(c*(a*x-I)*(a*x+I))^(1/2)*arctan(a*x)*(2*arctan(a*x)^2*x^2*a^2-3*arc
tan(a*x)*a*x+8*arctan(a*x)^2+6)-1/2*c*(c*(a*x-I)*(a*x+I))^(1/2)*(2*arctan(
a*x)^3*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*arctan(a*x)^3*ln(1-(1+I*a*x)/(a
^2*x^2+1)^(1/2))-6*I*arctan(a*x)^2*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))
+6*I*arctan(a*x)^2*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-7*arctan(a*x)^2*
ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+7*arctan(a*x)^2*ln(1-I*(1+I*a*x)/(a^2*
x^2+1)^(1/2))+14*I*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-1
4*I*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+12*arctan(a*x)*po
lylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-12*arctan(a*x)*polylog(3,(1+I*a*x)/(
a^2*x^2+1)^(1/2))+12*I*polylog(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-12*I*polylo
g(4,(1+I*a*x)/(a^2*x^2+1)^(1/2))-4*I*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))-1
4*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+14*polylog(3,I*(1+I*a*x)/(a^2*
x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)
```

## Fricas [F]

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}\arctan{(ax)}^3}{x}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3/x,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^3/x, x)
```

## Sympy [F]

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x}\,dx = \int \frac{(c(a^2x^2+1))^{\frac{3}{2}}\operatorname{atan}^3{(ax)}}{x}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)**3/x,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**3/x, x)
```

## Maxima [F]

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}\arctan{(ax)}^3}{x}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3/x,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^3/x, x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x}\,dx = \int \frac{\operatorname{atan}(a\,x)^3\,(c\,a^2\,x^2+c)^{3/2}}{x}\,dx$$

input
```
int((atan(a*x)^3*(c + a^2*c*x^2)^(3/2))/x,x)
```

output
```
int((atan(a*x)^3*(c + a^2*c*x^2)^(3/2))/x, x)
```

## Reduce [F]

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x}\,dx = \sqrt{c}\,c\left(\int \frac{\sqrt{a^2x^2+1}\,atan(ax)^3}{x}dx + \left(\int \sqrt{a^2x^2+1}\,atan(ax)^3\,xdx\right)a^2\right)$$

input
```
int((a^2*c*x^2+c)^(3/2)*atan(a*x)^3/x,x)
```

output
```
sqrt(c)*c*(int((sqrt(a**2*x**2 + 1)*atan(a*x)**3)/x,x) + int(sqrt(a**2*x**
2 + 1)*atan(a*x)**3*x,x)*a**2)
```

# 3.425 $\int \frac{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3}{x^2}\,dx$

## Optimal result

Integrand size = 24, antiderivative size = 901

$$
\begin{aligned}
\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x^2}\,dx = & -\frac{3}{2}ac\sqrt{c+a^2cx^2}\arctan(ax)^2 \\
& -\frac{c\sqrt{c+a^2cx^2}\arctan(ax)^3}{x}+\frac{1}{2}a^2cx\sqrt{c+a^2cx^2}\arctan(ax)^3 \\
& -\frac{3iac^2\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^3}{\sqrt{c+a^2cx^2}} \\
& -\frac{6iac^2\sqrt{1+a^2x^2}\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} \\
& -\frac{6ac^2\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} \\
& +\frac{6iac^2\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} \\
& +\frac{9iac^2\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)}{2\sqrt{c+a^2cx^2}} \\
& -\frac{9iac^2\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{PolyLog}\left(2,ie^{i\arctan(ax)}\right)}{2\sqrt{c+a^2cx^2}} \\
& -\frac{6iac^2\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} \\
& +\frac{3iac^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} \\
& -\frac{3iac^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} \\
& -\frac{6ac^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,-e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} \\
& -\frac{9ac^2\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} \\
& +\frac{9ac^2\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(3,ie^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} \\
& +\frac{6ac^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} \\
& -\frac{9iac^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(4,-ie^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} \\
& +\frac{9iac^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(4,ie^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}
\end{aligned}
$$

output
```
-3/2*a*c*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2-c*(a^2*c*x^2+c)^(1/2)*arctan(a*
x)^3/x+1/2*a^2*c*x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^3+9/2*I*a*c^2*(a^2*x^2+
1)^(1/2)*arctan(a*x)^2*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^
2+c)^(1/2)+9*I*a*c^2*(a^2*x^2+1)^(1/2)*polylog(4,I*(1+I*a*x)/(a^2*x^2+1)^(
1/2))/(a^2*c*x^2+c)^(1/2)-6*a*c^2*(a^2*x^2+1)^(1/2)*arctan(a*x)^2*arctanh(
(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)-6*I*a*c^2*(a^2*x^2+1)^(1/
2)*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)-
6*I*a*c^2*(a^2*x^2+1)^(1/2)*arctan(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*x)^(
1/2))/(a^2*c*x^2+c)^(1/2)-9*I*a*c^2*(a^2*x^2+1)^(1/2)*polylog(4,-I*(1+I*a*
x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)+3*I*a*c^2*(a^2*x^2+1)^(1/2)*poly
log(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/(a^2*c*x^2+c)^(1/2)-3*I*a*c^2*(a
^2*x^2+1)^(1/2)*polylog(2,I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/(a^2*c*x^2+c)
^(1/2)-3*I*a*c^2*(a^2*x^2+1)^(1/2)*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))*arc
tan(a*x)^3/(a^2*c*x^2+c)^(1/2)-6*a*c^2*(a^2*x^2+1)^(1/2)*polylog(3,-(1+I*a
*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)-9*a*c^2*(a^2*x^2+1)^(1/2)*arcta
n(a*x)*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)+9*a*c
^2*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/
(a^2*c*x^2+c)^(1/2)+6*a*c^2*(a^2*x^2+1)^(1/2)*polylog(3,(1+I*a*x)/(a^2*x^2
+1)^(1/2))/(a^2*c*x^2+c)^(1/2)+6*I*a*c^2*(a^2*x^2+1)^(1/2)*arctan(a*x)*pol
ylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)-9/2*I*a*c^2*(a...
```

## Mathematica [A] (warning: unable to verify)

Time = 4.35 (sec) , antiderivative size = 1387, normalized size of antiderivative = 1.54

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x^2}\,dx = \text{Too large to display}$$

input
```
Integrate[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3)/x^2,x]
```

output
```
(a*c*Sqrt[c + a^2*c*x^2]*((-7*I)*Pi^4*Sqrt[1 + a^2*x^2] - (8*I)*Pi^3*Sqrt[
1 + a^2*x^2]*ArcTan[a*x] - (384*I)*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*
x])]*ArcTan[a*x] - 96*ArcTan[a*x]^2 - 96*a^2*x^2*ArcTan[a*x]^2 + (24*I)*Pi
^2*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2 - (64*ArcTan[a*x]^3)/(a*x) - 32*a*x*Arc
Tan[a*x]^3 + 32*a^3*x^3*ArcTan[a*x]^3 - (32*I)*Pi*Sqrt[1 + a^2*x^2]*ArcTan
[a*x]^3 - (64*I)*Sqrt[1 + a^2*x^2]*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^3
 + (16*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^4 + 48*Pi^2*Sqrt[1 + a^2*x^2]*ArcT
an[a*x]*Log[1 - I/E^(I*ArcTan[a*x])] - 96*Pi*Sqrt[1 + a^2*x^2]*ArcTan[a*x]
^2*Log[1 - I/E^(I*ArcTan[a*x])] - 8*Pi^3*Sqrt[1 + a^2*x^2]*Log[1 + I/E^(I*
ArcTan[a*x])] + 64*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^3*Log[1 + I/E^(I*ArcTan[a
*x])] + 192*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*Log[1 - E^(I*ArcTan[a*x])] + 8
*Pi^3*Sqrt[1 + a^2*x^2]*Log[1 + I*E^(I*ArcTan[a*x])] - 48*Pi^2*Sqrt[1 + a^
2*x^2]*ArcTan[a*x]*Log[1 + I*E^(I*ArcTan[a*x])] + 96*Pi*Sqrt[1 + a^2*x^2]*
ArcTan[a*x]^2*Log[1 + I*E^(I*ArcTan[a*x])] - 64*Sqrt[1 + a^2*x^2]*ArcTan[a
*x]^3*Log[1 + I*E^(I*ArcTan[a*x])] - 192*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*L
og[1 + E^(I*ArcTan[a*x])] + 8*Pi^3*Sqrt[1 + a^2*x^2]*Log[2*Sqrt[1 + a^2*x^
2]*Sin[(Pi + 2*ArcTan[a*x])/4]^2] + (192*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^
2*PolyLog[2, (-I)/E^(I*ArcTan[a*x])] + (48*I)*Pi*Sqrt[1 + a^2*x^2]*(Pi - 4
*ArcTan[a*x])*PolyLog[2, I/E^(I*ArcTan[a*x])] + (384*I)*Sqrt[1 + a^2*x^2]*
ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])] + (192*I)*Sqrt[1 + a^2*x^2]*...
```

## Rubi [A] (verified)

Time = 6.40 (sec) , antiderivative size = 768, normalized size of antiderivative = 0.85, number of steps used = 26, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 1.042$, Rules used = {5485, 5415, 5425, 5421, 5423, 3042, 4669, 3011, 5485, 5425, 5423, 3042, 4669, 3011, 5479, 5493, 5491, 3042, 4671, 3011, 2720, 7143, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3 \left(a^2cx^2+c\right)^{3/2}}{x^2}dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\int \sqrt{a^2cx^2+c}\arctan(ax)^3dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^2}dx$$

$$\Big\downarrow \text{5415}$$

$$a^2c\left(3c\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}c\int \frac{\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^3\sqrt{a^2cx^2+c}-\frac{3\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a}\right)+$$
$$c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^2}dx$$

$\downarrow$ 5425

$$a^2c\left(\frac{3c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+\frac{c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^3}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^3\sqrt{a^2cx^2+c}-\frac{3\arctan(ax)^2\sqrt{a}}{2a}\right.$$
$$c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^2}dx$$

$\downarrow$ 5421

$$c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^2}dx+$$
$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^3}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\right.}{a}\right.}{\sqrt{a^2cx^2+c}}\right.$$

$\downarrow$ 5423

$$c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^2}dx+$$
$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\int \sqrt{a^2x^2+1}\arctan(ax)^3d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2\right.}{}\right.}{\sqrt{a^2cx^2+c}}\right.$$

$\downarrow$ 3042

$$c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^2}dx+$$
$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\int \arctan(ax)^3\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\right.}{\sqrt{a^2c}}\right.$$

$\downarrow$ 4669

$$c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^2}dx+$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(-3\int \arctan(ax)^2\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+3\int \arctan(ax)^2\log\left(1+ie^{i\arctan(ax)}\right)d\right.}{2a\sqrt{a^2cx^2+c}}\right.$$

$$\Big\downarrow \text{3011}$$

$$c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^2}dx+$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int \arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan\right.\right.}{}\right.$$

$$\Big\downarrow \text{5485}$$

$$c\left(a^2c\int \frac{\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^3}{x^2\sqrt{a^2cx^2+c}}dx\right)+$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int \arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan\right.\right.}{}\right.$$

$$\Big\downarrow \text{5425}$$

$$c\left(\frac{a^2c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^3}{\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)^3}{x^2\sqrt{a^2cx^2+c}}dx\right)+$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int \arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan\right.\right.}{}\right.$$

$$\Big\downarrow \text{5423}$$

$$c\left(\frac{ac\sqrt{a^2x^2+1}\int \sqrt{a^2x^2+1}\arctan(ax)^3d\arctan(ax)}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)^3}{x^2\sqrt{a^2cx^2+c}}dx\right)+$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int \arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan\right.\right.}{}\right.$$

$$\Big\downarrow \text{ 3042}$$

$$c\left(c\int \frac{\arctan(ax)^3}{x^2\sqrt{a^2cx^2+c}}dx+\frac{ac\sqrt{a^2x^2+1}\int \arctan(ax)^3\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{\sqrt{a^2cx^2+c}}\right)+$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int \arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arcta\right.\right.}{}\right.$$

$$\Big\downarrow \text{ 4669}$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int \arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arcta\right.\right.}{}\right.$$

$$c\left(c\int \frac{\arctan(ax)^3}{x^2\sqrt{a^2cx^2+c}}dx+\frac{ac\sqrt{a^2x^2+1}\left(-3\int \arctan(ax)^2\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+3\int \arctan(ax)^2\right.}{\sqrt{a^2cx^2+c}}\right.$$

$$\Big\downarrow \text{ 3011}$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int \arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arcta\right.\right.}{}\right.$$

$$c\left(c\int \frac{\arctan(ax)^3}{x^2\sqrt{a^2cx^2+c}}dx+\frac{ac\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int \arctan(ax)\operatorname{PolyLog}\left(\right.\right.\right.}{}\right.$$

$$\Big\downarrow \text{ 5479}$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int \arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arcta\right.\right.}{}\right.$$

$$c\left(c\left(3a\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}\right)+\frac{ac\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(a\right.\right.\right.}{}\right.$$

$$\Big\downarrow \text{ 5493}$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\operatorname{arcta}\right.\right.}{}\right.$$

$$c\left(c\left(\frac{3a\sqrt{a^2x^2+1}\int\frac{\arctan(ax)^2}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}\right)+\frac{ac\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie\right.\right.\right.}{}\right.\right.$$

$\downarrow$ 5491

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\operatorname{arcta}\right.\right.}{}\right.$$

$$c\left(c\left(\frac{3a\sqrt{a^2x^2+1}\int\frac{\sqrt{a^2x^2+1}\arctan(ax)^2}{ax}d\arctan(ax)}{\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}\right)+\frac{ac\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)\right.\right.}{}\right.\right.$$

$\downarrow$ 3042

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\operatorname{arcta}\right.\right.}{}\right.$$

$$c\left(c\left(\frac{3a\sqrt{a^2x^2+1}\int\arctan(ax)^2\csc(\arctan(ax))d\arctan(ax)}{\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}\right)+\frac{ac\sqrt{a^2x^2+1}\left(3\left(i\right.\right.}{}\right.\right.$$

$\downarrow$ 4671

$$c\left(c\left(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(-2\int\arctan(ax)\log\left(1-e^{i\arctan(ax)}\right)d\arctan(ax)+2\int\operatorname{arctan}\right.}{\sqrt{a^2cx}}\right.\right.$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\operatorname{arcta}\right.\right.}{}\right.$$

$\downarrow$ 3011

$$c\left(c\left(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-i\int\operatorname{PolyLog}\left(2,-e^{i a\right.\right.\right.$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\operatorname{arcta}\right.\right.\right.$$

$\downarrow$ 2720

$$c\left(c\left(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyL}\right.\right.\right.$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\operatorname{arcta}\right.\right.\right.$$

$\downarrow$ 7143

$$c\left(\frac{ac\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\operatorname{arctan}\right.\right.\right.$$

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\operatorname{arcta}\right.\right.\right.$$

$\downarrow$ 7163

$$c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^3-\frac{3\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyL}}{}\right.}{\sqrt{a^2cx^2+c}}\right.$$

$$c\left(c\left(\frac{3a\sqrt{a^2x^2+1}\left(-2\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)\arctan(ax)^2+2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\operatorname{PolyLog}\left(\right.\right.\right.}{\sqrt{a^2cx^2+c}}\right.\right.$$

$\downarrow$ 2720

$$c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^3-\frac{3\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyL}}{}\right.}{\sqrt{a^2cx^2+c}}\right.$$

$$c\left(c\left(\frac{3a\sqrt{a^2x^2+1}\left(-2\text{arctanh}\left(e^{i\arctan(ax)}\right)\arctan(ax)^2+2\left(i\arctan(ax)\,\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\text{PolyLog}\left(\right.\right.\right.}{\sqrt{a^2cx^2+c}}\right.\right.$$

$$\Big\downarrow \text{7143}$$

$$c\left(c\left(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(-2\arctan(ax)^2\text{arctanh}\left(e^{i\arctan(ax)}\right)+2\left(i\arctan(ax)\,\text{PolyLog}\left(\right.\right.\right.}{}\right.\right.$$

$$a^2c\left(\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\,\text{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{\sqrt{a^2cx^2+c}}+\frac{c\sqrt{a^2x^2+1}\left(3\left(i\right.\right.}{}\right.$$

input
```
Int[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3)/x^2,x]
```

output
```
a^2*c*((-3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(2*a) + (x*Sqrt[c + a^2*c*x^
2]*ArcTan[a*x]^3)/2 + (3*c*Sqrt[1 + a^2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[S
qrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/a + (I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/
Sqrt[1 - I*a*x]])/a - (I*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/
a))/Sqrt[c + a^2*c*x^2] + (c*Sqrt[1 + a^2*x^2]*((-2*I)*ArcTan[E^(I*ArcTan[
a*x])]*ArcTan[a*x]^3 + 3*(I*ArcTan[a*x]^2*PolyLog[2, (-I)*E^(I*ArcTan[a*x]
)] - (2*I)*((-I)*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] + PolyLog[
4, (-I)*E^(I*ArcTan[a*x])])) - 3*(I*ArcTan[a*x]^2*PolyLog[2, I*E^(I*ArcTan
[a*x])] - (2*I)*((-I)*ArcTan[a*x]*PolyLog[3, I*E^(I*ArcTan[a*x])] + PolyLo
g[4, I*E^(I*ArcTan[a*x])]))))/(2*a*Sqrt[c + a^2*c*x^2])) + c*(c*(-((Sqrt[c
 + a^2*c*x^2]*ArcTan[a*x]^3)/(c*x)) + (3*a*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*
x]^2*ArcTanh[E^(I*ArcTan[a*x])] + 2*(I*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan
[a*x])] - PolyLog[3, -E^(I*ArcTan[a*x])]) - 2*(I*ArcTan[a*x]*PolyLog[2, E^
(I*ArcTan[a*x])] - PolyLog[3, E^(I*ArcTan[a*x])])))/Sqrt[c + a^2*c*x^2]) +
 (a*c*Sqrt[1 + a^2*x^2]*((-2*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^3 +
3*(I*ArcTan[a*x]^2*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - (2*I)*((-I)*ArcTan
[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] + PolyLog[4, (-I)*E^(I*ArcTan[a*x
])])) - 3*(I*ArcTan[a*x]^2*PolyLog[2, I*E^(I*ArcTan[a*x])] - (2*I)*((-I)*A
rcTan[a*x]*PolyLog[3, I*E^(I*ArcTan[a*x])] + PolyLog[4, I*E^(I*ArcTan[a*x]
)]))))/Sqrt[c + a^2*c*x^2])
```

### Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5415
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTan[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x])^p,
x], x] + Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*(
a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[q, 0] && GtQ[p, 1]
```

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5491
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Simp[1/Sqrt[d]   Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTa
n[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] &&
GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
 + b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

## Maple [A] (verified)

Time = 5.79 (sec) , antiderivative size = 602, normalized size of antiderivative = 0.67

| method | result |
|---|---|
| default | $\frac{c\sqrt{c(ax-i)(ax+i)}\,\arctan(ax)^2\,(x^2a^2\arctan(ax)-3ax-2\arctan(ax))}{2x} - \frac{3ica\sqrt{c(ax-i)(ax+i)}\,\left(-i\arctan(ax)^3\ln\left(1+\frac{i(iax+1)}{\sqrt{a^2x^2+1}}\right.\right.}{}$ |

input
```
int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*c*(c*(a*x-I)*(a*x+I))^(1/2)*arctan(a*x)^2*(x^2*a^2*arctan(a*x)-3*a*x-2
*arctan(a*x))/x-3/2*I*c*a*(c*(a*x-I)*(a*x+I))^(1/2)*(-I*arctan(a*x)^3*ln(1
+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+4*I*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))
-2*I*arctan(a*x)^2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+I*arctan(a*x)^3*ln(1-
I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I*arctan(a*x)*polylog(3,-I*(1+I*a*x)/(a^2
*x^2+1)^(1/2))-2*I*arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*arcta
n(a*x)^2*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-4*I*polylog(3,-(1+I*a*x
)/(a^2*x^2+1)^(1/2))+6*I*arctan(a*x)*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/
2))+3*arctan(a*x)^2*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-4*arctan(a*x)
*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*I*arctan(a*x)*ln(1-I*(1+I*a*x)/
(a^2*x^2+1)^(1/2))+4*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*
I*arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*polylog(2,-I*(1+I*a*x)
/(a^2*x^2+1)^(1/2))+6*polylog(4,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*polylog(
2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*polylog(4,I*(1+I*a*x)/(a^2*x^2+1)^(1/2)
))/(a^2*x^2+1)^(1/2)
```

## Fricas [F]

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x^2}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)^3}{x^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3/x^2,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^3/x^2, x)
```

## Sympy [F]

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x^2}\,dx = \int \frac{(c(a^2x^2+1))^{\frac{3}{2}}\operatorname{atan}^3(ax)}{x^2}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)**3/x**2,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**3/x**2, x)
```

## Maxima [F]

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x^2}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)^3}{x^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3/x^2,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^3/x^2, x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3/x^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x^2}\,dx = \int \frac{\mathrm{atan}(a\,x)^3\,(c\,a^2\,x^2+c)^{3/2}}{x^2}\,dx$$

input
```
int((atan(a*x)^3*(c + a^2*c*x^2)^(3/2))/x^2,x)
```

output
```
int((atan(a*x)^3*(c + a^2*c*x^2)^(3/2))/x^2, x)
```

## Reduce [F]

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x^2}\,dx = \sqrt{c}\,c\left(\int \frac{\sqrt{a^2x^2+1}\,atan(ax)^3}{x^2}dx + \left(\int \sqrt{a^2x^2+1}\,atan(ax)^3\,dx\right)a^2\right)$$

input
```
int((a^2*c*x^2+c)^(3/2)*atan(a*x)^3/x^2,x)
```

output
```
sqrt(c)*c*(int((sqrt(a**2*x**2 + 1)*atan(a*x)**3)/x**2,x) + int(sqrt(a**2*
x**2 + 1)*atan(a*x)**3,x)*a**2)
```

# 3.426 $\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x^3}\,dx$

## Optimal result

Integrand size = 24, antiderivative size = 919

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x^3}\,dx = -\frac{3ac\sqrt{c+a^2cx^2}\arctan(ax)^2}{2x} + \frac{6ia^2c^2\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2}{\sqrt{c+a^2cx^2}} + a^2c\sqrt{c+a^2cx^2}\arctan(ax)^3 - \frac{c\sqrt{c+a^2cx^2}\arctan(ax)^3}{2x^2} - \frac{3a^2c^2\sqrt{1+a^2x^2}\arctan(ax)^3\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} - \frac{6a^2c^2\sqrt{1+a^2x^2}\arctan(ax)\operatorname{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} + \frac{9ia^2c^2\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)}{2\sqrt{c+a^2cx^2}} - \frac{6ia^2c^2\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} + \frac{6ia^2c^2\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,ie^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} - \frac{9ia^2c^2\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{PolyLog}\left(2,e^{i\arctan(ax)}\right)}{2\sqrt{c+a^2cx^2}} + \frac{3ia^2c^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} - \frac{3ia^2c^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} - \frac{9a^2c^2\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(3,-e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} + \frac{6a^2c^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} - \frac{6a^2c^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,ie^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} + \frac{9a^2c^2\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(3,e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} - \frac{9ia^2c^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(4,-e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} + \frac{9ia^2c^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(4,e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}$$

output
```
-3/2*a*c*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2/x+9/2*I*a^2*c^2*(a^2*x^2+1)^(1/
2)*arctan(a*x)^2*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/
2)+a^2*c*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^3-1/2*c*(a^2*c*x^2+c)^(1/2)*arcta
n(a*x)^3/x^2-3*a^2*c^2*(a^2*x^2+1)^(1/2)*arctan(a*x)^3*arctanh((1+I*a*x)/(
a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)-6*a^2*c^2*(a^2*x^2+1)^(1/2)*arctan(a
*x)*arctanh((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/(a^2*c*x^2+c)^(1/2)-9*I*a^2*c
^2*(a^2*x^2+1)^(1/2)*polylog(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)
^(1/2)-3*I*a^2*c^2*(a^2*x^2+1)^(1/2)*polylog(2,(1+I*a*x)^(1/2)/(1-I*a*x)^(
1/2))/(a^2*c*x^2+c)^(1/2)-9/2*I*a^2*c^2*(a^2*x^2+1)^(1/2)*arctan(a*x)^2*po
lylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)-6*I*a^2*c^2*(a^2*
x^2+1)^(1/2)*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*
x^2+c)^(1/2)+3*I*a^2*c^2*(a^2*x^2+1)^(1/2)*polylog(2,-(1+I*a*x)^(1/2)/(1-I
*a*x)^(1/2))/(a^2*c*x^2+c)^(1/2)+6*I*a^2*c^2*(a^2*x^2+1)^(1/2)*arctan((1+I
*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2)-9*a^2*c^2*(a^2*
x^2+1)^(1/2)*arctan(a*x)*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^
2+c)^(1/2)+6*a^2*c^2*(a^2*x^2+1)^(1/2)*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^
(1/2))/(a^2*c*x^2+c)^(1/2)-6*a^2*c^2*(a^2*x^2+1)^(1/2)*polylog(3,I*(1+I*a*
x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)+9*a^2*c^2*(a^2*x^2+1)^(1/2)*arct
an(a*x)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)+9*I*a^2
*c^2*(a^2*x^2+1)^(1/2)*polylog(4,(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^...
```

## Mathematica [A] (warning: unable to verify)

Time = 5.99 (sec) , antiderivative size = 691, normalized size of antiderivative = 0.75

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x^3}\,dx = \text{Too large to display}$$

input
```
Integrate[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3)/x^3,x]
```

output
```
(a^2*c*Sqrt[c + a^2*c*x^2]*(-12*ArcTan[a*x]^2 - (3*I)*Pi^4*Cot[ArcTan[a*x]
/2] + (6*I)*ArcTan[a*x]^4*Cot[ArcTan[a*x]/2] - 12*ArcTan[a*x]^2*Cot[ArcTan
[a*x]/2]^2 + 8*a*x*ArcTan[a*x]^3*Csc[ArcTan[a*x]/2]^2 - 2*ArcTan[a*x]^3*Co
t[ArcTan[a*x]/2]*Csc[ArcTan[a*x]/2]^2 + 24*ArcTan[a*x]^3*Cot[ArcTan[a*x]/2
]*Log[1 - E^((-I)*ArcTan[a*x])] + 48*ArcTan[a*x]*Cot[ArcTan[a*x]/2]*Log[1
- E^(I*ArcTan[a*x])] - 48*ArcTan[a*x]^2*Cot[ArcTan[a*x]/2]*Log[1 - I*E^(I*
ArcTan[a*x])] + 48*ArcTan[a*x]^2*Cot[ArcTan[a*x]/2]*Log[1 + I*E^(I*ArcTan[
a*x])] - 48*ArcTan[a*x]*Cot[ArcTan[a*x]/2]*Log[1 + E^(I*ArcTan[a*x])] - 24
*ArcTan[a*x]^3*Cot[ArcTan[a*x]/2]*Log[1 + E^(I*ArcTan[a*x])] + (72*I)*ArcT
an[a*x]^2*Cot[ArcTan[a*x]/2]*PolyLog[2, E^((-I)*ArcTan[a*x])] + (24*I)*(2
+ 3*ArcTan[a*x]^2)*Cot[ArcTan[a*x]/2]*PolyLog[2, -E^(I*ArcTan[a*x])] - (96
*I)*ArcTan[a*x]*Cot[ArcTan[a*x]/2]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + (9
6*I)*ArcTan[a*x]*Cot[ArcTan[a*x]/2]*PolyLog[2, I*E^(I*ArcTan[a*x])] - (48*
I)*Cot[ArcTan[a*x]/2]*PolyLog[2, E^(I*ArcTan[a*x])] + 144*ArcTan[a*x]*Cot[
ArcTan[a*x]/2]*PolyLog[3, E^((-I)*ArcTan[a*x])] - 144*ArcTan[a*x]*Cot[ArcT
an[a*x]/2]*PolyLog[3, -E^(I*ArcTan[a*x])] + 96*Cot[ArcTan[a*x]/2]*PolyLog[
3, (-I)*E^(I*ArcTan[a*x])] - 96*Cot[ArcTan[a*x]/2]*PolyLog[3, I*E^(I*ArcTa
n[a*x])] - (144*I)*Cot[ArcTan[a*x]/2]*PolyLog[4, E^((-I)*ArcTan[a*x])] - (
144*I)*Cot[ArcTan[a*x]/2]*PolyLog[4, -E^(I*ArcTan[a*x])] + 2*ArcTan[a*x]^3
*Csc[ArcTan[a*x]/2]*Sec[ArcTan[a*x]/2])*Tan[ArcTan[a*x]/2])/(16*Sqrt[1 ...
```

## Rubi [A] (verified)

Time = 9.24 (sec) , antiderivative size = 925, normalized size of antiderivative = 1.01, number of steps used = 27, number of rules used = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 1.083$, Rules used = {5485, 5485, 5465, 5425, 5423, 3042, 4669, 3011, 2720, 5493, 5491, 3042, 4671, 3011, 5497, 5479, 5493, 5489, 5491, 3042, 4671, 3011, 7143, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3 \left(a^2cx^2+c\right)^{3/2}}{x^3}dx$$

$$\Big\downarrow \text{ 5485}$$

$$a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x}dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^3}dx$$

$$\Big\downarrow \text{ 5485}$$

$$a^2c\left(a^2c\int \frac{x\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx\right)+c\left(a^2c\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^3}{x^3\sqrt{a^2cx^2+c}}dx\right)$$

$$\Big\downarrow \text{5465}$$

$$a^2c\left(a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a}\right)+c\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx\right)+c\left(a^2c\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^3}{x^3\sqrt{a^2cx^2+c}}dx\right)$$

$$\Big\downarrow \text{5425}$$

$$a^2c\left(a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{a\sqrt{a^2cx^2+c}}\right)+c\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx\right)+c\left(a^2c\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^3}{x^3\sqrt{a^2cx^2+c}}dx\right)$$

$$\Big\downarrow \text{5423}$$

$$a^2c\left(a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\int \sqrt{a^2x^2+1}\arctan(ax)^2d\arctan(ax)}{a^2\sqrt{a^2cx^2+c}}\right)+c\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx\right.$$
$$c\left(a^2c\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^3}{x^3\sqrt{a^2cx^2+c}}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$a^2c\left(c\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx+a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\,\mathrm{ar}}{a^2\sqrt{a^2cx^2+c}}\right.\right.$$
$$c\left(a^2c\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^3}{x^3\sqrt{a^2cx^2+c}}dx\right)$$

$$\Big\downarrow \text{4669}$$

$$c\left(a^2c\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^3}{x^3\sqrt{a^2cx^2+c}}dx\right)+$$
$$a^2c\left(c\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx+a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(-2\int \arctan(ax)\log\left(1-ie^{i\arctan(ax)}\right)\right.}{}\right.\right.$$

$$\Big\downarrow \text{3011}$$

$$c\left(a^2c\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^3}{x^3\sqrt{a^2cx^2+c}}dx\right)+$$
$$a^2c\left(c\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx+a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\big(2\big(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(a}\right.}{}\right.\right.$$

$\downarrow$ 2720

$$c\left(a^2c\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^3}{x^3\sqrt{a^2cx^2+c}}dx\right)+$$
$$a^2c\left(c\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx+a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\big(2\big(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(a}\right.}{}\right.\right.$$

$\downarrow$ 5493

$$c\left(\frac{a^2c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^3}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)^3}{x^3\sqrt{a^2cx^2+c}}dx\right)+$$
$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^3}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\big(2\big(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i}\right.}{}\right.\right.$$

$\downarrow$ 5491

$$c\left(\frac{a^2c\sqrt{a^2x^2+1}\int \frac{\sqrt{a^2x^2+1}\arctan(ax)^3}{ax}d\arctan(ax)}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)^3}{x^3\sqrt{a^2cx^2+c}}dx\right)+$$
$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\int \frac{\sqrt{a^2x^2+1}\arctan(ax)^3}{ax}d\arctan(ax)}{\sqrt{a^2cx^2+c}}+a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\big(2\big(i\arctan(ax)}{}\right.\right.$$

$\downarrow$ 3042

$$c\left(\frac{a^2c\sqrt{a^2x^2+1}\int \arctan(ax)^3\csc(\arctan(ax))d\arctan(ax)}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)^3}{x^3\sqrt{a^2cx^2+c}}dx\right)+$$
$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\int \arctan(ax)^3\csc(\arctan(ax))d\arctan(ax)}{\sqrt{a^2cx^2+c}}+a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\big(2\big(i}{}\right.\right.$$

$\downarrow$ 4671

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(-3\int \arctan(ax)^2\log\left(1-e^{i\arctan(ax)}\right)d\arctan(ax)+3\int \arctan(ax)^2\log\left(1+e^{i\arctan(ax)}\right)d\,\mathrm{ar}\right.}{\sqrt{a^2cx^2+c}}\right.$$
$$c\left(c\int \frac{\arctan(ax)^3}{x^3\sqrt{a^2cx^2+c}}dx+\frac{a^2c\sqrt{a^2x^2+1}\left(-3\int \arctan(ax)^2\log\left(1-e^{i\arctan(ax)}\right)d\arctan(ax)+3\int \arctan(ax)^2\right.}{\sqrt{a^2cx^2+c}}\right.$$

$\downarrow$ 3011

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\int\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)d\arctan\right.\right.}{}\right.$$

$$c\left(c\int\frac{\arctan(ax)^3}{x^3\sqrt{a^2cx^2+c}}dx+\frac{a^2c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\int\arctan(ax)\operatorname{PolyLog}\left(\right.\right.\right.}{}\right.$$

$\Big\downarrow$ 5497

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\int\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)d\arctan\right.\right.}{}\right.$$

$$c\left(c\left(\frac{3}{2}a\int\frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx-\frac{1}{2}a^2\int\frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{2cx^2}\right)+\frac{a^2c\sqrt{a^2x^2+1}\left(3\left(i\arctan\right.\right.}{}\right.$$

$\Big\downarrow$ 5479

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\int\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)d\arctan\right.\right.}{}\right.$$

$$c\left(c\left(\frac{3}{2}a\left(2a\int\frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}\right)-\frac{1}{2}a^2\int\frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^3\sqrt{a^2cx^2+}}{2cx^2}\right.\right.$$

$\Big\downarrow$ 5493

$$a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\int\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)d\arctan\right.\right.}{}\right.$$

$$c\left(c\left(\frac{3}{2}a\left(\frac{2a\sqrt{a^2x^2+1}\int\frac{\arctan(ax)}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}\right)-\frac{a^2\sqrt{a^2x^2+1}\int\frac{\arctan(ax)^3}{x\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}-\frac{\arctan(ax}{2}\right.\right.$$

$\Big\downarrow$ 5489

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2+2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,\right.\right.\right.}{}\right.\right.$$

$$c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)\arctan(ax)^3+3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\int\arctan(a\right.\right.}{}\right.$$

$\Big\downarrow$ 5491

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2+2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,\right.\right.\right.}{}\right.\right.$$
$$c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)\arctan(ax)^3+3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\int\arctan(a\right.\right.}{}\right.$$

↓ 3042

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2+2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,\right.\right.\right.}{}\right.\right.$$
$$c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)\arctan(ax)^3+3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\int\arctan(a\right.\right.}{}\right.$$

↓ 4671

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2+2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,\right.\right.\right.}{}\right.\right.$$
$$c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)\arctan(ax)^3+3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\int\arctan(a\right.\right.}{}\right.$$

↓ 3011

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2+2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,\right.\right.\right.}{}\right.\right.$$
$$c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)\arctan(ax)^3+3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\int\arctan(a\right.\right.}{}\right.$$

↓ 7143

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2+2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,\right.\right.\right.}{}\right.\right.$$
$$c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)\arctan(ax)^3+3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\int\arctan(a\right.\right.}{}\right.$$

↓ 7163

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2+2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,\right.\right.\right.}{}\right.\right.$$

$$c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\text{arctanh}\left(e^{i\arctan(ax)}\right)\arctan(ax)^3+3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\left(i\int \operatorname{PolyLo}\right.\right.\right.}{}\right.$$

$\downarrow$ 2720

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2+2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,\right.\right.\right.}{}\right.\right.$$

$$c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\text{arctanh}\left(e^{i\arctan(ax)}\right)\arctan(ax)^3+3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\left(\int e^{-i\arctan}\right.\right.\right.}{}\right.$$

$\downarrow$ 7143

$$c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2+2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,\right.\right.\right.}{}\right.\right.$$

$$c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\text{arctanh}\left(e^{i\arctan(ax)}\right)\arctan(ax)^3+3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\left(\operatorname{PolyLog}\left(\right.\right.\right.\right.}{}\right.$$

input
```
Int[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3)/x^3,x]
```

output
```
a^2*c*(a^2*c*((Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/(a^2*c) - (3*Sqrt[1 + a^
2*x^2]*((-2*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 + 2*(I*ArcTan[a*x]*
PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - PolyLog[3, (-I)*E^(I*ArcTan[a*x])]) -
 2*(I*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] - PolyLog[3, I*E^(I*ArcT
an[a*x])])))/(a^2*Sqrt[c + a^2*c*x^2])) + (c*Sqrt[1 + a^2*x^2]*(-2*ArcTan[
a*x]^3*ArcTanh[E^(I*ArcTan[a*x])] + 3*(I*ArcTan[a*x]^2*PolyLog[2, -E^(I*Ar
cTan[a*x])] - (2*I)*((-I)*ArcTan[a*x]*PolyLog[3, -E^(I*ArcTan[a*x])] + Pol
yLog[4, -E^(I*ArcTan[a*x])])) - 3*(I*ArcTan[a*x]^2*PolyLog[2, E^(I*ArcTan[
a*x])] - (2*I)*((-I)*ArcTan[a*x]*PolyLog[3, E^(I*ArcTan[a*x])] + PolyLog[4
, E^(I*ArcTan[a*x])]))))/Sqrt[c + a^2*c*x^2]) + c*((a^2*c*Sqrt[1 + a^2*x^2
]*(-2*ArcTan[a*x]^3*ArcTanh[E^(I*ArcTan[a*x])] + 3*(I*ArcTan[a*x]^2*PolyLo
g[2, -E^(I*ArcTan[a*x])] - (2*I)*((-I)*ArcTan[a*x]*PolyLog[3, -E^(I*ArcTan
[a*x])] + PolyLog[4, -E^(I*ArcTan[a*x])])) - 3*(I*ArcTan[a*x]^2*PolyLog[2,
 E^(I*ArcTan[a*x])] - (2*I)*((-I)*ArcTan[a*x]*PolyLog[3, E^(I*ArcTan[a*x])
] + PolyLog[4, E^(I*ArcTan[a*x])]))))/Sqrt[c + a^2*c*x^2] + c*(-1/2*(Sqrt[
c + a^2*c*x^2]*ArcTan[a*x]^3)/(c*x^2) + (3*a*(-((Sqrt[c + a^2*c*x^2]*ArcTa
n[a*x]^2)/(c*x)) + (2*a*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]*ArcTanh[Sqrt[1 +
 I*a*x]/Sqrt[1 - I*a*x]] + I*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x])
] - I*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]]))/Sqrt[c + a^2*c*x^2]))/
2 - (a^2*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]^3*ArcTanh[E^(I*ArcTan[a*x])]...
```

## Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5489
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sq
rt[1 - I*c*x]], x] + (Simp[I*(b/Sqrt[d])*PolyLog[2, -Sqrt[1 + I*c*x]/Sqrt[1
 - I*c*x]], x] - Simp[I*(b/Sqrt[d])*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x]], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

rule 5491
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Simp[1/Sqrt[d]   Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTa
n[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] &&
GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5497
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)^(m
 + 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[c^2*((m
+ 2)/(f^2*(m + 1)))   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x
^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
 && LtQ[m, -1] && NeQ[m, -2]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

## Maple [A] (verified)

Time = 5.76 (sec) , antiderivative size = 592, normalized size of antiderivative = 0.64

| method | result |
|---|---|
| default | $\frac{c\sqrt{c(ax-i)(ax+i)}\,\arctan(ax)^{2}\left(2x^{2}a^{2}\arctan(ax)-3ax-\arctan(ax)\right)}{2x^{2}}-\frac{3ca^{2}\sqrt{c(ax-i)(ax+i)}\left(\arctan(ax)^{3}\ln\left(1+\frac{iax+1}{\sqrt{a^{2}x^{2}+1}}\right)\right.}{}$ |

input
```
int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*c*(c*(a*x-I)*(a*x+I))^(1/2)*arctan(a*x)^2*(2*x^2*a^2*arctan(a*x)-3*a*x
-arctan(a*x))/x^2-3/2*c*a^2*(c*(a*x-I)*(a*x+I))^(1/2)*(arctan(a*x)^3*ln(1+
(1+I*a*x)/(a^2*x^2+1)^(1/2))-arctan(a*x)^3*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2
))+6*I*polylog(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I*polylog(4,(1+I*a*x)/(a^
2*x^2+1)^(1/2))+2*arctan(a*x)^2*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*arct
an(a*x)^2*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+4*I*arctan(a*x)*polylog(2,-I
*(1+I*a*x)/(a^2*x^2+1)^(1/2))+3*I*arctan(a*x)^2*polylog(2,(1+I*a*x)/(a^2*x
^2+1)^(1/2))+2*arctan(a*x)*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*arctan(a*x)
*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*arctan(a*x)*ln(1-(1+I*a*x)/(a^2
*x^2+1)^(1/2))-6*arctan(a*x)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*I*ar
ctan(a*x)^2*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*I*polylog(2,-(1+I*a*
x)/(a^2*x^2+1)^(1/2))-4*I*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1
/2))+2*I*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+4*polylog(3,I*(1+I*a*x)/(a
^2*x^2+1)^(1/2))-4*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^
(1/2)
```

## Fricas [F]

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x^3}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)^3}{x^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3/x^3,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^3/x^3, x)
```

## Sympy [F]

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x^3}\,dx = \int \frac{(c(a^2x^2+1))^{\frac{3}{2}}\operatorname{atan}^3(ax)}{x^3}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)**3/x**3,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**3/x**3, x)
```

## Maxima [F]

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x^3}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)^3}{x^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3/x^3,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^3/x^3, x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3/x^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x^3}\,dx = \int \frac{\mathrm{atan}(a\,x)^3\,(c\,a^2\,x^2+c)^{3/2}}{x^3}\,dx$$

input
```
int((atan(a*x)^3*(c + a^2*c*x^2)^(3/2))/x^3,x)
```

output
```
int((atan(a*x)^3*(c + a^2*c*x^2)^(3/2))/x^3, x)
```

## Reduce [F]

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x^3}\,dx = \sqrt{c}\,c\left(\int \frac{\sqrt{a^2x^2+1}\,atan(ax)^3}{x^3}dx + \left(\int \frac{\sqrt{a^2x^2+1}\,atan(ax)^3}{x}dx\right)a^2\right)$$

input
```
int((a^2*c*x^2+c)^(3/2)*atan(a*x)^3/x^3,x)
```

output

```
sqrt(c)*c*(int((sqrt(a**2*x**2 + 1)*atan(a*x)**3)/x**3,x) + int((sqrt(a**2
*x**2 + 1)*atan(a*x)**3)/x,x)*a**2)
```

# 3.427 $\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x^4}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 788

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x^4}\,dx = -\frac{a^2c\sqrt{c+a^2cx^2}\arctan(ax)}{x}$$
$$-\frac{ac\sqrt{c+a^2cx^2}\arctan(ax)^2}{2x^2}-\frac{a^2c\sqrt{c+a^2cx^2}\arctan(ax)^3}{x}$$
$$-\frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{3x^3}-\frac{2ia^3c^2\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^3}{\sqrt{c+a^2cx^2}}$$
$$-\frac{7a^3c^2\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}$$
$$-a^3c^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c+a^2cx^2}}{\sqrt{c}}\right)+\frac{7ia^3c^2\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}+\frac{3ia^3c^2\sqrt{1+a^2x}}{}$$

output
```
-a^2*c*(a^2*c*x^2+c)^(1/2)*arctan(a*x)/x-1/2*a*c*(a^2*c*x^2+c)^(1/2)*arcta
n(a*x)^2/x^2-a^2*c*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^3/x-1/3*(a^2*c*x^2+c)^(
3/2)*arctan(a*x)^3/x^3+7*I*a^3*c^2*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(2
,-(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)-7*a^3*c^2*(a^2*x^2+1)^(
1/2)*arctan(a*x)^2*arctanh((1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2
)-a^3*c^(3/2)*arctanh((a^2*c*x^2+c)^(1/2)/c^(1/2))-2*I*a^3*c^2*(a^2*x^2+1)
^(1/2)*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^3/(a^2*c*x^2+c)^(1/
2)-3*I*a^3*c^2*(a^2*x^2+1)^(1/2)*arctan(a*x)^2*polylog(2,I*(1+I*a*x)/(a^2*
x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)-7*I*a^3*c^2*(a^2*x^2+1)^(1/2)*arctan(a*x
)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)-6*I*a^3*c^2*(
a^2*x^2+1)^(1/2)*polylog(4,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(
1/2)-7*a^3*c^2*(a^2*x^2+1)^(1/2)*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))/(
a^2*c*x^2+c)^(1/2)-6*a^3*c^2*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(3,-I*(1
+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)+6*a^3*c^2*(a^2*x^2+1)^(1/2)
*arctan(a*x)*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)+
7*a^3*c^2*(a^2*x^2+1)^(1/2)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*
x^2+c)^(1/2)+3*I*a^3*c^2*(a^2*x^2+1)^(1/2)*arctan(a*x)^2*polylog(2,-I*(1+I
*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)+6*I*a^3*c^2*(a^2*x^2+1)^(1/2)
*polylog(4,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (warning: unable to verify)

Time = 9.06 (sec) , antiderivative size = 1508, normalized size of antiderivative = 1.91

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x^4}\,dx = \text{Too large to display}$$

input
```
Integrate[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3)/x^4,x]
```

output
```
(a^3*c*Sqrt[c*(1 + a^2*x^2)]*Csc[ArcTan[a*x]/2]*(((-7*I)*a*Pi^4*x)/Sqrt[1
+ a^2*x^2] - ((8*I)*a*Pi^3*x*ArcTan[a*x])/Sqrt[1 + a^2*x^2] + ((24*I)*a*Pi
^2*x*ArcTan[a*x]^2)/Sqrt[1 + a^2*x^2] - 64*ArcTan[a*x]^3 - ((32*I)*a*Pi*x*
ArcTan[a*x]^3)/Sqrt[1 + a^2*x^2] + ((16*I)*a*x*ArcTan[a*x]^4)/Sqrt[1 + a^2
*x^2] + (48*a*Pi^2*x*ArcTan[a*x]*Log[1 - I/E^(I*ArcTan[a*x])])/Sqrt[1 + a^
2*x^2] - (96*a*Pi*x*ArcTan[a*x]^2*Log[1 - I/E^(I*ArcTan[a*x])])/Sqrt[1 + a
^2*x^2] - (8*a*Pi^3*x*Log[1 + I/E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (6
4*a*x*ArcTan[a*x]^3*Log[1 + I/E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (192
*a*x*ArcTan[a*x]^2*Log[1 - E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (8*a*Pi
^3*x*Log[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - (48*a*Pi^2*x*ArcTan
[a*x]*Log[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (96*a*Pi*x*ArcTan[
a*x]^2*Log[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - (64*a*x*ArcTan[a*
x]^3*Log[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - (192*a*x*ArcTan[a*x
]^2*Log[1 + E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (8*a*Pi^3*x*Log[Tan[(P
i + 2*ArcTan[a*x])/4]])/Sqrt[1 + a^2*x^2] + ((192*I)*a*x*ArcTan[a*x]^2*Pol
yLog[2, (-I)/E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + ((48*I)*a*Pi*x*(Pi -
4*ArcTan[a*x])*PolyLog[2, I/E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + ((384*
I)*a*x*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + ((4
8*I)*a*Pi^2*x*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - ((19
2*I)*a*Pi*x*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/Sqrt[1 + a^...
```

## Rubi [A] (verified)

Time = 8.45 (sec) , antiderivative size = 755, normalized size of antiderivative = 0.96, number of steps used = 31, number of rules used = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 1.250$, Rules used = {5485, 5479, 5485, 5425, 5423, 3042, 4669, 3011, 5479, 5493, 5491, 3042, 4671, 3011, 2720, 5497, 5479, 243, 73, 221, 5493, 5491, 3042, 4671, 3011, 2720, 7143, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3 \left(a^2cx^2+c\right)^{3/2}}{x^4} \, dx$$

$$\Big\downarrow \text{ 5485}$$

$$a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^2}dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^4}dx$$

$$\Big\downarrow \text{5479}$$

$$a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^2}dx+c\left(a\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^3}dx-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)$$

$$\Big\downarrow \text{5485}$$

$$a^2c\left(a^2c\int \frac{\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^3}{x^2\sqrt{a^2cx^2+c}}dx\right)+c\left(a\left(a^2c\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx\right)-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)$$

$$\Big\downarrow \text{5425}$$

$$a^2c\left(\frac{a^2c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^3}{\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)^3}{x^2\sqrt{a^2cx^2+c}}dx\right)+c\left(a\left(a^2c\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx\right)-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)$$

$$\Big\downarrow \text{5423}$$

$$a^2c\left(\frac{ac\sqrt{a^2x^2+1}\int \sqrt{a^2x^2+1}\arctan(ax)^3d\arctan(ax)}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)^3}{x^2\sqrt{a^2cx^2+c}}dx\right)+c\left(a\left(a^2c\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx\right)-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^2c\left(c\int \frac{\arctan(ax)^3}{x^2\sqrt{a^2cx^2+c}}dx+\frac{ac\sqrt{a^2x^2+1}\int \arctan(ax)^3\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{\sqrt{a^2cx^2+c}}\right)+c\left(a\left(a^2c\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx\right)-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)$$

$$\Big\downarrow \text{4669}$$

$$c\left(a\left(a^2c\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx\right)-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)+a^2c\left(c\int \frac{\arctan(ax)^3}{x^2\sqrt{a^2cx^2+c}}dx+\frac{ac\sqrt{a^2x^2+1}\left(-3\int \arctan(ax)^2\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+3\int \arctan(ax\right.}{\sqrt{a^2cx^2+}}\right.$$

$$\Big\downarrow \text{3011}$$

$$c\left(a\left(a^2c\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx\right)-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)+$$
$$a^2c\left(c\int \frac{\arctan(ax)^3}{x^2\sqrt{a^2cx^2+c}}dx+\frac{ac\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int \arctan(ax)\operatorname{PolyLog}\right.\right.}{}\right.$$

$$\Big\downarrow \ 5479$$

$$c\left(a\left(a^2c\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx\right)-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)+$$
$$a^2c\left(c\left(3a\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}\right)+\frac{ac\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan}\right.\right.\right.}{}\right.$$

$$\Big\downarrow \ 5493$$

$$c\left(a\left(\frac{a^2c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx\right)-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)+$$
$$a^2c\left(c\left(\frac{3a\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}\right)+\frac{ac\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-\right.\right.\right.}{}\right.$$

$$\Big\downarrow \ 5491$$

$$c\left(a\left(\frac{a^2c\sqrt{a^2x^2+1}\int \frac{\sqrt{a^2x^2+1}\arctan(ax)^2}{ax}d\arctan(ax)}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx\right)-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)$$
$$a^2c\left(c\left(\frac{3a\sqrt{a^2x^2+1}\int \frac{\sqrt{a^2x^2+1}\arctan(ax)^2}{ax}d\arctan(ax)}{\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}\right)+\frac{ac\sqrt{a^2x^2+1}\left(3\left(i\arctan(a\right.\right.}{}\right.$$

$$\Big\downarrow \ 3042$$

$$c\left(a\left(\frac{a^2c\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc(\arctan(ax))d\arctan(ax)}{\sqrt{a^2cx^2+c}}+c\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx\right)-\frac{\arctan(ax)^3\left(a^2cx^2\right.}{3cx^3}\right.$$
$$a^2c\left(c\left(\frac{3a\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc(\arctan(ax))d\arctan(ax)}{\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}\right)+\frac{ac\sqrt{a^2x^2+1}\left(3\right.}{}\right.$$

$$\Big\downarrow \ 4671$$

$$a^2c\left(c\left(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(-2\int\arctan(ax)\log\left(1-e^{i\arctan(ax)}\right)d\arctan(ax)+2\int\text{arcta}\right.}{\sqrt{a^2}}\right.\right.$$

$$c\left(-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}+a\left(c\int\frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx+\frac{a^2c\sqrt{a^2x^2+1}\left(-2\int\arctan(ax)\log\left(1-e^{i\arctan(ax)}\right.\right.}{}\right.\right.$$

$\big\downarrow$ 3011

$$a^2c\left(c\left(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-i\int\operatorname{PolyLog}\left(2,-e\right.\right.\right.}{}\right.\right.$$

$$c\left(-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}+a\left(c\int\frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx+\frac{a^2c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\,\text{arcta}}\right.\right.\right.}{}\right.\right.$$

$\big\downarrow$ 2720

$$a^2c\left(c\left(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{Pol}\right.\right.}{}\right.\right.$$

$$c\left(-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}+a\left(c\int\frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx+\frac{a^2c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\,\text{arcta}}\right.\right.\right.}{}\right.\right.$$

$\big\downarrow$ 5497

$$a^2c\left(c\left(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{Pol}\right.\right.}{}\right.\right.$$

$$c\left(-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}+a\left(c\left(-\frac{1}{2}a^2\int\frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+a\int\frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^2\sqrt{a^2cx^2}}{2cx^2}\right.\right.\right.$$

$\big\downarrow$ 5479

$$a^2c\left(c\left(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{Pol}\right.\right.}{}\right.\right.$$

$$c\left(-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}+a\left(c\left(-\frac{1}{2}a^2\int\frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+a\left(a\int\frac{1}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2c}}{cx}\right.\right.\right.\right.$$

$\big\downarrow$ 243

$$a^2c\left(c\left(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{Pol}\right.\right.}{}\right.\right.$$
$$c\left(-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}+a\left(c\left(-\frac{1}{2}a^2\int\frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+a\left(\frac{1}{2}a\int\frac{1}{x^2\sqrt{a^2cx^2+c}}dx^2-\frac{\arctan(ax)\sqrt{}}{cx}\right.\right.\right.\right.$$

$\Big\downarrow$ 73

$$a^2c\left(c\left(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{Pol}\right.\right.}{}\right.\right.$$
$$c\left(-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}+a\left(c\left(-\frac{1}{2}a^2\int\frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+a\left(\frac{\int\frac{1}{\frac{x^4}{a^2c}-\frac{1}{a^2}}d\sqrt{a^2cx^2+c}}{ac}-\frac{\arctan(ax)\sqrt{a}}{cx}\right.\right.\right.\right.$$

$\Big\downarrow$ 221

$$a^2c\left(c\left(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{Pol}\right.\right.}{}\right.\right.$$
$$c\left(-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}+a\left(c\left(-\frac{1}{2}a^2\int\frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+a\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\operatorname{arctanh}\left(\right.}{\sqrt{c}}\right.\right.\right.\right.$$

$\Big\downarrow$ 5493

$$a^2c\left(c\left(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{Pol}\right.\right.}{}\right.\right.$$
$$c\left(-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}+a\left(c\left(-\frac{a^2\sqrt{a^2x^2+1}\int\frac{\arctan(ax)^2}{x\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+a\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\operatorname{arcta}}{}\right.\right.\right.\right.$$

$\Big\downarrow$ 5491

$$a^2c\left(c\left(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{Pol}\right.\right.}{}\right.\right.$$
$$c\left(-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}+a\left(c\left(-\frac{a^2\sqrt{a^2x^2+1}\int\frac{\sqrt{a^2x^2+1}\arctan(ax)^2}{ax}d\arctan(ax)}{2\sqrt{a^2cx^2+c}}+a\left(-\frac{\arctan(ax)\sqrt{a}}{cx}\right.\right.\right.\right.$$

$\Big\downarrow$ 3042

$$a^2c\left(c\left(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{Pol}\right.\right.}{}\right.\right.$$

$$c\left(-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}+a\left(c\left(-\frac{a^2\sqrt{a^2x^2+1}\int\arctan(ax)^2\csc(\arctan(ax))d\arctan(ax)}{2\sqrt{a^2cx^2+c}}+a\left(-\frac{\operatorname{arct}}{}\right.\right.\right.\right.$$

$\downarrow$ 4671

$$c\left(\frac{ac\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^3+3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int\operatorname{arctan}\right.\right.}{}\right.$$

$$c\left(a\left(\frac{c\sqrt{a^2x^2+1}\left(-2\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)\arctan(ax)^2+2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\operatorname{arctan}}\right.\right.}{}\right.\right.$$

$\downarrow$ 3011

$$c\left(\frac{ac\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^3+3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int\operatorname{arctan}\right.\right.}{}\right.$$

$$c\left(a\left(\frac{c\sqrt{a^2x^2+1}\left(-2\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)\arctan(ax)^2+2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\operatorname{arctan}}\right.\right.}{}\right.\right.$$

$\downarrow$ 2720

$$c\left(\frac{ac\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^3+3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int\operatorname{arctan}\right.\right.}{}\right.$$

$$c\left(a\left(\frac{c\sqrt{a^2x^2+1}\left(-2\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)\arctan(ax)^2+2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\operatorname{arctan}}\right.\right.}{}\right.\right.$$

$\downarrow$ 7143

$$a^2c\left(\frac{ac\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\operatorname{arc}\right.\right.}{}\right.$$

$$c\left(-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}+a\left(\frac{a^2c\sqrt{a^2x^2+1}\left(-2\arctan(ax)^2\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)+2\left(i\arctan(ax)\operatorname{Poly}\right.\right.}{}\right.\right.$$

$\downarrow$ 7163

$$a^2c\left(\frac{ac\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\left(i\int\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)d\arctan(ax)-\right.\right.\right.}{}\right.$$

$$c\left(-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}+a\left(\frac{a^2c\sqrt{a^2x^2+1}\left(-2\arctan(ax)^2\text{arctanh}\left(e^{i\arctan(ax)}\right)+2\left(i\arctan(ax)\operatorname{Poly}\right.\right.}{}\right.\right.$$

$$\Big\downarrow \text{2720}$$

$$c\left(c\left(\frac{3a\sqrt{a^2x^2+1}\left(-2\text{arctanh}\left(e^{i\arctan(ax)}\right)\arctan(ax)^2+2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\operatorname{PolyLog}\left(\right.\right.\right.}{\sqrt{a^2cx^2+c}}\right.\right.$$

$$c\left(a\left(\frac{c\sqrt{a^2x^2+1}\left(-2\text{arctanh}\left(e^{i\arctan(ax)}\right)\arctan(ax)^2+2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\operatorname{PolyLog}\left(3\right.\right.\right.}{\sqrt{a^2cx^2+c}}\right.\right.$$

$$\Big\downarrow \text{7143}$$

$$a^2c\left(c\left(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(-2\arctan(ax)^2\text{arctanh}\left(e^{i\arctan(ax)}\right)+2\left(i\arctan(ax)\operatorname{PolyLo}\right.\right.}{}\right.\right.$$

$$c\left(-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}+a\left(\frac{a^2c\sqrt{a^2x^2+1}\left(-2\arctan(ax)^2\text{arctanh}\left(e^{i\arctan(ax)}\right)+2\left(i\arctan(ax)\operatorname{Poly}\right.\right.}{}\right.\right.$$

input
```
Int[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3)/x^4,x]
```

output
```
c*(-1/3*((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3)/(c*x^3) + a*(c*(-1/2*(Sqrt[c
 + a^2*c*x^2]*ArcTan[a*x]^2)/(c*x^2) + a*(-((Sqrt[c + a^2*c*x^2]*ArcTan[a*
x])/(c*x)) - (a*ArcTanh[Sqrt[c + a^2*c*x^2]/Sqrt[c]])/Sqrt[c]) - (a^2*Sqrt
[1 + a^2*x^2]*(-2*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])] + 2*(I*ArcTan[a
*x]*PolyLog[2, -E^(I*ArcTan[a*x])] - PolyLog[3, -E^(I*ArcTan[a*x])]) - 2*(
I*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])] - PolyLog[3, E^(I*ArcTan[a*x])
])))/(2*Sqrt[c + a^2*c*x^2])) + (a^2*c*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]^2
*ArcTanh[E^(I*ArcTan[a*x])] + 2*(I*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x
])] - PolyLog[3, -E^(I*ArcTan[a*x])]) - 2*(I*ArcTan[a*x]*PolyLog[2, E^(I*A
rcTan[a*x])] - PolyLog[3, E^(I*ArcTan[a*x])])))/Sqrt[c + a^2*c*x^2])) + a^
2*c*(c*(-((Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/(c*x)) + (3*a*Sqrt[1 + a^2*x
^2]*(-2*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])] + 2*(I*ArcTan[a*x]*PolyLo
g[2, -E^(I*ArcTan[a*x])] - PolyLog[3, -E^(I*ArcTan[a*x])]) - 2*(I*ArcTan[a
*x]*PolyLog[2, E^(I*ArcTan[a*x])] - PolyLog[3, E^(I*ArcTan[a*x])])))/Sqrt[
c + a^2*c*x^2]) + (a*c*Sqrt[1 + a^2*x^2]*((-2*I)*ArcTan[E^(I*ArcTan[a*x])]
*ArcTan[a*x]^3 + 3*(I*ArcTan[a*x]^2*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - (
2*I)*((-I)*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] + PolyLog[4, (-I
)*E^(I*ArcTan[a*x])])) - 3*(I*ArcTan[a*x]^2*PolyLog[2, I*E^(I*ArcTan[a*x])
] - (2*I)*((-I)*ArcTan[a*x]*PolyLog[3, I*E^(I*ArcTan[a*x])] + PolyLog[4, I
*E^(I*ArcTan[a*x])]))))/Sqrt[c + a^2*c*x^2])
```

### Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5485
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[d   Int[(f*x)^m*(d + e*x^2)^(q - 1)*(a +
 b*ArcTan[c*x])^p, x], x] + Simp[c^2*(d/f^2)   Int[(f*x)^(m + 2)*(d + e*x^2
)^(q - 1)*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[e, c^2*d] && GtQ[q, 0] && IGtQ[p, 0] && (RationalQ[m] || (EqQ[p, 1]
&& IntegerQ[q]))
```

rule 5491
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Simp[1/Sqrt[d]   Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTa
n[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] &&
GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5497
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)^(m
 + 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[c^2*((m
+ 2)/(f^2*(m + 1)))   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x
^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
 && LtQ[m, -1] && NeQ[m, -2]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

## Maple [A] (verified)

Time = 5.79 (sec) , antiderivative size = 557, normalized size of antiderivative = 0.71

| method | result |
|---|---|
| default | $-\frac{c\sqrt{c(ax-i)(ax+i)}\,\arctan(ax)\left(8\arctan(ax)^2x^2a^2+6a^2x^2+3\arctan(ax)ax+2\arctan(ax)^2\right)}{6x^3}+\frac{ica^3\sqrt{c(ax-i)(ax+i)}\,\left(12i\,\mathrm{arc}\right.}{}$ |

input
```
int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/6*c*(c*(a*x-I)*(a*x+I))^(1/2)*arctan(a*x)*(8*arctan(a*x)^2*x^2*a^2+6*a^
2*x^2+3*arctan(a*x)*a*x+2*arctan(a*x)^2)/x^3+1/2*I*c*a^3*(c*(a*x-I)*(a*x+I
))^(1/2)*(12*I*arctan(a*x)*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*I*l
n(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*I*arctan(a*x)^3*ln(1+I*(1+I*a*x)/(a^2*x
^2+1)^(1/2))+14*I*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*arctan(a*x)^2*
polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-14*I*polylog(3,(1+I*a*x)/(a^2*x^2
+1)^(1/2))+6*arctan(a*x)^2*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-12*I*
arctan(a*x)*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+7*I*arctan(a*x)^2*ln(
1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+14*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^
2+1)^(1/2))-2*I*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-1)-14*arctan(a*x)*polylog(2
,(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*I*arctan(a*x)^3*ln(1-I*(1+I*a*x)/(a^2*x^2+
1)^(1/2))-7*I*arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))+12*polylog(4
,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-12*polylog(4,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2
)))/(a^2*x^2+1)^(1/2)
```

## Fricas [F]

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x^4}\,dx=\int \frac{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)^3}{x^4}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3/x^4,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^3/x^4, x)
```

## Sympy [F]

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x^4}\,dx=\int \frac{(c(a^2x^2+1))^{\frac{3}{2}}\operatorname{atan}^3(ax)}{x^4}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)**3/x**4,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**3/x**4, x)
```

## Maxima [F]

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x^4}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}\arctan{(ax)}^3}{x^4}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3/x^4,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^3/x^4, x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^3/x^4,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x^4}\,dx = \int \frac{\mathrm{atan}(a\,x)^3\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}{x^4}\,dx$$

input
```
int((atan(a*x)^3*(c + a^2*c*x^2)^(3/2))/x^4,x)
```

output
```
int((atan(a*x)^3*(c + a^2*c*x^2)^(3/2))/x^4, x)
```

## Reduce [F]

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^3}{x^4}\,dx = \sqrt{c}\,c\left(\int \frac{\sqrt{a^2x^2+1}\,atan(ax)^3}{x^4}dx + \left(\int \frac{\sqrt{a^2x^2+1}\,atan(ax)^3}{x^2}dx\right)a^2\right)$$

input
```
int((a^2*c*x^2+c)^(3/2)*atan(a*x)^3/x^4,x)
```

output
```
sqrt(c)*c*(int((sqrt(a**2*x**2 + 1)*atan(a*x)**3)/x**4,x) + int((sqrt(a**2
*x**2 + 1)*atan(a*x)**3)/x**2,x)*a**2)
```

# 3.428 $\int x^2(c+a^2cx^2)^{5/2}\arctan(ax)^3\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 1019

$$\int x^2\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3\,dx = \text{Too large to display}$$

output
```
13/6720*c^2*(a^2*c*x^2+c)^(1/2)/a^3-3/560*c*(a^2*c*x^2+c)^(3/2)/a^3-1/280*
(a^2*c*x^2+c)^(5/2)/a^3+43/1344*c^2*x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)/a^2+
29/560*c^2*x^3*(a^2*c*x^2+c)^(1/2)*arctan(a*x)+1/56*a^2*c^2*x^5*(a^2*c*x^2
+c)^(1/2)*arctan(a*x)+1373/13440*c^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2/a^3
-737/6720*c^2*x^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2/a-83/560*a*c^2*x^4*(a^
2*c*x^2+c)^(1/2)*arctan(a*x)^2-3/56*a^3*c^2*x^6*(a^2*c*x^2+c)^(1/2)*arctan
(a*x)^2+5/128*c^2*x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^3/a^2+59/192*c^2*x^3*(
a^2*c*x^2+c)^(1/2)*arctan(a*x)^3+17/48*a^2*c^2*x^5*(a^2*c*x^2+c)^(1/2)*arc
tan(a*x)^3+1/8*a^4*c^2*x^7*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^3-15/64*I*c^3*(
a^2*x^2+1)^(1/2)*polylog(4,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a^3/(a^2*c*x^2+c
)^(1/2)-15/128*I*c^3*(a^2*x^2+1)^(1/2)*arctan(a*x)^2*polylog(2,-I*(1+I*a*x
)/(a^2*x^2+1)^(1/2))/a^3/(a^2*c*x^2+c)^(1/2)+5/64*I*c^3*(a^2*x^2+1)^(1/2)*
arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^3/a^3/(a^2*c*x^2+c)^(1/2)-
397/1680*I*c^3*(a^2*x^2+1)^(1/2)*polylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1
/2))/a^3/(a^2*c*x^2+c)^(1/2)+15/128*I*c^3*(a^2*x^2+1)^(1/2)*arctan(a*x)^2*
polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a^3/(a^2*c*x^2+c)^(1/2)+397/1680*
I*c^3*(a^2*x^2+1)^(1/2)*polylog(2,I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/a^3/(
a^2*c*x^2+c)^(1/2)+15/64*c^3*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(3,-I*(1
+I*a*x)/(a^2*x^2+1)^(1/2))/a^3/(a^2*c*x^2+c)^(1/2)-15/64*c^3*(a^2*x^2+1)^(
1/2)*arctan(a*x)*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a^3/(a^2*c*x^...
```

## Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 6517 vs. 2(1019) = 2038.

Time = 24.58 (sec) , antiderivative size = 6517, normalized size of antiderivative = 6.40

$$\int x^2 (c + a^2 c x^2)^{5/2} \arctan(ax)^3 \, dx = \text{Result too large to show}$$

input
```
Integrate[x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3,x]
```

output
```
Result too large to show
```

## Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arctan(ax)^3 \left(a^2cx^2+c\right)^{5/2} dx$$

$$\Big\downarrow \text{5485}$$

$$c\int x^2\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3dx+a^2c\int x^4\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3dx$$

$$\Big\downarrow \text{5485}$$

$$c\left(c\int x^2\sqrt{a^2cx^2+c}\arctan(ax)^3dx+a^2c\int x^4\sqrt{a^2cx^2+c}\arctan(ax)^3dx\right)+ a^2c\left(a^2c\int x^6\sqrt{a^2cx^2+c}\arctan(ax)^3dx+c\int x^4\sqrt{a^2cx^2+c}\arctan(ax)^3dx\right)$$

$$\Big\downarrow \text{5485}$$

$$c\left(c\left(c\int \frac{x^2\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx+a^2c\int \frac{x^4\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx\right)+a^2c\left(a^2c\int \frac{x^6\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx+c\int \frac{x^4\arctan(ax)^3}{\sqrt{a^2cx^2+c}}d\right.\right.$$
$$a^2c\left(a^2c\left(a^2c\int \frac{x^8\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx+c\int \frac{x^6\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx\right)+c\left(a^2c\int \frac{x^6\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx+c\int \frac{x^4\arctan(ax)^3}{\sqrt{a^2cx^2+c}}\right.\right.$$

$$\Big\downarrow \text{5487}$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^7}{8a^2c}-\frac{3\int \frac{x^7\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{8a}-\frac{7\int \frac{x^6\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{8a^2}\right)a^2+c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax}{6a^2c}\right.\right.\right.$$
$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\int \frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{5\int \frac{x^4\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)a^2+c\left(\frac{x^3\sqrt{a^2cx^2+c}\arctan(a}{4a^2c}\right.\right.\right.$$

$$\Big\downarrow \text{5425}$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^7}{8a^2c}-\frac{3\int \frac{x^7\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{8a}-\frac{7\int \frac{x^6\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{8a^2}\right)a^2+c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax}{6a^2c}\right.\right.\right.$$
$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\int \frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{5\int \frac{x^4\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)a^2+c\left(\frac{x^3\sqrt{a^2cx^2+c}\arctan(a}{4a^2c}\right.\right.\right.$$

$$\Big\downarrow \text{5423}$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^7}{8a^2c}-\frac{3\int\frac{x^7\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{8a}-\frac{7\int\frac{x^6\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{8a^2}\right)a^2+c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax}{6a^2c}\right.\right.\right.$$
$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\int\frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{5\int\frac{x^4\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)a^2+c\left(\frac{x^3\sqrt{a^2cx^2+c}\arctan(a}{4a^2c}\right.\right.\right.$$

$\downarrow$ 3042

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^7}{8a^2c}-\frac{3\int\frac{x^7\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{8a}-\frac{7\int\frac{x^6\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{8a^2}\right)a^2+c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax}{6a^2c}\right.\right.\right.$$
$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\int\frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{5\int\frac{x^4\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)a^2+c\left(\frac{x^3\sqrt{a^2cx^2+c}\arctan(a}{4a^2c}\right.\right.\right.$$

$\downarrow$ 4669

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^7}{8a^2c}-\frac{3\int\frac{x^7\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{8a}-\frac{7\int\frac{x^6\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{8a^2}\right)a^2+c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax}{6a^2c}\right.\right.\right.$$
$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\int\frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{5\int\frac{x^4\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)a^2+c\left(\frac{x^3\sqrt{a^2cx^2+c}\arctan(a}{4a^2c}\right.\right.\right.$$

$\downarrow$ 3011

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^7}{8a^2c}-\frac{3\int\frac{x^7\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{8a}-\frac{7\int\frac{x^6\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{8a^2}\right)a^2+c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax}{6a^2c}\right.\right.\right.$$
$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\int\frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{5\int\frac{x^4\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)a^2+c\left(\frac{x^3\sqrt{a^2cx^2+c}\arctan(a}{4a^2c}\right.\right.\right.$$

$\downarrow$ 5465

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^7}{8a^2c}-\frac{3\int\frac{x^7\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{8a}-\frac{7\int\frac{x^6\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{8a^2}\right)a^2+c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax}{6a^2c}\right.\right.\right.$$
$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\int\frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{5\int\frac{x^4\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)a^2+c\left(\frac{x^3\sqrt{a^2cx^2+c}\arctan(a}{4a^2c}\right.\right.\right.$$

$$\Big\downarrow \text{5425}$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^7}{8a^2c}-\frac{3\int\frac{x^7\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{8a}-\frac{7\int\frac{x^6\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{8a^2}\right)a^2+c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax}{6a^2c}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\int\frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{5\int\frac{x^4\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)a^2+c\left(\frac{x^3\sqrt{a^2cx^2+c}\arctan(a}{4a^2c}\right.\right.\right.$$

$$\Big\downarrow \text{5421}$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^7}{8a^2c}-\frac{3\int\frac{x^7\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{8a}-\frac{7\int\frac{x^6\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{8a^2}\right)a^2+c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax}{6a^2c}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\int\frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{5\int\frac{x^4\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{6a^2}\right)a^2+c\left(\frac{x^3\sqrt{a^2cx^2+c}\arctan(a}{4a^2c}\right.\right.\right.$$

$$\Big\downarrow \text{5487}$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^7}{8a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\int\frac{x^6\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{7a}-\frac{6\int\frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{7a^2}\right)}{8a}-\frac{7\left(\sqrt{a^2c\right.}{}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\int\frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int\frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a^2}}{2a}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+}}{4}\right.}{}\right.\right.\right.$$

$$\Big\downarrow \text{5425}$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^7}{8a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\int\frac{x^6\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{7a}-\frac{6\int\frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{7a^2}\right)}{8a}-\frac{7\left(\sqrt{a^2c}\right.}{}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\int\frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int\frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a^2}}{2a}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+}}{4}\right.}{}\right.\right.\right.$$

$\downarrow$ 5423

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^7}{8a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\int\frac{x^6\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{7a}-\frac{6\int\frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{7a^2}\right)}{8a}-\frac{7\left(\sqrt{a^2c}\right.}{}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\int\frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int\frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a^2}}{2a}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+}}{4}\right.}{}\right.\right.\right.$$

$\downarrow$ 3042

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^7}{8a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\int\frac{x^6\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{7a}-\frac{6\int\frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{7a^2}\right)}{8a}-\frac{7\left(\sqrt{a^2c\right.}{}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\int\frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int\frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a^2}}{2a}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+\right.}{4}}{}\right.\right.\right.$$

$\downarrow$ 4669

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^7}{8a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\int\frac{x^6\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{7a}-\frac{6\int\frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{7a^2}\right)}{8a}-\frac{7\left(\sqrt{a^2c\right.}{}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\int\frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int\frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a^2}}{2a}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+\right.}{4}}{}\right.\right.\right.$$

$\downarrow$ 3011

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^7}{8a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\int\frac{x^6\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{7a}-\frac{6\int\frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{7a^2}\right)}{8a}-\frac{7\left(\sqrt{a^2c\right.}{}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\int\frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int\frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a^2}}{2a}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+}}{4}\right.}{}\right.\right.\right.$$

$\downarrow$ 5465

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^7}{8a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\int\frac{x^6\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{7a}-\frac{6\int\frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{7a^2}\right)}{8a}-\frac{7\left(\sqrt{a^2c\right.}{}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\int\frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int\frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a^2}}{2a}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+}}{4}\right.}{}\right.\right.\right.$$

$\downarrow$ 5425

$$c\left(c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^7}{8a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\int\frac{x^6\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{7a}-\frac{6\int\frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{7a^2}\right)}{8a}-\frac{7\left(\frac{\sqrt{a^2c\cdots}}{\cdots}\right.}{\cdots}\right.\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\int\frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a}-\frac{4\int\frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a^2}}{2a}-\frac{5\left(\frac{x^3\sqrt{a^2cx^2+\cdots}}{4\cdots}\right.}{\cdots}\right.\right.\right.$$

$$\Big\downarrow \text{5421}$$

$$c\Bigg( c\Bigg( c\Bigg( \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^7}{8a^2c} - \frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c} - \frac{2\int\frac{x^6\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{7a} - \frac{6\int\frac{x^5\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{7a^2}\right)}{8a} - \frac{7\Big(\sqrt{a^2c}}{}$$

$$c\Bigg( c\Bigg( c\Bigg( \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c} - \frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c} - \frac{2\int\frac{x^4\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{5a} - \frac{4\int\frac{x^3\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{5a^2}}{2a} - \frac{5\Big(\frac{x^3\sqrt{a^2cx^2+}}{4}}{}$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^7}{8a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\int\frac{x^5}{\sqrt{a^2cx^2+c}}dx}{6a}-\frac{5\int\frac{x^4\arctan(ax}{\sqrt{a^2cx^2+c}}}{6a^2}\right.}{7a}\right.}{8a}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\int\frac{x^3}{\sqrt{a^2cx^2+c}}dx}{4a}-\frac{3\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right.}{5a}}{2a}\right.\right.\right.$$

↓ 241

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^7}{8a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\int\frac{x^5}{\sqrt{a^2cx^2+c}}dx}{6a}-\frac{5\int\frac{x^4\arctan(ax}{\sqrt{a^2cx^2+c}}}{6a^2}\right.}{7a}\right.}{8a}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\int\frac{x^3}{\sqrt{a^2cx^2+c}}dx}{4a}-\frac{3\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right.}{5a}}{2a}\right.\right.\right.$$

↓ 243

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^7}{8a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\int\frac{x^4}{\sqrt{a^2cx^2+c}}dx^2}{12a}-\frac{5\int\frac{x^4\arctan(a}{\sqrt{a^2cx^2+}}}{6a^2}\right.}{7a}\right.}{8a}\right.\right.\right.$$

$$\left(\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\int\frac{x^2}{\sqrt{a^2cx^2+c}}dx^2}{8a}-\frac{3\int\frac{x^2\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{4a^2}\right.}{5a}}{2a}\right.\right.\right.$$

↓ 53

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\,\arctan(ax)^3x^7}{8a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\,\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\,\arctan(ax)x^5}{6a^2c}-\frac{\int\left(\frac{\left(a^2cx^2+c\right)^{3/2}}{a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}+\right.}{12a}\right.}{7a}\right.}{}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\,\arctan(ax)^3x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\,\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\,\arctan(ax)x^3}{4a^2c}-\frac{\int\left(\frac{\sqrt{a^2cx^2+c}}{a^2c}-\frac{1}{a^2\sqrt{a^2cx^2+c}}\right)dx^2}{8a}-\right.}{5a}}{2}\right.\right.\right.$$

↓ 2009

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^7}{8a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}}{12a}\right.}{7a}\right.}{}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}-\frac{3\int \frac{x}{}}{}\right.}{5a}}{2a}\right.\right.\right.$$

↓ 5425

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^7}{8a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}}{12a}\right.}{7a}\right.}{}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}-\frac{3\int\frac{x}{}}{}\right.}{5a}}{2a}\right.\right.\right.$$

↓ 5421

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^7}{8a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}}{12a}\right.}{7a}\right.}{}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}-\frac{3\int\frac{x}{}}{}\right.}{5a}}{2a}\right.\right.\right.$$

↓ 5423

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^7}{8a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}}{12a}\right.}{7a}\right.}{}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}-\frac{3\int\frac{x}{}}{}\right.}{5a}}{2a}\right.\right.\right.$$

$\downarrow$ 3042

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^7}{8a^2c}-\frac{3\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^6}{7a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^5}{6a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{5/2}}{5a^6c^3}-\frac{4\left(a^2cx^2+c\right)^{3/2}}{3a^6c^2}}{12a}\right.}{7a}\right.}{}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3x^5}{6a^2c}-\frac{\frac{\sqrt{a^2cx^2+c}\arctan(ax)^2x^4}{5a^2c}-\frac{2\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)x^3}{4a^2c}-\frac{\frac{2\left(a^2cx^2+c\right)^{3/2}}{3a^4c^2}-\frac{2\sqrt{a^2cx^2+c}}{a^4c}}{8a}-\frac{3\int\frac{x}{}}{}\right.}{5a}}{2a}\right.\right.\right.$$

input
```
Int[x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3,x]
```

output
```
$Aborted
```

## Maple [A] (verified)

Time = 14.62 (sec) , antiderivative size = 566, normalized size of antiderivative = 0.56

| method | result |
|---|---|
| default | $\frac{c^2\sqrt{c(ax-i)(ax+i)}\,\left(1680\arctan(ax)^3a^7x^7-720a^6x^6\arctan(ax)^2+4760\arctan(ax)^3a^5x^5+240\arctan(ax)a^5x^5-1992a^4\arctan(a\right.}{}$ |

input
```
int(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/13440*c^2/a^3*(c*(a*x-I)*(a*x+I))^(1/2)*(1680*arctan(a*x)^3*a^7*x^7-720*
a^6*x^6*arctan(a*x)^2+4760*arctan(a*x)^3*a^5*x^5+240*arctan(a*x)*a^5*x^5-1
992*a^4*arctan(a*x)^2*x^4+4130*arctan(a*x)^3*a^3*x^3-48*a^4*x^4+696*arctan
(a*x)*x^3*a^3-1474*arctan(a*x)^2*x^2*a^2+525*x*arctan(a*x)^3*a-168*a^2*x^2
+430*arctan(a*x)*a*x+1373*arctan(a*x)^2-94)-1/13440*c^2*(c*(a*x-I)*(a*x+I)
)^(1/2)*(525*arctan(a*x)^3*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-525*arctan(
a*x)^3*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-1575*I*arctan(a*x)^2*polylog(2,
I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+1575*I*arctan(a*x)^2*polylog(2,-I*(1+I*a*x)
/(a^2*x^2+1)^(1/2))+3176*arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+3
150*arctan(a*x)*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-3176*arctan(a*x)*
ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-3150*arctan(a*x)*polylog(3,-I*(1+I*a*x
)/(a^2*x^2+1)^(1/2))+3150*I*polylog(4,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-3150*
I*polylog(4,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+3176*I*dilog(1+I*(1+I*a*x)/(a^
2*x^2+1)^(1/2))-3176*I*dilog(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/a^3/(a^2*x^
2+1)^(1/2)
```

## Fricas [F]

$$\int x^2\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3\,dx=\int\left(a^2cx^2+c\right)^{\frac{5}{2}}x^2\arctan\left(ax\right)^3\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2)*sqrt(a^2*c*x^2 + c)*arcta
n(a*x)^3, x)
```

## Sympy [F]

$$\int x^2 (c + a^2 cx^2)^{5/2} \arctan(ax)^3 \, dx = \int x^2 \left(c(a^2x^2 + 1)\right)^{\frac{5}{2}} \operatorname{atan}^3(ax) \, dx$$

input
```
integrate(x**2*(a**2*c*x**2+c)**(5/2)*atan(a*x)**3,x)
```

output
```
Integral(x**2*(c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**3, x)
```

## Maxima [F]

$$\int x^2 (c + a^2 cx^2)^{5/2} \arctan(ax)^3 \, dx = \int (a^2cx^2 + c)^{\frac{5}{2}} x^2 \arctan(ax)^3 \, dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)*x^2*arctan(a*x)^3, x)
```

## Giac [F]

$$\int x^2 (c + a^2 cx^2)^{5/2} \arctan(ax)^3 \, dx = \int (a^2cx^2 + c)^{\frac{5}{2}} x^2 \arctan(ax)^3 \, dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)*x^2*arctan(a*x)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int x^2 \left(c+a^2 c x^2\right)^{5/2} \arctan(ax)^3 \, dx = \int x^2 \operatorname{atan}(a\,x)^3 \left(c\,a^2\,x^2+c\right)^{5/2} dx$$

input
```
int(x^2*atan(a*x)^3*(c + a^2*c*x^2)^(5/2),x)
```

output
```
int(x^2*atan(a*x)^3*(c + a^2*c*x^2)^(5/2), x)
```

## Reduce [F]

$$\int x^2 \left(c+a^2 c x^2\right)^{5/2} \arctan(ax)^3 \, dx = \sqrt{c}\,c^2 \left(\left(\int \sqrt{a^2x^2+1}\,atan(ax)^3\,x^6dx\right) a^4 + 2\left(\int \sqrt{a^2x^2+1}\,atan(ax)^3\,x^4dx\right) a^2 + \int \sqrt{a^2x^2+1}\,atan(ax)^3\,x^2dx\right)$$

input
```
int(x^2*(a^2*c*x^2+c)^(5/2)*atan(a*x)^3,x)
```

output
```
sqrt(c)*c**2*(int(sqrt(a**2*x**2 + 1)*atan(a*x)**3*x**6,x)*a**4 + 2*int(sq
rt(a**2*x**2 + 1)*atan(a*x)**3*x**4,x)*a**2 + int(sqrt(a**2*x**2 + 1)*atan
(a*x)**3*x**2,x))
```

# 3.429 $\int x(c+a^2cx^2)^{5/2}\arctan(ax)^3\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 561

$$\begin{aligned}\int x(c+a^2cx^2)^{5/2}\arctan(ax)^3\,dx &= -\frac{17c^2x\sqrt{c+a^2cx^2}}{420a}-\frac{cx(c+a^2cx^2)^{3/2}}{140a}\\
&+\frac{15c^2\sqrt{c+a^2cx^2}\arctan(ax)}{56a^2}+\frac{5c(c+a^2cx^2)^{3/2}\arctan(ax)}{84a^2}\\
&+\frac{(c+a^2cx^2)^{5/2}\arctan(ax)}{35a^2}-\frac{15c^2x\sqrt{c+a^2cx^2}\arctan(ax)^2}{112a}\\
&-\frac{5cx(c+a^2cx^2)^{3/2}\arctan(ax)^2}{56a}-\frac{x(c+a^2cx^2)^{5/2}\arctan(ax)^2}{14a}\\
&+\frac{15ic^3\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2}{56a^2\sqrt{c+a^2cx^2}}\\
&+\frac{(c+a^2cx^2)^{7/2}\arctan(ax)^3}{7a^2c}-\frac{37c^{5/2}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right)}{120a^2}\\
&-\frac{15ic^3\sqrt{1+a^2x^2}\arctan(ax)\,\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)}{56a^2\sqrt{c+a^2cx^2}}\\
&+\frac{15ic^3\sqrt{1+a^2x^2}\arctan(ax)\,\text{PolyLog}\left(2,ie^{i\arctan(ax)}\right)}{56a^2\sqrt{c+a^2cx^2}}\\
&+\frac{15c^3\sqrt{1+a^2x^2}\,\text{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)}{56a^2\sqrt{c+a^2cx^2}}\\
&-\frac{15c^3\sqrt{1+a^2x^2}\,\text{PolyLog}\left(3,ie^{i\arctan(ax)}\right)}{56a^2\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
-17/420*c^2*x*(a^2*c*x^2+c)^(1/2)/a-1/140*c*x*(a^2*c*x^2+c)^(3/2)/a+15/56*
c^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)/a^2+5/84*c*(a^2*c*x^2+c)^(3/2)*arctan(
a*x)/a^2+1/35*(a^2*c*x^2+c)^(5/2)*arctan(a*x)/a^2-15/112*c^2*x*(a^2*c*x^2+
c)^(1/2)*arctan(a*x)^2/a-5/56*c*x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2/a-1/14
*x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^2/a+15/56*I*c^3*(a^2*x^2+1)^(1/2)*arcta
n((1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^2/a^2/(a^2*c*x^2+c)^(1/2)+1/7*(
a^2*c*x^2+c)^(7/2)*arctan(a*x)^3/a^2/c-37/120*c^(5/2)*arctanh(a*c^(1/2)*x/
(a^2*c*x^2+c)^(1/2))/a^2-15/56*I*c^3*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog
(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a^2/(a^2*c*x^2+c)^(1/2)+15/56*I*c^3*(a^
2*x^2+1)^(1/2)*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a^2/(a
^2*c*x^2+c)^(1/2)+15/56*c^3*(a^2*x^2+1)^(1/2)*polylog(3,-I*(1+I*a*x)/(a^2*
x^2+1)^(1/2))/a^2/(a^2*c*x^2+c)^(1/2)-15/56*c^3*(a^2*x^2+1)^(1/2)*polylog(
3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a^2/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (warning: unable to verify)

Time = 3.85 (sec) , antiderivative size = 717, normalized size of antiderivative = 1.28

$$\int x(c+a^2cx^2)^{5/2}\arctan(ax)^3\,dx = \text{Too large to display}$$

input
```
Integrate[x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3,x]
```

output
```
(c^2*Sqrt[c + a^2*c*x^2]*(-53760*(ArcCoth[(a*x)/Sqrt[1 + a^2*x^2]] - I*Arc
Tan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 + I*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*
ArcTan[a*x])] - I*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] - PolyLog[3,
 (-I)*E^(I*ArcTan[a*x])] + PolyLog[3, I*E^(I*ArcTan[a*x])]) - 64*(259*ArcC
oth[(a*x)/Sqrt[1 + a^2*x^2]] - (309*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*
x]^2 + (309*I)*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - (309*I)*Ar
cTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] - 309*PolyLog[3, (-I)*E^(I*ArcTa
n[a*x])] + 309*PolyLog[3, I*E^(I*ArcTan[a*x])]) + 4480*(1 + a^2*x^2)^(3/2)
*ArcTan[a*x]*(6 + 4*ArcTan[a*x]^2 + 6*Cos[2*ArcTan[a*x]] - 3*ArcTan[a*x]*S
in[2*ArcTan[a*x]]) + 112*(48*(10*ArcCoth[(a*x)/Sqrt[1 + a^2*x^2]] - (11*I)
*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 + (11*I)*ArcTan[a*x]*PolyLog[2, (
-I)*E^(I*ArcTan[a*x])] - (11*I)*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])
] - 11*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] + 11*PolyLog[3, I*E^(I*ArcTan[a*
x])]) - (1 + a^2*x^2)^(5/2)*((48*a*x)/(1 + a^2*x^2)^2 + 32*ArcTan[a*x]^3*(
-1 + 5*Cos[2*ArcTan[a*x]]) + 6*ArcTan[a*x]*(25 + 36*Cos[2*ArcTan[a*x]] + 1
1*Cos[4*ArcTan[a*x]]) + ArcTan[a*x]^2*(6*Sin[2*ArcTan[a*x]] - 33*Sin[4*Arc
Tan[a*x]]))) + (1 + a^2*x^2)^(7/2)*(64*ArcTan[a*x]^3*(57 - 28*Cos[2*ArcTan
[a*x]] + 35*Cos[4*ArcTan[a*x]]) + (8*ArcTan[a*x]*(647 + 764*Cos[2*ArcTan[a
*x]] + 309*Cos[4*ArcTan[a*x]]))/(1 + a^2*x^2) + 4*(101*Sin[2*ArcTan[a*x]]
+ 88*Sin[4*ArcTan[a*x]] + 25*Sin[6*ArcTan[a*x]]) - 3*ArcTan[a*x]^2*(211...
```

## Rubi [A] (verified)

Time = 2.11 (sec) , antiderivative size = 523, normalized size of antiderivative = 0.93, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.909$, Rules used = {5465, 5415, 211, 211, 224, 219, 5415, 211, 224, 219, 5415, 224, 219, 5425, 5423, 3042, 4669, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \arctan(ax)^3 \left(a^2cx^2+c\right)^{5/2} dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{\arctan(ax)^3 \left(a^2cx^2+c\right)^{7/2}}{7a^2c}-\frac{3\int \left(a^2cx^2+c\right)^{5/2}\arctan(ax)^2dx}{7a}$$

$$\Big\downarrow \text{5415}$$

$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{7/2}}{7a^2c}-\frac{3\left(\frac{5}{6}c\int\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2dx+\frac{1}{15}c\int\left(a^2cx^2+c\right)^{3/2}dx+\frac{1}{6}x\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}-\frac{\arctan(ax)\left(a^2cx\right.}{15a}\right.}{7a}$$

$\Big\downarrow$ 211

$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{7/2}}{7a^2c}-\frac{3\left(\frac{5}{6}c\int\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2dx+\frac{1}{15}c\left(\frac{3}{4}c\int\sqrt{a^2cx^2+c}dx+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\right)+\frac{1}{6}x\arctan(ax)^2\left(a^2cx^2\right.\right.}{7a}$$

$\Big\downarrow$ 211

$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{7/2}}{7a^2c}-\frac{3\left(\frac{5}{6}c\int\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2dx+\frac{1}{15}c\left(\frac{3}{4}c\left(\frac{1}{2}c\int\frac{1}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\right)+\frac{1}{6}x\,a\right.}{7a}$$

$\Big\downarrow$ 224

$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{7/2}}{7a^2c}-\frac{3\left(\frac{5}{6}c\int\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2dx+\frac{1}{15}c\left(\frac{3}{4}c\left(\frac{1}{2}c\int\frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\left(a^2cx^2+c\right)^{3/}\right.\right.}{7a}$$

$\Big\downarrow$ 219

$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{7/2}}{7a^2c}-\frac{3\left(\frac{5}{6}c\int\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2dx+\frac{1}{6}x\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}{15a}+\frac{1}{15}c\left(\frac{3}{4}c\left(\frac{\sqrt{c}\,\mathrm{arc}}{}\right.\right.\right.}{7a}$$

$\Big\downarrow$ 5415

$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{7/2}}{7a^2c}-\frac{3\left(\frac{5}{6}c\left(\frac{3}{4}c\int\sqrt{a^2cx^2+c}\arctan(ax)^2dx+\frac{1}{6}c\int\sqrt{a^2cx^2+c}dx+\frac{1}{4}x\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}-\frac{\arctan(ax)\left(a^2cx^2+\right.}{6a}\right.\right.}{}$$

$\Big\downarrow$ 211

$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{7/2}}{7a^2c}-$$
$$3\left(\frac{5}{6}c\left(\frac{3}{4}c\int\sqrt{a^2cx^2+c}\arctan(ax)^2dx+\frac{1}{6}c\left(\frac{1}{2}c\int\frac{1}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\arctan(ax)^2\left(a^2cx^2+c\right)^{3/}\right.\right.$$

$\downarrow$ 224

$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{7/2}}{7a^2c}-$$
$$3\left(\frac{5}{6}c\left(\frac{3}{4}c\int\sqrt{a^2cx^2+c}\arctan(ax)^2dx+\frac{1}{6}c\left(\frac{1}{2}c\int\frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)+\frac{1}{4}x\arctan(ax)^2\left(a^2c\right.\right.\right.$$

$\downarrow$ 219

$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{7/2}}{7a^2c}-$$
$$3\left(\frac{5}{6}c\left(\frac{3}{4}c\int\sqrt{a^2cx^2+c}\arctan(ax)^2dx+\frac{1}{4}x\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}-\frac{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}{6a}+\frac{1}{6}c\left(\frac{\sqrt{c}\text{arctanh}}{}\right.\right.\right.$$

$\downarrow$ 5415

$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{7/2}}{7a^2c}-$$
$$3\left(\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{1}{2}c\int\frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int\frac{1}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}\right)+\frac{1}{4}x\arctan(\right.\right.$$

$\downarrow$ 224

$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{7/2}}{7a^2c}-$$
$$3\left(\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{1}{2}c\int\frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int\frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}\right)+\frac{1}{4}x\right.\right.$$

$\downarrow$ 219

$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{7/2}}{7a^2c}-$$
$$3\left(\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{1}{2}c\int\frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}\right)+\frac{1}{4}x\right.\right.$$

$\downarrow$ 5425

$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{7/2}}{7a^2c}-$$
$$3\left(\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\int\frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}\right)+\right.\right.$$

$\downarrow$ 5423

$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{7/2}}{7a^2c}-$$
$$3\left(\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\int\sqrt{a^2x^2+1}\arctan(ax)^2d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\frac{\sqrt{c}\text{arctan}}{}\right.\right.\right.$$

$\downarrow$ 3042

$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{7/2}}{7a^2c}-$$
$$3\left(\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\int\arctan(ax)^2\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\right.\right.\right.$$

$\downarrow$ 4669

$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{7/2}}{7a^2c}-$$
$$3\left(\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\int\arctan(ax)\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+2\int\arctan(ax)\log\left(1+ie^{i\arctan(ax)}\right)d\arctan(ax)-2i\arctan\left(e^{i\,\text{ar}}\right.\right.}{2a\sqrt{a^2cx^2+c}}\right.\right.\right.$$

$\downarrow$ 3011

$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{7/2}}{7a^2c}-$$

$$3\left(\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-i\int\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(ax)\right)-2\left(i\arctan(ax)\operatorname{PolyLog}(2,i\right.\right.}{2a\sqrt{a^2cx^2+c}}\right.\right.\right.$$

$$\Big\downarrow \text{2720}$$

$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{7/2}}{7a^2c}-$$

$$3\left(\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)-2\left(i\arctan(ax\right.\right.}{2a\sqrt{a^2cx^2}}\right.\right.\right.$$

$$\Big\downarrow \text{7143}$$

$$\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{7/2}}{7a^2c}-$$

$$3\left(\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)\right)-2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,ie^{i\arctan(ax)}\right)-\mathrm{F}\right.\right.}{2a\sqrt{a^2cx^2+c}}\right.\right.\right.$$

input
```
Int[x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3,x]
```

output
```
((c + a^2*c*x^2)^(7/2)*ArcTan[a*x]^3)/(7*a^2*c) - (3*(-1/15*((c + a^2*c*x^
2)^(5/2)*ArcTan[a*x])/a + (x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2)/6 + (c*(
(x*(c + a^2*c*x^2)^(3/2))/4 + (3*c*((x*Sqrt[c + a^2*c*x^2])/2 + (Sqrt[c]*A
rcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]])/(2*a)))/4))/15 + (5*c*(-1/6*((c
 + a^2*c*x^2)^(3/2)*ArcTan[a*x])/a + (x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^
2)/4 + (c*((x*Sqrt[c + a^2*c*x^2])/2 + (Sqrt[c]*ArcTanh[(a*Sqrt[c]*x)/Sqrt
[c + a^2*c*x^2]])/(2*a)))/6 + (3*c*(-((Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/a)
 + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/2 + (Sqrt[c]*ArcTanh[(a*Sqrt[c]*x
)/Sqrt[c + a^2*c*x^2]])/a + (c*Sqrt[1 + a^2*x^2]*((-2*I)*ArcTan[E^(I*ArcTa
n[a*x])]*ArcTan[a*x]^2 + 2*(I*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x]
)] - PolyLog[3, (-I)*E^(I*ArcTan[a*x])]) - 2*(I*ArcTan[a*x]*PolyLog[2, I*E
^(I*ArcTan[a*x])] - PolyLog[3, I*E^(I*ArcTan[a*x])])))/(2*a*Sqrt[c + a^2*c
*x^2])))/4))/6))/(7*a)
```

## Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5415
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTan[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x])^p,
x], x] + Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*(
a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[q, 0] && GtQ[p, 1]
```

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

## Maple [A] (verified)

Time = 9.70 (sec) , antiderivative size = 477, normalized size of antiderivative = 0.85

| method | result |
|---|---|
| default | $\frac{c^2\sqrt{c(ax-i)(ax+i)}\left(240\arctan(ax)^3a^6x^6-120a^5\arctan(ax)^2x^5+720\arctan(ax)^3a^4x^4+48x^4\arctan(ax)a^4-390a^3\arctan(ax)^2x\right.}{1680a^2}$ |

input

```
int(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

output

```
1/1680*c^2/a^2*(c*(a*x-I)*(a*x+I))^(1/2)*(240*arctan(a*x)^3*a^6*x^6-120*a^
5*arctan(a*x)^2*x^5+720*arctan(a*x)^3*a^4*x^4+48*x^4*arctan(a*x)*a^4-390*a
^3*arctan(a*x)^2*x^3+720*arctan(a*x)^3*a^2*x^2-12*a^3*x^3+196*x^2*a^2*arct
an(a*x)-495*a*arctan(a*x)^2*x+240*arctan(a*x)^3-80*a*x+598*arctan(a*x))+5/
112*c^2*(c*(a*x-I)*(a*x+I))^(1/2)*(I*arctan(a*x)^3-3*arctan(a*x)^2*ln(1-I*
(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^
2+1)^(1/2))-6*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/a^2/(a^2*x^2+1)^(1
/2)+5/112*c^2*(c*(a*x-I)*(a*x+I))^(1/2)*(-I*arctan(a*x)^3+3*arctan(a*x)^2*
ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I*arctan(a*x)*polylog(2,-I*(1+I*a*x)
/(a^2*x^2+1)^(1/2))+6*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/a^2/(a^2*
x^2+1)^(1/2)+37/60*I*c^2/a^2*(c*(a*x-I)*(a*x+I))^(1/2)*arctan((1+I*a*x)/(a
^2*x^2+1)^(1/2))/(a^2*x^2+1)^(1/2)
```

## Fricas [F]

$$\int x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3\,dx=\int\left(a^2cx^2+c\right)^{\frac{5}{2}}x\arctan\left(ax\right)^3\,dx$$

input

```
integrate(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^3,x, algorithm="fricas")
```

output

```
integral((a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^2*x)*sqrt(a^2*c*x^2 + c)*arctan(
a*x)^3, x)
```

## Sympy [F]

$$\int x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3\,dx=\int x\big(c\big(a^2x^2+1\big)\big)^{\frac{5}{2}}\operatorname{atan}^3\left(ax\right)dx$$

input
```
integrate(x*(a**2*c*x**2+c)**(5/2)*atan(a*x)**3,x)
```

output
```
Integral(x*(c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**3, x)
```

## Maxima [F]

$$\int x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3\,dx=\int\left(a^2cx^2+c\right)^{\frac{5}{2}}x\arctan\left(ax\right)^3\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)*x*arctan(a*x)^3, x)
```

## Giac [F(-2)]

Exception generated.

$$\int x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3\,dx=\int x\,\mathrm{atan}(a\,x)^3\,\left(c\,a^2\,x^2+c\right)^{5/2}\,dx$$

input
```
int(x*atan(a*x)^3*(c + a^2*c*x^2)^(5/2),x)
```

output
```
int(x*atan(a*x)^3*(c + a^2*c*x^2)^(5/2), x)
```

## Reduce [F]

$$\int x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3\,dx=\sqrt{c}\,c^2\left(\left(\int\sqrt{a^2x^2+1}\,atan(ax)^3\,x^5dx\right)a^4+2\left(\int\sqrt{a^2x^2+1}\,atan(ax)^3\,x^3dx\right)a^2+\int\sqrt{a^2x^2+1}\,atan(ax)^3\,xdx\right)$$

input
```
int(x*(a^2*c*x^2+c)^(5/2)*atan(a*x)^3,x)
```

output
```
sqrt(c)*c**2*(int(sqrt(a**2*x**2 + 1)*atan(a*x)**3*x**5,x)*a**4 + 2*int(sq
rt(a**2*x**2 + 1)*atan(a*x)**3*x**3,x)*a**2 + int(sqrt(a**2*x**2 + 1)*atan
(a*x)**3*x,x))
```

# 3.430 $\int (c + a^2cx^2)^{5/2} \arctan(ax)^3 \, dx$



## Optimal result

Integrand size = 21, antiderivative size = 870

$$\int (c + a^2cx^2)^{5/2} \arctan(ax)^3 \, dx = -\frac{17c^2\sqrt{c + a^2cx^2}}{60a}$$

$$- \frac{c(c + a^2cx^2)^{3/2}}{60a} + \frac{17}{60}c^2x\sqrt{c + a^2cx^2} \arctan(ax)$$

$$+ \frac{1}{20}cx(c+a^2cx^2)^{3/2} \arctan(ax) - \frac{15c^2\sqrt{c + a^2cx^2} \arctan(ax)^2}{16a} - \frac{5c(c + a^2cx^2)^{3/2} \arctan(ax)^2}{24a} - \frac{(c + a^2cx^2}{}$$

output
```
-17/60*c^2*(a^2*c*x^2+c)^(1/2)/a-1/60*c*(a^2*c*x^2+c)^(3/2)/a+17/60*c^2*x*
(a^2*c*x^2+c)^(1/2)*arctan(a*x)+1/20*c*x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)-1
5/16*c^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2/a-5/24*c*(a^2*c*x^2+c)^(3/2)*ar
ctan(a*x)^2/a-1/10*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^2/a+5/16*c^2*x*(a^2*c*x
^2+c)^(1/2)*arctan(a*x)^3+5/24*c*x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3+1/6*x
*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^3-5/8*I*c^3*(a^2*x^2+1)^(1/2)*arctan((1+I
*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^3/a/(a^2*c*x^2+c)^(1/2)-15/16*I*c^3*(
a^2*x^2+1)^(1/2)*arctan(a*x)^2*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a/
(a^2*c*x^2+c)^(1/2)-259/120*I*c^3*(a^2*x^2+1)^(1/2)*polylog(2,I*(1+I*a*x)^
(1/2)/(1-I*a*x)^(1/2))/a/(a^2*c*x^2+c)^(1/2)+259/120*I*c^3*(a^2*x^2+1)^(1/
2)*polylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/a/(a^2*c*x^2+c)^(1/2)-15/
8*I*c^3*(a^2*x^2+1)^(1/2)*polylog(4,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a/(a^2
*c*x^2+c)^(1/2)-259/60*I*c^3*(a^2*x^2+1)^(1/2)*arctan(a*x)*arctan((1+I*a*x
)^(1/2)/(1-I*a*x)^(1/2))/a/(a^2*c*x^2+c)^(1/2)-15/8*c^3*(a^2*x^2+1)^(1/2)*
arctan(a*x)*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a/(a^2*c*x^2+c)^(1/2
)+15/8*c^3*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)
^(1/2))/a/(a^2*c*x^2+c)^(1/2)+15/8*I*c^3*(a^2*x^2+1)^(1/2)*polylog(4,I*(1+
I*a*x)/(a^2*x^2+1)^(1/2))/a/(a^2*c*x^2+c)^(1/2)+15/16*I*c^3*(a^2*x^2+1)^(1
/2)*arctan(a*x)^2*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a/(a^2*c*x^2+c
)^(1/2)
```

## Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 4281 vs. $2(870) = 1740$.

Time = 18.80 (sec) , antiderivative size = 4281, normalized size of antiderivative = 4.92

$$\int \left(c+a^2cx^2\right)^{5/2} \arctan(ax)^3 \, dx = \text{Result too large to show}$$

input
```
Integrate[(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3,x]
```

output
```
((-1/2*I)*c^2*Sqrt[c*(1 + a^2*x^2)]*(12*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a
*x] - (3*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2 + I*a*x*Sqrt[1 + a^2*x^2]*ArcT
an[a*x]^3 + 2*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^3 - 3*(2 + ArcTan[a*x]
^2)*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + 3*(2 + ArcTan[a*x]^2)*PolyLog[2,
I*E^(I*ArcTan[a*x])] - (6*I)*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])
] + (6*I)*ArcTan[a*x]*PolyLog[3, I*E^(I*ArcTan[a*x])] + 6*PolyLog[4, (-I)*
E^(I*ArcTan[a*x])] - 6*PolyLog[4, I*E^(I*ArcTan[a*x])]))/(a*Sqrt[1 + a^2*x
^2]) + (2*c^2*((Sqrt[c*(1 + a^2*x^2)]*(-1 + ArcTan[a*x]^2))/(4*Sqrt[1 + a^
2*x^2]) + (Sqrt[c*(1 + a^2*x^2)]*(-(ArcTan[a*x]*(Log[1 - I*E^(I*ArcTan[a*x
])] - Log[1 + I*E^(I*ArcTan[a*x])])) - I*(PolyLog[2, (-I)*E^(I*ArcTan[a*x]
)] - PolyLog[2, I*E^(I*ArcTan[a*x])])))/(2*Sqrt[1 + a^2*x^2]) + (Sqrt[c*(1
 + a^2*x^2)]*(-1/8*(Pi^3*Log[Cot[(Pi/2 - ArcTan[a*x])/2]]) - (3*Pi^2*((Pi/
2 - ArcTan[a*x])*(Log[1 - E^(I*(Pi/2 - ArcTan[a*x]))] - Log[1 + E^(I*(Pi/2
 - ArcTan[a*x]))]) + I*(PolyLog[2, -E^(I*(Pi/2 - ArcTan[a*x]))] - PolyLog[
2, E^(I*(Pi/2 - ArcTan[a*x]))])))/4 + (3*Pi*((Pi/2 - ArcTan[a*x])^2*(Log[1
 - E^(I*(Pi/2 - ArcTan[a*x]))] - Log[1 + E^(I*(Pi/2 - ArcTan[a*x]))]) + (2
*I)*(Pi/2 - ArcTan[a*x])*(PolyLog[2, -E^(I*(Pi/2 - ArcTan[a*x]))] - PolyLo
g[2, E^(I*(Pi/2 - ArcTan[a*x]))]) + 2*(-PolyLog[3, -E^(I*(Pi/2 - ArcTan[a*
x]))] + PolyLog[3, E^(I*(Pi/2 - ArcTan[a*x]))])))/2 - 8*((I/64)*(Pi/2 - Ar
cTan[a*x])^4 + (I/4)*(Pi/2 + (-1/2*Pi + ArcTan[a*x])/2)^4 - ((Pi/2 - Ar...
```

## Rubi [A] (verified)

Time = 4.11 (sec) , antiderivative size = 954, normalized size of antiderivative = 1.10, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {5415, 5413, 5413, 5415, 5413, 5415, 5425, 5421, 5423, 3042, 4669, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(ax)^3 \left(a^2cx^2+c\right)^{5/2} dx$$

$$\Big\downarrow \text{5415}$$

$$\frac{1}{5}c\int \left(a^2cx^2+c\right)^{3/2}\arctan(ax)dx+\frac{5}{6}c\int \left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3dx+\frac{1}{6}x\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}}{10a}$$

$$\Big\downarrow \text{5413}$$

$$\frac{1}{5}c\left(\frac{3}{4}c\int \sqrt{a^2cx^2+c}\arctan(ax)dx+\frac{1}{4}x\arctan(ax)\left(a^2cx^2+c\right)^{3/2}-\frac{\left(a^2cx^2+c\right)^{3/2}}{12a}\right)+\frac{5}{6}c\int \left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3dx+\frac{1}{6}x\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}}{10a}$$

$$\Big\downarrow \text{5413}$$

$$\frac{1}{5}c\left(\frac{3}{4}c\left(\frac{1}{2}c\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)+\frac{1}{4}x\arctan(ax)\left(a^2cx^2+c\right)^{3/2}-\frac{(\cdots}{\cdots}\right.\frac{5}{6}c\int \left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3dx+\frac{1}{6}x\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}}{10a}$$

$$\Big\downarrow \text{5415}$$

$$\frac{1}{5}c\left(\frac{3}{4}c\left(\frac{1}{2}c\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)+\frac{1}{4}x\arctan(ax)\left(a^2cx^2+c\right)^{3/2}-\frac{(\cdots}{\cdots}\right.\frac{5}{6}c\left(\frac{1}{2}c\int \sqrt{a^2cx^2+c}\arctan(ax)dx+\frac{3}{4}c\int \sqrt{a^2cx^2+c}\arctan(ax)^3dx+\frac{1}{4}x\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}-\frac{\text{arct}\cdots}{\cdots}\right.\frac{1}{6}x\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}}{10a}$$

$$\Big\downarrow \text{5413}$$

$$\frac{1}{5}c\left(\frac{3}{4}c\left(\frac{1}{2}c\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)+\frac{1}{4}x\arctan(ax)\left(a^2cx^2+c\right)^{3/2}-\frac{(\cdots}{\cdots}\right.\frac{5}{6}c\left(\frac{1}{2}c\left(\frac{1}{2}c\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)+\frac{3}{4}c\int \sqrt{a^2cx^2+c}\arctan(ax)^3dx\cdots\right.\frac{1}{6}x\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}}{10a}$$

$$\Big\downarrow \text{5415}$$

$$\frac{1}{5}c\left(\frac{3}{4}c\left(\frac{1}{2}c\int\frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)+\frac{1}{4}x\arctan(ax)\left(a^2cx^2+c\right)^{3/2}-\frac{(\right.$$
$$\frac{5}{6}c\left(\frac{1}{2}c\left(\frac{1}{2}c\int\frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)+\frac{3}{4}c\left(3c\int\frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}c\int\right.\right.$$
$$\frac{1}{6}x\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}}{10a}$$

$\downarrow$ 5425

$$\frac{1}{5}c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\int\frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)+\frac{1}{4}x\arctan(ax)\left(a^2cx^2+c\right)^{3}\right.$$
$$\frac{5}{6}c\left(\frac{1}{2}c\left(\frac{c\sqrt{a^2x^2+1}\int\frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)+\frac{3}{4}c\left(\frac{3c\sqrt{a^2x^2+1}\int\frac{\arctan(a}{\sqrt{a^2x^2+}}}{\sqrt{a^2cx^2+c}}\right.\right.$$
$$\frac{1}{6}x\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}-\frac{\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}}{10a}$$

$\downarrow$ 5421

$$\frac{1}{6}x\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^3-\frac{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^2}{10a}+$$
$$\frac{1}{5}c\left(\frac{1}{4}x\arctan(ax)\left(a^2cx^2+c\right)^{3/2}-\frac{\left(a^2cx^2+c\right)^{3/2}}{12a}+\frac{3}{4}c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(}{}\right.}{}\right.\right.$$
$$\frac{5}{6}c\left(\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3-\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{4a}+\frac{1}{2}c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+}}{}\right.\right.$$

$\downarrow$ 5423

$$\frac{1}{6}x\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^3-\frac{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^2}{10a}+$$
$$\frac{1}{5}c\left(\frac{1}{4}x\arctan(ax)\left(a^2cx^2+c\right)^{3/2}-\frac{\left(a^2cx^2+c\right)^{3/2}}{12a}+\frac{3}{4}c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(\right.}{}\right.}{}\right.\right.$$
$$\frac{5}{6}c\left(\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3-\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{4a}+\frac{1}{2}c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+}}{}\right.\right.$$

$\downarrow$ 3042

$$\frac{1}{6}x\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^3-\frac{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^2}{10a}+$$
$$\frac{1}{5}c\left(\frac{1}{4}x\arctan(ax)\left(a^2cx^2+c\right)^{3/2}-\frac{\left(a^2cx^2+c\right)^{3/2}}{12a}+\frac{3}{4}c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(\right.}{}\right.}{}\right.\right.$$
$$\frac{5}{6}c\left(\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3-\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{4a}+\frac{1}{2}c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+}}{}\right.\right.$$

$\downarrow$ 4669

$$\frac{1}{6}x\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^3-\frac{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^2}{10a}+$$
$$\frac{1}{5}c\left(\frac{1}{4}x\arctan(ax)\left(a^2cx^2+c\right)^{3/2}-\frac{\left(a^2cx^2+c\right)^{3/2}}{12a}+\frac{3}{4}c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(\right.}{}\right.}{}\right.\right.$$
$$\frac{5}{6}c\left(\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3-\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{4a}+\frac{1}{2}c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+}}{}\right.\right.$$

$\downarrow$ 3011

$$\frac{1}{6}x\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^3-\frac{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^2}{10a}+$$
$$\frac{1}{5}c\left(\frac{1}{4}x\arctan(ax)\left(a^2cx^2+c\right)^{3/2}-\frac{\left(a^2cx^2+c\right)^{3/2}}{12a}+\frac{3}{4}c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(}{}\right.}{}\right.\right.$$
$$\frac{5}{6}c\left(\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3-\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{4a}+\frac{1}{2}c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+}}{}\right.\right.$$

$\downarrow$ 7163

$$\frac{1}{6}x\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^3-\frac{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^2}{10a}+$$
$$\frac{1}{5}c\left(\frac{1}{4}x\arctan(ax)\left(a^2cx^2+c\right)^{3/2}-\frac{\left(a^2cx^2+c\right)^{3/2}}{12a}+\frac{3}{4}c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(}{}\right.}{}\right.\right.$$
$$\frac{5}{6}c\left(\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3-\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{4a}+\frac{1}{2}c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+}}{}\right.\right.$$

$\downarrow$ 2720

$$\frac{1}{6}x\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^3-\frac{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^2}{10a}+$$
$$\frac{1}{5}c\left(\frac{1}{4}x\arctan(ax)\left(a^2cx^2+c\right)^{3/2}-\frac{\left(a^2cx^2+c\right)^{3/2}}{12a}+\frac{3}{4}c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(}{}\right.}{}\right.\right.$$
$$\frac{5}{6}c\left(\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3-\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{4a}+\frac{1}{2}c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+}}{}\right.\right.$$

$\downarrow$ 7143

$$\frac{1}{6}x(a^2cx^2+c)^{5/2}\arctan(ax)^3-\frac{(a^2cx^2+c)^{5/2}\arctan(ax)^2}{10a}+$$

$$\frac{1}{5}c\left(\frac{1}{4}x\arctan(ax)\left(a^2cx^2+c\right)^{3/2}-\frac{(a^2cx^2+c)^{3/2}}{12a}+\frac{3}{4}c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(}{}\right.}{}\right.\right.$$

$$\frac{5}{6}c\left(\frac{1}{4}x(a^2cx^2+c)^{3/2}\arctan(ax)^3-\frac{(a^2cx^2+c)^{3/2}\arctan(ax)^2}{4a}+\frac{1}{2}c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+}}{}\right.\right.$$

input
```
Int[(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3,x]
```

output
```
-1/10*((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2)/a + (x*(c + a^2*c*x^2)^(5/2)*A
rcTan[a*x]^3)/6 + (c*(-1/12*(c + a^2*c*x^2)^(3/2)/a + (x*(c + a^2*c*x^2)^(
3/2)*ArcTan[a*x])/4 + (3*c*(-1/2*Sqrt[c + a^2*c*x^2]/a + (x*Sqrt[c + a^2*c
*x^2]*ArcTan[a*x])/2 + (c*Sqrt[1 + a^2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sq
rt[1 + I*a*x]/Sqrt[1 - I*a*x]])/a + (I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/S
qrt[1 - I*a*x]])/a - (I*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a
))/(2*Sqrt[c + a^2*c*x^2])))/4))/5 + (5*c*(-1/4*((c + a^2*c*x^2)^(3/2)*Arc
Tan[a*x]^2)/a + (x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3)/4 + (c*(-1/2*Sqrt[
c + a^2*c*x^2]/a + (x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/2 + (c*Sqrt[1 + a^2
*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/a + (I
*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a - (I*PolyLog[2, (I*
Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/(2*Sqrt[c + a^2*c*x^2])))/2 + (3*c*
((-3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(2*a) + (x*Sqrt[c + a^2*c*x^2]*Arc
Tan[a*x]^3)/2 + (3*c*Sqrt[1 + a^2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan[Sqrt[1
+ I*a*x]/Sqrt[1 - I*a*x]])/a + (I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x])/Sqrt[1
 - I*a*x]])/a - (I*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]])/a))/Sq
rt[c + a^2*c*x^2] + (c*Sqrt[1 + a^2*x^2]*((-2*I)*ArcTan[E^(I*ArcTan[a*x])]
*ArcTan[a*x]^3 + 3*(I*ArcTan[a*x]^2*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - (
2*I)*((-I)*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] + PolyLog[4, (-I
)*E^(I*ArcTan[a*x])])) - 3*(I*ArcTan[a*x]^2*PolyLog[2, I*E^(I*ArcTan[a*...
```

### Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5413
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_.), x_Symbo
l] :> Simp[(-b)*((d + e*x^2)^q/(2*c*q*(2*q + 1))), x] + (Simp[x*(d + e*x^2)
^q*((a + b*ArcTan[c*x])/(2*q + 1)), x] + Simp[2*d*(q/(2*q + 1))   Int[(d +
e*x^2)^(q - 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
 EqQ[e, c^2*d] && GtQ[q, 0]
```

rule 5415
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(-b)*p*(d + e*x^2)^q*((a + b*ArcTan[c*x])^(p - 1)/(2*c*q*(2
*q + 1))), x] + (Simp[x*(d + e*x^2)^q*((a + b*ArcTan[c*x])^p/(2*q + 1)), x]
 + Simp[2*d*(q/(2*q + 1))   Int[(d + e*x^2)^(q - 1)*(a + b*ArcTan[c*x])^p,
x], x] + Simp[b^2*d*p*((p - 1)/(2*q*(2*q + 1)))   Int[(d + e*x^2)^(q - 1)*(
a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e,
c^2*d] && GtQ[q, 0] && GtQ[p, 1]
```

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

## Maple [A] (verified)

Time = 7.55 (sec) , antiderivative size = 518, normalized size of antiderivative = 0.60

| method | result |
|---|---|
| default | $\frac{c^{2}\sqrt{c(ax-i)(ax+i)}\,\left(40\arctan(ax)^{3}a^{5}x^{5}-24a^{4}\arctan(ax)^{2}x^{4}+130\arctan(ax)^{3}a^{3}x^{3}+12\arctan(ax)x^{3}a^{3}-98\arctan(ax)^{2}x^{2}a^{2}+1}{240a}$ |

input

```
int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

output

```
1/240*c^2/a*(c*(a*x-I)*(a*x+I))^(1/2)*(40*arctan(a*x)^3*a^5*x^5-24*a^4*arc
tan(a*x)^2*x^4+130*arctan(a*x)^3*a^3*x^3+12*arctan(a*x)*x^3*a^3-98*arctan(
a*x)^2*x^2*a^2+165*x*arctan(a*x)^3*a-4*a^2*x^2+80*arctan(a*x)*a*x-299*arct
an(a*x)^2-72)-1/240*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/2)/a*(75*arct
an(a*x)^3*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-75*arctan(a*x)^3*ln(1-I*(1+I
*a*x)/(a^2*x^2+1)^(1/2))-225*I*arctan(a*x)^2*polylog(2,-I*(1+I*a*x)/(a^2*x
^2+1)^(1/2))+225*I*arctan(a*x)^2*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+
518*arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+450*arctan(a*x)*polylo
g(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-518*arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*
x^2+1)^(1/2))-450*arctan(a*x)*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+450
*I*polylog(4,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-450*I*polylog(4,I*(1+I*a*x)/(
a^2*x^2+1)^(1/2))-518*I*dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+518*I*dilog
(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))*c^2
```

## Fricas [F]

$$\int \left(c+a^{2}cx^{2}\right)^{5/2}\arctan(ax)^{3}\,dx=\int \left(a^{2}cx^{2}+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^{3}\,dx$$

input

```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3,x, algorithm="fricas")
```

output

```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*arctan(a*
x)^3, x)
```

## Sympy [F]

$$\int \left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3\,dx=\int \left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}\operatorname{atan}^3\left(ax\right)dx$$

input
```
integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)**3,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**3, x)
```

## Maxima [F]

$$\int \left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3\,dx=\int \left(a^2cx^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^3\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^3, x)
```

## Giac [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3\,dx=\int \operatorname{atan}(a\,x)^3\,\left(c\,a^2\,x^2+c\right)^{5/2}\,dx$$

input
```
int(atan(a*x)^3*(c + a^2*c*x^2)^(5/2),x)
```

output
```
int(atan(a*x)^3*(c + a^2*c*x^2)^(5/2), x)
```

## Reduce [F]

$$\int \left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3\,dx=\sqrt{c}\,c^2\left(\left(\left(\int \sqrt{a^2x^2+1}\,atan(ax)^3\,x^4dx\right)a^4+2\left(\int \sqrt{a^2x^2+1}\,atan(ax)^3\,x^2dx\right)a^2+\int \sqrt{a^2x^2+1}\,atan(ax)^3\,dx\right)\right.$$

input
```
int((a^2*c*x^2+c)^(5/2)*atan(a*x)^3,x)
```

output
```
sqrt(c)*c**2*(int(sqrt(a**2*x**2 + 1)*atan(a*x)**3*x**4,x)*a**4 + 2*int(sq
rt(a**2*x**2 + 1)*atan(a*x)**3*x**2,x)*a**2 + int(sqrt(a**2*x**2 + 1)*atan
(a*x)**3,x))
```

# 3.431 $\int \frac{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3}{x}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 845

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^3}{x}\,dx = -\frac{1}{20}ac^2x\sqrt{c+a^2cx^2}+\frac{29}{20}c^2\sqrt{c+a^2cx^2}\arctan(ax)$$
$$+\frac{1}{10}c\left(c+a^2cx^2\right)^{3/2}\arctan(ax)-\frac{29}{40}ac^2x\sqrt{c+a^2cx^2}\arctan(ax)^2$$
$$-\frac{3}{20}acx\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2+\frac{149ic^3\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2}{20\sqrt{c+a^2cx^2}}+c^2\sqrt{c+a^2cx^2}\,\text{arcta}$$

output
```
-1/20*a*c^2*x*(a^2*c*x^2+c)^(1/2)+29/20*c^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x
)+1/10*c*(a^2*c*x^2+c)^(3/2)*arctan(a*x)-29/40*a*c^2*x*(a^2*c*x^2+c)^(1/2)
*arctan(a*x)^2-3/20*a*c*x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^2+3*I*c^3*(a^2*x
^2+1)^(1/2)*arctan(a*x)^2*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x
^2+c)^(1/2)+c^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^3+1/3*c*(a^2*c*x^2+c)^(3/2
)*arctan(a*x)^3+1/5*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^3-2*c^3*(a^2*x^2+1)^(1
/2)*arctan(a*x)^3*arctanh((1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)
-3/2*c^(5/2)*arctanh(a*c^(1/2)*x/(a^2*c*x^2+c)^(1/2))+149/20*I*c^3*(a^2*x^
2+1)^(1/2)*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^2/(a^2*c*x^2+c)
^(1/2)+149/20*I*c^3*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a
^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)-149/20*I*c^3*(a^2*x^2+1)^(1/2)*arctan
(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)-6*I*c^
3*(a^2*x^2+1)^(1/2)*polylog(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^
(1/2)-6*c^3*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(3,-(1+I*a*x)/(a^2*x^2+1)
^(1/2))/(a^2*c*x^2+c)^(1/2)+149/20*c^3*(a^2*x^2+1)^(1/2)*polylog(3,-I*(1+I
*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)-149/20*c^3*(a^2*x^2+1)^(1/2)*
polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)+6*c^3*(a^2*x^
2+1)^(1/2)*arctan(a*x)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c
)^(1/2)-3*I*c^3*(a^2*x^2+1)^(1/2)*arctan(a*x)^2*polylog(2,(1+I*a*x)/(a^2*x
^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)+6*I*c^3*(a^2*x^2+1)^(1/2)*polylog(4,(1...
```

## Mathematica [A] (warning: unable to verify)

Time = 4.81 (sec) , antiderivative size = 723, normalized size of antiderivative = 0.86

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^3}{x}\,dx = \text{Too large to display}$$

input
```
Integrate[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3)/x,x]
```

output
```
(c^2*Sqrt[c + a^2*c*x^2]*((-120*I)*Pi^4 - 1440*ArcCoth[(a*x)/Sqrt[1 + a^2*
x^2]] + 960*(1 + a^2*x^2)^(3/2)*ArcTan[a*x] - 150*(1 + a^2*x^2)^(5/2)*ArcT
an[a*x] + (1392*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 + 960*Sqrt[1 +
a^2*x^2]*ArcTan[a*x]^3 + 640*(1 + a^2*x^2)^(3/2)*ArcTan[a*x]^3 + 32*(1 + a
^2*x^2)^(5/2)*ArcTan[a*x]^3 + (240*I)*ArcTan[a*x]^4 + 960*(1 + a^2*x^2)^(3
/2)*ArcTan[a*x]*Cos[2*ArcTan[a*x]] - 216*(1 + a^2*x^2)^(5/2)*ArcTan[a*x]*C
os[2*ArcTan[a*x]] - 160*(1 + a^2*x^2)^(5/2)*ArcTan[a*x]^3*Cos[2*ArcTan[a*x
]] - 66*(1 + a^2*x^2)^(5/2)*ArcTan[a*x]*Cos[4*ArcTan[a*x]] + 960*ArcTan[a*
x]^3*Log[1 - E^((-I)*ArcTan[a*x])] - 2880*ArcTan[a*x]^2*Log[1 - I*E^(I*Arc
Tan[a*x])] + 2880*ArcTan[a*x]^2*Log[1 + I*E^(I*ArcTan[a*x])] - 960*ArcTan[
a*x]^3*Log[1 + E^(I*ArcTan[a*x])] + (2880*I)*ArcTan[a*x]^2*PolyLog[2, E^((
-I)*ArcTan[a*x])] + (2880*I)*ArcTan[a*x]^2*PolyLog[2, -E^(I*ArcTan[a*x])]
- (7152*I)*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + (7152*I)*ArcTa
n[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] + 5760*ArcTan[a*x]*PolyLog[3, E^((-
I)*ArcTan[a*x])] - 5760*ArcTan[a*x]*PolyLog[3, -E^(I*ArcTan[a*x])] + 7152*
PolyLog[3, (-I)*E^(I*ArcTan[a*x])] - 7152*PolyLog[3, I*E^(I*ArcTan[a*x])]
- (5760*I)*PolyLog[4, E^((-I)*ArcTan[a*x])] - (5760*I)*PolyLog[4, -E^(I*Ar
cTan[a*x])] - 12*(1 + a^2*x^2)^(5/2)*Sin[2*ArcTan[a*x]] - 480*(1 + a^2*x^2
)^(3/2)*ArcTan[a*x]^2*Sin[2*ArcTan[a*x]] - 6*(1 + a^2*x^2)^(5/2)*ArcTan[a*
x]^2*Sin[2*ArcTan[a*x]] - 6*(1 + a^2*x^2)^(5/2)*Sin[4*ArcTan[a*x]] + 33...
```

## Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}{x}dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\int x\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3dx+c\int \frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{x}dx$$

$$\Big\downarrow \text{5465}$$

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{3\int \left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2dx}{5a}\right)+ c\int \frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{x}dx$$

$\Big\downarrow$ 5415

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{3\left(\frac{3}{4}c\int\sqrt{a^2cx^2+c}\arctan(ax)^2dx+\frac{1}{6}c\int\sqrt{a^2cx^2+c}dx+\frac{1}{4}x\arctan(ax)^2\left(c\right.\right.}{5a}\right.$$

$$c\int\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{x}dx$$

$\Big\downarrow$ 211

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{3\left(\frac{3}{4}c\int\sqrt{a^2cx^2+c}\arctan(ax)^2dx+\frac{1}{6}c\left(\frac{1}{2}c\int\frac{1}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\sqrt{a^2cx^2+c}\right)\right.}{5a}\right.$$

$$c\int\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{x}dx$$

$\Big\downarrow$ 224

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{3\left(\frac{3}{4}c\int\sqrt{a^2cx^2+c}\arctan(ax)^2dx+\frac{1}{6}c\left(\frac{1}{2}c\int\frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}+\frac{1}{2}x\sqrt{a^2c\right.\right.}{5a}\right.$$

$$c\int\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{x}dx$$

$\Big\downarrow$ 219

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{3\left(\frac{3}{4}c\int\sqrt{a^2cx^2+c}\arctan(ax)^2dx+\frac{1}{4}x\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}-\frac{\arctan(}{}\right.}{5a}\right.$$

$$c\int\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{x}dx$$

$\Big\downarrow$ 5415

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{3\left(\frac{3}{4}c\left(\frac{1}{2}c\int\frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int\frac{1}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\right.\right.}{}\right.$$

$$c\int\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{x}dx$$

$\downarrow$ 224

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{3\left(\frac{3}{4}c\left(\frac{1}{2}c\int\frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int\frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2}\right.\right.}{}\right.$$

$$c\int\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{x}dx$$

$\downarrow$ 219

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{3\left(\frac{3}{4}c\left(\frac{1}{2}c\int\frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\right.\right.}{}\right.$$

$$c\int\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{x}dx$$

$\downarrow$ 5425

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{3\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\int\frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2}}{a}\right.\right.}{}\right.$$

$$c\int\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{x}dx$$

$\downarrow$ 5423

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{3\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\int\sqrt{a^2x^2+1}\arctan(ax)^2d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}\right.\right.}{}\right.$$

$$c\int\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{x}dx$$

$\downarrow$ 3042

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{3\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\int\arctan(ax)^2\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2c\right.\right.}{}\right.$$

$$c\int\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{x}dx$$

$\downarrow$ 4669

$$c\int\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{x}dx+$$

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{3\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(-2\int\arctan(ax)\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+2\int\arctan(ax)\log\left(1+ie\right.\right.}{2a\sqrt{a^2cx^2+c}}\right.\right.}{}\right.$$

$\downarrow$ 3011

$$c\int\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{x}dx+$$

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{3\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-i\int\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\right.\right.}{}\right.\right.}{}\right.$$

$\downarrow$ 2720

$$c\int \frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{x}dx+$$

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{3\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}(2(i\arctan(ax)\operatorname{PolyLog}(2,-ie^{i\arctan(ax)})-\int e^{-i\arctan(ax)}\operatorname{PolyLog}(2,-ie^{i}\right.\right.}{}\right.$$

$\downarrow$ 5485

$$c\left(a^2c\int x\sqrt{a^2cx^2+c}\arctan(ax)^3dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x}dx\right)+$$

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{3\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}(2(i\arctan(ax)\operatorname{PolyLog}(2,-ie^{i\arctan(ax)})-\int e^{-i\arctan(ax)}\operatorname{PolyLog}(2,-ie^{i}\right.\right.}{}\right.$$

$\downarrow$ 5465

$$c\left(a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\int\sqrt{a^2cx^2+c}\arctan(ax)^2dx}{a}\right)+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x}dx\right)+$$

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{3\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}(2(i\arctan(ax)\operatorname{PolyLog}(2,-ie^{i\arctan(ax)})-\int e^{-i\arctan(ax)}\operatorname{PolyLog}(2,-ie^{i}\right.\right.}{}\right.$$

$\downarrow$ 5415

$$c\left(a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{1}{2}c\int\frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int\frac{1}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(}{}}{a}\right.\right.$$

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{3\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}(2(i\arctan(ax)\operatorname{PolyLog}(2,-ie^{i\arctan(ax)})-\int e^{-i\arctan(ax)}\operatorname{PolyLog}(2,-ie^{i}\right.\right.}{}\right.$$

$\downarrow$ 224

$$c\left(a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{1}{2}c\int\frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int\frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}\cdots}{a}\cdots\right.\right.$$

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{3\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\cdots}\right.\right.\right.}{\cdots}\right.\right.}{\cdots}\right.$$

$\big\downarrow$ 219

$$c\left(a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{1}{2}c\int\frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\frac{\sqrt{c}\cdots}{\cdots}\cdots}{a}\cdots\right.\right.$$

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{3\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\cdots}\right.\right.\right.}{\cdots}\right.\right.}{\cdots}\right.$$

$\big\downarrow$ 5425

$$c\left(a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{c\sqrt{a^2x^2+1}\int\frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+\cdots}{a}\cdots\right.\right.$$

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{3\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\cdots}\right.\right.\right.}{\cdots}\right.\right.}{\cdots}\right.$$

$\big\downarrow$ 5423

$$c\left(a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{c\sqrt{a^2x^2+1}\int\sqrt{a^2x^2+1}\arctan(ax)^2d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\mathrm{arc}}{}}{a}\right.\right.$$

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{3\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}(2(i\arctan(ax)\operatorname{PolyLog}(2,-ie^{i\arctan(ax)})-\int e^{-i\arctan(ax)}\operatorname{PolyLog}(2,-ie^{i}}{}\right.\right.}{}\right.$$

$\downarrow$ 3042

$$c\left(a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{c\sqrt{a^2x^2+1}\int\arctan(ax)^2\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+}}{a}\right.\right.$$

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{3\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}(2(i\arctan(ax)\operatorname{PolyLog}(2,-ie^{i\arctan(ax)})-\int e^{-i\arctan(ax)}\operatorname{PolyLog}(2,-ie^{i}}{}\right.\right.}{}\right.$$

$\downarrow$ 4669

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{3\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}(2(i\arctan(ax)\operatorname{PolyLog}(2,-ie^{i\arctan(ax)})-\int e^{-i\arctan(ax)}\operatorname{PolyLog}(2,-ie^{i}}{}\right.\right.}{}\right.$$

$$c\left(c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x}dx+a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{c\sqrt{a^2x^2+1}\left(-2\int\arctan(ax)\log\left(1-ie^{i\arctan(ax)}\right)\right.}{}\right.\right.$$

$\downarrow$ 3011

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{3\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\right.\right.}}{}\right.\right.}{}\right.$$

$$c\left(c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x}dx+a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(\right.\right.}}{}\right.\right.$$

$\downarrow$ 2720

$$a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{3\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\right.\right.}}{}\right.\right.}{}\right.$$

$$c\left(c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x}dx+a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(\right.\right.}}{}\right.\right.$$

$\downarrow$ 5485

$$c\left(\frac{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^3}{5a^2c}-\frac{3\left(\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2-\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}{6a}+\frac{1}{6}c\left(\frac{1}{2}\sqrt{a^2cx^2+c}x\right.\right.}{}\right.$$

$$c\left(c\left(\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{3a^2c}-\frac{\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^2-\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a}+\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}+\frac{c}{}}{}\right.\right.$$

$\downarrow$ 5465

$$c\left(\frac{(a^2cx^2+c)^{5/2}\arctan(ax)^3}{5a^2c}-\frac{3\left(\frac{1}{4}x(a^2cx^2+c)^{3/2}\arctan(ax)^2-\frac{(a^2cx^2+c)^{3/2}\arctan(ax)}{6a}+\frac{1}{6}c\left(\frac{1}{2}\sqrt{a^2cx^2+c}x\right.\right.}{}\right.$$

$$c\left(c\left(\frac{(a^2cx^2+c)^{3/2}\arctan(ax)^3}{3a^2c}-\frac{\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^2-\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a}+\frac{\sqrt{c}\,\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}+\frac{c}{}}{}\right.\right.$$

$\Big\downarrow$ 5425

$$c\left(\frac{(a^2cx^2+c)^{5/2}\arctan(ax)^3}{5a^2c}-\frac{3\left(\frac{1}{4}x(a^2cx^2+c)^{3/2}\arctan(ax)^2-\frac{(a^2cx^2+c)^{3/2}\arctan(ax)}{6a}+\frac{1}{6}c\left(\frac{1}{2}\sqrt{a^2cx^2+c}x\right.\right.}{}\right.$$

$$c\left(c\left(\frac{(a^2cx^2+c)^{3/2}\arctan(ax)^3}{3a^2c}-\frac{\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^2-\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a}+\frac{\sqrt{c}\,\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}+\frac{c}{}}{}\right.\right.$$

$\Big\downarrow$ 5423

$$c\left(\frac{(a^2cx^2+c)^{5/2}\arctan(ax)^3}{5a^2c}-\frac{3\left(\frac{1}{4}x(a^2cx^2+c)^{3/2}\arctan(ax)^2-\frac{(a^2cx^2+c)^{3/2}\arctan(ax)}{6a}+\frac{1}{6}c\left(\frac{1}{2}\sqrt{a^2cx^2+c}x\right.\right.}{}\right.$$

$$c\left(c\left(\frac{(a^2cx^2+c)^{3/2}\arctan(ax)^3}{3a^2c}-\frac{\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^2-\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a}+\frac{\sqrt{c}\,\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}+\frac{c}{}}{}\right.\right.$$

input
```
Int[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3)/x,x]
```

output
```
$Aborted
```

## Maple [A] (verified)

Time = 8.79 (sec) , antiderivative size = 562, normalized size of antiderivative = 0.67

| method | result |
|---|---|
| default | $\frac{c^2\sqrt{c(ax-i)(ax+i)}\,\left(24\arctan(ax)^3a^4x^4-18a^3\arctan(ax)^2x^3+88\arctan(ax)^3a^2x^2+12x^2a^2\arctan(ax)-105a\arctan(ax)^2x+184}{120}$ |

input
```
int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3/x,x,method=_RETURNVERBOSE)
```

output
```
1/120*c^2*(c*(a*x-I)*(a*x+I))^(1/2)*(24*arctan(a*x)^3*a^4*x^4-18*a^3*arcta
n(a*x)^2*x^3+88*arctan(a*x)^3*a^2*x^2+12*x^2*a^2*arctan(a*x)-105*a*arctan(
a*x)^2*x+184*arctan(a*x)^3-6*a*x+186*arctan(a*x))+1/40*c^2*(c*(a*x-I)*(a*x
+I))^(1/2)*(-40*arctan(a*x)^3*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+120*I*arct
an(a*x)^2*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+40*arctan(a*x)^3*ln(1-(1
+I*a*x)/(a^2*x^2+1)^(1/2))-120*I*arctan(a*x)^2*polylog(2,(1+I*a*x)/(a^2*x^
2+1)^(1/2))+149*arctan(a*x)^2*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-298*I*ar
ctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-149*arctan(a*x)^2*ln(1
-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+298*I*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a
^2*x^2+1)^(1/2))-240*arctan(a*x)*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-2
40*I*polylog(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+240*arctan(a*x)*polylog(3,(1+
I*a*x)/(a^2*x^2+1)^(1/2))+240*I*polylog(4,(1+I*a*x)/(a^2*x^2+1)^(1/2))+120
*I*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))+298*polylog(3,-I*(1+I*a*x)/(a^2*x^2
+1)^(1/2))-298*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)
```

## Fricas [F]

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^3}{x}\,dx=\int \frac{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)^3}{x}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3/x,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*arctan(a*
x)^3/x, x)
```

## Sympy [F]

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^3}{x}\,dx = \int \frac{(c(a^2x^2+1))^{\frac{5}{2}}\operatorname{atan}^3(ax)}{x}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)**3/x,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**3/x, x)
```

## Maxima [F]

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^3}{x}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)^3}{x}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3/x,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^3/x, x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^3}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^3}{x}\,dx = \int \frac{\mathrm{atan}(a\,x)^3\,(c\,a^2\,x^2+c)^{5/2}}{x}\,dx$$

input
```
int((atan(a*x)^3*(c + a^2*c*x^2)^(5/2))/x,x)
```

output
```
int((atan(a*x)^3*(c + a^2*c*x^2)^(5/2))/x, x)
```

## Reduce [F]

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^3}{x}\,dx = \sqrt{c}\,c^2\left(\int \frac{\sqrt{a^2x^2+1}\,atan(ax)^3}{x}dx + \left(\int \sqrt{a^2x^2+1}\,atan(ax)^3\,x^3dx\right)a^4 + 2\left(\int \sqrt{a^2x^2+1}\,atan(ax)^3\,xdx\right)a^2\right)$$

input
```
int((a^2*c*x^2+c)^(5/2)*atan(a*x)^3/x,x)
```

output
```
sqrt(c)*c**2*(int((sqrt(a**2*x**2 + 1)*atan(a*x)**3)/x,x) + int(sqrt(a**2*
x**2 + 1)*atan(a*x)**3*x**3,x)*a**4 + 2*int(sqrt(a**2*x**2 + 1)*atan(a*x)*
*3*x,x)*a**2)
```

# 3.432 $\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^3}{x^2}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 1027

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^3}{x^2}\,dx = \text{Too large to display}$$

output
```
-1/4*a*c^2*(a^2*c*x^2+c)^(1/2)+1/4*a^2*c^2*x*(a^2*c*x^2+c)^(1/2)*arctan(a*
x)-21/8*a*c^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2-1/4*a*c*(a^2*c*x^2+c)^(3/2
)*arctan(a*x)^2-c^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^3/x+7/8*a^2*c^2*x*(a^2
*c*x^2+c)^(1/2)*arctan(a*x)^3+1/4*a^2*c*x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^
3-15/4*I*a*c^3*(a^2*x^2+1)^(1/2)*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))*arcta
n(a*x)^3/(a^2*c*x^2+c)^(1/2)-11/2*I*a*c^3*(a^2*x^2+1)^(1/2)*polylog(2,I*(1
+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/(a^2*c*x^2+c)^(1/2)-6*a*c^3*(a^2*x^2+1)^(1/
2)*arctan(a*x)^2*arctanh((1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)+
45/4*I*a*c^3*(a^2*x^2+1)^(1/2)*polylog(4,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a
^2*c*x^2+c)^(1/2)+6*I*a*c^3*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(2,-(1+I*
a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)+11/2*I*a*c^3*(a^2*x^2+1)^(1/2)
*polylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/(a^2*c*x^2+c)^(1/2)-45/4*I*
a*c^3*(a^2*x^2+1)^(1/2)*polylog(4,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x
^2+c)^(1/2)-6*I*a*c^3*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(2,(1+I*a*x)/(a
^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)-11*I*a*c^3*(a^2*x^2+1)^(1/2)*arctan(a
*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/(a^2*c*x^2+c)^(1/2)-6*a*c^3*(a
^2*x^2+1)^(1/2)*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2
)-45/4*a*c^3*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(3,-I*(1+I*a*x)/(a^2*x^2
+1)^(1/2))/(a^2*c*x^2+c)^(1/2)+45/4*a*c^3*(a^2*x^2+1)^(1/2)*arctan(a*x)*po
lylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)+6*a*c^3*(a^2...
```

## Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 3267 vs. $2(1027) = 2054$.

Time = 14.47 (sec) , antiderivative size = 3267, normalized size of antiderivative = 3.18

$$\int \frac{(c + a^2cx^2)^{5/2} \arctan(ax)^3}{x^2} dx = \text{Result too large to show}$$

input
```
Integrate[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3)/x^2,x]
```

output
```
((-I)*a*c^2*Sqrt[c*(1 + a^2*x^2)]*(12*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x
] - (3*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2 + I*a*x*Sqrt[1 + a^2*x^2]*ArcTan
[a*x]^3 + 2*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^3 - 3*(2 + ArcTan[a*x]^2
)*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + 3*(2 + ArcTan[a*x]^2)*PolyLog[2, I*
E^(I*ArcTan[a*x])] - (6*I)*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])]
+ (6*I)*ArcTan[a*x]*PolyLog[3, I*E^(I*ArcTan[a*x])] + 6*PolyLog[4, (-I)*E^
(I*ArcTan[a*x])] - 6*PolyLog[4, I*E^(I*ArcTan[a*x])]))/Sqrt[1 + a^2*x^2] +
 (a*c^2*Sqrt[c*(1 + a^2*x^2)]*Csc[ArcTan[a*x]/2]*(((-7*I)*a*Pi^4*x)/Sqrt[1
 + a^2*x^2] - ((8*I)*a*Pi^3*x*ArcTan[a*x])/Sqrt[1 + a^2*x^2] + ((24*I)*a*P
i^2*x*ArcTan[a*x]^2)/Sqrt[1 + a^2*x^2] - 64*ArcTan[a*x]^3 - ((32*I)*a*Pi*x
*ArcTan[a*x]^3)/Sqrt[1 + a^2*x^2] + ((16*I)*a*x*ArcTan[a*x]^4)/Sqrt[1 + a^
2*x^2] + (48*a*Pi^2*x*ArcTan[a*x]*Log[1 - I/E^(I*ArcTan[a*x])])/Sqrt[1 + a
^2*x^2] - (96*a*Pi*x*ArcTan[a*x]^2*Log[1 - I/E^(I*ArcTan[a*x])])/Sqrt[1 +
a^2*x^2] - (8*a*Pi^3*x*Log[1 + I/E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (
64*a*x*ArcTan[a*x]^3*Log[1 + I/E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (19
2*a*x*ArcTan[a*x]^2*Log[1 - E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (8*a*P
i^3*x*Log[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - (48*a*Pi^2*x*ArcTa
n[a*x]*Log[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (96*a*Pi*x*ArcTan
[a*x]^2*Log[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - (64*a*x*ArcTan[a
*x]^3*Log[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - (192*a*x*ArcTan...
```

## Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3 \left(a^2cx^2+c\right)^{5/2}}{x^2} \, dx$$

$$\Big\downarrow \text{ 5485}$$

$$a^2c\int \left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3dx+c\int \frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{x^2}dx$$

$$\Big\downarrow \text{ 5415}$$

$$a^2c\left(\frac{1}{2}c\int \sqrt{a^2cx^2+c}\arctan(ax)dx+\frac{3}{4}c\int \sqrt{a^2cx^2+c}\arctan(ax)^3dx+\frac{1}{4}x\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}-\text{arc}\right.$$
$$c\int \frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{x^2}dx$$

$$\Big\downarrow \text{ 5413}$$

$$a^2c\left(\frac{1}{2}c\left(\frac{1}{2}c\int\frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)+\frac{3}{4}c\int\sqrt{a^2cx^2+c}\arctan(ax)^3dx\right.$$
$$c\int\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{x^2}dx$$

$$\Big\downarrow \text{ 5415}$$

$$a^2c\left(\frac{1}{2}c\left(\frac{1}{2}c\int\frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)+\frac{3}{4}c\left(3c\int\frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}c\int\right.\right.$$
$$c\int\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{x^2}dx$$

$$\Big\downarrow \text{ 5425}$$

$$a^2c\left(\frac{1}{2}c\left(\frac{c\sqrt{a^2x^2+1}\int\frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)\sqrt{a^2cx^2+c}-\frac{\sqrt{a^2cx^2+c}}{2a}\right)+\frac{3}{4}c\left(\frac{3c\sqrt{a^2x^2+1}\int\frac{\arctan(a}{\sqrt{a^2x^2-}}}{\sqrt{a^2cx^2+c}}\right.\right.$$
$$c\int\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{x^2}dx$$

$$\Big\downarrow \text{ 5421}$$

$$c\int\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{x^2}dx+$$
$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\int\frac{\arctan(ax)^3}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyI}}{}\right.}{\sqrt{a^2cx^2+c}}\right.\right.$$

$$\Big\downarrow \text{ 5423}$$

$$c\int\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{x^2}dx+$$
$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\int\sqrt{a^2x^2+1}\arctan(ax)^3d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{Poly}}{}\right.}{\sqrt{a^2cx^2+c}}\right.\right.$$

$$\Big\downarrow \text{ 3042}$$

$$c\int \frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{x^2}dx+$$

$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\int \arctan(ax)^3\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+i}}{\sqrt{1-i}}\right.}{a}\right.}{}\right.\right.$$

$$\Big\downarrow \text{4669}$$

$$c\int \frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{x^2}dx+$$

$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(-3\int \arctan(ax)^2\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+3\int \arctan(ax)^2\log\left(1+ie^{i\arctan(a\right.\right.}{2a\sqrt{a^2cx^2+c}}\right.\right.$$

$$\Big\downarrow \text{3011}$$

$$c\int \frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{x^2}dx+$$

$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int \arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\right.\right.}{}\right.\right.$$

$$\Big\downarrow \text{5485}$$

$$c\left(a^2c\int \sqrt{a^2cx^2+c}\arctan(ax)^3dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^2}dx\right)+$$

$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int \arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\right.\right.}{}\right.\right.$$

$$\Big\downarrow \text{5415}$$

$$c\left(a^2c\left(3c\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}c\int \frac{\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^3\sqrt{a^2cx^2+c}-\frac{3\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a}\right)\right.$$

$$a^2c\left(\frac{3}{4}c\left(\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int \arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\right.\right.}{}\right.\right.$$

$\downarrow$ 5425

$$c\left(\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3-\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{4a}+\frac{1}{2}c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}}{}\right.\right.$$

$$c\left(c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^3-\frac{3\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a}+\frac{3c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+\frac{c\sqrt{a^2x^2+1}\int \frac{\text{arc}}{\sqrt{}}}{2\sqrt{a^2cx^2}}\right.\right.$$

$\downarrow$ 5421

$$c\left(\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3-\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{4a}+\frac{1}{2}c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}}{}\right.\right.$$

$$c\left(c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^3-\frac{3\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{Po}}{}\right.}{\sqrt{a^2cx^2+}}\right.\right.$$

$\downarrow$ 5423

$$c\left(\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3-\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{4a}+\frac{1}{2}c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}}{}\right.\right.$$

$$c\left(c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^3-\frac{3\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{Po}}{}\right.}{\sqrt{a^2cx^2+}}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$c\left(\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3-\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{4a}+\frac{1}{2}c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}}{\right.}\right.\right.$$

$$c\left(c\left(\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^3-\frac{3\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{Po}}{\right.}}{\sqrt{a^2cx^2+}}\right.\right.\right.$$

$$\Big\downarrow \text{4669}$$

$$c\left(\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3-\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{4a}+\frac{1}{2}c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}}{\right.}\right.\right.$$

$$c\left(c\left(\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^3-\frac{3\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{Po}}{\right.}}{\sqrt{a^2cx^2+}}\right.\right.\right.$$

$$\Big\downarrow \text{3011}$$

$$c\left(\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3-\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{4a}+\frac{1}{2}c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}}{\right.}\right.\right.$$

$$c\left(c\left(\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^3-\frac{3\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{Po}}{\right.}}{\sqrt{a^2cx^2+}}\right.\right.\right.$$

$$\Big\downarrow \text{5485}$$

$$c\left(\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3-\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{4a}+\frac{1}{2}c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}}{\phantom{x}}\right.\right.$$

$$c\left(c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^3-\frac{3\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{Pc}}{}\right.}{\sqrt{a^2cx^2+}}\right.\right.$$

$\big\downarrow$ 5425

$$c\left(\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3-\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{4a}+\frac{1}{2}c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}}{\phantom{x}}\right.\right.$$

$$c\left(c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^3-\frac{3\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{Pc}}{}\right.}{\sqrt{a^2cx^2+}}\right.\right.$$

$\big\downarrow$ 5423

$$c\left(\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3-\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{4a}+\frac{1}{2}c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}}{\phantom{x}}\right.\right.$$

$$c\left(c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^3-\frac{3\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{Pc}}{}\right.}{\sqrt{a^2cx^2+}}\right.\right.$$

$\big\downarrow$ 3042

$$c\left(\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3-\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{4a}+\frac{1}{2}c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}}{\cdots}\right.\right.$$

$$c\left(c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^3-\frac{3\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{Po}\cdots}{\cdots}\right.}{\sqrt{a^2cx^2+\cdots}}\right.\right.$$

$\downarrow$ 4669

$$c\left(\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3-\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{4a}+\frac{1}{2}c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}}{\cdots}\right.\right.$$

$$c\left(c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^3-\frac{3\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{Po}\cdots}{\cdots}\right.}{\sqrt{a^2cx^2+\cdots}}\right.\right.$$

$\downarrow$ 3011

$$c\left(\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3-\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{4a}+\frac{1}{2}c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}}{\cdots}\right.\right.$$

$$c\left(c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^3-\frac{3\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{Po}\cdots}{\cdots}\right.}{\sqrt{a^2cx^2+\cdots}}\right.\right.$$

$\downarrow$ 5479

$$c\left(\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3-\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{4a}+\frac{1}{2}c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}}{\cdots}\right.\right.$$

$$c\left(c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^3-\frac{3\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{Po}\cdots}{\cdots}\right.}{\sqrt{a^2cx^2+\cdots}}\right.\right.$$

$$\Big\downarrow \text{5493}$$

$$c\left(\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3-\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{4a}+\frac{1}{2}c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}}{\cdots}\right.\right.$$

$$c\left(c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^3-\frac{3\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{Po}\cdots}{\cdots}\right.}{\sqrt{a^2cx^2+\cdots}}\right.\right.$$

$$\Big\downarrow \text{5491}$$

$$c\left(\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3-\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{4a}+\frac{1}{2}c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}}{\cdots}\right.\right.$$

$$c\left(c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^3-\frac{3\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{Po}\cdots}{\cdots}\right.}{\sqrt{a^2cx^2+\cdots}}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$c\left(\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3-\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{4a}+\frac{1}{2}c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}}{\right.\right.$$

$$c\left(c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^3-\frac{3\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{Po}}{}\right.}{\sqrt{a^2cx^2+}}\right.\right.$$

$$\Big\downarrow \text{4671}$$

$$c\left(\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3-\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{4a}+\frac{1}{2}c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}}{\right.\right.$$

$$c\left(c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^3-\frac{3\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{Po}}{}\right.}{\sqrt{a^2cx^2+}}\right.\right.$$

$$\Big\downarrow \text{3011}$$

$$c\left(\frac{1}{4}x\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3-\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}{4a}+\frac{1}{2}c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)+\frac{c\sqrt{a^2x^2+1}}{\right.\right.$$

$$c\left(c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^3-\frac{3\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{Po}}{}\right.}{\sqrt{a^2cx^2+}}\right.\right.$$

input
```
Int[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3)/x^2,x]
```

output
```
$Aborted
```

## Maple [A] (verified)

Time = 8.97 (sec) , antiderivative size = 655, normalized size of antiderivative = 0.64

| method | result |
|---|---|
| default | $\frac{c^2\sqrt{c(ax-i)(ax+i)}\,\left(2\arctan(ax)^3a^4x^4-2a^3\arctan(ax)^2x^3+9\arctan(ax)^3a^2x^2+2x^2a^2\arctan(ax)-23a\arctan(ax)^2x-8\arctan(\right.}{8x}$ |

input
```
int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*c^2*(c*(a*x-I)*(a*x+I))^(1/2)*(2*arctan(a*x)^3*a^4*x^4-2*a^3*arctan(a*
x)^2*x^3+9*arctan(a*x)^3*a^2*x^2+2*x^2*a^2*arctan(a*x)-23*a*arctan(a*x)^2*
x-8*arctan(a*x)^3-2*a*x)/x+1/8*I*c^2*a*(c*(a*x-I)*(a*x+I))^(1/2)*(15*I*arc
tan(a*x)^3*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-48*I*polylog(3,(1+I*a*x)/(a
^2*x^2+1)^(1/2))+24*I*arctan(a*x)^2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-15*I
*arctan(a*x)^3*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-45*arctan(a*x)^2*polylo
g(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+45*arctan(a*x)^2*polylog(2,-I*(1+I*a*x)
/(a^2*x^2+1)^(1/2))+90*I*arctan(a*x)*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1
/2))+44*I*arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+48*I*polylog(3,-
(1+I*a*x)/(a^2*x^2+1)^(1/2))-90*I*arctan(a*x)*polylog(3,I*(1+I*a*x)/(a^2*x
^2+1)^(1/2))+48*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-48*arc
tan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-44*I*arctan(a*x)*ln(1-I*(1
+I*a*x)/(a^2*x^2+1)^(1/2))-24*I*arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(
1/2))-44*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+90*polylog(4,I*(1+I*a*x)
/(a^2*x^2+1)^(1/2))+44*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-90*polylo
g(4,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)
```

## Fricas [F]

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^3}{x^2}\,dx=\int \frac{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)^3}{x^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3/x^2,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*arctan(a*
x)^3/x^2, x)
```

## Sympy [F]

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^3}{x^2}\,dx = \int \frac{(c(a^2x^2+1))^{\frac{5}{2}}\operatorname{atan}^3(ax)}{x^2}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)**3/x**2,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**3/x**2, x)
```

## Maxima [F]

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^3}{x^2}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)^3}{x^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3/x^2,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^3/x^2, x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^3}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3/x^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^3}{x^2}\,dx = \int \frac{\mathrm{atan}(a\,x)^3\,(c\,a^2\,x^2+c)^{5/2}}{x^2}\,dx$$

input
```
int((atan(a*x)^3*(c + a^2*c*x^2)^(5/2))/x^2,x)
```

output
```
int((atan(a*x)^3*(c + a^2*c*x^2)^(5/2))/x^2, x)
```

## Reduce [F]

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^3}{x^2}\,dx = \sqrt{c}\,c^2\left(\int \frac{\sqrt{a^2x^2+1}\,atan(ax)^3}{x^2}dx + \left(\int \sqrt{a^2x^2+1}\,atan(ax)^3\,x^2dx\right)a^4 + 2\left(\int \sqrt{a^2x^2+1}\,atan(ax)^3\,dx\right)a^2\right)$$

input
```
int((a^2*c*x^2+c)^(5/2)*atan(a*x)^3/x^2,x)
```

output
```
sqrt(c)*c**2*(int((sqrt(a**2*x**2 + 1)*atan(a*x)**3)/x**2,x) + int(sqrt(a*
*2*x**2 + 1)*atan(a*x)**3*x**2,x)*a**4 + 2*int(sqrt(a**2*x**2 + 1)*atan(a*
x)**3,x)*a**2)
```

# 3.433 $\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^3}{x^3}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 1043

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^3}{x^3}\,dx = \text{Too large to display}$$

output
```
a^2*c^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)-3/2*a*c^2*(a^2*c*x^2+c)^(1/2)*arct
an(a*x)^2/x-1/2*a^3*c^2*x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2+3*I*a^2*c^3*(a
^2*x^2+1)^(1/2)*polylog(2,-(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/(a^2*c*x^2+c)^
(1/2)+2*a^2*c^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^3-1/2*c^2*(a^2*c*x^2+c)^(1
/2)*arctan(a*x)^3/x^2+1/3*a^2*c*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3-5*a^2*c^
3*(a^2*x^2+1)^(1/2)*arctan(a*x)^3*arctanh((1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^
2*c*x^2+c)^(1/2)-6*a^2*c^3*(a^2*x^2+1)^(1/2)*arctan(a*x)*arctanh((1+I*a*x)
^(1/2)/(1-I*a*x)^(1/2))/(a^2*c*x^2+c)^(1/2)-a^2*c^(5/2)*arctanh(a*c^(1/2)*
x/(a^2*c*x^2+c)^(1/2))-15/2*I*a^2*c^3*(a^2*x^2+1)^(1/2)*arctan(a*x)^2*poly
log(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)+15*I*a^2*c^3*(a^2*x
^2+1)^(1/2)*polylog(4,(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)+15/
2*I*a^2*c^3*(a^2*x^2+1)^(1/2)*arctan(a*x)^2*polylog(2,-(1+I*a*x)/(a^2*x^2+
1)^(1/2))/(a^2*c*x^2+c)^(1/2)+13*I*a^2*c^3*(a^2*x^2+1)^(1/2)*arctan((1+I*a
*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^2/(a^2*c*x^2+c)^(1/2)-15*I*a^2*c^3*(a^2
*x^2+1)^(1/2)*polylog(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)-
3*I*a^2*c^3*(a^2*x^2+1)^(1/2)*polylog(2,(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/(
a^2*c*x^2+c)^(1/2)-15*a^2*c^3*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(3,-(1+
I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)+13*a^2*c^3*(a^2*x^2+1)^(1/2)
*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)-13*a^2*c^3*
(a^2*x^2+1)^(1/2)*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c...
```

## Mathematica [A] (warning: unable to verify)

Time = 7.68 (sec) , antiderivative size = 934, normalized size of antiderivative = 0.90

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^3}{x^3}\,dx = \text{Too large to display}$$

input
```
Integrate[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3)/x^3,x]
```

output
```
(a^2*c^2*Sqrt[c + a^2*c*x^2]*(-36*ArcTan[a*x]^2 - (15*I)*Pi^4*Cot[ArcTan[a
*x]/2] - 48*ArcCoth[(a*x)/Sqrt[1 + a^2*x^2]]*Cot[ArcTan[a*x]/2] + (48*I)*A
rcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2*Cot[ArcTan[a*x]/2] + (30*I)*ArcTan[
a*x]^4*Cot[ArcTan[a*x]/2] - 36*ArcTan[a*x]^2*Cot[ArcTan[a*x]/2]^2 + 12*a*x
*ArcTan[a*x]*Csc[ArcTan[a*x]/2]^2 + 12*a^3*x^3*ArcTan[a*x]*Csc[ArcTan[a*x]
/2]^2 + 56*a*x*ArcTan[a*x]^3*Csc[ArcTan[a*x]/2]^2 + 8*a^3*x^3*ArcTan[a*x]^
3*Csc[ArcTan[a*x]/2]^2 + 12*a*x*ArcTan[a*x]*Cos[2*ArcTan[a*x]]*Csc[ArcTan[
a*x]/2]^2 + 12*a^3*x^3*ArcTan[a*x]*Cos[2*ArcTan[a*x]]*Csc[ArcTan[a*x]/2]^2
 - 6*ArcTan[a*x]^3*Cot[ArcTan[a*x]/2]*Csc[ArcTan[a*x]/2]^2 + 120*ArcTan[a*
x]^3*Cot[ArcTan[a*x]/2]*Log[1 - E^((-I)*ArcTan[a*x])] + 144*ArcTan[a*x]*Co
t[ArcTan[a*x]/2]*Log[1 - E^(I*ArcTan[a*x])] - 288*ArcTan[a*x]^2*Cot[ArcTan
[a*x]/2]*Log[1 - I*E^(I*ArcTan[a*x])] + 288*ArcTan[a*x]^2*Cot[ArcTan[a*x]/
2]*Log[1 + I*E^(I*ArcTan[a*x])] - 144*ArcTan[a*x]*Cot[ArcTan[a*x]/2]*Log[1
 + E^(I*ArcTan[a*x])] - 120*ArcTan[a*x]^3*Cot[ArcTan[a*x]/2]*Log[1 + E^(I*
ArcTan[a*x])] + (360*I)*ArcTan[a*x]^2*Cot[ArcTan[a*x]/2]*PolyLog[2, E^((-I
)*ArcTan[a*x])] + (72*I)*(2 + 5*ArcTan[a*x]^2)*Cot[ArcTan[a*x]/2]*PolyLog[
2, -E^(I*ArcTan[a*x])] - (624*I)*ArcTan[a*x]*Cot[ArcTan[a*x]/2]*PolyLog[2,
 (-I)*E^(I*ArcTan[a*x])] + (624*I)*ArcTan[a*x]*Cot[ArcTan[a*x]/2]*PolyLog[
2, I*E^(I*ArcTan[a*x])] - (144*I)*Cot[ArcTan[a*x]/2]*PolyLog[2, E^(I*ArcTa
n[a*x])] + 720*ArcTan[a*x]*Cot[ArcTan[a*x]/2]*PolyLog[3, E^((-I)*ArcTan...
```

## Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3 \left(a^2cx^2+c\right)^{5/2}}{x^3}\,dx$$

$$\Big\downarrow \text{ 5485}$$

$$a^2c\int \frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{x}dx+c\int \frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{x^3}dx$$

$$\Big\downarrow \text{ 5485}$$

$$a^2c\left(a^2c\int x\sqrt{a^2cx^2+c}\arctan(ax)^3dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x}dx\right)+c\left(a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x}dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^3}dx\right)$$

$\downarrow$ 5465

$$a^2c\left(a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\int\sqrt{a^2cx^2+c}\arctan(ax)^2dx}{a}\right)+c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x}dx\right)+$$
$$c\left(a^2c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x}dx+c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^3}dx\right)$$

$\downarrow$ 5415

$$a^2c\left(a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{1}{2}c\int\frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int\frac{1}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\text{arct}}{}}{a}\right.\right.$$
$$c\left(a^2c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x}dx+c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^3}dx\right)$$

$\downarrow$ 224

$$a^2c\left(a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{1}{2}c\int\frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+c\int\frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+}}{a}\right.\right.$$
$$c\left(a^2c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x}dx+c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^3}dx\right)$$

$\downarrow$ 219

$$a^2c\left(a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{1}{2}c\int\frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}+}{a}\right.\right.$$
$$c\left(a^2c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x}dx+c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^3}dx\right)$$

$\downarrow$ 5425

$$a^2c\left(a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{c\sqrt{a^2x^2+1}\int\frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a}\right.\right.\right.$$
$$c\left(a^2c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x}dx+c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^3}dx\right)$$

$\Big\downarrow$ 5423

$$a^2c\left(a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{c\sqrt{a^2x^2+1}\int\sqrt{a^2x^2+1}\arctan(ax)^2d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2+c}-\right.\right.\right.$$
$$c\left(a^2c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x}dx+c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^3}dx\right)$$

$\Big\downarrow$ 3042

$$a^2c\left(a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\frac{c\sqrt{a^2x^2+1}\int\arctan(ax)^2\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^2\sqrt{a^2cx^2}\right.\right.\right.$$
$$c\left(a^2c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x}dx+c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^3}dx\right)$$

$\Big\downarrow$ 4669

$$c\left(a^2c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x}dx+c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^3}dx\right)+$$
$$a^2c\left(c\int\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x}dx+a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{c\sqrt{a^2x^2+1}\left(-2\int\arctan(ax)\log\left(1-ie^{i\arctan(ax)}\right)\right.}{}\right.\right.$$

$\Big\downarrow$ 3011

$$c\left(a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x}dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^3}dx\right)+$$
$$a^2c\left(c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x}dx+a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan}\right.\right.\right.}{}\right.\right.$$

$\downarrow$ 2720

$$c\left(a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x}dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^3}dx\right)+$$
$$a^2c\left(c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x}dx+a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan}\right.\right.\right.}{}\right.\right.$$

$\downarrow$ 5485

$$c\left(a^2c\left(a^2c\int \frac{x\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx\right)+c\left(a^2c\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^3}{x^3\sqrt{a^2cx^2+c}}dx\right)\right)+$$
$$a^2c\left(c\left(a^2c\int \frac{x\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx\right)+a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan\right.\right.}{}\right.\right.$$

$\downarrow$ 5465

$$c\left(a^2c\left(a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a}\right)+c\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx\right)+c\left(a^2c\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx\right.\right.$$
$$a^2c\left(c\left(a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a}\right)+c\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx\right)+a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2\right.}{3a^2c}\right.\right.$$

$\downarrow$ 5425

$$c\left(a^2c\left(a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\int\frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{a\sqrt{a^2cx^2+c}}\right)+c\int\frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx\right)+c\left(a^2c\int\frac{\arctan}{x\sqrt{a^2}}\right.\right.$$
$$a^2c\left(c\left(a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\int\frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{a\sqrt{a^2cx^2+c}}\right)+c\int\frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx\right)+a^2c\left(\frac{\arctan(a}{}\right.\right.$$

$\downarrow$ 5423

$$c\left(a^2c\left(a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\int\sqrt{a^2x^2+1}\arctan(ax)^2d\arctan(ax)}{a^2\sqrt{a^2cx^2+c}}\right)+c\int\frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}\right.\right.$$
$$a^2c\left(c\left(a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\int\sqrt{a^2x^2+1}\arctan(ax)^2d\arctan(ax)}{a^2\sqrt{a^2cx^2+c}}\right)+c\int\frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}\right.\right.$$

$\downarrow$ 3042

$$c\left(a^2c\left(c\int\frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx+a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\int\arctan(ax)^2\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d}{a^2\sqrt{a^2cx^2+c}}\right.\right.\right.$$
$$a^2c\left(c\left(c\int\frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx+a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\int\arctan(ax)^2\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d}{a^2\sqrt{a^2cx^2+c}}\right.\right.\right.$$

$\downarrow$ 4669

$$a^2c\left(a^2c\left(\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{c\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan}\right.\right.\right.}{}\right.\right.$$
$$c\left(c\left(a^2c\int\frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx+c\int\frac{\arctan(ax)^3}{x^3\sqrt{a^2cx^2+c}}dx\right)+a^2c\left(c\int\frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx+a^2c\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+}}{a^2c}\right.\right.\right.$$

$\downarrow$ 3011

$$c\left(c\left(\frac{(a^2cx^2+c)^{3/2}\arctan(ax)^3}{3a^2c}-\frac{\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^2-\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a}+\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}+\frac{c}{}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2+2\left(i\arctan(ax)\operatorname{PolyLog}\right.\right.}{}\right.\right.\right.$$

$\downarrow$ 2720

$$c\left(c\left(\frac{(a^2cx^2+c)^{3/2}\arctan(ax)^3}{3a^2c}-\frac{\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^2-\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a}+\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}+\frac{c}{}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2+2\left(i\arctan(ax)\operatorname{PolyLog}\right.\right.}{}\right.\right.\right.$$

$\downarrow$ 5493

$$c\left(c\left(\frac{(a^2cx^2+c)^{3/2}\arctan(ax)^3}{3a^2c}-\frac{\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^2-\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a}+\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}+\frac{c}{}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2+2\left(i\arctan(ax)\operatorname{PolyLog}\right.\right.}{}\right.\right.\right.$$

$\downarrow$ 5491

$$c\left(c\left(\frac{(a^2cx^2+c)^{3/2}\arctan(ax)^3}{3a^2c}-\frac{\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^2-\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a}+\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}+\frac{c}{}\right.\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2+2\left(i\arctan(ax)\operatorname{PolyLog}\right.\right.}{}\right.\right.\right.$$

$\downarrow$ 3042

$$c\left(c\left(\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{3a^2c}-\frac{\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^2-\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a}+\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}+\cdots}{\cdots}\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2+2\left(i\arctan(ax)\operatorname{PolyLog}\cdots\right.\right.}{\cdots}\right.\right.\right.$$

$\downarrow$ 4671

$$c\left(c\left(\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{3a^2c}-\frac{\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^2-\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a}+\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}+\cdots}{\cdots}\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2+2\left(i\arctan(ax)\operatorname{PolyLog}\cdots\right.\right.}{\cdots}\right.\right.\right.$$

$\downarrow$ 3011

$$c\left(c\left(\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{3a^2c}-\frac{\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^2-\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a}+\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}+\cdots}{\cdots}\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2+2\left(i\arctan(ax)\operatorname{PolyLog}\cdots\right.\right.}{\cdots}\right.\right.\right.$$

$\downarrow$ 5497

$$c\left(c\left(\frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{3a^2c}-\frac{\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^2-\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a}+\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}+\cdots}{\cdots}\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2+2\left(i\arctan(ax)\operatorname{PolyLog}\cdots\right.\right.}{\cdots}\right.\right.\right.$$

$\downarrow$ 5479

$$c\left(c\left(\frac{(a^2cx^2+c)^{3/2}\arctan(ax)^3}{3a^2c}-\frac{\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^2-\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a}+\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}+\frac{c}{}}{}\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2+2\left(i\arctan(ax)\text{PolyLog}\right.\right.}{}\right.\right.\right.$$

$\downarrow$ 5493

$$c\left(c\left(\frac{(a^2cx^2+c)^{3/2}\arctan(ax)^3}{3a^2c}-\frac{\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^2-\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a}+\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}+\frac{c}{}}{}\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2+2\left(i\arctan(ax)\text{PolyLog}\right.\right.}{}\right.\right.\right.$$

$\downarrow$ 5489

$$c\left(c\left(\frac{(a^2cx^2+c)^{3/2}\arctan(ax)^3}{3a^2c}-\frac{\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^2-\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a}+\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}+\frac{c}{}}{}\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2+2\left(i\arctan(ax)\text{PolyLog}\right.\right.}{}\right.\right.\right.$$

$\downarrow$ 5491

$$c\left(c\left(\frac{(a^2cx^2+c)^{3/2}\arctan(ax)^3}{3a^2c}-\frac{\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^2-\frac{\sqrt{a^2cx^2+c}\arctan(ax)}{a}+\frac{\sqrt{c}\text{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a}+\frac{c}{}}{}\right.\right.$$

$$c\left(c\left(c\left(\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2+2\left(i\arctan(ax)\text{PolyLog}\right.\right.}{}\right.\right.\right.$$

input
```
Int[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3)/x^3,x]
```

output
```
$Aborted
```

## Maple [A] (verified)

Time = 8.63 (sec) , antiderivative size = 660, normalized size of antiderivative = 0.63

| method | result |
|---|---|
| default | $\frac{c^2\sqrt{c(ax-i)(ax+i)}\,\arctan(ax)\left(2a^4\arctan(ax)^2x^4-3\arctan(ax)x^3a^3+14\arctan(ax)^2x^2a^2+6a^2x^2-9\arctan(ax)ax-3\arctan(ax\right.}{6x^2}$ |

input
```
int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/6*c^2*(c*(a*x-I)*(a*x+I))^(1/2)*arctan(a*x)*(2*a^4*arctan(a*x)^2*x^4-3*a
rctan(a*x)*x^3*a^3+14*arctan(a*x)^2*x^2*a^2+6*a^2*x^2-9*arctan(a*x)*a*x-3*
arctan(a*x)^2)/x^2-1/2*c^2*a^2*(c*(a*x-I)*(a*x+I))^(1/2)*(5*arctan(a*x)^3*
ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-5*arctan(a*x)^3*ln(1-(1+I*a*x)/(a^2*x^2+
1)^(1/2))+30*I*polylog(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-30*I*polylog(4,(1+I
*a*x)/(a^2*x^2+1)^(1/2))+13*arctan(a*x)^2*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/
2))-13*arctan(a*x)^2*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-4*I*arctan((1+I*a
*x)/(a^2*x^2+1)^(1/2))+26*I*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)
^(1/2))+6*arctan(a*x)*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+30*arctan(a*x)*pol
ylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*arctan(a*x)*ln(1-(1+I*a*x)/(a^2*x^2
+1)^(1/2))-30*arctan(a*x)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))+15*I*arct
an(a*x)^2*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-15*I*arctan(a*x)^2*polylo
g(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/
2))-26*I*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*polylog(
2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+26*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))
-26*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)
```

## Fricas [F]

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^3}{x^3}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)^3}{x^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3/x^3,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*arctan(a*
x)^3/x^3, x)
```

## Sympy [F]

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^3}{x^3}\,dx = \int \frac{(c(a^2x^2+1))^{\frac{5}{2}}\operatorname{atan}^3(ax)}{x^3}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)**3/x**3,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**3/x**3, x)
```

## Maxima [F]

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^3}{x^3}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)^3}{x^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3/x^3,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^3/x^3, x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^3}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3/x^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^3}{x^3}\,dx = \int \frac{\operatorname{atan}(a\,x)^3\,(c\,a^2\,x^2+c)^{5/2}}{x^3}\,dx$$

input
```
int((atan(a*x)^3*(c + a^2*c*x^2)^(5/2))/x^3,x)
```

output
```
int((atan(a*x)^3*(c + a^2*c*x^2)^(5/2))/x^3, x)
```

## Reduce [F]

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^3}{x^3}\,dx = \sqrt{c}\,c^2\left(\int \frac{\sqrt{a^2x^2+1}\,atan(ax)^3}{x^3}dx + 2\left(\int \frac{\sqrt{a^2x^2+1}\,atan(ax)^3}{x}dx\right)a^2 + \left(\int \sqrt{a^2x^2+1}\,atan(ax)^3\,xdx\right)a^4\right)$$

input
```
int((a^2*c*x^2+c)^(5/2)*atan(a*x)^3/x^3,x)
```

output
```
sqrt(c)*c**2*(int((sqrt(a**2*x**2 + 1)*atan(a*x)**3)/x**3,x) + 2*int((sqrt
(a**2*x**2 + 1)*atan(a*x)**3)/x,x)*a**2 + int(sqrt(a**2*x**2 + 1)*atan(a*x
)**3*x,x)*a**4)
```

# 3.434 $\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^3}{x^4}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 1061

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^3}{x^4}\,dx = \text{Too large to display}$$

output
```
-a^2*c^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)/x-3/2*a^3*c^2*(a^2*c*x^2+c)^(1/2)
*arctan(a*x)^2-1/2*a*c^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2/x^2-2*a^2*c^2*(
a^2*c*x^2+c)^(1/2)*arctan(a*x)^3/x+1/2*a^4*c^2*x*(a^2*c*x^2+c)^(1/2)*arcta
n(a*x)^3-1/3*c*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3/x^3-6*I*a^3*c^3*(a^2*x^2+
1)^(1/2)*arctan(a*x)*arctan((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/(a^2*c*x^2+c)
^(1/2)+15/2*I*a^3*c^3*(a^2*x^2+1)^(1/2)*arctan(a*x)^2*polylog(2,-I*(1+I*a*
x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)-13*a^3*c^3*(a^2*x^2+1)^(1/2)*arc
tan(a*x)^2*arctanh((1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)-a^3*c^
(5/2)*arctanh((a^2*c*x^2+c)^(1/2)/c^(1/2))-15/2*I*a^3*c^3*(a^2*x^2+1)^(1/2
)*arctan(a*x)^2*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/
2)-13*I*a^3*c^3*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2
+1)^(1/2))/(a^2*c*x^2+c)^(1/2)+13*I*a^3*c^3*(a^2*x^2+1)^(1/2)*arctan(a*x)*
polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)+3*I*a^3*c^3*(a
^2*x^2+1)^(1/2)*polylog(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/(a^2*c*x^2+c
)^(1/2)-15*I*a^3*c^3*(a^2*x^2+1)^(1/2)*polylog(4,-I*(1+I*a*x)/(a^2*x^2+1)^
(1/2))/(a^2*c*x^2+c)^(1/2)-3*I*a^3*c^3*(a^2*x^2+1)^(1/2)*polylog(2,I*(1+I*
a*x)^(1/2)/(1-I*a*x)^(1/2))/(a^2*c*x^2+c)^(1/2)-13*a^3*c^3*(a^2*x^2+1)^(1/
2)*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)-15*a^3*c^3*
(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a
^2*c*x^2+c)^(1/2)+15*a^3*c^3*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(3,I*...
```

## Mathematica [A] (warning: unable to verify)

Time = 9.37 (sec) , antiderivative size = 1771, normalized size of antiderivative = 1.67

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^3}{x^4}\,dx = \text{Too large to display}$$

input
```
Integrate[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3)/x^4,x]
```

output
```
((-1/2*I)*a^3*c^2*Sqrt[c*(1 + a^2*x^2)]*(12*ArcTan[E^(I*ArcTan[a*x])]*ArcT
an[a*x] - (3*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2 + I*a*x*Sqrt[1 + a^2*x^2]*
ArcTan[a*x]^3 + 2*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^3 - 3*(2 + ArcTan[
a*x]^2)*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + 3*(2 + ArcTan[a*x]^2)*PolyLog
[2, I*E^(I*ArcTan[a*x])] - (6*I)*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a
*x])] + (6*I)*ArcTan[a*x]*PolyLog[3, I*E^(I*ArcTan[a*x])] + 6*PolyLog[4, (
-I)*E^(I*ArcTan[a*x])] - 6*PolyLog[4, I*E^(I*ArcTan[a*x])]))/Sqrt[1 + a^2*
x^2] + (a^3*c^2*Sqrt[c*(1 + a^2*x^2)]*Csc[ArcTan[a*x]/2]*(((-7*I)*a*Pi^4*x
)/Sqrt[1 + a^2*x^2] - ((8*I)*a*Pi^3*x*ArcTan[a*x])/Sqrt[1 + a^2*x^2] + ((2
4*I)*a*Pi^2*x*ArcTan[a*x]^2)/Sqrt[1 + a^2*x^2] - 64*ArcTan[a*x]^3 - ((32*I
)*a*Pi*x*ArcTan[a*x]^3)/Sqrt[1 + a^2*x^2] + ((16*I)*a*x*ArcTan[a*x]^4)/Sqr
t[1 + a^2*x^2] + (48*a*Pi^2*x*ArcTan[a*x]*Log[1 - I/E^(I*ArcTan[a*x])])/Sq
rt[1 + a^2*x^2] - (96*a*Pi*x*ArcTan[a*x]^2*Log[1 - I/E^(I*ArcTan[a*x])])/S
qrt[1 + a^2*x^2] - (8*a*Pi^3*x*Log[1 + I/E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*
x^2] + (64*a*x*ArcTan[a*x]^3*Log[1 + I/E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^
2] + (192*a*x*ArcTan[a*x]^2*Log[1 - E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2]
+ (8*a*Pi^3*x*Log[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - (48*a*Pi^2
*x*ArcTan[a*x]*Log[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (96*a*Pi*
x*ArcTan[a*x]^2*Log[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - (64*a*x*
ArcTan[a*x]^3*Log[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - (192*a*...
```

## Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3 \left(a^2cx^2+c\right)^{5/2}}{x^4}\,dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\int \frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{x^2}dx+c\int \frac{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}{x^4}dx$$

$$\Big\downarrow \text{5485}$$

$$a^2c\left(a^2c\int \sqrt{a^2cx^2+c}\arctan(ax)^3dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^2}dx\right)+ c\left(a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^2}dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^4}dx\right)$$

$$\Big\downarrow \text{ 5415}$$

$$a^2c\left(a^2c\left(3c\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}c\int \frac{\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^3\sqrt{a^2cx^2+c}-\frac{3\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a}\right.\right.$$
$$c\left(a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^2}dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^4}dx\right)$$

$$\Big\downarrow \text{ 5425}$$

$$a^2c\left(a^2c\left(\frac{3c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+\frac{c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^3}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{1}{2}x\arctan(ax)^3\sqrt{a^2cx^2+c}-\frac{3\arctan(ax)}{}\right.\right.$$
$$c\left(a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^2}dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^4}dx\right)$$

$$\Big\downarrow \text{ 5421}$$

$$c\left(a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^2}dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^4}dx\right)+$$
$$a^2c\left(c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^2}dx+a^2c\left(\frac{c\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^3}{\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+}}{\sqrt{1-}}\right)}{a}\right.}{}\right.\right.$$

$$\Big\downarrow \text{ 5423}$$

$$c\left(a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^2}dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^4}dx\right)+$$
$$a^2c\left(c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^2}dx+a^2c\left(\frac{c\sqrt{a^2x^2+1}\int \sqrt{a^2x^2+1}\arctan(ax)^3d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{3c\sqrt{a^2x^2+1}\left(\right.}{}\right.\right.$$

$$\Big\downarrow \text{ 3042}$$

$$c\left(a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^2}dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^4}dx\right)+$$
$$a^2c\left(c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^2}dx+a^2c\left(\frac{c\sqrt{a^2x^2+1}\int \arctan(ax)^3\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{2a\sqrt{a^2cx^2+c}}+\frac{3c\surd}{}\right.\right.$$

$$\Big\downarrow \ 4669$$

$$c\left(a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^2}dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^4}dx\right)+$$
$$a^2c\left(c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^2}dx+a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(-3\int \arctan(ax)^2\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+\right.}{}\right.\right.$$

$$\Big\downarrow \ 3011$$

$$c\left(a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^2}dx+c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^4}dx\right)+$$
$$a^2c\left(c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^2}dx+a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int \operatorname{arcta}\right.\right.}{}\right.\right.$$

$$\Big\downarrow \ 5479$$

$$c\left(a^2c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^2}dx+c\left(a\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^2}{x^3}dx-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)\right)+$$
$$a^2c\left(c\int \frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{x^2}dx+a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int \operatorname{arcta}\right.\right.}{}\right.\right.$$

$$\Big\downarrow \ 5485$$

$$c\left(a^2c\left(a^2c\int \frac{\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^3}{x^2\sqrt{a^2cx^2+c}}dx\right)+c\left(a\left(a^2c\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx\right)\right.\right.$$
$$a^2c\left(c\left(a^2c\int \frac{\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx+c\int \frac{\arctan(ax)^3}{x^2\sqrt{a^2cx^2+c}}dx\right)+a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\operatorname{arct}}\right.\right.\right.}{}\right.\right.$$

$$\Big\downarrow \ 5425$$

$$c\left(a^2c\left(\frac{a^2c\sqrt{a^2x^2+1}\int\frac{\arctan(ax)^3}{\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+c\int\frac{\arctan(ax)^3}{x^2\sqrt{a^2cx^2+c}}dx\right)+c\left(a\left(a^2c\int\frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+c\int\frac{\arctan(a}{x^3\sqrt{a^2cx^2}}\right.\right.\right.$$

$$a^2c\left(c\left(\frac{a^2c\sqrt{a^2x^2+1}\int\frac{\arctan(ax)^3}{\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}+c\int\frac{\arctan(ax)^3}{x^2\sqrt{a^2cx^2+c}}dx\right)+a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2\right.\right.\right.}{}\right.\right.$$

$\downarrow$ 5423

$$c\left(a^2c\left(\frac{ac\sqrt{a^2x^2+1}\int\sqrt{a^2x^2+1}\arctan(ax)^3d\arctan(ax)}{\sqrt{a^2cx^2+c}}+c\int\frac{\arctan(ax)^3}{x^2\sqrt{a^2cx^2+c}}dx\right)+c\left(a\left(a^2c\int\frac{\arctan(ax)}{x\sqrt{a^2cx^2+}}\right.\right.\right.$$

$$a^2c\left(c\left(\frac{ac\sqrt{a^2x^2+1}\int\sqrt{a^2x^2+1}\arctan(ax)^3d\arctan(ax)}{\sqrt{a^2cx^2+c}}+c\int\frac{\arctan(ax)^3}{x^2\sqrt{a^2cx^2+c}}dx\right)+a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(3\left(i\,\mathrm{ar}\right.\right.}{}\right.\right.$$

$\downarrow$ 3042

$$c\left(a^2c\left(c\int\frac{\arctan(ax)^3}{x^2\sqrt{a^2cx^2+c}}dx+\frac{ac\sqrt{a^2x^2+1}\int\arctan(ax)^3\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{\sqrt{a^2cx^2+c}}\right)+c\left(a\left(a^2c\int\frac{}{x}\right.\right.\right.$$

$$a^2c\left(c\left(c\int\frac{\arctan(ax)^3}{x^2\sqrt{a^2cx^2+c}}dx+\frac{ac\sqrt{a^2x^2+1}\int\arctan(ax)^3\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{\sqrt{a^2cx^2+c}}\right)+a^2c\left(\frac{c\sqrt{a^2x^2}}{}\right.\right.$$

$\downarrow$ 4669

$$a^2c\left(a^2c\left(\frac{c\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\right.\right.}{}\right.\right.$$

$$c\left(c\left(a\left(a^2c\int\frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+c\int\frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx\right)-\frac{\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}{3cx^3}\right)+a^2c\left(c\int\frac{\arctan(ax}{x^2\sqrt{a^2cx^2}}\right.\right.$$

$\downarrow$ 3011

$$c\left(c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^3-\frac{3\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{Po}}{}\right.}{\sqrt{a^2cx^2+}}\right.\right.$$

$$c\left(c\left(c\int\frac{\arctan(ax)^3}{x^2\sqrt{a^2cx^2+c}}dx+\frac{ac\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^3+3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-\right.\right.\right.}{}\right.\right.$$

$$\Big\downarrow \text{5479}$$

$$c\left(c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^3-\frac{3\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{Po}}{}\right.}{\sqrt{a^2cx^2+}}\right.\right.$$

$$c\left(c\left(c\left(3a\int\frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx-\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{cx}\right)+\frac{ac\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^3\right.}{}\right.\right.$$

$$\Big\downarrow \text{5493}$$

$$c\left(c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^3-\frac{3\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{Po}}{}\right.}{\sqrt{a^2cx^2+}}\right.\right.$$

$$c\left(c\left(c\left(\frac{3a\sqrt{a^2x^2+1}\int\frac{\arctan(ax)^2}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}-\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{cx}\right)+\frac{ac\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\operatorname{arct}\right.}{}\right.\right.$$

$$\Big\downarrow \text{5491}$$

$$c\left(c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^3-\frac{3\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{Po}}{}\right.}{\sqrt{a^2cx^2+}}\right.\right.$$

$$c\left(c\left(c\left(\frac{3a\sqrt{a^2x^2+1}\int\frac{\sqrt{a^2x^2+1}\arctan(ax)^2}{ax}d\arctan(ax)}{\sqrt{a^2cx^2+c}}-\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{cx}\right)+\frac{ac\sqrt{a^2x^2+1}\left(-2i\arctan\right.}{}\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$c\left(c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^3-\frac{3\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{Po}}{}\right.}{\sqrt{a^2cx^2+}}\right.\right.$$

$$c\left(c\left(c\left(\frac{3a\sqrt{a^2x^2+1}\int\arctan(ax)^2\csc(\arctan(ax))d\arctan(ax)}{\sqrt{a^2cx^2+c}}-\frac{\sqrt{a^2cx^2+c}\arctan(ax)^3}{cx}\right)+\frac{ac\sqrt{a^2x^2+1}(}{}\right.\right.\right.$$

$$\Big\downarrow \; 4671$$

$$c\left(c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^3-\frac{3\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{Po}}{}\right.}{\sqrt{a^2cx^2+}}\right.\right.$$

$$c\left(c\left(c\left(\frac{3a\sqrt{a^2x^2+1}\left(-2\text{arctanh}\left(e^{i\arctan(ax)}\right)\arctan(ax)^2-2\int\arctan(ax)\log\left(1-e^{i\arctan(ax)}\right)d\arctan(ax)+\right.}{\sqrt{a^2cx^2+c}}\right.\right.\right.$$

$$\Big\downarrow \; 3011$$

$$c\left(c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^3-\frac{3\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{Po}}{}\right.}{\sqrt{a^2cx^2+}}\right.\right.$$

$$c\left(c\left(\frac{ac\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^3+3\left(i\arctan(ax)^2\,\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int\text{arc}\right.\right.}{}\right.\right.$$

$$\Big\downarrow \; 2720$$

$$c\left(c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^3-\frac{3\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\text{Po}}{}\right.}{\sqrt{a^2cx^2+}}\right.\right.$$

$$c\left(c\left(\frac{ac\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^3+3\left(i\arctan(ax)^2\,\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int\text{arc}\right.\right.}{}\right.\right.$$

$$\Big\downarrow \; 5497$$

$$c\left(c\left(\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^3-\frac{3\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{Po}}{}\right.}{\sqrt{a^2cx^2+}}\right.\right.\right.$$

$$c\left(c\left(\frac{ac\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^3+3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int\operatorname{arc}\right.\right.}{}\right.\right.$$

$\downarrow$ 5479

$$c\left(c\left(\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^3-\frac{3\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{Po}}{}\right.}{\sqrt{a^2cx^2+}}\right.\right.\right.$$

$$c\left(c\left(\frac{ac\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^3+3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int\operatorname{arc}\right.\right.}{}\right.\right.$$

$\downarrow$ 243

$$c\left(c\left(\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^3-\frac{3\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{Po}}{}\right.}{\sqrt{a^2cx^2+}}\right.\right.\right.$$

$$c\left(c\left(\frac{ac\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^3+3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int\operatorname{arc}\right.\right.}{}\right.\right.$$

$\downarrow$ 73

$$c\left(c\left(\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^3-\frac{3\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{Po}}{}\right.}{\sqrt{a^2cx^2+}}\right.\right.\right.$$

$$c\left(c\left(\frac{ac\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^3+3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int\operatorname{arc}\right.\right.}{}\right.\right.$$

$\downarrow$ 221

$$c\left(c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^3-\frac{3\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{Po}}{}\right.}{\sqrt{a^2cx^2+}}\right.\right.$$

$$c\left(c\left(\frac{ac\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^3+3\left(i\arctan(ax)^2\,\mathrm{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int\mathrm{arc}\right.\right.}{}\right.\right.$$

$\downarrow$ 5493

$$c\left(c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^3-\frac{3\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{Po}}{}\right.}{\sqrt{a^2cx^2+}}\right.\right.$$

$$c\left(c\left(\frac{ac\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^3+3\left(i\arctan(ax)^2\,\mathrm{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int\mathrm{arc}\right.\right.}{}\right.\right.$$

$\downarrow$ 5491

$$c\left(c\left(\frac{1}{2}x\sqrt{a^2cx^2+c}\arctan(ax)^3-\frac{3\sqrt{a^2cx^2+c}\arctan(ax)^2}{2a}+\frac{3c\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}+\frac{i\,\mathrm{Po}}{}\right.}{\sqrt{a^2cx^2+}}\right.\right.$$

$$c\left(c\left(\frac{ac\sqrt{a^2x^2+1}\left(-2i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^3+3\left(i\arctan(ax)^2\,\mathrm{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int\mathrm{arc}\right.\right.}{}\right.\right.$$

input
```
Int[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3)/x^4,x]
```

output
```
$Aborted
```

## Maple [A] (verified)

Time = 8.79 (sec) , antiderivative size = 699, normalized size of antiderivative = 0.66

| method | result |
|---|---|
| default | $\frac{c^2 \sqrt{c(ax-i)(ax+i)}\, \arctan(ax) \left(3a^4 \arctan(ax)^2 x^4 - 9 \arctan(ax) x^3 a^3 - 14 \arctan(ax)^2 x^2 a^2 - 6a^2x^2 - 3\arctan(ax) ax - 2\arctan(ax\right.}{6x^3}$ |

input
```
int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/6*c^2*(c*(a*x-I)*(a*x+I))^(1/2)*arctan(a*x)*(3*a^4*arctan(a*x)^2*x^4-9*a
rctan(a*x)*x^3*a^3-14*arctan(a*x)^2*x^2*a^2-6*a^2*x^2-3*arctan(a*x)*a*x-2*
arctan(a*x)^2)/x^3+1/2*I*c^2*a^3*(c*(a*x-I)*(a*x+I))^(1/2)*(5*I*arctan(a*x
)^3*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-26*I*polylog(3,(1+I*a*x)/(a^2*x^2+
1)^(1/2))+13*I*arctan(a*x)^2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-5*I*arctan(
a*x)^3*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-15*arctan(a*x)^2*polylog(2,I*(1
+I*a*x)/(a^2*x^2+1)^(1/2))+15*arctan(a*x)^2*polylog(2,-I*(1+I*a*x)/(a^2*x^
2+1)^(1/2))+30*I*arctan(a*x)*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*I
*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x
^2+1)^(1/2))+26*I*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-26*arctan(a*x)*p
olylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+26*arctan(a*x)*polylog(2,-(1+I*a*x)/
(a^2*x^2+1)^(1/2))-30*I*arctan(a*x)*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2
))-2*I*ln((1+I*a*x)/(a^2*x^2+1)^(1/2)-1)-6*I*arctan(a*x)*ln(1-I*(1+I*a*x)/
(a^2*x^2+1)^(1/2))-13*I*arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*
polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+30*polylog(4,I*(1+I*a*x)/(a^2*x^2
+1)^(1/2))+6*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-30*polylog(4,-I*(1+
I*a*x)/(a^2*x^2+1)^(1/2)))/(a^2*x^2+1)^(1/2)
```

## Fricas [F]

$$\int \frac{(c+a^2cx^2)^{5/2} \arctan(ax)^3}{x^4}\, dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}} \arctan(ax)^3}{x^4}\, dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3/x^4,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*arctan(a*
x)^3/x^4, x)
```

## Sympy [F]

$$\int \frac{(c + a^2cx^2)^{5/2} \arctan(ax)^3}{x^4} \, dx = \int \frac{(c(a^2x^2 + 1))^{\frac{5}{2}} \operatorname{atan}^3(ax)}{x^4} \, dx$$

input
```
integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)**3/x**4,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**3/x**4, x)
```

## Maxima [F]

$$\int \frac{(c + a^2cx^2)^{5/2} \arctan(ax)^3}{x^4} \, dx = \int \frac{(a^2cx^2 + c)^{\frac{5}{2}} \arctan(ax)^3}{x^4} \, dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3/x^4,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^3/x^4, x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{(c + a^2cx^2)^{5/2} \arctan(ax)^3}{x^4} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^3/x^4,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^3}{x^4}\,dx = \int \frac{\mathrm{atan}(a\,x)^3\,(c\,a^2\,x^2+c)^{5/2}}{x^4}\,dx$$

input
```
int((atan(a*x)^3*(c + a^2*c*x^2)^(5/2))/x^4,x)
```

output
```
int((atan(a*x)^3*(c + a^2*c*x^2)^(5/2))/x^4, x)
```

## Reduce [F]

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^3}{x^4}\,dx = \sqrt{c}\,c^2\left(\int \frac{\sqrt{a^2x^2+1}\,atan(ax)^3}{x^4}dx + 2\left(\int \frac{\sqrt{a^2x^2+1}\,atan(ax)^3}{x^2}dx\right)a^2 + \left(\int \sqrt{a^2x^2+1}\,atan(ax)^3\,dx\right)a^4\right)$$

input
```
int((a^2*c*x^2+c)^(5/2)*atan(a*x)^3/x^4,x)
```

output
```
sqrt(c)*c**2*(int((sqrt(a**2*x**2 + 1)*atan(a*x)**3)/x**4,x) + 2*int((sqrt
(a**2*x**2 + 1)*atan(a*x)**3)/x**2,x)*a**2 + int(sqrt(a**2*x**2 + 1)*atan(
a*x)**3,x)*a**4)
```

# 3.435 $\int \frac{x^3 \arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 408

$$\begin{aligned}\int \frac{x^3\arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx &= \frac{\sqrt{c+a^2cx^2}\arctan(ax)}{a^4c} - \frac{x\sqrt{c+a^2cx^2}\arctan(ax)^2}{2a^3c} \\ &\quad - \frac{5i\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2}{a^4\sqrt{c+a^2cx^2}} \\ &\quad - \frac{2\sqrt{c+a^2cx^2}\arctan(ax)^3}{3a^4c} \\ &\quad + \frac{x^2\sqrt{c+a^2cx^2}\arctan(ax)^3}{3a^2c} - \frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{c+a^2cx^2}}\right)}{a^4\sqrt{c}} \\ &\quad + \frac{5i\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)}{a^4\sqrt{c+a^2cx^2}} \\ &\quad - \frac{5i\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,ie^{i\arctan(ax)}\right)}{a^4\sqrt{c+a^2cx^2}} \\ &\quad - \frac{5\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)}{a^4\sqrt{c+a^2cx^2}} \\ &\quad + \frac{5\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,ie^{i\arctan(ax)}\right)}{a^4\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
(a^2*c*x^2+c)^(1/2)*arctan(a*x)/a^4/c-1/2*x*(a^2*c*x^2+c)^(1/2)*arctan(a*x
)^2/a^3/c-5*I*(a^2*x^2+1)^(1/2)*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan
(a*x)^2/a^4/(a^2*c*x^2+c)^(1/2)-2/3*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^3/a^4/
c+1/3*x^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^3/a^2/c-arctanh(a*c^(1/2)*x/(a^2
*c*x^2+c)^(1/2))/a^4/c^(1/2)+5*I*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(2,-
I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a^4/(a^2*c*x^2+c)^(1/2)-5*I*(a^2*x^2+1)^(1/
2)*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a^4/(a^2*c*x^2+c)^
(1/2)-5*(a^2*x^2+1)^(1/2)*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a^4/(a
^2*c*x^2+c)^(1/2)+5*(a^2*x^2+1)^(1/2)*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1
/2))/a^4/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (warning: unable to verify)

Time = 0.61 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.53

$$\int \frac{x^3 \arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{c+a^2cx^2}\left(-\frac{12\left(\coth^{-1}\left(\frac{ax}{\sqrt{1+a^2x^2}}\right)+5i\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2-5i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)+5i\arctan(ax)\operatorname{Po}\right.}{\sqrt{1+a^2x^2}}\right.}{}$$

input
```
Integrate[(x^3*ArcTan[a*x]^3)/Sqrt[c + a^2*c*x^2],x]
```

output
```
(Sqrt[c + a^2*c*x^2]*((-12*(ArcCoth[(a*x)/Sqrt[1 + a^2*x^2]] + (5*I)*ArcTa
n[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 - (5*I)*ArcTan[a*x]*PolyLog[2, (-I)*E^(
I*ArcTan[a*x])] + (5*I)*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] + 5*Po
lyLog[3, (-I)*E^(I*ArcTan[a*x])] - 5*PolyLog[3, I*E^(I*ArcTan[a*x])]))/Sqr
t[1 + a^2*x^2] - (1 + a^2*x^2)*ArcTan[a*x]*(-6 + 2*ArcTan[a*x]^2 + 6*(-1 +
 ArcTan[a*x]^2)*Cos[2*ArcTan[a*x]] + 3*ArcTan[a*x]*Sin[2*ArcTan[a*x]])))/(
12*a^4*c)
```

## Rubi [A] (verified)

Time = 3.66 (sec) , antiderivative size = 460, normalized size of antiderivative = 1.13, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.792, Rules used = {5487, 5465, 5425, 5423, 3042, 4669, 3011, 2720, 5487, 5425, 5423, 3042, 4669, 3011, 2720, 5465, 224, 219, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{ 5487}$$

$$-\frac{\int \frac{x^2 \arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{2\int \frac{x \arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2 \arctan(ax)^3\sqrt{a^2cx^2+c}}{3a^2c}$$

$$\Big\downarrow \text{ 5465}$$

$$-\frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a}\right)}{3a^2}-\frac{\int \frac{x^2 \arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a}+\frac{x^2 \arctan(ax)^3\sqrt{a^2cx^2+c}}{3a^2c}$$

$$\Big\downarrow \text{ 5425}$$

$$-\frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{a\sqrt{a^2cx^2+c}}\right)}{3a^2}-\frac{\int \frac{x^2 \arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a}+\frac{x^2 \arctan(ax)^3\sqrt{a^2cx^2+c}}{3a^2c}$$

$$\Big\downarrow \text{ 5423}$$

$$-\frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\int \sqrt{a^2x^2+1}\arctan(ax)^2d\arctan(ax)}{a^2\sqrt{a^2cx^2+c}}\right)}{3a^2}-\frac{\int \frac{x^2 \arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a}+\frac{x^2 \arctan(ax)^3\sqrt{a^2cx^2+c}}{3a^2c}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\int \frac{x^2 \arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{a^2\sqrt{a^2cx^2+c}}\right)}{3a^2}+\frac{x^2 \arctan(ax)^3\sqrt{a^2cx^2+c}}{3a^2c}$$

$$\Big\downarrow \; 4669$$

$$\frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(-2\int \arctan(ax)\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+2\int \arctan(ax)\log\left(1+ie^{i\arctan(ax)}\right)d\arctan(ax)-\right.}{a^2\sqrt{a^2cx^2+c}}\right.}{3a^2}-\frac{\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{x^2\arctan(ax)^3\sqrt{a^2cx^2+c}}{3a^2c}$$

$$\Big\downarrow \; 3011$$

$$-\frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-i\int \operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(ax)\right)-2\left(i\arctan\right.\right.}{a^2\sqrt{a^2\ldots}}\right.}{3a^2}\frac{\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a}+\frac{x^2\arctan(ax)^3\sqrt{a^2cx^2+c}}{3a^2c}$$

$$\Big\downarrow \; 2720$$

$$-\frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right.\right.}{\ldots}\right.}{3a\ldots}\frac{\int \frac{x^2\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a}+\frac{x^2\arctan(ax)^3\sqrt{a^2cx^2+c}}{3a^2c}$$

$$\Big\downarrow \; 5487$$

$$-\frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right.\right.}{\ldots}\right.}{3a\ldots}\frac{-\frac{\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a^2}+\frac{x\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a^2c}}{a}+\frac{x^2\arctan(ax)^3\sqrt{a^2cx^2+c}}{3a^2c}$$

$$\Big\downarrow \; 5425$$

$$-\frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right.\right.}{\ldots}\right.}{3a\ldots}\frac{-\frac{\int \frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{2a^2\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a^2c}}{a}+\frac{x^2\arctan(ax)^3\sqrt{a^2cx^2+c}}{3a^2c}$$

$$\Big\downarrow \; 5423$$

$$-\frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right.\right.}{3a}\right.}{\frac{-\frac{\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{\sqrt{a^2x^2+1}\int\sqrt{a^2x^2+1}\arctan(ax)^2d\arctan(ax)}{2a^3\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a^2c}}{a}}+\frac{x^2\arctan(ax)^3\sqrt{a^2cx^2+c}}{3a^2c}$$

$\downarrow$ 3042

$$-\frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right.\right.}{3a}\right.}{\frac{-\frac{\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{\sqrt{a^2x^2+1}\int\arctan(ax)^2\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{2a^3\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a^2c}}{a}}+\frac{x^2\arctan(ax)^3\sqrt{a^2cx^2+c}}{3a^2c}$$

$\downarrow$ 4669

$$-\frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right.\right.}{3a}\right.}{\frac{-\frac{\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{\sqrt{a^2x^2+1}\left(-2\int\arctan(ax)\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+2\int\arctan(ax)\log\left(1+ie^{i\arctan(ax)}\right)d\arctan(ax)-2i\,\mathrm{arcta}\right.}{2a^3\sqrt{a^2cx^2+c}}}{a}}$$

$$\frac{x^2\arctan(ax)^3\sqrt{a^2cx^2+c}}{3a^2c}$$

$\downarrow$ 3011

$$-\frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right.\right.}{3a}\right.}{\frac{-\frac{\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-i\int\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(ax)\right)-2\left(i\arctan(ax)\,\mathrm{PolyL}\right.\right.}{2a^3\sqrt{a^2cx^2+c}}}{a}}$$

$$\frac{x^2\arctan(ax)^3\sqrt{a^2cx^2+c}}{3a^2c}$$

$\downarrow$ 2720

$$-\frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\cdots\right.\right.}{3a\cdots}}{-\frac{\int\frac{x\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}-\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)-2\left(i\arctan\cdots\right.\right.}{2a^3\cdots}}$$

$$\frac{x^2\arctan(ax)^3\sqrt{a^2cx^2+c}}{3a^2c}$$

$$\Big\downarrow \ 5465$$

$$-\frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\cdots\right.\right.}{3a\cdots}}{-\frac{\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\int\frac{1}{\sqrt{a^2cx^2+c}}dx}{a}}{a}-\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\cdots}\right.\right.}{\cdots}}$$

$$\frac{x^2\arctan(ax)^3\sqrt{a^2cx^2+c}}{3a^2c}$$

$$\Big\downarrow \ 224$$

$$-\frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\cdots\right.\right.}{3a\cdots}}{-\frac{\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\int\frac{1}{1-\frac{a^2cx^2}{a^2cx^2+c}}d\frac{x}{\sqrt{a^2cx^2+c}}}{a}}{a}-\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\,\mathrm{arc}\cdots}\right.\right.\right.}{\cdots}}$$

$$\frac{x^2\arctan(ax)^3\sqrt{a^2cx^2+c}}{3a^2c}$$

$$\Big\downarrow \ 219$$

$$-\frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\cdots\right.\right.}{3a\cdots}}{-\frac{\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)-2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,ie\cdots\right.\right.\right.}{2a^3\sqrt{a^2cx^2+c}}}$$

$$\frac{x^2\arctan(ax)^3\sqrt{a^2cx^2+c}}{3a^2c}$$

$$\Big\downarrow \ 7143$$

$$-\frac{2\left(\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}(2(i\arctan(ax)\operatorname{PolyLog}(2,-ie^{i\arctan(ax)})-\operatorname{PolyLog}(3,-ie^{i\arctan(ax)}))-2(i\arctan(ax)\operatorname{PolyLog}(2}{a^2\sqrt{a^2cx^2+c}}\right.}{3a^2}$$

$$\frac{x^2\arctan(ax)^3\sqrt{a^2cx^2+c}}{3a^2c}-$$

$$\frac{-\frac{\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{a^2c}-\frac{\operatorname{arctanh}\left(\frac{a\sqrt{c}x}{\sqrt{a^2cx^2+c}}\right)}{a^2\sqrt{c}}}{a}+\frac{x\arctan(ax)^2\sqrt{a^2cx^2+c}}{2a^2c}-\frac{\sqrt{a^2x^2+1}(2(i\arctan(ax)\operatorname{PolyLog}(2,-ie^{i\arctan(ax)})-\operatorname{PolyLo}}{}}{a}$$

input
```
Int[(x^3*ArcTan[a*x]^3)/Sqrt[c + a^2*c*x^2],x]
```

output
```
(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/(3*a^2*c) - ((x*Sqrt[c + a^2*c*x^2
]*ArcTan[a*x]^2)/(2*a^2*c) - ((Sqrt[c + a^2*c*x^2]*ArcTan[a*x])/(a^2*c) -
ArcTanh[(a*Sqrt[c]*x)/Sqrt[c + a^2*c*x^2]]/(a^2*Sqrt[c]))/a - (Sqrt[1 + a^
2*x^2]*((-2*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 + 2*(I*ArcTan[a*x]*
PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - PolyLog[3, (-I)*E^(I*ArcTan[a*x])]) -
 2*(I*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] - PolyLog[3, I*E^(I*ArcT
an[a*x])])))/(2*a^3*Sqrt[c + a^2*c*x^2]))/a - (2*((Sqrt[c + a^2*c*x^2]*Arc
Tan[a*x]^3)/(a^2*c) - (3*Sqrt[1 + a^2*x^2]*((-2*I)*ArcTan[E^(I*ArcTan[a*x]
)]*ArcTan[a*x]^2 + 2*(I*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - P
olyLog[3, (-I)*E^(I*ArcTan[a*x])]) - 2*(I*ArcTan[a*x]*PolyLog[2, I*E^(I*Ar
cTan[a*x])] - PolyLog[3, I*E^(I*ArcTan[a*x])])))/(a^2*Sqrt[c + a^2*c*x^2])
))/(3*a^2)
```

## Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5487
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcTan[c*x])^p/(c^2*d*m)), x] + (-Simp[b*f*(p/(c*m))   Int[(f*x)^(m - 1)*((
a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[f^2*((m - 1)/(c^
2*m))   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2]), x], x])
/; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

## Maple [A] (verified)

Time = 4.65 (sec) , antiderivative size = 382, normalized size of antiderivative = 0.94

| method | result |
|---|---|
| default | $\frac{\left(2\arctan(ax)^2x^2a^2-3\arctan(ax)ax-4\arctan(ax)^2+6\right)\arctan(ax)\sqrt{c(ax-i)(ax+i)}}{6c\,a^4}-\frac{5\left(i\arctan(ax)^3-3\arctan(ax)^2\ln\left(1-\right.\right.}{}$ |

input
```
int(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*arctan(a*x)^2*x^2*a^2-3*arctan(a*x)*a*x-4*arctan(a*x)^2+6)*arctan(a
*x)*(c*(a*x-I)*(a*x+I))^(1/2)/c/a^4-5/6*(I*arctan(a*x)^3-3*arctan(a*x)^2*l
n(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*arctan(a*x)*polylog(2,I*(1+I*a*x)/(
a^2*x^2+1)^(1/2))-6*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))*(c*(a*x-I)*(
a*x+I))^(1/2)/(a^2*x^2+1)^(1/2)/a^4/c+5/6*(I*arctan(a*x)^3-3*arctan(a*x)^2
*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*arctan(a*x)*polylog(2,-I*(1+I*a*x
)/(a^2*x^2+1)^(1/2))-6*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2)))*(c*(a*x-
I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/2)/a^4/c+2*I*arctan((1+I*a*x)/(a^2*x^2+1)
^(1/2))*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/2)/a^4/c
```

## Fricas [F]

$$\int \frac{x^3 \arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^3 \arctan(ax)^3}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(x^3*arctan(a*x)^3/sqrt(a^2*c*x^2 + c), x)
```

## Sympy [F]

$$\int \frac{x^3 \arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^3 \operatorname{atan}^3(ax)}{\sqrt{c(a^2x^2+1)}}\,dx$$

input
```
integrate(x**3*atan(a*x)**3/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x**3*atan(a*x)**3/sqrt(c*(a**2*x**2 + 1)), x)
```

## Maxima [F]

$$\int \frac{x^3 \arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^3 \arctan\left(ax\right)^3}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3*arctan(a*x)^3/sqrt(a^2*c*x^2 + c), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x^3 \arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^3\,\mathrm{atan}(a\,x)^3}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int((x^3*atan(a*x)^3)/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int((x^3*atan(a*x)^3)/(c + a^2*c*x^2)^(1/2), x)
```

## Reduce [F]

$$\int \frac{x^3 \arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \frac{\int \frac{atan(ax)^3x^3}{\sqrt{a^2x^2+1}}dx}{\sqrt{c}}$$

input

```
int(x^3*atan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

output

```
int((atan(a*x)**3*x**3)/sqrt(a**2*x**2 + 1),x)/sqrt(c)
```

## 3.436 $\int \frac{x^2 \arctan(ax)^3}{\sqrt{c+a^2cx^2}} \, dx$

## Optimal result

Integrand size = 24, antiderivative size = 625

$$\int \frac{x^2 \arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = -\frac{3\sqrt{c+a^2cx^2}\arctan(ax)^2}{2a^3c} + \frac{x\sqrt{c+a^2cx^2}\arctan(ax)^3}{2a^2c} + \frac{i\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^3}{a^3\sqrt{c+a^2cx^2}} - \frac{6i\sqrt{1+a^2x^2}\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a^3\sqrt{c+a^2cx^2}} - \frac{3i\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)}{2a^3\sqrt{c+a^2cx^2}} + \frac{3i\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{PolyLog}\left(2,ie^{i\arctan(ax)}\right)}{2a^3\sqrt{c+a^2cx^2}} + \frac{3i\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,-\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a^3\sqrt{c+a^2cx^2}} - \frac{3i\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,\frac{i\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a^3\sqrt{c+a^2cx^2}} + \frac{3\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)}{a^3\sqrt{c+a^2cx^2}} - \frac{3\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(3,ie^{i\arctan(ax)}\right)}{a^3\sqrt{c+a^2cx^2}} + \frac{3i\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(4,-ie^{i\arctan(ax)}\right)}{a^3\sqrt{c+a^2cx^2}} - \frac{3i\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(4,ie^{i\arctan(ax)}\right)}{a^3\sqrt{c+a^2cx^2}}$$

output
```
-3/2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2/a^3/c+1/2*x*(a^2*c*x^2+c)^(1/2)*arc
tan(a*x)^3/a^2/c+I*(a^2*x^2+1)^(1/2)*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))*a
rctan(a*x)^3/a^3/(a^2*c*x^2+c)^(1/2)-6*I*(a^2*x^2+1)^(1/2)*arctan(a*x)*arc
tan((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/a^3/(a^2*c*x^2+c)^(1/2)-3/2*I*(a^2*x^
2+1)^(1/2)*arctan(a*x)^2*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a^3/(a^
2*c*x^2+c)^(1/2)+3/2*I*(a^2*x^2+1)^(1/2)*arctan(a*x)^2*polylog(2,I*(1+I*a*
x)/(a^2*x^2+1)^(1/2))/a^3/(a^2*c*x^2+c)^(1/2)+3*I*(a^2*x^2+1)^(1/2)*polylo
g(2,-I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/a^3/(a^2*c*x^2+c)^(1/2)-3*I*(a^2*x
^2+1)^(1/2)*polylog(2,I*(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/a^3/(a^2*c*x^2+c)
^(1/2)+3*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^
(1/2))/a^3/(a^2*c*x^2+c)^(1/2)-3*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(3,I
*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a^3/(a^2*c*x^2+c)^(1/2)+3*I*(a^2*x^2+1)^(1/2
)*polylog(4,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a^3/(a^2*c*x^2+c)^(1/2)-3*I*(a
^2*x^2+1)^(1/2)*polylog(4,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a^3/(a^2*c*x^2+c)
^(1/2)
```

## Mathematica [A] (warning: unable to verify)

Time = 4.07 (sec) , antiderivative size = 812, normalized size of antiderivative = 1.30

$$\int \frac{x^2 \arctan(ax)^3}{\sqrt{c + a^2 c x^2}} \, dx = \text{Too large to display}$$

input
```
Integrate[(x^2*ArcTan[a*x]^3)/Sqrt[c + a^2*c*x^2],x]
```

output
```
(Sqrt[c*(1 + a^2*x^2)]*(((7*I)/32)*Pi^4 + (I/4)*Pi^3*ArcTan[a*x] - 6*ArcTa
n[a*x]^2 - ((3*I)/4)*Pi^2*ArcTan[a*x]^2 + I*Pi*ArcTan[a*x]^3 - (I/2)*ArcTa
n[a*x]^4 - (3*Pi^2*ArcTan[a*x]*Log[1 - I/E^(I*ArcTan[a*x])])/2 + 3*Pi*ArcT
an[a*x]^2*Log[1 - I/E^(I*ArcTan[a*x])] + (Pi^3*Log[1 + I/E^(I*ArcTan[a*x])
])/4 - 2*ArcTan[a*x]^3*Log[1 + I/E^(I*ArcTan[a*x])] + 12*ArcTan[a*x]*Log[1
 - I*E^(I*ArcTan[a*x])] - (Pi^3*Log[1 + I*E^(I*ArcTan[a*x])])/4 - 12*ArcTa
n[a*x]*Log[1 + I*E^(I*ArcTan[a*x])] + (3*Pi^2*ArcTan[a*x]*Log[1 + I*E^(I*A
rcTan[a*x])])/2 - 3*Pi*ArcTan[a*x]^2*Log[1 + I*E^(I*ArcTan[a*x])] + 2*ArcT
an[a*x]^3*Log[1 + I*E^(I*ArcTan[a*x])] - (Pi^3*Log[Tan[(Pi + 2*ArcTan[a*x]
)/4]])/4 - (6*I)*ArcTan[a*x]^2*PolyLog[2, (-I)/E^(I*ArcTan[a*x])] - ((3*I)
/2)*Pi*(Pi - 4*ArcTan[a*x])*PolyLog[2, I/E^(I*ArcTan[a*x])] + (12*I)*PolyL
og[2, (-I)*E^(I*ArcTan[a*x])] - ((3*I)/2)*Pi^2*PolyLog[2, (-I)*E^(I*ArcTan
[a*x])] + (6*I)*Pi*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - (6*I)*
ArcTan[a*x]^2*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - (12*I)*PolyLog[2, I*E^(
I*ArcTan[a*x])] - 12*ArcTan[a*x]*PolyLog[3, (-I)/E^(I*ArcTan[a*x])] + 6*Pi
*PolyLog[3, I/E^(I*ArcTan[a*x])] - 6*Pi*PolyLog[3, (-I)*E^(I*ArcTan[a*x])]
 + 12*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] + (12*I)*PolyLog[4, (
-I)/E^(I*ArcTan[a*x])] + (12*I)*PolyLog[4, (-I)*E^(I*ArcTan[a*x])] + ArcTa
n[a*x]^3/(Cos[ArcTan[a*x]/2] - Sin[ArcTan[a*x]/2])^2 - (6*ArcTan[a*x]^2*Si
n[ArcTan[a*x]/2])/(Cos[ArcTan[a*x]/2] - Sin[ArcTan[a*x]/2]) - ArcTan[a*...
```

## Rubi [A] (verified)

Time = 2.13 (sec) , antiderivative size = 415, normalized size of antiderivative = 0.66, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5487, 5425, 5423, 3042, 4669, 3011, 5465, 5425, 5421, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5487}$$

$$-\frac{3\int \frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{\int \frac{\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{2a^2}+\frac{x\arctan(ax)^3\sqrt{a^2cx^2+c}}{2a^2c}$$

$$\Big\downarrow \text{5425}$$

$$-\frac{3\int \frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^3}{\sqrt{a^2x^2+1}}dx}{2a^2\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^3\sqrt{a^2cx^2+c}}{2a^2c}$$

$$\Big\downarrow \text{5423}$$

$$-\frac{3\int \frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{\sqrt{a^2x^2+1}\int \sqrt{a^2x^2+1}\arctan(ax)^3d\arctan(ax)}{2a^3\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^3\sqrt{a^2cx^2+c}}{2a^2c}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\int \frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{\sqrt{a^2x^2+1}\int \arctan(ax)^3\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{2a^3\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^3\sqrt{a^2cx^2+c}}{2a^2c}$$

$$\Big\downarrow \text{4669}$$

$$-\frac{3\int \frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{\sqrt{a^2x^2+1}\left(-3\int \arctan(ax)^2\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+3\int \arctan(ax)^2\log\left(1+ie^{i\arctan(ax)}\right)d\arctan\right.}{2a^3\sqrt{a^2cx^2+c}}$$
$$\frac{x\arctan(ax)^3\sqrt{a^2cx^2+c}}{2a^2c}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{3\int \frac{x\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int \arctan(ax)\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(ax)\right.\right.}{}$$
$$\frac{x\arctan(ax)^3\sqrt{a^2cx^2+c}}{2a^2c}$$

$$\Big\downarrow \text{5465}$$

$$-\frac{3\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\int \frac{\arctan(ax)}{\sqrt{a^2cx^2+c}}dx}{a}\right)}{2a}-\frac{\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int \arctan(ax)\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(ax)\right.\right.}{}$$
$$\frac{x\arctan(ax)^3\sqrt{a^2cx^2+c}}{2a^2c}$$

$$\Big\downarrow \text{5425}$$

$$-\frac{3\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\int\frac{\arctan(ax)}{\sqrt{a^2x^2+1}}dx}{a\sqrt{a^2cx^2+c}}\right)}{2a}-$$

$$\frac{\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(ax)\right.\right.}{}$$

$$\frac{x\arctan(ax)^3\sqrt{a^2cx^2+c}}{2a^2c}$$

$$\Big\downarrow \text{5421}$$

$$-\frac{\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(a\right.\right.}{}$$

$$\frac{3\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{a\sqrt{a^2cx^2+c}}\right)}{2a}+$$

$$\frac{x\arctan(ax)^3\sqrt{a^2cx^2+c}}{2a^2c}$$

$$\Big\downarrow \text{7163}$$

$$-\frac{\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\left(i\int\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)d\arctan(ax)-i\operatorname{arc}\right.\right.\right.}{}$$

$$\frac{3\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{a\sqrt{a^2cx^2+c}}\right)}{2a}+$$

$$\frac{x\arctan(ax)^3\sqrt{a^2cx^2+c}}{2a^2c}$$

$$\Big\downarrow \text{2720}$$

$$-\frac{\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\left(\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)de^{i\arctan(}\right.\right.\right.}{}$$

$$\frac{3\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{a\sqrt{a^2cx^2+c}}\right)}{2a}+$$

$$\frac{x\arctan(ax)^3\sqrt{a^2cx^2+c}}{2a^2c}$$

$$\Big\downarrow \text{7143}$$

$$-\frac{3\left(\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{a^2c}-\frac{2\sqrt{a^2x^2+1}\left(-\frac{2i\arctan(ax)\arctan\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{a}+\frac{i\operatorname{PolyLog}\left(2,-\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}-\frac{i\operatorname{PolyLog}\left(2,\frac{i\sqrt{iax+1}}{\sqrt{1-iax}}\right)}{a}\right)}{a\sqrt{a^2cx^2+c}}\right)}{2a}+\frac{x\arctan(ax)^3\sqrt{a^2cx^2+c}}{2a^2c}-$$

$$\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\left(\operatorname{PolyLog}\left(4,-ie^{i\arctan(ax)}\right)-i\arctan(ax)\operatorname{PolyLog}\left(\right.\right.\right.\right.$$

input
```
Int[(x^2*ArcTan[a*x]^3)/Sqrt[c + a^2*c*x^2],x]
```

output
```
(x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/(2*a^2*c) - (3*((Sqrt[c + a^2*c*x^2]
*ArcTan[a*x]^2)/(a^2*c) - (2*Sqrt[1 + a^2*x^2]*(((-2*I)*ArcTan[a*x]*ArcTan
[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]])/a + (I*PolyLog[2, ((-I)*Sqrt[1 + I*a*x]
)/Sqrt[1 - I*a*x]])/a - (I*PolyLog[2, (I*Sqrt[1 + I*a*x])/Sqrt[1 - I*a*x]]
)/a))/(a*Sqrt[c + a^2*c*x^2])))/(2*a) - (Sqrt[1 + a^2*x^2]*((-2*I)*ArcTan[
E^(I*ArcTan[a*x])]*ArcTan[a*x]^3 + 3*(I*ArcTan[a*x]^2*PolyLog[2, (-I)*E^(I
*ArcTan[a*x])] - (2*I)*((-I)*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])
] + PolyLog[4, (-I)*E^(I*ArcTan[a*x])])) - 3*(I*ArcTan[a*x]^2*PolyLog[2, I
*E^(I*ArcTan[a*x])] - (2*I)*((-I)*ArcTan[a*x]*PolyLog[3, I*E^(I*ArcTan[a*x
])] + PolyLog[4, I*E^(I*ArcTan[a*x])]))))/(2*a^3*Sqrt[c + a^2*c*x^2])
```

## Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5421
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
 :> Simp[-2*I*(a + b*ArcTan[c*x])*(ArcTan[Sqrt[1 + I*c*x]/Sqrt[1 - I*c*x]]/
(c*Sqrt[d])), x] + (Simp[I*b*(PolyLog[2, (-I)*(Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x])]/(c*Sqrt[d])), x] - Simp[I*b*(PolyLog[2, I*(Sqrt[1 + I*c*x]/Sqrt[1 - I
*c*x])]/(c*Sqrt[d])), x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
GtQ[d, 0]
```

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5487
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[f*(f*x)^(m - 1)*Sqrt[d + e*x^2]*((a + b*
ArcTan[c*x])^p/(c^2*d*m)), x] + (-Simp[b*f*(p/(c*m))   Int[(f*x)^(m - 1)*((
a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[f^2*((m - 1)/(c^
2*m))   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x^2]), x], x])
/; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0] && GtQ[m, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

## Maple [A] (verified)

Time = 5.02 (sec) , antiderivative size = 428, normalized size of antiderivative = 0.68

| method | result |
|---|---|
| default | $\frac{(\arctan(ax)ax-3)\arctan(ax)^2\sqrt{c(ax-i)(ax+i)}}{2c\,a^3}-\frac{\left(\arctan(ax)^3\ln\left(1-\frac{i(iax+1)}{\sqrt{a^2x^2+1}}\right)-3i\arctan(ax)^2\operatorname{polylog}\left(2,\frac{i(iax+1)}{\sqrt{a^2x^2+1}}\right)-\operatorname{arct}\right.}{}$ |

input
```
int(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(arctan(a*x)*a*x-3)*arctan(a*x)^2*(c*(a*x-I)*(a*x+I))^(1/2)/c/a^3-1/2*
(arctan(a*x)^3*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*I*arctan(a*x)^2*polyl
og(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-arctan(a*x)^3*ln(1+I*(1+I*a*x)/(a^2*x^
2+1)^(1/2))+3*I*arctan(a*x)^2*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*
arctan(a*x)*ln(1-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*arctan(a*x)*polylog(3,I*
(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*polylog(4,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+
6*arctan(a*x)*ln(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*arctan(a*x)*polylog(3,
-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I*polylog(4,-I*(1+I*a*x)/(a^2*x^2+1)^(1/
2))-6*I*dilog(1+I*(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*dilog(1-I*(1+I*a*x)/(a^
2*x^2+1)^(1/2)))*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/2)/a^3/c
```

## Fricas [F]

$$\int \frac{x^2 \arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^2 \arctan\left(ax\right)^3}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(x^2*arctan(a*x)^3/sqrt(a^2*c*x^2 + c), x)
```

## Sympy [F]

$$\int \frac{x^2 \arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^2 \operatorname{atan}^3\left(ax\right)}{\sqrt{c\left(a^2x^2+1\right)}}\,dx$$

input
```
integrate(x**2*atan(a*x)**3/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x**2*atan(a*x)**3/sqrt(c*(a**2*x**2 + 1)), x)
```

## Maxima [F]

$$\int \frac{x^2 \arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^2 \arctan\left(ax\right)^3}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2*arctan(a*x)^3/sqrt(a^2*c*x^2 + c), x)
```

## Giac [F]

$$\int \frac{x^2 \arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^2 \arctan\left(ax\right)^3}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2*arctan(a*x)^3/sqrt(a^2*c*x^2 + c), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^2\,\mathrm{atan}(a\,x)^3}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int((x^2*atan(a*x)^3)/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int((x^2*atan(a*x)^3)/(c + a^2*c*x^2)^(1/2), x)
```

## Reduce [F]

$$\int \frac{x^2 \arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \frac{\int \frac{atan(ax)^3x^2}{\sqrt{a^2x^2+1}}dx}{\sqrt{c}}$$

input
```
int(x^2*atan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int((atan(a*x)**3*x**2)/sqrt(a**2*x**2 + 1),x)/sqrt(c)
```

# 3.437 $\int \frac{x \arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 283

$$\begin{aligned}\int \frac{x \arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = {} & \frac{6i\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2}{a^2\sqrt{c+a^2cx^2}} \\ & + \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{a^2c} \\ & - \frac{6i\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)}{a^2\sqrt{c+a^2cx^2}} \\ & + \frac{6i\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,ie^{i\arctan(ax)}\right)}{a^2\sqrt{c+a^2cx^2}} \\ & + \frac{6\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)}{a^2\sqrt{c+a^2cx^2}} \\ & - \frac{6\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,ie^{i\arctan(ax)}\right)}{a^2\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
6*I*(a^2*x^2+1)^(1/2)*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^2/a^
2/(a^2*c*x^2+c)^(1/2)+(a^2*c*x^2+c)^(1/2)*arctan(a*x)^3/a^2/c-6*I*(a^2*x^2
+1)^(1/2)*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a^2/(a^2*c
*x^2+c)^(1/2)+6*I*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2
*x^2+1)^(1/2))/a^2/(a^2*c*x^2+c)^(1/2)+6*(a^2*x^2+1)^(1/2)*polylog(3,-I*(1
+I*a*x)/(a^2*x^2+1)^(1/2))/a^2/(a^2*c*x^2+c)^(1/2)-6*(a^2*x^2+1)^(1/2)*pol
ylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a^2/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.59

$$\int \frac{x\arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{c(1+a^2x^2)}\left(\arctan(ax)^3-\frac{3\left(\arctan(ax)^2\left(\log\left(1-ie^{i\arctan(ax)}\right)-\log\left(1+ie^{i\arctan(ax)}\right)\right)+2i\arctan(ax)\left(\operatorname{PolyLog}\left(2,-ie^{i\arctan(a}\right.\right.\right.}{\sqrt{1+a^2x^2}}}{a^2c}$$

input
```
Integrate[(x*ArcTan[a*x]^3)/Sqrt[c + a^2*c*x^2],x]
```

output
```
(Sqrt[c*(1 + a^2*x^2)]*(ArcTan[a*x]^3 - (3*(ArcTan[a*x]^2*(Log[1 - I*E^(I*
ArcTan[a*x])] - Log[1 + I*E^(I*ArcTan[a*x])]) + (2*I)*ArcTan[a*x]*(PolyLog
[2, (-I)*E^(I*ArcTan[a*x])] - PolyLog[2, I*E^(I*ArcTan[a*x])]) - 2*PolyLog
[3, (-I)*E^(I*ArcTan[a*x])] + 2*PolyLog[3, I*E^(I*ArcTan[a*x])]))/Sqrt[1 +
 a^2*x^2]))/(a^2*c)
```

## Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.61, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {5465, 5425, 5423, 3042, 4669, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\arctan(ax)^3}{\sqrt{a^2cx^2+c}}\,dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a}$$

$$\Big\downarrow \text{5425}$$

$$\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{a\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \; 5423$$

$$\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\int\sqrt{a^2x^2+1}\arctan(ax)^2d\arctan(ax)}{a^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\int\arctan(ax)^2\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{a^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \; 4669$$

$$\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(-2\int\arctan(ax)\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+2\int\arctan(ax)\log\left(1+ie^{i\arctan(ax)}\right)d\arctan(\right.}{a^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \; 3011$$

$$\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-i\int\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(ax)\right)-2\left(i\,\mathrm{arcta}\right.\right.}{a^2\sqrt{a}}$$

$$\Big\downarrow \; 2720$$

$$\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)\right.}{}$$

$$\Big\downarrow \; 7143$$

$$\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)\right)-2\left(i\arctan(ax)\operatorname{PolyLog}\left(\right.\right.\right.}{a^2\sqrt{a^2cx^2+c}}$$

input
```
Int[(x*ArcTan[a*x]^3)/Sqrt[c + a^2*c*x^2],x]
```

output
```
(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/(a^2*c) - (3*Sqrt[1 + a^2*x^2]*((-2*I)
*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 + 2*(I*ArcTan[a*x]*PolyLog[2, (-I
)*E^(I*ArcTan[a*x])] - PolyLog[3, (-I)*E^(I*ArcTan[a*x])]) - 2*(I*ArcTan[a
*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] - PolyLog[3, I*E^(I*ArcTan[a*x])])))/(
a^2*Sqrt[c + a^2*c*x^2])
```

### Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

## Maple [F]

$$\int \frac{x \arctan\left(ax\right)^{3}}{\sqrt{a^{2} c \,x^{2}+c}}dx$$

input
```
int(x*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(x*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

## Fricas [F]

$$\int \frac{x \arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x \arctan\left(ax\right)^{3}}{\sqrt{a^{2} c x^{2} + c}}\,dx$$

input
```
integrate(x*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(x*arctan(a*x)^3/sqrt(a^2*c*x^2 + c), x)
```

## Sympy [F]

$$\int \frac{x \arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x \operatorname{atan}^3(ax)}{\sqrt{c(a^2x^2+1)}}\,dx$$

input
```
integrate(x*atan(a*x)**3/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x*atan(a*x)**3/sqrt(c*(a**2*x**2 + 1)), x)
```

## Maxima [F]

$$\int \frac{x \arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x \arctan(ax)^3}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(x*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x*arctan(a*x)^3/sqrt(a^2*c*x^2 + c), x)
```

## Giac [F]

$$\int \frac{x \arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x \arctan(ax)^3}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(x*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x*arctan(a*x)^3/sqrt(a^2*c*x^2 + c), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x\arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x\,\mathrm{atan}(a\,x)^3}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int((x*atan(a*x)^3)/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int((x*atan(a*x)^3)/(c + a^2*c*x^2)^(1/2), x)
```

## Reduce [F]

$$\int \frac{x\arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \frac{\int \frac{atan(ax)^3x}{\sqrt{a^2x^2+1}}dx}{\sqrt{c}}$$

input
```
int(x*atan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int((atan(a*x)**3*x)/sqrt(a**2*x**2 + 1),x)/sqrt(c)
```

# 3.438 $\int \frac{\arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 368

$$\int \frac{\arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = -\frac{2i\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^3}{a\sqrt{c+a^2cx^2}} + \frac{3i\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)}{a\sqrt{c+a^2cx^2}} - \frac{3i\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{PolyLog}\left(2,ie^{i\arctan(ax)}\right)}{a\sqrt{c+a^2cx^2}} - \frac{6\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)}{a\sqrt{c+a^2cx^2}} + \frac{6\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(3,ie^{i\arctan(ax)}\right)}{a\sqrt{c+a^2cx^2}} - \frac{6i\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(4,-ie^{i\arctan(ax)}\right)}{a\sqrt{c+a^2cx^2}} + \frac{6i\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(4,ie^{i\arctan(ax)}\right)}{a\sqrt{c+a^2cx^2}}$$

output
```
-2*I*(a^2*x^2+1)^(1/2)*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^3/a
/(a^2*c*x^2+c)^(1/2)+3*I*(a^2*x^2+1)^(1/2)*arctan(a*x)^2*polylog(2,-I*(1+I
*a*x)/(a^2*x^2+1)^(1/2))/a/(a^2*c*x^2+c)^(1/2)-3*I*(a^2*x^2+1)^(1/2)*arcta
n(a*x)^2*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a/(a^2*c*x^2+c)^(1/2)-6*
(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a/
(a^2*c*x^2+c)^(1/2)+6*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(3,I*(1+I*a*x)/
(a^2*x^2+1)^(1/2))/a/(a^2*c*x^2+c)^(1/2)-6*I*(a^2*x^2+1)^(1/2)*polylog(4,-
I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a/(a^2*c*x^2+c)^(1/2)+6*I*(a^2*x^2+1)^(1/2)
*polylog(4,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.52

$$\int \frac{\arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = -\frac{i\sqrt{c\left(1+a^2x^2\right)}\left(2\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^3-3\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)+3\operatorname{arcta}\right.}{}$$

input
```
Integrate[ArcTan[a*x]^3/Sqrt[c + a^2*c*x^2],x]
```

output
```
((-I)*Sqrt[c*(1 + a^2*x^2)]*(2*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^3 - 3
*ArcTan[a*x]^2*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + 3*ArcTan[a*x]^2*PolyLo
g[2, I*E^(I*ArcTan[a*x])] - (6*I)*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[
a*x])] + (6*I)*ArcTan[a*x]*PolyLog[3, I*E^(I*ArcTan[a*x])] + 6*PolyLog[4,
(-I)*E^(I*ArcTan[a*x])] - 6*PolyLog[4, I*E^(I*ArcTan[a*x])]))/(a*c*Sqrt[1
+ a^2*x^2])
```

## Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.55, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {5425, 5423, 3042, 4669, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx$$

$\downarrow$ 5425

$$\frac{\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^3}{\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}$$

$\downarrow$ 5423

$$\frac{\sqrt{a^2x^2+1}\int \sqrt{a^2x^2+1}\arctan(ax)^3d\arctan(ax)}{a\sqrt{a^2cx^2+c}}$$

$\downarrow$ 3042

$$\frac{\sqrt{a^2x^2+1}\int \arctan(ax)^3\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{a\sqrt{a^2cx^2+c}}$$

$\downarrow$ 4669

$$\frac{\sqrt{a^2x^2+1}\left(-3\int \arctan(ax)^2\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+3\int \arctan(ax)^2\log\left(1+ie^{i\arctan(ax)}\right)d\arctan\right.}{a\sqrt{a^2cx^2+c}}$$

$\downarrow$ 3011

$$\frac{\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int \arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(ax)\right.\right.}{}$$

$\downarrow$ 7163

$$\frac{\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\left(i\int \operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)d\arctan(ax)-i\arctan\right.\right.\right.}{}$$

$\downarrow$ 2720

$$\frac{\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\left(\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right.\right.\right.}{}$$

$\downarrow$ 7143

$$\frac{\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\left(\operatorname{PolyLog}\left(4,-ie^{i\arctan(ax)}\right)-i\arctan(ax)\operatorname{PolyLog}\left(\right.\right.\right.\right.}{}$$

input
```
Int[ArcTan[a*x]^3/Sqrt[c + a^2*c*x^2],x]
```

output
```
(Sqrt[1 + a^2*x^2]*((-2*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^3 + 3*(I*
ArcTan[a*x]^2*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - (2*I)*((-I)*ArcTan[a*x]
*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] + PolyLog[4, (-I)*E^(I*ArcTan[a*x])]))
 - 3*(I*ArcTan[a*x]^2*PolyLog[2, I*E^(I*ArcTan[a*x])] - (2*I)*((-I)*ArcTan
[a*x]*PolyLog[3, I*E^(I*ArcTan[a*x])] + PolyLog[4, I*E^(I*ArcTan[a*x])])))
)/(a*Sqrt[c + a^2*c*x^2])
```

## Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

## Maple [F]

$$\int \frac{\arctan(ax)^3}{\sqrt{a^2c\,x^2+c}}dx$$

input
```
int(arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

## Fricas [F]

$$\int \frac{\arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan(ax)^3}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(arctan(a*x)^3/sqrt(a^2*c*x^2 + c), x)
```

## Sympy [F]

$$\int \frac{\arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\operatorname{atan}^3(ax)}{\sqrt{c(a^2x^2+1)}}\,dx$$

input
```
integrate(atan(a*x)**3/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(atan(a*x)**3/sqrt(c*(a**2*x**2 + 1)), x)
```

## Maxima [F]

$$\int \frac{\arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan(ax)^3}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)^3/sqrt(a^2*c*x^2 + c), x)
```

## Giac [F]

$$\int \frac{\arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan(ax)^3}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^3/sqrt(a^2*c*x^2 + c), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\mathrm{atan}(a\,x)^3}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(atan(a*x)^3/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(atan(a*x)^3/(c + a^2*c*x^2)^(1/2), x)
```

## Reduce [F]

$$\int \frac{\arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \frac{\int \frac{atan(ax)^3}{\sqrt{a^2x^2+1}}dx}{\sqrt{c}}$$

input
```
int(atan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(atan(a*x)**3/sqrt(a**2*x**2 + 1),x)/sqrt(c)
```

# 3.439 $\int \frac{\arctan(ax)^3}{x\sqrt{c+a^2cx^2}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 327

$$\begin{aligned}\int \frac{\arctan(ax)^3}{x\sqrt{c+a^2cx^2}}\,dx = & -\frac{2\sqrt{1+a^2x^2}\arctan(ax)^3\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} \\ & +\frac{3i\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} \\ & -\frac{3i\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{PolyLog}\left(2,e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} \\ & -\frac{6\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(3,-e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} \\ & +\frac{6\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(3,e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} \\ & -\frac{6i\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(4,-e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} \\ & +\frac{6i\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(4,e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
-2*(a^2*x^2+1)^(1/2)*arctan(a*x)^3*arctanh((1+I*a*x)/(a^2*x^2+1)^(1/2))/(a
^2*c*x^2+c)^(1/2)+3*I*(a^2*x^2+1)^(1/2)*arctan(a*x)^2*polylog(2,-(1+I*a*x)
/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)-3*I*(a^2*x^2+1)^(1/2)*arctan(a*x)^
2*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)-6*(a^2*x^2+1)
^(1/2)*arctan(a*x)*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(
1/2)+6*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2)
)/(a^2*c*x^2+c)^(1/2)-6*I*(a^2*x^2+1)^(1/2)*polylog(4,-(1+I*a*x)/(a^2*x^2+
1)^(1/2))/(a^2*c*x^2+c)^(1/2)+6*I*(a^2*x^2+1)^(1/2)*polylog(4,(1+I*a*x)/(a
^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.64

$$\int \frac{\arctan(ax)^3}{x\sqrt{c+a^2cx^2}}\,dx = -\frac{i\sqrt{1+a^2x^2}\left(\pi^4-2\arctan(ax)^4+8i\arctan(ax)^3\log\left(1-e^{-i\arctan(ax)}\right)-8i\arctan(ax)^3\log\left(1+e^{i\arctan(ax)}\right)\right.}{}$$

input
```
Integrate[ArcTan[a*x]^3/(x*Sqrt[c + a^2*c*x^2]),x]
```

output
```
((-1/8*I)*Sqrt[1 + a^2*x^2]*(Pi^4 - 2*ArcTan[a*x]^4 + (8*I)*ArcTan[a*x]^3*
Log[1 - E^((-I)*ArcTan[a*x])] - (8*I)*ArcTan[a*x]^3*Log[1 + E^(I*ArcTan[a*
x])] - 24*ArcTan[a*x]^2*PolyLog[2, E^((-I)*ArcTan[a*x])] - 24*ArcTan[a*x]^
2*PolyLog[2, -E^(I*ArcTan[a*x])] + (48*I)*ArcTan[a*x]*PolyLog[3, E^((-I)*A
rcTan[a*x])] - (48*I)*ArcTan[a*x]*PolyLog[3, -E^(I*ArcTan[a*x])] + 48*Poly
Log[4, E^((-I)*ArcTan[a*x])] + 48*PolyLog[4, -E^(I*ArcTan[a*x])]))/Sqrt[c*
(1 + a^2*x^2)]
```

## Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.54, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5493, 5491, 3042, 4671, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \ 5493$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^3}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \ 5491$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{\sqrt{a^2x^2+1}\arctan(ax)^3}{ax}d\arctan(ax)}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\sqrt{a^2x^2+1}\int \arctan(ax)^3\csc(\arctan(ax))d\arctan(ax)}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \ 4671$$

$$\frac{\sqrt{a^2x^2+1}\left(-3\int \arctan(ax)^2\log\left(1-e^{i\arctan(ax)}\right)d\arctan(ax)+3\int \arctan(ax)^2\log\left(1+e^{i\arctan(ax)}\right)d\arctan(a\right.}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \ 3011$$

$$\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\int \arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)d\arctan(ax)\right)\right.$$

$$\Big\downarrow \ 7163$$

$$\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\left(i\int \operatorname{PolyLog}\left(3,-e^{i\arctan(ax)}\right)d\arctan(ax)-i\arctan\right.\right.\right.$$

$$\Big\downarrow \ 2720$$

$$\frac{\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\left(\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(3,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right.\right.\right.}{}$$

$$\Big\downarrow \text{7143}$$

$$\frac{\sqrt{a^2x^2+1}\left(-2\arctan(ax)^3\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)+3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\left(\operatorname{PolyLog}\left(4,-\right.\right.\right.\right.}{}$$

input
```
Int[ArcTan[a*x]^3/(x*Sqrt[c + a^2*c*x^2]),x]
```

output
```
(Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]^3*ArcTanh[E^(I*ArcTan[a*x])] + 3*(I*Arc
Tan[a*x]^2*PolyLog[2, -E^(I*ArcTan[a*x])] - (2*I)*((-I)*ArcTan[a*x]*PolyLo
g[3, -E^(I*ArcTan[a*x])] + PolyLog[4, -E^(I*ArcTan[a*x])])) - 3*(I*ArcTan[
a*x]^2*PolyLog[2, E^(I*ArcTan[a*x])] - (2*I)*((-I)*ArcTan[a*x]*PolyLog[3,
E^(I*ArcTan[a*x])] + PolyLog[4, E^(I*ArcTan[a*x])]))))/Sqrt[c + a^2*c*x^2]
```

## Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5491
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Simp[1/Sqrt[d]   Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTa
n[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] &&
GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

## Maple [A] (verified)

Time = 4.78 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.80

| method | result |
|---|---|
| default | $\frac{i\left(i\arctan(ax)^{3}\ln\left(1+\frac{iax+1}{\sqrt{a^{2}x^{2}+1}}\right)-i\arctan(ax)^{3}\ln\left(1-\frac{iax+1}{\sqrt{a^{2}x^{2}+1}}\right)+3\arctan(ax)^{2}\operatorname{polylog}\left(2,-\frac{iax+1}{\sqrt{a^{2}x^{2}+1}}\right)-3\arctan(ax)^{2}\operatorname{polylog} \ldots}{}$ |

input
```
int(arctan(a*x)^3/x/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
I*(I*arctan(a*x)^3*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*arctan(a*x)^3*ln(1-
(1+I*a*x)/(a^2*x^2+1)^(1/2))+3*arctan(a*x)^2*polylog(2,-(1+I*a*x)/(a^2*x^2
+1)^(1/2))-3*arctan(a*x)^2*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*arct
an(a*x)*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I*arctan(a*x)*polylog(3,
(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*polylog(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*p
olylog(4,(1+I*a*x)/(a^2*x^2+1)^(1/2)))*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+
1)^(1/2)/c
```

## Fricas [F]

$$\int \frac{\arctan(ax)^3}{x\sqrt{c+a^2cx^2}}\,dx=\int \frac{\arctan{(ax)}^3}{\sqrt{a^2cx^2+cx}}\,dx$$

input
```
integrate(arctan(a*x)^3/x/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3/(a^2*c*x^3 + c*x), x)
```

## Sympy [F]

$$\int \frac{\arctan(ax)^3}{x\sqrt{c+a^2cx^2}}\,dx=\int \frac{\operatorname{atan}^3{(ax)}}{x\sqrt{c\,(a^2x^2+1)}}\,dx$$

input
```
integrate(atan(a*x)**3/x/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(atan(a*x)**3/(x*sqrt(c*(a**2*x**2 + 1))), x)
```

## Maxima [F]

$$\int \frac{\arctan(ax)^3}{x\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan(ax)^3}{\sqrt{a^2cx^2+c}x}\,dx$$

input
```
integrate(arctan(a*x)^3/x/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)^3/(sqrt(a^2*c*x^2 + c)*x), x)
```

## Giac [F]

$$\int \frac{\arctan(ax)^3}{x\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan(ax)^3}{\sqrt{a^2cx^2+c}x}\,dx$$

input
```
integrate(arctan(a*x)^3/x/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^3/(sqrt(a^2*c*x^2 + c)*x), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^3}{x\sqrt{c+a^2cx^2}}\,dx = \int \frac{\mathrm{atan}(a\,x)^3}{x\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(atan(a*x)^3/(x*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(atan(a*x)^3/(x*(c + a^2*c*x^2)^(1/2)), x)
```

## Reduce [F]

$$\int \frac{\arctan(ax)^3}{x\sqrt{c+a^2cx^2}}\,dx = \frac{\int \frac{atan(ax)^3}{\sqrt{a^2x^2+1}\,x}dx}{\sqrt{c}}$$

input
```
int(atan(a*x)^3/x/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(atan(a*x)**3/(sqrt(a**2*x**2 + 1)*x),x)/sqrt(c)
```

# 3.440 $\int \frac{\arctan(ax)^3}{x^2\sqrt{c+a^2cx^2}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 260

$$\begin{aligned}\int \frac{\arctan(ax)^3}{x^2\sqrt{c+a^2cx^2}}\,dx = & -\frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{cx} \\ & -\frac{6a\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} \\ & +\frac{6ia\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} \\ & -\frac{6ia\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} \\ & -\frac{6a\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,-e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} \\ & +\frac{6a\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}\end{aligned}$$

output

```
-(a^2*c*x^2+c)^(1/2)*arctan(a*x)^3/c/x-6*a*(a^2*x^2+1)^(1/2)*arctan(a*x)^2
*arctanh((1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)+6*I*a*(a^2*x^2+1
)^(1/2)*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^
(1/2)-6*I*a*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^
(1/2))/(a^2*c*x^2+c)^(1/2)-6*a*(a^2*x^2+1)^(1/2)*polylog(3,-(1+I*a*x)/(a^2
*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)+6*a*(a^2*x^2+1)^(1/2)*polylog(3,(1+I*a*
x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.67

$$\int \frac{\arctan(ax)^3}{x^2\sqrt{c+a^2cx^2}}\,dx = -\frac{a\sqrt{1+a^2x^2}\left(\frac{\sqrt{1+a^2x^2}\arctan(ax)^3}{ax} - 3\arctan(ax)^2\log\left(1-e^{i\arctan(ax)}\right) + 3\arctan(ax)^2\log\left(1+e^{i\arctan(ax)}\right)\right.}{}$$

input
```
Integrate[ArcTan[a*x]^3/(x^2*Sqrt[c + a^2*c*x^2]),x]
```

output
```
-((a*Sqrt[1 + a^2*x^2]*((Sqrt[1 + a^2*x^2]*ArcTan[a*x]^3)/(a*x) - 3*ArcTan
[a*x]^2*Log[1 - E^(I*ArcTan[a*x])] + 3*ArcTan[a*x]^2*Log[1 + E^(I*ArcTan[a
*x])] - (6*I)*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])] + (6*I)*ArcTan[a*
x]*PolyLog[2, E^(I*ArcTan[a*x])] + 6*PolyLog[3, -E^(I*ArcTan[a*x])] - 6*Po
lyLog[3, E^(I*ArcTan[a*x])]))/Sqrt[c*(1 + a^2*x^2)])
```

## Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.60, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5479, 5493, 5491, 3042, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3}{x^2\sqrt{a^2cx^2+c}}\,dx$$

$$\Big\downarrow \text{5479}$$

$$3a\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx - \frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}$$

$$\Big\downarrow \text{5493}$$

$$\frac{3a\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}} - \frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}$$

$$\Big\downarrow \text{5491}$$

$$\frac{3a\sqrt{a^2x^2+1}\int \frac{\sqrt{a^2x^2+1}\arctan(ax)^2}{ax}d\arctan(ax)}{\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3a\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc(\arctan(ax))d\arctan(ax)}{\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}$$

$$\Big\downarrow \text{4671}$$

$$-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(-2\int \arctan(ax)\log\left(1-e^{i\arctan(ax)}\right)d\arctan(ax)+2\int \arctan(ax)\log\left(1+e^{i\arctan(ax)}\right)d\arctan(\right.}{\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-i\int \operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)d\arctan(ax)\right)-2\left(i\arcta\right.\right.}{\sqrt{a^2c}}$$

$$\Big\downarrow \text{2720}$$

$$-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)\right.}{}$$

$$\Big\downarrow \text{7143}$$

$$-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(-2\arctan(ax)^2\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)+2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\operatorname{PolyLog}\left(3,-e^{\right.\right.\right.}}{\sqrt{a^2cx^2+c}}$$

input
```
Int[ArcTan[a*x]^3/(x^2*Sqrt[c + a^2*c*x^2]),x]
```

output
```
-((Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/(c*x)) + (3*a*Sqrt[1 + a^2*x^2]*(-2*
ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])] + 2*(I*ArcTan[a*x]*PolyLog[2, -E^
(I*ArcTan[a*x])] - PolyLog[3, -E^(I*ArcTan[a*x])]) - 2*(I*ArcTan[a*x]*Poly
Log[2, E^(I*ArcTan[a*x])] - PolyLog[3, E^(I*ArcTan[a*x])])))/Sqrt[c + a^2*
c*x^2]
```

## Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5491
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Simp[1/Sqrt[d]   Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTa
n[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] &&
GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

## Maple [A] (verified)

Time = 4.70 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.88

| method | result |
|---|---|
| default | $-\frac{\arctan(ax)^3\sqrt{c(ax-i)(ax+i)}}{cx}-\frac{3a\left(\arctan(ax)^2\ln\left(1+\frac{iax+1}{\sqrt{a^2x^2+1}}\right)-2i\arctan(ax)\operatorname{polylog}\left(2,-\frac{iax+1}{\sqrt{a^2x^2+1}}\right)-\arctan(ax)^2\ln\left(\right.\right.}{}$ |

input
```
int(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-arctan(a*x)^3*(c*(a*x-I)*(a*x+I))^(1/2)/c/x-3*a*(arctan(a*x)^2*ln(1+(1+I*
a*x)/(a^2*x^2+1)^(1/2))-2*I*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(
1/2))-arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*I*arctan(a*x)*poly
log(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2
))-2*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2)))*(c*(a*x-I)*(a*x+I))^(1/2)/(a^
2*x^2+1)^(1/2)/c
```

## Fricas [F]

$$\int \frac{\arctan(ax)^3}{x^2\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan{(ax)}^3}{\sqrt{a^2cx^2+c}x^2}\,dx$$

input
```
integrate(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3/(a^2*c*x^4 + c*x^2), x)
```

## Sympy [F]

$$\int \frac{\arctan(ax)^3}{x^2\sqrt{c+a^2cx^2}}\,dx = \int \frac{\operatorname{atan}^3{(ax)}}{x^2\sqrt{c(a^2x^2+1)}}\,dx$$

input
```
integrate(atan(a*x)**3/x**2/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(atan(a*x)**3/(x**2*sqrt(c*(a**2*x**2 + 1))), x)
```

## Maxima [F]

$$\int \frac{\arctan(ax)^3}{x^2\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan(ax)^3}{\sqrt{a^2cx^2+c}x^2}\,dx$$

input
```
integrate(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)^3/(sqrt(a^2*c*x^2 + c)*x^2), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^3}{x^2\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^3}{x^2\sqrt{c+a^2cx^2}}\,dx = \int \frac{\mathrm{atan}(a\,x)^3}{x^2\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(atan(a*x)^3/(x^2*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(atan(a*x)^3/(x^2*(c + a^2*c*x^2)^(1/2)), x)
```

## Reduce [F]

$$\int \frac{\arctan(ax)^3}{x^2\sqrt{c+a^2cx^2}}\,dx = \frac{\int \frac{atan(ax)^3}{\sqrt{a^2x^2+1}\,x^2}dx}{\sqrt{c}}$$

input
```
int(atan(a*x)^3/x^2/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(atan(a*x)**3/(sqrt(a**2*x**2 + 1)*x**2),x)/sqrt(c)
```

# 3.441 $\int \frac{\arctan(ax)^3}{x^3\sqrt{c+a^2cx^2}}\,dx$

## Optimal result

Integrand size = 24, antiderivative size = 597

$$\int \frac{\arctan(ax)^3}{x^3\sqrt{c+a^2cx^2}}\,dx = -\frac{3a\sqrt{c+a^2cx^2}\arctan(ax)^2}{2cx} - \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{2cx^2} + \frac{a^2\sqrt{1+a^2x^2}\arctan(ax)^3\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} - \frac{6a^2\sqrt{1+a^2x^2}\arctan(ax)\operatorname{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} - \frac{3ia^2\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)}{2\sqrt{c+a^2cx^2}} + \frac{3ia^2\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{PolyLog}\left(2,e^{i\arctan(ax)}\right)}{2\sqrt{c+a^2cx^2}} + \frac{3ia^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,-\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} - \frac{3ia^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(2,\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)}{\sqrt{c+a^2cx^2}} + \frac{3a^2\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(3,-e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} - \frac{3a^2\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(3,e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} + \frac{3ia^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(4,-e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}} - \frac{3ia^2\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(4,e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}$$

output
```
-3/2*a*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^2/c/x-1/2*(a^2*c*x^2+c)^(1/2)*arcta
n(a*x)^3/c/x^2+a^2*(a^2*x^2+1)^(1/2)*arctan(a*x)^3*arctanh((1+I*a*x)/(a^2*
x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)-6*a^2*(a^2*x^2+1)^(1/2)*arctan(a*x)*arct
anh((1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/(a^2*c*x^2+c)^(1/2)-3/2*I*a^2*(a^2*x^
2+1)^(1/2)*arctan(a*x)^2*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^
2+c)^(1/2)+3/2*I*a^2*(a^2*x^2+1)^(1/2)*arctan(a*x)^2*polylog(2,(1+I*a*x)/(
a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)+3*I*a^2*(a^2*x^2+1)^(1/2)*polylog(2,
-(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/(a^2*c*x^2+c)^(1/2)-3*I*a^2*(a^2*x^2+1)^
(1/2)*polylog(2,(1+I*a*x)^(1/2)/(1-I*a*x)^(1/2))/(a^2*c*x^2+c)^(1/2)+3*a^2
*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^
2*c*x^2+c)^(1/2)-3*a^2*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(3,(1+I*a*x)/(
a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)+3*I*a^2*(a^2*x^2+1)^(1/2)*polylog(4,
-(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)-3*I*a^2*(a^2*x^2+1)^(1/2
)*polylog(4,(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (warning: unable to verify)

Time = 2.73 (sec) , antiderivative size = 345, normalized size of antiderivative = 0.58

$$\int \frac{\arctan(ax)^3}{x^3\sqrt{c+a^2cx^2}}\,dx = \frac{a^2\sqrt{1+a^2x^2}\left(i\pi^4-2i\arctan(ax)^4-12\arctan(ax)^2\cot\left(\frac{1}{2}\arctan(ax)\right)-2\arctan(ax)^3\csc^2\left(\frac{1}{2}\arctan(a\right.\right.}{}$$

input
```
Integrate[ArcTan[a*x]^3/(x^3*Sqrt[c + a^2*c*x^2]),x]
```

output
```
(a^2*Sqrt[1 + a^2*x^2]*(I*Pi^4 - (2*I)*ArcTan[a*x]^4 - 12*ArcTan[a*x]^2*Co
t[ArcTan[a*x]/2] - 2*ArcTan[a*x]^3*Csc[ArcTan[a*x]/2]^2 - 8*ArcTan[a*x]^3*
Log[1 - E^((-I)*ArcTan[a*x])] + 48*ArcTan[a*x]*Log[1 - E^(I*ArcTan[a*x])]
- 48*ArcTan[a*x]*Log[1 + E^(I*ArcTan[a*x])] + 8*ArcTan[a*x]^3*Log[1 + E^(I
*ArcTan[a*x])] - (24*I)*ArcTan[a*x]^2*PolyLog[2, E^((-I)*ArcTan[a*x])] - (
24*I)*(-2 + ArcTan[a*x]^2)*PolyLog[2, -E^(I*ArcTan[a*x])] - (48*I)*PolyLog
[2, E^(I*ArcTan[a*x])] - 48*ArcTan[a*x]*PolyLog[3, E^((-I)*ArcTan[a*x])] +
 48*ArcTan[a*x]*PolyLog[3, -E^(I*ArcTan[a*x])] + (48*I)*PolyLog[4, E^((-I)
*ArcTan[a*x])] + (48*I)*PolyLog[4, -E^(I*ArcTan[a*x])] + 2*ArcTan[a*x]^3*S
ec[ArcTan[a*x]/2]^2 - 12*ArcTan[a*x]^2*Tan[ArcTan[a*x]/2]))/(16*Sqrt[c*(1
+ a^2*x^2)])
```

## Rubi [A] (verified)

Time = 2.09 (sec) , antiderivative size = 375, normalized size of antiderivative = 0.63, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {5497, 5479, 5493, 5489, 5491, 3042, 4671, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3}{x^3\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5497}$$

$$\frac{3}{2}a\int \frac{\arctan(ax)^2}{x^2\sqrt{a^2cx^2+c}}dx-\frac{1}{2}a^2\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{2cx^2}$$

$$\Big\downarrow \text{5479}$$

$$\frac{3}{2}a\left(2a\int \frac{\arctan(ax)}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}\right)-\frac{1}{2}a^2\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{2cx^2}$$

$$\Big\downarrow \text{5493}$$

$$\frac{3}{2}a\left(\frac{2a\sqrt{a^2x^2+1}\int \frac{\arctan(ax)}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}\right)-\frac{a^2\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^3}{x\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{2cx^2}$$

$$\Big\downarrow \text{5489}$$

$$-\frac{a^2\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^3}{x\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+\frac{3}{2}a\left(-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}+\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{P}\right.}{\sqrt{a^2cx^2+c}}\right.-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{2cx^2}$$

$$\Big\downarrow \text{5491}$$

$$-\frac{a^2\sqrt{a^2x^2+1}\int \frac{\sqrt{a^2x^2+1}\arctan(ax)^3}{ax}d\arctan(ax)}{2\sqrt{a^2cx^2+c}}+$$
$$\frac{3}{2}a\left(-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}+\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{P}\right.}{\sqrt{a^2cx^2+c}}\right.$$
$$\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{2cx^2}$$

$\downarrow$ 3042

$$-\frac{a^2\sqrt{a^2x^2+1}\int \arctan(ax)^3\csc(\arctan(ax))d\arctan(ax)}{2\sqrt{a^2cx^2+c}}+$$
$$\frac{3}{2}a\left(-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}+\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{P}\right.}{\sqrt{a^2cx^2+c}}\right.$$
$$\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{2cx^2}$$

$\downarrow$ 4671

$$-\frac{a^2\sqrt{a^2x^2+1}\left(-3\int \arctan(ax)^2\log\left(1-e^{i\arctan(ax)}\right)d\arctan(ax)+3\int \arctan(ax)^2\log\left(1+e^{i\arctan(ax)}\right)d\,\text{arct}\right.}{2\sqrt{a^2cx^2+c}}$$
$$\frac{3}{2}a\left(-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}+\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{P}\right.}{\sqrt{a^2cx^2+c}}\right.$$
$$\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{2cx^2}$$

$\downarrow$ 3011

$$-\frac{a^2\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\,\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\int \arctan(ax)\,\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)d\arctan(a\right.\right.}{}$$
$$\frac{3}{2}a\left(-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}+\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\text{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\,\text{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\,\text{P}\right.}{\sqrt{a^2cx^2+c}}\right.$$
$$\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{2cx^2}$$

$\downarrow$ 7163

$$-\frac{a^2\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\left(i\int\operatorname{PolyLog}\left(3,-e^{i\arctan(ax)}\right)d\arctan(ax)-i\operatorname{arc}\right.\right.\right.}{}$$

$$\frac{3}{2}a\left(-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}+\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\operatorname{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\operatorname{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\operatorname{F}\right.}{\sqrt{a^2cx^2+c}}\right.$$

$$\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{2cx^2}$$

$$\Big\downarrow \text{2720}$$

$$-\frac{a^2\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\left(\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(3,-e^{i\arctan(ax)}\right)de^{i\arctan}\right.\right.\right.}{}$$

$$\frac{3}{2}a\left(-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}+\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\operatorname{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\operatorname{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\operatorname{F}\right.}{\sqrt{a^2cx^2+c}}\right.$$

$$\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{2cx^2}$$

$$\Big\downarrow \text{7143}$$

$$\frac{3}{2}a\left(-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{cx}+\frac{2a\sqrt{a^2x^2+1}\left(-2\arctan(ax)\operatorname{arctanh}\left(\frac{\sqrt{1+iax}}{\sqrt{1-iax}}\right)+i\operatorname{PolyLog}\left(2,-\frac{\sqrt{iax+1}}{\sqrt{1-iax}}\right)-i\operatorname{F}\right.}{\sqrt{a^2cx^2+c}}\right.$$

$$\frac{a^2\sqrt{a^2x^2+1}\left(-2\arctan(ax)^3\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)+3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\left(\operatorname{PolyLog}\left(4,\right.\right.\right.\right.}{}$$

$$\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{2cx^2}$$

input

```
Int[ArcTan[a*x]^3/(x^3*Sqrt[c + a^2*c*x^2]),x]
```

output
```
-1/2*(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/(c*x^2) + (3*a*(-((Sqrt[c + a^2*c
*x^2]*ArcTan[a*x]^2)/(c*x)) + (2*a*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]*ArcTa
nh[Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]] + I*PolyLog[2, -(Sqrt[1 + I*a*x]/Sqrt[
1 - I*a*x])] - I*PolyLog[2, Sqrt[1 + I*a*x]/Sqrt[1 - I*a*x]]))/Sqrt[c + a^
2*c*x^2]))/2 - (a^2*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]^3*ArcTanh[E^(I*ArcTa
n[a*x])] + 3*(I*ArcTan[a*x]^2*PolyLog[2, -E^(I*ArcTan[a*x])] - (2*I)*((-I)
*ArcTan[a*x]*PolyLog[3, -E^(I*ArcTan[a*x])] + PolyLog[4, -E^(I*ArcTan[a*x]
)])) - 3*(I*ArcTan[a*x]^2*PolyLog[2, E^(I*ArcTan[a*x])] - (2*I)*((-I)*ArcT
an[a*x]*PolyLog[3, E^(I*ArcTan[a*x])] + PolyLog[4, E^(I*ArcTan[a*x])]))))/
(2*Sqrt[c + a^2*c*x^2])
```

## Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5489
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]), x_
Symbol] :> Simp[(-2/Sqrt[d])*(a + b*ArcTan[c*x])*ArcTanh[Sqrt[1 + I*c*x]/Sq
rt[1 - I*c*x]], x] + (Simp[I*(b/Sqrt[d])*PolyLog[2, -Sqrt[1 + I*c*x]/Sqrt[1
 - I*c*x]], x] - Simp[I*(b/Sqrt[d])*PolyLog[2, Sqrt[1 + I*c*x]/Sqrt[1 - I*c
*x]], x]) /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[d, 0]
```

rule 5491
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Simp[1/Sqrt[d]   Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTa
n[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] &&
GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5497
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)^(m
 + 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[c^2*((m
+ 2)/(f^2*(m + 1)))   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x
^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
 && LtQ[m, -1] && NeQ[m, -2]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

## Maple [A] (verified)

Time = 4.69 (sec) , antiderivative size = 410, normalized size of antiderivative = 0.69

| method | result |
|---|---|
| default | $-\frac{(3ax+\arctan(ax))\arctan(ax)^2\sqrt{c(ax-i)(ax+i)}}{2c\,x^2}-\frac{i a^2\left(i\arctan(ax)^3\ln\left(1+\frac{iax+1}{\sqrt{a^2x^2+1}}\right)-i\arctan(ax)^3\ln\left(1-\frac{iax+1}{\sqrt{a^2x^2+1}}\right)+3\right.}{}$ |

input
```
int(arctan(a*x)^3/x^3/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(3*a*x+arctan(a*x))*arctan(a*x)^2*(c*(a*x-I)*(a*x+I))^(1/2)/c/x^2-1/2
*I*a^2*(I*arctan(a*x)^3*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*arctan(a*x)^3*
ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))+3*arctan(a*x)^2*polylog(2,-(1+I*a*x)/(a^
2*x^2+1)^(1/2))-3*arctan(a*x)^2*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I
*arctan(a*x)*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*arctan(a*x)*polylog(3,-
(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*arctan(a*x)*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1
/2))-6*I*arctan(a*x)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*polylog(2,-(
1+I*a*x)/(a^2*x^2+1)^(1/2))-6*polylog(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*po
lylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*polylog(4,(1+I*a*x)/(a^2*x^2+1)^(1/
2)))*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/2)/c
```

## Fricas [F]

$$\int \frac{\arctan(ax)^3}{x^3\sqrt{c+a^2cx^2}}\,dx=\int \frac{\arctan{(ax)}^3}{\sqrt{a^2cx^2+cx^3}}\,dx$$

input
```
integrate(arctan(a*x)^3/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3/(a^2*c*x^5 + c*x^3), x)
```

## Sympy [F]

$$\int \frac{\arctan(ax)^3}{x^3\sqrt{c+a^2cx^2}}\,dx = \int \frac{\operatorname{atan}^3(ax)}{x^3\sqrt{c(a^2x^2+1)}}\,dx$$

input
```
integrate(atan(a*x)**3/x**3/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(atan(a*x)**3/(x**3*sqrt(c*(a**2*x**2 + 1))), x)
```

## Maxima [F]

$$\int \frac{\arctan(ax)^3}{x^3\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan(ax)^3}{\sqrt{a^2cx^2+c}x^3}\,dx$$

input
```
integrate(arctan(a*x)^3/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)^3/(sqrt(a^2*c*x^2 + c)*x^3), x)
```

## Giac [F]

$$\int \frac{\arctan(ax)^3}{x^3\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan(ax)^3}{\sqrt{a^2cx^2+c}x^3}\,dx$$

input
```
integrate(arctan(a*x)^3/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^3/(sqrt(a^2*c*x^2 + c)*x^3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^3}{x^3\sqrt{c+a^2cx^2}}\,dx = \int \frac{\operatorname{atan}(a\,x)^3}{x^3\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(atan(a*x)^3/(x^3*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(atan(a*x)^3/(x^3*(c + a^2*c*x^2)^(1/2)), x)
```

## Reduce [F]

$$\int \frac{\arctan(ax)^3}{x^3\sqrt{c+a^2cx^2}}\,dx = \frac{\int \frac{atan(ax)^3}{\sqrt{a^2x^2+1}\,x^3}dx}{\sqrt{c}}$$

input
```
int(atan(a*x)^3/x^3/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(atan(a*x)**3/(sqrt(a**2*x**2 + 1)*x**3),x)/sqrt(c)
```

# 3.442 $\int \frac{\arctan(ax)^3}{x^4\sqrt{c+a^2cx^2}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 396

$$\begin{aligned}\int \frac{\arctan(ax)^3}{x^4\sqrt{c+a^2cx^2}}\,dx = &-\frac{a^2\sqrt{c+a^2cx^2}\arctan(ax)}{cx}-\frac{a\sqrt{c+a^2cx^2}\arctan(ax)^2}{2cx^2}\\&-\frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{3cx^3}+\frac{2a^2\sqrt{c+a^2cx^2}\arctan(ax)^3}{3cx}\\&+\frac{5a^3\sqrt{1+a^2x^2}\arctan(ax)^2\text{arctanh}\left(e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}\\&-\frac{a^3\text{arctanh}\left(\frac{\sqrt{c+a^2cx^2}}{\sqrt{c}}\right)}{\sqrt{c}}\\&-\frac{5ia^3\sqrt{1+a^2x^2}\arctan(ax)\,\text{PolyLog}\left(2,-e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}\\&+\frac{5ia^3\sqrt{1+a^2x^2}\arctan(ax)\,\text{PolyLog}\left(2,e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}\\&+\frac{5a^3\sqrt{1+a^2x^2}\,\text{PolyLog}\left(3,-e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}\\&-\frac{5a^3\sqrt{1+a^2x^2}\,\text{PolyLog}\left(3,e^{i\arctan(ax)}\right)}{\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
-a^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)/c/x-1/2*a*(a^2*c*x^2+c)^(1/2)*arctan(
a*x)^2/c/x^2-1/3*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^3/c/x^3+2/3*a^2*(a^2*c*x^
2+c)^(1/2)*arctan(a*x)^3/c/x+5*a^3*(a^2*x^2+1)^(1/2)*arctan(a*x)^2*arctanh
((1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)-a^3*arctanh((a^2*c*x^2+c
)^(1/2)/c^(1/2))/c^(1/2)-5*I*a^3*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(2,-
(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)+5*I*a^3*(a^2*x^2+1)^(1/2)
*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c)^(1/2)+5*
a^3*(a^2*x^2+1)^(1/2)*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^2*c*x^2+c
)^(1/2)-5*a^3*(a^2*x^2+1)^(1/2)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))/(a^
2*c*x^2+c)^(1/2)
```

## Mathematica [A] (warning: unable to verify)

Time = 3.99 (sec) , antiderivative size = 343, normalized size of antiderivative = 0.87

$$\int \frac{\arctan(ax)^3}{x^4\sqrt{c+a^2cx^2}}\,dx = \frac{a^3\sqrt{c(1+a^2x^2)}\left(-12\arctan(ax)\cot\left(\frac{1}{2}\arctan(ax)\right)+10\arctan(ax)^3\cot\left(\frac{1}{2}\arctan(ax)\right)-3\arctan(ax\right.}{}$$

input
```
Integrate[ArcTan[a*x]^3/(x^4*Sqrt[c + a^2*c*x^2]),x]
```

output
```
(a^3*Sqrt[c*(1 + a^2*x^2)]*(-12*ArcTan[a*x]*Cot[ArcTan[a*x]/2] + 10*ArcTan
[a*x]^3*Cot[ArcTan[a*x]/2] - 3*ArcTan[a*x]^2*Csc[ArcTan[a*x]/2]^2 - (a*x*A
rcTan[a*x]^3*Csc[ArcTan[a*x]/2]^4)/(2*Sqrt[1 + a^2*x^2]) - 60*ArcTan[a*x]^
2*Log[1 - E^(I*ArcTan[a*x])] + 60*ArcTan[a*x]^2*Log[1 + E^(I*ArcTan[a*x])]
 + 24*Log[Tan[ArcTan[a*x]/2]] - (120*I)*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTa
n[a*x])] + (120*I)*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])] + 120*PolyLog
[3, -E^(I*ArcTan[a*x])] - 120*PolyLog[3, E^(I*ArcTan[a*x])] + 3*ArcTan[a*x
]^2*Sec[ArcTan[a*x]/2]^2 - (8*(1 + a^2*x^2)^(3/2)*ArcTan[a*x]^3*Sin[ArcTan
[a*x]/2]^4)/(a^3*x^3) - 12*ArcTan[a*x]*Tan[ArcTan[a*x]/2] + 10*ArcTan[a*x]
^3*Tan[ArcTan[a*x]/2]))/(24*c*Sqrt[1 + a^2*x^2])
```

## Rubi [A] (verified)

Time = 3.76 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.06, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {5497, 5479, 5493, 5491, 3042, 4671, 3011, 2720, 5497, 5479, 243, 73, 221, 5493, 5491, 3042, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3}{x^4\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5497}$$

$$-\frac{2}{3}a^2\int \frac{\arctan(ax)^3}{x^2\sqrt{a^2cx^2+c}}dx+a\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{3cx^3}$$

$$\Big\downarrow \text{5479}$$

$$-\frac{2}{3}a^2\left(3a\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}\right)+a\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{3cx^3}$$

$$\Big\downarrow \text{5493}$$

$$-\frac{2}{3}a^2\left(\frac{3a\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}\right)+a\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{3cx^3}$$

$$\Big\downarrow \text{5491}$$

$$-\frac{2}{3}a^2\left(\frac{3a\sqrt{a^2x^2+1}\int \frac{\sqrt{a^2x^2+1}\arctan(ax)^2}{ax}d\arctan(ax)}{\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}\right)+a\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{3cx^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2}{3}a^2\left(\frac{3a\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc(\arctan(ax))d\arctan(ax)}{\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}\right)+a\int \frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{3cx^3}$$

$\Big\downarrow$ 4671

$$-\frac{2}{3}a^2\left(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(-2\int \arctan(ax)\log\left(1-e^{i\arctan(ax)}\right)d\arctan(ax)+2\int \arcta\right.}{\sqrt{a^2}}\right.$$

$$a\int\frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{3cx^3}$$

$\Big\downarrow$ 3011

$$-\frac{2}{3}a^2\left(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-i\int \operatorname{PolyLog}\left(2,-e\right.\right.\right.}{}\right.$$

$$a\int\frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{3cx^3}$$

$\Big\downarrow$ 2720

$$-\frac{2}{3}a^2\left(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{Pol}\right.\right.}{}\right.$$

$$a\int\frac{\arctan(ax)^2}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{3cx^3}$$

$\Big\downarrow$ 5497

$$-\frac{2}{3}a^2\left(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{Pol}\right.\right.}{}\right.$$

$$a\left(-\frac{1}{2}a^2\int\frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+a\int\frac{\arctan(ax)}{x^2\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{2cx^2}\right)-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{3cx^3}$$

$\Big\downarrow$ 5479

$$-\frac{2}{3}a^2\left(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{Pol}\right.\right.}{}\right.$$

$$a\left(-\frac{1}{2}a^2\int\frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+a\left(a\int\frac{1}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right)-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{2cx^2}\right)-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{3cx^3}$$

$\Big\downarrow$ 243

$$-\frac{2}{3}a^2\left(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{Pol}\right.\right.}{}\right.$$
$$a\left(-\frac{1}{2}a^2\int\frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+a\left(\frac{1}{2}a\int\frac{1}{x^2\sqrt{a^2cx^2+c}}dx^2-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right)-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{2cx^2}\right.$$
$$\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{3cx^3}$$

$\downarrow$ 73

$$-\frac{2}{3}a^2\left(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{Pol}\right.\right.}{}\right.$$
$$a\left(-\frac{1}{2}a^2\int\frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+a\left(\frac{\int\frac{1}{\frac{x^4}{a^2c}-\frac{1}{a^2}}d\sqrt{a^2cx^2+c}}{ac}-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}\right)-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{2cx^2}\right)$$
$$\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{3cx^3}$$

$\downarrow$ 221

$$-\frac{2}{3}a^2\left(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{Pol}\right.\right.}{}\right.$$
$$a\left(-\frac{1}{2}a^2\int\frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx+a\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\operatorname{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{\arctan(ax)^2\sqrt{a^2cx^2+c}}{2cx^2}\right.$$
$$\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{3cx^3}$$

$\downarrow$ 5493

$$-\frac{2}{3}a^2\left(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{Pol}\right.\right.}{}\right.$$
$$a\left(-\frac{a^2\sqrt{a^2x^2+1}\int\frac{\arctan(ax)^2}{x\sqrt{a^2x^2+1}}dx}{2\sqrt{a^2cx^2+c}}+a\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\operatorname{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{\arctan(ax)^2\sqrt{a^2cx}}{2cx^2}\right.$$
$$\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{3cx^3}$$

$\downarrow$ 5491

$$-\frac{2}{3}a^2\left(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{Pol}\right.\right.}{}\right.$$

$$a\left(-\frac{a^2\sqrt{a^2x^2+1}\int\frac{\sqrt{a^2x^2+1}\arctan(ax)^2}{ax}d\arctan(ax)}{2\sqrt{a^2cx^2+c}}+a\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\operatorname{arctanh}\left(\frac{\sqrt{a^2cx^2+c}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\right.$$

$$\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{3cx^3}$$

$\downarrow$ 3042

$$-\frac{2}{3}a^2\left(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{Pol}\right.\right.}{}\right.$$

$$a\left(-\frac{a^2\sqrt{a^2x^2+1}\int\arctan(ax)^2\csc(\arctan(ax))d\arctan(ax)}{2\sqrt{a^2cx^2+c}}+a\left(-\frac{\arctan(ax)\sqrt{a^2cx^2+c}}{cx}-\frac{a\operatorname{arctanh}\left(\frac{\sqrt{a^2cx^2}}{\sqrt{c}}\right.}{\sqrt{c}}\right.\right.$$

$$\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{3cx^3}$$

$\downarrow$ 4671

$$-\frac{2}{3}a^2\left(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{Pol}\right.\right.}{}\right.$$

$$a\left(-\frac{a^2\sqrt{a^2x^2+1}\left(-2\int\arctan(ax)\log\left(1-e^{i\arctan(ax)}\right)d\arctan(ax)+2\int\arctan(ax)\log\left(1+e^{i\arctan(ax)}\right)d\operatorname{arc}\right.}{2\sqrt{a^2cx^2+c}}\right.$$

$$\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{3cx^3}$$

$\downarrow$ 3011

$$a\left(-\frac{a^2\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-i\int\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)d\arctan(ax)\right)-2\left(i\right.\right.}{2}\right.$$

$$\frac{2}{3}a^2\left(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyI}\right.\right.}{}\right.$$

$$\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{3cx^3}$$

$\downarrow$ 2720

$$-\frac{2}{3}a^2\left(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{Pol}\right.\right.}{}\right.$$

$$a\left(-\frac{a^2\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(}\right.\right.}{}\right.$$

$$\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{3cx^3}$$

$\big\downarrow$ 7143

$$-\frac{2}{3}a^2\left(-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(-2\arctan(ax)^2\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)+2\left(i\arctan(ax)\operatorname{PolyLog}\right.\right.}{}\right.$$

$$a\left(-\frac{a^2\sqrt{a^2x^2+1}\left(-2\arctan(ax)^2\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)+2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\operatorname{PolyLog}\left(3\right.\right.\right.}{2\sqrt{a^2cx^2+c}}\right.$$

$$\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{3cx^3}$$

input
```
Int[ArcTan[a*x]^3/(x^4*Sqrt[c + a^2*c*x^2]),x]
```

output
```
-1/3*(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/(c*x^3) - (2*a^2*(-((Sqrt[c + a^2
*c*x^2]*ArcTan[a*x]^3)/(c*x)) + (3*a*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]^2*A
rcTanh[E^(I*ArcTan[a*x])] + 2*(I*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])
] - PolyLog[3, -E^(I*ArcTan[a*x])]) - 2*(I*ArcTan[a*x]*PolyLog[2, E^(I*Arc
Tan[a*x])] - PolyLog[3, E^(I*ArcTan[a*x])])))/Sqrt[c + a^2*c*x^2]))/3 + a*
(-1/2*(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)/(c*x^2) + a*(-((Sqrt[c + a^2*c*x
^2]*ArcTan[a*x])/(c*x)) - (a*ArcTanh[Sqrt[c + a^2*c*x^2]/Sqrt[c]])/Sqrt[c]
) - (a^2*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])] +
2*(I*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])] - PolyLog[3, -E^(I*ArcTan[
a*x])]) - 2*(I*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])] - PolyLog[3, E^(I
*ArcTan[a*x])])))/(2*Sqrt[c + a^2*c*x^2]))
```

### Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5491
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Simp[1/Sqrt[d]   Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTa
n[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] &&
GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5497
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(f*x)^(m + 1)*Sqrt[d + e*x^2]*((a + b*Ar
cTan[c*x])^p/(d*f*(m + 1))), x] + (-Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)^(m
 + 1)*((a + b*ArcTan[c*x])^(p - 1)/Sqrt[d + e*x^2]), x], x] - Simp[c^2*((m
+ 2)/(f^2*(m + 1)))   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/Sqrt[d + e*x
^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
 && LtQ[m, -1] && NeQ[m, -2]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

## Maple [A] (verified)

Time = 4.85 (sec) , antiderivative size = 324, normalized size of antiderivative = 0.82

| method | result |
|---|---|
| default | $\frac{\left(4\arctan(ax)^2x^2a^2-6a^2x^2-3\arctan(ax)ax-2\arctan(ax)^2\right)\arctan(ax)\sqrt{c(ax-i)(ax+i)}}{6c\,x^3}-\frac{2a^3\,\mathrm{arctanh}\left(\frac{iax+1}{\sqrt{a^2x^2+1}}\right)\sqrt{c(ax-i}}{\sqrt{a^2x^2+1}\,c}$ |

input
```
int(arctan(a*x)^3/x^4/(a^2*c*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(4*arctan(a*x)^2*x^2*a^2-6*a^2*x^2-3*arctan(a*x)*a*x-2*arctan(a*x)^2)*
arctan(a*x)*(c*(a*x-I)*(a*x+I))^(1/2)/c/x^3-2*a^3*arctanh((1+I*a*x)/(a^2*x
^2+1)^(1/2))*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/2)/c+5/2*a^3*(arctan
(a*x)^2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*I*arctan(a*x)*polylog(2,-(1+I*
a*x)/(a^2*x^2+1)^(1/2))-arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*
I*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*polylog(3,-(1+I*a*x
)/(a^2*x^2+1)^(1/2))-2*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2)))*(c*(a*x-I)*
(a*x+I))^(1/2)/(a^2*x^2+1)^(1/2)/c
```

## Fricas [F]

$$\int \frac{\arctan(ax)^3}{x^4\sqrt{c+a^2cx^2}}\,dx=\int \frac{\arctan\left(ax\right)^3}{\sqrt{a^2cx^2+cx^4}}\,dx$$

input
```
integrate(arctan(a*x)^3/x^4/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3/(a^2*c*x^6 + c*x^4), x)
```

## Sympy [F]

$$\int \frac{\arctan(ax)^3}{x^4\sqrt{c+a^2cx^2}}\,dx = \int \frac{\operatorname{atan}^3(ax)}{x^4\sqrt{c\,(a^2x^2+1)}}\,dx$$

input
```
integrate(atan(a*x)**3/x**4/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(atan(a*x)**3/(x**4*sqrt(c*(a**2*x**2 + 1))), x)
```

## Maxima [F]

$$\int \frac{\arctan(ax)^3}{x^4\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan(ax)^3}{\sqrt{a^2cx^2+c}\,x^4}\,dx$$

input
```
integrate(arctan(a*x)^3/x^4/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)^3/(sqrt(a^2*c*x^2 + c)*x^4), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^3}{x^4\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^3/x^4/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^3}{x^4\sqrt{c+a^2cx^2}}\,dx = \int \frac{\mathrm{atan}(a\,x)^3}{x^4\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(atan(a*x)^3/(x^4*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(atan(a*x)^3/(x^4*(c + a^2*c*x^2)^(1/2)), x)
```

## Reduce [F]

$$\int \frac{\arctan(ax)^3}{x^4\sqrt{c+a^2cx^2}}\,dx = \frac{\int \frac{atan(ax)^3}{\sqrt{a^2x^2+1}\,x^4}dx}{\sqrt{c}}$$

input
```
int(atan(a*x)^3/x^4/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(atan(a*x)**3/(sqrt(a**2*x**2 + 1)*x**4),x)/sqrt(c)
```

# 3.443 $\int \frac{x^3 \arctan(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 403

$$
\begin{aligned}
\int \frac{x^3 \arctan(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx &= \frac{6x}{a^3c\sqrt{c+a^2cx^2}}-\frac{6\arctan(ax)}{a^4c\sqrt{c+a^2cx^2}}-\frac{3x\arctan(ax)^2}{a^3c\sqrt{c+a^2cx^2}} \\
&+\frac{6i\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2}{a^4c\sqrt{c+a^2cx^2}}+\frac{\arctan(ax)^3}{a^4c\sqrt{c+a^2cx^2}} \\
&+\frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{a^4c^2}-\frac{6i\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)}{a^4c\sqrt{c+a^2cx^2}} \\
&+\frac{6i\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,ie^{i\arctan(ax)}\right)}{a^4c\sqrt{c+a^2cx^2}} \\
&+\frac{6\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)}{a^4c\sqrt{c+a^2cx^2}} \\
&-\frac{6\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,ie^{i\arctan(ax)}\right)}{a^4c\sqrt{c+a^2cx^2}}
\end{aligned}
$$

output
```
6*x/a^3/c/(a^2*c*x^2+c)^(1/2)-6*arctan(a*x)/a^4/c/(a^2*c*x^2+c)^(1/2)-3*x*
arctan(a*x)^2/a^3/c/(a^2*c*x^2+c)^(1/2)+6*I*(a^2*x^2+1)^(1/2)*arctan((1+I*
a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^2/a^4/c/(a^2*c*x^2+c)^(1/2)+arctan(a*x
)^3/a^4/c/(a^2*c*x^2+c)^(1/2)+(a^2*c*x^2+c)^(1/2)*arctan(a*x)^3/a^4/c^2-6*
I*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/
a^4/c/(a^2*c*x^2+c)^(1/2)+6*I*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(2,I*(1
+I*a*x)/(a^2*x^2+1)^(1/2))/a^4/c/(a^2*c*x^2+c)^(1/2)+6*(a^2*x^2+1)^(1/2)*p
olylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a^4/c/(a^2*c*x^2+c)^(1/2)-6*(a^2*
x^2+1)^(1/2)*polylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a^4/c/(a^2*c*x^2+c)^
(1/2)
```

## Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 308, normalized size of antiderivative = 0.76

$$\int \frac{x^3 \arctan(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{1+a^2x^2}\left(\frac{6ax}{\sqrt{1+a^2x^2}} - 3\sqrt{1+a^2x^2}\arctan(ax) - \frac{3ax\arctan(ax)^2}{\sqrt{1+a^2x^2}} + \frac{3}{2}\sqrt{1+a^2x^2}\arctan\right.}{}$$

input
```
Integrate[(x^3*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(3/2),x]
```

output
```
(Sqrt[1 + a^2*x^2]*((6*a*x)/Sqrt[1 + a^2*x^2] - 3*Sqrt[1 + a^2*x^2]*ArcTan
[a*x] - (3*a*x*ArcTan[a*x]^2)/Sqrt[1 + a^2*x^2] + (3*Sqrt[1 + a^2*x^2]*Arc
Tan[a*x]^3)/2 - 3*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Cos[2*ArcTan[a*x]] + (Sqrt
[1 + a^2*x^2]*ArcTan[a*x]^3*Cos[2*ArcTan[a*x]])/2 - 3*ArcTan[a*x]^2*Log[1
- I*E^(I*ArcTan[a*x])] + 3*ArcTan[a*x]^2*Log[1 + I*E^(I*ArcTan[a*x])] - (6
*I)*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + (6*I)*ArcTan[a*x]*Pol
yLog[2, I*E^(I*ArcTan[a*x])] + 6*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] - 6*Po
lyLog[3, I*E^(I*ArcTan[a*x])]))/(a^4*c*Sqrt[c*(1 + a^2*x^2)])
```

## Rubi [A] (verified)

Time = 1.64 (sec) , antiderivative size = 291, normalized size of antiderivative = 0.72, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {5499, 5465, 5425, 5423, 3042, 4669, 3011, 2720, 5433, 208, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \text{5499}$$

$$\frac{\int \frac{x\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{a^2c}-\frac{\int \frac{x\arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx}{a^2}$$

$$\Big\downarrow \text{5465}$$

$$\frac{\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\int \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a}}{a^2c}-\frac{\frac{3\int \frac{\arctan(ax)^2}{\left(a^2cx^2+c\right)^{3/2}}dx}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}}{a^2}$$

$$\Big\downarrow \text{5425}$$

$$\frac{\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{a\sqrt{a^2cx^2+c}}}{a^2c}-\frac{\frac{3\int \frac{\arctan(ax)^2}{\left(a^2cx^2+c\right)^{3/2}}dx}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}}{a^2}$$

$$\Big\downarrow \text{5423}$$

$$\frac{\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\int \sqrt{a^2x^2+1}\arctan(ax)^2d\arctan(ax)}{a^2\sqrt{a^2cx^2+c}}}{a^2c}-\frac{\frac{3\int \frac{\arctan(ax)^2}{\left(a^2cx^2+c\right)^{3/2}}dx}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{a^2\sqrt{a^2cx^2+c}}}{a^2c}-\frac{\frac{3\int \frac{\arctan(ax)^2}{\left(a^2cx^2+c\right)^{3/2}}dx}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}}{a^2}$$

$$\Big\downarrow \text{4669}$$

$$-\frac{\frac{3\int \frac{\arctan(ax)^2}{\left(a^2cx^2+c\right)^{3/2}}dx}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}}{a^2}+\frac{\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(-2\int \arctan(ax)\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+2\int \arctan(ax)\log\left(1+ie^{i\arctan(ax)}\right)d\arctan(ax)-2i\right.}{a^2\sqrt{a^2cx^2+c}}}{a^2c}$$

$\downarrow$ 3011

$$-\frac{\frac{3\int \frac{\arctan(ax)^2}{\left(a^2cx^2+c\right)^{3/2}}dx}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}}{a^2}+\frac{\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-i\int \operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(ax)\right)-2\left(i\arctan(ax)\right.\right.}{a^2\sqrt{a^2cx^2+}}}{a^2c}$$

$\downarrow$ 2720

$$-\frac{\frac{3\int \frac{\arctan(ax)^2}{\left(a^2cx^2+c\right)^{3/2}}dx}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}}{a^2}+\frac{\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)-2\left(\right.\right.}{a^2c}$$

$\downarrow$ 5433

$$-\frac{\frac{3\left(-2\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}}dx+\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}}{a^2}+\frac{\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)-2\left(\right.\right.}{a^2c}$$

$\downarrow$ 208

$$-\frac{\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}}{a^2}+\frac{\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)-2\left(\right.\right.}{a^2c}$$

$\downarrow$ 7143

$$-\frac{\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}}{a^2}+\frac{\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)\right)-2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,ie^{i\,\mathrm{ar}}\right.\right.\right.}{a^2\sqrt{a^2cx^2+c}}}{a^2c}$$

input
```
Int[(x^3*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(3/2),x]
```

output
```
-((-(ArcTan[a*x]^3/(a^2*c*Sqrt[c + a^2*c*x^2])) + (3*((-2*x)/(c*Sqrt[c + a
^2*c*x^2]) + (2*ArcTan[a*x])/(a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x]^2)
/(c*Sqrt[c + a^2*c*x^2])))/a)/a^2) + ((Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/
(a^2*c) - (3*Sqrt[1 + a^2*x^2]*((-2*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*
x]^2 + 2*(I*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - PolyLog[3, (-
I)*E^(I*ArcTan[a*x])]) - 2*(I*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])]
- PolyLog[3, I*E^(I*ArcTan[a*x])])))/(a^2*Sqrt[c + a^2*c*x^2]))/(a^2*c)
```

## Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5433
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x] - Simp[b^2*p*(p
- 1)   Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2), x], x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5499
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ
[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

## Maple [F]

$$\int \frac{x^3 \arctan{(ax)}^3}{(a^2 c\,x^2 + c)^{\frac{3}{2}}} dx$$

input
```
int(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x)
```

output
```
int(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x)
```

## Fricas [F]

$$\int \frac{x^3 \arctan(ax)^3}{(c + a^2cx^2)^{3/2}} \, dx = \int \frac{x^3 \arctan{(ax)}^3}{(a^2cx^2 + c)^{\frac{3}{2}}} \, dx$$

input
```
integrate(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^3*arctan(a*x)^3/(a^4*c^2*x^4 + 2*a^2*c^2*x^
2 + c^2), x)
```

## Sympy [F]

$$\int \frac{x^3 \arctan(ax)^3}{(c + a^2cx^2)^{3/2}} \, dx = \int \frac{x^3 \operatorname{atan}^3{(ax)}}{(c\,(a^2x^2 + 1))^{\frac{3}{2}}} \, dx$$

input
```
integrate(x**3*atan(a*x)**3/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(x**3*atan(a*x)**3/(c*(a**2*x**2 + 1))**(3/2), x)
```

## Maxima [F]

$$\int \frac{x^3 \arctan(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^3 \arctan{(ax)}^3}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^3*arctan(a*x)^3/(a^2*c*x^2 + c)^(3/2), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x^3 \arctan(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \arctan(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^3\,\mathrm{atan}(a\,x)^3}{{(c\,a^2\,x^2+c)}^{3/2}}\,dx$$

input
```
int((x^3*atan(a*x)^3)/(c + a^2*c*x^2)^(3/2),x)
```

output
```
int((x^3*atan(a*x)^3)/(c + a^2*c*x^2)^(3/2), x)
```

## Reduce [F]

$$\int \frac{x^3 \arctan(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\int \frac{atan(ax)^3x^3}{\sqrt{a^2x^2+1}\,a^2x^2+\sqrt{a^2x^2+1}}dx}{\sqrt{c}\,c}$$

input
```
int(x^3*atan(a*x)^3/(a^2*c*x^2+c)^(3/2),x)
```

output
```
int((atan(a*x)**3*x**3)/(sqrt(a**2*x**2 + 1)*a**2*x**2 + sqrt(a**2*x**2 +
1)),x)/(sqrt(c)*c)
```

# 3.444 $\int \frac{x^2 \arctan(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 495

$$\begin{aligned}
\int \frac{x^2 \arctan(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx &= \frac{6}{a^3c\sqrt{c+a^2cx^2}}+\frac{6x\arctan(ax)}{a^2c\sqrt{c+a^2cx^2}}-\frac{3\arctan(ax)^2}{a^3c\sqrt{c+a^2cx^2}} \\
&-\frac{x\arctan(ax)^3}{a^2c\sqrt{c+a^2cx^2}}-\frac{2i\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^3}{a^3c\sqrt{c+a^2cx^2}} \\
&+\frac{3i\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)}{a^3c\sqrt{c+a^2cx^2}} \\
&-\frac{3i\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{PolyLog}\left(2,ie^{i\arctan(ax)}\right)}{a^3c\sqrt{c+a^2cx^2}} \\
&-\frac{6\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)}{a^3c\sqrt{c+a^2cx^2}} \\
&+\frac{6\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(3,ie^{i\arctan(ax)}\right)}{a^3c\sqrt{c+a^2cx^2}} \\
&-\frac{6i\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(4,-ie^{i\arctan(ax)}\right)}{a^3c\sqrt{c+a^2cx^2}} \\
&+\frac{6i\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(4,ie^{i\arctan(ax)}\right)}{a^3c\sqrt{c+a^2cx^2}}
\end{aligned}$$

output
```
6/a^3/c/(a^2*c*x^2+c)^(1/2)+6*x*arctan(a*x)/a^2/c/(a^2*c*x^2+c)^(1/2)-3*ar
ctan(a*x)^2/a^3/c/(a^2*c*x^2+c)^(1/2)-x*arctan(a*x)^3/a^2/c/(a^2*c*x^2+c)^
(1/2)-2*I*(a^2*x^2+1)^(1/2)*arctan((1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x
)^3/a^3/c/(a^2*c*x^2+c)^(1/2)+3*I*(a^2*x^2+1)^(1/2)*arctan(a*x)^2*polylog(
2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a^3/c/(a^2*c*x^2+c)^(1/2)-3*I*(a^2*x^2+1
)^(1/2)*arctan(a*x)^2*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a^3/c/(a^2*
c*x^2+c)^(1/2)-6*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(3,-I*(1+I*a*x)/(a^2
*x^2+1)^(1/2))/a^3/c/(a^2*c*x^2+c)^(1/2)+6*(a^2*x^2+1)^(1/2)*arctan(a*x)*p
olylog(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a^3/c/(a^2*c*x^2+c)^(1/2)-6*I*(a^2
*x^2+1)^(1/2)*polylog(4,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a^3/c/(a^2*c*x^2+c
)^(1/2)+6*I*(a^2*x^2+1)^(1/2)*polylog(4,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a^3
/c/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 639, normalized size of antiderivative = 1.29

$$\int \frac{x^2 \arctan(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx =$$

$$-\frac{\sqrt{1+a^2x^2}\left(7i\pi^4-\frac{384}{\sqrt{1+a^2x^2}}+8i\pi^3\arctan(ax)-\frac{384ax\arctan(ax)}{\sqrt{1+a^2x^2}}-24i\pi^2\arctan(ax)^2+\frac{192\arctan(ax)^2}{\sqrt{1+a^2x^2}}+32i\right.}{}$$

input
```
Integrate[(x^2*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(3/2),x]
```

output
```
-1/64*(Sqrt[1 + a^2*x^2]*((7*I)*Pi^4 - 384/Sqrt[1 + a^2*x^2] + (8*I)*Pi^3*
ArcTan[a*x] - (384*a*x*ArcTan[a*x])/Sqrt[1 + a^2*x^2] - (24*I)*Pi^2*ArcTan
[a*x]^2 + (192*ArcTan[a*x]^2)/Sqrt[1 + a^2*x^2] + (32*I)*Pi*ArcTan[a*x]^3
+ (64*a*x*ArcTan[a*x]^3)/Sqrt[1 + a^2*x^2] - (16*I)*ArcTan[a*x]^4 - 48*Pi^
2*ArcTan[a*x]*Log[1 - I/E^(I*ArcTan[a*x])] + 96*Pi*ArcTan[a*x]^2*Log[1 - I
/E^(I*ArcTan[a*x])] + 8*Pi^3*Log[1 + I/E^(I*ArcTan[a*x])] - 64*ArcTan[a*x]
^3*Log[1 + I/E^(I*ArcTan[a*x])] - 8*Pi^3*Log[1 + I*E^(I*ArcTan[a*x])] + 48
*Pi^2*ArcTan[a*x]*Log[1 + I*E^(I*ArcTan[a*x])] - 96*Pi*ArcTan[a*x]^2*Log[1
 + I*E^(I*ArcTan[a*x])] + 64*ArcTan[a*x]^3*Log[1 + I*E^(I*ArcTan[a*x])] -
8*Pi^3*Log[Tan[(Pi + 2*ArcTan[a*x])/4]] - (192*I)*ArcTan[a*x]^2*PolyLog[2,
 (-I)/E^(I*ArcTan[a*x])] - (48*I)*Pi*(Pi - 4*ArcTan[a*x])*PolyLog[2, I/E^(
I*ArcTan[a*x])] - (48*I)*Pi^2*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] + (192*I)
*Pi*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - (192*I)*ArcTan[a*x]^2
*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - 384*ArcTan[a*x]*PolyLog[3, (-I)/E^(I
*ArcTan[a*x])] + 192*Pi*PolyLog[3, I/E^(I*ArcTan[a*x])] - 192*Pi*PolyLog[3
, (-I)*E^(I*ArcTan[a*x])] + 384*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a*
x])] + (384*I)*PolyLog[4, (-I)/E^(I*ArcTan[a*x])] + (384*I)*PolyLog[4, (-I
)*E^(I*ArcTan[a*x])]))/(a^3*c*Sqrt[c*(1 + a^2*x^2)])
```

## Rubi [A] (verified)

Time = 1.59 (sec) , antiderivative size = 311, normalized size of antiderivative = 0.63, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {5499, 5425, 5423, 3042, 4669, 3011, 5433, 5429, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \arctan(ax)^3}{(a^2cx^2+c)^{3/2}} \, dx$$

$$\Big\downarrow \text{5499}$$

$$\frac{\int \frac{\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{a^2c} - \frac{\int \frac{\arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx}{a^2}$$

$$\Big\downarrow \text{5425}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^3}{\sqrt{a^2x^2+1}}dx}{a^2c\sqrt{a^2cx^2+c}} - \frac{\int \frac{\arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx}{a^2}$$

$$\Big\downarrow \text{5423}$$

$$\frac{\sqrt{a^2x^2+1}\int \sqrt{a^2x^2+1}\arctan(ax)^3d\arctan(ax)}{a^3c\sqrt{a^2cx^2+c}}-\frac{\int \frac{\arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a^2x^2+1}\int \arctan(ax)^3\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{a^3c\sqrt{a^2cx^2+c}}-\frac{\int \frac{\arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx}{a^2}$$

$$\Big\downarrow \text{4669}$$

$$-\frac{\int \frac{\arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx}{a^2}+\frac{\sqrt{a^2x^2+1}\left(-3\int \arctan(ax)^2\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+3\int \arctan(ax)^2\log\left(1+ie^{i\arctan(ax)}\right)d\arctan\right.}{a^3c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{\int \frac{\arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx}{a^2}+\frac{\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int \arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(ax)\right.\right.}{}$$

$$\Big\downarrow \text{5433}$$

$$-\frac{-6\int \frac{\arctan(ax)}{(a^2cx^2+c)^{3/2}}dx+\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}}}{a^2}+\frac{\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int \arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(ax)\right.\right.}{}$$

$$\Big\downarrow \text{5429}$$

$$-\frac{\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}}-6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a^2}+\frac{\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int \arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(ax)\right.\right.}{}$$

$$\Big\downarrow \text{7163}$$

$$-\frac{\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}}-6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a^2}+\frac{\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\left(i\int \operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)d\arctan(ax)-i\,\mathrm{arcta}\right.\right.\right.}{}$$

$$\Big\downarrow \text{2720}$$

$$-\frac{\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}}-6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a^2}+$$
$$\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\left(\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right.\right.\right.$$

$\downarrow$ 7143

$$-\frac{\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}}-6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a^2}+$$
$$\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\left(\operatorname{PolyLog}\left(4,-ie^{i\arctan(ax)}\right)-i\arctan(ax)\operatorname{PolyLog}\left(\right.\right.\right.\right.$$

input
```
Int[(x^2*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(3/2),x]
```

output
```
-(((3*ArcTan[a*x]^2)/(a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x]^3)/(c*Sqrt
[c + a^2*c*x^2]) - 6*(1/(a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x])/(c*Sqr
t[c + a^2*c*x^2])))/a^2) + (Sqrt[1 + a^2*x^2]*((-2*I)*ArcTan[E^(I*ArcTan[a
*x])]*ArcTan[a*x]^3 + 3*(I*ArcTan[a*x]^2*PolyLog[2, (-I)*E^(I*ArcTan[a*x])
] - (2*I)*((-I)*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] + PolyLog[4
, (-I)*E^(I*ArcTan[a*x])])) - 3*(I*ArcTan[a*x]^2*PolyLog[2, I*E^(I*ArcTan[
a*x])] - (2*I)*((-I)*ArcTan[a*x]*PolyLog[3, I*E^(I*ArcTan[a*x])] + PolyLog
[4, I*E^(I*ArcTan[a*x])]))))/(a^3*c*Sqrt[c + a^2*c*x^2])
```

## Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5429
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTan[c*x])/(d*Sqr
t[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

rule 5433
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x] - Simp[b^2*p*(p
- 1)   Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2), x], x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

rule 5499
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ
[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

## Maple [F]

$$\int \frac{x^{2} \arctan \left(a x \right)^{3}}{\left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}d x$$

input
```
int(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x)
```

output
```
int(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x)
```

## Fricas [F]

$$\int \frac{x^2 \arctan(ax)^3}{(c+a^2cx^2)^{3/2}}\, dx = \int \frac{x^{2} \arctan \left(a x\right)^{3}}{{\left(a^{2} c x^{2} + c\right)}^{\frac{3}{2}}} \,d x$$

input
```
integrate(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^2*arctan(a*x)^3/(a^4*c^2*x^4 + 2*a^2*c^2*x^
2 + c^2), x)
```

## Sympy [F]

$$\int \frac{x^2 \arctan(ax)^3}{(c+a^2cx^2)^{3/2}} \, dx = \int \frac{x^2 \operatorname{atan}^3{(ax)}}{(c(a^2x^2+1))^{\frac{3}{2}}} \, dx$$

input
```
integrate(x**2*atan(a*x)**3/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(x**2*atan(a*x)**3/(c*(a**2*x**2 + 1))**(3/2), x)
```

## Maxima [F]

$$\int \frac{x^2 \arctan(ax)^3}{(c+a^2cx^2)^{3/2}} \, dx = \int \frac{x^2 \arctan{(ax)}^3}{(a^2cx^2+c)^{\frac{3}{2}}} \, dx$$

input
```
integrate(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^2*arctan(a*x)^3/(a^2*c*x^2 + c)^(3/2), x)
```

## Giac [F]

$$\int \frac{x^2 \arctan(ax)^3}{(c+a^2cx^2)^{3/2}} \, dx = \int \frac{x^2 \arctan{(ax)}^3}{(a^2cx^2+c)^{\frac{3}{2}}} \, dx$$

input
```
integrate(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^2*arctan(a*x)^3/(a^2*c*x^2 + c)^(3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \arctan(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^2\,\mathrm{atan}(a\,x)^3}{(c\,a^2\,x^2+c)^{3/2}}\,dx$$

input
```
int((x^2*atan(a*x)^3)/(c + a^2*c*x^2)^(3/2),x)
```

output
```
int((x^2*atan(a*x)^3)/(c + a^2*c*x^2)^(3/2), x)
```

## Reduce [F]

$$\int \frac{x^2 \arctan(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\int \frac{atan(ax)^3x^2}{\sqrt{a^2x^2+1}\,a^2x^2+\sqrt{a^2x^2+1}}dx}{\sqrt{c}\,c}$$

input
```
int(x^2*atan(a*x)^3/(a^2*c*x^2+c)^(3/2),x)
```

output
```
int((atan(a*x)**3*x**2)/(sqrt(a**2*x**2 + 1)*a**2*x**2 + sqrt(a**2*x**2 +
1)),x)/(sqrt(c)*c)
```

# 3.445 $\int \frac{x\arctan(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 107

$$\int \frac{x\arctan(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx = -\frac{6x}{ac\sqrt{c+a^2cx^2}} + \frac{6\arctan(ax)}{a^2c\sqrt{c+a^2cx^2}} + \frac{3x\arctan(ax)^2}{ac\sqrt{c+a^2cx^2}} - \frac{\arctan(ax)^3}{a^2c\sqrt{c+a^2cx^2}}$$

output
```
-6*x/a/c/(a^2*c*x^2+c)^(1/2)+6*arctan(a*x)/a^2/c/(a^2*c*x^2+c)^(1/2)+3*x*a
rctan(a*x)^2/a/c/(a^2*c*x^2+c)^(1/2)-arctan(a*x)^3/a^2/c/(a^2*c*x^2+c)^(1/
2)
```

## Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.57

$$\int \frac{x\arctan(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{c+a^2cx^2}\left(-6ax+6\arctan(ax)+3ax\arctan(ax)^2-\arctan(ax)^3\right)}{a^2c^2(1+a^2x^2)}$$

input
```
Integrate[(x*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(3/2),x]
```

output
```
(Sqrt[c + a^2*c*x^2]*(-6*a*x + 6*ArcTan[a*x] + 3*a*x*ArcTan[a*x]^2 - ArcTa
n[a*x]^3))/(a^2*c^2*(1 + a^2*x^2))
```

## Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {5465, 5433, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \arctan(ax)^3}{(a^2cx^2+c)^{3/2}} \, dx$$

$$\Big\downarrow \text{ 5465}$$

$$\frac{3\int \frac{\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{a} - \frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 5433}$$

$$\frac{3\left(-2\int \frac{1}{(a^2cx^2+c)^{3/2}}dx + \frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}} + \frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}\right)}{a} - \frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 208}$$

$$\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}} + \frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}} - \frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a} - \frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}$$

input
```
Int[(x*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(3/2),x]
```

output
```
-(ArcTan[a*x]^3/(a^2*c*Sqrt[c + a^2*c*x^2])) + (3*((-2*x)/(c*Sqrt[c + a^2*
c*x^2]) + (2*ArcTan[a*x])/(a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x]^2)/(c
*Sqrt[c + a^2*c*x^2])))/a
```

### Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 5433
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x] - Simp[b^2*p*(p
- 1)   Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2), x], x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.67 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.25

| method | result |
|---|---|
| default | $-\frac{\left(\arctan(ax)^3-6\arctan(ax)+3i\arctan(ax)^2-6i\right)(iax+1)\sqrt{c(ax-i)(ax+i)}}{2(a^2x^2+1)a^2c^2}+\frac{\sqrt{c(ax-i)(ax+i)}\,(iax-1)\left(\arctan(ax)^3-6\,\mathrm{arcta}\right.}{2(a^2x^2+1)a^2c^2}$ |
| orering | $-\frac{4(a^2x^2+1)(6a^6x^6-a^4x^4+a^2x^2-2)\arctan(ax)^3}{a^4x^2(a^2cx^2+c)^{\frac{3}{2}}}-\frac{2(a^2x^2+1)^2(18a^4x^4-7a^2x^2+4)\left(\frac{\arctan(ax)^3}{(a^2cx^2+c)^{\frac{3}{2}}}+\frac{3x\arctan(ax)^2a}{(a^2cx^2+c)^{\frac{3}{2}}(a^2x^2+1)}-\right.}{x^2a^4}$ |

input
```
int(x*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(arctan(a*x)^3-6*arctan(a*x)+3*I*arctan(a*x)^2-6*I)*(1+I*a*x)*(c*(a*x
-I)*(a*x+I))^(1/2)/(a^2*x^2+1)/a^2/c^2+1/2*(c*(a*x-I)*(a*x+I))^(1/2)*(I*a*
x-1)*(arctan(a*x)^3-6*arctan(a*x)-3*I*arctan(a*x)^2+6*I)/(a^2*x^2+1)/a^2/c
^2
```

## Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.58

$$\int \frac{x \arctan(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{a^2cx^2+c}\left(3\,ax\arctan\left(ax\right)^2-\arctan\left(ax\right)^3-6\,ax+6\,\arctan\left(ax\right)\right)}{a^4c^2x^2+a^2c^2}$$

input
```
integrate(x*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
sqrt(a^2*c*x^2 + c)*(3*a*x*arctan(a*x)^2 - arctan(a*x)^3 - 6*a*x + 6*arcta
n(a*x))/(a^4*c^2*x^2 + a^2*c^2)
```

## Sympy [F]

$$\int \frac{x \arctan(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x\,\mathrm{atan}^3\left(ax\right)}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*atan(a*x)**3/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(x*atan(a*x)**3/(c*(a**2*x**2 + 1))**(3/2), x)
```

## Maxima [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.92

$$\int \frac{x \arctan(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx = \sqrt{c}\left(\frac{3\,x\arctan\left(ax\right)^2}{\sqrt{a^2x^2+1}ac^2}-\frac{\arctan\left(ax\right)^3}{\sqrt{a^2x^2+1}a^2c^2}-\frac{6\left(\frac{x}{\sqrt{a^2x^2+1}}-\frac{\arctan(ax)}{\sqrt{a^2x^2+1}a}\right)}{ac^2}\right)$$

input
```
integrate(x*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
sqrt(c)*(3*x*arctan(a*x)^2/(sqrt(a^2*x^2 + 1)*a*c^2) - arctan(a*x)^3/(sqrt
(a^2*x^2 + 1)*a^2*c^2) - 6*(x/sqrt(a^2*x^2 + 1) - arctan(a*x)/(sqrt(a^2*x^
2 + 1)*a))/(a*c^2))
```

## Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.93

$$\int \frac{x\arctan(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx = \frac{3\,x\arctan{(ax)}^2}{\sqrt{a^2cx^2+c}ac} - \frac{\arctan{(ax)}^3}{\sqrt{a^2cx^2+c}a^2c} - \frac{6\,x}{\sqrt{a^2cx^2+c}ac} + \frac{6\arctan{(ax)}}{\sqrt{a^2cx^2+c}a^2c}$$

input
```
integrate(x*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
3*x*arctan(a*x)^2/(sqrt(a^2*c*x^2 + c)*a*c) - arctan(a*x)^3/(sqrt(a^2*c*x^
2 + c)*a^2*c) - 6*x/(sqrt(a^2*c*x^2 + c)*a*c) + 6*arctan(a*x)/(sqrt(a^2*c*
x^2 + c)*a^2*c)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x\arctan(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x\,\mathrm{atan}(a\,x)^3}{\left(c\,a^2\,x^2+c\right)^{3/2}}\,dx$$

input
```
int((x*atan(a*x)^3)/(c + a^2*c*x^2)^(3/2),x)
```

output
```
int((x*atan(a*x)^3)/(c + a^2*c*x^2)^(3/2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.92

$$\int \frac{x\arctan(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{c}\left(-\sqrt{a^2x^2+1}\,atan(ax)^3+3\sqrt{a^2x^2+1}\,atan(ax)^2\,ax+6\sqrt{a^2x^2+1}\,atan(ax)-6\right.}{a^2c^2\left(a^2x^2+1\right)}$$

input
```
int(x*atan(a*x)^3/(a^2*c*x^2+c)^(3/2),x)
```

output
```
(sqrt(c)*( - sqrt(a**2*x**2 + 1)*atan(a*x)**3 + 3*sqrt(a**2*x**2 + 1)*atan
(a*x)**2*a*x + 6*sqrt(a**2*x**2 + 1)*atan(a*x) - 6*sqrt(a**2*x**2 + 1)*a*x
 - 6*a**2*x**2 - 6))/(a**2*c**2*(a**2*x**2 + 1))
```

# 3.446 $\int \frac{\arctan(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 100

$$\int \frac{\arctan(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx = -\frac{6}{ac\sqrt{c+a^2cx^2}} - \frac{6x\arctan(ax)}{c\sqrt{c+a^2cx^2}} + \frac{3\arctan(ax)^2}{ac\sqrt{c+a^2cx^2}} + \frac{x\arctan(ax)^3}{c\sqrt{c+a^2cx^2}}$$

output
```
-6/a/c/(a^2*c*x^2+c)^(1/2)-6*x*arctan(a*x)/c/(a^2*c*x^2+c)^(1/2)+3*arctan(
a*x)^2/a/c/(a^2*c*x^2+c)^(1/2)+x*arctan(a*x)^3/c/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.56

$$\int \frac{\arctan(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{c+a^2cx^2}(-6-6ax\arctan(ax)+3\arctan(ax)^2+ax\arctan(ax)^3)}{c^2(a+a^3x^2)}$$

input
```
Integrate[ArcTan[a*x]^3/(c + a^2*c*x^2)^(3/2),x]
```

output
```
(Sqrt[c + a^2*c*x^2]*(-6 - 6*a*x*ArcTan[a*x] + 3*ArcTan[a*x]^2 + a*x*ArcTa
n[a*x]^3))/(c^2*(a + a^3*x^2))
```

## Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.01, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5433, 5429}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \text{5433}$$

$$-6\int \frac{\arctan(ax)}{(a^2cx^2+c)^{3/2}}dx+\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5429}$$

$$\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}}-6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)$$

input
```
Int[ArcTan[a*x]^3/(c + a^2*c*x^2)^(3/2),x]
```

output
```
(3*ArcTan[a*x]^2)/(a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x]^3)/(c*Sqrt[c
+ a^2*c*x^2]) - 6*(1/(a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x])/(c*Sqrt[c
 + a^2*c*x^2]))
```

### Defintions of rubi rules used

rule 5429
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTan[c*x])/(d*Sqr
t[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

rule 5433
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x] - Simp[b^2*p*(p
- 1)   Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2), x], x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.38 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.32

| method | result |
|---|---|
| default | $\frac{\left(\arctan(ax)^3-6\arctan(ax)+3i\arctan(ax)^2-6i\right)(ax-i)\sqrt{c(ax-i)(ax+i)}}{2(a^2x^2+1)a\,c^2}+\frac{\sqrt{c(ax-i)(ax+i)}\,(ax+i)\left(\arctan(ax)^3-6\arctan(a\right.}{2(a^2x^2+1)a\,c^2}$ |
| orering | $\frac{\left(-72a^4x^5-104a^2x^3-32x\right)\arctan(ax)^3}{(a^2c\,x^2+c)^{\frac{3}{2}}}-\frac{2\left(a^2x^2+1\right)^2\left(34a^2x^2+5\right)\left(\frac{3\arctan(ax)^2a}{\left(a^2c\,x^2+c\right)^{\frac{3}{2}}\left(a^2x^2+1\right)}-\frac{3\arctan(ax)^3cx\,a^2}{\left(a^2c\,x^2+c\right)^{\frac{5}{2}}}\right)}{a^2}-\frac{16\left(a^2x^2+\right.}{}$ |

input
```
int(arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(arctan(a*x)^3-6*arctan(a*x)+3*I*arctan(a*x)^2-6*I)*(a*x-I)*(c*(a*x-I)
*(a*x+I))^(1/2)/(a^2*x^2+1)/a/c^2+1/2*(c*(a*x-I)*(a*x+I))^(1/2)*(a*x+I)*(a
rctan(a*x)^3-6*arctan(a*x)-3*I*arctan(a*x)^2+6*I)/(a^2*x^2+1)/a/c^2
```

## Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.58

$$\int \frac{\arctan(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx=\frac{\sqrt{a^2cx^2+c}\left(ax\arctan(ax)^3-6\,ax\arctan(ax)+3\arctan(ax)^2-6\right)}{a^3c^2x^2+ac^2}$$

input
```
integrate(arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
sqrt(a^2*c*x^2 + c)*(a*x*arctan(a*x)^3 - 6*a*x*arctan(a*x) + 3*arctan(a*x)
^2 - 6)/(a^3*c^2*x^2 + a*c^2)
```

## Sympy [F]

$$\int \frac{\arctan(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\operatorname{atan}^3(ax)}{(c(a^2x^2+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(atan(a*x)**3/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(atan(a*x)**3/(c*(a**2*x**2 + 1))**(3/2), x)
```

## Maxima [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.99

$$\int \frac{\arctan(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx = \frac{x\arctan(ax)^3}{\sqrt{a^2cx^2+cc}} - \frac{3\,a\left(\frac{2\,x\arctan(ax)}{\sqrt{a^2x^2+1}ac} - \frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}a^2c} + \frac{2}{\sqrt{a^2x^2+1}a^2c}\right)}{\sqrt{c}}$$

input
```
integrate(arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
x*arctan(a*x)^3/(sqrt(a^2*c*x^2 + c)*c) - 3*a*(2*x*arctan(a*x)/(sqrt(a^2*x
^2 + 1)*a*c) - arctan(a*x)^2/(sqrt(a^2*x^2 + 1)*a^2*c) + 2/(sqrt(a^2*x^2 +
 1)*a^2*c))/sqrt(c)
```

## Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.99

$$\int \frac{\arctan(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx = \frac{x\arctan(ax)^3}{\sqrt{a^2cx^2+c}c} - 3a\left(\frac{2x\arctan(ax)}{\sqrt{a^2cx^2+c}ac} - \frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}a^2c} + \frac{2}{\sqrt{a^2cx^2+c}a^2c}\right)$$

input
```
integrate(arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
x*arctan(a*x)^3/(sqrt(a^2*c*x^2 + c)*c) - 3*a*(2*x*arctan(a*x)/(sqrt(a^2*c
*x^2 + c)*a*c) - arctan(a*x)^2/(sqrt(a^2*c*x^2 + c)*a^2*c) + 2/(sqrt(a^2*c
*x^2 + c)*a^2*c))
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\mathrm{atan}(a\,x)^3}{(c\,a^2\,x^2+c)^{3/2}}\,dx$$

input
```
int(atan(a*x)^3/(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(atan(a*x)^3/(c + a^2*c*x^2)^(3/2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.46

$$\int \frac{\arctan(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{c}\left(12atan\left(\sqrt{a^2x^2+1}+ax\right)a^2x^2+12atan\left(\sqrt{a^2x^2+1}+ax\right)+\sqrt{a^2x^2+1}\,atan(a\right.}{}$$

input
```
int(atan(a*x)^3/(a^2*c*x^2+c)^(3/2),x)
```

output
```
(sqrt(c)*(12*atan(sqrt(a**2*x**2 + 1) + a*x)*a**2*x**2 + 12*atan(sqrt(a**2
*x**2 + 1) + a*x) + sqrt(a**2*x**2 + 1)*atan(a*x)**3*a*x + 3*sqrt(a**2*x**
2 + 1)*atan(a*x)**2 - 6*sqrt(a**2*x**2 + 1)*atan(a*x)*a*x - 6*atan(a*x)*a*
*2*x**2 - 6*atan(a*x) - 6*sqrt(a**2*x**2 + 1)))/(a*c**2*(a**2*x**2 + 1))
```

# 3.447 $\int \frac{\arctan(ax)^3}{x(c+a^2cx^2)^{3/2}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 443

$$
\begin{aligned}
\int \frac{\arctan(ax)^3}{x\left(c+a^2cx^2\right)^{3/2}}\,dx &= \frac{6ax}{c\sqrt{c+a^2cx^2}} - \frac{6\arctan(ax)}{c\sqrt{c+a^2cx^2}} - \frac{3ax\arctan(ax)^2}{c\sqrt{c+a^2cx^2}} \\
&+ \frac{\arctan(ax)^3}{c\sqrt{c+a^2cx^2}} - \frac{2\sqrt{1+a^2x^2}\arctan(ax)^3\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)}{c\sqrt{c+a^2cx^2}} \\
&+ \frac{3i\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)}{c\sqrt{c+a^2cx^2}} \\
&- \frac{3i\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{PolyLog}\left(2,e^{i\arctan(ax)}\right)}{c\sqrt{c+a^2cx^2}} \\
&- \frac{6\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(3,-e^{i\arctan(ax)}\right)}{c\sqrt{c+a^2cx^2}} \\
&+ \frac{6\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(3,e^{i\arctan(ax)}\right)}{c\sqrt{c+a^2cx^2}} \\
&- \frac{6i\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(4,-e^{i\arctan(ax)}\right)}{c\sqrt{c+a^2cx^2}} \\
&+ \frac{6i\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(4,e^{i\arctan(ax)}\right)}{c\sqrt{c+a^2cx^2}}
\end{aligned}
$$

output
```
6*a*x/c/(a^2*c*x^2+c)^(1/2)-6*arctan(a*x)/c/(a^2*c*x^2+c)^(1/2)-3*a*x*arct
an(a*x)^2/c/(a^2*c*x^2+c)^(1/2)+arctan(a*x)^3/c/(a^2*c*x^2+c)^(1/2)-2*(a^2
*x^2+1)^(1/2)*arctan(a*x)^3*arctanh((1+I*a*x)/(a^2*x^2+1)^(1/2))/c/(a^2*c*
x^2+c)^(1/2)+3*I*(a^2*x^2+1)^(1/2)*arctan(a*x)^2*polylog(2,-(1+I*a*x)/(a^2
*x^2+1)^(1/2))/c/(a^2*c*x^2+c)^(1/2)-3*I*(a^2*x^2+1)^(1/2)*arctan(a*x)^2*p
olylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))/c/(a^2*c*x^2+c)^(1/2)-6*(a^2*x^2+1)^
(1/2)*arctan(a*x)*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))/c/(a^2*c*x^2+c)^
(1/2)+6*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2
))/c/(a^2*c*x^2+c)^(1/2)-6*I*(a^2*x^2+1)^(1/2)*polylog(4,-(1+I*a*x)/(a^2*x
^2+1)^(1/2))/c/(a^2*c*x^2+c)^(1/2)+6*I*(a^2*x^2+1)^(1/2)*polylog(4,(1+I*a*
x)/(a^2*x^2+1)^(1/2))/c/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 295, normalized size of antiderivative = 0.67

$$\int \frac{\arctan(ax)^3}{x\,(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{1+a^2x^2}\Big(-i\pi^4+\frac{48ax}{\sqrt{1+a^2x^2}}-\frac{48\arctan(ax)}{\sqrt{1+a^2x^2}}-\frac{24ax\arctan(ax)^2}{\sqrt{1+a^2x^2}}+\frac{8\arctan(ax)^3}{\sqrt{1+a^2x^2}}+2i\,\mathrm{arcta}}{}$$

input
```
Integrate[ArcTan[a*x]^3/(x*(c + a^2*c*x^2)^(3/2)),x]
```

output
```
(Sqrt[1 + a^2*x^2]*((-I)*Pi^4 + (48*a*x)/Sqrt[1 + a^2*x^2] - (48*ArcTan[a*
x])/Sqrt[1 + a^2*x^2] - (24*a*x*ArcTan[a*x]^2)/Sqrt[1 + a^2*x^2] + (8*ArcT
an[a*x]^3)/Sqrt[1 + a^2*x^2] + (2*I)*ArcTan[a*x]^4 + 8*ArcTan[a*x]^3*Log[1
 - E^((-I)*ArcTan[a*x])] - 8*ArcTan[a*x]^3*Log[1 + E^(I*ArcTan[a*x])] + (2
4*I)*ArcTan[a*x]^2*PolyLog[2, E^((-I)*ArcTan[a*x])] + (24*I)*ArcTan[a*x]^2
*PolyLog[2, -E^(I*ArcTan[a*x])] + 48*ArcTan[a*x]*PolyLog[3, E^((-I)*ArcTan
[a*x])] - 48*ArcTan[a*x]*PolyLog[3, -E^(I*ArcTan[a*x])] - (48*I)*PolyLog[4
, E^((-I)*ArcTan[a*x])] - (48*I)*PolyLog[4, -E^(I*ArcTan[a*x])]))/(8*c*Sqr
t[c*(1 + a^2*x^2)])
```

## Rubi [A] (verified)

Time = 1.83 (sec) , antiderivative size = 293, normalized size of antiderivative = 0.66, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5501, 5465, 5433, 208, 5493, 5491, 3042, 4671, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3}{x\left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx}{c}-a^2\int \frac{x\arctan(ax)^3}{\left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx}{c}-a^2\left(\frac{3\int \frac{\arctan(ax)^2}{\left(a^2cx^2+c\right)^{3/2}}dx}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \text{5433}$$

$$\frac{\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx}{c}-a^2\left(\frac{3\left(-2\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}}dx+\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \text{208}$$

$$\frac{\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx}{c}-a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \text{5493}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^3}{x\sqrt{a^2x^2+1}}dx}{c\sqrt{a^2cx^2+c}}-a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \text{5491}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{\sqrt{a^2x^2+1}\arctan(ax)^3}{ax}d\arctan(ax)}{c\sqrt{a^2cx^2+c}}-a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a^2x^2+1}\int \arctan(ax)^3\csc(\arctan(ax))d\arctan(ax)}{c\sqrt{a^2cx^2+c}}-a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \text{4671}$$

$$-a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)+\frac{\sqrt{a^2x^2+1}\left(-3\int \arctan(ax)^2\log\left(1-e^{i\arctan(ax)}\right)d\arctan(ax)+3\int \arctan(ax)^2\log\left(1+e^{i\arctan(ax)}\right)d\arctan(a\right.}{c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3011}$$

$$-a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)+\frac{\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\int \arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)d\arctan(ax)\right)\right.}{}$$

$$\Big\downarrow \text{7163}$$

$$-a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)+\frac{\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\left(i\int \operatorname{PolyLog}\left(3,-e^{i\arctan(ax)}\right)d\arctan(ax)-i\arctan\right.\right.\right.}{}$$

$$\Big\downarrow \text{2720}$$

$$-a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)+\frac{\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\left(\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(3,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right.\right.\right.}{}$$

$$\Big\downarrow \text{7143}$$

$$-a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)+$$
$$\sqrt{a^2x^2+1}\left(-2\arctan(ax)^3\text{arctanh}\left(e^{i\arctan(ax)}\right)+3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\left(\operatorname{PolyLog}\left(4,-\right.\right.\right.\right.$$

input
```
Int[ArcTan[a*x]^3/(x*(c + a^2*c*x^2)^(3/2)),x]
```

output
```
-(a^2*(-(ArcTan[a*x]^3/(a^2*c*Sqrt[c + a^2*c*x^2])) + (3*((-2*x)/(c*Sqrt[c
 + a^2*c*x^2]) + (2*ArcTan[a*x])/(a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x
]^2)/(c*Sqrt[c + a^2*c*x^2])))/a)) + (Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]^3*
ArcTanh[E^(I*ArcTan[a*x])] + 3*(I*ArcTan[a*x]^2*PolyLog[2, -E^(I*ArcTan[a*
x])] - (2*I)*((-I)*ArcTan[a*x]*PolyLog[3, -E^(I*ArcTan[a*x])] + PolyLog[4,
 -E^(I*ArcTan[a*x])])) - 3*(I*ArcTan[a*x]^2*PolyLog[2, E^(I*ArcTan[a*x])]
- (2*I)*((-I)*ArcTan[a*x]*PolyLog[3, E^(I*ArcTan[a*x])] + PolyLog[4, E^(I*
ArcTan[a*x])]))))/(c*Sqrt[c + a^2*c*x^2])
```

## Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5433
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x] - Simp[b^2*p*(p
- 1)   Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2), x], x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5491
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Simp[1/Sqrt[d]   Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTa
n[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] &&
GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

## Maple [A] (verified)

Time = 4.80 (sec) , antiderivative size = 388, normalized size of antiderivative = 0.88

| method | result |
|---|---|
| default | $\frac{\left(\arctan(ax)^3-6\arctan(ax)+3i\arctan(ax)^2-6i\right)(iax+1)\sqrt{c(ax-i)(ax+i)}}{2(a^2x^2+1)c^2}-\frac{\sqrt{c(ax-i)(ax+i)}\,(iax-1)\left(\arctan(ax)^3-6\arctan(\right.}{2(a^2x^2+1)c^2}$ |

input
```
int(arctan(a*x)^3/x/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(arctan(a*x)^3-6*arctan(a*x)+3*I*arctan(a*x)^2-6*I)*(1+I*a*x)*(c*(a*x-
I)*(a*x+I))^(1/2)/(a^2*x^2+1)/c^2-1/2*(c*(a*x-I)*(a*x+I))^(1/2)*(I*a*x-1)*
(arctan(a*x)^3-6*arctan(a*x)-3*I*arctan(a*x)^2+6*I)/(a^2*x^2+1)/c^2+I*(I*a
rctan(a*x)^3*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*arctan(a*x)^3*ln(1-(1+I*a
*x)/(a^2*x^2+1)^(1/2))+3*arctan(a*x)^2*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1
/2))-3*arctan(a*x)^2*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*arctan(a*x
)*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I*arctan(a*x)*polylog(3,(1+I*a
*x)/(a^2*x^2+1)^(1/2))-6*polylog(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*polylog
(4,(1+I*a*x)/(a^2*x^2+1)^(1/2)))*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/
2)/c^2
```

## Fricas [F]

$$\int \frac{\arctan(ax)^3}{x\left(c+a^2cx^2\right)^{3/2}}\,dx=\int \frac{\arctan\left(ax\right)^3}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(arctan(a*x)^3/x/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3/(a^4*c^2*x^5 + 2*a^2*c^2*x^3 +
c^2*x), x)
```

## Sympy [F]

$$\int \frac{\arctan(ax)^3}{x\left(c+a^2cx^2\right)^{3/2}}\,dx=\int \frac{\operatorname{atan}^3\left(ax\right)}{x\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(atan(a*x)**3/x/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(atan(a*x)**3/(x*(c*(a**2*x**2 + 1))**(3/2)), x)
```

## Maxima [F]

$$\int \frac{\arctan(ax)^3}{x\left(c+a^2cx^2\right)^{3/2}}\,dx = \int \frac{\arctan\left(ax\right)^3}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(arctan(a*x)^3/x/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)^3/((a^2*c*x^2 + c)^(3/2)*x), x)
```

## Giac [F]

$$\int \frac{\arctan(ax)^3}{x\left(c+a^2cx^2\right)^{3/2}}\,dx = \int \frac{\arctan\left(ax\right)^3}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(arctan(a*x)^3/x/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^3/((a^2*c*x^2 + c)^(3/2)*x), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^3}{x\left(c+a^2cx^2\right)^{3/2}}\,dx = \int \frac{\mathrm{atan}\left(a\,x\right)^3}{x\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

input
```
int(atan(a*x)^3/(x*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(atan(a*x)^3/(x*(c + a^2*c*x^2)^(3/2)), x)
```

## Reduce [F]

$$\int \frac{\arctan(ax)^3}{x\,(c+a^2cx^2)^{3/2}}\,dx = \frac{\int \frac{atan(ax)^3}{\sqrt{a^2x^2+1}\,a^2x^3+\sqrt{a^2x^2+1}\,x}dx}{\sqrt{c}\,c}$$

input
```
int(atan(a*x)^3/x/(a^2*c*x^2+c)^(3/2),x)
```

output
```
int(atan(a*x)**3/(sqrt(a**2*x**2 + 1)*a**2*x**3 + sqrt(a**2*x**2 + 1)*x),x
)/(sqrt(c)*c)
```

# 3.448 $\int \frac{\arctan(ax)^3}{x^2(c+a^2cx^2)^{3/2}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 377

$$\begin{aligned}\int \frac{\arctan(ax)^3}{x^2\,(c+a^2cx^2)^{3/2}}\,dx &= \frac{6a}{c\sqrt{c+a^2cx^2}}+\frac{6a^2x\arctan(ax)}{c\sqrt{c+a^2cx^2}} \\ &-\frac{3a\arctan(ax)^2}{c\sqrt{c+a^2cx^2}}-\frac{a^2x\arctan(ax)^3}{c\sqrt{c+a^2cx^2}}-\frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{c^2x} \\ &-\frac{6a\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)}{c\sqrt{c+a^2cx^2}} \\ &+\frac{6ia\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)}{c\sqrt{c+a^2cx^2}} \\ &-\frac{6ia\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,e^{i\arctan(ax)}\right)}{c\sqrt{c+a^2cx^2}} \\ &-\frac{6a\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,-e^{i\arctan(ax)}\right)}{c\sqrt{c+a^2cx^2}} \\ &+\frac{6a\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,e^{i\arctan(ax)}\right)}{c\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
6*a/c/(a^2*c*x^2+c)^(1/2)+6*a^2*x*arctan(a*x)/c/(a^2*c*x^2+c)^(1/2)-3*a*ar
ctan(a*x)^2/c/(a^2*c*x^2+c)^(1/2)-a^2*x*arctan(a*x)^3/c/(a^2*c*x^2+c)^(1/2
)-(a^2*c*x^2+c)^(1/2)*arctan(a*x)^3/c^2/x-6*a*(a^2*x^2+1)^(1/2)*arctan(a*x
)^2*arctanh((1+I*a*x)/(a^2*x^2+1)^(1/2))/c/(a^2*c*x^2+c)^(1/2)+6*I*a*(a^2*
x^2+1)^(1/2)*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))/c/(a^2*c*
x^2+c)^(1/2)-6*I*a*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2*
x^2+1)^(1/2))/c/(a^2*c*x^2+c)^(1/2)-6*a*(a^2*x^2+1)^(1/2)*polylog(3,-(1+I*
a*x)/(a^2*x^2+1)^(1/2))/c/(a^2*c*x^2+c)^(1/2)+6*a*(a^2*x^2+1)^(1/2)*polylo
g(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))/c/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (warning: unable to verify)

Time = 1.08 (sec) , antiderivative size = 301, normalized size of antiderivative = 0.80

$$\int \frac{\arctan(ax)^3}{x^2\left(c+a^2cx^2\right)^{3/2}}\,dx = \frac{a\Big(12+12ax\arctan(ax)-6\arctan(ax)^2-2ax\arctan(ax)^3-\frac{1}{2}ax\arctan(ax)^3\,\mathrm{c}$$

input
```
Integrate[ArcTan[a*x]^3/(x^2*(c + a^2*c*x^2)^(3/2)),x]
```

output
```
(a*(12 + 12*a*x*ArcTan[a*x] - 6*ArcTan[a*x]^2 - 2*a*x*ArcTan[a*x]^3 - (a*x
*ArcTan[a*x]^3*Csc[ArcTan[a*x]/2]^2)/2 + 6*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2
*Log[1 - E^(I*ArcTan[a*x])] - 6*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*Log[1 + E^
(I*ArcTan[a*x])] + (12*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, -E^(I*A
rcTan[a*x])] - (12*I)*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan
[a*x])] - 12*Sqrt[1 + a^2*x^2]*PolyLog[3, -E^(I*ArcTan[a*x])] + 12*Sqrt[1
+ a^2*x^2]*PolyLog[3, E^(I*ArcTan[a*x])] - (2*(1 + a^2*x^2)*ArcTan[a*x]^3*
Sin[ArcTan[a*x]/2]^2)/(a*x)))/(2*c*Sqrt[c + a^2*c*x^2])
```

## Rubi [A] (verified)

Time = 1.82 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.71, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.458$, Rules used = {5501, 5433, 5429, 5479, 5493, 5491, 3042, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3}{x^2 (a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{\arctan(ax)^3}{x^2\sqrt{a^2cx^2+c}}dx}{c} - a^2 \int \frac{\arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \text{5433}$$

$$\frac{\int \frac{\arctan(ax)^3}{x^2\sqrt{a^2cx^2+c}}dx}{c} - a^2 \left(-6\int \frac{\arctan(ax)}{(a^2cx^2+c)^{3/2}}dx + \frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}} + \frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \text{5429}$$

$$\frac{\int \frac{\arctan(ax)^3}{x^2\sqrt{a^2cx^2+c}}dx}{c} - a^2 \left(\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}} + \frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}} - 6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}} + \frac{1}{ac\sqrt{a^2cx^2+c}}\right)\right)$$

$$\Big\downarrow \text{5479}$$

$$\frac{3a\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx - \frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}}{c} - a^2 \left(\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}} + \frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}} - 6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}} + \frac{1}{ac\sqrt{a^2cx^2+c}}\right)\right)$$

$$\Big\downarrow \text{5493}$$

$$\frac{\frac{3a\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}} - \frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}}{c} - a^2 \left(\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}} + \frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}} - 6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}} + \frac{1}{ac\sqrt{a^2cx^2+c}}\right)\right)$$

$$\Big\downarrow \text{5491}$$

$$\frac{\frac{3a\sqrt{a^2x^2+1}\int \frac{\sqrt{a^2x^2+1}\arctan(ax)^2}{ax}d\arctan(ax)}{\sqrt{a^2cx^2+c}} - \frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}}{c} - a^2 \left(\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}} + \frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}} - 6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}} + \frac{1}{ac\sqrt{a^2cx^2+c}}\right)\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{3a\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc(\arctan(ax))d\arctan(ax)}{\sqrt{a^2cx^2+c}} - \frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}}{c} - a^2 \left(\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}} + \frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}} - 6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}} + \frac{1}{ac\sqrt{a^2cx^2+c}}\right)\right)$$

$$\Big\downarrow \text{4671}$$

$$-a^2\left(\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}}-6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)\right)+\frac{-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(-2\int\arctan(ax)\log\left(1-e^{i\arctan(ax)}\right)d\arctan(ax)+2\int\arctan(ax)\log\left(1+e^{i\arctan(ax)}\right)d\arctan(ax)-\right.}{\sqrt{a^2cx^2+c}}}{c}$$

$$\Big\downarrow \text{3011}$$

$$-a^2\left(\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}}-6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)\right)+\frac{-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-i\int\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)d\arctan(ax)\right)-2\left(i\arctan(ax\right.\right.}{\sqrt{a^2cx^2}}}{c}$$

$$\Big\downarrow \text{2720}$$

$$-a^2\left(\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}}-6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)\right)+\frac{-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)de^{i\arctan(ax)}\right)-\right.}{}}{c}$$

$$\Big\downarrow \text{7143}$$

$$-a^2\left(\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}}-6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)\right)+\frac{-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(-2\arctan(ax)^2\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)+2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-\operatorname{PolyLog}\left(3,-e^{i a}\right.\right.\right.}{\sqrt{a^2cx^2+c}}}{c}$$

input
```
Int[ArcTan[a*x]^3/(x^2*(c + a^2*c*x^2)^(3/2)),x]
```

output
```
-(a^2*((3*ArcTan[a*x]^2)/(a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x]^3)/(c*
Sqrt[c + a^2*c*x^2]) - 6*(1/(a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x])/(c
*Sqrt[c + a^2*c*x^2])))) + (-((Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/(c*x)) +
 (3*a*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*x])] + 2*(
I*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])] - PolyLog[3, -E^(I*ArcTan[a*x
])]) - 2*(I*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])] - PolyLog[3, E^(I*Ar
cTan[a*x])])))/Sqrt[c + a^2*c*x^2])/c
```

## Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5429
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTan[c*x])/(d*Sqr
t[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

rule 5433
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x] - Simp[b^2*p*(p
- 1)   Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2), x], x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5491
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Simp[1/Sqrt[d]   Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTa
n[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] &&
GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

## Maple [A] (verified)

Time = 4.65 (sec) , antiderivative size = 356, normalized size of antiderivative = 0.94

| method | result |
|---|---|
| default | $-\frac{a\left(\arctan(ax)^{3}-6\arctan(ax)+3i\arctan(ax)^{2}-6i\right)(ax-i)\sqrt{c(ax-i)(ax+i)}}{2(a^{2}x^{2}+1)c^{2}}-\frac{\sqrt{c(ax-i)(ax+i)}\,(ax+i)\left(\arctan(ax)^{3}-6\,\mathrm{arcta}\right.}{2(a^{2}x^{2}+1)c^{2}}$ |

input
```
int(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*a*(arctan(a*x)^3-6*arctan(a*x)+3*I*arctan(a*x)^2-6*I)*(a*x-I)*(c*(a*x
-I)*(a*x+I))^(1/2)/(a^2*x^2+1)/c^2-1/2*(c*(a*x-I)*(a*x+I))^(1/2)*(a*x+I)*(
arctan(a*x)^3-6*arctan(a*x)-3*I*arctan(a*x)^2+6*I)*a/(a^2*x^2+1)/c^2-arcta
n(a*x)^3*(c*(a*x-I)*(a*x+I))^(1/2)/c^2/x-3*a*(arctan(a*x)^2*ln(1+(1+I*a*x)
/(a^2*x^2+1)^(1/2))-2*I*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2)
)-arctan(a*x)^2*ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*I*arctan(a*x)*polylog(
2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-2
*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2)))*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^
2+1)^(1/2)/c^2
```

## Fricas [F]

$$\int \frac{\arctan(ax)^3}{x^2 (c+a^2cx^2)^{3/2}}\,dx = \int \frac{\arctan\left(ax\right)^3}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3/(a^4*c^2*x^6 + 2*a^2*c^2*x^4 +
c^2*x^2), x)
```

## Sympy [F]

$$\int \frac{\arctan(ax)^3}{x^2 (c+a^2cx^2)^{3/2}}\,dx = \int \frac{\operatorname{atan}^3\left(ax\right)}{x^2\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(atan(a*x)**3/x**2/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(atan(a*x)**3/(x**2*(c*(a**2*x**2 + 1))**(3/2)), x)
```

## Maxima [F]

$$\int \frac{\arctan(ax)^3}{x^2(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\arctan(ax)^3}{(a^2cx^2+c)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)^3/((a^2*c*x^2 + c)^(3/2)*x^2), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^3}{x^2(c+a^2cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^3}{x^2(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\mathrm{atan}(a\,x)^3}{x^2\,(c\,a^2\,x^2+c)^{3/2}}\,dx$$

input
```
int(atan(a*x)^3/(x^2*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(atan(a*x)^3/(x^2*(c + a^2*c*x^2)^(3/2)), x)
```

## Reduce [F]

$$\int \frac{\arctan(ax)^3}{x^2 (c + a^2cx^2)^{3/2}} \, dx = \frac{\int \frac{atan(ax)^3}{\sqrt{a^2x^2+1}\, a^2x^4+\sqrt{a^2x^2+1}\, x^2} dx}{\sqrt{c}\, c}$$

input
```
int(atan(a*x)^3/x^2/(a^2*c*x^2+c)^(3/2),x)
```

output
```
int(atan(a*x)**3/(sqrt(a**2*x**2 + 1)*a**2*x**4 + sqrt(a**2*x**2 + 1)*x**2
),x)/(sqrt(c)*c)
```

# 3.449 $\int \frac{x^5 \arctan(ax)^3}{(c+a^2cx^2)^{5/2}} dx$



## Optimal result

Integrand size = 24, antiderivative size = 534

$$\begin{aligned}\int \frac{x^5 \arctan(ax)^3}{(c+a^2cx^2)^{5/2}} dx &= \frac{2x^3}{27a^3c(c+a^2cx^2)^{3/2}} + \frac{94x}{9a^5c^2\sqrt{c+a^2cx^2}} \\ &- \frac{2x^2\arctan(ax)}{9a^4c(c+a^2cx^2)^{3/2}} - \frac{94\arctan(ax)}{9a^6c^2\sqrt{c+a^2cx^2}} - \frac{x^3\arctan(ax)^2}{3a^3c(c+a^2cx^2)^{3/2}} \\ &- \frac{5x\arctan(ax)^2}{a^5c^2\sqrt{c+a^2cx^2}} + \frac{6i\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^2}{a^6c^2\sqrt{c+a^2cx^2}} \\ &+ \frac{x^2\arctan(ax)^3}{3a^4c(c+a^2cx^2)^{3/2}} + \frac{5\arctan(ax)^3}{3a^6c^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{a^6c^3} \\ &- \frac{6i\sqrt{1+a^2x^2}\arctan(ax)\,\text{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)}{a^6c^2\sqrt{c+a^2cx^2}} \\ &+ \frac{6i\sqrt{1+a^2x^2}\arctan(ax)\,\text{PolyLog}\left(2,ie^{i\arctan(ax)}\right)}{a^6c^2\sqrt{c+a^2cx^2}} \\ &+ \frac{6\sqrt{1+a^2x^2}\,\text{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)}{a^6c^2\sqrt{c+a^2cx^2}} \\ &- \frac{6\sqrt{1+a^2x^2}\,\text{PolyLog}\left(3,ie^{i\arctan(ax)}\right)}{a^6c^2\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
2/27*x^3/a^3/c/(a^2*c*x^2+c)^(3/2)+94/9*x/a^5/c^2/(a^2*c*x^2+c)^(1/2)-2/9*
x^2*arctan(a*x)/a^4/c/(a^2*c*x^2+c)^(3/2)-94/9*arctan(a*x)/a^6/c^2/(a^2*c*
x^2+c)^(1/2)-1/3*x^3*arctan(a*x)^2/a^3/c/(a^2*c*x^2+c)^(3/2)-5*x*arctan(a*
x)^2/a^5/c^2/(a^2*c*x^2+c)^(1/2)+6*I*(a^2*x^2+1)^(1/2)*arctan((1+I*a*x)/(a
^2*x^2+1)^(1/2))*arctan(a*x)^2/a^6/c^2/(a^2*c*x^2+c)^(1/2)+1/3*x^2*arctan(
a*x)^3/a^4/c/(a^2*c*x^2+c)^(3/2)+5/3*arctan(a*x)^3/a^6/c^2/(a^2*c*x^2+c)^(
1/2)+(a^2*c*x^2+c)^(1/2)*arctan(a*x)^3/a^6/c^3-6*I*(a^2*x^2+1)^(1/2)*arcta
n(a*x)*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a^6/c^2/(a^2*c*x^2+c)^(1/
2)+6*I*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/
2))/a^6/c^2/(a^2*c*x^2+c)^(1/2)+6*(a^2*x^2+1)^(1/2)*polylog(3,-I*(1+I*a*x)
/(a^2*x^2+1)^(1/2))/a^6/c^2/(a^2*c*x^2+c)^(1/2)-6*(a^2*x^2+1)^(1/2)*polylo
g(3,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a^6/c^2/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (warning: unable to verify)

Time = 1.62 (sec) , antiderivative size = 367, normalized size of antiderivative = 0.69

$$\int \frac{x^5 \arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx =$$

$$-\frac{(1+a^2x^2)^2 \left(1134\arctan(ax) - 405\arctan(ax)^3 + 1128\arctan(ax)\cos(2\arctan(ax)) - 180\arctan(ax)^3\right.}{}$$

input
```
Integrate[(x^5*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(5/2),x]
```

output
```
-1/216*((1 + a^2*x^2)^2*(1134*ArcTan[a*x] - 405*ArcTan[a*x]^3 + 1128*ArcTa
n[a*x]*Cos[2*ArcTan[a*x]] - 180*ArcTan[a*x]^3*Cos[2*ArcTan[a*x]] - 6*ArcTa
n[a*x]*Cos[4*ArcTan[a*x]] + 9*ArcTan[a*x]^3*Cos[4*ArcTan[a*x]] + (648*ArcT
an[a*x]^2*Log[1 - I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - (648*ArcTan[a*
x]^2*Log[1 + I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + ((1296*I)*ArcTan[a*
x]*PolyLog[2, (-I)*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - ((1296*I)*ArcTa
n[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] - (1296*PolyLog[
3, (-I)*E^(I*ArcTan[a*x])])/Sqrt[1 + a^2*x^2] + (1296*PolyLog[3, I*E^(I*Ar
cTan[a*x])])/Sqrt[1 + a^2*x^2] - 1132*Sin[2*ArcTan[a*x]] + 558*ArcTan[a*x]
^2*Sin[2*ArcTan[a*x]] + 2*Sin[4*ArcTan[a*x]] - 9*ArcTan[a*x]^2*Sin[4*ArcTa
n[a*x]]))/(a^6*c*(c*(1 + a^2*x^2))^(3/2))
```

## Rubi [A] (verified)

Time = 4.44 (sec) , antiderivative size = 609, normalized size of antiderivative = 1.14, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.792$, Rules used = {5499, 5475, 5465, 5433, 208, 5473, 5465, 208, 5499, 5465, 5425, 5423, 3042, 4669, 3011, 2720, 5433, 208, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 \arctan(ax)^3}{(a^2cx^2+c)^{5/2}} \, dx$$

$$\Big\downarrow \text{5499}$$

$$\frac{\int \frac{x^3 \arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx}{a^2c} - \frac{\int \frac{x^3 \arctan(ax)^3}{(a^2cx^2+c)^{5/2}}dx}{a^2}$$

$$\Big\downarrow \text{5475}$$

$$\frac{\int \frac{x^3 \arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx}{a^2c} - \frac{\frac{2\int \frac{x \arctan(ax)^3}{\left(a^2cx^2+c\right)^{3/2}}dx}{3a^2c} - \frac{2}{3}\int \frac{x^3 \arctan(ax)}{(a^2cx^2+c)^{5/2}}dx - \frac{x^2 \arctan(ax)^3}{3a^2c(a^2cx^2+c)^{3/2}} + \frac{x^3 \arctan(ax)^2}{3ac(a^2cx^2+c)^{3/2}}}{a^2}$$

$$\Big\downarrow \text{5465}$$

$$\frac{\int \frac{x^3 \arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx}{a^2c} - \frac{\frac{2\left(\frac{3\int \frac{\arctan(ax)^2}{\left(a^2cx^2+c\right)^{3/2}}dx}{a} - \frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c} - \frac{2}{3}\int \frac{x^3 \arctan(ax)}{(a^2cx^2+c)^{5/2}}dx - \frac{x^2 \arctan(ax)^3}{3a^2c(a^2cx^2+c)^{3/2}} + \frac{x^3 \arctan(ax)^2}{3ac(a^2cx^2+c)^{3/2}}}{a^2}$$

$$\Big\downarrow \text{5433}$$

$$\frac{\int \frac{x^3 \arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx}{a^2c} - \frac{\frac{2\left(\frac{3\left(-2\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}}dx + \frac{x \arctan(ax)^2}{c\sqrt{a^2cx^2+c}} + \frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}\right)}{a} - \frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c} - \frac{2}{3}\int \frac{x^3 \arctan(ax)}{(a^2cx^2+c)^{5/2}}dx - \frac{x^2 \arctan(ax)^3}{3a^2c(a^2cx^2+c)^{3/2}} + \frac{x^3 \arctan(ax)^2}{3ac(a^2cx^2+c)^{3/2}}}{a^2}$$

$$\Big\downarrow \text{208}$$

$$\frac{\int \frac{x^3 \arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx}{a^2c} - \frac{-\frac{2}{3}\int \frac{x^3\arctan(ax)}{(a^2cx^2+c)^{5/2}}dx - \frac{x^2\arctan(ax)^3}{3a^2c(a^2cx^2+c)^{3/2}} + \frac{2\left(\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a} - \frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c} + \frac{x^3\arctan(ax)^2}{3ac(a^2cx^2+c)^{3/2}}}{a^2}$$

$\downarrow$ 5473

$$\frac{\int \frac{x^3 \arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx}{a^2c} - \frac{-\frac{2}{3}\left(\frac{2\int \frac{x\arctan(ax)}{(a^2cx^2+c)^{3/2}}dx}{3a^2c} - \frac{x^2\arctan(ax)}{3a^2c(a^2cx^2+c)^{3/2}} + \frac{x^3}{9ac(a^2cx^2+c)^{3/2}}\right) - \frac{x^2\arctan(ax)^3}{3a^2c(a^2cx^2+c)^{3/2}} + \frac{2\left(\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2}}\right.}{a}\right.}{3a^2c}}{a^2}$$

$\downarrow$ 5465

$$\frac{\int \frac{x^3 \arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx}{a^2c} - \frac{-\frac{2}{3}\left(\frac{2\left(\frac{\int \frac{1}{(a^2cx^2+c)^{3/2}}dx}{a} - \frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c} - \frac{x^2\arctan(ax)}{3a^2c(a^2cx^2+c)^{3/2}} + \frac{x^3}{9ac(a^2cx^2+c)^{3/2}}\right) - \frac{x^2\arctan(ax)^3}{3a^2c(a^2cx^2+c)^{3/2}} + \frac{2\left(\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2a}{ac}\right.}{}\right.}{}}{a^2}$$

$\downarrow$ 208

$$\frac{\int \frac{x^3 \arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx}{a^2c} - \frac{-\frac{x^2\arctan(ax)^3}{3a^2c(a^2cx^2+c)^{3/2}} + \frac{2\left(\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a} - \frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c} + \frac{x^3\arctan(ax)^2}{3ac(a^2cx^2+c)^{3/2}} - \frac{2}{3}\left(-\frac{x^2\arctan(ax)}{3a^2c(a^2cx^2+c)^{3/2}}\right.}{a^2}$$

$\downarrow$ 5499

$$\frac{\frac{\int \frac{x\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{a^2c}-\frac{\int \frac{x\arctan(ax)^3}{\left(a^2cx^2+c\right)^{3/2}}dx}{a^2}}{a^2c}-$$

$$\frac{-\frac{x^2\arctan(ax)^3}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{x^3\arctan(ax)^2}{3ac(a^2cx^2+c)^{3/2}}-\frac{2}{3}\left(-\frac{x^2\arctan(ax)}{3a^2c(a^2cx^2+c)^{3/2}}-\right.}{a^2}$$

$\downarrow$ 5465

$$\frac{\frac{\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\int\frac{\arctan(ax)^2}{\sqrt{a^2cx^2+c}}dx}{a}}{a^2c}-\frac{\frac{3\int\frac{\arctan(ax)^2}{\left(a^2cx^2+c\right)^{3/2}}dx}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-$$

$$\frac{-\frac{x^2\arctan(ax)^3}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{x^3\arctan(ax)^2}{3ac(a^2cx^2+c)^{3/2}}-\frac{2}{3}\left(-\frac{x^2\arctan(ax)}{3a^2c(a^2cx^2+c)^{3/2}}-\right.}{a^2}$$

$\downarrow$ 5425

$$\frac{\frac{\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\int\frac{\arctan(ax)^2}{\sqrt{a^2x^2+1}}dx}{a\sqrt{a^2cx^2+c}}}{a^2c}-\frac{\frac{3\int\frac{\arctan(ax)^2}{\left(a^2cx^2+c\right)^{3/2}}dx}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-$$

$$\frac{-\frac{x^2\arctan(ax)^3}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{x^3\arctan(ax)^2}{3ac(a^2cx^2+c)^{3/2}}-\frac{2}{3}\left(-\frac{x^2\arctan(ax)}{3a^2c(a^2cx^2+c)^{3/2}}-\right.}{a^2}$$

$\downarrow$ 5423

$$\frac{\frac{\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\int\sqrt{a^2x^2+1}\arctan(ax)^2d\arctan(ax)}{a^2\sqrt{a^2cx^2+c}}}{a^2c}-\frac{\frac{3\int\frac{\arctan(ax)^2}{\left(a^2cx^2+c\right)^{3/2}}dx}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-$$

$$\frac{-\frac{x^2\arctan(ax)^3}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{x^3\arctan(ax)^2}{3ac(a^2cx^2+c)^{3/2}}-\frac{2}{3}\left(-\frac{x^2\arctan(ax)}{3a^2c(a^2cx^2+c)^{3/2}}-\right.}{a^2}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{a^2\sqrt{a^2cx^2+c}}}{a^2c}-\frac{\frac{3\int\frac{\arctan(ax)^2}{\left(a^2cx^2+c\right)^{3/2}}dx}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-$$

$$\frac{-\frac{x^2\arctan(ax)^3}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{x^3\arctan(ax)^2}{3ac(a^2cx^2+c)^{3/2}}-\frac{2}{3}\left(-\frac{x^2\arctan(ax)}{3a^2c(a^2cx^2+c)^{3/2}}-\right.}{a^2}$$

$\downarrow$ 4669

$$-\frac{-\frac{x^2\arctan(ax)^3}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{x^3\arctan(ax)^2}{3ac(a^2cx^2+c)^{3/2}}-\frac{2}{3}\left(-\frac{x^2\arctan(ax)}{3a^2c(a^2cx^2+c)^{3/2}}\right.}{a^2}$$

$$\frac{-\frac{\frac{3\int\frac{\arctan(ax)^2}{\left(a^2cx^2+c\right)^{3/2}}dx}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}}{a^2}+\frac{\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(-2\int\arctan(ax)\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+2\int\arctan(ax)\log\left(1+i\right.\right.}{a^2\sqrt{a^2cx^2+c}}}{a^2c}}{a^2c}$$

$\downarrow$ 3011

$$-\frac{-\frac{x^2\arctan(ax)^3}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{x^3\arctan(ax)^2}{3ac(a^2cx^2+c)^{3/2}}-\frac{2}{3}\left(-\frac{x^2\arctan(ax)}{3a^2c(a^2cx^2+c)^{3/2}}\right.}{a^2}$$

$$\frac{-\frac{\frac{3\int\frac{\arctan(ax)^2}{\left(a^2cx^2+c\right)^{3/2}}dx}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}}{a^2}+\frac{\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-i\int\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)\right.\right.}{}}{}}{a^2c}$$

$\downarrow$ 2720

$$-\frac{-\frac{x^2\arctan(ax)^3}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{x^3\arctan(ax)^2}{3ac(a^2cx^2+c)^{3/2}}-\frac{2}{3}\left(-\frac{x^2\arctan(ax)}{3a^2c(a^2cx^2+c)^{3/2}}\right.}{a^2}$$

$$-\frac{\frac{3\int\frac{\arctan(ax)^2}{\left(a^2cx^2+c\right)^{3/2}}dx}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}}{a^2}+\frac{\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e^{-i\arctan(ax)}\operatorname{PolyLog}\left(2,-\right.\right.\right.}{}}{}$$

$\downarrow$ 5433

$$-\frac{-\frac{x^2\arctan(ax)^3}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{x^3\arctan(ax)^2}{3ac(a^2cx^2+c)^{3/2}}-\frac{2}{3}\left(-\frac{x^2\arctan(ax)}{3a^2c(a^2cx^2+c)^{3/2}}\right.}{a^2}$$

$$-\frac{\frac{3\left(-2\int\frac{1}{(a^2cx^2+c)^{3/2}}dx+\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}}{a^2}+\frac{\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan}\right.\right.\right.}{}}{}$$

$\downarrow$ 208

$$-\frac{-\frac{x^2\arctan(ax)^3}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{x^3\arctan(ax)^2}{3ac(a^2cx^2+c)^{3/2}}-\frac{2}{3}\left(-\frac{x^2\arctan(ax)}{3a^2c(a^2cx^2+c)^{3/2}}\right.}{a^2}$$

$$-\frac{\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}}{a^2}+\frac{\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\int e\right.\right.}{}}{}$$

$\downarrow$ 7143

$$-\frac{-\frac{x^2\arctan(ax)^3}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{x^3\arctan(ax)^2}{3ac(a^2cx^2+c)^{3/2}}-\frac{2}{3}\left(-\frac{x^2\arctan(ax)}{3a^2c(a^2cx^2+c)^{3/2}}\right.}{a^2}$$

$$-\frac{\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}}{a^2}+\frac{\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-\operatorname{Po}\right.\right.}{}}{a^2c}$$

input
```
Int[(x^5*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(5/2),x]
```

output
```
-(((x^3*ArcTan[a*x]^2)/(3*a*c*(c + a^2*c*x^2)^(3/2)) - (x^2*ArcTan[a*x]^3)
/(3*a^2*c*(c + a^2*c*x^2)^(3/2)) - (2*(x^3/(9*a*c*(c + a^2*c*x^2)^(3/2)) -
 (x^2*ArcTan[a*x])/(3*a^2*c*(c + a^2*c*x^2)^(3/2)) + (2*(x/(a*c*Sqrt[c + a
^2*c*x^2]) - ArcTan[a*x]/(a^2*c*Sqrt[c + a^2*c*x^2])))/(3*a^2*c)))/3 + (2*
(-(ArcTan[a*x]^3/(a^2*c*Sqrt[c + a^2*c*x^2])) + (3*((-2*x)/(c*Sqrt[c + a^2
*c*x^2]) + (2*ArcTan[a*x])/(a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x]^2)/(
c*Sqrt[c + a^2*c*x^2])))/a))/(3*a^2*c))/a^2) + (-((-(ArcTan[a*x]^3/(a^2*c*
Sqrt[c + a^2*c*x^2])) + (3*((-2*x)/(c*Sqrt[c + a^2*c*x^2]) + (2*ArcTan[a*x
])/(a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x]^2)/(c*Sqrt[c + a^2*c*x^2])))
/a)/a^2) + ((Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3)/(a^2*c) - (3*Sqrt[1 + a^2*
x^2]*((-2*I)*ArcTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^2 + 2*(I*ArcTan[a*x]*Po
lyLog[2, (-I)*E^(I*ArcTan[a*x])] - PolyLog[3, (-I)*E^(I*ArcTan[a*x])]) - 2
*(I*ArcTan[a*x]*PolyLog[2, I*E^(I*ArcTan[a*x])] - PolyLog[3, I*E^(I*ArcTan
[a*x])])))/(a^2*Sqrt[c + a^2*c*x^2]))/(a^2*c))/(a^2*c)
```

## Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5433
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x] - Simp[b^2*p*(p
- 1)   Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2), x], x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5473
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[b*(f*x)^m*((d + e*x^2)^(q + 1)/(c*d*m^2)), x] +
 (-Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(c^2*d*m))
, x] + Simp[f^2*((m - 1)/(c^2*d*m))   Int[(f*x)^(m - 2)*(d + e*x^2)^(q + 1)
*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2
*d] && EqQ[m + 2*q + 2, 0] && LtQ[q, -1]
```

rule 5475
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_)*((d_) + (e_.)
*(x_)^2)^(q_), x_Symbol] :> Simp[b*p*(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*Ar
cTan[c*x])^(p - 1)/(c*d*m^2)), x] + (-Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(q +
 1)*((a + b*ArcTan[c*x])^p/(c^2*d*m)), x] + Simp[f^2*((m - 1)/(c^2*d*m))
Int[(f*x)^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p, x], x] - Simp[
b^2*p*((p - 1)/m^2)   Int[(f*x)^m*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2)
, x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && EqQ[m + 2*
q + 2, 0] && LtQ[q, -1] && GtQ[p, 1]
```

rule 5499
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ
[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

## Maple [F]

$$\int \frac{x^5 \arctan (ax)^3}{(a^2 c\,x^2 + c)^{\frac{5}{2}}} dx$$

input
```
int(x^5*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x)
```

output
```
int(x^5*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x)
```

## Fricas [F]

$$\int \frac{x^5 \arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^5 \arctan{(ax)}^3}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^5*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^5*arctan(a*x)^3/(a^6*c^3*x^6 + 3*a^4*c^3*x^
4 + 3*a^2*c^3*x^2 + c^3), x)
```

## Sympy [F]

$$\int \frac{x^5 \arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^5 \operatorname{atan}^3{(ax)}}{(c\,(a^2x^2+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**5*atan(a*x)**3/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(x**5*atan(a*x)**3/(c*(a**2*x**2 + 1))**(5/2), x)
```

## Maxima [F]

$$\int \frac{x^5 \arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^5 \arctan{(ax)}^3}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^5*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(x^5*arctan(a*x)^3/(a^2*c*x^2 + c)^(5/2), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x^5 \arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^5*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^5 \arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^5 \operatorname{atan}(a\,x)^3}{(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int((x^5*atan(a*x)^3)/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int((x^5*atan(a*x)^3)/(c + a^2*c*x^2)^(5/2), x)
```

## Reduce [F]

$$\int \frac{x^5 \arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \frac{\int \frac{atan(ax)^3x^5}{\sqrt{a^2x^2+1}\,a^4x^4+2\sqrt{a^2x^2+1}\,a^2x^2+\sqrt{a^2x^2+1}}dx}{\sqrt{c}\,c^2}$$

input
```
int(x^5*atan(a*x)^3/(a^2*c*x^2+c)^(5/2),x)
```

output
```
int((atan(a*x)**3*x**5)/(sqrt(a**2*x**2 + 1)*a**4*x**4 + 2*sqrt(a**2*x**2
+ 1)*a**2*x**2 + sqrt(a**2*x**2 + 1)),x)/(sqrt(c)*c**2)
```

# 3.450 $\int \frac{x^4 \arctan(ax)^3}{\left(c+a^2cx^2\right)^{5/2}}\,dx$

## Optimal result

Integrand size = 24, antiderivative size = 622

$$\int \frac{x^4 \arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = -\frac{2}{27a^5c\,(c+a^2cx^2)^{3/2}} + \frac{68}{9a^5c^2\sqrt{c+a^2cx^2}} + \frac{2x^3\arctan(ax)}{9a^2c\,(c+a^2cx^2)^{3/2}} + \frac{22x\arctan(ax)}{3a^4c^2\sqrt{c+a^2cx^2}} - \frac{x^2\arctan(ax)^2}{3a^3c\,(c+a^2cx^2)^{3/2}} - \frac{11\arctan(ax)^2}{3a^5c^2\sqrt{c+a^2cx^2}} - \frac{x^3\arctan(ax)^3}{3a^2c\,(c+a^2cx^2)^{3/2}} - \frac{x\arctan(ax)^3}{a^4c^2\sqrt{c+a^2cx^2}} - \frac{2i\sqrt{1+a^2x^2}\arctan\left(e^{i\arctan(ax)}\right)\arctan(ax)^3}{a^5c^2\sqrt{c+a^2cx^2}} + \frac{3i\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)}{a^5c^2\sqrt{c+a^2cx^2}} - \frac{3i\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{PolyLog}\left(2,ie^{i\arctan(ax)}\right)}{a^5c^2\sqrt{c+a^2cx^2}} - \frac{6\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(3,-ie^{i\arctan(ax)}\right)}{a^5c^2\sqrt{c+a^2cx^2}} + \frac{6\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(3,ie^{i\arctan(ax)}\right)}{a^5c^2\sqrt{c+a^2cx^2}} - \frac{6i\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(4,-ie^{i\arctan(ax)}\right)}{a^5c^2\sqrt{c+a^2cx^2}} + \frac{6i\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(4,ie^{i\arctan(ax)}\right)}{a^5c^2\sqrt{c+a^2cx^2}}$$

output
```
-2/27/a^5/c/(a^2*c*x^2+c)^(3/2)+68/9/a^5/c^2/(a^2*c*x^2+c)^(1/2)+2/9*x^3*a
rctan(a*x)/a^2/c/(a^2*c*x^2+c)^(3/2)+22/3*x*arctan(a*x)/a^4/c^2/(a^2*c*x^2
+c)^(1/2)-1/3*x^2*arctan(a*x)^2/a^3/c/(a^2*c*x^2+c)^(3/2)-11/3*arctan(a*x)
^2/a^5/c^2/(a^2*c*x^2+c)^(1/2)-1/3*x^3*arctan(a*x)^3/a^2/c/(a^2*c*x^2+c)^(
3/2)-x*arctan(a*x)^3/a^4/c^2/(a^2*c*x^2+c)^(1/2)-2*I*(a^2*x^2+1)^(1/2)*arc
tan((1+I*a*x)/(a^2*x^2+1)^(1/2))*arctan(a*x)^3/a^5/c^2/(a^2*c*x^2+c)^(1/2)
+3*I*(a^2*x^2+1)^(1/2)*arctan(a*x)^2*polylog(2,-I*(1+I*a*x)/(a^2*x^2+1)^(1
/2))/a^5/c^2/(a^2*c*x^2+c)^(1/2)-3*I*(a^2*x^2+1)^(1/2)*arctan(a*x)^2*polyl
og(2,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a^5/c^2/(a^2*c*x^2+c)^(1/2)-6*(a^2*x^2
+1)^(1/2)*arctan(a*x)*polylog(3,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a^5/c^2/(a
^2*c*x^2+c)^(1/2)+6*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(3,I*(1+I*a*x)/(a
^2*x^2+1)^(1/2))/a^5/c^2/(a^2*c*x^2+c)^(1/2)-6*I*(a^2*x^2+1)^(1/2)*polylog
(4,-I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a^5/c^2/(a^2*c*x^2+c)^(1/2)+6*I*(a^2*x^
2+1)^(1/2)*polylog(4,I*(1+I*a*x)/(a^2*x^2+1)^(1/2))/a^5/c^2/(a^2*c*x^2+c)^
(1/2)
```

## Mathematica [A] (verified)

Time = 2.14 (sec) , antiderivative size = 691, normalized size of antiderivative = 1.11

$$\int \frac{x^4 \arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx =$$
$$-\frac{\sqrt{c(1+a^2x^2)}\left(189i\pi^4 - \frac{12960}{\sqrt{1+a^2x^2}} + 216i\pi^3\arctan(ax) - \frac{12960ax\arctan(ax)}{\sqrt{1+a^2x^2}} - 648i\pi^2\arctan(ax)^2 + \frac{6480\,\mathrm{arcta}}{\sqrt{1+a}}\right.}{}$$

input
```
Integrate[(x^4*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(5/2),x]
```

output
```
-1/1728*(Sqrt[c*(1 + a^2*x^2)]*((189*I)*Pi^4 - 12960/Sqrt[1 + a^2*x^2] + (
216*I)*Pi^3*ArcTan[a*x] - (12960*a*x*ArcTan[a*x])/Sqrt[1 + a^2*x^2] - (648
*I)*Pi^2*ArcTan[a*x]^2 + (6480*ArcTan[a*x]^2)/Sqrt[1 + a^2*x^2] + (864*I)*
Pi*ArcTan[a*x]^3 + (2160*a*x*ArcTan[a*x]^3)/Sqrt[1 + a^2*x^2] - (432*I)*Ar
cTan[a*x]^4 + 32*Cos[3*ArcTan[a*x]] - 144*ArcTan[a*x]^2*Cos[3*ArcTan[a*x]]
 - 1296*Pi^2*ArcTan[a*x]*Log[1 - I/E^(I*ArcTan[a*x])] + 2592*Pi*ArcTan[a*x
]^2*Log[1 - I/E^(I*ArcTan[a*x])] + 216*Pi^3*Log[1 + I/E^(I*ArcTan[a*x])] -
 1728*ArcTan[a*x]^3*Log[1 + I/E^(I*ArcTan[a*x])] - 216*Pi^3*Log[1 + I*E^(I
*ArcTan[a*x])] + 1296*Pi^2*ArcTan[a*x]*Log[1 + I*E^(I*ArcTan[a*x])] - 2592
*Pi*ArcTan[a*x]^2*Log[1 + I*E^(I*ArcTan[a*x])] + 1728*ArcTan[a*x]^3*Log[1
+ I*E^(I*ArcTan[a*x])] - 216*Pi^3*Log[Tan[(Pi + 2*ArcTan[a*x])/4]] - (5184
*I)*ArcTan[a*x]^2*PolyLog[2, (-I)/E^(I*ArcTan[a*x])] - (1296*I)*Pi*(Pi - 4
*ArcTan[a*x])*PolyLog[2, I/E^(I*ArcTan[a*x])] - (1296*I)*Pi^2*PolyLog[2, (
-I)*E^(I*ArcTan[a*x])] + (5184*I)*Pi*ArcTan[a*x]*PolyLog[2, (-I)*E^(I*ArcT
an[a*x])] - (5184*I)*ArcTan[a*x]^2*PolyLog[2, (-I)*E^(I*ArcTan[a*x])] - 10
368*ArcTan[a*x]*PolyLog[3, (-I)/E^(I*ArcTan[a*x])] + 5184*Pi*PolyLog[3, I/
E^(I*ArcTan[a*x])] - 5184*Pi*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] + 10368*Ar
cTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[a*x])] + (10368*I)*PolyLog[4, (-I)/E
^(I*ArcTan[a*x])] + (10368*I)*PolyLog[4, (-I)*E^(I*ArcTan[a*x])] + 96*ArcT
an[a*x]*Sin[3*ArcTan[a*x]] - 144*ArcTan[a*x]^3*Sin[3*ArcTan[a*x]]))/(a^...
```

## Rubi [A] (verified)

Time = 4.06 (sec) , antiderivative size = 563, normalized size of antiderivative = 0.91, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.792$, Rules used = {5499, 5479, 5475, 243, 53, 2009, 5465, 5429, 5499, 5425, 5423, 3042, 4669, 3011, 5433, 5429, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \arctan(ax)^3}{(a^2cx^2+c)^{5/2}} dx$$

$$\Big\downarrow \text{5499}$$

$$\frac{\int \frac{x^2 \arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx}{a^2c} - \frac{\int \frac{x^2 \arctan(ax)^3}{(a^2cx^2+c)^{5/2}}dx}{a^2}$$

$$\Big\downarrow \text{5479}$$

$$\frac{\int \frac{x^2 \arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx}{a^2c}-\frac{\frac{x^3 \arctan(ax)^3}{3c(a^2cx^2+c)^{3/2}}-a\int \frac{x^3 \arctan(ax)^2}{(a^2cx^2+c)^{5/2}}dx}{a^2}$$

$$\Big\downarrow \; 5475$$

$$\frac{\int \frac{x^2 \arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx}{a^2c}-\frac{\frac{x^3 \arctan(ax)^3}{3c(a^2cx^2+c)^{3/2}}-a\left(\frac{2\int \frac{x \arctan(ax)^2}{\left(a^2cx^2+c\right)^{3/2}}dx}{3a^2c}-\frac{2}{9}\int \frac{x^3}{(a^2cx^2+c)^{5/2}}dx-\frac{x^2 \arctan(ax)^2}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2x^3 \arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}\right)}{a^2}$$

$$\Big\downarrow \; 243$$

$$\frac{\int \frac{x^2 \arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx}{a^2c}-\frac{\frac{x^3 \arctan(ax)^3}{3c(a^2cx^2+c)^{3/2}}-a\left(\frac{2\int \frac{x \arctan(ax)^2}{\left(a^2cx^2+c\right)^{3/2}}dx}{3a^2c}-\frac{1}{9}\int \frac{x^2}{(a^2cx^2+c)^{5/2}}dx^2-\frac{x^2 \arctan(ax)^2}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2x^3 \arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}\right)}{a^2}$$

$$\Big\downarrow \; 53$$

$$\frac{\int \frac{x^2 \arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx}{a^2c}-\frac{\frac{x^3 \arctan(ax)^3}{3c(a^2cx^2+c)^{3/2}}-a\left(\frac{2\int \frac{x \arctan(ax)^2}{\left(a^2cx^2+c\right)^{3/2}}dx}{3a^2c}-\frac{1}{9}\int \left(\frac{1}{a^2c(a^2cx^2+c)^{3/2}}-\frac{1}{a^2(a^2cx^2+c)^{5/2}}\right)dx^2-\frac{x^2 \arctan(ax)^2}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2x^3 \arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}\right)}{a^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{\int \frac{x^2 \arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx}{a^2c}-\frac{\frac{x^3 \arctan(ax)^3}{3c(a^2cx^2+c)^{3/2}}-a\left(\frac{2\int \frac{x \arctan(ax)^2}{\left(a^2cx^2+c\right)^{3/2}}dx}{3a^2c}-\frac{x^2 \arctan(ax)^2}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2x^3 \arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}+\frac{1}{9}\left(\frac{2}{a^4c^2\sqrt{a^2cx^2+c}}-\frac{2}{3a^4c(a^2cx^2+c)^{3/2}}\right)\right)}{a^2}$$

$$\Big\downarrow \; 5465$$

$$\frac{\frac{x^3\arctan(ax)^3}{3c(a^2cx^2+c)^{3/2}}-a\left(\frac{2\left(\frac{2\int\frac{\arctan(ax)}{(a^2cx^2+c)^{3/2}}dx}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{x^2\arctan(ax)^2}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2x^3\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}+\frac{1}{9}\left(\frac{2}{a^4c^2\sqrt{a^2cx^2+c}}-\frac{}{3a^4c(}\right.\right.}{a^2}\quad\frac{\int\frac{x^2\arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx}{a^2c}-$$

$\downarrow$ 5429

$$\frac{\frac{x^3\arctan(ax)^3}{3c(a^2cx^2+c)^{3/2}}-a\left(-\frac{x^2\arctan(ax)^2}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{2x^3\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}+\frac{1}{9}\left(\frac{2}{a^4c^2\sqrt{a^2cx^2}}\right.\right.}{a^2}\quad\frac{\int\frac{x^2\arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx}{a^2c}-$$

$\downarrow$ 5499

$$\frac{\frac{x^3\arctan(ax)^3}{3c(a^2cx^2+c)^{3/2}}-a\left(-\frac{x^2\arctan(ax)^2}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{2x^3\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}+\frac{1}{9}\left(\frac{2}{a^4c^2\sqrt{a^2cx^2}}\right.\right.}{a^2}\quad\frac{\frac{\int\frac{\arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx}{a^2c}-\frac{\int\frac{\arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx}{a^2}}{a^2c}-$$

$\downarrow$ 5425

$$\frac{\frac{x^3\arctan(ax)^3}{3c(a^2cx^2+c)^{3/2}}-a\left(-\frac{x^2\arctan(ax)^2}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{2x^3\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}+\frac{1}{9}\left(\frac{2}{a^4c^2\sqrt{a^2cx^2}}\right.\right.}{a^2}\quad\frac{\frac{\sqrt{a^2x^2+1}\int\frac{\arctan(ax)^3}{\sqrt{a^2x^2+1}}dx}{a^2c\sqrt{a^2cx^2+c}}-\frac{\int\frac{\arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx}{a^2}}{a^2c}-$$

$\downarrow$ 5423

$$\frac{\frac{\sqrt{a^2x^2+1}\int \sqrt{a^2x^2+1}\arctan(ax)^3 d\arctan(ax)}{a^3c\sqrt{a^2cx^2+c}}-\frac{\int \frac{\arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx}{a^2}}{a^2c}-$$

$$\frac{\frac{x^3\arctan(ax)^3}{3c(a^2cx^2+c)^{3/2}}-a\left(-\frac{x^2\arctan(ax)^2}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{2x^3\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}+\frac{1}{9}\left(\frac{2}{a^4c^2\sqrt{a^2cx^2-}}\right.\right.}{a^2}$$

$\downarrow$ 3042

$$\frac{\frac{\sqrt{a^2x^2+1}\int \arctan(ax)^3\csc\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{a^3c\sqrt{a^2cx^2+c}}-\frac{\int \frac{\arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx}{a^2}}{a^2c}-$$

$$\frac{\frac{x^3\arctan(ax)^3}{3c(a^2cx^2+c)^{3/2}}-a\left(-\frac{x^2\arctan(ax)^2}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{2x^3\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}+\frac{1}{9}\left(\frac{2}{a^4c^2\sqrt{a^2cx^2-}}\right.\right.}{a^2}$$

$\downarrow$ 4669

$$-\frac{\frac{x^3\arctan(ax)^3}{3c(a^2cx^2+c)^{3/2}}-a\left(-\frac{x^2\arctan(ax)^2}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{2x^3\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}+\frac{1}{9}\left(\frac{2}{a^4c^2\sqrt{a^2cx}}\right.\right.}{a^2}$$

$$\frac{-\frac{\int \frac{\arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx}{a^2}+\frac{\sqrt{a^2x^2+1}\left(-3\int \arctan(ax)^2\log\left(1-ie^{i\arctan(ax)}\right)d\arctan(ax)+3\int \arctan(ax)^2\log\left(1+ie^{i\arctan(ax)}\right)d\arctan(ax)-2i\,a\right.}{a^3c\sqrt{a^2cx^2+c}}}{a^2c}$$

$\downarrow$ 3011

$$-\frac{\frac{x^3\arctan(ax)^3}{3c(a^2cx^2+c)^{3/2}}-a\left(-\frac{x^2\arctan(ax)^2}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{2x^3\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}+\frac{1}{9}\left(\frac{2}{a^4c^2\sqrt{a^2cx}}\right.\right.}{a^2}$$

$$\frac{-\frac{\int \frac{\arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx}{a^2}+\frac{\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int \arctan(ax)\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)d\arctan(ax)\right)-3(i\,a\right.}{a}}{a^2c}$$

$\downarrow$ 5433

$$-\frac{\frac{x^3\arctan(ax)^3}{3c(a^2cx^2+c)^{3/2}}-a\left(-\frac{x^2\arctan(ax)^2}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{2x^3\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}+\frac{1}{9}\left(\frac{2}{a^4c^2\sqrt{a^2cx^2+c}}\right.\right.}{a^2}$$

$$-\frac{-6\int\frac{\arctan(ax)}{(a^2cx^2+c)^{3/2}}dx+\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}}}{a^2}+\frac{\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int\arctan(ax)\operatorname{PolyLog}(2,-i\right.\right.}{}$$

$\Big\downarrow$ 5429

$$-\frac{\frac{x^3\arctan(ax)^3}{3c(a^2cx^2+c)^{3/2}}-a\left(-\frac{x^2\arctan(ax)^2}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{2x^3\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}+\frac{1}{9}\left(\frac{2}{a^4c^2\sqrt{a^2cx^2+c}}\right.\right.}{a^2}$$

$$-\frac{\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}}-6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a^2}+\frac{\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\int\arctan(ax)\operatorname{PolyL}\right.\right.}{}$$

$\Big\downarrow$ 7163

$$-\frac{\frac{x^3\arctan(ax)^3}{3c(a^2cx^2+c)^{3/2}}-a\left(-\frac{x^2\arctan(ax)^2}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{2x^3\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}+\frac{1}{9}\left(\frac{2}{a^4c^2\sqrt{a^2cx^2+c}}\right.\right.}{a^2}$$

$$-\frac{\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}}-6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a^2}+\frac{\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\left(i\int\operatorname{PolyLog}(3,-ie\right.\right.\right.}{}$$

$\Big\downarrow$ 2720

$$-\frac{\frac{x^3\arctan(ax)^3}{3c(a^2cx^2+c)^{3/2}}-a\left(-\frac{x^2\arctan(ax)^2}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{2x^3\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}+\frac{1}{9}\left(\frac{2}{a^4c^2\sqrt{a^2cx^2+c}}\right.\right.}{a^2}$$

$$-\frac{\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}}-6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a^2}+\frac{\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arctan(ax)}\right)-2i\left(\int e^{-i\arctan(ax)}\operatorname{Po}\right.\right.\right.}{}$$

$$\downarrow 7143$$

$$-\frac{\frac{x^3\arctan(ax)^3}{3c(a^2cx^2+c)^{3/2}}-a\left(-\frac{x^2\arctan(ax)^2}{3a^2c(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{2x^3\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}+\frac{1}{9}\left(\frac{2}{a^4c^2\sqrt{a^2cx^2+c}}\right.\right.}{a^2}$$

$$-\frac{\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}}-6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a^2}+\frac{\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}(2,-ie^{i\arctan(ax)})-2i\left(\operatorname{PolyLog}(4,-ie^{i\operatorname{arc}}\right.\right.\right.}{}$$

input
```
Int[(x^4*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(5/2),x]
```

output
```
-(((x^3*ArcTan[a*x]^3)/(3*c*(c + a^2*c*x^2)^(3/2)) - a*((-2/(3*a^4*c*(c +
a^2*c*x^2)^(3/2)) + 2/(a^4*c^2*Sqrt[c + a^2*c*x^2]))/9 + (2*x^3*ArcTan[a*x
])/(9*a*c*(c + a^2*c*x^2)^(3/2)) - (x^2*ArcTan[a*x]^2)/(3*a^2*c*(c + a^2*c
*x^2)^(3/2)) + (2*(-(ArcTan[a*x]^2/(a^2*c*Sqrt[c + a^2*c*x^2])) + (2*(1/(a
*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x])/(c*Sqrt[c + a^2*c*x^2])))/a))/(3
*a^2*c)))/a^2) + (-(((3*ArcTan[a*x]^2)/(a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcT
an[a*x]^3)/(c*Sqrt[c + a^2*c*x^2]) - 6*(1/(a*c*Sqrt[c + a^2*c*x^2]) + (x*A
rcTan[a*x])/(c*Sqrt[c + a^2*c*x^2])))/a^2) + (Sqrt[1 + a^2*x^2]*((-2*I)*Ar
cTan[E^(I*ArcTan[a*x])]*ArcTan[a*x]^3 + 3*(I*ArcTan[a*x]^2*PolyLog[2, (-I)
*E^(I*ArcTan[a*x])] - (2*I)*((-I)*ArcTan[a*x]*PolyLog[3, (-I)*E^(I*ArcTan[
a*x])] + PolyLog[4, (-I)*E^(I*ArcTan[a*x])])) - 3*(I*ArcTan[a*x]^2*PolyLog
[2, I*E^(I*ArcTan[a*x])] - (2*I)*((-I)*ArcTan[a*x]*PolyLog[3, I*E^(I*ArcTa
n[a*x])] + PolyLog[4, I*E^(I*ArcTan[a*x])]))))/(a^3*c*Sqrt[c + a^2*c*x^2])
)/(a^2*c)
```

## Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4669
```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Si
mp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
 x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x
))], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 5423
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[1/(c*Sqrt[d])   Subst[Int[(a + b*x)^p*Sec[x], x], x, ArcTan[
c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] && Gt
Q[d, 0]
```

rule 5425
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan[c*x])^
p/Sqrt[1 + c^2*x^2], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &
& IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5429
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTan[c*x])/(d*Sqr
t[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

rule 5433
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x] - Simp[b^2*p*(p
- 1)   Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2), x], x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5475
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_)*((d_) + (e_.)
*(x_)^2)^(q_), x_Symbol] :> Simp[b*p*(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*Ar
cTan[c*x])^(p - 1)/(c*d*m^2)), x] + (-Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(q +
 1)*((a + b*ArcTan[c*x])^p/(c^2*d*m)), x] + Simp[f^2*((m - 1)/(c^2*d*m))
Int[(f*x)^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p, x], x] - Simp[
b^2*p*((p - 1)/m^2)   Int[(f*x)^m*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2)
, x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && EqQ[m + 2*
q + 2, 0] && LtQ[q, -1] && GtQ[p, 1]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5499
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ
[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

## Maple [F]

$$\int \frac{x^4 \arctan(ax)^3}{(a^2c\,x^2+c)^{\frac{5}{2}}}dx$$

input
```
int(x^4*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x)
```

output
```
int(x^4*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x)
```

## Fricas [F]

$$\int \frac{x^4 \arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^4 \arctan(ax)^3}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^4*arctan(a*x)^3/(a^6*c^3*x^6 + 3*a^4*c^3*x^
4 + 3*a^2*c^3*x^2 + c^3), x)
```

## Sympy [F]

$$\int \frac{x^4 \arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^4 \operatorname{atan}^3(ax)}{(c(a^2x^2+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**4*atan(a*x)**3/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(x**4*atan(a*x)**3/(c*(a**2*x**2 + 1))**(5/2), x)
```

## Maxima [F]

$$\int \frac{x^4 \arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^4 \arctan(ax)^3}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(x^4*arctan(a*x)^3/(a^2*c*x^2 + c)^(5/2), x)
```

## Giac [F]

$$\int \frac{x^4 \arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^4 \arctan(ax)^3}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
integrate(x^4*arctan(a*x)^3/(a^2*c*x^2 + c)^(5/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^4 \arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^4 \operatorname{atan}(a\,x)^3}{(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int((x^4*atan(a*x)^3)/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int((x^4*atan(a*x)^3)/(c + a^2*c*x^2)^(5/2), x)
```

## Reduce [F]

$$\int \frac{x^4 \arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \frac{\int \frac{atan(ax)^3x^4}{\sqrt{a^2x^2+1}\,a^4x^4+2\sqrt{a^2x^2+1}\,a^2x^2+\sqrt{a^2x^2+1}}dx}{\sqrt{c}\,c^2}$$

input
```
int(x^4*atan(a*x)^3/(a^2*c*x^2+c)^(5/2),x)
```

output
```
int((atan(a*x)**3*x**4)/(sqrt(a**2*x**2 + 1)*a**4*x**4 + 2*sqrt(a**2*x**2
+ 1)*a**2*x**2 + sqrt(a**2*x**2 + 1)),x)/(sqrt(c)*c**2)
```

# 3.451 $\int \frac{x^3 \arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 237

$$\begin{aligned}\int \frac{x^3 \arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = & -\frac{2x^3}{27ac\,(c+a^2cx^2)^{3/2}}-\frac{40x}{9a^3c^2\sqrt{c+a^2cx^2}} \\ & +\frac{2x^2\arctan(ax)}{9a^2c\,(c+a^2cx^2)^{3/2}}+\frac{40\arctan(ax)}{9a^4c^2\sqrt{c+a^2cx^2}}+\frac{x^3\arctan(ax)^2}{3ac\,(c+a^2cx^2)^{3/2}} \\ & +\frac{2x\arctan(ax)^2}{a^3c^2\sqrt{c+a^2cx^2}}-\frac{x^2\arctan(ax)^3}{3a^2c\,(c+a^2cx^2)^{3/2}}-\frac{2\arctan(ax)^3}{3a^4c^2\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
-2/27*x^3/a/c/(a^2*c*x^2+c)^(3/2)-40/9*x/a^3/c^2/(a^2*c*x^2+c)^(1/2)+2/9*x
^2*arctan(a*x)/a^2/c/(a^2*c*x^2+c)^(3/2)+40/9*arctan(a*x)/a^4/c^2/(a^2*c*x
^2+c)^(1/2)+1/3*x^3*arctan(a*x)^2/a/c/(a^2*c*x^2+c)^(3/2)+2*x*arctan(a*x)^
2/a^3/c^2/(a^2*c*x^2+c)^(1/2)-1/3*x^2*arctan(a*x)^3/a^2/c/(a^2*c*x^2+c)^(3
/2)-2/3*arctan(a*x)^3/a^4/c^2/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.44

$$\int \frac{x^3 \arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \frac{\sqrt{c+a^2cx^2}\left(-2ax(60+61a^2x^2)+6(20+21a^2x^2)\arctan(ax)+9ax(6+7a^2x^2)\operatorname{ar}\right.}{27a^4c^3\left(1+a^2x^2\right)^2}$$

input
```
Integrate[(x^3*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(Sqrt[c + a^2*c*x^2]*(-2*a*x*(60 + 61*a^2*x^2) + 6*(20 + 21*a^2*x^2)*ArcTa
n[a*x] + 9*a*x*(6 + 7*a^2*x^2)*ArcTan[a*x]^2 - 9*(2 + 3*a^2*x^2)*ArcTan[a*
x]^3))/(27*a^4*c^3*(1 + a^2*x^2)^2)
```

## Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.29, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {5475, 5465, 5433, 208, 5473, 5465, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \arctan(ax)^3}{(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{5475}$$

$$\frac{2\int \frac{x\arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx}{3a^2c} - \frac{2}{3}\int \frac{x^3\arctan(ax)}{(a^2cx^2+c)^{5/2}}dx - \frac{x^2\arctan(ax)^3}{3a^2c\,(a^2cx^2+c)^{3/2}} + \frac{x^3\arctan(ax)^2}{3ac\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5465}$$

$$\frac{2\left(\frac{3\int \frac{\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{a} - \frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c} - \frac{2}{3}\int \frac{x^3\arctan(ax)}{(a^2cx^2+c)^{5/2}}dx - \frac{x^2\arctan(ax)^3}{3a^2c\,(a^2cx^2+c)^{3/2}} + \frac{x^3\arctan(ax)^2}{3ac\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5433}$$

$$\frac{2\left(\frac{3\left(-2\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}}dx+\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{2}{3}\int \frac{x^3\arctan(ax)}{\left(a^2cx^2+c\right)^{5/2}}dx-\frac{x^2\arctan(ax)^3}{3a^2c\left(a^2cx^2+c\right)^{3/2}}+\frac{x^3\arctan(ax)^2}{3ac\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{208}$$

$$-\frac{2}{3}\int \frac{x^3\arctan(ax)}{\left(a^2cx^2+c\right)^{5/2}}dx-\frac{x^2\arctan(ax)^3}{3a^2c\left(a^2cx^2+c\right)^{3/2}}+\frac{2\left(\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{x^3\arctan(ax)^2}{3ac\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{5473}$$

$$-\frac{2}{3}\left(\frac{2\int \frac{x\arctan(ax)}{\left(a^2cx^2+c\right)^{3/2}}dx}{3a^2c}-\frac{x^2\arctan(ax)}{3a^2c\left(a^2cx^2+c\right)^{3/2}}+\frac{x^3}{9ac\left(a^2cx^2+c\right)^{3/2}}\right)-\frac{x^2\arctan(ax)^3}{3a^2c\left(a^2cx^2+c\right)^{3/2}}+\frac{2\left(\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{x^3\arctan(ax)^2}{3ac\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{5465}$$

$$-\frac{2}{3}\left(\frac{2\left(\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}}dx}{a}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{x^2\arctan(ax)}{3a^2c\left(a^2cx^2+c\right)^{3/2}}+\frac{x^3}{9ac\left(a^2cx^2+c\right)^{3/2}}\right)-\frac{x^2\arctan(ax)^3}{3a^2c\left(a^2cx^2+c\right)^{3/2}}+\frac{2\left(\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{x^3\arctan(ax)^2}{3ac\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{208}$$

$$-\frac{x^2\arctan(ax)^3}{3a^2c\left(a^2cx^2+c\right)^{3/2}}+\frac{2\left(\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{x^3\arctan(ax)^2}{3ac\left(a^2cx^2+c\right)^{3/2}}-\frac{2}{3}\left(-\frac{x^2\arctan(ax)}{3a^2c\left(a^2cx^2+c\right)^{3/2}}+\frac{2\left(\frac{x}{ac\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{x^3}{9ac\left(a^2cx^2+c\right)^{3/2}}\right)$$

input
```
Int[(x^3*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(x^3*ArcTan[a*x]^2)/(3*a*c*(c + a^2*c*x^2)^(3/2)) - (x^2*ArcTan[a*x]^3)/(3
*a^2*c*(c + a^2*c*x^2)^(3/2)) - (2*(x^3/(9*a*c*(c + a^2*c*x^2)^(3/2)) - (x
^2*ArcTan[a*x])/(3*a^2*c*(c + a^2*c*x^2)^(3/2)) + (2*(x/(a*c*Sqrt[c + a^2*
c*x^2]) - ArcTan[a*x]/(a^2*c*Sqrt[c + a^2*c*x^2])))/(3*a^2*c)))/3 + (2*(-(
ArcTan[a*x]^3/(a^2*c*Sqrt[c + a^2*c*x^2])) + (3*((-2*x)/(c*Sqrt[c + a^2*c*
x^2]) + (2*ArcTan[a*x])/(a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x]^2)/(c*S
qrt[c + a^2*c*x^2])))/a))/(3*a^2*c)
```

### Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 5433
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x] - Simp[b^2*p*(p
- 1)   Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2), x], x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5473
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)
^2)^(q_), x_Symbol] :> Simp[b*(f*x)^m*((d + e*x^2)^(q + 1)/(c*d*m^2)), x] +
 (-Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(c^2*d*m))
, x] + Simp[f^2*((m - 1)/(c^2*d*m))   Int[(f*x)^(m - 2)*(d + e*x^2)^(q + 1)
*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[e, c^2
*d] && EqQ[m + 2*q + 2, 0] && LtQ[q, -1]
```

rule 5475
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_)*((d_) + (e_.)
*(x_)^2)^(q_), x_Symbol] :> Simp[b*p*(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*Ar
cTan[c*x])^(p - 1)/(c*d*m^2)), x] + (-Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(q +
 1)*((a + b*ArcTan[c*x])^p/(c^2*d*m)), x] + Simp[f^2*((m - 1)/(c^2*d*m))
Int[(f*x)^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p, x], x] - Simp[
b^2*p*((p - 1)/m^2)   Int[(f*x)^m*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2)
, x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && EqQ[m + 2*
q + 2, 0] && LtQ[q, -1] && GtQ[p, 1]
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.86 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.32

| method | result |
|---|---|
| default | $-\frac{\left(9i\arctan(ax)^2+9\arctan(ax)^3-2i-6\arctan(ax)\right)\left(ia^3x^3+3a^2x^2-3iax-1\right)\sqrt{c(ax-i)(ax+i)}}{216(a^2x^2+1)^2a^4c^3}-\frac{3\left(\arctan(ax)^3-6\arctan(ax)\right.}{}$ |
| orering | Expression too large to display |

input
```
int(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/216*(9*I*arctan(a*x)^2+9*arctan(a*x)^3-2*I-6*arctan(a*x))*(I*a^3*x^3+3*
a^2*x^2-3*I*a*x-1)*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^2/a^4/c^3-3/8*(ar
ctan(a*x)^3-6*arctan(a*x)+3*I*arctan(a*x)^2-6*I)*(1+I*a*x)*(c*(a*x-I)*(a*x
+I))^(1/2)/c^3/a^4/(a^2*x^2+1)+3/8*(c*(a*x-I)*(a*x+I))^(1/2)*(I*a*x-1)*(ar
ctan(a*x)^3-6*arctan(a*x)-3*I*arctan(a*x)^2+6*I)/c^3/a^4/(a^2*x^2+1)+1/216
*(c*(a*x-I)*(a*x+I))^(1/2)*(I*a^3*x^3-3*a^2*x^2-3*I*a*x+1)*(-9*I*arctan(a*
x)^2+9*arctan(a*x)^3+2*I-6*arctan(a*x))/c^3/a^4/(a^4*x^4+2*a^2*x^2+1)
```

## Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.48

$$\int \frac{x^3 \arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = -\frac{\left(122\,a^3x^3 + 9\left(3\,a^2x^2+2\right)\arctan\left(ax\right)^3 - 9\left(7\,a^3x^3+6\,ax\right)\arctan\left(ax\right)^2 + 120\,ax - 6\left(21\,a^2x^2+20\right)\mathrm{arct}}{27\left(a^8c^3x^4+2\,a^6c^3x^2+a^4c^3\right)}$$

input
```
integrate(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
-1/27*(122*a^3*x^3 + 9*(3*a^2*x^2 + 2)*arctan(a*x)^3 - 9*(7*a^3*x^3 + 6*a*
x)*arctan(a*x)^2 + 120*a*x - 6*(21*a^2*x^2 + 20)*arctan(a*x))*sqrt(a^2*c*x
^2 + c)/(a^8*c^3*x^4 + 2*a^6*c^3*x^2 + a^4*c^3)
```

## Sympy [F]

$$\int \frac{x^3 \arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^3 \operatorname{atan}^3\left(ax\right)}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**3*atan(a*x)**3/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(x**3*atan(a*x)**3/(c*(a**2*x**2 + 1))**(5/2), x)
```

## Maxima [F]

$$\int \frac{x^3 \arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^3 \arctan(ax)^3}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(x^3*arctan(a*x)^3/(a^2*c*x^2 + c)^(5/2), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x^3 \arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^3 \,\mathrm{atan}(a\,x)^3}{(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int((x^3*atan(a*x)^3)/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int((x^3*atan(a*x)^3)/(c + a^2*c*x^2)^(5/2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.86

$$\int \frac{x^3 \arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \frac{\sqrt{c}\left(-27\sqrt{a^2x^2+1}\,atan(ax)^3\,a^2x^2-18\sqrt{a^2x^2+1}\,atan(ax)^3+63\sqrt{a^2x^2+1}\,atan(\right.}{}$$

input
```
int(x^3*atan(a*x)^3/(a^2*c*x^2+c)^(5/2),x)
```

output
```
(sqrt(c)*( - 27*sqrt(a**2*x**2 + 1)*atan(a*x)**3*a**2*x**2 - 18*sqrt(a**2*
x**2 + 1)*atan(a*x)**3 + 63*sqrt(a**2*x**2 + 1)*atan(a*x)**2*a**3*x**3 + 5
4*sqrt(a**2*x**2 + 1)*atan(a*x)**2*a*x + 126*sqrt(a**2*x**2 + 1)*atan(a*x)
*a**2*x**2 + 120*sqrt(a**2*x**2 + 1)*atan(a*x) - 122*sqrt(a**2*x**2 + 1)*a
**3*x**3 - 120*sqrt(a**2*x**2 + 1)*a*x + 38*a**4*x**4 + 76*a**2*x**2 + 38)
)/(27*a**4*c**3*(a**4*x**4 + 2*a**2*x**2 + 1))
```

# 3.452 $\int \frac{x^2 \arctan(ax)^3}{(c+a^2cx^2)^{5/2}} \, dx$



## Optimal result

Integrand size = 24, antiderivative size = 199

$$\int \frac{x^2 \arctan(ax)^3}{(c+a^2cx^2)^{5/2}} \, dx = \frac{2}{27a^3c\,(c+a^2cx^2)^{3/2}} - \frac{14}{9a^3c^2\sqrt{c+a^2cx^2}} - \frac{2x^3\arctan(ax)}{9c\,(c+a^2cx^2)^{3/2}} - \frac{4x\arctan(ax)}{3a^2c^2\sqrt{c+a^2cx^2}} + \frac{x^2\arctan(ax)^2}{3ac\,(c+a^2cx^2)^{3/2}} + \frac{2\arctan(ax)^2}{3a^3c^2\sqrt{c+a^2cx^2}} + \frac{x^3\arctan(ax)^3}{3c\,(c+a^2cx^2)^{3/2}}$$

output
```
2/27/a^3/c/(a^2*c*x^2+c)^(3/2)-14/9/a^3/c^2/(a^2*c*x^2+c)^(1/2)-2/9*x^3*ar
ctan(a*x)/c/(a^2*c*x^2+c)^(3/2)-4/3*x*arctan(a*x)/a^2/c^2/(a^2*c*x^2+c)^(1
/2)+1/3*x^2*arctan(a*x)^2/a/c/(a^2*c*x^2+c)^(3/2)+2/3*arctan(a*x)^2/a^3/c^
2/(a^2*c*x^2+c)^(1/2)+1/3*x^3*arctan(a*x)^3/c/(a^2*c*x^2+c)^(3/2)
```

## Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.48

$$\int \frac{x^2 \arctan(ax)^3}{(c+a^2cx^2)^{5/2}} \, dx = \frac{\sqrt{c+a^2cx^2}(-40-42a^2x^2-6ax(6+7a^2x^2)\arctan(ax)+9(2+3a^2x^2)\arctan(ax}{27a^3c^3\,(1+a^2x^2)^2}$$

input
```
Integrate[(x^2*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(Sqrt[c + a^2*c*x^2]*(-40 - 42*a^2*x^2 - 6*a*x*(6 + 7*a^2*x^2)*ArcTan[a*x]
 + 9*(2 + 3*a^2*x^2)*ArcTan[a*x]^2 + 9*a^3*x^3*ArcTan[a*x]^3))/(27*a^3*c^3
*(1 + a^2*x^2)^2)
```

## Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.20, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {5479, 5475, 243, 53, 2009, 5465, 5429}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \arctan(ax)^3}{(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{5479}$$

$$\frac{x^3\arctan(ax)^3}{3c(a^2cx^2+c)^{3/2}} - a\int \frac{x^3\arctan(ax)^2}{(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{5475}$$

$$\frac{x^3\arctan(ax)^3}{3c(a^2cx^2+c)^{3/2}} - a\left(\frac{2\int \frac{x\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{3a^2c} - \frac{2}{9}\int \frac{x^3}{(a^2cx^2+c)^{5/2}}dx - \frac{x^2\arctan(ax)^2}{3a^2c(a^2cx^2+c)^{3/2}} + \frac{2x^3\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}\right)$$

$$\Big\downarrow \text{243}$$

$$\frac{x^3\arctan(ax)^3}{3c(a^2cx^2+c)^{3/2}} - a\left(\frac{2\int \frac{x\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{3a^2c} - \frac{1}{9}\int \frac{x^2}{(a^2cx^2+c)^{5/2}}dx^2 - \frac{x^2\arctan(ax)^2}{3a^2c(a^2cx^2+c)^{3/2}} + \frac{2x^3\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}\right)$$

$$\Big\downarrow \text{53}$$

$$\frac{x^3\arctan(ax)^3}{3c\left(a^2cx^2+c\right)^{3/2}}-a\left(\frac{2\int\frac{x\arctan(ax)^2}{\left(a^2cx^2+c\right)^{3/2}}dx}{3a^2c}-\frac{1}{9}\int\left(\frac{1}{a^2c\left(a^2cx^2+c\right)^{3/2}}-\frac{1}{a^2\left(a^2cx^2+c\right)^{5/2}}\right)dx^2-\frac{x^2\arctan(ax)^2}{3a^2c\left(a^2cx^2+c\right)^{3/2}}+\frac{2x^3\arctan}{9ac\left(a^2cx^2+\right.}\right.$$

$\big\downarrow$ 2009

$$\frac{x^3\arctan(ax)^3}{3c\left(a^2cx^2+c\right)^{3/2}}-a\left(\frac{2\int\frac{x\arctan(ax)^2}{\left(a^2cx^2+c\right)^{3/2}}dx}{3a^2c}-\frac{x^2\arctan(ax)^2}{3a^2c\left(a^2cx^2+c\right)^{3/2}}+\frac{2x^3\arctan(ax)}{9ac\left(a^2cx^2+c\right)^{3/2}}+\frac{1}{9}\left(\frac{2}{a^4c^2\sqrt{a^2cx^2+c}}-\frac{2}{3a^4c\left(a^2cx^2+c\right)^{3/2}}\right)\right.$$

$\big\downarrow$ 5465

$$\frac{x^3\arctan(ax)^3}{3c\left(a^2cx^2+c\right)^{3/2}}-a\left(\frac{2\left(\frac{2\int\frac{\arctan(ax)}{\left(a^2cx^2+c\right)^{3/2}}dx}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{x^2\arctan(ax)^2}{3a^2c\left(a^2cx^2+c\right)^{3/2}}+\frac{2x^3\arctan(ax)}{9ac\left(a^2cx^2+c\right)^{3/2}}+\frac{1}{9}\left(\frac{2}{a^4c^2\sqrt{a^2cx^2+c}}-\frac{}{3a}\right.\right.$$

$\big\downarrow$ 5429

$$\frac{x^3\arctan(ax)^3}{3c\left(a^2cx^2+c\right)^{3/2}}-a\left(-\frac{x^2\arctan(ax)^2}{3a^2c\left(a^2cx^2+c\right)^{3/2}}+\frac{2\left(\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^2}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}+\frac{2x^3\arctan(ax)}{9ac\left(a^2cx^2+c\right)^{3/2}}+\frac{1}{9}\left(\frac{2}{a^4c^2\sqrt{a^2cx^2}}\right.\right.$$

input
```
Int[(x^2*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(x^3*ArcTan[a*x]^3)/(3*c*(c + a^2*c*x^2)^(3/2)) - a*((-2/(3*a^4*c*(c + a^2
*c*x^2)^(3/2)) + 2/(a^4*c^2*Sqrt[c + a^2*c*x^2]))/9 + (2*x^3*ArcTan[a*x])/
(9*a*c*(c + a^2*c*x^2)^(3/2)) - (x^2*ArcTan[a*x]^2)/(3*a^2*c*(c + a^2*c*x^
2)^(3/2)) + (2*(-(ArcTan[a*x]^2/(a^2*c*Sqrt[c + a^2*c*x^2])) + (2*(1/(a*c*
Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x])/(c*Sqrt[c + a^2*c*x^2])))/a))/(3*a^
2*c))
```

**Defintions of rubi rules used**

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5429
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTan[c*x])/(d*Sqr
t[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5475
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_)*((d_) + (e_.)
*(x_)^2)^(q_), x_Symbol] :> Simp[b*p*(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*Ar
cTan[c*x])^(p - 1)/(c*d*m^2)), x] + (-Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(q +
 1)*((a + b*ArcTan[c*x])^p/(c^2*d*m)), x] + Simp[f^2*((m - 1)/(c^2*d*m))
Int[(f*x)^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p, x], x] - Simp[
b^2*p*((p - 1)/m^2)   Int[(f*x)^m*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2)
, x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && EqQ[m + 2*
q + 2, 0] && LtQ[q, -1] && GtQ[p, 1]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.57 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.55

| method | result |
|---|---|
| default | $\frac{\left(9i\arctan(ax)^2+9\arctan(ax)^3-2i-6\arctan(ax)\right)(a^3x^3-3ia^2x^2-3ax+i)\sqrt{c(ax-i)(ax+i)}}{216(a^2x^2+1)^2a^3c^3}+\frac{\left(\arctan(ax)^3-6\arctan(ax)+3i\,a\right.}{8c^3}$ |
| orering | Expression too large to display |

input
```
int(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/216*(9*I*arctan(a*x)^2+9*arctan(a*x)^3-2*I-6*arctan(a*x))*(a^3*x^3-3*I*a
^2*x^2-3*a*x+I)*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^2/a^3/c^3+1/8*(arcta
n(a*x)^3-6*arctan(a*x)+3*I*arctan(a*x)^2-6*I)*(a*x-I)*(c*(a*x-I)*(a*x+I))^
(1/2)/c^3/a^3/(a^2*x^2+1)+1/8*(c*(a*x-I)*(a*x+I))^(1/2)*(a*x+I)*(arctan(a*
x)^3-6*arctan(a*x)-3*I*arctan(a*x)^2+6*I)/c^3/a^3/(a^2*x^2+1)+1/216*(-9*I*
arctan(a*x)^2+9*arctan(a*x)^3+2*I-6*arctan(a*x))*(c*(a*x-I)*(a*x+I))^(1/2)
*(a^3*x^3+3*I*a^2*x^2-3*a*x-I)/(a^4*x^4+2*a^2*x^2+1)/a^3/c^3
```

## Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.53

$$\int \frac{x^2 \arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \frac{\left(9\,a^3x^3\arctan\left(ax\right)^3 - 42\,a^2x^2 + 9\left(3\,a^2x^2+2\right)\arctan\left(ax\right)^2 - 6\left(7\,a^3x^3+6\,ax\right)\mathrm{arc}\right.}{27\left(a^7c^3x^4+2\,a^5c^3x^2+a^3c^3\right)}$$

input
```
integrate(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
1/27*(9*a^3*x^3*arctan(a*x)^3 - 42*a^2*x^2 + 9*(3*a^2*x^2 + 2)*arctan(a*x)
^2 - 6*(7*a^3*x^3 + 6*a*x)*arctan(a*x) - 40)*sqrt(a^2*c*x^2 + c)/(a^7*c^3*
x^4 + 2*a^5*c^3*x^2 + a^3*c^3)
```

## Sympy [F]

$$\int \frac{x^2 \arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^2\operatorname{atan}^3(ax)}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**2*atan(a*x)**3/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(x**2*atan(a*x)**3/(c*(a**2*x**2 + 1))**(5/2), x)
```

## Maxima [F]

$$\int \frac{x^2 \arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^2\arctan\left(ax\right)^3}{\left(a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(x^2*arctan(a*x)^3/(a^2*c*x^2 + c)^(5/2), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x^2 \arctan(ax)^3}{(c+a^2cx^2)^{5/2}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \arctan(ax)^3}{(c+a^2cx^2)^{5/2}} \, dx = \int \frac{x^2 \operatorname{atan}(a\,x)^3}{(c\,a^2\,x^2+c)^{5/2}} \, dx$$

input
```
int((x^2*atan(a*x)^3)/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int((x^2*atan(a*x)^3)/(c + a^2*c*x^2)^(5/2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.30

$$\int \frac{x^2 \arctan(ax)^3}{(c+a^2cx^2)^{5/2}} \, dx = \frac{\sqrt{c}\left(84atan\left(\sqrt{a^2x^2+1}+ax\right)a^4x^4+168atan\left(\sqrt{a^2x^2+1}+ax\right)a^2x^2+84atan\left(\sqrt{}\right.\right.}{}$$

input
```
int(x^2*atan(a*x)^3/(a^2*c*x^2+c)^(5/2),x)
```

output
```
(sqrt(c)*(84*atan(sqrt(a**2*x**2 + 1) + a*x)*a**4*x**4 + 168*atan(sqrt(a**
2*x**2 + 1) + a*x)*a**2*x**2 + 84*atan(sqrt(a**2*x**2 + 1) + a*x) + 9*sqrt
(a**2*x**2 + 1)*atan(a*x)**3*a**3*x**3 + 27*sqrt(a**2*x**2 + 1)*atan(a*x)*
*2*a**2*x**2 + 18*sqrt(a**2*x**2 + 1)*atan(a*x)**2 - 42*sqrt(a**2*x**2 + 1
)*atan(a*x)*a**3*x**3 - 36*sqrt(a**2*x**2 + 1)*atan(a*x)*a*x - 42*atan(a*x
)*a**4*x**4 - 84*atan(a*x)*a**2*x**2 - 42*atan(a*x) - 42*sqrt(a**2*x**2 +
1)*a**2*x**2 - 40*sqrt(a**2*x**2 + 1)))/(27*a**3*c**3*(a**4*x**4 + 2*a**2*
x**2 + 1))
```

# 3.453 $\int \frac{x \arctan(ax)^3}{(c+a^2cx^2)^{5/2}} \, dx$



## Optimal result

Integrand size = 22, antiderivative size = 199

$$\int \frac{x \arctan(ax)^3}{(c+a^2cx^2)^{5/2}} \, dx = -\frac{2x}{27ac(c+a^2cx^2)^{3/2}} - \frac{40x}{27ac^2\sqrt{c+a^2cx^2}} + \frac{2\arctan(ax)}{9a^2c(c+a^2cx^2)^{3/2}} + \frac{4\arctan(ax)}{3a^2c^2\sqrt{c+a^2cx^2}} + \frac{x\arctan(ax)^2}{3ac(c+a^2cx^2)^{3/2}} + \frac{2x\arctan(ax)^2}{3ac^2\sqrt{c+a^2cx^2}} - \frac{\arctan(ax)^3}{3a^2c(c+a^2cx^2)^{3/2}}$$

output
```
-2/27*x/a/c/(a^2*c*x^2+c)^(3/2)-40/27*x/a/c^2/(a^2*c*x^2+c)^(1/2)+2/9*arct
an(a*x)/a^2/c/(a^2*c*x^2+c)^(3/2)+4/3*arctan(a*x)/a^2/c^2/(a^2*c*x^2+c)^(1
/2)+1/3*x*arctan(a*x)^2/a/c/(a^2*c*x^2+c)^(3/2)+2/3*x*arctan(a*x)^2/a/c^2/
(a^2*c*x^2+c)^(1/2)-1/3*arctan(a*x)^3/a^2/c/(a^2*c*x^2+c)^(3/2)
```

## Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.46

$$\int \frac{x\arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \frac{\sqrt{c+a^2cx^2}(-2ax(21+20a^2x^2)+6(7+6a^2x^2)\arctan(ax)+9ax(3+2a^2x^2)\operatorname{arcta}}{27c^3\left(a+a^3x^2\right)^2}$$

input
```
Integrate[(x*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(Sqrt[c + a^2*c*x^2]*(-2*a*x*(21 + 20*a^2*x^2) + 6*(7 + 6*a^2*x^2)*ArcTan[
a*x] + 9*a*x*(3 + 2*a^2*x^2)*ArcTan[a*x]^2 - 9*ArcTan[a*x]^3))/(27*c^3*(a
+ a^3*x^2)^2)
```

## Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {5465, 5435, 209, 208, 5433, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\arctan(ax)^3}{(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{\int \frac{\arctan(ax)^2}{(a^2cx^2+c)^{5/2}}dx}{a}-\frac{\arctan(ax)^3}{3a^2c\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5435}$$

$$\frac{\frac{2\int \frac{\arctan(ax)^2}{\left(a^2cx^2+c\right)^{3/2}}dx}{3c}-\frac{2}{9}\int \frac{1}{(a^2cx^2+c)^{5/2}}dx+\frac{x\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}}+\frac{2\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}}{a}-\frac{\arctan(ax)^3}{3a^2c\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{209}$$

$$\frac{\frac{2\int \frac{\arctan(ax)^2}{\left(a^2cx^2+c\right)^{3/2}}dx}{3c}-\frac{2}{9}\left(\frac{2\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}}dx}{3c}+\frac{x}{3c(a^2cx^2+c)^{3/2}}\right)+\frac{x\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}}+\frac{2\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}}{a}-\frac{\arctan(ax)^3}{3a^2c\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \; 208$$

$$\frac{\frac{2\int \frac{\arctan(ax)^2}{\left(a^2cx^2+c\right)^{3/2}}dx}{3c}+\frac{x\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}}+\frac{2\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}-\frac{2}{9}\left(\frac{2x}{3c^2\sqrt{a^2cx^2+c}}+\frac{x}{3c(a^2cx^2+c)^{3/2}}\right)}{a}-\frac{\arctan(ax)^3}{3a^2c\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \; 5433$$

$$\frac{\frac{2\left(-2\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}}dx+\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{x\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}}+\frac{2\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}-\frac{2}{9}\left(\frac{2x}{3c^2\sqrt{a^2cx^2+c}}+\frac{x}{3c(a^2cx^2+c)^{3/2}}\right)}{a}-\frac{\arctan(ax)^3}{3a^2c\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \; 208$$

$$\frac{\frac{x\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}}+\frac{2\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{3c}-\frac{2}{9}\left(\frac{2x}{3c^2\sqrt{a^2cx^2+c}}+\frac{x}{3c(a^2cx^2+c)^{3/2}}\right)}{a}-\frac{\arctan(ax)^3}{3a^2c\left(a^2cx^2+c\right)^{3/2}}$$

input
```
Int[(x*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(5/2),x]
```

output
```
-1/3*ArcTan[a*x]^3/(a^2*c*(c + a^2*c*x^2)^(3/2)) + ((-2*(x/(3*c*(c + a^2*c
*x^2)^(3/2)) + (2*x)/(3*c^2*Sqrt[c + a^2*c*x^2])))/9 + (2*ArcTan[a*x])/(9*
a*c*(c + a^2*c*x^2)^(3/2)) + (x*ArcTan[a*x]^2)/(3*c*(c + a^2*c*x^2)^(3/2))
 + (2*((-2*x)/(c*Sqrt[c + a^2*c*x^2]) + (2*ArcTan[a*x])/(a*c*Sqrt[c + a^2*
c*x^2]) + (x*ArcTan[a*x]^2)/(c*Sqrt[c + a^2*c*x^2])))/(3*c))/a
```

### Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 5433
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x] - Simp[b^2*p*(p
- 1)   Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2), x], x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

rule 5435
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[b*p*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p - 1)/(4*c*d*
(q + 1)^2)), x] + (-Simp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*d*
(q + 1))), x] + Simp[(2*q + 3)/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p, x], x] - Simp[b^2*p*((p - 1)/(4*(q + 1)^2))   Int[(d + e
*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 4.29 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.57

| method | result |
|---|---|
| default | $\frac{\left(9i\arctan(ax)^2+9\arctan(ax)^3-2i-6\arctan(ax)\right)\left(ia^3x^3+3a^2x^2-3iax-1\right)\sqrt{c(ax-i)(ax+i)}}{216(a^2x^2+1)^2a^2c^3}-\frac{\left(\arctan(ax)^3-6\arctan(ax)+3i\right.}{8c}$ |
| orering | $-\frac{4(a^2x^2+1)\left(600a^8x^8+562a^6x^6-37a^4x^4+27a^2x^2-42\right)\arctan(ax)^3}{81a^4x^2(a^2cx^2+c)^{\frac{5}{2}}}-\frac{2(a^2x^2+1)^2\left(900a^6x^6+716a^4x^4-177a^2x^2+84\right)\left(\frac{\arctan}{(a^2cx}\right.}{81x^2}$ |

input
```
int(x*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/216*(9*I*arctan(a*x)^2+9*arctan(a*x)^3-2*I-6*arctan(a*x))*(I*a^3*x^3+3*a
^2*x^2-3*I*a*x-1)*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^2/a^2/c^3-1/8*(arc
tan(a*x)^3-6*arctan(a*x)+3*I*arctan(a*x)^2-6*I)*(1+I*a*x)*(c*(a*x-I)*(a*x+
I))^(1/2)/c^3/a^2/(a^2*x^2+1)+1/8*(c*(a*x-I)*(a*x+I))^(1/2)*(I*a*x-1)*(arc
tan(a*x)^3-6*arctan(a*x)-3*I*arctan(a*x)^2+6*I)/c^3/a^2/(a^2*x^2+1)-1/216*
(c*(a*x-I)*(a*x+I))^(1/2)*(I*a^3*x^3-3*a^2*x^2-3*I*a*x+1)*(-9*I*arctan(a*x
)^2+9*arctan(a*x)^3+2*I-6*arctan(a*x))/a^2/c^3/(a^4*x^4+2*a^2*x^2+1)
```

## Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.52

$$\int \frac{x\arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx =$$
$$-\frac{\left(40\,a^3x^3-9\left(2\,a^3x^3+3\,ax\right)\arctan\left(ax\right)^2+9\,\arctan\left(ax\right)^3+42\,ax-6\left(6\,a^2x^2+7\right)\arctan\left(ax\right)\right)\sqrt{a^2cx^2+}}{27\left(a^6c^3x^4+2\,a^4c^3x^2+a^2c^3\right)}$$

input
```
integrate(x*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
-1/27*(40*a^3*x^3 - 9*(2*a^3*x^3 + 3*a*x)*arctan(a*x)^2 + 9*arctan(a*x)^3
+ 42*a*x - 6*(6*a^2*x^2 + 7)*arctan(a*x))*sqrt(a^2*c*x^2 + c)/(a^6*c^3*x^4
 + 2*a^4*c^3*x^2 + a^2*c^3)
```

## Sympy [F]

$$\int \frac{x\arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x\,\mathrm{atan}^3\,(ax)}{\left(c\,(a^2x^2+1)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*atan(a*x)**3/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(x*atan(a*x)**3/(c*(a**2*x**2 + 1))**(5/2), x)
```

## Maxima [F]

$$\int \frac{x\arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x\arctan\,(ax)^3}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(x*arctan(a*x)^3/(a^2*c*x^2 + c)^(5/2), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x\arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x\arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x\,\mathrm{atan}(a\,x)^3}{(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int((x*atan(a*x)^3)/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int((x*atan(a*x)^3)/(c + a^2*c*x^2)^(5/2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.90

$$\int \frac{x\arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \frac{\sqrt{c}\left(-9\sqrt{a^2x^2+1}\,atan(ax)^3+18\sqrt{a^2x^2+1}\,atan(ax)^2\,a^3x^3+27\sqrt{a^2x^2+1}\,atan(a\right.}{}$$

input
```
int(x*atan(a*x)^3/(a^2*c*x^2+c)^(5/2),x)
```

output
```
(sqrt(c)*( - 9*sqrt(a**2*x**2 + 1)*atan(a*x)**3 + 18*sqrt(a**2*x**2 + 1)*a
tan(a*x)**2*a**3*x**3 + 27*sqrt(a**2*x**2 + 1)*atan(a*x)**2*a*x + 36*sqrt(
a**2*x**2 + 1)*atan(a*x)*a**2*x**2 + 42*sqrt(a**2*x**2 + 1)*atan(a*x) - 40
*sqrt(a**2*x**2 + 1)*a**3*x**3 - 42*sqrt(a**2*x**2 + 1)*a*x + 16*a**4*x**4
 + 32*a**2*x**2 + 16))/(27*a**2*c**3*(a**4*x**4 + 2*a**2*x**2 + 1))
```

# 3.454 $\int \frac{\arctan(ax)^3}{(c+a^2cx^2)^{5/2}} \, dx$



## Optimal result

Integrand size = 21, antiderivative size = 215

$$\int \frac{\arctan(ax)^3}{(c+a^2cx^2)^{5/2}} \, dx = -\frac{2}{27ac(c+a^2cx^2)^{3/2}} - \frac{40}{9ac^2\sqrt{c+a^2cx^2}} - \frac{2x\arctan(ax)}{9c(c+a^2cx^2)^{3/2}} - \frac{40x\arctan(ax)}{9c^2\sqrt{c+a^2cx^2}} + \frac{\arctan(ax)^2}{3ac(c+a^2cx^2)^{3/2}} + \frac{2\arctan(ax)^2}{ac^2\sqrt{c+a^2cx^2}} + \frac{x\arctan(ax)^3}{3c(c+a^2cx^2)^{3/2}} + \frac{2x\arctan(ax)^3}{3c^2\sqrt{c+a^2cx^2}}$$

output
```
-2/27/a/c/(a^2*c*x^2+c)^(3/2)-40/9/a/c^2/(a^2*c*x^2+c)^(1/2)-2/9*x*arctan(
a*x)/c/(a^2*c*x^2+c)^(3/2)-40/9*x*arctan(a*x)/c^2/(a^2*c*x^2+c)^(1/2)+1/3*
arctan(a*x)^2/a/c/(a^2*c*x^2+c)^(3/2)+2*arctan(a*x)^2/a/c^2/(a^2*c*x^2+c)^
(1/2)+1/3*x*arctan(a*x)^3/c/(a^2*c*x^2+c)^(3/2)+2/3*x*arctan(a*x)^3/c^2/(a
^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.48

$$\int \frac{\arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \frac{\sqrt{c+a^2cx^2}(-2(61+60a^2x^2)-6ax(21+20a^2x^2)\arctan(ax)+9(7+6a^2x^2)\,\text{arcta}}{27ac^3\left(1+a^2x^2\right)^2}$$

input

```
Integrate[ArcTan[a*x]^3/(c + a^2*c*x^2)^(5/2),x]
```

output

```
(Sqrt[c + a^2*c*x^2]*(-2*(61 + 60*a^2*x^2) - 6*a*x*(21 + 20*a^2*x^2)*ArcTa
n[a*x] + 9*(7 + 6*a^2*x^2)*ArcTan[a*x]^2 + 9*a*x*(3 + 2*a^2*x^2)*ArcTan[a*
x]^3))/(27*a*c^3*(1 + a^2*x^2)^2)
```

## Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.27, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5435, 5431, 5429, 5433, 5429}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3}{(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{5435}$$

$$-\frac{2}{3}\int \frac{\arctan(ax)}{(a^2cx^2+c)^{5/2}}dx+\frac{2\int \frac{\arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx}{3c}+\frac{x\arctan(ax)^3}{3c\,(a^2cx^2+c)^{3/2}}+\frac{\arctan(ax)^2}{3ac\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5431}$$

$$-\frac{2}{3}\left(\frac{2\int \frac{\arctan(ax)}{(a^2cx^2+c)^{3/2}}dx}{3c}+\frac{x\arctan(ax)}{3c\,(a^2cx^2+c)^{3/2}}+\frac{1}{9ac\,(a^2cx^2+c)^{3/2}}\right)+\frac{2\int \frac{\arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx}{3c}+\frac{x\arctan(ax)^3}{3c\,(a^2cx^2+c)^{3/2}}+\frac{\arctan(ax)^2}{3ac\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5429}$$

$$\frac{2\int \frac{\arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx}{3c}+\frac{x\arctan(ax)^3}{3c\,(a^2cx^2+c)^{3/2}}+\frac{\arctan(ax)^2}{3ac\,(a^2cx^2+c)^{3/2}}-\frac{2}{3}\left(\frac{x\arctan(ax)}{3c\,(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{1}{9ac\,(a^2cx^2+c)^{3/2}}\right)$$

$$\Big\downarrow \text{5433}$$

$$\frac{2\left(-6\int \frac{\arctan(ax)}{(a^2cx^2+c)^{3/2}}dx+\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{x\arctan(ax)^3}{3c\,(a^2cx^2+c)^{3/2}}+\frac{\arctan(ax)^2}{3ac\,(a^2cx^2+c)^{3/2}}-\frac{2}{3}\left(\frac{x\arctan(ax)}{3c\,(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{1}{9ac\,(a^2cx^2+c)^{3/2}}\right)$$

$$\Big\downarrow \text{5429}$$

$$\frac{x\arctan(ax)^3}{3c\,(a^2cx^2+c)^{3/2}}+\frac{\arctan(ax)^2}{3ac\,(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}}-6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)\right)}{3c}-\frac{2}{3}\left(\frac{x\arctan(ax)}{3c\,(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{1}{9ac\,(a^2cx^2+c)^{3/2}}\right)$$

input
```
Int[ArcTan[a*x]^3/(c + a^2*c*x^2)^(5/2),x]
```

output
```
ArcTan[a*x]^2/(3*a*c*(c + a^2*c*x^2)^(3/2)) + (x*ArcTan[a*x]^3)/(3*c*(c +
a^2*c*x^2)^(3/2)) + (2*((3*ArcTan[a*x]^2)/(a*c*Sqrt[c + a^2*c*x^2]) + (x*A
rcTan[a*x]^3)/(c*Sqrt[c + a^2*c*x^2]) - 6*(1/(a*c*Sqrt[c + a^2*c*x^2]) + (
x*ArcTan[a*x])/(c*Sqrt[c + a^2*c*x^2]))))/(3*c) - (2*(1/(9*a*c*(c + a^2*c*
x^2)^(3/2)) + (x*ArcTan[a*x])/(3*c*(c + a^2*c*x^2)^(3/2)) + (2*(1/(a*c*Sqr
t[c + a^2*c*x^2]) + (x*ArcTan[a*x])/(c*Sqrt[c + a^2*c*x^2])))/(3*c)))/3
```

### Defintions of rubi rules used

rule 5429
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTan[c*x])/(d*Sqr
t[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

rule 5431
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbol
] :> Simp[b*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (-Simp[x*(d + e*x
^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*d*(q + 1))), x] + Simp[(2*q + 3)/(2*d*(
q + 1))   Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a,
 b, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -3/2]
```

rule 5433
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x] - Simp[b^2*p*(p
- 1)   Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2), x], x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

rule 5435
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[b*p*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p - 1)/(4*c*d*
(q + 1)^2)), x] + (-Simp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*d*
(q + 1))), x] + Simp[(2*q + 3)/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p, x], x] - Simp[b^2*p*((p - 1)/(4*(q + 1)^2))   Int[(d + e
*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.77 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.43

| method | result |
|---|---|
| default | $-\frac{\left(9i\arctan(ax)^2+9\arctan(ax)^3-2i-6\arctan(ax)\right)\left(a^3x^3-3ia^2x^2-3ax+i\right)\sqrt{c(ax-i)(ax+i)}}{216(a^2x^2+1)^2 a c^3}+\frac{3\left(\arctan(ax)^3-6\arctan(ax)+\right.}{}$ |
| orering | $\frac{\left(-\frac{1600}{9}a^6x^7-\frac{11392}{27}a^4x^5-\frac{24880}{81}a^2x^3-\frac{5104}{81}x\right)\arctan(ax)^3}{(a^2cx^2+c)^{\frac{5}{2}}}-\frac{2(a^2x^2+1)^2(4140a^4x^4+4704a^2x^2+487)\left(\frac{3\arctan(ax)^2a}{(a^2cx^2+c)^{\frac{5}{2}}(a^2x^2+1)}\right.}{81a^2}$ |

input
```
int(arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/216*(9*I*arctan(a*x)^2+9*arctan(a*x)^3-2*I-6*arctan(a*x))*(a^3*x^3-3*I*
a^2*x^2-3*a*x+I)*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^2/a/c^3+3/8*(arctan
(a*x)^3-6*arctan(a*x)+3*I*arctan(a*x)^2-6*I)*(a*x-I)*(c*(a*x-I)*(a*x+I))^(
1/2)/c^3/a/(a^2*x^2+1)+3/8*(c*(a*x-I)*(a*x+I))^(1/2)*(a*x+I)*(arctan(a*x)^
3-6*arctan(a*x)-3*I*arctan(a*x)^2+6*I)/c^3/a/(a^2*x^2+1)-1/216*(-9*I*arcta
n(a*x)^2+9*arctan(a*x)^3+2*I-6*arctan(a*x))*(c*(a*x-I)*(a*x+I))^(1/2)*(a^3
*x^3+3*I*a^2*x^2-3*a*x-I)/(a^4*x^4+2*a^2*x^2+1)/a/c^3
```

## Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.52

$$\int \frac{\arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = -\frac{\sqrt{a^2cx^2+c}\left(120\,a^2x^2-9\,(2\,a^3x^3+3\,ax)\arctan(ax)^3-9\,(6\,a^2x^2+7)\arctan(ax)^2+6\,(20\,a^3x^3+21\,ax)\right.}{27\,(a^5c^3x^4+2\,a^3c^3x^2+ac^3)}$$

input
```
integrate(arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
-1/27*sqrt(a^2*c*x^2 + c)*(120*a^2*x^2 - 9*(2*a^3*x^3 + 3*a*x)*arctan(a*x)
^3 - 9*(6*a^2*x^2 + 7)*arctan(a*x)^2 + 6*(20*a^3*x^3 + 21*a*x)*arctan(a*x)
 + 122)/(a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)
```

## Sympy [F]

$$\int \frac{\arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\operatorname{atan}^3(ax)}{(c(a^2x^2+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(atan(a*x)**3/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(atan(a*x)**3/(c*(a**2*x**2 + 1))**(5/2), x)
```

## Maxima [F]

$$\int \frac{\arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\arctan(ax)^3}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)^3/(a^2*c*x^2 + c)^(5/2), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^3/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\mathrm{atan}(a\,x)^3}{(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int(atan(a*x)^3/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int(atan(a*x)^3/(c + a^2*c*x^2)^(5/2), x)
```

## Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.30

$$\int \frac{\arctan(ax)^3}{(c+a^2cx^2)^{5/2}}\,dx = \frac{\sqrt{c}\left(240atan\left(\sqrt{a^2x^2+1}+ax\right)a^4x^4+480atan\left(\sqrt{a^2x^2+1}+ax\right)a^2x^2+240atan(\cdots$$

input
```
int(atan(a*x)^3/(a^2*c*x^2+c)^(5/2),x)
```

output
```
(sqrt(c)*(240*atan(sqrt(a**2*x**2 + 1) + a*x)*a**4*x**4 + 480*atan(sqrt(a*
*2*x**2 + 1) + a*x)*a**2*x**2 + 240*atan(sqrt(a**2*x**2 + 1) + a*x) + 18*s
qrt(a**2*x**2 + 1)*atan(a*x)**3*a**3*x**3 + 27*sqrt(a**2*x**2 + 1)*atan(a*
x)**3*a*x + 54*sqrt(a**2*x**2 + 1)*atan(a*x)**2*a**2*x**2 + 63*sqrt(a**2*x
**2 + 1)*atan(a*x)**2 - 120*sqrt(a**2*x**2 + 1)*atan(a*x)*a**3*x**3 - 126*
sqrt(a**2*x**2 + 1)*atan(a*x)*a*x - 120*atan(a*x)*a**4*x**4 - 240*atan(a*x
)*a**2*x**2 - 120*atan(a*x) - 120*sqrt(a**2*x**2 + 1)*a**2*x**2 - 122*sqrt
(a**2*x**2 + 1)))/(27*a*c**3*(a**4*x**4 + 2*a**2*x**2 + 1))
```

# 3.455 $\int \frac{\arctan(ax)^3}{x(c+a^2cx^2)^{5/2}}\,dx$

## Optimal result

Integrand size = 24, antiderivative size = 553

$$\int \frac{\arctan(ax)^3}{x\left(c+a^2cx^2\right)^{5/2}}\,dx = \frac{2ax}{27c\left(c+a^2cx^2\right)^{3/2}}+\frac{202ax}{27c^2\sqrt{c+a^2cx^2}} - \frac{2\arctan(ax)}{9c\left(c+a^2cx^2\right)^{3/2}}-\frac{22\arctan(ax)}{3c^2\sqrt{c+a^2cx^2}}-\frac{ax\arctan(ax)^2}{3c\left(c+a^2cx^2\right)^{3/2}} - \frac{11ax\arctan(ax)^2}{3c^2\sqrt{c+a^2cx^2}}+\frac{\arctan(ax)^3}{3c\left(c+a^2cx^2\right)^{3/2}}+\frac{\arctan(ax)^3}{c^2\sqrt{c+a^2cx^2}} - \frac{2\sqrt{1+a^2x^2}\arctan(ax)^3\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)}{c^2\sqrt{c+a^2cx^2}} + \frac{3i\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)}{c^2\sqrt{c+a^2cx^2}} - \frac{3i\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{PolyLog}\left(2,e^{i\arctan(ax)}\right)}{c^2\sqrt{c+a^2cx^2}} - \frac{6\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(3,-e^{i\arctan(ax)}\right)}{c^2\sqrt{c+a^2cx^2}} + \frac{6\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(3,e^{i\arctan(ax)}\right)}{c^2\sqrt{c+a^2cx^2}} - \frac{6i\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(4,-e^{i\arctan(ax)}\right)}{c^2\sqrt{c+a^2cx^2}} + \frac{6i\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(4,e^{i\arctan(ax)}\right)}{c^2\sqrt{c+a^2cx^2}}$$

output
```
2/27*a*x/c/(a^2*c*x^2+c)^(3/2)+202/27*a*x/c^2/(a^2*c*x^2+c)^(1/2)-2/9*arct
an(a*x)/c/(a^2*c*x^2+c)^(3/2)-22/3*arctan(a*x)/c^2/(a^2*c*x^2+c)^(1/2)-1/3
*a*x*arctan(a*x)^2/c/(a^2*c*x^2+c)^(3/2)-11/3*a*x*arctan(a*x)^2/c^2/(a^2*c
*x^2+c)^(1/2)+1/3*arctan(a*x)^3/c/(a^2*c*x^2+c)^(3/2)+arctan(a*x)^3/c^2/(a
^2*c*x^2+c)^(1/2)-2*(a^2*x^2+1)^(1/2)*arctan(a*x)^3*arctanh((1+I*a*x)/(a^2
*x^2+1)^(1/2))/c^2/(a^2*c*x^2+c)^(1/2)+3*I*(a^2*x^2+1)^(1/2)*arctan(a*x)^2
*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))/c^2/(a^2*c*x^2+c)^(1/2)-3*I*(a^2*
x^2+1)^(1/2)*arctan(a*x)^2*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))/c^2/(a^2
*c*x^2+c)^(1/2)-6*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(3,-(1+I*a*x)/(a^2*
x^2+1)^(1/2))/c^2/(a^2*c*x^2+c)^(1/2)+6*(a^2*x^2+1)^(1/2)*arctan(a*x)*poly
log(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))/c^2/(a^2*c*x^2+c)^(1/2)-6*I*(a^2*x^2+1)
^(1/2)*polylog(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2))/c^2/(a^2*c*x^2+c)^(1/2)+6*I
*(a^2*x^2+1)^(1/2)*polylog(4,(1+I*a*x)/(a^2*x^2+1)^(1/2))/c^2/(a^2*c*x^2+c
)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 347, normalized size of antiderivative = 0.63

$$\int \frac{\arctan(ax)^3}{x\left(c+a^2cx^2\right)^{5/2}}\,dx = \frac{\left(1+a^2x^2\right)^{3/2}\left(-27i\pi^4+\frac{1620ax}{\sqrt{1+a^2x^2}}-\frac{1620\arctan(ax)}{\sqrt{1+a^2x^2}}-\frac{810ax\arctan(ax)^2}{\sqrt{1+a^2x^2}}+\frac{270\arctan(ax)^3}{\sqrt{1+a^2x^2}}-\right.}{}$$

input
```
Integrate[ArcTan[a*x]^3/(x*(c + a^2*c*x^2)^(5/2)),x]
```

output
```
((1 + a^2*x^2)^(3/2)*((-27*I)*Pi^4 + (1620*a*x)/Sqrt[1 + a^2*x^2] - (1620*
ArcTan[a*x])/Sqrt[1 + a^2*x^2] - (810*a*x*ArcTan[a*x]^2)/Sqrt[1 + a^2*x^2]
 + (270*ArcTan[a*x]^3)/Sqrt[1 + a^2*x^2] + (54*I)*ArcTan[a*x]^4 - 12*ArcTa
n[a*x]*Cos[3*ArcTan[a*x]] + 18*ArcTan[a*x]^3*Cos[3*ArcTan[a*x]] + 216*ArcT
an[a*x]^3*Log[1 - E^((-I)*ArcTan[a*x])] - 216*ArcTan[a*x]^3*Log[1 + E^(I*A
rcTan[a*x])] + (648*I)*ArcTan[a*x]^2*PolyLog[2, E^((-I)*ArcTan[a*x])] + (6
48*I)*ArcTan[a*x]^2*PolyLog[2, -E^(I*ArcTan[a*x])] + 1296*ArcTan[a*x]*Poly
Log[3, E^((-I)*ArcTan[a*x])] - 1296*ArcTan[a*x]*PolyLog[3, -E^(I*ArcTan[a*
x])] - (1296*I)*PolyLog[4, E^((-I)*ArcTan[a*x])] - (1296*I)*PolyLog[4, -E^
(I*ArcTan[a*x])] + 4*Sin[3*ArcTan[a*x]] - 18*ArcTan[a*x]^2*Sin[3*ArcTan[a*
x]]))/(216*c*(c*(1 + a^2*x^2))^(3/2))
```

## Rubi [A] (verified)

Time = 3.55 (sec) , antiderivative size = 523, normalized size of antiderivative = 0.95, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.792$, Rules used = {5501, 5465, 5435, 209, 208, 5433, 208, 5501, 5465, 5433, 208, 5493, 5491, 3042, 4671, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3}{x(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{\arctan(ax)^3}{x(a^2cx^2+c)^{3/2}}dx}{c} - a^2 \int \frac{x\arctan(ax)^3}{(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{\int \frac{\arctan(ax)^3}{x(a^2cx^2+c)^{3/2}}dx}{c} - a^2\left(\frac{\int \frac{\arctan(ax)^2}{(a^2cx^2+c)^{5/2}}dx}{a} - \frac{\arctan(ax)^3}{3a^2c(a^2cx^2+c)^{3/2}}\right)$$

$$\Big\downarrow \text{5435}$$

$$\frac{\int \frac{\arctan(ax)^3}{x(a^2cx^2+c)^{3/2}}dx}{c} - a^2\left(\frac{\frac{2\int \frac{\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{3c} - \frac{2}{9}\int \frac{1}{(a^2cx^2+c)^{5/2}}dx + \frac{x\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}} + \frac{2\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}}{a} - \frac{\arctan(ax)^3}{3a^2c(a^2cx^2+c)^{3/2}}\right)$$

$$\Big\downarrow \text{209}$$

$$\frac{\int \frac{\arctan(ax)^3}{x(a^2cx^2+c)^{3/2}}dx}{c} - a^2\left(\frac{\frac{2\int \frac{\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{3c} - \frac{2}{9}\left(\frac{2\int \frac{1}{(a^2cx^2+c)^{3/2}}dx}{3c} + \frac{x}{3c(a^2cx^2+c)^{3/2}}\right) + \frac{x\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}} + \frac{2\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}}{a} - \frac{\arctan(ax)^3}{3a^2c(a^2cx^2+c)^{3/2}}\right)$$

$$\Big\downarrow \ 208$$

$$\frac{\int \frac{\arctan(ax)^3}{x(a^2cx^2+c)^{3/2}}dx}{c} - a^2\left(\frac{\frac{2\int \frac{\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{3c} + \frac{x\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}} + \frac{2\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}} - \frac{2}{9}\left(\frac{2x}{3c^2\sqrt{a^2cx^2+c}} + \frac{x}{3c(a^2cx^2+c)^{3/2}}\right)}{a} - \frac{\arctan(ax)^3}{3a^2c\,(a^2cx^2+c)^{3/2}}\right)$$

$$\Big\downarrow \ 5433$$

$$\frac{\int \frac{\arctan(ax)^3}{x(a^2cx^2+c)^{3/2}}dx}{c} - a^2\left(\frac{\frac{2\left(-2\int \frac{1}{(a^2cx^2+c)^{3/2}}dx + \frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}} + \frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}\right)}{3c} + \frac{x\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}} + \frac{2\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}} - \frac{2}{9}\left(\frac{2x}{3c^2\sqrt{a^2cx^2+c}} + \frac{x}{3c(a^2cx^2+c)^{3}}\right.}{a}\right.$$

$$\Big\downarrow \ 208$$

$$\frac{\int \frac{\arctan(ax)^3}{x(a^2cx^2+c)^{3/2}}dx}{c} - a^2\left(\frac{\frac{x\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}} + \frac{2\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}} + \frac{2\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}} + \frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}} - \frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{3c} - \frac{2}{9}\left(\frac{2x}{3c^2\sqrt{a^2cx^2+c}} + \frac{x}{3c(a^2cx^2+c)^{3/2}}\right)}{a} - \frac{}{3a}\right.$$

$$\Big\downarrow \ 5501$$

$$\frac{\frac{\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx}{c} - a^2\int \frac{x\arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx}{c} - a^2\left(\frac{\frac{x\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}} + \frac{2\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}} + \frac{2\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}} + \frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}} - \frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{3c} - \frac{2}{9}\left(\frac{2x}{3c^2\sqrt{a^2cx^2+c}} + \frac{x}{3c(a^2cx^2+c)^{3/2}}\right)}{a} - \frac{}{3a}\right.$$

$$\Big\downarrow \ 5465$$

$$\frac{\frac{\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx}{c} - a^2\left(\frac{3\int \frac{\arctan(ax)^2}{(a^2cx^2+c)^{3/2}}dx}{a} - \frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)}{c} - a^2\left(\frac{\frac{x\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}} + \frac{2\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}} + \frac{2\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{3c} - \frac{2}{9}\left(\frac{2x}{3c^2\sqrt{a^2cx^2+c}}+\frac{x}{3c(a^2cx^2+c)^{3/2}}\right)}{a} - \frac{\cdots}{3c\cdots}\right.$$

$\downarrow$ 5433

$$\frac{\frac{\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx}{c} - a^2\left(\frac{3\left(-2\int \frac{1}{(a^2cx^2+c)^{3/2}}dx+\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}\right)}{a} - \frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)}{c} - a^2\left(\frac{\frac{x\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}} + \frac{2\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}} + \frac{2\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{3c} - \frac{2}{9}\left(\frac{2x}{3c^2\sqrt{a^2cx^2+c}}+\frac{x}{3c(a^2cx^2+c)^{3/2}}\right)}{a} - \frac{\cdots}{3c\cdots}\right.$$

$\downarrow$ 208

$$\frac{\frac{\int \frac{\arctan(ax)^3}{x\sqrt{a^2cx^2+c}}dx}{c} - a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a} - \frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)}{c} - a^2\left(\frac{\frac{x\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}} + \frac{2\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}} + \frac{2\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{3c} - \frac{2}{9}\left(\frac{2x}{3c^2\sqrt{a^2cx^2+c}}+\frac{x}{3c(a^2cx^2+c)^{3/2}}\right)}{a} - \frac{\cdots}{3c\cdots}\right.$$

$\downarrow$ 5493

$$\frac{\frac{\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^3}{x\sqrt{a^2x^2+1}}dx}{c\sqrt{a^2cx^2+c}} - a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a} - \frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)}{c} - a^2\left(\frac{\frac{x\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}} + \frac{2\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}} + \frac{2\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{3c} - \frac{2}{9}\left(\frac{2x}{3c^2\sqrt{a^2cx^2+c}}+\frac{x}{3c(a^2cx^2+c)^{3/2}}\right)}{a} - \frac{\cdots}{3c\cdots}\right.$$

$\downarrow$ 5491

$$\frac{\frac{\sqrt{a^2x^2+1}\int \frac{\sqrt{a^2x^2+1}\arctan(ax)^3}{ax}d\arctan(ax)}{c\sqrt{a^2cx^2+c}}-a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)}{c}-$$

$$a^2\left(\frac{\frac{x\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}}+\frac{2\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{3c}-\frac{2}{9}\left(\frac{2x}{3c^2\sqrt{a^2cx^2+c}}+\frac{x}{3c(a^2cx^2+c)^{3/2}}\right)}{a}-\frac{}{3c}\right.$$

$\downarrow$ 3042

$$\frac{\frac{\sqrt{a^2x^2+1}\int \arctan(ax)^3\csc(\arctan(ax))d\arctan(ax)}{c\sqrt{a^2cx^2+c}}-a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)}{c}-$$

$$a^2\left(\frac{\frac{x\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}}+\frac{2\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{3c}-\frac{2}{9}\left(\frac{2x}{3c^2\sqrt{a^2cx^2+c}}+\frac{x}{3c(a^2cx^2+c)^{3/2}}\right)}{a}-\frac{}{3c}\right.$$

$\downarrow$ 4671

$$-a^2\left(\frac{\frac{x\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}}+\frac{2\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{3c}-\frac{2}{9}\left(\frac{2x}{3c^2\sqrt{a^2cx^2+c}}+\frac{x}{3c(a^2cx^2+c)^{3/2}}\right)}{a}-\right.$$

$$\frac{-a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)+\frac{\sqrt{a^2x^2+1}\left(-3\int \arctan(ax)^2\log\left(1-e^{i\arctan(ax)}\right)d\arctan(ax)+3\right.}{}}{c}$$

$\downarrow$ 3011

$$-a^2\left(\frac{\frac{x\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}}+\frac{2\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{3c}-\frac{2}{9}\left(\frac{2x}{3c^2\sqrt{a^2cx^2+c}}+\frac{x}{3c(a^2cx^2+c)^{3/2}}\right)}{a}-\right.$$

$$-a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)+\frac{\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)-2i\int \arctan\right.\right.}{}$$

$$\Big\downarrow \; 7163$$

$$-a^2\left(\frac{\frac{x\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}}+\frac{2\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{3c}-\frac{2}{9}\left(\frac{2x}{3c^2\sqrt{a^2cx^2+c}}+\frac{x}{3c(a^2cx^2+c)^{3/2}}\right)}{a}-\right.$$

$$\frac{-a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)+\frac{\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}(2,-e^{i\arctan(ax)})-2i\left(i\int \operatorname{Poly}\right.\right.\right.}{}}{}$$

$$\Big\downarrow \; 2720$$

$$-a^2\left(\frac{\frac{x\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}}+\frac{2\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{3c}-\frac{2}{9}\left(\frac{2x}{3c^2\sqrt{a^2cx^2+c}}+\frac{x}{3c(a^2cx^2+c)^{3/2}}\right)}{a}-\right.$$

$$\frac{-a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)+\frac{\sqrt{a^2x^2+1}\left(3\left(i\arctan(ax)^2\operatorname{PolyLog}(2,-e^{i\arctan(ax)})-2i\left(\int e^{-i a}\right.\right.\right.}{}}{}$$

$$\Big\downarrow \; 7143$$

$$-a^2\left(\frac{\frac{x\arctan(ax)^2}{3c(a^2cx^2+c)^{3/2}}+\frac{2\arctan(ax)}{9ac(a^2cx^2+c)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{3c}-\frac{2}{9}\left(\frac{2x}{3c^2\sqrt{a^2cx^2+c}}+\frac{x}{3c(a^2cx^2+c)^{3/2}}\right)}{a}-\right.$$

$$\frac{-a^2\left(\frac{3\left(\frac{x\arctan(ax)^2}{c\sqrt{a^2cx^2+c}}+\frac{2\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{\arctan(ax)^3}{a^2c\sqrt{a^2cx^2+c}}\right)+\frac{\sqrt{a^2x^2+1}\left(-2\arctan(ax)^3\operatorname{arctanh}(e^{i\arctan(ax)})+3\left(i\arctan(a\right.\right.}{}}{}$$

input
```
Int[ArcTan[a*x]^3/(x*(c + a^2*c*x^2)^(5/2)),x]
```

output
```
-(a^2*(-1/3*ArcTan[a*x]^3/(a^2*c*(c + a^2*c*x^2)^(3/2)) + ((-2*(x/(3*c*(c
+ a^2*c*x^2)^(3/2)) + (2*x)/(3*c^2*Sqrt[c + a^2*c*x^2])))/9 + (2*ArcTan[a*
x])/(9*a*c*(c + a^2*c*x^2)^(3/2)) + (x*ArcTan[a*x]^2)/(3*c*(c + a^2*c*x^2)
^(3/2)) + (2*((-2*x)/(c*Sqrt[c + a^2*c*x^2]) + (2*ArcTan[a*x])/(a*c*Sqrt[c
 + a^2*c*x^2]) + (x*ArcTan[a*x]^2)/(c*Sqrt[c + a^2*c*x^2])))/(3*c))/a)) +
(-(a^2*(-(ArcTan[a*x]^3/(a^2*c*Sqrt[c + a^2*c*x^2])) + (3*((-2*x)/(c*Sqrt[
c + a^2*c*x^2]) + (2*ArcTan[a*x])/(a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*
x]^2)/(c*Sqrt[c + a^2*c*x^2])))/a)) + (Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]^3
*ArcTanh[E^(I*ArcTan[a*x])] + 3*(I*ArcTan[a*x]^2*PolyLog[2, -E^(I*ArcTan[a
*x])] - (2*I)*((-I)*ArcTan[a*x]*PolyLog[3, -E^(I*ArcTan[a*x])] + PolyLog[4
, -E^(I*ArcTan[a*x])])) - 3*(I*ArcTan[a*x]^2*PolyLog[2, E^(I*ArcTan[a*x])]
 - (2*I)*((-I)*ArcTan[a*x]*PolyLog[3, E^(I*ArcTan[a*x])] + PolyLog[4, E^(I
*ArcTan[a*x])]))))/(c*Sqrt[c + a^2*c*x^2]))/c
```

## Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5433
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x] - Simp[b^2*p*(p
- 1)   Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2), x], x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

rule 5435
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[b*p*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p - 1)/(4*c*d*
(q + 1)^2)), x] + (-Simp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*d*
(q + 1))), x] + Simp[(2*q + 3)/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p, x], x] - Simp[b^2*p*((p - 1)/(4*(q + 1)^2))   Int[(d + e
*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5491
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Simp[1/Sqrt[d]   Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTa
n[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] &&
GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

## Maple [A] (verified)

Time = 5.13 (sec) , antiderivative size = 560, normalized size of antiderivative = 1.01

| method | result |
|---|---|
| default | $-\frac{\left(9i\arctan(ax)^2+9\arctan(ax)^3-2i-6\arctan(ax)\right)\left(ia^3x^3+3a^2x^2-3iax-1\right)\sqrt{c(ax-i)(ax+i)}}{216c^3(a^2x^2+1)^2}+\frac{5\left(\arctan(ax)^3-6\arctan(ax)\right)}{}$ |

input
```
int(arctan(a*x)^3/x/(a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/216*(9*I*arctan(a*x)^2+9*arctan(a*x)^3-2*I-6*arctan(a*x))*(I*a^3*x^3+3*
a^2*x^2-3*I*a*x-1)*(c*(a*x-I)*(a*x+I))^(1/2)/c^3/(a^2*x^2+1)^2+5/8*(arctan
(a*x)^3-6*arctan(a*x)+3*I*arctan(a*x)^2-6*I)*(1+I*a*x)*(c*(a*x-I)*(a*x+I))
^(1/2)/(a^2*x^2+1)/c^3-5/8*(c*(a*x-I)*(a*x+I))^(1/2)*(I*a*x-1)*(arctan(a*x
)^3-6*arctan(a*x)-3*I*arctan(a*x)^2+6*I)/(a^2*x^2+1)/c^3+1/216*(-9*I*arcta
n(a*x)^2+9*arctan(a*x)^3+2*I-6*arctan(a*x))*(c*(a*x-I)*(a*x+I))^(1/2)*(I*a
^3*x^3-3*a^2*x^2-3*I*a*x+1)/(a^4*x^4+2*a^2*x^2+1)/c^3+I*(I*arctan(a*x)^3*l
n(1+(1+I*a*x)/(a^2*x^2+1)^(1/2))-I*arctan(a*x)^3*ln(1-(1+I*a*x)/(a^2*x^2+1
)^(1/2))+3*arctan(a*x)^2*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-3*arctan(
a*x)^2*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*I*arctan(a*x)*polylog(3,-(
1+I*a*x)/(a^2*x^2+1)^(1/2))-6*I*arctan(a*x)*polylog(3,(1+I*a*x)/(a^2*x^2+1
)^(1/2))-6*polylog(4,-(1+I*a*x)/(a^2*x^2+1)^(1/2))+6*polylog(4,(1+I*a*x)/(
a^2*x^2+1)^(1/2)))*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/2)/c^3
```

## Fricas [F]

$$\int \frac{\arctan(ax)^3}{x\left(c+a^2cx^2\right)^{5/2}}\,dx=\int \frac{\arctan\left(ax\right)^3}{\left(a^2cx^2+c\right)^{\frac{5}{2}}x}\,dx$$

input
```
integrate(arctan(a*x)^3/x/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3/(a^6*c^3*x^7 + 3*a^4*c^3*x^5 +
3*a^2*c^3*x^3 + c^3*x), x)
```

## Sympy [F]

$$\int \frac{\arctan(ax)^3}{x\left(c+a^2cx^2\right)^{5/2}}\,dx=\int \frac{\operatorname{atan}^3\left(ax\right)}{x\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(atan(a*x)**3/x/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(atan(a*x)**3/(x*(c*(a**2*x**2 + 1))**(5/2)), x)
```

## Maxima [F]

$$\int \frac{\arctan(ax)^3}{x\left(c+a^2cx^2\right)^{5/2}}\,dx = \int \frac{\arctan\left(ax\right)^3}{\left(a^2cx^2+c\right)^{\frac{5}{2}}x}\,dx$$

input
```
integrate(arctan(a*x)^3/x/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)^3/((a^2*c*x^2 + c)^(5/2)*x), x)
```

## Giac [F]

$$\int \frac{\arctan(ax)^3}{x\left(c+a^2cx^2\right)^{5/2}}\,dx = \int \frac{\arctan\left(ax\right)^3}{\left(a^2cx^2+c\right)^{\frac{5}{2}}x}\,dx$$

input
```
integrate(arctan(a*x)^3/x/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^3/((a^2*c*x^2 + c)^(5/2)*x), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^3}{x\left(c+a^2cx^2\right)^{5/2}}\,dx = \int \frac{\mathrm{atan}\left(a\,x\right)^3}{x\,\left(c\,a^2\,x^2+c\right)^{5/2}}\,dx$$

input
```
int(atan(a*x)^3/(x*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(atan(a*x)^3/(x*(c + a^2*c*x^2)^(5/2)), x)
```

## Reduce [F]

$$\int \frac{\arctan(ax)^3}{x\,(c+a^2cx^2)^{5/2}}\,dx = \frac{\int \frac{atan(ax)^3}{\sqrt{a^2x^2+1}\,a^4x^5+2\sqrt{a^2x^2+1}\,a^2x^3+\sqrt{a^2x^2+1}\,x}dx}{\sqrt{c}\,c^2}$$

input
```
int(atan(a*x)^3/x/(a^2*c*x^2+c)^(5/2),x)
```

output
```
int(atan(a*x)**3/(sqrt(a**2*x**2 + 1)*a**4*x**5 + 2*sqrt(a**2*x**2 + 1)*a*
*2*x**3 + sqrt(a**2*x**2 + 1)*x),x)/(sqrt(c)*c**2)
```

# 3.456 $\int \frac{\arctan(ax)^3}{x^2(c+a^2cx^2)^{5/2}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 493

$$\begin{aligned}\int \frac{\arctan(ax)^3}{x^2(c+a^2cx^2)^{5/2}}\,dx &= \frac{2a}{27c(c+a^2cx^2)^{3/2}}+\frac{94a}{9c^2\sqrt{c+a^2cx^2}}\\
&+\frac{2a^2x\arctan(ax)}{9c(c+a^2cx^2)^{3/2}}+\frac{94a^2x\arctan(ax)}{9c^2\sqrt{c+a^2cx^2}}-\frac{a\arctan(ax)^2}{3c(c+a^2cx^2)^{3/2}}\\
&-\frac{5a\arctan(ax)^2}{c^2\sqrt{c+a^2cx^2}}-\frac{a^2x\arctan(ax)^3}{3c(c+a^2cx^2)^{3/2}}-\frac{5a^2x\arctan(ax)^3}{3c^2\sqrt{c+a^2cx^2}}\\
&-\frac{\sqrt{c+a^2cx^2}\arctan(ax)^3}{c^3x}-\frac{6a\sqrt{1+a^2x^2}\arctan(ax)^2\operatorname{arctanh}\left(e^{i\arctan(ax)}\right)}{c^2\sqrt{c+a^2cx^2}}\\
&+\frac{6ia\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,-e^{i\arctan(ax)}\right)}{c^2\sqrt{c+a^2cx^2}}\\
&-\frac{6ia\sqrt{1+a^2x^2}\arctan(ax)\operatorname{PolyLog}\left(2,e^{i\arctan(ax)}\right)}{c^2\sqrt{c+a^2cx^2}}\\
&-\frac{6a\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,-e^{i\arctan(ax)}\right)}{c^2\sqrt{c+a^2cx^2}}\\
&+\frac{6a\sqrt{1+a^2x^2}\operatorname{PolyLog}\left(3,e^{i\arctan(ax)}\right)}{c^2\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
2/27*a/c/(a^2*c*x^2+c)^(3/2)+94/9*a/c^2/(a^2*c*x^2+c)^(1/2)+2/9*a^2*x*arct
an(a*x)/c/(a^2*c*x^2+c)^(3/2)+94/9*a^2*x*arctan(a*x)/c^2/(a^2*c*x^2+c)^(1/
2)-1/3*a*arctan(a*x)^2/c/(a^2*c*x^2+c)^(3/2)-5*a*arctan(a*x)^2/c^2/(a^2*c*
x^2+c)^(1/2)-1/3*a^2*x*arctan(a*x)^3/c/(a^2*c*x^2+c)^(3/2)-5/3*a^2*x*arcta
n(a*x)^3/c^2/(a^2*c*x^2+c)^(1/2)-(a^2*c*x^2+c)^(1/2)*arctan(a*x)^3/c^3/x-6
*a*(a^2*x^2+1)^(1/2)*arctan(a*x)^2*arctanh((1+I*a*x)/(a^2*x^2+1)^(1/2))/c^
2/(a^2*c*x^2+c)^(1/2)+6*I*a*(a^2*x^2+1)^(1/2)*arctan(a*x)*polylog(2,-(1+I*
a*x)/(a^2*x^2+1)^(1/2))/c^2/(a^2*c*x^2+c)^(1/2)-6*I*a*(a^2*x^2+1)^(1/2)*ar
ctan(a*x)*polylog(2,(1+I*a*x)/(a^2*x^2+1)^(1/2))/c^2/(a^2*c*x^2+c)^(1/2)-6
*a*(a^2*x^2+1)^(1/2)*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))/c^2/(a^2*c*x^
2+c)^(1/2)+6*a*(a^2*x^2+1)^(1/2)*polylog(3,(1+I*a*x)/(a^2*x^2+1)^(1/2))/c^
2/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (warning: unable to verify)

Time = 1.70 (sec) , antiderivative size = 399, normalized size of antiderivative = 0.81

$$\int \frac{\arctan(ax)^3}{x^2\left(c+a^2cx^2\right)^{5/2}}\,dx =$$
$$-\frac{a\left(-1134-1134ax\arctan(ax)+567\arctan(ax)^2+189ax\arctan(ax)^3-2\sqrt{1+a^2x^2}\cos(3\arctan(ax))\right)}{}$$

input
```
Integrate[ArcTan[a*x]^3/(x^2*(c + a^2*c*x^2)^(5/2)),x]
```

output
```
-1/108*(a*(-1134 - 1134*a*x*ArcTan[a*x] + 567*ArcTan[a*x]^2 + 189*a*x*ArcT
an[a*x]^3 - 2*Sqrt[1 + a^2*x^2]*Cos[3*ArcTan[a*x]] + 9*Sqrt[1 + a^2*x^2]*A
rcTan[a*x]^2*Cos[3*ArcTan[a*x]] + 27*a*x*ArcTan[a*x]^3*Csc[ArcTan[a*x]/2]^
2 - 324*Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*Log[1 - E^(I*ArcTan[a*x])] + 324*S
qrt[1 + a^2*x^2]*ArcTan[a*x]^2*Log[1 + E^(I*ArcTan[a*x])] - (648*I)*Sqrt[1
 + a^2*x^2]*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])] + (648*I)*Sqrt[1 +
a^2*x^2]*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])] + 648*Sqrt[1 + a^2*x^2]
*PolyLog[3, -E^(I*ArcTan[a*x])] - 648*Sqrt[1 + a^2*x^2]*PolyLog[3, E^(I*Ar
cTan[a*x])] - 6*Sqrt[1 + a^2*x^2]*ArcTan[a*x]*Sin[3*ArcTan[a*x]] + 9*Sqrt[
1 + a^2*x^2]*ArcTan[a*x]^3*Sin[3*ArcTan[a*x]] + 54*Sqrt[1 + a^2*x^2]*ArcTa
n[a*x]^3*Tan[ArcTan[a*x]/2]))/(c^2*Sqrt[c + a^2*c*x^2])
```

## Rubi [A] (verified)

Time = 4.29 (sec) , antiderivative size = 552, normalized size of antiderivative = 1.12, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.708, Rules used = {5501, 5435, 5431, 5429, 5433, 5429, 5501, 5433, 5429, 5479, 5493, 5491, 3042, 4671, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^3}{x^2\left(a^2cx^2+c\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 5501}$$

$$\frac{\int \frac{\arctan(ax)^3}{x^2(a^2cx^2+c)^{3/2}}dx}{c}-a^2\int \frac{\arctan(ax)^3}{\left(a^2cx^2+c\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 5435}$$

$$\frac{\int \frac{\arctan(ax)^3}{x^2(a^2cx^2+c)^{3/2}}dx}{c}-a^2\left(-\frac{2}{3}\int \frac{\arctan(ax)}{\left(a^2cx^2+c\right)^{5/2}}dx+\frac{2\int \frac{\arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx}{3c}+\frac{x\arctan(ax)^3}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{\arctan(ax)^2}{3ac\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\Big\downarrow \text{ 5431}$$

$$\frac{\int \frac{\arctan(ax)^3}{x^2(a^2cx^2+c)^{3/2}}dx}{c}-a^2\left(-\frac{2}{3}\left(\frac{2\int \frac{\arctan(ax)}{(a^2cx^2+c)^{3/2}}dx}{3c}+\frac{x\arctan(ax)}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{1}{9ac\left(a^2cx^2+c\right)^{3/2}}\right)+\frac{2\int \frac{\arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx}{3c}+\frac{x\arctan(ax)^3}{3c\left(a^2cx^2+c\right)^{3/2}}\right.$$

$$\Big\downarrow \text{ 5429}$$

$$\frac{\int \frac{\arctan(ax)^3}{x^2(a^2cx^2+c)^{3/2}}dx}{c}-a^2\left(\frac{2\int \frac{\arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx}{3c}+\frac{x\arctan(ax)^3}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{\arctan(ax)^2}{3ac\left(a^2cx^2+c\right)^{3/2}}-\frac{2}{3}\left(\frac{x\arctan(ax)}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2}}\right.}{3c}\right.\right.$$

$$\Big\downarrow \text{ 5433}$$

$$\frac{\int \frac{\arctan(ax)^3}{x^2(a^2cx^2+c)^{3/2}}dx}{c} - a^2\left(\frac{2\left(-6\int \frac{\arctan(ax)}{(a^2cx^2+c)^{3/2}}dx + \frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}} + \frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}}\right)}{3c} + \frac{x\arctan(ax)^3}{3c(a^2cx^2+c)^{3/2}} + \frac{\arctan(ax)^2}{3ac(a^2cx^2+c)^{3/2}} - \frac{2}{3}\left(\frac{x\arctan}{3c(a^2cx}\right.\right.$$

↓ 5429

$$\frac{\int \frac{\arctan(ax)^3}{x^2(a^2cx^2+c)^{3/2}}dx}{c} - a^2\left(\frac{x\arctan(ax)^3}{3c(a^2cx^2+c)^{3/2}} + \frac{\arctan(ax)^2}{3ac(a^2cx^2+c)^{3/2}} + \frac{2\left(\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}} + \frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}} - 6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}} + \frac{1}{ac\sqrt{a^2cx^2+c}}\right)\right)}{3c} - \frac{2}{3}\left(\right.\right.$$

↓ 5501

$$\frac{\frac{\int \frac{\arctan(ax)^3}{x^2\sqrt{a^2cx^2+c}}dx}{c} - a^2\int \frac{\arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx}{c} - a^2\left(\frac{x\arctan(ax)^3}{3c(a^2cx^2+c)^{3/2}} + \frac{\arctan(ax)^2}{3ac(a^2cx^2+c)^{3/2}} + \frac{2\left(\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}} + \frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}} - 6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}} + \frac{1}{ac\sqrt{a^2cx^2+c}}\right)\right)}{3c} - \frac{2}{3}\left(\right.\right.$$

↓ 5433

$$\frac{\frac{\int \frac{\arctan(ax)^3}{x^2\sqrt{a^2cx^2+c}}dx}{c} - a^2\left(-6\int \frac{\arctan(ax)}{(a^2cx^2+c)^{3/2}}dx + \frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}} + \frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}}\right)}{c} - a^2\left(\frac{x\arctan(ax)^3}{3c(a^2cx^2+c)^{3/2}} + \frac{\arctan(ax)^2}{3ac(a^2cx^2+c)^{3/2}} + \frac{2\left(\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}} + \frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}} - 6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}} + \frac{1}{ac\sqrt{a^2cx^2+c}}\right)\right)}{3c} - \frac{2}{3}\left(\right.\right.$$

↓ 5429

$$\frac{\frac{\int \frac{\arctan(ax)^3}{x^2\sqrt{a^2cx^2+c}}dx}{c} - a^2\left(\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}} + \frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}} - 6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}} + \frac{1}{ac\sqrt{a^2cx^2+c}}\right)\right)}{c} - a^2\left(\frac{x\arctan(ax)^3}{3c(a^2cx^2+c)^{3/2}} + \frac{\arctan(ax)^2}{3ac(a^2cx^2+c)^{3/2}} + \frac{2\left(\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}} + \frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}} - 6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}} + \frac{1}{ac\sqrt{a^2cx^2+c}}\right)\right)}{3c} - \frac{2}{3}\left(\right.\right.$$

↓ 5479

$$\frac{\frac{3a\int \frac{\arctan(ax)^2}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}}{c}-a^2\left(\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}}-6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)\right)}{c}-$$

$$a^2\left(\frac{x\arctan(ax)^3}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{\arctan(ax)^2}{3ac\left(a^2cx^2+c\right)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}}-6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)\right)}{3c}-\frac{2}{3}\right.$$

$\big\downarrow$ 5493

$$\frac{\frac{\frac{3a\sqrt{a^2x^2+1}\int \frac{\arctan(ax)^2}{x\sqrt{a^2x^2+1}}dx}{\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}}{c}-a^2\left(\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}}-6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)\right)}{c}-$$

$$a^2\left(\frac{x\arctan(ax)^3}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{\arctan(ax)^2}{3ac\left(a^2cx^2+c\right)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}}-6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)\right)}{3c}-\frac{2}{3}\right.$$

$\big\downarrow$ 5491

$$\frac{\frac{\frac{3a\sqrt{a^2x^2+1}\int \frac{\sqrt{a^2x^2+1}\arctan(ax)^2}{ax}d\arctan(ax)}{\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}}{c}-a^2\left(\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}}-6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2}}\right.\right.}{c}$$

$$a^2\left(\frac{x\arctan(ax)^3}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{\arctan(ax)^2}{3ac\left(a^2cx^2+c\right)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}}-6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)\right)}{3c}-\frac{2}{3}\right.$$

$\big\downarrow$ 3042

$$\frac{\frac{\frac{3a\sqrt{a^2x^2+1}\int \arctan(ax)^2\csc(\arctan(ax))d\arctan(ax)}{\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}}{c}-a^2\left(\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}}-6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{}{ac\sqrt{a}}\right.\right.}{c}$$

$$a^2\left(\frac{x\arctan(ax)^3}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{\arctan(ax)^2}{3ac\left(a^2cx^2+c\right)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}}-6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)\right)}{3c}-\frac{2}{3}\right.$$

$\big\downarrow$ 4671

$$-a^2\left(\frac{x\arctan(ax)^3}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{\arctan(ax)^2}{3ac\left(a^2cx^2+c\right)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}}-6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)\right)}{3c}-\frac{2}{3}\right.$$

$$\frac{-a^2\left(\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}}-6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)\right)+\frac{-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(-2\int \arctan(ax)\log\left(1-e\right.\right.}{}}{c}}{}$$

$\downarrow$ 3011

$$-a^2\left(\frac{x\arctan(ax)^3}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{\arctan(ax)^2}{3ac\left(a^2cx^2+c\right)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}}-6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)\right)}{3c}-\frac{2}{3}\right.$$

$$\overline{-a^2\left(\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}}-6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)\right)+\frac{-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(\right.\right.\right.}{}}{}}$$

$\downarrow$ 2720

$$-a^2\left(\frac{x\arctan(ax)^3}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{\arctan(ax)^2}{3ac\left(a^2cx^2+c\right)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}}-6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)\right)}{3c}-\frac{2}{3}\right.$$

$$\overline{-a^2\left(\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}}-6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)\right)+\frac{-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(2\left(i\arctan(ax)\operatorname{PolyLog}\left(\right.\right.\right.}{}}{}}$$

$\downarrow$ 7143

$$-a^2\left(\frac{x\arctan(ax)^3}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{\arctan(ax)^2}{3ac\left(a^2cx^2+c\right)^{3/2}}+\frac{2\left(\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}}-6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)\right)}{3c}-\frac{2}{3}\right.$$

$$\overline{-a^2\left(\frac{x\arctan(ax)^3}{c\sqrt{a^2cx^2+c}}+\frac{3\arctan(ax)^2}{ac\sqrt{a^2cx^2+c}}-6\left(\frac{x\arctan(ax)}{c\sqrt{a^2cx^2+c}}+\frac{1}{ac\sqrt{a^2cx^2+c}}\right)\right)+\frac{-\frac{\arctan(ax)^3\sqrt{a^2cx^2+c}}{cx}+\frac{3a\sqrt{a^2x^2+1}\left(-2\arctan(ax)^2\arctan\right.}{}}{}}$$

input
```
Int[ArcTan[a*x]^3/(x^2*(c + a^2*c*x^2)^(5/2)),x]
```

output
```
-(a^2*(ArcTan[a*x]^2/(3*a*c*(c + a^2*c*x^2)^(3/2)) + (x*ArcTan[a*x]^3)/(3*
c*(c + a^2*c*x^2)^(3/2)) + (2*((3*ArcTan[a*x]^2)/(a*c*Sqrt[c + a^2*c*x^2])
 + (x*ArcTan[a*x]^3)/(c*Sqrt[c + a^2*c*x^2]) - 6*(1/(a*c*Sqrt[c + a^2*c*x^
2]) + (x*ArcTan[a*x])/(c*Sqrt[c + a^2*c*x^2]))))/(3*c) - (2*(1/(9*a*c*(c +
 a^2*c*x^2)^(3/2)) + (x*ArcTan[a*x])/(3*c*(c + a^2*c*x^2)^(3/2)) + (2*(1/(
a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x])/(c*Sqrt[c + a^2*c*x^2])))/(3*c)
))/3)) + (-(a^2*((3*ArcTan[a*x]^2)/(a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a
*x]^3)/(c*Sqrt[c + a^2*c*x^2]) - 6*(1/(a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTa
n[a*x])/(c*Sqrt[c + a^2*c*x^2])))) + (-((Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3
)/(c*x)) + (3*a*Sqrt[1 + a^2*x^2]*(-2*ArcTan[a*x]^2*ArcTanh[E^(I*ArcTan[a*
x])] + 2*(I*ArcTan[a*x]*PolyLog[2, -E^(I*ArcTan[a*x])] - PolyLog[3, -E^(I*
ArcTan[a*x])]) - 2*(I*ArcTan[a*x]*PolyLog[2, E^(I*ArcTan[a*x])] - PolyLog[
3, E^(I*ArcTan[a*x])])))/Sqrt[c + a^2*c*x^2])/c)/c
```

## Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4671
```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[-
2*(c + d*x)^m*(ArcTanh[E^(I*(e + f*x))]/f), x] + (-Simp[d*(m/f)   Int[(c +
d*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Simp[d*(m/f)   Int[(c + d*x
)^(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IG
tQ[m, 0]
```

rule 5429
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/((d_) + (e_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[b/(c*d*Sqrt[d + e*x^2]), x] + Simp[x*((a + b*ArcTan[c*x])/(d*Sqr
t[d + e*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d]
```

rule 5431
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(q_), x_Symbol
] :> Simp[b*((d + e*x^2)^(q + 1)/(4*c*d*(q + 1)^2)), x] + (-Simp[x*(d + e*x
^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*d*(q + 1))), x] + Simp[(2*q + 3)/(2*d*(
q + 1))   Int[(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x]), x], x]) /; FreeQ[{a,
 b, c, d, e}, x] && EqQ[e, c^2*d] && LtQ[q, -1] && NeQ[q, -3/2]
```

rule 5433
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x] - Simp[b^2*p*(p
- 1)   Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2), x], x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

rule 5435
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[b*p*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p - 1)/(4*c*d*
(q + 1)^2)), x] + (-Simp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*d*
(q + 1))), x] + Simp[(2*q + 3)/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p, x], x] - Simp[b^2*p*((p - 1)/(4*(q + 1)^2))   Int[(d + e
*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5491
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2]
), x_Symbol] :> Simp[1/Sqrt[d]   Subst[Int[(a + b*x)^p*Csc[x], x], x, ArcTa
n[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0] &&
GtQ[d, 0]
```

rule 5493
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*Sqrt[(d_) + (e_.)*(x_)^2
]), x_Symbol] :> Simp[Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2]   Int[(a + b*ArcTan
[c*x])^p/(x*Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[
e, c^2*d] && IGtQ[p, 0] &&  !GtQ[d, 0]
```

rule 5501
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/d   Int[x^m*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c
*x])^p, x], x] - Simp[e/d   Int[x^(m + 2)*(d + e*x^2)^q*(a + b*ArcTan[c*x])
^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ[p, 2*
q] && LtQ[q, -1] && ILtQ[m, 0] && NeQ[p, -1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

## Maple [A] (verified)

Time = 4.93 (sec) , antiderivative size = 528, normalized size of antiderivative = 1.07

| method | result |
|---|---|
| default | $\frac{a\left(9i\arctan(ax)^2+9\arctan(ax)^3-2i-6\arctan(ax)\right)\left(a^3x^3-3ia^2x^2-3ax+i\right)\sqrt{c(ax-i)(ax+i)}}{216c^3(a^2x^2+1)^2}-\frac{7a\left(\arctan(ax)^3-6\arctan(ax)+\right.}{}$ |

input
```
int(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/216*a*(9*I*arctan(a*x)^2+9*arctan(a*x)^3-2*I-6*arctan(a*x))*(a^3*x^3-3*I
*a^2*x^2-3*a*x+I)*(c*(a*x-I)*(a*x+I))^(1/2)/c^3/(a^2*x^2+1)^2-7/8*a*(arcta
n(a*x)^3-6*arctan(a*x)+3*I*arctan(a*x)^2-6*I)*(a*x-I)*(c*(a*x-I)*(a*x+I))^
(1/2)/(a^2*x^2+1)/c^3-7/8*(c*(a*x-I)*(a*x+I))^(1/2)*(a*x+I)*(arctan(a*x)^3
-6*arctan(a*x)-3*I*arctan(a*x)^2+6*I)*a/(a^2*x^2+1)/c^3+1/216*(c*(a*x-I)*(
a*x+I))^(1/2)*(a^3*x^3+3*I*a^2*x^2-3*a*x-I)*(-9*I*arctan(a*x)^2+9*arctan(a
*x)^3+2*I-6*arctan(a*x))*a/c^3/(a^4*x^4+2*a^2*x^2+1)-arctan(a*x)^3*(c*(a*x
-I)*(a*x+I))^(1/2)/c^3/x-3*a*(arctan(a*x)^2*ln(1+(1+I*a*x)/(a^2*x^2+1)^(1/
2))-2*I*arctan(a*x)*polylog(2,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-arctan(a*x)^2*
ln(1-(1+I*a*x)/(a^2*x^2+1)^(1/2))+2*I*arctan(a*x)*polylog(2,(1+I*a*x)/(a^2
*x^2+1)^(1/2))+2*polylog(3,-(1+I*a*x)/(a^2*x^2+1)^(1/2))-2*polylog(3,(1+I*
a*x)/(a^2*x^2+1)^(1/2)))*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/2)/c^3
```

## Fricas [F]

$$\int \frac{\arctan(ax)^3}{x^2\left(c+a^2cx^2\right)^{5/2}}\,dx=\int \frac{\arctan\left(ax\right)^3}{\left(a^2cx^2+c\right)^{\frac{5}{2}}x^2}\,dx$$

input
```
integrate(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3/(a^6*c^3*x^8 + 3*a^4*c^3*x^6 +
3*a^2*c^3*x^4 + c^3*x^2), x)
```

## Sympy [F]

$$\int \frac{\arctan(ax)^3}{x^2\left(c+a^2cx^2\right)^{5/2}}\,dx=\int \frac{\operatorname{atan}^3\left(ax\right)}{x^2\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(atan(a*x)**3/x**2/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(atan(a*x)**3/(x**2*(c*(a**2*x**2 + 1))**(5/2)), x)
```

## Maxima [F]

$$\int \frac{\arctan(ax)^3}{x^2 (c + a^2cx^2)^{5/2}}\, dx = \int \frac{\arctan{(ax)}^3}{(a^2cx^2 + c)^{\frac{5}{2}} x^2}\, dx$$

input
```
integrate(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
integrate(arctan(a*x)^3/((a^2*c*x^2 + c)^(5/2)*x^2), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^3}{x^2 (c + a^2cx^2)^{5/2}}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^3/x^2/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^3}{x^2 (c + a^2cx^2)^{5/2}}\, dx = \int \frac{\mathrm{atan}(a\,x)^3}{x^2\,(c\,a^2\,x^2 + c)^{5/2}}\, dx$$

input
```
int(atan(a*x)^3/(x^2*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(atan(a*x)^3/(x^2*(c + a^2*c*x^2)^(5/2)), x)
```

## Reduce [F]

$$\int \frac{\arctan(ax)^3}{x^2(c+a^2cx^2)^{5/2}}\,dx = \frac{\int \frac{atan(ax)^3}{\sqrt{a^2x^2+1}\,a^4x^6+2\sqrt{a^2x^2+1}\,a^2x^4+\sqrt{a^2x^2+1}\,x^2}dx}{\sqrt{c}\,c^2}$$

input
```
int(atan(a*x)^3/x^2/(a^2*c*x^2+c)^(5/2),x)
```

output
```
int(atan(a*x)**3/(sqrt(a**2*x**2 + 1)*a**4*x**6 + 2*sqrt(a**2*x**2 + 1)*a*
*2*x**4 + sqrt(a**2*x**2 + 1)*x**2),x)/(sqrt(c)*c**2)
```

# 3.457 $\int x^m(c+a^2cx^2)^2 \arctan(ax)^3 \,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int x^m\left(c+a^2cx^2\right)^2 \arctan(ax)^3 \,dx = \mathrm{Int}\left(x^m\left(c+a^2cx^2\right)^2 \arctan(ax)^3, x\right)$$

output
```
Defer(Int)(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^3,x)
```

## Mathematica [N/A]

Not integrable

Time = 1.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int x^m\left(c+a^2cx^2\right)^2 \arctan(ax)^3 \,dx = \int x^m\left(c+a^2cx^2\right)^2 \arctan(ax)^3 \,dx$$

input
```
Integrate[x^m*(c + a^2*c*x^2)^2*ArcTan[a*x]^3,x]
```

output
```
Integrate[x^m*(c + a^2*c*x^2)^2*ArcTan[a*x]^3, x]
```

## Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \arctan(ax)^3 \left(a^2cx^2+c\right)^2 dx$$

$$\Big\downarrow \text{5560}$$

$$\int x^m \arctan(ax)^3 \left(a^2cx^2+c\right)^2 dx$$

input
```
Int[x^m*(c + a^2*c*x^2)^2*ArcTan[a*x]^3,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 4.64 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x^m \left(a^2c\,x^2+c\right)^2 \arctan\left(ax\right)^3 dx$$

input
```
int(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^3,x)
```

output
```
int(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

$$\int x^m (c + a^2cx^2)^2 \arctan(ax)^3 \, dx = \int (a^2cx^2 + c)^2 x^m \arctan(ax)^3 \, dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*x^m*arctan(a*x)^3, x)
```

## Sympy [N/A]

Not integrable

Time = 27.59 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.41

$$\int x^m (c + a^2cx^2)^2 \arctan(ax)^3 \, dx = c^2 \left( \int x^m \operatorname{atan}^3(ax) \, dx + \int 2a^2x^2x^m \operatorname{atan}^3(ax) \, dx + \int a^4x^4x^m \operatorname{atan}^3(ax) \, dx \right)$$

input
```
integrate(x**m*(a**2*c*x**2+c)**2*atan(a*x)**3,x)
```

output
```
c**2*(Integral(x**m*atan(a*x)**3, x) + Integral(2*a**2*x**2*x**m*atan(a*x)
**3, x) + Integral(a**4*x**4*x**m*atan(a*x)**3, x))
```

## Maxima [N/A]

Not integrable

Time = 13.90 (sec) , antiderivative size = 952, normalized size of antiderivative = 43.27

$$\int x^m (c+a^2cx^2)^2 \arctan(ax)^3 \, dx = \int (a^2cx^2+c)^2 x^m \arctan(ax)^3 \, dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^3,x, algorithm="maxima")
```

output
```
1/32*(4*((a^4*c^2*m^2 + 4*a^4*c^2*m + 3*a^4*c^2)*x^5 + 2*(a^2*c^2*m^2 + 6*
a^2*c^2*m + 5*a^2*c^2)*x^3 + (c^2*m^2 + 8*c^2*m + 15*c^2)*x)*x^m*arctan(a*
x)^3 - 3*((a^4*c^2*m^2 + 4*a^4*c^2*m + 3*a^4*c^2)*x^5 + 2*(a^2*c^2*m^2 + 6
*a^2*c^2*m + 5*a^2*c^2)*x^3 + (c^2*m^2 + 8*c^2*m + 15*c^2)*x)*x^m*arctan(a
*x)*log(a^2*x^2 + 1)^2 + 32*(m^3 + 9*m^2 + 23*m + 15)*integrate(1/32*(28*(
(a^6*c^2*m^3 + 9*a^6*c^2*m^2 + 23*a^6*c^2*m + 15*a^6*c^2)*x^6 + c^2*m^3 +
3*(a^4*c^2*m^3 + 9*a^4*c^2*m^2 + 23*a^4*c^2*m + 15*a^4*c^2)*x^4 + 9*c^2*m^
2 + 23*c^2*m + 3*(a^2*c^2*m^3 + 9*a^2*c^2*m^2 + 23*a^2*c^2*m + 15*a^2*c^2)
*x^2 + 15*c^2)*x^m*arctan(a*x)^3 - 12*((a^5*c^2*m^2 + 4*a^5*c^2*m + 3*a^5*
c^2)*x^5 + 2*(a^3*c^2*m^2 + 6*a^3*c^2*m + 5*a^3*c^2)*x^3 + (a*c^2*m^2 + 8*
a*c^2*m + 15*a*c^2)*x)*x^m*arctan(a*x)^2 + 12*((a^6*c^2*m^2 + 4*a^6*c^2*m
+ 3*a^6*c^2)*x^6 + 2*(a^4*c^2*m^2 + 6*a^4*c^2*m + 5*a^4*c^2)*x^4 + (a^2*c^
2*m^2 + 8*a^2*c^2*m + 15*a^2*c^2)*x^2)*x^m*arctan(a*x)*log(a^2*x^2 + 1) +
3*(((a^6*c^2*m^3 + 9*a^6*c^2*m^2 + 23*a^6*c^2*m + 15*a^6*c^2)*x^6 + c^2*m^
3 + 3*(a^4*c^2*m^3 + 9*a^4*c^2*m^2 + 23*a^4*c^2*m + 15*a^4*c^2)*x^4 + 9*c^
2*m^2 + 23*c^2*m + 3*(a^2*c^2*m^3 + 9*a^2*c^2*m^2 + 23*a^2*c^2*m + 15*a^2*
c^2)*x^2 + 15*c^2)*x^m*arctan(a*x) + ((a^5*c^2*m^2 + 4*a^5*c^2*m + 3*a^5*c
^2)*x^5 + 2*(a^3*c^2*m^2 + 6*a^3*c^2*m + 5*a^3*c^2)*x^3 + (a*c^2*m^2 + 8*a
*c^2*m + 15*a*c^2)*x)*x^m)*log(a^2*x^2 + 1)^2)/(m^3 + (a^2*m^3 + 9*a^2*m^2
 + 23*a^2*m + 15*a^2)*x^2 + 9*m^2 + 23*m + 15), x))/(m^3 + 9*m^2 + 23*m...
```

## Giac [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int x^m (c+a^2cx^2)^2 \arctan(ax)^3 \, dx = \int (a^2cx^2+c)^2 x^m \arctan(ax)^3 \, dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^2*x^m*arctan(a*x)^3, x)
```

## Mupad [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int x^m (c+a^2cx^2)^2 \arctan(ax)^3 \, dx = \int x^m \operatorname{atan}(a\,x)^3 \left(c\,a^2\,x^2+c\right)^2 dx$$

input
```
int(x^m*atan(a*x)^3*(c + a^2*c*x^2)^2,x)
```

output
```
int(x^m*atan(a*x)^3*(c + a^2*c*x^2)^2, x)
```

## Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 3236, normalized size of antiderivative = 147.09

$$\int x^m (c+a^2cx^2)^2 \arctan(ax)^3 \, dx = \text{Too large to display}$$

input
```
int(x^m*(a^2*c*x^2+c)^2*atan(a*x)^3,x)
```

output
```
(c**2*(x**m*atan(a*x)**3*a**5*m**5*x**5 + 10*x**m*atan(a*x)**3*a**5*m**4*x
**5 + 35*x**m*atan(a*x)**3*a**5*m**3*x**5 + 50*x**m*atan(a*x)**3*a**5*m**2
*x**5 + 24*x**m*atan(a*x)**3*a**5*m*x**5 + 2*x**m*atan(a*x)**3*a**3*m**5*x
**3 + 24*x**m*atan(a*x)**3*a**3*m**4*x**3 + 98*x**m*atan(a*x)**3*a**3*m**3
*x**3 + 156*x**m*atan(a*x)**3*a**3*m**2*x**3 + 80*x**m*atan(a*x)**3*a**3*m
*x**3 + x**m*atan(a*x)**3*a*m**5*x + 14*x**m*atan(a*x)**3*a*m**4*x + 71*x*
*m*atan(a*x)**3*a*m**3*x + 154*x**m*atan(a*x)**3*a*m**2*x + 120*x**m*atan(
a*x)**3*a*m*x - 3*x**m*atan(a*x)**2*a**4*m**4*x**4 - 18*x**m*atan(a*x)**2*
a**4*m**3*x**4 - 33*x**m*atan(a*x)**2*a**4*m**2*x**4 - 18*x**m*atan(a*x)**
2*a**4*m*x**4 - 3*x**m*atan(a*x)**2*a**2*m**4*x**2 - 36*x**m*atan(a*x)**2*
a**2*m**3*x**2 - 117*x**m*atan(a*x)**2*a**2*m**2*x**2 - 84*x**m*atan(a*x)*
*2*a**2*m*x**2 - 24*x**m*atan(a*x)**2*m**2 - 144*x**m*atan(a*x)**2*m - 192
*x**m*atan(a*x)**2 + 6*x**m*atan(a*x)*a**3*m**3*x**3 + 18*x**m*atan(a*x)*a
**3*m**2*x**3 + 12*x**m*atan(a*x)*a**3*m*x**3 + 36*x**m*atan(a*x)*a*m**2*x
 + 132*x**m*atan(a*x)*a*m*x - 6*x**m*a**2*m**2*x**2 - 6*x**m*a**2*m*x**2 +
 6*x**m*m**2 - 18*x**m*m - 120*x**m - 6*int(x**m/(a**2*m**5*x**3 + 15*a**2
*m**4*x**3 + 85*a**2*m**3*x**3 + 225*a**2*m**2*x**3 + 274*a**2*m*x**3 + 12
0*a**2*x**3 + m**5*x + 15*m**4*x + 85*m**3*x + 225*m**2*x + 274*m*x + 120*
x),x)*m**8 - 72*int(x**m/(a**2*m**5*x**3 + 15*a**2*m**4*x**3 + 85*a**2*m**
3*x**3 + 225*a**2*m**2*x**3 + 274*a**2*m*x**3 + 120*a**2*x**3 + m**5*x ...
```

# 3.458 $\int x^m(c+a^2cx^2)\arctan(ax)^3\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 20

$$\int x^m(c+a^2cx^2)\arctan(ax)^3\,dx=\text{Int}\left(x^m(c+a^2cx^2)\arctan(ax)^3,x\right)$$

output
```
Defer(Int)(x^m*(a^2*c*x^2+c)*arctan(a*x)^3,x)
```

## Mathematica [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x^m(c+a^2cx^2)\arctan(ax)^3\,dx=\int x^m(c+a^2cx^2)\arctan(ax)^3\,dx$$

input
```
Integrate[x^m*(c + a^2*c*x^2)*ArcTan[a*x]^3,x]
```

output
```
Integrate[x^m*(c + a^2*c*x^2)*ArcTan[a*x]^3, x]
```

## Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \arctan(ax)^3 \left(a^2cx^2+c\right) \, dx$$
$$\Big\downarrow \ 5560$$
$$\int x^m \arctan(ax)^3 \left(a^2cx^2+c\right) dx$$

input
```
Int[x^m*(c + a^2*c*x^2)*ArcTan[a*x]^3,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 6.71 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int x^m \left(a^2c\,x^2+c\right) \arctan\left(ax\right)^3 dx$$

input
```
int(x^m*(a^2*c*x^2+c)*arctan(a*x)^3,x)
```

output
```
int(x^m*(a^2*c*x^2+c)*arctan(a*x)^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x^m(c+a^2cx^2)\arctan(ax)^3\,dx=\int (a^2cx^2+c)x^m\arctan(ax)^3\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)*arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)*x^m*arctan(a*x)^3, x)
```

## Sympy [N/A]

Not integrable

Time = 12.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.55

$$\int x^m(c+a^2cx^2)\arctan(ax)^3\,dx=c\left(\int x^m\,\mathrm{atan}^3(ax)\,dx+\int a^2x^2x^m\,\mathrm{atan}^3(ax)\,dx\right)$$

input
```
integrate(x**m*(a**2*c*x**2+c)*atan(a*x)**3,x)
```

output
```
c*(Integral(x**m*atan(a*x)**3, x) + Integral(a**2*x**2*x**m*atan(a*x)**3,
x))
```

## Maxima [N/A]

Not integrable

Time = 6.64 (sec) , antiderivative size = 435, normalized size of antiderivative = 21.75

$$\int x^m(c+a^2cx^2)\arctan(ax)^3\,dx=\int (a^2cx^2+c)x^m\arctan(ax)^3\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)*arctan(a*x)^3,x, algorithm="maxima")
```

output
```
1/32*(4*((a^2*c*m + a^2*c)*x^3 + (c*m + 3*c)*x)*x^m*arctan(a*x)^3 - 3*((a^
2*c*m + a^2*c)*x^3 + (c*m + 3*c)*x)*x^m*arctan(a*x)*log(a^2*x^2 + 1)^2 + 3
2*(m^2 + 4*m + 3)*integrate(1/32*(28*((a^4*c*m^2 + 4*a^4*c*m + 3*a^4*c)*x^
4 + c*m^2 + 2*(a^2*c*m^2 + 4*a^2*c*m + 3*a^2*c)*x^2 + 4*c*m + 3*c)*x^m*arc
tan(a*x)^3 - 12*((a^3*c*m + a^3*c)*x^3 + (a*c*m + 3*a*c)*x)*x^m*arctan(a*x
)^2 + 12*((a^4*c*m + a^4*c)*x^4 + (a^2*c*m + 3*a^2*c)*x^2)*x^m*arctan(a*x)
*log(a^2*x^2 + 1) + 3*(((a^4*c*m^2 + 4*a^4*c*m + 3*a^4*c)*x^4 + c*m^2 + 2*
(a^2*c*m^2 + 4*a^2*c*m + 3*a^2*c)*x^2 + 4*c*m + 3*c)*x^m*arctan(a*x) + ((a
^3*c*m + a^3*c)*x^3 + (a*c*m + 3*a*c)*x)*x^m)*log(a^2*x^2 + 1)^2)/((a^2*m^
2 + 4*a^2*m + 3*a^2)*x^2 + m^2 + 4*m + 3), x))/(m^2 + 4*m + 3)
```

## Giac [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x^m \left(c+a^2cx^2\right) \arctan(ax)^3 \, dx = \int \left(a^2cx^2+c\right)x^m \arctan\left(ax\right)^3 \, dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)*arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)*x^m*arctan(a*x)^3, x)
```

## Mupad [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x^m \left(c+a^2cx^2\right) \arctan(ax)^3 \, dx = \int x^m \operatorname{atan}(a\,x)^3 \left(c\,a^2\,x^2+c\right) \, dx$$

input
```
int(x^m*atan(a*x)^3*(c + a^2*c*x^2),x)
```

output
```
int(x^m*atan(a*x)^3*(c + a^2*c*x^2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 1269, normalized size of antiderivative = 63.45

$$\int x^m \left(c + a^2 c x^2\right) \arctan(ax)^3 \, dx = \text{Too large to display}$$

input
```
int(x^m*(a^2*c*x^2+c)*atan(a*x)^3,x)
```

output
```
(c*(x**m*atan(a*x)**3*a**3*m**3*x**3 + 3*x**m*atan(a*x)**3*a**3*m**2*x**3
+ 2*x**m*atan(a*x)**3*a**3*m*x**3 + x**m*atan(a*x)**3*a*m**3*x + 5*x**m*at
an(a*x)**3*a*m**2*x + 6*x**m*atan(a*x)**3*a*m*x - 3*x**m*atan(a*x)**2*a**2
*m**2*x**2 - 3*x**m*atan(a*x)**2*a**2*m*x**2 - 6*x**m*atan(a*x)**2*m - 12*
x**m*atan(a*x)**2 + 6*x**m*atan(a*x)*a*m*x - 6*x**m + 6*int(x**m/(a**2*m**
3*x**3 + 6*a**2*m**2*x**3 + 11*a**2*m*x**3 + 6*a**2*x**3 + m**3*x + 6*m**2
*x + 11*m*x + 6*x),x)*m**4 + 36*int(x**m/(a**2*m**3*x**3 + 6*a**2*m**2*x**
3 + 11*a**2*m*x**3 + 6*a**2*x**3 + m**3*x + 6*m**2*x + 11*m*x + 6*x),x)*m*
*3 + 66*int(x**m/(a**2*m**3*x**3 + 6*a**2*m**2*x**3 + 11*a**2*m*x**3 + 6*a
**2*x**3 + m**3*x + 6*m**2*x + 11*m*x + 6*x),x)*m**2 + 36*int(x**m/(a**2*m
**3*x**3 + 6*a**2*m**2*x**3 + 11*a**2*m*x**3 + 6*a**2*x**3 + m**3*x + 6*m*
*2*x + 11*m*x + 6*x),x)*m - 6*int((x**m*atan(a*x))/(a**2*m**3*x**2 + 6*a**
2*m**2*x**2 + 11*a**2*m*x**2 + 6*a**2*x**2 + m**3 + 6*m**2 + 11*m + 6),x)*
a*m**5 - 30*int((x**m*atan(a*x))/(a**2*m**3*x**2 + 6*a**2*m**2*x**2 + 11*a
**2*m*x**2 + 6*a**2*x**2 + m**3 + 6*m**2 + 11*m + 6),x)*a*m**4 - 6*int((x*
*m*atan(a*x))/(a**2*m**3*x**2 + 6*a**2*m**2*x**2 + 11*a**2*m*x**2 + 6*a**2
*x**2 + m**3 + 6*m**2 + 11*m + 6),x)*a*m**3 + 174*int((x**m*atan(a*x))/(a*
*2*m**3*x**2 + 6*a**2*m**2*x**2 + 11*a**2*m*x**2 + 6*a**2*x**2 + m**3 + 6*
m**2 + 11*m + 6),x)*a*m**2 + 300*int((x**m*atan(a*x))/(a**2*m**3*x**2 + 6*
a**2*m**2*x**2 + 11*a**2*m*x**2 + 6*a**2*x**2 + m**3 + 6*m**2 + 11*m + ...
```

# 3.459 $\int \frac{x^m \arctan(ax)^3}{c+a^2cx^2}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^m \arctan(ax)^3}{c+a^2cx^2}\,dx = \mathrm{Int}\left(\frac{x^m \arctan(ax)^3}{c+a^2cx^2},x\right)$$

output
```
Defer(Int)(x^m*arctan(a*x)^3/(a^2*c*x^2+c),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m \arctan(ax)^3}{c+a^2cx^2}\,dx = \int \frac{x^m \arctan(ax)^3}{c+a^2cx^2}\,dx$$

input
```
Integrate[(x^m*ArcTan[a*x]^3)/(c + a^2*c*x^2),x]
```

output
```
Integrate[(x^m*ArcTan[a*x]^3)/(c + a^2*c*x^2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \arctan(ax)^3}{a^2cx^2+c} \, dx$$

$$\Big\downarrow \text{5560}$$

$$\int \frac{x^m \arctan(ax)^3}{a^2cx^2+c}dx$$

input
```
Int[(x^m*ArcTan[a*x]^3)/(c + a^2*c*x^2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 2.37 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan\left(ax\right)^3}{a^2c\,x^2+c}dx$$

input
```
int(x^m*arctan(a*x)^3/(a^2*c*x^2+c),x)
```

output
```
int(x^m*arctan(a*x)^3/(a^2*c*x^2+c),x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m \arctan(ax)^3}{c+a^2cx^2}\,dx = \int \frac{x^m \arctan{(ax)}^3}{a^2cx^2+c}\,dx$$

input
```
integrate(x^m*arctan(a*x)^3/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
integral(x^m*arctan(a*x)^3/(a^2*c*x^2 + c), x)
```

## Sympy [N/A]

Not integrable

Time = 2.93 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x^m \arctan(ax)^3}{c+a^2cx^2}\,dx = \frac{\int \frac{x^m \operatorname{atan}^3{(ax)}}{a^2x^2+1}\,dx}{c}$$

input
```
integrate(x**m*atan(a*x)**3/(a**2*c*x**2+c),x)
```

output
```
Integral(x**m*atan(a*x)**3/(a**2*x**2 + 1), x)/c
```

## Maxima [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m \arctan(ax)^3}{c+a^2cx^2}\,dx = \int \frac{x^m \arctan{(ax)}^3}{a^2cx^2+c}\,dx$$

input
```
integrate(x^m*arctan(a*x)^3/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
integrate(x^m*arctan(a*x)^3/(a^2*c*x^2 + c), x)
```

## Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m \arctan(ax)^3}{c + a^2cx^2} \, dx = \int \frac{x^m \arctan{(ax)}^3}{a^2cx^2 + c} \, dx$$

input
```
integrate(x^m*arctan(a*x)^3/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
integrate(x^m*arctan(a*x)^3/(a^2*c*x^2 + c), x)
```

## Mupad [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m \arctan(ax)^3}{c + a^2cx^2} \, dx = \int \frac{x^m \operatorname{atan}(a\,x)^3}{c\,a^2\,x^2 + c} \, dx$$

input
```
int((x^m*atan(a*x)^3)/(c + a^2*c*x^2),x)
```

output
```
int((x^m*atan(a*x)^3)/(c + a^2*c*x^2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

$$\int \frac{x^m \arctan(ax)^3}{c+a^2cx^2}\,dx = \frac{\int \frac{x^m atan(ax)^3}{a^2x^2+1}dx}{c}$$

input
```
int(x^m*atan(a*x)^3/(a^2*c*x^2+c),x)
```

output
```
int((x**m*atan(a*x)**3)/(a**2*x**2 + 1),x)/c
```

# 3.460 $\int \frac{x^m \arctan(ax)^3}{(c+a^2cx^2)^2} dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^m \arctan(ax)^3}{(c+a^2cx^2)^2} dx = \text{Int}\left(\frac{x^m \arctan(ax)^3}{(c+a^2cx^2)^2}, x\right)$$

output
```
Defer(Int)(x^m*arctan(a*x)^3/(a^2*c*x^2+c)^2,x)
```

## Mathematica [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m \arctan(ax)^3}{(c+a^2cx^2)^2} dx = \int \frac{x^m \arctan(ax)^3}{(c+a^2cx^2)^2} dx$$

input
```
Integrate[(x^m*ArcTan[a*x]^3)/(c + a^2*c*x^2)^2,x]
```

output
```
Integrate[(x^m*ArcTan[a*x]^3)/(c + a^2*c*x^2)^2, x]
```

## Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \arctan(ax)^3}{(a^2cx^2+c)^2} dx$$

$$\Big\downarrow \ 5560$$

$$\int \frac{x^m \arctan(ax)^3}{(a^2cx^2+c)^2} dx$$

input
```
Int[(x^m*ArcTan[a*x]^3)/(c + a^2*c*x^2)^2,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 3.91 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan(ax)^3}{(a^2c\,x^2+c)^2} dx$$

input
```
int(x^m*arctan(a*x)^3/(a^2*c*x^2+c)^2,x)
```

output
```
int(x^m*arctan(a*x)^3/(a^2*c*x^2+c)^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.77

$$\int \frac{x^m \arctan(ax)^3}{(c+a^2cx^2)^2}\,dx = \int \frac{x^m \arctan{(ax)}^3}{(a^2cx^2+c)^2}\,dx$$

input
```
integrate(x^m*arctan(a*x)^3/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
integral(x^m*arctan(a*x)^3/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2), x)
```

## Sympy [N/A]

Not integrable

Time = 4.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.41

$$\int \frac{x^m \arctan(ax)^3}{(c+a^2cx^2)^2}\,dx = \frac{\int \frac{x^m \operatorname{atan}^3{(ax)}}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

input
```
integrate(x**m*atan(a*x)**3/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(x**m*atan(a*x)**3/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

## Maxima [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m \arctan(ax)^3}{(c+a^2cx^2)^2}\,dx = \int \frac{x^m \arctan{(ax)}^3}{(a^2cx^2+c)^2}\,dx$$

input
```
integrate(x^m*arctan(a*x)^3/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
integrate(x^m*arctan(a*x)^3/(a^2*c*x^2 + c)^2, x)
```

## Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m \arctan(ax)^3}{(c+a^2cx^2)^2}\,dx = \int \frac{x^m \arctan{(ax)}^3}{(a^2cx^2+c)^2}\,dx$$

input
```
integrate(x^m*arctan(a*x)^3/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate(x^m*arctan(a*x)^3/(a^2*c*x^2 + c)^2, x)
```

## Mupad [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m \arctan(ax)^3}{(c+a^2cx^2)^2}\,dx = \int \frac{x^m\,\mathrm{atan}(a\,x)^3}{(c\,a^2\,x^2+c)^2}\,dx$$

input
```
int((x^m*atan(a*x)^3)/(c + a^2*c*x^2)^2,x)
```

output
```
int((x^m*atan(a*x)^3)/(c + a^2*c*x^2)^2, x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.59

$$\int \frac{x^m \arctan(ax)^3}{(c+a^2cx^2)^2} \, dx = \frac{\int \frac{x^m atan(ax)^3}{a^4x^4+2a^2x^2+1} dx}{c^2}$$

input
```
int(x^m*atan(a*x)^3/(a^2*c*x^2+c)^2,x)
```

output
```
int((x**m*atan(a*x)**3)/(a**4*x**4 + 2*a**2*x**2 + 1),x)/c**2
```

# 3.461 $\int x^m(c+a^2cx^2)^{3/2}\arctan(ax)^3\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int x^m\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3\,dx=\operatorname{Int}\left(x^m\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3,x\right)$$

output
```
Defer(Int)(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x)
```

## Mathematica [N/A]

Not integrable

Time = 0.99 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int x^m\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3\,dx=\int x^m\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3\,dx$$

input
```
Integrate[x^m*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3,x]
```

output
```
Integrate[x^m*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3, x]
```

## Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \arctan(ax)^3 \left(a^2cx^2+c\right)^{3/2} \, dx$$

$$\Big\downarrow \ 5560$$

$$\int x^m \arctan(ax)^3 \left(a^2cx^2+c\right)^{3/2}dx$$

input
```
Int[x^m*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 8.35 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int x^m \left(a^2 c\, x^2+c\right)^{\frac{3}{2}} \arctan\left(a x\right)^3 dx$$

input
```
int(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x)
```

output
```
int(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^m \left(c+a^2 c x^2\right)^{3/2} \arctan(ax)^3 \, dx = \int \left(a^2 c x^2 + c\right)^{\frac{3}{2}} x^m \arctan\left(ax\right)^3 \, dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^(3/2)*x^m*arctan(a*x)^3, x)
```

## Sympy [F(-1)]

Timed out.

$$\int x^m \left(c+a^2 c x^2\right)^{3/2} \arctan(ax)^3 \, dx = \text{Timed out}$$

input
```
integrate(x**m*(a**2*c*x**2+c)**(3/2)*atan(a*x)**3,x)
```

output
```
Timed out
```

## Maxima [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^m \left(c+a^2 c x^2\right)^{3/2} \arctan(ax)^3 \, dx = \int \left(a^2 c x^2 + c\right)^{\frac{3}{2}} x^m \arctan\left(ax\right)^3 \, dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*x^m*arctan(a*x)^3, x)
```

## Giac [F(-2)]

Exception generated.

$$\int x^m \left(c+a^2cx^2\right)^{3/2} \arctan(ax)^3 \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^m \left(c+a^2cx^2\right)^{3/2} \arctan(ax)^3 \, dx = \int x^m \operatorname{atan}(a\,x)^3 \left(c\,a^2\,x^2+c\right)^{3/2} \, dx$$

input
```
int(x^m*atan(a*x)^3*(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(x^m*atan(a*x)^3*(c + a^2*c*x^2)^(3/2), x)
```

## Reduce [N/A]

Not integrable

Time = 186.86 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.33

$$\int x^m \left(c+a^2cx^2\right)^{3/2} \arctan(ax)^3 \, dx = \sqrt{c}\,c\left(\left(\int x^m\sqrt{a^2x^2+1}\,atan(ax)^3\,x^2dx\right)a^2 + \int x^m\sqrt{a^2x^2+1}\,atan(ax)^3\,dx\right)$$

input
```
int(x^m*(a^2*c*x^2+c)^(3/2)*atan(a*x)^3,x)
```

output
```
sqrt(c)*c*(int(x**m*sqrt(a**2*x**2 + 1)*atan(a*x)**3*x**2,x)*a**2 + int(x*
*m*sqrt(a**2*x**2 + 1)*atan(a*x)**3,x))
```

# 3.462 $\int x^m \sqrt{c+a^2cx^2} \arctan(ax)^3 \, dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int x^m \sqrt{c+a^2cx^2} \arctan(ax)^3 \, dx = \text{Int}\left(x^m \sqrt{c+a^2cx^2} \arctan(ax)^3, x\right)$$

output
```
Defer(Int)(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^3,x)
```

## Mathematica [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int x^m \sqrt{c+a^2cx^2} \arctan(ax)^3 \, dx = \int x^m \sqrt{c+a^2cx^2} \arctan(ax)^3 \, dx$$

input
```
Integrate[x^m*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3,x]
```

output
```
Integrate[x^m*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3, x]
```

## Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \arctan(ax)^3 \sqrt{a^2cx^2+c}\,dx$$

$$\Big\downarrow \text{5560}$$

$$\int x^m \arctan(ax)^3 \sqrt{a^2cx^2+c}dx$$

input
```
Int[x^m*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 7.92 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int x^m \sqrt{a^2c\,x^2+c}\,\arctan\left(ax\right)^3 dx$$

input
```
int(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^3,x)
```

output
```
int(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^m \sqrt{c+a^2cx^2} \arctan(ax)^3 \, dx = \int \sqrt{a^2cx^2+c}x^m \arctan(ax)^3 \, dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^m*arctan(a*x)^3, x)
```

## Sympy [N/A]

Not integrable

Time = 55.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^m \sqrt{c+a^2cx^2} \arctan(ax)^3 \, dx = \int x^m \sqrt{c(a^2x^2+1)} \operatorname{atan}^3(ax) \, dx$$

input
```
integrate(x**m*(a**2*c*x**2+c)**(1/2)*atan(a*x)**3,x)
```

output
```
Integral(x**m*sqrt(c*(a**2*x**2 + 1))*atan(a*x)**3, x)
```

## Maxima [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^m \sqrt{c+a^2cx^2} \arctan(ax)^3 \, dx = \int \sqrt{a^2cx^2+c}x^m \arctan(ax)^3 \, dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*x^m*arctan(a*x)^3, x)
```

## Giac [F(-2)]

Exception generated.

$$\int x^m \sqrt{c + a^2 c x^2} \arctan(ax)^3 \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^m \sqrt{c + a^2 c x^2} \arctan(ax)^3 \, dx = \int x^m \operatorname{atan}(a\,x)^3 \sqrt{c\,a^2\,x^2 + c} \, dx$$

input
```
int(x^m*atan(a*x)^3*(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(x^m*atan(a*x)^3*(c + a^2*c*x^2)^(1/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int x^m \sqrt{c+a^2cx^2} \arctan(ax)^3 \, dx = \sqrt{c} \left( \int x^m \sqrt{a^2x^2+1} \, atan(ax)^3 \, dx \right)$$

input
```
int(x^m*(a^2*c*x^2+c)^(1/2)*atan(a*x)^3,x)
```

output
```
sqrt(c)*int(x**m*sqrt(a**2*x**2 + 1)*atan(a*x)**3,x)
```

# 3.463 $\int \frac{x^m \arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m \arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \text{Int}\left(\frac{x^m \arctan(ax)^3}{\sqrt{c+a^2cx^2}},x\right)$$

output
```
Defer(Int)(x^m*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^m \arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx$$

input
```
Integrate[(x^m*ArcTan[a*x]^3)/Sqrt[c + a^2*c*x^2],x]
```

output
```
Integrate[(x^m*ArcTan[a*x]^3)/Sqrt[c + a^2*c*x^2], x]
```

## Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \arctan(ax)^3}{\sqrt{a^2cx^2+c}} \, dx$$

$$\Big\downarrow \ 5560$$

$$\int \frac{x^m \arctan(ax)^3}{\sqrt{a^2cx^2+c}}dx$$

input
```
Int[(x^m*ArcTan[a*x]^3)/Sqrt[c + a^2*c*x^2],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 7.86 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m \arctan(ax)^3}{\sqrt{a^2c\,x^2+c}}dx$$

input
```
int(x^m*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(x^m*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan(ax)^3}{\sqrt{c+a^2cx^2}} \, dx = \int \frac{x^m \arctan{(ax)}^3}{\sqrt{a^2cx^2+c}} \, dx$$

input
```
integrate(x^m*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(x^m*arctan(a*x)^3/sqrt(a^2*c*x^2 + c), x)
```

## Sympy [N/A]

Not integrable

Time = 37.64 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan(ax)^3}{\sqrt{c+a^2cx^2}} \, dx = \int \frac{x^m \operatorname{atan}^3{(ax)}}{\sqrt{c(a^2x^2+1)}} \, dx$$

input
```
integrate(x**m*atan(a*x)**3/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x**m*atan(a*x)**3/sqrt(c*(a**2*x**2 + 1)), x)
```

## Maxima [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan(ax)^3}{\sqrt{c+a^2cx^2}} \, dx = \int \frac{x^m \arctan{(ax)}^3}{\sqrt{a^2cx^2+c}} \, dx$$

input
```
integrate(x^m*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^m*arctan(a*x)^3/sqrt(a^2*c*x^2 + c), x)
```

## Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^m \arctan{(ax)}^3}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(x^m*arctan(a*x)^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^m*arctan(a*x)^3/sqrt(a^2*c*x^2 + c), x)
```

## Mupad [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^m\,\mathrm{atan}(a\,x)^3}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int((x^m*atan(a*x)^3)/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int((x^m*atan(a*x)^3)/(c + a^2*c*x^2)^(1/2), x)
```

## Reduce [N/A]

Not integrable

Time = 200.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan(ax)^3}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^m atan(ax)^3}{\sqrt{a^2c\,x^2+c}}dx$$

input
```
int(x^m*atan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(x^m*atan(a*x)^3/(a^2*c*x^2+c)^(1/2),x)
```

# 3.464 $\int \frac{x^m \arctan(ax)^3}{\left(c+a^2cx^2\right)^{3/2}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m \arctan(ax)^3}{\left(c+a^2cx^2\right)^{3/2}}\,dx = \text{Int}\left(\frac{x^m \arctan(ax)^3}{\left(c+a^2cx^2\right)^{3/2}},x\right)$$

output
```
Defer(Int)(x^m*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \arctan(ax)^3}{\left(c+a^2cx^2\right)^{3/2}}\,dx = \int \frac{x^m \arctan(ax)^3}{\left(c+a^2cx^2\right)^{3/2}}\,dx$$

input
```
Integrate[(x^m*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(3/2),x]
```

output
```
Integrate[(x^m*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(3/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \text{5560}$$

$$\int \frac{x^m \arctan(ax)^3}{(a^2cx^2+c)^{3/2}}dx$$

input
```
Int[(x^m*ArcTan[a*x]^3)/(c + a^2*c*x^2)^(3/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 8.47 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m \arctan\left(ax\right)^3}{\left(a^2c\,x^2+c\right)^{\frac{3}{2}}}dx$$

input
```
int(x^m*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x)
```

output
```
int(x^m*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.12

$$\int \frac{x^m \arctan(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^m \arctan(ax)^3}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^m*arctan(a*x)^3/(a^4*c^2*x^4 + 2*a^2*c^2*x^
2 + c^2), x)
```

## Sympy [N/A]

Not integrable

Time = 29.60 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^m \operatorname{atan}^3(ax)}{(c(a^2x^2+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**m*atan(a*x)**3/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(x**m*atan(a*x)**3/(c*(a**2*x**2 + 1))**(3/2), x)
```

## Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^m \arctan(ax)^3}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^m*arctan(a*x)^3/(a^2*c*x^2 + c)^(3/2), x)
```

## Giac [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^m \arctan{(ax)}^3}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m*arctan(a*x)^3/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^m*arctan(a*x)^3/(a^2*c*x^2 + c)^(3/2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.85 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^m\,\mathrm{atan}(a\,x)^3}{(c\,a^2\,x^2+c)^{3/2}}\,dx$$

input
```
int((x^m*atan(a*x)^3)/(c + a^2*c*x^2)^(3/2),x)
```

output
```
int((x^m*atan(a*x)^3)/(c + a^2*c*x^2)^(3/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.08

$$\int \frac{x^m \arctan(ax)^3}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\int \frac{x^m atan(ax)^3}{\sqrt{a^2x^2+1}\,a^2x^2+\sqrt{a^2x^2+1}}dx}{\sqrt{c}\,c}$$

input
```
int(x^m*atan(a*x)^3/(a^2*c*x^2+c)^(3/2),x)
```

output
```
int((x**m*atan(a*x)**3)/(sqrt(a**2*x**2 + 1)*a**2*x**2 + sqrt(a**2*x**2 +
1)),x)/(sqrt(c)*c)
```

# 3.465 $\int \frac{x(c+a^2cx^2)}{\arctan(ax)}\,dx$



## Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{x(c+a^2cx^2)}{\arctan(ax)}\,dx = \text{Int}\left(\frac{x(c+a^2cx^2)}{\arctan(ax)},x\right)$$

output
```
Defer(Int)(x*(a^2*c*x^2+c)/arctan(a*x),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x(c+a^2cx^2)}{\arctan(ax)}\,dx = \int \frac{x(c+a^2cx^2)}{\arctan(ax)}\,dx$$

input
```
Integrate[(x*(c + a^2*c*x^2))/ArcTan[a*x],x]
```

output
```
Integrate[(x*(c + a^2*c*x^2))/ArcTan[a*x], x]
```

## Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a^2cx^2+c)}{\arctan(ax)}\,dx$$

$$\Big\downarrow \; 5560$$

$$\int \frac{x(a^2cx^2+c)}{\arctan(ax)}dx$$

input
```
Int[(x*(c + a^2*c*x^2))/ArcTan[a*x],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 39.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x(a^2c\,x^2+c)}{\arctan\left(ax\right)}dx$$

input
```
int(x*(a^2*c*x^2+c)/arctan(a*x),x)
```

output
```
int(x*(a^2*c*x^2+c)/arctan(a*x),x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.17

$$\int \frac{x(c+a^2cx^2)}{\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)x}{\arctan{(ax)}}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)/arctan(a*x),x, algorithm="fricas")
```

output
```
integral((a^2*c*x^3 + c*x)/arctan(a*x), x)
```

## Sympy [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{x(c+a^2cx^2)}{\arctan(ax)}\,dx = c\left(\int \frac{x}{\operatorname{atan}{(ax)}}\,dx + \int \frac{a^2x^3}{\operatorname{atan}{(ax)}}\,dx\right)$$

input
```
integrate(x*(a**2*c*x**2+c)/atan(a*x),x)
```

output
```
c*(Integral(x/atan(a*x), x) + Integral(a**2*x**3/atan(a*x), x))
```

## Maxima [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x(c+a^2cx^2)}{\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)x}{\arctan{(ax)}}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)*x/arctan(a*x), x)
```

## Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x(c+a^2cx^2)}{\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)x}{\arctan(ax)}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)/arctan(a*x),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)*x/arctan(a*x), x)
```

## Mupad [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x(c+a^2cx^2)}{\arctan(ax)}\,dx = \int \frac{x\,(c\,a^2\,x^2+c)}{\mathrm{atan}\,(a\,x)}\,dx$$

input
```
int((x*(c + a^2*c*x^2))/atan(a*x),x)
```

output
```
int((x*(c + a^2*c*x^2))/atan(a*x), x)
```

## Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.61

$$\int \frac{x(c+a^2cx^2)}{\arctan(ax)}\,dx = c\left(\left(\int \frac{x^3}{atan\,(ax)}dx\right)a^2 + \int \frac{x}{atan\,(ax)}dx\right)$$

input
```
int(x*(a^2*c*x^2+c)/atan(a*x),x)
```

output
```
c*(int(x**3/atan(a*x),x)*a**2 + int(x/atan(a*x),x))
```

# 3.466 $\int \frac{c+a^2cx^2}{\arctan(ax)}\,dx$



## Optimal result

Integrand size = 17, antiderivative size = 17

$$\int \frac{c+a^2cx^2}{\arctan(ax)}\,dx = \text{Int}\left(\frac{c+a^2cx^2}{\arctan(ax)},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)/arctan(a*x),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{c+a^2cx^2}{\arctan(ax)}\,dx = \int \frac{c+a^2cx^2}{\arctan(ax)}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)/ArcTan[a*x],x]
```

output
```
Integrate[(c + a^2*c*x^2)/ArcTan[a*x], x]
```

## Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2cx^2+c}{\arctan(ax)}dx$$

$$\Big\downarrow \; 5560$$

$$\int \frac{a^2cx^2+c}{\arctan(ax)}dx$$

input
```
Int[(c + a^2*c*x^2)/ArcTan[a*x],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 17.76 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{a^2c\,x^2+c}{\arctan\left(ax\right)}dx$$

input
```
int((a^2*c*x^2+c)/arctan(a*x),x)
```

output
```
int((a^2*c*x^2+c)/arctan(a*x),x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{c+a^2cx^2}{\arctan(ax)}\,dx = \int \frac{a^2cx^2+c}{\arctan(ax)}\,dx$$

input
```
integrate((a^2*c*x^2+c)/arctan(a*x),x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)/arctan(a*x), x)
```

## Sympy [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.29

$$\int \frac{c+a^2cx^2}{\arctan(ax)}\,dx = c\left(\int \frac{a^2x^2}{\operatorname{atan}(ax)}\,dx + \int \frac{1}{\operatorname{atan}(ax)}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)/atan(a*x),x)
```

output
```
c*(Integral(a**2*x**2/atan(a*x), x) + Integral(1/atan(a*x), x))
```

## Maxima [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{c+a^2cx^2}{\arctan(ax)}\,dx = \int \frac{a^2cx^2+c}{\arctan(ax)}\,dx$$

input
```
integrate((a^2*c*x^2+c)/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)/arctan(a*x), x)
```

## Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{c+a^2cx^2}{\arctan(ax)}\,dx=\int \frac{a^2cx^2+c}{\arctan(ax)}\,dx$$

input
```
integrate((a^2*c*x^2+c)/arctan(a*x),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)/arctan(a*x), x)
```

## Mupad [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{c+a^2cx^2}{\arctan(ax)}\,dx=\int \frac{c\,a^2\,x^2+c}{\mathrm{atan}\left(a\,x\right)}\,dx$$

input
```
int((c + a^2*c*x^2)/atan(a*x),x)
```

output
```
int((c + a^2*c*x^2)/atan(a*x), x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 68, normalized size of antiderivative = 4.00

$$\int \frac{c+a^2cx^2}{\arctan(ax)}\,dx = \frac{c\left(\left(\int \frac{x^4}{atan(ax)a^2x^2+atan(ax)}dx\right)a^5+2\left(\int \frac{x^2}{atan(ax)a^2x^2+atan(ax)}dx\right)a^3+\log(atan(ax))\right)}{a}$$

input
```
int((a^2*c*x^2+c)/atan(a*x),x)
```

output
```
(c*(int(x**4/(atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**5 + 2*int(x**2/(atan(
a*x)*a**2*x**2 + atan(a*x)),x)*a**3 + log(atan(a*x))))/a
```

# 3.467 $\int \frac{c+a^2cx^2}{x\arctan(ax)}\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{c+a^2cx^2}{x\arctan(ax)}\,dx=\text{Int}\left(\frac{c+a^2cx^2}{x\arctan(ax)},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)/x/arctan(a*x),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{c+a^2cx^2}{x\arctan(ax)}\,dx=\int \frac{c+a^2cx^2}{x\arctan(ax)}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)/(x*ArcTan[a*x]),x]
```

output
```
Integrate[(c + a^2*c*x^2)/(x*ArcTan[a*x]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2cx^2+c}{x\arctan(ax)}\,dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int \frac{a^2cx^2+c}{x\arctan(ax)}dx$$

input
```
Int[(c + a^2*c*x^2)/(x*ArcTan[a*x]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 58.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{a^2c\,x^2+c}{x\arctan\left(ax\right)}dx$$

input
```
int((a^2*c*x^2+c)/x/arctan(a*x),x)
```

output
```
int((a^2*c*x^2+c)/x/arctan(a*x),x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{c+a^2cx^2}{x\arctan(ax)}\,dx = \int \frac{a^2cx^2+c}{x\arctan{(ax)}}\,dx$$

input
```
integrate((a^2*c*x^2+c)/x/arctan(a*x),x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)/(x*arctan(a*x)), x)
```

## Sympy [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{c+a^2cx^2}{x\arctan(ax)}\,dx = c\left(\int \frac{1}{x\,\mathrm{atan}\,(ax)}\,dx + \int \frac{a^2x}{\mathrm{atan}\,(ax)}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)/x/atan(a*x),x)
```

output
```
c*(Integral(1/(x*atan(a*x)), x) + Integral(a**2*x/atan(a*x), x))
```

## Maxima [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{c+a^2cx^2}{x\arctan(ax)}\,dx = \int \frac{a^2cx^2+c}{x\arctan{(ax)}}\,dx$$

input
```
integrate((a^2*c*x^2+c)/x/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)/(x*arctan(a*x)), x)
```

## Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{c+a^2cx^2}{x\arctan(ax)}\,dx = \int \frac{a^2cx^2+c}{x\arctan{(ax)}}\,dx$$

input
```
integrate((a^2*c*x^2+c)/x/arctan(a*x),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)/(x*arctan(a*x)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{c+a^2cx^2}{x\arctan(ax)}\,dx = \int \frac{c\,a^2\,x^2+c}{x\,\mathrm{atan}\left(a\,x\right)}\,dx$$

input
```
int((c + a^2*c*x^2)/(x*atan(a*x)),x)
```

output
```
int((c + a^2*c*x^2)/(x*atan(a*x)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.45

$$\int \frac{c + a^2cx^2}{x \arctan(ax)}\,dx = c\left(\left(\int \frac{x}{atan\,(ax)}dx\right)a^2 + \int \frac{1}{atan\,(ax)\,x}dx\right)$$

input
```
int((a^2*c*x^2+c)/x/atan(a*x),x)
```

output
```
c*(int(x/atan(a*x),x)*a**2 + int(1/(atan(a*x)*x),x))
```

# 3.468 $\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)}\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)}\,dx = \text{Int}\left(\frac{x(c+a^2cx^2)^2}{\arctan(ax)},x\right)$$

output
```
Defer(Int)(x*(a^2*c*x^2+c)^2/arctan(a*x),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)}\,dx = \int \frac{x(c+a^2cx^2)^2}{\arctan(ax)}\,dx$$

input
```
Integrate[(x*(c + a^2*c*x^2)^2)/ArcTan[a*x],x]
```

output
```
Integrate[(x*(c + a^2*c*x^2)^2)/ArcTan[a*x], x]
```

## Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a^2cx^2+c)^2}{\arctan(ax)}dx$$

$$\Big\downarrow \text{5560}$$

$$\int \frac{x(a^2cx^2+c)^2}{\arctan(ax)}dx$$

input
```
Int[(x*(c + a^2*c*x^2)^2)/ArcTan[a*x],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 43.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{x(a^2c\,x^2+c)^2}{\arctan\left(ax\right)}dx$$

input
```
int(x*(a^2*c*x^2+c)^2/arctan(a*x),x)
```

output
```
int(x*(a^2*c*x^2+c)^2/arctan(a*x),x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

$$\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^2x}{\arctan(ax)}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^2*x)/arctan(a*x), x)
```

## Sympy [N/A]

Not integrable

Time = 0.85 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.95

$$\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)}\,dx = c^2\left(\int \frac{x}{\operatorname{atan}(ax)}\,dx + \int \frac{2a^2x^3}{\operatorname{atan}(ax)}\,dx + \int \frac{a^4x^5}{\operatorname{atan}(ax)}\,dx\right)$$

input
```
integrate(x*(a**2*c*x**2+c)**2/atan(a*x),x)
```

output
```
c**2*(Integral(x/atan(a*x), x) + Integral(2*a**2*x**3/atan(a*x), x) + Inte
gral(a**4*x**5/atan(a*x), x))
```

## Maxima [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^2x}{\arctan(ax)}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^2*x/arctan(a*x), x)
```

## Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^2x}{\arctan(ax)}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^2*x/arctan(a*x), x)
```

## Mupad [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)}\,dx = \int \frac{x\,(c\,a^2\,x^2+c)^2}{\mathrm{atan}\,(a\,x)}\,dx$$

input
```
int((x*(c + a^2*c*x^2)^2)/atan(a*x),x)
```

output
```
int((x*(c + a^2*c*x^2)^2)/atan(a*x), x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.40

$$\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)}\,dx = c^2\left(\left(\int \frac{x^5}{atan\,(ax)}dx\right)a^4 + 2\left(\int \frac{x^3}{atan\,(ax)}dx\right)a^2 + \int \frac{x}{atan\,(ax)}dx\right)$$

input
```
int(x*(a^2*c*x^2+c)^2/atan(a*x),x)
```

output
```
c**2*(int(x**5/atan(a*x),x)*a**4 + 2*int(x**3/atan(a*x),x)*a**2 + int(x/at
an(a*x),x))
```

# 3.469 $\int \frac{(c+a^2cx^2)^2}{\arctan(ax)} dx$



## Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \frac{(c+a^2cx^2)^2}{\arctan(ax)} dx = \text{Int}\left(\frac{(c+a^2cx^2)^2}{\arctan(ax)},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^2/arctan(a*x),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{(c+a^2cx^2)^2}{\arctan(ax)} dx = \int \frac{(c+a^2cx^2)^2}{\arctan(ax)} dx$$

input
```
Integrate[(c + a^2*c*x^2)^2/ArcTan[a*x],x]
```

output
```
Integrate[(c + a^2*c*x^2)^2/ArcTan[a*x], x]
```

## Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^2}{\arctan(ax)}dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{(a^2cx^2+c)^2}{\arctan(ax)}dx$$

input
```
Int[(c + a^2*c*x^2)^2/ArcTan[a*x],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 87.74 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{(a^2c\,x^2+c)^2}{\arctan(ax)}dx$$

input
```
int((a^2*c*x^2+c)^2/arctan(a*x),x)
```

output
```
int((a^2*c*x^2+c)^2/arctan(a*x),x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.79

$$\int \frac{(c+a^2cx^2)^2}{\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^2}{\arctan(ax)}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)/arctan(a*x), x)
```

## Sympy [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.05

$$\int \frac{(c+a^2cx^2)^2}{\arctan(ax)}\,dx = c^2\left(\int \frac{2a^2x^2}{\operatorname{atan}(ax)}\,dx + \int \frac{a^4x^4}{\operatorname{atan}(ax)}\,dx + \int \frac{1}{\operatorname{atan}(ax)}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**2/atan(a*x),x)
```

output
```
c**2*(Integral(2*a**2*x**2/atan(a*x), x) + Integral(a**4*x**4/atan(a*x), x
) + Integral(1/atan(a*x), x))
```

## Maxima [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{(c+a^2cx^2)^2}{\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^2}{\arctan(ax)}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^2/arctan(a*x), x)
```

## Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{(c+a^2cx^2)^2}{\arctan(ax)}\,dx=\int \frac{(a^2cx^2+c)^2}{\arctan\left(ax\right)}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^2/arctan(a*x), x)
```

## Mupad [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{(c+a^2cx^2)^2}{\arctan(ax)}\,dx=\int \frac{{\left(c\,a^2\,x^2+c\right)}^2}{\mathrm{atan}\left(a\,x\right)}\,dx$$

input
```
int((c + a^2*c*x^2)^2/atan(a*x),x)
```

output
```
int((c + a^2*c*x^2)^2/atan(a*x), x)
```

## Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 99, normalized size of antiderivative = 5.21

$$\int \frac{(c+a^2cx^2)^2}{\arctan(ax)}\,dx$$
$$= \frac{c^2\left(\left(\int \frac{x^6}{atan(ax)a^2x^2+atan(ax)}dx\right)a^7+3\left(\int \frac{x^4}{atan(ax)a^2x^2+atan(ax)}dx\right)a^5+3\left(\int \frac{x^2}{atan(ax)a^2x^2+atan(ax)}dx\right)a^3+\log\right.}{a}$$

input
```
int((a^2*c*x^2+c)^2/atan(a*x),x)
```

output
```
(c**2*(int(x**6/(atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**7 + 3*int(x**4/(at
an(a*x)*a**2*x**2 + atan(a*x)),x)*a**5 + 3*int(x**2/(atan(a*x)*a**2*x**2 +
 atan(a*x)),x)*a**3 + log(atan(a*x))))/a
```

# 3.470 $\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)}\,dx = \operatorname{Int}\left(\frac{(c+a^2cx^2)^2}{x\arctan(ax)},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^2/x/arctan(a*x),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)}\,dx = \int \frac{(c+a^2cx^2)^2}{x\arctan(ax)}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^2/(x*ArcTan[a*x]),x]
```

output
```
Integrate[(c + a^2*c*x^2)^2/(x*ArcTan[a*x]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^2}{x\arctan(ax)}dx$$

$$\Big\downarrow \text{5560}$$

$$\int \frac{(a^2cx^2+c)^2}{x\arctan(ax)}dx$$

input
```
Int[(c + a^2*c*x^2)^2/(x*ArcTan[a*x]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 87.64 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a^2c\,x^2+c)^2}{x\arctan(ax)}dx$$

input
```
int((a^2*c*x^2+c)^2/x/arctan(a*x),x)
```

output
```
int((a^2*c*x^2+c)^2/x/arctan(a*x),x)
```

## Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

$$\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^2}{x\arctan(ax)}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2/x/arctan(a*x),x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)/(x*arctan(a*x)), x)
```

## Sympy [N/A]

Not integrable

Time = 1.63 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.77

$$\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)}\,dx = c^2\left(\int \frac{1}{x\,\mathrm{atan}\,(ax)}\,dx + \int \frac{2a^2x}{\mathrm{atan}\,(ax)}\,dx + \int \frac{a^4x^3}{\mathrm{atan}\,(ax)}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**2/x/atan(a*x),x)
```

output
```
c**2*(Integral(1/(x*atan(a*x)), x) + Integral(2*a**2*x/atan(a*x), x) + Int
egral(a**4*x**3/atan(a*x), x))
```

## Maxima [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^2}{x\arctan(ax)}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2/x/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^2/(x*arctan(a*x)), x)
```

## Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^2}{x\arctan(ax)}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2/x/arctan(a*x),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^2/(x*arctan(a*x)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)}\,dx = \int \frac{(c\,a^2\,x^2+c)^2}{x\,\mathrm{atan}(a\,x)}\,dx$$

input
```
int((c + a^2*c*x^2)^2/(x*atan(a*x)),x)
```

output
```
int((c + a^2*c*x^2)^2/(x*atan(a*x)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.18

$$\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)}\,dx = c^2\left(\left(\int \frac{x^3}{atan\,(ax)}dx\right)a^4+2\left(\int \frac{x}{atan\,(ax)}dx\right)a^2 + \int \frac{1}{atan\,(ax)\,x}dx\right)$$

input
```
int((a^2*c*x^2+c)^2/x/atan(a*x),x)
```

output
```
c**2*(int(x**3/atan(a*x),x)*a**4 + 2*int(x/atan(a*x),x)*a**2 + int(1/(atan
(a*x)*x),x))
```

# 3.471 $\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)}\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)}\,dx = \operatorname{Int}\left(\frac{x(c+a^2cx^2)^3}{\arctan(ax)},x\right)$$

output
```
Defer(Int)(x*(a^2*c*x^2+c)^3/arctan(a*x),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)}\,dx = \int \frac{x(c+a^2cx^2)^3}{\arctan(ax)}\,dx$$

input
```
Integrate[(x*(c + a^2*c*x^2)^3)/ArcTan[a*x],x]
```

output
```
Integrate[(x*(c + a^2*c*x^2)^3)/ArcTan[a*x], x]
```

## Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a^2cx^2+c)^3}{\arctan(ax)}dx$$
$$\big\downarrow \ 5560$$
$$\int \frac{x(a^2cx^2+c)^3}{\arctan(ax)}dx$$

input
```
Int[(x*(c + a^2*c*x^2)^3)/ArcTan[a*x],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 103.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{x(a^2c\,x^2+c)^3}{\arctan\left(ax\right)}dx$$

input
```
int(x*(a^2*c*x^2+c)^3/arctan(a*x),x)
```

output
```
int(x*(a^2*c*x^2+c)^3/arctan(a*x),x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.35

$$\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^3x}{\arctan(ax)}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="fricas")
```

output
```
integral((a^6*c^3*x^7 + 3*a^4*c^3*x^5 + 3*a^2*c^3*x^3 + c^3*x)/arctan(a*x)
, x)
```

## Sympy [N/A]

Not integrable

Time = 1.15 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.70

$$\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)}\,dx = c^3\left(\int \frac{x}{\operatorname{atan}(ax)}\,dx + \int \frac{3a^2x^3}{\operatorname{atan}(ax)}\,dx + \int \frac{3a^4x^5}{\operatorname{atan}(ax)}\,dx + \int \frac{a^6x^7}{\operatorname{atan}(ax)}\,dx\right)$$

input
```
integrate(x*(a**2*c*x**2+c)**3/atan(a*x),x)
```

output
```
c**3*(Integral(x/atan(a*x), x) + Integral(3*a**2*x**3/atan(a*x), x) + Inte
gral(3*a**4*x**5/atan(a*x), x) + Integral(a**6*x**7/atan(a*x), x))
```

## Maxima [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^3x}{\arctan(ax)}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^3*x/arctan(a*x), x)
```

## Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^3x}{\arctan(ax)}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^3*x/arctan(a*x), x)
```

## Mupad [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)}\,dx = \int \frac{x\,(c\,a^2\,x^2+c)^3}{\mathrm{atan}\,(a\,x)}\,dx$$

input
```
int((x*(c + a^2*c*x^2)^3)/atan(a*x),x)
```

output
```
int((x*(c + a^2*c*x^2)^3)/atan(a*x), x)
```

## Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 65, normalized size of antiderivative = 3.25

$$\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)}\,dx = c^3\left(\left(\int \frac{x^7}{atan\,(ax)}dx\right)a^6 + 3\left(\int \frac{x^5}{atan\,(ax)}dx\right)a^4 + 3\left(\int \frac{x^3}{atan\,(ax)}dx\right)a^2 + \int \frac{x}{atan\,(ax)}dx\right)$$

input
```
int(x*(a^2*c*x^2+c)^3/atan(a*x),x)
```

output
```
c**3*(int(x**7/atan(a*x),x)*a**6 + 3*int(x**5/atan(a*x),x)*a**4 + 3*int(x*
*3/atan(a*x),x)*a**2 + int(x/atan(a*x),x))
```

# 3.472 $\int \frac{(c+a^2cx^2)^3}{\arctan(ax)}\,dx$



## Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \frac{(c+a^2cx^2)^3}{\arctan(ax)}\,dx = \text{Int}\left(\frac{(c+a^2cx^2)^3}{\arctan(ax)},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^3/arctan(a*x),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{(c+a^2cx^2)^3}{\arctan(ax)}\,dx = \int \frac{(c+a^2cx^2)^3}{\arctan(ax)}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^3/ArcTan[a*x],x]
```

output
```
Integrate[(c + a^2*c*x^2)^3/ArcTan[a*x], x]
```

## Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^3}{\arctan(ax)}dx$$

$$\Big\downarrow \text{5560}$$

$$\int \frac{(a^2cx^2+c)^3}{\arctan(ax)}dx$$

input
```
Int[(c + a^2*c*x^2)^3/ArcTan[a*x],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 104.84 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{(a^2c\,x^2+c)^3}{\arctan\,(ax)}dx$$

input
```
int((a^2*c*x^2+c)^3/arctan(a*x),x)
```

output
```
int((a^2*c*x^2+c)^3/arctan(a*x),x)
```

## Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.37

$$\int \frac{(c+a^2cx^2)^3}{\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^3}{\arctan(ax)}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="fricas")
```

output
```
integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)/arctan(a*x),
x)
```

## Sympy [N/A]

Not integrable

Time = 1.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.84

$$\int \frac{(c+a^2cx^2)^3}{\arctan(ax)}\,dx = c^3\left(\int \frac{3a^2x^2}{\operatorname{atan}(ax)}\,dx + \int \frac{3a^4x^4}{\operatorname{atan}(ax)}\,dx + \int \frac{a^6x^6}{\operatorname{atan}(ax)}\,dx + \int \frac{1}{\operatorname{atan}(ax)}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**3/atan(a*x),x)
```

output
```
c**3*(Integral(3*a**2*x**2/atan(a*x), x) + Integral(3*a**4*x**4/atan(a*x),
 x) + Integral(a**6*x**6/atan(a*x), x) + Integral(1/atan(a*x), x))
```

## Maxima [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{(c+a^2cx^2)^3}{\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^3}{\arctan(ax)}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^3/arctan(a*x), x)
```

## Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{(c+a^2cx^2)^3}{\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^3}{\arctan(ax)}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^3/arctan(a*x), x)
```

## Mupad [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{(c+a^2cx^2)^3}{\arctan(ax)}\,dx = \int \frac{(c\,a^2\,x^2+c)^3}{\mathrm{atan}(a\,x)}\,dx$$

input
```
int((c + a^2*c*x^2)^3/atan(a*x),x)
```

output
```
int((c + a^2*c*x^2)^3/atan(a*x), x)
```

## Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 128, normalized size of antiderivative = 6.74

$$\int \frac{(c+a^2cx^2)^3}{\arctan(ax)}\,dx$$
$$= \frac{c^3\left(\left(\int \frac{x^8}{atan(ax)a^2x^2+atan(ax)}dx\right)a^9+4\left(\int \frac{x^6}{atan(ax)a^2x^2+atan(ax)}dx\right)a^7+6\left(\int \frac{x^4}{atan(ax)a^2x^2+atan(ax)}dx\right)a^5+4\left(\int\right.\right.}{a}$$

input
```
int((a^2*c*x^2+c)^3/atan(a*x),x)
```

output
```
(c**3*(int(x**8/(atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**9 + 4*int(x**6/(at
an(a*x)*a**2*x**2 + atan(a*x)),x)*a**7 + 6*int(x**4/(atan(a*x)*a**2*x**2 +
 atan(a*x)),x)*a**5 + 4*int(x**2/(atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**3
 + log(atan(a*x))))/a
```

# 3.473 $\int \frac{\left(c+a^2cx^2\right)^3}{x\arctan(ax)}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\left(c+a^2cx^2\right)^3}{x\arctan(ax)}\,dx=\operatorname{Int}\left(\frac{\left(c+a^2cx^2\right)^3}{x\arctan(ax)},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^3/x/arctan(a*x),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\left(c+a^2cx^2\right)^3}{x\arctan(ax)}\,dx=\int \frac{\left(c+a^2cx^2\right)^3}{x\arctan(ax)}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^3/(x*ArcTan[a*x]),x]
```

output
```
Integrate[(c + a^2*c*x^2)^3/(x*ArcTan[a*x]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^3}{x\arctan(ax)}\,dx$$

$$\Big\downarrow \ 5560$$

$$\int \frac{(a^2cx^2+c)^3}{x\arctan(ax)}dx$$

input
```
Int[(c + a^2*c*x^2)^3/(x*ArcTan[a*x]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 123.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a^2c\,x^2+c)^3}{x\arctan\left(ax\right)}dx$$

input
```
int((a^2*c*x^2+c)^3/x/arctan(a*x),x)
```

output
```
int((a^2*c*x^2+c)^3/x/arctan(a*x),x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.18

$$\int \frac{(c+a^2cx^2)^3}{x\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^3}{x\arctan(ax)}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3/x/arctan(a*x),x, algorithm="fricas")
```

output
```
integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)/(x*arctan(a*x
)), x)
```

## Sympy [N/A]

Not integrable

Time = 2.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.45

$$\int \frac{(c+a^2cx^2)^3}{x\arctan(ax)}\,dx = c^3\left(\int \frac{1}{x\,\mathrm{atan}\,(ax)}\,dx + \int \frac{3a^2x}{\mathrm{atan}\,(ax)}\,dx + \int \frac{3a^4x^3}{\mathrm{atan}\,(ax)}\,dx + \int \frac{a^6x^5}{\mathrm{atan}\,(ax)}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**3/x/atan(a*x),x)
```

output
```
c**3*(Integral(1/(x*atan(a*x)), x) + Integral(3*a**2*x/atan(a*x), x) + Int
egral(3*a**4*x**3/atan(a*x), x) + Integral(a**6*x**5/atan(a*x), x))
```

## Maxima [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(c+a^2cx^2)^3}{x\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^3}{x\arctan(ax)}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3/x/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^3/(x*arctan(a*x)), x)
```

## Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(c+a^2cx^2)^3}{x\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^3}{x\arctan(ax)}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3/x/arctan(a*x),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^3/(x*arctan(a*x)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(c+a^2cx^2)^3}{x\arctan(ax)}\,dx = \int \frac{(c\,a^2\,x^2+c)^3}{x\,\mathrm{atan}(a\,x)}\,dx$$

input
```
int((c + a^2*c*x^2)^3/(x*atan(a*x)),x)
```

output
```
int((c + a^2*c*x^2)^3/(x*atan(a*x)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.95

$$\int \frac{(c + a^2cx^2)^3}{x \arctan(ax)} dx = c^3 \left( \left( \int \frac{x^5}{atan(ax)} dx \right) a^6 + 3 \left( \int \frac{x^3}{atan(ax)} dx \right) a^4 + 3 \left( \int \frac{x}{atan(ax)} dx \right) a^2 + \int \frac{1}{atan(ax)\, x} dx \right)$$

input
```
int((a^2*c*x^2+c)^3/x/atan(a*x),x)
```

output
```
c**3*(int(x**5/atan(a*x),x)*a**6 + 3*int(x**3/atan(a*x),x)*a**4 + 3*int(x/
atan(a*x),x)*a**2 + int(1/(atan(a*x)*x),x))
```

# 3.474 $\int \frac{x^2}{(c+a^2cx^2)\arctan(ax)}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^2}{(c+a^2cx^2)\arctan(ax)}\,dx=\text{Int}\left(\frac{x^2}{(c+a^2cx^2)\arctan(ax)},x\right)$$

output
```
Defer(Int)(x^2/(a^2*c*x^2+c)/arctan(a*x),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^2}{(c+a^2cx^2)\arctan(ax)}\,dx=\int \frac{x^2}{(c+a^2cx^2)\arctan(ax)}\,dx$$

input
```
Integrate[x^2/((c + a^2*c*x^2)*ArcTan[a*x]),x]
```

output
```
Integrate[x^2/((c + a^2*c*x^2)*ArcTan[a*x]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\arctan(ax)\left(a^2cx^2+c\right)}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x^2}{\arctan(ax)\left(a^2cx^2+c\right)}dx$$

input
```
Int[x^2/((c + a^2*c*x^2)*ArcTan[a*x]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.87 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{\left(a^2c\,x^2+c\right)\arctan\left(ax\right)}dx$$

input
```
int(x^2/(a^2*c*x^2+c)/arctan(a*x),x)
```

output
```
int(x^2/(a^2*c*x^2+c)/arctan(a*x),x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^2}{(c+a^2cx^2)\arctan(ax)}\,dx = \int \frac{x^2}{(a^2cx^2+c)\arctan(ax)}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(x^2/((a^2*c*x^2 + c)*arctan(a*x)), x)
```

## Sympy [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(c+a^2cx^2)\arctan(ax)}\,dx = \frac{\int \frac{x^2}{a^2x^2\operatorname{atan}(ax)+\operatorname{atan}(ax)}\,dx}{c}$$

input
```
integrate(x**2/(a**2*c*x**2+c)/atan(a*x),x)
```

output
```
Integral(x**2/(a**2*x**2*atan(a*x) + atan(a*x)), x)/c
```

## Maxima [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^2}{(c+a^2cx^2)\arctan(ax)}\,dx = \int \frac{x^2}{(a^2cx^2+c)\arctan(ax)}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(x^2/((a^2*c*x^2 + c)*arctan(a*x)), x)
```

## Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^2}{(c+a^2cx^2)\arctan(ax)}\,dx=\int \frac{x^2}{(a^2cx^2+c)\arctan(ax)}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)/arctan(a*x),x, algorithm="giac")
```

output
```
integrate(x^2/((a^2*c*x^2 + c)*arctan(a*x)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^2}{(c+a^2cx^2)\arctan(ax)}\,dx=\int \frac{x^2}{\mathrm{atan}\left(a\,x\right)\,\left(c\,a^2\,x^2+c\right)}\,dx$$

input
```
int(x^2/(atan(a*x)*(c + a^2*c*x^2)),x)
```

output
```
int(x^2/(atan(a*x)*(c + a^2*c*x^2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.27

$$\int \frac{x^2}{(c+a^2cx^2)\arctan(ax)}\,dx = \frac{\int \frac{x^2}{atan(ax)a^2x^2+atan(ax)}dx}{c}$$

input
```
int(x^2/(a^2*c*x^2+c)/atan(a*x),x)
```

output
```
int(x**2/(atan(a*x)*a**2*x**2 + atan(a*x)),x)/c
```

# 3.475 $\int \frac{x}{(c+a^2cx^2)\arctan(ax)}\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{x}{(c+a^2cx^2)\arctan(ax)}\,dx = \text{Int}\left(\frac{x}{(c+a^2cx^2)\arctan(ax)},x\right)$$

output
```
Defer(Int)(x/(a^2*c*x^2+c)/arctan(a*x),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x}{(c+a^2cx^2)\arctan(ax)}\,dx = \int \frac{x}{(c+a^2cx^2)\arctan(ax)}\,dx$$

input
```
Integrate[x/((c + a^2*c*x^2)*ArcTan[a*x]),x]
```

output
```
Integrate[x/((c + a^2*c*x^2)*ArcTan[a*x]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\arctan(ax)\left(a^2cx^2+c\right)}dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int \frac{x}{\arctan(ax)\left(a^2cx^2+c\right)}dx$$

input
```
Int[x/((c + a^2*c*x^2)*ArcTan[a*x]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{x}{\left(a^2c\,x^2+c\right)\arctan\left(ax\right)}dx$$

input
```
int(x/(a^2*c*x^2+c)/arctan(a*x),x)
```

output
```
int(x/(a^2*c*x^2+c)/arctan(a*x),x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x}{(c+a^2cx^2)\arctan(ax)}\,dx = \int \frac{x}{(a^2cx^2+c)\arctan(ax)}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(x/((a^2*c*x^2 + c)*arctan(a*x)), x)
```

## Sympy [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{x}{(c+a^2cx^2)\arctan(ax)}\,dx = \frac{\int \frac{x}{a^2x^2\operatorname{atan}(ax)+\operatorname{atan}(ax)}\,dx}{c}$$

input
```
integrate(x/(a**2*c*x**2+c)/atan(a*x),x)
```

output
```
Integral(x/(a**2*x**2*atan(a*x) + atan(a*x)), x)/c
```

## Maxima [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x}{(c+a^2cx^2)\arctan(ax)}\,dx = \int \frac{x}{(a^2cx^2+c)\arctan(ax)}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(x/((a^2*c*x^2 + c)*arctan(a*x)), x)
```

## Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x}{(c+a^2cx^2)\arctan(ax)}\,dx = \int \frac{x}{(a^2cx^2+c)\arctan(ax)}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)/arctan(a*x),x, algorithm="giac")
```

output
```
integrate(x/((a^2*c*x^2 + c)*arctan(a*x)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x}{(c+a^2cx^2)\arctan(ax)}\,dx = \int \frac{x}{\mathrm{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)}\,dx$$

input
```
int(x/(atan(a*x)*(c + a^2*c*x^2)),x)
```

output
```
int(x/(atan(a*x)*(c + a^2*c*x^2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30

$$\int \frac{x}{(c+a^2cx^2)\arctan(ax)}\,dx = \frac{\int \frac{x}{atan(ax)a^2x^2+atan(ax)}dx}{c}$$

input
```
int(x/(a^2*c*x^2+c)/atan(a*x),x)
```

output
```
int(x/(atan(a*x)*a**2*x**2 + atan(a*x)),x)/c
```

# 3.476 $\int \frac{1}{(c+a^2cx^2)\arctan(ax)}\,dx$



## Optimal result

Integrand size = 19, antiderivative size = 12

$$\int \frac{1}{(c+a^2cx^2)\arctan(ax)}\,dx = \frac{\log(\arctan(ax))}{ac}$$

output
```
ln(arctan(a*x))/a/c
```

## Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{(c+a^2cx^2)\arctan(ax)}\,dx = \frac{\log(\arctan(ax))}{ac}$$

input
```
Integrate[1/((c + a^2*c*x^2)*ArcTan[a*x]),x]
```

output
```
Log[ArcTan[a*x]]/(a*c)
```

## Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {5417}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arctan(ax)\left(a^2cx^2+c\right)}dx$$
$$\Big\downarrow \ 5417$$
$$\frac{\log(\arctan(ax))}{ac}$$

input
```
Int[1/((c + a^2*c*x^2)*ArcTan[a*x]),x]
```

output
```
Log[ArcTan[a*x]]/(a*c)
```

### Defintions of rubi rules used

rule 5417
```
Int[1/(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)), x_Symbol]
 :> Simp[Log[RemoveContent[a + b*ArcTan[c*x], x]]/(b*c*d), x] /; FreeQ[{a,
b, c, d, e}, x] && EqQ[e, c^2*d]
```

## Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\ln(\arctan(ax))}{ac}$ | 13 |
| default | $\frac{\ln(\arctan(ax))}{ac}$ | 13 |
| parallelrisch | $\frac{\ln(\arctan(ax))}{ac}$ | 13 |
| risch | $\frac{\ln(-\ln(-iax+1)+\ln(iax+1))}{ac}$ | 28 |

input
```
int(1/(a^2*c*x^2+c)/arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
ln(arctan(a*x))/a/c
```

## Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{(c+a^2cx^2)\arctan(ax)}\,dx = \frac{\log(\arctan(ax))}{ac}$$

input
```
integrate(1/(a^2*c*x^2+c)/arctan(a*x),x, algorithm="fricas")
```

output
```
log(arctan(a*x))/(a*c)
```

## Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{(c+a^2cx^2)\arctan(ax)}\,dx = \frac{\log(\operatorname{atan}(ax))}{ac}$$

input
```
integrate(1/(a**2*c*x**2+c)/atan(a*x),x)
```

output
```
log(atan(a*x))/(a*c)
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int \frac{1}{(c+a^2cx^2)\arctan(ax)}\,dx = \frac{\log(2\,|\arctan(ax)|)}{ac}$$

input
```
integrate(1/(a^2*c*x^2+c)/arctan(a*x),x, algorithm="maxima")
```

output
```
log(2*abs(arctan(a*x)))/(a*c)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{1}{(c+a^2cx^2)\arctan(ax)}\,dx = \frac{\log(|\arctan(ax)|)}{ac}$$

input
```
integrate(1/(a^2*c*x^2+c)/arctan(a*x),x, algorithm="giac")
```

output
```
log(abs(arctan(a*x)))/(a*c)
```

## Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{(c+a^2cx^2)\arctan(ax)}\,dx = \frac{\ln(\mathrm{atan}(a\,x))}{a\,c}$$

input
```
int(1/(atan(a*x)*(c + a^2*c*x^2)),x)
```

output
```
log(atan(a*x))/(a*c)
```

## Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{(c+a^2cx^2)\arctan(ax)}\,dx = \frac{\log(atan(ax))}{ac}$$

input
```
int(1/(a^2*c*x^2+c)/atan(a*x),x)
```

output
```
log(atan(a*x))/(a*c)
```

# 3.477 $\int \frac{1}{x(c+a^2cx^2)\arctan(ax)}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x\,(c+a^2cx^2)\arctan(ax)}\,dx = \text{Int}\left(\frac{1}{x\,(c+a^2cx^2)\arctan(ax)},x\right)$$

output
```
Defer(Int)(1/x/(a^2*c*x^2+c)/arctan(a*x),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\,(c+a^2cx^2)\arctan(ax)}\,dx = \int \frac{1}{x\,(c+a^2cx^2)\arctan(ax)}\,dx$$

input
```
Integrate[1/(x*(c + a^2*c*x^2)*ArcTan[a*x]),x]
```

output
```
Integrate[1/(x*(c + a^2*c*x^2)*ArcTan[a*x]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x \arctan(ax) \left(a^2cx^2+c\right)} dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{1}{x \arctan(ax) \left(a^2cx^2+c\right)}dx$$

input
```
Int[1/(x*(c + a^2*c*x^2)*ArcTan[a*x]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \left(a^{2} c \,x^{2}+c \right) \arctan \left(a x \right)}dx$$

input
```
int(1/x/(a^2*c*x^2+c)/arctan(a*x),x)
```

output
```
int(1/x/(a^2*c*x^2+c)/arctan(a*x),x)
```

## Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{1}{x\left(c+a^2cx^2\right)\arctan(ax)}\,dx=\int \frac{1}{(a^2cx^2+c)x\arctan\left(ax\right)}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(1/((a^2*c*x^3 + c*x)*arctan(a*x)), x)
```

## Sympy [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)\arctan(ax)}\,dx=\frac{\int \frac{1}{a^2x^3\operatorname{atan}\left(ax\right)+x\operatorname{atan}\left(ax\right)}\,dx}{c}$$

input
```
integrate(1/x/(a**2*c*x**2+c)/atan(a*x),x)
```

output
```
Integral(1/(a**2*x**3*atan(a*x) + x*atan(a*x)), x)/c
```

## Maxima [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\left(c+a^2cx^2\right)\arctan(ax)}\,dx=\int \frac{1}{(a^2cx^2+c)x\arctan\left(ax\right)}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)*x*arctan(a*x)), x)
```

## Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x(c+a^2cx^2)\arctan(ax)}\,dx=\int \frac{1}{(a^2cx^2+c)x\arctan(ax)}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)/arctan(a*x),x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)*x*arctan(a*x)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x(c+a^2cx^2)\arctan(ax)}\,dx=\int \frac{1}{x\,\mathrm{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)}\,dx$$

input
```
int(1/(x*atan(a*x)*(c + a^2*c*x^2)),x)
```

output
```
int(1/(x*atan(a*x)*(c + a^2*c*x^2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{1}{x\left(c+a^2cx^2\right)\arctan(ax)}\,dx = \frac{\int \frac{1}{atan(ax)a^2x^3+atan(ax)x}dx}{c}$$

input
```
int(1/x/(a^2*c*x^2+c)/atan(a*x),x)
```

output
```
int(1/(atan(a*x)*a**2*x**3 + atan(a*x)*x),x)/c
```

# 3.478 $\int \frac{1}{x^2(c+a^2cx^2)\arctan(ax)}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x^2 (c+a^2cx^2)\arctan(ax)}\,dx = \text{Int}\left(\frac{1}{x^2 (c+a^2cx^2)\arctan(ax)},x\right)$$

output
```
Defer(Int)(1/x^2/(a^2*c*x^2+c)/arctan(a*x),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2 (c+a^2cx^2)\arctan(ax)}\,dx = \int \frac{1}{x^2 (c+a^2cx^2)\arctan(ax)}\,dx$$

input
```
Integrate[1/(x^2*(c + a^2*c*x^2)*ArcTan[a*x]),x]
```

output
```
Integrate[1/(x^2*(c + a^2*c*x^2)*ArcTan[a*x]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \arctan(ax) (a^2cx^2+c)} \, dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{1}{x^2 \arctan(ax) (a^2cx^2+c)}dx$$

input
```
Int[1/(x^2*(c + a^2*c*x^2)*ArcTan[a*x]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (a^2c\,x^2+c) \arctan(ax)}dx$$

input
```
int(1/x^2/(a^2*c*x^2+c)/arctan(a*x),x)
```

output
```
int(1/x^2/(a^2*c*x^2+c)/arctan(a*x),x)
```

## Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^2 (c+a^2cx^2) \arctan(ax)} \, dx = \int \frac{1}{(a^2cx^2+c)x^2 \arctan(ax)} \, dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(1/((a^2*c*x^4 + c*x^2)*arctan(a*x)), x)
```

## Sympy [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2 (c+a^2cx^2) \arctan(ax)} \, dx = \frac{\int \frac{1}{a^2x^4 \operatorname{atan}(ax)+x^2 \operatorname{atan}(ax)} \, dx}{c}$$

input
```
integrate(1/x**2/(a**2*c*x**2+c)/atan(a*x),x)
```

output
```
Integral(1/(a**2*x**4*atan(a*x) + x**2*atan(a*x)), x)/c
```

## Maxima [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2 (c+a^2cx^2) \arctan(ax)} \, dx = \int \frac{1}{(a^2cx^2+c)x^2 \arctan(ax)} \, dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)*x^2*arctan(a*x)), x)
```

## Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)\arctan(ax)}\,dx=\int \frac{1}{(a^2cx^2+c)x^2\arctan\left(ax\right)}\,dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)/arctan(a*x),x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)*x^2*arctan(a*x)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)\arctan(ax)}\,dx=\int \frac{1}{x^2\,\mathrm{atan}\left(a\,x\right)\,\left(c\,a^2\,x^2+c\right)}\,dx$$

input
```
int(1/(x^2*atan(a*x)*(c + a^2*c*x^2)),x)
```

output
```
int(1/(x^2*atan(a*x)*(c + a^2*c*x^2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.27

$$\int \frac{1}{x^2 (c + a^2 c x^2) \arctan(ax)} \, dx = \frac{\int \frac{1}{atan(ax)a^2x^4 + atan(ax)x^2} dx}{c}$$

input
```
int(1/x^2/(a^2*c*x^2+c)/atan(a*x),x)
```

output
```
int(1/(atan(a*x)*a**2*x**4 + atan(a*x)*x**2),x)/c
```

# 3.479 $\int \frac{x^4}{(c+a^2cx^2)^2\arctan(ax)}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^4}{(c+a^2cx^2)^2\arctan(ax)}\,dx = \text{Int}\left(\frac{x^4}{(c+a^2cx^2)^2\arctan(ax)},x\right)$$

output
```
Defer(Int)(x^4/(a^2*c*x^2+c)^2/arctan(a*x),x)
```

## Mathematica [N/A]

Not integrable

Time = 2.64 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^4}{(c+a^2cx^2)^2\arctan(ax)}\,dx = \int \frac{x^4}{(c+a^2cx^2)^2\arctan(ax)}\,dx$$

input
```
Integrate[x^4/((c + a^2*c*x^2)^2*ArcTan[a*x]),x]
```

output
```
Integrate[x^4/((c + a^2*c*x^2)^2*ArcTan[a*x]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\arctan(ax)\left(a^2cx^2+c\right)^2}\,dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x^4}{\arctan(ax)\left(a^2cx^2+c\right)^2}dx$$

input
```
Int[x^4/((c + a^2*c*x^2)^2*ArcTan[a*x]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 5.70 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{\left(a^2c\,x^2+c\right)^2\arctan\left(ax\right)}dx$$

input
```
int(x^4/(a^2*c*x^2+c)^2/arctan(a*x),x)
```

output
```
int(x^4/(a^2*c*x^2+c)^2/arctan(a*x),x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.77

$$\int \frac{x^4}{(c+a^2cx^2)^2 \arctan(ax)}\,dx = \int \frac{x^4}{(a^2cx^2+c)^2 \arctan(ax)}\,dx$$

input
```
integrate(x^4/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(x^4/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)), x)
```

## Sympy [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

$$\int \frac{x^4}{(c+a^2cx^2)^2 \arctan(ax)}\,dx = \frac{\int \frac{x^4}{a^4x^4 \operatorname{atan}(ax)+2a^2x^2 \operatorname{atan}(ax)+\operatorname{atan}(ax)}\,dx}{c^2}$$

input
```
integrate(x**4/(a**2*c*x**2+c)**2/atan(a*x),x)
```

output
```
Integral(x**4/(a**4*x**4*atan(a*x) + 2*a**2*x**2*atan(a*x) + atan(a*x)), x
)/c**2
```

## Maxima [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^4}{(c+a^2cx^2)^2 \arctan(ax)}\,dx = \int \frac{x^4}{(a^2cx^2+c)^2 \arctan(ax)}\,dx$$

input
```
integrate(x^4/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(x^4/((a^2*c*x^2 + c)^2*arctan(a*x)), x)
```

## Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^4}{(c+a^2cx^2)^2 \arctan(ax)} \, dx = \int \frac{x^4}{(a^2cx^2+c)^2 \arctan(ax)} \, dx$$

input
```
integrate(x^4/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="giac")
```

output
```
integrate(x^4/((a^2*c*x^2 + c)^2*arctan(a*x)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^4}{(c+a^2cx^2)^2 \arctan(ax)} \, dx = \int \frac{x^4}{\mathrm{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^2} \, dx$$

input
```
int(x^4/(atan(a*x)*(c + a^2*c*x^2)^2),x)
```

output
```
int(x^4/(atan(a*x)*(c + a^2*c*x^2)^2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.82

$$\int \frac{x^4}{(c+a^2cx^2)^2 \arctan(ax)} \, dx = \frac{\int \frac{x^4}{atan(ax)a^4x^4+2atan(ax)a^2x^2+atan(ax)}dx}{c^2}$$

input
```
int(x^4/(a^2*c*x^2+c)^2/atan(a*x),x)
```

output
```
int(x**4/(atan(a*x)*a**4*x**4 + 2*atan(a*x)*a**2*x**2 + atan(a*x)),x)/c**2
```

# 3.480 $\int \frac{x^3}{(c+a^2cx^2)^2 \arctan(ax)} dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^3}{(c+a^2cx^2)^2 \arctan(ax)} dx = \text{Int}\left(\frac{x^3}{(c+a^2cx^2)^2 \arctan(ax)}, x\right)$$

output
```
Defer(Int)(x^3/(a^2*c*x^2+c)^2/arctan(a*x),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.44 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^3}{(c+a^2cx^2)^2 \arctan(ax)} dx = \int \frac{x^3}{(c+a^2cx^2)^2 \arctan(ax)} dx$$

input
```
Integrate[x^3/((c + a^2*c*x^2)^2*ArcTan[a*x]),x]
```

output
```
Integrate[x^3/((c + a^2*c*x^2)^2*ArcTan[a*x]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\arctan(ax)\left(a^2cx^2+c\right)^2}\,dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x^3}{\arctan(ax)\left(a^2cx^2+c\right)^2}dx$$

input
```
Int[x^3/((c + a^2*c*x^2)^2*ArcTan[a*x]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 5.46 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{\left(a^2c\,x^2+c\right)^2\arctan\left(ax\right)}dx$$

input
```
int(x^3/(a^2*c*x^2+c)^2/arctan(a*x),x)
```

output
```
int(x^3/(a^2*c*x^2+c)^2/arctan(a*x),x)
```

## Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.77

$$\int \frac{x^3}{(c+a^2cx^2)^2\arctan(ax)}\,dx = \int \frac{x^3}{(a^2cx^2+c)^2\arctan(ax)}\,dx$$

input
```
integrate(x^3/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(x^3/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)), x)
```

## Sympy [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

$$\int \frac{x^3}{(c+a^2cx^2)^2\arctan(ax)}\,dx = \frac{\int \frac{x^3}{a^4x^4\operatorname{atan}(ax)+2a^2x^2\operatorname{atan}(ax)+\operatorname{atan}(ax)}\,dx}{c^2}$$

input
```
integrate(x**3/(a**2*c*x**2+c)**2/atan(a*x),x)
```

output
```
Integral(x**3/(a**4*x**4*atan(a*x) + 2*a**2*x**2*atan(a*x) + atan(a*x)), x
)/c**2
```

## Maxima [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^3}{(c+a^2cx^2)^2\arctan(ax)}\,dx = \int \frac{x^3}{(a^2cx^2+c)^2\arctan(ax)}\,dx$$

input
```
integrate(x^3/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(x^3/((a^2*c*x^2 + c)^2*arctan(a*x)), x)
```

## Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^3}{(c+a^2cx^2)^2 \arctan(ax)} \, dx = \int \frac{x^3}{(a^2cx^2+c)^2 \arctan(ax)} \, dx$$

input
```
integrate(x^3/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="giac")
```

output
```
integrate(x^3/((a^2*c*x^2 + c)^2*arctan(a*x)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^3}{(c+a^2cx^2)^2 \arctan(ax)} \, dx = \int \frac{x^3}{\mathrm{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^2} \, dx$$

input
```
int(x^3/(atan(a*x)*(c + a^2*c*x^2)^2),x)
```

output
```
int(x^3/(atan(a*x)*(c + a^2*c*x^2)^2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.82

$$\int \frac{x^3}{(c+a^2cx^2)^2 \arctan(ax)}\,dx = \frac{\int \frac{x^3}{atan(ax)a^4x^4+2atan(ax)a^2x^2+atan(ax)}dx}{c^2}$$

input
```
int(x^3/(a^2*c*x^2+c)^2/atan(a*x),x)
```

output
```
int(x**3/(atan(a*x)*a**4*x**4 + 2*atan(a*x)*a**2*x**2 + atan(a*x)),x)/c**2
```

# 3.481 $\int \frac{x^2}{(c+a^2cx^2)^2 \arctan(ax)} \, dx$



## Optimal result

Integrand size = 22, antiderivative size = 33

$$\int \frac{x^2}{(c+a^2cx^2)^2 \arctan(ax)} \, dx = -\frac{\text{CosIntegral}(2\arctan(ax))}{2a^3c^2} + \frac{\log(\arctan(ax))}{2a^3c^2}$$

output
```
-1/2*Ci(2*arctan(a*x))/a^3/c^2+1/2*ln(arctan(a*x))/a^3/c^2
```

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{x^2}{(c+a^2cx^2)^2 \arctan(ax)} \, dx = \frac{-\text{CosIntegral}(2\arctan(ax)) + \log(\arctan(ax))}{2a^3c^2}$$

input
```
Integrate[x^2/((c + a^2*c*x^2)^2*ArcTan[a*x]),x]
```

output
```
(-CosIntegral[2*ArcTan[a*x]] + Log[ArcTan[a*x]])/(2*a^3*c^2)
```

## Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {5505, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\arctan(ax)\left(a^2cx^2+c\right)^2}dx$$

$$\Big\downarrow \text{5505}$$

$$\frac{\int \frac{a^2x^2}{(a^2x^2+1)\arctan(ax)}d\arctan(ax)}{a^3c^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sin(\arctan(ax))^2}{\arctan(ax)}d\arctan(ax)}{a^3c^2}$$

$$\Big\downarrow \text{3793}$$

$$\frac{\int \left(\frac{1}{2\arctan(ax)}-\frac{\cos(2\arctan(ax))}{2\arctan(ax)}\right)d\arctan(ax)}{a^3c^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{2}\log(\arctan(ax))-\frac{1}{2}\text{CosIntegral}(2\arctan(ax))}{a^3c^2}$$

input
```
Int[x^2/((c + a^2*c*x^2)^2*ArcTan[a*x]),x]
```

output
```
(-1/2*CosIntegral[2*ArcTan[a*x]] + Log[ArcTan[a*x]]/2)/(a^3*c^2)
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

## Maple [A] (verified)

Time = 4.78 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.73

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\ln(\arctan(ax))-\operatorname{Ci}(2\arctan(ax))}{2a^3c^2}$ | 24 |
| default | $\frac{\ln(\arctan(ax))-\operatorname{Ci}(2\arctan(ax))}{2a^3c^2}$ | 24 |

input
```
int(x^2/(a^2*c*x^2+c)^2/arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/2/a^3*(ln(arctan(a*x))-Ci(2*arctan(a*x)))/c^2
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.24

$$\int \frac{x^2}{(c+a^2cx^2)^2 \arctan(ax)} \, dx = \frac{2 \log(\arctan(ax)) - \text{log\_integral}\left(-\frac{a^2x^2+2i\,ax-1}{a^2x^2+1}\right) - \text{log\_integral}\left(-\frac{a^2x^2-2i\,ax-1}{a^2x^2+1}\right)}{4\,a^3c^2}$$

input
```
integrate(x^2/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="fricas")
```

output
```
1/4*(2*log(arctan(a*x)) - log_integral(-(a^2*x^2 + 2*I*a*x - 1)/(a^2*x^2 +
 1)) - log_integral(-(a^2*x^2 - 2*I*a*x - 1)/(a^2*x^2 + 1)))/(a^3*c^2)
```

## Sympy [F]

$$\int \frac{x^2}{(c+a^2cx^2)^2 \arctan(ax)} \, dx = \frac{\int \frac{x^2}{a^4x^4 \operatorname{atan}(ax) + 2a^2x^2 \operatorname{atan}(ax) + \operatorname{atan}(ax)} \, dx}{c^2}$$

input
```
integrate(x**2/(a**2*c*x**2+c)**2/atan(a*x),x)
```

output
```
Integral(x**2/(a**4*x**4*atan(a*x) + 2*a**2*x**2*atan(a*x) + atan(a*x)), x
)/c**2
```

## Maxima [F]

$$\int \frac{x^2}{\left(c+a^2cx^2\right)^2 \arctan(ax)}\,dx = \int \frac{x^2}{\left(a^2cx^2+c\right)^2 \arctan\left(ax\right)}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(x^2/((a^2*c*x^2 + c)^2*arctan(a*x)), x)
```

## Giac [F]

$$\int \frac{x^2}{\left(c+a^2cx^2\right)^2 \arctan(ax)}\,dx = \int \frac{x^2}{\left(a^2cx^2+c\right)^2 \arctan\left(ax\right)}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="giac")
```

output
```
integrate(x^2/((a^2*c*x^2 + c)^2*arctan(a*x)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\left(c+a^2cx^2\right)^2 \arctan(ax)}\,dx = \int \frac{x^2}{\mathrm{atan}\left(a\,x\right)\,\left(c\,a^2\,x^2+c\right)^2}\,dx$$

input
```
int(x^2/(atan(a*x)*(c + a^2*c*x^2)^2),x)
```

output
```
int(x^2/(atan(a*x)*(c + a^2*c*x^2)^2), x)
```

## Reduce [F]

$$\int \frac{x^2}{(c+a^2cx^2)^2 \arctan(ax)}\,dx$$
$$= \frac{-\left(\int \frac{1}{atan(ax)a^4x^4+2atan(ax)a^2x^2+atan(ax)}dx\right)a+\log(atan(ax))}{a^3c^2}$$

input
```
int(x^2/(a^2*c*x^2+c)^2/atan(a*x),x)
```

output
```
( - int(1/(atan(a*x)*a**4*x**4 + 2*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a +
 log(atan(a*x)))/(a**3*c**2)
```

# 3.482 $\int \frac{x}{(c+a^2cx^2)^2 \arctan(ax)} \, dx$



## Optimal result

Integrand size = 20, antiderivative size = 17

$$\int \frac{x}{(c+a^2cx^2)^2 \arctan(ax)} \, dx = \frac{\text{Si}(2\arctan(ax))}{2a^2c^2}$$

output
```
1/2*Si(2*arctan(a*x))/a^2/c^2
```

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{x}{(c+a^2cx^2)^2 \arctan(ax)} \, dx = \frac{\text{Si}(2\arctan(ax))}{2a^2c^2}$$

input
```
Integrate[x/((c + a^2*c*x^2)^2*ArcTan[a*x]),x]
```

output
```
SinIntegral[2*ArcTan[a*x]]/(2*a^2*c^2)
```

## Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5505, 4906, 27, 3042, 3780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\arctan(ax)\left(a^2cx^2+c\right)^2}dx$$

$$\Big\downarrow \text{ 5505}$$

$$\frac{\int \frac{ax}{(a^2x^2+1)\arctan(ax)}d\arctan(ax)}{a^2c^2}$$

$$\Big\downarrow \text{ 4906}$$

$$\frac{\int \frac{\sin(2\arctan(ax))}{2\arctan(ax)}d\arctan(ax)}{a^2c^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{2a^2c^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{2a^2c^2}$$

$$\Big\downarrow \text{ 3780}$$

$$\frac{\text{Si}(2\arctan(ax))}{2a^2c^2}$$

input
```
Int[x/((c + a^2*c*x^2)^2*ArcTan[a*x]),x]
```

output
```
SinIntegral[2*ArcTan[a*x]]/(2*a^2*c^2)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

## Maple [A] (verified)

Time = 4.44 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\mathrm{Si}(2\arctan(ax))}{2a^2c^2}$ | 16 |
| default | $\frac{\mathrm{Si}(2\arctan(ax))}{2a^2c^2}$ | 16 |

input
```
int(x/(a^2*c*x^2+c)^2/arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*Si(2*arctan(a*x))/a^2/c^2
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 67, normalized size of antiderivative = 3.94

$$\int \frac{x}{(c+a^2cx^2)^2 \arctan(ax)}\,dx = \frac{i \text{ log\_integral}\left(-\frac{a^2x^2+2i\,ax-1}{a^2x^2+1}\right) - i \text{ log\_integral}\left(-\frac{a^2x^2-2i\,ax-1}{a^2x^2+1}\right)}{4\,a^2c^2}$$

input
```
integrate(x/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="fricas")
```

output
```
1/4*(I*log_integral(-(a^2*x^2 + 2*I*a*x - 1)/(a^2*x^2 + 1)) - I*log_integr
al(-(a^2*x^2 - 2*I*a*x - 1)/(a^2*x^2 + 1)))/(a^2*c^2)
```

## Sympy [F]

$$\int \frac{x}{(c+a^2cx^2)^2 \arctan(ax)}\,dx = \frac{\int \frac{x}{a^4x^4 \operatorname{atan}(ax)+2a^2x^2 \operatorname{atan}(ax)+\operatorname{atan}(ax)}\,dx}{c^2}$$

input
```
integrate(x/(a**2*c*x**2+c)**2/atan(a*x),x)
```

output
```
Integral(x/(a**4*x**4*atan(a*x) + 2*a**2*x**2*atan(a*x) + atan(a*x)), x)/c
**2
```

## Maxima [F]

$$\int \frac{x}{(c+a^2cx^2)^2 \arctan(ax)} dx = \int \frac{x}{(a^2cx^2+c)^2 \arctan(ax)} dx$$

input
```
integrate(x/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(x/((a^2*c*x^2 + c)^2*arctan(a*x)), x)
```

## Giac [F]

$$\int \frac{x}{(c+a^2cx^2)^2 \arctan(ax)} dx = \int \frac{x}{(a^2cx^2+c)^2 \arctan(ax)} dx$$

input
```
integrate(x/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="giac")
```

output
```
integrate(x/((a^2*c*x^2 + c)^2*arctan(a*x)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(c+a^2cx^2)^2 \arctan(ax)} dx = \int \frac{x}{\mathrm{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^2} dx$$

input
```
int(x/(atan(a*x)*(c + a^2*c*x^2)^2),x)
```

output
```
int(x/(atan(a*x)*(c + a^2*c*x^2)^2), x)
```

## Reduce [F]

$$\int \frac{x}{\left(c+a^2cx^2\right)^2\arctan(ax)}\,dx = \frac{\int \frac{x}{atan(ax)a^4x^4+2atan(ax)a^2x^2+atan(ax)}dx}{c^2}$$

input
```
int(x/(a^2*c*x^2+c)^2/atan(a*x),x)
```

output
```
int(x/(atan(a*x)*a**4*x**4 + 2*atan(a*x)*a**2*x**2 + atan(a*x)),x)/c**2
```

# 3.483 $\int \frac{1}{(c+a^2cx^2)^2\arctan(ax)}\,dx$



## Optimal result

Integrand size = 19, antiderivative size = 33

$$\int \frac{1}{(c+a^2cx^2)^2\arctan(ax)}\,dx = \frac{\text{CosIntegral}(2\arctan(ax))}{2ac^2} + \frac{\log(\arctan(ax))}{2ac^2}$$

output
```
1/2*Ci(2*arctan(a*x))/a/c^2+1/2*ln(arctan(a*x))/a/c^2
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.70

$$\int \frac{1}{(c+a^2cx^2)^2\arctan(ax)}\,dx = \frac{\text{CosIntegral}(2\arctan(ax)) + \log(\arctan(ax))}{2ac^2}$$

input
```
Integrate[1/((c + a^2*c*x^2)^2*ArcTan[a*x]),x]
```

output
```
(CosIntegral[2*ArcTan[a*x]] + Log[ArcTan[a*x]])/(2*a*c^2)
```

## Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {5439, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arctan(ax)\left(a^2cx^2+c\right)^2}dx$$

$$\Big\downarrow \text{5439}$$

$$\frac{\int \frac{1}{(a^2x^2+1)\arctan(ax)}d\arctan(ax)}{ac^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^2}{\arctan(ax)}d\arctan(ax)}{ac^2}$$

$$\Big\downarrow \text{3793}$$

$$\frac{\int \left(\frac{\cos(2\arctan(ax))}{2\arctan(ax)}+\frac{1}{2\arctan(ax)}\right)d\arctan(ax)}{ac^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{ac^2}$$

input
```
Int[1/((c + a^2*c*x^2)^2*ArcTan[a*x]),x]
```

output
```
(CosIntegral[2*ArcTan[a*x]]/2 + Log[ArcTan[a*x]]/2)/(a*c^2)
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

## Maple [A] (verified)

Time = 4.50 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.67

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\ln(\arctan(ax))+\text{Ci}(2\arctan(ax))}{2a\,c^2}$ | 22 |
| default | $\frac{\ln(\arctan(ax))+\text{Ci}(2\arctan(ax))}{2a\,c^2}$ | 22 |

input
```
int(1/(a^2*c*x^2+c)^2/arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/2/a*(ln(arctan(a*x))+Ci(2*arctan(a*x)))/c^2
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.12

$$\int \frac{1}{(c+a^2cx^2)^2 \arctan(ax)}\,dx = \frac{2\log(\arctan(ax)) + \text{log\_integral}\left(-\frac{a^2x^2+2i\,ax-1}{a^2x^2+1}\right) + \text{log\_integral}\left(-\frac{a^2x^2-2i\,ax-1}{a^2x^2+1}\right)}{4\,ac^2}$$

input
```
integrate(1/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="fricas")
```

output
```
1/4*(2*log(arctan(a*x)) + log_integral(-(a^2*x^2 + 2*I*a*x - 1)/(a^2*x^2 +
 1)) + log_integral(-(a^2*x^2 - 2*I*a*x - 1)/(a^2*x^2 + 1)))/(a*c^2)
```

## Sympy [F]

$$\int \frac{1}{(c+a^2cx^2)^2 \arctan(ax)}\,dx = \frac{\int \frac{1}{a^4x^4\,\text{atan}\,(ax)+2a^2x^2\,\text{atan}\,(ax)+\text{atan}\,(ax)}\,dx}{c^2}$$

input
```
integrate(1/(a**2*c*x**2+c)**2/atan(a*x),x)
```

output
```
Integral(1/(a**4*x**4*atan(a*x) + 2*a**2*x**2*atan(a*x) + atan(a*x)), x)/c
**2
```

## Maxima [F]

$$\int \frac{1}{(c+a^2cx^2)^2 \arctan(ax)}\,dx = \int \frac{1}{(a^2cx^2+c)^2 \arctan(ax)}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)^2*arctan(a*x)), x)
```

## Giac [F]

$$\int \frac{1}{(c+a^2cx^2)^2 \arctan(ax)}\,dx = \int \frac{1}{(a^2cx^2+c)^2 \arctan(ax)}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^2*arctan(a*x)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+a^2cx^2)^2 \arctan(ax)}\,dx = \int \frac{1}{\operatorname{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^2}\,dx$$

input
```
int(1/(atan(a*x)*(c + a^2*c*x^2)^2),x)
```

output
```
int(1/(atan(a*x)*(c + a^2*c*x^2)^2), x)
```

## Reduce [F]

$$\int \frac{1}{\left(c+a^2cx^2\right)^2 \arctan(ax)}\,dx = \frac{\int \frac{1}{atan(ax)a^4x^4+2atan(ax)a^2x^2+atan(ax)}dx}{c^2}$$

input
```
int(1/(a^2*c*x^2+c)^2/atan(a*x),x)
```

output
```
int(1/(atan(a*x)*a**4*x**4 + 2*atan(a*x)*a**2*x**2 + atan(a*x)),x)/c**2
```

# 3.484 $\int \frac{1}{x(c+a^2cx^2)^2 \arctan(ax)}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2 \arctan(ax)}\,dx = \text{Int}\left(\frac{1}{x\left(c+a^2cx^2\right)^2 \arctan(ax)},x\right)$$

output
```
Defer(Int)(1/x/(a^2*c*x^2+c)^2/arctan(a*x),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2 \arctan(ax)}\,dx = \int \frac{1}{x\left(c+a^2cx^2\right)^2 \arctan(ax)}\,dx$$

input
```
Integrate[1/(x*(c + a^2*c*x^2)^2*ArcTan[a*x]),x]
```

output
```
Integrate[1/(x*(c + a^2*c*x^2)^2*ArcTan[a*x]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x \arctan(ax) \left(a^2cx^2+c\right)^2} \, dx$$

$$\big\downarrow \text{ 5560}$$

$$\int \frac{1}{x \arctan(ax) \left(a^2cx^2+c\right)^2}dx$$

input
```
Int[1/(x*(c + a^2*c*x^2)^2*ArcTan[a*x]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 8.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \left(a^2c\,x^2+c\right)^2 \arctan\left(ax\right)}dx$$

input
```
int(1/x/(a^2*c*x^2+c)^2/arctan(a*x),x)
```

output
```
int(1/x/(a^2*c*x^2+c)^2/arctan(a*x),x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.73

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\arctan(ax)}\,dx=\int \frac{1}{\left(a^2cx^2+c\right)^2x\arctan\left(ax\right)}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(1/((a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^2*x)*arctan(a*x)), x)
```

## Sympy [N/A]

Not integrable

Time = 0.94 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\arctan(ax)}\,dx=\frac{\int \frac{1}{a^4x^5\,\mathrm{atan}\,(ax)+2a^2x^3\,\mathrm{atan}\,(ax)+x\,\mathrm{atan}\,(ax)}\,dx}{c^2}$$

input
```
integrate(1/x/(a**2*c*x**2+c)**2/atan(a*x),x)
```

output
```
Integral(1/(a**4*x**5*atan(a*x) + 2*a**2*x**3*atan(a*x) + x*atan(a*x)), x)
/c**2
```

## Maxima [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\arctan(ax)}\,dx=\int \frac{1}{\left(a^2cx^2+c\right)^2x\arctan\left(ax\right)}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)^2*x*arctan(a*x)), x)
```

## Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\arctan(ax)}\,dx = \int \frac{1}{\left(a^2cx^2+c\right)^2x\arctan\left(ax\right)}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^2*x*arctan(a*x)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\arctan(ax)}\,dx = \int \frac{1}{x\,\mathrm{atan}\left(a\,x\right)\,\left(c\,a^2\,x^2+c\right)^2}\,dx$$

input
```
int(1/(x*atan(a*x)*(c + a^2*c*x^2)^2),x)
```

output
```
int(1/(x*atan(a*x)*(c + a^2*c*x^2)^2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.73

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\arctan(ax)}\,dx = \frac{\int \frac{1}{atan(ax)a^4x^5+2atan(ax)a^2x^3+atan(ax)x}dx}{c^2}$$

input
```
int(1/x/(a^2*c*x^2+c)^2/atan(a*x),x)
```

output
```
int(1/(atan(a*x)*a**4*x**5 + 2*atan(a*x)*a**2*x**3 + atan(a*x)*x),x)/c**2
```

# 3.485 $\int \frac{1}{x^2(c+a^2cx^2)^2 \arctan(ax)} dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x^2 (c+a^2cx^2)^2 \arctan(ax)} dx = \text{Int}\left(\frac{1}{x^2 (c+a^2cx^2)^2 \arctan(ax)}, x\right)$$

output
```
Defer(Int)(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.84 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2 (c+a^2cx^2)^2 \arctan(ax)} dx = \int \frac{1}{x^2 (c+a^2cx^2)^2 \arctan(ax)} dx$$

input
```
Integrate[1/(x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]),x]
```

output
```
Integrate[1/(x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \arctan(ax) \left(a^2cx^2+c\right)^2} \, dx$$

$$\Big\downarrow \; 5560$$

$$\int \frac{1}{x^2 \arctan(ax) \left(a^2cx^2+c\right)^2}dx$$

input
```
Int[1/(x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 3.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 \left(a^2c\,x^2+c\right)^2 \arctan\left(ax\right)}dx$$

input
```
int(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x),x)
```

output
```
int(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x),x)
```

## Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.82

$$\int \frac{1}{x^2 (c + a^2cx^2)^2 \arctan(ax)} \, dx = \int \frac{1}{(a^2cx^2 + c)^2 x^2 \arctan(ax)} \, dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(1/((a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2)*arctan(a*x)), x)
```

## Sympy [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.77

$$\int \frac{1}{x^2 (c + a^2cx^2)^2 \arctan(ax)} \, dx = \frac{\int \frac{1}{a^4x^6 \operatorname{atan}(ax) + 2a^2x^4 \operatorname{atan}(ax) + x^2 \operatorname{atan}(ax)} \, dx}{c^2}$$

input
```
integrate(1/x**2/(a**2*c*x**2+c)**2/atan(a*x),x)
```

output
```
Integral(1/(a**4*x**6*atan(a*x) + 2*a**2*x**4*atan(a*x) + x**2*atan(a*x)),
 x)/c**2
```

## Maxima [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2 (c + a^2cx^2)^2 \arctan(ax)} \, dx = \int \frac{1}{(a^2cx^2 + c)^2 x^2 \arctan(ax)} \, dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)^2*x^2*arctan(a*x)), x)
```

## Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2 (c + a^2cx^2)^2 \arctan(ax)} \, dx = \int \frac{1}{(a^2cx^2 + c)^2 x^2 \arctan(ax)} \, dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^2*x^2*arctan(a*x)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2 (c + a^2cx^2)^2 \arctan(ax)} \, dx = \int \frac{1}{x^2 \operatorname{atan}(a\,x) \,(c\,a^2\,x^2 + c)^2} \, dx$$

input
```
int(1/(x^2*atan(a*x)*(c + a^2*c*x^2)^2),x)
```

output
```
int(1/(x^2*atan(a*x)*(c + a^2*c*x^2)^2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.82

$$\int \frac{1}{x^2 (c+a^2cx^2)^2 \arctan(ax)}\,dx = \frac{\int \frac{1}{atan(ax)a^4x^6+2atan(ax)a^2x^4+atan(ax)x^2}dx}{c^2}$$

input
```
int(1/x^2/(a^2*c*x^2+c)^2/atan(a*x),x)
```

output
```
int(1/(atan(a*x)*a**4*x**6 + 2*atan(a*x)*a**2*x**4 + atan(a*x)*x**2),x)/c*
*2
```

# 3.486 $\int \frac{x^6}{(c+a^2cx^2)^3\arctan(ax)}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^6}{(c+a^2cx^2)^3\arctan(ax)}\,dx = \text{Int}\left(\frac{x^6}{(c+a^2cx^2)^3\arctan(ax)},x\right)$$

output
```
Defer(Int)(x^6/(a^2*c*x^2+c)^3/arctan(a*x),x)
```

## Mathematica [N/A]

Not integrable

Time = 5.54 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^6}{(c+a^2cx^2)^3\arctan(ax)}\,dx = \int \frac{x^6}{(c+a^2cx^2)^3\arctan(ax)}\,dx$$

input
```
Integrate[x^6/((c + a^2*c*x^2)^3*ArcTan[a*x]),x]
```

output
```
Integrate[x^6/((c + a^2*c*x^2)^3*ArcTan[a*x]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6}{\arctan(ax)\,(a^2cx^2+c)^3}\,dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{x^6}{\arctan(ax)\,(a^2cx^2+c)^3}dx$$

input
```
Int[x^6/((c + a^2*c*x^2)^3*ArcTan[a*x]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 1.67 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^6}{(a^2c\,x^2+c)^3\arctan\,(ax)}dx$$

input
```
int(x^6/(a^2*c*x^2+c)^3/arctan(a*x),x)
```

output
```
int(x^6/(a^2*c*x^2+c)^3/arctan(a*x),x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.27

$$\int \frac{x^6}{(c+a^2cx^2)^3 \arctan(ax)}\,dx = \int \frac{x^6}{(a^2cx^2+c)^3 \arctan{(ax)}}\,dx$$

input
```
integrate(x^6/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(x^6/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a
*x)), x)
```

## Sympy [N/A]

Not integrable

Time = 0.87 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.32

$$\int \frac{x^6}{(c+a^2cx^2)^3 \arctan(ax)}\,dx = \frac{\int \frac{x^6}{a^6x^6\operatorname{atan}{(ax)}+3a^4x^4\operatorname{atan}{(ax)}+3a^2x^2\operatorname{atan}{(ax)}+\operatorname{atan}{(ax)}}\,dx}{c^3}$$

input
```
integrate(x**6/(a**2*c*x**2+c)**3/atan(a*x),x)
```

output
```
Integral(x**6/(a**6*x**6*atan(a*x) + 3*a**4*x**4*atan(a*x) + 3*a**2*x**2*a
tan(a*x) + atan(a*x)), x)/c**3
```

## Maxima [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^6}{(c+a^2cx^2)^3\arctan(ax)}\,dx = \int \frac{x^6}{(a^2cx^2+c)^3\arctan(ax)}\,dx$$

input
```
integrate(x^6/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(x^6/((a^2*c*x^2 + c)^3*arctan(a*x)), x)
```

## Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^6}{(c+a^2cx^2)^3\arctan(ax)}\,dx = \int \frac{x^6}{(a^2cx^2+c)^3\arctan(ax)}\,dx$$

input
```
integrate(x^6/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="giac")
```

output
```
integrate(x^6/((a^2*c*x^2 + c)^3*arctan(a*x)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^6}{(c+a^2cx^2)^3\arctan(ax)}\,dx = \int \frac{x^6}{\mathrm{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^3}\,dx$$

input
```
int(x^6/(atan(a*x)*(c + a^2*c*x^2)^3),x)
```

output
```
int(x^6/(atan(a*x)*(c + a^2*c*x^2)^3), x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.36

$$\int \frac{x^6}{(c+a^2cx^2)^3\arctan(ax)}\,dx = \frac{\int \frac{x^6}{atan(ax)a^6x^6+3atan(ax)a^4x^4+3atan(ax)a^2x^2+atan(ax)}dx}{c^3}$$

input
```
int(x^6/(a^2*c*x^2+c)^3/atan(a*x),x)
```

output
```
int(x**6/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x
**2 + atan(a*x)),x)/c**3
```

# 3.487 $\int \frac{x^5}{(c+a^2cx^2)^3 \arctan(ax)} \, dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^5}{(c+a^2cx^2)^3 \arctan(ax)} \, dx = \text{Int}\left(\frac{x^5}{(c+a^2cx^2)^3 \arctan(ax)}, x\right)$$

output
```
Defer(Int)(x^5/(a^2*c*x^2+c)^3/arctan(a*x),x)
```

## Mathematica [N/A]

Not integrable

Time = 5.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^5}{(c+a^2cx^2)^3 \arctan(ax)} \, dx = \int \frac{x^5}{(c+a^2cx^2)^3 \arctan(ax)} \, dx$$

input
```
Integrate[x^5/((c + a^2*c*x^2)^3*ArcTan[a*x]),x]
```

output
```
Integrate[x^5/((c + a^2*c*x^2)^3*ArcTan[a*x]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\arctan(ax)\left(a^2cx^2+c\right)^3}\,dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int \frac{x^5}{\arctan(ax)\left(a^2cx^2+c\right)^3}dx$$

input
```
Int[x^5/((c + a^2*c*x^2)^3*ArcTan[a*x]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 4.83 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{\left(a^2c\,x^2+c\right)^3\arctan\left(ax\right)}dx$$

input
```
int(x^5/(a^2*c*x^2+c)^3/arctan(a*x),x)
```

output
```
int(x^5/(a^2*c*x^2+c)^3/arctan(a*x),x)
```

## Fricas [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.27

$$\int \frac{x^5}{(c+a^2cx^2)^3\arctan(ax)}\,dx = \int \frac{x^5}{(a^2cx^2+c)^3\arctan(ax)}\,dx$$

input
```
integrate(x^5/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(x^5/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a
*x)), x)
```

## Sympy [N/A]

Not integrable

Time = 0.89 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.32

$$\int \frac{x^5}{(c+a^2cx^2)^3\arctan(ax)}\,dx = \frac{\int \frac{x^5}{a^6x^6\operatorname{atan}(ax)+3a^4x^4\operatorname{atan}(ax)+3a^2x^2\operatorname{atan}(ax)+\operatorname{atan}(ax)}\,dx}{c^3}$$

input
```
integrate(x**5/(a**2*c*x**2+c)**3/atan(a*x),x)
```

output
```
Integral(x**5/(a**6*x**6*atan(a*x) + 3*a**4*x**4*atan(a*x) + 3*a**2*x**2*a
tan(a*x) + atan(a*x)), x)/c**3
```

## Maxima [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^5}{(c+a^2cx^2)^3 \arctan(ax)} \, dx = \int \frac{x^5}{(a^2cx^2+c)^3 \arctan(ax)} \, dx$$

input
```
integrate(x^5/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(x^5/((a^2*c*x^2 + c)^3*arctan(a*x)), x)
```

## Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^5}{(c+a^2cx^2)^3 \arctan(ax)} \, dx = \int \frac{x^5}{(a^2cx^2+c)^3 \arctan(ax)} \, dx$$

input
```
integrate(x^5/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="giac")
```

output
```
integrate(x^5/((a^2*c*x^2 + c)^3*arctan(a*x)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^5}{(c+a^2cx^2)^3 \arctan(ax)} \, dx = \int \frac{x^5}{\mathrm{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^3} \, dx$$

input
```
int(x^5/(atan(a*x)*(c + a^2*c*x^2)^3),x)
```

output
```
int(x^5/(atan(a*x)*(c + a^2*c*x^2)^3), x)
```

## Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.36

$$\int \frac{x^5}{\left(c+a^2cx^2\right)^3\arctan(ax)}\,dx=\frac{\int \frac{x^5}{atan(ax)a^6x^6+3atan(ax)a^4x^4+3atan(ax)a^2x^2+atan(ax)}dx}{c^3}$$

input
```
int(x^5/(a^2*c*x^2+c)^3/atan(a*x),x)
```

output
```
int(x**5/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x
**2 + atan(a*x)),x)/c**3
```

# 3.488 $\int \frac{x^4}{(c+a^2cx^2)^3 \arctan(ax)} dx$



## Optimal result

Integrand size = 22, antiderivative size = 50

$$\int \frac{x^4}{(c+a^2cx^2)^3 \arctan(ax)} dx = -\frac{\text{CosIntegral}(2\arctan(ax))}{2a^5c^3} + \frac{\text{CosIntegral}(4\arctan(ax))}{8a^5c^3} + \frac{3\log(\arctan(ax))}{8a^5c^3}$$

output
```
-1/2*Ci(2*arctan(a*x))/a^5/c^3+1/8*Ci(4*arctan(a*x))/a^5/c^3+3/8*ln(arctan
(a*x))/a^5/c^3
```

## Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.68

$$\int \frac{x^4}{(c+a^2cx^2)^3 \arctan(ax)} dx = \frac{-4\,\text{CosIntegral}(2\arctan(ax)) + \text{CosIntegral}(4\arctan(ax)) + 3\log(\arctan(ax))}{8a^5c^3}$$

input
```
Integrate[x^4/((c + a^2*c*x^2)^3*ArcTan[a*x]),x]
```

output
```
(-4*CosIntegral[2*ArcTan[a*x]] + CosIntegral[4*ArcTan[a*x]] + 3*Log[ArcTan
[a*x]])/(8*a^5*c^3)
```

## Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {5505, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\arctan(ax)\left(a^2cx^2+c\right)^3}dx$$

$$\Big\downarrow \ 5505$$

$$\frac{\int \frac{a^4x^4}{(a^2x^2+1)^2\arctan(ax)}d\arctan(ax)}{a^5c^3}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \frac{\sin(\arctan(ax))^4}{\arctan(ax)}d\arctan(ax)}{a^5c^3}$$

$$\Big\downarrow \ 3793$$

$$\frac{\int \left(-\frac{\cos(2\arctan(ax))}{2\arctan(ax)}+\frac{\cos(4\arctan(ax))}{8\arctan(ax)}+\frac{3}{8\arctan(ax)}\right)d\arctan(ax)}{a^5c^3}$$

$$\Big\downarrow \ 2009$$

$$\frac{-\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{8}\text{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^5c^3}$$

input
```
Int[x^4/((c + a^2*c*x^2)^3*ArcTan[a*x]),x]
```

output
```
(-1/2*CosIntegral[2*ArcTan[a*x]] + CosIntegral[4*ArcTan[a*x]]/8 + (3*Log[A
rcTan[a*x]])/8)/(a^5*c^3)
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

## Maple [A] (verified)

Time = 5.34 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.66

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{3\ln(\arctan(ax))-4\ \mathrm{Ci}(2\arctan(ax))+\mathrm{Ci}(4\arctan(ax))}{8a^5c^3}$ | 33 |
| default | $\frac{3\ln(\arctan(ax))-4\ \mathrm{Ci}(2\arctan(ax))+\mathrm{Ci}(4\arctan(ax))}{8a^5c^3}$ | 33 |

input
```
int(x^4/(a^2*c*x^2+c)^3/arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/8/a^5*(3*ln(arctan(a*x))-4*Ci(2*arctan(a*x))+Ci(4*arctan(a*x)))/c^3
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 174, normalized size of antiderivative = 3.48

$$\int \frac{x^4}{(c+a^2cx^2)^3 \arctan(ax)}\,dx = \frac{6 \log(\arctan(ax)) + \operatorname{log\_integral}\left(\frac{a^4x^4+4i\,a^3x^3-6\,a^2x^2-4i\,ax+1}{a^4x^4+2\,a^2x^2+1}\right) + \operatorname{log\_integral}\left(\frac{a^4x^4-4i\,a^3x^3-6\,a^2x^2+4i\,ax+1}{a^4x^4+2\,a^2x^2+1}\right)}{16\,a^5c^3}$$

input
```
integrate(x^4/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="fricas")
```

output
```
1/16*(6*log(arctan(a*x)) + log_integral((a^4*x^4 + 4*I*a^3*x^3 - 6*a^2*x^2
 - 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) + log_integral((a^4*x^4 - 4*I*a
^3*x^3 - 6*a^2*x^2 + 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) - 4*log_integ
ral(-(a^2*x^2 + 2*I*a*x - 1)/(a^2*x^2 + 1)) - 4*log_integral(-(a^2*x^2 - 2
*I*a*x - 1)/(a^2*x^2 + 1)))/(a^5*c^3)
```

## Sympy [F]

$$\int \frac{x^4}{(c+a^2cx^2)^3 \arctan(ax)}\,dx = \frac{\int \frac{x^4}{a^6x^6 \operatorname{atan}(ax)+3a^4x^4 \operatorname{atan}(ax)+3a^2x^2 \operatorname{atan}(ax)+\operatorname{atan}(ax)}\,dx}{c^3}$$

input
```
integrate(x**4/(a**2*c*x**2+c)**3/atan(a*x),x)
```

output
```
Integral(x**4/(a**6*x**6*atan(a*x) + 3*a**4*x**4*atan(a*x) + 3*a**2*x**2*a
tan(a*x) + atan(a*x)), x)/c**3
```

## Maxima [F]

$$\int \frac{x^4}{(c+a^2cx^2)^3 \arctan(ax)}\,dx = \int \frac{x^4}{(a^2cx^2+c)^3 \arctan(ax)}\,dx$$

input
```
integrate(x^4/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(x^4/((a^2*c*x^2 + c)^3*arctan(a*x)), x)
```

## Giac [F]

$$\int \frac{x^4}{(c+a^2cx^2)^3 \arctan(ax)}\,dx = \int \frac{x^4}{(a^2cx^2+c)^3 \arctan(ax)}\,dx$$

input
```
integrate(x^4/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="giac")
```

output
```
integrate(x^4/((a^2*c*x^2 + c)^3*arctan(a*x)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(c+a^2cx^2)^3 \arctan(ax)}\,dx = \int \frac{x^4}{\mathrm{atan}(a\,x)\,(c\,a^2\,x^2+c)^3}\,dx$$

input
```
int(x^4/(atan(a*x)*(c + a^2*c*x^2)^3),x)
```

output
```
int(x^4/(atan(a*x)*(c + a^2*c*x^2)^3), x)
```

## Reduce [F]

$$\int \frac{x^4}{(c+a^2cx^2)^3 \arctan(ax)}\,dx$$
$$= \frac{-2\left(\int \frac{x^2}{atan(ax)a^6x^6+3atan(ax)a^4x^4+3atan(ax)a^2x^2+atan(ax)}dx\right)a^3 - \left(\int \frac{1}{atan(ax)a^6x^6+3atan(ax)a^4x^4+3atan(ax)a^2x^2+atan(ax)}dx\right)a + \log(atan(ax))}{a^5c^3}$$

input
```
int(x^4/(a^2*c*x^2+c)^3/atan(a*x),x)
```

output
```
( - 2*int(x**2/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*
a**2*x**2 + atan(a*x)),x)*a**3 - int(1/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*
a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a + log(atan(a*x)))/(a**
5*c**3)
```

# 3.489 $\int \frac{x^3}{(c+a^2cx^2)^3 \arctan(ax)} dx$



## Optimal result

Integrand size = 22, antiderivative size = 35

$$\int \frac{x^3}{(c+a^2cx^2)^3 \arctan(ax)} dx = \frac{\text{Si}(2\arctan(ax))}{4a^4c^3} - \frac{\text{Si}(4\arctan(ax))}{8a^4c^3}$$

output
```
1/4*Si(2*arctan(a*x))/a^4/c^3-1/8*Si(4*arctan(a*x))/a^4/c^3
```

## Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{x^3}{(c+a^2cx^2)^3 \arctan(ax)} dx = -\frac{-2\text{Si}(2\arctan(ax)) + \text{Si}(4\arctan(ax))}{8a^4c^3}$$

input
```
Integrate[x^3/((c + a^2*c*x^2)^3*ArcTan[a*x]),x]
```

output
```
-1/8*(-2*SinIntegral[2*ArcTan[a*x]] + SinIntegral[4*ArcTan[a*x]])/(a^4*c^3
)
```

## Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {5505, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\arctan(ax)\,(a^2cx^2+c)^3}dx$$

$$\Big\downarrow \text{5505}$$

$$\frac{\int \frac{a^3x^3}{(a^2x^2+1)^2\arctan(ax)}d\arctan(ax)}{a^4c^3}$$

$$\Big\downarrow \text{4906}$$

$$\frac{\int \left(\frac{\sin(2\arctan(ax))}{4\arctan(ax)}-\frac{\sin(4\arctan(ax))}{8\arctan(ax)}\right)d\arctan(ax)}{a^4c^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{4}\text{Si}(2\arctan(ax))-\frac{1}{8}\text{Si}(4\arctan(ax))}{a^4c^3}$$

input
```
Int[x^3/((c + a^2*c*x^2)^3*ArcTan[a*x]),x]
```

output
```
(SinIntegral[2*ArcTan[a*x]]/4 - SinIntegral[4*ArcTan[a*x]]/8)/(a^4*c^3)
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

## Maple [A] (verified)

Time = 4.84 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{2\,\mathrm{Si}(2\arctan(ax))-\mathrm{Si}(4\arctan(ax))}{8a^4c^3}$ | 28 |
| default | $\frac{2\,\mathrm{Si}(2\arctan(ax))-\mathrm{Si}(4\arctan(ax))}{8a^4c^3}$ | 28 |

input
```
int(x^3/(a^2*c*x^2+c)^3/arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/8/a^4*(2*Si(2*arctan(a*x))-Si(4*arctan(a*x)))/c^3
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 171, normalized size of antiderivative = 4.89

$$\int \frac{x^3}{(c+a^2cx^2)^3\arctan(ax)}\,dx = \frac{-i\,\mathrm{log\_integral}\left(\frac{a^4x^4+4i\,a^3x^3-6\,a^2x^2-4i\,ax+1}{a^4x^4+2\,a^2x^2+1}\right)+i\,\mathrm{log\_integral}\left(\frac{a^4x^4-4i\,a^3x^3-6\,a^2x^2+4i\,ax+1}{a^4x^4+2\,a^2x^2+1}\right)+2i\,\mathrm{log\_integr}}{16\,a^4c^3}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="fricas")
```

output
```
1/16*(-I*log_integral((a^4*x^4 + 4*I*a^3*x^3 - 6*a^2*x^2 - 4*I*a*x + 1)/(a
^4*x^4 + 2*a^2*x^2 + 1)) + I*log_integral((a^4*x^4 - 4*I*a^3*x^3 - 6*a^2*x
^2 + 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) + 2*I*log_integral(-(a^2*x^2
+ 2*I*a*x - 1)/(a^2*x^2 + 1)) - 2*I*log_integral(-(a^2*x^2 - 2*I*a*x - 1)/
(a^2*x^2 + 1)))/(a^4*c^3)
```

## Sympy [F]

$$\int \frac{x^3}{(c+a^2cx^2)^3\arctan(ax)}\,dx = \frac{\int \frac{x^3}{a^6x^6\operatorname{atan}(ax)+3a^4x^4\operatorname{atan}(ax)+3a^2x^2\operatorname{atan}(ax)+\operatorname{atan}(ax)}\,dx}{c^3}$$

input
```
integrate(x**3/(a**2*c*x**2+c)**3/atan(a*x),x)
```

output
```
Integral(x**3/(a**6*x**6*atan(a*x) + 3*a**4*x**4*atan(a*x) + 3*a**2*x**2*a
tan(a*x) + atan(a*x)), x)/c**3
```

## Maxima [F]

$$\int \frac{x^3}{(c+a^2cx^2)^3\arctan(ax)}\,dx = \int \frac{x^3}{(a^2cx^2+c)^3\arctan(ax)}\,dx$$

input
```
integrate(x^3/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(x^3/((a^2*c*x^2 + c)^3*arctan(a*x)), x)
```

## Giac [F]

$$\int \frac{x^3}{(c+a^2cx^2)^3 \arctan(ax)}\,dx = \int \frac{x^3}{(a^2cx^2+c)^3 \arctan(ax)}\,dx$$

input
```
integrate(x^3/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="giac")
```

output
```
integrate(x^3/((a^2*c*x^2 + c)^3*arctan(a*x)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(c+a^2cx^2)^3 \arctan(ax)}\,dx = \int \frac{x^3}{\mathrm{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^3}\,dx$$

input
```
int(x^3/(atan(a*x)*(c + a^2*c*x^2)^3),x)
```

output
```
int(x^3/(atan(a*x)*(c + a^2*c*x^2)^3), x)
```

## Reduce [F]

$$\int \frac{x^3}{(c+a^2cx^2)^3 \arctan(ax)}\,dx = \frac{\int \frac{x^3}{atan(ax)a^6x^6+3atan(ax)a^4x^4+3atan(ax)a^2x^2+atan(ax)}dx}{c^3}$$

input
```
int(x^3/(a^2*c*x^2+c)^3/atan(a*x),x)
```

output
```
int(x**3/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x
**2 + atan(a*x)),x)/c**3
```

# 3.490 $\int \frac{x^2}{(c+a^2cx^2)^3 \arctan(ax)} dx$



## Optimal result

Integrand size = 22, antiderivative size = 33

$$\int \frac{x^2}{(c+a^2cx^2)^3 \arctan(ax)} dx = -\frac{\text{CosIntegral}(4\arctan(ax))}{8a^3c^3} + \frac{\log(\arctan(ax))}{8a^3c^3}$$

output
```
-1/8*Ci(4*arctan(a*x))/a^3/c^3+1/8*ln(arctan(a*x))/a^3/c^3
```

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.76

$$\int \frac{x^2}{(c+a^2cx^2)^3 \arctan(ax)} dx = \frac{-\text{CosIntegral}(4\arctan(ax)) + \log(\arctan(ax))}{8a^3c^3}$$

input
```
Integrate[x^2/((c + a^2*c*x^2)^3*ArcTan[a*x]),x]
```

output
```
(-CosIntegral[4*ArcTan[a*x]] + Log[ArcTan[a*x]])/(8*a^3*c^3)
```

## Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {5505, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\arctan(ax)\,(a^2cx^2+c)^3}dx$$

$$\Big\downarrow \text{5505}$$

$$\frac{\int \frac{a^2x^2}{(a^2x^2+1)^2\arctan(ax)}d\arctan(ax)}{a^3c^3}$$

$$\Big\downarrow \text{4906}$$

$$\frac{\int \left(\frac{1}{8\arctan(ax)}-\frac{\cos(4\arctan(ax))}{8\arctan(ax)}\right)d\arctan(ax)}{a^3c^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{8}\log(\arctan(ax))-\frac{1}{8}\text{CosIntegral}(4\arctan(ax))}{a^3c^3}$$

input
```
Int[x^2/((c + a^2*c*x^2)^3*ArcTan[a*x]),x]
```

output
```
(-1/8*CosIntegral[4*ArcTan[a*x]] + Log[ArcTan[a*x]]/8)/(a^3*c^3)
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

## Maple [A] (verified)

Time = 4.50 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.73

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{\ln(\arctan(ax))-\operatorname{Ci}(4\arctan(ax))}{8a^3c^3}$ | 24 |
| default | $\frac{\ln(\arctan(ax))-\operatorname{Ci}(4\arctan(ax))}{8a^3c^3}$ | 24 |

input
```
int(x^2/(a^2*c*x^2+c)^3/arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/8/a^3*(ln(arctan(a*x))-Ci(4*arctan(a*x)))/c^3
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 120, normalized size of antiderivative = 3.64

$$\int \frac{x^2}{(c+a^2cx^2)^3 \arctan(ax)}\,dx = \frac{2\log(\arctan(ax)) - \mathrm{log\_integral}\left(\frac{a^4x^4+4i\,a^3x^3-6\,a^2x^2-4i\,ax+1}{a^4x^4+2\,a^2x^2+1}\right) - \mathrm{log\_integral}\left(\frac{a^4x^4-4i\,a^3x^3-6\,a^2x^2+4i\,ax+1}{a^4x^4+2\,a^2x^2+1}\right)}{16\,a^3c^3}$$

input
```
integrate(x^2/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="fricas")
```

output
```
1/16*(2*log(arctan(a*x)) - log_integral((a^4*x^4 + 4*I*a^3*x^3 - 6*a^2*x^2
 - 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) - log_integral((a^4*x^4 - 4*I*a
^3*x^3 - 6*a^2*x^2 + 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)))/(a^3*c^3)
```

## Sympy [F]

$$\int \frac{x^2}{(c+a^2cx^2)^3 \arctan(ax)}\,dx = \frac{\int \frac{x^2}{a^6x^6\,\mathrm{atan}\,(ax)+3a^4x^4\,\mathrm{atan}\,(ax)+3a^2x^2\,\mathrm{atan}\,(ax)+\mathrm{atan}\,(ax)}\,dx}{c^3}$$

input
```
integrate(x**2/(a**2*c*x**2+c)**3/atan(a*x),x)
```

output
```
Integral(x**2/(a**6*x**6*atan(a*x) + 3*a**4*x**4*atan(a*x) + 3*a**2*x**2*a
tan(a*x) + atan(a*x)), x)/c**3
```

## Maxima [F]

$$\int \frac{x^2}{(c+a^2cx^2)^3 \arctan(ax)}\,dx = \int \frac{x^2}{(a^2cx^2+c)^3 \arctan(ax)}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(x^2/((a^2*c*x^2 + c)^3*arctan(a*x)), x)
```

## Giac [F]

$$\int \frac{x^2}{(c+a^2cx^2)^3 \arctan(ax)}\,dx = \int \frac{x^2}{(a^2cx^2+c)^3 \arctan(ax)}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="giac")
```

output
```
integrate(x^2/((a^2*c*x^2 + c)^3*arctan(a*x)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(c+a^2cx^2)^3 \arctan(ax)} \, dx = \int \frac{x^2}{\mathrm{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^3} \, dx$$

input
```
int(x^2/(atan(a*x)*(c + a^2*c*x^2)^3),x)
```

output
```
int(x^2/(atan(a*x)*(c + a^2*c*x^2)^3), x)
```

## Reduce [F]

$$\int \frac{x^2}{(c+a^2cx^2)^3 \arctan(ax)} \, dx = \frac{\int \frac{x^2}{atan(ax)a^6x^6+3atan(ax)a^4x^4+3atan(ax)a^2x^2+atan(ax)}dx}{c^3}$$

input
```
int(x^2/(a^2*c*x^2+c)^3/atan(a*x),x)
```

output
```
int(x**2/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x
**2 + atan(a*x)),x)/c**3
```

# 3.491 $\int \frac{x}{(c+a^2cx^2)^3\arctan(ax)}\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 35

$$\int \frac{x}{(c+a^2cx^2)^3\arctan(ax)}\,dx = \frac{\text{Si}(2\arctan(ax))}{4a^2c^3} + \frac{\text{Si}(4\arctan(ax))}{8a^2c^3}$$

output
```
1/4*Si(2*arctan(a*x))/a^2/c^3+1/8*Si(4*arctan(a*x))/a^2/c^3
```

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{x}{(c+a^2cx^2)^3\arctan(ax)}\,dx = \frac{2\text{Si}(2\arctan(ax)) + \text{Si}(4\arctan(ax))}{8a^2c^3}$$

input
```
Integrate[x/((c + a^2*c*x^2)^3*ArcTan[a*x]),x]
```

output
```
(2*SinIntegral[2*ArcTan[a*x]] + SinIntegral[4*ArcTan[a*x]])/(8*a^2*c^3)
```

## Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {5505, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\arctan(ax)\left(a^2cx^2+c\right)^3}dx$$

$$\Big\downarrow \text{5505}$$

$$\frac{\int \frac{ax}{(a^2x^2+1)^2\arctan(ax)}d\arctan(ax)}{a^2c^3}$$

$$\Big\downarrow \text{4906}$$

$$\frac{\int \left(\frac{\sin(2\arctan(ax))}{4\arctan(ax)}+\frac{\sin(4\arctan(ax))}{8\arctan(ax)}\right)d\arctan(ax)}{a^2c^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{4}\text{Si}(2\arctan(ax))+\frac{1}{8}\text{Si}(4\arctan(ax))}{a^2c^3}$$

input
```
Int[x/((c + a^2*c*x^2)^3*ArcTan[a*x]),x]
```

output
```
(SinIntegral[2*ArcTan[a*x]]/4 + SinIntegral[4*ArcTan[a*x]]/8)/(a^2*c^3)
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

## Maple [A] (verified)

Time = 4.94 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.74

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{2\,\mathrm{Si}(2\arctan(ax))+\mathrm{Si}(4\arctan(ax))}{8a^2c^3}$ | 26 |
| default | $\frac{2\,\mathrm{Si}(2\arctan(ax))+\mathrm{Si}(4\arctan(ax))}{8a^2c^3}$ | 26 |

input
```
int(x/(a^2*c*x^2+c)^3/arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/8/a^2*(2*Si(2*arctan(a*x))+Si(4*arctan(a*x)))/c^3
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 171, normalized size of antiderivative = 4.89

$$\int \frac{x}{(c+a^2cx^2)^3\arctan(ax)}\,dx = \frac{i\ \mathrm{log\_integral}\left(\frac{a^4x^4+4i\,a^3x^3-6\,a^2x^2-4i\,ax+1}{a^4x^4+2\,a^2x^2+1}\right)-i\ \mathrm{log\_integral}\left(\frac{a^4x^4-4i\,a^3x^3-6\,a^2x^2+4i\,ax+1}{a^4x^4+2\,a^2x^2+1}\right)+2i\ \mathrm{log\_integral}}{16\,a^2c^3}$$

input
```
integrate(x/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="fricas")
```

output
```
1/16*(I*log_integral((a^4*x^4 + 4*I*a^3*x^3 - 6*a^2*x^2 - 4*I*a*x + 1)/(a^
4*x^4 + 2*a^2*x^2 + 1)) - I*log_integral((a^4*x^4 - 4*I*a^3*x^3 - 6*a^2*x^
2 + 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) + 2*I*log_integral(-(a^2*x^2 +
 2*I*a*x - 1)/(a^2*x^2 + 1)) - 2*I*log_integral(-(a^2*x^2 - 2*I*a*x - 1)/(
a^2*x^2 + 1)))/(a^2*c^3)
```

## Sympy [F]

$$\int \frac{x}{(c+a^2cx^2)^3 \arctan(ax)}\,dx = \frac{\int \frac{x}{a^6x^6 \operatorname{atan}(ax)+3a^4x^4 \operatorname{atan}(ax)+3a^2x^2 \operatorname{atan}(ax)+\operatorname{atan}(ax)}\,dx}{c^3}$$

input
```
integrate(x/(a**2*c*x**2+c)**3/atan(a*x),x)
```

output
```
Integral(x/(a**6*x**6*atan(a*x) + 3*a**4*x**4*atan(a*x) + 3*a**2*x**2*atan
(a*x) + atan(a*x)), x)/c**3
```

## Maxima [F]

$$\int \frac{x}{(c+a^2cx^2)^3 \arctan(ax)}\,dx = \int \frac{x}{(a^2cx^2+c)^3 \arctan(ax)}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(x/((a^2*c*x^2 + c)^3*arctan(a*x)), x)
```

## Giac [F]

$$\int \frac{x}{(c+a^2cx^2)^3 \arctan(ax)}\,dx = \int \frac{x}{(a^2cx^2+c)^3 \arctan(ax)}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="giac")
```

output
```
integrate(x/((a^2*c*x^2 + c)^3*arctan(a*x)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(c+a^2cx^2)^3 \arctan(ax)}\,dx = \int \frac{x}{\mathrm{atan}(a\,x)\,(c\,a^2\,x^2+c)^3}\,dx$$

input
```
int(x/(atan(a*x)*(c + a^2*c*x^2)^3),x)
```

output
```
int(x/(atan(a*x)*(c + a^2*c*x^2)^3), x)
```

## Reduce [F]

$$\int \frac{x}{(c+a^2cx^2)^3 \arctan(ax)}\,dx = \frac{\int \frac{x}{atan(ax)a^6x^6+3atan(ax)a^4x^4+3atan(ax)a^2x^2+atan(ax)}dx}{c^3}$$

input
```
int(x/(a^2*c*x^2+c)^3/atan(a*x),x)
```

output
```
int(x/(atan(a*x)*a**6*x***6 + 3*atan(a*x)*a**4*x***4 + 3*atan(a*x)*a**2*x***2
 + atan(a*x)),x)/c**3
```

# 3.492 $\int \frac{1}{(c+a^2cx^2)^3 \arctan(ax)} \, dx$



## Optimal result

Integrand size = 19, antiderivative size = 50

$$\int \frac{1}{(c+a^2cx^2)^3 \arctan(ax)} \, dx = \frac{\text{CosIntegral}(2\arctan(ax))}{2ac^3} + \frac{\text{CosIntegral}(4\arctan(ax))}{8ac^3} + \frac{3\log(\arctan(ax))}{8ac^3}$$

output
```
1/2*Ci(2*arctan(a*x))/a/c^3+1/8*Ci(4*arctan(a*x))/a/c^3+3/8*ln(arctan(a*x)
)/a/c^3
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.68

$$\int \frac{1}{(c+a^2cx^2)^3 \arctan(ax)} \, dx = \frac{4\,\text{CosIntegral}(2\arctan(ax)) + \text{CosIntegral}(4\arctan(ax)) + 3\log(\arctan(ax))}{8ac^3}$$

input
```
Integrate[1/((c + a^2*c*x^2)^3*ArcTan[a*x]),x]
```

output
```
(4*CosIntegral[2*ArcTan[a*x]] + CosIntegral[4*ArcTan[a*x]] + 3*Log[ArcTan[
a*x]])/(8*a*c^3)
```

## Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.78, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {5439, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arctan(ax)\left(a^2cx^2+c\right)^3}dx$$

$$\Big\downarrow \text{5439}$$

$$\frac{\int \frac{1}{(a^2x^2+1)^2\arctan(ax)}d\arctan(ax)}{ac^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^4}{\arctan(ax)}d\arctan(ax)}{ac^3}$$

$$\Big\downarrow \text{3793}$$

$$\frac{\int \left(\frac{\cos(2\arctan(ax))}{2\arctan(ax)}+\frac{\cos(4\arctan(ax))}{8\arctan(ax)}+\frac{3}{8\arctan(ax)}\right)d\arctan(ax)}{ac^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{8}\text{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{ac^3}$$

input
```
Int[1/((c + a^2*c*x^2)^3*ArcTan[a*x]),x]
```

output
```
(CosIntegral[2*ArcTan[a*x]]/2 + CosIntegral[4*ArcTan[a*x]]/8 + (3*Log[ArcT
an[a*x]])/8)/(a*c^3)
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

## Maple [A] (verified)

Time = 5.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.66

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{3 \ln(\arctan(ax))+4 \operatorname{Ci}(2 \arctan(ax))+\operatorname{Ci}(4 \arctan(ax))}{8 a c^3}$ | 33 |
| default | $\frac{3 \ln(\arctan(ax))+4 \operatorname{Ci}(2 \arctan(ax))+\operatorname{Ci}(4 \arctan(ax))}{8 a c^3}$ | 33 |

input
```
int(1/(a^2*c*x^2+c)^3/arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/8/a*(3*ln(arctan(a*x))+4*Ci(2*arctan(a*x))+Ci(4*arctan(a*x)))/c^3
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 174, normalized size of antiderivative = 3.48

$$\int \frac{1}{(c+a^2cx^2)^3 \arctan(ax)}\,dx = \frac{6\log(\arctan(ax)) + \mathrm{log\_integral}\left(\frac{a^4x^4+4i\,a^3x^3-6\,a^2x^2-4i\,ax+1}{a^4x^4+2\,a^2x^2+1}\right) + \mathrm{log\_integral}\left(\frac{a^4x^4-4i\,a^3x^3-6\,a^2x^2+4i\,ax+1}{a^4x^4+2\,a^2x^2+1}\right)}{16\,ac^3}$$

input
```
integrate(1/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="fricas")
```

output
```
1/16*(6*log(arctan(a*x)) + log_integral((a^4*x^4 + 4*I*a^3*x^3 - 6*a^2*x^2
 - 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) + log_integral((a^4*x^4 - 4*I*a
^3*x^3 - 6*a^2*x^2 + 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) + 4*log_integ
ral(-(a^2*x^2 + 2*I*a*x - 1)/(a^2*x^2 + 1)) + 4*log_integral(-(a^2*x^2 - 2
*I*a*x - 1)/(a^2*x^2 + 1)))/(a*c^3)
```

## Sympy [F]

$$\int \frac{1}{(c+a^2cx^2)^3 \arctan(ax)}\,dx = \frac{\int \frac{1}{a^6x^6\,\mathrm{atan}\,(ax)+3a^4x^4\,\mathrm{atan}\,(ax)+3a^2x^2\,\mathrm{atan}\,(ax)+\mathrm{atan}\,(ax)}\,dx}{c^3}$$

input
```
integrate(1/(a**2*c*x**2+c)**3/atan(a*x),x)
```

output
```
Integral(1/(a**6*x**6*atan(a*x) + 3*a**4*x**4*atan(a*x) + 3*a**2*x**2*atan
(a*x) + atan(a*x)), x)/c**3
```

## Maxima [F]

$$\int \frac{1}{(c+a^2cx^2)^3 \arctan(ax)}\,dx = \int \frac{1}{(a^2cx^2+c)^3 \arctan(ax)}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)^3*arctan(a*x)), x)
```

## Giac [F]

$$\int \frac{1}{(c+a^2cx^2)^3 \arctan(ax)}\,dx = \int \frac{1}{(a^2cx^2+c)^3 \arctan(ax)}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^3*arctan(a*x)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+a^2cx^2)^3 \arctan(ax)}\,dx = \int \frac{1}{\mathrm{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^3}\,dx$$

input
```
int(1/(atan(a*x)*(c + a^2*c*x^2)^3),x)
```

output
```
int(1/(atan(a*x)*(c + a^2*c*x^2)^3), x)
```

## Reduce [F]

$$\int \frac{1}{\left(c+a^2cx^2\right)^3\arctan(ax)}\,dx=\frac{\int \frac{1}{atan(ax)a^6x^6+3atan(ax)a^4x^4+3atan(ax)a^2x^2+atan(ax)}dx}{c^3}$$

input
```
int(1/(a^2*c*x^2+c)^3/atan(a*x),x)
```

output
```
int(1/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2
 + atan(a*x)),x)/c**3
```

# 3.493 $\int \frac{1}{x(c+a^2cx^2)^3 \arctan(ax)} \, dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x\,(c+a^2cx^2)^3 \arctan(ax)} \, dx = \text{Int}\left(\frac{1}{x\,(c+a^2cx^2)^3 \arctan(ax)}, x\right)$$

output
```
Defer(Int)(1/x/(a^2*c*x^2+c)^3/arctan(a*x),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\,(c+a^2cx^2)^3 \arctan(ax)} \, dx = \int \frac{1}{x\,(c+a^2cx^2)^3 \arctan(ax)} \, dx$$

input
```
Integrate[1/(x*(c + a^2*c*x^2)^3*ArcTan[a*x]),x]
```

output
```
Integrate[1/(x*(c + a^2*c*x^2)^3*ArcTan[a*x]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x \arctan(ax) (a^2cx^2+c)^3} \, dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{1}{x \arctan(ax) (a^2cx^2+c)^3} dx$$

input
```
Int[1/(x*(c + a^2*c*x^2)^3*ArcTan[a*x]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 5.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\,(a^2c\,x^2+c)^3 \arctan\,(ax)} dx$$

input
```
int(1/x/(a^2*c*x^2+c)^3/arctan(a*x),x)
```

output
```
int(1/x/(a^2*c*x^2+c)^3/arctan(a*x),x)
```

## Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.23

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\arctan(ax)}\,dx=\int \frac{1}{\left(a^2cx^2+c\right)^3x\arctan\left(ax\right)}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(1/((a^6*c^3*x^7 + 3*a^4*c^3*x^5 + 3*a^2*c^3*x^3 + c^3*x)*arctan(a
*x)), x)
```

## Sympy [N/A]

Not integrable

Time = 1.23 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.32

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\arctan(ax)}\,dx=\frac{\int \frac{1}{a^6x^7\operatorname{atan}\left(ax\right)+3a^4x^5\operatorname{atan}\left(ax\right)+3a^2x^3\operatorname{atan}\left(ax\right)+x\operatorname{atan}\left(ax\right)}\,dx}{c^3}$$

input
```
integrate(1/x/(a**2*c*x**2+c)**3/atan(a*x),x)
```

output
```
Integral(1/(a**6*x**7*atan(a*x) + 3*a**4*x**5*atan(a*x) + 3*a**2*x**3*atan
(a*x) + x*atan(a*x)), x)/c**3
```

## Maxima [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\arctan(ax)}\,dx=\int \frac{1}{\left(a^2cx^2+c\right)^3x\arctan\left(ax\right)}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)^3*x*arctan(a*x)), x)
```

## Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\arctan(ax)}\,dx=\int \frac{1}{\left(a^2cx^2+c\right)^3x\arctan\left(ax\right)}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^3*x*arctan(a*x)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\arctan(ax)}\,dx=\int \frac{1}{x\,\mathrm{atan}\left(a\,x\right)\,{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

input
```
int(1/(x*atan(a*x)*(c + a^2*c*x^2)^3),x)
```

output
```
int(1/(x*atan(a*x)*(c + a^2*c*x^2)^3), x)
```

## Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.27

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\arctan(ax)}\,dx=\frac{\int \frac{1}{atan(ax)a^6x^7+3atan(ax)a^4x^5+3atan(ax)a^2x^3+atan(ax)x}dx}{c^3}$$

input
```
int(1/x/(a^2*c*x^2+c)^3/atan(a*x),x)
```

output
```
int(1/(atan(a*x)*a**6*x**7 + 3*atan(a*x)*a**4*x**5 + 3*atan(a*x)*a**2*x**3
 + atan(a*x)*x),x)/c**3
```

# 3.494 $\int \frac{1}{x^2(c+a^2cx^2)^3 \arctan(ax)} dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x^2 (c + a^2cx^2)^3 \arctan(ax)} dx = \text{Int}\left(\frac{1}{x^2 (c + a^2cx^2)^3 \arctan(ax)}, x\right)$$

output
```
Defer(Int)(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.95 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2 (c + a^2cx^2)^3 \arctan(ax)} dx = \int \frac{1}{x^2 (c + a^2cx^2)^3 \arctan(ax)} dx$$

input
```
Integrate[1/(x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]),x]
```

output
```
Integrate[1/(x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \arctan(ax) \left(a^2cx^2+c\right)^3} \, dx$$
$$\Big\downarrow \ 5560$$
$$\int \frac{1}{x^2 \arctan(ax) \left(a^2cx^2+c\right)^3}dx$$

input
```
Int[1/(x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 5.37 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 \left(a^2c\,x^2+c\right)^3 \arctan\left(ax\right)}dx$$

input
```
int(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x),x)
```

output
```
int(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x),x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.32

$$\int \frac{1}{x^2 (c+a^2cx^2)^3 \arctan(ax)} \, dx = \int \frac{1}{(a^2cx^2+c)^3 x^2 \arctan(ax)} \, dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(1/((a^6*c^3*x^8 + 3*a^4*c^3*x^6 + 3*a^2*c^3*x^4 + c^3*x^2)*arctan
(a*x)), x)
```

## Sympy [N/A]

Not integrable

Time = 1.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.41

$$\int \frac{1}{x^2 (c+a^2cx^2)^3 \arctan(ax)} \, dx = \frac{\int \frac{1}{a^6x^8 \operatorname{atan}(ax)+3a^4x^6 \operatorname{atan}(ax)+3a^2x^4 \operatorname{atan}(ax)+x^2 \operatorname{atan}(ax)} \, dx}{c^3}$$

input
```
integrate(1/x**2/(a**2*c*x**2+c)**3/atan(a*x),x)
```

output
```
Integral(1/(a**6*x**8*atan(a*x) + 3*a**4*x**6*atan(a*x) + 3*a**2*x**4*atan
(a*x) + x**2*atan(a*x)), x)/c**3
```

## Maxima [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2 (c + a^2cx^2)^3 \arctan(ax)} \, dx = \int \frac{1}{(a^2cx^2 + c)^3 x^2 \arctan(ax)} \, dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)^3*x^2*arctan(a*x)), x)
```

## Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2 (c + a^2cx^2)^3 \arctan(ax)} \, dx = \int \frac{1}{(a^2cx^2 + c)^3 x^2 \arctan(ax)} \, dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^3*x^2*arctan(a*x)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2 (c + a^2cx^2)^3 \arctan(ax)} \, dx = \int \frac{1}{x^2 \operatorname{atan}(a\,x)\,\left(c\,a^2\,x^2 + c\right)^3} \, dx$$

input
```
int(1/(x^2*atan(a*x)*(c + a^2*c*x^2)^3),x)
```

output
```
int(1/(x^2*atan(a*x)*(c + a^2*c*x^2)^3), x)
```

## Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.36

$$\int \frac{1}{x^2 (c+a^2cx^2)^3 \arctan(ax)} \, dx = \frac{\int \frac{1}{atan(ax)a^6x^8+3atan(ax)a^4x^6+3atan(ax)a^2x^4+atan(ax)x^2} dx}{c^3}$$

input
```
int(1/x^2/(a^2*c*x^2+c)^3/atan(a*x),x)
```

output
```
int(1/(atan(a*x)*a**6*x**8 + 3*atan(a*x)*a**4*x**6 + 3*atan(a*x)*a**2*x**4
 + atan(a*x)*x**2),x)/c**3
```

# 3.495 $\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)}\,dx = \operatorname{Int}\left(\frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)},x\right)$$

output
```
Defer(Int)(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.62 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)}\,dx = \int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)}\,dx$$

input
```
Integrate[(x*Sqrt[c + a^2*c*x^2])/ArcTan[a*x],x]
```

output
```
Integrate[(x*Sqrt[c + a^2*c*x^2])/ArcTan[a*x], x]
```

## Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{a^2cx^2+c}}{\arctan(ax)}\,dx$$

$$\Big\downarrow \ 5560$$

$$\int \frac{x\sqrt{a^2cx^2+c}}{\arctan(ax)}dx$$

input
```
Int[(x*Sqrt[c + a^2*c*x^2])/ArcTan[a*x],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 16.76 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x\sqrt{a^2c\,x^2+c}}{\arctan\left(ax\right)}dx$$

input
```
int(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x),x)
```

output
```
int(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x),x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)}\,dx=\int \frac{\sqrt{a^2cx^2+cx}}{\arctan(ax)}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x/arctan(a*x), x)
```

## Sympy [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)}\,dx=\int \frac{x\sqrt{c(a^2x^2+1)}}{\operatorname{atan}(ax)}\,dx$$

input
```
integrate(x*(a**2*c*x**2+c)**(1/2)/atan(a*x),x)
```

output
```
Integral(x*sqrt(c*(a**2*x**2 + 1))/atan(a*x), x)
```

## Maxima [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)}\,dx=\int \frac{\sqrt{a^2cx^2+cx}}{\arctan(ax)}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*x/arctan(a*x), x)
```

## Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)}\,dx = \int \frac{\sqrt{a^2cx^2+c}x}{\arctan(ax)}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x),x, algorithm="giac")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*x/arctan(a*x), x)
```

## Mupad [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)}\,dx = \int \frac{x\sqrt{c\,a^2\,x^2+c}}{\mathrm{atan}\left(a\,x\right)}\,dx$$

input
```
int((x*(c + a^2*c*x^2)^(1/2))/atan(a*x),x)
```

output
```
int((x*(c + a^2*c*x^2)^(1/2))/atan(a*x), x)
```

## Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)}\,dx = \sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,x}{atan\,(ax)}dx\right)$$

input
```
int(x*(a^2*c*x^2+c)^(1/2)/atan(a*x),x)
```

output
```
sqrt(c)*int((sqrt(a**2*x**2 + 1)*x)/atan(a*x),x)
```

# 3.496 $\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)}\,dx = \operatorname{Int}\left(\frac{\sqrt{c+a^2cx^2}}{\arctan(ax)},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^(1/2)/arctan(a*x),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)}\,dx = \int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)}\,dx$$

input
```
Integrate[Sqrt[c + a^2*c*x^2]/ArcTan[a*x],x]
```

output
```
Integrate[Sqrt[c + a^2*c*x^2]/ArcTan[a*x], x]
```

## Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2cx^2+c}}{\arctan(ax)}dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int \frac{\sqrt{a^2cx^2+c}}{\arctan(ax)}dx$$

input
```
Int[Sqrt[c + a^2*c*x^2]/ArcTan[a*x],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 7.41 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{a^2c\,x^2+c}}{\arctan\left(ax\right)}dx$$

input
```
int((a^2*c*x^2+c)^(1/2)/arctan(a*x),x)
```

output
```
int((a^2*c*x^2+c)^(1/2)/arctan(a*x),x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)}\,dx = \int \frac{\sqrt{a^2cx^2+c}}{\arctan\left(ax\right)}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)/arctan(a*x), x)
```

## Sympy [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)}\,dx = \int \frac{\sqrt{c\left(a^2x^2+1\right)}}{\operatorname{atan}\left(ax\right)}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(1/2)/atan(a*x),x)
```

output
```
Integral(sqrt(c*(a**2*x**2 + 1))/atan(a*x), x)
```

## Maxima [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)}\,dx = \int \frac{\sqrt{a^2cx^2+c}}{\arctan\left(ax\right)}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)/arctan(a*x), x)
```

## Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)}\,dx=\int \frac{\sqrt{a^2cx^2+c}}{\arctan\left(ax\right)}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/arctan(a*x),x, algorithm="giac")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)/arctan(a*x), x)
```

## Mupad [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)}\,dx=\int \frac{\sqrt{c\,a^2\,x^2+c}}{\mathrm{atan}\left(a\,x\right)}\,dx$$

input
```
int((c + a^2*c*x^2)^(1/2)/atan(a*x),x)
```

output
```
int((c + a^2*c*x^2)^(1/2)/atan(a*x), x)
```

## Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)}\,dx = \sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}}{atan\,(ax)}dx\right)$$

input
```
int((a^2*c*x^2+c)^(1/2)/atan(a*x),x)
```

output
```
sqrt(c)*int(sqrt(a**2*x**2 + 1)/atan(a*x),x)
```

# 3.497 $\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)}\,dx = \text{Int}\left(\frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^(1/2)/x/arctan(a*x),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)}\,dx = \int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)}\,dx$$

input
```
Integrate[Sqrt[c + a^2*c*x^2]/(x*ArcTan[a*x]),x]
```

output
```
Integrate[Sqrt[c + a^2*c*x^2]/(x*ArcTan[a*x]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2cx^2+c}}{x\arctan(ax)}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{\sqrt{a^2cx^2+c}}{x\arctan(ax)}dx$$

input
```
Int[Sqrt[c + a^2*c*x^2]/(x*ArcTan[a*x]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 58.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{a^2c\,x^2+c}}{x\arctan\left(ax\right)}dx$$

input
```
int((a^2*c*x^2+c)^(1/2)/x/arctan(a*x),x)
```

output
```
int((a^2*c*x^2+c)^(1/2)/x/arctan(a*x),x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)}\,dx = \int \frac{\sqrt{a^2cx^2+c}}{x\arctan{(ax)}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/x/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)/(x*arctan(a*x)), x)
```

## Sympy [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)}\,dx = \int \frac{\sqrt{c\,(a^2x^2+1)}}{x\,\mathrm{atan}\,(ax)}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(1/2)/x/atan(a*x),x)
```

output
```
Integral(sqrt(c*(a**2*x**2 + 1))/(x*atan(a*x)), x)
```

## Maxima [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)}\,dx = \int \frac{\sqrt{a^2cx^2+c}}{x\arctan{(ax)}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/x/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)/(x*arctan(a*x)), x)
```

## Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)}\,dx = \int \frac{\sqrt{a^2cx^2+c}}{x\arctan{(ax)}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/x/arctan(a*x),x, algorithm="giac")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)/(x*arctan(a*x)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)}\,dx = \int \frac{\sqrt{c\,a^2\,x^2+c}}{x\,\mathrm{atan}\left(a\,x\right)}\,dx$$

input
```
int((c + a^2*c*x^2)^(1/2)/(x*atan(a*x)),x)
```

output
```
int((c + a^2*c*x^2)^(1/2)/(x*atan(a*x)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)}\,dx = \sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}}{atan\,(ax)\,x}dx\right)$$

input
```
int((a^2*c*x^2+c)^(1/2)/x/atan(a*x),x)
```

output
```
sqrt(c)*int(sqrt(a**2*x**2 + 1)/(atan(a*x)*x),x)
```

# 3.498 $\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)}\,dx = \operatorname{Int}\left(\frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)},x\right)$$

output
```
Defer(Int)(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.69 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)}\,dx = \int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)}\,dx$$

input
```
Integrate[(x*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x],x]
```

output
```
Integrate[(x*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x], x]
```

## Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a^2cx^2+c\right)^{3/2}}{\arctan(ax)}dx$$

$$\Big\downarrow \text{5560}$$

$$\int \frac{x\left(a^2cx^2+c\right)^{3/2}}{\arctan(ax)}dx$$

input
```
Int[(x*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 34.92 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x(a^2c\,x^2+c)^{\frac{3}{2}}}{\arctan\left(ax\right)}dx$$

input
```
int(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

output
```
int(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.50

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)}\,dx=\int \frac{(a^2cx^2+c)^{\frac{3}{2}}x}{\arctan{(ax)}}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="fricas")
```

output
```
integral((a^2*c*x^3 + c*x)*sqrt(a^2*c*x^2 + c)/arctan(a*x), x)
```

## Sympy [N/A]

Not integrable

Time = 3.53 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)}\,dx=\int \frac{x(c(a^2x^2+1))^{\frac{3}{2}}}{\operatorname{atan}{(ax)}}\,dx$$

input
```
integrate(x*(a**2*c*x**2+c)**(3/2)/atan(a*x),x)
```

output
```
Integral(x*(c*(a**2*x**2 + 1))**(3/2)/atan(a*x), x)
```

## Maxima [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)}\,dx=\int \frac{(a^2cx^2+c)^{\frac{3}{2}}x}{\arctan{(ax)}}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*x/arctan(a*x), x)
```

## Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}x}{\arctan(ax)}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*x/arctan(a*x), x)
```

## Mupad [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)}\,dx = \int \frac{x\,(c\,a^2\,x^2+c)^{3/2}}{\mathrm{atan}\,(a\,x)}\,dx$$

input
```
int((x*(c + a^2*c*x^2)^(3/2))/atan(a*x),x)
```

output
```
int((x*(c + a^2*c*x^2)^(3/2))/atan(a*x), x)
```

## Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.32

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)}\,dx = \sqrt{c}\,c\left(\left(\int \frac{\sqrt{a^2x^2+1}\,x^3}{atan\,(ax)}dx\right)a^2+\int \frac{\sqrt{a^2x^2+1}\,x}{atan\,(ax)}dx\right)$$

input
```
int(x*(a^2*c*x^2+c)^(3/2)/atan(a*x),x)
```

output
```
sqrt(c)*c*(int((sqrt(a**2*x**2 + 1)*x**3)/atan(a*x),x)*a**2 + int((sqrt(a*
*2*x**2 + 1)*x)/atan(a*x),x))
```

# 3.499 $\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)}\,dx=\operatorname{Int}\left(\frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)}\,dx=\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^(3/2)/ArcTan[a*x],x]
```

output
```
Integrate[(c + a^2*c*x^2)^(3/2)/ArcTan[a*x], x]
```

## Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2cx^2+c\right)^{3/2}}{\arctan(ax)}\,dx$$

$$\Big\downarrow \; 5560$$

$$\int \frac{\left(a^2cx^2+c\right)^{3/2}}{\arctan(ax)}dx$$

input
```
Int[(c + a^2*c*x^2)^(3/2)/ArcTan[a*x],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 36.40 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{\left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}{\arctan \left(a x \right)}d x$$

input
```
int((a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

output
```
int((a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}}{\arctan(ax)}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^(3/2)/arctan(a*x), x)
```

## Sympy [N/A]

Not integrable

Time = 2.42 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)}\,dx = \int \frac{(c(a^2x^2+1))^{\frac{3}{2}}}{\operatorname{atan}(ax)}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(3/2)/atan(a*x),x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(3/2)/atan(a*x), x)
```

## Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}}{\arctan(ax)}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)/arctan(a*x), x)
```

## Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}}{\arctan(ax)}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)/arctan(a*x), x)
```

## Mupad [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)}\,dx = \int \frac{(c\,a^2\,x^2+c)^{3/2}}{\mathrm{atan}(a\,x)}\,dx$$

input
```
int((c + a^2*c*x^2)^(3/2)/atan(a*x),x)
```

output
```
int((c + a^2*c*x^2)^(3/2)/atan(a*x), x)
```

## Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.38

$$\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)}\,dx = \sqrt{c}\,c\left(\int \frac{\sqrt{a^2x^2+1}}{atan\,(ax)}dx + \left(\int \frac{\sqrt{a^2x^2+1}\,x^2}{atan\,(ax)}dx\right)a^2\right)$$

input
```
int((a^2*c*x^2+c)^(3/2)/atan(a*x),x)
```

output
```
sqrt(c)*c*(int(sqrt(a**2*x**2 + 1)/atan(a*x),x) + int((sqrt(a**2*x**2 + 1)
*x**2)/atan(a*x),x)*a**2)
```

# 3.500 $\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)}\,dx = \text{Int}\left(\frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^(3/2)/x/arctan(a*x),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)}\,dx = \int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^(3/2)/(x*ArcTan[a*x]),x]
```

output
```
Integrate[(c + a^2*c*x^2)^(3/2)/(x*ArcTan[a*x]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^{3/2}}{x\arctan(ax)}dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int \frac{(a^2cx^2+c)^{3/2}}{x\arctan(ax)}dx$$

input
```
Int[(c + a^2*c*x^2)^(3/2)/(x*ArcTan[a*x]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 73.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{(a^2c\,x^2+c)^{\frac{3}{2}}}{x\arctan(ax)}dx$$

input
```
int((a^2*c*x^2+c)^(3/2)/x/arctan(a*x),x)
```

output
```
int((a^2*c*x^2+c)^(3/2)/x/arctan(a*x),x)
```

## Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}}{x\arctan(ax)}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/x/arctan(a*x),x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^(3/2)/(x*arctan(a*x)), x)
```

## Sympy [N/A]

Not integrable

Time = 4.90 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)}\,dx = \int \frac{(c(a^2x^2+1))^{\frac{3}{2}}}{x\operatorname{atan}(ax)}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(3/2)/x/atan(a*x),x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(3/2)/(x*atan(a*x)), x)
```

## Maxima [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}}{x\arctan(ax)}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/x/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)/(x*arctan(a*x)), x)
```

## Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)}\,dx=\int \frac{(a^2cx^2+c)^{\frac{3}{2}}}{x\arctan{(ax)}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/x/arctan(a*x),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)/(x*arctan(a*x)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)}\,dx=\int \frac{(c\,a^2\,x^2+c)^{3/2}}{x\,\mathrm{atan}\left(a\,x\right)}\,dx$$

input
```
int((c + a^2*c*x^2)^(3/2)/(x*atan(a*x)),x)
```

output
```
int((c + a^2*c*x^2)^(3/2)/(x*atan(a*x)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.12

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)}\,dx = \sqrt{c}\,c\left(\int \frac{\sqrt{a^2x^2+1}}{atan\left(ax\right)x}dx + \left(\int \frac{\sqrt{a^2x^2+1}\,x}{atan\left(ax\right)}dx\right)a^2\right)$$

input
```
int((a^2*c*x^2+c)^(3/2)/x/atan(a*x),x)
```

output
```
sqrt(c)*c*(int(sqrt(a**2*x**2 + 1)/(atan(a*x)*x),x) + int((sqrt(a**2*x**2
+ 1)*x)/atan(a*x),x)*a**2)
```

# 3.501 $\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)}\,dx = \operatorname{Int}\left(\frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)},x\right)$$

output
```
Defer(Int)(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.73 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)}\,dx = \int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)}\,dx$$

input
```
Integrate[(x*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x],x]
```

output
```
Integrate[(x*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x], x]
```

## Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a^2cx^2+c\right)^{5/2}}{\arctan(ax)}dx$$

$$\Big\downarrow \text{5560}$$

$$\int \frac{x\left(a^2cx^2+c\right)^{5/2}}{\arctan(ax)}dx$$

input
```
Int[(x*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 41.86 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x(a^2c\,x^2+c)^{\frac{5}{2}}}{\arctan\left(ax\right)}dx$$

input
```
int(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

output
```
int(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.18

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}x}{\arctan(ax)}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^2*x)*sqrt(a^2*c*x^2 + c)/arctan(
a*x), x)
```

## Sympy [N/A]

Not integrable

Time = 12.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)}\,dx = \int \frac{x(c(a^2x^2+1))^{\frac{5}{2}}}{\operatorname{atan}(ax)}\,dx$$

input
```
integrate(x*(a**2*c*x**2+c)**(5/2)/atan(a*x),x)
```

output
```
Integral(x*(c*(a**2*x**2 + 1))**(5/2)/atan(a*x), x)
```

## Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}x}{\arctan(ax)}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)*x/arctan(a*x), x)
```

## Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)}\,dx=\int \frac{(a^2cx^2+c)^{\frac{5}{2}}x}{\arctan(ax)}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)*x/arctan(a*x), x)
```

## Mupad [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)}\,dx=\int \frac{x\,(c\,a^2\,x^2+c)^{5/2}}{\mathrm{atan}\,(a\,x)}\,dx$$

input
```
int((x*(c + a^2*c*x^2)^(5/2))/atan(a*x),x)
```

output
```
int((x*(c + a^2*c*x^2)^(5/2))/atan(a*x), x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 80, normalized size of antiderivative = 3.64

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)}\,dx = \sqrt{c}\,c^2\left(\left(\int \frac{\sqrt{a^2x^2+1}\,x^5}{atan(ax)}dx\right)a^4 + 2\left(\int \frac{\sqrt{a^2x^2+1}\,x^3}{atan(ax)}dx\right)a^2 + \int \frac{\sqrt{a^2x^2+1}\,x}{atan(ax)}dx\right)$$

input
```
int(x*(a^2*c*x^2+c)^(5/2)/atan(a*x),x)
```

output
```
sqrt(c)*c**2*(int((sqrt(a**2*x**2 + 1)*x**5)/atan(a*x),x)*a**4 + 2*int((sq
rt(a**2*x**2 + 1)*x**3)/atan(a*x),x)*a**2 + int((sqrt(a**2*x**2 + 1)*x)/at
an(a*x),x))
```

# 3.502 $\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)}\,dx = \text{Int}\left(\frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)}\,dx = \int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^(5/2)/ArcTan[a*x],x]
```

output
```
Integrate[(c + a^2*c*x^2)^(5/2)/ArcTan[a*x], x]
```

## Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2cx^2+c\right)^{5/2}}{\arctan(ax)}\,dx$$

$$\Big\downarrow \ 5560$$

$$\int \frac{\left(a^2cx^2+c\right)^{5/2}}{\arctan(ax)}dx$$

input
```
Int[(c + a^2*c*x^2)^(5/2)/ArcTan[a*x],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 47.55 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{\left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}}}{\arctan \left(a x \right)}d x$$

input
```
int((a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

output
```
int((a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.19

$$\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}}{\arctan(ax)}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)/arctan(a*
x), x)
```

## Sympy [N/A]

Not integrable

Time = 8.58 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)}\,dx = \int \frac{(c(a^2x^2+1))^{\frac{5}{2}}}{\operatorname{atan}(ax)}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(5/2)/atan(a*x),x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(5/2)/atan(a*x), x)
```

## Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}}{\arctan(ax)}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)/arctan(a*x), x)
```

## Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)}\,dx=\int \frac{(a^2cx^2+c)^{\frac{5}{2}}}{\arctan(ax)}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)/arctan(a*x), x)
```

## Mupad [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)}\,dx=\int \frac{(c\,a^2\,x^2+c)^{5/2}}{\mathrm{atan}\left(a\,x\right)}\,dx$$

input
```
int((c + a^2*c*x^2)^(5/2)/atan(a*x),x)
```

output
```
int((c + a^2*c*x^2)^(5/2)/atan(a*x), x)
```

## Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 79, normalized size of antiderivative = 3.76

$$\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)}\,dx = \sqrt{c}\,c^2\left(\int \frac{\sqrt{a^2x^2+1}}{atan\,(ax)}dx + \left(\int \frac{\sqrt{a^2x^2+1}\,x^4}{atan\,(ax)}dx\right)a^4 + 2\left(\int \frac{\sqrt{a^2x^2+1}\,x^2}{atan\,(ax)}dx\right)a^2\right)$$

input

```
int((a^2*c*x^2+c)^(5/2)/atan(a*x),x)
```

output

```
sqrt(c)*c**2*(int(sqrt(a**2*x**2 + 1)/atan(a*x),x) + int((sqrt(a**2*x**2 +
 1)*x**4)/atan(a*x),x)*a**4 + 2*int((sqrt(a**2*x**2 + 1)*x**2)/atan(a*x),x
)*a**2)
```

# 3.503 $\int \frac{\left(c+a^2cx^2\right)^{5/2}}{x\arctan(ax)}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\left(c+a^2cx^2\right)^{5/2}}{x\arctan(ax)}\,dx=\text{Int}\left(\frac{\left(c+a^2cx^2\right)^{5/2}}{x\arctan(ax)},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^(5/2)/x/arctan(a*x),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.36 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\left(c+a^2cx^2\right)^{5/2}}{x\arctan(ax)}\,dx=\int \frac{\left(c+a^2cx^2\right)^{5/2}}{x\arctan(ax)}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^(5/2)/(x*ArcTan[a*x]),x]
```

output
```
Integrate[(c + a^2*c*x^2)^(5/2)/(x*ArcTan[a*x]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^{5/2}}{x\arctan(ax)}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{(a^2cx^2+c)^{5/2}}{x\arctan(ax)}dx$$

input
```
Int[(c + a^2*c*x^2)^(5/2)/(x*ArcTan[a*x]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 107.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{(a^2c\,x^2+c)^{\frac{5}{2}}}{x\arctan(ax)}dx$$

input
```
int((a^2*c*x^2+c)^(5/2)/x/arctan(a*x),x)
```

output
```
int((a^2*c*x^2+c)^(5/2)/x/arctan(a*x),x)
```

## Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.04

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}}{x\arctan(ax)}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/x/arctan(a*x),x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)/(x*arctan
(a*x)), x)
```

## Sympy [N/A]

Not integrable

Time = 11.98 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)}\,dx = \int \frac{(c(a^2x^2+1))^{\frac{5}{2}}}{x\,\mathrm{atan}(ax)}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(5/2)/x/atan(a*x),x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(5/2)/(x*atan(a*x)), x)
```

## Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}}{x\arctan(ax)}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/x/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)/(x*arctan(a*x)), x)
```

## Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}}{x\arctan(ax)}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/x/arctan(a*x),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)/(x*arctan(a*x)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)}\,dx = \int \frac{(c\,a^2\,x^2+c)^{5/2}}{x\,\mathrm{atan}(a\,x)}\,dx$$

input
```
int((c + a^2*c*x^2)^(5/2)/(x*atan(a*x)),x)
```

output
```
int((c + a^2*c*x^2)^(5/2)/(x*atan(a*x)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 80, normalized size of antiderivative = 3.33

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)}\,dx = \sqrt{c}\,c^2\left(\int \frac{\sqrt{a^2x^2+1}}{atan\,(ax)\,x}dx + \left(\int \frac{\sqrt{a^2x^2+1}\,x^3}{atan\,(ax)}dx\right)a^4 + 2\left(\int \frac{\sqrt{a^2x^2+1}\,x}{atan\,(ax)}dx\right)a^2\right)$$

input
```
int((a^2*c*x^2+c)^(5/2)/x/atan(a*x),x)
```

output
```
sqrt(c)*c**2*(int(sqrt(a**2*x**2 + 1)/(atan(a*x)*x),x) + int((sqrt(a**2*x*
*2 + 1)*x**3)/atan(a*x),x)*a**4 + 2*int((sqrt(a**2*x**2 + 1)*x)/atan(a*x),
x)*a**2)
```

# 3.504 $\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)}\,dx = \text{Int}\left(\frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)},x\right)$$

output
```
Defer(Int)(x/(a^2*c*x^2+c)^(1/2)/arctan(a*x),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)}\,dx = \int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)}\,dx$$

input
```
Integrate[x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]),x]
```

output
```
Integrate[x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\arctan(ax)\sqrt{a^2cx^2+c}}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x}{\arctan(ax)\sqrt{a^2cx^2+c}}dx$$

input
```
Int[x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 1.67 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x}{\sqrt{a^2c\,x^2+c}\,\arctan\left(ax\right)}dx$$

input
```
int(x/(a^2*c*x^2+c)^(1/2)/arctan(a*x),x)
```

output
```
int(x/(a^2*c*x^2+c)^(1/2)/arctan(a*x),x)
```

## Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)}\,dx = \int \frac{x}{\sqrt{a^2cx^2+c}\arctan{(ax)}}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)^(1/2)/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(x/(sqrt(a^2*c*x^2 + c)*arctan(a*x)), x)
```

## Sympy [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)}\,dx = \int \frac{x}{\sqrt{c\left(a^2x^2+1\right)}\operatorname{atan}{(ax)}}\,dx$$

input
```
integrate(x/(a**2*c*x**2+c)**(1/2)/atan(a*x),x)
```

output
```
Integral(x/(sqrt(c*(a**2*x**2 + 1))*atan(a*x)), x)
```

## Maxima [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)}\,dx = \int \frac{x}{\sqrt{a^2cx^2+c}\arctan{(ax)}}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)^(1/2)/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(a^2*c*x^2 + c)*arctan(a*x)), x)
```

## Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)}\,dx = \int \frac{x}{\sqrt{a^2cx^2+c}\arctan{(ax)}}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)^(1/2)/arctan(a*x),x, algorithm="giac")
```

output
```
integrate(x/(sqrt(a^2*c*x^2 + c)*arctan(a*x)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)}\,dx = \int \frac{x}{\mathrm{atan}\left(a\,x\right)\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(x/(atan(a*x)*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(x/(atan(a*x)*(c + a^2*c*x^2)^(1/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

$$\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)}\,dx = \frac{\int \frac{x}{\sqrt{a^2x^2+1}\,atan(ax)}dx}{\sqrt{c}}$$

input
```
int(x/(a^2*c*x^2+c)^(1/2)/atan(a*x),x)
```

output
```
int(x/(sqrt(a**2*x**2 + 1)*atan(a*x)),x)/sqrt(c)
```

# 3.505 $\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)}\,dx = \text{Int}\left(\frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)},x\right)$$

output
```
Defer(Int)(1/(a^2*c*x^2+c)^(1/2)/arctan(a*x),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)}\,dx = \int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)}\,dx$$

input
```
Integrate[1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]),x]
```

output
```
Integrate[1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arctan(ax)\sqrt{a^2cx^2+c}}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{1}{\arctan(ax)\sqrt{a^2cx^2+c}}dx$$

input
```
Int[1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 4.58 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1}{\sqrt{a^2c\,x^2+c}\,\arctan\left(ax\right)}dx$$

input
```
int(1/(a^2*c*x^2+c)^(1/2)/arctan(a*x),x)
```

output
```
int(1/(a^2*c*x^2+c)^(1/2)/arctan(a*x),x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)}\,dx = \int \frac{1}{\sqrt{a^2cx^2+c}\arctan{(ax)}}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(1/2)/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(1/(sqrt(a^2*c*x^2 + c)*arctan(a*x)), x)
```

## Sympy [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)}\,dx = \int \frac{1}{\sqrt{c(a^2x^2+1)}\operatorname{atan}{(ax)}}\,dx$$

input
```
integrate(1/(a**2*c*x**2+c)**(1/2)/atan(a*x),x)
```

output
```
Integral(1/(sqrt(c*(a**2*x**2 + 1))*atan(a*x)), x)
```

## Maxima [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)}\,dx = \int \frac{1}{\sqrt{a^2cx^2+c}\arctan{(ax)}}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(1/2)/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a^2*c*x^2 + c)*arctan(a*x)), x)
```

## Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)}\,dx = \int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(1/2)/arctan(a*x),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(a^2*c*x^2 + c)*arctan(a*x)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)}\,dx = \int \frac{1}{\mathrm{atan}(a\,x)\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(1/(atan(a*x)*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(1/(atan(a*x)*(c + a^2*c*x^2)^(1/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.24

$$\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)}\,dx = \frac{\int \frac{1}{\sqrt{a^2x^2+1}\,atan(ax)}dx}{\sqrt{c}}$$

input
```
int(1/(a^2*c*x^2+c)^(1/2)/atan(a*x),x)
```

output
```
int(1/(sqrt(a**2*x**2 + 1)*atan(a*x)),x)/sqrt(c)
```

# 3.506 $\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)}\,dx = \operatorname{Int}\left(\frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)},x\right)$$

output
```
Defer(Int)(1/x/(a^2*c*x^2+c)^(1/2)/arctan(a*x),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)}\,dx = \int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)}\,dx$$

input
```
Integrate[1/(x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]),x]
```

output
```
Integrate[1/(x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x \arctan(ax)\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int \frac{1}{x \arctan(ax)\sqrt{a^2cx^2+c}}dx$$

input
```
Int[1/(x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 5.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\sqrt{a^2c\,x^2+c}\,\arctan\left(ax\right)}dx$$

input
```
int(1/x/(a^2*c*x^2+c)^(1/2)/arctan(a*x),x)
```

output
```
int(1/x/(a^2*c*x^2+c)^(1/2)/arctan(a*x),x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)}\,dx = \int \frac{1}{\sqrt{a^2cx^2+cx}\arctan(ax)}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(1/2)/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)/((a^2*c*x^3 + c*x)*arctan(a*x)), x)
```

## Sympy [N/A]

Not integrable

Time = 0.93 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)}\,dx = \int \frac{1}{x\sqrt{c(a^2x^2+1)}\,\mathrm{atan}(ax)}\,dx$$

input
```
integrate(1/x/(a**2*c*x**2+c)**(1/2)/atan(a*x),x)
```

output
```
Integral(1/(x*sqrt(c*(a**2*x**2 + 1))*atan(a*x)), x)
```

## Maxima [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)}\,dx = \int \frac{1}{\sqrt{a^2cx^2+cx}\arctan(ax)}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(1/2)/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a^2*c*x^2 + c)*x*arctan(a*x)), x)
```

## Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)}\,dx = \int \frac{1}{\sqrt{a^2cx^2+c}x\arctan(ax)}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(1/2)/arctan(a*x),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(a^2*c*x^2 + c)*x*arctan(a*x)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)}\,dx = \int \frac{1}{x\,\mathrm{atan}(a\,x)\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(1/(x*atan(a*x)*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(1/(x*atan(a*x)*(c + a^2*c*x^2)^(1/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)}\,dx = \frac{\int \frac{1}{\sqrt{a^2x^2+1}\,atan(ax)x}dx}{\sqrt{c}}$$

input
```
int(1/x/(a^2*c*x^2+c)^(1/2)/atan(a*x),x)
```

output
```
int(1/(sqrt(a**2*x**2 + 1)*atan(a*x)*x),x)/sqrt(c)
```

# 3.507 $\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx = \text{Int}\left(\frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)},x\right)$$

output
```
Defer(Int)(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

## Mathematica [N/A]

Not integrable

Time = 4.81 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx = \int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx$$

input
```
Integrate[x^3/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]),x]
```

output
```
Integrate[x^3/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \; 5560$$

$$\int \frac{x^3}{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}dx$$

input
```
Int[x^3/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 2.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^3}{\left(a^2c\,x^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)}dx$$

input
```
int(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

output
```
int(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.12

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx = \int \frac{x^3}{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)}\,dx$$

input
```
integrate(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^3/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arct
an(a*x)), x)
```

## Sympy [N/A]

Not integrable

Time = 1.54 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx = \int \frac{x^3}{(c(a^2x^2+1))^{\frac{3}{2}}\operatorname{atan}(ax)}\,dx$$

input
```
integrate(x**3/(a**2*c*x**2+c)**(3/2)/atan(a*x),x)
```

output
```
Integral(x**3/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)), x)
```

## Maxima [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx = \int \frac{x^3}{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)}\,dx$$

input
```
integrate(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(x^3/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx = \int \frac{x^3}{\mathrm{atan}\left(a\,x\right)\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

input
```
int(x^3/(atan(a*x)*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(x^3/(atan(a*x)*(c + a^2*c*x^2)^(3/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.21

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx = \frac{\int \frac{x^3}{\sqrt{a^2x^2+1}\,atan(ax)a^2x^2+\sqrt{a^2x^2+1}\,atan(ax)}dx}{\sqrt{c}\,c}$$

input
```
int(x^3/(a^2*c*x^2+c)^(3/2)/atan(a*x),x)
```

output
```
int(x**3/(sqrt(a**2*x**2 + 1)*atan(a*x)*a**2*x**2 + sqrt(a**2*x**2 + 1)*at
an(a*x)),x)/(sqrt(c)*c)
```

# 3.508 $\int \frac{x^2}{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^2}{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)}\,dx=\text{Int}\left(\frac{x^2}{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)},x\right)$$

output
```
Defer(Int)(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

## Mathematica [F(-1)]

Timed out.

$$\int \frac{x^2}{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)}\,dx=\text{\$Aborted}$$

input
```
Integrate[x^2/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]),x]
```

output
```
$Aborted
```

## Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{x^2}{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}dx$$

input
```
Int[x^2/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 2.37 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^2}{\left(a^2c\,x^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)}dx$$

input
```
int(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

output
```
int(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

## Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.12

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx = \int \frac{x^2}{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^2/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arct
an(a*x)), x)
```

## Sympy [N/A]

Not integrable

Time = 1.54 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx = \int \frac{x^2}{(c(a^2x^2+1))^{\frac{3}{2}}\operatorname{atan}(ax)}\,dx$$

input
```
integrate(x**2/(a**2*c*x**2+c)**(3/2)/atan(a*x),x)
```

output
```
Integral(x**2/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)), x)
```

## Maxima [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx = \int \frac{x^2}{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(x^2/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)), x)
```

## Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx = \int \frac{x^2}{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="giac")
```

output
```
integrate(x^2/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx = \int \frac{x^2}{\mathrm{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^{3/2}}\,dx$$

input
```
int(x^2/(atan(a*x)*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(x^2/(atan(a*x)*(c + a^2*c*x^2)^(3/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.21

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx = \frac{\int \frac{x^2}{\sqrt{a^2x^2+1}\,atan(ax)a^2x^2+\sqrt{a^2x^2+1}\,atan(ax)}dx}{\sqrt{c}\,c}$$

input
```
int(x^2/(a^2*c*x^2+c)^(3/2)/atan(a*x),x)
```

output
```
int(x**2/(sqrt(a**2*x**2 + 1)*atan(a*x)*a**2*x**2 + sqrt(a**2*x**2 + 1)*at
an(a*x)),x)/(sqrt(c)*c)
```

# 3.509 $\int \frac{x}{(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 39

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx = \frac{\sqrt{1+a^2x^2}\,\mathrm{Si}(\arctan(ax))}{a^2c\sqrt{c+a^2cx^2}}$$

output
```
(a^2*x^2+1)^(1/2)*Si(arctan(a*x))/a^2/c/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx = \frac{\sqrt{1+a^2x^2}\,\mathrm{Si}(\arctan(ax))}{a^2c\sqrt{c\,(1+a^2x^2)}}$$

input
```
Integrate[x/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]),x]
```

output
```
(Sqrt[1 + a^2*x^2]*SinIntegral[ArcTan[a*x]])/(a^2*c*Sqrt[c*(1 + a^2*x^2)])
```

## Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {5506, 5505, 3042, 3780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{5506}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{x}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}dx}{c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5505}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{ax}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{\sin(\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3780}$$

$$\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}$$

input
```
Int[x/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]),x]
```

output
```
(Sqrt[1 + a^2*x^2]*SinIntegral[ArcTan[a*x]])/(a^2*c*Sqrt[c + a^2*c*x^2])
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.92 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.10

| method | result | size |
|---|---|---|
| default | $-\frac{\operatorname{csgn}(\arctan(ax))\pi\sqrt{c(ax-i)(ax+i)}}{2\sqrt{a^2x^2+1}\,a^2c^2}+\frac{\operatorname{Si}(\arctan(ax))\sqrt{c(ax-i)(ax+i)}}{\sqrt{a^2x^2+1}\,a^2c^2}$ | 82 |

input
```
int(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*csgn(arctan(a*x))*Pi*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/2)/a^2/
c^2+Si(arctan(a*x))*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/2)/a^2/c^2
```

## Fricas [F]

$$\int \frac{x}{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)}\,dx = \int \frac{x}{\left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan
(a*x)), x)
```

## Sympy [F]

$$\int \frac{x}{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)}\,dx = \int \frac{x}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}\left(ax\right)}\,dx$$

input
```
integrate(x/(a**2*c*x**2+c)**(3/2)/atan(a*x),x)
```

output
```
Integral(x/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)), x)
```

## Maxima [F]

$$\int \frac{x}{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)}\,dx = \int \frac{x}{\left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(x/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx = \int \frac{x}{\mathrm{atan}\left(a\,x\right)\,\left(c\,a^2\,x^2+c\right)^{3/2}}\,dx$$

input
```
int(x/(atan(a*x)*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(x/(atan(a*x)*(c + a^2*c*x^2)^(3/2)), x)
```

## Reduce [F]

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx = \frac{\int \frac{x}{\sqrt{a^2x^2+1}\,atan(ax)a^2x^2+\sqrt{a^2x^2+1}\,atan(ax)}dx}{\sqrt{c}\,c}$$

input
```
int(x/(a^2*c*x^2+c)^(3/2)/atan(a*x),x)
```

output
```
int(x/(sqrt(a**2*x**2 + 1)*atan(a*x)*a**2*x**2 + sqrt(a**2*x**2 + 1)*atan(
a*x)),x)/(sqrt(c)*c)
```

# 3.510 $\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 39

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx = \frac{\sqrt{1+a^2x^2}\,\text{CosIntegral}(\arctan(ax))}{ac\sqrt{c+a^2cx^2}}$$

output
```
(a^2*x^2+1)^(1/2)*Ci(arctan(a*x))/a/c/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx = \frac{\sqrt{1+a^2x^2}\,\text{CosIntegral}(\arctan(ax))}{ac\sqrt{c(1+a^2x^2)}}$$

input
```
Integrate[1/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]),x]
```

output
```
(Sqrt[1 + a^2*x^2]*CosIntegral[ArcTan[a*x]])/(a*c*Sqrt[c*(1 + a^2*x^2)])
```

## Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {5440, 5439, 3042, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{5440}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{1}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}dx}{c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5439}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)}{\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3783}$$

$$\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}$$

input
```
Int[1/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]),x]
```

output
```
(Sqrt[1 + a^2*x^2]*CosIntegral[ArcTan[a*x]])/(a*c*Sqrt[c + a^2*c*x^2])
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5440
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])   Int[(1 +
c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.69 (sec) , antiderivative size = 136, normalized size of antiderivative = 3.49

| method | result |
|---|---|
| default | $-\frac{i\,\mathrm{csgn}(\arctan(ax))\,\mathrm{csgn}(i\arctan(ax))\pi\sqrt{c(ax-i)(ax+i)}}{2\sqrt{a^2x^2+1}\,a\,c^2}+\frac{i\,\mathrm{csgn}(i\arctan(ax))\pi\sqrt{c(ax-i)(ax+i)}}{2\sqrt{a^2x^2+1}\,a\,c^2}+\frac{\mathrm{Ci}(\arctan(ax))\sqrt{c(ax-}}{\sqrt{a^2x^2+1}\,a\,c^2}$ |

input
```
int(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*csgn(arctan(a*x))*csgn(I*arctan(a*x))*Pi*(c*(a*x-I)*(a*x+I))^(1/2)/
(a^2*x^2+1)^(1/2)/a/c^2+1/2*I*csgn(I*arctan(a*x))*Pi*(c*(a*x-I)*(a*x+I))^(
1/2)/(a^2*x^2+1)^(1/2)/a/c^2+Ci(arctan(a*x))*(c*(a*x-I)*(a*x+I))^(1/2)/(a^
2*x^2+1)^(1/2)/a/c^2
```

## Fricas [F]

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx=\int \frac{1}{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a
*x)), x)
```

## Sympy [F]

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx=\int \frac{1}{(c(a^2x^2+1))^{\frac{3}{2}}\operatorname{atan}(ax)}\,dx$$

input
```
integrate(1/(a**2*c*x**2+c)**(3/2)/atan(a*x),x)
```

output
```
Integral(1/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)), x)
```

## Maxima [F]

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx=\int \frac{1}{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)), x)
```

## Giac [F]

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx = \int \frac{1}{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx = \int \frac{1}{\operatorname{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^{3/2}}\,dx$$

input
```
int(1/(atan(a*x)*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(1/(atan(a*x)*(c + a^2*c*x^2)^(3/2)), x)
```

## Reduce [F]

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx = \frac{\int \frac{1}{\sqrt{a^2x^2+1}\,atan(ax)a^2x^2+\sqrt{a^2x^2+1}\,atan(ax)}dx}{\sqrt{c}\,c}$$

input
```
int(1/(a^2*c*x^2+c)^(3/2)/atan(a*x),x)
```

output
```
int(1/(sqrt(a**2*x**2 + 1)*atan(a*x)*a**2*x**2 + sqrt(a**2*x**2 + 1)*atan(
a*x)),x)/(sqrt(c)*c)
```

# 3.511 $\int \frac{1}{x(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)}\,dx = \text{Int}\left(\frac{1}{x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)},x\right)$$

output

```
Defer(Int)(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)}\,dx = \int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)}\,dx$$

input

```
Integrate[1/(x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]),x]
```

output

```
Integrate[1/(x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x \arctan(ax) (a^2cx^2+c)^{3/2}} \, dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int \frac{1}{x \arctan(ax) (a^2cx^2+c)^{3/2}}dx$$

input
```
Int[1/(x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 2.82 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x \left(a^2 c\, x^2+c\right)^{\frac{3}{2}} \arctan\left(a x\right)}dx$$

input
```
int(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

output
```
int(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.08

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)}\,dx=\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x\arctan\left(ax\right)}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)/((a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^2*x)*arctan
(a*x)), x)
```

## Sympy [N/A]

Not integrable

Time = 3.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)}\,dx=\int \frac{1}{x\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}\left(ax\right)}\,dx$$

input
```
integrate(1/x/(a**2*c*x**2+c)**(3/2)/atan(a*x),x)
```

output
```
Integral(1/(x*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x)), x)
```

## Maxima [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)}\,dx=\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x\arctan\left(ax\right)}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(3/2)*x*arctan(a*x)), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx = \int \frac{1}{x\,\mathrm{atan}\left(a\,x\right)\,\left(c\,a^2\,x^2+c\right)^{3/2}}\,dx$$

input
```
int(1/(x*atan(a*x)*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(1/(x*atan(a*x)*(c + a^2*c*x^2)^(3/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.08

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)}\,dx=\frac{\int \frac{1}{\sqrt{a^2x^2+1}\,atan(ax)a^2x^3+\sqrt{a^2x^2+1}\,atan(ax)x}dx}{\sqrt{c}\,c}$$

input
```
int(1/x/(a^2*c*x^2+c)^(3/2)/atan(a*x),x)
```

output
```
int(1/(sqrt(a**2*x**2 + 1)*atan(a*x)*a**2*x**3 + sqrt(a**2*x**2 + 1)*atan(
a*x)*x),x)/(sqrt(c)*c)
```

# 3.512 $\int \frac{1}{x^2(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^{3/2}\arctan(ax)}\,dx = \text{Int}\left(\frac{1}{x^2\left(c+a^2cx^2\right)^{3/2}\arctan(ax)},x\right)$$

output
```
Defer(Int)(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^{3/2}\arctan(ax)}\,dx = \int \frac{1}{x^2\left(c+a^2cx^2\right)^{3/2}\arctan(ax)}\,dx$$

input
```
Integrate[1/(x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]),x]
```

output
```
Integrate[1/(x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \arctan(ax) \left(a^2cx^2+c\right)^{3/2}} \, dx$$
$$\Big\downarrow \ 5560$$
$$\int \frac{1}{x^2 \arctan(ax) \left(a^2cx^2+c\right)^{3/2}}dx$$

input
```
Int[1/(x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 2.97 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^2 \left(a^2 c\, x^2+c\right)^{\frac{3}{2}} \arctan\left(a x\right)}dx$$

input
```
int(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

output
```
int(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

## Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.17

$$\int \frac{1}{x^2 \left(c+a^2 c x^2\right)^{3/2} \arctan(ax)} \, dx = \int \frac{1}{(a^2cx^2+c)^{\frac{3}{2}} x^2 \arctan{(ax)}} \, dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)/((a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2)*arct
an(a*x)), x)
```

## Sympy [N/A]

Not integrable

Time = 3.48 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 \left(c+a^2 c x^2\right)^{3/2} \arctan(ax)} \, dx = \int \frac{1}{x^2 \left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}} \operatorname{atan}{(ax)}} \, dx$$

input
```
integrate(1/x**2/(a**2*c*x**2+c)**(3/2)/atan(a*x),x)
```

output
```
Integral(1/(x**2*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x)), x)
```

## Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 \left(c+a^2 c x^2\right)^{3/2} \arctan(ax)} \, dx = \int \frac{1}{(a^2cx^2+c)^{\frac{3}{2}} x^2 \arctan{(ax)}} \, dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(3/2)*x^2*arctan(a*x)), x)
```

## Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (c+a^2cx^2)^{3/2} \arctan(ax)} \, dx = \int \frac{1}{(a^2cx^2+c)^{\frac{3}{2}} x^2 \arctan(ax)} \, dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(3/2)*x^2*arctan(a*x)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (c+a^2cx^2)^{3/2} \arctan(ax)} \, dx = \int \frac{1}{x^2 \operatorname{atan}(a\,x)\, (c\,a^2\,x^2+c)^{3/2}} \, dx$$

input
```
int(1/(x^2*atan(a*x)*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(1/(x^2*atan(a*x)*(c + a^2*c*x^2)^(3/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.17

$$\int \frac{1}{x^2 (c+a^2cx^2)^{3/2} \arctan(ax)} \, dx = \frac{\int \frac{1}{\sqrt{a^2x^2+1}\, atan(ax)a^2x^4+\sqrt{a^2x^2+1}\, atan(ax)x^2}dx}{\sqrt{c}\, c}$$

input
```
int(1/x^2/(a^2*c*x^2+c)^(3/2)/atan(a*x),x)
```

output
```
int(1/(sqrt(a**2*x**2 + 1)*atan(a*x)*a**2*x**4 + sqrt(a**2*x**2 + 1)*atan(
a*x)*x**2),x)/(sqrt(c)*c)
```

# 3.513 $\int \frac{x^5}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^5}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \operatorname{Int}\left(\frac{x^5}{(c+a^2cx^2)^{5/2}\arctan(ax)},x\right)$$

output
```
Defer(Int)(x^5/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

## Mathematica [N/A]

Not integrable

Time = 5.88 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^5}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \int \frac{x^5}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx$$

input
```
Integrate[x^5/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]),x]
```

output
```
Integrate[x^5/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}\,dx$$

$$\Big\downarrow \; 5560$$

$$\int \frac{x^5}{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}dx$$

input
```
Int[x^5/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 2.57 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^5}{\left(a^2c\,x^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)}dx$$

input
```
int(x^5/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

output
```
int(x^5/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

## Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.58

$$\int \frac{x^5}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \int \frac{x^5}{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)}\,dx$$

input
```
integrate(x^5/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^5/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3
*x^2 + c^3)*arctan(a*x)), x)
```

## Sympy [N/A]

Not integrable

Time = 4.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^5}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \int \frac{x^5}{(c(a^2x^2+1))^{\frac{5}{2}}\operatorname{atan}(ax)}\,dx$$

input
```
integrate(x**5/(a**2*c*x**2+c)**(5/2)/atan(a*x),x)
```

output
```
Integral(x**5/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)), x)
```

## Maxima [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \int \frac{x^5}{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)}\,dx$$

input
```
integrate(x^5/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(x^5/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x^5}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^5/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \int \frac{x^5}{\mathrm{atan}\left(a\,x\right)\,{\left(c\,a^2\,x^2+c\right)}^{5/2}} \,d x$$

input
```
int(x^5/(atan(a*x)*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(x^5/(atan(a*x)*(c + a^2*c*x^2)^(5/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 75, normalized size of antiderivative = 3.12

$$\int \frac{x^5}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \frac{\int \frac{x^5}{\sqrt{a^2x^2+1}\,atan(ax)a^4x^4+2\sqrt{a^2x^2+1}\,atan(ax)a^2x^2+\sqrt{a^2x^2+1}\,atan(ax)}dx}{\sqrt{c}\,c^2}$$

input
```
int(x^5/(a^2*c*x^2+c)^(5/2)/atan(a*x),x)
```

output
```
int(x**5/(sqrt(a**2*x**2 + 1)*atan(a*x)*a**4*x**4 + 2*sqrt(a**2*x**2 + 1)*
atan(a*x)*a**2*x**2 + sqrt(a**2*x**2 + 1)*atan(a*x)),x)/(sqrt(c)*c**2)
```

# 3.514 $\int \frac{x^4}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^4}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \text{Int}\left(\frac{x^4}{(c+a^2cx^2)^{5/2}\arctan(ax)},x\right)$$

output
```
Defer(Int)(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

## Mathematica [F(-1)]

Timed out.

$$\int \frac{x^4}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \$\text{Aborted}$$

input
```
Integrate[x^4/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]),x]
```

output
```
$Aborted
```

## Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}dx$$
$$\Big\downarrow \ 5560$$
$$\int \frac{x^4}{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}dx$$

input
```
Int[x^4/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 2.49 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^4}{\left(a^2c\,x^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)}dx$$

input
```
int(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

output
```
int(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

## Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.58

$$\int \frac{x^4}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx=\int \frac{x^4}{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)}\,dx$$

input
```
integrate(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^4/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3
*x^2 + c^3)*arctan(a*x)), x)
```

## Sympy [N/A]

Not integrable

Time = 4.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^4}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx=\int \frac{x^4}{(c(a^2x^2+1))^{\frac{5}{2}}\operatorname{atan}(ax)}\,dx$$

input
```
integrate(x**4/(a**2*c*x**2+c)**(5/2)/atan(a*x),x)
```

output
```
Integral(x**4/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)), x)
```

## Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx=\int \frac{x^4}{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)}\,dx$$

input
```
integrate(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(x^4/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)), x)
```

## Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx=\int \frac{x^4}{(a^2cx^2+c)^{\frac{5}{2}}\arctan{(ax)}}\,dx$$

input
```
integrate(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="giac")
```

output
```
integrate(x^4/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx=\int \frac{x^4}{\mathrm{atan}\left(a\,x\right)\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

input
```
int(x^4/(atan(a*x)*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(x^4/(atan(a*x)*(c + a^2*c*x^2)^(5/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 75, normalized size of antiderivative = 3.12

$$\int \frac{x^4}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \frac{\int \frac{x^4}{\sqrt{a^2x^2+1}\,atan(ax)a^4x^4+2\sqrt{a^2x^2+1}\,atan(ax)a^2x^2+\sqrt{a^2x^2+1}\,atan(ax)}dx}{\sqrt{c}\,c^2}$$

input
```
int(x^4/(a^2*c*x^2+c)^(5/2)/atan(a*x),x)
```

output
```
int(x**4/(sqrt(a**2*x**2 + 1)*atan(a*x)*a**4*x**4 + 2*sqrt(a**2*x**2 + 1)*
atan(a*x)*a**2*x**2 + sqrt(a**2*x**2 + 1)*atan(a*x)),x)/(sqrt(c)*c**2)
```

# 3.515 $\int \frac{x^3}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 87

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \frac{3\sqrt{1+a^2x^2}\,\mathrm{Si}(\arctan(ax))}{4a^4c^2\sqrt{c+a^2cx^2}} - \frac{\sqrt{1+a^2x^2}\,\mathrm{Si}(3\arctan(ax))}{4a^4c^2\sqrt{c+a^2cx^2}}$$

output
```
3/4*(a^2*x^2+1)^(1/2)*Si(arctan(a*x))/a^4/c^2/(a^2*c*x^2+c)^(1/2)-1/4*(a^2
*x^2+1)^(1/2)*Si(3*arctan(a*x))/a^4/c^2/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.63

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \frac{(1+a^2x^2)^{3/2}\,(3\mathrm{Si}(\arctan(ax)) - \mathrm{Si}(3\arctan(ax)))}{4a^4c\,(c\,(1+a^2x^2))^{3/2}}$$

input
```
Integrate[x^3/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]),x]
```

output
```
((1 + a^2*x^2)^(3/2)*(3*SinIntegral[ArcTan[a*x]] - SinIntegral[3*ArcTan[a*
x]]))/(4*a^4*c*(c*(1 + a^2*x^2))^(3/2))
```

## Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.63, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {5506, 5505, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 5506}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{x^3}{\left(a^2x^2+1\right)^{5/2}\arctan(ax)}dx}{c^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 5505}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{a^3x^3}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}d\arctan(ax)}{a^4c^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{\sin(\arctan(ax))^3}{\arctan(ax)}d\arctan(ax)}{a^4c^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 3793}$$

$$\frac{\sqrt{a^2x^2+1}\int \left(\frac{3ax}{4\sqrt{a^2x^2+1}\arctan(ax)}-\frac{\sin(3\arctan(ax))}{4\arctan(ax)}\right)d\arctan(ax)}{a^4c^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{a^2x^2+1}\left(\frac{3}{4}\text{Si}(\arctan(ax))-\frac{1}{4}\text{Si}(3\arctan(ax))\right)}{a^4c^2\sqrt{a^2cx^2+c}}$$

input
```
Int[x^3/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]),x]
```

output
```
(Sqrt[1 + a^2*x^2]*((3*SinIntegral[ArcTan[a*x]])/4 - SinIntegral[3*ArcTan[
a*x]]/4))/(a^4*c^2*Sqrt[c + a^2*c*x^2])
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.22 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.44

| method | result | size |
|---|---|---|
| default | $-\frac{\operatorname{csgn}\left(\arctan\left(a x\right)\right) \pi \sqrt{c \left(a x -i\right) \left(a x +i\right)}}{4 \sqrt{a^{2} x^{2}+1}\, a^{4} c^{3}}-\frac{\operatorname{Si}\left(3 \arctan\left(a x\right)\right) \sqrt{c \left(a x -i\right) \left(a x +i\right)}}{4 \sqrt{a^{2} x^{2}+1}\, a^{4} c^{3}}+\frac{3 \,\operatorname{Si}\left(\arctan\left(a x\right)\right) \sqrt{c \left(a x -i\right) \left(a x +i\right)}}{4 \sqrt{a^{2} x^{2}+1}\, a^{4} c^{3}}$ | 125 |

input
```
int(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
-1/4*csgn(arctan(a*x))*Pi*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/2)/a^4/
c^3-1/4*Si(3*arctan(a*x))*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/2)/a^4/
c^3+3/4*Si(arctan(a*x))*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/2)/a^4/c^
3
```

## Fricas [F]

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx=\int \frac{x^3}{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)}\,dx$$

input
```
integrate(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^3/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3
*x^2 + c^3)*arctan(a*x)), x)
```

## Sympy [F]

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx=\int \frac{x^3}{(c(a^2x^2+1))^{\frac{5}{2}}\operatorname{atan}(ax)}\,dx$$

input
```
integrate(x**3/(a**2*c*x**2+c)**(5/2)/atan(a*x),x)
```

output
```
Integral(x**3/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)), x)
```

## Maxima [F]

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \int \frac{x^3}{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)}\,dx$$

input
```
integrate(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(x^3/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \int \frac{x^3}{\operatorname{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^{5/2}}\,dx$$

input
```
int(x^3/(atan(a*x)*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(x^3/(atan(a*x)*(c + a^2*c*x^2)^(5/2)), x)
```

## Reduce [F]

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \frac{\int \frac{x^3}{\sqrt{a^2x^2+1}\,atan(ax)a^4x^4+2\sqrt{a^2x^2+1}\,atan(ax)a^2x^2+\sqrt{a^2x^2+1}\,atan(ax)}dx}{\sqrt{c}\,c^2}$$

input
```
int(x^3/(a^2*c*x^2+c)^(5/2)/atan(a*x),x)
```

output
```
int(x**3/(sqrt(a**2*x**2 + 1)*atan(a*x)*a**4*x**4 + 2*sqrt(a**2*x**2 + 1)*
atan(a*x)*a**2*x**2 + sqrt(a**2*x**2 + 1)*atan(a*x)),x)/(sqrt(c)*c**2)
```

# 3.516 $\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 87

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \frac{\sqrt{1+a^2x^2}\,\text{CosIntegral}(\arctan(ax))}{4a^3c^2\sqrt{c+a^2cx^2}} - \frac{\sqrt{1+a^2x^2}\,\text{CosIntegral}(3\arctan(ax))}{4a^3c^2\sqrt{c+a^2cx^2}}$$

output
```
1/4*(a^2*x^2+1)^(1/2)*Ci(arctan(a*x))/a^3/c^2/(a^2*c*x^2+c)^(1/2)-1/4*(a^2
*x^2+1)^(1/2)*Ci(3*arctan(a*x))/a^3/c^2/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.61

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \frac{\sqrt{1+a^2x^2}(\text{CosIntegral}(\arctan(ax)) - \text{CosIntegral}(3\arctan(ax)))}{4a^3c^2\sqrt{c(1+a^2x^2)}}$$

input
```
Integrate[x^2/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]),x]
```

output
```
(Sqrt[1 + a^2*x^2]*(CosIntegral[ArcTan[a*x]] - CosIntegral[3*ArcTan[a*x]])
)/(4*a^3*c^2*Sqrt[c*(1 + a^2*x^2)])
```

## Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.63, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5506, 5505, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}dx$$

$$\Big\downarrow \text{5506}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{x^2}{(a^2x^2+1)^{5/2}\arctan(ax)}dx}{c^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5505}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{a^2x^2}{(a^2x^2+1)^{3/2}\arctan(ax)}d\arctan(ax)}{a^3c^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{4906}$$

$$\frac{\sqrt{a^2x^2+1}\int \left(\frac{1}{4\sqrt{a^2x^2+1}\arctan(ax)}-\frac{\cos(3\arctan(ax))}{4\arctan(ax)}\right)d\arctan(ax)}{a^3c^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2x^2+1}\left(\frac{1}{4}\text{CosIntegral}(\arctan(ax))-\frac{1}{4}\text{CosIntegral}(3\arctan(ax))\right)}{a^3c^2\sqrt{a^2cx^2+c}}$$

input
```
Int[x^2/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]),x]
```

output
```
(Sqrt[1 + a^2*x^2]*(CosIntegral[ArcTan[a*x]]/4 - CosIntegral[3*ArcTan[a*x]
]/4))/(a^3*c^2*Sqrt[c + a^2*c*x^2])
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.40 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.97

| method | result | size |
|---|---|---|
| default | $-\frac{\operatorname{Ci}\left(3 \arctan \left(a x\right)\right) \sqrt{c \left(a x -i\right) \left(a x +i\right)}}{4 \sqrt{a^{2} x^{2}+1}\, a^{3} c^{3}}+\frac{\operatorname{Ci}\left(\arctan \left(a x\right)\right) \sqrt{c \left(a x -i\right) \left(a x +i\right)}}{4 \sqrt{a^{2} x^{2}+1}\, a^{3} c^{3}}$ | 84 |

input
```
int(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
-1/4*Ci(3*arctan(a*x))*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/2)/a^3/c^3
+1/4*Ci(arctan(a*x))*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/2)/a^3/c^3
```

## Fricas [F]

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \int \frac{x^2}{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^2/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3
*x^2 + c^3)*arctan(a*x)), x)
```

## Sympy [F]

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \int \frac{x^2}{(c(a^2x^2+1))^{\frac{5}{2}}\operatorname{atan}(ax)}\,dx$$

input
```
integrate(x**2/(a**2*c*x**2+c)**(5/2)/atan(a*x),x)
```

output
```
Integral(x**2/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)), x)
```

## Maxima [F]

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \int \frac{x^2}{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(x^2/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)), x)
```

## Giac [F]

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \int \frac{x^2}{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="giac")
```

output
```
integrate(x^2/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \int \frac{x^2}{\operatorname{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^{5/2}}\,dx$$

input
```
int(x^2/(atan(a*x)*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(x^2/(atan(a*x)*(c + a^2*c*x^2)^(5/2)), x)
```

## Reduce [F]

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \frac{\int \frac{x^2}{\sqrt{a^2x^2+1}\,atan(ax)a^4x^4+2\sqrt{a^2x^2+1}\,atan(ax)a^2x^2+\sqrt{a^2x^2+1}\,atan(ax)}dx}{\sqrt{c}\,c^2}$$

input
```
int(x^2/(a^2*c*x^2+c)^(5/2)/atan(a*x),x)
```

output
```
int(x**2/(sqrt(a**2*x**2 + 1)*atan(a*x)*a**4*x**4 + 2*sqrt(a**2*x**2 + 1)*
atan(a*x)*a**2*x**2 + sqrt(a**2*x**2 + 1)*atan(a*x)),x)/(sqrt(c)*c**2)
```

# 3.517 $\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 87

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx=\frac{\sqrt{1+a^2x^2}\,\mathrm{Si}(\arctan(ax))}{4a^2c^2\sqrt{c+a^2cx^2}}+\frac{\sqrt{1+a^2x^2}\,\mathrm{Si}(3\arctan(ax))}{4a^2c^2\sqrt{c+a^2cx^2}}$$

output
```
1/4*(a^2*x^2+1)^(1/2)*Si(arctan(a*x))/a^2/c^2/(a^2*c*x^2+c)^(1/2)+1/4*(a^2
*x^2+1)^(1/2)*Si(3*arctan(a*x))/a^2/c^2/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.59

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx=\frac{(1+a^2x^2)^{3/2}\,(\mathrm{Si}(\arctan(ax))+\mathrm{Si}(3\arctan(ax)))}{4a^2c\,(c\,(1+a^2x^2))^{3/2}}$$

input
```
Integrate[x/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]),x]
```

output
```
((1 + a^2*x^2)^(3/2)*(SinIntegral[ArcTan[a*x]] + SinIntegral[3*ArcTan[a*x]
]))/(4*a^2*c*(c*(1 + a^2*x^2))^(3/2))
```

## Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.63, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {5506, 5505, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}dx$$

$$\Big\downarrow \text{5506}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{x}{\left(a^2x^2+1\right)^{5/2}\arctan(ax)}dx}{c^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5505}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{ax}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{4906}$$

$$\frac{\sqrt{a^2x^2+1}\int \left(\frac{ax}{4\sqrt{a^2x^2+1}\arctan(ax)}+\frac{\sin(3\arctan(ax))}{4\arctan(ax)}\right)d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2x^2+1}\left(\frac{1}{4}\text{Si}(\arctan(ax))+\frac{1}{4}\text{Si}(3\arctan(ax))\right)}{a^2c^2\sqrt{a^2cx^2+c}}$$

input
```
Int[x/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]),x]
```

output
```
(Sqrt[1 + a^2*x^2]*(SinIntegral[ArcTan[a*x]]/4 + SinIntegral[3*ArcTan[a*x]
]/4))/(a^2*c^2*Sqrt[c + a^2*c*x^2])
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.51 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.44

| method | result | size |
|---|---|---|
| default | $-\frac{\operatorname{csgn}\left(\arctan\left(a x\right)\right) \pi \sqrt{c\left(a x-i\right)\left(a x+i\right)}}{4 \sqrt{a^{2} x^{2}+1}\, a^{2} c^{3}}+\frac{\operatorname{Si}\left(3 \arctan\left(a x\right)\right) \sqrt{c\left(a x-i\right)\left(a x+i\right)}}{4 \sqrt{a^{2} x^{2}+1}\, a^{2} c^{3}}+\frac{\operatorname{Si}\left(\arctan\left(a x\right)\right) \sqrt{c\left(a x-i\right)\left(a x+i\right)}}{4 \sqrt{a^{2} x^{2}+1}\, a^{2} c^{3}}$ | 125 |

input
```
int(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
-1/4*csgn(arctan(a*x))*Pi*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/2)/a^2/
c^3+1/4*Si(3*arctan(a*x))*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/2)/a^2/
c^3+1/4*Si(arctan(a*x))*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/2)/a^2/c^
3
```

## Fricas [F]

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \int \frac{x}{(a^2cx^2+c)^{\frac{5}{2}}\arctan{(ax)}}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x
^2 + c^3)*arctan(a*x)), x)
```

## Sympy [F]

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \int \frac{x}{(c(a^2x^2+1))^{\frac{5}{2}}\operatorname{atan}{(ax)}}\,dx$$

input
```
integrate(x/(a**2*c*x**2+c)**(5/2)/atan(a*x),x)
```

output
```
Integral(x/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)), x)
```

## Maxima [F]

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \int \frac{x}{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(x/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \int \frac{x}{\mathrm{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^{5/2}}\,dx$$

input
```
int(x/(atan(a*x)*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(x/(atan(a*x)*(c + a^2*c*x^2)^(5/2)), x)
```

## Reduce [F]

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \frac{\int \frac{x}{\sqrt{a^2x^2+1}\,atan(ax)a^4x^4+2\sqrt{a^2x^2+1}\,atan(ax)a^2x^2+\sqrt{a^2x^2+1}\,atan(ax)}dx}{\sqrt{c}\,c^2}$$

input
```
int(x/(a^2*c*x^2+c)^(5/2)/atan(a*x),x)
```

output
```
int(x/(sqrt(a**2*x**2 + 1)*atan(a*x)*a**4*x**4 + 2*sqrt(a**2*x**2 + 1)*ata
n(a*x)*a**2*x**2 + sqrt(a**2*x**2 + 1)*atan(a*x)),x)/(sqrt(c)*c**2)
```

# 3.518 $\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 87

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \frac{3\sqrt{1+a^2x^2}\,\text{CosIntegral}(\arctan(ax))}{4ac^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+a^2x^2}\,\text{CosIntegral}(3\arctan(ax))}{4ac^2\sqrt{c+a^2cx^2}}$$

output
```
3/4*(a^2*x^2+1)^(1/2)*Ci(arctan(a*x))/a/c^2/(a^2*c*x^2+c)^(1/2)+1/4*(a^2*x
^2+1)^(1/2)*Ci(3*arctan(a*x))/a/c^2/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.61

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \frac{(1+a^2x^2)^{3/2}\,(3\,\text{CosIntegral}(\arctan(ax))+\text{CosIntegral}(3\arctan(ax)))}{4ac\,(c\,(1+a^2x^2))^{3/2}}$$

input
```
Integrate[1/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]),x]
```

output
```
((1 + a^2*x^2)^(3/2)*(3*CosIntegral[ArcTan[a*x]] + CosIntegral[3*ArcTan[a*
x]]))/(4*a*c*(c*(1 + a^2*x^2))^(3/2))
```

## Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.63, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5440, 5439, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arctan(ax)\left(a^2cx^2+c\right)^{5/2}}dx$$

$$\Big\downarrow \text{5440}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{1}{\left(a^2x^2+1\right)^{5/2}\arctan(ax)}dx}{c^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5439}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{1}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^3}{\arctan(ax)}d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3793}$$

$$\frac{\sqrt{a^2x^2+1}\int \left(\frac{\cos(3\arctan(ax))}{4\arctan(ax)}+\frac{3}{4\sqrt{a^2x^2+1}\arctan(ax)}\right)d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2x^2+1}\left(\frac{3}{4}\text{CosIntegral}(\arctan(ax))+\frac{1}{4}\text{CosIntegral}(3\arctan(ax))\right)}{ac^2\sqrt{a^2cx^2+c}}$$

input
```
Int[1/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]),x]
```

output
```
(Sqrt[1 + a^2*x^2]*((3*CosIntegral[ArcTan[a*x]])/4 + CosIntegral[3*ArcTan[
a*x]]/4))/(a*c^2*Sqrt[c + a^2*c*x^2])
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5440
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])   Int[(1 +
c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.76 (sec) , antiderivative size = 179, normalized size of antiderivative = 2.06

| method | result |
|---|---|
| default | $-\frac{i \,\mathrm{csgn}(\arctan(ax))\,\mathrm{csgn}(i\arctan(ax))\pi\sqrt{c(ax-i)(ax+i)}}{2\sqrt{a^2x^2+1}\,a\,c^3}+\frac{i\,\mathrm{csgn}(i\arctan(ax))\pi\sqrt{c(ax-i)(ax+i)}}{2\sqrt{a^2x^2+1}\,a\,c^3}+\frac{\mathrm{Ci}(3\arctan(ax))\sqrt{c(ax}}{4\sqrt{a^2x^2+1}\,a\,c}$ |

input
```
int(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*csgn(arctan(a*x))*csgn(I*arctan(a*x))*Pi*(c*(a*x-I)*(a*x+I))^(1/2)/
(a^2*x^2+1)^(1/2)/a/c^3+1/2*I*csgn(I*arctan(a*x))*Pi*(c*(a*x-I)*(a*x+I))^(
1/2)/(a^2*x^2+1)^(1/2)/a/c^3+1/4*Ci(3*arctan(a*x))*(c*(a*x-I)*(a*x+I))^(1/
2)/(a^2*x^2+1)^(1/2)/a/c^3+3/4*Ci(arctan(a*x))*(c*(a*x-I)*(a*x+I))^(1/2)/(
a^2*x^2+1)^(1/2)/a/c^3
```

## Fricas [F]

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \int \frac{1}{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2
 + c^3)*arctan(a*x)), x)
```

## Sympy [F]

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \int \frac{1}{(c(a^2x^2+1))^{\frac{5}{2}}\operatorname{atan}(ax)}\,dx$$

input
```
integrate(1/(a**2*c*x**2+c)**(5/2)/atan(a*x),x)
```

output
```
Integral(1/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)), x)
```

## Maxima [F]

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \int \frac{1}{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)), x)
```

## Giac [F]

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \int \frac{1}{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \int \frac{1}{\mathrm{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^{5/2}}\,dx$$

input
```
int(1/(atan(a*x)*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(1/(atan(a*x)*(c + a^2*c*x^2)^(5/2)), x)
```

## Reduce [F]

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \frac{\int \frac{1}{\sqrt{a^2x^2+1}\,atan(ax)a^4x^4+2\sqrt{a^2x^2+1}\,atan(ax)a^2x^2+\sqrt{a^2x^2+1}\,atan(ax)}dx}{\sqrt{c}\,c^2}$$

input
```
int(1/(a^2*c*x^2+c)^(5/2)/atan(a*x),x)
```

output
```
int(1/(sqrt(a**2*x**2 + 1)*atan(a*x)*a**4*x**4 + 2*sqrt(a**2*x**2 + 1)*ata
n(a*x)*a**2*x**2 + sqrt(a**2*x**2 + 1)*atan(a*x)),x)/(sqrt(c)*c**2)
```

# 3.519 $\int \frac{1}{x(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)}\,dx=\operatorname{Int}\left(\frac{1}{x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)},x\right)$$

output
```
Defer(Int)(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.52 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)}\,dx=\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)}\,dx$$

input
```
Integrate[1/(x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]),x]
```

output
```
Integrate[1/(x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x \arctan(ax) \left(a^2cx^2+c\right)^{5/2}} \, dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int \frac{1}{x \arctan(ax) \left(a^2cx^2+c\right)^{5/2}}dx$$

input
```
Int[1/(x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 3.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x \left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}} \arctan \left(a x \right)}d x$$

input
```
int(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

output
```
int(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.54

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)}\,dx=\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{5}{2}}x\arctan\left(ax\right)}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)/((a^6*c^3*x^7 + 3*a^4*c^3*x^5 + 3*a^2*c^3*x^3
 + c^3*x)*arctan(a*x)), x)
```

## Sympy [N/A]

Not integrable

Time = 7.68 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)}\,dx=\int \frac{1}{x\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}\operatorname{atan}\left(ax\right)}\,dx$$

input
```
integrate(1/x/(a**2*c*x**2+c)**(5/2)/atan(a*x),x)
```

output
```
Integral(1/(x*(c*(a**2*x**2 + 1))**(5/2)*atan(a*x)), x)
```

## Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)}\,dx=\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{5}{2}}x\arctan\left(ax\right)}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(5/2)*x*arctan(a*x)), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)}\,dx = \int \frac{1}{x\,\mathrm{atan}\left(a\,x\right)\,\left(c\,a^2\,x^2+c\right)^{5/2}}\,dx$$

input
```
int(1/(x*atan(a*x)*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(1/(x*atan(a*x)*(c + a^2*c*x^2)^(5/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 72, normalized size of antiderivative = 3.00

$$\int \frac{1}{x\,(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \frac{\int \frac{1}{\sqrt{a^2x^2+1}\,atan(ax)a^4x^5+2\sqrt{a^2x^2+1}\,atan(ax)a^2x^3+\sqrt{a^2x^2+1}\,atan(ax)x}dx}{\sqrt{c}\,c^2}$$

input
```
int(1/x/(a^2*c*x^2+c)^(5/2)/atan(a*x),x)
```

output
```
int(1/(sqrt(a**2*x**2 + 1)*atan(a*x)*a**4*x**5 + 2*sqrt(a**2*x**2 + 1)*ata
n(a*x)*a**2*x**3 + sqrt(a**2*x**2 + 1)*atan(a*x)*x),x)/(sqrt(c)*c**2)
```

# 3.520 $\int \frac{1}{x^2\left(c+a^2cx^2\right)^{5/2}\arctan(ax)}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^{5/2}\arctan(ax)}\,dx=\text{Int}\left(\frac{1}{x^2\left(c+a^2cx^2\right)^{5/2}\arctan(ax)},x\right)$$

output
```
Defer(Int)(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.75 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^{5/2}\arctan(ax)}\,dx=\int \frac{1}{x^2\left(c+a^2cx^2\right)^{5/2}\arctan(ax)}\,dx$$

input
```
Integrate[1/(x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]),x]
```

output
```
Integrate[1/(x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \arctan(ax) \left(a^2cx^2+c\right)^{5/2}} \, dx$$

$$\Big\downarrow \; 5560$$

$$\int \frac{1}{x^2 \arctan(ax) \left(a^2cx^2+c\right)^{5/2}}dx$$

input
```
Int[1/(x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 3.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^2 \left(a^2 c\,x^2+c\right)^{\frac{5}{2}} \arctan\left(ax\right)}dx$$

input
```
int(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

output
```
int(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.62

$$\int \frac{1}{x^2 (c+a^2cx^2)^{5/2} \arctan(ax)} \, dx = \int \frac{1}{(a^2cx^2+c)^{\frac{5}{2}} x^2 \arctan(ax)} \, dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)/((a^6*c^3*x^8 + 3*a^4*c^3*x^6 + 3*a^2*c^3*x^4
 + c^3*x^2)*arctan(a*x)), x)
```

## Sympy [N/A]

Not integrable

Time = 10.76 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (c+a^2cx^2)^{5/2} \arctan(ax)} \, dx = \int \frac{1}{x^2 (c(a^2x^2+1))^{\frac{5}{2}} \operatorname{atan}(ax)} \, dx$$

input
```
integrate(1/x**2/(a**2*c*x**2+c)**(5/2)/atan(a*x),x)
```

output
```
Integral(1/(x**2*(c*(a**2*x**2 + 1))**(5/2)*atan(a*x)), x)
```

## Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (c+a^2cx^2)^{5/2} \arctan(ax)} \, dx = \int \frac{1}{(a^2cx^2+c)^{\frac{5}{2}} x^2 \arctan(ax)} \, dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(5/2)*x^2*arctan(a*x)), x)
```

## Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (c+a^2cx^2)^{5/2} \arctan(ax)} \, dx = \int \frac{1}{(a^2cx^2+c)^{\frac{5}{2}} x^2 \arctan(ax)} \, dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(5/2)*x^2*arctan(a*x)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (c+a^2cx^2)^{5/2} \arctan(ax)} \, dx = \int \frac{1}{x^2 \operatorname{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^{5/2}} \, dx$$

input
```
int(1/(x^2*atan(a*x)*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(1/(x^2*atan(a*x)*(c + a^2*c*x^2)^(5/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 74, normalized size of antiderivative = 3.08

$$\int \frac{1}{x^2 (c+a^2cx^2)^{5/2} \arctan(ax)} \, dx = \frac{\int \frac{1}{\sqrt{a^2x^2+1}\, atan(ax) a^4x^6 + 2\sqrt{a^2x^2+1}\, atan(ax) a^2x^4 + \sqrt{a^2x^2+1}\, atan(ax) x^2} dx}{\sqrt{c}\, c^2}$$

input

```
int(1/x^2/(a^2*c*x^2+c)^(5/2)/atan(a*x),x)
```

output

```
int(1/(sqrt(a**2*x**2 + 1)*atan(a*x)*a**4*x**6 + 2*sqrt(a**2*x**2 + 1)*ata
n(a*x)*a**2*x**4 + sqrt(a**2*x**2 + 1)*atan(a*x)*x**2),x)/(sqrt(c)*c**2)
```

# 3.521 $\int \frac{x^m(c+a^2cx^2)^3}{\arctan(ax)}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^m(c+a^2cx^2)^3}{\arctan(ax)}\,dx = \text{Int}\left(\frac{x^m(c+a^2cx^2)^3}{\arctan(ax)},x\right)$$

output
```
Defer(Int)(x^m*(a^2*c*x^2+c)^3/arctan(a*x),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m(c+a^2cx^2)^3}{\arctan(ax)}\,dx = \int \frac{x^m(c+a^2cx^2)^3}{\arctan(ax)}\,dx$$

input
```
Integrate[(x^m*(c + a^2*c*x^2)^3)/ArcTan[a*x],x]
```

output
```
Integrate[(x^m*(c + a^2*c*x^2)^3)/ArcTan[a*x], x]
```

## Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m (a^2cx^2+c)^3}{\arctan(ax)}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x^m (a^2cx^2+c)^3}{\arctan(ax)}dx$$

input
```
Int[(x^m*(c + a^2*c*x^2)^3)/ArcTan[a*x],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 5.88 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m (a^2c\,x^2+c)^3}{\arctan(ax)}dx$$

input
```
int(x^m*(a^2*c*x^2+c)^3/arctan(a*x),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^3/arctan(a*x),x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.18

$$\int \frac{x^m (c + a^2 c x^2)^3}{\arctan(ax)}\,dx = \int \frac{(a^2 c x^2 + c)^3 x^m}{\arctan(ax)}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="fricas")
```

output
```
integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*x^m/arctan(a*
x), x)
```

## Sympy [N/A]

Not integrable

Time = 10.01 (sec) , antiderivative size = 66, normalized size of antiderivative = 3.00

$$\int \frac{x^m (c + a^2 c x^2)^3}{\arctan(ax)}\,dx = c^3 \left( \int \frac{x^m}{\operatorname{atan}(ax)}\,dx + \int \frac{3a^2 x^2 x^m}{\operatorname{atan}(ax)}\,dx + \int \frac{3a^4 x^4 x^m}{\operatorname{atan}(ax)}\,dx + \int \frac{a^6 x^6 x^m}{\operatorname{atan}(ax)}\,dx \right)$$

input
```
integrate(x**m*(a**2*c*x**2+c)**3/atan(a*x),x)
```

output
```
c**3*(Integral(x**m/atan(a*x), x) + Integral(3*a**2*x**2*x**m/atan(a*x), x
) + Integral(3*a**4*x**4*x**m/atan(a*x), x) + Integral(a**6*x**6*x**m/atan
(a*x), x))
```

## Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m(c+a^2cx^2)^3}{\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^3x^m}{\arctan(ax)}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^3*x^m/arctan(a*x), x)
```

## Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m(c+a^2cx^2)^3}{\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^3x^m}{\arctan(ax)}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^3*x^m/arctan(a*x), x)
```

## Mupad [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m(c+a^2cx^2)^3}{\arctan(ax)}\,dx = \int \frac{x^m\,(c\,a^2\,x^2+c)^3}{\mathrm{atan}\,(a\,x)}\,dx$$

input
```
int((x^m*(c + a^2*c*x^2)^3)/atan(a*x),x)
```

output
```
int((x^m*(c + a^2*c*x^2)^3)/atan(a*x), x)
```

## Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 76, normalized size of antiderivative = 3.45

$$\int \frac{x^m(c+a^2cx^2)^3}{\arctan(ax)}\,dx = c^3\left(\int \frac{x^m}{atan\,(ax)}dx + \left(\int \frac{x^m x^6}{atan\,(ax)}dx\right)a^6 + 3\left(\int \frac{x^m x^4}{atan\,(ax)}dx\right)a^4 + 3\left(\int \frac{x^m x^2}{atan\,(ax)}dx\right)a^2\right)$$

input
```
int(x^m*(a^2*c*x^2+c)^3/atan(a*x),x)
```

output
```
c**3*(int(x**m/atan(a*x),x) + int((x**m*x**6)/atan(a*x),x)*a**6 + 3*int((x
**m*x**4)/atan(a*x),x)*a**4 + 3*int((x**m*x**2)/atan(a*x),x)*a**2)
```

# 3.522 $\int \frac{x^m (c+a^2 c x^2)^2}{\arctan (a x)} \, dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^m (c+a^2 c x^2)^2}{\arctan (a x)} \, dx = \text{Int}\left(\frac{x^m (c+a^2 c x^2)^2}{\arctan (a x)},x\right)$$

output
```
Defer(Int)(x^m*(a^2*c*x^2+c)^2/arctan(a*x),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m (c+a^2 c x^2)^2}{\arctan (a x)} \, dx = \int \frac{x^m (c+a^2 c x^2)^2}{\arctan (a x)} \, dx$$

input
```
Integrate[(x^m*(c + a^2*c*x^2)^2)/ArcTan[a*x],x]
```

output
```
Integrate[(x^m*(c + a^2*c*x^2)^2)/ArcTan[a*x], x]
```

## Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m\left(a^2cx^2+c\right)^2}{\arctan(ax)}dx$$

$$\Big\downarrow \text{5560}$$

$$\int \frac{x^m\left(a^2cx^2+c\right)^2}{\arctan(ax)}dx$$

input
```
Int[(x^m*(c + a^2*c*x^2)^2)/ArcTan[a*x],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 5.47 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m\left(a^2c\,x^2+c\right)^2}{\arctan\left(ax\right)}dx$$

input
```
int(x^m*(a^2*c*x^2+c)^2/arctan(a*x),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^2/arctan(a*x),x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

$$\int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^2x^m}{\arctan(ax)}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*x^m/arctan(a*x), x)
```

## Sympy [N/A]

Not integrable

Time = 4.63 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.18

$$\int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)}\,dx = c^2\left(\int \frac{x^m}{\operatorname{atan}(ax)}\,dx + \int \frac{2a^2x^2x^m}{\operatorname{atan}(ax)}\,dx + \int \frac{a^4x^4x^m}{\operatorname{atan}(ax)}\,dx\right)$$

input
```
integrate(x**m*(a**2*c*x**2+c)**2/atan(a*x),x)
```

output
```
c**2*(Integral(x**m/atan(a*x), x) + Integral(2*a**2*x**2*x**m/atan(a*x), x
) + Integral(a**4*x**4*x**m/atan(a*x), x))
```

## Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^2x^m}{\arctan(ax)}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^2*x^m/arctan(a*x), x)
```

## Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^2x^m}{\arctan(ax)}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^2*x^m/arctan(a*x), x)
```

## Mupad [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)}\,dx = \int \frac{x^m\,(c\,a^2\,x^2+c)^2}{\mathrm{atan}\,(a\,x)}\,dx$$

input
```
int((x^m*(c + a^2*c*x^2)^2)/atan(a*x),x)
```

output
```
int((x^m*(c + a^2*c*x^2)^2)/atan(a*x), x)
```

## Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.55

$$\int \frac{x^m (c+a^2 c x^2)^2}{\arctan(ax)} \, dx = c^2 \left( \int \frac{x^m}{atan\,(ax)} dx + \left( \int \frac{x^m x^4}{atan\,(ax)} dx \right) a^4 + 2 \left( \int \frac{x^m x^2}{atan\,(ax)} dx \right) a^2 \right)$$

input
```
int(x^m*(a^2*c*x^2+c)^2/atan(a*x),x)
```

output
```
c**2*(int(x**m/atan(a*x),x) + int((x**m*x**4)/atan(a*x),x)*a**4 + 2*int((x
**m*x**2)/atan(a*x),x)*a**2)
```

# 3.523 $\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)}\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)}\,dx = \text{Int}\left(\frac{x^m(c+a^2cx^2)}{\arctan(ax)},x\right)$$

output
```
Defer(Int)(x^m*(a^2*c*x^2+c)/arctan(a*x),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)}\,dx = \int \frac{x^m(c+a^2cx^2)}{\arctan(ax)}\,dx$$

input
```
Integrate[(x^m*(c + a^2*c*x^2))/ArcTan[a*x],x]
```

output
```
Integrate[(x^m*(c + a^2*c*x^2))/ArcTan[a*x], x]
```

## Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m (a^2 c x^2 + c)}{\arctan(ax)} \, dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{x^m (a^2 c x^2 + c)}{\arctan(ax)} dx$$

input
```
Int[(x^m*(c + a^2*c*x^2))/ArcTan[a*x],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 7.83 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{x^m (a^2 c\, x^2 + c)}{\arctan(ax)} dx$$

input
```
int(x^m*(a^2*c*x^2+c)/arctan(a*x),x)
```

output
```
int(x^m*(a^2*c*x^2+c)/arctan(a*x),x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)}\,dx=\int \frac{(a^2cx^2+c)x^m}{\arctan(ax)}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)/arctan(a*x),x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)*x^m/arctan(a*x), x)
```

## Sympy [N/A]

Not integrable

Time = 2.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.35

$$\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)}\,dx=c\left(\int \frac{x^m}{\operatorname{atan}(ax)}\,dx+\int \frac{a^2x^2x^m}{\operatorname{atan}(ax)}\,dx\right)$$

input
```
integrate(x**m*(a**2*c*x**2+c)/atan(a*x),x)
```

output
```
c*(Integral(x**m/atan(a*x), x) + Integral(a**2*x**2*x**m/atan(a*x), x))
```

## Maxima [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)}\,dx=\int \frac{(a^2cx^2+c)x^m}{\arctan(ax)}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)*x^m/arctan(a*x), x)
```

## Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)}\,dx=\int \frac{(a^2cx^2+c)x^m}{\arctan(ax)}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)/arctan(a*x),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)*x^m/arctan(a*x), x)
```

## Mupad [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)}\,dx=\int \frac{x^m\,(c\,a^2\,x^2+c)}{\mathrm{atan}\,(a\,x)}\,dx$$

input
```
int((x^m*(c + a^2*c*x^2))/atan(a*x),x)
```

output
```
int((x^m*(c + a^2*c*x^2))/atan(a*x), x)
```

## Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.70

$$\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)}\,dx = c\left(\int \frac{x^m}{atan\,(ax)}dx + \left(\int \frac{x^m x^2}{atan\,(ax)}dx\right)a^2\right)$$

input
```
int(x^m*(a^2*c*x^2+c)/atan(a*x),x)
```

output
```
c*(int(x**m/atan(a*x),x) + int((x**m*x**2)/atan(a*x),x)*a**2)
```

# 3.524 $\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)}\,dx = \text{Int}\left(\frac{x^m}{(c+a^2cx^2)\arctan(ax)},x\right)$$

output
```
Defer(Int)(x^m/(a^2*c*x^2+c)/arctan(a*x),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)}\,dx = \int \frac{x^m}{(c+a^2cx^2)\arctan(ax)}\,dx$$

input
```
Integrate[x^m/((c + a^2*c*x^2)*ArcTan[a*x]),x]
```

output
```
Integrate[x^m/((c + a^2*c*x^2)*ArcTan[a*x]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\arctan(ax)\,(a^2cx^2+c)}\,dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x^m}{\arctan(ax)\,(a^2cx^2+c)}dx$$

input
```
Int[x^m/((c + a^2*c*x^2)*ArcTan[a*x]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 2.86 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(a^2c\,x^2+c)\arctan\,(ax)}dx$$

input
```
int(x^m/(a^2*c*x^2+c)/arctan(a*x),x)
```

output
```
int(x^m/(a^2*c*x^2+c)/arctan(a*x),x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)}\,dx = \int \frac{x^m}{(a^2cx^2+c)\arctan{(ax)}}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(x^m/((a^2*c*x^2 + c)*arctan(a*x)), x)
```

## Sympy [N/A]

Not integrable

Time = 1.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)}\,dx = \frac{\int \frac{x^m}{a^2x^2\operatorname{atan}{(ax)}+\operatorname{atan}{(ax)}}\,dx}{c}$$

input
```
integrate(x**m/(a**2*c*x**2+c)/atan(a*x),x)
```

output
```
Integral(x**m/(a**2*x**2*atan(a*x) + atan(a*x)), x)/c
```

## Maxima [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)}\,dx = \int \frac{x^m}{(a^2cx^2+c)\arctan{(ax)}}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(x^m/((a^2*c*x^2 + c)*arctan(a*x)), x)
```

## Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)}\,dx = \int \frac{x^m}{(a^2cx^2+c)\arctan(ax)}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)/arctan(a*x),x, algorithm="giac")
```

output
```
integrate(x^m/((a^2*c*x^2 + c)*arctan(a*x)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)}\,dx = \int \frac{x^m}{\mathrm{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)}\,dx$$

input
```
int(x^m/(atan(a*x)*(c + a^2*c*x^2)),x)
```

output
```
int(x^m/(atan(a*x)*(c + a^2*c*x^2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.27

$$\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)}\,dx = \frac{\int \frac{x^m}{atan(ax)a^2x^2+atan(ax)}dx}{c}$$

input
```
int(x^m/(a^2*c*x^2+c)/atan(a*x),x)
```

output
```
int(x**m/(atan(a*x)*a**2*x**2 + atan(a*x)),x)/c
```

# 3.525 $\int \frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)}\,dx = \text{Int}\left(\frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)},x\right)$$

output
```
Defer(Int)(x^m/(a^2*c*x^2+c)^2/arctan(a*x),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)}\,dx = \int \frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)}\,dx$$

input
```
Integrate[x^m/((c + a^2*c*x^2)^2*ArcTan[a*x]),x]
```

output
```
Integrate[x^m/((c + a^2*c*x^2)^2*ArcTan[a*x]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\arctan(ax)\left(a^2cx^2+c\right)^2}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x^m}{\arctan(ax)\left(a^2cx^2+c\right)^2}dx$$

input
```
Int[x^m/((c + a^2*c*x^2)^2*ArcTan[a*x]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 4.82 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\left(a^2c\,x^2+c\right)^2\arctan\left(ax\right)}dx$$

input
```
int(x^m/(a^2*c*x^2+c)^2/arctan(a*x),x)
```

output
```
int(x^m/(a^2*c*x^2+c)^2/arctan(a*x),x)
```

## Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.77

$$\int \frac{x^m}{(c+a^2cx^2)^2\arctan(ax)}\,dx = \int \frac{x^m}{(a^2cx^2+c)^2\arctan(ax)}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(x^m/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)), x)
```

## Sympy [N/A]

Not integrable

Time = 3.49 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

$$\int \frac{x^m}{(c+a^2cx^2)^2\arctan(ax)}\,dx = \frac{\int \frac{x^m}{a^4x^4\operatorname{atan}(ax)+2a^2x^2\operatorname{atan}(ax)+\operatorname{atan}(ax)}\,dx}{c^2}$$

input
```
integrate(x**m/(a**2*c*x**2+c)**2/atan(a*x),x)
```

output
```
Integral(x**m/(a**4*x**4*atan(a*x) + 2*a**2*x**2*atan(a*x) + atan(a*x)), x
)/c**2
```

## Maxima [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m}{(c+a^2cx^2)^2\arctan(ax)}\,dx = \int \frac{x^m}{(a^2cx^2+c)^2\arctan(ax)}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(x^m/((a^2*c*x^2 + c)^2*arctan(a*x)), x)
```

## Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m}{(c+a^2cx^2)^2\arctan(ax)}\,dx=\int \frac{x^m}{(a^2cx^2+c)^2\arctan{(ax)}}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^2/arctan(a*x),x, algorithm="giac")
```

output
```
integrate(x^m/((a^2*c*x^2 + c)^2*arctan(a*x)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m}{(c+a^2cx^2)^2\arctan(ax)}\,dx=\int \frac{x^m}{\mathrm{atan}\left(a\,x\right)\,{\left(c\,a^2\,x^2+c\right)}^2}\,dx$$

input
```
int(x^m/(atan(a*x)*(c + a^2*c*x^2)^2),x)
```

output
```
int(x^m/(atan(a*x)*(c + a^2*c*x^2)^2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.82

$$\int \frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)} \, dx = \frac{\int \frac{x^m}{atan(ax)a^4x^4+2atan(ax)a^2x^2+atan(ax)} dx}{c^2}$$

input
```
int(x^m/(a^2*c*x^2+c)^2/atan(a*x),x)
```

output
```
int(x**m/(atan(a*x)*a**4*x**4 + 2*atan(a*x)*a**2*x**2 + atan(a*x)),x)/c**2
```

# 3.526 $\int \frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)} \, dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)} \, dx = \text{Int}\left(\frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)}, x\right)$$

output
```
Defer(Int)(x^m/(a^2*c*x^2+c)^3/arctan(a*x),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)} \, dx = \int \frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)} \, dx$$

input
```
Integrate[x^m/((c + a^2*c*x^2)^3*ArcTan[a*x]),x]
```

output
```
Integrate[x^m/((c + a^2*c*x^2)^3*ArcTan[a*x]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\arctan(ax)\left(a^2cx^2+c\right)^3}\,dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{x^m}{\arctan(ax)\left(a^2cx^2+c\right)^3}dx$$

input
```
Int[x^m/((c + a^2*c*x^2)^3*ArcTan[a*x]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 4.81 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\left(a^2c\,x^2+c\right)^3\arctan\left(ax\right)}dx$$

input
```
int(x^m/(a^2*c*x^2+c)^3/arctan(a*x),x)
```

output
```
int(x^m/(a^2*c*x^2+c)^3/arctan(a*x),x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.27

$$\int \frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)} \, dx = \int \frac{x^m}{(a^2cx^2+c)^3 \arctan(ax)} \, dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(x^m/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a
*x)), x)
```

## Sympy [N/A]

Not integrable

Time = 8.69 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.32

$$\int \frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)} \, dx = \frac{\int \frac{x^m}{a^6x^6 \operatorname{atan}(ax)+3a^4x^4 \operatorname{atan}(ax)+3a^2x^2 \operatorname{atan}(ax)+\operatorname{atan}(ax)} \, dx}{c^3}$$

input
```
integrate(x**m/(a**2*c*x**2+c)**3/atan(a*x),x)
```

output
```
Integral(x**m/(a**6*x**6*atan(a*x) + 3*a**4*x**4*atan(a*x) + 3*a**2*x**2*a
tan(a*x) + atan(a*x)), x)/c**3
```

## Maxima [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m}{(c+a^2cx^2)^3\arctan(ax)}\,dx = \int \frac{x^m}{(a^2cx^2+c)^3\arctan(ax)}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(x^m/((a^2*c*x^2 + c)^3*arctan(a*x)), x)
```

## Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m}{(c+a^2cx^2)^3\arctan(ax)}\,dx = \int \frac{x^m}{(a^2cx^2+c)^3\arctan(ax)}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^3/arctan(a*x),x, algorithm="giac")
```

output
```
integrate(x^m/((a^2*c*x^2 + c)^3*arctan(a*x)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m}{(c+a^2cx^2)^3\arctan(ax)}\,dx = \int \frac{x^m}{\mathrm{atan}(a\,x)\,(c\,a^2\,x^2+c)^3}\,dx$$

input
```
int(x^m/(atan(a*x)*(c + a^2*c*x^2)^3),x)
```

output
```
int(x^m/(atan(a*x)*(c + a^2*c*x^2)^3), x)
```

## Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.36

$$\int \frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)}\,dx = \frac{\int \frac{x^m}{atan(ax)a^6x^6+3atan(ax)a^4x^4+3atan(ax)a^2x^2+atan(ax)}dx}{c^3}$$

input
```
int(x^m/(a^2*c*x^2+c)^3/atan(a*x),x)
```

output
```
int(x**m/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x
**2 + atan(a*x)),x)/c**3
```

# 3.527 $\int \frac{x^m(c+a^2cx^2)^{5/2}}{\arctan(ax)} dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\arctan(ax)} dx = \text{Int}\left(\frac{x^m(c+a^2cx^2)^{5/2}}{\arctan(ax)}, x\right)$$

output
```
Defer(Int)(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\arctan(ax)} dx = \int \frac{x^m(c+a^2cx^2)^{5/2}}{\arctan(ax)} dx$$

input
```
Integrate[(x^m*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x],x]
```

output
```
Integrate[(x^m*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x], x]
```

## Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \left(a^2 c x^2+c\right)^{5/2}}{\arctan (a x)} \, dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x^m \left(a^2 c x^2+c\right)^{5/2}}{\arctan (a x)}dx$$

input
```
Int[(x^m*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 7.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^{m} \left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}}}{\arctan \left(a x \right)}d x$$

input
```
int(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.04

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}x^m}{\arctan(ax)}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*x^m/arcta
n(a*x), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\arctan(ax)}\,dx = \text{Timed out}$$

input
```
integrate(x**m*(a**2*c*x**2+c)**(5/2)/atan(a*x),x)
```

output
```
Timed out
```

## Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}x^m}{\arctan(ax)}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)*x^m/arctan(a*x), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\arctan(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\arctan(ax)}\,dx = \int \frac{x^m\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}{\mathrm{atan}\left(a\,x\right)}\,dx$$

input
```
int((x^m*(c + a^2*c*x^2)^(5/2))/atan(a*x),x)
```

output
```
int((x^m*(c + a^2*c*x^2)^(5/2))/atan(a*x), x)
```

## Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 88, normalized size of antiderivative = 3.67

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\arctan(ax)}\,dx = \sqrt{c}\,c^2\left(\left(\int \frac{x^m\sqrt{a^2x^2+1}\,x^4}{atan\,(ax)}dx\right)a^4 + 2\left(\int \frac{x^m\sqrt{a^2x^2+1}\,x^2}{atan\,(ax)}dx\right)a^2 + \int \frac{x^m\sqrt{a^2x^2+1}}{atan\,(ax)}dx\right)$$

input
```
int(x^m*(a^2*c*x^2+c)^(5/2)/atan(a*x),x)
```

output
```
sqrt(c)*c**2*(int((x**m*sqrt(a**2*x**2 + 1)*x**4)/atan(a*x),x)*a**4 + 2*in
t((x**m*sqrt(a**2*x**2 + 1)*x**2)/atan(a*x),x)*a**2 + int((x**m*sqrt(a**2*
x**2 + 1))/atan(a*x),x))
```

# 3.528 $\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)}\,dx = \text{Int}\left(\frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)},x\right)$$

output
```
Defer(Int)(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)}\,dx = \int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)}\,dx$$

input
```
Integrate[(x^m*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x],x]
```

output
```
Integrate[(x^m*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x], x]
```

## Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \left(a^2 c x^2+c\right)^{3/2}}{\arctan (a x)} \, dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{x^m \left(a^2 c x^2+c\right)^{3/2}}{\arctan (a x)}dx$$

input
```
Int[(x^m*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 7.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^{m} \left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}{\arctan \left(a x \right)}d x$$

input
```
int(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}x^m}{\arctan(ax)}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^(3/2)*x^m/arctan(a*x), x)
```

## Sympy [N/A]

Not integrable

Time = 92.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)}\,dx = \int \frac{x^m(c(a^2x^2+1))^{\frac{3}{2}}}{\operatorname{atan}(ax)}\,dx$$

input
```
integrate(x**m*(a**2*c*x**2+c)**(3/2)/atan(a*x),x)
```

output
```
Integral(x**m*(c*(a**2*x**2 + 1))**(3/2)/atan(a*x), x)
```

## Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}x^m}{\arctan(ax)}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*x^m/arctan(a*x), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)}\,dx = \int \frac{x^m\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}{\mathrm{atan}\left(a\,x\right)}\,dx$$

input
```
int((x^m*(c + a^2*c*x^2)^(3/2))/atan(a*x),x)
```

output
```
int((x^m*(c + a^2*c*x^2)^(3/2))/atan(a*x), x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.33

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)}\,dx = \sqrt{c}\,c\left(\left(\int \frac{x^m\sqrt{a^2x^2+1}\,x^2}{atan\,(ax)}dx\right)a^2+\int \frac{x^m\sqrt{a^2x^2+1}}{atan\,(ax)}dx\right)$$

input
```
int(x^m*(a^2*c*x^2+c)^(3/2)/atan(a*x),x)
```

output
```
sqrt(c)*c*(int((x**m*sqrt(a**2*x**2 + 1)*x**2)/atan(a*x),x)*a**2 + int((x*
*m*sqrt(a**2*x**2 + 1))/atan(a*x),x))
```

# 3.529 $\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)}\,dx=\text{Int}\left(\frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)},x\right)$$

output
```
Defer(Int)(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)}\,dx=\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)}\,dx$$

input
```
Integrate[(x^m*Sqrt[c + a^2*c*x^2])/ArcTan[a*x],x]
```

output
```
Integrate[(x^m*Sqrt[c + a^2*c*x^2])/ArcTan[a*x], x]
```

## Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m\sqrt{a^2cx^2+c}}{\arctan(ax)}dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int \frac{x^m\sqrt{a^2cx^2+c}}{\arctan(ax)}dx$$

input
```
Int[(x^m*Sqrt[c + a^2*c*x^2])/ArcTan[a*x],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 6.91 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m\sqrt{a^2c\,x^2+c}}{\arctan\left(ax\right)}dx$$

input
```
int(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x),x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)}\,dx = \int \frac{\sqrt{a^2cx^2+c}\,x^m}{\arctan(ax)}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^m/arctan(a*x), x)
```

## Sympy [N/A]

Not integrable

Time = 5.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)}\,dx = \int \frac{x^m\sqrt{c\,(a^2x^2+1)}}{\operatorname{atan}(ax)}\,dx$$

input
```
integrate(x**m*(a**2*c*x**2+c)**(1/2)/atan(a*x),x)
```

output
```
Integral(x**m*sqrt(c*(a**2*x**2 + 1))/atan(a*x), x)
```

## Maxima [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)}\,dx = \int \frac{\sqrt{a^2cx^2+c}\,x^m}{\arctan(ax)}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*x^m/arctan(a*x), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)}\,dx = \int \frac{x^m\,\sqrt{c\,a^2\,x^2+c}}{\mathrm{atan}\left(a\,x\right)} \,d x$$

input
```
int((x^m*(c + a^2*c*x^2)^(1/2))/atan(a*x),x)
```

output
```
int((x^m*(c + a^2*c*x^2)^(1/2))/atan(a*x), x)
```

## Reduce [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)}\,dx = \sqrt{c}\left(\int \frac{x^m\sqrt{a^2x^2+1}}{atan\,(ax)}dx\right)$$

input
```
int(x^m*(a^2*c*x^2+c)^(1/2)/atan(a*x),x)
```

output
```
sqrt(c)*int((x**m*sqrt(a**2*x**2 + 1))/atan(a*x),x)
```

# 3.530 $\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)}\,dx = \mathrm{Int}\left(\frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)},x\right)$$

output
```
Defer(Int)(x^m/(a^2*c*x^2+c)^(1/2)/arctan(a*x),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)}\,dx = \int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)}\,dx$$

input
```
Integrate[x^m/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]),x]
```

output
```
Integrate[x^m/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\arctan(ax)\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int \frac{x^m}{\arctan(ax)\sqrt{a^2cx^2+c}}dx$$

input
```
Int[x^m/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 6.82 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{\sqrt{a^2c\,x^2+c}\,\arctan\left(ax\right)}dx$$

input
```
int(x^m/(a^2*c*x^2+c)^(1/2)/arctan(a*x),x)
```

output
```
int(x^m/(a^2*c*x^2+c)^(1/2)/arctan(a*x),x)
```

## Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)}\,dx = \int \frac{x^m}{\sqrt{a^2cx^2+c}\arctan{(ax)}}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(1/2)/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(x^m/(sqrt(a^2*c*x^2 + c)*arctan(a*x)), x)
```

## Sympy [N/A]

Not integrable

Time = 6.47 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)}\,dx = \int \frac{x^m}{\sqrt{c\,(a^2x^2+1)}\,\mathrm{atan}\,(ax)}\,dx$$

input
```
integrate(x**m/(a**2*c*x**2+c)**(1/2)/atan(a*x),x)
```

output
```
Integral(x**m/(sqrt(c*(a**2*x**2 + 1))*atan(a*x)), x)
```

## Maxima [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)}\,dx = \int \frac{x^m}{\sqrt{a^2cx^2+c}\arctan{(ax)}}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(1/2)/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(x^m/(sqrt(a^2*c*x^2 + c)*arctan(a*x)), x)
```

## Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)}\,dx = \int \frac{x^m}{\sqrt{a^2cx^2+c}\arctan{(ax)}}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(1/2)/arctan(a*x),x, algorithm="giac")
```

output
```
integrate(x^m/(sqrt(a^2*c*x^2 + c)*arctan(a*x)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)}\,dx = \int \frac{x^m}{\mathrm{atan}\left(a\,x\right)\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(x^m/(atan(a*x)*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(x^m/(atan(a*x)*(c + a^2*c*x^2)^(1/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)}\,dx=\frac{\int \frac{x^m}{\sqrt{a^2x^2+1}\,atan(ax)}dx}{\sqrt{c}}$$

input
```
int(x^m/(a^2*c*x^2+c)^(1/2)/atan(a*x),x)
```

output
```
int(x**m/(sqrt(a**2*x**2 + 1)*atan(a*x)),x)/sqrt(c)
```

# 3.531 $\int \frac{x^m}{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)}\,dx=\text{Int}\left(\frac{x^m}{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)},x\right)$$

output
```
Defer(Int)(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)}\,dx=\int \frac{x^m}{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)}\,dx$$

input
```
Integrate[x^m/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]),x]
```

output
```
Integrate[x^m/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x^m}{\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}dx$$

input
```
Int[x^m/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 7.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{\left(a^2c\,x^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)}dx$$

input
```
int(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

output
```
int(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.12

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx = \int \frac{x^m}{(a^2cx^2+c)^{\frac{3}{2}}\arctan{(ax)}}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^m/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arct
an(a*x)), x)
```

## Sympy [N/A]

Not integrable

Time = 28.73 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx = \int \frac{x^m}{(c\,(a^2x^2+1))^{\frac{3}{2}}\operatorname{atan}{(ax)}}\,dx$$

input
```
integrate(x**m/(a**2*c*x**2+c)**(3/2)/atan(a*x),x)
```

output
```
Integral(x**m/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)), x)
```

## Maxima [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx = \int \frac{x^m}{(a^2cx^2+c)^{\frac{3}{2}}\arctan{(ax)}}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(x^m/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)), x)
```

## Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx = \int \frac{x^m}{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x),x, algorithm="giac")
```

output
```
integrate(x^m/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.85 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx = \int \frac{x^m}{\operatorname{atan}(a\,x)\,\left(c\,a^2\,x^2+c\right)^{3/2}}\,dx$$

input
```
int(x^m/(atan(a*x)*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(x^m/(atan(a*x)*(c + a^2*c*x^2)^(3/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.21

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)}\,dx = \frac{\int \frac{x^m}{\sqrt{a^2x^2+1}\,atan(ax)a^2x^2+\sqrt{a^2x^2+1}\,atan(ax)}dx}{\sqrt{c}\,c}$$

input
```
int(x^m/(a^2*c*x^2+c)^(3/2)/atan(a*x),x)
```

output
```
int(x**m/(sqrt(a**2*x**2 + 1)*atan(a*x)*a**2*x**2 + sqrt(a**2*x**2 + 1)*at
an(a*x)),x)/(sqrt(c)*c)
```

# 3.532 $\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \text{Int}\left(\frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)},x\right)$$

output
```
Defer(Int)(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx$$

input
```
Integrate[x^m/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]),x]
```

output
```
Integrate[x^m/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\arctan(ax) \left(a^2cx^2+c\right)^{5/2}} \, dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int \frac{x^m}{\arctan(ax) \left(a^2cx^2+c\right)^{5/2}}dx$$

input
```
Int[x^m/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 7.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{\left(a^2 c\,x^2+c\right)^{\frac{5}{2}} \arctan\left(a x\right)}dx$$

input
```
int(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

output
```
int(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x)
```

## Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.58

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \int \frac{x^m}{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^m/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3
*x^2 + c^3)*arctan(a*x)), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \text{Timed out}$$

input
```
integrate(x**m/(a**2*c*x**2+c)**(5/2)/atan(a*x),x)
```

output
```
Timed out
```

## Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \int \frac{x^m}{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="maxima")
```

output
```
integrate(x^m/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)), x)
```

## Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \int \frac{x^m}{(a^2cx^2+c)^{\frac{5}{2}}\arctan{(ax)}}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x),x, algorithm="giac")
```

output
```
integrate(x^m/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.87 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \int \frac{x^m}{\mathrm{atan}\left(a\,x\right)\,\left(c\,a^2\,x^2+c\right)^{5/2}}\,dx$$

input
```
int(x^m/(atan(a*x)*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(x^m/(atan(a*x)*(c + a^2*c*x^2)^(5/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 75, normalized size of antiderivative = 3.12

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)}\,dx = \frac{\int \frac{x^m}{\sqrt{a^2x^2+1}\,atan(ax)a^4x^4+2\sqrt{a^2x^2+1}\,atan(ax)a^2x^2+\sqrt{a^2x^2+1}\,atan(ax)}dx}{\sqrt{c}\,c^2}$$

input
```
int(x^m/(a^2*c*x^2+c)^(5/2)/atan(a*x),x)
```

output
```
int(x**m/(sqrt(a**2*x**2 + 1)*atan(a*x)*a**4*x**4 + 2*sqrt(a**2*x**2 + 1)*
atan(a*x)*a**2*x**2 + sqrt(a**2*x**2 + 1)*atan(a*x)),x)/(sqrt(c)*c**2)
```

# 3.533 $\int \frac{x(c+a^2cx^2)}{\arctan(ax)^2}\,dx$



## Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{x(c+a^2cx^2)}{\arctan(ax)^2}\,dx = \text{Int}\left(\frac{x(c+a^2cx^2)}{\arctan(ax)^2},x\right)$$

output
```
Defer(Int)(x*(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

## Mathematica [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x(c+a^2cx^2)}{\arctan(ax)^2}\,dx = \int \frac{x(c+a^2cx^2)}{\arctan(ax)^2}\,dx$$

input
```
Integrate[(x*(c + a^2*c*x^2))/ArcTan[a*x]^2,x]
```

output
```
Integrate[(x*(c + a^2*c*x^2))/ArcTan[a*x]^2, x]
```

## Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a^2cx^2+c)}{\arctan(ax)^2} \, dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{x(a^2cx^2+c)}{\arctan(ax)^2}dx$$

input
```
Int[(x*(c + a^2*c*x^2))/ArcTan[a*x]^2,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 43.96 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x(a^2c\,x^2+c)}{\arctan\left(ax\right)^2}dx$$

input
```
int(x*(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

output
```
int(x*(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.17

$$\int \frac{x(c+a^2cx^2)}{\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)x}{\arctan{(ax)}^2}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^3 + c*x)/arctan(a*x)^2, x)
```

## Sympy [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.44

$$\int \frac{x(c+a^2cx^2)}{\arctan(ax)^2}\,dx = c\left(\int \frac{x}{\operatorname{atan}^2{(ax)}}\,dx + \int \frac{a^2x^3}{\operatorname{atan}^2{(ax)}}\,dx\right)$$

input
```
integrate(x*(a**2*c*x**2+c)/atan(a*x)**2,x)
```

output
```
c*(Integral(x/atan(a*x)**2, x) + Integral(a**2*x**3/atan(a*x)**2, x))
```

## Maxima [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 67, normalized size of antiderivative = 3.72

$$\int \frac{x(c+a^2cx^2)}{\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)x}{\arctan{(ax)}^2}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-(a^4*c*x^5 + 2*a^2*c*x^3 + c*x - arctan(a*x)*integrate((5*a^4*c*x^4 + 6*a
^2*c*x^2 + c)/arctan(a*x), x))/(a*arctan(a*x))
```

## Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x(c+a^2cx^2)}{\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)x}{\arctan{(ax)}^2}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)*x/arctan(a*x)^2, x)
```

## Mupad [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x(c+a^2cx^2)}{\arctan(ax)^2}\,dx = \int \frac{x\,(c\,a^2\,x^2+c)}{\mathrm{atan}\left(a\,x\right)^2}\,dx$$

input
```
int((x*(c + a^2*c*x^2))/atan(a*x)^2,x)
```

output
```
int((x*(c + a^2*c*x^2))/atan(a*x)^2, x)
```

## Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.61

$$\int \frac{x(c+a^2cx^2)}{\arctan(ax)^2}\,dx = c\left(\left(\int \frac{x^3}{atan\,(ax)^2}dx\right)a^2 + \int \frac{x}{atan\,(ax)^2}dx\right)$$

input
```
int(x*(a^2*c*x^2+c)/atan(a*x)^2,x)
```

output
```
c*(int(x**3/atan(a*x)**2,x)*a**2 + int(x/atan(a*x)**2,x))
```

# 3.534 $\int \frac{c+a^2cx^2}{\arctan(ax)^2}\,dx$



## Optimal result

Integrand size = 17, antiderivative size = 17

$$\int \frac{c+a^2cx^2}{\arctan(ax)^2}\,dx=\text{Int}\left(\frac{c+a^2cx^2}{\arctan(ax)^2},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)/arctan(a*x)^2,x)
```

## Mathematica [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{c+a^2cx^2}{\arctan(ax)^2}\,dx=\int \frac{c+a^2cx^2}{\arctan(ax)^2}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)/ArcTan[a*x]^2,x]
```

output
```
Integrate[(c + a^2*c*x^2)/ArcTan[a*x]^2, x]
```

## Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2cx^2+c}{\arctan(ax)^2}\,dx$$

$$\Big\downarrow \; 5560$$

$$\int \frac{a^2cx^2+c}{\arctan(ax)^2}dx$$

input
```
Int[(c + a^2*c*x^2)/ArcTan[a*x]^2,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 51.48 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{a^2c\,x^2+c}{\arctan\left(ax\right)^2}dx$$

input
```
int((a^2*c*x^2+c)/arctan(a*x)^2,x)
```

output
```
int((a^2*c*x^2+c)/arctan(a*x)^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{c+a^2cx^2}{\arctan(ax)^2}\,dx = \int \frac{a^2cx^2+c}{\arctan\left(ax\right)^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)/arctan(a*x)^2, x)
```

## Sympy [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.53

$$\int \frac{c+a^2cx^2}{\arctan(ax)^2}\,dx = c\left(\int \frac{a^2x^2}{\operatorname{atan}^2\left(ax\right)}\,dx + \int \frac{1}{\operatorname{atan}^2\left(ax\right)}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)/atan(a*x)**2,x)
```

output
```
c*(Integral(a**2*x**2/atan(a*x)**2, x) + Integral(atan(a*x)**(-2), x))
```

## Maxima [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 3.53

$$\int \frac{c+a^2cx^2}{\arctan(ax)^2}\,dx = \int \frac{a^2cx^2+c}{\arctan\left(ax\right)^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-(a^4*c*x^4 + 2*a^2*c*x^2 - a*arctan(a*x)*integrate(4*(a^3*c*x^3 + a*c*x)/
arctan(a*x), x) + c)/(a*arctan(a*x))
```

## Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{c+a^2cx^2}{\arctan(ax)^2}\,dx = \int \frac{a^2cx^2+c}{\arctan\left(ax\right)^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)/arctan(a*x)^2, x)
```

## Mupad [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{c+a^2cx^2}{\arctan(ax)^2}\,dx = \int \frac{c\,a^2\,x^2+c}{\mathrm{atan}\left(a\,x\right)^2}\,dx$$

input
```
int((c + a^2*c*x^2)/atan(a*x)^2,x)
```

output
```
int((c + a^2*c*x^2)/atan(a*x)^2, x)
```

## Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

$$\int \frac{c + a^2cx^2}{\arctan(ax)^2} \, dx = c\left(\left(\int \frac{x^2}{atan\left(ax\right)^2}dx\right)a^2 + \int \frac{1}{atan\left(ax\right)^2}dx\right)$$

input
```
int((a^2*c*x^2+c)/atan(a*x)^2,x)
```

output
```
c*(int(x**2/atan(a*x)**2,x)*a**2 + int(1/atan(a*x)**2,x))
```

# 3.535 $\int \frac{c+a^2cx^2}{x\arctan(ax)^2}\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{c+a^2cx^2}{x\arctan(ax)^2}\,dx=\mathrm{Int}\left(\frac{c+a^2cx^2}{x\arctan(ax)^2},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)/x/arctan(a*x)^2,x)
```

## Mathematica [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{c+a^2cx^2}{x\arctan(ax)^2}\,dx=\int \frac{c+a^2cx^2}{x\arctan(ax)^2}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)/(x*ArcTan[a*x]^2),x]
```

output
```
Integrate[(c + a^2*c*x^2)/(x*ArcTan[a*x]^2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2cx^2+c}{x\arctan(ax)^2}\,dx$$
$$\Big\downarrow \ 5560$$
$$\int \frac{a^2cx^2+c}{x\arctan(ax)^2}dx$$

input
```
Int[(c + a^2*c*x^2)/(x*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 54.80 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{a^2c\,x^2+c}{x\arctan\left(ax\right)^2}dx$$

input
```
int((a^2*c*x^2+c)/x/arctan(a*x)^2,x)
```

output
```
int((a^2*c*x^2+c)/x/arctan(a*x)^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{c+a^2cx^2}{x\arctan(ax)^2}\,dx = \int \frac{a^2cx^2+c}{x\arctan{(ax)}^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)/x/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)/(x*arctan(a*x)^2), x)
```

## Sympy [N/A]

Not integrable

Time = 0.92 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30

$$\int \frac{c+a^2cx^2}{x\arctan(ax)^2}\,dx = c\left(\int \frac{1}{x\,\mathrm{atan}^2{(ax)}}\,dx + \int \frac{a^2x}{\mathrm{atan}^2{(ax)}}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)/x/atan(a*x)**2,x)
```

output
```
c*(Integral(1/(x*atan(a*x)**2), x) + Integral(a**2*x/atan(a*x)**2, x))
```

## Maxima [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 74, normalized size of antiderivative = 3.70

$$\int \frac{c+a^2cx^2}{x\arctan(ax)^2}\,dx = \int \frac{a^2cx^2+c}{x\arctan{(ax)}^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)/x/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-(a^4*c*x^4 + 2*a^2*c*x^2 - x*arctan(a*x)*integrate((3*a^4*c*x^4 + 2*a^2*c
*x^2 - c)/(x^2*arctan(a*x)), x) + c)/(a*x*arctan(a*x))
```

## Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{c+a^2cx^2}{x\arctan(ax)^2}\,dx=\int \frac{a^2cx^2+c}{x\arctan{(ax)}^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)/x/arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)/(x*arctan(a*x)^2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{c+a^2cx^2}{x\arctan(ax)^2}\,dx=\int \frac{c\,a^2\,x^2+c}{x\,\mathrm{atan}{(a\,x)}^2}\,dx$$

input
```
int((c + a^2*c*x^2)/(x*atan(a*x)^2),x)
```

output
```
int((c + a^2*c*x^2)/(x*atan(a*x)^2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.45

$$\int \frac{c+a^2cx^2}{x\arctan(ax)^2}\,dx = c\left(\left(\int \frac{x}{atan\,(ax)^2}dx\right)a^2+\int \frac{1}{atan\,(ax)^2\,x}dx\right)$$

input
```
int((a^2*c*x^2+c)/x/atan(a*x)^2,x)
```

output
```
c*(int(x/atan(a*x)**2,x)*a**2 + int(1/(atan(a*x)**2*x),x))
```

# 3.536 $\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^2}\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^2}\,dx = \operatorname{Int}\left(\frac{x(c+a^2cx^2)^2}{\arctan(ax)^2},x\right)$$

output
```
Defer(Int)(x*(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

## Mathematica [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^2}\,dx = \int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^2}\,dx$$

input
```
Integrate[(x*(c + a^2*c*x^2)^2)/ArcTan[a*x]^2,x]
```

output
```
Integrate[(x*(c + a^2*c*x^2)^2)/ArcTan[a*x]^2, x]
```

## Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a^2cx^2+c\right)^2}{\arctan(ax)^2}dx$$
$$\Big\downarrow \ 5560$$
$$\int \frac{x\left(a^2cx^2+c\right)^2}{\arctan(ax)^2}dx$$

input
```
Int[(x*(c + a^2*c*x^2)^2)/ArcTan[a*x]^2,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 31.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{x\left(a^2c\,x^2+c\right)^2}{\arctan\left(ax\right)^2}dx$$

input
```
int(x*(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

output
```
int(x*(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

$$\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^2}\,dx = \int \frac{\left(a^2cx^2+c\right)^2x}{\arctan\left(ax\right)^2}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^2*x)/arctan(a*x)^2, x)
```

## Sympy [N/A]

Not integrable

Time = 0.93 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.20

$$\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^2}\,dx = c^2\left(\int \frac{x}{\operatorname{atan}^2(ax)}\,dx + \int \frac{2a^2x^3}{\operatorname{atan}^2(ax)}\,dx + \int \frac{a^4x^5}{\operatorname{atan}^2(ax)}\,dx\right)$$

input
```
integrate(x*(a**2*c*x**2+c)**2/atan(a*x)**2,x)
```

output
```
c**2*(Integral(x/atan(a*x)**2, x) + Integral(2*a**2*x**3/atan(a*x)**2, x)
+ Integral(a**4*x**5/atan(a*x)**2, x))
```

## Maxima [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 101, normalized size of antiderivative = 5.05

$$\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^2}\,dx = \int \frac{\left(a^2cx^2+c\right)^2x}{\arctan\left(ax\right)^2}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-(a^6*c^2*x^7 + 3*a^4*c^2*x^5 + 3*a^2*c^2*x^3 + c^2*x - arctan(a*x)*integr
ate((7*a^6*c^2*x^6 + 15*a^4*c^2*x^4 + 9*a^2*c^2*x^2 + c^2)/arctan(a*x), x)
)/(a*arctan(a*x))
```

## Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^2x}{\arctan(ax)^2}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^2*x/arctan(a*x)^2, x)
```

## Mupad [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^2}\,dx = \int \frac{x\,(c\,a^2\,x^2+c)^2}{\mathrm{atan}\,(a\,x)^2}\,dx$$

input
```
int((x*(c + a^2*c*x^2)^2)/atan(a*x)^2,x)
```

output
```
int((x*(c + a^2*c*x^2)^2)/atan(a*x)^2, x)
```

## Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.40

$$\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^2}\,dx = c^2\left(\left(\int \frac{x^5}{atan(ax)^2}dx\right)a^4 + 2\left(\int \frac{x^3}{atan(ax)^2}dx\right)a^2 + \int \frac{x}{atan(ax)^2}dx\right)$$

input
```
int(x*(a^2*c*x^2+c)^2/atan(a*x)^2,x)
```

output
```
c**2*(int(x**5/atan(a*x)**2,x)*a**4 + 2*int(x**3/atan(a*x)**2,x)*a**2 + in
t(x/atan(a*x)**2,x))
```

# 3.537 $\int \frac{(c+a^2cx^2)^2}{\arctan(ax)^2} dx$



## Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \frac{(c+a^2cx^2)^2}{\arctan(ax)^2} dx = \text{Int}\left(\frac{(c+a^2cx^2)^2}{\arctan(ax)^2}, x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

## Mathematica [N/A]

Not integrable

Time = 0.84 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{(c+a^2cx^2)^2}{\arctan(ax)^2} dx = \int \frac{(c+a^2cx^2)^2}{\arctan(ax)^2} dx$$

input
```
Integrate[(c + a^2*c*x^2)^2/ArcTan[a*x]^2,x]
```

output
```
Integrate[(c + a^2*c*x^2)^2/ArcTan[a*x]^2, x]
```

## Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^2}{\arctan(ax)^2}dx$$
$$\Big\downarrow \ 5560$$
$$\int \frac{(a^2cx^2+c)^2}{\arctan(ax)^2}dx$$

input
```
Int[(c + a^2*c*x^2)^2/ArcTan[a*x]^2,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 125.51 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{(a^2c\,x^2+c)^2}{\arctan(ax)^2}dx$$

input
```
int((a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

output
```
int((a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.79

$$\int \frac{(c+a^2cx^2)^2}{\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^2}{\arctan(ax)^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)/arctan(a*x)^2, x)
```

## Sympy [N/A]

Not integrable

Time = 0.84 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.32

$$\int \frac{(c+a^2cx^2)^2}{\arctan(ax)^2}\,dx = c^2\left(\int \frac{2a^2x^2}{\operatorname{atan}^2(ax)}\,dx + \int \frac{a^4x^4}{\operatorname{atan}^2(ax)}\,dx + \int \frac{1}{\operatorname{atan}^2(ax)}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**2/atan(a*x)**2,x)
```

output
```
c**2*(Integral(2*a**2*x**2/atan(a*x)**2, x) + Integral(a**4*x**4/atan(a*x)
**2, x) + Integral(atan(a*x)**(-2), x))
```

## Maxima [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 92, normalized size of antiderivative = 4.84

$$\int \frac{(c+a^2cx^2)^2}{\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^2}{\arctan(ax)^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-(a^6*c^2*x^6 + 3*a^4*c^2*x^4 + 3*a^2*c^2*x^2 - a*arctan(a*x)*integrate(6*
(a^5*c^2*x^5 + 2*a^3*c^2*x^3 + a*c^2*x)/arctan(a*x), x) + c^2)/(a*arctan(a
*x))
```

## Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{(c+a^2cx^2)^2}{\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^2}{\arctan\left(ax\right)^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^2/arctan(a*x)^2, x)
```

## Mupad [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{(c+a^2cx^2)^2}{\arctan(ax)^2}\,dx = \int \frac{\left(c\,a^2\,x^2+c\right)^2}{\mathrm{atan}\left(a\,x\right)^2}\,dx$$

input
```
int((c + a^2*c*x^2)^2/atan(a*x)^2,x)
```

output
```
int((c + a^2*c*x^2)^2/atan(a*x)^2, x)
```

## Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.42

$$\int \frac{(c+a^2cx^2)^2}{\arctan(ax)^2}\,dx = c^2\left(\left(\int \frac{x^4}{atan(ax)^2}dx\right)a^4 + 2\left(\int \frac{x^2}{atan(ax)^2}dx\right)a^2 + \int \frac{1}{atan(ax)^2}dx\right)$$

input
```
int((a^2*c*x^2+c)^2/atan(a*x)^2,x)
```

output
```
c**2*(int(x**4/atan(a*x)**2,x)*a**4 + 2*int(x**2/atan(a*x)**2,x)*a**2 + in
t(1/atan(a*x)**2,x))
```

# 3.538 $\int \frac{(c+a^2cx^2)^2}{x \arctan(ax)^2} \, dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{(c+a^2cx^2)^2}{x \arctan(ax)^2} \, dx = \text{Int}\left(\frac{(c+a^2cx^2)^2}{x \arctan(ax)^2}, x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^2/x/arctan(a*x)^2,x)
```

## Mathematica [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(c+a^2cx^2)^2}{x \arctan(ax)^2} \, dx = \int \frac{(c+a^2cx^2)^2}{x \arctan(ax)^2} \, dx$$

input
```
Integrate[(c + a^2*c*x^2)^2/(x*ArcTan[a*x]^2),x]
```

output
```
Integrate[(c + a^2*c*x^2)^2/(x*ArcTan[a*x]^2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^2}{x\arctan(ax)^2}\,dx$$

$$\Big\downarrow \; 5560$$

$$\int \frac{(a^2cx^2+c)^2}{x\arctan(ax)^2}dx$$

input
```
Int[(c + a^2*c*x^2)^2/(x*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 197.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a^2c\,x^2+c)^2}{x\arctan\left(ax\right)^2}dx$$

input
```
int((a^2*c*x^2+c)^2/x/arctan(a*x)^2,x)
```

output
```
int((a^2*c*x^2+c)^2/x/arctan(a*x)^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

$$\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)^2}\,dx=\int \frac{(a^2cx^2+c)^2}{x\arctan(ax)^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2/x/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)/(x*arctan(a*x)^2), x)
```

## Sympy [N/A]

Not integrable

Time = 1.71 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.00

$$\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)^2}\,dx=c^2\left(\int \frac{1}{x\,\mathrm{atan}^2(ax)}\,dx+\int \frac{2a^2x}{\mathrm{atan}^2(ax)}\,dx+\int \frac{a^4x^3}{\mathrm{atan}^2(ax)}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**2/x/atan(a*x)**2,x)
```

output
```
c**2*(Integral(1/(x*atan(a*x)**2), x) + Integral(2*a**2*x/atan(a*x)**2, x)
 + Integral(a**4*x**3/atan(a*x)**2, x))
```

## Maxima [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 108, normalized size of antiderivative = 4.91

$$\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)^2}\,dx=\int \frac{(a^2cx^2+c)^2}{x\arctan(ax)^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2/x/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-(a^6*c^2*x^6 + 3*a^4*c^2*x^4 + 3*a^2*c^2*x^2 - x*arctan(a*x)*integrate((5
*a^6*c^2*x^6 + 9*a^4*c^2*x^4 + 3*a^2*c^2*x^2 - c^2)/(x^2*arctan(a*x)), x)
+ c^2)/(a*x*arctan(a*x))
```

## Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^2}{x\arctan{(ax)}^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2/x/arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^2/(x*arctan(a*x)^2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)^2}\,dx = \int \frac{(c\,a^2\,x^2+c)^2}{x\,\mathrm{atan}\left(a\,x\right)^2}\,dx$$

input
```
int((c + a^2*c*x^2)^2/(x*atan(a*x)^2),x)
```

output
```
int((c + a^2*c*x^2)^2/(x*atan(a*x)^2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.18

$$\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)^2}\,dx = c^2\left(\left(\int \frac{x^3}{atan\,(ax)^2}dx\right)a^4 + 2\left(\int \frac{x}{atan\,(ax)^2}dx\right)a^2 + \int \frac{1}{atan\,(ax)^2\,x}dx\right)$$

input
```
int((a^2*c*x^2+c)^2/x/atan(a*x)^2,x)
```

output
```
c**2*(int(x**3/atan(a*x)**2,x)*a**4 + 2*int(x/atan(a*x)**2,x)*a**2 + int(1
/(atan(a*x)**2*x),x))
```

# 3.539 $\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)^2} dx$



## Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)^2} dx = \text{Int}\left(\frac{x(c+a^2cx^2)^3}{\arctan(ax)^2}, x\right)$$

output
```
Defer(Int)(x*(a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

## Mathematica [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)^2} dx = \int \frac{x(c+a^2cx^2)^3}{\arctan(ax)^2} dx$$

input
```
Integrate[(x*(c + a^2*c*x^2)^3)/ArcTan[a*x]^2,x]
```

output
```
Integrate[(x*(c + a^2*c*x^2)^3)/ArcTan[a*x]^2, x]
```

## Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a^2cx^2+c\right)^3}{\arctan(ax)^2}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x\left(a^2cx^2+c\right)^3}{\arctan(ax)^2}dx$$

input
```
Int[(x*(c + a^2*c*x^2)^3)/ArcTan[a*x]^2,x]
```

output
```
$Aborted
```

## Maple [F(-1)]

Timed out.

$$\int \frac{x(a^2c\,x^2+c)^3}{\arctan\left(ax\right)^2}dx$$

input
```
int(x*(a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

output
```
int(x*(a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.35

$$\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^3x}{\arctan(ax)^2}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^6*c^3*x^7 + 3*a^4*c^3*x^5 + 3*a^2*c^3*x^3 + c^3*x)/arctan(a*x)
^2, x)
```

## Sympy [N/A]

Not integrable

Time = 1.27 (sec) , antiderivative size = 61, normalized size of antiderivative = 3.05

$$\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)^2}\,dx = c^3\left(\int \frac{x}{\operatorname{atan}^2(ax)}\,dx + \int \frac{3a^2x^3}{\operatorname{atan}^2(ax)}\,dx + \int \frac{3a^4x^5}{\operatorname{atan}^2(ax)}\,dx + \int \frac{a^6x^7}{\operatorname{atan}^2(ax)}\,dx\right)$$

input
```
integrate(x*(a**2*c*x**2+c)**3/atan(a*x)**2,x)
```

output
```
c**3*(Integral(x/atan(a*x)**2, x) + Integral(3*a**2*x**3/atan(a*x)**2, x)
+ Integral(3*a**4*x**5/atan(a*x)**2, x) + Integral(a**6*x**7/atan(a*x)**2,
 x))
```

## Maxima [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 123, normalized size of antiderivative = 6.15

$$\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^3x}{\arctan{(ax)}^2}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-(a^8*c^3*x^9 + 4*a^6*c^3*x^7 + 6*a^4*c^3*x^5 + 4*a^2*c^3*x^3 + c^3*x - ar
ctan(a*x)*integrate((9*a^8*c^3*x^8 + 28*a^6*c^3*x^6 + 30*a^4*c^3*x^4 + 12*
a^2*c^3*x^2 + c^3)/arctan(a*x), x))/(a*arctan(a*x))
```

## Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^3x}{\arctan{(ax)}^2}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^3*x/arctan(a*x)^2, x)
```

## Mupad [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)^2}\,dx = \int \frac{x\,(c\,a^2\,x^2+c)^3}{\mathrm{atan}\,(a\,x)^2}\,dx$$

input
```
int((x*(c + a^2*c*x^2)^3)/atan(a*x)^2,x)
```

output
```
int((x*(c + a^2*c*x^2)^3)/atan(a*x)^2, x)
```

## Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 65, normalized size of antiderivative = 3.25

$$\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)^2}\,dx = c^3\left(\left(\int \frac{x^7}{atan\,(ax)^2}dx\right)a^6+3\left(\int \frac{x^5}{atan\,(ax)^2}dx\right)a^4 + 3\left(\int \frac{x^3}{atan\,(ax)^2}dx\right)a^2+\int \frac{x}{atan\,(ax)^2}dx\right)$$

input
```
int(x*(a^2*c*x^2+c)^3/atan(a*x)^2,x)
```

output
```
c**3*(int(x**7/atan(a*x)**2,x)*a**6 + 3*int(x**5/atan(a*x)**2,x)*a**4 + 3*
int(x**3/atan(a*x)**2,x)*a**2 + int(x/atan(a*x)**2,x))
```

# 3.540 $\int \frac{(c+a^2cx^2)^3}{\arctan(ax)^2} \, dx$



## Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \frac{(c+a^2cx^2)^3}{\arctan(ax)^2} \, dx = \text{Int}\left(\frac{(c+a^2cx^2)^3}{\arctan(ax)^2}, x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

## Mathematica [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{(c+a^2cx^2)^3}{\arctan(ax)^2} \, dx = \int \frac{(c+a^2cx^2)^3}{\arctan(ax)^2} \, dx$$

input
```
Integrate[(c + a^2*c*x^2)^3/ArcTan[a*x]^2,x]
```

output
```
Integrate[(c + a^2*c*x^2)^3/ArcTan[a*x]^2, x]
```

## Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^3}{\arctan(ax)^2}\,dx$$

$$\Big\downarrow \; 5560$$

$$\int \frac{(a^2cx^2+c)^3}{\arctan(ax)^2}dx$$

input
```
Int[(c + a^2*c*x^2)^3/ArcTan[a*x]^2,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 178.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{(a^2c\,x^2+c)^3}{\arctan\left(ax\right)^2}dx$$

input
```
int((a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

output
```
int((a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.37

$$\int \frac{(c+a^2cx^2)^3}{\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^3}{\arctan(ax)^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)/arctan(a*x)^2
, x)
```

## Sympy [N/A]

Not integrable

Time = 1.13 (sec) , antiderivative size = 61, normalized size of antiderivative = 3.21

$$\int \frac{(c+a^2cx^2)^3}{\arctan(ax)^2}\,dx = c^3\left(\int \frac{3a^2x^2}{\operatorname{atan}^2(ax)}\,dx + \int \frac{3a^4x^4}{\operatorname{atan}^2(ax)}\,dx + \int \frac{a^6x^6}{\operatorname{atan}^2(ax)}\,dx + \int \frac{1}{\operatorname{atan}^2(ax)}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**3/atan(a*x)**2,x)
```

output
```
c**3*(Integral(3*a**2*x**2/atan(a*x)**2, x) + Integral(3*a**4*x**4/atan(a*
x)**2, x) + Integral(a**6*x**6/atan(a*x)**2, x) + Integral(atan(a*x)**(-2)
, x))
```

## Maxima [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 114, normalized size of antiderivative = 6.00

$$\int \frac{(c+a^2cx^2)^3}{\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^3}{\arctan{(ax)}^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-(a^8*c^3*x^8 + 4*a^6*c^3*x^6 + 6*a^4*c^3*x^4 + 4*a^2*c^3*x^2 + c^3 - a*ar
ctan(a*x)*integrate(8*(a^7*c^3*x^7 + 3*a^5*c^3*x^5 + 3*a^3*c^3*x^3 + a*c^3
*x)/arctan(a*x), x))/(a*arctan(a*x))
```

## Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{(c+a^2cx^2)^3}{\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^3}{\arctan{(ax)}^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^3/arctan(a*x)^2, x)
```

## Mupad [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{(c+a^2cx^2)^3}{\arctan(ax)^2}\,dx = \int \frac{\left(c\,a^2\,x^2+c\right)^3}{\mathrm{atan}\left(a\,x\right)^2}\,dx$$

input
```
int((c + a^2*c*x^2)^3/atan(a*x)^2,x)
```

output
```
int((c + a^2*c*x^2)^3/atan(a*x)^2, x)
```

## Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 63, normalized size of antiderivative = 3.32

$$\int \frac{(c+a^2cx^2)^3}{\arctan(ax)^2}\,dx = c^3\left(\left(\int \frac{x^6}{atan\left(ax\right)^2}dx\right)a^6+3\left(\int \frac{x^4}{atan\left(ax\right)^2}dx\right)a^4 +3\left(\int \frac{x^2}{atan\left(ax\right)^2}dx\right)a^2+\int \frac{1}{atan\left(ax\right)^2}dx\right)$$

input
```
int((a^2*c*x^2+c)^3/atan(a*x)^2,x)
```

output
```
c**3*(int(x**6/atan(a*x)**2,x)*a**6 + 3*int(x**4/atan(a*x)**2,x)*a**4 + 3*
int(x**2/atan(a*x)**2,x)*a**2 + int(1/atan(a*x)**2,x))
```

# 3.541 $\int \frac{(c+a^2cx^2)^3}{x\arctan(ax)^2}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{(c+a^2cx^2)^3}{x\arctan(ax)^2}\,dx = \text{Int}\left(\frac{(c+a^2cx^2)^3}{x\arctan(ax)^2},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^3/x/arctan(a*x)^2,x)
```

## Mathematica [N/A]

Not integrable

Time = 0.85 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(c+a^2cx^2)^3}{x\arctan(ax)^2}\,dx = \int \frac{(c+a^2cx^2)^3}{x\arctan(ax)^2}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^3/(x*ArcTan[a*x]^2),x]
```

output
```
Integrate[(c + a^2*c*x^2)^3/(x*ArcTan[a*x]^2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^3}{x\arctan(ax)^2}\,dx$$

$$\Big\downarrow \text{5560}$$

$$\int \frac{(a^2cx^2+c)^3}{x\arctan(ax)^2}dx$$

input
```
Int[(c + a^2*c*x^2)^3/(x*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

## Maple [F(-1)]

Timed out.

$$\int \frac{(a^2c\,x^2+c)^3}{x\arctan\left(ax\right)^2}dx$$

input
```
int((a^2*c*x^2+c)^3/x/arctan(a*x)^2,x)
```

output
```
int((a^2*c*x^2+c)^3/x/arctan(a*x)^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.18

$$\int \frac{(c+a^2cx^2)^3}{x\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^3}{x\arctan{(ax)}^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3/x/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)/(x*arctan(a*x
)^2), x)
```

## Sympy [N/A]

Not integrable

Time = 2.22 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.77

$$\int \frac{(c+a^2cx^2)^3}{x\arctan(ax)^2}\,dx = c^3\left(\int \frac{1}{x\,\mathrm{atan}^2{(ax)}}\,dx + \int \frac{3a^2x}{\mathrm{atan}^2{(ax)}}\,dx + \int \frac{3a^4x^3}{\mathrm{atan}^2{(ax)}}\,dx + \int \frac{a^6x^5}{\mathrm{atan}^2{(ax)}}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**3/x/atan(a*x)**2,x)
```

output
```
c**3*(Integral(1/(x*atan(a*x)**2), x) + Integral(3*a**2*x/atan(a*x)**2, x)
 + Integral(3*a**4*x**3/atan(a*x)**2, x) + Integral(a**6*x**5/atan(a*x)**2
, x))
```

## Maxima [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 130, normalized size of antiderivative = 5.91

$$\int \frac{(c+a^2cx^2)^3}{x\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^3}{x\arctan{(ax)}^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3/x/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-(a^8*c^3*x^8 + 4*a^6*c^3*x^6 + 6*a^4*c^3*x^4 + 4*a^2*c^3*x^2 + c^3 - x*ar
ctan(a*x)*integrate((7*a^8*c^3*x^8 + 20*a^6*c^3*x^6 + 18*a^4*c^3*x^4 + 4*a
^2*c^3*x^2 - c^3)/(x^2*arctan(a*x)), x))/(a*x*arctan(a*x))
```

## Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(c+a^2cx^2)^3}{x\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^3}{x\arctan{(ax)}^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3/x/arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^3/(x*arctan(a*x)^2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(c+a^2cx^2)^3}{x\arctan(ax)^2}\,dx=\int \frac{{\left(c\,a^2\,x^2+c\right)}^3}{x\,{\mathrm{atan}\left(a\,x\right)}^2}\,dx$$

input
```
int((c + a^2*c*x^2)^3/(x*atan(a*x)^2),x)
```

output
```
int((c + a^2*c*x^2)^3/(x*atan(a*x)^2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.95

$$\int \frac{(c+a^2cx^2)^3}{x\arctan(ax)^2}\,dx=c^3\left(\left(\int \frac{x^5}{atan(ax)^2}dx\right)a^6+3\left(\int \frac{x^3}{atan(ax)^2}dx\right)a^4+3\left(\int \frac{x}{atan(ax)^2}dx\right)a^2+\int \frac{1}{atan(ax)^2x}dx\right)$$

input
```
int((a^2*c*x^2+c)^3/x/atan(a*x)^2,x)
```

output
```
c**3*(int(x**5/atan(a*x)**2,x)*a**6 + 3*int(x**3/atan(a*x)**2,x)*a**4 + 3*
int(x/atan(a*x)**2,x)*a**2 + int(1/(atan(a*x)**2*x),x))
```

# 3.542 $\int \frac{x^3}{(c+a^2cx^2)\arctan(ax)^2}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^3}{(c+a^2cx^2)\arctan(ax)^2}\,dx = -\frac{x^3}{ac\arctan(ax)}+\frac{3\mathrm{Int}\left(\frac{x^2}{\arctan(ax)},x\right)}{ac}$$

output
```
-x^3/a/c/arctan(a*x)+3*Defer(Int)(x^2/arctan(a*x),x)/a/c
```

## Mathematica [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^3}{(c+a^2cx^2)\arctan(ax)^2}\,dx = \int \frac{x^3}{(c+a^2cx^2)\arctan(ax)^2}\,dx$$

input
```
Integrate[x^3/((c + a^2*c*x^2)*ArcTan[a*x]^2),x]
```

output
```
Integrate[x^3/((c + a^2*c*x^2)*ArcTan[a*x]^2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\arctan(ax)^2\,(a^2cx^2+c)}dx$$

$$\Big\downarrow \text{ 5461}$$

$$\frac{3\int \frac{x^2}{\arctan(ax)}dx}{ac}-\frac{x^3}{ac\arctan(ax)}$$

$$\Big\downarrow \text{ 5377}$$

$$\frac{3\int \frac{x^2}{\arctan(ax)}dx}{ac}-\frac{x^3}{ac\arctan(ax)}$$

input
```
Int[x^3/((c + a^2*c*x^2)*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 74.54 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(a^2c\,x^2+c)\arctan\,(ax)^2}dx$$

input
```
int(x^3/(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

output
```
int(x^3/(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^3}{(c+a^2cx^2)\arctan(ax)^2}\,dx = \int \frac{x^3}{(a^2cx^2+c)\arctan{(ax)}^2}\,dx$$

input
```
integrate(x^3/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(x^3/((a^2*c*x^2 + c)*arctan(a*x)^2), x)
```

## Sympy [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{x^3}{(c+a^2cx^2)\arctan(ax)^2}\,dx = \frac{\int \frac{x^3}{a^2x^2\operatorname{atan}^2{(ax)}+\operatorname{atan}^2{(ax)}}\,dx}{c}$$

input
```
integrate(x**3/(a**2*c*x**2+c)/atan(a*x)**2,x)
```

output
```
Integral(x**3/(a**2*x**2*atan(a*x)**2 + atan(a*x)**2), x)/c
```

## Maxima [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.64

$$\int \frac{x^3}{(c+a^2cx^2)\arctan(ax)^2}\,dx = \int \frac{x^3}{(a^2cx^2+c)\arctan{(ax)}^2}\,dx$$

input
```
integrate(x^3/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-(x^3 - 3*arctan(a*x)*integrate(x^2/arctan(a*x), x))/(a*c*arctan(a*x))
```

## Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^3}{(c+a^2cx^2)\arctan(ax)^2}\,dx = \int \frac{x^3}{(a^2cx^2+c)\arctan{(ax)}^2}\,dx$$

input
```
integrate(x^3/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate(x^3/((a^2*c*x^2 + c)*arctan(a*x)^2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^3}{(c+a^2cx^2)\arctan(ax)^2}\,dx = \int \frac{x^3}{\mathrm{atan}{(a\,x)}^2\,\left(c\,a^2\,x^2+c\right)}\,dx$$

input
```
int(x^3/(atan(a*x)^2*(c + a^2*c*x^2)),x)
```

output
```
int(x^3/(atan(a*x)^2*(c + a^2*c*x^2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.45

$$\int \frac{x^3}{(c+a^2cx^2)\arctan(ax)^2}\,dx = \frac{\int \frac{x^3}{atan(ax)^2a^2x^2+atan(ax)^2}dx}{c}$$

input
```
int(x^3/(a^2*c*x^2+c)/atan(a*x)^2,x)
```

output
```
int(x**3/(atan(a*x)**2*a**2*x**2 + atan(a*x)**2),x)/c
```

# 3.543 $\int \frac{x^2}{(c+a^2cx^2)\arctan(ax)^2}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^2}{(c+a^2cx^2)\arctan(ax)^2}\,dx = -\frac{x^2}{ac\arctan(ax)} + \frac{2\text{Int}\left(\frac{x}{\arctan(ax)},x\right)}{ac}$$

output
```
-x^2/a/c/arctan(a*x)+2*Defer(Int)(x/arctan(a*x),x)/a/c
```

## Mathematica [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^2}{(c+a^2cx^2)\arctan(ax)^2}\,dx = \int \frac{x^2}{(c+a^2cx^2)\arctan(ax)^2}\,dx$$

input
```
Integrate[x^2/((c + a^2*c*x^2)*ArcTan[a*x]^2),x]
```

output
```
Integrate[x^2/((c + a^2*c*x^2)*ArcTan[a*x]^2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\arctan(ax)^2 (a^2cx^2+c)}dx$$
$$\Big\downarrow \text{ 5461}$$
$$\frac{2\int \frac{x}{\arctan(ax)}dx}{ac} - \frac{x^2}{ac\arctan(ax)}$$
$$\Big\downarrow \text{ 5377}$$
$$\frac{2\int \frac{x}{\arctan(ax)}dx}{ac} - \frac{x^2}{ac\arctan(ax)}$$

input
```
Int[x^2/((c + a^2*c*x^2)*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 6.46 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(a^2c\,x^2+c)\arctan\left(ax\right)^2}dx$$

input
```
int(x^2/(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

output
```
int(x^2/(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^2}{(c+a^2cx^2)\arctan(ax)^2}\,dx = \int \frac{x^2}{(a^2cx^2+c)\arctan{(ax)}^2}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(x^2/((a^2*c*x^2 + c)*arctan(a*x)^2), x)
```

## Sympy [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{x^2}{(c+a^2cx^2)\arctan(ax)^2}\,dx = \frac{\int \frac{x^2}{a^2x^2\operatorname{atan}^2(ax)+\operatorname{atan}^2(ax)}\,dx}{c}$$

input
```
integrate(x**2/(a**2*c*x**2+c)/atan(a*x)**2,x)
```

output
```
Integral(x**2/(a**2*x**2*atan(a*x)**2 + atan(a*x)**2), x)/c
```

## Maxima [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.55

$$\int \frac{x^2}{(c+a^2cx^2)\arctan(ax)^2}\,dx = \int \frac{x^2}{(a^2cx^2+c)\arctan{(ax)}^2}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-(x^2 - 2*arctan(a*x)*integrate(x/arctan(a*x), x))/(a*c*arctan(a*x))
```

## Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^2}{(c+a^2cx^2)\arctan(ax)^2}\,dx = \int \frac{x^2}{(a^2cx^2+c)\arctan\left(ax\right)^2}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate(x^2/((a^2*c*x^2 + c)*arctan(a*x)^2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^2}{(c+a^2cx^2)\arctan(ax)^2}\,dx = \int \frac{x^2}{\mathrm{atan}\left(a\,x\right)^2\,\left(c\,a^2\,x^2+c\right)}\,dx$$

input
```
int(x^2/(atan(a*x)^2*(c + a^2*c*x^2)),x)
```

output
```
int(x^2/(atan(a*x)^2*(c + a^2*c*x^2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.45

$$\int \frac{x^2}{(c+a^2cx^2)\arctan(ax)^2}\,dx = \frac{\int \frac{x^2}{atan(ax)^2a^2x^2+atan(ax)^2}dx}{c}$$

input
```
int(x^2/(a^2*c*x^2+c)/atan(a*x)^2,x)
```

output
```
int(x**2/(atan(a*x)**2*a**2*x**2 + atan(a*x)**2),x)/c
```

# 3.544 $\int \frac{x}{(c+a^2cx^2)\arctan(ax)^2}\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{x}{(c+a^2cx^2)\arctan(ax)^2}\,dx = -\frac{x}{ac\arctan(ax)} + \frac{\mathrm{Int}\left(\frac{1}{\arctan(ax)},x\right)}{ac}$$

output
```
-x/a/c/arctan(a*x)+Defer(Int)(1/arctan(a*x),x)/a/c
```

## Mathematica [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x}{(c+a^2cx^2)\arctan(ax)^2}\,dx = \int \frac{x}{(c+a^2cx^2)\arctan(ax)^2}\,dx$$

input
```
Integrate[x/((c + a^2*c*x^2)*ArcTan[a*x]^2),x]
```

output
```
Integrate[x/((c + a^2*c*x^2)*ArcTan[a*x]^2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\arctan(ax)^2\left(a^2cx^2+c\right)}dx$$

$$\Big\downarrow \text{5457}$$

$$\frac{\int \frac{1}{\arctan(ax)}dx}{ac}-\frac{x}{ac\arctan(ax)}$$

$$\Big\downarrow \text{5353}$$

$$\frac{\int \frac{1}{\arctan(ax)}dx}{ac}-\frac{x}{ac\arctan(ax)}$$

input
```
Int[x/((c + a^2*c*x^2)*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 6.40 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{x}{\left(a^2c\,x^2+c\right)\arctan\left(ax\right)^2}dx$$

input
```
int(x/(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

output
```
int(x/(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x}{(c+a^2cx^2)\arctan(ax)^2}\,dx = \int \frac{x}{(a^2cx^2+c)\arctan{(ax)}^2}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(x/((a^2*c*x^2 + c)*arctan(a*x)^2), x)
```

## Sympy [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{x}{(c+a^2cx^2)\arctan(ax)^2}\,dx = \frac{\int \frac{x}{a^2x^2\operatorname{atan}^2{(ax)}+\operatorname{atan}^2{(ax)}}\,dx}{c}$$

input
```
integrate(x/(a**2*c*x**2+c)/atan(a*x)**2,x)
```

output
```
Integral(x/(a**2*x**2*atan(a*x)**2 + atan(a*x)**2), x)/c
```

## Maxima [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.50

$$\int \frac{x}{(c+a^2cx^2)\arctan(ax)^2}\,dx = \int \frac{x}{(a^2cx^2+c)\arctan{(ax)}^2}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
(arctan(a*x)*integrate(1/arctan(a*x), x) - x)/(a*c*arctan(a*x))
```

## Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x}{(c+a^2cx^2)\arctan(ax)^2}\,dx = \int \frac{x}{(a^2cx^2+c)\arctan{(ax)}^2}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate(x/((a^2*c*x^2 + c)*arctan(a*x)^2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x}{(c+a^2cx^2)\arctan(ax)^2}\,dx = \int \frac{x}{\mathrm{atan}\left(a\,x\right)^2\,\left(c\,a^2\,x^2+c\right)}\,dx$$

input
```
int(x/(atan(a*x)^2*(c + a^2*c*x^2)),x)
```

output
```
int(x/(atan(a*x)^2*(c + a^2*c*x^2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.50

$$\int \frac{x}{(c+a^2cx^2)\arctan(ax)^2}\,dx = \frac{\int \frac{x}{atan(ax)^2a^2x^2+atan(ax)^2}dx}{c}$$

input
```
int(x/(a^2*c*x^2+c)/atan(a*x)^2,x)
```

output
```
int(x/(atan(a*x)**2*a**2*x**2 + atan(a*x)**2),x)/c
```

# 3.545 $\int \frac{1}{(c+a^2cx^2)\arctan(ax)^2}\,dx$



## Optimal result

Integrand size = 19, antiderivative size = 14

$$\int \frac{1}{(c+a^2cx^2)\arctan(ax)^2}\,dx = -\frac{1}{ac\arctan(ax)}$$

output
```
-1/a/c/arctan(a*x)
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{(c+a^2cx^2)\arctan(ax)^2}\,dx = -\frac{1}{ac\arctan(ax)}$$

input
```
Integrate[1/((c + a^2*c*x^2)*ArcTan[a*x]^2),x]
```

output
```
-(1/(a*c*ArcTan[a*x]))
```

## Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arctan(ax)^2\left(a^2cx^2+c\right)}dx$$
$$\Big\downarrow \text{5419}$$
$$-\frac{1}{ac\arctan(ax)}$$

input
```
Int[1/((c + a^2*c*x^2)*ArcTan[a*x]^2),x]
```

output
```
-(1/(a*c*ArcTan[a*x]))
```

### Defintions of rubi rules used

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

## Maple [A] (verified)

Time = 1.36 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

| method | result | size |
|---|---|---|
| derivativedivides | $-\frac{1}{ac\arctan(ax)}$ | 15 |
| default | $-\frac{1}{ac\arctan(ax)}$ | 15 |
| parallelrisch | $-\frac{1}{ac\arctan(ax)}$ | 15 |
| risch | $\frac{2i}{ac(\ln(-iax+1)-\ln(iax+1))}$ | 31 |

input
```
int(1/(a^2*c*x^2+c)/arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/a/c/arctan(a*x)
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{(c+a^2cx^2)\arctan(ax)^2}\,dx = -\frac{1}{ac\arctan{(ax)}}$$

input
```
integrate(1/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
-1/(a*c*arctan(a*x))
```

## Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{1}{(c+a^2cx^2)\arctan(ax)^2}\,dx = -\frac{1}{ac\operatorname{atan}{(ax)}}$$

input
```
integrate(1/(a**2*c*x**2+c)/atan(a*x)**2,x)
```

output
```
-1/(a*c*atan(a*x))
```

## Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{(c+a^2cx^2)\arctan(ax)^2}\,dx = -\frac{1}{ac\arctan(ax)}$$

input
```
integrate(1/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-1/(a*c*arctan(a*x))
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{(c+a^2cx^2)\arctan(ax)^2}\,dx = -\frac{1}{ac\arctan(ax)}$$

input
```
integrate(1/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
-1/(a*c*arctan(a*x))
```

## Mupad [B] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{(c+a^2cx^2)\arctan(ax)^2}\,dx = -\frac{1}{a\,c\,\mathrm{atan}\,(a\,x)}$$

input
```
int(1/(atan(a*x)^2*(c + a^2*c*x^2)),x)
```

output
```
-1/(a*c*atan(a*x))
```

## Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{(c+a^2cx^2)\arctan(ax)^2}\,dx = -\frac{1}{atan\,(ax)\,ac}$$

input
```
int(1/(a^2*c*x^2+c)/atan(a*x)^2,x)
```

output
```
( - 1)/(atan(a*x)*a*c)
```

# 3.546 $\int \frac{1}{x(c+a^2cx^2)\arctan(ax)^2} dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x(c+a^2cx^2)\arctan(ax)^2} dx = -\frac{1}{acx\arctan(ax)} - \frac{\text{Int}\left(\frac{1}{x^2\arctan(ax)}, x\right)}{ac}$$

output
```
-1/a/c/x/arctan(a*x)-Defer(Int)(1/x^2/arctan(a*x),x)/a/c
```

## Mathematica [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x(c+a^2cx^2)\arctan(ax)^2} dx = \int \frac{1}{x(c+a^2cx^2)\arctan(ax)^2} dx$$

input
```
Integrate[1/(x*(c + a^2*c*x^2)*ArcTan[a*x]^2),x]
```

output
```
Integrate[1/(x*(c + a^2*c*x^2)*ArcTan[a*x]^2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\arctan(ax)^2(a^2cx^2+c)}dx$$

$$\Big\downarrow \text{5461}$$

$$-\frac{\int \frac{1}{x^2\arctan(ax)}dx}{ac}-\frac{1}{acx\arctan(ax)}$$

$$\Big\downarrow \text{5377}$$

$$-\frac{\int \frac{1}{x^2\arctan(ax)}dx}{ac}-\frac{1}{acx\arctan(ax)}$$

input
```
Int[1/(x*(c + a^2*c*x^2)*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 6.45 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a^2c\,x^2+c\right)\arctan\left(ax\right)^2}dx$$

input
```
int(1/x/(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

output
```
int(1/x/(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{1}{x\left(c+a^2cx^2\right)\arctan(ax)^2}\,dx=\int \frac{1}{(a^2cx^2+c)x\arctan\left(ax\right)^2}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(1/((a^2*c*x^3 + c*x)*arctan(a*x)^2), x)
```

## Sympy [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{1}{x\left(c+a^2cx^2\right)\arctan(ax)^2}\,dx=\frac{\int \frac{1}{a^2x^3\operatorname{atan}^2\left(ax\right)+x\operatorname{atan}^2\left(ax\right)}\,dx}{c}$$

input
```
integrate(1/x/(a**2*c*x**2+c)/atan(a*x)**2,x)
```

output
```
Integral(1/(a**2*x**3*atan(a*x)**2 + x*atan(a*x)**2), x)/c
```

## Maxima [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

$$\int \frac{1}{x\left(c+a^2cx^2\right)\arctan(ax)^2}\,dx=\int \frac{1}{(a^2cx^2+c)x\arctan\left(ax\right)^2}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-(x*arctan(a*x)*integrate(1/(x^2*arctan(a*x)), x) + 1)/(a*c*x*arctan(a*x))
```

## Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x(c+a^2cx^2)\arctan(ax)^2}\,dx = \int \frac{1}{(a^2cx^2+c)x\arctan(ax)^2}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)*x*arctan(a*x)^2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x(c+a^2cx^2)\arctan(ax)^2}\,dx = \int \frac{1}{x\,\mathrm{atan}(a\,x)^2\,(c\,a^2\,x^2+c)}\,dx$$

input
```
int(1/(x*atan(a*x)^2*(c + a^2*c*x^2)),x)
```

output
```
int(1/(x*atan(a*x)^2*(c + a^2*c*x^2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.36

$$\int \frac{1}{x\left(c+a^2cx^2\right)\arctan(ax)^2}\,dx=\frac{\int \frac{1}{atan(ax)^2a^2x^3+atan(ax)^2x}dx}{c}$$

input
```
int(1/x/(a^2*c*x^2+c)/atan(a*x)^2,x)
```

output
```
int(1/(atan(a*x)**2*a**2*x**3 + atan(a*x)**2*x),x)/c
```

# 3.547 $\int \frac{1}{x^2(c+a^2cx^2)\arctan(ax)^2}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x^2(c+a^2cx^2)\arctan(ax)^2}\,dx = -\frac{1}{acx^2\arctan(ax)} - \frac{2\text{Int}\left(\frac{1}{x^3\arctan(ax)},x\right)}{ac}$$

output
```
-1/a/c/x^2/arctan(a*x)-2*Defer(Int)(1/x^3/arctan(a*x),x)/a/c
```

## Mathematica [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2(c+a^2cx^2)\arctan(ax)^2}\,dx = \int \frac{1}{x^2(c+a^2cx^2)\arctan(ax)^2}\,dx$$

input
```
Integrate[1/(x^2*(c + a^2*c*x^2)*ArcTan[a*x]^2),x]
```

output
```
Integrate[1/(x^2*(c + a^2*c*x^2)*ArcTan[a*x]^2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \arctan(ax)^2 (a^2cx^2+c)}dx$$

$$\Big\downarrow \text{ 5461}$$

$$-\frac{2\int \frac{1}{x^3 \arctan(ax)}dx}{ac}-\frac{1}{acx^2 \arctan(ax)}$$

$$\Big\downarrow \text{ 5377}$$

$$-\frac{2\int \frac{1}{x^3 \arctan(ax)}dx}{ac}-\frac{1}{acx^2 \arctan(ax)}$$

input
```
Int[1/(x^2*(c + a^2*c*x^2)*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 6.51 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (a^2c\,x^2+c) \arctan(ax)^2}dx$$

input
```
int(1/x^2/(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

output
```
int(1/x^2/(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^2 (c + a^2 c x^2) \arctan(ax)^2} \, dx = \int \frac{1}{(a^2 c x^2 + c) x^2 \arctan(ax)^2} \, dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(1/((a^2*c*x^4 + c*x^2)*arctan(a*x)^2), x)
```

## Sympy [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

$$\int \frac{1}{x^2 (c + a^2 c x^2) \arctan(ax)^2} \, dx = \frac{\int \frac{1}{a^2 x^4 \operatorname{atan}^2(ax) + x^2 \operatorname{atan}^2(ax)} \, dx}{c}$$

input
```
integrate(1/x**2/(a**2*c*x**2+c)/atan(a*x)**2,x)
```

output
```
Integral(1/(a**2*x**4*atan(a*x)**2 + x**2*atan(a*x)**2), x)/c
```

## Maxima [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.82

$$\int \frac{1}{x^2 (c + a^2 c x^2) \arctan(ax)^2} \, dx = \int \frac{1}{(a^2 c x^2 + c) x^2 \arctan(ax)^2} \, dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-(2*x^2*arctan(a*x)*integrate(1/(x^3*arctan(a*x)), x) + 1)/(a*c*x^2*arctan
(a*x))
```

## Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2 (c + a^2cx^2) \arctan(ax)^2} \, dx = \int \frac{1}{(a^2cx^2 + c)x^2 \arctan{(ax)}^2} \, dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)*x^2*arctan(a*x)^2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2 (c + a^2cx^2) \arctan(ax)^2} \, dx = \int \frac{1}{x^2 \operatorname{atan}{(a\,x)}^2 \,(c\,a^2\,x^2 + c)} \, dx$$

input
```
int(1/(x^2*atan(a*x)^2*(c + a^2*c*x^2)),x)
```

output
```
int(1/(x^2*atan(a*x)^2*(c + a^2*c*x^2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.45

$$\int \frac{1}{x^2 (c + a^2 c x^2) \arctan(ax)^2} \, dx = \frac{\int \frac{1}{atan(ax)^2 a^2 x^4 + atan(ax)^2 x^2} dx}{c}$$

input
```
int(1/x^2/(a^2*c*x^2+c)/atan(a*x)^2,x)
```

output
```
int(1/(atan(a*x)**2*a**2*x**4 + atan(a*x)**2*x**2),x)/c
```

# 3.548 $\int \frac{1}{x^3(c+a^2cx^2)\arctan(ax)^2}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)\arctan(ax)^2}\,dx=-\frac{1}{acx^3\arctan(ax)}-\frac{3\text{Int}\left(\frac{1}{x^4\arctan(ax)},x\right)}{ac}$$

output
```
-1/a/c/x^3/arctan(a*x)-3*Defer(Int)(1/x^4/arctan(a*x),x)/a/c
```

## Mathematica [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)\arctan(ax)^2}\,dx=\int \frac{1}{x^3\left(c+a^2cx^2\right)\arctan(ax)^2}\,dx$$

input
```
Integrate[1/(x^3*(c + a^2*c*x^2)*ArcTan[a*x]^2),x]
```

output
```
Integrate[1/(x^3*(c + a^2*c*x^2)*ArcTan[a*x]^2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \arctan(ax)^2 (a^2cx^2+c)} \, dx$$

$$\Big\downarrow \text{5461}$$

$$-\frac{3\int \frac{1}{x^4 \arctan(ax)}dx}{ac} - \frac{1}{acx^3 \arctan(ax)}$$

$$\Big\downarrow \text{5377}$$

$$-\frac{3\int \frac{1}{x^4 \arctan(ax)}dx}{ac} - \frac{1}{acx^3 \arctan(ax)}$$

input
```
Int[1/(x^3*(c + a^2*c*x^2)*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 71.45 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3 \left(a^2 c\, x^2+c\right) \arctan \left(a x\right)^2} dx$$

input
```
int(1/x^3/(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

output
```
int(1/x^3/(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^3 (c + a^2cx^2) \arctan(ax)^2} \, dx = \int \frac{1}{(a^2cx^2 + c)x^3 \arctan{(ax)}^2} \, dx$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(1/((a^2*c*x^5 + c*x^3)*arctan(a*x)^2), x)
```

## Sympy [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

$$\int \frac{1}{x^3 (c + a^2cx^2) \arctan(ax)^2} \, dx = \frac{\int \frac{1}{a^2x^5 \operatorname{atan}^2{(ax)} + x^3 \operatorname{atan}^2{(ax)}} \, dx}{c}$$

input
```
integrate(1/x**3/(a**2*c*x**2+c)/atan(a*x)**2,x)
```

output
```
Integral(1/(a**2*x**5*atan(a*x)**2 + x**3*atan(a*x)**2), x)/c
```

## Maxima [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.82

$$\int \frac{1}{x^3 (c + a^2cx^2) \arctan(ax)^2} \, dx = \int \frac{1}{(a^2cx^2 + c)x^3 \arctan{(ax)}^2} \, dx$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-(3*x^3*arctan(a*x)*integrate(1/(x^4*arctan(a*x)), x) + 1)/(a*c*x^3*arctan
(a*x))
```

## Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^3 (c + a^2cx^2) \arctan(ax)^2} \, dx = \int \frac{1}{(a^2cx^2 + c)x^3 \arctan(ax)^2} \, dx$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)*x^3*arctan(a*x)^2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^3 (c + a^2cx^2) \arctan(ax)^2} \, dx = \int \frac{1}{x^3 \operatorname{atan}(a\,x)^2 \, (c\,a^2\,x^2 + c)} \, dx$$

input
```
int(1/(x^3*atan(a*x)^2*(c + a^2*c*x^2)),x)
```

output
```
int(1/(x^3*atan(a*x)^2*(c + a^2*c*x^2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.45

$$\int \frac{1}{x^3 (c + a^2 c x^2) \arctan(ax)^2} \, dx = \frac{\int \frac{1}{atan(ax)^2 a^2 x^5 + atan(ax)^2 x^3} dx}{c}$$

input
```
int(1/x^3/(a^2*c*x^2+c)/atan(a*x)^2,x)
```

output
```
int(1/(atan(a*x)**2*a**2*x**5 + atan(a*x)**2*x**3),x)/c
```

# 3.549 $\int \frac{1}{x^4(c+a^2cx^2)\arctan(ax)^2}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x^4 (c+a^2cx^2) \arctan(ax)^2}\,dx = -\frac{1}{acx^4\arctan(ax)} - \frac{4\text{Int}\left(\frac{1}{x^5\arctan(ax)},x\right)}{ac}$$

output
```
-1/a/c/x^4/arctan(a*x)-4*Defer(Int)(1/x^5/arctan(a*x),x)/a/c
```

## Mathematica [N/A]

Not integrable

Time = 0.94 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^4 (c+a^2cx^2) \arctan(ax)^2}\,dx = \int \frac{1}{x^4 (c+a^2cx^2) \arctan(ax)^2}\,dx$$

input
```
Integrate[1/(x^4*(c + a^2*c*x^2)*ArcTan[a*x]^2),x]
```

output
```
Integrate[1/(x^4*(c + a^2*c*x^2)*ArcTan[a*x]^2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \arctan(ax)^2 (a^2cx^2+c)} \, dx$$

$$\Big\downarrow \text{ 5461}$$

$$-\frac{4\int \frac{1}{x^5 \arctan(ax)}dx}{ac}-\frac{1}{acx^4 \arctan(ax)}$$

$$\Big\downarrow \text{ 5377}$$

$$-\frac{4\int \frac{1}{x^5 \arctan(ax)}dx}{ac}-\frac{1}{acx^4 \arctan(ax)}$$

input
```
Int[1/(x^4*(c + a^2*c*x^2)*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 64.55 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4 \left(a^2 c\, x^2+c\right) \arctan\left(a x\right)^2}dx$$

input
```
int(1/x^4/(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

output
```
int(1/x^4/(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^4 (c + a^2 c x^2) \arctan(ax)^2} \, dx = \int \frac{1}{(a^2 c x^2 + c) x^4 \arctan(ax)^2} \, dx$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(1/((a^2*c*x^6 + c*x^4)*arctan(a*x)^2), x)
```

## Sympy [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

$$\int \frac{1}{x^4 (c + a^2 c x^2) \arctan(ax)^2} \, dx = \frac{\int \frac{1}{a^2 x^6 \operatorname{atan}^2(ax) + x^4 \operatorname{atan}^2(ax)} \, dx}{c}$$

input
```
integrate(1/x**4/(a**2*c*x**2+c)/atan(a*x)**2,x)
```

output
```
Integral(1/(a**2*x**6*atan(a*x)**2 + x**4*atan(a*x)**2), x)/c
```

## Maxima [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.82

$$\int \frac{1}{x^4 (c + a^2 c x^2) \arctan(ax)^2} \, dx = \int \frac{1}{(a^2 c x^2 + c) x^4 \arctan(ax)^2} \, dx$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-(4*x^4*arctan(a*x)*integrate(1/(x^5*arctan(a*x)), x) + 1)/(a*c*x^4*arctan
(a*x))
```

## Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^4 (c + a^2cx^2) \arctan(ax)^2} \, dx = \int \frac{1}{(a^2cx^2 + c)x^4 \arctan{(ax)}^2} \, dx$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)*x^4*arctan(a*x)^2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^4 (c + a^2cx^2) \arctan(ax)^2} \, dx = \int \frac{1}{x^4 \,\mathrm{atan}\left(a\,x\right)^2\,\left(c\,a^2\,x^2+c\right)} \, dx$$

input
```
int(1/(x^4*atan(a*x)^2*(c + a^2*c*x^2)),x)
```

output
```
int(1/(x^4*atan(a*x)^2*(c + a^2*c*x^2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.45

$$\int \frac{1}{x^4 (c + a^2 c x^2) \arctan(ax)^2} \, dx = \frac{\int \frac{1}{atan(ax)^2 a^2 x^6 + atan(ax)^2 x^4} dx}{c}$$

input
```
int(1/x^4/(a^2*c*x^2+c)/atan(a*x)^2,x)
```

output
```
int(1/(atan(a*x)**2*a**2*x**6 + atan(a*x)**2*x**4),x)/c
```

# 3.550 $\int \frac{x^3}{\left(c+a^2cx^2\right)^2\arctan(ax)^2}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^2\arctan(ax)^2}\,dx = -\frac{x}{a^3c^2\arctan(ax)}+\frac{x}{a^3c^2\left(1+a^2x^2\right)\arctan(ax)} - \frac{\text{CosIntegral}(2\arctan(ax))}{a^4c^2}+\frac{\text{Int}\left(\frac{1}{\arctan(ax)},x\right)}{a^3c^2}$$

output
```
-x/a^3/c^2/arctan(a*x)+x/a^3/c^2/(a^2*x^2+1)/arctan(a*x)-Ci(2*arctan(a*x))
/a^4/c^2+Defer(Int)(1/arctan(a*x),x)/a^3/c^2
```

## Mathematica [N/A]

Not integrable

Time = 2.77 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^2\arctan(ax)^2}\,dx = \int \frac{x^3}{\left(c+a^2cx^2\right)^2\arctan(ax)^2}\,dx$$

input
```
Integrate[x^3/((c + a^2*c*x^2)^2*ArcTan[a*x]^2),x]
```

output
```
Integrate[x^3/((c + a^2*c*x^2)^2*ArcTan[a*x]^2), x]
```

## Rubi [N/A]

Not integrable

Time = 1.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\arctan(ax)^2 (a^2cx^2+c)^2}dx$$

$$\Big\downarrow \text{5499}$$

$$\frac{\int \frac{x}{c(a^2x^2+1)\arctan(ax)^2}dx}{a^2c} - \frac{\int \frac{x}{c^2(a^2x^2+1)^2\arctan(ax)^2}dx}{a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{x}{(a^2x^2+1)\arctan(ax)^2}dx}{a^2c^2} - \frac{\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^2}dx}{a^2c^2}$$

$$\Big\downarrow \text{5457}$$

$$\frac{\frac{\int \frac{1}{\arctan(ax)}dx}{a} - \frac{x}{a\arctan(ax)}}{a^2c^2} - \frac{\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^2}dx}{a^2c^2}$$

$$\Big\downarrow \text{5353}$$

$$\frac{\frac{\int \frac{1}{\arctan(ax)}dx}{a} - \frac{x}{a\arctan(ax)}}{a^2c^2} - \frac{\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^2}dx}{a^2c^2}$$

$$\Big\downarrow \text{5503}$$

$$\frac{\frac{\int \frac{1}{\arctan(ax)}dx}{a} - \frac{x}{a\arctan(ax)}}{a^2c^2} - \frac{\frac{\int \frac{1}{(a^2x^2+1)^2\arctan(ax)}dx}{a} - a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx - \frac{x}{a(a^2x^2+1)\arctan(ax)}}{a^2c^2}$$

$$\Big\downarrow \; 5439$$

$$\frac{\frac{\frac{\int \frac{1}{\arctan(ax)}dx}{a}-\frac{x}{a\arctan(ax)}}{a^2c^2}-\frac{-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\int \frac{1}{(a^2x^2+1)\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}}{a^2c^2}}{}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\int \frac{1}{\arctan(ax)}dx}{a}-\frac{x}{a\arctan(ax)}}{a^2c^2}-\frac{-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^2}{\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}}{a^2c^2}$$

$$\Big\downarrow \; 3793$$

$$\frac{\frac{\int \frac{1}{\arctan(ax)}dx}{a}-\frac{x}{a\arctan(ax)}}{a^2c^2}-\frac{-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\int \left(\frac{\cos(2\arctan(ax))}{2\arctan(ax)}+\frac{1}{2\arctan(ax)}\right)d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}}{a^2c^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{\int \frac{1}{\arctan(ax)}dx}{a}-\frac{x}{a\arctan(ax)}}{a^2c^2}-\frac{-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}}{a^2c^2}$$

$$\Big\downarrow \; 5505$$

$$\frac{\frac{\int \frac{1}{\arctan(ax)}dx}{a}-\frac{x}{a\arctan(ax)}}{a^2c^2}-\frac{-\frac{\int \frac{a^2x^2}{(a^2x^2+1)\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}}{a^2c^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\int \frac{1}{\arctan(ax)}dx}{a}-\frac{x}{a\arctan(ax)}}{a^2c^2}-\frac{-\frac{\int \frac{\sin(\arctan(ax))^2}{\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}}{a^2c^2}$$

$$\Big\downarrow \text{3793}$$

$$\frac{\frac{\int \frac{1}{\arctan(ax)}dx}{a} - \frac{x}{a\arctan(ax)}}{a^2c^2} - \frac{-\frac{\int \left(\frac{1}{2\arctan(ax)} - \frac{\cos(2\arctan(ax))}{2\arctan(ax)}\right)d\arctan(ax)}{a^2} + \frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax)) + \frac{1}{2}\log(\arctan(ax))}{a^2} - \frac{x}{a(a^2x^2+1)\arctan(ax)}}{a^2c^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\int \frac{1}{\arctan(ax)}dx}{a} - \frac{x}{a\arctan(ax)}}{a^2c^2} - \frac{-\frac{\frac{1}{2}\log(\arctan(ax)) - \frac{1}{2}\text{CosIntegral}(2\arctan(ax))}{a^2} + \frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax)) + \frac{1}{2}\log(\arctan(ax))}{a^2} - \frac{x}{a(a^2x^2+1)\arctan(ax)}}{a^2c^2}$$

input
```
Int[x^3/((c + a^2*c*x^2)^2*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 7.97 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{\left(a^2c\,x^2+c\right)^2 \arctan\left(ax\right)^2}dx$$

input
```
int(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

output
```
int(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.77

$$\int \frac{x^3}{(c+a^2cx^2)^2 \arctan(ax)^2} \, dx = \int \frac{x^3}{(a^2cx^2+c)^2 \arctan(ax)^2} \, dx$$

input
```
integrate(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(x^3/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)^2), x)
```

## Sympy [N/A]

Not integrable

Time = 0.77 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.91

$$\int \frac{x^3}{(c+a^2cx^2)^2 \arctan(ax)^2} \, dx = \frac{\int \frac{x^3}{a^4x^4 \operatorname{atan}^2(ax)+2a^2x^2 \operatorname{atan}^2(ax)+\operatorname{atan}^2(ax)} \, dx}{c^2}$$

input
```
integrate(x**3/(a**2*c*x**2+c)**2/atan(a*x)**2,x)
```

output
```
Integral(x**3/(a**4*x**4*atan(a*x)**2 + 2*a**2*x**2*atan(a*x)**2 + atan(a*
x)**2), x)/c**2
```

## Maxima [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 103, normalized size of antiderivative = 4.68

$$\int \frac{x^3}{(c+a^2cx^2)^2 \arctan(ax)^2} \, dx = \int \frac{x^3}{(a^2cx^2+c)^2 \arctan(ax)^2} \, dx$$

input
```
integrate(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-(x^3 - (a^3*c^2*x^2 + a*c^2)*arctan(a*x)*integrate((a^2*x^4 + 3*x^2)/((a^
5*c^2*x^4 + 2*a^3*c^2*x^2 + a*c^2)*arctan(a*x)), x))/((a^3*c^2*x^2 + a*c^2
)*arctan(a*x))
```

## Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^3}{(c+a^2cx^2)^2 \arctan(ax)^2} \, dx = \int \frac{x^3}{(a^2cx^2+c)^2 \arctan(ax)^2} \, dx$$

input
```
integrate(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate(x^3/((a^2*c*x^2 + c)^2*arctan(a*x)^2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^3}{(c+a^2cx^2)^2 \arctan(ax)^2} \, dx = \int \frac{x^3}{\mathrm{atan}\left(a\,x\right)^2\,{\left(c\,a^2\,x^2+c\right)}^2} \, dx$$

input
```
int(x^3/(atan(a*x)^2*(c + a^2*c*x^2)^2),x)
```

output
```
int(x^3/(atan(a*x)^2*(c + a^2*c*x^2)^2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.09

$$\int \frac{x^3}{(c+a^2cx^2)^2 \arctan(ax)^2}\,dx = \frac{\int \frac{x^3}{atan(ax)^2a^4x^4+2atan(ax)^2a^2x^2+atan(ax)^2}dx}{c^2}$$

input
```
int(x^3/(a^2*c*x^2+c)^2/atan(a*x)^2,x)
```

output
```
int(x**3/(atan(a*x)**2*a**4*x**4 + 2*atan(a*x)**2*a**2*x**2 + atan(a*x)**2
),x)/c**2
```

# 3.551 $\int \frac{x^2}{\left(c+a^2cx^2\right)^2 \arctan(ax)^2}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 43

$$\int \frac{x^2}{\left(c+a^2cx^2\right)^2 \arctan(ax)^2}\,dx = -\frac{x^2}{ac^2\left(1+a^2x^2\right)\arctan(ax)} + \frac{\operatorname{Si}(2\arctan(ax))}{a^3c^2}$$

output
```
-x^2/a/c^2/(a^2*x^2+1)/arctan(a*x)+Si(2*arctan(a*x))/a^3/c^2
```

## Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

$$\int \frac{x^2}{\left(c+a^2cx^2\right)^2 \arctan(ax)^2}\,dx = \frac{-\frac{a^2x^2}{(1+a^2x^2)\arctan(ax)} + \operatorname{Si}(2\arctan(ax))}{a^3c^2}$$

input
```
Integrate[x^2/((c + a^2*c*x^2)^2*ArcTan[a*x]^2),x]
```

output
```
(-((a^2*x^2)/((1 + a^2*x^2)*ArcTan[a*x])) + SinIntegral[2*ArcTan[a*x]])/(a
^3*c^2)
```

## Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {5477, 27, 5505, 4906, 27, 3042, 3780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\arctan(ax)^2 (a^2cx^2+c)^2}dx$$

$$\Big\downarrow \text{5477}$$

$$\frac{2\int \frac{x}{c^2(a^2x^2+1)^2\arctan(ax)}dx}{a}-\frac{x^2}{ac^2(a^2x^2+1)\arctan(ax)}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\int \frac{x}{(a^2x^2+1)^2\arctan(ax)}dx}{ac^2}-\frac{x^2}{ac^2(a^2x^2+1)\arctan(ax)}$$

$$\Big\downarrow \text{5505}$$

$$\frac{2\int \frac{ax}{(a^2x^2+1)\arctan(ax)}d\arctan(ax)}{a^3c^2}-\frac{x^2}{ac^2(a^2x^2+1)\arctan(ax)}$$

$$\Big\downarrow \text{4906}$$

$$\frac{2\int \frac{\sin(2\arctan(ax))}{2\arctan(ax)}d\arctan(ax)}{a^3c^2}-\frac{x^2}{ac^2(a^2x^2+1)\arctan(ax)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a^3c^2}-\frac{x^2}{ac^2(a^2x^2+1)\arctan(ax)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a^3c^2}-\frac{x^2}{ac^2(a^2x^2+1)\arctan(ax)}$$

$$\Big\downarrow \text{3780}$$

$$\frac{\text{Si}(2\arctan(ax))}{a^3c^2}-\frac{x^2}{ac^2(a^2x^2+1)\arctan(ax)}$$

input
```
Int[x^2/((c + a^2*c*x^2)^2*ArcTan[a*x]^2),x]
```

output
```
-(x^2/(a*c^2*(1 + a^2*x^2)*ArcTan[a*x])) + SinIntegral[2*ArcTan[a*x]]/(a^3
*c^2)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5477
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*ArcT
an[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp[f*(m/(b*c*(p + 1)))   Int[(f*x
)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1
]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

## Maple [A] (verified)

Time = 8.29 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{2\,\mathrm{Si}(2\arctan(ax))\arctan(ax)+\cos(2\arctan(ax))-1}{2a^3c^2\arctan(ax)}$ | 37 |
| default | $\frac{2\,\mathrm{Si}(2\arctan(ax))\arctan(ax)+\cos(2\arctan(ax))-1}{2a^3c^2\arctan(ax)}$ | 37 |

input
```
int(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/a^3/c^2*(2*Si(2*arctan(a*x))*arctan(a*x)+cos(2*arctan(a*x))-1)/arctan(
a*x)
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.86

$$\int \frac{x^2}{(c+a^2cx^2)^2\arctan(ax)^2}\,dx = -\frac{2\,a^2x^2-(i\,a^2x^2+i)\arctan(ax)\,\mathrm{log\_integral}\left(-\frac{a^2x^2+2i\,ax-1}{a^2x^2+1}\right)-(-i\,a^2x^2-i)\arctan(ax)\,\mathrm{log\_integra}}{2\,(a^5c^2x^2+a^3c^2)\arctan(ax)}$$

input
```
integrate(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
-1/2*(2*a^2*x^2 - (I*a^2*x^2 + I)*arctan(a*x)*log_integral(-(a^2*x^2 + 2*I
*a*x - 1)/(a^2*x^2 + 1)) - (-I*a^2*x^2 - I)*arctan(a*x)*log_integral(-(a^2
*x^2 - 2*I*a*x - 1)/(a^2*x^2 + 1)))/((a^5*c^2*x^2 + a^3*c^2)*arctan(a*x))
```

## Sympy [F]

$$\int \frac{x^2}{(c+a^2cx^2)^2 \arctan(ax)^2}\,dx = \frac{\int \frac{x^2}{a^4x^4\operatorname{atan}^2(ax)+2a^2x^2\operatorname{atan}^2(ax)+\operatorname{atan}^2(ax)}\,dx}{c^2}$$

input
```
integrate(x**2/(a**2*c*x**2+c)**2/atan(a*x)**2,x)
```

output
```
Integral(x**2/(a**4*x**4*atan(a*x)**2 + 2*a**2*x**2*atan(a*x)**2 + atan(a*
x)**2), x)/c**2
```

## Maxima [F]

$$\int \frac{x^2}{(c+a^2cx^2)^2 \arctan(ax)^2}\,dx = \int \frac{x^2}{(a^2cx^2+c)^2 \arctan(ax)^2}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
(4*(a^3*c^2*x^2 + a*c^2)*arctan(a*x)*integrate(1/2*x/((a^5*c^2*x^4 + 2*a^3
*c^2*x^2 + a*c^2)*arctan(a*x)), x) - x^2)/((a^3*c^2*x^2 + a*c^2)*arctan(a*
x))
```

## Giac [F]

$$\int \frac{x^2}{(c+a^2cx^2)^2 \arctan(ax)^2} \, dx = \int \frac{x^2}{(a^2cx^2+c)^2 \arctan(ax)^2} \, dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate(x^2/((a^2*c*x^2 + c)^2*arctan(a*x)^2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(c+a^2cx^2)^2 \arctan(ax)^2} \, dx = \int \frac{x^2}{\mathrm{atan}(a\,x)^2\,(c\,a^2\,x^2+c)^2} \, dx$$

input
```
int(x^2/(atan(a*x)^2*(c + a^2*c*x^2)^2),x)
```

output
```
int(x^2/(atan(a*x)^2*(c + a^2*c*x^2)^2), x)
```

## Reduce [F]

$$\int \frac{x^2}{(c+a^2cx^2)^2 \arctan(ax)^2} \, dx$$
$$= \frac{2atan(ax)\left(\int \frac{x}{atan(ax)a^4x^4+2atan(ax)a^2x^2+atan(ax)}dx\right)a^2x^2 + 2atan(ax)\left(\int \frac{x}{atan(ax)a^4x^4+2atan(ax)a^2x^2+atan(ax)}dx\right.}{atan(ax)\,a\,c^2\,(a^2x^2+1)}$$

input
```
int(x^2/(a^2*c*x^2+c)^2/atan(a*x)^2,x)
```

output
```
(2*atan(a*x)*int(x/(atan(a*x)*a**4*x**4 + 2*atan(a*x)*a**2*x**2 + atan(a*x
)),x)*a**2*x**2 + 2*atan(a*x)*int(x/(atan(a*x)*a**4*x**4 + 2*atan(a*x)*a**
2*x**2 + atan(a*x)),x) - x**2)/(atan(a*x)*a*c**2*(a**2*x**2 + 1))
```

# 3.552 $\int \frac{x}{\left(c+a^2cx^2\right)^2 \arctan(ax)^2} dx$



## Optimal result

Integrand size = 20, antiderivative size = 41

$$\int \frac{x}{(c+a^2cx^2)^2 \arctan(ax)^2} dx = -\frac{x}{ac^2(1+a^2x^2)\arctan(ax)} + \frac{\text{CosIntegral}(2\arctan(ax))}{a^2c^2}$$

output
```
-x/a/c^2/(a^2*x^2+1)/arctan(a*x)+Ci(2*arctan(a*x))/a^2/c^2
```

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int \frac{x}{(c+a^2cx^2)^2 \arctan(ax)^2} dx = \frac{-\frac{ax}{(1+a^2x^2)\arctan(ax)} + \text{CosIntegral}(2\arctan(ax))}{a^2c^2}$$

input
```
Integrate[x/((c + a^2*c*x^2)^2*ArcTan[a*x]^2),x]
```

output
```
(-((a*x)/((1 + a^2*x^2)*ArcTan[a*x])) + CosIntegral[2*ArcTan[a*x]])/(a^2*c
^2)
```

## Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 84 vs. $2(41) = 82$.

Time = 0.85 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.05, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5503, 27, 5439, 3042, 3793, 2009, 5505, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\arctan(ax)^2 (a^2cx^2+c)^2}dx$$

$$\Big\downarrow \text{5503}$$

$$\frac{\int \frac{1}{c^2(a^2x^2+1)^2\arctan(ax)}dx}{a}-a\int \frac{x^2}{c^2(a^2x^2+1)^2\arctan(ax)}dx-\frac{x}{ac^2(a^2x^2+1)\arctan(ax)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{1}{(a^2x^2+1)^2\arctan(ax)}dx}{ac^2}-\frac{a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx}{c^2}-\frac{x}{ac^2(a^2x^2+1)\arctan(ax)}$$

$$\Big\downarrow \text{5439}$$

$$-\frac{a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx}{c^2}+\frac{\int \frac{1}{(a^2x^2+1)\arctan(ax)}d\arctan(ax)}{a^2c^2}-\frac{x}{ac^2(a^2x^2+1)\arctan(ax)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx}{c^2}+\frac{\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^2}{\arctan(ax)}d\arctan(ax)}{a^2c^2}-\frac{x}{ac^2(a^2x^2+1)\arctan(ax)}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx}{c^2}+\frac{\int \left(\frac{\cos(2\arctan(ax))}{2\arctan(ax)}+\frac{1}{2\arctan(ax)}\right)d\arctan(ax)}{a^2c^2}-\frac{x}{ac^2(a^2x^2+1)\arctan(ax)}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx}{c^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2c^2}-\frac{x}{ac^2\left(a^2x^2+1\right)\arctan(ax)}$$

$\Big\downarrow$ 5505

$$-\frac{\int \frac{a^2x^2}{(a^2x^2+1)\arctan(ax)}d\arctan(ax)}{a^2c^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2c^2}-\frac{x}{ac^2\left(a^2x^2+1\right)\arctan(ax)}$$

$\Big\downarrow$ 3042

$$-\frac{\int \frac{\sin(\arctan(ax))^2}{\arctan(ax)}d\arctan(ax)}{a^2c^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2c^2}-\frac{x}{ac^2\left(a^2x^2+1\right)\arctan(ax)}$$

$\Big\downarrow$ 3793

$$-\frac{\int \left(\frac{1}{2\arctan(ax)}-\frac{\cos(2\arctan(ax))}{2\arctan(ax)}\right)d\arctan(ax)}{a^2c^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2c^2}-\frac{x}{ac^2\left(a^2x^2+1\right)\arctan(ax)}$$

$\Big\downarrow$ 2009

$$-\frac{\frac{1}{2}\log(\arctan(ax))-\frac{1}{2}\text{CosIntegral}(2\arctan(ax))}{a^2c^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2c^2}-\frac{x}{ac^2\left(a^2x^2+1\right)\arctan(ax)}$$

input
```
Int[x/((c + a^2*c*x^2)^2*ArcTan[a*x]^2),x]
```

output
```
-(x/(a*c^2*(1 + a^2*x^2)*ArcTan[a*x])) - (-1/2*CosIntegral[2*ArcTan[a*x]]
+ Log[ArcTan[a*x]]/2)/(a^2*c^2) + (CosIntegral[2*ArcTan[a*x]]/2 + Log[ArcT
an[a*x]]/2)/(a^2*c^2)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

## Maple [A] (verified)

Time = 8.49 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

| method | result | size |
|---|---|---|
| derivativedivides | $-\frac{-2\,\mathrm{Ci}(2\arctan(ax))\arctan(ax)+\sin(2\arctan(ax))}{2a^2c^2\arctan(ax)}$ | 36 |
| default | $-\frac{-2\,\mathrm{Ci}(2\arctan(ax))\arctan(ax)+\sin(2\arctan(ax))}{2a^2c^2\arctan(ax)}$ | 36 |

input
```
int(x/(a^2*c*x^2+c)^2/arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/a^2/c^2*(-2*Ci(2*arctan(a*x))*arctan(a*x)+sin(2*arctan(a*x)))/arctan(
a*x)
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.80

$$\int \frac{x}{(c+a^2cx^2)^2\arctan(ax)^2}\,dx = \frac{(a^2x^2+1)\arctan(ax)\,\mathrm{log\_integral}\left(-\frac{a^2x^2+2i\,ax-1}{a^2x^2+1}\right)+(a^2x^2+1)\arctan(ax)\,\mathrm{log\_integral}\left(-\frac{a^2x^2-2i\,ax-}{a^2x^2+1}\right.}{2\left(a^4c^2x^2+a^2c^2\right)\arctan(ax)}$$

input
```
integrate(x/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
1/2*((a^2*x^2 + 1)*arctan(a*x)*log_integral(-(a^2*x^2 + 2*I*a*x - 1)/(a^2*
x^2 + 1)) + (a^2*x^2 + 1)*arctan(a*x)*log_integral(-(a^2*x^2 - 2*I*a*x - 1
)/(a^2*x^2 + 1)) - 2*a*x)/((a^4*c^2*x^2 + a^2*c^2)*arctan(a*x))
```

## Sympy [F]

$$\int \frac{x}{(c+a^2cx^2)^2 \arctan(ax)^2}\,dx = \frac{\int \frac{x}{a^4x^4 \operatorname{atan}^2(ax)+2a^2x^2 \operatorname{atan}^2(ax)+\operatorname{atan}^2(ax)}\,dx}{c^2}$$

input
```
integrate(x/(a**2*c*x**2+c)**2/atan(a*x)**2,x)
```

output
```
Integral(x/(a**4*x**4*atan(a*x)**2 + 2*a**2*x**2*atan(a*x)**2 + atan(a*x)*
*2), x)/c**2
```

## Maxima [F]

$$\int \frac{x}{(c+a^2cx^2)^2 \arctan(ax)^2}\,dx = \int \frac{x}{(a^2cx^2+c)^2 \arctan(ax)^2}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-((a^3*c^2*x^2 + a*c^2)*arctan(a*x)*integrate((a^2*x^2 - 1)/((a^5*c^2*x^4
+ 2*a^3*c^2*x^2 + a*c^2)*arctan(a*x)), x) + x)/((a^3*c^2*x^2 + a*c^2)*arct
an(a*x))
```

## Giac [F]

$$\int \frac{x}{(c+a^2cx^2)^2 \arctan(ax)^2}\,dx = \int \frac{x}{(a^2cx^2+c)^2 \arctan(ax)^2}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate(x/((a^2*c*x^2 + c)^2*arctan(a*x)^2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(c+a^2cx^2)^2\arctan(ax)^2}\,dx = \int \frac{x}{\mathrm{atan}\left(a\,x\right)^2\,{\left(c\,a^2\,x^2+c\right)}^2}\,dx$$

input
```
int(x/(atan(a*x)^2*(c + a^2*c*x^2)^2),x)
```

output
```
int(x/(atan(a*x)^2*(c + a^2*c*x^2)^2), x)
```

## Reduce [F]

$$\int \frac{x}{(c+a^2cx^2)^2\arctan(ax)^2}\,dx = \frac{\int \frac{x}{atan(ax)^2a^4x^4+2atan(ax)^2a^2x^2+atan(ax)^2}dx}{c^2}$$

input
```
int(x/(a^2*c*x^2+c)^2/atan(a*x)^2,x)
```

output
```
int(x/(atan(a*x)**2*a**4*x**4 + 2*atan(a*x)**2*a**2*x**2 + atan(a*x)**2),x
)/c**2
```

# 3.553 $\int \frac{1}{(c+a^2cx^2)^2 \arctan(ax)^2}\,dx$



## Optimal result

Integrand size = 19, antiderivative size = 41

$$\int \frac{1}{(c+a^2cx^2)^2 \arctan(ax)^2}\,dx = -\frac{1}{ac^2(1+a^2x^2)\arctan(ax)} - \frac{\mathrm{Si}(2\arctan(ax))}{ac^2}$$

output
```
-1/a/c^2/(a^2*x^2+1)/arctan(a*x)-Si(2*arctan(a*x))/a/c^2
```

## Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int \frac{1}{(c+a^2cx^2)^2 \arctan(ax)^2}\,dx = -\frac{\frac{1}{\arctan(ax)+a^2x^2\arctan(ax)} + \mathrm{Si}(2\arctan(ax))}{ac^2}$$

input
```
Integrate[1/((c + a^2*c*x^2)^2*ArcTan[a*x]^2),x]
```

output
```
-(((ArcTan[a*x] + a^2*x^2*ArcTan[a*x])^(-1) + SinIntegral[2*ArcTan[a*x]])/
(a*c^2))
```

## Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {5437, 27, 5505, 4906, 27, 3042, 3780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arctan(ax)^2 (a^2cx^2+c)^2}dx$$

$$\Big\downarrow \text{5437}$$

$$-2a\int \frac{x}{c^2(a^2x^2+1)^2\arctan(ax)}dx-\frac{1}{ac^2(a^2x^2+1)\arctan(ax)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2a\int \frac{x}{(a^2x^2+1)^2\arctan(ax)}dx}{c^2}-\frac{1}{ac^2(a^2x^2+1)\arctan(ax)}$$

$$\Big\downarrow \text{5505}$$

$$-\frac{2\int \frac{ax}{(a^2x^2+1)\arctan(ax)}d\arctan(ax)}{ac^2}-\frac{1}{ac^2(a^2x^2+1)\arctan(ax)}$$

$$\Big\downarrow \text{4906}$$

$$-\frac{2\int \frac{\sin(2\arctan(ax))}{2\arctan(ax)}d\arctan(ax)}{ac^2}-\frac{1}{ac^2(a^2x^2+1)\arctan(ax)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{ac^2}-\frac{1}{ac^2(a^2x^2+1)\arctan(ax)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{ac^2}-\frac{1}{ac^2(a^2x^2+1)\arctan(ax)}$$

$$\Big\downarrow \text{3780}$$

$$-\frac{1}{ac^2(a^2x^2+1)\arctan(ax)}-\frac{\text{Si}(2\arctan(ax))}{ac^2}$$

input
```
Int[1/((c + a^2*c*x^2)^2*ArcTan[a*x]^2),x]
```

output
```
-(1/(a*c^2*(1 + a^2*x^2)*ArcTan[a*x])) - SinIntegral[2*ArcTan[a*x]]/(a*c^2
)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

## Maple [A] (verified)

Time = 8.05 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

| method | result | size |
|---|---|---|
| derivativedivides | $-\frac{2\,\mathrm{Si}(2\arctan(ax))\arctan(ax)+\cos(2\arctan(ax))+1}{2a\,c^{2}\arctan(ax)}$ | 37 |
| default | $-\frac{2\,\mathrm{Si}(2\arctan(ax))\arctan(ax)+\cos(2\arctan(ax))+1}{2a\,c^{2}\arctan(ax)}$ | 37 |

input
```
int(1/(a^2*c*x^2+c)^2/arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/a/c^2*(2*Si(2*arctan(a*x))*arctan(a*x)+cos(2*arctan(a*x))+1)/arctan(a
*x)
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.73

$$\int \frac{1}{(c+a^2cx^2)^2\arctan(ax)^2}\,dx = \frac{(-i\,a^2x^2-i)\arctan(ax)\,\mathrm{log\_integral}\left(-\frac{a^2x^2+2i\,ax-1}{a^2x^2+1}\right)+(i\,a^2x^2+i)\arctan(ax)\,\mathrm{log\_integral}\left(-\frac{a^2x^2-2i}{a^2x^2}\right.}{2(a^3c^2x^2+ac^2)\arctan(ax)}$$

input
```
integrate(1/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
1/2*((-I*a^2*x^2 - I)*arctan(a*x)*log_integral(-(a^2*x^2 + 2*I*a*x - 1)/(a
^2*x^2 + 1)) + (I*a^2*x^2 + I)*arctan(a*x)*log_integral(-(a^2*x^2 - 2*I*a*
x - 1)/(a^2*x^2 + 1)) - 2)/((a^3*c^2*x^2 + a*c^2)*arctan(a*x))
```

## Sympy [F]

$$\int \frac{1}{\left(c+a^2cx^2\right)^2\arctan(ax)^2}\,dx=\frac{\int \frac{1}{a^4x^4\operatorname{atan}^2(ax)+2a^2x^2\operatorname{atan}^2(ax)+\operatorname{atan}^2(ax)}\,dx}{c^2}$$

input
```
integrate(1/(a**2*c*x**2+c)**2/atan(a*x)**2,x)
```

output
```
Integral(1/(a**4*x**4*atan(a*x)**2 + 2*a**2*x**2*atan(a*x)**2 + atan(a*x)*
*2), x)/c**2
```

## Maxima [F]

$$\int \frac{1}{\left(c+a^2cx^2\right)^2\arctan(ax)^2}\,dx=\int \frac{1}{\left(a^2cx^2+c\right)^2\arctan(ax)^2}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-(4*(a^4*c^2*x^2 + a^2*c^2)*arctan(a*x)*integrate(1/2*x/((a^4*c^2*x^4 + 2*
a^2*c^2*x^2 + c^2)*arctan(a*x)), x) + 1)/((a^3*c^2*x^2 + a*c^2)*arctan(a*x
))
```

## Giac [F]

$$\int \frac{1}{\left(c+a^2cx^2\right)^2\arctan(ax)^2}\,dx=\int \frac{1}{\left(a^2cx^2+c\right)^2\arctan(ax)^2}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^2*arctan(a*x)^2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(c+a^2cx^2\right)^2\arctan(ax)^2}\,dx=\int \frac{1}{\mathrm{atan}\left(a\,x\right)^2\,{\left(c\,a^2\,x^2+c\right)}^2}\,dx$$

input
```
int(1/(atan(a*x)^2*(c + a^2*c*x^2)^2),x)
```

output
```
int(1/(atan(a*x)^2*(c + a^2*c*x^2)^2), x)
```

## Reduce [F]

$$\int \frac{1}{\left(c+a^2cx^2\right)^2\arctan(ax)^2}\,dx$$
$$=\frac{-2atan(ax)\left(\int \frac{x}{atan(ax)a^4x^4+2atan(ax)a^2x^2+atan(ax)}dx\right)a^4x^2-2atan(ax)\left(\int \frac{x}{atan(ax)a^4x^4+2atan(ax)a^2x^2+atan(ax)}\right.}{atan\left(ax\right)a\,c^2\left(a^2x^2+1\right)}$$

input
```
int(1/(a^2*c*x^2+c)^2/atan(a*x)^2,x)
```

output
```
( - 2*atan(a*x)*int(x/(atan(a*x)*a**4*x**4 + 2*atan(a*x)*a**2*x**2 + atan(
a*x)),x)*a**4*x**2 - 2*atan(a*x)*int(x/(atan(a*x)*a**4*x**4 + 2*atan(a*x)*
a**2*x**2 + atan(a*x)),x)*a**2 - 1)/(atan(a*x)*a*c**2*(a**2*x**2 + 1))
```

# 3.554 $\int \frac{1}{x(c+a^2cx^2)^2 \arctan(ax)^2} \, dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2 \arctan(ax)^2} \, dx = -\frac{1}{ac^2x\arctan(ax)} + \frac{ax}{c^2(1+a^2x^2)\arctan(ax)} - \frac{\text{CosIntegral}(2\arctan(ax))}{c^2} - \frac{\text{Int}\left(\frac{1}{x^2\arctan(ax)}, x\right)}{ac^2}$$

output
```
-1/a/c^2/x/arctan(a*x)+a*x/c^2/(a^2*x^2+1)/arctan(a*x)-Ci(2*arctan(a*x))/c
^2-Defer(Int)(1/x^2/arctan(a*x),x)/a/c^2
```

## Mathematica [N/A]

Not integrable

Time = 1.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2 \arctan(ax)^2} \, dx = \int \frac{1}{x\left(c+a^2cx^2\right)^2 \arctan(ax)^2} \, dx$$

input
```
Integrate[1/(x*(c + a^2*c*x^2)^2*ArcTan[a*x]^2),x]
```

output
```
Integrate[1/(x*(c + a^2*c*x^2)^2*ArcTan[a*x]^2), x]
```

## Rubi [N/A]

Not integrable

Time = 1.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x \arctan(ax)^2 (a^2cx^2+c)^2} dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{1}{cx(a^2x^2+1)\arctan(ax)^2}dx}{c} - a^2 \int \frac{x}{c^2(a^2x^2+1)^2 \arctan(ax)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{1}{x(a^2x^2+1)\arctan(ax)^2}dx}{c^2} - \frac{a^2 \int \frac{x}{(a^2x^2+1)^2 \arctan(ax)^2}dx}{c^2}$$

$$\Big\downarrow \text{5461}$$

$$\frac{-\frac{\int \frac{1}{x^2\arctan(ax)}dx}{a} - \frac{1}{ax\arctan(ax)}}{c^2} - \frac{a^2 \int \frac{x}{(a^2x^2+1)^2 \arctan(ax)^2}dx}{c^2}$$

$$\Big\downarrow \text{5377}$$

$$\frac{-\frac{\int \frac{1}{x^2\arctan(ax)}dx}{a} - \frac{1}{ax\arctan(ax)}}{c^2} - \frac{a^2 \int \frac{x}{(a^2x^2+1)^2 \arctan(ax)^2}dx}{c^2}$$

$$\Big\downarrow \text{5503}$$

$$\frac{-\frac{\int \frac{1}{x^2\arctan(ax)}dx}{a} - \frac{1}{ax\arctan(ax)}}{c^2} - \frac{a^2 \left(\frac{\int \frac{1}{(a^2x^2+1)^2\arctan(ax)}dx}{a} - a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx - \frac{x}{a(a^2x^2+1)\arctan(ax)}\right)}{c^2}$$

$$\Big\downarrow \text{5439}$$

$$\frac{\frac{-\frac{\int \frac{1}{x^2\arctan(ax)}dx}{a}-\frac{1}{ax\arctan(ax)}}{c^2}-\frac{a^2\left(-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\int \frac{1}{\left(a^2x^2+1\right)\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)}{c^2}}{}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{\int \frac{1}{x^2\arctan(ax)}dx}{a}-\frac{1}{ax\arctan(ax)}}{c^2}-\frac{a^2\left(-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^2}{\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)}{c^2}$$

$$\Big\downarrow \text{3793}$$

$$\frac{-\frac{\int \frac{1}{x^2\arctan(ax)}dx}{a}-\frac{1}{ax\arctan(ax)}}{c^2}-\frac{a^2\left(-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\int \left(\frac{\cos(2\arctan(ax))}{2\arctan(ax)}+\frac{1}{2\arctan(ax)}\right)d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)}{c^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{\int \frac{1}{x^2\arctan(ax)}dx}{a}-\frac{1}{ax\arctan(ax)}}{c^2}-\frac{a^2\left(-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)}{c^2}$$

$$\Big\downarrow \text{5505}$$

$$\frac{-\frac{\int \frac{1}{x^2\arctan(ax)}dx}{a}-\frac{1}{ax\arctan(ax)}}{c^2}-\frac{a^2\left(-\frac{\int \frac{a^2x^2}{\left(a^2x^2+1\right)\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)}{c^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{\int \frac{1}{x^2\arctan(ax)}dx}{a}-\frac{1}{ax\arctan(ax)}}{c^2}-\frac{a^2\left(-\frac{\int \frac{\sin(\arctan(ax))^2}{\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)}{c^2}$$

$$\Big\downarrow \text{3793}$$

$$\frac{-\frac{\int \frac{1}{x^2\arctan(ax)}dx}{a}-\frac{1}{ax\arctan(ax)}}{c^2}-\frac{a^2\left(-\frac{\int \left(\frac{1}{2\arctan(ax)}-\frac{\cos(2\arctan(ax))}{2\arctan(ax)}\right)d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)}{c^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{\int \frac{1}{x^2\arctan(ax)}dx}{a}-\frac{1}{ax\arctan(ax)}}{c^2}-\frac{a^2\left(-\frac{\frac{1}{2}\log(\arctan(ax))-\frac{1}{2}\text{CosIntegral}(2\arctan(ax))}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)}{c^2}$$

input
```
Int[1/(x*(c + a^2*c*x^2)^2*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 4.66 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\,(a^2c\,x^2+c)^2\arctan\,(ax)^2}dx$$

input
```
int(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

output
```
int(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.73

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\arctan(ax)^2}\,dx=\int \frac{1}{\left(a^2cx^2+c\right)^2x\arctan\left(ax\right)^2}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(1/((a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^2*x)*arctan(a*x)^2), x)
```

## Sympy [N/A]

Not integrable

Time = 1.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.91

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\arctan(ax)^2}\,dx=\frac{\int \frac{1}{a^4x^5\operatorname{atan}^2(ax)+2a^2x^3\operatorname{atan}^2(ax)+x\operatorname{atan}^2(ax)}\,dx}{c^2}$$

input
```
integrate(1/x/(a**2*c*x**2+c)**2/atan(a*x)**2,x)
```

output
```
Integral(1/(a**4*x**5*atan(a*x)**2 + 2*a**2*x**3*atan(a*x)**2 + x*atan(a*x
)**2), x)/c**2
```

## Maxima [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 102, normalized size of antiderivative = 4.64

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\arctan(ax)^2}\,dx=\int \frac{1}{\left(a^2cx^2+c\right)^2x\arctan\left(ax\right)^2}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-((a^3*c^2*x^3 + a*c^2*x)*arctan(a*x)*integrate((3*a^2*x^2 + 1)/((a^5*c^2*
x^6 + 2*a^3*c^2*x^4 + a*c^2*x^2)*arctan(a*x)), x) + 1)/((a^3*c^2*x^3 + a*c
^2*x)*arctan(a*x))
```

## Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\arctan(ax)^2}\,dx=\int \frac{1}{\left(a^2cx^2+c\right)^2x\arctan\left(ax\right)^2}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^2*x*arctan(a*x)^2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\arctan(ax)^2}\,dx=\int \frac{1}{x\,\mathrm{atan}\left(a\,x\right)^2\,\left(c\,a^2\,x^2+c\right)^2}\,dx$$

input
```
int(1/(x*atan(a*x)^2*(c + a^2*c*x^2)^2),x)
```

output
```
int(1/(x*atan(a*x)^2*(c + a^2*c*x^2)^2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\arctan(ax)^2}\,dx=\frac{\int \frac{1}{atan(ax)^2a^4x^5+2atan(ax)^2a^2x^3+atan(ax)^2x}dx}{c^2}$$

input
```
int(1/x/(a^2*c*x^2+c)^2/atan(a*x)^2,x)
```

output
```
int(1/(atan(a*x)**2*a**4*x**5 + 2*atan(a*x)**2*a**2*x**3 + atan(a*x)**2*x)
,x)/c**2
```

# 3.555 $\int \frac{1}{x^2(c+a^2cx^2)^2\arctan(ax)^2}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x^2(c+a^2cx^2)^2\arctan(ax)^2}\,dx = -\frac{1}{ac^2x^2\arctan(ax)} + \frac{a}{c^2(1+a^2x^2)\arctan(ax)} + \frac{a\text{Si}(2\arctan(ax))}{c^2} - \frac{2\text{Int}\left(\frac{1}{x^3\arctan(ax)},x\right)}{ac^2}$$

output
```
-1/a/c^2/x^2/arctan(a*x)+a/c^2/(a^2*x^2+1)/arctan(a*x)+a*Si(2*arctan(a*x))
/c^2-2*Defer(Int)(1/x^3/arctan(a*x),x)/a/c^2
```

## Mathematica [N/A]

Not integrable

Time = 2.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2(c+a^2cx^2)^2\arctan(ax)^2}\,dx = \int \frac{1}{x^2(c+a^2cx^2)^2\arctan(ax)^2}\,dx$$

input
```
Integrate[1/(x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^2),x]
```

output
```
Integrate[1/(x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \arctan(ax)^2 (a^2cx^2+c)^2}dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{1}{cx^2(a^2x^2+1)\arctan(ax)^2}dx}{c} - a^2\int \frac{1}{c^2(a^2x^2+1)^2\arctan(ax)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{1}{x^2(a^2x^2+1)\arctan(ax)^2}dx}{c^2} - \frac{a^2\int \frac{1}{(a^2x^2+1)^2\arctan(ax)^2}dx}{c^2}$$

$$\Big\downarrow \text{5437}$$

$$\frac{\int \frac{1}{x^2(a^2x^2+1)\arctan(ax)^2}dx}{c^2} - \frac{a^2\left(-2a\int \frac{x}{(a^2x^2+1)^2\arctan(ax)}dx - \frac{1}{a(a^2x^2+1)\arctan(ax)}\right)}{c^2}$$

$$\Big\downarrow \text{5461}$$

$$\frac{-\frac{2\int \frac{1}{x^3\arctan(ax)}dx}{a} - \frac{1}{ax^2\arctan(ax)}}{c^2} - \frac{a^2\left(-2a\int \frac{x}{(a^2x^2+1)^2\arctan(ax)}dx - \frac{1}{a(a^2x^2+1)\arctan(ax)}\right)}{c^2}$$

$$\Big\downarrow \text{5377}$$

$$\frac{-\frac{2\int \frac{1}{x^3\arctan(ax)}dx}{a} - \frac{1}{ax^2\arctan(ax)}}{c^2} - \frac{a^2\left(-2a\int \frac{x}{(a^2x^2+1)^2\arctan(ax)}dx - \frac{1}{a(a^2x^2+1)\arctan(ax)}\right)}{c^2}$$

$$\Big\downarrow \text{5505}$$

$$\frac{-\frac{2\int \frac{1}{x^3\arctan(ax)}dx}{a}-\frac{1}{ax^2\arctan(ax)}}{c^2}-\frac{a^2\left(-\frac{2\int \frac{ax}{(a^2x^2+1)\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a(a^2x^2+1)\arctan(ax)}\right)}{c^2}$$

$\big\downarrow$ 4906

$$\frac{-\frac{2\int \frac{1}{x^3\arctan(ax)}dx}{a}-\frac{1}{ax^2\arctan(ax)}}{c^2}-\frac{a^2\left(-\frac{2\int \frac{\sin(2\arctan(ax))}{2\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a(a^2x^2+1)\arctan(ax)}\right)}{c^2}$$

$\big\downarrow$ 27

$$\frac{-\frac{2\int \frac{1}{x^3\arctan(ax)}dx}{a}-\frac{1}{ax^2\arctan(ax)}}{c^2}-\frac{a^2\left(-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a(a^2x^2+1)\arctan(ax)}\right)}{c^2}$$

$\big\downarrow$ 3042

$$\frac{-\frac{2\int \frac{1}{x^3\arctan(ax)}dx}{a}-\frac{1}{ax^2\arctan(ax)}}{c^2}-\frac{a^2\left(-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a(a^2x^2+1)\arctan(ax)}\right)}{c^2}$$

$\big\downarrow$ 3780

$$\frac{-\frac{2\int \frac{1}{x^3\arctan(ax)}dx}{a}-\frac{1}{ax^2\arctan(ax)}}{c^2}-\frac{a^2\left(-\frac{1}{a(a^2x^2+1)\arctan(ax)}-\frac{\text{Si}(2\arctan(ax))}{a}\right)}{c^2}$$

input
```
Int[1/(x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 5.95 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\left(a^2c\,x^2+c\right)^2\arctan\left(ax\right)^2}dx$$

input
```
int(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

output
```
int(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.82

$$\int \frac{1}{x^2 (c + a^2 c x^2)^2 \arctan(ax)^2} \, dx = \int \frac{1}{(a^2 c x^2 + c)^2 x^2 \arctan(ax)^2} \, dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(1/((a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2)*arctan(a*x)^2), x)
```

## Sympy [N/A]

Not integrable

Time = 1.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.00

$$\int \frac{1}{x^2 (c + a^2 c x^2)^2 \arctan(ax)^2} \, dx = \frac{\int \frac{1}{a^4 x^6 \operatorname{atan}^2(ax) + 2a^2 x^4 \operatorname{atan}^2(ax) + x^2 \operatorname{atan}^2(ax)} \, dx}{c^2}$$

input
```
integrate(1/x**2/(a**2*c*x**2+c)**2/atan(a*x)**2,x)
```

output
```
Integral(1/(a**4*x**6*atan(a*x)**2 + 2*a**2*x**4*atan(a*x)**2 + x**2*atan(
a*x)**2), x)/c**2
```

## Maxima [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 107, normalized size of antiderivative = 4.86

$$\int \frac{1}{x^2 (c + a^2cx^2)^2 \arctan(ax)^2} \, dx = \int \frac{1}{(a^2cx^2 + c)^2 x^2 \arctan(ax)^2} \, dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-((a^3*c^2*x^4 + a*c^2*x^2)*arctan(a*x)*integrate(2*(2*a^2*x^2 + 1)/((a^5*
c^2*x^7 + 2*a^3*c^2*x^5 + a*c^2*x^3)*arctan(a*x)), x) + 1)/((a^3*c^2*x^4 +
 a*c^2*x^2)*arctan(a*x))
```

## Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2 (c + a^2cx^2)^2 \arctan(ax)^2} \, dx = \int \frac{1}{(a^2cx^2 + c)^2 x^2 \arctan(ax)^2} \, dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^2*x^2*arctan(a*x)^2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2 (c + a^2cx^2)^2 \arctan(ax)^2} \, dx = \int \frac{1}{x^2 \,\mathrm{atan}(a\,x)^2 \,(c\,a^2\,x^2 + c)^2} \, dx$$

input
```
int(1/(x^2*atan(a*x)^2*(c + a^2*c*x^2)^2),x)
```

output
```
int(1/(x^2*atan(a*x)^2*(c + a^2*c*x^2)^2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.09

$$\int \frac{1}{x^2 (c + a^2cx^2)^2 \arctan(ax)^2} \, dx = \frac{\int \frac{1}{atan(ax)^2 a^4 x^6 + 2atan(ax)^2 a^2 x^4 + atan(ax)^2 x^2} dx}{c^2}$$

input
```
int(1/x^2/(a^2*c*x^2+c)^2/atan(a*x)^2,x)
```

output
```
int(1/(atan(a*x)**2*a**4*x**6 + 2*atan(a*x)**2*a**2*x**4 + atan(a*x)**2*x*
*2),x)/c**2
```

# 3.556 $\int \frac{1}{x^3(c+a^2cx^2)^2 \arctan(ax)^2} \, dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x^3 (c+a^2cx^2)^2 \arctan(ax)^2} \, dx = -\frac{1}{ac^2x^3 \arctan(ax)} + \frac{a}{c^2x \arctan(ax)} - \frac{a^3x}{c^2 (1+a^2x^2) \arctan(ax)} + \frac{a^2 \operatorname{CosIntegral}(2\arctan(ax))}{c^2} - \frac{3\mathrm{Int}\left(\frac{1}{x^4 \arctan(ax)}, x\right)}{ac^2} + \frac{a\mathrm{Int}\left(\frac{1}{x^2 \arctan(ax)}, x\right)}{c^2}$$

output
```
-1/a/c^2/x^3/arctan(a*x)+a/c^2/x/arctan(a*x)-a^3*x/c^2/(a^2*x^2+1)/arctan(
a*x)+a^2*Ci(2*arctan(a*x))/c^2-3*Defer(Int)(1/x^4/arctan(a*x),x)/a/c^2+a*D
efer(Int)(1/x^2/arctan(a*x),x)/c^2
```

## Mathematica [N/A]

Not integrable

Time = 2.43 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^3 (c+a^2cx^2)^2 \arctan(ax)^2} \, dx = \int \frac{1}{x^3 (c+a^2cx^2)^2 \arctan(ax)^2} \, dx$$

input
```
Integrate[1/(x^3*(c + a^2*c*x^2)^2*ArcTan[a*x]^2),x]
```

output
```
Integrate[1/(x^3*(c + a^2*c*x^2)^2*ArcTan[a*x]^2), x]
```

## Rubi [N/A]

Not integrable

Time = 1.79 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \arctan(ax)^2 (a^2cx^2+c)^2} \, dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{1}{cx^3(a^2x^2+1)\arctan(ax)^2}dx}{c} - a^2 \int \frac{1}{c^2x\left(a^2x^2+1\right)^2 \arctan(ax)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{1}{x^3(a^2x^2+1)\arctan(ax)^2}dx}{c^2} - \frac{a^2 \int \frac{1}{x(a^2x^2+1)^2\arctan(ax)^2}dx}{c^2}$$

$$\Big\downarrow \text{5461}$$

$$\frac{-\frac{3\int \frac{1}{x^4\arctan(ax)}dx}{a} - \frac{1}{ax^3\arctan(ax)}}{c^2} - \frac{a^2 \int \frac{1}{x(a^2x^2+1)^2\arctan(ax)^2}dx}{c^2}$$

$\downarrow$ 5377

$$\frac{-\frac{3\int \frac{1}{x^4\arctan(ax)}dx}{a}-\frac{1}{ax^3\arctan(ax)}}{c^2}-\frac{a^2\int \frac{1}{x(a^2x^2+1)^2\arctan(ax)^2}dx}{c^2}$$

$\downarrow$ 5501

$$\frac{-\frac{3\int \frac{1}{x^4\arctan(ax)}dx}{a}-\frac{1}{ax^3\arctan(ax)}}{c^2}-\frac{a^2\left(\int \frac{1}{x(a^2x^2+1)\arctan(ax)^2}dx-a^2\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^2}dx\right)}{c^2}$$

$\downarrow$ 5461

$$\frac{-\frac{3\int \frac{1}{x^4\arctan(ax)}dx}{a}-\frac{1}{ax^3\arctan(ax)}}{c^2}-\frac{a^2\left(a^2\left(-\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^2}dx\right)-\frac{\int \frac{1}{x^2\arctan(ax)}dx}{a}-\frac{1}{ax\arctan(ax)}\right)}{c^2}$$

$\downarrow$ 5377

$$\frac{-\frac{3\int \frac{1}{x^4\arctan(ax)}dx}{a}-\frac{1}{ax^3\arctan(ax)}}{c^2}-\frac{a^2\left(a^2\left(-\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^2}dx\right)-\frac{\int \frac{1}{x^2\arctan(ax)}dx}{a}-\frac{1}{ax\arctan(ax)}\right)}{c^2}$$

$\downarrow$ 5503

$$\frac{-\frac{3\int \frac{1}{x^4\arctan(ax)}dx}{a}-\frac{1}{ax^3\arctan(ax)}}{c^2}-\frac{a^2\left(-\left(a^2\left(\frac{\int \frac{1}{(a^2x^2+1)^2\arctan(ax)}dx}{a}-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)\right)\right)-\frac{\int \frac{1}{x^2\arctan(ax)}dx}{a}-\frac{1}{ax\arctan(ax)}}{c^2}$$

$\downarrow$ 5439

$$\frac{-\frac{3\int \frac{1}{x^4\arctan(ax)}dx}{a}-\frac{1}{ax^3\arctan(ax)}}{c^2}-\frac{a^2\left(-\left(a^2\left(-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\int \frac{1}{(a^2x^2+1)\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)\right)\right)-\frac{\int \frac{1}{x^2\arctan(ax)}dx}{a}-}{c^2}$$

$\downarrow$ 3042

$$\frac{a^2\left(-\left(a^2\left(-a\int\frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\int\frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^2}{\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{\int\frac{1}{x^2\arctan(ax)}dx}{a}-\right.}{c^2}\quad\frac{-\frac{3\int\frac{1}{x^4\arctan(ax)}dx}{a}-\frac{1}{ax^3\arctan(ax)}}{c^2}-$$

$\downarrow$ 3793

$$\frac{a^2\left(-\left(a^2\left(-a\int\frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\int\left(\frac{\cos(2\arctan(ax))}{2\arctan(ax)}+\frac{1}{2\arctan(ax)}\right)d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{\int\frac{1}{x^2\arctan(ax)}}{a}\right.}{c^2}\quad\frac{-\frac{3\int\frac{1}{x^4\arctan(ax)}dx}{a}-\frac{1}{ax^3\arctan(ax)}}{c^2}-$$

$\downarrow$ 2009

$$\frac{a^2\left(-\left(a^2\left(-a\int\frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{\int\frac{1}{x^2\arctan}}{a}\right.}{c^2}\quad\frac{-\frac{3\int\frac{1}{x^4\arctan(ax)}dx}{a}-\frac{1}{ax^3\arctan(ax)}}{c^2}-$$

$\downarrow$ 5505

$$\frac{a^2\left(-\left(a^2\left(-\frac{\int\frac{a^2x^2}{(a^2x^2+1)\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{\int}{}\right.}{c^2}\quad\frac{-\frac{3\int\frac{1}{x^4\arctan(ax)}dx}{a}-\frac{1}{ax^3\arctan(ax)}}{c^2}-$$

$\downarrow$ 3042

$$\frac{a^2\left(-\left(a^2\left(-\frac{\int\frac{\sin(\arctan(ax))^2}{\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{\int\frac{}{x^2\arctan}}{}\right.}{c^2}\quad\frac{-\frac{3\int\frac{1}{x^4\arctan(ax)}dx}{a}-\frac{1}{ax^3\arctan(ax)}}{c^2}-$$

$\downarrow$ 3793

$$\frac{-\frac{3\int \frac{1}{x^4 \arctan(ax)}dx}{a} - \frac{1}{ax^3\arctan(ax)}}{c^2} - \frac{a^2\left(-\left(a^2\left(-\frac{\int \left(\frac{1}{2\arctan(ax)} - \frac{\cos(2\arctan(ax))}{2\arctan(ax)}\right)d\arctan(ax)}{a^2} + \frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax)) + \frac{1}{2}\log(\arctan(ax))}{a^2} - \frac{x}{a(a^2x^2+1)\arctan(ax)}\right.\right.\right.}{c^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{3\int \frac{1}{x^4 \arctan(ax)}dx}{a} - \frac{1}{ax^3\arctan(ax)}}{c^2} - \frac{a^2\left(-\frac{\int \frac{1}{x^2\arctan(ax)}dx}{a} - \left(a^2\left(-\frac{\frac{1}{2}\log(\arctan(ax)) - \frac{1}{2}\text{CosIntegral}(2\arctan(ax))}{a^2} + \frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax)) + \frac{1}{2}\log(\arctan(ax))}{a^2} - \right.\right.\right.}{c^2}$$

input
```
Int[1/(x^3*(c + a^2*c*x^2)^2*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 74.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3 (a^2c\,x^2 + c)^2 \arctan(ax)^2}dx$$

input
```
int(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

output
```
int(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.82

$$\int \frac{1}{x^3 (c + a^2 c x^2)^2 \arctan(ax)^2} \, dx = \int \frac{1}{(a^2 c x^2 + c)^2 x^3 \arctan{(ax)}^2} \, dx$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(1/((a^4*c^2*x^7 + 2*a^2*c^2*x^5 + c^2*x^3)*arctan(a*x)^2), x)
```

## Sympy [N/A]

Not integrable

Time = 1.07 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.00

$$\int \frac{1}{x^3 (c + a^2 c x^2)^2 \arctan(ax)^2} \, dx = \frac{\int \frac{1}{a^4 x^7 \operatorname{atan}^2{(ax)} + 2a^2 x^5 \operatorname{atan}^2{(ax)} + x^3 \operatorname{atan}^2{(ax)}} \, dx}{c^2}$$

input
```
integrate(1/x**3/(a**2*c*x**2+c)**2/atan(a*x)**2,x)
```

output
```
Integral(1/(a**4*x**7*atan(a*x)**2 + 2*a**2*x**5*atan(a*x)**2 + x**3*atan(
a*x)**2), x)/c**2
```

## Maxima [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 106, normalized size of antiderivative = 4.82

$$\int \frac{1}{x^3 (c + a^2 c x^2)^2 \arctan(ax)^2} \, dx = \int \frac{1}{(a^2 c x^2 + c)^2 x^3 \arctan{(ax)}^2} \, dx$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-((a^3*c^2*x^5 + a*c^2*x^3)*arctan(a*x)*integrate((5*a^2*x^2 + 3)/((a^5*c^
2*x^8 + 2*a^3*c^2*x^6 + a*c^2*x^4)*arctan(a*x)), x) + 1)/((a^3*c^2*x^5 + a
*c^2*x^3)*arctan(a*x))
```

## Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^3 (c + a^2cx^2)^2 \arctan(ax)^2} \, dx = \int \frac{1}{(a^2cx^2 + c)^2 x^3 \arctan(ax)^2} \, dx$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^2*x^3*arctan(a*x)^2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^3 (c + a^2cx^2)^2 \arctan(ax)^2} \, dx = \int \frac{1}{x^3 \operatorname{atan}(a\,x)^2 \,(c\,a^2\,x^2 + c)^2} \, dx$$

input
```
int(1/(x^3*atan(a*x)^2*(c + a^2*c*x^2)^2),x)
```

output
```
int(1/(x^3*atan(a*x)^2*(c + a^2*c*x^2)^2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.09

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^2\arctan(ax)^2}\,dx = \frac{\int \frac{1}{atan(ax)^2a^4x^7+2atan(ax)^2a^2x^5+atan(ax)^2x^3}dx}{c^2}$$

input
```
int(1/x^3/(a^2*c*x^2+c)^2/atan(a*x)^2,x)
```

output
```
int(1/(atan(a*x)**2*a**4*x**7 + 2*atan(a*x)**2*a**2*x**5 + atan(a*x)**2*x*
*3),x)/c**2
```

# 3.557 $\int \frac{1}{x^4(c+a^2cx^2)^2 \arctan(ax)^2} dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x^4 (c+a^2cx^2)^2 \arctan(ax)^2} dx = -\frac{1}{ac^2x^4 \arctan(ax)} + \frac{a}{c^2x^2 \arctan(ax)} - \frac{a^3}{c^2 (1+a^2x^2) \arctan(ax)} - \frac{a^3 \mathrm{Si}(2\arctan(ax))}{c^2} - \frac{4\mathrm{Int}\left(\frac{1}{x^5 \arctan(ax)}, x\right)}{ac^2} + \frac{2a\mathrm{Int}\left(\frac{1}{x^3 \arctan(ax)}, x\right)}{c^2}$$

output
```
-1/a/c^2/x^4/arctan(a*x)+a/c^2/x^2/arctan(a*x)-a^3/c^2/(a^2*x^2+1)/arctan(
a*x)-a^3*Si(2*arctan(a*x))/c^2-4*Defer(Int)(1/x^5/arctan(a*x),x)/a/c^2+2*a
*Defer(Int)(1/x^3/arctan(a*x),x)/c^2
```

## Mathematica [N/A]

Not integrable

Time = 2.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^4 (c+a^2cx^2)^2 \arctan(ax)^2} dx = \int \frac{1}{x^4 (c+a^2cx^2)^2 \arctan(ax)^2} dx$$

input
```
Integrate[1/(x^4*(c + a^2*c*x^2)^2*ArcTan[a*x]^2),x]
```

output
```
Integrate[1/(x^4*(c + a^2*c*x^2)^2*ArcTan[a*x]^2), x]
```

## Rubi [N/A]

Not integrable

Time = 1.43 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \arctan(ax)^2 \left(a^2cx^2+c\right)^2}dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{1}{cx^4(a^2x^2+1)\arctan(ax)^2}dx}{c}-a^2\int \frac{1}{c^2x^2\left(a^2x^2+1\right)^2\arctan(ax)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{1}{x^4(a^2x^2+1)\arctan(ax)^2}dx}{c^2}-\frac{a^2\int \frac{1}{x^2(a^2x^2+1)^2\arctan(ax)^2}dx}{c^2}$$

$$\Big\downarrow \text{5461}$$

$$\frac{-\frac{4\int \frac{1}{x^5\arctan(ax)}dx}{a}-\frac{1}{ax^4\arctan(ax)}}{c^2}-\frac{a^2\int \frac{1}{x^2(a^2x^2+1)^2\arctan(ax)^2}dx}{c^2}$$

$$\Big\downarrow \text{5377}$$

$$\frac{-\frac{4\int \frac{1}{x^5\arctan(ax)}dx}{a}-\frac{1}{ax^4\arctan(ax)}}{c^2}-\frac{a^2\int \frac{1}{x^2(a^2x^2+1)^2\arctan(ax)^2}dx}{c^2}$$

$$\Big\downarrow \text{5501}$$

$$\frac{-\frac{4\int \frac{1}{x^5\arctan(ax)}dx}{a}-\frac{1}{ax^4\arctan(ax)}}{c^2}-\frac{a^2\left(\int \frac{1}{x^2(a^2x^2+1)\arctan(ax)^2}dx-a^2\int \frac{1}{(a^2x^2+1)^2\arctan(ax)^2}dx\right)}{c^2}$$

$$\Big\downarrow \ 5437$$

$$\frac{-\frac{4\int \frac{1}{x^5 \arctan(ax)}dx}{a}-\frac{1}{ax^4\arctan(ax)}}{c^2}-\frac{a^2\left(\int \frac{1}{x^2(a^2x^2+1)\arctan(ax)^2}dx-a^2\left(-2a\int \frac{x}{(a^2x^2+1)^2\arctan(ax)}dx-\frac{1}{a(a^2x^2+1)\arctan(ax)}\right)\right)}{c^2}$$

$$\Big\downarrow \ 5461$$

$$\frac{-\frac{4\int \frac{1}{x^5 \arctan(ax)}dx}{a}-\frac{1}{ax^4\arctan(ax)}}{c^2}-\frac{a^2\left(-\left(a^2\left(-2a\int \frac{x}{(a^2x^2+1)^2\arctan(ax)}dx-\frac{1}{a(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{2\int \frac{1}{x^3\arctan(ax)}dx}{a}-\frac{1}{ax^2\arctan(ax)}\right)}{c^2}$$

$$\Big\downarrow \ 5377$$

$$\frac{-\frac{4\int \frac{1}{x^5 \arctan(ax)}dx}{a}-\frac{1}{ax^4\arctan(ax)}}{c^2}-\frac{a^2\left(-\left(a^2\left(-2a\int \frac{x}{(a^2x^2+1)^2\arctan(ax)}dx-\frac{1}{a(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{2\int \frac{1}{x^3\arctan(ax)}dx}{a}-\frac{1}{ax^2\arctan(ax)}\right)}{c^2}$$

$$\Big\downarrow \ 5505$$

$$\frac{-\frac{4\int \frac{1}{x^5 \arctan(ax)}dx}{a}-\frac{1}{ax^4\arctan(ax)}}{c^2}-\frac{a^2\left(-\left(a^2\left(-\frac{2\int \frac{ax}{\left(a^2x^2+1\right)\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{2\int \frac{1}{x^3\arctan(ax)}dx}{a}-\frac{1}{ax^2\arctan(ax)}\right)}{c^2}$$

$$\Big\downarrow \ 4906$$

$$\frac{-\frac{4\int \frac{1}{x^5 \arctan(ax)}dx}{a}-\frac{1}{ax^4\arctan(ax)}}{c^2}-\frac{a^2\left(-\left(a^2\left(-\frac{2\int \frac{\sin(2\arctan(ax))}{2\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{2\int \frac{1}{x^3\arctan(ax)}dx}{a}-\frac{1}{ax^2\arctan(ax)}\right)}{c^2}$$

$$\Big\downarrow \ 27$$

$$\frac{-\frac{4\int \frac{1}{x^5 \arctan(ax)}dx}{a} - \frac{1}{ax^4 \arctan(ax)}}{c^2} - \frac{a^2\left(-\left(a^2\left(-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a} - \frac{1}{a(a^2x^2+1)\arctan(ax)}\right)\right) - \frac{2\int \frac{1}{x^3\arctan(ax)}dx}{a} - \frac{1}{ax^2\arctan(ax)}\right)}{c^2}$$

$\downarrow$ 3042

$$\frac{-\frac{4\int \frac{1}{x^5 \arctan(ax)}dx}{a} - \frac{1}{ax^4 \arctan(ax)}}{c^2} - \frac{a^2\left(-\left(a^2\left(-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a} - \frac{1}{a(a^2x^2+1)\arctan(ax)}\right)\right) - \frac{2\int \frac{1}{x^3\arctan(ax)}dx}{a} - \frac{1}{ax^2\arctan(ax)}\right)}{c^2}$$

$\downarrow$ 3780

$$\frac{-\frac{4\int \frac{1}{x^5 \arctan(ax)}dx}{a} - \frac{1}{ax^4 \arctan(ax)}}{c^2} - \frac{a^2\left(-\frac{2\int \frac{1}{x^3\arctan(ax)}dx}{a} - \left(a^2\left(-\frac{1}{a(a^2x^2+1)\arctan(ax)} - \frac{\text{Si}(2\arctan(ax))}{a}\right)\right) - \frac{1}{ax^2\arctan(ax)}\right)}{c^2}$$

input
```
Int[1/(x^4*(c + a^2*c*x^2)^2*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 81.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4 \left(a^2c\,x^2+c\right)^2 \arctan\left(ax\right)^2}dx$$

input
```
int(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

output
```
int(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.82

$$\int \frac{1}{x^4 (c+a^2cx^2)^2 \arctan(ax)^2} \, dx = \int \frac{1}{(a^2cx^2+c)^2 x^4 \arctan(ax)^2} \, dx$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(1/((a^4*c^2*x^8 + 2*a^2*c^2*x^6 + c^2*x^4)*arctan(a*x)^2), x)
```

## Sympy [N/A]

Not integrable

Time = 1.16 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.00

$$\int \frac{1}{x^4 (c+a^2cx^2)^2 \arctan(ax)^2} \, dx = \frac{\int \frac{1}{a^4x^8 \operatorname{atan}^2(ax)+2a^2x^6 \operatorname{atan}^2(ax)+x^4 \operatorname{atan}^2(ax)} \, dx}{c^2}$$

input
```
integrate(1/x**4/(a**2*c*x**2+c)**2/atan(a*x)**2,x)
```

output
```
Integral(1/(a**4*x**8*atan(a*x)**2 + 2*a**2*x**6*atan(a*x)**2 + x**4*atan(
a*x)**2), x)/c**2
```

## Maxima [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 107, normalized size of antiderivative = 4.86

$$\int \frac{1}{x^4 (c+a^2cx^2)^2 \arctan(ax)^2} \, dx = \int \frac{1}{(a^2cx^2+c)^2 x^4 \arctan(ax)^2} \, dx$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-((a^3*c^2*x^6 + a*c^2*x^4)*arctan(a*x)*integrate(2*(3*a^2*x^2 + 2)/((a^5*
c^2*x^9 + 2*a^3*c^2*x^7 + a*c^2*x^5)*arctan(a*x)), x) + 1)/((a^3*c^2*x^6 +
 a*c^2*x^4)*arctan(a*x))
```

## Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^2\arctan(ax)^2}\,dx=\int \frac{1}{\left(a^2cx^2+c\right)^2x^4\arctan\left(ax\right)^2}\,dx$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^2*x^4*arctan(a*x)^2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^2\arctan(ax)^2}\,dx=\int \frac{1}{x^4\,\mathrm{atan}\left(a\,x\right)^2\,\left(c\,a^2\,x^2+c\right)^2}\,dx$$

input
```
int(1/(x^4*atan(a*x)^2*(c + a^2*c*x^2)^2),x)
```

output
```
int(1/(x^4*atan(a*x)^2*(c + a^2*c*x^2)^2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.09

$$\int \frac{1}{x^4 (c + a^2 c x^2)^2 \arctan(ax)^2} \, dx = \frac{\int \frac{1}{atan(ax)^2 a^4 x^8 + 2atan(ax)^2 a^2 x^6 + atan(ax)^2 x^4} dx}{c^2}$$

input
```
int(1/x^4/(a^2*c*x^2+c)^2/atan(a*x)^2,x)
```

output
```
int(1/(atan(a*x)**2*a**4*x**8 + 2*atan(a*x)**2*a**2*x**6 + atan(a*x)**2*x*
*4),x)/c**2
```

# 3.558 $\int \frac{x^3}{(c+a^2cx^2)^3 \arctan(ax)^2} \, dx$



## Optimal result

Integrand size = 22, antiderivative size = 86

$$\int \frac{x^3}{(c+a^2cx^2)^3 \arctan(ax)^2} \, dx = \frac{x}{a^3c^3(1+a^2x^2)^2 \arctan(ax)} - \frac{x}{a^3c^3(1+a^2x^2)\arctan(ax)} + \frac{\text{CosIntegral}(2\arctan(ax))}{2a^4c^3} - \frac{\text{CosIntegral}(4\arctan(ax))}{2a^4c^3}$$

output
```
x/a^3/c^3/(a^2*x^2+1)^2/arctan(a*x)-x/a^3/c^3/(a^2*x^2+1)/arctan(a*x)+1/2*
Ci(2*arctan(a*x))/a^4/c^3-1/2*Ci(4*arctan(a*x))/a^4/c^3
```

## Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.97

$$\int \frac{x^3}{(c+a^2cx^2)^3\arctan(ax)^2}\,dx = \frac{-2a^3x^3+(1+a^2x^2)^2\arctan(ax)\,\text{CosIntegral}(2\arctan(ax))-(1+a^2x^2)^2\arctan(ax)\,\text{CosIntegral}(4\arctan(ax))}{2a^4c^3\,(1+a^2x^2)^2\arctan(ax)}$$

input
```
Integrate[x^3/((c + a^2*c*x^2)^3*ArcTan[a*x]^2),x]
```

output
```
(-2*a^3*x^3 + (1 + a^2*x^2)^2*ArcTan[a*x]*CosIntegral[2*ArcTan[a*x]] - (1
+ a^2*x^2)^2*ArcTan[a*x]*CosIntegral[4*ArcTan[a*x]])/(2*a^4*c^3*(1 + a^2*x
^2)^2*ArcTan[a*x])
```

## Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 177 vs. 2(86) = 172.

Time = 1.67 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.06, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {5499, 27, 5503, 5439, 3042, 3793, 2009, 5505, 3042, 3793, 2009, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\arctan(ax)^2\,(a^2cx^2+c)^3}\,dx$$

$$\Big\downarrow \text{5499}$$

$$\frac{\int \frac{x}{c^2(a^2x^2+1)^2\arctan(ax)^2}dx}{a^2c}-\frac{\int \frac{x}{c^3(a^2x^2+1)^3\arctan(ax)^2}dx}{a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^2}dx}{a^2c^3}-\frac{\int \frac{x}{(a^2x^2+1)^3\arctan(ax)^2}dx}{a^2c^3}$$

$$\Big\downarrow \text{5503}$$

$$\frac{\frac{\int \frac{1}{\left(a^2x^2+1\right)^2 \arctan(ax)}dx}{a} - a\int \frac{x^2}{(a^2x^2+1)^2 \arctan(ax)}dx - \frac{x}{a(a^2x^2+1)\arctan(ax)}}{a^2c^3} - \frac{\frac{\int \frac{1}{\left(a^2x^2+1\right)^3 \arctan(ax)}dx}{a} - 3a\int \frac{x^2}{(a^2x^2+1)^3 \arctan(ax)}dx - \frac{x}{a(a^2x^2+1)^2\arctan(ax)}}{a^2c^3}$$

$$\Big\downarrow \; 5439$$

$$\frac{-a\int \frac{x^2}{(a^2x^2+1)^2 \arctan(ax)}dx + \frac{\int \frac{1}{\left(a^2x^2+1\right)\arctan(ax)}d\arctan(ax)}{a^2} - \frac{x}{a(a^2x^2+1)\arctan(ax)}}{a^2c^3} - \frac{-3a\int \frac{x^2}{(a^2x^2+1)^3 \arctan(ax)}dx + \frac{\int \frac{1}{\left(a^2x^2+1\right)^2\arctan(ax)}d\arctan(ax)}{a^2} - \frac{x}{a(a^2x^2+1)^2\arctan(ax)}}{a^2c^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{-a\int \frac{x^2}{(a^2x^2+1)^2 \arctan(ax)}dx + \frac{\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^2}{\arctan(ax)}d\arctan(ax)}{a^2} - \frac{x}{a(a^2x^2+1)\arctan(ax)}}{a^2c^3} - \frac{-3a\int \frac{x^2}{(a^2x^2+1)^3 \arctan(ax)}dx + \frac{\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^4}{\arctan(ax)}d\arctan(ax)}{a^2} - \frac{x}{a(a^2x^2+1)^2\arctan(ax)}}{a^2c^3}$$

$$\Big\downarrow \; 3793$$

$$\frac{-a\int \frac{x^2}{(a^2x^2+1)^2 \arctan(ax)}dx + \frac{\int \left(\frac{\cos(2\arctan(ax))}{2\arctan(ax)}+\frac{1}{2\arctan(ax)}\right)d\arctan(ax)}{a^2} - \frac{x}{a(a^2x^2+1)\arctan(ax)}}{a^2c^3} - \frac{-3a\int \frac{x^2}{(a^2x^2+1)^3 \arctan(ax)}dx + \frac{\int \left(\frac{\cos(2\arctan(ax))}{2\arctan(ax)}+\frac{\cos(4\arctan(ax))}{8\arctan(ax)}+\frac{3}{8\arctan(ax)}\right)d\arctan(ax)}{a^2} - \frac{x}{a(a^2x^2+1)^2\arctan(ax)}}{a^2c^3}$$

$$\Big\downarrow \; 2009$$

$$\frac{-a\int \frac{x^2}{(a^2x^2+1)^2 \arctan(ax)}dx + \frac{\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2} - \frac{x}{a(a^2x^2+1)\arctan(ax)}}{a^2c^3} - \frac{-3a\int \frac{x^2}{(a^2x^2+1)^3 \arctan(ax)}dx + \frac{\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))+\frac{1}{8}\operatorname{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2} - \frac{x}{a(a^2x^2+1)^2\arctan(ax)}}{a^2c^3}$$

$$\Big\downarrow \; 5505$$

$$\frac{-\frac{\int \frac{a^2x^2}{\left(a^2x^2+1\right)\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}}{a^2c^3}-\frac{-\frac{3\int \frac{a^2x^2}{\left(a^2x^2+1\right)^2\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{8}\text{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)^2\,\text{arcta}}}{a^2c^3}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{-\frac{\int \frac{\sin(\arctan(ax))^2}{\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}}{a^2c^3}-\frac{-\frac{3\int \frac{a^2x^2}{\left(a^2x^2+1\right)^2\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{8}\text{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)^2\,\text{arcta}}}{a^2c^3}$$

$$\Big\downarrow \text{ 3793}$$

$$\frac{-\frac{\int \left(\frac{1}{2\arctan(ax)}-\frac{\cos(2\arctan(ax))}{2\arctan(ax)}\right)d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}}{a^2c^3}-\frac{-\frac{3\int \frac{a^2x^2}{\left(a^2x^2+1\right)^2\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{8}\text{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)^2\,\text{arcta}}}{a^2c^3}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\frac{\frac{1}{2}\log(\arctan(ax))-\frac{1}{2}\text{CosIntegral}(2\arctan(ax))}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}}{a^2c^3}-\frac{-\frac{3\int \frac{a^2x^2}{\left(a^2x^2+1\right)^2\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{8}\text{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)^2\,\text{arcta}}}{a^2c^3}$$

$$\Big\downarrow \text{ 4906}$$

$$\frac{-\frac{\frac{1}{2}\log(\arctan(ax))-\frac{1}{2}\text{CosIntegral}(2\arctan(ax))}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}}{a^2c^3}-\frac{-\frac{3\int \left(\frac{1}{8\arctan(ax)}-\frac{\cos(4\arctan(ax))}{8\arctan(ax)}\right)d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{8}\text{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}-\frac{}{a(a^2x^2}}{a^2c^3}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{-\frac{\frac{1}{2}\log(\arctan(ax))-\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))}{a^2}+\frac{\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}}{a^2c^3}-\frac{-\frac{3\left(\frac{1}{8}\log(\arctan(ax))-\frac{1}{8}\operatorname{CosIntegral}(4\arctan(ax))\right)}{a^2}+\frac{\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))+\frac{1}{8}\operatorname{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}-\frac{}{a(a}}{a^2c^3}$$

input
```
Int[x^3/((c + a^2*c*x^2)^3*ArcTan[a*x]^2),x]
```

output
```
-((-(x/(a*(1 + a^2*x^2)^2*ArcTan[a*x])) - (3*(-1/8*CosIntegral[4*ArcTan[a*
x]] + Log[ArcTan[a*x]]/8))/a^2 + (CosIntegral[2*ArcTan[a*x]]/2 + CosIntegr
al[4*ArcTan[a*x]]/8 + (3*Log[ArcTan[a*x]])/8)/a^2)/(a^2*c^3)) + (-(x/(a*(1
 + a^2*x^2)*ArcTan[a*x])) - (-1/2*CosIntegral[2*ArcTan[a*x]] + Log[ArcTan[
a*x]]/2)/a^2 + (CosIntegral[2*ArcTan[a*x]]/2 + Log[ArcTan[a*x]]/2)/a^2)/(a
^2*c^3)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5499
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ
[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

## Maple [A] (verified)

Time = 9.82 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.70

| method | result | size |
|---|---|---|
| derivativedivides | $-\frac{4\,\mathrm{Ci}(4\arctan(ax))\arctan(ax)-4\,\mathrm{Ci}(2\arctan(ax))\arctan(ax)+2\sin(2\arctan(ax))-\sin(4\arctan(ax))}{8a^{4}c^{3}\arctan(ax)}$ | 60 |
| default | $-\frac{4\,\mathrm{Ci}(4\arctan(ax))\arctan(ax)-4\,\mathrm{Ci}(2\arctan(ax))\arctan(ax)+2\sin(2\arctan(ax))-\sin(4\arctan(ax))}{8a^{4}c^{3}\arctan(ax)}$ | 60 |

input
```
int(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/8/a^4/c^3*(4*Ci(4*arctan(a*x))*arctan(a*x)-4*Ci(2*arctan(a*x))*arctan(a
*x)+2*sin(2*arctan(a*x))-sin(4*arctan(a*x)))/arctan(a*x)
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 292, normalized size of antiderivative = 3.40

$$\int \frac{x^3}{(c+a^2cx^2)^3 \arctan(ax)^2} dx = -\frac{4a^3x^3 + (a^4x^4 + 2a^2x^2 + 1)\arctan(ax)\,\mathrm{log\_integral}\left(\frac{a^4x^4 + 4i\,a^3x^3 - 6\,a^2x^2 - 4i\,ax + 1}{a^4x^4 + 2\,a^2x^2 + 1}\right) + (a^4x^4 + 2\,a^2x^2 + 1}{}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
-1/4*(4*a^3*x^3 + (a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x)*log_integral((a^4*
x^4 + 4*I*a^3*x^3 - 6*a^2*x^2 - 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) +
(a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x)*log_integral((a^4*x^4 - 4*I*a^3*x^3
- 6*a^2*x^2 + 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) - (a^4*x^4 + 2*a^2*x
^2 + 1)*arctan(a*x)*log_integral(-(a^2*x^2 + 2*I*a*x - 1)/(a^2*x^2 + 1)) -
 (a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x)*log_integral(-(a^2*x^2 - 2*I*a*x -
1)/(a^2*x^2 + 1)))/((a^8*c^3*x^4 + 2*a^6*c^3*x^2 + a^4*c^3)*arctan(a*x))
```

## Sympy [F]

$$\int \frac{x^3}{(c+a^2cx^2)^3 \arctan(ax)^2} dx = \frac{\int \frac{x^3}{a^6x^6 \operatorname{atan}^2(ax) + 3a^4x^4 \operatorname{atan}^2(ax) + 3a^2x^2 \operatorname{atan}^2(ax) + \operatorname{atan}^2(ax)} dx}{c^3}$$

input
```
integrate(x**3/(a**2*c*x**2+c)**3/atan(a*x)**2,x)
```

output
```
Integral(x**3/(a**6*x**6*atan(a*x)**2 + 3*a**4*x**4*atan(a*x)**2 + 3*a**2*
x**2*atan(a*x)**2 + atan(a*x)**2), x)/c**3
```

## Maxima [F]

$$\int \frac{x^3}{(c+a^2cx^2)^3\arctan(ax)^2}\,dx=\int \frac{x^3}{(a^2cx^2+c)^3\arctan{(ax)}^2}\,dx$$

input
```
integrate(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-(x^3 + (a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)*arctan(a*x)*integrate((a^2*x
^4 - 3*x^2)/((a^7*c^3*x^6 + 3*a^5*c^3*x^4 + 3*a^3*c^3*x^2 + a*c^3)*arctan(
a*x)), x))/((a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)*arctan(a*x))
```

## Giac [F]

$$\int \frac{x^3}{(c+a^2cx^2)^3\arctan(ax)^2}\,dx=\int \frac{x^3}{(a^2cx^2+c)^3\arctan{(ax)}^2}\,dx$$

input
```
integrate(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate(x^3/((a^2*c*x^2 + c)^3*arctan(a*x)^2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(c+a^2cx^2)^3\arctan(ax)^2}\,dx=\int \frac{x^3}{\mathrm{atan}\left(a\,x\right)^2\,{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

input
```
int(x^3/(atan(a*x)^2*(c + a^2*c*x^2)^3),x)
```

output
```
int(x^3/(atan(a*x)^2*(c + a^2*c*x^2)^3), x)
```

## Reduce [F]

$$\int \frac{x^3}{(c+a^2cx^2)^3 \arctan(ax)^2} \, dx = \frac{\int \frac{x^3}{atan(ax)^2 a^6 x^6 + 3atan(ax)^2 a^4 x^4 + 3atan(ax)^2 a^2 x^2 + atan(ax)^2} dx}{c^3}$$

input
```
int(x^3/(a^2*c*x^2+c)^3/atan(a*x)^2,x)
```

output
```
int(x**3/(atan(a*x)**2*a**6*x**6 + 3*atan(a*x)**2*a**4*x**4 + 3*atan(a*x)*
*2*a**2*x**2 + atan(a*x)**2),x)/c**3
```

# 3.559 $\int \frac{x^2}{(c+a^2cx^2)^3 \arctan(ax)^2} dx$



## Optimal result

Integrand size = 22, antiderivative size = 67

$$\int \frac{x^2}{(c+a^2cx^2)^3 \arctan(ax)^2} dx = \frac{1}{a^3c^3(1+a^2x^2)^2 \arctan(ax)} - \frac{1}{a^3c^3(1+a^2x^2)\arctan(ax)} + \frac{\mathrm{Si}(4\arctan(ax))}{2a^3c^3}$$

output
```
1/a^3/c^3/(a^2*x^2+1)^2/arctan(a*x)-1/a^3/c^3/(a^2*x^2+1)/arctan(a*x)+1/2*
Si(4*arctan(a*x))/a^3/c^3
```

## Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.88

$$\int \frac{x^2}{(c+a^2cx^2)^3 \arctan(ax)^2} dx = \frac{-2a^2x^2 + (1+a^2x^2)^2 \arctan(ax)\mathrm{Si}(4\arctan(ax))}{2a^3c^3(1+a^2x^2)^2 \arctan(ax)}$$

input
```
Integrate[x^2/((c + a^2*c*x^2)^3*ArcTan[a*x]^2),x]
```

output
```
(-2*a^2*x^2 + (1 + a^2*x^2)^2*ArcTan[a*x]*SinIntegral[4*ArcTan[a*x]])/(2*a
^3*c^3*(1 + a^2*x^2)^2*ArcTan[a*x])
```

## Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.52, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {5499, 27, 5437, 5505, 4906, 27, 2009, 3042, 3780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\arctan(ax)^2\left(a^2cx^2+c\right)^3}dx$$

$$\Big\downarrow \text{5499}$$

$$\frac{\int \frac{1}{c^2(a^2x^2+1)^2\arctan(ax)^2}dx}{a^2c}-\frac{\int \frac{1}{c^3(a^2x^2+1)^3\arctan(ax)^2}dx}{a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{1}{(a^2x^2+1)^2\arctan(ax)^2}dx}{a^2c^3}-\frac{\int \frac{1}{(a^2x^2+1)^3\arctan(ax)^2}dx}{a^2c^3}$$

$$\Big\downarrow \text{5437}$$

$$\frac{-2a\int \frac{x}{(a^2x^2+1)^2\arctan(ax)}dx-\frac{1}{a(a^2x^2+1)\arctan(ax)}}{a^2c^3}-\frac{-4a\int \frac{x}{(a^2x^2+1)^3\arctan(ax)}dx-\frac{1}{a(a^2x^2+1)^2\arctan(ax)}}{a^2c^3}$$

$$\Big\downarrow \text{5505}$$

$$\frac{-\frac{2\int \frac{ax}{\left(a^2x^2+1\right)\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a(a^2x^2+1)\arctan(ax)}}{a^2c^3}-\frac{-\frac{4\int \frac{ax}{\left(a^2x^2+1\right)^2\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a(a^2x^2+1)^2\arctan(ax)}}{a^2c^3}$$

$$\Big\downarrow \text{4906}$$

$$\frac{-\frac{2\int \frac{\sin(2\arctan(ax))}{2\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a(a^2x^2+1)\arctan(ax)}}{a^2c^3}-\frac{-\frac{4\int \left(\frac{\sin(2\arctan(ax))}{4\arctan(ax)}+\frac{\sin(4\arctan(ax))}{8\arctan(ax)}\right)d\arctan(ax)}{a}-\frac{1}{a(a^2x^2+1)^2\arctan(ax)}}{a^2c^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a(a^2x^2+1)\arctan(ax)}}{a^2c^3}-\frac{-\frac{4\int \left(\frac{\sin(2\arctan(ax))}{4\arctan(ax)}+\frac{\sin(4\arctan(ax))}{8\arctan(ax)}\right)d\arctan(ax)}{a}-\frac{1}{a(a^2x^2+1)^2\arctan(ax)}}{a^2c^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a(a^2x^2+1)\arctan(ax)}}{a^2c^3}-\frac{-\frac{1}{a(a^2x^2+1)^2\arctan(ax)}-\frac{4\left(\frac{1}{4}\text{Si}(2\arctan(ax))+\frac{1}{8}\text{Si}(4\arctan(ax))\right)}{a}}{a^2c^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a(a^2x^2+1)\arctan(ax)}}{a^2c^3}-\frac{-\frac{1}{a(a^2x^2+1)^2\arctan(ax)}-\frac{4\left(\frac{1}{4}\text{Si}(2\arctan(ax))+\frac{1}{8}\text{Si}(4\arctan(ax))\right)}{a}}{a^2c^3}$$

$$\Big\downarrow \text{3780}$$

$$\frac{-\frac{1}{a(a^2x^2+1)\arctan(ax)}-\frac{\text{Si}(2\arctan(ax))}{a}}{a^2c^3}-\frac{-\frac{1}{a(a^2x^2+1)^2\arctan(ax)}-\frac{4\left(\frac{1}{4}\text{Si}(2\arctan(ax))+\frac{1}{8}\text{Si}(4\arctan(ax))\right)}{a}}{a^2c^3}$$

input
```
Int[x^2/((c + a^2*c*x^2)^3*ArcTan[a*x]^2),x]
```

output
```
(-(1/(a*(1 + a^2*x^2)*ArcTan[a*x])) - SinIntegral[2*ArcTan[a*x]]/a)/(a^2*c
^3) - (-(1/(a*(1 + a^2*x^2)^2*ArcTan[a*x])) - (4*(SinIntegral[2*ArcTan[a*x
]]/4 + SinIntegral[4*ArcTan[a*x]]/8))/a)/(a^2*c^3)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5499
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ
[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

## Maple [A] (verified)

Time = 8.57 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.55

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{4\,\mathrm{Si}(4\arctan(ax))\arctan(ax)+\cos(4\arctan(ax))-1}{8a^3c^3\arctan(ax)}$ | 37 |
| default | $\frac{4\,\mathrm{Si}(4\arctan(ax))\arctan(ax)+\cos(4\arctan(ax))-1}{8a^3c^3\arctan(ax)}$ | 37 |

input
```
int(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8/a^3/c^3*(4*Si(4*arctan(a*x))*arctan(a*x)+cos(4*arctan(a*x))-1)/arctan(
a*x)
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.93

$$\int \frac{x^2}{(c+a^2cx^2)^3\arctan(ax)^2}\,dx = -\frac{4\,a^2x^2-(i\,a^4x^4+2i\,a^2x^2+i)\arctan(ax)\,\mathrm{log\_integral}\left(\frac{a^4x^4+4i\,a^3x^3-6\,a^2x^2-4i\,ax+1}{a^4x^4+2\,a^2x^2+1}\right)-(-i\,a^4x^4-2i\,a^2\cdots}{4\,(a^7c^3x^4+2\,a^5c^3x^2+a^3c^3)\arctan(ax)}$$

input
```
integrate(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
-1/4*(4*a^2*x^2 - (I*a^4*x^4 + 2*I*a^2*x^2 + I)*arctan(a*x)*log_integral((
a^4*x^4 + 4*I*a^3*x^3 - 6*a^2*x^2 - 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)
) - (-I*a^4*x^4 - 2*I*a^2*x^2 - I)*arctan(a*x)*log_integral((a^4*x^4 - 4*I
*a^3*x^3 - 6*a^2*x^2 + 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)))/((a^7*c^3*
x^4 + 2*a^5*c^3*x^2 + a^3*c^3)*arctan(a*x))
```

## Sympy [F]

$$\int \frac{x^2}{(c+a^2cx^2)^3\arctan(ax)^2}\,dx = \frac{\int \frac{x^2}{a^6x^6\operatorname{atan}^2(ax)+3a^4x^4\operatorname{atan}^2(ax)+3a^2x^2\operatorname{atan}^2(ax)+\operatorname{atan}^2(ax)}\,dx}{c^3}$$

input
```
integrate(x**2/(a**2*c*x**2+c)**3/atan(a*x)**2,x)
```

output
```
Integral(x**2/(a**6*x**6*atan(a*x)**2 + 3*a**4*x**4*atan(a*x)**2 + 3*a**2*
x**2*atan(a*x)**2 + atan(a*x)**2), x)/c**3
```

## Maxima [F]

$$\int \frac{x^2}{(c+a^2cx^2)^3\arctan(ax)^2}\,dx = \int \frac{x^2}{(a^2cx^2+c)^3\arctan(ax)^2}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-((a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)*arctan(a*x)*integrate(2*(a^2*x^3 -
 x)/((a^7*c^3*x^6 + 3*a^5*c^3*x^4 + 3*a^3*c^3*x^2 + a*c^3)*arctan(a*x)), x
) + x^2)/((a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)*arctan(a*x))
```

## Giac [F]

$$\int \frac{x^2}{(c+a^2cx^2)^3 \arctan(ax)^2} dx = \int \frac{x^2}{(a^2cx^2+c)^3 \arctan(ax)^2} dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate(x^2/((a^2*c*x^2 + c)^3*arctan(a*x)^2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(c+a^2cx^2)^3 \arctan(ax)^2} dx = \int \frac{x^2}{\mathrm{atan}(a\,x)^2\,(c\,a^2\,x^2+c)^3} dx$$

input
```
int(x^2/(atan(a*x)^2*(c + a^2*c*x^2)^3),x)
```

output
```
int(x^2/(atan(a*x)^2*(c + a^2*c*x^2)^3), x)
```

## Reduce [F]

$$\int \frac{x^2}{(c+a^2cx^2)^3 \arctan(ax)^2} dx = \frac{\int \frac{x^2}{atan(ax)^2a^6x^6+3atan(ax)^2a^4x^4+3atan(ax)^2a^2x^2+atan(ax)^2} dx}{c^3}$$

input
```
int(x^2/(a^2*c*x^2+c)^3/atan(a*x)^2,x)
```

output
```
int(x**2/(atan(a*x)**2*a**6*x**6 + 3*atan(a*x)**2*a**4*x**4 + 3*atan(a*x)*
*2*a**2*x**2 + atan(a*x)**2),x)/c**3
```

# 3.560 $\int \frac{x}{(c+a^2cx^2)^3\arctan(ax)^2}\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 61

$$\int \frac{x}{(c+a^2cx^2)^3\arctan(ax)^2}\,dx = -\frac{x}{ac^3(1+a^2x^2)^2\arctan(ax)} + \frac{\text{CosIntegral}(2\arctan(ax))}{2a^2c^3} + \frac{\text{CosIntegral}(4\arctan(ax))}{2a^2c^3}$$

output
```
-x/a/c^3/(a^2*x^2+1)^2/arctan(a*x)+1/2*Ci(2*arctan(a*x))/a^2/c^3+1/2*Ci(4*
arctan(a*x))/a^2/c^3
```

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.23

$$\int \frac{x}{(c+a^2cx^2)^3\arctan(ax)^2}\,dx = \frac{-2ax+(1+a^2x^2)^2\arctan(ax)\,\text{CosIntegral}(2\arctan(ax))+(1+a^2x^2)^2\arctan(ax)\,\text{CosIntegral}(4\,\text{arcta}}{2c^3(a+a^3x^2)^2\arctan(ax)}$$

input
```
Integrate[x/((c + a^2*c*x^2)^3*ArcTan[a*x]^2),x]
```

output
```
(-2*a*x + (1 + a^2*x^2)^2*ArcTan[a*x]*CosIntegral[2*ArcTan[a*x]] + (1 + a^
2*x^2)^2*ArcTan[a*x]*CosIntegral[4*ArcTan[a*x]])/(2*c^3*(a + a^3*x^2)^2*Ar
cTan[a*x])
```

## Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.56, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {5503, 27, 5439, 3042, 3793, 2009, 5505, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\arctan(ax)^2 (a^2cx^2+c)^3}dx$$

$$\Big\downarrow \text{5503}$$

$$\frac{\int \frac{1}{c^3(a^2x^2+1)^3\arctan(ax)}dx}{a}-3a\int \frac{x^2}{c^3(a^2x^2+1)^3\arctan(ax)}dx-\frac{x}{ac^3(a^2x^2+1)^2\arctan(ax)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{1}{(a^2x^2+1)^3\arctan(ax)}dx}{ac^3}-\frac{3a\int \frac{x^2}{(a^2x^2+1)^3\arctan(ax)}dx}{c^3}-\frac{x}{ac^3(a^2x^2+1)^2\arctan(ax)}$$

$$\Big\downarrow \text{5439}$$

$$-\frac{3a\int \frac{x^2}{(a^2x^2+1)^3\arctan(ax)}dx}{c^3}+\frac{\int \frac{1}{(a^2x^2+1)^2\arctan(ax)}d\arctan(ax)}{a^2c^3}-\frac{x}{ac^3(a^2x^2+1)^2\arctan(ax)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3a\int \frac{x^2}{(a^2x^2+1)^3\arctan(ax)}dx}{c^3}+\frac{\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^4}{\arctan(ax)}d\arctan(ax)}{a^2c^3}-\frac{x}{ac^3(a^2x^2+1)^2\arctan(ax)}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{3a\int \frac{x^2}{(a^2x^2+1)^3\arctan(ax)}dx}{c^3}+\frac{\int \left(\frac{\cos(2\arctan(ax))}{2\arctan(ax)}+\frac{\cos(4\arctan(ax))}{8\arctan(ax)}+\frac{3}{8\arctan(ax)}\right)d\arctan(ax)}{a^2c^3}-\frac{x}{ac^3(a^2x^2+1)^2\arctan(ax)}$$

$$\Big\downarrow \textbf{2009}$$

$$-\frac{3a\int \frac{x^2}{(a^2x^2+1)^3\arctan(ax)}dx}{c^3}+\frac{\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))+\frac{1}{8}\operatorname{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2c^3}-\frac{x}{ac^3\left(a^2x^2+1\right)^2\arctan(ax)}$$

$$\Big\downarrow \textbf{5505}$$

$$-\frac{3\int \frac{a^2x^2}{(a^2x^2+1)^2\arctan(ax)}d\arctan(ax)}{a^2c^3}+\frac{\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))+\frac{1}{8}\operatorname{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2c^3}-\frac{x}{ac^3\left(a^2x^2+1\right)^2\arctan(ax)}$$

$$\Big\downarrow \textbf{4906}$$

$$-\frac{3\int \left(\frac{1}{8\arctan(ax)}-\frac{\cos(4\arctan(ax))}{8\arctan(ax)}\right)d\arctan(ax)}{a^2c^3}+\frac{\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))+\frac{1}{8}\operatorname{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2c^3}-\frac{x}{ac^3\left(a^2x^2+1\right)^2\arctan(ax)}$$

$$\Big\downarrow \textbf{2009}$$

$$-\frac{3\left(\frac{1}{8}\log(\arctan(ax))-\frac{1}{8}\operatorname{CosIntegral}(4\arctan(ax))\right)}{a^2c^3}+\frac{\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))+\frac{1}{8}\operatorname{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2c^3}-\frac{x}{ac^3\left(a^2x^2+1\right)^2\arctan(ax)}$$

input
```
Int[x/((c + a^2*c*x^2)^3*ArcTan[a*x]^2),x]
```

output
```
-(x/(a*c^3*(1 + a^2*x^2)^2*ArcTan[a*x])) - (3*(-1/8*CosIntegral[4*ArcTan[a
*x]] + Log[ArcTan[a*x]]/8))/(a^2*c^3) + (CosIntegral[2*ArcTan[a*x]]/2 + Co
sIntegral[4*ArcTan[a*x]]/8 + (3*Log[ArcTan[a*x]])/8)/(a^2*c^3)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

## Maple [A] (verified)

Time = 9.21 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.98

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{4\,\mathrm{Ci}(2\arctan(ax))\arctan(ax)+4\,\mathrm{Ci}(4\arctan(ax))\arctan(ax)-2\sin(2\arctan(ax))-\sin(4\arctan(ax))}{8a^2c^3\arctan(ax)}$ | 60 |
| default | $\frac{4\,\mathrm{Ci}(2\arctan(ax))\arctan(ax)+4\,\mathrm{Ci}(4\arctan(ax))\arctan(ax)-2\sin(2\arctan(ax))-\sin(4\arctan(ax))}{8a^2c^3\arctan(ax)}$ | 60 |

input
```
int(x/(a^2*c*x^2+c)^3/arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8/a^2/c^3*(4*Ci(2*arctan(a*x))*arctan(a*x)+4*Ci(4*arctan(a*x))*arctan(a*
x)-2*sin(2*arctan(a*x))-sin(4*arctan(a*x)))/arctan(a*x)
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 286, normalized size of antiderivative = 4.69

$$\int \frac{x}{(c+a^2cx^2)^3\arctan(ax)^2}\,dx = \frac{(a^4x^4+2\,a^2x^2+1)\arctan{(ax)}\,\mathrm{log\_integral}\left(\frac{a^4x^4+4i\,a^3x^3-6\,a^2x^2-4i\,ax+1}{a^4x^4+2\,a^2x^2+1}\right)+(a^4x^4+2\,a^2x^2+1)\arctan{(ax)}}{}$$

input
```
integrate(x/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
1/4*((a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x)*log_integral((a^4*x^4 + 4*I*a^3
*x^3 - 6*a^2*x^2 - 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) + (a^4*x^4 + 2*
a^2*x^2 + 1)*arctan(a*x)*log_integral((a^4*x^4 - 4*I*a^3*x^3 - 6*a^2*x^2 +
 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) + (a^4*x^4 + 2*a^2*x^2 + 1)*arcta
n(a*x)*log_integral(-(a^2*x^2 + 2*I*a*x - 1)/(a^2*x^2 + 1)) + (a^4*x^4 + 2
*a^2*x^2 + 1)*arctan(a*x)*log_integral(-(a^2*x^2 - 2*I*a*x - 1)/(a^2*x^2 +
 1)) - 4*a*x)/((a^6*c^3*x^4 + 2*a^4*c^3*x^2 + a^2*c^3)*arctan(a*x))
```

## Sympy [F]

$$\int \frac{x}{(c+a^2cx^2)^3\arctan(ax)^2}\,dx = \frac{\int \frac{x}{a^6x^6\operatorname{atan}^2(ax)+3a^4x^4\operatorname{atan}^2(ax)+3a^2x^2\operatorname{atan}^2(ax)+\operatorname{atan}^2(ax)}\,dx}{c^3}$$

input
```
integrate(x/(a**2*c*x**2+c)**3/atan(a*x)**2,x)
```

output
```
Integral(x/(a**6*x**6*atan(a*x)**2 + 3*a**4*x**4*atan(a*x)**2 + 3*a**2*x**
2*atan(a*x)**2 + atan(a*x)**2), x)/c**3
```

## Maxima [F]

$$\int \frac{x}{(c+a^2cx^2)^3\arctan(ax)^2}\,dx = \int \frac{x}{(a^2cx^2+c)^3\arctan(ax)^2}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-((a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)*arctan(a*x)*integrate((3*a^2*x^2 -
 1)/((a^7*c^3*x^6 + 3*a^5*c^3*x^4 + 3*a^3*c^3*x^2 + a*c^3)*arctan(a*x)), x
) + x)/((a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)*arctan(a*x))
```

## Giac [F]

$$\int \frac{x}{(c+a^2cx^2)^3 \arctan(ax)^2}\,dx = \int \frac{x}{(a^2cx^2+c)^3 \arctan(ax)^2}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate(x/((a^2*c*x^2 + c)^3*arctan(a*x)^2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(c+a^2cx^2)^3 \arctan(ax)^2}\,dx = \int \frac{x}{\operatorname{atan}(a\,x)^2\,(c\,a^2\,x^2+c)^3}\,dx$$

input
```
int(x/(atan(a*x)^2*(c + a^2*c*x^2)^3),x)
```

output
```
int(x/(atan(a*x)^2*(c + a^2*c*x^2)^3), x)
```

## Reduce [F]

$$\int \frac{x}{(c+a^2cx^2)^3 \arctan(ax)^2}\,dx = \frac{\int \frac{x}{atan(ax)^2a^6x^6+3atan(ax)^2a^4x^4+3atan(ax)^2a^2x^2+atan(ax)^2}dx}{c^3}$$

input
```
int(x/(a^2*c*x^2+c)^3/atan(a*x)^2,x)
```

output
```
int(x/(atan(a*x)**2*a**6*x**6 + 3*atan(a*x)**2*a**4*x**4 + 3*atan(a*x)**2*
a**2*x**2 + atan(a*x)**2),x)/c**3
```

# 3.561 $\int \frac{1}{(c+a^2cx^2)^3 \arctan(ax)^2} \, dx$



## Optimal result

Integrand size = 19, antiderivative size = 58

$$\int \frac{1}{(c+a^2cx^2)^3 \arctan(ax)^2} \, dx = -\frac{1}{ac^3 (1+a^2x^2)^2 \arctan(ax)} - \frac{\text{Si}(2\arctan(ax))}{ac^3} - \frac{\text{Si}(4\arctan(ax))}{2ac^3}$$

output
```
-1/a/c^3/(a^2*x^2+1)^2/arctan(a*x)-Si(2*arctan(a*x))/a/c^3-1/2*Si(4*arctan
(a*x))/a/c^3
```

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.78

$$\int \frac{1}{(c+a^2cx^2)^3 \arctan(ax)^2} \, dx = -\frac{\frac{1}{(1+a^2x^2)^2 \arctan(ax)} + \text{Si}(2\arctan(ax)) + \frac{1}{2}\text{Si}(4\arctan(ax))}{ac^3}$$

input
```
Integrate[1/((c + a^2*c*x^2)^3*ArcTan[a*x]^2),x]
```

output
```
-((1/((1 + a^2*x^2)^2*ArcTan[a*x]) + SinIntegral[2*ArcTan[a*x]] + SinInteg
ral[4*ArcTan[a*x]]/2)/(a*c^3))
```

## Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {5437, 27, 5505, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arctan(ax)^2 (a^2cx^2+c)^3}dx$$

$$\Big\downarrow \text{5437}$$

$$-4a\int \frac{x}{c^3 (a^2x^2+1)^3 \arctan(ax)}dx-\frac{1}{ac^3 (a^2x^2+1)^2 \arctan(ax)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{4a\int \frac{x}{(a^2x^2+1)^3 \arctan(ax)}dx}{c^3}-\frac{1}{ac^3 (a^2x^2+1)^2 \arctan(ax)}$$

$$\Big\downarrow \text{5505}$$

$$-\frac{4\int \frac{ax}{(a^2x^2+1)^2 \arctan(ax)}d\arctan(ax)}{ac^3}-\frac{1}{ac^3 (a^2x^2+1)^2 \arctan(ax)}$$

$$\Big\downarrow \text{4906}$$

$$-\frac{4\int \left(\frac{\sin(2\arctan(ax))}{4\arctan(ax)}+\frac{\sin(4\arctan(ax))}{8\arctan(ax)}\right)d\arctan(ax)}{ac^3}-\frac{1}{ac^3 (a^2x^2+1)^2 \arctan(ax)}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{ac^3 (a^2x^2+1)^2 \arctan(ax)}-\frac{4\left(\frac{1}{4}\text{Si}(2\arctan(ax))+\frac{1}{8}\text{Si}(4\arctan(ax))\right)}{ac^3}$$

input
```
Int[1/((c + a^2*c*x^2)^3*ArcTan[a*x]^2),x]
```

output
```
-(1/(a*c^3*(1 + a^2*x^2)^2*ArcTan[a*x])) - (4*(SinIntegral[2*ArcTan[a*x]]/
4 + SinIntegral[4*ArcTan[a*x]]/8))/(a*c^3)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

## Maple [A] (verified)

Time = 8.73 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.02

| method | result | size |
|---|---|---|
| derivativedivides | $-\frac{8\,\mathrm{Si}(2\arctan(ax))\arctan(ax)+4\,\mathrm{Si}(4\arctan(ax))\arctan(ax)+\cos(4\arctan(ax))+4\cos(2\arctan(ax))+3}{8a\,c^{3}\arctan(ax)}$ | 59 |
| default | $-\frac{8\,\mathrm{Si}(2\arctan(ax))\arctan(ax)+4\,\mathrm{Si}(4\arctan(ax))\arctan(ax)+\cos(4\arctan(ax))+4\cos(2\arctan(ax))+3}{8a\,c^{3}\arctan(ax)}$ | 59 |

input
```
int(1/(a^2*c*x^2+c)^3/arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/8/a/c^3*(8*Si(2*arctan(a*x))*arctan(a*x)+4*Si(4*arctan(a*x))*arctan(a*x
)+cos(4*arctan(a*x))+4*cos(2*arctan(a*x))+3)/arctan(a*x)
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 287, normalized size of antiderivative = 4.95

$$\int \frac{1}{(c+a^2cx^2)^3\arctan(ax)^2}\,dx$$
$$= \frac{(-i\,a^4x^4-2i\,a^2x^2-i)\arctan(ax)\,\mathrm{log\_integral}\left(\frac{a^4x^4+4i\,a^3x^3-6\,a^2x^2-4i\,ax+1}{a^4x^4+2\,a^2x^2+1}\right)+(i\,a^4x^4+2i\,a^2x^2+i)\,\mathrm{arcta}}{}$$

input
```
integrate(1/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
1/4*((-I*a^4*x^4 - 2*I*a^2*x^2 - I)*arctan(a*x)*log_integral((a^4*x^4 + 4*
I*a^3*x^3 - 6*a^2*x^2 - 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) + (I*a^4*x
^4 + 2*I*a^2*x^2 + I)*arctan(a*x)*log_integral((a^4*x^4 - 4*I*a^3*x^3 - 6*
a^2*x^2 + 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) - 2*(I*a^4*x^4 + 2*I*a^2
*x^2 + I)*arctan(a*x)*log_integral(-(a^2*x^2 + 2*I*a*x - 1)/(a^2*x^2 + 1))
 - 2*(-I*a^4*x^4 - 2*I*a^2*x^2 - I)*arctan(a*x)*log_integral(-(a^2*x^2 - 2
*I*a*x - 1)/(a^2*x^2 + 1)) - 4)/((a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)*arc
tan(a*x))
```

## Sympy [F]

$$\int \frac{1}{(c+a^2cx^2)^3 \arctan(ax)^2}\,dx = \frac{\int \frac{1}{a^6x^6 \operatorname{atan}^2(ax)+3a^4x^4 \operatorname{atan}^2(ax)+3a^2x^2 \operatorname{atan}^2(ax)+\operatorname{atan}^2(ax)}\,dx}{c^3}$$

input
```
integrate(1/(a**2*c*x**2+c)**3/atan(a*x)**2,x)
```

output
```
Integral(1/(a**6*x**6*atan(a*x)**2 + 3*a**4*x**4*atan(a*x)**2 + 3*a**2*x**
2*atan(a*x)**2 + atan(a*x)**2), x)/c**3
```

## Maxima [F]

$$\int \frac{1}{(c+a^2cx^2)^3 \arctan(ax)^2}\,dx = \int \frac{1}{(a^2cx^2+c)^3 \arctan(ax)^2}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-(8*(a^6*c^3*x^4 + 2*a^4*c^3*x^2 + a^2*c^3)*arctan(a*x)*integrate(1/2*x/((
a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a*x)), x) + 1)/(
(a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)*arctan(a*x))
```

## Giac [F]

$$\int \frac{1}{(c+a^2cx^2)^3 \arctan(ax)^2}\,dx = \int \frac{1}{(a^2cx^2+c)^3 \arctan(ax)^2}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^3*arctan(a*x)^2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+a^2cx^2)^3 \arctan(ax)^2}\,dx = \int \frac{1}{\mathrm{atan}\,(a\,x)^2\,(c\,a^2\,x^2+c)^3}\,dx$$

input
```
int(1/(atan(a*x)^2*(c + a^2*c*x^2)^3),x)
```

output
```
int(1/(atan(a*x)^2*(c + a^2*c*x^2)^3), x)
```

## Reduce [F]

$$\int \frac{1}{(c+a^2cx^2)^3 \arctan(ax)^2}\,dx = \frac{-4atan(ax)\left(\int \frac{x}{atan(ax)a^6x^6+3atan(ax)a^4x^4+3atan(ax)a^2x^2+atan(ax)}dx\right)a^6x^4-8atan(ax)\left(\int \frac{}{atan(ax)a^6x^6+3atan(ax}\right.}{atan\,(ax)\,a\,c^3\,(a}$$

input
```
int(1/(a^2*c*x^2+c)^3/atan(a*x)^2,x)
```

output
```
( - 4*atan(a*x)*int(x/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*ata
n(a*x)*a**2*x**2 + atan(a*x)),x)*a**6*x**4 - 8*atan(a*x)*int(x/(atan(a*x)*
a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*
a**4*x**2 - 4*atan(a*x)*int(x/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4
 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**2 - 1)/(atan(a*x)*a*c**3*(a**4
*x**4 + 2*a**2*x**2 + 1))
```

# 3.562 $\int \frac{1}{x(c+a^2cx^2)^3 \arctan(ax)^2} dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x(c+a^2cx^2)^3 \arctan(ax)^2} dx = -\frac{1}{ac^3x\arctan(ax)} + \frac{ax}{c^3(1+a^2x^2)^2\arctan(ax)} + \frac{ax}{c^3(1+a^2x^2)\arctan(ax)} - \frac{3\,\mathrm{CosIntegral}(2\arctan(ax))}{2c^3} - \frac{\mathrm{CosIntegral}(4\arctan(ax))}{2c^3} - \frac{\mathrm{Int}\left(\frac{1}{x^2\arctan(ax)}, x\right)}{ac^3}$$

output
```
-1/a/c^3/x/arctan(a*x)+a*x/c^3/(a^2*x^2+1)^2/arctan(a*x)+a*x/c^3/(a^2*x^2+
1)/arctan(a*x)-3/2*Ci(2*arctan(a*x))/c^3-1/2*Ci(4*arctan(a*x))/c^3-Defer(I
nt)(1/x^2/arctan(a*x),x)/a/c^3
```

## Mathematica [N/A]

Not integrable

Time = 1.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\arctan(ax)^2}\,dx=\int \frac{1}{x\left(c+a^2cx^2\right)^3\arctan(ax)^2}\,dx$$

input
```
Integrate[1/(x*(c + a^2*c*x^2)^3*ArcTan[a*x]^2),x]
```

output
```
Integrate[1/(x*(c + a^2*c*x^2)^3*ArcTan[a*x]^2), x]
```

## Rubi [N/A]

Not integrable

Time = 2.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\arctan(ax)^2\left(a^2cx^2+c\right)^3}\,dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{1}{c^2x(a^2x^2+1)^2\arctan(ax)^2}dx}{c}-a^2\int \frac{x}{c^3\left(a^2x^2+1\right)^3\arctan(ax)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{1}{x(a^2x^2+1)^2\arctan(ax)^2}dx}{c^3}-\frac{a^2\int \frac{x}{(a^2x^2+1)^3\arctan(ax)^2}dx}{c^3}$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{1}{x(a^2x^2+1)\arctan(ax)^2}dx-a^2\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^2}dx}{c^3}-\frac{a^2\int \frac{x}{(a^2x^2+1)^3\arctan(ax)^2}dx}{c^3}$$

$$\Big\downarrow \text{5461}$$

$$\frac{a^2\left(-\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^2}dx\right)-\frac{\int \frac{1}{x^2\arctan(ax)}dx}{a}-\frac{1}{ax\arctan(ax)}}{c^3}-\frac{a^2\int \frac{x}{(a^2x^2+1)^3\arctan(ax)^2}dx}{c^3}$$

$$\Big\downarrow \text{5377}$$

$$\frac{a^2\left(-\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^2}dx\right)-\frac{\int \frac{1}{x^2\arctan(ax)}dx}{a}-\frac{1}{ax\arctan(ax)}}{c^3}-\frac{a^2\int \frac{x}{(a^2x^2+1)^3\arctan(ax)^2}dx}{c^3}$$

$$\Big\downarrow \text{5503}$$

$$\frac{-\left(a^2\left(\frac{\int \frac{1}{\left(a^2x^2+1\right)^2\arctan(ax)}dx}{a}-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{\int \frac{1}{x^2\arctan(ax)}dx}{a}-\frac{1}{ax\arctan(ax)}}{c^3}$$
$$\frac{a^2\left(\frac{\int \frac{1}{\left(a^2x^2+1\right)^3\arctan(ax)}dx}{a}-3a\int \frac{x^2}{(a^2x^2+1)^3\arctan(ax)}dx-\frac{x}{a(a^2x^2+1)^2\arctan(ax)}\right)}{c^3}$$

$$\Big\downarrow \text{5439}$$

$$\frac{-\left(a^2\left(-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\int \frac{1}{\left(a^2x^2+1\right)\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{\int \frac{1}{x^2\arctan(ax)}dx}{a}-\frac{}{ax\,\text{ar}}}{c^3}$$
$$\frac{a^2\left(-3a\int \frac{x^2}{(a^2x^2+1)^3\arctan(ax)}dx+\frac{\int \frac{1}{\left(a^2x^2+1\right)^2\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)^2\arctan(ax)}\right)}{c^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\left(a^2\left(-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^2}{\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{\int \frac{1}{x^2\arctan(ax)}dx}{a}-\frac{}{ax\,\text{a}}}{c^3}$$
$$\frac{a^2\left(-3a\int \frac{x^2}{(a^2x^2+1)^3\arctan(ax)}dx+\frac{\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^4}{\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)^2\arctan(ax)}\right)}{c^3}$$

$$\Big\downarrow \text{3793}$$

$$\frac{-\left(a^2\left(-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\int\left(\frac{\cos(2\arctan(ax))}{2\arctan(ax)}+\frac{1}{2\arctan(ax)}\right)d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{\int\frac{1}{x^2\arctan(ax)}}{a}}{c^3}$$

$$\frac{a^2\left(-3a\int \frac{x^2}{(a^2x^2+1)^3\arctan(ax)}dx+\frac{\int\left(\frac{\cos(2\arctan(ax))}{2\arctan(ax)}+\frac{\cos(4\arctan(ax))}{8\arctan(ax)}+\frac{3}{8\arctan(ax)}\right)d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)^2\arctan(ax)}\right)}{c^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\left(a^2\left(-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{\int\frac{1}{x^2\arctan(ax)}}{a}}{c^3}$$

$$\frac{a^2\left(-3a\int \frac{x^2}{(a^2x^2+1)^3\arctan(ax)}dx+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{8}\text{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)^2\arctan}\right.}{c^3}$$

$$\Big\downarrow \text{5505}$$

$$\frac{-\left(a^2\left(-\frac{\int\frac{a^2x^2}{(a^2x^2+1)\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{\int\frac{}{x^2\arctan}}{}}{c^3}$$

$$\frac{a^2\left(-\frac{3\int\frac{a^2x^2}{(a^2x^2+1)^2\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{8}\text{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)^2}\right.}{c^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\left(a^2\left(-\frac{\int\frac{\sin(\arctan(ax))^2}{\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{\int\frac{1}{x^2\arctan(ax)}}{a}}{c^3}$$

$$\frac{a^2\left(-\frac{3\int\frac{a^2x^2}{(a^2x^2+1)^2\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{8}\text{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)^2}\right.}{c^3}$$

$$\Big\downarrow \text{3793}$$

$$\frac{-\left(a^2\left(-\frac{\int \left(\frac{1}{2\arctan(ax)}-\frac{\cos(2\arctan(ax))}{2\arctan(ax)}\right)d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)\right)}{c^3}$$

$$\frac{a^2\left(-\frac{3\int \frac{a^2x^2}{(a^2x^2+1)^2\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{8}\text{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)^2}\right.}{c^3}$$

$\Big\downarrow$ 2009

$$\frac{-\frac{\int \frac{1}{x^2\arctan(ax)}dx}{a}-\left(a^2\left(-\frac{\frac{1}{2}\log(\arctan(ax))-\frac{1}{2}\text{CosIntegral}(2\arctan(ax))}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{}{a(a^2x^2}\right.\right.}{c^3}$$

$$\frac{a^2\left(-\frac{3\int \frac{a^2x^2}{(a^2x^2+1)^2\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{8}\text{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)^2}\right.}{c^3}$$

$\Big\downarrow$ 4906

$$\frac{-\frac{\int \frac{1}{x^2\arctan(ax)}dx}{a}-\left(a^2\left(-\frac{\frac{1}{2}\log(\arctan(ax))-\frac{1}{2}\text{CosIntegral}(2\arctan(ax))}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{}{a(a^2x^2}\right.\right.}{c^3}$$

$$\frac{a^2\left(-\frac{3\int \left(\frac{1}{8\arctan(ax)}-\frac{\cos(4\arctan(ax))}{8\arctan(ax)}\right)d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{8}\text{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}-\frac{}{a(}\right.}{c^3}$$

$\Big\downarrow$ 2009

$$\frac{-\frac{\int \frac{1}{x^2\arctan(ax)}dx}{a}-\left(a^2\left(-\frac{\frac{1}{2}\log(\arctan(ax))-\frac{1}{2}\text{CosIntegral}(2\arctan(ax))}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{}{a(a^2x^2}\right.\right.}{c^3}$$

$$\frac{a^2\left(-\frac{3\left(\frac{1}{8}\log(\arctan(ax))-\frac{1}{8}\text{CosIntegral}(4\arctan(ax))\right)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{8}\text{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}-\right.}{c^3}$$

input
```
Int[1/(x*(c + a^2*c*x^2)^3*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 11.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a^{2} c\,x^{2}+c\right)^{3} \arctan\left(a x\right)^{2}}dx$$

input
```
int(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

output
```
int(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.23

$$\int \frac{1}{x\left(c+a^{2} c x^{2}\right)^{3} \arctan(ax)^{2}}\, dx = \int \frac{1}{\left(a^{2} c x^{2}+c\right)^{3} x \arctan\left(a x\right)^{2}}\, dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(1/((a^6*c^3*x^7 + 3*a^4*c^3*x^5 + 3*a^2*c^3*x^3 + c^3*x)*arctan(a
*x)^2), x)
```

## Sympy [N/A]

Not integrable

Time = 1.47 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.64

$$\int \frac{1}{x\left(c+a^{2} c x^{2}\right)^{3} \arctan(ax)^{2}}\, dx = \frac{\int \frac{1}{a^{6} x^{7} \operatorname{atan}^{2}{\left(a x \right)} + 3 a^{4} x^{5} \operatorname{atan}^{2}{\left(a x \right)} + 3 a^{2} x^{3} \operatorname{atan}^{2}{\left(a x \right)} + x \operatorname{atan}^{2}{\left(a x \right)}}\, dx}{c^{3}}$$

input
```
integrate(1/x/(a**2*c*x**2+c)**3/atan(a*x)**2,x)
```

output
```
Integral(1/(a**6*x**7*atan(a*x)**2 + 3*a**4*x**5*atan(a*x)**2 + 3*a**2*x**
3*atan(a*x)**2 + x*atan(a*x)**2), x)/c**3
```

## Maxima [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 135, normalized size of antiderivative = 6.14

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\arctan(ax)^2}\,dx=\int \frac{1}{\left(a^2cx^2+c\right)^3x\arctan\left(ax\right)^2}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-((a^5*c^3*x^5 + 2*a^3*c^3*x^3 + a*c^3*x)*arctan(a*x)*integrate((5*a^2*x^2
 + 1)/((a^7*c^3*x^8 + 3*a^5*c^3*x^6 + 3*a^3*c^3*x^4 + a*c^3*x^2)*arctan(a*
x)), x) + 1)/((a^5*c^3*x^5 + 2*a^3*c^3*x^3 + a*c^3*x)*arctan(a*x))
```

## Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\arctan(ax)^2}\,dx=\int \frac{1}{\left(a^2cx^2+c\right)^3x\arctan\left(ax\right)^2}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^3*x*arctan(a*x)^2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\arctan(ax)^2}\,dx=\int \frac{1}{x\,\mathrm{atan}\left(a\,x\right)^2\,{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

input
```
int(1/(x*atan(a*x)^2*(c + a^2*c*x^2)^3),x)
```

output
```
int(1/(x*atan(a*x)^2*(c + a^2*c*x^2)^3), x)
```

## Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.64

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\arctan(ax)^2}\,dx=\frac{\int \frac{1}{atan(ax)^2a^6x^7+3atan(ax)^2a^4x^5+3atan(ax)^2a^2x^3+atan(ax)^2x}dx}{c^3}$$

input
```
int(1/x/(a^2*c*x^2+c)^3/atan(a*x)^2,x)
```

output
```
int(1/(atan(a*x)**2*a**6*x**7 + 3*atan(a*x)**2*a**4*x**5 + 3*atan(a*x)**2*
a**2*x**3 + atan(a*x)**2*x),x)/c**3
```

# 3.563 $\int \frac{1}{x^2(c+a^2cx^2)^3 \arctan(ax)^2} dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x^2 (c+a^2cx^2)^3 \arctan(ax)^2} dx = -\frac{1}{ac^3x^2 \arctan(ax)} + \frac{a}{c^3 (1+a^2x^2)^2 \arctan(ax)} + \frac{a}{c^3 (1+a^2x^2) \arctan(ax)} + \frac{2a\mathrm{Si}(2\arctan(ax))}{c^3} + \frac{a\mathrm{Si}(4\arctan(ax))}{2c^3} - \frac{2\mathrm{Int}\left(\frac{1}{x^3\arctan(ax)}, x\right)}{ac^3}$$

output
```
-1/a/c^3/x^2/arctan(a*x)+a/c^3/(a^2*x^2+1)^2/arctan(a*x)+a/c^3/(a^2*x^2+1)
/arctan(a*x)+2*a*Si(2*arctan(a*x))/c^3+1/2*a*Si(4*arctan(a*x))/c^3-2*Defer
(Int)(1/x^3/arctan(a*x),x)/a/c^3
```

## Mathematica [N/A]

Not integrable

Time = 1.64 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2 (c+a^2cx^2)^3 \arctan(ax)^2} dx = \int \frac{1}{x^2 (c+a^2cx^2)^3 \arctan(ax)^2} dx$$

input
```
Integrate[1/(x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^2),x]
```

output
```
Integrate[1/(x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^2), x]
```

## Rubi [N/A]

Not integrable

Time = 1.57 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \arctan(ax)^2 (a^2cx^2+c)^3} dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{1}{c^2x^2(a^2x^2+1)^2 \arctan(ax)^2}dx}{c} - a^2 \int \frac{1}{c^3 (a^2x^2+1)^3 \arctan(ax)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{1}{x^2(a^2x^2+1)^2 \arctan(ax)^2}dx}{c^3} - \frac{a^2 \int \frac{1}{(a^2x^2+1)^3 \arctan(ax)^2}dx}{c^3}$$

$$\Big\downarrow \text{5437}$$

$$\frac{\int \frac{1}{x^2(a^2x^2+1)^2 \arctan(ax)^2}dx}{c^3} - \frac{a^2 \left(-4a \int \frac{x}{(a^2x^2+1)^3 \arctan(ax)}dx - \frac{1}{a(a^2x^2+1)^2 \arctan(ax)}\right)}{c^3}$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{1}{x^2(a^2x^2+1) \arctan(ax)^2}dx - a^2 \int \frac{1}{(a^2x^2+1)^2 \arctan(ax)^2}dx}{c^3} - \frac{a^2 \left(-4a \int \frac{x}{(a^2x^2+1)^3 \arctan(ax)}dx - \frac{1}{a(a^2x^2+1)^2 \arctan(ax)}\right)}{c^3}$$

$$\Big\downarrow \text{5437}$$

$$\frac{\int \frac{1}{x^2(a^2x^2+1)\arctan(ax)^2}dx - a^2\left(-2a\int \frac{x}{(a^2x^2+1)^2\arctan(ax)}dx - \frac{1}{a(a^2x^2+1)\arctan(ax)}\right)}{c^3} - \frac{a^2\left(-4a\int \frac{x}{(a^2x^2+1)^3\arctan(ax)}dx - \frac{1}{a(a^2x^2+1)^2\arctan(ax)}\right)}{c^3}$$

$$\Big\downarrow \text{ 5461}$$

$$\frac{-\left(a^2\left(-2a\int \frac{x}{(a^2x^2+1)^2\arctan(ax)}dx - \frac{1}{a(a^2x^2+1)\arctan(ax)}\right)\right) - \frac{2\int \frac{1}{x^3\arctan(ax)}dx}{a} - \frac{1}{ax^2\arctan(ax)}}{c^3} - \frac{a^2\left(-4a\int \frac{x}{(a^2x^2+1)^3\arctan(ax)}dx - \frac{1}{a(a^2x^2+1)^2\arctan(ax)}\right)}{c^3}$$

$$\Big\downarrow \text{ 5377}$$

$$\frac{-\left(a^2\left(-2a\int \frac{x}{(a^2x^2+1)^2\arctan(ax)}dx - \frac{1}{a(a^2x^2+1)\arctan(ax)}\right)\right) - \frac{2\int \frac{1}{x^3\arctan(ax)}dx}{a} - \frac{1}{ax^2\arctan(ax)}}{c^3} - \frac{a^2\left(-4a\int \frac{x}{(a^2x^2+1)^3\arctan(ax)}dx - \frac{1}{a(a^2x^2+1)^2\arctan(ax)}\right)}{c^3}$$

$$\Big\downarrow \text{ 5505}$$

$$\frac{-\left(a^2\left(-\frac{2\int \frac{ax}{(a^2x^2+1)\arctan(ax)}d\arctan(ax)}{a} - \frac{1}{a(a^2x^2+1)\arctan(ax)}\right)\right) - \frac{2\int \frac{1}{x^3\arctan(ax)}dx}{a} - \frac{1}{ax^2\arctan(ax)}}{c^3} - \frac{a^2\left(-\frac{4\int \frac{ax}{(a^2x^2+1)^2\arctan(ax)}d\arctan(ax)}{a} - \frac{1}{a(a^2x^2+1)^2\arctan(ax)}\right)}{c^3}$$

$$\Big\downarrow \text{ 4906}$$

$$\frac{-\left(a^2\left(-\frac{2\int \frac{\sin(2\arctan(ax))}{2\arctan(ax)}d\arctan(ax)}{a} - \frac{1}{a(a^2x^2+1)\arctan(ax)}\right)\right) - \frac{2\int \frac{1}{x^3\arctan(ax)}dx}{a} - \frac{1}{ax^2\arctan(ax)}}{c^3} - \frac{a^2\left(-\frac{4\int \left(\frac{\sin(2\arctan(ax))}{4\arctan(ax)} + \frac{\sin(4\arctan(ax))}{8\arctan(ax)}\right)d\arctan(ax)}{a} - \frac{1}{a(a^2x^2+1)^2\arctan(ax)}\right)}{c^3}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{-\left(a^2\left(-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{2\int \frac{1}{x^3\arctan(ax)}dx}{a}-\frac{1}{ax^2\arctan(ax)}}{c^3}-\frac{a^2\left(-\frac{4\int \left(\frac{\sin(2\arctan(ax))}{4\arctan(ax)}+\frac{\sin(4\arctan(ax))}{8\arctan(ax)}\right)d\arctan(ax)}{a}-\frac{1}{a(a^2x^2+1)^2\arctan(ax)}\right)}{c^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\left(a^2\left(-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{2\int \frac{1}{x^3\arctan(ax)}dx}{a}-\frac{1}{ax^2\arctan(ax)}}{c^3}-\frac{a^2\left(-\frac{1}{a(a^2x^2+1)^2\arctan(ax)}-\frac{4\left(\frac{1}{4}\text{Si}(2\arctan(ax))+\frac{1}{8}\text{Si}(4\arctan(ax))\right)}{a}\right)}{c^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\left(a^2\left(-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{2\int \frac{1}{x^3\arctan(ax)}dx}{a}-\frac{1}{ax^2\arctan(ax)}}{c^3}-\frac{a^2\left(-\frac{1}{a(a^2x^2+1)^2\arctan(ax)}-\frac{4\left(\frac{1}{4}\text{Si}(2\arctan(ax))+\frac{1}{8}\text{Si}(4\arctan(ax))\right)}{a}\right)}{c^3}$$

$$\Big\downarrow \text{3780}$$

$$\frac{-\frac{2\int \frac{1}{x^3\arctan(ax)}dx}{a}-\left(a^2\left(-\frac{1}{a(a^2x^2+1)\arctan(ax)}-\frac{\text{Si}(2\arctan(ax))}{a}\right)\right)-\frac{1}{ax^2\arctan(ax)}}{c^3}-\frac{a^2\left(-\frac{1}{a(a^2x^2+1)^2\arctan(ax)}-\frac{4\left(\frac{1}{4}\text{Si}(2\arctan(ax))+\frac{1}{8}\text{Si}(4\arctan(ax))\right)}{a}\right)}{c^3}$$

input
```
Int[1/(x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 5.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (a^2 c\, x^2 + c)^3 \arctan (a x)^2} dx$$

input
```
int(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

output
```
int(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.32

$$\int \frac{1}{x^2 (c+a^2 c x^2)^3 \arctan(ax)^2} \, dx = \int \frac{1}{(a^2 c x^2 + c)^3 x^2 \arctan (ax)^2} \, dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(1/((a^6*c^3*x^8 + 3*a^4*c^3*x^6 + 3*a^2*c^3*x^4 + c^3*x^2)*arctan
(a*x)^2), x)
```

## Sympy [N/A]

Not integrable

Time = 1.49 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.73

$$\int \frac{1}{x^2 (c+a^2 c x^2)^3 \arctan(ax)^2} \, dx = \frac{\int \frac{1}{a^6 x^8 \operatorname{atan}^2(ax) + 3a^4 x^6 \operatorname{atan}^2(ax) + 3a^2 x^4 \operatorname{atan}^2(ax) + x^2 \operatorname{atan}^2(ax)} \, dx}{c^3}$$

input
```
integrate(1/x**2/(a**2*c*x**2+c)**3/atan(a*x)**2,x)
```

output
```
Integral(1/(a**6*x**8*atan(a*x)**2 + 3*a**4*x**6*atan(a*x)**2 + 3*a**2*x**
4*atan(a*x)**2 + x**2*atan(a*x)**2), x)/c**3
```

## Maxima [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 140, normalized size of antiderivative = 6.36

$$\int \frac{1}{x^2 (c+a^2cx^2)^3 \arctan(ax)^2} \, dx = \int \frac{1}{(a^2cx^2+c)^3 x^2 \arctan(ax)^2} \, dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-((a^5*c^3*x^6 + 2*a^3*c^3*x^4 + a*c^3*x^2)*arctan(a*x)*integrate(2*(3*a^2
*x^2 + 1)/((a^7*c^3*x^9 + 3*a^5*c^3*x^7 + 3*a^3*c^3*x^5 + a*c^3*x^3)*arcta
n(a*x)), x) + 1)/((a^5*c^3*x^6 + 2*a^3*c^3*x^4 + a*c^3*x^2)*arctan(a*x))
```

## Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2 (c+a^2cx^2)^3 \arctan(ax)^2} \, dx = \int \frac{1}{(a^2cx^2+c)^3 x^2 \arctan(ax)^2} \, dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^3*x^2*arctan(a*x)^2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2 (c+a^2cx^2)^3 \arctan(ax)^2} dx = \int \frac{1}{x^2 \operatorname{atan}(a x)^2 (c a^2 x^2 + c)^3} dx$$

input
```
int(1/(x^2*atan(a*x)^2*(c + a^2*c*x^2)^3),x)
```

output
```
int(1/(x^2*atan(a*x)^2*(c + a^2*c*x^2)^3), x)
```

## Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.73

$$\int \frac{1}{x^2 (c+a^2cx^2)^3 \arctan(ax)^2} dx = \frac{\int \frac{1}{atan(ax)^2 a^6 x^8 + 3atan(ax)^2 a^4 x^6 + 3atan(ax)^2 a^2 x^4 + atan(ax)^2 x^2} dx}{c^3}$$

input
```
int(1/x^2/(a^2*c*x^2+c)^3/atan(a*x)^2,x)
```

output
```
int(1/(atan(a*x)**2*a**6*x**8 + 3*atan(a*x)**2*a**4*x**6 + 3*atan(a*x)**2*
a**2*x**4 + atan(a*x)**2*x**2),x)/c**3
```

# 3.564 $\int \frac{1}{x^3 (c+a^2cx^2)^3 \arctan(ax)^2} dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\begin{aligned}\int \frac{1}{x^3 (c+a^2cx^2)^3 \arctan(ax)^2} dx = &-\frac{1}{ac^3x^3\arctan(ax)} + \frac{2a}{c^3x\arctan(ax)} \\ &- \frac{a^3x}{c^3(1+a^2x^2)^2\arctan(ax)} \\ &- \frac{2a^3x}{c^3(1+a^2x^2)\arctan(ax)} \\ &+ \frac{5a^2\,\text{CosIntegral}(2\arctan(ax))}{2c^3} \\ &+ \frac{a^2\,\text{CosIntegral}(4\arctan(ax))}{2c^3} \\ &- \frac{3\text{Int}\left(\frac{1}{x^4\arctan(ax)},x\right)}{ac^3} + \frac{2a\text{Int}\left(\frac{1}{x^2\arctan(ax)},x\right)}{c^3}\end{aligned}$$

output
```
-1/a/c^3/x^3/arctan(a*x)+2*a/c^3/x/arctan(a*x)-a^3*x/c^3/(a^2*x^2+1)^2/arc
tan(a*x)-2*a^3*x/c^3/(a^2*x^2+1)/arctan(a*x)+5/2*a^2*Ci(2*arctan(a*x))/c^3
+1/2*a^2*Ci(4*arctan(a*x))/c^3-3*Defer(Int)(1/x^4/arctan(a*x),x)/a/c^3+2*a
*Defer(Int)(1/x^2/arctan(a*x),x)/c^3
```

## Mathematica [N/A]

Not integrable

Time = 2.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^3 (c+a^2cx^2)^3 \arctan(ax)^2}\, dx = \int \frac{1}{x^3 (c+a^2cx^2)^3 \arctan(ax)^2}\, dx$$

input
```
Integrate[1/(x^3*(c + a^2*c*x^2)^3*ArcTan[a*x]^2),x]
```

output
```
Integrate[1/(x^3*(c + a^2*c*x^2)^3*ArcTan[a*x]^2), x]
```

## Rubi [N/A]

Not integrable

Time = 4.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \arctan(ax)^2 (a^2cx^2+c)^3}dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{1}{c^2x^3(a^2x^2+1)^2 \arctan(ax)^2}dx}{c} - a^2 \int \frac{1}{c^3x (a^2x^2+1)^3 \arctan(ax)^2}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{1}{x^3(a^2x^2+1)^2 \arctan(ax)^2}dx}{c^3} - \frac{a^2 \int \frac{1}{x(a^2x^2+1)^3 \arctan(ax)^2}dx}{c^3}$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{1}{x^3(a^2x^2+1)\arctan(ax)^2}dx - a^2\int \frac{1}{x(a^2x^2+1)^2\arctan(ax)^2}dx}{c^3} - \frac{a^2\left(\int \frac{1}{x(a^2x^2+1)^2\arctan(ax)^2}dx - a^2\int \frac{x}{(a^2x^2+1)^3\arctan(ax)^2}dx\right)}{c^3}$$

$$\Big\downarrow \text{5461}$$

$$\frac{a^2\left(-\int \frac{1}{x(a^2x^2+1)^2\arctan(ax)^2}dx\right) - \frac{3\int \frac{1}{x^4\arctan(ax)}dx}{a} - \frac{1}{ax^3\arctan(ax)}}{c^3} - \frac{a^2\left(\int \frac{1}{x(a^2x^2+1)^2\arctan(ax)^2}dx - a^2\int \frac{x}{(a^2x^2+1)^3\arctan(ax)^2}dx\right)}{c^3}$$

$$\Big\downarrow \text{5377}$$

$$\frac{a^2\left(-\int \frac{1}{x(a^2x^2+1)^2\arctan(ax)^2}dx\right) - \frac{3\int \frac{1}{x^4\arctan(ax)}dx}{a} - \frac{1}{ax^3\arctan(ax)}}{c^3} - \frac{a^2\left(\int \frac{1}{x(a^2x^2+1)^2\arctan(ax)^2}dx - a^2\int \frac{x}{(a^2x^2+1)^3\arctan(ax)^2}dx\right)}{c^3}$$

$$\Big\downarrow \text{5501}$$

$$\frac{-\left(a^2\left(\int \frac{1}{x(a^2x^2+1)\arctan(ax)^2}dx - a^2\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^2}dx\right)\right) - \frac{3\int \frac{1}{x^4\arctan(ax)}dx}{a} - \frac{1}{ax^3\arctan(ax)}}{c^3} - \frac{a^2\left(a^2\left(-\int \frac{x}{(a^2x^2+1)^3\arctan(ax)^2}dx\right) - a^2\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^2}dx + \int \frac{1}{x(a^2x^2+1)\arctan(ax)^2}dx\right)}{c^3}$$

$$\Big\downarrow \text{5461}$$

$$\frac{-\left(a^2\left(a^2\left(-\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^2}dx\right) - \frac{\int \frac{1}{x^2\arctan(ax)}dx}{a} - \frac{1}{ax\arctan(ax)}\right)\right) - \frac{3\int \frac{1}{x^4\arctan(ax)}dx}{a} - \frac{1}{ax^3\arctan(ax)}}{c^3} - \frac{a^2\left(a^2\left(-\int \frac{x}{(a^2x^2+1)^3\arctan(ax)^2}dx\right) - a^2\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^2}dx - \frac{\int \frac{1}{x^2\arctan(ax)}dx}{a} - \frac{1}{ax\arctan(ax)}\right)}{c^3}$$

$$\Big\downarrow \text{5377}$$

$$\frac{-\left(a^2\left(a^2\left(-\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^2}dx\right) - \frac{\int \frac{1}{x^2\arctan(ax)}dx}{a} - \frac{1}{ax\arctan(ax)}\right)\right) - \frac{3\int \frac{1}{x^4\arctan(ax)}dx}{a} - \frac{1}{ax^3\arctan(ax)}}{c^3} - \frac{a^2\left(a^2\left(-\int \frac{x}{(a^2x^2+1)^3\arctan(ax)^2}dx\right) - a^2\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^2}dx - \frac{\int \frac{1}{x^2\arctan(ax)}dx}{a} - \frac{1}{ax\arctan(ax)}\right)}{c^3}$$

$\downarrow$ 5503

$$-\frac{\left(a^2\left(-\left(a^2\left(\frac{\int \frac{1}{(a^2x^2+1)^2\arctan(ax)}dx}{a}-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)\right)\right)-\frac{\int \frac{1}{x^2\arctan(ax)}dx}{a}-\frac{}{ax\,\mathrm{a}}}{c^3}$$

$$\frac{a^2\left(-\left(a^2\left(\frac{\int \frac{1}{(a^2x^2+1)^3\arctan(ax)}dx}{a}-3a\int \frac{x^2}{(a^2x^2+1)^3\arctan(ax)}dx-\frac{x}{a(a^2x^2+1)^2\arctan(ax)}\right)\right)\right)-a^2\left(\frac{\int \frac{1}{(a^2x^2+1)^2\arctan(ax)}}{a}\right.}{c^3}$$

$\downarrow$ 5439

$$-\frac{\left(a^2\left(-\left(a^2\left(-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\int \frac{1}{(a^2x^2+1)\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)\right)\right)-\frac{\int \frac{1}{x^2\arctan(ax)}d}{a}}{c^3}$$

$$\frac{a^2\left(-\left(a^2\left(-3a\int \frac{x^2}{(a^2x^2+1)^3\arctan(ax)}dx+\frac{\int \frac{1}{(a^2x^2+1)^2\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)^2\arctan(ax)}\right)\right)\right)-a^2\left(-a\int \frac{}{(a^2}\right.}{c^3}$$

$\downarrow$ 3042

$$-\frac{\left(a^2\left(-\left(a^2\left(-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^2}{\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)\right)\right)-\frac{\int \frac{1}{x^2\arctan(ax)}}{a}}{c^3}$$

$$\frac{a^2\left(-\left(a^2\left(-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^2}{\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)\right)\right)-a^2\left(-3a\int \frac{}{(a^2x^2}\right.}{c^3}$$

$\downarrow$ 3793

$$-\frac{\left(a^2\left(-\left(a^2\left(-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\int \left(\frac{\cos(2\arctan(ax))}{2\arctan(ax)}+\frac{1}{2\arctan(ax)}\right)d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)\right)\right)-\frac{\int \frac{}{x^2}}{}}{c^3}$$

$$\frac{a^2\left(-\left(a^2\left(-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\int \left(\frac{\cos(2\arctan(ax))}{2\arctan(ax)}+\frac{1}{2\arctan(ax)}\right)d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)\right)\right)-a^2\left(-3a\right.}{}$$

$\downarrow$ 2009

$$\frac{-\left(a^2\left(-\left(a^2\left(-a\int\frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{\int\frac{}{x^2 a}}{}\right.\right.}{c^3}$$

$$a^2\left(-\left(a^2\left(-3a\int\frac{x^2}{(a^2x^2+1)^3\arctan(ax)}dx+\frac{\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))+\frac{1}{8}\operatorname{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}-\frac{}{a(a^2x^2+}\right.\right.\right.$$

$\downarrow$ 5505

$$\frac{-\left(a^2\left(-\left(a^2\left(-\frac{\int\frac{a^2x^2}{(a^2x^2+1)\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)\right)\right.\right.}{c^3}$$

$$a^2\left(-\left(a^2\left(-\frac{3\int\frac{a^2x^2}{(a^2x^2+1)^2\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))+\frac{1}{8}\operatorname{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}-\frac{}{a}\right.\right.\right.$$

$\downarrow$ 3042

$$\frac{-\left(a^2\left(-\left(a^2\left(-\frac{\int\frac{\sin(\arctan(ax))^2}{\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{\int\frac{}{x}}{}\right.\right.}{c^3}$$

$$a^2\left(-\left(a^2\left(-\frac{3\int\frac{a^2x^2}{(a^2x^2+1)^2\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))+\frac{1}{8}\operatorname{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}-\frac{}{a}\right.\right.\right.$$

$\downarrow$ 3793

$$\frac{-\left(a^2\left(-\left(a^2\left(-\frac{\int\left(\frac{1}{2\arctan(ax)}-\frac{\cos(2\arctan(ax))}{2\arctan(ax)}\right)d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(}\right.\right.\right.\right.}{c^3}$$

$$a^2\left(-\left(a^2\left(-\frac{3\int\frac{a^2x^2}{(a^2x^2+1)^2\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))+\frac{1}{8}\operatorname{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}-\frac{}{a}\right.\right.\right.$$

$\downarrow$ 2009

$$-\frac{\left(a^2\left(-\frac{\int \frac{1}{x^2\arctan(ax)}dx}{a}-\left(a^2\left(-\frac{\frac{1}{2}\log(\arctan(ax))-\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))}{a^2}+\frac{\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}\right.\right.\right.\right.}{c^3}$$

$$a^2\left(-a^2\left(-\frac{3\int \frac{a^2x^2}{(a^2x^2+1)^2\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))+\frac{1}{8}\operatorname{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}-\frac{}{a(a^2\right.\right.$$

$$\Big\downarrow \; 4906$$

$$-\frac{\left(a^2\left(-\frac{\int \frac{1}{x^2\arctan(ax)}dx}{a}-\left(a^2\left(-\frac{\frac{1}{2}\log(\arctan(ax))-\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))}{a^2}+\frac{\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}\right.\right.\right.\right.}{c^3}$$

$$a^2\left(-a^2\left(-\frac{3\int \left(\frac{1}{8\arctan(ax)}-\frac{\cos(4\arctan(ax))}{8\arctan(ax)}\right)d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))+\frac{1}{8}\operatorname{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax\right.\right.}{a^2}$$

$$\Big\downarrow \; 2009$$

$$-\frac{\left(a^2\left(-\frac{\int \frac{1}{x^2\arctan(ax)}dx}{a}-\left(a^2\left(-\frac{\frac{1}{2}\log(\arctan(ax))-\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))}{a^2}+\frac{\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}\right.\right.\right.\right.}{c^3}$$

$$a^2\left(-\frac{\int \frac{1}{x^2\arctan(ax)}dx}{a}-\left(a^2\left(-\frac{3\left(\frac{1}{8}\log(\arctan(ax))-\frac{1}{8}\operatorname{CosIntegral}(4\arctan(ax))\right)}{a^2}+\frac{\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))+\frac{1}{8}\operatorname{CosIntegral}(4\arct\right.\right.\right.}{a^2}$$

input
```
Int[1/(x^3*(c + a^2*c*x^2)^3*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 60.39 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3\left(a^2c\,x^2+c\right)^3\arctan\left(ax\right)^2}dx$$

input
```
int(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

output
```
int(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.32

$$\int \frac{1}{x^3 (c+a^2cx^2)^3 \arctan(ax)^2} \, dx = \int \frac{1}{(a^2cx^2+c)^3 x^3 \arctan{(ax)}^2} \, dx$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(1/((a^6*c^3*x^9 + 3*a^4*c^3*x^7 + 3*a^2*c^3*x^5 + c^3*x^3)*arctan
(a*x)^2), x)
```

## Sympy [N/A]

Not integrable

Time = 1.56 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.73

$$\int \frac{1}{x^3 (c+a^2cx^2)^3 \arctan(ax)^2} \, dx = \frac{\int \frac{1}{a^6x^9 \operatorname{atan}^2{(ax)} + 3a^4x^7 \operatorname{atan}^2{(ax)} + 3a^2x^5 \operatorname{atan}^2{(ax)} + x^3 \operatorname{atan}^2{(ax)}} \, dx}{c^3}$$

input
```
integrate(1/x**3/(a**2*c*x**2+c)**3/atan(a*x)**2,x)
```

output
```
Integral(1/(a**6*x**9*atan(a*x)**2 + 3*a**4*x**7*atan(a*x)**2 + 3*a**2*x**
5*atan(a*x)**2 + x**3*atan(a*x)**2), x)/c**3
```

## Maxima [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 139, normalized size of antiderivative = 6.32

$$\int \frac{1}{x^3 (c + a^2 c x^2)^3 \arctan(ax)^2} \, dx = \int \frac{1}{(a^2 c x^2 + c)^3 x^3 \arctan{(ax)}^2} \, dx$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-((a^5*c^3*x^7 + 2*a^3*c^3*x^5 + a*c^3*x^3)*arctan(a*x)*integrate((7*a^2*x
^2 + 3)/((a^7*c^3*x^10 + 3*a^5*c^3*x^8 + 3*a^3*c^3*x^6 + a*c^3*x^4)*arctan
(a*x)), x) + 1)/((a^5*c^3*x^7 + 2*a^3*c^3*x^5 + a*c^3*x^3)*arctan(a*x))
```

## Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^3 (c + a^2 c x^2)^3 \arctan(ax)^2} \, dx = \int \frac{1}{(a^2 c x^2 + c)^3 x^3 \arctan{(ax)}^2} \, dx$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^3*x^3*arctan(a*x)^2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^3 (c + a^2 c x^2)^3 \arctan(ax)^2} \, dx = \int \frac{1}{x^3 \operatorname{atan}{(a\,x)}^2 \, {(c\,a^2\,x^2 + c)}^3} \, dx$$

input
```
int(1/(x^3*atan(a*x)^2*(c + a^2*c*x^2)^3),x)
```

output
```
int(1/(x^3*atan(a*x)^2*(c + a^2*c*x^2)^3), x)
```

## Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.73

$$\int \frac{1}{x^3 (c + a^2 c x^2)^3 \arctan(ax)^2} \, dx = \frac{\int \frac{1}{atan(ax)^2 a^6 x^9 + 3atan(ax)^2 a^4 x^7 + 3atan(ax)^2 a^2 x^5 + atan(ax)^2 x^3} dx}{c^3}$$

input
```
int(1/x^3/(a^2*c*x^2+c)^3/atan(a*x)^2,x)
```

output
```
int(1/(atan(a*x)**2*a**6*x**9 + 3*atan(a*x)**2*a**4*x**7 + 3*atan(a*x)**2*
a**2*x**5 + atan(a*x)**2*x**3),x)/c**3
```

# 3.565 $\int \frac{1}{x^4(c+a^2cx^2)^3\arctan(ax)^2}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\begin{aligned}\int \frac{1}{x^4(c+a^2cx^2)^3\arctan(ax)^2}\,dx =& -\frac{1}{ac^3x^4\arctan(ax)}+\frac{2a}{c^3x^2\arctan(ax)}\\ &-\frac{a^3}{c^3(1+a^2x^2)^2\arctan(ax)}\\ &-\frac{2a^3}{c^3(1+a^2x^2)\arctan(ax)}\\ &-\frac{3a^3\text{Si}(2\arctan(ax))}{c^3}-\frac{a^3\text{Si}(4\arctan(ax))}{2c^3}\\ &-\frac{4\text{Int}\left(\frac{1}{x^5\arctan(ax)},x\right)}{ac^3}+\frac{4a\text{Int}\left(\frac{1}{x^3\arctan(ax)},x\right)}{c^3}\end{aligned}$$

output
```
-1/a/c^3/x^4/arctan(a*x)+2*a/c^3/x^2/arctan(a*x)-a^3/c^3/(a^2*x^2+1)^2/arc
tan(a*x)-2*a^3/c^3/(a^2*x^2+1)/arctan(a*x)-3*a^3*Si(2*arctan(a*x))/c^3-1/2
*a^3*Si(4*arctan(a*x))/c^3-4*Defer(Int)(1/x^5/arctan(a*x),x)/a/c^3+4*a*Def
er(Int)(1/x^3/arctan(a*x),x)/c^3
```

## Mathematica [N/A]

Not integrable

Time = 2.68 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^4 (c+a^2cx^2)^3 \arctan(ax)^2} dx = \int \frac{1}{x^4 (c+a^2cx^2)^3 \arctan(ax)^2} dx$$

input
```
Integrate[1/(x^4*(c + a^2*c*x^2)^3*ArcTan[a*x]^2),x]
```

output
```
Integrate[1/(x^4*(c + a^2*c*x^2)^3*ArcTan[a*x]^2), x]
```

## Rubi [N/A]

Not integrable

Time = 3.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \arctan(ax)^2 (a^2cx^2+c)^3} dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{1}{c^2x^4(a^2x^2+1)^2 \arctan(ax)^2} dx}{c} - a^2 \int \frac{1}{c^3x^2 (a^2x^2+1)^3 \arctan(ax)^2} dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{1}{x^4(a^2x^2+1)^2 \arctan(ax)^2} dx}{c^3} - \frac{a^2 \int \frac{1}{x^2(a^2x^2+1)^3 \arctan(ax)^2} dx}{c^3}$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{1}{x^4(a^2x^2+1)\arctan(ax)^2}dx - a^2\int \frac{1}{x^2(a^2x^2+1)^2\arctan(ax)^2}dx}{c^3} - \frac{a^2\left(\int \frac{1}{x^2(a^2x^2+1)^2\arctan(ax)^2}dx - a^2\int \frac{1}{(a^2x^2+1)^3\arctan(ax)^2}dx\right)}{c^3}$$

$$\Big\downarrow \text{5437}$$

$$\frac{\int \frac{1}{x^4(a^2x^2+1)\arctan(ax)^2}dx - a^2\int \frac{1}{x^2(a^2x^2+1)^2\arctan(ax)^2}dx}{c^3} - \frac{a^2\left(\int \frac{1}{x^2(a^2x^2+1)^2\arctan(ax)^2}dx - a^2\left(-4a\int \frac{x}{(a^2x^2+1)^3\arctan(ax)}dx - \frac{1}{a(a^2x^2+1)^2\arctan(ax)}\right)\right)}{c^3}$$

$$\Big\downarrow \text{5461}$$

$$\frac{a^2\left(-\int \frac{1}{x^2(a^2x^2+1)^2\arctan(ax)^2}dx\right) - \frac{4\int \frac{1}{x^5\arctan(ax)}dx}{a} - \frac{1}{ax^4\arctan(ax)}}{c^3} - \frac{a^2\left(\int \frac{1}{x^2(a^2x^2+1)^2\arctan(ax)^2}dx - a^2\left(-4a\int \frac{x}{(a^2x^2+1)^3\arctan(ax)}dx - \frac{1}{a(a^2x^2+1)^2\arctan(ax)}\right)\right)}{c^3}$$

$$\Big\downarrow \text{5377}$$

$$\frac{a^2\left(-\int \frac{1}{x^2(a^2x^2+1)^2\arctan(ax)^2}dx\right) - \frac{4\int \frac{1}{x^5\arctan(ax)}dx}{a} - \frac{1}{ax^4\arctan(ax)}}{c^3} - \frac{a^2\left(\int \frac{1}{x^2(a^2x^2+1)^2\arctan(ax)^2}dx - a^2\left(-4a\int \frac{x}{(a^2x^2+1)^3\arctan(ax)}dx - \frac{1}{a(a^2x^2+1)^2\arctan(ax)}\right)\right)}{c^3}$$

$$\Big\downarrow \text{5501}$$

$$\frac{-\left(a^2\left(\int \frac{1}{x^2(a^2x^2+1)\arctan(ax)^2}dx - a^2\int \frac{1}{(a^2x^2+1)^2\arctan(ax)^2}dx\right)\right) - \frac{4\int \frac{1}{x^5\arctan(ax)}dx}{a} - \frac{1}{ax^4\arctan(ax)}}{c^3} - \frac{a^2\left(a^2\left(-\int \frac{1}{(a^2x^2+1)^2\arctan(ax)^2}dx\right) - a^2\left(-4a\int \frac{x}{(a^2x^2+1)^3\arctan(ax)}dx - \frac{1}{a(a^2x^2+1)^2\arctan(ax)}\right) + \int \frac{1}{x^2(a^2x^2+1)\arctan(a}\right.}{c^3}$$

$$\Big\downarrow \text{5437}$$

$$\frac{-\left(a^2\left(\int \frac{1}{x^2(a^2x^2+1)\arctan(ax)^2}dx - a^2\left(-2a\int \frac{x}{(a^2x^2+1)^2\arctan(ax)}dx - \frac{1}{a(a^2x^2+1)\arctan(ax)}\right)\right)\right) - \frac{4\int \frac{1}{x^5\arctan(ax)}dx}{a} - \frac{}{a}}{c^3} - \frac{a^2\left(-\left(a^2\left(-4a\int \frac{x}{(a^2x^2+1)^3\arctan(ax)}dx - \frac{1}{a(a^2x^2+1)^2\arctan(ax)}\right)\right) - a^2\left(-2a\int \frac{x}{(a^2x^2+1)^2\arctan(ax)}dx - \frac{1}{a(a^2x^2+1)\arctan}\right.\right.}{c^3}$$

$$\Big\downarrow \text{5461}$$

$$\frac{-\left(a^2\left(-\left(a^2\left(-2a\int \frac{x}{(a^2x^2+1)^2\arctan(ax)}dx-\frac{1}{a(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{2\int \frac{1}{x^3\arctan(ax)}dx}{a}-\frac{1}{ax^2\arctan(ax)}\right)\right)-\frac{4\int \frac{}{x^5\,\mathrm{arc}}}{c}}{c^3}$$

$$\frac{a^2\left(-\left(a^2\left(-4a\int \frac{x}{(a^2x^2+1)^3\arctan(ax)}dx-\frac{1}{a(a^2x^2+1)^2\arctan(ax)}\right)\right)-a^2\left(-2a\int \frac{x}{(a^2x^2+1)^2\arctan(ax)}dx-\frac{1}{a(a^2x^2+1)\,\mathrm{arcta}}\right.\right.}{c^3}$$

$\Big\downarrow$ 5377

$$\frac{-\left(a^2\left(-\left(a^2\left(-2a\int \frac{x}{(a^2x^2+1)^2\arctan(ax)}dx-\frac{1}{a(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{2\int \frac{1}{x^3\arctan(ax)}dx}{a}-\frac{1}{ax^2\arctan(ax)}\right)\right)-\frac{4\int \frac{}{x^5\,\mathrm{arc}}}{c}}{c^3}$$

$$\frac{a^2\left(-\left(a^2\left(-4a\int \frac{x}{(a^2x^2+1)^3\arctan(ax)}dx-\frac{1}{a(a^2x^2+1)^2\arctan(ax)}\right)\right)-a^2\left(-2a\int \frac{x}{(a^2x^2+1)^2\arctan(ax)}dx-\frac{1}{a(a^2x^2+1)\,\mathrm{arcta}}\right.\right.}{c^3}$$

$\Big\downarrow$ 5505

$$\frac{-\left(a^2\left(-\left(a^2\left(-\frac{2\int \frac{ax}{(a^2x^2+1)\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{2\int \frac{1}{x^3\arctan(ax)}dx}{a}-\frac{1}{ax^2\arctan(ax)}\right)\right)-\frac{4}{}}{c^3}$$

$$\frac{a^2\left(-\left(a^2\left(-\frac{4\int \frac{ax}{(a^2x^2+1)^2\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a(a^2x^2+1)^2\arctan(ax)}\right)\right)-a^2\left(-\frac{2\int \frac{ax}{(a^2x^2+1)\arctan(ax)}d\arctan(ax)}{a}-\frac{}{a(a}\right.\right.}{c^3}$$

$\Big\downarrow$ 4906

$$\frac{-\left(a^2\left(-\left(a^2\left(-\frac{2\int \frac{\sin(2\arctan(ax))}{2\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{2\int \frac{1}{x^3\arctan(ax)}dx}{a}-\frac{1}{ax^2\arctan(ax)}\right)\right)-\frac{4\int \frac{}{x^5\,\mathrm{a}}}{}}{c^3}$$

$$\frac{a^2\left(-\left(a^2\left(-\frac{2\int \frac{\sin(2\arctan(ax))}{2\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a(a^2x^2+1)\arctan(ax)}\right)\right)-a^2\left(-\frac{4\int \left(\frac{\sin(2\arctan(ax))}{4\arctan(ax)}+\frac{\sin(4\arctan(ax))}{8\arctan(ax)}\right)d\arctan(ax)}{a}\right.\right.}{c^3}$$

$\Big\downarrow$ 27

$$\frac{-\left(a^2\left(-\left(a^2\left(-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{2\int \frac{1}{x^3\arctan(ax)}dx}{a}-\frac{1}{ax^2\arctan(ax)}\right)\right)-\frac{4\int \frac{}{x^5\,\mathrm{arc}}}{c}}{c^3}$$

$$\frac{a^2\left(-\left(a^2\left(-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a(a^2x^2+1)\arctan(ax)}\right)\right)-a^2\left(-\frac{4\int \left(\frac{\sin(2\arctan(ax))}{4\arctan(ax)}+\frac{\sin(4\arctan(ax))}{8\arctan(ax)}\right)d\arctan(ax)}{a}\right.\right.}{c^3}$$

$\Big\downarrow$ 2009

$$\frac{-\left(a^2\left(-\left(a^2\left(-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{2\int \frac{1}{x^3\arctan(ax)}dx}{a}-\frac{1}{ax^2\arctan(ax)}\right)\right)-\frac{4\int \frac{1}{x^5\arctan(ax)}}{}}{c^3}$$

$$\frac{a^2\left(-a^2\left(-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a(a^2x^2+1)\arctan(ax)}\right)-\frac{2\int \frac{1}{x^3\arctan(ax)}dx}{a}-\left(a^2\left(-\frac{1}{a(a^2x^2+1)^2\arctan(ax)}-\frac{4\left(\frac{1}{4}\right.}{}\right.\right.\right.}{c^3}$$

$\Big\downarrow$ 3042

$$\frac{-\left(a^2\left(-\left(a^2\left(-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{2\int \frac{1}{x^3\arctan(ax)}dx}{a}-\frac{1}{ax^2\arctan(ax)}\right)\right)-\frac{4\int \frac{1}{x^5\arctan(ax)}}{}}{c^3}$$

$$\frac{a^2\left(-a^2\left(-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a(a^2x^2+1)\arctan(ax)}\right)-\frac{2\int \frac{1}{x^3\arctan(ax)}dx}{a}-\left(a^2\left(-\frac{1}{a(a^2x^2+1)^2\arctan(ax)}-\frac{4\left(\frac{1}{4}\right.}{}\right.\right.\right.}{c^3}$$

$\Big\downarrow$ 3780

$$\frac{-\left(a^2\left(-\frac{2\int \frac{1}{x^3\arctan(ax)}dx}{a}-\left(a^2\left(-\frac{1}{a(a^2x^2+1)\arctan(ax)}-\frac{\text{Si}(2\arctan(ax))}{a}\right)\right)-\frac{1}{ax^2\arctan(ax)}\right)\right)-\frac{4\int \frac{1}{x^5\arctan(ax)}dx}{a}-}{c^3}$$

$$\frac{a^2\left(-\frac{2\int \frac{1}{x^3\arctan(ax)}dx}{a}-\left(a^2\left(-\frac{1}{a(a^2x^2+1)\arctan(ax)}-\frac{\text{Si}(2\arctan(ax))}{a}\right)\right)-a^2\left(-\frac{1}{a(a^2x^2+1)^2\arctan(ax)}-\frac{4\left(\frac{1}{4}\text{Si}(2\arctan(\right.}{}\right.\right.}{c^3}$$

input
```
Int[1/(x^4*(c + a^2*c*x^2)^3*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 54.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4 \left(a^2 c\, x^2+c\right)^3 \arctan\left(a x\right)^2}dx$$

input
```
int(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

output
```
int(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.32

$$\int \frac{1}{x^4 (c+a^2cx^2)^3 \arctan(ax)^2}\, dx = \int \frac{1}{\left(a^2cx^2+c\right)^3 x^4 \arctan\left(ax\right)^2}\, dx$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(1/((a^6*c^3*x^10 + 3*a^4*c^3*x^8 + 3*a^2*c^3*x^6 + c^3*x^4)*arcta
n(a*x)^2), x)
```

## Sympy [N/A]

Not integrable

Time = 1.80 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.73

$$\int \frac{1}{x^4 (c+a^2cx^2)^3 \arctan(ax)^2}\, dx = \frac{\int \frac{1}{a^6x^{10}\operatorname{atan}^2(ax)+3a^4x^8\operatorname{atan}^2(ax)+3a^2x^6\operatorname{atan}^2(ax)+x^4\operatorname{atan}^2(ax)}\, dx}{c^3}$$

input
```
integrate(1/x**4/(a**2*c*x**2+c)**3/atan(a*x)**2,x)
```

output
```
Integral(1/(a**6*x**10*atan(a*x)**2 + 3*a**4*x**8*atan(a*x)**2 + 3*a**2*x*
*6*atan(a*x)**2 + x**4*atan(a*x)**2), x)/c**3
```

## Maxima [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 140, normalized size of antiderivative = 6.36

$$\int \frac{1}{x^4 (c+a^2cx^2)^3 \arctan(ax)^2} \, dx = \int \frac{1}{(a^2cx^2+c)^3 x^4 \arctan(ax)^2} \, dx$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-((a^5*c^3*x^8 + 2*a^3*c^3*x^6 + a*c^3*x^4)*arctan(a*x)*integrate(4*(2*a^2
*x^2 + 1)/((a^7*c^3*x^11 + 3*a^5*c^3*x^9 + 3*a^3*c^3*x^7 + a*c^3*x^5)*arct
an(a*x)), x) + 1)/((a^5*c^3*x^8 + 2*a^3*c^3*x^6 + a*c^3*x^4)*arctan(a*x))
```

## Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^4 (c+a^2cx^2)^3 \arctan(ax)^2} \, dx = \int \frac{1}{(a^2cx^2+c)^3 x^4 \arctan(ax)^2} \, dx$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^3*x^4*arctan(a*x)^2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^4 (c+a^2cx^2)^3 \arctan(ax)^2} \, dx = \int \frac{1}{x^4 \operatorname{atan}(a\,x)^2 \left(c\,a^2\,x^2+c\right)^3} \, dx$$

input
```
int(1/(x^4*atan(a*x)^2*(c + a^2*c*x^2)^3),x)
```

output
```
int(1/(x^4*atan(a*x)^2*(c + a^2*c*x^2)^3), x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.73

$$\int \frac{1}{x^4 (c+a^2cx^2)^3 \arctan(ax)^2} \, dx = \frac{\int \frac{1}{atan(ax)^2 a^6 x^{10}+3atan(ax)^2 a^4 x^8+3atan(ax)^2 a^2 x^6+atan(ax)^2 x^4}dx}{c^3}$$

input
```
int(1/x^4/(a^2*c*x^2+c)^3/atan(a*x)^2,x)
```

output
```
int(1/(atan(a*x)**2*a**6*x**10 + 3*atan(a*x)**2*a**4*x**8 + 3*atan(a*x)**2
*a**2*x**6 + atan(a*x)**2*x**4),x)/c**3
```

# 3.566 $\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^2}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^2}\,dx=\text{Int}\left(\frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^2},x\right)$$

output
```
Defer(Int)(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x)
```

## Mathematica [N/A]

Not integrable

Time = 1.38 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^2}\,dx=\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^2}\,dx$$

input
```
Integrate[(x*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^2,x]
```

output
```
Integrate[(x*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^2, x]
```

## Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{a^2cx^2+c}}{\arctan(ax)^2} \, dx$$

$$\Big\downarrow \ 5560$$

$$\int \frac{x\sqrt{a^2cx^2+c}}{\arctan(ax)^2}dx$$

input
```
Int[(x*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^2,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 34.52 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x\sqrt{a^2c\,x^2+c}}{\arctan\left(ax\right)^2}dx$$

input
```
int(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x)
```

output
```
int(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^2}\,dx = \int \frac{\sqrt{a^2cx^2+cx}}{\arctan(ax)^2}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x/arctan(a*x)^2, x)
```

## Sympy [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^2}\,dx = \int \frac{x\sqrt{c(a^2x^2+1)}}{\operatorname{atan}^2(ax)}\,dx$$

input
```
integrate(x*(a**2*c*x**2+c)**(1/2)/atan(a*x)**2,x)
```

output
```
Integral(x*sqrt(c*(a**2*x**2 + 1))/atan(a*x)**2, x)
```

## Maxima [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^2}\,dx = \int \frac{\sqrt{a^2cx^2+cx}}{\arctan(ax)^2}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*x/arctan(a*x)^2, x)
```

## Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^2}\,dx = \int \frac{\sqrt{a^2cx^2+c}x}{\arctan(ax)^2}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*x/arctan(a*x)^2, x)
```

## Mupad [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^2}\,dx = \int \frac{x\sqrt{c\,a^2\,x^2+c}}{\mathrm{atan}(a\,x)^2}\,dx$$

input
```
int((x*(c + a^2*c*x^2)^(1/2))/atan(a*x)^2,x)
```

output
```
int((x*(c + a^2*c*x^2)^(1/2))/atan(a*x)^2, x)
```

## Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^2}\,dx = \sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,x}{atan\,(ax)^2}dx\right)$$

input
```
int(x*(a^2*c*x^2+c)^(1/2)/atan(a*x)^2,x)
```

output
```
sqrt(c)*int((sqrt(a**2*x**2 + 1)*x)/atan(a*x)**2,x)
```

# 3.567 $\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^2}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^2}\,dx=\operatorname{Int}\left(\frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^2},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x)
```

## Mathematica [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^2}\,dx=\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^2}\,dx$$

input
```
Integrate[Sqrt[c + a^2*c*x^2]/ArcTan[a*x]^2,x]
```

output
```
Integrate[Sqrt[c + a^2*c*x^2]/ArcTan[a*x]^2, x]
```

## Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2cx^2+c}}{\arctan(ax)^2}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{\sqrt{a^2cx^2+c}}{\arctan(ax)^2}dx$$

input
```
Int[Sqrt[c + a^2*c*x^2]/ArcTan[a*x]^2,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 21.10 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{a^2c\,x^2+c}}{\arctan\left(ax\right)^2}dx$$

input
```
int((a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x)
```

output
```
int((a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^2}\,dx = \int \frac{\sqrt{a^2cx^2+c}}{\arctan\left(ax\right)^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)/arctan(a*x)^2, x)
```

## Sympy [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^2}\,dx = \int \frac{\sqrt{c\left(a^2x^2+1\right)}}{\operatorname{atan}^2\left(ax\right)}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(1/2)/atan(a*x)**2,x)
```

output
```
Integral(sqrt(c*(a**2*x**2 + 1))/atan(a*x)**2, x)
```

## Maxima [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^2}\,dx = \int \frac{\sqrt{a^2cx^2+c}}{\arctan\left(ax\right)^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)/arctan(a*x)^2, x)
```

## Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^2}\,dx = \int \frac{\sqrt{a^2cx^2+c}}{\arctan\left(ax\right)^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)/arctan(a*x)^2, x)
```

## Mupad [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^2}\,dx = \int \frac{\sqrt{c\,a^2\,x^2+c}}{\mathrm{atan}\left(a\,x\right)^2}\,dx$$

input
```
int((c + a^2*c*x^2)^(1/2)/atan(a*x)^2,x)
```

output
```
int((c + a^2*c*x^2)^(1/2)/atan(a*x)^2, x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^2}\,dx = \sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}}{atan\,(ax)^2}dx\right)$$

input
```
int((a^2*c*x^2+c)^(1/2)/atan(a*x)^2,x)
```

output
```
sqrt(c)*int(sqrt(a**2*x**2 + 1)/atan(a*x)**2,x)
```

# 3.568 $\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^2}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^2}\,dx = \text{Int}\left(\frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^2},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^2,x)
```

## Mathematica [N/A]

Not integrable

Time = 2.95 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^2}\,dx = \int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^2}\,dx$$

input
```
Integrate[Sqrt[c + a^2*c*x^2]/(x*ArcTan[a*x]^2),x]
```

output
```
Integrate[Sqrt[c + a^2*c*x^2]/(x*ArcTan[a*x]^2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2cx^2+c}}{x\arctan(ax)^2}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{\sqrt{a^2cx^2+c}}{x\arctan(ax)^2}dx$$

input
```
Int[Sqrt[c + a^2*c*x^2]/(x*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 86.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{a^2c\,x^2+c}}{x\arctan\left(ax\right)^2}dx$$

input
```
int((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^2,x)
```

output
```
int((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^2}\,dx = \int \frac{\sqrt{a^2cx^2+c}}{x\arctan\left(ax\right)^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)/(x*arctan(a*x)^2), x)
```

## Sympy [N/A]

Not integrable

Time = 1.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^2}\,dx = \int \frac{\sqrt{c\left(a^2x^2+1\right)}}{x\,\mathrm{atan}^2\left(ax\right)}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(1/2)/x/atan(a*x)**2,x)
```

output
```
Integral(sqrt(c*(a**2*x**2 + 1))/(x*atan(a*x)**2), x)
```

## Maxima [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^2}\,dx = \int \frac{\sqrt{a^2cx^2+c}}{x\arctan\left(ax\right)^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)/(x*arctan(a*x)^2), x)
```

## Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^2}\,dx = \int \frac{\sqrt{a^2cx^2+c}}{x\arctan\left(ax\right)^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)/(x*arctan(a*x)^2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^2}\,dx = \int \frac{\sqrt{c\,a^2\,x^2+c}}{x\,\mathrm{atan}\left(a\,x\right)^2}\,dx$$

input
```
int((c + a^2*c*x^2)^(1/2)/(x*atan(a*x)^2),x)
```

output
```
int((c + a^2*c*x^2)^(1/2)/(x*atan(a*x)^2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^2}\,dx = \sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}}{atan\,(ax)^2\,x}dx\right)$$

input
```
int((a^2*c*x^2+c)^(1/2)/x/atan(a*x)^2,x)
```

output
```
sqrt(c)*int(sqrt(a**2*x**2 + 1)/(atan(a*x)**2*x),x)
```

# 3.569 $\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^2} dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^2} dx = \text{Int}\left(\frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^2}, x\right)$$

output
```
Defer(Int)(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

## Mathematica [N/A]

Not integrable

Time = 3.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^2} dx = \int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^2} dx$$

input
```
Integrate[(x*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^2,x]
```

output
```
Integrate[(x*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^2, x]
```

## Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a^2cx^2+c\right)^{3/2}}{\arctan(ax)^2}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x\left(a^2cx^2+c\right)^{3/2}}{\arctan(ax)^2}dx$$

input
```
Int[(x*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^2,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 62.34 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x(a^2c\,x^2+c)^{\frac{3}{2}}}{\arctan\left(ax\right)^2}dx$$

input
```
int(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

output
```
int(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.50

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}x}{\arctan(ax)^2}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^3 + c*x)*sqrt(a^2*c*x^2 + c)/arctan(a*x)^2, x)
```

## Sympy [N/A]

Not integrable

Time = 4.86 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^2}\,dx = \int \frac{x(c(a^2x^2+1))^{\frac{3}{2}}}{\operatorname{atan}^2(ax)}\,dx$$

input
```
integrate(x*(a**2*c*x**2+c)**(3/2)/atan(a*x)**2,x)
```

output
```
Integral(x*(c*(a**2*x**2 + 1))**(3/2)/atan(a*x)**2, x)
```

## Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}x}{\arctan(ax)^2}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*x/arctan(a*x)^2, x)
```

## Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}x}{\arctan{(ax)}^2}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*x/arctan(a*x)^2, x)
```

## Mupad [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^2}\,dx = \int \frac{x\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}{{\mathrm{atan}\left(a\,x\right)}^2}\,dx$$

input
```
int((x*(c + a^2*c*x^2)^(3/2))/atan(a*x)^2,x)
```

output
```
int((x*(c + a^2*c*x^2)^(3/2))/atan(a*x)^2, x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.32

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^2}\,dx = \sqrt{c}\,c\left(\left(\int \frac{\sqrt{a^2x^2+1}\,x^3}{atan\,(ax)^2}dx\right)a^2+\int \frac{\sqrt{a^2x^2+1}\,x}{atan\,(ax)^2}dx\right)$$

input
```
int(x*(a^2*c*x^2+c)^(3/2)/atan(a*x)^2,x)
```

output
```
sqrt(c)*c*(int((sqrt(a**2*x**2 + 1)*x**3)/atan(a*x)**2,x)*a**2 + int((sqrt
(a**2*x**2 + 1)*x)/atan(a*x)**2,x))
```

# 3.570 $\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^2} \, dx$



## Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^2} \, dx = \text{Int}\left(\frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^2}, x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

## Mathematica [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^2} \, dx = \int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^2} \, dx$$

input
```
Integrate[(c + a^2*c*x^2)^(3/2)/ArcTan[a*x]^2,x]
```

output
```
Integrate[(c + a^2*c*x^2)^(3/2)/ArcTan[a*x]^2, x]
```

## Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^{3/2}}{\arctan(ax)^2}dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int \frac{(a^2cx^2+c)^{3/2}}{\arctan(ax)^2}dx$$

input
```
Int[(c + a^2*c*x^2)^(3/2)/ArcTan[a*x]^2,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 53.34 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{(a^2c\,x^2+c)^{\frac{3}{2}}}{\arctan\left(ax\right)^2}dx$$

input
```
int((a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

output
```
int((a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{(c + a^2cx^2)^{3/2}}{\arctan(ax)^2} \, dx = \int \frac{(a^2cx^2 + c)^{\frac{3}{2}}}{\arctan(ax)^2} \, dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^(3/2)/arctan(a*x)^2, x)
```

## Sympy [N/A]

Not integrable

Time = 3.36 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{(c + a^2cx^2)^{3/2}}{\arctan(ax)^2} \, dx = \int \frac{(c(a^2x^2 + 1))^{\frac{3}{2}}}{\operatorname{atan}^2(ax)} \, dx$$

input
```
integrate((a**2*c*x**2+c)**(3/2)/atan(a*x)**2,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(3/2)/atan(a*x)**2, x)
```

## Maxima [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{(c + a^2cx^2)^{3/2}}{\arctan(ax)^2} \, dx = \int \frac{(a^2cx^2 + c)^{\frac{3}{2}}}{\arctan(ax)^2} \, dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)/arctan(a*x)^2, x)
```

## Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}}{\arctan(ax)^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)/arctan(a*x)^2, x)
```

## Mupad [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^2}\,dx = \int \frac{(c\,a^2\,x^2+c)^{3/2}}{\mathrm{atan}\left(a\,x\right)^2}\,dx$$

input
```
int((c + a^2*c*x^2)^(3/2)/atan(a*x)^2,x)
```

output
```
int((c + a^2*c*x^2)^(3/2)/atan(a*x)^2, x)
```

## Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.38

$$\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^2}\,dx = \sqrt{c}\,c\left(\int \frac{\sqrt{a^2x^2+1}}{atan\,(ax)^2}dx + \left(\int \frac{\sqrt{a^2x^2+1}\,x^2}{atan\,(ax)^2}dx\right)a^2\right)$$

input
```
int((a^2*c*x^2+c)^(3/2)/atan(a*x)^2,x)
```

output
```
sqrt(c)*c*(int(sqrt(a**2*x**2 + 1)/atan(a*x)**2,x) + int((sqrt(a**2*x**2 +
 1)*x**2)/atan(a*x)**2,x)*a**2)
```

# 3.571 $\int \frac{\left(c+a^2 c x^2\right)^{3/2}}{x \arctan (a x)^2} \, dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\left(c+a^2 c x^2\right)^{3/2}}{x \arctan (a x)^2} \, dx=\text{Int}\left(\frac{\left(c+a^2 c x^2\right)^{3/2}}{x \arctan (a x)^2},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^2,x)
```

## Mathematica [N/A]

Not integrable

Time = 3.45 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\left(c+a^2 c x^2\right)^{3/2}}{x \arctan (a x)^2} \, dx=\int \frac{\left(c+a^2 c x^2\right)^{3/2}}{x \arctan (a x)^2} \, dx$$

input
```
Integrate[(c + a^2*c*x^2)^(3/2)/(x*ArcTan[a*x]^2),x]
```

output
```
Integrate[(c + a^2*c*x^2)^(3/2)/(x*ArcTan[a*x]^2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2cx^2+c\right)^{3/2}}{x\arctan(ax)^2}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{\left(a^2cx^2+c\right)^{3/2}}{x\arctan(ax)^2}dx$$

input
```
Int[(c + a^2*c*x^2)^(3/2)/(x*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 132.99 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}{x \arctan \left(a x \right)^{2}}d x$$

input
```
int((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^2,x)
```

output
```
int((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}}{x\arctan{(ax)}^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^(3/2)/(x*arctan(a*x)^2), x)
```

## Sympy [N/A]

Not integrable

Time = 5.88 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)^2}\,dx = \int \frac{(c(a^2x^2+1))^{\frac{3}{2}}}{x\operatorname{atan}^2{(ax)}}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(3/2)/x/atan(a*x)**2,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(3/2)/(x*atan(a*x)**2), x)
```

## Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}}{x\arctan{(ax)}^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)/(x*arctan(a*x)^2), x)
```

## Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)^2}\,dx=\int \frac{(a^2cx^2+c)^{\frac{3}{2}}}{x\arctan{(ax)}^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)/(x*arctan(a*x)^2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)^2}\,dx=\int \frac{(c\,a^2\,x^2+c)^{3/2}}{x\,\mathrm{atan}\left(a\,x\right)^2}\,dx$$

input
```
int((c + a^2*c*x^2)^(3/2)/(x*atan(a*x)^2),x)
```

output
```
int((c + a^2*c*x^2)^(3/2)/(x*atan(a*x)^2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.12

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)^2}\,dx = \sqrt{c}\,c\left(\int \frac{\sqrt{a^2x^2+1}}{atan\,(ax)^2\,x}dx+\left(\int \frac{\sqrt{a^2x^2+1}\,x}{atan\,(ax)^2}dx\right)a^2\right)$$

input
```
int((a^2*c*x^2+c)^(3/2)/x/atan(a*x)^2,x)
```

output
```
sqrt(c)*c*(int(sqrt(a**2*x**2 + 1)/(atan(a*x)**2*x),x) + int((sqrt(a**2*x*
*2 + 1)*x)/atan(a*x)**2,x)*a**2)
```

# 3.572 $\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^2}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^2}\,dx = \operatorname{Int}\left(\frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^2},x\right)$$

output
```
Defer(Int)(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

## Mathematica [N/A]

Not integrable

Time = 1.86 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^2}\,dx = \int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^2}\,dx$$

input
```
Integrate[(x*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^2,x]
```

output
```
Integrate[(x*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^2, x]
```

## Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a^2cx^2+c\right)^{5/2}}{\arctan(ax)^2}dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int \frac{x\left(a^2cx^2+c\right)^{5/2}}{\arctan(ax)^2}dx$$

input
```
Int[(x*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^2,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 84.91 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x(a^2c\,x^2+c)^{\frac{5}{2}}}{\arctan\left(ax\right)^2}dx$$

input
```
int(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

output
```
int(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.18

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}x}{\arctan{(ax)}^2}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^2*x)*sqrt(a^2*c*x^2 + c)/arctan(
a*x)^2, x)
```

## Sympy [N/A]

Not integrable

Time = 19.88 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^2}\,dx = \int \frac{x(c(a^2x^2+1))^{\frac{5}{2}}}{\operatorname{atan}^2{(ax)}}\,dx$$

input
```
integrate(x*(a**2*c*x**2+c)**(5/2)/atan(a*x)**2,x)
```

output
```
Integral(x*(c*(a**2*x**2 + 1))**(5/2)/atan(a*x)**2, x)
```

## Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}x}{\arctan{(ax)}^2}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)*x/arctan(a*x)^2, x)
```

## Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}x}{\arctan(ax)^2}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)*x/arctan(a*x)^2, x)
```

## Mupad [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^2}\,dx = \int \frac{x\,(c\,a^2\,x^2+c)^{5/2}}{\mathrm{atan}\,(a\,x)^2}\,dx$$

input
```
int((x*(c + a^2*c*x^2)^(5/2))/atan(a*x)^2,x)
```

output
```
int((x*(c + a^2*c*x^2)^(5/2))/atan(a*x)^2, x)
```

## Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 80, normalized size of antiderivative = 3.64

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^2}\,dx = \sqrt{c}\,c^2\left(\left(\int \frac{\sqrt{a^2x^2+1}\,x^5}{atan\,(ax)^2}dx\right)a^4 + 2\left(\int \frac{\sqrt{a^2x^2+1}\,x^3}{atan\,(ax)^2}dx\right)a^2 + \int \frac{\sqrt{a^2x^2+1}\,x}{atan\,(ax)^2}dx\right)$$

input
```
int(x*(a^2*c*x^2+c)^(5/2)/atan(a*x)^2,x)
```

output
```
sqrt(c)*c**2*(int((sqrt(a**2*x**2 + 1)*x**5)/atan(a*x)**2,x)*a**4 + 2*int(
(sqrt(a**2*x**2 + 1)*x**3)/atan(a*x)**2,x)*a**2 + int((sqrt(a**2*x**2 + 1)
*x)/atan(a*x)**2,x))
```

# 3.573 $\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^2}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^2}\,dx = \text{Int}\left(\frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^2},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

## Mathematica [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^2}\,dx = \int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^2}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^(5/2)/ArcTan[a*x]^2,x]
```

output
```
Integrate[(c + a^2*c*x^2)^(5/2)/ArcTan[a*x]^2, x]
```

## Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^{5/2}}{\arctan(ax)^2}dx$$

$$\Big\downarrow \text{5560}$$

$$\int \frac{(a^2cx^2+c)^{5/2}}{\arctan(ax)^2}dx$$

input
```
Int[(c + a^2*c*x^2)^(5/2)/ArcTan[a*x]^2,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 69.84 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{(a^2c\,x^2+c)^{\frac{5}{2}}}{\arctan\left(ax\right)^2}dx$$

input
```
int((a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

output
```
int((a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.19

$$\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}}{\arctan{(ax)}^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)/arctan(a*
x)^2, x)
```

## Sympy [N/A]

Not integrable

Time = 12.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^2}\,dx = \int \frac{(c(a^2x^2+1))^{\frac{5}{2}}}{\operatorname{atan}^2{(ax)}}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(5/2)/atan(a*x)**2,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(5/2)/atan(a*x)**2, x)
```

## Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}}{\arctan{(ax)}^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)/arctan(a*x)^2, x)
```

## Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}}{\arctan\left(ax\right)^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)/arctan(a*x)^2, x)
```

## Mupad [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^2}\,dx = \int \frac{{\left(c\,a^2\,x^2+c\right)}^{5/2}}{{\mathrm{atan}\left(a\,x\right)}^2}\,dx$$

input
```
int((c + a^2*c*x^2)^(5/2)/atan(a*x)^2,x)
```

output
```
int((c + a^2*c*x^2)^(5/2)/atan(a*x)^2, x)
```

## Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 79, normalized size of antiderivative = 3.76

$$\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^2}\,dx = \sqrt{c}\,c^2\left(\int \frac{\sqrt{a^2x^2+1}}{atan(ax)^2}dx + \left(\int \frac{\sqrt{a^2x^2+1}\,x^4}{atan(ax)^2}dx\right)a^4 + 2\left(\int \frac{\sqrt{a^2x^2+1}\,x^2}{atan(ax)^2}dx\right)a^2\right)$$

input

```
int((a^2*c*x^2+c)^(5/2)/atan(a*x)^2,x)
```

output

```
sqrt(c)*c**2*(int(sqrt(a**2*x**2 + 1)/atan(a*x)**2,x) + int((sqrt(a**2*x**
2 + 1)*x**4)/atan(a*x)**2,x)*a**4 + 2*int((sqrt(a**2*x**2 + 1)*x**2)/atan(
a*x)**2,x)*a**2)
```

# 3.574 $\int \frac{\left(c+a^2cx^2\right)^{5/2}}{x\arctan(ax)^2}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\left(c+a^2cx^2\right)^{5/2}}{x\arctan(ax)^2}\,dx=\text{Int}\left(\frac{\left(c+a^2cx^2\right)^{5/2}}{x\arctan(ax)^2},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^2,x)
```

## Mathematica [N/A]

Not integrable

Time = 1.97 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\left(c+a^2cx^2\right)^{5/2}}{x\arctan(ax)^2}\,dx=\int \frac{\left(c+a^2cx^2\right)^{5/2}}{x\arctan(ax)^2}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^(5/2)/(x*ArcTan[a*x]^2),x]
```

output
```
Integrate[(c + a^2*c*x^2)^(5/2)/(x*ArcTan[a*x]^2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^{5/2}}{x\arctan(ax)^2}dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{(a^2cx^2+c)^{5/2}}{x\arctan(ax)^2}dx$$

input
```
Int[(c + a^2*c*x^2)^(5/2)/(x*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 179.77 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{(a^2c\,x^2+c)^{\frac{5}{2}}}{x\arctan\left(ax\right)^2}dx$$

input
```
int((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^2,x)
```

output
```
int((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.04

$$\int \frac{(c + a^2cx^2)^{5/2}}{x \arctan(ax)^2} \, dx = \int \frac{(a^2cx^2 + c)^{\frac{5}{2}}}{x \arctan(ax)^2} \, dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)/(x*arctan
(a*x)^2), x)
```

## Sympy [N/A]

Not integrable

Time = 14.64 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{(c + a^2cx^2)^{5/2}}{x \arctan(ax)^2} \, dx = \int \frac{(c(a^2x^2 + 1))^{\frac{5}{2}}}{x \operatorname{atan}^2(ax)} \, dx$$

input
```
integrate((a**2*c*x**2+c)**(5/2)/x/atan(a*x)**2,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(5/2)/(x*atan(a*x)**2), x)
```

## Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c + a^2cx^2)^{5/2}}{x \arctan(ax)^2} \, dx = \int \frac{(a^2cx^2 + c)^{\frac{5}{2}}}{x \arctan(ax)^2} \, dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)/(x*arctan(a*x)^2), x)
```

## Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}}{x\arctan{(ax)}^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)/(x*arctan(a*x)^2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)^2}\,dx = \int \frac{{(c\,a^2\,x^2+c)}^{5/2}}{x\,{\mathrm{atan}\left(a\,x\right)}^2}\,dx$$

input
```
int((c + a^2*c*x^2)^(5/2)/(x*atan(a*x)^2),x)
```

output
```
int((c + a^2*c*x^2)^(5/2)/(x*atan(a*x)^2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 80, normalized size of antiderivative = 3.33

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)^2}\,dx = \sqrt{c}\,c^2\left(\int \frac{\sqrt{a^2x^2+1}}{atan\,(ax)^2\,x}dx + \left(\int \frac{\sqrt{a^2x^2+1}\,x^3}{atan\,(ax)^2}dx\right)a^4 + 2\left(\int \frac{\sqrt{a^2x^2+1}\,x}{atan\,(ax)^2}dx\right)a^2\right)$$

input
```
int((a^2*c*x^2+c)^(5/2)/x/atan(a*x)^2,x)
```

output
```
sqrt(c)*c**2*(int(sqrt(a**2*x**2 + 1)/(atan(a*x)**2*x),x) + int((sqrt(a**2
*x**2 + 1)*x**3)/atan(a*x)**2,x)*a**4 + 2*int((sqrt(a**2*x**2 + 1)*x)/atan
(a*x)**2,x)*a**2)
```

# 3.575 $\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^2}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^2}\,dx=\text{Int}\left(\frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^2},x\right)$$

output
```
Defer(Int)(x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x)
```

## Mathematica [N/A]

Not integrable

Time = 0.94 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^2}\,dx=\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^2}\,dx$$

input
```
Integrate[x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2),x]
```

output
```
Integrate[x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\arctan(ax)^2\sqrt{a^2cx^2+c}}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{x}{\arctan(ax)^2\sqrt{a^2cx^2+c}}dx$$

input
```
Int[x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 2.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x}{\sqrt{a^2c\,x^2+c}\,\arctan\left(ax\right)^2}dx$$

input
```
int(x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x)
```

output
```
int(x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^2}\,dx = \int \frac{x}{\sqrt{a^2cx^2+c}\arctan{(ax)}^2}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(x/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2), x)
```

## Sympy [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^2}\,dx = \int \frac{x}{\sqrt{c\left(a^2x^2+1\right)}\operatorname{atan}^2{(ax)}}\,dx$$

input
```
integrate(x/(a**2*c*x**2+c)**(1/2)/atan(a*x)**2,x)
```

output
```
Integral(x/(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**2), x)
```

## Maxima [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^2}\,dx = \int \frac{x}{\sqrt{a^2cx^2+c}\arctan{(ax)}^2}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2), x)
```

## Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^2}\,dx=\int \frac{x}{\sqrt{a^2cx^2+c}\arctan\left(ax\right)^2}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate(x/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^2}\,dx=\int \frac{x}{\mathrm{atan}\left(a\,x\right)^2\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(x/(atan(a*x)^2*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(x/(atan(a*x)^2*(c + a^2*c*x^2)^(1/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

$$\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^2}\,dx = \frac{\int \frac{x}{\sqrt{a^2x^2+1}\,atan(ax)^2}dx}{\sqrt{c}}$$

input
```
int(x/(a^2*c*x^2+c)^(1/2)/atan(a*x)^2,x)
```

output
```
int(x/(sqrt(a**2*x**2 + 1)*atan(a*x)**2),x)/sqrt(c)
```

# 3.576 $\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^2}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^2}\,dx=\text{Int}\left(\frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^2},x\right)$$

output
```
Defer(Int)(1/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x)
```

## Mathematica [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^2}\,dx=\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^2}\,dx$$

input
```
Integrate[1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2),x]
```

output
```
Integrate[1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arctan(ax)^2\sqrt{a^2cx^2+c}}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{1}{\arctan(ax)^2\sqrt{a^2cx^2+c}}dx$$

input
```
Int[1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 5.47 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1}{\sqrt{a^2c\,x^2+c}\,\arctan\left(ax\right)^2}dx$$

input
```
int(1/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x)
```

output
```
int(1/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^2}\,dx = \int \frac{1}{\sqrt{a^2cx^2+c}\arctan\left(ax\right)^2}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(1/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2), x)
```

## Sympy [N/A]

Not integrable

Time = 0.92 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^2}\,dx = \int \frac{1}{\sqrt{c\left(a^2x^2+1\right)}\operatorname{atan}^2\left(ax\right)}\,dx$$

input
```
integrate(1/(a**2*c*x**2+c)**(1/2)/atan(a*x)**2,x)
```

output
```
Integral(1/(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**2), x)
```

## Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^2}\,dx = \int \frac{1}{\sqrt{a^2cx^2+c}\arctan\left(ax\right)^2}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2), x)
```

## Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^2}\,dx = \int \frac{1}{\sqrt{a^2cx^2+c}\arctan\left(ax\right)^2}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate(1/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^2}\,dx = \int \frac{1}{\mathrm{atan}\left(a\,x\right)^2\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(1/(atan(a*x)^2*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(1/(atan(a*x)^2*(c + a^2*c*x^2)^(1/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.24

$$\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^2}\,dx=\frac{\int \frac{1}{\sqrt{a^2x^2+1}\,atan(ax)^2}dx}{\sqrt{c}}$$

input
```
int(1/(a^2*c*x^2+c)^(1/2)/atan(a*x)^2,x)
```

output
```
int(1/(sqrt(a**2*x**2 + 1)*atan(a*x)**2),x)/sqrt(c)
```

# 3.577 $\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)^2}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)^2}\,dx = -\frac{\sqrt{c+a^2cx^2}}{acx\arctan(ax)} - \frac{\mathrm{Int}\left(\frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)},x\right)}{a}$$

output
```
-(a^2*c*x^2+c)^(1/2)/a/c/x/arctan(a*x)-Defer(Int)(1/x^2/(a^2*c*x^2+c)^(1/2
)/arctan(a*x),x)/a
```

## Mathematica [N/A]

Not integrable

Time = 1.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)^2}\,dx = \int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)^2}\,dx$$

input
```
Integrate[1/(x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2),x]
```

output
```
Integrate[1/(x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x \arctan(ax)^2\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5477}$$

$$-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}$$

$$\Big\downarrow \text{5560}$$

$$-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}$$

input
```
Int[1/(x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 6.42 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\sqrt{a^2c\,x^2+c}\,\arctan\left(ax\right)^2}dx$$

input
```
int(1/x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x)
```

output
```
int(1/x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)^2}\,dx = \int \frac{1}{\sqrt{a^2cx^2+c}x\arctan{(ax)}^2}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)/((a^2*c*x^3 + c*x)*arctan(a*x)^2), x)
```

## Sympy [N/A]

Not integrable

Time = 1.63 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)^2}\,dx = \int \frac{1}{x\sqrt{c\left(a^2x^2+1\right)}\operatorname{atan}^2{(ax)}}\,dx$$

input
```
integrate(1/x/(a**2*c*x**2+c)**(1/2)/atan(a*x)**2,x)
```

output
```
Integral(1/(x*sqrt(c*(a**2*x**2 + 1))*atan(a*x)**2), x)
```

## Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)^2}\,dx = \int \frac{1}{\sqrt{a^2cx^2+c}x\arctan{(ax)}^2}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a^2*c*x^2 + c)*x*arctan(a*x)^2), x)
```

## Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)^2}\,dx = \int \frac{1}{\sqrt{a^2cx^2+c}x\arctan{(ax)}^2}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate(1/(sqrt(a^2*c*x^2 + c)*x*arctan(a*x)^2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)^2}\,dx = \int \frac{1}{x\,\mathrm{atan}\left(a\,x\right)^2\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(1/(x*atan(a*x)^2*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(1/(x*atan(a*x)^2*(c + a^2*c*x^2)^(1/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)^2}\,dx = \frac{\int \frac{1}{\sqrt{a^2x^2+1}\,atan(ax)^2x}dx}{\sqrt{c}}$$

input
```
int(1/x/(a^2*c*x^2+c)^(1/2)/atan(a*x)^2,x)
```

output
```
int(1/(sqrt(a**2*x**2 + 1)*atan(a*x)**2*x),x)/sqrt(c)
```

# 3.578 $\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx = \frac{x}{a^3c\sqrt{c+a^2cx^2}\arctan(ax)} - \frac{\sqrt{1+a^2x^2}\,\mathrm{CosIntegral}(\arctan(ax))}{a^4c\sqrt{c+a^2cx^2}} + \frac{\mathrm{Int}\left(\frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^2},x\right)}{a^2c}$$

output
```
x/a^3/c/(a^2*c*x^2+c)^(1/2)/arctan(a*x)-(a^2*x^2+1)^(1/2)*Ci(arctan(a*x))/
a^4/c/(a^2*c*x^2+c)^(1/2)+Defer(Int)(x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x
)/a^2/c
```

## Mathematica [N/A]

Not integrable

Time = 9.55 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx = \int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx$$

input
```
Integrate[x^3/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2),x]
```

output
```
Integrate[x^3/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2), x]
```

## Rubi [N/A]

Not integrable

Time = 1.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{5499}$$

$$\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{a^2c}-\frac{\int \frac{x}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}dx}{a^2}$$

$$\Big\downarrow \text{5477}$$

$$\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{a^2c}-\frac{\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}dx}{a}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2}$$

$$\Big\downarrow \text{5440}$$

$$\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{a^2c}-\frac{\frac{\sqrt{a^2x^2+1}\int \frac{1}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}dx}{ac\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2}$$

$$\Big\downarrow \text{5439}$$

$$\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{a^2c}-\frac{\frac{\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{a^2c}-\frac{\frac{\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)}{\arctan(ax)}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2}$$

$$\Big\downarrow \text{3783}$$

$$\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{a^2c} - \frac{\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2}$$

$$\Big\downarrow \text{5560}$$

$$\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{a^2c} - \frac{\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2}$$

input
```
Int[x^3/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 10.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^3}{(a^2c\,x^2+c)^{\frac{3}{2}}\arctan(ax)^2}dx$$

input
```
int(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

output
```
int(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.12

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx = \int \frac{x^3}{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)^2}\,dx$$

input
```
integrate(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^3/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arct
an(a*x)^2), x)
```

## Sympy [N/A]

Not integrable

Time = 2.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx=\int \frac{x^3}{(c\,(a^2x^2+1))^{\frac{3}{2}}\operatorname{atan}^2(ax)}\,dx$$

input
```
integrate(x**3/(a**2*c*x**2+c)**(3/2)/atan(a*x)**2,x)
```

output
```
Integral(x**3/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**2), x)
```

## Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx=\int \frac{x^3}{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)^2}\,dx$$

input
```
integrate(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(x^3/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^2), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx = \int \frac{x^3}{\mathrm{atan}\left(a\,x\right)^2\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

input
```
int(x^3/(atan(a*x)^2*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(x^3/(atan(a*x)^2*(c + a^2*c*x^2)^(3/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.38

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx = \frac{\int \frac{x^3}{\sqrt{a^2x^2+1}\,atan(ax)^2a^2x^2+\sqrt{a^2x^2+1}\,atan(ax)^2}dx}{\sqrt{c}\,c}$$

input
```
int(x^3/(a^2*c*x^2+c)^(3/2)/atan(a*x)^2,x)
```

output
```
int(x**3/(sqrt(a**2*x**2 + 1)*atan(a*x)**2*a**2*x**2 + sqrt(a**2*x**2 + 1)
*atan(a*x)**2),x)/(sqrt(c)*c)
```

# 3.579 $\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx = \frac{1}{a^3c\sqrt{c+a^2cx^2}\arctan(ax)} + \frac{\sqrt{1+a^2x^2}\operatorname{Si}(\arctan(ax))}{a^3c\sqrt{c+a^2cx^2}} + \frac{\operatorname{Int}\left(\frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^2},x\right)}{a^2c}$$

output
```
1/a^3/c/(a^2*c*x^2+c)^(1/2)/arctan(a*x)+(a^2*x^2+1)^(1/2)*Si(arctan(a*x))/
a^3/c/(a^2*c*x^2+c)^(1/2)+Defer(Int)(1/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x
)/a^2/c
```

## Mathematica [N/A]

Not integrable

Time = 6.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx = \int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx$$

input
```
Integrate[x^2/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2),x]
```

output
```
Integrate[x^2/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2), x]
```

## Rubi [N/A]

Not integrable

Time = 1.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{5499}$$

$$\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{a^2c}-\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}dx}{a^2}$$

$$\Big\downarrow \text{5437}$$

$$\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{a^2c}-\frac{-a\int \frac{x}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}dx-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2}$$

$$\Big\downarrow \text{5506}$$

$$\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{a^2c}-\frac{-\frac{a\sqrt{a^2x^2+1}\int \frac{x}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}dx}{c\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2}$$

$$\Big\downarrow \text{5505}$$

$$\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{a^2c}-\frac{-\frac{\sqrt{a^2x^2+1}\int \frac{ax}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{a^2c}-\frac{-\frac{\sqrt{a^2x^2+1}\int \frac{\sin(\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2}$$

$$\Big\downarrow \text{3780}$$

$$\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{a^2c} - \frac{-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}} - \frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2}$$

$$\Big\downarrow \text{5560}$$

$$\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{a^2c} - \frac{-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}} - \frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2}$$

input
```
Int[x^2/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 10.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^2}{(a^2c\,x^2+c)^{\frac{3}{2}}\arctan(ax)^2}dx$$

input
```
int(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

output
```
int(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.12

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx = \int \frac{x^2}{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)^2}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^2/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arct
an(a*x)^2), x)
```

## Sympy [N/A]

Not integrable

Time = 2.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx = \int \frac{x^2}{(c\,(a^2x^2+1))^{\frac{3}{2}}\,\mathrm{atan}^2\,(ax)}\,dx$$

input
```
integrate(x**2/(a**2*c*x**2+c)**(3/2)/atan(a*x)**2,x)
```

output
```
Integral(x**2/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**2), x)
```

## Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx = \int \frac{x^2}{(a^2cx^2+c)^{\frac{3}{2}}\arctan\,(ax)^2}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(x^2/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^2), x)
```

## Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx = \int \frac{x^2}{(a^2cx^2+c)^{\frac{3}{2}}\arctan{(ax)}^2}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate(x^2/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx = \int \frac{x^2}{\mathrm{atan}\left(a\,x\right)^2\,\left(c\,a^2\,x^2+c\right)^{3/2}}\,dx$$

input
```
int(x^2/(atan(a*x)^2*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(x^2/(atan(a*x)^2*(c + a^2*c*x^2)^(3/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.38

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx = \frac{\int \frac{x^2}{\sqrt{a^2x^2+1}\,atan(ax)^2a^2x^2+\sqrt{a^2x^2+1}\,atan(ax)^2}dx}{\sqrt{c}\,c}$$

input
```
int(x^2/(a^2*c*x^2+c)^(3/2)/atan(a*x)^2,x)
```

output
```
int(x**2/(sqrt(a**2*x**2 + 1)*atan(a*x)**2*a**2*x**2 + sqrt(a**2*x**2 + 1)
*atan(a*x)**2),x)/(sqrt(c)*c)
```

# 3.580 $\int \frac{x}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 69

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx = -\frac{x}{ac\sqrt{c+a^2cx^2}\arctan(ax)} + \frac{\sqrt{1+a^2x^2}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{c+a^2cx^2}}$$

output
```
-x/a/c/(a^2*c*x^2+c)^(1/2)/arctan(a*x)+(a^2*x^2+1)^(1/2)*Ci(arctan(a*x))/a
^2/c/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.80

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx = \frac{-ax+\sqrt{1+a^2x^2}\arctan(ax)\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{c+a^2cx^2}\arctan(ax)}$$

input
```
Integrate[x/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2),x]
```

output
```
(-(a*x) + Sqrt[1 + a^2*x^2]*ArcTan[a*x]*CosIntegral[ArcTan[a*x]])/(a^2*c*S
qrt[c + a^2*c*x^2]*ArcTan[a*x])
```

## Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {5477, 5440, 5439, 3042, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{5477}$$

$$\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}dx}{a}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5440}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{1}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}dx}{ac\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5439}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)}{\arctan(ax)}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3783}$$

$$\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}$$

input
```
Int[x/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2),x]
```

output
```
-(x/(a*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])) + (Sqrt[1 + a^2*x^2]*CosIntegra
l[ArcTan[a*x]])/(a^2*c*Sqrt[c + a^2*c*x^2])
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5440
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])   Int[(1 +
c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

rule 5477
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*ArcT
an[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp[f*(m/(b*c*(p + 1)))   Int[(f*x
)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1
]
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 6.60 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.83

| method | result |
|---|---|
| default | $-\frac{\left(\arctan(ax)\,\mathrm{expIntegral}_1(i\arctan(ax))a^2x^2+\arctan(ax)\,\mathrm{expIntegral}_1(-i\arctan(ax))a^2x^2+2\sqrt{a^2x^2+1}\,ax+\mathrm{expIntegral}_1(i\arct}{2(a^2x^2+1)^{\frac{3}{2}}\arctan(ax)a^2c^2}$ |

input
```
int(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(arctan(a*x)*Ei(1,I*arctan(a*x))*a^2*x^2+arctan(a*x)*Ei(1,-I*arctan(a
*x))*a^2*x^2+2*(a^2*x^2+1)^(1/2)*a*x+Ei(1,I*arctan(a*x))*arctan(a*x)+Ei(1,
-I*arctan(a*x))*arctan(a*x))/(a^2*x^2+1)^(3/2)*(c*(a*x-I)*(a*x+I))^(1/2)/a
rctan(a*x)/a^2/c^2
```

## Fricas [F]

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx=\int \frac{x}{(a^2cx^2+c)^{\frac{3}{2}}\arctan{(ax)}^2}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan
(a*x)^2), x)
```

## Sympy [F]

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx = \int \frac{x}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^2\left(ax\right)}\,dx$$

input
```
integrate(x/(a**2*c*x**2+c)**(3/2)/atan(a*x)**2,x)
```

output
```
Integral(x/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**2), x)
```

## Maxima [F]

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx = \int \frac{x}{\left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^2}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(x/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^2), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2}\,dx=\int \frac{x}{\mathrm{atan}\left(a\,x\right)^2\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

input
```
int(x/(atan(a*x)^2*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(x/(atan(a*x)^2*(c + a^2*c*x^2)^(3/2)), x)
```

## Reduce [F]

$$\int \frac{x}{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2}\,dx=\frac{\int \frac{x}{\sqrt{a^2x^2+1}\,atan(ax)^2a^2x^2+\sqrt{a^2x^2+1}\,atan(ax)^2}dx}{\sqrt{c}\,c}$$

input
```
int(x/(a^2*c*x^2+c)^(3/2)/atan(a*x)^2,x)
```

output
```
int(x/(sqrt(a**2*x**2 + 1)*atan(a*x)**2*a**2*x**2 + sqrt(a**2*x**2 + 1)*at
an(a*x)**2),x)/(sqrt(c)*c)
```

# 3.581 $\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 69

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx = -\frac{1}{ac\sqrt{c+a^2cx^2}\arctan(ax)} - \frac{\sqrt{1+a^2x^2}\,\mathrm{Si}(\arctan(ax))}{ac\sqrt{c+a^2cx^2}}$$

output
```
-1/a/c/(a^2*c*x^2+c)^(1/2)/arctan(a*x)-(a^2*x^2+1)^(1/2)*Si(arctan(a*x))/a
/c/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.77

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx = -\frac{1+\sqrt{1+a^2x^2}\arctan(ax)\mathrm{Si}(\arctan(ax))}{ac\sqrt{c+a^2cx^2}\arctan(ax)}$$

input
```
Integrate[1/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2),x]
```

output
```
-((1 + Sqrt[1 + a^2*x^2]*ArcTan[a*x]*SinIntegral[ArcTan[a*x]])/(a*c*Sqrt[c
 + a^2*c*x^2]*ArcTan[a*x]))
```

## Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {5437, 5506, 5505, 3042, 3780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arctan(ax)^2 \left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{5437}$$

$$-a\int \frac{x}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}dx-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5506}$$

$$-\frac{a\sqrt{a^2x^2+1}\int \frac{x}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}dx}{c\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5505}$$

$$-\frac{\sqrt{a^2x^2+1}\int \frac{ax}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sqrt{a^2x^2+1}\int \frac{\sin(\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3780}$$

$$-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}$$

input
```
Int[1/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2),x]
```

output
```
-(1/(a*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])) - (Sqrt[1 + a^2*x^2]*SinIntegra
l[ArcTan[a*x]])/(a*c*Sqrt[c + a^2*c*x^2])
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 6.21 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.86

| method | result |
|---|---|
| default | $\frac{i\left(\arctan(ax)\,\mathrm{expIntegral}_1(i\arctan(ax))a^2x^2-\arctan(ax)\,\mathrm{expIntegral}_1(-i\arctan(ax))a^2x^2+2i\sqrt{a^2x^2+1}+\mathrm{expIntegral}_1(i\arctan(\right.}{2(a^2x^2+1)^{\frac{3}{2}}\arctan(ax)a\,c^2}$ |

input
```
int(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*I*(arctan(a*x)*Ei(1,I*arctan(a*x))*a^2*x^2-arctan(a*x)*Ei(1,-I*arctan(
a*x))*a^2*x^2+2*I*(a^2*x^2+1)^(1/2)+Ei(1,I*arctan(a*x))*arctan(a*x)-Ei(1,-
I*arctan(a*x))*arctan(a*x))/(a^2*x^2+1)^(3/2)*(c*(a*x-I)*(a*x+I))^(1/2)/ar
ctan(a*x)/a/c^2
```

## Fricas [F]

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx = \int \frac{1}{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)^2}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a
*x)^2), x)
```

## Sympy [F]

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx = \int \frac{1}{(c(a^2x^2+1))^{\frac{3}{2}}\operatorname{atan}^2(ax)}\,dx$$

input
```
integrate(1/(a**2*c*x**2+c)**(3/2)/atan(a*x)**2,x)
```

output
```
Integral(1/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**2), x)
```

## Maxima [F]

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx = \int \frac{1}{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)^2}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^2), x)
```

## Giac [F]

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx = \int \frac{1}{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)^2}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx = \int \frac{1}{\mathrm{atan}(a\,x)^2\,(c\,a^2\,x^2+c)^{3/2}}\,dx$$

input
```
int(1/(atan(a*x)^2*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(1/(atan(a*x)^2*(c + a^2*c*x^2)^(3/2)), x)
```

## Reduce [F]

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx = \frac{\int \frac{1}{\sqrt{a^2x^2+1}\,atan(ax)^2a^2x^2+\sqrt{a^2x^2+1}\,atan(ax)^2}dx}{\sqrt{c}\,c}$$

input
```
int(1/(a^2*c*x^2+c)^(3/2)/atan(a*x)^2,x)
```

output
```
int(1/(sqrt(a**2*x**2 + 1)*atan(a*x)**2*a**2*x**2 + sqrt(a**2*x**2 + 1)*at
an(a*x)**2),x)/(sqrt(c)*c)
```

# 3.582 $\int \frac{1}{x(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx = \frac{ax}{c\sqrt{c+a^2cx^2}\arctan(ax)} - \frac{\sqrt{c+a^2cx^2}}{ac^2x\arctan(ax)} - \frac{\sqrt{1+a^2x^2}\,\mathrm{CosIntegral}(\arctan(ax))}{c\sqrt{c+a^2cx^2}} - \frac{\mathrm{Int}\left(\frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)},x\right)}{ac}$$

output
```
a*x/c/(a^2*c*x^2+c)^(1/2)/arctan(a*x)-(a^2*c*x^2+c)^(1/2)/a/c^2/x/arctan(a
*x)-(a^2*x^2+1)^(1/2)*Ci(arctan(a*x))/c/(a^2*c*x^2+c)^(1/2)-Defer(Int)(1/x
^2/(a^2*c*x^2+c)^(1/2)/arctan(a*x),x)/a/c
```

## Mathematica [N/A]

Not integrable

Time = 1.82 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx = \int \frac{1}{x(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx$$

input
```
Integrate[1/(x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2),x]
```

output
```
Integrate[1/(x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2), x]
```

## Rubi [N/A]

Not integrable

Time = 1.47 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x \arctan(ax)^2 \left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 5501}$$

$$\frac{\int \frac{1}{x\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\int \frac{x}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}dx$$

$$\Big\downarrow \text{ 5477}$$

$$\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c}-a^2\left(\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}dx}{a}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \text{ 5440}$$

$$\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c}-a^2\left(\frac{\sqrt{a^2x^2+1}\int \frac{1}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}dx}{ac\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \text{ 5439}$$

$$\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c}-a^2\left(\frac{\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)$$

$\Big\downarrow$ 3042

$$\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c}-a^2\left(\frac{\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)}{\arctan(ax)}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)$$

$\Big\downarrow$ 3783

$$\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c}-a^2\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)$$

$\Big\downarrow$ 5560

$$\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c}-a^2\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)$$

input
```
Int[1/(x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 11.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\,(a^2c\,x^2+c)^{\frac{3}{2}} \arctan\left(ax\right)^2}dx$$

input
```
int(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

output
```
int(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.08

$$\int \frac{1}{x\,(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx = \int \frac{1}{(a^2cx^2+c)^{\frac{3}{2}}x\arctan\left(ax\right)^2}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)/((a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^2*x)*arctan
(a*x)^2), x)
```

## Sympy [N/A]

Not integrable

Time = 4.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\,(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx = \int \frac{1}{x\,(c\,(a^2x^2+1))^{\frac{3}{2}}\operatorname{atan}^2\left(ax\right)}\,dx$$

input
```
integrate(1/x/(a**2*c*x**2+c)**(3/2)/atan(a*x)**2,x)
```

output
```
Integral(1/(x*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**2), x)
```

## Maxima [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2}\,dx = \int \frac{1}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x\arctan\left(ax\right)^2}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(3/2)*x*arctan(a*x)^2), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2}\,dx=\int \frac{1}{x\,\mathrm{atan}\left(a\,x\right)^2\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

input
```
int(1/(x*atan(a*x)^2*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(1/(x*atan(a*x)^2*(c + a^2*c*x^2)^(3/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.25

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2}\,dx=\frac{\int \frac{1}{\sqrt{a^2x^2+1}\,atan(ax)^2a^2x^3+\sqrt{a^2x^2+1}\,atan(ax)^2x}dx}{\sqrt{c}\,c}$$

input
```
int(1/x/(a^2*c*x^2+c)^(3/2)/atan(a*x)^2,x)
```

output
```
int(1/(sqrt(a**2*x**2 + 1)*atan(a*x)**2*a**2*x***3 + sqrt(a**2*x**2 + 1)*at
an(a*x)**2*x),x)/(sqrt(c)*c)
```

# 3.583 $\int \frac{1}{x^2(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2}\,dx = \frac{a}{c\sqrt{c+a^2cx^2}\arctan(ax)} + \frac{a\sqrt{1+a^2x^2}\,\mathrm{Si}(\arctan(ax))}{c\sqrt{c+a^2cx^2}} + \frac{\mathrm{Int}\left(\frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^2},x\right)}{c}$$

output
```
a/c/(a^2*c*x^2+c)^(1/2)/arctan(a*x)+a*(a^2*x^2+1)^(1/2)*Si(arctan(a*x))/c/
(a^2*c*x^2+c)^(1/2)+Defer(Int)(1/x^2/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x)/
c
```

## Mathematica [N/A]

Not integrable

Time = 3.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2}\,dx = \int \frac{1}{x^2\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2}\,dx$$

input
```
Integrate[1/(x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2),x]
```

output
```
Integrate[1/(x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2), x]
```

## Rubi [N/A]

Not integrable

Time = 1.39 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \arctan(ax)^2 \left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 5501}$$

$$\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}dx$$

$$\Big\downarrow \text{ 5437}$$

$$\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\left(-a\int \frac{x}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}dx-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \text{ 5506}$$

$$\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\left(-\frac{a\sqrt{a^2x^2+1}\int \frac{x}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}dx}{c\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \text{ 5505}$$

$$\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\left(-\frac{\sqrt{a^2x^2+1}\int \frac{ax}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\left(-\frac{\sqrt{a^2x^2+1}\int \frac{\sin(\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}} - \frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \text{3780}$$

$$\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\left(-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}} - \frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \text{5560}$$

$$\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\left(-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}} - \frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)$$

input
```
Int[1/(x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 12.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^2\left(a^2c\,x^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^2}dx$$

input
```
int(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

output
```
int(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.17

$$\int \frac{1}{x^2 (c+a^2cx^2)^{3/2} \arctan(ax)^2} \, dx = \int \frac{1}{(a^2cx^2+c)^{\frac{3}{2}} x^2 \arctan{(ax)}^2} \, dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)/((a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2)*arct
an(a*x)^2), x)
```

## Sympy [N/A]

Not integrable

Time = 5.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^2 (c+a^2cx^2)^{3/2} \arctan(ax)^2} \, dx = \int \frac{1}{x^2 (c (a^2x^2+1))^{\frac{3}{2}} \operatorname{atan}^2{(ax)}} \, dx$$

input
```
integrate(1/x**2/(a**2*c*x**2+c)**(3/2)/atan(a*x)**2,x)
```

output
```
Integral(1/(x**2*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**2), x)
```

## Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (c+a^2cx^2)^{3/2} \arctan(ax)^2} \, dx = \int \frac{1}{(a^2cx^2+c)^{\frac{3}{2}} x^2 \arctan{(ax)}^2} \, dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(3/2)*x^2*arctan(a*x)^2), x)
```

## Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (c + a^2cx^2)^{3/2} \arctan(ax)^2} \, dx = \int \frac{1}{(a^2cx^2 + c)^{\frac{3}{2}} x^2 \arctan(ax)^2} \, dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(3/2)*x^2*arctan(a*x)^2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (c + a^2cx^2)^{3/2} \arctan(ax)^2} \, dx = \int \frac{1}{x^2 \operatorname{atan}(a\,x)^2 \,(c\,a^2\,x^2 + c)^{3/2}} \, dx$$

input
```
int(1/(x^2*atan(a*x)^2*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(1/(x^2*atan(a*x)^2*(c + a^2*c*x^2)^(3/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.33

$$\int \frac{1}{x^2 (c+a^2cx^2)^{3/2} \arctan(ax)^2} \, dx = \frac{\int \frac{1}{\sqrt{a^2x^2+1}\, atan(ax)^2 a^2x^4+\sqrt{a^2x^2+1}\, atan(ax)^2 x^2} dx}{\sqrt{c}\, c}$$

input
```
int(1/x^2/(a^2*c*x^2+c)^(3/2)/atan(a*x)^2,x)
```

output
```
int(1/(sqrt(a**2*x**2 + 1)*atan(a*x)**2*a**2*x**4 + sqrt(a**2*x**2 + 1)*at
an(a*x)**2*x**2),x)/(sqrt(c)*c)
```

# 3.584 $\int \frac{1}{x^3(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\begin{aligned}\int \frac{1}{x^3\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2}\,dx =& -\frac{a^3x}{c\sqrt{c+a^2cx^2}\arctan(ax)} \\ &+\frac{a\sqrt{c+a^2cx^2}}{c^2x\arctan(ax)}+\frac{a^2\sqrt{1+a^2x^2}\,\text{CosIntegral}(\arctan(ax))}{c\sqrt{c+a^2cx^2}} \\ &+\frac{\text{Int}\left(\frac{1}{x^3\sqrt{c+a^2cx^2}\arctan(ax)^2},x\right)}{c}+\frac{a\text{Int}\left(\frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)},x\right)}{c}\end{aligned}$$

output
```
-a^3*x/c/(a^2*c*x^2+c)^(1/2)/arctan(a*x)+a*(a^2*c*x^2+c)^(1/2)/c^2/x/arcta
n(a*x)+a^2*(a^2*x^2+1)^(1/2)*Ci(arctan(a*x))/c/(a^2*c*x^2+c)^(1/2)+Defer(I
nt)(1/x^3/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x)/c+a*Defer(Int)(1/x^2/(a^2*c
*x^2+c)^(1/2)/arctan(a*x),x)/c
```

## Mathematica [N/A]

Not integrable

Time = 3.60 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^3 \left(c+a^2 c x^2\right)^{3/2} \arctan (a x)^2} \, dx=\int \frac{1}{x^3 \left(c+a^2 c x^2\right)^{3/2} \arctan (a x)^2} \, dx$$

input
```
Integrate[1/(x^3*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2),x]
```

output
```
Integrate[1/(x^3*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2), x]
```

## Rubi [N/A]

Not integrable

Time = 2.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \arctan (a x)^2 \left(a^2 c x^2+c\right)^{3/2}} \, dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{1}{x^3 \sqrt{a^2 c x^2+c} \arctan (a x)^2}dx}{c}-a^2 \int \frac{1}{x \left(a^2 c x^2+c\right)^{3/2} \arctan (a x)^2}dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{1}{x^3 \sqrt{a^2 c x^2+c} \arctan (a x)^2}dx}{c}-a^2 \left(\frac{\int \frac{1}{x \sqrt{a^2 c x^2+c} \arctan (a x)^2}dx}{c}-a^2 \int \frac{x}{\left(a^2 c x^2+c\right)^{3/2} \arctan (a x)^2}dx\right)$$

$$\Big\downarrow \text{5477}$$

$$\frac{\int \frac{1}{x^3\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\left(\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a} - \frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c} - a^2\left(\frac{\int \frac{1}{(a^2cx^2+c)^{3/2}\arctan(ax)}dx}{a} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)\right)$$

$\big\downarrow$ 5440

$$\frac{\int \frac{1}{x^3\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\left(\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a} - \frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c} - a^2\left(\frac{\sqrt{a^2x^2+1}\int \frac{1}{(a^2x^2+1)^{3/2}\arctan(ax)}dx}{ac\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+\cdots}}\right.\right.$$

$\big\downarrow$ 5439

$$\frac{\int \frac{1}{x^3\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\left(\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a} - \frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c} - a^2\left(\frac{\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{\cdots}}\right.\right.$$

$\big\downarrow$ 3042

$$\frac{\int \frac{1}{x^3\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\left(\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a} - \frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c} - a^2\left(\frac{\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)}{\arctan(ax)}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a\cdots}}\right.\right.$$

$\big\downarrow$ 3783

$$\frac{\int \frac{1}{x^3\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\left(\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a} - \frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c} - a^2\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2\cdots}}\right.\right.$$

$\big\downarrow$ 5560

$$\frac{\int \frac{1}{x^3\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\left(\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a} - \frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c} - a^2\left(\frac{\sqrt{a^2x^2+1}\,\mathrm{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2 -}}\right.\right.$$

input
```
Int[1/(x^3*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 22.89 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^3 \left(a^2 c\, x^2+c\right)^{\frac{3}{2}} \arctan\left(a x\right)^2}dx$$

input
```
int(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

output
```
int(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.17

$$\int \frac{1}{x^3 (c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx = \int \frac{1}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x^3\arctan\left(ax\right)^2}\,dx$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)/((a^4*c^2*x^7 + 2*a^2*c^2*x^5 + c^2*x^3)*arct
an(a*x)^2), x)
```

## Sympy [N/A]

Not integrable

Time = 6.93 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^3 (c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx=\int \frac{1}{x^3\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^2\left(ax\right)}\,dx$$

input
```
integrate(1/x**3/(a**2*c*x**2+c)**(3/2)/atan(a*x)**2,x)
```

output
```
Integral(1/(x**3*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**2), x)
```

## Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3 (c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx=\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x^3\arctan\left(ax\right)^2}\,dx$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(3/2)*x^3*arctan(a*x)^2), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x^3 (c+a^2cx^2)^{3/2} \arctan(ax)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3 (c+a^2cx^2)^{3/2} \arctan(ax)^2}\,dx = \int \frac{1}{x^3\,\mathrm{atan}\,(a\,x)^2\,(c\,a^2\,x^2+c)^{3/2}}\,dx$$

input
```
int(1/(x^3*atan(a*x)^2*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(1/(x^3*atan(a*x)^2*(c + a^2*c*x^2)^(3/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.33

$$\int \frac{1}{x^3 (c+a^2cx^2)^{3/2} \arctan(ax)^2}\,dx = \frac{\int \frac{1}{\sqrt{a^2x^2+1}\,atan(ax)^2a^2x^5+\sqrt{a^2x^2+1}\,atan(ax)^2x^3}dx}{\sqrt{c}\,c}$$

input
```
int(1/x^3/(a^2*c*x^2+c)^(3/2)/atan(a*x)^2,x)
```

output
```
int(1/(sqrt(a**2*x**2 + 1)*atan(a*x)**2*a**2*x**5 + sqrt(a**2*x**2 + 1)*at
an(a*x)**2*x**3),x)/(sqrt(c)*c)
```

# 3.585 $\int \frac{1}{x^4(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2}\,dx = -\frac{a^3}{c\sqrt{c+a^2cx^2}\arctan(ax)} - \frac{a^3\sqrt{1+a^2x^2}\,\mathrm{Si}(\arctan(ax))}{c\sqrt{c+a^2cx^2}} + \frac{\mathrm{Int}\left(\frac{1}{x^4\sqrt{c+a^2cx^2}\arctan(ax)^2},x\right)}{c} - \frac{a^2\mathrm{Int}\left(\frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^2},x\right)}{c}$$

output
```
-a^3/c/(a^2*c*x^2+c)^(1/2)/arctan(a*x)-a^3*(a^2*x^2+1)^(1/2)*Si(arctan(a*x
))/c/(a^2*c*x^2+c)^(1/2)+Defer(Int)(1/x^4/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^
2,x)/c-a^2*Defer(Int)(1/x^2/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x)/c
```

## Mathematica [N/A]

Not integrable

Time = 4.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^4 (c+a^2cx^2)^{3/2} \arctan(ax)^2}\,dx = \int \frac{1}{x^4 (c+a^2cx^2)^{3/2} \arctan(ax)^2}\,dx$$

input
```
Integrate[1/(x^4*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2),x]
```

output
```
Integrate[1/(x^4*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2), x]
```

## Rubi [N/A]

Not integrable

Time = 2.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \arctan(ax)^2 (a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{1}{x^4\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\int \frac{1}{x^2 (a^2cx^2+c)^{3/2} \arctan(ax)^2}dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{1}{x^4\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\left(\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\int \frac{1}{(a^2cx^2+c)^{3/2}\arctan(ax)^2}dx\right)$$

$$\Big\downarrow \text{5437}$$

$$\frac{\int \frac{1}{x^4\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\left(\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\left(-a\int \frac{x}{(a^2cx^2+c)^{3/2}\arctan(ax)}dx - \frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)\right)$$

$$\Big\downarrow \text{5506}$$

$$\frac{\int \frac{1}{x^4\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\left(\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\left(-\frac{a\sqrt{a^2x^2+1}\int \frac{x}{(a^2x^2+1)^{3/2}\arctan(ax)}dx}{c\sqrt{a^2cx^2+c}} - \frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)\right)$$

$$\Big\downarrow \text{5505}$$

$$\frac{\int \frac{1}{x^4\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\left(\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\left(-\frac{\sqrt{a^2x^2+1}\int \frac{ax}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}} - \frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{x^4\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\left(\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\left(-\frac{\sqrt{a^2x^2+1}\int \frac{\sin(\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}} - \frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)\right)$$

$$\Big\downarrow \text{3780}$$

$$\frac{\int \frac{1}{x^4\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\left(\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\left(-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}} - \frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)\right)$$

$$\Big\downarrow \text{5560}$$

$$\frac{\int \frac{1}{x^4\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\left(\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\left(-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}} - \frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)\right)$$

input
```
Int[1/(x^4*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 36.55 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^4 (a^2c\,x^2+c)^{\frac{3}{2}} \arctan(ax)^2} dx$$

input
```
int(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

output
```
int(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.17

$$\int \frac{1}{x^4 (c+a^2cx^2)^{3/2} \arctan(ax)^2} dx = \int \frac{1}{(a^2cx^2+c)^{\frac{3}{2}} x^4 \arctan(ax)^2} dx$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)/((a^4*c^2*x^8 + 2*a^2*c^2*x^6 + c^2*x^4)*arct
an(a*x)^2), x)
```

## Sympy [N/A]

Not integrable

Time = 10.75 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^4 (c+a^2cx^2)^{3/2} \arctan(ax)^2} \, dx = \int \frac{1}{x^4 (c (a^2x^2+1))^{\frac{3}{2}} \operatorname{atan}^2 (ax)} \, dx$$

input
```
integrate(1/x**4/(a**2*c*x**2+c)**(3/2)/atan(a*x)**2,x)
```

output
```
Integral(1/(x**4*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**2), x)
```

## Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4 (c+a^2cx^2)^{3/2} \arctan(ax)^2} \, dx = \int \frac{1}{(a^2cx^2+c)^{\frac{3}{2}} x^4 \arctan (ax)^2} \, dx$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(3/2)*x^4*arctan(a*x)^2), x)
```

## Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4 (c+a^2cx^2)^{3/2} \arctan(ax)^2} \, dx = \int \frac{1}{(a^2cx^2+c)^{\frac{3}{2}} x^4 \arctan (ax)^2} \, dx$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(3/2)*x^4*arctan(a*x)^2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2}\,dx=\int \frac{1}{x^4\,\mathrm{atan}\left(a\,x\right)^2\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

input
```
int(1/(x^4*atan(a*x)^2*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(1/(x^4*atan(a*x)^2*(c + a^2*c*x^2)^(3/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.33

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2}\,dx=\frac{\int \frac{1}{\sqrt{a^2x^2+1}\,atan(ax)^2a^2x^6+\sqrt{a^2x^2+1}\,atan(ax)^2x^4}dx}{\sqrt{c}\,c}$$

input
```
int(1/x^4/(a^2*c*x^2+c)^(3/2)/atan(a*x)^2,x)
```

output
```
int(1/(sqrt(a**2*x**2 + 1)*atan(a*x)**2*a**2*x**6 + sqrt(a**2*x**2 + 1)*at
an(a*x)**2*x**4),x)/(sqrt(c)*c)
```

# 3.586 $\int \frac{x^5}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^5}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = \frac{x^3}{a^3c\,(c+a^2cx^2)^{3/2}\arctan(ax)} + \frac{x}{a^5c^2\sqrt{c+a^2cx^2}\arctan(ax)} - \frac{7\sqrt{1+a^2x^2}\,\text{CosIntegral}(\arctan(ax))}{4a^6c^2\sqrt{c+a^2cx^2}} + \frac{3\sqrt{1+a^2x^2}\,\text{CosIntegral}(3\arctan(ax))}{4a^6c^2\sqrt{c+a^2cx^2}} + \frac{\text{Int}\left(\frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^2},x\right)}{a^4c^2}$$

output

```
x^3/a^3/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)+x/a^5/c^2/(a^2*c*x^2+c)^(1/2)/ar
ctan(a*x)-7/4*(a^2*x^2+1)^(1/2)*Ci(arctan(a*x))/a^6/c^2/(a^2*c*x^2+c)^(1/2
)+3/4*(a^2*x^2+1)^(1/2)*Ci(3*arctan(a*x))/a^6/c^2/(a^2*c*x^2+c)^(1/2)+Defe
r(Int)(x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x)/a^4/c^2
```

## Mathematica [N/A]

Not integrable

Time = 11.60 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^5}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = \int \frac{x^5}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx$$

input
```
Integrate[x^5/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

output
```
Integrate[x^5/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2), x]
```

## Rubi [N/A]

Not integrable

Time = 3.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\arctan(ax)^2(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{5499}$$

$$\frac{\int \frac{x^3}{(a^2cx^2+c)^{3/2}\arctan(ax)^2}dx}{a^2c}-\frac{\int \frac{x^3}{(a^2cx^2+c)^{5/2}\arctan(ax)^2}dx}{a^2}$$

$$\Big\downarrow \text{5477}$$

$$\frac{\int \frac{x^3}{(a^2cx^2+c)^{3/2}\arctan(ax)^2}dx}{a^2c}-\frac{\frac{3\int \frac{x^2}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)}dx}{a}-\frac{x^3}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}}{a^2}$$

$$\Big\downarrow \text{5499}$$

$$\frac{\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{a^2c}-\frac{\int \frac{x}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}dx}{a^2}}{a^2c}-\frac{\frac{3\int \frac{x^2}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)}dx}{a}-\frac{x^3}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}}{a^2}$$

$$\Big\downarrow \text{ 5477}$$

$$\frac{\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{a^2c}-\frac{\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}dx}{a}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-\frac{\frac{3\int \frac{x^2}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)}dx}{a}-\frac{x^3}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}}{a^2}$$

$$\Big\downarrow \text{ 5440}$$

$$\frac{\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{a^2c}-\frac{\frac{\sqrt{a^2x^2+1}\int \frac{1}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}dx}{ac\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-\frac{\frac{3\int \frac{x^2}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)}dx}{a}-\frac{x^3}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}}{a^2}$$

$$\Big\downarrow \text{ 5439}$$

$$\frac{\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{a^2c}-\frac{\frac{\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-\frac{\frac{3\int \frac{x^2}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)}dx}{a}-\frac{x^3}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}}{a^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{a^2c}-\frac{\frac{\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)}{\arctan(ax)}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-\frac{\frac{3\int \frac{x^2}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)}dx}{a}-\frac{x^3}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}}{a^2}$$

$$\Big\downarrow \text{ 3783}$$

$$\frac{\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{a^2c}-\frac{\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-\frac{\frac{3\int \frac{x^2}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)}dx}{a}-\frac{x^3}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}}{a^2}$$

$\Big\downarrow$ 5506

$$\frac{\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{a^2c}-\frac{\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-\frac{\frac{3\sqrt{a^2x^2+1}\int \frac{x^2}{\left(a^2x^2+1\right)^{5/2}\arctan(ax)}dx}{ac^2\sqrt{a^2cx^2+c}}-\frac{x^3}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}}{a^2}$$

$\Big\downarrow$ 5505

$$\frac{\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{a^2c}-\frac{\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-\frac{\frac{3\sqrt{a^2x^2+1}\int \frac{a^2x^2}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}d\arctan(ax)}{a^4c^2\sqrt{a^2cx^2+c}}-\frac{x^3}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}}{a^2}$$

$\Big\downarrow$ 4906

$$\frac{\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{a^2c}-\frac{\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-\frac{\frac{3\sqrt{a^2x^2+1}\int \left(\frac{1}{4\sqrt{a^2x^2+1}\arctan(ax)}-\frac{\cos(3\arctan(ax))}{4\arctan(ax)}\right)d\arctan(ax)}{a^4c^2\sqrt{a^2cx^2+c}}-\frac{x^3}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}}{a^2}$$

$\Big\downarrow$ 2009

$$\frac{\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{a^2c}-\frac{\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-\frac{\frac{3\sqrt{a^2x^2+1}\left(\frac{1}{4}\text{CosIntegral}(\arctan(ax))-\frac{1}{4}\text{CosIntegral}(3\arctan(ax))\right)}{a^4c^2\sqrt{a^2cx^2+c}}-\frac{x^3}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}}{a^2}$$

$\Big\downarrow$ 5560

$$\frac{\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{a^2c} - \frac{\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2}}{a^2c} - \frac{\frac{3\sqrt{a^2x^2+1}\left(\frac{1}{4}\,\text{CosIntegral}(\arctan(ax)) - \frac{1}{4}\,\text{CosIntegral}(3\arctan(ax))\right)}{a^4c^2\sqrt{a^2cx^2+c}} - \frac{x^3}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}}{a^2}$$

input
```
Int[x^5/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 7.73 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^5}{(a^2c\,x^2+c)^{\frac{5}{2}}\arctan(ax)^2}dx$$

input
```
int(x^5/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

output
```
int(x^5/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.58

$$\int \frac{x^5}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = \int \frac{x^5}{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)^2}\,dx$$

input
```
integrate(x^5/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^5/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3
*x^2 + c^3)*arctan(a*x)^2), x)
```

## Sympy [N/A]

Not integrable

Time = 6.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx=\int \frac{x^5}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}\operatorname{atan}^2(ax)}\,dx$$

input
```
integrate(x**5/(a**2*c*x**2+c)**(5/2)/atan(a*x)**2,x)
```

output
```
Integral(x**5/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**2), x)
```

## Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx=\int \frac{x^5}{\left(a^2cx^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^2}\,dx$$

input
```
integrate(x^5/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(x^5/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^2), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x^5}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^5/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = \int \frac{x^5}{\mathrm{atan}\left(a\,x\right)^2\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

input
```
int(x^5/(atan(a*x)^2*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(x^5/(atan(a*x)^2*(c + a^2*c*x^2)^(5/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 81, normalized size of antiderivative = 3.38

$$\int \frac{x^5}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = \frac{\int \frac{x^5}{\sqrt{a^2x^2+1}\,atan(ax)^2a^4x^4+2\sqrt{a^2x^2+1}\,atan(ax)^2a^2x^2+\sqrt{a^2x^2+1}\,atan(ax)^2}dx}{\sqrt{c}\,c^2}$$

input
```
int(x^5/(a^2*c*x^2+c)^(5/2)/atan(a*x)^2,x)
```

output
```
int(x**5/(sqrt(a**2*x**2 + 1)*atan(a*x)**2*a**4*x**4 + 2*sqrt(a**2*x**2 +
1)*atan(a*x)**2*a**2*x**2 + sqrt(a**2*x**2 + 1)*atan(a*x)**2),x)/(sqrt(c)*
c**2)
```

# 3.587 $\int \frac{x^4}{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\begin{aligned}\int \frac{x^4}{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2}\,dx &= -\frac{1}{a^5c\left(c+a^2cx^2\right)^{3/2}\arctan(ax)} \\ &+ \frac{2}{a^5c^2\sqrt{c+a^2cx^2}\arctan(ax)} + \frac{5\sqrt{1+a^2x^2}\,\mathrm{Si}(\arctan(ax))}{4a^5c^2\sqrt{c+a^2cx^2}} \\ &- \frac{3\sqrt{1+a^2x^2}\,\mathrm{Si}(3\arctan(ax))}{4a^5c^2\sqrt{c+a^2cx^2}} + \frac{\mathrm{Int}\left(\frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^2},x\right)}{a^4c^2}\end{aligned}$$

output

```
-1/a^5/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)+2/a^5/c^2/(a^2*c*x^2+c)^(1/2)/arc
tan(a*x)+5/4*(a^2*x^2+1)^(1/2)*Si(arctan(a*x))/a^5/c^2/(a^2*c*x^2+c)^(1/2)
-3/4*(a^2*x^2+1)^(1/2)*Si(3*arctan(a*x))/a^5/c^2/(a^2*c*x^2+c)^(1/2)+Defer
(Int)(1/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x)/a^4/c^2
```

## Mathematica [N/A]

Not integrable

Time = 10.84 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^4}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = \int \frac{x^4}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx$$

input
```
Integrate[x^4/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

output
```
Integrate[x^4/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2), x]
```

## Rubi [N/A]

Not integrable

Time = 2.31 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\arctan(ax)^2(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{5499}$$

$$\frac{\int \frac{x^2}{(a^2cx^2+c)^{3/2}\arctan(ax)^2}dx}{a^2c}-\frac{\int \frac{x^2}{(a^2cx^2+c)^{5/2}\arctan(ax)^2}dx}{a^2}$$

$$\Big\downarrow \text{5499}$$

$$\frac{\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{a^2c}-\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}dx}{a^2}}{a^2c}-\frac{\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}dx}{a^2c}-\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^2}dx}{a^2}}{a^2}$$

$\downarrow$ 5437

$$\frac{\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{a^2c}-\frac{-a\int \frac{x}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}dx-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-\frac{\frac{-a\int \frac{x}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}dx-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-3a\int \frac{x}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)}dx-\frac{1}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}}{a^2}}{a^2}$$

$\downarrow$ 5506

$$\frac{\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{a^2c}-\frac{-\frac{a\sqrt{a^2x^2+1}\int \frac{x}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}dx}{c\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-\frac{\frac{-\frac{a\sqrt{a^2x^2+1}\int \frac{x}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}dx}{c\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3a\sqrt{a^2x^2+1}\int \frac{x}{\left(a^2x^2+1\right)^{5/2}\arctan(ax)}dx}{c^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}}{a^2}}{a^2}$$

$\downarrow$ 5505

$$\frac{\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{a^2c}-\frac{-\frac{\sqrt{a^2x^2+1}\int \frac{ax}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-\frac{\frac{-\frac{\sqrt{a^2x^2+1}\int \frac{ax}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3\sqrt{a^2x^2+1}\int \frac{ax}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}}{a^2}}{a^2}$$

$\downarrow$ 3042

$$\frac{\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{a^2c}-\frac{-\frac{\sqrt{a^2x^2+1}\int \frac{\sin(\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-\frac{\frac{-\frac{\sqrt{a^2x^2+1}\int \frac{\sin(\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3\sqrt{a^2x^2+1}\int \frac{ax}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}}{a^2}}{a^2}$$

$\downarrow$ 3780

$$\frac{\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{a^2c}-\frac{-\frac{\sqrt{a^2x^2+1}\,\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-\frac{\frac{-\frac{\sqrt{a^2x^2+1}\,\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3\sqrt{a^2x^2+1}\int \frac{ax}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}}{a^2}}{a^2}$$

$$\Big\downarrow \; 4906$$

$$\frac{\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{a^2c} - \frac{-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}} - \frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2}}{a^2c} - \frac{\frac{-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c} - \frac{-\frac{3\sqrt{a^2x^2+1}\int\left(\frac{ax}{4\sqrt{a^2x^2+1}\arctan(ax)}+\frac{\sin(3\arctan(ax))}{4\arctan(ax)}\right)d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}} - \frac{1}{ac\arctan(ax)\left(a^2cx^2+c\right)}}{a^2}}{a^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{a^2c} - \frac{-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}} - \frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2}}{a^2c} - \frac{\frac{-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c} - \frac{-\frac{3\sqrt{a^2x^2+1}\left(\frac{1}{4}\text{Si}(\arctan(ax))+\frac{1}{4}\text{Si}(3\arctan(ax))\right)}{ac^2\sqrt{a^2cx^2+c}} - \frac{1}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}}{a^2}}{a^2}$$

$$\Big\downarrow \; 5560$$

$$\frac{\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{a^2c} - \frac{-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}} - \frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2}}{a^2c} - \frac{\frac{-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c} - \frac{-\frac{3\sqrt{a^2x^2+1}\left(\frac{1}{4}\text{Si}(\arctan(ax))+\frac{1}{4}\text{Si}(3\arctan(ax))\right)}{ac^2\sqrt{a^2cx^2+c}} - \frac{1}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}}{a^2}}{a^2}$$

input
```
Int[x^4/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 8.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^4}{\left(a^2c\,x^2+c\right)^{\frac{5}{2}} \arctan\left(ax\right)^2}dx$$

input
```
int(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

output
```
int(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.58

$$\int \frac{x^4}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = \int \frac{x^4}{(a^2cx^2+c)^{\frac{5}{2}}\arctan\left(ax\right)^2}\,dx$$

input
```
integrate(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^4/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3
*x^2 + c^3)*arctan(a*x)^2), x)
```

## Sympy [N/A]

Not integrable

Time = 6.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = \int \frac{x^4}{(c\left(a^2x^2+1\right))^{\frac{5}{2}}\operatorname{atan}^2\left(ax\right)}\,dx$$

input
```
integrate(x**4/(a**2*c*x**2+c)**(5/2)/atan(a*x)**2,x)
```

output
```
Integral(x**4/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**2), x)
```

## Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = \int \frac{x^4}{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)^2}\,dx$$

input
```
integrate(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(x^4/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^2), x)
```

## Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = \int \frac{x^4}{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)^2}\,dx$$

input
```
integrate(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate(x^4/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = \int \frac{x^4}{\mathrm{atan}\left(a\,x\right)^2\,\left(c\,a^2\,x^2+c\right)^{5/2}}\,dx$$

input
```
int(x^4/(atan(a*x)^2*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(x^4/(atan(a*x)^2*(c + a^2*c*x^2)^(5/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 81, normalized size of antiderivative = 3.38

$$\int \frac{x^4}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = \frac{\int \frac{x^4}{\sqrt{a^2x^2+1}\,atan(ax)^2a^4x^4+2\sqrt{a^2x^2+1}\,atan(ax)^2a^2x^2+\sqrt{a^2x^2+1}\,atan(ax)^2}dx}{\sqrt{c}\,c^2}$$

input
```
int(x^4/(a^2*c*x^2+c)^(5/2)/atan(a*x)^2,x)
```

output
```
int(x**4/(sqrt(a**2*x**2 + 1)*atan(a*x)**2*a**4*x**4 + 2*sqrt(a**2*x**2 +
1)*atan(a*x)**2*a**2*x**2 + sqrt(a**2*x**2 + 1)*atan(a*x)**2),x)/(sqrt(c)*
c**2)
```

# 3.588 $\int \frac{x^3}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 118

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = -\frac{x^3}{ac\,(c+a^2cx^2)^{3/2}\arctan(ax)} + \frac{3\sqrt{1+a^2x^2}\,\text{CosIntegral}(\arctan(ax))}{4a^4c^2\sqrt{c+a^2cx^2}} - \frac{3\sqrt{1+a^2x^2}\,\text{CosIntegral}(3\arctan(ax))}{4a^4c^2\sqrt{c+a^2cx^2}}$$

output
```
-x^3/a/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)+3/4*(a^2*x^2+1)^(1/2)*Ci(arctan(a
*x))/a^4/c^2/(a^2*c*x^2+c)^(1/2)-3/4*(a^2*x^2+1)^(1/2)*Ci(3*arctan(a*x))/a
^4/c^2/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.69

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = \frac{-\frac{4a^3cx^3}{(1+a^2x^2)\arctan(ax)} + 3c\sqrt{1+a^2x^2}(\text{CosIntegral}(\arctan(ax)) - \text{CosInteg}}{4a^4c^3\sqrt{c+a^2cx^2}}$$

input
```
Integrate[x^3/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

output
```
((-4*a^3*c*x^3)/((1 + a^2*x^2)*ArcTan[a*x]) + 3*c*Sqrt[1 + a^2*x^2]*(CosIn
tegral[ArcTan[a*x]] - CosIntegral[3*ArcTan[a*x]]))/(4*a^4*c^3*Sqrt[c + a^2
*c*x^2])
```

## Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.75, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {5477, 5506, 5505, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\arctan(ax)^2 \left(a^2cx^2+c\right)^{5/2}}dx$$

$$\Big\downarrow \text{5477}$$

$$\frac{3\int \frac{x^2}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)}dx}{a}-\frac{x^3}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{5506}$$

$$\frac{3\sqrt{a^2x^2+1}\int \frac{x^2}{\left(a^2x^2+1\right)^{5/2}\arctan(ax)}dx}{ac^2\sqrt{a^2cx^2+c}}-\frac{x^3}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{5505}$$

$$\frac{3\sqrt{a^2x^2+1}\int \frac{a^2x^2}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}d\arctan(ax)}{a^4c^2\sqrt{a^2cx^2+c}}-\frac{x^3}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{4906}$$

$$\frac{3\sqrt{a^2x^2+1}\int \left(\frac{1}{4\sqrt{a^2x^2+1}\arctan(ax)}-\frac{\cos(3\arctan(ax))}{4\arctan(ax)}\right)d\arctan(ax)}{a^4c^2\sqrt{a^2cx^2+c}}-\frac{x^3}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\sqrt{a^2x^2+1}\left(\frac{1}{4}\operatorname{CosIntegral}(\arctan(ax))-\frac{1}{4}\operatorname{CosIntegral}(3\arctan(ax))\right)}{a^4c^2\sqrt{a^2cx^2+c}}-\frac{x^3}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}$$

input
```
Int[x^3/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

output
```
-(x^3/(a*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x])) + (3*Sqrt[1 + a^2*x^2]*(Cos
Integral[ArcTan[a*x]]/4 - CosIntegral[3*ArcTan[a*x]]/4))/(a^4*c^2*Sqrt[c +
 a^2*c*x^2])
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5477
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*ArcT
an[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp[f*(m/(b*c*(p + 1)))   Int[(f*x
)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1
]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 9.81 (sec) , antiderivative size = 309, normalized size of antiderivative = 2.62

| method | result |
|---|---|
| default | $-\frac{\left(3\arctan(ax)\,\mathrm{expIntegral}_1(-i\arctan(ax))a^4x^4-3\arctan(ax)\,\mathrm{expIntegral}_1(3i\arctan(ax))a^4x^4-3\arctan(ax)\,\mathrm{expIntegral}_1(-3i\right.}{}$ |

input
```
int(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/8*(3*arctan(a*x)*Ei(1,-I*arctan(a*x))*a^4*x^4-3*arctan(a*x)*Ei(1,3*I*ar
ctan(a*x))*a^4*x^4-3*arctan(a*x)*Ei(1,-3*I*arctan(a*x))*a^4*x^4+3*arctan(a
*x)*Ei(1,I*arctan(a*x))*a^4*x^4+8*(a^2*x^2+1)^(1/2)*a^3*x^3+6*arctan(a*x)*
Ei(1,-I*arctan(a*x))*a^2*x^2-6*arctan(a*x)*Ei(1,3*I*arctan(a*x))*a^2*x^2-6
*arctan(a*x)*Ei(1,-3*I*arctan(a*x))*a^2*x^2+6*arctan(a*x)*Ei(1,I*arctan(a*
x))*a^2*x^2+3*Ei(1,-I*arctan(a*x))*arctan(a*x)-3*Ei(1,3*I*arctan(a*x))*arc
tan(a*x)-3*Ei(1,-3*I*arctan(a*x))*arctan(a*x)+3*Ei(1,I*arctan(a*x))*arctan
(a*x))/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(a*x+I))^(1/2)/arctan(a*x)/a^4/c^3/(a^
4*x^4+2*a^2*x^2+1)
```

## Fricas [F]

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx=\int \frac{x^3}{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)^2}\,dx$$

input
```
integrate(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^3/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3
*x^2 + c^3)*arctan(a*x)^2), x)
```

## Sympy [F]

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx=\int \frac{x^3}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}\operatorname{atan}^2\left(ax\right)}\,dx$$

input
```
integrate(x**3/(a**2*c*x**2+c)**(5/2)/atan(a*x)**2,x)
```

output
```
Integral(x**3/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**2), x)
```

## Maxima [F]

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx=\int \frac{x^3}{\left(a^2cx^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^2}\,dx$$

input
```
integrate(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(x^3/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^2), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx=\int \frac{x^3}{\mathrm{atan}\left(a\,x\right)^2\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

input
```
int(x^3/(atan(a*x)^2*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(x^3/(atan(a*x)^2*(c + a^2*c*x^2)^(5/2)), x)
```

## Reduce [F]

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx=\frac{\int \frac{x^3}{\sqrt{a^2x^2+1}\,atan(ax)^2a^4x^4+2\sqrt{a^2x^2+1}\,atan(ax)^2a^2x^2+\sqrt{a^2x^2+1}\,atan(ax)^2}dx}{\sqrt{c}\,c^2}$$

input
```
int(x^3/(a^2*c*x^2+c)^(5/2)/atan(a*x)^2,x)
```

output
```
int(x**3/(sqrt(a**2*x**2 + 1)*atan(a*x)**2*a**4*x**4 + 2*sqrt(a**2*x**2 +
1)*atan(a*x)**2*a**2*x**2 + sqrt(a**2*x**2 + 1)*atan(a*x)**2),x)/(sqrt(c)*
c**2)
```

# 3.589 $\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 142

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = \frac{1}{a^3c(c+a^2cx^2)^{3/2}\arctan(ax)} - \frac{1}{a^3c^2\sqrt{c+a^2cx^2}\arctan(ax)} - \frac{\sqrt{1+a^2x^2}\,\text{Si}(\arctan(ax))}{4a^3c^2\sqrt{c+a^2cx^2}} + \frac{3\sqrt{1+a^2x^2}\,\text{Si}(3\arctan(ax))}{4a^3c^2\sqrt{c+a^2cx^2}}$$

output
```
1/a^3/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)-1/a^3/c^2/(a^2*c*x^2+c)^(1/2)/arct
an(a*x)-1/4*(a^2*x^2+1)^(1/2)*Si(arctan(a*x))/a^3/c^2/(a^2*c*x^2+c)^(1/2)+
3/4*(a^2*x^2+1)^(1/2)*Si(3*arctan(a*x))/a^3/c^2/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.70

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = -\frac{4a^2x^2+(1+a^2x^2)^{3/2}\arctan(ax)\text{Si}(\arctan(ax))-3(1+a^2x^2)^{3/2}\arctan(ax)\text{Si}(3\arctan(ax))}{4a^3c^2(1+a^2x^2)\sqrt{c+a^2cx^2}\arctan(ax)}$$

input
```
Integrate[x^2/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

output
```
-1/4*(4*a^2*x^2 + (1 + a^2*x^2)^(3/2)*ArcTan[a*x]*SinIntegral[ArcTan[a*x]]
 - 3*(1 + a^2*x^2)^(3/2)*ArcTan[a*x]*SinIntegral[3*ArcTan[a*x]])/(a^3*c^2*
(1 + a^2*x^2)*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])
```

## Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.18, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5499, 5437, 5506, 5505, 3042, 3780, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\arctan(ax)^2(a^2cx^2+c)^{5/2}}\,dx$$

$$\Big\downarrow \; 5499$$

$$\frac{\int \frac{1}{(a^2cx^2+c)^{3/2}\arctan(ax)^2}dx}{a^2c}-\frac{\int \frac{1}{(a^2cx^2+c)^{5/2}\arctan(ax)^2}dx}{a^2}$$

$$\Big\downarrow \; 5437$$

$$\frac{-a\int \frac{x}{(a^2cx^2+c)^{3/2}\arctan(ax)}dx-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-3a\int \frac{x}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx-\frac{1}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}}{a^2}$$

$$\Big\downarrow \; 5506$$

$$\frac{-\frac{a\sqrt{a^2x^2+1}\int \frac{x}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}dx}{c\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3a\sqrt{a^2x^2+1}\int \frac{x}{\left(a^2x^2+1\right)^{5/2}\arctan(ax)}dx}{c^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}}{a^2}$$

$$\Big\downarrow \; 5505$$

$$\frac{-\frac{\sqrt{a^2x^2+1}\int \frac{ax}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3\sqrt{a^2x^2+1}\int \frac{ax}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}}{a^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{\sqrt{a^2x^2+1}\int \frac{\sin(\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3\sqrt{a^2x^2+1}\int \frac{ax}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}}{a^2}$$

$$\Big\downarrow \; 3780$$

$$\frac{-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3\sqrt{a^2x^2+1}\int \frac{ax}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}}{a^2}$$

$$\Big\downarrow \; 4906$$

$$\frac{-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3\sqrt{a^2x^2+1}\int \left(\frac{ax}{4\sqrt{a^2x^2+1}\arctan(ax)}+\frac{\sin(3\arctan(ax))}{4\arctan(ax)}\right)d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}}{a^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\frac{\sqrt{a^2x^2+1}\,\mathrm{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3\sqrt{a^2x^2+1}\left(\frac{1}{4}\mathrm{Si}(\arctan(ax))+\frac{1}{4}\mathrm{Si}(3\arctan(ax))\right)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}}{a^2}$$

input
```
Int[x^2/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

output
```
(-(1/(a*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])) - (Sqrt[1 + a^2*x^2]*SinIntegr
al[ArcTan[a*x]])/(a*c*Sqrt[c + a^2*c*x^2]))/(a^2*c) - (-(1/(a*c*(c + a^2*c
*x^2)^(3/2)*ArcTan[a*x])) - (3*Sqrt[1 + a^2*x^2]*(SinIntegral[ArcTan[a*x]]
/4 + SinIntegral[3*ArcTan[a*x]]/4))/(a*c^2*Sqrt[c + a^2*c*x^2]))/a^2
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5499
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ
[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 8.70 (sec) , antiderivative size = 309, normalized size of antiderivative = 2.18

| method | result |
|---|---|
| default | $-\frac{i\left(3\arctan(ax)\,\mathrm{expIntegral}_1(3i\arctan(ax))a^4x^4+\arctan(ax)\,\mathrm{expIntegral}_1(-i\arctan(ax))a^4x^4-\arctan(ax)\,\mathrm{expIntegral}_1(i\arcta\right.}{}$ |

input
```
int(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/8*I*(3*arctan(a*x)*Ei(1,3*I*arctan(a*x))*a^4*x^4+arctan(a*x)*Ei(1,-I*ar
ctan(a*x))*a^4*x^4-arctan(a*x)*Ei(1,I*arctan(a*x))*a^4*x^4-3*arctan(a*x)*E
i(1,-3*I*arctan(a*x))*a^4*x^4+6*arctan(a*x)*Ei(1,3*I*arctan(a*x))*a^2*x^2+
2*arctan(a*x)*Ei(1,-I*arctan(a*x))*a^2*x^2-2*arctan(a*x)*Ei(1,I*arctan(a*x
))*a^2*x^2-6*arctan(a*x)*Ei(1,-3*I*arctan(a*x))*a^2*x^2-8*I*(a^2*x^2+1)^(1
/2)*a^2*x^2+3*Ei(1,3*I*arctan(a*x))*arctan(a*x)+Ei(1,-I*arctan(a*x))*arcta
n(a*x)-Ei(1,I*arctan(a*x))*arctan(a*x)-3*Ei(1,-3*I*arctan(a*x))*arctan(a*x
))/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(a*x+I))^(1/2)/arctan(a*x)/a^3/c^3/(a^4*x^
4+2*a^2*x^2+1)
```

## Fricas [F]

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = \int \frac{x^2}{(a^2cx^2+c)^{\frac{5}{2}}\arctan{(ax)}^2}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^2/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3
*x^2 + c^3)*arctan(a*x)^2), x)
```

## Sympy [F]

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = \int \frac{x^2}{(c(a^2x^2+1))^{\frac{5}{2}}\operatorname{atan}^2{(ax)}}\,dx$$

input
```
integrate(x**2/(a**2*c*x**2+c)**(5/2)/atan(a*x)**2,x)
```

output
```
Integral(x**2/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**2), x)
```

## Maxima [F]

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = \int \frac{x^2}{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)^2}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(x^2/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^2), x)
```

## Giac [F]

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = \int \frac{x^2}{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)^2}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate(x^2/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = \int \frac{x^2}{\mathrm{atan}(a\,x)^2\,(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int(x^2/(atan(a*x)^2*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(x^2/(atan(a*x)^2*(c + a^2*c*x^2)^(5/2)), x)
```

## Reduce [F]

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = \frac{\int \frac{x^2}{\sqrt{a^2x^2+1}\,atan(ax)^2a^4x^4+2\sqrt{a^2x^2+1}\,atan(ax)^2a^2x^2+\sqrt{a^2x^2+1}\,atan(ax)^2}dx}{\sqrt{c}\,c^2}$$

input
```
int(x^2/(a^2*c*x^2+c)^(5/2)/atan(a*x)^2,x)
```

output
```
int(x**2/(sqrt(a**2*x**2 + 1)*atan(a*x)**2*a**4*x**4 + 2*sqrt(a**2*x**2 +
1)*atan(a*x)**2*a**2*x**2 + sqrt(a**2*x**2 + 1)*atan(a*x)**2),x)/(sqrt(c)*
c**2)
```

# 3.590 $\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 116

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = -\frac{x}{ac\,(c+a^2cx^2)^{3/2}\arctan(ax)} + \frac{\sqrt{1+a^2x^2}\,\text{CosIntegral}(\arctan(ax))}{4a^2c^2\sqrt{c+a^2cx^2}} + \frac{3\sqrt{1+a^2x^2}\,\text{CosIntegral}(3\arctan(ax))}{4a^2c^2\sqrt{c+a^2cx^2}}$$

output
```
-x/a/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)+1/4*(a^2*x^2+1)^(1/2)*Ci(arctan(a*x
))/a^2/c^2/(a^2*c*x^2+c)^(1/2)+3/4*(a^2*x^2+1)^(1/2)*Ci(3*arctan(a*x))/a^2
/c^2/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.82

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = \frac{-4ax+(1+a^2x^2)^{3/2}\arctan(ax)\,\text{CosIntegral}(\arctan(ax))+3(1+a^2x}{4a^2c^2\,(1+a^2x^2)\,\sqrt{c+a^2cx^2}\,\text{arcta}}$$

input
```
Integrate[x/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

output
```
(-4*a*x + (1 + a^2*x^2)^(3/2)*ArcTan[a*x]*CosIntegral[ArcTan[a*x]] + 3*(1
+ a^2*x^2)^(3/2)*ArcTan[a*x]*CosIntegral[3*ArcTan[a*x]])/(4*a^2*c^2*(1 + a
^2*x^2)*Sqrt[c + a^2*c*x^2]*ArcTan[a*x])
```

## Rubi [A] (verified)

Time = 1.65 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.22, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {5503, 5440, 5439, 3042, 3793, 2009, 5506, 5505, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\arctan(ax)^2 \left(a^2cx^2+c\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 5503}$$

$$\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)}dx}{a}-2a\int \frac{x^2}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)}dx-\frac{x}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{ 5440}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{1}{\left(a^2x^2+1\right)^{5/2}\arctan(ax)}dx}{ac^2\sqrt{a^2cx^2+c}}-2a\int \frac{x^2}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)}dx-\frac{x}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{ 5439}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{1}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}-2a\int \frac{x^2}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)}dx-\frac{x}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^3}{\arctan(ax)}d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}-2a\int \frac{x^2}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)}dx-\frac{x}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{3793}$$

$$\frac{\sqrt{a^2x^2+1}\int \left(\frac{\cos(3\arctan(ax))}{4\arctan(ax)}+\frac{3}{4\sqrt{a^2x^2+1}\arctan(ax)}\right)d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}-2a\int \frac{x^2}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx-\frac{x}{ac\arctan(ax)\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$-2a\int \frac{x^2}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx+\frac{\sqrt{a^2x^2+1}\left(\frac{3}{4}\text{CosIntegral}(\arctan(ax))+\frac{1}{4}\text{CosIntegral}(3\arctan(ax))\right)}{a^2c^2\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5506}$$

$$-\frac{2a\sqrt{a^2x^2+1}\int \frac{x^2}{(a^2x^2+1)^{5/2}\arctan(ax)}dx}{c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2x^2+1}\left(\frac{3}{4}\text{CosIntegral}(\arctan(ax))+\frac{1}{4}\text{CosIntegral}(3\arctan(ax))\right)}{a^2c^2\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5505}$$

$$-\frac{2\sqrt{a^2x^2+1}\int \frac{a^2x^2}{(a^2x^2+1)^{3/2}\arctan(ax)}d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2x^2+1}\left(\frac{3}{4}\text{CosIntegral}(\arctan(ax))+\frac{1}{4}\text{CosIntegral}(3\arctan(ax))\right)}{a^2c^2\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{4906}$$

$$-\frac{2\sqrt{a^2x^2+1}\int \left(\frac{1}{4\sqrt{a^2x^2+1}\arctan(ax)}-\frac{\cos(3\arctan(ax))}{4\arctan(ax)}\right)d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2x^2+1}\left(\frac{3}{4}\text{CosIntegral}(\arctan(ax))+\frac{1}{4}\text{CosIntegral}(3\arctan(ax))\right)}{a^2c^2\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2\sqrt{a^2x^2+1}\left(\frac{1}{4}\operatorname{CosIntegral}(\arctan(ax))-\frac{1}{4}\operatorname{CosIntegral}(3\arctan(ax))\right)}{a^2c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2x^2+1}\left(\frac{3}{4}\operatorname{CosIntegral}(\arctan(ax))+\frac{1}{4}\operatorname{CosIntegral}(3\arctan(ax))\right)}{a^2c^2\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}$$

input
```
Int[x/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

output
```
-(x/(a*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x])) - (2*Sqrt[1 + a^2*x^2]*(CosIn
tegral[ArcTan[a*x]]/4 - CosIntegral[3*ArcTan[a*x]]/4))/(a^2*c^2*Sqrt[c + a
^2*c*x^2]) + (Sqrt[1 + a^2*x^2]*((3*CosIntegral[ArcTan[a*x]])/4 + CosInteg
ral[3*ArcTan[a*x]]/4))/(a^2*c^2*Sqrt[c + a^2*c*x^2])
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5440
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])   Int[(1 +
c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 8.84 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.59

| method | result |
|---|---|
| default | $-\frac{(3\arctan(ax)\,\mathrm{expIntegral}_1(3i\arctan(ax))a^4x^4+\arctan(ax)\,\mathrm{expIntegral}_1(i\arctan(ax))a^4x^4+3\arctan(ax)\,\mathrm{expIntegral}_1(-3i\arctan$ |

input
```
int(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/8*(3*arctan(a*x)*Ei(1,3*I*arctan(a*x))*a^4*x^4+arctan(a*x)*Ei(1,I*arcta
n(a*x))*a^4*x^4+3*arctan(a*x)*Ei(1,-3*I*arctan(a*x))*a^4*x^4+arctan(a*x)*E
i(1,-I*arctan(a*x))*a^4*x^4+6*arctan(a*x)*Ei(1,3*I*arctan(a*x))*a^2*x^2+2*
arctan(a*x)*Ei(1,I*arctan(a*x))*a^2*x^2+6*arctan(a*x)*Ei(1,-3*I*arctan(a*x
))*a^2*x^2+2*arctan(a*x)*Ei(1,-I*arctan(a*x))*a^2*x^2+8*(a^2*x^2+1)^(1/2)*
a*x+3*Ei(1,3*I*arctan(a*x))*arctan(a*x)+Ei(1,I*arctan(a*x))*arctan(a*x)+3*
Ei(1,-3*I*arctan(a*x))*arctan(a*x)+Ei(1,-I*arctan(a*x))*arctan(a*x))/(a^2*
x^2+1)^(1/2)*(c*(a*x-I)*(a*x+I))^(1/2)/arctan(a*x)/a^2/c^3/(a^4*x^4+2*a^2*
x^2+1)
```

## Fricas [F]

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = \int \frac{x}{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)^2}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x
^2 + c^3)*arctan(a*x)^2), x)
```

## Sympy [F]

$$\int \frac{x}{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2}\,dx = \int \frac{x}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}\operatorname{atan}^2\left(ax\right)}\,dx$$

input
```
integrate(x/(a**2*c*x**2+c)**(5/2)/atan(a*x)**2,x)
```

output
```
Integral(x/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**2), x)
```

## Maxima [F]

$$\int \frac{x}{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2}\,dx = \int \frac{x}{\left(a^2cx^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^2}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(x/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^2), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x}{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2}\,dx=\int \frac{x}{\mathrm{atan}\left(a\,x\right)^2\,\left(c\,a^2\,x^2+c\right)^{5/2}}\,dx$$

input
```
int(x/(atan(a*x)^2*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(x/(atan(a*x)^2*(c + a^2*c*x^2)^(5/2)), x)
```

## Reduce [F]

$$\int \frac{x}{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2}\,dx=\frac{\int \frac{x}{\sqrt{a^2x^2+1}\,atan(ax)^2a^4x^4+2\sqrt{a^2x^2+1}\,atan(ax)^2a^2x^2+\sqrt{a^2x^2+1}\,atan(ax)^2}dx}{\sqrt{c}\,c^2}$$

input
```
int(x/(a^2*c*x^2+c)^(5/2)/atan(a*x)^2,x)
```

output
```
int(x/(sqrt(a**2*x**2 + 1)*atan(a*x)**2*a**4*x**4 + 2*sqrt(a**2*x**2 + 1)*
atan(a*x)**2*a**2*x**2 + sqrt(a**2*x**2 + 1)*atan(a*x)**2),x)/(sqrt(c)*c**
2)
```

# 3.591 $\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 115

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = -\frac{1}{ac\,(c+a^2cx^2)^{3/2}\arctan(ax)} - \frac{3\sqrt{1+a^2x^2}\operatorname{Si}(\arctan(ax))}{4ac^2\sqrt{c+a^2cx^2}} - \frac{3\sqrt{1+a^2x^2}\operatorname{Si}(3\arctan(ax))}{4ac^2\sqrt{c+a^2cx^2}}$$

output
```
-1/a/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)-3/4*(a^2*x^2+1)^(1/2)*Si(arctan(a*x
))/a/c^2/(a^2*c*x^2+c)^(1/2)-3/4*(a^2*x^2+1)^(1/2)*Si(3*arctan(a*x))/a/c^2
/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.53

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = \frac{-\frac{4}{\arctan(ax)} - 3(1+a^2x^2)^{3/2}\left(\operatorname{Si}(\arctan(ax)) + \operatorname{Si}(3\arctan(ax))\right)}{4ac\,(c+a^2cx^2)^{3/2}}$$

input
```
Integrate[1/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

output
```
(-4/ArcTan[a*x] - 3*(1 + a^2*x^2)^(3/2)*(SinIntegral[ArcTan[a*x]] + SinInt
egral[3*ArcTan[a*x]]))/(4*a*c*(c + a^2*c*x^2)^(3/2))
```

## Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.74, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {5437, 5506, 5505, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arctan(ax)^2 (a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{5437}$$

$$-3a\int \frac{x}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx-\frac{1}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5506}$$

$$-\frac{3a\sqrt{a^2x^2+1}\int \frac{x}{(a^2x^2+1)^{5/2}\arctan(ax)}dx}{c^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5505}$$

$$-\frac{3\sqrt{a^2x^2+1}\int \frac{ax}{(a^2x^2+1)^{3/2}\arctan(ax)}d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{4906}$$

$$-\frac{3\sqrt{a^2x^2+1}\int \left(\frac{ax}{4\sqrt{a^2x^2+1}\arctan(ax)}+\frac{\sin(3\arctan(ax))}{4\arctan(ax)}\right)d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3\sqrt{a^2x^2+1}\left(\frac{1}{4}\text{Si}(\arctan(ax))+\frac{1}{4}\text{Si}(3\arctan(ax))\right)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}$$

input
```
Int[1/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

output
```
-(1/(a*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x])) - (3*Sqrt[1 + a^2*x^2]*(SinIn
tegral[ArcTan[a*x]]/4 + SinIntegral[3*ArcTan[a*x]]/4))/(a*c^2*Sqrt[c + a^2
*c*x^2])
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 9.15 (sec) , antiderivative size = 305, normalized size of antiderivative = 2.65

| method | result |
|---|---|
| default | $-\frac{i\left(3\arctan(ax)\,\mathrm{expIntegral}_1(-i\arctan(ax))a^4x^4-3\arctan(ax)\,\mathrm{expIntegral}_1(i\arctan(ax))a^4x^4-3\arctan(ax)\,\mathrm{expIntegral}_1(3i\,\mathrm{ar}\right.}{}$ |

input
```
int(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/8*I*(3*arctan(a*x)*Ei(1,-I*arctan(a*x))*a^4*x^4-3*arctan(a*x)*Ei(1,I*ar
ctan(a*x))*a^4*x^4-3*arctan(a*x)*Ei(1,3*I*arctan(a*x))*a^4*x^4+3*arctan(a*
x)*Ei(1,-3*I*arctan(a*x))*a^4*x^4+6*arctan(a*x)*Ei(1,-I*arctan(a*x))*a^2*x
^2-6*arctan(a*x)*Ei(1,I*arctan(a*x))*a^2*x^2-6*arctan(a*x)*Ei(1,3*I*arctan
(a*x))*a^2*x^2+6*arctan(a*x)*Ei(1,-3*I*arctan(a*x))*a^2*x^2+3*Ei(1,-I*arct
an(a*x))*arctan(a*x)-3*Ei(1,I*arctan(a*x))*arctan(a*x)-3*Ei(1,3*I*arctan(a
*x))*arctan(a*x)+3*Ei(1,-3*I*arctan(a*x))*arctan(a*x)-8*I*(a^2*x^2+1)^(1/2
))/(a^2*x^2+1)^(1/2)*(c*(a*x-I)*(a*x+I))^(1/2)/arctan(a*x)/a/c^3/(a^4*x^4+
2*a^2*x^2+1)
```

## Fricas [F]

$$\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2}\,dx=\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^2}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2
 + c^3)*arctan(a*x)^2), x)
```

## Sympy [F]

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = \int \frac{1}{(c(a^2x^2+1))^{\frac{5}{2}}\operatorname{atan}^2(ax)}\,dx$$

input
```
integrate(1/(a**2*c*x**2+c)**(5/2)/atan(a*x)**2,x)
```

output
```
Integral(1/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**2), x)
```

## Maxima [F]

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = \int \frac{1}{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)^2}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^2), x)
```

## Giac [F]

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = \int \frac{1}{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)^2}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx=\int \frac{1}{\mathrm{atan}\left(a\,x\right)^2\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

input
```
int(1/(atan(a*x)^2*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(1/(atan(a*x)^2*(c + a^2*c*x^2)^(5/2)), x)
```

## Reduce [F]

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx=\frac{\int \frac{1}{\sqrt{a^2x^2+1}\,atan(ax)^2a^4x^4+2\sqrt{a^2x^2+1}\,atan(ax)^2a^2x^2+\sqrt{a^2x^2+1}\,atan(ax)^2}dx}{\sqrt{c}\,c^2}$$

input
```
int(1/(a^2*c*x^2+c)^(5/2)/atan(a*x)^2,x)
```

output
```
int(1/(sqrt(a**2*x**2 + 1)*atan(a*x)**2*a**4*x**4 + 2*sqrt(a**2*x**2 + 1)*
atan(a*x)**2*a**2*x**2 + sqrt(a**2*x**2 + 1)*atan(a*x)**2),x)/(sqrt(c)*c**
2)
```

# 3.592 $\int \frac{1}{x(c+a^2cx^2)^{5/2}\arctan(ax)^2} dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x(c+a^2cx^2)^{5/2}\arctan(ax)^2} dx = \frac{ax}{c(c+a^2cx^2)^{3/2}\arctan(ax)} + \frac{ax}{c^2\sqrt{c+a^2cx^2}\arctan(ax)} - \frac{\sqrt{c+a^2cx^2}}{ac^3x\arctan(ax)} - \frac{5\sqrt{1+a^2x^2}\,\text{CosIntegral}(\arctan(ax))}{4c^2\sqrt{c+a^2cx^2}} - \frac{3\sqrt{1+a^2x^2}\,\text{CosIntegral}(3\arctan(ax))}{4c^2\sqrt{c+a^2cx^2}} - \frac{\text{Int}\left(\frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)},x\right)}{ac^2}$$

output
```
a*x/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)+a*x/c^2/(a^2*c*x^2+c)^(1/2)/arctan(a
*x)-(a^2*c*x^2+c)^(1/2)/a/c^3/x/arctan(a*x)-5/4*(a^2*x^2+1)^(1/2)*Ci(arcta
n(a*x))/c^2/(a^2*c*x^2+c)^(1/2)-3/4*(a^2*x^2+1)^(1/2)*Ci(3*arctan(a*x))/c^
2/(a^2*c*x^2+c)^(1/2)-Defer(Int)(1/x^2/(a^2*c*x^2+c)^(1/2)/arctan(a*x),x)/
a/c^2
```

## Mathematica [N/A]

Not integrable

Time = 1.87 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2}\,dx=\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2}\,dx$$

input
```
Integrate[1/(x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

output
```
Integrate[1/(x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2), x]
```

## Rubi [N/A]

Not integrable

Time = 4.76 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\arctan(ax)^2\left(a^2cx^2+c\right)^{5/2}}\,dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{1}{x(a^2cx^2+c)^{3/2}\arctan(ax)^2}dx}{c}-a^2\int \frac{x}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^2}dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\frac{\int \frac{1}{x\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\int \frac{x}{(a^2cx^2+c)^{3/2}\arctan(ax)^2}dx}{c}-a^2\int \frac{x}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^2}dx$$

$$\Big\downarrow \text{5477}$$

$$\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c}-a^2\left(\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}dx}{a}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c}-a^2\int \frac{x}{(a^2cx^2+c)^{5/2}\arctan(ax)^2}dx$$

$$\Big\downarrow \text{5440}$$

$$\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c}-a^2\left(\frac{\sqrt{a^2x^2+1}\int \frac{1}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}dx}{ac\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c}-a^2\int \frac{x}{(a^2cx^2+c)^{5/2}\arctan(ax)^2}dx$$

$$\Big\downarrow \text{5439}$$

$$\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c}-a^2\left(\frac{\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c}-a^2\int \frac{x}{(a^2cx^2+c)^{5/2}\arctan(ax)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c}-a^2\left(\frac{\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)}{\arctan(ax)}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c}-a^2\int \frac{x}{(a^2cx^2+c)^{5/2}\arctan(ax)^2}dx$$

$$\Big\downarrow \text{3783}$$

$$\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c}-a^2\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c}-a^2\int \frac{x}{(a^2cx^2+c)^{5/2}\arctan(ax)^2}dx$$

$$\Big\downarrow \text{5503}$$

$$\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c}-a^2\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c}-a^2\left(\frac{\int\frac{1}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx}{a}-2a\int\frac{x^2}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx-\frac{x}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\Big\downarrow \text{ 5440}$$

$$\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c}-a^2\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c}-a^2\left(\frac{\sqrt{a^2x^2+1}\int\frac{1}{(a^2x^2+1)^{5/2}\arctan(ax)}dx}{ac^2\sqrt{a^2cx^2+c}}-2a\int\frac{x^2}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx-\frac{x}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\Big\downarrow \text{ 5439}$$

$$\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c}-a^2\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c}-a^2\left(\frac{\sqrt{a^2x^2+1}\int\frac{1}{(a^2x^2+1)^{3/2}\arctan(ax)}d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}-2a\int\frac{x^2}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx-\frac{x}{ac\arctan(ax)\left(a^2cx^2+\cdots\right.}\right.$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c}-a^2\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c}-a^2\left(\frac{\sqrt{a^2x^2+1}\int\frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^3}{\arctan(ax)}d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}-2a\int\frac{x^2}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx-\frac{x}{ac\arctan(ax)\left(a^2cx^2+c\cdots\right.}\right.$$

$$\Big\downarrow \text{ 3793}$$

$$\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c}-a^2\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c}-a^2\left(\frac{\sqrt{a^2x^2+1}\int\left(\frac{\cos(3\arctan(ax))}{4\arctan(ax)}+\frac{3}{4\sqrt{a^2x^2+1}\arctan(ax)}\right)d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}-2a\int\frac{x^2}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx-\cdots\right.$$

$\Big\downarrow$ 2009

$$\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c}-a^2\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c}-$$

$$a^2\left(-2a\int \frac{x^2}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx+\frac{\sqrt{a^2x^2+1}\left(\frac{3}{4}\text{CosIntegral}(\arctan(ax))+\frac{1}{4}\text{CosIntegral}(3\arctan(ax))\right)}{a^2c^2\sqrt{a^2cx^2+c}}\right.$$

$\Big\downarrow$ 5506

$$\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c}-a^2\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c}-$$

$$a^2\left(-\frac{2a\sqrt{a^2x^2+1}\int \frac{x^2}{(a^2x^2+1)^{5/2}\arctan(ax)}dx}{c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2x^2+1}\left(\frac{3}{4}\text{CosIntegral}(\arctan(ax))+\frac{1}{4}\text{CosIntegral}(3\arctan(\right.}{a^2c^2\sqrt{a^2cx^2+c}}\right.$$

$\Big\downarrow$ 5505

$$\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c}-a^2\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c}-$$

$$a^2\left(-\frac{2\sqrt{a^2x^2+1}\int \frac{a^2x^2}{(a^2x^2+1)^{3/2}\arctan(ax)}d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2x^2+1}\left(\frac{3}{4}\text{CosIntegral}(\arctan(ax))+\frac{1}{4}\text{CosIntegral}(\right.}{a^2c^2\sqrt{a^2cx^2+c}}\right.$$

$\Big\downarrow$ 4906

$$\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c}-a^2\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c}-$$

$$a^2\left(-\frac{2\sqrt{a^2x^2+1}\int \left(\frac{1}{4\sqrt{a^2x^2+1}\arctan(ax)}-\frac{\cos(3\arctan(ax))}{4\arctan(ax)}\right)d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2x^2+1}\left(\frac{3}{4}\text{CosIntegral}(\arctan(ax)\right.}{a^2c^2\sqrt{a^2cx}}\right.$$

$\Big\downarrow$ 2009

$$\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c}-a^2\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c}-$$

$$a^2\left(-\frac{2\sqrt{a^2x^2+1}\left(\frac{1}{4}\text{CosIntegral}(\arctan(ax))-\frac{1}{4}\text{CosIntegral}(3\arctan(ax))\right)}{a^2c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2x^2+1}\left(\frac{3}{4}\text{CosIntegral}(\text{arcta}\right.}{a^2c^2}\right.$$

$$\Big\downarrow \text{5560}$$

$$\frac{-\frac{\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}}{c}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c}-a^2\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c}-$$
$$a^2\left(-\frac{2\sqrt{a^2x^2+1}\left(\frac{1}{4}\text{CosIntegral}(\arctan(ax))-\frac{1}{4}\text{CosIntegral}(3\arctan(ax))\right)}{a^2c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2x^2+1}\left(\frac{3}{4}\text{CosIntegral}(\text{arcta}\right.}{a^2c^2}\right.$$

input
```
Int[1/(x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 12.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\left(a^2c\,x^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^2}dx$$

input
```
int(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

output
```
int(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.54

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2}\,dx=\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{5}{2}}x\arctan\left(ax\right)^2}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)/((a^6*c^3*x^7 + 3*a^4*c^3*x^5 + 3*a^2*c^3*x^3
 + c^3*x)*arctan(a*x)^2), x)
```

## Sympy [N/A]

Not integrable

Time = 11.95 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2}\,dx=\int \frac{1}{x\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}\operatorname{atan}^2\left(ax\right)}\,dx$$

input
```
integrate(1/x/(a**2*c*x**2+c)**(5/2)/atan(a*x)**2,x)
```

output
```
Integral(1/(x*(c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**2), x)
```

## Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2}\,dx=\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{5}{2}}x\arctan\left(ax\right)^2}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(5/2)*x*arctan(a*x)^2), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2}\,dx = \int \frac{1}{x\,\mathrm{atan}\left(a\,x\right)^2\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

input
```
int(1/(x*atan(a*x)^2*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(1/(x*atan(a*x)^2*(c + a^2*c*x^2)^(5/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 78, normalized size of antiderivative = 3.25

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2}\,dx = \frac{\int \frac{1}{\sqrt{a^2x^2+1}\,atan(ax)^2a^4x^5+2\sqrt{a^2x^2+1}\,atan(ax)^2a^2x^3+\sqrt{a^2x^2+1}\,atan(ax)^2x}dx}{\sqrt{c}\,c^2}$$

input
```
int(1/x/(a^2*c*x^2+c)^(5/2)/atan(a*x)^2,x)
```

output
```
int(1/(sqrt(a**2*x**2 + 1)*atan(a*x)**2*a**4*x**5 + 2*sqrt(a**2*x**2 + 1)*
atan(a*x)**2*a**2*x**3 + sqrt(a**2*x**2 + 1)*atan(a*x)**2*x),x)/(sqrt(c)*c
**2)
```

# 3.593 $\int \frac{1}{x^2(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x^2(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = \frac{a}{c(c+a^2cx^2)^{3/2}\arctan(ax)} + \frac{a}{c^2\sqrt{c+a^2cx^2}\arctan(ax)} + \frac{7a\sqrt{1+a^2x^2}\operatorname{Si}(\arctan(ax))}{4c^2\sqrt{c+a^2cx^2}} + \frac{3a\sqrt{1+a^2x^2}\operatorname{Si}(3\arctan(ax))}{4c^2\sqrt{c+a^2cx^2}} + \frac{\operatorname{Int}\left(\frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^2},x\right)}{c^2}$$

output
```
a/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)+a/c^2/(a^2*c*x^2+c)^(1/2)/arctan(a*x)+
7/4*a*(a^2*x^2+1)^(1/2)*Si(arctan(a*x))/c^2/(a^2*c*x^2+c)^(1/2)+3/4*a*(a^2
*x^2+1)^(1/2)*Si(3*arctan(a*x))/c^2/(a^2*c*x^2+c)^(1/2)+Defer(Int)(1/x^2/(
a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x)/c^2
```

## Mathematica [N/A]

Not integrable

Time = 4.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^2 (c+a^2cx^2)^{5/2} \arctan(ax)^2} \, dx = \int \frac{1}{x^2 (c+a^2cx^2)^{5/2} \arctan(ax)^2} \, dx$$

input
```
Integrate[1/(x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

output
```
Integrate[1/(x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2), x]
```

## Rubi [N/A]

Not integrable

Time = 2.51 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \arctan(ax)^2 (a^2cx^2+c)^{5/2}} \, dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{1}{x^2(a^2cx^2+c)^{3/2}\arctan(ax)^2}dx}{c} - a^2 \int \frac{1}{(a^2cx^2+c)^{5/2}\arctan(ax)^2}dx$$

$$\Big\downarrow \text{5437}$$

$$\frac{\int \frac{1}{x^2(a^2cx^2+c)^{3/2}\arctan(ax)^2}dx}{c} - a^2\left(-3a\int \frac{x}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx - \frac{1}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}\right)$$

$$\Big\downarrow \text{5501}$$

$$\frac{\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}dx}{c}-a^2\left(-3a\int \frac{x}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)}dx-\frac{1}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\Big\downarrow \text{5437}$$

$$\frac{\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\left(-a\int \frac{x}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}dx-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c}-a^2\left(-3a\int \frac{x}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)}dx-\frac{1}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\Big\downarrow \text{5506}$$

$$\frac{\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\left(-\frac{a\sqrt{a^2x^2+1}\int \frac{x}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}dx}{c\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c}-a^2\left(-\frac{3a\sqrt{a^2x^2+1}\int \frac{x}{\left(a^2x^2+1\right)^{5/2}\arctan(ax)}dx}{c^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\Big\downarrow \text{5505}$$

$$\frac{\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\left(-\frac{\sqrt{a^2x^2+1}\int \frac{ax}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c}-a^2\left(-\frac{3\sqrt{a^2x^2+1}\int \frac{ax}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\left(-\frac{\sqrt{a^2x^2+1}\int \frac{\sin(\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c}-a^2\left(-\frac{3\sqrt{a^2x^2+1}\int \frac{ax}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\Big\downarrow \text{3780}$$

$$\frac{\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\left(-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c}-a^2\left(-\frac{3\sqrt{a^2x^2+1}\int \frac{ax}{(a^2x^2+1)^{3/2}\arctan(ax)}d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\Big\downarrow \text{4906}$$

$$\frac{\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\left(-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c}-a^2\left(-\frac{3\sqrt{a^2x^2+1}\int \left(\frac{ax}{4\sqrt{a^2x^2+1}\arctan(ax)}+\frac{\sin(3\arctan(ax))}{4\arctan(ax)}\right)d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\left(-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c}-a^2\left(-\frac{3\sqrt{a^2x^2+1}\left(\frac{1}{4}\text{Si}(\arctan(ax))+\frac{1}{4}\text{Si}(3\arctan(ax))\right)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\Big\downarrow \text{5560}$$

$$\frac{\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\left(-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c}-a^2\left(-\frac{3\sqrt{a^2x^2+1}\left(\frac{1}{4}\text{Si}(\arctan(ax))+\frac{1}{4}\text{Si}(3\arctan(ax))\right)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}\right)$$

input
```
Int[1/(x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 8.77 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^{2} \left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}} \arctan \left(a x \right)^{2}}d x$$

input
```
int(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

output
```
int(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.62

$$\int \frac{1}{x^2 \left(c+a^2 c x^2\right)^{5/2} \arctan(a x)^2} \, dx=\int { \frac {1}{{\left (a^{2} c x^{2} + c\right )}^{\frac {5}{2}} x^{2} \arctan \left (a x\right )^{2}} \,d x }$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)/((a^6*c^3*x^8 + 3*a^4*c^3*x^6 + 3*a^2*c^3*x^4
 + c^3*x^2)*arctan(a*x)^2), x)
```

## Sympy [N/A]

Not integrable

Time = 15.34 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^2 \left(c+a^2 c x^2\right)^{5/2} \arctan(a x)^2} \, dx=\int \frac {1}{x^{2} \left (c \left (a^{2} x^{2} + 1\right )\right )^{\frac {5}{2}} \operatorname {atan}^{2}{\left (a x \right )}}\, dx$$

input
```
integrate(1/x**2/(a**2*c*x**2+c)**(5/2)/atan(a*x)**2,x)
```

output
```
Integral(1/(x**2*(c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**2), x)
```

## Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (c + a^2 c x^2)^{5/2} \arctan(ax)^2} \, dx = \int \frac{1}{(a^2 c x^2 + c)^{\frac{5}{2}} x^2 \arctan{(ax)}^2} \, dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(5/2)*x^2*arctan(a*x)^2), x)
```

## Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (c + a^2 c x^2)^{5/2} \arctan(ax)^2} \, dx = \int \frac{1}{(a^2 c x^2 + c)^{\frac{5}{2}} x^2 \arctan{(ax)}^2} \, dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(5/2)*x^2*arctan(a*x)^2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (c+a^2cx^2)^{5/2} \arctan(ax)^2} dx = \int \frac{1}{x^2 \operatorname{atan}(a\,x)^2 \left(c\,a^2\,x^2+c\right)^{5/2}} dx$$

input
```
int(1/(x^2*atan(a*x)^2*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(1/(x^2*atan(a*x)^2*(c + a^2*c*x^2)^(5/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 80, normalized size of antiderivative = 3.33

$$\int \frac{1}{x^2 (c+a^2cx^2)^{5/2} \arctan(ax)^2} dx = \frac{\int \frac{1}{\sqrt{a^2x^2+1}\,atan(ax)^2a^4x^6+2\sqrt{a^2x^2+1}\,atan(ax)^2a^2x^4+\sqrt{a^2x^2+1}\,atan(ax)^2x^2}dx}{\sqrt{c}\,c^2}$$

input
```
int(1/x^2/(a^2*c*x^2+c)^(5/2)/atan(a*x)^2,x)
```

output
```
int(1/(sqrt(a**2*x**2 + 1)*atan(a*x)**2*a**4*x**6 + 2*sqrt(a**2*x**2 + 1)*
atan(a*x)**2*a**2*x**4 + sqrt(a**2*x**2 + 1)*atan(a*x)**2*x**2),x)/(sqrt(c
)*c**2)
```

# 3.594 $\int \frac{1}{x^3 (c+a^2 c x^2)^{5/2} \arctan(ax)^2} dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x^3 (c+a^2 c x^2)^{5/2} \arctan(ax)^2} dx = -\frac{a^3 x}{c (c+a^2 c x^2)^{3/2} \arctan(ax)} - \frac{2a^3 x}{c^2 \sqrt{c+a^2 c x^2} \arctan(ax)} + \frac{2a\sqrt{c+a^2 c x^2}}{c^3 x \arctan(ax)} + \frac{9a^2 \sqrt{1+a^2 x^2}\, \text{CosIntegral}(\arctan(ax))}{4c^2 \sqrt{c+a^2 c x^2}} + \frac{3a^2 \sqrt{1+a^2 x^2}\, \text{CosIntegral}(3\arctan(ax))}{4c^2 \sqrt{c+a^2 c x^2}} + \frac{\text{Int}\left(\frac{1}{x^3 \sqrt{c+a^2 c x^2} \arctan(ax)^2}, x\right)}{c^2} + \frac{2a\text{Int}\left(\frac{1}{x^2 \sqrt{c+a^2 c x^2} \arctan(ax)}, x\right)}{c^2}$$

output

```
-a^3*x/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)-2*a^3*x/c^2/(a^2*c*x^2+c)^(1/2)/a
rctan(a*x)+2*a*(a^2*c*x^2+c)^(1/2)/c^3/x/arctan(a*x)+9/4*a^2*(a^2*x^2+1)^(
1/2)*Ci(arctan(a*x))/c^2/(a^2*c*x^2+c)^(1/2)+3/4*a^2*(a^2*x^2+1)^(1/2)*Ci(
3*arctan(a*x))/c^2/(a^2*c*x^2+c)^(1/2)+Defer(Int)(1/x^3/(a^2*c*x^2+c)^(1/2
)/arctan(a*x)^2,x)/c^2+2*a*Defer(Int)(1/x^2/(a^2*c*x^2+c)^(1/2)/arctan(a*x
),x)/c^2
```

## Mathematica [N/A]

Not integrable

Time = 5.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^3 (c+a^2cx^2)^{5/2} \arctan(ax)^2} dx = \int \frac{1}{x^3 (c+a^2cx^2)^{5/2} \arctan(ax)^2} dx$$

input
```
Integrate[1/(x^3*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

output
```
Integrate[1/(x^3*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2), x]
```

## Rubi [N/A]

Not integrable

Time = 9.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \arctan(ax)^2 (a^2cx^2+c)^{5/2}} dx$$

$$\Big\downarrow \text{ 5501}$$

$$\frac{\int \frac{1}{x^3(a^2cx^2+c)^{3/2}\arctan(ax)^2}dx}{c} - a^2 \int \frac{1}{x (a^2cx^2+c)^{5/2} \arctan(ax)^2} dx$$

$$\Big\downarrow \text{ 5501}$$

$$\frac{\frac{\int \frac{1}{x^3\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2 \int \frac{1}{x(a^2cx^2+c)^{3/2}\arctan(ax)^2}dx}{c} - a^2 \left( \frac{\int \frac{1}{x(a^2cx^2+c)^{3/2}\arctan(ax)^2}dx}{c} - a^2 \int \frac{x}{(a^2cx^2+c)^{5/2}\arctan(ax)^2} dx \right)$$

$$\downarrow \text{5501}$$

$$\frac{\frac{\int \frac{1}{x^3\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\left(\frac{\int \frac{1}{x\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\int \frac{x}{(a^2cx^2+c)^{3/2}\arctan(ax)^2}dx\right)}{c} - a^2\left(\frac{\frac{\int \frac{1}{x\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\int \frac{x}{(a^2cx^2+c)^{3/2}\arctan(ax)^2}dx}{c} - a^2\int \frac{x}{(a^2cx^2+c)^{5/2}\arctan(ax)^2}dx\right)$$

$$\downarrow \text{5477}$$

$$\frac{\frac{\int \frac{1}{x^3\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\left(\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a} - \frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c} - a^2\left(\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}dx}{a} - \frac{x}{ac\arctan(ax)\sqrt{a^2\cdots}}\cdots\right.\right.}{c} \cdots$$

$$a^2\left(\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a} - \frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c} - a^2\left(\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}dx}{a} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c} - a^2\int \frac{\cdots}{(a^2cx^2+\cdots}\cdots\right.$$

$$\downarrow \text{5440}$$

$$\frac{\frac{\int \frac{1}{x^3\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\left(\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a} - \frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c} - a^2\left(\frac{\sqrt{a^2x^2+1}\int \frac{1}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}dx}{ac\sqrt{a^2cx^2+c}} - \frac{\cdots}{ac\arctan\cdots}\cdots\right.\right.}{c} \cdots$$

$$a^2\left(\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a} - \frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c} - a^2\left(\frac{\sqrt{a^2x^2+1}\int \frac{1}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}dx}{ac\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c} - a^2\int \frac{\cdots}{(a\cdots}\cdots\right.$$

$$\downarrow \text{5439}$$

$$\frac{\frac{\int \frac{1}{x^3\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\left(\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c}-a^2\left(\frac{\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{}{ac\ldots}\right.\right.}{c}$$

$$a^2\left(\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c}-a^2\left(\frac{\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c}-a^2\ldots\right.$$

$\downarrow$ 3042

$$\frac{\frac{\int \frac{1}{x^3\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\left(\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c}-a^2\left(\frac{\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)}{\arctan(ax)}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{}{ac\ldots}\right.\right.}{c}$$

$$a^2\left(\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c}-a^2\left(\frac{\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)}{\arctan(ax)}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c}-a^2\ldots\right.$$

$\downarrow$ 3783

$$\frac{\frac{\int \frac{1}{x^3\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\left(\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c}-a^2\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax\ldots}\right.\right.}{c}$$

$$a^2\left(\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c}-a^2\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c}-a^2\int\frac{}{(a^2cx\ldots}\right.$$

$\downarrow$ 5503

$$\frac{\frac{\int \frac{1}{x^3\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\left(\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}}{c} - \frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)} - a^2\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax\cdots}\right.\right.}{c}$$

$$a^2\left(\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}}{c} - \frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)} - a^2\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c} - a^2\left(\frac{\int \cdots}{(a^2cx\cdots}\right.\right.$$

$$\Big\downarrow \; 5440$$

$$\frac{\frac{\int \frac{1}{x^3\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\left(\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}}{c} - \frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)} - a^2\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax\cdots}\right.\right.}{c}$$

$$a^2\left(\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}}{c} - \frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)} - a^2\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c} - a^2\left(\frac{\sqrt{a^2x^2\cdots}}{\cdots}\right.\right.$$

$$\Big\downarrow \; 5439$$

$$\frac{\frac{\int \frac{1}{x^3\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\left(\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}}{c} - \frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)} - a^2\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax\cdots}\right.\right.}{c}$$

$$a^2\left(\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}}{c} - \frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)} - a^2\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c} - a^2\left(\frac{\sqrt{a^2x^2\cdots}}{\cdots}\right.\right.$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\int \frac{1}{x^3\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\left(\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}}{c} - \frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)} - a^2\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax\cdots}\right.\right.}{c}$$

$$a^2\left(\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}}{c} - \frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)} - a^2\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c} - a^2\left(\frac{\sqrt{a^2x^2\cdots}}{\cdots}\right.\right.$$

$\downarrow$ 3793

$$\frac{\frac{\int \frac{1}{x^3\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\left(\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}}{c} - \frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)} - a^2\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\ldots}\right.\right.}{c}$$

$$a^2\left(\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}}{c} - \frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)} - a^2\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c} - a^2\left(\frac{\sqrt{a^2x^2}\ldots}{\ldots}\right.\right.$$

$\downarrow$ 2009

$$\frac{\frac{\int \frac{1}{x^3\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\left(\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}}{c} - \frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)} - a^2\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\ldots}\right.\right.}{c}$$

$$a^2\left(\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}}{c} - \frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)} - a^2\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c} - a^2\left(-2a\int\ldots\right.\right.$$

$\downarrow$ 5506

$$\frac{\frac{\int \frac{1}{x^3\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\left(\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}}{c} - \frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)} - a^2\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\ldots}\right.\right.}{c}$$

$$a^2\left(\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}}{c} - \frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)} - a^2\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c} - a^2\left(-\frac{2a\sqrt{\ldots}}{\ldots}\right.\right.$$

$\downarrow$ 5505

$$\frac{\frac{\int \frac{1}{x^3\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\left(\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c}-a^2\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax}\right.\right.}{c}$$

$$a^2\left(\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c}-a^2\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c}-a^2\left(-\frac{2\sqrt{a^2}}{}\right.\right.$$

$\downarrow$ 4906

$$\frac{\frac{\int \frac{1}{x^3\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\left(\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c}-a^2\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax}\right.\right.}{c}$$

$$a^2\left(\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c}-a^2\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c}-a^2\left(-\frac{2\sqrt{a^2}}{}\right.\right.$$

$\downarrow$ 2009

$$\frac{\frac{\int \frac{1}{x^3\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\left(\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c}-a^2\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax}\right.\right.}{c}$$

$$a^2\left(\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c}-a^2\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c}-a^2\left(-\frac{2\sqrt{a^2}}{}\right.\right.$$

$\downarrow$ 5560

$$\frac{\frac{\int \frac{1}{x^3\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\left(\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c}-a^2\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax}\right.\right.}{c}$$

$$a^2\left(\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)}dx}{a}-\frac{\sqrt{a^2cx^2+c}}{acx\arctan(ax)}}{c}-a^2\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c}-a^2\left(-\frac{2\sqrt{a^2}}{}\right.\right.$$

input
```
Int[1/(x^3*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 22.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^{3} \left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}} \arctan \left(a x \right)^{2}}d x$$

input
```
int(1/x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

output
```
int(1/x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.62

$$\int \frac{1}{x^3 \left(c+a^2 c x^2\right)^{5/2} \arctan (a x)^2} \, dx=\int { \frac {1}{{\left (a^{2} c x^{2} + c\right )}^{\frac {5}{2}} x^{3} \arctan \left (a x\right )^{2}} \,d x }$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)/((a^6*c^3*x^9 + 3*a^4*c^3*x^7 + 3*a^2*c^3*x^5
 + c^3*x^3)*arctan(a*x)^2), x)
```

## Sympy [N/A]

Not integrable

Time = 19.81 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^3 (c+a^2cx^2)^{5/2} \arctan(ax)^2} \, dx = \int \frac{1}{x^3 \left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}} \operatorname{atan}^2(ax)} \, dx$$

input
```
integrate(1/x**3/(a**2*c*x**2+c)**(5/2)/atan(a*x)**2,x)
```

output
```
Integral(1/(x**3*(c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**2), x)
```

## Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3 (c+a^2cx^2)^{5/2} \arctan(ax)^2} \, dx = \int \frac{1}{\left(a^2cx^2+c\right)^{\frac{5}{2}} x^3 \arctan(ax)^2} \, dx$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(5/2)*x^3*arctan(a*x)^2), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x^3 (c+a^2cx^2)^{5/2} \arctan(ax)^2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3 (c+a^2cx^2)^{5/2} \arctan(ax)^2} \, dx = \int \frac{1}{x^3 \operatorname{atan}(a\,x)^2 \, (c\,a^2\,x^2+c)^{5/2}} \, dx$$

input
```
int(1/(x^3*atan(a*x)^2*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(1/(x^3*atan(a*x)^2*(c + a^2*c*x^2)^(5/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 80, normalized size of antiderivative = 3.33

$$\int \frac{1}{x^3 (c+a^2cx^2)^{5/2} \arctan(ax)^2} \, dx = \frac{\int \frac{1}{\sqrt{a^2x^2+1}\, atan(ax)^2 a^4x^7+2\sqrt{a^2x^2+1}\, atan(ax)^2 a^2x^5+\sqrt{a^2x^2+1}\, atan(ax)^2 x^3}dx}{\sqrt{c}\, c^2}$$

input
```
int(1/x^3/(a^2*c*x^2+c)^(5/2)/atan(a*x)^2,x)
```

output
```
int(1/(sqrt(a**2*x**2 + 1)*atan(a*x)**2*a**4*x**7 + 2*sqrt(a**2*x**2 + 1)*
atan(a*x)**2*a**2*x**5 + sqrt(a**2*x**2 + 1)*atan(a*x)**2*x**3),x)/(sqrt(c
)*c**2)
```

# 3.595 $\int \frac{1}{x^4(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\begin{aligned}\int \frac{1}{x^4\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2}\,dx = &-\frac{a^3}{c\left(c+a^2cx^2\right)^{3/2}\arctan(ax)}-\frac{2a^3}{c^2\sqrt{c+a^2cx^2}\arctan(ax)}\\ &-\frac{11a^3\sqrt{1+a^2x^2}\operatorname{Si}(\arctan(ax))}{4c^2\sqrt{c+a^2cx^2}}-\frac{3a^3\sqrt{1+a^2x^2}\operatorname{Si}(3\arctan(ax))}{4c^2\sqrt{c+a^2cx^2}}\\ &+\frac{\operatorname{Int}\left(\frac{1}{x^4\sqrt{c+a^2cx^2}\arctan(ax)^2},x\right)}{c^2}-\frac{2a^2\operatorname{Int}\left(\frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^2},x\right)}{c^2}\end{aligned}$$

output
```
-a^3/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)-2*a^3/c^2/(a^2*c*x^2+c)^(1/2)/arcta
n(a*x)-11/4*a^3*(a^2*x^2+1)^(1/2)*Si(arctan(a*x))/c^2/(a^2*c*x^2+c)^(1/2)-
3/4*a^3*(a^2*x^2+1)^(1/2)*Si(3*arctan(a*x))/c^2/(a^2*c*x^2+c)^(1/2)+Defer(
Int)(1/x^4/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x)/c^2-2*a^2*Defer(Int)(1/x^2
/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x)/c^2
```

## Mathematica [N/A]

Not integrable

Time = 6.61 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^4 (c + a^2cx^2)^{5/2} \arctan(ax)^2} \, dx = \int \frac{1}{x^4 (c + a^2cx^2)^{5/2} \arctan(ax)^2} \, dx$$

input
```
Integrate[1/(x^4*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

output
```
Integrate[1/(x^4*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2), x]
```

## Rubi [N/A]

Not integrable

Time = 5.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \arctan(ax)^2 (a^2cx^2 + c)^{5/2}} \, dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{1}{x^4 (a^2cx^2+c)^{3/2} \arctan(ax)^2} dx}{c} - a^2 \int \frac{1}{x^2 (a^2cx^2 + c)^{5/2} \arctan(ax)^2} dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\frac{\int \frac{1}{x^4 \sqrt{a^2cx^2+c} \arctan(ax)^2} dx}{c} - a^2 \int \frac{1}{x^2 (a^2cx^2+c)^{3/2} \arctan(ax)^2} dx}{c} - a^2 \left( \frac{\int \frac{1}{x^2 (a^2cx^2+c)^{3/2} \arctan(ax)^2} dx}{c} - a^2 \int \frac{1}{(a^2cx^2 + c)^{5/2} \arctan(ax)^2} dx \right)$$

$\big\downarrow$ 5437

$$\frac{\frac{\int \frac{1}{x^4\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\int \frac{1}{x^2(a^2cx^2+c)^{3/2}\arctan(ax)^2}dx}{c}-a^2\left(\frac{\int \frac{1}{x^2(a^2cx^2+c)^{3/2}\arctan(ax)^2}dx}{c}-a^2\left(-3a\int \frac{x}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx-\frac{1}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}\right)\right)$$

$\big\downarrow$ 5501

$$\frac{\frac{\int \frac{1}{x^4\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\left(\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\int \frac{1}{(a^2cx^2+c)^{3/2}\arctan(ax)^2}dx\right)}{c}-a^2\left(\frac{\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\int \frac{1}{(a^2cx^2+c)^{3/2}\arctan(ax)^2}dx}{c}-a^2\left(-3a\int \frac{x}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx-\frac{}{ac\arctan\ldots}\right.\right.$$

$\big\downarrow$ 5437

$$\frac{\frac{\int \frac{1}{x^4\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\left(\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\left(-a\int \frac{x}{(a^2cx^2+c)^{3/2}\arctan(ax)}dx-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)\right)}{c}-a^2\left(\frac{\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\left(-a\int \frac{x}{(a^2cx^2+c)^{3/2}\arctan(ax)}dx-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c}-a^2\left(-3a\int \frac{}{(a^2cx^2+c)^{5\ldots}}\right.\right.$$

$\big\downarrow$ 5506

$$\frac{\frac{\int \frac{1}{x^4\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\left(\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\left(-\frac{a\sqrt{a^2x^2+1}\int \frac{x}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}dx}{c\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2\ldots}}\right.\right.}{c}-a^2\left(\frac{\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\left(-\frac{a\sqrt{a^2x^2+1}\int \frac{x}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}dx}{c\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c}-a^2\left(-\frac{3a\sqrt{a^2x^2+\ldots}}{}\right.\right.$$

$\big\downarrow$ 5505

$$\frac{\frac{\int \frac{1}{x^4\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\left(\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\left(-\frac{\sqrt{a^2x^2+1}\int \frac{ax}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a}\cdots}\right.\right.}{c}$$

$$a^2\left(\frac{\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\left(-\frac{\sqrt{a^2x^2+1}\int \frac{ax}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c}-a^2\left(-\frac{3\sqrt{a^2x^2}\cdots}{\cdots}\right.\right.$$

$\downarrow$ 3042

$$\frac{\frac{\int \frac{1}{x^4\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\left(\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\left(-\frac{\sqrt{a^2x^2+1}\int \frac{\sin(\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2}\cdots}\right.\right.}{c}$$

$$a^2\left(\frac{\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\left(-\frac{\sqrt{a^2x^2+1}\int \frac{\sin(\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c}-a^2\left(-\frac{3\sqrt{a^2x^2+1}\cdots}{\cdots}\right.\right.$$

$\downarrow$ 3780

$$\frac{\frac{\int \frac{1}{x^4\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\left(\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\left(-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)\right)}{c}-$$

$$a^2\left(\frac{\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\left(-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c}-a^2\left(-\frac{3\sqrt{a^2x^2+1}\int \frac{\cdots}{(a^2x^2+1)^{3/\cdots}}}{ac^2\sqrt{a^2\cdots}}\right.\right.$$

$\downarrow$ 4906

$$\frac{\frac{\int \frac{1}{x^4\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\left(\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\left(-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)\right)}{c}-$$

$$a^2\left(\frac{\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c}-a^2\left(-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c}-a^2\left(-\frac{3\sqrt{a^2x^2+1}\int \left(\frac{\cdots}{4\sqrt{a^2x^2+\cdots}}\right.}{\cdots}\right.\right.$$

$\downarrow$ 2009

$$\frac{\frac{\int \frac{1}{x^4\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\left(\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\left(-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}} - \frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)\right)}{c} - a^2\left(\frac{\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\left(-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}} - \frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c} - a^2\left(-\frac{3\sqrt{a^2x^2+1}\left(\frac{1}{4}\text{Si}(\arctan\right.}{ac^2\sqrt{}}\right.\right.$$

$\Big\downarrow$ 5560

$$\frac{\frac{\int \frac{1}{x^4\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\left(\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\left(-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}} - \frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)\right)}{c} - a^2\left(\frac{\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{c} - a^2\left(-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}} - \frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)}{c} - a^2\left(-\frac{3\sqrt{a^2x^2+1}\left(\frac{1}{4}\text{Si}(\arctan\right.}{ac^2\sqrt{}}\right.\right.$$

input
```
Int[1/(x^4*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 34.59 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^4\left(a^2c\,x^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^2}dx$$

input
```
int(1/x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

output
```
int(1/x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.62

$$\int \frac{1}{x^4 (c+a^2cx^2)^{5/2} \arctan(ax)^2} \, dx = \int \frac{1}{(a^2cx^2+c)^{\frac{5}{2}} x^4 \arctan{(ax)}^2} \, dx$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)/((a^6*c^3*x^10 + 3*a^4*c^3*x^8 + 3*a^2*c^3*x^
6 + c^3*x^4)*arctan(a*x)^2), x)
```

## Sympy [N/A]

Not integrable

Time = 26.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^4 (c+a^2cx^2)^{5/2} \arctan(ax)^2} \, dx = \int \frac{1}{x^4 (c (a^2x^2+1))^{\frac{5}{2}} \operatorname{atan}^2{(ax)}} \, dx$$

input
```
integrate(1/x**4/(a**2*c*x**2+c)**(5/2)/atan(a*x)**2,x)
```

output
```
Integral(1/(x**4*(c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**2), x)
```

## Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4 (c+a^2cx^2)^{5/2} \arctan(ax)^2} \, dx = \int \frac{1}{(a^2cx^2+c)^{\frac{5}{2}} x^4 \arctan{(ax)}^2} \, dx$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(5/2)*x^4*arctan(a*x)^2), x)
```

## Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4 (c + a^2cx^2)^{5/2} \arctan(ax)^2} \, dx = \int \frac{1}{(a^2cx^2 + c)^{\frac{5}{2}} x^4 \arctan(ax)^2} \, dx$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(5/2)*x^4*arctan(a*x)^2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4 (c + a^2cx^2)^{5/2} \arctan(ax)^2} \, dx = \int \frac{1}{x^4 \operatorname{atan}(a\,x)^2 \,(c\,a^2\,x^2 + c)^{5/2}} \, dx$$

input
```
int(1/(x^4*atan(a*x)^2*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(1/(x^4*atan(a*x)^2*(c + a^2*c*x^2)^(5/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 3.33

$$\int \frac{1}{x^4 (c+a^2 c x^2)^{5/2} \arctan(ax)^2}\,dx = \frac{\int \frac{1}{\sqrt{a^2x^2+1}\,atan(ax)^2 a^4x^8+2\sqrt{a^2x^2+1}\,atan(ax)^2 a^2x^6+\sqrt{a^2x^2+1}\,atan(ax)^2 x^4}dx}{\sqrt{c}\,c^2}$$

input
```
int(1/x^4/(a^2*c*x^2+c)^(5/2)/atan(a*x)^2,x)
```

output
```
int(1/(sqrt(a**2*x**2 + 1)*atan(a*x)**2*a**4*x**8 + 2*sqrt(a**2*x**2 + 1)*
atan(a*x)**2*a**2*x**6 + sqrt(a**2*x**2 + 1)*atan(a*x)**2*x**4),x)/(sqrt(c
)*c**2)
```

# 3.596 $\int \frac{\sqrt{fx}}{(d+c^2dx^2)^2(a+b\arctan(cx))^2}\,dx$



## Optimal result

Integrand size = 30, antiderivative size = 30

$$\int \frac{\sqrt{fx}}{(d+c^2dx^2)^2(a+b\arctan(cx))^2}\,dx = \text{Int}\left(\frac{\sqrt{fx}}{(d+c^2dx^2)^2(a+b\arctan(cx))^2},x\right)$$

output
```
Defer(Int)((f*x)^(1/2)/(c^2*d*x^2+d)^2/(a+b*arctan(c*x))^2,x)
```

## Mathematica [N/A]

Not integrable

Time = 49.87 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{fx}}{(d+c^2dx^2)^2(a+b\arctan(cx))^2}\,dx = \int \frac{\sqrt{fx}}{(d+c^2dx^2)^2(a+b\arctan(cx))^2}\,dx$$

input
```
Integrate[Sqrt[f*x]/((d + c^2*d*x^2)^2*(a + b*ArcTan[c*x])^2),x]
```

output
```
Integrate[Sqrt[f*x]/((d + c^2*d*x^2)^2*(a + b*ArcTan[c*x])^2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{fx}}{(c^2dx^2+d)^2 (a+b\arctan(cx))^2} \, dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{\sqrt{fx}}{(c^2dx^2+d)^2 (a+b\arctan(cx))^2}dx$$

input
```
Int[Sqrt[f*x]/((d + c^2*d*x^2)^2*(a + b*ArcTan[c*x])^2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 4.81 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{fx}}{(c^2d\,x^2+d)^2 (a+b\arctan(cx))^2}dx$$

input
```
int((f*x)^(1/2)/(c^2*d*x^2+d)^2/(a+b*arctan(c*x))^2,x)
```

output
```
int((f*x)^(1/2)/(c^2*d*x^2+d)^2/(a+b*arctan(c*x))^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 125, normalized size of antiderivative = 4.17

$$\int \frac{\sqrt{fx}}{(d+c^2dx^2)^2 (a+b\arctan(cx))^2}\,dx = \int \frac{\sqrt{fx}}{(c^2dx^2+d)^2(b\arctan(cx)+a)^2}\,dx$$

input
```
integrate((f*x)^(1/2)/(c^2*d*x^2+d)^2/(a+b*arctan(c*x))^2,x, algorithm="fr
icas")
```

output
```
integral(sqrt(f*x)/(a^2*c^4*d^2*x^4 + 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c
^4*d^2*x^4 + 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arctan(c*x)^2 + 2*(a*b*c^4*d^2*x
^4 + 2*a*b*c^2*d^2*x^2 + a*b*d^2)*arctan(c*x)), x)
```

## Sympy [N/A]

Not integrable

Time = 100.40 (sec) , antiderivative size = 126, normalized size of antiderivative = 4.20

$$\int \frac{\sqrt{fx}}{(d+c^2dx^2)^2 (a+b\arctan(cx))^2}\,dx$$
$$= \frac{\int \frac{\sqrt{fx}}{a^2c^4x^4+2a^2c^2x^2+a^2+2abc^4x^4\operatorname{atan}(cx)+4abc^2x^2\operatorname{atan}(cx)+2ab\operatorname{atan}(cx)+b^2c^4x^4\operatorname{atan}^2(cx)+2b^2c^2x^2\operatorname{atan}^2(cx)+b^2\operatorname{atan}^2(cx)}\,dx}{d^2}$$

input
```
integrate((f*x)**(1/2)/(c**2*d*x**2+d)**2/(a+b*atan(c*x))**2,x)
```

output
```
Integral(sqrt(f*x)/(a**2*c**4*x**4 + 2*a**2*c**2*x**2 + a**2 + 2*a*b*c**4*
x**4*atan(c*x) + 4*a*b*c**2*x**2*atan(c*x) + 2*a*b*atan(c*x) + b**2*c**4*x
**4*atan(c*x)**2 + 2*b**2*c**2*x**2*atan(c*x)**2 + b**2*atan(c*x)**2), x)/
d**2
```

## Maxima [N/A]

Not integrable

Time = 1.43 (sec) , antiderivative size = 371, normalized size of antiderivative = 12.37

$$\int \frac{\sqrt{fx}}{(d+c^2dx^2)^2 (a+b\arctan(cx))^2}\,dx = \int \frac{\sqrt{fx}}{(c^2dx^2+d)^2(b\arctan(cx)+a)^2}\,dx$$

input
```
integrate((f*x)^(1/2)/(c^2*d*x^2+d)^2/(a+b*arctan(c*x))^2,x, algorithm="ma
xima")
```

output
```
1/2*(2*(a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^2*d^2*x^2 + b^2*d^2)*arctan(c*x
)^2 + 2*(a*b*c^2*d^2*x^2 + a*b*d^2)*arctan(c*x))*sqrt(f)*integrate(1/4*(a*
c^2*x^2 + 4*b*c*x + (b*c^2*x^2 + b)*arctan(c*x) + a)*sqrt(x)/(a^3*c^4*d^2*
x^4 + 2*a^3*c^2*d^2*x^2 + a^3*d^2 + (b^3*c^4*d^2*x^4 + 2*b^3*c^2*d^2*x^2 +
 b^3*d^2)*arctan(c*x)^3 + 3*(a*b^2*c^4*d^2*x^4 + 2*a*b^2*c^2*d^2*x^2 + a*b
^2*d^2)*arctan(c*x)^2 + 3*(a^2*b*c^4*d^2*x^4 + 2*a^2*b*c^2*d^2*x^2 + a^2*b
*d^2)*arctan(c*x)), x) + sqrt(f)*x^(3/2))/(a^2*c^2*d^2*x^2 + a^2*d^2 + (b^
2*c^2*d^2*x^2 + b^2*d^2)*arctan(c*x)^2 + 2*(a*b*c^2*d^2*x^2 + a*b*d^2)*arc
tan(c*x))
```

## Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{fx}}{(d+c^2dx^2)^2 (a+b\arctan(cx))^2}\,dx = \int \frac{\sqrt{fx}}{(c^2dx^2+d)^2(b\arctan(cx)+a)^2}\,dx$$

input
```
integrate((f*x)^(1/2)/(c^2*d*x^2+d)^2/(a+b*arctan(c*x))^2,x, algorithm="gi
ac")
```

output
```
integrate(sqrt(f*x)/((c^2*d*x^2 + d)^2*(b*arctan(c*x) + a)^2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{fx}}{(d+c^2dx^2)^2 (a+b\arctan(cx))^2}\,dx = \int \frac{\sqrt{f\,x}}{(a+b\,\mathrm{atan}\,(c\,x))^2\,(d\,c^2\,x^2+d)^2}\,dx$$

input
```
int((f*x)^(1/2)/((a + b*atan(c*x))^2*(d + c^2*d*x^2)^2),x)
```

output
```
int((f*x)^(1/2)/((a + b*atan(c*x))^2*(d + c^2*d*x^2)^2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 586, normalized size of antiderivative = 19.53

$$\int \frac{\sqrt{fx}}{(d+c^2dx^2)^2 (a+b\arctan(cx))^2}\,dx = \text{Too large to display}$$

input
```
int((f*x)^(1/2)/(c^2*d*x^2+d)^2/(a+b*atan(c*x))^2,x)
```

output
```
(sqrt(f)*(atan(c*x)*int(sqrt(x)/(atan(c*x)*b*c**4*x**5 + 2*atan(c*x)*b*c**
2*x**3 + atan(c*x)*b*x + a*c**4*x**5 + 2*a*c**2*x**3 + a*x),x)*b*c**2*x**2
 + atan(c*x)*int(sqrt(x)/(atan(c*x)*b*c**4*x**5 + 2*atan(c*x)*b*c**2*x**3
+ atan(c*x)*b*x + a*c**4*x**5 + 2*a*c**2*x**3 + a*x),x)*b - 3*atan(c*x)*in
t((sqrt(x)*x)/(atan(c*x)*b*c**4*x**4 + 2*atan(c*x)*b*c**2*x**2 + atan(c*x)
*b + a*c**4*x**4 + 2*a*c**2*x**2 + a),x)*b*c**4*x**2 - 3*atan(c*x)*int((sq
rt(x)*x)/(atan(c*x)*b*c**4*x**4 + 2*atan(c*x)*b*c**2*x**2 + atan(c*x)*b +
a*c**4*x**4 + 2*a*c**2*x**2 + a),x)*b*c**2 - 2*sqrt(x) + int(sqrt(x)/(atan
(c*x)*b*c**4*x**5 + 2*atan(c*x)*b*c**2*x**3 + atan(c*x)*b*x + a*c**4*x**5
+ 2*a*c**2*x**3 + a*x),x)*a*c**2*x**2 + int(sqrt(x)/(atan(c*x)*b*c**4*x**5
 + 2*atan(c*x)*b*c**2*x**3 + atan(c*x)*b*x + a*c**4*x**5 + 2*a*c**2*x**3 +
 a*x),x)*a - 3*int((sqrt(x)*x)/(atan(c*x)*b*c**4*x**4 + 2*atan(c*x)*b*c**2
*x**2 + atan(c*x)*b + a*c**4*x**4 + 2*a*c**2*x**2 + a),x)*a*c**4*x**2 - 3*
int((sqrt(x)*x)/(atan(c*x)*b*c**4*x**4 + 2*atan(c*x)*b*c**2*x**2 + atan(c*
x)*b + a*c**4*x**4 + 2*a*c**2*x**2 + a),x)*a*c**2))/(2*b*c*d**2*(atan(c*x)
*b*c**2*x**2 + atan(c*x)*b + a*c**2*x**2 + a))
```

# 3.597 $\int \frac{x^m(c+a^2cx^2)^3}{\arctan(ax)^2}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^m(c+a^2cx^2)^3}{\arctan(ax)^2}\,dx = \text{Int}\left(\frac{x^m(c+a^2cx^2)^3}{\arctan(ax)^2},x\right)$$

output
```
Defer(Int)(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

## Mathematica [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m(c+a^2cx^2)^3}{\arctan(ax)^2}\,dx = \int \frac{x^m(c+a^2cx^2)^3}{\arctan(ax)^2}\,dx$$

input
```
Integrate[(x^m*(c + a^2*c*x^2)^3)/ArcTan[a*x]^2,x]
```

output
```
Integrate[(x^m*(c + a^2*c*x^2)^3)/ArcTan[a*x]^2, x]
```

## Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m (a^2cx^2+c)^3}{\arctan(ax)^2}\,dx$$

$$\Big\downarrow \ 5560$$

$$\int \frac{x^m (a^2cx^2+c)^3}{\arctan(ax)^2}dx$$

input
```
Int[(x^m*(c + a^2*c*x^2)^3)/ArcTan[a*x]^2,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 7.92 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m (a^2c\,x^2+c)^3}{\arctan\left(ax\right)^2}dx$$

input
```
int(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

output
```
int(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.18

$$\int \frac{x^m(c+a^2cx^2)^3}{\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^3x^m}{\arctan{(ax)}^2}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*x^m/arctan(a*
x)^2, x)
```

## Sympy [N/A]

Not integrable

Time = 12.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 3.32

$$\int \frac{x^m(c+a^2cx^2)^3}{\arctan(ax)^2}\,dx = c^3\left(\int \frac{x^m}{\operatorname{atan}^2{(ax)}}\,dx + \int \frac{3a^2x^2x^m}{\operatorname{atan}^2{(ax)}}\,dx + \int \frac{3a^4x^4x^m}{\operatorname{atan}^2{(ax)}}\,dx + \int \frac{a^6x^6x^m}{\operatorname{atan}^2{(ax)}}\,dx\right)$$

input
```
integrate(x**m*(a**2*c*x**2+c)**3/atan(a*x)**2,x)
```

output
```
c**3*(Integral(x**m/atan(a*x)**2, x) + Integral(3*a**2*x**2*x**m/atan(a*x)
**2, x) + Integral(3*a**4*x**4*x**m/atan(a*x)**2, x) + Integral(a**6*x**6*
x**m/atan(a*x)**2, x))
```

## Maxima [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 177, normalized size of antiderivative = 8.05

$$\int \frac{x^m(c+a^2cx^2)^3}{\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^3x^m}{\arctan(ax)^2}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-((a^8*c^3*x^8 + 4*a^6*c^3*x^6 + 6*a^4*c^3*x^4 + 4*a^2*c^3*x^2 + c^3)*x^m
- arctan(a*x)*integrate(((a^8*c^3*m + 8*a^8*c^3)*x^8 + 4*(a^6*c^3*m + 6*a^
6*c^3)*x^6 + 6*(a^4*c^3*m + 4*a^4*c^3)*x^4 + c^3*m + 4*(a^2*c^3*m + 2*a^2*
c^3)*x^2)*x^m/(x*arctan(a*x)), x))/(a*arctan(a*x))
```

## Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m(c+a^2cx^2)^3}{\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^3x^m}{\arctan(ax)^2}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^3*x^m/arctan(a*x)^2, x)
```

## Mupad [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m(c+a^2cx^2)^3}{\arctan(ax)^2}\,dx = \int \frac{x^m\,\left(c\,a^2\,x^2+c\right)^3}{\mathrm{atan}\left(a\,x\right)^2}\,dx$$

input
```
int((x^m*(c + a^2*c*x^2)^3)/atan(a*x)^2,x)
```

output
```
int((x^m*(c + a^2*c*x^2)^3)/atan(a*x)^2, x)
```

## Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 76, normalized size of antiderivative = 3.45

$$\int \frac{x^m(c+a^2cx^2)^3}{\arctan(ax)^2}\,dx = c^3\left(\int \frac{x^m}{atan\,(ax)^2}dx + \left(\int \frac{x^m x^6}{atan\,(ax)^2}dx\right)a^6 + 3\left(\int \frac{x^m x^4}{atan\,(ax)^2}dx\right)a^4 + 3\left(\int \frac{x^m x^2}{atan\,(ax)^2}dx\right)a^2\right)$$

input
```
int(x^m*(a^2*c*x^2+c)^3/atan(a*x)^2,x)
```

output
```
c**3*(int(x**m/atan(a*x)**2,x) + int((x**m*x**6)/atan(a*x)**2,x)*a**6 + 3*
int((x**m*x**4)/atan(a*x)**2,x)*a**4 + 3*int((x**m*x**2)/atan(a*x)**2,x)*a
**2)
```

# 3.598 $\int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)^2}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)^2}\,dx = \operatorname{Int}\left(\frac{x^m(c+a^2cx^2)^2}{\arctan(ax)^2},x\right)$$

output
```
Defer(Int)(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

## Mathematica [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)^2}\,dx = \int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)^2}\,dx$$

input
```
Integrate[(x^m*(c + a^2*c*x^2)^2)/ArcTan[a*x]^2,x]
```

output
```
Integrate[(x^m*(c + a^2*c*x^2)^2)/ArcTan[a*x]^2, x]
```

## Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \left(a^2 c x^2+c\right)^2}{\arctan(a x)^2} \, dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{x^m \left(a^2 c x^2+c\right)^2}{\arctan(a x)^2}dx$$

input
```
Int[(x^m*(c + a^2*c*x^2)^2)/ArcTan[a*x]^2,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 10.49 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^{m} \left(a^{2} c \,x^{2}+c \right)^{2}}{\arctan \left(a x \right)^{2}}d x$$

input
```
int(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

output
```
int(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

$$\int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^2x^m}{\arctan(ax)^2}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*x^m/arctan(a*x)^2, x)
```

## Sympy [N/A]

Not integrable

Time = 5.68 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.41

$$\int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)^2}\,dx = c^2\left(\int \frac{x^m}{\operatorname{atan}^2(ax)}\,dx + \int \frac{2a^2x^2x^m}{\operatorname{atan}^2(ax)}\,dx + \int \frac{a^4x^4x^m}{\operatorname{atan}^2(ax)}\,dx\right)$$

input
```
integrate(x**m*(a**2*c*x**2+c)**2/atan(a*x)**2,x)
```

output
```
c**2*(Integral(x**m/atan(a*x)**2, x) + Integral(2*a**2*x**2*x**m/atan(a*x)
**2, x) + Integral(a**4*x**4*x**m/atan(a*x)**2, x))
```

## Maxima [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 144, normalized size of antiderivative = 6.55

$$\int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^2x^m}{\arctan(ax)^2}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-((a^6*c^2*x^6 + 3*a^4*c^2*x^4 + 3*a^2*c^2*x^2 + c^2)*x^m - arctan(a*x)*in
tegrate(((a^6*c^2*m + 6*a^6*c^2)*x^6 + 3*(a^4*c^2*m + 4*a^4*c^2)*x^4 + c^2
*m + 3*(a^2*c^2*m + 2*a^2*c^2)*x^2)*x^m/(x*arctan(a*x)), x))/(a*arctan(a*x
))
```

## Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^2x^m}{\arctan(ax)^2}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^2*x^m/arctan(a*x)^2, x)
```

## Mupad [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)^2}\,dx = \int \frac{x^m\,(c\,a^2\,x^2+c)^2}{\mathrm{atan}\,(a\,x)^2}\,dx$$

input
```
int((x^m*(c + a^2*c*x^2)^2)/atan(a*x)^2,x)
```

output
```
int((x^m*(c + a^2*c*x^2)^2)/atan(a*x)^2, x)
```

## Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.55

$$\int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)^2}\,dx = c^2\left(\int \frac{x^m}{atan\left(ax\right)^2}dx + \left(\int \frac{x^m x^4}{atan\left(ax\right)^2}dx\right)a^4 + 2\left(\int \frac{x^m x^2}{atan\left(ax\right)^2}dx\right)a^2\right)$$

input
```
int(x^m*(a^2*c*x^2+c)^2/atan(a*x)^2,x)
```

output
```
c**2*(int(x**m/atan(a*x)**2,x) + int((x**m*x**4)/atan(a*x)**2,x)*a**4 + 2*
int((x**m*x**2)/atan(a*x)**2,x)*a**2)
```

# 3.599 $\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)^2}\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)^2}\,dx = \mathrm{Int}\left(\frac{x^m(c+a^2cx^2)}{\arctan(ax)^2},x\right)$$

output
```
Defer(Int)(x^m*(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

## Mathematica [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)^2}\,dx = \int \frac{x^m(c+a^2cx^2)}{\arctan(ax)^2}\,dx$$

input
```
Integrate[(x^m*(c + a^2*c*x^2))/ArcTan[a*x]^2,x]
```

output
```
Integrate[(x^m*(c + a^2*c*x^2))/ArcTan[a*x]^2, x]
```

## Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m (a^2cx^2+c)}{\arctan(ax)^2}dx$$
$$\Big\downarrow \ 5560$$
$$\int \frac{x^m (a^2cx^2+c)}{\arctan(ax)^2}dx$$

input
```
Int[(x^m*(c + a^2*c*x^2))/ArcTan[a*x]^2,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 6.73 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{x^m (a^2c\,x^2+c)}{\arctan(ax)^2}dx$$

input
```
int(x^m*(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

output
```
int(x^m*(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)x^m}{\arctan\left(ax\right)^2}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)*x^m/arctan(a*x)^2, x)
```

## Sympy [N/A]

Not integrable

Time = 3.46 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.55

$$\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)^2}\,dx = c\left(\int \frac{x^m}{\operatorname{atan}^2\left(ax\right)}\,dx + \int \frac{a^2x^2x^m}{\operatorname{atan}^2\left(ax\right)}\,dx\right)$$

input
```
integrate(x**m*(a**2*c*x**2+c)/atan(a*x)**2,x)
```

output
```
c*(Integral(x**m/atan(a*x)**2, x) + Integral(a**2*x**2*x**m/atan(a*x)**2,
x))
```

## Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 95, normalized size of antiderivative = 4.75

$$\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)x^m}{\arctan\left(ax\right)^2}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
-((a^4*c*x^4 + 2*a^2*c*x^2 + c)*x^m - arctan(a*x)*integrate(((a^4*c*m + 4*
a^4*c)*x^4 + 2*(a^2*c*m + 2*a^2*c)*x^2 + c*m)*x^m/(x*arctan(a*x)), x))/(a*
arctan(a*x))
```

## Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)x^m}{\arctan{(ax)}^2}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)*x^m/arctan(a*x)^2, x)
```

## Mupad [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)^2}\,dx = \int \frac{x^m\,\left(c\,a^2\,x^2+c\right)}{\mathrm{atan}\left(a\,x\right)^2}\,dx$$

input
```
int((x^m*(c + a^2*c*x^2))/atan(a*x)^2,x)
```

output
```
int((x^m*(c + a^2*c*x^2))/atan(a*x)^2, x)
```

## Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.70

$$\int \frac{x^m(c + a^2cx^2)}{\arctan(ax)^2}\, dx = c\left(\int \frac{x^m}{atan\,(ax)^2}dx + \left(\int \frac{x^m x^2}{atan\,(ax)^2}dx\right) a^2\right)$$

input
```
int(x^m*(a^2*c*x^2+c)/atan(a*x)^2,x)
```

output
```
c*(int(x**m/atan(a*x)**2,x) + int((x**m*x**2)/atan(a*x)**2,x)*a**2)
```

# 3.600 $\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^2}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^2}\,dx = -\frac{x^m}{ac\arctan(ax)} + \frac{m\mathrm{Int}\left(\frac{x^{-1+m}}{\arctan(ax)},x\right)}{ac}$$

output
```
-x^m/a/c/arctan(a*x)+m*Defer(Int)(x^(-1+m)/arctan(a*x),x)/a/c
```

## Mathematica [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^2}\,dx = \int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^2}\,dx$$

input
```
Integrate[x^m/((c + a^2*c*x^2)*ArcTan[a*x]^2),x]
```

output
```
Integrate[x^m/((c + a^2*c*x^2)*ArcTan[a*x]^2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\arctan(ax)^2 (a^2cx^2+c)} \, dx$$

$$\Big\downarrow \text{5461}$$

$$\frac{m\int \frac{x^{m-1}}{\arctan(ax)}dx}{ac} - \frac{x^m}{ac\arctan(ax)}$$

$$\Big\downarrow \text{5377}$$

$$\frac{m\int \frac{x^{m-1}}{\arctan(ax)}dx}{ac} - \frac{x^m}{ac\arctan(ax)}$$

input
```
Int[x^m/((c + a^2*c*x^2)*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 4.41 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(a^2c\,x^2+c)\arctan\,(ax)^2}dx$$

input
```
int(x^m/(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

output
```
int(x^m/(a^2*c*x^2+c)/arctan(a*x)^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^2}\,dx = \int \frac{x^m}{(a^2cx^2+c)\arctan{(ax)}^2}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(x^m/((a^2*c*x^2 + c)*arctan(a*x)^2), x)
```

## Sympy [N/A]

Not integrable

Time = 3.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^2}\,dx = \frac{\int \frac{x^m}{a^2x^2\operatorname{atan}^2{(ax)}+\operatorname{atan}^2{(ax)}}\,dx}{c}$$

input
```
integrate(x**m/(a**2*c*x**2+c)/atan(a*x)**2,x)
```

output
```
Integral(x**m/(a**2*x**2*atan(a*x)**2 + atan(a*x)**2), x)/c
```

## Maxima [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.82

$$\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^2}\,dx = \int \frac{x^m}{(a^2cx^2+c)\arctan{(ax)}^2}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
(m*arctan(a*x)*integrate(x^m/(x*arctan(a*x)), x) - x^m)/(a*c*arctan(a*x))
```

## Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^2}\,dx=\int \frac{x^m}{(a^2cx^2+c)\arctan{(ax)}^2}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate(x^m/((a^2*c*x^2 + c)*arctan(a*x)^2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^2}\,dx=\int \frac{x^m}{\mathrm{atan}\left(a\,x\right)^2\,\left(c\,a^2\,x^2+c\right)}\,dx$$

input
```
int(x^m/(atan(a*x)^2*(c + a^2*c*x^2)),x)
```

output
```
int(x^m/(atan(a*x)^2*(c + a^2*c*x^2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.45

$$\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^2}\,dx = \frac{\int \frac{x^m}{atan(ax)^2a^2x^2+atan(ax)^2}dx}{c}$$

input
```
int(x^m/(a^2*c*x^2+c)/atan(a*x)^2,x)
```

output
```
int(x**m/(atan(a*x)**2*a**2*x**2 + atan(a*x)**2),x)/c
```

# 3.601 $\int \frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)^2} \, dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)^2} \, dx = \text{Int}\left(\frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)^2}, x\right)$$

output
```
Defer(Int)(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

## Mathematica [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)^2} \, dx = \int \frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)^2} \, dx$$

input
```
Integrate[x^m/((c + a^2*c*x^2)^2*ArcTan[a*x]^2),x]
```

output
```
Integrate[x^m/((c + a^2*c*x^2)^2*ArcTan[a*x]^2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\arctan(ax)^2\,(a^2cx^2+c)^2}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x^m}{\arctan(ax)^2\,(a^2cx^2+c)^2}dx$$

input
```
Int[x^m/((c + a^2*c*x^2)^2*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 8.92 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(a^2c\,x^2+c)^2\arctan\left(ax\right)^2}dx$$

input
```
int(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

output
```
int(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.77

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^2\arctan(ax)^2}\,dx=\int \frac{x^m}{\left(a^2cx^2+c\right)^2\arctan\left(ax\right)^2}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(x^m/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)^2), x)
```

## Sympy [N/A]

Not integrable

Time = 8.95 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.91

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^2\arctan(ax)^2}\,dx=\frac{\int \frac{x^m}{a^4x^4\operatorname{atan}^2(ax)+2a^2x^2\operatorname{atan}^2(ax)+\operatorname{atan}^2(ax)}\,dx}{c^2}$$

input
```
integrate(x**m/(a**2*c*x**2+c)**2/atan(a*x)**2,x)
```

output
```
Integral(x**m/(a**4*x**4*atan(a*x)**2 + 2*a**2*x**2*atan(a*x)**2 + atan(a*
x)**2), x)/c**2
```

## Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 111, normalized size of antiderivative = 5.05

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^2\arctan(ax)^2}\,dx=\int \frac{x^m}{\left(a^2cx^2+c\right)^2\arctan\left(ax\right)^2}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
((a^3*c^2*x^2 + a*c^2)*arctan(a*x)*integrate(((a^2*m - 2*a^2)*x^2 + m)*x^m
/((a^5*c^2*x^5 + 2*a^3*c^2*x^3 + a*c^2*x)*arctan(a*x)), x) - x^m)/((a^3*c^
2*x^2 + a*c^2)*arctan(a*x))
```

## Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)^2} \, dx = \int \frac{x^m}{(a^2cx^2+c)^2 \arctan(ax)^2} \, dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate(x^m/((a^2*c*x^2 + c)^2*arctan(a*x)^2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)^2} \, dx = \int \frac{x^m}{\mathrm{atan}(a\,x)^2\,(c\,a^2\,x^2+c)^2} \, dx$$

input
```
int(x^m/(atan(a*x)^2*(c + a^2*c*x^2)^2),x)
```

output
```
int(x^m/(atan(a*x)^2*(c + a^2*c*x^2)^2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.09

$$\int \frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)^2}\,dx = \frac{\int \frac{x^m}{atan(ax)^2a^4x^4+2atan(ax)^2a^2x^2+atan(ax)^2}dx}{c^2}$$

input
```
int(x^m/(a^2*c*x^2+c)^2/atan(a*x)^2,x)
```

output
```
int(x**m/(atan(a*x)**2*a**4*x**4 + 2*atan(a*x)**2*a**2*x**2 + atan(a*x)**2
),x)/c**2
```

# 3.602 $\int \frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)^2} \, dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)^2} \, dx = \text{Int}\left(\frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)^2}, x\right)$$

output
```
Defer(Int)(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

## Mathematica [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)^2} \, dx = \int \frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)^2} \, dx$$

input
```
Integrate[x^m/((c + a^2*c*x^2)^3*ArcTan[a*x]^2),x]
```

output
```
Integrate[x^m/((c + a^2*c*x^2)^3*ArcTan[a*x]^2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\arctan(ax)^2 (a^2cx^2+c)^3} dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{x^m}{\arctan(ax)^2 (a^2cx^2+c)^3} dx$$

input
```
Int[x^m/((c + a^2*c*x^2)^3*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 9.75 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(a^2c\,x^2+c)^3 \arctan(ax)^2} dx$$

input
```
int(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

output
```
int(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.27

$$\int \frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)^2} \, dx = \int \frac{x^m}{(a^2cx^2+c)^3 \arctan{(ax)}^2} \, dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(x^m/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a
*x)^2), x)
```

## Sympy [N/A]

Not integrable

Time = 26.60 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.64

$$\int \frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)^2} \, dx = \frac{\int \frac{x^m}{a^6x^6 \operatorname{atan}^2{(ax)}+3a^4x^4 \operatorname{atan}^2{(ax)}+3a^2x^2 \operatorname{atan}^2{(ax)}+\operatorname{atan}^2{(ax)}} \, dx}{c^3}$$

input
```
integrate(x**m/(a**2*c*x**2+c)**3/atan(a*x)**2,x)
```

output
```
Integral(x**m/(a**6*x**6*atan(a*x)**2 + 3*a**4*x**4*atan(a*x)**2 + 3*a**2*
x**2*atan(a*x)**2 + atan(a*x)**2), x)/c**3
```

## Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 144, normalized size of antiderivative = 6.55

$$\int \frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)^2} \, dx = \int \frac{x^m}{(a^2cx^2+c)^3 \arctan{(ax)}^2} \, dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
((a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)*arctan(a*x)*integrate(((a^2*m - 4*a
^2)*x^2 + m)*x^m/((a^7*c^3*x^7 + 3*a^5*c^3*x^5 + 3*a^3*c^3*x^3 + a*c^3*x)*
arctan(a*x)), x) - x^m)/((a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)*arctan(a*x)
)
```

## Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)^2} \, dx = \int \frac{x^m}{(a^2cx^2+c)^3 \arctan{(ax)}^2} \, dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate(x^m/((a^2*c*x^2 + c)^3*arctan(a*x)^2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m}{(c+a^2cx^2)^3\arctan(ax)^2}\,dx = \int \frac{x^m}{\mathrm{atan}\left(a\,x\right)^2\,\left(c\,a^2\,x^2+c\right)^3}\,dx$$

input
```
int(x^m/(atan(a*x)^2*(c + a^2*c*x^2)^3),x)
```

output
```
int(x^m/(atan(a*x)^2*(c + a^2*c*x^2)^3), x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.73

$$\int \frac{x^m}{(c+a^2cx^2)^3\arctan(ax)^2}\,dx = \frac{\int \frac{x^m}{atan(ax)^2a^6x^6+3atan(ax)^2a^4x^4+3atan(ax)^2a^2x^2+atan(ax)^2}dx}{c^3}$$

input
```
int(x^m/(a^2*c*x^2+c)^3/atan(a*x)^2,x)
```

output
```
int(x**m/(atan(a*x)**2*a**6*x**6 + 3*atan(a*x)**2*a**4*x**4 + 3*atan(a*x)*
*2*a**2*x**2 + atan(a*x)**2),x)/c**3
```

# 3.603 $\int \frac{x^m(c+a^2cx^2)^{5/2}}{\arctan(ax)^2}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\arctan(ax)^2}\,dx = \text{Int}\left(\frac{x^m(c+a^2cx^2)^{5/2}}{\arctan(ax)^2},x\right)$$

output
```
Defer(Int)(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

## Mathematica [N/A]

Not integrable

Time = 1.65 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\arctan(ax)^2}\,dx = \int \frac{x^m(c+a^2cx^2)^{5/2}}{\arctan(ax)^2}\,dx$$

input
```
Integrate[(x^m*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^2,x]
```

output
```
Integrate[(x^m*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^2, x]
```

## Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m (a^2 c x^2 + c)^{5/2}}{\arctan(ax)^2} \, dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{x^m (a^2 c x^2 + c)^{5/2}}{\arctan(ax)^2} dx$$

input
```
Int[(x^m*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^2,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 13.62 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m (a^2 c \, x^2 + c)^{\frac{5}{2}}}{\arctan \left(a x\right)^2} dx$$

input
```
int(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

output
```
int(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.04

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}x^m}{\arctan{(ax)}^2}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*x^m/arcta
n(a*x)^2, x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\arctan(ax)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**m*(a**2*c*x**2+c)**(5/2)/atan(a*x)**2,x)
```

output
```
Timed out
```

## Maxima [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}x^m}{\arctan{(ax)}^2}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)*x^m/arctan(a*x)^2, x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\arctan(ax)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\arctan(ax)^2}\,dx = \int \frac{x^m\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}{{\mathrm{atan}\left(a\,x\right)}^2}\,dx$$

input
```
int((x^m*(c + a^2*c*x^2)^(5/2))/atan(a*x)^2,x)
```

output
```
int((x^m*(c + a^2*c*x^2)^(5/2))/atan(a*x)^2, x)
```

## Reduce [N/A]

Not integrable

Time = 15.10 (sec) , antiderivative size = 88, normalized size of antiderivative = 3.67

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\arctan(ax)^2}\,dx = \sqrt{c}\,c^2\left(\left(\int \frac{x^m\sqrt{a^2x^2+1}\,x^4}{atan\,(ax)^2}dx\right)a^4 + 2\left(\int \frac{x^m\sqrt{a^2x^2+1}\,x^2}{atan\,(ax)^2}dx\right)a^2 + \int \frac{x^m\sqrt{a^2x^2+1}}{atan\,(ax)^2}dx\right)$$

input
```
int(x^m*(a^2*c*x^2+c)^(5/2)/atan(a*x)^2,x)
```

output
```
sqrt(c)*c**2*(int((x**m*sqrt(a**2*x**2 + 1)*x**4)/atan(a*x)**2,x)*a**4 + 2
*int((x**m*sqrt(a**2*x**2 + 1)*x**2)/atan(a*x)**2,x)*a**2 + int((x**m*sqrt
(a**2*x**2 + 1))/atan(a*x)**2,x))
```

# 3.604 $\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)^2}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)^2}\,dx = \operatorname{Int}\left(\frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)^2},x\right)$$

output
```
Defer(Int)(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

## Mathematica [N/A]

Not integrable

Time = 0.87 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)^2}\,dx = \int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)^2}\,dx$$

input
```
Integrate[(x^m*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^2,x]
```

output
```
Integrate[(x^m*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^2, x]
```

## Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \left(a^2 c x^2+c\right)^{3/2}}{\arctan (a x)^2} \, dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x^m \left(a^2 c x^2+c\right)^{3/2}}{\arctan (a x)^2}dx$$

input
```
Int[(x^m*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^2,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 12.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^{m} \left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}{\arctan \left(a x \right)^{2}}d x$$

input
```
int(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

output
```
int(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}x^m}{\arctan(ax)^2}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^(3/2)*x^m/arctan(a*x)^2, x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**m*(a**2*c*x**2+c)**(3/2)/atan(a*x)**2,x)
```

output
```
Timed out
```

## Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)^2}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}x^m}{\arctan(ax)^2}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*x^m/arctan(a*x)^2, x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)^2}\,dx = \int \frac{x^m\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}{{\mathrm{atan}\left(a\,x\right)}^2}\,dx$$

input
```
int((x^m*(c + a^2*c*x^2)^(3/2))/atan(a*x)^2,x)
```

output
```
int((x^m*(c + a^2*c*x^2)^(3/2))/atan(a*x)^2, x)
```

## Reduce [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.33

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)^2}\,dx = \sqrt{c}\,c\left(\left(\int \frac{x^m\sqrt{a^2x^2+1}\,x^2}{atan\left(ax\right)^2}dx\right)a^2+\int \frac{x^m\sqrt{a^2x^2+1}}{atan\left(ax\right)^2}dx\right)$$

input
```
int(x^m*(a^2*c*x^2+c)^(3/2)/atan(a*x)^2,x)
```

output
```
sqrt(c)*c*(int((x**m*sqrt(a**2*x**2 + 1)*x**2)/atan(a*x)**2,x)*a**2 + int(
(x**m*sqrt(a**2*x**2 + 1))/atan(a*x)**2,x))
```

# 3.605 $\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^2}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^2}\,dx = \text{Int}\left(\frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^2},x\right)$$

output
```
Defer(Int)(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x)
```

## Mathematica [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^2}\,dx = \int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^2}\,dx$$

input
```
Integrate[(x^m*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^2,x]
```

output
```
Integrate[(x^m*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^2, x]
```

## Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m\sqrt{a^2cx^2+c}}{\arctan(ax)^2}\,dx$$

$$\Big\downarrow \text{5560}$$

$$\int \frac{x^m\sqrt{a^2cx^2+c}}{\arctan(ax)^2}dx$$

input
```
Int[(x^m*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^2,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 11.77 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m\sqrt{a^2c\,x^2+c}}{\arctan\left(ax\right)^2}dx$$

input
```
int(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x)
```

output
```
int(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^2}\,dx=\int \frac{\sqrt{a^2cx^2+c}x^m}{\arctan\left(ax\right)^2}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^m/arctan(a*x)^2, x)
```

## Sympy [N/A]

Not integrable

Time = 5.88 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^2}\,dx=\int \frac{x^m\sqrt{c\left(a^2x^2+1\right)}}{\operatorname{atan}^2\left(ax\right)}\,dx$$

input
```
integrate(x**m*(a**2*c*x**2+c)**(1/2)/atan(a*x)**2,x)
```

output
```
Integral(x**m*sqrt(c*(a**2*x**2 + 1))/atan(a*x)**2, x)
```

## Maxima [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^2}\,dx=\int \frac{\sqrt{a^2cx^2+c}x^m}{\arctan\left(ax\right)^2}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*x^m/arctan(a*x)^2, x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^2}\,dx = \int \frac{x^m\,\sqrt{c\,a^2\,x^2+c}}{\mathrm{atan}\left(a\,x\right)^2}\,dx$$

input
```
int((x^m*(c + a^2*c*x^2)^(1/2))/atan(a*x)^2,x)
```

output
```
int((x^m*(c + a^2*c*x^2)^(1/2))/atan(a*x)^2, x)
```

## Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^2}\,dx=\sqrt{c}\left(\int \frac{x^m\sqrt{a^2x^2+1}}{atan\,(ax)^2}dx\right)$$

input
```
int(x^m*(a^2*c*x^2+c)^(1/2)/atan(a*x)^2,x)
```

output
```
sqrt(c)*int((x**m*sqrt(a**2*x**2 + 1))/atan(a*x)**2,x)
```

# 3.606 $\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^2}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^2}\,dx = \text{Int}\left(\frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^2},x\right)$$

output
```
Defer(Int)(x^m/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x)
```

## Mathematica [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^2}\,dx = \int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^2}\,dx$$

input
```
Integrate[x^m/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2),x]
```

output
```
Integrate[x^m/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\arctan(ax)^2\sqrt{a^2cx^2+c}}dx$$
$$\Big\downarrow \ 5560$$
$$\int \frac{x^m}{\arctan(ax)^2\sqrt{a^2cx^2+c}}dx$$

input
```
Int[x^m/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 9.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{\sqrt{a^2c\,x^2+c}\,\arctan\left(ax\right)^2}dx$$

input
```
int(x^m/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x)
```

output
```
int(x^m/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^2}\,dx = \int \frac{x^m}{\sqrt{a^2cx^2+c}\arctan\left(ax\right)^2}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(x^m/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2), x)
```

## Sympy [N/A]

Not integrable

Time = 14.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^2}\,dx = \int \frac{x^m}{\sqrt{c\left(a^2x^2+1\right)}\operatorname{atan}^2\left(ax\right)}\,dx$$

input
```
integrate(x**m/(a**2*c*x**2+c)**(1/2)/atan(a*x)**2,x)
```

output
```
Integral(x**m/(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**2), x)
```

## Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^2}\,dx = \int \frac{x^m}{\sqrt{a^2cx^2+c}\arctan\left(ax\right)^2}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(x^m/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2), x)
```

## Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^2}\,dx = \int \frac{x^m}{\sqrt{a^2cx^2+c}\arctan\left(ax\right)^2}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate(x^m/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^2}\,dx = \int \frac{x^m}{\mathrm{atan}\left(a\,x\right)^2\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(x^m/(atan(a*x)^2*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(x^m/(atan(a*x)^2*(c + a^2*c*x^2)^(1/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^2}\,dx = \frac{\int \frac{x^m}{\sqrt{a^2x^2+1}\,atan(ax)^2}dx}{\sqrt{c}}$$

input
```
int(x^m/(a^2*c*x^2+c)^(1/2)/atan(a*x)^2,x)
```

output
```
int(x**m/(sqrt(a**2*x**2 + 1)*atan(a*x)**2),x)/sqrt(c)
```

# 3.607 $\int \frac{x^m}{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2}\,dx=\text{Int}\left(\frac{x^m}{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2},x\right)$$

output
```
Defer(Int)(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

## Mathematica [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2}\,dx=\int \frac{x^m}{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2}\,dx$$

input
```
Integrate[x^m/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2),x]
```

output
```
Integrate[x^m/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\arctan(ax)^2 \left(a^2cx^2+c\right)^{3/2}} \, dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x^m}{\arctan(ax)^2 \left(a^2cx^2+c\right)^{3/2}}dx$$

input
```
Int[x^m/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 13.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{\left(a^2c\,x^2+c\right)^{\frac{3}{2}} \arctan\left(ax\right)^2}dx$$

input
```
int(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

output
```
int(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.12

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx = \int \frac{x^m}{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)^2}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^m/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arct
an(a*x)^2), x)
```

## Sympy [N/A]

Not integrable

Time = 52.90 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx = \int \frac{x^m}{(c(a^2x^2+1))^{\frac{3}{2}}\operatorname{atan}^2(ax)}\,dx$$

input
```
integrate(x**m/(a**2*c*x**2+c)**(3/2)/atan(a*x)**2,x)
```

output
```
Integral(x**m/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**2), x)
```

## Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx = \int \frac{x^m}{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)^2}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(x^m/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^2), x)
```

## Giac [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2}\,dx=\int \frac{x^m}{\left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^2}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate(x^m/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.85 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2}\,dx=\int \frac{x^m}{\mathrm{atan}\left(a\,x\right)^2\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

input
```
int(x^m/(atan(a*x)^2*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(x^m/(atan(a*x)^2*(c + a^2*c*x^2)^(3/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.38

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^2}\,dx = \frac{\int \frac{x^m}{\sqrt{a^2x^2+1}\,atan(ax)^2a^2x^2+\sqrt{a^2x^2+1}\,atan(ax)^2}dx}{\sqrt{c}\,c}$$

input
```
int(x^m/(a^2*c*x^2+c)^(3/2)/atan(a*x)^2,x)
```

output
```
int(x**m/(sqrt(a**2*x**2 + 1)*atan(a*x)**2*a**2*x**2 + sqrt(a**2*x**2 + 1)
*atan(a*x)**2),x)/(sqrt(c)*c)
```

# 3.608 $\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = \text{Int}\left(\frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^2},x\right)$$

output
```
Defer(Int)(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

## Mathematica [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = \int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx$$

input
```
Integrate[x^m/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

output
```
Integrate[x^m/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\arctan(ax)^2 (a^2cx^2+c)^{5/2}} dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{x^m}{\arctan(ax)^2 (a^2cx^2+c)^{5/2}} dx$$

input
```
Int[x^m/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 13.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{(a^2c\,x^2+c)^{\frac{5}{2}} \arctan(ax)^2} dx$$

input
```
int(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

output
```
int(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.58

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2}\,dx=\int \frac{x^m}{\left(a^2cx^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^2}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^m/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3
*x^2 + c^3)*arctan(a*x)^2), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2}\,dx=\text{Timed out}$$

input
```
integrate(x**m/(a**2*c*x**2+c)**(5/2)/atan(a*x)**2,x)
```

output
```
Timed out
```

## Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^2}\,dx=\int \frac{x^m}{\left(a^2cx^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^2}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="maxima")
```

output
```
integrate(x^m/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^2), x)
```

## Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = \int \frac{x^m}{(a^2cx^2+c)^{\frac{5}{2}}\arctan{(ax)}^2}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^2,x, algorithm="giac")
```

output
```
integrate(x^m/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.89 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = \int \frac{x^m}{\mathrm{atan}\left(a\,x\right)^2\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

input
```
int(x^m/(atan(a*x)^2*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(x^m/(atan(a*x)^2*(c + a^2*c*x^2)^(5/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 81, normalized size of antiderivative = 3.38

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^2}\,dx = \frac{\int \frac{x^m}{\sqrt{a^2x^2+1}\,atan(ax)^2a^4x^4+2\sqrt{a^2x^2+1}\,atan(ax)^2a^2x^2+\sqrt{a^2x^2+1}\,atan(ax)^2}dx}{\sqrt{c}\,c^2}$$

input
```
int(x^m/(a^2*c*x^2+c)^(5/2)/atan(a*x)^2,x)
```

output
```
int(x**m/(sqrt(a**2*x**2 + 1)*atan(a*x)**2*a**4*x**4 + 2*sqrt(a**2*x**2 +
1)*atan(a*x)**2*a**2*x**2 + sqrt(a**2*x**2 + 1)*atan(a*x)**2),x)/(sqrt(c)*
c**2)
```

# 3.609 $\int \frac{x(c+a^2cx^2)}{\arctan(ax)^3}\,dx$



## Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{x(c+a^2cx^2)}{\arctan(ax)^3}\,dx = \mathrm{Int}\left(\frac{x(c+a^2cx^2)}{\arctan(ax)^3},x\right)$$

output
```
Defer(Int)(x*(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

## Mathematica [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x(c+a^2cx^2)}{\arctan(ax)^3}\,dx = \int \frac{x(c+a^2cx^2)}{\arctan(ax)^3}\,dx$$

input
```
Integrate[(x*(c + a^2*c*x^2))/ArcTan[a*x]^3,x]
```

output
```
Integrate[(x*(c + a^2*c*x^2))/ArcTan[a*x]^3, x]
```

## Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a^2cx^2+c)}{\arctan(ax)^3} dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x(a^2cx^2+c)}{\arctan(ax)^3} dx$$

input
```
Int[(x*(c + a^2*c*x^2))/ArcTan[a*x]^3,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 135.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{x(a^2c\,x^2+c)}{\arctan\left(ax\right)^3} dx$$

input
```
int(x*(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

output
```
int(x*(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.17

$$\int \frac{x(c+a^2cx^2)}{\arctan(ax)^3}\,dx=\int \frac{(a^2cx^2+c)x}{\arctan(ax)^3}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^3 + c*x)/arctan(a*x)^3, x)
```

## Sympy [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.44

$$\int \frac{x(c+a^2cx^2)}{\arctan(ax)^3}\,dx=c\left(\int \frac{x}{\operatorname{atan}^3(ax)}\,dx+\int \frac{a^2x^3}{\operatorname{atan}^3(ax)}\,dx\right)$$

input
```
integrate(x*(a**2*c*x**2+c)/atan(a*x)**3,x)
```

output
```
c*(Integral(x/atan(a*x)**3, x) + Integral(a**2*x**3/atan(a*x)**3, x))
```

## Maxima [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 110, normalized size of antiderivative = 6.11

$$\int \frac{x(c+a^2cx^2)}{\arctan(ax)^3}\,dx=\int \frac{(a^2cx^2+c)x}{\arctan(ax)^3}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
-1/2*(a^5*c*x^5 + 2*a^3*c*x^3 - 2*a^2*arctan(a*x)^2*integrate((15*a^4*c*x^
5 + 22*a^2*c*x^3 + 7*c*x)/arctan(a*x), x) + a*c*x + (5*a^6*c*x^6 + 11*a^4*
c*x^4 + 7*a^2*c*x^2 + c)*arctan(a*x))/(a^2*arctan(a*x)^2)
```

## Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x(c+a^2cx^2)}{\arctan(ax)^3}\,dx = \int \frac{(a^2cx^2+c)x}{\arctan{(ax)}^3}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)*x/arctan(a*x)^3, x)
```

## Mupad [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{x(c+a^2cx^2)}{\arctan(ax)^3}\,dx = \int \frac{x\,\left(c\,a^2\,x^2+c\right)}{{\mathrm{atan}\left(a\,x\right)}^3}\,dx$$

input
```
int((x*(c + a^2*c*x^2))/atan(a*x)^3,x)
```

output
```
int((x*(c + a^2*c*x^2))/atan(a*x)^3, x)
```

## Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.61

$$\int \frac{x(c+a^2cx^2)}{\arctan(ax)^3}\,dx = c\left(\left(\int \frac{x^3}{atan\,(ax)^3}dx\right)a^2+\int \frac{x}{atan\,(ax)^3}dx\right)$$

input
```
int(x*(a^2*c*x^2+c)/atan(a*x)^3,x)
```

output
```
c*(int(x**3/atan(a*x)**3,x)*a**2 + int(x/atan(a*x)**3,x))
```

# 3.610 $\int \frac{c+a^2cx^2}{\arctan(ax)^3}\,dx$



## Optimal result

Integrand size = 17, antiderivative size = 17

$$\int \frac{c+a^2cx^2}{\arctan(ax)^3}\,dx=\mathrm{Int}\left(\frac{c+a^2cx^2}{\arctan(ax)^3},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)/arctan(a*x)^3,x)
```

## Mathematica [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{c+a^2cx^2}{\arctan(ax)^3}\,dx=\int \frac{c+a^2cx^2}{\arctan(ax)^3}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)/ArcTan[a*x]^3,x]
```

output
```
Integrate[(c + a^2*c*x^2)/ArcTan[a*x]^3, x]
```

## Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2cx^2+c}{\arctan(ax)^3}\,dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{a^2cx^2+c}{\arctan(ax)^3}dx$$

input
```
Int[(c + a^2*c*x^2)/ArcTan[a*x]^3,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 62.57 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{a^2c\,x^2+c}{\arctan\left(ax\right)^3}dx$$

input
```
int((a^2*c*x^2+c)/arctan(a*x)^3,x)
```

output
```
int((a^2*c*x^2+c)/arctan(a*x)^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{c+a^2cx^2}{\arctan(ax)^3}\,dx = \int \frac{a^2cx^2+c}{\arctan{(ax)}^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)/arctan(a*x)^3, x)
```

## Sympy [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.53

$$\int \frac{c+a^2cx^2}{\arctan(ax)^3}\,dx = c\left(\int \frac{a^2x^2}{\operatorname{atan}^3{(ax)}}\,dx + \int \frac{1}{\operatorname{atan}^3{(ax)}}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)/atan(a*x)**3,x)
```

output
```
c*(Integral(a**2*x**2/atan(a*x)**3, x) + Integral(atan(a*x)**(-3), x))
```

## Maxima [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 97, normalized size of antiderivative = 5.71

$$\int \frac{c+a^2cx^2}{\arctan(ax)^3}\,dx = \int \frac{a^2cx^2+c}{\arctan{(ax)}^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
-1/2*(a^4*c*x^4 + 2*a^2*c*x^2 - 2*a*arctan(a*x)^2*integrate(2*(5*a^4*c*x^4
 + 6*a^2*c*x^2 + c)/arctan(a*x), x) + 4*(a^5*c*x^5 + 2*a^3*c*x^3 + a*c*x)*
arctan(a*x) + c)/(a*arctan(a*x)^2)
```

## Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{c+a^2cx^2}{\arctan(ax)^3}\,dx = \int \frac{a^2cx^2+c}{\arctan{(ax)}^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)/arctan(a*x)^3, x)
```

## Mupad [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.12

$$\int \frac{c+a^2cx^2}{\arctan(ax)^3}\,dx = \int \frac{c\,a^2\,x^2+c}{\mathrm{atan}\left(a\,x\right)^3}\,dx$$

input
```
int((c + a^2*c*x^2)/atan(a*x)^3,x)
```

output
```
int((c + a^2*c*x^2)/atan(a*x)^3, x)
```

## Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

$$\int \frac{c+a^2cx^2}{\arctan(ax)^3}\,dx = c\left(\left(\int \frac{x^2}{atan\,(ax)^3}dx\right)a^2 + \int \frac{1}{atan\,(ax)^3}dx\right)$$

input
```
int((a^2*c*x^2+c)/atan(a*x)^3,x)
```

output
```
c*(int(x**2/atan(a*x)**3,x)*a**2 + int(1/atan(a*x)**3,x))
```

# 3.611 $\int \frac{c+a^2cx^2}{x\arctan(ax)^3}\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{c+a^2cx^2}{x\arctan(ax)^3}\,dx = \operatorname{Int}\left(\frac{c+a^2cx^2}{x\arctan(ax)^3},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)/x/arctan(a*x)^3,x)
```

## Mathematica [N/A]

Not integrable

Time = 1.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{c+a^2cx^2}{x\arctan(ax)^3}\,dx = \int \frac{c+a^2cx^2}{x\arctan(ax)^3}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)/(x*ArcTan[a*x]^3),x]
```

output
```
Integrate[(c + a^2*c*x^2)/(x*ArcTan[a*x]^3), x]
```

## Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2cx^2+c}{x\arctan(ax)^3}\,dx$$

$$\Big\downarrow \text{5560}$$

$$\int \frac{a^2cx^2+c}{x\arctan(ax)^3}dx$$

input
```
Int[(c + a^2*c*x^2)/(x*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 177.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{a^2c\,x^2+c}{x\arctan\left(ax\right)^3}dx$$

input
```
int((a^2*c*x^2+c)/x/arctan(a*x)^3,x)
```

output
```
int((a^2*c*x^2+c)/x/arctan(a*x)^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{c+a^2cx^2}{x\arctan(ax)^3}\,dx=\int \frac{a^2cx^2+c}{x\arctan{(ax)}^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)/x/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)/(x*arctan(a*x)^3), x)
```

## Sympy [N/A]

Not integrable

Time = 0.92 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30

$$\int \frac{c+a^2cx^2}{x\arctan(ax)^3}\,dx=c\left(\int \frac{1}{x\,\mathrm{atan}^3{(ax)}}\,dx+\int \frac{a^2x}{\mathrm{atan}^3{(ax)}}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)/x/atan(a*x)**3,x)
```

output
```
c*(Integral(1/(x*atan(a*x)**3), x) + Integral(a**2*x/atan(a*x)**3, x))
```

## Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 114, normalized size of antiderivative = 5.70

$$\int \frac{c+a^2cx^2}{x\arctan(ax)^3}\,dx=\int \frac{a^2cx^2+c}{x\arctan{(ax)}^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)/x/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
-1/2*(a^5*c*x^5 + 2*a^3*c*x^3 - 2*x^2*arctan(a*x)^2*integrate((6*a^6*c*x^6
 + 5*a^4*c*x^4 + c)/(x^3*arctan(a*x)), x) + a*c*x + (3*a^6*c*x^6 + 5*a^4*c
*x^4 + a^2*c*x^2 - c)*arctan(a*x))/(a^2*x^2*arctan(a*x)^2)
```

## Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{c+a^2cx^2}{x\arctan(ax)^3}\,dx = \int \frac{a^2cx^2+c}{x\arctan{(ax)}^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)/x/arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)/(x*arctan(a*x)^3), x)
```

## Mupad [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{c+a^2cx^2}{x\arctan(ax)^3}\,dx = \int \frac{c\,a^2\,x^2+c}{x\,\mathrm{atan}\left(a\,x\right)^3}\,dx$$

input
```
int((c + a^2*c*x^2)/(x*atan(a*x)^3),x)
```

output
```
int((c + a^2*c*x^2)/(x*atan(a*x)^3), x)
```

## Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.45

$$\int \frac{c+a^2cx^2}{x\arctan(ax)^3}\,dx = c\left(\left(\int \frac{x}{atan\left(ax\right)^3}dx\right)a^2+\int \frac{1}{atan\left(ax\right)^3 x}dx\right)$$

input
```
int((a^2*c*x^2+c)/x/atan(a*x)^3,x)
```

output
```
c*(int(x/atan(a*x)**3,x)*a**2 + int(1/(atan(a*x)**3*x),x))
```

# 3.612 $\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^3}\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^3}\,dx = \operatorname{Int}\left(\frac{x(c+a^2cx^2)^2}{\arctan(ax)^3},x\right)$$

output
```
Defer(Int)(x*(a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

## Mathematica [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^3}\,dx = \int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^3}\,dx$$

input
```
Integrate[(x*(c + a^2*c*x^2)^2)/ArcTan[a*x]^3,x]
```

output
```
Integrate[(x*(c + a^2*c*x^2)^2)/ArcTan[a*x]^3, x]
```

## Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a^2cx^2+c\right)^2}{\arctan(ax)^3}dx$$
$$\Big\downarrow \ 5560$$
$$\int \frac{x\left(a^2cx^2+c\right)^2}{\arctan(ax)^3}dx$$

input
```
Int[(x*(c + a^2*c*x^2)^2)/ArcTan[a*x]^3,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 166.87 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{x(a^2c\,x^2+c)^2}{\arctan\left(ax\right)^3}dx$$

input
```
int(x*(a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

output
```
int(x*(a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

$$\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^3}\,dx = \int \frac{(a^2cx^2+c)^2x}{\arctan(ax)^3}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^2*x)/arctan(a*x)^3, x)
```

## Sympy [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.20

$$\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^3}\,dx = c^2\left(\int \frac{x}{\operatorname{atan}^3(ax)}\,dx + \int \frac{2a^2x^3}{\operatorname{atan}^3(ax)}\,dx + \int \frac{a^4x^5}{\operatorname{atan}^3(ax)}\,dx\right)$$

input
```
integrate(x*(a**2*c*x**2+c)**2/atan(a*x)**3,x)
```

output
```
c**2*(Integral(x/atan(a*x)**3, x) + Integral(2*a**2*x**3/atan(a*x)**3, x)
+ Integral(a**4*x**5/atan(a*x)**3, x))
```

## Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 164, normalized size of antiderivative = 8.20

$$\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^3}\,dx = \int \frac{(a^2cx^2+c)^2x}{\arctan(ax)^3}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
-1/2*(a^7*c^2*x^7 + 3*a^5*c^2*x^5 + 3*a^3*c^2*x^3 - 2*a^2*arctan(a*x)^2*in
tegrate(2*(14*a^6*c^2*x^7 + 33*a^4*c^2*x^5 + 24*a^2*c^2*x^3 + 5*c^2*x)/arc
tan(a*x), x) + a*c^2*x + (7*a^8*c^2*x^8 + 22*a^6*c^2*x^6 + 24*a^4*c^2*x^4
+ 10*a^2*c^2*x^2 + c^2)*arctan(a*x))/(a^2*arctan(a*x)^2)
```

## Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^3}\,dx = \int \frac{(a^2cx^2+c)^2x}{\arctan{(ax)}^3}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^2*x/arctan(a*x)^3, x)
```

## Mupad [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^3}\,dx = \int \frac{x\,(c\,a^2\,x^2+c)^2}{\mathrm{atan}\left(a\,x\right)^3}\,dx$$

input
```
int((x*(c + a^2*c*x^2)^2)/atan(a*x)^3,x)
```

output
```
int((x*(c + a^2*c*x^2)^2)/atan(a*x)^3, x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.40

$$\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^3}\,dx = c^2\left(\left(\int \frac{x^5}{atan\,(ax)^3}dx\right)a^4 + 2\left(\int \frac{x^3}{atan\,(ax)^3}dx\right)a^2 + \int \frac{x}{atan\,(ax)^3}dx\right)$$

input
```
int(x*(a^2*c*x^2+c)^2/atan(a*x)^3,x)
```

output
```
c**2*(int(x**5/atan(a*x)**3,x)*a**4 + 2*int(x**3/atan(a*x)**3,x)*a**2 + in
t(x/atan(a*x)**3,x))
```

# 3.613 $\int \frac{(c+a^2cx^2)^2}{\arctan(ax)^3}\,dx$



## Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \frac{(c+a^2cx^2)^2}{\arctan(ax)^3}\,dx = \text{Int}\left(\frac{(c+a^2cx^2)^2}{\arctan(ax)^3},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

## Mathematica [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{(c+a^2cx^2)^2}{\arctan(ax)^3}\,dx = \int \frac{(c+a^2cx^2)^2}{\arctan(ax)^3}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^2/ArcTan[a*x]^3,x]
```

output
```
Integrate[(c + a^2*c*x^2)^2/ArcTan[a*x]^3, x]
```

## Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2cx^2+c\right)^2}{\arctan(ax)^3}\,dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{\left(a^2cx^2+c\right)^2}{\arctan(ax)^3}dx$$

input
```
Int[(c + a^2*c*x^2)^2/ArcTan[a*x]^3,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 178.30 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{\left(a^{2} c \,x^{2}+c \right)^{2}}{\arctan \left(a x \right)^{3}}d x$$

input
```
int((a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

output
```
int((a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.79

$$\int \frac{(c + a^2cx^2)^2}{\arctan(ax)^3} \, dx = \int \frac{(a^2cx^2 + c)^2}{\arctan(ax)^3} \, dx$$

input
```
integrate((a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)/arctan(a*x)^3, x)
```

## Sympy [N/A]

Not integrable

Time = 0.86 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.32

$$\int \frac{(c + a^2cx^2)^2}{\arctan(ax)^3} \, dx = c^2 \left( \int \frac{2a^2x^2}{\operatorname{atan}^3(ax)} \, dx + \int \frac{a^4x^4}{\operatorname{atan}^3(ax)} \, dx + \int \frac{1}{\operatorname{atan}^3(ax)} \, dx \right)$$

input
```
integrate((a**2*c*x**2+c)**2/atan(a*x)**3,x)
```

output
```
c**2*(Integral(2*a**2*x**2/atan(a*x)**3, x) + Integral(a**4*x**4/atan(a*x)
**3, x) + Integral(atan(a*x)**(-3), x))
```

## Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 148, normalized size of antiderivative = 7.79

$$\int \frac{(c + a^2cx^2)^2}{\arctan(ax)^3} \, dx = \int \frac{(a^2cx^2 + c)^2}{\arctan(ax)^3} \, dx$$

input
```
integrate((a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
-1/2*(a^6*c^2*x^6 + 3*a^4*c^2*x^4 + 3*a^2*c^2*x^2 - 2*a*arctan(a*x)^2*inte
grate(3*(7*a^6*c^2*x^6 + 15*a^4*c^2*x^4 + 9*a^2*c^2*x^2 + c^2)/arctan(a*x)
, x) + c^2 + 6*(a^7*c^2*x^7 + 3*a^5*c^2*x^5 + 3*a^3*c^2*x^3 + a*c^2*x)*arc
tan(a*x))/(a*arctan(a*x)^2)
```

## Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{(c+a^2cx^2)^2}{\arctan(ax)^3}\,dx=\int \frac{(a^2cx^2+c)^2}{\arctan{(ax)}^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^2/arctan(a*x)^3, x)
```

## Mupad [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{(c+a^2cx^2)^2}{\arctan(ax)^3}\,dx=\int \frac{(c\,a^2\,x^2+c)^2}{\mathrm{atan}\left(a\,x\right)^3}\,dx$$

input
```
int((c + a^2*c*x^2)^2/atan(a*x)^3,x)
```

output
```
int((c + a^2*c*x^2)^2/atan(a*x)^3, x)
```

## Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.42

$$\int \frac{(c+a^2cx^2)^2}{\arctan(ax)^3}\,dx = c^2\left(\left(\int \frac{x^4}{atan\,(ax)^3}dx\right)a^4 + 2\left(\int \frac{x^2}{atan\,(ax)^3}dx\right)a^2 + \int \frac{1}{atan\,(ax)^3}dx\right)$$

input
```
int((a^2*c*x^2+c)^2/atan(a*x)^3,x)
```

output
```
c**2*(int(x**4/atan(a*x)**3,x)*a**4 + 2*int(x**2/atan(a*x)**3,x)*a**2 + in
t(1/atan(a*x)**3,x))
```

# 3.614 $\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)^3}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)^3}\,dx = \text{Int}\left(\frac{(c+a^2cx^2)^2}{x\arctan(ax)^3},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^2/x/arctan(a*x)^3,x)
```

## Mathematica [N/A]

Not integrable

Time = 0.86 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)^3}\,dx = \int \frac{(c+a^2cx^2)^2}{x\arctan(ax)^3}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^2/(x*ArcTan[a*x]^3),x]
```

output
```
Integrate[(c + a^2*c*x^2)^2/(x*ArcTan[a*x]^3), x]
```

## Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^2}{x\arctan(ax)^3}\,dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int \frac{(a^2cx^2+c)^2}{x\arctan(ax)^3}dx$$

input
```
Int[(c + a^2*c*x^2)^2/(x*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 51.72 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a^2c\,x^2+c)^2}{x\arctan\left(ax\right)^3}dx$$

input
```
int((a^2*c*x^2+c)^2/x/arctan(a*x)^3,x)
```

output
```
int((a^2*c*x^2+c)^2/x/arctan(a*x)^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

$$\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)^3}\,dx = \int \frac{(a^2cx^2+c)^2}{x\arctan(ax)^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2/x/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)/(x*arctan(a*x)^3), x)
```

## Sympy [N/A]

Not integrable

Time = 1.68 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.00

$$\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)^3}\,dx = c^2\left(\int \frac{1}{x\,\mathrm{atan}^3(ax)}\,dx + \int \frac{2a^2x}{\mathrm{atan}^3(ax)}\,dx + \int \frac{a^4x^3}{\mathrm{atan}^3(ax)}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**2/x/atan(a*x)**3,x)
```

output
```
c**2*(Integral(1/(x*atan(a*x)**3), x) + Integral(2*a**2*x/atan(a*x)**3, x)
 + Integral(a**4*x**3/atan(a*x)**3, x))
```

## Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 168, normalized size of antiderivative = 7.64

$$\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)^3}\,dx = \int \frac{(a^2cx^2+c)^2}{x\arctan(ax)^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2/x/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
-1/2*(a^7*c^2*x^7 + 3*a^5*c^2*x^5 + 3*a^3*c^2*x^3 - 2*x^2*arctan(a*x)^2*in
tegrate((15*a^8*c^2*x^8 + 28*a^6*c^2*x^6 + 12*a^4*c^2*x^4 + c^2)/(x^3*arct
an(a*x)), x) + a*c^2*x + (5*a^8*c^2*x^8 + 14*a^6*c^2*x^6 + 12*a^4*c^2*x^4
+ 2*a^2*c^2*x^2 - c^2)*arctan(a*x))/(a^2*x^2*arctan(a*x)^2)
```

## Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(c + a^2cx^2)^2}{x \arctan(ax)^3} \, dx = \int \frac{(a^2cx^2 + c)^2}{x \arctan(ax)^3} \, dx$$

input
```
integrate((a^2*c*x^2+c)^2/x/arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^2/(x*arctan(a*x)^3), x)
```

## Mupad [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(c + a^2cx^2)^2}{x \arctan(ax)^3} \, dx = \int \frac{(c\,a^2\,x^2 + c)^2}{x\,\mathrm{atan}(a\,x)^3} \, dx$$

input
```
int((c + a^2*c*x^2)^2/(x*atan(a*x)^3),x)
```

output
```
int((c + a^2*c*x^2)^2/(x*atan(a*x)^3), x)
```

## Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.18

$$\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)^3}\,dx = c^2\left(\left(\int \frac{x^3}{atan(ax)^3}dx\right)a^4 + 2\left(\int \frac{x}{atan(ax)^3}dx\right)a^2 + \int \frac{1}{atan(ax)^3\,x}dx\right)$$

input
```
int((a^2*c*x^2+c)^2/x/atan(a*x)^3,x)
```

output
```
c**2*(int(x**3/atan(a*x)**3,x)*a**4 + 2*int(x/atan(a*x)**3,x)*a**2 + int(1
/(atan(a*x)**3*x),x))
```

# 3.615 $\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)^3} dx$



## Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)^3} dx = \text{Int}\left(\frac{x(c+a^2cx^2)^3}{\arctan(ax)^3}, x\right)$$

output
```
Defer(Int)(x*(a^2*c*x^2+c)^3/arctan(a*x)^3,x)
```

## Mathematica [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)^3} dx = \int \frac{x(c+a^2cx^2)^3}{\arctan(ax)^3} dx$$

input
```
Integrate[(x*(c + a^2*c*x^2)^3)/ArcTan[a*x]^3,x]
```

output
```
Integrate[(x*(c + a^2*c*x^2)^3)/ArcTan[a*x]^3, x]
```

## Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a^2cx^2+c)^3}{\arctan(ax)^3}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{x(a^2cx^2+c)^3}{\arctan(ax)^3}dx$$

input
```
Int[(x*(c + a^2*c*x^2)^3)/ArcTan[a*x]^3,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 15.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{x(a^2c\,x^2+c)^3}{\arctan(ax)^3}dx$$

input
```
int(x*(a^2*c*x^2+c)^3/arctan(a*x)^3,x)
```

output
```
int(x*(a^2*c*x^2+c)^3/arctan(a*x)^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.35

$$\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)^3}\,dx = \int \frac{(a^2cx^2+c)^3x}{\arctan(ax)^3}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^6*c^3*x^7 + 3*a^4*c^3*x^5 + 3*a^2*c^3*x^3 + c^3*x)/arctan(a*x)
^3, x)
```

## Sympy [N/A]

Not integrable

Time = 1.23 (sec) , antiderivative size = 61, normalized size of antiderivative = 3.05

$$\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)^3}\,dx = c^3\left(\int \frac{x}{\operatorname{atan}^3(ax)}\,dx + \int \frac{3a^2x^3}{\operatorname{atan}^3(ax)}\,dx + \int \frac{3a^4x^5}{\operatorname{atan}^3(ax)}\,dx + \int \frac{a^6x^7}{\operatorname{atan}^3(ax)}\,dx\right)$$

input
```
integrate(x*(a**2*c*x**2+c)**3/atan(a*x)**3,x)
```

output
```
c**3*(Integral(x/atan(a*x)**3, x) + Integral(3*a**2*x**3/atan(a*x)**3, x)
+ Integral(3*a**4*x**5/atan(a*x)**3, x) + Integral(a**6*x**7/atan(a*x)**3,
 x))
```

## Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 196, normalized size of antiderivative = 9.80

$$\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)^3}\,dx = \int \frac{(a^2cx^2+c)^3x}{\arctan{(ax)}^3}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
-1/2*(a^9*c^3*x^9 + 4*a^7*c^3*x^7 + 6*a^5*c^3*x^5 + 4*a^3*c^3*x^3 + a*c^3*
x - 2*a^2*arctan(a*x)^2*integrate((45*a^8*c^3*x^9 + 148*a^6*c^3*x^7 + 174*
a^4*c^3*x^5 + 84*a^2*c^3*x^3 + 13*c^3*x)/arctan(a*x), x) + (9*a^10*c^3*x^1
0 + 37*a^8*c^3*x^8 + 58*a^6*c^3*x^6 + 42*a^4*c^3*x^4 + 13*a^2*c^3*x^2 + c^
3)*arctan(a*x))/(a^2*arctan(a*x)^2)
```

## Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)^3}\,dx = \int \frac{(a^2cx^2+c)^3x}{\arctan{(ax)}^3}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^3*x/arctan(a*x)^3, x)
```

## Mupad [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)^3}\,dx = \int \frac{x\,\left(c\,a^2\,x^2+c\right)^3}{\mathrm{atan}\left(a\,x\right)^3}\,dx$$

input
```
int((x*(c + a^2*c*x^2)^3)/atan(a*x)^3,x)
```

output
```
int((x*(c + a^2*c*x^2)^3)/atan(a*x)^3, x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 65, normalized size of antiderivative = 3.25

$$\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)^3}\,dx = c^3\left(\left(\int \frac{x^7}{atan\left(ax\right)^3}dx\right)a^6+3\left(\int \frac{x^5}{atan\left(ax\right)^3}dx\right)a^4 + 3\left(\int \frac{x^3}{atan\left(ax\right)^3}dx\right)a^2+\int \frac{x}{atan\left(ax\right)^3}dx\right)$$

input
```
int(x*(a^2*c*x^2+c)^3/atan(a*x)^3,x)
```

output
```
c**3*(int(x**7/atan(a*x)**3,x)*a**6 + 3*int(x**5/atan(a*x)**3,x)*a**4 + 3*
int(x**3/atan(a*x)**3,x)*a**2 + int(x/atan(a*x)**3,x))
```

# 3.616 $\int \frac{(c+a^2cx^2)^3}{\arctan(ax)^3}\,dx$



## Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \frac{(c+a^2cx^2)^3}{\arctan(ax)^3}\,dx = \text{Int}\left(\frac{(c+a^2cx^2)^3}{\arctan(ax)^3},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^3/arctan(a*x)^3,x)
```

## Mathematica [N/A]

Not integrable

Time = 0.84 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{(c+a^2cx^2)^3}{\arctan(ax)^3}\,dx = \int \frac{(c+a^2cx^2)^3}{\arctan(ax)^3}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^3/ArcTan[a*x]^3,x]
```

output
```
Integrate[(c + a^2*c*x^2)^3/ArcTan[a*x]^3, x]
```

## Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^3}{\arctan(ax)^3}dx$$

$$\Big\downarrow \text{5560}$$

$$\int \frac{(a^2cx^2+c)^3}{\arctan(ax)^3}dx$$

input
```
Int[(c + a^2*c*x^2)^3/ArcTan[a*x]^3,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 1.68 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{(a^2c\,x^2+c)^3}{\arctan(ax)^3}dx$$

input
```
int((a^2*c*x^2+c)^3/arctan(a*x)^3,x)
```

output
```
int((a^2*c*x^2+c)^3/arctan(a*x)^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.37

$$\int \frac{(c+a^2cx^2)^3}{\arctan(ax)^3}\,dx = \int \frac{(a^2cx^2+c)^3}{\arctan(ax)^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)/arctan(a*x)^3
, x)
```

## Sympy [N/A]

Not integrable

Time = 1.15 (sec) , antiderivative size = 61, normalized size of antiderivative = 3.21

$$\int \frac{(c+a^2cx^2)^3}{\arctan(ax)^3}\,dx = c^3\left(\int \frac{3a^2x^2}{\operatorname{atan}^3(ax)}\,dx + \int \frac{3a^4x^4}{\operatorname{atan}^3(ax)}\,dx + \int \frac{a^6x^6}{\operatorname{atan}^3(ax)}\,dx + \int \frac{1}{\operatorname{atan}^3(ax)}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**3/atan(a*x)**3,x)
```

output
```
c**3*(Integral(3*a**2*x**2/atan(a*x)**3, x) + Integral(3*a**4*x**4/atan(a*
x)**3, x) + Integral(a**6*x**6/atan(a*x)**3, x) + Integral(atan(a*x)**(-3)
, x))
```

## Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 181, normalized size of antiderivative = 9.53

$$\int \frac{(c+a^2cx^2)^3}{\arctan(ax)^3}\,dx = \int \frac{(a^2cx^2+c)^3}{\arctan(ax)^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
-1/2*(a^8*c^3*x^8 + 4*a^6*c^3*x^6 + 6*a^4*c^3*x^4 + 4*a^2*c^3*x^2 - 2*a*ar
ctan(a*x)^2*integrate(4*(9*a^8*c^3*x^8 + 28*a^6*c^3*x^6 + 30*a^4*c^3*x^4 +
 12*a^2*c^3*x^2 + c^3)/arctan(a*x), x) + c^3 + 8*(a^9*c^3*x^9 + 4*a^7*c^3*
x^7 + 6*a^5*c^3*x^5 + 4*a^3*c^3*x^3 + a*c^3*x)*arctan(a*x))/(a*arctan(a*x)
^2)
```

## Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{(c+a^2cx^2)^3}{\arctan(ax)^3}\,dx = \int \frac{(a^2cx^2+c)^3}{\arctan(ax)^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^3/arctan(a*x)^3, x)
```

## Mupad [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{(c+a^2cx^2)^3}{\arctan(ax)^3}\,dx = \int \frac{{\left(c\,a^2\,x^2+c\right)}^3}{{\mathrm{atan}\left(a\,x\right)}^3}\,dx$$

input
```
int((c + a^2*c*x^2)^3/atan(a*x)^3,x)
```

output
```
int((c + a^2*c*x^2)^3/atan(a*x)^3, x)
```

## Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 63, normalized size of antiderivative = 3.32

$$\int \frac{(c+a^2cx^2)^3}{\arctan(ax)^3}\,dx = c^3\left(\left(\int \frac{x^6}{atan\left(ax\right)^3}dx\right)a^6+3\left(\int \frac{x^4}{atan\left(ax\right)^3}dx\right)a^4 +3\left(\int \frac{x^2}{atan\left(ax\right)^3}dx\right)a^2+\int \frac{1}{atan\left(ax\right)^3}dx\right)$$

input
```
int((a^2*c*x^2+c)^3/atan(a*x)^3,x)
```

output
```
c**3*(int(x**6/atan(a*x)**3,x)*a**6 + 3*int(x**4/atan(a*x)**3,x)*a**4 + 3*
int(x**2/atan(a*x)**3,x)*a**2 + int(1/atan(a*x)**3,x))
```

# 3.617 $\int \frac{\left(c+a^2cx^2\right)^3}{x\arctan(ax)^3}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\left(c+a^2cx^2\right)^3}{x\arctan(ax)^3}\,dx=\text{Int}\left(\frac{\left(c+a^2cx^2\right)^3}{x\arctan(ax)^3},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^3/x/arctan(a*x)^3,x)
```

## Mathematica [N/A]

Not integrable

Time = 0.86 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\left(c+a^2cx^2\right)^3}{x\arctan(ax)^3}\,dx=\int \frac{\left(c+a^2cx^2\right)^3}{x\arctan(ax)^3}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^3/(x*ArcTan[a*x]^3),x]
```

output
```
Integrate[(c + a^2*c*x^2)^3/(x*ArcTan[a*x]^3), x]
```

## Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^3}{x\arctan(ax)^3}dx$$

$$\Big\downarrow \text{5560}$$

$$\int \frac{(a^2cx^2+c)^3}{x\arctan(ax)^3}dx$$

input
```
Int[(c + a^2*c*x^2)^3/(x*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 30.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(a^2c\,x^2+c)^3}{x\arctan\left(ax\right)^3}dx$$

input
```
int((a^2*c*x^2+c)^3/x/arctan(a*x)^3,x)
```

output
```
int((a^2*c*x^2+c)^3/x/arctan(a*x)^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.18

$$\int \frac{(c+a^2cx^2)^3}{x\arctan(ax)^3}\,dx = \int \frac{(a^2cx^2+c)^3}{x\arctan(ax)^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3/x/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)/(x*arctan(a*x
)^3), x)
```

## Sympy [N/A]

Not integrable

Time = 2.21 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.77

$$\int \frac{(c+a^2cx^2)^3}{x\arctan(ax)^3}\,dx = c^3\left(\int \frac{1}{x\operatorname{atan}^3(ax)}\,dx + \int \frac{3a^2x}{\operatorname{atan}^3(ax)}\,dx + \int \frac{3a^4x^3}{\operatorname{atan}^3(ax)}\,dx + \int \frac{a^6x^5}{\operatorname{atan}^3(ax)}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**3/x/atan(a*x)**3,x)
```

output
```
c**3*(Integral(1/(x*atan(a*x)**3), x) + Integral(3*a**2*x/atan(a*x)**3, x)
 + Integral(3*a**4*x**3/atan(a*x)**3, x) + Integral(a**6*x**5/atan(a*x)**3
, x))
```

## Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 201, normalized size of antiderivative = 9.14

$$\int \frac{\left(c+a^2cx^2\right)^3}{x\arctan(ax)^3}\,dx=\int \frac{\left(a^2cx^2+c\right)^3}{x\arctan\left(ax\right)^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3/x/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
-1/2*(a^9*c^3*x^9 + 4*a^7*c^3*x^7 + 6*a^5*c^3*x^5 + 4*a^3*c^3*x^3 + a*c^3*
x - 2*x^2*arctan(a*x)^2*integrate((28*a^10*c^3*x^10 + 81*a^8*c^3*x^8 + 76*
a^6*c^3*x^6 + 22*a^4*c^3*x^4 + c^3)/(x^3*arctan(a*x)), x) + (7*a^10*c^3*x^
10 + 27*a^8*c^3*x^8 + 38*a^6*c^3*x^6 + 22*a^4*c^3*x^4 + 3*a^2*c^3*x^2 - c^
3)*arctan(a*x))/(a^2*x^2*arctan(a*x)^2)
```

## Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\left(c+a^2cx^2\right)^3}{x\arctan(ax)^3}\,dx=\int \frac{\left(a^2cx^2+c\right)^3}{x\arctan\left(ax\right)^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3/x/arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^3/(x*arctan(a*x)^3), x)
```

## Mupad [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(c+a^2cx^2)^3}{x\arctan(ax)^3}\,dx=\int \frac{{\left(c\,a^2\,x^2+c\right)}^3}{x\,{\mathrm{atan}\left(a\,x\right)}^3} \,d x$$

input
```
int((c + a^2*c*x^2)^3/(x*atan(a*x)^3),x)
```

output
```
int((c + a^2*c*x^2)^3/(x*atan(a*x)^3), x)
```

## Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.95

$$\int \frac{(c+a^2cx^2)^3}{x\arctan(ax)^3}\,dx=c^3\left(\left(\int \frac{x^5}{atan(ax)^3}dx\right)a^6+3\left(\int \frac{x^3}{atan(ax)^3}dx\right)a^4+3\left(\int \frac{x}{atan(ax)^3}dx\right)a^2+\int \frac{1}{atan(ax)^3x}dx\right)$$

input
```
int((a^2*c*x^2+c)^3/x/atan(a*x)^3,x)
```

output
```
c**3*(int(x**5/atan(a*x)**3,x)*a**6 + 3*int(x**3/atan(a*x)**3,x)*a**4 + 3*
int(x/atan(a*x)**3,x)*a**2 + int(1/(atan(a*x)**3*x),x))
```

# 3.618 $\int \frac{x^3}{(c+a^2cx^2)\arctan(ax)^3}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^3}{(c+a^2cx^2)\arctan(ax)^3}\,dx = -\frac{x^3}{2ac\arctan(ax)^2} + \frac{3\mathrm{Int}\left(\frac{x^2}{\arctan(ax)^2},x\right)}{2ac}$$

output
```
-1/2*x^3/a/c/arctan(a*x)^2+3/2*Defer(Int)(x^2/arctan(a*x)^2,x)/a/c
```

## Mathematica [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^3}{(c+a^2cx^2)\arctan(ax)^3}\,dx = \int \frac{x^3}{(c+a^2cx^2)\arctan(ax)^3}\,dx$$

input
```
Integrate[x^3/((c + a^2*c*x^2)*ArcTan[a*x]^3),x]
```

output
```
Integrate[x^3/((c + a^2*c*x^2)*ArcTan[a*x]^3), x]
```

## Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\arctan(ax)^3 (a^2cx^2+c)}dx$$
$$\Big\downarrow \text{ 5461}$$
$$\frac{3\int \frac{x^2}{\arctan(ax)^2}dx}{2ac}-\frac{x^3}{2ac\arctan(ax)^2}$$
$$\Big\downarrow \text{ 5377}$$
$$\frac{3\int \frac{x^2}{\arctan(ax)^2}dx}{2ac}-\frac{x^3}{2ac\arctan(ax)^2}$$

input
```
Int[x^3/((c + a^2*c*x^2)*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 7.79 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(a^2c\,x^2+c)\arctan\left(ax\right)^3}dx$$

input
```
int(x^3/(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

output
```
int(x^3/(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^3}{(c+a^2cx^2)\arctan(ax)^3}\,dx = \int \frac{x^3}{(a^2cx^2+c)\arctan{(ax)}^3}\,dx$$

input
```
integrate(x^3/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(x^3/((a^2*c*x^2 + c)*arctan(a*x)^3), x)
```

## Sympy [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{x^3}{(c+a^2cx^2)\arctan(ax)^3}\,dx = \frac{\int \frac{x^3}{a^2x^2\operatorname{atan}^3{(ax)}+\operatorname{atan}^3{(ax)}}\,dx}{c}$$

input
```
integrate(x**3/(a**2*c*x**2+c)/atan(a*x)**3,x)
```

output
```
Integral(x**3/(a**2*x**2*atan(a*x)**3 + atan(a*x)**3), x)/c
```

## Maxima [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.95

$$\int \frac{x^3}{(c+a^2cx^2)\arctan(ax)^3}\,dx = \int \frac{x^3}{(a^2cx^2+c)\arctan(ax)^3}\,dx$$

input
```
integrate(x^3/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
-1/2*(a*x^3 - 2*arctan(a*x)^2*integrate(3*(2*a^2*x^3 + x)/arctan(a*x), x)
+ 3*(a^2*x^4 + x^2)*arctan(a*x))/(a^2*c*arctan(a*x)^2)
```

## Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^3}{(c+a^2cx^2)\arctan(ax)^3}\,dx = \int \frac{x^3}{(a^2cx^2+c)\arctan(ax)^3}\,dx$$

input
```
integrate(x^3/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate(x^3/((a^2*c*x^2 + c)*arctan(a*x)^3), x)
```

## Mupad [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^3}{(c+a^2cx^2)\arctan(ax)^3}\,dx = \int \frac{x^3}{\mathrm{atan}\left(a\,x\right)^3\,\left(c\,a^2\,x^2+c\right)}\,dx$$

input
```
int(x^3/(atan(a*x)^3*(c + a^2*c*x^2)),x)
```

output
```
int(x^3/(atan(a*x)^3*(c + a^2*c*x^2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.45

$$\int \frac{x^3}{(c+a^2cx^2)\arctan(ax)^3}\,dx = \frac{\int \frac{x^3}{atan(ax)^3a^2x^2+atan(ax)^3}dx}{c}$$

input
```
int(x^3/(a^2*c*x^2+c)/atan(a*x)^3,x)
```

output
```
int(x**3/(atan(a*x)**3*a**2*x**2 + atan(a*x)**3),x)/c
```

# 3.619 $\int \frac{x^2}{(c+a^2cx^2)\arctan(ax)^3}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^2}{(c+a^2cx^2)\arctan(ax)^3}\,dx = -\frac{x^2}{2ac\arctan(ax)^2} + \frac{\text{Int}\left(\frac{x}{\arctan(ax)^2},x\right)}{ac}$$

output
```
-1/2*x^2/a/c/arctan(a*x)^2+Defer(Int)(x/arctan(a*x)^2,x)/a/c
```

## Mathematica [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^2}{(c+a^2cx^2)\arctan(ax)^3}\,dx = \int \frac{x^2}{(c+a^2cx^2)\arctan(ax)^3}\,dx$$

input
```
Integrate[x^2/((c + a^2*c*x^2)*ArcTan[a*x]^3),x]
```

output
```
Integrate[x^2/((c + a^2*c*x^2)*ArcTan[a*x]^3), x]
```

## Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\arctan(ax)^3\left(a^2cx^2+c\right)}dx$$

$$\Big\downarrow \; 5461$$

$$\frac{\int \frac{x}{\arctan(ax)^2}dx}{ac}-\frac{x^2}{2ac\arctan(ax)^2}$$

$$\Big\downarrow \; 5377$$

$$\frac{\int \frac{x}{\arctan(ax)^2}dx}{ac}-\frac{x^2}{2ac\arctan(ax)^2}$$

input
```
Int[x^2/((c + a^2*c*x^2)*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{\left(a^2c\,x^2+c\right)\arctan\left(ax\right)^3}dx$$

input
```
int(x^2/(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

output
```
int(x^2/(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^2}{(c+a^2cx^2)\arctan(ax)^3}\,dx = \int \frac{x^2}{(a^2cx^2+c)\arctan{(ax)}^3}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(x^2/((a^2*c*x^2 + c)*arctan(a*x)^3), x)
```

## Sympy [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{x^2}{(c+a^2cx^2)\arctan(ax)^3}\,dx = \frac{\int \frac{x^2}{a^2x^2\operatorname{atan}^3{(ax)}+\operatorname{atan}^3{(ax)}}\,dx}{c}$$

input
```
integrate(x**2/(a**2*c*x**2+c)/atan(a*x)**3,x)
```

output
```
Integral(x**2/(a**2*x**2*atan(a*x)**3 + atan(a*x)**3), x)/c
```

## Maxima [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.82

$$\int \frac{x^2}{(c+a^2cx^2)\arctan(ax)^3}\,dx = \int \frac{x^2}{(a^2cx^2+c)\arctan{(ax)}^3}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
-1/2*(a*x^2 - 2*arctan(a*x)^2*integrate((3*a^2*x^2 + 1)/arctan(a*x), x) +
2*(a^2*x^3 + x)*arctan(a*x))/(a^2*c*arctan(a*x)^2)
```

## Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^2}{(c+a^2cx^2)\arctan(ax)^3}\,dx = \int \frac{x^2}{(a^2cx^2+c)\arctan{(ax)}^3}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate(x^2/((a^2*c*x^2 + c)*arctan(a*x)^3), x)
```

## Mupad [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^2}{(c+a^2cx^2)\arctan(ax)^3}\,dx = \int \frac{x^2}{\mathrm{atan}\left(a\,x\right)^3\,\left(c\,a^2\,x^2+c\right)}\,dx$$

input
```
int(x^2/(atan(a*x)^3*(c + a^2*c*x^2)),x)
```

output
```
int(x^2/(atan(a*x)^3*(c + a^2*c*x^2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.45

$$\int \frac{x^2}{(c+a^2cx^2)\arctan(ax)^3}\,dx = \frac{\int \frac{x^2}{atan(ax)^3a^2x^2+atan(ax)^3}dx}{c}$$

input
```
int(x^2/(a^2*c*x^2+c)/atan(a*x)^3,x)
```

output
```
int(x**2/(atan(a*x)**3*a**2*x**2 + atan(a*x)**3),x)/c
```

# 3.620 $\int \frac{x}{(c+a^2cx^2)\arctan(ax)^3}\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{x}{(c+a^2cx^2)\arctan(ax)^3}\,dx = -\frac{x}{2ac\arctan(ax)^2}+\frac{\operatorname{Int}\left(\frac{1}{\arctan(ax)^2},x\right)}{2ac}$$

output
```
-1/2*x/a/c/arctan(a*x)^2+1/2*Defer(Int)(1/arctan(a*x)^2,x)/a/c
```

## Mathematica [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x}{(c+a^2cx^2)\arctan(ax)^3}\,dx = \int \frac{x}{(c+a^2cx^2)\arctan(ax)^3}\,dx$$

input
```
Integrate[x/((c + a^2*c*x^2)*ArcTan[a*x]^3),x]
```

output
```
Integrate[x/((c + a^2*c*x^2)*ArcTan[a*x]^3), x]
```

## Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\arctan(ax)^3 (a^2cx^2+c)}dx$$
$$\Big\downarrow \text{ 5457}$$
$$\frac{\int \frac{1}{\arctan(ax)^2}dx}{2ac}-\frac{x}{2ac\arctan(ax)^2}$$
$$\Big\downarrow \text{ 5353}$$
$$\frac{\int \frac{1}{\arctan(ax)^2}dx}{2ac}-\frac{x}{2ac\arctan(ax)^2}$$

input
```
Int[x/((c + a^2*c*x^2)*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{x}{(a^2c\,x^2+c)\arctan{(ax)}^3}dx$$

input
```
int(x/(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

output
```
int(x/(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x}{(c+a^2cx^2)\arctan(ax)^3}\,dx = \int \frac{x}{(a^2cx^2+c)\arctan{(ax)}^3}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(x/((a^2*c*x^2 + c)*arctan(a*x)^3), x)
```

## Sympy [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{x}{(c+a^2cx^2)\arctan(ax)^3}\,dx = \frac{\int \frac{x}{a^2x^2\operatorname{atan}^3{(ax)}+\operatorname{atan}^3{(ax)}}\,dx}{c}$$

input
```
integrate(x/(a**2*c*x**2+c)/atan(a*x)**3,x)
```

output
```
Integral(x/(a**2*x**2*atan(a*x)**3 + atan(a*x)**3), x)/c
```

## Maxima [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.75

$$\int \frac{x}{(c+a^2cx^2)\arctan(ax)^3}\,dx = \int \frac{x}{(a^2cx^2+c)\arctan{(ax)}^3}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
1/2*(2*a^2*arctan(a*x)^2*integrate(x/arctan(a*x), x) - a*x - (a^2*x^2 + 1)
*arctan(a*x))/(a^2*c*arctan(a*x)^2)
```

## Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x}{(c+a^2cx^2)\arctan(ax)^3}\,dx = \int \frac{x}{(a^2cx^2+c)\arctan{(ax)}^3}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate(x/((a^2*c*x^2 + c)*arctan(a*x)^3), x)
```

## Mupad [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x}{(c+a^2cx^2)\arctan(ax)^3}\,dx = \int \frac{x}{\mathrm{atan}\left(a\,x\right)^3\,\left(c\,a^2\,x^2+c\right)}\,dx$$

input
```
int(x/(atan(a*x)^3*(c + a^2*c*x^2)),x)
```

output
```
int(x/(atan(a*x)^3*(c + a^2*c*x^2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.50

$$\int \frac{x}{(c+a^2cx^2)\arctan(ax)^3}\,dx = \frac{\int \frac{x}{atan(ax)^3a^2x^2+atan(ax)^3}dx}{c}$$

input
```
int(x/(a^2*c*x^2+c)/atan(a*x)^3,x)
```

output
```
int(x/(atan(a*x)**3*a**2*x**2 + atan(a*x)**3),x)/c
```

# 3.621 $\int \frac{1}{(c+a^2cx^2)\arctan(ax)^3}\,dx$



## Optimal result

Integrand size = 19, antiderivative size = 16

$$\int \frac{1}{(c+a^2cx^2)\arctan(ax)^3}\,dx = -\frac{1}{2ac\arctan(ax)^2}$$

output
```
-1/2/a/c/arctan(a*x)^2
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{(c+a^2cx^2)\arctan(ax)^3}\,dx = -\frac{1}{2ac\arctan(ax)^2}$$

input
```
Integrate[1/((c + a^2*c*x^2)*ArcTan[a*x]^3),x]
```

output
```
-1/2*1/(a*c*ArcTan[a*x]^2)
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arctan(ax)^3 (a^2cx^2+c)} dx$$
$$\Big\downarrow \text{5419}$$
$$-\frac{1}{2ac\arctan(ax)^2}$$

input
```
Int[1/((c + a^2*c*x^2)*ArcTan[a*x]^3),x]
```

output
```
-1/2*1/(a*c*ArcTan[a*x]^2)
```

### Defintions of rubi rules used

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

## Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

| method | result | size |
|---|---|---|
| derivativedivides | $-\frac{1}{2ac\arctan(ax)^{2}}$ | 15 |
| default | $-\frac{1}{2ac\arctan(ax)^{2}}$ | 15 |
| parallelrisch | $-\frac{1}{2ac\arctan(ax)^{2}}$ | 15 |
| risch | $\frac{2}{ac(\ln(-iax+1)-\ln(iax+1))^{2}}$ | 30 |

input
```
int(1/(a^2*c*x^2+c)/arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2/a/c/arctan(a*x)^2
```

## Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{(c+a^2cx^2)\arctan(ax)^3}\,dx = -\frac{1}{2\,ac\arctan\left(ax\right)^2}$$

input
```
integrate(1/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
-1/2/(a*c*arctan(a*x)^2)
```

## Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{(c+a^2cx^2)\arctan(ax)^3}\,dx = -\frac{1}{2ac\operatorname{atan}^2\left(ax\right)}$$

input
```
integrate(1/(a**2*c*x**2+c)/atan(a*x)**3,x)
```

output
```
-1/(2*a*c*atan(a*x)**2)
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{(c+a^2cx^2)\arctan(ax)^3}\,dx = -\frac{1}{2\,ac\arctan{(ax)}^2}$$

input
```
integrate(1/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
-1/2/(a*c*arctan(a*x)^2)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{(c+a^2cx^2)\arctan(ax)^3}\,dx = -\frac{1}{2\,ac\arctan{(ax)}^2}$$

input
```
integrate(1/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
-1/2/(a*c*arctan(a*x)^2)
```

## Mupad [B] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{(c+a^2cx^2)\arctan(ax)^3}\,dx = -\frac{1}{2\,a\,c\,\mathrm{atan}\left(a\,x\right)^2}$$

input
```
int(1/(atan(a*x)^3*(c + a^2*c*x^2)),x)
```

output
```
-1/(2*a*c*atan(a*x)^2)
```

## Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{(c+a^2cx^2)\arctan(ax)^3}\,dx = -\frac{1}{2atan\left(ax\right)^2ac}$$

input
```
int(1/(a^2*c*x^2+c)/atan(a*x)^3,x)
```

output
```
( - 1)/(2*atan(a*x)**2*a*c)
```

# 3.622 $\int \frac{1}{x(c+a^2cx^2)\arctan(ax)^3}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x(c+a^2cx^2)\arctan(ax)^3}\,dx=-\frac{1}{2acx\arctan(ax)^2}-\frac{\text{Int}\left(\frac{1}{x^2\arctan(ax)^2},x\right)}{2ac}$$

output
```
-1/2/a/c/x/arctan(a*x)^2-1/2*Defer(Int)(1/x^2/arctan(a*x)^2,x)/a/c
```

## Mathematica [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x(c+a^2cx^2)\arctan(ax)^3}\,dx=\int \frac{1}{x(c+a^2cx^2)\arctan(ax)^3}\,dx$$

input
```
Integrate[1/(x*(c + a^2*c*x^2)*ArcTan[a*x]^3),x]
```

output
```
Integrate[1/(x*(c + a^2*c*x^2)*ArcTan[a*x]^3), x]
```

## Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x \arctan(ax)^3 (a^2cx^2+c)} \, dx$$

$$\Big\downarrow \text{5461}$$

$$-\frac{\int \frac{1}{x^2 \arctan(ax)^2}dx}{2ac} - \frac{1}{2acx \arctan(ax)^2}$$

$$\Big\downarrow \text{5377}$$

$$-\frac{\int \frac{1}{x^2 \arctan(ax)^2}dx}{2ac} - \frac{1}{2acx \arctan(ax)^2}$$

input
```
Int[1/(x*(c + a^2*c*x^2)*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \left(a^2 c\, x^2+c\right) \arctan \left(a x \right)^3}dx$$

input
```
int(1/x/(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

output
```
int(1/x/(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{1}{x(c+a^2cx^2)\arctan(ax)^3}\,dx = \int \frac{1}{(a^2cx^2+c)x\arctan(ax)^3}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(1/((a^2*c*x^3 + c*x)*arctan(a*x)^3), x)
```

## Sympy [N/A]

Not integrable

Time = 0.83 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{1}{x(c+a^2cx^2)\arctan(ax)^3}\,dx = \frac{\int \frac{1}{a^2x^3\operatorname{atan}^3(ax)+x\operatorname{atan}^3(ax)}\,dx}{c}$$

input
```
integrate(1/x/(a**2*c*x**2+c)/atan(a*x)**3,x)
```

output
```
Integral(1/(a**2*x**3*atan(a*x)**3 + x*atan(a*x)**3), x)/c
```

## Maxima [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.68

$$\int \frac{1}{x(c+a^2cx^2)\arctan(ax)^3}\,dx = \int \frac{1}{(a^2cx^2+c)x\arctan(ax)^3}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
1/2*(2*x^2*arctan(a*x)^2*integrate(1/(x^3*arctan(a*x)), x) - a*x + (a^2*x^
2 + 1)*arctan(a*x))/(a^2*c*x^2*arctan(a*x)^2)
```

## Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\left(c+a^2cx^2\right)\arctan(ax)^3}\,dx=\int \frac{1}{(a^2cx^2+c)x\arctan\left(ax\right)^3}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)*x*arctan(a*x)^3), x)
```

## Mupad [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\left(c+a^2cx^2\right)\arctan(ax)^3}\,dx=\int \frac{1}{x\,\mathrm{atan}\left(a\,x\right)^3\,\left(c\,a^2\,x^2+c\right)}\,dx$$

input
```
int(1/(x*atan(a*x)^3*(c + a^2*c*x^2)),x)
```

output
```
int(1/(x*atan(a*x)^3*(c + a^2*c*x^2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.36

$$\int \frac{1}{x\left(c+a^2cx^2\right)\arctan(ax)^3}\,dx = \frac{\int \frac{1}{atan(ax)^3a^2x^3+atan(ax)^3x}dx}{c}$$

input

```
int(1/x/(a^2*c*x^2+c)/atan(a*x)^3,x)
```

output

```
int(1/(atan(a*x)**3*a**2*x**3 + atan(a*x)**3*x),x)/c
```

# 3.623 $\int \frac{1}{x^2(c+a^2cx^2)\arctan(ax)^3}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x^2(c+a^2cx^2)\arctan(ax)^3}\,dx = -\frac{1}{2acx^2\arctan(ax)^2} - \frac{\operatorname{Int}\left(\frac{1}{x^3\arctan(ax)^2},x\right)}{ac}$$

output
```
-1/2/a/c/x^2/arctan(a*x)^2-Defer(Int)(1/x^3/arctan(a*x)^2,x)/a/c
```

## Mathematica [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2(c+a^2cx^2)\arctan(ax)^3}\,dx = \int \frac{1}{x^2(c+a^2cx^2)\arctan(ax)^3}\,dx$$

input
```
Integrate[1/(x^2*(c + a^2*c*x^2)*ArcTan[a*x]^3),x]
```

output
```
Integrate[1/(x^2*(c + a^2*c*x^2)*ArcTan[a*x]^3), x]
```

## Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \arctan(ax)^3 (a^2cx^2+c)} \, dx$$

$$\Big\downarrow \text{5461}$$

$$-\frac{\int \frac{1}{x^3 \arctan(ax)^2}dx}{ac} - \frac{1}{2acx^2 \arctan(ax)^2}$$

$$\Big\downarrow \text{5377}$$

$$-\frac{\int \frac{1}{x^3 \arctan(ax)^2}dx}{ac} - \frac{1}{2acx^2 \arctan(ax)^2}$$

input
```
Int[1/(x^2*(c + a^2*c*x^2)*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (a^2c\,x^2+c) \arctan(ax)^3} dx$$

input
```
int(1/x^2/(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

output
```
int(1/x^2/(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^2 (c + a^2cx^2) \arctan(ax)^3} \, dx = \int \frac{1}{(a^2cx^2 + c)x^2 \arctan{(ax)}^3} \, dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(1/((a^2*c*x^4 + c*x^2)*arctan(a*x)^3), x)
```

## Sympy [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

$$\int \frac{1}{x^2 (c + a^2cx^2) \arctan(ax)^3} \, dx = \frac{\int \frac{1}{a^2x^4 \operatorname{atan}^3{(ax)} + x^2 \operatorname{atan}^3{(ax)}} \, dx}{c}$$

input
```
integrate(1/x**2/(a**2*c*x**2+c)/atan(a*x)**3,x)
```

output
```
Integral(1/(a**2*x**4*atan(a*x)**3 + x**2*atan(a*x)**3), x)/c
```

## Maxima [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 69, normalized size of antiderivative = 3.14

$$\int \frac{1}{x^2 (c + a^2cx^2) \arctan(ax)^3} \, dx = \int \frac{1}{(a^2cx^2 + c)x^2 \arctan{(ax)}^3} \, dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
1/2*(2*x^3*arctan(a*x)^2*integrate((a^2*x^2 + 3)/(x^4*arctan(a*x)), x) - a
*x + 2*(a^2*x^2 + 1)*arctan(a*x))/(a^2*c*x^3*arctan(a*x)^2)
```

## Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2 (c + a^2 c x^2) \arctan(ax)^3} \, dx = \int \frac{1}{(a^2 c x^2 + c) x^2 \arctan(ax)^3} \, dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)*x^2*arctan(a*x)^3), x)
```

## Mupad [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2 (c + a^2 c x^2) \arctan(ax)^3} \, dx = \int \frac{1}{x^2 \operatorname{atan}(a\,x)^3 \, (c\,a^2\,x^2 + c)} \, dx$$

input
```
int(1/(x^2*atan(a*x)^3*(c + a^2*c*x^2)),x)
```

output
```
int(1/(x^2*atan(a*x)^3*(c + a^2*c*x^2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.45

$$\int \frac{1}{x^2 (c+a^2cx^2) \arctan(ax)^3} \, dx = \frac{\int \frac{1}{atan(ax)^3 a^2 x^4 + atan(ax)^3 x^2} dx}{c}$$

input
```
int(1/x^2/(a^2*c*x^2+c)/atan(a*x)^3,x)
```

output
```
int(1/(atan(a*x)**3*a**2*x**4 + atan(a*x)**3*x**2),x)/c
```

# 3.624 $\int \frac{1}{x^3(c+a^2cx^2)\arctan(ax)^3}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x^3(c+a^2cx^2)\arctan(ax)^3}\,dx=-\frac{1}{2acx^3\arctan(ax)^2}-\frac{3\text{Int}\left(\frac{1}{x^4\arctan(ax)^2},x\right)}{2ac}$$

output
```
-1/2/a/c/x^3/arctan(a*x)^2-3/2*Defer(Int)(1/x^4/arctan(a*x)^2,x)/a/c
```

## Mathematica [N/A]

Not integrable

Time = 0.98 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^3(c+a^2cx^2)\arctan(ax)^3}\,dx=\int \frac{1}{x^3(c+a^2cx^2)\arctan(ax)^3}\,dx$$

input
```
Integrate[1/(x^3*(c + a^2*c*x^2)*ArcTan[a*x]^3),x]
```

output
```
Integrate[1/(x^3*(c + a^2*c*x^2)*ArcTan[a*x]^3), x]
```

## Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \arctan(ax)^3 (a^2cx^2+c)}dx$$

$$\Big\downarrow \text{5461}$$

$$-\frac{3\int \frac{1}{x^4 \arctan(ax)^2}dx}{2ac}-\frac{1}{2acx^3 \arctan(ax)^2}$$

$$\Big\downarrow \text{5377}$$

$$-\frac{3\int \frac{1}{x^4 \arctan(ax)^2}dx}{2ac}-\frac{1}{2acx^3 \arctan(ax)^2}$$

input
```
Int[1/(x^3*(c + a^2*c*x^2)*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 7.84 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3 (a^2c\,x^2+c) \arctan(ax)^3}dx$$

input
```
int(1/x^3/(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

output
```
int(1/x^3/(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^3 (c + a^2 c x^2) \arctan(ax)^3} \, dx = \int \frac{1}{(a^2 c x^2 + c) x^3 \arctan{(ax)}^3} \, dx$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(1/((a^2*c*x^5 + c*x^3)*arctan(a*x)^3), x)
```

## Sympy [N/A]

Not integrable

Time = 0.77 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

$$\int \frac{1}{x^3 (c + a^2 c x^2) \arctan(ax)^3} \, dx = \frac{\int \frac{1}{a^2 x^5 \operatorname{atan}^3{(ax)} + x^3 \operatorname{atan}^3{(ax)}} \, dx}{c}$$

input
```
integrate(1/x**3/(a**2*c*x**2+c)/atan(a*x)**3,x)
```

output
```
Integral(1/(a**2*x**5*atan(a*x)**3 + x**3*atan(a*x)**3), x)/c
```

## Maxima [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 70, normalized size of antiderivative = 3.18

$$\int \frac{1}{x^3 (c + a^2 c x^2) \arctan(ax)^3} \, dx = \int \frac{1}{(a^2 c x^2 + c) x^3 \arctan{(ax)}^3} \, dx$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
1/2*(2*x^4*arctan(a*x)^2*integrate(3*(a^2*x^2 + 2)/(x^5*arctan(a*x)), x) -
 a*x + 3*(a^2*x^2 + 1)*arctan(a*x))/(a^2*c*x^4*arctan(a*x)^2)
```

## Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)\arctan(ax)^3}\,dx=\int \frac{1}{(a^2cx^2+c)x^3\arctan{(ax)}^3}\,dx$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)*x^3*arctan(a*x)^3), x)
```

## Mupad [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)\arctan(ax)^3}\,dx=\int \frac{1}{x^3\,\mathrm{atan}\left(a\,x\right)^3\,\left(c\,a^2\,x^2+c\right)}\,dx$$

input
```
int(1/(x^3*atan(a*x)^3*(c + a^2*c*x^2)),x)
```

output
```
int(1/(x^3*atan(a*x)^3*(c + a^2*c*x^2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.45

$$\int \frac{1}{x^3 (c+a^2cx^2) \arctan(ax)^3} \, dx = \frac{\int \frac{1}{atan(ax)^3 a^2 x^5 + atan(ax)^3 x^3} dx}{c}$$

input
```
int(1/x^3/(a^2*c*x^2+c)/atan(a*x)^3,x)
```

output
```
int(1/(atan(a*x)**3*a**2*x**5 + atan(a*x)**3*x**3),x)/c
```

# 3.625 $\int \frac{1}{x^4(c+a^2cx^2)\arctan(ax)^3}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x^4(c+a^2cx^2)\arctan(ax)^3}\,dx = -\frac{1}{2acx^4\arctan(ax)^2} - \frac{2\text{Int}\left(\frac{1}{x^5\arctan(ax)^2},x\right)}{ac}$$

output
```
-1/2/a/c/x^4/arctan(a*x)^2-2*Defer(Int)(1/x^5/arctan(a*x)^2,x)/a/c
```

## Mathematica [N/A]

Not integrable

Time = 1.83 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^4(c+a^2cx^2)\arctan(ax)^3}\,dx = \int \frac{1}{x^4(c+a^2cx^2)\arctan(ax)^3}\,dx$$

input
```
Integrate[1/(x^4*(c + a^2*c*x^2)*ArcTan[a*x]^3),x]
```

output
```
Integrate[1/(x^4*(c + a^2*c*x^2)*ArcTan[a*x]^3), x]
```

## Rubi [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \arctan(ax)^3 (a^2cx^2+c)} \, dx$$

$$\Big\downarrow \text{ 5461}$$

$$-\frac{2\int \frac{1}{x^5 \arctan(ax)^2}dx}{ac} - \frac{1}{2acx^4 \arctan(ax)^2}$$

$$\Big\downarrow \text{ 5377}$$

$$-\frac{2\int \frac{1}{x^5 \arctan(ax)^2}dx}{ac} - \frac{1}{2acx^4 \arctan(ax)^2}$$

input
```
Int[1/(x^4*(c + a^2*c*x^2)*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 7.80 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4 (a^2c\,x^2+c) \arctan(ax)^3} dx$$

input
```
int(1/x^4/(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

output
```
int(1/x^4/(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^4 (c+a^2cx^2) \arctan(ax)^3} dx = \int \frac{1}{(a^2cx^2+c)x^4 \arctan(ax)^3} dx$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(1/((a^2*c*x^6 + c*x^4)*arctan(a*x)^3), x)
```

## Sympy [N/A]

Not integrable

Time = 0.84 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

$$\int \frac{1}{x^4 (c+a^2cx^2) \arctan(ax)^3} dx = \frac{\int \frac{1}{a^2x^6 \operatorname{atan}^3(ax)+x^4 \operatorname{atan}^3(ax)} dx}{c}$$

input
```
integrate(1/x**4/(a**2*c*x**2+c)/atan(a*x)**3,x)
```

output
```
Integral(1/(a**2*x**6*atan(a*x)**3 + x**4*atan(a*x)**3), x)/c
```

## Maxima [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 71, normalized size of antiderivative = 3.23

$$\int \frac{1}{x^4 (c+a^2cx^2) \arctan(ax)^3} dx = \int \frac{1}{(a^2cx^2+c)x^4 \arctan(ax)^3} dx$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
1/2*(2*x^5*arctan(a*x)^2*integrate(2*(3*a^2*x^2 + 5)/(x^6*arctan(a*x)), x)
 - a*x + 4*(a^2*x^2 + 1)*arctan(a*x))/(a^2*c*x^5*arctan(a*x)^2)
```

## Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^4 (c + a^2cx^2) \arctan(ax)^3} \, dx = \int \frac{1}{(a^2cx^2 + c)x^4 \arctan{(ax)}^3} \, dx$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)*x^4*arctan(a*x)^3), x)
```

## Mupad [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^4 (c + a^2cx^2) \arctan(ax)^3} \, dx = \int \frac{1}{x^4 \operatorname{atan}{(a\,x)}^3 \, (c\,a^2\,x^2 + c)} \, dx$$

input
```
int(1/(x^4*atan(a*x)^3*(c + a^2*c*x^2)),x)
```

output
```
int(1/(x^4*atan(a*x)^3*(c + a^2*c*x^2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.45

$$\int \frac{1}{x^4 (c+a^2cx^2) \arctan(ax)^3}\,dx = \frac{\int \frac{1}{atan(ax)^3 a^2x^6+atan(ax)^3 x^4}dx}{c}$$

input
```
int(1/x^4/(a^2*c*x^2+c)/atan(a*x)^3,x)
```

output
```
int(1/(atan(a*x)**3*a**2*x**6 + atan(a*x)**3*x**4),x)/c
```

# 3.626 $\int \frac{x^3}{(c+a^2cx^2)^2 \arctan(ax)^3} \, dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^3}{(c+a^2cx^2)^2 \arctan(ax)^3} \, dx = -\frac{x}{2a^3c^2 \arctan(ax)^2} + \frac{x}{2a^3c^2 (1+a^2x^2) \arctan(ax)^2} + \frac{1-a^2x^2}{2a^4c^2 (1+a^2x^2) \arctan(ax)} + \frac{\text{Si}(2\arctan(ax))}{a^4c^2} + \frac{\text{Int}\left(\frac{1}{\arctan(ax)^2}, x\right)}{2a^3c^2}$$

output

```
-1/2*x/a^3/c^2/arctan(a*x)^2+1/2*x/a^3/c^2/(a^2*x^2+1)/arctan(a*x)^2+1/2*(
-a^2*x^2+1)/a^4/c^2/(a^2*x^2+1)/arctan(a*x)+Si(2*arctan(a*x))/a^4/c^2+1/2*
Defer(Int)(1/arctan(a*x)^2,x)/a^3/c^2
```

## Mathematica [N/A]

Not integrable

Time = 5.68 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^3}{(c+a^2cx^2)^2 \arctan(ax)^3}\,dx = \int \frac{x^3}{(c+a^2cx^2)^2 \arctan(ax)^3}\,dx$$

input
```
Integrate[x^3/((c + a^2*c*x^2)^2*ArcTan[a*x]^3),x]
```

output
```
Integrate[x^3/((c + a^2*c*x^2)^2*ArcTan[a*x]^3), x]
```

## Rubi [N/A]

Not integrable

Time = 0.86 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\arctan(ax)^3 (a^2cx^2+c)^2}\,dx$$

$$\Big\downarrow \text{5499}$$

$$\frac{\int \frac{x}{c(a^2x^2+1)\arctan(ax)^3}dx}{a^2c} - \frac{\int \frac{x}{c^2(a^2x^2+1)^2\arctan(ax)^3}dx}{a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{x}{(a^2x^2+1)\arctan(ax)^3}dx}{a^2c^2} - \frac{\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^3}dx}{a^2c^2}$$

$$\Big\downarrow \text{5457}$$

$$\frac{\frac{\int \frac{1}{\arctan(ax)^2}dx}{2a} - \frac{x}{2a\arctan(ax)^2}}{a^2c^2} - \frac{\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^3}dx}{a^2c^2}$$

$$\Big\downarrow \; 5353$$

$$\frac{\frac{\int \frac{1}{\arctan(ax)^2}dx}{2a}-\frac{x}{2a\arctan(ax)^2}}{a^2c^2}-\frac{\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^3}dx}{a^2c^2}$$

$$\Big\downarrow \; 5467$$

$$\frac{\frac{\int \frac{1}{\arctan(ax)^2}dx}{2a}-\frac{x}{2a\arctan(ax)^2}}{a^2c^2}-\frac{-2\int \frac{x}{(a^2x^2+1)^2\arctan(ax)}dx-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}}{a^2c^2}$$

$$\Big\downarrow \; 5505$$

$$\frac{\frac{\int \frac{1}{\arctan(ax)^2}dx}{2a}-\frac{x}{2a\arctan(ax)^2}}{a^2c^2}-\frac{-\frac{2\int \frac{ax}{\left(a^2x^2+1\right)\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}}{a^2c^2}$$

$$\Big\downarrow \; 4906$$

$$\frac{\frac{\int \frac{1}{\arctan(ax)^2}dx}{2a}-\frac{x}{2a\arctan(ax)^2}}{a^2c^2}-\frac{-\frac{2\int \frac{\sin(2\arctan(ax))}{2\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}}{a^2c^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{1}{\arctan(ax)^2}dx}{2a}-\frac{x}{2a\arctan(ax)^2}}{a^2c^2}-\frac{-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}}{a^2c^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\int \frac{1}{\arctan(ax)^2}dx}{2a}-\frac{x}{2a\arctan(ax)^2}}{a^2c^2}-\frac{-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}}{a^2c^2}$$

$$\Big\downarrow \; 3780$$

$$\frac{\frac{\int \frac{1}{\arctan(ax)^2}dx}{2a} - \frac{x}{2a\arctan(ax)^2}}{a^2c^2} - \frac{-\frac{\text{Si}(2\arctan(ax))}{a^2} - \frac{x}{2a(a^2x^2+1)\arctan(ax)^2} - \frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}}{a^2c^2}$$

input
```
Int[x^3/((c + a^2*c*x^2)^2*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 8.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{\left(a^2c\,x^2+c\right)^2 \arctan\left(ax\right)^3}dx$$

input
```
int(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

output
```
int(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.77

$$\int \frac{x^3}{(c+a^2cx^2)^2 \arctan(ax)^3}\,dx = \int \frac{x^3}{\left(a^2cx^2+c\right)^2 \arctan\left(ax\right)^3}\,dx$$

input
```
integrate(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(x^3/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)^3), x)
```

## Sympy [N/A]

Not integrable

Time = 0.83 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.91

$$\int \frac{x^3}{(c+a^2cx^2)^2 \arctan(ax)^3} \, dx = \frac{\int \frac{x^3}{a^4x^4 \operatorname{atan}^3(ax)+2a^2x^2 \operatorname{atan}^3(ax)+\operatorname{atan}^3(ax)} \, dx}{c^2}$$

input
```
integrate(x**3/(a**2*c*x**2+c)**2/atan(a*x)**3,x)
```

output
```
Integral(x**3/(a**4*x**4*atan(a*x)**3 + 2*a**2*x**2*atan(a*x)**3 + atan(a*
x)**3), x)/c**2
```

## Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 137, normalized size of antiderivative = 6.23

$$\int \frac{x^3}{(c+a^2cx^2)^2 \arctan(ax)^3} \, dx = \int \frac{x^3}{(a^2cx^2+c)^2 \arctan(ax)^3} \, dx$$

input
```
integrate(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
-1/2*(a*x^3 - 2*(a^4*c^2*x^2 + a^2*c^2)*arctan(a*x)^2*integrate((a^4*x^5 +
 2*a^2*x^3 + 3*x)/((a^6*c^2*x^4 + 2*a^4*c^2*x^2 + a^2*c^2)*arctan(a*x)), x
) + (a^2*x^4 + 3*x^2)*arctan(a*x))/((a^4*c^2*x^2 + a^2*c^2)*arctan(a*x)^2)
```

## Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^3}{(c+a^2cx^2)^2 \arctan(ax)^3} \, dx = \int \frac{x^3}{(a^2cx^2+c)^2 \arctan(ax)^3} \, dx$$

input
```
integrate(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate(x^3/((a^2*c*x^2 + c)^2*arctan(a*x)^3), x)
```

## Mupad [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^3}{(c+a^2cx^2)^2 \arctan(ax)^3} \, dx = \int \frac{x^3}{\mathrm{atan}(a\,x)^3 \,(c\,a^2\,x^2+c)^2} \, dx$$

input
```
int(x^3/(atan(a*x)^3*(c + a^2*c*x^2)^2),x)
```

output
```
int(x^3/(atan(a*x)^3*(c + a^2*c*x^2)^2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.09

$$\int \frac{x^3}{(c+a^2cx^2)^2 \arctan(ax)^3} \, dx = \frac{\int \frac{x^3}{atan(ax)^3 a^4 x^4 + 2atan(ax)^3 a^2 x^2 + atan(ax)^3} dx}{c^2}$$

input
```
int(x^3/(a^2*c*x^2+c)^2/atan(a*x)^3,x)
```

output
```
int(x**3/(atan(a*x)**3*a**4*x**4 + 2*atan(a*x)**3*a**2*x**2 + atan(a*x)**3
),x)/c**2
```

# 3.627 $\int \frac{x^2}{(c+a^2cx^2)^2 \arctan(ax)^3} dx$



## Optimal result

Integrand size = 22, antiderivative size = 71

$$\int \frac{x^2}{(c+a^2cx^2)^2 \arctan(ax)^3} dx = -\frac{x^2}{2ac^2(1+a^2x^2)\arctan(ax)^2} - \frac{x}{a^2c^2(1+a^2x^2)\arctan(ax)} + \frac{\text{CosIntegral}(2\arctan(ax))}{a^3c^2}$$

output
```
-1/2*x^2/a/c^2/(a^2*x^2+1)/arctan(a*x)^2-x/a^2/c^2/(a^2*x^2+1)/arctan(a*x)
+Ci(2*arctan(a*x))/a^3/c^2
```

## Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.72

$$\int \frac{x^2}{(c+a^2cx^2)^2 \arctan(ax)^3} dx = \frac{-\frac{ax(ax+2\arctan(ax))}{(1+a^2x^2)\arctan(ax)^2} + 2\,\text{CosIntegral}(2\arctan(ax))}{2a^3c^2}$$

input
```
Integrate[x^2/((c + a^2*c*x^2)^2*ArcTan[a*x]^3),x]
```

output
```
(-((a*x*(a*x + 2*ArcTan[a*x]))/((1 + a^2*x^2)*ArcTan[a*x]^2)) + 2*CosInteg
ral[2*ArcTan[a*x]])/(2*a^3*c^2)
```

## Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.59, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5477, 27, 5503, 5439, 3042, 3793, 2009, 5505, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\arctan(ax)^3 (a^2cx^2+c)^2}dx$$

$$\Big\downarrow \text{5477}$$

$$\frac{\int \frac{x}{c^2(a^2x^2+1)^2\arctan(ax)^2}dx}{a}-\frac{x^2}{2ac^2(a^2x^2+1)\arctan(ax)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^2}dx}{ac^2}-\frac{x^2}{2ac^2(a^2x^2+1)\arctan(ax)^2}$$

$$\Big\downarrow \text{5503}$$

$$\frac{\frac{\int \frac{1}{\left(a^2x^2+1\right)^2\arctan(ax)}dx}{a}-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx-\frac{x}{a(a^2x^2+1)\arctan(ax)}}{ac^2}-\frac{x^2}{2ac^2(a^2x^2+1)\arctan(ax)^2}$$

$$\Big\downarrow \text{5439}$$

$$\frac{-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\int \frac{1}{\left(a^2x^2+1\right)\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}}{ac^2}-\frac{x^2}{2ac^2(a^2x^2+1)\arctan(ax)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^2}{\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}}{ac^2}-\frac{x^2}{2ac^2(a^2x^2+1)\arctan(ax)^2}$$

$$\Big\downarrow \text{3793}$$

$$\frac{-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\int \left(\frac{\cos(2\arctan(ax))}{2\arctan(ax)}+\frac{1}{2\arctan(ax)}\right)d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}}{ac^2}-\frac{x^2}{2ac^2(a^2x^2+1)\arctan(ax)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}}{ac^2}-\frac{x^2}{2ac^2(a^2x^2+1)\arctan(ax)^2}$$

$$\Big\downarrow \text{5505}$$

$$\frac{-\frac{\int \frac{a^2x^2}{\left(a^2x^2+1\right)\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}}{ac^2}-\frac{x^2}{2ac^2(a^2x^2+1)\arctan(ax)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{\int \frac{\sin(\arctan(ax))^2}{\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}}{ac^2}-\frac{x^2}{2ac^2(a^2x^2+1)\arctan(ax)^2}$$

$$\Big\downarrow \text{3793}$$

$$\frac{-\frac{\int \left(\frac{1}{2\arctan(ax)}-\frac{\cos(2\arctan(ax))}{2\arctan(ax)}\right)d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}}{ac^2}-\frac{x^2}{2ac^2(a^2x^2+1)\arctan(ax)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{\frac{1}{2}\log(\arctan(ax))-\frac{1}{2}\text{CosIntegral}(2\arctan(ax))}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}}{ac^2}-\frac{x^2}{2ac^2(a^2x^2+1)\arctan(ax)^2}$$

input
```
Int[x^2/((c + a^2*c*x^2)^2*ArcTan[a*x]^3),x]
```

output
```
-1/2*x^2/(a*c^2*(1 + a^2*x^2)*ArcTan[a*x]^2) + (-(x/(a*(1 + a^2*x^2)*ArcTa
n[a*x])) - (-1/2*CosIntegral[2*ArcTan[a*x]] + Log[ArcTan[a*x]]/2)/a^2 + (C
osIntegral[2*ArcTan[a*x]]/2 + Log[ArcTan[a*x]]/2)/a^2)/(a*c^2)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5477
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*ArcT
an[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp[f*(m/(b*c*(p + 1)))   Int[(f*x
)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1
]
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

## Maple [A] (verified)

Time = 7.81 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.73

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{4\,\operatorname{Ci}(2\arctan(ax))\arctan(ax)^2-2\sin(2\arctan(ax))\arctan(ax)+\cos(2\arctan(ax))-1}{4a^3c^2\arctan(ax)^2}$ | 52 |
| default | $\frac{4\,\operatorname{Ci}(2\arctan(ax))\arctan(ax)^2-2\sin(2\arctan(ax))\arctan(ax)+\cos(2\arctan(ax))-1}{4a^3c^2\arctan(ax)^2}$ | 52 |

input
```
int(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4/a^3/c^2*(4*Ci(2*arctan(a*x))*arctan(a*x)^2-2*sin(2*arctan(a*x))*arctan
(a*x)+cos(2*arctan(a*x))-1)/arctan(a*x)^2
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.86

$$\int \frac{x^2}{(c+a^2cx^2)^2 \arctan(ax)^3} dx = -\frac{a^2x^2 - (a^2x^2+1)\arctan(ax)^2 \operatorname{log\_integral}\left(-\frac{a^2x^2+2i\,ax-1}{a^2x^2+1}\right) - (a^2x^2+1)\arctan(ax)^2 \operatorname{log\_integral}\left(\right.}{2\left(a^5c^2x^2+a^3c^2\right)\arctan(ax)^2}$$

input
```
integrate(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
-1/2*(a^2*x^2 - (a^2*x^2 + 1)*arctan(a*x)^2*log_integral(-(a^2*x^2 + 2*I*a
*x - 1)/(a^2*x^2 + 1)) - (a^2*x^2 + 1)*arctan(a*x)^2*log_integral(-(a^2*x^
2 - 2*I*a*x - 1)/(a^2*x^2 + 1)) + 2*a*x*arctan(a*x))/((a^5*c^2*x^2 + a^3*c
^2)*arctan(a*x)^2)
```

## Sympy [F]

$$\int \frac{x^2}{(c+a^2cx^2)^2 \arctan(ax)^3} dx = \frac{\int \frac{x^2}{a^4x^4\operatorname{atan}^3(ax)+2a^2x^2\operatorname{atan}^3(ax)+\operatorname{atan}^3(ax)}\,dx}{c^2}$$

input
```
integrate(x**2/(a**2*c*x**2+c)**2/atan(a*x)**3,x)
```

output
```
Integral(x**2/(a**4*x**4*atan(a*x)**3 + 2*a**2*x**2*atan(a*x)**3 + atan(a*
x)**3), x)/c**2
```

## Maxima [F]

$$\int \frac{x^2}{(c+a^2cx^2)^2 \arctan(ax)^3} \, dx = \int \frac{x^2}{(a^2cx^2+c)^2 \arctan(ax)^3} \, dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
-1/2*(2*(a^4*c^2*x^2 + a^2*c^2)*arctan(a*x)^2*integrate((a^2*x^2 - 1)/((a^
6*c^2*x^4 + 2*a^4*c^2*x^2 + a^2*c^2)*arctan(a*x)), x) + a*x^2 + 2*x*arctan
(a*x))/((a^4*c^2*x^2 + a^2*c^2)*arctan(a*x)^2)
```

## Giac [F]

$$\int \frac{x^2}{(c+a^2cx^2)^2 \arctan(ax)^3} \, dx = \int \frac{x^2}{(a^2cx^2+c)^2 \arctan(ax)^3} \, dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate(x^2/((a^2*c*x^2 + c)^2*arctan(a*x)^3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(c+a^2cx^2)^2 \arctan(ax)^3} \, dx = \int \frac{x^2}{\mathrm{atan}(a\,x)^3\,(c\,a^2\,x^2+c)^2} \, dx$$

input
```
int(x^2/(atan(a*x)^3*(c + a^2*c*x^2)^2),x)
```

output
```
int(x^2/(atan(a*x)^3*(c + a^2*c*x^2)^2), x)
```

## Reduce [F]

$$\int \frac{x^2}{(c+a^2cx^2)^2 \arctan(ax)^3}\,dx$$
$$= \frac{2atan(ax)^2 \left(\int \frac{x}{atan(ax)^2a^4x^4+2atan(ax)^2a^2x^2+atan(ax)^2}dx\right) a^2x^2 + 2atan(ax)^2 \left(\int \frac{x}{atan(ax)^2a^4x^4+2atan(ax)^2a^2x^2+ata}\right.}{2atan\,(ax)^2\,a\,c^2\,(a^2x^2+1)}$$

input

```
int(x^2/(a^2*c*x^2+c)^2/atan(a*x)^3,x)
```

output

```
(2*atan(a*x)**2*int(x/(atan(a*x)**2*a**4*x**4 + 2*atan(a*x)**2*a**2*x**2 +
 atan(a*x)**2),x)*a**2*x**2 + 2*atan(a*x)**2*int(x/(atan(a*x)**2*a**4*x**4
 + 2*atan(a*x)**2*a**2*x**2 + atan(a*x)**2),x) - x**2)/(2*atan(a*x)**2*a*c
**2*(a**2*x**2 + 1))
```

# 3.628 $\int \frac{x}{(c+a^2cx^2)^2 \arctan(ax)^3} dx$



## Optimal result

Integrand size = 20, antiderivative size = 81

$$\int \frac{x}{(c+a^2cx^2)^2 \arctan(ax)^3} dx = -\frac{x}{2ac^2(1+a^2x^2)\arctan(ax)^2} - \frac{1-a^2x^2}{2a^2c^2(1+a^2x^2)\arctan(ax)} - \frac{\text{Si}(2\arctan(ax))}{a^2c^2}$$

output
```
-1/2*x/a/c^2/(a^2*x^2+1)/arctan(a*x)^2-1/2*(-a^2*x^2+1)/a^2/c^2/(a^2*x^2+1
)/arctan(a*x)-Si(2*arctan(a*x))/a^2/c^2
```

## Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.86

$$\int \frac{x}{(c+a^2cx^2)^2 \arctan(ax)^3} dx = \frac{-ax + (-1+a^2x^2)\arctan(ax) - 2(1+a^2x^2)\arctan(ax)^2\text{Si}(2\arctan(ax))}{2a^2c^2(1+a^2x^2)\arctan(ax)^2}$$

input
```
Integrate[x/((c + a^2*c*x^2)^2*ArcTan[a*x]^3),x]
```

output
```
(-(a*x) + (-1 + a^2*x^2)*ArcTan[a*x] - 2*(1 + a^2*x^2)*ArcTan[a*x]^2*SinIn
tegral[2*ArcTan[a*x]])/(2*a^2*c^2*(1 + a^2*x^2)*ArcTan[a*x]^2)
```

## Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {5467, 27, 5505, 4906, 27, 3042, 3780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\arctan(ax)^3 (a^2cx^2+c)^2}dx$$

$$\Big\downarrow \text{5467}$$

$$-2\int \frac{x}{c^2 (a^2x^2+1)^2 \arctan(ax)}dx-\frac{x}{2ac^2 (a^2x^2+1) \arctan(ax)^2}-\frac{1-a^2x^2}{2a^2c^2 (a^2x^2+1) \arctan(ax)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2\int \frac{x}{(a^2x^2+1)^2 \arctan(ax)}dx}{c^2}-\frac{x}{2ac^2 (a^2x^2+1) \arctan(ax)^2}-\frac{1-a^2x^2}{2a^2c^2 (a^2x^2+1) \arctan(ax)}$$

$$\Big\downarrow \text{5505}$$

$$-\frac{2\int \frac{ax}{(a^2x^2+1) \arctan(ax)}d\arctan(ax)}{a^2c^2}-\frac{x}{2ac^2 (a^2x^2+1) \arctan(ax)^2}-\frac{1-a^2x^2}{2a^2c^2 (a^2x^2+1) \arctan(ax)}$$

$$\Big\downarrow \text{4906}$$

$$-\frac{2\int \frac{\sin(2\arctan(ax))}{2\arctan(ax)}d\arctan(ax)}{a^2c^2}-\frac{x}{2ac^2 (a^2x^2+1) \arctan(ax)^2}-\frac{1-a^2x^2}{2a^2c^2 (a^2x^2+1) \arctan(ax)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a^2c^2}-\frac{x}{2ac^2 (a^2x^2+1) \arctan(ax)^2}-\frac{1-a^2x^2}{2a^2c^2 (a^2x^2+1) \arctan(ax)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a^2c^2}-\frac{x}{2ac^2\left(a^2x^2+1\right)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2c^2\left(a^2x^2+1\right)\arctan(ax)}$$

$$\Big\downarrow \text{3780}$$

$$-\frac{\text{Si}(2\arctan(ax))}{a^2c^2}-\frac{x}{2ac^2\left(a^2x^2+1\right)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2c^2\left(a^2x^2+1\right)\arctan(ax)}$$

input
```
Int[x/((c + a^2*c*x^2)^2*ArcTan[a*x]^3),x]
```

output
```
-1/2*x/(a*c^2*(1 + a^2*x^2)*ArcTan[a*x]^2) - (1 - a^2*x^2)/(2*a^2*c^2*(1 +
 a^2*x^2)*ArcTan[a*x]) - SinIntegral[2*ArcTan[a*x]]/(a^2*c^2)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5467
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*(x_))/((d_) + (e_.)*(x_)^2)^2,
 x_Symbol] :> Simp[x*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)*(d + e*x^2
))), x] + (-Simp[(1 - c^2*x^2)*((a + b*ArcTan[c*x])^(p + 2)/(b^2*e*(p + 1)*
(p + 2)*(d + e*x^2))), x] - Simp[4/(b^2*(p + 1)*(p + 2))   Int[x*((a + b*Ar
cTan[c*x])^(p + 2)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[e, c^2*d] && LtQ[p, -1] && NeQ[p, -2]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

## Maple [A] (verified)

Time = 7.51 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.63

| method | result | size |
|---|---|---|
| derivativedivides | $-\frac{4\,\mathrm{Si}(2\arctan(ax))\arctan(ax)^{2}+2\cos(2\arctan(ax))\arctan(ax)+\sin(2\arctan(ax))}{4a^{2}c^{2}\arctan(ax)^{2}}$ | 51 |
| default | $-\frac{4\,\mathrm{Si}(2\arctan(ax))\arctan(ax)^{2}+2\cos(2\arctan(ax))\arctan(ax)+\sin(2\arctan(ax))}{4a^{2}c^{2}\arctan(ax)^{2}}$ | 51 |

input
```
int(x/(a^2*c*x^2+c)^2/arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4/a^2/c^2*(4*Si(2*arctan(a*x))*arctan(a*x)^2+2*cos(2*arctan(a*x))*arcta
n(a*x)+sin(2*arctan(a*x)))/arctan(a*x)^2
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.67

$$\int \frac{x}{(c+a^2cx^2)^2 \arctan(ax)^3} dx = \frac{(-i a^2x^2 - i) \arctan(ax)^2 \log\_integral\left(-\frac{a^2x^2+2i ax-1}{a^2x^2+1}\right) + (i a^2x^2 + i) \arctan(ax)^2 \log\_integral\left(-\frac{a^2x^2-}{a^2}\right.}{2(a^4c^2x^2 + a^2c^2) \arctan(ax)^2}$$

input
```
integrate(x/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
1/2*((-I*a^2*x^2 - I)*arctan(a*x)^2*log_integral(-(a^2*x^2 + 2*I*a*x - 1)/
(a^2*x^2 + 1)) + (I*a^2*x^2 + I)*arctan(a*x)^2*log_integral(-(a^2*x^2 - 2*
I*a*x - 1)/(a^2*x^2 + 1)) - a*x + (a^2*x^2 - 1)*arctan(a*x))/((a^4*c^2*x^2
 + a^2*c^2)*arctan(a*x)^2)
```

## Sympy [F]

$$\int \frac{x}{(c+a^2cx^2)^2 \arctan(ax)^3} dx = \frac{\int \frac{x}{a^4x^4 \operatorname{atan}^3(ax)+2a^2x^2 \operatorname{atan}^3(ax)+\operatorname{atan}^3(ax)} dx}{c^2}$$

input
```
integrate(x/(a**2*c*x**2+c)**2/atan(a*x)**3,x)
```

output
```
Integral(x/(a**4*x**4*atan(a*x)**3 + 2*a**2*x**2*atan(a*x)**3 + atan(a*x)*
*3), x)/c**2
```

## Maxima [F]

$$\int \frac{x}{(c+a^2cx^2)^2\arctan(ax)^3}\,dx = \int \frac{x}{(a^2cx^2+c)^2\arctan(ax)^3}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
-1/2*(8*(a^4*c^2*x^2 + a^2*c^2)*arctan(a*x)^2*integrate(1/2*x/((a^4*c^2*x^
4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)), x) + a*x - (a^2*x^2 - 1)*arctan(a*x
))/((a^4*c^2*x^2 + a^2*c^2)*arctan(a*x)^2)
```

## Giac [F]

$$\int \frac{x}{(c+a^2cx^2)^2\arctan(ax)^3}\,dx = \int \frac{x}{(a^2cx^2+c)^2\arctan(ax)^3}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate(x/((a^2*c*x^2 + c)^2*arctan(a*x)^3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(c+a^2cx^2)^2\arctan(ax)^3}\,dx = \int \frac{x}{\mathrm{atan}(a\,x)^3\,(c\,a^2\,x^2+c)^2}\,dx$$

input
```
int(x/(atan(a*x)^3*(c + a^2*c*x^2)^2),x)
```

output
```
int(x/(atan(a*x)^3*(c + a^2*c*x^2)^2), x)
```

## Reduce [F]

$$\int \frac{x}{(c+a^2cx^2)^2\arctan(ax)^3}\,dx$$
$$= \frac{-4atan(ax)^2\left(\int \frac{x}{atan(ax)a^4x^4+2atan(ax)a^2x^2+atan(ax)}dx\right)a^4x^2-4atan(ax)^2\left(\int \frac{x}{atan(ax)a^4x^4+2atan(ax)a^2x^2+atan(a}\right.}{2atan\,(ax)^2\,a^2c^2\,(a^2x^2+1)}$$

input
```
int(x/(a^2*c*x^2+c)^2/atan(a*x)^3,x)
```

output
```
( - 4*atan(a*x)**2*int(x/(atan(a*x)*a**4*x**4 + 2*atan(a*x)*a**2*x**2 + at
an(a*x)),x)*a**4*x**2 - 4*atan(a*x)**2*int(x/(atan(a*x)*a**4*x**4 + 2*atan
(a*x)*a**2*x**2 + atan(a*x)),x)*a**2 + atan(a*x)*a**2*x**2 - atan(a*x) - a
*x)/(2*atan(a*x)**2*a**2*c**2*(a**2*x**2 + 1))
```

# 3.629 $\int \frac{1}{(c+a^2cx^2)^2 \arctan(ax)^3} dx$



## Optimal result

Integrand size = 19, antiderivative size = 65

$$\int \frac{1}{(c+a^2cx^2)^2 \arctan(ax)^3} dx = -\frac{1}{2ac^2(1+a^2x^2)\arctan(ax)^2} + \frac{x}{c^2(1+a^2x^2)\arctan(ax)} - \frac{\text{CosIntegral}(2\arctan(ax))}{ac^2}$$

output
```
-1/2/a/c^2/(a^2*x^2+1)/arctan(a*x)^2+x/c^2/(a^2*x^2+1)/arctan(a*x)-Ci(2*ar
ctan(a*x))/a/c^2
```

## Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.89

$$\int \frac{1}{(c+a^2cx^2)^2 \arctan(ax)^3} dx = \frac{-1+2ax\arctan(ax)-2(1+a^2x^2)\arctan(ax)^2\,\text{CosIntegral}(2\arctan(ax))}{2c^2(a+a^3x^2)\arctan(ax)^2}$$

input
```
Integrate[1/((c + a^2*c*x^2)^2*ArcTan[a*x]^3),x]
```

output
```
(-1 + 2*a*x*ArcTan[a*x] - 2*(1 + a^2*x^2)*ArcTan[a*x]^2*CosIntegral[2*ArcT
an[a*x]])/(2*c^2*(a + a^3*x^2)*ArcTan[a*x]^2)
```

## Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.68, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.579, Rules used = {5437, 27, 5503, 5439, 3042, 3793, 2009, 5505, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arctan(ax)^3\left(a^2cx^2+c\right)^2}dx$$

$$\Big\downarrow \text{5437}$$

$$-a\int \frac{x}{c^2\left(a^2x^2+1\right)^2\arctan(ax)^2}dx-\frac{1}{2ac^2\left(a^2x^2+1\right)\arctan(ax)^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\int \frac{x}{\left(a^2x^2+1\right)^2\arctan(ax)^2}dx}{c^2}-\frac{1}{2ac^2\left(a^2x^2+1\right)\arctan(ax)^2}$$

$$\Big\downarrow \text{5503}$$

$$-\frac{a\left(\frac{\int \frac{1}{\left(a^2x^2+1\right)^2\arctan(ax)}dx}{a}-a\int \frac{x^2}{\left(a^2x^2+1\right)^2\arctan(ax)}dx-\frac{x}{a\left(a^2x^2+1\right)\arctan(ax)}\right)}{c^2}-\frac{1}{2ac^2\left(a^2x^2+1\right)\arctan(ax)^2}$$

$$\Big\downarrow \text{5439}$$

$$-\frac{a\left(-a\int \frac{x^2}{\left(a^2x^2+1\right)^2\arctan(ax)}dx+\frac{\int \frac{1}{\left(a^2x^2+1\right)\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{a\left(a^2x^2+1\right)\arctan(ax)}\right)}{c^2}-\frac{1}{2ac^2\left(a^2x^2+1\right)\arctan(ax)^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\left(-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^2}{\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)}{c^2}-\frac{1}{2ac^2\left(a^2x^2+1\right)\arctan(ax)^2}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{a\left(-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\int \left(\frac{\cos(2\arctan(ax))}{2\arctan(ax)}+\frac{1}{2\arctan(ax)}\right)d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)}{c^2}-\frac{1}{2ac^2\left(a^2x^2+1\right)\arctan(ax)^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a\left(-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)}{c^2}-\frac{1}{2ac^2\left(a^2x^2+1\right)\arctan(ax)^2}$$

$$\Big\downarrow \text{5505}$$

$$-\frac{a\left(-\frac{\int \frac{a^2x^2}{\left(a^2x^2+1\right)\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)}{c^2}-\frac{1}{2ac^2\left(a^2x^2+1\right)\arctan(ax)^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\left(-\frac{\int \frac{\sin(\arctan(ax))^2}{\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)}{c^2}-\frac{1}{2ac^2\left(a^2x^2+1\right)\arctan(ax)^2}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{a\left(-\frac{\int \left(\frac{1}{2\arctan(ax)}-\frac{\cos(2\arctan(ax))}{2\arctan(ax)}\right)d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)}{c^2}-\frac{1}{2ac^2\left(a^2x^2+1\right)\arctan(ax)^2}$$

$$\Big\downarrow \; 2009$$

$$-\frac{a\left(-\frac{\frac{1}{2}\log(\arctan(ax))-\frac{1}{2}\text{CosIntegral}(2\arctan(ax))}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)}{c^2}-\frac{1}{2ac^2\left(a^2x^2+1\right)\arctan(ax)^2}$$

input
```
Int[1/((c + a^2*c*x^2)^2*ArcTan[a*x]^3),x]
```

output
```
-1/2*1/(a*c^2*(1 + a^2*x^2)*ArcTan[a*x]^2) - (a*(-(x/(a*(1 + a^2*x^2)*ArcT
an[a*x])) - (-1/2*CosIntegral[2*ArcTan[a*x]] + Log[ArcTan[a*x]]/2)/a^2 + (
CosIntegral[2*ArcTan[a*x]]/2 + Log[ArcTan[a*x]]/2)/a^2))/c^2
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

## Maple [A] (verified)

Time = 3.38 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.80

| method | result | size |
|---|---|---|
| derivativedivides | $-\frac{4\,\mathrm{Ci}(2\arctan(ax))\arctan(ax)^2-2\sin(2\arctan(ax))\arctan(ax)+\cos(2\arctan(ax))+1}{4a\,c^2\arctan(ax)^2}$ | 52 |
| default | $-\frac{4\,\mathrm{Ci}(2\arctan(ax))\arctan(ax)^2-2\sin(2\arctan(ax))\arctan(ax)+\cos(2\arctan(ax))+1}{4a\,c^2\arctan(ax)^2}$ | 52 |

input
```
int(1/(a^2*c*x^2+c)^2/arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4/a/c^2*(4*Ci(2*arctan(a*x))*arctan(a*x)^2-2*sin(2*arctan(a*x))*arctan(
a*x)+cos(2*arctan(a*x))+1)/arctan(a*x)^2
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.88

$$\int \frac{1}{(c+a^2cx^2)^2\arctan(ax)^3}\,dx = -\frac{(a^2x^2+1)\arctan{(ax)}^2\operatorname{log\_integral}\left(-\frac{a^2x^2+2i\,ax-1}{a^2x^2+1}\right)+(a^2x^2+1)\arctan{(ax)}^2\operatorname{log\_integral}\left(-\frac{a^2x^2-2}{a^2x^2}\right)}{2\,(a^3c^2x^2+ac^2)\arctan{(ax)}^2}$$

input
```
integrate(1/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
-1/2*((a^2*x^2 + 1)*arctan(a*x)^2*log_integral(-(a^2*x^2 + 2*I*a*x - 1)/(a
^2*x^2 + 1)) + (a^2*x^2 + 1)*arctan(a*x)^2*log_integral(-(a^2*x^2 - 2*I*a*
x - 1)/(a^2*x^2 + 1)) - 2*a*x*arctan(a*x) + 1)/((a^3*c^2*x^2 + a*c^2)*arct
an(a*x)^2)
```

## Sympy [F]

$$\int \frac{1}{(c+a^2cx^2)^2\arctan(ax)^3}\,dx = \frac{\int \frac{1}{a^4x^4\operatorname{atan}^3{(ax)}+2a^2x^2\operatorname{atan}^3{(ax)}+\operatorname{atan}^3{(ax)}}\,dx}{c^2}$$

input
```
integrate(1/(a**2*c*x**2+c)**2/atan(a*x)**3,x)
```

output
```
Integral(1/(a**4*x**4*atan(a*x)**3 + 2*a**2*x**2*atan(a*x)**3 + atan(a*x)*
*3), x)/c**2
```

## Maxima [F]

$$\int \frac{1}{(c+a^2cx^2)^2 \arctan(ax)^3} \, dx = \int \frac{1}{(a^2cx^2+c)^2 \arctan(ax)^3} \, dx$$

input
```
integrate(1/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
1/2*(2*(a^3*c^2*x^2 + a*c^2)*arctan(a*x)^2*integrate((a^2*x^2 - 1)/((a^4*c
^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)), x) + 2*a*x*arctan(a*x) - 1)/((
a^3*c^2*x^2 + a*c^2)*arctan(a*x)^2)
```

## Giac [F]

$$\int \frac{1}{(c+a^2cx^2)^2 \arctan(ax)^3} \, dx = \int \frac{1}{(a^2cx^2+c)^2 \arctan(ax)^3} \, dx$$

input
```
integrate(1/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^2*arctan(a*x)^3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+a^2cx^2)^2 \arctan(ax)^3} \, dx = \int \frac{1}{\mathrm{atan}(a\,x)^3\,(c\,a^2\,x^2+c)^2} \, dx$$

input
```
int(1/(atan(a*x)^3*(c + a^2*c*x^2)^2),x)
```

output
```
int(1/(atan(a*x)^3*(c + a^2*c*x^2)^2), x)
```

## Reduce [F]

$$\int \frac{1}{(c+a^2cx^2)^2 \arctan(ax)^3} \, dx$$
$$= \frac{-2atan(ax)^2 \left(\int \frac{x}{atan(ax)^2a^4x^4+2atan(ax)^2a^2x^2+atan(ax)^2}dx\right)a^4x^2 - 2atan(ax)^2 \left(\int \frac{x}{atan(ax)^2a^4x^4+2atan(ax)^2a^2x^2+a}\right.}{2atan(ax)^2 a c^2 (a^2x^2+1)}$$

input
```
int(1/(a^2*c*x^2+c)^2/atan(a*x)^3,x)
```

output
```
( - 2*atan(a*x)**2*int(x/(atan(a*x)**2*a**4*x**4 + 2*atan(a*x)**2*a**2*x**
2 + atan(a*x)**2),x)*a**4*x**2 - 2*atan(a*x)**2*int(x/(atan(a*x)**2*a**4*x
**4 + 2*atan(a*x)**2*a**2*x**2 + atan(a*x)**2),x)*a**2 - 1)/(2*atan(a*x)**
2*a*c**2*(a**2*x**2 + 1))
```

# 3.630 $\int \frac{1}{x(c+a^2cx^2)^2\arctan(ax)^3}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x(c+a^2cx^2)^2\arctan(ax)^3}\,dx = -\frac{1}{2ac^2x\arctan(ax)^2} + \frac{ax}{2c^2(1+a^2x^2)\arctan(ax)^2} + \frac{1-a^2x^2}{2c^2(1+a^2x^2)\arctan(ax)} + \frac{\mathrm{Si}(2\arctan(ax))}{c^2} - \frac{\mathrm{Int}\left(\frac{1}{x^2\arctan(ax)^2},x\right)}{2ac^2}$$

output
```
-1/2/a/c^2/x/arctan(a*x)^2+1/2*a*x/c^2/(a^2*x^2+1)/arctan(a*x)^2+1/2*(-a^2
*x^2+1)/c^2/(a^2*x^2+1)/arctan(a*x)+Si(2*arctan(a*x))/c^2-1/2*Defer(Int)(1
/x^2/arctan(a*x)^2,x)/a/c^2
```

## Mathematica [N/A]

Not integrable

Time = 1.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x(c+a^2cx^2)^2\arctan(ax)^3}\,dx = \int \frac{1}{x(c+a^2cx^2)^2\arctan(ax)^3}\,dx$$

input
```
Integrate[1/(x*(c + a^2*c*x^2)^2*ArcTan[a*x]^3),x]
```

output
```
Integrate[1/(x*(c + a^2*c*x^2)^2*ArcTan[a*x]^3), x]
```

## Rubi [N/A]

Not integrable

Time = 0.90 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x \arctan(ax)^3 \left(a^2cx^2+c\right)^2}dx$$

$$\Big\downarrow \text{ 5501}$$

$$\frac{\int \frac{1}{cx(a^2x^2+1)\arctan(ax)^3}dx}{c}-a^2\int \frac{x}{c^2\left(a^2x^2+1\right)^2\arctan(ax)^3}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{1}{x(a^2x^2+1)\arctan(ax)^3}dx}{c^2}-\frac{a^2\int \frac{x}{\left(a^2x^2+1\right)^2\arctan(ax)^3}dx}{c^2}$$

$$\Big\downarrow \text{ 5461}$$

$$\frac{-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{1}{2ax\arctan(ax)^2}}{c^2}-\frac{a^2\int \frac{x}{\left(a^2x^2+1\right)^2\arctan(ax)^3}dx}{c^2}$$

$$\Big\downarrow \text{ 5377}$$

$$\frac{-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{1}{2ax\arctan(ax)^2}}{c^2}-\frac{a^2\int \frac{x}{\left(a^2x^2+1\right)^2\arctan(ax)^3}dx}{c^2}$$

$$\Big\downarrow \text{ 5467}$$

$$\frac{-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{1}{2ax\arctan(ax)^2}}{c^2}-\frac{a^2\left(-2\int \frac{x}{(a^2x^2+1)^2\arctan(ax)}dx-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\right)}{c^2}$$

$\downarrow$ 5505

$$\frac{-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{1}{2ax\arctan(ax)^2}}{c^2}-\frac{a^2\left(-\frac{2\int \frac{ax}{(a^2x^2+1)\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\right)}{c^2}$$

$\downarrow$ 4906

$$\frac{-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{1}{2ax\arctan(ax)^2}}{c^2}-\frac{a^2\left(-\frac{2\int \frac{\sin(2\arctan(ax))}{2\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\right)}{c^2}$$

$\downarrow$ 27

$$\frac{-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{1}{2ax\arctan(ax)^2}}{c^2}-\frac{a^2\left(-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\right)}{c^2}$$

$\downarrow$ 3042

$$\frac{-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{1}{2ax\arctan(ax)^2}}{c^2}-\frac{a^2\left(-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\right)}{c^2}$$

$\downarrow$ 3780

$$\frac{-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{1}{2ax\arctan(ax)^2}}{c^2}-\frac{a^2\left(-\frac{\text{Si}(2\arctan(ax))}{a^2}-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\right)}{c^2}$$

input
```
Int[1/(x*(c + a^2*c*x^2)^2*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 1.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a^{2} c\,x^{2}+c\right)^{2} \arctan\left(a x\right)^{3}}dx$$

input
```
int(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

output
```
int(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.73

$$\int \frac{1}{x\left(c+a^{2} c x^{2}\right)^{2} \arctan(ax)^{3}}\, dx = \int \frac{1}{\left(a^{2} c x^{2}+c\right)^{2} x \arctan\left(ax\right)^{3}}\, dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(1/((a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^2*x)*arctan(a*x)^3), x)
```

## Sympy [N/A]

Not integrable

Time = 1.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.91

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\arctan(ax)^3}\,dx=\frac{\int \frac{1}{a^4x^5\operatorname{atan}^3(ax)+2a^2x^3\operatorname{atan}^3(ax)+x\operatorname{atan}^3(ax)}\,dx}{c^2}$$

input
```
integrate(1/x/(a**2*c*x**2+c)**2/atan(a*x)**3,x)
```

output
```
Integral(1/(a**4*x**5*atan(a*x)**3 + 2*a**2*x**3*atan(a*x)**3 + x*atan(a*x
)**3), x)/c**2
```

## Maxima [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 141, normalized size of antiderivative = 6.41

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\arctan(ax)^3}\,dx=\int \frac{1}{\left(a^2cx^2+c\right)^2x\arctan\left(ax\right)^3}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
1/2*(2*(a^4*c^2*x^4 + a^2*c^2*x^2)*arctan(a*x)^2*integrate((3*a^4*x^4 + 2*
a^2*x^2 + 1)/((a^6*c^2*x^7 + 2*a^4*c^2*x^5 + a^2*c^2*x^3)*arctan(a*x)), x)
 - a*x + (3*a^2*x^2 + 1)*arctan(a*x))/((a^4*c^2*x^4 + a^2*c^2*x^2)*arctan(
a*x)^2)
```

## Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x(c+a^2cx^2)^2 \arctan(ax)^3} \, dx = \int \frac{1}{(a^2cx^2+c)^2 x \arctan(ax)^3} \, dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^2*x*arctan(a*x)^3), x)
```

## Mupad [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x(c+a^2cx^2)^2 \arctan(ax)^3} \, dx = \int \frac{1}{x \operatorname{atan}(a x)^3 (c a^2 x^2 + c)^2} \, dx$$

input
```
int(1/(x*atan(a*x)^3*(c + a^2*c*x^2)^2),x)
```

output
```
int(1/(x*atan(a*x)^3*(c + a^2*c*x^2)^2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.00

$$\int \frac{1}{x(c+a^2cx^2)^2 \arctan(ax)^3} \, dx = \frac{\int \frac{1}{atan(ax)^3 a^4 x^5 + 2atan(ax)^3 a^2 x^3 + atan(ax)^3 x} dx}{c^2}$$

input
```
int(1/x/(a^2*c*x^2+c)^2/atan(a*x)^3,x)
```

output
```
int(1/(atan(a*x)**3*a**4*x**5 + 2*atan(a*x)**3*a**2*x**3 + atan(a*x)**3*x)
,x)/c**2
```

# 3.631 $\int \frac{1}{x^2(c+a^2cx^2)^2 \arctan(ax)^3} dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x^2 (c+a^2cx^2)^2 \arctan(ax)^3} dx = -\frac{1}{2ac^2x^2 \arctan(ax)^2} + \frac{a}{2c^2 (1+a^2x^2) \arctan(ax)^2} - \frac{a^2x}{c^2 (1+a^2x^2) \arctan(ax)} + \frac{a \operatorname{CosIntegral}(2\arctan(ax))}{c^2} - \frac{\operatorname{Int}\left(\frac{1}{x^3 \arctan(ax)^2}, x\right)}{ac^2}$$

output

```
-1/2/a/c^2/x^2/arctan(a*x)^2+1/2*a/c^2/(a^2*x^2+1)/arctan(a*x)^2-a^2*x/c^2
/(a^2*x^2+1)/arctan(a*x)+a*Ci(2*arctan(a*x))/c^2-Defer(Int)(1/x^3/arctan(a
*x)^2,x)/a/c^2
```

## Mathematica [N/A]

Not integrable

Time = 1.91 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2 (c+a^2cx^2)^2 \arctan(ax)^3} \, dx = \int \frac{1}{x^2 (c+a^2cx^2)^2 \arctan(ax)^3} \, dx$$

input
```
Integrate[1/(x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^3),x]
```

output
```
Integrate[1/(x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^3), x]
```

## Rubi [N/A]

Not integrable

Time = 1.50 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \arctan(ax)^3 (a^2cx^2+c)^2} \, dx$$

$$\downarrow \text{5501}$$

$$\frac{\int \frac{1}{cx^2(a^2x^2+1)\arctan(ax)^3}dx}{c} - a^2 \int \frac{1}{c^2 (a^2x^2+1)^2 \arctan(ax)^3}dx$$

$$\downarrow \text{27}$$

$$\frac{\int \frac{1}{x^2(a^2x^2+1)\arctan(ax)^3}dx}{c^2} - \frac{a^2 \int \frac{1}{(a^2x^2+1)^2\arctan(ax)^3}dx}{c^2}$$

$$\downarrow \text{5437}$$

$$\frac{\int \frac{1}{x^2(a^2x^2+1)\arctan(ax)^3}dx}{c^2} - \frac{a^2 \left(-a \int \frac{x}{(a^2x^2+1)^2\arctan(ax)^2}dx - \frac{1}{2a(a^2x^2+1)\arctan(ax)^2}\right)}{c^2}$$

$$\Big\downarrow \text{ 5461}$$

$$\frac{-\frac{\int \frac{1}{x^3\arctan(ax)^2}dx}{a}-\frac{1}{2ax^2\arctan(ax)^2}}{c^2}-\frac{a^2\left(-a\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^2}dx-\frac{1}{2a(a^2x^2+1)\arctan(ax)^2}\right)}{c^2}$$

$$\Big\downarrow \text{ 5377}$$

$$\frac{-\frac{\int \frac{1}{x^3\arctan(ax)^2}dx}{a}-\frac{1}{2ax^2\arctan(ax)^2}}{c^2}-\frac{a^2\left(-a\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^2}dx-\frac{1}{2a(a^2x^2+1)\arctan(ax)^2}\right)}{c^2}$$

$$\Big\downarrow \text{ 5503}$$

$$\frac{-\frac{\int \frac{1}{x^3\arctan(ax)^2}dx}{a}-\frac{1}{2ax^2\arctan(ax)^2}}{c^2}-\frac{a^2\left(-a\left(\frac{\int \frac{1}{(a^2x^2+1)^2\arctan(ax)}dx}{a}-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)-\frac{1}{2a(a^2x^2+1)\arctan(ax)^2}\right)}{c^2}$$

$$\Big\downarrow \text{ 5439}$$

$$\frac{-\frac{\int \frac{1}{x^3\arctan(ax)^2}dx}{a}-\frac{1}{2ax^2\arctan(ax)^2}}{c^2}-\frac{a^2\left(-a\left(-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\int \frac{1}{(a^2x^2+1)\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)-\frac{1}{2a(a^2x^2+1)\arctan(ax)^2}\right)}{c^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{-\frac{\int \frac{1}{x^3\arctan(ax)^2}dx}{a}-\frac{1}{2ax^2\arctan(ax)^2}}{c^2}-\frac{a^2\left(-a\left(-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^2}{\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)-\frac{1}{2a(a^2x^2+1)\arctan(ax)^2}\right)}{c^2}$$

$$\Big\downarrow \text{ 3793}$$

$$\frac{-\frac{\int \frac{1}{x^3\arctan(ax)^2}dx}{a}-\frac{1}{2ax^2\arctan(ax)^2}}{c^2}-\frac{a^2\left(-a\left(-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\int \left(\frac{\cos(2\arctan(ax))}{2\arctan(ax)}+\frac{1}{2\arctan(ax)}\right)d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)-\frac{1}{2a(a^2x^2+1)\arc}\right)}{c^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{-\frac{\int \frac{1}{x^3 \arctan(ax)^2}dx}{a}-\frac{1}{2ax^2\arctan(ax)^2}}{c^2}-}{\phantom{x}}$$

$$\frac{a^2\left(-a\left(-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)-\frac{1}{2a(a^2x^2+1)\,\text{arc}}\right.}{c^2}$$

$$\Big\downarrow \text{5505}$$

$$\frac{\frac{-\frac{\int \frac{1}{x^3 \arctan(ax)^2}dx}{a}-\frac{1}{2ax^2\arctan(ax)^2}}{c^2}-}{\phantom{x}}$$

$$\frac{a^2\left(-a\left(-\frac{\int \frac{a^2x^2}{(a^2x^2+1)\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)-\frac{}{2a(a^2x^2}\right.}{c^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{-\frac{\int \frac{1}{x^3 \arctan(ax)^2}dx}{a}-\frac{1}{2ax^2\arctan(ax)^2}}{c^2}-}{\phantom{x}}$$

$$\frac{a^2\left(-a\left(-\frac{\int \frac{\sin(\arctan(ax))^2}{\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)-\frac{1}{2a(a^2x^2+1)\,\text{a}}\right.}{c^2}$$

$$\Big\downarrow \text{3793}$$

$$\frac{\frac{-\frac{\int \frac{1}{x^3 \arctan(ax)^2}dx}{a}-\frac{1}{2ax^2\arctan(ax)^2}}{c^2}-}{\phantom{x}}$$

$$\frac{a^2\left(-a\left(-\frac{\int \left(\frac{1}{2\arctan(ax)}-\frac{\cos(2\arctan(ax))}{2\arctan(ax)}\right)d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)-\right.}{c^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{-\frac{\int \frac{1}{x^3 \arctan(ax)^2}dx}{a}-\frac{1}{2ax^2\arctan(ax)^2}}{c^2}-}{\phantom{x}}$$

$$\frac{a^2\left(-a\left(-\frac{\frac{1}{2}\log(\arctan(ax))-\frac{1}{2}\text{CosIntegral}(2\arctan(ax))}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)-\right.}{c^2}$$

input
```
Int[1/(x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 1.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{2} \left(a^{2} c \,x^{2}+c \right)^{2} \arctan \left(a x \right)^{3}}d x$$

input
```
int(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

output
```
int(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.82

$$\int \frac{1}{x^2 (c+a^2cx^2)^2 \arctan(ax)^3} \, dx = \int \frac{1}{{\left(a^{2} c x^{2} + c\right)}^{2} x^{2} \arctan \left(a x\right)^{3}} \, dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(1/((a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2)*arctan(a*x)^3), x)
```

## Sympy [N/A]

Not integrable

Time = 1.11 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.00

$$\int \frac{1}{x^2 (c+a^2cx^2)^2 \arctan(ax)^3} \, dx = \frac{\int \frac{1}{a^{4} x^{6} \operatorname{atan}^{3}{\left(a x \right)} + 2 a^{2} x^{4} \operatorname{atan}^{3}{\left(a x \right)} + x^{2} \operatorname{atan}^{3}{\left(a x \right)}}\, dx}{c^{2}}$$

input
```
integrate(1/x**2/(a**2*c*x**2+c)**2/atan(a*x)**3,x)
```

output
```
Integral(1/(a**4*x**6*atan(a*x)**3 + 2*a**2*x**4*atan(a*x)**3 + x**2*atan(
a*x)**3), x)/c**2
```

## Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 142, normalized size of antiderivative = 6.45

$$\int \frac{1}{x^2 (c+a^2cx^2)^2 \arctan(ax)^3} \, dx = \int \frac{1}{(a^2cx^2+c)^2 x^2 \arctan(ax)^3} \, dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
1/2*(2*(a^4*c^2*x^5 + a^2*c^2*x^3)*arctan(a*x)^2*integrate((6*a^4*x^4 + 7*
a^2*x^2 + 3)/((a^6*c^2*x^8 + 2*a^4*c^2*x^6 + a^2*c^2*x^4)*arctan(a*x)), x)
 - a*x + 2*(2*a^2*x^2 + 1)*arctan(a*x))/((a^4*c^2*x^5 + a^2*c^2*x^3)*arcta
n(a*x)^2)
```

## Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2 (c+a^2cx^2)^2 \arctan(ax)^3} \, dx = \int \frac{1}{(a^2cx^2+c)^2 x^2 \arctan(ax)^3} \, dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^2*x^2*arctan(a*x)^3), x)
```

## Mupad [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2 (c + a^2cx^2)^2 \arctan(ax)^3} \, dx = \int \frac{1}{x^2 \operatorname{atan}(a\,x)^3 \left(c\,a^2\,x^2 + c\right)^2} \, dx$$

input
```
int(1/(x^2*atan(a*x)^3*(c + a^2*c*x^2)^2),x)
```

output
```
int(1/(x^2*atan(a*x)^3*(c + a^2*c*x^2)^2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.09

$$\int \frac{1}{x^2 (c + a^2cx^2)^2 \arctan(ax)^3} \, dx = \frac{\int \frac{1}{atan(ax)^3 a^4 x^6 + 2atan(ax)^3 a^2 x^4 + atan(ax)^3 x^2} dx}{c^2}$$

input
```
int(1/x^2/(a^2*c*x^2+c)^2/atan(a*x)^3,x)
```

output
```
int(1/(atan(a*x)**3*a**4*x**6 + 2*atan(a*x)**3*a**2*x**4 + atan(a*x)**3*x*
*2),x)/c**2
```

# 3.632 $\int \frac{1}{x^3(c+a^2cx^2)^2 \arctan(ax)^3} dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\begin{aligned}\int \frac{1}{x^3 (c+a^2cx^2)^2 \arctan(ax)^3} dx = & -\frac{1}{2ac^2x^3 \arctan(ax)^2} + \frac{a}{2c^2x \arctan(ax)^2} \\ & - \frac{a^3x}{2c^2 (1+a^2x^2) \arctan(ax)^2} \\ & - \frac{a^2(1-a^2x^2)}{2c^2 (1+a^2x^2) \arctan(ax)} - \frac{a^2 \mathrm{Si}(2\arctan(ax))}{c^2} \\ & - \frac{3\mathrm{Int}\left(\frac{1}{x^4 \arctan(ax)^2}, x\right)}{2ac^2} + \frac{a\mathrm{Int}\left(\frac{1}{x^2 \arctan(ax)^2}, x\right)}{2c^2}\end{aligned}$$

output
```
-1/2/a/c^2/x^3/arctan(a*x)^2+1/2*a/c^2/x/arctan(a*x)^2-1/2*a^3*x/c^2/(a^2*
x^2+1)/arctan(a*x)^2-1/2*a^2*(-a^2*x^2+1)/c^2/(a^2*x^2+1)/arctan(a*x)-a^2*
Si(2*arctan(a*x))/c^2-3/2*Defer(Int)(1/x^4/arctan(a*x)^2,x)/a/c^2+1/2*a*De
fer(Int)(1/x^2/arctan(a*x)^2,x)/c^2
```

## Mathematica [N/A]

Not integrable

Time = 1.48 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^3 (c+a^2cx^2)^2 \arctan(ax)^3} dx = \int \frac{1}{x^3 (c+a^2cx^2)^2 \arctan(ax)^3} dx$$

input
```
Integrate[1/(x^3*(c + a^2*c*x^2)^2*ArcTan[a*x]^3),x]
```

output
```
Integrate[1/(x^3*(c + a^2*c*x^2)^2*ArcTan[a*x]^3), x]
```

## Rubi [N/A]

Not integrable

Time = 1.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \arctan(ax)^3 (a^2cx^2+c)^2} dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{1}{cx^3(a^2x^2+1)\arctan(ax)^3}dx}{c} - a^2 \int \frac{1}{c^2x (a^2x^2+1)^2 \arctan(ax)^3}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{1}{x^3(a^2x^2+1)\arctan(ax)^3}dx}{c^2} - \frac{a^2 \int \frac{1}{x(a^2x^2+1)^2\arctan(ax)^3}dx}{c^2}$$

$$\Big\downarrow \text{5461}$$

$$\frac{-\frac{3\int \frac{1}{x^4\arctan(ax)^2}dx}{2a} - \frac{1}{2ax^3\arctan(ax)^2}}{c^2} - \frac{a^2 \int \frac{1}{x(a^2x^2+1)^2\arctan(ax)^3}dx}{c^2}$$

$$\Big\downarrow \text{5377}$$

$$\frac{-\frac{3\int \frac{1}{x^4\arctan(ax)^2}dx}{2a}-\frac{1}{2ax^3\arctan(ax)^2}}{c^2}-\frac{a^2\int \frac{1}{x(a^2x^2+1)^2\arctan(ax)^3}dx}{c^2}$$

$$\Big\downarrow \text{5501}$$

$$\frac{-\frac{3\int \frac{1}{x^4\arctan(ax)^2}dx}{2a}-\frac{1}{2ax^3\arctan(ax)^2}}{c^2}-\frac{a^2\left(\int \frac{1}{x(a^2x^2+1)\arctan(ax)^3}dx-a^2\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^3}dx\right)}{c^2}$$

$$\Big\downarrow \text{5461}$$

$$\frac{-\frac{3\int \frac{1}{x^4\arctan(ax)^2}dx}{2a}-\frac{1}{2ax^3\arctan(ax)^2}}{c^2}-\frac{a^2\left(a^2\left(-\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^3}dx\right)-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{1}{2ax\arctan(ax)^2}\right)}{c^2}$$

$$\Big\downarrow \text{5377}$$

$$\frac{-\frac{3\int \frac{1}{x^4\arctan(ax)^2}dx}{2a}-\frac{1}{2ax^3\arctan(ax)^2}}{c^2}-\frac{a^2\left(a^2\left(-\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^3}dx\right)-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{1}{2ax\arctan(ax)^2}\right)}{c^2}$$

$$\Big\downarrow \text{5467}$$

$$\frac{-\frac{3\int \frac{1}{x^4\arctan(ax)^2}dx}{2a}-\frac{1}{2ax^3\arctan(ax)^2}}{c^2}-\frac{a^2\left(-\left(a^2\left(-2\int \frac{x}{(a^2x^2+1)^2\arctan(ax)}dx-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{1}{2ax\arctan(ax)^2}\right)}{c^2}$$

$$\Big\downarrow \text{5505}$$

$$\frac{-\frac{3\int \frac{1}{x^4\arctan(ax)^2}dx}{2a}-\frac{1}{2ax^3\arctan(ax)^2}}{c^2}-\frac{a^2\left(-\left(a^2\left(-\frac{2\int \frac{ax}{(a^2x^2+1)\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\right.}{c^2}$$

$$\Big\downarrow \text{4906}$$

$$\frac{\frac{-\frac{3\int \frac{1}{x^4 \arctan(ax)^2}dx}{2a}-\frac{1}{2ax^3\arctan(ax)^2}}{c^2}-}{}$$

$$\frac{a^2\left(-\left(a^2\left(-\frac{2\int \frac{\sin(2\arctan(ax))}{2\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{}{2ax\,a}\right.}{c^2}$$

$$\downarrow \text{27}$$

$$\frac{\frac{-\frac{3\int \frac{1}{x^4 \arctan(ax)^2}dx}{2a}-\frac{1}{2ax^3\arctan(ax)^2}}{c^2}-}{}$$

$$\frac{a^2\left(-\left(a^2\left(-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{}{2ax\,\mathrm{arc}}\right.}{c^2}$$

$$\downarrow \text{3042}$$

$$\frac{\frac{-\frac{3\int \frac{1}{x^4 \arctan(ax)^2}dx}{2a}-\frac{1}{2ax^3\arctan(ax)^2}}{c^2}-}{}$$

$$\frac{a^2\left(-\left(a^2\left(-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{}{2ax\,\mathrm{arc}}\right.}{c^2}$$

$$\downarrow \text{3780}$$

$$\frac{\frac{-\frac{3\int \frac{1}{x^4 \arctan(ax)^2}dx}{2a}-\frac{1}{2ax^3\arctan(ax)^2}}{c^2}-}{}$$

$$\frac{a^2\left(-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\left(a^2\left(-\frac{\mathrm{Si}(2\arctan(ax))}{a^2}-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{1}{2ax\arctan(ax)^2}\right)}{c^2}$$

input
```
Int[1/(x^3*(c + a^2*c*x^2)^2*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 8.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^{3} \left(a^{2} c \,x^{2}+c \right)^{2} \arctan \left(a x \right)^{3}}d x$$

input
```
int(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

output
```
int(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.82

$$\int \frac{1}{x^3 \left(c+a^2 c x^2\right)^2 \arctan(a x)^3} \, dx = \int \frac{1}{{\left(a^{2} c x^{2} + c\right)}^{2} x^{3} \arctan \left(a x\right)^{3}} \,d x$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(1/((a^4*c^2*x^7 + 2*a^2*c^2*x^5 + c^2*x^3)*arctan(a*x)^3), x)
```

## Sympy [N/A]

Not integrable

Time = 1.13 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.00

$$\int \frac{1}{x^3 \left(c+a^2 c x^2\right)^2 \arctan(a x)^3} \, dx = \frac{\int \frac{1}{a^{4} x^{7} \operatorname{atan}^{3}{\left(a x \right)} + 2 a^{2} x^{5} \operatorname{atan}^{3}{\left(a x \right)} + x^{3} \operatorname{atan}^{3}{\left(a x \right)}}\, dx}{c^{2}}$$

input
```
integrate(1/x**3/(a**2*c*x**2+c)**2/atan(a*x)**3,x)
```

output
```
Integral(1/(a**4*x**7*atan(a*x)**3 + 2*a**2*x**5*atan(a*x)**3 + x**3*atan(
a*x)**3), x)/c**2
```

## Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 142, normalized size of antiderivative = 6.45

$$\int \frac{1}{x^3 (c+a^2cx^2)^2 \arctan(ax)^3} \, dx = \int \frac{1}{(a^2cx^2+c)^2 x^3 \arctan(ax)^3} \, dx$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
1/2*(2*(a^4*c^2*x^6 + a^2*c^2*x^4)*arctan(a*x)^2*integrate(2*(5*a^4*x^4 +
7*a^2*x^2 + 3)/((a^6*c^2*x^9 + 2*a^4*c^2*x^7 + a^2*c^2*x^5)*arctan(a*x)),
x) - a*x + (5*a^2*x^2 + 3)*arctan(a*x))/((a^4*c^2*x^6 + a^2*c^2*x^4)*arcta
n(a*x)^2)
```

## Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^3 (c+a^2cx^2)^2 \arctan(ax)^3} \, dx = \int \frac{1}{(a^2cx^2+c)^2 x^3 \arctan(ax)^3} \, dx$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^2*x^3*arctan(a*x)^3), x)
```

## Mupad [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^3 (c+a^2cx^2)^2 \arctan(ax)^3} dx = \int \frac{1}{x^3 \operatorname{atan}(a\,x)^3 \left(c\,a^2\,x^2+c\right)^2} dx$$

input
```
int(1/(x^3*atan(a*x)^3*(c + a^2*c*x^2)^2),x)
```

output
```
int(1/(x^3*atan(a*x)^3*(c + a^2*c*x^2)^2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.09

$$\int \frac{1}{x^3 (c+a^2cx^2)^2 \arctan(ax)^3} dx = \frac{\int \frac{1}{atan(ax)^3 a^4 x^7+2 atan(ax)^3 a^2 x^5+atan(ax)^3 x^3} dx}{c^2}$$

input
```
int(1/x^3/(a^2*c*x^2+c)^2/atan(a*x)^3,x)
```

output
```
int(1/(atan(a*x)**3*a**4*x**7 + 2*atan(a*x)**3*a**2*x**5 + atan(a*x)**3*x*
*3),x)/c**2
```

# 3.633 $\int \frac{1}{x^4(c+a^2cx^2)^2 \arctan(ax)^3} dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x^4 (c + a^2cx^2)^2 \arctan(ax)^3} dx = -\frac{1}{2ac^2x^4 \arctan(ax)^2} + \frac{a}{2c^2x^2 \arctan(ax)^2} - \frac{a^3}{2c^2 (1 + a^2x^2) \arctan(ax)^2} + \frac{a^4x}{c^2 (1 + a^2x^2) \arctan(ax)} - \frac{a^3 \operatorname{CosIntegral}(2 \arctan(ax))}{c^2} - \frac{2\operatorname{Int}\left(\frac{1}{x^5 \arctan(ax)^2}, x\right)}{ac^2} + \frac{a\operatorname{Int}\left(\frac{1}{x^3 \arctan(ax)^2}, x\right)}{c^2}$$

output
```
-1/2/a/c^2/x^4/arctan(a*x)^2+1/2*a/c^2/x^2/arctan(a*x)^2-1/2*a^3/c^2/(a^2*
x^2+1)/arctan(a*x)^2+a^4*x/c^2/(a^2*x^2+1)/arctan(a*x)-a^3*Ci(2*arctan(a*x
))/c^2-2*Defer(Int)(1/x^5/arctan(a*x)^2,x)/a/c^2+a*Defer(Int)(1/x^3/arctan
(a*x)^2,x)/c^2
```

## Mathematica [N/A]

Not integrable

Time = 5.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^4 \left(c+a^2 c x^2\right)^2 \arctan(ax)^3} \, dx=\int \frac{1}{x^4 \left(c+a^2 c x^2\right)^2 \arctan(ax)^3} \, dx$$

input
```
Integrate[1/(x^4*(c + a^2*c*x^2)^2*ArcTan[a*x]^3),x]
```

output
```
Integrate[1/(x^4*(c + a^2*c*x^2)^2*ArcTan[a*x]^3), x]
```

## Rubi [N/A]

Not integrable

Time = 2.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \arctan(ax)^3 \left(a^2 c x^2+c\right)^2} \, dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{1}{c x^4 (a^2x^2+1) \arctan(ax)^3}dx}{c}-a^2 \int \frac{1}{c^2 x^2 \left(a^2 x^2+1\right)^2 \arctan(ax)^3}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{1}{x^4 (a^2x^2+1) \arctan(ax)^3}dx}{c^2}-\frac{a^2 \int \frac{1}{x^2 \left(a^2 x^2+1\right)^2 \arctan(ax)^3}dx}{c^2}$$

$$\Big\downarrow \text{5461}$$

$$\frac{-\frac{2 \int \frac{1}{x^5 \arctan(ax)^2}dx}{a}-\frac{1}{2 a x^4 \arctan(ax)^2}}{c^2}-\frac{a^2 \int \frac{1}{x^2 \left(a^2 x^2+1\right)^2 \arctan(ax)^3}dx}{c^2}$$

$$\Big\downarrow \text{5377}$$

$$\frac{-\frac{2\int \frac{1}{x^5 \arctan(ax)^2}dx}{a}-\frac{1}{2ax^4\arctan(ax)^2}}{c^2}-\frac{a^2\int \frac{1}{x^2(a^2x^2+1)^2\arctan(ax)^3}dx}{c^2}$$

$$\Big\downarrow \text{5501}$$

$$\frac{-\frac{2\int \frac{1}{x^5 \arctan(ax)^2}dx}{a}-\frac{1}{2ax^4\arctan(ax)^2}}{c^2}-\frac{a^2\left(\int \frac{1}{x^2(a^2x^2+1)\arctan(ax)^3}dx-a^2\int \frac{1}{(a^2x^2+1)^2\arctan(ax)^3}dx\right)}{c^2}$$

$$\Big\downarrow \text{5437}$$

$$\frac{-\frac{2\int \frac{1}{x^5 \arctan(ax)^2}dx}{a}-\frac{1}{2ax^4\arctan(ax)^2}}{c^2}-\frac{a^2\left(\int \frac{1}{x^2(a^2x^2+1)\arctan(ax)^3}dx-a^2\left(-a\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^2}dx-\frac{1}{2a(a^2x^2+1)\arctan(ax)^2}\right)\right)}{c^2}$$

$$\Big\downarrow \text{5461}$$

$$\frac{-\frac{2\int \frac{1}{x^5 \arctan(ax)^2}dx}{a}-\frac{1}{2ax^4\arctan(ax)^2}}{c^2}-\frac{a^2\left(-\left(a^2\left(-a\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^2}dx-\frac{1}{2a(a^2x^2+1)\arctan(ax)^2}\right)\right)-\frac{\int \frac{1}{x^3\arctan(ax)^2}dx}{a}-\frac{1}{2ax^2\arctan(ax)^2}\right)}{c^2}$$

$$\Big\downarrow \text{5377}$$

$$\frac{-\frac{2\int \frac{1}{x^5 \arctan(ax)^2}dx}{a}-\frac{1}{2ax^4\arctan(ax)^2}}{c^2}-\frac{a^2\left(-\left(a^2\left(-a\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^2}dx-\frac{1}{2a(a^2x^2+1)\arctan(ax)^2}\right)\right)-\frac{\int \frac{1}{x^3\arctan(ax)^2}dx}{a}-\frac{1}{2ax^2\arctan(ax)^2}\right)}{c^2}$$

$$\Big\downarrow \text{5503}$$

$$\frac{-\frac{2\int \frac{1}{x^5 \arctan(ax)^2}dx}{a}-\frac{1}{2ax^4\arctan(ax)^2}}{c^2}-\frac{a^2\left(-\left(a^2\left(-a\left(\frac{\int \frac{1}{(a^2x^2+1)^2\arctan(ax)}dx}{a}-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)-\frac{1}{2a(a^2x^2+1)\arctan(ax)^2}\right)\right.\right.}{c^2}$$

$$\Big\downarrow \text{5439}$$

$$\frac{-\frac{2\int \frac{1}{x^5\arctan(ax)^2}dx}{a}-\frac{1}{2ax^4\arctan(ax)^2}}{c^2}-\frac{a^2\left(-\left(a^2\left(-a\left(-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\int \frac{1}{(a^2x^2+1)\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)-\frac{1}{2a(a^2x^2+1)\operatorname{arcta}}\right.\right.\right.}{c^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{2\int \frac{1}{x^5\arctan(ax)^2}dx}{a}-\frac{1}{2ax^4\arctan(ax)^2}}{c^2}-\frac{a^2\left(-\left(a^2\left(-a\left(-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^2}{\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)-\frac{1}{2a(a^2x^2+1)\operatorname{arct}}\right.\right.\right.}{c^2}$$

$$\Big\downarrow \text{3793}$$

$$\frac{-\frac{2\int \frac{1}{x^5\arctan(ax)^2}dx}{a}-\frac{1}{2ax^4\arctan(ax)^2}}{c^2}-\frac{a^2\left(-\left(a^2\left(-a\left(-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\int \left(\frac{\cos(2\arctan(ax))}{2\arctan(ax)}+\frac{1}{2\arctan(ax)}\right)d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)-\frac{}{2a(a^2}\right.\right.\right.}{c^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{2\int \frac{1}{x^5\arctan(ax)^2}dx}{a}-\frac{1}{2ax^4\arctan(ax)^2}}{c^2}-\frac{a^2\left(-\left(a^2\left(-a\left(-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)-\frac{}{2a(a^2x}\right.\right.\right.}{c^2}$$

$$\Big\downarrow \text{5505}$$

$$\frac{-\frac{2\int \frac{1}{x^5\arctan(ax)^2}dx}{a}-\frac{1}{2ax^4\arctan(ax)^2}}{c^2}-\frac{a^2\left(-\left(a^2\left(-a\left(-\frac{\int \frac{a^2x^2}{(a^2x^2+1)\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)-\right.\right.\right.}{c^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{2\int \frac{1}{x^5 \arctan(ax)^2}dx}{a} - \frac{1}{2ax^4\arctan(ax)^2}}{c^2} - \frac{a^2\left(-\left(a^2\left(-a\left(-\frac{\int \frac{\sin(\arctan(ax))^2}{\arctan(ax)}d\arctan(ax)}{a^2} + \frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2} - \frac{x}{a(a^2x^2+1)\arctan(ax)}\right) - \frac{}{2a(}\right.\right.\right.}{c^2}$$

$\downarrow$ 3793

$$\frac{-\frac{2\int \frac{1}{x^5 \arctan(ax)^2}dx}{a} - \frac{1}{2ax^4\arctan(ax)^2}}{c^2} - \frac{a^2\left(-\left(a^2\left(-a\left(-\frac{\int \left(\frac{1}{2\arctan(ax)} - \frac{\cos(2\arctan(ax))}{2\arctan(ax)}\right)d\arctan(ax)}{a^2} + \frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2} - \frac{x}{a(a^2x^2+1)\text{arcta}}\right.\right.\right.}{c^2}$$

$\downarrow$ 2009

$$\frac{-\frac{2\int \frac{1}{x^5 \arctan(ax)^2}dx}{a} - \frac{1}{2ax^4\arctan(ax)^2}}{c^2} - \frac{a^2\left(-\frac{\int \frac{1}{x^3\arctan(ax)^2}dx}{a} - \left(a^2\left(-a\left(-\frac{\frac{1}{2}\log(\arctan(ax))-\frac{1}{2}\text{CosIntegral}(2\arctan(ax))}{a^2} + \frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax)}{a^2}\right.\right.\right.\right.}{c^2}$$

input
```
Int[1/(x^4*(c + a^2*c*x^2)^2*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 8.72 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4\left(a^2c\,x^2+c\right)^2\arctan\left(ax\right)^3}dx$$

input
```
int(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

output
```
int(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.82

$$\int \frac{1}{x^4 \left(c+a^2 c x^2\right)^2 \arctan(ax)^3} \, dx = \int \frac{1}{\left(a^2 c x^2 + c\right)^2 x^4 \arctan\left(ax\right)^3} \, dx$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(1/((a^4*c^2*x^8 + 2*a^2*c^2*x^6 + c^2*x^4)*arctan(a*x)^3), x)
```

## Sympy [N/A]

Not integrable

Time = 1.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.00

$$\int \frac{1}{x^4 \left(c+a^2 c x^2\right)^2 \arctan(ax)^3} \, dx = \frac{\int \frac{1}{a^4 x^8 \operatorname{atan}^3(ax) + 2a^2 x^6 \operatorname{atan}^3(ax) + x^4 \operatorname{atan}^3(ax)} \, dx}{c^2}$$

input
```
integrate(1/x**4/(a**2*c*x**2+c)**2/atan(a*x)**3,x)
```

output
```
Integral(1/(a**4*x**8*atan(a*x)**3 + 2*a**2*x**6*atan(a*x)**3 + x**4*atan(
a*x)**3), x)/c**2
```

## Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 142, normalized size of antiderivative = 6.45

$$\int \frac{1}{x^4 \left(c+a^2 c x^2\right)^2 \arctan(ax)^3} \, dx = \int \frac{1}{\left(a^2 c x^2 + c\right)^2 x^4 \arctan\left(ax\right)^3} \, dx$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
1/2*(2*(a^4*c^2*x^7 + a^2*c^2*x^5)*arctan(a*x)^2*integrate((15*a^4*x^4 + 2
3*a^2*x^2 + 10)/((a^6*c^2*x^10 + 2*a^4*c^2*x^8 + a^2*c^2*x^6)*arctan(a*x))
, x) - a*x + 2*(3*a^2*x^2 + 2)*arctan(a*x))/((a^4*c^2*x^7 + a^2*c^2*x^5)*a
rctan(a*x)^2)
```

## Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^4(c+a^2cx^2)^2\arctan(ax)^3}\,dx=\int \frac{1}{(a^2cx^2+c)^2x^4\arctan(ax)^3}\,dx$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^2*x^4*arctan(a*x)^3), x)
```

## Mupad [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^4(c+a^2cx^2)^2\arctan(ax)^3}\,dx=\int \frac{1}{x^4\,\mathrm{atan}\left(a\,x\right)^3\,{\left(c\,a^2\,x^2+c\right)}^2}\,dx$$

input
```
int(1/(x^4*atan(a*x)^3*(c + a^2*c*x^2)^2),x)
```

output
```
int(1/(x^4*atan(a*x)^3*(c + a^2*c*x^2)^2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.09

$$\int \frac{1}{x^4 (c+a^2cx^2)^2 \arctan(ax)^3} \, dx = \frac{\int \frac{1}{atan(ax)^3a^4x^8+2atan(ax)^3a^2x^6+atan(ax)^3x^4}dx}{c^2}$$

input
```
int(1/x^4/(a^2*c*x^2+c)^2/atan(a*x)^3,x)
```

output
```
int(1/(atan(a*x)**3*a**4*x**8 + 2*atan(a*x)**3*a**2*x**6 + atan(a*x)**3*x*
*4),x)/c**2
```

# 3.634 $\int \frac{x^3}{(c+a^2cx^2)^3 \arctan(ax)^3} dx$



## Optimal result

Integrand size = 22, antiderivative size = 177

$$\int \frac{x^3}{(c+a^2cx^2)^3 \arctan(ax)^3} dx = \frac{x}{2a^3c^3(1+a^2x^2)^2 \arctan(ax)^2} - \frac{x}{2a^3c^3(1+a^2x^2)\arctan(ax)^2} + \frac{2}{a^4c^3(1+a^2x^2)^2 \arctan(ax)} - \frac{3}{2a^4c^3(1+a^2x^2)\arctan(ax)} - \frac{1-a^2x^2}{2a^4c^3(1+a^2x^2)\arctan(ax)} - \frac{\mathrm{Si}(2\arctan(ax))}{2a^4c^3} + \frac{\mathrm{Si}(4\arctan(ax))}{a^4c^3}$$

output

```
1/2*x/a^3/c^3/(a^2*x^2+1)^2/arctan(a*x)^2-1/2*x/a^3/c^3/(a^2*x^2+1)/arctan
(a*x)^2+2/a^4/c^3/(a^2*x^2+1)^2/arctan(a*x)-3/2/a^4/c^3/(a^2*x^2+1)/arctan
(a*x)-1/2*(-a^2*x^2+1)/a^4/c^3/(a^2*x^2+1)/arctan(a*x)-1/2*Si(2*arctan(a*x
))/a^4/c^3+Si(4*arctan(a*x))/a^4/c^3
```

## Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.41

$$\int \frac{x^3}{(c+a^2cx^2)^3 \arctan(ax)^3}\,dx = \frac{\frac{a^2x^2(-ax+(-3+a^2x^2)\arctan(ax))}{(1+a^2x^2)^2\arctan(ax)^2} - \mathrm{Si}(2\arctan(ax)) + 2\mathrm{Si}(4\arctan(ax))}{2a^4c^3}$$

input
```
Integrate[x^3/((c + a^2*c*x^2)^3*ArcTan[a*x]^3),x]
```

output
```
((a^2*x^2*(-(a*x) + (-3 + a^2*x^2)*ArcTan[a*x]))/((1 + a^2*x^2)^2*ArcTan[a
*x]^2) - SinIntegral[2*ArcTan[a*x]] + 2*SinIntegral[4*ArcTan[a*x]])/(2*a^4
*c^3)
```

## Rubi [A] (verified)

Time = 2.05 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.54, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {5499, 27, 5467, 5503, 5437, 5499, 5437, 5505, 4906, 27, 2009, 3042, 3780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\arctan(ax)^3 (a^2cx^2+c)^3}\,dx$$

$$\Big\downarrow \text{5499}$$

$$\frac{\int \frac{x}{c^2(a^2x^2+1)^2\arctan(ax)^3}dx}{a^2c} - \frac{\int \frac{x}{c^3(a^2x^2+1)^3\arctan(ax)^3}dx}{a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^3}dx}{a^2c^3} - \frac{\int \frac{x}{(a^2x^2+1)^3\arctan(ax)^3}dx}{a^2c^3}$$

$$\Big\downarrow \text{5467}$$

$$\frac{-2\int \frac{x}{(a^2x^2+1)^2\arctan(ax)}dx-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}}{a^2c^3}-\frac{\int \frac{x}{(a^2x^2+1)^3\arctan(ax)^3}dx}{a^2c^3}$$

$\Big\downarrow$ 5503

$$\frac{-2\int \frac{x}{(a^2x^2+1)^2\arctan(ax)}dx-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}}{a^2c^3}-\frac{\frac{\int \frac{1}{\left(a^2x^2+1\right)^3\arctan(ax)^2}dx}{2a}-\frac{3}{2}a\int \frac{x^2}{(a^2x^2+1)^3\arctan(ax)^2}dx-\frac{x}{2a(a^2x^2+1)^2\arctan(ax)^2}}{a^2c^3}$$

$\Big\downarrow$ 5437

$$\frac{-2\int \frac{x}{(a^2x^2+1)^2\arctan(ax)}dx-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}}{a^2c^3}-\frac{-\frac{3}{2}a\int \frac{x^2}{(a^2x^2+1)^3\arctan(ax)^2}dx+\frac{-4a\int \frac{x}{\left(a^2x^2+1\right)^3\arctan(ax)}dx-\frac{1}{a\left(a^2x^2+1\right)^2\arctan(ax)}}{2a}-\frac{x}{2a(a^2x^2+1)^2\arctan(ax)^2}}{a^2c^3}$$

$\Big\downarrow$ 5499

$$\frac{-2\int \frac{x}{(a^2x^2+1)^2\arctan(ax)}dx-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}}{a^2c^3}-\frac{-\frac{3}{2}a\left(\frac{\int \frac{1}{\left(a^2x^2+1\right)^2\arctan(ax)^2}dx}{a^2}-\frac{\int \frac{1}{\left(a^2x^2+1\right)^3\arctan(ax)^2}dx}{a^2}\right)+\frac{-4a\int \frac{x}{\left(a^2x^2+1\right)^3\arctan(ax)}dx-\frac{1}{a\left(a^2x^2+1\right)^2\arctan(ax)}}{2a}-\frac{\quad}{2a(a^2x^2+1)}}{a^2c^3}$$

$\Big\downarrow$ 5437

$$\frac{-2\int \frac{x}{(a^2x^2+1)^2\arctan(ax)}dx-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}}{a^2c^3}-\frac{\frac{-4a\int \frac{x}{\left(a^2x^2+1\right)^3\arctan(ax)}dx-\frac{1}{a\left(a^2x^2+1\right)^2\arctan(ax)}}{2a}-\frac{3}{2}a\left(\frac{-2a\int \frac{x}{\left(a^2x^2+1\right)^2\arctan(ax)}dx-\frac{1}{a\left(a^2x^2+1\right)\arctan(ax)}}{a^2}-\frac{-4a\int \frac{x}{\left(a^2x^2+1\right)^3\text{arct}}}{}\right.}{a^2c^3}$$

$\Big\downarrow$ 5505

$$\frac{-\frac{2\int \frac{ax}{\left(a^2x^2+1\right)\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}}{a^2c^3}-$$

$$\frac{\frac{-\frac{4\int \frac{ax}{\left(a^2x^2+1\right)^2\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a\left(a^2x^2+1\right)^2\arctan(ax)}}{2a}-\frac{3}{2}a\left(\frac{-\frac{2\int \frac{ax}{\left(a^2x^2+1\right)\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a\left(a^2x^2+1\right)\arctan(ax)}}{a^2}-\frac{-\frac{4\int}{}}{}\right.}{a^2c^3}$$

$\downarrow$ 4906

$$\frac{-\frac{2\int \frac{\sin(2\arctan(ax))}{2\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}}{a^2c^3}-$$

$$\frac{\frac{-\frac{4\int \left(\frac{\sin(2\arctan(ax))}{4\arctan(ax)}+\frac{\sin(4\arctan(ax))}{8\arctan(ax)}\right)d\arctan(ax)}{a}-\frac{1}{a\left(a^2x^2+1\right)^2\arctan(ax)}}{2a}-\frac{3}{2}a\left(\frac{-\frac{2\int \frac{\sin(2\arctan(ax))}{2\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a\left(a^2x^2+1\right)\arctan(ax)}}{a^2}-\right.}{a^2c^3}$$

$\downarrow$ 27

$$\frac{-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}}{a^2c^3}-$$

$$\frac{\frac{-\frac{4\int \left(\frac{\sin(2\arctan(ax))}{4\arctan(ax)}+\frac{\sin(4\arctan(ax))}{8\arctan(ax)}\right)d\arctan(ax)}{a}-\frac{1}{a\left(a^2x^2+1\right)^2\arctan(ax)}}{2a}-\frac{3}{2}a\left(\frac{-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a\left(a^2x^2+1\right)\arctan(ax)}}{a^2}-\right.}{a^2c^3}$$

$\downarrow$ 2009

$$\frac{-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}}{a^2c^3}-$$

$$\frac{-\frac{3}{2}a\left(\frac{-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a\left(a^2x^2+1\right)\arctan(ax)}}{a^2}-\frac{-\frac{1}{a\left(a^2x^2+1\right)^2\arctan(ax)}-\frac{4\left(\frac{1}{4}\text{Si}(2\arctan(ax))+\frac{1}{8}\text{Si}(4\arctan(ax))\right)}{a}}{a^2}\right)+\frac{-}{a(}}{a^2c^3}$$

$\downarrow$ 3042

$$\frac{\frac{-\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}}{a^2c^3}-$$

$$\frac{-\frac{3}{2}a\left(\frac{-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a\left(a^2x^2+1\right)\arctan(ax)}}{a^2}-\frac{-\frac{1}{a\left(a^2x^2+1\right)^2\arctan(ax)}-\frac{4\left(\frac{1}{4}\text{Si}(2\arctan(ax))+\frac{1}{8}\text{Si}(4\arctan(ax))\right)}{a}}{a^2}\right)+\frac{-\frac{}{a(}}{}}{a^2c^3}$$

$$\Big\downarrow \; 3780$$

$$\frac{-\frac{\text{Si}(2\arctan(ax))}{a^2}-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}}{a^2c^3}-$$

$$\frac{-\frac{3}{2}a\left(\frac{-\frac{1}{a\left(a^2x^2+1\right)\arctan(ax)}-\frac{\text{Si}(2\arctan(ax))}{a}}{a^2}-\frac{-\frac{1}{a\left(a^2x^2+1\right)^2\arctan(ax)}-\frac{4\left(\frac{1}{4}\text{Si}(2\arctan(ax))+\frac{1}{8}\text{Si}(4\arctan(ax))\right)}{a}}{a^2}\right)+\frac{-\frac{1}{a\left(a^2x^2+1\right)^2\text{arct}}}{}}{a^2c^3}$$

input
```
Int[x^3/((c + a^2*c*x^2)^3*ArcTan[a*x]^3),x]
```

output
```
(-1/2*x/(a*(1 + a^2*x^2)*ArcTan[a*x]^2) - (1 - a^2*x^2)/(2*a^2*(1 + a^2*x^
2)*ArcTan[a*x]) - SinIntegral[2*ArcTan[a*x]]/a^2)/(a^2*c^3) - (-1/2*x/(a*(
1 + a^2*x^2)^2*ArcTan[a*x]^2) - (3*a*((-(1/(a*(1 + a^2*x^2)*ArcTan[a*x]))
- SinIntegral[2*ArcTan[a*x]]/a)/a^2 - (-(1/(a*(1 + a^2*x^2)^2*ArcTan[a*x])
) - (4*(SinIntegral[2*ArcTan[a*x]]/4 + SinIntegral[4*ArcTan[a*x]]/8))/a)/a
^2))/2 + (-(1/(a*(1 + a^2*x^2)^2*ArcTan[a*x])) - (4*(SinIntegral[2*ArcTan[
a*x]]/4 + SinIntegral[4*ArcTan[a*x]]/8))/a)/(2*a))/(a^2*c^3)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5467
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*(x_))/((d_) + (e_.)*(x_)^2)^2,
 x_Symbol] :> Simp[x*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)*(d + e*x^2
))), x] + (-Simp[(1 - c^2*x^2)*((a + b*ArcTan[c*x])^(p + 2)/(b^2*e*(p + 1)*
(p + 2)*(d + e*x^2))), x] - Simp[4/(b^2*(p + 1)*(p + 2))   Int[x*((a + b*Ar
cTan[c*x])^(p + 2)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[e, c^2*d] && LtQ[p, -1] && NeQ[p, -2]
```

rule 5499
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ
[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

## Maple [A] (verified)

Time = 8.79 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.51

| method | result |
|---|---|
| derivativedivides | $-\frac{8\,\mathrm{Si}(2\arctan(ax))\arctan(ax)^2-16\,\mathrm{Si}(4\arctan(ax))\arctan(ax)^2+4\cos(2\arctan(ax))\arctan(ax)-4\arctan(ax)\cos}{16a^4c^3\arctan(ax)^2}$ |
| default | $-\frac{8\,\mathrm{Si}(2\arctan(ax))\arctan(ax)^2-16\,\mathrm{Si}(4\arctan(ax))\arctan(ax)^2+4\cos(2\arctan(ax))\arctan(ax)-4\arctan(ax)\cos}{16a^4c^3\arctan(ax)^2}$ |

input
```
int(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/16/a^4/c^3*(8*Si(2*arctan(a*x))*arctan(a*x)^2-16*Si(4*arctan(a*x))*arct
an(a*x)^2+4*cos(2*arctan(a*x))*arctan(a*x)-4*arctan(a*x)*cos(4*arctan(a*x)
)+2*sin(2*arctan(a*x))-sin(4*arctan(a*x)))/arctan(a*x)^2
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.85

$$\int \frac{x^3}{(c+a^2cx^2)^3 \arctan(ax)^3} dx =$$
$$-\frac{2a^3x^3 + 2(-ia^4x^4 - 2ia^2x^2 - i)\arctan(ax)^2 \text{log\_integral}\left(\frac{a^4x^4+4ia^3x^3-6a^2x^2-4iax+1}{a^4x^4+2a^2x^2+1}\right) + 2(ia^4x^4 + 2}{}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
-1/4*(2*a^3*x^3 + 2*(-I*a^4*x^4 - 2*I*a^2*x^2 - I)*arctan(a*x)^2*log_integ
ral((a^4*x^4 + 4*I*a^3*x^3 - 6*a^2*x^2 - 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2
 + 1)) + 2*(I*a^4*x^4 + 2*I*a^2*x^2 + I)*arctan(a*x)^2*log_integral((a^4*x
^4 - 4*I*a^3*x^3 - 6*a^2*x^2 + 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) - (
-I*a^4*x^4 - 2*I*a^2*x^2 - I)*arctan(a*x)^2*log_integral(-(a^2*x^2 + 2*I*a
*x - 1)/(a^2*x^2 + 1)) - (I*a^4*x^4 + 2*I*a^2*x^2 + I)*arctan(a*x)^2*log_i
ntegral(-(a^2*x^2 - 2*I*a*x - 1)/(a^2*x^2 + 1)) - 2*(a^4*x^4 - 3*a^2*x^2)*
arctan(a*x))/((a^8*c^3*x^4 + 2*a^6*c^3*x^2 + a^4*c^3)*arctan(a*x)^2)
```

## Sympy [F]

$$\int \frac{x^3}{(c+a^2cx^2)^3 \arctan(ax)^3} dx = \frac{\int \frac{x^3}{a^6x^6 \operatorname{atan}^3(ax)+3a^4x^4 \operatorname{atan}^3(ax)+3a^2x^2 \operatorname{atan}^3(ax)+\operatorname{atan}^3(ax)} dx}{c^3}$$

input
```
integrate(x**3/(a**2*c*x**2+c)**3/atan(a*x)**3,x)
```

output
```
Integral(x**3/(a**6*x**6*atan(a*x)**3 + 3*a**4*x**4*atan(a*x)**3 + 3*a**2*
x**2*atan(a*x)**3 + atan(a*x)**3), x)/c**3
```

## Maxima [F]

$$\int \frac{x^3}{(c+a^2cx^2)^3 \arctan(ax)^3} \, dx = \int \frac{x^3}{(a^2cx^2+c)^3 \arctan(ax)^3} \, dx$$

input
```
integrate(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
-1/2*(a*x^3 + 2*(a^6*c^3*x^4 + 2*a^4*c^3*x^2 + a^2*c^3)*arctan(a*x)^2*inte
grate((5*a^2*x^3 - 3*x)/((a^8*c^3*x^6 + 3*a^6*c^3*x^4 + 3*a^4*c^3*x^2 + a^
2*c^3)*arctan(a*x)), x) - (a^2*x^4 - 3*x^2)*arctan(a*x))/((a^6*c^3*x^4 + 2
*a^4*c^3*x^2 + a^2*c^3)*arctan(a*x)^2)
```

## Giac [F]

$$\int \frac{x^3}{(c+a^2cx^2)^3 \arctan(ax)^3} \, dx = \int \frac{x^3}{(a^2cx^2+c)^3 \arctan(ax)^3} \, dx$$

input
```
integrate(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate(x^3/((a^2*c*x^2 + c)^3*arctan(a*x)^3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(c+a^2cx^2)^3 \arctan(ax)^3} \, dx = \int \frac{x^3}{\mathrm{atan}(a\,x)^3\,(c\,a^2\,x^2+c)^3} \, dx$$

input
```
int(x^3/(atan(a*x)^3*(c + a^2*c*x^2)^3),x)
```

output
```
int(x^3/(atan(a*x)^3*(c + a^2*c*x^2)^3), x)
```

## Reduce [F]

$$\int \frac{x^3}{(c+a^2cx^2)^3 \arctan(ax)^3}\,dx = \frac{\int \frac{x^3}{atan(ax)^3a^6x^6+3atan(ax)^3a^4x^4+3atan(ax)^3a^2x^2+atan(ax)^3}dx}{c^3}$$

input
```
int(x^3/(a^2*c*x^2+c)^3/atan(a*x)^3,x)
```

output
```
int(x**3/(atan(a*x)**3*a**6*x**6 + 3*atan(a*x)**3*a**4*x**4 + 3*atan(a*x)*
*3*a**2*x**2 + atan(a*x)**3),x)/c**3
```

# 3.635 $\int \frac{x^2}{(c+a^2cx^2)^3 \arctan(ax)^3} dx$



## Optimal result

Integrand size = 22, antiderivative size = 120

$$\int \frac{x^2}{(c+a^2cx^2)^3 \arctan(ax)^3} dx = \frac{1}{2a^3c^3(1+a^2x^2)^2 \arctan(ax)^2} - \frac{1}{2a^3c^3(1+a^2x^2)\arctan(ax)^2} - \frac{2x}{a^2c^3(1+a^2x^2)^2 \arctan(ax)} + \frac{x}{a^2c^3(1+a^2x^2)\arctan(ax)} + \frac{\text{CosIntegral}(4\arctan(ax))}{a^3c^3}$$

output
```
1/2/a^3/c^3/(a^2*x^2+1)^2/arctan(a*x)^2-1/2/a^3/c^3/(a^2*x^2+1)/arctan(a*x
)^2-2*x/a^2/c^3/(a^2*x^2+1)^2/arctan(a*x)+x/a^2/c^3/(a^2*x^2+1)/arctan(a*x
)+Ci(4*arctan(a*x))/a^3/c^3
```

## Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.50

$$\int \frac{x^2}{(c+a^2cx^2)^3 \arctan(ax)^3}\,dx = \frac{\frac{ax(-ax+2(-1+a^2x^2)\arctan(ax))}{(1+a^2x^2)^2\arctan(ax)^2}+2\,\mathrm{CosIntegral}(4\arctan(ax))}{2a^3c^3}$$

input
```
Integrate[x^2/((c + a^2*c*x^2)^3*ArcTan[a*x]^3),x]
```

output
```
((a*x*(-(a*x) + 2*(-1 + a^2*x^2)*ArcTan[a*x]))/((1 + a^2*x^2)^2*ArcTan[a*x
]^2) + 2*CosIntegral[4*ArcTan[a*x]])/(2*a^3*c^3)
```

## Rubi [A] (verified)

Time = 2.00 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.94, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {5499, 27, 5437, 5503, 5439, 3042, 3793, 2009, 5505, 3042, 3793, 2009, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\arctan(ax)^3 (a^2cx^2+c)^3}\,dx$$

$$\Big\downarrow \text{5499}$$

$$\frac{\int \frac{1}{c^2(a^2x^2+1)^2\arctan(ax)^3}dx}{a^2c}-\frac{\int \frac{1}{c^3(a^2x^2+1)^3\arctan(ax)^3}dx}{a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{1}{(a^2x^2+1)^2\arctan(ax)^3}dx}{a^2c^3}-\frac{\int \frac{1}{(a^2x^2+1)^3\arctan(ax)^3}dx}{a^2c^3}$$

$$\Big\downarrow \text{5437}$$

$$\frac{-a\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^2}dx-\frac{1}{2a(a^2x^2+1)\arctan(ax)^2}}{a^2c^3}-\frac{-2a\int \frac{x}{(a^2x^2+1)^3\arctan(ax)^2}dx-\frac{1}{2a(a^2x^2+1)^2\arctan(ax)^2}}{a^2c^3}$$

$\downarrow$ 5503

$$\frac{-a\left(\frac{\int \frac{1}{(a^2x^2+1)^2\arctan(ax)}dx}{a}-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)-\frac{1}{2a(a^2x^2+1)\arctan(ax)^2}}{a^2c^3}-\frac{-2a\left(\frac{\int \frac{1}{(a^2x^2+1)^3\arctan(ax)}dx}{a}-3a\int \frac{x^2}{(a^2x^2+1)^3\arctan(ax)}dx-\frac{x}{a(a^2x^2+1)^2\arctan(ax)}\right)-\frac{1}{2a(a^2x^2+1)^2\arctan(ax)^2}}{a^2c^3}$$

$\downarrow$ 5439

$$\frac{-a\left(-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\int \frac{1}{(a^2x^2+1)\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)-\frac{1}{2a(a^2x^2+1)\arctan(ax)^2}}{a^2c^3}-\frac{-2a\left(-3a\int \frac{x^2}{(a^2x^2+1)^3\arctan(ax)}dx+\frac{\int \frac{1}{(a^2x^2+1)^2\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)^2\arctan(ax)}\right)-\frac{1}{2a(a^2x^2+1)^2\arctan(ax)^2}}{a^2c^3}$$

$\downarrow$ 3042

$$\frac{-a\left(-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^2}{\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)-\frac{1}{2a(a^2x^2+1)\arctan(ax)^2}}{a^2c^3}-\frac{-2a\left(-3a\int \frac{x^2}{(a^2x^2+1)^3\arctan(ax)}dx+\frac{\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^4}{\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)^2\arctan(ax)}\right)-\frac{1}{2a(a^2x^2+1)^2\arctan(ax)^2}}{a^2c^3}$$

$\downarrow$ 3793

$$\frac{-a\left(-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\int \left(\frac{\cos(2\arctan(ax))}{2\arctan(ax)}+\frac{1}{2\arctan(ax)}\right)d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)-\frac{1}{2a(a^2x^2+1)\arctan(a\cdots}}{a^2c^3}$$

$$\frac{-2a\left(-3a\int \frac{x^2}{(a^2x^2+1)^3\arctan(ax)}dx+\frac{\int \left(\frac{\cos(2\arctan(ax))}{2\arctan(ax)}+\frac{\cos(4\arctan(ax))}{8\arctan(ax)}+\frac{3}{8\arctan(ax)}\right)d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)^2\arctan(ax)}\right)-\cdots}{a^2c^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-a\left(-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)-\frac{1}{2a(a^2x^2+1)\arctan(\cdots}}{a^2c^3}$$

$$\frac{-2a\left(-3a\int \frac{x^2}{(a^2x^2+1)^3\arctan(ax)}dx+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{8}\text{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)^2\text{arc}\cdots}\cdots\right.}{a^2c^3}$$

$$\Big\downarrow \text{5505}$$

$$\frac{-a\left(-\frac{\int \frac{a^2x^2}{(a^2x^2+1)\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)-\frac{1}{2a(a^2x^2+1)\text{ar}\cdots}}{a^2c^3}$$

$$\frac{-2a\left(-\frac{3\int \frac{a^2x^2}{(a^2x^2+1)^2\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{8}\text{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}-\frac{\cdots}{a(a^2x^2+\cdots}\cdots\right.}{a^2c^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-a\left(-\frac{\int \frac{\sin(\arctan(ax))^2}{\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)-\frac{1}{2a(a^2x^2+1)\arctan\cdots}}{a^2c^3}$$

$$\frac{-2a\left(-\frac{3\int \frac{a^2x^2}{(a^2x^2+1)^2\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{8}\text{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}-\frac{\cdots}{a(a^2x^2+\cdots}\cdots\right.}{a^2c^3}$$

$$\Big\downarrow \text{3793}$$

$$\frac{-a\left(-\frac{\int\left(\frac{1}{2\arctan(ax)}-\frac{\cos(2\arctan(ax))}{2\arctan(ax)}\right)d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)-\frac{\cdots}{2a(\cdots}}{a^2c^3}$$

$$\frac{-2a\left(-\frac{3\int\frac{a^2x^2}{(a^2x^2+1)^2\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{8}\text{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}-\frac{\cdots}{a(a^2x^2+\cdots}\cdots\right.}{a^2c^3}$$

$\Big\downarrow$ 2009

$$\frac{-a\left(-\frac{\frac{1}{2}\log(\arctan(ax))-\frac{1}{2}\text{CosIntegral}(2\arctan(ax))}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)-\frac{\cdots}{2a\cdots}}{a^2c^3}$$

$$\frac{-2a\left(-\frac{3\int\frac{a^2x^2}{(a^2x^2+1)^2\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{8}\text{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}-\frac{\cdots}{a(a^2x^2+\cdots}\cdots\right.}{a^2c^3}$$

$\Big\downarrow$ 4906

$$\frac{-a\left(-\frac{\frac{1}{2}\log(\arctan(ax))-\frac{1}{2}\text{CosIntegral}(2\arctan(ax))}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)-\frac{\cdots}{2a\cdots}}{a^2c^3}$$

$$\frac{-2a\left(-\frac{3\int\left(\frac{1}{8\arctan(ax)}-\frac{\cos(4\arctan(ax))}{8\arctan(ax)}\right)d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{8}\text{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}-\cdots\right.}{a^2c^3}$$

$\Big\downarrow$ 2009

$$\frac{-a\left(-\frac{\frac{1}{2}\log(\arctan(ax))-\frac{1}{2}\text{CosIntegral}(2\arctan(ax))}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)-\frac{\cdots}{2a\cdots}}{a^2c^3}$$

$$\frac{-2a\left(-\frac{3\left(\frac{1}{8}\log(\arctan(ax))-\frac{1}{8}\text{CosIntegral}(4\arctan(ax))\right)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{8}\text{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}\cdots\right.}{a^2c^3}$$

input
```
Int[x^2/((c + a^2*c*x^2)^3*ArcTan[a*x]^3),x]
```

output
```
-((-1/2*1/(a*(1 + a^2*x^2)^2*ArcTan[a*x]^2) - 2*a*(-(x/(a*(1 + a^2*x^2)^2*
ArcTan[a*x])) - (3*(-1/8*CosIntegral[4*ArcTan[a*x]] + Log[ArcTan[a*x]]/8))
/a^2 + (CosIntegral[2*ArcTan[a*x]]/2 + CosIntegral[4*ArcTan[a*x]]/8 + (3*L
og[ArcTan[a*x]])/8)/a^2))/(a^2*c^3)) + (-1/2*1/(a*(1 + a^2*x^2)*ArcTan[a*x
]^2) - a*(-(x/(a*(1 + a^2*x^2)*ArcTan[a*x])) - (-1/2*CosIntegral[2*ArcTan[
a*x]] + Log[ArcTan[a*x]]/2)/a^2 + (CosIntegral[2*ArcTan[a*x]]/2 + Log[ArcT
an[a*x]]/2)/a^2))/(a^2*c^3)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5499
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ
[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

## Maple [A] (verified)

Time = 7.66 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.43

| method | result | size |
|---|---|---|
| derivativedivides | $\frac{16\,\mathrm{Ci}(4\arctan(ax))\arctan(ax)^2-4\arctan(ax)\sin(4\arctan(ax))+\cos(4\arctan(ax))-1}{16a^3c^3\arctan(ax)^2}$ | 52 |
| default | $\frac{16\,\mathrm{Ci}(4\arctan(ax))\arctan(ax)^2-4\arctan(ax)\sin(4\arctan(ax))+\cos(4\arctan(ax))-1}{16a^3c^3\arctan(ax)^2}$ | 52 |

input
```
int(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/16/a^3/c^3*(16*Ci(4*arctan(a*x))*arctan(a*x)^2-4*arctan(a*x)*sin(4*arcta
n(a*x))+cos(4*arctan(a*x))-1)/arctan(a*x)^2
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.79

$$\int \frac{x^2}{(c+a^2cx^2)^3 \arctan(ax)^3} dx = -\frac{a^2x^2 - (a^4x^4 + 2a^2x^2 + 1)\arctan(ax)^2 \operatorname{log\_integral}\left(\frac{a^4x^4+4i a^3x^3-6a^2x^2-4i ax+1}{a^4x^4+2a^2x^2+1}\right) - (a^4x^4 + 2a^2x^2 + 1}{2(a^7c^3x^4 + 2a^5c^3x^2 + a^3c^3)\operatorname{ar}}$$

input
```
integrate(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
-1/2*(a^2*x^2 - (a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x)^2*log_integral((a^4*
x^4 + 4*I*a^3*x^3 - 6*a^2*x^2 - 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) -
(a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x)^2*log_integral((a^4*x^4 - 4*I*a^3*x^
3 - 6*a^2*x^2 + 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) - 2*(a^3*x^3 - a*x
)*arctan(a*x))/((a^7*c^3*x^4 + 2*a^5*c^3*x^2 + a^3*c^3)*arctan(a*x)^2)
```

## Sympy [F]

$$\int \frac{x^2}{(c+a^2cx^2)^3 \arctan(ax)^3} dx = \frac{\int \frac{x^2}{a^6x^6 \operatorname{atan}^3(ax)+3a^4x^4 \operatorname{atan}^3(ax)+3a^2x^2 \operatorname{atan}^3(ax)+\operatorname{atan}^3(ax)} dx}{c^3}$$

input
```
integrate(x**2/(a**2*c*x**2+c)**3/atan(a*x)**3,x)
```

output
```
Integral(x**2/(a**6*x**6*atan(a*x)**3 + 3*a**4*x**4*atan(a*x)**3 + 3*a**2*
x**2*atan(a*x)**3 + atan(a*x)**3), x)/c**3
```

## Maxima [F]

$$\int \frac{x^2}{\left(c+a^2cx^2\right)^3 \arctan(ax)^3}\,dx = \int \frac{x^2}{\left(a^2cx^2+c\right)^3 \arctan\left(ax\right)^3}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
1/2*(2*(a^6*c^3*x^4 + 2*a^4*c^3*x^2 + a^2*c^3)*arctan(a*x)^2*integrate((a^
4*x^4 - 6*a^2*x^2 + 1)/((a^8*c^3*x^6 + 3*a^6*c^3*x^4 + 3*a^4*c^3*x^2 + a^2
*c^3)*arctan(a*x)), x) - a*x^2 + 2*(a^2*x^3 - x)*arctan(a*x))/((a^6*c^3*x^
4 + 2*a^4*c^3*x^2 + a^2*c^3)*arctan(a*x)^2)
```

## Giac [F]

$$\int \frac{x^2}{\left(c+a^2cx^2\right)^3 \arctan(ax)^3}\,dx = \int \frac{x^2}{\left(a^2cx^2+c\right)^3 \arctan\left(ax\right)^3}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate(x^2/((a^2*c*x^2 + c)^3*arctan(a*x)^3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\left(c+a^2cx^2\right)^3 \arctan(ax)^3}\,dx = \int \frac{x^2}{\mathrm{atan}\left(a\,x\right)^3\,\left(c\,a^2\,x^2+c\right)^3}\,dx$$

input
```
int(x^2/(atan(a*x)^3*(c + a^2*c*x^2)^3),x)
```

output
```
int(x^2/(atan(a*x)^3*(c + a^2*c*x^2)^3), x)
```

## Reduce [F]

$$\int \frac{x^2}{(c+a^2cx^2)^3 \arctan(ax)^3}\,dx = \frac{\int \frac{x^2}{atan(ax)^3a^6x^6+3atan(ax)^3a^4x^4+3atan(ax)^3a^2x^2+atan(ax)^3}dx}{c^3}$$

input
```
int(x^2/(a^2*c*x^2+c)^3/atan(a*x)^3,x)
```

output
```
int(x**2/(atan(a*x)**3*a**6*x**6 + 3*atan(a*x)**3*a**4*x**4 + 3*atan(a*x)*
*3*a**2*x**2 + atan(a*x)**3),x)/c**3
```

# 3.636 $\int \frac{x}{(c+a^2cx^2)^3 \arctan(ax)^3} dx$



## Optimal result

Integrand size = 20, antiderivative size = 113

$$\int \frac{x}{(c+a^2cx^2)^3 \arctan(ax)^3} dx = -\frac{x}{2ac^3(1+a^2x^2)^2 \arctan(ax)^2} - \frac{2}{a^2c^3(1+a^2x^2)^2 \arctan(ax)} + \frac{3}{2a^2c^3(1+a^2x^2)\arctan(ax)} - \frac{\text{Si}(2\arctan(ax))}{2a^2c^3} - \frac{\text{Si}(4\arctan(ax))}{a^2c^3}$$

output
```
-1/2*x/a/c^3/(a^2*x^2+1)^2/arctan(a*x)^2-2/a^2/c^3/(a^2*x^2+1)^2/arctan(a*
x)+3/2/a^2/c^3/(a^2*x^2+1)/arctan(a*x)-1/2*Si(2*arctan(a*x))/a^2/c^3-Si(4*
arctan(a*x))/a^2/c^3
```

## Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.87

$$\int \frac{x}{(c+a^2cx^2)^3 \arctan(ax)^3} \, dx = -\frac{ax+\arctan(ax)-3a^2x^2\arctan(ax)+(1+a^2x^2)^2\arctan(ax)^2\text{Si}(2\arctan(ax))+2(1+a^2x^2)^2\arctan}{2a^2c^3(1+a^2x^2)^2\arctan(ax)^2}$$

input
```
Integrate[x/((c + a^2*c*x^2)^3*ArcTan[a*x]^3),x]
```

output
```
-1/2*(a*x + ArcTan[a*x] - 3*a^2*x^2*ArcTan[a*x] + (1 + a^2*x^2)^2*ArcTan[a
*x]^2*SinIntegral[2*ArcTan[a*x]] + 2*(1 + a^2*x^2)^2*ArcTan[a*x]^2*SinInte
gral[4*ArcTan[a*x]])/(a^2*c^3*(1 + a^2*x^2)^2*ArcTan[a*x]^2)
```

## Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.72, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {5503, 27, 5437, 5499, 5437, 5505, 4906, 27, 2009, 3042, 3780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\arctan(ax)^3(a^2cx^2+c)^3} \, dx$$

$$\Big\downarrow \text{ 5503}$$

$$\frac{\int \frac{1}{c^3(a^2x^2+1)^3\arctan(ax)^2}dx}{2a}-\frac{3}{2}a\int \frac{x^2}{c^3(a^2x^2+1)^3\arctan(ax)^2}dx-\frac{x}{2ac^3(a^2x^2+1)^2\arctan(ax)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{1}{(a^2x^2+1)^3\arctan(ax)^2}dx}{2ac^3}-\frac{3a\int \frac{x^2}{(a^2x^2+1)^3\arctan(ax)^2}dx}{2c^3}-\frac{x}{2ac^3(a^2x^2+1)^2\arctan(ax)^2}$$

$$\Big\downarrow \text{ 5437}$$

$$-\frac{3a\int \frac{x^2}{(a^2x^2+1)^3\arctan(ax)^2}dx}{2c^3}+\frac{-4a\int \frac{x}{(a^2x^2+1)^3\arctan(ax)}dx-\frac{1}{a(a^2x^2+1)^2\arctan(ax)}}{2ac^3}-\frac{x}{2ac^3\left(a^2x^2+1\right)^2\arctan(ax)^2}$$

$$\Big\downarrow \text{5499}$$

$$-\frac{3a\left(\frac{\int \frac{1}{\left(a^2x^2+1\right)^2\arctan(ax)^2}dx}{a^2}-\frac{\int \frac{1}{\left(a^2x^2+1\right)^3\arctan(ax)^2}dx}{a^2}\right)}{2c^3}+\frac{-4a\int \frac{x}{(a^2x^2+1)^3\arctan(ax)}dx-\frac{1}{a(a^2x^2+1)^2\arctan(ax)}}{2ac^3}-\frac{x}{2ac^3\left(a^2x^2+1\right)^2\arctan(ax)^2}$$

$$\Big\downarrow \text{5437}$$

$$\frac{-4a\int \frac{x}{(a^2x^2+1)^3\arctan(ax)}dx-\frac{1}{a(a^2x^2+1)^2\arctan(ax)}}{2ac^3}-\frac{3a\left(\frac{-2a\int \frac{x}{\left(a^2x^2+1\right)^2\arctan(ax)}dx-\frac{1}{a\left(a^2x^2+1\right)\arctan(ax)}}{a^2}-\frac{-4a\int \frac{x}{\left(a^2x^2+1\right)^3\arctan(ax)}dx-\frac{1}{a\left(a^2x^2+1\right)^2\arctan(ax)}}{a^2}\right)}{2c^3}-\frac{x}{2ac^3\left(a^2x^2+1\right)^2\arctan(ax)^2}$$

$$\Big\downarrow \text{5505}$$

$$\frac{-\frac{4\int \frac{ax}{\left(a^2x^2+1\right)^2\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a(a^2x^2+1)^2\arctan(ax)}}{2ac^3}-\frac{3a\left(\frac{-\frac{2\int \frac{ax}{\left(a^2x^2+1\right)\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a\left(a^2x^2+1\right)\arctan(ax)}}{a^2}-\frac{-\frac{4\int \frac{ax}{\left(a^2x^2+1\right)^2\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a\left(a^2x^2+1\right)^2\arctan(ax)}}{a^2}\right)}{2c^3}-\frac{x}{2ac^3\left(a^2x^2+1\right)^2\arctan(ax)^2}$$

$$\Big\downarrow \text{4906}$$

$$\frac{3a\left(\frac{-\frac{2\int \frac{\sin(2\arctan(ax))}{2\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a\left(a^2x^2+1\right)\arctan(ax)}}{a^2}-\frac{-\frac{4\int\left(\frac{\sin(2\arctan(ax))}{4\arctan(ax)}+\frac{\sin(4\arctan(ax))}{8\arctan(ax)}\right)d\arctan(ax)}{a}-\frac{1}{a\left(a^2x^2+1\right)^2\arctan(ax)}}{a^2}\right)}{2c^3}\frac{-\frac{4\int\left(\frac{\sin(2\arctan(ax))}{4\arctan(ax)}+\frac{\sin(4\arctan(ax))}{8\arctan(ax)}\right)d\arctan(ax)}{a}-\frac{1}{a(a^2x^2+1)^2\arctan(ax)}}{2ac^3}-\frac{x}{2ac^3\left(a^2x^2+1\right)^2\arctan(ax)^2}$$

$\downarrow$ 27

$$\frac{3a\left(\frac{-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a\left(a^2x^2+1\right)\arctan(ax)}}{a^2}-\frac{-\frac{4\int\left(\frac{\sin(2\arctan(ax))}{4\arctan(ax)}+\frac{\sin(4\arctan(ax))}{8\arctan(ax)}\right)d\arctan(ax)}{a}-\frac{1}{a\left(a^2x^2+1\right)^2\arctan(ax)}}{a^2}\right)}{2c^3}\frac{-\frac{4\int\left(\frac{\sin(2\arctan(ax))}{4\arctan(ax)}+\frac{\sin(4\arctan(ax))}{8\arctan(ax)}\right)d\arctan(ax)}{a}-\frac{1}{a(a^2x^2+1)^2\arctan(ax)}}{2ac^3}-\frac{x}{2ac^3\left(a^2x^2+1\right)^2\arctan(ax)^2}$$

$\downarrow$ 2009

$$-\frac{3a\left(\frac{-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a\left(a^2x^2+1\right)\arctan(ax)}}{a^2}-\frac{-\frac{1}{a\left(a^2x^2+1\right)^2\arctan(ax)}-\frac{4\left(\frac{1}{4}\text{Si}(2\arctan(ax))+\frac{1}{8}\text{Si}(4\arctan(ax))\right)}{a}}{a^2}\right)}{2c^3}+\frac{-\frac{1}{a(a^2x^2+1)^2\arctan(ax)}-\frac{4\left(\frac{1}{4}\text{Si}(2\arctan(ax))+\frac{1}{8}\text{Si}(4\arctan(ax))\right)}{a}}{2ac^3}-\frac{x}{2ac^3\left(a^2x^2+1\right)^2\arctan(ax)^2}$$

$\downarrow$ 3042

$$-\frac{3a\left(\frac{-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a\left(a^2x^2+1\right)\arctan(ax)}}{a^2}-\frac{-\frac{1}{a\left(a^2x^2+1\right)^2\arctan(ax)}-\frac{4\left(\frac{1}{4}\text{Si}(2\arctan(ax))+\frac{1}{8}\text{Si}(4\arctan(ax))\right)}{a}}{a^2}\right)}{2c^3}+\frac{-\frac{1}{a(a^2x^2+1)^2\arctan(ax)}-\frac{4\left(\frac{1}{4}\text{Si}(2\arctan(ax))+\frac{1}{8}\text{Si}(4\arctan(ax))\right)}{a}}{2ac^3}-\frac{x}{2ac^3\left(a^2x^2+1\right)^2\arctan(ax)^2}$$

$\downarrow$ 3780

$$-\frac{3a\left(\frac{-\frac{1}{a\left(a^2x^2+1\right)\arctan(ax)}-\frac{\operatorname{Si}(2\arctan(ax))}{a}}{a^2}-\frac{-\frac{1}{a\left(a^2x^2+1\right)^2\arctan(ax)}-\frac{4\left(\frac{1}{4}\operatorname{Si}(2\arctan(ax))+\frac{1}{8}\operatorname{Si}(4\arctan(ax))\right)}{a}}{a^2}\right)}{2c^3}+\frac{-\frac{1}{a(a^2x^2+1)^2\arctan(ax)}-\frac{4\left(\frac{1}{4}\operatorname{Si}(2\arctan(ax))+\frac{1}{8}\operatorname{Si}(4\arctan(ax))\right)}{a}}{2ac^3}-\frac{x}{2ac^3\left(a^2x^2+1\right)^2\arctan(ax)^2}$$

input
```
Int[x/((c + a^2*c*x^2)^3*ArcTan[a*x]^3),x]
```

output
```
-1/2*x/(a*c^3*(1 + a^2*x^2)^2*ArcTan[a*x]^2) - (3*a*((-(1/(a*(1 + a^2*x^2)
*ArcTan[a*x])) - SinIntegral[2*ArcTan[a*x]]/a)/a^2 - (-(1/(a*(1 + a^2*x^2)
^2*ArcTan[a*x])) - (4*(SinIntegral[2*ArcTan[a*x]]/4 + SinIntegral[4*ArcTan
[a*x]]/8))/a)/a^2))/(2*c^3) + (-(1/(a*(1 + a^2*x^2)^2*ArcTan[a*x])) - (4*(
SinIntegral[2*ArcTan[a*x]]/4 + SinIntegral[4*ArcTan[a*x]]/8))/a)/(2*a*c^3)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5499
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ
[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

## Maple [A] (verified)

Time = 6.33 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.78

| method | result |
|---|---|
| derivativedivides | $-\frac{8\,\mathrm{Si}(2\arctan(ax))\arctan(ax)^2+16\,\mathrm{Si}(4\arctan(ax))\arctan(ax)^2+4\cos(2\arctan(ax))\arctan(ax)+4\arctan(ax)\cos}{16a^2c^3\arctan(ax)^2}$ |
| default | $-\frac{8\,\mathrm{Si}(2\arctan(ax))\arctan(ax)^2+16\,\mathrm{Si}(4\arctan(ax))\arctan(ax)^2+4\cos(2\arctan(ax))\arctan(ax)+4\arctan(ax)\cos}{16a^2c^3\arctan(ax)^2}$ |

input
```
int(x/(a^2*c*x^2+c)^3/arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/16/a^2/c^3*(8*Si(2*arctan(a*x))*arctan(a*x)^2+16*Si(4*arctan(a*x))*arct
an(a*x)^2+4*cos(2*arctan(a*x))*arctan(a*x)+4*arctan(a*x)*cos(4*arctan(a*x)
)+2*sin(2*arctan(a*x))+sin(4*arctan(a*x)))/arctan(a*x)^2
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.81

$$\int \frac{x}{(c+a^2cx^2)^3\arctan(ax)^3}\,dx = -\frac{2\,(i\,a^4x^4+2i\,a^2x^2+i)\arctan(ax)^2\,\mathrm{log\_integral}\left(\frac{a^4x^4+4i\,a^3x^3-6\,a^2x^2-4i\,ax+1}{a^4x^4+2\,a^2x^2+1}\right)+2\,(-i\,a^4x^4-2i\,a^2x^2-}{}$$

input
```
integrate(x/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
-1/4*(2*(I*a^4*x^4 + 2*I*a^2*x^2 + I)*arctan(a*x)^2*log_integral((a^4*x^4
+ 4*I*a^3*x^3 - 6*a^2*x^2 - 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) + 2*(-
I*a^4*x^4 - 2*I*a^2*x^2 - I)*arctan(a*x)^2*log_integral((a^4*x^4 - 4*I*a^3
*x^3 - 6*a^2*x^2 + 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) - (-I*a^4*x^4 -
 2*I*a^2*x^2 - I)*arctan(a*x)^2*log_integral(-(a^2*x^2 + 2*I*a*x - 1)/(a^2
*x^2 + 1)) - (I*a^4*x^4 + 2*I*a^2*x^2 + I)*arctan(a*x)^2*log_integral(-(a^
2*x^2 - 2*I*a*x - 1)/(a^2*x^2 + 1)) + 2*a*x - 2*(3*a^2*x^2 - 1)*arctan(a*x
))/((a^6*c^3*x^4 + 2*a^4*c^3*x^2 + a^2*c^3)*arctan(a*x)^2)
```

## Sympy [F]

$$\int \frac{x}{(c+a^2cx^2)^3\arctan(ax)^3}\,dx = \frac{\int \frac{x}{a^6x^6\operatorname{atan}^3(ax)+3a^4x^4\operatorname{atan}^3(ax)+3a^2x^2\operatorname{atan}^3(ax)+\operatorname{atan}^3(ax)}\,dx}{c^3}$$

input
```
integrate(x/(a**2*c*x**2+c)**3/atan(a*x)**3,x)
```

output
```
Integral(x/(a**6*x**6*atan(a*x)**3 + 3*a**4*x**4*atan(a*x)**3 + 3*a**2*x**
2*atan(a*x)**3 + atan(a*x)**3), x)/c**3
```

## Maxima [F]

$$\int \frac{x}{(c+a^2cx^2)^3\arctan(ax)^3}\,dx = \int \frac{x}{(a^2cx^2+c)^3\arctan(ax)^3}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
1/2*(2*(a^6*c^3*x^4 + 2*a^4*c^3*x^2 + a^2*c^3)*arctan(a*x)^2*integrate((3*
a^2*x^3 - 5*x)/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan
(a*x)), x) - a*x + (3*a^2*x^2 - 1)*arctan(a*x))/((a^6*c^3*x^4 + 2*a^4*c^3*
x^2 + a^2*c^3)*arctan(a*x)^2)
```

## Giac [F]

$$\int \frac{x}{(c+a^2cx^2)^3\arctan(ax)^3}\,dx = \int \frac{x}{(a^2cx^2+c)^3\arctan(ax)^3}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate(x/((a^2*c*x^2 + c)^3*arctan(a*x)^3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(c+a^2cx^2)^3 \arctan(ax)^3} \, dx = \int \frac{x}{\operatorname{atan}(a\,x)^3\,(c\,a^2\,x^2+c)^3} \, dx$$

input
```
int(x/(atan(a*x)^3*(c + a^2*c*x^2)^3),x)
```

output
```
int(x/(atan(a*x)^3*(c + a^2*c*x^2)^3), x)
```

## Reduce [F]

$$\int \frac{x}{(c+a^2cx^2)^3 \arctan(ax)^3} \, dx = \frac{-3atan(ax)^2 \left(\int \frac{x^2}{atan(ax)^2a^6x^6+3atan(ax)^2a^4x^4+3atan(ax)^2a^2x^2+atan(ax)^2}dx\right)a^7x^4 - 6atan(ax)^2 \left(\int \frac{}{atan(ax)^2a^6x^6+3}\right.}{}$$

input
```
int(x/(a^2*c*x^2+c)^3/atan(a*x)^3,x)
```

output
```
( - 3*atan(a*x)**2*int(x**2/(atan(a*x)**2*a**6*x**6 + 3*atan(a*x)**2*a**4*
x**4 + 3*atan(a*x)**2*a**2*x**2 + atan(a*x)**2),x)*a**7*x**4 - 6*atan(a*x)
**2*int(x**2/(atan(a*x)**2*a**6*x**6 + 3*atan(a*x)**2*a**4*x**4 + 3*atan(a
*x)**2*a**2*x**2 + atan(a*x)**2),x)*a**5*x**2 - 3*atan(a*x)**2*int(x**2/(a
tan(a*x)**2*a**6*x**6 + 3*atan(a*x)**2*a**4*x**4 + 3*atan(a*x)**2*a**2*x**
2 + atan(a*x)**2),x)*a**3 - 4*atan(a*x)**2*int(x/(atan(a*x)*a**6*x**6 + 3*
atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**6*x**4 - 8*
atan(a*x)**2*int(x/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a
*x)*a**2*x**2 + atan(a*x)),x)*a**4*x**2 - 4*atan(a*x)**2*int(x/(atan(a*x)*
a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*
a**2 - atan(a*x) - a*x)/(2*atan(a*x)**2*a**2*c**3*(a**4*x**4 + 2*a**2*x**2
 + 1))
```

# 3.637 $\int \frac{1}{(c+a^2cx^2)^3 \arctan(ax)^3} dx$



## Optimal result

Integrand size = 19, antiderivative size = 81

$$\int \frac{1}{(c+a^2cx^2)^3 \arctan(ax)^3} dx = -\frac{1}{2ac^3(1+a^2x^2)^2 \arctan(ax)^2} + \frac{2x}{c^3(1+a^2x^2)^2 \arctan(ax)} - \frac{\text{CosIntegral}(2\arctan(ax))}{ac^3} - \frac{\text{CosIntegral}(4\arctan(ax))}{ac^3}$$

output
```
-1/2/a/c^3/(a^2*x^2+1)^2/arctan(a*x)^2+2*x/c^3/(a^2*x^2+1)^2/arctan(a*x)-C
i(2*arctan(a*x))/a/c^3-Ci(4*arctan(a*x))/a/c^3
```

## Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+a^2cx^2)^3 \arctan(ax)^3} \, dx =$$
$$-\frac{1 - 4ax \arctan(ax) + 2(1+a^2x^2)^2 \arctan(ax)^2 \operatorname{CosIntegral}(2\arctan(ax)) + 2(1+a^2x^2)^2 \arctan(ax)^2}{2ac^3 (1+a^2x^2)^2 \arctan(ax)^2}$$

input
```
Integrate[1/((c + a^2*c*x^2)^3*ArcTan[a*x]^3),x]
```

output
```
-1/2*(1 - 4*a*x*ArcTan[a*x] + 2*(1 + a^2*x^2)^2*ArcTan[a*x]^2*CosIntegral[
2*ArcTan[a*x]] + 2*(1 + a^2*x^2)^2*ArcTan[a*x]^2*CosIntegral[4*ArcTan[a*x]
])/(a*c^3*(1 + a^2*x^2)^2*ArcTan[a*x]^2)
```

## Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.48, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {5437, 27, 5503, 5439, 3042, 3793, 2009, 5505, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arctan(ax)^3 (a^2cx^2+c)^3} \, dx$$

$$\Big\downarrow \; 5437$$

$$-2a \int \frac{x}{c^3 (a^2x^2+1)^3 \arctan(ax)^2} dx - \frac{1}{2ac^3 (a^2x^2+1)^2 \arctan(ax)^2}$$

$$\Big\downarrow \; 27$$

$$-\frac{2a \int \frac{x}{(a^2x^2+1)^3 \arctan(ax)^2} dx}{c^3} - \frac{1}{2ac^3 (a^2x^2+1)^2 \arctan(ax)^2}$$

$$\Big\downarrow \; 5503$$

$$-\frac{2a\left(\frac{\int \frac{1}{\left(a^2x^2+1\right)^3\arctan(ax)}dx}{a}-3a\int \frac{x^2}{(a^2x^2+1)^3\arctan(ax)}dx-\frac{x}{a(a^2x^2+1)^2\arctan(ax)}\right)}{c^3}-\frac{1}{2ac^3\left(a^2x^2+1\right)^2\arctan(ax)^2}$$

$$\Big\downarrow \; 5439$$

$$-\frac{2a\left(-3a\int \frac{x^2}{(a^2x^2+1)^3\arctan(ax)}dx+\frac{\int \frac{1}{\left(a^2x^2+1\right)^2\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)^2\arctan(ax)}\right)}{c^3}-\frac{1}{2ac^3\left(a^2x^2+1\right)^2\arctan(ax)^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{2a\left(-3a\int \frac{x^2}{(a^2x^2+1)^3\arctan(ax)}dx+\frac{\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^4}{\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)^2\arctan(ax)}\right)}{c^3}-\frac{1}{2ac^3\left(a^2x^2+1\right)^2\arctan(ax)^2}$$

$$\Big\downarrow \; 3793$$

$$-\frac{2a\left(-3a\int \frac{x^2}{(a^2x^2+1)^3\arctan(ax)}dx+\frac{\int \left(\frac{\cos(2\arctan(ax))}{2\arctan(ax)}+\frac{\cos(4\arctan(ax))}{8\arctan(ax)}+\frac{3}{8\arctan(ax)}\right)d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)^2\arctan(ax)}\right)}{c^3}-\frac{1}{2ac^3\left(a^2x^2+1\right)^2\arctan(ax)^2}$$

$$\Big\downarrow \; 2009$$

$$-\frac{2a\left(-3a\int \frac{x^2}{(a^2x^2+1)^3\arctan(ax)}dx+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{8}\text{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)^2\text{arc}}\right.}{c^3}$$
$$\frac{1}{2ac^3\left(a^2x^2+1\right)^2\arctan(ax)^2}$$

$$\Big\downarrow \; 5505$$

$$-\frac{2a\left(-\frac{3\int \frac{a^2x^2}{\left(a^2x^2+1\right)^2\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))+\frac{1}{8}\operatorname{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}-\frac{}{a(a^2x^2+1}\right.}{c^3}$$

$$\frac{1}{2ac^3\left(a^2x^2+1\right)^2\arctan(ax)^2}$$

$\Big\downarrow$ 4906

$$-\frac{2a\left(-\frac{3\int\left(\frac{1}{8\arctan(ax)}-\frac{\cos(4\arctan(ax))}{8\arctan(ax)}\right)d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))+\frac{1}{8}\operatorname{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}-\right.}{c^3}$$

$$\frac{1}{2ac^3\left(a^2x^2+1\right)^2\arctan(ax)^2}$$

$\Big\downarrow$ 2009

$$-\frac{2a\left(-\frac{3\left(\frac{1}{8}\log(\arctan(ax))-\frac{1}{8}\operatorname{CosIntegral}(4\arctan(ax))\right)}{a^2}+\frac{\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))+\frac{1}{8}\operatorname{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}\right.}{c^3}$$

$$\frac{1}{2ac^3\left(a^2x^2+1\right)^2\arctan(ax)^2}$$

input
```
Int[1/((c + a^2*c*x^2)^3*ArcTan[a*x]^3),x]
```

output
```
-1/2*1/(a*c^3*(1 + a^2*x^2)^2*ArcTan[a*x]^2) - (2*a*(-(x/(a*(1 + a^2*x^2)^
2*ArcTan[a*x])) - (3*(-1/8*CosIntegral[4*ArcTan[a*x]] + Log[ArcTan[a*x]]/8
))/a^2 + (CosIntegral[2*ArcTan[a*x]]/2 + CosIntegral[4*ArcTan[a*x]]/8 + (3
*Log[ArcTan[a*x]])/8)/a^2))/c^3
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

## Maple [A] (verified)

Time = 6.80 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.10

| method | result |
|---|---|
| derivativedivides | $-\frac{16\,\mathrm{Ci}(2\arctan(ax))\arctan(ax)^2+16\,\mathrm{Ci}(4\arctan(ax))\arctan(ax)^2-8\sin(2\arctan(ax))\arctan(ax)-4\arctan(ax)\,\mathrm{s}}{16a\,c^3\arctan(ax)^2}$ |
| default | $-\frac{16\,\mathrm{Ci}(2\arctan(ax))\arctan(ax)^2+16\,\mathrm{Ci}(4\arctan(ax))\arctan(ax)^2-8\sin(2\arctan(ax))\arctan(ax)-4\arctan(ax)\,\mathrm{s}}{16a\,c^3\arctan(ax)^2}$ |

input
```
int(1/(a^2*c*x^2+c)^3/arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/16/a/c^3*(16*Ci(2*arctan(a*x))*arctan(a*x)^2+16*Ci(4*arctan(a*x))*arcta
n(a*x)^2-8*sin(2*arctan(a*x))*arctan(a*x)-4*arctan(a*x)*sin(4*arctan(a*x))
+4*cos(2*arctan(a*x))+cos(4*arctan(a*x))+3)/arctan(a*x)^2
```

## Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 297, normalized size of antiderivative = 3.67

$$\int \frac{1}{(c+a^2cx^2)^3\arctan(ax)^3}\,dx =$$

$$-\frac{(a^4x^4+2\,a^2x^2+1)\arctan(ax)^2\,\mathrm{log\_integral}\left(\frac{a^4x^4+4i\,a^3x^3-6\,a^2x^2-4i\,ax+1}{a^4x^4+2\,a^2x^2+1}\right)+(a^4x^4+2\,a^2x^2+1)\arctan}{}$$

input
```
integrate(1/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
-1/2*((a^4*x^4 + 2*a^2*x^2 + 1)*arctan(a*x)^2*log_integral((a^4*x^4 + 4*I*
a^3*x^3 - 6*a^2*x^2 - 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) + (a^4*x^4 +
 2*a^2*x^2 + 1)*arctan(a*x)^2*log_integral((a^4*x^4 - 4*I*a^3*x^3 - 6*a^2*
x^2 + 4*I*a*x + 1)/(a^4*x^4 + 2*a^2*x^2 + 1)) + (a^4*x^4 + 2*a^2*x^2 + 1)*
arctan(a*x)^2*log_integral(-(a^2*x^2 + 2*I*a*x - 1)/(a^2*x^2 + 1)) + (a^4*
x^4 + 2*a^2*x^2 + 1)*arctan(a*x)^2*log_integral(-(a^2*x^2 - 2*I*a*x - 1)/(
a^2*x^2 + 1)) - 4*a*x*arctan(a*x) + 1)/((a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c
^3)*arctan(a*x)^2)
```

## Sympy [F]

$$\int \frac{1}{\left(c+a^2cx^2\right)^3\arctan(ax)^3}\,dx=\frac{\int \frac{1}{a^6x^6\operatorname{atan}^3(ax)+3a^4x^4\operatorname{atan}^3(ax)+3a^2x^2\operatorname{atan}^3(ax)+\operatorname{atan}^3(ax)}\,dx}{c^3}$$

input
```
integrate(1/(a**2*c*x**2+c)**3/atan(a*x)**3,x)
```

output
```
Integral(1/(a**6*x**6*atan(a*x)**3 + 3*a**4*x**4*atan(a*x)**3 + 3*a**2*x**
2*atan(a*x)**3 + atan(a*x)**3), x)/c**3
```

## Maxima [F]

$$\int \frac{1}{\left(c+a^2cx^2\right)^3\arctan(ax)^3}\,dx=\int \frac{1}{\left(a^2cx^2+c\right)^3\arctan(ax)^3}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
1/2*(2*(a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)*arctan(a*x)^2*integrate(2*(3*
a^2*x^2 - 1)/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a
*x)), x) + 4*a*x*arctan(a*x) - 1)/((a^5*c^3*x^4 + 2*a^3*c^3*x^2 + a*c^3)*a
rctan(a*x)^2)
```

## Giac [F]

$$\int \frac{1}{(c+a^2cx^2)^3 \arctan(ax)^3}\,dx = \int \frac{1}{(a^2cx^2+c)^3 \arctan(ax)^3}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^3*arctan(a*x)^3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+a^2cx^2)^3 \arctan(ax)^3}\,dx = \int \frac{1}{\mathrm{atan}(a\,x)^3\,(c\,a^2\,x^2+c)^3}\,dx$$

input
```
int(1/(atan(a*x)^3*(c + a^2*c*x^2)^3),x)
```

output
```
int(1/(atan(a*x)^3*(c + a^2*c*x^2)^3), x)
```

## Reduce [F]

$$\int \frac{1}{(c+a^2cx^2)^3 \arctan(ax)^3}\,dx$$
$$= \frac{-4atan(ax)^2\left(\int \frac{x}{atan(ax)^2a^6x^6+3atan(ax)^2a^4x^4+3atan(ax)^2a^2x^2+atan(ax)^2}dx\right)a^6x^4-8atan(ax)^2\left(\int \frac{}{atan(ax)^2a^6x^6+3}\right.}{2atan(ax)}$$

input
```
int(1/(a^2*c*x^2+c)^3/atan(a*x)^3,x)
```

output
```
( - 4*atan(a*x)**2*int(x/(atan(a*x)**2*a**6*x**6 + 3*atan(a*x)**2*a**4*x**
4 + 3*atan(a*x)**2*a**2*x**2 + atan(a*x)**2),x)*a**6*x**4 - 8*atan(a*x)**2
*int(x/(atan(a*x)**2*a**6*x**6 + 3*atan(a*x)**2*a**4*x**4 + 3*atan(a*x)**2
*a**2*x**2 + atan(a*x)**2),x)*a**4*x**2 - 4*atan(a*x)**2*int(x/(atan(a*x)*
*2*a**6*x**6 + 3*atan(a*x)**2*a**4*x**4 + 3*atan(a*x)**2*a**2*x**2 + atan(
a*x)**2),x)*a**2 - 1)/(2*atan(a*x)**2*a*c**3*(a**4*x**4 + 2*a**2*x**2 + 1)
)
```

# 3.638 $\int \frac{1}{x(c+a^2cx^2)^3 \arctan(ax)^3} \, dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{1}{x(c+a^2cx^2)^3 \arctan(ax)^3} \, dx = -\frac{1}{2ac^3x\arctan(ax)^2} + \frac{ax}{2c^3(1+a^2x^2)^2\arctan(ax)^2} + \frac{ax}{2c^3(1+a^2x^2)\arctan(ax)^2} + \frac{2}{c^3(1+a^2x^2)^2\arctan(ax)} - \frac{3}{2c^3(1+a^2x^2)\arctan(ax)} + \frac{1-a^2x^2}{2c^3(1+a^2x^2)\arctan(ax)} + \frac{3\text{Si}(2\arctan(ax))}{2c^3} + \frac{\text{Si}(4\arctan(ax))}{c^3} - \frac{\text{Int}\left(\frac{1}{x^2\arctan(ax)^2}, x\right)}{2ac^3}$$

output
```
-1/2/a/c^3/x/arctan(a*x)^2+1/2*a*x/c^3/(a^2*x^2+1)^2/arctan(a*x)^2+1/2*a*x
/c^3/(a^2*x^2+1)/arctan(a*x)^2+2/c^3/(a^2*x^2+1)^2/arctan(a*x)-3/2/c^3/(a^
2*x^2+1)/arctan(a*x)+1/2*(-a^2*x^2+1)/c^3/(a^2*x^2+1)/arctan(a*x)+3/2*Si(2
*arctan(a*x))/c^3+Si(4*arctan(a*x))/c^3-1/2*Defer(Int)(1/x^2/arctan(a*x)^2
,x)/a/c^3
```

## Mathematica [N/A]

Not integrable

Time = 1.84 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\arctan(ax)^3}\,dx=\int \frac{1}{x\left(c+a^2cx^2\right)^3\arctan(ax)^3}\,dx$$

input
```
Integrate[1/(x*(c + a^2*c*x^2)^3*ArcTan[a*x]^3),x]
```

output
```
Integrate[1/(x*(c + a^2*c*x^2)^3*ArcTan[a*x]^3), x]
```

## Rubi [N/A]

Not integrable

Time = 2.69 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\arctan(ax)^3\left(a^2cx^2+c\right)^3}dx$$

$$\Big\downarrow \text{ 5501}$$

$$\frac{\int \frac{1}{c^2x(a^2x^2+1)^2\arctan(ax)^3}dx}{c}-a^2\int \frac{x}{c^3\left(a^2x^2+1\right)^3\arctan(ax)^3}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{1}{x(a^2x^2+1)^2\arctan(ax)^3}dx}{c^3}-\frac{a^2\int \frac{x}{(a^2x^2+1)^3\arctan(ax)^3}dx}{c^3}$$

$$\Big\downarrow \text{ 5501}$$

$$\frac{\int \frac{1}{x(a^2x^2+1)\arctan(ax)^3}dx-a^2\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^3}dx}{c^3}-\frac{a^2\int \frac{x}{(a^2x^2+1)^3\arctan(ax)^3}dx}{c^3}$$

$$\Big\downarrow \text{5461}$$

$$\frac{a^2\left(-\int \frac{x}{\left(a^2x^2+1\right)^2\arctan(ax)^3}dx\right)-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{1}{2ax\arctan(ax)^2}}{c^3}-\frac{a^2\int \frac{x}{\left(a^2x^2+1\right)^3\arctan(ax)^3}dx}{c^3}$$

$$\Big\downarrow \text{5377}$$

$$\frac{a^2\left(-\int \frac{x}{\left(a^2x^2+1\right)^2\arctan(ax)^3}dx\right)-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{1}{2ax\arctan(ax)^2}}{c^3}-\frac{a^2\int \frac{x}{\left(a^2x^2+1\right)^3\arctan(ax)^3}dx}{c^3}$$

$$\Big\downarrow \text{5467}$$

$$\frac{-\left(a^2\left(-2\int \frac{x}{\left(a^2x^2+1\right)^2\arctan(ax)}dx-\frac{x}{2a\left(a^2x^2+1\right)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2\left(a^2x^2+1\right)\arctan(ax)}\right)\right)-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{1}{2ax\arctan(ax)^2}}{c^3}-\frac{a^2\int \frac{x}{\left(a^2x^2+1\right)^3\arctan(ax)^3}dx}{c^3}$$

$$\Big\downarrow \text{5503}$$

$$\frac{-\left(a^2\left(-2\int \frac{x}{\left(a^2x^2+1\right)^2\arctan(ax)}dx-\frac{x}{2a\left(a^2x^2+1\right)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2\left(a^2x^2+1\right)\arctan(ax)}\right)\right)-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{1}{2ax\arctan(ax)^2}}{c^3}-\frac{a^2\left(\frac{\int \frac{1}{\left(a^2x^2+1\right)^3\arctan(ax)^2}dx}{2a}-\frac{3}{2}a\int \frac{x^2}{\left(a^2x^2+1\right)^3\arctan(ax)^2}dx-\frac{x}{2a\left(a^2x^2+1\right)^2\arctan(ax)^2}\right)}{c^3}$$

$$\Big\downarrow \text{5437}$$

$$\frac{-\left(a^2\left(-2\int \frac{x}{\left(a^2x^2+1\right)^2\arctan(ax)}dx-\frac{x}{2a\left(a^2x^2+1\right)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2\left(a^2x^2+1\right)\arctan(ax)}\right)\right)-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{1}{2ax\arctan(ax)^2}}{c^3}-\frac{a^2\left(-\frac{3}{2}a\int \frac{x^2}{\left(a^2x^2+1\right)^3\arctan(ax)^2}dx+\frac{-4a\int \frac{x}{\left(a^2x^2+1\right)^3\arctan(ax)}dx-\frac{1}{a\left(a^2x^2+1\right)^2\arctan(ax)}}{2a}-\frac{x}{2a\left(a^2x^2+1\right)^2\arctan(ax)^2}\right)}{c^3}$$

$$\Big\downarrow \text{5499}$$

$$\frac{-\left(a^2\left(-2\int \frac{x}{(a^2x^2+1)^2\arctan(ax)}dx-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{1}{2ax\arctan(ax}}{c^3}$$

$$\frac{a^2\left(-\frac{3}{2}a\left(\frac{\int \frac{1}{\left(a^2x^2+1\right)^2\arctan(ax)^2}dx}{a^2}-\frac{\int \frac{1}{\left(a^2x^2+1\right)^3\arctan(ax)^2}dx}{a^2}\right)+\frac{-4a\int \frac{x}{\left(a^2x^2+1\right)^3\arctan(ax)}dx-\frac{1}{a\left(a^2x^2+1\right)^2\arctan(ax)}}{2a}-\frac{}{2a(a^2}\right.}{c^3}$$

$\downarrow$ 5437

$$\frac{-\left(a^2\left(-2\int \frac{x}{(a^2x^2+1)^2\arctan(ax)}dx-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{1}{2ax\arctan(ax}}{c^3}$$

$$\frac{a^2\left(\frac{-4a\int \frac{x}{\left(a^2x^2+1\right)^3\arctan(ax)}dx-\frac{1}{a\left(a^2x^2+1\right)^2\arctan(ax)}}{2a}-\frac{3}{2}a\left(\frac{-2a\int \frac{x}{\left(a^2x^2+1\right)^2\arctan(ax)}dx-\frac{1}{a\left(a^2x^2+1\right)\arctan(ax)}}{a^2}-\frac{-4a\int \frac{}{\left(a^2x^2+1\right.}}{}\right.\right.}{c^3}$$

$\downarrow$ 5505

$$\frac{-\left(a^2\left(-\frac{2\int \frac{ax}{\left(a^2x^2+1\right)\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{}{2ax\,a}}{c^3}$$

$$\frac{a^2\left(\frac{-\frac{4\int \frac{ax}{\left(a^2x^2+1\right)^2\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a\left(a^2x^2+1\right)^2\arctan(ax)}}{2a}-\frac{3}{2}a\left(\frac{-\frac{2\int \frac{ax}{\left(a^2x^2+1\right)\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a\left(a^2x^2+1\right)\arctan(ax)}}{a^2}-\frac{-}{}\right.\right.}{c^3}$$

$\downarrow$ 4906

$$\frac{-\left(a^2\left(-\frac{2\int \frac{\sin(2\arctan(ax))}{2\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{1}{2ax\arctan}}{c^3}$$

$$\frac{a^2\left(\frac{-\frac{4\int \left(\frac{\sin(2\arctan(ax))}{4\arctan(ax)}+\frac{\sin(4\arctan(ax))}{8\arctan(ax)}\right)d\arctan(ax)}{a}-\frac{1}{a\left(a^2x^2+1\right)^2\arctan(ax)}}{2a}-\frac{3}{2}a\left(\frac{-\frac{2\int \frac{\sin(2\arctan(ax))}{2\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a\left(a^2x^2+1\right)\arctan(}}{a^2}\right.\right.}{c^3}$$

$\downarrow$ 27

$$\frac{-\left(a^2\left(-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{1}{2ax\arctan(}}{c^3}$$

$$\frac{a^2\left(\frac{-\frac{4\int\left(\frac{\sin(2\arctan(ax))}{4\arctan(ax)}+\frac{\sin(4\arctan(ax))}{8\arctan(ax)}\right)d\arctan(ax)}{a}-\frac{1}{a\left(a^2x^2+1\right)^2\arctan(ax)}}{2a}-\frac{3}{2}a\left(\frac{-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a\left(a^2x^2+1\right)\arctan(a}}{a^2}\right.\right.}{c^3}$$

$\downarrow$ 2009

$$\frac{-\left(a^2\left(-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{1}{2ax\arctan(}}{c^3}$$

$$\frac{a^2\left(-\frac{3}{2}a\left(\frac{-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a\left(a^2x^2+1\right)\arctan(ax)}}{a^2}-\frac{-\frac{1}{a\left(a^2x^2+1\right)^2\arctan(ax)}-\frac{4\left(\frac{1}{4}\text{Si}(2\arctan(ax))+\frac{1}{8}\text{Si}(4\arctan(ax))\right)}{a}}{a^2}\right)+}{c^3}$$

$\downarrow$ 3042

$$\frac{-\left(a^2\left(-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{1}{2ax\arctan(}}{c^3}$$

$$\frac{a^2\left(-\frac{3}{2}a\left(\frac{-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a\left(a^2x^2+1\right)\arctan(ax)}}{a^2}-\frac{-\frac{1}{a\left(a^2x^2+1\right)^2\arctan(ax)}-\frac{4\left(\frac{1}{4}\text{Si}(2\arctan(ax))+\frac{1}{8}\text{Si}(4\arctan(ax))\right)}{a}}{a^2}\right)+}{c^3}$$

$\downarrow$ 3780

$$\frac{-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\left(a^2\left(-\frac{\text{Si}(2\arctan(ax))}{a^2}-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{1}{2ax\arctan(ax)^2}}{c^3}-$$

$$\frac{a^2\left(-\frac{3}{2}a\left(\frac{-\frac{1}{a\left(a^2x^2+1\right)\arctan(ax)}-\frac{\text{Si}(2\arctan(ax))}{a}}{a^2}-\frac{-\frac{1}{a\left(a^2x^2+1\right)^2\arctan(ax)}-\frac{4\left(\frac{1}{4}\text{Si}(2\arctan(ax))+\frac{1}{8}\text{Si}(4\arctan(ax))\right)}{a}}{a^2}\right)+\frac{-}{a\left(a^2x^2+1\right)}}{c^3}$$

input

```
Int[1/(x*(c + a^2*c*x^2)^3*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 4.59 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a^2c\,x^2+c\right)^3\arctan\left(ax\right)^3}dx$$

input
```
int(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^3,x)
```

output
```
int(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.23

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\arctan(ax)^3}\,dx=\int \frac{1}{\left(a^2cx^2+c\right)^3x\arctan\left(ax\right)^3}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(1/((a^6*c^3*x^7 + 3*a^4*c^3*x^5 + 3*a^2*c^3*x^3 + c^3*x)*arctan(a
*x)^3), x)
```

## Sympy [N/A]

Not integrable

Time = 1.59 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.64

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\arctan(ax)^3}\,dx = \frac{\int \frac{1}{a^6x^7\operatorname{atan}^3(ax)+3a^4x^5\operatorname{atan}^3(ax)+3a^2x^3\operatorname{atan}^3(ax)+x\operatorname{atan}^3(ax)}\,dx}{c^3}$$

input
```
integrate(1/x/(a**2*c*x**2+c)**3/atan(a*x)**3,x)
```

output
```
Integral(1/(a**6*x**7*atan(a*x)**3 + 3*a**4*x**5*atan(a*x)**3 + 3*a**2*x**
3*atan(a*x)**3 + x*atan(a*x)**3), x)/c**3
```

## Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 174, normalized size of antiderivative = 7.91

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\arctan(ax)^3}\,dx = \int \frac{1}{\left(a^2cx^2+c\right)^3x\arctan\left(ax\right)^3}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
1/2*(2*(a^6*c^3*x^6 + 2*a^4*c^3*x^4 + a^2*c^3*x^2)*arctan(a*x)^2*integrate
((10*a^4*x^4 + 3*a^2*x^2 + 1)/((a^8*c^3*x^9 + 3*a^6*c^3*x^7 + 3*a^4*c^3*x^
5 + a^2*c^3*x^3)*arctan(a*x)), x) - a*x + (5*a^2*x^2 + 1)*arctan(a*x))/((a
^6*c^3*x^6 + 2*a^4*c^3*x^4 + a^2*c^3*x^2)*arctan(a*x)^2)
```

## Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\arctan(ax)^3}\,dx=\int \frac{1}{\left(a^2cx^2+c\right)^3x\arctan\left(ax\right)^3}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^3*x*arctan(a*x)^3), x)
```

## Mupad [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\arctan(ax)^3}\,dx=\int \frac{1}{x\,\mathrm{atan}\left(a\,x\right)^3\,\left(c\,a^2\,x^2+c\right)^3}\,dx$$

input
```
int(1/(x*atan(a*x)^3*(c + a^2*c*x^2)^3),x)
```

output
```
int(1/(x*atan(a*x)^3*(c + a^2*c*x^2)^3), x)
```

## Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.64

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\arctan(ax)^3}\,dx=\frac{\int \frac{1}{atan(ax)^3a^6x^7+3atan(ax)^3a^4x^5+3atan(ax)^3a^2x^3+atan(ax)^3x}dx}{c^3}$$

input
```
int(1/x/(a^2*c*x^2+c)^3/atan(a*x)^3,x)
```

output
```
int(1/(atan(a*x)**3*a**6*x**7 + 3*atan(a*x)**3*a**4*x**5 + 3*atan(a*x)**3*
a**2*x**3 + atan(a*x)**3*x),x)/c**3
```

# 3.639 $\int \frac{1}{x^2(c+a^2cx^2)^3\arctan(ax)^3}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\begin{aligned}
\int \frac{1}{x^2(c+a^2cx^2)^3\arctan(ax)^3}\,dx = &-\frac{1}{2ac^3x^2\arctan(ax)^2}+\frac{a}{2c^3(1+a^2x^2)^2\arctan(ax)^2}\\
&+\frac{a}{2c^3(1+a^2x^2)\arctan(ax)^2}\\
&-\frac{2a^2x}{c^3(1+a^2x^2)^2\arctan(ax)}\\
&-\frac{a^2x}{c^3(1+a^2x^2)\arctan(ax)}\\
&+\frac{2a\,\mathrm{CosIntegral}(2\arctan(ax))}{c^3}\\
&+\frac{a\,\mathrm{CosIntegral}(4\arctan(ax))}{c^3}\\
&-\frac{\mathrm{Int}\left(\frac{1}{x^3\arctan(ax)^2},x\right)}{ac^3}
\end{aligned}$$

output

```
-1/2/a/c^3/x^2/arctan(a*x)^2+1/2*a/c^3/(a^2*x^2+1)^2/arctan(a*x)^2+1/2*a/c
^3/(a^2*x^2+1)/arctan(a*x)^2-2*a^2*x/c^3/(a^2*x^2+1)^2/arctan(a*x)-a^2*x/c
^3/(a^2*x^2+1)/arctan(a*x)+2*a*Ci(2*arctan(a*x))/c^3+a*Ci(4*arctan(a*x))/c
^3-Defer(Int)(1/x^3/arctan(a*x)^2,x)/a/c^3
```

## Mathematica [N/A]

Not integrable

Time = 2.70 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2 \left(c+a^2 c x^2\right)^3 \arctan(ax)^3} \, dx=\int \frac{1}{x^2 \left(c+a^2 c x^2\right)^3 \arctan(ax)^3} \, dx$$

input
```
Integrate[1/(x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^3),x]
```

output
```
Integrate[1/(x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^3), x]
```

## Rubi [N/A]

Not integrable

Time = 2.90 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \arctan(ax)^3 \left(a^2 c x^2+c\right)^3} \, dx$$

$$\Big\downarrow \text{ 5501}$$

$$\frac{\int \frac{1}{c^2 x^2 \left(a^2 x^2+1\right)^2 \arctan(ax)^3}dx}{c}-a^2 \int \frac{1}{c^3 \left(a^2 x^2+1\right)^3 \arctan(ax)^3}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{1}{x^2 \left(a^2 x^2+1\right)^2 \arctan(ax)^3}dx}{c^3}-\frac{a^2 \int \frac{1}{\left(a^2 x^2+1\right)^3 \arctan(ax)^3}dx}{c^3}$$

$$\Big\downarrow \text{ 5437}$$

$$\frac{\int \frac{1}{x^2 \left(a^2 x^2+1\right)^2 \arctan(ax)^3}dx}{c^3}-\frac{a^2 \left(-2 a \int \frac{x}{\left(a^2 x^2+1\right)^3 \arctan(ax)^2}dx-\frac{1}{2 a \left(a^2 x^2+1\right)^2 \arctan(ax)^2}\right)}{c^3}$$

$\downarrow$ 5501

$$\frac{\int \frac{1}{x^2(a^2x^2+1)\arctan(ax)^3}dx-a^2\int \frac{1}{(a^2x^2+1)^2\arctan(ax)^3}dx}{c^3}-\frac{a^2\left(-2a\int \frac{x}{(a^2x^2+1)^3\arctan(ax)^2}dx-\frac{1}{2a(a^2x^2+1)^2\arctan(ax)^2}\right)}{c^3}$$

$\downarrow$ 5437

$$\frac{\int \frac{1}{x^2(a^2x^2+1)\arctan(ax)^3}dx-a^2\left(-a\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^2}dx-\frac{1}{2a(a^2x^2+1)\arctan(ax)^2}\right)}{c^3}-\frac{a^2\left(-2a\int \frac{x}{(a^2x^2+1)^3\arctan(ax)^2}dx-\frac{1}{2a(a^2x^2+1)^2\arctan(ax)^2}\right)}{c^3}$$

$\downarrow$ 5461

$$\frac{-\left(a^2\left(-a\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^2}dx-\frac{1}{2a(a^2x^2+1)\arctan(ax)^2}\right)\right)-\frac{\int \frac{1}{x^3\arctan(ax)^2}dx}{a}-\frac{1}{2ax^2\arctan(ax)^2}}{c^3}-\frac{a^2\left(-2a\int \frac{x}{(a^2x^2+1)^3\arctan(ax)^2}dx-\frac{1}{2a(a^2x^2+1)^2\arctan(ax)^2}\right)}{c^3}$$

$\downarrow$ 5377

$$\frac{-\left(a^2\left(-a\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^2}dx-\frac{1}{2a(a^2x^2+1)\arctan(ax)^2}\right)\right)-\frac{\int \frac{1}{x^3\arctan(ax)^2}dx}{a}-\frac{1}{2ax^2\arctan(ax)^2}}{c^3}-\frac{a^2\left(-2a\int \frac{x}{(a^2x^2+1)^3\arctan(ax)^2}dx-\frac{1}{2a(a^2x^2+1)^2\arctan(ax)^2}\right)}{c^3}$$

$\downarrow$ 5503

$$\frac{-\left(a^2\left(-a\left(\frac{\int \frac{1}{(a^2x^2+1)^2\arctan(ax)}dx}{a}-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)-\frac{1}{2a(a^2x^2+1)\arctan(ax)^2}\right)\right)-}{c^3}$$

$$\frac{a^2\left(-2a\left(\frac{\int \frac{1}{(a^2x^2+1)^3\arctan(ax)}dx}{a}-3a\int \frac{x^2}{(a^2x^2+1)^3\arctan(ax)}dx-\frac{x}{a(a^2x^2+1)^2\arctan(ax)}\right)-\frac{1}{2a(a^2x^2+1)^2\arctan(ax)^2}\right)}{c^3}$$

$\downarrow$ 5439

$$-\frac{\left(a^2\left(-a\left(-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\int \frac{1}{(a^2x^2+1)\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)-\frac{1}{2a(a^2x^2+1)\arctan(ax}\right.\right.}{c^3}$$

$$\frac{a^2\left(-2a\left(-3a\int \frac{x^2}{(a^2x^2+1)^3\arctan(ax)}dx+\frac{\int \frac{1}{(a^2x^2+1)^2\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)^2\arctan(ax)}\right)-\frac{1}{2a(a^2x^2+1)^2\arctan(}\right.}{c^3}$$

$\downarrow$ 3042

$$-\frac{\left(a^2\left(-a\left(-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^2}{\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)-\frac{1}{2a(a^2x^2+1)\arctan(a}\right.\right.}{c^3}$$

$$\frac{a^2\left(-2a\left(-3a\int \frac{x^2}{(a^2x^2+1)^3\arctan(ax)}dx+\frac{\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^4}{\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)^2\arctan(ax)}\right)-\frac{1}{2a(a^2x^2+1)^2\arctan(a}\right.}{c^3}$$

$\downarrow$ 3793

$$-\frac{\left(a^2\left(-a\left(-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\int \left(\frac{\cos(2\arctan(ax))}{2\arctan(ax)}+\frac{1}{2\arctan(ax)}\right)d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)-\frac{}{2a(a^2x^2+1}\right.\right.}{c^3}$$

$$\frac{a^2\left(-2a\left(-3a\int \frac{x^2}{(a^2x^2+1)^3\arctan(ax)}dx+\frac{\int \left(\frac{\cos(2\arctan(ax))}{2\arctan(ax)}+\frac{\cos(4\arctan(ax))}{8\arctan(ax)}+\frac{3}{8\arctan(ax)}\right)d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)^2\arctan(ax)}\right.\right.}{c^3}$$

$\downarrow$ 2009

$$-\frac{\left(a^2\left(-a\left(-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)-\frac{}{2a(a^2x^2+1}\right.\right.}{c^3}$$

$$\frac{a^2\left(-2a\left(-3a\int \frac{x^2}{(a^2x^2+1)^3\arctan(ax)}dx+\frac{\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))+\frac{1}{8}\operatorname{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}-\frac{}{a(a^2x^2+1)}\right.\right.}{c^3}$$

$\downarrow$ 5505

$$-\frac{\left(a^2\left(-a\left(-\frac{\int \frac{a^2x^2}{\left(a^2x^2+1\right)\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)-\frac{}{2a(\cdots}\right.\right.}{c^3}$$

$$\frac{a^2\left(-2a\left(-\frac{3\int \frac{a^2x^2}{\left(a^2x^2+1\right)^2\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{8}\text{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}-\frac{}{a(a\cdots}\right.\right.}{c^3}$$

$\downarrow$ 3042

$$-\frac{\left(a^2\left(-a\left(-\frac{\int \frac{\sin(\arctan(ax))^2}{\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)-\frac{}{2a(a^2x^2+\cdots}\right.\right.}{c^3}$$

$$\frac{a^2\left(-2a\left(-\frac{3\int \frac{a^2x^2}{\left(a^2x^2+1\right)^2\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{8}\text{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}-\frac{}{a(a\cdots}\right.\right.}{c^3}$$

$\downarrow$ 3793

$$-\frac{\left(a^2\left(-a\left(-\frac{\int \left(\frac{1}{2\arctan(ax)}-\frac{\cos(2\arctan(ax))}{2\arctan(ax)}\right)d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\cdots\right.\right.}{c^3}$$

$$\frac{a^2\left(-2a\left(-\frac{3\int \frac{a^2x^2}{\left(a^2x^2+1\right)^2\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{8}\text{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}-\frac{}{a(a\cdots}\right.\right.}{c^3}$$

$\downarrow$ 2009

$$-\frac{\int \frac{1}{x^3\arctan(ax)^2}dx}{a}-\frac{\left(a^2\left(-a\left(-\frac{\frac{1}{2}\log(\arctan(ax))-\frac{1}{2}\text{CosIntegral}(2\arctan(ax))}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\cdots\right.\right.}{c^3}$$

$$\frac{a^2\left(-2a\left(-\frac{3\int \frac{a^2x^2}{\left(a^2x^2+1\right)^2\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{8}\text{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}-\frac{}{a(a\cdots}\right.\right.}{c^3}$$

$\downarrow$ 4906

$$\frac{-\frac{\int \frac{1}{x^3 \arctan(ax)^2}dx}{a} - \left(a^2\left(-a\left(-\frac{\frac{1}{2}\log(\arctan(ax))-\frac{1}{2}\text{CosIntegral}(2\arctan(ax))}{a^2} + \frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2} - \right.\right.\right.}{c^3}$$

$$\frac{a^2\left(-2a\left(-\frac{3\int\left(\frac{1}{8\arctan(ax)}-\frac{\cos(4\arctan(ax))}{8\arctan(ax)}\right)d\arctan(ax)}{a^2} + \frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{8}\text{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax\right.\right.}{c^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{\int \frac{1}{x^3 \arctan(ax)^2}dx}{a} - \left(a^2\left(-a\left(-\frac{\frac{1}{2}\log(\arctan(ax))-\frac{1}{2}\text{CosIntegral}(2\arctan(ax))}{a^2} + \frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2} - \right.\right.\right.}{c^3}$$

$$\frac{a^2\left(-2a\left(-\frac{3\left(\frac{1}{8}\log(\arctan(ax))-\frac{1}{8}\text{CosIntegral}(4\arctan(ax))\right)}{a^2} + \frac{\frac{1}{2}\text{CosIntegral}(2\arctan(ax))+\frac{1}{8}\text{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(\right.\right.}{c^3}$$

input
```
Int[1/(x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.87 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 \left(a^2 c\, x^2 + c\right)^3 \arctan\left(ax\right)^3}dx$$

input
```
int(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^3,x)
```

output
```
int(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.32

$$\int \frac{1}{x^2 (c + a^2 c x^2)^3 \arctan(ax)^3} \, dx = \int \frac{1}{(a^2 c x^2 + c)^3 x^2 \arctan(ax)^3} \, dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(1/((a^6*c^3*x^8 + 3*a^4*c^3*x^6 + 3*a^2*c^3*x^4 + c^3*x^2)*arctan
(a*x)^3), x)
```

## Sympy [N/A]

Not integrable

Time = 1.63 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.73

$$\int \frac{1}{x^2 (c + a^2 c x^2)^3 \arctan(ax)^3} \, dx = \frac{\int \frac{1}{a^6 x^8 \operatorname{atan}^3(ax) + 3a^4 x^6 \operatorname{atan}^3(ax) + 3a^2 x^4 \operatorname{atan}^3(ax) + x^2 \operatorname{atan}^3(ax)} \, dx}{c^3}$$

input
```
integrate(1/x**2/(a**2*c*x**2+c)**3/atan(a*x)**3,x)
```

output
```
Integral(1/(a**6*x**8*atan(a*x)**3 + 3*a**4*x**6*atan(a*x)**3 + 3*a**2*x**
4*atan(a*x)**3 + x**2*atan(a*x)**3), x)/c**3
```

## Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 175, normalized size of antiderivative = 7.95

$$\int \frac{1}{x^2 (c + a^2cx^2)^3 \arctan(ax)^3} \, dx = \int \frac{1}{(a^2cx^2 + c)^3 x^2 \arctan(ax)^3} \, dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
1/2*(2*(a^6*c^3*x^7 + 2*a^4*c^3*x^5 + a^2*c^3*x^3)*arctan(a*x)^2*integrate
((15*a^4*x^4 + 10*a^2*x^2 + 3)/((a^8*c^3*x^10 + 3*a^6*c^3*x^8 + 3*a^4*c^3*
x^6 + a^2*c^3*x^4)*arctan(a*x)), x) - a*x + 2*(3*a^2*x^2 + 1)*arctan(a*x))
/((a^6*c^3*x^7 + 2*a^4*c^3*x^5 + a^2*c^3*x^3)*arctan(a*x)^2)
```

## Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2 (c + a^2cx^2)^3 \arctan(ax)^3} \, dx = \int \frac{1}{(a^2cx^2 + c)^3 x^2 \arctan(ax)^3} \, dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^3*x^2*arctan(a*x)^3), x)
```

## Mupad [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^2 (c+a^2cx^2)^3 \arctan(ax)^3} \, dx = \int \frac{1}{x^2 \operatorname{atan}(a\,x)^3 \left(c\,a^2\,x^2+c\right)^3} \, dx$$

input
```
int(1/(x^2*atan(a*x)^3*(c + a^2*c*x^2)^3),x)
```

output
```
int(1/(x^2*atan(a*x)^3*(c + a^2*c*x^2)^3), x)
```

## Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.73

$$\int \frac{1}{x^2 (c+a^2cx^2)^3 \arctan(ax)^3} \, dx = \frac{\int \frac{1}{atan(ax)^3 a^6 x^8 + 3atan(ax)^3 a^4 x^6 + 3atan(ax)^3 a^2 x^4 + atan(ax)^3 x^2} dx}{c^3}$$

input
```
int(1/x^2/(a^2*c*x^2+c)^3/atan(a*x)^3,x)
```

output
```
int(1/(atan(a*x)**3*a**6*x**8 + 3*atan(a*x)**3*a**4*x**6 + 3*atan(a*x)**3*
a**2*x**4 + atan(a*x)**3*x**2),x)/c**3
```

# 3.640 $\int \frac{1}{x^3(c+a^2cx^2)^3 \arctan(ax)^3} dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\begin{aligned}\int \frac{1}{x^3 (c+a^2cx^2)^3 \arctan(ax)^3} dx &= -\frac{1}{2ac^3x^3 \arctan(ax)^2} + \frac{a}{c^3x \arctan(ax)^2} \\ &\quad - \frac{a^3x}{2c^3 (1+a^2x^2)^2 \arctan(ax)^2} \\ &\quad - \frac{a^3x}{c^3 (1+a^2x^2) \arctan(ax)^2} \\ &\quad - \frac{2a^2}{c^3 (1+a^2x^2)^2 \arctan(ax)} \\ &\quad + \frac{3a^2}{2c^3 (1+a^2x^2) \arctan(ax)} \\ &\quad - \frac{a^2(1-a^2x^2)}{c^3 (1+a^2x^2) \arctan(ax)} \\ &\quad - \frac{5a^2 \mathrm{Si}(2\arctan(ax))}{2c^3} - \frac{a^2 \mathrm{Si}(4\arctan(ax))}{c^3} \\ &\quad - \frac{3\mathrm{Int}\left(\frac{1}{x^4 \arctan(ax)^2}, x\right)}{2ac^3} + \frac{a\mathrm{Int}\left(\frac{1}{x^2 \arctan(ax)^2}, x\right)}{c^3}\end{aligned}$$

output
```
-1/2/a/c^3/x^3/arctan(a*x)^2+a/c^3/x/arctan(a*x)^2-1/2*a^3*x/c^3/(a^2*x^2+
1)^2/arctan(a*x)^2-a^3*x/c^3/(a^2*x^2+1)/arctan(a*x)^2-2*a^2/c^3/(a^2*x^2+
1)^2/arctan(a*x)+3/2*a^2/c^3/(a^2*x^2+1)/arctan(a*x)-a^2*(-a^2*x^2+1)/c^3/
(a^2*x^2+1)/arctan(a*x)-5/2*a^2*Si(2*arctan(a*x))/c^3-a^2*Si(4*arctan(a*x)
)/c^3-3/2*Defer(Int)(1/x^4/arctan(a*x)^2,x)/a/c^3+a*Defer(Int)(1/x^2/arcta
n(a*x)^2,x)/c^3
```

## Mathematica [N/A]

Not integrable

Time = 3.43 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^3 (c+a^2cx^2)^3 \arctan(ax)^3} \, dx = \int \frac{1}{x^3 (c+a^2cx^2)^3 \arctan(ax)^3} \, dx$$

input
```
Integrate[1/(x^3*(c + a^2*c*x^2)^3*ArcTan[a*x]^3),x]
```

output
```
Integrate[1/(x^3*(c + a^2*c*x^2)^3*ArcTan[a*x]^3), x]
```

## Rubi [N/A]

Not integrable

Time = 4.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \arctan(ax)^3 (a^2cx^2+c)^3} \, dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{1}{c^2x^3(a^2x^2+1)^2\arctan(ax)^3}dx}{c} - a^2 \int \frac{1}{c^3x\,(a^2x^2+1)^3 \arctan(ax)^3}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{1}{x^3(a^2x^2+1)^2\arctan(ax)^3}dx}{c^3}-\frac{a^2\int \frac{1}{x(a^2x^2+1)^3\arctan(ax)^3}dx}{c^3}$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{1}{x^3(a^2x^2+1)\arctan(ax)^3}dx-a^2\int \frac{1}{x(a^2x^2+1)^2\arctan(ax)^3}dx}{c^3}-\frac{a^2\left(\int \frac{1}{x(a^2x^2+1)^2\arctan(ax)^3}dx-a^2\int \frac{x}{(a^2x^2+1)^3\arctan(ax)^3}dx\right)}{c^3}$$

$$\Big\downarrow \text{5461}$$

$$\frac{a^2\left(-\int \frac{1}{x(a^2x^2+1)^2\arctan(ax)^3}dx\right)-\frac{3\int \frac{1}{x^4\arctan(ax)^2}dx}{2a}-\frac{1}{2ax^3\arctan(ax)^2}}{c^3}-\frac{a^2\left(\int \frac{1}{x(a^2x^2+1)^2\arctan(ax)^3}dx-a^2\int \frac{x}{(a^2x^2+1)^3\arctan(ax)^3}dx\right)}{c^3}$$

$$\Big\downarrow \text{5377}$$

$$\frac{a^2\left(-\int \frac{1}{x(a^2x^2+1)^2\arctan(ax)^3}dx\right)-\frac{3\int \frac{1}{x^4\arctan(ax)^2}dx}{2a}-\frac{1}{2ax^3\arctan(ax)^2}}{c^3}-\frac{a^2\left(\int \frac{1}{x(a^2x^2+1)^2\arctan(ax)^3}dx-a^2\int \frac{x}{(a^2x^2+1)^3\arctan(ax)^3}dx\right)}{c^3}$$

$$\Big\downarrow \text{5501}$$

$$\frac{-\left(a^2\left(\int \frac{1}{x(a^2x^2+1)\arctan(ax)^3}dx-a^2\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^3}dx\right)\right)-\frac{3\int \frac{1}{x^4\arctan(ax)^2}dx}{2a}-\frac{1}{2ax^3\arctan(ax)^2}}{c^3}-\frac{a^2\left(a^2\left(-\int \frac{x}{(a^2x^2+1)^3\arctan(ax)^3}dx\right)-a^2\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^3}dx+\int \frac{1}{x(a^2x^2+1)\arctan(ax)^3}dx\right)}{c^3}$$

$$\Big\downarrow \text{5461}$$

$$\frac{-\left(a^2\left(a^2\left(-\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^3}dx\right)-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{1}{2ax\arctan(ax)^2}\right)\right)-\frac{3\int \frac{1}{x^4\arctan(ax)^2}dx}{2a}-\frac{1}{2ax^3\arctan(ax)^2}}{c^3}-\frac{a^2\left(a^2\left(-\int \frac{x}{(a^2x^2+1)^3\arctan(ax)^3}dx\right)-a^2\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^3}dx-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{1}{2ax\arctan(ax)^2}\right)}{c^3}$$

$$\Big\downarrow \text{5377}$$

$$-\frac{\left(a^2\left(a^2\left(-\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^3}dx\right)-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{1}{2ax\arctan(ax)^2}\right)\right)-\frac{3\int \frac{1}{x^4\arctan(ax)^2}dx}{2a}-\frac{1}{2ax^3\arctan(ax)^2}}{c^3}$$
$$\frac{a^2\left(a^2\left(-\int \frac{x}{(a^2x^2+1)^3\arctan(ax)^3}dx\right)-a^2\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^3}dx-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{1}{2ax\arctan(ax)^2}\right)}{c^3}$$

$\downarrow$ 5467

$$-\frac{\left(a^2\left(-\left(a^2\left(-2\int \frac{x}{(a^2x^2+1)^2\arctan(ax)}dx-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{}{2ax}\right.}{c^3}$$
$$\frac{a^2\left(a^2\left(-\int \frac{x}{(a^2x^2+1)^3\arctan(ax)^3}dx\right)-a^2\left(-2\int \frac{x}{(a^2x^2+1)^2\arctan(ax)}dx-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\right.\right.}{c^3}$$

$\downarrow$ 5503

$$-\frac{\left(a^2\left(-\left(a^2\left(-2\int \frac{x}{(a^2x^2+1)^2\arctan(ax)}dx-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{}{2ax}\right.}{c^3}$$
$$\frac{a^2\left(-\left(a^2\left(\frac{\int \frac{1}{(a^2x^2+1)^3\arctan(ax)^2}dx}{2a}-\frac{3}{2}a\int \frac{x^2}{(a^2x^2+1)^3\arctan(ax)^2}dx-\frac{x}{2a(a^2x^2+1)^2\arctan(ax)^2}\right)\right)\right)-a^2\left(-2\int \frac{x}{(a^2x^2+1)^2 a}\right.}{c^3}$$

$\downarrow$ 5437

$$-\frac{\left(a^2\left(-\left(a^2\left(-2\int \frac{x}{(a^2x^2+1)^2\arctan(ax)}dx-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{}{2ax}\right.}{c^3}$$
$$\frac{a^2\left(-\left(a^2\left(-\frac{3}{2}a\int \frac{x^2}{(a^2x^2+1)^3\arctan(ax)^2}dx+\frac{-4a\int \frac{x}{(a^2x^2+1)^3\arctan(ax)}dx-\frac{1}{a(a^2x^2+1)^2\arctan(ax)}}{2a}-\frac{x}{2a(a^2x^2+1)^2\arctan(ax)^2}\right)\right.\right.}{}$$

$\downarrow$ 5499

$$-\frac{\left(a^2\left(-\left(a^2\left(-2\int \frac{x}{(a^2x^2+1)^2\arctan(ax)}dx-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{}{2ax}\right.}{c^3}$$
$$\frac{a^2\left(-\left(a^2\left(-\frac{3}{2}a\left(\frac{\int \frac{1}{(a^2x^2+1)^2\arctan(ax)^2}dx}{a^2}-\frac{\int \frac{1}{(a^2x^2+1)^3\arctan(ax)^2}dx}{a^2}\right)+\frac{-4a\int \frac{x}{(a^2x^2+1)^3\arctan(ax)}dx-\frac{1}{a(a^2x^2+1)^2\arctan(ax)}}{2a}\right.\right.\right.}{}$$

$$\downarrow \text{5437}$$

$$\frac{-\left(a^2\left(-\left(a^2\left(-2\int \frac{x}{(a^2x^2+1)^2\arctan(ax)}dx-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{}{2ax}\right.\right.}{c^3}$$

$$a^2\left(-\left(a^2\left(-2\int \frac{x}{(a^2x^2+1)^2\arctan(ax)}dx-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\right)\right)-a^2\left(\frac{-4a\int \frac{x}{(a^2x^2+1)^3\arctan(a}}{}\right.\right.$$

$$\downarrow \text{5505}$$

$$\frac{-\left(a^2\left(-\left(a^2\left(-\frac{2\int \frac{ax}{(a^2x^2+1)\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}\right.\right.}{c^3}$$

$$a^2\left(-\left(a^2\left(-\frac{2\int \frac{ax}{(a^2x^2+1)\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\right)\right)-a^2\left(\frac{-\frac{4\int \frac{ax}{(a^2x^2+1)^2}}{}}{}\right.\right.$$

$$\downarrow \text{4906}$$

$$\frac{-\left(a^2\left(-\left(a^2\left(-\frac{2\int \frac{\sin(2\arctan(ax))}{2\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\right.\right.}{c^3}$$

$$a^2\left(-\left(a^2\left(-\frac{2\int \frac{\sin(2\arctan(ax))}{2\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\right)\right)-a^2\left(\frac{-\frac{4\int\left(\frac{\sin(2\arctan(ax))}{4\arctan(ax)}\right.}{}}{}\right.\right.$$

$$\downarrow \text{27}$$

$$\frac{-\left(a^2\left(-\left(a^2\left(-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{}{2a}\right.\right.}{c^3}$$

$$a^2\left(-\left(a^2\left(-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\right)\right)-a^2\left(\frac{-\frac{4\int\left(\frac{\sin(2\arctan(ax))}{4\arctan(ax)}+\right.}{}}{}\right.\right.$$

$$\downarrow \text{2009}$$

$$\frac{-\left(a^2\left(-\left(a^2\left(-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{}{2a}\right.\right.}{c^3}$$

$$a^2\left(-a^2\left(-\frac{3}{2}a\left(\frac{-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a\left(a^2x^2+1\right)\arctan(ax)}}{a^2}-\frac{-\frac{1}{a\left(a^2x^2+1\right)^2\arctan(ax)}-\frac{4\left(\frac{1}{4}\text{Si}(2\arctan(ax))+\frac{1}{8}\text{Si}(4\arctan(ax))\right)}{a}}{a^2}\right.\right.\right.$$

$\downarrow$ 3042

$$\frac{-\left(a^2\left(-\left(a^2\left(-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\frac{}{2a}\right.\right.}{c^3}$$

$$a^2\left(-a^2\left(-\frac{3}{2}a\left(\frac{-\frac{\int \frac{\sin(2\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{a}-\frac{1}{a\left(a^2x^2+1\right)\arctan(ax)}}{a^2}-\frac{-\frac{1}{a\left(a^2x^2+1\right)^2\arctan(ax)}-\frac{4\left(\frac{1}{4}\text{Si}(2\arctan(ax))+\frac{1}{8}\text{Si}(4\arctan(ax))\right)}{a}}{a^2}\right.\right.\right.$$

$\downarrow$ 3780

$$\frac{-\left(a^2\left(-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\left(a^2\left(-\frac{\text{Si}(2\arctan(ax))}{a^2}-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\right)\right)-\frac{1}{2ax\arctan(ax)^2}\right.\right.}{c^3}$$

$$a^2\left(-\frac{\int \frac{1}{x^2\arctan(ax)^2}dx}{2a}-\left(a^2\left(-\frac{\text{Si}(2\arctan(ax))}{a^2}-\frac{x}{2a(a^2x^2+1)\arctan(ax)^2}-\frac{1-a^2x^2}{2a^2(a^2x^2+1)\arctan(ax)}\right)\right)-a^2\left(-\frac{3}{2}a\left(\frac{-\frac{}{a\left(a^2\right.}}{}\right.\right.\right.$$

input
```
Int[1/(x^3*(c + a^2*c*x^2)^3*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 7.99 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3 (a^2c\,x^2+c)^3 \arctan(ax)^3}dx$$

input
```
int(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^3,x)
```

output
```
int(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.32

$$\int \frac{1}{x^3 (c+a^2cx^2)^3 \arctan(ax)^3}\,dx = \int \frac{1}{(a^2cx^2+c)^3 x^3 \arctan(ax)^3}\,dx$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(1/((a^6*c^3*x^9 + 3*a^4*c^3*x^7 + 3*a^2*c^3*x^5 + c^3*x^3)*arctan
(a*x)^3), x)
```

## Sympy [N/A]

Not integrable

Time = 1.71 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.73

$$\int \frac{1}{x^3 (c+a^2cx^2)^3 \arctan(ax)^3}\,dx = \frac{\int \frac{1}{a^6x^9\operatorname{atan}^3(ax)+3a^4x^7\operatorname{atan}^3(ax)+3a^2x^5\operatorname{atan}^3(ax)+x^3\operatorname{atan}^3(ax)}\,dx}{c^3}$$

input
```
integrate(1/x**3/(a**2*c*x**2+c)**3/atan(a*x)**3,x)
```

output
```
Integral(1/(a**6*x**9*atan(a*x)**3 + 3*a**4*x**7*atan(a*x)**3 + 3*a**2*x**
5*atan(a*x)**3 + x**3*atan(a*x)**3), x)/c**3
```

## Maxima [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 174, normalized size of antiderivative = 7.91

$$\int \frac{1}{x^3 (c+a^2cx^2)^3 \arctan(ax)^3} \, dx = \int \frac{1}{(a^2cx^2+c)^3 x^3 \arctan(ax)^3} \, dx$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
1/2*(2*(a^6*c^3*x^8 + 2*a^4*c^3*x^6 + a^2*c^3*x^4)*arctan(a*x)^2*integrate
((21*a^4*x^4 + 19*a^2*x^2 + 6)/((a^8*c^3*x^11 + 3*a^6*c^3*x^9 + 3*a^4*c^3*
x^7 + a^2*c^3*x^5)*arctan(a*x)), x) - a*x + (7*a^2*x^2 + 3)*arctan(a*x))/(
(a^6*c^3*x^8 + 2*a^4*c^3*x^6 + a^2*c^3*x^4)*arctan(a*x)^2)
```

## Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^3 (c+a^2cx^2)^3 \arctan(ax)^3} \, dx = \int \frac{1}{(a^2cx^2+c)^3 x^3 \arctan(ax)^3} \, dx$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^3*x^3*arctan(a*x)^3), x)
```

## Mupad [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^3 (c+a^2cx^2)^3 \arctan(ax)^3} \, dx = \int \frac{1}{x^3 \operatorname{atan}(a\,x)^3 \,(c\,a^2\,x^2+c)^3} \, dx$$

input
```
int(1/(x^3*atan(a*x)^3*(c + a^2*c*x^2)^3),x)
```

output
```
int(1/(x^3*atan(a*x)^3*(c + a^2*c*x^2)^3), x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.73

$$\int \frac{1}{x^3 (c+a^2cx^2)^3 \arctan(ax)^3} \, dx = \frac{\int \frac{1}{atan(ax)^3 a^6 x^9+3 atan(ax)^3 a^4 x^7+3 atan(ax)^3 a^2 x^5+atan(ax)^3 x^3} dx}{c^3}$$

input
```
int(1/x^3/(a^2*c*x^2+c)^3/atan(a*x)^3,x)
```

output
```
int(1/(atan(a*x)**3*a**6*x**9 + 3*atan(a*x)**3*a**4*x**7 + 3*atan(a*x)**3*
a**2*x**5 + atan(a*x)**3*x**3),x)/c**3
```

# 3.641 $\int \frac{1}{x^4(c+a^2cx^2)^3 \arctan(ax)^3} \, dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\begin{aligned}\int \frac{1}{x^4 (c+a^2cx^2)^3 \arctan(ax)^3} \, dx = &-\frac{1}{2ac^3x^4 \arctan(ax)^2} + \frac{a}{c^3x^2 \arctan(ax)^2} \\ &- \frac{a^3}{2c^3 (1+a^2x^2)^2 \arctan(ax)^2} \\ &- \frac{a^3}{c^3 (1+a^2x^2) \arctan(ax)^2} \\ &+ \frac{2a^4x}{c^3 (1+a^2x^2)^2 \arctan(ax)} \\ &+ \frac{2a^4x}{c^3 (1+a^2x^2) \arctan(ax)} \\ &- \frac{3a^3 \operatorname{CosIntegral}(2\arctan(ax))}{c^3} \\ &- \frac{a^3 \operatorname{CosIntegral}(4\arctan(ax))}{c^3} \\ &- \frac{2\text{Int}\left(\frac{1}{x^5 \arctan(ax)^2}, x\right)}{ac^3} + \frac{2a\text{Int}\left(\frac{1}{x^3 \arctan(ax)^2}, x\right)}{c^3}\end{aligned}$$

output
```
-1/2/a/c^3/x^4/arctan(a*x)^2+a/c^3/x^2/arctan(a*x)^2-1/2*a^3/c^3/(a^2*x^2+
1)^2/arctan(a*x)^2-a^3/c^3/(a^2*x^2+1)/arctan(a*x)^2+2*a^4*x/c^3/(a^2*x^2+
1)^2/arctan(a*x)+2*a^4*x/c^3/(a^2*x^2+1)/arctan(a*x)-3*a^3*Ci(2*arctan(a*x
))/c^3-a^3*Ci(4*arctan(a*x))/c^3-2*Defer(Int)(1/x^5/arctan(a*x)^2,x)/a/c^3
+2*a*Defer(Int)(1/x^3/arctan(a*x)^2,x)/c^3
```

## Mathematica [N/A]

Not integrable

Time = 7.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^4 (c+a^2cx^2)^3 \arctan(ax)^3} dx = \int \frac{1}{x^4 (c+a^2cx^2)^3 \arctan(ax)^3} dx$$

input
```
Integrate[1/(x^4*(c + a^2*c*x^2)^3*ArcTan[a*x]^3),x]
```

output
```
Integrate[1/(x^4*(c + a^2*c*x^2)^3*ArcTan[a*x]^3), x]
```

## Rubi [N/A]

Not integrable

Time = 4.96 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \arctan(ax)^3 (a^2cx^2+c)^3} dx$$
$$\Big\downarrow \text{ 5501}$$
$$\frac{\int \frac{1}{c^2x^4(a^2x^2+1)^2 \arctan(ax)^3}dx}{c} - a^2 \int \frac{1}{c^3x^2 (a^2x^2+1)^3 \arctan(ax)^3}dx$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{1}{x^4(a^2x^2+1)^2\arctan(ax)^3}dx}{c^3}-\frac{a^2\int \frac{1}{x^2(a^2x^2+1)^3\arctan(ax)^3}dx}{c^3}$$

$$\Big\downarrow \ 5501$$

$$\frac{\int \frac{1}{x^4(a^2x^2+1)\arctan(ax)^3}dx-a^2\int \frac{1}{x^2(a^2x^2+1)^2\arctan(ax)^3}dx}{c^3}-\frac{a^2\left(\int \frac{1}{x^2(a^2x^2+1)^2\arctan(ax)^3}dx-a^2\int \frac{1}{(a^2x^2+1)^3\arctan(ax)^3}dx\right)}{c^3}$$

$$\Big\downarrow \ 5437$$

$$\frac{\int \frac{1}{x^4(a^2x^2+1)\arctan(ax)^3}dx-a^2\int \frac{1}{x^2(a^2x^2+1)^2\arctan(ax)^3}dx}{c^3}-\frac{a^2\left(\int \frac{1}{x^2(a^2x^2+1)^2\arctan(ax)^3}dx-a^2\left(-2a\int \frac{x}{(a^2x^2+1)^3\arctan(ax)^2}dx-\frac{1}{2a(a^2x^2+1)^2\arctan(ax)^2}\right)\right)}{c^3}$$

$$\Big\downarrow \ 5461$$

$$\frac{a^2\left(-\int \frac{1}{x^2(a^2x^2+1)^2\arctan(ax)^3}dx\right)-\frac{2\int \frac{1}{x^5\arctan(ax)^2}dx}{a}-\frac{1}{2ax^4\arctan(ax)^2}}{c^3}-\frac{a^2\left(\int \frac{1}{x^2(a^2x^2+1)^2\arctan(ax)^3}dx-a^2\left(-2a\int \frac{x}{(a^2x^2+1)^3\arctan(ax)^2}dx-\frac{1}{2a(a^2x^2+1)^2\arctan(ax)^2}\right)\right)}{c^3}$$

$$\Big\downarrow \ 5377$$

$$\frac{a^2\left(-\int \frac{1}{x^2(a^2x^2+1)^2\arctan(ax)^3}dx\right)-\frac{2\int \frac{1}{x^5\arctan(ax)^2}dx}{a}-\frac{1}{2ax^4\arctan(ax)^2}}{c^3}-\frac{a^2\left(\int \frac{1}{x^2(a^2x^2+1)^2\arctan(ax)^3}dx-a^2\left(-2a\int \frac{x}{(a^2x^2+1)^3\arctan(ax)^2}dx-\frac{1}{2a(a^2x^2+1)^2\arctan(ax)^2}\right)\right)}{c^3}$$

$$\Big\downarrow \ 5501$$

$$\frac{-\left(a^2\left(\int \frac{1}{x^2(a^2x^2+1)\arctan(ax)^3}dx-a^2\int \frac{1}{(a^2x^2+1)^2\arctan(ax)^3}dx\right)\right)-\frac{2\int \frac{1}{x^5\arctan(ax)^2}dx}{a}-\frac{1}{2ax^4\arctan(ax)^2}}{c^3}-\frac{a^2\left(a^2\left(-\int \frac{1}{(a^2x^2+1)^2\arctan(ax)^3}dx\right)-a^2\left(-2a\int \frac{x}{(a^2x^2+1)^3\arctan(ax)^2}dx-\frac{1}{2a(a^2x^2+1)^2\arctan(ax)^2}\right)+\int \frac{1}{x^2(a^2x^2+1)\text{arcta}\ldots}\right.}{c^3}$$

$$\Big\downarrow \ 5437$$

$$-\frac{\left(a^2\left(\int \frac{1}{x^2(a^2x^2+1)\arctan(ax)^3}dx - a^2\left(-a\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^2}dx - \frac{1}{2a(a^2x^2+1)\arctan(ax)^2}\right)\right)\right) - \frac{2\int \frac{1}{x^5\arctan(ax)^2}dx}{a} - }{c^3}$$

$$\frac{a^2\left(-\left(a^2\left(-2a\int \frac{x}{(a^2x^2+1)^3\arctan(ax)^2}dx - \frac{1}{2a(a^2x^2+1)^2\arctan(ax)^2}\right)\right) - a^2\left(-a\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^2}dx - \frac{1}{2a(a^2x^2+1)\,\mathrm{ar}}\right.\right.}{c^3}$$

$\big\downarrow$ 5461

$$-\frac{\left(a^2\left(-\left(a^2\left(-a\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^2}dx - \frac{1}{2a(a^2x^2+1)\arctan(ax)^2}\right)\right) - \frac{\int \frac{1}{x^3\arctan(ax)^2}dx}{a} - \frac{1}{2ax^2\arctan(ax)^2}\right)\right) - \frac{2\int \frac{}{x^5}}{}}{c^3}$$

$$\frac{a^2\left(-\left(a^2\left(-2a\int \frac{x}{(a^2x^2+1)^3\arctan(ax)^2}dx - \frac{1}{2a(a^2x^2+1)^2\arctan(ax)^2}\right)\right) - a^2\left(-a\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^2}dx - \frac{1}{2a(a^2x^2+1)\,\mathrm{a}}\right.\right.}{c^3}$$

$\big\downarrow$ 5377

$$-\frac{\left(a^2\left(-\left(a^2\left(-a\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^2}dx - \frac{1}{2a(a^2x^2+1)\arctan(ax)^2}\right)\right) - \frac{\int \frac{1}{x^3\arctan(ax)^2}dx}{a} - \frac{1}{2ax^2\arctan(ax)^2}\right)\right) - \frac{2\int \frac{}{x^5}}{}}{c^3}$$

$$\frac{a^2\left(-\left(a^2\left(-2a\int \frac{x}{(a^2x^2+1)^3\arctan(ax)^2}dx - \frac{1}{2a(a^2x^2+1)^2\arctan(ax)^2}\right)\right) - a^2\left(-a\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^2}dx - \frac{1}{2a(a^2x^2+1)\,\mathrm{a}}\right.\right.}{c^3}$$

$\big\downarrow$ 5503

$$-\frac{\left(a^2\left(-\left(a^2\left(-a\left(\frac{\int \frac{1}{(a^2x^2+1)^2\arctan(ax)}dx}{a} - a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx - \frac{x}{a(a^2x^2+1)\arctan(ax)}\right) - \frac{1}{2a(a^2x^2+1)\arctan(ax}\right.\right.\right.\right.}{c^3}$$

$$\frac{a^2\left(-\left(a^2\left(-2a\left(\frac{\int \frac{1}{(a^2x^2+1)^3\arctan(ax)}dx}{a} - 3a\int \frac{x^2}{(a^2x^2+1)^3\arctan(ax)}dx - \frac{x}{a(a^2x^2+1)^2\arctan(ax)}\right) - \frac{1}{2a(a^2x^2+1)^2\arctan(a}\right.\right.\right.}{}$$

$\big\downarrow$ 5439

$$-\frac{\left(a^2\left(-\left(a^2\left(-a\left(-a\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx + \frac{\int \frac{1}{(a^2x^2+1)\arctan(ax)}d\arctan(ax)}{a^2} - \frac{x}{a(a^2x^2+1)\arctan(ax)}\right) - \frac{1}{2a(a^2x^2+1)}\right.\right.\right.\right.}{c^3}$$

$$\frac{a^2\left(-\left(a^2\left(-2a\left(-3a\int \frac{x^2}{(a^2x^2+1)^3\arctan(ax)}dx + \frac{\int \frac{1}{(a^2x^2+1)^2\arctan(ax)}d\arctan(ax)}{a^2} - \frac{x}{a(a^2x^2+1)^2\arctan(ax)}\right) - \frac{}{2a(a^2x^2+}\right.\right.\right.}{}$$

$\Big\downarrow$ 3042

$$-\frac{\left(a^2\left(-\left(a^2\left(-a\left(-a\int\frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\int\frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^2}{\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)-\frac{}{2a(a^2x^2+1}\right.\right.\right.\right.}{c^3}$$
$$a^2\left(-\left(a^2\left(-a\left(-a\int\frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\int\frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^2}{\arctan(ax)}d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)-\frac{1}{2a(a^2x^2+1)\,\mathrm{arct}}\right.\right.\right.$$

$\Big\downarrow$ 3793

$$-\frac{\left(a^2\left(-\left(a^2\left(-a\left(-a\int\frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\int\left(\frac{\cos(2\arctan(ax))}{2\arctan(ax)}+\frac{1}{2\arctan(ax)}\right)d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)-\right.\right.\right.\right.}{c^3}$$
$$a^2\left(-\left(a^2\left(-a\left(-a\int\frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\int\left(\frac{\cos(2\arctan(ax))}{2\arctan(ax)}+\frac{1}{2\arctan(ax)}\right)d\arctan(ax)}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)-\frac{}{2a(a^2}\right.\right.\right.$$

$\Big\downarrow$ 2009

$$-\frac{\left(a^2\left(-\left(a^2\left(-a\left(-a\int\frac{x^2}{(a^2x^2+1)^2\arctan(ax)}dx+\frac{\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)-\frac{}{2a}\right.\right.\right.\right.}{c^3}$$
$$a^2\left(-\left(a^2\left(-2a\left(-3a\int\frac{x^2}{(a^2x^2+1)^3\arctan(ax)}dx+\frac{\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))+\frac{1}{8}\operatorname{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}-\frac{}{a(}\right.\right.\right.\right.$$

$\Big\downarrow$ 5505

$$-\frac{\left(a^2\left(-\left(a^2\left(-a\left(-\frac{\int\frac{a^2x^2}{\left(a^2x^2+1\right)\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right.\right.\right.\right.\right.}{c^3}$$
$$a^2\left(-\left(a^2\left(-2a\left(-\frac{3\int\frac{a^2x^2}{\left(a^2x^2+1\right)^2\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))+\frac{1}{8}\operatorname{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax\right.\right.\right.\right.}{a^2}$$

$\Big\downarrow$ 3042

$$-\left(a^2\left(-\left(a^2\left(-a\left(-\frac{\int \frac{\sin(\arctan(ax))^2}{\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\arctan(ax)}\right)-\right.\right.\right.\right.$$

$$c^3$$

$$a^2\left(-\left(a^2\left(-2a\left(-\frac{3\int \frac{a^2x^2}{(a^2x^2+1)^2\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))+\frac{1}{8}\operatorname{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax\right.\right.\right.$$

$$\Big\downarrow \; 3793$$

$$-\left(a^2\left(-\left(a^2\left(-a\left(-\frac{\int\left(\frac{1}{2\arctan(ax)}-\frac{\cos(2\arctan(ax))}{2\arctan(ax)}\right)d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan(ax))}{a^2}-\frac{x}{a(a^2x^2+1)\,a}\right.\right.\right.\right.$$

$$c^3$$

$$a^2\left(-\left(a^2\left(-2a\left(-\frac{3\int \frac{a^2x^2}{(a^2x^2+1)^2\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))+\frac{1}{8}\operatorname{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax\right.\right.\right.$$

$$\Big\downarrow \; 2009$$

$$-\left(a^2\left(-\frac{\int \frac{1}{x^3\arctan(ax)^2}dx}{a}-\left(a^2\left(-a\left(-\frac{\frac{1}{2}\log(\arctan(ax))-\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))}{a^2}+\frac{\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan}{a^2}\right.\right.\right.\right.$$

$$c^3$$

$$a^2\left(-a^2\left(-2a\left(-\frac{3\int \frac{a^2x^2}{(a^2x^2+1)^2\arctan(ax)}d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))+\frac{1}{8}\operatorname{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arctan(ax))}{a^2}\right.\right.\right.$$

$$\Big\downarrow \; 4906$$

$$-\left(a^2\left(-\frac{\int \frac{1}{x^3\arctan(ax)^2}dx}{a}-\left(a^2\left(-a\left(-\frac{\frac{1}{2}\log(\arctan(ax))-\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))}{a^2}+\frac{\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan}{a^2}\right.\right.\right.\right.$$

$$c^3$$

$$a^2\left(-a^2\left(-2a\left(-\frac{3\int\left(\frac{1}{8\arctan(ax)}-\frac{\cos(4\arctan(ax))}{8\arctan(ax)}\right)d\arctan(ax)}{a^2}+\frac{\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))+\frac{1}{8}\operatorname{CosIntegral}(4\arctan(ax))+\frac{3}{8}\log(\arc}{a^2}\right.\right.\right.$$

$$\Big\downarrow \; 2009$$

$$-\frac{\left(a^2\left(-\frac{\int \frac{1}{x^3\arctan(ax)^2}dx}{a}-\left(a^2\left(-a\left(-\frac{\frac{1}{2}\log(\arctan(ax))-\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))}{a^2}+\frac{\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))+\frac{1}{2}\log(\arctan}{a^2}\right.\right.\right.\right.}{c^3}$$

$$\frac{a^2\left(-\frac{\int \frac{1}{x^3\arctan(ax)^2}dx}{a}-\left(a^2\left(-2a\left(-\frac{3\left(\frac{1}{8}\log(\arctan(ax))-\frac{1}{8}\operatorname{CosIntegral}(4\arctan(ax))\right)}{a^2}+\frac{\frac{1}{2}\operatorname{CosIntegral}(2\arctan(ax))+\frac{1}{8}\operatorname{CosIntegr}}{a^2}\right.\right.\right.\right.}{}$$

input
```
Int[1/(x^4*(c + a^2*c*x^2)^3*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 8.46 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4\left(a^2c\,x^2+c\right)^3\arctan\left(ax\right)^3}dx$$

input
```
int(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^3,x)
```

output
```
int(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.32

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^3\arctan(ax)^3}\,dx=\int \frac{1}{\left(a^2cx^2+c\right)^3x^4\arctan\left(ax\right)^3}\,dx$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(1/((a^6*c^3*x^10 + 3*a^4*c^3*x^8 + 3*a^2*c^3*x^6 + c^3*x^4)*arcta
n(a*x)^3), x)
```

## Sympy [N/A]

Not integrable

Time = 1.95 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.73

$$\int \frac{1}{x^4 (c+a^2cx^2)^3 \arctan(ax)^3} \, dx = \frac{\int \frac{1}{a^6x^{10} \operatorname{atan}^3(ax)+3a^4x^8 \operatorname{atan}^3(ax)+3a^2x^6 \operatorname{atan}^3(ax)+x^4 \operatorname{atan}^3(ax)} \, dx}{c^3}$$

input
```
integrate(1/x**4/(a**2*c*x**2+c)**3/atan(a*x)**3,x)
```

output
```
Integral(1/(a**6*x**10*atan(a*x)**3 + 3*a**4*x**8*atan(a*x)**3 + 3*a**2*x*
*6*atan(a*x)**3 + x**4*atan(a*x)**3), x)/c**3
```

## Maxima [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 176, normalized size of antiderivative = 8.00

$$\int \frac{1}{x^4 (c+a^2cx^2)^3 \arctan(ax)^3} \, dx = \int \frac{1}{(a^2cx^2+c)^3 x^4 \arctan(ax)^3} \, dx$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
1/2*(2*(a^6*c^3*x^9 + 2*a^4*c^3*x^7 + a^2*c^3*x^5)*arctan(a*x)^2*integrate
(2*(14*a^4*x^4 + 15*a^2*x^2 + 5)/((a^8*c^3*x^12 + 3*a^6*c^3*x^10 + 3*a^4*c
^3*x^8 + a^2*c^3*x^6)*arctan(a*x)), x) - a*x + 4*(2*a^2*x^2 + 1)*arctan(a*
x))/((a^6*c^3*x^9 + 2*a^4*c^3*x^7 + a^2*c^3*x^5)*arctan(a*x)^2)
```

## Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^4 (c+a^2cx^2)^3 \arctan(ax)^3} dx = \int \frac{1}{(a^2cx^2+c)^3 x^4 \arctan(ax)^3} dx$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^3*x^4*arctan(a*x)^3), x)
```

## Mupad [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{x^4 (c+a^2cx^2)^3 \arctan(ax)^3} dx = \int \frac{1}{x^4 \operatorname{atan}(a\,x)^3 (c\,a^2\,x^2+c)^3} dx$$

input
```
int(1/(x^4*atan(a*x)^3*(c + a^2*c*x^2)^3),x)
```

output
```
int(1/(x^4*atan(a*x)^3*(c + a^2*c*x^2)^3), x)
```

## Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.73

$$\int \frac{1}{x^4 (c+a^2cx^2)^3 \arctan(ax)^3} dx = \frac{\int \frac{1}{atan(ax)^3 a^6 x^{10}+3 atan(ax)^3 a^4 x^8+3 atan(ax)^3 a^2 x^6+atan(ax)^3 x^4} dx}{c^3}$$

input
```
int(1/x^4/(a^2*c*x^2+c)^3/atan(a*x)^3,x)
```

output
```
int(1/(atan(a*x)**3*a**6*x**10 + 3*atan(a*x)**3*a**4*x**8 + 3*atan(a*x)**3
*a**2*x**6 + atan(a*x)**3*x**4),x)/c**3
```

# 3.642 $\int \left(\frac{x^3}{(1+a^2x^2)\arctan(ax)^3}-\frac{3x^2}{2a\arctan(ax)^2}\right)\,dx$



## Optimal result

Integrand size = 38, antiderivative size = 16

$$\int \left(\frac{x^3}{(1+a^2x^2)\arctan(ax)^3}-\frac{3x^2}{2a\arctan(ax)^2}\right)\,dx=-\frac{x^3}{2a\arctan(ax)^2}$$

output
```
-1/2*x^3/a/arctan(a*x)^2
```

## Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \left(\frac{x^3}{(1+a^2x^2)\arctan(ax)^3}-\frac{3x^2}{2a\arctan(ax)^2}\right)\,dx=-\frac{x^3}{2a\arctan(ax)^2}$$

input
```
Integrate[x^3/((1 + a^2*x^2)*ArcTan[a*x]^3) - (3*x^2)/(2*a*ArcTan[a*x]^2),
x]
```

output
```
-1/2*x^3/(a*ArcTan[a*x]^2)
```

## Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.026, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{x^3}{(a^2x^2+1)\arctan(ax)^3}-\frac{3x^2}{2a\arctan(ax)^2}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{x^3}{2a\arctan(ax)^2}$$

input
```
Int[x^3/((1 + a^2*x^2)*ArcTan[a*x]^3) - (3*x^2)/(2*a*ArcTan[a*x]^2),x]
```

output
```
-1/2*x^3/(a*ArcTan[a*x]^2)
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

## Maple [A] (verified)

Time = 43.36 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

| method | result |
|---|---|
| parallelrisch | $-\frac{x^{3}}{2a\arctan(ax)^{2}}$ |
| risch | $\frac{2x^{3}}{a(\ln(-iax+1)-\ln(iax+1))^{2}}$ |
| derivativedivides | $\frac{-\frac{i}{a^{3}\arctan(ax)^{2}}-\frac{(a^{2}x^{2}-iax-1)(ax+i)}{a^{3}\arctan(ax)^{2}}}{2a}$ |
| default | $-\frac{x^{3}}{2a\arctan(ax)^{2}}-\frac{x^{2}}{2a^{2}\arctan(ax)^{3}}-\frac{1}{2a^{4}\arctan(ax)^{3}}-\frac{3x}{2a^{3}\arctan(ax)^{4}}-\frac{3i}{2a^{4}\arctan(ax)^{4}}+\frac{(-i\arctan(}{}$ |
| parts | $-\frac{x^{3}}{2a\arctan(ax)^{2}}-\frac{x^{2}}{2a^{2}\arctan(ax)^{3}}-\frac{1}{2a^{4}\arctan(ax)^{3}}-\frac{3x}{2a^{3}\arctan(ax)^{4}}-\frac{3i}{2a^{4}\arctan(ax)^{4}}+\frac{(-i\arctan(}{}$ |

input
```
int(x^3/(a^2*x^2+1)/arctan(a*x)^3-3/2*x^2/a/arctan(a*x)^2,x,method=_RETURN
VERBOSE)
```

output
```
-1/2*x^3/a/arctan(a*x)^2
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \left(\frac{x^3}{(1+a^2x^2)\arctan(ax)^3}-\frac{3x^2}{2a\arctan(ax)^2}\right) dx = -\frac{x^3}{2\,a\arctan\left(ax\right)^2}$$

input
```
integrate(x^3/(a^2*x^2+1)/arctan(a*x)^3-3/2*x^2/a/arctan(a*x)^2,x, algorit
hm="fricas")
```

output
```
-1/2*x^3/(a*arctan(a*x)^2)
```

## Sympy [F]

$$\int \left(\frac{x^3}{(1+a^2x^2)\arctan(ax)^3}-\frac{3x^2}{2a\arctan(ax)^2}\right)dx = -\frac{\int\left(-\frac{2ax^3}{a^2x^2\operatorname{atan}^3(ax)+\operatorname{atan}^3(ax)}\right)dx+\int\frac{3x^2\operatorname{atan}(ax)}{a^2x^2\operatorname{atan}^3(ax)+\operatorname{atan}^3(ax)}\,dx+\int\frac{3a^2x^4\operatorname{atan}(ax)}{a^2x^2\operatorname{atan}^3(ax)+\operatorname{atan}^3(ax)}\,dx}{2a}$$

input
```
integrate(x**3/(a**2*x**2+1)/atan(a*x)**3-3/2*x**2/a/atan(a*x)**2,x)
```

output
```
-(Integral(-2*a*x**3/(a**2*x**2*atan(a*x)**3 + atan(a*x)**3), x) + Integra
l(3*x**2*atan(a*x)/(a**2*x**2*atan(a*x)**3 + atan(a*x)**3), x) + Integral(
3*a**2*x**4*atan(a*x)/(a**2*x**2*atan(a*x)**3 + atan(a*x)**3), x))/(2*a)
```

## Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \left(\frac{x^3}{(1+a^2x^2)\arctan(ax)^3}-\frac{3x^2}{2a\arctan(ax)^2}\right)dx = -\frac{x^3}{2\,a\arctan(ax)^2}$$

input
```
integrate(x^3/(a^2*x^2+1)/arctan(a*x)^3-3/2*x^2/a/arctan(a*x)^2,x, algorit
hm="maxima")
```

output
```
-1/2*x^3/(a*arctan(a*x)^2)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \left(\frac{x^3}{(1+a^2x^2)\arctan(ax)^3}-\frac{3x^2}{2a\arctan(ax)^2}\right)dx = -\frac{x^3}{2\,a\arctan(ax)^2}$$

input
```
integrate(x^3/(a^2*x^2+1)/arctan(a*x)^3-3/2*x^2/a/arctan(a*x)^2,x, algorit
hm="giac")
```

output
```
-1/2*x^3/(a*arctan(a*x)^2)
```

## Mupad [B] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \left(\frac{x^3}{(1+a^2x^2)\arctan(ax)^3}-\frac{3x^2}{2a\arctan(ax)^2}\right)dx=-\frac{x^3}{2\,a\,\mathrm{atan}\left(a\,x\right)^2}$$

input
```
int(x^3/(atan(a*x)^3*(a^2*x^2 + 1)) - (3*x^2)/(2*a*atan(a*x)^2),x)
```

output
```
-x^3/(2*a*atan(a*x)^2)
```

## Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \left(\frac{x^3}{(1+a^2x^2)\arctan(ax)^3}-\frac{3x^2}{2a\arctan(ax)^2}\right)dx=-\frac{x^3}{2atan\left(ax\right)^2a}$$

input
```
int(x^3/(a^2*x^2+1)/atan(a*x)^3-3/2*x^2/a/atan(a*x)^2,x)
```

output
```
( - x**3)/(2*atan(a*x)**2*a)
```

# 3.643 $\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^3}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^3}\,dx = \text{Int}\left(\frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^3},x\right)$$

output
```
Defer(Int)(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x)
```

## Mathematica [N/A]

Not integrable

Time = 1.59 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^3}\,dx = \int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^3}\,dx$$

input
```
Integrate[(x*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^3,x]
```

output
```
Integrate[(x*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^3, x]
```

## Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{a^2cx^2+c}}{\arctan(ax)^3}\,dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int \frac{x\sqrt{a^2cx^2+c}}{\arctan(ax)^3}dx$$

input
```
Int[(x*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^3,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 10.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x\sqrt{a^2c\,x^2+c}}{\arctan\left(ax\right)^3}dx$$

input
```
int(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x)
```

output
```
int(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^3}\,dx = \int \frac{\sqrt{a^2cx^2+cx}}{\arctan\left(ax\right)^3}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x/arctan(a*x)^3, x)
```

## Sympy [N/A]

Not integrable

Time = 1.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^3}\,dx = \int \frac{x\sqrt{c\left(a^2x^2+1\right)}}{\operatorname{atan}^3\left(ax\right)}\,dx$$

input
```
integrate(x*(a**2*c*x**2+c)**(1/2)/atan(a*x)**3,x)
```

output
```
Integral(x*sqrt(c*(a**2*x**2 + 1))/atan(a*x)**3, x)
```

## Maxima [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^3}\,dx = \int \frac{\sqrt{a^2cx^2+cx}}{\arctan\left(ax\right)^3}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*x/arctan(a*x)^3, x)
```

## Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^3}\,dx = \int \frac{\sqrt{a^2cx^2+c}x}{\arctan\left(ax\right)^3}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*x/arctan(a*x)^3, x)
```

## Mupad [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^3}\,dx = \int \frac{x\,\sqrt{c\,a^2\,x^2+c}}{\mathrm{atan}\left(a\,x\right)^3}\,dx$$

input
```
int((x*(c + a^2*c*x^2)^(1/2))/atan(a*x)^3,x)
```

output
```
int((x*(c + a^2*c*x^2)^(1/2))/atan(a*x)^3, x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^3}\,dx = \sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,x}{atan\,(ax)^3}dx\right)$$

input

```
int(x*(a^2*c*x^2+c)^(1/2)/atan(a*x)^3,x)
```

output

```
sqrt(c)*int((sqrt(a**2*x**2 + 1)*x)/atan(a*x)**3,x)
```

# 3.644 $\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^3} dx$



## Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^3} dx = \text{Int}\left(\frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^3}, x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x)
```

## Mathematica [N/A]

Not integrable

Time = 0.85 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^3} dx = \int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^3} dx$$

input
```
Integrate[Sqrt[c + a^2*c*x^2]/ArcTan[a*x]^3,x]
```

output
```
Integrate[Sqrt[c + a^2*c*x^2]/ArcTan[a*x]^3, x]
```

## Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2cx^2+c}}{\arctan(ax)^3}dx$$

$$\Big\downarrow \text{5560}$$

$$\int \frac{\sqrt{a^2cx^2+c}}{\arctan(ax)^3}dx$$

input
```
Int[Sqrt[c + a^2*c*x^2]/ArcTan[a*x]^3,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 9.41 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{a^2c\,x^2+c}}{\arctan\left(ax\right)^3}dx$$

input
```
int((a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x)
```

output
```
int((a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^3} \, dx = \int \frac{\sqrt{a^2cx^2+c}}{\arctan\left(ax\right)^3} \, dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)/arctan(a*x)^3, x)
```

## Sympy [N/A]

Not integrable

Time = 0.94 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^3} \, dx = \int \frac{\sqrt{c\left(a^2x^2+1\right)}}{\operatorname{atan}^3\left(ax\right)} \, dx$$

input
```
integrate((a**2*c*x**2+c)**(1/2)/atan(a*x)**3,x)
```

output
```
Integral(sqrt(c*(a**2*x**2 + 1))/atan(a*x)**3, x)
```

## Maxima [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^3} \, dx = \int \frac{\sqrt{a^2cx^2+c}}{\arctan\left(ax\right)^3} \, dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)/arctan(a*x)^3, x)
```

## Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^3}\,dx = \int \frac{\sqrt{a^2cx^2+c}}{\arctan{(ax)}^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)/arctan(a*x)^3, x)
```

## Mupad [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^3}\,dx = \int \frac{\sqrt{c\,a^2\,x^2+c}}{\mathrm{atan}\left(a\,x\right)^3}\,dx$$

input
```
int((c + a^2*c*x^2)^(1/2)/atan(a*x)^3,x)
```

output
```
int((c + a^2*c*x^2)^(1/2)/atan(a*x)^3, x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^3}\,dx = \sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}}{atan\,(ax)^3}dx\right)$$

input
```
int((a^2*c*x^2+c)^(1/2)/atan(a*x)^3,x)
```

output
```
sqrt(c)*int(sqrt(a**2*x**2 + 1)/atan(a*x)**3,x)
```

# 3.645 $\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^3}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^3}\,dx=\text{Int}\left(\frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^3},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^3,x)
```

## Mathematica [N/A]

Not integrable

Time = 2.99 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^3}\,dx=\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^3}\,dx$$

input
```
Integrate[Sqrt[c + a^2*c*x^2]/(x*ArcTan[a*x]^3),x]
```

output
```
Integrate[Sqrt[c + a^2*c*x^2]/(x*ArcTan[a*x]^3), x]
```

## Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2cx^2+c}}{x\arctan(ax)^3}\,dx$$

$$\Big\downarrow \text{5560}$$

$$\int \frac{\sqrt{a^2cx^2+c}}{x\arctan(ax)^3}dx$$

input
```
Int[Sqrt[c + a^2*c*x^2]/(x*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 20.57 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{a^2c\,x^2+c}}{x\arctan\left(ax\right)^3}dx$$

input
```
int((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^3,x)
```

output
```
int((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^3}\,dx = \int \frac{\sqrt{a^2cx^2+c}}{x\arctan{(ax)}^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)/(x*arctan(a*x)^3), x)
```

## Sympy [N/A]

Not integrable

Time = 1.42 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^3}\,dx = \int \frac{\sqrt{c\,(a^2x^2+1)}}{x\,\mathrm{atan}^3{(ax)}}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(1/2)/x/atan(a*x)**3,x)
```

output
```
Integral(sqrt(c*(a**2*x**2 + 1))/(x*atan(a*x)**3), x)
```

## Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^3}\,dx = \int \frac{\sqrt{a^2cx^2+c}}{x\arctan{(ax)}^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)/(x*arctan(a*x)^3), x)
```

## Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^3}\,dx=\int \frac{\sqrt{a^2cx^2+c}}{x\arctan\left(ax\right)^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)/(x*arctan(a*x)^3), x)
```

## Mupad [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^3}\,dx=\int \frac{\sqrt{c\,a^2\,x^2+c}}{x\,\mathrm{atan}\left(a\,x\right)^3}\,dx$$

input
```
int((c + a^2*c*x^2)^(1/2)/(x*atan(a*x)^3),x)
```

output
```
int((c + a^2*c*x^2)^(1/2)/(x*atan(a*x)^3), x)
```

## Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^3}\,dx = \sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}}{atan\,(ax)^3\,x}dx\right)$$

input
```
int((a^2*c*x^2+c)^(1/2)/x/atan(a*x)^3,x)
```

output
```
sqrt(c)*int(sqrt(a**2*x**2 + 1)/(atan(a*x)**3*x),x)
```

# 3.646 $\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^3} dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^3} dx = \text{Int}\left(\frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^3}, x\right)$$

output
```
Defer(Int)(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

## Mathematica [N/A]

Not integrable

Time = 5.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^3} dx = \int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^3} dx$$

input
```
Integrate[(x*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^3,x]
```

output
```
Integrate[(x*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^3, x]
```

## Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a^2cx^2+c\right)^{3/2}}{\arctan(ax)^3}dx$$
$$\Big\downarrow \ 5560$$
$$\int \frac{x\left(a^2cx^2+c\right)^{3/2}}{\arctan(ax)^3}dx$$

input
```
Int[(x*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^3,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 13.98 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x\left(a^2c\,x^2+c\right)^{\frac{3}{2}}}{\arctan\left(ax\right)^3}dx$$

input
```
int(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

output
```
int(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.50

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^3}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}x}{\arctan(ax)^3}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^3 + c*x)*sqrt(a^2*c*x^2 + c)/arctan(a*x)^3, x)
```

## Sympy [N/A]

Not integrable

Time = 5.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^3}\,dx = \int \frac{x(c(a^2x^2+1))^{\frac{3}{2}}}{\operatorname{atan}^3(ax)}\,dx$$

input
```
integrate(x*(a**2*c*x**2+c)**(3/2)/atan(a*x)**3,x)
```

output
```
Integral(x*(c*(a**2*x**2 + 1))**(3/2)/atan(a*x)**3, x)
```

## Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^3}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}x}{\arctan(ax)^3}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*x/arctan(a*x)^3, x)
```

## Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^3}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}x}{\arctan(ax)^3}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*x/arctan(a*x)^3, x)
```

## Mupad [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^3}\,dx = \int \frac{x\,(c\,a^2\,x^2+c)^{3/2}}{\mathrm{atan}\,(a\,x)^3}\,dx$$

input
```
int((x*(c + a^2*c*x^2)^(3/2))/atan(a*x)^3,x)
```

output
```
int((x*(c + a^2*c*x^2)^(3/2))/atan(a*x)^3, x)
```

## Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.32

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^3}\,dx = \sqrt{c}\,c\left(\left(\int \frac{\sqrt{a^2x^2+1}\,x^3}{atan\,(ax)^3}dx\right)a^2+\int \frac{\sqrt{a^2x^2+1}\,x}{atan\,(ax)^3}dx\right)$$

input

```
int(x*(a^2*c*x^2+c)^(3/2)/atan(a*x)^3,x)
```

output

```
sqrt(c)*c*(int((sqrt(a**2*x**2 + 1)*x**3)/atan(a*x)**3,x)*a**2 + int((sqrt
(a**2*x**2 + 1)*x)/atan(a*x)**3,x))
```

# 3.647 $\int \frac{\left(c+a^2cx^2\right)^{3/2}}{\arctan(ax)^3}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{\left(c+a^2cx^2\right)^{3/2}}{\arctan(ax)^3}\,dx = \operatorname{Int}\left(\frac{\left(c+a^2cx^2\right)^{3/2}}{\arctan(ax)^3},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

## Mathematica [N/A]

Not integrable

Time = 0.89 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{\left(c+a^2cx^2\right)^{3/2}}{\arctan(ax)^3}\,dx = \int \frac{\left(c+a^2cx^2\right)^{3/2}}{\arctan(ax)^3}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^(3/2)/ArcTan[a*x]^3,x]
```

output
```
Integrate[(c + a^2*c*x^2)^(3/2)/ArcTan[a*x]^3, x]
```

## Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^{3/2}}{\arctan(ax)^3}dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{(a^2cx^2+c)^{3/2}}{\arctan(ax)^3}dx$$

input
```
Int[(c + a^2*c*x^2)^(3/2)/ArcTan[a*x]^3,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 12.33 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{(a^2c\,x^2+c)^{\frac{3}{2}}}{\arctan\left(ax\right)^3}dx$$

input
```
int((a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

output
```
int((a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^3}\,dx=\int \frac{(a^2cx^2+c)^{\frac{3}{2}}}{\arctan(ax)^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^(3/2)/arctan(a*x)^3, x)
```

## Sympy [N/A]

Not integrable

Time = 3.51 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^3}\,dx=\int \frac{(c(a^2x^2+1))^{\frac{3}{2}}}{\operatorname{atan}^3(ax)}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(3/2)/atan(a*x)**3,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(3/2)/atan(a*x)**3, x)
```

## Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^3}\,dx=\int \frac{(a^2cx^2+c)^{\frac{3}{2}}}{\arctan(ax)^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)/arctan(a*x)^3, x)
```

## Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^3}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}}{\arctan(ax)^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)/arctan(a*x)^3, x)
```

## Mupad [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^3}\,dx = \int \frac{(c\,a^2\,x^2+c)^{3/2}}{\mathrm{atan}(a\,x)^3}\,dx$$

input
```
int((c + a^2*c*x^2)^(3/2)/atan(a*x)^3,x)
```

output
```
int((c + a^2*c*x^2)^(3/2)/atan(a*x)^3, x)
```

## Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.38

$$\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^3}\,dx = \sqrt{c}\,c\left(\int \frac{\sqrt{a^2x^2+1}}{atan\,(ax)^3}dx + \left(\int \frac{\sqrt{a^2x^2+1}\,x^2}{atan\,(ax)^3}dx\right)a^2\right)$$

input
```
int((a^2*c*x^2+c)^(3/2)/atan(a*x)^3,x)
```

output
```
sqrt(c)*c*(int(sqrt(a**2*x**2 + 1)/atan(a*x)**3,x) + int((sqrt(a**2*x**2 +
 1)*x**2)/atan(a*x)**3,x)*a**2)
```

# 3.648 $\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)^3}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)^3}\,dx = \text{Int}\left(\frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)^3},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^3,x)
```

## Mathematica [N/A]

Not integrable

Time = 3.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)^3}\,dx = \int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)^3}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^(3/2)/(x*ArcTan[a*x]^3),x]
```

output
```
Integrate[(c + a^2*c*x^2)^(3/2)/(x*ArcTan[a*x]^3), x]
```

## Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2cx^2+c\right)^{3/2}}{x\arctan(ax)^3}\,dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{\left(a^2cx^2+c\right)^{3/2}}{x\arctan(ax)^3}dx$$

input
```
Int[(c + a^2*c*x^2)^(3/2)/(x*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 23.78 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}{x \arctan \left(a x \right)^{3}}d x$$

input
```
int((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^3,x)
```

output
```
int((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)^3}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}}{x\arctan{(ax)}^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^(3/2)/(x*arctan(a*x)^3), x)
```

## Sympy [N/A]

Not integrable

Time = 7.46 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)^3}\,dx = \int \frac{(c(a^2x^2+1))^{\frac{3}{2}}}{x\operatorname{atan}^3{(ax)}}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(3/2)/x/atan(a*x)**3,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(3/2)/(x*atan(a*x)**3), x)
```

## Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)^3}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}}{x\arctan{(ax)}^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)/(x*arctan(a*x)^3), x)
```

## Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)^3}\,dx=\int \frac{(a^2cx^2+c)^{\frac{3}{2}}}{x\arctan{(ax)}^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)/(x*arctan(a*x)^3), x)
```

## Mupad [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)^3}\,dx=\int \frac{(c\,a^2\,x^2+c)^{3/2}}{x\,\mathrm{atan}\left(a\,x\right)^3}\,dx$$

input
```
int((c + a^2*c*x^2)^(3/2)/(x*atan(a*x)^3),x)
```

output
```
int((c + a^2*c*x^2)^(3/2)/(x*atan(a*x)^3), x)
```

## Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.12

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)^3}\,dx=\sqrt{c}\,c\left(\int \frac{\sqrt{a^2x^2+1}}{atan\left(ax\right)^3x}dx+\left(\int \frac{\sqrt{a^2x^2+1}\,x}{atan\left(ax\right)^3}dx\right)a^2\right)$$

input
```
int((a^2*c*x^2+c)^(3/2)/x/atan(a*x)^3,x)
```

output
```
sqrt(c)*c*(int(sqrt(a**2*x**2 + 1)/(atan(a*x)**3*x),x) + int((sqrt(a**2*x*
*2 + 1)*x)/atan(a*x)**3,x)*a**2)
```

# 3.649 $\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^3}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^3}\,dx = \text{Int}\left(\frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^3},x\right)$$

output
```
Defer(Int)(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

## Mathematica [N/A]

Not integrable

Time = 2.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^3}\,dx = \int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^3}\,dx$$

input
```
Integrate[(x*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^3,x]
```

output
```
Integrate[(x*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^3, x]
```

## Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a^2cx^2+c\right)^{5/2}}{\arctan(ax)^3}dx$$

$$\Big\downarrow \ 5560$$

$$\int \frac{x\left(a^2cx^2+c\right)^{5/2}}{\arctan(ax)^3}dx$$

input
```
Int[(x*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^3,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 16.82 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x(a^2c\,x^2+c)^{\frac{5}{2}}}{\arctan\left(ax\right)^3}dx$$

input
```
int(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

output
```
int(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.18

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^3}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}x}{\arctan(ax)^3}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^2*x)*sqrt(a^2*c*x^2 + c)/arctan(
a*x)^3, x)
```

## Sympy [N/A]

Not integrable

Time = 20.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^3}\,dx = \int \frac{x(c(a^2x^2+1))^{\frac{5}{2}}}{\operatorname{atan}^3(ax)}\,dx$$

input
```
integrate(x*(a**2*c*x**2+c)**(5/2)/atan(a*x)**3,x)
```

output
```
Integral(x*(c*(a**2*x**2 + 1))**(5/2)/atan(a*x)**3, x)
```

## Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^3}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}x}{\arctan(ax)^3}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)*x/arctan(a*x)^3, x)
```

## Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^3}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}x}{\arctan(ax)^3}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)*x/arctan(a*x)^3, x)
```

## Mupad [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^3}\,dx = \int \frac{x\,(c\,a^2\,x^2+c)^{5/2}}{\mathrm{atan}(a\,x)^3}\,dx$$

input
```
int((x*(c + a^2*c*x^2)^(5/2))/atan(a*x)^3,x)
```

output
```
int((x*(c + a^2*c*x^2)^(5/2))/atan(a*x)^3, x)
```

## Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 3.64

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^3}\,dx = \sqrt{c}\,c^2\left(\left(\int \frac{\sqrt{a^2x^2+1}\,x^5}{atan\,(ax)^3}dx\right)a^4 + 2\left(\int \frac{\sqrt{a^2x^2+1}\,x^3}{atan\,(ax)^3}dx\right)a^2 + \int \frac{\sqrt{a^2x^2+1}\,x}{atan\,(ax)^3}dx\right)$$

input
```
int(x*(a^2*c*x^2+c)^(5/2)/atan(a*x)^3,x)
```

output
```
sqrt(c)*c**2*(int((sqrt(a**2*x**2 + 1)*x**5)/atan(a*x)**3,x)*a**4 + 2*int(
(sqrt(a**2*x**2 + 1)*x**3)/atan(a*x)**3,x)*a**2 + int((sqrt(a**2*x**2 + 1)
*x)/atan(a*x)**3,x))
```

# 3.650 $\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^3}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^3}\,dx=\text{Int}\left(\frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^3},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

## Mathematica [N/A]

Not integrable

Time = 0.95 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^3}\,dx=\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^3}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^(5/2)/ArcTan[a*x]^3,x]
```

output
```
Integrate[(c + a^2*c*x^2)^(5/2)/ArcTan[a*x]^3, x]
```

## Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^{5/2}}{\arctan(ax)^3} \, dx$$

$$\Big\downarrow \; 5560$$

$$\int \frac{(a^2cx^2+c)^{5/2}}{\arctan(ax)^3}dx$$

input
```
Int[(c + a^2*c*x^2)^(5/2)/ArcTan[a*x]^3,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 16.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{(a^2c\,x^2+c)^{\frac{5}{2}}}{\arctan\left(ax\right)^3}dx$$

input
```
int((a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

output
```
int((a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.19

$$\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^3}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}}{\arctan{(ax)}^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)/arctan(a*
x)^3, x)
```

## Sympy [N/A]

Not integrable

Time = 12.37 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.95

$$\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^3}\,dx = \int \frac{(c(a^2x^2+1))^{\frac{5}{2}}}{\operatorname{atan}^3{(ax)}}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(5/2)/atan(a*x)**3,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(5/2)/atan(a*x)**3, x)
```

## Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^3}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}}{\arctan{(ax)}^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)/arctan(a*x)^3, x)
```

## Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^3}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}}{\arctan{(ax)}^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)/arctan(a*x)^3, x)
```

## Mupad [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^3}\,dx = \int \frac{(c\,a^2\,x^2+c)^{5/2}}{\mathrm{atan}\left(a\,x\right)^3}\,dx$$

input
```
int((c + a^2*c*x^2)^(5/2)/atan(a*x)^3,x)
```

output
```
int((c + a^2*c*x^2)^(5/2)/atan(a*x)^3, x)
```

## Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 79, normalized size of antiderivative = 3.76

$$\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^3}\,dx = \sqrt{c}\,c^2\left(\int \frac{\sqrt{a^2x^2+1}}{atan\,(ax)^3}dx + \left(\int \frac{\sqrt{a^2x^2+1}\,x^4}{atan\,(ax)^3}dx\right)a^4 + 2\left(\int \frac{\sqrt{a^2x^2+1}\,x^2}{atan\,(ax)^3}dx\right)a^2\right)$$

input
```
int((a^2*c*x^2+c)^(5/2)/atan(a*x)^3,x)
```

output
```
sqrt(c)*c**2*(int(sqrt(a**2*x**2 + 1)/atan(a*x)**3,x) + int((sqrt(a**2*x**
2 + 1)*x**4)/atan(a*x)**3,x)*a**4 + 2*int((sqrt(a**2*x**2 + 1)*x**2)/atan(
a*x)**3,x)*a**2)
```

# 3.651 $\int \frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)^3}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)^3}\,dx = \operatorname{Int}\left(\frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)^3},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^3,x)
```

## Mathematica [N/A]

Not integrable

Time = 2.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)^3}\,dx = \int \frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)^3}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^(5/2)/(x*ArcTan[a*x]^3),x]
```

output
```
Integrate[(c + a^2*c*x^2)^(5/2)/(x*ArcTan[a*x]^3), x]
```

## Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^{5/2}}{x\arctan(ax)^3}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{(a^2cx^2+c)^{5/2}}{x\arctan(ax)^3}dx$$

input
```
Int[(c + a^2*c*x^2)^(5/2)/(x*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 31.80 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{(a^2c\,x^2+c)^{\frac{5}{2}}}{x\arctan\left(ax\right)^3}dx$$

input
```
int((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^3,x)
```

output
```
int((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.04

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)^3}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}}{x\arctan{(ax)}^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)/(x*arctan
(a*x)^3), x)
```

## Sympy [N/A]

Not integrable

Time = 15.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)^3}\,dx = \int \frac{(c(a^2x^2+1))^{\frac{5}{2}}}{x\,\mathrm{atan}^3{(ax)}}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(5/2)/x/atan(a*x)**3,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(5/2)/(x*atan(a*x)**3), x)
```

## Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)^3}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}}{x\arctan{(ax)}^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)/(x*arctan(a*x)^3), x)
```

## Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)^3}\,dx=\int \frac{(a^2cx^2+c)^{\frac{5}{2}}}{x\arctan{(ax)}^3}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)/(x*arctan(a*x)^3), x)
```

## Mupad [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)^3}\,dx=\int \frac{{(c\,a^2\,x^2+c)}^{5/2}}{x\,\mathrm{atan}{(a\,x)}^3}\,dx$$

input
```
int((c + a^2*c*x^2)^(5/2)/(x*atan(a*x)^3),x)
```

output
```
int((c + a^2*c*x^2)^(5/2)/(x*atan(a*x)^3), x)
```

## Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 80, normalized size of antiderivative = 3.33

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)^3}\,dx = \sqrt{c}\,c^2\left(\int \frac{\sqrt{a^2x^2+1}}{atan\,(ax)^3\,x}dx + \left(\int \frac{\sqrt{a^2x^2+1}\,x^3}{atan\,(ax)^3}dx\right)a^4 + 2\left(\int \frac{\sqrt{a^2x^2+1}\,x}{atan\,(ax)^3}dx\right)a^2\right)$$

input
```
int((a^2*c*x^2+c)^(5/2)/x/atan(a*x)^3,x)
```

output
```
sqrt(c)*c**2*(int(sqrt(a**2*x**2 + 1)/(atan(a*x)**3*x),x) + int((sqrt(a**2
*x**2 + 1)*x**3)/atan(a*x)**3,x)*a**4 + 2*int((sqrt(a**2*x**2 + 1)*x)/atan
(a*x)**3,x)*a**2)
```

# 3.652 $\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx = \text{Int}\left(\frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^3},x\right)$$

output
```
Defer(Int)(x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x)
```

## Mathematica [N/A]

Not integrable

Time = 1.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx = \int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx$$

input
```
Integrate[x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3),x]
```

output
```
Integrate[x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3), x]
```

## Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\arctan(ax)^3\sqrt{a^2cx^2+c}}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x}{\arctan(ax)^3\sqrt{a^2cx^2+c}}dx$$

input
```
Int[x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 2.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x}{\sqrt{a^2c\,x^2+c}\,\arctan\left(ax\right)^3}dx$$

input
```
int(x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x)
```

output
```
int(x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx = \int \frac{x}{\sqrt{a^2cx^2+c}\arctan{(ax)}^3}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(x/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3), x)
```

## Sympy [N/A]

Not integrable

Time = 1.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx = \int \frac{x}{\sqrt{c\,(a^2x^2+1)}\,\mathrm{atan}^3\,(ax)}\,dx$$

input
```
integrate(x/(a**2*c*x**2+c)**(1/2)/atan(a*x)**3,x)
```

output
```
Integral(x/(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**3), x)
```

## Maxima [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx = \int \frac{x}{\sqrt{a^2cx^2+c}\arctan{(ax)}^3}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3), x)
```

## Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx=\int \frac{x}{\sqrt{a^2cx^2+c}\arctan\left(ax\right)^3}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate(x/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3), x)
```

## Mupad [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx=\int \frac{x}{\mathrm{atan}\left(a\,x\right)^3\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(x/(atan(a*x)^3*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(x/(atan(a*x)^3*(c + a^2*c*x^2)^(1/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

$$\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx = \frac{\int \frac{x}{\sqrt{a^2x^2+1}\,atan(ax)^3}dx}{\sqrt{c}}$$

input
```
int(x/(a^2*c*x^2+c)^(1/2)/atan(a*x)^3,x)
```

output
```
int(x/(sqrt(a**2*x**2 + 1)*atan(a*x)**3),x)/sqrt(c)
```

# 3.653 $\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx=\text{Int}\left(\frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^3},x\right)$$

output
```
Defer(Int)(1/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x)
```

## Mathematica [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx=\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx$$

input
```
Integrate[1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3),x]
```

output
```
Integrate[1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3), x]
```

## Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arctan(ax)^3\sqrt{a^2cx^2+c}}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{1}{\arctan(ax)^3\sqrt{a^2cx^2+c}}dx$$

input
```
Int[1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 1.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{1}{\sqrt{a^2c\,x^2+c}\,\arctan\left(ax\right)^3}dx$$

input
```
int(1/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x)
```

output
```
int(1/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx=\int \frac{1}{\sqrt{a^2cx^2+c}\arctan\left(ax\right)^3}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(1/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3), x)
```

## Sympy [N/A]

Not integrable

Time = 1.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.05

$$\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx=\int \frac{1}{\sqrt{c\left(a^2x^2+1\right)}\operatorname{atan}^3\left(ax\right)}\,dx$$

input
```
integrate(1/(a**2*c*x**2+c)**(1/2)/atan(a*x)**3,x)
```

output
```
Integral(1/(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**3), x)
```

## Maxima [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx=\int \frac{1}{\sqrt{a^2cx^2+c}\arctan\left(ax\right)^3}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3), x)
```

## Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx = \int \frac{1}{\sqrt{a^2cx^2+c}\arctan\left(ax\right)^3}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate(1/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3), x)
```

## Mupad [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx = \int \frac{1}{\mathrm{atan}\left(a\,x\right)^3\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(1/(atan(a*x)^3*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(1/(atan(a*x)^3*(c + a^2*c*x^2)^(1/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.24

$$\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx = \frac{\int \frac{1}{\sqrt{a^2x^2+1}\,atan(ax)^3}dx}{\sqrt{c}}$$

input
```
int(1/(a^2*c*x^2+c)^(1/2)/atan(a*x)^3,x)
```

output
```
int(1/(sqrt(a**2*x**2 + 1)*atan(a*x)**3),x)/sqrt(c)
```

# 3.654 $\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx=-\frac{\sqrt{c+a^2cx^2}}{2acx\arctan(ax)^2}-\frac{\operatorname{Int}\left(\frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^2},x\right)}{2a}$$

output
```
-1/2*(a^2*c*x^2+c)^(1/2)/a/c/x/arctan(a*x)^2-1/2*Defer(Int)(1/x^2/(a^2*c*x
^2+c)^(1/2)/arctan(a*x)^2,x)/a
```

## Mathematica [N/A]

Not integrable

Time = 3.32 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx=\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx$$

input
```
Integrate[1/(x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3),x]
```

output
```
Integrate[1/(x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3), x]
```

## Rubi [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x \arctan(ax)^3\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5477}$$

$$-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{2a}-\frac{\sqrt{a^2cx^2+c}}{2acx\arctan(ax)^2}$$

$$\Big\downarrow \text{5560}$$

$$-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{2a}-\frac{\sqrt{a^2cx^2+c}}{2acx\arctan(ax)^2}$$

input
```
Int[1/(x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 5.39 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\sqrt{a^2c\,x^2+c}\,\arctan\left(ax\right)^3}dx$$

input
```
int(1/x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x)
```

output
```
int(1/x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx = \int \frac{1}{\sqrt{a^2cx^2+c}x\arctan\left(ax\right)^3}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)/((a^2*c*x^3 + c*x)*arctan(a*x)^3), x)
```

## Sympy [N/A]

Not integrable

Time = 2.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx = \int \frac{1}{x\sqrt{c\left(a^2x^2+1\right)}\operatorname{atan}^3\left(ax\right)}\,dx$$

input
```
integrate(1/x/(a**2*c*x**2+c)**(1/2)/atan(a*x)**3,x)
```

output
```
Integral(1/(x*sqrt(c*(a**2*x**2 + 1))*atan(a*x)**3), x)
```

## Maxima [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx = \int \frac{1}{\sqrt{a^2cx^2+c}x\arctan{(ax)}^3}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a^2*c*x^2 + c)*x*arctan(a*x)^3), x)
```

## Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx = \int \frac{1}{\sqrt{a^2cx^2+c}x\arctan{(ax)}^3}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate(1/(sqrt(a^2*c*x^2 + c)*x*arctan(a*x)^3), x)
```

## Mupad [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx = \int \frac{1}{x\,\mathrm{atan}\left(a\,x\right)^3\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(1/(x*atan(a*x)^3*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(1/(x*atan(a*x)^3*(c + a^2*c*x^2)^(1/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx = \frac{\int \frac{1}{\sqrt{a^2x^2+1}\,atan(ax)^3x}dx}{\sqrt{c}}$$

input
```
int(1/x/(a^2*c*x^2+c)^(1/2)/atan(a*x)^3,x)
```

output
```
int(1/(sqrt(a**2*x**2 + 1)*atan(a**3*x),x)/sqrt(c)
```

# 3.655 $\int \frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx=\text{Int}\left(\frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^3},x\right)$$

output
```
Defer(Int)(1/x^2/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x)
```

## Mathematica [N/A]

Not integrable

Time = 3.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx=\int \frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx$$

input
```
Integrate[1/(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3),x]
```

output
```
Integrate[1/(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3), x]
```

## Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \arctan(ax)^3\sqrt{a^2cx^2+c}}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{1}{x^2 \arctan(ax)^3\sqrt{a^2cx^2+c}}dx$$

input
```
Int[1/(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 3.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^2\sqrt{a^2c\,x^2+c}\,\arctan\left(ax\right)^3}dx$$

input
```
int(1/x^2/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x)
```

output
```
int(1/x^2/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx = \int \frac{1}{\sqrt{a^2cx^2+c}x^2\arctan(ax)^3}\,dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)/((a^2*c*x^4 + c*x^2)*arctan(a*x)^3), x)
```

## Sympy [N/A]

Not integrable

Time = 3.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx = \int \frac{1}{x^2\sqrt{c(a^2x^2+1)}\operatorname{atan}^3(ax)}\,dx$$

input
```
integrate(1/x**2/(a**2*c*x**2+c)**(1/2)/atan(a*x)**3,x)
```

output
```
Integral(1/(x**2*sqrt(c*(a**2*x**2 + 1))*atan(a*x)**3), x)
```

## Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx = \int \frac{1}{\sqrt{a^2cx^2+c}x^2\arctan(ax)^3}\,dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a^2*c*x^2 + c)*x^2*arctan(a*x)^3), x)
```

## Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx = \int \frac{1}{\sqrt{a^2cx^2+c}x^2\arctan\left(ax\right)^3}\,dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate(1/(sqrt(a^2*c*x^2 + c)*x^2*arctan(a*x)^3), x)
```

## Mupad [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx = \int \frac{1}{x^2\,\mathrm{atan}\left(a\,x\right)^3\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(1/(x^2*atan(a*x)^3*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(1/(x^2*atan(a*x)^3*(c + a^2*c*x^2)^(1/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int \frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx = \frac{\int \frac{1}{\sqrt{a^2x^2+1}\,atan(ax)^3x^2}dx}{\sqrt{c}}$$

input
```
int(1/x^2/(a^2*c*x^2+c)^(1/2)/atan(a*x)^3,x)
```

output
```
int(1/(sqrt(a**2*x**2 + 1)*atan(a*x)**3*x**2),x)/sqrt(c)
```

# 3.656 $\int \frac{1}{x^3\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x^3\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx = \operatorname{Int}\left(\frac{1}{x^3\sqrt{c+a^2cx^2}\arctan(ax)^3},x\right)$$

output
```
Defer(Int)(1/x^3/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x)
```

## Mathematica [N/A]

Not integrable

Time = 4.54 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^3\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx = \int \frac{1}{x^3\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx$$

input
```
Integrate[1/(x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3),x]
```

output
```
Integrate[1/(x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3), x]
```

## Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \arctan(ax)^3\sqrt{a^2cx^2+c}}dx$$
$$\Big\downarrow \ 5560$$
$$\int \frac{1}{x^3 \arctan(ax)^3\sqrt{a^2cx^2+c}}dx$$

input
```
Int[1/(x^3*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 10.86 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^3\sqrt{a^2c\,x^2+c}\,\arctan\left(ax\right)^3}dx$$

input
```
int(1/x^3/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x)
```

output
```
int(1/x^3/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{1}{x^3\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx = \int \frac{1}{\sqrt{a^2cx^2+c}x^3\arctan(ax)^3}\,dx$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)/((a^2*c*x^5 + c*x^3)*arctan(a*x)^3), x)
```

## Sympy [N/A]

Not integrable

Time = 4.58 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^3\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx = \int \frac{1}{x^3\sqrt{c(a^2x^2+1)}\operatorname{atan}^3(ax)}\,dx$$

input
```
integrate(1/x**3/(a**2*c*x**2+c)**(1/2)/atan(a*x)**3,x)
```

output
```
Integral(1/(x**3*sqrt(c*(a**2*x**2 + 1))*atan(a*x)**3), x)
```

## Maxima [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx = \int \frac{1}{\sqrt{a^2cx^2+c}x^3\arctan(ax)^3}\,dx$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a^2*c*x^2 + c)*x^3*arctan(a*x)^3), x)
```

## Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx = \int \frac{1}{\sqrt{a^2cx^2+c}x^3\arctan{(ax)}^3}\,dx$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate(1/(sqrt(a^2*c*x^2 + c)*x^3*arctan(a*x)^3), x)
```

## Mupad [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx = \int \frac{1}{x^3\,\mathrm{atan}\left(a\,x\right)^3\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(1/(x^3*atan(a*x)^3*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(1/(x^3*atan(a*x)^3*(c + a^2*c*x^2)^(1/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int \frac{1}{x^3\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx = \frac{\int \frac{1}{\sqrt{a^2x^2+1}\,atan(ax)^3x^3}dx}{\sqrt{c}}$$

input
```
int(1/x^3/(a^2*c*x^2+c)^(1/2)/atan(a*x)^3,x)
```

output
```
int(1/(sqrt(a**2*x**2 + 1)*atan(a*x)**3*x**3),x)/sqrt(c)
```

# 3.657 $\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx = \frac{x}{2a^3c\sqrt{c+a^2cx^2}\arctan(ax)^2} + \frac{1}{2a^4c\sqrt{c+a^2cx^2}\arctan(ax)} + \frac{\sqrt{1+a^2x^2}\text{Si}(\arctan(ax))}{2a^4c\sqrt{c+a^2cx^2}} + \frac{\text{Int}\left(\frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^3},x\right)}{a^2c}$$

output
```
1/2*x/a^3/c/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2+1/2/a^4/c/(a^2*c*x^2+c)^(1/2
)/arctan(a*x)+1/2*(a^2*x^2+1)^(1/2)*Si(arctan(a*x))/a^4/c/(a^2*c*x^2+c)^(1
/2)+Defer(Int)(x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x)/a^2/c
```

## Mathematica [N/A]

Not integrable

Time = 5.96 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx = \int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx$$

input
```
Integrate[x^3/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3),x]
```

output
```
Integrate[x^3/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3), x]
```

## Rubi [N/A]

Not integrable

Time = 1.51 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\arctan(ax)^3 (a^2cx^2+c)^{3/2}}\,dx$$

$$\Big\downarrow \text{5499}$$

$$\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c} - \frac{\int \frac{x}{(a^2cx^2+c)^{3/2}\arctan(ax)^3}dx}{a^2}$$

$$\Big\downarrow \text{5477}$$

$$\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c} - \frac{\frac{\int \frac{1}{(a^2cx^2+c)^{3/2}\arctan(ax)^2}dx}{2a} - \frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}$$

$$\Big\downarrow \text{5437}$$

$$\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c}-\frac{\frac{-a\int \frac{x}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}dx-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a}-\frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}$$

$\Big\downarrow$ 5506

$$\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c}-\frac{\frac{-\frac{a\sqrt{a^2x^2+1}\int \frac{x}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}dx}{c\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a}-\frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}$$

$\Big\downarrow$ 5505

$$\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c}-\frac{\frac{-\frac{\sqrt{a^2x^2+1}\int \frac{ax}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a}-\frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}$$

$\Big\downarrow$ 3042

$$\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c}-\frac{\frac{-\frac{\sqrt{a^2x^2+1}\int \frac{\sin(\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a}-\frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}$$

$\Big\downarrow$ 3780

$$\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c}-\frac{\frac{-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a}-\frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}$$

$\Big\downarrow$ 5560

$$\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c}-\frac{\frac{-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a}-\frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}$$

input
```
Int[x^3/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 2.58 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^3}{\left(a^2 c\,x^2+c\right)^{\frac{3}{2}} \arctan\left(a x\right)^3}dx$$

input
```
int(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

output
```
int(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.12

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx=\int \frac{x^3}{\left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^3}\,dx$$

input
```
integrate(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^3/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arct
an(a*x)^3), x)
```

## Sympy [N/A]

Not integrable

Time = 2.93 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx=\int \frac{x^3}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^3\left(ax\right)}\,dx$$

input
```
integrate(x**3/(a**2*c*x**2+c)**(3/2)/atan(a*x)**3,x)
```

output
```
Integral(x**3/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**3), x)
```

## Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx = \int \frac{x^3}{(a^2cx^2+c)^{\frac{3}{2}}\arctan{(ax)}^3}\,dx$$

input
```
integrate(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(x^3/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^3), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx = \int \frac{x^3}{\mathrm{atan}\left(a\,x\right)^3\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

input
```
int(x^3/(atan(a*x)^3*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(x^3/(atan(a*x)^3*(c + a^2*c*x^2)^(3/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.38

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx = \frac{\int \frac{x^3}{\sqrt{a^2x^2+1}\,atan(ax)^3a^2x^2+\sqrt{a^2x^2+1}\,atan(ax)^3}dx}{\sqrt{c}\,c}$$

input
```
int(x^3/(a^2*c*x^2+c)^(3/2)/atan(a*x)^3,x)
```

output
```
int(x**3/(sqrt(a**2*x**2 + 1)*atan(a*x)**3*a**2*x**2 + sqrt(a**2*x**2 + 1)
*atan(a*x)**3),x)/(sqrt(c)*c)
```

# 3.658 $\int \frac{x^2}{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^2}{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3}\,dx = \frac{1}{2a^3c\sqrt{c+a^2cx^2}\arctan(ax)^2} - \frac{x}{2a^2c\sqrt{c+a^2cx^2}\arctan(ax)} + \frac{\sqrt{1+a^2x^2}\,\text{CosIntegral}(\arctan(ax))}{2a^3c\sqrt{c+a^2cx^2}} + \frac{\text{Int}\left(\frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^3},x\right)}{a^2c}$$

output
```
1/2/a^3/c/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2-1/2*x/a^2/c/(a^2*c*x^2+c)^(1/2
)/arctan(a*x)+1/2*(a^2*x^2+1)^(1/2)*Ci(arctan(a*x))/a^3/c/(a^2*c*x^2+c)^(1
/2)+Defer(Int)(1/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x)/a^2/c
```

## Mathematica [N/A]

Not integrable

Time = 4.72 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx = \int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx$$

input
```
Integrate[x^2/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3),x]
```

output
```
Integrate[x^2/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3), x]
```

## Rubi [N/A]

Not integrable

Time = 1.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\arctan(ax)^3\,(a^2cx^2+c)^{3/2}}\,dx$$

$$\Big\downarrow \text{5499}$$

$$\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c}-\frac{\int \frac{1}{(a^2cx^2+c)^{3/2}\arctan(ax)^3}dx}{a^2}$$

$$\Big\downarrow \text{5437}$$

$$\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c}-\frac{-\frac{1}{2}a\int \frac{x}{(a^2cx^2+c)^{3/2}\arctan(ax)^2}dx-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}$$

$$\Big\downarrow \text{5477}$$

$$\frac{\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c}-\frac{-\frac{1}{2}a\left(\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}dx}{a}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}}{}$$

$$\Big\downarrow \text{5440}$$

$$\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c}-\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\int \frac{1}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}dx}{ac\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}$$

$$\Big\downarrow \text{5439}$$

$$\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c}-\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c}-\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)}{\arctan(ax)}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}$$

$$\Big\downarrow \text{3783}$$

$$\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c}-\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}$$

$$\Big\downarrow \text{5560}$$

$$\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c}-\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}$$

input
```
Int[x^2/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 1.43 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^2}{(a^2c\,x^2+c)^{\frac{3}{2}} \arctan(ax)^3}dx$$

input
```
int(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

output
```
int(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.12

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx = \int \frac{x^2}{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)^3}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^2/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arct
an(a*x)^3), x)
```

## Sympy [N/A]

Not integrable

Time = 2.90 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx = \int \frac{x^2}{(c\,(a^2x^2+1))^{\frac{3}{2}}\operatorname{atan}^3(ax)}\,dx$$

input
```
integrate(x**2/(a**2*c*x**2+c)**(3/2)/atan(a*x)**3,x)
```

output
```
Integral(x**2/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**3), x)
```

## Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx = \int \frac{x^2}{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)^3}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(x^2/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^3), x)
```

## Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx = \int \frac{x^2}{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)^3}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate(x^2/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^3), x)
```

## Mupad [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx = \int \frac{x^2}{\mathrm{atan}\left(a\,x\right)^3\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

input
```
int(x^2/(atan(a*x)^3*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(x^2/(atan(a*x)^3*(c + a^2*c*x^2)^(3/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.38

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx = \frac{\int \frac{x^2}{\sqrt{a^2x^2+1}\,atan(ax)^3a^2x^2+\sqrt{a^2x^2+1}\,atan(ax)^3}dx}{\sqrt{c}\,c}$$

input
```
int(x^2/(a^2*c*x^2+c)^(3/2)/atan(a*x)^3,x)
```

output
```
int(x**2/(sqrt(a**2*x**2 + 1)*atan(a*x)**3*a**2*x**2 + sqrt(a**2*x**2 + 1)
*atan(a*x)**3),x)/(sqrt(c)*c)
```

# 3.659 $\int \frac{x}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 104

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx = -\frac{x}{2ac\sqrt{c+a^2cx^2}\arctan(ax)^2} - \frac{1}{2a^2c\sqrt{c+a^2cx^2}\arctan(ax)} - \frac{\sqrt{1+a^2x^2}\,\text{Si}(\arctan(ax))}{2a^2c\sqrt{c+a^2cx^2}}$$

output
```
-1/2*x/a/c/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2-1/2/a^2/c/(a^2*c*x^2+c)^(1/2)
/arctan(a*x)-1/2*(a^2*x^2+1)^(1/2)*Si(arctan(a*x))/a^2/c/(a^2*c*x^2+c)^(1/
2)
```

## Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.61

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx = -\frac{ax+\arctan(ax)+\sqrt{1+a^2x^2}\arctan(ax)^2\text{Si}(\arctan(ax))}{2a^2c\sqrt{c+a^2cx^2}\arctan(ax)^2}$$

input
```
Integrate[x/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3),x]
```

output
```
-1/2*(a*x + ArcTan[a*x] + Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*SinIntegral[ArcT
an[a*x]])/(a^2*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)
```

## Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {5477, 5437, 5506, 5505, 3042, 3780}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\arctan(ax)^3 \left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{5477}$$

$$\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}dx}{2a}-\frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5437}$$

$$\frac{-a\int \frac{x}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}dx-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a}-\frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5506}$$

$$\frac{-\frac{a\sqrt{a^2x^2+1}\int \frac{x}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}dx}{c\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a}-\frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5505}$$

$$\frac{-\frac{\sqrt{a^2x^2+1}\int \frac{ax}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a}-\frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{\sqrt{a^2x^2+1}\int \frac{\sin(\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a}-\frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3780}$$

$$\frac{-\frac{\sqrt{a^2x^2+1}\operatorname{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a}-\frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}$$

input
```
Int[x/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3),x]
```

output
```
-1/2*x/(a*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2) + (-(1/(a*c*Sqrt[c + a^2*c*
x^2]*ArcTan[a*x])) - (Sqrt[1 + a^2*x^2]*SinIntegral[ArcTan[a*x]])/(a*c*Sqr
t[c + a^2*c*x^2]))/(2*a)
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5477
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*ArcT
an[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp[f*(m/(b*c*(p + 1)))   Int[(f*x
)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1
]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.95 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.50

| method | result |
|---|---|
| default | $\frac{i\left(\arctan(ax)^2 \operatorname{expIntegral}_1(i\arctan(ax))a^2x^2-\arctan(ax)^2 \operatorname{expIntegral}_1(-i\arctan(ax))a^2x^2+\operatorname{expIntegral}_1(i\arctan(ax))\arctan(\right.}{4(a^2x^2+1)^{\frac{3}{2}}\arctan(ax)^2}$ |

input
```
int(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*I*(arctan(a*x)^2*Ei(1,I*arctan(a*x))*a^2*x^2-arctan(a*x)^2*Ei(1,-I*arc
tan(a*x))*a^2*x^2+Ei(1,I*arctan(a*x))*arctan(a*x)^2-Ei(1,-I*arctan(a*x))*a
rctan(a*x)^2+2*I*(a^2*x^2+1)^(1/2)*a*x+2*I*arctan(a*x)*(a^2*x^2+1)^(1/2))*
(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(3/2)/arctan(a*x)^2/a^2/c^2
```

## Fricas [F]

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx = \int \frac{x}{(a^2cx^2+c)^{\frac{3}{2}}\arctan{(ax)}^3}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan
(a*x)^3), x)
```

## Sympy [F]

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx = \int \frac{x}{(c(a^2x^2+1))^{\frac{3}{2}}\operatorname{atan}^3{(ax)}}\,dx$$

input
```
integrate(x/(a**2*c*x**2+c)**(3/2)/atan(a*x)**3,x)
```

output
```
Integral(x/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**3), x)
```

## Maxima [F]

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx = \int \frac{x}{(a^2cx^2+c)^{\frac{3}{2}}\arctan{(ax)}^3}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(x/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^3), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx = \int \frac{x}{\mathrm{atan}\left(a\,x\right)^3\,\left(c\,a^2\,x^2+c\right)^{3/2}}\,dx$$

input
```
int(x/(atan(a*x)^3*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(x/(atan(a*x)^3*(c + a^2*c*x^2)^(3/2)), x)
```

## Reduce [F]

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx = \frac{\int \frac{x}{\sqrt{a^2x^2+1}\,atan(ax)^3a^2x^2+\sqrt{a^2x^2+1}\,atan(ax)^3}dx}{\sqrt{c}\,c}$$

input
```
int(x/(a^2*c*x^2+c)^(3/2)/atan(a*x)^3,x)
```

output
```
int(x/(sqrt(a**2*x**2 + 1)*atan(a*x)**3*a**2*x**2 + sqrt(a**2*x**2 + 1)*at
an(a*x)**3),x)/(sqrt(c)*c)
```

# 3.660 $\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 101

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx = -\frac{1}{2ac\sqrt{c+a^2cx^2}\arctan(ax)^2} + \frac{x}{2c\sqrt{c+a^2cx^2}\arctan(ax)} - \frac{\sqrt{1+a^2x^2}\,\mathrm{CosIntegral}(\arctan(ax))}{2ac\sqrt{c+a^2cx^2}}$$

output
```
-1/2/a/c/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2+1/2*x/c/(a^2*c*x^2+c)^(1/2)/arc
tan(a*x)-1/2*(a^2*x^2+1)^(1/2)*Ci(arctan(a*x))/a/c/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.64

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx = \frac{-1+ax\arctan(ax)-\sqrt{1+a^2x^2}\arctan(ax)^2\,\mathrm{CosIntegral}(\arctan(ax))}{2ac\sqrt{c+a^2cx^2}\arctan(ax)^2}$$

input
```
Integrate[1/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3),x]
```

output
```
(-1 + a*x*ArcTan[a*x] - Sqrt[1 + a^2*x^2]*ArcTan[a*x]^2*CosIntegral[ArcTan
[a*x]])/(2*a*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)
```

## Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5437, 5477, 5440, 5439, 3042, 3783}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arctan(ax)^3 (a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \text{5437}$$

$$-\frac{1}{2}a\int \frac{x}{(a^2cx^2+c)^{3/2}\arctan(ax)^2}dx-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5477}$$

$$-\frac{1}{2}a\left(\frac{\int \frac{1}{(a^2cx^2+c)^{3/2}\arctan(ax)}dx}{a}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5440}$$

$$-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\int \frac{1}{(a^2x^2+1)^{3/2}\arctan(ax)}dx}{ac\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5439}$$

$$-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\int\frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)}{\arctan(ax)}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \; 3783$$

$$-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}$$

input
```
Int[1/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3),x]
```

output
```
-1/2*1/(a*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2) - (a*(-(x/(a*c*Sqrt[c + a^2
*c*x^2]*ArcTan[a*x])) + (Sqrt[1 + a^2*x^2]*CosIntegral[ArcTan[a*x]])/(a^2*
c*Sqrt[c + a^2*c*x^2])))/2
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5440
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])   Int[(1 +
c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

rule 5477
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*ArcT
an[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp[f*(m/(b*c*(p + 1)))   Int[(f*x
)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1
]
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.50 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.50

| method | result |
|---|---|
| default | $\frac{\left(\arctan(ax)^2 \operatorname{expIntegral}_1(i\arctan(ax))a^2x^2+\arctan(ax)^2 \operatorname{expIntegral}_1(-i\arctan(ax))a^2x^2+2\arctan(ax)\sqrt{a^2x^2+1}\,ax+\operatorname{expInte}\right.}{4(a^2x^2+1)^{\frac{3}{2}}\arctan(ax)^2 c}$ |

input
```
int(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*(arctan(a*x)^2*Ei(1,I*arctan(a*x))*a^2*x^2+arctan(a*x)^2*Ei(1,-I*arcta
n(a*x))*a^2*x^2+2*arctan(a*x)*(a^2*x^2+1)^(1/2)*a*x+Ei(1,I*arctan(a*x))*ar
ctan(a*x)^2+Ei(1,-I*arctan(a*x))*arctan(a*x)^2-2*(a^2*x^2+1)^(1/2))*(c*(a*
x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(3/2)/arctan(a*x)^2/c^2/a
```

## Fricas [F]

$$\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3}\,dx=\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^3}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a
*x)^3), x)
```

## Sympy [F]

$$\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3}\,dx=\int \frac{1}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^3\left(ax\right)}\,dx$$

input
```
integrate(1/(a**2*c*x**2+c)**(3/2)/atan(a*x)**3,x)
```

output
```
Integral(1/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**3), x)
```

## Maxima [F]

$$\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3}\,dx=\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^3}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^3), x)
```

## Giac [F]

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx=\int \frac{1}{(a^2cx^2+c)^{\frac{3}{2}}\arctan{(ax)}^3}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx=\int \frac{1}{\mathrm{atan}{(a\,x)}^3\,{(c\,a^2\,x^2+c)}^{3/2}}\,dx$$

input
```
int(1/(atan(a*x)^3*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(1/(atan(a*x)^3*(c + a^2*c*x^2)^(3/2)), x)
```

## Reduce [F]

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx=\frac{\int \frac{1}{\sqrt{a^2x^2+1}\,atan(ax)^3a^2x^2+\sqrt{a^2x^2+1}\,atan(ax)^3}dx}{\sqrt{c}\,c}$$

input
```
int(1/(a^2*c*x^2+c)^(3/2)/atan(a*x)^3,x)
```

output
```
int(1/(sqrt(a**2*x**2 + 1)*atan(a**x)**3*a**2*x**2 + sqrt(a**2*x**2 + 1)*at
an(a*x)**3),x)/(sqrt(c)*c)
```

# 3.661 $\int \frac{1}{x(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3}\,dx = \frac{ax}{2c\sqrt{c+a^2cx^2}\arctan(ax)^2} - \frac{\sqrt{c+a^2cx^2}}{2ac^2x\arctan(ax)^2} + \frac{1}{2c\sqrt{c+a^2cx^2}\arctan(ax)} + \frac{\sqrt{1+a^2x^2}\,\mathrm{Si}(\arctan(ax))}{2c\sqrt{c+a^2cx^2}} - \frac{\mathrm{Int}\left(\frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^2},x\right)}{2ac}$$

output
```
1/2*a*x/c/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2-1/2*(a^2*c*x^2+c)^(1/2)/a/c^2/
x/arctan(a*x)^2+1/2/c/(a^2*c*x^2+c)^(1/2)/arctan(a*x)+1/2*(a^2*x^2+1)^(1/2
)*Si(arctan(a*x))/c/(a^2*c*x^2+c)^(1/2)-1/2*Defer(Int)(1/x^2/(a^2*c*x^2+c)
^(1/2)/arctan(a*x)^2,x)/a/c
```

## Mathematica [N/A]

Not integrable

Time = 2.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3}\,dx=\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3}\,dx$$

input
```
Integrate[1/(x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3),x]
```

output
```
Integrate[1/(x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3), x]
```

## Rubi [N/A]

Not integrable

Time = 1.84 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\arctan(ax)^3\left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 5501}$$

$$\frac{\int \frac{1}{x\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c}-a^2\int \frac{x}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}dx$$

$$\Big\downarrow \text{ 5477}$$

$$\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{2a}-\frac{\sqrt{a^2cx^2+c}}{2acx\arctan(ax)^2}}{c}-a^2\left(\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}dx}{2a}-\frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \; 5437$$

$$\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{2a}-\frac{\sqrt{a^2cx^2+c}}{2acx\arctan(ax)^2}}{c}-a^2\left(\frac{-a\int \frac{x}{(a^2cx^2+c)^{3/2}\arctan(ax)}dx-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a}-\frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \; 5506$$

$$\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{2a}-\frac{\sqrt{a^2cx^2+c}}{2acx\arctan(ax)^2}}{c}-a^2\left(\frac{-\frac{a\sqrt{a^2x^2+1}\int \frac{x}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}dx}{c\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a}-\frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \; 5505$$

$$\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{2a}-\frac{\sqrt{a^2cx^2+c}}{2acx\arctan(ax)^2}}{c}-a^2\left(\frac{-\frac{\sqrt{a^2x^2+1}\int \frac{ax}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a}-\frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{2a}-\frac{\sqrt{a^2cx^2+c}}{2acx\arctan(ax)^2}}{c}-a^2\left(\frac{-\frac{\sqrt{a^2x^2+1}\int \frac{\sin(\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a}-\frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \; 3780$$

$$\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{2a}-\frac{\sqrt{a^2cx^2+c}}{2acx\arctan(ax)^2}}{c}-a^2\left(\frac{-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a}-\frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \; 5560$$

$$\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{2a}-\frac{\sqrt{a^2cx^2+c}}{2acx\arctan(ax)^2}}{c}-a^2\left(\frac{-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a}-\frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}\right)$$

input
```
Int[1/(x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 2.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\left(a^2c\,x^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^3}dx$$

input
```
int(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

output
```
int(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.08

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3}\,dx=\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x\arctan\left(ax\right)^3}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)/((a^4*c^2*x^5 + 2*a^2*c^2*x^3 + c^2*x)*arctan
(a*x)^3), x)
```

## Sympy [N/A]

Not integrable

Time = 5.43 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\,(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx = \int \frac{1}{x\,(c\,(a^2x^2+1))^{\frac{3}{2}}\operatorname{atan}^3(ax)}\,dx$$

input
```
integrate(1/x/(a**2*c*x**2+c)**(3/2)/atan(a*x)**3,x)
```

output
```
Integral(1/(x*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**3), x)
```

## Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\,(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx = \int \frac{1}{(a^2cx^2+c)^{\frac{3}{2}}x\arctan(ax)^3}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(3/2)*x*arctan(a*x)^3), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3}\,dx = \int \frac{1}{x\,\mathrm{atan}\left(a\,x\right)^3\,\left(c\,a^2\,x^2+c\right)^{3/2}}\,dx$$

input
```
int(1/(x*atan(a*x)^3*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(1/(x*atan(a*x)^3*(c + a^2*c*x^2)^(3/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.25

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3}\,dx = \frac{\int \frac{1}{\sqrt{a^2x^2+1}\,atan(ax)^3a^2x^3+\sqrt{a^2x^2+1}\,atan(ax)^3x}dx}{\sqrt{c}\,c}$$

input
```
int(1/x/(a^2*c*x^2+c)^(3/2)/atan(a*x)^3,x)
```

output
```
int(1/(sqrt(a**2*x**2 + 1)*atan(a*x)**3*a**2*x**3 + sqrt(a**2*x**2 + 1)*at
an(a*x)**3*x),x)/(sqrt(c)*c)
```

# 3.662 $\int \frac{1}{x^2(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\begin{aligned}\int \frac{1}{x^2\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3}\,dx &= \frac{a}{2c\sqrt{c+a^2cx^2}\arctan(ax)^2}\\ &-\frac{a^2x}{2c\sqrt{c+a^2cx^2}\arctan(ax)}+\frac{a\sqrt{1+a^2x^2}\,\mathrm{CosIntegral}(\arctan(ax))}{2c\sqrt{c+a^2cx^2}}\\ &+\frac{\mathrm{Int}\left(\frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^3},x\right)}{c}\end{aligned}$$

output
```
1/2*a/c/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2-1/2*a^2*x/c/(a^2*c*x^2+c)^(1/2)/
arctan(a*x)+1/2*a*(a^2*x^2+1)^(1/2)*Ci(arctan(a*x))/c/(a^2*c*x^2+c)^(1/2)+
Defer(Int)(1/x^2/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x)/c
```

## Mathematica [N/A]

Not integrable

Time = 2.35 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^2 (c+a^2cx^2)^{3/2} \arctan(ax)^3} \, dx = \int \frac{1}{x^2 (c+a^2cx^2)^{3/2} \arctan(ax)^3} \, dx$$

input
```
Integrate[1/(x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3),x]
```

output
```
Integrate[1/(x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3), x]
```

## Rubi [N/A]

Not integrable

Time = 1.63 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \arctan(ax)^3 (a^2cx^2+c)^{3/2}} \, dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c} - a^2 \int \frac{1}{(a^2cx^2+c)^{3/2}\arctan(ax)^3}dx$$

$$\Big\downarrow \text{5437}$$

$$\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c} - a^2 \left(-\frac{1}{2}a \int \frac{x}{(a^2cx^2+c)^{3/2}\arctan(ax)^2}dx - \frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}\right)$$

$$\Big\downarrow \text{5477}$$

$$a^2\left(-\frac{1}{2}a\left(\frac{\int \frac{1}{(a^2cx^2+c)^{3/2}\arctan(ax)}dx}{a}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}\right)\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c}-$$

$\downarrow$ 5440

$$a^2\left(-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\int \frac{1}{(a^2x^2+1)^{3/2}\arctan(ax)}dx}{ac\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}\right)\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c}-$$

$\downarrow$ 5439

$$a^2\left(-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2}}\right)\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c}-$$

$\downarrow$ 3042

$$a^2\left(-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)}{\arctan(ax)}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+}}\right)\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c}-$$

$\downarrow$ 3783

$$a^2\left(-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}\right)\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c}-$$

$\downarrow$ 5560

$$a^2\left(-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}\right)\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c}-$$

input
```
Int[1/(x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 2.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^{2} \left(a^{2} c\, x^{2}+c \right)^{\frac{3}{2}} \arctan \left(a x \right)^{3}}d x$$

input
```
int(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

output
```
int(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.17

$$\int \frac{1}{x^2 \left(c+a^2 c x^2\right)^{3/2} \arctan(a x)^3} \, dx=\int \frac{1}{{\left(a^{2} c x^{2} + c\right)}^{\frac{3}{2}} x^{2} \arctan \left(a x\right)^{3}} \,d x$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)/((a^4*c^2*x^6 + 2*a^2*c^2*x^4 + c^2*x^2)*arct
an(a*x)^3), x)
```

## Sympy [N/A]

Not integrable

Time = 6.83 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^2 (c + a^2 c x^2)^{3/2} \arctan(ax)^3} \, dx = \int \frac{1}{x^2 (c (a^2 x^2 + 1))^{\frac{3}{2}} \operatorname{atan}^3(ax)} \, dx$$

input
```
integrate(1/x**2/(a**2*c*x**2+c)**(3/2)/atan(a*x)**3,x)
```

output
```
Integral(1/(x**2*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**3), x)
```

## Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (c + a^2 c x^2)^{3/2} \arctan(ax)^3} \, dx = \int \frac{1}{(a^2 c x^2 + c)^{\frac{3}{2}} x^2 \arctan(ax)^3} \, dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(3/2)*x^2*arctan(a*x)^3), x)
```

## Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (c + a^2 c x^2)^{3/2} \arctan(ax)^3} \, dx = \int \frac{1}{(a^2 c x^2 + c)^{\frac{3}{2}} x^2 \arctan(ax)^3} \, dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(3/2)*x^2*arctan(a*x)^3), x)
```

## Mupad [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (c + a^2cx^2)^{3/2} \arctan(ax)^3} dx = \int \frac{1}{x^2 \operatorname{atan}(a x)^3 (c a^2 x^2 + c)^{3/2}} dx$$

input
```
int(1/(x^2*atan(a*x)^3*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(1/(x^2*atan(a*x)^3*(c + a^2*c*x^2)^(3/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.33

$$\int \frac{1}{x^2 (c + a^2cx^2)^{3/2} \arctan(ax)^3} dx = \frac{\int \frac{1}{\sqrt{a^2x^2+1}\, atan(ax)^3 a^2x^4 + \sqrt{a^2x^2+1}\, atan(ax)^3 x^2} dx}{\sqrt{c}\, c}$$

input
```
int(1/x^2/(a^2*c*x^2+c)^(3/2)/atan(a*x)^3,x)
```

output
```
int(1/(sqrt(a**2*x**2 + 1)*atan(a*x)**3*a**2*x**4 + sqrt(a**2*x**2 + 1)*at
an(a*x)**3*x**2),x)/(sqrt(c)*c)
```

# 3.663 $\int \frac{1}{x^3(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x^3(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx = -\frac{a^3x}{2c\sqrt{c+a^2cx^2}\arctan(ax)^2} + \frac{a\sqrt{c+a^2cx^2}}{2c^2x\arctan(ax)^2} - \frac{a^2}{2c\sqrt{c+a^2cx^2}\arctan(ax)} - \frac{a^2\sqrt{1+a^2x^2}\operatorname{Si}(\arctan(ax))}{2c\sqrt{c+a^2cx^2}} + \frac{\operatorname{Int}\left(\frac{1}{x^3\sqrt{c+a^2cx^2}\arctan(ax)^3},x\right)}{c} + \frac{a\operatorname{Int}\left(\frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^2},x\right)}{2c}$$

output
```
-1/2*a^3*x/c/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2+1/2*a*(a^2*c*x^2+c)^(1/2)/c
^2/x/arctan(a*x)^2-1/2*a^2/c/(a^2*c*x^2+c)^(1/2)/arctan(a*x)-1/2*a^2*(a^2*
x^2+1)^(1/2)*Si(arctan(a*x))/c/(a^2*c*x^2+c)^(1/2)+Defer(Int)(1/x^3/(a^2*c
*x^2+c)^(1/2)/arctan(a*x)^3,x)/c+1/2*a*Defer(Int)(1/x^2/(a^2*c*x^2+c)^(1/2
)/arctan(a*x)^2,x)/c
```

## Mathematica [N/A]

Not integrable

Time = 2.84 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^3 (c+a^2cx^2)^{3/2} \arctan(ax)^3} \, dx = \int \frac{1}{x^3 (c+a^2cx^2)^{3/2} \arctan(ax)^3} \, dx$$

input
```
Integrate[1/(x^3*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3),x]
```

output
```
Integrate[1/(x^3*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3), x]
```

## Rubi [N/A]

Not integrable

Time = 2.82 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \arctan(ax)^3 (a^2cx^2+c)^{3/2}} \, dx$$

$$\Big\downarrow \text{ 5501}$$

$$\frac{\int \frac{1}{x^3\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c} - a^2\int \frac{1}{x (a^2cx^2+c)^{3/2}\arctan(ax)^3}dx$$

$$\Big\downarrow \text{ 5501}$$

$$\frac{\int \frac{1}{x^3\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c} - a^2\left(\frac{\int \frac{1}{x\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c} - a^2\int \frac{x}{(a^2cx^2+c)^{3/2}\arctan(ax)^3}dx\right)$$

$$\Big\downarrow \text{ 5477}$$

$$\frac{\int \frac{1}{x^3\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c} - a^2\left(\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{2a} - \frac{\sqrt{a^2cx^2+c}}{2acx\arctan(ax)^2}}{c} - a^2\left(\frac{\int \frac{1}{(a^2cx^2+c)^{3/2}\arctan(ax)^2}dx}{2a} - \frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}\right)\right.$$

$\downarrow$ 5437

$$\frac{\int \frac{1}{x^3\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c} - a^2\left(\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{2a} - \frac{\sqrt{a^2cx^2+c}}{2acx\arctan(ax)^2}}{c} - a^2\left(\frac{-a\int \frac{x}{(a^2cx^2+c)^{3/2}\arctan(ax)}dx - \frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a} - \frac{}{2ac\,a}\right.\right.$$

$\downarrow$ 5506

$$\frac{\int \frac{1}{x^3\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c} - a^2\left(\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{2a} - \frac{\sqrt{a^2cx^2+c}}{2acx\arctan(ax)^2}}{c} - a^2\left(\frac{-\frac{a\sqrt{a^2x^2+1}\int \frac{x}{(a^2x^2+1)^{3/2}\arctan(ax)}dx}{c\sqrt{a^2cx^2+c}} - \frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a} - \right.\right.$$

$\downarrow$ 5505

$$\frac{\int \frac{1}{x^3\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c} - a^2\left(\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{2a} - \frac{\sqrt{a^2cx^2+c}}{2acx\arctan(ax)^2}}{c} - a^2\left(\frac{-\frac{\sqrt{a^2x^2+1}\int \frac{ax}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}} - \frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a}\right.\right.$$

$\downarrow$ 3042

$$\frac{\int \frac{1}{x^3\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c} - a^2\left(\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{2a} - \frac{\sqrt{a^2cx^2+c}}{2acx\arctan(ax)^2}}{c} - a^2\left(\frac{-\frac{\sqrt{a^2x^2+1}\int \frac{\sin(\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}} - \frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a} - \right.\right.$$

$\downarrow$ 3780

$$\frac{\int \frac{1}{x^3\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c} - a^2\left(\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{2a} - \frac{\sqrt{a^2cx^2+c}}{2acx\arctan(ax)^2}}{c} - a^2\left(\frac{-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}} - \frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a} - \frac{}{2ac\arctan(}\right.\right.$$

$\downarrow$ 5560

$$\frac{\int \frac{1}{x^3\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c} - a^2\left(\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{2a} - \frac{\sqrt{a^2cx^2+c}}{2acx\arctan(ax)^2}}{c} - a^2\left(\frac{-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}} - \frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a} - \frac{}{2ac\arctan(}\right.\right.$$

input
```
Int[1/(x^3*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 9.55 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^3\left(a^2c\,x^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^3}dx$$

input
```
int(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

output
```
int(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.17

$$\int \frac{1}{x^3 (c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx = \int \frac{1}{(a^2cx^2+c)^{\frac{3}{2}}x^3\arctan(ax)^3}\,dx$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)/((a^4*c^2*x^7 + 2*a^2*c^2*x^5 + c^2*x^3)*arct
an(a*x)^3), x)
```

## Sympy [N/A]

Not integrable

Time = 10.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^3 (c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx = \int \frac{1}{x^3(c(a^2x^2+1))^{\frac{3}{2}}\operatorname{atan}^3(ax)}\,dx$$

input
```
integrate(1/x**3/(a**2*c*x**2+c)**(3/2)/atan(a*x)**3,x)
```

output
```
Integral(1/(x**3*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**3), x)
```

## Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3 (c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx = \int \frac{1}{(a^2cx^2+c)^{\frac{3}{2}}x^3\arctan(ax)^3}\,dx$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(3/2)*x^3*arctan(a*x)^3), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x^3 (c+a^2cx^2)^{3/2} \arctan(ax)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3 (c+a^2cx^2)^{3/2} \arctan(ax)^3}\,dx = \int \frac{1}{x^3\,\mathrm{atan}\left(a\,x\right)^3\,{\left(c\,a^2\,x^2+c\right)}^{3/2}} \,d x$$

input
```
int(1/(x^3*atan(a*x)^3*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(1/(x^3*atan(a*x)^3*(c + a^2*c*x^2)^(3/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.33

$$\int \frac{1}{x^3 (c+a^2cx^2)^{3/2} \arctan(ax)^3} dx = \frac{\int \frac{1}{\sqrt{a^2x^2+1}\, atan(ax)^3 a^2x^5+\sqrt{a^2x^2+1}\, atan(ax)^3 x^3} dx}{\sqrt{c}\, c}$$

input
```
int(1/x^3/(a^2*c*x^2+c)^(3/2)/atan(a*x)^3,x)
```

output
```
int(1/(sqrt(a**2*x**2 + 1)*atan(a*x)**3*a**2*x**5 + sqrt(a**2*x**2 + 1)*at
an(a*x)**3*x**3),x)/(sqrt(c)*c)
```

# 3.664 $\int \frac{1}{x^4(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x^4(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx = -\frac{a^3}{2c\sqrt{c+a^2cx^2}\arctan(ax)^2} + \frac{a^4x}{2c\sqrt{c+a^2cx^2}\arctan(ax)} - \frac{a^3\sqrt{1+a^2x^2}\,\mathrm{CosIntegral}(\arctan(ax))}{2c\sqrt{c+a^2cx^2}} + \frac{\mathrm{Int}\left(\frac{1}{x^4\sqrt{c+a^2cx^2}\arctan(ax)^3},x\right)}{c} - \frac{a^2\mathrm{Int}\left(\frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^3},x\right)}{c}$$

output
```
-1/2*a^3/c/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^2+1/2*a^4*x/c/(a^2*c*x^2+c)^(1/
2)/arctan(a*x)-1/2*a^3*(a^2*x^2+1)^(1/2)*Ci(arctan(a*x))/c/(a^2*c*x^2+c)^(
1/2)+Defer(Int)(1/x^4/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x)/c-a^2*Defer(Int
)(1/x^2/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x)/c
```

## Mathematica [N/A]

Not integrable

Time = 5.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^4(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx=\int \frac{1}{x^4(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx$$

input
```
Integrate[1/(x^4*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3),x]
```

output
```
Integrate[1/(x^4*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3), x]
```

## Rubi [N/A]

Not integrable

Time = 2.69 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\arctan(ax)^3(a^2cx^2+c)^{3/2}}\,dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{1}{x^4\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c}-a^2\int \frac{1}{x^2(a^2cx^2+c)^{3/2}\arctan(ax)^3}dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{1}{x^4\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c}-a^2\left(\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c}-a^2\int \frac{1}{(a^2cx^2+c)^{3/2}\arctan(ax)^3}dx\right)$$

$$\Big\downarrow \text{5437}$$

$$a^2\left(\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c}-a^2\left(-\frac{1}{2}a\int \frac{x}{(a^2cx^2+c)^{3/2}\arctan(ax)^2}dx-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}\right)\right)$$

$$\frac{\int \frac{1}{x^4\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c}-$$

$\big\downarrow$ 5477

$$\frac{\int \frac{1}{x^4\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c}-$$

$$a^2\left(\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c}-a^2\left(-\frac{1}{2}a\left(\frac{\int \frac{1}{(a^2cx^2+c)^{3/2}\arctan(ax)}dx}{a}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{}{2ac\arctan}\right.\right.$$

$\big\downarrow$ 5440

$$\frac{\int \frac{1}{x^4\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c}-$$

$$a^2\left(\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c}-a^2\left(-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\int \frac{1}{(a^2x^2+1)^{3/2}\arctan(ax)}dx}{ac\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\right.\right.$$

$\big\downarrow$ 5439

$$\frac{\int \frac{1}{x^4\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c}-$$

$$a^2\left(\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c}-a^2\left(-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2}}\right.\right.\right.$$

$\big\downarrow$ 3042

$$\frac{\int \frac{1}{x^4\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c}-$$

$$a^2\left(\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c}-a^2\left(-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)}{\arctan(ax)}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+}}\right.\right.\right.$$

$\big\downarrow$ 3783

$$\frac{\int \frac{1}{x^4\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c}-$$

$$a^2\left(\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c}-a^2\left(-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\right.\right.$$

$\big\downarrow$ 5560

$$\frac{\int \frac{1}{x^4\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c} -$$

$$a^2\left(\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c} - a^2\left(-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right) -\right.\right.$$

input
```
Int[1/(x^4*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 10.57 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^4\left(a^2c\,x^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^3}dx$$

input
```
int(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

output
```
int(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.17

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^3}\,dx = \int \frac{1}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x^4\arctan\left(ax\right)^3}\,dx$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)/((a^4*c^2*x^8 + 2*a^2*c^2*x^6 + c^2*x^4)*arct
an(a*x)^3), x)
```

## Sympy [N/A]

Not integrable

Time = 14.47 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^4 (c+a^2cx^2)^{3/2} \arctan(ax)^3} \, dx = \int \frac{1}{x^4 (c (a^2x^2+1))^{\frac{3}{2}} \operatorname{atan}^3 (ax)} \, dx$$

input
```
integrate(1/x**4/(a**2*c*x**2+c)**(3/2)/atan(a*x)**3,x)
```

output
```
Integral(1/(x**4*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**3), x)
```

## Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4 (c+a^2cx^2)^{3/2} \arctan(ax)^3} \, dx = \int \frac{1}{(a^2cx^2+c)^{\frac{3}{2}} x^4 \arctan(ax)^3} \, dx$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(3/2)*x^4*arctan(a*x)^3), x)
```

## Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4 (c+a^2cx^2)^{3/2} \arctan(ax)^3} \, dx = \int \frac{1}{(a^2cx^2+c)^{\frac{3}{2}} x^4 \arctan(ax)^3} \, dx$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(3/2)*x^4*arctan(a*x)^3), x)
```

## Mupad [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4 (c + a^2cx^2)^{3/2} \arctan(ax)^3} \, dx = \int \frac{1}{x^4 \operatorname{atan}(a\,x)^3 \left(c\,a^2\,x^2 + c\right)^{3/2}} \, dx$$

input
```
int(1/(x^4*atan(a*x)^3*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(1/(x^4*atan(a*x)^3*(c + a^2*c*x^2)^(3/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.33

$$\int \frac{1}{x^4 (c + a^2cx^2)^{3/2} \arctan(ax)^3} \, dx = \frac{\int \frac{1}{\sqrt{a^2x^2+1}\, atan(ax)^3 a^2x^6 + \sqrt{a^2x^2+1}\, atan(ax)^3 x^4} dx}{\sqrt{c}\, c}$$

input
```
int(1/x^4/(a^2*c*x^2+c)^(3/2)/atan(a*x)^3,x)
```

output
```
int(1/(sqrt(a**2*x**2 + 1)*atan(a*x)**3*a**2*x**6 + sqrt(a**2*x**2 + 1)*at
an(a*x)**3*x**4),x)/(sqrt(c)*c)
```

# 3.665 $\int \frac{x^5}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\begin{aligned}\int \frac{x^5}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx &= \frac{x^3}{2a^3c\,(c+a^2cx^2)^{3/2}\arctan(ax)^2} \\ &+ \frac{x}{2a^5c^2\sqrt{c+a^2cx^2}\arctan(ax)^2} - \frac{3}{2a^6c\,(c+a^2cx^2)^{3/2}\arctan(ax)} \\ &+ \frac{2}{a^6c^2\sqrt{c+a^2cx^2}\arctan(ax)} + \frac{7\sqrt{1+a^2x^2}\,\mathrm{Si}(\arctan(ax))}{8a^6c^2\sqrt{c+a^2cx^2}} \\ &- \frac{9\sqrt{1+a^2x^2}\,\mathrm{Si}(3\arctan(ax))}{8a^6c^2\sqrt{c+a^2cx^2}} + \frac{\mathrm{Int}\left(\frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^3},x\right)}{a^4c^2}\end{aligned}$$

output
```
1/2*x^3/a^3/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2+1/2*x/a^5/c^2/(a^2*c*x^2+c
)^(1/2)/arctan(a*x)^2-3/2/a^6/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)+2/a^6/c^2/
(a^2*c*x^2+c)^(1/2)/arctan(a*x)+7/8*(a^2*x^2+1)^(1/2)*Si(arctan(a*x))/a^6/
c^2/(a^2*c*x^2+c)^(1/2)-9/8*(a^2*x^2+1)^(1/2)*Si(3*arctan(a*x))/a^6/c^2/(a
^2*c*x^2+c)^(1/2)+Defer(Int)(x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x)/a^4/c^
2
```

## Mathematica [N/A]

Not integrable

Time = 7.51 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^5}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = \int \frac{x^5}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx$$

input
```
Integrate[x^5/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3),x]
```

output
```
Integrate[x^5/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3), x]
```

## Rubi [N/A]

Not integrable

Time = 3.99 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\arctan(ax)^3\,(a^2cx^2+c)^{5/2}}\,dx$$

$$\Big\downarrow \text{5499}$$

$$\frac{\int \frac{x^3}{(a^2cx^2+c)^{3/2}\arctan(ax)^3}dx}{a^2c} - \frac{\int \frac{x^3}{(a^2cx^2+c)^{5/2}\arctan(ax)^3}dx}{a^2}$$

$$\Big\downarrow \text{5477}$$

$$\frac{\int \frac{x^3}{(a^2cx^2+c)^{3/2}\arctan(ax)^3}dx}{a^2c} - \frac{\frac{3\int \frac{x^2}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^2}dx}{2a} - \frac{x^3}{2ac\arctan(ax)^2(a^2cx^2+c)^{3/2}}}{a^2}$$

$$\Big\downarrow \text{5499}$$

$$\frac{\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c}-\frac{\int \frac{x}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}dx}{a^2}}{a^2c}-\frac{\frac{3\left(\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}dx}{a^2c}-\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^2}dx}{a^2}\right)}{2a}-\frac{x^3}{2ac\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}}{a^2}$$

$\Big\downarrow$ 5437

$$\frac{\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c}-\frac{\int \frac{x}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}dx}{a^2}}{a^2c}-\frac{\frac{3\left(\frac{-a\int \frac{x}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}dx-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-3a\int \frac{x}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)}dx-\frac{1}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}}{a^2}\right)}{2a}-\frac{x^3}{2ac\arctan(ax)^2}}{a^2}$$

$\Big\downarrow$ 5477

$$\frac{\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c}-\frac{\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}dx}{2a}-\frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-\frac{\frac{3\left(\frac{-a\int \frac{x}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}dx-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-3a\int \frac{x}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)}dx-\frac{1}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}}{a^2}\right)}{2a}-\frac{x^3}{2ac\arctan(ax)^2}}{a^2}$$

$\Big\downarrow$ 5437

$$\frac{\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c}-\frac{\frac{-a\int \frac{x}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}dx-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a}-\frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-\frac{\frac{3\left(\frac{-a\int \frac{x}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}dx-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-3a\int \frac{x}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)}dx-\frac{1}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}}{a^2}\right)}{2a}-\frac{x^3}{2ac\arctan(ax)^2}}{a^2}$$

$\Big\downarrow$ 5506

$$\frac{\frac{\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c}-\frac{\frac{-\frac{a\sqrt{a^2x^2+1}\int \frac{x}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}dx}{c\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a}-\frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-\frac{3\left(\frac{-\frac{a\sqrt{a^2x^2+1}\int \frac{x}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}dx}{c\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3a\sqrt{a^2x^2+1}\int \frac{x}{\left(a^2x^2+1\right)^{5/2}\arctan(ax)}dx}{c^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}}{a^2}\right)}{2a}-}{a^2}$$

$\big\downarrow$ 5505

$$\frac{\frac{\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c}-\frac{\frac{-\frac{\sqrt{a^2x^2+1}\int \frac{ax}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a}-\frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-\frac{3\left(\frac{-\frac{\sqrt{a^2x^2+1}\int \frac{ax}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3\sqrt{a^2x^2+1}\int \frac{ax}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\left(a^2cx^2+\right.}}{a^2}\right.}{2a}}{a^2}$$

$\big\downarrow$ 3042

$$\frac{\frac{\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c}-\frac{\frac{-\frac{\sqrt{a^2x^2+1}\int \frac{\sin(\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a}-\frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-\frac{3\left(\frac{-\frac{\sqrt{a^2x^2+1}\int \frac{\sin(\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3\sqrt{a^2x^2+1}\int \frac{ax}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/}}}{a^2}\right.}{2a}}{a^2}$$

$\big\downarrow$ 3780

$$\frac{\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c}-\frac{\frac{-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a}-\frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-\frac{3\left(\frac{-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3\sqrt{a^2x^2+1}\int \frac{ax}{(a^2x^2+1)^{3/2}\arctan(ax)}d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}}{a^2}\right)}{2a}-\frac{}{2ac\,\text{ar}}$$
$$a^2$$

$\downarrow$ 4906

$$\frac{\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c}-\frac{\frac{-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a}-\frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-\frac{3\left(\frac{-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3\sqrt{a^2x^2+1}\int \left(\frac{ax}{4\sqrt{a^2x^2+1}\arctan(ax)}+\frac{\sin(3\arctan(ax))}{4\arctan(ax)}\right)d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)(a^2cx^2+c}}{a^2}\right.}{2a}$$
$$a^2$$

$\downarrow$ 2009

$$\frac{\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c}-\frac{\frac{-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a}-\frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-\frac{3\left(\frac{-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3\sqrt{a^2x^2+1}\left(\frac{1}{4}\text{Si}(\arctan(ax))+\frac{1}{4}\text{Si}(3\arctan(ax))\right)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}}{a^2}\right)}{2a}-\frac{}{2ac\,\text{ar}}$$
$$a^2$$

$\downarrow$ 5560

$$\frac{\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c}-\frac{\frac{-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a}-\frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-\frac{3\left(\frac{-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3\sqrt{a^2x^2+1}\left(\frac{1}{4}\text{Si}(\arctan(ax))+\frac{1}{4}\text{Si}(3\arctan(ax))\right)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}}{a^2}\right)}{2a}-\frac{}{2ac\,\text{ar}}$$
$$a^2$$

input
```
Int[x^5/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 3.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^5}{\left(a^2 c\,x^2+c\right)^{\frac{5}{2}} \arctan\left(a x\right)^{3}}d x$$

input
```
int(x^5/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

output
```
int(x^5/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.58

$$\int \frac{x^5}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx=\int \frac{x^{5}}{{\left(a^{2} c x^{2} + c\right)}^{\frac{5}{2}} \arctan\left(a x\right)^{3}}\,{d x}$$

input
```
integrate(x^5/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^5/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3
*x^2 + c^3)*arctan(a*x)^3), x)
```

## Sympy [N/A]

Not integrable

Time = 9.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = \int \frac{x^5}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}\operatorname{atan}^3(ax)}\,dx$$

input
```
integrate(x**5/(a**2*c*x**2+c)**(5/2)/atan(a*x)**3,x)
```

output
```
Integral(x**5/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**3), x)
```

## Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = \int \frac{x^5}{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)^3}\,dx$$

input
```
integrate(x^5/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(x^5/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^3), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x^5}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^5/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.77 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = \int \frac{x^5}{\mathrm{atan}\left(a\,x\right)^3\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

input
```
int(x^5/(atan(a*x)^3*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(x^5/(atan(a*x)^3*(c + a^2*c*x^2)^(5/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 81, normalized size of antiderivative = 3.38

$$\int \frac{x^5}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = \frac{\int \frac{x^5}{\sqrt{a^2x^2+1}\,atan(ax)^3a^4x^4+2\sqrt{a^2x^2+1}\,atan(ax)^3a^2x^2+\sqrt{a^2x^2+1}\,atan(ax)^3}dx}{\sqrt{c}\,c^2}$$

input
```
int(x^5/(a^2*c*x^2+c)^(5/2)/atan(a*x)^3,x)
```

output
```
int(x**5/(sqrt(a**2*x**2 + 1)*atan(a*x)**3*a**4*x**4 + 2*sqrt(a**2*x**2 +
1)*atan(a*x)**3*a**2*x**2 + sqrt(a**2*x**2 + 1)*atan(a*x)**3),x)/(sqrt(c)*
c**2)
```

# 3.666 $\int \frac{x^4}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^4}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = -\frac{1}{2a^5c\,(c+a^2cx^2)^{3/2}\arctan(ax)^2} + \frac{1}{a^5c^2\sqrt{c+a^2cx^2}\arctan(ax)^2} + \frac{3x}{2a^4c\,(c+a^2cx^2)^{3/2}\arctan(ax)} - \frac{x}{a^4c^2\sqrt{c+a^2cx^2}\arctan(ax)} + \frac{5\sqrt{1+a^2x^2}\,\mathrm{CosIntegral}(\arctan(ax))}{8a^5c^2\sqrt{c+a^2cx^2}} - \frac{9\sqrt{1+a^2x^2}\,\mathrm{CosIntegral}(3\arctan(ax))}{8a^5c^2\sqrt{c+a^2cx^2}} + \frac{\mathrm{Int}\left(\frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^3},x\right)}{a^4c^2}$$

output
```
-1/2/a^5/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2+1/a^5/c^2/(a^2*c*x^2+c)^(1/2)
/arctan(a*x)^2+3/2*x/a^4/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)-x/a^4/c^2/(a^2*
c*x^2+c)^(1/2)/arctan(a*x)+5/8*(a^2*x^2+1)^(1/2)*Ci(arctan(a*x))/a^5/c^2/(
a^2*c*x^2+c)^(1/2)-9/8*(a^2*x^2+1)^(1/2)*Ci(3*arctan(a*x))/a^5/c^2/(a^2*c*
x^2+c)^(1/2)+Defer(Int)(1/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x)/a^4/c^2
```

## Mathematica [N/A]

Not integrable

Time = 6.84 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^4}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = \int \frac{x^4}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx$$

input
```
Integrate[x^4/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3),x]
```

output
```
Integrate[x^4/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3), x]
```

## Rubi [N/A]

Not integrable

Time = 5.31 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\arctan(ax)^3(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{5499}$$

$$\frac{\int \frac{x^2}{(a^2cx^2+c)^{3/2}\arctan(ax)^3}dx}{a^2c}-\frac{\int \frac{x^2}{(a^2cx^2+c)^{5/2}\arctan(ax)^3}dx}{a^2}$$

$$\Big\downarrow \text{5499}$$

$$\frac{\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c}-\frac{\int \frac{1}{(a^2cx^2+c)^{3/2}\arctan(ax)^3}dx}{a^2}}{a^2c}-\frac{\frac{\int \frac{1}{(a^2cx^2+c)^{3/2}\arctan(ax)^3}dx}{a^2c}-\frac{\int \frac{1}{(a^2cx^2+c)^{5/2}\arctan(ax)^3}dx}{a^2}}{a^2}$$

$\Big\downarrow$ 5437

$$\frac{\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c}-\frac{-\frac{1}{2}a\int \frac{x}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}dx-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-\frac{\frac{-\frac{1}{2}a\int \frac{x}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}dx-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3}{2}a\int \frac{x}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^2}dx-\frac{1}{2ac\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}}{a^2}}{a^2}$$

$\Big\downarrow$ 5477

$$\frac{\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c}-\frac{-\frac{1}{2}a\left(\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}dx}{a}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-\frac{\frac{-\frac{1}{2}a\left(\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}dx}{a}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3}{2}a\int \frac{x}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^2}dx-\frac{1}{2ac\arctan(ax)^2\cdots}}{a^2}}{a^2}$$

$\Big\downarrow$ 5440

$$\frac{\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c}-\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\int \frac{1}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}dx}{ac\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-\frac{\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\int \frac{1}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}dx}{ac\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3}{2}a\int \frac{x}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^2}dx-\frac{\cdots}{2ac\,\mathrm{ar}\cdots}}{a^2}}{a^2}$$

$\Big\downarrow$ 5439

$$\frac{\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c}-\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-\frac{\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3}{2}a\int \frac{x}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^2}dx-\cdots}{a^2}}{a^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c} - \frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)}{\arctan(ax)}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right) - \frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}}{a^2c} - $$

$$\frac{\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)}{\arctan(ax)}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right) - \frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2c} - \frac{-\frac{3}{2}a\int \frac{x}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^2}dx - \cdots}{a^2}}{a^2}$$

$\downarrow$ 3783

$$\frac{\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c} - \frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right) - \frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}}{a^2c} - $$

$$\frac{\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right) - \frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2c} - \frac{-\frac{3}{2}a\int \frac{x}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^2}dx - \frac{\cdots}{2ac\arctan(ax\cdots}}{a^2}}{a^2}$$

$\downarrow$ 5503

$$\frac{\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c} - \frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right) - \frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}}{a^2c} - $$

$$\frac{\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right) - \frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2c} - \frac{-\frac{3}{2}a\left(\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)}dx}{a} - 2a\int \frac{\cdots}{\left(a^2cx^2\cdots\right.}\cdots\right.}{a^2}}{a^2}$$

$\downarrow$ 5440

$$\frac{\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c} - \frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right) - \frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}}{a^2c} - $$

$$\frac{\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right) - \frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2c} - \frac{-\frac{3}{2}a\left(\frac{\sqrt{a^2x^2+1}\int \frac{1}{\left(a^2x^2+1\right)^{5/2}\arctan(ax)}dx}{ac^2\sqrt{a^2cx^2+c}} - 2a\cdots\right.}{a^2}}{a^2}$$

$\downarrow$ 5439

$$\frac{\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c}-\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\mathrm{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-$$

$$\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\mathrm{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3}{2}a\left(\frac{\sqrt{a^2x^2+1}\int\frac{1}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}d\arctan(\ldots}{a^2c^2\sqrt{a^2cx^2+c}}\right.}{a^2}$$

$\downarrow$ 3042

$$\frac{\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c}-\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\mathrm{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-$$

$$\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\mathrm{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3}{2}a\left(\frac{\sqrt{a^2x^2+1}\int\frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^3}{\arctan(ax)}d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}\right.}{a^2}$$

$\downarrow$ 3793

$$\frac{\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c}-\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\mathrm{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-$$

$$\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\mathrm{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3}{2}a\left(\frac{\sqrt{a^2x^2+1}\int\left(\frac{\cos(3\arctan(ax))}{4\arctan(ax)}+\frac{3}{4\sqrt{a^2x^2+1}\,\mathrm{ar}\ldots}\right.}{a^2c^2\sqrt{a^2cx^2+c}}\right.}{a^2}$$

$\downarrow$ 2009

$$\frac{\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c}-\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\mathrm{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-$$

$$\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\mathrm{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3}{2}a\left(-2a\int\frac{x^2}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)}dx+\frac{\sqrt{a^2x^2\ldots}}{\ldots}\right.}{a^2}$$

$\downarrow$ 5506

$$\frac{\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c}-\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-$$

$$\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3}{2}a\left(-\frac{2a\sqrt{a^2x^2+1}\int \frac{x^2}{\left(a^2x^2+1\right)^{5/2}\arctan(ax)}dx}{c^2\sqrt{a^2cx^2+c}}+\right.}{a^2}$$

$$\Big\downarrow \text{5505}$$

$$\frac{\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c}-\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-$$

$$\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3}{2}a\left(-\frac{2\sqrt{a^2x^2+1}\int \frac{a^2x^2}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}d\,\text{arct}}{a^2c^2\sqrt{a^2cx^2+c}}\right.}{a^2}$$

$$\Big\downarrow \text{4906}$$

$$\frac{\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c}-\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-$$

$$\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3}{2}a\left(-\frac{2\sqrt{a^2x^2+1}\int \left(\frac{1}{4\sqrt{a^2x^2+1}\arctan(ax)}-\frac{\cos(3}{4\,a}\right.}{a^2c^2\sqrt{a^2cx^2+c}}\right.}{}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c}-\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}}{a^2c}-$$

$$\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3}{2}a\left(-\frac{2\sqrt{a^2x^2+1}\left(\frac{1}{4}\,\text{CosIntegral}(\arctan(ax))-\frac{1}{4}\,\text{Co}\right.}{a^2c^2\sqrt{a^2cx^2+c}}\right.}{}$$

$$\Big\downarrow \text{5560}$$

$$\frac{\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{a^2c} - \frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\mathrm{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right) - \frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2}}{a^2c} -$$

$$\frac{\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\mathrm{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right) - \frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2c} - \frac{-\frac{3}{2}a\left(-\frac{2\sqrt{a^2x^2+1}\left(\frac{1}{4}\,\mathrm{CosIntegral}(\arctan(ax)) - \frac{1}{4}\,\mathrm{Co}\right.}{a^2c^2\sqrt{a^2cx^2+c}}\right.}{}}{}$$

input
```
Int[x^4/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 2.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^4}{(a^2c\,x^2+c)^{\frac{5}{2}}\arctan(ax)^3}dx$$

input
```
int(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

output
```
int(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.58

$$\int \frac{x^4}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = \int \frac{x^4}{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)^3}\,dx$$

input
```
integrate(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^4/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3
*x^2 + c^3)*arctan(a*x)^3), x)
```

## Sympy [N/A]

Not integrable

Time = 8.99 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = \int \frac{x^4}{(c\,(a^2x^2+1))^{\frac{5}{2}}\operatorname{atan}^3{(ax)}}\,dx$$

input
```
integrate(x**4/(a**2*c*x**2+c)**(5/2)/atan(a*x)**3,x)
```

output
```
Integral(x**4/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**3), x)
```

## Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = \int \frac{x^4}{(a^2cx^2+c)^{\frac{5}{2}}\arctan{(ax)}^3}\,dx$$

input
```
integrate(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(x^4/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^3), x)
```

## Giac [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = \int \frac{x^4}{(a^2cx^2+c)^{\frac{5}{2}}\arctan{(ax)}^3}\,dx$$

input
```
integrate(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate(x^4/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^3), x)
```

## Mupad [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = \int \frac{x^4}{\mathrm{atan}\left(a\,x\right)^3\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

input
```
int(x^4/(atan(a*x)^3*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(x^4/(atan(a*x)^3*(c + a^2*c*x^2)^(5/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 81, normalized size of antiderivative = 3.38

$$\int \frac{x^4}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = \frac{\int \frac{x^4}{\sqrt{a^2x^2+1}\,atan(ax)^3a^4x^4+2\sqrt{a^2x^2+1}\,atan(ax)^3a^2x^2+\sqrt{a^2x^2+1}\,atan(ax)^3}dx}{\sqrt{c}\,c^2}$$

input
```
int(x^4/(a^2*c*x^2+c)^(5/2)/atan(a*x)^3,x)
```

output
```
int(x**4/(sqrt(a**2*x**2 + 1)*atan(a*x)**3*a**4*x**4 + 2*sqrt(a**2*x**2 +
1)*atan(a*x)**3*a**2*x**2 + sqrt(a**2*x**2 + 1)*atan(a*x)**3),x)/(sqrt(c)*
c**2)
```

# 3.667 $\int \frac{x^3}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 180

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = -\frac{x^3}{2ac\,(c+a^2cx^2)^{3/2}\arctan(ax)^2} + \frac{3}{2a^4c\,(c+a^2cx^2)^{3/2}\arctan(ax)} - \frac{3}{2a^4c^2\sqrt{c+a^2cx^2}\arctan(ax)} - \frac{3\sqrt{1+a^2x^2}\,\mathrm{Si}(\arctan(ax))}{8a^4c^2\sqrt{c+a^2cx^2}} + \frac{9\sqrt{1+a^2x^2}\,\mathrm{Si}(3\arctan(ax))}{8a^4c^2\sqrt{c+a^2cx^2}}$$

output

```
-1/2*x^3/a/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2+3/2/a^4/c/(a^2*c*x^2+c)^(3/
2)/arctan(a*x)-3/2/a^4/c^2/(a^2*c*x^2+c)^(1/2)/arctan(a*x)-3/8*(a^2*x^2+1)
^(1/2)*Si(arctan(a*x))/a^4/c^2/(a^2*c*x^2+c)^(1/2)+9/8*(a^2*x^2+1)^(1/2)*S
i(3*arctan(a*x))/a^4/c^2/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.63

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = \frac{-4a^2x^2(ax+3\arctan(ax))-3(1+a^2x^2)^{3/2}\arctan(ax)^2\text{Si}(\arctan(ax)}{8a^4c^2(1+a^2x^2)\sqrt{c+a^2cx^2}\,\text{arct}}$$

input
```
Integrate[x^3/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3),x]
```

output
```
(-4*a^2*x^2*(a*x + 3*ArcTan[a*x]) - 3*(1 + a^2*x^2)^(3/2)*ArcTan[a*x]^2*Si
nIntegral[ArcTan[a*x]] + 9*(1 + a^2*x^2)^(3/2)*ArcTan[a*x]^2*SinIntegral[3
*ArcTan[a*x]])/(8*a^4*c^2*(1 + a^2*x^2)*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)
```

## Rubi [A] (verified)

Time = 1.48 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.16, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5477, 5499, 5437, 5506, 5505, 3042, 3780, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\arctan(ax)^3(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{5477}$$

$$\frac{3\int \frac{x^2}{(a^2cx^2+c)^{5/2}\arctan(ax)^2}dx}{2a}-\frac{x^3}{2ac\arctan(ax)^2(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5499}$$

$$\frac{3\left(\frac{\int \frac{1}{(a^2cx^2+c)^{3/2}\arctan(ax)^2}dx}{a^2c}-\frac{\int \frac{1}{(a^2cx^2+c)^{5/2}\arctan(ax)^2}dx}{a^2}\right)}{2a}-\frac{x^3}{2ac\arctan(ax)^2(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5437}$$

$$\frac{3\left(\frac{-a\int \frac{x}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}dx-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-3a\int \frac{x}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)}dx-\frac{1}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}}{a^2}\right)}{2a}-\frac{x^3}{2ac\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}$$

$\Big\downarrow$ 5506

$$\frac{3\left(\frac{-\frac{a\sqrt{a^2x^2+1}\int \frac{x}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}dx}{c\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3a\sqrt{a^2x^2+1}\int \frac{x}{\left(a^2x^2+1\right)^{5/2}\arctan(ax)}dx}{c^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}}{a^2}\right)}{2a}-\frac{x^3}{2ac\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}$$

$\Big\downarrow$ 5505

$$\frac{3\left(\frac{-\frac{\sqrt{a^2x^2+1}\int \frac{ax}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3\sqrt{a^2x^2+1}\int \frac{ax}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}}{a^2}\right)}{2a}-\frac{x^3}{2ac\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{3\left(\frac{-\frac{\sqrt{a^2x^2+1}\int \frac{\sin(\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3\sqrt{a^2x^2+1}\int \frac{ax}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}}{a^2}\right)}{2a}-\frac{x^3}{2ac\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}$$

$\Big\downarrow$ 3780

$$\frac{3\left(\frac{-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3\sqrt{a^2x^2+1}\int\frac{ax}{(a^2x^2+1)^{3/2}\arctan(ax)}d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}}{a^2}\right)}{2a}-\frac{x^3}{2ac\arctan(ax)^2(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{4906}$$

$$\frac{3\left(\frac{-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3\sqrt{a^2x^2+1}\int\left(\frac{ax}{4\sqrt{a^2x^2+1}\arctan(ax)}+\frac{\sin(3\arctan(ax))}{4\arctan(ax)}\right)d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)(a^2cx}}{a^2}\right.}{2a}$$
$$\frac{x^3}{2ac\arctan(ax)^2(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\left(\frac{-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3\sqrt{a^2x^2+1}\left(\frac{1}{4}\text{Si}(\arctan(ax))+\frac{1}{4}\text{Si}(3\arctan(ax))\right)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}}{a^2}\right)}{2a}-\frac{x^3}{2ac\arctan(ax)^2(a^2cx^2+c)^{3/2}}$$

input
```
Int[x^3/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3),x]
```

output
```
-1/2*x^3/(a*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2) + (3*((-(1/(a*c*Sqrt[c
+ a^2*c*x^2]*ArcTan[a*x])) - (Sqrt[1 + a^2*x^2]*SinIntegral[ArcTan[a*x]])/
(a*c*Sqrt[c + a^2*c*x^2]))/(a^2*c) - (-(1/(a*c*(c + a^2*c*x^2)^(3/2)*ArcTa
n[a*x])) - (3*Sqrt[1 + a^2*x^2]*(SinIntegral[ArcTan[a*x]]/4 + SinIntegral[
3*ArcTan[a*x]]/4))/(a*c^2*Sqrt[c + a^2*c*x^2]))/a^2))/(2*a)
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5477
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*ArcT
an[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp[f*(m/(b*c*(p + 1)))   Int[(f*x
)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1
]
```

rule 5499
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ
[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 6.78 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.99

| method | result |
|---|---|
| default | $\frac{i\left(3\arctan(ax)^2 \operatorname{expIntegral}_1(i\arctan(ax))a^4x^4-3\arctan(ax)^2 \operatorname{expIntegral}_1(-i\arctan(ax))a^4x^4-9\arctan(ax)^2 \operatorname{expIntegral}_1(3i a \ldots\right.}{\ldots}$ |

input
```
int(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/16*I*(3*arctan(a*x)^2*Ei(1,I*arctan(a*x))*a^4*x^4-3*arctan(a*x)^2*Ei(1,-
I*arctan(a*x))*a^4*x^4-9*arctan(a*x)^2*Ei(1,3*I*arctan(a*x))*a^4*x^4+9*arc
tan(a*x)^2*Ei(1,-3*I*arctan(a*x))*a^4*x^4+6*arctan(a*x)^2*Ei(1,I*arctan(a*
x))*a^2*x^2-6*arctan(a*x)^2*Ei(1,-I*arctan(a*x))*a^2*x^2-18*arctan(a*x)^2*
Ei(1,3*I*arctan(a*x))*a^2*x^2+18*arctan(a*x)^2*Ei(1,-3*I*arctan(a*x))*a^2*
x^2+8*I*(a^2*x^2+1)^(1/2)*a^3*x^3+24*I*arctan(a*x)*(a^2*x^2+1)^(1/2)*a^2*x
^2+3*Ei(1,I*arctan(a*x))*arctan(a*x)^2-3*Ei(1,-I*arctan(a*x))*arctan(a*x)^
2-9*Ei(1,3*I*arctan(a*x))*arctan(a*x)^2+9*Ei(1,-3*I*arctan(a*x))*arctan(a*
x)^2)*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/2)/arctan(a*x)^2/c^3/a^4/(a
^4*x^4+2*a^2*x^2+1)
```

## Fricas [F]

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = \int \frac{x^3}{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)^3}\,dx$$

input
```
integrate(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^3/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3
*x^2 + c^3)*arctan(a*x)^3), x)
```

## Sympy [F]

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = \int \frac{x^3}{(c(a^2x^2+1))^{\frac{5}{2}}\operatorname{atan}^3(ax)}\,dx$$

input
```
integrate(x**3/(a**2*c*x**2+c)**(5/2)/atan(a*x)**3,x)
```

output
```
Integral(x**3/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**3), x)
```

## Maxima [F]

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = \int \frac{x^3}{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)^3}\,dx$$

input
```
integrate(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(x^3/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^3), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3}\,dx=\int \frac{x^3}{\mathrm{atan}\left(a\,x\right)^3\,\left(c\,a^2\,x^2+c\right)^{5/2}}\,dx$$

input
```
int(x^3/(atan(a*x)^3*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(x^3/(atan(a*x)^3*(c + a^2*c*x^2)^(5/2)), x)
```

## Reduce [F]

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3}\,dx=\frac{\int \frac{x^3}{\sqrt{a^2x^2+1}\,atan(ax)^3a^4x^4+2\sqrt{a^2x^2+1}\,atan(ax)^3a^2x^2+\sqrt{a^2x^2+1}\,atan(ax)^3}dx}{\sqrt{c}\,c^2}$$

input
```
int(x^3/(a^2*c*x^2+c)^(5/2)/atan(a*x)^3,x)
```

output
```
int(x**3/(sqrt(a**2*x**2 + 1)*atan(a*x)**3*a**4*x**4 + 2*sqrt(a**2*x**2 +
1)*atan(a*x)**3*a**2*x**2 + sqrt(a**2*x**2 + 1)*atan(a*x)**3),x)/(sqrt(c)*
c**2)
```

# 3.668 $\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 209

$$\begin{aligned}\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx &= \frac{1}{2a^3c(c+a^2cx^2)^{3/2}\arctan(ax)^2} \\ &- \frac{1}{2a^3c^2\sqrt{c+a^2cx^2}\arctan(ax)^2} - \frac{3x}{2a^2c(c+a^2cx^2)^{3/2}\arctan(ax)} \\ &+ \frac{x}{2a^2c^2\sqrt{c+a^2cx^2}\arctan(ax)} - \frac{\sqrt{1+a^2x^2}\,\text{CosIntegral}(\arctan(ax))}{8a^3c^2\sqrt{c+a^2cx^2}} \\ &+ \frac{9\sqrt{1+a^2x^2}\,\text{CosIntegral}(3\arctan(ax))}{8a^3c^2\sqrt{c+a^2cx^2}}\end{aligned}$$

output

```
1/2/a^3/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2-1/2/a^3/c^2/(a^2*c*x^2+c)^(1/2
)/arctan(a*x)^2-3/2*x/a^2/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)+1/2*x/a^2/c^2/
(a^2*c*x^2+c)^(1/2)/arctan(a*x)-1/8*(a^2*x^2+1)^(1/2)*Ci(arctan(a*x))/a^3/
c^2/(a^2*c*x^2+c)^(1/2)+9/8*(a^2*x^2+1)^(1/2)*Ci(3*arctan(a*x))/a^3/c^2/(a
^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.57

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = \frac{4ax(-ax+(-2+a^2x^2)\arctan(ax)) - (1+a^2x^2)^{3/2}\arctan(ax)^2\,\mathrm{CosI}}{8a^3c^2(1+a^2x^2)\sqrt{}}$$

input
```
Integrate[x^2/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3),x]
```

output
```
(4*a*x*(-(a*x) + (-2 + a^2*x^2)*ArcTan[a*x]) - (1 + a^2*x^2)^(3/2)*ArcTan[
a*x]^2*CosIntegral[ArcTan[a*x]] + 9*(1 + a^2*x^2)^(3/2)*ArcTan[a*x]^2*CosI
ntegral[3*ArcTan[a*x]])/(8*a^3*c^2*(1 + a^2*x^2)*Sqrt[c + a^2*c*x^2]*ArcTa
n[a*x]^2)
```

## Rubi [A] (verified)

Time = 3.30 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.41, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.708$, Rules used = {5499, 5437, 5477, 5440, 5439, 3042, 3783, 5503, 5440, 5439, 3042, 3793, 2009, 5506, 5505, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\arctan(ax)^3(a^2cx^2+c)^{5/2}}\,dx$$

$$\Big\downarrow \ 5499$$

$$\frac{\int \frac{1}{(a^2cx^2+c)^{3/2}\arctan(ax)^3}dx}{a^2c} - \frac{\int \frac{1}{(a^2cx^2+c)^{5/2}\arctan(ax)^3}dx}{a^2}$$

$$\Big\downarrow \ 5437$$

$$\frac{-\frac{1}{2}a\int \frac{x}{(a^2cx^2+c)^{3/2}\arctan(ax)^2}dx - \frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2c} - \frac{-\frac{3}{2}a\int \frac{x}{(a^2cx^2+c)^{5/2}\arctan(ax)^2}dx - \frac{1}{2ac\arctan(ax)^2(a^2cx^2+c)^{3/2}}}{a^2}$$

$$\Big\downarrow \ 5477$$

$$\frac{-\frac{1}{2}a\left(\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}dx}{a}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3}{2}a\int \frac{x}{(a^2cx^2+c)^{5/2}\arctan(ax)^2}dx-\frac{1}{2ac\arctan(ax)^2(a^2cx^2+c)^{3/2}}}{a^2}$$

$$\Big\downarrow \ 5440$$

$$\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\int \frac{1}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}dx}{ac\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3}{2}a\int \frac{x}{(a^2cx^2+c)^{5/2}\arctan(ax)^2}dx-\frac{1}{2ac\arctan(ax)^2(a^2cx^2+c)^{3/2}}}{a^2}$$

$$\Big\downarrow \ 5439$$

$$\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3}{2}a\int \frac{x}{(a^2cx^2+c)^{5/2}\arctan(ax)^2}dx-\frac{1}{2ac\arctan(ax)^2(a^2cx^2+c)^{3/2}}}{a^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)}{\arctan(ax)}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3}{2}a\int \frac{x}{(a^2cx^2+c)^{5/2}\arctan(ax)^2}dx-\frac{1}{2ac\arctan(ax)^2(a^2cx^2+c)^{3/2}}}{a^2}$$

$$\Big\downarrow \ 3783$$

$$\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3}{2}a\int \frac{x}{(a^2cx^2+c)^{5/2}\arctan(ax)^2}dx-\frac{1}{2ac\arctan(ax)^2(a^2cx^2+c)^{3/2}}}{a^2}$$

$$\Big\downarrow \ 5503$$

$$\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2c}-$$

$$\frac{-\frac{3}{2}a\left(\frac{\int\frac{1}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx}{a}-2a\int\frac{x^2}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx-\frac{x}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}\right)-\frac{1}{2ac\arctan(ax)^2(a^2cx^2+c)^{3/2}}}{a^2}$$

$\downarrow$ 5440

$$\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2c}-$$

$$\frac{-\frac{3}{2}a\left(\frac{\sqrt{a^2x^2+1}\int\frac{1}{(a^2x^2+1)^{5/2}\arctan(ax)}dx}{ac^2\sqrt{a^2cx^2+c}}-2a\int\frac{x^2}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx-\frac{x}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}\right)-\frac{1}{2ac\arctan(ax)^2(a^2\cdots}}{a^2}$$

$\downarrow$ 5439

$$\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2c}-$$

$$\frac{-\frac{3}{2}a\left(\frac{\sqrt{a^2x^2+1}\int\frac{1}{(a^2x^2+1)^{3/2}\arctan(ax)}d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}-2a\int\frac{x^2}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx-\frac{x}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}\right)-\frac{\cdots}{2ac\arctan\cdots}}{a^2}$$

$\downarrow$ 3042

$$\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2c}-$$

$$\frac{-\frac{3}{2}a\left(\frac{\sqrt{a^2x^2+1}\int\frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^3}{\arctan(ax)}d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}-2a\int\frac{x^2}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx-\frac{x}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}\right)-\frac{\cdots}{2ac\arctan(a\cdots}}{a^2}$$

$\downarrow$ 3793

$$\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2c}-$$

$$\frac{-\frac{3}{2}a\left(\frac{\sqrt{a^2x^2+1}\int\left(\frac{\cos(3\arctan(ax))}{4\arctan(ax)}+\frac{3}{4\sqrt{a^2x^2+1}\arctan(ax)}\right)d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}-2a\int\frac{x^2}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx-\frac{x}{ac\arctan(ax)(a^2cx^2+c)^{3/\cdots}}\cdots\right.}{a^2}$$

$\downarrow$ 2009

$$\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2c}-$$
$$\frac{-\frac{3}{2}a\left(-2a\int\frac{x^2}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx+\frac{\sqrt{a^2x^2+1}\left(\frac{3}{4}\,\text{CosIntegral}(\arctan(ax))+\frac{1}{4}\,\text{CosIntegral}(3\arctan(ax))\right)}{a^2c^2\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)(a^2cx^2+\ldots}\right.}{a^2}$$

$\big\downarrow$ 5506

$$\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2c}-$$
$$\frac{-\frac{3}{2}a\left(-\frac{2a\sqrt{a^2x^2+1}\int\frac{x^2}{(a^2x^2+1)^{5/2}\arctan(ax)}dx}{c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2x^2+1}\left(\frac{3}{4}\,\text{CosIntegral}(\arctan(ax))+\frac{1}{4}\,\text{CosIntegral}(3\arctan(ax))\right)}{a^2c^2\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)(\ldots}\right.}{a^2}$$

$\big\downarrow$ 5505

$$\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2c}-$$
$$\frac{-\frac{3}{2}a\left(-\frac{2\sqrt{a^2x^2+1}\int\frac{a^2x^2}{(a^2x^2+1)^{3/2}\arctan(ax)}d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2x^2+1}\left(\frac{3}{4}\,\text{CosIntegral}(\arctan(ax))+\frac{1}{4}\,\text{CosIntegral}(3\arctan(ax))\right)}{a^2c^2\sqrt{a^2cx^2+c}}-\frac{\ldots}{ac\,\text{arct}\ldots}\right.}{a^2}$$

$\big\downarrow$ 4906

$$\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2c}-$$
$$\frac{-\frac{3}{2}a\left(-\frac{2\sqrt{a^2x^2+1}\int\left(\frac{1}{4\sqrt{a^2x^2+1}\arctan(ax)}-\frac{\cos(3\arctan(ax))}{4\arctan(ax)}\right)d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2x^2+1}\left(\frac{3}{4}\,\text{CosIntegral}(\arctan(ax))+\frac{1}{4}\,\text{CosIntegral}(3\,\text{arcta}\ldots\right.}{a^2c^2\sqrt{a^2cx^2+c}}\right.}{a^2}$$

$\big\downarrow$ 2009

$$\frac{-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}}{a^2c}-$$
$$\frac{-\frac{3}{2}a\left(-\frac{2\sqrt{a^2x^2+1}\left(\frac{1}{4}\,\text{CosIntegral}(\arctan(ax))-\frac{1}{4}\,\text{CosIntegral}(3\arctan(ax))\right)}{a^2c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2x^2+1}\left(\frac{3}{4}\,\text{CosIntegral}(\arctan(ax))+\frac{1}{4}\,\text{CosIntegral}(3\,\text{arct}\ldots\right.}{a^2c^2\sqrt{a^2cx^2+c}}\right.}{a^2}$$

input
```
Int[x^2/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3),x]
```

output
```
(-1/2*1/(a*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2) - (a*(-(x/(a*c*Sqrt[c + a^
2*c*x^2]*ArcTan[a*x])) + (Sqrt[1 + a^2*x^2]*CosIntegral[ArcTan[a*x]])/(a^2
*c*Sqrt[c + a^2*c*x^2])))/2)/(a^2*c) - (-1/2*1/(a*c*(c + a^2*c*x^2)^(3/2)*
ArcTan[a*x]^2) - (3*a*(-(x/(a*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x])) - (2*S
qrt[1 + a^2*x^2]*(CosIntegral[ArcTan[a*x]]/4 - CosIntegral[3*ArcTan[a*x]]/
4))/(a^2*c^2*Sqrt[c + a^2*c*x^2]) + (Sqrt[1 + a^2*x^2]*((3*CosIntegral[Arc
Tan[a*x]])/4 + CosIntegral[3*ArcTan[a*x]]/4))/(a^2*c^2*Sqrt[c + a^2*c*x^2]
)))/2)/a^2
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3783
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5440
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])   Int[(1 +
c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

rule 5477
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*ArcT
an[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp[f*(m/(b*c*(p + 1)))   Int[(f*x
)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1
]
```

rule 5499
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ
[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.82 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.78

| method | result |
|---|---|
| default | $\frac{\left(\arctan(ax)^2 \operatorname{expIntegral}_1(i\arctan(ax))a^4x^4+\arctan(ax)^2 \operatorname{expIntegral}_1(-i\arctan(ax))a^4x^4-9\arctan(ax)^2 \operatorname{expIntegral}_1(3i \arcta\right.}{}$ |

input
```
int(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/16*(arctan(a*x)^2*Ei(1,I*arctan(a*x))*a^4*x^4+arctan(a*x)^2*Ei(1,-I*arct
an(a*x))*a^4*x^4-9*arctan(a*x)^2*Ei(1,3*I*arctan(a*x))*a^4*x^4-9*arctan(a*
x)^2*Ei(1,-3*I*arctan(a*x))*a^4*x^4+8*arctan(a*x)*(a^2*x^2+1)^(1/2)*a^3*x^
3+2*arctan(a*x)^2*Ei(1,I*arctan(a*x))*a^2*x^2+2*arctan(a*x)^2*Ei(1,-I*arct
an(a*x))*a^2*x^2-18*arctan(a*x)^2*Ei(1,3*I*arctan(a*x))*a^2*x^2-18*arctan(
a*x)^2*Ei(1,-3*I*arctan(a*x))*a^2*x^2-8*a^2*x^2*(a^2*x^2+1)^(1/2)-16*arcta
n(a*x)*(a^2*x^2+1)^(1/2)*a*x+Ei(1,I*arctan(a*x))*arctan(a*x)^2+Ei(1,-I*arc
tan(a*x))*arctan(a*x)^2-9*Ei(1,3*I*arctan(a*x))*arctan(a*x)^2-9*Ei(1,-3*I*
arctan(a*x))*arctan(a*x)^2)*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/2)/ar
ctan(a*x)^2/c^3/a^3/(a^4*x^4+2*a^2*x^2+1)
```

## Fricas [F]

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = \int \frac{x^2}{(a^2cx^2+c)^{\frac{5}{2}}\arctan{(ax)}^3}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^2/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3
*x^2 + c^3)*arctan(a*x)^3), x)
```

## Sympy [F]

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = \int \frac{x^2}{(c(a^2x^2+1))^{\frac{5}{2}}\operatorname{atan}^3{(ax)}}\,dx$$

input
```
integrate(x**2/(a**2*c*x**2+c)**(5/2)/atan(a*x)**3,x)
```

output
```
Integral(x**2/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**3), x)
```

## Maxima [F]

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx=\int \frac{x^2}{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)^3}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(x^2/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^3), x)
```

## Giac [F]

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx=\int \frac{x^2}{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)^3}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate(x^2/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx=\int \frac{x^2}{\mathrm{atan}(a\,x)^3\,(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int(x^2/(atan(a*x)^3*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(x^2/(atan(a*x)^3*(c + a^2*c*x^2)^(5/2)), x)
```

## Reduce [F]

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = \frac{\int \frac{x^2}{\sqrt{a^2x^2+1}\,atan(ax)^3a^4x^4+2\sqrt{a^2x^2+1}\,atan(ax)^3a^2x^2+\sqrt{a^2x^2+1}\,atan(ax)^3}dx}{\sqrt{c}\,c^2}$$

input
```
int(x^2/(a^2*c*x^2+c)^(5/2)/atan(a*x)^3,x)
```

output
```
int(x**2/(sqrt(a**2*x**2 + 1)*atan(a*x)**3*a**4*x**4 + 2*sqrt(a**2*x**2 +
1)*atan(a*x)**3*a**2*x**2 + sqrt(a**2*x**2 + 1)*atan(a*x)**3),x)/(sqrt(c)*
c**2)
```

# 3.669 $\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 175

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = -\frac{x}{2ac\,(c+a^2cx^2)^{3/2}\arctan(ax)^2} - \frac{3}{2a^2c\,(c+a^2cx^2)^{3/2}\arctan(ax)} + \frac{1}{a^2c^2\sqrt{c+a^2cx^2}\arctan(ax)} - \frac{\sqrt{1+a^2x^2}\,\mathrm{Si}(\arctan(ax))}{8a^2c^2\sqrt{c+a^2cx^2}} - \frac{9\sqrt{1+a^2x^2}\,\mathrm{Si}(3\arctan(ax))}{8a^2c^2\sqrt{c+a^2cx^2}}$$

output
```
-1/2*x/a/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2-3/2/a^2/c/(a^2*c*x^2+c)^(3/2)
/arctan(a*x)+1/a^2/c^2/(a^2*c*x^2+c)^(1/2)/arctan(a*x)-1/8*(a^2*x^2+1)^(1/
2)*Si(arctan(a*x))/a^2/c^2/(a^2*c*x^2+c)^(1/2)-9/8*(a^2*x^2+1)^(1/2)*Si(3*
arctan(a*x))/a^2/c^2/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.67

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = \frac{-4ax-4\arctan(ax)+8a^2x^2\arctan(ax)-(1+a^2x^2)^{3/2}\arctan(ax)^2\mathrm{S}}{8a^2c^2\,(1+a^2x^2)\,\sqrt{c+a^2c}}$$

input
```
Integrate[x/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3),x]
```

output
```
(-4*a*x - 4*ArcTan[a*x] + 8*a^2*x^2*ArcTan[a*x] - (1 + a^2*x^2)^(3/2)*ArcT
an[a*x]^2*SinIntegral[ArcTan[a*x]] - 9*(1 + a^2*x^2)^(3/2)*ArcTan[a*x]^2*S
inIntegral[3*ArcTan[a*x]])/(8*a^2*c^2*(1 + a^2*x^2)*Sqrt[c + a^2*c*x^2]*Ar
cTan[a*x]^2)
```

## Rubi [A] (verified)

Time = 2.26 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.68, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {5503, 5437, 5499, 5437, 5506, 5505, 3042, 3780, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\arctan(ax)^3\,(a^2cx^2+c)^{5/2}}\,dx$$

$$\Big\downarrow \; 5503$$

$$\frac{\int \frac{1}{(a^2cx^2+c)^{5/2}\arctan(ax)^2}dx}{2a}-a\int \frac{x^2}{(a^2cx^2+c)^{5/2}\arctan(ax)^2}dx-\frac{x}{2ac\arctan(ax)^2\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \; 5437$$

$$-a\int \frac{x^2}{(a^2cx^2+c)^{5/2}\arctan(ax)^2}dx+\frac{-3a\int \frac{x}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx-\frac{1}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}}{2a}-\frac{x}{2ac\arctan(ax)^2\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \; 5499$$

$$-a\left(\frac{\int \frac{1}{(a^2cx^2+c)^{3/2}\arctan(ax)^2}dx}{a^2c}-\frac{\int \frac{1}{(a^2cx^2+c)^{5/2}\arctan(ax)^2}dx}{a^2}\right)+\frac{-3a\int \frac{x}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx-\frac{1}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}}{2a}-\frac{x}{2ac\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}$$

$\Big\downarrow$ 5437

$$\frac{-3a\int \frac{x}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx-\frac{1}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}}{2a}-a\left(\frac{-a\int \frac{x}{(a^2cx^2+c)^{3/2}\arctan(ax)}dx-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-3a\int \frac{x}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx-\frac{1}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}}{a^2}\right)-\frac{x}{2ac\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}$$

$\Big\downarrow$ 5506

$$\frac{-\frac{3a\sqrt{a^2x^2+1}\int \frac{x}{\left(a^2x^2+1\right)^{5/2}\arctan(ax)}dx}{c^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}}{2a}-a\left(\frac{-\frac{a\sqrt{a^2x^2+1}\int \frac{x}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}dx}{c\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3a\sqrt{a^2x^2+1}\int \frac{x}{\left(a^2x^2+1\right)^{5/2}\arctan(ax)}dx}{c^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}}{a^2}\right)-\frac{x}{2ac\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}$$

$\Big\downarrow$ 5505

$$\frac{-\frac{3\sqrt{a^2x^2+1}\int \frac{ax}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}}{2a}-a\left(\frac{-\frac{\sqrt{a^2x^2+1}\int \frac{ax}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3\sqrt{a^2x^2+1}\int \frac{ax}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}}{a^2}\right)-\frac{x}{2ac\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{-\frac{3\sqrt{a^2x^2+1}\int \frac{ax}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}}{2a}-$$

$$a\left(\frac{-\frac{\sqrt{a^2x^2+1}\int \frac{\sin(\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3\sqrt{a^2x^2+1}\int \frac{ax}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{}{ac\arctan}}{a^2}\right.$$

$$\frac{x}{2ac\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}$$

$\downarrow$ 3780

$$-a\left(\frac{-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3\sqrt{a^2x^2+1}\int \frac{ax}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)(a^2cx^2}}{a^2}\right.$$

$$\frac{-\frac{3\sqrt{a^2x^2+1}\int \frac{ax}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}}{2a}-\frac{x}{2ac\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}$$

$\downarrow$ 4906

$$-a\left(\frac{-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3\sqrt{a^2x^2+1}\int \left(\frac{ax}{4\sqrt{a^2x^2+1}\arctan(ax)}+\frac{\sin(3\arctan(ax))}{4\arctan(ax)}\right)d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{}{ac}}{a^2}\right.$$

$$\frac{-\frac{3\sqrt{a^2x^2+1}\int \left(\frac{ax}{4\sqrt{a^2x^2+1}\arctan(ax)}+\frac{\sin(3\arctan(ax))}{4\arctan(ax)}\right)d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}}{2a}-\frac{x}{2ac\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}$$

$\downarrow$ 2009

$$-a\left(\frac{-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3\sqrt{a^2x^2+1}\left(\frac{1}{4}\text{Si}(\arctan(ax))+\frac{1}{4}\text{Si}(3\arctan(ax))\right)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)(a^2c}}{a^2}\right.$$

$$\frac{-\frac{3\sqrt{a^2x^2+1}\left(\frac{1}{4}\text{Si}(\arctan(ax))+\frac{1}{4}\text{Si}(3\arctan(ax))\right)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}}{2a}-\frac{x}{2ac\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}$$

input
```
Int[x/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3),x]
```

output
```
-1/2*x/(a*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2) - a*((-(1/(a*c*Sqrt[c + a
^2*c*x^2]*ArcTan[a*x])) - (Sqrt[1 + a^2*x^2]*SinIntegral[ArcTan[a*x]])/(a*
c*Sqrt[c + a^2*c*x^2]))/(a^2*c) - (-(1/(a*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a
*x])) - (3*Sqrt[1 + a^2*x^2]*(SinIntegral[ArcTan[a*x]]/4 + SinIntegral[3*A
rcTan[a*x]]/4))/(a*c^2*Sqrt[c + a^2*c*x^2]))/a^2) + (-(1/(a*c*(c + a^2*c*x
^2)^(3/2)*ArcTan[a*x])) - (3*Sqrt[1 + a^2*x^2]*(SinIntegral[ArcTan[a*x]]/4
 + SinIntegral[3*ArcTan[a*x]]/4))/(a*c^2*Sqrt[c + a^2*c*x^2]))/(2*a)
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3780
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5499
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2
)^(q_), x_Symbol] :> Simp[1/e   Int[x^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*Ar
cTan[c*x])^p, x], x] - Simp[d/e   Int[x^(m - 2)*(d + e*x^2)^q*(a + b*ArcTan
[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && IntegersQ
[p, 2*q] && LtQ[q, -1] && IGtQ[m, 1] && NeQ[p, -1]
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 6.75 (sec) , antiderivative size = 371, normalized size of antiderivative = 2.12

| method | result |
|---|---|
| default | $\frac{i\left(\arctan(ax)^2 \operatorname{expIntegral}_1(i\arctan(ax))a^4x^4 - \arctan(ax)^2 \operatorname{expIntegral}_1(-i\arctan(ax))a^4x^4 + 9\arctan(ax)^2 \operatorname{expIntegral}_1(3i \operatorname{arct}\right.}{}$ |

input
```
int(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/16*I*(arctan(a*x)^2*Ei(1,I*arctan(a*x))*a^4*x^4-arctan(a*x)^2*Ei(1,-I*ar
ctan(a*x))*a^4*x^4+9*arctan(a*x)^2*Ei(1,3*I*arctan(a*x))*a^4*x^4-9*arctan(
a*x)^2*Ei(1,-3*I*arctan(a*x))*a^4*x^4+2*arctan(a*x)^2*Ei(1,I*arctan(a*x))*
a^2*x^2-2*arctan(a*x)^2*Ei(1,-I*arctan(a*x))*a^2*x^2+18*arctan(a*x)^2*Ei(1
,3*I*arctan(a*x))*a^2*x^2-18*arctan(a*x)^2*Ei(1,-3*I*arctan(a*x))*a^2*x^2-
16*I*arctan(a*x)*(a^2*x^2+1)^(1/2)*a^2*x^2+Ei(1,I*arctan(a*x))*arctan(a*x)
^2-Ei(1,-I*arctan(a*x))*arctan(a*x)^2+9*Ei(1,3*I*arctan(a*x))*arctan(a*x)^
2-9*Ei(1,-3*I*arctan(a*x))*arctan(a*x)^2+8*I*(a^2*x^2+1)^(1/2)*a*x+8*I*arc
tan(a*x)*(a^2*x^2+1)^(1/2))*(c*(a*x-I)*(a*x+I))^(1/2)/(a^2*x^2+1)^(1/2)/ar
ctan(a*x)^2/a^2/c^3/(a^4*x^4+2*a^2*x^2+1)
```

## Fricas [F]

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = \int \frac{x}{(a^2cx^2+c)^{\frac{5}{2}}\arctan{(ax)}^3}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x
^2 + c^3)*arctan(a*x)^3), x)
```

## Sympy [F]

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = \int \frac{x}{(c\,(a^2x^2+1))^{\frac{5}{2}}\,\mathrm{atan}^3\,(ax)}\,dx$$

input
```
integrate(x/(a**2*c*x**2+c)**(5/2)/atan(a*x)**3,x)
```

output
```
Integral(x/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**3), x)
```

## Maxima [F]

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = \int \frac{x}{(a^2cx^2+c)^{\frac{5}{2}}\arctan\,(ax)^3}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(x/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^3), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx=\int \frac{x}{\mathrm{atan}\left(a\,x\right)^3\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

input
```
int(x/(atan(a*x)^3*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(x/(atan(a*x)^3*(c + a^2*c*x^2)^(5/2)), x)
```

## Reduce [F]

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx=\frac{\int \frac{x}{\sqrt{a^2x^2+1}\,atan(ax)^3a^4x^4+2\sqrt{a^2x^2+1}\,atan(ax)^3a^2x^2+\sqrt{a^2x^2+1}\,atan(ax)^3}dx}{\sqrt{c}\,c^2}$$

input
```
int(x/(a^2*c*x^2+c)^(5/2)/atan(a*x)^3,x)
```

output
```
int(x/(sqrt(a**2*x**2 + 1)*atan(a*x)**3*a**4*x**4 + 2*sqrt(a**2*x**2 + 1)*
atan(a*x)**3*a**2*x**2 + sqrt(a**2*x**2 + 1)*atan(a*x)**3),x)/(sqrt(c)*c**
2)
```

# 3.670 $\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 145

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = -\frac{1}{2ac\,(c+a^2cx^2)^{3/2}\arctan(ax)^2} + \frac{3x}{2c\,(c+a^2cx^2)^{3/2}\arctan(ax)} - \frac{3\sqrt{1+a^2x^2}\,\mathrm{CosIntegral}(\arctan(ax))}{8ac^2\sqrt{c+a^2cx^2}} - \frac{9\sqrt{1+a^2x^2}\,\mathrm{CosIntegral}(3\arctan(ax))}{8ac^2\sqrt{c+a^2cx^2}}$$

output
```
-1/2/a/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2+3/2*x/c/(a^2*c*x^2+c)^(3/2)/arc
tan(a*x)-3/8*(a^2*x^2+1)^(1/2)*Ci(arctan(a*x))/a/c^2/(a^2*c*x^2+c)^(1/2)-9
/8*(a^2*x^2+1)^(1/2)*Ci(3*arctan(a*x))/a/c^2/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.70

$$\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3}\,dx = \frac{-4+12ax\arctan(ax)-3\left(1+a^2x^2\right)^{3/2}\arctan(ax)^2\operatorname{CosIntegral}(\arctan}{8c^2\left(a+a^3x^2\right)\sqrt{c+a^2c}}$$

input
```
Integrate[1/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3),x]
```

output
```
(-4 + 12*a*x*ArcTan[a*x] - 3*(1 + a^2*x^2)^(3/2)*ArcTan[a*x]^2*CosIntegral
[ArcTan[a*x]] - 9*(1 + a^2*x^2)^(3/2)*ArcTan[a*x]^2*CosIntegral[3*ArcTan[a
*x]])/(8*c^2*(a + a^3*x^2)*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^2)
```

## Rubi [A] (verified)

Time = 1.83 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.22, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {5437, 5503, 5440, 5439, 3042, 3793, 2009, 5506, 5505, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}dx$$

$$\Big\downarrow \text{5437}$$

$$-\frac{3}{2}a\int \frac{x}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^2}dx-\frac{1}{2ac\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{5503}$$

$$-\frac{3}{2}a\left(\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)}dx}{a}-2a\int \frac{x^2}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)}dx-\frac{x}{ac\arctan(ax)\left(a^2cx^2+c\right)^{3/2}}\right)-\frac{1}{2ac\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{5440}$$

$$-\frac{3}{2}a\left(\frac{\sqrt{a^2x^2+1}\int \frac{1}{(a^2x^2+1)^{5/2}\arctan(ax)}dx}{ac^2\sqrt{a^2cx^2+c}}-2a\int \frac{x^2}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx-\frac{x}{ac\arctan(ax)\,(a^2cx^2+c)^{3/2}}\right)$$
$$\frac{1}{2ac\arctan(ax)^2\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5439}$$

$$-\frac{3}{2}a\left(\frac{\sqrt{a^2x^2+1}\int \frac{1}{(a^2x^2+1)^{3/2}\arctan(ax)}d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}-2a\int \frac{x^2}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx-\frac{x}{ac\arctan(ax)\,(a^2cx\right.$$
$$\frac{1}{2ac\arctan(ax)^2\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3}{2}a\left(\frac{\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^3}{\arctan(ax)}d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}-2a\int \frac{x^2}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx-\frac{x}{ac\arctan(ax)\,(a^2cx^2+\right.$$
$$\frac{1}{2ac\arctan(ax)^2\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{3}{2}a\left(\frac{\sqrt{a^2x^2+1}\int \left(\frac{\cos(3\arctan(ax))}{4\arctan(ax)}+\frac{3}{4\sqrt{a^2x^2+1}\arctan(ax)}\right)d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}-2a\int \frac{x^2}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx-\right.$$
$$\frac{1}{2ac\arctan(ax)^2\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3}{2}a\left(-2a\int \frac{x^2}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx+\frac{\sqrt{a^2x^2+1}\left(\frac{3}{4}\text{CosIntegral}(\arctan(ax))+\frac{1}{4}\text{CosIntegral}(3\arctan(a\right.}{a^2c^2\sqrt{a^2cx^2+c}}\right.$$
$$\frac{1}{2ac\arctan(ax)^2\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5506}$$

$$-\frac{3}{2}a\left(-\frac{2a\sqrt{a^2x^2+1}\int \frac{x^2}{(a^2x^2+1)^{5/2}\arctan(ax)}dx}{c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2x^2+1}\left(\frac{3}{4}\operatorname{CosIntegral}(\arctan(ax))+\frac{1}{4}\operatorname{CosIntegral}(3\arcta\right.}{a^2c^2\sqrt{a^2cx^2+c}}\right.$$
$$\frac{1}{2ac\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}$$

$\big\downarrow$ 5505

$$-\frac{3}{2}a\left(-\frac{2\sqrt{a^2x^2+1}\int \frac{a^2x^2}{(a^2x^2+1)^{3/2}\arctan(ax)}d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2x^2+1}\left(\frac{3}{4}\operatorname{CosIntegral}(\arctan(ax))+\frac{1}{4}\operatorname{CosIntegra}\right.}{a^2c^2\sqrt{a^2cx^2+c}}\right.$$
$$\frac{1}{2ac\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}$$

$\big\downarrow$ 4906

$$-\frac{3}{2}a\left(-\frac{2\sqrt{a^2x^2+1}\int \left(\frac{1}{4\sqrt{a^2x^2+1}\arctan(ax)}-\frac{\cos(3\arctan(ax))}{4\arctan(ax)}\right)d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2x^2+1}\left(\frac{3}{4}\operatorname{CosIntegral}(\arctan(a\right.}{a^2c^2\sqrt{a^2}}\right.$$
$$\frac{1}{2ac\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}$$

$\big\downarrow$ 2009

$$-\frac{3}{2}a\left(-\frac{2\sqrt{a^2x^2+1}\left(\frac{1}{4}\operatorname{CosIntegral}(\arctan(ax))-\frac{1}{4}\operatorname{CosIntegral}(3\arctan(ax))\right)}{a^2c^2\sqrt{a^2cx^2+c}}+\frac{\sqrt{a^2x^2+1}\left(\frac{3}{4}\operatorname{CosIntegral}(\operatorname{arc}\right.}{a^2}\right.$$
$$\frac{1}{2ac\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}$$

input
```
Int[1/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3),x]
```

output
```
-1/2*1/(a*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^2) - (3*a*(-(x/(a*c*(c + a^2
*c*x^2)^(3/2)*ArcTan[a*x])) - (2*Sqrt[1 + a^2*x^2]*(CosIntegral[ArcTan[a*x
]]/4 - CosIntegral[3*ArcTan[a*x]]/4))/(a^2*c^2*Sqrt[c + a^2*c*x^2]) + (Sqr
t[1 + a^2*x^2]*((3*CosIntegral[ArcTan[a*x]])/4 + CosIntegral[3*ArcTan[a*x]
]/4))/(a^2*c^2*Sqrt[c + a^2*c*x^2])))/2
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5440
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])   Int[(1 +
c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

## Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.79 (sec) , antiderivative size = 346, normalized size of antiderivative = 2.39

| method | result |
|---|---|
| default | $\frac{\left(3 \arctan(ax)^2 \operatorname{expIntegral}_1(i \arctan(ax)) a^4 x^4 + 3 \arctan(ax)^2 \operatorname{expIntegral}_1(-i \arctan(ax)) a^4 x^4 + 9 \arctan(ax)^2 \operatorname{expIntegral}_1(3i \operatorname{ar}\right.}{}$ |

input
```
int(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/16*(3*arctan(a*x)^2*Ei(1,I*arctan(a*x))*a^4*x^4+3*arctan(a*x)^2*Ei(1,-I*
arctan(a*x))*a^4*x^4+9*arctan(a*x)^2*Ei(1,3*I*arctan(a*x))*a^4*x^4+9*arcta
n(a*x)^2*Ei(1,-3*I*arctan(a*x))*a^4*x^4+6*arctan(a*x)^2*Ei(1,I*arctan(a*x)
)*a^2*x^2+6*arctan(a*x)^2*Ei(1,-I*arctan(a*x))*a^2*x^2+18*arctan(a*x)^2*Ei
(1,3*I*arctan(a*x))*a^2*x^2+18*arctan(a*x)^2*Ei(1,-3*I*arctan(a*x))*a^2*x^
2+24*arctan(a*x)*(a^2*x^2+1)^(1/2)*a*x+3*Ei(1,I*arctan(a*x))*arctan(a*x)^2
+3*Ei(1,-I*arctan(a*x))*arctan(a*x)^2+9*Ei(1,3*I*arctan(a*x))*arctan(a*x)^
2+9*Ei(1,-3*I*arctan(a*x))*arctan(a*x)^2-8*(a^2*x^2+1)^(1/2))*(c*(a*x-I)*(
a*x+I))^(1/2)/(a^2*x^2+1)^(1/2)/arctan(a*x)^2/a/c^3/(a^4*x^4+2*a^2*x^2+1)
```

## Fricas [F]

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = \int \frac{1}{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)^3}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2
 + c^3)*arctan(a*x)^3), x)
```

## Sympy [F]

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = \int \frac{1}{(c(a^2x^2+1))^{\frac{5}{2}}\operatorname{atan}^3(ax)}\,dx$$

input
```
integrate(1/(a**2*c*x**2+c)**(5/2)/atan(a*x)**3,x)
```

output
```
Integral(1/((c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**3), x)
```

## Maxima [F]

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = \int \frac{1}{(a^2cx^2+c)^{\frac{5}{2}}\arctan{(ax)}^3}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^3), x)
```

## Giac [F]

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = \int \frac{1}{(a^2cx^2+c)^{\frac{5}{2}}\arctan{(ax)}^3}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = \int \frac{1}{\mathrm{atan}\left(a\,x\right)^3\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

input
```
int(1/(atan(a*x)^3*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(1/(atan(a*x)^3*(c + a^2*c*x^2)^(5/2)), x)
```

## Reduce [F]

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = \frac{\int \frac{1}{\sqrt{a^2x^2+1}\,atan(ax)^3a^4x^4+2\sqrt{a^2x^2+1}\,atan(ax)^3a^2x^2+\sqrt{a^2x^2+1}\,atan(ax)^3}dx}{\sqrt{c}\,c^2}$$

input
```
int(1/(a^2*c*x^2+c)^(5/2)/atan(a*x)^3,x)
```

output
```
int(1/(sqrt(a**2*x**2 + 1)*atan(a*x)**3*a**4*x**4 + 2*sqrt(a**2*x**2 + 1)*
atan(a*x)**3*a**2*x**2 + sqrt(a**2*x**2 + 1)*atan(a*x)**3),x)/(sqrt(c)*c**
2)
```

# 3.671 $\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3}\,dx = \frac{ax}{2c\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2} + \frac{ax}{2c^2\sqrt{c+a^2cx^2}\arctan(ax)^2} - \frac{\sqrt{c+a^2cx^2}}{2ac^3x\arctan(ax)^2} + \frac{3}{2c\left(c+a^2cx^2\right)^{3/2}\arctan(ax)} - \frac{1}{2c^2\sqrt{c+a^2cx^2}\arctan(ax)} + \frac{5\sqrt{1+a^2x^2}\,\mathrm{Si}(\arctan(ax))}{8c^2\sqrt{c+a^2cx^2}} + \frac{9\sqrt{1+a^2x^2}\,\mathrm{Si}(3\arctan(ax))}{8c^2\sqrt{c+a^2cx^2}} - \frac{\mathrm{Int}\left(\frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^2},x\right)}{2ac^2}$$

output
```
1/2*a*x/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2+1/2*a*x/c^2/(a^2*c*x^2+c)^(1/2
)/arctan(a*x)^2-1/2*(a^2*c*x^2+c)^(1/2)/a/c^3/x/arctan(a*x)^2+3/2/c/(a^2*c
*x^2+c)^(3/2)/arctan(a*x)-1/2/c^2/(a^2*c*x^2+c)^(1/2)/arctan(a*x)+5/8*(a^2
*x^2+1)^(1/2)*Si(arctan(a*x))/c^2/(a^2*c*x^2+c)^(1/2)+9/8*(a^2*x^2+1)^(1/2
)*Si(3*arctan(a*x))/c^2/(a^2*c*x^2+c)^(1/2)-1/2*Defer(Int)(1/x^2/(a^2*c*x^
2+c)^(1/2)/arctan(a*x)^2,x)/a/c^2
```

## Mathematica [N/A]

Not integrable

Time = 3.36 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3}\,dx=\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3}\,dx$$

input
```
Integrate[1/(x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3),x]
```

output
```
Integrate[1/(x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3), x]
```

## Rubi [N/A]

Not integrable

Time = 5.44 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\arctan(ax)^3\left(a^2cx^2+c\right)^{5/2}}\,dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{1}{x\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}dx}{c}-a^2\int \frac{x}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^3}dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\frac{\int \frac{1}{x\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c}-a^2\int \frac{x}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^3}dx}{c}-a^2\int \frac{x}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^3}dx$$

$$\Big\downarrow \text{5477}$$

$$\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{2a}-\frac{\sqrt{a^2cx^2+c}}{2acx\arctan(ax)^2}}{c}-a^2\left(\frac{\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^2}dx}{2a}-\frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}\right)}{c}-$$
$$a^2\int \frac{x}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^3}dx$$

$\downarrow$ 5437

$$\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{2a}-\frac{\sqrt{a^2cx^2+c}}{2acx\arctan(ax)^2}}{c}-a^2\left(\frac{-a\int \frac{x}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}dx-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a}-\frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+}}\right.}{c}$$
$$a^2\int \frac{x}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^3}dx$$

$\downarrow$ 5503

$$\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{2a}-\frac{\sqrt{a^2cx^2+c}}{2acx\arctan(ax)^2}}{c}-a^2\left(\frac{-a\int \frac{x}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}dx-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a}-\frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+}}\right.}{c}$$
$$a^2\left(\frac{\int \frac{1}{(a^2cx^2+c)^{5/2}\arctan(ax)^2}dx}{2a}-a\int \frac{x^2}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^2}dx-\frac{x}{2ac\arctan(ax)^2\left(a^2cx^2+c\right)^{3/2}}\right)$$

$\downarrow$ 5437

$$\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{2a}-\frac{\sqrt{a^2cx^2+c}}{2acx\arctan(ax)^2}}{c}-a^2\left(\frac{-a\int \frac{x}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}dx-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a}-\frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+}}\right.}{c}$$
$$a^2\left(-a\int \frac{x^2}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^2}dx+\frac{-3a\int \frac{x}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx-\frac{1}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}}{2a}-\frac{}{2ac\arctan(}\right.$$

$\downarrow$ 5499

$$\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{2a}-\frac{\sqrt{a^2cx^2+c}}{2acx\arctan(ax)^2}}{c}-a^2\left(\frac{-a\int \frac{x}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}dx-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a}-\frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+}}\right.}{c}$$

$$a^2\left(-a\left(\frac{\int \frac{1}{(a^2cx^2+c)^{3/2}\arctan(ax)^2}dx}{a^2c}-\frac{\int \frac{1}{(a^2cx^2+c)^{5/2}\arctan(ax)^2}dx}{a^2}\right)+\frac{-3a\int \frac{x}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx-\frac{}{ac\arctan(ax)}}{2a}\right.$$

$\downarrow$ 5437

$$\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{2a}-\frac{\sqrt{a^2cx^2+c}}{2acx\arctan(ax)^2}}{c}-a^2\left(\frac{-a\int \frac{x}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)}dx-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a}-\frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+}}\right.}{c}$$

$$a^2\left(\frac{-3a\int \frac{x}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx-\frac{1}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}}{2a}-a\left(\frac{-a\int \frac{x}{(a^2cx^2+c)^{3/2}\arctan(ax)}dx-\frac{1}{ac\arctan(ax)\sqrt{a^2cx}}}{a^2c}\right.\right.$$

$\downarrow$ 5506

$$\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{2a}-\frac{\sqrt{a^2cx^2+c}}{2acx\arctan(ax)^2}}{c}-a^2\left(\frac{-\frac{a\sqrt{a^2x^2+1}\int \frac{x}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}dx}{c\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a}-\frac{x}{2ac\arctan(ax)^2\sqrt{}}\right.}{c}$$

$$a^2\left(\frac{-\frac{3a\sqrt{a^2x^2+1}\int \frac{x}{\left(a^2x^2+1\right)^{5/2}\arctan(ax)}dx}{c^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}}{2a}-a\left(\frac{-\frac{a\sqrt{a^2x^2+1}\int \frac{x}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}dx}{c\sqrt{a^2cx^2+c}}-\frac{}{ac\arctan}}{a^2c}\right.\right.$$

$\downarrow$ 5505

$$\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{2a}-\frac{\sqrt{a^2cx^2+c}}{2acx\arctan(ax)^2}}{c}-a^2\left(\frac{-\frac{\sqrt{a^2x^2+1}\int \frac{ax}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a}-\frac{}{2ac\arctan(a}\right.}{c}$$

$$a^2\left(\frac{-\frac{3\sqrt{a^2x^2+1}\int \frac{ax}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}}{2a}-a\left(\frac{-\frac{\sqrt{a^2x^2+1}\int \frac{ax}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax}{ac\sqrt{a^2cx^2+c}}}{a^2c}\right.\right.$$

$\downarrow$ 3042

$$\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{2a}-\frac{\sqrt{a^2cx^2+c}}{2acx\arctan(ax)^2}}{c}-a^2\left(\frac{-\frac{\sqrt{a^2x^2+1}\int\frac{\sin(\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a}-\frac{x}{2ac\arctan(ax)^2\sqrt{c}}\right.}{c}$$

$$a^2\left(\frac{-\frac{3\sqrt{a^2x^2+1}\int\frac{ax}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)(a^2cx^2+c)^{3/2}}}{2a}-a\left(\frac{-\frac{\sqrt{a^2x^2+1}\int\frac{\sin(\arctan(ax))}{\arctan(ax)}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-}{a^2c}\right.\right.$$

$\downarrow$ 3780

$$\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{2a}-\frac{\sqrt{a^2cx^2+c}}{2acx\arctan(ax)^2}}{c}-a^2\left(\frac{-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a}-\frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}\right)}{c}-$$

$$a^2\left(-a\left(\frac{-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3\sqrt{a^2x^2+1}\int\frac{ax}{\left(a^2x^2+1\right)^{3/2}\arctan(ax)}d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)(}}{a^2}\right.\right.$$

$\downarrow$ 4906

$$\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{2a}-\frac{\sqrt{a^2cx^2+c}}{2acx\arctan(ax)^2}}{c}-a^2\left(\frac{-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a}-\frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}\right)}{c}-$$

$$a^2\left(-a\left(\frac{-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3\sqrt{a^2x^2+1}\int\left(\frac{ax}{4\sqrt{a^2x^2+1}\arctan(ax)}+\frac{\sin(3\arctan(ax))}{4\arctan(ax)}\right)d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}}{a^2}\right.\right.$$

$\downarrow$ 2009

$$\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{2a}-\frac{\sqrt{a^2cx^2+c}}{2acx\arctan(ax)^2}}{c}-a^2\left(\frac{-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a}-\frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}\right)}{c}-$$

$$a^2\left(-a\left(\frac{-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3\sqrt{a^2x^2+1}\left(\frac{1}{4}\text{Si}(\arctan(ax))+\frac{1}{4}\text{Si}(3\arctan(ax))\right)}{ac^2\sqrt{a^2cx^2+c}}-\frac{}{ac\arctan(ax}}{a^2}\right.\right.$$

$$\Big\downarrow \text{5560}$$

$$\frac{\frac{-\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^2}dx}{2a}}{c}-\frac{\sqrt{a^2cx^2+c}}{2acx\arctan(ax)^2}-a^2\left(\frac{-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{2a}-\frac{x}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}\right)}{c}-$$

$$a^2\left(-a\left(\frac{-\frac{\sqrt{a^2x^2+1}\text{Si}(\arctan(ax))}{ac\sqrt{a^2cx^2+c}}-\frac{1}{ac\arctan(ax)\sqrt{a^2cx^2+c}}}{a^2c}-\frac{-\frac{3\sqrt{a^2x^2+1}\left(\frac{1}{4}\text{Si}(\arctan(ax))+\frac{1}{4}\text{Si}(3\arctan(ax))\right)}{ac^2\sqrt{a^2cx^2+c}}-\frac{}{ac\arctan(ax}}{a^2}\right.\right.$$

input
```
Int[1/(x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 3.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\left(a^2c\,x^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^3}dx$$

input
```
int(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

output
```
int(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.54

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3}\,dx=\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{5}{2}}x\arctan\left(ax\right)^3}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)/((a^6*c^3*x^7 + 3*a^4*c^3*x^5 + 3*a^2*c^3*x^3
 + c^3*x)*arctan(a*x)^3), x)
```

## Sympy [N/A]

Not integrable

Time = 15.82 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3}\,dx=\int \frac{1}{x\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}\operatorname{atan}^3\left(ax\right)}\,dx$$

input
```
integrate(1/x/(a**2*c*x**2+c)**(5/2)/atan(a*x)**3,x)
```

output
```
Integral(1/(x*(c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**3), x)
```

## Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3}\,dx=\int \frac{1}{\left(a^2cx^2+c\right)^{\frac{5}{2}}x\arctan\left(ax\right)^3}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(5/2)*x*arctan(a*x)^3), x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3}\,dx = \int \frac{1}{x\,\mathrm{atan}\left(a\,x\right)^3\,\left(c\,a^2\,x^2+c\right)^{5/2}}\,dx$$

input
```
int(1/(x*atan(a*x)^3*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(1/(x*atan(a*x)^3*(c + a^2*c*x^2)^(5/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 78, normalized size of antiderivative = 3.25

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3}\,dx = \frac{\int \frac{1}{\sqrt{a^2x^2+1}\,atan(ax)^3a^4x^5+2\sqrt{a^2x^2+1}\,atan(ax)^3a^2x^3+\sqrt{a^2x^2+1}\,atan(ax)^3x}dx}{\sqrt{c}\,c^2}$$

input
```
int(1/x/(a^2*c*x^2+c)^(5/2)/atan(a*x)^3,x)
```

output

```
int(1/(sqrt(a**2*x**2 + 1)*atan(a*x)**3*a**4*x**5 + 2*sqrt(a**2*x**2 + 1)*
atan(a*x)**3*a**2*x**3 + sqrt(a**2*x**2 + 1)*atan(a*x)**3*x),x)/(sqrt(c)*c
**2)
```

# 3.672 $\int \frac{1}{x^2\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\begin{aligned}\int \frac{1}{x^2\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^3}\,dx &= \frac{a}{2c\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^2}\\ &+\frac{a}{2c^2\sqrt{c+a^2cx^2}\arctan(ax)^2}-\frac{3a^2x}{2c\left(c+a^2cx^2\right)^{3/2}\arctan(ax)}\\ &-\frac{a^2x}{2c^2\sqrt{c+a^2cx^2}\arctan(ax)}+\frac{7a\sqrt{1+a^2x^2}\operatorname{CosIntegral}(\arctan(ax))}{8c^2\sqrt{c+a^2cx^2}}\\ &+\frac{9a\sqrt{1+a^2x^2}\operatorname{CosIntegral}(3\arctan(ax))}{8c^2\sqrt{c+a^2cx^2}}+\frac{\operatorname{Int}\left(\frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^3},x\right)}{c^2}\end{aligned}$$

output
```
1/2*a/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^2+1/2*a/c^2/(a^2*c*x^2+c)^(1/2)/ar
ctan(a*x)^2-3/2*a^2*x/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)-1/2*a^2*x/c^2/(a^2
*c*x^2+c)^(1/2)/arctan(a*x)+7/8*a*(a^2*x^2+1)^(1/2)*Ci(arctan(a*x))/c^2/(a
^2*c*x^2+c)^(1/2)+9/8*a*(a^2*x^2+1)^(1/2)*Ci(3*arctan(a*x))/c^2/(a^2*c*x^2
+c)^(1/2)+Defer(Int)(1/x^2/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x)/c^2
```

## Mathematica [N/A]

Not integrable

Time = 3.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^2 (c+a^2cx^2)^{5/2} \arctan(ax)^3} dx = \int \frac{1}{x^2 (c+a^2cx^2)^{5/2} \arctan(ax)^3} dx$$

input
```
Integrate[1/(x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3),x]
```

output
```
Integrate[1/(x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3), x]
```

## Rubi [N/A]

Not integrable

Time = 5.59 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \arctan(ax)^3 (a^2cx^2+c)^{5/2}} dx$$

$$\Big\downarrow \text{5501}$$

$$\frac{\int \frac{1}{x^2(a^2cx^2+c)^{3/2} \arctan(ax)^3} dx}{c} - a^2 \int \frac{1}{(a^2cx^2+c)^{5/2} \arctan(ax)^3} dx$$

$$\Big\downarrow \text{5437}$$

$$\frac{\int \frac{1}{x^2(a^2cx^2+c)^{3/2} \arctan(ax)^3} dx}{c} - a^2 \left(-\frac{3}{2} a \int \frac{x}{(a^2cx^2+c)^{5/2} \arctan(ax)^2} dx - \frac{1}{2ac \arctan(ax)^2 (a^2cx^2+c)^{3/2}}\right)$$

$$\Big\downarrow \text{5501}$$

$$\frac{\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c}-a^2\int \frac{1}{(a^2cx^2+c)^{3/2}\arctan(ax)^3}dx}{c}-a^2\left(-\frac{3}{2}a\int \frac{x}{(a^2cx^2+c)^{5/2}\arctan(ax)^2}dx-\frac{1}{2ac\arctan(ax)^2(a^2cx^2+c)^{3/2}}\right)$$

$\downarrow$ 5437

$$\frac{\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c}-a^2\left(-\frac{1}{2}a\int \frac{x}{(a^2cx^2+c)^{3/2}\arctan(ax)^2}dx-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}\right)}{c}-a^2\left(-\frac{3}{2}a\int \frac{x}{(a^2cx^2+c)^{5/2}\arctan(ax)^2}dx-\frac{1}{2ac\arctan(ax)^2(a^2cx^2+c)^{3/2}}\right)$$

$\downarrow$ 5477

$$\frac{\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c}-a^2\left(-\frac{1}{2}a\left(\frac{\int \frac{1}{(a^2cx^2+c)^{3/2}\arctan(ax)}dx}{a}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}\right)}{c}-a^2\left(-\frac{3}{2}a\int \frac{x}{(a^2cx^2+c)^{5/2}\arctan(ax)^2}dx-\frac{1}{2ac\arctan(ax)^2(a^2cx^2+c)^{3/2}}\right)$$

$\downarrow$ 5440

$$\frac{\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c}-a^2\left(-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\int \frac{1}{(a^2x^2+1)^{3/2}\arctan(ax)}dx}{ac\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2}}\right.}{c}$$
$$a^2\left(-\frac{3}{2}a\int \frac{x}{(a^2cx^2+c)^{5/2}\arctan(ax)^2}dx-\frac{1}{2ac\arctan(ax)^2(a^2cx^2+c)^{3/2}}\right)$$

$\downarrow$ 5439

$$\frac{\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c}-a^2\left(-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}\arctan(ax)}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right)-\frac{1}{2ac\arctan(ax)^2\sqrt{}}\right.}{c}$$
$$a^2\left(-\frac{3}{2}a\int \frac{x}{(a^2cx^2+c)^{5/2}\arctan(ax)^2}dx-\frac{1}{2ac\arctan(ax)^2(a^2cx^2+c)^{3/2}}\right)$$

$\downarrow$ 3042

$$\frac{\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c} - a^2\left(-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)}{\arctan(ax)}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right) - \frac{1}{2ac\arctan(ax)^2\sqrt{}}\right.}{c}$$

$$a^2\left(-\frac{3}{2}a\int \frac{x}{(a^2cx^2+c)^{5/2}\arctan(ax)^2}dx - \frac{1}{2ac\arctan(ax)^2\,(a^2cx^2+c)^{3/2}}\right)$$

$$\Big\downarrow \; 3783$$

$$\frac{\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c} - a^2\left(-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right) - \frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}\right)}{c}$$

$$a^2\left(-\frac{3}{2}a\int \frac{x}{(a^2cx^2+c)^{5/2}\arctan(ax)^2}dx - \frac{1}{2ac\arctan(ax)^2\,(a^2cx^2+c)^{3/2}}\right)$$

$$\Big\downarrow \; 5503$$

$$\frac{\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c} - a^2\left(-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right) - \frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}\right)}{c}$$

$$a^2\left(-\frac{3}{2}a\left(\frac{\int \frac{1}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx}{a} - 2a\int \frac{x^2}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx - \frac{x}{ac\arctan(ax)\,(a^2cx^2+c)^{3/2}}\right) - \frac{}{2a}\right.$$

$$\Big\downarrow \; 5440$$

$$\frac{\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c} - a^2\left(-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right) - \frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}\right)}{c}$$

$$a^2\left(-\frac{3}{2}a\left(\frac{\sqrt{a^2x^2+1}\int \frac{1}{(a^2x^2+1)^{5/2}\arctan(ax)}dx}{ac^2\sqrt{a^2cx^2+c}} - 2a\int \frac{x^2}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx - \frac{x}{ac\arctan(ax)\,(a^2cx^2+c)}\right.\right.$$

$$\Big\downarrow \; 5439$$

$$\frac{\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c} - a^2\left(-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right) - \frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}\right)}{c}$$

$$a^2\left(-\frac{3}{2}a\left(\frac{\sqrt{a^2x^2+1}\int \frac{1}{(a^2x^2+1)^{3/2}\arctan(ax)}d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}} - 2a\int \frac{x^2}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx - \frac{x}{ac\arctan(ax)\,(}\right.\right.$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c} - a^2\left(-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right) - \frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}\right)}{c}$$

$$a^2\left(-\frac{3}{2}a\left(\frac{\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^3}{\arctan(ax)}d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}} - 2a\int \frac{x^2}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx - \frac{x}{ac\arctan(ax)\,(a^2c\right.\right.$$

$\big\downarrow$ 3793

$$\frac{\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c} - a^2\left(-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right) - \frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}\right)}{c}$$

$$a^2\left(-\frac{3}{2}a\left(\frac{\sqrt{a^2x^2+1}\int \left(\frac{\cos(3\arctan(ax))}{4\arctan(ax)} + \frac{3}{4\sqrt{a^2x^2+1}\arctan(ax)}\right)d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}} - 2a\int \frac{x^2}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx\right.\right.$$

$\big\downarrow$ 2009

$$\frac{\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c} - a^2\left(-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right) - \frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}\right)}{c}$$

$$a^2\left(-\frac{3}{2}a\left(-2a\int \frac{x^2}{(a^2cx^2+c)^{5/2}\arctan(ax)}dx + \frac{\sqrt{a^2x^2+1}\left(\frac{3}{4}\text{CosIntegral}(\arctan(ax)) + \frac{1}{4}\text{CosIntegral}(3\,\text{arcta}\right.}{a^2c^2\sqrt{a^2cx^2+c}}\right.\right.$$

$\big\downarrow$ 5506

$$\frac{\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c} - a^2\left(-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right) - \frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}\right)}{c}$$

$$a^2\left(-\frac{3}{2}a\left(-\frac{2a\sqrt{a^2x^2+1}\int \frac{x^2}{(a^2x^2+1)^{5/2}\arctan(ax)}dx}{c^2\sqrt{a^2cx^2+c}} + \frac{\sqrt{a^2x^2+1}\left(\frac{3}{4}\text{CosIntegral}(\arctan(ax)) + \frac{1}{4}\text{CosIntegral}(3\,\text{a}\right.}{a^2c^2\sqrt{a^2cx^2+c}}\right.\right.$$

$\big\downarrow$ 5505

$$\frac{\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c} - a^2\left(-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right) - \frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}\right)}{c}$$

$$a^2\left(-\frac{3}{2}a\left(-\frac{2\sqrt{a^2x^2+1}\int \frac{a^2x^2}{(a^2x^2+1)^{3/2}\arctan(ax)}d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}} + \frac{\sqrt{a^2x^2+1}\left(\frac{3}{4}\text{CosIntegral}(\arctan(ax)) + \frac{1}{4}\text{CosInt}\right.}{a^2c^2\sqrt{a^2cx^2+c}}\right.\right.$$

$\big\downarrow$ 4906

$$\frac{\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c} - a^2\left(-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right) - \frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}\right)}{c}$$

$$a^2\left(-\frac{3}{2}a\left(-\frac{2\sqrt{a^2x^2+1}\int\left(\frac{1}{4\sqrt{a^2x^2+1}\arctan(ax)} - \frac{\cos(3\arctan(ax))}{4\arctan(ax)}\right)d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}} + \frac{\sqrt{a^2x^2+1}\left(\frac{3}{4}\,\text{CosIntegral}(\text{arcta}\right.}{a^2c}\right.\right.$$

$\Big\downarrow$ 2009

$$\frac{\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c} - a^2\left(-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right) - \frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}\right)}{c}$$

$$a^2\left(-\frac{3}{2}a\left(-\frac{2\sqrt{a^2x^2+1}\left(\frac{1}{4}\,\text{CosIntegral}(\arctan(ax)) - \frac{1}{4}\,\text{CosIntegral}(3\arctan(ax))\right)}{a^2c^2\sqrt{a^2cx^2+c}} + \frac{\sqrt{a^2x^2+1}\left(\frac{3}{4}\,\text{CosIntegra}\right.}{}\right.\right.$$

$\Big\downarrow$ 5560

$$\frac{\frac{\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^3}dx}{c} - a^2\left(-\frac{1}{2}a\left(\frac{\sqrt{a^2x^2+1}\,\text{CosIntegral}(\arctan(ax))}{a^2c\sqrt{a^2cx^2+c}} - \frac{x}{ac\arctan(ax)\sqrt{a^2cx^2+c}}\right) - \frac{1}{2ac\arctan(ax)^2\sqrt{a^2cx^2+c}}\right)}{c}$$

$$a^2\left(-\frac{3}{2}a\left(-\frac{2\sqrt{a^2x^2+1}\left(\frac{1}{4}\,\text{CosIntegral}(\arctan(ax)) - \frac{1}{4}\,\text{CosIntegral}(3\arctan(ax))\right)}{a^2c^2\sqrt{a^2cx^2+c}} + \frac{\sqrt{a^2x^2+1}\left(\frac{3}{4}\,\text{CosIntegra}\right.}{}\right.\right.$$

input
```
Int[1/(x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 2.55 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^2\left(a^2c\,x^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^3}dx$$

input
```
int(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

output
```
int(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.62

$$\int \frac{1}{x^2 (c+a^2cx^2)^{5/2} \arctan(ax)^3}\,dx = \int \frac{1}{(a^2cx^2+c)^{\frac{5}{2}} x^2 \arctan(ax)^3}\,dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)/((a^6*c^3*x^8 + 3*a^4*c^3*x^6 + 3*a^2*c^3*x^4
 + c^3*x^2)*arctan(a*x)^3), x)
```

## Sympy [N/A]

Not integrable

Time = 20.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^2 (c+a^2cx^2)^{5/2} \arctan(ax)^3}\,dx = \int \frac{1}{x^2 (c(a^2x^2+1))^{\frac{5}{2}} \operatorname{atan}^3(ax)}\,dx$$

input
```
integrate(1/x**2/(a**2*c*x**2+c)**(5/2)/atan(a*x)**3,x)
```

output
```
Integral(1/(x**2*(c*(a**2*x**2 + 1))**(5/2)*atan(a*x)**3), x)
```

## Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (c+a^2cx^2)^{5/2} \arctan(ax)^3} \, dx = \int \frac{1}{(a^2cx^2+c)^{\frac{5}{2}} x^2 \arctan(ax)^3} \, dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(5/2)*x^2*arctan(a*x)^3), x)
```

## Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (c+a^2cx^2)^{5/2} \arctan(ax)^3} \, dx = \int \frac{1}{(a^2cx^2+c)^{\frac{5}{2}} x^2 \arctan(ax)^3} \, dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(5/2)*x^2*arctan(a*x)^3), x)
```

## Mupad [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (c+a^2cx^2)^{5/2} \arctan(ax)^3} \, dx = \int \frac{1}{x^2 \,\mathrm{atan}(a\,x)^3 \,(c\,a^2\,x^2+c)^{5/2}} \, dx$$

input
```
int(1/(x^2*atan(a*x)^3*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(1/(x^2*atan(a*x)^3*(c + a^2*c*x^2)^(5/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 80, normalized size of antiderivative = 3.33

$$\int \frac{1}{x^2 (c+a^2cx^2)^{5/2} \arctan(ax)^3}\,dx = \frac{\int \frac{1}{\sqrt{a^2x^2+1}\,atan(ax)^3 a^4x^6+2\sqrt{a^2x^2+1}\,atan(ax)^3 a^2x^4+\sqrt{a^2x^2+1}\,atan(ax)^3 x^2}dx}{\sqrt{c}\,c^2}$$

input
```
int(1/x^2/(a^2*c*x^2+c)^(5/2)/atan(a*x)^3,x)
```

output
```
int(1/(sqrt(a**2*x**2 + 1)*atan(a*x)**3*a**4*x**6 + 2*sqrt(a**2*x**2 + 1)*
atan(a*x)**3*a**2*x**4 + sqrt(a**2*x**2 + 1)*atan(a*x)**3*x**2),x)/(sqrt(c
)*c**2)
```

# 3.673 $\int \frac{x^m(c+a^2cx^2)^3}{\arctan(ax)^3}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^m(c+a^2cx^2)^3}{\arctan(ax)^3}\,dx = \text{Int}\left(\frac{x^m(c+a^2cx^2)^3}{\arctan(ax)^3},x\right)$$

output
```
Defer(Int)(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^3,x)
```

## Mathematica [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m(c+a^2cx^2)^3}{\arctan(ax)^3}\,dx = \int \frac{x^m(c+a^2cx^2)^3}{\arctan(ax)^3}\,dx$$

input
```
Integrate[(x^m*(c + a^2*c*x^2)^3)/ArcTan[a*x]^3,x]
```

output
```
Integrate[(x^m*(c + a^2*c*x^2)^3)/ArcTan[a*x]^3, x]
```

## Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \left(a^2 c x^2+c\right)^3}{\arctan(a x)^3} \, dx$$

$$\Big\downarrow \text{5560}$$

$$\int \frac{x^m \left(a^2 c x^2+c\right)^3}{\arctan(a x)^3}dx$$

input
```
Int[(x^m*(c + a^2*c*x^2)^3)/ArcTan[a*x]^3,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 8.67 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m (a^2 c\,x^2+c)^3}{\arctan\left(a x\right)^3}dx$$

input
```
int(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^3,x)
```

output
```
int(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.18

$$\int \frac{x^m(c+a^2cx^2)^3}{\arctan(ax)^3}\,dx = \int \frac{(a^2cx^2+c)^3x^m}{\arctan(ax)^3}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*x^m/arctan(a*
x)^3, x)
```

## Sympy [N/A]

Not integrable

Time = 12.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 3.32

$$\int \frac{x^m(c+a^2cx^2)^3}{\arctan(ax)^3}\,dx = c^3\left(\int \frac{x^m}{\operatorname{atan}^3(ax)}\,dx + \int \frac{3a^2x^2x^m}{\operatorname{atan}^3(ax)}\,dx + \int \frac{3a^4x^4x^m}{\operatorname{atan}^3(ax)}\,dx + \int \frac{a^6x^6x^m}{\operatorname{atan}^3(ax)}\,dx\right)$$

input
```
integrate(x**m*(a**2*c*x**2+c)**3/atan(a*x)**3,x)
```

output
```
c**3*(Integral(x**m/atan(a*x)**3, x) + Integral(3*a**2*x**2*x**m/atan(a*x)
**3, x) + Integral(3*a**4*x**4*x**m/atan(a*x)**3, x) + Integral(a**6*x**6*
x**m/atan(a*x)**3, x))
```

## Maxima [N/A]

Not integrable

Time = 1.34 (sec) , antiderivative size = 399, normalized size of antiderivative = 18.14

$$\int \frac{x^m(c+a^2cx^2)^3}{\arctan(ax)^3} dx = \int \frac{(a^2cx^2+c)^3 x^m}{\arctan(ax)^3} dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
1/2*(x*arctan(a*x)^2*integrate(((a^10*c^3*m^2 + 17*a^10*c^3*m + 72*a^10*c^
3)*x^10 + (5*a^8*c^3*m^2 + 67*a^8*c^3*m + 224*a^8*c^3)*x^8 + 2*(5*a^6*c^3*
m^2 + 49*a^6*c^3*m + 120*a^6*c^3)*x^6 + c^3*m^2 + 2*(5*a^4*c^3*m^2 + 31*a^
4*c^3*m + 48*a^4*c^3)*x^4 - c^3*m + (5*a^2*c^3*m^2 + 13*a^2*c^3*m + 8*a^2*
c^3)*x^2)*x^m/(x^2*arctan(a*x)), x) - ((a^10*c^3*m + 8*a^10*c^3)*x^10 + (5
*a^8*c^3*m + 32*a^8*c^3)*x^8 + 2*(5*a^6*c^3*m + 24*a^6*c^3)*x^6 + 2*(5*a^4
*c^3*m + 16*a^4*c^3)*x^4 + c^3*m + (5*a^2*c^3*m + 8*a^2*c^3)*x^2)*x^m*arct
an(a*x) - (a^9*c^3*x^9 + 4*a^7*c^3*x^7 + 6*a^5*c^3*x^5 + 4*a^3*c^3*x^3 + a
*c^3*x)*x^m)/(a^2*x*arctan(a*x)^2)
```

## Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m(c+a^2cx^2)^3}{\arctan(ax)^3} dx = \int \frac{(a^2cx^2+c)^3 x^m}{\arctan(ax)^3} dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^3*x^m/arctan(a*x)^3, x)
```

## Mupad [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m(c+a^2cx^2)^3}{\arctan(ax)^3}\,dx = \int \frac{x^m\,\left(c\,a^2\,x^2+c\right)^3}{\mathrm{atan}\left(a\,x\right)^3}\,dx$$

input
```
int((x^m*(c + a^2*c*x^2)^3)/atan(a*x)^3,x)
```

output
```
int((x^m*(c + a^2*c*x^2)^3)/atan(a*x)^3, x)
```

## Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 76, normalized size of antiderivative = 3.45

$$\int \frac{x^m(c+a^2cx^2)^3}{\arctan(ax)^3}\,dx = c^3\left(\int \frac{x^m}{atan\,(ax)^3}dx+\left(\int \frac{x^m x^6}{atan\,(ax)^3}dx\right)a^6 + 3\left(\int \frac{x^m x^4}{atan\,(ax)^3}dx\right)a^4+3\left(\int \frac{x^m x^2}{atan\,(ax)^3}dx\right)a^2\right)$$

input
```
int(x^m*(a^2*c*x^2+c)^3/atan(a*x)^3,x)
```

output
```
c**3*(int(x**m/atan(a*x)**3,x) + int((x**m*x**6)/atan(a*x)**3,x)*a**6 + 3*
int((x**m*x**4)/atan(a*x)**3,x)*a**4 + 3*int((x**m*x**2)/atan(a*x)**3,x)*a
**2)
```

# 3.674 $\int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)^3} dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)^3} dx = \text{Int}\left(\frac{x^m(c+a^2cx^2)^2}{\arctan(ax)^3}, x\right)$$

output
```
Defer(Int)(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

## Mathematica [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)^3} dx = \int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)^3} dx$$

input
```
Integrate[(x^m*(c + a^2*c*x^2)^2)/ArcTan[a*x]^3,x]
```

output
```
Integrate[(x^m*(c + a^2*c*x^2)^2)/ArcTan[a*x]^3, x]
```

## Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \left(a^2 c x^2+c\right)^2}{\arctan(ax)^3} \, dx$$

$$\Big\downarrow \ 5560$$

$$\int \frac{x^m \left(a^2 c x^2+c\right)^2}{\arctan(ax)^3}dx$$

input
```
Int[(x^m*(c + a^2*c*x^2)^2)/ArcTan[a*x]^3,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 4.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m {\left(a^2 c\,x^2+c\right)}^2}{\arctan \left(a x\right)^3}dx$$

input
```
int(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

output
```
int(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

$$\int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)^3}\,dx = \int \frac{(a^2cx^2+c)^2x^m}{\arctan(ax)^3}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*x^m/arctan(a*x)^3, x)
```

## Sympy [N/A]

Not integrable

Time = 5.99 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.41

$$\int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)^3}\,dx = c^2\left(\int \frac{x^m}{\operatorname{atan}^3(ax)}\,dx + \int \frac{2a^2x^2x^m}{\operatorname{atan}^3(ax)}\,dx + \int \frac{a^4x^4x^m}{\operatorname{atan}^3(ax)}\,dx\right)$$

input
```
integrate(x**m*(a**2*c*x**2+c)**2/atan(a*x)**3,x)
```

output
```
c**2*(Integral(x**m/atan(a*x)**3, x) + Integral(2*a**2*x**2*x**m/atan(a*x)
**3, x) + Integral(a**4*x**4*x**m/atan(a*x)**3, x))
```

## Maxima [N/A]

Not integrable

Time = 1.62 (sec) , antiderivative size = 333, normalized size of antiderivative = 15.14

$$\int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)^3}\,dx = \int \frac{(a^2cx^2+c)^2x^m}{\arctan(ax)^3}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
1/2*(x*arctan(a*x)^2*integrate(((a^8*c^2*m^2 + 13*a^8*c^2*m + 42*a^8*c^2)*
x^8 + 2*(2*a^6*c^2*m^2 + 19*a^6*c^2*m + 45*a^6*c^2)*x^6 + 6*(a^4*c^2*m^2 +
 6*a^4*c^2*m + 9*a^4*c^2)*x^4 + c^2*m^2 - c^2*m + 2*(2*a^2*c^2*m^2 + 5*a^2
*c^2*m + 3*a^2*c^2)*x^2)*x^m/(x^2*arctan(a*x)), x) - ((a^8*c^2*m + 6*a^8*c
^2)*x^8 + 2*(2*a^6*c^2*m + 9*a^6*c^2)*x^6 + 6*(a^4*c^2*m + 3*a^4*c^2)*x^4
+ c^2*m + 2*(2*a^2*c^2*m + 3*a^2*c^2)*x^2)*x^m*arctan(a*x) - (a^7*c^2*x^7
+ 3*a^5*c^2*x^5 + 3*a^3*c^2*x^3 + a*c^2*x)*x^m)/(a^2*x*arctan(a*x)^2)
```

## Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)^3}\,dx = \int \frac{(a^2cx^2+c)^2x^m}{\arctan{(ax)}^3}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^2*x^m/arctan(a*x)^3, x)
```

## Mupad [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)^3}\,dx = \int \frac{x^m\,{\left(c\,a^2\,x^2+c\right)}^2}{{\mathrm{atan}\left(a\,x\right)}^3}\,dx$$

input
```
int((x^m*(c + a^2*c*x^2)^2)/atan(a*x)^3,x)
```

output
```
int((x^m*(c + a^2*c*x^2)^2)/atan(a*x)^3, x)
```

## Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.55

$$\int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)^3}\,dx = c^2\left(\int \frac{x^m}{atan\left(ax\right)^3}dx + \left(\int \frac{x^m x^4}{atan\left(ax\right)^3}dx\right)a^4 + 2\left(\int \frac{x^m x^2}{atan\left(ax\right)^3}dx\right)a^2\right)$$

input
```
int(x^m*(a^2*c*x^2+c)^2/atan(a*x)^3,x)
```

output
```
c**2*(int(x**m/atan(a*x)**3,x) + int((x**m*x**4)/atan(a*x)**3,x)*a**4 + 2*
int((x**m*x**2)/atan(a*x)**3,x)*a**2)
```

# 3.675 $\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)^3}\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)^3}\,dx = \text{Int}\left(\frac{x^m(c+a^2cx^2)}{\arctan(ax)^3},x\right)$$

output
```
Defer(Int)(x^m*(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

## Mathematica [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)^3}\,dx = \int \frac{x^m(c+a^2cx^2)}{\arctan(ax)^3}\,dx$$

input
```
Integrate[(x^m*(c + a^2*c*x^2))/ArcTan[a*x]^3,x]
```

output
```
Integrate[(x^m*(c + a^2*c*x^2))/ArcTan[a*x]^3, x]
```

## Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m (a^2cx^2+c)}{\arctan(ax)^3} \, dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int \frac{x^m (a^2cx^2+c)}{\arctan(ax)^3} dx$$

input
```
Int[(x^m*(c + a^2*c*x^2))/ArcTan[a*x]^3,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 3.77 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{x^m(a^2c\,x^2+c)}{\arctan\left(ax\right)^3}dx$$

input
```
int(x^m*(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

output
```
int(x^m*(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)^3}\,dx = \int \frac{(a^2cx^2+c)x^m}{\arctan{(ax)}^3}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)*x^m/arctan(a*x)^3, x)
```

## Sympy [N/A]

Not integrable

Time = 4.45 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.55

$$\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)^3}\,dx = c\left(\int \frac{x^m}{\operatorname{atan}^3{(ax)}}\,dx + \int \frac{a^2x^2x^m}{\operatorname{atan}^3{(ax)}}\,dx\right)$$

input
```
integrate(x**m*(a**2*c*x**2+c)/atan(a*x)**3,x)
```

output
```
c*(Integral(x**m/atan(a*x)**3, x) + Integral(a**2*x**2*x**m/atan(a*x)**3,
x))
```

## Maxima [N/A]

Not integrable

Time = 1.00 (sec) , antiderivative size = 221, normalized size of antiderivative = 11.05

$$\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)^3}\,dx = \int \frac{(a^2cx^2+c)x^m}{\arctan{(ax)}^3}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
1/2*(x*arctan(a*x)^2*integrate(((a^6*c*m^2 + 9*a^6*c*m + 20*a^6*c)*x^6 + (
3*a^4*c*m^2 + 17*a^4*c*m + 24*a^4*c)*x^4 + c*m^2 + (3*a^2*c*m^2 + 7*a^2*c*
m + 4*a^2*c)*x^2 - c*m)*x^m/(x^2*arctan(a*x)), x) - ((a^6*c*m + 4*a^6*c)*x
^6 + (3*a^4*c*m + 8*a^4*c)*x^4 + (3*a^2*c*m + 4*a^2*c)*x^2 + c*m)*x^m*arct
an(a*x) - (a^5*c*x^5 + 2*a^3*c*x^3 + a*c*x)*x^m)/(a^2*x*arctan(a*x)^2)
```

## Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)^3}\,dx = \int \frac{(a^2cx^2+c)x^m}{\arctan{(ax)}^3}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)*x^m/arctan(a*x)^3, x)
```

## Mupad [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)^3}\,dx = \int \frac{x^m\,\left(c\,a^2\,x^2+c\right)}{\mathrm{atan}\left(a\,x\right)^3}\,dx$$

input
```
int((x^m*(c + a^2*c*x^2))/atan(a*x)^3,x)
```

output
```
int((x^m*(c + a^2*c*x^2))/atan(a*x)^3, x)
```

## Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.70

$$\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)^3}\,dx = c\left(\int \frac{x^m}{atan\,(ax)^3}dx + \left(\int \frac{x^m x^2}{atan\,(ax)^3}dx\right)a^2\right)$$

input
```
int(x^m*(a^2*c*x^2+c)/atan(a*x)^3,x)
```

output
```
c*(int(x**m/atan(a*x)**3,x) + int((x**m*x**2)/atan(a*x)**3,x)*a**2)
```

# 3.676 $\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^3}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^3}\,dx = -\frac{x^m}{2ac\arctan(ax)^2} + \frac{m\text{Int}\left(\frac{x^{-1+m}}{\arctan(ax)^2},x\right)}{2ac}$$

output
```
-1/2*x^m/a/c/arctan(a*x)^2+1/2*m*Defer(Int)(x^(-1+m)/arctan(a*x)^2,x)/a/c
```

## Mathematica [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^3}\,dx = \int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^3}\,dx$$

input
```
Integrate[x^m/((c + a^2*c*x^2)*ArcTan[a*x]^3),x]
```

output
```
Integrate[x^m/((c + a^2*c*x^2)*ArcTan[a*x]^3), x]
```

## Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\arctan(ax)^3 (a^2cx^2+c)} \, dx$$

$$\Big\downarrow \text{ 5461}$$

$$\frac{m\int \frac{x^{m-1}}{\arctan(ax)^2}dx}{2ac}-\frac{x^m}{2ac\arctan(ax)^2}$$

$$\Big\downarrow \text{ 5377}$$

$$\frac{m\int \frac{x^{m-1}}{\arctan(ax)^2}dx}{2ac}-\frac{x^m}{2ac\arctan(ax)^2}$$

input
```
Int[x^m/((c + a^2*c*x^2)*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 3.59 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(a^2c\,x^2+c)\arctan\left(ax\right)^3}dx$$

input
```
int(x^m/(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

output
```
int(x^m/(a^2*c*x^2+c)/arctan(a*x)^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^3}\,dx = \int \frac{x^m}{(a^2cx^2+c)\arctan{(ax)}^3}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(x^m/((a^2*c*x^2 + c)*arctan(a*x)^3), x)
```

## Sympy [N/A]

Not integrable

Time = 4.92 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^3}\,dx = \frac{\int \frac{x^m}{a^2x^2\operatorname{atan}^3{(ax)}+\operatorname{atan}^3{(ax)}}\,dx}{c}$$

input
```
integrate(x**m/(a**2*c*x**2+c)/atan(a*x)**3,x)
```

output
```
Integral(x**m/(a**2*x**2*atan(a*x)**3 + atan(a*x)**3), x)/c
```

## Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 91, normalized size of antiderivative = 4.14

$$\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^3}\,dx = \int \frac{x^m}{(a^2cx^2+c)\arctan{(ax)}^3}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
1/2*(x*arctan(a*x)^2*integrate(((a^2*m^2 + a^2*m)*x^2 + m^2 - m)*x^m/(x^2*
arctan(a*x)), x) - a*x*x^m - (a^2*m*x^2 + m)*x^m*arctan(a*x))/(a^2*c*x*arc
tan(a*x)^2)
```

## Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^3}\,dx = \int \frac{x^m}{(a^2cx^2+c)\arctan{(ax)}^3}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate(x^m/((a^2*c*x^2 + c)*arctan(a*x)^3), x)
```

## Mupad [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^3}\,dx = \int \frac{x^m}{\mathrm{atan}\left(a\,x\right)^3\,\left(c\,a^2\,x^2+c\right)}\,dx$$

input
```
int(x^m/(atan(a*x)^3*(c + a^2*c*x^2)),x)
```

output
```
int(x^m/(atan(a*x)^3*(c + a^2*c*x^2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.45

$$\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^3}\,dx = \frac{\int \frac{x^m}{atan(ax)^3a^2x^2+atan(ax)^3}dx}{c}$$

input
```
int(x^m/(a^2*c*x^2+c)/atan(a*x)^3,x)
```

output
```
int(x**m/(atan(a*x)**3*a**2*x**2 + atan(a*x)**3),x)/c
```

# 3.677 $\int \frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)^3} dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)^3} dx = \operatorname{Int}\left(\frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)^3}, x\right)$$

output
```
Defer(Int)(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

## Mathematica [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)^3} dx = \int \frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)^3} dx$$

input
```
Integrate[x^m/((c + a^2*c*x^2)^2*ArcTan[a*x]^3),x]
```

output
```
Integrate[x^m/((c + a^2*c*x^2)^2*ArcTan[a*x]^3), x]
```

## Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\arctan(ax)^3 \left(a^2cx^2+c\right)^2} \, dx$$

$$\Big\downarrow \ 5560$$

$$\int \frac{x^m}{\arctan(ax)^3 \left(a^2cx^2+c\right)^2}dx$$

input
```
Int[x^m/((c + a^2*c*x^2)^2*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 4.71 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\left(a^2c\,x^2+c\right)^2 \arctan\left(ax\right)^3}dx$$

input
```
int(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

output
```
int(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.77

$$\int \frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)^3} \, dx = \int \frac{x^m}{(a^2cx^2+c)^2 \arctan(ax)^3} \, dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(x^m/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)^3), x)
```

## Sympy [N/A]

Not integrable

Time = 13.95 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.91

$$\int \frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)^3} \, dx = \frac{\int \frac{x^m}{a^4x^4 \operatorname{atan}^3(ax)+2a^2x^2 \operatorname{atan}^3(ax)+\operatorname{atan}^3(ax)} \, dx}{c^2}$$

input
```
integrate(x**m/(a**2*c*x**2+c)**2/atan(a*x)**3,x)
```

output
```
Integral(x**m/(a**4*x**4*atan(a*x)**3 + 2*a**2*x**2*atan(a*x)**3 + atan(a*
x)**3), x)/c**2
```

## Maxima [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 191, normalized size of antiderivative = 8.68

$$\int \frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)^3} \, dx = \int \frac{x^m}{(a^2cx^2+c)^2 \arctan(ax)^3} \, dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
1/2*(2*(a^4*c^2*x^3 + a^2*c^2*x)*arctan(a*x)^2*integrate(1/2*((a^4*m^2 - 3
*a^4*m + 2*a^4)*x^4 + 2*(a^2*m^2 - 2*a^2*m - a^2)*x^2 + m^2 - m)*x^m/((a^6
*c^2*x^6 + 2*a^4*c^2*x^4 + a^2*c^2*x^2)*arctan(a*x)), x) - a*x*x^m - ((a^2
*m - 2*a^2)*x^2 + m)*x^m*arctan(a*x))/((a^4*c^2*x^3 + a^2*c^2*x)*arctan(a*
x)^2)
```

## Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m}{(c+a^2cx^2)^2\arctan(ax)^3}\,dx = \int \frac{x^m}{(a^2cx^2+c)^2\arctan{(ax)}^3}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate(x^m/((a^2*c*x^2 + c)^2*arctan(a*x)^3), x)
```

## Mupad [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m}{(c+a^2cx^2)^2\arctan(ax)^3}\,dx = \int \frac{x^m}{\mathrm{atan}\left(a\,x\right)^3\,{\left(c\,a^2\,x^2+c\right)}^2}\,dx$$

input
```
int(x^m/(atan(a*x)^3*(c + a^2*c*x^2)^2),x)
```

output
```
int(x^m/(atan(a*x)^3*(c + a^2*c*x^2)^2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 363, normalized size of antiderivative = 16.50

$$\int \frac{x^m}{(c+a^2cx^2)^2\arctan(ax)^3}\,dx$$
$$= \frac{atan(ax)^2\left(\int \frac{x^m}{atan(ax)^2a^4x^5+2atan(ax)^2a^2x^3+atan(ax)^2x}dx\right)a^2m\,x^2+atan(ax)^2\left(\int \frac{x^m}{atan(ax)^2a^4x^5+2atan(ax)^2a^2x^3+at}\right.}{}$$

input
```
int(x^m/(a^2*c*x^2+c)^2/atan(a*x)^3,x)
```

output
```
(atan(a*x)**2*int(x**m/(atan(a*x)**2*a**4*x**5 + 2*atan(a*x)**2*a**2*x**3
+ atan(a*x)**2*x),x)*a**2*m*x**2 + atan(a*x)**2*int(x**m/(atan(a*x)**2*a**
4*x**5 + 2*atan(a*x)**2*a**2*x**3 + atan(a*x)**2*x),x)*m + atan(a*x)**2*in
t((x**m*x)/(atan(a*x)**2*a**4*x**4 + 2*atan(a*x)**2*a**2*x**2 + atan(a*x)*
*2),x)*a**4*m*x**2 - 2*atan(a*x)**2*int((x**m*x)/(atan(a*x)**2*a**4*x**4 +
 2*atan(a*x)**2*a**2*x**2 + atan(a*x)**2),x)*a**4*x**2 + atan(a*x)**2*int(
(x**m*x)/(atan(a*x)**2*a**4*x**4 + 2*atan(a*x)**2*a**2*x**2 + atan(a*x)**2
),x)*a**2*m - 2*atan(a*x)**2*int((x**m*x)/(atan(a*x)**2*a**4*x**4 + 2*atan
(a*x)**2*a**2*x**2 + atan(a*x)**2),x)*a**2 - x**m)/(2*atan(a*x)**2*a*c**2*
(a**2*x**2 + 1))
```

# 3.678 $\int \frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)^3} \, dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)^3} \, dx = \text{Int}\left(\frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)^3}, x\right)$$

output
```
Defer(Int)(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^3,x)
```

## Mathematica [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)^3} \, dx = \int \frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)^3} \, dx$$

input
```
Integrate[x^m/((c + a^2*c*x^2)^3*ArcTan[a*x]^3),x]
```

output
```
Integrate[x^m/((c + a^2*c*x^2)^3*ArcTan[a*x]^3), x]
```

## Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\arctan(ax)^3 (a^2cx^2+c)^3} dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{x^m}{\arctan(ax)^3 (a^2cx^2+c)^3} dx$$

input
```
Int[x^m/((c + a^2*c*x^2)^3*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 8.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(a^2c\,x^2+c)^3 \arctan(ax)^3} dx$$

input
```
int(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^3,x)
```

output
```
int(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.27

$$\int \frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)^3} \, dx = \int \frac{x^m}{(a^2cx^2+c)^3 \arctan(ax)^3} \, dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(x^m/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a
*x)^3), x)
```

## Sympy [N/A]

Not integrable

Time = 39.39 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.64

$$\int \frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)^3} \, dx = \frac{\int \frac{x^m}{a^6x^6 \operatorname{atan}^3(ax)+3a^4x^4 \operatorname{atan}^3(ax)+3a^2x^2 \operatorname{atan}^3(ax)+\operatorname{atan}^3(ax)} \, dx}{c^3}$$

input
```
integrate(x**m/(a**2*c*x**2+c)**3/atan(a*x)**3,x)
```

output
```
Integral(x**m/(a**6*x**6*atan(a*x)**3 + 3*a**4*x**4*atan(a*x)**3 + 3*a**2*
x**2*atan(a*x)**3 + atan(a*x)**3), x)/c**3
```

## Maxima [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 224, normalized size of antiderivative = 10.18

$$\int \frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)^3} \, dx = \int \frac{x^m}{(a^2cx^2+c)^3 \arctan{(ax)}^3} \, dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
1/2*(2*(a^6*c^3*x^5 + 2*a^4*c^3*x^3 + a^2*c^3*x)*arctan(a*x)^2*integrate(1
/2*((a^4*m^2 - 7*a^4*m + 12*a^4)*x^4 + 2*(a^2*m^2 - 4*a^2*m - 2*a^2)*x^2 +
 m^2 - m)*x^m/((a^8*c^3*x^8 + 3*a^6*c^3*x^6 + 3*a^4*c^3*x^4 + a^2*c^3*x^2)
*arctan(a*x)), x) - a*x*x^m - ((a^2*m - 4*a^2)*x^2 + m)*x^m*arctan(a*x))/(
(a^6*c^3*x^5 + 2*a^4*c^3*x^3 + a^2*c^3*x)*arctan(a*x)^2)
```

## Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)^3} \, dx = \int \frac{x^m}{(a^2cx^2+c)^3 \arctan{(ax)}^3} \, dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate(x^m/((a^2*c*x^2 + c)^3*arctan(a*x)^3), x)
```

## Mupad [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)^3}\,dx = \int \frac{x^m}{\mathrm{atan}\left(a\,x\right)^3\,{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

input
```
int(x^m/(atan(a*x)^3*(c + a^2*c*x^2)^3),x)
```

output
```
int(x^m/(atan(a*x)^3*(c + a^2*c*x^2)^3), x)
```

## Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 671, normalized size of antiderivative = 30.50

$$\int \frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)^3}\,dx = \text{Too large to display}$$

input
```
int(x^m/(a^2*c*x^2+c)^3/atan(a*x)^3,x)
```

output
```
(atan(a*x)**2*int(x**m/(atan(a*x)**2*a**6*x**7 + 3*atan(a*x)**2*a**4*x**5
+ 3*atan(a*x)**2*a**2*x**3 + atan(a*x)**2*x),x)*a**4*m*x**4 + 2*atan(a*x)*
*2*int(x**m/(atan(a*x)**2*a**6*x**7 + 3*atan(a*x)**2*a**4*x**5 + 3*atan(a*
x)**2*a**2*x**3 + atan(a*x)**2*x),x)*a**2*m*x**2 + atan(a*x)**2*int(x**m/(
atan(a*x)**2*a**6*x**7 + 3*atan(a*x)**2*a**4*x**5 + 3*atan(a*x)**2*a**2*x*
*3 + atan(a*x)**2*x),x)*m + atan(a*x)**2*int((x**m*x)/(atan(a*x)**2*a**6*x
**6 + 3*atan(a*x)**2*a**4*x**4 + 3*atan(a*x)**2*a**2*x**2 + atan(a*x)**2),
x)*a**6*m*x**4 - 4*atan(a*x)**2*int((x**m*x)/(atan(a*x)**2*a**6*x**6 + 3*a
tan(a*x)**2*a**4*x**4 + 3*atan(a*x)**2*a**2*x**2 + atan(a*x)**2),x)*a**6*x
**4 + 2*atan(a*x)**2*int((x**m*x)/(atan(a*x)**2*a**6*x**6 + 3*atan(a*x)**2
*a**4*x**4 + 3*atan(a*x)**2*a**2*x**2 + atan(a*x)**2),x)*a**4*m*x**2 - 8*a
tan(a*x)**2*int((x**m*x)/(atan(a*x)**2*a**6*x**6 + 3*atan(a*x)**2*a**4*x**
4 + 3*atan(a*x)**2*a**2*x**2 + atan(a*x)**2),x)*a**4*x**2 + atan(a*x)**2*i
nt((x**m*x)/(atan(a*x)**2*a**6*x**6 + 3*atan(a*x)**2*a**4*x**4 + 3*atan(a*
x)**2*a**2*x**2 + atan(a*x)**2),x)*a**2*m - 4*atan(a*x)**2*int((x**m*x)/(a
tan(a*x)**2*a**6*x**6 + 3*atan(a*x)**2*a**4*x**4 + 3*atan(a*x)**2*a**2*x**
2 + atan(a*x)**2),x)*a**2 - x**m)/(2*atan(a*x)**2*a*c**3*(a**4*x**4 + 2*a*
*2*x**2 + 1))
```

# 3.679 $\int \frac{x^m (c+a^2cx^2)^{5/2}}{\arctan(ax)^3} \, dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m (c+a^2cx^2)^{5/2}}{\arctan(ax)^3} \, dx = \text{Int}\left(\frac{x^m (c+a^2cx^2)^{5/2}}{\arctan(ax)^3}, x\right)$$

output
```
Defer(Int)(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

## Mathematica [N/A]

Not integrable

Time = 2.70 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m (c+a^2cx^2)^{5/2}}{\arctan(ax)^3} \, dx = \int \frac{x^m (c+a^2cx^2)^{5/2}}{\arctan(ax)^3} \, dx$$

input
```
Integrate[(x^m*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^3,x]
```

output
```
Integrate[(x^m*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^3, x]
```

## Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \left(a^2cx^2+c\right)^{5/2}}{\arctan(ax)^3} \, dx$$

$$\Big\downarrow \; 5560$$

$$\int \frac{x^m \left(a^2cx^2+c\right)^{5/2}}{\arctan(ax)^3}dx$$

input
```
Int[(x^m*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^3,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 6.45 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m (a^2c\,x^2+c)^{\frac{5}{2}}}{\arctan\left(ax\right)^3}dx$$

input
```
int(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

output
```
int(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.04

$$\int \frac{x^m (c + a^2 c x^2)^{5/2}}{\arctan(ax)^3} \, dx = \int \frac{(a^2 c x^2 + c)^{\frac{5}{2}} x^m}{\arctan(ax)^3} \, dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*x^m/arcta
n(a*x)^3, x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^m (c + a^2 c x^2)^{5/2}}{\arctan(ax)^3} \, dx = \text{Timed out}$$

input
```
integrate(x**m*(a**2*c*x**2+c)**(5/2)/atan(a*x)**3,x)
```

output
```
Timed out
```

## Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m (c + a^2 c x^2)^{5/2}}{\arctan(ax)^3} \, dx = \int \frac{(a^2 c x^2 + c)^{\frac{5}{2}} x^m}{\arctan(ax)^3} \, dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)*x^m/arctan(a*x)^3, x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\arctan(ax)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\arctan(ax)^3}\,dx = \int \frac{x^m\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}{{\mathrm{atan}\left(a\,x\right)}^3}\,dx$$

input
```
int((x^m*(c + a^2*c*x^2)^(5/2))/atan(a*x)^3,x)
```

output
```
int((x^m*(c + a^2*c*x^2)^(5/2))/atan(a*x)^3, x)
```

## Reduce [N/A]

Not integrable

Time = 168.73 (sec) , antiderivative size = 88, normalized size of antiderivative = 3.67

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\arctan(ax)^3}\,dx = \sqrt{c}\,c^2\left(\left(\int \frac{x^m\sqrt{a^2x^2+1}\,x^4}{atan\,(ax)^3}dx\right)a^4 + 2\left(\int \frac{x^m\sqrt{a^2x^2+1}\,x^2}{atan\,(ax)^3}dx\right)a^2 + \int \frac{x^m\sqrt{a^2x^2+1}}{atan\,(ax)^3}dx\right)$$

input
```
int(x^m*(a^2*c*x^2+c)^(5/2)/atan(a*x)^3,x)
```

output
```
sqrt(c)*c**2*(int((x**m*sqrt(a**2*x**2 + 1)*x**4)/atan(a*x)**3,x)*a**4 + 2
*int((x**m*sqrt(a**2*x**2 + 1)*x**2)/atan(a*x)**3,x)*a**2 + int((x**m*sqrt
(a**2*x**2 + 1))/atan(a*x)**3,x))
```

# 3.680 $\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)^3} dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)^3} dx = \text{Int}\left(\frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)^3}, x\right)$$

output
```
Defer(Int)(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

## Mathematica [N/A]

Not integrable

Time = 1.40 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)^3} dx = \int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)^3} dx$$

input
```
Integrate[(x^m*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^3,x]
```

output
```
Integrate[(x^m*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^3, x]
```

## Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \left(a^2 c x^2+c\right)^{3/2}}{\arctan (a x)^3} \, dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x^m \left(a^2 c x^2+c\right)^{3/2}}{\arctan (a x)^3}dx$$

input
```
Int[(x^m*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^3,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 5.44 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^{m} \left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}{\arctan \left(a x \right)^{3}}d x$$

input
```
int(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

output
```
int(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)^3}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}x^m}{\arctan{(ax)}^3}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^(3/2)*x^m/arctan(a*x)^3, x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**m*(a**2*c*x**2+c)**(3/2)/atan(a*x)**3,x)
```

output
```
Timed out
```

## Maxima [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)^3}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}x^m}{\arctan{(ax)}^3}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*x^m/arctan(a*x)^3, x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)^3}\,dx = \int \frac{x^m\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}{{\mathrm{atan}\left(a\,x\right)}^3}\,dx$$

input
```
int((x^m*(c + a^2*c*x^2)^(3/2))/atan(a*x)^3,x)
```

output
```
int((x^m*(c + a^2*c*x^2)^(3/2))/atan(a*x)^3, x)
```

## Reduce [N/A]

Not integrable

Time = 8.15 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.33

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)^3}\,dx = \sqrt{c}\,c\left(\left(\int \frac{x^m\sqrt{a^2x^2+1}\,x^2}{atan\,(ax)^3}dx\right)a^2+\int \frac{x^m\sqrt{a^2x^2+1}}{atan\,(ax)^3}dx\right)$$

input
```
int(x^m*(a^2*c*x^2+c)^(3/2)/atan(a*x)^3,x)
```

output
```
sqrt(c)*c*(int((x**m*sqrt(a**2*x**2 + 1)*x**2)/atan(a*x)**3,x)*a**2 + int(
(x**m*sqrt(a**2*x**2 + 1))/atan(a*x)**3,x))
```

# 3.681 $\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^3}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^3}\,dx=\operatorname{Int}\left(\frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^3},x\right)$$

output
```
Defer(Int)(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x)
```

## Mathematica [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^3}\,dx=\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^3}\,dx$$

input
```
Integrate[(x^m*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^3,x]
```

output
```
Integrate[(x^m*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^3, x]
```

## Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m\sqrt{a^2cx^2+c}}{\arctan(ax)^3}dx$$
$$\Big\downarrow \ 5560$$
$$\int \frac{x^m\sqrt{a^2cx^2+c}}{\arctan(ax)^3}dx$$

input
```
Int[(x^m*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^3,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 4.77 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m\sqrt{a^2c\,x^2+c}}{\arctan\left(ax\right)^3}dx$$

input
```
int(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x)
```

output
```
int(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^3}\,dx = \int \frac{\sqrt{a^2cx^2+c}x^m}{\arctan{(ax)}^3}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^m/arctan(a*x)^3, x)
```

## Sympy [N/A]

Not integrable

Time = 20.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^3}\,dx = \int \frac{x^m\sqrt{c\,(a^2x^2+1)}}{\operatorname{atan}^3{(ax)}}\,dx$$

input
```
integrate(x**m*(a**2*c*x**2+c)**(1/2)/atan(a*x)**3,x)
```

output
```
Integral(x**m*sqrt(c*(a**2*x**2 + 1))/atan(a*x)**3, x)
```

## Maxima [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^3}\,dx = \int \frac{\sqrt{a^2cx^2+c}x^m}{\arctan{(ax)}^3}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*x^m/arctan(a*x)^3, x)
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^3}\,dx = \int \frac{x^m\,\sqrt{c\,a^2\,x^2+c}}{{\mathrm{atan}\left(a\,x\right)}^3}\,dx$$

input
```
int((x^m*(c + a^2*c*x^2)^(1/2))/atan(a*x)^3,x)
```

output
```
int((x^m*(c + a^2*c*x^2)^(1/2))/atan(a*x)^3, x)
```

## Reduce [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^3}\,dx = \sqrt{c}\left(\int \frac{x^m\sqrt{a^2x^2+1}}{atan\,(ax)^3}dx\right)$$

input
```
int(x^m*(a^2*c*x^2+c)^(1/2)/atan(a*x)^3,x)
```

output
```
sqrt(c)*int((x**m*sqrt(a**2*x**2 + 1))/atan(a*x)**3,x)
```

# 3.682 $\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx = \text{Int}\left(\frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^3},x\right)$$

output
```
Defer(Int)(x^m/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x)
```

## Mathematica [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx = \int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx$$

input
```
Integrate[x^m/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3),x]
```

output
```
Integrate[x^m/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3), x]
```

## Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\arctan(ax)^3\sqrt{a^2cx^2+c}}\,dx$$

$$\Big\downarrow \; 5560$$

$$\int \frac{x^m}{\arctan(ax)^3\sqrt{a^2cx^2+c}}dx$$

input
```
Int[x^m/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 4.98 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{\sqrt{a^2c\,x^2+c}\,\arctan\left(ax\right)^3}dx$$

input
```
int(x^m/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x)
```

output
```
int(x^m/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx = \int \frac{x^m}{\sqrt{a^2cx^2+c}\arctan{(ax)}^3}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(x^m/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3), x)
```

## Sympy [N/A]

Not integrable

Time = 28.85 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx = \int \frac{x^m}{\sqrt{c\,(a^2x^2+1)}\operatorname{atan}^3{(ax)}}\,dx$$

input
```
integrate(x**m/(a**2*c*x**2+c)**(1/2)/atan(a*x)**3,x)
```

output
```
Integral(x**m/(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**3), x)
```

## Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx = \int \frac{x^m}{\sqrt{a^2cx^2+c}\arctan{(ax)}^3}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(x^m/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3), x)
```

## Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx = \int \frac{x^m}{\sqrt{a^2cx^2+c}\arctan\left(ax\right)^3}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate(x^m/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^3), x)
```

## Mupad [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx = \int \frac{x^m}{\mathrm{atan}\left(a\,x\right)^3\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(x^m/(atan(a*x)^3*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(x^m/(atan(a*x)^3*(c + a^2*c*x^2)^(1/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^3}\,dx = \frac{\int \frac{x^m}{\sqrt{a^2x^2+1}\,atan(ax)^3}dx}{\sqrt{c}}$$

input
```
int(x^m/(a^2*c*x^2+c)^(1/2)/atan(a*x)^3,x)
```

output
```
int(x**m/(sqrt(a**2*x**2 + 1)*atan(a*x)**3),x)/sqrt(c)
```

# 3.683 $\int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx = \operatorname{Int}\left(\frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^3},x\right)$$

output
```
Defer(Int)(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

## Mathematica [N/A]

Not integrable

Time = 0.94 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx = \int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx$$

input
```
Integrate[x^m/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3),x]
```

output
```
Integrate[x^m/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3), x]
```

## Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\arctan(ax)^3 (a^2cx^2+c)^{3/2}} \, dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{x^m}{\arctan(ax)^3 (a^2cx^2+c)^{3/2}} dx$$

input
```
Int[x^m/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 5.93 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{(a^2c\,x^2+c)^{\frac{3}{2}} \arctan(ax)^3} dx$$

input
```
int(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

output
```
int(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.12

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx = \int \frac{x^m}{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)^3}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^m/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arct
an(a*x)^3), x)
```

## Sympy [N/A]

Not integrable

Time = 88.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx = \int \frac{x^m}{(c(a^2x^2+1))^{\frac{3}{2}}\operatorname{atan}^3(ax)}\,dx$$

input
```
integrate(x**m/(a**2*c*x**2+c)**(3/2)/atan(a*x)**3,x)
```

output
```
Integral(x**m/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**3), x)
```

## Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx = \int \frac{x^m}{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)^3}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(x^m/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^3), x)
```

## Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx = \int \frac{x^m}{(a^2cx^2+c)^{\frac{3}{2}}\arctan{(ax)}^3}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate(x^m/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^3), x)
```

## Mupad [N/A]

Not integrable

Time = 0.84 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx = \int \frac{x^m}{\mathrm{atan}\left(a\,x\right)^3\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

input
```
int(x^m/(atan(a*x)^3*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(x^m/(atan(a*x)^3*(c + a^2*c*x^2)^(3/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.38

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^3}\,dx = \frac{\int \frac{x^m}{\sqrt{a^2x^2+1}\,atan(ax)^3a^2x^2+\sqrt{a^2x^2+1}\,atan(ax)^3}dx}{\sqrt{c}\,c}$$

input
```
int(x^m/(a^2*c*x^2+c)^(3/2)/atan(a*x)^3,x)
```

output
```
int(x**m/(sqrt(a**2*x**2 + 1)*atan(a*x)**3*a**2*x**2 + sqrt(a**2*x**2 + 1)
*atan(a*x)**3),x)/(sqrt(c)*c)
```

# 3.684 $\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = \text{Int}\left(\frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^3},x\right)$$

output
```
Defer(Int)(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

## Mathematica [N/A]

Not integrable

Time = 0.98 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = \int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx$$

input
```
Integrate[x^m/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3),x]
```

output
```
Integrate[x^m/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3), x]
```

## Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\arctan(ax)^3 \left(a^2cx^2+c\right)^{5/2}} \, dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{x^m}{\arctan(ax)^3 \left(a^2cx^2+c\right)^{5/2}}dx$$

input
```
Int[x^m/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^3),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 5.88 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{\left(a^2c\,x^2+c\right)^{\frac{5}{2}} \arctan\left(ax\right)^3}dx$$

input
```
int(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

output
```
int(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.58

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = \int \frac{x^m}{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)^3}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^m/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3
*x^2 + c^3)*arctan(a*x)^3), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**m/(a**2*c*x**2+c)**(5/2)/atan(a*x)**3,x)
```

output
```
Timed out
```

## Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = \int \frac{x^m}{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)^3}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="maxima")
```

output
```
integrate(x^m/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^3), x)
```

## Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = \int \frac{x^m}{(a^2cx^2+c)^{\frac{5}{2}}\arctan{(ax)}^3}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^3,x, algorithm="giac")
```

output
```
integrate(x^m/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^3), x)
```

## Mupad [N/A]

Not integrable

Time = 0.86 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = \int \frac{x^m}{\mathrm{atan}\left(a\,x\right)^3\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

input
```
int(x^m/(atan(a*x)^3*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(x^m/(atan(a*x)^3*(c + a^2*c*x^2)^(5/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 3.38

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^3}\,dx = \frac{\int \frac{x^m}{\sqrt{a^2x^2+1}\,atan(ax)^3a^4x^4+2\sqrt{a^2x^2+1}\,atan(ax)^3a^2x^2+\sqrt{a^2x^2+1}\,atan(ax)^3}dx}{\sqrt{c}\,c^2}$$

input
```
int(x^m/(a^2*c*x^2+c)^(5/2)/atan(a*x)^3,x)
```

output

```
int(x**m/(sqrt(a**2*x**2 + 1)*atan(a*x)**3*a**4*x**4 + 2*sqrt(a**2*x**2 +
1)*atan(a*x)**3*a**2*x**2 + sqrt(a**2*x**2 + 1)*atan(a*x)**3),x)/(sqrt(c)*
c**2)
```

# 3.685 $\int x^m(c+a^2cx^2)\sqrt{\arctan(ax)}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int x^m(c+a^2cx^2)\sqrt{\arctan(ax)}\,dx = \text{Int}\left(x^m(c+a^2cx^2)\sqrt{\arctan(ax)},x\right)$$

output
```
Defer(Int)(x^m*(a^2*c*x^2+c)*arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.92 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int x^m(c+a^2cx^2)\sqrt{\arctan(ax)}\,dx = \int x^m(c+a^2cx^2)\sqrt{\arctan(ax)}\,dx$$

input
```
Integrate[x^m*(c + a^2*c*x^2)*Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[x^m*(c + a^2*c*x^2)*Sqrt[ArcTan[a*x]], x]
```

## Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sqrt{\arctan(ax)}\left(a^2cx^2+c\right)\,dx$$

$$\Big\downarrow \text{5560}$$

$$\int x^m \sqrt{\arctan(ax)}\left(a^2cx^2+c\right)dx$$

input
```
Int[x^m*(c + a^2*c*x^2)*Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 8.48 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int x^{m} \left(a^{2} c \,x^{2}+c \right) \sqrt{\arctan \left(a x \right)}dx$$

input
```
int(x^m*(a^2*c*x^2+c)*arctan(a*x)^(1/2),x)
```

output
```
int(x^m*(a^2*c*x^2+c)*arctan(a*x)^(1/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x^m \left(c+a^2cx^2\right) \sqrt{\arctan(ax)}\, dx = \int (a^2cx^2+c)x^m \sqrt{\arctan{(ax)}}\, dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)*arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)*x^m*sqrt(arctan(a*x)), x)
```

## Sympy [N/A]

Not integrable

Time = 27.75 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.55

$$\int x^m \left(c+a^2cx^2\right) \sqrt{\arctan(ax)}\, dx = c\left(\int x^m \sqrt{\operatorname{atan}{(ax)}}\, dx + \int a^2x^2x^m \sqrt{\operatorname{atan}{(ax)}}\, dx\right)$$

input
```
integrate(x**m*(a**2*c*x**2+c)*atan(a*x)**(1/2),x)
```

output
```
c*(Integral(x**m*sqrt(atan(a*x)), x) + Integral(a**2*x**2*x**m*sqrt(atan(a
*x)), x))
```

## Maxima [F(-2)]

Exception generated.

$$\int x^m \left(c+a^2cx^2\right) \sqrt{\arctan(ax)}\, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)*arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x^m(c+a^2cx^2)\sqrt{\arctan(ax)}\,dx = \int (a^2cx^2+c)x^m\sqrt{\arctan(ax)}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)*arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)*x^m*sqrt(arctan(a*x)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x^m(c+a^2cx^2)\sqrt{\arctan(ax)}\,dx = \int x^m\sqrt{\mathrm{atan}(a\,x)}\,\left(c\,a^2\,x^2+c\right)\,dx$$

input
```
int(x^m*atan(a*x)^(1/2)*(c + a^2*c*x^2),x)
```

output
```
int(x^m*atan(a*x)^(1/2)*(c + a^2*c*x^2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 725, normalized size of antiderivative = 32.95

$$\int x^m(c+a^2cx^2)\sqrt{\arctan(ax)}\,dx = \text{Too large to display}$$

input

```
int(x^m*(a^2*c*x^2+c)*atan(a*x)^(1/2),x)
```

output

```
(c*(2*x**m*sqrt(atan(a*x))*a**2*m*x**3 + 2*x**m*sqrt(atan(a*x))*a**2*x**3
+ 2*x**m*sqrt(atan(a*x))*m*x + 6*x**m*sqrt(atan(a*x))*x - int((x**m*sqrt(a
tan(a*x))*x**3)/(atan(a*x)*a**2*m**2*x**2 + 4*atan(a*x)*a**2*m*x**2 + 3*at
an(a*x)*a**2*x**2 + atan(a*x)*m**2 + 4*atan(a*x)*m + 3*atan(a*x)),x)*a**3*
m**3 - 5*int((x**m*sqrt(atan(a*x))*x**3)/(atan(a*x)*a**2*m**2*x**2 + 4*ata
n(a*x)*a**2*m*x**2 + 3*atan(a*x)*a**2*x**2 + atan(a*x)*m**2 + 4*atan(a*x)*
m + 3*atan(a*x)),x)*a**3*m**2 - 7*int((x**m*sqrt(atan(a*x))*x**3)/(atan(a*
x)*a**2*m**2*x**2 + 4*atan(a*x)*a**2*m*x**2 + 3*atan(a*x)*a**2*x**2 + atan
(a*x)*m**2 + 4*atan(a*x)*m + 3*atan(a*x)),x)*a**3*m - 3*int((x**m*sqrt(ata
n(a*x))*x**3)/(atan(a*x)*a**2*m**2*x**2 + 4*atan(a*x)*a**2*m*x**2 + 3*atan
(a*x)*a**2*x**2 + atan(a*x)*m**2 + 4*atan(a*x)*m + 3*atan(a*x)),x)*a**3 -
int((x**m*sqrt(atan(a*x))*x)/(atan(a*x)*a**2*m**2*x**2 + 4*atan(a*x)*a**2*
m*x**2 + 3*atan(a*x)*a**2*x**2 + atan(a*x)*m**2 + 4*atan(a*x)*m + 3*atan(a
*x)),x)*a*m**3 - 7*int((x**m*sqrt(atan(a*x))*x)/(atan(a*x)*a**2*m**2*x**2
+ 4*atan(a*x)*a**2*m*x**2 + 3*atan(a*x)*a**2*x**2 + atan(a*x)*m**2 + 4*ata
n(a*x)*m + 3*atan(a*x)),x)*a*m**2 - 15*int((x**m*sqrt(atan(a*x))*x)/(atan(
a*x)*a**2*m**2*x**2 + 4*atan(a*x)*a**2*m*x**2 + 3*atan(a*x)*a**2*x**2 + at
an(a*x)*m**2 + 4*atan(a*x)*m + 3*atan(a*x)),x)*a*m - 9*int((x**m*sqrt(atan
(a*x))*x)/(atan(a*x)*a**2*m**2*x**2 + 4*atan(a*x)*a**2*m*x**2 + 3*atan(a*x
)*a**2*x**2 + atan(a*x)*m**2 + 4*atan(a*x)*m + 3*atan(a*x)),x)*a))/(2*(...
```

# 3.686 $\int x(c+a^2cx^2)\sqrt{\arctan(ax)}\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 20

$$\int x(c+a^2cx^2)\sqrt{\arctan(ax)}\,dx=\frac{c(1+a^2x^2)^2\sqrt{\arctan(ax)}}{4a^2}-\frac{\text{Int}\left(\frac{c+a^2cx^2}{\sqrt{\arctan(ax)}},x\right)}{8a}$$

output
```
1/4*c*(a^2*x^2+1)^2*arctan(a*x)^(1/2)/a^2-1/8*Defer(Int)((a^2*c*x^2+c)/arc
tan(a*x)^(1/2),x)/a
```

## Mathematica [N/A]

Not integrable

Time = 0.95 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x(c+a^2cx^2)\sqrt{\arctan(ax)}\,dx=\int x(c+a^2cx^2)\sqrt{\arctan(ax)}\,dx$$

input
```
Integrate[x*(c + a^2*c*x^2)*Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[x*(c + a^2*c*x^2)*Sqrt[ArcTan[a*x]], x]
```

## Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)\,dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{c\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}{4a^2}-\frac{\int \frac{c\left(a^2x^2+1\right)}{\sqrt{\arctan(ax)}}dx}{8a}$$

$$\Big\downarrow \text{27}$$

$$\frac{c\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}{4a^2}-\frac{c\int \frac{a^2x^2+1}{\sqrt{\arctan(ax)}}dx}{8a}$$

$$\Big\downarrow \text{5560}$$

$$\frac{c\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}{4a^2}-\frac{c\int \frac{a^2x^2+1}{\sqrt{\arctan(ax)}}dx}{8a}$$

input
```
Int[x*(c + a^2*c*x^2)*Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 4.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int x\left(a^{2} c\,x^{2}+c\right) \sqrt{\arctan \left(a x\right)}dx$$

input
```
int(x*(a^2*c*x^2+c)*arctan(a*x)^(1/2),x)
```

output
```
int(x*(a^2*c*x^2+c)*arctan(a*x)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int x\left(c+a^2cx^2\right) \sqrt{\arctan(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)*arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 1.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.45

$$\int x\left(c+a^2cx^2\right) \sqrt{\arctan(ax)}\,dx = c\left(\int x\sqrt{\operatorname{atan}\left(ax\right)}\,dx+\int a^{2}x^{3}\sqrt{\operatorname{atan}\left(ax\right)}\,dx\right)$$

input
```
integrate(x*(a**2*c*x**2+c)*atan(a*x)**(1/2),x)
```

output
```
c*(Integral(x*sqrt(atan(a*x)), x) + Integral(a**2*x**3*sqrt(atan(a*x)), x)
)
```

## Maxima [F(-2)]

Exception generated.

$$\int x(c+a^2cx^2)\sqrt{\arctan(ax)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a^2*c*x^2+c)*arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int x(c+a^2cx^2)\sqrt{\arctan(ax)}\,dx = \int (a^2cx^2+c)x\sqrt{\arctan(ax)}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)*arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)*x*sqrt(arctan(a*x)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int x(c+a^2cx^2)\sqrt{\arctan(ax)}\,dx = \int x\,\sqrt{\mathrm{atan}\,(a\,x)}\,\left(c\,a^2\,x^2+c\right)\,dx$$

input
```
int(x*atan(a*x)^(1/2)*(c + a^2*c*x^2),x)
```

output
```
int(x*atan(a*x)^(1/2)*(c + a^2*c*x^2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 93, normalized size of antiderivative = 4.65

$$\int x(c+a^2cx^2)\sqrt{\arctan(ax)}\,dx$$
$$= \frac{c\left(2\sqrt{atan\,(ax)}\,a^2x^4+4\sqrt{atan\,(ax)}\,x^2-\left(\int\frac{\sqrt{atan(ax)}\,x^4}{atan(ax)a^2x^2+atan(ax)}dx\right)a^3-2\left(\int\frac{\sqrt{atan(ax)}\,x^2}{atan(ax)a^2x^2+atan(ax)}dx\right)a\right)}{8}$$

input
```
int(x*(a^2*c*x^2+c)*atan(a*x)^(1/2),x)
```

output
```
(c*(2*sqrt(atan(a*x))*a**2*x**4 + 4*sqrt(atan(a*x))*x**2 - int((sqrt(atan(
a*x))*x**4)/(atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**3 - 2*int((sqrt(atan(a
*x))*x**2)/(atan(a*x)*a**2*x**2 + atan(a*x)),x)*a))/8
```

# 3.687 $\int (c+a^2cx^2) \sqrt{\arctan(ax)}\,dx$



## Optimal result

Integrand size = 19, antiderivative size = 19

$$\int (c+a^2cx^2) \sqrt{\arctan(ax)}\,dx = \text{Int}\left((c+a^2cx^2) \sqrt{\arctan(ax)},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)*arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.55 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int (c+a^2cx^2) \sqrt{\arctan(ax)}\,dx = \int (c+a^2cx^2) \sqrt{\arctan(ax)}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)*Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[(c + a^2*c*x^2)*Sqrt[ArcTan[a*x]], x]
```

## Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\arctan(ax)}(a^2cx^2+c)\ dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \sqrt{\arctan(ax)}(a^2cx^2+c)dx$$

input
```
Int[(c + a^2*c*x^2)*Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 2.72 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \left(a^2c\,x^2+c\right)\sqrt{\arctan\left(ax\right)}dx$$

input
```
int((a^2*c*x^2+c)*arctan(a*x)^(1/2),x)
```

output
```
int((a^2*c*x^2+c)*arctan(a*x)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int (c+a^2cx^2)\sqrt{\arctan(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 0.85 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.42

$$\int (c+a^2cx^2)\sqrt{\arctan(ax)}\,dx = c\left(\int a^2x^2\sqrt{\operatorname{atan}(ax)}\,dx + \int \sqrt{\operatorname{atan}(ax)}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)*atan(a*x)**(1/2),x)
```

output
```
c*(Integral(a**2*x**2*sqrt(atan(a*x)), x) + Integral(sqrt(atan(a*x)), x))
```

## Maxima [F(-2)]

Exception generated.

$$\int (c+a^2cx^2)\sqrt{\arctan(ax)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int (c+a^2cx^2) \sqrt{\arctan(ax)}\,dx = \int (a^2cx^2+c)\sqrt{\arctan(ax)}\,dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)*sqrt(arctan(a*x)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int (c+a^2cx^2) \sqrt{\arctan(ax)}\,dx = \int \sqrt{\mathrm{atan}(a\,x)}\,\left(c\,a^2\,x^2+c\right)\,dx$$

input
```
int(atan(a*x)^(1/2)*(c + a^2*c*x^2),x)
```

output
```
int(atan(a*x)^(1/2)*(c + a^2*c*x^2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 89, normalized size of antiderivative = 4.68

$$\int (c+a^2cx^2) \sqrt{\arctan(ax)}\,dx = \frac{c\left(2\sqrt{atan(ax)}\,a^2x^3+6\sqrt{atan(ax)}\,x-\left(\int \frac{\sqrt{atan(ax)}\,x^3}{atan(ax)a^2x^2+atan(ax)}dx\right)a^3-3\left(\int \frac{\sqrt{atan(ax)}\,x}{atan(ax)a^2x^2+atan(ax)}dx\right)a\right)}{6}$$

input
```
int((a^2*c*x^2+c)*atan(a*x)^(1/2),x)
```

output
```
(c*(2*sqrt(atan(a*x))*a**2*x**3 + 6*sqrt(atan(a*x))*x - int((sqrt(atan(a*x
))*x**3)/(atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**3 - 3*int((sqrt(atan(a*x)
)*x)/(atan(a*x)*a**2*x**2 + atan(a*x)),x)*a))/6
```

# 3.688 $\int \frac{(c+a^2cx^2)\sqrt{\arctan(ax)}}{x}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{(c+a^2cx^2)\sqrt{\arctan(ax)}}{x}\,dx = \text{Int}\left(\frac{(c+a^2cx^2)\sqrt{\arctan(ax)}}{x},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)*arctan(a*x)^(1/2)/x,x)
```

## Mathematica [N/A]

Not integrable

Time = 0.85 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(c+a^2cx^2)\sqrt{\arctan(ax)}}{x}\,dx = \int \frac{(c+a^2cx^2)\sqrt{\arctan(ax)}}{x}\,dx$$

input
```
Integrate[((c + a^2*c*x^2)*Sqrt[ArcTan[a*x]])/x,x]
```

output
```
Integrate[((c + a^2*c*x^2)*Sqrt[ArcTan[a*x]])/x, x]
```

## Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\arctan(ax)}(a^2cx^2+c)}{x}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{\sqrt{\arctan(ax)}(a^2cx^2+c)}{x}dx$$

input
```
Int[((c + a^2*c*x^2)*Sqrt[ArcTan[a*x]])/x,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 3.67 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{(a^2c\,x^2+c)\sqrt{\arctan(ax)}}{x}dx$$

input
```
int((a^2*c*x^2+c)*arctan(a*x)^(1/2)/x,x)
```

output
```
int((a^2*c*x^2+c)*arctan(a*x)^(1/2)/x,x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{(c + a^2cx^2)\sqrt{\arctan(ax)}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^(1/2)/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 1.59 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

$$\int \frac{(c + a^2cx^2)\sqrt{\arctan(ax)}}{x}\,dx = c\left(\int \frac{\sqrt{\operatorname{atan}(ax)}}{x}\,dx + \int a^2x\sqrt{\operatorname{atan}(ax)}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)*atan(a*x)**(1/2)/x,x)
```

output
```
c*(Integral(sqrt(atan(a*x))/x, x) + Integral(a**2*x*sqrt(atan(a*x)), x))
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(c + a^2cx^2)\sqrt{\arctan(ax)}}{x}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^(1/2)/x,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)\sqrt{\arctan(ax)}}{x}\,dx = \int \frac{(a^2cx^2+c)\sqrt{\arctan(ax)}}{x}\,dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^(1/2)/x,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)*sqrt(arctan(a*x))/x, x)
```

## Mupad [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)\sqrt{\arctan(ax)}}{x}\,dx = \int \frac{\sqrt{\mathrm{atan}\left(a\,x\right)}\,\left(c\,a^2\,x^2+c\right)}{x}\,dx$$

input
```
int((atan(a*x)^(1/2)*(c + a^2*c*x^2))/x,x)
```

output
```
int((atan(a*x)^(1/2)*(c + a^2*c*x^2))/x, x)
```

## Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.91

$$\int \frac{(c + a^2cx^2)\sqrt{\arctan(ax)}}{x}\,dx$$
$$= \frac{c\left(2\sqrt{atan(ax)}\,a^2x^2 + 4\left(\int \frac{\sqrt{atan(ax)}}{x}dx\right) - \left(\int \frac{\sqrt{atan(ax)}\,x^2}{atan(ax)a^2x^2+atan(ax)}dx\right)a^3\right)}{4}$$

input
```
int((a^2*c*x^2+c)*atan(a*x)^(1/2)/x,x)
```

output
```
(c*(2*sqrt(atan(a*x))*a**2*x**2 + 4*int(sqrt(atan(a*x))/x,x) - int((sqrt(a
tan(a*x))*x**2)/(atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**3))/4
```

# 3.689 $\int x^m (c+a^2 c x^2)^2 \sqrt{\arctan(ax)}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int x^m (c+a^2 c x^2)^2 \sqrt{\arctan(ax)}\,dx = \text{Int}\left(x^m (c+a^2 c x^2)^2 \sqrt{\arctan(ax)},x\right)$$

output
```
Defer(Int)(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int x^m (c+a^2 c x^2)^2 \sqrt{\arctan(ax)}\,dx = \int x^m (c+a^2 c x^2)^2 \sqrt{\arctan(ax)}\,dx$$

input
```
Integrate[x^m*(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[x^m*(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]], x]
```

## Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sqrt{\arctan(ax)} \left(a^2 c x^2 + c\right)^2 \, dx$$

$$\Big\downarrow \ 5560$$

$$\int x^m \sqrt{\arctan(ax)} \left(a^2 c x^2 + c\right)^2 \, dx$$

input
```
Int[x^m*(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 7.95 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int x^m \left(a^2 c \, x^2 + c\right)^2 \sqrt{\arctan\left(a x\right)} dx$$

input
```
int(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^(1/2),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^(1/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int x^m \left(c+a^2cx^2\right)^2 \sqrt{\arctan(ax)}\,dx = \int \left(a^2cx^2+c\right)^2 x^m \sqrt{\arctan\left(ax\right)}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*x^m*sqrt(arctan(a*x)), x)
```

## Sympy [N/A]

Not integrable

Time = 92.31 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.42

$$\int x^m \left(c+a^2cx^2\right)^2 \sqrt{\arctan(ax)}\,dx = c^2\left(\int x^m\sqrt{\operatorname{atan}\left(ax\right)}\,dx + \int 2a^2x^2x^m\sqrt{\operatorname{atan}\left(ax\right)}\,dx + \int a^4x^4x^m\sqrt{\operatorname{atan}\left(ax\right)}\,dx\right)$$

input
```
integrate(x**m*(a**2*c*x**2+c)**2*atan(a*x)**(1/2),x)
```

output
```
c**2*(Integral(x**m*sqrt(atan(a*x)), x) + Integral(2*a**2*x**2*x**m*sqrt(a
tan(a*x)), x) + Integral(a**4*x**4*x**m*sqrt(atan(a*x)), x))
```

## Maxima [F(-2)]

Exception generated.

$$\int x^m (c + a^2 c x^2)^2 \sqrt{\arctan(ax)}\, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^m (c + a^2 c x^2)^2 \sqrt{\arctan(ax)}\, dx = \int (a^2 c x^2 + c)^2 x^m \sqrt{\arctan(ax)}\, dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^2*x^m*sqrt(arctan(a*x)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^m (c + a^2 c x^2)^2 \sqrt{\arctan(ax)}\, dx = \int x^m \sqrt{\mathrm{atan}(a\,x)}\, (c\,a^2\,x^2 + c)^2\, dx$$

input
```
int(x^m*atan(a*x)^(1/2)*(c + a^2*c*x^2)^2,x)
```

output
```
int(x^m*atan(a*x)^(1/2)*(c + a^2*c*x^2)^2, x)
```

## Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 2086, normalized size of antiderivative = 86.92

$$\int x^m \left(c+a^2cx^2\right)^2 \sqrt{\arctan(ax)}\, dx = \text{Too large to display}$$

input
```
int(x^m*(a^2*c*x^2+c)^2*atan(a*x)^(1/2),x)
```

output
```
(c**2*(2*x**m*sqrt(atan(a*x))*a**4*m**2*x**5 + 8*x**m*sqrt(atan(a*x))*a**4
*m*x**5 + 6*x**m*sqrt(atan(a*x))*a**4*x**5 + 4*x**m*sqrt(atan(a*x))*a**2*m
**2*x**3 + 24*x**m*sqrt(atan(a*x))*a**2*m*x**3 + 20*x**m*sqrt(atan(a*x))*a
**2*x**3 + 2*x**m*sqrt(atan(a*x))*m**2*x + 16*x**m*sqrt(atan(a*x))*m*x + 3
0*x**m*sqrt(atan(a*x))*x - int((x**m*sqrt(atan(a*x))*x**5)/(atan(a*x)*a**2
*m**3*x**2 + 9*atan(a*x)*a**2*m**2*x**2 + 23*atan(a*x)*a**2*m*x**2 + 15*at
an(a*x)*a**2*x**2 + atan(a*x)*m**3 + 9*atan(a*x)*m**2 + 23*atan(a*x)*m + 1
5*atan(a*x)),x)*a**5*m**5 - 13*int((x**m*sqrt(atan(a*x))*x**5)/(atan(a*x)*
a**2*m**3*x**2 + 9*atan(a*x)*a**2*m**2*x**2 + 23*atan(a*x)*a**2*m*x**2 + 1
5*atan(a*x)*a**2*x**2 + atan(a*x)*m**3 + 9*atan(a*x)*m**2 + 23*atan(a*x)*m
 + 15*atan(a*x)),x)*a**5*m**4 - 62*int((x**m*sqrt(atan(a*x))*x**5)/(atan(a
*x)*a**2*m**3*x**2 + 9*atan(a*x)*a**2*m**2*x**2 + 23*atan(a*x)*a**2*m*x**2
 + 15*atan(a*x)*a**2*x**2 + atan(a*x)*m**3 + 9*atan(a*x)*m**2 + 23*atan(a*
x)*m + 15*atan(a*x)),x)*a**5*m**3 - 134*int((x**m*sqrt(atan(a*x))*x**5)/(a
tan(a*x)*a**2*m**3*x**2 + 9*atan(a*x)*a**2*m**2*x**2 + 23*atan(a*x)*a**2*m
*x**2 + 15*atan(a*x)*a**2*x**2 + atan(a*x)*m**3 + 9*atan(a*x)*m**2 + 23*at
an(a*x)*m + 15*atan(a*x)),x)*a**5*m**2 - 129*int((x**m*sqrt(atan(a*x))*x**
5)/(atan(a*x)*a**2*m**3*x**2 + 9*atan(a*x)*a**2*m**2*x**2 + 23*atan(a*x)*a
**2*m*x**2 + 15*atan(a*x)*a**2*x**2 + atan(a*x)*m**3 + 9*atan(a*x)*m**2 +
23*atan(a*x)*m + 15*atan(a*x)),x)*a**5*m - 45*int((x**m*sqrt(atan(a*x))...
```

# 3.690 $\int x(c+a^2cx^2)^2 \sqrt{\arctan(ax)}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int x\left(c+a^2cx^2\right)^2\sqrt{\arctan(ax)}\,dx=\frac{c^2\left(1+a^2x^2\right)^3\sqrt{\arctan(ax)}}{6a^2}-\frac{\operatorname{Int}\left(\frac{\left(c+a^2cx^2\right)^2}{\sqrt{\arctan(ax)}},x\right)}{12a}$$

output
```
1/6*c^2*(a^2*x^2+1)^3*arctan(a*x)^(1/2)/a^2-1/12*Defer(Int)((a^2*c*x^2+c)^
2/arctan(a*x)^(1/2),x)/a
```

## Mathematica [N/A]

Not integrable

Time = 0.77 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int x\left(c+a^2cx^2\right)^2\sqrt{\arctan(ax)}\,dx=\int x\left(c+a^2cx^2\right)^2\sqrt{\arctan(ax)}\,dx$$

input
```
Integrate[x*(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[x*(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]], x]
```

## Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^2\,dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{c^2\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}{6a^2}-\frac{\int \frac{c^2\left(a^2x^2+1\right)^2}{\sqrt{\arctan(ax)}}dx}{12a}$$

$$\Big\downarrow \text{27}$$

$$\frac{c^2\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}{6a^2}-\frac{c^2\int \frac{\left(a^2x^2+1\right)^2}{\sqrt{\arctan(ax)}}dx}{12a}$$

$$\Big\downarrow \text{5560}$$

$$\frac{c^2\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}{6a^2}-\frac{c^2\int \frac{\left(a^2x^2+1\right)^2}{\sqrt{\arctan(ax)}}dx}{12a}$$

input
```
Int[x*(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 2.97 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int x\left(a^{2} c \,x^{2}+c \right)^{2} \sqrt{\arctan \left(a x \right)}d x$$

input
```
int(x*(a^2*c*x^2+c)^2*arctan(a*x)^(1/2),x)
```

output
```
int(x*(a^2*c*x^2+c)^2*arctan(a*x)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int x\left(c+a^2cx^2\right)^2\sqrt{\arctan(ax)}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^2*arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 2.33 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.23

$$\int x\left(c+a^2cx^2\right)^2\sqrt{\arctan(ax)}\,dx=c^{2}\left(\int x\sqrt{\operatorname{atan}{\left(ax \right)}}\,dx+\int 2a^{2}x^{3}\sqrt{\operatorname{atan}{\left(ax \right)}}\,dx+\int a^{4}x^{5}\sqrt{\operatorname{atan}{\left(ax \right)}}\,dx\right)$$

input
```
integrate(x*(a**2*c*x**2+c)**2*atan(a*x)**(1/2),x)
```

output
```
c**2*(Integral(x*sqrt(atan(a*x)), x) + Integral(2*a**2*x**3*sqrt(atan(a*x)
), x) + Integral(a**4*x**5*sqrt(atan(a*x)), x))
```

## Maxima [F(-2)]

Exception generated.

$$\int x(c+a^2cx^2)^2\sqrt{\arctan(ax)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^2*arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x(c+a^2cx^2)^2\sqrt{\arctan(ax)}\,dx = \int (a^2cx^2+c)^2x\sqrt{\arctan(ax)}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^2*arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^2*x*sqrt(arctan(a*x)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x\left(c+a^2cx^2\right)^2\sqrt{\arctan(ax)}\,dx=\int x\,\sqrt{\mathrm{atan}\left(a\,x\right)}\,{\left(c\,a^2\,x^2+c\right)}^2\,dx$$

input
```
int(x*atan(a*x)^(1/2)*(c + a^2*c*x^2)^2,x)
```

output
```
int(x*atan(a*x)^(1/2)*(c + a^2*c*x^2)^2, x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 142, normalized size of antiderivative = 6.45

$$\int x\left(c+a^2cx^2\right)^2\sqrt{\arctan(ax)}\,dx = \frac{c^2\left(2\sqrt{atan\left(ax\right)}\,a^4x^6+6\sqrt{atan\left(ax\right)}\,a^2x^4+6\sqrt{atan\left(ax\right)}\,x^2-\left(\int\frac{\sqrt{atan(ax)}\,x^6}{atan(ax)a^2x^2+atan(ax)}dx\right)a^5-3\left(\int\frac{}{atan}\right.\right.}{12}$$

input
```
int(x*(a^2*c*x^2+c)^2*atan(a*x)^(1/2),x)
```

output
```
(c**2*(2*sqrt(atan(a*x))*a**4*x**6 + 6*sqrt(atan(a*x))*a**2*x**4 + 6*sqrt(
atan(a*x))*x**2 - int((sqrt(atan(a*x))*x**6)/(atan(a*x)*a**2*x**2 + atan(a
*x)),x)*a**5 - 3*int((sqrt(atan(a*x))*x**4)/(atan(a*x)*a**2*x**2 + atan(a*
x)),x)*a**3 - 3*int((sqrt(atan(a*x))*x**2)/(atan(a*x)*a**2*x**2 + atan(a*x
)),x)*a))/12
```

# 3.691 $\int (c+a^2cx^2)^2 \sqrt{\arctan(ax)}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 21

$$\int (c+a^2cx^2)^2 \sqrt{\arctan(ax)}\,dx = \operatorname{Int}\left((c+a^2cx^2)^2 \sqrt{\arctan(ax)},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^2*arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.86 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int (c+a^2cx^2)^2 \sqrt{\arctan(ax)}\,dx = \int (c+a^2cx^2)^2 \sqrt{\arctan(ax)}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]], x]
```

## Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^2\,dx$$

$$\Big\downarrow \text{5560}$$

$$\int \sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^2dx$$

input
```
Int[(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 1.89 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \left(a^2c\,x^2+c\right)^2\sqrt{\arctan\left(ax\right)}dx$$

input
```
int((a^2*c*x^2+c)^2*arctan(a*x)^(1/2),x)
```

output
```
int((a^2*c*x^2+c)^2*arctan(a*x)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^2 \sqrt{\arctan(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 1.72 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.29

$$\int \left(c+a^2cx^2\right)^2 \sqrt{\arctan(ax)}\,dx = c^2\left(\int 2a^2x^2\sqrt{\operatorname{atan}(ax)}\,dx + \int a^4x^4\sqrt{\operatorname{atan}(ax)}\,dx + \int \sqrt{\operatorname{atan}(ax)}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**2*atan(a*x)**(1/2),x)
```

output
```
c**2*(Integral(2*a**2*x**2*sqrt(atan(a*x)), x) + Integral(a**4*x**4*sqrt(a
tan(a*x)), x) + Integral(sqrt(atan(a*x)), x))
```

## Maxima [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^2 \sqrt{\arctan(ax)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \left(c+a^2cx^2\right)^2 \sqrt{\arctan(ax)}\,dx = \int \left(a^2cx^2+c\right)^2\sqrt{\arctan\left(ax\right)}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^2*sqrt(arctan(a*x)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \left(c+a^2cx^2\right)^2 \sqrt{\arctan(ax)}\,dx = \int \sqrt{\mathrm{atan}\left(a\,x\right)}\,\left(c\,a^2\,x^2+c\right)^2\,dx$$

input
```
int(atan(a*x)^(1/2)*(c + a^2*c*x^2)^2,x)
```

output
```
int(atan(a*x)^(1/2)*(c + a^2*c*x^2)^2, x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 138, normalized size of antiderivative = 6.57

$$\int \left(c+a^2cx^2\right)^2\sqrt{\arctan(ax)}\,dx$$
$$=\frac{c^2\left(6\sqrt{atan\left(ax\right)}\,a^4x^5+20\sqrt{atan\left(ax\right)}\,a^2x^3+30\sqrt{atan\left(ax\right)}\,x-3\left(\int\frac{\sqrt{atan(ax)}\,x^5}{atan(ax)a^2x^2+atan(ax)}dx\right)a^5-10\left(\int\right.\right.}{30}$$

input
```
int((a^2*c*x^2+c)^2*atan(a*x)^(1/2),x)
```

output
```
(c**2*(6*sqrt(atan(a*x))*a**4*x**5 + 20*sqrt(atan(a*x))*a**2*x**3 + 30*sqr
t(atan(a*x))*x - 3*int((sqrt(atan(a*x))*x**5)/(atan(a*x)*a**2*x**2 + atan(
a*x)),x)*a**5 - 10*int((sqrt(atan(a*x))*x**3)/(atan(a*x)*a**2*x**2 + atan(
a*x)),x)*a**3 - 15*int((sqrt(atan(a*x))*x)/(atan(a*x)*a**2*x**2 + atan(a*x
)),x)*a))/30
```

# 3.692 $\int \frac{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}{x}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}{x}\,dx = \text{Int}\left(\frac{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}{x},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^2*arctan(a*x)^(1/2)/x,x)
```

## Mathematica [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}{x}\,dx = \int \frac{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}{x}\,dx$$

input
```
Integrate[((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]])/x,x]
```

output
```
Integrate[((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]])/x, x]
```

## Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^2}{x}\,dx$$

$$\Big\downarrow \ 5560$$

$$\int \frac{\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^2}{x}dx$$

input
```
Int[((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]])/x,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 2.45 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\left(a^2c\,x^2+c\right)^2\sqrt{\arctan\left(ax\right)}}{x}dx$$

input
```
int((a^2*c*x^2+c)^2*arctan(a*x)^(1/2)/x,x)
```

output
```
int((a^2*c*x^2+c)^2*arctan(a*x)^(1/2)/x,x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{(c + a^2cx^2)^2 \sqrt{\arctan(ax)}}{x} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^(1/2)/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 2.53 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.00

$$\int \frac{(c + a^2cx^2)^2 \sqrt{\arctan(ax)}}{x} \, dx = c^2 \left( \int \frac{\sqrt{\operatorname{atan}(ax)}}{x} \, dx + \int 2a^2x\sqrt{\operatorname{atan}(ax)} \, dx + \int a^4x^3\sqrt{\operatorname{atan}(ax)} \, dx \right)$$

input
```
integrate((a**2*c*x**2+c)**2*atan(a*x)**(1/2)/x,x)
```

output
```
c**2*(Integral(sqrt(atan(a*x))/x, x) + Integral(2*a**2*x*sqrt(atan(a*x)),
x) + Integral(a**4*x**3*sqrt(atan(a*x)), x))
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}{x}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^(1/2)/x,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}{x}\,dx = \int \frac{(a^2cx^2+c)^2\sqrt{\arctan(ax)}}{x}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^(1/2)/x,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^2*sqrt(arctan(a*x))/x, x)
```

## Mupad [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}{x}\,dx = \int \frac{\sqrt{\mathrm{atan}\,(a\,x)}\,(c\,a^2\,x^2+c)^2}{x}\,dx$$

input
```
int((atan(a*x)^(1/2)*(c + a^2*c*x^2)^2)/x,x)
```

output
```
int((atan(a*x)^(1/2)*(c + a^2*c*x^2)^2)/x, x)
```

## Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 113, normalized size of antiderivative = 4.71

$$\int \frac{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}{x}\,dx = \frac{c^2\left(2\sqrt{atan(ax)}\,a^4x^4+8\sqrt{atan(ax)}\,a^2x^2+8\left(\int \frac{\sqrt{atan(ax)}}{x}dx\right)-\left(\int \frac{\sqrt{atan(ax)}\,x^4}{atan(ax)a^2x^2+atan(ax)}dx\right)a^5-4\left(\int \frac{\sqrt{atan(ax)}\,x^2}{atan(ax)a^2x^2+atan(ax)}dx\right)a^3\right)}{8}$$

input
```
int((a^2*c*x^2+c)^2*atan(a*x)^(1/2)/x,x)
```

output
```
(c**2*(2*sqrt(atan(a*x))*a**4*x**4 + 8*sqrt(atan(a*x))*a**2*x**2 + 8*int(s
qrt(atan(a*x))/x,x) - int((sqrt(atan(a*x))*x**4)/(atan(a*x)*a**2*x**2 + at
an(a*x)),x)*a**5 - 4*int((sqrt(atan(a*x))*x**2)/(atan(a*x)*a**2*x**2 + ata
n(a*x)),x)*a**3))/8
```

# 3.693 $\int x^m (c+a^2 c x^2)^3 \sqrt{\arctan(ax)}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int x^m (c+a^2 c x^2)^3 \sqrt{\arctan(ax)}\,dx = \mathrm{Int}\left(x^m (c+a^2 c x^2)^3 \sqrt{\arctan(ax)},x\right)$$

output
```
Defer(Int)(x^m*(a^2*c*x^2+c)^3*arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int x^m (c+a^2 c x^2)^3 \sqrt{\arctan(ax)}\,dx = \int x^m (c+a^2 c x^2)^3 \sqrt{\arctan(ax)}\,dx$$

input
```
Integrate[x^m*(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[x^m*(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]], x]
```

## Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sqrt{\arctan(ax)} \left(a^2cx^2+c\right)^3 \, dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int x^m \sqrt{\arctan(ax)} \left(a^2cx^2+c\right)^3 dx$$

input
```
Int[x^m*(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 7.96 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int x^m \left(a^2c\,x^2+c\right)^3 \sqrt{\arctan\left(ax\right)}dx$$

input
```
int(x^m*(a^2*c*x^2+c)^3*arctan(a*x)^(1/2),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^3*arctan(a*x)^(1/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.00

$$\int x^m (c + a^2cx^2)^3 \sqrt{\arctan(ax)}\,dx = \int (a^2cx^2 + c)^3 x^m \sqrt{\arctan(ax)}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^3*arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*x^m*sqrt(arct
an(a*x)), x)
```

## Sympy [F(-1)]

Timed out.

$$\int x^m (c + a^2cx^2)^3 \sqrt{\arctan(ax)}\,dx = \text{Timed out}$$

input
```
integrate(x**m*(a**2*c*x**2+c)**3*atan(a*x)**(1/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int x^m (c + a^2cx^2)^3 \sqrt{\arctan(ax)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^3*arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^m (c+a^2cx^2)^3 \sqrt{\arctan(ax)}\,dx = \int (a^2cx^2+c)^3 x^m \sqrt{\arctan(ax)}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^3*arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^3*x^m*sqrt(arctan(a*x)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^m (c+a^2cx^2)^3 \sqrt{\arctan(ax)}\,dx = \int x^m \sqrt{\mathrm{atan}(a\,x)}\,(c\,a^2\,x^2+c)^3\,dx$$

input
```
int(x^m*atan(a*x)^(1/2)*(c + a^2*c*x^2)^3,x)
```

output
```
int(x^m*atan(a*x)^(1/2)*(c + a^2*c*x^2)^3, x)
```

## Reduce [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 4495, normalized size of antiderivative = 187.29

$$\int x^m (c+a^2cx^2)^3 \sqrt{\arctan(ax)}\,dx = \text{Too large to display}$$

input
```
int(x^m*(a^2*c*x^2+c)^3*atan(a*x)^(1/2),x)
```

output
```
(c**3*(2*x**m*sqrt(atan(a*x))*a**6*m**3*x**7 + 18*x**m*sqrt(atan(a*x))*a**
6*m**2*x**7 + 46*x**m*sqrt(atan(a*x))*a**6*m*x**7 + 30*x**m*sqrt(atan(a*x)
)*a**6*x**7 + 6*x**m*sqrt(atan(a*x))*a**4*m**3*x**5 + 66*x**m*sqrt(atan(a*
x))*a**4*m**2*x**5 + 186*x**m*sqrt(atan(a*x))*a**4*m*x**5 + 126*x**m*sqrt(
atan(a*x))*a**4*x**5 + 6*x**m*sqrt(atan(a*x))*a**2*m**3*x**3 + 78*x**m*sqr
t(atan(a*x))*a**2*m**2*x**3 + 282*x**m*sqrt(atan(a*x))*a**2*m*x**3 + 210*x
**m*sqrt(atan(a*x))*a**2*x**3 + 2*x**m*sqrt(atan(a*x))*m**3*x + 30*x**m*sq
rt(atan(a*x))*m**2*x + 142*x**m*sqrt(atan(a*x))*m*x + 210*x**m*sqrt(atan(a
*x))*x - int((x**m*sqrt(atan(a*x))*x**7)/(atan(a*x)*a**2*m**4*x**2 + 16*at
an(a*x)*a**2*m**3*x**2 + 86*atan(a*x)*a**2*m**2*x**2 + 176*atan(a*x)*a**2*
m*x**2 + 105*atan(a*x)*a**2*x**2 + atan(a*x)*m**4 + 16*atan(a*x)*m**3 + 86
*atan(a*x)*m**2 + 176*atan(a*x)*m + 105*atan(a*x)),x)*a**7*m**7 - 25*int((
x**m*sqrt(atan(a*x))*x**7)/(atan(a*x)*a**2*m**4*x**2 + 16*atan(a*x)*a**2*m
**3*x**2 + 86*atan(a*x)*a**2*m**2*x**2 + 176*atan(a*x)*a**2*m*x**2 + 105*a
tan(a*x)*a**2*x**2 + atan(a*x)*m**4 + 16*atan(a*x)*m**3 + 86*atan(a*x)*m**
2 + 176*atan(a*x)*m + 105*atan(a*x)),x)*a**7*m**6 - 253*int((x**m*sqrt(ata
n(a*x))*x**7)/(atan(a*x)*a**2*m**4*x**2 + 16*atan(a*x)*a**2*m**3*x**2 + 86
*atan(a*x)*a**2*m**2*x**2 + 176*atan(a*x)*a**2*m*x**2 + 105*atan(a*x)*a**2
*x**2 + atan(a*x)*m**4 + 16*atan(a*x)*m**3 + 86*atan(a*x)*m**2 + 176*atan(
a*x)*m + 105*atan(a*x)),x)*a**7*m**5 - 1333*int((x**m*sqrt(atan(a*x))*x...
```

# 3.694 $\int x(c+a^2cx^2)^3 \sqrt{\arctan(ax)}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int x(c+a^2cx^2)^3 \sqrt{\arctan(ax)}\,dx = \frac{c^3(1+a^2x^2)^4\sqrt{\arctan(ax)}}{8a^2} - \frac{\mathrm{Int}\left(\frac{(c+a^2cx^2)^3}{\sqrt{\arctan(ax)}},x\right)}{16a}$$

output
```
1/8*c^3*(a^2*x^2+1)^4*arctan(a*x)^(1/2)/a^2-1/16*Defer(Int)((a^2*c*x^2+c)^
3/arctan(a*x)^(1/2),x)/a
```

## Mathematica [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int x(c+a^2cx^2)^3 \sqrt{\arctan(ax)}\,dx = \int x(c+a^2cx^2)^3 \sqrt{\arctan(ax)}\,dx$$

input
```
Integrate[x*(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[x*(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]], x]
```

## Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^3\,dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{c^3(a^2x^2+1)^4\sqrt{\arctan(ax)}}{8a^2}-\frac{\int \frac{c^3(a^2x^2+1)^3}{\sqrt{\arctan(ax)}}dx}{16a}$$

$$\Big\downarrow \text{27}$$

$$\frac{c^3(a^2x^2+1)^4\sqrt{\arctan(ax)}}{8a^2}-\frac{c^3\int \frac{(a^2x^2+1)^3}{\sqrt{\arctan(ax)}}dx}{16a}$$

$$\Big\downarrow \text{5560}$$

$$\frac{c^3(a^2x^2+1)^4\sqrt{\arctan(ax)}}{8a^2}-\frac{c^3\int \frac{(a^2x^2+1)^3}{\sqrt{\arctan(ax)}}dx}{16a}$$

input
```
Int[x*(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 3.41 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int x\left(a^{2} c \,x^{2}+c \right)^{3} \sqrt{\arctan \left(a x \right)}d x$$

input
```
int(x*(a^2*c*x^2+c)^3*arctan(a*x)^(1/2),x)
```

output
```
int(x*(a^2*c*x^2+c)^3*arctan(a*x)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int x(c+a^2cx^2)^3 \sqrt{\arctan(ax)}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^3*arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 4.99 (sec) , antiderivative size = 68, normalized size of antiderivative = 3.09

$$\int x(c+a^2cx^2)^3 \sqrt{\arctan(ax)}\, dx = c^{3} \left(\int x \sqrt{\operatorname{atan}{\left(a x \right)}}\, dx + \int 3 a^{2} x^{3} \sqrt{\operatorname{atan}{\left(a x \right)}}\, dx + \int 3 a^{4} x^{5} \sqrt{\operatorname{atan}{\left(a x \right)}}\, dx + \int a^{6} x^{7} \sqrt{\operatorname{atan}{\left(a x \right)}}\, dx\right)$$

input
```
integrate(x*(a**2*c*x**2+c)**3*atan(a*x)**(1/2),x)
```

output
```
c**3*(Integral(x*sqrt(atan(a*x)), x) + Integral(3*a**2*x**3*sqrt(atan(a*x)
), x) + Integral(3*a**4*x**5*sqrt(atan(a*x)), x) + Integral(a**6*x**7*sqrt
(atan(a*x)), x))
```

## Maxima [F(-2)]

Exception generated.

$$\int x(c+a^2cx^2)^3\sqrt{\arctan(ax)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^3*arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x(c+a^2cx^2)^3\sqrt{\arctan(ax)}\,dx = \int (a^2cx^2+c)^3x\sqrt{\arctan(ax)}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^3*arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^3*x*sqrt(arctan(a*x)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x\left(c+a^2cx^2\right)^3\sqrt{\arctan(ax)}\,dx=\int x\,\sqrt{\mathrm{atan}\left(a\,x\right)}\,{\left(c\,a^2\,x^2+c\right)}^3\,dx$$

input
```
int(x*atan(a*x)^(1/2)*(c + a^2*c*x^2)^3,x)
```

output
```
int(x*atan(a*x)^(1/2)*(c + a^2*c*x^2)^3, x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 189, normalized size of antiderivative = 8.59

$$\int x\left(c+a^2cx^2\right)^3\sqrt{\arctan(ax)}\,dx$$
$$=\frac{c^3\left(2\sqrt{atan\left(ax\right)}\,a^6x^8+8\sqrt{atan\left(ax\right)}\,a^4x^6+12\sqrt{atan\left(ax\right)}\,a^2x^4+8\sqrt{atan\left(ax\right)}\,x^2-\left(\int\frac{\sqrt{atan(ax)}\,x^8}{atan(ax)a^2x^2+ata}\right.\right.}{}$$

input
```
int(x*(a^2*c*x^2+c)^3*atan(a*x)^(1/2),x)
```

output
```
(c**3*(2*sqrt(atan(a*x))*a**6*x**8 + 8*sqrt(atan(a*x))*a**4*x**6 + 12*sqrt
(atan(a*x))*a**2*x**4 + 8*sqrt(atan(a*x))*x**2 - int((sqrt(atan(a*x))*x**8
)/(atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**7 - 4*int((sqrt(atan(a*x))*x**6)
/(atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**5 - 6*int((sqrt(atan(a*x))*x**4)/
(atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**3 - 4*int((sqrt(atan(a*x))*x**2)/(
atan(a*x)*a**2*x**2 + atan(a*x)),x)*a))/16
```

# 3.695 $\int (c+a^2cx^2)^3 \sqrt{\arctan(ax)}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 21

$$\int (c+a^2cx^2)^3 \sqrt{\arctan(ax)}\,dx = \operatorname{Int}\left((c+a^2cx^2)^3 \sqrt{\arctan(ax)},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^3*arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.92 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int (c+a^2cx^2)^3 \sqrt{\arctan(ax)}\,dx = \int (c+a^2cx^2)^3 \sqrt{\arctan(ax)}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]], x]
```

## Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^3\,dx$$

$$\Big\downarrow \ 5560$$

$$\int \sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^3dx$$

input
```
Int[(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 2.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \left(a^2c\,x^2+c\right)^3\sqrt{\arctan\left(ax\right)}dx$$

input
```
int((a^2*c*x^2+c)^3*arctan(a*x)^(1/2),x)
```

output
```
int((a^2*c*x^2+c)^3*arctan(a*x)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^3 \sqrt{\arctan(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 3.78 (sec) , antiderivative size = 66, normalized size of antiderivative = 3.14

$$\int \left(c+a^2cx^2\right)^3 \sqrt{\arctan(ax)}\,dx = c^3\left(\int 3a^2x^2\sqrt{\operatorname{atan}(ax)}\,dx + \int 3a^4x^4\sqrt{\operatorname{atan}(ax)}\,dx + \int a^6x^6\sqrt{\operatorname{atan}(ax)}\,dx + \int \sqrt{\operatorname{atan}(ax)}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**3*atan(a*x)**(1/2),x)
```

output
```
c**3*(Integral(3*a**2*x**2*sqrt(atan(a*x)), x) + Integral(3*a**4*x**4*sqrt
(atan(a*x)), x) + Integral(a**6*x**6*sqrt(atan(a*x)), x) + Integral(sqrt(a
tan(a*x)), x))
```

## Maxima [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^3 \sqrt{\arctan(ax)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \left(c+a^2cx^2\right)^3 \sqrt{\arctan(ax)}\,dx = \int \left(a^2cx^2+c\right)^3 \sqrt{\arctan\left(ax\right)}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^3*sqrt(arctan(a*x)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \left(c+a^2cx^2\right)^3 \sqrt{\arctan(ax)}\,dx = \int \sqrt{\mathrm{atan}\left(a\,x\right)}\,\left(c\,a^2\,x^2+c\right)^3\,dx$$

input
```
int(atan(a*x)^(1/2)*(c + a^2*c*x^2)^3,x)
```

output
```
int(atan(a*x)^(1/2)*(c + a^2*c*x^2)^3, x)
```

## Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 185, normalized size of antiderivative = 8.81

$$\int \left(c+a^2cx^2\right)^3\sqrt{\arctan(ax)}\,dx$$
$$= \frac{c^3\left(10\sqrt{atan\,(ax)}\,a^6x^7+42\sqrt{atan\,(ax)}\,a^4x^5+70\sqrt{atan\,(ax)}\,a^2x^3+70\sqrt{atan\,(ax)}\,x-5\left(\int \frac{\sqrt{atan(a}}{atan(ax)a^2x^2}\right.\right.}{}$$

input
```
int((a^2*c*x^2+c)^3*atan(a*x)^(1/2),x)
```

output
```
(c**3*(10*sqrt(atan(a*x))*a**6*x**7 + 42*sqrt(atan(a*x))*a**4*x**5 + 70*sq
rt(atan(a*x))*a**2*x**3 + 70*sqrt(atan(a*x))*x - 5*int((sqrt(atan(a*x))*x*
*7)/(atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**7 - 21*int((sqrt(atan(a*x))*x*
*5)/(atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**5 - 35*int((sqrt(atan(a*x))*x*
*3)/(atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**3 - 35*int((sqrt(atan(a*x))*x)
/(atan(a*x)*a**2*x**2 + atan(a*x)),x)*a))/70
```

# 3.696 $\int \frac{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}{x}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}{x}\,dx = \text{Int}\left(\frac{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}{x},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^3*arctan(a*x)^(1/2)/x,x)
```

## Mathematica [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}{x}\,dx = \int \frac{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}{x}\,dx$$

input
```
Integrate[((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]])/x,x]
```

output
```
Integrate[((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]])/x, x]
```

## Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^3}{x}\,dx$$

$$\Big\downarrow \ 5560$$

$$\int \frac{\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^3}{x}dx$$

input
```
Int[((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]])/x,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 3.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\left(a^2c\,x^2+c\right)^3\sqrt{\arctan\left(ax\right)}}{x}dx$$

input
```
int((a^2*c*x^2+c)^3*arctan(a*x)^(1/2)/x,x)
```

output
```
int((a^2*c*x^2+c)^3*arctan(a*x)^(1/2)/x,x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^3 \sqrt{\arctan(ax)}}{x} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^(1/2)/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 4.21 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.75

$$\int \frac{(c+a^2cx^2)^3 \sqrt{\arctan(ax)}}{x} \, dx = c^3 \left( \int \frac{\sqrt{\operatorname{atan}(ax)}}{x} \, dx + \int 3a^2 x \sqrt{\operatorname{atan}(ax)} \, dx + \int 3a^4 x^3 \sqrt{\operatorname{atan}(ax)} \, dx + \int a^6 x^5 \sqrt{\operatorname{atan}(ax)} \, dx \right)$$

input
```
integrate((a**2*c*x**2+c)**3*atan(a*x)**(1/2)/x,x)
```

output
```
c**3*(Integral(sqrt(atan(a*x))/x, x) + Integral(3*a**2*x*sqrt(atan(a*x)),
x) + Integral(3*a**4*x**3*sqrt(atan(a*x)), x) + Integral(a**6*x**5*sqrt(at
an(a*x)), x))
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}{x}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^(1/2)/x,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}{x}\,dx = \int \frac{(a^2cx^2+c)^3\sqrt{\arctan(ax)}}{x}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^(1/2)/x,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^3*sqrt(arctan(a*x))/x, x)
```

## Mupad [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}{x}\,dx = \int \frac{\sqrt{\mathrm{atan}(a\,x)}\,(c\,a^2\,x^2+c)^3}{x}\,dx$$

input
```
int((atan(a*x)^(1/2)*(c + a^2*c*x^2)^3)/x,x)
```

output
```
int((atan(a*x)^(1/2)*(c + a^2*c*x^2)^3)/x, x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 160, normalized size of antiderivative = 6.67

$$\int \frac{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}{x}\,dx$$
$$= \frac{c^3\left(4\sqrt{atan(ax)}\,a^6x^6+18\sqrt{atan(ax)}\,a^4x^4+36\sqrt{atan(ax)}\,a^2x^2+24\left(\int\frac{\sqrt{atan(ax)}}{x}dx\right)-2\left(\int\frac{\sqrt{atan}}{atan(ax)a^2x}\right.\right.}{24}$$

input
```
int((a^2*c*x^2+c)^3*atan(a*x)^(1/2)/x,x)
```

output
```
(c**3*(4*sqrt(atan(a*x))*a**6*x**6 + 18*sqrt(atan(a*x))*a**4*x**4 + 36*sqr
t(atan(a*x))*a**2*x**2 + 24*int(sqrt(atan(a*x))/x,x) - 2*int((sqrt(atan(a*
x))*x**6)/(atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**7 - 9*int((sqrt(atan(a*x
))*x**4)/(atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**5 - 18*int((sqrt(atan(a*x
))*x**2)/(atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**3))/24
```

# 3.697 $\int \frac{x^m \sqrt{\arctan(ax)}}{c+a^2cx^2} \, dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m \sqrt{\arctan(ax)}}{c+a^2cx^2} \, dx = \text{Int}\left(\frac{x^m \sqrt{\arctan(ax)}}{c+a^2cx^2}, x\right)$$

output
```
Defer(Int)(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \sqrt{\arctan(ax)}}{c+a^2cx^2} \, dx = \int \frac{x^m \sqrt{\arctan(ax)}}{c+a^2cx^2} \, dx$$

input
```
Integrate[(x^m*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2),x]
```

output
```
Integrate[(x^m*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \sqrt{\arctan(ax)}}{a^2cx^2+c} \, dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x^m \sqrt{\arctan(ax)}}{a^2cx^2+c}dx$$

input
```
Int[(x^m*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m \sqrt{\arctan(ax)}}{a^2c\,x^2+c}dx$$

input
```
int(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x)
```

output
```
int(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x)
```

## Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \sqrt{\arctan(ax)}}{c+a^2cx^2}\,dx = \int \frac{x^m \sqrt{\arctan(ax)}}{a^2cx^2+c}\,dx$$

input
```
integrate(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
integral(x^m*sqrt(arctan(a*x))/(a^2*c*x^2 + c), x)
```

## Sympy [N/A]

Not integrable

Time = 4.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m \sqrt{\arctan(ax)}}{c+a^2cx^2}\,dx = \frac{\int \frac{x^m \sqrt{\operatorname{atan}(ax)}}{a^2x^2+1}\,dx}{c}$$

input
```
integrate(x**m*atan(a*x)**(1/2)/(a**2*c*x**2+c),x)
```

output
```
Integral(x**m*sqrt(atan(a*x))/(a**2*x**2 + 1), x)/c
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m \sqrt{\arctan(ax)}}{c+a^2cx^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \sqrt{\arctan(ax)}}{c + a^2 c x^2} \, dx = \int \frac{x^m \sqrt{\arctan(ax)}}{a^2 c x^2 + c} \, dx$$

input
```
integrate(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
integrate(x^m*sqrt(arctan(a*x))/(a^2*c*x^2 + c), x)
```

## Mupad [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \sqrt{\arctan(ax)}}{c + a^2 c x^2} \, dx = \int \frac{x^m \sqrt{\mathrm{atan}(a\,x)}}{c\,a^2\,x^2 + c} \, dx$$

input
```
int((x^m*atan(a*x)^(1/2))/(c + a^2*c*x^2),x)
```

output
```
int((x^m*atan(a*x)^(1/2))/(c + a^2*c*x^2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \sqrt{\arctan(ax)}}{c + a^2 c x^2} \, dx = \frac{\int \frac{x^m \sqrt{atan(ax)}}{a^2 x^2 + 1} dx}{c}$$

input
```
int(x^m*atan(a*x)^(1/2)/(a^2*c*x^2+c),x)
```

output
```
int((x**m*sqrt(atan(a*x)))/(a**2*x**2 + 1),x)/c
```

# 3.698 $\int \frac{x^3\sqrt{\arctan(ax)}}{c+a^2cx^2}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^3\sqrt{\arctan(ax)}}{c+a^2cx^2}\,dx = -\frac{2x\arctan(ax)^{3/2}}{3a^3c}+\frac{\mathrm{Int}\left(x\sqrt{\arctan(ax)},x\right)}{a^2c}+\frac{2\mathrm{Int}\left(\arctan(ax)^{3/2},x\right)}{3a^3c}$$

output
```
-2/3*x*arctan(a*x)^(3/2)/a^3/c+Defer(Int)(x*arctan(a*x)^(1/2),x)/a^2/c+2/3
*Defer(Int)(arctan(a*x)^(3/2),x)/a^3/c
```

## Mathematica [N/A]

Not integrable

Time = 2.44 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^3\sqrt{\arctan(ax)}}{c+a^2cx^2}\,dx = \int \frac{x^3\sqrt{\arctan(ax)}}{c+a^2cx^2}\,dx$$

input
```
Integrate[(x^3*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2),x]
```

output
```
Integrate[(x^3*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\sqrt{\arctan(ax)}}{a^2cx^2+c}dx$$

$$\Big\downarrow \text{5451}$$

$$\frac{\int x\sqrt{\arctan(ax)}dx}{a^2c}-\frac{\int \frac{x\sqrt{\arctan(ax)}}{c(a^2x^2+1)}dx}{a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int x\sqrt{\arctan(ax)}dx}{a^2c}-\frac{\int \frac{x\sqrt{\arctan(ax)}}{a^2x^2+1}dx}{a^2c}$$

$$\Big\downarrow \text{5377}$$

$$\frac{\int x\sqrt{\arctan(ax)}dx}{a^2c}-\frac{\int \frac{x\sqrt{\arctan(ax)}}{a^2x^2+1}dx}{a^2c}$$

$$\Big\downarrow \text{5457}$$

$$\frac{\int x\sqrt{\arctan(ax)}dx}{a^2c}-\frac{\frac{2x\arctan(ax)^{3/2}}{3a}-\frac{2\int \arctan(ax)^{3/2}dx}{3a}}{a^2c}$$

$$\Big\downarrow \text{5353}$$

$$\frac{\int x\sqrt{\arctan(ax)}dx}{a^2c}-\frac{\frac{2x\arctan(ax)^{3/2}}{3a}-\frac{2\int \arctan(ax)^{3/2}dx}{3a}}{a^2c}$$

input
```
Int[(x^3*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^3 \sqrt{\arctan(ax)}}{a^2 c\, x^2 + c} dx$$

input
```
int(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x)
```

output
```
int(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3 \sqrt{\arctan(ax)}}{c + a^2 c x^2} dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^3\sqrt{\arctan(ax)}}{c+a^2cx^2}\,dx = \frac{\int \frac{x^3\sqrt{\operatorname{atan}(ax)}}{a^2x^2+1}\,dx}{c}$$

input
```
integrate(x**3*atan(a*x)**(1/2)/(a**2*c*x**2+c),x)
```

output
```
Integral(x**3*sqrt(atan(a*x))/(a**2*x**2 + 1), x)/c
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3\sqrt{\arctan(ax)}}{c+a^2cx^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^3\sqrt{\arctan(ax)}}{c+a^2cx^2}\,dx = \int \frac{x^3\sqrt{\arctan(ax)}}{a^2cx^2+c}\,dx$$

input
```
integrate(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
integrate(x^3*sqrt(arctan(a*x))/(a^2*c*x^2 + c), x)
```

## Mupad [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^3\sqrt{\arctan(ax)}}{c+a^2cx^2}\,dx = \int \frac{x^3\,\sqrt{\mathrm{atan}\left(a\,x\right)}}{c\,a^2\,x^2+c}\,dx$$

input
```
int((x^3*atan(a*x)^(1/2))/(c + a^2*c*x^2),x)
```

output
```
int((x^3*atan(a*x)^(1/2))/(c + a^2*c*x^2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^3\sqrt{\arctan(ax)}}{c+a^2cx^2}\,dx = \frac{\int \frac{\sqrt{atan(ax)}\,x^3}{a^2x^2+1}dx}{c}$$

input
```
int(x^3*atan(a*x)^(1/2)/(a^2*c*x^2+c),x)
```

output
```
int((sqrt(atan(a*x))*x**3)/(a**2*x**2 + 1),x)/c
```

# 3.699 $\int \frac{x^2\sqrt{\arctan(ax)}}{c+a^2cx^2}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^2\sqrt{\arctan(ax)}}{c+a^2cx^2}\,dx = -\frac{2\arctan(ax)^{3/2}}{3a^3c}+\frac{\text{Int}\left(\sqrt{\arctan(ax)},x\right)}{a^2c}$$

output
```
-2/3*arctan(a*x)^(3/2)/a^3/c+Defer(Int)(arctan(a*x)^(1/2),x)/a^2/c
```

## Mathematica [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^2\sqrt{\arctan(ax)}}{c+a^2cx^2}\,dx = \int \frac{x^2\sqrt{\arctan(ax)}}{c+a^2cx^2}\,dx$$

input
```
Integrate[(x^2*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2),x]
```

output
```
Integrate[(x^2*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{\arctan(ax)}}{a^2cx^2+c}dx$$

$$\Big\downarrow \text{ 5451}$$

$$\frac{\int \sqrt{\arctan(ax)}dx}{a^2c}-\frac{\int \frac{\sqrt{\arctan(ax)}}{c(a^2x^2+1)}dx}{a^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \sqrt{\arctan(ax)}dx}{a^2c}-\frac{\int \frac{\sqrt{\arctan(ax)}}{a^2x^2+1}dx}{a^2c}$$

$$\Big\downarrow \text{ 5353}$$

$$\frac{\int \sqrt{\arctan(ax)}dx}{a^2c}-\frac{\int \frac{\sqrt{\arctan(ax)}}{a^2x^2+1}dx}{a^2c}$$

$$\Big\downarrow \text{ 5419}$$

$$\frac{\int \sqrt{\arctan(ax)}dx}{a^2c}-\frac{2\arctan(ax)^{3/2}}{3a^3c}$$

input
```
Int[(x^2*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^2 \sqrt{\arctan\left(ax\right)}}{a^2 c\, x^2 + c} dx$$

input
```
int(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x)
```

output
```
int(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2 \sqrt{\arctan(ax)}}{c + a^2 c x^2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^2 \sqrt{\arctan(ax)}}{c + a^2 c x^2} \, dx = \frac{\int \frac{x^2 \sqrt{\operatorname{atan}(ax)}}{a^2 x^2 + 1} \, dx}{c}$$

input
```
integrate(x**2*atan(a*x)**(1/2)/(a**2*c*x**2+c),x)
```

output
```
Integral(x**2*sqrt(atan(a*x))/(a**2*x**2 + 1), x)/c
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2\sqrt{\arctan(ax)}}{c+a^2cx^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^2\sqrt{\arctan(ax)}}{c+a^2cx^2}\,dx = \int \frac{x^2\sqrt{\arctan(ax)}}{a^2cx^2+c}\,dx$$

input
```
integrate(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
integrate(x^2*sqrt(arctan(a*x))/(a^2*c*x^2 + c), x)
```

## Mupad [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^2\sqrt{\arctan(ax)}}{c+a^2cx^2}\,dx = \int \frac{x^2\,\sqrt{\mathrm{atan}\left(a\,x\right)}}{c\,a^2\,x^2+c}\,dx$$

input
```
int((x^2*atan(a*x)^(1/2))/(c + a^2*c*x^2),x)
```

output
```
int((x^2*atan(a*x)^(1/2))/(c + a^2*c*x^2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.54

$$\int \frac{x^2\sqrt{\arctan(ax)}}{c+a^2cx^2}\,dx = \frac{-4\sqrt{atan\left(ax\right)}\,atan(ax)+6\sqrt{atan\left(ax\right)}\,ax-3\left(\int \frac{\sqrt{atan(ax)}\,x}{atan(ax)a^2x^2+atan(ax)}dx\right)a^2}{6a^3c}$$

input
```
int(x^2*atan(a*x)^(1/2)/(a^2*c*x^2+c),x)
```

output
```
( - 4*sqrt(atan(a*x))*atan(a*x) + 6*sqrt(atan(a*x))*a*x - 3*int((sqrt(atan
(a*x))*x)/(atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**2)/(6*a**3*c)
```

# 3.700 $\int \frac{x\sqrt{\arctan(ax)}}{c+a^2cx^2}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x\sqrt{\arctan(ax)}}{c+a^2cx^2}\,dx = \frac{2x\arctan(ax)^{3/2}}{3ac} - \frac{2\mathrm{Int}\left(\arctan(ax)^{3/2},x\right)}{3ac}$$

output
```
2/3*x*arctan(a*x)^(3/2)/a/c-2/3*Defer(Int)(arctan(a*x)^(3/2),x)/a/c
```

## Mathematica [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x\sqrt{\arctan(ax)}}{c+a^2cx^2}\,dx = \int \frac{x\sqrt{\arctan(ax)}}{c+a^2cx^2}\,dx$$

input
```
Integrate[(x*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2),x]
```

output
```
Integrate[(x*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{\arctan(ax)}}{a^2cx^2+c}dx$$

$$\Big\downarrow \text{ 5457}$$

$$\frac{2x\arctan(ax)^{3/2}}{3ac}-\frac{2\int \arctan(ax)^{3/2}dx}{3ac}$$

$$\Big\downarrow \text{ 5353}$$

$$\frac{2x\arctan(ax)^{3/2}}{3ac}-\frac{2\int \arctan(ax)^{3/2}dx}{3ac}$$

input
```
Int[(x*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x\sqrt{\arctan\left(ax\right)}}{a^2c\,x^2+c}dx$$

input
```
int(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x)
```

output
```
int(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x\sqrt{\arctan(ax)}}{c+a^2cx^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x\sqrt{\arctan(ax)}}{c+a^2cx^2}\,dx = \frac{\int \frac{x\sqrt{\operatorname{atan}(ax)}}{a^2x^2+1}\,dx}{c}$$

input
```
integrate(x*atan(a*x)**(1/2)/(a**2*c*x**2+c),x)
```

output
```
Integral(x*sqrt(atan(a*x))/(a**2*x**2 + 1), x)/c
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x\sqrt{\arctan(ax)}}{c+a^2cx^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x\sqrt{\arctan(ax)}}{c+a^2cx^2}\,dx=\int \frac{x\sqrt{\arctan{(ax)}}}{a^2cx^2+c}\,dx$$

input
```
integrate(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
integrate(x*sqrt(arctan(a*x))/(a^2*c*x^2 + c), x)
```

## Mupad [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x\sqrt{\arctan(ax)}}{c+a^2cx^2}\,dx=\int \frac{x\,\sqrt{\mathrm{atan}\left(a\,x\right)}}{c\,a^2\,x^2+c}\,dx$$

input
```
int((x*atan(a*x)^(1/2))/(c + a^2*c*x^2),x)
```

output
```
int((x*atan(a*x)^(1/2))/(c + a^2*c*x^2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x\sqrt{\arctan(ax)}}{c+a^2cx^2}\,dx=\frac{\int \frac{\sqrt{atan(ax)}\,x}{a^2x^2+1}dx}{c}$$

input
```
int(x*atan(a*x)^(1/2)/(a^2*c*x^2+c),x)
```

output
```
int((sqrt(atan(a*x))*x)/(a**2*x**2 + 1),x)/c
```

# 3.701 $\int \frac{\sqrt{\arctan(ax)}}{c+a^2cx^2}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 18

$$\int \frac{\sqrt{\arctan(ax)}}{c+a^2cx^2}\,dx=\frac{2\arctan(ax)^{3/2}}{3ac}$$

output
```
2/3*arctan(a*x)^(3/2)/a/c
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\arctan(ax)}}{c+a^2cx^2}\,dx=\frac{2\arctan(ax)^{3/2}}{3ac}$$

input
```
Integrate[Sqrt[ArcTan[a*x]]/(c + a^2*c*x^2),x]
```

output
```
(2*ArcTan[a*x]^(3/2))/(3*a*c)
```

## Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\arctan(ax)}}{a^2cx^2+c}\,dx$$
$$\Big\downarrow \text{5419}$$
$$\frac{2\arctan(ax)^{3/2}}{3ac}$$

input
```
Int[Sqrt[ArcTan[a*x]]/(c + a^2*c*x^2),x]
```

output
```
(2*ArcTan[a*x]^(3/2))/(3*a*c)
```

### Defintions of rubi rules used

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

## Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

| method | result | size |
|---|---|---|
| default | $\frac{2\arctan(ax)^{\frac{3}{2}}}{3ac}$ | 15 |

input
```
int(arctan(a*x)^(1/2)/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
2/3*arctan(a*x)^(3/2)/a/c
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{\arctan(ax)}}{c+a^2cx^2}\,dx = \frac{2\arctan{(ax)}^{\frac{3}{2}}}{3\,ac}$$

input
```
integrate(arctan(a*x)^(1/2)/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
2/3*arctan(a*x)^(3/2)/(a*c)
```

## Sympy [F]

$$\int \frac{\sqrt{\arctan(ax)}}{c+a^2cx^2}\,dx = \frac{\int \frac{\sqrt{\operatorname{atan}{(ax)}}}{a^2x^2+1}\,dx}{c}$$

input
```
integrate(atan(a*x)**(1/2)/(a**2*c*x**2+c),x)
```

output
```
Integral(sqrt(atan(a*x))/(a**2*x**2 + 1), x)/c
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{c+a^2cx^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(1/2)/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{\arctan(ax)}}{c+a^2cx^2}\,dx=\frac{2\arctan{(ax)}^{\frac{3}{2}}}{3\,ac}$$

input
```
integrate(arctan(a*x)^(1/2)/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
2/3*arctan(a*x)^(3/2)/(a*c)
```

## Mupad [B] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{\arctan(ax)}}{c+a^2cx^2}\,dx=\frac{2\,\mathrm{atan}(a\,x)^{3/2}}{3\,a\,c}$$

input
```
int(atan(a*x)^(1/2)/(c + a^2*c*x^2),x)
```

output
```
(2*atan(a*x)^(3/2))/(3*a*c)
```

## Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{\arctan(ax)}}{c+a^2cx^2}\,dx=\frac{2\sqrt{atan\,(ax)}\,atan(ax)}{3ac}$$

input
```
int(atan(a*x)^(1/2)/(a^2*c*x^2+c),x)
```

output
```
(2*sqrt(atan(a*x))*atan(a*x))/(3*a*c)
```

# 3.702 $\int \frac{\sqrt{\arctan(ax)}}{x(c+a^2cx^2)}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\sqrt{\arctan(ax)}}{x(c+a^2cx^2)}\,dx = -\frac{2i\arctan(ax)^{3/2}}{3c} + \frac{i\text{Int}\left(\frac{\sqrt{\arctan(ax)}}{x(i+ax)},x\right)}{c}$$

output
```
-2/3*I*arctan(a*x)^(3/2)/c+I*Defer(Int)(arctan(a*x)^(1/2)/x/(I+a*x),x)/c
```

## Mathematica [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{\arctan(ax)}}{x(c+a^2cx^2)}\,dx = \int \frac{\sqrt{\arctan(ax)}}{x(c+a^2cx^2)}\,dx$$

input
```
Integrate[Sqrt[ArcTan[a*x]]/(x*(c + a^2*c*x^2)),x]
```

output
```
Integrate[Sqrt[ArcTan[a*x]]/(x*(c + a^2*c*x^2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\arctan(ax)}}{x(a^2cx^2+c)}dx$$

$$\Big\downarrow \text{ 5459}$$

$$\frac{i\int \frac{\sqrt{\arctan(ax)}}{x(ax+i)}dx}{c}-\frac{2i\arctan(ax)^{3/2}}{3c}$$

$$\Big\downarrow \text{ 5560}$$

$$\frac{i\int \frac{\sqrt{\arctan(ax)}}{x(ax+i)}dx}{c}-\frac{2i\arctan(ax)^{3/2}}{3c}$$

input
```
Int[Sqrt[ArcTan[a*x]]/(x*(c + a^2*c*x^2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{\arctan(ax)}}{x(a^2c\,x^2+c)}dx$$

input
```
int(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c),x)
```

output
```
int(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{x\left(c+a^2cx^2\right)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{\arctan(ax)}}{x\left(c+a^2cx^2\right)}\,dx = \frac{\int \frac{\sqrt{\operatorname{atan}(ax)}}{a^2x^3+x}\,dx}{c}$$

input
```
integrate(atan(a*x)**(1/2)/x/(a**2*c*x**2+c),x)
```

output
```
Integral(sqrt(atan(a*x))/(a**2*x**3 + x), x)/c
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{x\left(c+a^2cx^2\right)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\arctan(ax)}}{x\left(c+a^2cx^2\right)}\,dx = \int \frac{\sqrt{\arctan\left(ax\right)}}{(a^2cx^2+c)x}\,dx$$

input
```
integrate(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
integrate(sqrt(arctan(a*x))/((a^2*c*x^2 + c)*x), x)
```

## Mupad [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\arctan(ax)}}{x\left(c+a^2cx^2\right)}\,dx = \int \frac{\sqrt{\mathrm{atan}\left(a\,x\right)}}{x\,\left(c\,a^2\,x^2+c\right)}\,dx$$

input
```
int(atan(a*x)^(1/2)/(x*(c + a^2*c*x^2)),x)
```

output
```
int(atan(a*x)^(1/2)/(x*(c + a^2*c*x^2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{\arctan(ax)}}{x\left(c+a^2cx^2\right)}\,dx = \frac{\int \frac{\sqrt{atan(ax)}}{a^2x^3+x}dx}{c}$$

input
```
int(atan(a*x)^(1/2)/x/(a^2*c*x^2+c),x)
```

output
```
int(sqrt(atan(a*x))/(a**2*x**3 + x),x)/c
```

# 3.703 $\int \frac{\sqrt{\arctan(ax)}}{x^2(c+a^2cx^2)} dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\sqrt{\arctan(ax)}}{x^2\left(c+a^2cx^2\right)} dx = -\frac{2a\arctan(ax)^{3/2}}{3c} + \frac{\operatorname{Int}\left(\frac{\sqrt{\arctan(ax)}}{x^2},x\right)}{c}$$

output
```
-2/3*a*arctan(a*x)^(3/2)/c+Defer(Int)(arctan(a*x)^(1/2)/x^2,x)/c
```

## Mathematica [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{\arctan(ax)}}{x^2\left(c+a^2cx^2\right)} dx = \int \frac{\sqrt{\arctan(ax)}}{x^2\left(c+a^2cx^2\right)} dx$$

input
```
Integrate[Sqrt[ArcTan[a*x]]/(x^2*(c + a^2*c*x^2)),x]
```

output
```
Integrate[Sqrt[ArcTan[a*x]]/(x^2*(c + a^2*c*x^2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\arctan(ax)}}{x^2 (a^2cx^2+c)}dx$$

$$\Big\downarrow \text{5453}$$

$$\frac{\int \frac{\sqrt{\arctan(ax)}}{x^2}dx}{c} - a^2 \int \frac{\sqrt{\arctan(ax)}}{c (a^2x^2+1)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{\arctan(ax)}}{x^2}dx}{c} - \frac{a^2 \int \frac{\sqrt{\arctan(ax)}}{a^2x^2+1}dx}{c}$$

$$\Big\downarrow \text{5377}$$

$$\frac{\int \frac{\sqrt{\arctan(ax)}}{x^2}dx}{c} - \frac{a^2 \int \frac{\sqrt{\arctan(ax)}}{a^2x^2+1}dx}{c}$$

$$\Big\downarrow \text{5419}$$

$$\frac{\int \frac{\sqrt{\arctan(ax)}}{x^2}dx}{c} - \frac{2a \arctan(ax)^{3/2}}{3c}$$

input
```
Int[Sqrt[ArcTan[a*x]]/(x^2*(c + a^2*c*x^2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{\arctan (ax)}}{x^2 (a^2c\,x^2 + c)}dx$$

input
```
int(arctan(a*x)^(1/2)/x^2/(a^2*c*x^2+c),x)
```

output
```
int(arctan(a*x)^(1/2)/x^2/(a^2*c*x^2+c),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{x^2 (c + a^2cx^2)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(1/2)/x^2/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 0.90 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{\arctan(ax)}}{x^2 (c + a^2cx^2)}\,dx = \frac{\int \frac{\sqrt{\operatorname{atan}(ax)}}{a^2x^4+x^2}\,dx}{c}$$

input
```
integrate(atan(a*x)**(1/2)/x**2/(a**2*c*x**2+c),x)
```

output
```
Integral(sqrt(atan(a*x))/(a**2*x**4 + x**2), x)/c
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{x^2\left(c+a^2cx^2\right)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(1/2)/x^2/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\arctan(ax)}}{x^2\left(c+a^2cx^2\right)}\,dx = \int \frac{\sqrt{\arctan\left(ax\right)}}{(a^2cx^2+c)x^2}\,dx$$

input
```
integrate(arctan(a*x)^(1/2)/x^2/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
integrate(sqrt(arctan(a*x))/((a^2*c*x^2 + c)*x^2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\arctan(ax)}}{x^2\,(c+a^2cx^2)}\,dx=\int \frac{\sqrt{\mathrm{atan}\,(a\,x)}}{x^2\,\left(c\,a^2\,x^2+c\right)}\,dx$$

input
```
int(atan(a*x)^(1/2)/(x^2*(c + a^2*c*x^2)),x)
```

output
```
int(atan(a*x)^(1/2)/(x^2*(c + a^2*c*x^2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.54

$$\int \frac{\sqrt{\arctan(ax)}}{x^2\,(c+a^2cx^2)}\,dx$$
$$=\frac{-4\sqrt{atan\,(ax)}\,atan(ax)\,ax-6\sqrt{atan\,(ax)}+3\left(\int \frac{\sqrt{atan(ax)}}{atan(ax)a^2x^3+atan(ax)x}dx\right)ax}{6cx}$$

input
```
int(atan(a*x)^(1/2)/x^2/(a^2*c*x^2+c),x)
```

output
```
( - 4*sqrt(atan(a*x))*atan(a*x)*a*x - 6*sqrt(atan(a*x)) + 3*int(sqrt(atan(
a*x))/(atan(a*x)*a**2*x**3 + atan(a*x)*x),x)*a*x)/(6*c*x)
```

# 3.704 $\int \frac{\sqrt{\arctan(ax)}}{x^3(c+a^2cx^2)}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\sqrt{\arctan(ax)}}{x^3(c+a^2cx^2)}\,dx = \frac{2ia^2\arctan(ax)^{3/2}}{3c} + \frac{\operatorname{Int}\left(\frac{\sqrt{\arctan(ax)}}{x^3},x\right)}{c} - \frac{ia^2\operatorname{Int}\left(\frac{\sqrt{\arctan(ax)}}{x(i+ax)},x\right)}{c}$$

output
```
2/3*I*a^2*arctan(a*x)^(3/2)/c+Defer(Int)(arctan(a*x)^(1/2)/x^3,x)/c-I*a^2*
Defer(Int)(arctan(a*x)^(1/2)/x/(I+a*x),x)/c
```

## Mathematica [N/A]

Not integrable

Time = 1.65 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{\arctan(ax)}}{x^3(c+a^2cx^2)}\,dx = \int \frac{\sqrt{\arctan(ax)}}{x^3(c+a^2cx^2)}\,dx$$

input
```
Integrate[Sqrt[ArcTan[a*x]]/(x^3*(c + a^2*c*x^2)),x]
```

output
```
Integrate[Sqrt[ArcTan[a*x]]/(x^3*(c + a^2*c*x^2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\arctan(ax)}}{x^3(a^2cx^2+c)}dx$$

$$\Big\downarrow \text{5453}$$

$$\frac{\int \frac{\sqrt{\arctan(ax)}}{x^3}dx}{c} - a^2\int \frac{\sqrt{\arctan(ax)}}{cx(a^2x^2+1)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{\arctan(ax)}}{x^3}dx}{c} - \frac{a^2\int \frac{\sqrt{\arctan(ax)}}{x(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \text{5377}$$

$$\frac{\int \frac{\sqrt{\arctan(ax)}}{x^3}dx}{c} - \frac{a^2\int \frac{\sqrt{\arctan(ax)}}{x(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \text{5459}$$

$$\frac{\int \frac{\sqrt{\arctan(ax)}}{x^3}dx}{c} - \frac{a^2\left(i\int \frac{\sqrt{\arctan(ax)}}{x(ax+i)}dx - \frac{2}{3}i\arctan(ax)^{3/2}\right)}{c}$$

$$\Big\downarrow \text{5560}$$

$$\frac{\int \frac{\sqrt{\arctan(ax)}}{x^3}dx}{c} - \frac{a^2\left(i\int \frac{\sqrt{\arctan(ax)}}{x(ax+i)}dx - \frac{2}{3}i\arctan(ax)^{3/2}\right)}{c}$$

input
```
Int[Sqrt[ArcTan[a*x]]/(x^3*(c + a^2*c*x^2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{\arctan(ax)}}{x^3(a^2c\,x^2+c)}dx$$

input
```
int(arctan(a*x)^(1/2)/x^3/(a^2*c*x^2+c),x)
```

output
```
int(arctan(a*x)^(1/2)/x^3/(a^2*c*x^2+c),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{x^3(c+a^2cx^2)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(1/2)/x^3/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 1.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{\arctan(ax)}}{x^3(c+a^2cx^2)}\,dx = \frac{\int \frac{\sqrt{\operatorname{atan}(ax)}}{a^2x^5+x^3}\,dx}{c}$$

input
```
integrate(atan(a*x)**(1/2)/x**3/(a**2*c*x**2+c),x)
```

output
```
Integral(sqrt(atan(a*x))/(a**2*x**5 + x**3), x)/c
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{x^3\left(c+a^2cx^2\right)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(1/2)/x^3/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\arctan(ax)}}{x^3\left(c+a^2cx^2\right)}\,dx = \int \frac{\sqrt{\arctan\left(ax\right)}}{(a^2cx^2+c)x^3}\,dx$$

input
```
integrate(arctan(a*x)^(1/2)/x^3/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
integrate(sqrt(arctan(a*x))/((a^2*c*x^2 + c)*x^3), x)
```

## Mupad [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\arctan(ax)}}{x^3\,(c+a^2cx^2)}\,dx = \int \frac{\sqrt{\mathrm{atan}\left(a\,x\right)}}{x^3\,\left(c\,a^2\,x^2+c\right)}\,dx$$

input
```
int(atan(a*x)^(1/2)/(x^3*(c + a^2*c*x^2)),x)
```

output
```
int(atan(a*x)^(1/2)/(x^3*(c + a^2*c*x^2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

$$\int \frac{\sqrt{\arctan(ax)}}{x^3\,(c+a^2cx^2)}\,dx = \frac{\int \frac{\sqrt{atan(ax)}}{a^2x^5+x^3}dx}{c}$$

input
```
int(atan(a*x)^(1/2)/x^3/(a^2*c*x^2+c),x)
```

output
```
int(sqrt(atan(a*x))/(a**2*x**5 + x**3),x)/c
```

# 3.705 $\int \frac{\sqrt{\arctan(ax)}}{x^4(c+a^2cx^2)}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\sqrt{\arctan(ax)}}{x^4\left(c+a^2cx^2\right)}\,dx=\frac{2a^3\arctan(ax)^{3/2}}{3c}+\frac{\operatorname{Int}\left(\frac{\sqrt{\arctan(ax)}}{x^4},x\right)}{c}-\frac{a^2\operatorname{Int}\left(\frac{\sqrt{\arctan(ax)}}{x^2},x\right)}{c}$$

output
```
2/3*a^3*arctan(a*x)^(3/2)/c+Defer(Int)(arctan(a*x)^(1/2)/x^4,x)/c-a^2*Defe
r(Int)(arctan(a*x)^(1/2)/x^2,x)/c
```

## Mathematica [N/A]

Not integrable

Time = 4.50 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{\arctan(ax)}}{x^4\left(c+a^2cx^2\right)}\,dx=\int \frac{\sqrt{\arctan(ax)}}{x^4\left(c+a^2cx^2\right)}\,dx$$

input
```
Integrate[Sqrt[ArcTan[a*x]]/(x^4*(c + a^2*c*x^2)),x]
```

output
```
Integrate[Sqrt[ArcTan[a*x]]/(x^4*(c + a^2*c*x^2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\arctan(ax)}}{x^4(a^2cx^2+c)}dx$$

$$\Big\downarrow \text{ 5453}$$

$$\frac{\int \frac{\sqrt{\arctan(ax)}}{x^4}dx}{c} - a^2\int \frac{\sqrt{\arctan(ax)}}{cx^2(a^2x^2+1)}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{\sqrt{\arctan(ax)}}{x^4}dx}{c} - \frac{a^2\int \frac{\sqrt{\arctan(ax)}}{x^2(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \text{ 5377}$$

$$\frac{\int \frac{\sqrt{\arctan(ax)}}{x^4}dx}{c} - \frac{a^2\int \frac{\sqrt{\arctan(ax)}}{x^2(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \text{ 5453}$$

$$\frac{\int \frac{\sqrt{\arctan(ax)}}{x^4}dx}{c} - \frac{a^2\left(\int \frac{\sqrt{\arctan(ax)}}{x^2}dx - a^2\int \frac{\sqrt{\arctan(ax)}}{a^2x^2+1}dx\right)}{c}$$

$$\Big\downarrow \text{ 5377}$$

$$\frac{\int \frac{\sqrt{\arctan(ax)}}{x^4}dx}{c} - \frac{a^2\left(\int \frac{\sqrt{\arctan(ax)}}{x^2}dx - a^2\int \frac{\sqrt{\arctan(ax)}}{a^2x^2+1}dx\right)}{c}$$

$$\Big\downarrow \text{ 5419}$$

$$\frac{\int \frac{\sqrt{\arctan(ax)}}{x^4}dx}{c} - \frac{a^2\left(\int \frac{\sqrt{\arctan(ax)}}{x^2}dx - \frac{2}{3}a\arctan(ax)^{3/2}\right)}{c}$$

input
```
Int[Sqrt[ArcTan[a*x]]/(x^4*(c + a^2*c*x^2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{\arctan(ax)}}{x^4 (a^2 c x^2 + c)} dx$$

input
```
int(arctan(a*x)^(1/2)/x^4/(a^2*c*x^2+c),x)
```

output
```
int(arctan(a*x)^(1/2)/x^4/(a^2*c*x^2+c),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{x^4 (c + a^2 c x^2)} dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(1/2)/x^4/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 1.58 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{\arctan(ax)}}{x^4 (c + a^2cx^2)} \, dx = \frac{\int \frac{\sqrt{\operatorname{atan}(ax)}}{a^2x^6+x^4} \, dx}{c}$$

input
```
integrate(atan(a*x)**(1/2)/x**4/(a**2*c*x**2+c),x)
```

output
```
Integral(sqrt(atan(a*x))/(a**2*x**6 + x**4), x)/c
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{x^4 (c + a^2cx^2)} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(1/2)/x^4/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\arctan(ax)}}{x^4 (c + a^2cx^2)} \, dx = \int \frac{\sqrt{\arctan(ax)}}{(a^2cx^2 + c)x^4} \, dx$$

input
```
integrate(arctan(a*x)^(1/2)/x^4/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
integrate(sqrt(arctan(a*x))/((a^2*c*x^2 + c)*x^4), x)
```

## Mupad [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\arctan(ax)}}{x^4\left(c+a^2cx^2\right)}\,dx=\int \frac{\sqrt{\mathrm{atan}\left(a\,x\right)}}{x^4\,\left(c\,a^2\,x^2+c\right)} \,d x$$

input
```
int(atan(a*x)^(1/2)/(x^4*(c + a^2*c*x^2)),x)
```

output
```
int(atan(a*x)^(1/2)/(x^4*(c + a^2*c*x^2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 117, normalized size of antiderivative = 4.88

$$\int \frac{\sqrt{\arctan(ax)}}{x^4\left(c+a^2cx^2\right)}\,dx = \frac{4\sqrt{atan\left(ax\right)}\,atan(ax)\,a^3x^3+6\sqrt{atan\left(ax\right)}\,a^2x^2-2\sqrt{atan\left(ax\right)}+\left(\int \frac{\sqrt{atan(ax)}}{atan(ax)a^2x^5+atan(ax)x^3}dx\right)a\,x^3-3}{6c\,x^3}$$

input
```
int(atan(a*x)^(1/2)/x^4/(a^2*c*x^2+c),x)
```

output
```
(4*sqrt(atan(a*x))*atan(a*x)*a**3*x**3 + 6*sqrt(atan(a*x))*a**2*x**2 - 2*s
qrt(atan(a*x)) + int(sqrt(atan(a*x))/(atan(a*x)*a**2*x**5 + atan(a*x)*x**3
),x)*a*x**3 - 3*int(sqrt(atan(a*x))/(atan(a*x)*a**2*x**3 + atan(a*x)*x),x)
*a**3*x**3)/(6*c*x**3)
```

# 3.706 $\int \frac{x^m \sqrt{\arctan(ax)}}{(c+a^2cx^2)^2} \, dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m \sqrt{\arctan(ax)}}{(c+a^2cx^2)^2} \, dx = \text{Int}\left(\frac{x^m \sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}, x\right)$$

output
```
Defer(Int)(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x)
```

## Mathematica [N/A]

Not integrable

Time = 1.44 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \sqrt{\arctan(ax)}}{(c+a^2cx^2)^2} \, dx = \int \frac{x^m \sqrt{\arctan(ax)}}{(c+a^2cx^2)^2} \, dx$$

input
```
Integrate[(x^m*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^2,x]
```

output
```
Integrate[(x^m*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^2, x]
```

## Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \sqrt{\arctan(ax)}}{(a^2cx^2+c)^2} \, dx$$
$$\Big\downarrow \ 5560$$
$$\int \frac{x^m \sqrt{\arctan(ax)}}{(a^2cx^2+c)^2}dx$$

input
```
Int[(x^m*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^2,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m \sqrt{\arctan(ax)}}{(a^2c\,x^2+c)^2}dx$$

input
```
int(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x)
```

output
```
int(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.62

$$\int \frac{x^m \sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \int \frac{x^m \sqrt{\arctan(ax)}}{(a^2cx^2+c)^2}\,dx$$

input
```
integrate(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
integral(x^m*sqrt(arctan(a*x))/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2), x)
```

## Sympy [N/A]

Not integrable

Time = 23.39 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.33

$$\int \frac{x^m \sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \frac{\int \frac{x^m \sqrt{\operatorname{atan}(ax)}}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

input
```
integrate(x**m*atan(a*x)**(1/2)/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(x**m*sqrt(atan(a*x))/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m \sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x^m\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,2,1,0,0]%%%} / %%%{1,[0,0,0,1,2]%%%} Error: Bad Argum
ent Value
```

## Mupad [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \int \frac{x^m\,\sqrt{\mathrm{atan}\left(a\,x\right)}}{{\left(c\,a^2\,x^2+c\right)}^2}\,dx$$

input
```
int((x^m*atan(a*x)^(1/2))/(c + a^2*c*x^2)^2,x)
```

output
```
int((x^m*atan(a*x)^(1/2))/(c + a^2*c*x^2)^2, x)
```

## Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.42

$$\int \frac{x^m \sqrt{\arctan(ax)}}{\left(c + a^2 c x^2\right)^2} \, dx = \frac{\int \frac{x^m \sqrt{atan(ax)}}{a^4 x^4 + 2a^2 x^2 + 1} dx}{c^2}$$

input

```
int(x^m*atan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x)
```

output

```
int((x**m*sqrt(atan(a*x)))/(a**4*x**4 + 2*a**2*x**2 + 1),x)/c**2
```

# 3.707 $\int \frac{x^3\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^3\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \text{Int}\left(\frac{x^3\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2},x\right)$$

output
```
Defer(Int)(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x)
```

## Mathematica [N/A]

Not integrable

Time = 2.92 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^3\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \int \frac{x^3\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx$$

input
```
Integrate[(x^3*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^2,x]
```

output
```
Integrate[(x^3*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^2, x]
```

## Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\sqrt{\arctan(ax)}}{(a^2cx^2+c)^2}dx$$

$$\Big\downarrow \text{5560}$$

$$\int \frac{x^3\sqrt{\arctan(ax)}}{(a^2cx^2+c)^2}dx$$

input
```
Int[(x^3*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^2,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^3\sqrt{\arctan(ax)}}{(a^2c\,x^2+c)^2}dx$$

input
```
int(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x)
```

output
```
int(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 0.92 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.33

$$\int \frac{x^3\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \frac{\int \frac{x^3\sqrt{\operatorname{atan}(ax)}}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

input
```
integrate(x**3*atan(a*x)**(1/2)/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(x**3*sqrt(atan(a*x))/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 1.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^3\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \int \frac{x^3\sqrt{\arctan{(ax)}}}{(a^2cx^2+c)^2}\,dx$$

input
```
integrate(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate(x^3*sqrt(arctan(a*x))/(a^2*c*x^2 + c)^2, x)
```

## Mupad [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^3\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \int \frac{x^3\,\sqrt{\mathrm{atan}\left(a\,x\right)}}{{\left(c\,a^2\,x^2+c\right)}^2}\,dx$$

input
```
int((x^3*atan(a*x)^(1/2))/(c + a^2*c*x^2)^2,x)
```

output
```
int((x^3*atan(a*x)^(1/2))/(c + a^2*c*x^2)^2, x)
```

## Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.42

$$\int \frac{x^3 \sqrt{\arctan(ax)}}{(c + a^2cx^2)^2} \, dx = \frac{\int \frac{\sqrt{atan(ax)}\, x^3}{a^4x^4+2a^2x^2+1} dx}{c^2}$$

input

```
int(x^3*atan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x)
```

output

```
int((sqrt(atan(a*x))*x**3)/(a**4*x**4 + 2*a**2*x**2 + 1),x)/c**2
```

# 3.708 $\int \frac{x^2\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 80

$$\int \frac{x^2\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx=-\frac{x\sqrt{\arctan(ax)}}{2a^2c^2(1+a^2x^2)}+\frac{\arctan(ax)^{3/2}}{3a^3c^2}+\frac{\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{8a^3c^2}$$

output
```
-1/2*x*arctan(a*x)^(1/2)/a^2/c^2/(a^2*x^2+1)+1/3*arctan(a*x)^(3/2)/a^3/c^2
+1/8*Pi^(1/2)*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2))/a^3/c^2
```

## Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.82

$$\int \frac{x^2\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx =\frac{4\sqrt{\arctan(ax)}\left(-\frac{3ax}{1+a^2x^2}+2\arctan(ax)\right)+3\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{24a^3c^2}$$

input
```
Integrate[(x^2*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^2,x]
```

output
```
(4*Sqrt[ArcTan[a*x]]*((-3*a*x)/(1 + a^2*x^2) + 2*ArcTan[a*x]) + 3*Sqrt[Pi]
*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(24*a^3*c^2)
```

## Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5471, 27, 5505, 4906, 27, 3042, 3786, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{\arctan(ax)}}{(a^2cx^2+c)^2}dx$$

$$\Big\downarrow \text{5471}$$

$$\frac{\int \frac{x}{c^2(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx}{4a}+\frac{\arctan(ax)^{3/2}}{3a^3c^2}-\frac{x\sqrt{\arctan(ax)}}{2a^2c^2(a^2x^2+1)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{x}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx}{4ac^2}+\frac{\arctan(ax)^{3/2}}{3a^3c^2}-\frac{x\sqrt{\arctan(ax)}}{2a^2c^2(a^2x^2+1)}$$

$$\Big\downarrow \text{5505}$$

$$\frac{\int \frac{ax}{(a^2x^2+1)\sqrt{\arctan(ax)}}d\arctan(ax)}{4a^3c^2}+\frac{\arctan(ax)^{3/2}}{3a^3c^2}-\frac{x\sqrt{\arctan(ax)}}{2a^2c^2(a^2x^2+1)}$$

$$\Big\downarrow \text{4906}$$

$$\frac{\int \frac{\sin(2\arctan(ax))}{2\sqrt{\arctan(ax)}}d\arctan(ax)}{4a^3c^2}+\frac{\arctan(ax)^{3/2}}{3a^3c^2}-\frac{x\sqrt{\arctan(ax)}}{2a^2c^2(a^2x^2+1)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sin(2\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)}{8a^3c^2}+\frac{\arctan(ax)^{3/2}}{3a^3c^2}-\frac{x\sqrt{\arctan(ax)}}{2a^2c^2(a^2x^2+1)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sin(2\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)}{8a^3c^2}+\frac{\arctan(ax)^{3/2}}{3a^3c^2}-\frac{x\sqrt{\arctan(ax)}}{2a^2c^2\left(a^2x^2+1\right)}$$

$$\Big\downarrow \; 3786$$

$$\frac{\int \sin(2\arctan(ax))d\sqrt{\arctan(ax)}}{4a^3c^2}+\frac{\arctan(ax)^{3/2}}{3a^3c^2}-\frac{x\sqrt{\arctan(ax)}}{2a^2c^2\left(a^2x^2+1\right)}$$

$$\Big\downarrow \; 3832$$

$$\frac{\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{8a^3c^2}+\frac{\arctan(ax)^{3/2}}{3a^3c^2}-\frac{x\sqrt{\arctan(ax)}}{2a^2c^2\left(a^2x^2+1\right)}$$

input
```
Int[(x^2*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^2,x]
```

output
```
-1/2*(x*Sqrt[ArcTan[a*x]])/(a^2*c^2*(1 + a^2*x^2)) + ArcTan[a*x]^(3/2)/(3*
a^3*c^2) + (Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(8*a^3*c^2)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5471
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^2)/((d_) + (e_.)*(x_)^2)
^2, x_Symbol] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(2*b*c^3*d^2*(p + 1)), x]
 + (-Simp[x*((a + b*ArcTan[c*x])^p/(2*c^2*d*(d + e*x^2))), x] + Simp[b*(p/(
2*c))   Int[x*((a + b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

## Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.75

| method | result | size |
|---|---|---|
| default | $\frac{3\sqrt{\arctan(ax)}\,\sqrt{\pi}\,\operatorname{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+8\arctan(ax)^2-6\arctan(ax)\sin(2\arctan(ax))}{24a^3c^2\sqrt{\arctan(ax)}}$ | 60 |

input
```
int(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/24/a^3/c^2*(3*arctan(a*x)^(1/2)*Pi^(1/2)*FresnelS(2*arctan(a*x)^(1/2)/Pi
^(1/2))+8*arctan(a*x)^2-6*arctan(a*x)*sin(2*arctan(a*x)))/arctan(a*x)^(1/2
)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{x^2\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \frac{\int \frac{x^2\sqrt{\operatorname{atan}(ax)}}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

input
```
integrate(x**2*atan(a*x)**(1/2)/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(x**2*sqrt(atan(a*x))/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.14

$$\int \frac{x^2\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx$$
$$= \frac{\arctan(ax)^{\frac{3}{2}}}{3a^3c^2} - \frac{-(i-1)\sqrt{\pi}\,\text{erf}\left((i-1)\sqrt{\arctan(ax)}\right) - 4i\sqrt{\arctan(ax)}e^{(2i\arctan(ax))}}{32a^3c^2} - \frac{(i+1)\sqrt{\pi}\,\text{erf}\left(-(i+1)\sqrt{\arctan(ax)}\right) + 4i\sqrt{\arctan(ax)}e^{(-2i\arctan(ax))}}{32a^3c^2}$$

input
```
integrate(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
1/3*arctan(a*x)^(3/2)/(a^3*c^2) - 1/32*(-(I - 1)*sqrt(pi)*erf((I - 1)*sqrt
(arctan(a*x))) - 4*I*sqrt(arctan(a*x))*e^(2*I*arctan(a*x)))/(a^3*c^2) - 1/
32*((I + 1)*sqrt(pi)*erf(-(I + 1)*sqrt(arctan(a*x))) + 4*I*sqrt(arctan(a*x
))*e^(-2*I*arctan(a*x)))/(a^3*c^2)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \int \frac{x^2\sqrt{\mathrm{atan}(a\,x)}}{(c\,a^2\,x^2+c)^2}\,dx$$

input
```
int((x^2*atan(a*x)^(1/2))/(c + a^2*c*x^2)^2,x)
```

output
```
int((x^2*atan(a*x)^(1/2))/(c + a^2*c*x^2)^2, x)
```

## Reduce [F]

$$\int \frac{x^2 \sqrt{\arctan(ax)}}{(c + a^2 c x^2)^2} \, dx = \frac{2\sqrt{atan\left(ax\right)}\, atan(ax) - 3\left(\int \frac{\sqrt{atan(ax)}}{a^4x^4+2a^2x^2+1} dx\right) a}{3a^3c^2}$$

input
```
int(x^2*atan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x)
```

output
```
(2*sqrt(atan(a*x))*atan(a*x) - 3*int(sqrt(atan(a*x))/(a**4*x**4 + 2*a**2*x
**2 + 1),x)*a)/(3*a**3*c**2)
```

# 3.709 $\int \frac{x\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 79

$$\int \frac{x\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \frac{\sqrt{\arctan(ax)}}{4a^2c^2} - \frac{\sqrt{\arctan(ax)}}{2a^2c^2(1+a^2x^2)} + \frac{\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{8a^2c^2}$$

output
```
1/4*arctan(a*x)^(1/2)/a^2/c^2-1/2*arctan(a*x)^(1/2)/a^2/c^2/(a^2*x^2+1)+1/
8*Pi^(1/2)*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))/a^2/c^2
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.22 (sec), antiderivative size = 136, normalized size of antiderivative = 1.72

$$\int \frac{x\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx$$

$$= \frac{4\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right) + \frac{\frac{16(-1+a^2x^2)\arctan(ax)}{1+a^2x^2} - i\sqrt{2}\sqrt{-i\arctan(ax)}\,\Gamma\left(\frac{1}{2},-2i\arctan(ax)\right) + i\sqrt{2}\sqrt{i\arctan(ax)}\,\Gamma\left(\frac{1}{2},2i\,\text{arcta}\right.}{\sqrt{\arctan(ax)}}}{64a^2c^2}$$

input
```
Integrate[(x*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^2,x]
```

output
```
(4*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]] + ((16*(-1 + a^2*x^2)
*ArcTan[a*x])/(1 + a^2*x^2) - I*Sqrt[2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2,
(-2*I)*ArcTan[a*x]] + I*Sqrt[2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (2*I)*ArcTa
n[a*x]])/Sqrt[ArcTan[a*x]])/(64*a^2*c^2)
```

## Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {5465, 27, 5439, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{\arctan(ax)}}{(a^2cx^2+c)^2}dx$$

$$\Big\downarrow \ 5465$$

$$\frac{\int \frac{1}{c^2(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx}{4a} - \frac{\sqrt{\arctan(ax)}}{2a^2c^2(a^2x^2+1)}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{1}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx}{4ac^2} - \frac{\sqrt{\arctan(ax)}}{2a^2c^2(a^2x^2+1)}$$

$$\Big\downarrow \ 5439$$

$$\frac{\int \frac{1}{(a^2x^2+1)\sqrt{\arctan(ax)}}d\arctan(ax)}{4a^2c^2} - \frac{\sqrt{\arctan(ax)}}{2a^2c^2(a^2x^2+1)}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^2}{\sqrt{\arctan(ax)}}d\arctan(ax)}{4a^2c^2} - \frac{\sqrt{\arctan(ax)}}{2a^2c^2(a^2x^2+1)}$$

$$\Big\downarrow \ 3793$$

$$\frac{\int \left(\frac{\cos(2\arctan(ax))}{2\sqrt{\arctan(ax)}}+\frac{1}{2\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{4a^2c^2} - \frac{\sqrt{\arctan(ax)}}{2a^2c^2(a^2x^2+1)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\sqrt{\arctan(ax)}}{4a^2c^2}-\frac{\sqrt{\arctan(ax)}}{2a^2c^2\,(a^2x^2+1)}$$

input
```
Int[(x*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^2,x]
```

output
```
-1/2*Sqrt[ArcTan[a*x]]/(a^2*c^2*(1 + a^2*x^2)) + (Sqrt[ArcTan[a*x]] + (Sqr
t[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/2)/(4*a^2*c^2)
```

**Defintions of rubi rules used**

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

## Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.58

| method | result | size |
|---|---|---|
| default | $-\frac{2\cos(2\arctan(ax))\sqrt{\arctan(ax)}\,\sqrt{\pi}-\pi\ \mathrm{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{8a^{2}c^{2}\sqrt{\pi}}$ | 46 |

input
```
int(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/8/a^2/c^2*(2*cos(2*arctan(a*x))*arctan(a*x)^(1/2)*Pi^(1/2)-Pi*FresnelC(
2*arctan(a*x)^(1/2)/Pi^(1/2)))/Pi^(1/2)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{x\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \frac{\int \frac{x\sqrt{\operatorname{atan}(ax)}}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

input
```
integrate(x*atan(a*x)**(1/2)/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(x*sqrt(atan(a*x))/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F]

$$\int \frac{x\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \int \frac{x\sqrt{\arctan(ax)}}{(a^2cx^2+c)^2}\,dx$$

input
```
integrate(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate(x*sqrt(arctan(a*x))/(a^2*c*x^2 + c)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \int \frac{x\,\sqrt{\mathrm{atan}\,(a\,x)}}{\left(c\,a^2\,x^2+c\right)^2}\,dx$$

input
```
int((x*atan(a*x)^(1/2))/(c + a^2*c*x^2)^2,x)
```

output
```
int((x*atan(a*x)^(1/2))/(c + a^2*c*x^2)^2, x)
```

## Reduce [F]

$$\int \frac{x\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \frac{2\sqrt{atan\,(ax)}\,x^2-\left(\int \frac{\sqrt{atan(ax)}\,x^2}{atan(ax)a^4x^4+2atan(ax)a^2x^2+atan(ax)}dx\right)a^3x^2-\left(\int \frac{\sqrt{atan(ax)}\,x^2}{atan(ax)a^4x^4+2atan(ax)a^2x^2+atan(ax)}dx\right)}{4c^2\,(a^2x^2+1)}$$

input
```
int(x*atan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x)
```

output
```
(2*sqrt(atan(a*x))*x**2 - int((sqrt(atan(a*x))*x**2)/(atan(a*x)*a**4*x**4
+ 2*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**3*x**2 - int((sqrt(atan(a*x))*x
**2)/(atan(a*x)*a**4*x**4 + 2*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a)/(4*c*
*2*(a**2*x**2 + 1))
```

# 3.710 $\int \frac{\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 77

$$\int \frac{\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \frac{x\sqrt{\arctan(ax)}}{2c^2(1+a^2x^2)} + \frac{\arctan(ax)^{3/2}}{3ac^2} - \frac{\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{8ac^2}$$

output
```
1/2*x*arctan(a*x)^(1/2)/c^2/(a^2*x^2+1)+1/3*arctan(a*x)^(3/2)/a/c^2-1/8*Pi
^(1/2)*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2))/a/c^2
```

## Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \frac{4\sqrt{\arctan(ax)}\left(\frac{3x}{1+a^2x^2}+\frac{2\arctan(ax)}{a}\right) - \frac{3\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{a}}{24c^2}$$

input
```
Integrate[Sqrt[ArcTan[a*x]]/(c + a^2*c*x^2)^2,x]
```

output
```
(4*Sqrt[ArcTan[a*x]]*((3*x)/(1 + a^2*x^2) + (2*ArcTan[a*x])/a) - (3*Sqrt[P
i]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/a)/(24*c^2)
```

## Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {5427, 27, 5505, 4906, 27, 3042, 3786, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\arctan(ax)}}{(a^2cx^2+c)^2}dx$$

$$\Big\downarrow \text{5427}$$

$$-\frac{1}{4}a\int \frac{x}{c^2(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx+\frac{x\sqrt{\arctan(ax)}}{2c^2(a^2x^2+1)}+\frac{\arctan(ax)^{3/2}}{3ac^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\int \frac{x}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx}{4c^2}+\frac{x\sqrt{\arctan(ax)}}{2c^2(a^2x^2+1)}+\frac{\arctan(ax)^{3/2}}{3ac^2}$$

$$\Big\downarrow \text{5505}$$

$$-\frac{\int \frac{ax}{(a^2x^2+1)\sqrt{\arctan(ax)}}d\arctan(ax)}{4ac^2}+\frac{x\sqrt{\arctan(ax)}}{2c^2(a^2x^2+1)}+\frac{\arctan(ax)^{3/2}}{3ac^2}$$

$$\Big\downarrow \text{4906}$$

$$-\frac{\int \frac{\sin(2\arctan(ax))}{2\sqrt{\arctan(ax)}}d\arctan(ax)}{4ac^2}+\frac{x\sqrt{\arctan(ax)}}{2c^2(a^2x^2+1)}+\frac{\arctan(ax)^{3/2}}{3ac^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\sin(2\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)}{8ac^2}+\frac{x\sqrt{\arctan(ax)}}{2c^2(a^2x^2+1)}+\frac{\arctan(ax)^{3/2}}{3ac^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sin(2\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)}{8ac^2}+\frac{x\sqrt{\arctan(ax)}}{2c^2(a^2x^2+1)}+\frac{\arctan(ax)^{3/2}}{3ac^2}$$

$$\Big\downarrow \text{3786}$$

$$-\frac{\int \sin(2\arctan(ax))d\sqrt{\arctan(ax)}}{4ac^2}+\frac{x\sqrt{\arctan(ax)}}{2c^2(a^2x^2+1)}+\frac{\arctan(ax)^{3/2}}{3ac^2}$$

$$\Big\downarrow \text{3832}$$

$$\frac{x\sqrt{\arctan(ax)}}{2c^2(a^2x^2+1)}-\frac{\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{8ac^2}+\frac{\arctan(ax)^{3/2}}{3ac^2}$$

input
```
Int[Sqrt[ArcTan[a*x]]/(c + a^2*c*x^2)^2,x]
```

output
```
(x*Sqrt[ArcTan[a*x]])/(2*c^2*(1 + a^2*x^2)) + ArcTan[a*x]^(3/2)/(3*a*c^2)
- (Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(8*a*c^2)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

## Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.78

| method | result | size |
|---|---|---|
| default | $\frac{8\arctan(ax)^{2}+6\arctan(ax)\sin(2\arctan(ax))-3\sqrt{\arctan(ax)}\,\sqrt{\pi}\ \mathrm{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{24a\,c^{2}\sqrt{\arctan(ax)}}$ | 60 |

input
```
int(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/24/a/c^2/arctan(a*x)^(1/2)*(8*arctan(a*x)^2+6*arctan(a*x)*sin(2*arctan(a
*x))-3*arctan(a*x)^(1/2)*Pi^(1/2)*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2)))
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \frac{\int \frac{\sqrt{\operatorname{atan}(ax)}}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

input
```
integrate(atan(a*x)**(1/2)/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(sqrt(atan(a*x))/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.18

$$\int \frac{\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \frac{\arctan(ax)^{\frac{3}{2}}}{3\,ac^2} - \frac{(i-1)\sqrt{\pi}\operatorname{erf}\left((i-1)\sqrt{\arctan(ax)}\right) + 4i\sqrt{\arctan(ax)}e^{(2i\arctan(ax))}}{32\,ac^2} - \frac{-(i+1)\sqrt{\pi}\operatorname{erf}\left(-(i+1)\sqrt{\arctan(ax)}\right) - 4i\sqrt{\arctan(ax)}e^{(-2i\arctan(ax))}}{32\,ac^2}$$

input
```
integrate(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
1/3*arctan(a*x)^(3/2)/(a*c^2) - 1/32*((I - 1)*sqrt(pi)*erf((I - 1)*sqrt(ar
ctan(a*x))) + 4*I*sqrt(arctan(a*x))*e^(2*I*arctan(a*x)))/(a*c^2) - 1/32*(-
(I + 1)*sqrt(pi)*erf(-(I + 1)*sqrt(arctan(a*x))) - 4*I*sqrt(arctan(a*x))*e
^(-2*I*arctan(a*x)))/(a*c^2)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\arctan(ax)}}{(c+a^2cx^2)^2}\,dx = \int \frac{\sqrt{\mathrm{atan}\left(a\,x\right)}}{{\left(c\,a^2\,x^2+c\right)}^2}\,dx$$

input
```
int(atan(a*x)^(1/2)/(c + a^2*c*x^2)^2,x)
```

output
```
int(atan(a*x)^(1/2)/(c + a^2*c*x^2)^2, x)
```

## Reduce [F]

$$\int \frac{\sqrt{\arctan(ax)}}{(c + a^2cx^2)^2}\,dx = \frac{\int \frac{\sqrt{atan(ax)}}{a^4x^4+2a^2x^2+1}dx}{c^2}$$

input
```
int(atan(a*x)^(1/2)/(a^2*c*x^2+c)^2,x)
```

output
```
int(sqrt(atan(a*x))/(a**4*x**4 + 2*a**2*x**2 + 1),x)/c**2
```

# 3.711 $\int \frac{\sqrt{\arctan(ax)}}{x(c+a^2cx^2)^2} dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\sqrt{\arctan(ax)}}{x\left(c+a^2cx^2\right)^2} dx = \text{Int}\left(\frac{\sqrt{\arctan(ax)}}{x\left(c+a^2cx^2\right)^2}, x\right)$$

output
```
Defer(Int)(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^2,x)
```

## Mathematica [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{\arctan(ax)}}{x\left(c+a^2cx^2\right)^2} dx = \int \frac{\sqrt{\arctan(ax)}}{x\left(c+a^2cx^2\right)^2} dx$$

input
```
Integrate[Sqrt[ArcTan[a*x]]/(x*(c + a^2*c*x^2)^2),x]
```

output
```
Integrate[Sqrt[ArcTan[a*x]]/(x*(c + a^2*c*x^2)^2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\arctan(ax)}}{x\left(a^2cx^2+c\right)^2}dx$$

$$\Big\downarrow \; 5560$$

$$\int \frac{\sqrt{\arctan(ax)}}{x\left(a^2cx^2+c\right)^2}dx$$

input
```
Int[Sqrt[ArcTan[a*x]]/(x*(c + a^2*c*x^2)^2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{\arctan\left(ax\right)}}{x\left(a^2c\,x^2+c\right)^2}dx$$

input
```
int(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^2,x)
```

output
```
int(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^2,x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{x\left(c+a^2cx^2\right)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 1.36 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int \frac{\sqrt{\arctan(ax)}}{x\left(c+a^2cx^2\right)^2}\,dx = \frac{\int \frac{\sqrt{\operatorname{atan}(ax)}}{a^4x^5+2a^2x^3+x}\,dx}{c^2}$$

input
```
integrate(atan(a*x)**(1/2)/x/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(sqrt(atan(a*x))/(a**4*x**5 + 2*a**2*x**3 + x), x)/c**2
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{x\left(c+a^2cx^2\right)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\arctan(ax)}}{x\left(c+a^2cx^2\right)^2}\,dx=\int \frac{\sqrt{\arctan\left(ax\right)}}{\left(a^2cx^2+c\right)^2x}\,dx$$

input
```
integrate(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(arctan(a*x))/((a^2*c*x^2 + c)^2*x), x)
```

## Mupad [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\arctan(ax)}}{x\left(c+a^2cx^2\right)^2}\,dx=\int \frac{\sqrt{\mathrm{atan}\left(a\,x\right)}}{x\,\left(c\,a^2\,x^2+c\right)^2}\,dx$$

input
```
int(atan(a*x)^(1/2)/(x*(c + a^2*c*x^2)^2),x)
```

output
```
int(atan(a*x)^(1/2)/(x*(c + a^2*c*x^2)^2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.29

$$\int \frac{\sqrt{\arctan(ax)}}{x\left(c+a^2cx^2\right)^2}\,dx=\frac{\int \frac{\sqrt{atan(ax)}}{a^4x^5+2a^2x^3+x}dx}{c^2}$$

input
```
int(atan(a*x)^(1/2)/x/(a^2*c*x^2+c)^2,x)
```

output
```
int(sqrt(atan(a*x))/(a**4*x**5 + 2*a**2*x**3 + x),x)/c**2
```

# 3.712 $\int \frac{x^m \sqrt{\arctan(ax)}}{(c+a^2cx^2)^3} dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m \sqrt{\arctan(ax)}}{(c+a^2cx^2)^3} dx = \text{Int}\left(\frac{x^m \sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}, x\right)$$

output
```
Defer(Int)(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x)
```

## Mathematica [N/A]

Not integrable

Time = 1.49 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \sqrt{\arctan(ax)}}{(c+a^2cx^2)^3} dx = \int \frac{x^m \sqrt{\arctan(ax)}}{(c+a^2cx^2)^3} dx$$

input
```
Integrate[(x^m*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^3,x]
```

output
```
Integrate[(x^m*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^3, x]
```

## Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \sqrt{\arctan(ax)}}{(a^2cx^2+c)^3} \, dx$$

$$\Big\downarrow \; 5560$$

$$\int \frac{x^m \sqrt{\arctan(ax)}}{(a^2cx^2+c)^3}dx$$

input
```
Int[(x^m*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^3,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m \sqrt{\arctan(ax)}}{(a^2c\,x^2+c)^3}dx$$

input
```
int(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x)
```

output
```
int(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.08

$$\int \frac{x^m \sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \int \frac{x^m \sqrt{\arctan{(ax)}}}{(a^2cx^2+c)^3}\,dx$$

input
```
integrate(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
integral(x^m*sqrt(arctan(a*x))/(a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^
2 + c^3), x)
```

## Sympy [N/A]

Not integrable

Time = 98.66 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.71

$$\int \frac{x^m \sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \frac{\int \frac{x^m \sqrt{\operatorname{atan}{(ax)}}}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

input
```
integrate(x**m*atan(a*x)**(1/2)/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(x**m*sqrt(atan(a*x))/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1),
 x)/c**3
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m \sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x^m \sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,2,1,0,0]%%%} / %%%{1,[0,0,0,1,3]%%%} Error: Bad Argum
ent Value
```

## Mupad [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \int \frac{x^m\,\sqrt{\mathrm{atan}\left(a\,x\right)}}{{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

input
```
int((x^m*atan(a*x)^(1/2))/(c + a^2*c*x^2)^3,x)
```

output
```
int((x^m*atan(a*x)^(1/2))/(c + a^2*c*x^2)^3, x)
```

## Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.75

$$\int \frac{x^m \sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \frac{\int \frac{x^m\sqrt{atan(ax)}}{a^6x^6+3a^4x^4+3a^2x^2+1}dx}{c^3}$$

input
```
int(x^m*atan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x)
```

output
```
int((x**m*sqrt(atan(a*x)))/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1),x)/
c**3
```

# 3.713 $\int \frac{x^5 \sqrt{\arctan(ax)}}{(c+a^2cx^2)^3} \, dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^5 \sqrt{\arctan(ax)}}{(c+a^2cx^2)^3} \, dx = \text{Int}\left(\frac{x^5 \sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}, x\right)$$

output
```
Defer(Int)(x^5*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x)
```

## Mathematica [N/A]

Not integrable

Time = 3.85 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^5 \sqrt{\arctan(ax)}}{(c+a^2cx^2)^3} \, dx = \int \frac{x^5 \sqrt{\arctan(ax)}}{(c+a^2cx^2)^3} \, dx$$

input
```
Integrate[(x^5*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^3,x]
```

output
```
Integrate[(x^5*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^3, x]
```

## Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5\sqrt{\arctan(ax)}}{(a^2cx^2+c)^3}dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{x^5\sqrt{\arctan(ax)}}{(a^2cx^2+c)^3}dx$$

input
```
Int[(x^5*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^3,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^5\sqrt{\arctan(ax)}}{(a^2c\,x^2+c)^3}dx$$

input
```
int(x^5*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x)
```

output
```
int(x^5*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x^5\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^5*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 1.71 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.71

$$\int \frac{x^5\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \frac{\int \frac{x^5\sqrt{\operatorname{atan}(ax)}}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

input
```
integrate(x**5*atan(a*x)**(1/2)/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(x**5*sqrt(atan(a*x))/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1),
 x)/c**3
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^5*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 2.40 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^5\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx=\int \frac{x^5\sqrt{\arctan{(ax)}}}{(a^2cx^2+c)^3}\,dx$$

input
```
integrate(x^5*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
integrate(x^5*sqrt(arctan(a*x))/(a^2*c*x^2 + c)^3, x)
```

## Mupad [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^5\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx=\int \frac{x^5\,\sqrt{\mathrm{atan}\left(a\,x\right)}}{{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

input
```
int((x^5*atan(a*x)^(1/2))/(c + a^2*c*x^2)^3,x)
```

output
```
int((x^5*atan(a*x)^(1/2))/(c + a^2*c*x^2)^3, x)
```

## Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.75

$$\int \frac{x^5\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \frac{\int \frac{\sqrt{atan(ax)}\,x^5}{a^6x^6+3a^4x^4+3a^2x^2+1}dx}{c^3}$$

input
```
int(x^5*atan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x)
```

output
```
int((sqrt(atan(a*x))*x***5)/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1),x)/
c**3
```

# 3.714 $\int \frac{x^4\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 139

$$\int \frac{x^4\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \frac{\arctan(ax)^{3/2}}{4a^5c^3} - \frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{64a^5c^3} + \frac{\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{8a^5c^3} - \frac{\sqrt{\arctan(ax)}\sin(2\arctan(ax))}{4a^5c^3} + \frac{\sqrt{\arctan(ax)}\sin(4\arctan(ax))}{32a^5c^3}$$

output

```
1/4*arctan(a*x)^(3/2)/a^5/c^3-1/128*2^(1/2)*Pi^(1/2)*FresnelS(2*2^(1/2)/Pi
^(1/2)*arctan(a*x)^(1/2))/a^5/c^3+1/8*Pi^(1/2)*FresnelS(2*arctan(a*x)^(1/2
)/Pi^(1/2))/a^5/c^3-1/4*arctan(a*x)^(1/2)*sin(2*arctan(a*x))/a^5/c^3+1/32*
arctan(a*x)^(1/2)*sin(4*arctan(a*x))/a^5/c^3
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.41 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.30

$$\int \frac{x^4\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx$$
$$= \frac{-\frac{96ax\arctan(ax)}{(1+a^2x^2)^2} - \frac{160a^3x^3\arctan(ax)}{(1+a^2x^2)^2} + 64\arctan(ax)^2 - 8\sqrt{2}\sqrt{-i\arctan(ax)}\Gamma\left(\frac{1}{2},-2i\arctan(ax)\right) - 8\sqrt{2}\sqrt{}}{256a^5c^3}$$

input
```
Integrate[(x^4*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^3,x]
```

output
```
((-96*a*x*ArcTan[a*x])/(1 + a^2*x^2)^2 - (160*a^3*x^3*ArcTan[a*x])/(1 + a^
2*x^2)^2 + 64*ArcTan[a*x]^2 - 8*Sqrt[2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2,
(-2*I)*ArcTan[a*x]] - 8*Sqrt[2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (2*I)*ArcTa
n[a*x]] + Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-4*I)*ArcTan[a*x]] + Sqrt[I*A
rcTan[a*x]]*Gamma[1/2, (4*I)*ArcTan[a*x]])/(256*a^5*c^3*Sqrt[ArcTan[a*x]])
```

## Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.83, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5505, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\sqrt{\arctan(ax)}}{(a^2cx^2+c)^3}\,dx$$

$$\Big\downarrow \; 5505$$

$$\frac{\int \frac{a^4x^4\sqrt{\arctan(ax)}}{(a^2x^2+1)^2}d\arctan(ax)}{a^5c^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \sqrt{\arctan(ax)}\sin(\arctan(ax))^4 d\arctan(ax)}{a^5c^3}$$

$$\Big\downarrow \text{3793}$$

$$\frac{\int \left(-\frac{1}{2}\sqrt{\arctan(ax)}\cos(2\arctan(ax))+\frac{1}{8}\sqrt{\arctan(ax)}\cos(4\arctan(ax))+\frac{3}{8}\sqrt{\arctan(ax)}\right)d\arctan(ax)}{a^5c^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{64}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{8}\sqrt{\pi}\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{1}{4}\arctan(ax)^{3/2}-\frac{1}{4}\sqrt{\arctan(ax)}\sin(2}{a^5c^3}$$

input
```
Int[(x^4*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^3,x]
```

output
```
(ArcTan[a*x]^(3/2)/4 - (Sqrt[Pi/2]*FresnelS[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]
])/64 + (Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/8 - (Sqrt[ArcT
an[a*x]]*Sin[2*ArcTan[a*x]])/4 + (Sqrt[ArcTan[a*x]]*Sin[4*ArcTan[a*x]])/32
)/(a^5*c^3)
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

## Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.73

| method | result |
|---|---|
| default | $\frac{-\sqrt{2}\,\sqrt{\arctan(ax)}\,\sqrt{\pi}\,\mathrm{FresnelS}\left(\frac{2\sqrt{2}\,\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+16\sqrt{\arctan(ax)}\,\sqrt{\pi}\,\mathrm{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+32\arctan(ax)^2-32\arctan(ax}{128a^5c^3\sqrt{\arctan(ax)}}$ |

input
```
int(x^4*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/128/a^5/c^3*(-2^(1/2)*arctan(a*x)^(1/2)*Pi^(1/2)*FresnelS(2*2^(1/2)/Pi^(
1/2)*arctan(a*x)^(1/2))+16*arctan(a*x)^(1/2)*Pi^(1/2)*FresnelS(2*arctan(a*
x)^(1/2)/Pi^(1/2))+32*arctan(a*x)^2-32*arctan(a*x)*sin(2*arctan(a*x))+4*ar
ctan(a*x)*sin(4*arctan(a*x)))/arctan(a*x)^(1/2)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x^4\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^4*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{x^4\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \frac{\int \frac{x^4\sqrt{\operatorname{atan}(ax)}}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

input
```
integrate(x**4*atan(a*x)**(1/2)/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(x**4*sqrt(atan(a*x))/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1),
 x)/c**3
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^4*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.29

$$\int \frac{x^4\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx$$
$$= \frac{\arctan(ax)^{\frac{3}{2}}}{4a^5c^3}$$
$$- \frac{(i-1)\sqrt{2}\sqrt{\pi}\operatorname{erf}\left((i-1)\sqrt{2}\sqrt{\arctan(ax)}\right)+8i\sqrt{\arctan(ax)}e^{(4i\arctan(ax))}}{512a^5c^3}$$
$$- \frac{-(i+1)\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-(i+1)\sqrt{2}\sqrt{\arctan(ax)}\right)-8i\sqrt{\arctan(ax)}e^{(-4i\arctan(ax))}}{512a^5c^3}$$
$$- \frac{-(i-1)\sqrt{\pi}\operatorname{erf}\left((i-1)\sqrt{\arctan(ax)}\right)-4i\sqrt{\arctan(ax)}e^{(2i\arctan(ax))}}{32a^5c^3}$$
$$- \frac{(i+1)\sqrt{\pi}\operatorname{erf}\left(-(i+1)\sqrt{\arctan(ax)}\right)+4i\sqrt{\arctan(ax)}e^{(-2i\arctan(ax))}}{32a^5c^3}$$

input
```
integrate(x^4*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
1/4*arctan(a*x)^(3/2)/(a^5*c^3) - 1/512*((I - 1)*sqrt(2)*sqrt(pi)*erf((I -
 1)*sqrt(2)*sqrt(arctan(a*x))) + 8*I*sqrt(arctan(a*x))*e^(4*I*arctan(a*x))
)/(a^5*c^3) - 1/512*(-(I + 1)*sqrt(2)*sqrt(pi)*erf(-(I + 1)*sqrt(2)*sqrt(a
rctan(a*x))) - 8*I*sqrt(arctan(a*x))*e^(-4*I*arctan(a*x)))/(a^5*c^3) - 1/3
2*(-(I - 1)*sqrt(pi)*erf((I - 1)*sqrt(arctan(a*x))) - 4*I*sqrt(arctan(a*x)
)*e^(2*I*arctan(a*x)))/(a^5*c^3) - 1/32*((I + 1)*sqrt(pi)*erf(-(I + 1)*sqr
t(arctan(a*x))) + 4*I*sqrt(arctan(a*x))*e^(-2*I*arctan(a*x)))/(a^5*c^3)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^4\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \int \frac{x^4\sqrt{\mathrm{atan}(a\,x)}}{(c\,a^2\,x^2+c)^3}\,dx$$

input
```
int((x^4*atan(a*x)^(1/2))/(c + a^2*c*x^2)^3,x)
```

output
```
int((x^4*atan(a*x)^(1/2))/(c + a^2*c*x^2)^3, x)
```

## Reduce [F]

$$\int \frac{x^4 \sqrt{\arctan(ax)}}{(c + a^2 c x^2)^3} \, dx$$
$$= \frac{2\sqrt{atan\left(ax\right)}\, atan(ax)\, a^4 x^4 + 4\sqrt{atan\left(ax\right)}\, atan(ax)\, a^2 x^2 + 2\sqrt{atan\left(ax\right)}\, atan(ax) + 2\sqrt{atan\left(ax\right)}\, ax -}{}$$

input
```
int(x^4*atan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x)
```

output
```
(2*sqrt(atan(a*x))*atan(a*x)*a**4*x**4 + 4*sqrt(atan(a*x))*atan(a*x)*a**2*
x**2 + 2*sqrt(atan(a*x))*atan(a*x) + 2*sqrt(atan(a*x))*a*x - 5*int(sqrt(at
an(a*x))/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1),x)*a**5*x**4 - 10*int
(sqrt(atan(a*x))/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1),x)*a**3*x**2
- 5*int(sqrt(atan(a*x))/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1),x)*a -
 int((sqrt(atan(a*x))*x)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*
atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**6*x**4 - 2*int((sqrt(atan(a*x))*x)/
(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + ata
n(a*x)),x)*a**4*x**2 - int((sqrt(atan(a*x))*x)/(atan(a*x)*a**6*x**6 + 3*at
an(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**2)/(3*a**5*c*
*3*(a**4*x**4 + 2*a**2*x**2 + 1))
```

# 3.715 $\int \frac{x^3\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 118

$$\int \frac{x^3\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = -\frac{3\sqrt{\arctan(ax)}}{32a^4c^3} + \frac{x^4\sqrt{\arctan(ax)}}{4c^3(1+a^2x^2)^2} - \frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{64a^4c^3} + \frac{\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{16a^4c^3}$$

output

```
-3/32*arctan(a*x)^(1/2)/a^4/c^3+1/4*x^4*arctan(a*x)^(1/2)/c^3/(a^2*x^2+1)^
2-1/128*2^(1/2)*Pi^(1/2)*FresnelC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))/a^
4/c^3+1/16*Pi^(1/2)*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))/a^4/c^3
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.95

$$\int \frac{x^3\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx$$

$$= \frac{-10\sqrt{2\pi}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+80\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{\frac{64\left(-3-6a^2x^2+5a^4x^4\right)\arctan(ax)}{\left(1+a^2x^2\right)^2}-12i\sqrt{}}{}}{}$$

input
```
Integrate[(x^3*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^3,x]
```

output
```
(-10*Sqrt[2*Pi]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]] + 80*Sqrt[Pi]*Fre
snelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]] + ((64*(-3 - 6*a^2*x^2 + 5*a^4*x^4)*
ArcTan[a*x])/(1 + a^2*x^2)^2 - (12*I)*Sqrt[2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma
[1/2, (-2*I)*ArcTan[a*x]] + (12*I)*Sqrt[2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2,
(2*I)*ArcTan[a*x]] + (3*I)*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-4*I)*ArcTan
[a*x]] - (3*I)*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (4*I)*ArcTan[a*x]])/Sqrt[Arc
Tan[a*x]])/(2048*a^4*c^3)
```

## Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5479, 27, 5505, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\sqrt{\arctan(ax)}}{(a^2cx^2+c)^3}\,dx$$

$$\Big\downarrow \text{5479}$$

$$\frac{x^4\sqrt{\arctan(ax)}}{4c^3(a^2x^2+1)^2}-\frac{1}{8}a\int \frac{x^4}{c^3(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{x^4\sqrt{\arctan(ax)}}{4c^3\left(a^2x^2+1\right)^2}-\frac{a\int \frac{x^4}{\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{8c^3}$$

$$\Big\downarrow \text{5505}$$

$$\frac{x^4\sqrt{\arctan(ax)}}{4c^3\left(a^2x^2+1\right)^2}-\frac{\int \frac{a^4x^4}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{8a^4c^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{x^4\sqrt{\arctan(ax)}}{4c^3\left(a^2x^2+1\right)^2}-\frac{\int \frac{\sin(\arctan(ax))^4}{\sqrt{\arctan(ax)}}d\arctan(ax)}{8a^4c^3}$$

$$\Big\downarrow \text{3793}$$

$$\frac{x^4\sqrt{\arctan(ax)}}{4c^3\left(a^2x^2+1\right)^2}-\frac{\int \left(-\frac{\cos(2\arctan(ax))}{2\sqrt{\arctan(ax)}}+\frac{\cos(4\arctan(ax))}{8\sqrt{\arctan(ax)}}+\frac{3}{8\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{8a^4c^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^4\sqrt{\arctan(ax)}}{4c^3\left(a^2x^2+1\right)^2}-\frac{\frac{1}{8}\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{2}\sqrt{\pi}\operatorname{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\arctan(ax)}}{8a^4c^3}$$

input
```
Int[(x^3*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^3,x]
```

output
```
(x^4*Sqrt[ArcTan[a*x]])/(4*c^3*(1 + a^2*x^2)^2) - ((3*Sqrt[ArcTan[a*x]])/4
 + (Sqrt[Pi/2]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/8 - (Sqrt[Pi]*Fre
snelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/2)/(8*a^4*c^3)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

## Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.80

| method | result |
|---|---|
| default | $\frac{-\sqrt{2}\,\sqrt{\arctan(ax)}\,\sqrt{\pi}\,\mathrm{FresnelC}\left(\frac{2\sqrt{2}\,\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+4\arctan(ax)\cos(4\arctan(ax))-16\arctan(ax)\cos(2\arctan(ax))+8\sqrt{\arctan(}}{128a^4c^3\sqrt{\arctan(ax)}}$ |

input
```
int(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/128/a^4/c^3/arctan(a*x)^(1/2)*(-2^(1/2)*arctan(a*x)^(1/2)*Pi^(1/2)*Fresn
elC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))+4*arctan(a*x)*cos(4*arctan(a*x))
-16*arctan(a*x)*cos(2*arctan(a*x))+8*arctan(a*x)^(1/2)*Pi^(1/2)*FresnelC(2
*arctan(a*x)^(1/2)/Pi^(1/2)))
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{x^3\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \frac{\int \frac{x^3\sqrt{\operatorname{atan}(ax)}}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

input
```
integrate(x**3*atan(a*x)**(1/2)/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(x**3*sqrt(atan(a*x))/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1),
 x)/c**3
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F]

$$\int \frac{x^3\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \int \frac{x^3\sqrt{\arctan(ax)}}{(a^2cx^2+c)^3}\,dx$$

input
```
integrate(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
integrate(x^3*sqrt(arctan(a*x))/(a^2*c*x^2 + c)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\sqrt{\arctan(ax)}}{\left(c+a^2cx^2\right)^3}\,dx=\int \frac{x^3\,\sqrt{\mathrm{atan}\left(a\,x\right)}}{{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

input
```
int((x^3*atan(a*x)^(1/2))/(c + a^2*c*x^2)^3,x)
```

output
```
int((x^3*atan(a*x)^(1/2))/(c + a^2*c*x^2)^3, x)
```

## Reduce [F]

$$\int \frac{x^3\sqrt{\arctan(ax)}}{\left(c+a^2cx^2\right)^3}\,dx$$
$$=\frac{2\sqrt{atan(ax)}\,x^4-\left(\int \frac{\sqrt{atan(ax)}\,x^4}{atan(ax)a^6x^6+3atan(ax)a^4x^4+3atan(ax)a^2x^2+atan(ax)}dx\right)a^5x^4-2\left(\int \frac{\sqrt{atan(}}{atan(ax)a^6x^6+3atan(ax)a^4x}\right.}{8c^3\left(a^4x^4+2a^2x^2+\right.}$$

input
```
int(x^3*atan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x)
```

output
```
(2*sqrt(atan(a*x))*x**4 - int((sqrt(atan(a*x))*x**4)/(atan(a*x)*a**6*x**6
+ 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**5*x**4
- 2*int((sqrt(atan(a*x))*x**4)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**
4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**3*x**2 - int((sqrt(atan(a*x))
*x**4)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**
2 + atan(a*x)),x)*a)/(8*c**3*(a**4*x**4 + 2*a**2*x**2 + 1))
```

# 3.716 $\int \frac{x^2\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 83

$$\int \frac{x^2\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \frac{\arctan(ax)^{3/2}}{12a^3c^3}+\frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{64a^3c^3} - \frac{\sqrt{\arctan(ax)}\sin(4\arctan(ax))}{32a^3c^3}$$

output
```
1/12*arctan(a*x)^(3/2)/a^3/c^3+1/128*2^(1/2)*Pi^(1/2)*FresnelS(2*2^(1/2)/P
i^(1/2)*arctan(a*x)^(1/2))/a^3/c^3-1/32*arctan(a*x)^(1/2)*sin(4*arctan(a*x
))/a^3/c^3
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.70

$$\int \frac{x^2\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \frac{32\arctan(ax)\left(3ax(-1+a^2x^2)+2(1+a^2x^2)^2\arctan(ax)\right)-3(1+a^2x^2)^2\sqrt{-i\arctan(ax)}\,\Gamma\left(\frac{1}{2},-4i\,\text{ar}\right.}{768a^3c^3\left(1+a^2x^2\right)^2\sqrt{\arctan(ax)}}$$

input
```
Integrate[(x^2*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^3,x]
```

output
```
(32*ArcTan[a*x]*(3*a*x*(-1 + a^2*x^2) + 2*(1 + a^2*x^2)^2*ArcTan[a*x]) - 3
*(1 + a^2*x^2)^2*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-4*I)*ArcTan[a*x]] - 3
*(1 + a^2*x^2)^2*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (4*I)*ArcTan[a*x]])/(768*a
^3*c^3*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]])
```

## Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.87, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {5505, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{\arctan(ax)}}{(a^2cx^2+c)^3}\,dx$$

$$\Big\downarrow \ 5505$$

$$\frac{\int \frac{a^2x^2\sqrt{\arctan(ax)}}{(a^2x^2+1)^2}d\arctan(ax)}{a^3c^3}$$

$$\Big\downarrow \ 4906$$

$$\frac{\int \left(\frac{1}{8}\sqrt{\arctan(ax)}-\frac{1}{8}\sqrt{\arctan(ax)}\cos(4\arctan(ax))\right)d\arctan(ax)}{a^3c^3}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{1}{64}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{12}\arctan(ax)^{3/2}-\frac{1}{32}\sqrt{\arctan(ax)}\sin(4\arctan(ax))}{a^3c^3}$$

input
```
Int[(x^2*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^3,x]
```

output
```
(ArcTan[a*x]^(3/2)/12 + (Sqrt[Pi/2]*FresnelS[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]
]])/64 - (Sqrt[ArcTan[a*x]]*Sin[4*ArcTan[a*x]])/32)/(a^3*c^3)
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

## Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.80

| method | result | size |
|---|---|---|
| default | $\frac{3\sqrt{2}\,\sqrt{\arctan(ax)}\,\sqrt{\pi}\,\operatorname{FresnelS}\left(\frac{2\sqrt{2}\,\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+32\arctan(ax)^{2}-12\arctan(ax)\sin(4\arctan(ax))}{384a^{3}c^{3}\sqrt{\arctan(ax)}}$ | 66 |

input
```
int(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/384/a^3/c^3*(3*2^(1/2)*arctan(a*x)^(1/2)*Pi^(1/2)*FresnelS(2*2^(1/2)/Pi^
(1/2)*arctan(a*x)^(1/2))+32*arctan(a*x)^2-12*arctan(a*x)*sin(4*arctan(a*x)
))/arctan(a*x)^(1/2)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{x^2\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \frac{\int \frac{x^2\sqrt{\operatorname{atan}(ax)}}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

input
```
integrate(x**2*atan(a*x)**(1/2)/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(x**2*sqrt(atan(a*x))/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1),
 x)/c**3
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.24

$$\int \frac{x^2\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx$$
$$= \frac{\arctan(ax)^{\frac{3}{2}}}{12\,a^3c^3} - \frac{-(i-1)\,\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left((i-1)\,\sqrt{2}\sqrt{\arctan(ax)}\right) - 8i\,\sqrt{\arctan(ax)}e^{(4i\,\arctan(ax))}}{512\,a^3c^3} - \frac{(i+1)\,\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-(i+1)\,\sqrt{2}\sqrt{\arctan(ax)}\right) + 8i\,\sqrt{\arctan(ax)}e^{(-4i\,\arctan(ax))}}{512\,a^3c^3}$$

input
```
integrate(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
1/12*arctan(a*x)^(3/2)/(a^3*c^3) - 1/512*(-(I - 1)*sqrt(2)*sqrt(pi)*erf((I
 - 1)*sqrt(2)*sqrt(arctan(a*x))) - 8*I*sqrt(arctan(a*x))*e^(4*I*arctan(a*x
)))/(a^3*c^3) - 1/512*((I + 1)*sqrt(2)*sqrt(pi)*erf(-(I + 1)*sqrt(2)*sqrt(
arctan(a*x))) + 8*I*sqrt(arctan(a*x))*e^(-4*I*arctan(a*x)))/(a^3*c^3)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \int \frac{x^2\,\sqrt{\mathrm{atan}\left(a\,x\right)}}{{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

input
```
int((x^2*atan(a*x)^(1/2))/(c + a^2*c*x^2)^3,x)
```

output
```
int((x^2*atan(a*x)^(1/2))/(c + a^2*c*x^2)^3, x)
```

## Reduce [F]

$$\int \frac{x^2\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx$$
$$= \frac{-2\sqrt{atan\,(ax)}\,x+2\left(\int \frac{\sqrt{atan(ax)}}{a^6x^6+3a^4x^4+3a^2x^2+1}dx\right)a^4x^4+4\left(\int \frac{\sqrt{atan(ax)}}{a^6x^6+3a^4x^4+3a^2x^2+1}dx\right)a^2x^2+2\left(\int \frac{\sqrt{atan(a}}{a^6x^6+3a^4x^4+3}\right.}{}$$

input
```
int(x^2*atan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x)
```

output
```
( - 2*sqrt(atan(a*x))*x + 2*int(sqrt(atan(a*x))/(a**6*x**6 + 3*a**4*x**4 +
 3*a**2*x**2 + 1),x)*a**4*x**4 + 4*int(sqrt(atan(a*x))/(a**6*x**6 + 3*a**4
*x**4 + 3*a**2*x**2 + 1),x)*a**2*x**2 + 2*int(sqrt(atan(a*x))/(a**6*x**6 +
 3*a**4*x**4 + 3*a**2*x**2 + 1),x) + int((sqrt(atan(a*x))*x)/(atan(a*x)*a*
*6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a*
*5*x**4 + 2*int((sqrt(atan(a*x))*x)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**
4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**3*x**2 + int((sqrt(atan(
a*x))*x)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x
**2 + atan(a*x)),x)*a)/(6*a**2*c**3*(a**4*x**4 + 2*a**2*x**2 + 1))
```

# 3.717 $\int \frac{x\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 118

$$\int \frac{x\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \frac{3\sqrt{\arctan(ax)}}{32a^2c^3} - \frac{\sqrt{\arctan(ax)}}{4a^2c^3(1+a^2x^2)^2} + \frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{64a^2c^3} + \frac{\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{16a^2c^3}$$

output
```
3/32*arctan(a*x)^(1/2)/a^2/c^3-1/4*arctan(a*x)^(1/2)/a^2/c^3/(a^2*x^2+1)^2
+1/128*2^(1/2)*Pi^(1/2)*FresnelC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))/a^2
/c^3+1/16*Pi^(1/2)*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))/a^2/c^3
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.95

$$\int \frac{x\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx$$

$$= \frac{-6\sqrt{2\pi}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+48\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{\frac{64\left(-5+6a^2x^2+3a^4x^4\right)\arctan(ax)}{\left(1+a^2x^2\right)^2}-20i\sqrt{2}}{}}{}$$

input
```
Integrate[(x*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^3,x]
```

output
```
(-6*Sqrt[2*Pi]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]] + 48*Sqrt[Pi]*Fres
nelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]] + ((64*(-5 + 6*a^2*x^2 + 3*a^4*x^4)*A
rcTan[a*x])/(1 + a^2*x^2)^2 - (20*I)*Sqrt[2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[
1/2, (-2*I)*ArcTan[a*x]] + (20*I)*Sqrt[2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (
2*I)*ArcTan[a*x]] - (11*I)*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-4*I)*ArcTan
[a*x]] + (11*I)*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (4*I)*ArcTan[a*x]])/Sqrt[Ar
cTan[a*x]])/(2048*a^2*c^3)
```

## Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {5465, 27, 5439, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{\arctan(ax)}}{(a^2cx^2+c)^3}\,dx$$

$$\Big\downarrow \; 5465$$

$$\frac{\int \frac{1}{c^3(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{8a}-\frac{\sqrt{\arctan(ax)}}{4a^2c^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\int \frac{1}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{8ac^3}-\frac{\sqrt{\arctan(ax)}}{4a^2c^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \textbf{5439}$$

$$\frac{\int \frac{1}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{8a^2c^3}-\frac{\sqrt{\arctan(ax)}}{4a^2c^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^4}{\sqrt{\arctan(ax)}}d\arctan(ax)}{8a^2c^3}-\frac{\sqrt{\arctan(ax)}}{4a^2c^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \textbf{3793}$$

$$\frac{\int \left(\frac{\cos(2\arctan(ax))}{2\sqrt{\arctan(ax)}}+\frac{\cos(4\arctan(ax))}{8\sqrt{\arctan(ax)}}+\frac{3}{8\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{8a^2c^3}-\frac{\sqrt{\arctan(ax)}}{4a^2c^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \textbf{2009}$$

$$\frac{\frac{1}{8}\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\pi}\operatorname{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\arctan(ax)}}{8a^2c^3}-\frac{\sqrt{\arctan(ax)}}{4a^2c^3\left(a^2x^2+1\right)^2}$$

input
```
Int[(x*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^3,x]
```

output
```
-1/4*Sqrt[ArcTan[a*x]]/(a^2*c^3*(1 + a^2*x^2)^2) + ((3*Sqrt[ArcTan[a*x]])/
4 + (Sqrt[Pi/2]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/8 + (Sqrt[Pi]*Fr
esnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/2)/(8*a^2*c^3)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

## Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.80

| method | result |
|---|---|
| default | $-\frac{-\sqrt{2}\,\sqrt{\arctan(ax)}\,\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{2}\,\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+4\arctan(ax)\cos(4\arctan(ax))+16\arctan(ax)\cos(2\arctan(ax))-8\sqrt{\text{arcta}}}{128a^2c^3\sqrt{\arctan(ax)}}$ |

input
```
int(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/128/a^2/c^3/arctan(a*x)^(1/2)*(-2^(1/2)*arctan(a*x)^(1/2)*Pi^(1/2)*Fres
nelC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))+4*arctan(a*x)*cos(4*arctan(a*x)
)+16*arctan(a*x)*cos(2*arctan(a*x))-8*arctan(a*x)^(1/2)*Pi^(1/2)*FresnelC(
2*arctan(a*x)^(1/2)/Pi^(1/2)))
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{x\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \frac{\int \frac{x\sqrt{\operatorname{atan}(ax)}}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

input
```
integrate(x*atan(a*x)**(1/2)/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(x*sqrt(atan(a*x))/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x)
/c**3
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F]

$$\int \frac{x\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \int \frac{x\sqrt{\arctan(ax)}}{(a^2cx^2+c)^3}\,dx$$

input
```
integrate(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
integrate(x*sqrt(arctan(a*x))/(a^2*c*x^2 + c)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \int \frac{x\,\sqrt{\mathrm{atan}(a\,x)}}{(c\,a^2\,x^2+c)^3}\,dx$$

input
```
int((x*atan(a*x)^(1/2))/(c + a^2*c*x^2)^3,x)
```

output
```
int((x*atan(a*x)^(1/2))/(c + a^2*c*x^2)^3, x)
```

## Reduce [F]

$$\int \frac{x\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx$$
$$= \frac{2\sqrt{atan\,(ax)}\,a^2x^4+4\sqrt{atan\,(ax)}\,x^2-\left(\int \frac{\sqrt{atan(ax)}\,x^4}{atan(ax)a^6x^6+3atan(ax)a^4x^4+3atan(ax)a^2x^2+atan(ax)}dx\right)a^7x^4-2\left(\int \right.}{}$$

input
```
int(x*atan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x)
```

output
```
(2*sqrt(atan(a*x))*a**2*x**4 + 4*sqrt(atan(a*x))*x**2 - int((sqrt(atan(a*x
))*x**4)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x
**2 + atan(a*x)),x)*a**7*x**4 - 2*int((sqrt(atan(a*x))*x**4)/(atan(a*x)*a*
*6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a*
*5*x**2 - int((sqrt(atan(a*x))*x**4)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a*
*4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**3 - 2*int((sqrt(atan(a*
x))*x**2)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*
x**2 + atan(a*x)),x)*a**5*x**4 - 4*int((sqrt(atan(a*x))*x**2)/(atan(a*x)*a
**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a
**3*x**2 - 2*int((sqrt(atan(a*x))*x**2)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)
*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a)/(8*c**3*(a**4*x**4 +
 2*a**2*x**2 + 1))
```

# 3.718 $\int \frac{\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 139

$$\int \frac{\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \frac{\arctan(ax)^{3/2}}{4ac^3} - \frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{64ac^3} - \frac{\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{8ac^3} + \frac{\sqrt{\arctan(ax)}\sin(2\arctan(ax))}{4ac^3} + \frac{\sqrt{\arctan(ax)}\sin(4\arctan(ax))}{32ac^3}$$

output
```
1/4*arctan(a*x)^(3/2)/a/c^3-1/128*2^(1/2)*Pi^(1/2)*FresnelS(2*2^(1/2)/Pi^(
1/2)*arctan(a*x)^(1/2))/a/c^3-1/8*Pi^(1/2)*FresnelS(2*arctan(a*x)^(1/2)/Pi
^(1/2))/a/c^3+1/4*arctan(a*x)^(1/2)*sin(2*arctan(a*x))/a/c^3+1/32*arctan(a
*x)^(1/2)*sin(4*arctan(a*x))/a/c^3
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.38

$$\int \frac{\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx$$
$$= \frac{\frac{160x\arctan(ax)}{(1+a^2x^2)^2}+\frac{96a^2x^3\arctan(ax)}{(1+a^2x^2)^2}+\frac{64\arctan(ax)^2}{a}+\frac{8\sqrt{2}\sqrt{-i\arctan(ax)}\Gamma\left(\frac{1}{2},-2i\arctan(ax)\right)}{a}+\frac{8\sqrt{2}\sqrt{i\arctan(ax)}\Gamma\left(\frac{1}{2},2i\arctan\right.}{a}}{256c^3\sqrt{\arctan(ax)}}$$

input
```
Integrate[Sqrt[ArcTan[a*x]]/(c + a^2*c*x^2)^3,x]
```

output
```
((160*x*ArcTan[a*x])/(1 + a^2*x^2)^2 + (96*a^2*x^3*ArcTan[a*x])/(1 + a^2*x
^2)^2 + (64*ArcTan[a*x]^2)/a + (8*Sqrt[2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2
, (-2*I)*ArcTan[a*x]])/a + (8*Sqrt[2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (2*I)
*ArcTan[a*x]])/a + (Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-4*I)*ArcTan[a*x]])
/a + (Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (4*I)*ArcTan[a*x]])/a)/(256*c^3*Sqrt[
ArcTan[a*x]])
```

## Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.83, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {5439, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\arctan(ax)}}{(a^2cx^2+c)^3}\,dx$$

$$\Big\downarrow \text{5439}$$

$$\frac{\int \frac{\sqrt{\arctan(ax)}}{(a^2x^2+1)^2}d\arctan(ax)}{ac^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{\arctan(ax)}\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^4d\arctan(ax)}{ac^3}$$

$\big\downarrow$ 3793

$$\frac{\int \left(\frac{1}{2}\sqrt{\arctan(ax)}\cos(2\arctan(ax))+\frac{1}{8}\sqrt{\arctan(ax)}\cos(4\arctan(ax))+\frac{3}{8}\sqrt{\arctan(ax)}\right)d\arctan(ax)}{ac^3}$$

$\big\downarrow$ 2009

$$\frac{-\frac{1}{64}\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{8}\sqrt{\pi}\operatorname{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{1}{4}\arctan(ax)^{3/2}+\frac{1}{4}\sqrt{\arctan(ax)}\sin(2}{ac^3}$$

input
```
Int[Sqrt[ArcTan[a*x]]/(c + a^2*c*x^2)^3,x]
```

output
```
(ArcTan[a*x]^(3/2)/4 - (Sqrt[Pi/2]*FresnelS[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]
])/64 - (Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/8 + (Sqrt[ArcT
an[a*x]]*Sin[2*ArcTan[a*x]])/4 + (Sqrt[ArcTan[a*x]]*Sin[4*ArcTan[a*x]])/32
)/(a*c^3)
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

## Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.73

| method | result |
|---|---|
| default | $\frac{-\sqrt{2}\,\sqrt{\arctan(ax)}\,\sqrt{\pi}\,\mathrm{FresnelS}\left(\frac{2\sqrt{2}\,\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+32\arctan(ax)^2+32\arctan(ax)\sin(2\arctan(ax))+4\arctan(ax)\sin(4\arctan(a}{128a\,c^3\sqrt{\arctan(ax)}}$ |

input
```
int(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/128/a/c^3/arctan(a*x)^(1/2)*(-2^(1/2)*arctan(a*x)^(1/2)*Pi^(1/2)*Fresnel
S(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))+32*arctan(a*x)^2+32*arctan(a*x)*si
n(2*arctan(a*x))+4*arctan(a*x)*sin(4*arctan(a*x))-16*arctan(a*x)^(1/2)*Pi^
(1/2)*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2)))
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \frac{\int \frac{\sqrt{\operatorname{atan}(ax)}}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

input
```
integrate(atan(a*x)**(1/2)/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(sqrt(atan(a*x))/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x)/c
**3
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.29

$$\int \frac{\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \frac{\arctan(ax)^{\frac{3}{2}}}{4ac^3} - \frac{(i-1)\sqrt{2}\sqrt{\pi}\operatorname{erf}\left((i-1)\sqrt{2}\sqrt{\arctan(ax)}\right)+8i\sqrt{\arctan(ax)}e^{(4i\arctan(ax))}}{512ac^3} - \frac{-(i+1)\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-(i+1)\sqrt{2}\sqrt{\arctan(ax)}\right)-8i\sqrt{\arctan(ax)}e^{(-4i\arctan(ax))}}{512ac^3} - \frac{(i-1)\sqrt{\pi}\operatorname{erf}\left((i-1)\sqrt{\arctan(ax)}\right)+4i\sqrt{\arctan(ax)}e^{(2i\arctan(ax))}}{32ac^3} - \frac{-(i+1)\sqrt{\pi}\operatorname{erf}\left(-(i+1)\sqrt{\arctan(ax)}\right)-4i\sqrt{\arctan(ax)}e^{(-2i\arctan(ax))}}{32ac^3}$$

input
```
integrate(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
1/4*arctan(a*x)^(3/2)/(a*c^3) - 1/512*((I - 1)*sqrt(2)*sqrt(pi)*erf((I - 1
)*sqrt(2)*sqrt(arctan(a*x))) + 8*I*sqrt(arctan(a*x))*e^(4*I*arctan(a*x)))/
(a*c^3) - 1/512*(-(I + 1)*sqrt(2)*sqrt(pi)*erf(-(I + 1)*sqrt(2)*sqrt(arcta
n(a*x))) - 8*I*sqrt(arctan(a*x))*e^(-4*I*arctan(a*x)))/(a*c^3) - 1/32*((I
- 1)*sqrt(pi)*erf((I - 1)*sqrt(arctan(a*x))) + 4*I*sqrt(arctan(a*x))*e^(2*
I*arctan(a*x)))/(a*c^3) - 1/32*(-(I + 1)*sqrt(pi)*erf(-(I + 1)*sqrt(arctan
(a*x))) - 4*I*sqrt(arctan(a*x))*e^(-2*I*arctan(a*x)))/(a*c^3)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \int \frac{\sqrt{\mathrm{atan}(a\,x)}}{{(c\,a^2\,x^2+c)}^3}\,dx$$

input
```
int(atan(a*x)^(1/2)/(c + a^2*c*x^2)^3,x)
```

output
```
int(atan(a*x)^(1/2)/(c + a^2*c*x^2)^3, x)
```

## Reduce [F]

$$\int \frac{\sqrt{\arctan(ax)}}{(c+a^2cx^2)^3}\,dx = \frac{\int \frac{\sqrt{atan(ax)}}{a^6x^6+3a^4x^4+3a^2x^2+1}dx}{c^3}$$

input
```
int(atan(a*x)^(1/2)/(a^2*c*x^2+c)^3,x)
```

output
```
int(sqrt(atan(a*x))/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1),x)/c**3
```

# 3.719 $\int \frac{\sqrt{\arctan(ax)}}{x(c+a^2cx^2)^3}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\sqrt{\arctan(ax)}}{x(c+a^2cx^2)^3}\,dx = \text{Int}\left(\frac{\sqrt{\arctan(ax)}}{x(c+a^2cx^2)^3},x\right)$$

output
```
Defer(Int)(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^3,x)
```

## Mathematica [N/A]

Not integrable

Time = 1.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{\arctan(ax)}}{x(c+a^2cx^2)^3}\,dx = \int \frac{\sqrt{\arctan(ax)}}{x(c+a^2cx^2)^3}\,dx$$

input
```
Integrate[Sqrt[ArcTan[a*x]]/(x*(c + a^2*c*x^2)^3),x]
```

output
```
Integrate[Sqrt[ArcTan[a*x]]/(x*(c + a^2*c*x^2)^3), x]
```

## Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\arctan(ax)}}{x\left(a^2cx^2+c\right)^3}dx$$

$$\Big\downarrow \; 5560$$

$$\int \frac{\sqrt{\arctan(ax)}}{x\left(a^2cx^2+c\right)^3}dx$$

input
```
Int[Sqrt[ArcTan[a*x]]/(x*(c + a^2*c*x^2)^3),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{\arctan\left(ax\right)}}{x\left(a^2c\,x^2+c\right)^3}dx$$

input
```
int(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^3,x)
```

output
```
int(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^3,x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{x\left(c+a^2cx^2\right)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 2.68 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{\sqrt{\arctan(ax)}}{x\left(c+a^2cx^2\right)^3}\,dx = \frac{\int \frac{\sqrt{\operatorname{atan}(ax)}}{a^6x^7+3a^4x^5+3a^2x^3+x}\,dx}{c^3}$$

input
```
integrate(atan(a*x)**(1/2)/x/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(sqrt(atan(a*x))/(a**6*x**7 + 3*a**4*x**5 + 3*a**2*x**3 + x), x)/c
**3
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{x\left(c+a^2cx^2\right)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\arctan(ax)}}{x\left(c+a^2cx^2\right)^3}\,dx=\int \frac{\sqrt{\arctan\left(ax\right)}}{\left(a^2cx^2+c\right)^3x}\,dx$$

input
```
integrate(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
integrate(sqrt(arctan(a*x))/((a^2*c*x^2 + c)^3*x), x)
```

## Mupad [N/A]

Not integrable

Time = 0.77 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\arctan(ax)}}{x\left(c+a^2cx^2\right)^3}\,dx=\int \frac{\sqrt{\mathrm{atan}\left(a\,x\right)}}{x\,\left(c\,a^2\,x^2+c\right)^3}\,dx$$

input
```
int(atan(a*x)^(1/2)/(x*(c + a^2*c*x^2)^3),x)
```

output
```
int(atan(a*x)^(1/2)/(x*(c + a^2*c*x^2)^3), x)
```

## Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.62

$$\int \frac{\sqrt{\arctan(ax)}}{x\left(c+a^2cx^2\right)^3}\,dx = \frac{\int \frac{\sqrt{atan(ax)}}{a^6x^7+3a^4x^5+3a^2x^3+x}dx}{c^3}$$

input
```
int(atan(a*x)^(1/2)/x/(a^2*c*x^2+c)^3,x)
```

output
```
int(sqrt(atan(a*x))/(a**6*x**7 + 3*a**4*x**5 + 3*a**2*x**3 + x),x)/c**3
```

# 3.720 $\int x^m \sqrt{c+a^2 c x^2} \sqrt{\arctan (a x)} \, dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int x^m \sqrt{c+a^2 c x^2} \sqrt{\arctan (a x)} \, dx=\text{Int}\left(x^m \sqrt{c+a^2 c x^2} \sqrt{\arctan (a x)},x\right)$$

output
```
Defer(Int)(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.31 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int x^m \sqrt{c+a^2 c x^2} \sqrt{\arctan (a x)} \, dx=\int x^m \sqrt{c+a^2 c x^2} \sqrt{\arctan (a x)} \, dx$$

input
```
Integrate[x^m*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[x^m*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]], x]
```

## Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}\,dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int x^m \sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}dx$$

input
```
Int[x^m*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int x^m \sqrt{a^2c\,x^2+c}\,\sqrt{\arctan\left(ax\right)}dx$$

input
```
int(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int x^m\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}\,dx = \int \sqrt{a^2cx^2+c}\,x^m\sqrt{\arctan{(ax)}}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^m*sqrt(arctan(a*x)), x)
```

## Sympy [N/A]

Not integrable

Time = 54.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int x^m\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}\,dx = \int x^m\sqrt{c\left(a^2x^2+1\right)}\sqrt{\operatorname{atan}{(ax)}}\,dx$$

input
```
integrate(x**m*(a**2*c*x**2+c)**(1/2)*atan(a*x)**(1/2),x)
```

output
```
Integral(x**m*sqrt(c*(a**2*x**2 + 1))*sqrt(atan(a*x)), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int x^m\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int x^m\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int x^m\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}\,dx = \int x^m\,\sqrt{\mathrm{atan}\left(a\,x\right)}\,\sqrt{c\,a^2\,x^2+c}\,dx$$

input
```
int(x^m*atan(a*x)^(1/2)*(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(x^m*atan(a*x)^(1/2)*(c + a^2*c*x^2)^(1/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int x^m\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}\,dx = \sqrt{c}\left(\int x^m\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}dx\right)$$

input
```
int(x^m*(a^2*c*x^2+c)^(1/2)*atan(a*x)^(1/2),x)
```

output
```
sqrt(c)*int(x**m*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)),x)
```

# 3.721 $\int x^2\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int x^2\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}\,dx=\operatorname{Int}\left(x^2\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)},x\right)$$

output
```
Defer(Int)(x^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 4.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int x^2\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}\,dx=\int x^2\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}\,dx$$

input
```
Integrate[x^2*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[x^2*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]], x]
```

## Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}\,dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int x^2\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}dx$$

input
```
Int[x^2*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int x^2\sqrt{a^2c\,x^2+c}\,\sqrt{\arctan\left(ax\right)}dx$$

input
```
int(x^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x)
```

output
```
int(x^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int x^2\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 5.40 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int x^2\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}\,dx = \int x^2\sqrt{c\left(a^2x^2+1\right)}\sqrt{\operatorname{atan}\left(ax\right)}\,dx$$

input
```
integrate(x**2*(a**2*c*x**2+c)**(1/2)*atan(a*x)**(1/2),x)
```

output
```
Integral(x**2*sqrt(c*(a**2*x**2 + 1))*sqrt(atan(a*x)), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int x^2\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int x^2\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}\,dx = \int \sqrt{a^2cx^2+c}x^2\sqrt{\arctan{(ax)}}\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*x^2*sqrt(arctan(a*x)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int x^2\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}\,dx = \int x^2\,\sqrt{\mathrm{atan}\left(a\,x\right)}\,\sqrt{c\,a^2\,x^2+c}\,dx$$

input
```
int(x^2*atan(a*x)^(1/2)*(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(x^2*atan(a*x)^(1/2)*(c + a^2*c*x^2)^(1/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int x^2\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}\,dx = \sqrt{c}\left(\int \sqrt{a^2x^2+1}\,\sqrt{atan\left(ax\right)}\,x^2dx\right)$$

input
```
int(x^2*(a^2*c*x^2+c)^(1/2)*atan(a*x)^(1/2),x)
```

output
```
sqrt(c)*int(sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**2,x)
```

# 3.722 $\int x\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int x\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}\,dx=\frac{\left(c+a^2cx^2\right)^{3/2}\sqrt{\arctan(ax)}}{3a^2c}-\frac{\text{Int}\left(\frac{\sqrt{c+a^2cx^2}}{\sqrt{\arctan(ax)}},x\right)}{6a}$$

output
```
1/3*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2)/a^2/c-1/6*Defer(Int)((a^2*c*x^2+
c)^(1/2)/arctan(a*x)^(1/2),x)/a
```

## Mathematica [N/A]

Not integrable

Time = 3.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int x\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}\,dx=\int x\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}\,dx$$

input
```
Integrate[x*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[x*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]], x]
```

## Rubi [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}\,dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\int \frac{\sqrt{a^2cx^2+c}}{\sqrt{\arctan(ax)}}dx}{6a}$$

$$\Big\downarrow \text{5560}$$

$$\frac{\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\int \frac{\sqrt{a^2cx^2+c}}{\sqrt{\arctan(ax)}}dx}{6a}$$

input
```
Int[x*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int x\sqrt{a^2c\,x^2+c}\,\sqrt{\arctan\left(ax\right)}dx$$

input
```
int(x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x)
```

output
```
int(x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int x\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 2.67 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}\,dx = \int x\sqrt{c\,(a^2x^2+1)}\sqrt{\operatorname{atan}\,(ax)}\,dx$$

input
```
integrate(x*(a**2*c*x**2+c)**(1/2)*atan(a*x)**(1/2),x)
```

output
```
Integral(x*sqrt(c*(a**2*x**2 + 1))*sqrt(atan(a*x)), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int x\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int x\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int x\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}\,dx = \int x\,\sqrt{\mathrm{atan}\left(a\,x\right)}\,\sqrt{c\,a^2\,x^2+c}\,dx$$

input
```
int(x*atan(a*x)^(1/2)*(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(x*atan(a*x)^(1/2)*(c + a^2*c*x^2)^(1/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 75, normalized size of antiderivative = 3.12

$$\int x\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}\,dx = \frac{\sqrt{c}\left(2\sqrt{a^2x^2+1}\,\sqrt{atan\left(ax\right)}\,a^2x^2+2\sqrt{a^2x^2+1}\,\sqrt{atan\left(ax\right)}-\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}}{atan(ax)}dx\right)a\right)}{6a^2}$$

input
```
int(x*(a^2*c*x^2+c)^(1/2)*atan(a*x)^(1/2),x)
```

output
```
(sqrt(c)*(2*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*a**2*x**2 + 2*sqrt(a**2*x*
*2 + 1)*sqrt(atan(a*x)) - int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/atan(a
*x),x)*a))/(6*a**2)
```

# 3.723 $\int \sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}\,dx = \text{Int}\left(\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}\,dx = \int \sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}\,dx$$

input
```
Integrate[Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]], x]
```

## Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}\,dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int \sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}dx$$

input
```
Int[Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \sqrt{a^2c\,x^2+c}\,\sqrt{\arctan\left(ax\right)}dx$$

input
```
int((a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x)
```

output
```
int((a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 1.37 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}\,dx = \int \sqrt{c\,(a^2x^2+1)}\sqrt{\operatorname{atan}{(ax)}}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(1/2)*atan(a*x)**(1/2),x)
```

output
```
Integral(sqrt(c*(a**2*x**2 + 1))*sqrt(atan(a*x)), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int \sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}\,dx = \int \sqrt{\mathrm{atan}\left(a\,x\right)}\,\sqrt{c\,a^2\,x^2+c}\,dx$$

input
```
int(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(1/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}\,dx = \sqrt{c}\left(\int \sqrt{a^2x^2+1}\,\sqrt{atan(ax)}dx\right)$$

input
```
int((a^2*c*x^2+c)^(1/2)*atan(a*x)^(1/2),x)
```

output
```
sqrt(c)*int(sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)),x)
```

# 3.724 $\int x^m (c+a^2cx^2)^{3/2} \sqrt{\arctan(ax)}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int x^m \left(c+a^2cx^2\right)^{3/2} \sqrt{\arctan(ax)}\,dx = \operatorname{Int}\left(x^m \left(c+a^2cx^2\right)^{3/2} \sqrt{\arctan(ax)},x\right)$$

output
```
Defer(Int)(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int x^m \left(c+a^2cx^2\right)^{3/2} \sqrt{\arctan(ax)}\,dx = \int x^m \left(c+a^2cx^2\right)^{3/2} \sqrt{\arctan(ax)}\,dx$$

input
```
Integrate[x^m*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[x^m*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]], x]
```

## Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sqrt{\arctan(ax)} \left(a^2cx^2+c\right)^{3/2} \, dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int x^m \sqrt{\arctan(ax)} \left(a^2cx^2+c\right)^{3/2} dx$$

input
```
Int[x^m*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int x^m \left(a^2 c\,x^2+c\right)^{\frac{3}{2}} \sqrt{\arctan\left(a x\right)}dx$$

input
```
int(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int x^m \left(c+a^2 c x^2\right)^{3/2} \sqrt{\arctan(ax)}\, dx = \int \left(a^2 c x^2 + c\right)^{\frac{3}{2}} x^m \sqrt{\arctan\left(ax\right)}\, dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^(3/2)*x^m*sqrt(arctan(a*x)), x)
```

## Sympy [F(-1)]

Timed out.

$$\int x^m \left(c+a^2 c x^2\right)^{3/2} \sqrt{\arctan(ax)}\, dx = \text{Timed out}$$

input
```
integrate(x**m*(a**2*c*x**2+c)**(3/2)*atan(a*x)**(1/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int x^m \left(c+a^2 c x^2\right)^{3/2} \sqrt{\arctan(ax)}\, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int x^m (c+a^2cx^2)^{3/2} \sqrt{\arctan(ax)}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int x^m (c+a^2cx^2)^{3/2} \sqrt{\arctan(ax)}\, dx = \int x^m \sqrt{\mathrm{atan}\,(a\,x)}\,{\left(c\,a^2\,x^2+c\right)}^{3/2}\,dx$$

input
```
int(x^m*atan(a*x)^(1/2)*(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(x^m*atan(a*x)^(1/2)*(c + a^2*c*x^2)^(3/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.08

$$\int x^m (c + a^2cx^2)^{3/2} \sqrt{\arctan(ax)}\, dx = \sqrt{c}\, c\left(\left(\int x^m\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}\,x^2dx\right)a^2 + \int x^m\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}dx\right)$$

input
```
int(x^m*(a^2*c*x^2+c)^(3/2)*atan(a*x)^(1/2),x)
```

output
```
sqrt(c)*c*(int(x**m*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**2,x)*a**2 + int
(x**m*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)),x))
```

# 3.725 $\int x^2(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int x^2\left(c+a^2cx^2\right)^{3/2}\sqrt{\arctan(ax)}\,dx=\text{Int}\left(x^2\left(c+a^2cx^2\right)^{3/2}\sqrt{\arctan(ax)},x\right)$$

output
```
Defer(Int)(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 4.33 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int x^2\left(c+a^2cx^2\right)^{3/2}\sqrt{\arctan(ax)}\,dx=\int x^2\left(c+a^2cx^2\right)^{3/2}\sqrt{\arctan(ax)}\,dx$$

input
```
Integrate[x^2*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[x^2*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]], x]
```

## Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}\,dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int x^2\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}dx$$

input
```
Int[x^2*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int x^2\left(a^2c\,x^2+c\right)^{\frac{3}{2}}\sqrt{\arctan\left(ax\right)}dx$$

input
```
int(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x)
```

output
```
int(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int x^2 (c + a^2 cx^2)^{3/2} \sqrt{\arctan(ax)}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 100.36 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int x^2 (c + a^2 cx^2)^{3/2} \sqrt{\arctan(ax)}\, dx = \int x^2 \left(c(a^2x^2 + 1)\right)^{\frac{3}{2}} \sqrt{\operatorname{atan}(ax)}\, dx$$

input
```
integrate(x**2*(a**2*c*x**2+c)**(3/2)*atan(a*x)**(1/2),x)
```

output
```
Integral(x**2*(c*(a**2*x**2 + 1))**(3/2)*sqrt(atan(a*x)), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int x^2 (c + a^2 cx^2)^{3/2} \sqrt{\arctan(ax)}\, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int x^2(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}\,dx=\int (a^2cx^2+c)^{\frac{3}{2}}x^2\sqrt{\arctan(ax)}\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*x^2*sqrt(arctan(a*x)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int x^2(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}\,dx=\int x^2\sqrt{\mathrm{atan}\,(a\,x)}\,(c\,a^2\,x^2+c)^{3/2}\,dx$$

input
```
int(x^2*atan(a*x)^(1/2)*(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(x^2*atan(a*x)^(1/2)*(c + a^2*c*x^2)^(3/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.96

$$\int x^2(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}\,dx=\sqrt{c}\,c\left(\left(\int\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,x^4dx\right)a^2+\int\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,x^2dx\right)$$

input
```
int(x^2*(a^2*c*x^2+c)^(3/2)*atan(a*x)^(1/2),x)
```

output
```
sqrt(c)*c*(int(sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**4,x)*a**2 + int(sqrt
(a**2*x**2 + 1)*sqrt(atan(a*x))*x**2,x))
```

# 3.726 $\int x(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int x\left(c+a^2cx^2\right)^{3/2}\sqrt{\arctan(ax)}\,dx=\frac{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}{5a^2c}-\frac{\operatorname{Int}\left(\frac{(c+a^2cx^2)^{3/2}}{\sqrt{\arctan(ax)}},x\right)}{10a}$$

output
```
1/5*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2)/a^2/c-1/10*Defer(Int)((a^2*c*x^2
+c)^(3/2)/arctan(a*x)^(1/2),x)/a
```

## Mathematica [N/A]

Not integrable

Time = 4.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int x\left(c+a^2cx^2\right)^{3/2}\sqrt{\arctan(ax)}\,dx=\int x\left(c+a^2cx^2\right)^{3/2}\sqrt{\arctan(ax)}\,dx$$

input
```
Integrate[x*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[x*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]], x]
```

## Rubi [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}\,dx$$

$$\Big\downarrow \text{ 5465}$$

$$\frac{\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{\int \frac{\left(a^2cx^2+c\right)^{3/2}}{\sqrt{\arctan(ax)}}dx}{10a}$$

$$\Big\downarrow \text{ 5560}$$

$$\frac{\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{\int \frac{\left(a^2cx^2+c\right)^{3/2}}{\sqrt{\arctan(ax)}}dx}{10a}$$

input
```
Int[x*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int x\left(a^2c\,x^2+c\right)^{\frac{3}{2}}\sqrt{\arctan\left(ax\right)}dx$$

input
```
int(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x)
```

output
```
int(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int x(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 51.82 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}\,dx = \int x\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\sqrt{\operatorname{atan}(ax)}\,dx$$

input
```
integrate(x*(a**2*c*x**2+c)**(3/2)*atan(a*x)**(1/2),x)
```

output
```
Integral(x*(c*(a**2*x**2 + 1))**(3/2)*sqrt(atan(a*x)), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int x(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int x(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int x(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}\,dx = \int x\,\sqrt{\mathrm{atan}\left(a\,x\right)}\,{\left(c\,a^2\,x^2+c\right)}^{3/2}\,dx$$

input
```
int(x*atan(a*x)^(1/2)*(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(x*atan(a*x)^(1/2)*(c + a^2*c*x^2)^(3/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 131, normalized size of antiderivative = 5.46

$$\int x(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}\,dx = \frac{\sqrt{c}\,c\Big(2\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}\,a^4x^4+4\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}\,a^2x^2+2\sqrt{a^2x^2}}{1}$$

input
```
int(x*(a^2*c*x^2+c)^(3/2)*atan(a*x)^(1/2),x)
```

output
```
(sqrt(c)*c*(2*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*a**4*x**4 + 4*sqrt(a**2*
x**2 + 1)*sqrt(atan(a*x))*a**2*x**2 + 2*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)
) - int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**2)/atan(a*x),x)*a**3 - int
((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/atan(a*x),x)*a))/(10*a**2)
```

# 3.727 $\int (c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 23

$$\int (c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}\,dx = \text{Int}\Big((c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)},x\Big)$$

output
```
Defer(Int)((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.62 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}\,dx = \int (c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]], x]
```

## Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}\,dx$$

$$\Big\downarrow \; 5560$$

$$\int \sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}dx$$

input
```
Int[(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \left(a^2c\,x^2+c\right)^{\frac{3}{2}}\sqrt{\arctan\left(ax\right)}dx$$

input
```
int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x)
```

output
```
int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^{3/2}\sqrt{\arctan(ax)}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 27.98 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \left(c+a^2cx^2\right)^{3/2}\sqrt{\arctan(ax)}\,dx=\int \left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\sqrt{\operatorname{atan}\left(ax\right)}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)**(1/2),x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(3/2)*sqrt(atan(a*x)), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^{3/2}\sqrt{\arctan(ax)}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^{3/2}\sqrt{\arctan(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \left(c+a^2cx^2\right)^{3/2}\sqrt{\arctan(ax)}\,dx = \int \sqrt{\mathrm{atan}\left(a\,x\right)}\,{\left(c\,a^2\,x^2+c\right)}^{3/2} \,dx$$

input
```
int(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(3/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.09

$$\int \left(c+a^2cx^2\right)^{3/2}\sqrt{\arctan(ax)}\,dx = \sqrt{c}\,c\left(\left(\int \sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,x^2dx\right)a^2 + \int \sqrt{a^2x^2+1}\,\sqrt{atan(ax)}dx\right)$$

input
```
int((a^2*c*x^2+c)^(3/2)*atan(a*x)^(1/2),x)
```

output
```
sqrt(c)*c*(int(sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**2,x)*a**2 + int(sqrt
(a**2*x**2 + 1)*sqrt(atan(a*x)),x))
```

# 3.728 $\int x^m(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int x^m(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}\,dx = \text{Int}\left(x^m(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)},x\right)$$

output
```
Defer(Int)(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.56 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int x^m(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}\,dx = \int x^m(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}\,dx$$

input
```
Integrate[x^m*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[x^m*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]], x]
```

## Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sqrt{\arctan(ax)} \left(a^2cx^2+c\right)^{5/2} \, dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int x^m \sqrt{\arctan(ax)} \left(a^2cx^2+c\right)^{5/2} dx$$

input
```
Int[x^m*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int x^m \left(a^2 c\,x^2+c\right)^{\frac{5}{2}} \sqrt{\arctan\left(a x\right)}dx$$

input
```
int(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.88

$$\int x^m \left(c+a^2cx^2\right)^{5/2}\sqrt{\arctan(ax)}\,dx=\int \left(a^2cx^2+c\right)^{\frac{5}{2}}x^m\sqrt{\arctan\left(ax\right)}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*x^m*sqrt(
arctan(a*x)), x)
```

## Sympy [F(-1)]

Timed out.

$$\int x^m \left(c+a^2cx^2\right)^{5/2}\sqrt{\arctan(ax)}\,dx=\text{Timed out}$$

input
```
integrate(x**m*(a**2*c*x**2+c)**(5/2)*atan(a*x)**(1/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int x^m \left(c+a^2cx^2\right)^{5/2}\sqrt{\arctan(ax)}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int x^m (c + a^2cx^2)^{5/2} \sqrt{\arctan(ax)}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int x^m (c + a^2cx^2)^{5/2} \sqrt{\arctan(ax)}\, dx = \int x^m \sqrt{\mathrm{atan}\,(a\,x)}\, (c\,a^2\,x^2 + c)^{5/2}\, dx$$

input
```
int(x^m*atan(a*x)^(1/2)*(c + a^2*c*x^2)^(5/2),x)
```

output
```
int(x^m*atan(a*x)^(1/2)*(c + a^2*c*x^2)^(5/2), x)
```

## Reduce [N/A]

Not integrable

Time = 1.84 (sec) , antiderivative size = 85, normalized size of antiderivative = 3.27

$$\int x^m (c + a^2cx^2)^{5/2} \sqrt{\arctan(ax)}\, dx = \sqrt{c}\, c^2 \left( \left( \int x^m \sqrt{a^2x^2+1}\, \sqrt{atan\,(ax)}\, x^4 dx \right) a^4 + 2 \left( \int x^m \sqrt{a^2x^2+1}\, \sqrt{atan\,(ax)}\, x^2 dx \right) a^2 + \int x^m \sqrt{a^2x^2+1}\, \sqrt{atan\,(ax)} dx \right)$$

input
```
int(x^m*(a^2*c*x^2+c)^(5/2)*atan(a*x)^(1/2),x)
```

output
```
sqrt(c)*c**2*(int(x**m*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**4,x)*a**4 +
2*int(x**m*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**2,x)*a**2 + int(x**m*sqr
t(a**2*x**2 + 1)*sqrt(atan(a*x)),x))
```

# 3.729 $\int x^2(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int x^2\left(c+a^2cx^2\right)^{5/2}\sqrt{\arctan(ax)}\,dx=\text{Int}\left(x^2\left(c+a^2cx^2\right)^{5/2}\sqrt{\arctan(ax)},x\right)$$

output
```
Defer(Int)(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 3.61 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int x^2\left(c+a^2cx^2\right)^{5/2}\sqrt{\arctan(ax)}\,dx=\int x^2\left(c+a^2cx^2\right)^{5/2}\sqrt{\arctan(ax)}\,dx$$

input
```
Integrate[x^2*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[x^2*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]], x]
```

## Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sqrt{\arctan(ax)} \left(a^2 c x^2 + c\right)^{5/2} \, dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int x^2 \sqrt{\arctan(ax)} \left(a^2 c x^2 + c\right)^{5/2} dx$$

input
```
Int[x^2*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int x^{2} \left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}} \sqrt{\arctan \left(a x \right)}dx$$

input
```
int(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x)
```

output
```
int(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int x^2 (c + a^2 c x^2)^{5/2} \sqrt{\arctan(ax)}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int x^2 (c + a^2 c x^2)^{5/2} \sqrt{\arctan(ax)}\, dx = \text{Timed out}$$

input
```
integrate(x**2*(a**2*c*x**2+c)**(5/2)*atan(a*x)**(1/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int x^2 (c + a^2 c x^2)^{5/2} \sqrt{\arctan(ax)}\, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 1.41 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int x^2 (c + a^2 c x^2)^{5/2} \sqrt{\arctan(ax)}\, dx = \int (a^2 c x^2 + c)^{\frac{5}{2}} x^2 \sqrt{\arctan(ax)}\, dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)*x^2*sqrt(arctan(a*x)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int x^2 (c + a^2 c x^2)^{5/2} \sqrt{\arctan(ax)}\, dx = \int x^2 \sqrt{\mathrm{atan}(a x)} (c a^2 x^2 + c)^{5/2}\, dx$$

input
```
int(x^2*atan(a*x)^(1/2)*(c + a^2*c*x^2)^(5/2),x)
```

output
```
int(x^2*atan(a*x)^(1/2)*(c + a^2*c*x^2)^(5/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 79, normalized size of antiderivative = 3.04

$$\int x^2 (c + a^2 c x^2)^{5/2} \sqrt{\arctan(ax)}\, dx = \sqrt{c}\, c^2 \left( \left( \int \sqrt{a^2x^2+1} \sqrt{atan(ax)}\, x^6 dx \right) a^4 + 2 \left( \int \sqrt{a^2x^2+1} \sqrt{atan(ax)}\, x^4 dx \right) a^2 + \int \sqrt{a^2x^2+1} \sqrt{atan(ax)}\, x^2 dx \right)$$

input
```
int(x^2*(a^2*c*x^2+c)^(5/2)*atan(a*x)^(1/2),x)
```

output
```
sqrt(c)*c**2*(int(sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**6,x)*a**4 + 2*int
(sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**4,x)*a**2 + int(sqrt(a**2*x**2 + 1
)*sqrt(atan(a*x))*x**2,x))
```

# 3.730 $\int x(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int x(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}\,dx=\frac{(c+a^2cx^2)^{7/2}\sqrt{\arctan(ax)}}{7a^2c}-\frac{\text{Int}\left(\frac{(c+a^2cx^2)^{5/2}}{\sqrt{\arctan(ax)}},x\right)}{14a}$$

output
```
1/7*(a^2*c*x^2+c)^(7/2)*arctan(a*x)^(1/2)/a^2/c-1/14*Defer(Int)((a^2*c*x^2
+c)^(5/2)/arctan(a*x)^(1/2),x)/a
```

## Mathematica [N/A]

Not integrable

Time = 3.67 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int x(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}\,dx=\int x(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}\,dx$$

input
```
Integrate[x*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[x*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]], x]
```

## Rubi [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{5/2}\,dx$$

$$\Big\downarrow \text{ 5465}$$

$$\frac{\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{7/2}}{7a^2c}-\frac{\int \frac{\left(a^2cx^2+c\right)^{5/2}}{\sqrt{\arctan(ax)}}dx}{14a}$$

$$\Big\downarrow \text{ 5560}$$

$$\frac{\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{7/2}}{7a^2c}-\frac{\int \frac{\left(a^2cx^2+c\right)^{5/2}}{\sqrt{\arctan(ax)}}dx}{14a}$$

input
```
Int[x*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int x\left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}} \sqrt{\arctan \left(a x \right)}dx$$

input
```
int(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x)
```

output
```
int(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int x(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int x(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}\,dx = \text{Timed out}$$

input
```
integrate(x*(a**2*c*x**2+c)**(5/2)*atan(a*x)**(1/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int x(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int x(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int x(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}\,dx = \int x\,\sqrt{\mathrm{atan}\left(a\,x\right)}\,{\left(c\,a^2\,x^2+c\right)}^{5/2} \,d x$$

input
```
int(x*atan(a*x)^(1/2)*(c + a^2*c*x^2)^(5/2),x)
```

output
```
int(x*atan(a*x)^(1/2)*(c + a^2*c*x^2)^(5/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 188, normalized size of antiderivative = 7.83

$$\int x(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}\,dx = \frac{\sqrt{c}\,c^2\Big(2\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}\,a^6x^6+6\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}\,a^4x^4+6\sqrt{a^2}}{}$$

input
```
int(x*(a^2*c*x^2+c)^(5/2)*atan(a*x)^(1/2),x)
```

output
```
(sqrt(c)*c**2*(2*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*a**6*x**6 + 6*sqrt(a*
*2*x**2 + 1)*sqrt(atan(a*x))*a**4*x**4 + 6*sqrt(a**2*x**2 + 1)*sqrt(atan(a
*x))*a**2*x**2 + 2*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)) - int((sqrt(a**2*x*
*2 + 1)*sqrt(atan(a*x))*x**4)/atan(a*x),x)*a**5 - 2*int((sqrt(a**2*x**2 +
1)*sqrt(atan(a*x))*x**2)/atan(a*x),x)*a**3 - int((sqrt(a**2*x**2 + 1)*sqrt
(atan(a*x)))/atan(a*x),x)*a))/(14*a**2)
```

# 3.731 $\int (c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 23

$$\int (c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}\,dx = \text{Int}\Big((c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)},x\Big)$$

output
```
Defer(Int)((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}\,dx = \int (c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]], x]
```

## Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{5/2}\,dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int \sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{5/2}\,dx$$

input
```
Int[(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \left(a^2c\,x^2+c\right)^{\frac{5}{2}}\sqrt{\arctan\left(ax\right)}dx$$

input
```
int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x)
```

output
```
int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \left(c + a^2 c x^2\right)^{5/2} \sqrt{\arctan(ax)}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \left(c + a^2 c x^2\right)^{5/2} \sqrt{\arctan(ax)}\, dx = \text{Timed out}$$

input
```
integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)**(1/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \left(c + a^2 c x^2\right)^{5/2} \sqrt{\arctan(ax)}\, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^{5/2}\sqrt{\arctan(ax)}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \left(c+a^2cx^2\right)^{5/2}\sqrt{\arctan(ax)}\,dx=\int \sqrt{\mathrm{atan}\left(a\,x\right)}\,{\left(c\,a^2\,x^2+c\right)}^{5/2}\,dx$$

input
```
int(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(5/2),x)
```

output
```
int(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(5/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 76, normalized size of antiderivative = 3.30

$$\int \left(c+a^2cx^2\right)^{5/2}\sqrt{\arctan(ax)}\,dx=\sqrt{c}\,c^2\left(\left(\left(\int \sqrt{a^2x^2+1}\,\sqrt{atan\left(ax\right)}\,x^4dx\right)a^4+2\left(\int \sqrt{a^2x^2+1}\,\sqrt{atan\left(ax\right)}\,x^2dx\right)a^2+\int \sqrt{a^2x^2+1}\,\sqrt{atan\left(ax\right)}dx\right)\right.$$

input
```
int((a^2*c*x^2+c)^(5/2)*atan(a*x)^(1/2),x)
```

output
```
sqrt(c)*c**2*(int(sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**4,x)*a**4 + 2*int
(sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**2,x)*a**2 + int(sqrt(a**2*x**2 + 1
)*sqrt(atan(a*x)),x))
```

# 3.732 $\int \frac{x^m\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^m\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \text{Int}\left(\frac{x^m\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}},x\right)$$

output
```
Defer(Int)(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.17 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^m\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^m\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx$$

input
```
Integrate[(x^m*Sqrt[ArcTan[a*x]])/Sqrt[c + a^2*c*x^2],x]
```

output
```
Integrate[(x^m*Sqrt[ArcTan[a*x]])/Sqrt[c + a^2*c*x^2], x]
```

## Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}}\,dx$$

$$\Big\downarrow \ 5560$$

$$\int \frac{x^m \sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}}dx$$

input
```
Int[(x^m*Sqrt[ArcTan[a*x]])/Sqrt[c + a^2*c*x^2],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^m \sqrt{\arctan(ax)}}{\sqrt{a^2c\,x^2+c}}dx$$

input
```
int(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^m\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(x^m*sqrt(arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

## Sympy [N/A]

Not integrable

Time = 28.80 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{x^m\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^m\sqrt{\operatorname{atan}(ax)}}{\sqrt{c\,(a^2x^2+1)}}\,dx$$

input
```
integrate(x**m*atan(a*x)**(1/2)/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x**m*sqrt(atan(a*x))/sqrt(c*(a**2*x**2 + 1)), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m \sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^m \sqrt{\arctan{(ax)}}}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^m*sqrt(arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

## Mupad [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m \sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^m\,\sqrt{\mathrm{atan}\left(a\,x\right)}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int((x^m*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int((x^m*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(1/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.46

$$\int \frac{x^m \sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{c}\left(\int \frac{x^m \sqrt{a^2x^2+1}\,\sqrt{atan(ax)}}{a^2x^2+1}dx\right)}{c}$$

input
```
int(x^m*atan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x)
```

output
```
(sqrt(c)*int((x**m*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(a**2*x**2 + 1),x)
)/c
```

# 3.733 $\int \frac{x^3\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^3\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx=-\frac{2\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}{3a^4c}+\frac{x^2\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}{3a^2c}+\frac{\operatorname{Int}\left(\frac{1}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}},x\right)}{3a^3}-\frac{\operatorname{Int}\left(\frac{x^2}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}},x\right)}{6a}$$

output
```
-2/3*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2)/a^4/c+1/3*x^2*(a^2*c*x^2+c)^(1/
2)*arctan(a*x)^(1/2)/a^2/c+1/3*Defer(Int)(1/(a^2*c*x^2+c)^(1/2)/arctan(a*x
)^(1/2),x)/a^3-1/6*Defer(Int)(x^2/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
/a
```

## Mathematica [N/A]

Not integrable

Time = 4.10 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^3\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx=\int \frac{x^3\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx$$

input
```
Integrate[(x^3*Sqrt[ArcTan[a*x]])/Sqrt[c + a^2*c*x^2],x]
```

output
```
Integrate[(x^3*Sqrt[ArcTan[a*x]])/Sqrt[c + a^2*c*x^2], x]
```

## Rubi [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5487}$$

$$-\frac{\int \frac{x^2}{\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx}{6a}-\frac{2\int \frac{x\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}{3a^2c}$$

$$\Big\downarrow \text{5465}$$

$$-\frac{2\left(\frac{\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}{a^2c}-\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx}{2a}\right)}{3a^2}-\frac{\int \frac{x^2}{\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx}{6a}+\frac{x^2\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}{3a^2c}$$

$$\Big\downarrow \text{5560}$$

$$-\frac{2\left(\frac{\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}{a^2c}-\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx}{2a}\right)}{3a^2}-\frac{\int \frac{x^2}{\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx}{6a}+\frac{x^2\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}{3a^2c}$$

input
```
Int[(x^3*Sqrt[ArcTan[a*x]])/Sqrt[c + a^2*c*x^2],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^3 \sqrt{\arctan (ax)}}{\sqrt{a^2 c\,x^2 + c}}dx$$

input
```
int(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3 \sqrt{\arctan(ax)}}{\sqrt{c + a^2 cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 7.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{x^3 \sqrt{\arctan(ax)}}{\sqrt{c + a^2 cx^2}}\,dx = \int \frac{x^3 \sqrt{\operatorname{atan}(ax)}}{\sqrt{c (a^2 x^2 + 1)}}\,dx$$

input
```
integrate(x**3*atan(a*x)**(1/2)/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x**3*sqrt(atan(a*x))/sqrt(c*(a**2*x**2 + 1)), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3 \sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x^3 \sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^3\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^3\,\sqrt{\mathrm{atan}\left(a\,x\right)}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int((x^3*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int((x^3*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(1/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 134, normalized size of antiderivative = 5.15

$$\int \frac{x^3\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{c}\left(2\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,a^2x^2 - 4\sqrt{a^2x^2+1}\,\sqrt{atan(ax)} - \left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,x^2}{atan(ax)a^2x^2+atan(ax)}dx\right)a^3 + 2\left(\int \frac{\sqrt{a}}{atan}\right.\right.}{6a^4c}$$

input
```
int(x^3*atan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x)
```

output
```
(sqrt(c)*(2*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*a**2*x**2 - 4*sqrt(a**2*x*
*2 + 1)*sqrt(atan(a*x)) - int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**2)/(
atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**3 + 2*int((sqrt(a**2*x**2 + 1)*sqrt
(atan(a*x)))/(atan(a*x)*a**2*x**2 + atan(a*x)),x)*a))/(6*a**4*c)
```

# 3.734 $\int \frac{x^2\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^2\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{x\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}{2a^2c} - \frac{\text{Int}\left(\frac{x}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}},x\right)}{4a} - \frac{\text{Int}\left(\frac{\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}},x\right)}{2a^2}$$

output
```
1/2*x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2)/a^2/c-1/4*Defer(Int)(x/(a^2*c*
x^2+c)^(1/2)/arctan(a*x)^(1/2),x)/a-1/2*Defer(Int)(arctan(a*x)^(1/2)/(a^2*
c*x^2+c)^(1/2),x)/a^2
```

## Mathematica [N/A]

Not integrable

Time = 2.92 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^2\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^2\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx$$

input
```
Integrate[(x^2*Sqrt[ArcTan[a*x]])/Sqrt[c + a^2*c*x^2],x]
```

output
```
Integrate[(x^2*Sqrt[ArcTan[a*x]])/Sqrt[c + a^2*c*x^2], x]
```

## Rubi [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \; 5487$$

$$-\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx}{4a}-\frac{\int \frac{\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}}dx}{2a^2}+\frac{x\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}{2a^2c}$$

$$\Big\downarrow \; 5560$$

$$-\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx}{4a}-\frac{\int \frac{\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}}dx}{2a^2}+\frac{x\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}{2a^2c}$$

input
```
Int[(x^2*Sqrt[ArcTan[a*x]])/Sqrt[c + a^2*c*x^2],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^2 \sqrt{\arctan (ax)}}{\sqrt{a^2 c\,x^2 + c}}dx$$

input
```
int(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2 \sqrt{\arctan(ax)}}{\sqrt{c + a^2cx^2}}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 3.64 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{x^2 \sqrt{\arctan(ax)}}{\sqrt{c + a^2cx^2}}\, dx = \int \frac{x^2 \sqrt{\operatorname{atan}{(ax)}}}{\sqrt{c\,(a^2x^2 + 1)}}\, dx$$

input
```
integrate(x**2*atan(a*x)**(1/2)/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x**2*sqrt(atan(a*x))/sqrt(c*(a**2*x**2 + 1)), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^2\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^2\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2*sqrt(arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

## Mupad [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^2\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^2\,\sqrt{\mathrm{atan}\left(a\,x\right)}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int((x^2*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int((x^2*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(1/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.46

$$\int \frac{x^2\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,x^2}{a^2x^2+1}dx\right)}{c}$$

input
```
int(x^2*atan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x)
```

output
```
(sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**2)/(a**2*x**2 + 1),x)
)/c
```

# 3.735 $\int \frac{x\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx=\frac{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}{a^2c}-\frac{\text{Int}\left(\frac{1}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}},x\right)}{2a}$$

output
```
(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2)/a^2/c-1/2*Defer(Int)(1/(a^2*c*x^2+c)
^(1/2)/arctan(a*x)^(1/2),x)/a
```

## Mathematica [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx=\int \frac{x\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx$$

input
```
Integrate[(x*Sqrt[ArcTan[a*x]])/Sqrt[c + a^2*c*x^2],x]
```

output
```
Integrate[(x*Sqrt[ArcTan[a*x]])/Sqrt[c + a^2*c*x^2], x]
```

## Rubi [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{ 5465}$$

$$\frac{\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}{a^2c}-\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx}{2a}$$

$$\Big\downarrow \text{ 5560}$$

$$\frac{\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}{a^2c}-\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx}{2a}$$

input
```
Int[(x*Sqrt[ArcTan[a*x]])/Sqrt[c + a^2*c*x^2],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{x\sqrt{\arctan\left(ax\right)}}{\sqrt{a^{2}c\,x^{2}+c}}dx$$

input
```
int(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 1.74 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x\sqrt{\operatorname{atan}(ax)}}{\sqrt{c(a^2x^2+1)}}\,dx$$

input
```
integrate(x*atan(a*x)**(1/2)/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x*sqrt(atan(a*x))/sqrt(c*(a**2*x**2 + 1)), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x*sqrt(arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

## Mupad [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x\,\sqrt{\mathrm{atan}\left(a\,x\right)}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int((x*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int((x*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(1/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.79

$$\int \frac{x\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{c}\left(2\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}-\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}}{atan(ax)a^2x^2+atan(ax)}dx\right)a\right)}{2a^2c}$$

input
```
int(x*atan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x)
```

output
```
(sqrt(c)*(2*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)) - int((sqrt(a**2*x**2 + 1)
*sqrt(atan(a*x)))/(atan(a*x)*a**2*x**2 + atan(a*x)),x)*a))/(2*a**2*c)
```

# 3.736 $\int \frac{\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx=\text{Int}\left(\frac{\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}},x\right)$$

output
```
Defer(Int)(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx=\int \frac{\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx$$

input
```
Integrate[Sqrt[ArcTan[a*x]]/Sqrt[c + a^2*c*x^2],x]
```

output
```
Integrate[Sqrt[ArcTan[a*x]]/Sqrt[c + a^2*c*x^2], x]
```

## Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int \frac{\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}}dx$$

input
```
Int[Sqrt[ArcTan[a*x]]/Sqrt[c + a^2*c*x^2],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{\arctan (ax)}}{\sqrt{a^2c\,x^2+c}}dx$$

input
```
int(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{\sqrt{c + a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 0.83 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{\arctan(ax)}}{\sqrt{c + a^2cx^2}}\,dx = \int \frac{\sqrt{\operatorname{atan}(ax)}}{\sqrt{c\,(a^2x^2 + 1)}}\,dx$$

input
```
integrate(atan(a*x)**(1/2)/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(atan(a*x))/sqrt(c*(a**2*x**2 + 1)), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{\sqrt{c + a^2cx^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\sqrt{\arctan{(ax)}}}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(arctan(a*x))/sqrt(a^2*c*x^2 + c), x)
```

## Mupad [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\sqrt{\mathrm{atan}\left(a\,x\right)}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(atan(a*x)^(1/2)/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(atan(a*x)^(1/2)/(c + a^2*c*x^2)^(1/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.52

$$\int \frac{\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}}{a^2x^2+1}dx\right)}{c}$$

input
```
int(atan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x)
```

output
```
(sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(a**2*x**2 + 1),x))/c
```

# 3.737 $\int \frac{\sqrt{\arctan(ax)}}{x\sqrt{c+a^2cx^2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\sqrt{\arctan(ax)}}{x\sqrt{c+a^2cx^2}}\,dx = \text{Int}\left(\frac{\sqrt{\arctan(ax)}}{x\sqrt{c+a^2cx^2}},x\right)$$

output
```
Defer(Int)(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.35 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{\arctan(ax)}}{x\sqrt{c+a^2cx^2}}\,dx = \int \frac{\sqrt{\arctan(ax)}}{x\sqrt{c+a^2cx^2}}\,dx$$

input
```
Integrate[Sqrt[ArcTan[a*x]]/(x*Sqrt[c + a^2*c*x^2]),x]
```

output
```
Integrate[Sqrt[ArcTan[a*x]]/(x*Sqrt[c + a^2*c*x^2]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\arctan(ax)}}{x\sqrt{a^2cx^2+c}}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{\sqrt{\arctan(ax)}}{x\sqrt{a^2cx^2+c}}dx$$

input
```
Int[Sqrt[ArcTan[a*x]]/(x*Sqrt[c + a^2*c*x^2]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{\arctan(ax)}}{x\sqrt{a^2c\,x^2+c}}dx$$

input
```
int(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{x\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 1.75 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{\arctan(ax)}}{x\sqrt{c+a^2cx^2}}\,dx = \int \frac{\sqrt{\operatorname{atan}(ax)}}{x\sqrt{c\,(a^2x^2+1)}}\,dx$$

input
```
integrate(atan(a*x)**(1/2)/x/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(atan(a*x))/(x*sqrt(c*(a**2*x**2 + 1))), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{x\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{\arctan(ax)}}{x\sqrt{c+a^2cx^2}}\,dx = \int \frac{\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+cx}}\,dx$$

input
```
integrate(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(arctan(a*x))/(sqrt(a^2*c*x^2 + c)*x), x)
```

## Mupad [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{\arctan(ax)}}{x\sqrt{c+a^2cx^2}}\,dx = \int \frac{\sqrt{\mathrm{atan}\left(a\,x\right)}}{x\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(atan(a*x)^(1/2)/(x*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(atan(a*x)^(1/2)/(x*(c + a^2*c*x^2)^(1/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.35

$$\int \frac{\sqrt{\arctan(ax)}}{x\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}}{a^2x^3+x}dx\right)}{c}$$

input
```
int(atan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(1/2),x)
```

output
```
(sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(a**2*x**3 + x),x))/c
```

# 3.738 $\int \frac{\sqrt{\arctan(ax)}}{x^2\sqrt{c+a^2cx^2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\sqrt{\arctan(ax)}}{x^2\sqrt{c+a^2cx^2}}\,dx = -\frac{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}{cx}+\frac{1}{2}a\text{Int}\left(\frac{1}{x\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}},x\right)$$

output
```
-(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2)/c/x+1/2*a*Defer(Int)(1/x/(a^2*c*x^2
+c)^(1/2)/arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.99 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{\arctan(ax)}}{x^2\sqrt{c+a^2cx^2}}\,dx = \int \frac{\sqrt{\arctan(ax)}}{x^2\sqrt{c+a^2cx^2}}\,dx$$

input
```
Integrate[Sqrt[ArcTan[a*x]]/(x^2*Sqrt[c + a^2*c*x^2]),x]
```

output
```
Integrate[Sqrt[ArcTan[a*x]]/(x^2*Sqrt[c + a^2*c*x^2]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\arctan(ax)}}{x^2\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{ 5479}$$

$$\frac{1}{2}a\int \frac{1}{x\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx-\frac{\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}{cx}$$

$$\Big\downarrow \text{ 5560}$$

$$\frac{1}{2}a\int \frac{1}{x\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx-\frac{\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}{cx}$$

input
```
Int[Sqrt[ArcTan[a*x]]/(x^2*Sqrt[c + a^2*c*x^2]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{\arctan(ax)}}{x^2\sqrt{a^2c\,x^2+c}}dx$$

input
```
int(arctan(a*x)^(1/2)/x^2/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(arctan(a*x)^(1/2)/x^2/(a^2*c*x^2+c)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{x^2\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(1/2)/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 4.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\arctan(ax)}}{x^2\sqrt{c+a^2cx^2}}\,dx = \int \frac{\sqrt{\operatorname{atan}(ax)}}{x^2\sqrt{c(a^2x^2+1)}}\,dx$$

input
```
integrate(atan(a*x)**(1/2)/x**2/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(atan(a*x))/(x**2*sqrt(c*(a**2*x**2 + 1))), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{x^2\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(1/2)/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{\arctan(ax)}}{x^2\sqrt{c+a^2cx^2}}\,dx = \int \frac{\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}x^2}\,dx$$

input
```
integrate(arctan(a*x)^(1/2)/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(arctan(a*x))/(sqrt(a^2*c*x^2 + c)*x^2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{\arctan(ax)}}{x^2\sqrt{c+a^2cx^2}}\,dx=\int \frac{\sqrt{\mathrm{atan}\left(a\,x\right)}}{x^2\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(atan(a*x)^(1/2)/(x^2*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(atan(a*x)^(1/2)/(x^2*(c + a^2*c*x^2)^(1/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.65

$$\int \frac{\sqrt{\arctan(ax)}}{x^2\sqrt{c+a^2cx^2}}\,dx=\frac{\sqrt{c}\left(-2\sqrt{a^2x^2+1}\,\sqrt{atan\left(ax\right)}+\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan\left(ax\right)}}{atan\left(ax\right)a^2x^3+atan\left(ax\right)x}dx\right)ax\right)}{2cx}$$

input
```
int(atan(a*x)^(1/2)/x^2/(a^2*c*x^2+c)^(1/2),x)
```

output
```
(sqrt(c)*( - 2*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)) + int((sqrt(a**2*x**2 +
 1)*sqrt(atan(a*x)))/(atan(a*x)*a**2*x**3 + atan(a*x)*x),x)*a*x))/(2*c*x)
```

# 3.739 $\int \frac{\sqrt{\arctan(ax)}}{x^3\sqrt{c+a^2cx^2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\sqrt{\arctan(ax)}}{x^3\sqrt{c+a^2cx^2}}\,dx = -\frac{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}{2cx^2} + \frac{1}{4}a\text{Int}\left(\frac{1}{x^2\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}},x\right) - \frac{1}{2}a^2\text{Int}\left(\frac{\sqrt{\arctan(ax)}}{x\sqrt{c+a^2cx^2}},x\right)$$

output
```
-1/2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2)/c/x^2+1/4*a*Defer(Int)(1/x^2/(a
^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)-1/2*a^2*Defer(Int)(arctan(a*x)^(1/2
)/x/(a^2*c*x^2+c)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 7.15 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{\arctan(ax)}}{x^3\sqrt{c+a^2cx^2}}\,dx = \int \frac{\sqrt{\arctan(ax)}}{x^3\sqrt{c+a^2cx^2}}\,dx$$

input
```
Integrate[Sqrt[ArcTan[a*x]]/(x^3*Sqrt[c + a^2*c*x^2]),x]
```

output
```
Integrate[Sqrt[ArcTan[a*x]]/(x^3*Sqrt[c + a^2*c*x^2]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\arctan(ax)}}{x^3\sqrt{a^2cx^2+c}}\,dx$$

$$\Big\downarrow \text{5497}$$

$$-\frac{1}{2}a^2\int \frac{\sqrt{\arctan(ax)}}{x\sqrt{a^2cx^2+c}}dx+\frac{1}{4}a\int \frac{1}{x^2\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx-\frac{\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}{2cx^2}$$

$$\Big\downarrow \text{5560}$$

$$-\frac{1}{2}a^2\int \frac{\sqrt{\arctan(ax)}}{x\sqrt{a^2cx^2+c}}dx+\frac{1}{4}a\int \frac{1}{x^2\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx-\frac{\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}{2cx^2}$$

input
```
Int[Sqrt[ArcTan[a*x]]/(x^3*Sqrt[c + a^2*c*x^2]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{\arctan (ax)}}{x^3\sqrt{a^2c\,x^2+c}}dx$$

input
```
int(arctan(a*x)^(1/2)/x^3/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(arctan(a*x)^(1/2)/x^3/(a^2*c*x^2+c)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{x^3\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(1/2)/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 7.43 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\arctan(ax)}}{x^3\sqrt{c+a^2cx^2}}\,dx = \int \frac{\sqrt{\operatorname{atan}(ax)}}{x^3\sqrt{c(a^2x^2+1)}}\,dx$$

input
```
integrate(atan(a*x)**(1/2)/x**3/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(atan(a*x))/(x**3*sqrt(c*(a**2*x**2 + 1))), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{x^3\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(1/2)/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{\arctan(ax)}}{x^3\sqrt{c+a^2cx^2}}\,dx = \int \frac{\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}x^3}\,dx$$

input
```
integrate(arctan(a*x)^(1/2)/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(arctan(a*x))/(sqrt(a^2*c*x^2 + c)*x^3), x)
```

## Mupad [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{\arctan(ax)}}{x^3\sqrt{c+a^2cx^2}}\,dx = \int \frac{\sqrt{\mathrm{atan}\left(a\,x\right)}}{x^3\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(atan(a*x)^(1/2)/(x^3*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(atan(a*x)^(1/2)/(x^3*(c + a^2*c*x^2)^(1/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.42

$$\int \frac{\sqrt{\arctan(ax)}}{x^3\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}}{a^2x^5+x^3}dx\right)}{c}$$

input
```
int(atan(a*x)^(1/2)/x^3/(a^2*c*x^2+c)^(1/2),x)
```

output
```
(sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(a**2*x**5 + x**3),x))/
c
```

# 3.740 $\int \frac{\sqrt{\arctan(ax)}}{x^4\sqrt{c+a^2cx^2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\sqrt{\arctan(ax)}}{x^4\sqrt{c+a^2cx^2}}\,dx = -\frac{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}{3cx^3} + \frac{2a^2\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}{3cx} + \frac{1}{6}a\text{Int}\left(\frac{1}{x^3\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}},x\right) - \frac{1}{3}a^3\text{Int}\left(\frac{1}{x\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}},x\right)$$

output
```
-1/3*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2)/c/x^3+2/3*a^2*(a^2*c*x^2+c)^(1/
2)*arctan(a*x)^(1/2)/c/x+1/6*a*Defer(Int)(1/x^3/(a^2*c*x^2+c)^(1/2)/arctan
(a*x)^(1/2),x)-1/3*a^3*Defer(Int)(1/x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2
),x)
```

## Mathematica [N/A]

Not integrable

Time = 13.80 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{\arctan(ax)}}{x^4\sqrt{c+a^2cx^2}}\,dx = \int \frac{\sqrt{\arctan(ax)}}{x^4\sqrt{c+a^2cx^2}}\,dx$$

input
```
Integrate[Sqrt[ArcTan[a*x]]/(x^4*Sqrt[c + a^2*c*x^2]),x]
```

output
```
Integrate[Sqrt[ArcTan[a*x]]/(x^4*Sqrt[c + a^2*c*x^2]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\arctan(ax)}}{x^4\sqrt{a^2cx^2+c}}\,dx$$

$$\Big\downarrow \text{5497}$$

$$-\frac{2}{3}a^2\int \frac{\sqrt{\arctan(ax)}}{x^2\sqrt{a^2cx^2+c}}dx+\frac{1}{6}a\int \frac{1}{x^3\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx-\frac{\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}{3cx^3}$$

$$\Big\downarrow \text{5479}$$

$$-\frac{2}{3}a^2\left(\frac{1}{2}a\int \frac{1}{x\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx-\frac{\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}{cx}\right)+\frac{1}{6}a\int \frac{1}{x^3\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx-\frac{\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}{3cx^3}$$

$$\Big\downarrow \text{5560}$$

$$-\frac{2}{3}a^2\left(\frac{1}{2}a\int\frac{1}{x\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx-\frac{\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}{cx}\right)+\frac{1}{6}a\int\frac{1}{x^3\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx-\frac{\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}{3cx^3}$$

input
```
Int[Sqrt[ArcTan[a*x]]/(x^4*Sqrt[c + a^2*c*x^2]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int\frac{\sqrt{\arctan(ax)}}{x^4\sqrt{a^2c\,x^2+c}}dx$$

input
```
int(arctan(a*x)^(1/2)/x^4/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(arctan(a*x)^(1/2)/x^4/(a^2*c*x^2+c)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int\frac{\sqrt{\arctan(ax)}}{x^4\sqrt{c+a^2cx^2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(1/2)/x^4/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 15.80 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\arctan(ax)}}{x^4\sqrt{c+a^2cx^2}}\,dx = \int \frac{\sqrt{\operatorname{atan}(ax)}}{x^4\sqrt{c\left(a^2x^2+1\right)}}\,dx$$

input
```
integrate(atan(a*x)**(1/2)/x**4/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(atan(a*x))/(x**4*sqrt(c*(a**2*x**2 + 1))), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{x^4\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(1/2)/x^4/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{\arctan(ax)}}{x^4\sqrt{c+a^2cx^2}}\,dx = \int \frac{\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+cx^4}}\,dx$$

input
```
integrate(arctan(a*x)^(1/2)/x^4/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(arctan(a*x))/(sqrt(a^2*c*x^2 + c)*x^4), x)
```

## Mupad [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{\arctan(ax)}}{x^4\sqrt{c+a^2cx^2}}\,dx = \int \frac{\sqrt{\mathrm{atan}\,(a\,x)}}{x^4\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(atan(a*x)^(1/2)/(x^4*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(atan(a*x)^(1/2)/(x^4*(c + a^2*c*x^2)^(1/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 142, normalized size of antiderivative = 5.46

$$\int \frac{\sqrt{\arctan(ax)}}{x^4\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{c}\left(4\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}\,a^2x^2 - 2\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)} + \left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}}{atan(ax)a^2x^5+atan(ax)x^3}dx\right)a\,x^3 - 2\left(\int\right.\right.}{6c\,x^3}$$

input
```
int(atan(a*x)^(1/2)/x^4/(a^2*c*x^2+c)^(1/2),x)
```

output
```
(sqrt(c)*(4*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*a**2*x**2 - 2*sqrt(a**2*x*
*2 + 1)*sqrt(atan(a*x)) + int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(atan(
a*x)*a**2*x**5 + atan(a*x)*x**3),x)*a*x**3 - 2*int((sqrt(a**2*x**2 + 1)*sq
rt(atan(a*x)))/(atan(a*x)*a**2*x**3 + atan(a*x)*x),x)*a**3*x**3))/(6*c*x**
3)
```

# 3.741 $\int \frac{x^m\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^m\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Int}\left(\frac{x^m\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}},x\right)$$

output
```
Defer(Int)(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.33 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^m\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^m\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx$$

input
```
Integrate[(x^m*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(3/2),x]
```

output
```
Integrate[(x^m*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(3/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m\sqrt{\arctan(ax)}}{(a^2cx^2+c)^{3/2}}\,dx$$

$$\Big\downarrow \; 5560$$

$$\int \frac{x^m\sqrt{\arctan(ax)}}{(a^2cx^2+c)^{3/2}}dx$$

input
```
Int[(x^m*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(3/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^m\sqrt{\arctan(ax)}}{(a^2c\,x^2+c)^{\frac{3}{2}}}dx$$

input
```
int(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x)
```

output
```
int(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.96

$$\int \frac{x^m \sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}} \, dx = \int \frac{x^m \sqrt{\arctan(ax)}}{(a^2cx^2+c)^{\frac{3}{2}}} \, dx$$

input
```
integrate(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^m*sqrt(arctan(a*x))/(a^4*c^2*x^4 + 2*a^2*c^
2*x^2 + c^2), x)
```

## Sympy [N/A]

Not integrable

Time = 72.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{x^m \sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}} \, dx = \int \frac{x^m \sqrt{\operatorname{atan}(ax)}}{(c(a^2x^2+1))^{\frac{3}{2}}} \, dx$$

input
```
integrate(x**m*atan(a*x)**(1/2)/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(x**m*sqrt(atan(a*x))/(c*(a**2*x**2 + 1))**(3/2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m \sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^m\sqrt{\arctan{(ax)}}}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^m*sqrt(arctan(a*x))/(a^2*c*x^2 + c)^(3/2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^m\,\sqrt{\mathrm{atan}\left(a\,x\right)}}{{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

input
```
int((x^m*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(3/2),x)
```

output
```
int((x^m*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(3/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.77

$$\int \frac{x^m \sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{c}\left(\int \frac{x^m\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}}{a^4x^4+2a^2x^2+1}dx\right)}{c^2}$$

input

```
int(x^m*atan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x)
```

output

```
(sqrt(c)*int((x**m*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(a**4*x**4 + 2*a**
2*x**2 + 1),x))/c**2
```

# 3.742 $\int \frac{x^3 \sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}} \, dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^3 \sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}} \, dx = \text{Int}\left(\frac{x^3 \sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}, x\right)$$

output
```
Defer(Int)(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 6.72 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^3 \sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}} \, dx = \int \frac{x^3 \sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}} \, dx$$

input
```
Integrate[(x^3*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(3/2),x]
```

output
```
Integrate[(x^3*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(3/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\sqrt{\arctan(ax)}}{(a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \text{5560}$$

$$\int \frac{x^3\sqrt{\arctan(ax)}}{(a^2cx^2+c)^{3/2}}dx$$

input
```
Int[(x^3*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(3/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^3\sqrt{\arctan(ax)}}{(a^2c\,x^2+c)^{\frac{3}{2}}}dx$$

input
```
int(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x)
```

output
```
int(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 8.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{x^3\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^3\sqrt{\operatorname{atan}(ax)}}{(c(a^2x^2+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3*atan(a*x)**(1/2)/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(x**3*sqrt(atan(a*x))/(c*(a**2*x**2 + 1))**(3/2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x^3 \sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^3 \sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}} \, dx = \int \frac{x^3 \sqrt{\mathrm{atan}\left(a\,x\right)}}{{\left(c\,a^2\,x^2+c\right)}^{3/2}} \,d x$$

input
```
int((x^3*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(3/2),x)
```

output
```
int((x^3*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(3/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 284, normalized size of antiderivative = 10.92

$$\int \frac{x^3 \sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{c}\left(2\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}\,a^2x^2+4\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}-\left(\int \frac{\sqrt{a^2x^2+1}}{atan(ax)a^4x^4+2at}\right.\right.}{}$$

input
```
int(x^3*atan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x)
```

output
```
(sqrt(c)*(2*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*a**2*x**2 + 4*sqrt(a**2*x*
*2 + 1)*sqrt(atan(a*x)) - int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**2)/(
atan(a*x)*a**4*x**4 + 2*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**5*x**2 - in
t((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**2)/(atan(a*x)*a**4*x**4 + 2*atan
(a*x)*a**2*x**2 + atan(a*x)),x)*a**3 - 2*int((sqrt(a**2*x**2 + 1)*sqrt(ata
n(a*x)))/(atan(a*x)*a**4*x**4 + 2*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**3
*x**2 - 2*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(atan(a*x)*a**4*x**4 +
 2*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a))/(2*a**4*c**2*(a**2*x**2 + 1))
```

# 3.743 $\int \frac{x^2 \sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}} \, dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^2 \sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}} \, dx = \text{Int}\left(\frac{x^2 \sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}},x\right)$$

output
```
Defer(Int)(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 3.81 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^2 \sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}} \, dx = \int \frac{x^2 \sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}} \, dx$$

input
```
Integrate[(x^2*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(3/2),x]
```

output
```
Integrate[(x^2*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(3/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{\arctan(ax)}}{(a^2cx^2+c)^{3/2}}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x^2\sqrt{\arctan(ax)}}{(a^2cx^2+c)^{3/2}}dx$$

input
```
Int[(x^2*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(3/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^2\sqrt{\arctan\left(ax\right)}}{\left(a^2c\,x^2+c\right)^{\frac{3}{2}}}dx$$

input
```
int(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x)
```

output
```
int(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 4.42 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{x^2\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^2\sqrt{\operatorname{atan}(ax)}}{(c(a^2x^2+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2*atan(a*x)**(1/2)/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(x**2*sqrt(atan(a*x))/(c*(a**2*x**2 + 1))**(3/2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^2\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^2\sqrt{\arctan\left(ax\right)}}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^2*sqrt(arctan(a*x))/(a^2*c*x^2 + c)^(3/2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^2\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^2\,\sqrt{\mathrm{atan}\left(a\,x\right)}}{{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

input
```
int((x^2*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(3/2),x)
```

output
```
int((x^2*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(3/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.77

$$\int \frac{x^2\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,x^2}{a^4x^4+2a^2x^2+1}dx\right)}{c^2}$$

input
```
int(x^2*atan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x)
```

output
```
(sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**2)/(a**4*x**4 + 2*a**
2*x**2 + 1),x))/c**2
```

# 3.744 $\int \frac{x\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 93

$$\int \frac{x\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx=-\frac{\sqrt{\arctan(ax)}}{a^2c\sqrt{c+a^2cx^2}}+\frac{\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{a^2c\sqrt{c+a^2cx^2}}$$

output
```
-arctan(a*x)^(1/2)/a^2/c/(a^2*c*x^2+c)^(1/2)+1/2*2^(1/2)*Pi^(1/2)*(a^2*x^2
+1)^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))/a^2/c/(a^2*c*x^2+c)
^(1/2)
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.30

$$\int \frac{x\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx=\frac{-4\arctan(ax)-i\sqrt{1+a^2x^2}\sqrt{-i\arctan(ax)}\Gamma\left(\frac{1}{2},-i\arctan(ax)\right)+i\sqrt{1+a^2x^2}\sqrt{}}{4a^2c\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}$$

input
```
Integrate[(x*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(3/2),x]
```

output
```
(-4*ArcTan[a*x] - I*Sqrt[1 + a^2*x^2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-
I)*ArcTan[a*x]] + I*Sqrt[1 + a^2*x^2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, I*Arc
Tan[a*x]])/(4*a^2*c*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])
```

## Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5465, 5440, 5439, 3042, 3785, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{\arctan(ax)}}{(a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \text{ 5465}$$

$$\frac{\int \frac{1}{(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx}{2a}-\frac{\sqrt{\arctan(ax)}}{a^2c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 5440}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{1}{(a^2x^2+1)^{3/2}\sqrt{\arctan(ax)}}dx}{2ac\sqrt{a^2cx^2+c}}-\frac{\sqrt{\arctan(ax)}}{a^2c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 5439}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}d\arctan(ax)}{2a^2c\sqrt{a^2cx^2+c}}-\frac{\sqrt{\arctan(ax)}}{a^2c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)}{\sqrt{\arctan(ax)}}d\arctan(ax)}{2a^2c\sqrt{a^2cx^2+c}}-\frac{\sqrt{\arctan(ax)}}{a^2c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 3785}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}}d\sqrt{\arctan(ax)}}{a^2c\sqrt{a^2cx^2+c}}-\frac{\sqrt{\arctan(ax)}}{a^2c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 3833}$$

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{a^2c\sqrt{a^2cx^2+c}}-\frac{\sqrt{\arctan(ax)}}{a^2c\sqrt{a^2cx^2+c}}$$

input
```
Int[(x*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(3/2),x]
```

output
```
-(Sqrt[ArcTan[a*x]]/(a^2*c*Sqrt[c + a^2*c*x^2])) + (Sqrt[Pi/2]*Sqrt[1 + a^
2*x^2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(a^2*c*Sqrt[c + a^2*c*x^2])
```

## Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5440
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])   Int[(1 +
c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

## Maple [F]

$$\int \frac{x\sqrt{\arctan\left(ax\right)}}{\left(a^{2}c\,x^{2}+c\right)^{\frac{3}{2}}}dx$$

input
```
int(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x)
```

output
```
int(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{x\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x\sqrt{\operatorname{atan}(ax)}}{(c(a^2x^2+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*atan(a*x)**(1/2)/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(x*sqrt(atan(a*x))/(c*(a**2*x**2 + 1))**(3/2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.85

$$\int \frac{x\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx =$$
$$-\frac{(i+1)\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{\arctan(ax)}\right)-(i-1)\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{\arctan(ax)}\right)+4}{8\,a^2c^{\frac{3}{2}}}$$

input
```
integrate(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
-1/8*((I + 1)*sqrt(2)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(arctan(a*x))
) - (I - 1)*sqrt(2)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(2)*sqrt(arctan(a*x)))
 + 4*sqrt(arctan(a*x))*e^(I*arctan(a*x)) + 4*sqrt(arctan(a*x))*e^(-I*arcta
n(a*x)))/(a^2*c^(3/2))
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x\,\sqrt{\mathrm{atan}\left(a\,x\right)}}{{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

input
```
int((x*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(3/2),x)
```

output
```
int((x*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(3/2), x)
```

## Reduce [F]

$$\int \frac{x\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{c}\left(-2\sqrt{a^2x^2+1}\,\sqrt{atan\left(ax\right)}+\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}}{atan(ax)a^4x^4+2atan(ax)a^2x^2+atan(ax)}dx\right)a^3x^2+\left(\int\right.\right.}{2a^2c^2\left(a^2x^2+1\right)}$$

input
```
int(x*atan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x)
```

output
```
(sqrt(c)*( - 2*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)) + int((sqrt(a**2*x**2 +
 1)*sqrt(atan(a*x)))/(atan(a*x)*a**4*x**4 + 2*atan(a*x)*a**2*x**2 + atan(a
*x)),x)*a**3*x**2 + int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(atan(a*x)*a
**4*x**4 + 2*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a))/(2*a**2*c**2*(a**2*x*
*2 + 1))
```

# 3.745 $\int \frac{\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 91

$$\int \frac{\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{x\sqrt{\arctan(ax)}}{c\sqrt{c+a^2cx^2}} - \frac{\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{ac\sqrt{c+a^2cx^2}}$$

output
```
x*arctan(a*x)^(1/2)/c/(a^2*c*x^2+c)^(1/2)-1/2*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)
^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))/a/c/(a^2*c*x^2+c)^(1/2
)
```

## Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{2ax\sqrt{\arctan(ax)} - \sqrt{2\pi}\sqrt{1+a^2x^2}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{2ac\sqrt{c+a^2cx^2}}$$

input
```
Integrate[Sqrt[ArcTan[a*x]]/(c + a^2*c*x^2)^(3/2),x]
```

output
```
(2*a*x*Sqrt[ArcTan[a*x]] - Sqrt[2*Pi]*Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[2/Pi
]*Sqrt[ArcTan[a*x]]])/(2*a*c*Sqrt[c + a^2*c*x^2])
```

## Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {5440, 5439, 3042, 3777, 25, 3042, 3786, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\arctan(ax)}}{(a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \text{ 5440}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{\sqrt{\arctan(ax)}}{(a^2x^2+1)^{3/2}}dx}{c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 5439}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{a^2x^2+1}\int \sqrt{\arctan(ax)}\sin\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 3777}$$

$$\frac{\sqrt{a^2x^2+1}\left(\frac{1}{2}\int -\frac{ax}{\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}d\arctan(ax)+\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}\right)}{ac\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\sqrt{a^2x^2+1}\left(\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}-\frac{1}{2}\int \frac{ax}{\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}d\arctan(ax)\right)}{ac\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{a^2x^2+1}\left(\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}-\frac{1}{2}\int\frac{\sin(\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)\right)}{ac\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3786}$$

$$\frac{\sqrt{a^2x^2+1}\left(\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}-\int\frac{ax}{\sqrt{a^2x^2+1}}d\sqrt{\arctan(ax)}\right)}{ac\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3832}$$

$$\frac{\sqrt{a^2x^2+1}\left(\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}-\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{ac\sqrt{a^2cx^2+c}}$$

input
```
Int[Sqrt[ArcTan[a*x]]/(c + a^2*c*x^2)^(3/2),x]
```

output
```
(Sqrt[1 + a^2*x^2]*((a*x*Sqrt[ArcTan[a*x]])/Sqrt[1 + a^2*x^2] - Sqrt[Pi/2]
*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]]))/(a*c*Sqrt[c + a^2*c*x^2])
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5440
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])   Int[(1 +
c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

## Maple [F]

$$\int \frac{\sqrt{\arctan\left(ax\right)}}{\left(a^{2} c\,x^{2}+c\right)^{\frac{3}{2}}}dx$$

input
```
int(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x)
```

output
```
int(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\sqrt{\operatorname{atan}(ax)}}{(c(a^2x^2+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(atan(a*x)**(1/2)/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(sqrt(atan(a*x))/(c*(a**2*x**2 + 1))**(3/2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{3/2}}\,dx =$$
$$-\frac{(i-1)\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{\arctan(ax)}\right)-(i+1)\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{\arctan(ax)}\right)+4i}{8ac^{\frac{3}{2}}}$$

input
```
integrate(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
-1/8*((I - 1)*sqrt(2)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(arctan(a*x))
) - (I + 1)*sqrt(2)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(2)*sqrt(arctan(a*x)))
 + 4*I*sqrt(arctan(a*x))*e^(I*arctan(a*x)) - 4*I*sqrt(arctan(a*x))*e^(-I*a
rctan(a*x)))/(a*c^(3/2))
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\arctan(ax)}}{\left(c+a^2cx^2\right)^{3/2}}\,dx = \int \frac{\sqrt{\mathrm{atan}\left(a\,x\right)}}{{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

input
```
int(atan(a*x)^(1/2)/(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(atan(a*x)^(1/2)/(c + a^2*c*x^2)^(3/2), x)
```

## Reduce [F]

$$\int \frac{\sqrt{\arctan(ax)}}{\left(c+a^2cx^2\right)^{3/2}}\,dx = \frac{\sqrt{c}\left(2\sqrt{a^2x^2+1}\,\sqrt{atan\left(ax\right)}\,x-\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,x}{atan(ax)a^4x^4+2atan(ax)a^2x^2+atan(ax)}dx\right)a^3x^2-\left(\int\right.}{2c^2\left(a^2x^2+1\right)}$$

input
```
int(atan(a*x)^(1/2)/(a^2*c*x^2+c)^(3/2),x)
```

output
```
(sqrt(c)*(2*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x - int((sqrt(a**2*x**2 +
1)*sqrt(atan(a*x))*x)/(atan(a*x)*a**4*x**4 + 2*atan(a*x)*a**2*x**2 + atan(
a*x)),x)*a**3*x**2 - int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x)/(atan(a*x
)*a**4*x**4 + 2*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a))/(2*c**2*(a**2*x**2
 + 1))
```

# 3.746 $\int \frac{\sqrt{\arctan(ax)}}{x(c+a^2cx^2)^{3/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\sqrt{\arctan(ax)}}{x\left(c+a^2cx^2\right)^{3/2}}\,dx=\text{Int}\left(\frac{\sqrt{\arctan(ax)}}{x\left(c+a^2cx^2\right)^{3/2}},x\right)$$

output
```
Defer(Int)(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(3/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 2.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{\arctan(ax)}}{x\left(c+a^2cx^2\right)^{3/2}}\,dx=\int \frac{\sqrt{\arctan(ax)}}{x\left(c+a^2cx^2\right)^{3/2}}\,dx$$

input
```
Integrate[Sqrt[ArcTan[a*x]]/(x*(c + a^2*c*x^2)^(3/2)),x]
```

output
```
Integrate[Sqrt[ArcTan[a*x]]/(x*(c + a^2*c*x^2)^(3/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\arctan(ax)}}{x\left(a^2cx^2+c\right)^{3/2}}\,dx$$

$$\Big\downarrow \; 5560$$

$$\int \frac{\sqrt{\arctan(ax)}}{x\left(a^2cx^2+c\right)^{3/2}}dx$$

input
```
Int[Sqrt[ArcTan[a*x]]/(x*(c + a^2*c*x^2)^(3/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{\arctan\left(ax\right)}}{x\left(a^{2}c\,x^{2}+c\right)^{\frac{3}{2}}}dx$$

input
```
int(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(3/2),x)
```

output
```
int(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{x\,(c+a^2cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 7.81 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{\arctan(ax)}}{x\,(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\sqrt{\operatorname{atan}(ax)}}{x\,(c\,(a^2x^2+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(atan(a*x)**(1/2)/x/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(sqrt(atan(a*x))/(x*(c*(a**2*x**2 + 1))**(3/2)), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{x\,(c+a^2cx^2)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{\arctan(ax)}}{x\left(c+a^2cx^2\right)^{3/2}}\,dx=\int \frac{\sqrt{\arctan\left(ax\right)}}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(arctan(a*x))/((a^2*c*x^2 + c)^(3/2)*x), x)
```

## Mupad [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{\arctan(ax)}}{x\left(c+a^2cx^2\right)^{3/2}}\,dx=\int \frac{\sqrt{\mathrm{atan}\left(a\,x\right)}}{x\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

input
```
int(atan(a*x)^(1/2)/(x*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(atan(a*x)^(1/2)/(x*(c + a^2*c*x^2)^(3/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.65

$$\int \frac{\sqrt{\arctan(ax)}}{x\left(c+a^2cx^2\right)^{3/2}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}}{a^4x^5+2a^2x^3+x}dx\right)}{c^2}$$

input
```
int(atan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(3/2),x)
```

output
```
(sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(a**4*x**5 + 2*a**2*x**
3 + x),x))/c**2
```

# 3.747 $\int \frac{\sqrt{\arctan(ax)}}{x^2(c+a^2cx^2)^{3/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\sqrt{\arctan(ax)}}{x^2(c+a^2cx^2)^{3/2}}\,dx = \text{Int}\left(\frac{\sqrt{\arctan(ax)}}{x^2(c+a^2cx^2)^{3/2}},x\right)$$

output
```
Defer(Int)(arctan(a*x)^(1/2)/x^2/(a^2*c*x^2+c)^(3/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 4.10 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{\arctan(ax)}}{x^2(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\sqrt{\arctan(ax)}}{x^2(c+a^2cx^2)^{3/2}}\,dx$$

input
```
Integrate[Sqrt[ArcTan[a*x]]/(x^2*(c + a^2*c*x^2)^(3/2)),x]
```

output
```
Integrate[Sqrt[ArcTan[a*x]]/(x^2*(c + a^2*c*x^2)^(3/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\arctan(ax)}}{x^2\left(a^2cx^2+c\right)^{3/2}}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{\sqrt{\arctan(ax)}}{x^2\left(a^2cx^2+c\right)^{3/2}}dx$$

input
```
Int[Sqrt[ArcTan[a*x]]/(x^2*(c + a^2*c*x^2)^(3/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{\arctan\left(ax\right)}}{x^2\left(a^2c\,x^2+c\right)^{\frac{3}{2}}}dx$$

input
```
int(arctan(a*x)^(1/2)/x^2/(a^2*c*x^2+c)^(3/2),x)
```

output
```
int(arctan(a*x)^(1/2)/x^2/(a^2*c*x^2+c)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{x^2 (c + a^2cx^2)^{3/2}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(1/2)/x^2/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 14.93 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\arctan(ax)}}{x^2 (c + a^2cx^2)^{3/2}} \, dx = \int \frac{\sqrt{\operatorname{atan}(ax)}}{x^2 (c (a^2x^2 + 1))^{\frac{3}{2}}} \, dx$$

input
```
integrate(atan(a*x)**(1/2)/x**2/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(sqrt(atan(a*x))/(x**2*(c*(a**2*x**2 + 1))**(3/2)), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{x^2 (c + a^2cx^2)^{3/2}} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(1/2)/x^2/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{\arctan(ax)}}{x^2 (c + a^2cx^2)^{3/2}} \, dx = \int \frac{\sqrt{\arctan(ax)}}{(a^2cx^2 + c)^{\frac{3}{2}} x^2} \, dx$$

input
```
integrate(arctan(a*x)^(1/2)/x^2/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(arctan(a*x))/((a^2*c*x^2 + c)^(3/2)*x^2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{\arctan(ax)}}{x^2 (c + a^2cx^2)^{3/2}} \, dx = \int \frac{\sqrt{\operatorname{atan}(a\,x)}}{x^2\,(c\,a^2\,x^2 + c)^{3/2}} \, dx$$

input
```
int(atan(a*x)^(1/2)/(x^2*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(atan(a*x)^(1/2)/(x^2*(c + a^2*c*x^2)^(3/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 284, normalized size of antiderivative = 10.92

$$\int \frac{\sqrt{\arctan(ax)}}{x^2 (c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{c}\left(-4\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,a^2x^2-2\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}+2\left(\int \frac{\sqrt{a^2}}{atan(ax)a^4x^4}\right.\right.}{}$$

input
```
int(atan(a*x)^(1/2)/x^2/(a^2*c*x^2+c)^(3/2),x)
```

output
```
(sqrt(c)*( - 4*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*a**2*x**2 - 2*sqrt(a**2
*x**2 + 1)*sqrt(atan(a*x)) + 2*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x)
/(atan(a*x)*a**4*x**4 + 2*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**5*x**3 +
2*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x)/(atan(a*x)*a**4*x**4 + 2*ata
n(a*x)*a**2*x**2 + atan(a*x)),x)*a**3*x + int((sqrt(a**2*x**2 + 1)*sqrt(at
an(a*x)))/(atan(a*x)*a**4*x**5 + 2*atan(a*x)*a**2*x**3 + atan(a*x)*x),x)*a
**3*x**3 + int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(atan(a*x)*a**4*x**5
+ 2*atan(a*x)*a**2*x**3 + atan(a*x)*x),x)*a*x))/(2*c**2*x*(a**2*x**2 + 1))
```

# 3.748 $\int \frac{x^m \sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}} \, dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^m \sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}} \, dx = \text{Int}\left(\frac{x^m \sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}, x\right)$$

output
```
Defer(Int)(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 2.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^m \sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}} \, dx = \int \frac{x^m \sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}} \, dx$$

input
```
Integrate[(x^m*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(5/2),x]
```

output
```
Integrate[(x^m*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(5/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \sqrt{\arctan(ax)}}{(a^2cx^2+c)^{5/2}} \, dx$$

$$\Big\downarrow \text{5560}$$

$$\int \frac{x^m \sqrt{\arctan(ax)}}{(a^2cx^2+c)^{5/2}}dx$$

input
```
Int[(x^m*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(5/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^m \sqrt{\arctan (ax)}}{(a^2c\,x^2+c)^{\frac{5}{2}}}dx$$

input
```
int(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x)
```

output
```
int(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.38

$$\int \frac{x^m\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^m\sqrt{\arctan(ax)}}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^m*sqrt(arctan(a*x))/(a^6*c^3*x^6 + 3*a^4*c^
3*x^4 + 3*a^2*c^3*x^2 + c^3), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^m\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**m*atan(a*x)**(1/2)/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^m\sqrt{\arctan(ax)}}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^m*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
integrate(x^m*sqrt(arctan(a*x))/(a^2*c*x^2 + c)^(5/2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^m\sqrt{\mathrm{atan}\,(a\,x)}}{(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int((x^m*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int((x^m*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(5/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.08

$$\int \frac{x^m\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{\sqrt{c}\left(\int \frac{x^m\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}}{a^6x^6+3a^4x^4+3a^2x^2+1}dx\right)}{c^3}$$

input
```
int(x^m*atan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x)
```

output
```
(sqrt(c)*int((x**m*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(a**6*x**6 + 3*a**
4*x**4 + 3*a**2*x**2 + 1),x))/c**3
```

# 3.749 $\int \frac{x^4\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^4\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Int}\left(\frac{x^4\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}},x\right)$$

output
```
Defer(Int)(x^4*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 6.42 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^4\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^4\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx$$

input
```
Integrate[(x^4*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(5/2),x]
```

output
```
Integrate[(x^4*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(5/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\sqrt{\arctan(ax)}}{(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{5560}$$

$$\int \frac{x^4\sqrt{\arctan(ax)}}{(a^2cx^2+c)^{5/2}}dx$$

input
```
Int[(x^4*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(5/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^4\sqrt{\arctan(ax)}}{(a^2c\,x^2+c)^{\frac{5}{2}}}dx$$

input
```
int(x^4*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x)
```

output
```
int(x^4*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x^4\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^4*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 33.87 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{x^4\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^4\sqrt{\operatorname{atan}(ax)}}{(c\,(a^2x^2+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**4*atan(a*x)**(1/2)/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(x**4*sqrt(atan(a*x))/(c*(a**2*x**2 + 1))**(5/2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^4*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-1)]

Timed out.

$$\int \frac{x^4 \sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x^4*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

## Mupad [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^4 \sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^4\,\sqrt{\mathrm{atan}\,(a\,x)}}{{(c\,a^2\,x^2+c)}^{5/2}}\,dx$$

input
```
int((x^4*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int((x^4*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(5/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.08

$$\int \frac{x^4\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,x^4}{a^6x^6+3a^4x^4+3a^2x^2+1}dx\right)}{c^3}$$

input
```
int(x^4*atan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x)
```

output
```
(sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**4)/(a**6*x**6 + 3*a**
4*x**4 + 3*a**2*x**2 + 1),x))/c**3
```

# 3.750 $\int \frac{x^3\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 215

$$\begin{aligned}\int \frac{x^3\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = & -\frac{3\sqrt{\arctan(ax)}}{4a^4c^2\sqrt{c+a^2cx^2}} \\ & + \frac{\sqrt{1+a^2x^2}\sqrt{\arctan(ax)}\cos(3\arctan(ax))}{12a^4c^2\sqrt{c+a^2cx^2}} \\ & + \frac{3\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{4a^4c^2\sqrt{c+a^2cx^2}} \\ & - \frac{\sqrt{\frac{\pi}{6}}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)}{12a^4c^2\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
-3/4*arctan(a*x)^(1/2)/a^4/c^2/(a^2*c*x^2+c)^(1/2)+1/12*(a^2*x^2+1)^(1/2)*
arctan(a*x)^(1/2)*cos(3*arctan(a*x))/a^4/c^2/(a^2*c*x^2+c)^(1/2)+3/8*2^(1/
2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))
/a^4/c^2/(a^2*c*x^2+c)^(1/2)-1/72*6^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)*Fresn
elC(6^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))/a^4/c^2/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.51

$$\int \frac{x^3\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{-96\arctan(ax)-144a^2x^2\arctan(ax)-27i(1+a^2x^2)^{3/2}\sqrt{-i\arctan(ax)}\Gamma\left(\frac{1}{2},-\right.}{}$$

input
```
Integrate[(x^3*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(-96*ArcTan[a*x] - 144*a^2*x^2*ArcTan[a*x] - (27*I)*(1 + a^2*x^2)^(3/2)*Sq
rt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-I)*ArcTan[a*x]] + (27*I)*(1 + a^2*x^2)^(
3/2)*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, I*ArcTan[a*x]] + I*Sqrt[3 + 3*a^2*x^2]
*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-3*I)*ArcTan[a*x]] + I*a^2*x^2*Sqrt[3
+ 3*a^2*x^2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-3*I)*ArcTan[a*x]] - I*Sqr
t[3 + 3*a^2*x^2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (3*I)*ArcTan[a*x]] - I*a^2
*x^2*Sqrt[3 + 3*a^2*x^2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (3*I)*ArcTan[a*x]]
)/(144*a^4*c^2*(1 + a^2*x^2)*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])
```

## Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.67, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {5506, 5505, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\sqrt{\arctan(ax)}}{(a^2cx^2+c)^{5/2}}\,dx$$

$$\Big\downarrow \text{5506}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{x^3\sqrt{\arctan(ax)}}{(a^2x^2+1)^{5/2}}dx}{c^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5505}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{a^3x^3\sqrt{\arctan(ax)}}{(a^2x^2+1)^{3/2}}d\arctan(ax)}{a^4c^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{a^2x^2+1}\int \sqrt{\arctan(ax)}\sin(\arctan(ax))^3d\arctan(ax)}{a^4c^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 3793}$$

$$\frac{\sqrt{a^2x^2+1}\int \left(\frac{3ax\sqrt{\arctan(ax)}}{4\sqrt{a^2x^2+1}}-\frac{1}{4}\sqrt{\arctan(ax)}\sin(3\arctan(ax))\right)d\arctan(ax)}{a^4c^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{a^2x^2+1}\left(-\frac{3\sqrt{\arctan(ax)}}{4\sqrt{a^2x^2+1}}+\frac{3}{4}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{12}\sqrt{\frac{\pi}{6}}\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{12}\sqrt{\text{ar}}\right.}{a^4c^2\sqrt{a^2cx^2+c}}$$

input
```
Int[(x^3*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(Sqrt[1 + a^2*x^2]*((-3*Sqrt[ArcTan[a*x]])/(4*Sqrt[1 + a^2*x^2]) + (Sqrt[A
rcTan[a*x]]*Cos[3*ArcTan[a*x]])/12 + (3*Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Sqr
t[ArcTan[a*x]]])/4 - (Sqrt[Pi/6]*FresnelC[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/1
2))/(a^4*c^2*Sqrt[c + a^2*c*x^2])
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

## Maple [F]

$$\int \frac{x^3 \sqrt{\arctan (ax)}}{(a^2 c x^2 + c)^{\frac{5}{2}}} dx$$

input
```
int(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x)
```

output
```
int(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3 \sqrt{\arctan(ax)}}{(c + a^2 c x^2)^{5/2}} dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{x^3\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^3\sqrt{\operatorname{atan}(ax)}}{(c(a^2x^2+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**3*atan(a*x)**(1/2)/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(x**3*sqrt(atan(a*x))/(c*(a**2*x**2 + 1))**(5/2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x^3\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^3\,\sqrt{\mathrm{atan}\left(a\,x\right)}}{{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

input
```
int((x^3*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int((x^3*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(5/2), x)
```

## Reduce [F]

$$\int \frac{x^3\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{\sqrt{c}\left(-6\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,a^2x^2-4\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}+3\left(\int \frac{}{atan(ax)a^6x^6+}\right.\right.}{}$$

input
```
int(x^3*atan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x)
```

output
```
(sqrt(c)*( - 6*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*a**2*x**2 - 4*sqrt(a**2
*x**2 + 1)*sqrt(atan(a*x)) + 3*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x*
*2)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 +
 atan(a*x)),x)*a**7*x**4 + 6*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**2
)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + a
tan(a*x)),x)*a**5*x**2 + 3*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**2)/
(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + ata
n(a*x)),x)*a**3 + 2*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(atan(a*x)*a
**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a
**5*x**4 + 4*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(atan(a*x)*a**6*x**
6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**3*x**
2 + 2*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(atan(a*x)*a**6*x**6 + 3*a
tan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a))/(6*a**4*c**
3*(a**4*x**4 + 2*a**2*x**2 + 1))
```

# 3.751 $\int \frac{x^2\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 163

$$\int \frac{x^2\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{x^3\sqrt{\arctan(ax)}}{3c\,(c+a^2cx^2)^{3/2}} - \frac{\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{4a^3c^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{\frac{\pi}{6}}\sqrt{1+a^2x^2}\,\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)}{12a^3c^2\sqrt{c+a^2cx^2}}$$

output
```
1/3*x^3*arctan(a*x)^(1/2)/c/(a^2*c*x^2+c)^(3/2)-1/8*2^(1/2)*Pi^(1/2)*(a^2*
x^2+1)^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))/a^3/c^2/(a^2*c*x
^2+c)^(1/2)+1/72*6^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)*FresnelS(6^(1/2)/Pi^(1
/2)*arctan(a*x)^(1/2))/a^3/c^2/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.82

$$\int \frac{x^2\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{24a^3x^3\sqrt{\arctan(ax)}-9\sqrt{2\pi}(1+a^2x^2)^{3/2}\operatorname{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\sqrt{6\pi}(1}{72a^3c^2(1+a^2x^2)\sqrt{c+a^2cx^2}}$$

input
```
Integrate[(x^2*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(24*a^3*x^3*Sqrt[ArcTan[a*x]] - 9*Sqrt[2*Pi]*(1 + a^2*x^2)^(3/2)*FresnelS[
Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]] + Sqrt[6*Pi]*(1 + a^2*x^2)^(3/2)*FresnelS[Sq
rt[6/Pi]*Sqrt[ArcTan[a*x]]])/(72*a^3*c^2*(1 + a^2*x^2)*Sqrt[c + a^2*c*x^2]
)
```

## Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.83, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {5479, 5506, 5505, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{\arctan(ax)}}{(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{5479}$$

$$\frac{x^3\sqrt{\arctan(ax)}}{3c(a^2cx^2+c)^{3/2}}-\frac{1}{6}a\int \frac{x^3}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx$$

$$\Big\downarrow \text{5506}$$

$$\frac{x^3\sqrt{\arctan(ax)}}{3c(a^2cx^2+c)^{3/2}}-\frac{a\sqrt{a^2x^2+1}\int \frac{x^3}{(a^2x^2+1)^{5/2}\sqrt{\arctan(ax)}}dx}{6c^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5505}$$

$$\frac{x^3\sqrt{\arctan(ax)}}{3c\,(a^2cx^2+c)^{3/2}}-\frac{\sqrt{a^2x^2+1}\int\frac{a^3x^3}{(a^2x^2+1)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{6a^3c^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \; 3042$$

$$\frac{x^3\sqrt{\arctan(ax)}}{3c\,(a^2cx^2+c)^{3/2}}-\frac{\sqrt{a^2x^2+1}\int\frac{\sin(\arctan(ax))^3}{\sqrt{\arctan(ax)}}d\arctan(ax)}{6a^3c^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \; 3793$$

$$\frac{x^3\sqrt{\arctan(ax)}}{3c\,(a^2cx^2+c)^{3/2}}-\frac{\sqrt{a^2x^2+1}\int\left(\frac{3ax}{4\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}-\frac{\sin(3\arctan(ax))}{4\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{6a^3c^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \; 2009$$

$$\frac{x^3\sqrt{\arctan(ax)}}{3c\,(a^2cx^2+c)^{3/2}}-\frac{\sqrt{a^2x^2+1}\left(\frac{3}{2}\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{2}\sqrt{\frac{\pi}{6}}\,\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{6a^3c^2\sqrt{a^2cx^2+c}}$$

input
```
Int[(x^2*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(x^3*Sqrt[ArcTan[a*x]])/(3*c*(c + a^2*c*x^2)^(3/2)) - (Sqrt[1 + a^2*x^2]*(
(3*Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/2 - (Sqrt[Pi/6]*Fres
nelS[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/2))/(6*a^3*c^2*Sqrt[c + a^2*c*x^2])
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

## Maple [F]

$$\int \frac{x^2\sqrt{\arctan\left(ax\right)}}{\left(a^2c\,x^2+c\right)^{\frac{5}{2}}}dx$$

input
```
int(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x)
```

output
```
int(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{x^2\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^2\sqrt{\operatorname{atan}(ax)}}{(c(a^2x^2+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**2*atan(a*x)**(1/2)/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(x**2*sqrt(atan(a*x))/(c*(a**2*x**2 + 1))**(5/2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F]

$$\int \frac{x^2\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^2\sqrt{\arctan(ax)}}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
integrate(x^2*sqrt(arctan(a*x))/(a^2*c*x^2 + c)^(5/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^2\sqrt{\mathrm{atan}\,(a\,x)}}{(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int((x^2*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int((x^2*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(5/2), x)
```

## Reduce [F]

$$\int \frac{x^2\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{\sqrt{c}\left(2\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}\,x^3-\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,x^3}{atan(ax)a^6x^6+3atan(ax)a^4x^4+3atan(ax)a^2x^2+atan(ax)}\right.\right.}{}$$

input
```
int(x^2*atan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x)
```

output
```
(sqrt(c)*(2*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**3 - int((sqrt(a**2*x**2
 + 1)*sqrt(atan(a*x))*x**3)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 +
 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**5*x**4 - 2*int((sqrt(a**2*x**2 +
 1)*sqrt(atan(a*x))*x**3)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3
*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**3*x**2 - int((sqrt(a**2*x**2 + 1)*
sqrt(atan(a*x))*x**3)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*ata
n(a*x)*a**2*x**2 + atan(a*x)),x)*a))/(6*c**3*(a**4*x**4 + 2*a**2*x**2 + 1)
)
```

# 3.752 $\int \frac{x\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 163

$$\int \frac{x\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = -\frac{\sqrt{\arctan(ax)}}{3a^2c\,(c+a^2cx^2)^{3/2}} + \frac{\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{4a^2c^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{\frac{\pi}{6}}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)}{12a^2c^2\sqrt{c+a^2cx^2}}$$

output
```
-1/3*arctan(a*x)^(1/2)/a^2/c/(a^2*c*x^2+c)^(3/2)+1/8*2^(1/2)*Pi^(1/2)*(a^2
*x^2+1)^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))/a^2/c^2/(a^2*c*
x^2+c)^(1/2)+1/72*6^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)*FresnelC(6^(1/2)/Pi^(
1/2)*arctan(a*x)^(1/2))/a^2/c^2/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.02

$$\int \frac{x\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{-48\arctan(ax) - i(1+a^2x^2)^{3/2}\left(9\sqrt{-i\arctan(ax)}\Gamma\left(\frac{1}{2},-i\arctan(ax)\right) - 9\sqrt{i\arc}\right.}{14}$$

input
```
Integrate[(x*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(-48*ArcTan[a*x] - I*(1 + a^2*x^2)^(3/2)*(9*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1
/2, (-I)*ArcTan[a*x]] - 9*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, I*ArcTan[a*x]] +
Sqrt[3]*(Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-3*I)*ArcTan[a*x]] - Sqrt[I*Ar
cTan[a*x]]*Gamma[1/2, (3*I)*ArcTan[a*x]])))/(144*a^2*c*(c + a^2*c*x^2)^(3/
2)*Sqrt[ArcTan[a*x]])
```

## Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.83, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5465, 5440, 5439, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{\arctan(ax)}}{(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{\int \frac{1}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx}{6a} - \frac{\sqrt{\arctan(ax)}}{3a^2c\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5440}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{1}{(a^2x^2+1)^{5/2}\sqrt{\arctan(ax)}}dx}{6ac^2\sqrt{a^2cx^2+c}} - \frac{\sqrt{\arctan(ax)}}{3a^2c\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \; 5439$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{1}{(a^2x^2+1)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{6a^2c^2\sqrt{a^2cx^2+c}}-\frac{\sqrt{\arctan(ax)}}{3a^2c\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^3}{\sqrt{\arctan(ax)}}d\arctan(ax)}{6a^2c^2\sqrt{a^2cx^2+c}}-\frac{\sqrt{\arctan(ax)}}{3a^2c\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \; 3793$$

$$\frac{\sqrt{a^2x^2+1}\int \left(\frac{\cos(3\arctan(ax))}{4\sqrt{\arctan(ax)}}+\frac{3}{4\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{6a^2c^2\sqrt{a^2cx^2+c}}-\frac{\sqrt{\arctan(ax)}}{3a^2c\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \; 2009$$

$$\frac{\sqrt{a^2x^2+1}\left(\frac{3}{2}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{6}}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{6a^2c^2\sqrt{a^2cx^2+c}}-\frac{\sqrt{\arctan(ax)}}{3a^2c\,(a^2cx^2+c)^{3/2}}$$

input
```
Int[(x*Sqrt[ArcTan[a*x]])/(c + a^2*c*x^2)^(5/2),x]
```

output
```
-1/3*Sqrt[ArcTan[a*x]]/(a^2*c*(c + a^2*c*x^2)^(3/2)) + (Sqrt[1 + a^2*x^2]*
((3*Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/2 + (Sqrt[Pi/6]*Fre
snelC[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/2))/(6*a^2*c^2*Sqrt[c + a^2*c*x^2])
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5440
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])   Int[(1 +
c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

## Maple [F]

$$\int \frac{x\sqrt{\arctan(ax)}}{(a^2c\,x^2+c)^{\frac{5}{2}}}dx$$

input
```
int(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x)
```

output
```
int(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{x\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x\sqrt{\operatorname{atan}(ax)}}{(c(a^2x^2+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*atan(a*x)**(1/2)/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(x*sqrt(atan(a*x))/(c*(a**2*x**2 + 1))**(5/2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F]

$$\int \frac{x\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x\sqrt{\arctan(ax)}}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
integrate(x*sqrt(arctan(a*x))/(a^2*c*x^2 + c)^(5/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x\,\sqrt{\mathrm{atan}\,(a\,x)}}{(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int((x*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int((x*atan(a*x)^(1/2))/(c + a^2*c*x^2)^(5/2), x)
```

## Reduce [F]

$$\int \frac{x\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{\sqrt{c}\left(-2\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}+\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}}{atan(ax)a^6x^6+3atan(ax)a^4x^4+3atan(ax)a^2x^2+atan(ax)}dx\right.\right.}{}$$

input
```
int(x*atan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x)
```

output
```
(sqrt(c)*( - 2*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)) + int((sqrt(a**2*x**2 +
 1)*sqrt(atan(a*x)))/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan
(a*x)*a**2*x**2 + atan(a*x)),x)*a**5*x**4 + 2*int((sqrt(a**2*x**2 + 1)*sqr
t(atan(a*x)))/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a
**2*x**2 + atan(a*x)),x)*a**3*x**2 + int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*
x)))/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2
+ atan(a*x)),x)*a))/(6*a**2*c**3*(a**4*x**4 + 2*a**2*x**2 + 1))
```

# 3.753 $\int \frac{\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 213

$$\int \frac{\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{3x\sqrt{\arctan(ax)}}{4c^2\sqrt{c+a^2cx^2}} - \frac{3\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{4ac^2\sqrt{c+a^2cx^2}} - \frac{\sqrt{\frac{\pi}{6}}\sqrt{1+a^2x^2}\,\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)}{12ac^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+a^2x^2}\sqrt{\arctan(ax)}\sin(3\arctan(ax))}{12ac^2\sqrt{c+a^2cx^2}}$$

output

```
3/4*x*arctan(a*x)^(1/2)/c^2/(a^2*c*x^2+c)^(1/2)-3/8*2^(1/2)*Pi^(1/2)*(a^2*
x^2+1)^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))/a/c^2/(a^2*c*x^2
+c)^(1/2)-1/72*6^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)*FresnelS(6^(1/2)/Pi^(1/2
)*arctan(a*x)^(1/2))/a/c^2/(a^2*c*x^2+c)^(1/2)+1/12*(a^2*x^2+1)^(1/2)*arct
an(a*x)^(1/2)*sin(3*arctan(a*x))/a/c^2/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{24ax(3+2a^2x^2)\sqrt{\arctan(ax)} - 27\sqrt{2\pi}(1+a^2x^2)^{3/2}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{72c^2\,(a+a^3x^2)\,\sqrt{c+a^2cx^2}}$$

input
```
Integrate[Sqrt[ArcTan[a*x]]/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(24*a*x*(3 + 2*a^2*x^2)*Sqrt[ArcTan[a*x]] - 27*Sqrt[2*Pi]*(1 + a^2*x^2)^(3
/2)*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]] - Sqrt[6*Pi]*(1 + a^2*x^2)^(3/2
)*FresnelS[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/(72*c^2*(a + a^3*x^2)*Sqrt[c + a
^2*c*x^2])
```

## Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.68, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {5440, 5439, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\arctan(ax)}}{(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{5440}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{\sqrt{\arctan(ax)}}{(a^2x^2+1)^{5/2}}dx}{c^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5439}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{\sqrt{\arctan(ax)}}{(a^2x^2+1)^{3/2}}d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a^2x^2+1}\int \sqrt{\arctan(ax)}\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^3d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3793}$$

$$\frac{\sqrt{a^2x^2+1}\int \left(\frac{1}{4}\sqrt{\arctan(ax)}\cos(3\arctan(ax))+\frac{3\sqrt{\arctan(ax)}}{4\sqrt{a^2x^2+1}}\right)d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2x^2+1}\left(\frac{3ax\sqrt{\arctan(ax)}}{4\sqrt{a^2x^2+1}}-\frac{3}{4}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{12}\sqrt{\frac{\pi}{6}}\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{12}\sqrt{\text{ar}}\right.}{ac^2\sqrt{a^2cx^2+c}}$$

input
```
Int[Sqrt[ArcTan[a*x]]/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(Sqrt[1 + a^2*x^2]*((3*a*x*Sqrt[ArcTan[a*x]])/(4*Sqrt[1 + a^2*x^2]) - (3*S
qrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/4 - (Sqrt[Pi/6]*FresnelS
[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/12 + (Sqrt[ArcTan[a*x]]*Sin[3*ArcTan[a*x]]
)/12))/(a*c^2*Sqrt[c + a^2*c*x^2])
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5440
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])   Int[(1 +
c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

## Maple [F]

$$\int \frac{\sqrt{\arctan(ax)}}{(a^2 c\, x^2+c)^{\frac{5}{2}}}dx$$

input
```
int(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x)
```

output
```
int(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx=\int \frac{\sqrt{\operatorname{atan}(ax)}}{(c(a^2x^2+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(atan(a*x)**(1/2)/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(sqrt(atan(a*x))/(c*(a**2*x**2 + 1))**(5/2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F]

$$\int \frac{\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx=\int \frac{\sqrt{\arctan(ax)}}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
integrate(sqrt(arctan(a*x))/(a^2*c*x^2 + c)^(5/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\sqrt{\mathrm{atan}\,(a\,x)}}{(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int(atan(a*x)^(1/2)/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int(atan(a*x)^(1/2)/(c + a^2*c*x^2)^(5/2), x)
```

## Reduce [F]

$$\int \frac{\sqrt{\arctan(ax)}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{\sqrt{c}\left(4\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}\,a^2x^3+6\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}\,x-2\left(\int \frac{}{atan(ax)a^6x^6+3a}\right.\right.}{}$$

input
```
int(atan(a*x)^(1/2)/(a^2*c*x^2+c)^(5/2),x)
```

output
```
(sqrt(c)*(4*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*a**2*x**3 + 6*sqrt(a**2*x*
*2 + 1)*sqrt(atan(a*x))*x - 2*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**
3)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 +
atan(a*x)),x)*a**7*x**4 - 4*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**3)
/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + at
an(a*x)),x)*a**5*x**2 - 2*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**3)/(
atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan
(a*x)),x)*a**3 - 3*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x)/(atan(a*x)*
a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*
a**5*x**4 - 6*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x)/(atan(a*x)*a**6*
x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**3*
x**2 - 3*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x)/(atan(a*x)*a**6*x**6
+ 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a))/(6*c**
3*(a**4*x**4 + 2*a**2*x**2 + 1))
```

# 3.754 $\int \frac{\sqrt{\arctan(ax)}}{x(c+a^2cx^2)^{5/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\sqrt{\arctan(ax)}}{x(c+a^2cx^2)^{5/2}}\,dx = \text{Int}\left(\frac{\sqrt{\arctan(ax)}}{x(c+a^2cx^2)^{5/2}},x\right)$$

output
```
Defer(Int)(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(5/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 3.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{\arctan(ax)}}{x(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\sqrt{\arctan(ax)}}{x(c+a^2cx^2)^{5/2}}\,dx$$

input
```
Integrate[Sqrt[ArcTan[a*x]]/(x*(c + a^2*c*x^2)^(5/2)),x]
```

output
```
Integrate[Sqrt[ArcTan[a*x]]/(x*(c + a^2*c*x^2)^(5/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\arctan(ax)}}{x\left(a^2cx^2+c\right)^{5/2}}dx$$

$$\Big\downarrow \text{5560}$$

$$\int \frac{\sqrt{\arctan(ax)}}{x\left(a^2cx^2+c\right)^{5/2}}dx$$

input
```
Int[Sqrt[ArcTan[a*x]]/(x*(c + a^2*c*x^2)^(5/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{\arctan\left(ax\right)}}{x\left(a^{2}c\,x^{2}+c\right)^{\frac{5}{2}}}dx$$

input
```
int(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(5/2),x)
```

output
```
int(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{x\left(c+a^2cx^2\right)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 49.31 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{\arctan(ax)}}{x\left(c+a^2cx^2\right)^{5/2}}\,dx = \int \frac{\sqrt{\operatorname{atan}(ax)}}{x\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(atan(a*x)**(1/2)/x/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(sqrt(atan(a*x))/(x*(c*(a**2*x**2 + 1))**(5/2)), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arctan(ax)}}{x\left(c+a^2cx^2\right)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 3.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{\arctan(ax)}}{x(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\sqrt{\arctan(ax)}}{(a^2cx^2+c)^{\frac{5}{2}}x}\,dx$$

input
```
integrate(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
integrate(sqrt(arctan(a*x))/((a^2*c*x^2 + c)^(5/2)*x), x)
```

## Mupad [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{\arctan(ax)}}{x(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\sqrt{\mathrm{atan}\left(a\,x\right)}}{x\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

input
```
int(atan(a*x)^(1/2)/(x*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(atan(a*x)^(1/2)/(x*(c + a^2*c*x^2)^(5/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.96

$$\int \frac{\sqrt{\arctan(ax)}}{x\left(c+a^2cx^2\right)^{5/2}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}}{a^6x^7+3a^4x^5+3a^2x^3+x}dx\right)}{c^3}$$

input

```
int(atan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(5/2),x)
```

output

```
(sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(a**6*x**7 + 3*a**4*x**
5 + 3*a**2*x**3 + x),x))/c**3
```

# 3.755 $\int x^m(c+a^2cx^2)\arctan(ax)^{3/2}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int x^m(c+a^2cx^2)\arctan(ax)^{3/2}\,dx=\text{Int}\left(x^m(c+a^2cx^2)\arctan(ax)^{3/2},x\right)$$

output
```
Defer(Int)(x^m*(a^2*c*x^2+c)*arctan(a*x)^(3/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int x^m(c+a^2cx^2)\arctan(ax)^{3/2}\,dx=\int x^m(c+a^2cx^2)\arctan(ax)^{3/2}\,dx$$

input
```
Integrate[x^m*(c + a^2*c*x^2)*ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[x^m*(c + a^2*c*x^2)*ArcTan[a*x]^(3/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \arctan(ax)^{3/2} \left(a^2cx^2+c\right) \, dx$$

$$\Big\downarrow \text{5560}$$

$$\int x^m \arctan(ax)^{3/2} \left(a^2cx^2+c\right) dx$$

input
```
Int[x^m*(c + a^2*c*x^2)*ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int x^{m} \left(a^{2} c \,x^{2}+c \right) \arctan \left(a x \right)^{\frac{3}{2}}d x$$

input
```
int(x^m*(a^2*c*x^2+c)*arctan(a*x)^(3/2),x)
```

output
```
int(x^m*(a^2*c*x^2+c)*arctan(a*x)^(3/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x^m (c + a^2 c x^2) \arctan(ax)^{3/2} \, dx = \int (a^2 c x^2 + c) x^m \arctan(ax)^{\frac{3}{2}} \, dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)*arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)*x^m*arctan(a*x)^(3/2), x)
```

## Sympy [F(-1)]

Timed out.

$$\int x^m (c + a^2 c x^2) \arctan(ax)^{3/2} \, dx = \text{Timed out}$$

input
```
integrate(x**m*(a**2*c*x**2+c)*atan(a*x)**(3/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int x^m (c + a^2 c x^2) \arctan(ax)^{3/2} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)*arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x^m (c + a^2 c x^2) \arctan(ax)^{3/2} \, dx = \int (a^2 c x^2 + c) x^m \arctan(ax)^{\frac{3}{2}} \, dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)*arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)*x^m*arctan(a*x)^(3/2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.86 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x^m (c + a^2 c x^2) \arctan(ax)^{3/2} \, dx = \int x^m \operatorname{atan}(a\,x)^{3/2} \left(c\,a^2\,x^2 + c\right) \, dx$$

input
```
int(x^m*atan(a*x)^(3/2)*(c + a^2*c*x^2),x)
```

output
```
int(x^m*atan(a*x)^(3/2)*(c + a^2*c*x^2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 1635, normalized size of antiderivative = 74.32

$$\int x^m (c + a^2 c x^2) \arctan(ax)^{3/2} \, dx = \text{Too large to display}$$

input
```
int(x^m*(a^2*c*x^2+c)*atan(a*x)^(3/2),x)
```

output

```
(c*(4*x**m*sqrt(atan(a*x))*atan(a*x)*a**3*m**3*x**3 + 12*x**m*sqrt(atan(a*
x))*atan(a*x)*a**3*m**2*x**3 + 8*x**m*sqrt(atan(a*x))*atan(a*x)*a**3*m*x**
3 + 4*x**m*sqrt(atan(a*x))*atan(a*x)*a*m**3*x + 20*x**m*sqrt(atan(a*x))*at
an(a*x)*a*m**2*x + 24*x**m*sqrt(atan(a*x))*atan(a*x)*a*m*x - 6*x**m*sqrt(a
tan(a*x))*a**2*m**2*x**2 - 6*x**m*sqrt(atan(a*x))*a**2*m*x**2 - 12*x**m*sq
rt(atan(a*x))*m - 24*x**m*sqrt(atan(a*x)) + 3*int((x**m*sqrt(atan(a*x))*x*
*2)/(atan(a*x)*a**2*m**3*x**2 + 6*atan(a*x)*a**2*m**2*x**2 + 11*atan(a*x)*
a**2*m*x**2 + 6*atan(a*x)*a**2*x**2 + atan(a*x)*m**3 + 6*atan(a*x)*m**2 +
11*atan(a*x)*m + 6*atan(a*x)),x)*a**3*m**5 + 21*int((x**m*sqrt(atan(a*x))*
x**2)/(atan(a*x)*a**2*m**3*x**2 + 6*atan(a*x)*a**2*m**2*x**2 + 11*atan(a*x
)*a**2*m*x**2 + 6*atan(a*x)*a**2*x**2 + atan(a*x)*m**3 + 6*atan(a*x)*m**2
+ 11*atan(a*x)*m + 6*atan(a*x)),x)*a**3*m**4 + 51*int((x**m*sqrt(atan(a*x)
)*x**2)/(atan(a*x)*a**2*m**3*x**2 + 6*atan(a*x)*a**2*m**2*x**2 + 11*atan(a
*x)*a**2*m*x**2 + 6*atan(a*x)*a**2*x**2 + atan(a*x)*m**3 + 6*atan(a*x)*m**
2 + 11*atan(a*x)*m + 6*atan(a*x)),x)*a**3*m**3 + 51*int((x**m*sqrt(atan(a*
x))*x**2)/(atan(a*x)*a**2*m**3*x**2 + 6*atan(a*x)*a**2*m**2*x**2 + 11*atan
(a*x)*a**2*m*x**2 + 6*atan(a*x)*a**2*x**2 + atan(a*x)*m**3 + 6*atan(a*x)*m
**2 + 11*atan(a*x)*m + 6*atan(a*x)),x)*a**3*m**2 + 18*int((x**m*sqrt(atan(
a*x))*x**2)/(atan(a*x)*a**2*m**3*x**2 + 6*atan(a*x)*a**2*m**2*x**2 + 11*at
an(a*x)*a**2*m*x**2 + 6*atan(a*x)*a**2*x**2 + atan(a*x)*m**3 + 6*atan(a...
```

# 3.756 $\int x^2(c+a^2cx^2) \arctan(ax)^{3/2} dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int x^2(c+a^2cx^2) \arctan(ax)^{3/2} dx = \text{Int}\left(x^2(c+a^2cx^2) \arctan(ax)^{3/2}, x\right)$$

output
```
Defer(Int)(x^2*(a^2*c*x^2+c)*arctan(a*x)^(3/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 4.45 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int x^2(c+a^2cx^2) \arctan(ax)^{3/2} dx = \int x^2(c+a^2cx^2) \arctan(ax)^{3/2} dx$$

input
```
Integrate[x^2*(c + a^2*c*x^2)*ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[x^2*(c + a^2*c*x^2)*ArcTan[a*x]^(3/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arctan(ax)^{3/2} \left(a^2 c x^2 + c\right) \, dx$$

$$\Big\downarrow \text{5560}$$

$$\int x^2 \arctan(ax)^{3/2} \left(a^2 c x^2 + c\right) dx$$

input
```
Int[x^2*(c + a^2*c*x^2)*ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int x^2 \left(a^2 c\, x^2 + c\right) \arctan\left(a x\right)^{\frac{3}{2}} dx$$

input
```
int(x^2*(a^2*c*x^2+c)*arctan(a*x)^(3/2),x)
```

output
```
int(x^2*(a^2*c*x^2+c)*arctan(a*x)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int x^2(c+a^2cx^2)\arctan(ax)^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(a^2*c*x^2+c)*arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 7.51 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.41

$$\int x^2(c+a^2cx^2)\arctan(ax)^{3/2}\,dx = c\left(\int x^2\operatorname{atan}^{\frac{3}{2}}(ax)\,dx+\int a^2x^4\operatorname{atan}^{\frac{3}{2}}(ax)\,dx\right)$$

input
```
integrate(x**2*(a**2*c*x**2+c)*atan(a*x)**(3/2),x)
```

output
```
c*(Integral(x**2*atan(a*x)**(3/2), x) + Integral(a**2*x**4*atan(a*x)**(3/2
), x))
```

## Maxima [F(-2)]

Exception generated.

$$\int x^2(c+a^2cx^2)\arctan(ax)^{3/2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*(a^2*c*x^2+c)*arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x^2(c+a^2cx^2)\arctan(ax)^{3/2}\,dx=\int (a^2cx^2+c)x^2\arctan{(ax)}^{\frac{3}{2}}\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)*arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)*x^2*arctan(a*x)^(3/2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.77 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x^2(c+a^2cx^2)\arctan(ax)^{3/2}\,dx=\int x^2\,\mathrm{atan}(a\,x)^{3/2}\,\left(c\,a^2\,x^2+c\right)\,dx$$

input
```
int(x^2*atan(a*x)^(3/2)*(c + a^2*c*x^2),x)
```

output
```
int(x^2*atan(a*x)^(3/2)*(c + a^2*c*x^2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.68

$$\int x^2(c+a^2cx^2)\arctan(ax)^{3/2}\,dx = c\left(\left(\int\sqrt{atan\,(ax)}\,atan(ax)\,x^4dx\right)a^2 + \int\sqrt{atan\,(ax)}\,atan(ax)\,x^2dx\right)$$

input
```
int(x^2*(a^2*c*x^2+c)*atan(a*x)^(3/2),x)
```

output
```
c*(int(sqrt(atan(a*x))*atan(a*x)*x**4,x)*a**2 + int(sqrt(atan(a*x))*atan(a
*x)*x**2,x))
```

# 3.757 $\int x(c+a^2cx^2) \arctan(ax)^{3/2} \, dx$



## Optimal result

Integrand size = 20, antiderivative size = 20

$$\int x(c+a^2cx^2) \arctan(ax)^{3/2} \, dx = \frac{c(1+a^2x^2)^2 \arctan(ax)^{3/2}}{4a^2} - \frac{3\mathrm{Int}\left((c+a^2cx^2)\sqrt{\arctan(ax)}, x\right)}{8a}$$

output
```
1/4*c*(a^2*x^2+1)^2*arctan(a*x)^(3/2)/a^2-3/8*Defer(Int)((a^2*c*x^2+c)*arc
tan(a*x)^(1/2),x)/a
```

## Mathematica [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x(c+a^2cx^2) \arctan(ax)^{3/2} \, dx = \int x(c+a^2cx^2) \arctan(ax)^{3/2} \, dx$$

input
```
Integrate[x*(c + a^2*c*x^2)*ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[x*(c + a^2*c*x^2)*ArcTan[a*x]^(3/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \arctan(ax)^{3/2} \left(a^2cx^2+c\right) dx$$

$$\Big\downarrow \text{ 5465}$$

$$\frac{c\left(a^2x^2+1\right)^2 \arctan(ax)^{3/2}}{4a^2}-\frac{3\int c\left(a^2x^2+1\right)\sqrt{\arctan(ax)}dx}{8a}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{c\left(a^2x^2+1\right)^2 \arctan(ax)^{3/2}}{4a^2}-\frac{3c\int \left(a^2x^2+1\right)\sqrt{\arctan(ax)}dx}{8a}$$

$$\Big\downarrow \text{ 5560}$$

$$\frac{c\left(a^2x^2+1\right)^2 \arctan(ax)^{3/2}}{4a^2}-\frac{3c\int \left(a^2x^2+1\right)\sqrt{\arctan(ax)}dx}{8a}$$

input
```
Int[x*(c + a^2*c*x^2)*ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int x\left(a^{2} c\,x^{2}+c\right) \arctan \left(a x\right)^{\frac{3}{2}}d x$$

input
```
int(x*(a^2*c*x^2+c)*arctan(a*x)^(3/2),x)
```

output
```
int(x*(a^2*c*x^2+c)*arctan(a*x)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int x\left(c+a^2cx^2\right) \arctan(ax)^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)*arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 5.53 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.45

$$\int x\left(c+a^2cx^2\right) \arctan(ax)^{3/2}\,dx = c\left(\int x\operatorname{atan}^{\frac{3}{2}}{\left(ax\right)}\,dx + \int a^{2}x^{3}\operatorname{atan}^{\frac{3}{2}}{\left(ax\right)}\,dx\right)$$

input
```
integrate(x*(a**2*c*x**2+c)*atan(a*x)**(3/2),x)
```

output
```
c*(Integral(x*atan(a*x)**(3/2), x) + Integral(a**2*x**3*atan(a*x)**(3/2),
x))
```

## Maxima [F(-2)]

Exception generated.

$$\int x(c+a^2cx^2)\arctan(ax)^{3/2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a^2*c*x^2+c)*arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int x(c+a^2cx^2)\arctan(ax)^{3/2}\,dx = \int (a^2cx^2+c)x\arctan{(ax)}^{\frac{3}{2}}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)*arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)*x*arctan(a*x)^(3/2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int x(c+a^2cx^2)\arctan(ax)^{3/2}\,dx = \int x\,\mathrm{atan}(a\,x)^{3/2}\,\left(c\,a^2\,x^2+c\right)\,dx$$

input
```
int(x*atan(a*x)^(3/2)*(c + a^2*c*x^2),x)
```

output
```
int(x*atan(a*x)^(3/2)*(c + a^2*c*x^2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 139, normalized size of antiderivative = 6.95

$$\int x(c+a^2cx^2)\arctan(ax)^{3/2}\,dx = \frac{c\Big(4\sqrt{atan\,(ax)}\,atan(ax)\,a^4x^4+8\sqrt{atan\,(ax)}\,atan(ax)\,a^2x^2+4\sqrt{atan\,(ax)}\,a$$

input
```
int(x*(a^2*c*x^2+c)*atan(a*x)^(3/2),x)
```

output
```
(c*(4*sqrt(atan(a*x))*atan(a*x)*a**4*x**4 + 8*sqrt(atan(a*x))*atan(a*x)*a*
*2*x**2 + 4*sqrt(atan(a*x))*atan(a*x) - 2*sqrt(atan(a*x))*a**3*x**3 - 6*sq
rt(atan(a*x))*a*x + int((sqrt(atan(a*x))*x**3)/(atan(a*x)*a**2*x**2 + atan
(a*x)),x)*a**4 + 3*int((sqrt(atan(a*x))*x)/(atan(a*x)*a**2*x**2 + atan(a*x
)),x)*a**2))/(16*a**2)
```

# 3.758 $\int (c+a^2cx^2) \arctan(ax)^{3/2}\,dx$



## Optimal result

Integrand size = 19, antiderivative size = 19

$$\int (c+a^2cx^2) \arctan(ax)^{3/2}\,dx = -\frac{c(1+a^2x^2)\sqrt{\arctan(ax)}}{4a} + \frac{1}{3}cx(1+a^2x^2)\arctan(ax)^{3/2} + \frac{1}{8}c\text{Int}\left(\frac{1}{\sqrt{\arctan(ax)}},x\right) + \frac{2}{3}c\text{Int}\left(\arctan(ax)^{3/2},x\right)$$

output
```
-1/4*c*(a^2*x^2+1)*arctan(a*x)^(1/2)/a+1/3*c*x*(a^2*x^2+1)*arctan(a*x)^(3/
2)+1/8*c*Defer(Int)(1/arctan(a*x)^(1/2),x)+2/3*c*Defer(Int)(arctan(a*x)^(3
/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.92 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int (c+a^2cx^2) \arctan(ax)^{3/2}\,dx = \int (c+a^2cx^2) \arctan(ax)^{3/2}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)*ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[(c + a^2*c*x^2)*ArcTan[a*x]^(3/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(ax)^{3/2}\left(a^2cx^2+c\right)\,dx$$

$$\Big\downarrow \text{5415}$$

$$\frac{1}{8}c\int \frac{1}{\sqrt{\arctan(ax)}}dx+\frac{2}{3}c\int \arctan(ax)^{3/2}dx+\frac{1}{3}cx\left(a^2x^2+1\right)\arctan(ax)^{3/2}-\frac{c\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}{4a}$$

$$\Big\downarrow \text{5353}$$

$$\frac{1}{8}c\int \frac{1}{\sqrt{\arctan(ax)}}dx+\frac{2}{3}c\int \arctan(ax)^{3/2}dx+\frac{1}{3}cx\left(a^2x^2+1\right)\arctan(ax)^{3/2}-\frac{c\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}{4a}$$

input
```
Int[(c + a^2*c*x^2)*ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \left(a^{2} c \,x^{2}+c \right) \arctan \left(a x \right)^{\frac{3}{2}}d x$$

input
```
int((a^2*c*x^2+c)*arctan(a*x)^(3/2),x)
```

output
```
int((a^2*c*x^2+c)*arctan(a*x)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right) \arctan(ax)^{3/2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 3.38 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.42

$$\int \left(c+a^2cx^2\right) \arctan(ax)^{3/2} \, dx = c \left(\int a^{2} x^{2} \operatorname{atan}^{\frac{3}{2}}{\left(a x \right)}\, dx + \int \operatorname{atan}^{\frac{3}{2}}{\left(a x \right)}\, dx\right)$$

input
```
integrate((a**2*c*x**2+c)*atan(a*x)**(3/2),x)
```

output
```
c*(Integral(a**2*x**2*atan(a*x)**(3/2), x) + Integral(atan(a*x)**(3/2), x)
)
```

## Maxima [F(-2)]

Exception generated.

$$\int (c + a^2cx^2) \arctan(ax)^{3/2} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int (c + a^2cx^2) \arctan(ax)^{3/2} \, dx = \int (a^2cx^2 + c) \arctan(ax)^{\frac{3}{2}} \, dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)*arctan(a*x)^(3/2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int (c+a^2cx^2)\arctan(ax)^{3/2}\,dx=\int \mathrm{atan}(a\,x)^{3/2}\,\left(c\,a^2\,x^2+c\right)\,dx$$

input
```
int(atan(a*x)^(3/2)*(c + a^2*c*x^2),x)
```

output
```
int(atan(a*x)^(3/2)*(c + a^2*c*x^2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 101, normalized size of antiderivative = 5.32

$$\int (c+a^2cx^2)\arctan(ax)^{3/2}\,dx=\frac{c\left(8\sqrt{atan\,(ax)}\,atan(ax)\,a^2x^3+24\sqrt{atan\,(ax)}\,atan(ax)\,x-6\sqrt{atan\,(ax)}\,a\,x^2\right.}{24}$$

input
```
int((a^2*c*x^2+c)*atan(a*x)^(3/2),x)
```

output
```
(c*(8*sqrt(atan(a*x))*atan(a*x)*a**2*x**3 + 24*sqrt(atan(a*x))*atan(a*x)*x
 - 6*sqrt(atan(a*x))*a*x**2 + 3*int((sqrt(atan(a*x))*x**2)/(atan(a*x)*a**2
*x**2 + atan(a*x)),x)*a**2 - 24*int((sqrt(atan(a*x))*x)/(a**2*x**2 + 1),x)
*a))/24
```

# 3.759 $\int \frac{(c+a^2cx^2)\arctan(ax)^{3/2}}{x}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{(c+a^2cx^2)\arctan(ax)^{3/2}}{x}\,dx = \text{Int}\left(\frac{(c+a^2cx^2)\arctan(ax)^{3/2}}{x},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)*arctan(a*x)^(3/2)/x,x)
```

## Mathematica [N/A]

Not integrable

Time = 0.97 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(c+a^2cx^2)\arctan(ax)^{3/2}}{x}\,dx = \int \frac{(c+a^2cx^2)\arctan(ax)^{3/2}}{x}\,dx$$

input
```
Integrate[((c + a^2*c*x^2)*ArcTan[a*x]^(3/2))/x,x]
```

output
```
Integrate[((c + a^2*c*x^2)*ArcTan[a*x]^(3/2))/x, x]
```

## Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{3/2}\left(a^2cx^2+c\right)}{x}\,dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{\arctan(ax)^{3/2}\left(a^2cx^2+c\right)}{x}dx$$

input
```
Int[((c + a^2*c*x^2)*ArcTan[a*x]^(3/2))/x,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\left(a^2c\,x^2+c\right)\arctan\left(ax\right)^{\frac{3}{2}}}{x}dx$$

input
```
int((a^2*c*x^2+c)*arctan(a*x)^(3/2)/x,x)
```

output
```
int((a^2*c*x^2+c)*arctan(a*x)^(3/2)/x,x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{(c + a^2cx^2) \arctan(ax)^{3/2}}{x} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^(3/2)/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 3.82 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

$$\int \frac{(c + a^2cx^2) \arctan(ax)^{3/2}}{x} \, dx = c\left(\int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{x} \, dx + \int a^2 x \operatorname{atan}^{\frac{3}{2}}(ax) \, dx\right)$$

input
```
integrate((a**2*c*x**2+c)*atan(a*x)**(3/2)/x,x)
```

output
```
c*(Integral(atan(a*x)**(3/2)/x, x) + Integral(a**2*x*atan(a*x)**(3/2), x))
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(c + a^2cx^2) \arctan(ax)^{3/2}}{x} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^(3/2)/x,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)\arctan(ax)^{3/2}}{x}\,dx = \int \frac{(a^2cx^2+c)\arctan(ax)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^(3/2)/x,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)*arctan(a*x)^(3/2)/x, x)
```

## Mupad [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)\arctan(ax)^{3/2}}{x}\,dx = \int \frac{\mathrm{atan}(a\,x)^{3/2}\,(c\,a^2\,x^2+c)}{x}\,dx$$

input
```
int((atan(a*x)^(3/2)*(c + a^2*c*x^2))/x,x)
```

output
```
int((atan(a*x)^(3/2)*(c + a^2*c*x^2))/x, x)
```

## Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 90, normalized size of antiderivative = 4.09

$$\int \frac{(c+a^2cx^2)\arctan(ax)^{3/2}}{x}\,dx = \frac{c\Big(4\sqrt{atan(ax)}\,atan(ax)\,a^2x^2+4\sqrt{atan(ax)}\,atan(ax)-6\sqrt{atan(ax)}}{8}$$

input
```
int((a^2*c*x^2+c)*atan(a*x)^(3/2)/x,x)
```

output
```
(c*(4*sqrt(atan(a*x))*atan(a*x)*a**2*x**2 + 4*sqrt(atan(a*x))*atan(a*x) -
6*sqrt(atan(a*x))*a*x + 8*int((sqrt(atan(a*x))*atan(a*x))/x,x) + 3*int((sq
rt(atan(a*x))*x)/(atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**2))/8
```

# 3.760 $\int \frac{(c+a^2cx^2)\arctan(ax)^{3/2}}{x^2}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{(c+a^2cx^2)\arctan(ax)^{3/2}}{x^2}\,dx = \operatorname{Int}\left(\frac{(c+a^2cx^2)\arctan(ax)^{3/2}}{x^2},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)*arctan(a*x)^(3/2)/x^2,x)
```

## Mathematica [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(c+a^2cx^2)\arctan(ax)^{3/2}}{x^2}\,dx = \int \frac{(c+a^2cx^2)\arctan(ax)^{3/2}}{x^2}\,dx$$

input
```
Integrate[((c + a^2*c*x^2)*ArcTan[a*x]^(3/2))/x^2,x]
```

output
```
Integrate[((c + a^2*c*x^2)*ArcTan[a*x]^(3/2))/x^2, x]
```

## Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{3/2}\left(a^2cx^2+c\right)}{x^2}\,dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{\arctan(ax)^{3/2}\left(a^2cx^2+c\right)}{x^2}dx$$

input
```
Int[((c + a^2*c*x^2)*ArcTan[a*x]^(3/2))/x^2,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\left(a^{2} c\,x^{2}+c\right) \arctan \left(a x \right)^{\frac{3}{2}}}{x^{2}}d x$$

input
```
int((a^2*c*x^2+c)*arctan(a*x)^(3/2)/x^2,x)
```

output
```
int((a^2*c*x^2+c)*arctan(a*x)^(3/2)/x^2,x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{(c + a^2cx^2) \arctan(ax)^{3/2}}{x^2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^(3/2)/x^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 3.65 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

$$\int \frac{(c + a^2cx^2) \arctan(ax)^{3/2}}{x^2} \, dx = c\left(\int a^2 \operatorname{atan}^{\frac{3}{2}}(ax) \, dx + \int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{x^2} \, dx\right)$$

input
```
integrate((a**2*c*x**2+c)*atan(a*x)**(3/2)/x**2,x)
```

output
```
c*(Integral(a**2*atan(a*x)**(3/2), x) + Integral(atan(a*x)**(3/2)/x**2, x)
)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(c + a^2cx^2) \arctan(ax)^{3/2}}{x^2} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^(3/2)/x^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)\arctan(ax)^{3/2}}{x^2}\,dx=\int \frac{(a^2cx^2+c)\arctan{(ax)}^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^(3/2)/x^2,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)*arctan(a*x)^(3/2)/x^2, x)
```

## Mupad [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)\arctan(ax)^{3/2}}{x^2}\,dx=\int \frac{\mathrm{atan}(a\,x)^{3/2}\,(c\,a^2\,x^2+c)}{x^2}\,dx$$

input
```
int((atan(a*x)^(3/2)*(c + a^2*c*x^2))/x^2,x)
```

output
```
int((atan(a*x)^(3/2)*(c + a^2*c*x^2))/x^2, x)
```

## Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.68

$$\int \frac{(c+a^2cx^2)\arctan(ax)^{3/2}}{x^2}\,dx = \frac{c\left(-2\sqrt{atan\left(ax\right)}\,atan(ax)+3\left(\int \frac{\sqrt{atan(ax)}}{a^2x^3+x}dx\right)ax+2\left(\int \sqrt{atan\left(ax\right)}\,a\right.}{2x}$$

input
```
int((a^2*c*x^2+c)*atan(a*x)^(3/2)/x^2,x)
```

output
```
(c*( - 2*sqrt(atan(a*x))*atan(a*x) + 3*int(sqrt(atan(a*x))/(a**2*x**3 + x)
,x)*a*x + 2*int(sqrt(atan(a*x))*atan(a*x),x)*a**2*x))/(2*x)
```

# 3.761 $\int x^m (c+a^2cx^2)^2 \arctan(ax)^{3/2}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int x^m (c+a^2cx^2)^2 \arctan(ax)^{3/2}\,dx = \operatorname{Int}\left(x^m (c+a^2cx^2)^2 \arctan(ax)^{3/2}, x\right)$$

output
```
Defer(Int)(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int x^m (c+a^2cx^2)^2 \arctan(ax)^{3/2}\,dx = \int x^m (c+a^2cx^2)^2 \arctan(ax)^{3/2}\,dx$$

input
```
Integrate[x^m*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[x^m*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \arctan(ax)^{3/2} \left(a^2cx^2+c\right)^2 \, dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int x^m \arctan(ax)^{3/2} \left(a^2cx^2+c\right)^2 dx$$

input
```
Int[x^m*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int x^m \left(a^2 c\, x^2+c\right)^2 \arctan\left(a x\right)^{\frac{3}{2}} dx$$

input
```
int(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int x^m (c + a^2 c x^2)^2 \arctan(ax)^{3/2} \, dx = \int (a^2 c x^2 + c)^2 x^m \arctan(ax)^{\frac{3}{2}} \, dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*x^m*arctan(a*x)^(3/2), x)
```

## Sympy [F(-1)]

Timed out.

$$\int x^m (c + a^2 c x^2)^2 \arctan(ax)^{3/2} \, dx = \text{Timed out}$$

input
```
integrate(x**m*(a**2*c*x**2+c)**2*atan(a*x)**(3/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int x^m (c + a^2 c x^2)^2 \arctan(ax)^{3/2} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^m (c+a^2cx^2)^2 \arctan(ax)^{3/2}\,dx = \int (a^2cx^2+c)^2 x^m \arctan(ax)^{\frac{3}{2}}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^2*x^m*arctan(a*x)^(3/2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^m (c+a^2cx^2)^2 \arctan(ax)^{3/2}\,dx = \int x^m \operatorname{atan}(a\,x)^{3/2} (c\,a^2\,x^2+c)^2\,dx$$

input
```
int(x^m*atan(a*x)^(3/2)*(c + a^2*c*x^2)^2,x)
```

output
```
int(x^m*atan(a*x)^(3/2)*(c + a^2*c*x^2)^2, x)
```

## Reduce [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 5441, normalized size of antiderivative = 226.71

$$\int x^m (c+a^2cx^2)^2 \arctan(ax)^{3/2}\,dx = \text{Too large to display}$$

input
```
int(x^m*(a^2*c*x^2+c)^2*atan(a*x)^(3/2),x)
```

output

```
(c**2*(4*x**m*sqrt(atan(a*x))*atan(a*x)*a**5*m**5*x**5 + 40*x**m*sqrt(atan
(a*x))*atan(a*x)*a**5*m**4*x**5 + 140*x**m*sqrt(atan(a*x))*atan(a*x)*a**5*
m**3*x**5 + 200*x**m*sqrt(atan(a*x))*atan(a*x)*a**5*m**2*x**5 + 96*x**m*sq
rt(atan(a*x))*atan(a*x)*a**5*m*x**5 + 8*x**m*sqrt(atan(a*x))*atan(a*x)*a**
3*m**5*x**3 + 96*x**m*sqrt(atan(a*x))*atan(a*x)*a**3*m**4*x**3 + 392*x**m*
sqrt(atan(a*x))*atan(a*x)*a**3*m**3*x**3 + 624*x**m*sqrt(atan(a*x))*atan(a
*x)*a**3*m**2*x**3 + 320*x**m*sqrt(atan(a*x))*atan(a*x)*a**3*m*x**3 + 4*x*
*m*sqrt(atan(a*x))*atan(a*x)*a*m**5*x + 56*x**m*sqrt(atan(a*x))*atan(a*x)*
a*m**4*x + 284*x**m*sqrt(atan(a*x))*atan(a*x)*a*m**3*x + 616*x**m*sqrt(ata
n(a*x))*atan(a*x)*a*m**2*x + 480*x**m*sqrt(atan(a*x))*atan(a*x)*a*m*x - 6*
x**m*sqrt(atan(a*x))*a**4*m**4*x**4 - 36*x**m*sqrt(atan(a*x))*a**4*m**3*x*
*4 - 66*x**m*sqrt(atan(a*x))*a**4*m**2*x**4 - 36*x**m*sqrt(atan(a*x))*a**4
*m*x**4 - 6*x**m*sqrt(atan(a*x))*a**2*m**4*x**2 - 72*x**m*sqrt(atan(a*x))*
a**2*m**3*x**2 - 234*x**m*sqrt(atan(a*x))*a**2*m**2*x**2 - 168*x**m*sqrt(a
tan(a*x))*a**2*m*x**2 - 48*x**m*sqrt(atan(a*x))*m**2 - 288*x**m*sqrt(atan(
a*x))*m - 384*x**m*sqrt(atan(a*x)) + 3*int((x**m*sqrt(atan(a*x))*x**4)/(at
an(a*x)*a**2*m**5*x**2 + 15*atan(a*x)*a**2*m**4*x**2 + 85*atan(a*x)*a**2*m
**3*x**2 + 225*atan(a*x)*a**2*m**2*x**2 + 274*atan(a*x)*a**2*m*x**2 + 120*
atan(a*x)*a**2*x**2 + atan(a*x)*m**5 + 15*atan(a*x)*m**4 + 85*atan(a*x)*m*
*3 + 225*atan(a*x)*m**2 + 274*atan(a*x)*m + 120*atan(a*x)),x)*a**5*m**9...
```

# 3.762 $\int x^2(c+a^2cx^2)^2 \arctan(ax)^{3/2}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int x^2(c+a^2cx^2)^2 \arctan(ax)^{3/2}\,dx = \text{Int}\left(x^2(c+a^2cx^2)^2 \arctan(ax)^{3/2},x\right)$$

output
```
Defer(Int)(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 3.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int x^2(c+a^2cx^2)^2 \arctan(ax)^{3/2}\,dx = \int x^2(c+a^2cx^2)^2 \arctan(ax)^{3/2}\,dx$$

input
```
Integrate[x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arctan(ax)^{3/2} \left(a^2cx^2+c\right)^2 \, dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int x^2 \arctan(ax)^{3/2} \left(a^2cx^2+c\right)^2 dx$$

input
```
Int[x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int x^2 \left(a^2 c\,x^2+c\right)^2 \arctan\left(a x\right)^{\frac{3}{2}} dx$$

input
```
int(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x)
```

output
```
int(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int x^2 \left(c + a^2 c x^2\right)^2 \arctan(ax)^{3/2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 14.76 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.12

$$\int x^2 \left(c + a^2 c x^2\right)^2 \arctan(ax)^{3/2} \, dx = c^2 \left(\int x^2 \operatorname{atan}^{\frac{3}{2}}(ax) \, dx + \int 2a^2 x^4 \operatorname{atan}^{\frac{3}{2}}(ax) \, dx + \int a^4 x^6 \operatorname{atan}^{\frac{3}{2}}(ax) \, dx\right)$$

input
```
integrate(x**2*(a**2*c*x**2+c)**2*atan(a*x)**(3/2),x)
```

output
```
c**2*(Integral(x**2*atan(a*x)**(3/2), x) + Integral(2*a**2*x**4*atan(a*x)*
*(3/2), x) + Integral(a**4*x**6*atan(a*x)**(3/2), x))
```

## Maxima [F(-2)]

Exception generated.

$$\int x^2 (c + a^2 c x^2)^2 \arctan(ax)^{3/2} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^2 (c + a^2 c x^2)^2 \arctan(ax)^{3/2} \, dx = \int (a^2 c x^2 + c)^2 x^2 \arctan(ax)^{\frac{3}{2}} \, dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^2*x^2*arctan(a*x)^(3/2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^2 (c + a^2 c x^2)^2 \arctan(ax)^{3/2} \, dx = \int x^2 \operatorname{atan}(a\,x)^{3/2} \left(c\,a^2\,x^2 + c\right)^2 \, dx$$

input
```
int(x^2*atan(a*x)^(3/2)*(c + a^2*c*x^2)^2,x)
```

output
```
int(x^2*atan(a*x)^(3/2)*(c + a^2*c*x^2)^2, x)
```

## Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.46

$$\int x^2 \left(c+a^2 c x^2\right)^2 \arctan(ax)^{3/2} \, dx = c^2 \left( \left( \int \sqrt{atan\left(ax\right)}\, atan(ax)\, x^6 dx \right) a^4 + 2 \left( \int \sqrt{atan\left(ax\right)}\, atan(ax)\, x^4 dx \right) a^2 + \int \sqrt{atan\left(ax\right)}\, atan(ax)\, x^2 dx \right)$$

input
```
int(x^2*(a^2*c*x^2+c)^2*atan(a*x)^(3/2),x)
```

output
```
c**2*(int(sqrt(atan(a*x))*atan(a*x)*x**6,x)*a**4 + 2*int(sqrt(atan(a*x))*a
tan(a*x)*x**4,x)*a**2 + int(sqrt(atan(a*x))*atan(a*x)*x**2,x))
```

# 3.763 $\int x(c+a^2cx^2)^2 \arctan(ax)^{3/2}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int x(c+a^2cx^2)^2 \arctan(ax)^{3/2}\,dx = \frac{c^2(1+a^2x^2)^3 \arctan(ax)^{3/2}}{6a^2} - \frac{\mathrm{Int}\left((c+a^2cx^2)^2 \sqrt{\arctan(ax)},x\right)}{4a}$$

output
```
1/6*c^2*(a^2*x^2+1)^3*arctan(a*x)^(3/2)/a^2-1/4*Defer(Int)((a^2*c*x^2+c)^2
*arctan(a*x)^(1/2),x)/a
```

## Mathematica [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int x(c+a^2cx^2)^2 \arctan(ax)^{3/2}\,dx = \int x(c+a^2cx^2)^2 \arctan(ax)^{3/2}\,dx$$

input
```
Integrate[x*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[x*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \arctan(ax)^{3/2} \left(a^2cx^2+c\right)^2 dx$$

$$\Big\downarrow \text{ 5465}$$

$$\frac{c^2 \left(a^2x^2+1\right)^3 \arctan(ax)^{3/2}}{6a^2} - \frac{\int c^2 \left(a^2x^2+1\right)^2 \sqrt{\arctan(ax)}dx}{4a}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{c^2 \left(a^2x^2+1\right)^3 \arctan(ax)^{3/2}}{6a^2} - \frac{c^2 \int \left(a^2x^2+1\right)^2 \sqrt{\arctan(ax)}dx}{4a}$$

$$\Big\downarrow \text{ 5560}$$

$$\frac{c^2 \left(a^2x^2+1\right)^3 \arctan(ax)^{3/2}}{6a^2} - \frac{c^2 \int \left(a^2x^2+1\right)^2 \sqrt{\arctan(ax)}dx}{4a}$$

input
```
Int[x*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int x\left(a^{2} c\,x^{2}+c\right)^{2} \arctan\left(a x\right)^{\frac{3}{2}}d x$$

input
```
int(x*(a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x)
```

output
```
int(x*(a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int x(c+a^2cx^2)^2\arctan(ax)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 11.44 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.23

$$\int x(c+a^2cx^2)^2\arctan(ax)^{3/2}\,dx=c^2\left(\int x\operatorname{atan}^{\frac{3}{2}}(ax)\,dx+\int 2a^2x^3\operatorname{atan}^{\frac{3}{2}}(ax)\,dx+\int a^4x^5\operatorname{atan}^{\frac{3}{2}}(ax)\,dx\right)$$

input
```
integrate(x*(a**2*c*x**2+c)**2*atan(a*x)**(3/2),x)
```

output
```
c**2*(Integral(x*atan(a*x)**(3/2), x) + Integral(2*a**2*x**3*atan(a*x)**(3
/2), x) + Integral(a**4*x**5*atan(a*x)**(3/2), x))
```

## Maxima [F(-2)]

Exception generated.

$$\int x(c+a^2cx^2)^2 \arctan(ax)^{3/2} dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x(c+a^2cx^2)^2 \arctan(ax)^{3/2} dx = \int (a^2cx^2+c)^2 x \arctan(ax)^{\frac{3}{2}} dx$$

input
```
integrate(x*(a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^2*x*arctan(a*x)^(3/2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x\left(c+a^2cx^2\right)^2\arctan(ax)^{3/2}\,dx=\int x\,\mathrm{atan}(a\,x)^{3/2}\,\left(c\,a^2\,x^2+c\right)^2\,dx$$

input
```
int(x*atan(a*x)^(3/2)*(c + a^2*c*x^2)^2,x)
```

output
```
int(x*atan(a*x)^(3/2)*(c + a^2*c*x^2)^2, x)
```

## Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 206, normalized size of antiderivative = 9.36

$$\int x\left(c+a^2cx^2\right)^2\arctan(ax)^{3/2}\,dx=\frac{c^2\left(20\sqrt{atan\left(ax\right)}\,atan(ax)\,a^6x^6+60\sqrt{atan\left(ax\right)}\,atan(ax)\,a^4x^4+60\sqrt{atan\left(\right.}\right.}{}$$

input
```
int(x*(a^2*c*x^2+c)^2*atan(a*x)^(3/2),x)
```

output
```
(c**2*(20*sqrt(atan(a*x))*atan(a*x)*a**6*x**6 + 60*sqrt(atan(a*x))*atan(a*
x)*a**4*x**4 + 60*sqrt(atan(a*x))*atan(a*x)*a**2*x**2 + 20*sqrt(atan(a*x))
*atan(a*x) - 6*sqrt(atan(a*x))*a**5*x**5 - 20*sqrt(atan(a*x))*a**3*x**3 -
30*sqrt(atan(a*x))*a*x + 3*int((sqrt(atan(a*x))*x**5)/(atan(a*x)*a**2*x**2
 + atan(a*x)),x)*a**6 + 10*int((sqrt(atan(a*x))*x**3)/(atan(a*x)*a**2*x**2
 + atan(a*x)),x)*a**4 + 15*int((sqrt(atan(a*x))*x)/(atan(a*x)*a**2*x**2 +
atan(a*x)),x)*a**2))/(120*a**2)
```

# 3.764 $\int (c+a^2cx^2)^2 \arctan(ax)^{3/2}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 21

$$\int (c+a^2cx^2)^2 \arctan(ax)^{3/2}\,dx = -\frac{c^2(1+a^2x^2)\sqrt{\arctan(ax)}}{5a} - \frac{3c^2(1+a^2x^2)^2\sqrt{\arctan(ax)}}{40a} + \frac{4}{15}c^2x(1+a^2x^2)\arctan(ax)^{3/2} + \frac{1}{5}c^2x(1+a^2x^2)^2\arctan(ax)^{3/2} + \frac{1}{10}c^2\text{Int}\left(\frac{1}{\sqrt{\arctan(ax)}},x\right) + \frac{3}{80}c\text{Int}\left(\frac{c+a^2cx^2}{\sqrt{\arctan(ax)}},x\right) + \frac{8}{15}c^2\text{Int}\left(\arctan(ax)^{3/2},x\right)$$

output
```
-1/5*c^2*(a^2*x^2+1)*arctan(a*x)^(1/2)/a-3/40*c^2*(a^2*x^2+1)^2*arctan(a*x
)^(1/2)/a+4/15*c^2*x*(a^2*x^2+1)*arctan(a*x)^(3/2)+1/5*c^2*x*(a^2*x^2+1)^2
*arctan(a*x)^(3/2)+1/10*c^2*Defer(Int)(1/arctan(a*x)^(1/2),x)+3/80*c*Defer
(Int)((a^2*c*x^2+c)/arctan(a*x)^(1/2),x)+8/15*c^2*Defer(Int)(arctan(a*x)^(
3/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \left(c+a^2cx^2\right)^2 \arctan(ax)^{3/2}\,dx=\int \left(c+a^2cx^2\right)^2 \arctan(ax)^{3/2}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}=0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(ax)^{3/2}\left(a^2cx^2+c\right)^2dx$$

$$\Big\downarrow \text{5415}$$

$$\frac{3}{80}c\int \frac{c\left(a^2x^2+1\right)}{\sqrt{\arctan(ax)}}dx+\frac{4}{5}c\int c\left(a^2x^2+1\right)\arctan(ax)^{3/2}dx+\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}-\frac{3c^2\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}{40a}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{80}c^2\int \frac{a^2x^2+1}{\sqrt{\arctan(ax)}}dx+\frac{4}{5}c^2\int \left(a^2x^2+1\right)\arctan(ax)^{3/2}dx+\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}-\frac{3c^2\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}{40a}$$

$$\Big\downarrow \text{5415}$$

$$\frac{3}{80}c^2\int \frac{a^2x^2+1}{\sqrt{\arctan(ax)}}dx+\frac{4}{5}c^2\left(\frac{1}{8}\int \frac{1}{\sqrt{\arctan(ax)}}dx+\frac{2}{3}\int \arctan(ax)^{3/2}dx+\frac{1}{3}x(a^2x^2+1)\arctan(ax)^{3/2}-\frac{(a^2x^2+1)\sqrt{\arctan(ax)}}{4a}\right)+\frac{1}{5}c^2x(a^2x^2+1)^2\arctan(ax)^{3/2}-\frac{3c^2(a^2x^2+1)^2\sqrt{\arctan(ax)}}{40a}$$

$$\Big\downarrow \text{5353}$$

$$\frac{3}{80}c^2\int \frac{a^2x^2+1}{\sqrt{\arctan(ax)}}dx+\frac{4}{5}c^2\left(\frac{1}{8}\int \frac{1}{\sqrt{\arctan(ax)}}dx+\frac{2}{3}\int \arctan(ax)^{3/2}dx+\frac{1}{3}x(a^2x^2+1)\arctan(ax)^{3/2}-\frac{(a^2x^2+1)\sqrt{\arctan(ax)}}{4a}\right)+\frac{1}{5}c^2x(a^2x^2+1)^2\arctan(ax)^{3/2}-\frac{3c^2(a^2x^2+1)^2\sqrt{\arctan(ax)}}{40a}$$

$$\Big\downarrow \text{5560}$$

$$\frac{3}{80}c^2\int \frac{a^2x^2+1}{\sqrt{\arctan(ax)}}dx+\frac{4}{5}c^2\left(\frac{1}{8}\int \frac{1}{\sqrt{\arctan(ax)}}dx+\frac{2}{3}\int \arctan(ax)^{3/2}dx+\frac{1}{3}x(a^2x^2+1)\arctan(ax)^{3/2}-\frac{(a^2x^2+1)\sqrt{\arctan(ax)}}{4a}\right)+\frac{1}{5}c^2x(a^2x^2+1)^2\arctan(ax)^{3/2}-\frac{3c^2(a^2x^2+1)^2\sqrt{\arctan(ax)}}{40a}$$

input
```
Int[(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \left(a^2c\,x^2+c\right)^2\arctan\left(ax\right)^{\frac{3}{2}}dx$$

input
```
int((a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x)
```

output
```
int((a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int (c + a^2cx^2)^2 \arctan(ax)^{3/2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 7.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.29

$$\int (c + a^2cx^2)^2 \arctan(ax)^{3/2} \, dx = c^2 \left( \int 2a^2x^2 \operatorname{atan}^{\frac{3}{2}}(ax) \, dx + \int a^4x^4 \operatorname{atan}^{\frac{3}{2}}(ax) \, dx + \int \operatorname{atan}^{\frac{3}{2}}(ax) \, dx \right)$$

input
```
integrate((a**2*c*x**2+c)**2*atan(a*x)**(3/2),x)
```

output
```
c**2*(Integral(2*a**2*x**2*atan(a*x)**(3/2), x) + Integral(a**4*x**4*atan(
a*x)**(3/2), x) + Integral(atan(a*x)**(3/2), x))
```

## Maxima [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^2 \arctan(ax)^{3/2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \left(c+a^2cx^2\right)^2 \arctan(ax)^{3/2}\,dx = \int \left(a^2cx^2+c\right)^2 \arctan\left(ax\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^2*arctan(a*x)^(3/2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \left(c+a^2cx^2\right)^2 \arctan(ax)^{3/2}\,dx = \int \mathrm{atan}(a\,x)^{3/2}\,\left(c\,a^2\,x^2+c\right)^2\,dx$$

input
```
int(atan(a*x)^(3/2)*(c + a^2*c*x^2)^2,x)
```

output
```
int(atan(a*x)^(3/2)*(c + a^2*c*x^2)^2, x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 167, normalized size of antiderivative = 7.95

$$\int (c+a^2cx^2)^2 \arctan(ax)^{3/2}\,dx = \frac{c^2\Big(48\sqrt{atan\,(ax)}\,atan(ax)\,a^4x^5+160\sqrt{atan\,(ax)}\,atan(ax)\,a^2x^3+240\sqrt{ata$$

input
```
int((a^2*c*x^2+c)^2*atan(a*x)^(3/2),x)
```

output
```
(c**2*(48*sqrt(atan(a*x))*atan(a*x)*a**4*x**5 + 160*sqrt(atan(a*x))*atan(a
*x)*a**2*x**3 + 240*sqrt(atan(a*x))*atan(a*x)*x - 18*sqrt(atan(a*x))*a**3*
x**4 - 84*sqrt(atan(a*x))*a*x**2 + 9*int((sqrt(atan(a*x))*x**4)/(atan(a*x)
*a**2*x**2 + atan(a*x)),x)*a**4 + 42*int((sqrt(atan(a*x))*x**2)/(atan(a*x)
*a**2*x**2 + atan(a*x)),x)*a**2 - 192*int((sqrt(atan(a*x))*x)/(a**2*x**2 +
 1),x)*a))/240
```

# 3.765 $\int \frac{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}{x} \, dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}{x} \, dx = \text{Int}\left(\frac{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}{x}, x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^2*arctan(a*x)^(3/2)/x,x)
```

## Mathematica [N/A]

Not integrable

Time = 0.96 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}{x} \, dx = \int \frac{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}{x} \, dx$$

input
```
Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2))/x,x]
```

output
```
Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2))/x, x]
```

## Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^2}{x}dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int \frac{\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^2}{x}dx$$

input
```
Int[((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2))/x,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\left(a^{2} c \,x^{2}+c \right)^{2} \arctan \left(a x \right)^{\frac{3}{2}}}{x}d x$$

input
```
int((a^2*c*x^2+c)^2*arctan(a*x)^(3/2)/x,x)
```

output
```
int((a^2*c*x^2+c)^2*arctan(a*x)^(3/2)/x,x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}{x} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^(3/2)/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 7.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.00

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}{x} \, dx = c^2 \left( \int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{x} \, dx + \int 2a^2x \operatorname{atan}^{\frac{3}{2}}(ax) \, dx + \int a^4x^3 \operatorname{atan}^{\frac{3}{2}}(ax) \, dx \right)$$

input
```
integrate((a**2*c*x**2+c)**2*atan(a*x)**(3/2)/x,x)
```

output
```
c**2*(Integral(atan(a*x)**(3/2)/x, x) + Integral(2*a**2*x*atan(a*x)**(3/2)
, x) + Integral(a**4*x**3*atan(a*x)**(3/2), x))
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}{x}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^(3/2)/x,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}{x}\,dx = \int \frac{(a^2cx^2+c)^2 \arctan(ax)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^(3/2)/x,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^2*arctan(a*x)^(3/2)/x, x)
```

## Mupad [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}{x}\,dx = \int \frac{\mathrm{atan}(a\,x)^{3/2}\,(c\,a^2\,x^2+c)^2}{x}\,dx$$

input
```
int((atan(a*x)^(3/2)*(c + a^2*c*x^2)^2)/x,x)
```

output
```
int((atan(a*x)^(3/2)*(c + a^2*c*x^2)^2)/x, x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 155, normalized size of antiderivative = 6.46

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}{x}\,dx = \frac{c^2\left(4\sqrt{atan\,(ax)}\;atan(ax)\,a^4x^4+16\sqrt{atan\,(ax)}\;atan(ax)\,a^2x^2+12\sqrt{a}\right.}{}$$

input
```
int((a^2*c*x^2+c)^2*atan(a*x)^(3/2)/x,x)
```

output
```
(c**2*(4*sqrt(atan(a*x))*atan(a*x)*a**4*x**4 + 16*sqrt(atan(a*x))*atan(a*x
)*a**2*x**2 + 12*sqrt(atan(a*x))*atan(a*x) - 2*sqrt(atan(a*x))*a**3*x**3 -
 18*sqrt(atan(a*x))*a*x + 16*int((sqrt(atan(a*x))*atan(a*x))/x,x) + int((s
qrt(atan(a*x))*x**3)/(atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**4 + 9*int((sq
rt(atan(a*x))*x)/(atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**2))/16
```

# 3.766 $\int \frac{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}{x^2} dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}{x^2} dx = \text{Int}\left(\frac{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}{x^2}, x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^2*arctan(a*x)^(3/2)/x^2,x)
```

## Mathematica [N/A]

Not integrable

Time = 0.97 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}{x^2} dx = \int \frac{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}{x^2} dx$$

input
```
Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2))/x^2,x]
```

output
```
Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2))/x^2, x]
```

## Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^2}{x^2}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^2}{x^2}dx$$

input
```
Int[((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2))/x^2,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\left(a^2c\,x^2+c\right)^2\arctan\left(ax\right)^{\frac{3}{2}}}{x^2}dx$$

input
```
int((a^2*c*x^2+c)^2*arctan(a*x)^(3/2)/x^2,x)
```

output
```
int((a^2*c*x^2+c)^2*arctan(a*x)^(3/2)/x^2,x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}{x^2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^(3/2)/x^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 6.63 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.00

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}{x^2} \, dx = c^2 \left( \int 2a^2 \operatorname{atan}^{\frac{3}{2}}(ax) \, dx + \int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{x^2} \, dx + \int a^4x^2 \operatorname{atan}^{\frac{3}{2}}(ax) \, dx \right)$$

input
```
integrate((a**2*c*x**2+c)**2*atan(a*x)**(3/2)/x**2,x)
```

output
```
c**2*(Integral(2*a**2*atan(a*x)**(3/2), x) + Integral(atan(a*x)**(3/2)/x**
2, x) + Integral(a**4*x**2*atan(a*x)**(3/2), x))
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}{x^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^(3/2)/x^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}{x^2}\,dx = \int \frac{(a^2cx^2+c)^2 \arctan(ax)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^(3/2)/x^2,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^2*arctan(a*x)^(3/2)/x^2, x)
```

## Mupad [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}{x^2}\,dx = \int \frac{\mathrm{atan}(a\,x)^{3/2}\,(c\,a^2\,x^2+c)^2}{x^2}\,dx$$

input
```
int((atan(a*x)^(3/2)*(c + a^2*c*x^2)^2)/x^2,x)
```

output
```
int((atan(a*x)^(3/2)*(c + a^2*c*x^2)^2)/x^2, x)
```

## Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 151, normalized size of antiderivative = 6.29

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}{x^2}\,dx = \frac{c^2\left(8\sqrt{atan\,(ax)}\,atan(ax)\,a^4x^4 + 48\sqrt{atan\,(ax)}\,atan(ax)\,a^2x^2 - 24\sqrt{a}\right.}{}$$

input
```
int((a^2*c*x^2+c)^2*atan(a*x)^(3/2)/x^2,x)
```

output
```
(c**2*(8*sqrt(atan(a*x))*atan(a*x)*a**4*x**4 + 48*sqrt(atan(a*x))*atan(a*x
)*a**2*x**2 - 24*sqrt(atan(a*x))*atan(a*x) - 6*sqrt(atan(a*x))*a**3*x**3 +
 36*int(sqrt(atan(a*x))/(a**2*x**3 + x),x)*a*x + 3*int((sqrt(atan(a*x))*x*
*2)/(atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**4*x - 60*int((sqrt(atan(a*x))*
x)/(a**2*x**2 + 1),x)*a**3*x))/(24*x)
```

# 3.767 $\int x^m (c + a^2 c x^2)^3 \arctan(ax)^{3/2} \, dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int x^m \left(c + a^2 c x^2\right)^3 \arctan(ax)^{3/2} \, dx = \text{Int}\left(x^m \left(c + a^2 c x^2\right)^3 \arctan(ax)^{3/2}, x\right)$$

output
```
Defer(Int)(x^m*(a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int x^m \left(c + a^2 c x^2\right)^3 \arctan(ax)^{3/2} \, dx = \int x^m \left(c + a^2 c x^2\right)^3 \arctan(ax)^{3/2} \, dx$$

input
```
Integrate[x^m*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[x^m*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \arctan(ax)^{3/2} \left(a^2cx^2+c\right)^3 \, dx$$
$$\Big\downarrow \; 5560$$
$$\int x^m \arctan(ax)^{3/2} \left(a^2cx^2+c\right)^3 dx$$

input
```
Int[x^m*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int x^m \left(a^2 c\,x^2+c\right)^3 \arctan\left(a x\right)^{\frac{3}{2}} dx$$

input
```
int(x^m*(a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.00

$$\int x^m \left(c+a^2cx^2\right)^3 \arctan(ax)^{3/2}\,dx = \int \left(a^2cx^2+c\right)^3 x^m \arctan\left(ax\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*x^m*arctan(a*
x)^(3/2), x)
```

## Sympy [F(-1)]

Timed out.

$$\int x^m \left(c+a^2cx^2\right)^3 \arctan(ax)^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(x**m*(a**2*c*x**2+c)**3*atan(a*x)**(3/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int x^m \left(c+a^2cx^2\right)^3 \arctan(ax)^{3/2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 1.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^m (c+a^2cx^2)^3 \arctan(ax)^{3/2}\,dx = \int (a^2cx^2+c)^3 x^m \arctan(ax)^{\frac{3}{2}}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^3*x^m*arctan(a*x)^(3/2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^m (c+a^2cx^2)^3 \arctan(ax)^{3/2}\,dx = \int x^m \operatorname{atan}(a\,x)^{3/2} \left(c\,a^2\,x^2+c\right)^3 dx$$

input
```
int(x^m*atan(a*x)^(3/2)*(c + a^2*c*x^2)^3,x)
```

output
```
int(x^m*atan(a*x)^(3/2)*(c + a^2*c*x^2)^3, x)
```

## Reduce [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 12793, normalized size of antiderivative = 533.04

$$\int x^m (c+a^2cx^2)^3 \arctan(ax)^{3/2}\,dx = \text{Too large to display}$$

input
```
int(x^m*(a^2*c*x^2+c)^3*atan(a*x)^(3/2),x)
```

output
```
(c**3*(4*x**m*sqrt(atan(a*x))*atan(a*x)*a**7*m**7*x**7 + 84*x**m*sqrt(atan
(a*x))*atan(a*x)*a**7*m**6*x**7 + 700*x**m*sqrt(atan(a*x))*atan(a*x)*a**7*
m**5*x**7 + 2940*x**m*sqrt(atan(a*x))*atan(a*x)*a**7*m**4*x**7 + 6496*x**m
*sqrt(atan(a*x))*atan(a*x)*a**7*m**3*x**7 + 7056*x**m*sqrt(atan(a*x))*atan
(a*x)*a**7*m**2*x**7 + 2880*x**m*sqrt(atan(a*x))*atan(a*x)*a**7*m*x**7 + 1
2*x**m*sqrt(atan(a*x))*atan(a*x)*a**5*m**7*x**5 + 276*x**m*sqrt(atan(a*x))
*atan(a*x)*a**5*m**6*x**5 + 2484*x**m*sqrt(atan(a*x))*atan(a*x)*a**5*m**5*
x**5 + 11100*x**m*sqrt(atan(a*x))*atan(a*x)*a**5*m**4*x**5 + 25728*x**m*sq
rt(atan(a*x))*atan(a*x)*a**5*m**3*x**5 + 28944*x**m*sqrt(atan(a*x))*atan(a
*x)*a**5*m**2*x**5 + 12096*x**m*sqrt(atan(a*x))*atan(a*x)*a**5*m*x**5 + 12
*x**m*sqrt(atan(a*x))*atan(a*x)*a**3*m**7*x**3 + 300*x**m*sqrt(atan(a*x))*
atan(a*x)*a**3*m**6*x**3 + 2964*x**m*sqrt(atan(a*x))*atan(a*x)*a**3*m**5*x
**3 + 14628*x**m*sqrt(atan(a*x))*atan(a*x)*a**3*m**4*x**3 + 37344*x**m*sqr
t(atan(a*x))*atan(a*x)*a**3*m**3*x**3 + 45552*x**m*sqrt(atan(a*x))*atan(a*
x)*a**3*m**2*x**3 + 20160*x**m*sqrt(atan(a*x))*atan(a*x)*a**3*m*x**3 + 4*x
**m*sqrt(atan(a*x))*atan(a*x)*a*m**7*x + 108*x**m*sqrt(atan(a*x))*atan(a*x
)*a*m**6*x + 1180*x**m*sqrt(atan(a*x))*atan(a*x)*a*m**5*x + 6660*x**m*sqrt
(atan(a*x))*atan(a*x)*a*m**4*x + 20416*x**m*sqrt(atan(a*x))*atan(a*x)*a*m*
*3*x + 32112*x**m*sqrt(atan(a*x))*atan(a*x)*a*m**2*x + 20160*x**m*sqrt(ata
n(a*x))*atan(a*x)*a*m*x - 6*x**m*sqrt(atan(a*x))*a**6*m**6*x**6 - 90*x*...
```

# 3.768 $\int x^2(c+a^2cx^2)^3\arctan(ax)^{3/2}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int x^2\left(c+a^2cx^2\right)^3\arctan(ax)^{3/2}\,dx=\text{Int}\left(x^2\left(c+a^2cx^2\right)^3\arctan(ax)^{3/2},x\right)$$

output
```
Defer(Int)(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 2.96 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int x^2\left(c+a^2cx^2\right)^3\arctan(ax)^{3/2}\,dx=\int x^2\left(c+a^2cx^2\right)^3\arctan(ax)^{3/2}\,dx$$

input
```
Integrate[x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arctan(ax)^{3/2} \left(a^2cx^2+c\right)^3 \, dx$$
$$\Big\downarrow \text{5560}$$
$$\int x^2 \arctan(ax)^{3/2} \left(a^2cx^2+c\right)^3 \, dx$$

input
```
Int[x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int x^2 \left(a^2 c\,x^2+c\right)^3 \arctan\left(a x\right)^{\frac{3}{2}} dx$$

input
```
int(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x)
```

output
```
int(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int x^2(c + a^2cx^2)^3 \arctan(ax)^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 30.69 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.92

$$\int x^2(c + a^2cx^2)^3 \arctan(ax)^{3/2}\,dx = c^3\left(\int x^2 \operatorname{atan}^{\frac{3}{2}}(ax)\,dx + \int 3a^2x^4 \operatorname{atan}^{\frac{3}{2}}(ax)\,dx + \int 3a^4x^6 \operatorname{atan}^{\frac{3}{2}}(ax)\,dx + \int a^6x^8 \operatorname{atan}^{\frac{3}{2}}(ax)\,dx\right)$$

input
```
integrate(x**2*(a**2*c*x**2+c)**3*atan(a*x)**(3/2),x)
```

output
```
c**3*(Integral(x**2*atan(a*x)**(3/2), x) + Integral(3*a**2*x**4*atan(a*x)*
*(3/2), x) + Integral(3*a**4*x**6*atan(a*x)**(3/2), x) + Integral(a**6*x**
8*atan(a*x)**(3/2), x))
```

## Maxima [F(-2)]

Exception generated.

$$\int x^2 (c + a^2 c x^2)^3 \arctan(ax)^{3/2} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^2 (c + a^2 c x^2)^3 \arctan(ax)^{3/2} \, dx = \int (a^2 c x^2 + c)^3 x^2 \arctan(ax)^{\frac{3}{2}} \, dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^3*x^2*arctan(a*x)^(3/2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^2 (c + a^2 c x^2)^3 \arctan(ax)^{3/2} \, dx = \int x^2 \operatorname{atan}(a\,x)^{3/2} \left(c\,a^2\,x^2 + c\right)^3 \, dx$$

input
```
int(x^2*atan(a*x)^(3/2)*(c + a^2*c*x^2)^3,x)
```

output
```
int(x^2*atan(a*x)^(3/2)*(c + a^2*c*x^2)^3, x)
```

## Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 79, normalized size of antiderivative = 3.29

$$\int x^2(c+a^2cx^2)^3\arctan(ax)^{3/2}\,dx = c^3\left(\left(\int \sqrt{atan\,(ax)}\,atan(ax)\,x^8dx\right)a^6 + 3\left(\int \sqrt{atan\,(ax)}\,atan(ax)\,x^6dx\right)a^4 + 3\left(\int \sqrt{atan\,(ax)}\,atan(ax)\,x^4dx\right)a^2 + \int \sqrt{atan\,(ax)}\,atan(ax)\,x^2dx\right)$$

input
```
int(x^2*(a^2*c*x^2+c)^3*atan(a*x)^(3/2),x)
```

output
```
c**3*(int(sqrt(atan(a*x))*atan(a*x)*x**8,x)*a**6 + 3*int(sqrt(atan(a*x))*a
tan(a*x)*x**6,x)*a**4 + 3*int(sqrt(atan(a*x))*atan(a*x)*x**4,x)*a**2 + int
(sqrt(atan(a*x))*atan(a*x)*x**2,x))
```

# 3.769 $\int x(c+a^2cx^2)^3 \arctan(ax)^{3/2}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int x(c+a^2cx^2)^3 \arctan(ax)^{3/2}\,dx = \frac{c^3(1+a^2x^2)^4 \arctan(ax)^{3/2}}{8a^2} - \frac{3\text{Int}\left((c+a^2cx^2)^3\sqrt{\arctan(ax)},x\right)}{16a}$$

output
```
1/8*c^3*(a^2*x^2+1)^4*arctan(a*x)^(3/2)/a^2-3/16*Defer(Int)((a^2*c*x^2+c)^
3*arctan(a*x)^(1/2),x)/a
```

## Mathematica [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int x(c+a^2cx^2)^3 \arctan(ax)^{3/2}\,dx = \int x(c+a^2cx^2)^3 \arctan(ax)^{3/2}\,dx$$

input
```
Integrate[x*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[x*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \arctan(ax)^{3/2} \left(a^2cx^2+c\right)^3 dx$$

$$\Big\downarrow \text{ 5465}$$

$$\frac{c^3\left(a^2x^2+1\right)^4 \arctan(ax)^{3/2}}{8a^2}-\frac{3\int c^3\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}dx}{16a}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{c^3\left(a^2x^2+1\right)^4 \arctan(ax)^{3/2}}{8a^2}-\frac{3c^3\int \left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}dx}{16a}$$

$$\Big\downarrow \text{ 5560}$$

$$\frac{c^3\left(a^2x^2+1\right)^4 \arctan(ax)^{3/2}}{8a^2}-\frac{3c^3\int \left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}dx}{16a}$$

input
```
Int[x*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int x\left(a^{2} c\,x^{2}+c\right)^{3} \arctan\left(a x\right)^{\frac{3}{2}}d x$$

input
```
int(x*(a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x)
```

output
```
int(x*(a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int x\left(c+a^2cx^2\right)^3\arctan(ax)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 24.46 (sec) , antiderivative size = 68, normalized size of antiderivative = 3.09

$$\int x\left(c+a^2cx^2\right)^3\arctan(ax)^{3/2}\,dx=c^3\left(\int x\operatorname{atan}^{\frac{3}{2}}(ax)\,dx+\int 3a^2x^3\operatorname{atan}^{\frac{3}{2}}(ax)\,dx+\int 3a^4x^5\operatorname{atan}^{\frac{3}{2}}(ax)\,dx+\int a^6x^7\operatorname{atan}^{\frac{3}{2}}(ax)\,dx\right)$$

input
```
integrate(x*(a**2*c*x**2+c)**3*atan(a*x)**(3/2),x)
```

output
```
c**3*(Integral(x*atan(a*x)**(3/2), x) + Integral(3*a**2*x**3*atan(a*x)**(3
/2), x) + Integral(3*a**4*x**5*atan(a*x)**(3/2), x) + Integral(a**6*x**7*a
tan(a*x)**(3/2), x))
```

## Maxima [F(-2)]

Exception generated.

$$\int x(c+a^2cx^2)^3\arctan(ax)^{3/2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x(c+a^2cx^2)^3\arctan(ax)^{3/2}\,dx = \int (a^2cx^2+c)^3x\arctan(ax)^{\frac{3}{2}}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^3*x*arctan(a*x)^(3/2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x(c+a^2cx^2)^3 \arctan(ax)^{3/2}\,dx = \int x\,\mathrm{atan}(a\,x)^{3/2}\,\left(c\,a^2\,x^2+c\right)^3\,dx$$

input
```
int(x*atan(a*x)^(3/2)*(c + a^2*c*x^2)^3,x)
```

output
```
int(x*atan(a*x)^(3/2)*(c + a^2*c*x^2)^3, x)
```

## Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 270, normalized size of antiderivative = 12.27

$$\int x(c+a^2cx^2)^3 \arctan(ax)^{3/2}\,dx = \frac{c^3\left(140\sqrt{atan\,(ax)}\,atan(ax)\,a^8x^8+560\sqrt{atan\,(ax)}\,atan(ax)\,a^6x^6+840\sqrt{ata$$

input
```
int(x*(a^2*c*x^2+c)^3*atan(a*x)^(3/2),x)
```

output
```
(c**3*(140*sqrt(atan(a*x))*atan(a*x)*a**8*x**8 + 560*sqrt(atan(a*x))*atan(
a*x)*a**6*x**6 + 840*sqrt(atan(a*x))*atan(a*x)*a**4*x**4 + 560*sqrt(atan(a
*x))*atan(a*x)*a**2*x**2 + 140*sqrt(atan(a*x))*atan(a*x) - 30*sqrt(atan(a*
x))*a**7*x**7 - 126*sqrt(atan(a*x))*a**5*x**5 - 210*sqrt(atan(a*x))*a**3*x
**3 - 210*sqrt(atan(a*x))*a*x + 15*int((sqrt(atan(a*x))*x**7)/(atan(a*x)*a
**2*x**2 + atan(a*x)),x)*a**8 + 63*int((sqrt(atan(a*x))*x**5)/(atan(a*x)*a
**2*x**2 + atan(a*x)),x)*a**6 + 105*int((sqrt(atan(a*x))*x**3)/(atan(a*x)*
a**2*x**2 + atan(a*x)),x)*a**4 + 105*int((sqrt(atan(a*x))*x)/(atan(a*x)*a*
*2*x**2 + atan(a*x)),x)*a**2))/(1120*a**2)
```

# 3.770 $\int (c+a^2cx^2)^3 \arctan(ax)^{3/2} \, dx$



## Optimal result

Integrand size = 21, antiderivative size = 21

$$\int (c+a^2cx^2)^3 \arctan(ax)^{3/2} \, dx = -\frac{6c^3(1+a^2x^2)\sqrt{\arctan(ax)}}{35a} - \frac{9c^3(1+a^2x^2)^2\sqrt{\arctan(ax)}}{140a} - \frac{c^3(1+a^2x^2)^3\sqrt{\arctan(ax)}}{28a} + \frac{8}{35}c^3x(1+a^2x^2)\arctan(ax)^{3/2} + \frac{6}{35}c^3x(1+a^2x^2)^2\arctan(ax)^{3/2} + \frac{1}{7}c^3x(1+a^2x^2)^3\arctan(ax)^{3/2} + \frac{3}{35}c^3\text{Int}\left(\frac{1}{\sqrt{\arctan(ax)}},x\right) + \frac{9}{280}c^2\text{Int}\left(\frac{c+a^2cx^2}{\sqrt{\arctan(ax)}},x\right) + \frac{1}{56}c\text{Int}\left(\frac{(c}{\sqrt{}}\right.$$

output
```
-6/35*c^3*(a^2*x^2+1)*arctan(a*x)^(1/2)/a-9/140*c^3*(a^2*x^2+1)^2*arctan(a
*x)^(1/2)/a-1/28*c^3*(a^2*x^2+1)^3*arctan(a*x)^(1/2)/a+8/35*c^3*x*(a^2*x^2
+1)*arctan(a*x)^(3/2)+6/35*c^3*x*(a^2*x^2+1)^2*arctan(a*x)^(3/2)+1/7*c^3*x
*(a^2*x^2+1)^3*arctan(a*x)^(3/2)+3/35*c^3*Defer(Int)(1/arctan(a*x)^(1/2),x
)+9/280*c^2*Defer(Int)((a^2*c*x^2+c)/arctan(a*x)^(1/2),x)+1/56*c*Defer(Int
)((a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)+16/35*c^3*Defer(Int)(arctan(a*x)^(3
/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.14 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \left(c+a^2cx^2\right)^3\arctan(ax)^{3/2}\,dx=\int \left(c+a^2cx^2\right)^3\arctan(ax)^{3/2}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(ax)^{3/2}\left(a^2cx^2+c\right)^3\,dx$$

$$\Big\downarrow \text{5415}$$

$$\frac{1}{56}c\int \frac{c^2\left(a^2x^2+1\right)^2}{\sqrt{\arctan(ax)}}dx+\frac{6}{7}c\int c^2\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}dx+\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)^{3/2}-\frac{c^3\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}{28a}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{56}c^3\int \frac{\left(a^2x^2+1\right)^2}{\sqrt{\arctan(ax)}}dx+\frac{6}{7}c^3\int \left(a^2x^2+1\right)^2\arctan(ax)^{3/2}dx+\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)^{3/2}-\frac{c^3\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}{28a}$$

$$\downarrow \text{5415}$$

$$\frac{1}{56}c^3\int \frac{\left(a^2x^2+1\right)^2}{\sqrt{\arctan (ax)}}dx+$$
$$\frac{6}{7}c^3\left(\frac{3}{80}\int \frac{a^2x^2+1}{\sqrt{\arctan (ax)}}dx+\frac{4}{5}\int \left(a^2x^2+1\right)\arctan (ax)^{3/2}dx+\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan (ax)^{3/2}-\frac{3\left(a^2x^2+1\right)^2}{40}\right.$$
$$\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan (ax)^{3/2}-\frac{c^3\left(a^2x^2+1\right)^3\sqrt{\arctan (ax)}}{28a}$$

$$\downarrow \text{5415}$$

$$\frac{1}{56}c^3\int \frac{\left(a^2x^2+1\right)^2}{\sqrt{\arctan (ax)}}dx+$$
$$\frac{6}{7}c^3\left(\frac{3}{80}\int \frac{a^2x^2+1}{\sqrt{\arctan (ax)}}dx+\frac{4}{5}\left(\frac{1}{8}\int \frac{1}{\sqrt{\arctan (ax)}}dx+\frac{2}{3}\int \arctan (ax)^{3/2}dx+\frac{1}{3}x\left(a^2x^2+1\right)\arctan (ax)^{3/2}-\right.\right.$$
$$\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan (ax)^{3/2}-\frac{c^3\left(a^2x^2+1\right)^3\sqrt{\arctan (ax)}}{28a}$$

$$\downarrow \text{5353}$$

$$\frac{1}{56}c^3\int \frac{\left(a^2x^2+1\right)^2}{\sqrt{\arctan (ax)}}dx+$$
$$\frac{6}{7}c^3\left(\frac{3}{80}\int \frac{a^2x^2+1}{\sqrt{\arctan (ax)}}dx+\frac{4}{5}\left(\frac{1}{8}\int \frac{1}{\sqrt{\arctan (ax)}}dx+\frac{2}{3}\int \arctan (ax)^{3/2}dx+\frac{1}{3}x\left(a^2x^2+1\right)\arctan (ax)^{3/2}-\right.\right.$$
$$\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan (ax)^{3/2}-\frac{c^3\left(a^2x^2+1\right)^3\sqrt{\arctan (ax)}}{28a}$$

$$\downarrow \text{5560}$$

$$\frac{1}{56}c^3\int \frac{\left(a^2x^2+1\right)^2}{\sqrt{\arctan (ax)}}dx+$$
$$\frac{6}{7}c^3\left(\frac{3}{80}\int \frac{a^2x^2+1}{\sqrt{\arctan (ax)}}dx+\frac{4}{5}\left(\frac{1}{8}\int \frac{1}{\sqrt{\arctan (ax)}}dx+\frac{2}{3}\int \arctan (ax)^{3/2}dx+\frac{1}{3}x\left(a^2x^2+1\right)\arctan (ax)^{3/2}-\right.\right.$$
$$\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan (ax)^{3/2}-\frac{c^3\left(a^2x^2+1\right)^3\sqrt{\arctan (ax)}}{28a}$$

input
```
Int[(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \left(a^{2} c \,x^{2}+c \right)^{3} \arctan \left(a x \right)^{\frac{3}{2}}d x$$

input
```
int((a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x)
```

output
```
int((a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^3 \arctan(ax)^{3/2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 15.54 (sec) , antiderivative size = 66, normalized size of antiderivative = 3.14

$$\int \left(c+a^2cx^2\right)^3 \arctan(ax)^{3/2} \, dx = c^{3} \left(\int 3 a^{2} x^{2} \operatorname{atan}^{\frac{3}{2}}{\left(a x \right)}\, dx + \int 3 a^{4} x^{4} \operatorname{atan}^{\frac{3}{2}}{\left(a x \right)}\, dx + \int a^{6} x^{6} \operatorname{atan}^{\frac{3}{2}}{\left(a x \right)}\, dx + \int \operatorname{atan}^{\frac{3}{2}}{\left(a x \right)}\, dx\right)$$

input
```
integrate((a**2*c*x**2+c)**3*atan(a*x)**(3/2),x)
```

output
```
c**3*(Integral(3*a**2*x**2*atan(a*x)**(3/2), x) + Integral(3*a**4*x**4*ata
n(a*x)**(3/2), x) + Integral(a**6*x**6*atan(a*x)**(3/2), x) + Integral(ata
n(a*x)**(3/2), x))
```

## Maxima [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^3\arctan(ax)^{3/2}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \left(c+a^2cx^2\right)^3\arctan(ax)^{3/2}\,dx=\int \left(a^2cx^2+c\right)^3\arctan\left(ax\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^3*arctan(a*x)^(3/2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int (c+a^2cx^2)^3 \arctan(ax)^{3/2}\, dx = \int \mathrm{atan}(a\,x)^{3/2}\,\left(c\,a^2\,x^2+c\right)^3 dx$$

input
```
int(atan(a*x)^(3/2)*(c + a^2*c*x^2)^3,x)
```

output
```
int(atan(a*x)^(3/2)*(c + a^2*c*x^2)^3, x)
```

## Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 231, normalized size of antiderivative = 11.00

$$\int (c+a^2cx^2)^3 \arctan(ax)^{3/2}\, dx = \frac{c^3\left(40\sqrt{atan(ax)}\,atan(ax)\,a^6x^7+168\sqrt{atan(ax)}\,atan(ax)\,a^4x^5+280\sqrt{ata}\right.}{}$$

input
```
int((a^2*c*x^2+c)^3*atan(a*x)^(3/2),x)
```

output
```
(c**3*(40*sqrt(atan(a*x))*atan(a*x)*a**6*x**7 + 168*sqrt(atan(a*x))*atan(a
*x)*a**4*x**5 + 280*sqrt(atan(a*x))*atan(a*x)*a**2*x**3 + 280*sqrt(atan(a*
x))*atan(a*x)*x - 10*sqrt(atan(a*x))*a**5*x**6 - 48*sqrt(atan(a*x))*a**3*x
**4 - 114*sqrt(atan(a*x))*a*x**2 + 5*int((sqrt(atan(a*x))*x**6)/(atan(a*x)
*a**2*x**2 + atan(a*x)),x)*a**6 + 24*int((sqrt(atan(a*x))*x**4)/(atan(a*x)
*a**2*x**2 + atan(a*x)),x)*a**4 + 57*int((sqrt(atan(a*x))*x**2)/(atan(a*x)
*a**2*x**2 + atan(a*x)),x)*a**2 - 192*int((sqrt(atan(a*x))*x)/(a**2*x**2 +
 1),x)*a))/280
```

# 3.771 $\int \frac{(c+a^2cx^2)^3 \arctan(ax)^{3/2}}{x} \, dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^{3/2}}{x} \, dx = \text{Int}\left(\frac{(c+a^2cx^2)^3 \arctan(ax)^{3/2}}{x}, x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^3*arctan(a*x)^(3/2)/x,x)
```

## Mathematica [N/A]

Not integrable

Time = 0.94 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^{3/2}}{x} \, dx = \int \frac{(c+a^2cx^2)^3 \arctan(ax)^{3/2}}{x} \, dx$$

input
```
Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2))/x,x]
```

output
```
Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2))/x, x]
```

## Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{3/2} \left(a^2cx^2+c\right)^3}{x} \, dx$$

$$\Big\downarrow \ 5560$$

$$\int \frac{\arctan(ax)^{3/2} \left(a^2cx^2+c\right)^3}{x} dx$$

input
```
Int[((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2))/x,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\left(a^2c\,x^2+c\right)^3 \arctan\left(ax\right)^{\frac{3}{2}}}{x}dx$$

input
```
int((a^2*c*x^2+c)^3*arctan(a*x)^(3/2)/x,x)
```

output
```
int((a^2*c*x^2+c)^3*arctan(a*x)^(3/2)/x,x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^{3/2}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^(3/2)/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 14.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.75

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^{3/2}}{x}\,dx = c^3\left(\int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{x}\,dx + \int 3a^2x\operatorname{atan}^{\frac{3}{2}}(ax)\,dx + \int 3a^4x^3\operatorname{atan}^{\frac{3}{2}}(ax)\,dx + \int a^6x^5\operatorname{atan}^{\frac{3}{2}}(ax)\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**3*atan(a*x)**(3/2)/x,x)
```

output
```
c**3*(Integral(atan(a*x)**(3/2)/x, x) + Integral(3*a**2*x*atan(a*x)**(3/2)
, x) + Integral(3*a**4*x**3*atan(a*x)**(3/2), x) + Integral(a**6*x**5*atan
(a*x)**(3/2), x))
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^{3/2}}{x} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^(3/2)/x,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^{3/2}}{x} \, dx = \int \frac{(a^2cx^2+c)^3 \arctan(ax)^{\frac{3}{2}}}{x} \, dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^(3/2)/x,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^3*arctan(a*x)^(3/2)/x, x)
```

## Mupad [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^{3/2}}{x} \, dx = \int \frac{\mathrm{atan}(a\,x)^{3/2}\,(c\,a^2\,x^2+c)^3}{x} \, dx$$

input
```
int((atan(a*x)^(3/2)*(c + a^2*c*x^2)^3)/x,x)
```

output
```
int((atan(a*x)^(3/2)*(c + a^2*c*x^2)^3)/x, x)
```

## Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 220, normalized size of antiderivative = 9.17

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^{3/2}}{x}\,dx = \frac{c^3\left(40\sqrt{atan\,(ax)}\,atan(ax)\,a^6x^6 + 180\sqrt{atan\,(ax)}\,atan(ax)\,a^4x^4 + 360\right.}{}$$

input
```
int((a^2*c*x^2+c)^3*atan(a*x)^(3/2)/x,x)
```

output
```
(c**3*(40*sqrt(atan(a*x))*atan(a*x)*a**6*x**6 + 180*sqrt(atan(a*x))*atan(a
*x)*a**4*x**4 + 360*sqrt(atan(a*x))*atan(a*x)*a**2*x**2 + 220*sqrt(atan(a*
x))*atan(a*x) - 12*sqrt(atan(a*x))*a**5*x**5 - 70*sqrt(atan(a*x))*a**3*x**
3 - 330*sqrt(atan(a*x))*a*x + 240*int((sqrt(atan(a*x))*atan(a*x))/x,x) + 6
*int((sqrt(atan(a*x))*x**5)/(atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**6 + 35
*int((sqrt(atan(a*x))*x**3)/(atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**4 + 16
5*int((sqrt(atan(a*x))*x)/(atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**2))/240
```

# 3.772 $\int \frac{(c+a^2cx^2)^3 \arctan(ax)^{3/2}}{x^2} \, dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^{3/2}}{x^2} \, dx = \text{Int}\left(\frac{(c+a^2cx^2)^3 \arctan(ax)^{3/2}}{x^2}, x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^3*arctan(a*x)^(3/2)/x^2,x)
```

## Mathematica [N/A]

Not integrable

Time = 1.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^{3/2}}{x^2} \, dx = \int \frac{(c+a^2cx^2)^3 \arctan(ax)^{3/2}}{x^2} \, dx$$

input
```
Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2))/x^2,x]
```

output
```
Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2))/x^2, x]
```

## Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^3}{x^2}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^3}{x^2}dx$$

input
```
Int[((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2))/x^2,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\left(a^2c\,x^2+c\right)^3\arctan\left(ax\right)^{\frac{3}{2}}}{x^2}dx$$

input
```
int((a^2*c*x^2+c)^3*arctan(a*x)^(3/2)/x^2,x)
```

output
```
int((a^2*c*x^2+c)^3*arctan(a*x)^(3/2)/x^2,x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^{3/2}}{x^2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^(3/2)/x^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 13.25 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.75

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^{3/2}}{x^2} \, dx = c^3 \left( \int 3a^2 \operatorname{atan}^{\frac{3}{2}}(ax) \, dx + \int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{x^2} \, dx + \int 3a^4x^2 \operatorname{atan}^{\frac{3}{2}}(ax) \, dx + \int a^6x^4 \operatorname{atan}^{\frac{3}{2}}(ax) \, dx \right)$$

input
```
integrate((a**2*c*x**2+c)**3*atan(a*x)**(3/2)/x**2,x)
```

output
```
c**3*(Integral(3*a**2*atan(a*x)**(3/2), x) + Integral(atan(a*x)**(3/2)/x**
2, x) + Integral(3*a**4*x**2*atan(a*x)**(3/2), x) + Integral(a**6*x**4*ata
n(a*x)**(3/2), x))
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^{3/2}}{x^2} dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^(3/2)/x^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^{3/2}}{x^2} dx = \int \frac{(a^2cx^2+c)^3 \arctan(ax)^{\frac{3}{2}}}{x^2} dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^(3/2)/x^2,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^3*arctan(a*x)^(3/2)/x^2, x)
```

## Mupad [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^{3/2}}{x^2} dx = \int \frac{\mathrm{atan}(a\,x)^{3/2}\,(c\,a^2\,x^2+c)^3}{x^2} dx$$

input
```
int((atan(a*x)^(3/2)*(c + a^2*c*x^2)^3)/x^2,x)
```

output
```
int((atan(a*x)^(3/2)*(c + a^2*c*x^2)^3)/x^2, x)
```

## Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 216, normalized size of antiderivative = 9.00

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^{3/2}}{x^2} dx = \frac{c^3 \Big(16\sqrt{atan\,(ax)}\, atan(ax)\, a^6x^6 + 80\sqrt{atan\,(ax)}\, atan(ax)\, a^4x^4 + 240$$

input
```
int((a^2*c*x^2+c)^3*atan(a*x)^(3/2)/x^2,x)
```

output
```
(c**3*(16*sqrt(atan(a*x))*atan(a*x)*a**6*x**6 + 80*sqrt(atan(a*x))*atan(a*
x)*a**4*x**4 + 240*sqrt(atan(a*x))*atan(a*x)*a**2*x**2 - 80*sqrt(atan(a*x)
)*atan(a*x) - 6*sqrt(atan(a*x))*a**5*x**5 - 48*sqrt(atan(a*x))*a**3*x**3 +
 120*int(sqrt(atan(a*x))/(a**2*x**3 + x),x)*a*x + 3*int((sqrt(atan(a*x))*x
**4)/(atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**6*x + 24*int((sqrt(atan(a*x))
*x**2)/(atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**4*x - 264*int((sqrt(atan(a*
x))*x)/(a**2*x**2 + 1),x)*a**3*x))/(80*x)
```

# 3.773 $\int \frac{x^m \arctan(ax)^{3/2}}{c+a^2cx^2}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m \arctan(ax)^{3/2}}{c+a^2cx^2}\,dx = \text{Int}\left(\frac{x^m \arctan(ax)^{3/2}}{c+a^2cx^2},x\right)$$

output
```
Defer(Int)(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \arctan(ax)^{3/2}}{c+a^2cx^2}\,dx = \int \frac{x^m \arctan(ax)^{3/2}}{c+a^2cx^2}\,dx$$

input
```
Integrate[(x^m*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2),x]
```

output
```
Integrate[(x^m*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \arctan(ax)^{3/2}}{a^2cx^2+c} \, dx$$

$$\Big\downarrow \text{5560}$$

$$\int \frac{x^m \arctan(ax)^{3/2}}{a^2cx^2+c}dx$$

input
```
Int[(x^m*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m \arctan\left(ax\right)^{\frac{3}{2}}}{a^2c\,x^2+c}dx$$

input
```
int(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x)
```

output
```
int(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan(ax)^{3/2}}{c+a^2cx^2}\,dx = \int \frac{x^m \arctan(ax)^{\frac{3}{2}}}{a^2cx^2+c}\,dx$$

input
```
integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
integral(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2 + c), x)
```

## Sympy [N/A]

Not integrable

Time = 59.74 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m \arctan(ax)^{3/2}}{c+a^2cx^2}\,dx = \frac{\int \frac{x^m \operatorname{atan}^{\frac{3}{2}}(ax)}{a^2x^2+1}\,dx}{c}$$

input
```
integrate(x**m*atan(a*x)**(3/2)/(a**2*c*x**2+c),x)
```

output
```
Integral(x**m*atan(a*x)**(3/2)/(a**2*x**2 + 1), x)/c
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m \arctan(ax)^{3/2}}{c+a^2cx^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan(ax)^{3/2}}{c+a^2cx^2}\,dx = \int \frac{x^m \arctan(ax)^{\frac{3}{2}}}{a^2cx^2+c}\,dx$$

input
```
integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2 + c), x)
```

## Mupad [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan(ax)^{3/2}}{c+a^2cx^2}\,dx = \int \frac{x^m\,\mathrm{atan}(a\,x)^{3/2}}{c\,a^2\,x^2+c}\,dx$$

input
```
int((x^m*atan(a*x)^(3/2))/(c + a^2*c*x^2),x)
```

output
```
int((x^m*atan(a*x)^(3/2))/(c + a^2*c*x^2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.25

$$\int \frac{x^m \arctan(ax)^{3/2}}{c + a^2cx^2} \, dx = \frac{\int \frac{x^m \sqrt{atan(ax)}\, atan(ax)}{a^2x^2+1} dx}{c}$$

input
```
int(x^m*atan(a*x)^(3/2)/(a^2*c*x^2+c),x)
```

output
```
int((x**m*sqrt(atan(a*x))*atan(a*x))/(a**2*x**2 + 1),x)/c
```

# 3.774 $\int \frac{x^3 \arctan(ax)^{3/2}}{c+a^2cx^2} dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^3 \arctan(ax)^{3/2}}{c+a^2cx^2} dx = -\frac{2x \arctan(ax)^{5/2}}{5a^3c} + \frac{\text{Int}\left(x \arctan(ax)^{3/2}, x\right)}{a^2c} + \frac{2\text{Int}\left(\arctan(ax)^{5/2}, x\right)}{5a^3c}$$

output
```
-2/5*x*arctan(a*x)^(5/2)/a^3/c+Defer(Int)(x*arctan(a*x)^(3/2),x)/a^2/c+2/5
*Defer(Int)(arctan(a*x)^(5/2),x)/a^3/c
```

## Mathematica [N/A]

Not integrable

Time = 2.66 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^3 \arctan(ax)^{3/2}}{c+a^2cx^2} dx = \int \frac{x^3 \arctan(ax)^{3/2}}{c+a^2cx^2} dx$$

input
```
Integrate[(x^3*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2),x]
```

output
```
Integrate[(x^3*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \arctan(ax)^{3/2}}{a^2cx^2+c}dx$$

$$\Big\downarrow \text{5451}$$

$$\frac{\int x\arctan(ax)^{3/2}dx}{a^2c}-\frac{\int \frac{x\arctan(ax)^{3/2}}{c(a^2x^2+1)}dx}{a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int x\arctan(ax)^{3/2}dx}{a^2c}-\frac{\int \frac{x\arctan(ax)^{3/2}}{a^2x^2+1}dx}{a^2c}$$

$$\Big\downarrow \text{5377}$$

$$\frac{\int x\arctan(ax)^{3/2}dx}{a^2c}-\frac{\int \frac{x\arctan(ax)^{3/2}}{a^2x^2+1}dx}{a^2c}$$

$$\Big\downarrow \text{5457}$$

$$\frac{\int x\arctan(ax)^{3/2}dx}{a^2c}-\frac{\frac{2x\arctan(ax)^{5/2}}{5a}-\frac{2\int \arctan(ax)^{5/2}dx}{5a}}{a^2c}$$

$$\Big\downarrow \text{5353}$$

$$\frac{\int x\arctan(ax)^{3/2}dx}{a^2c}-\frac{\frac{2x\arctan(ax)^{5/2}}{5a}-\frac{2\int \arctan(ax)^{5/2}dx}{5a}}{a^2c}$$

input
```
Int[(x^3*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^3 \arctan(ax)^{\frac{3}{2}}}{a^2 c x^2 + c} dx$$

input
```
int(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x)
```

output
```
int(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3 \arctan(ax)^{3/2}}{c + a^2 c x^2} dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 2.35 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^3 \arctan(ax)^{3/2}}{c+a^2cx^2}\,dx = \frac{\int \frac{x^3 \operatorname{atan}^{\frac{3}{2}}(ax)}{a^2x^2+1}\,dx}{c}$$

input
```
integrate(x**3*atan(a*x)**(3/2)/(a**2*c*x**2+c),x)
```

output
```
Integral(x**3*atan(a*x)**(3/2)/(a**2*x**2 + 1), x)/c
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3 \arctan(ax)^{3/2}}{c+a^2cx^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^3 \arctan(ax)^{3/2}}{c+a^2cx^2}\,dx = \int \frac{x^3 \arctan(ax)^{\frac{3}{2}}}{a^2cx^2+c}\,dx$$

input
```
integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2 + c), x)
```

## Mupad [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^3 \arctan(ax)^{3/2}}{c + a^2cx^2}\,dx = \int \frac{x^3 \operatorname{atan}(a\,x)^{3/2}}{c\,a^2\,x^2 + c}\,dx$$

input
```
int((x^3*atan(a*x)^(3/2))/(c + a^2*c*x^2),x)
```

output
```
int((x^3*atan(a*x)^(3/2))/(c + a^2*c*x^2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.25

$$\int \frac{x^3 \arctan(ax)^{3/2}}{c + a^2cx^2}\,dx = \frac{\int \frac{\sqrt{atan(ax)}\,atan(ax)x^3}{a^2x^2+1}dx}{c}$$

input
```
int(x^3*atan(a*x)^(3/2)/(a^2*c*x^2+c),x)
```

output
```
int((sqrt(atan(a*x))*atan(a*x)*x**3)/(a**2*x**2 + 1),x)/c
```

# 3.775 $\int \frac{x^2 \arctan(ax)^{3/2}}{c+a^2cx^2} dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^2 \arctan(ax)^{3/2}}{c+a^2cx^2} \, dx = -\frac{2 \arctan(ax)^{5/2}}{5a^3c} + \frac{\operatorname{Int}\left(\arctan(ax)^{3/2}, x\right)}{a^2c}$$

output
```
-2/5*arctan(a*x)^(5/2)/a^3/c+Defer(Int)(arctan(a*x)^(3/2),x)/a^2/c
```

## Mathematica [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^2 \arctan(ax)^{3/2}}{c+a^2cx^2} \, dx = \int \frac{x^2 \arctan(ax)^{3/2}}{c+a^2cx^2} \, dx$$

input
```
Integrate[(x^2*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2),x]
```

output
```
Integrate[(x^2*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \arctan(ax)^{3/2}}{a^2cx^2+c} \, dx$$

$$\Big\downarrow \text{5451}$$

$$\frac{\int \arctan(ax)^{3/2}dx}{a^2c} - \frac{\int \frac{\arctan(ax)^{3/2}}{c(a^2x^2+1)}dx}{a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \arctan(ax)^{3/2}dx}{a^2c} - \frac{\int \frac{\arctan(ax)^{3/2}}{a^2x^2+1}dx}{a^2c}$$

$$\Big\downarrow \text{5353}$$

$$\frac{\int \arctan(ax)^{3/2}dx}{a^2c} - \frac{\int \frac{\arctan(ax)^{3/2}}{a^2x^2+1}dx}{a^2c}$$

$$\Big\downarrow \text{5419}$$

$$\frac{\int \arctan(ax)^{3/2}dx}{a^2c} - \frac{2\arctan(ax)^{5/2}}{5a^3c}$$

input
```
Int[(x^2*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^2 \arctan \left(a x\right)^{\frac{3}{2}}}{a^2 c\,x^2+c}dx$$

input
```
int(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x)
```

output
```
int(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2 \arctan(ax)^{3/2}}{c+a^2cx^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 1.56 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^2 \arctan(ax)^{3/2}}{c+a^2cx^2}\,dx = \frac{\int \frac{x^2 \operatorname{atan}^{\frac{3}{2}}(ax)}{a^2x^2+1}\,dx}{c}$$

input
```
integrate(x**2*atan(a*x)**(3/2)/(a**2*c*x**2+c),x)
```

output
```
Integral(x**2*atan(a*x)**(3/2)/(a**2*x**2 + 1), x)/c
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2 \arctan(ax)^{3/2}}{c+a^2cx^2} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^2 \arctan(ax)^{3/2}}{c+a^2cx^2} \, dx = \int \frac{x^2 \arctan\left(ax\right)^{\frac{3}{2}}}{a^2cx^2+c} \, dx$$

input
```
integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2 + c), x)
```

## Mupad [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^2 \arctan(ax)^{3/2}}{c+a^2cx^2}\,dx = \int \frac{x^2\,\mathrm{atan}(a\,x)^{3/2}}{c\,a^2\,x^2+c}\,dx$$

input
```
int((x^2*atan(a*x)^(3/2))/(c + a^2*c*x^2),x)
```

output
```
int((x^2*atan(a*x)^(3/2))/(c + a^2*c*x^2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.25

$$\int \frac{x^2 \arctan(ax)^{3/2}}{c+a^2cx^2}\,dx = \frac{\int \frac{\sqrt{atan(ax)}\,atan(ax)x^2}{a^2x^2+1}dx}{c}$$

input
```
int(x^2*atan(a*x)^(3/2)/(a^2*c*x^2+c),x)
```

output
```
int((sqrt(atan(a*x))*atan(a*x)*x**2)/(a**2*x**2 + 1),x)/c
```

# 3.776 $\int \frac{x \arctan(ax)^{3/2}}{c+a^2cx^2}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x \arctan(ax)^{3/2}}{c+a^2cx^2}\,dx = \frac{2x \arctan(ax)^{5/2}}{5ac} - \frac{2\text{Int}\left(\arctan(ax)^{5/2},x\right)}{5ac}$$

output
```
2/5*x*arctan(a*x)^(5/2)/a/c-2/5*Defer(Int)(arctan(a*x)^(5/2),x)/a/c
```

## Mathematica [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x \arctan(ax)^{3/2}}{c+a^2cx^2}\,dx = \int \frac{x \arctan(ax)^{3/2}}{c+a^2cx^2}\,dx$$

input
```
Integrate[(x*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2),x]
```

output
```
Integrate[(x*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \arctan(ax)^{3/2}}{a^2cx^2+c}dx$$
$$\Big\downarrow \text{5457}$$
$$\frac{2x\arctan(ax)^{5/2}}{5ac}-\frac{2\int \arctan(ax)^{5/2}dx}{5ac}$$
$$\Big\downarrow \text{5353}$$
$$\frac{2x\arctan(ax)^{5/2}}{5ac}-\frac{2\int \arctan(ax)^{5/2}dx}{5ac}$$

input
```
Int[(x*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x \arctan\left(ax\right)^{\frac{3}{2}}}{a^2c\,x^2+c}dx$$

input
```
int(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x)
```

output
```
int(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x\arctan(ax)^{3/2}}{c+a^2cx^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 1.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x\arctan(ax)^{3/2}}{c+a^2cx^2}\,dx = \frac{\int \frac{x\operatorname{atan}^{\frac{3}{2}}(ax)}{a^2x^2+1}\,dx}{c}$$

input
```
integrate(x*atan(a*x)**(3/2)/(a**2*c*x**2+c),x)
```

output
```
Integral(x*atan(a*x)**(3/2)/(a**2*x**2 + 1), x)/c
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x\arctan(ax)^{3/2}}{c+a^2cx^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x \arctan(ax)^{3/2}}{c+a^2cx^2}\,dx = \int \frac{x \arctan{(ax)}^{\frac{3}{2}}}{a^2cx^2+c}\,dx$$

input
```
integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2 + c), x)
```

## Mupad [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x \arctan(ax)^{3/2}}{c+a^2cx^2}\,dx = \int \frac{x\,\mathrm{atan}(a\,x)^{3/2}}{c\,a^2\,x^2+c}\,dx$$

input
```
int((x*atan(a*x)^(3/2))/(c + a^2*c*x^2),x)
```

output
```
int((x*atan(a*x)^(3/2))/(c + a^2*c*x^2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.27

$$\int \frac{x \arctan(ax)^{3/2}}{c+a^2cx^2}\,dx = \frac{\int \frac{\sqrt{atan(ax)}\,atan(ax)x}{a^2x^2+1}dx}{c}$$

input
```
int(x*atan(a*x)^(3/2)/(a^2*c*x^2+c),x)
```

output
```
int((sqrt(atan(a*x))*atan(a*x)*x)/(a**2*x**2 + 1),x)/c
```

# 3.777 $\int \frac{\arctan(ax)^{3/2}}{c+a^2cx^2}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 18

$$\int \frac{\arctan(ax)^{3/2}}{c+a^2cx^2}\,dx=\frac{2\arctan(ax)^{5/2}}{5ac}$$

output
```
2/5*arctan(a*x)^(5/2)/a/c
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax)^{3/2}}{c+a^2cx^2}\,dx=\frac{2\arctan(ax)^{5/2}}{5ac}$$

input
```
Integrate[ArcTan[a*x]^(3/2)/(c + a^2*c*x^2),x]
```

output
```
(2*ArcTan[a*x]^(5/2))/(5*a*c)
```

## Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{3/2}}{a^2cx^2+c}\,dx$$
$$\Big\downarrow \text{5419}$$
$$\frac{2\arctan(ax)^{5/2}}{5ac}$$

input
```
Int[ArcTan[a*x]^(3/2)/(c + a^2*c*x^2),x]
```

output
```
(2*ArcTan[a*x]^(5/2))/(5*a*c)
```

### Defintions of rubi rules used

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

## Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

| method | result | size |
|---|---|---|
| default | $\frac{2\arctan(ax)^{\frac{5}{2}}}{5ac}$ | 15 |

input
```
int(arctan(a*x)^(3/2)/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
2/5*arctan(a*x)^(5/2)/a/c
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{\arctan(ax)^{3/2}}{c+a^2cx^2}\,dx = \frac{2\arctan\left(ax\right)^{\frac{5}{2}}}{5\,ac}$$

input
```
integrate(arctan(a*x)^(3/2)/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
2/5*arctan(a*x)^(5/2)/(a*c)
```

## Sympy [F]

$$\int \frac{\arctan(ax)^{3/2}}{c+a^2cx^2}\,dx = \frac{\int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{a^2x^2+1}\,dx}{c}$$

input
```
integrate(atan(a*x)**(3/2)/(a**2*c*x**2+c),x)
```

output
```
Integral(atan(a*x)**(3/2)/(a**2*x**2 + 1), x)/c
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{c+a^2cx^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(3/2)/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{\arctan(ax)^{3/2}}{c+a^2cx^2}\,dx=\frac{2\arctan{(ax)}^{\frac{5}{2}}}{5\,ac}$$

input
```
integrate(arctan(a*x)^(3/2)/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
2/5*arctan(a*x)^(5/2)/(a*c)
```

## Mupad [B] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{\arctan(ax)^{3/2}}{c+a^2cx^2}\,dx=\frac{2\,\mathrm{atan}(a\,x)^{5/2}}{5\,a\,c}$$

input
```
int(atan(a*x)^(3/2)/(c + a^2*c*x^2),x)
```

output
```
(2*atan(a*x)^(5/2))/(5*a*c)
```

## Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{\arctan(ax)^{3/2}}{c+a^2cx^2}\,dx=\frac{2\sqrt{atan\,(ax)}\,atan(ax)^2}{5ac}$$

input
```
int(atan(a*x)^(3/2)/(a^2*c*x^2+c),x)
```

output
```
(2*sqrt(atan(a*x))*atan(a*x)**2)/(5*a*c)
```

# 3.778 $\int \frac{\arctan(ax)^{3/2}}{x(c+a^2cx^2)}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\arctan(ax)^{3/2}}{x\left(c+a^2cx^2\right)}\,dx=-\frac{2i\arctan(ax)^{5/2}}{5c}+\frac{i\mathrm{Int}\left(\frac{\arctan(ax)^{3/2}}{x(i+ax)},x\right)}{c}$$

output
```
-2/5*I*arctan(a*x)^(5/2)/c+I*Defer(Int)(arctan(a*x)^(3/2)/x/(I+a*x),x)/c
```

## Mathematica [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\arctan(ax)^{3/2}}{x\left(c+a^2cx^2\right)}\,dx=\int \frac{\arctan(ax)^{3/2}}{x\left(c+a^2cx^2\right)}\,dx$$

input
```
Integrate[ArcTan[a*x]^(3/2)/(x*(c + a^2*c*x^2)),x]
```

output
```
Integrate[ArcTan[a*x]^(3/2)/(x*(c + a^2*c*x^2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{3/2}}{x\left(a^2cx^2+c\right)}dx$$

$$\Big\downarrow \text{ 5459}$$

$$\frac{i\int \frac{\arctan(ax)^{3/2}}{x(ax+i)}dx}{c}-\frac{2i\arctan(ax)^{5/2}}{5c}$$

$$\Big\downarrow \text{ 5560}$$

$$\frac{i\int \frac{\arctan(ax)^{3/2}}{x(ax+i)}dx}{c}-\frac{2i\arctan(ax)^{5/2}}{5c}$$

input
```
Int[ArcTan[a*x]^(3/2)/(x*(c + a^2*c*x^2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\arctan\left(ax\right)^{\frac{3}{2}}}{x\left(a^2c\,x^2+c\right)}dx$$

input
```
int(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c),x)
```

output
```
int(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{x\,(c+a^2cx^2)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 1.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \frac{\arctan(ax)^{3/2}}{x\,(c+a^2cx^2)}\,dx = \frac{\int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{a^2x^3+x}\,dx}{c}$$

input
```
integrate(atan(a*x)**(3/2)/x/(a**2*c*x**2+c),x)
```

output
```
Integral(atan(a*x)**(3/2)/(a**2*x**3 + x), x)/c
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{x\,(c+a^2cx^2)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax)^{3/2}}{x\,(c+a^2cx^2)}\,dx = \int \frac{\arctan\left(ax\right)^{\frac{3}{2}}}{(a^2cx^2+c)x}\,dx$$

input
```
integrate(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^(3/2)/((a^2*c*x^2 + c)*x), x)
```

## Mupad [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax)^{3/2}}{x\,(c+a^2cx^2)}\,dx = \int \frac{\mathrm{atan}(a\,x)^{3/2}}{x\,\left(c\,a^2\,x^2+c\right)}\,dx$$

input
```
int(atan(a*x)^(3/2)/(x*(c + a^2*c*x^2)),x)
```

output
```
int(atan(a*x)^(3/2)/(x*(c + a^2*c*x^2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.12

$$\int \frac{\arctan(ax)^{3/2}}{x\,(c+a^2cx^2)}\,dx = \frac{\int \frac{\sqrt{atan(ax)}\,atan(ax)}{a^2x^3+x}dx}{c}$$

input
```
int(atan(a*x)^(3/2)/x/(a^2*c*x^2+c),x)
```

output
```
int((sqrt(atan(a*x))*atan(a*x))/(a**2*x**3 + x),x)/c
```

# 3.779 $\int \frac{\arctan(ax)^{3/2}}{x^2(c+a^2cx^2)}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\arctan(ax)^{3/2}}{x^2(c+a^2cx^2)}\,dx = -\frac{2a\arctan(ax)^{5/2}}{5c} + \frac{\operatorname{Int}\left(\frac{\arctan(ax)^{3/2}}{x^2},x\right)}{c}$$

output
```
-2/5*a*arctan(a*x)^(5/2)/c+Defer(Int)(arctan(a*x)^(3/2)/x^2,x)/c
```

## Mathematica [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\arctan(ax)^{3/2}}{x^2(c+a^2cx^2)}\,dx = \int \frac{\arctan(ax)^{3/2}}{x^2(c+a^2cx^2)}\,dx$$

input
```
Integrate[ArcTan[a*x]^(3/2)/(x^2*(c + a^2*c*x^2)),x]
```

output
```
Integrate[ArcTan[a*x]^(3/2)/(x^2*(c + a^2*c*x^2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{3/2}}{x^2\left(a^2cx^2+c\right)}dx$$

$$\Big\downarrow \text{5453}$$

$$\frac{\int \frac{\arctan(ax)^{3/2}}{x^2}dx}{c}-a^2\int \frac{\arctan(ax)^{3/2}}{c\left(a^2x^2+1\right)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\arctan(ax)^{3/2}}{x^2}dx}{c}-\frac{a^2\int \frac{\arctan(ax)^{3/2}}{a^2x^2+1}dx}{c}$$

$$\Big\downarrow \text{5377}$$

$$\frac{\int \frac{\arctan(ax)^{3/2}}{x^2}dx}{c}-\frac{a^2\int \frac{\arctan(ax)^{3/2}}{a^2x^2+1}dx}{c}$$

$$\Big\downarrow \text{5419}$$

$$\frac{\int \frac{\arctan(ax)^{3/2}}{x^2}dx}{c}-\frac{2a\arctan(ax)^{5/2}}{5c}$$

input
```
Int[ArcTan[a*x]^(3/2)/(x^2*(c + a^2*c*x^2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\arctan{(ax)}^{\frac{3}{2}}}{x^2\,(a^2c\,x^2+c)}dx$$

input
```
int(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c),x)
```

output
```
int(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{x^2\,(c+a^2cx^2)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 1.71 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{\arctan(ax)^{3/2}}{x^2\,(c+a^2cx^2)}\,dx = \frac{\int \frac{\operatorname{atan}^{\frac{3}{2}}{(ax)}}{a^2x^4+x^2}\,dx}{c}$$

input
```
integrate(atan(a*x)**(3/2)/x**2/(a**2*c*x**2+c),x)
```

output
```
Integral(atan(a*x)**(3/2)/(a**2*x**4 + x**2), x)/c
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{x^2\,(c+a^2cx^2)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax)^{3/2}}{x^2\,(c+a^2cx^2)}\,dx = \int \frac{\arctan\left(ax\right)^{\frac{3}{2}}}{(a^2cx^2+c)x^2}\,dx$$

input
```
integrate(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^(3/2)/((a^2*c*x^2 + c)*x^2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax)^{3/2}}{x^2 (c+a^2cx^2)} \, dx = \int \frac{\mathrm{atan}(a\,x)^{3/2}}{x^2\,\left(c\,a^2\,x^2+c\right)} \,dx$$

input
```
int(atan(a*x)^(3/2)/(x^2*(c + a^2*c*x^2)),x)
```

output
```
int(atan(a*x)^(3/2)/(x^2*(c + a^2*c*x^2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.42

$$\int \frac{\arctan(ax)^{3/2}}{x^2 (c+a^2cx^2)} \, dx = \frac{-4\sqrt{atan\,(ax)}\,atan(ax)^2\,ax-10\sqrt{atan\,(ax)}\,atan(ax)+15\left(\int \frac{\sqrt{atan(ax)}}{a^2x^3+x}dx\right)ax}{10cx}$$

input
```
int(atan(a*x)^(3/2)/x^2/(a^2*c*x^2+c),x)
```

output
```
( - 4*sqrt(atan(a*x))*atan(a*x)**2*a*x - 10*sqrt(atan(a*x))*atan(a*x) + 15
*int(sqrt(atan(a*x))/(a**2*x**3 + x),x)*a*x)/(10*c*x)
```

# 3.780 $\int \frac{\arctan(ax)^{3/2}}{x^3(c+a^2cx^2)}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\arctan(ax)^{3/2}}{x^3(c+a^2cx^2)}\,dx=\frac{2ia^2\arctan(ax)^{5/2}}{5c}+\frac{\text{Int}\left(\frac{\arctan(ax)^{3/2}}{x^3},x\right)}{c}-\frac{ia^2\text{Int}\left(\frac{\arctan(ax)^{3/2}}{x(i+ax)},x\right)}{c}$$

output
```
2/5*I*a^2*arctan(a*x)^(5/2)/c+Defer(Int)(arctan(a*x)^(3/2)/x^3,x)/c-I*a^2*
Defer(Int)(arctan(a*x)^(3/2)/x/(I+a*x),x)/c
```

## Mathematica [N/A]

Not integrable

Time = 1.35 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\arctan(ax)^{3/2}}{x^3(c+a^2cx^2)}\,dx=\int \frac{\arctan(ax)^{3/2}}{x^3(c+a^2cx^2)}\,dx$$

input
```
Integrate[ArcTan[a*x]^(3/2)/(x^3*(c + a^2*c*x^2)),x]
```

output
```
Integrate[ArcTan[a*x]^(3/2)/(x^3*(c + a^2*c*x^2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{3/2}}{x^3 (a^2cx^2+c)} dx$$

$$\Big\downarrow \text{5453}$$

$$\frac{\int \frac{\arctan(ax)^{3/2}}{x^3}dx}{c} - a^2 \int \frac{\arctan(ax)^{3/2}}{cx (a^2x^2+1)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\arctan(ax)^{3/2}}{x^3}dx}{c} - \frac{a^2 \int \frac{\arctan(ax)^{3/2}}{x(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \text{5377}$$

$$\frac{\int \frac{\arctan(ax)^{3/2}}{x^3}dx}{c} - \frac{a^2 \int \frac{\arctan(ax)^{3/2}}{x(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \text{5459}$$

$$\frac{\int \frac{\arctan(ax)^{3/2}}{x^3}dx}{c} - \frac{a^2 \left(i \int \frac{\arctan(ax)^{3/2}}{x(ax+i)}dx - \frac{2}{5} i \arctan(ax)^{5/2}\right)}{c}$$

$$\Big\downarrow \text{5560}$$

$$\frac{\int \frac{\arctan(ax)^{3/2}}{x^3}dx}{c} - \frac{a^2 \left(i \int \frac{\arctan(ax)^{3/2}}{x(ax+i)}dx - \frac{2}{5} i \arctan(ax)^{5/2}\right)}{c}$$

input
```
Int[ArcTan[a*x]^(3/2)/(x^3*(c + a^2*c*x^2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\arctan (ax)^{\frac{3}{2}}}{x^3 (a^2c\,x^2 + c)}dx$$

input
```
int(arctan(a*x)^(3/2)/x^3/(a^2*c*x^2+c),x)
```

output
```
int(arctan(a*x)^(3/2)/x^3/(a^2*c*x^2+c),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{x^3 (c + a^2cx^2)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(3/2)/x^3/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 2.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{\arctan(ax)^{3/2}}{x^3 (c + a^2cx^2)}\,dx = \frac{\int \frac{\operatorname{atan}^{\frac{3}{2}}{(ax)}}{a^2x^5+x^3}\,dx}{c}$$

input
```
integrate(atan(a*x)**(3/2)/x**3/(a**2*c*x**2+c),x)
```

output
```
Integral(atan(a*x)**(3/2)/(a**2*x**5 + x**3), x)/c
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{x^3\left(c+a^2cx^2\right)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(3/2)/x^3/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax)^{3/2}}{x^3\left(c+a^2cx^2\right)}\,dx = \int \frac{\arctan\left(ax\right)^{\frac{3}{2}}}{(a^2cx^2+c)x^3}\,dx$$

input
```
integrate(arctan(a*x)^(3/2)/x^3/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^(3/2)/((a^2*c*x^2 + c)*x^3), x)
```

## Mupad [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax)^{3/2}}{x^3 (c + a^2cx^2)} dx = \int \frac{\mathrm{atan}(a\,x)^{3/2}}{x^3\,\left(c\,a^2\,x^2 + c\right)} dx$$

input
```
int(atan(a*x)^(3/2)/(x^3*(c + a^2*c*x^2)),x)
```

output
```
int(atan(a*x)^(3/2)/(x^3*(c + a^2*c*x^2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int \frac{\arctan(ax)^{3/2}}{x^3 (c + a^2cx^2)} dx = \frac{\int \frac{\sqrt{atan(ax)}\,atan(ax)}{a^2x^5+x^3}dx}{c}$$

input
```
int(atan(a*x)^(3/2)/x^3/(a^2*c*x^2+c),x)
```

output
```
int((sqrt(atan(a*x))*atan(a*x))/(a**2*x**5 + x**3),x)/c
```

# 3.781 $\int \frac{\arctan(ax)^{3/2}}{x^4(c+a^2cx^2)}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\arctan(ax)^{3/2}}{x^4(c+a^2cx^2)}\,dx = \frac{2a^3\arctan(ax)^{5/2}}{5c} + \frac{\text{Int}\left(\frac{\arctan(ax)^{3/2}}{x^4},x\right)}{c} - \frac{a^2\text{Int}\left(\frac{\arctan(ax)^{3/2}}{x^2},x\right)}{c}$$

output
```
2/5*a^3*arctan(a*x)^(5/2)/c+Defer(Int)(arctan(a*x)^(3/2)/x^4,x)/c-a^2*Defe
r(Int)(arctan(a*x)^(3/2)/x^2,x)/c
```

## Mathematica [N/A]

Not integrable

Time = 3.48 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\arctan(ax)^{3/2}}{x^4(c+a^2cx^2)}\,dx = \int \frac{\arctan(ax)^{3/2}}{x^4(c+a^2cx^2)}\,dx$$

input
```
Integrate[ArcTan[a*x]^(3/2)/(x^4*(c + a^2*c*x^2)),x]
```

output
```
Integrate[ArcTan[a*x]^(3/2)/(x^4*(c + a^2*c*x^2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{3/2}}{x^4(a^2cx^2+c)}dx$$

$$\Big\downarrow \text{5453}$$

$$\frac{\int \frac{\arctan(ax)^{3/2}}{x^4}dx}{c}-a^2\int \frac{\arctan(ax)^{3/2}}{cx^2(a^2x^2+1)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\arctan(ax)^{3/2}}{x^4}dx}{c}-\frac{a^2\int \frac{\arctan(ax)^{3/2}}{x^2(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \text{5377}$$

$$\frac{\int \frac{\arctan(ax)^{3/2}}{x^4}dx}{c}-\frac{a^2\int \frac{\arctan(ax)^{3/2}}{x^2(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \text{5453}$$

$$\frac{\int \frac{\arctan(ax)^{3/2}}{x^4}dx}{c}-\frac{a^2\left(\int \frac{\arctan(ax)^{3/2}}{x^2}dx-a^2\int \frac{\arctan(ax)^{3/2}}{a^2x^2+1}dx\right)}{c}$$

$$\Big\downarrow \text{5377}$$

$$\frac{\int \frac{\arctan(ax)^{3/2}}{x^4}dx}{c}-\frac{a^2\left(\int \frac{\arctan(ax)^{3/2}}{x^2}dx-a^2\int \frac{\arctan(ax)^{3/2}}{a^2x^2+1}dx\right)}{c}$$

$$\Big\downarrow \text{5419}$$

$$\frac{\int \frac{\arctan(ax)^{3/2}}{x^4}dx}{c}-\frac{a^2\left(\int \frac{\arctan(ax)^{3/2}}{x^2}dx-\frac{2}{5}a\arctan(ax)^{5/2}\right)}{c}$$

input
```
Int[ArcTan[a*x]^(3/2)/(x^4*(c + a^2*c*x^2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\arctan(ax)^{\frac{3}{2}}}{x^4 (a^2 c x^2 + c)} dx$$

input
```
int(arctan(a*x)^(3/2)/x^4/(a^2*c*x^2+c),x)
```

output
```
int(arctan(a*x)^(3/2)/x^4/(a^2*c*x^2+c),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{x^4 (c + a^2 c x^2)} dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(3/2)/x^4/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 3.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{\arctan(ax)^{3/2}}{x^4 (c + a^2cx^2)} \, dx = \frac{\int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{a^2x^6+x^4} \, dx}{c}$$

input
```
integrate(atan(a*x)**(3/2)/x**4/(a**2*c*x**2+c),x)
```

output
```
Integral(atan(a*x)**(3/2)/(a**2*x**6 + x**4), x)/c
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{x^4 (c + a^2cx^2)} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(3/2)/x^4/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax)^{3/2}}{x^4 (c + a^2cx^2)} \, dx = \int \frac{\arctan(ax)^{\frac{3}{2}}}{(a^2cx^2 + c)x^4} \, dx$$

input
```
integrate(arctan(a*x)^(3/2)/x^4/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^(3/2)/((a^2*c*x^2 + c)*x^4), x)
```

## Mupad [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax)^{3/2}}{x^4(c+a^2cx^2)}\,dx=\int \frac{\mathrm{atan}(a\,x)^{3/2}}{x^4\,\left(c\,a^2\,x^2+c\right)}\,dx$$

input
```
int(atan(a*x)^(3/2)/(x^4*(c + a^2*c*x^2)),x)
```

output
```
int(atan(a*x)^(3/2)/(x^4*(c + a^2*c*x^2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 130, normalized size of antiderivative = 5.42

$$\int \frac{\arctan(ax)^{3/2}}{x^4(c+a^2cx^2)}\,dx=\frac{48\sqrt{atan\,(ax)}\,atan(ax)^2\,a^3x^3+120\sqrt{atan\,(ax)}\,atan(ax)\,a^2x^2-40\sqrt{atan\,(ax)}\,ata}{}$$

input
```
int(atan(a*x)^(3/2)/x^4/(a^2*c*x^2+c),x)
```

output
```
(48*sqrt(atan(a*x))*atan(a*x)**2*a**3*x**3 + 120*sqrt(atan(a*x))*atan(a*x)
*a**2*x**2 - 40*sqrt(atan(a*x))*atan(a*x) + 90*sqrt(atan(a*x))*a*x - 45*in
t(sqrt(atan(a*x))/(atan(a*x)*a**2*x**4 + atan(a*x)*x**2),x)*a**2*x**3 + 24
0*int(sqrt(atan(a*x))/(a**2*x**5 + x**3),x)*a*x**3)/(120*c*x**3)
```

# 3.782 $\int \frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^2} dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^2} dx = \text{Int}\left(\frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^2}, x\right)$$

output
```
Defer(Int)(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x)
```

## Mathematica [N/A]

Not integrable

Time = 1.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^2} dx = \int \frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^2} dx$$

input
```
Integrate[(x^m*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^2,x]
```

output
```
Integrate[(x^m*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^2, x]
```

## Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \arctan(ax)^{3/2}}{(a^2cx^2+c)^2} \, dx$$

$$\Big\downarrow \text{5560}$$

$$\int \frac{x^m \arctan(ax)^{3/2}}{(a^2cx^2+c)^2}dx$$

input
```
Int[(x^m*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^2,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m \arctan(ax)^{\frac{3}{2}}}{(a^2c\,x^2+c)^2}dx$$

input
```
int(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x)
```

output
```
int(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.62

$$\int \frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^2} \, dx = \int \frac{x^m \arctan{(ax)}^{\frac{3}{2}}}{(a^2cx^2+c)^2} \, dx$$

input
```
integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
integral(x^m*arctan(a*x)^(3/2)/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2), x)
```

## Sympy [N/A]

Not integrable

Time = 81.62 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.33

$$\int \frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^2} \, dx = \frac{\int \frac{x^m \operatorname{atan}^{\frac{3}{2}}(ax)}{a^4x^4+2a^2x^2+1} \, dx}{c^2}$$

input
```
integrate(x**m*atan(a*x)**(3/2)/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(x**m*atan(a*x)**(3/2)/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^2} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,2,1,0,0]%%%} / %%%{1,[0,0,0,1,2]%%%} Error: Bad Argum
ent Value
```

## Mupad [N/A]

Not integrable

Time = 0.90 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^2} \, dx = \int \frac{x^m \operatorname{atan}(a\,x)^{3/2}}{{\left(c\,a^2\,x^2+c\right)}^2} \, dx$$

input
```
int((x^m*atan(a*x)^(3/2))/(c + a^2*c*x^2)^2,x)
```

output
```
int((x^m*atan(a*x)^(3/2))/(c + a^2*c*x^2)^2, x)
```

## Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.58

$$\int \frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^2}\,dx = \frac{\int \frac{x^m \sqrt{atan(ax)}\,atan(ax)}{a^4x^4+2a^2x^2+1}dx}{c^2}$$

input
```
int(x^m*atan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x)
```

output
```
int((x**m*sqrt(atan(a*x))*atan(a*x))/(a**4*x**4 + 2*a**2*x**2 + 1),x)/c**2
```

# 3.783 $\int \frac{x^3 \arctan(ax)^{3/2}}{(c+a^2cx^2)^2}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^3 \arctan(ax)^{3/2}}{(c+a^2cx^2)^2}\,dx = \text{Int}\left(\frac{x^3 \arctan(ax)^{3/2}}{(c+a^2cx^2)^2},x\right)$$

output
```
Defer(Int)(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x)
```

## Mathematica [N/A]

Not integrable

Time = 3.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^3 \arctan(ax)^{3/2}}{(c+a^2cx^2)^2}\,dx = \int \frac{x^3 \arctan(ax)^{3/2}}{(c+a^2cx^2)^2}\,dx$$

input
```
Integrate[(x^3*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^2,x]
```

output
```
Integrate[(x^3*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^2, x]
```

## Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \arctan(ax)^{3/2}}{(a^2cx^2+c)^2} dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int \frac{x^3 \arctan(ax)^{3/2}}{(a^2cx^2+c)^2} dx$$

input
```
Int[(x^3*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^2,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^3 \arctan(ax)^{\frac{3}{2}}}{(a^2c\,x^2+c)^2} dx$$

input
```
int(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x)
```

output
```
int(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3 \arctan(ax)^{3/2}}{(c+a^2cx^2)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 3.23 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.33

$$\int \frac{x^3 \arctan(ax)^{3/2}}{(c+a^2cx^2)^2}\,dx = \frac{\int \frac{x^3 \operatorname{atan}^{\frac{3}{2}}(ax)}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

input
```
integrate(x**3*atan(a*x)**(3/2)/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(x**3*atan(a*x)**(3/2)/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3 \arctan(ax)^{3/2}}{(c+a^2cx^2)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 1.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^3 \arctan(ax)^{3/2}}{(c+a^2cx^2)^2} \, dx = \int \frac{x^3 \arctan{(ax)}^{\frac{3}{2}}}{(a^2cx^2+c)^2} \, dx$$

input
```
integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2 + c)^2, x)
```

## Mupad [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^3 \arctan(ax)^{3/2}}{(c+a^2cx^2)^2} \, dx = \int \frac{x^3 \operatorname{atan}(a\,x)^{3/2}}{(c\,a^2\,x^2+c)^2} \, dx$$

input
```
int((x^3*atan(a*x)^(3/2))/(c + a^2*c*x^2)^2,x)
```

output
```
int((x^3*atan(a*x)^(3/2))/(c + a^2*c*x^2)^2, x)
```

## Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.58

$$\int \frac{x^3 \arctan(ax)^{3/2}}{(c + a^2cx^2)^2} \, dx = \frac{\int \frac{\sqrt{atan(ax)}\, atan(ax)x^3}{a^4x^4+2a^2x^2+1} dx}{c^2}$$

input
```
int(x^3*atan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x)
```

output
```
int((sqrt(atan(a*x))*atan(a*x)*x**3)/(a**4*x**4 + 2*a**2*x**2 + 1),x)/c**2
```

# 3.784 $\int \frac{x^2 \arctan(ax)^{3/2}}{(c+a^2cx^2)^2} dx$



## Optimal result

Integrand size = 24, antiderivative size = 127

$$\int \frac{x^2 \arctan(ax)^{3/2}}{(c+a^2cx^2)^2} dx = \frac{3\sqrt{\arctan(ax)}}{16a^3c^2} - \frac{3\sqrt{\arctan(ax)}}{8a^3c^2(1+a^2x^2)} - \frac{x\arctan(ax)^{3/2}}{2a^2c^2(1+a^2x^2)} + \frac{\arctan(ax)^{5/2}}{5a^3c^2} + \frac{3\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{32a^3c^2}$$

output
```
3/16*arctan(a*x)^(1/2)/a^3/c^2-3/8*arctan(a*x)^(1/2)/a^3/c^2/(a^2*x^2+1)-1
/2*x*arctan(a*x)^(3/2)/a^2/c^2/(a^2*x^2+1)+1/5*arctan(a*x)^(5/2)/a^3/c^2+3
/32*Pi^(1/2)*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))/a^3/c^2
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.41 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.47

$$\int \frac{x^2 \arctan(ax)^{3/2}}{(c+a^2cx^2)^2} dx = \frac{\frac{16\sqrt{\arctan(ax)}(15(-1+a^2x^2)-40ax\arctan(ax)+16(1+a^2x^2)\arctan(ax)^2)}{1+a^2x^2} + 60\left(-2\sqrt{\arctan(ax)} \right.}{}$$

input
```
Integrate[(x^2*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^2,x]
```

output
```
((16*Sqrt[ArcTan[a*x]]*(15*(-1 + a^2*x^2) - 40*a*x*ArcTan[a*x] + 16*(1 + a
^2*x^2)*ArcTan[a*x]^2))/(1 + a^2*x^2) + 60*(-2*Sqrt[ArcTan[a*x]] + Sqrt[Pi
]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]]) + (15*(8*ArcTan[a*x] - I*Sqrt[
2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-2*I)*ArcTan[a*x]] + I*Sqrt[2]*Sqrt[
I*ArcTan[a*x]]*Gamma[1/2, (2*I)*ArcTan[a*x]]))/Sqrt[ArcTan[a*x]])/(1280*a^
3*c^2)
```

## Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {5471, 27, 5465, 5439, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \arctan(ax)^{3/2}}{(a^2cx^2+c)^2}dx$$

$$\Big\downarrow \text{ 5471}$$

$$\frac{3\int \frac{x\sqrt{\arctan(ax)}}{c^2(a^2x^2+1)^2}dx}{4a}+\frac{\arctan(ax)^{5/2}}{5a^3c^2}-\frac{x\arctan(ax)^{3/2}}{2a^2c^2(a^2x^2+1)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{3\int \frac{x\sqrt{\arctan(ax)}}{(a^2x^2+1)^2}dx}{4ac^2}+\frac{\arctan(ax)^{5/2}}{5a^3c^2}-\frac{x\arctan(ax)^{3/2}}{2a^2c^2(a^2x^2+1)}$$

$$\Big\downarrow \text{ 5465}$$

$$\frac{3\left(\frac{\int \frac{1}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx}{4a}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)}{4ac^2}+\frac{\arctan(ax)^{5/2}}{5a^3c^2}-\frac{x\arctan(ax)^{3/2}}{2a^2c^2(a^2x^2+1)}$$

$$\Big\downarrow \text{ 5439}$$

$$\frac{3\left(\frac{\int \frac{1}{\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}d\arctan(ax)}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)}{4ac^2}+\frac{\arctan(ax)^{5/2}}{5a^3c^2}-\frac{x\arctan(ax)^{3/2}}{2a^2c^2\left(a^2x^2+1\right)}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{3\left(\frac{\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^2}{\sqrt{\arctan(ax)}}d\arctan(ax)}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)}{4ac^2}+\frac{\arctan(ax)^{5/2}}{5a^3c^2}-\frac{x\arctan(ax)^{3/2}}{2a^2c^2\left(a^2x^2+1\right)}$$

$$\Big\downarrow \textbf{3793}$$

$$\frac{3\left(\frac{\int \left(\frac{\cos(2\arctan(ax))}{2\sqrt{\arctan(ax)}}+\frac{1}{2\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)}{4ac^2}+\frac{\arctan(ax)^{5/2}}{5a^3c^2}-\frac{x\arctan(ax)^{3/2}}{2a^2c^2\left(a^2x^2+1\right)}$$

$$\Big\downarrow \textbf{2009}$$

$$\frac{\arctan(ax)^{5/2}}{5a^3c^2}+\frac{3\left(\frac{\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\sqrt{\arctan(ax)}}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)}{4ac^2}-\frac{x\arctan(ax)^{3/2}}{2a^2c^2\left(a^2x^2+1\right)}$$

input
```
Int[(x^2*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^2,x]
```

output
```
-1/2*(x*ArcTan[a*x]^(3/2))/(a^2*c^2*(1 + a^2*x^2)) + ArcTan[a*x]^(5/2)/(5*
a^3*c^2) + (3*(-1/2*Sqrt[ArcTan[a*x]]/(a^2*(1 + a^2*x^2)) + (Sqrt[ArcTan[a
*x]] + (Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/2)/(4*a^2)))/(4
*a*c^2)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5471
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^2)/((d_) + (e_.)*(x_)^2)
^2, x_Symbol] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(2*b*c^3*d^2*(p + 1)), x]
 + (-Simp[x*((a + b*ArcTan[c*x])^p/(2*c^2*d*(d + e*x^2))), x] + Simp[b*(p/(
2*c))   Int[x*((a + b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

## Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.59

| method | result |
|---|---|
| default | $\frac{32\arctan(ax)^3-40\arctan(ax)^2\sin(2\arctan(ax))+15\sqrt{\arctan(ax)}\,\sqrt{\pi}\,\mathrm{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)-30\arctan(ax)\cos(2\arctan(ax))}{160a^3c^2\sqrt{\arctan(ax)}}$ |

input
```
int(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/160/a^3/c^2*(32*arctan(a*x)^3-40*arctan(a*x)^2*sin(2*arctan(a*x))+15*arc
tan(a*x)^(1/2)*Pi^(1/2)*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))-30*arctan(a
*x)*cos(2*arctan(a*x)))/arctan(a*x)^(1/2)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2 \arctan(ax)^{3/2}}{(c+a^2cx^2)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{x^2 \arctan(ax)^{3/2}}{(c+a^2cx^2)^2}\,dx = \frac{\int \frac{x^2 \operatorname{atan}^{\frac{3}{2}}(ax)}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

input
```
integrate(x**2*atan(a*x)**(3/2)/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(x**2*atan(a*x)**(3/2)/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2 \arctan(ax)^{3/2}}{(c+a^2cx^2)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F]

$$\int \frac{x^2 \arctan(ax)^{3/2}}{(c+a^2cx^2)^2}\,dx = \int \frac{x^2 \arctan(ax)^{\frac{3}{2}}}{(a^2cx^2+c)^2}\,dx$$

input
```
integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2 + c)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \arctan(ax)^{3/2}}{(c+a^2cx^2)^2}\,dx = \int \frac{x^2 \operatorname{atan}(a\,x)^{3/2}}{(c\,a^2\,x^2+c)^2}\,dx$$

input
```
int((x^2*atan(a*x)^(3/2))/(c + a^2*c*x^2)^2,x)
```

output
```
int((x^2*atan(a*x)^(3/2))/(c + a^2*c*x^2)^2, x)
```

## Reduce [F]

$$\int \frac{x^2 \arctan(ax)^{3/2}}{(c+a^2cx^2)^2}\,dx = \frac{16\sqrt{atan\,(ax)}\,atan(ax)^2\,a^2x^2 + 16\sqrt{atan\,(ax)}\,atan(ax)^2 - 40\sqrt{atan\,(ax)}\,atan(}{}$$

input

```
int(x^2*atan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x)
```

output

```
(16*sqrt(atan(a*x))*atan(a*x)**2*a**2*x**2 + 16*sqrt(atan(a*x))*atan(a*x)*
*2 - 40*sqrt(atan(a*x))*atan(a*x)*a*x + 30*sqrt(atan(a*x))*a**2*x**2 - 15*
int((sqrt(atan(a*x))*x**2)/(atan(a*x)*a**4*x**4 + 2*atan(a*x)*a**2*x**2 +
atan(a*x)),x)*a**5*x**2 - 15*int((sqrt(atan(a*x))*x**2)/(atan(a*x)*a**4*x*
*4 + 2*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**3)/(80*a**3*c**2*(a**2*x**2
+ 1))
```

# 3.785 $\int \frac{x \arctan(ax)^{3/2}}{\left(c+a^2cx^2\right)^2} dx$



## Optimal result

Integrand size = 22, antiderivative size = 109

$$\int \frac{x \arctan(ax)^{3/2}}{\left(c+a^2cx^2\right)^2} dx = \frac{3x\sqrt{\arctan(ax)}}{8ac^2(1+a^2x^2)} + \frac{\arctan(ax)^{3/2}}{4a^2c^2} - \frac{\arctan(ax)^{3/2}}{2a^2c^2(1+a^2x^2)} - \frac{3\sqrt{\pi}\,\mathrm{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{32a^2c^2}$$

output
```
3/8*x*arctan(a*x)^(1/2)/a/c^2/(a^2*x^2+1)+1/4*arctan(a*x)^(3/2)/a^2/c^2-1/
2*arctan(a*x)^(3/2)/a^2/c^2/(a^2*x^2+1)-3/32*Pi^(1/2)*FresnelS(2*arctan(a*
x)^(1/2)/Pi^(1/2))/a^2/c^2
```

## Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.69

$$\int \frac{x \arctan(ax)^{3/2}}{\left(c+a^2cx^2\right)^2} dx = \frac{\frac{4\sqrt{\arctan(ax)}(3ax+2(-1+a^2x^2)\arctan(ax))}{1+a^2x^2} - 3\sqrt{\pi}\,\mathrm{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{32a^2c^2}$$

input
```
Integrate[(x*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^2,x]
```

output
```
((4*Sqrt[ArcTan[a*x]]*(3*a*x + 2*(-1 + a^2*x^2)*ArcTan[a*x]))/(1 + a^2*x^2
) - 3*Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(32*a^2*c^2)
```

## Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {5465, 27, 5427, 5505, 4906, 27, 3042, 3786, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\arctan(ax)^{3/2}}{(a^2cx^2+c)^2}dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{3\int \frac{\sqrt{\arctan(ax)}}{c^2(a^2x^2+1)^2}dx}{4a}-\frac{\arctan(ax)^{3/2}}{2a^2c^2\,(a^2x^2+1)}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\int \frac{\sqrt{\arctan(ax)}}{(a^2x^2+1)^2}dx}{4ac^2}-\frac{\arctan(ax)^{3/2}}{2a^2c^2\,(a^2x^2+1)}$$

$$\Big\downarrow \text{5427}$$

$$\frac{3\left(-\frac{1}{4}a\int \frac{x}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx+\frac{x\sqrt{\arctan(ax)}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4ac^2}-\frac{\arctan(ax)^{3/2}}{2a^2c^2\,(a^2x^2+1)}$$

$$\Big\downarrow \text{5505}$$

$$\frac{3\left(-\frac{\int \frac{ax}{\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}d\arctan(ax)}{4a}+\frac{x\sqrt{\arctan(ax)}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4ac^2}-\frac{\arctan(ax)^{3/2}}{2a^2c^2\,(a^2x^2+1)}$$

$$\Big\downarrow \text{4906}$$

$$\frac{3\left(-\frac{\int \frac{\sin(2\arctan(ax))}{2\sqrt{\arctan(ax)}}d\arctan(ax)}{4a}+\frac{x\sqrt{\arctan(ax)}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4ac^2}-\frac{\arctan(ax)^{3/2}}{2a^2c^2\,(a^2x^2+1)}$$

$$\Big\downarrow \ 27$$

$$\frac{3\left(-\frac{\int \frac{\sin(2\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)}{8a}+\frac{x\sqrt{\arctan(ax)}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4ac^2}-\frac{\arctan(ax)^{3/2}}{2a^2c^2\left(a^2x^2+1\right)}$$

$$\Big\downarrow \ 3042$$

$$\frac{3\left(-\frac{\int \frac{\sin(2\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)}{8a}+\frac{x\sqrt{\arctan(ax)}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4ac^2}-\frac{\arctan(ax)^{3/2}}{2a^2c^2\left(a^2x^2+1\right)}$$

$$\Big\downarrow \ 3786$$

$$\frac{3\left(-\frac{\int \sin(2\arctan(ax))d\sqrt{\arctan(ax)}}{4a}+\frac{x\sqrt{\arctan(ax)}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4ac^2}-\frac{\arctan(ax)^{3/2}}{2a^2c^2\left(a^2x^2+1\right)}$$

$$\Big\downarrow \ 3832$$

$$\frac{3\left(\frac{x\sqrt{\arctan(ax)}}{2(a^2x^2+1)}-\frac{\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{8a}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4ac^2}-\frac{\arctan(ax)^{3/2}}{2a^2c^2\left(a^2x^2+1\right)}$$

input
```
Int[(x*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^2,x]
```

output
```
-1/2*ArcTan[a*x]^(3/2)/(a^2*c^2*(1 + a^2*x^2)) + (3*((x*Sqrt[ArcTan[a*x]])
/(2*(1 + a^2*x^2)) + ArcTan[a*x]^(3/2)/(3*a) - (Sqrt[Pi]*FresnelS[(2*Sqrt[
ArcTan[a*x]])/Sqrt[Pi]])/(8*a)))/(4*a*c^2)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

## Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.61

| method | result | size |
|---|---|---|
| default | $-\frac{8\arctan(ax)^{2}\cos(2\arctan(ax))+3\sqrt{\arctan(ax)}\,\sqrt{\pi}\,\mathrm{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)-6\arctan(ax)\sin(2\arctan(ax))}{32a^{2}c^{2}\sqrt{\arctan(ax)}}$ | 67 |

input
```
int(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/32/a^2/c^2*(8*arctan(a*x)^2*cos(2*arctan(a*x))+3*arctan(a*x)^(1/2)*Pi^(
1/2)*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2))-6*arctan(a*x)*sin(2*arctan(a*x
)))/arctan(a*x)^(1/2)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x\arctan(ax)^{3/2}}{(c+a^2cx^2)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{x\arctan(ax)^{3/2}}{(c+a^2cx^2)^2}\,dx = \frac{\int \frac{x\,\mathrm{atan}^{\frac{3}{2}}(ax)}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

input
```
integrate(x*atan(a*x)**(3/2)/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(x*atan(a*x)**(3/2)/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x \arctan(ax)^{3/2}}{(c + a^2cx^2)^2} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F]

$$\int \frac{x \arctan(ax)^{3/2}}{(c + a^2cx^2)^2} \, dx = \int \frac{x \arctan(ax)^{\frac{3}{2}}}{(a^2cx^2 + c)^2} \, dx$$

input
```
integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2 + c)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x \arctan(ax)^{3/2}}{(c + a^2cx^2)^2} \, dx = \int \frac{x \operatorname{atan}(a\,x)^{3/2}}{(c\,a^2\,x^2 + c)^2} \, dx$$

input
```
int((x*atan(a*x)^(3/2))/(c + a^2*c*x^2)^2,x)
```

output
```
int((x*atan(a*x)^(3/2))/(c + a^2*c*x^2)^2, x)
```

## Reduce [F]

$$\int \frac{x \arctan(ax)^{3/2}}{\left(c+a^2cx^2\right)^2}\,dx = \frac{-2\sqrt{atan\left(ax\right)}\,atan(ax)+3\left(\int \frac{\sqrt{atan(ax)}}{a^4x^4+2a^2x^2+1}dx\right)a^3x^2+3\left(\int \frac{\sqrt{atan(ax)}}{a^4x^4+2a^2x^2+1}dx\right)a}{4a^2c^2\left(a^2x^2+1\right)}$$

input
```
int(x*atan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x)
```

output
```
( - 2*sqrt(atan(a*x))*atan(a*x) + 3*int(sqrt(atan(a*x))/(a**4*x**4 + 2*a**
2*x**2 + 1),x)*a**3*x**2 + 3*int(sqrt(atan(a*x))/(a**4*x**4 + 2*a**2*x**2
+ 1),x)*a)/(4*a**2*c**2*(a**2*x**2 + 1))
```

# 3.786 $\int \frac{\arctan(ax)^{3/2}}{(c+a^2cx^2)^2}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 124

$$\int \frac{\arctan(ax)^{3/2}}{(c+a^2cx^2)^2}\,dx = -\frac{3\sqrt{\arctan(ax)}}{16ac^2}+\frac{3\sqrt{\arctan(ax)}}{8ac^2(1+a^2x^2)} + \frac{x\arctan(ax)^{3/2}}{2c^2(1+a^2x^2)}+\frac{\arctan(ax)^{5/2}}{5ac^2}-\frac{3\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{32ac^2}$$

output
```
-3/16*arctan(a*x)^(1/2)/a/c^2+3/8*arctan(a*x)^(1/2)/a/c^2/(a^2*x^2+1)+1/2*
x*arctan(a*x)^(3/2)/c^2/(a^2*x^2+1)+1/5*arctan(a*x)^(5/2)/a/c^2-3/32*Pi^(1
/2)*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))/a/c^2
```

## Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.73

$$\int \frac{\arctan(ax)^{3/2}}{(c+a^2cx^2)^2}\,dx = \frac{\frac{2\sqrt{\arctan(ax)}(15-15a^2x^2+40ax\arctan(ax)+16(1+a^2x^2)\arctan(ax)^2)}{1+a^2x^2}-15\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{160ac^2}$$

input
```
Integrate[ArcTan[a*x]^(3/2)/(c + a^2*c*x^2)^2,x]
```

output
```
((2*Sqrt[ArcTan[a*x]]*(15 - 15*a^2*x^2 + 40*a*x*ArcTan[a*x] + 16*(1 + a^2*
x^2)*ArcTan[a*x]^2))/(1 + a^2*x^2) - 15*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a
*x]])/Sqrt[Pi]])/(160*a*c^2)
```

## Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5427, 27, 5465, 5439, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{3/2}}{(a^2cx^2+c)^2}dx$$

$$\Big\downarrow \text{5427}$$

$$-\frac{3}{4}a\int \frac{x\sqrt{\arctan(ax)}}{c^2(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^{3/2}}{2c^2(a^2x^2+1)}+\frac{\arctan(ax)^{5/2}}{5ac^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3a\int \frac{x\sqrt{\arctan(ax)}}{(a^2x^2+1)^2}dx}{4c^2}+\frac{x\arctan(ax)^{3/2}}{2c^2(a^2x^2+1)}+\frac{\arctan(ax)^{5/2}}{5ac^2}$$

$$\Big\downarrow \text{5465}$$

$$-\frac{3a\left(\frac{\int \frac{1}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx}{4a}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)}{4c^2}+\frac{x\arctan(ax)^{3/2}}{2c^2(a^2x^2+1)}+\frac{\arctan(ax)^{5/2}}{5ac^2}$$

$$\Big\downarrow \text{5439}$$

$$-\frac{3a\left(\frac{\int \frac{1}{(a^2x^2+1)\sqrt{\arctan(ax)}}d\arctan(ax)}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)}{4c^2}+\frac{x\arctan(ax)^{3/2}}{2c^2(a^2x^2+1)}+\frac{\arctan(ax)^{5/2}}{5ac^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3a\left(\frac{\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^2}{\sqrt{\arctan(ax)}}d\arctan(ax)}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)}{4c^2}+\frac{x\arctan(ax)^{3/2}}{2c^2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^{5/2}}{5ac^2}$$

$$\Big\downarrow \textbf{3793}$$

$$-\frac{3a\left(\frac{\int \left(\frac{\cos(2\arctan(ax))}{2\sqrt{\arctan(ax)}}+\frac{1}{2\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)}{4c^2}+\frac{x\arctan(ax)^{3/2}}{2c^2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^{5/2}}{5ac^2}$$

$$\Big\downarrow \textbf{2009}$$

$$-\frac{3a\left(\frac{\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\sqrt{\arctan(ax)}}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)}{4c^2}+\frac{x\arctan(ax)^{3/2}}{2c^2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^{5/2}}{5ac^2}$$

input
```
Int[ArcTan[a*x]^(3/2)/(c + a^2*c*x^2)^2,x]
```

output
```
(x*ArcTan[a*x]^(3/2))/(2*c^2*(1 + a^2*x^2)) + ArcTan[a*x]^(5/2)/(5*a*c^2)
- (3*a*(-1/2*Sqrt[ArcTan[a*x]]/(a^2*(1 + a^2*x^2)) + (Sqrt[ArcTan[a*x]] +
(Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/2)/(4*a^2)))/(4*c^2)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

## Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.60

| method | result |
|---|---|
| default | $\frac{32 \arctan(ax)^3+40 \arctan(ax)^2 \sin(2 \arctan(ax))+30 \arctan(ax) \cos(2 \arctan(ax))-15 \sqrt{\arctan(ax)}\, \sqrt{\pi}\, \operatorname{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{160 a\, c^2 \sqrt{\arctan(ax)}}$ |

input
```
int(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/160/a/c^2/arctan(a*x)^(1/2)*(32*arctan(a*x)^3+40*arctan(a*x)^2*sin(2*arc
tan(a*x))+30*arctan(a*x)*cos(2*arctan(a*x))-15*arctan(a*x)^(1/2)*Pi^(1/2)*
FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2)))
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{(c+a^2cx^2)^2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{\arctan(ax)^{3/2}}{(c+a^2cx^2)^2} \, dx = \frac{\int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{a^4x^4+2a^2x^2+1} \, dx}{c^2}$$

input
```
integrate(atan(a*x)**(3/2)/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(atan(a*x)**(3/2)/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{(c+a^2cx^2)^2} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F]

$$\int \frac{\arctan(ax)^{3/2}}{(c+a^2cx^2)^2}\,dx = \int \frac{\arctan\left(ax\right)^{\frac{3}{2}}}{\left(a^2cx^2+c\right)^2}\,dx$$

input
```
integrate(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^(3/2)/(a^2*c*x^2 + c)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^{3/2}}{(c+a^2cx^2)^2}\,dx = \int \frac{\mathrm{atan}(a\,x)^{3/2}}{\left(c\,a^2\,x^2+c\right)^2}\,dx$$

input
```
int(atan(a*x)^(3/2)/(c + a^2*c*x^2)^2,x)
```

output
```
int(atan(a*x)^(3/2)/(c + a^2*c*x^2)^2, x)
```

## Reduce [F]

$$\int \frac{\arctan(ax)^{3/2}}{(c+a^2cx^2)^2}\,dx = \frac{16\sqrt{atan\left(ax\right)}\,atan(ax)^2\,a^2x^2+16\sqrt{atan\left(ax\right)}\,atan(ax)^2+40\sqrt{atan\left(ax\right)}\,atan(ax)}{}$$

input
```
int(atan(a*x)^(3/2)/(a^2*c*x^2+c)^2,x)
```

output
```
(16*sqrt(atan(a*x))*atan(a*x)**2*a**2*x**2 + 16*sqrt(atan(a*x))*atan(a*x)*
*2 + 40*sqrt(atan(a*x))*atan(a*x)*a*x - 30*sqrt(atan(a*x))*a**2*x**2 + 15*
int((sqrt(atan(a*x))*x**2)/(atan(a*x)*a**4*x**4 + 2*atan(a*x)*a**2*x**2 +
atan(a*x)),x)*a**5*x**2 + 15*int((sqrt(atan(a*x))*x**2)/(atan(a*x)*a**4*x*
*4 + 2*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**3)/(80*a*c**2*(a**2*x**2 + 1
))
```

# 3.787 $\int \frac{\arctan(ax)^{3/2}}{x(c+a^2cx^2)^2}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\arctan(ax)^{3/2}}{x\left(c+a^2cx^2\right)^2}\,dx=\text{Int}\left(\frac{\arctan(ax)^{3/2}}{x\left(c+a^2cx^2\right)^2},x\right)$$

output
```
Defer(Int)(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^2,x)
```

## Mathematica [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\arctan(ax)^{3/2}}{x\left(c+a^2cx^2\right)^2}\,dx=\int \frac{\arctan(ax)^{3/2}}{x\left(c+a^2cx^2\right)^2}\,dx$$

input
```
Integrate[ArcTan[a*x]^(3/2)/(x*(c + a^2*c*x^2)^2),x]
```

output
```
Integrate[ArcTan[a*x]^(3/2)/(x*(c + a^2*c*x^2)^2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{3/2}}{x\left(a^2cx^2+c\right)^2}dx$$

$$\Big\downarrow \text{5560}$$

$$\int \frac{\arctan(ax)^{3/2}}{x\left(a^2cx^2+c\right)^2}dx$$

input
```
Int[ArcTan[a*x]^(3/2)/(x*(c + a^2*c*x^2)^2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\arctan\left(ax\right)^{\frac{3}{2}}}{x\left(a^2c\,x^2+c\right)^2}dx$$

input
```
int(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^2,x)
```

output
```
int(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^2,x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{x\left(c+a^2cx^2\right)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 2.14 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int \frac{\arctan(ax)^{3/2}}{x\left(c+a^2cx^2\right)^2}\,dx = \frac{\int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{a^4x^5+2a^2x^3+x}\,dx}{c^2}$$

input
```
integrate(atan(a*x)**(3/2)/x/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(atan(a*x)**(3/2)/(a**4*x**5 + 2*a**2*x**3 + x), x)/c**2
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{x\left(c+a^2cx^2\right)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax)^{3/2}}{x\left(c+a^2cx^2\right)^2}\,dx=\int \frac{\arctan\left(ax\right)^{\frac{3}{2}}}{\left(a^2cx^2+c\right)^2x}\,dx$$

input
```
integrate(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^(3/2)/((a^2*c*x^2 + c)^2*x), x)
```

## Mupad [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax)^{3/2}}{x\left(c+a^2cx^2\right)^2}\,dx=\int \frac{\mathrm{atan}(a\,x)^{3/2}}{x\,\left(c\,a^2\,x^2+c\right)^2}\,dx$$

input
```
int(atan(a*x)^(3/2)/(x*(c + a^2*c*x^2)^2),x)
```

output
```
int(atan(a*x)^(3/2)/(x*(c + a^2*c*x^2)^2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int \frac{\arctan(ax)^{3/2}}{x\left(c+a^2cx^2\right)^2}\,dx = \frac{\int \frac{\sqrt{atan(ax)}\,atan(ax)}{a^4x^5+2a^2x^3+x}dx}{c^2}$$

input
```
int(atan(a*x)^(3/2)/x/(a^2*c*x^2+c)^2,x)
```

output
```
int((sqrt(atan(a*x))*atan(a*x))/(a**4*x**5 + 2*a**2*x**3 + x),x)/c**2
```

# 3.788 $\int \frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^3} dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^3} dx = \text{Int}\left(\frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^3}, x\right)$$

output
```
Defer(Int)(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x)
```

## Mathematica [N/A]

Not integrable

Time = 1.32 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^3} dx = \int \frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^3} dx$$

input
```
Integrate[(x^m*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^3,x]
```

output
```
Integrate[(x^m*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^3, x]
```

## Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \arctan(ax)^{3/2}}{(a^2cx^2+c)^3} dx$$
$$\Big\downarrow \ 5560$$
$$\int \frac{x^m \arctan(ax)^{3/2}}{(a^2cx^2+c)^3}dx$$

input
```
Int[(x^m*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^3,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m \arctan(ax)^{\frac{3}{2}}}{(a^2c\,x^2+c)^3}dx$$

input
```
int(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x)
```

output
```
int(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.08

$$\int \frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx = \int \frac{x^m \arctan(ax)^{\frac{3}{2}}}{(a^2cx^2+c)^3}\,dx$$

input
```
integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
integral(x^m*arctan(a*x)^(3/2)/(a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^
2 + c^3), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**m*atan(a*x)**(3/2)/(a**2*c*x**2+c)**3,x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,2,1,0,0]%%%} / %%%{1,[0,0,0,1,3]%%%} Error: Bad Argum
ent Value
```

## Mupad [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx = \int \frac{x^m\,\mathrm{atan}(a\,x)^{3/2}}{{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

input
```
int((x^m*atan(a*x)^(3/2))/(c + a^2*c*x^2)^3,x)
```

output
```
int((x^m*atan(a*x)^(3/2))/(c + a^2*c*x^2)^3, x)
```

## Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.92

$$\int \frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx = \frac{\int \frac{x^m \sqrt{atan(ax)}\,atan(ax)}{a^6x^6+3a^4x^4+3a^2x^2+1}dx}{c^3}$$

input
```
int(x^m*atan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x)
```

output
```
int((x**m*sqrt(atan(a*x))*atan(a*x))/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**
2 + 1),x)/c**3
```

# 3.789 $\int \frac{x^5 \arctan(ax)^{3/2}}{(c+a^2cx^2)^3} dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^5 \arctan(ax)^{3/2}}{(c+a^2cx^2)^3} dx = \text{Int}\left(\frac{x^5 \arctan(ax)^{3/2}}{(c+a^2cx^2)^3}, x\right)$$

output
```
Defer(Int)(x^5*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x)
```

## Mathematica [N/A]

Not integrable

Time = 4.71 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^5 \arctan(ax)^{3/2}}{(c+a^2cx^2)^3} dx = \int \frac{x^5 \arctan(ax)^{3/2}}{(c+a^2cx^2)^3} dx$$

input
```
Integrate[(x^5*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^3,x]
```

output
```
Integrate[(x^5*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^3, x]
```

## Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 \arctan(ax)^{3/2}}{(a^2cx^2+c)^3} dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int \frac{x^5 \arctan(ax)^{3/2}}{(a^2cx^2+c)^3} dx$$

input
```
Int[(x^5*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^3,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^5 \arctan(ax)^{\frac{3}{2}}}{(a^2c\,x^2+c)^3} dx$$

input
```
int(x^5*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x)
```

output
```
int(x^5*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x^5 \arctan(ax)^{3/2}}{(c+a^2cx^2)^3} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^5*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 8.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.71

$$\int \frac{x^5 \arctan(ax)^{3/2}}{(c+a^2cx^2)^3} \, dx = \frac{\int \frac{x^5 \operatorname{atan}^{\frac{3}{2}}(ax)}{a^6x^6+3a^4x^4+3a^2x^2+1} \, dx}{c^3}$$

input
```
integrate(x**5*atan(a*x)**(3/2)/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(x**5*atan(a*x)**(3/2)/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1)
, x)/c**3
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5 \arctan(ax)^{3/2}}{(c+a^2cx^2)^3} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^5*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 2.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^5 \arctan(ax)^{3/2}}{(c+a^2cx^2)^3} \, dx = \int \frac{x^5 \arctan{(ax)}^{\frac{3}{2}}}{(a^2cx^2+c)^3} \, dx$$

input
```
integrate(x^5*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
integrate(x^5*arctan(a*x)^(3/2)/(a^2*c*x^2 + c)^3, x)
```

## Mupad [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^5 \arctan(ax)^{3/2}}{(c+a^2cx^2)^3} \, dx = \int \frac{x^5 \,\mathrm{atan}(a\,x)^{3/2}}{(c\,a^2\,x^2+c)^3} \, dx$$

input
```
int((x^5*atan(a*x)^(3/2))/(c + a^2*c*x^2)^3,x)
```

output
```
int((x^5*atan(a*x)^(3/2))/(c + a^2*c*x^2)^3, x)
```

## Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.92

$$\int \frac{x^5 \arctan(ax)^{3/2}}{(c + a^2cx^2)^3} \, dx = \frac{\int \frac{\sqrt{atan(ax)}\, atan(ax)x^5}{a^6x^6+3a^4x^4+3a^2x^2+1} dx}{c^3}$$

input
```
int(x^5*atan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x)
```

output
```
int((sqrt(atan(a*x))*atan(a*x)*x**5)/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**
2 + 1),x)/c**3
```

# 3.790 $\int \frac{x^4 \arctan(ax)^{3/2}}{(c+a^2cx^2)^3} dx$



## Optimal result

Integrand size = 24, antiderivative size = 230

$$\int \frac{x^4 \arctan(ax)^{3/2}}{(c+a^2cx^2)^3} dx = \frac{27\sqrt{\arctan(ax)}}{256a^5c^3} + \frac{3x^4\sqrt{\arctan(ax)}}{32ac^3(1+a^2x^2)^2} - \frac{9\sqrt{\arctan(ax)}}{32a^5c^3(1+a^2x^2)} - \frac{x^3\arctan(ax)^{3/2}}{4a^2c^3(1+a^2x^2)^2} - \frac{3x\arctan(ax)^{3/2}}{8a^4c^3(1+a^2x^2)} + \frac{3\arctan(ax)^{5/2}}{20a^5c^3} - \frac{3\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{512a^5c^3} + \frac{3\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{32a^5c^3}$$

output
```
27/256*arctan(a*x)^(1/2)/a^5/c^3+3/32*x^4*arctan(a*x)^(1/2)/a/c^3/(a^2*x^2
+1)^2-9/32*arctan(a*x)^(1/2)/a^5/c^3/(a^2*x^2+1)-1/4*x^3*arctan(a*x)^(3/2)
/a^2/c^3/(a^2*x^2+1)^2-3/8*x*arctan(a*x)^(3/2)/a^4/c^3/(a^2*x^2+1)+3/20*ar
ctan(a*x)^(5/2)/a^5/c^3-3/1024*2^(1/2)*Pi^(1/2)*FresnelC(2*2^(1/2)/Pi^(1/2
)*arctan(a*x)^(1/2))/a^5/c^3+3/32*Pi^(1/2)*FresnelC(2*arctan(a*x)^(1/2)/Pi
^(1/2))/a^5/c^3
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.54

$$\int \frac{x^4 \arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx = \frac{\frac{64\sqrt{\arctan(ax)}\left(15(-15-6a^2x^2+17a^4x^4)-160ax(3+5a^2x^2)\arctan(ax)+192(1+a^2x^2)^2\arctan(ax)^2\right)}{(1+a^2x^2)^2} - 51$$

input
```
Integrate[(x^4*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^3,x]
```

output
```
((64*Sqrt[ArcTan[a*x]]*(15*(-15 - 6*a^2*x^2 + 17*a^4*x^4) - 160*a*x*(3 + 5
*a^2*x^2)*ArcTan[a*x] + 192*(1 + a^2*x^2)^2*ArcTan[a*x]^2))/(1 + a^2*x^2)^
2 - 510*(12*Sqrt[ArcTan[a*x]] + Sqrt[2*Pi]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcT
an[a*x]]] - 8*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]]) + 90*Sqrt
[ArcTan[a*x]]*(8 + Gamma[1/2, (-4*I)*ArcTan[a*x]]/Sqrt[(-I)*ArcTan[a*x]] +
 Gamma[1/2, (4*I)*ArcTan[a*x]]/Sqrt[I*ArcTan[a*x]]) + (225*(24*ArcTan[a*x]
 - (4*I)*Sqrt[2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-2*I)*ArcTan[a*x]] + (
4*I)*Sqrt[2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (2*I)*ArcTan[a*x]] - I*Sqrt[(-
I)*ArcTan[a*x]]*Gamma[1/2, (-4*I)*ArcTan[a*x]] + I*Sqrt[I*ArcTan[a*x]]*Gam
ma[1/2, (4*I)*ArcTan[a*x]]))/Sqrt[ArcTan[a*x]])/(81920*a^5*c^3)
```

## Rubi [A] (verified)

Time = 1.54 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.19, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5475, 27, 5471, 5465, 5439, 3042, 3793, 2009, 5505, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \arctan(ax)^{3/2}}{(a^2cx^2+c)^3}\,dx$$

$$\Big\downarrow \text{ 5475}$$

$$-\frac{3}{64}\int \frac{x^4}{c^3\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx+\frac{3\int \frac{x^2\arctan(ax)^{3/2}}{c^2\left(a^2x^2+1\right)^2}dx}{4a^2c}+\frac{3x^4\sqrt{\arctan(ax)}}{32ac^3\left(a^2x^2+1\right)^2}-\frac{x^3\arctan(ax)^{3/2}}{4a^2c^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\int \frac{x^2\arctan(ax)^{3/2}}{\left(a^2x^2+1\right)^2}dx}{4a^2c^3}-\frac{3\int \frac{x^4}{\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{64c^3}+\frac{3x^4\sqrt{\arctan(ax)}}{32ac^3\left(a^2x^2+1\right)^2}-\frac{x^3\arctan(ax)^{3/2}}{4a^2c^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \text{5471}$$

$$-\frac{3\int \frac{x^4}{\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{64c^3}+\frac{3\left(\frac{3\int \frac{x\sqrt{\arctan(ax)}}{\left(a^2x^2+1\right)^2}dx}{4a}+\frac{\arctan(ax)^{5/2}}{5a^3}-\frac{x\arctan(ax)^{3/2}}{2a^2\left(a^2x^2+1\right)}\right)}{4a^2c^3}+\frac{3x^4\sqrt{\arctan(ax)}}{32ac^3\left(a^2x^2+1\right)^2}-\frac{x^3\arctan(ax)^{3/2}}{4a^2c^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \text{5465}$$

$$-\frac{3\int \frac{x^4}{\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{64c^3}+\frac{3\left(\frac{3\left(\frac{\int \frac{1}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{4a}-\frac{\sqrt{\arctan(ax)}}{2a^2\left(a^2x^2+1\right)}\right)}{4a}+\frac{\arctan(ax)^{5/2}}{5a^3}-\frac{x\arctan(ax)^{3/2}}{2a^2\left(a^2x^2+1\right)}\right)}{4a^2c^3}+\frac{3x^4\sqrt{\arctan(ax)}}{32ac^3\left(a^2x^2+1\right)^2}-\frac{x^3\arctan(ax)^{3/2}}{4a^2c^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \text{5439}$$

$$-\frac{3\int \frac{x^4}{\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{64c^3}+\frac{3\left(\frac{3\left(\frac{\int \frac{1}{\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}d\arctan(ax)}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2\left(a^2x^2+1\right)}\right)}{4a}+\frac{\arctan(ax)^{5/2}}{5a^3}-\frac{x\arctan(ax)^{3/2}}{2a^2\left(a^2x^2+1\right)}\right)}{4a^2c^3}+\frac{3x^4\sqrt{\arctan(ax)}}{32ac^3\left(a^2x^2+1\right)^2}-\frac{x^3\arctan(ax)^{3/2}}{4a^2c^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\int \frac{x^4}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{64c^3}+\frac{3\left(\frac{3\left(\frac{\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^2}{\sqrt{\arctan(ax)}}d\arctan(ax)}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2\left(a^2x^2+1\right)}\right)}{4a}+\frac{\arctan(ax)^{5/2}}{5a^3}-\frac{x\arctan(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)}{4a^2c^3}+\frac{3x^4\sqrt{\arctan(ax)}}{32ac^3\left(a^2x^2+1\right)^2}-\frac{x^3\arctan(ax)^{3/2}}{4a^2c^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{3\int \frac{x^4}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{64c^3}+\frac{3\left(\frac{3\left(\frac{\int \left(\frac{\cos(2\arctan(ax))}{2\sqrt{\arctan(ax)}}+\frac{1}{2\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2\left(a^2x^2+1\right)}\right)}{4a}+\frac{\arctan(ax)^{5/2}}{5a^3}-\frac{x\arctan(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)}{4a^2c^3}+\frac{3x^4\sqrt{\arctan(ax)}}{32ac^3\left(a^2x^2+1\right)^2}-\frac{x^3\arctan(ax)^{3/2}}{4a^2c^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3\int \frac{x^4}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{64c^3}+\frac{3x^4\sqrt{\arctan(ax)}}{32ac^3\left(a^2x^2+1\right)^2}-\frac{x^3\arctan(ax)^{3/2}}{4a^2c^3\left(a^2x^2+1\right)^2}+\frac{3\left(\frac{\arctan(ax)^{5/2}}{5a^3}+\frac{3\left(\frac{\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\sqrt{\arctan(ax)}}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2\left(a^2x^2+1\right)}\right)}{4a}-\frac{x\arctan(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)}{4a^2c^3}$$

$$\Big\downarrow \text{5505}$$

$$-\frac{3\int \frac{a^4x^4}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{64a^5c^3}+\frac{3x^4\sqrt{\arctan(ax)}}{32ac^3\left(a^2x^2+1\right)^2}-\frac{x^3\arctan(ax)^{3/2}}{4a^2c^3\left(a^2x^2+1\right)^2}+\frac{3\left(\frac{\arctan(ax)^{5/2}}{5a^3}+\frac{3\left(\frac{\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\sqrt{\arctan(ax)}}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2\left(a^2x^2+1\right)}\right)}{4a}-\frac{x\arctan(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)}{4a^2c^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\int \frac{\sin(\arctan(ax))^4}{\sqrt{\arctan(ax)}}d\arctan(ax)}{64a^5c^3}+\frac{3x^4\sqrt{\arctan(ax)}}{32ac^3\left(a^2x^2+1\right)^2}-\frac{x^3\arctan(ax)^{3/2}}{4a^2c^3\left(a^2x^2+1\right)^2}+\frac{3\left(\frac{\arctan(ax)^{5/2}}{5a^3}+\frac{3\left(\frac{\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\sqrt{\arctan(ax)}}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2\left(a^2x^2+1\right)}\right)}{4a}-\frac{x\arctan(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)}{4a^2c^3}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{3\int \left(-\frac{\cos(2\arctan(ax))}{2\sqrt{\arctan(ax)}}+\frac{\cos(4\arctan(ax))}{8\sqrt{\arctan(ax)}}+\frac{3}{8\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{64a^5c^3}+\frac{3x^4\sqrt{\arctan(ax)}}{32ac^3\left(a^2x^2+1\right)^2}-\frac{x^3\arctan(ax)^{3/2}}{4a^2c^3\left(a^2x^2+1\right)^2}+\frac{3\left(\frac{\arctan(ax)^{5/2}}{5a^3}+\frac{3\left(\frac{\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\sqrt{\arctan(ax)}}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2\left(a^2x^2+1\right)}\right)}{4a}-\frac{x\arctan(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)}{4a^2c^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\arctan(ax)}\right)}{64a^5c^3}+\frac{3x^4\sqrt{\arctan(ax)}}{32ac^3\left(a^2x^2+1\right)^2}-\frac{x^3\arctan(ax)^{3/2}}{4a^2c^3\left(a^2x^2+1\right)^2}+\frac{3\left(\frac{\arctan(ax)^{5/2}}{5a^3}+\frac{3\left(\frac{\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\sqrt{\arctan(ax)}}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2\left(a^2x^2+1\right)}\right)}{4a}-\frac{x\arctan(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)}{4a^2c^3}$$

input
```
Int[(x^4*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^3,x]
```

output
```
(3*x^4*Sqrt[ArcTan[a*x]])/(32*a*c^3*(1 + a^2*x^2)^2) - (x^3*ArcTan[a*x]^(3
/2))/(4*a^2*c^3*(1 + a^2*x^2)^2) - (3*((3*Sqrt[ArcTan[a*x]])/4 + (Sqrt[Pi/
2]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/8 - (Sqrt[Pi]*FresnelC[(2*Sqr
t[ArcTan[a*x]])/Sqrt[Pi]])/2))/(64*a^5*c^3) + (3*(-1/2*(x*ArcTan[a*x]^(3/2
))/(a^2*(1 + a^2*x^2)) + ArcTan[a*x]^(5/2)/(5*a^3) + (3*(-1/2*Sqrt[ArcTan[
a*x]]/(a^2*(1 + a^2*x^2)) + (Sqrt[ArcTan[a*x]] + (Sqrt[Pi]*FresnelC[(2*Sqr
t[ArcTan[a*x]])/Sqrt[Pi]])/2)/(4*a^2)))/(4*a)))/(4*a^2*c^3)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5471
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^2)/((d_) + (e_.)*(x_)^2)
^2, x_Symbol] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(2*b*c^3*d^2*(p + 1)), x]
 + (-Simp[x*((a + b*ArcTan[c*x])^p/(2*c^2*d*(d + e*x^2))), x] + Simp[b*(p/(
2*c))   Int[x*((a + b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5475
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_)*((d_) + (e_.)
*(x_)^2)^(q_), x_Symbol] :> Simp[b*p*(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*Ar
cTan[c*x])^(p - 1)/(c*d*m^2)), x] + (-Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(q +
 1)*((a + b*ArcTan[c*x])^p/(c^2*d*m)), x] + Simp[f^2*((m - 1)/(c^2*d*m))
Int[(f*x)^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p, x], x] - Simp[
b^2*p*((p - 1)/m^2)   Int[(f*x)^m*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2)
, x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && EqQ[m + 2*
q + 2, 0] && LtQ[q, -1] && GtQ[p, 1]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

## Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.57

| method | result |
|---|---|
| default | $\frac{-15\sqrt{2}\,\sqrt{\arctan(ax)}\,\sqrt{\pi}\;\mathrm{FresnelC}\left(\frac{2\sqrt{2}\,\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+768\arctan(ax)^{3}-1280\arctan(ax)^{2}\sin(2\arctan(ax))+160\arctan(ax)^{2}\sin}{5120a^{5}c^{3}\sqrt{}}$ |

input
```
int(x^4*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/5120/a^5/c^3*(-15*2^(1/2)*arctan(a*x)^(1/2)*Pi^(1/2)*FresnelC(2*2^(1/2)/
Pi^(1/2)*arctan(a*x)^(1/2))+768*arctan(a*x)^3-1280*arctan(a*x)^2*sin(2*arc
tan(a*x))+160*arctan(a*x)^2*sin(4*arctan(a*x))+480*arctan(a*x)^(1/2)*Pi^(1
/2)*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))-960*arctan(a*x)*cos(2*arctan(a*
x))+60*arctan(a*x)*cos(4*arctan(a*x)))/arctan(a*x)^(1/2)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x^4 \arctan(ax)^{3/2}}{(c+a^2cx^2)^3} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^4*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{x^4 \arctan(ax)^{3/2}}{(c+a^2cx^2)^3} \, dx = \frac{\int \frac{x^4 \operatorname{atan}^{\frac{3}{2}}(ax)}{a^6x^6+3a^4x^4+3a^2x^2+1} \, dx}{c^3}$$

input
```
integrate(x**4*atan(a*x)**(3/2)/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(x**4*atan(a*x)**(3/2)/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1)
, x)/c**3
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4 \arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^4*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F]

$$\int \frac{x^4 \arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx = \int \frac{x^4 \arctan(ax)^{\frac{3}{2}}}{(a^2cx^2+c)^3}\,dx$$

input
```
integrate(x^4*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
integrate(x^4*arctan(a*x)^(3/2)/(a^2*c*x^2 + c)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^4 \arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx = \int \frac{x^4 \operatorname{atan}(a\,x)^{3/2}}{(c\,a^2\,x^2+c)^3}\,dx$$

input
```
int((x^4*atan(a*x)^(3/2))/(c + a^2*c*x^2)^3,x)
```

output
```
int((x^4*atan(a*x)^(3/2))/(c + a^2*c*x^2)^3, x)
```

## Reduce [F]

$$\int \frac{x^4 \arctan(ax)^{3/2}}{(c+a^2cx^2)^3} \, dx = \text{Too large to display}$$

input
```
int(x^4*atan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x)
```

output
```
(96*sqrt(atan(a*x))*atan(a*x)**2*a**4*x**4 + 192*sqrt(atan(a*x))*atan(a*x)
**2*a**2*x**2 + 96*sqrt(atan(a*x))*atan(a*x)**2 - 400*sqrt(atan(a*x))*atan
(a*x)*a**3*x**3 - 240*sqrt(atan(a*x))*atan(a*x)*a*x + 150*sqrt(atan(a*x))*
a**4*x**4 - 90*sqrt(atan(a*x)) + 45*int(sqrt(atan(a*x))/(atan(a*x)*a**6*x*
*6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**5*x*
*4 + 90*int(sqrt(atan(a*x))/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 +
 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**3*x**2 + 45*int(sqrt(atan(a*x))/
(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + ata
n(a*x)),x)*a - 75*int((sqrt(atan(a*x))*x**4)/(atan(a*x)*a**6*x**6 + 3*atan
(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**9*x**4 - 150*in
t((sqrt(atan(a*x))*x**4)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*
atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**7*x**2 - 75*int((sqrt(atan(a*x))*x*
*4)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 +
 atan(a*x)),x)*a**5)/(640*a**5*c**3*(a**4*x**4 + 2*a**2*x**2 + 1))
```

# 3.791 $\int \frac{x^3 \arctan(ax)^{3/2}}{(c+a^2cx^2)^3} \, dx$



## Optimal result

Integrand size = 24, antiderivative size = 168

$$\int \frac{x^3 \arctan(ax)^{3/2}}{(c+a^2cx^2)^3} \, dx = -\frac{3\arctan(ax)^{3/2}}{32a^4c^3} + \frac{x^4\arctan(ax)^{3/2}}{4c^3(1+a^2x^2)^2} + \frac{3\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{512a^4c^3} - \frac{3\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{64a^4c^3} + \frac{3\sqrt{\arctan(ax)}\sin(2\arctan(ax))}{32a^4c^3} - \frac{3\sqrt{\arctan(ax)}\sin(4\arctan(ax))}{256a^4c^3}$$

output
```
-3/32*arctan(a*x)^(3/2)/a^4/c^3+1/4*x^4*arctan(a*x)^(3/2)/c^3/(a^2*x^2+1)^
2+3/1024*2^(1/2)*Pi^(1/2)*FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))/a
^4/c^3-3/64*Pi^(1/2)*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2))/a^4/c^3+3/32*a
rctan(a*x)^(1/2)*sin(2*arctan(a*x))/a^4/c^3-3/256*arctan(a*x)^(1/2)*sin(4*
arctan(a*x))/a^4/c^3
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 350, normalized size of antiderivative = 2.08

$$\int \frac{x^3 \arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx = \sqrt{\arctan(ax)}\left(\frac{3x(3+5a^2x^2)}{64a^3c^3(1+a^2x^2)^2} + \frac{(-3-6a^2x^2+5a^4x^4)\arctan(ax)}{32a^4c^3(1+a^2x^2)^2}\right) - \frac{9\left(-2\sqrt{2}\sqrt{-i\arctan(ax)}\Gamma\left(\frac{1}{2},-2i\arctan(ax)\right) - 2\sqrt{2}\sqrt{i\arctan(ax)}\Gamma\left(\frac{1}{2},2i\arctan(ax)\right) - \sqrt{-i\arctan(\right.}{4096a^4c^3\sqrt{\arctan(ax)}} - \frac{15\left(-2\sqrt{2}\sqrt{-i\arctan(ax)}\Gamma\left(\frac{1}{2},-2i\arctan(ax)\right) - 2\sqrt{2}\sqrt{i\arctan(ax)}\Gamma\left(\frac{1}{2},2i\arctan(ax)\right) + \sqrt{-i\arctan\right.}{4096a^4c^3\sqrt{\arctan(ax)}}$$

input
```
Integrate[(x^3*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^3,x]
```

output
```
Sqrt[ArcTan[a*x]]*((3*x*(3 + 5*a^2*x^2))/(64*a^3*c^3*(1 + a^2*x^2)^2) + ((
-3 - 6*a^2*x^2 + 5*a^4*x^4)*ArcTan[a*x])/(32*a^4*c^3*(1 + a^2*x^2)^2)) - (
9*(-2*Sqrt[2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-2*I)*ArcTan[a*x]] - 2*Sq
rt[2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (2*I)*ArcTan[a*x]] - Sqrt[(-I)*ArcTan
[a*x]]*Gamma[1/2, (-4*I)*ArcTan[a*x]] - Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (4*
I)*ArcTan[a*x]]))/(4096*a^4*c^3*Sqrt[ArcTan[a*x]]) - (15*(-2*Sqrt[2]*Sqrt[
(-I)*ArcTan[a*x]]*Gamma[1/2, (-2*I)*ArcTan[a*x]] - 2*Sqrt[2]*Sqrt[I*ArcTan
[a*x]]*Gamma[1/2, (2*I)*ArcTan[a*x]] + Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (
-4*I)*ArcTan[a*x]] + Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (4*I)*ArcTan[a*x]]))/(
4096*a^4*c^3*Sqrt[ArcTan[a*x]])
```

## Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.89, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5479, 27, 5505, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \arctan(ax)^{3/2}}{(a^2cx^2+c)^3} \, dx$$

$\Big\downarrow$ 5479

$$\frac{x^4 \arctan(ax)^{3/2}}{4c^3 (a^2x^2+1)^2} - \frac{3}{8} a \int \frac{x^4 \sqrt{\arctan(ax)}}{c^3 (a^2x^2+1)^3} dx$$

$\Big\downarrow$ 27

$$\frac{x^4 \arctan(ax)^{3/2}}{4c^3 (a^2x^2+1)^2} - \frac{3a \int \frac{x^4 \sqrt{\arctan(ax)}}{(a^2x^2+1)^3} dx}{8c^3}$$

$\Big\downarrow$ 5505

$$\frac{x^4 \arctan(ax)^{3/2}}{4c^3 (a^2x^2+1)^2} - \frac{3 \int \frac{a^4x^4 \sqrt{\arctan(ax)}}{(a^2x^2+1)^2} d\arctan(ax)}{8a^4c^3}$$

$\Big\downarrow$ 3042

$$\frac{x^4 \arctan(ax)^{3/2}}{4c^3 (a^2x^2+1)^2} - \frac{3 \int \sqrt{\arctan(ax)} \sin(\arctan(ax))^4 d\arctan(ax)}{8a^4c^3}$$

$\Big\downarrow$ 3793

$$\frac{x^4 \arctan(ax)^{3/2}}{4c^3 (a^2x^2+1)^2} - \frac{3 \int \left(-\frac{1}{2} \sqrt{\arctan(ax)} \cos(2\arctan(ax)) + \frac{1}{8} \sqrt{\arctan(ax)} \cos(4\arctan(ax)) + \frac{3}{8} \sqrt{\arctan(ax)}\right) d\arctan(ax)}{8a^4c^3}$$

$\Big\downarrow$ 2009

$$\frac{x^4 \arctan(ax)^{3/2}}{4c^3 (a^2x^2+1)^2} - \frac{3\left(-\frac{1}{64} \sqrt{\frac{\pi}{2}} \text{FresnelS}\left(2\sqrt{\frac{2}{\pi}} \sqrt{\arctan(ax)}\right) + \frac{1}{8} \sqrt{\pi} \text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right) + \frac{1}{4} \arctan(ax)^{3/2} - \frac{1}{4} \sqrt{\arctan(ax)} \sin\right.}{8a^4c^3}$$

input
```
Int[(x^3*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^3,x]
```

output

```
(x^4*ArcTan[a*x]^(3/2))/(4*c^3*(1 + a^2*x^2)^2) - (3*(ArcTan[a*x]^(3/2)/4
- (Sqrt[Pi/2]*FresnelS[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/64 + (Sqrt[Pi]*Fre
snelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/8 - (Sqrt[ArcTan[a*x]]*Sin[2*ArcTan
[a*x]])/4 + (Sqrt[ArcTan[a*x]]*Sin[4*ArcTan[a*x]])/32))/(8*a^4*c^3)
```

### Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5479

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5505

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

## Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.74

| method | result |
|---|---|
| default | $\frac{3\sqrt{2}\,\sqrt{\arctan(ax)}\,\sqrt{\pi}\,\mathrm{FresnelS}\left(\frac{2\sqrt{2}\,\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+32\arctan(ax)^{2}\cos(4\arctan(ax))-128\arctan(ax)^{2}\cos(2\arctan(ax))-48\sqrt{\mathrm{ar}}}{1024a^{4}c^{3}\sqrt{\arctan(ax)}}$ |

input
```
int(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/1024/a^4/c^3*(3*2^(1/2)*arctan(a*x)^(1/2)*Pi^(1/2)*FresnelS(2*2^(1/2)/Pi
^(1/2)*arctan(a*x)^(1/2))+32*arctan(a*x)^2*cos(4*arctan(a*x))-128*arctan(a
*x)^2*cos(2*arctan(a*x))-48*arctan(a*x)^(1/2)*Pi^(1/2)*FresnelS(2*arctan(a
*x)^(1/2)/Pi^(1/2))-12*arctan(a*x)*sin(4*arctan(a*x))+96*arctan(a*x)*sin(2
*arctan(a*x)))/arctan(a*x)^(1/2)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3 \arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{x^3 \arctan(ax)^{3/2}}{(c+a^2cx^2)^3} \, dx = \frac{\int \frac{x^3 \operatorname{atan}^{\frac{3}{2}}(ax)}{a^6x^6+3a^4x^4+3a^2x^2+1} \, dx}{c^3}$$

input
```
integrate(x**3*atan(a*x)**(3/2)/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(x**3*atan(a*x)**(3/2)/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1)
, x)/c**3
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3 \arctan(ax)^{3/2}}{(c+a^2cx^2)^3} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F]

$$\int \frac{x^3 \arctan(ax)^{3/2}}{(c+a^2cx^2)^3} \, dx = \int \frac{x^3 \arctan(ax)^{\frac{3}{2}}}{(a^2cx^2+c)^3} \, dx$$

input
```
integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2 + c)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx = \int \frac{x^3\,\mathrm{atan}(a\,x)^{3/2}}{\left(c\,a^2\,x^2+c\right)^3}\,dx$$

input
```
int((x^3*atan(a*x)^(3/2))/(c + a^2*c*x^2)^3,x)
```

output
```
int((x^3*atan(a*x)^(3/2))/(c + a^2*c*x^2)^3, x)
```

## Reduce [F]

$$\int \frac{x^3 \arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx = \frac{-4\sqrt{atan\,(ax)}\,atan(ax)\,a^2x^2-2\sqrt{atan\,(ax)}\,atan(ax)-2\sqrt{atan\,(ax)}\,ax+5\Big(\int$$

input
```
int(x^3*atan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x)
```

output
```
( - 4*sqrt(atan(a*x))*atan(a*x)*a**2*x**2 - 2*sqrt(atan(a*x))*atan(a*x) -
2*sqrt(atan(a*x))*a*x + 5*int(sqrt(atan(a*x))/(a**6*x**6 + 3*a**4*x**4 + 3
*a**2*x**2 + 1),x)*a**5*x**4 + 10*int(sqrt(atan(a*x))/(a**6*x**6 + 3*a**4*
x**4 + 3*a**2*x**2 + 1),x)*a**3*x**2 + 5*int(sqrt(atan(a*x))/(a**6*x**6 +
3*a**4*x**4 + 3*a**2*x**2 + 1),x)*a + int((sqrt(atan(a*x))*x)/(atan(a*x)*a
**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a
**6*x**4 + 2*int((sqrt(atan(a*x))*x)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a*
*4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**4*x**2 + int((sqrt(atan
(a*x))*x)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*
x**2 + atan(a*x)),x)*a**2)/(8*a**4*c**3*(a**4*x**4 + 2*a**2*x**2 + 1))
```

# 3.792 $\int \frac{x^2 \arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 108

$$\int \frac{x^2 \arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx = \frac{\arctan(ax)^{5/2}}{20a^3c^3} - \frac{3\sqrt{\arctan(ax)}\cos(4\arctan(ax))}{256a^3c^3} + \frac{3\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{512a^3c^3} - \frac{\arctan(ax)^{3/2}\sin(4\arctan(ax))}{32a^3c^3}$$

output
```
1/20*arctan(a*x)^(5/2)/a^3/c^3-3/256*arctan(a*x)^(1/2)*cos(4*arctan(a*x))/
a^3/c^3+3/1024*2^(1/2)*Pi^(1/2)*FresnelC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1
/2))/a^3/c^3-1/32*arctan(a*x)^(3/2)*sin(4*arctan(a*x))/a^3/c^3
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 353, normalized size of antiderivative = 3.27

$$\int \frac{x^2 \arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx = \frac{\frac{64\sqrt{\arctan(ax)}\left(-15(1-6a^2x^2+a^4x^4)+160ax(-1+a^2x^2)\arctan(ax)+64(1+a^2x^2)^2\arctan(ax)^2\right)}{(1+a^2x^2)^2} + 30\Big(1}{}$$

input
```
Integrate[(x^2*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^3,x]
```

output
```
((64*Sqrt[ArcTan[a*x]]*(-15*(1 - 6*a^2*x^2 + a^4*x^4) + 160*a*x*(-1 + a^2*
x^2)*ArcTan[a*x] + 64*(1 + a^2*x^2)^2*ArcTan[a*x]^2))/(1 + a^2*x^2)^2 + 30
*(12*Sqrt[ArcTan[a*x]] + Sqrt[2*Pi]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]
]] - 8*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]]) - 90*Sqrt[ArcTan
[a*x]]*(8 + Gamma[1/2, (-4*I)*ArcTan[a*x]]/Sqrt[(-I)*ArcTan[a*x]] + Gamma[
1/2, (4*I)*ArcTan[a*x]]/Sqrt[I*ArcTan[a*x]]) + (15*(24*ArcTan[a*x] - (4*I)
*Sqrt[2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-2*I)*ArcTan[a*x]] + (4*I)*Sqr
t[2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (2*I)*ArcTan[a*x]] - I*Sqrt[(-I)*ArcTa
n[a*x]]*Gamma[1/2, (-4*I)*ArcTan[a*x]] + I*Sqrt[I*ArcTan[a*x]]*Gamma[1/2,
(4*I)*ArcTan[a*x]]))/Sqrt[ArcTan[a*x]])/(81920*a^3*c^3)
```

## Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.84, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5505, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \arctan(ax)^{3/2}}{(a^2cx^2+c)^3}\,dx$$

$$\Big\downarrow \text{5505}$$

$$\frac{\int \frac{a^2x^2\arctan(ax)^{3/2}}{(a^2x^2+1)^2}d\arctan(ax)}{a^3c^3}$$

$$\Big\downarrow \text{4906}$$

$$\frac{\int \left(\frac{1}{8}\arctan(ax)^{3/2}-\frac{1}{8}\arctan(ax)^{3/2}\cos(4\arctan(ax))\right)d\arctan(ax)}{a^3c^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{3}{512}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{20}\arctan(ax)^{5/2}-\frac{1}{32}\arctan(ax)^{3/2}\sin(4\arctan(ax))-\frac{3}{256}\sqrt{\arctan(a}}{a^3c^3}$$

input
```
Int[(x^2*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^3,x]
```

output
```
(ArcTan[a*x]^(5/2)/20 - (3*Sqrt[ArcTan[a*x]]*Cos[4*ArcTan[a*x]])/256 + (3*
Sqrt[Pi/2]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/512 - (ArcTan[a*x]^(3
/2)*Sin[4*ArcTan[a*x]])/32)/(a^3*c^3)
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

## Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.75

| method | result |
|---|---|
| default | $\frac{15\sqrt{2}\,\sqrt{\arctan(ax)}\,\sqrt{\pi}\ \mathrm{FresnelC}\left(\frac{2\sqrt{2}\,\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+256\arctan(ax)^3-160\arctan(ax)^2\sin(4\arctan(ax))-60\arctan(ax)\cos(4\arctan}{5120a^3c^3\sqrt{\arctan(ax)}}$ |

input
```
int(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/5120/a^3/c^3*(15*2^(1/2)*arctan(a*x)^(1/2)*Pi^(1/2)*FresnelC(2*2^(1/2)/P
i^(1/2)*arctan(a*x)^(1/2))+256*arctan(a*x)^3-160*arctan(a*x)^2*sin(4*arcta
n(a*x))-60*arctan(a*x)*cos(4*arctan(a*x)))/arctan(a*x)^(1/2)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2 \arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{x^2 \arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx = \frac{\int \frac{x^2 \operatorname{atan}^{\frac{3}{2}}(ax)}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

input
```
integrate(x**2*atan(a*x)**(3/2)/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(x**2*atan(a*x)**(3/2)/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1)
, x)/c**3
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2 \arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F]

$$\int \frac{x^2 \arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx = \int \frac{x^2 \arctan(ax)^{\frac{3}{2}}}{(a^2cx^2+c)^3}\,dx$$

input
```
integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2 + c)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx = \int \frac{x^2 \operatorname{atan}(a\,x)^{3/2}}{(c\,a^2\,x^2+c)^3}\,dx$$

input
```
int((x^2*atan(a*x)^(3/2))/(c + a^2*c*x^2)^3,x)
```

output
```
int((x^2*atan(a*x)^(3/2))/(c + a^2*c*x^2)^3, x)
```

## Reduce [F]

$$\int \frac{x^2 \arctan(ax)^{3/2}}{\left(c + a^2cx^2\right)^3} \, dx = \text{Too large to display}$$

input
```
int(x^2*atan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x)
```

output
```
(32*sqrt(atan(a*x))*atan(a*x)**2*a**4*x**4 + 64*sqrt(atan(a*x))*atan(a*x)*
*2*a**2*x**2 + 32*sqrt(atan(a*x))*atan(a*x)**2 + 80*sqrt(atan(a*x))*atan(a
*x)*a**3*x**3 - 80*sqrt(atan(a*x))*atan(a*x)*a*x - 30*sqrt(atan(a*x))*a**4
*x**4 - 30*sqrt(atan(a*x)) + 15*int(sqrt(atan(a*x))/(atan(a*x)*a**6*x**6 +
 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**5*x**4 +
 30*int(sqrt(atan(a*x))/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*a
tan(a*x)*a**2*x**2 + atan(a*x)),x)*a**3*x**2 + 15*int(sqrt(atan(a*x))/(ata
n(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*
x)),x)*a + 15*int((sqrt(atan(a*x))*x**4)/(atan(a*x)*a**6*x**6 + 3*atan(a*x
)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**9*x**4 + 30*int((sq
rt(atan(a*x))*x**4)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(
a*x)*a**2*x**2 + atan(a*x)),x)*a**7*x**2 + 15*int((sqrt(atan(a*x))*x**4)/(
atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan
(a*x)),x)*a**5)/(640*a**3*c**3*(a**4*x**4 + 2*a**2*x**2 + 1))
```

# 3.793 $\int \frac{x\arctan(ax)^{3/2}}{\left(c+a^2cx^2\right)^3}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 168

$$\int \frac{x\arctan(ax)^{3/2}}{\left(c+a^2cx^2\right)^3}\,dx = \frac{3\arctan(ax)^{3/2}}{32a^2c^3} - \frac{\arctan(ax)^{3/2}}{4a^2c^3\left(1+a^2x^2\right)^2} - \frac{3\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{512a^2c^3} - \frac{3\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{64a^2c^3} + \frac{3\sqrt{\arctan(ax)}\sin(2\arctan(ax))}{32a^2c^3} + \frac{3\sqrt{\arctan(ax)}\sin(4\arctan(ax))}{256a^2c^3}$$

output

```
3/32*arctan(a*x)^(3/2)/a^2/c^3-1/4*arctan(a*x)^(3/2)/a^2/c^3/(a^2*x^2+1)^2
-3/1024*2^(1/2)*Pi^(1/2)*FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))/a^
2/c^3-3/64*Pi^(1/2)*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2))/a^2/c^3+3/32*ar
ctan(a*x)^(1/2)*sin(2*arctan(a*x))/a^2/c^3+3/256*arctan(a*x)^(1/2)*sin(4*a
rctan(a*x))/a^2/c^3
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 347, normalized size of antiderivative = 2.07

$$\int \frac{x\arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx = \frac{480ax\arctan(ax)+288a^3x^3\arctan(ax)-320\arctan(ax)^2+384a^2x^2\arctan(ax)^2}{}$$

input
```
Integrate[(x*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^3,x]
```

output
```
(480*a*x*ArcTan[a*x] + 288*a^3*x^3*ArcTan[a*x] - 320*ArcTan[a*x]^2 + 384*a
^2*x^2*ArcTan[a*x]^2 + 192*a^4*x^4*ArcTan[a*x]^2 + 24*Sqrt[2]*(1 + a^2*x^2
)^2*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-2*I)*ArcTan[a*x]] + 24*Sqrt[2]*(1
+ a^2*x^2)^2*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (2*I)*ArcTan[a*x]] + 3*Sqrt[(-
I)*ArcTan[a*x]]*Gamma[1/2, (-4*I)*ArcTan[a*x]] + 6*a^2*x^2*Sqrt[(-I)*ArcTa
n[a*x]]*Gamma[1/2, (-4*I)*ArcTan[a*x]] + 3*a^4*x^4*Sqrt[(-I)*ArcTan[a*x]]*
Gamma[1/2, (-4*I)*ArcTan[a*x]] + 3*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (4*I)*Ar
cTan[a*x]] + 6*a^2*x^2*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (4*I)*ArcTan[a*x]] +
 3*a^4*x^4*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (4*I)*ArcTan[a*x]])/(2048*c^3*(a
 + a^3*x^2)^2*Sqrt[ArcTan[a*x]])
```

## Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.89, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {5465, 27, 5439, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\arctan(ax)^{3/2}}{(a^2cx^2+c)^3}dx$$

$$\Big\downarrow \; 5465$$

$$\frac{3\int \frac{\sqrt{\arctan(ax)}}{c^3(a^2x^2+1)^3}dx}{8a}-\frac{\arctan(ax)^{3/2}}{4a^2c^3(a^2x^2+1)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\int \frac{\sqrt{\arctan(ax)}}{(a^2x^2+1)^3}dx}{8ac^3}-\frac{\arctan(ax)^{3/2}}{4a^2c^3(a^2x^2+1)^2}$$

$$\Big\downarrow \text{5439}$$

$$\frac{3\int \frac{\sqrt{\arctan(ax)}}{(a^2x^2+1)^2}d\arctan(ax)}{8a^2c^3}-\frac{\arctan(ax)^{3/2}}{4a^2c^3(a^2x^2+1)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int \sqrt{\arctan(ax)}\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^4d\arctan(ax)}{8a^2c^3}-\frac{\arctan(ax)^{3/2}}{4a^2c^3(a^2x^2+1)^2}$$

$$\Big\downarrow \text{3793}$$

$$\frac{3\int \left(\frac{1}{2}\sqrt{\arctan(ax)}\cos(2\arctan(ax))+\frac{1}{8}\sqrt{\arctan(ax)}\cos(4\arctan(ax))+\frac{3}{8}\sqrt{\arctan(ax)}\right)d\arctan(ax)}{8a^2c^3}-\frac{\arctan(ax)^{3/2}}{4a^2c^3(a^2x^2+1)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\left(-\frac{1}{64}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{8}\sqrt{\pi}\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{1}{4}\arctan(ax)^{3/2}+\frac{1}{4}\sqrt{\arctan(ax)}\sin\right.}{8a^2c^3}$$

$$\frac{\arctan(ax)^{3/2}}{4a^2c^3(a^2x^2+1)^2}$$

input
```
Int[(x*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^3,x]
```

output
```
-1/4*ArcTan[a*x]^(3/2)/(a^2*c^3*(1 + a^2*x^2)^2) + (3*(ArcTan[a*x]^(3/2)/4
 - (Sqrt[Pi/2]*FresnelS[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/64 - (Sqrt[Pi]*Fr
esnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/8 + (Sqrt[ArcTan[a*x]]*Sin[2*ArcTa
n[a*x]])/4 + (Sqrt[ArcTan[a*x]]*Sin[4*ArcTan[a*x]])/32))/(8*a^2*c^3)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

## Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.74

| method | result |
|---|---|
| default | $-\frac{3\sqrt{2}\,\sqrt{\arctan(ax)}\,\sqrt{\pi}\,\mathrm{FresnelS}\left(\frac{2\sqrt{2}\,\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+32\arctan(ax)^2\cos(4\arctan(ax))+128\arctan(ax)^2\cos(2\arctan(ax))+48\sqrt{}}{1024a^2c^3\sqrt{\arctan(ax}}$ |

input
```
int(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/1024/a^2/c^3*(3*2^(1/2)*arctan(a*x)^(1/2)*Pi^(1/2)*FresnelS(2*2^(1/2)/P
i^(1/2)*arctan(a*x)^(1/2))+32*arctan(a*x)^2*cos(4*arctan(a*x))+128*arctan(
a*x)^2*cos(2*arctan(a*x))+48*arctan(a*x)^(1/2)*Pi^(1/2)*FresnelS(2*arctan(
a*x)^(1/2)/Pi^(1/2))-12*arctan(a*x)*sin(4*arctan(a*x))-96*arctan(a*x)*sin(
2*arctan(a*x)))/arctan(a*x)^(1/2)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x \arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{x \arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx = \frac{\int \frac{x \operatorname{atan}^{\frac{3}{2}}(ax)}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

input
```
integrate(x*atan(a*x)**(3/2)/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(x*atan(a*x)**(3/2)/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x
)/c**3
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x \arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F]

$$\int \frac{x \arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx = \int \frac{x \arctan(ax)^{\frac{3}{2}}}{(a^2cx^2+c)^3}\,dx$$

input
```
integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2 + c)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x \arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx = \int \frac{x\,\mathrm{atan}(a\,x)^{3/2}}{(c\,a^2\,x^2+c)^3}\,dx$$

input
```
int((x*atan(a*x)^(3/2))/(c + a^2*c*x^2)^3,x)
```

output
```
int((x*atan(a*x)^(3/2))/(c + a^2*c*x^2)^3, x)
```

## Reduce [F]

$$\int \frac{x \arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx = \frac{-2\sqrt{atan\,(ax)}\,atan(ax)+3\left(\int \frac{\sqrt{atan(ax)}}{a^6x^6+3a^4x^4+3a^2x^2+1}dx\right)a^5x^4+6\left(\int \frac{\sqrt{atan(ax)}}{a^6x^6+3a^4x^4+3a^2x^2+1}\right.}{8a^2c^3\left(a^4x^4+2a^2x^2+1\right)}$$

input
```
int(x*atan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x)
```

output
```
( - 2*sqrt(atan(a*x))*atan(a*x) + 3*int(sqrt(atan(a*x))/(a**6*x**6 + 3*a**
4*x**4 + 3*a**2*x**2 + 1),x)*a**5*x**4 + 6*int(sqrt(atan(a*x))/(a**6*x**6
+ 3*a**4*x**4 + 3*a**2*x**2 + 1),x)*a**3*x**2 + 3*int(sqrt(atan(a*x))/(a**
6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1),x)*a)/(8*a**2*c**3*(a**4*x**4 + 2*
a**2*x**2 + 1))
```

# 3.794 $\int \frac{\arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 219

$$\int \frac{\arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx = -\frac{45\sqrt{\arctan(ax)}}{256ac^3} + \frac{3\sqrt{\arctan(ax)}}{32ac^3(1+a^2x^2)^2} + \frac{9\sqrt{\arctan(ax)}}{32ac^3(1+a^2x^2)} + \frac{x\arctan(ax)^{3/2}}{4c^3(1+a^2x^2)^2} + \frac{3x\arctan(ax)^{3/2}}{8c^3(1+a^2x^2)} + \frac{3\arctan(ax)^{5/2}}{20ac^3} - \frac{3\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{512ac^3} - \frac{3\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{32ac^3}$$

output

```
-45/256*arctan(a*x)^(1/2)/a/c^3+3/32*arctan(a*x)^(1/2)/a/c^3/(a^2*x^2+1)^2
+9/32*arctan(a*x)^(1/2)/a/c^3/(a^2*x^2+1)+1/4*x*arctan(a*x)^(3/2)/c^3/(a^2
*x^2+1)^2+3/8*x*arctan(a*x)^(3/2)/c^3/(a^2*x^2+1)+3/20*arctan(a*x)^(5/2)/a
/c^3-3/1024*2^(1/2)*Pi^(1/2)*FresnelC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2)
)/a/c^3-3/32*Pi^(1/2)*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))/a/c^3
```

## Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.65

$$\int \frac{\arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx = \frac{\frac{4\sqrt{\arctan(ax)}\left(-15(-17+6a^2x^2+15a^4x^4)+160ax(5+3a^2x^2)\arctan(ax)+192(1+a^2x^2)^2\arctan(ax)^2\right)}{(1+a^2x^2)^2} - 15\sqrt{}}{5120ac^3}$$

input
```
Integrate[ArcTan[a*x]^(3/2)/(c + a^2*c*x^2)^3,x]
```

output
```
((4*Sqrt[ArcTan[a*x]]*(-15*(-17 + 6*a^2*x^2 + 15*a^4*x^4) + 160*a*x*(5 + 3
*a^2*x^2)*ArcTan[a*x] + 192*(1 + a^2*x^2)^2*ArcTan[a*x]^2))/(1 + a^2*x^2)^
2 - 15*Sqrt[2*Pi]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]] - 480*Sqrt[Pi]*
FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(5120*a*c^3)
```

## Rubi [A] (verified)

Time = 1.30 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.18, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5435, 27, 5427, 5439, 3042, 3793, 2009, 5465, 5439, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{3/2}}{(a^2cx^2+c)^3}\,dx$$

$$\Big\downarrow \text{5435}$$

$$-\frac{3}{64}\int \frac{1}{c^3(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx+\frac{3\int \frac{\arctan(ax)^{3/2}}{c^2(a^2x^2+1)^2}dx}{4c}+\frac{x\arctan(ax)^{3/2}}{4c^3(a^2x^2+1)^2}+\frac{3\sqrt{\arctan(ax)}}{32ac^3(a^2x^2+1)^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3\int \frac{1}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{64c^3}+\frac{3\int \frac{\arctan(ax)^{3/2}}{(a^2x^2+1)^2}dx}{4c^3}+\frac{x\arctan(ax)^{3/2}}{4c^3(a^2x^2+1)^2}+\frac{3\sqrt{\arctan(ax)}}{32ac^3(a^2x^2+1)^2}$$

$$\Big\downarrow \text{5427}$$

$$-\frac{3\int \frac{1}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{64c^3}+\frac{3\left(-\frac{3}{4}a\int \frac{x\sqrt{\arctan(ax)}}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^{3/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{5/2}}{5a}\right)}{4c^3}+\frac{x\arctan(ax)^{3/2}}{4c^3\left(a^2x^2+1\right)^2}+\frac{3\sqrt{\arctan(ax)}}{32ac^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \text{5439}$$

$$-\frac{3\int \frac{1}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{64ac^3}+\frac{3\left(-\frac{3}{4}a\int \frac{x\sqrt{\arctan(ax)}}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^{3/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{5/2}}{5a}\right)}{4c^3}+\frac{x\arctan(ax)^{3/2}}{4c^3\left(a^2x^2+1\right)^2}+\frac{3\sqrt{\arctan(ax)}}{32ac^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(-\frac{3}{4}a\int \frac{x\sqrt{\arctan(ax)}}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^{3/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{5/2}}{5a}\right)}{4c^3}-\frac{3\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^4}{\sqrt{\arctan(ax)}}d\arctan(ax)}{64ac^3}+\frac{x\arctan(ax)^{3/2}}{4c^3\left(a^2x^2+1\right)^2}+\frac{3\sqrt{\arctan(ax)}}{32ac^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \text{3793}$$

$$\frac{3\left(-\frac{3}{4}a\int \frac{x\sqrt{\arctan(ax)}}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^{3/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{5/2}}{5a}\right)}{4c^3}-\frac{3\int \left(\frac{\cos(2\arctan(ax))}{2\sqrt{\arctan(ax)}}+\frac{\cos(4\arctan(ax))}{8\sqrt{\arctan(ax)}}+\frac{3}{8\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{64ac^3}+\frac{x\arctan(ax)^{3/2}}{4c^3\left(a^2x^2+1\right)^2}+\frac{3\sqrt{\arctan(ax)}}{32ac^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\left(-\frac{3}{4}a\int \frac{x\sqrt{\arctan(ax)}}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^{3/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{5/2}}{5a}\right)}{4c^3}+\frac{x\arctan(ax)^{3/2}}{4c^3\left(a^2x^2+1\right)^2}+\frac{3\sqrt{\arctan(ax)}}{32ac^3\left(a^2x^2+1\right)^2}-\frac{3\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\arctan(ax)}\right)}{64ac^3}$$

$$\Big\downarrow \text{5465}$$

$$\frac{3\left(-\frac{3}{4}a\left(\frac{\int \frac{1}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{4a}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^{3/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{5/2}}{5a}\right)}{4c^3}+\frac{x\arctan(ax)^{3/2}}{4c^3\left(a^2x^2+1\right)^2}+\frac{3\sqrt{\arctan(ax)}}{32ac^3\left(a^2x^2+1\right)^2}-\frac{3\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\arctan(ax)}\right)}{64ac^3}$$

$$\Big\downarrow \text{5439}$$

$$\frac{3\left(-\frac{3}{4}a\left(\frac{\int \frac{1}{\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}d\arctan(ax)}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^{3/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{5/2}}{5a}\right)}{4c^3}+\frac{x\arctan(ax)^{3/2}}{4c^3\left(a^2x^2+1\right)^2}+\frac{3\sqrt{\arctan(ax)}}{32ac^3\left(a^2x^2+1\right)^2}-\frac{3\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\arctan(ax)}\right)}{64ac^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(-\frac{3}{4}a\left(\frac{\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^2}{\sqrt{\arctan(ax)}}d\arctan(ax)}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^{3/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{5/2}}{5a}\right)}{4c^3}+\frac{x\arctan(ax)^{3/2}}{4c^3\left(a^2x^2+1\right)^2}+\frac{3\sqrt{\arctan(ax)}}{32ac^3\left(a^2x^2+1\right)^2}-\frac{3\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\arctan(ax)}\right)}{64ac^3}$$

$$\Big\downarrow \text{3793}$$

$$\frac{3\left(-\frac{3}{4}a\left(\frac{\int \left(\frac{\cos(2\arctan(ax))}{2\sqrt{\arctan(ax)}}+\frac{1}{2\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^{3/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{5/2}}{5a}\right)}{4c^3}+\frac{x\arctan(ax)^{3/2}}{4c^3\left(a^2x^2+1\right)^2}+\frac{3\sqrt{\arctan(ax)}}{32ac^3\left(a^2x^2+1\right)^2}-\frac{3\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\arctan(ax)}\right)}{64ac^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\left(-\frac{3}{4}a\left(\frac{\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\sqrt{\arctan(ax)}}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^{3/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{5/2}}{5a}\right)}{4c^3}+\frac{x\arctan(ax)^{3/2}}{4c^3\left(a^2x^2+1\right)^2}+\frac{3\sqrt{\arctan(ax)}}{32ac^3\left(a^2x^2+1\right)^2}-\frac{3\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\arctan(ax)}\right)}{64ac^3}$$

input
```
Int[ArcTan[a*x]^(3/2)/(c + a^2*c*x^2)^3,x]
```

output
```
(3*Sqrt[ArcTan[a*x]])/(32*a*c^3*(1 + a^2*x^2)^2) + (x*ArcTan[a*x]^(3/2))/(
4*c^3*(1 + a^2*x^2)^2) - (3*((3*Sqrt[ArcTan[a*x]])/4 + (Sqrt[Pi/2]*Fresnel
C[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/8 + (Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a
*x]])/Sqrt[Pi]])/2))/(64*a*c^3) + (3*((x*ArcTan[a*x]^(3/2))/(2*(1 + a^2*x^
2)) + ArcTan[a*x]^(5/2)/(5*a) - (3*a*(-1/2*Sqrt[ArcTan[a*x]]/(a^2*(1 + a^2
*x^2)) + (Sqrt[ArcTan[a*x]] + (Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqr
t[Pi]])/2)/(4*a^2)))/4))/(4*c^3)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5435
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[b*p*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p - 1)/(4*c*d*
(q + 1)^2)), x] + (-Simp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*d*
(q + 1))), x] + Simp[(2*q + 3)/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p, x], x] - Simp[b^2*p*((p - 1)/(4*(q + 1)^2))   Int[(d + e
*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

## Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.60

| method | result |
|---|---|
| default | $\frac{768\arctan(ax)^3+1280\arctan(ax)^2\sin(2\arctan(ax))+160\arctan(ax)^2\sin(4\arctan(ax))-15\sqrt{2}\,\sqrt{\arctan(ax)}\,\sqrt{\pi}\ \mathrm{FresnelC}\left(\frac{2\sqrt{2}\,\sqrt{}}{}\right.}{5120a\,c^3\sqrt{}}$ |

input
```
int(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/5120/a/c^3/arctan(a*x)^(1/2)*(768*arctan(a*x)^3+1280*arctan(a*x)^2*sin(2
*arctan(a*x))+160*arctan(a*x)^2*sin(4*arctan(a*x))-15*2^(1/2)*arctan(a*x)^
(1/2)*Pi^(1/2)*FresnelC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))+960*arctan(a
*x)*cos(2*arctan(a*x))+60*arctan(a*x)*cos(4*arctan(a*x))-480*arctan(a*x)^(
1/2)*Pi^(1/2)*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2)))
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{\arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx = \frac{\int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

input
```
integrate(atan(a*x)**(3/2)/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(atan(a*x)**(3/2)/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x)/
c**3
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F]

$$\int \frac{\arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx = \int \frac{\arctan{(ax)}^{\frac{3}{2}}}{(a^2cx^2+c)^3}\,dx$$

input
```
integrate(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^(3/2)/(a^2*c*x^2 + c)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx = \int \frac{\mathrm{atan}(a\,x)^{3/2}}{(c\,a^2\,x^2+c)^3}\,dx$$

input
```
int(atan(a*x)^(3/2)/(c + a^2*c*x^2)^3,x)
```

output
```
int(atan(a*x)^(3/2)/(c + a^2*c*x^2)^3, x)
```

## Reduce [F]

$$\int \frac{\arctan(ax)^{3/2}}{(c+a^2cx^2)^3}\,dx = \text{Too large to display}$$

input
```
int(atan(a*x)^(3/2)/(a^2*c*x^2+c)^3,x)
```

output
```
(96*sqrt(atan(a*x))*atan(a*x)**2*a**4*x**4 + 192*sqrt(atan(a*x))*atan(a*x)
**2*a**2*x**2 + 96*sqrt(atan(a*x))*atan(a*x)**2 + 240*sqrt(atan(a*x))*atan
(a*x)*a**3*x**3 + 400*sqrt(atan(a*x))*atan(a*x)*a*x - 90*sqrt(atan(a*x))*a
**4*x**4 + 150*sqrt(atan(a*x)) - 75*int(sqrt(atan(a*x))/(atan(a*x)*a**6*x*
*6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**5*x*
*4 - 150*int(sqrt(atan(a*x))/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4
+ 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**3*x**2 - 75*int(sqrt(atan(a*x))
/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + at
an(a*x)),x)*a + 45*int((sqrt(atan(a*x))*x**4)/(atan(a*x)*a**6*x**6 + 3*ata
n(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**9*x**4 + 90*in
t((sqrt(atan(a*x))*x**4)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*
atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**7*x**2 + 45*int((sqrt(atan(a*x))*x*
*4)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 +
 atan(a*x)),x)*a**5)/(640*a*c**3*(a**4*x**4 + 2*a**2*x**2 + 1))
```

# 3.795 $\int \frac{\arctan(ax)^{3/2}}{x(c+a^2cx^2)^3} \, dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\arctan(ax)^{3/2}}{x\left(c+a^2cx^2\right)^3}\,dx = \text{Int}\left(\frac{\arctan(ax)^{3/2}}{x\left(c+a^2cx^2\right)^3},x\right)$$

output
```
Defer(Int)(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^3,x)
```

## Mathematica [N/A]

Not integrable

Time = 1.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\arctan(ax)^{3/2}}{x\left(c+a^2cx^2\right)^3}\,dx = \int \frac{\arctan(ax)^{3/2}}{x\left(c+a^2cx^2\right)^3}\,dx$$

input
```
Integrate[ArcTan[a*x]^(3/2)/(x*(c + a^2*c*x^2)^3),x]
```

output
```
Integrate[ArcTan[a*x]^(3/2)/(x*(c + a^2*c*x^2)^3), x]
```

## Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{3/2}}{x\left(a^2cx^2+c\right)^3}dx$$

$$\Big\downarrow \text{5560}$$

$$\int \frac{\arctan(ax)^{3/2}}{x\left(a^2cx^2+c\right)^3}dx$$

input
```
Int[ArcTan[a*x]^(3/2)/(x*(c + a^2*c*x^2)^3),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\arctan\left(ax\right)^{\frac{3}{2}}}{x\left(a^2c\,x^2+c\right)^3}dx$$

input
```
int(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^3,x)
```

output
```
int(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^3,x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{x\left(c+a^2cx^2\right)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 4.88 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{\arctan(ax)^{3/2}}{x\left(c+a^2cx^2\right)^3}\,dx = \frac{\int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{a^6x^7+3a^4x^5+3a^2x^3+x}\,dx}{c^3}$$

input
```
integrate(atan(a*x)**(3/2)/x/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(atan(a*x)**(3/2)/(a**6*x**7 + 3*a**4*x**5 + 3*a**2*x**3 + x), x)/
c**3
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{x\left(c+a^2cx^2\right)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax)^{3/2}}{x\,(c+a^2cx^2)^3}\,dx = \int \frac{\arctan{(ax)}^{\frac{3}{2}}}{(a^2cx^2+c)^3x}\,dx$$

input
```
integrate(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^(3/2)/((a^2*c*x^2 + c)^3*x), x)
```

## Mupad [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax)^{3/2}}{x\,(c+a^2cx^2)^3}\,dx = \int \frac{\mathrm{atan}(a\,x)^{3/2}}{x\,(c\,a^2\,x^2+c)^3}\,dx$$

input
```
int(atan(a*x)^(3/2)/(x*(c + a^2*c*x^2)^3),x)
```

output
```
int(atan(a*x)^(3/2)/(x*(c + a^2*c*x^2)^3), x)
```

## Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.79

$$\int \frac{\arctan(ax)^{3/2}}{x\left(c+a^2cx^2\right)^3}\,dx = \frac{\int \frac{\sqrt{atan(ax)}\,atan(ax)}{a^6x^7+3a^4x^5+3a^2x^3+x}dx}{c^3}$$

input
```
int(atan(a*x)^(3/2)/x/(a^2*c*x^2+c)^3,x)
```

output
```
int((sqrt(atan(a*x))*atan(a*x))/(a**6*x**7 + 3*a**4*x**5 + 3*a**2*x**3 + x
),x)/c**3
```

# 3.796 $\int x^m \sqrt{c+a^2cx^2} \arctan(ax)^{3/2}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int x^m \sqrt{c+a^2cx^2} \arctan(ax)^{3/2}\,dx = \text{Int}\left(x^m \sqrt{c+a^2cx^2} \arctan(ax)^{3/2},x\right)$$

output
```
Defer(Int)(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(3/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int x^m \sqrt{c+a^2cx^2} \arctan(ax)^{3/2}\,dx = \int x^m \sqrt{c+a^2cx^2} \arctan(ax)^{3/2}\,dx$$

input
```
Integrate[x^m*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[x^m*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \arctan(ax)^{3/2}\sqrt{a^2cx^2+c}\,dx$$

$$\Big\downarrow \text{5560}$$

$$\int x^m \arctan(ax)^{3/2}\sqrt{a^2cx^2+c}dx$$

input
```
Int[x^m*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int x^m \sqrt{a^2 c\,x^2+c}\,\arctan\left(ax\right)^{\frac{3}{2}}dx$$

input
```
int(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(3/2),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(3/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int x^m \sqrt{c + a^2 c x^2} \arctan(ax)^{3/2} \, dx = \int \sqrt{a^2 c x^2 + c} x^m \arctan(ax)^{\frac{3}{2}} \, dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^m*arctan(a*x)^(3/2), x)
```

## Sympy [F(-1)]

Timed out.

$$\int x^m \sqrt{c + a^2 c x^2} \arctan(ax)^{3/2} \, dx = \text{Timed out}$$

input
```
integrate(x**m*(a**2*c*x**2+c)**(1/2)*atan(a*x)**(3/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int x^m \sqrt{c + a^2 c x^2} \arctan(ax)^{3/2} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int x^m \sqrt{c+a^2cx^2} \arctan(ax)^{3/2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int x^m \sqrt{c+a^2cx^2} \arctan(ax)^{3/2} \, dx = \int x^m \operatorname{atan}(a\,x)^{3/2} \sqrt{c\,a^2\,x^2+c} \, dx$$

input
```
int(x^m*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(x^m*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int x^m \sqrt{c+a^2cx^2} \arctan(ax)^{3/2} \, dx = \sqrt{c} \left( \int x^m \sqrt{a^2x^2+1}\, \sqrt{atan(ax)}\, atan(ax) \, dx \right)$$

input
```
int(x^m*(a^2*c*x^2+c)^(1/2)*atan(a*x)^(3/2),x)
```

output
```
sqrt(c)*int(x**m*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x),x)
```

# 3.797 $\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}\,dx=\text{Int}\left(x^2\sqrt{c+a^2cx^2}\arctan(ax)^{3/2},x\right)$$

output
```
Defer(Int)(x^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(3/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 3.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}\,dx=\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}\,dx$$

input
```
Integrate[x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arctan(ax)^{3/2}\sqrt{a^2cx^2+c}\,dx$$

$$\Big\downarrow \text{5560}$$

$$\int x^2 \arctan(ax)^{3/2}\sqrt{a^2cx^2+c}dx$$

input
```
Int[x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int x^2\sqrt{a^2c\,x^2+c}\,\arctan\left(ax\right)^{\frac{3}{2}}dx$$

input
```
int(x^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(3/2),x)
```

output
```
int(x^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 70.67 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}\,dx = \int x^2\sqrt{c\,(a^2x^2+1)}\,\mathrm{atan}^{\frac{3}{2}}\,(ax)\,dx$$

input
```
integrate(x**2*(a**2*c*x**2+c)**(1/2)*atan(a*x)**(3/2),x)
```

output
```
Integral(x**2*sqrt(c*(a**2*x**2 + 1))*atan(a*x)**(3/2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}\,dx = \int \sqrt{a^2cx^2+c}x^2\arctan\left(ax\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*x^2*arctan(a*x)^(3/2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}\,dx = \int x^2\,\mathrm{atan}(a\,x)^{3/2}\,\sqrt{c\,a^2\,x^2+c}\,dx$$

input
```
int(x^2*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(x^2*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}\,dx = \sqrt{c}\left(\int\sqrt{a^2x^2+1}\,\sqrt{atan\left(ax\right)}\,atan(ax)\,x^2dx\right)$$

input
```
int(x^2*(a^2*c*x^2+c)^(1/2)*atan(a*x)^(3/2),x)
```

output
```
sqrt(c)*int(sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)*x**2,x)
```

# 3.798 $\int x\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int x\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}\,dx=\frac{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}{3a^2c}-\frac{\operatorname{Int}\left(\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)},x\right)}{2a}$$

output
```
1/3*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2)/a^2/c-1/2*Defer(Int)((a^2*c*x^2+
c)^(1/2)*arctan(a*x)^(1/2),x)/a
```

## Mathematica [N/A]

Not integrable

Time = 5.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int x\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}\,dx=\int x\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}\,dx$$

input
```
Integrate[x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \arctan(ax)^{3/2}\sqrt{a^2cx^2+c}\,dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\int\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}dx}{2a}$$

$$\Big\downarrow \text{5560}$$

$$\frac{\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{\int\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}dx}{2a}$$

input
```
Int[x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int x\sqrt{a^2c\,x^2+c}\,\arctan\left(ax\right)^{\frac{3}{2}}dx$$

input
```
int(x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(3/2),x)
```

output
```
int(x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int x\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 39.76 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}\,dx = \int x\sqrt{c\left(a^2x^2+1\right)}\operatorname{atan}^{\frac{3}{2}}(ax)\,dx$$

input
```
integrate(x*(a**2*c*x**2+c)**(1/2)*atan(a*x)**(3/2),x)
```

output
```
Integral(x*sqrt(c*(a**2*x**2 + 1))*atan(a*x)**(3/2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int x\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int x\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int x\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}\,dx = \int x\,\mathrm{atan}(a\,x)^{3/2}\,\sqrt{c\,a^2\,x^2+c}\,dx$$

input
```
int(x*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(x*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 149, normalized size of antiderivative = 6.21

$$\int x\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}\,dx = \frac{\sqrt{c}\left(8\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}\,atan(ax)\,a^2x^2+8\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}\right.}{}$$

input
```
int(x*(a^2*c*x^2+c)^(1/2)*atan(a*x)^(3/2),x)
```

output
```
(sqrt(c)*(8*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)*a**2*x**2 + 8*sq
rt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x) - 6*sqrt(a**2*x**2 + 1)*sqrt(a
tan(a*x))*a*x + 3*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x)/(atan(a*x)*a
**2*x**2 + atan(a*x)),x)*a**2 - 6*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))
)/(a**2*x**2 + 1),x)*a))/(24*a**2)
```

# 3.799 $\int \sqrt{c+a^2cx^2}\arctan(ax)^{3/2}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \sqrt{c+a^2cx^2}\arctan(ax)^{3/2}\,dx = -\frac{3\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}{4a}+\frac{1}{2}x\sqrt{c+a^2cx^2}\arctan(ax)^{3/2} + \frac{3}{8}c\text{Int}\left(\frac{1}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}},x\right)+\frac{1}{2}c\text{Int}\left(\frac{\arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}},x\right)$$

output
```
-3/4*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2)/a+1/2*x*(a^2*c*x^2+c)^(1/2)*arc
tan(a*x)^(3/2)+3/8*c*Defer(Int)(1/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
+1/2*c*Defer(Int)(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \sqrt{c+a^2cx^2}\arctan(ax)^{3/2}\,dx = \int \sqrt{c+a^2cx^2}\arctan(ax)^{3/2}\,dx$$

input
```
Integrate[Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(ax)^{3/2}\sqrt{a^2cx^2+c}\,dx$$

$$\Big\downarrow \text{5415}$$

$$\frac{3}{8}c\int \frac{1}{\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx+\frac{1}{2}c\int \frac{\arctan(ax)^{3/2}}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}-\frac{3\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}{4a}$$

$$\Big\downarrow \text{5560}$$

$$\frac{3}{8}c\int \frac{1}{\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx+\frac{1}{2}c\int \frac{\arctan(ax)^{3/2}}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}-\frac{3\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}{4a}$$

input
```
Int[Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \sqrt{a^2 c\,x^2+c}\,\arctan\left(a x\right)^{\frac{3}{2}}dx$$

input
```
int((a^2*c*x^2+c)^(1/2)*arctan(a*x)^(3/2),x)
```

output
```
int((a^2*c*x^2+c)^(1/2)*arctan(a*x)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \sqrt{c+a^2cx^2}\arctan(ax)^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(1/2)*arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 20.64 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \sqrt{c+a^2cx^2}\arctan(ax)^{3/2}\,dx = \int \sqrt{c\left(a^2x^2+1\right)}\,\operatorname{atan}^{\frac{3}{2}}\left(ax\right)dx$$

input
```
integrate((a**2*c*x**2+c)**(1/2)*atan(a*x)**(3/2),x)
```

output
```
Integral(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**(3/2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \sqrt{c+a^2cx^2}\arctan(ax)^{3/2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^(1/2)*arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int \sqrt{c+a^2cx^2}\arctan(ax)^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(1/2)*arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \sqrt{c+a^2cx^2}\arctan(ax)^{3/2}\,dx = \int \mathrm{atan}(a\,x)^{3/2}\,\sqrt{c\,a^2\,x^2+c}\,dx$$

input
```
int(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \sqrt{c + a^2cx^2}\arctan(ax)^{3/2}\,dx = \sqrt{c}\left(\int \sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}\,atan(ax)\,dx\right)$$

input
```
int((a^2*c*x^2+c)^(1/2)*atan(a*x)^(3/2),x)
```

output
```
sqrt(c)*int(sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x),x)
```

# 3.800 $\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}{x}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}{x}\,dx = \text{Int}\left(\frac{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}{x},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^(1/2)*arctan(a*x)^(3/2)/x,x)
```

## Mathematica [N/A]

Not integrable

Time = 3.67 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}{x}\,dx = \int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}{x}\,dx$$

input
```
Integrate[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2))/x,x]
```

output
```
Integrate[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2))/x, x]
```

## Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}{x}\,dx$$

$$\Big\downarrow \; 5560$$

$$\int \frac{\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}{x}dx$$

input
```
Int[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2))/x,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{a^2c\,x^2+c}\,\arctan\left(ax\right)^{\frac{3}{2}}}{x}dx$$

input
```
int((a^2*c*x^2+c)^(1/2)*arctan(a*x)^(3/2)/x,x)
```

output
```
int((a^2*c*x^2+c)^(1/2)*arctan(a*x)^(3/2)/x,x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(1/2)*arctan(a*x)^(3/2)/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 15.94 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}{x}\,dx = \int \frac{\sqrt{c\,(a^2x^2+1)}\,\operatorname{atan}^{\frac{3}{2}}(ax)}{x}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(1/2)*atan(a*x)**(3/2)/x,x)
```

output
```
Integral(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**(3/2)/x, x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}{x}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^(1/2)*arctan(a*x)^(3/2)/x,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(1/2)*arctan(a*x)^(3/2)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}{x}\,dx = \int \frac{\mathrm{atan}(a\,x)^{3/2}\,\sqrt{c\,a^2\,x^2+c}}{x}\,dx$$

input
```
int((atan(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2))/x,x)
```

output
```
int((atan(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2))/x, x)
```

## Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}{x}\,dx = \sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,atan(ax)}{x}dx\right)$$

input
```
int((a^2*c*x^2+c)^(1/2)*atan(a*x)^(3/2)/x,x)
```

output
```
sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x))/x,x)
```

# 3.801 $\int x^m(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int x^m\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{3/2}\,dx=\text{Int}\left(x^m\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{3/2},x\right)$$

output
```
Defer(Int)(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int x^m\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{3/2}\,dx=\int x^m\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{3/2}\,dx$$

input
```
Integrate[x^m*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[x^m*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \arctan(ax)^{3/2} \left(a^2cx^2+c\right)^{3/2} \, dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int x^m \arctan(ax)^{3/2} \left(a^2cx^2+c\right)^{3/2} dx$$

input
```
Int[x^m*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int x^m \left(a^2 c\,x^2+c\right)^{\frac{3}{2}} \arctan\left(a x\right)^{\frac{3}{2}} dx$$

input
```
int(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int x^m \left(c+a^2cx^2\right)^{3/2} \arctan(ax)^{3/2} \, dx = \int \left(a^2cx^2+c\right)^{\frac{3}{2}} x^m \arctan\left(ax\right)^{\frac{3}{2}} \, dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^(3/2)*x^m*arctan(a*x)^(3/2), x)
```

## Sympy [F(-1)]

Timed out.

$$\int x^m \left(c+a^2cx^2\right)^{3/2} \arctan(ax)^{3/2} \, dx = \text{Timed out}$$

input
```
integrate(x**m*(a**2*c*x**2+c)**(3/2)*atan(a*x)**(3/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int x^m \left(c+a^2cx^2\right)^{3/2} \arctan(ax)^{3/2} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int x^m \left(c+a^2cx^2\right)^{3/2} \arctan(ax)^{3/2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int x^m \left(c+a^2cx^2\right)^{3/2} \arctan(ax)^{3/2} \, dx = \int x^m \operatorname{atan}(a\,x)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^{3/2} \,d x$$

input
```
int(x^m*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(x^m*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.38

$$\int x^m (c +a^2cx^2)^{3/2} \arctan(ax)^{3/2} \, dx = \sqrt{c}\, c \left( \left( \int x^m \sqrt{a^2x^2+1}\, \sqrt{atan(ax)}\, atan(ax)\, x^2 dx \right) a^2 + \int x^m \sqrt{a^2x^2+1}\, \sqrt{\vphantom{x}}$$

input
```
int(x^m*(a^2*c*x^2+c)^(3/2)*atan(a*x)^(3/2),x)
```

output
```
sqrt(c)*c*(int(x**m*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)*x**2,x)*
a**2 + int(x**m*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x),x))
```

# 3.802 $\int x^2(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int x^2\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{3/2}\,dx=\text{Int}\left(x^2\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{3/2},x\right)$$

output
```
Defer(Int)(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 4.13 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int x^2\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{3/2}\,dx=\int x^2\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{3/2}\,dx$$

input
```
Integrate[x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arctan(ax)^{3/2} \left(a^2cx^2+c\right)^{3/2} \, dx$$

$$\Big\downarrow \ 5560$$

$$\int x^2 \arctan(ax)^{3/2} \left(a^2cx^2+c\right)^{3/2} dx$$

input
```
Int[x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int x^2 \left(a^2 c\, x^2+c\right)^{\frac{3}{2}} \arctan\left(a x\right)^{\frac{3}{2}} dx$$

input
```
int(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x)
```

output
```
int(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int x^2 (c + a^2 c x^2)^{3/2} \arctan(ax)^{3/2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int x^2 (c + a^2 c x^2)^{3/2} \arctan(ax)^{3/2} \, dx = \text{Timed out}$$

input
```
integrate(x**2*(a**2*c*x**2+c)**(3/2)*atan(a*x)**(3/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int x^2 (c + a^2 c x^2)^{3/2} \arctan(ax)^{3/2} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int x^2 (c + a^2 c x^2)^{3/2} \arctan(ax)^{3/2} \, dx = \int (a^2 c x^2 + c)^{\frac{3}{2}} x^2 \arctan(ax)^{\frac{3}{2}} \, dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*x^2*arctan(a*x)^(3/2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int x^2 (c + a^2 c x^2)^{3/2} \arctan(ax)^{3/2} \, dx = \int x^2 \operatorname{atan}(a\,x)^{3/2} \left(c\,a^2\,x^2 + c\right)^{3/2} dx$$

input
```
int(x^2*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(x^2*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.27

$$\int x^2 (c + a^2 c x^2)^{3/2} \arctan(ax)^{3/2} \, dx = \sqrt{c}\, c \left( \left( \int \sqrt{a^2x^2+1}\, \sqrt{atan(ax)}\, atan(ax)\, x^4 dx \right) a^2 + \int \sqrt{a^2x^2+1}\, \sqrt{atan}\right.$$

input
```
int(x^2*(a^2*c*x^2+c)^(3/2)*atan(a*x)^(3/2),x)
```

output
```
sqrt(c)*c*(int(sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)*x**4,x)*a**2
+ int(sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)*x**2,x))
```

# 3.803 $\int x(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int x(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}\,dx=\frac{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}{5a^2c}-\frac{3\mathrm{Int}\left((c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)},x\right)}{10a}$$

output
```
1/5*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2)/a^2/c-3/10*Defer(Int)((a^2*c*x^2
+c)^(3/2)*arctan(a*x)^(1/2),x)/a
```

## Mathematica [N/A]

Not integrable

Time = 2.73 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int x(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}\,dx=\int x(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}\,dx$$

input
```
Integrate[x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \arctan(ax)^{3/2} \left(a^2cx^2+c\right)^{3/2} dx$$

$$\Big\downarrow \ 5465$$

$$\frac{\arctan(ax)^{3/2} \left(a^2cx^2+c\right)^{5/2}}{5a^2c} - \frac{3\int \left(a^2cx^2+c\right)^{3/2} \sqrt{\arctan(ax)}dx}{10a}$$

$$\Big\downarrow \ 5560$$

$$\frac{\arctan(ax)^{3/2} \left(a^2cx^2+c\right)^{5/2}}{5a^2c} - \frac{3\int \left(a^2cx^2+c\right)^{3/2} \sqrt{\arctan(ax)}dx}{10a}$$

input
```
Int[x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int x\left(a^2c\,x^2+c\right)^{\frac{3}{2}} \arctan\left(ax\right)^{\frac{3}{2}}dx$$

input
```
int(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x)
```

output
```
int(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int x(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int x(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(x*(a**2*c*x**2+c)**(3/2)*atan(a*x)**(3/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int x(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{3/2}\,dx=\int x\,\mathrm{atan}(a\,x)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^{3/2}\,dx$$

input
```
int(x*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(x*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 244, normalized size of antiderivative = 10.17

$$\int x(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}\,dx = \frac{\sqrt{c}\,c\Big(32\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}\,atan(ax)\,a^4x^4+64\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}\,a}{}$$

input

```
int(x*(a^2*c*x^2+c)^(3/2)*atan(a*x)^(3/2),x)
```

output

```
(sqrt(c)*c*(32*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)*a**4*x**4 + 6
4*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)*a**2*x**2 + 32*sqrt(a**2*x
**2 + 1)*sqrt(atan(a*x))*atan(a*x) - 12*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)
)*a**3*x**3 - 30*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*a*x + 6*int((sqrt(a**
2*x**2 + 1)*sqrt(atan(a*x))*x**3)/(atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**
4 + 15*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x)/(atan(a*x)*a**2*x**2 +
atan(a*x)),x)*a**2 - 18*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(a**2*x*
*2 + 1),x)*a))/(160*a**2)
```

# 3.804 $\int (c+a^2cx^2)^{3/2} \arctan(ax)^{3/2}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 23

$$\int (c+a^2cx^2)^{3/2} \arctan(ax)^{3/2}\,dx = -\frac{9c\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}{16a} - \frac{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}{8a} + \frac{3}{8}cx\sqrt{c+a^2cx^2}\arctan(ax)^{3/2} + \frac{1}{4}x(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2} + \frac{9}{32}c^2\text{Int}\left(\frac{1}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}},x\right) +$$

output
```
-9/16*c*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2)/a-1/8*(a^2*c*x^2+c)^(3/2)*ar
ctan(a*x)^(1/2)/a+3/8*c*x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(3/2)+1/4*x*(a^2
*c*x^2+c)^(3/2)*arctan(a*x)^(3/2)+9/32*c^2*Defer(Int)(1/(a^2*c*x^2+c)^(1/2
)/arctan(a*x)^(1/2),x)+1/16*c*Defer(Int)((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(
1/2),x)+3/8*c^2*Defer(Int)(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.30 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{3/2}\,dx=\int \left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{3/2}\,dx$$

input

```
Integrate[(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2),x]
```

output

```
Integrate[(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}\,dx$$

$$\Big\downarrow \text{ 5415}$$

$$\frac{1}{16}c\int \frac{\sqrt{a^2cx^2+c}}{\sqrt{\arctan(ax)}}dx+\frac{3}{4}c\int \sqrt{a^2cx^2+c}\arctan(ax)^{3/2}dx+\frac{1}{4}x\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}-\frac{\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}{8a}$$

$$\Big\downarrow \text{ 5415}$$

$$\frac{1}{16}c\int \frac{\sqrt{a^2cx^2+c}}{\sqrt{\arctan(ax)}}dx+$$
$$\frac{3}{4}c\left(\frac{3}{8}c\int \frac{1}{\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx+\frac{1}{2}c\int \frac{\arctan(ax)^{3/2}}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}-\frac{3\sqrt{\arctan(ax}}{4}\right.$$
$$\frac{1}{4}x\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}-\frac{\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}{8a}$$

$$\Big\downarrow \text{ 5560}$$

$$\frac{1}{16}c\int \frac{\sqrt{a^2cx^2+c}}{\sqrt{\arctan(ax)}}dx+$$
$$\frac{3}{4}c\left(\frac{3}{8}c\int \frac{1}{\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx+\frac{1}{2}c\int \frac{\arctan(ax)^{3/2}}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}-\frac{3\sqrt{\arctan(ax}}{4}\right.$$
$$\frac{1}{4}x\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}-\frac{\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}{8a}$$

input
```
Int[(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \left(a^{2} c\,x^{2}+c\right)^{\frac{3}{2}} \arctan\left(a x\right)^{\frac{3}{2}}d x$$

input
```
int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x)
```

output
```
int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)**(3/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{3/2}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{3/2}\,dx=\int \mathrm{atan}(a\,x)^{3/2}\,\left(c\,a^2\,x^2+c\right)^{3/2}\,dx$$

input
```
int(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.43

$$\int \left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{3/2}\,dx=\sqrt{c}\,c\left(\left(\int\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}\,atan(ax)\,x^2dx\right)a^2+\int\sqrt{a^2x^2+1}\,\sqrt{atan$$

input
```
int((a^2*c*x^2+c)^(3/2)*atan(a*x)^(3/2),x)
```

output
```
sqrt(c)*c*(int(sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)*x**2,x)*a**2
+ int(sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x),x))
```

# 3.805 $\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}{x}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}{x}\,dx = \text{Int}\left(\frac{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}{x},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2)/x,x)
```

## Mathematica [N/A]

Not integrable

Time = 2.85 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}{x}\,dx = \int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}{x}\,dx$$

input
```
Integrate[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2))/x,x]
```

output
```
Integrate[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2))/x, x]
```

## Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{3/2} \left(a^2cx^2+c\right)^{3/2}}{x} \, dx$$

$$\Big\downarrow \; 5560$$

$$\int \frac{\arctan(ax)^{3/2} \left(a^2cx^2+c\right)^{3/2}}{x} dx$$

input
```
Int[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2))/x,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\left(a^2c\,x^2+c\right)^{\frac{3}{2}} \arctan\left(ax\right)^{\frac{3}{2}}}{x}dx$$

input
```
int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2)/x,x)
```

output
```
int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2)/x,x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2)/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 107.89 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}{x}\,dx = \int \frac{(c(a^2x^2+1))^{\frac{3}{2}}\operatorname{atan}^{\frac{3}{2}}(ax)}{x}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)**(3/2)/x,x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**(3/2)/x, x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}{x}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2)/x,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}{x}\,dx = \int \frac{\mathrm{atan}(a\,x)^{3/2}\,(c\,a^2\,x^2+c)^{3/2}}{x}\,dx$$

input
```
int((atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2))/x,x)
```

output
```
int((atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2))/x, x)
```

## Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 182, normalized size of antiderivative = 7.00

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}{x}\,dx = \frac{\sqrt{c}\,c\Big(8\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}\,atan(ax)\,a^2x^2+32\sqrt{a^2x^2+1}\,\sqrt{atan\,(}$$

input
```
int((a^2*c*x^2+c)^(3/2)*atan(a*x)^(3/2)/x,x)
```

output
```
(sqrt(c)*c*(8*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)*a**2*x**2 + 32
*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x) - 6*sqrt(a**2*x**2 + 1)*sqr
t(atan(a*x))*a*x + 24*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x))/
(a**2*x**3 + x),x) + 3*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x)/(atan(a
*x)*a**2*x**2 + atan(a*x)),x)*a**2 - 42*int((sqrt(a**2*x**2 + 1)*sqrt(atan
(a*x)))/(a**2*x**2 + 1),x)*a))/24
```

# 3.806 $\int x^m(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int x^m\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{3/2}\,dx=\operatorname{Int}\left(x^m\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{3/2},x\right)$$

output
```
Defer(Int)(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.51 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int x^m\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{3/2}\,dx=\int x^m\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{3/2}\,dx$$

input
```
Integrate[x^m*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[x^m*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \arctan(ax)^{3/2} \left(a^2cx^2+c\right)^{5/2} dx$$

$$\Big\downarrow \; 5560$$

$$\int x^m \arctan(ax)^{3/2} \left(a^2cx^2+c\right)^{5/2} dx$$

input
```
Int[x^m*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int x^m \left(a^2 c\, x^2+c\right)^{\frac{5}{2}} \arctan\left(a x\right)^{\frac{3}{2}} dx$$

input
```
int(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.88

$$\int x^m \left(c+a^2cx^2\right)^{5/2} \arctan(ax)^{3/2}\,dx = \int \left(a^2cx^2+c\right)^{\frac{5}{2}} x^m \arctan\left(ax\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*x^m*arcta
n(a*x)^(3/2), x)
```

## Sympy [F(-1)]

Timed out.

$$\int x^m \left(c+a^2cx^2\right)^{5/2} \arctan(ax)^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(x**m*(a**2*c*x**2+c)**(5/2)*atan(a*x)**(3/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int x^m \left(c+a^2cx^2\right)^{5/2} \arctan(ax)^{3/2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int x^m (c + a^2 c x^2)^{5/2} \arctan(ax)^{3/2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int x^m (c + a^2 c x^2)^{5/2} \arctan(ax)^{3/2} \, dx = \int x^m \operatorname{atan}(a\,x)^{3/2} \left(c\,a^2\,x^2 + c\right)^{5/2} dx$$

input
```
int(x^m*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2),x)
```

output
```
int(x^m*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2), x)
```

## Reduce [N/A]

Not integrable

Time = 17.44 (sec) , antiderivative size = 97, normalized size of antiderivative = 3.73

$$\int x^m (c + a^2 c x^2)^{5/2} \arctan(ax)^{3/2} \, dx = \sqrt{c}\, c^2 \left( \left( \int x^m \sqrt{a^2x^2+1}\, \sqrt{atan(ax)}\, atan(ax)\, x^4 dx \right) a^4 + 2 \left( \int x^m \sqrt{a^2x^2 +$$

input
```
int(x^m*(a^2*c*x^2+c)^(5/2)*atan(a*x)^(3/2),x)
```

output
```
sqrt(c)*c**2*(int(x**m*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)*x**4,
x)*a**4 + 2*int(x**m*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)*x**2,x)
*a**2 + int(x**m*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x),x))
```

# 3.807 $\int x^2(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int x^2\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{3/2}\,dx=\operatorname{Int}\left(x^2\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{3/2},x\right)$$

output
```
Defer(Int)(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 3.13 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int x^2\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{3/2}\,dx=\int x^2\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{3/2}\,dx$$

input
```
Integrate[x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arctan(ax)^{3/2} \left(a^2cx^2+c\right)^{5/2} dx$$

$$\Big\downarrow \text{5560}$$

$$\int x^2 \arctan(ax)^{3/2} \left(a^2cx^2+c\right)^{5/2} dx$$

input
```
Int[x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int x^{2} \left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}} \arctan \left(a x \right)^{\frac{3}{2}}d x$$

input
```
int(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x)
```

output
```
int(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int x^2 \left(c + a^2 c x^2\right)^{5/2} \arctan(ax)^{3/2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int x^2 \left(c + a^2 c x^2\right)^{5/2} \arctan(ax)^{3/2} \, dx = \text{Timed out}$$

input
```
integrate(x**2*(a**2*c*x**2+c)**(5/2)*atan(a*x)**(3/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int x^2 \left(c + a^2 c x^2\right)^{5/2} \arctan(ax)^{3/2} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 1.60 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int x^2(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}\,dx=\int(a^2cx^2+c)^{\frac{5}{2}}x^2\arctan(ax)^{\frac{3}{2}}\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)*x^2*arctan(a*x)^(3/2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int x^2(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}\,dx=\int x^2\,\mathrm{atan}(a\,x)^{3/2}\,(c\,a^2\,x^2+c)^{5/2}\,dx$$

input
```
int(x^2*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2),x)
```

output
```
int(x^2*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 91, normalized size of antiderivative = 3.50

$$\int x^2(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}\,dx=\sqrt{c}\,c^2\left(\left(\left(\int\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,atan(ax)\,x^6dx\right)a^4+2\left(\int\sqrt{a^2x^2+1}\,\sqrt{a\right.\right.\right.$$

input
```
int(x^2*(a^2*c*x^2+c)^(5/2)*atan(a*x)^(3/2),x)
```

output
```
sqrt(c)*c**2*(int(sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)*x**6,x)*a*
*4 + 2*int(sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)*x**4,x)*a**2 + in
t(sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)*x**2,x))
```

# 3.808 $\int x(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{3/2}\,dx=\frac{\left(c+a^2cx^2\right)^{7/2}\arctan(ax)^{3/2}}{7a^2c}-\frac{3\text{Int}\left(\left(c+a^2cx^2\right)^{5/2}\sqrt{\arctan(ax)},x\right)}{14a}$$

output
```
1/7*(a^2*c*x^2+c)^(7/2)*arctan(a*x)^(3/2)/a^2/c-3/14*Defer(Int)((a^2*c*x^2
+c)^(5/2)*arctan(a*x)^(1/2),x)/a
```

## Mathematica [N/A]

Not integrable

Time = 5.74 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{3/2}\,dx=\int x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{3/2}\,dx$$

input
```
Integrate[x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \arctan(ax)^{3/2} \left(a^2cx^2+c\right)^{5/2} dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{\arctan(ax)^{3/2} \left(a^2cx^2+c\right)^{7/2}}{7a^2c} - \frac{3\int \left(a^2cx^2+c\right)^{5/2} \sqrt{\arctan(ax)}dx}{14a}$$

$$\Big\downarrow \text{5560}$$

$$\frac{\arctan(ax)^{3/2} \left(a^2cx^2+c\right)^{7/2}}{7a^2c} - \frac{3\int \left(a^2cx^2+c\right)^{5/2} \sqrt{\arctan(ax)}dx}{14a}$$

input
```
Int[x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int x \left(a^2c\,x^2+c\right)^{\frac{5}{2}} \arctan\left(ax\right)^{\frac{3}{2}} dx$$

input
```
int(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x)
```

output
```
int(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int x(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int x(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(x*(a**2*c*x**2+c)**(5/2)*atan(a*x)**(3/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int x(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int x(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int x(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}\,dx = \int x\,\mathrm{atan}(a\,x)^{3/2}\,\left(c\,a^2\,x^2+c\right)^{5/2}\,dx$$

input
```
int(x*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2),x)
```

output
```
int(x*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 340, normalized size of antiderivative = 14.17

$$\int x(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}\,dx = \frac{\sqrt{c}\,c^2\Big(64\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}\,atan(ax)\,a^6x^6+192\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}$$

input
```
int(x*(a^2*c*x^2+c)^(5/2)*atan(a*x)^(3/2),x)
```

output
```
(sqrt(c)*c**2*(64*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)*a**6*x**6
+ 192*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)*a**4*x**4 + 192*sqrt(a
**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)*a**2*x**2 + 64*sqrt(a**2*x**2 + 1)
*sqrt(atan(a*x))*atan(a*x) - 16*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*a**5*x
**5 - 52*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*a**3*x**3 - 66*sqrt(a**2*x**2
 + 1)*sqrt(atan(a*x))*a*x + 8*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**
5)/(atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**6 + 26*int((sqrt(a**2*x**2 + 1)
*sqrt(atan(a*x))*x**3)/(atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**4 + 33*int(
(sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x)/(atan(a*x)*a**2*x**2 + atan(a*x)),
x)*a**2 - 30*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(a**2*x**2 + 1),x)*
a))/(448*a**2)
```

# 3.809 $\int (c + a^2cx^2)^{5/2} \arctan(ax)^{3/2}\, dx$



## Optimal result

Integrand size = 23, antiderivative size = 23

$$\int (c + a^2cx^2)^{5/2} \arctan(ax)^{3/2}\, dx = -\frac{15c^2\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}{32a} - \frac{5c(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}{48a} - \frac{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}{20a} + \frac{5}{16}c^2x\sqrt{c+a^2cx^2}\arctan(ax)^{3/2} + \frac{5}{24}cx(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2} + \frac{1}{6}x(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2} + \frac{15}{64}c$$

output

```
-15/32*c^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2)/a-5/48*c*(a^2*c*x^2+c)^(3
/2)*arctan(a*x)^(1/2)/a-1/20*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2)/a+5/16*
c^2*x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(3/2)+5/24*c*x*(a^2*c*x^2+c)^(3/2)*a
rctan(a*x)^(3/2)+1/6*x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2)+15/64*c^3*Def
er(Int)(1/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)+5/96*c^2*Defer(Int)((a^
2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)+1/40*c*Defer(Int)((a^2*c*x^2+c)^(3/2
)/arctan(a*x)^(1/2),x)+5/16*c^3*Defer(Int)(arctan(a*x)^(3/2)/(a^2*c*x^2+c)
^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{3/2}\,dx=\int \left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{3/2}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.90 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{5/2}\,dx$$

$$\Big\downarrow \text{5415}$$

$$\frac{1}{40}c\int \frac{\left(a^2cx^2+c\right)^{3/2}}{\sqrt{\arctan(ax)}}dx+\frac{5}{6}c\int \left(a^2cx^2+c\right)^{3/2}\arctan(ax)^{3/2}dx+\frac{1}{6}x\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{5/2}-\frac{\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{5/2}}{20a}$$

$$\Big\downarrow \text{5415}$$

$$\frac{1}{40}c\int \frac{\left(a^2cx^2+c\right)^{3/2}}{\sqrt{\arctan(ax)}}dx+$$
$$\frac{5}{6}c\left(\frac{1}{16}c\int \frac{\sqrt{a^2cx^2+c}}{\sqrt{\arctan(ax)}}dx+\frac{3}{4}c\int \sqrt{a^2cx^2+c}\arctan(ax)^{3/2}dx+\frac{1}{4}x\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}-\frac{\sqrt{\arctan}}{}\right.$$
$$\frac{1}{6}x\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{5/2}-\frac{\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{5/2}}{20a}$$

$$\Big\downarrow \text{5415}$$

$$\frac{1}{40}c\int \frac{\left(a^2cx^2+c\right)^{3/2}}{\sqrt{\arctan(ax)}}dx+$$
$$\frac{5}{6}c\left(\frac{1}{16}c\int \frac{\sqrt{a^2cx^2+c}}{\sqrt{\arctan(ax)}}dx+\frac{3}{4}c\left(\frac{3}{8}c\int \frac{1}{\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx+\frac{1}{2}c\int \frac{\arctan(ax)^{3/2}}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)\right.\right.$$
$$\frac{1}{6}x\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{5/2}-\frac{\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{5/2}}{20a}$$

$$\Big\downarrow \text{5560}$$

$$\frac{1}{40}c\int \frac{\left(a^2cx^2+c\right)^{3/2}}{\sqrt{\arctan(ax)}}dx+$$
$$\frac{5}{6}c\left(\frac{1}{16}c\int \frac{\sqrt{a^2cx^2+c}}{\sqrt{\arctan(ax)}}dx+\frac{3}{4}c\left(\frac{3}{8}c\int \frac{1}{\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx+\frac{1}{2}c\int \frac{\arctan(ax)^{3/2}}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)\right.\right.$$
$$\frac{1}{6}x\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{5/2}-\frac{\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{5/2}}{20a}$$

input
```
Int[(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \left(a^{2} c\,x^{2}+c\right)^{\frac{5}{2}} \arctan\left(a x\right)^{\frac{3}{2}}d x$$

input
```
int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x)
```

output
```
int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{3/2}\,dx = \text{Timed out}$$

input
```
integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)**(3/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{3/2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{3/2}\,dx=\int \mathrm{atan}(a\,x)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^{5/2}\,dx$$

input
```
int(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2),x)
```

output
```
int(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 88, normalized size of antiderivative = 3.83

$$\int (c+a^2cx^2)^{5/2} \arctan(ax)^{3/2} \, dx = \sqrt{c}\, c^2 \left( \left( \int \sqrt{a^2x^2+1}\, \sqrt{atan\,(ax)}\, atan(ax)\, x^4 dx \right) a^4 + 2 \left( \int \sqrt{a^2x^2+1}\, \sqrt{a$$

input
```
int((a^2*c*x^2+c)^(5/2)*atan(a*x)^(3/2),x)
```

output
```
sqrt(c)*c**2*(int(sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)*x**4,x)*a*
*4 + 2*int(sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)*x**2,x)*a**2 + in
t(sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x),x))
```

# 3.810 $\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}{x}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}{x}\,dx = \text{Int}\left(\frac{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}{x},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2)/x,x)
```

## Mathematica [N/A]

Not integrable

Time = 2.95 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}{x}\,dx = \int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}{x}\,dx$$

input
```
Integrate[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2))/x,x]
```

output
```
Integrate[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2))/x, x]
```

## Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{3/2} \left(a^2cx^2+c\right)^{5/2}}{x} \, dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int \frac{\arctan(ax)^{3/2} \left(a^2cx^2+c\right)^{5/2}}{x}dx$$

input
```
Int[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2))/x,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\left(a^2c\,x^2+c\right)^{\frac{5}{2}} \arctan\left(ax\right)^{\frac{3}{2}}}{x}dx$$

input
```
int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2)/x,x)
```

output
```
int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2)/x,x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2)/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}{x}\,dx = \text{Timed out}$$

input
```
integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)**(3/2)/x,x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}{x}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2)/x,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}{x}\,dx = \int \frac{\mathrm{atan}(a\,x)^{3/2}\,(c\,a^2\,x^2+c)^{5/2}}{x}\,dx$$

input
```
int((atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2))/x,x)
```

output
```
int((atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2))/x, x)
```

## Reduce [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 278, normalized size of antiderivative = 10.69

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}{x}\,dx = \frac{\sqrt{c}\,c^2\left(96\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}\,atan(ax)\,a^4x^4+352\sqrt{a^2x^2+1}\,\sqrt{ata}\right.}{}$$

input
```
int((a^2*c*x^2+c)^(5/2)*atan(a*x)^(3/2)/x,x)
```

output
```
(sqrt(c)*c**2*(96*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)*a**4*x**4
+ 352*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)*a**2*x**2 + 736*sqrt(a
**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x) - 36*sqrt(a**2*x**2 + 1)*sqrt(atan
(a*x))*a**3*x**3 - 210*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*a*x + 480*int((
sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x))/(a**2*x**3 + x),x) + 18*int
((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**3)/(atan(a*x)*a**2*x**2 + atan(a*
x)),x)*a**4 + 105*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x)/(atan(a*x)*a
**2*x**2 + atan(a*x)),x)*a**2 - 894*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x
)))/(a**2*x**2 + 1),x)*a))/480
```

# 3.811 $\int \frac{x^m \arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^m \arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx=\text{Int}\left(\frac{x^m \arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}},x\right)$$

output
```
Defer(Int)(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^m \arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx=\int \frac{x^m \arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx$$

input
```
Integrate[(x^m*ArcTan[a*x]^(3/2))/Sqrt[c + a^2*c*x^2],x]
```

output
```
Integrate[(x^m*ArcTan[a*x]^(3/2))/Sqrt[c + a^2*c*x^2], x]
```

## Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \arctan(ax)^{3/2}}{\sqrt{a^2cx^2+c}}\,dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int \frac{x^m \arctan(ax)^{3/2}}{\sqrt{a^2cx^2+c}}dx$$

input
```
Int[(x^m*ArcTan[a*x]^(3/2))/Sqrt[c + a^2*c*x^2],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^m \arctan\left(ax\right)^{\frac{3}{2}}}{\sqrt{a^2c\,x^2+c}}dx$$

input
```
int(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m \arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^m \arctan{(ax)}^{\frac{3}{2}}}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(x^m*arctan(a*x)^(3/2)/sqrt(a^2*c*x^2 + c), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^m \arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx = \text{Timed out}$$

input
```
integrate(x**m*atan(a*x)**(3/2)/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m \arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m \arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^m \arctan\left(ax\right)^{\frac{3}{2}}}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^m*arctan(a*x)^(3/2)/sqrt(a^2*c*x^2 + c), x)
```

## Mupad [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m \arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^m \operatorname{atan}(a\,x)^{3/2}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int((x^m*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int((x^m*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(1/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.62

$$\int \frac{x^m \arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{c}\left(\int \frac{x^m\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,atan(ax)}{a^2x^2+1}dx\right)}{c}$$

input
```
int(x^m*atan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

output

```
(sqrt(c)*int((x**m*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x))/(a**2*x*
*2 + 1),x))/c
```

# 3.812 $\int \frac{x^3 \arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^3 \arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx = -\frac{x\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}{4a^3c} - \frac{2\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}{3a^4c} + \frac{x^2\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}{3a^2c} + \frac{\operatorname{Int}\left(\frac{x}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}},x\right)}{8a^2} + \frac{5\operatorname{Int}\left(\frac{\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}},x\right)}{4a^3}$$

output
```
-1/4*x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2)/a^3/c-2/3*(a^2*c*x^2+c)^(1/2)
*arctan(a*x)^(3/2)/a^4/c+1/3*x^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(3/2)/a^2
/c+1/8*Defer(Int)(x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)/a^2+5/4*Defer
(Int)(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x)/a^3
```

## Mathematica [N/A]

Not integrable

Time = 3.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^3 \arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^3 \arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx$$

input
```
Integrate[(x^3*ArcTan[a*x]^(3/2))/Sqrt[c + a^2*c*x^2],x]
```

output
```
Integrate[(x^3*ArcTan[a*x]^(3/2))/Sqrt[c + a^2*c*x^2], x]
```

## Rubi [N/A]

Not integrable

Time = 1.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\arctan(ax)^{3/2}}{\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \textbf{5487}$$

$$-\frac{\int \frac{x^2\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}}dx}{2a}-\frac{2\int \frac{x\arctan(ax)^{3/2}}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}{3a^2c}$$

$$\Big\downarrow \textbf{5465}$$

$$-\frac{2\left(\frac{\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\int \frac{\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}}dx}{2a}\right)}{3a^2}-\frac{\int \frac{x^2\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}}dx}{2a}+\frac{x^2\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}{3a^2c}$$

$$\Big\downarrow \textbf{5487}$$

$$-\frac{-\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx}{4a}-\frac{\int \frac{\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}}dx}{2a^2}+\frac{x\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}{2a^2c}}{2a}-\frac{2\left(\frac{\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\int \frac{\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}}dx}{2a}\right)}{3a^2}+\frac{x^2\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}{3a^2c}$$

$$\Big\downarrow \textbf{5560}$$

$$-\frac{-\frac{\int \frac{x}{\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx}{4a}-\frac{\int \frac{\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}}dx}{2a^2}+\frac{x\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}{2a^2c}}{2a}-\frac{2\left(\frac{\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\int \frac{\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}}dx}{2a}\right)}{3a^2}+\frac{x^2\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}{3a^2c}$$

input
```
Int[(x^3*ArcTan[a*x]^(3/2))/Sqrt[c + a^2*c*x^2],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^{3} \arctan \left(a x \right)^{\frac{3}{2}}}{\sqrt{a^{2} c \,x^{2}+c}}d x$$

input
```
int(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3 \arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 96.49 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{x^3 \arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^3 \operatorname{atan}^{\frac{3}{2}}(ax)}{\sqrt{c(a^2x^2+1)}}\,dx$$

input
```
integrate(x**3*atan(a*x)**(3/2)/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x**3*atan(a*x)**(3/2)/sqrt(c*(a**2*x**2 + 1)), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3 \arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x^3 \arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^3 \arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^3\,\mathrm{atan}(a\,x)^{3/2}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int((x^3*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int((x^3*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(1/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 152, normalized size of antiderivative = 5.85

$$\int \frac{x^3 \arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{c}\left(8\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}\,atan(ax)\,a^2x^2-16\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}\,atan(ax)\right.}{}$$

input
```
int(x^3*atan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

output
```
(sqrt(c)*(8*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)*a**2*x**2 - 16*s
qrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x) - 6*sqrt(a**2*x**2 + 1)*sqrt(
atan(a*x))*a*x + 3*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x)/(atan(a*x)*
a**2*x**2 + atan(a*x)),x)*a**2 + 30*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x
)))/(a**2*x**2 + 1),x)*a))/(24*a**4*c)
```

# 3.813 $\int \frac{x^2 \arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^2 \arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx = -\frac{3\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}{4a^3c} + \frac{x\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}{2a^2c} + \frac{3\text{Int}\left(\frac{1}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}},x\right)}{8a^2} - \frac{\text{Int}\left(\frac{\arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}},x\right)}{2a^2}$$

output
```
-3/4*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2)/a^3/c+1/2*x*(a^2*c*x^2+c)^(1/2)
*arctan(a*x)^(3/2)/a^2/c+3/8*Defer(Int)(1/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^
(1/2),x)/a^2-1/2*Defer(Int)(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)/a^2
```

## Mathematica [N/A]

Not integrable

Time = 2.68 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^2 \arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^2 \arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx$$

input
```
Integrate[(x^2*ArcTan[a*x]^(3/2))/Sqrt[c + a^2*c*x^2],x]
```

output
```
Integrate[(x^2*ArcTan[a*x]^(3/2))/Sqrt[c + a^2*c*x^2], x]
```

## Rubi [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \arctan(ax)^{3/2}}{\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5487}$$

$$-\frac{3\int \frac{x\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}}dx}{4a}-\frac{\int \frac{\arctan(ax)^{3/2}}{\sqrt{a^2cx^2+c}}dx}{2a^2}+\frac{x\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}{2a^2c}$$

$$\Big\downarrow \text{5465}$$

$$-\frac{3\left(\frac{\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}{a^2c}-\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx}{2a}\right)}{4a}-\frac{\int \frac{\arctan(ax)^{3/2}}{\sqrt{a^2cx^2+c}}dx}{2a^2}+\frac{x\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}{2a^2c}$$

$$\Big\downarrow \text{5560}$$

$$-\frac{3\left(\frac{\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}{a^2c}-\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx}{2a}\right)}{4a}-\frac{\int \frac{\arctan(ax)^{3/2}}{\sqrt{a^2cx^2+c}}dx}{2a^2}+\frac{x\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}{2a^2c}$$

input
```
Int[(x^2*ArcTan[a*x]^(3/2))/Sqrt[c + a^2*c*x^2],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^2 \arctan(ax)^{\frac{3}{2}}}{\sqrt{a^2 c x^2 + c}} dx$$

input
```
int(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2 \arctan(ax)^{3/2}}{\sqrt{c + a^2 c x^2}} dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 50.87 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{x^2 \arctan(ax)^{3/2}}{\sqrt{c + a^2 c x^2}} dx = \int \frac{x^2 \operatorname{atan}^{\frac{3}{2}}(ax)}{\sqrt{c(a^2 x^2 + 1)}} dx$$

input
```
integrate(x**2*atan(a*x)**(3/2)/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x**2*atan(a*x)**(3/2)/sqrt(c*(a**2*x**2 + 1)), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2 \arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^2 \arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^2 \arctan(ax)^{\frac{3}{2}}}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2*arctan(a*x)^(3/2)/sqrt(a^2*c*x^2 + c), x)
```

## Mupad [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^2 \arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^2\,\mathrm{atan}(a\,x)^{3/2}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int((x^2*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int((x^2*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(1/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.62

$$\int \frac{x^2 \arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,atan(ax)x^2}{a^2x^2+1}dx\right)}{c}$$

input
```
int(x^2*atan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

output
```
(sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)*x**2)/(a**2*x*
*2 + 1),x))/c
```

# 3.814 $\int \frac{x\arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x\arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx=\frac{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}{a^2c}-\frac{3\text{Int}\left(\frac{\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}},x\right)}{2a}$$

output
```
(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(3/2)/a^2/c-3/2*Defer(Int)(arctan(a*x)^(1/
2)/(a^2*c*x^2+c)^(1/2),x)/a
```

## Mathematica [N/A]

Not integrable

Time = 0.86 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x\arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx=\int \frac{x\arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx$$

input
```
Integrate[(x*ArcTan[a*x]^(3/2))/Sqrt[c + a^2*c*x^2],x]
```

output
```
Integrate[(x*ArcTan[a*x]^(3/2))/Sqrt[c + a^2*c*x^2], x]
```

## Rubi [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \arctan(ax)^{3/2}}{\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\int \frac{\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}}dx}{2a}$$

$$\Big\downarrow \text{5560}$$

$$\frac{\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\int \frac{\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}}dx}{2a}$$

input
```
Int[(x*ArcTan[a*x]^(3/2))/Sqrt[c + a^2*c*x^2],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{x \arctan\left(ax\right)^{\frac{3}{2}}}{\sqrt{a^{2}c\,x^{2}+c}}dx$$

input
```
int(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x\arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 28.68 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x\arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x\operatorname{atan}^{\frac{3}{2}}(ax)}{\sqrt{c\,(a^2x^2+1)}}\,dx$$

input
```
integrate(x*atan(a*x)**(3/2)/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(x*atan(a*x)**(3/2)/sqrt(c*(a**2*x**2 + 1)), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x \arctan(ax)^{3/2}}{\sqrt{c + a^2cx^2}} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x \arctan(ax)^{3/2}}{\sqrt{c + a^2cx^2}} \, dx = \int \frac{x \arctan{(ax)}^{\frac{3}{2}}}{\sqrt{a^2cx^2 + c}} \, dx$$

input
```
integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x*arctan(a*x)^(3/2)/sqrt(a^2*c*x^2 + c), x)
```

## Mupad [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x \arctan(ax)^{3/2}}{\sqrt{c + a^2cx^2}} \, dx = \int \frac{x \,\mathrm{atan}(a\,x)^{3/2}}{\sqrt{c\,a^2\,x^2 + c}} \, dx$$

input
```
int((x*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int((x*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(1/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.67

$$\int \frac{x \arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\, dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,atan(ax)x}{a^2x^2+1}dx\right)}{c}$$

input
```
int(x*atan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

output
```
(sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)*x)/(a**2*x**2
+ 1),x))/c
```

# 3.815 $\int \frac{\arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx = \text{Int}\left(\frac{\arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}},x\right)$$

output
```
Defer(Int)(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx$$

input
```
Integrate[ArcTan[a*x]^(3/2)/Sqrt[c + a^2*c*x^2],x]
```

output
```
Integrate[ArcTan[a*x]^(3/2)/Sqrt[c + a^2*c*x^2], x]
```

## Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{3/2}}{\sqrt{a^2cx^2+c}}\,dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{\arctan(ax)^{3/2}}{\sqrt{a^2cx^2+c}}dx$$

input
```
Int[ArcTan[a*x]^(3/2)/Sqrt[c + a^2*c*x^2],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{\arctan\left(ax\right)^{\frac{3}{2}}}{\sqrt{a^2c\,x^2+c}}dx$$

input
```
int(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 10.71 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{\arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{\sqrt{c\,(a^2x^2+1)}}\,dx$$

input
```
integrate(atan(a*x)**(3/2)/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(atan(a*x)**(3/2)/sqrt(c*(a**2*x**2 + 1)), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan{(ax)}^{\frac{3}{2}}}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^(3/2)/sqrt(a^2*c*x^2 + c), x)
```

## Mupad [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\mathrm{atan}(a\,x)^{3/2}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(atan(a*x)^(3/2)/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(atan(a*x)^(3/2)/(c + a^2*c*x^2)^(1/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.70

$$\int \frac{\arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,atan(ax)}{a^2x^2+1}dx\right)}{c}$$

input
```
int(atan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)
```

output
```
(sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x))/(a**2*x**2 +
1),x))/c
```

# 3.816 $\int \frac{\arctan(ax)^{3/2}}{x\sqrt{c+a^2cx^2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\arctan(ax)^{3/2}}{x\sqrt{c+a^2cx^2}}\,dx = \operatorname{Int}\left(\frac{\arctan(ax)^{3/2}}{x\sqrt{c+a^2cx^2}},x\right)$$

output
```
Defer(Int)(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.92 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\arctan(ax)^{3/2}}{x\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan(ax)^{3/2}}{x\sqrt{c+a^2cx^2}}\,dx$$

input
```
Integrate[ArcTan[a*x]^(3/2)/(x*Sqrt[c + a^2*c*x^2]),x]
```

output
```
Integrate[ArcTan[a*x]^(3/2)/(x*Sqrt[c + a^2*c*x^2]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{3/2}}{x\sqrt{a^2cx^2+c}}\,dx$$

$$\Big\downarrow \ 5560$$

$$\int \frac{\arctan(ax)^{3/2}}{x\sqrt{a^2cx^2+c}}dx$$

input
```
Int[ArcTan[a*x]^(3/2)/(x*Sqrt[c + a^2*c*x^2]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\arctan\left(ax\right)^{\frac{3}{2}}}{x\sqrt{a^2c\,x^2+c}}dx$$

input
```
int(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{x\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 12.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\arctan(ax)^{3/2}}{x\sqrt{c+a^2cx^2}}\,dx = \int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{x\sqrt{c\,(a^2x^2+1)}}\,dx$$

input
```
integrate(atan(a*x)**(3/2)/x/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(atan(a*x)**(3/2)/(x*sqrt(c*(a**2*x**2 + 1))), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{x\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\arctan(ax)^{3/2}}{x\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan(ax)^{\frac{3}{2}}}{\sqrt{a^2cx^2+c}x}\,dx$$

input
```
integrate(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^(3/2)/(sqrt(a^2*c*x^2 + c)*x), x)
```

## Mupad [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\arctan(ax)^{3/2}}{x\sqrt{c+a^2cx^2}}\,dx = \int \frac{\mathrm{atan}(a\,x)^{3/2}}{x\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(atan(a*x)^(3/2)/(x*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(atan(a*x)^(3/2)/(x*(c + a^2*c*x^2)^(1/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.50

$$\int \frac{\arctan(ax)^{3/2}}{x\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,atan(ax)}{a^2x^3+x}dx\right)}{c}$$

input
```
int(atan(a*x)^(3/2)/x/(a^2*c*x^2+c)^(1/2),x)
```

output
```
(sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x))/(a**2*x**3 +
x),x))/c
```

# 3.817 $\int \frac{\arctan(ax)^{3/2}}{x^2\sqrt{c+a^2cx^2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\arctan(ax)^{3/2}}{x^2\sqrt{c+a^2cx^2}}\,dx = -\frac{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}{cx} + \frac{3}{2}a\text{Int}\left(\frac{\sqrt{\arctan(ax)}}{x\sqrt{c+a^2cx^2}},x\right)$$

output
```
-(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(3/2)/c/x+3/2*a*Defer(Int)(arctan(a*x)^(1
/2)/x/(a^2*c*x^2+c)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.95 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\arctan(ax)^{3/2}}{x^2\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan(ax)^{3/2}}{x^2\sqrt{c+a^2cx^2}}\,dx$$

input
```
Integrate[ArcTan[a*x]^(3/2)/(x^2*Sqrt[c + a^2*c*x^2]),x]
```

output
```
Integrate[ArcTan[a*x]^(3/2)/(x^2*Sqrt[c + a^2*c*x^2]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{3/2}}{x^2\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{ 5479}$$

$$\frac{3}{2}a\int \frac{\sqrt{\arctan(ax)}}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}{cx}$$

$$\Big\downarrow \text{ 5560}$$

$$\frac{3}{2}a\int \frac{\sqrt{\arctan(ax)}}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}{cx}$$

input
```
Int[ArcTan[a*x]^(3/2)/(x^2*Sqrt[c + a^2*c*x^2]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\arctan\left(ax\right)^{\frac{3}{2}}}{x^2\sqrt{a^2c\,x^2+c}}dx$$

input
```
int(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{x^2\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 15.65 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax)^{3/2}}{x^2\sqrt{c+a^2cx^2}}\,dx = \int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{x^2\sqrt{c(a^2x^2+1)}}\,dx$$

input
```
integrate(atan(a*x)**(3/2)/x**2/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(atan(a*x)**(3/2)/(x**2*sqrt(c*(a**2*x**2 + 1))), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{x^2\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\arctan(ax)^{3/2}}{x^2\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan(ax)^{\frac{3}{2}}}{\sqrt{a^2cx^2+cx^2}}\,dx$$

input
```
integrate(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^(3/2)/(sqrt(a^2*c*x^2 + c)*x^2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\arctan(ax)^{3/2}}{x^2\sqrt{c+a^2cx^2}}\,dx = \int \frac{\mathrm{atan}(a\,x)^{3/2}}{x^2\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(atan(a*x)^(3/2)/(x^2*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(atan(a*x)^(3/2)/(x^2*(c + a^2*c*x^2)^(1/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.50

$$\int \frac{\arctan(ax)^{3/2}}{x^2\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{c}\left(-2\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,atan(ax)+3\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}}{a^2x^3+x}dx\right)ax\right)}{2cx}$$

input
```
int(atan(a*x)^(3/2)/x^2/(a^2*c*x^2+c)^(1/2),x)
```

output

```
(sqrt(c)*( - 2*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x) + 3*int((sqrt
(a**2*x**2 + 1)*sqrt(atan(a*x)))/(a**2*x**3 + x),x)*a*x))/(2*c*x)
```

# 3.818 $\int \frac{\arctan(ax)^{3/2}}{x^3\sqrt{c+a^2cx^2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\arctan(ax)^{3/2}}{x^3\sqrt{c+a^2cx^2}}\,dx = -\frac{3a\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}{4cx} - \frac{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}{2cx^2} + \frac{3}{8}a^2\text{Int}\left(\frac{1}{x\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}},x\right) - \frac{1}{2}a^2\text{Int}\left(\frac{\arctan(ax)^{3/2}}{x\sqrt{c+a^2cx^2}},x\right)$$

output
```
-3/4*a*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2)/c/x-1/2*(a^2*c*x^2+c)^(1/2)*a
rctan(a*x)^(3/2)/c/x^2+3/8*a^2*Defer(Int)(1/x/(a^2*c*x^2+c)^(1/2)/arctan(a
*x)^(1/2),x)-1/2*a^2*Defer(Int)(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 6.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\arctan(ax)^{3/2}}{x^3\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan(ax)^{3/2}}{x^3\sqrt{c+a^2cx^2}}\,dx$$

input
```
Integrate[ArcTan[a*x]^(3/2)/(x^3*Sqrt[c + a^2*c*x^2]),x]
```

output
```
Integrate[ArcTan[a*x]^(3/2)/(x^3*Sqrt[c + a^2*c*x^2]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan (a x)^{3/2}}{x^3 \sqrt{a^2 c x^2+c}} \, dx$$

$$\Big\downarrow \text{5497}$$

$$-\frac{1}{2} a^2 \int \frac{\arctan (a x)^{3/2}}{x \sqrt{a^2 c x^2+c}}dx+\frac{3}{4} a \int \frac{\sqrt{\arctan (a x)}}{x^2 \sqrt{a^2 c x^2+c}}dx-\frac{\arctan (a x)^{3/2} \sqrt{a^2 c x^2+c}}{2 c x^2}$$

$$\Big\downarrow \text{5479}$$

$$-\frac{1}{2} a^2 \int \frac{\arctan (a x)^{3/2}}{x \sqrt{a^2 c x^2+c}}dx+\frac{3}{4} a \left(\frac{1}{2} a \int \frac{1}{x \sqrt{a^2 c x^2+c} \sqrt{\arctan (a x)}}dx-\frac{\sqrt{\arctan (a x)} \sqrt{a^2 c x^2+c}}{c x}\right)-\frac{\arctan (a x)^{3/2} \sqrt{a^2 c x^2+c}}{2 c x^2}$$

$$\Big\downarrow \text{5560}$$

$$-\frac{1}{2} a^2 \int \frac{\arctan (a x)^{3/2}}{x \sqrt{a^2 c x^2+c}}dx+\frac{3}{4} a \left(\frac{1}{2} a \int \frac{1}{x \sqrt{a^2 c x^2+c} \sqrt{\arctan (a x)}}dx-\frac{\sqrt{\arctan (a x)} \sqrt{a^2 c x^2+c}}{c x}\right)-\frac{\arctan (a x)^{3/2} \sqrt{a^2 c x^2+c}}{2 c x^2}$$

input
```
Int[ArcTan[a*x]^(3/2)/(x^3*Sqrt[c + a^2*c*x^2]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\arctan (ax)^{\frac{3}{2}}}{x^3\sqrt{a^2c\,x^2+c}}dx$$

input
```
int(arctan(a*x)^(3/2)/x^3/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(arctan(a*x)^(3/2)/x^3/(a^2*c*x^2+c)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{x^3\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(3/2)/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 28.46 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax)^{3/2}}{x^3\sqrt{c+a^2cx^2}}\,dx = \int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{x^3\sqrt{c\,(a^2x^2+1)}}\,dx$$

input
```
integrate(atan(a*x)**(3/2)/x**3/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(atan(a*x)**(3/2)/(x**3*sqrt(c*(a**2*x**2 + 1))), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{x^3\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(3/2)/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\arctan(ax)^{3/2}}{x^3\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan(ax)^{\frac{3}{2}}}{\sqrt{a^2cx^2+c}\,x^3}\,dx$$

input
```
integrate(arctan(a*x)^(3/2)/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^(3/2)/(sqrt(a^2*c*x^2 + c)*x^3), x)
```

## Mupad [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\arctan(ax)^{3/2}}{x^3\sqrt{c+a^2cx^2}}\,dx = \int \frac{\mathrm{atan}(a\,x)^{3/2}}{x^3\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(atan(a*x)^(3/2)/(x^3*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(atan(a*x)^(3/2)/(x^3*(c + a^2*c*x^2)^(1/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.58

$$\int \frac{\arctan(ax)^{3/2}}{x^3\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,atan(ax)}{a^2x^5+x^3}dx\right)}{c}$$

input
```
int(atan(a*x)^(3/2)/x^3/(a^2*c*x^2+c)^(1/2),x)
```

output
```
(sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x))/(a**2*x**5 +
x**3),x))/c
```

# 3.819 $\int \frac{\arctan(ax)^{3/2}}{x^4\sqrt{c+a^2cx^2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\begin{aligned}\int \frac{\arctan(ax)^{3/2}}{x^4\sqrt{c+a^2cx^2}}\,dx &= -\frac{a\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}{4cx^2} \\ &- \frac{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}{3cx^3} + \frac{2a^2\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}{3cx} \\ &+ \frac{1}{8}a^2\text{Int}\left(\frac{1}{x^2\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}},x\right) - \frac{5}{4}a^3\text{Int}\left(\frac{\sqrt{\arctan(ax)}}{x\sqrt{c+a^2cx^2}},x\right)\end{aligned}$$

output
```
-1/4*a*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2)/c/x^2-1/3*(a^2*c*x^2+c)^(1/2)
*arctan(a*x)^(3/2)/c/x^3+2/3*a^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(3/2)/c/x
+1/8*a^2*Defer(Int)(1/x^2/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)-5/4*a^3
*Defer(Int)(arctan(a*x)^(1/2)/x/(a^2*c*x^2+c)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 15.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\arctan(ax)^{3/2}}{x^4\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan(ax)^{3/2}}{x^4\sqrt{c+a^2cx^2}}\,dx$$

input
```
Integrate[ArcTan[a*x]^(3/2)/(x^4*Sqrt[c + a^2*c*x^2]),x]
```

output
```
Integrate[ArcTan[a*x]^(3/2)/(x^4*Sqrt[c + a^2*c*x^2]), x]
```

## Rubi [N/A]

Not integrable

Time = 1.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{3/2}}{x^4\sqrt{a^2cx^2+c}}\,dx$$

$$\Big\downarrow \text{ 5497}$$

$$-\frac{2}{3}a^2\int \frac{\arctan(ax)^{3/2}}{x^2\sqrt{a^2cx^2+c}}dx+\frac{1}{2}a\int \frac{\sqrt{\arctan(ax)}}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}{3cx^3}$$

$$\Big\downarrow \text{ 5479}$$

$$-\frac{2}{3}a^2\left(\frac{3}{2}a\int \frac{\sqrt{\arctan(ax)}}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}{cx}\right)+\frac{1}{2}a\int \frac{\sqrt{\arctan(ax)}}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}{3cx^3}$$

$$\Big\downarrow \text{ 5497}$$

$$-\frac{2}{3}a^2\left(\frac{3}{2}a\int\frac{\sqrt{\arctan(ax)}}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}{cx}\right)+\frac{1}{2}a\left(-\frac{1}{2}a^2\int\frac{\sqrt{\arctan(ax)}}{x\sqrt{a^2cx^2+c}}dx+\frac{1}{4}a\int\frac{1}{x^2\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx-\frac{\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}{2cx^2}\right)-\frac{\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}{3cx^3}$$

$$\Big\downarrow \text{5560}$$

$$-\frac{2}{3}a^2\left(\frac{3}{2}a\int\frac{\sqrt{\arctan(ax)}}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}{cx}\right)+\frac{1}{2}a\left(-\frac{1}{2}a^2\int\frac{\sqrt{\arctan(ax)}}{x\sqrt{a^2cx^2+c}}dx+\frac{1}{4}a\int\frac{1}{x^2\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx-\frac{\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}{2cx^2}\right)-\frac{\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}{3cx^3}$$

input
```
Int[ArcTan[a*x]^(3/2)/(x^4*Sqrt[c + a^2*c*x^2]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int\frac{\arctan(ax)^{\frac{3}{2}}}{x^4\sqrt{a^2c\,x^2+c}}dx$$

input
```
int(arctan(a*x)^(3/2)/x^4/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(arctan(a*x)^(3/2)/x^4/(a^2*c*x^2+c)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{x^4\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(3/2)/x^4/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 49.79 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax)^{3/2}}{x^4\sqrt{c+a^2cx^2}}\,dx = \int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{x^4\sqrt{c\,(a^2x^2+1)}}\,dx$$

input
```
integrate(atan(a*x)**(3/2)/x**4/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(atan(a*x)**(3/2)/(x**4*sqrt(c*(a**2*x**2 + 1))), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{x^4\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(3/2)/x^4/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.90 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\arctan(ax)^{3/2}}{x^4\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan{(ax)}^{\frac{3}{2}}}{\sqrt{a^2cx^2+cx^4}}\,dx$$

input
```
integrate(arctan(a*x)^(3/2)/x^4/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^(3/2)/(sqrt(a^2*c*x^2 + c)*x^4), x)
```

## Mupad [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\arctan(ax)^{3/2}}{x^4\sqrt{c+a^2cx^2}}\,dx = \int \frac{\mathrm{atan}(a\,x)^{3/2}}{x^4\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(atan(a*x)^(3/2)/(x^4*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(atan(a*x)^(3/2)/(x^4*(c + a^2*c*x^2)^(1/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 163, normalized size of antiderivative = 6.27

$$\int \frac{\arctan(ax)^{3/2}}{x^4\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{c}\left(4\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}\,atan(ax)\,a^2x^2-2\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}\,atan(ax)+6\right.}{}$$

input
```
int(atan(a*x)^(3/2)/x^4/(a^2*c*x^2+c)^(1/2),x)
```

output
```
(sqrt(c)*(4*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)*a**2*x**2 - 2*sq
rt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x) + 6*sqrt(a**2*x**2 + 1)*sqrt(a
tan(a*x))*a*x - 3*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(atan(a*x)*a**
2*x**4 + atan(a*x)*x**2),x)*a**2*x**3 + 15*int((sqrt(a**2*x**2 + 1)*sqrt(a
tan(a*x)))/(a**2*x**5 + x**3),x)*a*x**3))/(6*c*x**3)
```

# 3.820 $\int \frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Int}\left(\frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}},x\right)$$

output
```
Defer(Int)(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.48 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx$$

input
```
Integrate[(x^m*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(3/2),x]
```

output
```
Integrate[(x^m*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(3/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \arctan(ax)^{3/2}}{(a^2cx^2+c)^{3/2}} \, dx$$

$$\Big\downarrow \; 5560$$

$$\int \frac{x^m \arctan(ax)^{3/2}}{(a^2cx^2+c)^{3/2}}dx$$

input
```
Int[(x^m*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(3/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^m \arctan\left(ax\right)^{\frac{3}{2}}}{\left(a^2c\,x^2+c\right)^{\frac{3}{2}}}dx$$

input
```
int(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x)
```

output
```
int(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.96

$$\int \frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^m \arctan(ax)^{\frac{3}{2}}}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^m*arctan(a*x)^(3/2)/(a^4*c^2*x^4 + 2*a^2*c^
2*x^2 + c^2), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**m*atan(a*x)**(3/2)/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^m \arctan(ax)^{\frac{3}{2}}}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2 + c)^(3/2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^m\,\mathrm{atan}(a\,x)^{3/2}}{(c\,a^2\,x^2+c)^{3/2}}\,dx$$

input
```
int((x^m*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(3/2),x)
```

output
```
int((x^m*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(3/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.92

$$\int \frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{c}\left(\int \frac{x^m\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,atan(ax)}{a^4x^4+2a^2x^2+1}dx\right)}{c^2}$$

input
```
int(x^m*atan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x)
```

output
```
(sqrt(c)*int((x**m*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x))/(a**4*x*
*4 + 2*a**2*x**2 + 1),x))/c**2
```

# 3.821 $\int \frac{x^3 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^3 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Int}\left(\frac{x^3 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}},x\right)$$

output
```
Defer(Int)(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 5.96 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^3 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^3 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx$$

input
```
Integrate[(x^3*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(3/2),x]
```

output
```
Integrate[(x^3*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(3/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \arctan(ax)^{3/2}}{(a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \text{5560}$$

$$\int \frac{x^3 \arctan(ax)^{3/2}}{(a^2cx^2+c)^{3/2}}dx$$

input
```
Int[(x^3*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(3/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^3 \arctan (ax)^{\frac{3}{2}}}{(a^2c\,x^2+c)^{\frac{3}{2}}}dx$$

input
```
int(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x)
```

output
```
int(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 102.35 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{x^3 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^3 \operatorname{atan}^{\frac{3}{2}}(ax)}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3*atan(a*x)**(3/2)/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(x**3*atan(a*x)**(3/2)/(c*(a**2*x**2 + 1))**(3/2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x^3 \arctan(ax)^{3/2}}{(c + a^2cx^2)^{3/2}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^3 \arctan(ax)^{3/2}}{(c + a^2cx^2)^{3/2}} \, dx = \int \frac{x^3 \operatorname{atan}(a\,x)^{3/2}}{(c\,a^2\,x^2 + c)^{3/2}} \, dx$$

input
```
int((x^3*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(3/2),x)
```

output
```
int((x^3*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(3/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.92

$$\int \frac{x^3 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,atan(ax)x^3}{a^4x^4+2a^2x^2+1}dx\right)}{c^2}$$

input
```
int(x^3*atan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x)
```

output
```
(sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)*x**3)/(a**4*x*
*4 + 2*a**2*x**2 + 1),x))/c**2
```

# 3.822 $\int \frac{x^2 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}} \, dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^2 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}} \, dx = \text{Int}\left(\frac{x^2 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}, x\right)$$

output
```
Defer(Int)(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 4.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^2 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}} \, dx = \int \frac{x^2 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}} \, dx$$

input
```
Integrate[(x^2*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(3/2),x]
```

output
```
Integrate[(x^2*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(3/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \arctan(ax)^{3/2}}{(a^2cx^2+c)^{3/2}} dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int \frac{x^2 \arctan(ax)^{3/2}}{(a^2cx^2+c)^{3/2}} dx$$

input
```
Int[(x^2*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(3/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^2 \arctan(ax)^{\frac{3}{2}}}{(a^2c\,x^2+c)^{\frac{3}{2}}} dx$$

input
```
int(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x)
```

output
```
int(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 55.33 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{x^2 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^2 \operatorname{atan}^{\frac{3}{2}}(ax)}{(c(a^2x^2+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2*atan(a*x)**(3/2)/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(x**2*atan(a*x)**(3/2)/(c*(a**2*x**2 + 1))**(3/2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^2 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^2 \arctan\left(ax\right)^{\frac{3}{2}}}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2 + c)^(3/2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^2 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^2 \operatorname{atan}(a\,x)^{3/2}}{\left(c\,a^2\,x^2+c\right)^{3/2}}\,dx$$

input
```
int((x^2*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(3/2),x)
```

output
```
int((x^2*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(3/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.92

$$\int \frac{x^2 \arctan(ax)^{3/2}}{(c + a^2cx^2)^{3/2}}\, dx = \frac{\sqrt{c} \left( \int \frac{\sqrt{a^2x^2+1}\, \sqrt{atan(ax)}\, atan(ax)x^2}{a^4x^4+2a^2x^2+1} dx \right)}{c^2}$$

input
```
int(x^2*atan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x)
```

output
```
(sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)*x**2)/(a**4*x*
*4 + 2*a**2*x**2 + 1),x))/c**2
```

# 3.823 $\int \frac{x \arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}} \, dx$



## Optimal result

Integrand size = 24, antiderivative size = 129

$$\int \frac{x \arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}} \, dx = \frac{3x\sqrt{\arctan(ax)}}{2ac\sqrt{c+a^2cx^2}} - \frac{\arctan(ax)^{3/2}}{a^2c\sqrt{c+a^2cx^2}} - \frac{3\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{2a^2c\sqrt{c+a^2cx^2}}$$

output
```
3/2*x*arctan(a*x)^(1/2)/a/c/(a^2*c*x^2+c)^(1/2)-arctan(a*x)^(3/2)/a^2/c/(a
^2*c*x^2+c)^(1/2)-3/4*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)*FresnelS(2^(1/2)/
Pi^(1/2)*arctan(a*x)^(1/2))/a^2/c/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.99

$$\int \frac{x \arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}} \, dx = \frac{4(3ax-2\arctan(ax))\arctan(ax)+3\sqrt{1+a^2x^2}\sqrt{-i\arctan(ax)}\Gamma\left(\frac{1}{2},-i\arctan(a\right.}{8a^2c\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}$$

input
```
Integrate[(x*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(3/2),x]
```

output
```
(4*(3*a*x - 2*ArcTan[a*x])*ArcTan[a*x] + 3*Sqrt[1 + a^2*x^2]*Sqrt[(-I)*Arc
Tan[a*x]]*Gamma[1/2, (-I)*ArcTan[a*x]] + 3*Sqrt[1 + a^2*x^2]*Sqrt[I*ArcTan
[a*x]]*Gamma[1/2, I*ArcTan[a*x]])/(8*a^2*c*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan
[a*x]])
```

## Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.95, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5465, 5440, 5439, 3042, 3777, 25, 3042, 3786, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \arctan(ax)^{3/2}}{(a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \text{ 5465}$$

$$\frac{3\int \frac{\sqrt{\arctan(ax)}}{(a^2cx^2+c)^{3/2}}dx}{2a}-\frac{\arctan(ax)^{3/2}}{a^2c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 5440}$$

$$\frac{3\sqrt{a^2x^2+1}\int \frac{\sqrt{\arctan(ax)}}{(a^2x^2+1)^{3/2}}dx}{2ac\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^{3/2}}{a^2c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 5439}$$

$$\frac{3\sqrt{a^2x^2+1}\int \frac{\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}d\arctan(ax)}{2a^2c\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^{3/2}}{a^2c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{3\sqrt{a^2x^2+1}\int \sqrt{\arctan(ax)}\sin\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{2a^2c\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^{3/2}}{a^2c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 3777}$$

$$\frac{3\sqrt{a^2x^2+1}\left(\frac{1}{2}\int -\frac{ax}{\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}d\arctan(ax)+\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}\right)}{2a^2c\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^{3/2}}{a^2c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \ 25$$

$$\frac{3\sqrt{a^2x^2+1}\left(\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}-\frac{1}{2}\int \frac{ax}{\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}d\arctan(ax)\right)}{2a^2c\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^{3/2}}{a^2c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \ 3042$$

$$\frac{3\sqrt{a^2x^2+1}\left(\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}-\frac{1}{2}\int \frac{\sin(\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)\right)}{2a^2c\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^{3/2}}{a^2c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \ 3786$$

$$\frac{3\sqrt{a^2x^2+1}\left(\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}-\int \frac{ax}{\sqrt{a^2x^2+1}}d\sqrt{\arctan(ax)}\right)}{2a^2c\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^{3/2}}{a^2c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \ 3832$$

$$\frac{3\sqrt{a^2x^2+1}\left(\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}-\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{2a^2c\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^{3/2}}{a^2c\sqrt{a^2cx^2+c}}$$

input
```
Int[(x*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(3/2),x]
```

output
```
-(ArcTan[a*x]^(3/2)/(a^2*c*Sqrt[c + a^2*c*x^2])) + (3*Sqrt[1 + a^2*x^2]*((
a*x*Sqrt[ArcTan[a*x]])/Sqrt[1 + a^2*x^2] - Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*
Sqrt[ArcTan[a*x]]]))/(2*a^2*c*Sqrt[c + a^2*c*x^2])
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5440
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])   Int[(1 +
c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

## Maple [F]

$$\int \frac{x \arctan (ax)^{\frac{3}{2}}}{(a^2 c\, x^2 + c)^{\frac{3}{2}}} dx$$

input
```
int(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x)
```

output
```
int(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x \arctan(ax)^{3/2}}{(c + a^2 c x^2)^{3/2}}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{x \arctan(ax)^{3/2}}{(c + a^2 c x^2)^{3/2}}\, dx = \int \frac{x \operatorname{atan}^{\frac{3}{2}}(ax)}{(c\,(a^2x^2 + 1))^{\frac{3}{2}}}\, dx$$

input
```
integrate(x*atan(a*x)**(3/2)/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(x*atan(a*x)**(3/2)/(c*(a**2*x**2 + 1))**(3/2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x \arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F]

$$\int \frac{x \arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x \arctan(ax)^{\frac{3}{2}}}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2 + c)^(3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x \arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x \operatorname{atan}(a\,x)^{3/2}}{(c\,a^2\,x^2+c)^{3/2}}\,dx$$

input
```
int((x*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(3/2),x)
```

output
```
int((x*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(3/2), x)
```

## Reduce [F]

$$\int \frac{x \arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{c}\left(-4\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}\,atan(ax)+6\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}\,ax-3\left(\int \frac{}{atan}\right.\right.}{4a^2c^2\,(a}$$

input
```
int(x*atan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x)
```

output
```
(sqrt(c)*( - 4*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x) + 6*sqrt(a**2
*x**2 + 1)*sqrt(atan(a*x))*a*x - 3*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)
)*x)/(atan(a*x)*a**4*x**4 + 2*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**4*x**
2 - 3*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x)/(atan(a*x)*a**4*x**4 + 2
*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**2))/(4*a**2*c**2*(a**2*x**2 + 1))
```

# 3.824 $\int \frac{\arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 125

$$\int \frac{\arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{c+a^2cx^2}} + \frac{x\arctan(ax)^{3/2}}{c\sqrt{c+a^2cx^2}} - \frac{3\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{2ac\sqrt{c+a^2cx^2}}$$

output
```
3/2*arctan(a*x)^(1/2)/a/c/(a^2*c*x^2+c)^(1/2)+x*arctan(a*x)^(3/2)/c/(a^2*c
*x^2+c)^(1/2)-3/4*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)*FresnelC(2^(1/2)/Pi^(
1/2)*arctan(a*x)^(1/2))/a/c/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.69

$$\int \frac{\arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{2\sqrt{\arctan(ax)}(3+2ax\arctan(ax)) - 3\sqrt{2\pi}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{4ac\sqrt{c+a^2cx^2}}$$

input
```
Integrate[ArcTan[a*x]^(3/2)/(c + a^2*c*x^2)^(3/2),x]
```

output
```
(2*Sqrt[ArcTan[a*x]]*(3 + 2*a*x*ArcTan[a*x]) - 3*Sqrt[2*Pi]*Sqrt[1 + a^2*x
^2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(4*a*c*Sqrt[c + a^2*c*x^2])
```

## Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {5433, 5440, 5439, 3042, 3785, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{3/2}}{(a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \text{5433}$$

$$-\frac{3}{4}\int \frac{1}{(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5440}$$

$$-\frac{3\sqrt{a^2x^2+1}\int \frac{1}{(a^2x^2+1)^{3/2}\sqrt{\arctan(ax)}}dx}{4c\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5439}$$

$$-\frac{3\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}d\arctan(ax)}{4ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)}{\sqrt{\arctan(ax)}}d\arctan(ax)}{4ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3785}$$

$$-\frac{3\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}}d\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3833}$$

$$-\frac{3\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{2ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}$$

input
```
Int[ArcTan[a*x]^(3/2)/(c + a^2*c*x^2)^(3/2),x]
```

output
```
(3*Sqrt[ArcTan[a*x]])/(2*a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x]^(3/2))/
(c*Sqrt[c + a^2*c*x^2]) - (3*Sqrt[Pi/2]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[2/
Pi]*Sqrt[ArcTan[a*x]]])/(2*a*c*Sqrt[c + a^2*c*x^2])
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 5433
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x] - Simp[b^2*p*(p
- 1)   Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2), x], x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5440
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])   Int[(1 +
c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

## Maple [F]

$$\int \frac{\arctan (ax)^{\frac{3}{2}}}{(a^2 c\,x^2 + c)^{\frac{3}{2}}} dx$$

input
```
int(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x)
```

output
```
int(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{(c + a^2cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{\arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{(c\,(a^2x^2+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(atan(a*x)**(3/2)/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(atan(a*x)**(3/2)/(c*(a**2*x**2 + 1))**(3/2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F]

$$\int \frac{\arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\arctan(ax)^{\frac{3}{2}}}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^(3/2)/(a^2*c*x^2 + c)^(3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\mathrm{atan}(a\,x)^{3/2}}{(c\,a^2\,x^2+c)^{3/2}}\,dx$$

input
```
int(atan(a*x)^(3/2)/(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(atan(a*x)^(3/2)/(c + a^2*c*x^2)^(3/2), x)
```

## Reduce [F]

$$\int \frac{\arctan(ax)^{3/2}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{c}\left(4\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,atan(ax)\,ax+6\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}-3\left(\int \frac{}{atan(ax)}\right.\right.}{4a\,c^2\,(a^2x^2}$$

input
```
int(atan(a*x)^(3/2)/(a^2*c*x^2+c)^(3/2),x)
```

output
```
(sqrt(c)*(4*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)*a*x + 6*sqrt(a**
2*x**2 + 1)*sqrt(atan(a*x)) - 3*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/
(atan(a*x)*a**4*x**4 + 2*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**3*x**2 - 3
*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(atan(a*x)*a**4*x**4 + 2*atan(a
*x)*a**2*x**2 + atan(a*x)),x)*a))/(4*a*c**2*(a**2*x**2 + 1))
```

# 3.825 $\int \frac{\arctan(ax)^{3/2}}{x(c+a^2cx^2)^{3/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\arctan(ax)^{3/2}}{x\left(c+a^2cx^2\right)^{3/2}}\,dx=\mathrm{Int}\left(\frac{\arctan(ax)^{3/2}}{x\left(c+a^2cx^2\right)^{3/2}},x\right)$$

output
```
Defer(Int)(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^(3/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 3.09 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\arctan(ax)^{3/2}}{x\left(c+a^2cx^2\right)^{3/2}}\,dx=\int \frac{\arctan(ax)^{3/2}}{x\left(c+a^2cx^2\right)^{3/2}}\,dx$$

input
```
Integrate[ArcTan[a*x]^(3/2)/(x*(c + a^2*c*x^2)^(3/2)),x]
```

output
```
Integrate[ArcTan[a*x]^(3/2)/(x*(c + a^2*c*x^2)^(3/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{3/2}}{x\left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{5560}$$

$$\int \frac{\arctan(ax)^{3/2}}{x\left(a^2cx^2+c\right)^{3/2}}dx$$

input
```
Int[ArcTan[a*x]^(3/2)/(x*(c + a^2*c*x^2)^(3/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\arctan \left(a x\right)^{\frac{3}{2}}}{x \left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}d x$$

input
```
int(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^(3/2),x)
```

output
```
int(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{x\left(c+a^2cx^2\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 28.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\arctan(ax)^{3/2}}{x\left(c+a^2cx^2\right)^{3/2}}\,dx = \int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{x\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(atan(a*x)**(3/2)/x/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(atan(a*x)**(3/2)/(x*(c*(a**2*x**2 + 1))**(3/2)), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{x\left(c+a^2cx^2\right)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\arctan(ax)^{3/2}}{x\,(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\arctan{(ax)}^{\frac{3}{2}}}{(a^2cx^2+c)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^(3/2)/((a^2*c*x^2 + c)^(3/2)*x), x)
```

## Mupad [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\arctan(ax)^{3/2}}{x\,(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\mathrm{atan}(a\,x)^{3/2}}{x\,{(c\,a^2\,x^2+c)}^{3/2}}\,dx$$

input
```
int(atan(a*x)^(3/2)/(x*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(atan(a*x)^(3/2)/(x*(c + a^2*c*x^2)^(3/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.81

$$\int \frac{\arctan(ax)^{3/2}}{x\,(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,atan(ax)}{a^4x^5+2a^2x^3+x}dx\right)}{c^2}$$

input
```
int(atan(a*x)^(3/2)/x/(a^2*c*x^2+c)^(3/2),x)
```

output
```
(sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x))/(a**4*x**5 +
2*a**2*x**3 + x),x))/c**2
```

# 3.826 $\int \frac{\arctan(ax)^{3/2}}{x^2(c+a^2cx^2)^{3/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\arctan(ax)^{3/2}}{x^2\left(c+a^2cx^2\right)^{3/2}}\,dx=\text{Int}\left(\frac{\arctan(ax)^{3/2}}{x^2\left(c+a^2cx^2\right)^{3/2}},x\right)$$

output
```
Defer(Int)(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c)^(3/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 5.47 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\arctan(ax)^{3/2}}{x^2\left(c+a^2cx^2\right)^{3/2}}\,dx=\int \frac{\arctan(ax)^{3/2}}{x^2\left(c+a^2cx^2\right)^{3/2}}\,dx$$

input
```
Integrate[ArcTan[a*x]^(3/2)/(x^2*(c + a^2*c*x^2)^(3/2)),x]
```

output
```
Integrate[ArcTan[a*x]^(3/2)/(x^2*(c + a^2*c*x^2)^(3/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{3/2}}{x^2 \left(a^2cx^2+c\right)^{3/2}} \, dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{\arctan(ax)^{3/2}}{x^2 \left(a^2cx^2+c\right)^{3/2}}dx$$

input
```
Int[ArcTan[a*x]^(3/2)/(x^2*(c + a^2*c*x^2)^(3/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\arctan \left(a x \right)^{\frac{3}{2}}}{x^{2} \left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}d x$$

input
```
int(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c)^(3/2),x)
```

output
```
int(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{x^2 (c + a^2cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 46.81 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax)^{3/2}}{x^2 (c + a^2cx^2)^{3/2}}\,dx = \int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{x^2 (c(a^2x^2+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(atan(a*x)**(3/2)/x**2/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(atan(a*x)**(3/2)/(x**2*(c*(a**2*x**2 + 1))**(3/2)), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{x^2 (c + a^2cx^2)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\arctan(ax)^{3/2}}{x^2 (c+a^2cx^2)^{3/2}}\,dx = \int \frac{\arctan(ax)^{\frac{3}{2}}}{(a^2cx^2+c)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^(3/2)/((a^2*c*x^2 + c)^(3/2)*x^2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\arctan(ax)^{3/2}}{x^2 (c+a^2cx^2)^{3/2}}\,dx = \int \frac{\mathrm{atan}(a\,x)^{3/2}}{x^2\,(c\,a^2\,x^2+c)^{3/2}}\,dx$$

input
```
int(atan(a*x)^(3/2)/(x^2*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(atan(a*x)^(3/2)/(x^2*(c + a^2*c*x^2)^(3/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 285, normalized size of antiderivative = 10.96

$$\int \frac{\arctan(ax)^{3/2}}{x^2 \left(c + a^2 c x^2\right)^{3/2}}\,dx = \frac{\sqrt{c}\left(-4\sqrt{a^2x^2+1}\,\sqrt{atan\left(ax\right)}\,atan(ax)\,a^2x^2 - 2\sqrt{a^2x^2+1}\,\sqrt{atan\left(ax\right)}\,atan(ax\right.}{}$$

input
```
int(atan(a*x)^(3/2)/x^2/(a^2*c*x^2+c)^(3/2),x)
```

output
```
(sqrt(c)*( - 4*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)*a**2*x**2 - 2
*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x) - 6*sqrt(a**2*x**2 + 1)*sqr
t(atan(a*x))*a*x + 3*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(atan(a*x)*
a**4*x**4 + 2*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**4*x**3 + 3*int((sqrt(
a**2*x**2 + 1)*sqrt(atan(a*x)))/(atan(a*x)*a**4*x**4 + 2*atan(a*x)*a**2*x*
*2 + atan(a*x)),x)*a**2*x + 3*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(a
**4*x**5 + 2*a**2*x**3 + x),x)*a**3*x**3 + 3*int((sqrt(a**2*x**2 + 1)*sqrt
(atan(a*x)))/(a**4*x**5 + 2*a**2*x**3 + x),x)*a*x))/(2*c**2*x*(a**2*x**2 +
 1))
```

# 3.827 $\int \frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}} \, dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}} \, dx = \text{Int}\left(\frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}, x\right)$$

output
```
Defer(Int)(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 2.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}} \, dx = \int \frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}} \, dx$$

input
```
Integrate[(x^m*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(5/2),x]
```

output
```
Integrate[(x^m*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(5/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \arctan(ax)^{3/2}}{(a^2cx^2+c)^{5/2}} dx$$

$$\Big\downarrow \text{5560}$$

$$\int \frac{x^m \arctan(ax)^{3/2}}{(a^2cx^2+c)^{5/2}} dx$$

input
```
Int[(x^m*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(5/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^m \arctan\left(ax\right)^{\frac{3}{2}}}{\left(a^2c\,x^2+c\right)^{\frac{5}{2}}}dx$$

input
```
int(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x)
```

output
```
int(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.38

$$\int \frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}} \, dx = \int \frac{x^m \arctan(ax)^{\frac{3}{2}}}{(a^2cx^2+c)^{\frac{5}{2}}} \, dx$$

input
```
integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^m*arctan(a*x)^(3/2)/(a^6*c^3*x^6 + 3*a^4*c^
3*x^4 + 3*a^2*c^3*x^2 + c^3), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}} \, dx = \text{Timed out}$$

input
```
integrate(x**m*atan(a*x)**(3/2)/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^m \arctan(ax)^{\frac{3}{2}}}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
integrate(x^m*arctan(a*x)^(3/2)/(a^2*c*x^2 + c)^(5/2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^m \operatorname{atan}(a\,x)^{3/2}}{(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int((x^m*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int((x^m*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(5/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.23

$$\int \frac{x^m \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{\sqrt{c}\left(\int \frac{x^m\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,atan(ax)}{a^6x^6+3a^4x^4+3a^2x^2+1}dx\right)}{c^3}$$

input
```
int(x^m*atan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x)
```

output
```
(sqrt(c)*int((x**m*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x))/(a**6*x*
*6 + 3*a**4*x**4 + 3*a**2*x**2 + 1),x))/c**3
```

# 3.828 $\int \frac{x^5 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^5 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Int}\left(\frac{x^5 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}},x\right)$$

output
```
Defer(Int)(x^5*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 9.70 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^5 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^5 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx$$

input
```
Integrate[(x^5*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(5/2),x]
```

output
```
Integrate[(x^5*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(5/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 \arctan(ax)^{3/2}}{(a^2cx^2+c)^{5/2}} \, dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int \frac{x^5 \arctan(ax)^{3/2}}{(a^2cx^2+c)^{5/2}}dx$$

input
```
Int[(x^5*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(5/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^5 \arctan\left(ax\right)^{\frac{3}{2}}}{\left(a^2 c\,x^2+c\right)^{\frac{5}{2}}}dx$$

input
```
int(x^5*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x)
```

output
```
int(x^5*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x^5 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^5*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^5 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**5*atan(a*x)**(3/2)/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^5*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x^5 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^5*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^5 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^5\,\mathrm{atan}(a\,x)^{3/2}}{\left(c\,a^2\,x^2+c\right)^{5/2}}\,dx$$

input
```
int((x^5*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int((x^5*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(5/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.23

$$\int \frac{x^5 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,atan(ax)x^5}{a^6x^6+3a^4x^4+3a^2x^2+1}dx\right)}{c^3}$$

input
```
int(x^5*atan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x)
```

output
```
(sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)*x**5)/(a**6*x*
*6 + 3*a**4*x**4 + 3*a**2*x**2 + 1),x))/c**3
```

# 3.829 $\int \frac{x^4 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}} dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^4 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}} dx = \text{Int}\left(\frac{x^4 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}, x\right)$$

output
```
Defer(Int)(x^4*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 6.94 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^4 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}} dx = \int \frac{x^4 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}} dx$$

input
```
Integrate[(x^4*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(5/2),x]
```

output
```
Integrate[(x^4*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(5/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \arctan(ax)^{3/2}}{(a^2cx^2+c)^{5/2}} \, dx$$

$$\Big\downarrow \text{5560}$$

$$\int \frac{x^4 \arctan(ax)^{3/2}}{(a^2cx^2+c)^{5/2}}dx$$

input
```
Int[(x^4*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(5/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^4 \arctan\left(ax\right)^{\frac{3}{2}}}{\left(a^2 c\, x^2+c\right)^{\frac{5}{2}}}dx$$

input
```
int(x^4*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x)
```

output
```
int(x^4*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x^4 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^4*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^4 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**4*atan(a*x)**(3/2)/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^4*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-1)]

Timed out.

$$\int \frac{x^4 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\, dx = \text{Timed out}$$

input
```
integrate(x^4*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

## Mupad [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^4 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\, dx = \int \frac{x^4\, \mathrm{atan}(a\,x)^{3/2}}{(c\,a^2\,x^2+c)^{5/2}}\, dx$$

input
```
int((x^4*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int((x^4*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(5/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.23

$$\int \frac{x^4 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\, dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,atan(ax)x^4}{a^6x^6+3a^4x^4+3a^2x^2+1}dx\right)}{c^3}$$

input
```
int(x^4*atan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x)
```

output
```
(sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)*x**4)/(a**6*x*
*6 + 3*a**4*x**4 + 3*a**2*x**2 + 1),x))/c**3
```

# 3.830 $\int \frac{x^3 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 263

$$\int \frac{x^3 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{x^3\sqrt{\arctan(ax)}}{6ac\,(c+a^2cx^2)^{3/2}} + \frac{x\sqrt{\arctan(ax)}}{a^3c^2\sqrt{c+a^2cx^2}} - \frac{x^2\arctan(ax)^{3/2}}{3a^2c\,(c+a^2cx^2)^{3/2}} - \frac{2\arctan(ax)^{3/2}}{3a^4c^2\sqrt{c+a^2cx^2}} - \frac{9\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{8a^4c^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{\frac{\pi}{6}}\sqrt{1+a^2x^2}\,\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)}{24a^4c^2\sqrt{c+a^2cx^2}}$$

output
```
1/6*x^3*arctan(a*x)^(1/2)/a/c/(a^2*c*x^2+c)^(3/2)+x*arctan(a*x)^(1/2)/a^3/
c^2/(a^2*c*x^2+c)^(1/2)-1/3*x^2*arctan(a*x)^(3/2)/a^2/c/(a^2*c*x^2+c)^(3/2
)-2/3*arctan(a*x)^(3/2)/a^4/c^2/(a^2*c*x^2+c)^(1/2)-9/16*2^(1/2)*Pi^(1/2)*
(a^2*x^2+1)^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))/a^4/c^2/(a^
2*c*x^2+c)^(1/2)+1/144*6^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)*FresnelS(6^(1/2)
/Pi^(1/2)*arctan(a*x)^(1/2))/a^4/c^2/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.84 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.03

$$\int \frac{x^3 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{24\arctan(ax)\left(ax(6+7a^2x^2)-2(2+3a^2x^2)\arctan(ax)\right)-7\sqrt{6\pi}(1+a^2x^2)^{3/2}\,\surd}{}$$

input
```
Integrate[(x^3*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(24*ArcTan[a*x]*(a*x*(6 + 7*a^2*x^2) - 2*(2 + 3*a^2*x^2)*ArcTan[a*x]) - 7*
Sqrt[6*Pi]*(1 + a^2*x^2)^(3/2)*Sqrt[ArcTan[a*x]]*(3*Sqrt[3]*FresnelS[Sqrt[
2/Pi]*Sqrt[ArcTan[a*x]]] - FresnelS[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]]) + 3*(1
+ a^2*x^2)^(3/2)*(3*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-I)*ArcTan[a*x]] +
3*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, I*ArcTan[a*x]] + Sqrt[3]*(Sqrt[(-I)*ArcTa
n[a*x]]*Gamma[1/2, (-3*I)*ArcTan[a*x]] + Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (3
*I)*ArcTan[a*x]])))/(144*a^4*c*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]])
```

## Rubi [A] (verified)

Time = 2.54 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.16, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.577$, Rules used = {5475, 5465, 5440, 5439, 3042, 3777, 25, 3042, 3786, 3832, 5506, 5505, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \arctan(ax)^{3/2}}{(a^2cx^2+c)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 5475}$$

$$\frac{2\int \frac{x\arctan(ax)^{3/2}}{(a^2cx^2+c)^{3/2}}dx}{3a^2c} - \frac{1}{12}\int \frac{x^3}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx - \frac{x^2\arctan(ax)^{3/2}}{3a^2c(a^2cx^2+c)^{3/2}} + \frac{x^3\sqrt{\arctan(ax)}}{6ac(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \ 5465$$

$$\frac{2\left(\frac{3\int \frac{\sqrt{\arctan(ax)}}{(a^2cx^2+c)^{3/2}}dx}{2a} - \frac{\arctan(ax)^{3/2}}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c} - \frac{1}{12}\int \frac{x^3}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx - \frac{x^2\arctan(ax)^{3/2}}{3a^2c(a^2cx^2+c)^{3/2}} + \frac{x^3\sqrt{\arctan(ax)}}{6ac(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \ 5440$$

$$\frac{2\left(\frac{3\sqrt{a^2x^2+1}\int \frac{\sqrt{\arctan(ax)}}{(a^2x^2+1)^{3/2}}dx}{2ac\sqrt{a^2cx^2+c}} - \frac{\arctan(ax)^{3/2}}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c} - \frac{1}{12}\int \frac{x^3}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx - \frac{x^2\arctan(ax)^{3/2}}{3a^2c(a^2cx^2+c)^{3/2}} + \frac{x^3\sqrt{\arctan(ax)}}{6ac(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \ 5439$$

$$\frac{2\left(\frac{3\sqrt{a^2x^2+1}\int \frac{\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}d\arctan(ax)}{2a^2c\sqrt{a^2cx^2+c}} - \frac{\arctan(ax)^{3/2}}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c} - \frac{1}{12}\int \frac{x^3}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx - \frac{x^2\arctan(ax)^{3/2}}{3a^2c(a^2cx^2+c)^{3/2}} + \frac{x^3\sqrt{\arctan(ax)}}{6ac(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \ 3042$$

$$\frac{2\left(\frac{3\sqrt{a^2x^2+1}\int \sqrt{\arctan(ax)}\sin\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{2a^2c\sqrt{a^2cx^2+c}} - \frac{\arctan(ax)^{3/2}}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c} - \frac{1}{12}\int \frac{x^3}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx - \frac{x^2\arctan(ax)^{3/2}}{3a^2c(a^2cx^2+c)^{3/2}} + \frac{x^3\sqrt{\arctan(ax)}}{6ac(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \ 3777$$

$$\frac{2\left(\frac{3\sqrt{a^2x^2+1}\left(\frac{1}{2}\int -\frac{ax}{\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}d\arctan(ax)+\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}\right)}{2a^2c\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^{3/2}}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{1}{12}\int \frac{x^3}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{x^2\arctan(ax)^{3/2}}{3a^2c\,(a^2cx^2+c)^{3/2}}+\frac{x^3\sqrt{\arctan(ax)}}{6ac\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{2\left(\frac{3\sqrt{a^2x^2+1}\left(\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}-\frac{1}{2}\int \frac{ax}{\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}d\arctan(ax)\right)}{2a^2c\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^{3/2}}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{1}{12}\int \frac{x^3}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{x^2\arctan(ax)^{3/2}}{3a^2c\,(a^2cx^2+c)^{3/2}}+\frac{x^3\sqrt{\arctan(ax)}}{6ac\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2\left(\frac{3\sqrt{a^2x^2+1}\left(\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}-\frac{1}{2}\int \frac{\sin(\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)\right)}{2a^2c\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^{3/2}}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{1}{12}\int \frac{x^3}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{x^2\arctan(ax)^{3/2}}{3a^2c\,(a^2cx^2+c)^{3/2}}+\frac{x^3\sqrt{\arctan(ax)}}{6ac\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{ 3786}$$

$$\frac{2\left(\frac{3\sqrt{a^2x^2+1}\left(\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}-\int \frac{ax}{\sqrt{a^2x^2+1}}d\sqrt{\arctan(ax)}\right)}{2a^2c\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^{3/2}}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{1}{12}\int \frac{x^3}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{x^2\arctan(ax)^{3/2}}{3a^2c\,(a^2cx^2+c)^{3/2}}+\frac{x^3\sqrt{\arctan(ax)}}{6ac\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{ 3832}$$

$$-\frac{1}{12}\int \frac{x^3}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx+\frac{2\left(\frac{3\sqrt{a^2x^2+1}\left(\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}-\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{2a^2c\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^{3/2}}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{x^2\arctan(ax)^{3/2}}{3a^2c\,(a^2cx^2+c)^{3/2}}+\frac{x^3\sqrt{\arctan(ax)}}{6ac\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{ 5506}$$

$$-\frac{\sqrt{a^2x^2+1}\int \frac{x^3}{(a^2x^2+1)^{5/2}\sqrt{\arctan(ax)}}dx}{12c^2\sqrt{a^2cx^2+c}}+\frac{2\left(\frac{3\sqrt{a^2x^2+1}\left(\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}-\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{2a^2c\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^{3/2}}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{x^2\arctan(ax)^{3/2}}{3a^2c\left(a^2cx^2+c\right)^{3/2}}+\frac{x^3\sqrt{\arctan(ax)}}{6ac\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{5505}$$

$$-\frac{\sqrt{a^2x^2+1}\int \frac{a^3x^3}{(a^2x^2+1)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{12a^4c^2\sqrt{a^2cx^2+c}}+\frac{2\left(\frac{3\sqrt{a^2x^2+1}\left(\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}-\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{2a^2c\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^{3/2}}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{x^2\arctan(ax)^{3/2}}{3a^2c\left(a^2cx^2+c\right)^{3/2}}+\frac{x^3\sqrt{\arctan(ax)}}{6ac\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sqrt{a^2x^2+1}\int \frac{\sin(\arctan(ax))^3}{\sqrt{\arctan(ax)}}d\arctan(ax)}{12a^4c^2\sqrt{a^2cx^2+c}}+\frac{2\left(\frac{3\sqrt{a^2x^2+1}\left(\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}-\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{2a^2c\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^{3/2}}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{x^2\arctan(ax)^{3/2}}{3a^2c\left(a^2cx^2+c\right)^{3/2}}+\frac{x^3\sqrt{\arctan(ax)}}{6ac\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{\sqrt{a^2x^2+1}\int \left(\frac{3ax}{4\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}-\frac{\sin(3\arctan(ax))}{4\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{12a^4c^2\sqrt{a^2cx^2+c}}+\frac{2\left(\frac{3\sqrt{a^2x^2+1}\left(\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}-\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{2a^2c\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^{3/2}}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{x^2\arctan(ax)^{3/2}}{3a^2c\left(a^2cx^2+c\right)^{3/2}}+\frac{x^3\sqrt{\arctan(ax)}}{6ac\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\left(\frac{3\sqrt{a^2x^2+1}\left(\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}-\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{2a^2c\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^{3/2}}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{x^2\arctan(ax)^{3/2}}{3a^2c\left(a^2cx^2+c\right)^{3/2}}+\frac{x^3\sqrt{\arctan(ax)}}{6ac\left(a^2cx^2+c\right)^{3/2}}-\frac{\sqrt{a^2x^2+1}\left(\frac{3}{2}\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{2}\sqrt{\frac{\pi}{6}}\,\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{12a^4c^2\sqrt{a^2cx^2+c}}$$

input
```
Int[(x^3*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(x^3*Sqrt[ArcTan[a*x]])/(6*a*c*(c + a^2*c*x^2)^(3/2)) - (x^2*ArcTan[a*x]^(
3/2))/(3*a^2*c*(c + a^2*c*x^2)^(3/2)) + (2*(-(ArcTan[a*x]^(3/2)/(a^2*c*Sqr
t[c + a^2*c*x^2])) + (3*Sqrt[1 + a^2*x^2]*((a*x*Sqrt[ArcTan[a*x]])/Sqrt[1
+ a^2*x^2] - Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]]))/(2*a^2*c*
Sqrt[c + a^2*c*x^2])))/(3*a^2*c) - (Sqrt[1 + a^2*x^2]*((3*Sqrt[Pi/2]*Fresn
elS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/2 - (Sqrt[Pi/6]*FresnelS[Sqrt[6/Pi]*Sqr
t[ArcTan[a*x]]])/2))/(12*a^4*c^2*Sqrt[c + a^2*c*x^2])
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5440
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])   Int[(1 +
c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5475
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_)*((d_) + (e_.)
*(x_)^2)^(q_), x_Symbol] :> Simp[b*p*(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*Ar
cTan[c*x])^(p - 1)/(c*d*m^2)), x] + (-Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(q +
 1)*((a + b*ArcTan[c*x])^p/(c^2*d*m)), x] + Simp[f^2*((m - 1)/(c^2*d*m))
Int[(f*x)^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p, x], x] - Simp[
b^2*p*((p - 1)/m^2)   Int[(f*x)^m*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2)
, x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && EqQ[m + 2*
q + 2, 0] && LtQ[q, -1] && GtQ[p, 1]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

## Maple [F]

$$\int \frac{x^3 \arctan(ax)^{\frac{3}{2}}}{(a^2c\,x^2+c)^{\frac{5}{2}}}dx$$

input
```
int(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x)
```

output
```
int(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{x^3 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^3 \operatorname{atan}^{\frac{3}{2}}(ax)}{(c(a^2x^2+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**3*atan(a*x)**(3/2)/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(x**3*atan(a*x)**(3/2)/(c*(a**2*x**2 + 1))**(5/2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x^3 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^3\,\mathrm{atan}(a\,x)^{3/2}}{(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int((x^3*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int((x^3*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(5/2), x)
```

## Reduce [F]

$$\int \frac{x^3 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
int(x^3*atan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x)
```

output
```
(sqrt(c)*( - 12*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)*a**2*x**2 -
8*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x) + 14*sqrt(a**2*x**2 + 1)*s
qrt(atan(a*x))*a**3*x**3 + 12*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*a*x - 7*
int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**3)/(atan(a*x)*a**6*x**6 + 3*at
an(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**8*x**4 - 14*i
nt((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**3)/(atan(a*x)*a**6*x**6 + 3*ata
n(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**6*x**2 - 7*int
((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**3)/(atan(a*x)*a**6*x**6 + 3*atan(
a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**4 - 6*int((sqrt(
a**2*x**2 + 1)*sqrt(atan(a*x))*x)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*
x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**6*x**4 - 12*int((sqrt(a**2
*x**2 + 1)*sqrt(atan(a*x))*x)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4
 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**4*x**2 - 6*int((sqrt(a**2*x**2
 + 1)*sqrt(atan(a*x))*x)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*
atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**2))/(12*a**4*c**3*(a**4*x**4 + 2*a*
*2*x**2 + 1))
```

# 3.831 $\int \frac{x^2 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 247

$$\int \frac{x^2 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{3\sqrt{\arctan(ax)}}{8a^3c^2\sqrt{c+a^2cx^2}} + \frac{x^3\arctan(ax)^{3/2}}{3c(c+a^2cx^2)^{3/2}} - \frac{\sqrt{1+a^2x^2}\sqrt{\arctan(ax)}\cos(3\arctan(ax))}{24a^3c^2\sqrt{c+a^2cx^2}} - \frac{3\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{8a^3c^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{\frac{\pi}{6}}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)}{24a^3c^2\sqrt{c+a^2cx^2}}$$

output

```
3/8*arctan(a*x)^(1/2)/a^3/c^2/(a^2*c*x^2+c)^(1/2)+1/3*x^3*arctan(a*x)^(3/2
)/c/(a^2*c*x^2+c)^(3/2)-1/24*(a^2*x^2+1)^(1/2)*arctan(a*x)^(1/2)*cos(3*arc
tan(a*x))/a^3/c^2/(a^2*c*x^2+c)^(1/2)-3/16*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1
/2)*FresnelC(2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))/a^3/c^2/(a^2*c*x^2+c)^(1/
2)+1/144*6^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)*FresnelC(6^(1/2)/Pi^(1/2)*arct
an(a*x)^(1/2))/a^3/c^2/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.41 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.37

$$\int \frac{x^2 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{96\arctan(ax)+144a^2x^2\arctan(ax)+96a^3x^3\arctan(ax)^2+27i(1+a^2x^2)^{3/2}\sqrt{}}{}$$

input
```
Integrate[(x^2*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(96*ArcTan[a*x] + 144*a^2*x^2*ArcTan[a*x] + 96*a^3*x^3*ArcTan[a*x]^2 + (27
*I)*(1 + a^2*x^2)^(3/2)*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-I)*ArcTan[a*x]
] - (27*I)*(1 + a^2*x^2)^(3/2)*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, I*ArcTan[a*x
]] - I*Sqrt[3 + 3*a^2*x^2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-3*I)*ArcTan
[a*x]] - I*a^2*x^2*Sqrt[3 + 3*a^2*x^2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (
-3*I)*ArcTan[a*x]] + I*Sqrt[3 + 3*a^2*x^2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2,
(3*I)*ArcTan[a*x]] + I*a^2*x^2*Sqrt[3 + 3*a^2*x^2]*Sqrt[I*ArcTan[a*x]]*Gam
ma[1/2, (3*I)*ArcTan[a*x]])/(288*a^3*c^2*(1 + a^2*x^2)*Sqrt[c + a^2*c*x^2]
*Sqrt[ArcTan[a*x]])
```

## Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.72, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {5479, 5506, 5505, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \arctan(ax)^{3/2}}{(a^2cx^2+c)^{5/2}}\,dx$$

$$\Big\downarrow \text{5479}$$

$$\frac{x^3 \arctan(ax)^{3/2}}{3c\,(a^2cx^2+c)^{3/2}} - \frac{1}{2}a\int \frac{x^3\sqrt{\arctan(ax)}}{(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{5506}$$

$$\frac{x^3\arctan(ax)^{3/2}}{3c\left(a^2cx^2+c\right)^{3/2}}-\frac{a\sqrt{a^2x^2+1}\int\frac{x^3\sqrt{\arctan(ax)}}{\left(a^2x^2+1\right)^{5/2}}dx}{2c^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \; 5505$$

$$\frac{x^3\arctan(ax)^{3/2}}{3c\left(a^2cx^2+c\right)^{3/2}}-\frac{\sqrt{a^2x^2+1}\int\frac{a^3x^3\sqrt{\arctan(ax)}}{\left(a^2x^2+1\right)^{3/2}}d\arctan(ax)}{2a^3c^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \; 3042$$

$$\frac{x^3\arctan(ax)^{3/2}}{3c\left(a^2cx^2+c\right)^{3/2}}-\frac{\sqrt{a^2x^2+1}\int\sqrt{\arctan(ax)}\sin(\arctan(ax))^3d\arctan(ax)}{2a^3c^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \; 3793$$

$$\frac{x^3\arctan(ax)^{3/2}}{3c\left(a^2cx^2+c\right)^{3/2}}-\frac{\sqrt{a^2x^2+1}\int\left(\frac{3ax\sqrt{\arctan(ax)}}{4\sqrt{a^2x^2+1}}-\frac{1}{4}\sqrt{\arctan(ax)}\sin(3\arctan(ax))\right)d\arctan(ax)}{2a^3c^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \; 2009$$

$$\frac{x^3\arctan(ax)^{3/2}}{3c\left(a^2cx^2+c\right)^{3/2}}-\frac{\sqrt{a^2x^2+1}\left(-\frac{3\sqrt{\arctan(ax)}}{4\sqrt{a^2x^2+1}}+\frac{3}{4}\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{12}\sqrt{\frac{\pi}{6}}\operatorname{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{12}\sqrt{\text{ar}}\right.}{2a^3c^2\sqrt{a^2cx^2+c}}$$

input
```
Int[(x^2*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(x^3*ArcTan[a*x]^(3/2))/(3*c*(c + a^2*c*x^2)^(3/2)) - (Sqrt[1 + a^2*x^2]*(
(-3*Sqrt[ArcTan[a*x]])/(4*Sqrt[1 + a^2*x^2]) + (Sqrt[ArcTan[a*x]]*Cos[3*Ar
cTan[a*x]])/12 + (3*Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/4 -
 (Sqrt[Pi/6]*FresnelC[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/12))/(2*a^3*c^2*Sqrt[
c + a^2*c*x^2])
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

## Maple [F]

$$\int \frac{x^2 \arctan(ax)^{\frac{3}{2}}}{(a^2c\,x^2+c)^{\frac{5}{2}}}dx$$

input
```
int(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x)
```

output
```
int(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{x^2 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^2 \operatorname{atan}^{\frac{3}{2}}(ax)}{(c\,(a^2x^2+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**2*atan(a*x)**(3/2)/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(x**2*atan(a*x)**(3/2)/(c*(a**2*x**2 + 1))**(5/2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F]

$$\int \frac{x^2 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^2 \arctan(ax)^{\frac{3}{2}}}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
integrate(x^2*arctan(a*x)^(3/2)/(a^2*c*x^2 + c)^(5/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^2 \,\mathrm{atan}(a\,x)^{3/2}}{(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int((x^2*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int((x^2*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(5/2), x)
```

## Reduce [F]

$$\int \frac{x^2 \arctan(ax)^{3/2}}{(c + a^2cx^2)^{5/2}}\,dx = \frac{\sqrt{c}\left(4\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}\,atan(ax)\,a^3x^3 + 6\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}\,a^2x^2 + 4\sqrt{\phantom{x}}\right.}{}$$

input
```
int(x^2*atan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x)
```

output
```
(sqrt(c)*(4*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)*a**3*x**3 + 6*sq
rt(a**2*x**2 + 1)*sqrt(atan(a*x))*a**2*x**2 + 4*sqrt(a**2*x**2 + 1)*sqrt(a
tan(a*x)) - 3*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**2)/(atan(a*x)*a*
*6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a*
*7*x**4 - 6*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**2)/(atan(a*x)*a**6
*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**5
*x**2 - 3*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**2)/(atan(a*x)*a**6*x
**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**3 -
 2*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(atan(a*x)*a**6*x**6 + 3*atan
(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**5*x**4 - 4*int(
(sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a
**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**3*x**2 - 2*int((sqrt(a
**2*x**2 + 1)*sqrt(atan(a*x)))/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**
4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a))/(12*a**3*c**3*(a**4*x**4 + 2
*a**2*x**2 + 1))
```

# 3.832 $\int \frac{x\arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 248

$$\int \frac{x\arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{3x\sqrt{\arctan(ax)}}{8ac^2\sqrt{c+a^2cx^2}} - \frac{\arctan(ax)^{3/2}}{3a^2c\,(c+a^2cx^2)^{3/2}} - \frac{3\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{8a^2c^2\sqrt{c+a^2cx^2}} - \frac{\sqrt{\frac{\pi}{6}}\sqrt{1+a^2x^2}\,\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)}{24a^2c^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{1+a^2x^2}\sqrt{\arctan(ax)}\sin(3\arctan(ax))}{24a^2c^2\sqrt{c+a^2cx^2}}$$

output
```
3/8*x*arctan(a*x)^(1/2)/a/c^2/(a^2*c*x^2+c)^(1/2)-1/3*arctan(a*x)^(3/2)/a^
2/c/(a^2*c*x^2+c)^(3/2)-3/16*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)*FresnelS(2
^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))/a^2/c^2/(a^2*c*x^2+c)^(1/2)-1/144*6^(1/
2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)*FresnelS(6^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))
/a^2/c^2/(a^2*c*x^2+c)^(1/2)+1/24*(a^2*x^2+1)^(1/2)*arctan(a*x)^(1/2)*sin(
3*arctan(a*x))/a^2/c^2/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.83 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.05

$$\int \frac{x \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{48(3ax+2a^3x^3-2\arctan(ax))\arctan(ax)-4\sqrt{6\pi}(1+a^2x^2)^{3/2}\sqrt{\arctan(ax)}\big(3$$

input
```
Integrate[(x*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(48*(3*a*x + 2*a^3*x^3 - 2*ArcTan[a*x])*ArcTan[a*x] - 4*Sqrt[6*Pi]*(1 + a^
2*x^2)^(3/2)*Sqrt[ArcTan[a*x]]*(3*Sqrt[3]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[
a*x]]] - FresnelS[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]]) + 3*(1 + a^2*x^2)^(3/2)*(
3*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-I)*ArcTan[a*x]] + 3*Sqrt[I*ArcTan[a*
x]]*Gamma[1/2, I*ArcTan[a*x]] + Sqrt[3]*(Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2,
 (-3*I)*ArcTan[a*x]] + Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (3*I)*ArcTan[a*x]]))
)/(288*a^2*c*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]])
```

## Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.73, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5465, 5440, 5439, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \arctan(ax)^{3/2}}{(a^2cx^2+c)^{5/2}}\,dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{\int \frac{\sqrt{\arctan(ax)}}{(a^2cx^2+c)^{5/2}}dx}{2a} - \frac{\arctan(ax)^{3/2}}{3a^2c\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5440}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{\sqrt{\arctan(ax)}}{(a^2x^2+1)^{5/2}}dx}{2ac^2\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^{3/2}}{3a^2c\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \; 5439$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{\sqrt{\arctan(ax)}}{(a^2x^2+1)^{3/2}}d\arctan(ax)}{2a^2c^2\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^{3/2}}{3a^2c\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sqrt{a^2x^2+1}\int \sqrt{\arctan(ax)}\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^3d\arctan(ax)}{2a^2c^2\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^{3/2}}{3a^2c\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \; 3793$$

$$\frac{\sqrt{a^2x^2+1}\int \left(\frac{1}{4}\sqrt{\arctan(ax)}\cos(3\arctan(ax))+\frac{3\sqrt{\arctan(ax)}}{4\sqrt{a^2x^2+1}}\right)d\arctan(ax)}{2a^2c^2\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^{3/2}}{3a^2c\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \; 2009$$

$$\frac{\sqrt{a^2x^2+1}\left(\frac{3ax\sqrt{\arctan(ax)}}{4\sqrt{a^2x^2+1}}-\frac{3}{4}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{12}\sqrt{\frac{\pi}{6}}\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{12}\sqrt{\text{ar}}\right.}{2a^2c^2\sqrt{a^2cx^2+c}}$$

$$\frac{\arctan(ax)^{3/2}}{3a^2c\left(a^2cx^2+c\right)^{3/2}}$$

input
```
Int[(x*ArcTan[a*x]^(3/2))/(c + a^2*c*x^2)^(5/2),x]
```

output
```
-1/3*ArcTan[a*x]^(3/2)/(a^2*c*(c + a^2*c*x^2)^(3/2)) + (Sqrt[1 + a^2*x^2]*
((3*a*x*Sqrt[ArcTan[a*x]])/(4*Sqrt[1 + a^2*x^2]) - (3*Sqrt[Pi/2]*FresnelS[
Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/4 - (Sqrt[Pi/6]*FresnelS[Sqrt[6/Pi]*Sqrt[Ar
cTan[a*x]]])/12 + (Sqrt[ArcTan[a*x]]*Sin[3*ArcTan[a*x]])/12))/(2*a^2*c^2*S
qrt[c + a^2*c*x^2])
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5440
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])   Int[(1 +
c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

## Maple [F]

$$\int \frac{x \arctan (ax)^{\frac{3}{2}}}{(a^2 c\, x^2 + c)^{\frac{5}{2}}} dx$$

input
```
int(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x)
```

output
```
int(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x \arctan(ax)^{3/2}}{(c + a^2 c x^2)^{5/2}} dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{x \arctan(ax)^{3/2}}{(c + a^2 c x^2)^{5/2}} dx = \int \frac{x \operatorname{atan}^{\frac{3}{2}}(ax)}{(c (a^2 x^2 + 1))^{\frac{5}{2}}} dx$$

input
```
integrate(x*atan(a*x)**(3/2)/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(x*atan(a*x)**(3/2)/(c*(a**2*x**2 + 1))**(5/2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F]

$$\int \frac{x \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x \arctan(ax)^{\frac{3}{2}}}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
integrate(x*arctan(a*x)^(3/2)/(a^2*c*x^2 + c)^(5/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x \arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x\,\mathrm{atan}(a\,x)^{3/2}}{(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int((x*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int((x*atan(a*x)^(3/2))/(c + a^2*c*x^2)^(5/2), x)
```

## Reduce [F]

$$\int \frac{x \arctan(ax)^{3/2}}{(c + a^2cx^2)^{5/2}} \, dx = \text{Too large to display}$$

input
```
int(x*atan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x)
```

output
```
(sqrt(c)*( - 4*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x) + 4*sqrt(a**2
*x**2 + 1)*sqrt(atan(a*x))*a**3*x**3 + 6*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x
))*a*x - 2*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**3)/(atan(a*x)*a**6*
x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**8*
x**4 - 4*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**3)/(atan(a*x)*a**6*x*
*6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**6*x*
*2 - 2*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**3)/(atan(a*x)*a**6*x**6
 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**4 - 3*
int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x)/(atan(a*x)*a**6*x**6 + 3*atan(
a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**6*x**4 - 6*int((
sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*
a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**4*x**2 - 3*int((sqrt(
a**2*x**2 + 1)*sqrt(atan(a*x))*x)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*
x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**2))/(12*a**2*c**3*(a**4*x*
*4 + 2*a**2*x**2 + 1))
```

# 3.833 $\int \frac{\arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 252

$$\int \frac{\arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{\sqrt{\arctan(ax)}}{6ac\,(c+a^2cx^2)^{3/2}}+\frac{\sqrt{\arctan(ax)}}{ac^2\sqrt{c+a^2cx^2}}+\frac{x\arctan(ax)^{3/2}}{3c\,(c+a^2cx^2)^{3/2}}+\frac{2x\arctan(ax)^{3/2}}{3c^2\sqrt{c+a^2cx^2}}-\frac{9\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{8ac^2\sqrt{c+a^2cx^2}}-\frac{\sqrt{\frac{\pi}{6}}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)}{24ac^2\sqrt{c+a^2cx^2}}$$

output
```
1/6*arctan(a*x)^(1/2)/a/c/(a^2*c*x^2+c)^(3/2)+arctan(a*x)^(1/2)/a/c^2/(a^2
*c*x^2+c)^(1/2)+1/3*x*arctan(a*x)^(3/2)/c/(a^2*c*x^2+c)^(3/2)+2/3*x*arctan
(a*x)^(3/2)/c^2/(a^2*c*x^2+c)^(1/2)-9/16*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2
)*FresnelC(2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))/a/c^2/(a^2*c*x^2+c)^(1/2)-1
/144*6^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)*FresnelC(6^(1/2)/Pi^(1/2)*arctan(a
*x)^(1/2))/a/c^2/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.37

$$\int \frac{\arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{336\arctan(ax) + 288a^2x^2\arctan(ax) + 288ax\arctan(ax)^2 + 192a^3x^3\arctan(ax)^2 -}{}$$

input
```
Integrate[ArcTan[a*x]^(3/2)/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(336*ArcTan[a*x] + 288*a^2*x^2*ArcTan[a*x] + 288*a*x*ArcTan[a*x]^2 + 192*a
^3*x^3*ArcTan[a*x]^2 + (81*I)*(1 + a^2*x^2)^(3/2)*Sqrt[(-I)*ArcTan[a*x]]*G
amma[1/2, (-I)*ArcTan[a*x]] - (81*I)*(1 + a^2*x^2)^(3/2)*Sqrt[I*ArcTan[a*x
]]*Gamma[1/2, I*ArcTan[a*x]] + I*Sqrt[3 + 3*a^2*x^2]*Sqrt[(-I)*ArcTan[a*x]
]*Gamma[1/2, (-3*I)*ArcTan[a*x]] + I*a^2*x^2*Sqrt[3 + 3*a^2*x^2]*Sqrt[(-I)
*ArcTan[a*x]]*Gamma[1/2, (-3*I)*ArcTan[a*x]] - I*Sqrt[3 + 3*a^2*x^2]*Sqrt[
I*ArcTan[a*x]]*Gamma[1/2, (3*I)*ArcTan[a*x]] - I*a^2*x^2*Sqrt[3 + 3*a^2*x^
2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (3*I)*ArcTan[a*x]])/(288*c^2*(a + a^3*x^
2)*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])
```

## Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.18, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {5435, 5433, 5440, 5439, 3042, 3785, 3793, 2009, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{3/2}}{(a^2cx^2+c)^{5/2}}\,dx$$

$$\Big\downarrow \text{5435}$$

$$-\frac{1}{12}\int \frac{1}{\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx+\frac{2\int \frac{\arctan(ax)^{3/2}}{\left(a^2cx^2+c\right)^{3/2}}dx}{3c}+\frac{x\arctan(ax)^{3/2}}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{\sqrt{\arctan(ax)}}{6ac\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{5433}$$

$$-\frac{1}{12}\int \frac{1}{\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx+\frac{2\left(-\frac{3}{4}\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{x\arctan(ax)^{3/2}}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{\sqrt{\arctan(ax)}}{6ac\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{5440}$$

$$-\frac{\sqrt{a^2x^2+1}\int \frac{1}{\left(a^2x^2+1\right)^{5/2}\sqrt{\arctan(ax)}}dx}{12c^2\sqrt{a^2cx^2+c}}+\frac{2\left(-\frac{3\sqrt{a^2x^2+1}\int \frac{1}{\left(a^2x^2+1\right)^{3/2}\sqrt{\arctan(ax)}}dx}{4c\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{x\arctan(ax)^{3/2}}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{\sqrt{\arctan(ax)}}{6ac\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{5439}$$

$$-\frac{\sqrt{a^2x^2+1}\int \frac{1}{\left(a^2x^2+1\right)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{12ac^2\sqrt{a^2cx^2+c}}+\frac{2\left(-\frac{3\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}d\arctan(ax)}{4ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{x\arctan(ax)^{3/2}}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{\sqrt{\arctan(ax)}}{6ac\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^3}{\sqrt{\arctan(ax)}}d\arctan(ax)}{12ac^2\sqrt{a^2cx^2+c}}+\frac{2\left(-\frac{3\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)}{\sqrt{\arctan(ax)}}d\arctan(ax)}{4ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{x\arctan(ax)^{3/2}}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{\sqrt{\arctan(ax)}}{6ac\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \ 3785$$

$$-\frac{\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^3}{\sqrt{\arctan(ax)}}d\arctan(ax)}{12ac^2\sqrt{a^2cx^2+c}}+\frac{2\left(-\frac{3\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}}d\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{x\arctan(ax)^{3/2}}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{\sqrt{\arctan(ax)}}{6ac\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \ 3793$$

$$-\frac{\sqrt{a^2x^2+1}\int \left(\frac{\cos(3\arctan(ax))}{4\sqrt{\arctan(ax)}}+\frac{3}{4\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{12ac^2\sqrt{a^2cx^2+c}}+\frac{2\left(-\frac{3\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}}d\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{x\arctan(ax)^{3/2}}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{\sqrt{\arctan(ax)}}{6ac\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \ 2009$$

$$\frac{2\left(-\frac{3\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}}d\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}\right)}{3c}-\frac{\sqrt{a^2x^2+1}\left(\frac{3}{2}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{6}}\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{12ac^2\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{3/2}}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{\sqrt{\arctan(ax)}}{6ac\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \ 3833$$

$$-\frac{\sqrt{a^2x^2+1}\left(\frac{3}{2}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{6}}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{12ac^2\sqrt{a^2cx^2+c}}+\frac{2\left(-\frac{3\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{2ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{x\arctan(ax)^{3/2}}{3c\,(a^2cx^2+c)^{3/2}}+\frac{\sqrt{\arctan(ax)}}{6ac\,(a^2cx^2+c)^{3/2}}$$

input
```
Int[ArcTan[a*x]^(3/2)/(c + a^2*c*x^2)^(5/2),x]
```

output
```
Sqrt[ArcTan[a*x]]/(6*a*c*(c + a^2*c*x^2)^(3/2)) + (x*ArcTan[a*x]^(3/2))/(3
*c*(c + a^2*c*x^2)^(3/2)) + (2*((3*Sqrt[ArcTan[a*x]])/(2*a*c*Sqrt[c + a^2*
c*x^2]) + (x*ArcTan[a*x]^(3/2))/(c*Sqrt[c + a^2*c*x^2]) - (3*Sqrt[Pi/2]*Sq
rt[1 + a^2*x^2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(2*a*c*Sqrt[c + a^
2*c*x^2])))/(3*c) - (Sqrt[1 + a^2*x^2]*((3*Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*
Sqrt[ArcTan[a*x]]])/2 + (Sqrt[Pi/6]*FresnelC[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]]
)/2))/(12*a*c^2*Sqrt[c + a^2*c*x^2])
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 5433
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x] - Simp[b^2*p*(p
- 1)   Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2), x], x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

rule 5435
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[b*p*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p - 1)/(4*c*d*
(q + 1)^2)), x] + (-Simp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*d*
(q + 1))), x] + Simp[(2*q + 3)/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p, x], x] - Simp[b^2*p*((p - 1)/(4*(q + 1)^2))   Int[(d + e
*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5440
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])   Int[(1 +
c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

## Maple [F]

$$\int \frac{\arctan (ax)^{\frac{3}{2}}}{(a^2c\,x^2+c)^{\frac{5}{2}}}dx$$

input
```
int(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x)
```

output
```
int(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{\arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{(c(a^2x^2+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(atan(a*x)**(3/2)/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(atan(a*x)**(3/2)/(c*(a**2*x**2 + 1))**(5/2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F]

$$\int \frac{\arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\arctan\left(ax\right)^{\frac{3}{2}}}{\left(a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^(3/2)/(a^2*c*x^2 + c)^(5/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\mathrm{atan}(a\,x)^{3/2}}{{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

input
```
int(atan(a*x)^(3/2)/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int(atan(a*x)^(3/2)/(c + a^2*c*x^2)^(5/2), x)
```

## Reduce [F]

$$\int \frac{\arctan(ax)^{3/2}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
int(atan(a*x)^(3/2)/(a^2*c*x^2+c)^(5/2),x)
```

output

```
(sqrt(c)*(8*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)*a**3*x**3 + 12*s
qrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)*a*x + 12*sqrt(a**2*x**2 + 1)*
sqrt(atan(a*x))*a**2*x**2 + 14*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)) - 6*int
((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**2)/(atan(a*x)*a**6*x**6 + 3*atan(
a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**7*x**4 - 12*int(
(sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**2)/(atan(a*x)*a**6*x**6 + 3*atan(a
*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**5*x**2 - 6*int((s
qrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**2)/(atan(a*x)*a**6*x**6 + 3*atan(a*x
)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**3 - 7*int((sqrt(a**
2*x**2 + 1)*sqrt(atan(a*x)))/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4
+ 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**5*x**4 - 14*int((sqrt(a**2*x**2
 + 1)*sqrt(atan(a*x)))/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*at
an(a*x)*a**2*x**2 + atan(a*x)),x)*a**3*x**2 - 7*int((sqrt(a**2*x**2 + 1)*s
qrt(atan(a*x)))/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)
*a**2*x**2 + atan(a*x)),x)*a))/(12*a*c**3*(a**4*x**4 + 2*a**2*x**2 + 1))
```

# 3.834 $\int \frac{\arctan(ax)^{3/2}}{x(c+a^2cx^2)^{5/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\arctan(ax)^{3/2}}{x\left(c+a^2cx^2\right)^{5/2}}\,dx = \text{Int}\left(\frac{\arctan(ax)^{3/2}}{x\left(c+a^2cx^2\right)^{5/2}},x\right)$$

output
```
Defer(Int)(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^(5/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 3.93 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\arctan(ax)^{3/2}}{x\left(c+a^2cx^2\right)^{5/2}}\,dx = \int \frac{\arctan(ax)^{3/2}}{x\left(c+a^2cx^2\right)^{5/2}}\,dx$$

input
```
Integrate[ArcTan[a*x]^(3/2)/(x*(c + a^2*c*x^2)^(5/2)),x]
```

output
```
Integrate[ArcTan[a*x]^(3/2)/(x*(c + a^2*c*x^2)^(5/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{3/2}}{x\left(a^2cx^2+c\right)^{5/2}}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{\arctan(ax)^{3/2}}{x\left(a^2cx^2+c\right)^{5/2}}dx$$

input
```
Int[ArcTan[a*x]^(3/2)/(x*(c + a^2*c*x^2)^(5/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\arctan\left(ax\right)^{\frac{3}{2}}}{x\left(a^2c\,x^2+c\right)^{\frac{5}{2}}}dx$$

input
```
int(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^(5/2),x)
```

output
```
int(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{x\left(c+a^2cx^2\right)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 86.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\arctan(ax)^{3/2}}{x\left(c+a^2cx^2\right)^{5/2}}\,dx = \int \frac{\operatorname{atan}^{\frac{3}{2}}(ax)}{x\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(atan(a*x)**(3/2)/x/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(atan(a*x)**(3/2)/(x*(c*(a**2*x**2 + 1))**(5/2)), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{x\left(c+a^2cx^2\right)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 2.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\arctan(ax)^{3/2}}{x\,(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\arctan\left(ax\right)^{\frac{3}{2}}}{(a^2cx^2+c)^{\frac{5}{2}}x}\,dx$$

input
```
integrate(arctan(a*x)^(3/2)/x/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^(3/2)/((a^2*c*x^2 + c)^(5/2)*x), x)
```

## Mupad [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\arctan(ax)^{3/2}}{x\,(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\mathrm{atan}(a\,x)^{3/2}}{x\,(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int(atan(a*x)^(3/2)/(x*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(atan(a*x)^(3/2)/(x*(c + a^2*c*x^2)^(5/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.12

$$\int \frac{\arctan(ax)^{3/2}}{x\left(c+a^2cx^2\right)^{5/2}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,atan(ax)}{a^6x^7+3a^4x^5+3a^2x^3+x}dx\right)}{c^3}$$

input
```
int(atan(a*x)^(3/2)/x/(a^2*c*x^2+c)^(5/2),x)
```

output
```
(sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x))/(a**6*x**7 +
3*a**4*x**5 + 3*a**2*x**3 + x),x))/c**3
```

# 3.835 $\int \frac{\arctan(ax)^{3/2}}{x^2(c+a^2cx^2)^{5/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\arctan(ax)^{3/2}}{x^2(c+a^2cx^2)^{5/2}}\,dx=\text{Int}\left(\frac{\arctan(ax)^{3/2}}{x^2(c+a^2cx^2)^{5/2}},x\right)$$

output
```
Defer(Int)(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c)^(5/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 6.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\arctan(ax)^{3/2}}{x^2(c+a^2cx^2)^{5/2}}\,dx=\int \frac{\arctan(ax)^{3/2}}{x^2(c+a^2cx^2)^{5/2}}\,dx$$

input
```
Integrate[ArcTan[a*x]^(3/2)/(x^2*(c + a^2*c*x^2)^(5/2)),x]
```

output
```
Integrate[ArcTan[a*x]^(3/2)/(x^2*(c + a^2*c*x^2)^(5/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{3/2}}{x^2 \left(a^2cx^2+c\right)^{5/2}} \, dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{\arctan(ax)^{3/2}}{x^2 \left(a^2cx^2+c\right)^{5/2}}dx$$

input
```
Int[ArcTan[a*x]^(3/2)/(x^2*(c + a^2*c*x^2)^(5/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\arctan \left(a x\right)^{\frac{3}{2}}}{x^{2} \left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}}}d x$$

input
```
int(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c)^(5/2),x)
```

output
```
int(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{x^2 (c + a^2cx^2)^{5/2}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^{3/2}}{x^2 (c + a^2cx^2)^{5/2}} \, dx = \text{Timed out}$$

input
```
integrate(atan(a*x)**(3/2)/x**2/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{3/2}}{x^2 (c + a^2cx^2)^{5/2}} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 2.47 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\arctan(ax)^{3/2}}{x^2 (c + a^2cx^2)^{5/2}} dx = \int \frac{\arctan(ax)^{\frac{3}{2}}}{(a^2cx^2 + c)^{\frac{5}{2}} x^2} dx$$

input
```
integrate(arctan(a*x)^(3/2)/x^2/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^(3/2)/((a^2*c*x^2 + c)^(5/2)*x^2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\arctan(ax)^{3/2}}{x^2 (c + a^2cx^2)^{5/2}} dx = \int \frac{\mathrm{atan}(a\,x)^{3/2}}{x^2\,(c\,a^2\,x^2 + c)^{5/2}} dx$$

input
```
int(atan(a*x)^(3/2)/(x^2*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(atan(a*x)^(3/2)/(x^2*(c + a^2*c*x^2)^(5/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 715, normalized size of antiderivative = 27.50

$$\int \frac{\arctan(ax)^{3/2}}{x^2 (c + a^2cx^2)^{5/2}} dx = \text{Too large to display}$$

input
```
int(atan(a*x)^(3/2)/x^2/(a^2*c*x^2+c)^(5/2),x)
```

output
```
(sqrt(c)*( - 16*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)*a**4*x**4 -
24*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)*a**2*x**2 - 6*sqrt(a**2*x
**2 + 1)*sqrt(atan(a*x))*atan(a*x) - 24*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)
)*a**3*x**3 - 28*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*a*x + 12*int((sqrt(a*
*2*x**2 + 1)*sqrt(atan(a*x))*x**2)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4
*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**8*x**5 + 24*int((sqrt(a**
2*x**2 + 1)*sqrt(atan(a*x))*x**2)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*
x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**6*x**3 + 12*int((sqrt(a**2
*x**2 + 1)*sqrt(atan(a*x))*x**2)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x
**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**4*x + 14*int((sqrt(a**2*x**
2 + 1)*sqrt(atan(a*x)))/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*a
tan(a*x)*a**2*x**2 + atan(a*x)),x)*a**6*x**5 + 28*int((sqrt(a**2*x**2 + 1)
*sqrt(atan(a*x)))/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*
x)*a**2*x**2 + atan(a*x)),x)*a**4*x**3 + 14*int((sqrt(a**2*x**2 + 1)*sqrt(
atan(a*x)))/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**
2*x**2 + atan(a*x)),x)*a**2*x + 9*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))
)/(a**6*x**7 + 3*a**4*x**5 + 3*a**2*x**3 + x),x)*a**5*x**5 + 18*int((sqrt(
a**2*x**2 + 1)*sqrt(atan(a*x)))/(a**6*x**7 + 3*a**4*x**5 + 3*a**2*x**3 + x
),x)*a**3*x**3 + 9*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(a**6*x**7 +
3*a**4*x**5 + 3*a**2*x**3 + x),x)*a*x))/(6*c**3*x*(a**4*x**4 + 2*a**2*x...
```

# 3.836 $\int x^m(c+a^2cx^2)\arctan(ax)^{5/2}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int x^m\left(c+a^2cx^2\right)\arctan(ax)^{5/2}\,dx=\text{Int}\left(x^m\left(c+a^2cx^2\right)\arctan(ax)^{5/2},x\right)$$

output
```
Defer(Int)(x^m*(a^2*c*x^2+c)*arctan(a*x)^(5/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int x^m\left(c+a^2cx^2\right)\arctan(ax)^{5/2}\,dx=\int x^m\left(c+a^2cx^2\right)\arctan(ax)^{5/2}\,dx$$

input
```
Integrate[x^m*(c + a^2*c*x^2)*ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[x^m*(c + a^2*c*x^2)*ArcTan[a*x]^(5/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \arctan(ax)^{5/2} \left(a^2cx^2+c\right) \, dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int x^m \arctan(ax)^{5/2} \left(a^2cx^2+c\right)dx$$

input
```
Int[x^m*(c + a^2*c*x^2)*ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int x^{m} \left(a^{2} c \,x^{2}+c \right) \arctan \left(a x \right)^{\frac{5}{2}}d x$$

input
```
int(x^m*(a^2*c*x^2+c)*arctan(a*x)^(5/2),x)
```

output
```
int(x^m*(a^2*c*x^2+c)*arctan(a*x)^(5/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x^m (c + a^2 c x^2) \arctan(ax)^{5/2} \, dx = \int (a^2 c x^2 + c) x^m \arctan(ax)^{\frac{5}{2}} \, dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)*arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)*x^m*arctan(a*x)^(5/2), x)
```

## Sympy [F(-1)]

Timed out.

$$\int x^m (c + a^2 c x^2) \arctan(ax)^{5/2} \, dx = \text{Timed out}$$

input
```
integrate(x**m*(a**2*c*x**2+c)*atan(a*x)**(5/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int x^m (c + a^2 c x^2) \arctan(ax)^{5/2} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)*arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x^m (c + a^2 c x^2) \arctan(ax)^{5/2}\, dx = \int (a^2 c x^2 + c) x^m \arctan(ax)^{\frac{5}{2}}\, dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)*arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)*x^m*arctan(a*x)^(5/2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.90 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x^m (c + a^2 c x^2) \arctan(ax)^{5/2}\, dx = \int x^m \operatorname{atan}(a\,x)^{5/2}\,(c\,a^2\,x^2 + c)\, dx$$

input
```
int(x^m*atan(a*x)^(5/2)*(c + a^2*c*x^2),x)
```

output
```
int(x^m*atan(a*x)^(5/2)*(c + a^2*c*x^2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 1479, normalized size of antiderivative = 67.23

$$\int x^m (c + a^2 c x^2) \arctan(ax)^{5/2}\, dx = \text{Too large to display}$$

input
```
int(x^m*(a^2*c*x^2+c)*atan(a*x)^(5/2),x)
```

output
```
(c*(8*x**m*sqrt(atan(a*x))*atan(a*x)**2*a**3*m**3*x**3 + 24*x**m*sqrt(atan
(a*x))*atan(a*x)**2*a**3*m**2*x**3 + 16*x**m*sqrt(atan(a*x))*atan(a*x)**2*
a**3*m*x**3 + 8*x**m*sqrt(atan(a*x))*atan(a*x)**2*a*m**3*x + 40*x**m*sqrt(
atan(a*x))*atan(a*x)**2*a*m**2*x + 48*x**m*sqrt(atan(a*x))*atan(a*x)**2*a*
m*x - 20*x**m*sqrt(atan(a*x))*atan(a*x)*a**2*m**2*x**2 - 20*x**m*sqrt(atan
(a*x))*atan(a*x)*a**2*m*x**2 - 40*x**m*sqrt(atan(a*x))*atan(a*x)*m - 80*x*
*m*sqrt(atan(a*x))*atan(a*x) + 30*x**m*sqrt(atan(a*x))*a*m*x + 40*int((x**
m*sqrt(atan(a*x))*atan(a*x))/(a**2*m**3*x**3 + 6*a**2*m**2*x**3 + 11*a**2*
m*x**3 + 6*a**2*x**3 + m**3*x + 6*m**2*x + 11*m*x + 6*x),x)*m**5 + 320*int
((x**m*sqrt(atan(a*x))*atan(a*x))/(a**2*m**3*x**3 + 6*a**2*m**2*x**3 + 11*
a**2*m*x**3 + 6*a**2*x**3 + m**3*x + 6*m**2*x + 11*m*x + 6*x),x)*m**4 + 92
0*int((x**m*sqrt(atan(a*x))*atan(a*x))/(a**2*m**3*x**3 + 6*a**2*m**2*x**3
+ 11*a**2*m*x**3 + 6*a**2*x**3 + m**3*x + 6*m**2*x + 11*m*x + 6*x),x)*m**3
 + 1120*int((x**m*sqrt(atan(a*x))*atan(a*x))/(a**2*m**3*x**3 + 6*a**2*m**2
*x**3 + 11*a**2*m*x**3 + 6*a**2*x**3 + m**3*x + 6*m**2*x + 11*m*x + 6*x),x
)*m**2 + 480*int((x**m*sqrt(atan(a*x))*atan(a*x))/(a**2*m**3*x**3 + 6*a**2
*m**2*x**3 + 11*a**2*m*x**3 + 6*a**2*x**3 + m**3*x + 6*m**2*x + 11*m*x + 6
*x),x)*m - 15*int((x**m*sqrt(atan(a*x))*x)/(atan(a*x)*a**2*m**3*x**2 + 6*a
tan(a*x)*a**2*m**2*x**2 + 11*atan(a*x)*a**2*m*x**2 + 6*atan(a*x)*a**2*x**2
 + atan(a*x)*m**3 + 6*atan(a*x)*m**2 + 11*atan(a*x)*m + 6*atan(a*x)),x)...
```

# 3.837 $\int x^2(c+a^2cx^2)\arctan(ax)^{5/2}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int x^2(c+a^2cx^2)\arctan(ax)^{5/2}\,dx=\operatorname{Int}\left(x^2(c+a^2cx^2)\arctan(ax)^{5/2},x\right)$$

output
```
Defer(Int)(x^2*(a^2*c*x^2+c)*arctan(a*x)^(5/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 3.39 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int x^2(c+a^2cx^2)\arctan(ax)^{5/2}\,dx=\int x^2(c+a^2cx^2)\arctan(ax)^{5/2}\,dx$$

input
```
Integrate[x^2*(c + a^2*c*x^2)*ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[x^2*(c + a^2*c*x^2)*ArcTan[a*x]^(5/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arctan(ax)^{5/2} \left(a^2cx^2+c\right) \, dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int x^2 \arctan(ax)^{5/2} \left(a^2cx^2+c\right)dx$$

input
```
Int[x^2*(c + a^2*c*x^2)*ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int x^{2} \left(a^{2} c \,x^{2}+c \right) \arctan \left(a x \right)^{\frac{5}{2}}d x$$

input
```
int(x^2*(a^2*c*x^2+c)*arctan(a*x)^(5/2),x)
```

output
```
int(x^2*(a^2*c*x^2+c)*arctan(a*x)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int x^2(c+a^2cx^2)\arctan(ax)^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(a^2*c*x^2+c)*arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 35.96 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.41

$$\int x^2(c+a^2cx^2)\arctan(ax)^{5/2}\,dx = c\left(\int x^2\operatorname{atan}^{\frac{5}{2}}(ax)\,dx + \int a^2x^4\operatorname{atan}^{\frac{5}{2}}(ax)\,dx\right)$$

input
```
integrate(x**2*(a**2*c*x**2+c)*atan(a*x)**(5/2),x)
```

output
```
c*(Integral(x**2*atan(a*x)**(5/2), x) + Integral(a**2*x**4*atan(a*x)**(5/2
), x))
```

## Maxima [F(-2)]

Exception generated.

$$\int x^2(c+a^2cx^2)\arctan(ax)^{5/2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*(a^2*c*x^2+c)*arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x^2(c+a^2cx^2)\arctan(ax)^{5/2}\,dx=\int (a^2cx^2+c)x^2\arctan{(ax)}^{\frac{5}{2}}\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)*arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)*x^2*arctan(a*x)^(5/2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.77 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x^2(c+a^2cx^2)\arctan(ax)^{5/2}\,dx=\int x^2\,\mathrm{atan}(a\,x)^{5/2}\,\left(c\,a^2\,x^2+c\right)\,dx$$

input
```
int(x^2*atan(a*x)^(5/2)*(c + a^2*c*x^2),x)
```

output
```
int(x^2*atan(a*x)^(5/2)*(c + a^2*c*x^2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.86

$$\int x^2(c+a^2cx^2)\arctan(ax)^{5/2}\,dx = c\left(\left(\int \sqrt{atan\,(ax)}\,atan(ax)^2\,x^4dx\right)a^2 + \int \sqrt{atan\,(ax)}\,atan(ax)^2\,x^2dx\right)$$

input
```
int(x^2*(a^2*c*x^2+c)*atan(a*x)^(5/2),x)
```

output
```
c*(int(sqrt(atan(a*x))*atan(a*x)**2*x**4,x)*a**2 + int(sqrt(atan(a*x))*ata
n(a*x)**2*x**2,x))
```

# 3.838 $\int x(c+a^2cx^2)\arctan(ax)^{5/2}\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 20

$$\int x(c+a^2cx^2)\arctan(ax)^{5/2}\,dx=\frac{5c(1+a^2x^2)\sqrt{\arctan(ax)}}{32a^2}-\frac{5cx(1+a^2x^2)\arctan(ax)^{3/2}}{24a}+\frac{c(1+a^2x^2)^2\arctan(ax)^{5/2}}{4a^2}-\frac{5c\text{Int}\left(\frac{1}{\sqrt{\arctan(ax)}},x\right)}{64a}-\frac{5c\text{Int}\left(\arctan(ax)^{3/2},x\right)}{12a}$$

output
```
5/32*c*(a^2*x^2+1)*arctan(a*x)^(1/2)/a^2-5/24*c*x*(a^2*x^2+1)*arctan(a*x)^
(3/2)/a+1/4*c*(a^2*x^2+1)^2*arctan(a*x)^(5/2)/a^2-5/64*c*Defer(Int)(1/arct
an(a*x)^(1/2),x)/a-5/12*c*Defer(Int)(arctan(a*x)^(3/2),x)/a
```

## Mathematica [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int x(c+a^2cx^2)\arctan(ax)^{5/2}\,dx=\int x(c+a^2cx^2)\arctan(ax)^{5/2}\,dx$$

input
```
Integrate[x*(c + a^2*c*x^2)*ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[x*(c + a^2*c*x^2)*ArcTan[a*x]^(5/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \arctan(ax)^{5/2} \left(a^2cx^2+c\right) \, dx$$

$$\Big\downarrow \text{ 5465}$$

$$\frac{c\left(a^2x^2+1\right)^2 \arctan(ax)^{5/2}}{4a^2}-\frac{5\int c\left(a^2x^2+1\right) \arctan(ax)^{3/2}dx}{8a}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{c\left(a^2x^2+1\right)^2 \arctan(ax)^{5/2}}{4a^2}-\frac{5c\int \left(a^2x^2+1\right) \arctan(ax)^{3/2}dx}{8a}$$

$$\Big\downarrow \text{ 5415}$$

$$\frac{c\left(a^2x^2+1\right)^2 \arctan(ax)^{5/2}}{4a^2}-\frac{5c\left(\frac{1}{8}\int \frac{1}{\sqrt{\arctan(ax)}}dx+\frac{2}{3}\int \arctan(ax)^{3/2}dx+\frac{1}{3}x\left(a^2x^2+1\right) \arctan(ax)^{3/2}-\frac{\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}{4a}\right)}{8a}$$

$$\Big\downarrow \text{ 5353}$$

$$\frac{c\left(a^2x^2+1\right)^2 \arctan(ax)^{5/2}}{4a^2}-\frac{5c\left(\frac{1}{8}\int \frac{1}{\sqrt{\arctan(ax)}}dx+\frac{2}{3}\int \arctan(ax)^{3/2}dx+\frac{1}{3}x\left(a^2x^2+1\right) \arctan(ax)^{3/2}-\frac{\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}{4a}\right)}{8a}$$

input
```
Int[x*(c + a^2*c*x^2)*ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int x\left(a^{2} c\,x^{2}+c\right) \arctan\left(a x\right)^{\frac{5}{2}}d x$$

input
```
int(x*(a^2*c*x^2+c)*arctan(a*x)^(5/2),x)
```

output
```
int(x*(a^2*c*x^2+c)*arctan(a*x)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int x\left(c+a^2cx^2\right)\arctan(ax)^{5/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)*arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 22.37 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.45

$$\int x(c+a^2cx^2)\arctan(ax)^{5/2}\,dx=c\left(\int x\,\mathrm{atan}^{\frac{5}{2}}\left(ax\right)dx+\int a^2x^3\,\mathrm{atan}^{\frac{5}{2}}\left(ax\right)dx\right)$$

input
```
integrate(x*(a**2*c*x**2+c)*atan(a*x)**(5/2),x)
```

output
```
c*(Integral(x*atan(a*x)**(5/2), x) + Integral(a**2*x**3*atan(a*x)**(5/2),
x))
```

## Maxima [F(-2)]

Exception generated.

$$\int x(c+a^2cx^2)\arctan(ax)^{5/2}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a^2*c*x^2+c)*arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int x(c+a^2cx^2)\arctan(ax)^{5/2}\,dx=\int (a^2cx^2+c)x\arctan\left(ax\right)^{\frac{5}{2}}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)*arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)*x*arctan(a*x)^(5/2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int x(c+a^2cx^2)\arctan(ax)^{5/2}\,dx=\int x\,\mathrm{atan}(a\,x)^{5/2}\,\left(c\,a^2\,x^2+c\right)\,dx$$

input
```
int(x*atan(a*x)^(5/2)*(c + a^2*c*x^2),x)
```

output
```
int(x*atan(a*x)^(5/2)*(c + a^2*c*x^2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 160, normalized size of antiderivative = 8.00

$$\int x(c+a^2cx^2)\arctan(ax)^{5/2}\,dx=\frac{c\Big(48\sqrt{atan\,(ax)}\,atan(ax)^2\,a^4x^4+96\sqrt{atan\,(ax)}\,atan(ax)^2\,a^2x^2+48\sqrt{atan\,(}$$

input
```
int(x*(a^2*c*x^2+c)*atan(a*x)^(5/2),x)
```

output
```
(c*(48*sqrt(atan(a*x))*atan(a*x)**2*a**4*x**4 + 96*sqrt(atan(a*x))*atan(a*
x)**2*a**2*x**2 + 48*sqrt(atan(a*x))*atan(a*x)**2 - 40*sqrt(atan(a*x))*ata
n(a*x)*a**3*x**3 - 120*sqrt(atan(a*x))*atan(a*x)*a*x + 30*sqrt(atan(a*x))*
a**2*x**2 - 15*int((sqrt(atan(a*x))*x**2)/(atan(a*x)*a**2*x**2 + atan(a*x)
),x)*a**3 + 120*int((sqrt(atan(a*x))*x)/(a**2*x**2 + 1),x)*a**2))/(192*a**
2)
```

# 3.839 $\int \left(c+a^2cx^2\right) \arctan(ax)^{5/2} \, dx$



## Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \left(c+a^2cx^2\right) \arctan(ax)^{5/2} \, dx = -\frac{5c(1+a^2x^2)\arctan(ax)^{3/2}}{12a} + \frac{1}{3}cx\left(1+a^2x^2\right)\arctan(ax)^{5/2} + \frac{5}{8}c\text{Int}\left(\sqrt{\arctan(ax)},x\right) + \frac{2}{3}c\text{Int}\left(\arctan(ax)^{5/2},x\right)$$

output
```
-5/12*c*(a^2*x^2+1)*arctan(a*x)^(3/2)/a+1/3*c*x*(a^2*x^2+1)*arctan(a*x)^(5
/2)+5/8*c*Defer(Int)(arctan(a*x)^(1/2),x)+2/3*c*Defer(Int)(arctan(a*x)^(5/
2),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.93 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \left(c+a^2cx^2\right) \arctan(ax)^{5/2} \, dx = \int \left(c+a^2cx^2\right) \arctan(ax)^{5/2} \, dx$$

input
```
Integrate[(c + a^2*c*x^2)*ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[(c + a^2*c*x^2)*ArcTan[a*x]^(5/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(ax)^{5/2} \left(a^2cx^2+c\right) \, dx$$

$$\Big\downarrow \text{5415}$$

$$\frac{5}{8}c\int \sqrt{\arctan(ax)}dx+\frac{2}{3}c\int \arctan(ax)^{5/2}dx+\frac{1}{3}cx\left(a^2x^2+1\right)\arctan(ax)^{5/2}-\frac{5c\left(a^2x^2+1\right)\arctan(ax)^{3/2}}{12a}$$

$$\Big\downarrow \text{5353}$$

$$\frac{5}{8}c\int \sqrt{\arctan(ax)}dx+\frac{2}{3}c\int \arctan(ax)^{5/2}dx+\frac{1}{3}cx\left(a^2x^2+1\right)\arctan(ax)^{5/2}-\frac{5c\left(a^2x^2+1\right)\arctan(ax)^{3/2}}{12a}$$

input
```
Int[(c + a^2*c*x^2)*ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \left(a^{2} c \,x^{2}+c \right) \arctan \left(a x \right)^{\frac{5}{2}}d x$$

input
```
int((a^2*c*x^2+c)*arctan(a*x)^(5/2),x)
```

output
```
int((a^2*c*x^2+c)*arctan(a*x)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int (c+a^2cx^2) \arctan(ax)^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 17.79 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.42

$$\int (c+a^2cx^2) \arctan(ax)^{5/2}\,dx = c\left(\int a^{2} x^{2} \operatorname{atan}^{\frac{5}{2}}{\left(a x \right)}\, dx + \int \operatorname{atan}^{\frac{5}{2}}{\left(a x \right)}\, dx\right)$$

input
```
integrate((a**2*c*x**2+c)*atan(a*x)**(5/2),x)
```

output
```
c*(Integral(a**2*x**2*atan(a*x)**(5/2), x) + Integral(atan(a*x)**(5/2), x)
)
```

## Maxima [F(-2)]

Exception generated.

$$\int (c + a^2cx^2) \arctan(ax)^{5/2} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int (c + a^2cx^2) \arctan(ax)^{5/2} \, dx = \int (a^2cx^2 + c) \arctan(ax)^{\frac{5}{2}} \, dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)*arctan(a*x)^(5/2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \left(c+a^2cx^2\right) \arctan(ax)^{5/2}\,dx=\int \mathrm{atan}(a\,x)^{5/2}\,\left(c\,a^2\,x^2+c\right)\,dx$$

input
```
int(atan(a*x)^(5/2)*(c + a^2*c*x^2),x)
```

output
```
int(atan(a*x)^(5/2)*(c + a^2*c*x^2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 139, normalized size of antiderivative = 7.32

$$\int (c +a^2cx^2) \arctan(ax)^{5/2}\,dx=\frac{c\left(16\sqrt{atan\,(ax)}\,atan(ax)^2\,a^3x^3+48\sqrt{atan\,(ax)}\,atan(ax)^2\,ax-20\sqrt{atan\,(ax$$

input
```
int((a^2*c*x^2+c)*atan(a*x)^(5/2),x)
```

output
```
(c*(16*sqrt(atan(a*x))*atan(a*x)**2*a**3*x**3 + 48*sqrt(atan(a*x))*atan(a*
x)**2*a*x - 20*sqrt(atan(a*x))*atan(a*x)*a**2*x**2 - 20*sqrt(atan(a*x))*at
an(a*x) + 30*sqrt(atan(a*x))*a*x - 80*int((sqrt(atan(a*x))*atan(a*x)*x)/(a
**2*x**2 + 1),x)*a**2 - 15*int((sqrt(atan(a*x))*x)/(atan(a*x)*a**2*x**2 +
atan(a*x)),x)*a**2))/(48*a)
```

# 3.840 $\int \frac{(c+a^2cx^2)\arctan(ax)^{5/2}}{x}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{(c+a^2cx^2)\arctan(ax)^{5/2}}{x}\,dx = \text{Int}\left(\frac{(c+a^2cx^2)\arctan(ax)^{5/2}}{x},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)*arctan(a*x)^(5/2)/x,x)
```

## Mathematica [N/A]

Not integrable

Time = 1.45 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(c+a^2cx^2)\arctan(ax)^{5/2}}{x}\,dx = \int \frac{(c+a^2cx^2)\arctan(ax)^{5/2}}{x}\,dx$$

input
```
Integrate[((c + a^2*c*x^2)*ArcTan[a*x]^(5/2))/x,x]
```

output
```
Integrate[((c + a^2*c*x^2)*ArcTan[a*x]^(5/2))/x, x]
```

## Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)}{x}\,dx$$

$$\Big\downarrow \text{5560}$$

$$\int \frac{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)}{x}dx$$

input
```
Int[((c + a^2*c*x^2)*ArcTan[a*x]^(5/2))/x,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\left(a^2c\,x^2+c\right)\arctan\left(ax\right)^{\frac{5}{2}}}{x}dx$$

input
```
int((a^2*c*x^2+c)*arctan(a*x)^(5/2)/x,x)
```

output
```
int((a^2*c*x^2+c)*arctan(a*x)^(5/2)/x,x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{(c + a^2cx^2)\arctan(ax)^{5/2}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^(5/2)/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 13.59 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

$$\int \frac{(c + a^2cx^2)\arctan(ax)^{5/2}}{x}\,dx = c\left(\int \frac{\operatorname{atan}^{\frac{5}{2}}(ax)}{x}\,dx + \int a^2x\operatorname{atan}^{\frac{5}{2}}(ax)\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)*atan(a*x)**(5/2)/x,x)
```

output
```
c*(Integral(atan(a*x)**(5/2)/x, x) + Integral(a**2*x*atan(a*x)**(5/2), x))
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(c + a^2cx^2)\arctan(ax)^{5/2}}{x}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^(5/2)/x,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)\arctan(ax)^{5/2}}{x}\,dx = \int \frac{(a^2cx^2+c)\arctan{(ax)}^{\frac{5}{2}}}{x}\,dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^(5/2)/x,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)*arctan(a*x)^(5/2)/x, x)
```

## Mupad [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)\arctan(ax)^{5/2}}{x}\,dx = \int \frac{\mathrm{atan}(a\,x)^{5/2}\,(c\,a^2\,x^2+c)}{x}\,dx$$

input
```
int((atan(a*x)^(5/2)*(c + a^2*c*x^2))/x,x)
```

output
```
int((atan(a*x)^(5/2)*(c + a^2*c*x^2))/x, x)
```

## Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 142, normalized size of antiderivative = 6.45

$$\int \frac{(c+a^2cx^2)\arctan(ax)^{5/2}}{x}\,dx = \frac{c\Big(16\sqrt{atan\,(ax)}\,atan(ax)^2\,a^2x^2+16\sqrt{atan\,(ax)}\,atan(ax)^2-40\sqrt{atan}$$

input
```
int((a^2*c*x^2+c)*atan(a*x)^(5/2)/x,x)
```

output

```
(c*(16*sqrt(atan(a*x))*atan(a*x)**2*a**2*x**2 + 16*sqrt(atan(a*x))*atan(a*
x)**2 - 40*sqrt(atan(a*x))*atan(a*x)*a*x + 30*sqrt(atan(a*x))*a**2*x**2 +
32*int((sqrt(atan(a*x))*atan(a*x)**2)/x,x) - 60*int((sqrt(atan(a*x))*x**3)
/(a**2*x**2 + 1),x)*a**4 - 15*int((sqrt(atan(a*x))*x**2)/(atan(a*x)*a**2*x
**2 + atan(a*x)),x)*a**3))/32
```

# 3.841 $\int \frac{(c+a^2cx^2)\arctan(ax)^{5/2}}{x^2}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{(c+a^2cx^2)\arctan(ax)^{5/2}}{x^2}\,dx = \text{Int}\left(\frac{(c+a^2cx^2)\arctan(ax)^{5/2}}{x^2},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)*arctan(a*x)^(5/2)/x^2,x)
```

## Mathematica [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{(c+a^2cx^2)\arctan(ax)^{5/2}}{x^2}\,dx = \int \frac{(c+a^2cx^2)\arctan(ax)^{5/2}}{x^2}\,dx$$

input
```
Integrate[((c + a^2*c*x^2)*ArcTan[a*x]^(5/2))/x^2,x]
```

output
```
Integrate[((c + a^2*c*x^2)*ArcTan[a*x]^(5/2))/x^2, x]
```

## Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)}{x^2}dx$$

$$\Big\downarrow \text{5560}$$

$$\int \frac{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)}{x^2}dx$$

input
```
Int[((c + a^2*c*x^2)*ArcTan[a*x]^(5/2))/x^2,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{\left(a^2c\,x^2+c\right)\arctan\left(ax\right)^{\frac{5}{2}}}{x^2}dx$$

input
```
int((a^2*c*x^2+c)*arctan(a*x)^(5/2)/x^2,x)
```

output
```
int((a^2*c*x^2+c)*arctan(a*x)^(5/2)/x^2,x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)\arctan(ax)^{5/2}}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^(5/2)/x^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 17.32 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

$$\int \frac{(c+a^2cx^2)\arctan(ax)^{5/2}}{x^2}\,dx = c\left(\int a^2\operatorname{atan}^{\frac{5}{2}}(ax)\,dx + \int \frac{\operatorname{atan}^{\frac{5}{2}}(ax)}{x^2}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)*atan(a*x)**(5/2)/x**2,x)
```

output
```
c*(Integral(a**2*atan(a*x)**(5/2), x) + Integral(atan(a*x)**(5/2)/x**2, x)
)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)\arctan(ax)^{5/2}}{x^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^(5/2)/x^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)\arctan(ax)^{5/2}}{x^2}\,dx = \int \frac{(a^2cx^2+c)\arctan{(ax)}^{\frac{5}{2}}}{x^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)*arctan(a*x)^(5/2)/x^2,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)*arctan(a*x)^(5/2)/x^2, x)
```

## Mupad [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)\arctan(ax)^{5/2}}{x^2}\,dx = \int \frac{\mathrm{atan}(a\,x)^{5/2}\,(c\,a^2\,x^2+c)}{x^2}\,dx$$

input
```
int((atan(a*x)^(5/2)*(c + a^2*c*x^2))/x^2,x)
```

output
```
int((atan(a*x)^(5/2)*(c + a^2*c*x^2))/x^2, x)
```

## Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 67, normalized size of antiderivative = 3.05

$$\int \frac{(c+a^2cx^2)\arctan(ax)^{5/2}}{x^2}\,dx = \frac{c\left(-2\sqrt{atan\,(ax)}\,atan(ax)^2+5\left(\int \frac{\sqrt{atan(ax)}\,atan(ax)}{a^2x^3+x}dx\right)ax+2\left(\int \sqrt{ata\ldots\right.\right.}{2x}$$

input
```
int((a^2*c*x^2+c)*atan(a*x)^(5/2)/x^2,x)
```

output
```
(c*( - 2*sqrt(atan(a*x))*atan(a*x)**2 + 5*int((sqrt(atan(a*x))*atan(a*x))/
(a**2*x**3 + x),x)*a*x + 2*int(sqrt(atan(a*x))*atan(a*x)**2,x)*a**2*x))/(2
*x)
```

# 3.842 $\int x^m(c+a^2cx^2)^2 \arctan(ax)^{5/2}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int x^m\left(c+a^2cx^2\right)^2 \arctan(ax)^{5/2}\,dx = \operatorname{Int}\left(x^m\left(c+a^2cx^2\right)^2 \arctan(ax)^{5/2},x\right)$$

output
```
Defer(Int)(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int x^m\left(c+a^2cx^2\right)^2 \arctan(ax)^{5/2}\,dx = \int x^m\left(c+a^2cx^2\right)^2 \arctan(ax)^{5/2}\,dx$$

input
```
Integrate[x^m*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[x^m*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \arctan(ax)^{5/2} \left(a^2cx^2+c\right)^2 \, dx$$

$$\Big\downarrow \; 5560$$

$$\int x^m \arctan(ax)^{5/2} \left(a^2cx^2+c\right)^2 dx$$

input
```
Int[x^m*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int x^m \left(a^2 c\,x^2+c\right)^2 \arctan\left(a x\right)^{\frac{5}{2}} dx$$

input
```
int(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int x^m \left(c+a^2cx^2\right)^2 \arctan(ax)^{5/2}\,dx = \int \left(a^2cx^2+c\right)^2 x^m \arctan\left(ax\right)^{\frac{5}{2}}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*x^m*arctan(a*x)^(5/2), x)
```

## Sympy [F(-1)]

Timed out.

$$\int x^m \left(c+a^2cx^2\right)^2 \arctan(ax)^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(x**m*(a**2*c*x**2+c)**2*atan(a*x)**(5/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int x^m \left(c+a^2cx^2\right)^2 \arctan(ax)^{5/2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^m \left(c+a^2cx^2\right)^2 \arctan(ax)^{5/2}\,dx = \int \left(a^2cx^2+c\right)^2 x^m \arctan\left(ax\right)^{\frac{5}{2}}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^2*x^m*arctan(a*x)^(5/2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.77 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^m \left(c+a^2cx^2\right)^2 \arctan(ax)^{5/2}\,dx = \int x^m\,\mathrm{atan}(a\,x)^{5/2}\left(c\,a^2\,x^2+c\right)^2\,dx$$

input
```
int(x^m*atan(a*x)^(5/2)*(c + a^2*c*x^2)^2,x)
```

output
```
int(x^m*atan(a*x)^(5/2)*(c + a^2*c*x^2)^2, x)
```

## Reduce [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 4847, normalized size of antiderivative = 201.96

$$\int x^m \left(c+a^2cx^2\right)^2 \arctan(ax)^{5/2}\,dx = \text{Too large to display}$$

input
```
int(x^m*(a^2*c*x^2+c)^2*atan(a*x)^(5/2),x)
```

output
```
(c**2*(8*x**m*sqrt(atan(a*x))*atan(a*x)**2*a**5*m**5*x**5 + 80*x**m*sqrt(a
tan(a*x))*atan(a*x)**2*a**5*m**4*x**5 + 280*x**m*sqrt(atan(a*x))*atan(a*x)
**2*a**5*m**3*x**5 + 400*x**m*sqrt(atan(a*x))*atan(a*x)**2*a**5*m**2*x**5
+ 192*x**m*sqrt(atan(a*x))*atan(a*x)**2*a**5*m*x**5 + 16*x**m*sqrt(atan(a*
x))*atan(a*x)**2*a**3*m**5*x**3 + 192*x**m*sqrt(atan(a*x))*atan(a*x)**2*a*
*3*m**4*x**3 + 784*x**m*sqrt(atan(a*x))*atan(a*x)**2*a**3*m**3*x**3 + 1248
*x**m*sqrt(atan(a*x))*atan(a*x)**2*a**3*m**2*x**3 + 640*x**m*sqrt(atan(a*x
))*atan(a*x)**2*a**3*m*x**3 + 8*x**m*sqrt(atan(a*x))*atan(a*x)**2*a*m**5*x
 + 112*x**m*sqrt(atan(a*x))*atan(a*x)**2*a*m**4*x + 568*x**m*sqrt(atan(a*x
))*atan(a*x)**2*a*m**3*x + 1232*x**m*sqrt(atan(a*x))*atan(a*x)**2*a*m**2*x
 + 960*x**m*sqrt(atan(a*x))*atan(a*x)**2*a*m*x - 20*x**m*sqrt(atan(a*x))*a
tan(a*x)*a**4*m**4*x**4 - 120*x**m*sqrt(atan(a*x))*atan(a*x)*a**4*m**3*x**
4 - 220*x**m*sqrt(atan(a*x))*atan(a*x)*a**4*m**2*x**4 - 120*x**m*sqrt(atan
(a*x))*atan(a*x)*a**4*m*x**4 - 20*x**m*sqrt(atan(a*x))*atan(a*x)*a**2*m**4
*x**2 - 240*x**m*sqrt(atan(a*x))*atan(a*x)*a**2*m**3*x**2 - 780*x**m*sqrt(
atan(a*x))*atan(a*x)*a**2*m**2*x**2 - 560*x**m*sqrt(atan(a*x))*atan(a*x)*a
**2*m*x**2 - 160*x**m*sqrt(atan(a*x))*atan(a*x)*m**2 - 960*x**m*sqrt(atan(
a*x))*atan(a*x)*m - 1280*x**m*sqrt(atan(a*x))*atan(a*x) + 30*x**m*sqrt(ata
n(a*x))*a**3*m**3*x**3 + 90*x**m*sqrt(atan(a*x))*a**3*m**2*x**3 + 60*x**m*
sqrt(atan(a*x))*a**3*m*x**3 + 180*x**m*sqrt(atan(a*x))*a*m**2*x + 660*x...
```

# 3.843 $\int x^2(c+a^2cx^2)^2 \arctan(ax)^{5/2}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int x^2\left(c+a^2cx^2\right)^2 \arctan(ax)^{5/2}\,dx = \text{Int}\left(x^2\left(c+a^2cx^2\right)^2 \arctan(ax)^{5/2},x\right)$$

output
```
Defer(Int)(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 2.35 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int x^2\left(c+a^2cx^2\right)^2 \arctan(ax)^{5/2}\,dx = \int x^2\left(c+a^2cx^2\right)^2 \arctan(ax)^{5/2}\,dx$$

input
```
Integrate[x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arctan(ax)^{5/2} \left(a^2 c x^2+c\right)^2 \, dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int x^2 \arctan(ax)^{5/2} \left(a^2 c x^2+c\right)^2 dx$$

input
```
Int[x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int x^{2} \left(a^{2} c \,x^{2}+c \right)^{2} \arctan \left(a x \right)^{\frac{5}{2}}d x$$

input
```
int(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x)
```

output
```
int(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int x^2 \left(c+a^2cx^2\right)^2 \arctan(ax)^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 66.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.12

$$\int x^2 \left(c+a^2cx^2\right)^2 \arctan(ax)^{5/2}\,dx = c^2 \left(\int x^2 \operatorname{atan}^{\frac{5}{2}}(ax)\,dx + \int 2a^2x^4 \operatorname{atan}^{\frac{5}{2}}(ax)\,dx + \int a^4x^6 \operatorname{atan}^{\frac{5}{2}}(ax)\,dx\right)$$

input
```
integrate(x**2*(a**2*c*x**2+c)**2*atan(a*x)**(5/2),x)
```

output
```
c**2*(Integral(x**2*atan(a*x)**(5/2), x) + Integral(2*a**2*x**4*atan(a*x)*
*(5/2), x) + Integral(a**4*x**6*atan(a*x)**(5/2), x))
```

## Maxima [F(-2)]

Exception generated.

$$\int x^2 \left(c+a^2 c x^2\right)^2 \arctan(ax)^{5/2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^2 \left(c+a^2 c x^2\right)^2 \arctan(ax)^{5/2}\,dx = \int \left(a^2 c x^2 + c\right)^2 x^2 \arctan\left(ax\right)^{\frac{5}{2}}\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^2*x^2*arctan(a*x)^(5/2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^2 \left(c+a^2 c x^2\right)^2 \arctan(ax)^{5/2}\,dx = \int x^2\,\mathrm{atan}(a\,x)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^2\,dx$$

input
```
int(x^2*atan(a*x)^(5/2)*(c + a^2*c*x^2)^2,x)
```

output
```
int(x^2*atan(a*x)^(5/2)*(c + a^2*c*x^2)^2, x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.71

$$\int x^2 \left(c+a^2 c x^2\right)^2 \arctan(ax)^{5/2} \, dx = c^2 \left( \left( \int \sqrt{atan\,(ax)}\, atan(ax)^2\, x^6 dx \right) a^4 + 2 \left( \int \sqrt{atan\,(ax)}\, atan(ax)^2\, x^4 dx \right) a^2 + \int \sqrt{atan\,(ax)}\, atan(ax)^2\, x^2 dx \right)$$

input
```
int(x^2*(a^2*c*x^2+c)^2*atan(a*x)^(5/2),x)
```

output
```
c**2*(int(sqrt(atan(a*x))*atan(a*x)**2*x**6,x)*a**4 + 2*int(sqrt(atan(a*x)
)*atan(a*x)**2*x**4,x)*a**2 + int(sqrt(atan(a*x))*atan(a*x)**2*x**2,x))
```

# 3.844 $\int x(c+a^2cx^2)^2 \arctan(ax)^{5/2}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int x(c+a^2cx^2)^2 \arctan(ax)^{5/2}\,dx = \frac{c^2(1+a^2x^2)\sqrt{\arctan(ax)}}{12a^2} + \frac{c^2(1+a^2x^2)^2\sqrt{\arctan(ax)}}{32a^2} - \frac{c^2x(1+a^2x^2)\arctan(ax)^{3/2}}{9a} - \frac{c^2x(1+a^2x^2)^2\arctan(ax)^{3/2}}{12a} + \frac{c^2(1+a^2x^2)^3\arctan(ax)^{5/2}}{6a^2} - \frac{c^2\text{Int}\left(\frac{1}{\sqrt{\arctan(ax)}},x\right)}{24a} - \frac{c\text{Int}\left(\frac{c+a^2cx^2}{\sqrt{\arctan(ax)}},x\right)}{64a} - \frac{2c^2\text{Int}\left(\arctan(ax)^{3/2},x\right)}{9a}$$

output

```
1/12*c^2*(a^2*x^2+1)*arctan(a*x)^(1/2)/a^2+1/32*c^2*(a^2*x^2+1)^2*arctan(a
*x)^(1/2)/a^2-1/9*c^2*x*(a^2*x^2+1)*arctan(a*x)^(3/2)/a-1/12*c^2*x*(a^2*x^
2+1)^2*arctan(a*x)^(3/2)/a+1/6*c^2*(a^2*x^2+1)^3*arctan(a*x)^(5/2)/a^2-1/2
4*c^2*Defer(Int)(1/arctan(a*x)^(1/2),x)/a-1/64*c*Defer(Int)((a^2*c*x^2+c)/
arctan(a*x)^(1/2),x)/a-2/9*c^2*Defer(Int)(arctan(a*x)^(3/2),x)/a
```

## Mathematica [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int x(c+a^2cx^2)^2 \arctan(ax)^{5/2}\,dx=\int x(c+a^2cx^2)^2 \arctan(ax)^{5/2}\,dx$$

input
```
Integrate[x*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[x*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x \arctan(ax)^{5/2}\left(a^2cx^2+c\right)^2\,dx$$

$$\Big\downarrow \; 5465$$

$$\frac{c^2\left(a^2x^2+1\right)^3\arctan(ax)^{5/2}}{6a^2}-\frac{5\int c^2\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}dx}{12a}$$

$$\Big\downarrow \; 27$$

$$\frac{c^2\left(a^2x^2+1\right)^3\arctan(ax)^{5/2}}{6a^2}-\frac{5c^2\int \left(a^2x^2+1\right)^2\arctan(ax)^{3/2}dx}{12a}$$

$$\Big\downarrow \; 5415$$

$$\frac{c^2\left(a^2x^2+1\right)^3\arctan(ax)^{5/2}}{6a^2}-$$
$$\frac{5c^2\left(\frac{3}{80}\int\frac{a^2x^2+1}{\sqrt{\arctan(ax)}}dx+\frac{4}{5}\int\left(a^2x^2+1\right)\arctan(ax)^{3/2}dx+\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}-\frac{3\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}{40a}\right.}{12a}$$

$\downarrow$ 5415

$$\frac{c^2\left(a^2x^2+1\right)^3\arctan(ax)^{5/2}}{6a^2}-$$
$$\frac{5c^2\left(\frac{3}{80}\int\frac{a^2x^2+1}{\sqrt{\arctan(ax)}}dx+\frac{4}{5}\left(\frac{1}{8}\int\frac{1}{\sqrt{\arctan(ax)}}dx+\frac{2}{3}\int\arctan(ax)^{3/2}dx+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)^{3/2}-\frac{\left(a^2x^2+1\right)\sqrt{}}{4}\right.\right.}{12a}$$

$\downarrow$ 5353

$$\frac{c^2\left(a^2x^2+1\right)^3\arctan(ax)^{5/2}}{6a^2}-$$
$$\frac{5c^2\left(\frac{3}{80}\int\frac{a^2x^2+1}{\sqrt{\arctan(ax)}}dx+\frac{4}{5}\left(\frac{1}{8}\int\frac{1}{\sqrt{\arctan(ax)}}dx+\frac{2}{3}\int\arctan(ax)^{3/2}dx+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)^{3/2}-\frac{\left(a^2x^2+1\right)\sqrt{}}{4}\right.\right.}{12a}$$

$\downarrow$ 5560

$$\frac{c^2\left(a^2x^2+1\right)^3\arctan(ax)^{5/2}}{6a^2}-$$
$$\frac{5c^2\left(\frac{3}{80}\int\frac{a^2x^2+1}{\sqrt{\arctan(ax)}}dx+\frac{4}{5}\left(\frac{1}{8}\int\frac{1}{\sqrt{\arctan(ax)}}dx+\frac{2}{3}\int\arctan(ax)^{3/2}dx+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)^{3/2}-\frac{\left(a^2x^2+1\right)\sqrt{}}{4}\right.\right.}{12a}$$

input
```
Int[x*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int x\left(a^2c\,x^2+c\right)^2\arctan\left(ax\right)^{\frac{5}{2}}dx$$

input
```
int(x*(a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x)
```

output
```
int(x*(a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int x(c+a^2cx^2)^2 \arctan(ax)^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 40.50 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.23

$$\int x(c+a^2cx^2)^2 \arctan(ax)^{5/2}\,dx = c^2\left(\int x \operatorname{atan}^{\frac{5}{2}}(ax)\,dx + \int 2a^2x^3 \operatorname{atan}^{\frac{5}{2}}(ax)\,dx + \int a^4x^5 \operatorname{atan}^{\frac{5}{2}}(ax)\,dx\right)$$

input
```
integrate(x*(a**2*c*x**2+c)**2*atan(a*x)**(5/2),x)
```

output
```
c**2*(Integral(x*atan(a*x)**(5/2), x) + Integral(2*a**2*x**3*atan(a*x)**(5
/2), x) + Integral(a**4*x**5*atan(a*x)**(5/2), x))
```

## Maxima [F(-2)]

Exception generated.

$$\int x(c+a^2cx^2)^2\arctan(ax)^{5/2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x(c+a^2cx^2)^2\arctan(ax)^{5/2}\,dx = \int (a^2cx^2+c)^2x\arctan(ax)^{\frac{5}{2}}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^2*x*arctan(a*x)^(5/2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x(c+a^2cx^2)^2\arctan(ax)^{5/2}\,dx = \int x\,\mathrm{atan}(a\,x)^{5/2}\,(c\,a^2\,x^2+c)^2\,dx$$

input
```
int(x*atan(a*x)^(5/2)*(c + a^2*c*x^2)^2,x)
```

output
```
int(x*atan(a*x)^(5/2)*(c + a^2*c*x^2)^2, x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 245, normalized size of antiderivative = 11.14

$$\int x\left(c+a^2cx^2\right)^2 \arctan(ax)^{5/2}\,dx = \frac{c^2\left(96\sqrt{atan\,(ax)}\,atan(ax)^2\,a^6x^6+288\sqrt{atan\,(ax)}\,atan(ax)^2\,a^4x^4+288\sqrt{at\right.}{}$$

input
```
int(x*(a^2*c*x^2+c)^2*atan(a*x)^(5/2),x)
```

output
```
(c**2*(96*sqrt(atan(a*x))*atan(a*x)**2*a**6*x**6 + 288*sqrt(atan(a*x))*ata
n(a*x)**2*a**4*x**4 + 288*sqrt(atan(a*x))*atan(a*x)**2*a**2*x**2 + 96*sqrt
(atan(a*x))*atan(a*x)**2 - 48*sqrt(atan(a*x))*atan(a*x)*a**5*x**5 - 160*sq
rt(atan(a*x))*atan(a*x)*a**3*x**3 - 240*sqrt(atan(a*x))*atan(a*x)*a*x + 18
*sqrt(atan(a*x))*a**4*x**4 + 84*sqrt(atan(a*x))*a**2*x**2 - 9*int((sqrt(at
an(a*x))*x**4)/(atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**5 - 42*int((sqrt(at
an(a*x))*x**2)/(atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**3 + 192*int((sqrt(a
tan(a*x))*x)/(a**2*x**2 + 1),x)*a**2))/(576*a**2)
```

# 3.845 $\int \left(c+a^2cx^2\right)^2 \arctan(ax)^{5/2}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \left(c+a^2cx^2\right)^2 \arctan(ax)^{5/2}\,dx = -\frac{c^2(1+a^2x^2)\arctan(ax)^{3/2}}{3a} - \frac{c^2(1+a^2x^2)^2\arctan(ax)^{3/2}}{8a} + \frac{4}{15}c^2x\left(1+a^2x^2\right)\arctan(ax)^{5/2} + \frac{1}{5}c^2x\left(1+a^2x^2\right)^2\arctan(ax)^{5/2} + \frac{1}{2}c^2\text{Int}\left(\sqrt{\arctan(ax)},x\right) + \frac{3}{16}c\text{Int}\Big((c+$$

output
```
-1/3*c^2*(a^2*x^2+1)*arctan(a*x)^(3/2)/a-1/8*c^2*(a^2*x^2+1)^2*arctan(a*x)
^(3/2)/a+4/15*c^2*x*(a^2*x^2+1)*arctan(a*x)^(5/2)+1/5*c^2*x*(a^2*x^2+1)^2*
arctan(a*x)^(5/2)+1/2*c^2*Defer(Int)(arctan(a*x)^(1/2),x)+3/16*c*Defer(Int
)((a^2*c*x^2+c)*arctan(a*x)^(1/2),x)+8/15*c^2*Defer(Int)(arctan(a*x)^(5/2)
,x)
```

## Mathematica [N/A]

Not integrable

Time = 1.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int (c + a^2cx^2)^2 \arctan(ax)^{5/2}\,dx = \int (c + a^2cx^2)^2 \arctan(ax)^{5/2}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(ax)^{5/2}\left(a^2cx^2+c\right)^2\,dx$$

$$\Big\downarrow \text{5415}$$

$$\frac{3}{16}c\int c(a^2x^2+1)\sqrt{\arctan(ax)}dx+\frac{4}{5}c\int c(a^2x^2+1)\arctan(ax)^{5/2}dx+\frac{1}{5}c^2x(a^2x^2+1)^2\arctan(ax)^{5/2}-\frac{c^2(a^2x^2+1)^2\arctan(ax)^{3/2}}{8a}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{16}c^2\int (a^2x^2+1)\sqrt{\arctan(ax)}dx+\frac{4}{5}c^2\int (a^2x^2+1)\arctan(ax)^{5/2}dx+\frac{1}{5}c^2x(a^2x^2+1)^2\arctan(ax)^{5/2}-\frac{c^2(a^2x^2+1)^2\arctan(ax)^{3/2}}{8a}$$

$$\Big\downarrow \text{5415}$$

$$\frac{3}{16}c^2\int \left(a^2x^2+1\right)\sqrt{\arctan(ax)}dx+$$
$$\frac{4}{5}c^2\left(\frac{5}{8}\int \sqrt{\arctan(ax)}dx+\frac{2}{3}\int \arctan(ax)^{5/2}dx+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)^{5/2}-\frac{5\left(a^2x^2+1\right)\arctan(ax)^{3/2}}{12a}\right)$$
$$\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\arctan(ax)^{5/2}-\frac{c^2\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}{8a}$$

$$\Big\downarrow \text{5353}$$

$$\frac{3}{16}c^2\int \left(a^2x^2+1\right)\sqrt{\arctan(ax)}dx+$$
$$\frac{4}{5}c^2\left(\frac{5}{8}\int \sqrt{\arctan(ax)}dx+\frac{2}{3}\int \arctan(ax)^{5/2}dx+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)^{5/2}-\frac{5\left(a^2x^2+1\right)\arctan(ax)^{3/2}}{12a}\right)$$
$$\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\arctan(ax)^{5/2}-\frac{c^2\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}{8a}$$

$$\Big\downarrow \text{5560}$$

$$\frac{3}{16}c^2\int \left(a^2x^2+1\right)\sqrt{\arctan(ax)}dx+$$
$$\frac{4}{5}c^2\left(\frac{5}{8}\int \sqrt{\arctan(ax)}dx+\frac{2}{3}\int \arctan(ax)^{5/2}dx+\frac{1}{3}x\left(a^2x^2+1\right)\arctan(ax)^{5/2}-\frac{5\left(a^2x^2+1\right)\arctan(ax)^{3/2}}{12a}\right)$$
$$\frac{1}{5}c^2x\left(a^2x^2+1\right)^2\arctan(ax)^{5/2}-\frac{c^2\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}{8a}$$

input
```
Int[(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \left(a^2c\,x^2+c\right)^2\arctan\left(ax\right)^{\frac{5}{2}}dx$$

input
```
int((a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x)
```

output
```
int((a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^2 \arctan(ax)^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 33.34 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.29

$$\int \left(c+a^2cx^2\right)^2 \arctan(ax)^{5/2}\,dx = c^2\left(\int 2a^2x^2\operatorname{atan}^{\frac{5}{2}}(ax)\,dx + \int a^4x^4\operatorname{atan}^{\frac{5}{2}}(ax)\,dx + \int \operatorname{atan}^{\frac{5}{2}}(ax)\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**2*atan(a*x)**(5/2),x)
```

output
```
c**2*(Integral(2*a**2*x**2*atan(a*x)**(5/2), x) + Integral(a**4*x**4*atan(
a*x)**(5/2), x) + Integral(atan(a*x)**(5/2), x))
```

## Maxima [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^2 \arctan(ax)^{5/2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \left(c+a^2cx^2\right)^2 \arctan(ax)^{5/2}\,dx = \int \left(a^2cx^2+c\right)^2 \arctan\left(ax\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^2*arctan(a*x)^(5/2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \left(c+a^2cx^2\right)^2 \arctan(ax)^{5/2}\,dx = \int \mathrm{atan}(a\,x)^{5/2}\,\left(c\,a^2\,x^2+c\right)^2\,dx$$

input
```
int(atan(a*x)^(5/2)*(c + a^2*c*x^2)^2,x)
```

output
```
int(atan(a*x)^(5/2)*(c + a^2*c*x^2)^2, x)
```

## Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 224, normalized size of antiderivative = 10.67

$$\int (c+a^2cx^2)^2 \arctan(ax)^{5/2}\,dx = \frac{c^2\left(96\sqrt{atan(ax)}\,atan(ax)^2\,a^5x^5 + 320\sqrt{atan(ax)}\,atan(ax)^2\,a^3x^3 + 480\sqrt{at\cdots}\right.}{}$$

input
```
int((a^2*c*x^2+c)^2*atan(a*x)^(5/2),x)
```

output
```
(c**2*(96*sqrt(atan(a*x))*atan(a*x)**2*a**5*x**5 + 320*sqrt(atan(a*x))*ata
n(a*x)**2*a**3*x**3 + 480*sqrt(atan(a*x))*atan(a*x)**2*a*x - 60*sqrt(atan(
a*x))*atan(a*x)*a**4*x**4 - 280*sqrt(atan(a*x))*atan(a*x)*a**2*x**2 - 220*
sqrt(atan(a*x))*atan(a*x) + 30*sqrt(atan(a*x))*a**3*x**3 + 330*sqrt(atan(a
*x))*a*x - 640*int((sqrt(atan(a*x))*atan(a*x)*x)/(a**2*x**2 + 1),x)*a**2 -
 15*int((sqrt(atan(a*x))*x**3)/(atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**4 -
 165*int((sqrt(atan(a*x))*x)/(atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**2))/(
480*a)
```

# 3.846 $\int \frac{(c+a^2cx^2)^2 \arctan(ax)^{5/2}}{x} \, dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^{5/2}}{x} \, dx = \text{Int}\left(\frac{(c+a^2cx^2)^2 \arctan(ax)^{5/2}}{x}, x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^2*arctan(a*x)^(5/2)/x,x)
```

## Mathematica [N/A]

Not integrable

Time = 0.98 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^{5/2}}{x} \, dx = \int \frac{(c+a^2cx^2)^2 \arctan(ax)^{5/2}}{x} \, dx$$

input
```
Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2))/x,x]
```

output
```
Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2))/x, x]
```

## Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{5/2} \left(a^2cx^2+c\right)^2}{x} \, dx$$
$$\Big\downarrow \ 5560$$
$$\int \frac{\arctan(ax)^{5/2} \left(a^2cx^2+c\right)^2}{x} dx$$

input
```
Int[((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2))/x,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\left(a^2 c\,x^2+c\right)^2 \arctan\left(a x\right)^{\frac{5}{2}}}{x}dx$$

input
```
int((a^2*c*x^2+c)^2*arctan(a*x)^(5/2)/x,x)
```

output
```
int((a^2*c*x^2+c)^2*arctan(a*x)^(5/2)/x,x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\left(c+a^2cx^2\right)^2\arctan(ax)^{5/2}}{x}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^(5/2)/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 23.00 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.00

$$\int \frac{\left(c+a^2cx^2\right)^2\arctan(ax)^{5/2}}{x}\,dx=c^2\left(\int \frac{\operatorname{atan}^{\frac{5}{2}}(ax)}{x}\,dx+\int 2a^2x\operatorname{atan}^{\frac{5}{2}}(ax)\,dx+\int a^4x^3\operatorname{atan}^{\frac{5}{2}}(ax)\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**2*atan(a*x)**(5/2)/x,x)
```

output
```
c**2*(Integral(atan(a*x)**(5/2)/x, x) + Integral(2*a**2*x*atan(a*x)**(5/2)
, x) + Integral(a**4*x**3*atan(a*x)**(5/2), x))
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^{5/2}}{x} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^(5/2)/x,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^{5/2}}{x} \, dx = \int \frac{(a^2cx^2+c)^2 \arctan(ax)^{\frac{5}{2}}}{x} \, dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^(5/2)/x,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^2*arctan(a*x)^(5/2)/x, x)
```

## Mupad [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^{5/2}}{x} \, dx = \int \frac{\mathrm{atan}(a\,x)^{5/2}\,(c\,a^2\,x^2+c)^2}{x} \, dx$$

input
```
int((atan(a*x)^(5/2)*(c + a^2*c*x^2)^2)/x,x)
```

output
```
int((atan(a*x)^(5/2)*(c + a^2*c*x^2)^2)/x, x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 205, normalized size of antiderivative = 8.54

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^{5/2}}{x} dx = \frac{c^2 \left(48\sqrt{atan(ax)}\, atan(ax)^2 a^4x^4 + 192\sqrt{atan(ax)}\, atan(ax)^2 a^2x^2 + 1\right.}{}$$

input
```
int((a^2*c*x^2+c)^2*atan(a*x)^(5/2)/x,x)
```

output
```
(c**2*(48*sqrt(atan(a*x))*atan(a*x)**2*a**4*x**4 + 192*sqrt(atan(a*x))*ata
n(a*x)**2*a**2*x**2 + 144*sqrt(atan(a*x))*atan(a*x)**2 - 40*sqrt(atan(a*x)
)*atan(a*x)*a**3*x**3 - 360*sqrt(atan(a*x))*atan(a*x)*a*x + 390*sqrt(atan(
a*x))*a**2*x**2 + 192*int((sqrt(atan(a*x))*atan(a*x)**2)/x,x) - 720*int((s
qrt(atan(a*x))*x**3)/(a**2*x**2 + 1),x)*a**4 - 195*int((sqrt(atan(a*x))*x*
*2)/(atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**3 - 240*int((sqrt(atan(a*x))*x
)/(a**2*x**2 + 1),x)*a**2))/192
```

# 3.847 $\int \frac{(c+a^2cx^2)^2 \arctan(ax)^{5/2}}{x^2} \, dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^{5/2}}{x^2} \, dx = \text{Int}\left(\frac{(c+a^2cx^2)^2 \arctan(ax)^{5/2}}{x^2}, x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^2*arctan(a*x)^(5/2)/x^2,x)
```

## Mathematica [N/A]

Not integrable

Time = 1.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^{5/2}}{x^2} \, dx = \int \frac{(c+a^2cx^2)^2 \arctan(ax)^{5/2}}{x^2} \, dx$$

input
```
Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2))/x^2,x]
```

output
```
Integrate[((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2))/x^2, x]
```

## Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^2}{x^2}dx$$

$$\Big\downarrow \text{5560}$$

$$\int \frac{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^2}{x^2}dx$$

input
```
Int[((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2))/x^2,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\left(a^{2} c \,x^{2}+c \right)^{2} \arctan \left(a x \right)^{\frac{5}{2}}}{x^{2}}d x$$

input
```
int((a^2*c*x^2+c)^2*arctan(a*x)^(5/2)/x^2,x)
```

output
```
int((a^2*c*x^2+c)^2*arctan(a*x)^(5/2)/x^2,x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^{5/2}}{x^2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^(5/2)/x^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 27.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.00

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^{5/2}}{x^2} \, dx = c^2 \left( \int 2a^2 \operatorname{atan}^{\frac{5}{2}}(ax) \, dx + \int \frac{\operatorname{atan}^{\frac{5}{2}}(ax)}{x^2} \, dx + \int a^4x^2 \operatorname{atan}^{\frac{5}{2}}(ax) \, dx \right)$$

input
```
integrate((a**2*c*x**2+c)**2*atan(a*x)**(5/2)/x**2,x)
```

output
```
c**2*(Integral(2*a**2*atan(a*x)**(5/2), x) + Integral(atan(a*x)**(5/2)/x**
2, x) + Integral(a**4*x**2*atan(a*x)**(5/2), x))
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^2\arctan(ax)^{5/2}}{x^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^(5/2)/x^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^2\arctan(ax)^{5/2}}{x^2}\,dx = \int \frac{(a^2cx^2+c)^2\arctan(ax)^{\frac{5}{2}}}{x^2}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2*arctan(a*x)^(5/2)/x^2,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^2*arctan(a*x)^(5/2)/x^2, x)
```

## Mupad [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^2\arctan(ax)^{5/2}}{x^2}\,dx = \int \frac{\mathrm{atan}(a\,x)^{5/2}\,(c\,a^2\,x^2+c)^2}{x^2}\,dx$$

input
```
int((atan(a*x)^(5/2)*(c + a^2*c*x^2)^2)/x^2,x)
```

output
```
int((atan(a*x)^(5/2)*(c + a^2*c*x^2)^2)/x^2, x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 193, normalized size of antiderivative = 8.04

$$\int \frac{(c+a^2cx^2)^2 \arctan(ax)^{5/2}}{x^2}\,dx = \frac{c^2\left(16\sqrt{atan\,(ax)}\,atan(ax)^2\,a^4x^4+96\sqrt{atan\,(ax)}\,atan(ax)^2\,a^2x^2-48\right.}{}$$

input
```
int((a^2*c*x^2+c)^2*atan(a*x)^(5/2)/x^2,x)
```

output
```
(c**2*(16*sqrt(atan(a*x))*atan(a*x)**2*a**4*x**4 + 96*sqrt(atan(a*x))*atan
(a*x)**2*a**2*x**2 - 48*sqrt(atan(a*x))*atan(a*x)**2 - 20*sqrt(atan(a*x))*
atan(a*x)*a**3*x**3 - 20*sqrt(atan(a*x))*atan(a*x)*a*x + 30*sqrt(atan(a*x)
)*a**2*x**2 - 200*int((sqrt(atan(a*x))*atan(a*x)*x)/(a**2*x**2 + 1),x)*a**
3*x + 120*int((sqrt(atan(a*x))*atan(a*x))/(a**2*x**3 + x),x)*a*x - 15*int(
(sqrt(atan(a*x))*x)/(atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**3*x))/(48*x)
```

# 3.848 $\int x^m(c+a^2cx^2)^3 \arctan(ax)^{5/2}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int x^m\left(c+a^2cx^2\right)^3 \arctan(ax)^{5/2}\,dx=\text{Int}\left(x^m\left(c+a^2cx^2\right)^3 \arctan(ax)^{5/2},x\right)$$

output
```
Defer(Int)(x^m*(a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int x^m\left(c+a^2cx^2\right)^3 \arctan(ax)^{5/2}\,dx=\int x^m\left(c+a^2cx^2\right)^3 \arctan(ax)^{5/2}\,dx$$

input
```
Integrate[x^m*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[x^m*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \arctan(ax)^{5/2} \left(a^2cx^2+c\right)^3 dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int x^m \arctan(ax)^{5/2} \left(a^2cx^2+c\right)^3 dx$$

input
```
Int[x^m*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.94 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int x^m \left(a^2c\,x^2+c\right)^3 \arctan\left(ax\right)^{\frac{5}{2}}dx$$

input
```
int(x^m*(a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.00

$$\int x^m \left(c+a^2cx^2\right)^3 \arctan(ax)^{5/2}\,dx = \int \left(a^2cx^2+c\right)^3 x^m \arctan\left(ax\right)^{\frac{5}{2}}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*x^m*arctan(a*
x)^(5/2), x)
```

## Sympy [F(-1)]

Timed out.

$$\int x^m \left(c+a^2cx^2\right)^3 \arctan(ax)^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(x**m*(a**2*c*x**2+c)**3*atan(a*x)**(5/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int x^m \left(c+a^2cx^2\right)^3 \arctan(ax)^{5/2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.98 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^m \left(c+a^2cx^2\right)^3 \arctan(ax)^{5/2}\,dx = \int \left(a^2cx^2+c\right)^3 x^m \arctan\left(ax\right)^{\frac{5}{2}}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^3*x^m*arctan(a*x)^(5/2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^m \left(c+a^2cx^2\right)^3 \arctan(ax)^{5/2}\,dx = \int x^m\,\mathrm{atan}(a\,x)^{5/2}\left(c\,a^2\,x^2+c\right)^3\,dx$$

input
```
int(x^m*atan(a*x)^(5/2)*(c + a^2*c*x^2)^3,x)
```

output
```
int(x^m*atan(a*x)^(5/2)*(c + a^2*c*x^2)^3, x)
```

## Reduce [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 11573, normalized size of antiderivative = 482.21

$$\int x^m \left(c+a^2cx^2\right)^3 \arctan(ax)^{5/2}\,dx = \text{Too large to display}$$

input
```
int(x^m*(a^2*c*x^2+c)^3*atan(a*x)^(5/2),x)
```

output
```
(c**3*(8*x**m*sqrt(atan(a*x))*atan(a*x)**2*a**7*m**7*x**7 + 168*x**m*sqrt(
atan(a*x))*atan(a*x)**2*a**7*m**6*x**7 + 1400*x**m*sqrt(atan(a*x))*atan(a*
x)**2*a**7*m**5*x**7 + 5880*x**m*sqrt(atan(a*x))*atan(a*x)**2*a**7*m**4*x*
*7 + 12992*x**m*sqrt(atan(a*x))*atan(a*x)**2*a**7*m**3*x**7 + 14112*x**m*s
qrt(atan(a*x))*atan(a*x)**2*a**7*m**2*x**7 + 5760*x**m*sqrt(atan(a*x))*ata
n(a*x)**2*a**7*m*x**7 + 24*x**m*sqrt(atan(a*x))*atan(a*x)**2*a**5*m**7*x**
5 + 552*x**m*sqrt(atan(a*x))*atan(a*x)**2*a**5*m**6*x**5 + 4968*x**m*sqrt(
atan(a*x))*atan(a*x)**2*a**5*m**5*x**5 + 22200*x**m*sqrt(atan(a*x))*atan(a
*x)**2*a**5*m**4*x**5 + 51456*x**m*sqrt(atan(a*x))*atan(a*x)**2*a**5*m**3*
x**5 + 57888*x**m*sqrt(atan(a*x))*atan(a*x)**2*a**5*m**2*x**5 + 24192*x**m
*sqrt(atan(a*x))*atan(a*x)**2*a**5*m*x**5 + 24*x**m*sqrt(atan(a*x))*atan(a
*x)**2*a**3*m**7*x**3 + 600*x**m*sqrt(atan(a*x))*atan(a*x)**2*a**3*m**6*x*
*3 + 5928*x**m*sqrt(atan(a*x))*atan(a*x)**2*a**3*m**5*x**3 + 29256*x**m*sq
rt(atan(a*x))*atan(a*x)**2*a**3*m**4*x**3 + 74688*x**m*sqrt(atan(a*x))*ata
n(a*x)**2*a**3*m**3*x**3 + 91104*x**m*sqrt(atan(a*x))*atan(a*x)**2*a**3*m*
*2*x**3 + 40320*x**m*sqrt(atan(a*x))*atan(a*x)**2*a**3*m*x**3 + 8*x**m*sqr
t(atan(a*x))*atan(a*x)**2*a*m**7*x + 216*x**m*sqrt(atan(a*x))*atan(a*x)**2
*a*m**6*x + 2360*x**m*sqrt(atan(a*x))*atan(a*x)**2*a*m**5*x + 13320*x**m*s
qrt(atan(a*x))*atan(a*x)**2*a*m**4*x + 40832*x**m*sqrt(atan(a*x))*atan(a*x
)**2*a*m**3*x + 64224*x**m*sqrt(atan(a*x))*atan(a*x)**2*a*m**2*x + 4032...
```

# 3.849 $\int x^2(c+a^2cx^2)^3\arctan(ax)^{5/2}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int x^2\left(c+a^2cx^2\right)^3\arctan(ax)^{5/2}\,dx=\operatorname{Int}\left(x^2\left(c+a^2cx^2\right)^3\arctan(ax)^{5/2},x\right)$$

output
```
Defer(Int)(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 2.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int x^2\left(c+a^2cx^2\right)^3\arctan(ax)^{5/2}\,dx=\int x^2\left(c+a^2cx^2\right)^3\arctan(ax)^{5/2}\,dx$$

input
```
Integrate[x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arctan(ax)^{5/2} \left(a^2cx^2+c\right)^3 \, dx$$

$$\big\downarrow \ 5560$$

$$\int x^2 \arctan(ax)^{5/2} \left(a^2cx^2+c\right)^3 dx$$

input
```
Int[x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int x^2 \left(a^2 c \,x^2+c\right)^3 \arctan\left(a x\right)^{\frac{5}{2}}d x$$

input
```
int(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x)
```

output
```
int(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int x^2 (c + a^2 c x^2)^3 \arctan(ax)^{5/2}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int x^2 (c + a^2 c x^2)^3 \arctan(ax)^{5/2}\, dx = \text{Timed out}$$

input
```
integrate(x**2*(a**2*c*x**2+c)**3*atan(a*x)**(5/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int x^2 (c + a^2 c x^2)^3 \arctan(ax)^{5/2}\, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^2(c+a^2cx^2)^3\arctan(ax)^{5/2}\,dx=\int\left(a^2cx^2+c\right)^3x^2\arctan\left(ax\right)^{\frac{5}{2}}\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^3*x^2*arctan(a*x)^(5/2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int x^2(c+a^2cx^2)^3\arctan(ax)^{5/2}\,dx=\int x^2\,\mathrm{atan}(a\,x)^{5/2}\,\left(c\,a^2\,x^2+c\right)^3\,dx$$

input
```
int(x^2*atan(a*x)^(5/2)*(c + a^2*c*x^2)^3,x)
```

output
```
int(x^2*atan(a*x)^(5/2)*(c + a^2*c*x^2)^3, x)
```

## Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 87, normalized size of antiderivative = 3.62

$$\int x^2(c+a^2cx^2)^3\arctan(ax)^{5/2}\,dx=c^3\left(\left(\int\sqrt{atan\left(ax\right)}\,atan(ax)^2\,x^8dx\right)a^6+3\left(\int\sqrt{atan\left(ax\right)}\,atan(ax)^2\,x^6dx\right)a^4+3\left(\int\sqrt{atan\left(ax\right)}\,atan(ax)^2\,x^4dx\right)a^2+\int\sqrt{atan\left(ax\right)}\,atan(ax)^2\,x^2dx\right)$$

input
```
int(x^2*(a^2*c*x^2+c)^3*atan(a*x)^(5/2),x)
```

output
```
c**3*(int(sqrt(atan(a*x))*atan(a*x)**2*x**8,x)*a**6 + 3*int(sqrt(atan(a*x)
)*atan(a*x)**2*x**6,x)*a**4 + 3*int(sqrt(atan(a*x))*atan(a*x)**2*x**4,x)*a
**2 + int(sqrt(atan(a*x))*atan(a*x)**2*x**2,x))
```

# 3.850 $\int x(c+a^2cx^2)^3 \arctan(ax)^{5/2}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int x\left(c+a^2cx^2\right)^3\arctan(ax)^{5/2}\,dx=\frac{3c^3(1+a^2x^2)\sqrt{\arctan(ax)}}{56a^2}+\frac{9c^3(1+a^2x^2)^2\sqrt{\arctan(ax)}}{448a^2}+\frac{5c^3(1+a^2x^2)^3\sqrt{\arctan(ax)}}{448a^2}-\frac{c^3x(1+a^2x^2)\arctan(ax)^{3/2}}{14a}-\frac{3c^3x(1+a^2x^2)^2\arctan(ax)^{3/2}}{56a}-\frac{5c^3x(1+a^2x^2)^3\arctan(ax)^{3/2}}{112a}+\frac{c^3(1+a^2x^2)^4\arctan(ax)^{5/2}}{8a^2}-\frac{3c^3\text{Int}\left(\frac{1}{\sqrt{\arctan(ax)}},x\right)}{112a}-\frac{9c^2\text{Int}\left(\frac{c+a^2cx^2}{\sqrt{\arctan(ax)}},x\right)}{896a}-\frac{5c\text{Int}\left(\frac{(c+a^2cx^2)^2}{\sqrt{\arctan(ax)}},x\right)}{896a}-\frac{c^3\text{Int}\left(\arctan(ax)^{3/2},x\right)}{7a}$$

output
```
3/56*c^3*(a^2*x^2+1)*arctan(a*x)^(1/2)/a^2+9/448*c^3*(a^2*x^2+1)^2*arctan(
a*x)^(1/2)/a^2+5/448*c^3*(a^2*x^2+1)^3*arctan(a*x)^(1/2)/a^2-1/14*c^3*x*(a
^2*x^2+1)*arctan(a*x)^(3/2)/a-3/56*c^3*x*(a^2*x^2+1)^2*arctan(a*x)^(3/2)/a
-5/112*c^3*x*(a^2*x^2+1)^3*arctan(a*x)^(3/2)/a+1/8*c^3*(a^2*x^2+1)^4*arcta
n(a*x)^(5/2)/a^2-3/112*c^3*Defer(Int)(1/arctan(a*x)^(1/2),x)/a-9/896*c^2*D
efer(Int)((a^2*c*x^2+c)/arctan(a*x)^(1/2),x)/a-5/896*c*Defer(Int)((a^2*c*x
^2+c)^2/arctan(a*x)^(1/2),x)/a-1/7*c^3*Defer(Int)(arctan(a*x)^(3/2),x)/a
```

## Mathematica [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int x\left(c+a^2cx^2\right)^3\arctan(ax)^{5/2}\,dx=\int x\left(c+a^2cx^2\right)^3\arctan(ax)^{5/2}\,dx$$

input
```
Integrate[x*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[x*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.98 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^3\,dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{c^3\left(a^2x^2+1\right)^4\arctan(ax)^{5/2}}{8a^2}-\frac{5\int c^3\left(a^2x^2+1\right)^3\arctan(ax)^{3/2}dx}{16a}$$

$$\Big\downarrow \text{27}$$

$$\frac{c^3\left(a^2x^2+1\right)^4\arctan(ax)^{5/2}}{8a^2}-\frac{5c^3\int \left(a^2x^2+1\right)^3\arctan(ax)^{3/2}dx}{16a}$$

$$\Big\downarrow \text{5415}$$

$$\frac{c^3(a^2x^2+1)^4\arctan(ax)^{5/2}}{8a^2}-$$
$$\frac{5c^3\left(\frac{1}{56}\int\frac{(a^2x^2+1)^2}{\sqrt{\arctan(ax)}}dx+\frac{6}{7}\int(a^2x^2+1)^2\arctan(ax)^{3/2}dx+\frac{1}{7}x(a^2x^2+1)^3\arctan(ax)^{3/2}-\frac{(a^2x^2+1)^3\sqrt{\arctan(ax)}}{28a}\right.}{16a}$$

$\downarrow$ 5415

$$\frac{c^3(a^2x^2+1)^4\arctan(ax)^{5/2}}{8a^2}-$$
$$\frac{5c^3\left(\frac{1}{56}\int\frac{(a^2x^2+1)^2}{\sqrt{\arctan(ax)}}dx+\frac{6}{7}\left(\frac{3}{80}\int\frac{a^2x^2+1}{\sqrt{\arctan(ax)}}dx+\frac{4}{5}\int(a^2x^2+1)\arctan(ax)^{3/2}dx+\frac{1}{5}x(a^2x^2+1)^2\arctan(ax)^{3/}\right.\right.}{16a}$$

$\downarrow$ 5415

$$\frac{c^3(a^2x^2+1)^4\arctan(ax)^{5/2}}{8a^2}-$$
$$5c^3\left(\frac{1}{56}\int\frac{(a^2x^2+1)^2}{\sqrt{\arctan(ax)}}dx+\frac{6}{7}\left(\frac{3}{80}\int\frac{a^2x^2+1}{\sqrt{\arctan(ax)}}dx+\frac{4}{5}\left(\frac{1}{8}\int\frac{1}{\sqrt{\arctan(ax)}}dx+\frac{2}{3}\int\arctan(ax)^{3/2}dx+\frac{1}{3}x(a^2x^2+1)\text{ar}\right.\right.\right.$$

$\downarrow$ 5353

$$\frac{c^3(a^2x^2+1)^4\arctan(ax)^{5/2}}{8a^2}-$$
$$5c^3\left(\frac{1}{56}\int\frac{(a^2x^2+1)^2}{\sqrt{\arctan(ax)}}dx+\frac{6}{7}\left(\frac{3}{80}\int\frac{a^2x^2+1}{\sqrt{\arctan(ax)}}dx+\frac{4}{5}\left(\frac{1}{8}\int\frac{1}{\sqrt{\arctan(ax)}}dx+\frac{2}{3}\int\arctan(ax)^{3/2}dx+\frac{1}{3}x(a^2x^2+1)\text{ar}\right.\right.\right.$$

$\downarrow$ 5560

$$\frac{c^3(a^2x^2+1)^4\arctan(ax)^{5/2}}{8a^2}-$$
$$5c^3\left(\frac{1}{56}\int\frac{(a^2x^2+1)^2}{\sqrt{\arctan(ax)}}dx+\frac{6}{7}\left(\frac{3}{80}\int\frac{a^2x^2+1}{\sqrt{\arctan(ax)}}dx+\frac{4}{5}\left(\frac{1}{8}\int\frac{1}{\sqrt{\arctan(ax)}}dx+\frac{2}{3}\int\arctan(ax)^{3/2}dx+\frac{1}{3}x(a^2x^2+1)\text{ar}\right.\right.\right.$$

input
```
Int[x*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int x\left(a^{2} c \,x^{2}+c \right)^{3} \arctan \left(a x \right)^{\frac{5}{2}}d x$$

input
```
int(x*(a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x)
```

output
```
int(x*(a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int x(c+a^2cx^2)^3 \arctan(ax)^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 81.81 (sec) , antiderivative size = 68, normalized size of antiderivative = 3.09

$$\int x(c+a^2cx^2)^3 \arctan(ax)^{5/2}\,dx = c^{3} \left(\int x \operatorname{atan}^{\frac{5}{2}}{\left(a x \right)}\, dx + \int 3 a^{2} x^{3} \operatorname{atan}^{\frac{5}{2}}{\left(a x \right)}\, dx + \int 3 a^{4} x^{5} \operatorname{atan}^{\frac{5}{2}}{\left(a x \right)}\, dx + \int a^{6} x^{7} \operatorname{atan}^{\frac{5}{2}}{\left(a x \right)}\, dx\right)$$

input
```
integrate(x*(a**2*c*x**2+c)**3*atan(a*x)**(5/2),x)
```

output
```
c**3*(Integral(x*atan(a*x)**(5/2), x) + Integral(3*a**2*x**3*atan(a*x)**(5
/2), x) + Integral(3*a**4*x**5*atan(a*x)**(5/2), x) + Integral(a**6*x**7*a
tan(a*x)**(5/2), x))
```

## Maxima [F(-2)]

Exception generated.

$$\int x(c+a^2cx^2)^3 \arctan(ax)^{5/2} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x(c+a^2cx^2)^3 \arctan(ax)^{5/2} \, dx = \int (a^2cx^2+c)^3 x \arctan(ax)^{\frac{5}{2}} \, dx$$

input
```
integrate(x*(a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^3*x*arctan(a*x)^(5/2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int x\left(c+a^2cx^2\right)^3\arctan(ax)^{5/2}\,dx=\int x\,\mathrm{atan}(a\,x)^{5/2}\,\left(c\,a^2\,x^2+c\right)^3\,dx$$

input
```
int(x*atan(a*x)^(5/2)*(c + a^2*c*x^2)^3,x)
```

output
```
int(x*atan(a*x)^(5/2)*(c + a^2*c*x^2)^3, x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 328, normalized size of antiderivative = 14.91

$$\int x(c+a^2cx^2)^3\arctan(ax)^{5/2}\,dx=\frac{c^3\left(112\sqrt{atan\left(ax\right)}\,atan(ax)^2\,a^8x^8+448\sqrt{atan\left(ax\right)}\,atan(ax)^2\,a^6x^6+672\sqrt{}\right.}{}$$

input
```
int(x*(a^2*c*x^2+c)^3*atan(a*x)^(5/2),x)
```

output
```
(c**3*(112*sqrt(atan(a*x))*atan(a*x)**2*a**8*x**8 + 448*sqrt(atan(a*x))*at
an(a*x)**2*a**6*x**6 + 672*sqrt(atan(a*x))*atan(a*x)**2*a**4*x**4 + 448*sq
rt(atan(a*x))*atan(a*x)**2*a**2*x**2 + 112*sqrt(atan(a*x))*atan(a*x)**2 -
40*sqrt(atan(a*x))*atan(a*x)*a**7*x**7 - 168*sqrt(atan(a*x))*atan(a*x)*a**
5*x**5 - 280*sqrt(atan(a*x))*atan(a*x)*a**3*x**3 - 280*sqrt(atan(a*x))*ata
n(a*x)*a*x + 10*sqrt(atan(a*x))*a**6*x**6 + 48*sqrt(atan(a*x))*a**4*x**4 +
 114*sqrt(atan(a*x))*a**2*x**2 - 5*int((sqrt(atan(a*x))*x**6)/(atan(a*x)*a
**2*x**2 + atan(a*x)),x)*a**7 - 24*int((sqrt(atan(a*x))*x**4)/(atan(a*x)*a
**2*x**2 + atan(a*x)),x)*a**5 - 57*int((sqrt(atan(a*x))*x**2)/(atan(a*x)*a
**2*x**2 + atan(a*x)),x)*a**3 + 192*int((sqrt(atan(a*x))*x)/(a**2*x**2 + 1
),x)*a**2))/(896*a**2)
```

# 3.851 $\int (c+a^2cx^2)^3 \arctan(ax)^{5/2}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 21

$$\int (c+a^2cx^2)^3 \arctan(ax)^{5/2}\,dx = -\frac{2c^3(1+a^2x^2)\arctan(ax)^{3/2}}{7a} - \frac{3c^3(1+a^2x^2)^2\arctan(ax)^{3/2}}{28a} - \frac{5c^3(1+a^2x^2)^3\arctan(ax)^{3/2}}{84a} + \frac{8}{35}c^3x(1+a^2x^2)\arctan(ax)^{5/2} + \frac{6}{35}c^3x(1+a^2x^2)^2\arctan(ax)^{5/2} + \frac{1}{7}c^3x(1+a^2x^2)^3\arctan(ax)^{5/2} + \frac{3}{7}c^3\text{Int}($$

output
```
-2/7*c^3*(a^2*x^2+1)*arctan(a*x)^(3/2)/a-3/28*c^3*(a^2*x^2+1)^2*arctan(a*x
)^(3/2)/a-5/84*c^3*(a^2*x^2+1)^3*arctan(a*x)^(3/2)/a+8/35*c^3*x*(a^2*x^2+1
)*arctan(a*x)^(5/2)+6/35*c^3*x*(a^2*x^2+1)^2*arctan(a*x)^(5/2)+1/7*c^3*x*(
a^2*x^2+1)^3*arctan(a*x)^(5/2)+3/7*c^3*Defer(Int)(arctan(a*x)^(1/2),x)+9/5
6*c^2*Defer(Int)((a^2*c*x^2+c)*arctan(a*x)^(1/2),x)+5/56*c*Defer(Int)((a^2
*c*x^2+c)^2*arctan(a*x)^(1/2),x)+16/35*c^3*Defer(Int)(arctan(a*x)^(5/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \left(c+a^2cx^2\right)^3 \arctan(ax)^{5/2}\,dx=\int \left(c+a^2cx^2\right)^3 \arctan(ax)^{5/2}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(ax)^{5/2}\left(a^2cx^2+c\right)^3\,dx$$

$$\Big\downarrow \text{5415}$$

$$\frac{5}{56}c\int c^2\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}dx+\frac{6}{7}c\int c^2\left(a^2x^2+1\right)^2\arctan(ax)^{5/2}dx+\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)^{5/2}-\frac{5c^3\left(a^2x^2+1\right)^3\arctan(ax)^{3/2}}{84a}$$

$$\Big\downarrow \text{27}$$

$$\frac{5}{56}c^3\int \left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}dx+\frac{6}{7}c^3\int \left(a^2x^2+1\right)^2\arctan(ax)^{5/2}dx+\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)^{5/2}-\frac{5c^3\left(a^2x^2+1\right)^3\arctan(ax)^{3/2}}{84a}$$

$$\Big\downarrow \text{5415}$$

$$\frac{5}{56}c^3\int \left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}dx+$$
$$\frac{6}{7}c^3\left(\frac{3}{16}\int \left(a^2x^2+1\right)\sqrt{\arctan(ax)}dx+\frac{4}{5}\int \left(a^2x^2+1\right)\arctan(ax)^{5/2}dx+\frac{1}{5}x\left(a^2x^2+1\right)^2\arctan(ax)^{5/2}-\frac{(a^2\right.$$
$$\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)^{5/2}-\frac{5c^3\left(a^2x^2+1\right)^3\arctan(ax)^{3/2}}{84a}$$

$\downarrow$ 5415

$$\frac{5}{56}c^3\int \left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}dx+$$
$$\frac{6}{7}c^3\left(\frac{3}{16}\int \left(a^2x^2+1\right)\sqrt{\arctan(ax)}dx+\frac{4}{5}\left(\frac{5}{8}\int \sqrt{\arctan(ax)}dx+\frac{2}{3}\int \arctan(ax)^{5/2}dx+\frac{1}{3}x\left(a^2x^2+1\right)\text{arcta}\right.\right.$$
$$\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)^{5/2}-\frac{5c^3\left(a^2x^2+1\right)^3\arctan(ax)^{3/2}}{84a}$$

$\downarrow$ 5353

$$\frac{5}{56}c^3\int \left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}dx+$$
$$\frac{6}{7}c^3\left(\frac{3}{16}\int \left(a^2x^2+1\right)\sqrt{\arctan(ax)}dx+\frac{4}{5}\left(\frac{5}{8}\int \sqrt{\arctan(ax)}dx+\frac{2}{3}\int \arctan(ax)^{5/2}dx+\frac{1}{3}x\left(a^2x^2+1\right)\text{arcta}\right.\right.$$
$$\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)^{5/2}-\frac{5c^3\left(a^2x^2+1\right)^3\arctan(ax)^{3/2}}{84a}$$

$\downarrow$ 5560

$$\frac{5}{56}c^3\int \left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}dx+$$
$$\frac{6}{7}c^3\left(\frac{3}{16}\int \left(a^2x^2+1\right)\sqrt{\arctan(ax)}dx+\frac{4}{5}\left(\frac{5}{8}\int \sqrt{\arctan(ax)}dx+\frac{2}{3}\int \arctan(ax)^{5/2}dx+\frac{1}{3}x\left(a^2x^2+1\right)\text{arcta}\right.\right.$$
$$\frac{1}{7}c^3x\left(a^2x^2+1\right)^3\arctan(ax)^{5/2}-\frac{5c^3\left(a^2x^2+1\right)^3\arctan(ax)^{3/2}}{84a}$$

input
```
Int[(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \left(a^{2} c \,x^{2}+c \right)^{3} \arctan \left(a x \right)^{\frac{5}{2}}d x$$

input
```
int((a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x)
```

output
```
int((a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^3\arctan(ax)^{5/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 69.24 (sec) , antiderivative size = 66, normalized size of antiderivative = 3.14

$$\int \left(c+a^2cx^2\right)^3\arctan(ax)^{5/2}\,dx=c^{3}\left(\int 3a^{2}x^{2}\operatorname{atan}^{\frac{5}{2}}{\left(ax\right)}\,dx + \int 3a^{4}x^{4}\operatorname{atan}^{\frac{5}{2}}{\left(ax\right)}\,dx + \int a^{6}x^{6}\operatorname{atan}^{\frac{5}{2}}{\left(ax\right)}\,dx + \int \operatorname{atan}^{\frac{5}{2}}{\left(ax\right)}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**3*atan(a*x)**(5/2),x)
```

output
```
c**3*(Integral(3*a**2*x**2*atan(a*x)**(5/2), x) + Integral(3*a**4*x**4*ata
n(a*x)**(5/2), x) + Integral(a**6*x**6*atan(a*x)**(5/2), x) + Integral(ata
n(a*x)**(5/2), x))
```

## Maxima [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^3 \arctan(ax)^{5/2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \left(c+a^2cx^2\right)^3 \arctan(ax)^{5/2}\,dx = \int \left(a^2cx^2+c\right)^3 \arctan\left(ax\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^3*arctan(a*x)^(5/2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \left(c+a^2cx^2\right)^3 \arctan(ax)^{5/2}\, dx = \int \mathrm{atan}(a\,x)^{5/2}\,\left(c\,a^2\,x^2+c\right)^3\, dx$$

input
```
int(atan(a*x)^(5/2)*(c + a^2*c*x^2)^3,x)
```

output
```
int(atan(a*x)^(5/2)*(c + a^2*c*x^2)^3, x)
```

## Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 307, normalized size of antiderivative = 14.62

$$\int (c +a^2cx^2)^3 \arctan(ax)^{5/2}\, dx = \frac{c^3\Big(240\sqrt{atan\,(ax)}\;atan(ax)^2\,a^7x^7 + 1008\sqrt{atan\,(ax)}\;atan(ax)^2\,a^5x^5 + 1680$$

input
```
int((a^2*c*x^2+c)^3*atan(a*x)^(5/2),x)
```

output
```
(c**3*(240*sqrt(atan(a*x))*atan(a*x)**2*a**7*x**7 + 1008*sqrt(atan(a*x))*a
tan(a*x)**2*a**5*x**5 + 1680*sqrt(atan(a*x))*atan(a*x)**2*a**3*x**3 + 1680
*sqrt(atan(a*x))*atan(a*x)**2*a*x - 100*sqrt(atan(a*x))*atan(a*x)*a**6*x**
6 - 480*sqrt(atan(a*x))*atan(a*x)*a**4*x**4 - 1140*sqrt(atan(a*x))*atan(a*
x)*a**2*x**2 - 760*sqrt(atan(a*x))*atan(a*x) + 30*sqrt(atan(a*x))*a**5*x**
5 + 190*sqrt(atan(a*x))*a**3*x**3 + 1140*sqrt(atan(a*x))*a*x - 1920*int((s
qrt(atan(a*x))*atan(a*x)*x)/(a**2*x**2 + 1),x)*a**2 - 15*int((sqrt(atan(a*
x))*x**5)/(atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**6 - 95*int((sqrt(atan(a*
x))*x**3)/(atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**4 - 570*int((sqrt(atan(a
*x))*x)/(atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**2))/(1680*a)
```

# 3.852 $\int \frac{(c+a^2cx^2)^3 \arctan(ax)^{5/2}}{x} \, dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^{5/2}}{x} \, dx = \text{Int}\left(\frac{(c+a^2cx^2)^3 \arctan(ax)^{5/2}}{x}, x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^3*arctan(a*x)^(5/2)/x,x)
```

## Mathematica [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^{5/2}}{x} \, dx = \int \frac{(c+a^2cx^2)^3 \arctan(ax)^{5/2}}{x} \, dx$$

input
```
Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2))/x,x]
```

output
```
Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2))/x, x]
```

## Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^3}{x}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^3}{x}dx$$

input
```
Int[((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2))/x,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\left(a^2c\,x^2+c\right)^3\arctan\left(ax\right)^{\frac{5}{2}}}{x}dx$$

input
```
int((a^2*c*x^2+c)^3*arctan(a*x)^(5/2)/x,x)
```

output
```
int((a^2*c*x^2+c)^3*arctan(a*x)^(5/2)/x,x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^{5/2}}{x} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^(5/2)/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 45.15 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.75

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^{5/2}}{x} \, dx = c^3 \left( \int \frac{\operatorname{atan}^{\frac{5}{2}}(ax)}{x} \, dx + \int 3a^2x \operatorname{atan}^{\frac{5}{2}}(ax) \, dx + \int 3a^4x^3 \operatorname{atan}^{\frac{5}{2}}(ax) \, dx + \int a^6x^5 \operatorname{atan}^{\frac{5}{2}}(ax) \, dx \right)$$

input
```
integrate((a**2*c*x**2+c)**3*atan(a*x)**(5/2)/x,x)
```

output
```
c**3*(Integral(atan(a*x)**(5/2)/x, x) + Integral(3*a**2*x*atan(a*x)**(5/2)
, x) + Integral(3*a**4*x**3*atan(a*x)**(5/2), x) + Integral(a**6*x**5*atan
(a*x)**(5/2), x))
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^3\arctan(ax)^{5/2}}{x}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^(5/2)/x,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^3\arctan(ax)^{5/2}}{x}\,dx = \int \frac{(a^2cx^2+c)^3\arctan(ax)^{\frac{5}{2}}}{x}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^(5/2)/x,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^3*arctan(a*x)^(5/2)/x, x)
```

## Mupad [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^3\arctan(ax)^{5/2}}{x}\,dx = \int \frac{\mathrm{atan}(a\,x)^{5/2}\,(c\,a^2\,x^2+c)^3}{x}\,dx$$

input
```
int((atan(a*x)^(5/2)*(c + a^2*c*x^2)^3)/x,x)
```

output
```
int((atan(a*x)^(5/2)*(c + a^2*c*x^2)^3)/x, x)
```

## Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 288, normalized size of antiderivative = 12.00

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^{5/2}}{x} \, dx = \frac{c^3 \left(96\sqrt{atan\,(ax)}\, atan(ax)^2 a^6x^6 + 432\sqrt{atan\,(ax)}\, atan(ax)^2 a^4x^4 + 8\right.}{}$$

input
```
int((a^2*c*x^2+c)^3*atan(a*x)^(5/2)/x,x)
```

output
```
(c**3*(96*sqrt(atan(a*x))*atan(a*x)**2*a**6*x**6 + 432*sqrt(atan(a*x))*ata
n(a*x)**2*a**4*x**4 + 864*sqrt(atan(a*x))*atan(a*x)**2*a**2*x**2 + 528*sqr
t(atan(a*x))*atan(a*x)**2 - 48*sqrt(atan(a*x))*atan(a*x)*a**5*x**5 - 280*s
qrt(atan(a*x))*atan(a*x)*a**3*x**3 - 1320*sqrt(atan(a*x))*atan(a*x)*a*x +
18*sqrt(atan(a*x))*a**4*x**4 + 1794*sqrt(atan(a*x))*a**2*x**2 + 576*int((s
qrt(atan(a*x))*atan(a*x)**2)/x,x) - 9*int((sqrt(atan(a*x))*x**4)/(atan(a*x
)*a**2*x**2 + atan(a*x)),x)*a**5 - 3240*int((sqrt(atan(a*x))*x**3)/(a**2*x
**2 + 1),x)*a**4 - 897*int((sqrt(atan(a*x))*x**2)/(atan(a*x)*a**2*x**2 + a
tan(a*x)),x)*a**3 - 1608*int((sqrt(atan(a*x))*x)/(a**2*x**2 + 1),x)*a**2))
/576
```

# 3.853 $\int \frac{(c+a^2cx^2)^3 \arctan(ax)^{5/2}}{x^2} dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^{5/2}}{x^2} dx = \text{Int}\left(\frac{(c+a^2cx^2)^3 \arctan(ax)^{5/2}}{x^2}, x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^3*arctan(a*x)^(5/2)/x^2,x)
```

## Mathematica [N/A]

Not integrable

Time = 1.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^{5/2}}{x^2} dx = \int \frac{(c+a^2cx^2)^3 \arctan(ax)^{5/2}}{x^2} dx$$

input
```
Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2))/x^2,x]
```

output
```
Integrate[((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2))/x^2, x]
```

## Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^3}{x^2}dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int \frac{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^3}{x^2}dx$$

input
```
Int[((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2))/x^2,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\left(a^2c\,x^2+c\right)^3\arctan\left(ax\right)^{\frac{5}{2}}}{x^2}dx$$

input
```
int((a^2*c*x^2+c)^3*arctan(a*x)^(5/2)/x^2,x)
```

output
```
int((a^2*c*x^2+c)^3*arctan(a*x)^(5/2)/x^2,x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^{5/2}}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^(5/2)/x^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 51.73 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.75

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^{5/2}}{x^2}\,dx = c^3\left(\int 3a^2 \operatorname{atan}^{\frac{5}{2}}(ax)\,dx + \int \frac{\operatorname{atan}^{\frac{5}{2}}(ax)}{x^2}\,dx + \int 3a^4x^2 \operatorname{atan}^{\frac{5}{2}}(ax)\,dx + \int a^6x^4 \operatorname{atan}^{\frac{5}{2}}(ax)\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**3*atan(a*x)**(5/2)/x**2,x)
```

output
```
c**3*(Integral(3*a**2*atan(a*x)**(5/2), x) + Integral(atan(a*x)**(5/2)/x**
2, x) + Integral(3*a**4*x**2*atan(a*x)**(5/2), x) + Integral(a**6*x**4*ata
n(a*x)**(5/2), x))
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^{5/2}}{x^2} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^(5/2)/x^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^{5/2}}{x^2} \, dx = \int \frac{(a^2cx^2+c)^3 \arctan(ax)^{\frac{5}{2}}}{x^2} \, dx$$

input
```
integrate((a^2*c*x^2+c)^3*arctan(a*x)^(5/2)/x^2,x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^3*arctan(a*x)^(5/2)/x^2, x)
```

## Mupad [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^{5/2}}{x^2} \, dx = \int \frac{\mathrm{atan}(a\,x)^{5/2}\,\left(c\,a^2\,x^2+c\right)^3}{x^2} \, dx$$

input
```
int((atan(a*x)^(5/2)*(c + a^2*c*x^2)^3)/x^2,x)
```

output
```
int((atan(a*x)^(5/2)*(c + a^2*c*x^2)^3)/x^2, x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 277, normalized size of antiderivative = 11.54

$$\int \frac{(c+a^2cx^2)^3 \arctan(ax)^{5/2}}{x^2} \, dx = \frac{c^3 \left(32\sqrt{atan\,(ax)}\,atan(ax)^2\,a^6x^6 + 160\sqrt{atan\,(ax)}\,atan(ax)^2\,a^4x^4 + 4\right.}{}$$

input
```
int((a^2*c*x^2+c)^3*atan(a*x)^(5/2)/x^2,x)
```

output
```
(c**3*(32*sqrt(atan(a*x))*atan(a*x)**2*a**6*x**6 + 160*sqrt(atan(a*x))*ata
n(a*x)**2*a**4*x**4 + 480*sqrt(atan(a*x))*atan(a*x)**2*a**2*x**2 - 160*sqr
t(atan(a*x))*atan(a*x)**2 - 20*sqrt(atan(a*x))*atan(a*x)*a**5*x**5 - 160*s
qrt(atan(a*x))*atan(a*x)*a**3*x**3 - 140*sqrt(atan(a*x))*atan(a*x)*a*x + 1
0*sqrt(atan(a*x))*a**4*x**4 + 210*sqrt(atan(a*x))*a**2*x**2 - 880*int((sqr
t(atan(a*x))*atan(a*x)*x)/(a**2*x**2 + 1),x)*a**3*x + 400*int((sqrt(atan(a
*x))*atan(a*x))/(a**2*x**3 + x),x)*a*x - 5*int((sqrt(atan(a*x))*x**3)/(ata
n(a*x)*a**2*x**2 + atan(a*x)),x)*a**5*x - 105*int((sqrt(atan(a*x))*x)/(ata
n(a*x)*a**2*x**2 + atan(a*x)),x)*a**3*x))/(160*x)
```

# 3.854 $\int \frac{x^m \arctan(ax)^{5/2}}{c+a^2cx^2} dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m \arctan(ax)^{5/2}}{c+a^2cx^2} dx = \text{Int}\left(\frac{x^m \arctan(ax)^{5/2}}{c+a^2cx^2}, x\right)$$

output
```
Defer(Int)(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \arctan(ax)^{5/2}}{c+a^2cx^2} dx = \int \frac{x^m \arctan(ax)^{5/2}}{c+a^2cx^2} dx$$

input
```
Integrate[(x^m*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2),x]
```

output
```
Integrate[(x^m*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \arctan(ax)^{5/2}}{a^2cx^2+c} \, dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{x^m \arctan(ax)^{5/2}}{a^2cx^2+c}dx$$

input
```
Int[(x^m*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m \arctan\left(ax\right)^{\frac{5}{2}}}{a^2c\,x^2+c}dx$$

input
```
int(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x)
```

output
```
int(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x)
```

## Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan(ax)^{5/2}}{c+a^2cx^2}\,dx = \int \frac{x^m \arctan(ax)^{\frac{5}{2}}}{a^2cx^2+c}\,dx$$

input
```
integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
integral(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2 + c), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^m \arctan(ax)^{5/2}}{c+a^2cx^2}\,dx = \text{Timed out}$$

input
```
integrate(x**m*atan(a*x)**(5/2)/(a**2*c*x**2+c),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m \arctan(ax)^{5/2}}{c+a^2cx^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan(ax)^{5/2}}{c+a^2cx^2}\,dx = \int \frac{x^m \arctan(ax)^{\frac{5}{2}}}{a^2cx^2+c}\,dx$$

input
```
integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2 + c), x)
```

## Mupad [N/A]

Not integrable

Time = 0.77 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan(ax)^{5/2}}{c+a^2cx^2}\,dx = \int \frac{x^m \operatorname{atan}(a\,x)^{5/2}}{c\,a^2\,x^2+c}\,dx$$

input
```
int((x^m*atan(a*x)^(5/2))/(c + a^2*c*x^2),x)
```

output
```
int((x^m*atan(a*x)^(5/2))/(c + a^2*c*x^2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.33

$$\int \frac{x^m \arctan(ax)^{5/2}}{c+a^2cx^2}\,dx = \frac{\int \frac{x^m \sqrt{atan(ax)}\,atan(ax)^2}{a^2x^2+1}dx}{c}$$

input
```
int(x^m*atan(a*x)^(5/2)/(a^2*c*x^2+c),x)
```

output
```
int((x**m*sqrt(atan(a*x))*atan(a*x)**2)/(a**2*x**2 + 1),x)/c
```

# 3.855 $\int \frac{x^3 \arctan(ax)^{5/2}}{c+a^2cx^2} dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^3 \arctan(ax)^{5/2}}{c+a^2cx^2} dx = -\frac{2x \arctan(ax)^{7/2}}{7a^3c} + \frac{\mathrm{Int}\left(x \arctan(ax)^{5/2}, x\right)}{a^2c} + \frac{2\mathrm{Int}\left(\arctan(ax)^{7/2}, x\right)}{7a^3c}$$

output
```
-2/7*x*arctan(a*x)^(7/2)/a^3/c+Defer(Int)(x*arctan(a*x)^(5/2),x)/a^2/c+2/7
*Defer(Int)(arctan(a*x)^(7/2),x)/a^3/c
```

## Mathematica [N/A]

Not integrable

Time = 2.95 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^3 \arctan(ax)^{5/2}}{c+a^2cx^2} dx = \int \frac{x^3 \arctan(ax)^{5/2}}{c+a^2cx^2} dx$$

input
```
Integrate[(x^3*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2),x]
```

output
```
Integrate[(x^3*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \arctan(ax)^{5/2}}{a^2cx^2+c} \, dx$$

$$\Big\downarrow \ 5451$$

$$\frac{\int x \arctan(ax)^{5/2}dx}{a^2c} - \frac{\int \frac{x \arctan(ax)^{5/2}}{c(a^2x^2+1)}dx}{a^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\int x \arctan(ax)^{5/2}dx}{a^2c} - \frac{\int \frac{x \arctan(ax)^{5/2}}{a^2x^2+1}dx}{a^2c}$$

$$\Big\downarrow \ 5377$$

$$\frac{\int x \arctan(ax)^{5/2}dx}{a^2c} - \frac{\int \frac{x \arctan(ax)^{5/2}}{a^2x^2+1}dx}{a^2c}$$

$$\Big\downarrow \ 5457$$

$$\frac{\int x \arctan(ax)^{5/2}dx}{a^2c} - \frac{\frac{2x \arctan(ax)^{7/2}}{7a} - \frac{2\int \arctan(ax)^{7/2}dx}{7a}}{a^2c}$$

$$\Big\downarrow \ 5353$$

$$\frac{\int x \arctan(ax)^{5/2}dx}{a^2c} - \frac{\frac{2x \arctan(ax)^{7/2}}{7a} - \frac{2\int \arctan(ax)^{7/2}dx}{7a}}{a^2c}$$

input
```
Int[(x^3*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^3 \arctan(ax)^{\frac{5}{2}}}{a^2 c\, x^2 + c} dx$$

input
```
int(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x)
```

output
```
int(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3 \arctan(ax)^{5/2}}{c + a^2 c x^2} dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 8.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^3 \arctan(ax)^{5/2}}{c + a^2cx^2}\, dx = \frac{\int \frac{x^3 \operatorname{atan}^{\frac{5}{2}}(ax)}{a^2x^2+1}\, dx}{c}$$

input
```
integrate(x**3*atan(a*x)**(5/2)/(a**2*c*x**2+c),x)
```

output
```
Integral(x**3*atan(a*x)**(5/2)/(a**2*x**2 + 1), x)/c
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3 \arctan(ax)^{5/2}}{c + a^2cx^2}\, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^3 \arctan(ax)^{5/2}}{c + a^2cx^2}\, dx = \int \frac{x^3 \arctan(ax)^{\frac{5}{2}}}{a^2cx^2 + c}\, dx$$

input
```
integrate(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
integrate(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2 + c), x)
```

## Mupad [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^3 \arctan(ax)^{5/2}}{c+a^2cx^2}\,dx = \int \frac{x^3\,\mathrm{atan}(a\,x)^{5/2}}{c\,a^2\,x^2+c}\,dx$$

input
```
int((x^3*atan(a*x)^(5/2))/(c + a^2*c*x^2),x)
```

output
```
int((x^3*atan(a*x)^(5/2))/(c + a^2*c*x^2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.33

$$\int \frac{x^3 \arctan(ax)^{5/2}}{c+a^2cx^2}\,dx = \frac{\int \frac{\sqrt{atan(ax)}\,atan(ax)^2x^3}{a^2x^2+1}dx}{c}$$

input
```
int(x^3*atan(a*x)^(5/2)/(a^2*c*x^2+c),x)
```

output
```
int((sqrt(atan(a*x))*atan(a*x)**2*x**3)/(a**2*x**2 + 1),x)/c
```

# 3.856 $\int \frac{x^2 \arctan(ax)^{5/2}}{c+a^2cx^2} dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^2 \arctan(ax)^{5/2}}{c+a^2cx^2} dx = -\frac{2 \arctan(ax)^{7/2}}{7a^3c} + \frac{\operatorname{Int}\left(\arctan(ax)^{5/2}, x\right)}{a^2c}$$

output
```
-2/7*arctan(a*x)^(7/2)/a^3/c+Defer(Int)(arctan(a*x)^(5/2),x)/a^2/c
```

## Mathematica [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^2 \arctan(ax)^{5/2}}{c+a^2cx^2} dx = \int \frac{x^2 \arctan(ax)^{5/2}}{c+a^2cx^2} dx$$

input
```
Integrate[(x^2*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2),x]
```

output
```
Integrate[(x^2*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \arctan(ax)^{5/2}}{a^2cx^2+c}dx$$

$$\Big\downarrow \text{5451}$$

$$\frac{\int \arctan(ax)^{5/2}dx}{a^2c}-\frac{\int \frac{\arctan(ax)^{5/2}}{c(a^2x^2+1)}dx}{a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \arctan(ax)^{5/2}dx}{a^2c}-\frac{\int \frac{\arctan(ax)^{5/2}}{a^2x^2+1}dx}{a^2c}$$

$$\Big\downarrow \text{5353}$$

$$\frac{\int \arctan(ax)^{5/2}dx}{a^2c}-\frac{\int \frac{\arctan(ax)^{5/2}}{a^2x^2+1}dx}{a^2c}$$

$$\Big\downarrow \text{5419}$$

$$\frac{\int \arctan(ax)^{5/2}dx}{a^2c}-\frac{2\arctan(ax)^{7/2}}{7a^3c}$$

input
```
Int[(x^2*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^2 \arctan \left(a x\right)^{\frac{5}{2}}}{a^2 c\, x^2+c}dx$$

input
```
int(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x)
```

output
```
int(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2 \arctan(ax)^{5/2}}{c+a^2cx^2}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 6.68 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^2 \arctan(ax)^{5/2}}{c+a^2cx^2}\, dx = \frac{\int \frac{x^2 \operatorname{atan}^{\frac{5}{2}}(ax)}{a^2x^2+1}\, dx}{c}$$

input
```
integrate(x**2*atan(a*x)**(5/2)/(a**2*c*x**2+c),x)
```

output
```
Integral(x**2*atan(a*x)**(5/2)/(a**2*x**2 + 1), x)/c
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2 \arctan(ax)^{5/2}}{c + a^2cx^2} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^2 \arctan(ax)^{5/2}}{c + a^2cx^2} \, dx = \int \frac{x^2 \arctan(ax)^{\frac{5}{2}}}{a^2cx^2 + c} \, dx$$

input
```
integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2 + c), x)
```

## Mupad [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^2 \arctan(ax)^{5/2}}{c+a^2cx^2} \, dx = \int \frac{x^2 \operatorname{atan}(a\,x)^{5/2}}{c\,a^2\,x^2+c} \, dx$$

input
```
int((x^2*atan(a*x)^(5/2))/(c + a^2*c*x^2),x)
```

output
```
int((x^2*atan(a*x)^(5/2))/(c + a^2*c*x^2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.33

$$\int \frac{x^2 \arctan(ax)^{5/2}}{c+a^2cx^2} \, dx = \frac{\int \frac{\sqrt{atan(ax)}\, atan(ax)^2 x^2}{a^2x^2+1} dx}{c}$$

input
```
int(x^2*atan(a*x)^(5/2)/(a^2*c*x^2+c),x)
```

output
```
int((sqrt(atan(a*x))*atan(a*x)**2*x**2)/(a**2*x**2 + 1),x)/c
```

# 3.857 $\int \frac{x \arctan(ax)^{5/2}}{c+a^2cx^2}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x \arctan(ax)^{5/2}}{c+a^2cx^2}\,dx = \frac{2x \arctan(ax)^{7/2}}{7ac} - \frac{2\mathrm{Int}\left(\arctan(ax)^{7/2},x\right)}{7ac}$$

output
```
2/7*x*arctan(a*x)^(7/2)/a/c-2/7*Defer(Int)(arctan(a*x)^(7/2),x)/a/c
```

## Mathematica [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x \arctan(ax)^{5/2}}{c+a^2cx^2}\,dx = \int \frac{x \arctan(ax)^{5/2}}{c+a^2cx^2}\,dx$$

input
```
Integrate[(x*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2),x]
```

output
```
Integrate[(x*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \arctan(ax)^{5/2}}{a^2cx^2+c} \, dx$$

$$\Big\downarrow \; 5457$$

$$\frac{2x \arctan(ax)^{7/2}}{7ac}-\frac{2\int \arctan(ax)^{7/2}dx}{7ac}$$

$$\Big\downarrow \; 5353$$

$$\frac{2x \arctan(ax)^{7/2}}{7ac}-\frac{2\int \arctan(ax)^{7/2}dx}{7ac}$$

input
```
Int[(x*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x \arctan \left(a x \right)^{\frac{5}{2}}}{a^{2} c \,x^{2}+c}d x$$

input
```
int(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x)
```

output
```
int(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x \arctan(ax)^{5/2}}{c + a^2cx^2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 4.45 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x \arctan(ax)^{5/2}}{c + a^2cx^2} \, dx = \frac{\int \frac{x \operatorname{atan}^{\frac{5}{2}}(ax)}{a^2x^2+1} \, dx}{c}$$

input
```
integrate(x*atan(a*x)**(5/2)/(a**2*c*x**2+c),x)
```

output
```
Integral(x*atan(a*x)**(5/2)/(a**2*x**2 + 1), x)/c
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x \arctan(ax)^{5/2}}{c + a^2cx^2} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x \arctan(ax)^{5/2}}{c+a^2cx^2}\,dx = \int \frac{x \arctan{(ax)}^{\frac{5}{2}}}{a^2cx^2+c}\,dx$$

input
```
integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2 + c), x)
```

## Mupad [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x \arctan(ax)^{5/2}}{c+a^2cx^2}\,dx = \int \frac{x\,\mathrm{atan}(a\,x)^{5/2}}{c\,a^2\,x^2+c}\,dx$$

input
```
int((x*atan(a*x)^(5/2))/(c + a^2*c*x^2),x)
```

output
```
int((x*atan(a*x)^(5/2))/(c + a^2*c*x^2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.36

$$\int \frac{x \arctan(ax)^{5/2}}{c+a^2cx^2}\,dx = \frac{\int \frac{\sqrt{atan(ax)}\,atan(ax)^2 x}{a^2x^2+1}dx}{c}$$

input
```
int(x*atan(a*x)^(5/2)/(a^2*c*x^2+c),x)
```

output
```
int((sqrt(atan(a*x))*atan(a*x)**2*x)/(a**2*x**2 + 1),x)/c
```

# 3.858 $\int \frac{\arctan(ax)^{5/2}}{c+a^2cx^2} dx$



## Optimal result

Integrand size = 21, antiderivative size = 18

$$\int \frac{\arctan(ax)^{5/2}}{c+a^2cx^2} dx = \frac{2\arctan(ax)^{7/2}}{7ac}$$

output
```
2/7*arctan(a*x)^(7/2)/a/c
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax)^{5/2}}{c+a^2cx^2} dx = \frac{2\arctan(ax)^{7/2}}{7ac}$$

input
```
Integrate[ArcTan[a*x]^(5/2)/(c + a^2*c*x^2),x]
```

output
```
(2*ArcTan[a*x]^(7/2))/(7*a*c)
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{5/2}}{a^2cx^2+c}\,dx$$

$$\Big\downarrow \text{ 5419}$$

$$\frac{2\arctan(ax)^{7/2}}{7ac}$$

input
```
Int[ArcTan[a*x]^(5/2)/(c + a^2*c*x^2),x]
```

output
```
(2*ArcTan[a*x]^(7/2))/(7*a*c)
```

### Defintions of rubi rules used

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

## Maple [A] (verified)

Time = 1.74 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

| method | result | size |
|---|---|---|
| default | $\frac{2\arctan(ax)^{\frac{7}{2}}}{7ac}$ | 15 |

input
```
int(arctan(a*x)^(5/2)/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
2/7*arctan(a*x)^(7/2)/a/c
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{\arctan(ax)^{5/2}}{c+a^2cx^2}\,dx = \frac{2\arctan(ax)^{\frac{7}{2}}}{7\,ac}$$

input
```
integrate(arctan(a*x)^(5/2)/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
2/7*arctan(a*x)^(7/2)/(a*c)
```

## Sympy [A] (verification not implemented)

Time = 6.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{\arctan(ax)^{5/2}}{c+a^2cx^2}\,dx = \begin{cases} \frac{2\operatorname{atan}^{\frac{7}{2}}(ax)}{7ac} & \text{for } a \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate(atan(a*x)**(5/2)/(a**2*c*x**2+c),x)
```

output
```
Piecewise((2*atan(a*x)**(7/2)/(7*a*c), Ne(a, 0)), (0, True))
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{5/2}}{c+a^2cx^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(5/2)/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{\arctan(ax)^{5/2}}{c+a^2cx^2}\,dx = \frac{2\arctan{(ax)}^{\frac{7}{2}}}{7ac}$$

input
```
integrate(arctan(a*x)^(5/2)/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
2/7*arctan(a*x)^(7/2)/(a*c)
```

## Mupad [B] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{\arctan(ax)^{5/2}}{c+a^2cx^2}\,dx = \frac{2\,\mathrm{atan}(a\,x)^{7/2}}{7\,a\,c}$$

input
```
int(atan(a*x)^(5/2)/(c + a^2*c*x^2),x)
```

output
```
(2*atan(a*x)^(7/2))/(7*a*c)
```

## Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{\arctan(ax)^{5/2}}{c+a^2cx^2}\,dx = \frac{2\sqrt{atan\,(ax)}\,atan(ax)^3}{7ac}$$

input
```
int(atan(a*x)^(5/2)/(a^2*c*x^2+c),x)
```

output
```
(2*sqrt(atan(a*x))*atan(a*x)**3)/(7*a*c)
```

# 3.859 $\int \frac{\arctan(ax)^{5/2}}{x(c+a^2cx^2)}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\arctan(ax)^{5/2}}{x(c+a^2cx^2)}\,dx = -\frac{2i\arctan(ax)^{7/2}}{7c} + \frac{i\text{Int}\left(\frac{\arctan(ax)^{5/2}}{x(i+ax)},x\right)}{c}$$

output
```
-2/7*I*arctan(a*x)^(7/2)/c+I*Defer(Int)(arctan(a*x)^(5/2)/x/(I+a*x),x)/c
```

## Mathematica [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\arctan(ax)^{5/2}}{x(c+a^2cx^2)}\,dx = \int \frac{\arctan(ax)^{5/2}}{x(c+a^2cx^2)}\,dx$$

input
```
Integrate[ArcTan[a*x]^(5/2)/(x*(c + a^2*c*x^2)),x]
```

output
```
Integrate[ArcTan[a*x]^(5/2)/(x*(c + a^2*c*x^2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{5/2}}{x\,(a^2cx^2+c)}\,dx$$

$$\Big\downarrow \text{ 5459}$$

$$\frac{i\int \frac{\arctan(ax)^{5/2}}{x(ax+i)}dx}{c} - \frac{2i\arctan(ax)^{7/2}}{7c}$$

$$\Big\downarrow \text{ 5560}$$

$$\frac{i\int \frac{\arctan(ax)^{5/2}}{x(ax+i)}dx}{c} - \frac{2i\arctan(ax)^{7/2}}{7c}$$

input
```
Int[ArcTan[a*x]^(5/2)/(x*(c + a^2*c*x^2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\arctan\left(ax\right)^{\frac{5}{2}}}{x\left(a^{2}c\,x^{2}+c\right)}dx$$

input
```
int(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c),x)
```

output
```
int(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{5/2}}{x\,(c+a^2cx^2)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 3.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \frac{\arctan(ax)^{5/2}}{x\,(c+a^2cx^2)}\,dx = \frac{\int \frac{\operatorname{atan}^{\frac{5}{2}}(ax)}{a^2x^3+x}\,dx}{c}$$

input
```
integrate(atan(a*x)**(5/2)/x/(a**2*c*x**2+c),x)
```

output
```
Integral(atan(a*x)**(5/2)/(a**2*x**3 + x), x)/c
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{5/2}}{x\,(c+a^2cx^2)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax)^{5/2}}{x\,(c+a^2cx^2)}\,dx = \int \frac{\arctan(ax)^{\frac{5}{2}}}{(a^2cx^2+c)x}\,dx$$

input
```
integrate(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^(5/2)/((a^2*c*x^2 + c)*x), x)
```

## Mupad [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax)^{5/2}}{x\,(c+a^2cx^2)}\,dx = \int \frac{\mathrm{atan}(a\,x)^{5/2}}{x\,\left(c\,a^2\,x^2+c\right)}\,dx$$

input
```
int(atan(a*x)^(5/2)/(x*(c + a^2*c*x^2)),x)
```

output
```
int(atan(a*x)^(5/2)/(x*(c + a^2*c*x^2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int \frac{\arctan(ax)^{5/2}}{x\left(c+a^2cx^2\right)}\,dx = \frac{\int \frac{\sqrt{atan(ax)}\,atan(ax)^2}{a^2x^3+x}dx}{c}$$

input
```
int(atan(a*x)^(5/2)/x/(a^2*c*x^2+c),x)
```

output
```
int((sqrt(atan(a*x))*atan(a*x)**2)/(a**2*x**3 + x),x)/c
```

# 3.860 $\int \frac{\arctan(ax)^{5/2}}{x^2(c+a^2cx^2)}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\arctan(ax)^{5/2}}{x^2\left(c+a^2cx^2\right)}\,dx=-\frac{2a\arctan(ax)^{7/2}}{7c}+\frac{\operatorname{Int}\left(\frac{\arctan(ax)^{5/2}}{x^2},x\right)}{c}$$

output
```
-2/7*a*arctan(a*x)^(7/2)/c+Defer(Int)(arctan(a*x)^(5/2)/x^2,x)/c
```

## Mathematica [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\arctan(ax)^{5/2}}{x^2\left(c+a^2cx^2\right)}\,dx=\int \frac{\arctan(ax)^{5/2}}{x^2\left(c+a^2cx^2\right)}\,dx$$

input
```
Integrate[ArcTan[a*x]^(5/2)/(x^2*(c + a^2*c*x^2)),x]
```

output
```
Integrate[ArcTan[a*x]^(5/2)/(x^2*(c + a^2*c*x^2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{5/2}}{x^2(a^2cx^2+c)}dx$$

$$\Big\downarrow \text{5453}$$

$$\frac{\int \frac{\arctan(ax)^{5/2}}{x^2}dx}{c} - a^2\int \frac{\arctan(ax)^{5/2}}{c(a^2x^2+1)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\arctan(ax)^{5/2}}{x^2}dx}{c} - \frac{a^2\int \frac{\arctan(ax)^{5/2}}{a^2x^2+1}dx}{c}$$

$$\Big\downarrow \text{5377}$$

$$\frac{\int \frac{\arctan(ax)^{5/2}}{x^2}dx}{c} - \frac{a^2\int \frac{\arctan(ax)^{5/2}}{a^2x^2+1}dx}{c}$$

$$\Big\downarrow \text{5419}$$

$$\frac{\int \frac{\arctan(ax)^{5/2}}{x^2}dx}{c} - \frac{2a\arctan(ax)^{7/2}}{7c}$$

input
```
Int[ArcTan[a*x]^(5/2)/(x^2*(c + a^2*c*x^2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\arctan{(ax)}^{\frac{5}{2}}}{x^2 (a^2 c x^2 + c)} dx$$

input
```
int(arctan(a*x)^(5/2)/x^2/(a^2*c*x^2+c),x)
```

output
```
int(arctan(a*x)^(5/2)/x^2/(a^2*c*x^2+c),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{5/2}}{x^2 (c + a^2 c x^2)} dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(5/2)/x^2/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 4.50 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{\arctan(ax)^{5/2}}{x^2 (c + a^2 c x^2)} dx = \frac{\int \frac{\operatorname{atan}^{\frac{5}{2}}{(ax)}}{a^2 x^4 + x^2} dx}{c}$$

input
```
integrate(atan(a*x)**(5/2)/x**2/(a**2*c*x**2+c),x)
```

output
```
Integral(atan(a*x)**(5/2)/(a**2*x**4 + x**2), x)/c
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{5/2}}{x^2(c+a^2cx^2)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(5/2)/x^2/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax)^{5/2}}{x^2(c+a^2cx^2)}\,dx = \int \frac{\arctan(ax)^{\frac{5}{2}}}{(a^2cx^2+c)x^2}\,dx$$

input
```
integrate(arctan(a*x)^(5/2)/x^2/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^(5/2)/((a^2*c*x^2 + c)*x^2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax)^{5/2}}{x^2 (c+a^2cx^2)} \, dx = \int \frac{\mathrm{atan}(a\,x)^{5/2}}{x^2\,\left(c\,a^2\,x^2+c\right)} \,dx$$

input
```
int(atan(a*x)^(5/2)/(x^2*(c + a^2*c*x^2)),x)
```

output
```
int(atan(a*x)^(5/2)/(x^2*(c + a^2*c*x^2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.67

$$\int \frac{\arctan(ax)^{5/2}}{x^2 (c+a^2cx^2)} \, dx = \frac{-4\sqrt{atan(ax)}\,atan(ax)^3\,ax - 14\sqrt{atan(ax)}\,atan(ax)^2 + 35\left(\int \frac{\sqrt{atan(ax)}\,atan(ax)}{a^2x^3+x}dx\right.}{14cx}$$

input
```
int(atan(a*x)^(5/2)/x^2/(a^2*c*x^2+c),x)
```

output
```
( - 4*sqrt(atan(a*x))*atan(a*x)**3*a*x - 14*sqrt(atan(a*x))*atan(a*x)**2 +
 35*int((sqrt(atan(a*x))*atan(a*x))/(a**2*x**3 + x),x)*a*x)/(14*c*x)
```

# 3.861 $\int \frac{\arctan(ax)^{5/2}}{x^3(c+a^2cx^2)}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\arctan(ax)^{5/2}}{x^3(c+a^2cx^2)}\,dx = \frac{2ia^2\arctan(ax)^{7/2}}{7c} + \frac{\text{Int}\left(\frac{\arctan(ax)^{5/2}}{x^3},x\right)}{c} - \frac{ia^2\text{Int}\left(\frac{\arctan(ax)^{5/2}}{x(i+ax)},x\right)}{c}$$

output
```
2/7*I*a^2*arctan(a*x)^(7/2)/c+Defer(Int)(arctan(a*x)^(5/2)/x^3,x)/c-I*a^2*
Defer(Int)(arctan(a*x)^(5/2)/x/(I+a*x),x)/c
```

## Mathematica [N/A]

Not integrable

Time = 1.51 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\arctan(ax)^{5/2}}{x^3(c+a^2cx^2)}\,dx = \int \frac{\arctan(ax)^{5/2}}{x^3(c+a^2cx^2)}\,dx$$

input
```
Integrate[ArcTan[a*x]^(5/2)/(x^3*(c + a^2*c*x^2)),x]
```

output
```
Integrate[ArcTan[a*x]^(5/2)/(x^3*(c + a^2*c*x^2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{5/2}}{x^3\left(a^2cx^2+c\right)}dx$$

$$\Big\downarrow \text{ 5453}$$

$$\frac{\int \frac{\arctan(ax)^{5/2}}{x^3}dx}{c}-a^2\int \frac{\arctan(ax)^{5/2}}{cx\left(a^2x^2+1\right)}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{\arctan(ax)^{5/2}}{x^3}dx}{c}-\frac{a^2\int \frac{\arctan(ax)^{5/2}}{x(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \text{ 5377}$$

$$\frac{\int \frac{\arctan(ax)^{5/2}}{x^3}dx}{c}-\frac{a^2\int \frac{\arctan(ax)^{5/2}}{x(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \text{ 5459}$$

$$\frac{\int \frac{\arctan(ax)^{5/2}}{x^3}dx}{c}-\frac{a^2\left(i\int \frac{\arctan(ax)^{5/2}}{x(ax+i)}dx-\frac{2}{7}i\arctan(ax)^{7/2}\right)}{c}$$

$$\Big\downarrow \text{ 5560}$$

$$\frac{\int \frac{\arctan(ax)^{5/2}}{x^3}dx}{c}-\frac{a^2\left(i\int \frac{\arctan(ax)^{5/2}}{x(ax+i)}dx-\frac{2}{7}i\arctan(ax)^{7/2}\right)}{c}$$

input
```
Int[ArcTan[a*x]^(5/2)/(x^3*(c + a^2*c*x^2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\arctan(ax)^{\frac{5}{2}}}{x^3(a^2c\,x^2+c)}dx$$

input
```
int(arctan(a*x)^(5/2)/x^3/(a^2*c*x^2+c),x)
```

output
```
int(arctan(a*x)^(5/2)/x^3/(a^2*c*x^2+c),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{5/2}}{x^3(c+a^2cx^2)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(5/2)/x^3/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 4.52 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{\arctan(ax)^{5/2}}{x^3(c+a^2cx^2)}\,dx = \frac{\int \frac{\operatorname{atan}^{\frac{5}{2}}(ax)}{a^2x^5+x^3}\,dx}{c}$$

input
```
integrate(atan(a*x)**(5/2)/x**3/(a**2*c*x**2+c),x)
```

output
```
Integral(atan(a*x)**(5/2)/(a**2*x**5 + x**3), x)/c
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{5/2}}{x^3(c+a^2cx^2)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(5/2)/x^3/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax)^{5/2}}{x^3(c+a^2cx^2)}\,dx = \int \frac{\arctan(ax)^{\frac{5}{2}}}{(a^2cx^2+c)x^3}\,dx$$

input
```
integrate(arctan(a*x)^(5/2)/x^3/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^(5/2)/((a^2*c*x^2 + c)*x^3), x)
```

## Mupad [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax)^{5/2}}{x^3 (c+a^2cx^2)} \, dx = \int \frac{\mathrm{atan}(a\,x)^{5/2}}{x^3\,\left(c\,a^2\,x^2+c\right)} \,dx$$

input
```
int(atan(a*x)^(5/2)/(x^3*(c + a^2*c*x^2)),x)
```

output
```
int(atan(a*x)^(5/2)/(x^3*(c + a^2*c*x^2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.29

$$\int \frac{\arctan(ax)^{5/2}}{x^3 (c+a^2cx^2)} \, dx = \frac{\int \frac{\sqrt{atan(ax)}\,atan(ax)^2}{a^2x^5+x^3}dx}{c}$$

input
```
int(atan(a*x)^(5/2)/x^3/(a^2*c*x^2+c),x)
```

output
```
int((sqrt(atan(a*x))*atan(a*x)**2)/(a**2*x**5 + x**3),x)/c
```

# 3.862 $\int \frac{\arctan(ax)^{5/2}}{x^4(c+a^2cx^2)}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\arctan(ax)^{5/2}}{x^4(c+a^2cx^2)}\,dx = \frac{2a^3\arctan(ax)^{7/2}}{7c} + \frac{\text{Int}\left(\frac{\arctan(ax)^{5/2}}{x^4},x\right)}{c} - \frac{a^2\text{Int}\left(\frac{\arctan(ax)^{5/2}}{x^2},x\right)}{c}$$

output
```
2/7*a^3*arctan(a*x)^(7/2)/c+Defer(Int)(arctan(a*x)^(5/2)/x^4,x)/c-a^2*Defe
r(Int)(arctan(a*x)^(5/2)/x^2,x)/c
```

## Mathematica [N/A]

Not integrable

Time = 2.84 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\arctan(ax)^{5/2}}{x^4(c+a^2cx^2)}\,dx = \int \frac{\arctan(ax)^{5/2}}{x^4(c+a^2cx^2)}\,dx$$

input
```
Integrate[ArcTan[a*x]^(5/2)/(x^4*(c + a^2*c*x^2)),x]
```

output
```
Integrate[ArcTan[a*x]^(5/2)/(x^4*(c + a^2*c*x^2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{5/2}}{x^4\left(a^2cx^2+c\right)}dx$$

$$\Big\downarrow \text{5453}$$

$$\frac{\int \frac{\arctan(ax)^{5/2}}{x^4}dx}{c}-a^2\int \frac{\arctan(ax)^{5/2}}{cx^2\left(a^2x^2+1\right)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\arctan(ax)^{5/2}}{x^4}dx}{c}-\frac{a^2\int \frac{\arctan(ax)^{5/2}}{x^2(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \text{5377}$$

$$\frac{\int \frac{\arctan(ax)^{5/2}}{x^4}dx}{c}-\frac{a^2\int \frac{\arctan(ax)^{5/2}}{x^2(a^2x^2+1)}dx}{c}$$

$$\Big\downarrow \text{5453}$$

$$\frac{\int \frac{\arctan(ax)^{5/2}}{x^4}dx}{c}-\frac{a^2\left(\int \frac{\arctan(ax)^{5/2}}{x^2}dx-a^2\int \frac{\arctan(ax)^{5/2}}{a^2x^2+1}dx\right)}{c}$$

$$\Big\downarrow \text{5377}$$

$$\frac{\int \frac{\arctan(ax)^{5/2}}{x^4}dx}{c}-\frac{a^2\left(\int \frac{\arctan(ax)^{5/2}}{x^2}dx-a^2\int \frac{\arctan(ax)^{5/2}}{a^2x^2+1}dx\right)}{c}$$

$$\Big\downarrow \text{5419}$$

$$\frac{\int \frac{\arctan(ax)^{5/2}}{x^4}dx}{c}-\frac{a^2\left(\int \frac{\arctan(ax)^{5/2}}{x^2}dx-\frac{2}{7}a\arctan(ax)^{7/2}\right)}{c}$$

input
```
Int[ArcTan[a*x]^(5/2)/(x^4*(c + a^2*c*x^2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\arctan \left(a x\right)^{\frac{5}{2}}}{x^{4} \left(a^{2} c \,x^{2}+c \right)}d x$$

input
```
int(arctan(a*x)^(5/2)/x^4/(a^2*c*x^2+c),x)
```

output
```
int(arctan(a*x)^(5/2)/x^4/(a^2*c*x^2+c),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{5/2}}{x^4\left(c+a^2cx^2\right)}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(5/2)/x^4/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 6.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{\arctan(ax)^{5/2}}{x^4 (c + a^2cx^2)} \, dx = \frac{\int \frac{\operatorname{atan}^{\frac{5}{2}}(ax)}{a^2x^6+x^4} \, dx}{c}$$

input
```
integrate(atan(a*x)**(5/2)/x**4/(a**2*c*x**2+c),x)
```

output
```
Integral(atan(a*x)**(5/2)/(a**2*x**6 + x**4), x)/c
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{5/2}}{x^4 (c + a^2cx^2)} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(5/2)/x^4/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax)^{5/2}}{x^4 (c + a^2cx^2)} \, dx = \int \frac{\arctan(ax)^{\frac{5}{2}}}{(a^2cx^2 + c)x^4} \, dx$$

input
```
integrate(arctan(a*x)^(5/2)/x^4/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^(5/2)/((a^2*c*x^2 + c)*x^4), x)
```

## Mupad [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax)^{5/2}}{x^4\,(c+a^2cx^2)}\,dx = \int \frac{\mathrm{atan}(a\,x)^{5/2}}{x^4\,\left(c\,a^2\,x^2+c\right)}\,dx$$

input
```
int(atan(a*x)^(5/2)/(x^4*(c + a^2*c*x^2)),x)
```

output
```
int(atan(a*x)^(5/2)/(x^4*(c + a^2*c*x^2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 170, normalized size of antiderivative = 7.08

$$\int \frac{\arctan(ax)^{5/2}}{x^4\,(c+a^2cx^2)}\,dx = \frac{96\sqrt{atan\,(ax)}\,atan(ax)^3\,a^3x^3+336\sqrt{atan\,(ax)}\,atan(ax)^2\,a^2x^2-112\sqrt{atan\,(ax)}\,a}{}$$

input
```
int(atan(a*x)^(5/2)/x^4/(a^2*c*x^2+c),x)
```

output
```
(96*sqrt(atan(a*x))*atan(a*x)**3*a**3*x**3 + 336*sqrt(atan(a*x))*atan(a*x)
**2*a**2*x**2 - 112*sqrt(atan(a*x))*atan(a*x)**2 + 420*sqrt(atan(a*x))*ata
n(a*x)*a**3*x**3 + 420*sqrt(atan(a*x))*atan(a*x)*a*x + 630*sqrt(atan(a*x))
*a**2*x**2 - 315*int(sqrt(atan(a*x))/(atan(a*x)*a**2*x**3 + atan(a*x)*x),x
)*a**3*x**3 + 1120*int((sqrt(atan(a*x))*atan(a*x))/(a**2*x**5 + x**3),x)*a
*x**3)/(336*c*x**3)
```

# 3.863 $\int \frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^2} dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^2} dx = \text{Int}\left(\frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^2}, x\right)$$

output
```
Defer(Int)(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x)
```

## Mathematica [N/A]

Not integrable

Time = 1.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^2} dx = \int \frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^2} dx$$

input
```
Integrate[(x^m*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^2,x]
```

output
```
Integrate[(x^m*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^2, x]
```

## Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \arctan(ax)^{5/2}}{(a^2cx^2+c)^2}\,dx$$

$$\Big\downarrow \text{5560}$$

$$\int \frac{x^m \arctan(ax)^{5/2}}{(a^2cx^2+c)^2}dx$$

input
```
Int[(x^m*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^2,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m \arctan(ax)^{\frac{5}{2}}}{(a^2c\,x^2+c)^2}dx$$

input
```
int(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x)
```

output
```
int(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.62

$$\int \frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx = \int \frac{x^m \arctan(ax)^{\frac{5}{2}}}{(a^2cx^2+c)^2}\,dx$$

input
```
integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
integral(x^m*arctan(a*x)^(5/2)/(a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**m*atan(a*x)**(5/2)/(a**2*c*x**2+c)**2,x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,2,1,0,0]%%%} / %%%{1,[0,0,0,1,2]%%%} Error: Bad Argum
ent Value
```

## Mupad [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx = \int \frac{x^m\,\mathrm{atan}(a\,x)^{5/2}}{\left(c\,a^2\,x^2+c\right)^2}\,dx$$

input
```
int((x^m*atan(a*x)^(5/2))/(c + a^2*c*x^2)^2,x)
```

output
```
int((x^m*atan(a*x)^(5/2))/(c + a^2*c*x^2)^2, x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.67

$$\int \frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx = \frac{\int \frac{x^m \sqrt{atan(ax)}\,atan(ax)^2}{a^4x^4+2a^2x^2+1}dx}{c^2}$$

input
```
int(x^m*atan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x)
```

output
```
int((x**m*sqrt(atan(a*x))*atan(a*x)**2)/(a**4*x**4 + 2*a**2*x**2 + 1),x)/c
**2
```

# 3.864 $\int \frac{x^3 \arctan(ax)^{5/2}}{(c+a^2cx^2)^2} dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^3 \arctan(ax)^{5/2}}{(c+a^2cx^2)^2} dx = \operatorname{Int}\left(\frac{x^3 \arctan(ax)^{5/2}}{(c+a^2cx^2)^2}, x\right)$$

output
```
Defer(Int)(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x)
```

## Mathematica [N/A]

Not integrable

Time = 3.33 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^3 \arctan(ax)^{5/2}}{(c+a^2cx^2)^2} dx = \int \frac{x^3 \arctan(ax)^{5/2}}{(c+a^2cx^2)^2} dx$$

input
```
Integrate[(x^3*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^2,x]
```

output
```
Integrate[(x^3*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^2, x]
```

## Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \arctan(ax)^{5/2}}{(a^2cx^2+c)^2} dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{x^3 \arctan(ax)^{5/2}}{(a^2cx^2+c)^2} dx$$

input
```
Int[(x^3*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^2,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^3 \arctan(ax)^{\frac{5}{2}}}{(a^2c\,x^2+c)^2} dx$$

input
```
int(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x)
```

output
```
int(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3 \arctan(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 10.92 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.33

$$\int \frac{x^3 \arctan(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx = \frac{\int \frac{x^3 \operatorname{atan}^{\frac{5}{2}}(ax)}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

input
```
integrate(x**3*atan(a*x)**(5/2)/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(x**3*atan(a*x)**(5/2)/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3 \arctan(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 1.54 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^3 \arctan(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx = \int \frac{x^3 \arctan{(ax)}^{\frac{5}{2}}}{(a^2cx^2+c)^2}\,dx$$

input
```
integrate(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2 + c)^2, x)
```

## Mupad [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^3 \arctan(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx = \int \frac{x^3\,\mathrm{atan}(a\,x)^{5/2}}{(c\,a^2\,x^2+c)^2}\,dx$$

input
```
int((x^3*atan(a*x)^(5/2))/(c + a^2*c*x^2)^2,x)
```

output
```
int((x^3*atan(a*x)^(5/2))/(c + a^2*c*x^2)^2, x)
```

## Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.67

$$\int \frac{x^3 \arctan(ax)^{5/2}}{(c + a^2cx^2)^2} \, dx = \frac{\int \frac{\sqrt{atan(ax)}\, atan(ax)^2 x^3}{a^4x^4+2a^2x^2+1} dx}{c^2}$$

input
```
int(x^3*atan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x)
```

output
```
int((sqrt(atan(a*x))*atan(a*x)**2*x**3)/(a**4*x**4 + 2*a**2*x**2 + 1),x)/c
**2
```

# 3.865 $\int \frac{x^2 \arctan(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 157

$$\int \frac{x^2 \arctan(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx = \frac{15x\sqrt{\arctan(ax)}}{32a^2c^2(1+a^2x^2)} + \frac{5\arctan(ax)^{3/2}}{16a^3c^2} - \frac{5\arctan(ax)^{3/2}}{8a^3c^2(1+a^2x^2)} - \frac{x\arctan(ax)^{5/2}}{2a^2c^2(1+a^2x^2)} + \frac{\arctan(ax)^{7/2}}{7a^3c^2} - \frac{15\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{128a^3c^2}$$

output
```
15/32*x*arctan(a*x)^(1/2)/a^2/c^2/(a^2*x^2+1)+5/16*arctan(a*x)^(3/2)/a^3/c
^2-5/8*arctan(a*x)^(3/2)/a^3/c^2/(a^2*x^2+1)-1/2*x*arctan(a*x)^(5/2)/a^2/c
^2/(a^2*x^2+1)+1/7*arctan(a*x)^(7/2)/a^3/c^2-15/128*Pi^(1/2)*FresnelS(2*ar
ctan(a*x)^(1/2)/Pi^(1/2))/a^3/c^2
```

## Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.71

$$\int \frac{x^2 \arctan(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx = \frac{4\sqrt{\arctan(ax)}(105ax+70(-1+a^2x^2)\arctan(ax)-112ax\arctan(ax)^2+32(1-}{896a^3c^2(1+a^2x^2}$$

input
```
Integrate[(x^2*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^2,x]
```

output
```
(4*Sqrt[ArcTan[a*x]]*(105*a*x + 70*(-1 + a^2*x^2)*ArcTan[a*x] - 112*a*x*Ar
cTan[a*x]^2 + 32*(1 + a^2*x^2)*ArcTan[a*x]^3) - 105*Sqrt[Pi]*(1 + a^2*x^2)
*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(896*a^3*c^2*(1 + a^2*x^2))
```

## Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.03, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5471, 27, 5465, 5427, 5505, 4906, 27, 3042, 3786, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \arctan(ax)^{5/2}}{(a^2cx^2+c)^2}dx$$

$$\Big\downarrow \text{5471}$$

$$\frac{5\int \frac{x\arctan(ax)^{3/2}}{c^2(a^2x^2+1)^2}dx}{4a}+\frac{\arctan(ax)^{7/2}}{7a^3c^2}-\frac{x\arctan(ax)^{5/2}}{2a^2c^2(a^2x^2+1)}$$

$$\Big\downarrow \text{27}$$

$$\frac{5\int \frac{x\arctan(ax)^{3/2}}{(a^2x^2+1)^2}dx}{4ac^2}+\frac{\arctan(ax)^{7/2}}{7a^3c^2}-\frac{x\arctan(ax)^{5/2}}{2a^2c^2(a^2x^2+1)}$$

$$\Big\downarrow \text{5465}$$

$$\frac{5\left(\frac{3\int \frac{\sqrt{\arctan(ax)}}{\left(a^2x^2+1\right)^2}dx}{4a}-\frac{\arctan(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)}{4ac^2}+\frac{\arctan(ax)^{7/2}}{7a^3c^2}-\frac{x\arctan(ax)^{5/2}}{2a^2c^2(a^2x^2+1)}$$

$$\Big\downarrow \text{5427}$$

$$\frac{5\left(\frac{3\left(-\frac{1}{4}a\int \frac{x}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx+\frac{x\sqrt{\arctan(ax)}}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4a}-\frac{\arctan(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)}{4ac^2}+\frac{\arctan(ax)^{7/2}}{7a^3c^2}-\frac{x\arctan(ax)^{5/2}}{2a^2c^2\left(a^2x^2+1\right)}$$

$$\Big\downarrow \text{5505}$$

$$\frac{5\left(\frac{3\left(-\frac{\int \frac{ax}{\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}d\arctan(ax)}{4a}+\frac{x\sqrt{\arctan(ax)}}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4a}-\frac{\arctan(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)}{4ac^2}+\frac{\arctan(ax)^{7/2}}{7a^3c^2}-\frac{x\arctan(ax)^{5/2}}{2a^2c^2\left(a^2x^2+1\right)}$$

$$\Big\downarrow \; 4906$$

$$\frac{5\left(\frac{3\left(-\frac{\int \frac{\sin(2\arctan(ax))}{2\sqrt{\arctan(ax)}}d\arctan(ax)}{4a}+\frac{x\sqrt{\arctan(ax)}}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4a}-\frac{\arctan(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)}{4ac^2}+\frac{\arctan(ax)^{7/2}}{7a^3c^2}-\frac{x\arctan(ax)^{5/2}}{2a^2c^2\left(a^2x^2+1\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{5\left(\frac{3\left(-\frac{\int \frac{\sin(2\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)}{8a}+\frac{x\sqrt{\arctan(ax)}}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4a}-\frac{\arctan(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)}{4ac^2}+\frac{\arctan(ax)^{7/2}}{7a^3c^2}-\frac{x\arctan(ax)^{5/2}}{2a^2c^2\left(a^2x^2+1\right)}$$

$$\Big\downarrow \; 3042$$

$$\frac{5\left(\frac{3\left(-\frac{\int \frac{\sin(2\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)}{8a}+\frac{x\sqrt{\arctan(ax)}}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4a}-\frac{\arctan(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)}{4ac^2}+\frac{\arctan(ax)^{7/2}}{7a^3c^2}-\frac{x\arctan(ax)^{5/2}}{2a^2c^2\left(a^2x^2+1\right)}$$

$$\Big\downarrow \; 3786$$

$$\frac{5\left(\frac{3\left(-\frac{\int \sin(2\arctan(ax))d\sqrt{\arctan(ax)}}{4a}+\frac{x\sqrt{\arctan(ax)}}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4a}-\frac{\arctan(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)}{4ac^2}+\frac{\arctan(ax)^{7/2}}{7a^3c^2}-\frac{x\arctan(ax)^{5/2}}{2a^2c^2\left(a^2x^2+1\right)}$$

$$\Big\downarrow \text{3832}$$

$$\frac{\arctan(ax)^{7/2}}{7a^3c^2}+\frac{5\left(\frac{3\left(\frac{x\sqrt{\arctan(ax)}}{2(a^2x^2+1)}-\frac{\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{8a}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4a}-\frac{\arctan(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)}{4ac^2}-\frac{x\arctan(ax)^{5/2}}{2a^2c^2\left(a^2x^2+1\right)}$$

input
```
Int[(x^2*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^2,x]
```

output
```
-1/2*(x*ArcTan[a*x]^(5/2))/(a^2*c^2*(1 + a^2*x^2)) + ArcTan[a*x]^(7/2)/(7*
a^3*c^2) + (5*(-1/2*ArcTan[a*x]^(3/2)/(a^2*(1 + a^2*x^2)) + (3*((x*Sqrt[Ar
cTan[a*x]])/(2*(1 + a^2*x^2)) + ArcTan[a*x]^(3/2)/(3*a) - (Sqrt[Pi]*Fresne
lS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(8*a)))/(4*a)))/(4*a*c^2)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5471
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^2)/((d_) + (e_.)*(x_)^2)
^2, x_Symbol] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(2*b*c^3*d^2*(p + 1)), x]
 + (-Simp[x*((a + b*ArcTan[c*x])^p/(2*c^2*d*(d + e*x^2))), x] + Simp[b*(p/(
2*c))   Int[x*((a + b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

## Maple [A] (verified)

Time = 95.64 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.59

| method | result |
|---|---|
| default | $-\frac{-128\arctan(ax)^{\frac{7}{2}}\sqrt{\pi}+224\arctan(ax)^{\frac{5}{2}}\sqrt{\pi}\,\sin(2\arctan(ax))+280\arctan(ax)^{\frac{3}{2}}\sqrt{\pi}\,\cos(2\arctan(ax))-210\sqrt{\arctan(ax)}\,\sqrt{\pi}\,\mathrm{si}}{896a^3c^2\sqrt{\pi}}$ |

input
```
int(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/896/a^3/c^2/Pi^(1/2)*(-128*arctan(a*x)^(7/2)*Pi^(1/2)+224*arctan(a*x)^(
5/2)*Pi^(1/2)*sin(2*arctan(a*x))+280*arctan(a*x)^(3/2)*Pi^(1/2)*cos(2*arct
an(a*x))-210*arctan(a*x)^(1/2)*Pi^(1/2)*sin(2*arctan(a*x))+105*Pi*FresnelS
(2*arctan(a*x)^(1/2)/Pi^(1/2)))
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2\arctan(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{x^2\arctan(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx = \frac{\int \frac{x^2\operatorname{atan}^{\frac{5}{2}}(ax)}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

input
```
integrate(x**2*atan(a*x)**(5/2)/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(x**2*atan(a*x)**(5/2)/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2 \arctan(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F]

$$\int \frac{x^2 \arctan(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx = \int \frac{x^2 \arctan(ax)^{\frac{5}{2}}}{(a^2cx^2+c)^2}\,dx$$

input
```
integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2 + c)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \arctan(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx = \int \frac{x^2\,\mathrm{atan}(a\,x)^{5/2}}{(c\,a^2\,x^2+c)^2}\,dx$$

input
```
int((x^2*atan(a*x)^(5/2))/(c + a^2*c*x^2)^2,x)
```

output
```
int((x^2*atan(a*x)^(5/2))/(c + a^2*c*x^2)^2, x)
```

## Reduce [F]

$$\int \frac{x^2 \arctan(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx = \frac{16\sqrt{atan\,(ax)}\,atan(ax)^3\,a^2x^2 + 16\sqrt{atan\,(ax)}\,atan(ax)^3 - 56\sqrt{atan\,(ax)}\,atan(}{112}$$

input
```
int(x^2*atan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x)
```

output
```
(16*sqrt(atan(a*x))*atan(a*x)**3*a**2*x**2 + 16*sqrt(atan(a*x))*atan(a*x)*
*3 - 56*sqrt(atan(a*x))*atan(a*x)**2*a*x - 70*sqrt(atan(a*x))*atan(a*x) +
105*int(sqrt(atan(a*x))/(a**4*x**4 + 2*a**2*x**2 + 1),x)*a**3*x**2 + 105*i
nt(sqrt(atan(a*x))/(a**4*x**4 + 2*a**2*x**2 + 1),x)*a)/(112*a**3*c**2*(a**
2*x**2 + 1))
```

# 3.866 $\int \frac{x\arctan(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 156

$$\int \frac{x\arctan(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx = -\frac{15\sqrt{\arctan(ax)}}{64a^2c^2} + \frac{15\sqrt{\arctan(ax)}}{32a^2c^2(1+a^2x^2)} + \frac{5x\arctan(ax)^{3/2}}{8ac^2(1+a^2x^2)} + \frac{\arctan(ax)^{5/2}}{4a^2c^2} - \frac{\arctan(ax)^{5/2}}{2a^2c^2(1+a^2x^2)} - \frac{15\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{128a^2c^2}$$

output
```
-15/64*arctan(a*x)^(1/2)/a^2/c^2+15/32*arctan(a*x)^(1/2)/a^2/c^2/(a^2*x^2+
1)+5/8*x*arctan(a*x)^(3/2)/a/c^2/(a^2*x^2+1)+1/4*arctan(a*x)^(5/2)/a^2/c^2
-1/2*arctan(a*x)^(5/2)/a^2/c^2/(a^2*x^2+1)-15/128*Pi^(1/2)*FresnelC(2*arct
an(a*x)^(1/2)/Pi^(1/2))/a^2/c^2
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.50

$$\int \frac{x\arctan(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx = \frac{240\arctan(ax) - 240a^2x^2\arctan(ax) + 640ax\arctan(ax)^2 - 256\arctan(ax)^3 + 2}{}$$

input
```
Integrate[(x*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^2,x]
```

output
```
(240*ArcTan[a*x] - 240*a^2*x^2*ArcTan[a*x] + 640*a*x*ArcTan[a*x]^2 - 256*A
rcTan[a*x]^3 + 256*a^2*x^2*ArcTan[a*x]^3 - 60*Sqrt[Pi]*(1 + a^2*x^2)*Sqrt[
ArcTan[a*x]]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]] + (15*I)*Sqrt[2]*(1
+ a^2*x^2)*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-2*I)*ArcTan[a*x]] - (15*I)*
Sqrt[2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (2*I)*ArcTan[a*x]] - (15*I)*Sqrt[2]
*a^2*x^2*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (2*I)*ArcTan[a*x]])/(1024*a^2*c^2*
(1 + a^2*x^2)*Sqrt[ArcTan[a*x]])
```

## Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {5465, 27, 5427, 5465, 5439, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \arctan(ax)^{5/2}}{(a^2cx^2+c)^2}dx$$

$$\Big\downarrow \text{ 5465}$$

$$\frac{5\int \frac{\arctan(ax)^{3/2}}{c^2(a^2x^2+1)^2}dx}{4a}-\frac{\arctan(ax)^{5/2}}{2a^2c^2(a^2x^2+1)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{5\int \frac{\arctan(ax)^{3/2}}{(a^2x^2+1)^2}dx}{4ac^2}-\frac{\arctan(ax)^{5/2}}{2a^2c^2(a^2x^2+1)}$$

$$\Big\downarrow \text{ 5427}$$

$$\frac{5\left(-\frac{3}{4}a\int \frac{x\sqrt{\arctan(ax)}}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^{3/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{5/2}}{5a}\right)}{4ac^2}-\frac{\arctan(ax)^{5/2}}{2a^2c^2(a^2x^2+1)}$$

$$\Big\downarrow \text{ 5465}$$

$$\frac{5\left(-\frac{3}{4}a\left(\frac{\int \frac{1}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{4a}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^{3/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{5/2}}{5a}\right)}{4ac^2}-\frac{\arctan(ax)^{5/2}}{2a^2c^2\left(a^2x^2+1\right)}$$

$$\Big\downarrow \ 5439$$

$$\frac{5\left(-\frac{3}{4}a\left(\frac{\int \frac{1}{\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}d\arctan(ax)}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^{3/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{5/2}}{5a}\right)}{4ac^2}-\frac{\arctan(ax)^{5/2}}{2a^2c^2\left(a^2x^2+1\right)}$$

$$\Big\downarrow \ 3042$$

$$\frac{5\left(-\frac{3}{4}a\left(\frac{\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^2}{\sqrt{\arctan(ax)}}d\arctan(ax)}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^{3/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{5/2}}{5a}\right)}{4ac^2}-\frac{\arctan(ax)^{5/2}}{2a^2c^2\left(a^2x^2+1\right)}$$

$$\Big\downarrow \ 3793$$

$$\frac{5\left(-\frac{3}{4}a\left(\frac{\int \left(\frac{\cos(2\arctan(ax))}{2\sqrt{\arctan(ax)}}+\frac{1}{2\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^{3/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{5/2}}{5a}\right)}{4ac^2}-\frac{\arctan(ax)^{5/2}}{2a^2c^2\left(a^2x^2+1\right)}$$

$$\Big\downarrow \ 2009$$

$$\frac{5\left(-\frac{3}{4}a\left(\frac{\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\sqrt{\arctan(ax)}}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^{3/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{5/2}}{5a}\right)}{4ac^2}-\frac{\arctan(ax)^{5/2}}{2a^2c^2\left(a^2x^2+1\right)}$$

input
```
Int[(x*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^2,x]
```

output
```
-1/2*ArcTan[a*x]^(5/2)/(a^2*c^2*(1 + a^2*x^2)) + (5*((x*ArcTan[a*x]^(3/2))
/(2*(1 + a^2*x^2)) + ArcTan[a*x]^(5/2)/(5*a) - (3*a*(-1/2*Sqrt[ArcTan[a*x]
]/(a^2*(1 + a^2*x^2)) + (Sqrt[ArcTan[a*x]] + (Sqrt[Pi]*FresnelC[(2*Sqrt[Ar
cTan[a*x]])/Sqrt[Pi]])/2)/(4*a^2)))/4))/(4*a*c^2)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

## Maple [A] (verified)

Time = 96.56 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.53

| method | result |
|---|---|
| default | $-\frac{32\arctan(ax)^{\frac{5}{2}}\sqrt{\pi}\,\cos(2\arctan(ax))-40\arctan(ax)^{\frac{3}{2}}\sqrt{\pi}\,\sin(2\arctan(ax))-30\cos(2\arctan(ax))\sqrt{\arctan(ax)}\,\sqrt{\pi}+15\pi\ \mathrm{Fresn}}{128a^{2}c^{2}\sqrt{\pi}}$ |

input
```
int(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/128/a^2/c^2/Pi^(1/2)*(32*arctan(a*x)^(5/2)*Pi^(1/2)*cos(2*arctan(a*x))-
40*arctan(a*x)^(3/2)*Pi^(1/2)*sin(2*arctan(a*x))-30*cos(2*arctan(a*x))*arc
tan(a*x)^(1/2)*Pi^(1/2)+15*Pi*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2)))
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x\arctan(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{x \arctan(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx = \frac{\int \frac{x \operatorname{atan}^{\frac{5}{2}}(ax)}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

input
```
integrate(x*atan(a*x)**(5/2)/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(x*atan(a*x)**(5/2)/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x \arctan(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F]

$$\int \frac{x \arctan(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx = \int \frac{x \arctan(ax)^{\frac{5}{2}}}{(a^2cx^2+c)^2}\,dx$$

input
```
integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2 + c)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x \arctan(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx = \int \frac{x\,\mathrm{atan}(a\,x)^{5/2}}{(c\,a^2\,x^2+c)^2}\,dx$$

input
```
int((x*atan(a*x)^(5/2))/(c + a^2*c*x^2)^2,x)
```

output
```
int((x*atan(a*x)^(5/2))/(c + a^2*c*x^2)^2, x)
```

## Reduce [F]

$$\int \frac{x \arctan(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx = \frac{16\sqrt{atan\,(ax)}\,atan(ax)^2\,a^2x^2 - 16\sqrt{atan\,(ax)}\,atan(ax)^2 + 40\sqrt{atan\,(ax)}\,atan(a}{}$$

input
```
int(x*atan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x)
```

output
```
(16*sqrt(atan(a*x))*atan(a*x)**2*a**2*x**2 - 16*sqrt(atan(a*x))*atan(a*x)*
*2 + 40*sqrt(atan(a*x))*atan(a*x)*a*x - 30*sqrt(atan(a*x))*a**2*x**2 + 15*
int((sqrt(atan(a*x))*x**2)/(atan(a*x)*a**4*x**4 + 2*atan(a*x)*a**2*x**2 +
atan(a*x)),x)*a**5*x**2 + 15*int((sqrt(atan(a*x))*x**2)/(atan(a*x)*a**4*x*
*4 + 2*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**3)/(64*a**2*c**2*(a**2*x**2
+ 1))
```

# 3.867 $\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 151

$$\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx = -\frac{15x\sqrt{\arctan(ax)}}{32c^2(1+a^2x^2)} - \frac{5\arctan(ax)^{3/2}}{16ac^2} + \frac{5\arctan(ax)^{3/2}}{8ac^2(1+a^2x^2)} + \frac{x\arctan(ax)^{5/2}}{2c^2(1+a^2x^2)} + \frac{\arctan(ax)^{7/2}}{7ac^2} + \frac{15\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{128ac^2}$$

output
```
-15/32*x*arctan(a*x)^(1/2)/c^2/(a^2*x^2+1)-5/16*arctan(a*x)^(3/2)/a/c^2+5/
8*arctan(a*x)^(3/2)/a/c^2/(a^2*x^2+1)+1/2*x*arctan(a*x)^(5/2)/c^2/(a^2*x^2
+1)+1/7*arctan(a*x)^(7/2)/a/c^2+15/128*Pi^(1/2)*FresnelS(2*arctan(a*x)^(1/
2)/Pi^(1/2))/a/c^2
```

## Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.72

$$\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx = \frac{4\sqrt{\arctan(ax)}(-105ax-70(-1+a^2x^2)\arctan(ax)+112ax\arctan(ax)^2+32(1+}{896c^2(a+a^3x^2)}$$

input
```
Integrate[ArcTan[a*x]^(5/2)/(c + a^2*c*x^2)^2,x]
```

output
```
(4*Sqrt[ArcTan[a*x]]*(-105*a*x - 70*(-1 + a^2*x^2)*ArcTan[a*x] + 112*a*x*A
rcTan[a*x]^2 + 32*(1 + a^2*x^2)*ArcTan[a*x]^3) + 105*Sqrt[Pi]*(1 + a^2*x^2
)*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(896*c^2*(a + a^3*x^2))
```

## Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.03, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {5427, 27, 5465, 5427, 5505, 4906, 27, 3042, 3786, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{5/2}}{(a^2cx^2+c)^2}dx$$

$$\Big\downarrow \text{5427}$$

$$-\frac{5}{4}a\int \frac{x\arctan(ax)^{3/2}}{c^2(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^{5/2}}{2c^2(a^2x^2+1)}+\frac{\arctan(ax)^{7/2}}{7ac^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{5a\int \frac{x\arctan(ax)^{3/2}}{(a^2x^2+1)^2}dx}{4c^2}+\frac{x\arctan(ax)^{5/2}}{2c^2(a^2x^2+1)}+\frac{\arctan(ax)^{7/2}}{7ac^2}$$

$$\Big\downarrow \text{5465}$$

$$-\frac{5a\left(\frac{3\int \frac{\sqrt{\arctan(ax)}}{(a^2x^2+1)^2}dx}{4a}-\frac{\arctan(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)}{4c^2}+\frac{x\arctan(ax)^{5/2}}{2c^2(a^2x^2+1)}+\frac{\arctan(ax)^{7/2}}{7ac^2}$$

$$\Big\downarrow \text{5427}$$

$$-\frac{5a\left(\frac{3\left(-\frac{1}{4}a\int \frac{x}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx+\frac{x\sqrt{\arctan(ax)}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4a}-\frac{\arctan(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)}{4c^2}+\frac{x\arctan(ax)^{5/2}}{2c^2(a^2x^2+1)}+\frac{\arctan(ax)^{7/2}}{7ac^2}$$

$$\Big\downarrow \text{5505}$$

$$-\frac{5a\left(\frac{3\left(-\frac{\int \frac{ax}{\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}d\arctan(ax)}{4a}+\frac{x\sqrt{\arctan(ax)}}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4a}-\frac{\arctan(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)}{4c^2}+\frac{x\arctan(ax)^{5/2}}{2c^2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^{7/2}}{7ac^2}$$

$$\Big\downarrow \textbf{4906}$$

$$-\frac{5a\left(\frac{3\left(-\frac{\int \frac{\sin(2\arctan(ax))}{2\sqrt{\arctan(ax)}}d\arctan(ax)}{4a}+\frac{x\sqrt{\arctan(ax)}}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4a}-\frac{\arctan(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)}{4c^2}+\frac{x\arctan(ax)^{5/2}}{2c^2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^{7/2}}{7ac^2}$$

$$\Big\downarrow \textbf{27}$$

$$-\frac{5a\left(\frac{3\left(-\frac{\int \frac{\sin(2\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)}{8a}+\frac{x\sqrt{\arctan(ax)}}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4a}-\frac{\arctan(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)}{4c^2}+\frac{x\arctan(ax)^{5/2}}{2c^2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^{7/2}}{7ac^2}$$

$$\Big\downarrow \textbf{3042}$$

$$-\frac{5a\left(\frac{3\left(-\frac{\int \frac{\sin(2\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)}{8a}+\frac{x\sqrt{\arctan(ax)}}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4a}-\frac{\arctan(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)}{4c^2}+\frac{x\arctan(ax)^{5/2}}{2c^2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^{7/2}}{7ac^2}$$

$$\Big\downarrow \textbf{3786}$$

$$-\frac{5a\left(\frac{3\left(-\frac{\int \sin(2\arctan(ax))d\sqrt{\arctan(ax)}}{4a}+\frac{x\sqrt{\arctan(ax)}}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4a}-\frac{\arctan(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)}{4c^2}+\frac{x\arctan(ax)^{5/2}}{2c^2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^{7/2}}{7ac^2}$$

$$\Big\downarrow \text{3832}$$

$$-\frac{5a\left(\frac{3\left(\frac{x\sqrt{\arctan(ax)}}{2(a^2x^2+1)}-\frac{\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{8a}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4a}-\frac{\arctan(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)}{4c^2}+\frac{x\arctan(ax)^{5/2}}{2c^2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^{7/2}}{7ac^2}$$

input
```
Int[ArcTan[a*x]^(5/2)/(c + a^2*c*x^2)^2,x]
```

output
```
(x*ArcTan[a*x]^(5/2))/(2*c^2*(1 + a^2*x^2)) + ArcTan[a*x]^(7/2)/(7*a*c^2)
- (5*a*(-1/2*ArcTan[a*x]^(3/2)/(a^2*(1 + a^2*x^2)) + (3*((x*Sqrt[ArcTan[a*
x]])/(2*(1 + a^2*x^2)) + ArcTan[a*x]^(3/2)/(3*a) - (Sqrt[Pi]*FresnelS[(2*S
qrt[ArcTan[a*x]])/Sqrt[Pi]])/(8*a)))/(4*a)))/(4*c^2)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

## Maple [A] (verified)

Time = 96.92 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.62

| method | result |
|---|---|
| default | $\frac{128\arctan(ax)^{\frac{7}{2}}\sqrt{\pi}+224\arctan(ax)^{\frac{5}{2}}\sqrt{\pi}\,\sin(2\arctan(ax))+280\arctan(ax)^{\frac{3}{2}}\sqrt{\pi}\,\cos(2\arctan(ax))-210\sqrt{\arctan(ax)}\,\sqrt{\pi}\,\sin(2\,\mathrm{a}}{896a\,c^{2}\sqrt{\pi}}$ |

input
```
int(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/896/a/c^2/Pi^(1/2)*(128*arctan(a*x)^(7/2)*Pi^(1/2)+224*arctan(a*x)^(5/2)
*Pi^(1/2)*sin(2*arctan(a*x))+280*arctan(a*x)^(3/2)*Pi^(1/2)*cos(2*arctan(a
*x))-210*arctan(a*x)^(1/2)*Pi^(1/2)*sin(2*arctan(a*x))+105*Pi*FresnelS(2*a
rctan(a*x)^(1/2)/Pi^(1/2)))
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx = \frac{\int \frac{\operatorname{atan}^{\frac{5}{2}}(ax)}{a^4x^4+2a^2x^2+1}\,dx}{c^2}$$

input
```
integrate(atan(a*x)**(5/2)/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(atan(a*x)**(5/2)/(a**4*x**4 + 2*a**2*x**2 + 1), x)/c**2
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F]

$$\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx = \int \frac{\arctan(ax)^{\frac{5}{2}}}{(a^2cx^2+c)^2}\,dx$$

input
```
integrate(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^(5/2)/(a^2*c*x^2 + c)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx = \int \frac{\mathrm{atan}(a\,x)^{5/2}}{(c\,a^2\,x^2+c)^2}\,dx$$

input
```
int(atan(a*x)^(5/2)/(c + a^2*c*x^2)^2,x)
```

output
```
int(atan(a*x)^(5/2)/(c + a^2*c*x^2)^2, x)
```

## Reduce [F]

$$\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^2}\,dx = \frac{16\sqrt{atan(ax)}\,atan(ax)^3\,a^2x^2 + 16\sqrt{atan(ax)}\,atan(ax)^3 + 56\sqrt{atan(ax)}\,atan(ax)}{112a\,c}$$

input
```
int(atan(a*x)^(5/2)/(a^2*c*x^2+c)^2,x)
```

output
```
(16*sqrt(atan(a*x))*atan(a*x)**3*a**2*x**2 + 16*sqrt(atan(a*x))*atan(a*x)*
*3 + 56*sqrt(atan(a*x))*atan(a*x)**2*a*x + 70*sqrt(atan(a*x))*atan(a*x) -
105*int(sqrt(atan(a*x))/(a**4*x**4 + 2*a**2*x**2 + 1),x)*a**3*x**2 - 105*i
nt(sqrt(atan(a*x))/(a**4*x**4 + 2*a**2*x**2 + 1),x)*a)/(112*a*c**2*(a**2*x
**2 + 1))
```

# 3.868 $\int \frac{\arctan(ax)^{5/2}}{x(c+a^2cx^2)^2} dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\arctan(ax)^{5/2}}{x\,(c+a^2cx^2)^2}\,dx=\operatorname{Int}\left(\frac{\arctan(ax)^{5/2}}{x\,(c+a^2cx^2)^2},x\right)$$

output
```
Defer(Int)(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^2,x)
```

## Mathematica [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\arctan(ax)^{5/2}}{x\,(c+a^2cx^2)^2}\,dx=\int \frac{\arctan(ax)^{5/2}}{x\,(c+a^2cx^2)^2}\,dx$$

input
```
Integrate[ArcTan[a*x]^(5/2)/(x*(c + a^2*c*x^2)^2),x]
```

output
```
Integrate[ArcTan[a*x]^(5/2)/(x*(c + a^2*c*x^2)^2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{5/2}}{x\left(a^2cx^2+c\right)^2}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{\arctan(ax)^{5/2}}{x\left(a^2cx^2+c\right)^2}dx$$

input
```
Int[ArcTan[a*x]^(5/2)/(x*(c + a^2*c*x^2)^2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\arctan\left(ax\right)^{\frac{5}{2}}}{x\left(a^2c\,x^2+c\right)^2}dx$$

input
```
int(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^2,x)
```

output
```
int(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^2,x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{5/2}}{x\,(c+a^2cx^2)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 3.94 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int \frac{\arctan(ax)^{5/2}}{x\,(c+a^2cx^2)^2}\,dx = \frac{\int \frac{\operatorname{atan}^{\frac{5}{2}}(ax)}{a^4x^5+2a^2x^3+x}\,dx}{c^2}$$

input
```
integrate(atan(a*x)**(5/2)/x/(a**2*c*x**2+c)**2,x)
```

output
```
Integral(atan(a*x)**(5/2)/(a**4*x**5 + 2*a**2*x**3 + x), x)/c**2
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{5/2}}{x\,(c+a^2cx^2)^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^2,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax)^{5/2}}{x\left(c+a^2cx^2\right)^2}\,dx=\int \frac{\arctan\left(ax\right)^{\frac{5}{2}}}{\left(a^2cx^2+c\right)^2x}\,dx$$

input
```
integrate(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^2,x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^(5/2)/((a^2*c*x^2 + c)^2*x), x)
```

## Mupad [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax)^{5/2}}{x\left(c+a^2cx^2\right)^2}\,dx=\int \frac{\mathrm{atan}\left(a\,x\right)^{5/2}}{x\,\left(c\,a^2\,x^2+c\right)^2}\,dx$$

input
```
int(atan(a*x)^(5/2)/(x*(c + a^2*c*x^2)^2),x)
```

output
```
int(atan(a*x)^(5/2)/(x*(c + a^2*c*x^2)^2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{\arctan(ax)^{5/2}}{x\left(c+a^2cx^2\right)^2}\,dx = \frac{\int \frac{\sqrt{atan(ax)}\,atan(ax)^2}{a^4x^5+2a^2x^3+x}dx}{c^2}$$

input
```
int(atan(a*x)^(5/2)/x/(a^2*c*x^2+c)^2,x)
```

output
```
int((sqrt(atan(a*x))*atan(a*x)**2)/(a**4*x**5 + 2*a**2*x**3 + x),x)/c**2
```

# 3.869 $\int \frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^3} dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^3} dx = \text{Int}\left(\frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^3}, x\right)$$

output
```
Defer(Int)(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x)
```

## Mathematica [N/A]

Not integrable

Time = 1.32 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^3} dx = \int \frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^3} dx$$

input
```
Integrate[(x^m*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^3,x]
```

output
```
Integrate[(x^m*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^3, x]
```

## Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \arctan(ax)^{5/2}}{(a^2cx^2+c)^3} dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{x^m \arctan(ax)^{5/2}}{(a^2cx^2+c)^3} dx$$

input
```
Int[(x^m*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^3,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m \arctan(ax)^{\frac{5}{2}}}{(a^2c\,x^2+c)^3} dx$$

input
```
int(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x)
```

output
```
int(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.08

$$\int \frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx = \int \frac{x^m \arctan(ax)^{\frac{5}{2}}}{(a^2cx^2+c)^3}\,dx$$

input
```
integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
integral(x^m*arctan(a*x)^(5/2)/(a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^
2 + c^3), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**m*atan(a*x)**(5/2)/(a**2*c*x**2+c)**3,x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,2,1,0,0]%%%} / %%%{1,[0,0,0,1,3]%%%} Error: Bad Argum
ent Value
```

## Mupad [N/A]

Not integrable

Time = 0.83 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx = \int \frac{x^m\,\mathrm{atan}(a\,x)^{5/2}}{{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

input
```
int((x^m*atan(a*x)^(5/2))/(c + a^2*c*x^2)^3,x)
```

output
```
int((x^m*atan(a*x)^(5/2))/(c + a^2*c*x^2)^3, x)
```

## Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.00

$$\int \frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx = \frac{\int \frac{x^m \sqrt{atan(ax)}\,atan(ax)^2}{a^6x^6+3a^4x^4+3a^2x^2+1}dx}{c^3}$$

input
```
int(x^m*atan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x)
```

output
```
int((x**m*sqrt(atan(a*x))*atan(a*x)**2)/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*
x**2 + 1),x)/c**3
```

# 3.870 $\int \frac{x^5 \arctan(ax)^{5/2}}{(c+a^2cx^2)^3} dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^5 \arctan(ax)^{5/2}}{(c+a^2cx^2)^3} dx = \text{Int}\left(\frac{x^5 \arctan(ax)^{5/2}}{(c+a^2cx^2)^3}, x\right)$$

output
```
Defer(Int)(x^5*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x)
```

## Mathematica [N/A]

Not integrable

Time = 6.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^5 \arctan(ax)^{5/2}}{(c+a^2cx^2)^3} dx = \int \frac{x^5 \arctan(ax)^{5/2}}{(c+a^2cx^2)^3} dx$$

input
```
Integrate[(x^5*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^3,x]
```

output
```
Integrate[(x^5*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^3, x]
```

## Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 \arctan(ax)^{5/2}}{(a^2cx^2+c)^3} \, dx$$

$$\Big\downarrow \text{5560}$$

$$\int \frac{x^5 \arctan(ax)^{5/2}}{(a^2cx^2+c)^3}dx$$

input
```
Int[(x^5*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^3,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^5 \arctan \left(a x\right)^{\frac{5}{2}}}{\left(a^2 c \,x^2+c\right)^3}dx$$

input
```
int(x^5*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x)
```

output
```
int(x^5*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x^5 \arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^5*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 24.45 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.71

$$\int \frac{x^5 \arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx = \frac{\int \frac{x^5 \operatorname{atan}^{\frac{5}{2}}(ax)}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

input
```
integrate(x**5*atan(a*x)**(5/2)/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(x**5*atan(a*x)**(5/2)/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1)
, x)/c**3
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5 \arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^5*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 2.69 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^5 \arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx = \int \frac{x^5 \arctan{(ax)}^{\frac{5}{2}}}{(a^2cx^2+c)^3}\,dx$$

input
```
integrate(x^5*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
integrate(x^5*arctan(a*x)^(5/2)/(a^2*c*x^2 + c)^3, x)
```

## Mupad [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^5 \arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx = \int \frac{x^5\,\mathrm{atan}(a\,x)^{5/2}}{(c\,a^2\,x^2+c)^3}\,dx$$

input
```
int((x^5*atan(a*x)^(5/2))/(c + a^2*c*x^2)^3,x)
```

output
```
int((x^5*atan(a*x)^(5/2))/(c + a^2*c*x^2)^3, x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.00

$$\int \frac{x^5 \arctan(ax)^{5/2}}{(c + a^2cx^2)^3} \, dx = \frac{\int \frac{\sqrt{atan(ax)}\, atan(ax)^2 x^5}{a^6x^6+3a^4x^4+3a^2x^2+1} dx}{c^3}$$

input
```
int(x^5*atan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x)
```

output
```
int((sqrt(atan(a*x))*atan(a*x)**2*x**5)/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*
x**2 + 1),x)/c**3
```

# 3.871 $\int \frac{x^4 \arctan(ax)^{5/2}}{(c+a^2cx^2)^3} dx$



## Optimal result

Integrand size = 24, antiderivative size = 310

$$\begin{aligned}\int \frac{x^4 \arctan(ax)^{5/2}}{(c+a^2cx^2)^3} dx &= \frac{45x\sqrt{\arctan(ax)}}{128a^4c^3(1+a^2x^2)} \\ &+ \frac{45\arctan(ax)^{3/2}}{256a^5c^3} + \frac{5x^4\arctan(ax)^{3/2}}{32ac^3(1+a^2x^2)^2} - \frac{15\arctan(ax)^{3/2}}{32a^5c^3(1+a^2x^2)} \\ &- \frac{x^3\arctan(ax)^{5/2}}{4a^2c^3(1+a^2x^2)^2} - \frac{3x\arctan(ax)^{5/2}}{8a^4c^3(1+a^2x^2)} + \frac{3\arctan(ax)^{7/2}}{28a^5c^3} \\ &+ \frac{15\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{4096a^5c^3} - \frac{15\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{128a^5c^3} \\ &+ \frac{15\sqrt{\arctan(ax)}\sin(2\arctan(ax))}{256a^5c^3} - \frac{15\sqrt{\arctan(ax)}\sin(4\arctan(ax))}{2048a^5c^3}\end{aligned}$$

output

```
45/128*x*arctan(a*x)^(1/2)/a^4/c^3/(a^2*x^2+1)+45/256*arctan(a*x)^(3/2)/a^
5/c^3+5/32*x^4*arctan(a*x)^(3/2)/a/c^3/(a^2*x^2+1)^2-15/32*arctan(a*x)^(3/
2)/a^5/c^3/(a^2*x^2+1)-1/4*x^3*arctan(a*x)^(5/2)/a^2/c^3/(a^2*x^2+1)^2-3/8
*x*arctan(a*x)^(5/2)/a^4/c^3/(a^2*x^2+1)+3/28*arctan(a*x)^(7/2)/a^5/c^3+15
/8192*2^(1/2)*Pi^(1/2)*FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))/a^5/
c^3-15/128*Pi^(1/2)*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2))/a^5/c^3+15/256*
arctan(a*x)^(1/2)*sin(2*arctan(a*x))/a^5/c^3-15/2048*arctan(a*x)^(1/2)*sin
(4*arctan(a*x))/a^5/c^3
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.93

$$\int \frac{x^4 \arctan(ax)^{5/2}}{(c+a^2cx^2)^3} dx = \frac{50400ax \arctan(ax) + 57120a^3x^3 \arctan(ax) - 33600 \arctan(ax)^2 - 13440a^2x^2 \text{a}}{}$$

input
```
Integrate[(x^4*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^3,x]
```

output
```
(50400*a*x*ArcTan[a*x] + 57120*a^3*x^3*ArcTan[a*x] - 33600*ArcTan[a*x]^2 -
 13440*a^2*x^2*ArcTan[a*x]^2 + 38080*a^4*x^4*ArcTan[a*x]^2 - 43008*a*x*Arc
Tan[a*x]^3 - 71680*a^3*x^3*ArcTan[a*x]^3 + 12288*(1 + a^2*x^2)^2*ArcTan[a*
x]^4 + 3360*Sqrt[2]*(1 + a^2*x^2)^2*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-2*
I)*ArcTan[a*x]] + 3360*Sqrt[2]*(1 + a^2*x^2)^2*Sqrt[I*ArcTan[a*x]]*Gamma[1
/2, (2*I)*ArcTan[a*x]] - 105*(1 + a^2*x^2)^2*Sqrt[(-I)*ArcTan[a*x]]*Gamma[
1/2, (-4*I)*ArcTan[a*x]] - 105*(1 + a^2*x^2)^2*Sqrt[I*ArcTan[a*x]]*Gamma[1
/2, (4*I)*ArcTan[a*x]])/(114688*a^5*c^3*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]])
```

## Rubi [A] (verified)

Time = 1.83 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.12, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {5475, 27, 5471, 5465, 5427, 5505, 3042, 3793, 2009, 4906, 27, 3042, 3786, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \arctan(ax)^{5/2}}{(a^2cx^2+c)^3} dx$$

$$\Big\downarrow \text{5475}$$

$$-\frac{15}{64} \int \frac{x^4 \sqrt{\arctan(ax)}}{c^3 (a^2x^2+1)^3} dx + \frac{3 \int \frac{x^2 \arctan(ax)^{5/2}}{c^2(a^2x^2+1)^2} dx}{4a^2c} + \frac{5x^4 \arctan(ax)^{3/2}}{32ac^3 (a^2x^2+1)^2} - \frac{x^3 \arctan(ax)^{5/2}}{4a^2c^3 (a^2x^2+1)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\int \frac{x^2\arctan(ax)^{5/2}}{(a^2x^2+1)^2}dx}{4a^2c^3}-\frac{15\int \frac{x^4\sqrt{\arctan(ax)}}{(a^2x^2+1)^3}dx}{64c^3}+\frac{5x^4\arctan(ax)^{3/2}}{32ac^3\left(a^2x^2+1\right)^2}-\frac{x^3\arctan(ax)^{5/2}}{4a^2c^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \text{5471}$$

$$-\frac{15\int \frac{x^4\sqrt{\arctan(ax)}}{(a^2x^2+1)^3}dx}{64c^3}+\frac{3\left(\frac{5\int \frac{x\arctan(ax)^{3/2}}{\left(a^2x^2+1\right)^2}dx}{4a}+\frac{\arctan(ax)^{7/2}}{7a^3}-\frac{x\arctan(ax)^{5/2}}{2a^2(a^2x^2+1)}\right)}{4a^2c^3}+\frac{5x^4\arctan(ax)^{3/2}}{32ac^3\left(a^2x^2+1\right)^2}-\frac{x^3\arctan(ax)^{5/2}}{4a^2c^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \text{5465}$$

$$-\frac{15\int \frac{x^4\sqrt{\arctan(ax)}}{(a^2x^2+1)^3}dx}{64c^3}+\frac{3\left(\frac{5\left(\frac{3\int \frac{\sqrt{\arctan(ax)}}{\left(a^2x^2+1\right)^2}dx}{4a}-\frac{\arctan(ax)^{3/2}}{2a^2\left(a^2x^2+1\right)}\right)}{4a}+\frac{\arctan(ax)^{7/2}}{7a^3}-\frac{x\arctan(ax)^{5/2}}{2a^2(a^2x^2+1)}\right)}{4a^2c^3}+\frac{5x^4\arctan(ax)^{3/2}}{32ac^3\left(a^2x^2+1\right)^2}-\frac{x^3\arctan(ax)^{5/2}}{4a^2c^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \text{5427}$$

$$-\frac{15\int \frac{x^4\sqrt{\arctan(ax)}}{(a^2x^2+1)^3}dx}{64c^3}+\frac{3\left(\frac{5\left(\frac{3\left(-\frac{1}{4}a\int \frac{x}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx+\frac{x\sqrt{\arctan(ax)}}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4a}-\frac{\arctan(ax)^{3/2}}{2a^2\left(a^2x^2+1\right)}\right)}{4a}+\frac{\arctan(ax)^{7/2}}{7a^3}-\frac{x\arctan(ax)^{5/2}}{2a^2(a^2x^2+1)}\right)}{4a^2c^3}+\frac{5x^4\arctan(ax)^{3/2}}{32ac^3\left(a^2x^2+1\right)^2}-\frac{x^3\arctan(ax)^{5/2}}{4a^2c^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \text{5505}$$

$$\frac{3\left(\frac{5\left(\frac{3\left(-\frac{\int \frac{ax}{(a^2x^2+1)\sqrt{\arctan(ax)}}d\arctan(ax)}{4a}+\frac{x\sqrt{\arctan(ax)}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4a}-\frac{\arctan(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)}{4a}+\frac{\arctan(ax)^{7/2}}{7a^3}-\frac{x\arctan(ax)^{5/2}}{2a^2(a^2x^2+1)}\right)}{4a^2c^3}-$$

$$\frac{15\int \frac{a^4x^4\sqrt{\arctan(ax)}}{(a^2x^2+1)^2}d\arctan(ax)}{64a^5c^3}+\frac{5x^4\arctan(ax)^{3/2}}{32ac^3\,(a^2x^2+1)^2}-\frac{x^3\arctan(ax)^{5/2}}{4a^2c^3\,(a^2x^2+1)^2}$$

$\downarrow$ 3042

$$-\frac{15\int \sqrt{\arctan(ax)}\sin(\arctan(ax))^4d\arctan(ax)}{64a^5c^3}+$$

$$\frac{3\left(\frac{5\left(\frac{3\left(-\frac{\int \frac{ax}{(a^2x^2+1)\sqrt{\arctan(ax)}}d\arctan(ax)}{4a}+\frac{x\sqrt{\arctan(ax)}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4a}-\frac{\arctan(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)}{4a}+\frac{\arctan(ax)^{7/2}}{7a^3}-\frac{x\arctan(ax)^{5/2}}{2a^2(a^2x^2+1)}\right)}{4a^2c^3}+$$

$$\frac{5x^4\arctan(ax)^{3/2}}{32ac^3\,(a^2x^2+1)^2}-\frac{x^3\arctan(ax)^{5/2}}{4a^2c^3\,(a^2x^2+1)^2}$$

$\downarrow$ 3793

$$-\frac{15\int \left(-\frac{1}{2}\sqrt{\arctan(ax)}\cos(2\arctan(ax))+\frac{1}{8}\sqrt{\arctan(ax)}\cos(4\arctan(ax))+\frac{3}{8}\sqrt{\arctan(ax)}\right)d\arctan(ax)}{64a^5c^3}+\frac{3\left(\frac{5\left(\frac{3\left(-\frac{\int \frac{ax}{(a^2x^2+1)\sqrt{\arctan(ax)}}d\arctan(ax)}{4a}+\frac{x\sqrt{\arctan(ax)}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4a}-\frac{\arctan(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)}{4a}+\frac{\arctan(ax)^{7/2}}{7a^3}-\frac{x\arctan(ax)^{5/2}}{2a^2(a^2x^2+1)}\right)}{4a^2c^3}+\frac{5x^4\arctan(ax)^{3/2}}{32ac^3(a^2x^2+1)^2}-\frac{x^3\arctan(ax)^{5/2}}{4a^2c^3(a^2x^2+1)^2}$$

$\downarrow$ 2009

$$\frac{3\left(\frac{5\left(\frac{3\left(-\frac{\int \frac{ax}{(a^2x^2+1)\sqrt{\arctan(ax)}}d\arctan(ax)}{4a}+\frac{x\sqrt{\arctan(ax)}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4a}-\frac{\arctan(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)}{4a}+\frac{\arctan(ax)^{7/2}}{7a^3}-\frac{x\arctan(ax)^{5/2}}{2a^2(a^2x^2+1)}\right)}{4a^2c^3}-\frac{15\left(-\frac{1}{64}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{8}\sqrt{\pi}\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{1}{4}\arctan(ax)^{3/2}-\frac{1}{4}\sqrt{\arctan(ax)}\,\text{si}}{64a^5c^3}+\frac{5x^4\arctan(ax)^{3/2}}{32ac^3(a^2x^2+1)^2}-\frac{x^3\arctan(ax)^{5/2}}{4a^2c^3(a^2x^2+1)^2}$$

$\downarrow$ 4906

$$\frac{3\left(\frac{5\left(\frac{3\left(-\frac{\int \frac{\sin(2\arctan(ax))}{2\sqrt{\arctan(ax)}}d\arctan(ax)}{4a}+\frac{x\sqrt{\arctan(ax)}}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4a}-\frac{\arctan(ax)^{3/2}}{2a^2\left(a^2x^2+1\right)}\right)}{4a}+\frac{\arctan(ax)^{7/2}}{7a^3}-\frac{x\arctan(ax)^{5/2}}{2a^2(a^2x^2+1)}\right)}{4a^2c^3}-$$

$$\frac{15\left(-\frac{1}{64}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{8}\sqrt{\pi}\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{1}{4}\arctan(ax)^{3/2}-\frac{1}{4}\sqrt{\arctan(ax)}\text{si}}{64a^5c^3}$$

$$\frac{5x^4\arctan(ax)^{3/2}}{32ac^3\left(a^2x^2+1\right)^2}-\frac{x^3\arctan(ax)^{5/2}}{4a^2c^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{3\left(\frac{5\left(\frac{3\left(-\frac{\int \frac{\sin(2\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)}{8a}+\frac{x\sqrt{\arctan(ax)}}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4a}-\frac{\arctan(ax)^{3/2}}{2a^2\left(a^2x^2+1\right)}\right)}{4a}+\frac{\arctan(ax)^{7/2}}{7a^3}-\frac{x\arctan(ax)^{5/2}}{2a^2(a^2x^2+1)}\right)}{4a^2c^3}-$$

$$\frac{15\left(-\frac{1}{64}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{8}\sqrt{\pi}\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{1}{4}\arctan(ax)^{3/2}-\frac{1}{4}\sqrt{\arctan(ax)}\text{si}}{64a^5c^3}$$

$$\frac{5x^4\arctan(ax)^{3/2}}{32ac^3\left(a^2x^2+1\right)^2}-\frac{x^3\arctan(ax)^{5/2}}{4a^2c^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{5\left(\frac{3\left(-\frac{\int \frac{\sin(2\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)}{8a}+\frac{x\sqrt{\arctan(ax)}}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4a}-\frac{\arctan(ax)^{3/2}}{2a^2\left(a^2x^2+1\right)}\right)}{4a}+\frac{\arctan(ax)^{7/2}}{7a^3}-\frac{x\arctan(ax)^{5/2}}{2a^2(a^2x^2+1)}\right)}{4a^2c^3}-$$

$$\frac{15\left(-\frac{1}{64}\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{8}\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{1}{4}\arctan(ax)^{3/2}-\frac{1}{4}\sqrt{\arctan(ax)}\,\text{si}\right.}{64a^5c^3}$$

$$\frac{5x^4\arctan(ax)^{3/2}}{32ac^3\left(a^2x^2+1\right)^2}-\frac{x^3\arctan(ax)^{5/2}}{4a^2c^3\left(a^2x^2+1\right)^2}$$

$\downarrow$ 3786

$$\frac{3\left(\frac{5\left(\frac{3\left(-\frac{\int \sin(2\arctan(ax))d\sqrt{\arctan(ax)}}{4a}+\frac{x\sqrt{\arctan(ax)}}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4a}-\frac{\arctan(ax)^{3/2}}{2a^2\left(a^2x^2+1\right)}\right)}{4a}+\frac{\arctan(ax)^{7/2}}{7a^3}-\frac{x\arctan(ax)^{5/2}}{2a^2(a^2x^2+1)}\right)}{4a^2c^3}-$$

$$\frac{15\left(-\frac{1}{64}\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{8}\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{1}{4}\arctan(ax)^{3/2}-\frac{1}{4}\sqrt{\arctan(ax)}\,\text{si}\right.}{64a^5c^3}$$

$$\frac{5x^4\arctan(ax)^{3/2}}{32ac^3\left(a^2x^2+1\right)^2}-\frac{x^3\arctan(ax)^{5/2}}{4a^2c^3\left(a^2x^2+1\right)^2}$$

$\downarrow$ 3832

$$-\frac{15\left(-\frac{1}{64}\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{8}\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{1}{4}\arctan(ax)^{3/2}-\frac{1}{4}\sqrt{\arctan(ax)}\right.}{64a^5c^3}$$

$$\frac{5x^4\arctan(ax)^{3/2}}{32ac^3\left(a^2x^2+1\right)^2}-\frac{x^3\arctan(ax)^{5/2}}{4a^2c^3\left(a^2x^2+1\right)^2}+\frac{3\left(\frac{\arctan(ax)^{7/2}}{7a^3}+\frac{5\left(\frac{3\left(\frac{x\sqrt{\arctan(ax)}}{2\left(a^2x^2+1\right)}-\frac{\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{8a}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4a}-\frac{\arctan(ax)^{3/2}}{2a^2\left(a^2x^2+1\right)}\right)}{4a}-\frac{x\arctan(ax)^{5/2}}{2a^2(a^2x^2+1)}\right)}{4a^2c^3}$$

input
```
Int[(x^4*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^3,x]
```

output
```
(5*x^4*ArcTan[a*x]^(3/2))/(32*a*c^3*(1 + a^2*x^2)^2) - (x^3*ArcTan[a*x]^(5
/2))/(4*a^2*c^3*(1 + a^2*x^2)^2) + (3*(-1/2*(x*ArcTan[a*x]^(5/2))/(a^2*(1
+ a^2*x^2)) + ArcTan[a*x]^(7/2)/(7*a^3) + (5*(-1/2*ArcTan[a*x]^(3/2)/(a^2*
(1 + a^2*x^2)) + (3*((x*Sqrt[ArcTan[a*x]])/(2*(1 + a^2*x^2)) + ArcTan[a*x]
^(3/2)/(3*a) - (Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(8*a)))
/(4*a)))/(4*a)))/(4*a^2*c^3) - (15*(ArcTan[a*x]^(3/2)/4 - (Sqrt[Pi/2]*Fres
nelS[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/64 + (Sqrt[Pi]*FresnelS[(2*Sqrt[ArcT
an[a*x]])/Sqrt[Pi]])/8 - (Sqrt[ArcTan[a*x]]*Sin[2*ArcTan[a*x]])/4 + (Sqrt[
ArcTan[a*x]]*Sin[4*ArcTan[a*x]])/32))/(64*a^5*c^3)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5471
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^2)/((d_) + (e_.)*(x_)^2)
^2, x_Symbol] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(2*b*c^3*d^2*(p + 1)), x]
 + (-Simp[x*((a + b*ArcTan[c*x])^p/(2*c^2*d*(d + e*x^2))), x] + Simp[b*(p/(
2*c))   Int[x*((a + b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5475
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_)*((d_) + (e_.)
*(x_)^2)^(q_), x_Symbol] :> Simp[b*p*(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*Ar
cTan[c*x])^(p - 1)/(c*d*m^2)), x] + (-Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(q +
 1)*((a + b*ArcTan[c*x])^p/(c^2*d*m)), x] + Simp[f^2*((m - 1)/(c^2*d*m))
Int[(f*x)^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p, x], x] - Simp[
b^2*p*((p - 1)/m^2)   Int[(f*x)^m*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2)
, x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && EqQ[m + 2*
q + 2, 0] && LtQ[q, -1] && GtQ[p, 1]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

## Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.54

$$6144 \arctan (ax)^{\frac{7}{2}} \sqrt{\pi} - 14336 \arctan (ax)^{\frac{5}{2}} \sqrt{\pi} \sin (2 \arctan (ax)) + 1792 \arctan (ax)^{\frac{5}{2}} \sqrt{\pi} \sin (4 \arctan ($$

input
```
int(x^4*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x)
```

output
```
1/57344/a^5/c^3*(6144*arctan(a*x)^(7/2)*Pi^(1/2)-14336*arctan(a*x)^(5/2)*P
i^(1/2)*sin(2*arctan(a*x))+1792*arctan(a*x)^(5/2)*Pi^(1/2)*sin(4*arctan(a*
x))-17920*arctan(a*x)^(3/2)*Pi^(1/2)*cos(2*arctan(a*x))+1120*arctan(a*x)^(
3/2)*Pi^(1/2)*cos(4*arctan(a*x))+105*Pi*2^(1/2)*FresnelS(2*2^(1/2)/Pi^(1/2
)*arctan(a*x)^(1/2))+13440*arctan(a*x)^(1/2)*Pi^(1/2)*sin(2*arctan(a*x))-4
20*arctan(a*x)^(1/2)*Pi^(1/2)*sin(4*arctan(a*x))-6720*Pi*FresnelS(2*arctan
(a*x)^(1/2)/Pi^(1/2)))/Pi^(1/2)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x^4 \arctan(ax)^{5/2}}{(c+a^2cx^2)^3} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^4*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{x^4 \arctan(ax)^{5/2}}{(c+a^2cx^2)^3} \, dx = \frac{\int \frac{x^4 \operatorname{atan}^{\frac{5}{2}}(ax)}{a^6x^6+3a^4x^4+3a^2x^2+1} \, dx}{c^3}$$

input
```
integrate(x**4*atan(a*x)**(5/2)/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(x**4*atan(a*x)**(5/2)/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1)
, x)/c**3
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4 \arctan(ax)^{5/2}}{\left(c+a^2cx^2\right)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^4*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F]

$$\int \frac{x^4 \arctan(ax)^{5/2}}{\left(c+a^2cx^2\right)^3}\,dx = \int \frac{x^4 \arctan(ax)^{\frac{5}{2}}}{\left(a^2cx^2+c\right)^3}\,dx$$

input
```
integrate(x^4*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
integrate(x^4*arctan(a*x)^(5/2)/(a^2*c*x^2 + c)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^4 \arctan(ax)^{5/2}}{\left(c+a^2cx^2\right)^3}\,dx = \int \frac{x^4\,\mathrm{atan}(a\,x)^{5/2}}{\left(c\,a^2\,x^2+c\right)^3}\,dx$$

input
```
int((x^4*atan(a*x)^(5/2))/(c + a^2*c*x^2)^3,x)
```

output
```
int((x^4*atan(a*x)^(5/2))/(c + a^2*c*x^2)^3, x)
```

## Reduce [F]

$$\int \frac{x^4 \arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx = \frac{96\sqrt{atan\,(ax)}\,atan(ax)^3\,a^4x^4 + 192\sqrt{atan\,(ax)}\,atan(ax)^3\,a^2x^2 + 96\sqrt{atan\,(ax)}}{}$$

input
```
int(x^4*atan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x)
```

output
```
(96*sqrt(atan(a*x))*atan(a*x)**3*a**4*x**4 + 192*sqrt(atan(a*x))*atan(a*x)
**3*a**2*x**2 + 96*sqrt(atan(a*x))*atan(a*x)**3 - 560*sqrt(atan(a*x))*atan
(a*x)**2*a**3*x**3 - 336*sqrt(atan(a*x))*atan(a*x)**2*a*x - 700*sqrt(atan(
a*x))*atan(a*x)*a**2*x**2 - 560*sqrt(atan(a*x))*atan(a*x) - 350*sqrt(atan(
a*x))*a*x + 1190*int(sqrt(atan(a*x))/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**
2 + 1),x)*a**5*x**4 + 2380*int(sqrt(atan(a*x))/(a**6*x**6 + 3*a**4*x**4 +
3*a**2*x**2 + 1),x)*a**3*x**2 + 1190*int(sqrt(atan(a*x))/(a**6*x**6 + 3*a*
*4*x**4 + 3*a**2*x**2 + 1),x)*a + 175*int((sqrt(atan(a*x))*x)/(atan(a*x)*a
**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a
**6*x**4 + 350*int((sqrt(atan(a*x))*x)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*
a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**4*x**2 + 175*int((sqr
t(atan(a*x))*x)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)
*a**2*x**2 + atan(a*x)),x)*a**2)/(896*a**5*c**3*(a**4*x**4 + 2*a**2*x**2 +
 1))
```

# 3.872 $\int \frac{x^3 \arctan(ax)^{5/2}}{(c+a^2cx^2)^3} dx$



## Optimal result

Integrand size = 24, antiderivative size = 256

$$\int \frac{x^3 \arctan(ax)^{5/2}}{(c+a^2cx^2)^3} dx = -\frac{135\sqrt{\arctan(ax)}}{2048a^4c^3} - \frac{15x^4\sqrt{\arctan(ax)}}{256c^3(1+a^2x^2)^2} + \frac{45\sqrt{\arctan(ax)}}{256a^4c^3(1+a^2x^2)} + \frac{5x^3\arctan(ax)^{3/2}}{32ac^3(1+a^2x^2)^2} + \frac{15x\arctan(ax)^{3/2}}{64a^3c^3(1+a^2x^2)} - \frac{3\arctan(ax)^{5/2}}{32a^4c^3} + \frac{x^4\arctan(ax)^{5/2}}{4c^3(1+a^2x^2)^2} + \frac{15\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{4096a^4c^3} - \frac{15\sqrt{\pi}\operatorname{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{256a^4c^3}$$

output
```
-135/2048*arctan(a*x)^(1/2)/a^4/c^3-15/256*x^4*arctan(a*x)^(1/2)/c^3/(a^2*
x^2+1)^2+45/256*arctan(a*x)^(1/2)/a^4/c^3/(a^2*x^2+1)+5/32*x^3*arctan(a*x)
^(3/2)/a/c^3/(a^2*x^2+1)^2+15/64*x*arctan(a*x)^(3/2)/a^3/c^3/(a^2*x^2+1)-3
/32*arctan(a*x)^(5/2)/a^4/c^3+1/4*x^4*arctan(a*x)^(5/2)/c^3/(a^2*x^2+1)^2+
15/8192*2^(1/2)*Pi^(1/2)*FresnelC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))/a^
4/c^3-15/256*Pi^(1/2)*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))/a^4/c^3
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.40

$$\int \frac{x^3 \arctan(ax)^{5/2}}{(c+a^2cx^2)^3} dx = \frac{510\sqrt{2\pi}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right) + \frac{14400\arctan(ax)+5760a^2x^2\arctan(ax)-16320a^4x^4}{}}{}$$

input
```
Integrate[(x^3*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^3,x]
```

output
```
(510*Sqrt[2*Pi]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]] + (14400*ArcTan[a
*x] + 5760*a^2*x^2*ArcTan[a*x] - 16320*a^4*x^4*ArcTan[a*x] + 30720*a*x*Arc
Tan[a*x]^2 + 51200*a^3*x^3*ArcTan[a*x]^2 - 12288*ArcTan[a*x]^3 - 24576*a^2
*x^2*ArcTan[a*x]^3 + 20480*a^4*x^4*ArcTan[a*x]^3 - 4080*Sqrt[Pi]*(1 + a^2*
x^2)^2*Sqrt[ArcTan[a*x]]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]] + (900*I
)*Sqrt[2]*(1 + a^2*x^2)^2*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-2*I)*ArcTan[
a*x]] - (900*I)*Sqrt[2]*(1 + a^2*x^2)^2*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (2*
I)*ArcTan[a*x]] + (135*I)*(1 + a^2*x^2)^2*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2
, (-4*I)*ArcTan[a*x]] - (135*I)*(1 + a^2*x^2)^2*Sqrt[I*ArcTan[a*x]]*Gamma[
1/2, (4*I)*ArcTan[a*x]])/((1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]]))/(131072*a^4*
c^3)
```

## Rubi [A] (verified)

Time = 1.79 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.17, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.542$, Rules used = {5479, 27, 5475, 5471, 5465, 5439, 3042, 3793, 2009, 5505, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \arctan(ax)^{5/2}}{(a^2cx^2+c)^3} dx$$

$$\Big\downarrow \text{ 5479}$$

$$\frac{x^4\arctan(ax)^{5/2}}{4c^3\left(a^2x^2+1\right)^2}-\frac{5}{8}a\int\frac{x^4\arctan(ax)^{3/2}}{c^3\left(a^2x^2+1\right)^3}dx$$

$\Big\downarrow$ 27

$$\frac{x^4\arctan(ax)^{5/2}}{4c^3\left(a^2x^2+1\right)^2}-\frac{5a\int\frac{x^4\arctan(ax)^{3/2}}{\left(a^2x^2+1\right)^3}dx}{8c^3}$$

$\Big\downarrow$ 5475

$$\frac{x^4\arctan(ax)^{5/2}}{4c^3\left(a^2x^2+1\right)^2}-\frac{5a\left(\frac{3\int\frac{x^2\arctan(ax)^{3/2}}{\left(a^2x^2+1\right)^2}dx}{4a^2}-\frac{3}{64}\int\frac{x^4}{\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx+\frac{3x^4\sqrt{\arctan(ax)}}{32a\left(a^2x^2+1\right)^2}-\frac{x^3\arctan(ax)^{3/2}}{4a^2\left(a^2x^2+1\right)^2}\right)}{8c^3}$$

$\Big\downarrow$ 5471

$$\frac{x^4\arctan(ax)^{5/2}}{4c^3\left(a^2x^2+1\right)^2}-\frac{5a\left(-\frac{3}{64}\int\frac{x^4}{\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx+\frac{3\left(\frac{3\int\frac{x\sqrt{\arctan(ax)}}{\left(a^2x^2+1\right)^2}dx}{4a}+\frac{\arctan(ax)^{5/2}}{5a^3}-\frac{x\arctan(ax)^{3/2}}{2a^2\left(a^2x^2+1\right)}\right)}{4a^2}+\frac{3x^4\sqrt{\arctan(ax)}}{32a\left(a^2x^2+1\right)^2}-\frac{x^3\arctan(ax)^{3/}}{4a^2\left(a^2x^2+1\right)^2}\right.}{8c^3}$$

$\Big\downarrow$ 5465

$$\frac{x^4\arctan(ax)^{5/2}}{4c^3\left(a^2x^2+1\right)^2}-\frac{5a\left(-\frac{3}{64}\int\frac{x^4}{\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx+\frac{3\left(\frac{3\left(\frac{\int\frac{1}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{4a}-\frac{\sqrt{\arctan(ax)}}{2a^2\left(a^2x^2+1\right)}\right)}{4a}+\frac{\arctan(ax)^{5/2}}{5a^3}-\frac{x\arctan(ax)^{3/2}}{2a^2\left(a^2x^2+1\right)}\right)}{4a^2}+\frac{3x^4\sqrt{\text{arc}}}{32a(a^2x}\right.}{8c^3}$$

$\Big\downarrow$ 5439

$$\frac{5a\left(-\frac{3}{64}\int \frac{x^4}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx+\frac{\frac{x^4\arctan(ax)^{5/2}}{4c^3(a^2x^2+1)^2}-\frac{3\left(\frac{3\left(\frac{\int \frac{1}{\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}d\arctan(ax)}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2\left(a^2x^2+1\right)}\right)}{4a}+\frac{\arctan(ax)^{5/2}}{5a^3}-\frac{x\arctan(ax)^{3/2}}{2a^2\left(a^2x^2+1\right)}\right)}{4a^2}}{} + \frac{3\cdots}{}\right)}{8c^3}$$

$\downarrow$ 3042

$$\frac{5a\left(-\frac{3}{64}\int \frac{x^4}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx+\frac{\frac{x^4\arctan(ax)^{5/2}}{4c^3(a^2x^2+1)^2}-\frac{3\left(\frac{3\left(\frac{\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^2}{\sqrt{\arctan(ax)}}d\arctan(ax)}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2\left(a^2x^2+1\right)}\right)}{4a}+\frac{\arctan(ax)^{5/2}}{5a^3}-\frac{x\arctan(ax)^{3/2}}{2a^2\left(a^2x^2+1\right)}\right)}{4a^2}}{} + \frac{3x^4\cdots}{32\cdots}\right)}{8c^3}$$

$\downarrow$ 3793

$$\frac{5a\left(-\frac{3}{64}\int \frac{x^4}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx+\frac{\frac{x^4\arctan(ax)^{5/2}}{4c^3(a^2x^2+1)^2}-\frac{3\left(\frac{3\left(\frac{\int \left(\frac{\cos(2\arctan(ax))}{2\sqrt{\arctan(ax)}}+\frac{1}{2\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2\left(a^2x^2+1\right)}\right)}{4a}+\frac{\arctan(ax)^{5/2}}{5a^3}-\frac{x\arctan(ax)\cdots}{2a^2\left(a^2x^2+\cdots\right)}\cdots\right)}{4a^2}}{}\cdots\right)}{8c^3}$$

$\Big\downarrow$ 2009

$$\frac{x^4\arctan(ax)^{5/2}}{4c^3\left(a^2x^2+1\right)^2}-\frac{5a\left(-\frac{3}{64}\int\frac{x^4}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx+\frac{3x^4\sqrt{\arctan(ax)}}{32a(a^2x^2+1)^2}-\frac{x^3\arctan(ax)^{3/2}}{4a^2(a^2x^2+1)^2}+\frac{3\left(\frac{\arctan(ax)^{5/2}}{5a^3}+\frac{3\left(\frac{\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\sqrt{a}}{4a^2}\right.}{4a}\right.}{4a^2}\right.}{8c^3}$$

$\Big\downarrow$ 5505

$$\frac{x^4\arctan(ax)^{5/2}}{4c^3\left(a^2x^2+1\right)^2}-\frac{5a\left(-\frac{3\int\frac{a^4x^4}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{64a^5}+\frac{3x^4\sqrt{\arctan(ax)}}{32a(a^2x^2+1)^2}-\frac{x^3\arctan(ax)^{3/2}}{4a^2(a^2x^2+1)^2}+\frac{3\left(\frac{\arctan(ax)^{5/2}}{5a^3}+\frac{3\left(\frac{\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{4a^2}\right.}{4a}\right.}{4a}\right.}{8c^3}$$

$\Big\downarrow$ 3042

$$\frac{x^4\arctan(ax)^{5/2}}{4c^3\left(a^2x^2+1\right)^2}-\frac{5a\left(-\frac{3\int\frac{\sin(\arctan(ax))^4}{\sqrt{\arctan(ax)}}d\arctan(ax)}{64a^5}+\frac{3x^4\sqrt{\arctan(ax)}}{32a(a^2x^2+1)^2}-\frac{x^3\arctan(ax)^{3/2}}{4a^2(a^2x^2+1)^2}+\frac{3\left(\frac{\arctan(ax)^{5/2}}{5a^3}+\frac{3\left(\frac{\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\sqrt{\text{arct}}}{4a^2}\right.}{4a}\right.}{4a^2}\right.}{8c^3}$$

$\Big\downarrow$ 3793

$$\frac{x^4 \arctan(ax)^{5/2}}{4c^3 (a^2x^2+1)^2} - \frac{5a\left(-\frac{3\int\left(-\frac{\cos(2\arctan(ax))}{2\sqrt{\arctan(ax)}}+\frac{\cos(4\arctan(ax))}{8\sqrt{\arctan(ax)}}+\frac{3}{8\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{64a^5}+\frac{3x^4\sqrt{\arctan(ax)}}{32a(a^2x^2+1)^2}-\frac{x^3\arctan(ax)^{3/2}}{4a^2(a^2x^2+1)^2}+\frac{3\left(\frac{\arctan(ax)^{5/2}}{5a^3}+\right.}{}\right.}{8c^3}$$

$\Big\downarrow$ 2009

$$\frac{x^4 \arctan(ax)^{5/2}}{4c^3 (a^2x^2+1)^2} - \frac{5a\left(-\frac{3\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{2}\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\arctan(ax)}\right)}{64a^5}+\frac{3x^4\sqrt{\arctan(ax)}}{32a(a^2x^2+1)^2}-\frac{x^3\arctan(ax)^{3/2}}{4a^2(a^2x^2+1)^2}\right.}{8c^3}$$

input
```
Int[(x^3*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^3,x]
```

output
```
(x^4*ArcTan[a*x]^(5/2))/(4*c^3*(1 + a^2*x^2)^2) - (5*a*((3*x^4*Sqrt[ArcTan
[a*x]])/(32*a*(1 + a^2*x^2)^2) - (x^3*ArcTan[a*x]^(3/2))/(4*a^2*(1 + a^2*x
^2)^2) - (3*((3*Sqrt[ArcTan[a*x]])/4 + (Sqrt[Pi/2]*FresnelC[2*Sqrt[2/Pi]*S
qrt[ArcTan[a*x]]])/8 - (Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])
/2))/(64*a^5) + (3*(-1/2*(x*ArcTan[a*x]^(3/2))/(a^2*(1 + a^2*x^2)) + ArcTa
n[a*x]^(5/2)/(5*a^3) + (3*(-1/2*Sqrt[ArcTan[a*x]]/(a^2*(1 + a^2*x^2)) + (S
qrt[ArcTan[a*x]] + (Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/2)/
(4*a^2)))/(4*a)))/(4*a^2)))/(8*c^3)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5471
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^2)/((d_) + (e_.)*(x_)^2)
^2, x_Symbol] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(2*b*c^3*d^2*(p + 1)), x]
 + (-Simp[x*((a + b*ArcTan[c*x])^p/(2*c^2*d*(d + e*x^2))), x] + Simp[b*(p/(
2*c))   Int[x*((a + b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5475
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_)*((d_) + (e_.)
*(x_)^2)^(q_), x_Symbol] :> Simp[b*p*(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*Ar
cTan[c*x])^(p - 1)/(c*d*m^2)), x] + (-Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(q +
 1)*((a + b*ArcTan[c*x])^p/(c^2*d*m)), x] + Simp[f^2*((m - 1)/(c^2*d*m))
Int[(f*x)^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p, x], x] - Simp[
b^2*p*((p - 1)/m^2)   Int[(f*x)^m*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2)
, x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && EqQ[m + 2*
q + 2, 0] && LtQ[q, -1] && GtQ[p, 1]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

## Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.61

$$\frac{-1024 \arctan (ax)^{\frac{5}{2}} \sqrt{\pi} \cos (2 \arctan (ax)) + 256\sqrt{\pi} \arctan (ax)^{\frac{5}{2}} \cos (4 \arctan (ax)) + 1280 \arctan (ax)^{\frac{3}{2}}}{}$$

input
```
int(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x)
```

output
```
1/8192/a^4/c^3*(-1024*arctan(a*x)^(5/2)*Pi^(1/2)*cos(2*arctan(a*x))+256*Pi
^(1/2)*arctan(a*x)^(5/2)*cos(4*arctan(a*x))+1280*arctan(a*x)^(3/2)*Pi^(1/2
)*sin(2*arctan(a*x))-160*Pi^(1/2)*arctan(a*x)^(3/2)*sin(4*arctan(a*x))+15*
Pi*2^(1/2)*FresnelC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))+960*cos(2*arctan
(a*x))*arctan(a*x)^(1/2)*Pi^(1/2)-60*Pi^(1/2)*arctan(a*x)^(1/2)*cos(4*arct
an(a*x))-480*Pi*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2)))/Pi^(1/2)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3 \arctan(ax)^{5/2}}{(c+a^2cx^2)^3} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{x^3 \arctan(ax)^{5/2}}{(c+a^2cx^2)^3} \, dx = \frac{\int \frac{x^3 \operatorname{atan}^{\frac{5}{2}}(ax)}{a^6x^6+3a^4x^4+3a^2x^2+1} \, dx}{c^3}$$

input
```
integrate(x**3*atan(a*x)**(5/2)/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(x**3*atan(a*x)**(5/2)/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1)
, x)/c**3
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3 \arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F]

$$\int \frac{x^3 \arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx = \int \frac{x^3 \arctan{(ax)}^{\frac{5}{2}}}{(a^2cx^2+c)^3}\,dx$$

input
```
integrate(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
integrate(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2 + c)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx = \int \frac{x^3 \operatorname{atan}(a\,x)^{5/2}}{(c\,a^2\,x^2+c)^3}\,dx$$

input
```
int((x^3*atan(a*x)^(5/2))/(c + a^2*c*x^2)^3,x)
```

output
```
int((x^3*atan(a*x)^(5/2))/(c + a^2*c*x^2)^3, x)
```

## Reduce [F]

$$\int \frac{x^3 \arctan(ax)^{5/2}}{(c+a^2cx^2)^3} \, dx = \text{Too large to display}$$

input
```
int(x^3*atan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x)
```

output
```
(160*sqrt(atan(a*x))*atan(a*x)**2*a**4*x**4 - 192*sqrt(atan(a*x))*atan(a*x
)**2*a**2*x**2 - 96*sqrt(atan(a*x))*atan(a*x)**2 + 400*sqrt(atan(a*x))*ata
n(a*x)*a**3*x**3 + 240*sqrt(atan(a*x))*atan(a*x)*a*x - 150*sqrt(atan(a*x))
*a**4*x**4 + 90*sqrt(atan(a*x)) - 45*int(sqrt(atan(a*x))/(atan(a*x)*a**6*x
**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**5*x
**4 - 90*int(sqrt(atan(a*x))/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4
+ 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**3*x**2 - 45*int(sqrt(atan(a*x))
/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + at
an(a*x)),x)*a + 75*int((sqrt(atan(a*x))*x**4)/(atan(a*x)*a**6*x**6 + 3*ata
n(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**9*x**4 + 150*i
nt((sqrt(atan(a*x))*x**4)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3
*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**7*x**2 + 75*int((sqrt(atan(a*x))*x
**4)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2
+ atan(a*x)),x)*a**5)/(1024*a**4*c**3*(a**4*x**4 + 2*a**2*x**2 + 1))
```

# 3.873 $\int \frac{x^2 \arctan(ax)^{5/2}}{(c+a^2cx^2)^3} dx$



## Optimal result

Integrand size = 24, antiderivative size = 133

$$\int \frac{x^2 \arctan(ax)^{5/2}}{(c+a^2cx^2)^3} dx = \frac{\arctan(ax)^{7/2}}{28a^3c^3} - \frac{5\arctan(ax)^{3/2}\cos(4\arctan(ax))}{256a^3c^3} - \frac{15\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{4096a^3c^3} + \frac{15\sqrt{\arctan(ax)}\sin(4\arctan(ax))}{2048a^3c^3} - \frac{\arctan(ax)^{5/2}\sin(4\arctan(ax))}{32a^3c^3}$$

output
```
1/28*arctan(a*x)^(7/2)/a^3/c^3-5/256*arctan(a*x)^(3/2)*cos(4*arctan(a*x))/
a^3/c^3-15/8192*2^(1/2)*Pi^(1/2)*FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(
1/2))/a^3/c^3+15/2048*arctan(a*x)^(1/2)*sin(4*arctan(a*x))/a^3/c^3-1/32*ar
ctan(a*x)^(5/2)*sin(4*arctan(a*x))/a^3/c^3
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.39

$$\int \frac{x^2 \arctan(ax)^{5/2}}{(c+a^2cx^2)^3} \, dx = \frac{32 \arctan(ax) \left(-105ax(-1+a^2x^2) - 70(1-6a^2x^2+a^4x^4) \arctan(ax) + 448ax\right.}{}$$

input
```
Integrate[(x^2*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^3,x]
```

output
```
(32*ArcTan[a*x]*(-105*a*x*(-1 + a^2*x^2) - 70*(1 - 6*a^2*x^2 + a^4*x^4)*Ar
cTan[a*x] + 448*a*x*(-1 + a^2*x^2)*ArcTan[a*x]^2 + 128*(1 + a^2*x^2)^2*Arc
Tan[a*x]^3) + 105*(1 + a^2*x^2)^2*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-4*I)
*ArcTan[a*x]] + 105*(1 + a^2*x^2)^2*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (4*I)*A
rcTan[a*x]])/(114688*a^3*c^3*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]])
```

## Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.83, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {5505, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \arctan(ax)^{5/2}}{(a^2cx^2+c)^3} \, dx$$

$$\Big\downarrow \; 5505$$

$$\frac{\int \frac{a^2x^2 \arctan(ax)^{5/2}}{(a^2x^2+1)^2} d\arctan(ax)}{a^3c^3}$$

$$\Big\downarrow \; 4906$$

$$\frac{\int \left(\frac{1}{8}\arctan(ax)^{5/2} - \frac{1}{8}\arctan(ax)^{5/2}\cos(4\arctan(ax))\right) d\arctan(ax)}{a^3c^3}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\frac{15\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{4096}+\frac{1}{28}\arctan(ax)^{7/2}-\frac{1}{32}\arctan(ax)^{5/2}\sin(4\arctan(ax))+\frac{15\sqrt{\arctan(ax)}\sin(4\arctan(ax))}{2048}}{a^3c^3}$$

input
```
Int[(x^2*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^3,x]
```

output
```
(ArcTan[a*x]^(7/2)/28 - (5*ArcTan[a*x]^(3/2)*Cos[4*ArcTan[a*x]])/256 - (15
*Sqrt[Pi/2]*FresnelS[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/4096 + (15*Sqrt[ArcT
an[a*x]]*Sin[4*ArcTan[a*x]])/2048 - (ArcTan[a*x]^(5/2)*Sin[4*ArcTan[a*x]])
/32)/(a^3*c^3)
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

## Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.72

$$\frac{2048 \arctan (ax)^4 - 1792 \arctan (ax)^3 \sin (4 \arctan (ax)) - 105\sqrt{2}\,\sqrt{\arctan (ax)}\,\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{2}\,\sqrt{\arctan(}}{\sqrt{\pi}}\right.}{57344a^3c^3\sqrt{\arctan (ax)}}$$

input
```
int(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x)
```

output
```
1/57344/a^3/c^3*(2048*arctan(a*x)^4-1792*arctan(a*x)^3*sin(4*arctan(a*x))-
105*2^(1/2)*arctan(a*x)^(1/2)*Pi^(1/2)*FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(
a*x)^(1/2))-1120*arctan(a*x)^2*cos(4*arctan(a*x))+420*arctan(a*x)*sin(4*ar
ctan(a*x)))/arctan(a*x)^(1/2)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2 \arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{x^2 \arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx = \frac{\int \frac{x^2 \operatorname{atan}^{\frac{5}{2}}(ax)}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

input
```
integrate(x**2*atan(a*x)**(5/2)/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(x**2*atan(a*x)**(5/2)/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1)
, x)/c**3
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2 \arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F]

$$\int \frac{x^2 \arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx = \int \frac{x^2 \arctan(ax)^{\frac{5}{2}}}{(a^2cx^2+c)^3}\,dx$$

input
```
integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2 + c)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx = \int \frac{x^2\,\mathrm{atan}(a\,x)^{5/2}}{(c\,a^2\,x^2+c)^3}\,dx$$

input
```
int((x^2*atan(a*x)^(5/2))/(c + a^2*c*x^2)^3,x)
```

output
```
int((x^2*atan(a*x)^(5/2))/(c + a^2*c*x^2)^3, x)
```

## Reduce [F]

$$\int \frac{x^2 \arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx = \frac{32\sqrt{atan\,(ax)}\,atan(ax)^3\,a^4x^4 + 64\sqrt{atan\,(ax)}\,atan(ax)^3\,a^2x^2 + 32\sqrt{atan\,(ax)}\,a}{}$$

input
```
int(x^2*atan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x)
```

output
```
(32*sqrt(atan(a*x))*atan(a*x)**3*a**4*x**4 + 64*sqrt(atan(a*x))*atan(a*x)*
*3*a**2*x**2 + 32*sqrt(atan(a*x))*atan(a*x)**3 + 112*sqrt(atan(a*x))*atan(
a*x)**2*a**3*x**3 - 112*sqrt(atan(a*x))*atan(a*x)**2*a*x + 140*sqrt(atan(a
*x))*atan(a*x)*a**2*x**2 + 70*sqrt(atan(a*x))*a*x - 70*int(sqrt(atan(a*x))
/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1),x)*a**5*x**4 - 140*int(sqrt(a
tan(a*x))/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1),x)*a**3*x**2 - 70*in
t(sqrt(atan(a*x))/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1),x)*a - 35*in
t((sqrt(atan(a*x))*x)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*ata
n(a*x)*a**2*x**2 + atan(a*x)),x)*a**6*x**4 - 70*int((sqrt(atan(a*x))*x)/(a
tan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(
a*x)),x)*a**4*x**2 - 35*int((sqrt(atan(a*x))*x)/(atan(a*x)*a**6*x**6 + 3*a
tan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**2)/(896*a**3
*c**3*(a**4*x**4 + 2*a**2*x**2 + 1))
```

# 3.874 $\int \frac{x\arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 254

$$\int \frac{x\arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx = -\frac{225\sqrt{\arctan(ax)}}{2048a^2c^3} + \frac{15\sqrt{\arctan(ax)}}{256a^2c^3(1+a^2x^2)^2} + \frac{45\sqrt{\arctan(ax)}}{256a^2c^3(1+a^2x^2)} + \frac{5x\arctan(ax)^{3/2}}{32ac^3(1+a^2x^2)^2} + \frac{15x\arctan(ax)^{3/2}}{64ac^3(1+a^2x^2)} + \frac{3\arctan(ax)^{5/2}}{32a^2c^3} - \frac{\arctan(ax)^{5/2}}{4a^2c^3(1+a^2x^2)^2} - \frac{15\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{4096a^2c^3} - \frac{15\sqrt{\pi}\operatorname{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{256a^2c^3}$$

output
```
-225/2048*arctan(a*x)^(1/2)/a^2/c^3+15/256*arctan(a*x)^(1/2)/a^2/c^3/(a^2*
x^2+1)^2+45/256*arctan(a*x)^(1/2)/a^2/c^3/(a^2*x^2+1)+5/32*x*arctan(a*x)^(
3/2)/a/c^3/(a^2*x^2+1)^2+15/64*x*arctan(a*x)^(3/2)/a/c^3/(a^2*x^2+1)+3/32*
arctan(a*x)^(5/2)/a^2/c^3-1/4*arctan(a*x)^(5/2)/a^2/c^3/(a^2*x^2+1)^2-15/8
192*2^(1/2)*Pi^(1/2)*FresnelC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))/a^2/c^
3-15/256*Pi^(1/2)*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))/a^2/c^3
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.41

$$\int \frac{x \arctan(ax)^{5/2}}{\left(c+a^2cx^2\right)^3}\,dx = \frac{450\sqrt{2\pi}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{16320\arctan(ax)-5760a^2x^2\arctan(ax)-14400a^4x^4\,a}{}}{}$$

input
```
Integrate[(x*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^3,x]
```

output
```
(450*Sqrt[2*Pi]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]] + (16320*ArcTan[a
*x] - 5760*a^2*x^2*ArcTan[a*x] - 14400*a^4*x^4*ArcTan[a*x] + 51200*a*x*Arc
Tan[a*x]^2 + 30720*a^3*x^3*ArcTan[a*x]^2 - 20480*ArcTan[a*x]^3 + 24576*a^2
*x^2*ArcTan[a*x]^3 + 12288*a^4*x^4*ArcTan[a*x]^3 - 3600*Sqrt[Pi]*(1 + a^2*
x^2)^2*Sqrt[ArcTan[a*x]]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]] + (1020*
I)*Sqrt[2]*(1 + a^2*x^2)^2*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-2*I)*ArcTan
[a*x]] - (1020*I)*Sqrt[2]*(1 + a^2*x^2)^2*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (
2*I)*ArcTan[a*x]] + (345*I)*(1 + a^2*x^2)^2*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1
/2, (-4*I)*ArcTan[a*x]] - (345*I)*(1 + a^2*x^2)^2*Sqrt[I*ArcTan[a*x]]*Gamm
a[1/2, (4*I)*ArcTan[a*x]])/((1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]]))/(131072*a^
2*c^3)
```

## Rubi [A] (verified)

Time = 1.51 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.13, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$, Rules used = {5465, 27, 5435, 5427, 5439, 3042, 3793, 2009, 5465, 5439, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \arctan(ax)^{5/2}}{\left(a^2cx^2+c\right)^3}dx$$

$$\Big\downarrow \; 5465$$

$$\frac{5\int \frac{\arctan(ax)^{3/2}}{c^3(a^2x^2+1)^3}dx}{8a}-\frac{\arctan(ax)^{5/2}}{4a^2c^3(a^2x^2+1)^2}$$

$\downarrow$ 27

$$\frac{5\int \frac{\arctan(ax)^{3/2}}{(a^2x^2+1)^3}dx}{8ac^3}-\frac{\arctan(ax)^{5/2}}{4a^2c^3(a^2x^2+1)^2}$$

$\downarrow$ 5435

$$\frac{5\left(-\frac{3}{64}\int \frac{1}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx+\frac{3}{4}\int \frac{\arctan(ax)^{3/2}}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^{3/2}}{4(a^2x^2+1)^2}+\frac{3\sqrt{\arctan(ax)}}{32a(a^2x^2+1)^2}\right)}{8ac^3}-\frac{\arctan(ax)^{5/2}}{4a^2c^3(a^2x^2+1)^2}$$

$\downarrow$ 5427

$$\frac{5\left(-\frac{3}{64}\int \frac{1}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx+\frac{3}{4}\left(-\frac{3}{4}a\int \frac{x\sqrt{\arctan(ax)}}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^{3/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{5/2}}{5a}\right)+\frac{x\arctan(ax)^{3/2}}{4(a^2x^2+1)^2}+\frac{3\sqrt{a}}{32a(}\right.}{8ac^3}$$
$$\frac{\arctan(ax)^{5/2}}{4a^2c^3(a^2x^2+1)^2}$$

$\downarrow$ 5439

$$\frac{5\left(-\frac{3\int \frac{1}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{64a}+\frac{3}{4}\left(-\frac{3}{4}a\int \frac{x\sqrt{\arctan(ax)}}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^{3/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{5/2}}{5a}\right)+\frac{x\arctan(ax)^{3/2}}{4(a^2x^2+1)^2}+\right.}{8ac^3}$$
$$\frac{\arctan(ax)^{5/2}}{4a^2c^3(a^2x^2+1)^2}$$

$\downarrow$ 3042

$$\frac{5\left(\frac{3}{4}\left(-\frac{3}{4}a\int \frac{x\sqrt{\arctan(ax)}}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^{3/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{5/2}}{5a}\right)-\frac{3\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^4}{\sqrt{\arctan(ax)}}d\arctan(ax)}{64a}+\frac{x\arctan(ax)^{3/2}}{4(a^2x^2+1)^2}+\frac{3\sqrt{}}{32a}\right.}{8ac^3}$$
$$\frac{\arctan(ax)^{5/2}}{4a^2c^3(a^2x^2+1)^2}$$

$\downarrow$ 3793

$$\frac{5\left(\frac{3}{4}\left(-\frac{3}{4}a\int\frac{x\sqrt{\arctan(ax)}}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^{3/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{5/2}}{5a}\right)-\frac{3\int\left(\frac{\cos(2\arctan(ax))}{2\sqrt{\arctan(ax)}}+\frac{\cos(4\arctan(ax))}{8\sqrt{\arctan(ax)}}+\frac{3}{8\sqrt{\arctan(ax)}}\right)d\arctan(}{64a}\right.}{8ac^3}$$

$$\frac{\arctan(ax)^{5/2}}{4a^2c^3\left(a^2x^2+1\right)^2}$$

$\downarrow$ 2009

$$\frac{5\left(\frac{3}{4}\left(-\frac{3}{4}a\int\frac{x\sqrt{\arctan(ax)}}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^{3/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{5/2}}{5a}\right)+\frac{x\arctan(ax)^{3/2}}{4(a^2x^2+1)^2}+\frac{3\sqrt{\arctan(ax)}}{32a(a^2x^2+1)^2}-\frac{3\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\right.\right.}{}\right.}{8ac^3}$$

$$\frac{\arctan(ax)^{5/2}}{4a^2c^3\left(a^2x^2+1\right)^2}$$

$\downarrow$ 5465

$$\frac{5\left(\frac{3}{4}\left(-\frac{3}{4}a\left(\frac{\int\frac{1}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{4a}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^{3/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{5/2}}{5a}\right)+\frac{x\arctan(ax)^{3/2}}{4(a^2x^2+1)^2}+\frac{3\sqrt{\arctan(ax}}{32a(a^2x^2+1)}\right.}{8ac^3}$$

$$\frac{\arctan(ax)^{5/2}}{4a^2c^3\left(a^2x^2+1\right)^2}$$

$\downarrow$ 5439

$$\frac{5\left(\frac{3}{4}\left(-\frac{3}{4}a\left(\frac{\int\frac{1}{\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}d\arctan(ax)}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^{3/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{5/2}}{5a}\right)+\frac{x\arctan(ax)^{3/2}}{4(a^2x^2+1)^2}+\frac{3\sqrt{\mathrm{ar}}}{32a(}\right.}{8ac^3}$$

$$\frac{\arctan(ax)^{5/2}}{4a^2c^3\left(a^2x^2+1\right)^2}$$

$\downarrow$ 3042

$$\frac{5\left(\frac{3}{4}\left(-\frac{3}{4}a\left(\frac{\int\frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^2}{\sqrt{\arctan(ax)}}d\arctan(ax)}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^{3/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{5/2}}{5a}\right)+\frac{x\arctan(ax)^{3/2}}{4(a^2x^2+1)^2}+\frac{3\sqrt{\mathrm{arc}}}{32a(a^2}\right.}{8ac^3}$$

$$\frac{\arctan(ax)^{5/2}}{4a^2c^3\left(a^2x^2+1\right)^2}$$

$\downarrow$ 3793

$$\frac{5\left(\frac{3}{4}\left(-\frac{3}{4}a\left(\frac{\int\left(\frac{\cos(2\arctan(ax))}{2\sqrt{\arctan(ax)}}+\frac{1}{2\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^{3/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{5/2}}{5a}\right)+\frac{x\arctan(ax)}{4(a^2x^2+1)}\right.}{8ac^3}$$

$$\frac{\arctan(ax)^{5/2}}{4a^2c^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{5\left(\frac{3}{4}\left(-\frac{3}{4}a\left(\frac{\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\sqrt{\arctan(ax)}}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^{3/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{5/2}}{5a}\right)+\frac{x\arctan(ax)^{3/}}{4(a^2x^2+1)^2}\right.}{8ac^3}$$

$$\frac{\arctan(ax)^{5/2}}{4a^2c^3\left(a^2x^2+1\right)^2}$$

input
```
Int[(x*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^3,x]
```

output
```
-1/4*ArcTan[a*x]^(5/2)/(a^2*c^3*(1 + a^2*x^2)^2) + (5*((3*Sqrt[ArcTan[a*x]
])/(32*a*(1 + a^2*x^2)^2) + (x*ArcTan[a*x]^(3/2))/(4*(1 + a^2*x^2)^2) - (3
*((3*Sqrt[ArcTan[a*x]])/4 + (Sqrt[Pi/2]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[
a*x]]])/8 + (Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/2))/(64*a)
 + (3*((x*ArcTan[a*x]^(3/2))/(2*(1 + a^2*x^2)) + ArcTan[a*x]^(5/2)/(5*a) -
 (3*a*(-1/2*Sqrt[ArcTan[a*x]]/(a^2*(1 + a^2*x^2)) + (Sqrt[ArcTan[a*x]] + (
Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/2)/(4*a^2)))/4))/4))/(8
*a*c^3)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5435
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[b*p*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p - 1)/(4*c*d*
(q + 1)^2)), x] + (-Simp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*d*
(q + 1))), x] + Simp[(2*q + 3)/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p, x], x] - Simp[b^2*p*((p - 1)/(4*(q + 1)^2))   Int[(d + e
*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

## Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.62

$$-\frac{1024 \arctan(ax)^{\frac{5}{2}} \sqrt{\pi} \cos(2\arctan(ax)) + 256\sqrt{\pi} \arctan(ax)^{\frac{5}{2}} \cos(4\arctan(ax)) + 15\pi\sqrt{2}\, \text{FresnelC}\,(}{}$$

input
```
int(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x)
```

output
```
-1/8192/a^2/c^3/Pi^(1/2)*(1024*arctan(a*x)^(5/2)*Pi^(1/2)*cos(2*arctan(a*x
))+256*Pi^(1/2)*arctan(a*x)^(5/2)*cos(4*arctan(a*x))+15*Pi*2^(1/2)*Fresnel
C(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))-1280*arctan(a*x)^(3/2)*Pi^(1/2)*si
n(2*arctan(a*x))-160*Pi^(1/2)*arctan(a*x)^(3/2)*sin(4*arctan(a*x))-960*cos
(2*arctan(a*x))*arctan(a*x)^(1/2)*Pi^(1/2)-60*Pi^(1/2)*arctan(a*x)^(1/2)*c
os(4*arctan(a*x))+480*Pi*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2)))
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x \arctan(ax)^{5/2}}{(c + a^2cx^2)^3}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{x \arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx = \frac{\int \frac{x \operatorname{atan}^{\frac{5}{2}}(ax)}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

input
```
integrate(x*atan(a*x)**(5/2)/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(x*atan(a*x)**(5/2)/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x
)/c**3
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x \arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F]

$$\int \frac{x \arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx = \int \frac{x \arctan(ax)^{\frac{5}{2}}}{(a^2cx^2+c)^3}\,dx$$

input
```
integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2 + c)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x \arctan(ax)^{5/2}}{(c+a^2cx^2)^3} \, dx = \int \frac{x \operatorname{atan}(a\,x)^{5/2}}{(c\,a^2\,x^2+c)^3} \, dx$$

input
```
int((x*atan(a*x)^(5/2))/(c + a^2*c*x^2)^3,x)
```

output
```
int((x*atan(a*x)^(5/2))/(c + a^2*c*x^2)^3, x)
```

## Reduce [F]

$$\int \frac{x \arctan(ax)^{5/2}}{(c+a^2cx^2)^3} \, dx = \text{Too large to display}$$

input
```
int(x*atan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x)
```

output
```
(96*sqrt(atan(a*x))*atan(a*x)**2*a**4*x**4 + 192*sqrt(atan(a*x))*atan(a*x)
**2*a**2*x**2 - 160*sqrt(atan(a*x))*atan(a*x)**2 + 240*sqrt(atan(a*x))*ata
n(a*x)*a**3*x**3 + 400*sqrt(atan(a*x))*atan(a*x)*a*x - 90*sqrt(atan(a*x))*
a**4*x**4 + 150*sqrt(atan(a*x)) - 75*int(sqrt(atan(a*x))/(atan(a*x)*a**6*x
**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**5*x
**4 - 150*int(sqrt(atan(a*x))/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4
 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**3*x**2 - 75*int(sqrt(atan(a*x)
)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + a
tan(a*x)),x)*a + 45*int((sqrt(atan(a*x))*x**4)/(atan(a*x)*a**6*x**6 + 3*at
an(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**9*x**4 + 90*i
nt((sqrt(atan(a*x))*x**4)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3
*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**7*x**2 + 45*int((sqrt(atan(a*x))*x
**4)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2
+ atan(a*x)),x)*a**5)/(1024*a**2*c**3*(a**4*x**4 + 2*a**2*x**2 + 1))
```

# 3.875 $\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 296

$$\begin{aligned}\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx = &-\frac{45x\sqrt{\arctan(ax)}}{128c^3(1+a^2x^2)} \\ &-\frac{75\arctan(ax)^{3/2}}{256ac^3}+\frac{5\arctan(ax)^{3/2}}{32ac^3(1+a^2x^2)^2}+\frac{15\arctan(ax)^{3/2}}{32ac^3(1+a^2x^2)} \\ &+\frac{x\arctan(ax)^{5/2}}{4c^3(1+a^2x^2)^2}+\frac{3x\arctan(ax)^{5/2}}{8c^3(1+a^2x^2)}+\frac{3\arctan(ax)^{7/2}}{28ac^3} \\ &+\frac{15\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{4096ac^3}+\frac{15\sqrt{\pi}\operatorname{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{128ac^3} \\ &-\frac{15\sqrt{\arctan(ax)}\sin(2\arctan(ax))}{256ac^3}-\frac{15\sqrt{\arctan(ax)}\sin(4\arctan(ax))}{2048ac^3}\end{aligned}$$

output
```
-45/128*x*arctan(a*x)^(1/2)/c^3/(a^2*x^2+1)-75/256*arctan(a*x)^(3/2)/a/c^3
+5/32*arctan(a*x)^(3/2)/a/c^3/(a^2*x^2+1)^2+15/32*arctan(a*x)^(3/2)/a/c^3/
(a^2*x^2+1)+1/4*x*arctan(a*x)^(5/2)/c^3/(a^2*x^2+1)^2+3/8*x*arctan(a*x)^(5
/2)/c^3/(a^2*x^2+1)+3/28*arctan(a*x)^(7/2)/a/c^3+15/8192*2^(1/2)*Pi^(1/2)*
FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))/a/c^3+15/128*Pi^(1/2)*Fresn
elS(2*arctan(a*x)^(1/2)/Pi^(1/2))/a/c^3-15/256*arctan(a*x)^(1/2)*sin(2*arc
tan(a*x))/a/c^3-15/2048*arctan(a*x)^(1/2)*sin(4*arctan(a*x))/a/c^3
```

## Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.55

$$\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx = \frac{\frac{16\sqrt{\arctan(ax)}\left(-105ax(17+15a^2x^2)-70(-17+6a^2x^2+15a^4x^4)\arctan(ax)+448ax(5+3a^2x^2)\arctan(ax)^2+384}{(1+a^2x^2)^2}}{}$$

input
```
Integrate[ArcTan[a*x]^(5/2)/(c + a^2*c*x^2)^3,x]
```

output
```
((16*Sqrt[ArcTan[a*x]]*(-105*a*x*(17 + 15*a^2*x^2) - 70*(-17 + 6*a^2*x^2 +
 15*a^4*x^4)*ArcTan[a*x] + 448*a*x*(5 + 3*a^2*x^2)*ArcTan[a*x]^2 + 384*(1
+ a^2*x^2)^2*ArcTan[a*x]^3))/(1 + a^2*x^2)^2 + 105*Sqrt[2*Pi]*FresnelS[2*S
qrt[2/Pi]*Sqrt[ArcTan[a*x]]] + 6720*Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]]
)/Sqrt[Pi]])/(57344*a*c^3)
```

## Rubi [A] (verified)

Time = 1.73 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.11, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {5435, 27, 5427, 5439, 3042, 3793, 2009, 5465, 5427, 5505, 4906, 27, 3042, 3786, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{5/2}}{(a^2cx^2+c)^3}dx$$

$$\Big\downarrow \text{5435}$$

$$-\frac{15}{64}\int \frac{\sqrt{\arctan(ax)}}{c^3(a^2x^2+1)^3}dx+\frac{3\int \frac{\arctan(ax)^{5/2}}{c^2(a^2x^2+1)^2}dx}{4c}+\frac{x\arctan(ax)^{5/2}}{4c^3(a^2x^2+1)^2}+\frac{5\arctan(ax)^{3/2}}{32ac^3(a^2x^2+1)^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{15\int \frac{\sqrt{\arctan(ax)}}{(a^2x^2+1)^3}dx}{64c^3}+\frac{3\int \frac{\arctan(ax)^{5/2}}{(a^2x^2+1)^2}dx}{4c^3}+\frac{x\arctan(ax)^{5/2}}{4c^3(a^2x^2+1)^2}+\frac{5\arctan(ax)^{3/2}}{32ac^3(a^2x^2+1)^2}$$

$$\Big\downarrow \text{5427}$$

$$-\frac{15\int \frac{\sqrt{\arctan(ax)}}{(a^2x^2+1)^3}dx}{64c^3}+\frac{3\left(-\frac{5}{4}a\int \frac{x\arctan(ax)^{3/2}}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^{5/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{7/2}}{7a}\right)}{4c^3}+\frac{x\arctan(ax)^{5/2}}{4c^3\left(a^2x^2+1\right)^2}+\frac{5\arctan(ax)^{3/2}}{32ac^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \text{5439}$$

$$-\frac{15\int \frac{\sqrt{\arctan(ax)}}{(a^2x^2+1)^2}d\arctan(ax)}{64ac^3}+\frac{3\left(-\frac{5}{4}a\int \frac{x\arctan(ax)^{3/2}}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^{5/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{7/2}}{7a}\right)}{4c^3}+\frac{x\arctan(ax)^{5/2}}{4c^3\left(a^2x^2+1\right)^2}+\frac{5\arctan(ax)^{3/2}}{32ac^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(-\frac{5}{4}a\int \frac{x\arctan(ax)^{3/2}}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^{5/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{7/2}}{7a}\right)}{4c^3}-\frac{15\int \sqrt{\arctan(ax)}\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^4d\arctan(ax)}{64ac^3}+\frac{x\arctan(ax)^{5/2}}{4c^3\left(a^2x^2+1\right)^2}+\frac{5\arctan(ax)^{3/2}}{32ac^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \text{3793}$$

$$\frac{3\left(-\frac{5}{4}a\int \frac{x\arctan(ax)^{3/2}}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^{5/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{7/2}}{7a}\right)}{4c^3}-\frac{15\int \left(\frac{1}{2}\sqrt{\arctan(ax)}\cos(2\arctan(ax))+\frac{1}{8}\sqrt{\arctan(ax)}\cos(4\arctan(ax))+\frac{3}{8}\sqrt{\arctan(ax)}\right)d\arctan(ax)}{64ac^3}+\frac{x\arctan(ax)^{5/2}}{4c^3\left(a^2x^2+1\right)^2}+\frac{5\arctan(ax)^{3/2}}{32ac^3\left(a^2x^2+1\right)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\left(-\frac{5}{4}a\int \frac{x\arctan(ax)^{3/2}}{(a^2x^2+1)^2}dx+\frac{x\arctan(ax)^{5/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{7/2}}{7a}\right)}{4c^3}+\frac{x\arctan(ax)^{5/2}}{4c^3\left(a^2x^2+1\right)^2}+\frac{5\arctan(ax)^{3/2}}{32ac^3\left(a^2x^2+1\right)^2}-\frac{15\left(-\frac{1}{64}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{8}\sqrt{\pi}\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{1}{4}\arctan(ax)^{3/2}+\frac{1}{4}\sqrt{\arctan(ax)}\text{si}\right.}{64ac^3}$$

$$\Big\downarrow \text{5465}$$

$$\frac{3\left(-\frac{5}{4}a\left(\frac{3\int \frac{\sqrt{\arctan(ax)}}{\left(a^2x^2+1\right)^2}dx}{4a}-\frac{\arctan(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^{5/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{7/2}}{7a}\right)}{4c^3}+\frac{x\arctan(ax)^{5/2}}{4c^3\left(a^2x^2+1\right)^2}+\frac{5\arctan(ax)^{3/2}}{32ac^3\left(a^2x^2+1\right)^2}-\frac{15\left(-\frac{1}{64}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{8}\sqrt{\pi}\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{1}{4}\arctan(ax)^{3/2}+\frac{1}{4}\sqrt{\arctan(ax)}\text{si}}{64ac^3}$$

$\big\downarrow$ 5427

$$\frac{3\left(-\frac{5}{4}a\left(\frac{3\left(-\frac{1}{4}a\int \frac{x}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx+\frac{x\sqrt{\arctan(ax)}}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4a}-\frac{\arctan(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^{5/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{7/2}}{7a}\right)}{4c^3}+\frac{x\arctan(ax)^{5/2}}{4c^3\left(a^2x^2+1\right)^2}+\frac{5\arctan(ax)^{3/2}}{32ac^3\left(a^2x^2+1\right)^2}-\frac{15\left(-\frac{1}{64}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{8}\sqrt{\pi}\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{1}{4}\arctan(ax)^{3/2}+\frac{1}{4}\sqrt{\arctan(ax)}\text{si}}{64ac^3}$$

$\big\downarrow$ 5505

$$\frac{3\left(-\frac{5}{4}a\left(\frac{3\left(-\frac{\int \frac{ax}{\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}d\arctan(ax)}{4a}+\frac{x\sqrt{\arctan(ax)}}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4a}-\frac{\arctan(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^{5/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{7/}}{7a}}{4c^3}+\frac{x\arctan(ax)^{5/2}}{4c^3\left(a^2x^2+1\right)^2}+\frac{5\arctan(ax)^{3/2}}{32ac^3\left(a^2x^2+1\right)^2}-\frac{15\left(-\frac{1}{64}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{8}\sqrt{\pi}\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{1}{4}\arctan(ax)^{3/2}+\frac{1}{4}\sqrt{\arctan(ax)}\text{si}}{64ac^3}$$

$\big\downarrow$ 4906

$$\frac{3\left(-\frac{5}{4}a\left(\frac{3\left(-\frac{\int \frac{\sin(2\arctan(ax))}{2\sqrt{\arctan(ax)}}d\arctan(ax)}{4a}+\frac{x\sqrt{\arctan(ax)}}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4a}-\frac{\arctan(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^{5/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{7/2}}{7a}\right)}{4c^3}+$$

$$\frac{x\arctan(ax)^{5/2}}{4c^3\left(a^2x^2+1\right)^2}+\frac{5\arctan(ax)^{3/2}}{32ac^3\left(a^2x^2+1\right)^2}-$$

$$\frac{15\left(-\frac{1}{64}\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{8}\sqrt{\pi}\operatorname{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{1}{4}\arctan(ax)^{3/2}+\frac{1}{4}\sqrt{\arctan(ax)}\operatorname{si}\right.}{64ac^3}$$

$\Big\downarrow$ 27

$$\frac{3\left(-\frac{5}{4}a\left(\frac{3\left(-\frac{\int \frac{\sin(2\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)}{8a}+\frac{x\sqrt{\arctan(ax)}}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4a}-\frac{\arctan(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^{5/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{7/2}}{7a}\right)}{4c^3}+$$

$$\frac{x\arctan(ax)^{5/2}}{4c^3\left(a^2x^2+1\right)^2}+\frac{5\arctan(ax)^{3/2}}{32ac^3\left(a^2x^2+1\right)^2}-$$

$$\frac{15\left(-\frac{1}{64}\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{8}\sqrt{\pi}\operatorname{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{1}{4}\arctan(ax)^{3/2}+\frac{1}{4}\sqrt{\arctan(ax)}\operatorname{si}\right.}{64ac^3}$$

$\Big\downarrow$ 3042

$$\frac{3\left(-\frac{5}{4}a\left(\frac{3\left(-\frac{\int \frac{\sin(2\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)}{8a}+\frac{x\sqrt{\arctan(ax)}}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4a}-\frac{\arctan(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^{5/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{7/2}}{7a}\right)}{4c^3}+$$

$$\frac{x\arctan(ax)^{5/2}}{4c^3\left(a^2x^2+1\right)^2}+\frac{5\arctan(ax)^{3/2}}{32ac^3\left(a^2x^2+1\right)^2}-$$

$$\frac{15\left(-\frac{1}{64}\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{8}\sqrt{\pi}\operatorname{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{1}{4}\arctan(ax)^{3/2}+\frac{1}{4}\sqrt{\arctan(ax)}\operatorname{si}\right.}{64ac^3}$$

$\Big\downarrow$ 3786

$$\frac{3\left(-\frac{5}{4}a\left(\frac{3\left(-\frac{\int \sin(2\arctan(ax))d\sqrt{\arctan(ax)}}{4a}+\frac{x\sqrt{\arctan(ax)}}{2\left(a^2x^2+1\right)}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4a}-\frac{\arctan(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^{5/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{7/2}}{7a}\right)}{4c^3}+\frac{x\arctan(ax)^{5/2}}{4c^3\left(a^2x^2+1\right)^2}+\frac{5\arctan(ax)^{3/2}}{32ac^3\left(a^2x^2+1\right)^2}-\frac{15\left(-\frac{1}{64}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{8}\sqrt{\pi}\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{1}{4}\arctan(ax)^{3/2}+\frac{1}{4}\sqrt{\arctan(ax)}\sin\ldots\right.}{64ac^3}$$

$$\Big\downarrow \; 3832$$

$$\frac{3\left(-\frac{5}{4}a\left(\frac{3\left(\frac{x\sqrt{\arctan(ax)}}{2\left(a^2x^2+1\right)}-\frac{\sqrt{\pi}\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{8a}+\frac{\arctan(ax)^{3/2}}{3a}\right)}{4a}-\frac{\arctan(ax)^{3/2}}{2a^2(a^2x^2+1)}\right)+\frac{x\arctan(ax)^{5/2}}{2(a^2x^2+1)}+\frac{\arctan(ax)^{7/2}}{7a}\right)}{4c^3}+\frac{x\arctan(ax)^{5/2}}{4c^3\left(a^2x^2+1\right)^2}+\frac{5\arctan(ax)^{3/2}}{32ac^3\left(a^2x^2+1\right)^2}-\frac{15\left(-\frac{1}{64}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{8}\sqrt{\pi}\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{1}{4}\arctan(ax)^{3/2}+\frac{1}{4}\sqrt{\arctan(ax)}\sin\ldots\right.}{64ac^3}$$

input

```
Int[ArcTan[a*x]^(5/2)/(c + a^2*c*x^2)^3,x]
```

output

```
(5*ArcTan[a*x]^(3/2))/(32*a*c^3*(1 + a^2*x^2)^2) + (x*ArcTan[a*x]^(5/2))/(
4*c^3*(1 + a^2*x^2)^2) + (3*((x*ArcTan[a*x]^(5/2))/(2*(1 + a^2*x^2)) + Arc
Tan[a*x]^(7/2)/(7*a) - (5*a*(-1/2*ArcTan[a*x]^(3/2)/(a^2*(1 + a^2*x^2)) +
(3*((x*Sqrt[ArcTan[a*x]])/(2*(1 + a^2*x^2)) + ArcTan[a*x]^(3/2)/(3*a) - (S
qrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(8*a)))/(4*a)))/4))/(4*c
^3) - (15*(ArcTan[a*x]^(3/2)/4 - (Sqrt[Pi/2]*FresnelS[2*Sqrt[2/Pi]*Sqrt[Ar
cTan[a*x]]])/64 - (Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/8 +
(Sqrt[ArcTan[a*x]]*Sin[2*ArcTan[a*x]])/4 + (Sqrt[ArcTan[a*x]]*Sin[4*ArcTan
[a*x]])/32))/(64*a*c^3)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5427
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2)^2, x_Sym
bol] :> Simp[x*((a + b*ArcTan[c*x])^p/(2*d*(d + e*x^2))), x] + (Simp[(a + b
*ArcTan[c*x])^(p + 1)/(2*b*c*d^2*(p + 1)), x] - Simp[b*c*(p/2)   Int[x*((a
+ b*ArcTan[c*x])^(p - 1)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

rule 5435
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[b*p*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p - 1)/(4*c*d*
(q + 1)^2)), x] + (-Simp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*d*
(q + 1))), x] + Simp[(2*q + 3)/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p, x], x] - Simp[b^2*p*((p - 1)/(4*(q + 1)^2))   Int[(d + e
*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

## Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.57

$$6144 \arctan(ax)^{\frac{7}{2}} \sqrt{\pi} + 14336 \arctan(ax)^{\frac{5}{2}} \sqrt{\pi} \sin(2 \arctan(ax)) + 1792 \arctan(ax)^{\frac{5}{2}} \sqrt{\pi} \sin(4 \arctan($$

input
```
int(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x)
```

output
```
1/57344/a/c^3/Pi^(1/2)*(6144*arctan(a*x)^(7/2)*Pi^(1/2)+14336*arctan(a*x)^
(5/2)*Pi^(1/2)*sin(2*arctan(a*x))+1792*arctan(a*x)^(5/2)*Pi^(1/2)*sin(4*ar
ctan(a*x))+17920*arctan(a*x)^(3/2)*Pi^(1/2)*cos(2*arctan(a*x))+1120*arctan
(a*x)^(3/2)*Pi^(1/2)*cos(4*arctan(a*x))+105*Pi*2^(1/2)*FresnelS(2*2^(1/2)/
Pi^(1/2)*arctan(a*x)^(1/2))-13440*arctan(a*x)^(1/2)*Pi^(1/2)*sin(2*arctan(
a*x))-420*arctan(a*x)^(1/2)*Pi^(1/2)*sin(4*arctan(a*x))+6720*Pi*FresnelS(2
*arctan(a*x)^(1/2)/Pi^(1/2)))
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx = \frac{\int \frac{\operatorname{atan}^{\frac{5}{2}}(ax)}{a^6x^6+3a^4x^4+3a^2x^2+1}\,dx}{c^3}$$

input
```
integrate(atan(a*x)**(5/2)/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(atan(a*x)**(5/2)/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1), x)/
c**3
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F]

$$\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx = \int \frac{\arctan(ax)^{\frac{5}{2}}}{(a^2cx^2+c)^3}\,dx$$

input
```
integrate(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^(5/2)/(a^2*c*x^2 + c)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx = \int \frac{\mathrm{atan}(a\,x)^{5/2}}{(c\,a^2\,x^2+c)^3}\,dx$$

input
```
int(atan(a*x)^(5/2)/(c + a^2*c*x^2)^3,x)
```

output
```
int(atan(a*x)^(5/2)/(c + a^2*c*x^2)^3, x)
```

## Reduce [F]

$$\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^3}\,dx = \frac{96\sqrt{atan\,(ax)}\,atan(ax)^3\,a^4x^4+192\sqrt{atan\,(ax)}\,atan(ax)^3\,a^2x^2+96\sqrt{atan\,(ax)}\,ata}{}$$

input
```
int(atan(a*x)^(5/2)/(a^2*c*x^2+c)^3,x)
```

output
```
(96*sqrt(atan(a*x))*atan(a*x)**3*a**4*x**4 + 192*sqrt(atan(a*x))*atan(a*x)
**3*a**2*x**2 + 96*sqrt(atan(a*x))*atan(a*x)**3 + 336*sqrt(atan(a*x))*atan
(a*x)**2*a**3*x**3 + 560*sqrt(atan(a*x))*atan(a*x)**2*a*x + 420*sqrt(atan(
a*x))*atan(a*x)*a**2*x**2 + 560*sqrt(atan(a*x))*atan(a*x) + 210*sqrt(atan(
a*x))*a*x - 1050*int(sqrt(atan(a*x))/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**
2 + 1),x)*a**5*x**4 - 2100*int(sqrt(atan(a*x))/(a**6*x**6 + 3*a**4*x**4 +
3*a**2*x**2 + 1),x)*a**3*x**2 - 1050*int(sqrt(atan(a*x))/(a**6*x**6 + 3*a*
*4*x**4 + 3*a**2*x**2 + 1),x)*a - 105*int((sqrt(atan(a*x))*x)/(atan(a*x)*a
**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a
**6*x**4 - 210*int((sqrt(atan(a*x))*x)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*
a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**4*x**2 - 105*int((sqr
t(atan(a*x))*x)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)
*a**2*x**2 + atan(a*x)),x)*a**2)/(896*a*c**3*(a**4*x**4 + 2*a**2*x**2 + 1)
)
```

# 3.876 $\int \frac{\arctan(ax)^{5/2}}{x\left(c+a^2cx^2\right)^3} \, dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\arctan(ax)^{5/2}}{x\left(c+a^2cx^2\right)^3} \, dx=\text{Int}\left(\frac{\arctan(ax)^{5/2}}{x\left(c+a^2cx^2\right)^3},x\right)$$

output
```
Defer(Int)(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^3,x)
```

## Mathematica [N/A]

Not integrable

Time = 1.75 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\arctan(ax)^{5/2}}{x\left(c+a^2cx^2\right)^3} \, dx=\int \frac{\arctan(ax)^{5/2}}{x\left(c+a^2cx^2\right)^3} \, dx$$

input
```
Integrate[ArcTan[a*x]^(5/2)/(x*(c + a^2*c*x^2)^3),x]
```

output
```
Integrate[ArcTan[a*x]^(5/2)/(x*(c + a^2*c*x^2)^3), x]
```

## Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{5/2}}{x\left(a^2cx^2+c\right)^3}\,dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int \frac{\arctan(ax)^{5/2}}{x\left(a^2cx^2+c\right)^3}dx$$

input
```
Int[ArcTan[a*x]^(5/2)/(x*(c + a^2*c*x^2)^3),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\arctan\left(ax\right)^{\frac{5}{2}}}{x\left(a^2c\,x^2+c\right)^3}dx$$

input
```
int(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^3,x)
```

output
```
int(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^3,x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{5/2}}{x\left(c+a^2cx^2\right)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 8.41 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{\arctan(ax)^{5/2}}{x\left(c+a^2cx^2\right)^3}\,dx = \frac{\int \frac{\operatorname{atan}^{\frac{5}{2}}(ax)}{a^6x^7+3a^4x^5+3a^2x^3+x}\,dx}{c^3}$$

input
```
integrate(atan(a*x)**(5/2)/x/(a**2*c*x**2+c)**3,x)
```

output
```
Integral(atan(a*x)**(5/2)/(a**6*x**7 + 3*a**4*x**5 + 3*a**2*x**3 + x), x)/
c**3
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{5/2}}{x\left(c+a^2cx^2\right)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax)^{5/2}}{x\left(c+a^2cx^2\right)^3}\,dx=\int \frac{\arctan\left(ax\right)^{\frac{5}{2}}}{\left(a^2cx^2+c\right)^3x}\,dx$$

input
```
integrate(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^3,x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^(5/2)/((a^2*c*x^2 + c)^3*x), x)
```

## Mupad [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax)^{5/2}}{x\left(c+a^2cx^2\right)^3}\,dx=\int \frac{\mathrm{atan}(a\,x)^{5/2}}{x\,\left(c\,a^2\,x^2+c\right)^3}\,dx$$

input
```
int(atan(a*x)^(5/2)/(x*(c + a^2*c*x^2)^3),x)
```

output
```
int(atan(a*x)^(5/2)/(x*(c + a^2*c*x^2)^3), x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.88

$$\int \frac{\arctan(ax)^{5/2}}{x\left(c+a^2cx^2\right)^3}\,dx = \frac{\int \frac{\sqrt{atan(ax)}\,atan(ax)^2}{a^6x^7+3a^4x^5+3a^2x^3+x}dx}{c^3}$$

input
```
int(atan(a*x)^(5/2)/x/(a^2*c*x^2+c)^3,x)
```

output
```
int((sqrt(atan(a*x))*atan(a*x)**2)/(a**6*x**7 + 3*a**4*x**5 + 3*a**2*x**3
+ x),x)/c**3
```

# 3.877 $\int x^m \sqrt{c+a^2 c x^2} \arctan(ax)^{5/2} \, dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int x^m \sqrt{c+a^2 c x^2} \arctan(ax)^{5/2} \, dx = \operatorname{Int}\left(x^m \sqrt{c+a^2 c x^2} \arctan(ax)^{5/2}, x\right)$$

output
```
Defer(Int)(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(5/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.13 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int x^m \sqrt{c+a^2 c x^2} \arctan(ax)^{5/2} \, dx = \int x^m \sqrt{c+a^2 c x^2} \arctan(ax)^{5/2} \, dx$$

input
```
Integrate[x^m*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[x^m*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \arctan(ax)^{5/2}\sqrt{a^2cx^2+c}\,dx$$

$$\Big\downarrow \text{5560}$$

$$\int x^m \arctan(ax)^{5/2}\sqrt{a^2cx^2+c}dx$$

input
```
Int[x^m*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int x^m\sqrt{a^2c\,x^2+c}\,\arctan\left(ax\right)^{\frac{5}{2}}dx$$

input
```
int(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(5/2),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(5/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int x^m \sqrt{c+a^2cx^2} \arctan(ax)^{5/2}\, dx = \int \sqrt{a^2cx^2+c}x^m \arctan(ax)^{\frac{5}{2}}\, dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^m*arctan(a*x)^(5/2), x)
```

## Sympy [F(-1)]

Timed out.

$$\int x^m \sqrt{c+a^2cx^2} \arctan(ax)^{5/2}\, dx = \text{Timed out}$$

input
```
integrate(x**m*(a**2*c*x**2+c)**(1/2)*atan(a*x)**(5/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int x^m \sqrt{c+a^2cx^2} \arctan(ax)^{5/2}\, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int x^m\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int x^m\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}\,dx=\int x^m\,\mathrm{atan}(a\,x)^{5/2}\,\sqrt{c\,a^2\,x^2+c}\,dx$$

input
```
int(x^m*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(x^m*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2), x)
```

## Reduce [N/A]

Not integrable

Time = 200.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int x^m\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}\,dx=\int x^m\sqrt{a^2c\,x^2+c}\,atan(ax)^{\frac{5}{2}}\,dx$$

input
```
int(x^m*(a^2*c*x^2+c)^(1/2)*atan(a*x)^(5/2),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^(1/2)*atan(a*x)^(5/2),x)
```

# 3.878 $\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}\,dx = \mathrm{Int}\left(x^2\sqrt{c+a^2cx^2}\arctan(ax)^{5/2},x\right)$$

output
```
Defer(Int)(x^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(5/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 3.34 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}\,dx = \int x^2\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}\,dx$$

input
```
Integrate[x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arctan(ax)^{5/2}\sqrt{a^2cx^2+c}\,dx$$

$$\Big\downarrow \text{5560}$$

$$\int x^2 \arctan(ax)^{5/2}\sqrt{a^2cx^2+c}dx$$

input
```
Int[x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int x^2\sqrt{a^2c\,x^2+c}\,\arctan\left(ax\right)^{\frac{5}{2}}dx$$

input
```
int(x^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(5/2),x)
```

output
```
int(x^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(a**2*c*x**2+c)**(1/2)*atan(a*x)**(5/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}\,dx = \int \sqrt{a^2cx^2+c}x^2\arctan{(ax)}^{\frac{5}{2}}\,dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*x^2*arctan(a*x)^(5/2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}\,dx = \int x^2\,\mathrm{atan}(a\,x)^{5/2}\,\sqrt{c\,a^2\,x^2+c}\,dx$$

input
```
int(x^2*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(x^2*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int x^2\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}\,dx = \sqrt{c}\left(\int \sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}\,atan(ax)^2\,x^2dx\right)$$

input
```
int(x^2*(a^2*c*x^2+c)^(1/2)*atan(a*x)^(5/2),x)
```

output
```
sqrt(c)*int(sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)**2*x**2,x)
```

# 3.879 $\int x\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int x\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}\,dx=\frac{5\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}{8a^2}-\frac{5x\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}{12a}+\frac{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}{3a^2c}-\frac{5c\text{Int}\left(\frac{1}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}},x\right)}{16a}-\frac{5c\text{Int}\left(\frac{\arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}},x\right)}{12a}$$

output

```
5/8*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2)/a^2-5/12*x*(a^2*c*x^2+c)^(1/2)*a
rctan(a*x)^(3/2)/a+1/3*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2)/a^2/c-5/16*c*
Defer(Int)(1/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)/a-5/12*c*Defer(Int)(
arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)/a
```

## Mathematica [N/A]

Not integrable

Time = 3.96 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int x\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}\,dx=\int x\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}\,dx$$

input
```
Integrate[x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}\,dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{5\int\sqrt{a^2cx^2+c}\arctan(ax)^{3/2}dx}{6a}$$

$$\Big\downarrow \text{5415}$$

$$\frac{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{5\left(\frac{3}{8}c\int\frac{1}{\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx+\frac{1}{2}c\int\frac{\arctan(ax)^{3/2}}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}-\frac{3\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}{4a}\right)}{6a}$$

$$\Big\downarrow \text{5560}$$

$$\frac{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}}{3a^2c}-\frac{5\left(\frac{3}{8}c\int\frac{1}{\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx+\frac{1}{2}c\int\frac{\arctan(ax)^{3/2}}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}-\frac{3\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}{4a}\right)}{6a}$$

input
```
Int[x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int x\sqrt{a^2c\,x^2+c}\,\arctan\left(ax\right)^{\frac{5}{2}}dx$$

input
```
int(x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(5/2),x)
```

output
```
int(x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int x\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int x\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}\,dx=\text{Timed out}$$

input
```
integrate(x*(a**2*c*x**2+c)**(1/2)*atan(a*x)**(5/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int x\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int x\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int x\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}\,dx = \int x\,\mathrm{atan}(a\,x)^{5/2}\,\sqrt{c\,a^2\,x^2+c}\,dx$$

input
```
int(x*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(x*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 175, normalized size of antiderivative = 7.29

$$\int x\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}\,dx = \frac{\sqrt{c}\left(16\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}\,atan(ax)^2\,a^2x^2+16\sqrt{a^2x^2+1}\,\sqrt{atan\,(a}\right.}{}$$

input
```
int(x*(a^2*c*x^2+c)^(1/2)*atan(a*x)^(5/2),x)
```

output
```
(sqrt(c)*(16*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)**2*a**2*x**2 +
16*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)**2 - 20*sqrt(a**2*x**2 +
1)*sqrt(atan(a*x))*atan(a*x)*a*x + 30*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))
- 20*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x))/(a**2*x**2 + 1),x
)*a - 15*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(atan(a*x)*a**2*x**2 +
atan(a*x)),x)*a))/(48*a**2)
```

# 3.880 $\int \sqrt{c+a^2cx^2}\arctan(ax)^{5/2}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \sqrt{c+a^2cx^2}\arctan(ax)^{5/2}\,dx = -\frac{5\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}{4a}+\frac{1}{2}x\sqrt{c+a^2cx^2}\arctan(ax)^{5/2} + \frac{15}{8}c\text{Int}\left(\frac{\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}},x\right)+\frac{1}{2}c\text{Int}\left(\frac{\arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}},x\right)$$

output
```
-5/4*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(3/2)/a+1/2*x*(a^2*c*x^2+c)^(1/2)*arc
tan(a*x)^(5/2)+15/8*c*Defer(Int)(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x)+
1/2*c*Defer(Int)(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \sqrt{c+a^2cx^2}\arctan(ax)^{5/2}\,dx = \int \sqrt{c+a^2cx^2}\arctan(ax)^{5/2}\,dx$$

input
```
Integrate[Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(ax)^{5/2}\sqrt{a^2cx^2+c}\,dx$$

$$\Big\downarrow \text{5415}$$

$$\frac{15}{8}c\int \frac{\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}c\int \frac{\arctan(ax)^{5/2}}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}-\frac{5\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}{4a}$$

$$\Big\downarrow \text{5560}$$

$$\frac{15}{8}c\int \frac{\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}c\int \frac{\arctan(ax)^{5/2}}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}-\frac{5\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}{4a}$$

input
```
Int[Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \sqrt{a^2 c\, x^2 + c}\, \arctan\left(a x\right)^{\frac{5}{2}} dx$$

input
```
int((a^2*c*x^2+c)^(1/2)*arctan(a*x)^(5/2),x)
```

output
```
int((a^2*c*x^2+c)^(1/2)*arctan(a*x)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \sqrt{c + a^2 c x^2} \arctan(ax)^{5/2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(1/2)*arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \sqrt{c + a^2 c x^2} \arctan(ax)^{5/2} \, dx = \text{Timed out}$$

input
```
integrate((a**2*c*x**2+c)**(1/2)*atan(a*x)**(5/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \sqrt{c+a^2cx^2}\arctan(ax)^{5/2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^(1/2)*arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int \sqrt{c+a^2cx^2}\arctan(ax)^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(1/2)*arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \sqrt{c+a^2cx^2}\arctan(ax)^{5/2}\,dx = \int \mathrm{atan}(a\,x)^{5/2}\,\sqrt{c\,a^2\,x^2+c}\,dx$$

input
```
int(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \sqrt{c+a^2cx^2}\arctan(ax)^{5/2}\,dx=\sqrt{c}\left(\int \sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}\,atan(ax)^2\,dx\right)$$

input
```
int((a^2*c*x^2+c)^(1/2)*atan(a*x)^(5/2),x)
```

output
```
sqrt(c)*int(sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)**2,x)
```

# 3.881 $\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}{x}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}{x}\,dx = \text{Int}\left(\frac{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}{x},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^(1/2)*arctan(a*x)^(5/2)/x,x)
```

## Mathematica [N/A]

Not integrable

Time = 3.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}{x}\,dx = \int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}{x}\,dx$$

input
```
Integrate[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2))/x,x]
```

output
```
Integrate[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2))/x, x]
```

## Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}{x}\,dx$$

$$\Big\downarrow \; 5560$$

$$\int \frac{\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}{x}dx$$

input
```
Int[(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2))/x,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{a^2c\,x^2+c}\,\arctan\left(ax\right)^{\frac{5}{2}}}{x}dx$$

input
```
int((a^2*c*x^2+c)^(1/2)*arctan(a*x)^(5/2)/x,x)
```

output
```
int((a^2*c*x^2+c)^(1/2)*arctan(a*x)^(5/2)/x,x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(1/2)*arctan(a*x)^(5/2)/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}{x}\,dx = \text{Timed out}$$

input
```
integrate((a**2*c*x**2+c)**(1/2)*atan(a*x)**(5/2)/x,x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}{x}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^(1/2)*arctan(a*x)^(5/2)/x,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(1/2)*arctan(a*x)^(5/2)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}{x}\,dx = \int \frac{\mathrm{atan}(a\,x)^{5/2}\,\sqrt{c\,a^2\,x^2+c}}{x}\,dx$$

input
```
int((atan(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2))/x,x)
```

output
```
int((atan(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2))/x, x)
```

## Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \frac{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}{x}\,dx = \sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,atan(ax)^2}{x}dx\right)$$

input
```
int((a^2*c*x^2+c)^(1/2)*atan(a*x)^(5/2)/x,x)
```

output
```
sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)**2)/x,x)
```

# 3.882 $\int x^m(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int x^m\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{5/2}\,dx=\text{Int}\left(x^m\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{5/2},x\right)$$

output
```
Defer(Int)(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int x^m\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{5/2}\,dx=\int x^m\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{5/2}\,dx$$

input
```
Integrate[x^m*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[x^m*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \arctan(ax)^{5/2} \left(a^2cx^2+c\right)^{3/2} dx$$

$$\Big\downarrow \; 5560$$

$$\int x^m \arctan(ax)^{5/2} \left(a^2cx^2+c\right)^{3/2} dx$$

input
```
Int[x^m*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int x^m \left(a^2 c\,x^2+c\right)^{\frac{3}{2}} \arctan\left(a x\right)^{\frac{5}{2}} dx$$

input
```
int(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int x^m \left(c+a^2cx^2\right)^{3/2} \arctan(ax)^{5/2}\,dx = \int \left(a^2cx^2+c\right)^{\frac{3}{2}} x^m \arctan\left(ax\right)^{\frac{5}{2}}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^(3/2)*x^m*arctan(a*x)^(5/2), x)
```

## Sympy [F(-1)]

Timed out.

$$\int x^m \left(c+a^2cx^2\right)^{3/2} \arctan(ax)^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(x**m*(a**2*c*x**2+c)**(3/2)*atan(a*x)**(5/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int x^m \left(c+a^2cx^2\right)^{3/2} \arctan(ax)^{5/2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int x^m \left(c+a^2cx^2\right)^{3/2} \arctan(ax)^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int x^m \left(c+a^2cx^2\right)^{3/2} \arctan(ax)^{5/2}\,dx = \int x^m\,\mathrm{atan}(a\,x)^{5/2}\,\left(c\,a^2\,x^2+c\right)^{3/2}\,dx$$

input
```
int(x^m*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(x^m*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2), x)
```

## Reduce [N/A]

Not integrable

Time = 200.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int x^m \left(c+a^2cx^2\right)^{3/2} \arctan(ax)^{5/2}\,dx = \int x^m \left(a^2c\,x^2+c\right)^{\frac{3}{2}} atan(ax)^{\frac{5}{2}}\,dx$$

input
```
int(x^m*(a^2*c*x^2+c)^(3/2)*atan(a*x)^(5/2),x)
```

output

```
int(x^m*(a^2*c*x^2+c)^(3/2)*atan(a*x)^(5/2),x)
```

# 3.883 $\int x^2(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int x^2\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{5/2}\,dx=\text{Int}\left(x^2\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{5/2},x\right)$$

output
```
Defer(Int)(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 3.98 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int x^2\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{5/2}\,dx=\int x^2\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{5/2}\,dx$$

input
```
Integrate[x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arctan(ax)^{5/2} \left(a^2cx^2+c\right)^{3/2} dx$$

$$\Big\downarrow \; 5560$$

$$\int x^2 \arctan(ax)^{5/2} \left(a^2cx^2+c\right)^{3/2} dx$$

input
```
Int[x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int x^2 \left(a^2 c\,x^2+c\right)^{\frac{3}{2}} \arctan\left(a x\right)^{\frac{5}{2}} dx$$

input
```
int(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x)
```

output
```
int(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int x^2 (c + a^2 c x^2)^{3/2} \arctan(ax)^{5/2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int x^2 (c + a^2 c x^2)^{3/2} \arctan(ax)^{5/2} \, dx = \text{Timed out}$$

input
```
integrate(x**2*(a**2*c*x**2+c)**(3/2)*atan(a*x)**(5/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int x^2 (c + a^2 c x^2)^{3/2} \arctan(ax)^{5/2} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int x^2 \left(c+a^2 c x^2\right)^{3/2} \arctan(ax)^{5/2} \, dx = \int (a^2 c x^2 + c)^{\frac{3}{2}} x^2 \arctan(ax)^{\frac{5}{2}} \, dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*x^2*arctan(a*x)^(5/2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int x^2 \left(c+a^2 c x^2\right)^{3/2} \arctan(ax)^{5/2} \, dx = \int x^2 \operatorname{atan}(a\,x)^{5/2} \left(c\,a^2\,x^2+c\right)^{3/2} dx$$

input
```
int(x^2*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(x^2*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.42

$$\int x^2 (c + a^2 c x^2)^{3/2} \arctan(ax)^{5/2} \, dx = \sqrt{c}\, c \left( \left( \int \sqrt{a^2x^2+1}\, \sqrt{atan(ax)}\, atan(ax)^2\, x^4 dx \right) a^2 + \int \sqrt{a^2x^2+1}\, \sqrt{ata$$

input
```
int(x^2*(a^2*c*x^2+c)^(3/2)*atan(a*x)^(5/2),x)
```

output
```
sqrt(c)*c*(int(sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)**2*x**4,x)*a*
*2 + int(sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)**2*x**2,x))
```

# 3.884 $\int x(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int x(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}\,dx = \frac{9c\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}{32a^2} + \frac{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}{16a^2} - \frac{3cx\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}{16a} - \frac{x(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}{8a} + \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}{5a^2c} - \frac{9c^2\mathrm{Int}\left(\frac{1}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}},x\right)}{64a} - \frac{c\mathrm{Int}\left(\frac{\sqrt{c+a^2cx^2}}{\sqrt{\arctan(ax)}},x\right)}{32a} - \frac{3c^2\mathrm{Int}\left(\frac{\arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}},x\right)}{16a}$$

output
```
9/32*c*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2)/a^2+1/16*(a^2*c*x^2+c)^(3/2)*
arctan(a*x)^(1/2)/a^2-3/16*c*x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(3/2)/a-1/8
*x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(3/2)/a+1/5*(a^2*c*x^2+c)^(5/2)*arctan(
a*x)^(5/2)/a^2/c-9/64*c^2*Defer(Int)(1/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/
2),x)/a-1/32*c*Defer(Int)((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)/a-3/16*
c^2*Defer(Int)(arctan(a*x)^(3/2)/(a^2*c*x^2+c)^(1/2),x)/a
```

## Mathematica [N/A]

Not integrable

Time = 2.61 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{5/2}\,dx=\int x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{5/2}\,dx$$

input
```
Integrate[x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}\,dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{\int\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^{3/2}dx}{2a}$$

$$\Big\downarrow \text{5415}$$

$$\frac{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-$$
$$\frac{\frac{1}{16}c\int\frac{\sqrt{a^2cx^2+c}}{\sqrt{\arctan(ax)}}dx+\frac{3}{4}c\int\sqrt{a^2cx^2+c}\arctan(ax)^{3/2}dx+\frac{1}{4}x\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}-\frac{\sqrt{\arctan(ax)}(a^2cx^2+c)}{8a}}{2a}$$

$$\Big\downarrow \text{5415}$$

$$\frac{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{\frac{1}{16}c\int\frac{\sqrt{a^2cx^2+c}}{\sqrt{\arctan(ax)}}dx+\frac{3}{4}c\left(\frac{3}{8}c\int\frac{1}{\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx+\frac{1}{2}c\int\frac{\arctan(ax)^{3/2}}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}-\frac{3\sqrt{a}}{}\right.}{2a}$$

$\downarrow$ 5560

$$\frac{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^{5/2}}{5a^2c}-\frac{\frac{1}{16}c\int\frac{\sqrt{a^2cx^2+c}}{\sqrt{\arctan(ax)}}dx+\frac{3}{4}c\left(\frac{3}{8}c\int\frac{1}{\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx+\frac{1}{2}c\int\frac{\arctan(ax)^{3/2}}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}-\frac{3\sqrt{a}}{}\right.}{2a}$$

input
```
Int[x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int x\left(a^{2} c\,x^{2}+c\right)^{\frac{3}{2}} \arctan\left(a x\right)^{\frac{5}{2}}d x$$

input
```
int(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x)
```

output
```
int(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int x(c + a^2cx^2)^{3/2} \arctan(ax)^{5/2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int x(c + a^2cx^2)^{3/2} \arctan(ax)^{5/2} \, dx = \text{Timed out}$$

input
```
integrate(x*(a**2*c*x**2+c)**(3/2)*atan(a*x)**(5/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int x(c + a^2cx^2)^{3/2} \arctan(ax)^{5/2} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int x(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int x(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}\,dx = \int x\,\mathrm{atan}(a\,x)^{5/2}\,\left(c\,a^2\,x^2+c\right)^{3/2}\,dx$$

input
```
int(x*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(x*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 299, normalized size of antiderivative = 12.46

$$\int x(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}\,dx = \frac{\sqrt{c}\,c\Big(64\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}\,atan(ax)^2\,a^4x^4+128\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}}{}$$

input
```
int(x*(a^2*c*x^2+c)^(3/2)*atan(a*x)^(5/2),x)
```

output
```
(sqrt(c)*c*(64*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)**2*a**4*x**4
+ 128*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)**2*a**2*x**2 + 64*sqrt
(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)**2 - 40*sqrt(a**2*x**2 + 1)*sqrt
(atan(a*x))*atan(a*x)*a**3*x**3 - 100*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*
atan(a*x)*a*x + 20*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*a**2*x**2 + 110*sqr
t(a**2*x**2 + 1)*sqrt(atan(a*x)) - 60*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a
*x))*atan(a*x))/(a**2*x**2 + 1),x)*a - 10*int((sqrt(a**2*x**2 + 1)*sqrt(at
an(a*x))*x**2)/(atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**3 - 55*int((sqrt(a*
*2*x**2 + 1)*sqrt(atan(a*x)))/(atan(a*x)*a**2*x**2 + atan(a*x)),x)*a))/(32
0*a**2)
```

# 3.885 $\int \left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{5/2}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{5/2}\,dx =$$
$$-\frac{15c\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}{16a}-\frac{5\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{3/2}}{24a}$$
$$+\frac{3}{8}cx\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}+\frac{1}{4}x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{5/2}+\frac{45}{32}c^2\text{Int}\left(\frac{\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}},x\right)+\frac{5}{16}c\text{Int}\left(\sqrt{\right.$$

output
```
-15/16*c*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(3/2)/a-5/24*(a^2*c*x^2+c)^(3/2)*
arctan(a*x)^(3/2)/a+3/8*c*x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(5/2)+1/4*x*(a
^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2)+45/32*c^2*Defer(Int)(arctan(a*x)^(1/2)
/(a^2*c*x^2+c)^(1/2),x)+5/16*c*Defer(Int)((a^2*c*x^2+c)^(1/2)*arctan(a*x)^
(1/2),x)+3/8*c^2*Defer(Int)(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.53 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{5/2}\,dx=\int \left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{5/2}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}\,dx$$

$$\Big\downarrow \text{5415}$$

$$\frac{5}{16}c\int \sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}dx+\frac{3}{4}c\int \sqrt{a^2cx^2+c}\arctan(ax)^{5/2}dx+\frac{1}{4}x\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}-\frac{5\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}{24a}$$

$$\Big\downarrow \text{5415}$$

$$\frac{5}{16}c\int \sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}dx+$$
$$\frac{3}{4}c\left(\frac{15}{8}c\int \frac{\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}c\int \frac{\arctan(ax)^{5/2}}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}-\frac{5\arctan(ax)^{3/2}\sqrt{a^2cx^2}}{4a}\right.$$
$$\frac{1}{4}x\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}-\frac{5\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}{24a}$$

$$\Big\downarrow \; 5560$$

$$\frac{5}{16}c\int \sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}dx+$$
$$\frac{3}{4}c\left(\frac{15}{8}c\int \frac{\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}c\int \frac{\arctan(ax)^{5/2}}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}-\frac{5\arctan(ax)^{3/2}\sqrt{a^2cx^2}}{4a}\right.$$
$$\frac{1}{4}x\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}-\frac{5\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}{24a}$$

input
```
Int[(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \left(a^2c\,x^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^{\frac{5}{2}}dx$$

input
```
int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x)
```

output
```
int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int (c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int (c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)**(5/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int (c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{5/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{5/2}\,dx=\int \mathrm{atan}(a\,x)^{5/2}\,\left(c\,a^2\,x^2+c\right)^{3/2}\,dx$$

input
```
int(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.61

$$\int (c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}\,dx=\sqrt{c}\,c\left(\left(\int\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}\,atan(ax)^2\,x^2dx\right)a^2+\int\sqrt{a^2x^2+1}\,\sqrt{ata

input
```
int((a^2*c*x^2+c)^(3/2)*atan(a*x)^(5/2),x)
```

output
```
sqrt(c)*c*(int(sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)**2*x**2,x)*a*
*2 + int(sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)**2,x))
```

# 3.886 $\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}{x}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}{x}\,dx = \text{Int}\left(\frac{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}{x},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2)/x,x)
```

## Mathematica [N/A]

Not integrable

Time = 2.84 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}{x}\,dx = \int \frac{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}{x}\,dx$$

input
```
Integrate[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2))/x,x]
```

output
```
Integrate[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2))/x, x]
```

## Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}}{x}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}}{x}dx$$

input
```
Int[((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2))/x,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{(a^2c\,x^2+c)^{\frac{3}{2}}\arctan\left(ax\right)^{\frac{5}{2}}}{x}dx$$

input
```
int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2)/x,x)
```

output
```
int((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2)/x,x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{(c + a^2cx^2)^{3/2} \arctan(ax)^{5/2}}{x} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2)/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(c + a^2cx^2)^{3/2} \arctan(ax)^{5/2}}{x} \, dx = \text{Timed out}$$

input
```
integrate((a**2*c*x**2+c)**(3/2)*atan(a*x)**(5/2)/x,x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(c + a^2cx^2)^{3/2} \arctan(ax)^{5/2}}{x} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2)/x,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{(c + a^2cx^2)^{3/2} \arctan(ax)^{5/2}}{x} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)*arctan(a*x)^(5/2)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(c + a^2cx^2)^{3/2} \arctan(ax)^{5/2}}{x} \, dx = \int \frac{\mathrm{atan}(a\,x)^{5/2} \, (c\,a^2\,x^2 + c)^{3/2}}{x} \, dx$$

input
```
int((atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2))/x,x)
```

output
```
int((atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2))/x, x)
```

## Reduce [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 210, normalized size of antiderivative = 8.08

$$\int \frac{(c + a^2cx^2)^{3/2} \arctan(ax)^{5/2}}{x} \, dx = \frac{\sqrt{c}\,c\Big(16\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}\,atan(ax)^2\,a^2x^2 + 64\sqrt{a^2x^2+1}\,\sqrt{atan}}{}$$

input
```
int((a^2*c*x^2+c)^(3/2)*atan(a*x)^(5/2)/x,x)
```

output
```
(sqrt(c)*c*(16*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)**2*a**2*x**2
+ 64*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)**2 - 20*sqrt(a**2*x**2
+ 1)*sqrt(atan(a*x))*atan(a*x)*a*x + 30*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)
) - 140*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x))/(a**2*x**2 + 1
),x)*a + 48*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)**2)/(a**2*x
**3 + x),x) - 15*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(atan(a*x)*a**2
*x**2 + atan(a*x)),x)*a))/48
```

# 3.887 $\int x^m (c+a^2 c x^2)^{5/2} \arctan(ax)^{5/2} \, dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int x^m \left(c+a^2 c x^2\right)^{5/2} \arctan(ax)^{5/2} \, dx = \text{Int}\left(x^m \left(c+a^2 c x^2\right)^{5/2} \arctan(ax)^{5/2}, x\right)$$

output
```
Defer(Int)(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.54 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int x^m \left(c+a^2 c x^2\right)^{5/2} \arctan(ax)^{5/2} \, dx = \int x^m \left(c+a^2 c x^2\right)^{5/2} \arctan(ax)^{5/2} \, dx$$

input
```
Integrate[x^m*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[x^m*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \arctan(ax)^{5/2} \left(a^2cx^2+c\right)^{5/2} dx$$

$$\Big\downarrow \text{5560}$$

$$\int x^m \arctan(ax)^{5/2} \left(a^2cx^2+c\right)^{5/2} dx$$

input
```
Int[x^m*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int x^m \left(a^2c\,x^2+c\right)^{\frac{5}{2}} \arctan\left(ax\right)^{\frac{5}{2}} dx$$

input
```
int(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.88

$$\int x^m \left(c+a^2cx^2\right)^{5/2} \arctan(ax)^{5/2} \, dx = \int \left(a^2cx^2+c\right)^{\frac{5}{2}} x^m \arctan\left(ax\right)^{\frac{5}{2}} \, dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*x^m*arcta
n(a*x)^(5/2), x)
```

## Sympy [F(-1)]

Timed out.

$$\int x^m \left(c+a^2cx^2\right)^{5/2} \arctan(ax)^{5/2} \, dx = \text{Timed out}$$

input
```
integrate(x**m*(a**2*c*x**2+c)**(5/2)*atan(a*x)**(5/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int x^m \left(c+a^2cx^2\right)^{5/2} \arctan(ax)^{5/2} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int x^m \left(c+a^2cx^2\right)^{5/2} \arctan(ax)^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int x^m \left(c+a^2cx^2\right)^{5/2} \arctan(ax)^{5/2}\,dx = \int x^m\,\mathrm{atan}(a\,x)^{5/2}\,\left(c\,a^2\,x^2+c\right)^{5/2}\,dx$$

input
```
int(x^m*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2),x)
```

output
```
int(x^m*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2), x)
```

## Reduce [N/A]

Not integrable

Time = 200.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int x^m \left(c+a^2cx^2\right)^{5/2} \arctan(ax)^{5/2}\,dx = \int x^m \left(a^2c\,x^2+c\right)^{\frac{5}{2}} atan(ax)^{\frac{5}{2}}\,dx$$

input
```
int(x^m*(a^2*c*x^2+c)^(5/2)*atan(a*x)^(5/2),x)
```

output

```
int(x^m*(a^2*c*x^2+c)^(5/2)*atan(a*x)^(5/2),x)
```

# 3.888 $\int x^2(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int x^2\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{5/2}\,dx=\text{Int}\left(x^2\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{5/2},x\right)$$

output
```
Defer(Int)(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 3.13 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int x^2\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{5/2}\,dx=\int x^2\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{5/2}\,dx$$

input
```
Integrate[x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arctan(ax)^{5/2} \left(a^2cx^2+c\right)^{5/2} \, dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int x^2 \arctan(ax)^{5/2} \left(a^2cx^2+c\right)^{5/2} dx$$

input
```
Int[x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int x^{2} \left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}} \arctan \left(a x \right)^{\frac{5}{2}}d x$$

input
```
int(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x)
```

output
```
int(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int x^2 \left(c + a^2 c x^2\right)^{5/2} \arctan(ax)^{5/2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int x^2 \left(c + a^2 c x^2\right)^{5/2} \arctan(ax)^{5/2} \, dx = \text{Timed out}$$

input
```
integrate(x**2*(a**2*c*x**2+c)**(5/2)*atan(a*x)**(5/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int x^2 \left(c + a^2 c x^2\right)^{5/2} \arctan(ax)^{5/2} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 1.46 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int x^2 (c + a^2 c x^2)^{5/2} \arctan(ax)^{5/2} \, dx = \int (a^2 c x^2 + c)^{\frac{5}{2}} x^2 \arctan(ax)^{\frac{5}{2}} \, dx$$

input
```
integrate(x^2*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)*x^2*arctan(a*x)^(5/2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int x^2 (c + a^2 c x^2)^{5/2} \arctan(ax)^{5/2} \, dx = \int x^2 \operatorname{atan}(a\,x)^{5/2} \left(c\,a^2\,x^2 + c\right)^{5/2} dx$$

input
```
int(x^2*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2),x)
```

output
```
int(x^2*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 97, normalized size of antiderivative = 3.73

$$\int x^2 (c + a^2 c x^2)^{5/2} \arctan(ax)^{5/2} \, dx = \sqrt{c}\, c^2 \left( \left( \int \sqrt{a^2x^2+1}\, \sqrt{atan(ax)}\, atan(ax)^2\, x^6 dx \right) a^4 + 2 \left( \int \sqrt{a^2x^2+1}\, \sqrt{\right.\right.$$

input
```
int(x^2*(a^2*c*x^2+c)^(5/2)*atan(a*x)^(5/2),x)
```

output
```
sqrt(c)*c**2*(int(sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)**2*x**6,x)
*a**4 + 2*int(sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)**2*x**4,x)*a**
2 + int(sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)**2*x**2,x))
```

# 3.889 $\int x(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int x(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}\,dx = \frac{75c^2\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}{448a^2} + \frac{25c(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}{672a^2} + \frac{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}{56a^2} - \frac{25c^2x\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}{224a} - \frac{25cx(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}{336a} - \frac{5x(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}{84a} + \frac{(c+a^2cx^2)^{7/2}\arctan(ax)^{5/2}}{7a^2c} - \frac{75c^3\text{Int}\left(\frac{1}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}},x\right)}{896a} - \frac{25c^2\text{Int}\left(\frac{\sqrt{c+a^2cx^2}}{\sqrt{\arctan(ax)}},x\right)}{1344a} - \frac{c\text{Int}\left(\frac{(c+a^2cx^2)^{3/2}}{\sqrt{\arctan(ax)}},x\right)}{112a} - \frac{25c^3\text{Int}\left(\frac{\arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}},x\right)}{224a}$$

output
```
75/448*c^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2)/a^2+25/672*c*(a^2*c*x^2+c
)^(3/2)*arctan(a*x)^(1/2)/a^2+1/56*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(1/2)/a
^2-25/224*c^2*x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(3/2)/a-25/336*c*x*(a^2*c*
x^2+c)^(3/2)*arctan(a*x)^(3/2)/a-5/84*x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3
/2)/a+1/7*(a^2*c*x^2+c)^(7/2)*arctan(a*x)^(5/2)/a^2/c-75/896*c^3*Defer(Int
)(1/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)/a-25/1344*c^2*Defer(Int)((a^2
*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)/a-1/112*c*Defer(Int)((a^2*c*x^2+c)^(3
/2)/arctan(a*x)^(1/2),x)/a-25/224*c^3*Defer(Int)(arctan(a*x)^(3/2)/(a^2*c*
x^2+c)^(1/2),x)/a
```

## Mathematica [N/A]

Not integrable

Time = 4.65 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{5/2}\,dx=\int x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{5/2}\,dx$$

input
```
Integrate[x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2), x]
```

## Rubi [N/A]

Not integrable

Time = 1.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^{5/2}\,dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^{7/2}}{7a^2c}-\frac{5\int \left(a^2cx^2+c\right)^{5/2}\arctan(ax)^{3/2}dx}{14a}$$

$\big\downarrow$ 5415

$$\frac{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^{7/2}}{7a^2c}-$$
$$\frac{5\left(\frac{1}{40}c\int \frac{\left(a^2cx^2+c\right)^{3/2}}{\sqrt{\arctan(ax)}}dx+\frac{5}{6}c\int \left(a^2cx^2+c\right)^{3/2}\arctan(ax)^{3/2}dx+\frac{1}{6}x\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{5/2}-\frac{\sqrt{\arctan(ax)}(a}{20a}\right.}{14a}$$

$\big\downarrow$ 5415

$$\frac{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^{7/2}}{7a^2c}-$$
$$5\left(\frac{1}{40}c\int \frac{\left(a^2cx^2+c\right)^{3/2}}{\sqrt{\arctan(ax)}}dx+\frac{5}{6}c\left(\frac{1}{16}c\int \frac{\sqrt{a^2cx^2+c}}{\sqrt{\arctan(ax)}}dx+\frac{3}{4}c\int \sqrt{a^2cx^2+c}\arctan(ax)^{3/2}dx+\frac{1}{4}x\arctan(ax)^{3/2}\left(a^2cx^2\right.\right.\right.$$
$$14a$$

$\big\downarrow$ 5415

$$\frac{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^{7/2}}{7a^2c}-$$
$$5\left(\frac{1}{40}c\int \frac{\left(a^2cx^2+c\right)^{3/2}}{\sqrt{\arctan(ax)}}dx+\frac{5}{6}c\left(\frac{1}{16}c\int \frac{\sqrt{a^2cx^2+c}}{\sqrt{\arctan(ax)}}dx+\frac{3}{4}c\left(\frac{3}{8}c\int \frac{1}{\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx+\frac{1}{2}c\int \frac{\arctan(ax)^{3/2}}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\,\text{ar}\right.\right.\right.$$

$\big\downarrow$ 5560

$$\frac{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^{7/2}}{7a^2c}-$$
$$5\left(\frac{1}{40}c\int \frac{\left(a^2cx^2+c\right)^{3/2}}{\sqrt{\arctan(ax)}}dx+\frac{5}{6}c\left(\frac{1}{16}c\int \frac{\sqrt{a^2cx^2+c}}{\sqrt{\arctan(ax)}}dx+\frac{3}{4}c\left(\frac{3}{8}c\int \frac{1}{\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx+\frac{1}{2}c\int \frac{\arctan(ax)^{3/2}}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\,\text{ar}\right.\right.\right.$$

input
```
Int[x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int x\left(a^{2} c\,x^{2}+c\right)^{\frac{5}{2}} \arctan\left(a x\right)^{\frac{5}{2}}d x$$

input
```
int(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x)
```

output
```
int(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int x(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int x(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(x*(a**2*c*x**2+c)**(5/2)*atan(a*x)**(5/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int x(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int x(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int x(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}\,dx = \int x\,\mathrm{atan}(a\,x)^{5/2}\,\left(c\,a^2\,x^2+c\right)^{5/2}\,dx$$

input
```
int(x*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2),x)
```

output
```
int(x*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 424, normalized size of antiderivative = 17.67

$$\int x(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}\,dx = \frac{\sqrt{c}\,c^2\Big(384\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}\,atan(ax)^2\,a^6x^6+1152\sqrt{a^2x^2+1}\,\sqrt{atan\,(}$$

input
```
int(x*(a^2*c*x^2+c)^(5/2)*atan(a*x)^(5/2),x)
```

output
```
(sqrt(c)*c**2*(384*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)**2*a**6*x
**6 + 1152*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)**2*a**4*x**4 + 11
52*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)**2*a**2*x**2 + 384*sqrt(a
**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)**2 - 160*sqrt(a**2*x**2 + 1)*sqrt(
atan(a*x))*atan(a*x)*a**5*x**5 - 520*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*a
tan(a*x)*a**3*x**3 - 660*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)*a*x
 + 48*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*a**4*x**4 + 196*sqrt(a**2*x**2 +
 1)*sqrt(atan(a*x))*a**2*x**2 + 598*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)) -
300*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x))/(a**2*x**2 + 1),x)
*a - 24*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**4)/(atan(a*x)*a**2*x**
2 + atan(a*x)),x)*a**5 - 98*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**2)
/(atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**3 - 299*int((sqrt(a**2*x**2 + 1)*
sqrt(atan(a*x)))/(atan(a*x)*a**2*x**2 + atan(a*x)),x)*a))/(2688*a**2)
```

# 3.890 $\int (c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 23

$$\int (c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}\,dx = -\frac{25c^2\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}{32a}$$
$$-\frac{25c(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}{144a}-\frac{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}{12a}$$
$$+\frac{5}{16}c^2x\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}+\frac{5}{24}cx(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}+\frac{1}{6}x(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}+\frac{75}{64}c$$

output
```
-25/32*c^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(3/2)/a-25/144*c*(a^2*c*x^2+c)^
(3/2)*arctan(a*x)^(3/2)/a-1/12*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(3/2)/a+5/1
6*c^2*x*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(5/2)+5/24*c*x*(a^2*c*x^2+c)^(3/2)
*arctan(a*x)^(5/2)+1/6*x*(a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2)+75/64*c^3*D
efer(Int)(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(1/2),x)+25/96*c^2*Defer(Int)((a
^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2),x)+1/8*c*Defer(Int)((a^2*c*x^2+c)^(3/2
)*arctan(a*x)^(1/2),x)+5/16*c^3*Defer(Int)(arctan(a*x)^(5/2)/(a^2*c*x^2+c)
^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}\,dx=\int (c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.90 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(ax)^{5/2}\left(a^2cx^2+c\right)^{5/2}\,dx$$

$$\Big\downarrow \ 5415$$

$$\frac{1}{8}c\int \left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}dx+\frac{5}{6}c\int \left(a^2cx^2+c\right)^{3/2}\arctan(ax)^{5/2}dx+\frac{1}{6}x\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^{5/2}-\frac{\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{5/2}}{12a}$$

$$\Big\downarrow \ 5415$$

$$\frac{1}{8}c\int \left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}dx+$$
$$\frac{5}{6}c\left(\frac{5}{16}c\int \sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}dx+\frac{3}{4}c\int \sqrt{a^2cx^2+c}\arctan(ax)^{5/2}dx+\frac{1}{4}x\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}\right.$$
$$\frac{1}{6}x\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^{5/2}-\frac{\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{5/2}}{12a}$$

$$\Big\downarrow \text{5415}$$

$$\frac{1}{8}c\int \left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}dx+$$
$$\frac{5}{6}c\left(\frac{5}{16}c\int \sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}dx+\frac{3}{4}c\left(\frac{15}{8}c\int \frac{\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}c\int \frac{\arctan(ax)^{5/2}}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax\right.\right.$$
$$\frac{1}{6}x\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^{5/2}-\frac{\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{5/2}}{12a}$$

$$\Big\downarrow \text{5560}$$

$$\frac{1}{8}c\int \left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}dx+$$
$$\frac{5}{6}c\left(\frac{5}{16}c\int \sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}dx+\frac{3}{4}c\left(\frac{15}{8}c\int \frac{\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}c\int \frac{\arctan(ax)^{5/2}}{\sqrt{a^2cx^2+c}}dx+\frac{1}{2}x\arctan(ax\right.\right.$$
$$\frac{1}{6}x\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^{5/2}-\frac{\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{5/2}}{12a}$$

input
```
Int[(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \left(a^2c\,x^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^{\frac{5}{2}}dx$$

input
```
int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x)
```

output
```
int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \left(c + a^2 c x^2\right)^{5/2} \arctan(ax)^{5/2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \left(c + a^2 c x^2\right)^{5/2} \arctan(ax)^{5/2} \, dx = \text{Timed out}$$

input
```
integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)**(5/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \left(c + a^2 c x^2\right)^{5/2} \arctan(ax)^{5/2} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int \left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{5/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{5/2}\,dx=\int \operatorname{atan}(a\,x)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^{5/2}\,dx$$

input
```
int(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2),x)
```

output
```
int(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 94, normalized size of antiderivative = 4.09

$$\int (c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}\,dx = \sqrt{c}\,c^2\left(\left(\left(\int \sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}\,atan(ax)^2\,x^4dx\right)a^4+2\left(\int \sqrt{a^2x^2+1}\,\sqrt{\right.\right.\right.$$

input
```
int((a^2*c*x^2+c)^(5/2)*atan(a*x)^(5/2),x)
```

output
```
sqrt(c)*c**2*(int(sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)**2*x**4,x)
*a**4 + 2*int(sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)**2*x**2,x)*a**
2 + int(sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)**2,x))
```

# 3.891 $\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}{x}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}{x}\,dx = \text{Int}\left(\frac{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}{x},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2)/x,x)
```

## Mathematica [N/A]

Not integrable

Time = 3.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}{x}\,dx = \int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}{x}\,dx$$

input
```
Integrate[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2))/x,x]
```

output
```
Integrate[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2))/x, x]
```

## Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{5/2} \left(a^2cx^2+c\right)^{5/2}}{x}dx$$

$$\Big\downarrow \ 5560$$

$$\int \frac{\arctan(ax)^{5/2} \left(a^2cx^2+c\right)^{5/2}}{x}dx$$

input
```
Int[((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2))/x,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\left(a^2c\,x^2+c\right)^{\frac{5}{2}} \arctan\left(ax\right)^{\frac{5}{2}}}{x}dx$$

input
```
int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2)/x,x)
```

output
```
int((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2)/x,x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{5/2} \arctan(ax)^{5/2}}{x} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2)/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^{5/2} \arctan(ax)^{5/2}}{x} \, dx = \text{Timed out}$$

input
```
integrate((a**2*c*x**2+c)**(5/2)*atan(a*x)**(5/2)/x,x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{5/2} \arctan(ax)^{5/2}}{x} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2)/x,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)*arctan(a*x)^(5/2)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}{x}\,dx = \int \frac{\mathrm{atan}(a\,x)^{5/2}\,(c\,a^2\,x^2+c)^{5/2}}{x}\,dx$$

input
```
int((atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2))/x,x)
```

output
```
int((atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2))/x, x)
```

## Reduce [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 335, normalized size of antiderivative = 12.88

$$\int \frac{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}{x}\,dx = \frac{\sqrt{c}\,c^2\Big(192\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}\,atan(ax)^2\,a^4x^4+704\sqrt{a^2x^2+1}\,\sqrt{a}$$

input
```
int((a^2*c*x^2+c)^(5/2)*atan(a*x)^(5/2)/x,x)
```

output
```
(sqrt(c)*c**2*(192*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)**2*a**4*x
**4 + 704*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)**2*a**2*x**2 + 147
2*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)**2 - 120*sqrt(a**2*x**2 +
1)*sqrt(atan(a*x))*atan(a*x)*a**3*x**3 - 700*sqrt(a**2*x**2 + 1)*sqrt(atan
(a*x))*atan(a*x)*a*x + 60*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*a**2*x**2 +
930*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)) - 2980*int((sqrt(a**2*x**2 + 1)*sq
rt(atan(a*x))*atan(a*x))/(a**2*x**2 + 1),x)*a + 960*int((sqrt(a**2*x**2 +
1)*sqrt(atan(a*x))*atan(a*x)**2)/(a**2*x**3 + x),x) - 30*int((sqrt(a**2*x*
*2 + 1)*sqrt(atan(a*x))*x**2)/(atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**3 -
465*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(atan(a*x)*a**2*x**2 + atan(
a*x)),x)*a))/960
```

# 3.892 $\int \frac{x^m \arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^m \arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = \mathrm{Int}\left(\frac{x^m \arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}},x\right)$$

output
```
Defer(Int)(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^m \arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^m \arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx$$

input
```
Integrate[(x^m*ArcTan[a*x]^(5/2))/Sqrt[c + a^2*c*x^2],x]
```

output
```
Integrate[(x^m*ArcTan[a*x]^(5/2))/Sqrt[c + a^2*c*x^2], x]
```

## Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \arctan(ax)^{5/2}}{\sqrt{a^2cx^2+c}}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{x^m \arctan(ax)^{5/2}}{\sqrt{a^2cx^2+c}}dx$$

input
```
Int[(x^m*ArcTan[a*x]^(5/2))/Sqrt[c + a^2*c*x^2],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^m \arctan(ax)^{\frac{5}{2}}}{\sqrt{a^2c\,x^2+c}}dx$$

input
```
int(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m \arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^m \arctan(ax)^{\frac{5}{2}}}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(x^m*arctan(a*x)^(5/2)/sqrt(a^2*c*x^2 + c), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^m \arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = \text{Timed out}$$

input
```
integrate(x**m*atan(a*x)**(5/2)/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m \arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m \arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^m \arctan(ax)^{\frac{5}{2}}}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^m*arctan(a*x)^(5/2)/sqrt(a^2*c*x^2 + c), x)
```

## Mupad [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m \arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^m\,\mathrm{atan}(a\,x)^{5/2}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int((x^m*atan(a*x)^(5/2))/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int((x^m*atan(a*x)^(5/2))/(c + a^2*c*x^2)^(1/2), x)
```

## Reduce [N/A]

Not integrable

Time = 124.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.69

$$\int \frac{x^m \arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{c}\left(\int \frac{x^m\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,atan(ax)^2}{a^2x^2+1}dx\right)}{c}$$

input
```
int(x^m*atan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

output
```
(sqrt(c)*int((x**m*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)**2)/(a**2
*x**2 + 1),x))/c
```

# 3.893 $\int \frac{x^3 \arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^3 \arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = \frac{5\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}{8a^4c} - \frac{5x\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}{12a^3c} - \frac{2\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}{3a^4c} + \frac{x^2\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}{3a^2c} - \frac{5\text{Int}\left(\frac{1}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}},x\right)}{16a^3} + \frac{25\text{Int}\left(\frac{\arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}},x\right)}{12a^3}$$

output
```
5/8*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2)/a^4/c-5/12*x*(a^2*c*x^2+c)^(1/2)
*arctan(a*x)^(3/2)/a^3/c-2/3*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(5/2)/a^4/c+1
/3*x^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(5/2)/a^2/c-5/16*Defer(Int)(1/(a^2*
c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)/a^3+25/12*Defer(Int)(arctan(a*x)^(3/2)
/(a^2*c*x^2+c)^(1/2),x)/a^3
```

## Mathematica [N/A]

Not integrable

Time = 3.41 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^3 \arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^3 \arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx$$

input
```
Integrate[(x^3*ArcTan[a*x]^(5/2))/Sqrt[c + a^2*c*x^2],x]
```

output
```
Integrate[(x^3*ArcTan[a*x]^(5/2))/Sqrt[c + a^2*c*x^2], x]
```

## Rubi [N/A]

Not integrable

Time = 1.32 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \arctan(ax)^{5/2}}{\sqrt{a^2cx^2+c}}\,dx$$

$$\Big\downarrow \text{5487}$$

$$-\frac{5\int \frac{x^2\arctan(ax)^{3/2}}{\sqrt{a^2cx^2+c}}dx}{6a}-\frac{2\int \frac{x\arctan(ax)^{5/2}}{\sqrt{a^2cx^2+c}}dx}{3a^2}+\frac{x^2\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}{3a^2c}$$

$$\Big\downarrow \text{5465}$$

$$-\frac{2\left(\frac{\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}{a^2c}-\frac{5\int \frac{\arctan(ax)^{3/2}}{\sqrt{a^2cx^2+c}}dx}{2a}\right)}{3a^2}-\frac{5\int \frac{x^2\arctan(ax)^{3/2}}{\sqrt{a^2cx^2+c}}dx}{6a}+\frac{x^2\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}{3a^2c}$$

$$\Big\downarrow \text{5487}$$

$$-\frac{5\left(-\frac{3\int \frac{x\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}}dx}{4a}-\frac{\int \frac{\arctan(ax)^{3/2}}{\sqrt{a^2cx^2+c}}dx}{2a^2}+\frac{x\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}{2a^2c}\right)}{6a}-\frac{2\left(\frac{\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}{a^2c}-\frac{5\int \frac{\arctan(ax)^{3/2}}{\sqrt{a^2cx^2+c}}dx}{2a}\right)}{3a^2}+\frac{x^2\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}{3a^2c}$$

$$\Big\downarrow \text{ 5465}$$

$$-\frac{5\left(-\frac{3\left(\frac{\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}{a^2c}-\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx}{2a}\right)}{4a}-\frac{\int \frac{\arctan(ax)^{3/2}}{\sqrt{a^2cx^2+c}}dx}{2a^2}+\frac{x\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}{2a^2c}\right)}{6a}-\frac{2\left(\frac{\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}{a^2c}-\frac{5\int \frac{\arctan(ax)^{3/2}}{\sqrt{a^2cx^2+c}}dx}{2a}\right)}{3a^2}+\frac{x^2\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}{3a^2c}$$

$$\Big\downarrow \text{ 5560}$$

$$-\frac{5\left(-\frac{3\left(\frac{\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}{a^2c}-\frac{\int \frac{1}{\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx}{2a}\right)}{4a}-\frac{\int \frac{\arctan(ax)^{3/2}}{\sqrt{a^2cx^2+c}}dx}{2a^2}+\frac{x\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}{2a^2c}\right)}{6a}-\frac{2\left(\frac{\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}{a^2c}-\frac{5\int \frac{\arctan(ax)^{3/2}}{\sqrt{a^2cx^2+c}}dx}{2a}\right)}{3a^2}+\frac{x^2\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}{3a^2c}$$

input
```
Int[(x^3*ArcTan[a*x]^(5/2))/Sqrt[c + a^2*c*x^2],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^3 \arctan(ax)^{\frac{5}{2}}}{\sqrt{a^2 c\, x^2 + c}} dx$$

input
```
int(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3 \arctan(ax)^{5/2}}{\sqrt{c + a^2cx^2}} dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^3 \arctan(ax)^{5/2}}{\sqrt{c + a^2cx^2}} dx = \text{Timed out}$$

input
```
integrate(x**3*atan(a*x)**(5/2)/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3 \arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x^3 \arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^3 \arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^3\,\mathrm{atan}(a\,x)^{5/2}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int((x^3*atan(a*x)^(5/2))/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int((x^3*atan(a*x)^(5/2))/(c + a^2*c*x^2)^(1/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 178, normalized size of antiderivative = 6.85

$$\int \frac{x^3 \arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{c}\left(16\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}\,atan(ax)^2\,a^2x^2 - 32\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}\,atan(a$$

input
```
int(x^3*atan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

output
```
(sqrt(c)*(16*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)**2*a**2*x**2 -
32*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)**2 - 20*sqrt(a**2*x**2 +
1)*sqrt(atan(a*x))*atan(a*x)*a*x + 30*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))
+ 100*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x))/(a**2*x**2 + 1),
x)*a - 15*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(atan(a*x)*a**2*x**2 +
 atan(a*x)),x)*a))/(48*a**4*c)
```

# 3.894 $\int \frac{x^2 \arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^2 \arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = -\frac{5\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}{4a^3c} + \frac{x\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}{2a^2c} + \frac{15\text{Int}\left(\frac{\sqrt{\arctan(ax)}}{\sqrt{c+a^2cx^2}},x\right)}{8a^2} - \frac{\text{Int}\left(\frac{\arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}},x\right)}{2a^2}$$

output
```
-5/4*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(3/2)/a^3/c+1/2*x*(a^2*c*x^2+c)^(1/2)
*arctan(a*x)^(5/2)/a^2/c+15/8*Defer(Int)(arctan(a*x)^(1/2)/(a^2*c*x^2+c)^(
1/2),x)/a^2-1/2*Defer(Int)(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)/a^2
```

## Mathematica [N/A]

Not integrable

Time = 2.62 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^2 \arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^2 \arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx$$

input
```
Integrate[(x^2*ArcTan[a*x]^(5/2))/Sqrt[c + a^2*c*x^2],x]
```

output
```
Integrate[(x^2*ArcTan[a*x]^(5/2))/Sqrt[c + a^2*c*x^2], x]
```

## Rubi [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \arctan(ax)^{5/2}}{\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5487}$$

$$-\frac{5\int \frac{x\arctan(ax)^{3/2}}{\sqrt{a^2cx^2+c}}dx}{4a}-\frac{\int \frac{\arctan(ax)^{5/2}}{\sqrt{a^2cx^2+c}}dx}{2a^2}+\frac{x\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}{2a^2c}$$

$$\Big\downarrow \text{5465}$$

$$-\frac{5\left(\frac{\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\int \frac{\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}}dx}{2a}\right)}{4a}-\frac{\int \frac{\arctan(ax)^{5/2}}{\sqrt{a^2cx^2+c}}dx}{2a^2}+\frac{x\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}{2a^2c}$$

$$\Big\downarrow \text{5560}$$

$$-\frac{5\left(\frac{\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}{a^2c}-\frac{3\int \frac{\sqrt{\arctan(ax)}}{\sqrt{a^2cx^2+c}}dx}{2a}\right)}{4a}-\frac{\int \frac{\arctan(ax)^{5/2}}{\sqrt{a^2cx^2+c}}dx}{2a^2}+\frac{x\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}{2a^2c}$$

input
```
Int[(x^2*ArcTan[a*x]^(5/2))/Sqrt[c + a^2*c*x^2],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^2 \arctan{(ax)}^{\frac{5}{2}}}{\sqrt{a^2 c\,x^2 + c}}dx$$

input
```
int(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2 \arctan(ax)^{5/2}}{\sqrt{c + a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^2 \arctan(ax)^{5/2}}{\sqrt{c + a^2cx^2}}\,dx = \text{Timed out}$$

input
```
integrate(x**2*atan(a*x)**(5/2)/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2 \arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^2 \arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^2 \arctan\left(ax\right)^{\frac{5}{2}}}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2*arctan(a*x)^(5/2)/sqrt(a^2*c*x^2 + c), x)
```

## Mupad [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^2 \arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x^2\,\mathrm{atan}(a\,x)^{5/2}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int((x^2*atan(a*x)^(5/2))/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int((x^2*atan(a*x)^(5/2))/(c + a^2*c*x^2)^(1/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.69

$$\int \frac{x^2 \arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,atan(ax)^2x^2}{a^2x^2+1}dx\right)}{c}$$

input
```
int(x^2*atan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

output
```
(sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)**2*x**2)/(a**2
*x**2 + 1),x))/c
```

# 3.895 $\int \frac{x \arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x \arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}{a^2c} - \frac{5\text{Int}\left(\frac{\arctan(ax)^{3/2}}{\sqrt{c+a^2cx^2}},x\right)}{2a}$$

output
```
(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(5/2)/a^2/c-5/2*Defer(Int)(arctan(a*x)^(3/
2)/(a^2*c*x^2+c)^(1/2),x)/a
```

## Mathematica [N/A]

Not integrable

Time = 0.86 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x \arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x \arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx$$

input
```
Integrate[(x*ArcTan[a*x]^(5/2))/Sqrt[c + a^2*c*x^2],x]
```

output
```
Integrate[(x*ArcTan[a*x]^(5/2))/Sqrt[c + a^2*c*x^2], x]
```

## Rubi [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \arctan(ax)^{5/2}}{\sqrt{a^2cx^2+c}}dx$$
$$\Big\downarrow \text{5465}$$
$$\frac{\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}{a^2c}-\frac{5\int \frac{\arctan(ax)^{3/2}}{\sqrt{a^2cx^2+c}}dx}{2a}$$
$$\Big\downarrow \text{5560}$$
$$\frac{\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}{a^2c}-\frac{5\int \frac{\arctan(ax)^{3/2}}{\sqrt{a^2cx^2+c}}dx}{2a}$$

input
```
Int[(x*ArcTan[a*x]^(5/2))/Sqrt[c + a^2*c*x^2],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{x \arctan\left(ax\right)^{\frac{5}{2}}}{\sqrt{a^2c\,x^2+c}}dx$$

input
```
int(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x \arctan(ax)^{5/2}}{\sqrt{c + a^2 c x^2}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x \arctan(ax)^{5/2}}{\sqrt{c + a^2 c x^2}} \, dx = \text{Timed out}$$

input
```
integrate(x*atan(a*x)**(5/2)/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x \arctan(ax)^{5/2}}{\sqrt{c + a^2 c x^2}} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x \arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x \arctan{(ax)}^{\frac{5}{2}}}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(x*arctan(a*x)^(5/2)/sqrt(a^2*c*x^2 + c), x)
```

## Mupad [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x \arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{x\,\mathrm{atan}(a\,x)^{5/2}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int((x*atan(a*x)^(5/2))/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int((x*atan(a*x)^(5/2))/(c + a^2*c*x^2)^(1/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.75

$$\int \frac{x\arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,atan(ax)^2x}{a^2x^2+1}dx\right)}{c}$$

input
```
int(x*atan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

output
```
(sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)**2*x)/(a**2*x*
*2 + 1),x))/c
```

# 3.896 $\int \frac{\arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = \operatorname{Int}\left(\frac{\arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}},x\right)$$

output
```
Defer(Int)(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx$$

input
```
Integrate[ArcTan[a*x]^(5/2)/Sqrt[c + a^2*c*x^2],x]
```

output
```
Integrate[ArcTan[a*x]^(5/2)/Sqrt[c + a^2*c*x^2], x]
```

## Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{5/2}}{\sqrt{a^2cx^2+c}}\,dx$$
$$\Big\downarrow \ 5560$$
$$\int \frac{\arctan(ax)^{5/2}}{\sqrt{a^2cx^2+c}}dx$$

input
```
Int[ArcTan[a*x]^(5/2)/Sqrt[c + a^2*c*x^2],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{\arctan\left(ax\right)^{\frac{5}{2}}}{\sqrt{a^2c\,x^2+c}}dx$$

input
```
int(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 100.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{\arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\operatorname{atan}^{\frac{5}{2}}(ax)}{\sqrt{c\,(a^2x^2+1)}}\,dx$$

input
```
integrate(atan(a*x)**(5/2)/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(atan(a*x)**(5/2)/sqrt(c*(a**2*x**2 + 1)), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan{(ax)}^{\frac{5}{2}}}{\sqrt{a^2cx^2+c}}\,dx$$

input
```
integrate(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^(5/2)/sqrt(a^2*c*x^2 + c), x)
```

## Mupad [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = \int \frac{\mathrm{atan}(a\,x)^{5/2}}{\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(atan(a*x)^(5/2)/(c + a^2*c*x^2)^(1/2),x)
```

output
```
int(atan(a*x)^(5/2)/(c + a^2*c*x^2)^(1/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.78

$$\int \frac{\arctan(ax)^{5/2}}{\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,atan(ax)^2}{a^2x^2+1}dx\right)}{c}$$

input
```
int(atan(a*x)^(5/2)/(a^2*c*x^2+c)^(1/2),x)
```

output
```
(sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)**2)/(a**2*x**2
 + 1),x))/c
```

# 3.897 $\int \frac{\arctan(ax)^{5/2}}{x\sqrt{c+a^2cx^2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\arctan(ax)^{5/2}}{x\sqrt{c+a^2cx^2}}\,dx = \text{Int}\left(\frac{\arctan(ax)^{5/2}}{x\sqrt{c+a^2cx^2}},x\right)$$

output
```
Defer(Int)(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.92 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\arctan(ax)^{5/2}}{x\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan(ax)^{5/2}}{x\sqrt{c+a^2cx^2}}\,dx$$

input
```
Integrate[ArcTan[a*x]^(5/2)/(x*Sqrt[c + a^2*c*x^2]),x]
```

output
```
Integrate[ArcTan[a*x]^(5/2)/(x*Sqrt[c + a^2*c*x^2]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{5/2}}{x\sqrt{a^2cx^2+c}}\,dx$$

$$\Big\downarrow \; 5560$$

$$\int \frac{\arctan(ax)^{5/2}}{x\sqrt{a^2cx^2+c}}dx$$

input
```
Int[ArcTan[a*x]^(5/2)/(x*Sqrt[c + a^2*c*x^2]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\arctan\left(ax\right)^{\frac{5}{2}}}{x\sqrt{a^2c\,x^2+c}}dx$$

input
```
int(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{5/2}}{x\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 113.50 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\arctan(ax)^{5/2}}{x\sqrt{c+a^2cx^2}}\,dx = \int \frac{\operatorname{atan}^{\frac{5}{2}}(ax)}{x\sqrt{c(a^2x^2+1)}}\,dx$$

input
```
integrate(atan(a*x)**(5/2)/x/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Integral(atan(a*x)**(5/2)/(x*sqrt(c*(a**2*x**2 + 1))), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{5/2}}{x\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\arctan(ax)^{5/2}}{x\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan{(ax)}^{\frac{5}{2}}}{\sqrt{a^2cx^2+c}x}\,dx$$

input
```
integrate(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^(5/2)/(sqrt(a^2*c*x^2 + c)*x), x)
```

## Mupad [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\arctan(ax)^{5/2}}{x\sqrt{c+a^2cx^2}}\,dx = \int \frac{\mathrm{atan}(a\,x)^{5/2}}{x\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(atan(a*x)^(5/2)/(x*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(atan(a*x)^(5/2)/(x*(c + a^2*c*x^2)^(1/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.58

$$\int \frac{\arctan(ax)^{5/2}}{x\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,atan(ax)^2}{a^2x^3+x}dx\right)}{c}$$

input
```
int(atan(a*x)^(5/2)/x/(a^2*c*x^2+c)^(1/2),x)
```

output
```
(sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)**2)/(a**2*x**3
 + x),x))/c
```

# 3.898 $\int \frac{\arctan(ax)^{5/2}}{x^2\sqrt{c+a^2cx^2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\arctan(ax)^{5/2}}{x^2\sqrt{c+a^2cx^2}}\,dx = -\frac{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}{cx}+\frac{5}{2}a\text{Int}\left(\frac{\arctan(ax)^{3/2}}{x\sqrt{c+a^2cx^2}},x\right)$$

output
```
-(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(5/2)/c/x+5/2*a*Defer(Int)(arctan(a*x)^(3
/2)/x/(a^2*c*x^2+c)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 2.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\arctan(ax)^{5/2}}{x^2\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan(ax)^{5/2}}{x^2\sqrt{c+a^2cx^2}}\,dx$$

input
```
Integrate[ArcTan[a*x]^(5/2)/(x^2*Sqrt[c + a^2*c*x^2]),x]
```

output
```
Integrate[ArcTan[a*x]^(5/2)/(x^2*Sqrt[c + a^2*c*x^2]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{5/2}}{x^2\sqrt{a^2cx^2+c}}dx$$
$$\Big\downarrow \text{5479}$$
$$\frac{5}{2}a\int \frac{\arctan(ax)^{3/2}}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}{cx}$$
$$\Big\downarrow \text{5560}$$
$$\frac{5}{2}a\int \frac{\arctan(ax)^{3/2}}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}{cx}$$

input
```
Int[ArcTan[a*x]^(5/2)/(x^2*Sqrt[c + a^2*c*x^2]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\arctan\left(ax\right)^{\frac{5}{2}}}{x^{2}\sqrt{a^{2}c\,x^{2}+c}}dx$$

input
```
int(arctan(a*x)^(5/2)/x^2/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(arctan(a*x)^(5/2)/x^2/(a^2*c*x^2+c)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{5/2}}{x^2\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(5/2)/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^{5/2}}{x^2\sqrt{c+a^2cx^2}}\,dx = \text{Timed out}$$

input
```
integrate(atan(a*x)**(5/2)/x**2/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{5/2}}{x^2\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(5/2)/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\arctan(ax)^{5/2}}{x^2\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan{(ax)}^{\frac{5}{2}}}{\sqrt{a^2cx^2+c}x^2}\,dx$$

input
```
integrate(arctan(a*x)^(5/2)/x^2/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^(5/2)/(sqrt(a^2*c*x^2 + c)*x^2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\arctan(ax)^{5/2}}{x^2\sqrt{c+a^2cx^2}}\,dx = \int \frac{\mathrm{atan}(a\,x)^{5/2}}{x^2\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(atan(a*x)^(5/2)/(x^2*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(atan(a*x)^(5/2)/(x^2*(c + a^2*c*x^2)^(1/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.73

$$\int \frac{\arctan(ax)^{5/2}}{x^2\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{c}\left(-2\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}\,atan(ax)^2+5\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,atan(ax)}{a^2x^3+x}dx\right)ax\right)}{2cx}$$

input
```
int(atan(a*x)^(5/2)/x^2/(a^2*c*x^2+c)^(1/2),x)
```

output
```
(sqrt(c)*( - 2*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)**2 + 5*int((s
qrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x))/(a**2*x**3 + x),x)*a*x))/(2*
c*x)
```

# 3.899 $\int \frac{\arctan(ax)^{5/2}}{x^3\sqrt{c+a^2cx^2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\arctan(ax)^{5/2}}{x^3\sqrt{c+a^2cx^2}}\,dx = -\frac{5a\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}{4cx} - \frac{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}{2cx^2} + \frac{15}{8}a^2\mathrm{Int}\left(\frac{\sqrt{\arctan(ax)}}{x\sqrt{c+a^2cx^2}},x\right) - \frac{1}{2}a^2\mathrm{Int}\left(\frac{\arctan(ax)^{5/2}}{x\sqrt{c+a^2cx^2}},x\right)$$

output
```
-5/4*a*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(3/2)/c/x-1/2*(a^2*c*x^2+c)^(1/2)*a
rctan(a*x)^(5/2)/c/x^2+15/8*a^2*Defer(Int)(arctan(a*x)^(1/2)/x/(a^2*c*x^2+
c)^(1/2),x)-1/2*a^2*Defer(Int)(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 6.97 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\arctan(ax)^{5/2}}{x^3\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan(ax)^{5/2}}{x^3\sqrt{c+a^2cx^2}}\,dx$$

input
```
Integrate[ArcTan[a*x]^(5/2)/(x^3*Sqrt[c + a^2*c*x^2]),x]
```

output
```
Integrate[ArcTan[a*x]^(5/2)/(x^3*Sqrt[c + a^2*c*x^2]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.83 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{5/2}}{x^3\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5497}$$

$$\frac{5}{4}a\int \frac{\arctan(ax)^{3/2}}{x^2\sqrt{a^2cx^2+c}}dx-\frac{1}{2}a^2\int \frac{\arctan(ax)^{5/2}}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}{2cx^2}$$

$$\Big\downarrow \text{5479}$$

$$\frac{5}{4}a\left(\frac{3}{2}a\int \frac{\sqrt{\arctan(ax)}}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}{cx}\right)-\frac{1}{2}a^2\int \frac{\arctan(ax)^{5/2}}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}{2cx^2}$$

$$\Big\downarrow \text{5560}$$

$$\frac{5}{4}a\left(\frac{3}{2}a\int \frac{\sqrt{\arctan(ax)}}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}{cx}\right)-\frac{1}{2}a^2\int \frac{\arctan(ax)^{5/2}}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}{2cx^2}$$

input
```
Int[ArcTan[a*x]^(5/2)/(x^3*Sqrt[c + a^2*c*x^2]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\arctan{(ax)}^{\frac{5}{2}}}{x^3\sqrt{a^2c\,x^2+c}}dx$$

input
```
int(arctan(a*x)^(5/2)/x^3/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(arctan(a*x)^(5/2)/x^3/(a^2*c*x^2+c)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{5/2}}{x^3\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(5/2)/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^{5/2}}{x^3\sqrt{c+a^2cx^2}}\,dx = \text{Timed out}$$

input
```
integrate(atan(a*x)**(5/2)/x**3/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{5/2}}{x^3\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(5/2)/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\arctan(ax)^{5/2}}{x^3\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan\left(ax\right)^{\frac{5}{2}}}{\sqrt{a^2cx^2+c}x^3}\,dx$$

input
```
integrate(arctan(a*x)^(5/2)/x^3/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^(5/2)/(sqrt(a^2*c*x^2 + c)*x^3), x)
```

## Mupad [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\arctan(ax)^{5/2}}{x^3\sqrt{c+a^2cx^2}}\,dx = \int \frac{\mathrm{atan}(a\,x)^{5/2}}{x^3\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(atan(a*x)^(5/2)/(x^3*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(atan(a*x)^(5/2)/(x^3*(c + a^2*c*x^2)^(1/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.65

$$\int \frac{\arctan(ax)^{5/2}}{x^3\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,atan(ax)^2}{a^2x^5+x^3}dx\right)}{c}$$

input
```
int(atan(a*x)^(5/2)/x^3/(a^2*c*x^2+c)^(1/2),x)
```

output
```
(sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)**2)/(a**2*x**5
 + x**3),x))/c
```

# 3.900 $\int \frac{\arctan(ax)^{5/2}}{x^4\sqrt{c+a^2cx^2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\arctan(ax)^{5/2}}{x^4\sqrt{c+a^2cx^2}}\,dx = -\frac{5a^2\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}{8cx} - \frac{5a\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}{12cx^2} - \frac{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}{3cx^3} + \frac{2a^2\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}{3cx} + \frac{5}{16}a^3\text{Int}\left(\frac{1}{x\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}},x\right) - \frac{25}{12}a^3\text{Int}\left(\frac{\arctan(ax)^{3/2}}{x\sqrt{c+a^2cx^2}},x\right)$$

output
```
-5/8*a^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(1/2)/c/x-5/12*a*(a^2*c*x^2+c)^(1
/2)*arctan(a*x)^(3/2)/c/x^2-1/3*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(5/2)/c/x^
3+2/3*a^2*(a^2*c*x^2+c)^(1/2)*arctan(a*x)^(5/2)/c/x+5/16*a^3*Defer(Int)(1/
x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)-25/12*a^3*Defer(Int)(arctan(a*x
)^(3/2)/x/(a^2*c*x^2+c)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 14.87 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\arctan(ax)^{5/2}}{x^4\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan(ax)^{5/2}}{x^4\sqrt{c+a^2cx^2}}\,dx$$

input
```
Integrate[ArcTan[a*x]^(5/2)/(x^4*Sqrt[c + a^2*c*x^2]),x]
```

output
```
Integrate[ArcTan[a*x]^(5/2)/(x^4*Sqrt[c + a^2*c*x^2]), x]
```

## Rubi [N/A]

Not integrable

Time = 1.63 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{5/2}}{x^4\sqrt{a^2cx^2+c}}\,dx$$

$$\Big\downarrow \text{5497}$$

$$-\frac{2}{3}a^2\int \frac{\arctan(ax)^{5/2}}{x^2\sqrt{a^2cx^2+c}}dx+\frac{5}{6}a\int \frac{\arctan(ax)^{3/2}}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}{3cx^3}$$

$$\Big\downarrow \text{5479}$$

$$-\frac{2}{3}a^2\left(\frac{5}{2}a\int \frac{\arctan(ax)^{3/2}}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}{cx}\right)+\frac{5}{6}a\int \frac{\arctan(ax)^{3/2}}{x^3\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}{3cx^3}$$

$$\Big\downarrow \text{5497}$$

$$-\frac{2}{3}a^2\left(\frac{5}{2}a\int \frac{\arctan(ax)^{3/2}}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}{cx}\right)+ \frac{5}{6}a\left(-\frac{1}{2}a^2\int \frac{\arctan(ax)^{3/2}}{x\sqrt{a^2cx^2+c}}dx+\frac{3}{4}a\int \frac{\sqrt{\arctan(ax)}}{x^2\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}{2cx^2}\right)- \frac{\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}{3cx^3}$$

$\Big\downarrow$ 5479

$$-\frac{2}{3}a^2\left(\frac{5}{2}a\int \frac{\arctan(ax)^{3/2}}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}{cx}\right)+ \frac{5}{6}a\left(-\frac{1}{2}a^2\int \frac{\arctan(ax)^{3/2}}{x\sqrt{a^2cx^2+c}}dx+\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{x\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx-\frac{\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}{cx}\right)-\text{arcta}\right. \frac{\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}{3cx^3}$$

$\Big\downarrow$ 5560

$$-\frac{2}{3}a^2\left(\frac{5}{2}a\int \frac{\arctan(ax)^{3/2}}{x\sqrt{a^2cx^2+c}}dx-\frac{\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}{cx}\right)+ \frac{5}{6}a\left(-\frac{1}{2}a^2\int \frac{\arctan(ax)^{3/2}}{x\sqrt{a^2cx^2+c}}dx+\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{x\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx-\frac{\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}{cx}\right)-\text{arcta}\right. \frac{\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}{3cx^3}$$

input
```
Int[ArcTan[a*x]^(5/2)/(x^4*Sqrt[c + a^2*c*x^2]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\arctan(ax)^{\frac{5}{2}}}{x^4\sqrt{a^2c\,x^2+c}}dx$$

input
```
int(arctan(a*x)^(5/2)/x^4/(a^2*c*x^2+c)^(1/2),x)
```

output
```
int(arctan(a*x)^(5/2)/x^4/(a^2*c*x^2+c)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{5/2}}{x^4\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(5/2)/x^4/(a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^{5/2}}{x^4\sqrt{c+a^2cx^2}}\,dx = \text{Timed out}$$

input
```
integrate(atan(a*x)**(5/2)/x**4/(a**2*c*x**2+c)**(1/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{5/2}}{x^4\sqrt{c+a^2cx^2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(5/2)/x^4/(a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\arctan(ax)^{5/2}}{x^4\sqrt{c+a^2cx^2}}\,dx = \int \frac{\arctan\left(ax\right)^{\frac{5}{2}}}{\sqrt{a^2cx^2+c}x^4}\,dx$$

input
```
integrate(arctan(a*x)^(5/2)/x^4/(a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^(5/2)/(sqrt(a^2*c*x^2 + c)*x^4), x)
```

## Mupad [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\arctan(ax)^{5/2}}{x^4\sqrt{c+a^2cx^2}}\,dx = \int \frac{\mathrm{atan}(a\,x)^{5/2}}{x^4\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(atan(a*x)^(5/2)/(x^4*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(atan(a*x)^(5/2)/(x^4*(c + a^2*c*x^2)^(1/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 196, normalized size of antiderivative = 7.54

$$\int \frac{\arctan(ax)^{5/2}}{x^4\sqrt{c+a^2cx^2}}\,dx = \frac{\sqrt{c}\left(8\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}\,atan(ax)^2\,a^2x^2-4\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}\,atan(ax)^2+\right.}{}$$

input
```
int(atan(a*x)^(5/2)/x^4/(a^2*c*x^2+c)^(1/2),x)
```

output
```
(sqrt(c)*(8*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)**2*a**2*x**2 - 4
*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)**2 + 20*sqrt(a**2*x**2 + 1)
*sqrt(atan(a*x))*atan(a*x)*a*x + 30*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*a*
*2*x**2 + 50*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x))/(a**2*x**
5 + x**3),x)*a*x**3 - 15*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(atan(a
*x)*a**2*x**3 + atan(a*x)*x),x)*a**3*x**3))/(12*c*x**3)
```

# 3.901 $\int \frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = \operatorname{Int}\left(\frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^{3/2}},x\right)$$

output
```
Defer(Int)(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.50 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx$$

input
```
Integrate[(x^m*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(3/2),x]
```

output
```
Integrate[(x^m*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(3/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \arctan(ax)^{5/2}}{(a^2cx^2+c)^{3/2}} \, dx$$

$$\Big\downarrow \text{5560}$$

$$\int \frac{x^m \arctan(ax)^{5/2}}{(a^2cx^2+c)^{3/2}}dx$$

input
```
Int[(x^m*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(3/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^m \arctan\left(ax\right)^{\frac{5}{2}}}{\left(a^2c\,x^2+c\right)^{\frac{3}{2}}}dx$$

input
```
int(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x)
```

output
```
int(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.96

$$\int \frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^m \arctan(ax)^{\frac{5}{2}}}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^m*arctan(a*x)^(5/2)/(a^4*c^2*x^4 + 2*a^2*c^
2*x^2 + c^2), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**m*atan(a*x)**(5/2)/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^m \arctan(ax)^{\frac{5}{2}}}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2 + c)^(3/2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^m \operatorname{atan}(a\,x)^{5/2}}{(c\,a^2\,x^2+c)^{3/2}}\,dx$$

input
```
int((x^m*atan(a*x)^(5/2))/(c + a^2*c*x^2)^(3/2),x)
```

output
```
int((x^m*atan(a*x)^(5/2))/(c + a^2*c*x^2)^(3/2), x)
```

## Reduce [N/A]

Not integrable

Time = 1.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.00

$$\int \frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{c}\left(\int \frac{x^m\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,atan(ax)^2}{a^4x^4+2a^2x^2+1}dx\right)}{c^2}$$

input
```
int(x^m*atan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x)
```

output
```
(sqrt(c)*int((x**m*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)**2)/(a**4
*x**4 + 2*a**2*x**2 + 1),x))/c**2
```

# 3.902 $\int \frac{x^2 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{3/2}} dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^2 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{3/2}} dx = \operatorname{Int}\left(\frac{x^2 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}, x\right)$$

output
```
Defer(Int)(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 4.34 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^2 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{3/2}} dx = \int \frac{x^2 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{3/2}} dx$$

input
```
Integrate[(x^2*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(3/2),x]
```

output
```
Integrate[(x^2*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(3/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \arctan(ax)^{5/2}}{(a^2cx^2+c)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int \frac{x^2 \arctan(ax)^{5/2}}{(a^2cx^2+c)^{3/2}}dx$$

input
```
Int[(x^2*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(3/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^2 \arctan\left(ax\right)^{\frac{5}{2}}}{\left(a^2c\,x^2+c\right)^{\frac{3}{2}}}dx$$

input
```
int(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x)
```

output
```
int(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^2 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**2*atan(a*x)**(5/2)/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^2 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^2 \arctan\left(ax\right)^{\frac{5}{2}}}{\left(a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2 + c)^(3/2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^2 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x^2\,\mathrm{atan}(a\,x)^{5/2}}{{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

input
```
int((x^2*atan(a*x)^(5/2))/(c + a^2*c*x^2)^(3/2),x)
```

output
```
int((x^2*atan(a*x)^(5/2))/(c + a^2*c*x^2)^(3/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.00

$$\int \frac{x^2 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,atan(ax)^2x^2}{a^4x^4+2a^2x^2+1}dx\right)}{c^2}$$

input
```
int(x^2*atan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x)
```

output
```
(sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)**2*x**2)/(a**4
*x**4 + 2*a**2*x**2 + 1),x))/c**2
```

# 3.903 $\int \frac{x\arctan(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 161

$$\int \frac{x\arctan(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{15\sqrt{\arctan(ax)}}{4a^2c\sqrt{c+a^2cx^2}} + \frac{5x\arctan(ax)^{3/2}}{2ac\sqrt{c+a^2cx^2}} - \frac{\arctan(ax)^{5/2}}{a^2c\sqrt{c+a^2cx^2}} - \frac{15\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{4a^2c\sqrt{c+a^2cx^2}}$$

output
```
15/4*arctan(a*x)^(1/2)/a^2/c/(a^2*c*x^2+c)^(1/2)+5/2*x*arctan(a*x)^(3/2)/a
/c/(a^2*c*x^2+c)^(1/2)-arctan(a*x)^(5/2)/a^2/c/(a^2*c*x^2+c)^(1/2)-15/8*2^
(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/
2))/a^2/c/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.86

$$\int \frac{x\arctan(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{4\arctan(ax)\left(15+10ax\arctan(ax)-4\arctan(ax)^2\right)+15i\sqrt{1+a^2x^2}\sqrt{-i\arctan}}{16a^2c\sqrt{c+a^2cx^2}\sqrt{}}$$

input
```
Integrate[(x*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(3/2),x]
```

output
```
(4*ArcTan[a*x]*(15 + 10*a*x*ArcTan[a*x] - 4*ArcTan[a*x]^2) + (15*I)*Sqrt[1
 + a^2*x^2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-I)*ArcTan[a*x]] - (15*I)*S
qrt[1 + a^2*x^2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, I*ArcTan[a*x]])/(16*a^2*c*
Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])
```

## Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {5465, 5433, 5440, 5439, 3042, 3785, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \arctan(ax)^{5/2}}{(a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{5\int \frac{\arctan(ax)^{3/2}}{(a^2cx^2+c)^{3/2}}dx}{2a}-\frac{\arctan(ax)^{5/2}}{a^2c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5433}$$

$$\frac{5\left(-\frac{3}{4}\int \frac{1}{(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}\right)}{2a}-\frac{\arctan(ax)^{5/2}}{a^2c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5440}$$

$$\frac{5\left(-\frac{3\sqrt{a^2x^2+1}\int \frac{1}{(a^2x^2+1)^{3/2}\sqrt{\arctan(ax)}}dx}{4c\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}\right)}{2a}-\frac{\arctan(ax)^{5/2}}{a^2c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5439}$$

$$\frac{5\left(-\frac{3\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}d\arctan(ax)}{4ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}\right)}{2a}-\frac{\arctan(ax)^{5/2}}{a^2c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\left(-\frac{3\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)}{\sqrt{\arctan(ax)}}d\arctan(ax)}{4ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}\right)}{2a}-\frac{\arctan(ax)^{5/2}}{a^2c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3785}$$

$$\frac{5\left(-\frac{3\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}}d\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}\right)}{2a}-\frac{\arctan(ax)^{5/2}}{a^2c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3833}$$

$$\frac{5\left(-\frac{3\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{2ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}\right)}{2a}-\frac{\arctan(ax)^{5/2}}{a^2c\sqrt{a^2cx^2+c}}$$

input
```
Int[(x*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(3/2),x]
```

output
```
-(ArcTan[a*x]^(5/2)/(a^2*c*Sqrt[c + a^2*c*x^2])) + (5*((3*Sqrt[ArcTan[a*x]
])/(2*a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x]^(3/2))/(c*Sqrt[c + a^2*c*x
^2]) - (3*Sqrt[Pi/2]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x
]]])/(2*a*c*Sqrt[c + a^2*c*x^2])))/(2*a)
```

## Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 5433
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x] - Simp[b^2*p*(p
- 1)   Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2), x], x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5440
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])   Int[(1 +
c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

## Maple [F]

$$\int \frac{x \arctan(ax)^{\frac{5}{2}}}{(a^2 c x^2 + c)^{\frac{3}{2}}} dx$$

input
```
int(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x)
```

output
```
int(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x \arctan(ax)^{5/2}}{(c + a^2cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x \arctan(ax)^{5/2}}{(c + a^2cx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x*atan(a*x)**(5/2)/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x \arctan(ax)^{5/2}}{(c + a^2cx^2)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F]

$$\int \frac{x\arctan(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x\arctan{(ax)}^{\frac{5}{2}}}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2 + c)^(3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x\arctan(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{x\,\mathrm{atan}(a\,x)^{5/2}}{(c\,a^2\,x^2+c)^{3/2}}\,dx$$

input
```
int((x*atan(a*x)^(5/2))/(c + a^2*c*x^2)^(3/2),x)
```

output
```
int((x*atan(a*x)^(5/2))/(c + a^2*c*x^2)^(3/2), x)
```

## Reduce [F]

$$\int \frac{x\arctan(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{c}\left(-8\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}\,atan(ax)^2+20\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}\,atan(ax)\,ax\right.}{}$$

input
```
int(x*atan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x)
```

output
```
(sqrt(c)*( - 8*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)**2 + 20*sqrt(
a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)*a*x + 30*sqrt(a**2*x**2 + 1)*sqrt
(atan(a*x)) - 15*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(atan(a*x)*a**4
*x**4 + 2*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**3*x**2 - 15*int((sqrt(a**
2*x**2 + 1)*sqrt(atan(a*x)))/(atan(a*x)*a**4*x**4 + 2*atan(a*x)*a**2*x**2
+ atan(a*x)),x)*a))/(8*a**2*c**2*(a**2*x**2 + 1))
```

# 3.904 $\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 155

$$\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = -\frac{15x\sqrt{\arctan(ax)}}{4c\sqrt{c+a^2cx^2}}+\frac{5\arctan(ax)^{3/2}}{2ac\sqrt{c+a^2cx^2}}+\frac{x\arctan(ax)^{5/2}}{c\sqrt{c+a^2cx^2}}+\frac{15\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{4ac\sqrt{c+a^2cx^2}}$$

output
```
-15/4*x*arctan(a*x)^(1/2)/c/(a^2*c*x^2+c)^(1/2)+5/2*arctan(a*x)^(3/2)/a/c/
(a^2*c*x^2+c)^(1/2)+x*arctan(a*x)^(5/2)/c/(a^2*c*x^2+c)^(1/2)+15/8*2^(1/2)
*Pi^(1/2)*(a^2*x^2+1)^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))/a
/c/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.63

$$\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{2\sqrt{\arctan(ax)}(-15ax+10\arctan(ax)+4ax\arctan(ax)^2)+15\sqrt{2\pi}\sqrt{1+a^2x^2}\,\text{Fre}}{8ac\sqrt{c+a^2cx^2}}$$

input
```
Integrate[ArcTan[a*x]^(5/2)/(c + a^2*c*x^2)^(3/2),x]
```

output
```
(2*Sqrt[ArcTan[a*x]]*(-15*a*x + 10*ArcTan[a*x] + 4*a*x*ArcTan[a*x]^2) + 15
*Sqrt[2*Pi]*Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(8*a
*c*Sqrt[c + a^2*c*x^2])
```

## Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {5433, 5440, 5439, 3042, 3777, 25, 3042, 3786, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{5/2}}{(a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \text{ 5433}$$

$$-\frac{15}{4}\int \frac{\sqrt{\arctan(ax)}}{(a^2cx^2+c)^{3/2}}dx+\frac{x\arctan(ax)^{5/2}}{c\sqrt{a^2cx^2+c}}+\frac{5\arctan(ax)^{3/2}}{2ac\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 5440}$$

$$-\frac{15\sqrt{a^2x^2+1}\int \frac{\sqrt{\arctan(ax)}}{(a^2x^2+1)^{3/2}}dx}{4c\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{5/2}}{c\sqrt{a^2cx^2+c}}+\frac{5\arctan(ax)^{3/2}}{2ac\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 5439}$$

$$-\frac{15\sqrt{a^2x^2+1}\int \frac{\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}d\arctan(ax)}{4ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{5/2}}{c\sqrt{a^2cx^2+c}}+\frac{5\arctan(ax)^{3/2}}{2ac\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{15\sqrt{a^2x^2+1}\int \sqrt{\arctan(ax)}\sin\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{4ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{5/2}}{c\sqrt{a^2cx^2+c}}+\frac{5\arctan(ax)^{3/2}}{2ac\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 3777}$$

$$-\frac{15\sqrt{a^2x^2+1}\left(\frac{1}{2}\int -\frac{ax}{\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}d\arctan(ax)+\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}\right)}{4ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{5/2}}{c\sqrt{a^2cx^2+c}}+\frac{5\arctan(ax)^{3/2}}{2ac\sqrt{a^2cx^2+c}}$$

$\Big\downarrow$ 25

$$-\frac{15\sqrt{a^2x^2+1}\left(\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}-\frac{1}{2}\int \frac{ax}{\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}d\arctan(ax)\right)}{4ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{5/2}}{c\sqrt{a^2cx^2+c}}+\frac{5\arctan(ax)^{3/2}}{2ac\sqrt{a^2cx^2+c}}$$

$\Big\downarrow$ 3042

$$-\frac{15\sqrt{a^2x^2+1}\left(\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}-\frac{1}{2}\int \frac{\sin(\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)\right)}{4ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{5/2}}{c\sqrt{a^2cx^2+c}}+\frac{5\arctan(ax)^{3/2}}{2ac\sqrt{a^2cx^2+c}}$$

$\Big\downarrow$ 3786

$$-\frac{15\sqrt{a^2x^2+1}\left(\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}-\int \frac{ax}{\sqrt{a^2x^2+1}}d\sqrt{\arctan(ax)}\right)}{4ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{5/2}}{c\sqrt{a^2cx^2+c}}+\frac{5\arctan(ax)^{3/2}}{2ac\sqrt{a^2cx^2+c}}$$

$\Big\downarrow$ 3832

$$-\frac{15\sqrt{a^2x^2+1}\left(\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}-\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{4ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{5/2}}{c\sqrt{a^2cx^2+c}}+\frac{5\arctan(ax)^{3/2}}{2ac\sqrt{a^2cx^2+c}}$$

input
```
Int[ArcTan[a*x]^(5/2)/(c + a^2*c*x^2)^(3/2),x]
```

output
```
(5*ArcTan[a*x]^(3/2))/(2*a*c*Sqrt[c + a^2*c*x^2]) + (x*ArcTan[a*x]^(5/2))/
(c*Sqrt[c + a^2*c*x^2]) - (15*Sqrt[1 + a^2*x^2]*((a*x*Sqrt[ArcTan[a*x]])/S
qrt[1 + a^2*x^2] - Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]]))/(4*
a*c*Sqrt[c + a^2*c*x^2])
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 5433
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x] - Simp[b^2*p*(p
- 1)   Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2), x], x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5440
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])   Int[(1 +
c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

## Maple [F]

$$\int \frac{\arctan(ax)^{\frac{5}{2}}}{(a^2c\,x^2+c)^{\frac{3}{2}}}dx$$

input
```
int(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x)
```

output
```
int(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\operatorname{atan}^{\frac{5}{2}}(ax)}{(c\,(a^2x^2+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(atan(a*x)**(5/2)/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Integral(atan(a*x)**(5/2)/(c*(a**2*x**2 + 1))**(3/2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F]

$$\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\arctan(ax)^{\frac{5}{2}}}{(a^2cx^2+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^(5/2)/(a^2*c*x^2 + c)^(3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\mathrm{atan}(a\,x)^{5/2}}{(c\,a^2\,x^2+c)^{3/2}}\,dx$$

input
```
int(atan(a*x)^(5/2)/(c + a^2*c*x^2)^(3/2),x)
```

output
```
int(atan(a*x)^(5/2)/(c + a^2*c*x^2)^(3/2), x)
```

## Reduce [F]

$$\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^{3/2}}\,dx = \frac{\sqrt{c}\left(8\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}\,atan(ax)^2\,ax+20\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}\,atan(ax)-\right.}{}$$

input
```
int(atan(a*x)^(5/2)/(a^2*c*x^2+c)^(3/2),x)
```

output
```
(sqrt(c)*(8*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)**2*a*x + 20*sqrt
(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x) - 30*sqrt(a**2*x**2 + 1)*sqrt(at
an(a*x))*a*x + 15*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x)/(atan(a*x)*a
**4*x**4 + 2*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**4*x**2 + 15*int((sqrt(
a**2*x**2 + 1)*sqrt(atan(a*x))*x)/(atan(a*x)*a**4*x**4 + 2*atan(a*x)*a**2*
x**2 + atan(a*x)),x)*a**2))/(8*a*c**2*(a**2*x**2 + 1))
```

# 3.905 $\int \frac{\arctan(ax)^{5/2}}{x(c+a^2cx^2)^{3/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\arctan(ax)^{5/2}}{x(c+a^2cx^2)^{3/2}}\,dx = \text{Int}\left(\frac{\arctan(ax)^{5/2}}{x(c+a^2cx^2)^{3/2}},x\right)$$

output
```
Defer(Int)(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^(3/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 3.31 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\arctan(ax)^{5/2}}{x(c+a^2cx^2)^{3/2}}\,dx = \int \frac{\arctan(ax)^{5/2}}{x(c+a^2cx^2)^{3/2}}\,dx$$

input
```
Integrate[ArcTan[a*x]^(5/2)/(x*(c + a^2*c*x^2)^(3/2)),x]
```

output
```
Integrate[ArcTan[a*x]^(5/2)/(x*(c + a^2*c*x^2)^(3/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{5/2}}{x\left(a^2cx^2+c\right)^{3/2}}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{\arctan(ax)^{5/2}}{x\left(a^2cx^2+c\right)^{3/2}}dx$$

input
```
Int[ArcTan[a*x]^(5/2)/(x*(c + a^2*c*x^2)^(3/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\arctan\left(ax\right)^{\frac{5}{2}}}{x\left(a^2c\,x^2+c\right)^{\frac{3}{2}}}dx$$

input
```
int(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^(3/2),x)
```

output
```
int(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{5/2}}{x\left(c+a^2cx^2\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^{5/2}}{x\left(c+a^2cx^2\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(atan(a*x)**(5/2)/x/(a**2*c*x**2+c)**(3/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{5/2}}{x\left(c+a^2cx^2\right)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\arctan(ax)^{5/2}}{x\left(c+a^2cx^2\right)^{3/2}}\,dx = \int \frac{\arctan\left(ax\right)^{\frac{5}{2}}}{\left(a^2cx^2+c\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^(5/2)/((a^2*c*x^2 + c)^(3/2)*x), x)
```

## Mupad [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\arctan(ax)^{5/2}}{x\left(c+a^2cx^2\right)^{3/2}}\,dx = \int \frac{\mathrm{atan}(a\,x)^{5/2}}{x\,\left(c\,a^2\,x^2+c\right)^{3/2}}\,dx$$

input
```
int(atan(a*x)^(5/2)/(x*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(atan(a*x)^(5/2)/(x*(c + a^2*c*x^2)^(3/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.88

$$\int \frac{\arctan(ax)^{5/2}}{x\left(c+a^2cx^2\right)^{3/2}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,atan(ax)^2}{a^4x^5+2a^2x^3+x}dx\right)}{c^2}$$

input
```
int(atan(a*x)^(5/2)/x/(a^2*c*x^2+c)^(3/2),x)
```

output
```
(sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)**2)/(a**4*x**5
 + 2*a**2*x**3 + x),x))/c**2
```

# 3.906 $\int \frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Int}\left(\frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}},x\right)$$

output
```
Defer(Int)(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 2.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx$$

input
```
Integrate[(x^m*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(5/2),x]
```

output
```
Integrate[(x^m*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(5/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \arctan(ax)^{5/2}}{(a^2cx^2+c)^{5/2}}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x^m \arctan(ax)^{5/2}}{(a^2cx^2+c)^{5/2}}dx$$

input
```
Int[(x^m*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(5/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^m \arctan(ax)^{\frac{5}{2}}}{(a^2c\,x^2+c)^{\frac{5}{2}}}dx$$

input
```
int(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x)
```

output
```
int(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.38

$$\int \frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^m \arctan{(ax)}^{\frac{5}{2}}}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^m*arctan(a*x)^(5/2)/(a^6*c^3*x^6 + 3*a^4*c^
3*x^4 + 3*a^2*c^3*x^2 + c^3), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**m*atan(a*x)**(5/2)/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^m \arctan(ax)^{\frac{5}{2}}}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
integrate(x^m*arctan(a*x)^(5/2)/(a^2*c*x^2 + c)^(5/2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^m \operatorname{atan}(a\,x)^{5/2}}{(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int((x^m*atan(a*x)^(5/2))/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int((x^m*atan(a*x)^(5/2))/(c + a^2*c*x^2)^(5/2), x)
```

## Reduce [N/A]

Not integrable

Time = 10.37 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.31

$$\int \frac{x^m \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{\sqrt{c}\left(\int \frac{x^m\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,atan(ax)^2}{a^6x^6+3a^4x^4+3a^2x^2+1}dx\right)}{c^3}$$

input
```
int(x^m*atan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x)
```

output
```
(sqrt(c)*int((x**m*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)**2)/(a**6
*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1),x))/c**3
```

# 3.907 $\int \frac{x^4 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^4 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Int}\left(\frac{x^4 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}},x\right)$$

output
```
Defer(Int)(x^4*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 7.14 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^4 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^4 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx$$

input
```
Integrate[(x^4*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(5/2),x]
```

output
```
Integrate[(x^4*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(5/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \arctan(ax)^{5/2}}{(a^2cx^2+c)^{5/2}} \, dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int \frac{x^4 \arctan(ax)^{5/2}}{(a^2cx^2+c)^{5/2}}dx$$

input
```
Int[(x^4*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(5/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^4 \arctan\left(ax\right)^{\frac{5}{2}}}{\left(a^2 c\, x^2+c\right)^{\frac{5}{2}}}dx$$

input
```
int(x^4*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x)
```

output
```
int(x^4*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x^4 \arctan(ax)^{5/2}}{(c + a^2cx^2)^{5/2}}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^4*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^4 \arctan(ax)^{5/2}}{(c + a^2cx^2)^{5/2}}\, dx = \text{Timed out}$$

input
```
integrate(x**4*atan(a*x)**(5/2)/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4 \arctan(ax)^{5/2}}{(c + a^2cx^2)^{5/2}}\, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^4*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-1)]

Timed out.

$$\int \frac{x^4 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x^4*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

## Mupad [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^4 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^4\,\mathrm{atan}(a\,x)^{5/2}}{(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int((x^4*atan(a*x)^(5/2))/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int((x^4*atan(a*x)^(5/2))/(c + a^2*c*x^2)^(5/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.31

$$\int \frac{x^4 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,atan(ax)^2x^4}{a^6x^6+3a^4x^4+3a^2x^2+1}dx\right)}{c^3}$$

input
```
int(x^4*atan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x)
```

output
```
(sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)**2*x**4)/(a**6
*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1),x))/c**3
```

# 3.908 $\int \frac{x^3 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 350

$$\begin{aligned}\int \frac{x^3 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx &= \frac{45\sqrt{\arctan(ax)}}{16a^4c^2\sqrt{c+a^2cx^2}} + \frac{5x^3\arctan(ax)^{3/2}}{18ac\,(c+a^2cx^2)^{3/2}} \\ &+ \frac{5x\arctan(ax)^{3/2}}{3a^3c^2\sqrt{c+a^2cx^2}} - \frac{x^2\arctan(ax)^{5/2}}{3a^2c\,(c+a^2cx^2)^{3/2}} - \frac{2\arctan(ax)^{5/2}}{3a^4c^2\sqrt{c+a^2cx^2}} \\ &- \frac{5\sqrt{1+a^2x^2}\sqrt{\arctan(ax)}\cos(3\arctan(ax))}{144a^4c^2\sqrt{c+a^2cx^2}} \\ &- \frac{45\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{16a^4c^2\sqrt{c+a^2cx^2}} \\ &+ \frac{5\sqrt{\frac{\pi}{6}}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)}{144a^4c^2\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
45/16*arctan(a*x)^(1/2)/a^4/c^2/(a^2*c*x^2+c)^(1/2)+5/18*x^3*arctan(a*x)^(
3/2)/a/c/(a^2*c*x^2+c)^(3/2)+5/3*x*arctan(a*x)^(3/2)/a^3/c^2/(a^2*c*x^2+c)
^(1/2)-1/3*x^2*arctan(a*x)^(5/2)/a^2/c/(a^2*c*x^2+c)^(3/2)-2/3*arctan(a*x)
^(5/2)/a^4/c^2/(a^2*c*x^2+c)^(1/2)-5/144*(a^2*x^2+1)^(1/2)*arctan(a*x)^(1/
2)*cos(3*arctan(a*x))/a^4/c^2/(a^2*c*x^2+c)^(1/2)-45/32*2^(1/2)*Pi^(1/2)*(
a^2*x^2+1)^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))/a^4/c^2/(a^2
*c*x^2+c)^(1/2)+5/864*6^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)*FresnelC(6^(1/2)/
Pi^(1/2)*arctan(a*x)^(1/2))/a^4/c^2/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.06

$$\int \frac{x^3 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{4800\arctan(ax) + 5040a^2x^2\arctan(ax) + 2880ax\arctan(ax)^2 + 3360a^3x^3\arctan}{}$$

input
```
Integrate[(x^3*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(4800*ArcTan[a*x] + 5040*a^2*x^2*ArcTan[a*x] + 2880*a*x*ArcTan[a*x]^2 + 33
60*a^3*x^3*ArcTan[a*x]^2 - 1152*ArcTan[a*x]^3 - 1728*a^2*x^2*ArcTan[a*x]^3
 + (1215*I)*(1 + a^2*x^2)^(3/2)*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-I)*Arc
Tan[a*x]] - (1215*I)*(1 + a^2*x^2)^(3/2)*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, I*
ArcTan[a*x]] - (5*I)*Sqrt[3 + 3*a^2*x^2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2,
 (-3*I)*ArcTan[a*x]] - (5*I)*a^2*x^2*Sqrt[3 + 3*a^2*x^2]*Sqrt[(-I)*ArcTan[
a*x]]*Gamma[1/2, (-3*I)*ArcTan[a*x]] + (5*I)*Sqrt[3 + 3*a^2*x^2]*Sqrt[I*Ar
cTan[a*x]]*Gamma[1/2, (3*I)*ArcTan[a*x]] + (5*I)*a^2*x^2*Sqrt[3 + 3*a^2*x^
2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (3*I)*ArcTan[a*x]])/(1728*a^4*c^2*(1 + a
^2*x^2)*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])
```

## Rubi [A] (verified)

Time = 3.03 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.11, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5475, 5465, 5433, 5440, 5439, 3042, 3785, 3833, 5506, 5505, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \arctan(ax)^{5/2}}{(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{5475}$$

$$\frac{2\int \frac{x\arctan(ax)^{5/2}}{(a^2cx^2+c)^{3/2}}dx}{3a^2c}-\frac{5}{12}\int \frac{x^3\sqrt{\arctan(ax)}}{(a^2cx^2+c)^{5/2}}dx-\frac{x^2\arctan(ax)^{5/2}}{3a^2c\,(a^2cx^2+c)^{3/2}}+\frac{5x^3\arctan(ax)^{3/2}}{18ac\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5465}$$

$$\frac{2\left(\frac{5\int \frac{\arctan(ax)^{3/2}}{(a^2cx^2+c)^{3/2}}dx}{2a}-\frac{\arctan(ax)^{5/2}}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{5}{12}\int \frac{x^3\sqrt{\arctan(ax)}}{(a^2cx^2+c)^{5/2}}dx-\frac{x^2\arctan(ax)^{5/2}}{3a^2c\,(a^2cx^2+c)^{3/2}}+\frac{5x^3\arctan(ax)^{3/2}}{18ac\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5433}$$

$$\frac{2\left(\frac{5\left(-\frac{3}{4}\int \frac{1}{(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}\right)}{2a}-\frac{\arctan(ax)^{5/2}}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{5}{12}\int \frac{x^3\sqrt{\arctan(ax)}}{(a^2cx^2+c)^{5/2}}dx-\frac{x^2\arctan(ax)^{5/2}}{3a^2c\,(a^2cx^2+c)^{3/2}}+\frac{5x^3\arctan(ax)^{3/2}}{18ac\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5440}$$

$$\frac{2\left(\frac{5\left(-\frac{3\sqrt{a^2x^2+1}\int \frac{1}{\left(a^2x^2+1\right)^{3/2}\sqrt{\arctan(ax)}}dx}{4c\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}\right)}{2a}-\frac{\arctan(ax)^{5/2}}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{5}{12}\int \frac{x^3\sqrt{\arctan(ax)}}{(a^2cx^2+c)^{5/2}}dx-\frac{x^2\arctan(ax)^{5/2}}{3a^2c\left(a^2cx^2+c\right)^{3/2}}+\frac{5x^3\arctan(ax)^{3/2}}{18ac\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{5439}$$

$$\frac{2\left(\frac{5\left(-\frac{3\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}d\arctan(ax)}{4ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}\right)}{2a}-\frac{\arctan(ax)^{5/2}}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{5}{12}\int \frac{x^3\sqrt{\arctan(ax)}}{(a^2cx^2+c)^{5/2}}dx-\frac{x^2\arctan(ax)^{5/2}}{3a^2c\left(a^2cx^2+c\right)^{3/2}}+\frac{5x^3\arctan(ax)^{3/2}}{18ac\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\left(\frac{5\left(-\frac{3\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)}{\sqrt{\arctan(ax)}}d\arctan(ax)}{4ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}\right)}{2a}-\frac{\arctan(ax)^{5/2}}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{5}{12}\int \frac{x^3\sqrt{\arctan(ax)}}{(a^2cx^2+c)^{5/2}}dx-\frac{x^2\arctan(ax)^{5/2}}{3a^2c\left(a^2cx^2+c\right)^{3/2}}+\frac{5x^3\arctan(ax)^{3/2}}{18ac\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{3785}$$

$$\frac{2\left(\frac{5\left(-\frac{3\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}}d\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}\right)}{2a}-\frac{\arctan(ax)^{5/2}}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{5}{12}\int \frac{x^3\sqrt{\arctan(ax)}}{(a^2cx^2+c)^{5/2}}dx-\frac{x^2\arctan(ax)^{5/2}}{3a^2c\left(a^2cx^2+c\right)^{3/2}}+\frac{5x^3\arctan(ax)^{3/2}}{18ac\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{3833}$$

$$-\frac{5}{12}\int \frac{x^3\sqrt{\arctan(ax)}}{(a^2cx^2+c)^{5/2}}dx+\frac{2\left(\frac{5\left(-\frac{3\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{2ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}\right)}{2a}-\frac{\arctan(ax)^{5/2}}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{x^2\arctan(ax)^{5/2}}{3a^2c\,(a^2cx^2+c)^{3/2}}+\frac{5x^3\arctan(ax)^{3/2}}{18ac\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \; 5506$$

$$-\frac{5\sqrt{a^2x^2+1}\int \frac{x^3\sqrt{\arctan(ax)}}{(a^2x^2+1)^{5/2}}dx}{12c^2\sqrt{a^2cx^2+c}}+\frac{2\left(\frac{5\left(-\frac{3\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{2ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}\right)}{2a}-\frac{\arctan(ax)^{5/2}}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{x^2\arctan(ax)^{5/2}}{3a^2c\,(a^2cx^2+c)^{3/2}}+\frac{5x^3\arctan(ax)^{3/2}}{18ac\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \; 5505$$

$$-\frac{5\sqrt{a^2x^2+1}\int \frac{a^3x^3\sqrt{\arctan(ax)}}{(a^2x^2+1)^{3/2}}d\arctan(ax)}{12a^4c^2\sqrt{a^2cx^2+c}}+\frac{2\left(\frac{5\left(-\frac{3\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{2ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}\right)}{2a}-\frac{\arctan(ax)^{5/2}}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{x^2\arctan(ax)^{5/2}}{3a^2c\,(a^2cx^2+c)^{3/2}}+\frac{5x^3\arctan(ax)^{3/2}}{18ac\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{5\sqrt{a^2x^2+1}\int\sqrt{\arctan(ax)}\sin(\arctan(ax))^3d\arctan(ax)}{12a^4c^2\sqrt{a^2cx^2+c}}+\frac{2\left(\frac{5\left(-\frac{3\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{2ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}\right)}{2a}-\frac{\arctan(ax)^{5/2}}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{x^2\arctan(ax)^{5/2}}{3a^2c\left(a^2cx^2+c\right)^{3/2}}+\frac{5x^3\arctan(ax)^{3/2}}{18ac\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{5\sqrt{a^2x^2+1}\int\left(\frac{3ax\sqrt{\arctan(ax)}}{4\sqrt{a^2x^2+1}}-\frac{1}{4}\sqrt{\arctan(ax)}\sin(3\arctan(ax))\right)d\arctan(ax)}{12a^4c^2\sqrt{a^2cx^2+c}}+\frac{2\left(\frac{5\left(-\frac{3\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{2ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}\right)}{2a}-\frac{\arctan(ax)^{5/2}}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{x^2\arctan(ax)^{5/2}}{3a^2c\left(a^2cx^2+c\right)^{3/2}}+\frac{5x^3\arctan(ax)^{3/2}}{18ac\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\left(\frac{5\left(-\frac{3\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{2ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}\right)}{2a}-\frac{\arctan(ax)^{5/2}}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{x^2\arctan(ax)^{5/2}}{3a^2c\left(a^2cx^2+c\right)^{3/2}}+\frac{5x^3\arctan(ax)^{3/2}}{18ac\left(a^2cx^2+c\right)^{3/2}}-\frac{5\sqrt{a^2x^2+1}\left(-\frac{3\sqrt{\arctan(ax)}}{4\sqrt{a^2x^2+1}}+\frac{3}{4}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{12}\sqrt{\frac{\pi}{6}}\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{12}\sqrt{\right.}{12a^4c^2\sqrt{a^2cx^2+c}}$$

input
```
Int[(x^3*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(5*x^3*ArcTan[a*x]^(3/2))/(18*a*c*(c + a^2*c*x^2)^(3/2)) - (x^2*ArcTan[a*x
]^(5/2))/(3*a^2*c*(c + a^2*c*x^2)^(3/2)) + (2*(-(ArcTan[a*x]^(5/2)/(a^2*c*
Sqrt[c + a^2*c*x^2])) + (5*((3*Sqrt[ArcTan[a*x]])/(2*a*c*Sqrt[c + a^2*c*x^
2]) + (x*ArcTan[a*x]^(3/2))/(c*Sqrt[c + a^2*c*x^2]) - (3*Sqrt[Pi/2]*Sqrt[1
 + a^2*x^2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(2*a*c*Sqrt[c + a^2*c*
x^2])))/(2*a)))/(3*a^2*c) - (5*Sqrt[1 + a^2*x^2]*((-3*Sqrt[ArcTan[a*x]])/(
4*Sqrt[1 + a^2*x^2]) + (Sqrt[ArcTan[a*x]]*Cos[3*ArcTan[a*x]])/12 + (3*Sqrt
[Pi/2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/4 - (Sqrt[Pi/6]*FresnelC[Sq
rt[6/Pi]*Sqrt[ArcTan[a*x]]])/12))/(12*a^4*c^2*Sqrt[c + a^2*c*x^2])
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 5433
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x] - Simp[b^2*p*(p
- 1)   Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2), x], x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5440
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])   Int[(1 +
c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5475
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_)*((d_) + (e_.)
*(x_)^2)^(q_), x_Symbol] :> Simp[b*p*(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*Ar
cTan[c*x])^(p - 1)/(c*d*m^2)), x] + (-Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(q +
 1)*((a + b*ArcTan[c*x])^p/(c^2*d*m)), x] + Simp[f^2*((m - 1)/(c^2*d*m))
Int[(f*x)^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p, x], x] - Simp[
b^2*p*((p - 1)/m^2)   Int[(f*x)^m*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2)
, x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && EqQ[m + 2*
q + 2, 0] && LtQ[q, -1] && GtQ[p, 1]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

## Maple [F]

$$\int \frac{x^3 \arctan(ax)^{\frac{5}{2}}}{(a^2c\,x^2+c)^{\frac{5}{2}}}dx$$

input
```
int(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x)
```

output
```
int(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^3 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**3*atan(a*x)**(5/2)/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x^3 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^3\,\mathrm{atan}(a\,x)^{5/2}}{(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int((x^3*atan(a*x)^(5/2))/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int((x^3*atan(a*x)^(5/2))/(c + a^2*c*x^2)^(5/2), x)
```

## Reduce [F]

$$\int \frac{x^3 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
int(x^3*atan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x)
```

output
```
(sqrt(c)*( - 72*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)**2*a**2*x**2
 - 48*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)**2 + 140*sqrt(a**2*x**
2 + 1)*sqrt(atan(a*x))*atan(a*x)*a**3*x**3 + 120*sqrt(a**2*x**2 + 1)*sqrt(
atan(a*x))*atan(a*x)*a*x + 210*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*a**2*x*
*2 + 200*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)) - 105*int((sqrt(a**2*x**2 + 1
)*sqrt(atan(a*x))*x**2)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*a
tan(a*x)*a**2*x**2 + atan(a*x)),x)*a**7*x**4 - 210*int((sqrt(a**2*x**2 + 1
)*sqrt(atan(a*x))*x**2)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*a
tan(a*x)*a**2*x**2 + atan(a*x)),x)*a**5*x**2 - 105*int((sqrt(a**2*x**2 + 1
)*sqrt(atan(a*x))*x**2)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*a
tan(a*x)*a**2*x**2 + atan(a*x)),x)*a**3 - 100*int((sqrt(a**2*x**2 + 1)*sqr
t(atan(a*x)))/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a
**2*x**2 + atan(a*x)),x)*a**5*x**4 - 200*int((sqrt(a**2*x**2 + 1)*sqrt(ata
n(a*x)))/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x
**2 + atan(a*x)),x)*a**3*x**2 - 100*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x
)))/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 +
 atan(a*x)),x)*a))/(72*a**4*c**3*(a**4*x**4 + 2*a**2*x**2 + 1))
```

# 3.909 $\int \frac{x^2 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 295

$$\begin{aligned}\int \frac{x^2 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = &-\frac{5x^3\sqrt{\arctan(ax)}}{36c(c+a^2cx^2)^{3/2}}-\frac{5x\sqrt{\arctan(ax)}}{6a^2c^2\sqrt{c+a^2cx^2}}\\&+\frac{5x^2\arctan(ax)^{3/2}}{18ac(c+a^2cx^2)^{3/2}}+\frac{5\arctan(ax)^{3/2}}{9a^3c^2\sqrt{c+a^2cx^2}}+\frac{x^3\arctan(ax)^{5/2}}{3c(c+a^2cx^2)^{3/2}}\\&+\frac{15\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{16a^3c^2\sqrt{c+a^2cx^2}}\\&-\frac{5\sqrt{\frac{\pi}{6}}\sqrt{1+a^2x^2}\,\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)}{144a^3c^2\sqrt{c+a^2cx^2}}\end{aligned}$$

output

```
-5/36*x^3*arctan(a*x)^(1/2)/c/(a^2*c*x^2+c)^(3/2)-5/6*x*arctan(a*x)^(1/2)/
a^2/c^2/(a^2*c*x^2+c)^(1/2)+5/18*x^2*arctan(a*x)^(3/2)/a/c/(a^2*c*x^2+c)^(
3/2)+5/9*arctan(a*x)^(3/2)/a^3/c^2/(a^2*c*x^2+c)^(1/2)+1/3*x^3*arctan(a*x)
^(5/2)/c/(a^2*c*x^2+c)^(3/2)+15/32*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)*Fres
nelS(2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))/a^3/c^2/(a^2*c*x^2+c)^(1/2)-5/864
*6^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)*FresnelS(6^(1/2)/Pi^(1/2)*arctan(a*x)^
(1/2))/a^3/c^2/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.87 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.97

$$\int \frac{x^2 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{-24\arctan(ax)\left(5ax(6+7a^2x^2)-10(2+3a^2x^2)\arctan(ax)-12a^3x^3\arctan(ax\right.}{}$$

input
```
Integrate[(x^2*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(-24*ArcTan[a*x]*(5*a*x*(6 + 7*a^2*x^2) - 10*(2 + 3*a^2*x^2)*ArcTan[a*x] -
 12*a^3*x^3*ArcTan[a*x]^2) + 35*Sqrt[6*Pi]*(1 + a^2*x^2)^(3/2)*Sqrt[ArcTan
[a*x]]*(3*Sqrt[3]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]] - FresnelS[Sqrt[6
/Pi]*Sqrt[ArcTan[a*x]]]) - 15*(1 + a^2*x^2)^(3/2)*(3*Sqrt[(-I)*ArcTan[a*x]
]*Gamma[1/2, (-I)*ArcTan[a*x]] + 3*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, I*ArcTan
[a*x]] + Sqrt[3]*(Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-3*I)*ArcTan[a*x]] +
Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (3*I)*ArcTan[a*x]])))/(864*a^3*c*(c + a^2*c
*x^2)^(3/2)*Sqrt[ArcTan[a*x]])
```

## Rubi [A] (verified)

Time = 3.01 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.17, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {5479, 5475, 5465, 5440, 5439, 3042, 3777, 25, 3042, 3786, 3832, 5506, 5505, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \arctan(ax)^{5/2}}{(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{5479}$$

$$\frac{x^3 \arctan(ax)^{5/2}}{3c\,(a^2cx^2+c)^{3/2}}-\frac{5}{6}a\int \frac{x^3 \arctan(ax)^{3/2}}{(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{5475}$$

$$\frac{x^3 \arctan(ax)^{5/2}}{3c\left(a^2cx^2+c\right)^{3/2}}-\frac{5}{6}a\left(\frac{2\int \frac{x\arctan(ax)^{3/2}}{\left(a^2cx^2+c\right)^{3/2}}dx}{3a^2c}-\frac{1}{12}\int \frac{x^3}{\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{x^2\arctan(ax)^{3/2}}{3a^2c\left(a^2cx^2+c\right)^{3/2}}+\frac{x^3\sqrt{\arctan(ax)}}{6ac\left(a^2cx^2+c\right)^{3/2}}\right)$$

$$\Big\downarrow \text{5465}$$

$$\frac{x^3 \arctan(ax)^{5/2}}{3c\left(a^2cx^2+c\right)^{3/2}}-\frac{5}{6}a\left(\frac{2\left(\frac{3\int \frac{\sqrt{\arctan(ax)}}{\left(a^2cx^2+c\right)^{3/2}}dx}{2a}-\frac{\arctan(ax)^{3/2}}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{1}{12}\int \frac{x^3}{\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{x^2\arctan(ax)^{3/2}}{3a^2c\left(a^2cx^2+c\right)^{3/2}}+\frac{x^3\sqrt{}}{6ac\,(}\right.$$

$$\Big\downarrow \text{5440}$$

$$\frac{x^3 \arctan(ax)^{5/2}}{3c\left(a^2cx^2+c\right)^{3/2}}-\frac{5}{6}a\left(\frac{2\left(\frac{3\sqrt{a^2x^2+1}\int \frac{\sqrt{\arctan(ax)}}{\left(a^2x^2+1\right)^{3/2}}dx}{2ac\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^{3/2}}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{1}{12}\int \frac{x^3}{\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{x^2\arctan(ax)^{3/2}}{3a^2c\left(a^2cx^2+c\right)^{3/2}}\right.$$

$$\Big\downarrow \text{5439}$$

$$\frac{x^3 \arctan(ax)^{5/2}}{3c\left(a^2cx^2+c\right)^{3/2}}-\frac{5}{6}a\left(\frac{2\left(\frac{3\sqrt{a^2x^2+1}\int \frac{\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}d\arctan(ax)}{2a^2c\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^{3/2}}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{1}{12}\int \frac{x^3}{\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{x^2\arctan(ax}{3a^2c\,(a^2cx^2+}\right.$$

$$\Big\downarrow \text{3042}$$

$$\frac{x^3\arctan(ax)^{5/2}}{3c\left(a^2cx^2+c\right)^{3/2}}-\frac{5}{6}a\left(\frac{2\left(\frac{3\sqrt{a^2x^2+1}\int\sqrt{\arctan(ax)}\sin\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{2a^2c\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^{3/2}}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{1}{12}\int\frac{x^3}{\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx\right.$$

$\big\downarrow$ 3777

$$\frac{x^3\arctan(ax)^{5/2}}{3c\left(a^2cx^2+c\right)^{3/2}}-\frac{5}{6}a\left(\frac{2\left(\frac{3\sqrt{a^2x^2+1}\left(\frac{1}{2}\int-\frac{ax}{\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}d\arctan(ax)+\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}\right)}{2a^2c\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^{3/2}}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{1}{12}\int\frac{x^3}{\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}\right.$$

$\big\downarrow$ 25

$$\frac{x^3\arctan(ax)^{5/2}}{3c\left(a^2cx^2+c\right)^{3/2}}-\frac{5}{6}a\left(\frac{2\left(\frac{3\sqrt{a^2x^2+1}\left(\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}-\frac{1}{2}\int\frac{ax}{\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}d\arctan(ax)\right)}{2a^2c\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^{3/2}}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{1}{12}\int\frac{x^3}{\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(}}\right.$$

$\big\downarrow$ 3042

$$\frac{x^3\arctan(ax)^{5/2}}{3c\left(a^2cx^2+c\right)^{3/2}}-\frac{5}{6}a\left(\frac{2\left(\frac{3\sqrt{a^2x^2+1}\left(\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}-\frac{1}{2}\int\frac{\sin(\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)\right)}{2a^2c\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^{3/2}}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{1}{12}\int\frac{x^3}{\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}d\right.$$

$\big\downarrow$ 3786

$$\frac{x^3\arctan(ax)^{5/2}}{3c\left(a^2cx^2+c\right)^{3/2}}-\frac{5}{6}a\left(\frac{2\left(\frac{3\sqrt{a^2x^2+1}\left(\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}-\int\frac{ax}{\sqrt{a^2x^2+1}}d\sqrt{\arctan(ax)}\right)}{2a^2c\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^{3/2}}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{1}{12}\int\frac{x^3}{\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx-\right.$$

$\downarrow$ 3832

$$\frac{x^3\arctan(ax)^{5/2}}{3c\left(a^2cx^2+c\right)^{3/2}}-\frac{5}{6}a\left(-\frac{1}{12}\int\frac{x^3}{\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx+\frac{2\left(\frac{3\sqrt{a^2x^2+1}\left(\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}-\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{2a^2c\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2}}\right.}{3a^2c}\right.$$

$\downarrow$ 5506

$$\frac{x^3\arctan(ax)^{5/2}}{3c\left(a^2cx^2+c\right)^{3/2}}-\frac{5}{6}a\left(-\frac{\sqrt{a^2x^2+1}\int\frac{x^3}{\left(a^2x^2+1\right)^{5/2}\sqrt{\arctan(ax)}}dx}{12c^2\sqrt{a^2cx^2+c}}+\frac{2\left(\frac{3\sqrt{a^2x^2+1}\left(\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}-\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{2a^2c\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)}{a^2c\sqrt{a^2cx^2}}\right.}{3a^2c}\right.$$

$\downarrow$ 5505

$$\frac{x^3\arctan(ax)^{5/2}}{3c\left(a^2cx^2+c\right)^{3/2}}-\frac{5}{6}a\left(-\frac{\sqrt{a^2x^2+1}\int\frac{a^3x^3}{\left(a^2x^2+1\right)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{12a^4c^2\sqrt{a^2cx^2+c}}+\frac{2\left(\frac{3\sqrt{a^2x^2+1}\left(\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}-\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{2a^2c\sqrt{a^2cx^2+c}}-\right.}{3a^2c}\right.$$

$\downarrow$ 3042

$$\frac{x^3\arctan(ax)^{5/2}}{3c\left(a^2cx^2+c\right)^{3/2}}-$$
$$\frac{5}{6}a\left(-\frac{\sqrt{a^2x^2+1}\int\frac{\sin(\arctan(ax))^3}{\sqrt{\arctan(ax)}}d\arctan(ax)}{12a^4c^2\sqrt{a^2cx^2+c}}+\frac{2\left(\frac{3\sqrt{a^2x^2+1}\left(\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}-\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{2a^2c\sqrt{a^2cx^2+c}}-\frac{\arctan(}{a^2c\sqrt{a^2}}\right.}{3a^2c}\right.$$

$\downarrow$ 3793

$$\frac{x^3\arctan(ax)^{5/2}}{3c\left(a^2cx^2+c\right)^{3/2}}-$$
$$\frac{5}{6}a\left(-\frac{\sqrt{a^2x^2+1}\int\left(\frac{3ax}{4\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}-\frac{\sin(3\arctan(ax))}{4\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{12a^4c^2\sqrt{a^2cx^2+c}}+\frac{2\left(\frac{3\sqrt{a^2x^2+1}\left(\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}-\sqrt{\frac{\pi}{2}}\text{FresnelS}(\right.}{2a^2c\sqrt{a^2cx^2+c}}\right.}{3a^2c}\right.$$

$\downarrow$ 2009

$$\frac{x^3\arctan(ax)^{5/2}}{3c\left(a^2cx^2+c\right)^{3/2}}-$$
$$\frac{5}{6}a\left(\frac{2\left(\frac{3\sqrt{a^2x^2+1}\left(\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}-\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{2a^2c\sqrt{a^2cx^2+c}}-\frac{\arctan(ax)^{3/2}}{a^2c\sqrt{a^2cx^2+c}}\right)}{3a^2c}-\frac{x^2\arctan(ax)^{3/2}}{3a^2c\left(a^2cx^2+c\right)^{3/2}}+\frac{x^3\sqrt{\text{arcta}}}{6ac\left(a^2cx^2\right.}\right.$$

input
```
Int[(x^2*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(x^3*ArcTan[a*x]^(5/2))/(3*c*(c + a^2*c*x^2)^(3/2)) - (5*a*((x^3*Sqrt[ArcT
an[a*x]])/(6*a*c*(c + a^2*c*x^2)^(3/2)) - (x^2*ArcTan[a*x]^(3/2))/(3*a^2*c
*(c + a^2*c*x^2)^(3/2)) + (2*(-(ArcTan[a*x]^(3/2)/(a^2*c*Sqrt[c + a^2*c*x^
2])) + (3*Sqrt[1 + a^2*x^2]*((a*x*Sqrt[ArcTan[a*x]])/Sqrt[1 + a^2*x^2] - S
qrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]]))/(2*a^2*c*Sqrt[c + a^2*c
*x^2])))/(3*a^2*c) - (Sqrt[1 + a^2*x^2]*((3*Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]
*Sqrt[ArcTan[a*x]]])/2 - (Sqrt[Pi/6]*FresnelS[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]
])/2))/(12*a^4*c^2*Sqrt[c + a^2*c*x^2])))/6
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5440
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])   Int[(1 +
c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5475
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_)*((d_) + (e_.)
*(x_)^2)^(q_), x_Symbol] :> Simp[b*p*(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*Ar
cTan[c*x])^(p - 1)/(c*d*m^2)), x] + (-Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(q +
 1)*((a + b*ArcTan[c*x])^p/(c^2*d*m)), x] + Simp[f^2*((m - 1)/(c^2*d*m))
Int[(f*x)^(m - 2)*(d + e*x^2)^(q + 1)*(a + b*ArcTan[c*x])^p, x], x] - Simp[
b^2*p*((p - 1)/m^2)   Int[(f*x)^m*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 2)
, x], x]) /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[e, c^2*d] && EqQ[m + 2*
q + 2, 0] && LtQ[q, -1] && GtQ[p, 1]
```

rule 5479
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(q + 1)*((a +
 b*ArcTan[c*x])^p/(d*f*(m + 1))), x] - Simp[b*c*(p/(f*(m + 1)))   Int[(f*x)
^(m + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p - 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 3, 0] && GtQ[p, 0]
&& NeQ[m, -1]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

## Maple [F]

$$\int \frac{x^{2} \arctan \left(a x \right)^{\frac{5}{2}}}{\left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}}}d x$$

input
```
int(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x)
```

output
```
int(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^2 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**2*atan(a*x)**(5/2)/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F]

$$\int \frac{x^2 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^2 \arctan(ax)^{\frac{5}{2}}}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
integrate(x^2*arctan(a*x)^(5/2)/(a^2*c*x^2 + c)^(5/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x^2\,\mathrm{atan}(a\,x)^{5/2}}{(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int((x^2*atan(a*x)^(5/2))/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int((x^2*atan(a*x)^(5/2))/(c + a^2*c*x^2)^(5/2), x)
```

## Reduce [F]

$$\int \frac{x^2 \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
int(x^2*atan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x)
```

output
```
(sqrt(c)*(24*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)**2*a**3*x**3 +
60*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)*a**2*x**2 + 40*sqrt(a**2*
x**2 + 1)*sqrt(atan(a*x))*atan(a*x) - 70*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x
))*a**3*x**3 - 60*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*a*x + 35*int((sqrt(a
**2*x**2 + 1)*sqrt(atan(a*x))*x**3)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**
4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**8*x**4 + 70*int((sqrt(a*
*2*x**2 + 1)*sqrt(atan(a*x))*x**3)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4
*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**6*x**2 + 35*int((sqrt(a**
2*x**2 + 1)*sqrt(atan(a*x))*x**3)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*
x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**4 + 30*int((sqrt(a**2*x**2
 + 1)*sqrt(atan(a*x))*x)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*
atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**6*x**4 + 60*int((sqrt(a**2*x**2 + 1
)*sqrt(atan(a*x))*x)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan
(a*x)*a**2*x**2 + atan(a*x)),x)*a**4*x**2 + 30*int((sqrt(a**2*x**2 + 1)*sq
rt(atan(a*x))*x)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x
)*a**2*x**2 + atan(a*x)),x)*a**2))/(72*a**3*c**3*(a**4*x**4 + 2*a**2*x**2
+ 1))
```

# 3.910 $\int \frac{x \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 293

$$\begin{aligned}\int \frac{x \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx &= \frac{5\sqrt{\arctan(ax)}}{36a^2c\,(c+a^2cx^2)^{3/2}} + \frac{5\sqrt{\arctan(ax)}}{6a^2c^2\sqrt{c+a^2cx^2}} \\ &\quad + \frac{5x\arctan(ax)^{3/2}}{18ac\,(c+a^2cx^2)^{3/2}} + \frac{5x\arctan(ax)^{3/2}}{9ac^2\sqrt{c+a^2cx^2}} - \frac{\arctan(ax)^{5/2}}{3a^2c\,(c+a^2cx^2)^{3/2}} \\ &\quad - \frac{15\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{16a^2c^2\sqrt{c+a^2cx^2}} \\ &\quad - \frac{5\sqrt{\frac{\pi}{6}}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)}{144a^2c^2\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
5/36*arctan(a*x)^(1/2)/a^2/c/(a^2*c*x^2+c)^(3/2)+5/6*arctan(a*x)^(1/2)/a^2
/c^2/(a^2*c*x^2+c)^(1/2)+5/18*x*arctan(a*x)^(3/2)/a/c/(a^2*c*x^2+c)^(3/2)+
5/9*x*arctan(a*x)^(3/2)/a/c^2/(a^2*c*x^2+c)^(1/2)-1/3*arctan(a*x)^(5/2)/a^
2/c/(a^2*c*x^2+c)^(3/2)-15/32*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)*FresnelC(
2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))/a^2/c^2/(a^2*c*x^2+c)^(1/2)-5/864*6^(1
/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)*FresnelC(6^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2)
)/a^2/c^2/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.22

$$\int \frac{x \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{1680 \arctan(ax) + 1440a^2x^2 \arctan(ax) + 1440ax \arctan(ax)^2 + 960a^3x^3 \arctan(a$$

input
```
Integrate[(x*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(1680*ArcTan[a*x] + 1440*a^2*x^2*ArcTan[a*x] + 1440*a*x*ArcTan[a*x]^2 + 96
0*a^3*x^3*ArcTan[a*x]^2 - 576*ArcTan[a*x]^3 + (405*I)*(1 + a^2*x^2)^(3/2)*
Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-I)*ArcTan[a*x]] - (405*I)*(1 + a^2*x^2
)^(3/2)*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, I*ArcTan[a*x]] + (5*I)*Sqrt[3 + 3*a
^2*x^2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-3*I)*ArcTan[a*x]] + (5*I)*a^2*
x^2*Sqrt[3 + 3*a^2*x^2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-3*I)*ArcTan[a*
x]] - (5*I)*Sqrt[3 + 3*a^2*x^2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (3*I)*ArcTa
n[a*x]] - (5*I)*a^2*x^2*Sqrt[3 + 3*a^2*x^2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2,
 (3*I)*ArcTan[a*x]])/(1728*a^2*c^2*(1 + a^2*x^2)*Sqrt[c + a^2*c*x^2]*Sqrt[
ArcTan[a*x]])
```

## Rubi [A] (verified)

Time = 1.44 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.15, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5465, 5435, 5433, 5440, 5439, 3042, 3785, 3793, 2009, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \arctan(ax)^{5/2}}{(a^2cx^2+c)^{5/2}}\,dx$$

$$\Big\downarrow \text{5465}$$

$$\frac{5\int \frac{\arctan(ax)^{3/2}}{(a^2cx^2+c)^{5/2}}dx}{6a} - \frac{\arctan(ax)^{5/2}}{3a^2c\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5435}$$

$$\frac{5\left(-\frac{1}{12}\int \frac{1}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx+\frac{2\int \frac{\arctan(ax)^{3/2}}{\left(a^2cx^2+c\right)^{3/2}}dx}{3c}+\frac{x\arctan(ax)^{3/2}}{3c(a^2cx^2+c)^{3/2}}+\frac{\sqrt{\arctan(ax)}}{6ac(a^2cx^2+c)^{3/2}}\right)}{6a}-\frac{\arctan(ax)^{5/2}}{3a^2c\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{5433}$$

$$\frac{5\left(-\frac{1}{12}\int \frac{1}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx+\frac{2\left(-\frac{3}{4}\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{x\arctan(ax)^{3/2}}{3c(a^2cx^2+c)^{3/2}}+\cdots\right)}{6a}$$

$$\frac{\arctan(ax)^{5/2}}{3a^2c\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{5440}$$

$$\frac{5\left(-\frac{\sqrt{a^2x^2+1}\int \frac{1}{\left(a^2x^2+1\right)^{5/2}\sqrt{\arctan(ax)}}dx}{12c^2\sqrt{a^2cx^2+c}}+\frac{2\left(-\frac{3\sqrt{a^2x^2+1}\int \frac{1}{\left(a^2x^2+1\right)^{3/2}\sqrt{\arctan(ax)}}dx}{4c\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{x\arctan(\cdots}{3c(a^2cx^2+\cdots}\right.}{6a}$$

$$\frac{\arctan(ax)^{5/2}}{3a^2c\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{5439}$$

$$\frac{5\left(-\frac{\sqrt{a^2x^2+1}\int \frac{1}{\left(a^2x^2+1\right)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{12ac^2\sqrt{a^2cx^2+c}}+\frac{2\left(-\frac{3\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}d\arctan(ax)}{4ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}\right)}{3c}\cdots\right.}{6a}$$

$$\frac{\arctan(ax)^{5/2}}{3a^2c\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\left(-\frac{\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^3}{\sqrt{\arctan(ax)}}d\arctan(ax)}{12ac^2\sqrt{a^2cx^2+c}}+\frac{2\left(-\frac{3\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)}{\sqrt{\arctan(ax)}}d\arctan(ax)}{4ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{x a}{3c(}\right.}{6a}$$

$$\frac{\arctan(ax)^{5/2}}{3a^2c\left(a^2cx^2+c\right)^{3/2}}$$

$\downarrow$ 3785

$$\frac{5\left(-\frac{\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^3}{\sqrt{\arctan(ax)}}d\arctan(ax)}{12ac^2\sqrt{a^2cx^2+c}}+\frac{2\left(-\frac{3\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}}d\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{x\arctan(}{3c(a^2cx^2+}\right.}{6a}$$

$$\frac{\arctan(ax)^{5/2}}{3a^2c\left(a^2cx^2+c\right)^{3/2}}$$

$\downarrow$ 3793

$$\frac{5\left(-\frac{\sqrt{a^2x^2+1}\int \left(\frac{\cos(3\arctan(ax))}{4\sqrt{\arctan(ax)}}+\frac{3}{4\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{12ac^2\sqrt{a^2cx^2+c}}+\frac{2\left(-\frac{3\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}}d\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\text{arct}}}{2ac\sqrt{a^2}}\right.}{3c}\right.}{6a}$$

$$\frac{\arctan(ax)^{5/2}}{3a^2c\left(a^2cx^2+c\right)^{3/2}}$$

$\downarrow$ 2009

$$\frac{5\left(\frac{2\left(-\frac{3\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}}d\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{3/2}}{c\sqrt{a^2cx^2+c}}+\frac{3\sqrt{\arctan(ax)}}{2ac\sqrt{a^2cx^2+c}}\right)}{3c}-\frac{\sqrt{a^2x^2+1}\left(\frac{3}{2}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{6}}\text{Fresne}\right.}{12ac^2\sqrt{a^2cx^2+c}}\right.}{6a}$$

$$\frac{\arctan(ax)^{5/2}}{3a^2c\left(a^2cx^2+c\right)^{3/2}}$$

$\downarrow$ 3833

$$\frac{5\left(-\frac{\sqrt{a^2x^2+1}\left(\frac{3}{2}\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{6}}\operatorname{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{12ac^2\sqrt{a^2cx^2+c}}+\frac{2\left(-\frac{3\sqrt{\frac{\pi}{2}}\sqrt{a^2x^2+1}\operatorname{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{2ac\sqrt{a^2cx^2+c}}+\right.}{3c}\right.}{6a}$$

$$\frac{\arctan(ax)^{5/2}}{3a^2c\left(a^2cx^2+c\right)^{3/2}}$$

input
```
Int[(x*ArcTan[a*x]^(5/2))/(c + a^2*c*x^2)^(5/2),x]
```

output
```
-1/3*ArcTan[a*x]^(5/2)/(a^2*c*(c + a^2*c*x^2)^(3/2)) + (5*(Sqrt[ArcTan[a*x
]]/(6*a*c*(c + a^2*c*x^2)^(3/2)) + (x*ArcTan[a*x]^(3/2))/(3*c*(c + a^2*c*x
^2)^(3/2)) + (2*((3*Sqrt[ArcTan[a*x]])/(2*a*c*Sqrt[c + a^2*c*x^2]) + (x*Ar
cTan[a*x]^(3/2))/(c*Sqrt[c + a^2*c*x^2]) - (3*Sqrt[Pi/2]*Sqrt[1 + a^2*x^2]
*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(2*a*c*Sqrt[c + a^2*c*x^2])))/(3*
c) - (Sqrt[1 + a^2*x^2]*((3*Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x
]]])/2 + (Sqrt[Pi/6]*FresnelC[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/2))/(12*a*c^2
*Sqrt[c + a^2*c*x^2])))/(6*a)
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 5433
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x] - Simp[b^2*p*(p
- 1)   Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2), x], x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

rule 5435
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[b*p*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p - 1)/(4*c*d*
(q + 1)^2)), x] + (-Simp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*d*
(q + 1))), x] + Simp[(2*q + 3)/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p, x], x] - Simp[b^2*p*((p - 1)/(4*(q + 1)^2))   Int[(d + e
*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5440
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])   Int[(1 +
c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

## Maple [F]

$$\int \frac{x \arctan(ax)^{\frac{5}{2}}}{(a^2c\,x^2+c)^{\frac{5}{2}}}dx$$

input
```
int(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x)
```

output
```
int(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x*atan(a*x)**(5/2)/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x\arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F]

$$\int \frac{x\arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x\arctan\left(ax\right)^{\frac{5}{2}}}{\left(a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
integrate(x*arctan(a*x)^(5/2)/(a^2*c*x^2 + c)^(5/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x\arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{x\,\mathrm{atan}(a\,x)^{5/2}}{\left(c\,a^2\,x^2+c\right)^{5/2}}\,dx$$

input
```
int((x*atan(a*x)^(5/2))/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int((x*atan(a*x)^(5/2))/(c + a^2*c*x^2)^(5/2), x)
```

## Reduce [F]

$$\int \frac{x \arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
int(x*atan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x)
```

output
```
(sqrt(c)*( - 24*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)**2 + 40*sqrt
(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)*a**3*x**3 + 60*sqrt(a**2*x**2 +
1)*sqrt(atan(a*x))*atan(a*x)*a*x + 60*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*
a**2*x**2 + 70*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)) - 30*int((sqrt(a**2*x**
2 + 1)*sqrt(atan(a*x))*x**2)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4
+ 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**7*x**4 - 60*int((sqrt(a**2*x**2
 + 1)*sqrt(atan(a*x))*x**2)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 +
 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**5*x**2 - 30*int((sqrt(a**2*x**2
+ 1)*sqrt(atan(a*x))*x**2)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 +
3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**3 - 35*int((sqrt(a**2*x**2 + 1)*s
qrt(atan(a*x)))/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)
*a**2*x**2 + atan(a*x)),x)*a**5*x**4 - 70*int((sqrt(a**2*x**2 + 1)*sqrt(at
an(a*x)))/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*
x**2 + atan(a*x)),x)*a**3*x**2 - 35*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x
)))/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 +
 atan(a*x)),x)*a))/(72*a**2*c**3*(a**4*x**4 + 2*a**2*x**2 + 1))
```

# 3.911 $\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 337

$$\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = -\frac{45x\sqrt{\arctan(ax)}}{16c^2\sqrt{c+a^2cx^2}} + \frac{5\arctan(ax)^{3/2}}{18ac\,(c+a^2cx^2)^{3/2}} + \frac{5\arctan(ax)^{3/2}}{3ac^2\sqrt{c+a^2cx^2}} + \frac{x\arctan(ax)^{5/2}}{3c\,(c+a^2cx^2)^{3/2}} + \frac{2x\arctan(ax)^{5/2}}{3c^2\sqrt{c+a^2cx^2}} + \frac{45\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{16ac^2\sqrt{c+a^2cx^2}} + \frac{5\sqrt{\frac{\pi}{6}}\sqrt{1+a^2x^2}\,\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)}{144ac^2\sqrt{c+a^2cx^2}} - \frac{5\sqrt{1+a^2x^2}\sqrt{\arctan(ax)}\sin(3\arctan(ax))}{144ac^2\sqrt{c+a^2cx^2}}$$

output
```
-45/16*x*arctan(a*x)^(1/2)/c^2/(a^2*c*x^2+c)^(1/2)+5/18*arctan(a*x)^(3/2)/
a/c/(a^2*c*x^2+c)^(3/2)+5/3*arctan(a*x)^(3/2)/a/c^2/(a^2*c*x^2+c)^(1/2)+1/
3*x*arctan(a*x)^(5/2)/c/(a^2*c*x^2+c)^(3/2)+2/3*x*arctan(a*x)^(5/2)/c^2/(a
^2*c*x^2+c)^(1/2)+45/32*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)*FresnelS(2^(1/2
)/Pi^(1/2)*arctan(a*x)^(1/2))/a/c^2/(a^2*c*x^2+c)^(1/2)+5/864*6^(1/2)*Pi^(
1/2)*(a^2*x^2+1)^(1/2)*FresnelS(6^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))/a/c^2/
(a^2*c*x^2+c)^(1/2)-5/144*(a^2*x^2+1)^(1/2)*arctan(a*x)^(1/2)*sin(3*arctan
(a*x))/a/c^2/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.52

$$\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \frac{24\sqrt{\arctan(ax)}(-5ax(21+20a^2x^2)+10(7+6a^2x^2)\arctan(ax)+12ax(3+2a^2x^2}{}$$

input
```
Integrate[ArcTan[a*x]^(5/2)/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(24*Sqrt[ArcTan[a*x]]*(-5*a*x*(21 + 20*a^2*x^2) + 10*(7 + 6*a^2*x^2)*ArcTa
n[a*x] + 12*a*x*(3 + 2*a^2*x^2)*ArcTan[a*x]^2) + 1215*Sqrt[2*Pi]*(1 + a^2*
x^2)^(3/2)*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]] + 5*Sqrt[6*Pi]*(1 + a^2*
x^2)^(3/2)*FresnelS[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/(864*c^2*(a + a^3*x^2)*
Sqrt[c + a^2*c*x^2])
```

## Rubi [A] (verified)

Time = 1.42 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.10, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {5435, 5433, 5440, 5439, 3042, 3777, 25, 3042, 3786, 3793, 2009, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{5/2}}{(a^2cx^2+c)^{5/2}}\,dx$$

$$\Big\downarrow \text{5435}$$

$$-\frac{5}{12}\int \frac{\sqrt{\arctan(ax)}}{(a^2cx^2+c)^{5/2}}dx+\frac{2\int \frac{\arctan(ax)^{5/2}}{(a^2cx^2+c)^{3/2}}dx}{3c}+\frac{x\arctan(ax)^{5/2}}{3c\,(a^2cx^2+c)^{3/2}}+\frac{5\arctan(ax)^{3/2}}{18ac\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5433}$$

$$-\frac{5}{12}\int \frac{\sqrt{\arctan(ax)}}{(a^2cx^2+c)^{5/2}}dx+\frac{2\left(-\frac{15}{4}\int \frac{\sqrt{\arctan(ax)}}{(a^2cx^2+c)^{3/2}}dx+\frac{x\arctan(ax)^{5/2}}{c\sqrt{a^2cx^2+c}}+\frac{5\arctan(ax)^{3/2}}{2ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{x\arctan(ax)^{5/2}}{3c\,(a^2cx^2+c)^{3/2}}+\frac{5\arctan(ax)^{3/2}}{18ac\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5440}$$

$$-\frac{5\sqrt{a^2x^2+1}\int \frac{\sqrt{\arctan(ax)}}{(a^2x^2+1)^{5/2}}dx}{12c^2\sqrt{a^2cx^2+c}}+\frac{2\left(-\frac{15\sqrt{a^2x^2+1}\int \frac{\sqrt{\arctan(ax)}}{(a^2x^2+1)^{3/2}}dx}{4c\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{5/2}}{c\sqrt{a^2cx^2+c}}+\frac{5\arctan(ax)^{3/2}}{2ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{x\arctan(ax)^{5/2}}{3c\,(a^2cx^2+c)^{3/2}}+\frac{5\arctan(ax)^{3/2}}{18ac\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5439}$$

$$-\frac{5\sqrt{a^2x^2+1}\int \frac{\sqrt{\arctan(ax)}}{(a^2x^2+1)^{3/2}}d\arctan(ax)}{12ac^2\sqrt{a^2cx^2+c}}+\frac{2\left(-\frac{15\sqrt{a^2x^2+1}\int \frac{\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}d\arctan(ax)}{4ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{5/2}}{c\sqrt{a^2cx^2+c}}+\frac{5\arctan(ax)^{3/2}}{2ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{x\arctan(ax)^{5/2}}{3c\,(a^2cx^2+c)^{3/2}}+\frac{5\arctan(ax)^{3/2}}{18ac\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{5\sqrt{a^2x^2+1}\int \sqrt{\arctan(ax)}\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^3d\arctan(ax)}{12ac^2\sqrt{a^2cx^2+c}}+\frac{2\left(-\frac{15\sqrt{a^2x^2+1}\int \sqrt{\arctan(ax)}\sin\left(\arctan(ax)+\frac{\pi}{2}\right)d\arctan(ax)}{4ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{5/2}}{c\sqrt{a^2cx^2+c}}+\frac{5\arctan(ax)^{3/2}}{2ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{x\arctan(ax)^{5/2}}{3c\,(a^2cx^2+c)^{3/2}}+\frac{5\arctan(ax)^{3/2}}{18ac\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \; 3777$$

$$-\frac{5\sqrt{a^2x^2+1}\int \sqrt{\arctan(ax)}\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^3d\arctan(ax)}{12ac^2\sqrt{a^2cx^2+c}}+\frac{2\left(-\frac{15\sqrt{a^2x^2+1}\left(\frac{1}{2}\int -\frac{ax}{\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}d\arctan(ax)+\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}\right)}{4ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{5/2}}{c\sqrt{a^2cx^2+c}}+\frac{5\arctan(ax)^{3/2}}{2ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{x\arctan(ax)^{5/2}}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{5\arctan(ax)^{3/2}}{18ac\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \; 25$$

$$-\frac{5\sqrt{a^2x^2+1}\int \sqrt{\arctan(ax)}\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^3d\arctan(ax)}{12ac^2\sqrt{a^2cx^2+c}}+\frac{2\left(-\frac{15\sqrt{a^2x^2+1}\left(\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}-\frac{1}{2}\int \frac{ax}{\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}d\arctan(ax)\right)}{4ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{5/2}}{c\sqrt{a^2cx^2+c}}+\frac{5\arctan(ax)^{3/2}}{2ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{x\arctan(ax)^{5/2}}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{5\arctan(ax)^{3/2}}{18ac\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{5\sqrt{a^2x^2+1}\int \sqrt{\arctan(ax)}\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^3d\arctan(ax)}{12ac^2\sqrt{a^2cx^2+c}}+\frac{2\left(-\frac{15\sqrt{a^2x^2+1}\left(\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}-\frac{1}{2}\int \frac{\sin(\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)\right)}{4ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{5/2}}{c\sqrt{a^2cx^2+c}}+\frac{5\arctan(ax)^{3/2}}{2ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{x\arctan(ax)^{5/2}}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{5\arctan(ax)^{3/2}}{18ac\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \; 3786$$

$$-\frac{5\sqrt{a^2x^2+1}\int \sqrt{\arctan(ax)}\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^3d\arctan(ax)}{12ac^2\sqrt{a^2cx^2+c}}+\frac{2\left(-\frac{15\sqrt{a^2x^2+1}\left(\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}-\int \frac{ax}{\sqrt{a^2x^2+1}}d\sqrt{\arctan(ax)}\right)}{4ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{5/2}}{c\sqrt{a^2cx^2+c}}+\frac{5\arctan(ax)^{3/2}}{2ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{x\arctan(ax)^{5/2}}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{5\arctan(ax)^{3/2}}{18ac\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \; 3793$$

$$-\frac{5\sqrt{a^2x^2+1}\int\left(\frac{1}{4}\sqrt{\arctan(ax)}\cos(3\arctan(ax))+\frac{3\sqrt{\arctan(ax)}}{4\sqrt{a^2x^2+1}}\right)d\arctan(ax)}{12ac^2\sqrt{a^2cx^2+c}}+\frac{2\left(-\frac{15\sqrt{a^2x^2+1}\left(\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}-\int\frac{ax}{\sqrt{a^2x^2+1}}d\sqrt{\arctan(ax)}\right)}{4ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{5/2}}{c\sqrt{a^2cx^2+c}}+\frac{5\arctan(ax)^{3/2}}{2ac\sqrt{a^2cx^2+c}}\right)}{3c}+\frac{x\arctan(ax)^{5/2}}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{5\arctan(ax)^{3/2}}{18ac\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\left(-\frac{15\sqrt{a^2x^2+1}\left(\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}-\int\frac{ax}{\sqrt{a^2x^2+1}}d\sqrt{\arctan(ax)}\right)}{4ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{5/2}}{c\sqrt{a^2cx^2+c}}+\frac{5\arctan(ax)^{3/2}}{2ac\sqrt{a^2cx^2+c}}\right)}{3c}-\frac{5\sqrt{a^2x^2+1}\left(\frac{3ax\sqrt{\arctan(ax)}}{4\sqrt{a^2x^2+1}}-\frac{3}{4}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{12}\sqrt{\frac{\pi}{6}}\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{12}\sqrt{a\ldots}\right.}{12ac^2\sqrt{a^2cx^2+c}}$$

$$\frac{x\arctan(ax)^{5/2}}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{5\arctan(ax)^{3/2}}{18ac\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{3832}$$

$$-\frac{5\sqrt{a^2x^2+1}\left(\frac{3ax\sqrt{\arctan(ax)}}{4\sqrt{a^2x^2+1}}-\frac{3}{4}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{12}\sqrt{\frac{\pi}{6}}\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{12}\sqrt{\ldots}\right.}{12ac^2\sqrt{a^2cx^2+c}}$$

$$\frac{2\left(-\frac{15\sqrt{a^2x^2+1}\left(\frac{ax\sqrt{\arctan(ax)}}{\sqrt{a^2x^2+1}}-\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{4ac\sqrt{a^2cx^2+c}}+\frac{x\arctan(ax)^{5/2}}{c\sqrt{a^2cx^2+c}}+\frac{5\arctan(ax)^{3/2}}{2ac\sqrt{a^2cx^2+c}}\right)}{3c}+$$

$$\frac{x\arctan(ax)^{5/2}}{3c\left(a^2cx^2+c\right)^{3/2}}+\frac{5\arctan(ax)^{3/2}}{18ac\left(a^2cx^2+c\right)^{3/2}}$$

input
```
Int[ArcTan[a*x]^(5/2)/(c + a^2*c*x^2)^(5/2),x]
```

output
```
(5*ArcTan[a*x]^(3/2))/(18*a*c*(c + a^2*c*x^2)^(3/2)) + (x*ArcTan[a*x]^(5/2
))/(3*c*(c + a^2*c*x^2)^(3/2)) + (2*((5*ArcTan[a*x]^(3/2))/(2*a*c*Sqrt[c +
 a^2*c*x^2]) + (x*ArcTan[a*x]^(5/2))/(c*Sqrt[c + a^2*c*x^2]) - (15*Sqrt[1
+ a^2*x^2]*((a*x*Sqrt[ArcTan[a*x]])/Sqrt[1 + a^2*x^2] - Sqrt[Pi/2]*Fresnel
S[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]]))/(4*a*c*Sqrt[c + a^2*c*x^2])))/(3*c) - (5
*Sqrt[1 + a^2*x^2]*((3*a*x*Sqrt[ArcTan[a*x]])/(4*Sqrt[1 + a^2*x^2]) - (3*S
qrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/4 - (Sqrt[Pi/6]*FresnelS
[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/12 + (Sqrt[ArcTan[a*x]]*Sin[3*ArcTan[a*x]]
)/12))/(12*a*c^2*Sqrt[c + a^2*c*x^2])
```

### Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 5433
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)/((d_) + (e_.)*(x_)^2)^(3/2), x_
Symbol] :> Simp[b*p*((a + b*ArcTan[c*x])^(p - 1)/(c*d*Sqrt[d + e*x^2])), x]
 + (Simp[x*((a + b*ArcTan[c*x])^p/(d*Sqrt[d + e*x^2])), x] - Simp[b^2*p*(p
- 1)   Int[(a + b*ArcTan[c*x])^(p - 2)/(d + e*x^2)^(3/2), x], x]) /; FreeQ[
{a, b, c, d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 1]
```

rule 5435
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[b*p*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p - 1)/(4*c*d*
(q + 1)^2)), x] + (-Simp[x*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*d*
(q + 1))), x] + Simp[(2*q + 3)/(2*d*(q + 1))   Int[(d + e*x^2)^(q + 1)*(a +
 b*ArcTan[c*x])^p, x], x] - Simp[b^2*p*((p - 1)/(4*(q + 1)^2))   Int[(d + e
*x^2)^q*(a + b*ArcTan[c*x])^(p - 2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &
& EqQ[e, c^2*d] && LtQ[q, -1] && GtQ[p, 1] && NeQ[q, -3/2]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5440
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])   Int[(1 +
c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

## Maple [F]

$$\int \frac{\arctan\left(ax\right)^{\frac{5}{2}}}{\left(a^{2}c\,x^{2}+c\right)^{\frac{5}{2}}}dx$$

input
```
int(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x)
```

output
```
int(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\operatorname{atan}^{\frac{5}{2}}(ax)}{(c(a^2x^2+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(atan(a*x)**(5/2)/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Integral(atan(a*x)**(5/2)/(c*(a**2*x**2 + 1))**(5/2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F]

$$\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\arctan(ax)^{\frac{5}{2}}}{(a^2cx^2+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(arctan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^(5/2)/(a^2*c*x^2 + c)^(5/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\mathrm{atan}(a\,x)^{5/2}}{(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int(atan(a*x)^(5/2)/(c + a^2*c*x^2)^(5/2),x)
```

output
```
int(atan(a*x)^(5/2)/(c + a^2*c*x^2)^(5/2), x)
```

## Reduce [F]

$$\int \frac{\arctan(ax)^{5/2}}{(c+a^2cx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
int(atan(a*x)^(5/2)/(a^2*c*x^2+c)^(5/2),x)
```

output
```
(sqrt(c)*(48*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)**2*a**3*x**3 +
72*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)**2*a*x + 120*sqrt(a**2*x*
*2 + 1)*sqrt(atan(a*x))*atan(a*x)*a**2*x**2 + 140*sqrt(a**2*x**2 + 1)*sqrt
(atan(a*x))*atan(a*x) - 200*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*a**3*x**3
- 210*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*a*x + 100*int((sqrt(a**2*x**2 +
1)*sqrt(atan(a*x))*x**3)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*
atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**8*x**4 + 200*int((sqrt(a**2*x**2 +
1)*sqrt(atan(a*x))*x**3)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*
atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**6*x**2 + 100*int((sqrt(a**2*x**2 +
1)*sqrt(atan(a*x))*x**3)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*
atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**4 + 105*int((sqrt(a**2*x**2 + 1)*sq
rt(atan(a*x))*x)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x
)*a**2*x**2 + atan(a*x)),x)*a**6*x**4 + 210*int((sqrt(a**2*x**2 + 1)*sqrt(
atan(a*x))*x)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a
**2*x**2 + atan(a*x)),x)*a**4*x**2 + 105*int((sqrt(a**2*x**2 + 1)*sqrt(ata
n(a*x))*x)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2
*x**2 + atan(a*x)),x)*a**2))/(72*a*c**3*(a**4*x**4 + 2*a**2*x**2 + 1))
```

# 3.912 $\int \frac{\arctan(ax)^{5/2}}{x(c+a^2cx^2)^{5/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\arctan(ax)^{5/2}}{x(c+a^2cx^2)^{5/2}}\,dx = \mathrm{Int}\left(\frac{\arctan(ax)^{5/2}}{x(c+a^2cx^2)^{5/2}},x\right)$$

output
```
Defer(Int)(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^(5/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 5.48 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\arctan(ax)^{5/2}}{x(c+a^2cx^2)^{5/2}}\,dx = \int \frac{\arctan(ax)^{5/2}}{x(c+a^2cx^2)^{5/2}}\,dx$$

input
```
Integrate[ArcTan[a*x]^(5/2)/(x*(c + a^2*c*x^2)^(5/2)),x]
```

output
```
Integrate[ArcTan[a*x]^(5/2)/(x*(c + a^2*c*x^2)^(5/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^{5/2}}{x\left(a^2cx^2+c\right)^{5/2}}dx$$

$$\Big\downarrow \text{5560}$$

$$\int \frac{\arctan(ax)^{5/2}}{x\left(a^2cx^2+c\right)^{5/2}}dx$$

input
```
Int[ArcTan[a*x]^(5/2)/(x*(c + a^2*c*x^2)^(5/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\arctan\left(ax\right)^{\frac{5}{2}}}{x\left(a^2c\,x^2+c\right)^{\frac{5}{2}}}dx$$

input
```
int(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^(5/2),x)
```

output
```
int(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{5/2}}{x\left(c+a^2cx^2\right)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)^{5/2}}{x\left(c+a^2cx^2\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(atan(a*x)**(5/2)/x/(a**2*c*x**2+c)**(5/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^{5/2}}{x\left(c+a^2cx^2\right)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 2.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\arctan(ax)^{5/2}}{x\left(c+a^2cx^2\right)^{5/2}}\,dx=\int \frac{\arctan\left(ax\right)^{\frac{5}{2}}}{\left(a^2cx^2+c\right)^{\frac{5}{2}}x}\,dx$$

input
```
integrate(arctan(a*x)^(5/2)/x/(a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

output
```
integrate(arctan(a*x)^(5/2)/((a^2*c*x^2 + c)^(5/2)*x), x)
```

## Mupad [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\arctan(ax)^{5/2}}{x\left(c+a^2cx^2\right)^{5/2}}\,dx=\int \frac{\mathrm{atan}(a\,x)^{5/2}}{x\,\left(c\,a^2\,x^2+c\right)^{5/2}}\,dx$$

input
```
int(atan(a*x)^(5/2)/(x*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(atan(a*x)^(5/2)/(x*(c + a^2*c*x^2)^(5/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.19

$$\int \frac{\arctan(ax)^{5/2}}{x\left(c+a^2cx^2\right)^{5/2}}\,dx=\frac{\sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,atan(ax)^2}{a^6x^7+3a^4x^5+3a^2x^3+x}dx\right)}{c^3}$$

input
```
int(atan(a*x)^(5/2)/x/(a^2*c*x^2+c)^(5/2),x)
```

output
```
(sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)**2)/(a**6*x**7
 + 3*a**4*x**5 + 3*a**2*x**3 + x),x))/c**3
```

# 3.913 $\int \frac{x^m(c+a^2cx^2)}{\sqrt{\arctan(ax)}} \, dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^m(c+a^2cx^2)}{\sqrt{\arctan(ax)}} \, dx = \text{Int}\left(\frac{x^m(c+a^2cx^2)}{\sqrt{\arctan(ax)}}, x\right)$$

output
```
Defer(Int)(x^m*(a^2*c*x^2+c)/arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.92 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m(c+a^2cx^2)}{\sqrt{\arctan(ax)}} \, dx = \int \frac{x^m(c+a^2cx^2)}{\sqrt{\arctan(ax)}} \, dx$$

input
```
Integrate[(x^m*(c + a^2*c*x^2))/Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[(x^m*(c + a^2*c*x^2))/Sqrt[ArcTan[a*x]], x]
```

## Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \left(a^2 c x^2+c\right)}{\sqrt{\arctan (a x)}} \, dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{x^m \left(a^2 c x^2+c\right)}{\sqrt{\arctan (a x)}}dx$$

input
```
Int[(x^m*(c + a^2*c*x^2))/Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x^m (a^2 c\,x^2+c)}{\sqrt{\arctan \left(a x \right)}}dx$$

input
```
int(x^m*(a^2*c*x^2+c)/arctan(a*x)^(1/2),x)
```

output
```
int(x^m*(a^2*c*x^2+c)/arctan(a*x)^(1/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m(c+a^2cx^2)}{\sqrt{\arctan(ax)}}\,dx = \int \frac{(a^2cx^2+c)x^m}{\sqrt{\arctan(ax)}}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)*x^m/sqrt(arctan(a*x)), x)
```

## Sympy [N/A]

Not integrable

Time = 11.81 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.55

$$\int \frac{x^m(c+a^2cx^2)}{\sqrt{\arctan(ax)}}\,dx = c\left(\int \frac{x^m}{\sqrt{\operatorname{atan}(ax)}}\,dx + \int \frac{a^2x^2x^m}{\sqrt{\operatorname{atan}(ax)}}\,dx\right)$$

input
```
integrate(x**m*(a**2*c*x**2+c)/atan(a*x)**(1/2),x)
```

output
```
c*(Integral(x**m/sqrt(atan(a*x)), x) + Integral(a**2*x**2*x**m/sqrt(atan(a
*x)), x))
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m(c+a^2cx^2)}{\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m(c+a^2cx^2)}{\sqrt{\arctan(ax)}}\,dx = \int \frac{(a^2cx^2+c)x^m}{\sqrt{\arctan{(ax)}}}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)*x^m/sqrt(arctan(a*x)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.89 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m(c+a^2cx^2)}{\sqrt{\arctan(ax)}}\,dx = \int \frac{x^m\,\left(c\,a^2\,x^2+c\right)}{\sqrt{\mathrm{atan}\left(a\,x\right)}}\,dx$$

input
```
int((x^m*(c + a^2*c*x^2))/atan(a*x)^(1/2),x)
```

output
```
int((x^m*(c + a^2*c*x^2))/atan(a*x)^(1/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.00

$$\int \frac{x^m(c+a^2cx^2)}{\sqrt{\arctan(ax)}}\,dx = c\left(\left(\int \frac{x^m\sqrt{atan(ax)}\,x^2}{atan(ax)}dx\right)a^2+\int \frac{x^m\sqrt{atan(ax)}}{atan(ax)}dx\right)$$

input
```
int(x^m*(a^2*c*x^2+c)/atan(a*x)^(1/2),x)
```

output
```
c*(int((x**m*sqrt(atan(a*x))*x**2)/atan(a*x),x)*a**2 + int((x**m*sqrt(atan
(a*x)))/atan(a*x),x))
```

# 3.914 $\int \frac{x(c+a^2cx^2)}{\sqrt{\arctan(ax)}} dx$



## Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{x(c+a^2cx^2)}{\sqrt{\arctan(ax)}} dx = \text{Int}\left(\frac{x(c+a^2cx^2)}{\sqrt{\arctan(ax)}}, x\right)$$

output
```
Defer(Int)(x*(a^2*c*x^2+c)/arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x(c+a^2cx^2)}{\sqrt{\arctan(ax)}} dx = \int \frac{x(c+a^2cx^2)}{\sqrt{\arctan(ax)}} dx$$

input
```
Integrate[(x*(c + a^2*c*x^2))/Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[(x*(c + a^2*c*x^2))/Sqrt[ArcTan[a*x]], x]
```

## Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a^2cx^2+c)}{\sqrt{\arctan(ax)}}dx$$

$$\Big\downarrow \ 5560$$

$$\int \frac{x(a^2cx^2+c)}{\sqrt{\arctan(ax)}}dx$$

input
```
Int[(x*(c + a^2*c*x^2))/Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{x(a^2c\,x^2+c)}{\sqrt{\arctan\left(ax\right)}}dx$$

input
```
int(x*(a^2*c*x^2+c)/arctan(a*x)^(1/2),x)
```

output
```
int(x*(a^2*c*x^2+c)/arctan(a*x)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x(c + a^2cx^2)}{\sqrt{\arctan(ax)}}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 1.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.45

$$\int \frac{x(c + a^2cx^2)}{\sqrt{\arctan(ax)}}\, dx = c\left(\int \frac{x}{\sqrt{\operatorname{atan}(ax)}}\, dx + \int \frac{a^2x^3}{\sqrt{\operatorname{atan}(ax)}}\, dx\right)$$

input
```
integrate(x*(a**2*c*x**2+c)/atan(a*x)**(1/2),x)
```

output
```
c*(Integral(x/sqrt(atan(a*x)), x) + Integral(a**2*x**3/sqrt(atan(a*x)), x)
)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x(c + a^2cx^2)}{\sqrt{\arctan(ax)}}\, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{x(c+a^2cx^2)}{\sqrt{\arctan(ax)}}\,dx = \int \frac{(a^2cx^2+c)x}{\sqrt{\arctan(ax)}}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)*x/sqrt(arctan(a*x)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{x(c+a^2cx^2)}{\sqrt{\arctan(ax)}}\,dx = \int \frac{x\,(c\,a^2\,x^2+c)}{\sqrt{\mathrm{atan}\left(a\,x\right)}}\,dx$$

input
```
int((x*(c + a^2*c*x^2))/atan(a*x)^(1/2),x)
```

output
```
int((x*(c + a^2*c*x^2))/atan(a*x)^(1/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.95

$$\int \frac{x(c+a^2cx^2)}{\sqrt{\arctan(ax)}}\,dx = c\left(\left(\int \frac{\sqrt{atan\,(ax)}\,x^3}{atan\,(ax)}dx\right)a^2 + \int \frac{\sqrt{atan\,(ax)}\,x}{atan\,(ax)}dx\right)$$

input
```
int(x*(a^2*c*x^2+c)/atan(a*x)^(1/2),x)
```

output
```
c*(int((sqrt(atan(a*x))*x**3)/atan(a*x),x)*a**2 + int((sqrt(atan(a*x))*x)/
atan(a*x),x))
```

# 3.915 $\int \frac{c+a^2cx^2}{\sqrt{\arctan(ax)}}\,dx$



## Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \frac{c+a^2cx^2}{\sqrt{\arctan(ax)}}\,dx=\text{Int}\left(\frac{c+a^2cx^2}{\sqrt{\arctan(ax)}},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)/arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{c+a^2cx^2}{\sqrt{\arctan(ax)}}\,dx=\int \frac{c+a^2cx^2}{\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)/Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[(c + a^2*c*x^2)/Sqrt[ArcTan[a*x]], x]
```

## Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2cx^2+c}{\sqrt{\arctan(ax)}}\,dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{a^2cx^2+c}{\sqrt{\arctan(ax)}}dx$$

input
```
Int[(c + a^2*c*x^2)/Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{a^2c\,x^2+c}{\sqrt{\arctan\left(ax\right)}}dx$$

input
```
int((a^2*c*x^2+c)/arctan(a*x)^(1/2),x)
```

output
```
int((a^2*c*x^2+c)/arctan(a*x)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{c + a^2cx^2}{\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.53

$$\int \frac{c + a^2cx^2}{\sqrt{\arctan(ax)}}\,dx = c\left(\int \frac{a^2x^2}{\sqrt{\operatorname{atan}(ax)}}\,dx + \int \frac{1}{\sqrt{\operatorname{atan}(ax)}}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)/atan(a*x)**(1/2),x)
```

output
```
c*(Integral(a**2*x**2/sqrt(atan(a*x)), x) + Integral(1/sqrt(atan(a*x)), x)
)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{c + a^2cx^2}{\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{c+a^2cx^2}{\sqrt{\arctan(ax)}}\,dx = \int \frac{a^2cx^2+c}{\sqrt{\arctan(ax)}}\,dx$$

input
```
integrate((a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)/sqrt(arctan(a*x)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{c+a^2cx^2}{\sqrt{\arctan(ax)}}\,dx = \int \frac{c\,a^2\,x^2+c}{\sqrt{\mathrm{atan}\left(a\,x\right)}}\,dx$$

input
```
int((c + a^2*c*x^2)/atan(a*x)^(1/2),x)
```

output
```
int((c + a^2*c*x^2)/atan(a*x)^(1/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.00

$$\int \frac{c + a^2cx^2}{\sqrt{\arctan(ax)}}\,dx = c\left(\int \frac{\sqrt{atan\,(ax)}}{atan\,(ax)}dx + \left(\int \frac{\sqrt{atan\,(ax)}\,x^2}{atan\,(ax)}dx\right)a^2\right)$$

input
```
int((a^2*c*x^2+c)/atan(a*x)^(1/2),x)
```

output
```
c*(int(sqrt(atan(a*x))/atan(a*x),x) + int((sqrt(atan(a*x))*x**2)/atan(a*x)
,x)*a**2)
```

# 3.916 $\int \frac{c+a^2cx^2}{x\sqrt{\arctan(ax)}}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{c+a^2cx^2}{x\sqrt{\arctan(ax)}}\,dx = \text{Int}\left(\frac{c+a^2cx^2}{x\sqrt{\arctan(ax)}},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)/x/arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.90 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{c+a^2cx^2}{x\sqrt{\arctan(ax)}}\,dx = \int \frac{c+a^2cx^2}{x\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)/(x*Sqrt[ArcTan[a*x]]),x]
```

output
```
Integrate[(c + a^2*c*x^2)/(x*Sqrt[ArcTan[a*x]]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2cx^2+c}{x\sqrt{\arctan(ax)}}\,dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{a^2cx^2+c}{x\sqrt{\arctan(ax)}}dx$$

input
```
Int[(c + a^2*c*x^2)/(x*Sqrt[ArcTan[a*x]]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{a^2c\,x^2+c}{x\sqrt{\arctan\left(ax\right)}}dx$$

input
```
int((a^2*c*x^2+c)/x/arctan(a*x)^(1/2),x)
```

output
```
int((a^2*c*x^2+c)/x/arctan(a*x)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{c + a^2 c x^2}{x\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)/x/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 1.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.32

$$\int \frac{c + a^2 c x^2}{x\sqrt{\arctan(ax)}}\,dx = c\left(\int \frac{1}{x\sqrt{\operatorname{atan}(ax)}}\,dx + \int \frac{a^2 x}{\sqrt{\operatorname{atan}(ax)}}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)/x/atan(a*x)**(1/2),x)
```

output
```
c*(Integral(1/(x*sqrt(atan(a*x))), x) + Integral(a**2*x/sqrt(atan(a*x)), x
))
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{c + a^2 c x^2}{x\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)/x/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{c+a^2cx^2}{x\sqrt{\arctan(ax)}}\,dx = \int \frac{a^2cx^2+c}{x\sqrt{\arctan{(ax)}}}\,dx$$

input
```
integrate((a^2*c*x^2+c)/x/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)/(x*sqrt(arctan(a*x))), x)
```

## Mupad [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{c+a^2cx^2}{x\sqrt{\arctan(ax)}}\,dx = \int \frac{c\,a^2\,x^2+c}{x\,\sqrt{\mathrm{atan}\left(a\,x\right)}}\,dx$$

input
```
int((c + a^2*c*x^2)/(x*atan(a*x)^(1/2)),x)
```

output
```
int((c + a^2*c*x^2)/(x*atan(a*x)^(1/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.77

$$\int \frac{c+a^2cx^2}{x\sqrt{\arctan(ax)}}\,dx = c\left(\int \frac{\sqrt{atan\,(ax)}}{atan\,(ax)\,x}dx + \left(\int \frac{\sqrt{atan\,(ax)}\,x}{atan\,(ax)}dx\right)a^2\right)$$

input
```
int((a^2*c*x^2+c)/x/atan(a*x)^(1/2),x)
```

output
```
c*(int(sqrt(atan(a*x))/(atan(a*x)*x),x) + int((sqrt(atan(a*x))*x)/atan(a*x
),x)*a**2)
```

# 3.917 $\int \frac{x^m(c+a^2cx^2)^2}{\sqrt{\arctan(ax)}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m(c+a^2cx^2)^2}{\sqrt{\arctan(ax)}}\,dx = \text{Int}\left(\frac{x^m(c+a^2cx^2)^2}{\sqrt{\arctan(ax)}},x\right)$$

output
```
Defer(Int)(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m(c+a^2cx^2)^2}{\sqrt{\arctan(ax)}}\,dx = \int \frac{x^m(c+a^2cx^2)^2}{\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[(x^m*(c + a^2*c*x^2)^2)/Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[(x^m*(c + a^2*c*x^2)^2)/Sqrt[ArcTan[a*x]], x]
```

## Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \left(a^2 c x^2+c\right)^2}{\sqrt{\arctan (a x)}} \, dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{x^m \left(a^2 c x^2+c\right)^2}{\sqrt{\arctan (a x)}}dx$$

input
```
Int[(x^m*(c + a^2*c*x^2)^2)/Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m \left(a^2 c \,x^2+c \right)^2}{\sqrt{\arctan \left(a x \right)}}d x$$

input
```
int(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{x^m(c+a^2cx^2)^2}{\sqrt{\arctan(ax)}}\,dx = \int \frac{(a^2cx^2+c)^2x^m}{\sqrt{\arctan(ax)}}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*x^m/sqrt(arctan(a*x)), x)
```

## Sympy [N/A]

Not integrable

Time = 40.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.42

$$\int \frac{x^m(c+a^2cx^2)^2}{\sqrt{\arctan(ax)}}\,dx = c^2\left(\int \frac{x^m}{\sqrt{\operatorname{atan}(ax)}}\,dx+\int \frac{2a^2x^2x^m}{\sqrt{\operatorname{atan}(ax)}}\,dx+\int \frac{a^4x^4x^m}{\sqrt{\operatorname{atan}(ax)}}\,dx\right)$$

input
```
integrate(x**m*(a**2*c*x**2+c)**2/atan(a*x)**(1/2),x)
```

output
```
c**2*(Integral(x**m/sqrt(atan(a*x)), x) + Integral(2*a**2*x**2*x**m/sqrt(a
tan(a*x)), x) + Integral(a**4*x**4*x**m/sqrt(atan(a*x)), x))
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m(c+a^2cx^2)^2}{\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m(c+a^2cx^2)^2}{\sqrt{\arctan(ax)}}\,dx=\int \frac{(a^2cx^2+c)^2x^m}{\sqrt{\arctan(ax)}}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^2*x^m/sqrt(arctan(a*x)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.77 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m(c+a^2cx^2)^2}{\sqrt{\arctan(ax)}}\,dx=\int \frac{x^m\,(c\,a^2\,x^2+c)^2}{\sqrt{\mathrm{atan}\,(a\,x)}}\,dx$$

input
```
int((x^m*(c + a^2*c*x^2)^2)/atan(a*x)^(1/2),x)
```

output
```
int((x^m*(c + a^2*c*x^2)^2)/atan(a*x)^(1/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.96

$$\int \frac{x^m(c+a^2cx^2)^2}{\sqrt{\arctan(ax)}}\,dx = c^2\left(\left(\int \frac{x^m\sqrt{atan(ax)}\,x^4}{atan(ax)}dx\right)a^4 + 2\left(\int \frac{x^m\sqrt{atan(ax)}\,x^2}{atan(ax)}dx\right)a^2 + \int \frac{x^m\sqrt{atan(ax)}}{atan(ax)}dx\right)$$

input
```
int(x^m*(a^2*c*x^2+c)^2/atan(a*x)^(1/2),x)
```

output
```
c**2*(int((x**m*sqrt(atan(a*x))*x**4)/atan(a*x),x)*a**4 + 2*int((x**m*sqrt
(atan(a*x))*x**2)/atan(a*x),x)*a**2 + int((x**m*sqrt(atan(a*x)))/atan(a*x)
,x))
```

# 3.918 $\int \frac{x(c+a^2cx^2)^2}{\sqrt{\arctan(ax)}}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x(c+a^2cx^2)^2}{\sqrt{\arctan(ax)}}\,dx = \text{Int}\left(\frac{x(c+a^2cx^2)^2}{\sqrt{\arctan(ax)}},x\right)$$

output
```
Defer(Int)(x*(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x(c+a^2cx^2)^2}{\sqrt{\arctan(ax)}}\,dx = \int \frac{x(c+a^2cx^2)^2}{\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[(x*(c + a^2*c*x^2)^2)/Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[(x*(c + a^2*c*x^2)^2)/Sqrt[ArcTan[a*x]], x]
```

## Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a^2cx^2+c)^2}{\sqrt{\arctan(ax)}}dx$$

$$\Big\downarrow \textbf{5560}$$

$$\int \frac{x(a^2cx^2+c)^2}{\sqrt{\arctan(ax)}}dx$$

input
```
Int[(x*(c + a^2*c*x^2)^2)/Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x(a^2c\,x^2+c)^2}{\sqrt{\arctan\left(ax\right)}}dx$$

input
```
int(x*(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)
```

output
```
int(x*(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x(c+a^2cx^2)^2}{\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 1.80 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.23

$$\int \frac{x(c+a^2cx^2)^2}{\sqrt{\arctan(ax)}}\,dx = c^2\left(\int \frac{x}{\sqrt{\operatorname{atan}(ax)}}\,dx + \int \frac{2a^2x^3}{\sqrt{\operatorname{atan}(ax)}}\,dx + \int \frac{a^4x^5}{\sqrt{\operatorname{atan}(ax)}}\,dx\right)$$

input
```
integrate(x*(a**2*c*x**2+c)**2/atan(a*x)**(1/2),x)
```

output
```
c**2*(Integral(x/sqrt(atan(a*x)), x) + Integral(2*a**2*x**3/sqrt(atan(a*x)
), x) + Integral(a**4*x**5/sqrt(atan(a*x)), x))
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x(c+a^2cx^2)^2}{\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x(c+a^2cx^2)^2}{\sqrt{\arctan(ax)}}\,dx = \int \frac{(a^2cx^2+c)^2x}{\sqrt{\arctan(ax)}}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^2*x/sqrt(arctan(a*x)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x(c+a^2cx^2)^2}{\sqrt{\arctan(ax)}}\,dx = \int \frac{x\,(c\,a^2\,x^2+c)^2}{\sqrt{\mathrm{atan}\,(a\,x)}}\,dx$$

input
```
int((x*(c + a^2*c*x^2)^2)/atan(a*x)^(1/2),x)
```

output
```
int((x*(c + a^2*c*x^2)^2)/atan(a*x)^(1/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.86

$$\int \frac{x(c+a^2cx^2)^2}{\sqrt{\arctan(ax)}}\,dx = c^2\left(\left(\int \frac{\sqrt{atan\,(ax)}\,x^5}{atan\,(ax)}dx\right)a^4+2\left(\int \frac{\sqrt{atan\,(ax)}\,x^3}{atan\,(ax)}dx\right)a^2 + \int \frac{\sqrt{atan\,(ax)}\,x}{atan\,(ax)}dx\right)$$

input
```
int(x*(a^2*c*x^2+c)^2/atan(a*x)^(1/2),x)
```

output
```
c**2*(int((sqrt(atan(a*x))*x**5)/atan(a*x),x)*a**4 + 2*int((sqrt(atan(a*x)
)*x**3)/atan(a*x),x)*a**2 + int((sqrt(atan(a*x))*x)/atan(a*x),x))
```

# 3.919 $\int \frac{(c+a^2cx^2)^2}{\sqrt{\arctan(ax)}}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{(c+a^2cx^2)^2}{\sqrt{\arctan(ax)}}\,dx = \text{Int}\left(\frac{(c+a^2cx^2)^2}{\sqrt{\arctan(ax)}},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{(c+a^2cx^2)^2}{\sqrt{\arctan(ax)}}\,dx = \int \frac{(c+a^2cx^2)^2}{\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^2/Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[(c + a^2*c*x^2)^2/Sqrt[ArcTan[a*x]], x]
```

## Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^2}{\sqrt{\arctan(ax)}}dx$$

$$\Big\downarrow \ 5560$$

$$\int \frac{(a^2cx^2+c)^2}{\sqrt{\arctan(ax)}}dx$$

input
```
Int[(c + a^2*c*x^2)^2/Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{(a^2c\,x^2+c)^2}{\sqrt{\arctan(ax)}}dx$$

input
```
int((a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)
```

output
```
int((a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^2}{\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 1.38 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.33

$$\int \frac{(c+a^2cx^2)^2}{\sqrt{\arctan(ax)}}\,dx = c^2\left(\int \frac{2a^2x^2}{\sqrt{\operatorname{atan}(ax)}}\,dx + \int \frac{a^4x^4}{\sqrt{\operatorname{atan}(ax)}}\,dx + \int \frac{1}{\sqrt{\operatorname{atan}(ax)}}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**2/atan(a*x)**(1/2),x)
```

output
```
c**2*(Integral(2*a**2*x**2/sqrt(atan(a*x)), x) + Integral(a**4*x**4/sqrt(a
tan(a*x)), x) + Integral(1/sqrt(atan(a*x)), x))
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^2}{\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{(c + a^2cx^2)^2}{\sqrt{\arctan(ax)}}\,dx = \int \frac{(a^2cx^2 + c)^2}{\sqrt{\arctan(ax)}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^2/sqrt(arctan(a*x)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{(c + a^2cx^2)^2}{\sqrt{\arctan(ax)}}\,dx = \int \frac{(c\,a^2\,x^2 + c)^2}{\sqrt{\mathrm{atan}\left(a\,x\right)}}\,dx$$

input
```
int((c + a^2*c*x^2)^2/atan(a*x)^(1/2),x)
```

output
```
int((c + a^2*c*x^2)^2/atan(a*x)^(1/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.95

$$\int \frac{(c+a^2cx^2)^2}{\sqrt{\arctan(ax)}}\,dx = c^2\left(\int \frac{\sqrt{atan(ax)}}{atan(ax)}dx + \left(\int \frac{\sqrt{atan(ax)}\,x^4}{atan(ax)}dx\right)a^4 + 2\left(\int \frac{\sqrt{atan(ax)}\,x^2}{atan(ax)}dx\right)a^2\right)$$

input
```
int((a^2*c*x^2+c)^2/atan(a*x)^(1/2),x)
```

output
```
c**2*(int(sqrt(atan(a*x))/atan(a*x),x) + int((sqrt(atan(a*x))*x**4)/atan(a
*x),x)*a**4 + 2*int((sqrt(atan(a*x))*x**2)/atan(a*x),x)*a**2)
```

# 3.920 $\int \frac{(c+a^2cx^2)^2}{x\sqrt{\arctan(ax)}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{(c+a^2cx^2)^2}{x\sqrt{\arctan(ax)}}\,dx = \text{Int}\left(\frac{(c+a^2cx^2)^2}{x\sqrt{\arctan(ax)}},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^2/x/arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.86 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(c+a^2cx^2)^2}{x\sqrt{\arctan(ax)}}\,dx = \int \frac{(c+a^2cx^2)^2}{x\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^2/(x*Sqrt[ArcTan[a*x]]),x]
```

output
```
Integrate[(c + a^2*c*x^2)^2/(x*Sqrt[ArcTan[a*x]]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2cx^2+c\right)^2}{x\sqrt{\arctan(ax)}}\,dx$$

$$\Big\downarrow \text{5560}$$

$$\int \frac{\left(a^2cx^2+c\right)^2}{x\sqrt{\arctan(ax)}}dx$$

input
```
Int[(c + a^2*c*x^2)^2/(x*Sqrt[ArcTan[a*x]]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\left(a^2c\,x^2+c\right)^2}{x\sqrt{\arctan\left(ax\right)}}dx$$

input
```
int((a^2*c*x^2+c)^2/x/arctan(a*x)^(1/2),x)
```

output
```
int((a^2*c*x^2+c)^2/x/arctan(a*x)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^2}{x\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^2/x/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 2.30 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.04

$$\int \frac{(c+a^2cx^2)^2}{x\sqrt{\arctan(ax)}}\,dx = c^2\left(\int \frac{1}{x\sqrt{\operatorname{atan}(ax)}}\,dx + \int \frac{2a^2x}{\sqrt{\operatorname{atan}(ax)}}\,dx + \int \frac{a^4x^3}{\sqrt{\operatorname{atan}(ax)}}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**2/x/atan(a*x)**(1/2),x)
```

output
```
c**2*(Integral(1/(x*sqrt(atan(a*x))), x) + Integral(2*a**2*x/sqrt(atan(a*x
)), x) + Integral(a**4*x**3/sqrt(atan(a*x)), x))
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^2}{x\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^2/x/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^2}{x\sqrt{\arctan(ax)}}\,dx = \int \frac{(a^2cx^2+c)^2}{x\sqrt{\arctan(ax)}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2/x/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^2/(x*sqrt(arctan(a*x))), x)
```

## Mupad [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^2}{x\sqrt{\arctan(ax)}}\,dx = \int \frac{(c\,a^2\,x^2+c)^2}{x\,\sqrt{\mathrm{atan}\left(a\,x\right)}}\,dx$$

input
```
int((c + a^2*c*x^2)^2/(x*atan(a*x)^(1/2)),x)
```

output
```
int((c + a^2*c*x^2)^2/(x*atan(a*x)^(1/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.62

$$\int \frac{(c+a^2cx^2)^2}{x\sqrt{\arctan(ax)}}\,dx = c^2\left(\int \frac{\sqrt{atan\,(ax)}}{atan\,(ax)\,x}dx + \left(\int \frac{\sqrt{atan\,(ax)}\,x^3}{atan\,(ax)}dx\right)a^4 + 2\left(\int \frac{\sqrt{atan\,(ax)}\,x}{atan\,(ax)}dx\right)a^2\right)$$

input
```
int((a^2*c*x^2+c)^2/x/atan(a*x)^(1/2),x)
```

output
```
c**2*(int(sqrt(atan(a*x))/(atan(a*x)*x),x) + int((sqrt(atan(a*x))*x**3)/at
an(a*x),x)*a**4 + 2*int((sqrt(atan(a*x))*x)/atan(a*x),x)*a**2)
```

# 3.921 $\int \frac{x^m(c+a^2cx^2)^3}{\sqrt{\arctan(ax)}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m(c+a^2cx^2)^3}{\sqrt{\arctan(ax)}}\,dx = \text{Int}\left(\frac{x^m(c+a^2cx^2)^3}{\sqrt{\arctan(ax)}},x\right)$$

output
```
Defer(Int)(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m(c+a^2cx^2)^3}{\sqrt{\arctan(ax)}}\,dx = \int \frac{x^m(c+a^2cx^2)^3}{\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[(x^m*(c + a^2*c*x^2)^3)/Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[(x^m*(c + a^2*c*x^2)^3)/Sqrt[ArcTan[a*x]], x]
```

## Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \left(a^2 c x^2+c\right)^3}{\sqrt{\arctan (a x)}} \, dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x^m \left(a^2 c x^2+c\right)^3}{\sqrt{\arctan (a x)}}dx$$

input
```
Int[(x^m*(c + a^2*c*x^2)^3)/Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m (a^2 c\,x^2+c)^3}{\sqrt{\arctan \left(a x\right)}}dx$$

input
```
int(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.00

$$\int \frac{x^m(c+a^2cx^2)^3}{\sqrt{\arctan(ax)}}\,dx = \int \frac{(a^2cx^2+c)^3x^m}{\sqrt{\arctan(ax)}}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*x^m/sqrt(arct
an(a*x)), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^m(c+a^2cx^2)^3}{\sqrt{\arctan(ax)}}\,dx = \text{Timed out}$$

input
```
integrate(x**m*(a**2*c*x**2+c)**3/atan(a*x)**(1/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m(c+a^2cx^2)^3}{\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m(c+a^2cx^2)^3}{\sqrt{\arctan(ax)}}\,dx = \int \frac{(a^2cx^2+c)^3x^m}{\sqrt{\arctan{(ax)}}}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^3*x^m/sqrt(arctan(a*x)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m(c+a^2cx^2)^3}{\sqrt{\arctan(ax)}}\,dx = \int \frac{x^m\,{\left(c\,a^2\,x^2+c\right)}^3}{\sqrt{\mathrm{atan}\left(a\,x\right)}}\,dx$$

input
```
int((x^m*(c + a^2*c*x^2)^3)/atan(a*x)^(1/2),x)
```

output
```
int((x^m*(c + a^2*c*x^2)^3)/atan(a*x)^(1/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 96, normalized size of antiderivative = 4.00

$$\int \frac{x^m(c+a^2cx^2)^3}{\sqrt{\arctan(ax)}}\,dx = c^3\left(\left(\int \frac{x^m\sqrt{atan\,(ax)}\,x^6}{atan\,(ax)}dx\right)a^6 + 3\left(\int \frac{x^m\sqrt{atan\,(ax)}\,x^4}{atan\,(ax)}dx\right)a^4 + 3\left(\int \frac{x^m\sqrt{atan\,(ax)}\,x^2}{atan\,(ax)}dx\right)a^2 + \int \frac{x^m\sqrt{atan\,(ax)}}{atan\,(ax)}dx\right)$$

input
```
int(x^m*(a^2*c*x^2+c)^3/atan(a*x)^(1/2),x)
```

output
```
c**3*(int((x**m*sqrt(atan(a*x))*x**6)/atan(a*x),x)*a**6 + 3*int((x**m*sqrt
(atan(a*x))*x**4)/atan(a*x),x)*a**4 + 3*int((x**m*sqrt(atan(a*x))*x**2)/at
an(a*x),x)*a**2 + int((x**m*sqrt(atan(a*x)))/atan(a*x),x))
```

# 3.922 $\int \frac{x\left(c+a^2cx^2\right)^3}{\sqrt{\arctan(ax)}}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x(c+a^2cx^2)^3}{\sqrt{\arctan(ax)}}\,dx = \text{Int}\left(\frac{x(c+a^2cx^2)^3}{\sqrt{\arctan(ax)}},x\right)$$

output
```
Defer(Int)(x*(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x(c+a^2cx^2)^3}{\sqrt{\arctan(ax)}}\,dx = \int \frac{x(c+a^2cx^2)^3}{\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[(x*(c + a^2*c*x^2)^3)/Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[(x*(c + a^2*c*x^2)^3)/Sqrt[ArcTan[a*x]], x]
```

## Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a^2cx^2+c)^3}{\sqrt{\arctan(ax)}}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{x(a^2cx^2+c)^3}{\sqrt{\arctan(ax)}}dx$$

input
```
Int[(x*(c + a^2*c*x^2)^3)/Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x(a^2c\,x^2+c)^3}{\sqrt{\arctan\left(ax\right)}}dx$$

input
```
int(x*(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)
```

output
```
int(x*(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x(c + a^2cx^2)^3}{\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 3.54 (sec) , antiderivative size = 68, normalized size of antiderivative = 3.09

$$\int \frac{x(c + a^2cx^2)^3}{\sqrt{\arctan(ax)}}\,dx = c^3\left(\int \frac{x}{\sqrt{\operatorname{atan}(ax)}}\,dx + \int \frac{3a^2x^3}{\sqrt{\operatorname{atan}(ax)}}\,dx + \int \frac{3a^4x^5}{\sqrt{\operatorname{atan}(ax)}}\,dx + \int \frac{a^6x^7}{\sqrt{\operatorname{atan}(ax)}}\,dx\right)$$

input
```
integrate(x*(a**2*c*x**2+c)**3/atan(a*x)**(1/2),x)
```

output
```
c**3*(Integral(x/sqrt(atan(a*x)), x) + Integral(3*a**2*x**3/sqrt(atan(a*x)
), x) + Integral(3*a**4*x**5/sqrt(atan(a*x)), x) + Integral(a**6*x**7/sqrt
(atan(a*x)), x))
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x(c+a^2cx^2)^3}{\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x(c+a^2cx^2)^3}{\sqrt{\arctan(ax)}}\,dx = \int \frac{(a^2cx^2+c)^3x}{\sqrt{\arctan(ax)}}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^3*x/sqrt(arctan(a*x)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x(c+a^2cx^2)^3}{\sqrt{\arctan(ax)}}\,dx = \int \frac{x\,(c\,a^2\,x^2+c)^3}{\sqrt{\mathrm{atan}\,(a\,x)}}\,dx$$

input
```
int((x*(c + a^2*c*x^2)^3)/atan(a*x)^(1/2),x)
```

output
```
int((x*(c + a^2*c*x^2)^3)/atan(a*x)^(1/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 85, normalized size of antiderivative = 3.86

$$\int \frac{x(c+a^2cx^2)^3}{\sqrt{\arctan(ax)}}\,dx = c^3\left(\left(\int \frac{\sqrt{atan(ax)}\,x^7}{atan(ax)}dx\right)a^6 + 3\left(\int \frac{\sqrt{atan(ax)}\,x^5}{atan(ax)}dx\right)a^4 + 3\left(\int \frac{\sqrt{atan(ax)}\,x^3}{atan(ax)}dx\right)a^2 + \int \frac{\sqrt{atan(ax)}\,x}{atan(ax)}dx\right)$$

input
```
int(x*(a^2*c*x^2+c)^3/atan(a*x)^(1/2),x)
```

output
```
c**3*(int((sqrt(atan(a*x))*x**7)/atan(a*x),x)*a**6 + 3*int((sqrt(atan(a*x)
)*x**5)/atan(a*x),x)*a**4 + 3*int((sqrt(atan(a*x))*x**3)/atan(a*x),x)*a**2
 + int((sqrt(atan(a*x))*x)/atan(a*x),x))
```

# 3.923 $\int \frac{(c+a^2cx^2)^3}{\sqrt{\arctan(ax)}} \, dx$



## Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{(c+a^2cx^2)^3}{\sqrt{\arctan(ax)}} \, dx = \text{Int}\left(\frac{(c+a^2cx^2)^3}{\sqrt{\arctan(ax)}}, x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{(c+a^2cx^2)^3}{\sqrt{\arctan(ax)}} \, dx = \int \frac{(c+a^2cx^2)^3}{\sqrt{\arctan(ax)}} \, dx$$

input
```
Integrate[(c + a^2*c*x^2)^3/Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[(c + a^2*c*x^2)^3/Sqrt[ArcTan[a*x]], x]
```

## Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^3}{\sqrt{\arctan(ax)}}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{(a^2cx^2+c)^3}{\sqrt{\arctan(ax)}}dx$$

input
```
Int[(c + a^2*c*x^2)^3/Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{(a^2c\,x^2+c)^3}{\sqrt{\arctan\left(ax\right)}}dx$$

input
```
int((a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)
```

output
```
int((a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^3}{\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 2.67 (sec) , antiderivative size = 68, normalized size of antiderivative = 3.24

$$\int \frac{(c+a^2cx^2)^3}{\sqrt{\arctan(ax)}}\,dx = c^3\left(\int \frac{3a^2x^2}{\sqrt{\operatorname{atan}(ax)}}\,dx + \int \frac{3a^4x^4}{\sqrt{\operatorname{atan}(ax)}}\,dx + \int \frac{a^6x^6}{\sqrt{\operatorname{atan}(ax)}}\,dx + \int \frac{1}{\sqrt{\operatorname{atan}(ax)}}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**3/atan(a*x)**(1/2),x)
```

output
```
c**3*(Integral(3*a**2*x**2/sqrt(atan(a*x)), x) + Integral(3*a**4*x**4/sqrt
(atan(a*x)), x) + Integral(a**6*x**6/sqrt(atan(a*x)), x) + Integral(1/sqrt
(atan(a*x)), x))
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(c + a^2cx^2)^3}{\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{(c + a^2cx^2)^3}{\sqrt{\arctan(ax)}}\,dx = \int \frac{(a^2cx^2 + c)^3}{\sqrt{\arctan(ax)}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^3/sqrt(arctan(a*x)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{(c + a^2cx^2)^3}{\sqrt{\arctan(ax)}}\,dx = \int \frac{(c\,a^2\,x^2 + c)^3}{\sqrt{\mathrm{atan}(a\,x)}}\,dx$$

input
```
int((c + a^2*c*x^2)^3/atan(a*x)^(1/2),x)
```

output
```
int((c + a^2*c*x^2)^3/atan(a*x)^(1/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 84, normalized size of antiderivative = 4.00

$$\int \frac{(c+a^2cx^2)^3}{\sqrt{\arctan(ax)}}\,dx = c^3\left(\int \frac{\sqrt{atan\,(ax)}}{atan\,(ax)}dx + \left(\int \frac{\sqrt{atan\,(ax)}\,x^6}{atan\,(ax)}dx\right)a^6 + 3\left(\int \frac{\sqrt{atan\,(ax)}\,x^4}{atan\,(ax)}dx\right)a^4 + 3\left(\int \frac{\sqrt{atan\,(ax)}\,x^2}{atan\,(ax)}dx\right)a^2\right)$$

input
```
int((a^2*c*x^2+c)^3/atan(a*x)^(1/2),x)
```

output
```
c**3*(int(sqrt(atan(a*x))/atan(a*x),x) + int((sqrt(atan(a*x))*x**6)/atan(a
*x),x)*a**6 + 3*int((sqrt(atan(a*x))*x**4)/atan(a*x),x)*a**4 + 3*int((sqrt
(atan(a*x))*x**2)/atan(a*x),x)*a**2)
```

# 3.924 $\int \frac{(c+a^2cx^2)^3}{x\sqrt{\arctan(ax)}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{(c+a^2cx^2)^3}{x\sqrt{\arctan(ax)}}\,dx = \text{Int}\left(\frac{(c+a^2cx^2)^3}{x\sqrt{\arctan(ax)}},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^3/x/arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(c+a^2cx^2)^3}{x\sqrt{\arctan(ax)}}\,dx = \int \frac{(c+a^2cx^2)^3}{x\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^3/(x*Sqrt[ArcTan[a*x]]),x]
```

output
```
Integrate[(c + a^2*c*x^2)^3/(x*Sqrt[ArcTan[a*x]]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^3}{x\sqrt{\arctan(ax)}}\,dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{(a^2cx^2+c)^3}{x\sqrt{\arctan(ax)}}dx$$

input
```
Int[(c + a^2*c*x^2)^3/(x*Sqrt[ArcTan[a*x]]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{(a^2c\,x^2+c)^3}{x\sqrt{\arctan\left(ax\right)}}dx$$

input
```
int((a^2*c*x^2+c)^3/x/arctan(a*x)^(1/2),x)
```

output
```
int((a^2*c*x^2+c)^3/x/arctan(a*x)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^3}{x\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^3/x/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 3.51 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.83

$$\int \frac{(c+a^2cx^2)^3}{x\sqrt{\arctan(ax)}}\,dx = c^3\left(\int \frac{1}{x\sqrt{\operatorname{atan}(ax)}}\,dx + \int \frac{3a^2x}{\sqrt{\operatorname{atan}(ax)}}\,dx + \int \frac{3a^4x^3}{\sqrt{\operatorname{atan}(ax)}}\,dx + \int \frac{a^6x^5}{\sqrt{\operatorname{atan}(ax)}}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**3/x/atan(a*x)**(1/2),x)
```

output
```
c**3*(Integral(1/(x*sqrt(atan(a*x))), x) + Integral(3*a**2*x/sqrt(atan(a*x
)), x) + Integral(3*a**4*x**3/sqrt(atan(a*x)), x) + Integral(a**6*x**5/sqr
t(atan(a*x)), x))
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^3}{x\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^3/x/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^3}{x\sqrt{\arctan(ax)}}\,dx = \int \frac{(a^2cx^2+c)^3}{x\sqrt{\arctan(ax)}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3/x/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^3/(x*sqrt(arctan(a*x))), x)
```

## Mupad [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^3}{x\sqrt{\arctan(ax)}}\,dx = \int \frac{(c\,a^2\,x^2+c)^3}{x\,\sqrt{\mathrm{atan}\,(a\,x)}}\,dx$$

input
```
int((c + a^2*c*x^2)^3/(x*atan(a*x)^(1/2)),x)
```

output
```
int((c + a^2*c*x^2)^3/(x*atan(a*x)^(1/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 85, normalized size of antiderivative = 3.54

$$\int \frac{(c+a^2cx^2)^3}{x\sqrt{\arctan(ax)}}\,dx = c^3\left(\int \frac{\sqrt{atan\,(ax)}}{atan\,(ax)\,x}dx + \left(\int \frac{\sqrt{atan\,(ax)}\,x^5}{atan\,(ax)}dx\right)a^6 + 3\left(\int \frac{\sqrt{atan\,(ax)}\,x^3}{atan\,(ax)}dx\right)a^4 + 3\left(\int \frac{\sqrt{atan\,(ax)}\,x}{atan\,(ax)}dx\right)a^2\right)$$

input
```
int((a^2*c*x^2+c)^3/x/atan(a*x)^(1/2),x)
```

output
```
c**3*(int(sqrt(atan(a*x))/(atan(a*x)*x),x) + int((sqrt(atan(a*x))*x**5)/at
an(a*x),x)*a**6 + 3*int((sqrt(atan(a*x))*x**3)/atan(a*x),x)*a**4 + 3*int((
sqrt(atan(a*x))*x)/atan(a*x),x)*a**2)
```

# 3.925 $\int \frac{x^m}{(c+a^2cx^2)\sqrt{\arctan(ax)}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m}{(c+a^2cx^2)\sqrt{\arctan(ax)}}\,dx = \text{Int}\left(\frac{x^m}{(c+a^2cx^2)\sqrt{\arctan(ax)}},x\right)$$

output
```
Defer(Int)(x^m/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m}{(c+a^2cx^2)\sqrt{\arctan(ax)}}\,dx = \int \frac{x^m}{(c+a^2cx^2)\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[x^m/((c + a^2*c*x^2)*Sqrt[ArcTan[a*x]]),x]
```

output
```
Integrate[x^m/((c + a^2*c*x^2)*Sqrt[ArcTan[a*x]]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\sqrt{\arctan(ax)}\,(a^2cx^2+c)}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x^m}{\sqrt{\arctan(ax)}\,(a^2cx^2+c)}dx$$

input
```
Int[x^m/((c + a^2*c*x^2)*Sqrt[ArcTan[a*x]]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{(a^2c\,x^2+c)\,\sqrt{\arctan\left(ax\right)}}dx$$

input
```
int(x^m/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x)
```

output
```
int(x^m/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(c+a^2cx^2)\sqrt{\arctan(ax)}}\,dx = \int \frac{x^m}{(a^2cx^2+c)\sqrt{\arctan(ax)}}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
integral(x^m/((a^2*c*x^2 + c)*sqrt(arctan(a*x))), x)
```

## Sympy [N/A]

Not integrable

Time = 10.86 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int \frac{x^m}{(c+a^2cx^2)\sqrt{\arctan(ax)}}\,dx = \frac{\int \frac{x^m}{a^2x^2\sqrt{\operatorname{atan}(ax)}+\sqrt{\operatorname{atan}(ax)}}\,dx}{c}$$

input
```
integrate(x**m/(a**2*c*x**2+c)/atan(a*x)**(1/2),x)
```

output
```
Integral(x**m/(a**2*x**2*sqrt(atan(a*x)) + sqrt(atan(a*x))), x)/c
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m}{(c+a^2cx^2)\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(c+a^2cx^2)\sqrt{\arctan(ax)}}\,dx = \int \frac{x^m}{(a^2cx^2+c)\sqrt{\arctan(ax)}}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^m/((a^2*c*x^2 + c)*sqrt(arctan(a*x))), x)
```

## Mupad [N/A]

Not integrable

Time = 0.84 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(c+a^2cx^2)\sqrt{\arctan(ax)}}\,dx = \int \frac{x^m}{\sqrt{\mathrm{atan}(a\,x)}\,(c\,a^2\,x^2+c)}\,dx$$

input
```
int(x^m/(atan(a*x)^(1/2)*(c + a^2*c*x^2)),x)
```

output
```
int(x^m/(atan(a*x)^(1/2)*(c + a^2*c*x^2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.38

$$\int \frac{x^m}{(c+a^2cx^2)\sqrt{\arctan(ax)}}\,dx = \frac{\int \frac{x^m\sqrt{atan(ax)}}{atan(ax)a^2x^2+atan(ax)}dx}{c}$$

input
```
int(x^m/(a^2*c*x^2+c)/atan(a*x)^(1/2),x)
```

output
```
int((x**m*sqrt(atan(a*x)))/(atan(a*x)*a**2*x**2 + atan(a*x)),x)/c
```

# 3.926 $\int \frac{x}{(c+a^2cx^2)\sqrt{\arctan(ax)}}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x}{(c+a^2cx^2)\sqrt{\arctan(ax)}}\,dx = \frac{2x\sqrt{\arctan(ax)}}{ac} - \frac{2\text{Int}\left(\sqrt{\arctan(ax)},x\right)}{ac}$$

output
```
2*x*arctan(a*x)^(1/2)/a/c-2*Defer(Int)(arctan(a*x)^(1/2),x)/a/c
```

## Mathematica [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x}{(c+a^2cx^2)\sqrt{\arctan(ax)}}\,dx = \int \frac{x}{(c+a^2cx^2)\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[x/((c + a^2*c*x^2)*Sqrt[ArcTan[a*x]]),x]
```

output
```
Integrate[x/((c + a^2*c*x^2)*Sqrt[ArcTan[a*x]]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{\arctan(ax)}\,(a^2cx^2+c)}dx$$

$$\Big\downarrow \; 5457$$

$$\frac{2x\sqrt{\arctan(ax)}}{ac} - \frac{2\int \sqrt{\arctan(ax)}dx}{ac}$$

$$\Big\downarrow \; 5353$$

$$\frac{2x\sqrt{\arctan(ax)}}{ac} - \frac{2\int \sqrt{\arctan(ax)}dx}{ac}$$

input
```
Int[x/((c + a^2*c*x^2)*Sqrt[ArcTan[a*x]]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x}{(a^2c\,x^2+c)\,\sqrt{\arctan\left(ax\right)}}dx$$

input
```
int(x/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x)
```

output
```
int(x/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

$$\int \frac{x}{(c+a^2cx^2)\sqrt{\arctan(ax)}}\,dx = \frac{\int \frac{x}{a^2x^2\sqrt{\operatorname{atan}(ax)}+\sqrt{\operatorname{atan}(ax)}}\,dx}{c}$$

input
```
integrate(x/(a**2*c*x**2+c)/atan(a*x)**(1/2),x)
```

output
```
Integral(x/(a**2*x**2*sqrt(atan(a*x)) + sqrt(atan(a*x))), x)/c
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x}{(c+a^2cx^2)\sqrt{\arctan(ax)}}\,dx = \int \frac{x}{(a^2cx^2+c)\sqrt{\arctan(ax)}}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/((a^2*c*x^2 + c)*sqrt(arctan(a*x))), x)
```

## Mupad [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x}{(c+a^2cx^2)\sqrt{\arctan(ax)}}\,dx = \int \frac{x}{\sqrt{\mathrm{atan}\left(a\,x\right)}\,\left(c\,a^2\,x^2+c\right)}\,dx$$

input
```
int(x/(atan(a*x)^(1/2)*(c + a^2*c*x^2)),x)
```

output
```
int(x/(atan(a*x)^(1/2)*(c + a^2*c*x^2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.41

$$\int \frac{x}{(c+a^2cx^2)\sqrt{\arctan(ax)}}\,dx = \frac{\int \frac{\sqrt{atan(ax)}\,x}{atan(ax)a^2x^2+atan(ax)}dx}{c}$$

input
```
int(x/(a^2*c*x^2+c)/atan(a*x)^(1/2),x)
```

output
```
int((sqrt(atan(a*x))*x)/(atan(a*x)*a**2*x**2 + atan(a*x)),x)/c
```

# 3.927 $\int \frac{1}{(c+a^2cx^2)\sqrt{\arctan(ax)}}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 16

$$\int \frac{1}{(c+a^2cx^2)\sqrt{\arctan(ax)}}\,dx = \frac{2\sqrt{\arctan(ax)}}{ac}$$

output
```
2*arctan(a*x)^(1/2)/a/c
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{(c+a^2cx^2)\sqrt{\arctan(ax)}}\,dx = \frac{2\sqrt{\arctan(ax)}}{ac}$$

input
```
Integrate[1/((c + a^2*c*x^2)*Sqrt[ArcTan[a*x]]),x]
```

output
```
(2*Sqrt[ArcTan[a*x]])/(a*c)
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\arctan(ax)}\,(a^2cx^2+c)}\,dx$$
$$\Big\downarrow \text{5419}$$
$$\frac{2\sqrt{\arctan(ax)}}{ac}$$

input
```
Int[1/((c + a^2*c*x^2)*Sqrt[ArcTan[a*x]]),x]
```

output
```
(2*Sqrt[ArcTan[a*x]])/(a*c)
```

### Defintions of rubi rules used

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

## Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

| method | result | size |
|---|---|---|
| default | $\frac{2\sqrt{\arctan(ax)}}{ac}$ | 15 |

input
```
int(1/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*arctan(a*x)^(1/2)/a/c
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{(c+a^2cx^2)\sqrt{\arctan(ax)}}\,dx = \frac{2\sqrt{\arctan(ax)}}{ac}$$

input
```
integrate(1/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(arctan(a*x))/(a*c)
```

## Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{1}{(c+a^2cx^2)\sqrt{\arctan(ax)}}\,dx = \frac{2\sqrt{\operatorname{atan}(ax)}}{ac}$$

input
```
integrate(1/(a**2*c*x**2+c)/atan(a*x)**(1/2),x)
```

output
```
2*sqrt(atan(a*x))/(a*c)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(c+a^2cx^2)\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{(c+a^2cx^2)\sqrt{\arctan(ax)}}\,dx = \frac{2\sqrt{\arctan(ax)}}{ac}$$

input
```
integrate(1/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
2*sqrt(arctan(a*x))/(a*c)
```

## Mupad [B] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{(c+a^2cx^2)\sqrt{\arctan(ax)}}\,dx = \frac{2\sqrt{\mathrm{atan}\left(a\,x\right)}}{a\,c}$$

input
```
int(1/(atan(a*x)^(1/2)*(c + a^2*c*x^2)),x)
```

output
```
(2*atan(a*x)^(1/2))/(a*c)
```

## Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \frac{1}{(c+a^2cx^2)\sqrt{\arctan(ax)}}\,dx = \frac{2\sqrt{atan(ax)}}{ac}$$

input
```
int(1/(a^2*c*x^2+c)/atan(a*x)^(1/2),x)
```

output
```
(2*sqrt(atan(a*x)))/(a*c)
```

# 3.928 $\int \frac{1}{x(c+a^2cx^2)\sqrt{\arctan(ax)}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x(c+a^2cx^2)\sqrt{\arctan(ax)}}\,dx = \text{Int}\left(\frac{1}{x(c+a^2cx^2)\sqrt{\arctan(ax)}},x\right)$$

output
```
Defer(Int)(1/x/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x(c+a^2cx^2)\sqrt{\arctan(ax)}}\,dx = \int \frac{1}{x(c+a^2cx^2)\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[1/(x*(c + a^2*c*x^2)*Sqrt[ArcTan[a*x]]),x]
```

output
```
Integrate[1/(x*(c + a^2*c*x^2)*Sqrt[ArcTan[a*x]]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{1}{x\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)}dx$$

input
```
Int[1/(x*(c + a^2*c*x^2)*Sqrt[ArcTan[a*x]]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\left(a^2c\,x^2+c\right)\sqrt{\arctan\left(ax\right)}}dx$$

input
```
int(1/x/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x)
```

output
```
int(1/x/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(c+a^{2}cx^{2}\right)\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 1.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int \frac{1}{x\left(c+a^{2}cx^{2}\right)\sqrt{\arctan(ax)}}\,dx = \frac{\int \frac{1}{a^{2}x^{3}\sqrt{\operatorname{atan}(ax)}+x\sqrt{\operatorname{atan}(ax)}}\,dx}{c}$$

input
```
integrate(1/x/(a**2*c*x**2+c)/atan(a*x)**(1/2),x)
```

output
```
Integral(1/(a**2*x**3*sqrt(atan(a*x)) + x*sqrt(atan(a*x))), x)/c
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(c+a^{2}cx^{2}\right)\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)\sqrt{\arctan(ax)}}\,dx=\int \frac{1}{(a^2cx^2+c)x\sqrt{\arctan{(ax)}}}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)*x*sqrt(arctan(a*x))), x)
```

## Mupad [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)\sqrt{\arctan(ax)}}\,dx=\int \frac{1}{x\,\sqrt{\mathrm{atan}\left(a\,x\right)}\,\left(c\,a^2\,x^2+c\right)}\,dx$$

input
```
int(1/(x*atan(a*x)^(1/2)*(c + a^2*c*x^2)),x)
```

output
```
int(1/(x*atan(a*x)^(1/2)*(c + a^2*c*x^2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.33

$$\int \frac{1}{x\left(c+a^2cx^2\right)\sqrt{\arctan(ax)}}\,dx = \frac{\int \frac{\sqrt{atan(ax)}}{atan(ax)a^2x^3+atan(ax)x}dx}{c}$$

input
```
int(1/x/(a^2*c*x^2+c)/atan(a*x)^(1/2),x)
```

output
```
int(sqrt(atan(a*x))/(atan(a*x)*a**2*x**3 + atan(a*x)*x),x)/c
```

# 3.929 $\int \frac{x^m}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx = \text{Int}\left(\frac{x^m}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}},x\right)$$

output
```
Defer(Int)(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.43 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx = \int \frac{x^m}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[x^m/((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]),x]
```

output
```
Integrate[x^m/((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\sqrt{\arctan(ax)}\,(a^2cx^2+c)^2}\,dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x^m}{\sqrt{\arctan(ax)}\,(a^2cx^2+c)^2}dx$$

input
```
Int[x^m/((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{(a^2c\,x^2+c)^2\,\sqrt{\arctan\,(ax)}}dx$$

input
```
int(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)
```

output
```
int(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.62

$$\int \frac{x^m}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx = \int \frac{x^m}{(a^2cx^2+c)^2\sqrt{\arctan(ax)}}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
integral(x^m/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(arctan(a*x))), x)
```

## Sympy [N/A]

Not integrable

Time = 58.78 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.00

$$\int \frac{x^m}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx = \frac{\int \frac{x^m}{a^4x^4\sqrt{\operatorname{atan}(ax)}+2a^2x^2\sqrt{\operatorname{atan}(ax)}+\sqrt{\operatorname{atan}(ax)}}\,dx}{c^2}$$

input
```
integrate(x**m/(a**2*c*x**2+c)**2/atan(a*x)**(1/2),x)
```

output
```
Integral(x**m/(a**4*x**4*sqrt(atan(a*x)) + 2*a**2*x**2*sqrt(atan(a*x)) + s
qrt(atan(a*x))), x)/c**2
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x^m}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,1,0,0]%%%} / %%%{1,[0,0,1,2]%%%} Error: Bad Argument
Value
```

## Mupad [N/A]

Not integrable

Time = 0.85 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx = \int \frac{x^m}{\sqrt{\mathrm{atan}\left(a\,x\right)}\,{\left(c\,a^2\,x^2+c\right)}^2}\,dx$$

input
```
int(x^m/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^2),x)
```

output
```
int(x^m/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.88

$$\int \frac{x^m}{(c + a^2cx^2)^2 \sqrt{\arctan(ax)}} \, dx = \frac{\int \frac{x^m \sqrt{atan(ax)}}{atan(ax)a^4x^4 + 2atan(ax)a^2x^2 + atan(ax)} dx}{c^2}$$

input
```
int(x^m/(a^2*c*x^2+c)^2/atan(a*x)^(1/2),x)
```

output
```
int((x**m*sqrt(atan(a*x)))/(atan(a*x)*a**4*x**4 + 2*atan(a*x)*a**2*x**2 +
atan(a*x)),x)/c**2
```

# 3.930 $\int \frac{x^3}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^3}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx=\text{Int}\left(\frac{x^3}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}},x\right)$$

output
```
Defer(Int)(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 2.95 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^3}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx=\int \frac{x^3}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[x^3/((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]),x]
```

output
```
Integrate[x^3/((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{\arctan(ax)}\,(a^2cx^2+c)^2}\,dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int \frac{x^3}{\sqrt{\arctan(ax)}\,(a^2cx^2+c)^2}dx$$

input
```
Int[x^3/((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^3}{(a^2c\,x^2+c)^2\,\sqrt{\arctan\left(ax\right)}}dx$$

input
```
int(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)
```

output
```
int(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 1.62 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.00

$$\int \frac{x^3}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx = \frac{\int \frac{x^3}{a^4x^4\sqrt{\operatorname{atan}(ax)}+2a^2x^2\sqrt{\operatorname{atan}(ax)}+\sqrt{\operatorname{atan}(ax)}}\,dx}{c^2}$$

input
```
integrate(x**3/(a**2*c*x**2+c)**2/atan(a*x)**(1/2),x)
```

output
```
Integral(x**3/(a**4*x**4*sqrt(atan(a*x)) + 2*a**2*x**2*sqrt(atan(a*x)) + s
qrt(atan(a*x))), x)/c**2
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx = \int \frac{x^3}{(a^2cx^2+c)^2\sqrt{\arctan{(ax)}}}\,dx$$

input
```
integrate(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^3/((a^2*c*x^2 + c)^2*sqrt(arctan(a*x))), x)
```

## Mupad [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx = \int \frac{x^3}{\sqrt{\mathrm{atan}\left(a\,x\right)}\,{\left(c\,a^2\,x^2+c\right)}^2}\,dx$$

input
```
int(x^3/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^2),x)
```

output
```
int(x^3/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.88

$$\int \frac{x^3}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx = \frac{\int \frac{\sqrt{atan(ax)}\,x^3}{atan(ax)a^4x^4+2atan(ax)a^2x^2+atan(ax)}dx}{c^2}$$

input
```
int(x^3/(a^2*c*x^2+c)^2/atan(a*x)^(1/2),x)
```

output
```
int((sqrt(atan(a*x))*x**3)/(atan(a*x)*a**4*x**4 + 2*atan(a*x)*a**2*x**2 +
atan(a*x)),x)/c**2
```

# 3.931 $\int \frac{x^2}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 47

$$\int \frac{x^2}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx=\frac{\sqrt{\arctan(ax)}}{a^3c^2}-\frac{\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{2a^3c^2}$$

output
```
arctan(a*x)^(1/2)/a^3/c^2-1/2*Pi^(1/2)*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/
2))/a^3/c^2
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.60

$$\int \frac{x^2}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx = \frac{16\arctan(ax)-4\sqrt{\pi}\sqrt{\arctan(ax)}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+i\sqrt{2}\sqrt{-i\arctan(ax)}\Gamma\left(\frac{1}{2},-2i\arctan(ax)\right)}{16a^3c^2\sqrt{\arctan(ax)}}$$

input
```
Integrate[x^2/((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]),x]
```

output
```
(16*ArcTan[a*x] - 4*Sqrt[Pi]*Sqrt[ArcTan[a*x]]*FresnelC[(2*Sqrt[ArcTan[a*x
]])/Sqrt[Pi]] + I*Sqrt[2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-2*I)*ArcTan[
a*x]] - I*Sqrt[2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (2*I)*ArcTan[a*x]])/(16*a
^3*c^2*Sqrt[ArcTan[a*x]])
```

## Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5505, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{\arctan(ax)}\,(a^2cx^2+c)^2}dx$$

$$\Big\downarrow \text{5505}$$

$$\frac{\int \frac{a^2x^2}{(a^2x^2+1)\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3c^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sin(\arctan(ax))^2}{\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3c^2}$$

$$\Big\downarrow \text{3793}$$

$$\frac{\int \left(\frac{1}{2\sqrt{\arctan(ax)}}-\frac{\cos(2\arctan(ax))}{2\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^3c^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\arctan(ax)}-\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{a^3c^2}$$

input
```
Int[x^2/((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]),x]
```

output
```
(Sqrt[ArcTan[a*x]] - (Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/2
)/(a^3*c^2)
```

**Defintions of rubi rules used**

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

## Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.81

| method | result | size |
|---|---|---|
| default | $-\frac{\pi\,\mathrm{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)-2\sqrt{\arctan(ax)}\,\sqrt{\pi}}{2a^{3}c^{2}\sqrt{\pi}}$ | 38 |

input
```
int(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a^3/c^2/Pi^(1/2)*(Pi*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))-2*arctan(
a*x)^(1/2)*Pi^(1/2))
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{x^2}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx = \frac{\int \frac{x^2}{a^4x^4\sqrt{\operatorname{atan}(ax)}+2a^2x^2\sqrt{\operatorname{atan}(ax)}+\sqrt{\operatorname{atan}(ax)}}\,dx}{c^2}$$

input
```
integrate(x**2/(a**2*c*x**2+c)**2/atan(a*x)**(1/2),x)
```

output
```
Integral(x**2/(a**4*x**4*sqrt(atan(a*x)) + 2*a**2*x**2*sqrt(atan(a*x)) + s
qrt(atan(a*x))), x)/c**2
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.15

$$\int \frac{x^2}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx = \frac{(i+1)\sqrt{\pi}\,\mathrm{erf}\left((i-1)\sqrt{\arctan(ax)}\right)}{8a^3c^2} - \frac{(i-1)\sqrt{\pi}\,\mathrm{erf}\left(-(i+1)\sqrt{\arctan(ax)}\right)}{8a^3c^2} + \frac{\sqrt{\arctan(ax)}}{a^3c^2}$$

input
```
integrate(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
(1/8*I + 1/8)*sqrt(pi)*erf((I - 1)*sqrt(arctan(a*x)))/(a^3*c^2) - (1/8*I -
 1/8)*sqrt(pi)*erf(-(I + 1)*sqrt(arctan(a*x)))/(a^3*c^2) + sqrt(arctan(a*x
))/(a^3*c^2)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx = \int \frac{x^2}{\sqrt{\mathrm{atan}(a\,x)}\,(c\,a^2\,x^2+c)^2}\,dx$$

input
```
int(x^2/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^2),x)
```

output
```
int(x^2/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^2), x)
```

## Reduce [F]

$$\int \frac{x^2}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx = \frac{\int \frac{\sqrt{atan(ax)}\,x^2}{atan(ax)a^4x^4+2atan(ax)a^2x^2+atan(ax)}dx}{c^2}$$

input
```
int(x^2/(a^2*c*x^2+c)^2/atan(a*x)^(1/2),x)
```

output
```
int((sqrt(atan(a*x))*x**2)/(atan(a*x)*a**4*x**4 + 2*atan(a*x)*a**2*x**2 +
atan(a*x)),x)/c**2
```

# 3.932 $\int \frac{x}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 31

$$\int \frac{x}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx = \frac{\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{2a^2c^2}$$

output
```
1/2*Pi^(1/2)*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2))/a^2/c^2
```

## Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{x}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx = \frac{\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{2a^2c^2}$$

input
```
Integrate[x/((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]),x]
```

output
```
(Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(2*a^2*c^2)
```

## Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {5505, 4906, 27, 3042, 3786, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{\arctan(ax)}\,(a^2cx^2+c)^2}\,dx$$

$$\Big\downarrow \text{5505}$$

$$\frac{\int \frac{ax}{(a^2x^2+1)\sqrt{\arctan(ax)}}d\arctan(ax)}{a^2c^2}$$

$$\Big\downarrow \text{4906}$$

$$\frac{\int \frac{\sin(2\arctan(ax))}{2\sqrt{\arctan(ax)}}d\arctan(ax)}{a^2c^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sin(2\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)}{2a^2c^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sin(2\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)}{2a^2c^2}$$

$$\Big\downarrow \text{3786}$$

$$\frac{\int \sin(2\arctan(ax))d\sqrt{\arctan(ax)}}{a^2c^2}$$

$$\Big\downarrow \text{3832}$$

$$\frac{\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{2a^2c^2}$$

input
```
Int[x/((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]),x]
```

output
```
(Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(2*a^2*c^2)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

## Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

| method | result | size |
|---|---|---|
| default | $\frac{\sqrt{\pi}\,\mathrm{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{2a^2c^2}$ | 24 |

input
```
int(x/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*Pi^(1/2)*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2))/a^2/c^2
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{x}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx = \frac{\int \frac{x}{a^4x^4\sqrt{\operatorname{atan}(ax)}+2a^2x^2\sqrt{\operatorname{atan}(ax)}+\sqrt{\operatorname{atan}(ax)}}\,dx}{c^2}$$

input
```
integrate(x/(a**2*c*x**2+c)**2/atan(a*x)**(1/2),x)
```

output
```
Integral(x/(a**4*x**4*sqrt(atan(a*x)) + 2*a**2*x**2*sqrt(atan(a*x)) + sqrt
(atan(a*x))), x)/c**2
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F]

$$\int \frac{x}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx = \int \frac{x}{(a^2cx^2+c)^2\sqrt{\arctan(ax)}}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/((a^2*c*x^2 + c)^2*sqrt(arctan(a*x))), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx = \int \frac{x}{\sqrt{\mathrm{atan}(a\,x)}\,(c\,a^2\,x^2+c)^2}\,dx$$

input
```
int(x/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^2),x)
```

output
```
int(x/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^2), x)
```

## Reduce [F]

$$\int \frac{x}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx = \frac{\int \frac{\sqrt{atan(ax)}\,x}{atan(ax)a^4x^4+2atan(ax)a^2x^2+atan(ax)}dx}{c^2}$$

input
```
int(x/(a^2*c*x^2+c)^2/atan(a*x)^(1/2),x)
```

output
```
int((sqrt(atan(a*x))*x)/(atan(a*x)*a**4*x**4 + 2*atan(a*x)*a**2*x**2 + ata
n(a*x)),x)/c**2
```

# 3.933 $\int \frac{1}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 47

$$\int \frac{1}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx=\frac{\sqrt{\arctan(ax)}}{ac^2}+\frac{\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{2ac^2}$$

output
```
arctan(a*x)^(1/2)/a/c^2+1/2*Pi^(1/2)*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2)
)/a/c^2
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.60

$$\int \frac{1}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx = \frac{16\arctan(ax)+4\sqrt{\pi}\sqrt{\arctan(ax)}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)-i\sqrt{2}\sqrt{-i\arctan(ax)}\,\Gamma\left(\frac{1}{2},-2i\arctan(ax)\right)}{16ac^2\sqrt{\arctan(ax)}}$$

input
```
Integrate[1/((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]),x]
```

output
```
(16*ArcTan[a*x] + 4*Sqrt[Pi]*Sqrt[ArcTan[a*x]]*FresnelC[(2*Sqrt[ArcTan[a*x
]])/Sqrt[Pi]] - I*Sqrt[2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-2*I)*ArcTan[
a*x]] + I*Sqrt[2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (2*I)*ArcTan[a*x]])/(16*a
*c^2*Sqrt[ArcTan[a*x]])
```

## Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {5439, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\arctan(ax)}\,(a^2cx^2+c)^2}dx$$

$$\Big\downarrow \text{ 5439}$$

$$\frac{\int \frac{1}{(a^2x^2+1)\sqrt{\arctan(ax)}}d\arctan(ax)}{ac^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^2}{\sqrt{\arctan(ax)}}d\arctan(ax)}{ac^2}$$

$$\Big\downarrow \text{ 3793}$$

$$\frac{\int \left(\frac{\cos(2\arctan(ax))}{2\sqrt{\arctan(ax)}}+\frac{1}{2\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{ac^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\sqrt{\arctan(ax)}}{ac^2}$$

input
```
Int[1/((c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]),x]
```

output
```
(Sqrt[ArcTan[a*x]] + (Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/2
)/(a*c^2)
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

## Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.81

| method | result | size |
|---|---|---|
| default | $\frac{\pi\ \text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+2\sqrt{\arctan(ax)}\,\sqrt{\pi}}{2a\,c^{2}\sqrt{\pi}}$ | 38 |

input
```
int(1/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/a/c^2/Pi^(1/2)*(Pi*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))+2*arctan(a*x
)^(1/2)*Pi^(1/2))
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{1}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx = \frac{\int \frac{1}{a^4x^4\sqrt{\operatorname{atan}(ax)}+2a^2x^2\sqrt{\operatorname{atan}(ax)}+\sqrt{\operatorname{atan}(ax)}}\,dx}{c^2}$$

input
```
integrate(1/(a**2*c*x**2+c)**2/atan(a*x)**(1/2),x)
```

output
```
Integral(1/(a**4*x**4*sqrt(atan(a*x)) + 2*a**2*x**2*sqrt(atan(a*x)) + sqrt
(atan(a*x))), x)/c**2
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.09

$$\int \frac{1}{(c+a^2cx^2)^2 \sqrt{\arctan(ax)}}\,dx$$
$$= -\frac{(i+1)\sqrt{\pi}\operatorname{erf}\left((i-1)\sqrt{\arctan(ax)}\right)-(i-1)\sqrt{\pi}\operatorname{erf}\left(-(i+1)\sqrt{\arctan(ax)}\right)}{8\,ac^2} + \frac{\sqrt{\arctan(ax)}}{ac^2}$$

input
```
integrate(1/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
-1/8*((I + 1)*sqrt(pi)*erf((I - 1)*sqrt(arctan(a*x))) - (I - 1)*sqrt(pi)*e
rf(-(I + 1)*sqrt(arctan(a*x))))/(a*c^2) + sqrt(arctan(a*x))/(a*c^2)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx = \int \frac{1}{\sqrt{\mathrm{atan}\left(a\,x\right)}\,{\left(c\,a^2\,x^2+c\right)}^2}\,dx$$

input
```
int(1/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^2),x)
```

output
```
int(1/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^2), x)
```

## Reduce [F]

$$\int \frac{1}{(c+a^2cx^2)^2 \sqrt{\arctan(ax)}}\,dx = \frac{\int \frac{\sqrt{atan(ax)}}{atan(ax)a^4x^4+2atan(ax)a^2x^2+atan(ax)}dx}{c^2}$$

input
```
int(1/(a^2*c*x^2+c)^2/atan(a*x)^(1/2),x)
```

output
```
int(sqrt(atan(a*x))/(atan(a*x)*a**4*x**4 + 2*atan(a*x)*a**2*x**2 + atan(a*
x)),x)/c**2
```

# 3.934 $\int \frac{1}{x(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx=\text{Int}\left(\frac{1}{x(c+a^2cx^2)^2\sqrt{\arctan(ax)}},x\right)$$

output
```
Defer(Int)(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx=\int \frac{1}{x(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[1/(x*(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]),x]
```

output
```
Integrate[1/(x*(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^2}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{1}{x\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^2}dx$$

input
```
Int[1/(x*(c + a^2*c*x^2)^2*Sqrt[ArcTan[a*x]]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\left(a^2c\,x^2+c\right)^2\sqrt{\arctan\left(ax\right)}}dx$$

input
```
int(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)
```

output
```
int(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 2.18 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\sqrt{\arctan(ax)}}\,dx = \frac{\int \frac{1}{a^4x^5\sqrt{\operatorname{atan}(ax)}+2a^2x^3\sqrt{\operatorname{atan}(ax)}+x\sqrt{\operatorname{atan}(ax)}}\,dx}{c^2}$$

input
```
integrate(1/x/(a**2*c*x**2+c)**2/atan(a*x)**(1/2),x)
```

output
```
Integral(1/(a**4*x**5*sqrt(atan(a*x)) + 2*a**2*x**3*sqrt(atan(a*x)) + x*sq
rt(atan(a*x))), x)/c**2
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx=\int \frac{1}{(a^2cx^2+c)^2x\sqrt{\arctan{(ax)}}}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^2*x*sqrt(arctan(a*x))), x)
```

## Mupad [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx=\int \frac{1}{x\,\sqrt{\mathrm{atan}\left(a\,x\right)}\,{\left(c\,a^2\,x^2+c\right)}^2}\,dx$$

input
```
int(1/(x*atan(a*x)^(1/2)*(c + a^2*c*x^2)^2),x)
```

output
```
int(1/(x*atan(a*x)^(1/2)*(c + a^2*c*x^2)^2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.83

$$\int \frac{1}{x(c+a^2cx^2)^2\sqrt{\arctan(ax)}}\,dx = \frac{\int \frac{\sqrt{atan(ax)}}{atan(ax)a^4x^5+2atan(ax)a^2x^3+atan(ax)x}dx}{c^2}$$

input
```
int(1/x/(a^2*c*x^2+c)^2/atan(a*x)^(1/2),x)
```

output
```
int(sqrt(atan(a*x))/(atan(a*x)*a**4*x**5 + 2*atan(a*x)*a**2*x**3 + atan(a*
x)*x),x)/c**2
```

# 3.935 $\int \frac{x^m}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = \text{Int}\left(\frac{x^m}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}},x\right)$$

output
```
Defer(Int)(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.50 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = \int \frac{x^m}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[x^m/((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]),x]
```

output
```
Integrate[x^m/((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\sqrt{\arctan(ax)}\,(a^2cx^2+c)^3}\,dx$$

$$\Big\downarrow \ 5560$$

$$\int \frac{x^m}{\sqrt{\arctan(ax)}\,(a^2cx^2+c)^3}dx$$

input
```
Int[x^m/((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{(a^2c\,x^2+c)^3\,\sqrt{\arctan\,(ax)}}dx$$

input
```
int(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)
```

output
```
int(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.08

$$\int \frac{x^m}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = \int \frac{x^m}{(a^2cx^2+c)^3\sqrt{\arctan{(ax)}}}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
integral(x^m/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*sqrt(arc
tan(a*x))), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^m}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = \text{Timed out}$$

input
```
integrate(x**m/(a**2*c*x**2+c)**3/atan(a*x)**(1/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x^m}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{1,[0,1,0,0]%%%} / %%%{1,[0,0,1,3]%%%} Error: Bad Argument
Value
```

## Mupad [N/A]

Not integrable

Time = 0.90 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = \int \frac{x^m}{\sqrt{\mathrm{atan}\left(a\,x\right)}\,{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

input
```
int(x^m/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^3),x)
```

output
```
int(x^m/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^3), x)
```

## Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.38

$$\int \frac{x^m}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = \frac{\int \frac{x^m\sqrt{atan(ax)}}{atan(ax)a^6x^6+3atan(ax)a^4x^4+3atan(ax)a^2x^2+atan(ax)}dx}{c^3}$$

input
```
int(x^m/(a^2*c*x^2+c)^3/atan(a*x)^(1/2),x)
```

output
```
int((x**m*sqrt(atan(a*x)))/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 +
3*atan(a*x)*a**2*x**2 + atan(a*x)),x)/c**3
```

# 3.936 $\int \frac{x^5}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^5}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx=\text{Int}\left(\frac{x^5}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}},x\right)$$

output
```
Defer(Int)(x^5/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 3.49 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^5}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx=\int \frac{x^5}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[x^5/((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]),x]
```

output
```
Integrate[x^5/((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\sqrt{\arctan(ax)}\,(a^2cx^2+c)^3}\,dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{x^5}{\sqrt{\arctan(ax)}\,(a^2cx^2+c)^3}dx$$

input
```
Int[x^5/((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^5}{(a^2c\,x^2+c)^3\sqrt{\arctan\left(ax\right)}}dx$$

input
```
int(x^5/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)
```

output
```
int(x^5/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x^5}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^5/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 3.82 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.71

$$\int \frac{x^5}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = \frac{\int \frac{x^5}{a^6x^6\sqrt{\operatorname{atan}(ax)}+3a^4x^4\sqrt{\operatorname{atan}(ax)}+3a^2x^2\sqrt{\operatorname{atan}(ax)}+\sqrt{\operatorname{atan}(ax)}}\,dx}{c^3}$$

input
```
integrate(x**5/(a**2*c*x**2+c)**3/atan(a*x)**(1/2),x)
```

output
```
Integral(x**5/(a**6*x**6*sqrt(atan(a*x)) + 3*a**4*x**4*sqrt(atan(a*x)) + 3
*a**2*x**2*sqrt(atan(a*x)) + sqrt(atan(a*x))), x)/c**3
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{(c+a^2cx^2)^3 \sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^5/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 2.59 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{(c+a^2cx^2)^3 \sqrt{\arctan(ax)}}\,dx = \int \frac{x^5}{(a^2cx^2+c)^3 \sqrt{\arctan(ax)}}\,dx$$

input
```
integrate(x^5/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^5/((a^2*c*x^2 + c)^3*sqrt(arctan(a*x))), x)
```

## Mupad [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^5}{(c+a^2cx^2)^3 \sqrt{\arctan(ax)}}\,dx = \int \frac{x^5}{\sqrt{\mathrm{atan}(a\,x)}\,(c\,a^2\,x^2+c)^3}\,dx$$

input
```
int(x^5/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^3),x)
```

output
```
int(x^5/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^3), x)
```

## Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.38

$$\int \frac{x^5}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = \frac{\int \frac{\sqrt{atan(ax)}\,x^5}{atan(ax)a^6x^6+3atan(ax)a^4x^4+3atan(ax)a^2x^2+atan(ax)}dx}{c^3}$$

input
```
int(x^5/(a^2*c*x^2+c)^3/atan(a*x)^(1/2),x)
```

output
```
int((sqrt(atan(a*x))*x**5)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 +
3*atan(a*x)*a**2*x**2 + atan(a*x)),x)/c**3
```

# 3.937 $\int \frac{x^4}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 89

$$\int \frac{x^4}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = \frac{3\sqrt{\arctan(ax)}}{4a^5c^3}+\frac{\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{8a^5c^3}-\frac{\sqrt{\pi}\operatorname{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{2a^5c^3}$$

output
```
3/4*arctan(a*x)^(1/2)/a^5/c^3+1/16*2^(1/2)*Pi^(1/2)*FresnelC(2*2^(1/2)/Pi^
(1/2)*arctan(a*x)^(1/2))/a^5/c^3-1/2*Pi^(1/2)*FresnelC(2*arctan(a*x)^(1/2)
/Pi^(1/2))/a^5/c^3
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 230, normalized size of antiderivative = 2.58

$$\int \frac{x^4}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx$$
$$= \frac{10\sqrt{2\pi}\sqrt{\arctan(ax)^2}\operatorname{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-80\sqrt{\pi}\sqrt{\arctan(ax)^2}\operatorname{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+3}{}$$

input
```
Integrate[x^4/((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]),x]
```

output
```
(10*Sqrt[2*Pi]*Sqrt[ArcTan[a*x]^2]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]
] - 80*Sqrt[Pi]*Sqrt[ArcTan[a*x]^2]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi
]] + 3*Sqrt[ArcTan[a*x]]*(64*Sqrt[ArcTan[a*x]^2] + 4*Sqrt[2]*Sqrt[I*ArcTan
[a*x]]*Gamma[1/2, (-2*I)*ArcTan[a*x]] + 4*Sqrt[2]*Sqrt[(-I)*ArcTan[a*x]]*G
amma[1/2, (2*I)*ArcTan[a*x]] - Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (-4*I)*ArcTa
n[a*x]] - Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (4*I)*ArcTan[a*x]]))/(256*a^5*
c^3*Sqrt[ArcTan[a*x]^2])
```

## Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5505, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{\arctan(ax)}\,(a^2cx^2+c)^3}dx$$

$$\Big\downarrow \text{5505}$$

$$\frac{\int \frac{a^4x^4}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{a^5c^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sin(\arctan(ax))^4}{\sqrt{\arctan(ax)}}d\arctan(ax)}{a^5c^3}$$

$$\Big\downarrow \text{3793}$$

$$\frac{\int \left(-\frac{\cos(2\arctan(ax))}{2\sqrt{\arctan(ax)}}+\frac{\cos(4\arctan(ax))}{8\sqrt{\arctan(ax)}}+\frac{3}{8\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^5c^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{8}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\arctan(ax)}}{a^5c^3}$$

input
```
Int[x^4/((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]),x]
```

output
```
((3*Sqrt[ArcTan[a*x]])/4 + (Sqrt[Pi/2]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a
*x]]])/8 - (Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/2)/(a^5*c^3
)
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

## Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.66

| method | result | size |
|---|---|---|
| default | $\frac{\pi \sqrt{2}\, \operatorname{FresnelC}\left(\frac{2 \sqrt{2}\, \sqrt{\arctan \left(a x \right)}}{\sqrt{\pi}}\right)-8 \pi \,\operatorname{FresnelC}\left(\frac{2 \sqrt{\arctan \left(a x \right)}}{\sqrt{\pi}}\right)+12 \sqrt{\arctan \left(a x \right)}\, \sqrt{\pi}}{16 a^{5} c^{3} \sqrt{\pi}}$ | 59 |

input
```
int(x^4/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/16/a^5/c^3*(Pi*2^(1/2)*FresnelC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))-8*
Pi*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))+12*arctan(a*x)^(1/2)*Pi^(1/2))/P
i^(1/2)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x^4}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^4/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{x^4}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = \frac{\int \frac{x^4}{a^6x^6\sqrt{\operatorname{atan}(ax)}+3a^4x^4\sqrt{\operatorname{atan}(ax)}+3a^2x^2\sqrt{\operatorname{atan}(ax)}+\sqrt{\operatorname{atan}(ax)}}\,dx}{c^3}$$

input
```
integrate(x**4/(a**2*c*x**2+c)**3/atan(a*x)**(1/2),x)
```

output
```
Integral(x**4/(a**6*x**6*sqrt(atan(a*x)) + 3*a**4*x**4*sqrt(atan(a*x)) + 3
*a**2*x**2*sqrt(atan(a*x)) + sqrt(atan(a*x))), x)/c**3
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^4/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.20

$$\int \frac{x^4}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = -\frac{(i+1)\sqrt{2}\sqrt{\pi}\,\text{erf}\left((i-1)\sqrt{2}\sqrt{\arctan(ax)}\right)}{64\,a^5c^3} + \frac{(i-1)\sqrt{2}\sqrt{\pi}\,\text{erf}\left(-(i+1)\sqrt{2}\sqrt{\arctan(ax)}\right)}{64\,a^5c^3} + \frac{(i+1)\sqrt{\pi}\,\text{erf}\left((i-1)\sqrt{\arctan(ax)}\right)}{8\,a^5c^3} - \frac{(i-1)\sqrt{\pi}\,\text{erf}\left(-(i+1)\sqrt{\arctan(ax)}\right)}{8\,a^5c^3} + \frac{3\sqrt{\arctan(ax)}}{4\,a^5c^3}$$

input
```
integrate(x^4/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
-(1/64*I + 1/64)*sqrt(2)*sqrt(pi)*erf((I - 1)*sqrt(2)*sqrt(arctan(a*x)))/(
a^5*c^3) + (1/64*I - 1/64)*sqrt(2)*sqrt(pi)*erf(-(I + 1)*sqrt(2)*sqrt(arct
an(a*x)))/(a^5*c^3) + (1/8*I + 1/8)*sqrt(pi)*erf((I - 1)*sqrt(arctan(a*x))
)/(a^5*c^3) - (1/8*I - 1/8)*sqrt(pi)*erf(-(I + 1)*sqrt(arctan(a*x)))/(a^5*
c^3) + 3/4*sqrt(arctan(a*x))/(a^5*c^3)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = \int \frac{x^4}{\sqrt{\mathrm{atan}\,(a\,x)}\,(c\,a^2\,x^2+c)^3}\,dx$$

input
```
int(x^4/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^3),x)
```

output
```
int(x^4/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^3), x)
```

## Reduce [F]

$$\int \frac{x^4}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = \frac{\int \frac{\sqrt{atan(ax)}\,x^4}{atan(ax)a^6x^6+3atan(ax)a^4x^4+3atan(ax)a^2x^2+atan(ax)}dx}{c^3}$$

input
```
int(x^4/(a^2*c*x^2+c)^3/atan(a*x)^(1/2),x)
```

output
```
int((sqrt(atan(a*x))*x**4)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 +
3*atan(a*x)*a**2*x**2 + atan(a*x)),x)/c**3
```

# 3.938 $\int \frac{x^3}{(c+a^2cx^2)^3 \sqrt{\arctan(ax)}} \, dx$



## Optimal result

Integrand size = 24, antiderivative size = 71

$$\int \frac{x^3}{(c+a^2cx^2)^3 \sqrt{\arctan(ax)}} \, dx = -\frac{\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{8a^4c^3} + \frac{\sqrt{\pi} \operatorname{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{4a^4c^3}$$

output
```
-1/16*2^(1/2)*Pi^(1/2)*FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))/a^4/
c^3+1/4*Pi^(1/2)*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2))/a^4/c^3
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.85

$$\int \frac{x^3}{(c+a^2cx^2)^3 \sqrt{\arctan(ax)}} \, dx = \frac{-2\sqrt{2}\sqrt{-i\arctan(ax)}\Gamma\left(\frac{1}{2},-2i\arctan(ax)\right) - 2\sqrt{2}\sqrt{i\arctan(ax)}\Gamma\left(\frac{1}{2},2i\arctan(ax)\right) + \sqrt{-i\arctan(ax}}{32a^4c^3\sqrt{\arctan(ax)}}$$

input
```
Integrate[x^3/((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]),x]
```

output
```
(-2*Sqrt[2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-2*I)*ArcTan[a*x]] - 2*Sqrt
[2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (2*I)*ArcTan[a*x]] + Sqrt[(-I)*ArcTan[a
*x]]*Gamma[1/2, (-4*I)*ArcTan[a*x]] + Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (4*I)
*ArcTan[a*x]])/(32*a^4*c^3*Sqrt[ArcTan[a*x]])
```

## Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {5505, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{\arctan(ax)}\,(a^2cx^2+c)^3}\,dx$$

$$\Big\downarrow \ 5505$$

$$\frac{\int \frac{a^3x^3}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{a^4c^3}$$

$$\Big\downarrow \ 4906$$

$$\frac{\int \left(\frac{\sin(2\arctan(ax))}{4\sqrt{\arctan(ax)}}-\frac{\sin(4\arctan(ax))}{8\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^4c^3}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{1}{4}\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)-\frac{1}{8}\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{a^4c^3}$$

input
```
Int[x^3/((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]),x]
```

output
```
(-1/8*(Sqrt[Pi/2]*FresnelS[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]]) + (Sqrt[Pi]*Fr
esnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/4)/(a^4*c^3)
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

## Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.66

| method | result | size |
|---|---|---|
| default | $\frac{\sqrt{\pi}\left(-\sqrt{2}\,\operatorname{FresnelS}\left(\frac{2\sqrt{2}\,\sqrt{\arctan\left(a x\right)}}{\sqrt{\pi}}\right)+4\,\operatorname{FresnelS}\left(\frac{2\sqrt{\arctan\left(a x\right)}}{\sqrt{\pi}}\right)\right)}{16 a^{4} c^{3}}$ | 47 |

input
```
int(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/16/a^4/c^3*Pi^(1/2)*(-2^(1/2)*FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1
/2))+4*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2)))
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{x^3}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = \frac{\int \frac{x^3}{a^6x^6\sqrt{\operatorname{atan}(ax)}+3a^4x^4\sqrt{\operatorname{atan}(ax)}+3a^2x^2\sqrt{\operatorname{atan}(ax)}+\sqrt{\operatorname{atan}(ax)}}\,dx}{c^3}$$

input
```
integrate(x**3/(a**2*c*x**2+c)**3/atan(a*x)**(1/2),x)
```

output
```
Integral(x**3/(a**6*x**6*sqrt(atan(a*x)) + 3*a**4*x**4*sqrt(atan(a*x)) + 3
*a**2*x**2*sqrt(atan(a*x)) + sqrt(atan(a*x))), x)/c**3
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F]

$$\int \frac{x^3}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx=\int \frac{x^3}{(a^2cx^2+c)^3\sqrt{\arctan(ax)}}\,dx$$

input
```
integrate(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^3/((a^2*c*x^2 + c)^3*sqrt(arctan(a*x))), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx=\int \frac{x^3}{\sqrt{\mathrm{atan}(a\,x)}\,(c\,a^2\,x^2+c)^3}\,dx$$

input
```
int(x^3/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^3),x)
```

output
```
int(x^3/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^3), x)
```

## Reduce [F]

$$\int \frac{x^3}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx=\frac{\int \frac{\sqrt{atan(ax)}\,x^3}{atan(ax)a^6x^6+3atan(ax)a^4x^4+3atan(ax)a^2x^2+atan(ax)}dx}{c^3}$$

input
```
int(x^3/(a^2*c*x^2+c)^3/atan(a*x)^(1/2),x)
```

output
```
int((sqrt(atan(a*x))*x**3)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 +
3*atan(a*x)*a**2*x**2 + atan(a*x)),x)/c**3
```

# 3.939 $\int \frac{x^2}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 58

$$\int \frac{x^2}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx=\frac{\sqrt{\arctan(ax)}}{4a^3c^3}-\frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{8a^3c^3}$$

output
```
1/4*arctan(a*x)^(1/2)/a^3/c^3-1/16*2^(1/2)*Pi^(1/2)*FresnelC(2*2^(1/2)/Pi^
(1/2)*arctan(a*x)^(1/2))/a^3/c^3
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.41 (sec) , antiderivative size = 229, normalized size of antiderivative = 3.95

$$\int \frac{x^2}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx$$
$$=\frac{-2\sqrt{2\pi}\sqrt{\arctan(ax)^2}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+16\sqrt{\pi}\sqrt{\arctan(ax)^2}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\cdots}{}$$

input
```
Integrate[x^2/((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]),x]
```

output
```
(-2*Sqrt[2*Pi]*Sqrt[ArcTan[a*x]^2]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]
] + 16*Sqrt[Pi]*Sqrt[ArcTan[a*x]^2]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi
]] + Sqrt[ArcTan[a*x]]*(64*Sqrt[ArcTan[a*x]^2] + 4*Sqrt[2]*Sqrt[I*ArcTan[a
*x]]*Gamma[1/2, (-2*I)*ArcTan[a*x]] + 4*Sqrt[2]*Sqrt[(-I)*ArcTan[a*x]]*Gam
ma[1/2, (2*I)*ArcTan[a*x]] + 7*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (-4*I)*ArcTa
n[a*x]] + 7*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (4*I)*ArcTan[a*x]]))/(256*a^
3*c^3*Sqrt[ArcTan[a*x]^2])
```

## Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.91, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5505, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{\arctan(ax)}\,(a^2cx^2+c)^3}dx$$

$$\Big\downarrow \text{5505}$$

$$\frac{\int \frac{a^2x^2}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3c^3}$$

$$\Big\downarrow \text{4906}$$

$$\frac{\int \left(\frac{1}{8\sqrt{\arctan(ax)}}-\frac{\cos(4\arctan(ax))}{8\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^3c^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{4}\sqrt{\arctan(ax)}-\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{a^3c^3}$$

input
```
Int[x^2/((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]),x]
```

output
```
(Sqrt[ArcTan[a*x]]/4 - (Sqrt[Pi/2]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]
])/8)/(a^3*c^3)
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

## Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.81

| method | result | size |
|---|---|---|
| default | $-\frac{\sqrt{2}\,\left(-2\sqrt{2}\,\sqrt{\arctan\left(a x\right)}\,\sqrt{\pi}+\pi\,\operatorname{FresnelC}\left(\frac{2\sqrt{2}\,\sqrt{\arctan\left(a x\right)}}{\sqrt{\pi}}\right)\right)}{16 a^{3} c^{3} \sqrt{\pi}}$ | 47 |

input
```
int(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/16/a^3/c^3*2^(1/2)/Pi^(1/2)*(-2*2^(1/2)*arctan(a*x)^(1/2)*Pi^(1/2)+Pi*F
resnelC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2)))
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{x^2}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = \frac{\int \frac{x^2}{a^6x^6\sqrt{\operatorname{atan}(ax)}+3a^4x^4\sqrt{\operatorname{atan}(ax)}+3a^2x^2\sqrt{\operatorname{atan}(ax)}+\sqrt{\operatorname{atan}(ax)}}\,dx}{c^3}$$

input
```
integrate(x**2/(a**2*c*x**2+c)**3/atan(a*x)**(1/2),x)
```

output
```
Integral(x**2/(a**6*x**6*sqrt(atan(a*x)) + 3*a**4*x**4*sqrt(atan(a*x)) + 3
*a**2*x**2*sqrt(atan(a*x)) + sqrt(atan(a*x))), x)/c**3
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.16

$$\int \frac{x^2}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = \frac{(i+1)\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left((i-1)\sqrt{2}\sqrt{\arctan(ax)}\right)}{64\,a^3c^3} - \frac{(i-1)\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-(i+1)\sqrt{2}\sqrt{\arctan(ax)}\right)}{64\,a^3c^3} + \frac{\sqrt{\arctan(ax)}}{4\,a^3c^3}$$

input
```
integrate(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
(1/64*I + 1/64)*sqrt(2)*sqrt(pi)*erf((I - 1)*sqrt(2)*sqrt(arctan(a*x)))/(a
^3*c^3) - (1/64*I - 1/64)*sqrt(2)*sqrt(pi)*erf(-(I + 1)*sqrt(2)*sqrt(arcta
n(a*x)))/(a^3*c^3) + 1/4*sqrt(arctan(a*x))/(a^3*c^3)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = \int \frac{x^2}{\sqrt{\mathrm{atan}\left(a\,x\right)}\,{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

input
```
int(x^2/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^3),x)
```

output
```
int(x^2/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^3), x)
```

## Reduce [F]

$$\int \frac{x^2}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = \frac{\int \frac{\sqrt{atan(ax)}\,x^2}{atan(ax)a^6x^6+3atan(ax)a^4x^4+3atan(ax)a^2x^2+atan(ax)}dx}{c^3}$$

input
```
int(x^2/(a^2*c*x^2+c)^3/atan(a*x)^(1/2),x)
```

output
```
int((sqrt(atan(a*x))*x**2)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 +
3*atan(a*x)*a**2*x**2 + atan(a*x)),x)/c**3
```

# 3.940 $\int \frac{x}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 71

$$\int \frac{x}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = \frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{8a^2c^3} + \frac{\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{4a^2c^3}$$

output
```
1/16*2^(1/2)*Pi^(1/2)*FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))/a^2/c
^3+1/4*Pi^(1/2)*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2))/a^2/c^3
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.87

$$\int \frac{x}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = \frac{-2\sqrt{2}\sqrt{-i\arctan(ax)}\,\Gamma\left(\frac{1}{2},-2i\arctan(ax)\right) - 2\sqrt{2}\sqrt{i\arctan(ax)}\,\Gamma\left(\frac{1}{2},2i\arctan(ax)\right) - \sqrt{-i\arctan(ax}}{32a^2c^3\sqrt{\arctan(ax)}}$$

input
```
Integrate[x/((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]),x]
```

output
```
(-2*Sqrt[2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-2*I)*ArcTan[a*x]] - 2*Sqrt
[2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (2*I)*ArcTan[a*x]] - Sqrt[(-I)*ArcTan[a
*x]]*Gamma[1/2, (-4*I)*ArcTan[a*x]] - Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (4*I)
*ArcTan[a*x]])/(32*a^2*c^3*Sqrt[ArcTan[a*x]])
```

## Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {5505, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{\arctan(ax)}\,(a^2cx^2+c)^3}\,dx$$

$$\Big\downarrow \text{5505}$$

$$\frac{\int \frac{ax}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{a^2c^3}$$

$$\Big\downarrow \text{4906}$$

$$\frac{\int \left(\frac{\sin(2\arctan(ax))}{4\sqrt{\arctan(ax)}}+\frac{\sin(4\arctan(ax))}{8\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^2c^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{8}\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{4}\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{a^2c^3}$$

input
```
Int[x/((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]),x]
```

output
```
((Sqrt[Pi/2]*FresnelS[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/8 + (Sqrt[Pi]*Fresn
elS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/4)/(a^2*c^3)
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

## Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.65

| method | result | size |
|---|---|---|
| default | $\frac{\sqrt{\pi}\left(\sqrt{2}\,\operatorname{FresnelS}\left(\frac{2\sqrt{2}\,\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+4\,\operatorname{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)\right)}{16a^2c^3}$ | 46 |

input
```
int(x/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/16/a^2/c^3*Pi^(1/2)*(2^(1/2)*FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/
2))+4*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2)))
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{x}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = \frac{\int \frac{x}{a^6x^6\sqrt{\operatorname{atan}(ax)}+3a^4x^4\sqrt{\operatorname{atan}(ax)}+3a^2x^2\sqrt{\operatorname{atan}(ax)}+\sqrt{\operatorname{atan}(ax)}}\,dx}{c^3}$$

input
```
integrate(x/(a**2*c*x**2+c)**3/atan(a*x)**(1/2),x)
```

output
```
Integral(x/(a**6*x**6*sqrt(atan(a*x)) + 3*a**4*x**4*sqrt(atan(a*x)) + 3*a*
*2*x**2*sqrt(atan(a*x)) + sqrt(atan(a*x))), x)/c**3
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F]

$$\int \frac{x}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx=\int \frac{x}{(a^2cx^2+c)^3\sqrt{\arctan(ax)}}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/((a^2*c*x^2 + c)^3*sqrt(arctan(a*x))), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx=\int \frac{x}{\sqrt{\mathrm{atan}(a\,x)}\,(c\,a^2\,x^2+c)^3}\,dx$$

input
```
int(x/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^3),x)
```

output
```
int(x/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^3), x)
```

## Reduce [F]

$$\int \frac{x}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx=\frac{\int \frac{\sqrt{atan(ax)}\,x}{atan(ax)a^6x^6+3atan(ax)a^4x^4+3atan(ax)a^2x^2+atan(ax)}dx}{c^3}$$

input
```
int(x/(a^2*c*x^2+c)^3/atan(a*x)^(1/2),x)
```

output
```
int((sqrt(atan(a*x))*x)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*a
tan(a*x)*a**2*x**2 + atan(a*x)),x)/c**3
```

# 3.941 $\int \frac{1}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 89

$$\int \frac{1}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = \frac{3\sqrt{\arctan(ax)}}{4ac^3} + \frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{8ac^3} + \frac{\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{2ac^3}$$

output
```
3/4*arctan(a*x)^(1/2)/a/c^3+1/16*2^(1/2)*Pi^(1/2)*FresnelC(2*2^(1/2)/Pi^(1
/2)*arctan(a*x)^(1/2))/a/c^3+1/2*Pi^(1/2)*FresnelC(2*arctan(a*x)^(1/2)/Pi^
(1/2))/a/c^3
```

## Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.81

$$\int \frac{1}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = \frac{12\sqrt{\arctan(ax)} + \sqrt{2\pi}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right) + 8\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{16ac^3}$$

input
```
Integrate[1/((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]),x]
```

output
```
(12*Sqrt[ArcTan[a*x]] + Sqrt[2*Pi]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]
] + 8*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(16*a*c^3)
```

## Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {5439, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^3}dx$$

$$\Big\downarrow \text{5439}$$

$$\frac{\int \frac{1}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{ac^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^4}{\sqrt{\arctan(ax)}}d\arctan(ax)}{ac^3}$$

$$\Big\downarrow \text{3793}$$

$$\frac{\int \left(\frac{\cos(2\arctan(ax))}{2\sqrt{\arctan(ax)}}+\frac{\cos(4\arctan(ax))}{8\sqrt{\arctan(ax)}}+\frac{3}{8\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{ac^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\arctan(ax)}}{ac^3}$$

input
```
Int[1/((c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]),x]
```

output
```
((3*Sqrt[ArcTan[a*x]])/4 + (Sqrt[Pi/2]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a
*x]]])/8 + (Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/2)/(a*c^3)
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

## Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.66

| method | result | size |
|---|---|---|
| default | $\frac{\pi\sqrt{2}\,\operatorname{FresnelC}\left(\frac{2\sqrt{2}\,\sqrt{\arctan\left(a x\right)}}{\sqrt{\pi}}\right)+8\pi\,\operatorname{FresnelC}\left(\frac{2\sqrt{\arctan\left(a x\right)}}{\sqrt{\pi}}\right)+12\sqrt{\arctan\left(a x\right)}\,\sqrt{\pi}}{16 a\,c^{3}\sqrt{\pi}}$ | 59 |

input
```
int(1/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/16/a/c^3/Pi^(1/2)*(Pi*2^(1/2)*FresnelC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1
/2))+8*Pi*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))+12*arctan(a*x)^(1/2)*Pi^(
1/2))
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(c+a^2cx^2)^3 \sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{1}{(c+a^2cx^2)^3 \sqrt{\arctan(ax)}}\,dx = \frac{\int \frac{1}{a^6x^6\sqrt{\operatorname{atan}(ax)}+3a^4x^4\sqrt{\operatorname{atan}(ax)}+3a^2x^2\sqrt{\operatorname{atan}(ax)}+\sqrt{\operatorname{atan}(ax)}}\,dx}{c^3}$$

input
```
integrate(1/(a**2*c*x**2+c)**3/atan(a*x)**(1/2),x)
```

output
```
Integral(1/(a**6*x**6*sqrt(atan(a*x)) + 3*a**4*x**4*sqrt(atan(a*x)) + 3*a*
*2*x**2*sqrt(atan(a*x)) + sqrt(atan(a*x))), x)/c**3
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(c+a^2cx^2)^3 \sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.20

$$\int \frac{1}{(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = -\frac{(i+1)\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left((i-1)\sqrt{2}\sqrt{\arctan(ax)}\right)}{64\,ac^3} + \frac{(i-1)\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-(i+1)\sqrt{2}\sqrt{\arctan(ax)}\right)}{64\,ac^3} - \frac{(i+1)\sqrt{\pi}\,\mathrm{erf}\left((i-1)\sqrt{\arctan(ax)}\right)}{8\,ac^3} + \frac{(i-1)\sqrt{\pi}\,\mathrm{erf}\left(-(i+1)\sqrt{\arctan(ax)}\right)}{8\,ac^3} + \frac{3\sqrt{\arctan(ax)}}{4\,ac^3}$$

input
```
integrate(1/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
-(1/64*I + 1/64)*sqrt(2)*sqrt(pi)*erf((I - 1)*sqrt(2)*sqrt(arctan(a*x)))/(
a*c^3) + (1/64*I - 1/64)*sqrt(2)*sqrt(pi)*erf(-(I + 1)*sqrt(2)*sqrt(arctan
(a*x)))/(a*c^3) - (1/8*I + 1/8)*sqrt(pi)*erf((I - 1)*sqrt(arctan(a*x)))/(a
*c^3) + (1/8*I - 1/8)*sqrt(pi)*erf(-(I + 1)*sqrt(arctan(a*x)))/(a*c^3) + 3
/4*sqrt(arctan(a*x))/(a*c^3)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+a^2cx^2)^3 \sqrt{\arctan(ax)}}\,dx = \int \frac{1}{\sqrt{\operatorname{atan}(a\,x)}\,(c\,a^2\,x^2+c)^3}\,dx$$

input
```
int(1/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^3),x)
```

output
```
int(1/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^3), x)
```

## Reduce [F]

$$\int \frac{1}{(c+a^2cx^2)^3 \sqrt{\arctan(ax)}}\,dx = \frac{\int \frac{\sqrt{atan(ax)}}{atan(ax)a^6x^6+3atan(ax)a^4x^4+3atan(ax)a^2x^2+atan(ax)}dx}{c^3}$$

input
```
int(1/(a^2*c*x^2+c)^3/atan(a*x)^(1/2),x)
```

output
```
int(sqrt(atan(a*x))/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(
a*x)*a**2*x**2 + atan(a*x)),x)/c**3
```

# 3.942 $\int \frac{1}{x(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\sqrt{\arctan(ax)}}\,dx=\text{Int}\left(\frac{1}{x\left(c+a^2cx^2\right)^3\sqrt{\arctan(ax)}},x\right)$$

output
```
Defer(Int)(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.84 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\sqrt{\arctan(ax)}}\,dx=\int \frac{1}{x\left(c+a^2cx^2\right)^3\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[1/(x*(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]),x]
```

output
```
Integrate[1/(x*(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^3}dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int \frac{1}{x\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^3}dx$$

input
```
Int[1/(x*(c + a^2*c*x^2)^3*Sqrt[ArcTan[a*x]]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\left(a^2c\,x^2+c\right)^3\sqrt{\arctan\left(ax\right)}}dx$$

input
```
int(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)
```

output
```
int(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 4.60 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.71

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\sqrt{\arctan(ax)}}\,dx = \frac{\int \frac{1}{a^6x^7\sqrt{\operatorname{atan}(ax)}+3a^4x^5\sqrt{\operatorname{atan}(ax)}+3a^2x^3\sqrt{\operatorname{atan}(ax)}+x\sqrt{\operatorname{atan}(ax)}}\,dx}{c^3}$$

input
```
integrate(1/x/(a**2*c*x**2+c)**3/atan(a*x)**(1/2),x)
```

output
```
Integral(1/(a**6*x**7*sqrt(atan(a*x)) + 3*a**4*x**5*sqrt(atan(a*x)) + 3*a*
*2*x**3*sqrt(atan(a*x)) + x*sqrt(atan(a*x))), x)/c**3
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\sqrt{\arctan(ax)}}\,dx = \int \frac{1}{\left(a^2cx^2+c\right)^3 x\sqrt{\arctan\left(ax\right)}}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^3*x*sqrt(arctan(a*x))), x)
```

## Mupad [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\sqrt{\arctan(ax)}}\,dx = \int \frac{1}{x\,\sqrt{\mathrm{atan}\left(a\,x\right)}\,{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

input
```
int(1/(x*atan(a*x)^(1/2)*(c + a^2*c*x^2)^3),x)
```

output
```
int(1/(x*atan(a*x)^(1/2)*(c + a^2*c*x^2)^3), x)
```

## Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.33

$$\int \frac{1}{x(c+a^2cx^2)^3\sqrt{\arctan(ax)}}\,dx = \frac{\int \frac{\sqrt{atan(ax)}}{atan(ax)a^6x^7+3atan(ax)a^4x^5+3atan(ax)a^2x^3+atan(ax)x}dx}{c^3}$$

input
```
int(1/x/(a^2*c*x^2+c)^3/atan(a*x)^(1/2),x)
```

output
```
int(sqrt(atan(a*x))/(atan(a*x)*a**6*x**7 + 3*atan(a*x)*a**4*x**5 + 3*atan(
a*x)*a**2*x**3 + atan(a*x)*x),x)/c**3
```

# 3.943 $\int \frac{x^m\sqrt{c+a^2cx^2}}{\sqrt{\arctan(ax)}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\sqrt{\arctan(ax)}}\,dx = \text{Int}\left(\frac{x^m\sqrt{c+a^2cx^2}}{\sqrt{\arctan(ax)}},x\right)$$

output
```
Defer(Int)(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\sqrt{\arctan(ax)}}\,dx = \int \frac{x^m\sqrt{c+a^2cx^2}}{\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[(x^m*Sqrt[c + a^2*c*x^2])/Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[(x^m*Sqrt[c + a^2*c*x^2])/Sqrt[ArcTan[a*x]], x]
```

## Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m\sqrt{a^2cx^2+c}}{\sqrt{\arctan(ax)}}dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int \frac{x^m\sqrt{a^2cx^2+c}}{\sqrt{\arctan(ax)}}dx$$

input
```
Int[(x^m*Sqrt[c + a^2*c*x^2])/Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^m\sqrt{a^2c\,x^2+c}}{\sqrt{\arctan\left(ax\right)}}dx$$

input
```
int(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\sqrt{\arctan(ax)}}\,dx=\int \frac{\sqrt{a^2cx^2+c}x^m}{\sqrt{\arctan{(ax)}}}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^m/sqrt(arctan(a*x)), x)
```

## Sympy [N/A]

Not integrable

Time = 19.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\sqrt{\arctan(ax)}}\,dx=\int \frac{x^m\sqrt{c\left(a^2x^2+1\right)}}{\sqrt{\operatorname{atan}{(ax)}}}\,dx$$

input
```
integrate(x**m*(a**2*c*x**2+c)**(1/2)/atan(a*x)**(1/2),x)
```

output
```
Integral(x**m*sqrt(c*(a**2*x**2 + 1))/sqrt(atan(a*x)), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\sqrt{\arctan(ax)}}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\sqrt{\arctan(ax)}}\,dx = \int \frac{x^m\,\sqrt{c\,a^2\,x^2+c}}{\sqrt{\mathrm{atan}\left(a\,x\right)}}\,dx$$

input
```
int((x^m*(c + a^2*c*x^2)^(1/2))/atan(a*x)^(1/2),x)
```

output
```
int((x^m*(c + a^2*c*x^2)^(1/2))/atan(a*x)^(1/2), x)
```

## Reduce [N/A]

Not integrable

Time = 4.49 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\sqrt{\arctan(ax)}}\,dx = \sqrt{c}\left(\int \frac{x^m\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}}{atan\,(ax)}dx\right)$$

input
```
int(x^m*(a^2*c*x^2+c)^(1/2)/atan(a*x)^(1/2),x)
```

output
```
sqrt(c)*int((x**m*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/atan(a*x),x)
```

# 3.944 $\int \frac{x\sqrt{c+a^2cx^2}}{\sqrt{\arctan(ax)}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x\sqrt{c+a^2cx^2}}{\sqrt{\arctan(ax)}}\,dx=\text{Int}\left(\frac{x\sqrt{c+a^2cx^2}}{\sqrt{\arctan(ax)}},x\right)$$

output
```
Defer(Int)(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.76 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x\sqrt{c+a^2cx^2}}{\sqrt{\arctan(ax)}}\,dx=\int \frac{x\sqrt{c+a^2cx^2}}{\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[(x*Sqrt[c + a^2*c*x^2])/Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[(x*Sqrt[c + a^2*c*x^2])/Sqrt[ArcTan[a*x]], x]
```

## Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{a^2cx^2+c}}{\sqrt{\arctan(ax)}}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x\sqrt{a^2cx^2+c}}{\sqrt{\arctan(ax)}}dx$$

input
```
Int[(x*Sqrt[c + a^2*c*x^2])/Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{x\sqrt{a^2c\,x^2+c}}{\sqrt{\arctan\left(ax\right)}}dx$$

input
```
int(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

output
```
int(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x\sqrt{c+a^2cx^2}}{\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 1.44 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x\sqrt{c+a^2cx^2}}{\sqrt{\arctan(ax)}}\,dx = \int \frac{x\sqrt{c\,(a^2x^2+1)}}{\sqrt{\operatorname{atan}\,(ax)}}\,dx$$

input
```
integrate(x*(a**2*c*x**2+c)**(1/2)/atan(a*x)**(1/2),x)
```

output
```
Integral(x*sqrt(c*(a**2*x**2 + 1))/sqrt(atan(a*x)), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x\sqrt{c+a^2cx^2}}{\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 1.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x\sqrt{c+a^2cx^2}}{\sqrt{\arctan(ax)}}\,dx = \int \frac{\sqrt{a^2cx^2+cx}}{\sqrt{\arctan{(ax)}}}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*x/sqrt(arctan(a*x)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x\sqrt{c+a^2cx^2}}{\sqrt{\arctan(ax)}}\,dx = \int \frac{x\,\sqrt{c\,a^2\,x^2+c}}{\sqrt{\mathrm{atan}\left(a\,x\right)}}\,dx$$

input
```
int((x*(c + a^2*c*x^2)^(1/2))/atan(a*x)^(1/2),x)
```

output
```
int((x*(c + a^2*c*x^2)^(1/2))/atan(a*x)^(1/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.17

$$\int \frac{x\sqrt{c+a^2cx^2}}{\sqrt{\arctan(ax)}}\,dx = \sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}\,x}{atan\,(ax)}dx\right)$$

input
```
int(x*(a^2*c*x^2+c)^(1/2)/atan(a*x)^(1/2),x)
```

output
```
sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x)/atan(a*x),x)
```

# 3.945 $\int \frac{\sqrt{c+a^2cx^2}}{\sqrt{\arctan(ax)}}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\sqrt{c+a^2cx^2}}{\sqrt{\arctan(ax)}}\,dx = \text{Int}\left(\frac{\sqrt{c+a^2cx^2}}{\sqrt{\arctan(ax)}},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{c+a^2cx^2}}{\sqrt{\arctan(ax)}}\,dx = \int \frac{\sqrt{c+a^2cx^2}}{\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[Sqrt[c + a^2*c*x^2]/Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[Sqrt[c + a^2*c*x^2]/Sqrt[ArcTan[a*x]], x]
```

## Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2cx^2+c}}{\sqrt{\arctan(ax)}} \, dx$$

$$\Big\downarrow \text{5560}$$

$$\int \frac{\sqrt{a^2cx^2+c}}{\sqrt{\arctan(ax)}}dx$$

input
```
Int[Sqrt[c + a^2*c*x^2]/Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{a^2c\,x^2+c}}{\sqrt{\arctan\left(ax\right)}}dx$$

input
```
int((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

output
```
int((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+a^2cx^2}}{\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{c+a^2cx^2}}{\sqrt{\arctan(ax)}}\,dx = \int \frac{\sqrt{c\,(a^2x^2+1)}}{\sqrt{\operatorname{atan}\,(ax)}}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(1/2)/atan(a*x)**(1/2),x)
```

output
```
Integral(sqrt(c*(a**2*x**2 + 1))/sqrt(atan(a*x)), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+a^2cx^2}}{\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{c+a^2cx^2}}{\sqrt{\arctan(ax)}}\,dx = \int \frac{\sqrt{a^2cx^2+c}}{\sqrt{\arctan{(ax)}}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)/sqrt(arctan(a*x)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{c+a^2cx^2}}{\sqrt{\arctan(ax)}}\,dx = \int \frac{\sqrt{c\,a^2\,x^2+c}}{\sqrt{\mathrm{atan}\left(a\,x\right)}}\,dx$$

input
```
int((c + a^2*c*x^2)^(1/2)/atan(a*x)^(1/2),x)
```

output
```
int((c + a^2*c*x^2)^(1/2)/atan(a*x)^(1/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{c+a^2cx^2}}{\sqrt{\arctan(ax)}}\,dx = \sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}}{atan\,(ax)}dx\right)$$

input
```
int((a^2*c*x^2+c)^(1/2)/atan(a*x)^(1/2),x)
```

output
```
sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/atan(a*x),x)
```

# 3.946 $\int \frac{\sqrt{c+a^2cx^2}}{x\sqrt{\arctan(ax)}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\sqrt{c+a^2cx^2}}{x\sqrt{\arctan(ax)}}\,dx=\text{Int}\left(\frac{\sqrt{c+a^2cx^2}}{x\sqrt{\arctan(ax)}},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 4.15 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{c+a^2cx^2}}{x\sqrt{\arctan(ax)}}\,dx=\int \frac{\sqrt{c+a^2cx^2}}{x\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[Sqrt[c + a^2*c*x^2]/(x*Sqrt[ArcTan[a*x]]),x]
```

output
```
Integrate[Sqrt[c + a^2*c*x^2]/(x*Sqrt[ArcTan[a*x]]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2cx^2+c}}{x\sqrt{\arctan(ax)}}dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int \frac{\sqrt{a^2cx^2+c}}{x\sqrt{\arctan(ax)}}dx$$

input
```
Int[Sqrt[c + a^2*c*x^2]/(x*Sqrt[ArcTan[a*x]]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{a^2c\,x^2+c}}{x\sqrt{\arctan\left(ax\right)}}dx$$

input
```
int((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^(1/2),x)
```

output
```
int((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c + a^2 c x^2}}{x\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 1.68 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{c + a^2 c x^2}}{x\sqrt{\arctan(ax)}}\,dx = \int \frac{\sqrt{c\,(a^2x^2 + 1)}}{x\sqrt{\operatorname{atan}\,(ax)}}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(1/2)/x/atan(a*x)**(1/2),x)
```

output
```
Integral(sqrt(c*(a**2*x**2 + 1))/(x*sqrt(atan(a*x))), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c + a^2 c x^2}}{x\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{c+a^2cx^2}}{x\sqrt{\arctan(ax)}}\,dx = \int \frac{\sqrt{a^2cx^2+c}}{x\sqrt{\arctan{(ax)}}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)/(x*sqrt(arctan(a*x))), x)
```

## Mupad [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{c+a^2cx^2}}{x\sqrt{\arctan(ax)}}\,dx = \int \frac{\sqrt{c\,a^2\,x^2+c}}{x\,\sqrt{\mathrm{atan}\left(a\,x\right)}}\,dx$$

input
```
int((c + a^2*c*x^2)^(1/2)/(x*atan(a*x)^(1/2)),x)
```

output
```
int((c + a^2*c*x^2)^(1/2)/(x*atan(a*x)^(1/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \frac{\sqrt{c+a^2cx^2}}{x\sqrt{\arctan(ax)}}\,dx = \sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}}{atan\,(ax)\,x}dx\right)$$

input
```
int((a^2*c*x^2+c)^(1/2)/x/atan(a*x)^(1/2),x)
```

output
```
sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(atan(a*x)*x),x)
```

# 3.947 $\int \frac{x^m \left(c+a^2 c x^2\right)^{3/2}}{\sqrt{\arctan(ax)}} \, dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^m (c+a^2 c x^2)^{3/2}}{\sqrt{\arctan(ax)}} \, dx = \text{Int}\left(\frac{x^m (c+a^2 c x^2)^{3/2}}{\sqrt{\arctan(ax)}}, x\right)$$

output
```
Defer(Int)(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^m (c+a^2 c x^2)^{3/2}}{\sqrt{\arctan(ax)}} \, dx = \int \frac{x^m (c+a^2 c x^2)^{3/2}}{\sqrt{\arctan(ax)}} \, dx$$

input
```
Integrate[(x^m*(c + a^2*c*x^2)^(3/2))/Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[(x^m*(c + a^2*c*x^2)^(3/2))/Sqrt[ArcTan[a*x]], x]
```

## Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \left(a^2cx^2+c\right)^{3/2}}{\sqrt{\arctan(ax)}} \, dx$$

$$\Big\downarrow \; 5560$$

$$\int \frac{x^m \left(a^2cx^2+c\right)^{3/2}}{\sqrt{\arctan(ax)}}dx$$

input
```
Int[(x^m*(c + a^2*c*x^2)^(3/2))/Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^m (a^2 c\, x^2+c)^{\frac{3}{2}}}{\sqrt{\arctan \left(a x\right)}}dx$$

input
```
int(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\sqrt{\arctan(ax)}}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}x^m}{\sqrt{\arctan(ax)}}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^(3/2)*x^m/sqrt(arctan(a*x)), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\sqrt{\arctan(ax)}}\,dx = \text{Timed out}$$

input
```
integrate(x**m*(a**2*c*x**2+c)**(3/2)/atan(a*x)**(1/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\sqrt{\arctan(ax)}}\,dx = \int \frac{x^m\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}{\sqrt{\mathrm{atan}\left(a\,x\right)}}\,dx$$

input
```
int((x^m*(c + a^2*c*x^2)^(3/2))/atan(a*x)^(1/2),x)
```

output
```
int((x^m*(c + a^2*c*x^2)^(3/2))/atan(a*x)^(1/2), x)
```

## Reduce [N/A]

Not integrable

Time = 48.71 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.54

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\sqrt{\arctan(ax)}}\,dx = \sqrt{c}\,c\left(\left(\int \frac{x^m\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,x^2}{atan(ax)}dx\right)a^2 + \int \frac{x^m\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}}{atan(ax)}dx\right)$$

input
```
int(x^m*(a^2*c*x^2+c)^(3/2)/atan(a*x)^(1/2),x)
```

output
```
sqrt(c)*c*(int((x**m*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**2)/atan(a*x),x
)*a**2 + int((x**m*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/atan(a*x),x))
```

# 3.948 $\int \frac{x\left(c+a^2cx^2\right)^{3/2}}{\sqrt{\arctan(ax)}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\sqrt{\arctan(ax)}}\,dx=\text{Int}\left(\frac{x(c+a^2cx^2)^{3/2}}{\sqrt{\arctan(ax)}},x\right)$$

output
```
Defer(Int)(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 2.75 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\sqrt{\arctan(ax)}}\,dx=\int \frac{x(c+a^2cx^2)^{3/2}}{\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[(x*(c + a^2*c*x^2)^(3/2))/Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[(x*(c + a^2*c*x^2)^(3/2))/Sqrt[ArcTan[a*x]], x]
```

## Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a^2cx^2+c\right)^{3/2}}{\sqrt{\arctan(ax)}}\,dx$$

$$\Big\downarrow \ 5560$$

$$\int \frac{x\left(a^2cx^2+c\right)^{3/2}}{\sqrt{\arctan(ax)}}dx$$

input
```
Int[(x*(c + a^2*c*x^2)^(3/2))/Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{x(a^2c\,x^2+c)^{\frac{3}{2}}}{\sqrt{\arctan\left(ax\right)}}dx$$

input
```
int(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

output
```
int(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 35.82 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\sqrt{\arctan(ax)}}\,dx = \int \frac{x(c(a^2x^2+1))^{\frac{3}{2}}}{\sqrt{\operatorname{atan}(ax)}}\,dx$$

input
```
integrate(x*(a**2*c*x**2+c)**(3/2)/atan(a*x)**(1/2),x)
```

output
```
Integral(x*(c*(a**2*x**2 + 1))**(3/2)/sqrt(atan(a*x)), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\sqrt{\arctan(ax)}}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}x}{\sqrt{\arctan{(ax)}}}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*x/sqrt(arctan(a*x)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\sqrt{\arctan(ax)}}\,dx = \int \frac{x\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}{\sqrt{\mathrm{atan}\left(a\,x\right)}}\,dx$$

input
```
int((x*(c + a^2*c*x^2)^(3/2))/atan(a*x)^(1/2),x)
```

output
```
int((x*(c + a^2*c*x^2)^(3/2))/atan(a*x)^(1/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.54

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\sqrt{\arctan(ax)}}\,dx = \sqrt{c}\,c\left(\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}\,x^3}{atan\,(ax)}dx\right)a^2 + \int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}\,x}{atan\,(ax)}dx\right)$$

input
```
int(x*(a^2*c*x^2+c)^(3/2)/atan(a*x)^(1/2),x)
```

output
```
sqrt(c)*c*(int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**3)/atan(a*x),x)*a**
2 + int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x)/atan(a*x),x))
```

# 3.949 $\int \frac{(c+a^2cx^2)^{3/2}}{\sqrt{\arctan(ax)}}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(c+a^2cx^2)^{3/2}}{\sqrt{\arctan(ax)}}\,dx = \text{Int}\left(\frac{(c+a^2cx^2)^{3/2}}{\sqrt{\arctan(ax)}},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(c+a^2cx^2)^{3/2}}{\sqrt{\arctan(ax)}}\,dx = \int \frac{(c+a^2cx^2)^{3/2}}{\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^(3/2)/Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[(c + a^2*c*x^2)^(3/2)/Sqrt[ArcTan[a*x]], x]
```

## Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2cx^2+c\right)^{3/2}}{\sqrt{\arctan(ax)}}\,dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int \frac{\left(a^2cx^2+c\right)^{3/2}}{\sqrt{\arctan(ax)}}dx$$

input
```
Int[(c + a^2*c*x^2)^(3/2)/Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{\left(a^2c\,x^2+c\right)^{\frac{3}{2}}}{\sqrt{\arctan\left(ax\right)}}dx$$

input
```
int((a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

output
```
int((a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{3/2}}{\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 18.37 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{(c+a^2cx^2)^{3/2}}{\sqrt{\arctan(ax)}}\,dx = \int \frac{(c(a^2x^2+1))^{\frac{3}{2}}}{\sqrt{\operatorname{atan}(ax)}}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(3/2)/atan(a*x)**(1/2),x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(3/2)/sqrt(atan(a*x)), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{3/2}}{\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(c+a^2cx^2)^{3/2}}{\sqrt{\arctan(ax)}}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}}{\sqrt{\arctan(ax)}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)/sqrt(arctan(a*x)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(c+a^2cx^2)^{3/2}}{\sqrt{\arctan(ax)}}\,dx = \int \frac{(c\,a^2\,x^2+c)^{3/2}}{\sqrt{\mathrm{atan}\left(a\,x\right)}}\,dx$$

input
```
int((c + a^2*c*x^2)^(3/2)/atan(a*x)^(1/2),x)
```

output
```
int((c + a^2*c*x^2)^(3/2)/atan(a*x)^(1/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.61

$$\int \frac{(c+a^2cx^2)^{3/2}}{\sqrt{\arctan(ax)}}\,dx = \sqrt{c}\,c\left(\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}\,x^2}{atan\,(ax)}dx\right)a^2 + \int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}}{atan\,(ax)}dx\right)$$

input
```
int((a^2*c*x^2+c)^(3/2)/atan(a*x)^(1/2),x)
```

output
```
sqrt(c)*c*(int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**2)/atan(a*x),x)*a**
2 + int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/atan(a*x),x))
```

# 3.950 $\int \frac{(c+a^2cx^2)^{3/2}}{x\sqrt{\arctan(ax)}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\sqrt{\arctan(ax)}}\,dx = \text{Int}\left(\frac{(c+a^2cx^2)^{3/2}}{x\sqrt{\arctan(ax)}},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 4.72 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\sqrt{\arctan(ax)}}\,dx = \int \frac{(c+a^2cx^2)^{3/2}}{x\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^(3/2)/(x*Sqrt[ArcTan[a*x]]),x]
```

output
```
Integrate[(c + a^2*c*x^2)^(3/2)/(x*Sqrt[ArcTan[a*x]]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2cx^2+c\right)^{3/2}}{x\sqrt{\arctan(ax)}}\,dx$$

$$\Big\downarrow \; 5560$$

$$\int \frac{\left(a^2cx^2+c\right)^{3/2}}{x\sqrt{\arctan(ax)}}dx$$

input
```
Int[(c + a^2*c*x^2)^(3/2)/(x*Sqrt[ArcTan[a*x]]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\left(a^2c\,x^2+c\right)^{\frac{3}{2}}}{x\sqrt{\arctan\left(ax\right)}}dx$$

input
```
int((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^(1/2),x)
```

output
```
int((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 17.34 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\sqrt{\arctan(ax)}}\,dx = \int \frac{(c(a^2x^2+1))^{\frac{3}{2}}}{x\sqrt{\operatorname{atan}(ax)}}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(3/2)/x/atan(a*x)**(1/2),x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(3/2)/(x*sqrt(atan(a*x))), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.85 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\sqrt{\arctan(ax)}}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}}{x\sqrt{\arctan(ax)}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)/(x*sqrt(arctan(a*x))), x)
```

## Mupad [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\sqrt{\arctan(ax)}}\,dx = \int \frac{(c\,a^2\,x^2+c)^{3/2}}{x\,\sqrt{\mathrm{atan}\left(a\,x\right)}}\,dx$$

input
```
int((c + a^2*c*x^2)^(3/2)/(x*atan(a*x)^(1/2)),x)
```

output
```
int((c + a^2*c*x^2)^(3/2)/(x*atan(a*x)^(1/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.35

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\sqrt{\arctan(ax)}}\,dx = \sqrt{c}\,c\left(\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}\,x}{atan\,(ax)}dx\right)a^2 + \int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}}{atan\,(ax)\,x}dx\right)$$

input
```
int((a^2*c*x^2+c)^(3/2)/x/atan(a*x)^(1/2),x)
```

output
```
sqrt(c)*c*(int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x)/atan(a*x),x)*a**2 +
 int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(atan(a*x)*x),x))
```

# 3.951 $\int \frac{x^m (c+a^2cx^2)^{5/2}}{\sqrt{\arctan(ax)}} \, dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^m (c+a^2cx^2)^{5/2}}{\sqrt{\arctan(ax)}} \, dx = \text{Int}\left(\frac{x^m (c+a^2cx^2)^{5/2}}{\sqrt{\arctan(ax)}}, x\right)$$

output
```
Defer(Int)(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.31 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^m (c+a^2cx^2)^{5/2}}{\sqrt{\arctan(ax)}} \, dx = \int \frac{x^m (c+a^2cx^2)^{5/2}}{\sqrt{\arctan(ax)}} \, dx$$

input
```
Integrate[(x^m*(c + a^2*c*x^2)^(5/2))/Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[(x^m*(c + a^2*c*x^2)^(5/2))/Sqrt[ArcTan[a*x]], x]
```

## Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m (a^2cx^2+c)^{5/2}}{\sqrt{\arctan(ax)}} \, dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int \frac{x^m (a^2cx^2+c)^{5/2}}{\sqrt{\arctan(ax)}}dx$$

input
```
Int[(x^m*(c + a^2*c*x^2)^(5/2))/Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^m (a^2 c\, x^2+c)^{\frac{5}{2}}}{\sqrt{\arctan\left(a x\right)}}dx$$

input
```
int(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.88

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\sqrt{\arctan(ax)}}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}x^m}{\sqrt{\arctan(ax)}}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*x^m/sqrt(
arctan(a*x)), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\sqrt{\arctan(ax)}}\,dx = \text{Timed out}$$

input
```
integrate(x**m*(a**2*c*x**2+c)**(5/2)/atan(a*x)**(1/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\sqrt{\arctan(ax)}}\,dx = \int \frac{x^m\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}{\sqrt{\mathrm{atan}\left(a\,x\right)}}\,dx$$

input
```
int((x^m*(c + a^2*c*x^2)^(5/2))/atan(a*x)^(1/2),x)
```

output
```
int((x^m*(c + a^2*c*x^2)^(5/2))/atan(a*x)^(1/2), x)
```

## Reduce [N/A]

Not integrable

Time = 140.61 (sec) , antiderivative size = 103, normalized size of antiderivative = 3.96

$$\int \frac{x^m (c + a^2 c x^2)^{5/2}}{\sqrt{\arctan(ax)}}\, dx = \sqrt{c}\, c^2 \left( \left( \int \frac{x^m \sqrt{a^2 x^2 + 1}\, \sqrt{atan\,(ax)}\, x^4}{atan\,(ax)} dx \right) a^4 + 2 \left( \int \frac{x^m \sqrt{a^2 x^2 + 1}\, \sqrt{atan\,(ax)}\, x^2}{atan\,(ax)} dx \right) a^2 + \int \frac{x^m \sqrt{a^2 x^2 + 1}\, \sqrt{atan\,(ax)}}{atan\,(ax)} dx \right)$$

input
```
int(x^m*(a^2*c*x^2+c)^(5/2)/atan(a*x)^(1/2),x)
```

output
```
sqrt(c)*c**2*(int((x**m*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**4)/atan(a*x
),x)*a**4 + 2*int((x**m*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**2)/atan(a*x
),x)*a**2 + int((x**m*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/atan(a*x),x))
```

# 3.952 $\int \frac{x(c+a^2cx^2)^{5/2}}{\sqrt{\arctan(ax)}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\sqrt{\arctan(ax)}}\,dx = \text{Int}\left(\frac{x(c+a^2cx^2)^{5/2}}{\sqrt{\arctan(ax)}},x\right)$$

output
```
Defer(Int)(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 2.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\sqrt{\arctan(ax)}}\,dx = \int \frac{x(c+a^2cx^2)^{5/2}}{\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[(x*(c + a^2*c*x^2)^(5/2))/Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[(x*(c + a^2*c*x^2)^(5/2))/Sqrt[ArcTan[a*x]], x]
```

## Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a^2cx^2+c\right)^{5/2}}{\sqrt{\arctan(ax)}}dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int \frac{x\left(a^2cx^2+c\right)^{5/2}}{\sqrt{\arctan(ax)}}dx$$

input
```
Int[(x*(c + a^2*c*x^2)^(5/2))/Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{x(a^2c\,x^2+c)^{\frac{5}{2}}}{\sqrt{\arctan\left(ax\right)}}dx$$

input
```
int(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

output
```
int(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\sqrt{\arctan(ax)}}\,dx = \text{Timed out}$$

input
```
integrate(x*(a**2*c*x**2+c)**(5/2)/atan(a*x)**(1/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 1.50 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\sqrt{\arctan(ax)}}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}x}{\sqrt{\arctan{(ax)}}}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)*x/sqrt(arctan(a*x)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\sqrt{\arctan(ax)}}\,dx = \int \frac{x\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}{\sqrt{\mathrm{atan}\left(a\,x\right)}}\,dx$$

input
```
int((x*(c + a^2*c*x^2)^(5/2))/atan(a*x)^(1/2),x)
```

output
```
int((x*(c + a^2*c*x^2)^(5/2))/atan(a*x)^(1/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 95, normalized size of antiderivative = 3.96

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\sqrt{\arctan(ax)}}\,dx = \sqrt{c}\,c^2\left(\left(\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}\,x^5}{atan\,(ax)}dx\right)a^4 + 2\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}\,x^3}{atan\,(ax)}dx\right)a^2 + \int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}\,x}{atan\,(ax)}dx\right)\right.$$

input
```
int(x*(a^2*c*x^2+c)^(5/2)/atan(a*x)^(1/2),x)
```

output
```
sqrt(c)*c**2*(int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**5)/atan(a*x),x)*
a**4 + 2*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**3)/atan(a*x),x)*a**2
+ int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x)/atan(a*x),x))
```

# 3.953 $\int \frac{\left(c+a^2cx^2\right)^{5/2}}{\sqrt{\arctan(ax)}} \, dx$



## Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\left(c+a^2cx^2\right)^{5/2}}{\sqrt{\arctan(ax)}} \, dx=\text{Int}\left(\frac{\left(c+a^2cx^2\right)^{5/2}}{\sqrt{\arctan(ax)}},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\left(c+a^2cx^2\right)^{5/2}}{\sqrt{\arctan(ax)}} \, dx=\int \frac{\left(c+a^2cx^2\right)^{5/2}}{\sqrt{\arctan(ax)}} \, dx$$

input
```
Integrate[(c + a^2*c*x^2)^(5/2)/Sqrt[ArcTan[a*x]],x]
```

output
```
Integrate[(c + a^2*c*x^2)^(5/2)/Sqrt[ArcTan[a*x]], x]
```

## Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2cx^2+c\right)^{5/2}}{\sqrt{\arctan(ax)}}\,dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{\left(a^2cx^2+c\right)^{5/2}}{\sqrt{\arctan(ax)}}dx$$

input
```
Int[(c + a^2*c*x^2)^(5/2)/Sqrt[ArcTan[a*x]],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{\left(a^2c\,x^2+c\right)^{\frac{5}{2}}}{\sqrt{\arctan\left(ax\right)}}dx$$

input
```
int((a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

output
```
int((a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{5/2}}{\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^{5/2}}{\sqrt{\arctan(ax)}}\,dx = \text{Timed out}$$

input
```
integrate((a**2*c*x**2+c)**(5/2)/atan(a*x)**(1/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{5/2}}{\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 1.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(c+a^2cx^2)^{5/2}}{\sqrt{\arctan(ax)}}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}}{\sqrt{\arctan(ax)}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)/sqrt(arctan(a*x)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(c+a^2cx^2)^{5/2}}{\sqrt{\arctan(ax)}}\,dx = \int \frac{(c\,a^2\,x^2+c)^{5/2}}{\sqrt{\mathrm{atan}(a\,x)}}\,dx$$

input
```
int((c + a^2*c*x^2)^(5/2)/atan(a*x)^(1/2),x)
```

output
```
int((c + a^2*c*x^2)^(5/2)/atan(a*x)^(1/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 94, normalized size of antiderivative = 4.09

$$\int \frac{(c+a^2cx^2)^{5/2}}{\sqrt{\arctan(ax)}}\,dx = \sqrt{c}\,c^2\left(\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,x^4}{atan(ax)}dx\right)a^4 + 2\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,x^2}{atan(ax)}dx\right)a^2 + \int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}}{atan(ax)}dx\right)$$

input
```
int((a^2*c*x^2+c)^(5/2)/atan(a*x)^(1/2),x)
```

output
```
sqrt(c)*c**2*(int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**4)/atan(a*x),x)*
a**4 + 2*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**2)/atan(a*x),x)*a**2
+ int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/atan(a*x),x))
```

# 3.954 $\int \frac{(c+a^2cx^2)^{5/2}}{x\sqrt{\arctan(ax)}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\sqrt{\arctan(ax)}}\,dx = \text{Int}\left(\frac{(c+a^2cx^2)^{5/2}}{x\sqrt{\arctan(ax)}},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 3.95 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\sqrt{\arctan(ax)}}\,dx = \int \frac{(c+a^2cx^2)^{5/2}}{x\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^(5/2)/(x*Sqrt[ArcTan[a*x]]),x]
```

output
```
Integrate[(c + a^2*c*x^2)^(5/2)/(x*Sqrt[ArcTan[a*x]]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2cx^2+c\right)^{5/2}}{x\sqrt{\arctan(ax)}}\,dx$$

$$\Big\downarrow \; 5560$$

$$\int \frac{\left(a^2cx^2+c\right)^{5/2}}{x\sqrt{\arctan(ax)}}dx$$

input
```
Int[(c + a^2*c*x^2)^(5/2)/(x*Sqrt[ArcTan[a*x]]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\left(a^2c\,x^2+c\right)^{\frac{5}{2}}}{x\sqrt{\arctan\left(ax\right)}}dx$$

input
```
int((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^(1/2),x)
```

output
```
int((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{(c + a^2cx^2)^{5/2}}{x\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(c + a^2cx^2)^{5/2}}{x\sqrt{\arctan(ax)}}\,dx = \text{Timed out}$$

input
```
integrate((a**2*c*x**2+c)**(5/2)/x/atan(a*x)**(1/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(c + a^2cx^2)^{5/2}}{x\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 3.55 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\sqrt{\arctan(ax)}}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}}{x\sqrt{\arctan(ax)}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)/(x*sqrt(arctan(a*x))), x)
```

## Mupad [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\sqrt{\arctan(ax)}}\,dx = \int \frac{(c\,a^2\,x^2+c)^{5/2}}{x\,\sqrt{\mathrm{atan}\left(a\,x\right)}}\,dx$$

input
```
int((c + a^2*c*x^2)^(5/2)/(x*atan(a*x)^(1/2)),x)
```

output
```
int((c + a^2*c*x^2)^(5/2)/(x*atan(a*x)^(1/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 95, normalized size of antiderivative = 3.65

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\sqrt{\arctan(ax)}}\,dx = \sqrt{c}\,c^2\left(\left(\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,x^3}{atan(ax)}dx\right)a^4 + 2\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,x}{atan(ax)}dx\right)a^2 + \int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}}{atan(ax)\,x}dx\right)$$

input
```
int((a^2*c*x^2+c)^(5/2)/x/atan(a*x)^(1/2),x)
```

output
```
sqrt(c)*c**2*(int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**3)/atan(a*x),x)*
a**4 + 2*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x)/atan(a*x),x)*a**2 + i
nt((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(atan(a*x)*x),x))
```

# 3.955 $\int \frac{x^m}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx=\operatorname{Int}\left(\frac{x^m}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}},x\right)$$

output
```
Defer(Int)(x^m/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.05 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx=\int \frac{x^m}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[x^m/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]),x]
```

output
```
Integrate[x^m/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \; 5560$$

$$\int \frac{x^m}{\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}dx$$

input
```
Int[x^m/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^m}{\sqrt{a^2 c\,x^2+c}\,\sqrt{\arctan\left(a x\right)}}dx$$

input
```
int(x^m/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

output
```
int(x^m/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx = \int \frac{x^m}{\sqrt{a^2cx^2+c}\sqrt{\arctan{(ax)}}}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
integral(x^m/(sqrt(a^2*c*x^2 + c)*sqrt(arctan(a*x))), x)
```

## Sympy [N/A]

Not integrable

Time = 25.41 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx = \int \frac{x^m}{\sqrt{c\,(a^2x^2+1)}\sqrt{\operatorname{atan}{(ax)}}}\,dx$$

input
```
integrate(x**m/(a**2*c*x**2+c)**(1/2)/atan(a*x)**(1/2),x)
```

output
```
Integral(x**m/(sqrt(c*(a**2*x**2 + 1))*sqrt(atan(a*x))), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx = \int \frac{x^m}{\sqrt{a^2cx^2+c}\sqrt{\arctan{(ax)}}}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^m/(sqrt(a^2*c*x^2 + c)*sqrt(arctan(a*x))), x)
```

## Mupad [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx = \int \frac{x^m}{\sqrt{\mathrm{atan}\left(a\,x\right)}\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(x^m/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(x^m/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(1/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.73

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx = \frac{\sqrt{c}\left(\int \frac{x^m\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}}{atan(ax)a^2x^2+atan(ax)}dx\right)}{c}$$

input
```
int(x^m/(a^2*c*x^2+c)^(1/2)/atan(a*x)^(1/2),x)
```

output
```
(sqrt(c)*int((x**m*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(atan(a*x)*a**2*x*
*2 + atan(a*x)),x))/c
```

# 3.956 $\int \frac{x}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx = \text{Int}\left(\frac{x}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}},x\right)$$

output
```
Defer(Int)(x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.98 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx = \int \frac{x}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[x/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]),x]
```

output
```
Integrate[x/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x}{\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}dx$$

input
```
Int[x/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{x}{\sqrt{a^2c\,x^2+c}\,\sqrt{\arctan\left(ax\right)}}dx$$

input
```
int(x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

output
```
int(x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 1.75 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx = \int \frac{x}{\sqrt{c\left(a^2x^2+1\right)}\sqrt{\operatorname{atan}\left(ax\right)}}\,dx$$

input
```
integrate(x/(a**2*c*x**2+c)**(1/2)/atan(a*x)**(1/2),x)
```

output
```
Integral(x/(sqrt(c*(a**2*x**2 + 1))*sqrt(atan(a*x))), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx=\int \frac{x}{\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(x/(sqrt(a^2*c*x^2 + c)*sqrt(arctan(a*x))), x)
```

## Mupad [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx=\int \frac{x}{\sqrt{\mathrm{atan}(a\,x)}\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(x/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(x/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(1/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.79

$$\int \frac{x}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx=\frac{\sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,x}{atan(ax)a^2x^2+atan(ax)}dx\right)}{c}$$

input
```
int(x/(a^2*c*x^2+c)^(1/2)/atan(a*x)^(1/2),x)
```

output
```
(sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x)/(atan(a*x)*a**2*x**2
+ atan(a*x)),x))/c
```

# 3.957 $\int \frac{1}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{1}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx = \text{Int}\left(\frac{1}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}},x\right)$$

output
```
Defer(Int)(1/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx = \int \frac{1}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[1/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]),x]
```

output
```
Integrate[1/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{1}{\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}dx$$

input
```
Int[1/(Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{1}{\sqrt{a^2c\,x^2+c}\,\sqrt{\arctan\left(ax\right)}}dx$$

input
```
int(1/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

output
```
int(1/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 1.72 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{1}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx = \int \frac{1}{\sqrt{c\,(a^2x^2+1)}\sqrt{\operatorname{atan}\,(ax)}}\,dx$$

input
```
integrate(1/(a**2*c*x**2+c)**(1/2)/atan(a*x)**(1/2),x)
```

output
```
Integral(1/(sqrt(c*(a**2*x**2 + 1))*sqrt(atan(a*x))), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx = \int \frac{1}{\sqrt{a^2cx^2+c}\sqrt{\arctan{(ax)}}}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(a^2*c*x^2 + c)*sqrt(arctan(a*x))), x)
```

## Mupad [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx = \int \frac{1}{\sqrt{\mathrm{atan}\left(a\,x\right)}\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(1/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(1/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(1/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.83

$$\int \frac{1}{\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}}{atan(ax)a^2x^2+atan(ax)}dx\right)}{c}$$

input
```
int(1/(a^2*c*x^2+c)^(1/2)/atan(a*x)^(1/2),x)
```

output
```
(sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(atan(a*x)*a**2*x**2 +
atan(a*x)),x))/c
```

# 3.958 $\int \frac{1}{x\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx = \text{Int}\left(\frac{1}{x\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}},x\right)$$

output
```
Defer(Int)(1/x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx = \int \frac{1}{x\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[1/(x*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]),x]
```

output
```
Integrate[1/(x*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{1}{x\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}dx$$

input
```
Int[1/(x*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{1}{x\sqrt{a^2c\,x^2+c}\,\sqrt{\arctan\left(ax\right)}}dx$$

input
```
int(1/x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

output
```
int(1/x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 3.89 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx = \int \frac{1}{x\sqrt{c\,(a^2x^2+1)}\sqrt{\operatorname{atan}(ax)}}\,dx$$

input
```
integrate(1/x/(a**2*c*x**2+c)**(1/2)/atan(a*x)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(c*(a**2*x**2 + 1))*sqrt(atan(a*x))), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx = \int \frac{1}{\sqrt{a^2cx^2+c}x\sqrt{\arctan{(ax)}}}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(a^2*c*x^2 + c)*x*sqrt(arctan(a*x))), x)
```

## Mupad [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx = \int \frac{1}{x\,\sqrt{\mathrm{atan}\left(a\,x\right)}\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(1/(x*atan(a*x)^(1/2)*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(1/(x*atan(a*x)^(1/2)*(c + a^2*c*x^2)^(1/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.69

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}}{atan(ax)a^2x^3+atan(ax)x}dx\right)}{c}$$

input
```
int(1/x/(a^2*c*x^2+c)^(1/2)/atan(a*x)^(1/2),x)
```

output
```
(sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(atan(a*x)*a**2*x**3 +
atan(a*x)*x),x))/c
```

# 3.959 $\int \frac{x^m}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}\,dx = \text{Int}\left(\frac{x^m}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}},x\right)$$

output
```
Defer(Int)(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.30 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}\,dx = \int \frac{x^m}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[x^m/((c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]),x]
```

output
```
Integrate[x^m/((c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int \frac{x^m}{\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}dx$$

input
```
Int[x^m/((c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^m}{(a^2c\,x^2+c)^{\frac{3}{2}}\sqrt{\arctan\left(ax\right)}}dx$$

input
```
int(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

output
```
int(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.96

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^{3/2}\sqrt{\arctan(ax)}}\,dx = \int \frac{x^m}{\left(a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{\arctan\left(ax\right)}}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^m/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt
(arctan(a*x))), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^{3/2}\sqrt{\arctan(ax)}}\,dx = \text{Timed out}$$

input
```
integrate(x**m/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(1/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^{3/2}\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}\,dx = \int \frac{x^m}{(a^2cx^2+c)^{\frac{3}{2}}\sqrt{\arctan(ax)}}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^m/((a^2*c*x^2 + c)^(3/2)*sqrt(arctan(a*x))), x)
```

## Mupad [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}\,dx = \int \frac{x^m}{\sqrt{\mathrm{atan}(a\,x)}\,(c\,a^2\,x^2+c)^{3/2}}\,dx$$

input
```
int(x^m/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(x^m/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(3/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.19

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}\,dx = \frac{\sqrt{c}\left(\int \frac{x^m\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}}{atan(ax)a^4x^4+2atan(ax)a^2x^2+atan(ax)}dx\right)}{c^2}$$

input
```
int(x^m/(a^2*c*x^2+c)^(3/2)/atan(a*x)^(1/2),x)
```

output
```
(sqrt(c)*int((x**m*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(atan(a*x)*a**4*x*
*4 + 2*atan(a*x)*a**2*x**2 + atan(a*x)),x))/c**2
```

# 3.960 $\int \frac{x^2}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}\,dx = \text{Int}\left(\frac{x^2}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}},x\right)$$

output
```
Defer(Int)(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 10.82 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}\,dx = \int \frac{x^2}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[x^2/((c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]),x]
```

output
```
Integrate[x^2/((c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}\,dx$$

$$\Big\downarrow \ 5560$$

$$\int \frac{x^2}{\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}dx$$

input
```
Int[x^2/((c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^2}{\left(a^2c\,x^2+c\right)^{\frac{3}{2}}\sqrt{\arctan\left(ax\right)}}dx$$

input
```
int(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

output
```
int(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 7.77 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}\,dx = \int \frac{x^2}{(c\,(a^2x^2+1))^{\frac{3}{2}}\sqrt{\operatorname{atan}(ax)}}\,dx$$

input
```
integrate(x**2/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(1/2),x)
```

output
```
Integral(x**2/((c*(a**2*x**2 + 1))**(3/2)*sqrt(atan(a*x))), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}\,dx=\int \frac{x^2}{(a^2cx^2+c)^{\frac{3}{2}}\sqrt{\arctan(ax)}}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/((a^2*c*x^2 + c)^(3/2)*sqrt(arctan(a*x))), x)
```

## Mupad [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}\,dx=\int \frac{x^2}{\sqrt{\mathrm{atan}(a\,x)}\,(c\,a^2\,x^2+c)^{3/2}}\,dx$$

input
```
int(x^2/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(x^2/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(3/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.19

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,x^2}{atan(ax)a^4x^4+2atan(ax)a^2x^2+atan(ax)}dx\right)}{c^2}$$

input
```
int(x^2/(a^2*c*x^2+c)^(3/2)/atan(a*x)^(1/2),x)
```

output
```
(sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**2)/(atan(a*x)*a**4*x*
*4 + 2*atan(a*x)*a**2*x**2 + atan(a*x)),x))/c**2
```

# 3.961 $\int \frac{x}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 60

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}\,dx = \frac{\sqrt{2\pi}\sqrt{1+a^2x^2}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{a^2c\sqrt{c+a^2cx^2}}$$

output
```
2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)*arctan(a*x)^(
1/2))/a^2/c/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.62

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}\,dx = -\frac{\sqrt{1+a^2x^2}\left(\sqrt{-i\arctan(ax)}\,\Gamma\left(\frac{1}{2},-i\arctan(ax)\right)+\sqrt{i\arctan(ax)}\,\Gamma\left(\frac{1}{2},i\arctan(ax)\right)\right)}{2a^2c\sqrt{c(1+a^2x^2)}\sqrt{\arctan(ax)}}$$

input
```
Integrate[x/((c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]),x]
```

output
```
-1/2*(Sqrt[1 + a^2*x^2]*(Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-I)*ArcTan[a*x
]] + Sqrt[I*ArcTan[a*x]]*Gamma[1/2, I*ArcTan[a*x]]))/(a^2*c*Sqrt[c*(1 + a^
2*x^2)]*Sqrt[ArcTan[a*x]])
```

## Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {5506, 5505, 3042, 3786, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \text{ 5506}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{x}{(a^2x^2+1)^{3/2}\sqrt{\arctan(ax)}}dx}{c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 5505}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{ax}{\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{\sin(\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 3786}$$

$$\frac{2\sqrt{a^2x^2+1}\int \frac{ax}{\sqrt{a^2x^2+1}}d\sqrt{\arctan(ax)}}{a^2c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 3832}$$

$$\frac{\sqrt{2\pi}\sqrt{a^2x^2+1}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{a^2c\sqrt{a^2cx^2+c}}$$

input
```
Int[x/((c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]),x]
```

output
```
(Sqrt[2*Pi]*Sqrt[1 + a^2*x^2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(a^2
*c*Sqrt[c + a^2*c*x^2])
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

## Maple [F]

$$\int \frac{x}{(a^2c\,x^2+c)^{\frac{3}{2}}\sqrt{\arctan(ax)}}dx$$

input
```
int(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

output
```
int(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}\,dx = \int \frac{x}{(c\,(a^2x^2+1))^{\frac{3}{2}}\sqrt{\operatorname{atan}(ax)}}\,dx$$

input
```
integrate(x/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(1/2),x)
```

output
```
Integral(x/((c*(a**2*x**2 + 1))**(3/2)*sqrt(atan(a*x))), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}\,dx = \int \frac{x}{\sqrt{\mathrm{atan}\left(a\,x\right)}\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

input
```
int(x/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(x/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(3/2)), x)
```

## Reduce [F]

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,x}{atan(ax)a^4x^4+2atan(ax)a^2x^2+atan(ax)}dx\right)}{c^2}$$

input
```
int(x/(a^2*c*x^2+c)^(3/2)/atan(a*x)^(1/2),x)
```

output
```
(sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x)/(atan(a*x)*a**4*x***4
+ 2*atan(a*x)*a**2*x**2 + atan(a*x)),x))/c**2
```

# 3.962 $\int \frac{1}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 60

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}\,dx=\frac{\sqrt{2\pi}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{ac\sqrt{c+a^2cx^2}}$$

output
```
2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*arctan(a*x)^(
1/2))/a/c/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.02

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}\,dx=\frac{\sqrt{2\pi}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{ac\sqrt{c(1+a^2x^2)}}$$

input
```
Integrate[1/((c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]),x]
```

output
```
(Sqrt[2*Pi]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(a*c
*Sqrt[c*(1 + a^2*x^2)])
```

## Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {5440, 5439, 3042, 3785, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{5440}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{1}{(a^2x^2+1)^{3/2}\sqrt{\arctan(ax)}}dx}{c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5439}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)}{\sqrt{\arctan(ax)}}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3785}$$

$$\frac{2\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}}d\sqrt{\arctan(ax)}}{ac\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3833}$$

$$\frac{\sqrt{2\pi}\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{ac\sqrt{a^2cx^2+c}}$$

input
```
Int[1/((c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]),x]
```

output
```
(Sqrt[2*Pi]*Sqrt[1 + a^2*x^2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(a*c
*Sqrt[c + a^2*c*x^2])
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5440
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])   Int[(1 +
c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

## Maple [F]

$$\int \frac{1}{\left(a^{2} c\,x^{2}+c\right)^{\frac{3}{2}} \sqrt{\arctan\left(a x\right)}}dx$$

input
```
int(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

output
```
int(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\sqrt{\arctan(ax)}}\,dx = \int \frac{1}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\sqrt{\operatorname{atan}(ax)}}\,dx$$

input
```
integrate(1/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(1/2),x)
```

output
```
Integral(1/((c*(a**2*x**2 + 1))**(3/2)*sqrt(atan(a*x))), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+a^2cx^2\right)^{3/2}\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.82

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}\,dx = -\frac{(i+1)\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{\arctan(ax)}\right)-(i-1)\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{\arctan(ax)}\right)}{4\,ac^{\frac{3}{2}}}$$

input
```
integrate(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
-1/4*((I + 1)*sqrt(2)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(arctan(a*x))
) - (I - 1)*sqrt(2)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(2)*sqrt(arctan(a*x)))
)/(a*c^(3/2))
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}\,dx = \int \frac{1}{\sqrt{\mathrm{atan}\,(a\,x)}\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

input
```
int(1/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(1/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(3/2)), x)
```

## Reduce [F]

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}}{atan(ax)a^4x^4+2atan(ax)a^2x^2+atan(ax)}dx\right)}{c^2}$$

input
```
int(1/(a^2*c*x^2+c)^(3/2)/atan(a*x)^(1/2),x)
```

output
```
(sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(atan(a*x)*a**4*x**4 +
2*atan(a*x)*a**2*x**2 + atan(a*x)),x))/c**2
```

# 3.963 $\int \frac{1}{x(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{1}{x(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}\,dx = \text{Int}\left(\frac{1}{x(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}},x\right)$$

output
```
Defer(Int)(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.66 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{1}{x(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}\,dx = \int \frac{1}{x(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[1/(x*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]),x]
```

output
```
Integrate[1/(x*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}\,dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{1}{x\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}dx$$

input
```
Int[1/(x*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{1}{x\,(a^2c\,x^2+c)^{\frac{3}{2}}\,\sqrt{\arctan\,(ax)}}dx$$

input
```
int(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

output
```
int(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 20.46 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\sqrt{\arctan(ax)}}\,dx = \int \frac{1}{x\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\sqrt{\operatorname{atan}(ax)}}\,dx$$

input
```
integrate(1/x/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(1/2),x)
```

output
```
Integral(1/(x*(c*(a**2*x**2 + 1))**(3/2)*sqrt(atan(a*x))), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\sqrt{\arctan(ax)}}\,dx = \int \frac{1}{x\,\sqrt{\mathrm{atan}\left(a\,x\right)}\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

input
```
int(1/(x*atan(a*x)^(1/2)*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(1/(x*atan(a*x)^(1/2)*(c + a^2*c*x^2)^(3/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.15

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\sqrt{\arctan(ax)}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}}{atan(ax)a^4x^5+2atan(ax)a^2x^3+atan(ax)x}dx\right)}{c^2}$$

input
```
int(1/x/(a^2*c*x^2+c)^(3/2)/atan(a*x)^(1/2),x)
```

output
```
(sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(atan(a*x)*a**4*x**5 +
2*atan(a*x)*a**2*x**3 + atan(a*x)*x),x))/c**2
```

# 3.964 $\int \frac{x^m}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx = \text{Int}\left(\frac{x^m}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}},x\right)$$

output
```
Defer(Int)(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.83 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx = \int \frac{x^m}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[x^m/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]),x]
```

output
```
Integrate[x^m/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{5/2}}\,dx$$

$$\Big\downarrow \text{5560}$$

$$\int \frac{x^m}{\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{5/2}}dx$$

input
```
Int[x^m/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^m}{\left(a^2c\,x^2+c\right)^{\frac{5}{2}}\sqrt{\arctan\left(ax\right)}}dx$$

input
```
int(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

output
```
int(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.38

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx = \int \frac{x^m}{(a^2cx^2+c)^{\frac{5}{2}}\sqrt{\arctan{(ax)}}}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^m/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3
*x^2 + c^3)*sqrt(arctan(a*x))), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx = \text{Timed out}$$

input
```
integrate(x**m/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(1/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx=\int \frac{x^m}{(a^2cx^2+c)^{\frac{5}{2}}\sqrt{\arctan{(ax)}}}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^m/((a^2*c*x^2 + c)^(5/2)*sqrt(arctan(a*x))), x)
```

## Mupad [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx=\int \frac{x^m}{\sqrt{\mathrm{atan}\left(a\,x\right)}\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

input
```
int(x^m/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(x^m/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(5/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.65

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx=\frac{\sqrt{c}\left(\int \frac{x^m\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}}{atan(ax)a^6x^6+3atan(ax)a^4x^4+3atan(ax)a^2x^2+atan(ax)}dx\right)}{c^3}$$

input
```
int(x^m/(a^2*c*x^2+c)^(5/2)/atan(a*x)^(1/2),x)
```

output
```
(sqrt(c)*int((x**m*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(atan(a*x)*a**6*x*
*6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x))/c**3
```

# 3.965 $\int \frac{x^4}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^4}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx=\text{Int}\left(\frac{x^4}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}},x\right)$$

output
```
Defer(Int)(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 4.60 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^4}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx=\int \frac{x^4}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[x^4/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]),x]
```

output
```
Integrate[x^4/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{5/2}}\,dx$$

$$\Big\downarrow \text{5560}$$

$$\int \frac{x^4}{\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{5/2}}dx$$

input
```
Int[x^4/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^4}{(a^2c\,x^2+c)^{\frac{5}{2}}\sqrt{\arctan\left(ax\right)}}dx$$

input
```
int(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

output
```
int(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x^4}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 51.60 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx = \int \frac{x^4}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}\sqrt{\operatorname{atan}(ax)}}\,dx$$

input
```
integrate(x**4/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(1/2),x)
```

output
```
Integral(x**4/((c*(a**2*x**2 + 1))**(5/2)*sqrt(atan(a*x))), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-1)]

Timed out.

$$\int \frac{x^4}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx = \text{Timed out}$$

input
```
integrate(x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

## Mupad [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^4}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx = \int \frac{x^4}{\sqrt{\mathrm{atan}\left(a\,x\right)}\,{\left(c\,a^2\,x^2+c\right)}^{5/2}} \,d x$$

input
```
int(x^4/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(x^4/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(5/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.65

$$\int \frac{x^4}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,x^4}{atan(ax)a^6x^6+3atan(ax)a^4x^4+3atan(ax)a^2x^2+atan(ax)}dx\right)}{c^3}$$

input
```
int(x^4/(a^2*c*x^2+c)^(5/2)/atan(a*x)^(1/2),x)
```

output
```
(sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**4)/(atan(a*x)*a**6*x*
*6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x))/c**3
```

# 3.966 $\int \frac{x^3}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 131

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx = \frac{3\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{2a^4c^2\sqrt{c+a^2cx^2}} - \frac{\sqrt{\frac{\pi}{6}}\sqrt{1+a^2x^2}\,\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)}{2a^4c^2\sqrt{c+a^2cx^2}}$$

output
```
3/4*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)*arctan(a*
x)^(1/2))/a^4/c^2/(a^2*c*x^2+c)^(1/2)-1/12*6^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1
/2)*FresnelS(6^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))/a^4/c^2/(a^2*c*x^2+c)^(1/
2)
```

## Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.73

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx = \frac{\sqrt{\frac{\pi}{6}}(1+a^2x^2)^{3/2}\left(3\sqrt{3}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right) - \text{FresnelS}\left(\sqrt{\right.\right.}{2a^4c\,(c\,(1+a^2x^2))^{3/2}}$$

input
```
Integrate[x^3/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]),x]
```

output
```
(Sqrt[Pi/6]*(1 + a^2*x^2)^(3/2)*(3*Sqrt[3]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan
[a*x]]] - FresnelS[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]]))/(2*a^4*c*(c*(1 + a^2*x^
2))^(3/2))
```

## Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.76, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {5506, 5505, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{5506}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{x^3}{(a^2x^2+1)^{5/2}\sqrt{\arctan(ax)}}dx}{c^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5505}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{a^3x^3}{(a^2x^2+1)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{a^4c^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{\sin(\arctan(ax))^3}{\sqrt{\arctan(ax)}}d\arctan(ax)}{a^4c^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3793}$$

$$\frac{\sqrt{a^2x^2+1}\int \left(\frac{3ax}{4\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}-\frac{\sin(3\arctan(ax))}{4\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^4c^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2x^2+1}\left(\frac{3}{2}\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{2}\sqrt{\frac{\pi}{6}}\,\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{a^4c^2\sqrt{a^2cx^2+c}}$$

input
```
Int[x^3/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]),x]
```

output
```
(Sqrt[1 + a^2*x^2]*((3*Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/
2 - (Sqrt[Pi/6]*FresnelS[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/2))/(a^4*c^2*Sqrt[
c + a^2*c*x^2])
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

## Maple [F]

$$\int \frac{x^3}{\left(a^2c\,x^2+c\right)^{\frac{5}{2}}\sqrt{\arctan\left(ax\right)}}dx$$

input
```
int(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

output
```
int(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx = \int \frac{x^3}{\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}\sqrt{\operatorname{atan}\left(ax\right)}}\,dx$$

input
```
integrate(x**3/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(1/2),x)
```

output
```
Integral(x**3/((c*(a**2*x**2 + 1))**(5/2)*sqrt(atan(a*x))), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx = \int \frac{x^3}{\sqrt{\mathrm{atan}\left(a\,x\right)}\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

input
```
int(x^3/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(x^3/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(5/2)), x)
```

## Reduce [F]

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,x^3}{atan(ax)a^6x^6+3atan(ax)a^4x^4+3atan(ax)a^2x^2+atan(ax)}dx\right)}{c^3}$$

input
```
int(x^3/(a^2*c*x^2+c)^(5/2)/atan(a*x)^(1/2),x)
```

output
```
(sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**3)/(atan(a*x)*a**6*x*
*6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x))/c**3
```

# 3.967 $\int \frac{x^2}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 131

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx = \frac{\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{2a^3c^2\sqrt{c+a^2cx^2}} - \frac{\sqrt{\frac{\pi}{6}}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)}{2a^3c^2\sqrt{c+a^2cx^2}}$$

output
```
1/4*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*arctan(a*
x)^(1/2))/a^3/c^2/(a^2*c*x^2+c)^(1/2)-1/12*6^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1
/2)*FresnelC(6^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))/a^3/c^2/(a^2*c*x^2+c)^(1/
2)
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.21

$$\int \frac{x^2}{\left(c+a^2cx^2\right)^{5/2}\sqrt{\arctan(ax)}}\,dx =$$
$$-\frac{i\sqrt{1+a^2x^2}\left(3\sqrt{-i\arctan(ax)}\Gamma\left(\frac{1}{2},-i\arctan(ax)\right)-3\sqrt{i\arctan(ax)}\Gamma\left(\frac{1}{2},i\arctan(ax)\right)+\sqrt{3}\left(-\sqrt{-i\,\text{a}\right.\right.}{24a^3c^2\sqrt{c\left(1+a^2x^2\right)}\sqrt{\arctan(ax}}$$

input
```
Integrate[x^2/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]),x]
```

output
```
((-1/24*I)*Sqrt[1 + a^2*x^2]*(3*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-I)*Arc
Tan[a*x]] - 3*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, I*ArcTan[a*x]] + Sqrt[3]*(-(S
qrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-3*I)*ArcTan[a*x]]) + Sqrt[I*ArcTan[a*x]
]*Gamma[1/2, (3*I)*ArcTan[a*x]])))/(a^3*c^2*Sqrt[c*(1 + a^2*x^2)]*Sqrt[Arc
Tan[a*x]])
```

## Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.76, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5506, 5505, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{5/2}}dx$$
$$\Big\downarrow \text{ 5506}$$
$$\frac{\sqrt{a^2x^2+1}\int \frac{x^2}{\left(a^2x^2+1\right)^{5/2}\sqrt{\arctan(ax)}}dx}{c^2\sqrt{a^2cx^2+c}}$$
$$\Big\downarrow \text{ 5505}$$
$$\frac{\sqrt{a^2x^2+1}\int \frac{a^2x^2}{\left(a^2x^2+1\right)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3c^2\sqrt{a^2cx^2+c}}$$
$$\Big\downarrow \text{ 4906}$$

$$\frac{\sqrt{a^2x^2+1}\int\left(\frac{1}{4\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}-\frac{\cos(3\arctan(ax))}{4\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^3c^2\sqrt{a^2cx^2+c}}$$
$$\Big\downarrow \text{2009}$$
$$\frac{\sqrt{a^2x^2+1}\left(\frac{1}{2}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{2}\sqrt{\frac{\pi}{6}}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{a^3c^2\sqrt{a^2cx^2+c}}$$

input
```
Int[x^2/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]),x]
```

output
```
(Sqrt[1 + a^2*x^2]*((Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/2
- (Sqrt[Pi/6]*FresnelC[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/2))/(a^3*c^2*Sqrt[c
+ a^2*c*x^2])
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

## Maple [F]

$$\int \frac{x^2}{(a^2c\,x^2+c)^{\frac{5}{2}}\sqrt{\arctan(ax)}}dx$$

input
```
int(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

output
```
int(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx = \int \frac{x^2}{(c\,(a^2x^2+1))^{\frac{5}{2}}\sqrt{\operatorname{atan}(ax)}}\,dx$$

input
```
integrate(x**2/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(1/2),x)
```

output
```
Integral(x**2/((c*(a**2*x**2 + 1))**(5/2)*sqrt(atan(a*x))), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F]

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx = \int \frac{x^2}{(a^2cx^2+c)^{\frac{5}{2}}\sqrt{\arctan(ax)}}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/((a^2*c*x^2 + c)^(5/2)*sqrt(arctan(a*x))), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx = \int \frac{x^2}{\sqrt{\mathrm{atan}(a\,x)}\,(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int(x^2/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(x^2/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(5/2)), x)
```

## Reduce [F]

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,x^2}{atan(ax)a^6x^6+3atan(ax)a^4x^4+3atan(ax)a^2x^2+atan(ax)}dx\right)}{c^3}$$

input
```
int(x^2/(a^2*c*x^2+c)^(5/2)/atan(a*x)^(1/2),x)
```

output
```
(sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**2)/(atan(a*x)*a**6*x*
*6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x))/c**3
```

# 3.968 $\int \frac{x}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 131

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx = \frac{\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{2a^2c^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{\frac{\pi}{6}}\sqrt{1+a^2x^2}\,\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)}{2a^2c^2\sqrt{c+a^2cx^2}}$$

output
```
1/4*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)*arctan(a*
x)^(1/2))/a^2/c^2/(a^2*c*x^2+c)^(1/2)+1/12*6^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1
/2)*FresnelS(6^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))/a^2/c^2/(a^2*c*x^2+c)^(1/
2)
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.19

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx =$$
$$-\frac{(1+a^2x^2)^{3/2}\left(3\sqrt{-i\arctan(ax)}\Gamma\left(\frac{1}{2},-i\arctan(ax)\right)+3\sqrt{i\arctan(ax)}\Gamma\left(\frac{1}{2},i\arctan(ax)\right)+\sqrt{3}\left(\sqrt{-i\,a\right.\right.}{24a^2c\,(c\,(1+a^2x^2))^{3/2}\sqrt{\arctan(a}$$

input
```
Integrate[x/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]),x]
```

output
```
-1/24*((1 + a^2*x^2)^(3/2)*(3*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-I)*ArcTa
n[a*x]] + 3*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, I*ArcTan[a*x]] + Sqrt[3]*(Sqrt[
(-I)*ArcTan[a*x]]*Gamma[1/2, (-3*I)*ArcTan[a*x]] + Sqrt[I*ArcTan[a*x]]*Gam
ma[1/2, (3*I)*ArcTan[a*x]])))/(a^2*c*(c*(1 + a^2*x^2))^(3/2)*Sqrt[ArcTan[a
*x]])
```

## Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.76, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5506, 5505, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{5/2}}\,dx$$
$$\Big\downarrow \text{ 5506}$$
$$\frac{\sqrt{a^2x^2+1}\int \frac{x}{(a^2x^2+1)^{5/2}\sqrt{\arctan(ax)}}dx}{c^2\sqrt{a^2cx^2+c}}$$
$$\Big\downarrow \text{ 5505}$$
$$\frac{\sqrt{a^2x^2+1}\int \frac{ax}{(a^2x^2+1)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}$$
$$\Big\downarrow \text{ 4906}$$

$$\frac{\sqrt{a^2x^2+1}\int\left(\frac{ax}{4\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}+\frac{\sin(3\arctan(ax))}{4\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{a^2x^2+1}\left(\frac{1}{2}\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{6}}\,\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{a^2c^2\sqrt{a^2cx^2+c}}$$

input
```
Int[x/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]),x]
```

output
```
(Sqrt[1 + a^2*x^2]*((Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/2
+ (Sqrt[Pi/6]*FresnelS[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/2))/(a^2*c^2*Sqrt[c
+ a^2*c*x^2])
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

## Maple [F]

$$\int \frac{x}{\left(a^{2} c\,x^{2}+c \right)^{\frac{5}{2}} \sqrt{\arctan \left(a x \right)}}d x$$

input
```
int(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

output
```
int(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx = \int \frac{x}{\left(c\left(a^{2} x^{2} + 1\right)\right)^{\frac{5}{2}} \sqrt{\operatorname{atan}{\left(a x \right)}}}\, dx$$

input
```
integrate(x/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(1/2),x)
```

output
```
Integral(x/((c*(a**2*x**2 + 1))**(5/2)*sqrt(atan(a*x))), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx = \int \frac{x}{\sqrt{\mathrm{atan}\left(a\,x\right)}\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

input
```
int(x/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(x/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(5/2)), x)
```

## Reduce [F]

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,x}{atan(ax)a^6x^6+3atan(ax)a^4x^4+3atan(ax)a^2x^2+atan(ax)}dx\right)}{c^3}$$

input
```
int(x/(a^2*c*x^2+c)^(5/2)/atan(a*x)^(1/2),x)
```

output
```
(sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x)/(atan(a*x)*a**6*x**6
+ 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x))/c**3
```

# 3.969 $\int \frac{1}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 131

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx = \frac{3\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{2ac^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{\frac{\pi}{6}}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)}{2ac^2\sqrt{c+a^2cx^2}}$$

output
```
3/4*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*arctan(a*
x)^(1/2))/a/c^2/(a^2*c*x^2+c)^(1/2)+1/12*6^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2
)*FresnelC(6^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))/a/c^2/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.71

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx = \frac{\sqrt{\frac{\pi}{6}}(1+a^2x^2)^{3/2}\left(3\sqrt{3}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\text{FresnelC}\left(\sqrt{\right.\right.}{2ac\,(c\,(1+a^2x^2))^{3/2}}$$

input
```
Integrate[1/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]),x]
```

output
```
(Sqrt[Pi/6]*(1 + a^2*x^2)^(3/2)*(3*Sqrt[3]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan
[a*x]]] + FresnelC[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]]))/(2*a*c*(c*(1 + a^2*x^2)
)^(3/2))
```

## Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.76, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {5440, 5439, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{5/2}}\,dx$$

$$\Big\downarrow \ 5440$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{1}{(a^2x^2+1)^{5/2}\sqrt{\arctan(ax)}}dx}{c^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \ 5439$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{1}{(a^2x^2+1)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^3}{\sqrt{\arctan(ax)}}d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \ 3793$$

$$\frac{\sqrt{a^2x^2+1}\int \left(\frac{\cos(3\arctan(ax))}{4\sqrt{\arctan(ax)}}+\frac{3}{4\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \ 2009$$

$$\frac{\sqrt{a^2x^2+1}\left(\frac{3}{2}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{6}}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{ac^2\sqrt{a^2cx^2+c}}$$

input
```
Int[1/((c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]),x]
```

output
```
(Sqrt[1 + a^2*x^2]*((3*Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/
2 + (Sqrt[Pi/6]*FresnelC[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/2))/(a*c^2*Sqrt[c
+ a^2*c*x^2])
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5440
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])   Int[(1 +
c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

## Maple [F]

$$\int \frac{1}{\left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}} \sqrt{\arctan \left(a x \right)}}d x$$

input
```
int(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

output
```
int(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx = \int \frac{1}{\left(c \left(a^{2} x^{2} + 1\right)\right)^{\frac{5}{2}} \sqrt{\operatorname{atan}{\left(a x \right)}}}\, dx$$

input
```
integrate(1/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(1/2),x)
```

output
```
Integral(1/((c*(a**2*x**2 + 1))**(5/2)*sqrt(atan(a*x))), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F]

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx = \int \frac{1}{(a^2cx^2+c)^{\frac{5}{2}}\sqrt{\arctan(ax)}}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(5/2)*sqrt(arctan(a*x))), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx = \int \frac{1}{\sqrt{\mathrm{atan}(a\,x)}\,{(c\,a^2\,x^2+c)}^{5/2}}\,dx$$

input
```
int(1/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(1/(atan(a*x)^(1/2)*(c + a^2*c*x^2)^(5/2)), x)
```

## Reduce [F]

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}}{atan(ax)a^6x^6+3atan(ax)a^4x^4+3atan(ax)a^2x^2+atan(ax)}dx\right)}{c^3}$$

input
```
int(1/(a^2*c*x^2+c)^(5/2)/atan(a*x)^(1/2),x)
```

output
```
(sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(atan(a*x)*a**6*x**6 +
3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x))/c**3
```

# 3.970 $\int \frac{1}{x(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{1}{x\,(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx=\text{Int}\left(\frac{1}{x\,(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}},x\right)$$

output
```
Defer(Int)(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 2.33 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{1}{x\,(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx=\int \frac{1}{x\,(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}}\,dx$$

input
```
Integrate[1/(x*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]),x]
```

output
```
Integrate[1/(x*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int \frac{1}{x\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{5/2}}dx$$

input
```
Int[1/(x*(c + a^2*c*x^2)^(5/2)*Sqrt[ArcTan[a*x]]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{1}{x\left(a^2c\,x^2+c\right)^{\frac{5}{2}}\sqrt{\arctan\left(ax\right)}}dx$$

input
```
int(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

output
```
int(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 110.71 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\sqrt{\arctan(ax)}}\,dx = \int \frac{1}{x\left(c\left(a^2x^2+1\right)\right)^{\frac{5}{2}}\sqrt{\operatorname{atan}(ax)}}\,dx$$

input
```
integrate(1/x/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(1/2),x)
```

output
```
Integral(1/(x*(c*(a**2*x**2 + 1))**(5/2)*sqrt(atan(a*x))), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\sqrt{\arctan(ax)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\sqrt{\arctan(ax)}}\,dx = \int \frac{1}{x\,\sqrt{\mathrm{atan}\left(a\,x\right)}\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

input
```
int(1/(x*atan(a*x)^(1/2)*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(1/(x*atan(a*x)^(1/2)*(c + a^2*c*x^2)^(5/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.62

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\sqrt{\arctan(ax)}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}}{atan(ax)a^6x^7+3atan(ax)a^4x^5+3atan(ax)a^2x^3+atan(ax)x}dx\right)}{c^3}$$

input
```
int(1/x/(a^2*c*x^2+c)^(5/2)/atan(a*x)^(1/2),x)
```

output
```
(sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(atan(a*x)*a**6*x**7 +
3*atan(a*x)*a**4*x**5 + 3*atan(a*x)*a**2*x**3 + atan(a*x)*x),x))/c**3
```

# 3.971 $\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)^{3/2}}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)^{3/2}}\,dx = \text{Int}\left(\frac{x^m(c+a^2cx^2)}{\arctan(ax)^{3/2}},x\right)$$

output
```
Defer(Int)(x^m*(a^2*c*x^2+c)/arctan(a*x)^(3/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.86 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)^{3/2}}\,dx = \int \frac{x^m(c+a^2cx^2)}{\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[(x^m*(c + a^2*c*x^2))/ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[(x^m*(c + a^2*c*x^2))/ArcTan[a*x]^(3/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m (a^2cx^2+c)}{\arctan(ax)^{3/2}} \, dx$$

$$\Big\downarrow \; 5560$$

$$\int \frac{x^m (a^2cx^2+c)}{\arctan(ax)^{3/2}} dx$$

input
```
Int[(x^m*(c + a^2*c*x^2))/ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x^m (a^2c\,x^2+c)}{\arctan\left(ax\right)^{\frac{3}{2}}}dx$$

input
```
int(x^m*(a^2*c*x^2+c)/arctan(a*x)^(3/2),x)
```

output
```
int(x^m*(a^2*c*x^2+c)/arctan(a*x)^(3/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)^{3/2}}\,dx = \int \frac{(a^2cx^2+c)x^m}{\arctan{(ax)}^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)*x^m/arctan(a*x)^(3/2), x)
```

## Sympy [N/A]

Not integrable

Time = 30.33 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.55

$$\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)^{3/2}}\,dx = c\left(\int \frac{x^m}{\operatorname{atan}^{\frac{3}{2}}{(ax)}}\,dx + \int \frac{a^2x^2x^m}{\operatorname{atan}^{\frac{3}{2}}{(ax)}}\,dx\right)$$

input
```
integrate(x**m*(a**2*c*x**2+c)/atan(a*x)**(3/2),x)
```

output
```
c*(Integral(x**m/atan(a*x)**(3/2), x) + Integral(a**2*x**2*x**m/atan(a*x)*
*(3/2), x))
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)^{3/2}}\,dx = \int \frac{(a^2cx^2+c)x^m}{\arctan(ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)*x^m/arctan(a*x)^(3/2), x)
```

## Mupad [N/A]

Not integrable

Time = 1.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)^{3/2}}\,dx = \int \frac{x^m\,\left(c\,a^2\,x^2+c\right)}{\mathrm{atan}\left(a\,x\right)^{3/2}}\,dx$$

input
```
int((x^m*(c + a^2*c*x^2))/atan(a*x)^(3/2),x)
```

output
```
int((x^m*(c + a^2*c*x^2))/atan(a*x)^(3/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.00

$$\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)^{3/2}}\,dx = c\left(\left(\int \frac{x^m\sqrt{atan\,(ax)}\,x^2}{atan\,(ax)^2}dx\right)a^2+\int \frac{x^m\sqrt{atan\,(ax)}}{atan\,(ax)^2}dx\right)$$

input
```
int(x^m*(a^2*c*x^2+c)/atan(a*x)^(3/2),x)
```

output
```
c*(int((x**m*sqrt(atan(a*x))*x**2)/atan(a*x)**2,x)*a**2 + int((x**m*sqrt(a
tan(a*x)))/atan(a*x)**2,x))
```

# 3.972 $\int \frac{x(c+a^2cx^2)}{\arctan(ax)^{3/2}}\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{x(c+a^2cx^2)}{\arctan(ax)^{3/2}}\,dx = \text{Int}\left(\frac{x(c+a^2cx^2)}{\arctan(ax)^{3/2}},x\right)$$

output
```
Defer(Int)(x*(a^2*c*x^2+c)/arctan(a*x)^(3/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x(c+a^2cx^2)}{\arctan(ax)^{3/2}}\,dx = \int \frac{x(c+a^2cx^2)}{\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[(x*(c + a^2*c*x^2))/ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[(x*(c + a^2*c*x^2))/ArcTan[a*x]^(3/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a^2cx^2+c)}{\arctan(ax)^{3/2}}\,dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x(a^2cx^2+c)}{\arctan(ax)^{3/2}}dx$$

input
```
Int[(x*(c + a^2*c*x^2))/ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{x(a^2c\,x^2+c)}{\arctan\left(ax\right)^{\frac{3}{2}}}dx$$

input
```
int(x*(a^2*c*x^2+c)/arctan(a*x)^(3/2),x)
```

output
```
int(x*(a^2*c*x^2+c)/arctan(a*x)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x(c + a^2cx^2)}{\arctan(ax)^{3/2}}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 1.46 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.45

$$\int \frac{x(c + a^2cx^2)}{\arctan(ax)^{3/2}}\, dx = c\left(\int \frac{x}{\operatorname{atan}^{\frac{3}{2}}(ax)}\, dx + \int \frac{a^2x^3}{\operatorname{atan}^{\frac{3}{2}}(ax)}\, dx\right)$$

input
```
integrate(x*(a**2*c*x**2+c)/atan(a*x)**(3/2),x)
```

output
```
c*(Integral(x/atan(a*x)**(3/2), x) + Integral(a**2*x**3/atan(a*x)**(3/2),
x))
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x(c + a^2cx^2)}{\arctan(ax)^{3/2}}\, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{x(c+a^2cx^2)}{\arctan(ax)^{3/2}}\,dx = \int \frac{(a^2cx^2+c)x}{\arctan{(ax)}^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)*x/arctan(a*x)^(3/2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.84 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{x(c+a^2cx^2)}{\arctan(ax)^{3/2}}\,dx = \int \frac{x\,(c\,a^2\,x^2+c)}{\mathrm{atan}\left(a\,x\right)^{3/2}}\,dx$$

input
```
int((x*(c + a^2*c*x^2))/atan(a*x)^(3/2),x)
```

output
```
int((x*(c + a^2*c*x^2))/atan(a*x)^(3/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.95

$$\int \frac{x(c+a^2cx^2)}{\arctan(ax)^{3/2}}\,dx = c\left(\left(\int \frac{\sqrt{atan\,(ax)}\,x^3}{atan\,(ax)^2}dx\right)a^2+\int \frac{\sqrt{atan\,(ax)}\,x}{atan\,(ax)^2}dx\right)$$

input
```
int(x*(a^2*c*x^2+c)/atan(a*x)^(3/2),x)
```

output
```
c*(int((sqrt(atan(a*x))*x**3)/atan(a*x)**2,x)*a**2 + int((sqrt(atan(a*x))*
x)/atan(a*x)**2,x))
```

# 3.973 $\int \frac{c+a^2cx^2}{\arctan(ax)^{3/2}}\,dx$



## Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \frac{c+a^2cx^2}{\arctan(ax)^{3/2}}\,dx = \operatorname{Int}\left(\frac{c+a^2cx^2}{\arctan(ax)^{3/2}},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)/arctan(a*x)^(3/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{c+a^2cx^2}{\arctan(ax)^{3/2}}\,dx = \int \frac{c+a^2cx^2}{\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)/ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[(c + a^2*c*x^2)/ArcTan[a*x]^(3/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2cx^2+c}{\arctan(ax)^{3/2}}\,dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{a^2cx^2+c}{\arctan(ax)^{3/2}}dx$$

input
```
Int[(c + a^2*c*x^2)/ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{a^2c\,x^2+c}{\arctan\left(ax\right)^{\frac{3}{2}}}dx$$

input
```
int((a^2*c*x^2+c)/arctan(a*x)^(3/2),x)
```

output
```
int((a^2*c*x^2+c)/arctan(a*x)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{c + a^2cx^2}{\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 1.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.53

$$\int \frac{c + a^2cx^2}{\arctan(ax)^{3/2}}\,dx = c\left(\int \frac{a^2x^2}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx + \int \frac{1}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)/atan(a*x)**(3/2),x)
```

output
```
c*(Integral(a**2*x**2/atan(a*x)**(3/2), x) + Integral(atan(a*x)**(-3/2), x
))
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{c + a^2cx^2}{\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{c+a^2cx^2}{\arctan(ax)^{3/2}}\,dx=\int \frac{a^2cx^2+c}{\arctan{(ax)}^{\frac{3}{2}}}\,dx$$

input
```
integrate((a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)/arctan(a*x)^(3/2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{c+a^2cx^2}{\arctan(ax)^{3/2}}\,dx=\int \frac{c\,a^2\,x^2+c}{\mathrm{atan}\left(a\,x\right)^{3/2}}\,dx$$

input
```
int((c + a^2*c*x^2)/atan(a*x)^(3/2),x)
```

output
```
int((c + a^2*c*x^2)/atan(a*x)^(3/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.00

$$\int \frac{c+a^2cx^2}{\arctan(ax)^{3/2}}\,dx = c\left(\int \frac{\sqrt{atan\,(ax)}}{atan\,(ax)^2}dx + \left(\int \frac{\sqrt{atan\,(ax)}\,x^2}{atan\,(ax)^2}dx\right)a^2\right)$$

input

```
int((a^2*c*x^2+c)/atan(a*x)^(3/2),x)
```

output

```
c*(int(sqrt(atan(a*x))/atan(a*x)**2,x) + int((sqrt(atan(a*x))*x**2)/atan(a
*x)**2,x)*a**2)
```

# 3.974 $\int \frac{c+a^2cx^2}{x\arctan(ax)^{3/2}}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{c+a^2cx^2}{x\arctan(ax)^{3/2}}\,dx = \text{Int}\left(\frac{c+a^2cx^2}{x\arctan(ax)^{3/2}},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)/x/arctan(a*x)^(3/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{c+a^2cx^2}{x\arctan(ax)^{3/2}}\,dx = \int \frac{c+a^2cx^2}{x\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)/(x*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[(c + a^2*c*x^2)/(x*ArcTan[a*x]^(3/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2cx^2+c}{x\arctan(ax)^{3/2}}dx$$

$$\Big\downarrow \text{5560}$$

$$\int \frac{a^2cx^2+c}{x\arctan(ax)^{3/2}}dx$$

input
```
Int[(c + a^2*c*x^2)/(x*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{a^2c\,x^2+c}{x\arctan\left(ax\right)^{\frac{3}{2}}}dx$$

input
```
int((a^2*c*x^2+c)/x/arctan(a*x)^(3/2),x)
```

output
```
int((a^2*c*x^2+c)/x/arctan(a*x)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{c + a^2cx^2}{x \arctan(ax)^{3/2}} dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)/x/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 2.34 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.32

$$\int \frac{c + a^2cx^2}{x \arctan(ax)^{3/2}} dx = c\left(\int \frac{1}{x \operatorname{atan}^{\frac{3}{2}}(ax)} dx + \int \frac{a^2x}{\operatorname{atan}^{\frac{3}{2}}(ax)} dx\right)$$

input
```
integrate((a**2*c*x**2+c)/x/atan(a*x)**(3/2),x)
```

output
```
c*(Integral(1/(x*atan(a*x)**(3/2)), x) + Integral(a**2*x/atan(a*x)**(3/2),
 x))
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{c + a^2cx^2}{x \arctan(ax)^{3/2}} dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)/x/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{c+a^2cx^2}{x\arctan(ax)^{3/2}}\,dx=\int \frac{a^2cx^2+c}{x\arctan{(ax)}^{\frac{3}{2}}}\,dx$$

input
```
integrate((a^2*c*x^2+c)/x/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)/(x*arctan(a*x)^(3/2)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.85 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{c+a^2cx^2}{x\arctan(ax)^{3/2}}\,dx=\int \frac{c\,a^2\,x^2+c}{x\,\mathrm{atan}\left(a\,x\right)^{3/2}}\,dx$$

input
```
int((c + a^2*c*x^2)/(x*atan(a*x)^(3/2)),x)
```

output
```
int((c + a^2*c*x^2)/(x*atan(a*x)^(3/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.77

$$\int \frac{c+a^2cx^2}{x\arctan(ax)^{3/2}}\,dx = c\left(\int \frac{\sqrt{atan\left(ax\right)}}{atan\left(ax\right)^2 x}dx+\left(\int \frac{\sqrt{atan\left(ax\right)}\,x}{atan\left(ax\right)^2}dx\right)a^2\right)$$

input
```
int((a^2*c*x^2+c)/x/atan(a*x)^(3/2),x)
```

output
```
c*(int(sqrt(atan(a*x))/(atan(a*x)**2*x),x) + int((sqrt(atan(a*x))*x)/atan(
a*x)**2,x)*a**2)
```

# 3.975 $\int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)^{3/2}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)^{3/2}}\,dx = \operatorname{Int}\left(\frac{x^m(c+a^2cx^2)^2}{\arctan(ax)^{3/2}},x\right)$$

output
```
Defer(Int)(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)^{3/2}}\,dx = \int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[(x^m*(c + a^2*c*x^2)^2)/ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[(x^m*(c + a^2*c*x^2)^2)/ArcTan[a*x]^(3/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m (a^2cx^2+c)^2}{\arctan(ax)^{3/2}}dx$$

$$\Big\downarrow \text{5560}$$

$$\int \frac{x^m (a^2cx^2+c)^2}{\arctan(ax)^{3/2}}dx$$

input
```
Int[(x^m*(c + a^2*c*x^2)^2)/ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m (a^2c\,x^2+c)^2}{\arctan(ax)^{\frac{3}{2}}}dx$$

input
```
int(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)^{3/2}}\,dx = \int \frac{(a^2cx^2+c)^2x^m}{\arctan(ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*x^m/arctan(a*x)^(3/2), x)
```

## Sympy [N/A]

Not integrable

Time = 53.31 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.42

$$\int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)^{3/2}}\,dx = c^2\left(\int \frac{x^m}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx + \int \frac{2a^2x^2x^m}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx + \int \frac{a^4x^4x^m}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx\right)$$

input
```
integrate(x**m*(a**2*c*x**2+c)**2/atan(a*x)**(3/2),x)
```

output
```
c**2*(Integral(x**m/atan(a*x)**(3/2), x) + Integral(2*a**2*x**2*x**m/atan(
a*x)**(3/2), x) + Integral(a**4*x**4*x**m/atan(a*x)**(3/2), x))
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)^{3/2}}\,dx=\int \frac{(a^2cx^2+c)^2x^m}{\arctan{(ax)}^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^2*x^m/arctan(a*x)^(3/2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.90 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)^{3/2}}\,dx=\int \frac{x^m\,{\left(c\,a^2\,x^2+c\right)}^2}{{\mathrm{atan}\left(a\,x\right)}^{3/2}}\,dx$$

input
```
int((x^m*(c + a^2*c*x^2)^2)/atan(a*x)^(3/2),x)
```

output
```
int((x^m*(c + a^2*c*x^2)^2)/atan(a*x)^(3/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.96

$$\int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)^{3/2}}\,dx = c^2\left(\left(\int \frac{x^m\sqrt{atan(ax)}\,x^4}{atan(ax)^2}dx\right)a^4 + 2\left(\int \frac{x^m\sqrt{atan(ax)}\,x^2}{atan(ax)^2}dx\right)a^2 + \int \frac{x^m\sqrt{atan(ax)}}{atan(ax)^2}dx\right)$$

input
```
int(x^m*(a^2*c*x^2+c)^2/atan(a*x)^(3/2),x)
```

output
```
c**2*(int((x**m*sqrt(atan(a*x))*x**4)/atan(a*x)**2,x)*a**4 + 2*int((x**m*s
qrt(atan(a*x))*x**2)/atan(a*x)**2,x)*a**2 + int((x**m*sqrt(atan(a*x)))/ata
n(a*x)**2,x))
```

# 3.976 $\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^{3/2}}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^{3/2}}\,dx = \text{Int}\left(\frac{x(c+a^2cx^2)^2}{\arctan(ax)^{3/2}},x\right)$$

output
```
Defer(Int)(x*(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^{3/2}}\,dx = \int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[(x*(c + a^2*c*x^2)^2)/ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[(x*(c + a^2*c*x^2)^2)/ArcTan[a*x]^(3/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a^2cx^2+c)^2}{\arctan(ax)^{3/2}}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{x(a^2cx^2+c)^2}{\arctan(ax)^{3/2}}dx$$

input
```
Int[(x*(c + a^2*c*x^2)^2)/ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x(a^2c\,x^2+c)^2}{\arctan\left(ax\right)^{\frac{3}{2}}}dx$$

input
```
int(x*(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

output
```
int(x*(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 2.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.23

$$\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^{3/2}}\,dx = c^2\left(\int \frac{x}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx + \int \frac{2a^2x^3}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx + \int \frac{a^4x^5}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx\right)$$

input
```
integrate(x*(a**2*c*x**2+c)**2/atan(a*x)**(3/2),x)
```

output
```
c**2*(Integral(x/atan(a*x)**(3/2), x) + Integral(2*a**2*x**3/atan(a*x)**(3
/2), x) + Integral(a**4*x**5/atan(a*x)**(3/2), x))
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^{3/2}}\,dx = \int \frac{(a^2cx^2+c)^2x}{\arctan{(ax)}^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^2*x/arctan(a*x)^(3/2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^{3/2}}\,dx = \int \frac{x\,{\left(c\,a^2\,x^2+c\right)}^2}{{\mathrm{atan}\left(a\,x\right)}^{3/2}}\,dx$$

input
```
int((x*(c + a^2*c*x^2)^2)/atan(a*x)^(3/2),x)
```

output
```
int((x*(c + a^2*c*x^2)^2)/atan(a*x)^(3/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.86

$$\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^{3/2}}\,dx = c^2\left(\left(\int \frac{\sqrt{atan\,(ax)}\,x^5}{atan\,(ax)^2}dx\right)a^4 + 2\left(\int \frac{\sqrt{atan\,(ax)}\,x^3}{atan\,(ax)^2}dx\right)a^2 + \int \frac{\sqrt{atan\,(ax)}\,x}{atan\,(ax)^2}dx\right)$$

input
```
int(x*(a^2*c*x^2+c)^2/atan(a*x)^(3/2),x)
```

output
```
c**2*(int((sqrt(atan(a*x))*x**5)/atan(a*x)**2,x)*a**4 + 2*int((sqrt(atan(a
*x))*x**3)/atan(a*x)**2,x)*a**2 + int((sqrt(atan(a*x))*x)/atan(a*x)**2,x))
```

# 3.977 $\int \frac{(c+a^2cx^2)^2}{\arctan(ax)^{3/2}}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{(c+a^2cx^2)^2}{\arctan(ax)^{3/2}}\,dx = \text{Int}\left(\frac{(c+a^2cx^2)^2}{\arctan(ax)^{3/2}},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{(c+a^2cx^2)^2}{\arctan(ax)^{3/2}}\,dx = \int \frac{(c+a^2cx^2)^2}{\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^2/ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[(c + a^2*c*x^2)^2/ArcTan[a*x]^(3/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2cx^2+c\right)^2}{\arctan(ax)^{3/2}} \, dx$$

$$\Big\downarrow \text{5560}$$

$$\int \frac{\left(a^2cx^2+c\right)^2}{\arctan(ax)^{3/2}}dx$$

input
```
Int[(c + a^2*c*x^2)^2/ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{\left(a^2 c\, x^2+c\right)^2}{\arctan \left(a x\right)^{\frac{3}{2}}}dx$$

input
```
int((a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

output
```
int((a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^2}{\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 1.85 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.33

$$\int \frac{(c+a^2cx^2)^2}{\arctan(ax)^{3/2}}\,dx = c^2\left(\int \frac{2a^2x^2}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx + \int \frac{a^4x^4}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx + \int \frac{1}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**2/atan(a*x)**(3/2),x)
```

output
```
c**2*(Integral(2*a**2*x**2/atan(a*x)**(3/2), x) + Integral(a**4*x**4/atan(
a*x)**(3/2), x) + Integral(atan(a*x)**(-3/2), x))
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^2}{\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^2}{\arctan(ax)^{3/2}}\,dx=\int \frac{(a^2cx^2+c)^2}{\arctan{(ax)}^{\frac{3}{2}}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^2/arctan(a*x)^(3/2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^2}{\arctan(ax)^{3/2}}\,dx=\int \frac{{\left(c\,a^2\,x^2+c\right)}^2}{{\mathrm{atan}\left(a\,x\right)}^{3/2}}\,dx$$

input
```
int((c + a^2*c*x^2)^2/atan(a*x)^(3/2),x)
```

output
```
int((c + a^2*c*x^2)^2/atan(a*x)^(3/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.95

$$\int \frac{(c+a^2cx^2)^2}{\arctan(ax)^{3/2}}\,dx = c^2\left(\int \frac{\sqrt{atan(ax)}}{atan(ax)^2}dx + \left(\int \frac{\sqrt{atan(ax)}\,x^4}{atan(ax)^2}dx\right)a^4 + 2\left(\int \frac{\sqrt{atan(ax)}\,x^2}{atan(ax)^2}dx\right)a^2\right)$$

input

```
int((a^2*c*x^2+c)^2/atan(a*x)^(3/2),x)
```

output

```
c**2*(int(sqrt(atan(a*x))/atan(a*x)**2,x) + int((sqrt(atan(a*x))*x**4)/ata
n(a*x)**2,x)*a**4 + 2*int((sqrt(atan(a*x))*x**2)/atan(a*x)**2,x)*a**2)
```

# 3.978 $\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)^{3/2}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)^{3/2}}\,dx = \text{Int}\left(\frac{(c+a^2cx^2)^2}{x\arctan(ax)^{3/2}},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^2/x/arctan(a*x)^(3/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)^{3/2}}\,dx = \int \frac{(c+a^2cx^2)^2}{x\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^2/(x*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[(c + a^2*c*x^2)^2/(x*ArcTan[a*x]^(3/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^2}{x \arctan(ax)^{3/2}} dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{(a^2cx^2+c)^2}{x \arctan(ax)^{3/2}} dx$$

input
```
Int[(c + a^2*c*x^2)^2/(x*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{(a^2c\,x^2+c)^2}{x \arctan\left(ax\right)^{\frac{3}{2}}}dx$$

input
```
int((a^2*c*x^2+c)^2/x/arctan(a*x)^(3/2),x)
```

output
```
int((a^2*c*x^2+c)^2/x/arctan(a*x)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^2/x/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 2.79 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.04

$$\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)^{3/2}}\,dx = c^2\left(\int \frac{1}{x\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx + \int \frac{2a^2x}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx + \int \frac{a^4x^3}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**2/x/atan(a*x)**(3/2),x)
```

output
```
c**2*(Integral(1/(x*atan(a*x)**(3/2)), x) + Integral(2*a**2*x/atan(a*x)**(
3/2), x) + Integral(a**4*x**3/atan(a*x)**(3/2), x))
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^2/x/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)^{3/2}}\,dx = \int \frac{(a^2cx^2+c)^2}{x\arctan{(ax)}^{\frac{3}{2}}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2/x/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^2/(x*arctan(a*x)^(3/2)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)^{3/2}}\,dx = \int \frac{(c\,a^2\,x^2+c)^2}{x\,\mathrm{atan}\left(a\,x\right)^{3/2}}\,dx$$

input
```
int((c + a^2*c*x^2)^2/(x*atan(a*x)^(3/2)),x)
```

output
```
int((c + a^2*c*x^2)^2/(x*atan(a*x)^(3/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.62

$$\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)^{3/2}}\,dx = c^2\left(\int \frac{\sqrt{atan(ax)}}{atan(ax)^2 x}dx + \left(\int \frac{\sqrt{atan(ax)}\,x^3}{atan(ax)^2}dx\right)a^4 + 2\left(\int \frac{\sqrt{atan(ax)}\,x}{atan(ax)^2}dx\right)a^2\right)$$

input
```
int((a^2*c*x^2+c)^2/x/atan(a*x)^(3/2),x)
```

output
```
c**2*(int(sqrt(atan(a*x))/(atan(a*x)**2*x),x) + int((sqrt(atan(a*x))*x**3)
/atan(a*x)**2,x)*a**4 + 2*int((sqrt(atan(a*x))*x)/atan(a*x)**2,x)*a**2)
```

# 3.979 $\int \frac{x^m (c+a^2cx^2)^3}{\arctan(ax)^{3/2}} \, dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m (c+a^2cx^2)^3}{\arctan(ax)^{3/2}} \, dx = \text{Int}\left(\frac{x^m (c+a^2cx^2)^3}{\arctan(ax)^{3/2}}, x\right)$$

output
```
Defer(Int)(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m (c+a^2cx^2)^3}{\arctan(ax)^{3/2}} \, dx = \int \frac{x^m (c+a^2cx^2)^3}{\arctan(ax)^{3/2}} \, dx$$

input
```
Integrate[(x^m*(c + a^2*c*x^2)^3)/ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[(x^m*(c + a^2*c*x^2)^3)/ArcTan[a*x]^(3/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m\left(a^2cx^2+c\right)^3}{\arctan(ax)^{3/2}}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x^m\left(a^2cx^2+c\right)^3}{\arctan(ax)^{3/2}}dx$$

input
```
Int[(x^m*(c + a^2*c*x^2)^3)/ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 1.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m\left(a^2c\,x^2+c\right)^3}{\arctan\left(ax\right)^{\frac{3}{2}}}dx$$

input
```
int(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.00

$$\int \frac{x^m(c+a^2cx^2)^3}{\arctan(ax)^{3/2}}\,dx = \int \frac{(a^2cx^2+c)^3x^m}{\arctan(ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*x^m/arctan(a*
x)^(3/2), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^m(c+a^2cx^2)^3}{\arctan(ax)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**m*(a**2*c*x**2+c)**3/atan(a*x)**(3/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m(c+a^2cx^2)^3}{\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m(c+a^2cx^2)^3}{\arctan(ax)^{3/2}}\,dx = \int \frac{(a^2cx^2+c)^3x^m}{\arctan(ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^3*x^m/arctan(a*x)^(3/2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m(c+a^2cx^2)^3}{\arctan(ax)^{3/2}}\,dx = \int \frac{x^m\,(c\,a^2\,x^2+c)^3}{\mathrm{atan}\,(a\,x)^{3/2}}\,dx$$

input
```
int((x^m*(c + a^2*c*x^2)^3)/atan(a*x)^(3/2),x)
```

output
```
int((x^m*(c + a^2*c*x^2)^3)/atan(a*x)^(3/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 96, normalized size of antiderivative = 4.00

$$\int \frac{x^m(c+a^2cx^2)^3}{\arctan(ax)^{3/2}}\,dx = c^3\left(\left(\int \frac{x^m\sqrt{atan\,(ax)}\,x^6}{atan\,(ax)^2}dx\right)a^6 + 3\left(\int \frac{x^m\sqrt{atan\,(ax)}\,x^4}{atan\,(ax)^2}dx\right)a^4 + 3\left(\int \frac{x^m\sqrt{atan\,(ax)}\,x^2}{atan\,(ax)^2}dx\right)a^2 + \int \frac{x^m\sqrt{atan\,(ax)}}{atan\,(ax)^2}dx\right)$$

input
```
int(x^m*(a^2*c*x^2+c)^3/atan(a*x)^(3/2),x)
```

output
```
c**3*(int((x**m*sqrt(atan(a*x))*x**6)/atan(a*x)**2,x)*a**6 + 3*int((x**m*s
qrt(atan(a*x))*x**4)/atan(a*x)**2,x)*a**4 + 3*int((x**m*sqrt(atan(a*x))*x*
*2)/atan(a*x)**2,x)*a**2 + int((x**m*sqrt(atan(a*x)))/atan(a*x)**2,x))
```

# 3.980 $\int \frac{x\left(c+a^2cx^2\right)^3}{\arctan(ax)^{3/2}}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)^{3/2}}\,dx = \text{Int}\left(\frac{x(c+a^2cx^2)^3}{\arctan(ax)^{3/2}},x\right)$$

output
```
Defer(Int)(x*(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)^{3/2}}\,dx = \int \frac{x(c+a^2cx^2)^3}{\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[(x*(c + a^2*c*x^2)^3)/ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[(x*(c + a^2*c*x^2)^3)/ArcTan[a*x]^(3/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a^2cx^2+c\right)^3}{\arctan(ax)^{3/2}}dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int \frac{x\left(a^2cx^2+c\right)^3}{\arctan(ax)^{3/2}}dx$$

input
```
Int[(x*(c + a^2*c*x^2)^3)/ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x\left(a^2c\,x^2+c\right)^3}{\arctan\left(ax\right)^{\frac{3}{2}}}dx$$

input
```
int(x*(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)
```

output
```
int(x*(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 4.31 (sec) , antiderivative size = 68, normalized size of antiderivative = 3.09

$$\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)^{3/2}}\,dx = c^3\left(\int \frac{x}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx + \int \frac{3a^2x^3}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx + \int \frac{3a^4x^5}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx + \int \frac{a^6x^7}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx\right)$$

input
```
integrate(x*(a**2*c*x**2+c)**3/atan(a*x)**(3/2),x)
```

output
```
c**3*(Integral(x/atan(a*x)**(3/2), x) + Integral(3*a**2*x**3/atan(a*x)**(3
/2), x) + Integral(3*a**4*x**5/atan(a*x)**(3/2), x) + Integral(a**6*x**7/a
tan(a*x)**(3/2), x))
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x(c + a^2cx^2)^3}{\arctan(ax)^{3/2}} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x(c + a^2cx^2)^3}{\arctan(ax)^{3/2}} \, dx = \int \frac{(a^2cx^2 + c)^3 x}{\arctan(ax)^{\frac{3}{2}}} \, dx$$

input
```
integrate(x*(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^3*x/arctan(a*x)^(3/2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x(c + a^2cx^2)^3}{\arctan(ax)^{3/2}} \, dx = \int \frac{x \left(c\,a^2\,x^2 + c\right)^3}{\mathrm{atan}\left(a\,x\right)^{3/2}} \, dx$$

input
```
int((x*(c + a^2*c*x^2)^3)/atan(a*x)^(3/2),x)
```

output
```
int((x*(c + a^2*c*x^2)^3)/atan(a*x)^(3/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 85, normalized size of antiderivative = 3.86

$$\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)^{3/2}}\,dx = c^3\left(\left(\int \frac{\sqrt{atan\left(ax\right)}\,x^7}{atan\left(ax\right)^2}dx\right)a^6+3\left(\int \frac{\sqrt{atan\left(ax\right)}\,x^5}{atan\left(ax\right)^2}dx\right)a^4 +3\left(\int \frac{\sqrt{atan\left(ax\right)}\,x^3}{atan\left(ax\right)^2}dx\right)a^2+\int \frac{\sqrt{atan\left(ax\right)}\,x}{atan\left(ax\right)^2}dx\right)$$

input
```
int(x*(a^2*c*x^2+c)^3/atan(a*x)^(3/2),x)
```

output
```
c**3*(int((sqrt(atan(a*x))*x**7)/atan(a*x)**2,x)*a**6 + 3*int((sqrt(atan(a
*x))*x**5)/atan(a*x)**2,x)*a**4 + 3*int((sqrt(atan(a*x))*x**3)/atan(a*x)**
2,x)*a**2 + int((sqrt(atan(a*x))*x)/atan(a*x)**2,x))
```

# 3.981 $\int \frac{(c+a^2cx^2)^3}{\arctan(ax)^{3/2}}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{(c+a^2cx^2)^3}{\arctan(ax)^{3/2}}\,dx=\operatorname{Int}\left(\frac{(c+a^2cx^2)^3}{\arctan(ax)^{3/2}},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{(c+a^2cx^2)^3}{\arctan(ax)^{3/2}}\,dx=\int \frac{(c+a^2cx^2)^3}{\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^3/ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[(c + a^2*c*x^2)^3/ArcTan[a*x]^(3/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^3}{\arctan(ax)^{3/2}}\,dx$$

$$\Big\downarrow \ 5560$$

$$\int \frac{(a^2cx^2+c)^3}{\arctan(ax)^{3/2}}dx$$

input
```
Int[(c + a^2*c*x^2)^3/ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{(a^2c\,x^2+c)^3}{\arctan\left(ax\right)^{\frac{3}{2}}}dx$$

input
```
int((a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)
```

output
```
int((a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^3}{\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 3.32 (sec) , antiderivative size = 68, normalized size of antiderivative = 3.24

$$\int \frac{(c+a^2cx^2)^3}{\arctan(ax)^{3/2}}\,dx = c^3\left(\int \frac{3a^2x^2}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx + \int \frac{3a^4x^4}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx + \int \frac{a^6x^6}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx + \int \frac{1}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**3/atan(a*x)**(3/2),x)
```

output
```
c**3*(Integral(3*a**2*x**2/atan(a*x)**(3/2), x) + Integral(3*a**4*x**4/ata
n(a*x)**(3/2), x) + Integral(a**6*x**6/atan(a*x)**(3/2), x) + Integral(ata
n(a*x)**(-3/2), x))
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^3}{\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^3}{\arctan(ax)^{3/2}}\,dx = \int \frac{(a^2cx^2+c)^3}{\arctan(ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^3/arctan(a*x)^(3/2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^3}{\arctan(ax)^{3/2}}\,dx = \int \frac{(c\,a^2\,x^2+c)^3}{\mathrm{atan}(a\,x)^{3/2}}\,dx$$

input
```
int((c + a^2*c*x^2)^3/atan(a*x)^(3/2),x)
```

output
```
int((c + a^2*c*x^2)^3/atan(a*x)^(3/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 84, normalized size of antiderivative = 4.00

$$\int \frac{(c+a^2cx^2)^3}{\arctan(ax)^{3/2}}\,dx = c^3\left(\int \frac{\sqrt{atan\,(ax)}}{atan\,(ax)^2}dx + \left(\int \frac{\sqrt{atan\,(ax)}\,x^6}{atan\,(ax)^2}dx\right)a^6 + 3\left(\int \frac{\sqrt{atan\,(ax)}\,x^4}{atan\,(ax)^2}dx\right)a^4 + 3\left(\int \frac{\sqrt{atan\,(ax)}\,x^2}{atan\,(ax)^2}dx\right)a^2\right)$$

input
```
int((a^2*c*x^2+c)^3/atan(a*x)^(3/2),x)
```

output
```
c**3*(int(sqrt(atan(a*x))/atan(a*x)**2,x) + int((sqrt(atan(a*x))*x**6)/ata
n(a*x)**2,x)*a**6 + 3*int((sqrt(atan(a*x))*x**4)/atan(a*x)**2,x)*a**4 + 3*
int((sqrt(atan(a*x))*x**2)/atan(a*x)**2,x)*a**2)
```

# 3.982 $\int \frac{(c+a^2cx^2)^3}{x\arctan(ax)^{3/2}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{(c+a^2cx^2)^3}{x\arctan(ax)^{3/2}}\,dx = \text{Int}\left(\frac{(c+a^2cx^2)^3}{x\arctan(ax)^{3/2}},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^3/x/arctan(a*x)^(3/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.37 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(c+a^2cx^2)^3}{x\arctan(ax)^{3/2}}\,dx = \int \frac{(c+a^2cx^2)^3}{x\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^3/(x*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[(c + a^2*c*x^2)^3/(x*ArcTan[a*x]^(3/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^3}{x\arctan(ax)^{3/2}}dx$$

$$\Big\downarrow \text{5560}$$

$$\int \frac{(a^2cx^2+c)^3}{x\arctan(ax)^{3/2}}dx$$

input
```
Int[(c + a^2*c*x^2)^3/(x*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{(a^2c\,x^2+c)^3}{x\arctan(ax)^{\frac{3}{2}}}dx$$

input
```
int((a^2*c*x^2+c)^3/x/arctan(a*x)^(3/2),x)
```

output
```
int((a^2*c*x^2+c)^3/x/arctan(a*x)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{(c + a^2cx^2)^3}{x \arctan(ax)^{3/2}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^3/x/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 4.38 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.83

$$\int \frac{(c + a^2cx^2)^3}{x \arctan(ax)^{3/2}} \, dx = c^3 \left( \int \frac{1}{x \operatorname{atan}^{\frac{3}{2}}(ax)} \, dx + \int \frac{3a^2x}{\operatorname{atan}^{\frac{3}{2}}(ax)} \, dx + \int \frac{3a^4x^3}{\operatorname{atan}^{\frac{3}{2}}(ax)} \, dx + \int \frac{a^6x^5}{\operatorname{atan}^{\frac{3}{2}}(ax)} \, dx \right)$$

input
```
integrate((a**2*c*x**2+c)**3/x/atan(a*x)**(3/2),x)
```

output
```
c**3*(Integral(1/(x*atan(a*x)**(3/2)), x) + Integral(3*a**2*x/atan(a*x)**(
3/2), x) + Integral(3*a**4*x**3/atan(a*x)**(3/2), x) + Integral(a**6*x**5/
atan(a*x)**(3/2), x))
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(c + a^2cx^2)^3}{x \arctan(ax)^{3/2}} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^3/x/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c + a^2cx^2)^3}{x \arctan(ax)^{3/2}} \, dx = \int \frac{(a^2cx^2 + c)^3}{x \arctan(ax)^{\frac{3}{2}}} \, dx$$

input
```
integrate((a^2*c*x^2+c)^3/x/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^3/(x*arctan(a*x)^(3/2)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c + a^2cx^2)^3}{x \arctan(ax)^{3/2}} \, dx = \int \frac{(c\,a^2\,x^2 + c)^3}{x\,\mathrm{atan}\,(a\,x)^{3/2}} \, dx$$

input
```
int((c + a^2*c*x^2)^3/(x*atan(a*x)^(3/2)),x)
```

output
```
int((c + a^2*c*x^2)^3/(x*atan(a*x)^(3/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 85, normalized size of antiderivative = 3.54

$$\int \frac{(c+a^2cx^2)^3}{x\arctan(ax)^{3/2}}\,dx = c^3\left(\int \frac{\sqrt{atan\,(ax)}}{atan\,(ax)^2\,x}dx + \left(\int \frac{\sqrt{atan\,(ax)}\,x^5}{atan\,(ax)^2}dx\right)a^6 + 3\left(\int \frac{\sqrt{atan\,(ax)}\,x^3}{atan\,(ax)^2}dx\right)a^4 + 3\left(\int \frac{\sqrt{atan\,(ax)}\,x}{atan\,(ax)^2}dx\right)a^2\right)$$

input
```
int((a^2*c*x^2+c)^3/x/atan(a*x)^(3/2),x)
```

output
```
c**3*(int(sqrt(atan(a*x))/(atan(a*x)**2*x),x) + int((sqrt(atan(a*x))*x**5)
/atan(a*x)**2,x)*a**6 + 3*int((sqrt(atan(a*x))*x**3)/atan(a*x)**2,x)*a**4
+ 3*int((sqrt(atan(a*x))*x)/atan(a*x)**2,x)*a**2)
```

# 3.983 $\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^{3/2}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^{3/2}}\,dx = -\frac{2x^m}{ac\sqrt{\arctan(ax)}} + \frac{2m\text{Int}\left(\frac{x^{-1+m}}{\sqrt{\arctan(ax)}},x\right)}{ac}$$

output
```
-2*x^m/a/c/arctan(a*x)^(1/2)+2*m*Defer(Int)(x^(-1+m)/arctan(a*x)^(1/2),x)/
a/c
```

## Mathematica [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^{3/2}}\,dx = \int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[x^m/((c + a^2*c*x^2)*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[x^m/((c + a^2*c*x^2)*ArcTan[a*x]^(3/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\arctan(ax)^{3/2}\left(a^2cx^2+c\right)}dx$$
$$\Big\downarrow \text{ 5461}$$
$$\frac{2m\int \frac{x^{m-1}}{\sqrt{\arctan(ax)}}dx}{ac}-\frac{2x^m}{ac\sqrt{\arctan(ax)}}$$
$$\Big\downarrow \text{ 5377}$$
$$\frac{2m\int \frac{x^{m-1}}{\sqrt{\arctan(ax)}}dx}{ac}-\frac{2x^m}{ac\sqrt{\arctan(ax)}}$$

input
```
Int[x^m/((c + a^2*c*x^2)*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{\left(a^2c\,x^2+c\right)\arctan\left(ax\right)^{\frac{3}{2}}}dx$$

input
```
int(x^m/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x)
```

output
```
int(x^m/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^{3/2}}\,dx = \int \frac{x^m}{(a^2cx^2+c)\arctan(ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
integral(x^m/((a^2*c*x^2 + c)*arctan(a*x)^(3/2)), x)
```

## Sympy [N/A]

Not integrable

Time = 49.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^{3/2}}\,dx = \frac{\int \frac{x^m}{a^2x^2\operatorname{atan}^{\frac{3}{2}}(ax)+\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx}{c}$$

input
```
integrate(x**m/(a**2*c*x**2+c)/atan(a*x)**(3/2),x)
```

output
```
Integral(x**m/(a**2*x**2*atan(a*x)**(3/2) + atan(a*x)**(3/2)), x)/c
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^{3/2}}\,dx = \int \frac{x^m}{(a^2cx^2+c)\arctan(ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^m/((a^2*c*x^2 + c)*arctan(a*x)^(3/2)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.85 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^{3/2}}\,dx = \int \frac{x^m}{\mathrm{atan}(a\,x)^{3/2}\,(c\,a^2\,x^2+c)}\,dx$$

input
```
int(x^m/(atan(a*x)^(3/2)*(c + a^2*c*x^2)),x)
```

output
```
int(x^m/(atan(a*x)^(3/2)*(c + a^2*c*x^2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^{3/2}}\,dx = \frac{\int \frac{x^m\sqrt{atan(ax)}}{atan(ax)^2a^2x^2+atan(ax)^2}dx}{c}$$

input
```
int(x^m/(a^2*c*x^2+c)/atan(a*x)^(3/2),x)
```

output
```
int((x**m*sqrt(atan(a*x)))/(atan(a*x)**2*a**2*x**2 + atan(a*x)**2),x)/c
```

# 3.984 $\int \frac{x}{(c+a^2cx^2)\arctan(ax)^{3/2}}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x}{(c+a^2cx^2)\arctan(ax)^{3/2}}\,dx = -\frac{2x}{ac\sqrt{\arctan(ax)}}+\frac{2\text{Int}\left(\frac{1}{\sqrt{\arctan(ax)}},x\right)}{ac}$$

output
```
-2*x/a/c/arctan(a*x)^(1/2)+2*Defer(Int)(1/arctan(a*x)^(1/2),x)/a/c
```

## Mathematica [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x}{(c+a^2cx^2)\arctan(ax)^{3/2}}\,dx = \int \frac{x}{(c+a^2cx^2)\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[x/((c + a^2*c*x^2)*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[x/((c + a^2*c*x^2)*ArcTan[a*x]^(3/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\arctan(ax)^{3/2}\left(a^2cx^2+c\right)}dx$$

$$\Big\downarrow \text{5457}$$

$$\frac{2\int \frac{1}{\sqrt{\arctan(ax)}}dx}{ac}-\frac{2x}{ac\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{5353}$$

$$\frac{2\int \frac{1}{\sqrt{\arctan(ax)}}dx}{ac}-\frac{2x}{ac\sqrt{\arctan(ax)}}$$

input
```
Int[x/((c + a^2*c*x^2)*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x}{\left(a^{2} c \,x^{2}+c \right) \arctan \left(a x \right)^{\frac{3}{2}}}d x$$

input
```
int(x/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x)
```

output
```
int(x/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 1.33 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

$$\int \frac{x}{(c+a^2cx^2)\arctan(ax)^{3/2}}\,dx = \frac{\int \frac{x}{a^2x^2\operatorname{atan}^{\frac{3}{2}}(ax)+\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx}{c}$$

input
```
integrate(x/(a**2*c*x**2+c)/atan(a*x)**(3/2),x)
```

output
```
Integral(x/(a**2*x**2*atan(a*x)**(3/2) + atan(a*x)**(3/2)), x)/c
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x}{(c+a^2cx^2)\arctan(ax)^{3/2}}\,dx = \int \frac{x}{(a^2cx^2+c)\arctan(ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(x/((a^2*c*x^2 + c)*arctan(a*x)^(3/2)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x}{(c+a^2cx^2)\arctan(ax)^{3/2}}\,dx = \int \frac{x}{\mathrm{atan}(a\,x)^{3/2}\,(c\,a^2\,x^2+c)}\,dx$$

input
```
int(x/(atan(a*x)^(3/2)*(c + a^2*c*x^2)),x)
```

output
```
int(x/(atan(a*x)^(3/2)*(c + a^2*c*x^2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.59

$$\int \frac{x}{(c+a^2cx^2)\arctan(ax)^{3/2}}\,dx = \frac{\int \frac{\sqrt{atan(ax)}\,x}{atan(ax)^2a^2x^2+atan(ax)^2}dx}{c}$$

input
```
int(x/(a^2*c*x^2+c)/atan(a*x)^(3/2),x)
```

output
```
int((sqrt(atan(a*x))*x)/(atan(a*x)**2*a**2*x**2 + atan(a*x)**2),x)/c
```

# 3.985 $\int \frac{1}{(c+a^2cx^2)\arctan(ax)^{3/2}}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 16

$$\int \frac{1}{(c+a^2cx^2)\arctan(ax)^{3/2}}\,dx = -\frac{2}{ac\sqrt{\arctan(ax)}}$$

output
```
-2/a/c/arctan(a*x)^(1/2)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{(c+a^2cx^2)\arctan(ax)^{3/2}}\,dx = -\frac{2}{ac\sqrt{\arctan(ax)}}$$

input
```
Integrate[1/((c + a^2*c*x^2)*ArcTan[a*x]^(3/2)),x]
```

output
```
-2/(a*c*Sqrt[ArcTan[a*x]])
```

## Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arctan(ax)^{3/2}\left(a^2cx^2+c\right)}dx$$
$$\Big\downarrow \text{ 5419}$$
$$-\frac{2}{ac\sqrt{\arctan(ax)}}$$

input
```
Int[1/((c + a^2*c*x^2)*ArcTan[a*x]^(3/2)),x]
```

output
```
-2/(a*c*Sqrt[ArcTan[a*x]])
```

### Defintions of rubi rules used

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

## Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

| method | result | size |
|---|---|---|
| default | $-\frac{2}{ac\sqrt{\arctan(ax)}}$ | 15 |

input
```
int(1/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/a/c/arctan(a*x)^(1/2)
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{(c+a^2cx^2)\arctan(ax)^{3/2}}\,dx = -\frac{2}{ac\sqrt{\arctan{(ax)}}}$$

input
```
integrate(1/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
-2/(a*c*sqrt(arctan(a*x)))
```

## Sympy [A] (verification not implemented)

Time = 1.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{(c+a^2cx^2)\arctan(ax)^{3/2}}\,dx = -\frac{2}{ac\sqrt{\operatorname{atan}{(ax)}}}$$

input
```
integrate(1/(a**2*c*x**2+c)/atan(a*x)**(3/2),x)
```

output
```
-2/(a*c*sqrt(atan(a*x)))
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(c+a^2cx^2)\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{(c+a^2cx^2)\arctan(ax)^{3/2}}\,dx = -\frac{2}{ac\sqrt{\arctan{(ax)}}}$$

input
```
integrate(1/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
-2/(a*c*sqrt(arctan(a*x)))
```

## Mupad [B] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{(c+a^2cx^2)\arctan(ax)^{3/2}}\,dx = -\frac{2}{a\,c\,\sqrt{\mathrm{atan}\left(a\,x\right)}}$$

input
```
int(1/(atan(a*x)^(3/2)*(c + a^2*c*x^2)),x)
```

output
```
-2/(a*c*atan(a*x)^(1/2))
```

## Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.19

$$\int \frac{1}{(c+a^2cx^2)\arctan(ax)^{3/2}}\,dx = -\frac{2\sqrt{atan\left(ax\right)}}{atan\left(ax\right)ac}$$

input
```
int(1/(a^2*c*x^2+c)/atan(a*x)^(3/2),x)
```

output
```
( - 2*sqrt(atan(a*x)))/(atan(a*x)*a*c)
```

# 3.986 $\int \frac{1}{x(c+a^2cx^2)\arctan(ax)^{3/2}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x\left(c+a^2cx^2\right)\arctan(ax)^{3/2}}\,dx=-\frac{2}{acx\sqrt{\arctan(ax)}}-\frac{2\text{Int}\left(\frac{1}{x^2\sqrt{\arctan(ax)}},x\right)}{ac}$$

output
```
-2/a/c/x/arctan(a*x)^(1/2)-2*Defer(Int)(1/x^2/arctan(a*x)^(1/2),x)/a/c
```

## Mathematica [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x\left(c+a^2cx^2\right)\arctan(ax)^{3/2}}\,dx=\int \frac{1}{x\left(c+a^2cx^2\right)\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[1/(x*(c + a^2*c*x^2)*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[1/(x*(c + a^2*c*x^2)*ArcTan[a*x]^(3/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x \arctan(ax)^{3/2} (a^2cx^2+c)} dx$$
$$\Big\downarrow \text{5461}$$
$$-\frac{2\int \frac{1}{x^2\sqrt{\arctan(ax)}}dx}{ac}-\frac{2}{acx\sqrt{\arctan(ax)}}$$
$$\Big\downarrow \text{5377}$$
$$-\frac{2\int \frac{1}{x^2\sqrt{\arctan(ax)}}dx}{ac}-\frac{2}{acx\sqrt{\arctan(ax)}}$$

input
```
Int[1/(x*(c + a^2*c*x^2)*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\left(a^2c\,x^2+c\right)\arctan\left(ax\right)^{\frac{3}{2}}}dx$$

input
```
int(1/x/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x)
```

output
```
int(1/x/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(c+a^2cx^2\right)\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 1.75 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int \frac{1}{x\left(c+a^2cx^2\right)\arctan(ax)^{3/2}}\,dx = \frac{\int \frac{1}{a^2x^3\operatorname{atan}^{\frac{3}{2}}(ax)+x\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx}{c}$$

input
```
integrate(1/x/(a**2*c*x**2+c)/atan(a*x)**(3/2),x)
```

output
```
Integral(1/(a**2*x**3*atan(a*x)**(3/2) + x*atan(a*x)**(3/2)), x)/c
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(c+a^2cx^2\right)\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)\arctan(ax)^{3/2}}\,dx = \int \frac{1}{(a^2cx^2+c)x\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)*x*arctan(a*x)^(3/2)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)\arctan(ax)^{3/2}}\,dx = \int \frac{1}{x\,\mathrm{atan}\left(a\,x\right)^{3/2}\,\left(c\,a^2\,x^2+c\right)}\,dx$$

input
```
int(1/(x*atan(a*x)^(3/2)*(c + a^2*c*x^2)),x)
```

output
```
int(1/(x*atan(a*x)^(3/2)*(c + a^2*c*x^2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.50

$$\int \frac{1}{x(c+a^2cx^2)\arctan(ax)^{3/2}}\,dx = \frac{\int \frac{\sqrt{atan(ax)}}{atan(ax)^2a^2x^3+atan(ax)^2x}dx}{c}$$

input
```
int(1/x/(a^2*c*x^2+c)/atan(a*x)^(3/2),x)
```

output
```
int(sqrt(atan(a*x))/(atan(a*x)**2*a**2*x**3 + atan(a*x)**2*x),x)/c
```

# 3.987 $\int \frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)^{3/2}} \, dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)^{3/2}} \, dx = \text{Int}\left(\frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}, x\right)$$

output
```
Defer(Int)(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)^{3/2}} \, dx = \int \frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)^{3/2}} \, dx$$

input
```
Integrate[x^m/((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[x^m/((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\arctan(ax)^{3/2} (a^2cx^2+c)^2} dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x^m}{\arctan(ax)^{3/2} (a^2cx^2+c)^2} dx$$

input
```
Int[x^m/((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{(a^2c\,x^2+c)^2 \arctan(ax)^{\frac{3}{2}}} dx$$

input
```
int(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

output
```
int(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.62

$$\int \frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}\,dx = \int \frac{x^m}{(a^2cx^2+c)^2 \arctan(ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
integral(x^m/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)^(3/2)), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**m/(a**2*c*x**2+c)**2/atan(a*x)**(3/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}\,dx = \int \frac{x^m}{(a^2cx^2+c)^2 \arctan(ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^m/((a^2*c*x^2 + c)^2*arctan(a*x)^(3/2)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.87 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}\,dx = \int \frac{x^m}{\mathrm{atan}(a\,x)^{3/2}\,(c\,a^2\,x^2+c)^2}\,dx$$

input
```
int(x^m/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^2),x)
```

output
```
int(x^m/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 350, normalized size of antiderivative = 14.58

$$\int \frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}\,dx = \frac{2atan(ax)\left(\int \frac{x^m\sqrt{atan(ax)}\,x}{atan(ax)a^4x^4+2atan(ax)a^2x^2+atan(ax)}dx\right)a^4m\,x^2-4atan(ax)\,(}{}$$

input
```
int(x^m/(a^2*c*x^2+c)^2/atan(a*x)^(3/2),x)
```

output
```
(2*(atan(a*x)*int((x**m*sqrt(atan(a*x))*x)/(atan(a*x)*a**4*x**4 + 2*atan(a
*x)*a**2*x**2 + atan(a*x)),x)*a**4*m*x**2 - 2*atan(a*x)*int((x**m*sqrt(ata
n(a*x))*x)/(atan(a*x)*a**4*x**4 + 2*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a*
*4*x**2 + atan(a*x)*int((x**m*sqrt(atan(a*x))*x)/(atan(a*x)*a**4*x**4 + 2*
atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**2*m - 2*atan(a*x)*int((x**m*sqrt(at
an(a*x))*x)/(atan(a*x)*a**4*x**4 + 2*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a
**2 + atan(a*x)*int((x**m*sqrt(atan(a*x)))/(atan(a*x)*a**4*x**5 + 2*atan(a
*x)*a**2*x**3 + atan(a*x)*x),x)*a**2*m*x**2 + atan(a*x)*int((x**m*sqrt(ata
n(a*x)))/(atan(a*x)*a**4*x**5 + 2*atan(a*x)*a**2*x**3 + atan(a*x)*x),x)*m
- x**m*sqrt(atan(a*x))))/(atan(a*x)*a*c**2*(a**2*x**2 + 1))
```

# 3.988 $\int \frac{x^4}{(c+a^2cx^2)^2 \arctan(ax)^{3/2}} \, dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^4}{(c+a^2cx^2)^2 \arctan(ax)^{3/2}} \, dx = -\frac{2x^4}{ac^2(1+a^2x^2)\sqrt{\arctan(ax)}} + \frac{8\text{Int}\left(\frac{x^3}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}},x\right)}{a} + 4a\text{Int}\left(\frac{x^5}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}},x\right)$$

output
```
-2*x^4/a/c^2/(a^2*x^2+1)/arctan(a*x)^(1/2)+8*Defer(Int)(x^3/(a^2*c*x^2+c)^
2/arctan(a*x)^(1/2),x)/a+4*a*Defer(Int)(x^5/(a^2*c*x^2+c)^2/arctan(a*x)^(1
/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 6.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^4}{(c+a^2cx^2)^2 \arctan(ax)^{3/2}} \, dx = \int \frac{x^4}{(c+a^2cx^2)^2 \arctan(ax)^{3/2}} \, dx$$

input
```
Integrate[x^4/((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[x^4/((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\arctan(ax)^{3/2} (a^2cx^2+c)^2} \, dx$$

$$\Big\downarrow \text{5503}$$

$$4a \int \frac{x^5}{c^2 (a^2x^2+1)^2 \sqrt{\arctan(ax)}} dx + \frac{8 \int \frac{x^3}{c^2(a^2x^2+1)^2 \sqrt{\arctan(ax)}} dx}{a} - \frac{2x^4}{ac^2 (a^2x^2+1) \sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{4a \int \frac{x^5}{(a^2x^2+1)^2 \sqrt{\arctan(ax)}} dx}{c^2} + \frac{8 \int \frac{x^3}{(a^2x^2+1)^2 \sqrt{\arctan(ax)}} dx}{ac^2} - \frac{2x^4}{ac^2 (a^2x^2+1) \sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{5560}$$

$$\frac{4a \int \frac{x^5}{(a^2x^2+1)^2 \sqrt{\arctan(ax)}} dx}{c^2} + \frac{8 \int \frac{x^3}{(a^2x^2+1)^2 \sqrt{\arctan(ax)}} dx}{ac^2} - \frac{2x^4}{ac^2 (a^2x^2+1) \sqrt{\arctan(ax)}}$$

input
```
Int[x^4/((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^4}{\left(a^2 c\,x^2+c\right)^2 \arctan\left(a x\right)^{\frac{3}{2}}}dx$$

input
```
int(x^4/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

output
```
int(x^4/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x^4}{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^4/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 3.16 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.00

$$\int \frac{x^4}{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}\, dx = \frac{\int \frac{x^4}{a^4x^4 \operatorname{atan}^{\frac{3}{2}}(ax)+2a^2x^2 \operatorname{atan}^{\frac{3}{2}}(ax)+\operatorname{atan}^{\frac{3}{2}}(ax)}\, dx}{c^2}$$

input
```
integrate(x**4/(a**2*c*x**2+c)**2/atan(a*x)**(3/2),x)
```

output
```
Integral(x**4/(a**4*x**4*atan(a*x)**(3/2) + 2*a**2*x**2*atan(a*x)**(3/2) +
 atan(a*x)**(3/2)), x)/c**2
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4}{(c+a^2cx^2)^2 \arctan(ax)^{3/2}} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^4/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{(c+a^2cx^2)^2 \arctan(ax)^{3/2}} \, dx = \int \frac{x^4}{(a^2cx^2+c)^2 \arctan(ax)^{\frac{3}{2}}} \, dx$$

input
```
integrate(x^4/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^4/((a^2*c*x^2 + c)^2*arctan(a*x)^(3/2)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^4}{(c+a^2cx^2)^2 \arctan(ax)^{3/2}} \, dx = \int \frac{x^4}{\operatorname{atan}(a\,x)^{3/2}\,(c\,a^2\,x^2+c)^2} \, dx$$

input
```
int(x^4/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^2),x)
```

output
```
int(x^4/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.12

$$\int \frac{x^4}{(c+a^2cx^2)^2 \arctan(ax)^{3/2}} \, dx = \frac{\int \frac{\sqrt{atan(ax)}\,x^4}{atan(ax)^2 a^4 x^4+2 atan(ax)^2 a^2 x^2+atan(ax)^2} dx}{c^2}$$

input
```
int(x^4/(a^2*c*x^2+c)^2/atan(a*x)^(3/2),x)
```

output
```
int((sqrt(atan(a*x))*x***4)/(atan(a*x)**2*a**4*x***4 + 2*atan(a*x)**2*a**2*x
**2 + atan(a*x)**2),x)/c**2
```

# 3.989 $\int \frac{x^3}{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^3}{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}\,dx = -\frac{2x^3}{ac^2(1+a^2x^2)\sqrt{\arctan(ax)}} + \frac{6\sqrt{\arctan(ax)}}{a^4c^2} - \frac{3\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{a^4c^2} + 2a\text{Int}\left(\frac{x^4}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}},x\right)$$

output
```
-2*x^3/a/c^2/(a^2*x^2+1)/arctan(a*x)^(1/2)+6*arctan(a*x)^(1/2)/a^4/c^2-3*P
i^(1/2)*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))/a^4/c^2+2*a*Defer(Int)(x^4/
(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 5.59 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^3}{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}\,dx = \int \frac{x^3}{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[x^3/((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[x^3/((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.84 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^2}dx$$

$$\Big\downarrow \text{5503}$$

$$\frac{6\int \frac{x^2}{c^2\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{a}+2a\int \frac{x^4}{c^2\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx-\frac{2x^3}{ac^2\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{6\int \frac{x^2}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{ac^2}+\frac{2a\int \frac{x^4}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{c^2}-\frac{2x^3}{ac^2\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{5505}$$

$$\frac{2a\int \frac{x^4}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{c^2}+\frac{6\int \frac{a^2x^2}{\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}d\arctan(ax)}{a^4c^2}-\frac{2x^3}{ac^2\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{6\int \frac{\sin(\arctan(ax))^2}{\sqrt{\arctan(ax)}}d\arctan(ax)}{a^4c^2}+\frac{2a\int \frac{x^4}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{c^2}-\frac{2x^3}{ac^2\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{3793}$$

$$\frac{6\int \left(\frac{1}{2\sqrt{\arctan(ax)}}-\frac{\cos(2\arctan(ax))}{2\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^4c^2}+\frac{2a\int \frac{x^4}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx}{c^2}-\frac{2x^3}{ac^2\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \ 2009$$

$$\frac{2a\int \frac{x^4}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx}{c^2}+\frac{6\left(\sqrt{\arctan(ax)}-\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)\right)}{a^4c^2}-\frac{2x^3}{ac^2\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \ 5560$$

$$\frac{2a\int \frac{x^4}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx}{c^2}+\frac{6\left(\sqrt{\arctan(ax)}-\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)\right)}{a^4c^2}-\frac{2x^3}{ac^2\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}$$

input
```
Int[x^3/((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^3}{\left(a^2c\,x^2+c\right)^2 \arctan\left(ax\right)^{\frac{3}{2}}}dx$$

input
```
int(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

output
```
int(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3}{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 3.55 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.00

$$\int \frac{x^3}{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}\,dx = \frac{\int \frac{x^3}{a^4x^4 \operatorname{atan}^{\frac{3}{2}}(ax)+2a^2x^2 \operatorname{atan}^{\frac{3}{2}}(ax)+\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx}{c^2}$$

input
```
integrate(x**3/(a**2*c*x**2+c)**2/atan(a*x)**(3/2),x)
```

output
```
Integral(x**3/(a**4*x**4*atan(a*x)**(3/2) + 2*a**2*x**2*atan(a*x)**(3/2) +
 atan(a*x)**(3/2)), x)/c**2
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}\,dx = \int \frac{x^3}{(a^2cx^2+c)^2 \arctan{(ax)}^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^3/((a^2*c*x^2 + c)^2*arctan(a*x)^(3/2)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.77 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}\,dx = \int \frac{x^3}{\mathrm{atan}\left(a\,x\right)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^2}\,dx$$

input
```
int(x^3/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^2),x)
```

output
```
int(x^3/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.12

$$\int \frac{x^3}{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}\,dx = \frac{\int \frac{\sqrt{atan(ax)}\,x^3}{atan(ax)^2a^4x^4+2atan(ax)^2a^2x^2+atan(ax)^2}dx}{c^2}$$

input
```
int(x^3/(a^2*c*x^2+c)^2/atan(a*x)^(3/2),x)
```

output
```
int((sqrt(atan(a*x))*x**3)/(atan(a*x)**2*a**4*x**4 + 2*atan(a*x)**2*a**2*x
**2 + atan(a*x)**2),x)/c**2
```

# 3.990 $\int \frac{x^2}{(c+a^2cx^2)^2 \arctan(ax)^{3/2}} dx$



## Optimal result

Integrand size = 24, antiderivative size = 60

$$\int \frac{x^2}{(c+a^2cx^2)^2 \arctan(ax)^{3/2}} dx = -\frac{2x^2}{ac^2(1+a^2x^2)\sqrt{\arctan(ax)}} + \frac{2\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{a^3c^2}$$

output
```
-2*x^2/a/c^2/(a^2*x^2+1)/arctan(a*x)^(1/2)+2*Pi^(1/2)*FresnelS(2*arctan(a*
x)^(1/2)/Pi^(1/2))/a^3/c^2
```

## Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(c+a^2cx^2)^2 \arctan(ax)^{3/2}} dx = -\frac{2x^2}{ac^2(1+a^2x^2)\sqrt{\arctan(ax)}} + \frac{2\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{a^3c^2}$$

input
```
Integrate[x^2/((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
```

output
```
(-2*x^2)/(a*c^2*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]]) + (2*Sqrt[Pi]*FresnelS[(2
*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(a^3*c^2)
```

## Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5477, 27, 5505, 4906, 27, 3042, 3786, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^2}dx$$

$$\Big\downarrow \text{5477}$$

$$\frac{4\int \frac{x}{c^2(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx}{a}-\frac{2x^2}{ac^2\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{4\int \frac{x}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx}{ac^2}-\frac{2x^2}{ac^2\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{5505}$$

$$\frac{4\int \frac{ax}{(a^2x^2+1)\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3c^2}-\frac{2x^2}{ac^2\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{4906}$$

$$\frac{4\int \frac{\sin(2\arctan(ax))}{2\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3c^2}-\frac{2x^2}{ac^2\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\int \frac{\sin(2\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3c^2}-\frac{2x^2}{ac^2\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\int \frac{\sin(2\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3c^2}-\frac{2x^2}{ac^2\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}$$

$\Big\downarrow$ 3786

$$\frac{4\int \sin(2\arctan(ax))d\sqrt{\arctan(ax)}}{a^3c^2}-\frac{2x^2}{ac^2\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}$$

$\Big\downarrow$ 3832

$$\frac{2\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{a^3c^2}-\frac{2x^2}{ac^2\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}$$

input
```
Int[x^2/((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
```

output
```
(-2*x^2)/(a*c^2*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]]) + (2*Sqrt[Pi]*FresnelS[(2
*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/(a^3*c^2)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5477
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*ArcT
an[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp[f*(m/(b*c*(p + 1)))   Int[(f*x
)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1
]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

## Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.77

| method | result | size |
|---|---|---|
| default | $\frac{2\sqrt{\arctan \left(a x \right)}\, \sqrt{\pi}\, \operatorname{FresnelS}\left(\frac{2\sqrt{\arctan \left(a x \right)}}{\sqrt{\pi}}\right)+\cos \left(2 \arctan \left(a x \right)\right)-1}{a^{3} c^{2} \sqrt{\arctan \left(a x \right)}}$ | 46 |

input
```
int(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/a^3/c^2*(2*arctan(a*x)^(1/2)*Pi^(1/2)*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1
/2))+cos(2*arctan(a*x))-1)/arctan(a*x)^(1/2)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2}{\left(c+a^2cx^2\right)^2\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{x^2}{\left(c+a^2cx^2\right)^2\arctan(ax)^{3/2}}\,dx = \frac{\int \frac{x^2}{a^4x^4\operatorname{atan}^{\frac{3}{2}}(ax)+2a^2x^2\operatorname{atan}^{\frac{3}{2}}(ax)+\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx}{c^2}$$

input
```
integrate(x**2/(a**2*c*x**2+c)**2/atan(a*x)**(3/2),x)
```

output
```
Integral(x**2/(a**4*x**4*atan(a*x)**(3/2) + 2*a**2*x**2*atan(a*x)**(3/2) +
 atan(a*x)**(3/2)), x)/c**2
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{\left(c+a^2cx^2\right)^2\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F]

$$\int \frac{x^2}{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}\,dx = \int \frac{x^2}{(a^2cx^2+c)^2 \arctan(ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^2/((a^2*c*x^2 + c)^2*arctan(a*x)^(3/2)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}\,dx = \int \frac{x^2}{\mathrm{atan}(a\,x)^{3/2}\,(c\,a^2\,x^2+c)^2}\,dx$$

input
```
int(x^2/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^2),x)
```

output
```
int(x^2/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^2), x)
```

## Reduce [F]

$$\int \frac{x^2}{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}\,dx = \frac{\int \frac{\sqrt{atan(ax)}\,x^2}{atan(ax)^2a^4x^4+2atan(ax)^2a^2x^2+atan(ax)^2}dx}{c^2}$$

input
```
int(x^2/(a^2*c*x^2+c)^2/atan(a*x)^(3/2),x)
```

output
```
int((sqrt(atan(a*x))*x**2)/(atan(a*x)**2*a**4*x**4 + 2*atan(a*x)**2*a**2*x
**2 + atan(a*x)**2),x)/c**2
```

# 3.991 $\int \frac{x}{(c+a^2cx^2)^2 \arctan(ax)^{3/2}} dx$



## Optimal result

Integrand size = 22, antiderivative size = 138

$$\int \frac{x}{(c+a^2cx^2)^2 \arctan(ax)^{3/2}} dx = -\frac{2x}{ac^2(1+a^2x^2)\sqrt{\arctan(ax)}} + \frac{4\sqrt{\arctan(ax)}}{a^2c^2} - \frac{8\sqrt{\arctan(ax)}}{a^2c^2(1+a^2x^2)} + \frac{4(1-a^2x^2)\sqrt{\arctan(ax)}}{a^2c^2(1+a^2x^2)} + \frac{2\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{a^2c^2}$$

output
```
-2*x/a/c^2/(a^2*x^2+1)/arctan(a*x)^(1/2)+4*arctan(a*x)^(1/2)/a^2/c^2-8*arc
tan(a*x)^(1/2)/a^2/c^2/(a^2*x^2+1)+4*(-a^2*x^2+1)*arctan(a*x)^(1/2)/a^2/c^
2/(a^2*x^2+1)+2*Pi^(1/2)*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))/a^2/c^2
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.14

$$\int \frac{x}{(c+a^2cx^2)^2 \arctan(ax)^{3/2}} dx = \frac{-8ax + 4\sqrt{\pi}(1+a^2x^2)\sqrt{\arctan(ax)}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right) - i\sqrt{2}(1}{4}$$

input
```
Integrate[x/((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
```

output
```
(-8*a*x + 4*Sqrt[Pi]*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]]*FresnelC[(2*Sqrt[ArcT
an[a*x]])/Sqrt[Pi]] - I*Sqrt[2]*(1 + a^2*x^2)*Sqrt[(-I)*ArcTan[a*x]]*Gamma
[1/2, (-2*I)*ArcTan[a*x]] + I*Sqrt[2]*(1 + a^2*x^2)*Sqrt[I*ArcTan[a*x]]*Ga
mma[1/2, (2*I)*ArcTan[a*x]])/(4*a^2*c^2*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]])
```

## Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {5467, 27, 5465, 5439, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^2}dx$$

$$\Big\downarrow \text{ 5467}$$

$$16\int \frac{x\sqrt{\arctan(ax)}}{c^2\left(a^2x^2+1\right)^2}dx-\frac{2x}{ac^2\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}+\frac{4\left(1-a^2x^2\right)\sqrt{\arctan(ax)}}{a^2c^2\left(a^2x^2+1\right)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{16\int \frac{x\sqrt{\arctan(ax)}}{\left(a^2x^2+1\right)^2}dx}{c^2}-\frac{2x}{ac^2\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}+\frac{4\left(1-a^2x^2\right)\sqrt{\arctan(ax)}}{a^2c^2\left(a^2x^2+1\right)}$$

$$\Big\downarrow \text{ 5465}$$

$$\frac{16\left(\frac{\int \frac{1}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{4a}-\frac{\sqrt{\arctan(ax)}}{2a^2\left(a^2x^2+1\right)}\right)}{c^2}-\frac{2x}{ac^2\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}+\frac{4\left(1-a^2x^2\right)\sqrt{\arctan(ax)}}{a^2c^2\left(a^2x^2+1\right)}$$

$$\Big\downarrow \text{ 5439}$$

$$\frac{16\left(\frac{\int \frac{1}{(a^2x^2+1)\sqrt{\arctan(ax)}}d\arctan(ax)}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)}{c^2}-\frac{2x}{ac^2(a^2x^2+1)\sqrt{\arctan(ax)}}+\frac{4(1-a^2x^2)\sqrt{\arctan(ax)}}{a^2c^2(a^2x^2+1)}$$

$$\Big\downarrow \; 3042$$

$$\frac{16\left(\frac{\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^2}{\sqrt{\arctan(ax)}}d\arctan(ax)}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)}{c^2}-\frac{2x}{ac^2(a^2x^2+1)\sqrt{\arctan(ax)}}+\frac{4(1-a^2x^2)\sqrt{\arctan(ax)}}{a^2c^2(a^2x^2+1)}$$

$$\Big\downarrow \; 3793$$

$$\frac{16\left(\frac{\int \left(\frac{\cos(2\arctan(ax))}{2\sqrt{\arctan(ax)}}+\frac{1}{2\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)}{c^2}-\frac{2x}{ac^2(a^2x^2+1)\sqrt{\arctan(ax)}}+\frac{4(1-a^2x^2)\sqrt{\arctan(ax)}}{a^2c^2(a^2x^2+1)}$$

$$\Big\downarrow \; 2009$$

$$\frac{16\left(\frac{\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\sqrt{\arctan(ax)}}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)}{c^2}-\frac{2x}{ac^2(a^2x^2+1)\sqrt{\arctan(ax)}}+\frac{4(1-a^2x^2)\sqrt{\arctan(ax)}}{a^2c^2(a^2x^2+1)}$$

input
```
Int[x/((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
```

output
```
(-2*x)/(a*c^2*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]]) + (4*(1 - a^2*x^2)*Sqrt[Arc
Tan[a*x]])/(a^2*c^2*(1 + a^2*x^2)) + (16*(-1/2*Sqrt[ArcTan[a*x]]/(a^2*(1 +
 a^2*x^2)) + (Sqrt[ArcTan[a*x]] + (Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])
/Sqrt[Pi]])/2)/(4*a^2)))/c^2
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5467
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*(x_))/((d_) + (e_.)*(x_)^2)^2,
 x_Symbol] :> Simp[x*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)*(d + e*x^2
))), x] + (-Simp[(1 - c^2*x^2)*((a + b*ArcTan[c*x])^(p + 2)/(b^2*e*(p + 1)*
(p + 2)*(d + e*x^2))), x] - Simp[4/(b^2*(p + 1)*(p + 2))   Int[x*((a + b*Ar
cTan[c*x])^(p + 2)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[e, c^2*d] && LtQ[p, -1] && NeQ[p, -2]
```

## Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.33

| method | result | size |
|---|---|---|
| default | $-\frac{-2\sqrt{\arctan(ax)}\,\sqrt{\pi}\,\mathrm{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\sin(2\arctan(ax))}{a^2c^2\sqrt{\arctan(ax)}}$ | 46 |

input
```
int(x/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/a^2/c^2/arctan(a*x)^(1/2)*(-2*arctan(a*x)^(1/2)*Pi^(1/2)*FresnelC(2*arc
tan(a*x)^(1/2)/Pi^(1/2))+sin(2*arctan(a*x)))
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)^2\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{x}{(c+a^2cx^2)^2\arctan(ax)^{3/2}}\,dx = \frac{\int \frac{x}{a^4x^4\operatorname{atan}^{\frac{3}{2}}(ax)+2a^2x^2\operatorname{atan}^{\frac{3}{2}}(ax)+\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx}{c^2}$$

input
```
integrate(x/(a**2*c*x**2+c)**2/atan(a*x)**(3/2),x)
```

output
```
Integral(x/(a**4*x**4*atan(a*x)**(3/2) + 2*a**2*x**2*atan(a*x)**(3/2) + at
an(a*x)**(3/2)), x)/c**2
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{\left(c+a^2cx^2\right)^2\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F]

$$\int \frac{x}{\left(c+a^2cx^2\right)^2\arctan(ax)^{3/2}}\,dx = \int \frac{x}{\left(a^2cx^2+c\right)^2\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(x/((a^2*c*x^2 + c)^2*arctan(a*x)^(3/2)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(c+a^2cx^2\right)^2\arctan(ax)^{3/2}}\,dx = \int \frac{x}{\mathrm{atan}\left(a\,x\right)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^2}\,dx$$

input
```
int(x/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^2),x)
```

output
```
int(x/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^2), x)
```

## Reduce [F]

$$\int \frac{x}{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}\,dx = \frac{-4\sqrt{atan\,(ax)}\,atan(ax)\,a^2x^2 - 4\sqrt{atan\,(ax)}\,atan(ax) + 4atan(ax)\left(\int\right.}{}$$

input
```
int(x/(a^2*c*x^2+c)^2/atan(a*x)^(3/2),x)
```

output
```
(2*( - 2*sqrt(atan(a*x))*atan(a*x)*a**2*x**2 - 2*sqrt(atan(a*x))*atan(a*x)
 + 2*atan(a*x)*int(sqrt(atan(a*x))/(atan(a*x)*a**4*x**4 + 2*atan(a*x)*a**2
*x**2 + atan(a*x)),x)*a**3*x**2 + 2*atan(a*x)*int(sqrt(atan(a*x))/(atan(a*
x)*a**4*x**4 + 2*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a - sqrt(atan(a*x))*a
*x))/(atan(a*x)*a**2*c**2*(a**2*x**2 + 1))
```

# 3.992 $\int \frac{1}{\left(c+a^2cx^2\right)^2 \arctan(ax)^{3/2}}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 57

$$\int \frac{1}{\left(c+a^2cx^2\right)^2 \arctan(ax)^{3/2}}\,dx = -\frac{2}{ac^2\left(1+a^2x^2\right)\sqrt{\arctan(ax)}} - \frac{2\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{ac^2}$$

output
```
-2/a/c^2/(a^2*x^2+1)/arctan(a*x)^(1/2)-2*Pi^(1/2)*FresnelS(2*arctan(a*x)^(
1/2)/Pi^(1/2))/a/c^2
```

## Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(c+a^2cx^2\right)^2 \arctan(ax)^{3/2}}\,dx = \frac{-\frac{2}{(1+a^2x^2)\sqrt{\arctan(ax)}} - 2\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{ac^2}$$

input
```
Integrate[1/((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
```

output
```
(-2/((1 + a^2*x^2)*Sqrt[ArcTan[a*x]]) - 2*Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan
[a*x]])/Sqrt[Pi]])/(a*c^2)
```

## Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {5437, 27, 5505, 4906, 27, 3042, 3786, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^2}dx$$

$$\Big\downarrow \text{ 5437}$$

$$-4a\int \frac{x}{c^2\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx-\frac{2}{ac^2\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{4a\int \frac{x}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{c^2}-\frac{2}{ac^2\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{ 5505}$$

$$-\frac{4\int \frac{ax}{\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}d\arctan(ax)}{ac^2}-\frac{2}{ac^2\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{ 4906}$$

$$-\frac{4\int \frac{\sin(2\arctan(ax))}{2\sqrt{\arctan(ax)}}d\arctan(ax)}{ac^2}-\frac{2}{ac^2\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{2\int \frac{\sin(2\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)}{ac^2}-\frac{2}{ac^2\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{2\int \frac{\sin(2\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)}{ac^2}-\frac{2}{ac^2\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{ 3786}$$

$$-\frac{4\int \sin(2\arctan(ax))d\sqrt{\arctan(ax)}}{ac^2}-\frac{2}{ac^2\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{ 3832}$$

$$-\frac{2}{ac^2\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}-\frac{2\sqrt{\pi}\operatorname{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{ac^2}$$

input
```
Int[1/((c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
```

output
```
-2/(a*c^2*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]]) - (2*Sqrt[Pi]*FresnelS[(2*Sqrt[
ArcTan[a*x]])/Sqrt[Pi]])/(a*c^2)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

## Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82

| method | result | size |
|---|---|---|
| default | $-\frac{2\sqrt{\arctan\left(a x\right)}\,\sqrt{\pi}\,\operatorname{FresnelS}\left(\frac{2\sqrt{\arctan\left(a x\right)}}{\sqrt{\pi}}\right)+\cos\left(2\arctan\left(a x\right)\right)+1}{a\,c^{2}\sqrt{\arctan\left(a x\right)}}$ | 47 |

input
```
int(1/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/a/c^2*(2*arctan(a*x)^(1/2)*Pi^(1/2)*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/
2))+cos(2*arctan(a*x))+1)/arctan(a*x)^(1/2)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{1}{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}\,dx = \frac{\int \frac{1}{a^4x^4 \operatorname{atan}^{\frac{3}{2}}(ax)+2a^2x^2 \operatorname{atan}^{\frac{3}{2}}(ax)+\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx}{c^2}$$

input
```
integrate(1/(a**2*c*x**2+c)**2/atan(a*x)**(3/2),x)
```

output
```
Integral(1/(a**4*x**4*atan(a*x)**(3/2) + 2*a**2*x**2*atan(a*x)**(3/2) + at
an(a*x)**(3/2)), x)/c**2
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F]

$$\int \frac{1}{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}\,dx = \int \frac{1}{(a^2cx^2+c)^2 \arctan(ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^2*arctan(a*x)^(3/2)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}\,dx = \int \frac{1}{\mathrm{atan}(a\,x)^{3/2}\,(c\,a^2\,x^2+c)^2}\,dx$$

input
```
int(1/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^2),x)
```

output
```
int(1/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^2), x)
```

## Reduce [F]

$$\int \frac{1}{(c+a^2cx^2)^2 \arctan(ax)^{3/2}}\,dx = \frac{-\frac{16\sqrt{atan(ax)}\,atan(ax)^2a^2x^2}{3} - \frac{16\sqrt{atan(ax)}\,atan(ax)^2}{3} - 8\sqrt{atan\,(ax)}\,atan(ax)}{a}$$

input
```
int(1/(a^2*c*x^2+c)^2/atan(a*x)^(3/2),x)
```

output
```
(2*( - 8*sqrt(atan(a*x))*atan(a*x)**2*a**2*x**2 - 8*sqrt(atan(a*x))*atan(a
*x)**2 - 12*sqrt(atan(a*x))*atan(a*x)*a*x + 24*atan(a*x)*int(sqrt(atan(a*x
))/(a**4*x**4 + 2*a**2*x**2 + 1),x)*a**3*x**2 + 24*atan(a*x)*int(sqrt(atan
(a*x))/(a**4*x**4 + 2*a**2*x**2 + 1),x)*a - 3*sqrt(atan(a*x))))/(3*atan(a*
x)*a*c**2*(a**2*x**2 + 1))
```

# 3.993 $\int \frac{1}{x(c+a^2cx^2)^2 \arctan(ax)^{3/2}} dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x(c+a^2cx^2)^2 \arctan(ax)^{3/2}} dx = -\frac{2}{ac^2x(1+a^2x^2)\sqrt{\arctan(ax)}} - \frac{6\sqrt{\arctan(ax)}}{c^2} - \frac{3\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{c^2} - \frac{2\text{Int}\left(\frac{1}{x^2(c+a^2cx^2)^2\sqrt{\arctan(ax)}},x\right)}{a}$$

output
```
-2/a/c^2/x/(a^2*x^2+1)/arctan(a*x)^(1/2)-6*arctan(a*x)^(1/2)/c^2-3*Pi^(1/2
)*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))/c^2-2*Defer(Int)(1/x^2/(a^2*c*x^2
+c)^2/arctan(a*x)^(1/2),x)/a
```

## Mathematica [N/A]

Not integrable

Time = 2.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x(c+a^2cx^2)^2 \arctan(ax)^{3/2}} dx = \int \frac{1}{x(c+a^2cx^2)^2 \arctan(ax)^{3/2}} dx$$

input
```
Integrate[1/(x*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[1/(x*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x \arctan(ax)^{3/2} \left(a^2cx^2+c\right)^2}dx$$

$$\Big\downarrow \text{5503}$$

$$-6a\int \frac{1}{c^2\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx-\frac{2\int \frac{1}{c^2x^2\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ac^2x\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{6a\int \frac{1}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{c^2}-\frac{2\int \frac{1}{x^2\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{ac^2}-\frac{2}{ac^2x\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{5439}$$

$$-\frac{2\int \frac{1}{x^2\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{ac^2}-\frac{6\int \frac{1}{\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}d\arctan(ax)}{c^2}-\frac{2}{ac^2x\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\int \frac{1}{x^2\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{ac^2}-\frac{6\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^2}{\sqrt{\arctan(ax)}}d\arctan(ax)}{c^2}-\frac{2}{ac^2x\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{2\int \frac{1}{x^2(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx}{ac^2}-\frac{6\int \left(\frac{\cos(2\arctan(ax))}{2\sqrt{\arctan(ax)}}+\frac{1}{2\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{c^2}-\frac{2}{ac^2x\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2\int \frac{1}{x^2(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx}{ac^2}-\frac{2}{ac^2x\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}-\frac{6\left(\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\sqrt{\arctan(ax)}\right)}{c^2}$$

$$\Big\downarrow \text{5560}$$

$$-\frac{2\int \frac{1}{x^2(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx}{ac^2}-\frac{2}{ac^2x\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}-\frac{6\left(\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\sqrt{\arctan(ax)}\right)}{c^2}$$

input
```
Int[1/(x*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\left(a^2c\,x^2+c\right)^2\arctan\left(ax\right)^{\frac{3}{2}}}dx$$

input
```
int(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

output
```
int(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 4.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\arctan(ax)^{3/2}}\,dx = \frac{\int \frac{1}{a^4x^5\operatorname{atan}^{\frac{3}{2}}(ax)+2a^2x^3\operatorname{atan}^{\frac{3}{2}}(ax)+x\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx}{c^2}$$

input
```
integrate(1/x/(a**2*c*x**2+c)**2/atan(a*x)**(3/2),x)
```

output
```
Integral(1/(a**4*x**5*atan(a*x)**(3/2) + 2*a**2*x**3*atan(a*x)**(3/2) + x*
atan(a*x)**(3/2)), x)/c**2
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\arctan(ax)^{3/2}}\,dx=\int \frac{1}{\left(a^2cx^2+c\right)^2x\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^2*x*arctan(a*x)^(3/2)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\arctan(ax)^{3/2}}\,dx=\int \frac{1}{x\,\mathrm{atan}\left(a\,x\right)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^2}\,dx$$

input
```
int(1/(x*atan(a*x)^(3/2)*(c + a^2*c*x^2)^2),x)
```

output
```
int(1/(x*atan(a*x)^(3/2)*(c + a^2*c*x^2)^2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.08

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\arctan(ax)^{3/2}}\,dx = \frac{\int \frac{\sqrt{atan(ax)}}{atan(ax)^2a^4x^5+2atan(ax)^2a^2x^3+atan(ax)^2x}dx}{c^2}$$

input
```
int(1/x/(a^2*c*x^2+c)^2/atan(a*x)^(3/2),x)
```

output
```
int(sqrt(atan(a*x))/(atan(a*x)**2*a**4*x**5 + 2*atan(a*x)**2*a**2*x**3 + a
tan(a*x)**2*x),x)/c**2
```

# 3.994 $\int \frac{1}{x^2(c+a^2cx^2)^2 \arctan(ax)^{3/2}} \, dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x^2 (c+a^2cx^2)^2 \arctan(ax)^{3/2}} \, dx = -\frac{2}{ac^2x^2(1+a^2x^2)\sqrt{\arctan(ax)}} - \frac{4\text{Int}\left(\frac{1}{x^3(c+a^2cx^2)^2\sqrt{\arctan(ax)}}, x\right)}{a} - 8a\text{Int}\left(\frac{1}{x(c+a^2cx^2)^2\sqrt{\arctan(ax)}}, x\right)$$

output
```
-2/a/c^2/x^2/(a^2*x^2+1)/arctan(a*x)^(1/2)-4*Defer(Int)(1/x^3/(a^2*c*x^2+c
)^2/arctan(a*x)^(1/2),x)/a-8*a*Defer(Int)(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^
(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 3.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^2 (c+a^2cx^2)^2 \arctan(ax)^{3/2}} \, dx = \int \frac{1}{x^2 (c+a^2cx^2)^2 \arctan(ax)^{3/2}} \, dx$$

input
```
Integrate[1/(x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[1/(x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \arctan(ax)^{3/2} \left(a^2cx^2+c\right)^2}dx$$

$$\Big\downarrow \text{ 5503}$$

$$-8a\int \frac{1}{c^2x\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx-\frac{4\int \frac{1}{c^2x^3(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ac^2x^2\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{8a\int \frac{1}{x(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx}{c^2}-\frac{4\int \frac{1}{x^3(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx}{ac^2}-\frac{2}{ac^2x^2\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{ 5560}$$

$$-\frac{8a\int \frac{1}{x(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx}{c^2}-\frac{4\int \frac{1}{x^3(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx}{ac^2}-\frac{2}{ac^2x^2\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}$$

input
```
Int[1/(x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^{2} \left(a^{2} c \,x^{2}+c \right)^{2} \arctan \left(a x \right)^{\frac{3}{2}}}d x$$

input
```
int(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

output
```
int(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^2 (c+a^2cx^2)^2 \arctan(ax)^{3/2}}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 6.34 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.04

$$\int \frac{1}{x^2 (c+a^2cx^2)^2 \arctan(ax)^{3/2}}\, dx = \frac{\int \frac{1}{a^{4} x^{6} \operatorname{atan}^{\frac{3}{2}}{\left(a x \right)} + 2 a^{2} x^{4} \operatorname{atan}^{\frac{3}{2}}{\left(a x \right)} + x^{2} \operatorname{atan}^{\frac{3}{2}}{\left(a x \right)}}\, dx}{c^{2}}$$

input
```
integrate(1/x**2/(a**2*c*x**2+c)**2/atan(a*x)**(3/2),x)
```

output
```
Integral(1/(a**4*x**6*atan(a*x)**(3/2) + 2*a**2*x**4*atan(a*x)**(3/2) + x*
*2*atan(a*x)**(3/2)), x)/c**2
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^2 (c + a^2 c x^2)^2 \arctan(ax)^{3/2}} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (c + a^2 c x^2)^2 \arctan(ax)^{3/2}} \, dx = \int \frac{1}{(a^2 c x^2 + c)^2 x^2 \arctan(ax)^{\frac{3}{2}}} \, dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^2*x^2*arctan(a*x)^(3/2)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.83 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (c + a^2cx^2)^2 \arctan(ax)^{3/2}} \, dx = \int \frac{1}{x^2 \operatorname{atan}(a\,x)^{3/2} \left(c\,a^2\,x^2 + c\right)^2} \, dx$$

input
```
int(1/(x^2*atan(a*x)^(3/2)*(c + a^2*c*x^2)^2),x)
```

output
```
int(1/(x^2*atan(a*x)^(3/2)*(c + a^2*c*x^2)^2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.17

$$\int \frac{1}{x^2 (c + a^2cx^2)^2 \arctan(ax)^{3/2}} \, dx = \frac{\int \frac{\sqrt{atan(ax)}}{atan(ax)^2 a^4 x^6 + 2 atan(ax)^2 a^2 x^4 + atan(ax)^2 x^2} dx}{c^2}$$

input
```
int(1/x^2/(a^2*c*x^2+c)^2/atan(a*x)^(3/2),x)
```

output
```
int(sqrt(atan(a*x))/(atan(a*x)**2*a**4*x**6 + 2*atan(a*x)**2*a**2*x**4 + a
tan(a*x)**2*x**2),x)/c**2
```

# 3.995 $\int \frac{1}{x^3(c+a^2cx^2)^2 \arctan(ax)^{3/2}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x^3 (c+a^2cx^2)^2 \arctan(ax)^{3/2}}\,dx = -\frac{2}{ac^2x^3(1+a^2x^2)\sqrt{\arctan(ax)}} - \frac{6\text{Int}\left(\frac{1}{x^4(c+a^2cx^2)^2\sqrt{\arctan(ax)}},x\right)}{a} - 10a\text{Int}\left(\frac{1}{x^2(c+a^2cx^2)^2\sqrt{\arctan(ax)}},x\right)$$

output
```
-2/a/c^2/x^3/(a^2*x^2+1)/arctan(a*x)^(1/2)-6*Defer(Int)(1/x^4/(a^2*c*x^2+c
)^2/arctan(a*x)^(1/2),x)/a-10*a*Defer(Int)(1/x^2/(a^2*c*x^2+c)^2/arctan(a*
x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 6.45 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^3 (c+a^2cx^2)^2 \arctan(ax)^{3/2}}\,dx = \int \frac{1}{x^3 (c+a^2cx^2)^2 \arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[1/(x^3*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[1/(x^3*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \arctan(ax)^{3/2} (a^2cx^2+c)^2}dx$$
$$\Big\downarrow \text{5503}$$
$$-10a\int \frac{1}{c^2x^2(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx-\frac{6\int \frac{1}{c^2x^4(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ac^2x^3(a^2x^2+1)\sqrt{\arctan(ax)}}$$
$$\Big\downarrow \text{27}$$
$$-\frac{10a\int \frac{1}{x^2(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx}{c^2}-\frac{6\int \frac{1}{x^4(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx}{ac^2}-\frac{2}{ac^2x^3(a^2x^2+1)\sqrt{\arctan(ax)}}$$
$$\Big\downarrow \text{5560}$$
$$-\frac{10a\int \frac{1}{x^2(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx}{c^2}-\frac{6\int \frac{1}{x^4(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx}{ac^2}-\frac{2}{ac^2x^3(a^2x^2+1)\sqrt{\arctan(ax)}}$$

input
```
Int[1/(x^3*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^{3} \left(a^{2} c \,x^{2}+c \right)^{2} \arctan \left(a x \right)^{\frac{3}{2}}}d x$$

input
```
int(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

output
```
int(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^3 (c+a^2cx^2)^2 \arctan(ax)^{3/2}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 8.48 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.04

$$\int \frac{1}{x^3 (c+a^2cx^2)^2 \arctan(ax)^{3/2}} \, dx = \frac{\int \frac{1}{a^{4} x^{7} \operatorname{atan}^{\frac{3}{2}}{\left(a x \right)} + 2 a^{2} x^{5} \operatorname{atan}^{\frac{3}{2}}{\left(a x \right)} + x^{3} \operatorname{atan}^{\frac{3}{2}}{\left(a x \right)}}\, dx}{c^{2}}$$

input
```
integrate(1/x**3/(a**2*c*x**2+c)**2/atan(a*x)**(3/2),x)
```

output
```
Integral(1/(a**4*x**7*atan(a*x)**(3/2) + 2*a**2*x**5*atan(a*x)**(3/2) + x*
*3*atan(a*x)**(3/2)), x)/c**2
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^3 (c + a^2 c x^2)^2 \arctan(ax)^{3/2}} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3 (c + a^2 c x^2)^2 \arctan(ax)^{3/2}} \, dx = \int \frac{1}{(a^2 c x^2 + c)^2 x^3 \arctan(ax)^{\frac{3}{2}}} \, dx$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^2*x^3*arctan(a*x)^(3/2)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3 (c + a^2cx^2)^2 \arctan(ax)^{3/2}} \, dx = \int \frac{1}{x^3 \operatorname{atan}(a\,x)^{3/2} (c\,a^2\,x^2 + c)^2} \, dx$$

input
```
int(1/(x^3*atan(a*x)^(3/2)*(c + a^2*c*x^2)^2),x)
```

output
```
int(1/(x^3*atan(a*x)^(3/2)*(c + a^2*c*x^2)^2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.17

$$\int \frac{1}{x^3 (c + a^2cx^2)^2 \arctan(ax)^{3/2}} \, dx = \frac{\int \frac{\sqrt{atan(ax)}}{atan(ax)^2 a^4 x^7 + 2atan(ax)^2 a^2 x^5 + atan(ax)^2 x^3} dx}{c^2}$$

input
```
int(1/x^3/(a^2*c*x^2+c)^2/atan(a*x)^(3/2),x)
```

output
```
int(sqrt(atan(a*x))/(atan(a*x)**2*a**4*x**7 + 2*atan(a*x)**2*a**2*x**5 + a
tan(a*x)**2*x**3),x)/c**2
```

# 3.996 $\int \frac{1}{x^4\left(c+a^2cx^2\right)^2\arctan(ax)^{3/2}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^2\arctan(ax)^{3/2}}\,dx = -\frac{2}{ac^2x^4\left(1+a^2x^2\right)\sqrt{\arctan(ax)}} - \frac{8\text{Int}\left(\frac{1}{x^5\left(c+a^2cx^2\right)^2\sqrt{\arctan(ax)}},x\right)}{a} - 12a\text{Int}\left(\frac{1}{x^3\left(c+a^2cx^2\right)^2\sqrt{\arctan(ax)}},x\right)$$

output
```
-2/a/c^2/x^4/(a^2*x^2+1)/arctan(a*x)^(1/2)-8*Defer(Int)(1/x^5/(a^2*c*x^2+c
)^2/arctan(a*x)^(1/2),x)/a-12*a*Defer(Int)(1/x^3/(a^2*c*x^2+c)^2/arctan(a*
x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 6.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^2\arctan(ax)^{3/2}}\,dx = \int \frac{1}{x^4\left(c+a^2cx^2\right)^2\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[1/(x^4*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[1/(x^4*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \arctan(ax)^{3/2} (a^2cx^2+c)^2}dx$$

$$\Big\downarrow \text{5503}$$

$$-\frac{8\int \frac{1}{c^2x^5(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx}{a}-12a\int \frac{1}{c^2x^3(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx-\frac{2}{ac^2x^4(a^2x^2+1)\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{8\int \frac{1}{x^5(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx}{ac^2}-\frac{12a\int \frac{1}{x^3(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx}{c^2}-\frac{2}{ac^2x^4(a^2x^2+1)\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{5560}$$

$$-\frac{8\int \frac{1}{x^5(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx}{ac^2}-\frac{12a\int \frac{1}{x^3(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx}{c^2}-\frac{2}{ac^2x^4(a^2x^2+1)\sqrt{\arctan(ax)}}$$

input
```
Int[1/(x^4*(c + a^2*c*x^2)^2*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^4 \left(a^2 c\, x^2+c\right)^2 \arctan \left(a x\right)^{\frac{3}{2}}}dx$$

input
```
int(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

output
```
int(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^4 (c+a^2cx^2)^2 \arctan(ax)^{3/2}}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 11.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.04

$$\int \frac{1}{x^4 (c+a^2cx^2)^2 \arctan(ax)^{3/2}}\, dx = \frac{\int \frac{1}{a^4x^8 \operatorname{atan}^{\frac{3}{2}}(ax)+2a^2x^6 \operatorname{atan}^{\frac{3}{2}}(ax)+x^4 \operatorname{atan}^{\frac{3}{2}}(ax)}\, dx}{c^2}$$

input
```
integrate(1/x**4/(a**2*c*x**2+c)**2/atan(a*x)**(3/2),x)
```

output
```
Integral(1/(a**4*x**8*atan(a*x)**(3/2) + 2*a**2*x**6*atan(a*x)**(3/2) + x*
*4*atan(a*x)**(3/2)), x)/c**2
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^4 (c + a^2cx^2)^2 \arctan(ax)^{3/2}} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4 (c + a^2cx^2)^2 \arctan(ax)^{3/2}} \, dx = \int \frac{1}{(a^2cx^2 + c)^2 x^4 \arctan(ax)^{\frac{3}{2}}} \, dx$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^2*x^4*arctan(a*x)^(3/2)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4 \left(c+a^2cx^2\right)^2 \arctan(ax)^{3/2}}\,dx = \int \frac{1}{x^4\,\mathrm{atan}\left(a\,x\right)^{3/2}\,\left(c\,a^2\,x^2+c\right)^2}\,dx$$

input
```
int(1/(x^4*atan(a*x)^(3/2)*(c + a^2*c*x^2)^2),x)
```

output
```
int(1/(x^4*atan(a*x)^(3/2)*(c + a^2*c*x^2)^2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.17

$$\int \frac{1}{x^4 \left(c+a^2cx^2\right)^2 \arctan(ax)^{3/2}}\,dx = \frac{\int \frac{\sqrt{atan(ax)}}{atan(ax)^2a^4x^8+2atan(ax)^2a^2x^6+atan(ax)^2x^4}dx}{c^2}$$

input
```
int(1/x^4/(a^2*c*x^2+c)^2/atan(a*x)^(3/2),x)
```

output
```
int(sqrt(atan(a*x))/(atan(a*x)**2*a**4*x**8 + 2*atan(a*x)**2*a**2*x**6 + a
tan(a*x)**2*x**4),x)/c**2
```

# 3.997 $\int \frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)^{3/2}} \, dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)^{3/2}} \, dx = \operatorname{Int}\left(\frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)^{3/2}}, x\right)$$

output
```
Defer(Int)(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)^{3/2}} \, dx = \int \frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)^{3/2}} \, dx$$

input
```
Integrate[x^m/((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[x^m/((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^3}dx$$

$$\Big\downarrow \text{5560}$$

$$\int \frac{x^m}{\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^3}dx$$

input
```
Int[x^m/((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{\left(a^2c\,x^2+c\right)^3\arctan\left(ax\right)^{\frac{3}{2}}}dx$$

input
```
int(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)
```

output
```
int(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.08

$$\int \frac{x^m}{(c+a^2cx^2)^3\arctan(ax)^{3/2}}\,dx = \int \frac{x^m}{(a^2cx^2+c)^3\arctan{(ax)}^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
integral(x^m/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a
*x)^(3/2)), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^m}{(c+a^2cx^2)^3\arctan(ax)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**m/(a**2*c*x**2+c)**3/atan(a*x)**(3/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m}{(c+a^2cx^2)^3\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)^{3/2}}\,dx = \int \frac{x^m}{(a^2cx^2+c)^3 \arctan(ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^m/((a^2*c*x^2 + c)^3*arctan(a*x)^(3/2)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.93 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)^{3/2}}\,dx = \int \frac{x^m}{\mathrm{atan}(a\,x)^{3/2}\,(c\,a^2\,x^2+c)^3}\,dx$$

input
```
int(x^m/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^3),x)
```

output
```
int(x^m/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^3), x)
```

## Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 631, normalized size of antiderivative = 26.29

$$\int \frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(x^m/(a^2*c*x^2+c)^3/atan(a*x)^(3/2),x)
```

output
```
(2*(atan(a*x)*int((x**m*sqrt(atan(a*x))*x)/(atan(a*x)*a**6*x**6 + 3*atan(a
*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**6*m*x**4 - 4*atan
(a*x)*int((x**m*sqrt(atan(a*x))*x)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4
*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**6*x**4 + 2*atan(a*x)*int(
(x**m*sqrt(atan(a*x))*x)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*
atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**4*m*x**2 - 8*atan(a*x)*int((x**m*sq
rt(atan(a*x))*x)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x
)*a**2*x**2 + atan(a*x)),x)*a**4*x**2 + atan(a*x)*int((x**m*sqrt(atan(a*x)
)*x)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2
+ atan(a*x)),x)*a**2*m - 4*atan(a*x)*int((x**m*sqrt(atan(a*x))*x)/(atan(a*
x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),
x)*a**2 + atan(a*x)*int((x**m*sqrt(atan(a*x)))/(atan(a*x)*a**6*x**7 + 3*at
an(a*x)*a**4*x**5 + 3*atan(a*x)*a**2*x**3 + atan(a*x)*x),x)*a**4*m*x**4 +
2*atan(a*x)*int((x**m*sqrt(atan(a*x)))/(atan(a*x)*a**6*x**7 + 3*atan(a*x)*
a**4*x**5 + 3*atan(a*x)*a**2*x**3 + atan(a*x)*x),x)*a**2*m*x**2 + atan(a*x
)*int((x**m*sqrt(atan(a*x)))/(atan(a*x)*a**6*x**7 + 3*atan(a*x)*a**4*x**5
+ 3*atan(a*x)*a**2*x**3 + atan(a*x)*x),x)*m - x**m*sqrt(atan(a*x))))/(atan
(a*x)*a*c**3*(a**4*x**4 + 2*a**2*x**2 + 1))
```

# 3.998 $\int \frac{x^3}{\left(c+a^2cx^2\right)^3 \arctan(ax)^{3/2}} \, dx$



## Optimal result

Integrand size = 24, antiderivative size = 96

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^3 \arctan(ax)^{3/2}} \, dx = -\frac{2x^3}{ac^3\left(1+a^2x^2\right)^2\sqrt{\arctan(ax)}} - \frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{a^4c^3} + \frac{\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{a^4c^3}$$

output
```
-2*x^3/a/c^3/(a^2*x^2+1)^2/arctan(a*x)^(1/2)-1/2*2^(1/2)*Pi^(1/2)*FresnelC
(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))/a^4/c^3+Pi^(1/2)*FresnelC(2*arctan(
a*x)^(1/2)/Pi^(1/2))/a^4/c^3
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.54

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^3 \arctan(ax)^{3/2}} \, dx = \frac{-2\sqrt{2\pi}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right) + 16\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{16a}$$

input
```
Integrate[x^3/((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)),x]
```

output
```
(-2*Sqrt[2*Pi]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]] + 16*Sqrt[Pi]*Fres
nelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]] + ((-32*a^3*x^3)/(1 + a^2*x^2)^2 + (3
*I)*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-4*I)*ArcTan[a*x]] - (3*I)*Sqrt[I*A
rcTan[a*x]]*Gamma[1/2, (4*I)*ArcTan[a*x]])/Sqrt[ArcTan[a*x]])/(16*a^4*c^3)
```

## Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.71, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5503, 27, 5505, 3042, 3793, 2009, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\arctan(ax)^{3/2} \left(a^2cx^2+c\right)^3}dx$$

$$\Big\downarrow \text{5503}$$

$$\frac{6\int \frac{x^2}{c^3(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{a}-2a\int \frac{x^4}{c^3\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx-\frac{2x^3}{ac^3\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{6\int \frac{x^2}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{ac^3}-\frac{2a\int \frac{x^4}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{c^3}-\frac{2x^3}{ac^3\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{5505}$$

$$\frac{6\int \frac{a^2x^2}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{a^4c^3}-\frac{2\int \frac{a^4x^4}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{a^4c^3}-\frac{2x^3}{ac^3\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\int \frac{\sin(\arctan(ax))^4}{\sqrt{\arctan(ax)}}d\arctan(ax)}{a^4c^3}+\frac{6\int \frac{a^2x^2}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{a^4c^3}-\frac{2x^3}{ac^3\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{ 3793}$$

$$-\frac{2\int \left(-\frac{\cos(2\arctan(ax))}{2\sqrt{\arctan(ax)}}+\frac{\cos(4\arctan(ax))}{8\sqrt{\arctan(ax)}}+\frac{3}{8\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^4c^3}+\frac{6\int \frac{a^2x^2}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{a^4c^3}-\frac{2x^3}{ac^3\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{6\int \frac{a^2x^2}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{a^4c^3}-\frac{2\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{2}\sqrt{\pi}\operatorname{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\arctan(ax)}\right)}{a^4c^3}-\frac{2x^3}{ac^3\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{ 4906}$$

$$\frac{6\int \left(\frac{1}{8\sqrt{\arctan(ax)}}-\frac{\cos(4\arctan(ax))}{8\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^4c^3}-\frac{2\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{2}\sqrt{\pi}\operatorname{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\arctan(ax)}\right)}{a^4c^3}-\frac{2x^3}{ac^3\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{6\left(\frac{1}{4}\sqrt{\arctan(ax)}-\frac{1}{8}\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{a^4c^3}-\frac{2\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{2}\sqrt{\pi}\operatorname{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\arctan(ax)}\right)}{a^4c^3}-\frac{2x^3}{ac^3\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}$$

input

```
Int[x^3/((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)),x]
```

output
```
(-2*x^3)/(a*c^3*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]]) + (6*(Sqrt[ArcTan[a*x]]
/4 - (Sqrt[Pi/2]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/8))/(a^4*c^3) -
 (2*((3*Sqrt[ArcTan[a*x]])/4 + (Sqrt[Pi/2]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcT
an[a*x]]])/8 - (Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/2))/(a^
4*c^3)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

## Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.90

| method | result |
|---|---|
| default | $-\frac{2\sqrt{2}\,\sqrt{\arctan(ax)}\,\sqrt{\pi}\ \mathrm{FresnelC}\left(\frac{2\sqrt{2}\,\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)-4\sqrt{\arctan(ax)}\,\sqrt{\pi}\ \mathrm{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+2\sin(2\arctan(ax))-\sin(4\,\mathrm{ar}}{4a^4c^3\sqrt{\arctan(ax)}}$ |

input
```
int(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/a^4/c^3*(2*2^(1/2)*arctan(a*x)^(1/2)*Pi^(1/2)*FresnelC(2*2^(1/2)/Pi^(
1/2)*arctan(a*x)^(1/2))-4*arctan(a*x)^(1/2)*Pi^(1/2)*FresnelC(2*arctan(a*x
)^(1/2)/Pi^(1/2))+2*sin(2*arctan(a*x))-sin(4*arctan(a*x)))/arctan(a*x)^(1/
2)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3}{(c+a^2cx^2)^3\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{x^3}{(c+a^2cx^2)^3 \arctan(ax)^{3/2}}\,dx = \frac{\int \frac{x^3}{a^6x^6 \operatorname{atan}^{\frac{3}{2}}(ax)+3a^4x^4 \operatorname{atan}^{\frac{3}{2}}(ax)+3a^2x^2 \operatorname{atan}^{\frac{3}{2}}(ax)+\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx}{c^3}$$

input
```
integrate(x**3/(a**2*c*x**2+c)**3/atan(a*x)**(3/2),x)
```

output
```
Integral(x**3/(a**6*x**6*atan(a*x)**(3/2) + 3*a**4*x**4*atan(a*x)**(3/2) +
 3*a**2*x**2*atan(a*x)**(3/2) + atan(a*x)**(3/2)), x)/c**3
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(c+a^2cx^2)^3 \arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F]

$$\int \frac{x^3}{(c+a^2cx^2)^3 \arctan(ax)^{3/2}}\,dx = \int \frac{x^3}{(a^2cx^2+c)^3 \arctan(ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^3/((a^2*c*x^2 + c)^3*arctan(a*x)^(3/2)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(c+a^2cx^2)^3\arctan(ax)^{3/2}}\,dx=\int \frac{x^3}{\mathrm{atan}\left(a\,x\right)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

input
```
int(x^3/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^3),x)
```

output
```
int(x^3/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^3), x)
```

## Reduce [F]

$$\int \frac{x^3}{(c+a^2cx^2)^3\arctan(ax)^{3/2}}\,dx=\frac{\int \frac{\sqrt{atan(ax)}\,x^3}{atan(ax)^2a^6x^6+3atan(ax)^2a^4x^4+3atan(ax)^2a^2x^2+atan(ax)^2}dx}{c^3}$$

input
```
int(x^3/(a^2*c*x^2+c)^3/atan(a*x)^(3/2),x)
```

output
```
int((sqrt(atan(a*x))*x**3)/(atan(a*x)**2*a**6*x**6 + 3*atan(a*x)**2*a**4*x
**4 + 3*atan(a*x)**2*a**2*x**2 + atan(a*x)**2),x)/c**3
```

# 3.999 $\int \frac{x^2}{(c+a^2cx^2)^3 \arctan(ax)^{3/2}} \, dx$



## Optimal result

Integrand size = 24, antiderivative size = 67

$$\int \frac{x^2}{(c+a^2cx^2)^3 \arctan(ax)^{3/2}} \, dx = -\frac{2x^2}{ac^3(1+a^2x^2)^2\sqrt{\arctan(ax)}} + \frac{\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{a^3c^3}$$

output
```
-2*x^2/a/c^3/(a^2*x^2+1)^2/arctan(a*x)^(1/2)+1/2*2^(1/2)*Pi^(1/2)*FresnelS
(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))/a^3/c^3
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.67

$$\int \frac{x^2}{(c+a^2cx^2)^3 \arctan(ax)^{3/2}} \, dx = \frac{-8a^2x^2 - (1+a^2x^2)^2\sqrt{-i\arctan(ax)}\Gamma\left(\frac{1}{2}, -4i\arctan(ax)\right) - (1+a^2 \cdots}{4a^3c^3(1+a^2x^2)^2\sqrt{\arctan(ax)}}$$

input
```
Integrate[x^2/((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)),x]
```

output
```
(-8*a^2*x^2 - (1 + a^2*x^2)^2*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-4*I)*Arc
Tan[a*x]] - (1 + a^2*x^2)^2*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (4*I)*ArcTan[a*
x]])/(4*a^3*c^3*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]])
```

## Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 165 vs. $2(67) = 134$.

Time = 0.64 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.46, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {5503, 27, 5505, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\arctan(ax)^{3/2} (a^2cx^2+c)^3} dx$$

$$\Big\downarrow \text{5503}$$

$$\frac{4\int \frac{x}{c^3(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{a} - 4a\int \frac{x^3}{c^3(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx - \frac{2x^2}{ac^3(a^2x^2+1)^2\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{4\int \frac{x}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{ac^3} - \frac{4a\int \frac{x^3}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{c^3} - \frac{2x^2}{ac^3(a^2x^2+1)^2\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{5505}$$

$$\frac{4\int \frac{ax}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3c^3} - \frac{4\int \frac{a^3x^3}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3c^3} - \frac{2x^2}{ac^3(a^2x^2+1)^2\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{4906}$$

$$-\frac{4\int \left(\frac{\sin(2\arctan(ax))}{4\sqrt{\arctan(ax)}}-\frac{\sin(4\arctan(ax))}{8\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^3c^3}+\frac{4\int \left(\frac{\sin(2\arctan(ax))}{4\sqrt{\arctan(ax)}}+\frac{\sin(4\arctan(ax))}{8\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^3c^3}-\frac{2x^2}{ac^3\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \; 2009$$

$$-\frac{4\left(\frac{1}{4}\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)-\frac{1}{8}\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{a^3c^3}+\frac{4\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{4}\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)\right)}{a^3c^3}-\frac{2x^2}{ac^3\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}$$

input
```
Int[x^2/((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)),x]
```

output
```
(-2*x^2)/(a*c^3*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]]) - (4*(-1/8*(Sqrt[Pi/2]*
FresnelS[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]]) + (Sqrt[Pi]*FresnelS[(2*Sqrt[Arc
Tan[a*x]])/Sqrt[Pi]])/4))/(a^3*c^3) + (4*((Sqrt[Pi/2]*FresnelS[2*Sqrt[2/Pi
]*Sqrt[ArcTan[a*x]]])/8 + (Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi
]])/4))/(a^3*c^3)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

## Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.79

| method | result | size |
|---|---|---|
| default | $\frac{2\sqrt{2}\,\sqrt{\arctan(ax)}\,\sqrt{\pi}\,\operatorname{FresnelS}\left(\frac{2\sqrt{2}\,\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\cos(4\arctan(ax))-1}{4a^3c^3\sqrt{\arctan(ax)}}$ | 53 |

input
```
int(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4/a^3/c^3*(2*2^(1/2)*arctan(a*x)^(1/2)*Pi^(1/2)*FresnelS(2*2^(1/2)/Pi^(1
/2)*arctan(a*x)^(1/2))+cos(4*arctan(a*x))-1)/arctan(a*x)^(1/2)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2}{(c+a^2cx^2)^3\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{x^2}{(c + a^2cx^2)^3 \arctan(ax)^{3/2}} \, dx = \frac{\int \frac{x^2}{a^6x^6 \operatorname{atan}^{\frac{3}{2}}(ax) + 3a^4x^4 \operatorname{atan}^{\frac{3}{2}}(ax) + 3a^2x^2 \operatorname{atan}^{\frac{3}{2}}(ax) + \operatorname{atan}^{\frac{3}{2}}(ax)} \, dx}{c^3}$$

input
```
integrate(x**2/(a**2*c*x**2+c)**3/atan(a*x)**(3/2),x)
```

output
```
Integral(x**2/(a**6*x**6*atan(a*x)**(3/2) + 3*a**4*x**4*atan(a*x)**(3/2) +
 3*a**2*x**2*atan(a*x)**(3/2) + atan(a*x)**(3/2)), x)/c**3
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(c + a^2cx^2)^3 \arctan(ax)^{3/2}} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F]

$$\int \frac{x^2}{(c+a^2cx^2)^3\arctan(ax)^{3/2}}\,dx = \int \frac{x^2}{(a^2cx^2+c)^3\arctan(ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^2/((a^2*c*x^2 + c)^3*arctan(a*x)^(3/2)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(c+a^2cx^2)^3\arctan(ax)^{3/2}}\,dx = \int \frac{x^2}{\mathrm{atan}(a\,x)^{3/2}\,(c\,a^2\,x^2+c)^3}\,dx$$

input
```
int(x^2/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^3),x)
```

output
```
int(x^2/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^3), x)
```

## Reduce [F]

$$\int \frac{x^2}{(c+a^2cx^2)^3\arctan(ax)^{3/2}}\,dx = \frac{\int \frac{\sqrt{atan(ax)}\,x^2}{atan(ax)^2a^6x^6+3atan(ax)^2a^4x^4+3atan(ax)^2a^2x^2+atan(ax)^2}dx}{c^3}$$

input
```
int(x^2/(a^2*c*x^2+c)^3/atan(a*x)^(3/2),x)
```

output
```
int((sqrt(atan(a*x))*x**2)/(atan(a*x)**2*a**6*x**6 + 3*atan(a*x)**2*a**4*x
**4 + 3*atan(a*x)**2*a**2*x**2 + atan(a*x)**2),x)/c**3
```

# 3.1000 $\int \frac{x}{(c+a^2cx^2)^3 \arctan(ax)^{3/2}} \, dx$



## Optimal result

Integrand size = 22, antiderivative size = 93

$$\int \frac{x}{(c+a^2cx^2)^3 \arctan(ax)^{3/2}} \, dx = -\frac{2x}{ac^3 (1+a^2x^2)^2 \sqrt{\arctan(ax)}} + \frac{\sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{a^2c^3} + \frac{\sqrt{\pi} \operatorname{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{a^2c^3}$$

output
```
-2*x/a/c^3/(a^2*x^2+1)^2/arctan(a*x)^(1/2)+1/2*2^(1/2)*Pi^(1/2)*FresnelC(2
*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))/a^2/c^3+Pi^(1/2)*FresnelC(2*arctan(a*
x)^(1/2)/Pi^(1/2))/a^2/c^3
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.68

$$\int \frac{x}{(c+a^2cx^2)^3 \arctan(ax)^{3/2}} \, dx = \frac{-\frac{8ax}{(1+a^2x^2)^2} - i\sqrt{2}\sqrt{-i\arctan(ax)}\Gamma\left(\frac{1}{2}, -2i\arctan(ax)\right) + i\sqrt{2}\sqrt{i \arcta}}{}$$

input
```
Integrate[x/((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)),x]
```

output
```
((-8*a*x)/(1 + a^2*x^2)^2 - I*Sqrt[2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-
2*I)*ArcTan[a*x]] + I*Sqrt[2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (2*I)*ArcTan[
a*x]] - I*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-4*I)*ArcTan[a*x]] + I*Sqrt[I
*ArcTan[a*x]]*Gamma[1/2, (4*I)*ArcTan[a*x]])/(4*a^2*c^3*Sqrt[ArcTan[a*x]])
```

## Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.74, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {5503, 27, 5439, 3042, 3793, 2009, 5505, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^3}dx$$

$$\Big\downarrow \text{5503}$$

$$\frac{2\int \frac{1}{c^3\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{a}-6a\int \frac{x^2}{c^3\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx-\frac{2x}{ac^3\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\int \frac{1}{\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{ac^3}-\frac{6a\int \frac{x^2}{\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{c^3}-\frac{2x}{ac^3\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{5439}$$

$$-\frac{6a\int \frac{x^2}{\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{c^3}+\frac{2\int \frac{1}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{a^2c^3}-\frac{2x}{ac^3\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{6a\int \frac{x^2}{\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{c^3}+\frac{2\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^4}{\sqrt{\arctan(ax)}}d\arctan(ax)}{a^2c^3}-\frac{2x}{ac^3\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{6a\int \frac{x^2}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{c^3}+\frac{2\int \left(\frac{\cos(2\arctan(ax))}{2\sqrt{\arctan(ax)}}+\frac{\cos(4\arctan(ax))}{8\sqrt{\arctan(ax)}}+\frac{3}{8\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^2c^3}-\frac{2x}{ac^3(a^2x^2+1)^2\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{6a\int \frac{x^2}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{c^3}+\frac{2\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\arctan(ax)}\right)}{a^2c^3}-\frac{2x}{ac^3(a^2x^2+1)^2\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{5505}$$

$$-\frac{6\int \frac{a^2x^2}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{a^2c^3}+\frac{2\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\arctan(ax)}\right)}{a^2c^3}-\frac{2x}{ac^3(a^2x^2+1)^2\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{4906}$$

$$-\frac{6\int \left(\frac{1}{8\sqrt{\arctan(ax)}}-\frac{\cos(4\arctan(ax))}{8\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^2c^3}+\frac{2\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\arctan(ax)}\right)}{a^2c^3}-\frac{2x}{ac^3(a^2x^2+1)^2\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{6\left(\frac{1}{4}\sqrt{\arctan(ax)}-\frac{1}{8}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{a^2c^3}+\frac{2\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\arctan(ax)}\right)}{a^2c^3}-\frac{2x}{ac^3(a^2x^2+1)^2\sqrt{\arctan(ax)}}$$

input
```
Int[x/((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)),x]
```

output
```
(-2*x)/(a*c^3*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]]) - (6*(Sqrt[ArcTan[a*x]]/4
 - (Sqrt[Pi/2]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/8))/(a^2*c^3) + (
2*((3*Sqrt[ArcTan[a*x]])/4 + (Sqrt[Pi/2]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan
[a*x]]])/8 + (Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/2))/(a^2*
c^3)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

## Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.90

| method | result |
|---|---|
| default | $-\frac{-2\sqrt{2}\,\sqrt{\arctan(ax)}\,\sqrt{\pi}\,\mathrm{FresnelC}\left(\frac{2\sqrt{2}\,\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)-4\sqrt{\arctan(ax)}\,\sqrt{\pi}\,\mathrm{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+2\sin(2\arctan(ax))+\sin(4}{4a^2c^3\sqrt{\arctan(ax)}}$ |

input
```
int(x/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/a^2/c^3/arctan(a*x)^(1/2)*(-2*2^(1/2)*arctan(a*x)^(1/2)*Pi^(1/2)*Fres
nelC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))-4*arctan(a*x)^(1/2)*Pi^(1/2)*Fr
esnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))+2*sin(2*arctan(a*x))+sin(4*arctan(a*x
)))
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)^3 \arctan(ax)^{3/2}} dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{x}{(c+a^2cx^2)^3 \arctan(ax)^{3/2}} dx = \frac{\int \frac{x}{a^6x^6 \operatorname{atan}^{\frac{3}{2}}(ax)+3a^4x^4 \operatorname{atan}^{\frac{3}{2}}(ax)+3a^2x^2 \operatorname{atan}^{\frac{3}{2}}(ax)+\operatorname{atan}^{\frac{3}{2}}(ax)} dx}{c^3}$$

input
```
integrate(x/(a**2*c*x**2+c)**3/atan(a*x)**(3/2),x)
```

output
```
Integral(x/(a**6*x**6*atan(a*x)**(3/2) + 3*a**4*x**4*atan(a*x)**(3/2) + 3*
a**2*x**2*atan(a*x)**(3/2) + atan(a*x)**(3/2)), x)/c**3
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)^3 \arctan(ax)^{3/2}} dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F]

$$\int \frac{x}{\left(c+a^2cx^2\right)^3\arctan(ax)^{3/2}}\,dx=\int \frac{x}{\left(a^2cx^2+c\right)^3\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(x/((a^2*c*x^2 + c)^3*arctan(a*x)^(3/2)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(c+a^2cx^2\right)^3\arctan(ax)^{3/2}}\,dx=\int \frac{x}{\mathrm{atan}\left(a\,x\right)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

input
```
int(x/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^3),x)
```

output
```
int(x/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^3), x)
```

## Reduce [F]

$$\int \frac{x}{\left(c+a^2cx^2\right)^3\arctan(ax)^{3/2}}\,dx=\frac{2atan(ax)\left(\int \frac{\sqrt{atan(ax)}}{atan(ax)a^6x^6+3atan(ax)a^4x^4+3atan(ax)a^2x^2+atan(ax)}dx\right)a^4x^4+4

output
```
(2*(atan(a*x)*int(sqrt(atan(a*x))/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*
x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**4*x**4 + 2*atan(a*x)*int(s
qrt(atan(a*x))/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*
a**2*x**2 + atan(a*x)),x)*a**2*x**2 + atan(a*x)*int(sqrt(atan(a*x))/(atan(
a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)
),x) - 3*atan(a*x)*int((sqrt(atan(a*x))*x**2)/(atan(a*x)*a**6*x**6 + 3*ata
n(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**6*x**4 - 6*ata
n(a*x)*int((sqrt(atan(a*x))*x**2)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*
x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**4*x**2 - 3*atan(a*x)*int((
sqrt(atan(a*x))*x**2)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*ata
n(a*x)*a**2*x**2 + atan(a*x)),x)*a**2 - sqrt(atan(a*x))*x))/(atan(a*x)*a*c
**3*(a**4*x**4 + 2*a**2*x**2 + 1))
```

# 3.1001 $\int \frac{1}{(c+a^2cx^2)^3 \arctan(ax)^{3/2}} dx$



## Optimal result

Integrand size = 21, antiderivative size = 94

$$\int \frac{1}{(c+a^2cx^2)^3 \arctan(ax)^{3/2}} dx = -\frac{2}{ac^3(1+a^2x^2)^2\sqrt{\arctan(ax)}} - \frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{ac^3} - \frac{2\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{ac^3}$$

output
```
-2/a/c^3/(a^2*x^2+1)^2/arctan(a*x)^(1/2)-1/2*2^(1/2)*Pi^(1/2)*FresnelS(2*2
^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))/a/c^3-2*Pi^(1/2)*FresnelS(2*arctan(a*x)
^(1/2)/Pi^(1/2))/a/c^3
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.41 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.53

$$\int \frac{1}{(c+a^2cx^2)^3 \arctan(ax)^{3/2}} dx = \frac{-\frac{8}{(1+a^2x^2)^2} + 2\sqrt{2}\sqrt{-i\arctan(ax)}\Gamma\left(\frac{1}{2}, -2i\arctan(ax)\right) + 2\sqrt{2}\sqrt{i\arcta}}{}$$

input
```
Integrate[1/((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)),x]
```

output
```
(-8/(1 + a^2*x^2)^2 + 2*Sqrt[2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-2*I)*A
rcTan[a*x]] + 2*Sqrt[2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (2*I)*ArcTan[a*x]]
+ Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-4*I)*ArcTan[a*x]] + Sqrt[I*ArcTan[a*
x]]*Gamma[1/2, (4*I)*ArcTan[a*x]])/(4*a*c^3*Sqrt[ArcTan[a*x]])
```

## Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5437, 27, 5505, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^3}dx$$

$$\Big\downarrow \text{5437}$$

$$-8a\int \frac{x}{c^3\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx-\frac{2}{ac^3\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{8a\int \frac{x}{\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{c^3}-\frac{2}{ac^3\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{5505}$$

$$-\frac{8\int \frac{ax}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{ac^3}-\frac{2}{ac^3\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{4906}$$

$$-\frac{8\int \left(\frac{\sin(2\arctan(ax))}{4\sqrt{\arctan(ax)}}+\frac{\sin(4\arctan(ax))}{8\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{ac^3}-\frac{2}{ac^3\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2}{ac^3\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}-\frac{8\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{4}\sqrt{\pi}\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)\right)}{ac^3}$$

input
```
Int[1/((c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)),x]
```

output
```
-2/(a*c^3*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]]) - (8*((Sqrt[Pi/2]*FresnelS[2*
Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/8 + (Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]]
)/Sqrt[Pi]])/4))/(a*c^3)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

## Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.90

| method | result |
|---|---|
| default | $-\frac{2\sqrt{2}\,\sqrt{\arctan(ax)}\,\sqrt{\pi}\,\mathrm{FresnelS}\left(\frac{2\sqrt{2}\,\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+8\sqrt{\arctan(ax)}\,\sqrt{\pi}\,\mathrm{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+4\cos(2\arctan(ax))+\cos(4\,\mathrm{ar}}{4a\,c^{3}\sqrt{\arctan(ax)}}$ |

input
```
int(1/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/a/c^3*(2*2^(1/2)*arctan(a*x)^(1/2)*Pi^(1/2)*FresnelS(2*2^(1/2)/Pi^(1/
2)*arctan(a*x)^(1/2))+8*arctan(a*x)^(1/2)*Pi^(1/2)*FresnelS(2*arctan(a*x)^
(1/2)/Pi^(1/2))+4*cos(2*arctan(a*x))+cos(4*arctan(a*x))+3)/arctan(a*x)^(1/
2)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(c+a^2cx^2)^3\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{1}{(c+a^2cx^2)^3\arctan(ax)^{3/2}}\,dx = \frac{\int \frac{1}{a^6x^6\operatorname{atan}^{\frac{3}{2}}(ax)+3a^4x^4\operatorname{atan}^{\frac{3}{2}}(ax)+3a^2x^2\operatorname{atan}^{\frac{3}{2}}(ax)+\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx}{c^3}$$

input
```
integrate(1/(a**2*c*x**2+c)**3/atan(a*x)**(3/2),x)
```

output
```
Integral(1/(a**6*x**6*atan(a*x)**(3/2) + 3*a**4*x**4*atan(a*x)**(3/2) + 3*
a**2*x**2*atan(a*x)**(3/2) + atan(a*x)**(3/2)), x)/c**3
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(c+a^2cx^2)^3 \arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F]

$$\int \frac{1}{(c+a^2cx^2)^3 \arctan(ax)^{3/2}}\,dx = \int \frac{1}{(a^2cx^2+c)^3 \arctan(ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^3*arctan(a*x)^(3/2)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+a^2cx^2)^3 \arctan(ax)^{3/2}}\,dx = \int \frac{1}{\operatorname{atan}(a\,x)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

input
```
int(1/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^3),x)
```

output
```
int(1/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^3), x)
```

## Reduce [F]

$$\int \frac{1}{(c+a^2cx^2)^3 \arctan(ax)^{3/2}}\, dx = \text{Too large to display}$$

input
```
int(1/(a^2*c*x^2+c)^3/atan(a*x)^(3/2),x)
```

output
```
(2*( - 16*sqrt(atan(a*x))*atan(a*x)**2*a**4*x**4 - 32*sqrt(atan(a*x))*atan
(a*x)**2*a**2*x**2 - 16*sqrt(atan(a*x))*atan(a*x)**2 - 24*sqrt(atan(a*x))*
atan(a*x)*a**3*x**3 - 40*sqrt(atan(a*x))*atan(a*x)*a*x + 64*atan(a*x)*int(
sqrt(atan(a*x))/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1),x)*a**5*x**4 +
 128*atan(a*x)*int(sqrt(atan(a*x))/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2
+ 1),x)*a**3*x**2 + 64*atan(a*x)*int(sqrt(atan(a*x))/(a**6*x**6 + 3*a**4*x
**4 + 3*a**2*x**2 + 1),x)*a + 12*atan(a*x)*int((sqrt(atan(a*x))*x**3)/(ata
n(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*
x)),x)*a**8*x**4 + 24*atan(a*x)*int((sqrt(atan(a*x))*x**3)/(atan(a*x)*a**6
*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**6
*x**2 + 12*atan(a*x)*int((sqrt(atan(a*x))*x**3)/(atan(a*x)*a**6*x**6 + 3*a
tan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**4 - 5*sqrt(a
tan(a*x))))/(5*atan(a*x)*a*c**3*(a**4*x**4 + 2*a**2*x**2 + 1))
```

# 3.1002 $\int \frac{1}{x(c+a^2cx^2)^3 \arctan(ax)^{3/2}} \, dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x(c+a^2cx^2)^3 \arctan(ax)^{3/2}} \, dx = -\frac{2}{ac^3x(1+a^2x^2)^2\sqrt{\arctan(ax)}} - \frac{15\sqrt{\arctan(ax)}}{2c^3} - \frac{5\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{4c^3} - \frac{5\sqrt{\pi}\operatorname{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{c^3} - \frac{2\operatorname{Int}\left(\frac{1}{x^2(c+a^2cx^2)^3\sqrt{\arctan(ax)}},x\right)}{a}$$

output
```
-2/a/c^3/x/(a^2*x^2+1)^2/arctan(a*x)^(1/2)-15/2*arctan(a*x)^(1/2)/c^3-5/8*
2^(1/2)*Pi^(1/2)*FresnelC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))/c^3-5*Pi^(
1/2)*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))/c^3-2*Defer(Int)(1/x^2/(a^2*c*
x^2+c)^3/arctan(a*x)^(1/2),x)/a
```

## Mathematica [N/A]

Not integrable

Time = 2.73 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\arctan(ax)^{3/2}}\,dx=\int \frac{1}{x\left(c+a^2cx^2\right)^3\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[1/(x*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[1/(x*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.87 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^3}dx$$

$$\Big\downarrow \text{5503}$$

$$-10a\int \frac{1}{c^3\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx-\frac{2\int \frac{1}{c^3x^2\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ac^3x\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{10a\int \frac{1}{\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{c^3}-\frac{2\int \frac{1}{x^2\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{ac^3}-\frac{2}{ac^3x\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{5439}$$

$$-\frac{2\int \frac{1}{x^2(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{ac^3}-\frac{10\int \frac{1}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{c^3}-\frac{2}{ac^3x\,(a^2x^2+1)^2\sqrt{\arctan(ax)}}$$

$\Big\downarrow$ 3042

$$-\frac{2\int \frac{1}{x^2(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{ac^3}-\frac{10\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^4}{\sqrt{\arctan(ax)}}d\arctan(ax)}{c^3}-\frac{2}{ac^3x\,(a^2x^2+1)^2\sqrt{\arctan(ax)}}$$

$\Big\downarrow$ 3793

$$-\frac{2\int \frac{1}{x^2(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{ac^3}-\frac{10\int \left(\frac{\cos(2\arctan(ax))}{2\sqrt{\arctan(ax)}}+\frac{\cos(4\arctan(ax))}{8\sqrt{\arctan(ax)}}+\frac{3}{8\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{c^3}-\frac{2}{ac^3x\,(a^2x^2+1)^2\sqrt{\arctan(ax)}}$$

$\Big\downarrow$ 2009

$$-\frac{2\int \frac{1}{x^2(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{ac^3}-\frac{2}{ac^3x\,(a^2x^2+1)^2\sqrt{\arctan(ax)}}-\frac{10\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\arctan(ax)}\right)}{c^3}$$

$\Big\downarrow$ 5560

$$-\frac{2\int \frac{1}{x^2(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{ac^3}-\frac{2}{ac^3x\,(a^2x^2+1)^2\sqrt{\arctan(ax)}}-\frac{10\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\arctan(ax)}\right)}{c^3}$$

input
```
Int[1/(x*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\left(a^{2} c\,x^{2}+c\right)^{3} \arctan\left(a x\right)^{\frac{3}{2}}}d x$$

input
```
int(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)
```

output
```
int(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\arctan(ax)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 11.20 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.71

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\arctan(ax)^{3/2}}\,dx=\frac{\int \frac{1}{a^{6}x^{7}\operatorname{atan}^{\frac{3}{2}}{\left(ax\right)}+3a^{4}x^{5}\operatorname{atan}^{\frac{3}{2}}{\left(ax\right)}+3a^{2}x^{3}\operatorname{atan}^{\frac{3}{2}}{\left(ax\right)}+x\operatorname{atan}^{\frac{3}{2}}{\left(ax\right)}}\,dx}{c^{3}}$$

input
```
integrate(1/x/(a**2*c*x**2+c)**3/atan(a*x)**(3/2),x)
```

output
```
Integral(1/(a**6*x**7*atan(a*x)**(3/2) + 3*a**4*x**5*atan(a*x)**(3/2) + 3*
a**2*x**3*atan(a*x)**(3/2) + x*atan(a*x)**(3/2)), x)/c**3
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\arctan(ax)^{3/2}}\,dx = \int \frac{1}{\left(a^2cx^2+c\right)^3x\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^3*x*arctan(a*x)^(3/2)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\arctan(ax)^{3/2}}\,dx=\int \frac{1}{x\,\mathrm{atan}\left(a\,x\right)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

input
```
int(1/(x*atan(a*x)^(3/2)*(c + a^2*c*x^2)^3),x)
```

output
```
int(1/(x*atan(a*x)^(3/2)*(c + a^2*c*x^2)^3), x)
```

## Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.67

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\arctan(ax)^{3/2}}\,dx=\frac{\int \frac{\sqrt{atan(ax)}}{atan(ax)^2a^6x^7+3atan(ax)^2a^4x^5+3atan(ax)^2a^2x^3+atan(ax)^2x}dx}{c^3}$$

input
```
int(1/x/(a^2*c*x^2+c)^3/atan(a*x)^(3/2),x)
```

output
```
int(sqrt(atan(a*x))/(atan(a*x)**2*a**6*x**7 + 3*atan(a*x)**2*a**4*x**5 + 3
*atan(a*x)**2*a**2*x**3 + atan(a*x)**2*x),x)/c**3
```

# 3.1003 $\int \frac{1}{x^2(c+a^2cx^2)^3 \arctan(ax)^{3/2}} dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x^2 (c+a^2cx^2)^3 \arctan(ax)^{3/2}} dx = -\frac{2}{ac^3x^2 (1+a^2x^2)^2 \sqrt{\arctan(ax)}} - \frac{4\text{Int}\left(\frac{1}{x^3(c+a^2cx^2)^3\sqrt{\arctan(ax)}}, x\right)}{a} - 12a\text{Int}\left(\frac{1}{x (c+a^2cx^2)^3 \sqrt{\arctan(ax)}}, x\right)$$

output
```
-2/a/c^3/x^2/(a^2*x^2+1)^2/arctan(a*x)^(1/2)-4*Defer(Int)(1/x^3/(a^2*c*x^2
+c)^3/arctan(a*x)^(1/2),x)/a-12*a*Defer(Int)(1/x/(a^2*c*x^2+c)^3/arctan(a*
x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 4.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^2 (c+a^2cx^2)^3 \arctan(ax)^{3/2}} dx = \int \frac{1}{x^2 (c+a^2cx^2)^3 \arctan(ax)^{3/2}} dx$$

input
```
Integrate[1/(x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[1/(x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \arctan(ax)^{3/2} (a^2cx^2+c)^3}dx$$

$$\Big\downarrow \textbf{5503}$$

$$-12a\int \frac{1}{c^3x(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx-\frac{4\int \frac{1}{c^3x^3(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ac^3x^2(a^2x^2+1)^2\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \textbf{27}$$

$$-\frac{12a\int \frac{1}{x(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{c^3}-\frac{4\int \frac{1}{x^3(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{ac^3}-\frac{2}{ac^3x^2(a^2x^2+1)^2\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \textbf{5560}$$

$$-\frac{12a\int \frac{1}{x(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{c^3}-\frac{4\int \frac{1}{x^3(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{ac^3}-\frac{2}{ac^3x^2(a^2x^2+1)^2\sqrt{\arctan(ax)}}$$

input
```
Int[1/(x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^{2} \left(a^{2} c \,x^{2}+c \right)^{3} \arctan \left(a x \right)^{\frac{3}{2}}}d x$$

input
```
int(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)
```

output
```
int(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^2 (c+a^2cx^2)^3 \arctan(ax)^{3/2}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 16.27 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.75

$$\int \frac{1}{x^2 (c+a^2cx^2)^3 \arctan(ax)^{3/2}} \, dx = \frac{\int \frac{1}{a^{6} x^{8} \operatorname{atan}^{\frac{3}{2}}{\left(a x \right)} + 3 a^{4} x^{6} \operatorname{atan}^{\frac{3}{2}}{\left(a x \right)} + 3 a^{2} x^{4} \operatorname{atan}^{\frac{3}{2}}{\left(a x \right)} + x^{2} \operatorname{atan}^{\frac{3}{2}}{\left(a x \right)}}\, dx}{c^{3}}$$

input
```
integrate(1/x**2/(a**2*c*x**2+c)**3/atan(a*x)**(3/2),x)
```

output
```
Integral(1/(a**6*x**8*atan(a*x)**(3/2) + 3*a**4*x**6*atan(a*x)**(3/2) + 3*
a**2*x**4*atan(a*x)**(3/2) + x**2*atan(a*x)**(3/2)), x)/c**3
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^2 (c + a^2cx^2)^3 \arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (c + a^2cx^2)^3 \arctan(ax)^{3/2}}\,dx = \int \frac{1}{(a^2cx^2 + c)^3 x^2 \arctan(ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^3*x^2*arctan(a*x)^(3/2)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.84 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (c+a^2cx^2)^3 \arctan(ax)^{3/2}} \, dx = \int \frac{1}{x^2 \operatorname{atan}(a\,x)^{3/2} \left(c\,a^2\,x^2+c\right)^3} \, dx$$

input
```
int(1/(x^2*atan(a*x)^(3/2)*(c + a^2*c*x^2)^3),x)
```

output
```
int(1/(x^2*atan(a*x)^(3/2)*(c + a^2*c*x^2)^3), x)
```

## Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.75

$$\int \frac{1}{x^2 (c+a^2cx^2)^3 \arctan(ax)^{3/2}} \, dx = \frac{\int \frac{\sqrt{atan(ax)}}{atan(ax)^2 a^6 x^8 + 3atan(ax)^2 a^4 x^6 + 3atan(ax)^2 a^2 x^4 + atan(ax)^2 x^2} dx}{c^3}$$

input
```
int(1/x^2/(a^2*c*x^2+c)^3/atan(a*x)^(3/2),x)
```

output
```
int(sqrt(atan(a*x))/(atan(a*x)**2*a**6*x**8 + 3*atan(a*x)**2*a**4*x**6 + 3
*atan(a*x)**2*a**2*x**4 + atan(a*x)**2*x**2),x)/c**3
```

# 3.1004 $\int \frac{1}{x^3(c+a^2cx^2)^3 \arctan(ax)^{3/2}} dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x^3 (c+a^2cx^2)^3 \arctan(ax)^{3/2}} dx = -\frac{2}{ac^3x^3 (1+a^2x^2)^2 \sqrt{\arctan(ax)}} - \frac{6\text{Int}\left(\frac{1}{x^4(c+a^2cx^2)^3\sqrt{\arctan(ax)}}, x\right)}{a} - 14a\text{Int}\left(\frac{1}{x^2 (c+a^2cx^2)^3 \sqrt{\arctan(ax)}}, x\right)$$

output
```
-2/a/c^3/x^3/(a^2*x^2+1)^2/arctan(a*x)^(1/2)-6*Defer(Int)(1/x^4/(a^2*c*x^2
+c)^3/arctan(a*x)^(1/2),x)/a-14*a*Defer(Int)(1/x^2/(a^2*c*x^2+c)^3/arctan(
a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 6.68 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^3 (c+a^2cx^2)^3 \arctan(ax)^{3/2}} dx = \int \frac{1}{x^3 (c+a^2cx^2)^3 \arctan(ax)^{3/2}} dx$$

input
```
Integrate[1/(x^3*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[1/(x^3*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \arctan(ax)^{3/2} (a^2cx^2+c)^3} dx$$

$$\Big\downarrow \text{5503}$$

$$-14a \int \frac{1}{c^3x^2 (a^2x^2+1)^3 \sqrt{\arctan(ax)}} dx - \frac{6 \int \frac{1}{c^3x^4(a^2x^2+1)^3\sqrt{\arctan(ax)}} dx}{a} - \frac{2}{ac^3x^3 (a^2x^2+1)^2 \sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{14a \int \frac{1}{x^2(a^2x^2+1)^3\sqrt{\arctan(ax)}} dx}{c^3} - \frac{6 \int \frac{1}{x^4(a^2x^2+1)^3\sqrt{\arctan(ax)}} dx}{ac^3} - \frac{2}{ac^3x^3 (a^2x^2+1)^2 \sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{5560}$$

$$-\frac{14a \int \frac{1}{x^2(a^2x^2+1)^3\sqrt{\arctan(ax)}} dx}{c^3} - \frac{6 \int \frac{1}{x^4(a^2x^2+1)^3\sqrt{\arctan(ax)}} dx}{ac^3} - \frac{2}{ac^3x^3 (a^2x^2+1)^2 \sqrt{\arctan(ax)}}$$

input
```
Int[1/(x^3*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^{3} \left(a^{2} c \,x^{2}+c \right)^{3} \arctan \left(a x \right)^{\frac{3}{2}}}d x$$

input
```
int(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)
```

output
```
int(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^3 \left(c+a^2 c x^2\right)^3 \arctan(a x)^{3/2}} \, dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 20.67 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.75

$$\int \frac{1}{x^3 \left(c+a^2 c x^2\right)^3 \arctan(a x)^{3/2}} \, dx=\frac{\int \frac{1}{a^{6} x^{9} \operatorname{atan}^{\frac{3}{2}}{\left(a x \right)} + 3 a^{4} x^{7} \operatorname{atan}^{\frac{3}{2}}{\left(a x \right)} + 3 a^{2} x^{5} \operatorname{atan}^{\frac{3}{2}}{\left(a x \right)} + x^{3} \operatorname{atan}^{\frac{3}{2}}{\left(a x \right)}}\, dx}{c^{3}}$$

input
```
integrate(1/x**3/(a**2*c*x**2+c)**3/atan(a*x)**(3/2),x)
```

output
```
Integral(1/(a**6*x**9*atan(a*x)**(3/2) + 3*a**4*x**7*atan(a*x)**(3/2) + 3*
a**2*x**5*atan(a*x)**(3/2) + x**3*atan(a*x)**(3/2)), x)/c**3
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^3 (c + a^2cx^2)^3 \arctan(ax)^{3/2}} dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3 (c + a^2cx^2)^3 \arctan(ax)^{3/2}} dx = \int \frac{1}{(a^2cx^2 + c)^3 x^3 \arctan(ax)^{\frac{3}{2}}} dx$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^3*x^3*arctan(a*x)^(3/2)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.84 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3 (c+a^2cx^2)^3 \arctan(ax)^{3/2}} \, dx = \int \frac{1}{x^3 \operatorname{atan}(a\,x)^{3/2} \,(c\,a^2\,x^2+c)^3} \, dx$$

input
```
int(1/(x^3*atan(a*x)^(3/2)*(c + a^2*c*x^2)^3),x)
```

output
```
int(1/(x^3*atan(a*x)^(3/2)*(c + a^2*c*x^2)^3), x)
```

## Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.75

$$\int \frac{1}{x^3 (c+a^2cx^2)^3 \arctan(ax)^{3/2}} \, dx = \frac{\int \frac{\sqrt{atan(ax)}}{atan(ax)^2 a^6 x^9 + 3atan(ax)^2 a^4 x^7 + 3atan(ax)^2 a^2 x^5 + atan(ax)^2 x^3} dx}{c^3}$$

input
```
int(1/x^3/(a^2*c*x^2+c)^3/atan(a*x)^(3/2),x)
```

output
```
int(sqrt(atan(a*x))/(atan(a*x)**2*a**6*x**9 + 3*atan(a*x)**2*a**4*x**7 + 3
*atan(a*x)**2*a**2*x**5 + atan(a*x)**2*x**3),x)/c**3
```

# 3.1005 $\int \frac{1}{x^4(c+a^2cx^2)^3 \arctan(ax)^{3/2}} dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x^4 (c+a^2cx^2)^3 \arctan(ax)^{3/2}} dx = -\frac{2}{ac^3x^4 (1+a^2x^2)^2 \sqrt{\arctan(ax)}} - \frac{8\text{Int}\left(\frac{1}{x^5(c+a^2cx^2)^3\sqrt{\arctan(ax)}}, x\right)}{a} - 16a\text{Int}\left(\frac{1}{x^3 (c+a^2cx^2)^3 \sqrt{\arctan(ax)}}, x\right)$$

output
```
-2/a/c^3/x^4/(a^2*x^2+1)^2/arctan(a*x)^(1/2)-8*Defer(Int)(1/x^5/(a^2*c*x^2
+c)^3/arctan(a*x)^(1/2),x)/a-16*a*Defer(Int)(1/x^3/(a^2*c*x^2+c)^3/arctan(
a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 6.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^4 (c+a^2cx^2)^3 \arctan(ax)^{3/2}} dx = \int \frac{1}{x^4 (c+a^2cx^2)^3 \arctan(ax)^{3/2}} dx$$

input
```
Integrate[1/(x^4*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[1/(x^4*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \arctan(ax)^{3/2} \left(a^2cx^2+c\right)^3}dx$$

$$\Big\downarrow \text{5503}$$

$$-\frac{8\int \frac{1}{c^3x^5(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{a}-16a\int \frac{1}{c^3x^3\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx-\frac{2}{ac^3x^4\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{8\int \frac{1}{x^5(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{ac^3}-\frac{16a\int \frac{1}{x^3(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{c^3}-\frac{2}{ac^3x^4\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{5560}$$

$$-\frac{8\int \frac{1}{x^5(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{ac^3}-\frac{16a\int \frac{1}{x^3(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{c^3}-\frac{2}{ac^3x^4\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}$$

input
```
Int[1/(x^4*(c + a^2*c*x^2)^3*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^{4} \left(a^{2} c \,x^{2}+c \right)^{3} \arctan \left(a x \right)^{\frac{3}{2}}}d x$$

input
```
int(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)
```

output
```
int(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^4 \left(c+a^2 c x^2\right)^3 \arctan(ax)^{3/2}}\, dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 26.83 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.75

$$\int \frac{1}{x^4 \left(c+a^2 c x^2\right)^3 \arctan(ax)^{3/2}}\, dx=\frac{\int \frac{1}{a^{6} x^{10} \operatorname{atan}^{\frac{3}{2}}{\left(a x \right)} + 3 a^{4} x^{8} \operatorname{atan}^{\frac{3}{2}}{\left(a x \right)} + 3 a^{2} x^{6} \operatorname{atan}^{\frac{3}{2}}{\left(a x \right)} + x^{4} \operatorname{atan}^{\frac{3}{2}}{\left(a x \right)}}\, dx}{c^{3}}$$

input
```
integrate(1/x**4/(a**2*c*x**2+c)**3/atan(a*x)**(3/2),x)
```

output
```
Integral(1/(a**6*x**10*atan(a*x)**(3/2) + 3*a**4*x**8*atan(a*x)**(3/2) + 3
*a**2*x**6*atan(a*x)**(3/2) + x**4*atan(a*x)**(3/2)), x)/c**3
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^4 (c + a^2cx^2)^3 \arctan(ax)^{3/2}} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4 (c + a^2cx^2)^3 \arctan(ax)^{3/2}} \, dx = \int \frac{1}{(a^2cx^2 + c)^3 x^4 \arctan(ax)^{\frac{3}{2}}} \, dx$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^3*x^4*arctan(a*x)^(3/2)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.84 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4 (c+a^2cx^2)^3 \arctan(ax)^{3/2}}\,dx = \int \frac{1}{x^4\,\mathrm{atan}\left(a\,x\right)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

input
```
int(1/(x^4*atan(a*x)^(3/2)*(c + a^2*c*x^2)^3),x)
```

output
```
int(1/(x^4*atan(a*x)^(3/2)*(c + a^2*c*x^2)^3), x)
```

## Reduce [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.75

$$\int \frac{1}{x^4 (c+a^2cx^2)^3 \arctan(ax)^{3/2}}\,dx = \frac{\int \frac{\sqrt{atan(ax)}}{atan(ax)^2 a^6 x^{10}+3 atan(ax)^2 a^4 x^8+3 atan(ax)^2 a^2 x^6+atan(ax)^2 x^4}dx}{c^3}$$

input
```
int(1/x^4/(a^2*c*x^2+c)^3/atan(a*x)^(3/2),x)
```

output
```
int(sqrt(atan(a*x))/(atan(a*x)**2*a**6*x**10 + 3*atan(a*x)**2*a**4*x**8 +
3*atan(a*x)**2*a**2*x**6 + atan(a*x)**2*x**4),x)/c**3
```

# 3.1006 $\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^{3/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^{3/2}}\,dx=\text{Int}\left(\frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^{3/2}},x\right)$$

output
```
Defer(Int)(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.98 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^{3/2}}\,dx=\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[(x^m*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[(x^m*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^(3/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \sqrt{a^2 c x^2 + c}}{\arctan(ax)^{3/2}} \, dx$$

$$\Big\downarrow \ 5560$$

$$\int \frac{x^m \sqrt{a^2 c x^2 + c}}{\arctan(ax)^{3/2}} dx$$

input
```
Int[(x^m*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^m \sqrt{a^2 c \, x^2 + c}}{\arctan \left(a x \right)^{\frac{3}{2}}} dx$$

input
```
int(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^{3/2}}\,dx = \int \frac{\sqrt{a^2cx^2+c}\,x^m}{\arctan(ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^m/arctan(a*x)^(3/2), x)
```

## Sympy [N/A]

Not integrable

Time = 55.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^{3/2}}\,dx = \int \frac{x^m\sqrt{c\,(a^2x^2+1)}}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx$$

input
```
integrate(x**m*(a**2*c*x**2+c)**(1/2)/atan(a*x)**(3/2),x)
```

output
```
Integral(x**m*sqrt(c*(a**2*x**2 + 1))/atan(a*x)**(3/2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^{3/2}}\,dx = \int \frac{x^m\,\sqrt{c\,a^2\,x^2+c}}{{\mathrm{atan}\left(a\,x\right)}^{3/2}} \,d x$$

input
```
int((x^m*(c + a^2*c*x^2)^(1/2))/atan(a*x)^(3/2),x)
```

output
```
int((x^m*(c + a^2*c*x^2)^(1/2))/atan(a*x)^(3/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^{3/2}}\,dx = \sqrt{c}\left(\int \frac{x^m\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}}{atan\,(ax)^2}dx\right)$$

input
```
int(x^m*(a^2*c*x^2+c)^(1/2)/atan(a*x)^(3/2),x)
```

output
```
sqrt(c)*int((x**m*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/atan(a*x)**2,x)
```

# 3.1007 $\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^{3/2}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^{3/2}}\,dx=\text{Int}\left(\frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^{3/2}},x\right)$$

output
```
Defer(Int)(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 2.05 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^{3/2}}\,dx=\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[(x*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[(x*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^(3/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{a^2cx^2+c}}{\arctan(ax)^{3/2}} \, dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{x\sqrt{a^2cx^2+c}}{\arctan(ax)^{3/2}}dx$$

input
```
Int[(x*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{x\sqrt{a^2c\,x^2+c}}{\arctan\left(ax\right)^{\frac{3}{2}}}dx$$

input
```
int(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x)
```

output
```
int(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 3.39 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^{3/2}}\,dx = \int \frac{x\sqrt{c\left(a^2x^2+1\right)}}{\operatorname{atan}^{\frac{3}{2}}\left(ax\right)}\,dx$$

input
```
integrate(x*(a**2*c*x**2+c)**(1/2)/atan(a*x)**(3/2),x)
```

output
```
Integral(x*sqrt(c*(a**2*x**2 + 1))/atan(a*x)**(3/2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^{3/2}}\,dx = \int \frac{\sqrt{a^2cx^2+c}x}{\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*x/arctan(a*x)^(3/2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^{3/2}}\,dx = \int \frac{x\,\sqrt{c\,a^2\,x^2+c}}{\mathrm{atan}\left(a\,x\right)^{3/2}}\,dx$$

input
```
int((x*(c + a^2*c*x^2)^(1/2))/atan(a*x)^(3/2),x)
```

output
```
int((x*(c + a^2*c*x^2)^(1/2))/atan(a*x)^(3/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.17

$$\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^{3/2}}\,dx = \sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan\left(ax\right)}\,x}{atan\left(ax\right)^2}dx\right)$$

input
```
int(x*(a^2*c*x^2+c)^(1/2)/atan(a*x)^(3/2),x)
```

output
```
sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x)/atan(a*x)**2,x)
```

# 3.1008 $\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^{3/2}}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^{3/2}}\,dx = \text{Int}\left(\frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^{3/2}},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^{3/2}}\,dx = \int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[Sqrt[c + a^2*c*x^2]/ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[Sqrt[c + a^2*c*x^2]/ArcTan[a*x]^(3/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2cx^2+c}}{\arctan(ax)^{3/2}}\,dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{\sqrt{a^2cx^2+c}}{\arctan(ax)^{3/2}}dx$$

input
```
Int[Sqrt[c + a^2*c*x^2]/ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{a^2c\,x^2+c}}{\arctan\left(ax\right)^{\frac{3}{2}}}dx$$

input
```
int((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x)
```

output
```
int((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 4.81 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^{3/2}}\,dx = \int \frac{\sqrt{c\left(a^2x^2+1\right)}}{\operatorname{atan}^{\frac{3}{2}}\left(ax\right)}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(1/2)/atan(a*x)**(3/2),x)
```

output
```
Integral(sqrt(c*(a**2*x**2 + 1))/atan(a*x)**(3/2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^{3/2}}\,dx=\int \frac{\sqrt{a^2cx^2+c}}{\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)/arctan(a*x)^(3/2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^{3/2}}\,dx=\int \frac{\sqrt{c\,a^2\,x^2+c}}{\mathrm{atan}\left(a\,x\right)^{3/2}}\,dx$$

input
```
int((c + a^2*c*x^2)^(1/2)/atan(a*x)^(3/2),x)
```

output
```
int((c + a^2*c*x^2)^(1/2)/atan(a*x)^(3/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^{3/2}}\,dx=\sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan\left(ax\right)}}{atan\left(ax\right)^2}dx\right)$$

input
```
int((a^2*c*x^2+c)^(1/2)/atan(a*x)^(3/2),x)
```

output
```
sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/atan(a*x)**2,x)
```

# 3.1009 $\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^{3/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^{3/2}}\,dx = \operatorname{Int}\left(\frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^{3/2}},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^(3/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 5.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^{3/2}}\,dx = \int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[Sqrt[c + a^2*c*x^2]/(x*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[Sqrt[c + a^2*c*x^2]/(x*ArcTan[a*x]^(3/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2cx^2+c}}{x\arctan(ax)^{3/2}}dx$$

$$\Big\downarrow \text{5560}$$

$$\int \frac{\sqrt{a^2cx^2+c}}{x\arctan(ax)^{3/2}}dx$$

input
```
Int[Sqrt[c + a^2*c*x^2]/(x*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{a^2c\,x^2+c}}{x\arctan\left(ax\right)^{\frac{3}{2}}}dx$$

input
```
int((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^(3/2),x)
```

output
```
int((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 8.10 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^{3/2}}\,dx = \int \frac{\sqrt{c\,(a^2x^2+1)}}{x\,\mathrm{atan}^{\frac{3}{2}}\,(ax)}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(1/2)/x/atan(a*x)**(3/2),x)
```

output
```
Integral(sqrt(c*(a**2*x**2 + 1))/(x*atan(a*x)**(3/2)), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^{3/2}}\,dx = \int \frac{\sqrt{a^2cx^2+c}}{x\arctan{(ax)}^{\frac{3}{2}}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)/(x*arctan(a*x)^(3/2)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^{3/2}}\,dx = \int \frac{\sqrt{c\,a^2\,x^2+c}}{x\,\mathrm{atan}\left(a\,x\right)^{3/2}}\,dx$$

input
```
int((c + a^2*c*x^2)^(1/2)/(x*atan(a*x)^(3/2)),x)
```

output
```
int((c + a^2*c*x^2)^(1/2)/(x*atan(a*x)^(3/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^{3/2}}\,dx = \sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}}{atan\,(ax)^2\,x}dx\right)$$

input
```
int((a^2*c*x^2+c)^(1/2)/x/atan(a*x)^(3/2),x)
```

output
```
sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(atan(a*x)**2*x),x)
```

# 3.1010 $\int \frac{x^m\left(c+a^2cx^2\right)^{3/2}}{\arctan(ax)^{3/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)^{3/2}}\,dx=\operatorname{Int}\left(\frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)^{3/2}},x\right)$$

output
```
Defer(Int)(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)^{3/2}}\,dx=\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[(x^m*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[(x^m*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^(3/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \left(a^2 c x^2+c\right)^{3/2}}{\arctan (a x)^{3/2}} \, dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{x^m \left(a^2 c x^2+c\right)^{3/2}}{\arctan (a x)^{3/2}}dx$$

input
```
Int[(x^m*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^{m} \left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}{\arctan \left(a x \right)^{\frac{3}{2}}}d x$$

input
```
int(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)^{3/2}}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}x^m}{\arctan{(ax)}^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^(3/2)*x^m/arctan(a*x)^(3/2), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**m*(a**2*c*x**2+c)**(3/2)/atan(a*x)**(3/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)^{3/2}}\,dx = \int \frac{x^m\,\left(c\,a^2\,x^2+c\right)^{3/2}}{\mathrm{atan}\left(a\,x\right)^{3/2}}\,dx$$

input
```
int((x^m*(c + a^2*c*x^2)^(3/2))/atan(a*x)^(3/2),x)
```

output
```
int((x^m*(c + a^2*c*x^2)^(3/2))/atan(a*x)^(3/2), x)
```

## Reduce [N/A]

Not integrable

Time = 11.36 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.54

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)^{3/2}}\,dx = \sqrt{c}\,c\left(\left(\int \frac{x^m\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}\,x^2}{atan\,(ax)^2}dx\right)a^2+\int \frac{x^m\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}}{atan\,(ax)^2}dx\right.$$

input
```
int(x^m*(a^2*c*x^2+c)^(3/2)/atan(a*x)^(3/2),x)
```

output
```
sqrt(c)*c*(int((x**m*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**2)/atan(a*x)**
2,x)*a**2 + int((x**m*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/atan(a*x)**2,x)
)
```

# 3.1011 $\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^{3/2}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^{3/2}}\,dx = \text{Int}\left(\frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^{3/2}},x\right)$$

output
```
Defer(Int)(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 6.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^{3/2}}\,dx = \int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[(x*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[(x*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^(3/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a^2cx^2+c)^{3/2}}{\arctan(ax)^{3/2}}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{x(a^2cx^2+c)^{3/2}}{\arctan(ax)^{3/2}}dx$$

input
```
Int[(x*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{x(a^2c\,x^2+c)^{\frac{3}{2}}}{\arctan(ax)^{\frac{3}{2}}}dx$$

input
```
int(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

output
```
int(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 49.87 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^{3/2}}\,dx = \int \frac{x(c(a^2x^2+1))^{\frac{3}{2}}}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx$$

input
```
integrate(x*(a**2*c*x**2+c)**(3/2)/atan(a*x)**(3/2),x)
```

output
```
Integral(x*(c*(a**2*x**2 + 1))**(3/2)/atan(a*x)**(3/2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^{3/2}}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}x}{\arctan{(ax)}^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*x/arctan(a*x)^(3/2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^{3/2}}\,dx = \int \frac{x\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}{{\mathrm{atan}\left(a\,x\right)}^{3/2}}\,dx$$

input
```
int((x*(c + a^2*c*x^2)^(3/2))/atan(a*x)^(3/2),x)
```

output
```
int((x*(c + a^2*c*x^2)^(3/2))/atan(a*x)^(3/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.54

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^{3/2}}\,dx = \sqrt{c}\,c\left(\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,x^3}{atan(ax)^2}dx\right)a^2+\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,x}{atan(ax)^2}dx\right)$$

input
```
int(x*(a^2*c*x^2+c)^(3/2)/atan(a*x)^(3/2),x)
```

output
```
sqrt(c)*c*(int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**3)/atan(a*x)**2,x)*
a**2 + int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x)/atan(a*x)**2,x))
```

# 3.1012 $\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^{3/2}}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^{3/2}}\,dx = \text{Int}\left(\frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^{3/2}},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.09 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^{3/2}}\,dx = \int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^(3/2)/ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[(c + a^2*c*x^2)^(3/2)/ArcTan[a*x]^(3/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2cx^2+c\right)^{3/2}}{\arctan(ax)^{3/2}}dx$$

$$\Big\downarrow \text{5560}$$

$$\int \frac{\left(a^2cx^2+c\right)^{3/2}}{\arctan(ax)^{3/2}}dx$$

input
```
Int[(c + a^2*c*x^2)^(3/2)/ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{\left(a^2c\,x^2+c\right)^{\frac{3}{2}}}{\arctan\left(ax\right)^{\frac{3}{2}}}dx$$

input
```
int((a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

output
```
int((a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 28.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^{3/2}}\,dx = \int \frac{(c(a^2x^2+1))^{\frac{3}{2}}}{\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(3/2)/atan(a*x)**(3/2),x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(3/2)/atan(a*x)**(3/2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^{3/2}}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}}{\arctan(ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)/arctan(a*x)^(3/2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^{3/2}}\,dx = \int \frac{(c\,a^2\,x^2+c)^{3/2}}{\mathrm{atan}\left(a\,x\right)^{3/2}}\,dx$$

input
```
int((c + a^2*c*x^2)^(3/2)/atan(a*x)^(3/2),x)
```

output
```
int((c + a^2*c*x^2)^(3/2)/atan(a*x)^(3/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.61

$$\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^{3/2}}\,dx = \sqrt{c}\,c\left(\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,x^2}{atan(ax)^2}dx\right)a^2+\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}}{atan(ax)^2}dx\right)$$

input
```
int((a^2*c*x^2+c)^(3/2)/atan(a*x)^(3/2),x)
```

output
```
sqrt(c)*c*(int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**2)/atan(a*x)**2,x)*
a**2 + int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/atan(a*x)**2,x))
```

# 3.1013 $\int \frac{\left(c+a^2cx^2\right)^{3/2}}{x\arctan (ax)^{3/2}} \, dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\left(c+a^2cx^2\right)^{3/2}}{x\arctan (ax)^{3/2}} \, dx=\text{Int}\left(\frac{\left(c+a^2cx^2\right)^{3/2}}{x\arctan (ax)^{3/2}},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^(3/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 10.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\left(c+a^2cx^2\right)^{3/2}}{x\arctan (ax)^{3/2}} \, dx=\int \frac{\left(c+a^2cx^2\right)^{3/2}}{x\arctan (ax)^{3/2}} \, dx$$

input
```
Integrate[(c + a^2*c*x^2)^(3/2)/(x*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[(c + a^2*c*x^2)^(3/2)/(x*ArcTan[a*x]^(3/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^{3/2}}{x\arctan(ax)^{3/2}}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{(a^2cx^2+c)^{3/2}}{x\arctan(ax)^{3/2}}dx$$

input
```
Int[(c + a^2*c*x^2)^(3/2)/(x*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{(a^2c\,x^2+c)^{\frac{3}{2}}}{x\arctan(ax)^{\frac{3}{2}}}dx$$

input
```
int((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^(3/2),x)
```

output
```
int((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 45.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)^{3/2}}\,dx = \int \frac{(c(a^2x^2+1))^{\frac{3}{2}}}{x\,\mathrm{atan}^{\frac{3}{2}}(ax)}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(3/2)/x/atan(a*x)**(3/2),x)
```

output
```
Integral((c*(a**2*x**2 + 1))**(3/2)/(x*atan(a*x)**(3/2)), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)^{3/2}}\,dx=\int \frac{(a^2cx^2+c)^{\frac{3}{2}}}{x\arctan{(ax)}^{\frac{3}{2}}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)/(x*arctan(a*x)^(3/2)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)^{3/2}}\,dx=\int \frac{{\left(c\,a^2\,x^2+c\right)}^{3/2}}{x\,{\mathrm{atan}\left(a\,x\right)}^{3/2}}\,dx$$

input
```
int((c + a^2*c*x^2)^(3/2)/(x*atan(a*x)^(3/2)),x)
```

output
```
int((c + a^2*c*x^2)^(3/2)/(x*atan(a*x)^(3/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.35

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)^{3/2}}\,dx = \sqrt{c}\,c\left(\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}\,x}{atan\,(ax)^2}dx\right)a^2+\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}}{atan\,(ax)^2\,x}dx\right)$$

input
```
int((a^2*c*x^2+c)^(3/2)/x/atan(a*x)^(3/2),x)
```

output
```
sqrt(c)*c*(int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x)/atan(a*x)**2,x)*a**
2 + int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(atan(a*x)**2*x),x))
```

# 3.1014 $\int \frac{x^m\left(c+a^2cx^2\right)^{5/2}}{\arctan(ax)^{3/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\arctan(ax)^{3/2}}\,dx = \text{Int}\left(\frac{x^m(c+a^2cx^2)^{5/2}}{\arctan(ax)^{3/2}},x\right)$$

output
```
Defer(Int)(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.44 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\arctan(ax)^{3/2}}\,dx = \int \frac{x^m(c+a^2cx^2)^{5/2}}{\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[(x^m*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[(x^m*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^(3/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m (a^2cx^2+c)^{5/2}}{\arctan(ax)^{3/2}}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x^m (a^2cx^2+c)^{5/2}}{\arctan(ax)^{3/2}}dx$$

input
```
Int[(x^m*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 1.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^m (a^2c\,x^2+c)^{\frac{5}{2}}}{\arctan(ax)^{\frac{3}{2}}}dx$$

input
```
int(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.88

$$\int \frac{x^m (c + a^2 c x^2)^{5/2}}{\arctan(ax)^{3/2}} \, dx = \int \frac{(a^2 c x^2 + c)^{\frac{5}{2}} x^m}{\arctan(ax)^{\frac{3}{2}}} \, dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*x^m/arcta
n(a*x)^(3/2), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^m (c + a^2 c x^2)^{5/2}}{\arctan(ax)^{3/2}} \, dx = \text{Timed out}$$

input
```
integrate(x**m*(a**2*c*x**2+c)**(5/2)/atan(a*x)**(3/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m (c + a^2 c x^2)^{5/2}}{\arctan(ax)^{3/2}} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\arctan(ax)^{3/2}}\,dx = \int \frac{x^m\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}{{\mathrm{atan}\left(a\,x\right)}^{3/2}}\,dx$$

input
```
int((x^m*(c + a^2*c*x^2)^(5/2))/atan(a*x)^(3/2),x)
```

output
```
int((x^m*(c + a^2*c*x^2)^(5/2))/atan(a*x)^(3/2), x)
```

## Reduce [N/A]

Not integrable

Time = 147.93 (sec) , antiderivative size = 103, normalized size of antiderivative = 3.96

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\arctan(ax)^{3/2}}\,dx = \sqrt{c}\,c^2\left(\left(\left(\int \frac{x^m\sqrt{a^2x^2+1}\,\sqrt{atan\left(ax\right)}\,x^4}{atan\left(ax\right)^2}dx\right)a^4+2\left(\int \frac{x^m\sqrt{a^2x^2+1}\,\sqrt{atan\left(a\right.}}{atan\left(ax\right)^2}\right.\right.\right.$$

input
```
int(x^m*(a^2*c*x^2+c)^(5/2)/atan(a*x)^(3/2),x)
```

output
```
sqrt(c)*c**2*(int((x**m*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**4)/atan(a*x
)**2,x)*a**4 + 2*int((x**m*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**2)/atan(
a*x)**2,x)*a**2 + int((x**m*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/atan(a*x)
**2,x))
```

# 3.1015 $\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^{3/2}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^{3/2}}\,dx = \text{Int}\left(\frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^{3/2}},x\right)$$

output
```
Defer(Int)(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 3.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^{3/2}}\,dx = \int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[(x*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[(x*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^(3/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a^2cx^2+c)^{5/2}}{\arctan(ax)^{3/2}}\,dx$$
$$\Big\downarrow \ 5560$$
$$\int \frac{x(a^2cx^2+c)^{5/2}}{\arctan(ax)^{3/2}}dx$$

input
```
Int[(x*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{x(a^2c\,x^2+c)^{\frac{5}{2}}}{\arctan\left(ax\right)^{\frac{3}{2}}}dx$$

input
```
int(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

output
```
int(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x*(a**2*c*x**2+c)**(5/2)/atan(a*x)**(3/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^{3/2}}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}x}{\arctan{(ax)}^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)*x/arctan(a*x)^(3/2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^{3/2}}\,dx = \int \frac{x\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}{{\mathrm{atan}\left(a\,x\right)}^{3/2}}\,dx$$

input
```
int((x*(c + a^2*c*x^2)^(5/2))/atan(a*x)^(3/2),x)
```

output
```
int((x*(c + a^2*c*x^2)^(5/2))/atan(a*x)^(3/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 95, normalized size of antiderivative = 3.96

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^{3/2}}\,dx = \sqrt{c}\,c^2\left(\left(\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,x^5}{atan(ax)^2}dx\right)a^4+2\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,x^3}{atan(ax)^2}dx\right.\right.\right.$$

input
```
int(x*(a^2*c*x^2+c)^(5/2)/atan(a*x)^(3/2),x)
```

output
```
sqrt(c)*c**2*(int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**5)/atan(a*x)**2,
x)*a**4 + 2*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**3)/atan(a*x)**2,x)
*a**2 + int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x)/atan(a*x)**2,x))
```

# 3.1016 $\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^{3/2}} \, dx$



## Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^{3/2}} \, dx = \text{Int}\left(\frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^{3/2}}, x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.92 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^{3/2}} \, dx = \int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^{3/2}} \, dx$$

input
```
Integrate[(c + a^2*c*x^2)^(5/2)/ArcTan[a*x]^(3/2),x]
```

output
```
Integrate[(c + a^2*c*x^2)^(5/2)/ArcTan[a*x]^(3/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^{5/2}}{\arctan(ax)^{3/2}}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{(a^2cx^2+c)^{5/2}}{\arctan(ax)^{3/2}}dx$$

input
```
Int[(c + a^2*c*x^2)^(5/2)/ArcTan[a*x]^(3/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{(a^2c\,x^2+c)^{\frac{5}{2}}}{\arctan(ax)^{\frac{3}{2}}}dx$$

input
```
int((a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

output
```
int((a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((a**2*c*x**2+c)**(5/2)/atan(a*x)**(3/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^{3/2}}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}}{\arctan{(ax)}^{\frac{3}{2}}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)/arctan(a*x)^(3/2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^{3/2}}\,dx = \int \frac{(c\,a^2\,x^2+c)^{5/2}}{\mathrm{atan}\left(a\,x\right)^{3/2}}\,dx$$

input
```
int((c + a^2*c*x^2)^(5/2)/atan(a*x)^(3/2),x)
```

output
```
int((c + a^2*c*x^2)^(5/2)/atan(a*x)^(3/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 94, normalized size of antiderivative = 4.09

$$\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^{3/2}}\,dx = \sqrt{c}\,c^2\left(\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,x^4}{atan(ax)^2}dx\right)a^4+2\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,x^2}{atan(ax)^2}dx\right)\right.$$

input
```
int((a^2*c*x^2+c)^(5/2)/atan(a*x)^(3/2),x)
```

output
```
sqrt(c)*c**2*(int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**4)/atan(a*x)**2,
x)*a**4 + 2*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**2)/atan(a*x)**2,x)
*a**2 + int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/atan(a*x)**2,x))
```

# 3.1017 $\int \frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)^{3/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)^{3/2}}\,dx=\text{Int}\left(\frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)^{3/2}},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^(3/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 6.72 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)^{3/2}}\,dx=\int \frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^(5/2)/(x*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[(c + a^2*c*x^2)^(5/2)/(x*ArcTan[a*x]^(3/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^{5/2}}{x\arctan(ax)^{3/2}}dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{(a^2cx^2+c)^{5/2}}{x\arctan(ax)^{3/2}}dx$$

input
```
Int[(c + a^2*c*x^2)^(5/2)/(x*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{(a^2c\,x^2+c)^{\frac{5}{2}}}{x\arctan(ax)^{\frac{3}{2}}}dx$$

input
```
int((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^(3/2),x)
```

output
```
int((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((a**2*c*x**2+c)**(5/2)/x/atan(a*x)**(3/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)^{3/2}}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}}{x\arctan{(ax)}^{\frac{3}{2}}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)/(x*arctan(a*x)^(3/2)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)^{3/2}}\,dx = \int \frac{{(c\,a^2\,x^2+c)}^{5/2}}{x\,{\mathrm{atan}\left(a\,x\right)}^{3/2}}\,dx$$

input
```
int((c + a^2*c*x^2)^(5/2)/(x*atan(a*x)^(3/2)),x)
```

output
```
int((c + a^2*c*x^2)^(5/2)/(x*atan(a*x)^(3/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 95, normalized size of antiderivative = 3.65

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)^{3/2}}\,dx = \sqrt{c}\,c^2\left(\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}\,x^3}{atan\,(ax)^2}dx\right)a^4+2\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}\,x}{atan\,(ax)^2}dx\right.\right.$$

input
```
int((a^2*c*x^2+c)^(5/2)/x/atan(a*x)^(3/2),x)
```

output
```
sqrt(c)*c**2*(int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**3)/atan(a*x)**2,
x)*a**4 + 2*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x)/atan(a*x)**2,x)*a*
*2 + int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(atan(a*x)**2*x),x))
```

# 3.1018 $\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx = \text{Int}\left(\frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}},x\right)$$

output
```
Defer(Int)(x^m/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx = \int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[x^m/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[x^m/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{x^m}{\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}dx$$

input
```
Int[x^m/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^m}{\sqrt{a^2c\,x^2+c}\,\arctan\left(ax\right)^{\frac{3}{2}}}dx$$

input
```
int(x^m/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x)
```

output
```
int(x^m/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx = \int \frac{x^m}{\sqrt{a^2cx^2+c}\arctan{(ax)}^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
integral(x^m/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^(3/2)), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**m/(a**2*c*x**2+c)**(1/2)/atan(a*x)**(3/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx = \int \frac{x^m}{\sqrt{a^2cx^2+c}\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^m/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^(3/2)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx = \int \frac{x^m}{\mathrm{atan}\left(a\,x\right)^{3/2}\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(x^m/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(x^m/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.88

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx = \frac{\sqrt{c}\left(\int \frac{x^m\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}}{atan(ax)^2a^2x^2+atan(ax)^2}dx\right)}{c}$$

input
```
int(x^m/(a^2*c*x^2+c)^(1/2)/atan(a*x)^(3/2),x)
```

output
```
(sqrt(c)*int((x**m*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(atan(a*x)**2*a**2
*x**2 + atan(a*x)**2),x))/c
```

# 3.1019 $\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx = \text{Int}\left(\frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}},x\right)$$

output
```
Defer(Int)(x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.87 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx = \int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{x}{\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}dx$$

input
```
Int[x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{x}{\sqrt{a^2c\,x^2+c}\,\arctan\left(ax\right)^{\frac{3}{2}}}dx$$

input
```
int(x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x)
```

output
```
int(x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 7.70 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx = \int \frac{x}{\sqrt{c\,(a^2x^2+1)}\,\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx$$

input
```
integrate(x/(a**2*c*x**2+c)**(1/2)/atan(a*x)**(3/2),x)
```

output
```
Integral(x/(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**(3/2)), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx = \int \frac{x}{\sqrt{a^2cx^2+c}\arctan(ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(x/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^(3/2)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx = \int \frac{x}{\mathrm{atan}(a\,x)^{3/2}\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(x/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(x/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.96

$$\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,x}{atan(ax)^2a^2x^2+atan(ax)^2}dx\right)}{c}$$

input
```
int(x/(a^2*c*x^2+c)^(1/2)/atan(a*x)^(3/2),x)
```

output
```
(sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x)/(atan(a*x)**2*a**2*x*
*2 + atan(a*x)**2),x))/c
```

# 3.1020 $\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx = \text{Int}\left(\frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}},x\right)$$

output
```
Defer(Int)(1/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx = \int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int \frac{1}{\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}dx$$

input
```
Int[1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{1}{\sqrt{a^2c\,x^2+c}\,\arctan\left(ax\right)^{\frac{3}{2}}}dx$$

input
```
int(1/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x)
```

output
```
int(1/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 7.74 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx = \int \frac{1}{\sqrt{c\left(a^2x^2+1\right)}\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx$$

input
```
integrate(1/(a**2*c*x**2+c)**(1/2)/atan(a*x)**(3/2),x)
```

output
```
Integral(1/(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**(3/2)), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx = \int \frac{1}{\sqrt{a^2cx^2+c}\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^(3/2)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx = \int \frac{1}{\mathrm{atan}\left(a\,x\right)^{3/2}\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(1/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(1/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 79, normalized size of antiderivative = 3.43

$$\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx = \frac{2\sqrt{c}\left(atan(ax)\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,x}{atan(ax)a^2x^2+atan(ax)}dx\right)a^2-\sqrt{a^2x^2+1}\,\sqrt{atan\left(ax\right)}\right)}{atan\left(ax\right)ac}$$

input
```
int(1/(a^2*c*x^2+c)^(1/2)/atan(a*x)^(3/2),x)
```

output

```
(2*sqrt(c)*(atan(a*x)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x)/(atan(a*
x)*a**2*x**2 + atan(a*x)),x)*a**2 - sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))))/
(atan(a*x)*a*c)
```

# 3.1021 $\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx = -\frac{2\sqrt{c+a^2cx^2}}{acx\sqrt{\arctan(ax)}} - \frac{2\text{Int}\left(\frac{1}{x^2\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}},x\right)}{a}$$

output
```
-2*(a^2*c*x^2+c)^(1/2)/a/c/x/arctan(a*x)^(1/2)-2*Defer(Int)(1/x^2/(a^2*c*x
^2+c)^(1/2)/arctan(a*x)^(1/2),x)/a
```

## Mathematica [N/A]

Not integrable

Time = 3.41 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx = \int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[1/(x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[1/(x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x \arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5477}$$

$$-\frac{2\int \frac{1}{x^2\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx}{a}-\frac{2\sqrt{a^2cx^2+c}}{acx\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{5560}$$

$$-\frac{2\int \frac{1}{x^2\sqrt{a^2cx^2+c}\sqrt{\arctan(ax)}}dx}{a}-\frac{2\sqrt{a^2cx^2+c}}{acx\sqrt{\arctan(ax)}}$$

input
```
Int[1/(x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{1}{x\sqrt{a^2c\,x^2+c}\,\arctan\left(ax\right)^{\frac{3}{2}}}dx$$

input
```
int(1/x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x)
```

output
```
int(1/x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 19.61 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx = \int \frac{1}{x\sqrt{c\,(a^2x^2+1)}\,\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx$$

input
```
integrate(1/x/(a**2*c*x**2+c)**(1/2)/atan(a*x)**(3/2),x)
```

output
```
Integral(1/(x*sqrt(c*(a**2*x**2 + 1))*atan(a*x)**(3/2)), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx = \int \frac{1}{\sqrt{a^2cx^2+c}x\arctan\left(ax\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(a^2*c*x^2 + c)*x*arctan(a*x)^(3/2)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx = \int \frac{1}{x\,\mathrm{atan}\left(a\,x\right)^{3/2}\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(1/(x*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(1/(x*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.85

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}}{atan(ax)^2a^2x^3+atan(ax)^2x}dx\right)}{c}$$

input
```
int(1/x/(a^2*c*x^2+c)^(1/2)/atan(a*x)^(3/2),x)
```

output
```
(sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(atan(a*x)**2*a**2*x**3
 + atan(a*x)**2*x),x))/c
```

# 3.1022 $\int \frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx = \text{Int}\left(\frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}},x\right)$$

output
```
Defer(Int)(1/x^2/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 7.31 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx = \int \frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[1/(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[1/(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5560}$$

$$\int \frac{1}{x^2 \arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}dx$$

input
```
Int[1/(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^2\sqrt{a^2c\,x^2+c}\,\arctan\left(ax\right)^{\frac{3}{2}}}dx$$

input
```
int(1/x^2/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x)
```

output
```
int(1/x^2/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x, algorithm="fricas
")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 35.50 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx = \int \frac{1}{x^2\sqrt{c\,(a^2x^2+1)}\,\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx$$

input
```
integrate(1/x**2/(a**2*c*x**2+c)**(1/2)/atan(a*x)**(3/2),x)
```

output
```
Integral(1/(x**2*sqrt(c*(a**2*x**2 + 1))*atan(a*x)**(3/2)), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx = \int \frac{1}{\sqrt{a^2cx^2+c}x^2\arctan{(ax)}^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(a^2*c*x^2 + c)*x^2*arctan(a*x)^(3/2)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx = \int \frac{1}{x^2\,\mathrm{atan}\left(a\,x\right)^{3/2}\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(1/(x^2*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(1/(x^2*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(1/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.92

$$\int \frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}}{atan(ax)^2a^2x^4+atan(ax)^2x^2}dx\right)}{c}$$

input
```
int(1/x^2/(a^2*c*x^2+c)^(1/2)/atan(a*x)^(3/2),x)
```

output
```
(sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(atan(a*x)**2*a**2*x**4
 + atan(a*x)**2*x**2),x))/c
```

# 3.1023 $\int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx = \text{Int}\left(\frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}},x\right)$$

output
```
Defer(Int)(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.45 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx = \int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[x^m/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[x^m/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x^m}{\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}dx$$

input
```
Int[x^m/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^m}{\left(a^2c\,x^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^{\frac{3}{2}}}dx$$

input
```
int(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

output
```
int(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.96

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx = \int \frac{x^m}{(a^2cx^2+c)^{\frac{3}{2}}\arctan{(ax)}^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^m/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arct
an(a*x)^(3/2)), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**m/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(3/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx = \int \frac{x^m}{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^m/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^(3/2)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx = \int \frac{x^m}{\mathrm{atan}\,(a\,x)^{3/2}\,(c\,a^2\,x^2+c)^{3/2}}\,dx$$

input
```
int(x^m/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(x^m/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 422, normalized size of antiderivative = 16.23

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx = \frac{2\sqrt{c}\left(atan(ax)\left(\int \frac{x^m\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,x}{atan(ax)a^4x^4+2atan(ax)a^2x^2+atan(ax)}dx\right)a^4m\,x^2-atan$$

input
```
int(x^m/(a^2*c*x^2+c)^(3/2)/atan(a*x)^(3/2),x)
```

output
```
(2*sqrt(c)*(atan(a*x)*int((x**m*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x)/(at
an(a*x)*a**4*x**4 + 2*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**4*m*x**2 - at
an(a*x)*int((x**m*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x)/(atan(a*x)*a**4*x
**4 + 2*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**4*x**2 + atan(a*x)*int((x**
m*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x)/(atan(a*x)*a**4*x**4 + 2*atan(a*x
)*a**2*x**2 + atan(a*x)),x)*a**2*m - atan(a*x)*int((x**m*sqrt(a**2*x**2 +
1)*sqrt(atan(a*x))*x)/(atan(a*x)*a**4*x**4 + 2*atan(a*x)*a**2*x**2 + atan(
a*x)),x)*a**2 + atan(a*x)*int((x**m*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(
atan(a*x)*a**4*x**5 + 2*atan(a*x)*a**2*x**3 + atan(a*x)*x),x)*a**2*m*x**2
+ atan(a*x)*int((x**m*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(atan(a*x)*a**4
*x**5 + 2*atan(a*x)*a**2*x**3 + atan(a*x)*x),x)*m - x**m*sqrt(a**2*x**2 +
1)*sqrt(atan(a*x))))/(atan(a*x)*a*c**2*(a**2*x**2 + 1))
```

# 3.1024 $\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx = -\frac{2x^3}{ac\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}} + \frac{6\text{Int}\left(\frac{x^2}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}},x\right)}{a} + 4a\text{Int}\left(\frac{x^4}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}},x\right)$$

output
```
-2*x^3/a/c/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2)+6*Defer(Int)(x^2/(a^2*c*x
^2+c)^(3/2)/arctan(a*x)^(1/2),x)/a+4*a*Defer(Int)(x^4/(a^2*c*x^2+c)^(3/2)/
arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 5.92 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx = \int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[x^3/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[x^3/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{5503}$$

$$\frac{6\int \frac{x^2}{\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx}{a}+4a\int \frac{x^4}{\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{2x^3}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5560}$$

$$\frac{6\int \frac{x^2}{\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx}{a}+4a\int \frac{x^4}{\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{2x^3}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}$$

input
```
Int[x^3/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^3}{\left(a^2 c\, x^2+c\right)^{\frac{3}{2}} \arctan\left(a x\right)^{\frac{3}{2}}}dx$$

input
```
int(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

output
```
int(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2} \arctan(ax)^{3/2}}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 28.99 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2} \arctan(ax)^{3/2}}\, dx = \int \frac{x^3}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}} \operatorname{atan}^{\frac{3}{2}}\left(ax\right)}\, dx$$

input
```
integrate(x**3/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(3/2),x)
```

output
```
Integral(x**3/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**(3/2)), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx = \int \frac{x^3}{\mathrm{atan}\left(a\,x\right)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

input
```
int(x^3/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(x^3/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.42

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,x^3}{atan(ax)^2a^4x^4+2atan(ax)^2a^2x^2+atan(ax)^2}dx\right)}{c^2}$$

input
```
int(x^3/(a^2*c*x^2+c)^(3/2)/atan(a*x)^(3/2),x)
```

output
```
(sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**3)/(atan(a*x)**2*a**4
*x**4 + 2*atan(a*x)**2*a**2*x**2 + atan(a*x)**2),x))/c**2
```

# 3.1025 $\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx = -\frac{2x^2}{ac\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}} + \frac{4\sqrt{2\pi}\sqrt{1+a^2x^2}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{a^3c\sqrt{c+a^2cx^2}} + 2a\text{Int}\left(\frac{x^3}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}},x\right)$$

output
```
-2*x^2/a/c/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2)+4*2^(1/2)*Pi^(1/2)*(a^2*x
^2+1)^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))/a^3/c/(a^2*c*x^2+
c)^(1/2)+2*a*Defer(Int)(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 3.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx = \int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[x^2/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[x^2/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 1.46 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\arctan(ax)^{3/2}(a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \text{5503}$$

$$\frac{4\int \frac{x}{(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx}{a}+2a\int \frac{x^3}{(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{2x^2}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5506}$$

$$\frac{4\sqrt{a^2x^2+1}\int \frac{x}{(a^2x^2+1)^{3/2}\sqrt{\arctan(ax)}}dx}{ac\sqrt{a^2cx^2+c}}+2a\int \frac{x^3}{(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{2x^2}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5505}$$

$$2a\int \frac{x^3}{(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx+\frac{4\sqrt{a^2x^2+1}\int \frac{ax}{\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3c\sqrt{a^2cx^2+c}}-\frac{2x^2}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3042}$$

$$2a\int \frac{x^3}{(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx+\frac{4\sqrt{a^2x^2+1}\int \frac{\sin(\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3c\sqrt{a^2cx^2+c}}-\frac{2x^2}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3786}$$

$$2a\int \frac{x^3}{(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx+\frac{8\sqrt{a^2x^2+1}\int \frac{ax}{\sqrt{a^2x^2+1}}d\sqrt{\arctan(ax)}}{a^3c\sqrt{a^2cx^2+c}}-\frac{2x^2}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3832}$$

$$2a\int \frac{x^3}{(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{2x^2}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}+\frac{4\sqrt{2\pi}\sqrt{a^2x^2+1}\operatorname{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{a^3c\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5560}$$

$$2a\int \frac{x^3}{(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{2x^2}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}+\frac{4\sqrt{2\pi}\sqrt{a^2x^2+1}\operatorname{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{a^3c\sqrt{a^2cx^2+c}}$$

input
```
Int[x^2/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^2}{(a^2c\,x^2+c)^{\frac{3}{2}}\arctan\left(ax\right)^{\frac{3}{2}}}dx$$

input
```
int(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

output
```
int(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 28.77 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx=\int \frac{x^2}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx$$

input
```
integrate(x**2/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(3/2),x)
```

output
```
Integral(x**2/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**(3/2)), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx=\int \frac{x^2}{\left(a^2cx^2+c\right)^{\frac{3}{2}}\arctan(ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^2/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^(3/2)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx=\int \frac{x^2}{\mathrm{atan}\left(a\,x\right)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

input
```
int(x^2/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(x^2/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.42

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx=\frac{\sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,x^2}{atan(ax)^2a^4x^4+2atan(ax)^2a^2x^2+atan(ax)^2}dx\right)}{c^2}$$

input
```
int(x^2/(a^2*c*x^2+c)^(3/2)/atan(a*x)^(3/2),x)
```

output
```
(sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**2)/(atan(a*x)**2*a**4
*x**4 + 2*atan(a*x)**2*a**2*x**2 + atan(a*x)**2),x))/c**2
```

# 3.1026 $\int \frac{x}{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{3/2}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 93

$$\int \frac{x}{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{3/2}}\,dx=-\frac{2x}{ac\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}+\frac{2\sqrt{2\pi}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{a^2c\sqrt{c+a^2cx^2}}$$

output
```
-2*x/a/c/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2)+2*2^(1/2)*Pi^(1/2)*(a^2*x^2
+1)^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))/a^2/c/(a^2*c*x^2+c)
^(1/2)
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.25

$$\int \frac{x}{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{3/2}}\,dx=\frac{-2ax-i\sqrt{1+a^2x^2}\sqrt{-i\arctan(ax)}\,\Gamma\left(\frac{1}{2},-i\arctan(ax)\right)+i\sqrt{1+a^2}}{a^2c\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}$$

input
```
Integrate[x/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)),x]
```

output
```
(-2*a*x - I*Sqrt[1 + a^2*x^2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-I)*ArcTa
n[a*x]] + I*Sqrt[1 + a^2*x^2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, I*ArcTan[a*x]
])/(a^2*c*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])
```

## Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5477, 5440, 5439, 3042, 3785, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{5477}$$

$$\frac{2\int \frac{1}{(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2x}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5440}$$

$$\frac{2\sqrt{a^2x^2+1}\int \frac{1}{(a^2x^2+1)^{3/2}\sqrt{\arctan(ax)}}dx}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5439}$$

$$\frac{2\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{2x}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)}{\sqrt{\arctan(ax)}}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{2x}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3785}$$

$$\frac{4\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}}d\sqrt{\arctan(ax)}}{a^2c\sqrt{a^2cx^2+c}}-\frac{2x}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3833}$$

$$\frac{2\sqrt{2\pi}\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{a^2c\sqrt{a^2cx^2+c}}-\frac{2x}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}$$

input
```
Int[x/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)),x]
```

output
```
(-2*x)/(a*c*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]) + (2*Sqrt[2*Pi]*Sqrt[1
+ a^2*x^2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(a^2*c*Sqrt[c + a^2*c*x
^2])
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3785
```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5440
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])   Int[(1 +
c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

rule 5477
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*ArcT
an[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp[f*(m/(b*c*(p + 1)))   Int[(f*x
)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1
]
```

## Maple [F]

$$\int \frac{x}{\left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} \arctan \left(a x \right)^{\frac{3}{2}}}d x$$

input
```
int(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

output
```
int(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx = \int \frac{x}{\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx$$

input
```
integrate(x/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(3/2),x)
```

output
```
Integral(x/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**(3/2)), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx=\int \frac{x}{\operatorname{atan}(a\,x)^{3/2}\,(c\,a^2\,x^2+c)^{3/2}}\,dx$$

input
```
int(x/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(x/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2)), x)
```

## Reduce [F]

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx=\frac{2\sqrt{c}\left(atan(ax)\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}}{atan(ax)a^4x^4+2atan(ax)a^2x^2+atan(ax)}dx\right)a^2x^2+atan(a\right.}{atan\,(ax)\,a\,c}$$

input
```
int(x/(a^2*c*x^2+c)^(3/2)/atan(a*x)^(3/2),x)
```

output
```
(2*sqrt(c)*(atan(a*x)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(atan(a*x)
*a**4*x**4 + 2*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**2*x**2 + atan(a*x)*i
nt((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(atan(a*x)*a**4*x**4 + 2*atan(a*x
)*a**2*x**2 + atan(a*x)),x) - sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x))/(ata
n(a*x)*a*c**2*(a**2*x**2 + 1))
```

# 3.1027 $\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 92

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx = -\frac{2}{ac\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}} - \frac{2\sqrt{2\pi}\sqrt{1+a^2x^2}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{ac\sqrt{c+a^2cx^2}}$$

output
```
-2/a/c/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2)-2*2^(1/2)*Pi^(1/2)*(a^2*x^2+1
)^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))/a/c/(a^2*c*x^2+c)^(1/
2)
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.16

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx = \frac{-2+\sqrt{1+a^2x^2}\sqrt{-i\arctan(ax)}\,\Gamma\left(\frac{1}{2},-i\arctan(ax)\right)+\sqrt{1+a^2x^2}\sqrt{}}{ac\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}$$

input
```
Integrate[1/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)),x]
```

output
```
(-2 + Sqrt[1 + a^2*x^2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-I)*ArcTan[a*x]
] + Sqrt[1 + a^2*x^2]*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, I*ArcTan[a*x]])/(a*c*
Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])
```

## Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {5437, 5506, 5505, 3042, 3786, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{5437}$$

$$-2a\int \frac{x}{\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{2}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5506}$$

$$-\frac{2a\sqrt{a^2x^2+1}\int \frac{x}{\left(a^2x^2+1\right)^{3/2}\sqrt{\arctan(ax)}}dx}{c\sqrt{a^2cx^2+c}}-\frac{2}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5505}$$

$$-\frac{2\sqrt{a^2x^2+1}\int \frac{ax}{\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\sqrt{a^2x^2+1}\int \frac{\sin(\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3786}$$

$$-\frac{4\sqrt{a^2x^2+1}\int \frac{ax}{\sqrt{a^2x^2+1}}d\sqrt{\arctan(ax)}}{ac\sqrt{a^2cx^2+c}}-\frac{2}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3832}$$

$$-\frac{2\sqrt{2\pi}\sqrt{a^2x^2+1}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{ac\sqrt{a^2cx^2+c}}-\frac{2}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}$$

input
```
Int[1/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)),x]
```

output
```
-2/(a*c*Sqrt[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]) - (2*Sqrt[2*Pi]*Sqrt[1 + a^
2*x^2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(a*c*Sqrt[c + a^2*c*x^2])
```

## Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

## Maple [F]

$$\int \frac{1}{\left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} \arctan \left(a x \right)^{\frac{3}{2}}}d x$$

input
```
int(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

output
```
int(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx = \int \frac{1}{(c(a^2x^2+1))^{\frac{3}{2}}\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx$$

input
```
integrate(1/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(3/2),x)
```

output
```
Integral(1/((c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**(3/2)), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F]

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx = \int \frac{1}{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^(3/2)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx = \int \frac{1}{\mathrm{atan}\,(a\,x)^{3/2}\,(c\,a^2\,x^2+c)^{3/2}}\,dx$$

input
```
int(1/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(1/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2)), x)
```

## Reduce [F]

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx = -\frac{2\sqrt{c}\left(atan(ax)\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,x}{atan(ax)a^4x^4+2atan(ax)a^2x^2+atan(ax)}dx\right)a^4x^2+atan(ax)\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,x}{atan(ax)a^4x^4+2atan(ax)a^2x^2+atan(ax)}\right.\right.}{atan\,(ax)\,a\,c^2\,(a^2x^2+1)}$$

input
```
int(1/(a^2*c*x^2+c)^(3/2)/atan(a*x)^(3/2),x)
```

output
```
( - 2*sqrt(c)*(atan(a*x)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x)/(atan
(a*x)*a**4*x**4 + 2*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**4*x**2 + atan(a
*x)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x)/(atan(a*x)*a**4*x**4 + 2*a
tan(a*x)*a**2*x**2 + atan(a*x)),x)*a**2 + sqrt(a**2*x**2 + 1)*sqrt(atan(a*
x))))/(atan(a*x)*a*c**2*(a**2*x**2 + 1))
```

# 3.1028 $\int \frac{1}{x(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{1}{x(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx = -\frac{2}{acx\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}} - \frac{4\sqrt{2\pi}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{c\sqrt{c+a^2cx^2}} - \frac{2\text{Int}\left(\frac{1}{x^2(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}},x\right)}{a}$$

output
```
-2/a/c/x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2)-4*2^(1/2)*Pi^(1/2)*(a^2*x^2
+1)^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))/c/(a^2*c*x^2+c)^(1/
2)-2*Defer(Int)(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)/a
```

## Mathematica [N/A]

Not integrable

Time = 5.80 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{1}{x(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx = \int \frac{1}{x(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[1/(x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[1/(x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 1.44 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{5503}$$

$$-4a\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{2\int \frac{1}{x^2\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5440}$$

$$-\frac{4a\sqrt{a^2x^2+1}\int \frac{1}{\left(a^2x^2+1\right)^{3/2}\sqrt{\arctan(ax)}}dx}{c\sqrt{a^2cx^2+c}}-\frac{2\int \frac{1}{x^2\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5439}$$

$$-\frac{4\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}d\arctan(ax)}{c\sqrt{a^2cx^2+c}}-\frac{2\int \frac{1}{x^2\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\int \frac{1}{x^2\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{4\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)}{\sqrt{\arctan(ax)}}d\arctan(ax)}{c\sqrt{a^2cx^2+c}}-\frac{2}{acx\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \textbf{3785}$$

$$-\frac{8\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}}d\sqrt{\arctan(ax)}}{c\sqrt{a^2cx^2+c}}-\frac{2\int \frac{1}{x^2(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \textbf{3833}$$

$$-\frac{2\int \frac{1}{x^2(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{4\sqrt{2\pi}\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{c\sqrt{a^2cx^2+c}}-\frac{2}{acx\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \textbf{5560}$$

$$-\frac{2\int \frac{1}{x^2(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{4\sqrt{2\pi}\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{c\sqrt{a^2cx^2+c}}-\frac{2}{acx\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}$$

input
```
Int[1/(x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{1}{x\left(a^2c\,x^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^{\frac{3}{2}}}dx$$

input
```
int(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

output
```
int(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 62.81 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{3/2}}\,dx = \int \frac{1}{x\left(c\left(a^2x^2+1\right)\right)^{\frac{3}{2}}\operatorname{atan}^{\frac{3}{2}}(ax)}\,dx$$

input
```
integrate(1/x/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(3/2),x)
```

output
```
Integral(1/(x*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**(3/2)), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{3/2}}\,dx = \int \frac{1}{x\,\mathrm{atan}\left(a\,x\right)^{3/2}\,\left(c\,a^2\,x^2+c\right)^{3/2}}\,dx$$

input
```
int(1/(x*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(1/(x*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.38

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{3/2}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}}{atan(ax)^2a^4x^5+2atan(ax)^2a^2x^3+atan(ax)^2x}dx\right)}{c^2}$$

input
```
int(1/x/(a^2*c*x^2+c)^(3/2)/atan(a*x)^(3/2),x)
```

output
```
(sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(atan(a*x)**2*a**4*x**5
 + 2*atan(a*x)**2*a**2*x**3 + atan(a*x)**2*x),x))/c**2
```

# 3.1029 $\int \frac{1}{x^2(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{1}{x^2 (c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx = -\frac{2}{acx^2\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}} - \frac{4\text{Int}\left(\frac{1}{x^3(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}},x\right)}{a} - 6a\text{Int}\left(\frac{1}{x(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}},x\right)$$

output
```
-2/a/c/x^2/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2)-4*Defer(Int)(1/x^3/(a^2*c
*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)/a-6*a*Defer(Int)(1/x/(a^2*c*x^2+c)^(3/2
)/arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 10.17 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^2 (c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx = \int \frac{1}{x^2 (c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[1/(x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[1/(x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \arctan(ax)^{3/2} (a^2cx^2+c)^{3/2}}dx$$
$$\Big\downarrow \text{5503}$$
$$-6a\int \frac{1}{x(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{4\int \frac{1}{x^3(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^2\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}$$
$$\Big\downarrow \text{5560}$$
$$-6a\int \frac{1}{x(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{4\int \frac{1}{x^3(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^2\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}$$

input
```
Int[1/(x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^{2} \left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} \arctan \left(a x \right)^{\frac{3}{2}}}d x$$

input
```
int(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

output
```
int(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^2 (c+a^2cx^2)^{3/2} \arctan(ax)^{3/2}} \, dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="fricas
")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 104.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

$$\int \frac{1}{x^2 (c+a^2cx^2)^{3/2} \arctan(ax)^{3/2}} \, dx=\int \frac{1}{x^{2} \left(c \left(a^{2} x^{2} + 1\right)\right)^{\frac{3}{2}} \operatorname{atan}^{\frac{3}{2}}{\left(a x \right)}}\, dx$$

input
```
integrate(1/x**2/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(3/2),x)
```

output
```
Integral(1/(x**2*(c*(a**2*x**2 + 1))**(3/2)*atan(a*x)**(3/2)), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^2 (c+a^2cx^2)^{3/2} \arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^2 (c+a^2cx^2)^{3/2} \arctan(ax)^{3/2}}\,dx = \int \frac{1}{(a^2cx^2+c)^{\frac{3}{2}} x^2 \arctan(ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(3/2)*x^2*arctan(a*x)^(3/2)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^2 (c + a^2cx^2)^{3/2} \arctan(ax)^{3/2}} \, dx = \int \frac{1}{x^2 \,\mathrm{atan}\left(a\,x\right)^{3/2} \left(c\,a^2\,x^2+c\right)^{3/2}} \, dx$$

input
```
int(1/(x^2*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(1/(x^2*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.46

$$\int \frac{1}{x^2 (c + a^2cx^2)^{3/2} \arctan(ax)^{3/2}} \, dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}}{atan(ax)^2 a^4x^6+2atan(ax)^2 a^2x^4+atan(ax)^2 x^2}dx\right)}{c^2}$$

input
```
int(1/x^2/(a^2*c*x^2+c)^(3/2)/atan(a*x)^(3/2),x)
```

output
```
(sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(atan(a*x)**2*a**4*x**6
 + 2*atan(a*x)**2*a**2*x**4 + atan(a*x)**2*x**2),x))/c**2
```

# 3.1030 $\int \frac{1}{x^3(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{1}{x^3(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx = -\frac{2}{acx^3\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}} - \frac{6\text{Int}\left(\frac{1}{x^4(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}},x\right)}{a} - 8a\text{Int}\left(\frac{1}{x^2(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}},x\right)$$

output
```
-2/a/c/x^3/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2)-6*Defer(Int)(1/x^4/(a^2*c
*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)/a-8*a*Defer(Int)(1/x^2/(a^2*c*x^2+c)^(3
/2)/arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 13.50 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^3(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx = \int \frac{1}{x^3(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[1/(x^3*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[1/(x^3*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \arctan(ax)^{3/2} \left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{5503}$$

$$-8a\int \frac{1}{x^2 \left(a^2cx^2+c\right)^{3/2} \sqrt{\arctan(ax)}}dx-\frac{6\int \frac{1}{x^4\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^3\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5560}$$

$$-8a\int \frac{1}{x^2 \left(a^2cx^2+c\right)^{3/2} \sqrt{\arctan(ax)}}dx-\frac{6\int \frac{1}{x^4\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^3\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}$$

input
```
Int[1/(x^3*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^{3} \left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} \arctan \left(a x \right)^{\frac{3}{2}}}d x$$

input
```
int(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

output
```
int(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^3 (c+a^2cx^2)^{3/2} \arctan(ax)^{3/2}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="fricas
")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (c+a^2cx^2)^{3/2} \arctan(ax)^{3/2}} \, dx = \text{Timed out}$$

input
```
integrate(1/x**3/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(3/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^3 (c + a^2cx^2)^{3/2} \arctan(ax)^{3/2}} dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x^3 (c + a^2cx^2)^{3/2} \arctan(ax)^{3/2}} dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^3 (c + a^2cx^2)^{3/2} \arctan(ax)^{3/2}} dx = \int \frac{1}{x^3 \operatorname{atan}(a\,x)^{3/2} \left(c\,a^2\,x^2 + c\right)^{3/2}} dx$$

input
```
int(1/(x^3*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(1/(x^3*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.46

$$\int \frac{1}{x^3 (c+a^2cx^2)^{3/2} \arctan(ax)^{3/2}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}}{atan(ax)^2a^4x^7+2atan(ax)^2a^2x^5+atan(ax)^2x^3}dx\right)}{c^2}$$

input
```
int(1/x^3/(a^2*c*x^2+c)^(3/2)/atan(a*x)^(3/2),x)
```

output
```
(sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(atan(a*x)**2*a**4*x**7
 + 2*atan(a*x)**2*a**2*x**5 + atan(a*x)**2*x**3),x))/c**2
```

# 3.1031 $\int \frac{1}{x^4(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{1}{x^4 (c+a^2cx^2)^{3/2} \arctan(ax)^{3/2}}\,dx = -\frac{2}{acx^4\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}} - \frac{8\text{Int}\left(\frac{1}{x^5(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}},x\right)}{a} - 10a\text{Int}\left(\frac{1}{x^3(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}},x\right)$$

output
```
-2/a/c/x^4/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2)-8*Defer(Int)(1/x^5/(a^2*c
*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)/a-10*a*Defer(Int)(1/x^3/(a^2*c*x^2+c)^(
3/2)/arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 15.64 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^4 (c+a^2cx^2)^{3/2} \arctan(ax)^{3/2}}\,dx = \int \frac{1}{x^4 (c+a^2cx^2)^{3/2} \arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[1/(x^4*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[1/(x^4*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \arctan(ax)^{3/2} (a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \text{5503}$$

$$-\frac{8\int \frac{1}{x^5(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-10a\int \frac{1}{x^3(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{2}{acx^4\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5560}$$

$$-\frac{8\int \frac{1}{x^5(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-10a\int \frac{1}{x^3(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{2}{acx^4\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}$$

input
```
Int[1/(x^4*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^{4} \left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} \arctan \left(a x \right)^{\frac{3}{2}}}d x$$

input
```
int(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

output
```
int(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^4 (c+a^2cx^2)^{3/2} \arctan(ax)^{3/2}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="fricas
")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (c+a^2cx^2)^{3/2} \arctan(ax)^{3/2}} \, dx = \text{Timed out}$$

input
```
integrate(1/x**4/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(3/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^4 (c+a^2cx^2)^{3/2} \arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^4 (c+a^2cx^2)^{3/2} \arctan(ax)^{3/2}}\,dx = \int \frac{1}{(a^2cx^2+c)^{\frac{3}{2}} x^4 \arctan{(ax)}^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(3/2)*x^4*arctan(a*x)^(3/2)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^4 (c+a^2cx^2)^{3/2} \arctan(ax)^{3/2}}\,dx = \int \frac{1}{x^4\,\mathrm{atan}\left(a\,x\right)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

input
```
int(1/(x^4*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(1/(x^4*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(3/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.46

$$\int \frac{1}{x^4 (c+a^2cx^2)^{3/2} \arctan(ax)^{3/2}}\, dx = \frac{\sqrt{c} \left(\int \frac{\sqrt{a^2x^2+1}\, \sqrt{atan(ax)}}{atan(ax)^2 a^4x^8+2atan(ax)^2 a^2x^6+atan(ax)^2 x^4}dx\right)}{c^2}$$

input
```
int(1/x^4/(a^2*c*x^2+c)^(3/2)/atan(a*x)^(3/2),x)
```

output
```
(sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(atan(a*x)**2*a**4*x**8
 + 2*atan(a*x)**2*a**2*x**6 + atan(a*x)**2*x**4),x))/c**2
```

# 3.1032 $\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = \operatorname{Int}\left(\frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}},x\right)$$

output
```
Defer(Int)(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.57 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = \int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[x^m/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[x^m/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\arctan(ax)^{3/2} (a^2cx^2+c)^{5/2}} \, dx$$
$$\Big\downarrow \ 5560$$
$$\int \frac{x^m}{\arctan(ax)^{3/2} (a^2cx^2+c)^{5/2}}dx$$

input
```
Int[x^m/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^m}{(a^2c\,x^2+c)^{\frac{5}{2}} \arctan(ax)^{\frac{3}{2}}}dx$$

input
```
int(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

output
```
int(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.38

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = \int \frac{x^m}{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^m/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3
*x^2 + c^3)*arctan(a*x)^(3/2)), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**m/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(3/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = \int \frac{x^m}{(a^2cx^2+c)^{\frac{5}{2}}\arctan{(ax)}^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^m/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^(3/2)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = \int \frac{x^m}{\mathrm{atan}\left(a\,x\right)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

input
```
int(x^m/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(x^m/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 733, normalized size of antiderivative = 28.19

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(x^m/(a^2*c*x^2+c)^(5/2)/atan(a*x)^(3/2),x)
```

output

```
(2*sqrt(c)*(atan(a*x)*int((x**m*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x)/(at
an(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a
*x)),x)*a**6*m*x**4 - 3*atan(a*x)*int((x**m*sqrt(a**2*x**2 + 1)*sqrt(atan(
a*x))*x)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x
**2 + atan(a*x)),x)*a**6*x**4 + 2*atan(a*x)*int((x**m*sqrt(a**2*x**2 + 1)*
sqrt(atan(a*x))*x)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a
*x)*a**2*x**2 + atan(a*x)),x)*a**4*m*x**2 - 6*atan(a*x)*int((x**m*sqrt(a**
2*x**2 + 1)*sqrt(atan(a*x))*x)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**
4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**4*x**2 + atan(a*x)*int((x**m*
sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*
a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**2*m - 3*atan(a*x)*int
((x**m*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x)/(atan(a*x)*a**6*x**6 + 3*ata
n(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**2 + atan(a*x)*
int((x**m*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(atan(a*x)*a**6*x**7 + 3*at
an(a*x)*a**4*x**5 + 3*atan(a*x)*a**2*x**3 + atan(a*x)*x),x)*a**4*m*x**4 +
2*atan(a*x)*int((x**m*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(atan(a*x)*a**6
*x**7 + 3*atan(a*x)*a**4*x**5 + 3*atan(a*x)*a**2*x**3 + atan(a*x)*x),x)*a*
*2*m*x**2 + atan(a*x)*int((x**m*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(atan
(a*x)*a**6*x**7 + 3*atan(a*x)*a**4*x**5 + 3*atan(a*x)*a**2*x**3 + atan(a*x
)*x),x)*m - x**m*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))))/(atan(a*x)*a*c**...
```

# 3.1033 $\int \frac{x^3}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 160

$$\begin{aligned}\int \frac{x^3}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx &= -\frac{2x^3}{ac(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}\\ &+\frac{3\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{a^4c^2\sqrt{c+a^2cx^2}}\\ &-\frac{\sqrt{\frac{3\pi}{2}}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)}{a^4c^2\sqrt{c+a^2cx^2}}\end{aligned}$$

output
```
-2*x^3/a/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2)+3/2*2^(1/2)*Pi^(1/2)*(a^2
*x^2+1)^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))/a^4/c^2/(a^2*c*
x^2+c)^(1/2)-1/2*6^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)*FresnelC(6^(1/2)/Pi^(1
/2)*arctan(a*x)^(1/2))/a^4/c^2/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.14

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = \frac{-\frac{8a^3cx^3}{1+a^2x^2} - ic\sqrt{1+a^2x^2}\left(3\sqrt{-i\arctan(ax)}\Gamma\left(\frac{1}{2},-i\arctan(ax)\right) - 3\sqrt{\phantom{x}}\right.}{}$$

input
```
Integrate[x^3/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)),x]
```

output
```
((-8*a^3*c*x^3)/(1 + a^2*x^2) - I*c*Sqrt[1 + a^2*x^2]*(3*Sqrt[(-I)*ArcTan[
a*x]]*Gamma[1/2, (-I)*ArcTan[a*x]] - 3*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, I*Ar
cTan[a*x]] + Sqrt[3]*(-(Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-3*I)*ArcTan[a*
x]]) + Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (3*I)*ArcTan[a*x]])))/(4*a^4*c^3*Sqr
t[c + a^2*c*x^2]*Sqrt[ArcTan[a*x]])
```

## Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.84, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {5477, 5506, 5505, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\arctan(ax)^{3/2}(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{5477}$$

$$\frac{6\int \frac{x^2}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx}{a} - \frac{2x^3}{ac\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5506}$$

$$\frac{6\sqrt{a^2x^2+1}\int \frac{x^2}{(a^2x^2+1)^{5/2}\sqrt{\arctan(ax)}}dx}{ac^2\sqrt{a^2cx^2+c}} - \frac{2x^3}{ac\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5505}$$

$$\frac{6\sqrt{a^2x^2+1}\int \frac{a^2x^2}{(a^2x^2+1)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{a^4c^2\sqrt{a^2cx^2+c}}-\frac{2x^3}{ac\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{4906}$$

$$\frac{6\sqrt{a^2x^2+1}\int \left(\frac{1}{4\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}-\frac{\cos(3\arctan(ax))}{4\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^4c^2\sqrt{a^2cx^2+c}}-\frac{2x^3}{ac\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{6\sqrt{a^2x^2+1}\left(\frac{1}{2}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{2}\sqrt{\frac{\pi}{6}}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{a^4c^2\sqrt{a^2cx^2+c}}-\frac{2x^3}{ac\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}$$

input
```
Int[x^3/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)),x]
```

output
```
(-2*x^3)/(a*c*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]) + (6*Sqrt[1 + a^2*x
^2]*((Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/2 - (Sqrt[Pi/6]*F
resnelC[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/2))/(a^4*c^2*Sqrt[c + a^2*c*x^2])
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5477
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*ArcT
an[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp[f*(m/(b*c*(p + 1)))   Int[(f*x
)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1
]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

## Maple [F]

$$\int \frac{x^3}{(a^2c\,x^2+c)^{\frac{5}{2}} \arctan (ax)^{\frac{3}{2}}}dx$$

input
```
int(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

output
```
int(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**3/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(3/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = \int \frac{x^3}{\operatorname{atan}(a\,x)^{3/2}\,(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int(x^3/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(x^3/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2)), x)
```

## Reduce [F]

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,x^3}{atan(ax)^2a^6x^6+3atan(ax)^2a^4x^4+3atan(ax)^2a^2x^2+atan(ax)^2}dx\right)}{c^3}$$

input
```
int(x^3/(a^2*c*x^2+c)^(5/2)/atan(a*x)^(3/2),x)
```

output
```
(sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**3)/(atan(a*x)**2*a**6
*x**6 + 3*atan(a*x)**2*a**4*x**4 + 3*atan(a*x)**2*a**2*x**2 + atan(a*x)**2
),x))/c**3
```

# 3.1034 $\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 281

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = -\frac{2x^2}{ac(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}} - \frac{3\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{a^3c^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{2\pi}\sqrt{1+a^2x^2}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{a^3c^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{\frac{\pi}{6}}\sqrt{1+a^2x^2}\,\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)}{a^3c^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{\frac{2\pi}{3}}\sqrt{1+a^2x^2}\,\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)}{a^3c^2\sqrt{c+a^2cx^2}}$$

output
```
-2*x^2/a/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2)-1/2*2^(1/2)*Pi^(1/2)*(a^2
*x^2+1)^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))/a^3/c^2/(a^2*c*
x^2+c)^(1/2)+1/2*6^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)*FresnelS(6^(1/2)/Pi^(1
/2)*arctan(a*x)^(1/2))/a^3/c^2/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.86

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = \frac{-\frac{12a^2x^2}{\sqrt{\arctan(ax)}}+\sqrt{6\pi}(1+a^2x^2)^{3/2}\left(-3\sqrt{3}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right.\right.}{}$$

input
```
Integrate[x^2/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)),x]
```

output
```
((-12*a^2*x^2)/Sqrt[ArcTan[a*x]] + Sqrt[6*Pi]*(1 + a^2*x^2)^(3/2)*(-3*Sqrt
[3]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]] + FresnelS[Sqrt[6/Pi]*Sqrt[ArcT
an[a*x]]]) - ((1 + a^2*x^2)^(3/2)*(3*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-I
)*ArcTan[a*x]] + 3*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, I*ArcTan[a*x]] + Sqrt[3]
*(Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-3*I)*ArcTan[a*x]] + Sqrt[I*ArcTan[a*
x]]*Gamma[1/2, (3*I)*ArcTan[a*x]])))/Sqrt[ArcTan[a*x]])/(6*a^3*c*(c + a^2*
c*x^2)^(3/2))
```

## Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.83, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {5503, 5506, 5505, 3042, 3793, 2009, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\arctan(ax)^{3/2}(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{ 5503}$$

$$\frac{4\int \frac{x}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-2a\int \frac{x^3}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{2x^2}{ac\sqrt{\arctan(ax)}(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{ 5506}$$

$$\frac{4\sqrt{a^2x^2+1}\int \frac{x}{(a^2x^2+1)^{5/2}\sqrt{\arctan(ax)}}dx}{ac^2\sqrt{a^2cx^2+c}}-\frac{2a\sqrt{a^2x^2+1}\int \frac{x^3}{(a^2x^2+1)^{5/2}\sqrt{\arctan(ax)}}dx}{c^2\sqrt{a^2cx^2+c}}-\frac{2x^2}{ac\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{ 5505}$$

$$\frac{4\sqrt{a^2x^2+1}\int \frac{ax}{(a^2x^2+1)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3c^2\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}\int \frac{a^3x^3}{(a^2x^2+1)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3c^2\sqrt{a^2cx^2+c}}-\frac{2x^2}{ac\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{4\sqrt{a^2x^2+1}\int \frac{ax}{(a^2x^2+1)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3c^2\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}\int \frac{\sin(\arctan(ax))^3}{\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3c^2\sqrt{a^2cx^2+c}}-\frac{2x^2}{ac\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{ 3793}$$

$$\frac{4\sqrt{a^2x^2+1}\int \frac{ax}{(a^2x^2+1)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3c^2\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}\int \left(\frac{3ax}{4\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}-\frac{\sin(3\arctan(ax))}{4\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^3c^2\sqrt{a^2cx^2+c}}-\frac{2x^2}{ac\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{4\sqrt{a^2x^2+1}\int \frac{ax}{(a^2x^2+1)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3c^2\sqrt{a^2cx^2+c}}-\frac{2x^2}{ac\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}-\frac{2\sqrt{a^2x^2+1}\left(\frac{3}{2}\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{2}\sqrt{\frac{\pi}{6}}\,\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{a^3c^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 4906}$$

$$\frac{4\sqrt{a^2x^2+1}\int\left(\frac{ax}{4\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}+\frac{\sin(3\arctan(ax))}{4\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^3c^2\sqrt{a^2cx^2+c}}-\frac{2x^2}{ac\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}-\frac{2\sqrt{a^2x^2+1}\left(\frac{3}{2}\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{2}\sqrt{\frac{\pi}{6}}\operatorname{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{a^3c^2\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2x^2}{ac\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}-\frac{2\sqrt{a^2x^2+1}\left(\frac{3}{2}\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{2}\sqrt{\frac{\pi}{6}}\operatorname{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{a^3c^2\sqrt{a^2cx^2+c}}+\frac{4\sqrt{a^2x^2+1}\left(\frac{1}{2}\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{6}}\operatorname{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{a^3c^2\sqrt{a^2cx^2+c}}$$

input
```
Int[x^2/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)),x]
```

output
```
(-2*x^2)/(a*c*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]) - (2*Sqrt[1 + a^2*x
^2]*((3*Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/2 - (Sqrt[Pi/6]
*FresnelS[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/2))/(a^3*c^2*Sqrt[c + a^2*c*x^2])
 + (4*Sqrt[1 + a^2*x^2]*((Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]
])/2 + (Sqrt[Pi/6]*FresnelS[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/2))/(a^3*c^2*Sq
rt[c + a^2*c*x^2])
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

## Maple [F]

$$\int \frac{x^2}{(a^2c\,x^2+c)^{\frac{5}{2}} \arctan\left(ax\right)^{\frac{3}{2}}}dx$$

input
```
int(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

output
```
int(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**2/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(3/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F]

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = \int \frac{x^2}{(a^2cx^2+c)^{\frac{5}{2}}\arctan{(ax)}^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(x^2/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^(3/2)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = \int \frac{x^2}{\mathrm{atan}\left(a\,x\right)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

input
```
int(x^2/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(x^2/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2)), x)
```

## Reduce [F]

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,x^2}{atan(ax)^2a^6x^6+3atan(ax)^2a^4x^4+3atan(ax)^2a^2x^2+atan(ax)^2}dx\right)}{c^3}$$

input
```
int(x^2/(a^2*c*x^2+c)^(5/2)/atan(a*x)^(3/2),x)
```

output
```
(sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**2)/(atan(a*x)**2*a**6
*x**6 + 3*atan(a*x)**2*a**4*x**4 + 3*atan(a*x)**2*a**2*x**2 + atan(a*x)**2
),x))/c**3
```

# 3.1035 $\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 280

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = -\frac{2x}{ac(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}} + \frac{3\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{a^2c^2\sqrt{c+a^2cx^2}} - \frac{\sqrt{2\pi}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{a^2c^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{\frac{\pi}{6}}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)}{a^2c^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{\frac{2\pi}{3}}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)}{a^2c^2\sqrt{c+a^2cx^2}}$$

output
```
-2*x/a/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2)+1/2*2^(1/2)*Pi^(1/2)*(a^2*x
^2+1)^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))/a^2/c^2/(a^2*c*x^
2+c)^(1/2)+1/2*6^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)*FresnelC(6^(1/2)/Pi^(1/2
)*arctan(a*x)^(1/2))/a^2/c^2/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.07

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx =$$
$$-\frac{i\left(-8iax+(1+a^2x^2)^{3/2}\sqrt{-i\arctan(ax)}\Gamma\left(\frac{1}{2},-i\arctan(ax)\right)-(1+a^2x^2)^{3/2}\sqrt{i\arctan(ax)}\Gamma\left(\frac{1}{2},i\arcta\right.\right.}{}$$

input
```
Integrate[x/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)),x]
```

output
```
((-1/4*I)*((-8*I)*a*x + (1 + a^2*x^2)^(3/2)*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1
/2, (-I)*ArcTan[a*x]] - (1 + a^2*x^2)^(3/2)*Sqrt[I*ArcTan[a*x]]*Gamma[1/2,
 I*ArcTan[a*x]] + Sqrt[3 + 3*a^2*x^2]*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-
3*I)*ArcTan[a*x]] + a^2*x^2*Sqrt[3 + 3*a^2*x^2]*Sqrt[(-I)*ArcTan[a*x]]*Gam
ma[1/2, (-3*I)*ArcTan[a*x]] - Sqrt[3 + 3*a^2*x^2]*Sqrt[I*ArcTan[a*x]]*Gamm
a[1/2, (3*I)*ArcTan[a*x]] - a^2*x^2*Sqrt[3 + 3*a^2*x^2]*Sqrt[I*ArcTan[a*x]
]*Gamma[1/2, (3*I)*ArcTan[a*x]]))/(a^2*c^2*(1 + a^2*x^2)*Sqrt[c + a^2*c*x^
2]*Sqrt[ArcTan[a*x]])
```

## Rubi [A] (verified)

Time = 1.69 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.83, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5503, 5440, 5439, 3042, 3793, 2009, 5506, 5505, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\arctan(ax)^{3/2}\,(a^2cx^2+c)^{5/2}}\,dx$$

$$\Big\downarrow \text{5503}$$

$$\frac{2\int \frac{1}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-4a\int \frac{x^2}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{2x}{ac\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}$$

$\big\downarrow$ 5440

$$\frac{2\sqrt{a^2x^2+1}\int \frac{1}{(a^2x^2+1)^{5/2}\sqrt{\arctan(ax)}}dx}{ac^2\sqrt{a^2cx^2+c}}-4a\int \frac{x^2}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{2x}{ac\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}$$

$\big\downarrow$ 5439

$$\frac{2\sqrt{a^2x^2+1}\int \frac{1}{(a^2x^2+1)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}-4a\int \frac{x^2}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{2x}{ac\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}$$

$\big\downarrow$ 3042

$$\frac{2\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^3}{\sqrt{\arctan(ax)}}d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}-4a\int \frac{x^2}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{2x}{ac\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}$$

$\big\downarrow$ 3793

$$\frac{2\sqrt{a^2x^2+1}\int \left(\frac{\cos(3\arctan(ax))}{4\sqrt{\arctan(ax)}}+\frac{3}{4\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}-4a\int \frac{x^2}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{2x}{ac\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}$$

$\big\downarrow$ 2009

$$-4a\int \frac{x^2}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx+\frac{2\sqrt{a^2x^2+1}\left(\frac{3}{2}\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{6}}\operatorname{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{a^2c^2\sqrt{a^2cx^2+c}}-\frac{2x}{ac\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}$$

$\big\downarrow$ 5506

$$-\frac{4a\sqrt{a^2x^2+1}\int\frac{x^2}{(a^2x^2+1)^{5/2}\sqrt{\arctan(ax)}}dx}{c^2\sqrt{a^2cx^2+c}}+\frac{2\sqrt{a^2x^2+1}\left(\frac{3}{2}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{6}}\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{a^2c^2\sqrt{a^2cx^2+c}}-\frac{2x}{ac\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5505}$$

$$-\frac{4\sqrt{a^2x^2+1}\int\frac{a^2x^2}{(a^2x^2+1)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}+\frac{2\sqrt{a^2x^2+1}\left(\frac{3}{2}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{6}}\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{a^2c^2\sqrt{a^2cx^2+c}}-\frac{2x}{ac\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{4906}$$

$$-\frac{4\sqrt{a^2x^2+1}\int\left(\frac{1}{4\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}-\frac{\cos(3\arctan(ax))}{4\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}+\frac{2\sqrt{a^2x^2+1}\left(\frac{3}{2}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{6}}\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{a^2c^2\sqrt{a^2cx^2+c}}-\frac{2x}{ac\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{4\sqrt{a^2x^2+1}\left(\frac{1}{2}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{2}\sqrt{\frac{\pi}{6}}\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{a^2c^2\sqrt{a^2cx^2+c}}+\frac{2\sqrt{a^2x^2+1}\left(\frac{3}{2}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{6}}\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{a^2c^2\sqrt{a^2cx^2+c}}-\frac{2x}{ac\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}$$

input
```
Int[x/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)),x]
```

output
```
(-2*x)/(a*c*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]) - (4*Sqrt[1 + a^2*x^2
]*((Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/2 - (Sqrt[Pi/6]*Fre
snelC[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/2))/(a^2*c^2*Sqrt[c + a^2*c*x^2]) + (
2*Sqrt[1 + a^2*x^2]*((3*Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])
/2 + (Sqrt[Pi/6]*FresnelC[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/2))/(a^2*c^2*Sqrt
[c + a^2*c*x^2])
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5440
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])   Int[(1 +
c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

## Maple [F]

$$\int \frac{x}{\left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}} \arctan \left(a x \right)^{\frac{3}{2}}}d x$$

input
```
int(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

output
```
int(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(3/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = \int \frac{x}{\operatorname{atan}(a\,x)^{3/2}\,\left(c\,a^2\,x^2+c\right)^{5/2}}\,dx$$

input
```
int(x/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(x/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2)), x)
```

## Reduce [F]

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(x/(a^2*c*x^2+c)^(5/2)/atan(a*x)^(3/2),x)
```

output
```
(2*sqrt(c)*( - 2*atan(a*x)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**2)/
(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + ata
n(a*x)),x)*a**6*x**4 - 4*atan(a*x)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)
)*x**2)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x*
*2 + atan(a*x)),x)*a**4*x**2 - 2*atan(a*x)*int((sqrt(a**2*x**2 + 1)*sqrt(a
tan(a*x))*x**2)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)
*a**2*x**2 + atan(a*x)),x)*a**2 + atan(a*x)*int((sqrt(a**2*x**2 + 1)*sqrt(
atan(a*x)))/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**
2*x**2 + atan(a*x)),x)*a**4*x**4 + 2*atan(a*x)*int((sqrt(a**2*x**2 + 1)*sq
rt(atan(a*x)))/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*
a**2*x**2 + atan(a*x)),x)*a**2*x**2 + atan(a*x)*int((sqrt(a**2*x**2 + 1)*s
qrt(atan(a*x)))/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)
*a**2*x**2 + atan(a*x)),x) - sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x))/(atan
(a*x)*a*c**3*(a**4*x**4 + 2*a**2*x**2 + 1))
```

# 3.1036 $\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 157

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = -\frac{2}{ac(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}} - \frac{3\sqrt{\frac{\pi}{2}}\sqrt{1+a^2x^2}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{ac^2\sqrt{c+a^2cx^2}} - \frac{\sqrt{\frac{3\pi}{2}}\sqrt{1+a^2x^2}\,\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)}{ac^2\sqrt{c+a^2cx^2}}$$

output
```
-2/a/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2)-3/2*2^(1/2)*Pi^(1/2)*(a^2*x^2
+1)^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))/a/c^2/(a^2*c*x^2+c)
^(1/2)-1/2*6^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)*FresnelS(6^(1/2)/Pi^(1/2)*ar
ctan(a*x)^(1/2))/a/c^2/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.01

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = \frac{-8+(1+a^2x^2)^{3/2}\left(3\sqrt{-i\arctan(ax)}\Gamma\left(\frac{1}{2},-i\arctan(ax)\right)+3\sqrt{i\arc$$

input
```
Integrate[1/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)),x]
```

output
```
(-8 + (1 + a^2*x^2)^(3/2)*(3*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-I)*ArcTan
[a*x]] + 3*Sqrt[I*ArcTan[a*x]]*Gamma[1/2, I*ArcTan[a*x]] + Sqrt[3]*(Sqrt[(
-I)*ArcTan[a*x]]*Gamma[1/2, (-3*I)*ArcTan[a*x]] + Sqrt[I*ArcTan[a*x]]*Gamm
a[1/2, (3*I)*ArcTan[a*x]])))/(4*a*c*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]
])
```

## Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.83, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {5437, 5506, 5505, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arctan(ax)^{3/2}(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{5437}$$

$$-6a\int \frac{x}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{2}{ac\sqrt{\arctan(ax)}(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5506}$$

$$-\frac{6a\sqrt{a^2x^2+1}\int \frac{x}{(a^2x^2+1)^{5/2}\sqrt{\arctan(ax)}}dx}{c^2\sqrt{a^2cx^2+c}}-\frac{2}{ac\sqrt{\arctan(ax)}(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5505}$$

$$-\frac{6\sqrt{a^2x^2+1}\int \frac{ax}{(a^2x^2+1)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{2}{ac\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{4906}$$

$$-\frac{6\sqrt{a^2x^2+1}\int \left(\frac{ax}{4\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}+\frac{\sin(3\arctan(ax))}{4\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{2}{ac\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{6\sqrt{a^2x^2+1}\left(\frac{1}{2}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{6}}\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{ac^2\sqrt{a^2cx^2+c}}-\frac{2}{ac\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}$$

input
```
Int[1/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)),x]
```

output
```
-2/(a*c*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]) - (6*Sqrt[1 + a^2*x^2]*((
Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/2 + (Sqrt[Pi/6]*Fresnel
S[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/2))/(a*c^2*Sqrt[c + a^2*c*x^2])
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

## Maple [F]

$$\int \frac{1}{\left(a^{2} c\,x^{2}+c \right)^{\frac{5}{2}} \arctan \left(a x \right)^{\frac{3}{2}}}d x$$

input
```
int(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

output
```
int(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(3/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F]

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = \int \frac{1}{(a^2cx^2+c)^{\frac{5}{2}}\arctan{(ax)}^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^(3/2)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = \int \frac{1}{\mathrm{atan}\left(a\,x\right)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

input
```
int(1/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(1/(atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2)), x)
```

## Reduce [F]

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = \frac{2\sqrt{c}\left(-3atan(ax)\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,x}{atan(ax)a^6x^6+3atan(ax)a^4x^4+3atan(ax)a^2x^2+atan(ax)}dx\right)\right.}{}$$

input
```
int(1/(a^2*c*x^2+c)^(5/2)/atan(a*x)^(3/2),x)
```

output
```
(2*sqrt(c)*( - 3*atan(a*x)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x)/(at
an(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a
*x)),x)*a**6*x**4 - 6*atan(a*x)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x
)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + a
tan(a*x)),x)*a**4*x**2 - 3*atan(a*x)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*
x))*x)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**
2 + atan(a*x)),x)*a**2 - sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))))/(atan(a*x)*
a*c**3*(a**4*x**4 + 2*a**2*x**2 + 1))
```

# 3.1037 $\int \frac{1}{x(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{1}{x(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = -\frac{2}{acx(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}} - \frac{6\sqrt{2\pi}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{c^2\sqrt{c+a^2cx^2}} - \frac{2\sqrt{\frac{2\pi}{3}}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)}{c^2\sqrt{c+a^2cx^2}} - \frac{2\text{Int}\left(\frac{1}{x^2(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}},x\right)}{a}$$

output

```
-2/a/c/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2)-6*2^(1/2)*Pi^(1/2)*(a^2*x^2
+1)^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))/c^2/(a^2*c*x^2+c)^(
1/2)-2/3*6^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)*FresnelC(6^(1/2)/Pi^(1/2)*arct
an(a*x)^(1/2))/c^2/(a^2*c*x^2+c)^(1/2)-2*Defer(Int)(1/x^2/(a^2*c*x^2+c)^(5
/2)/arctan(a*x)^(1/2),x)/a
```

## Mathematica [N/A]

Not integrable

Time = 5.93 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{3/2}}\,dx=\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[1/(x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[1/(x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 1.49 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{5/2}}dx$$

$$\Big\downarrow \text{5503}$$

$$-8a\int \frac{1}{\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{2\int \frac{1}{x^2\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{5440}$$

$$-\frac{8a\sqrt{a^2x^2+1}\int \frac{1}{\left(a^2x^2+1\right)^{5/2}\sqrt{\arctan(ax)}}dx}{c^2\sqrt{a^2cx^2+c}}-\frac{2\int \frac{1}{x^2\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{5439}$$

$$-\frac{8\sqrt{a^2x^2+1}\int \frac{1}{(a^2x^2+1)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{c^2\sqrt{a^2cx^2+c}}-\frac{2\int \frac{1}{x^2(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{8\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^3}{\sqrt{\arctan(ax)}}d\arctan(ax)}{c^2\sqrt{a^2cx^2+c}}-\frac{2\int \frac{1}{x^2(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{8\sqrt{a^2x^2+1}\int \left(\frac{\cos(3\arctan(ax))}{4\sqrt{\arctan(ax)}}+\frac{3}{4\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{c^2\sqrt{a^2cx^2+c}}-\frac{2\int \frac{1}{x^2(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2\int \frac{1}{x^2(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{8\sqrt{a^2x^2+1}\left(\frac{3}{2}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{6}}\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{c^2\sqrt{a^2cx^2+c}}-\frac{2}{acx\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5560}$$

$$-\frac{2\int \frac{1}{x^2(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{8\sqrt{a^2x^2+1}\left(\frac{3}{2}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{6}}\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{c^2\sqrt{a^2cx^2+c}}-\frac{2}{acx\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}$$

input
```
Int[1/(x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{1}{x\left(a^2c\,x^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^{\frac{3}{2}}}dx$$

input
```
int(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

output
```
int(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x\,(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/x/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(3/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\,(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x\,(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\,(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = \int \frac{1}{x\,\mathrm{atan}\left(a\,x\right)^{3/2}\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

input
```
int(1/(x*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(1/(x*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.92

$$\int \frac{1}{x\,(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}}{atan(ax)^2a^6x^7+3atan(ax)^2a^4x^5+3atan(ax)^2a^2x^3+atan(ax)^2x}dx\right)}{c^3}$$

input
```
int(1/x/(a^2*c*x^2+c)^(5/2)/atan(a*x)^(3/2),x)
```

output
```
(sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(atan(a*x)**2*a**6*x**7
 + 3*atan(a*x)**2*a**4*x**5 + 3*atan(a*x)**2*a**2*x**3 + atan(a*x)**2*x),x
))/c**3
```

# 3.1038 $\int \frac{1}{x^2(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{1}{x^2(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = -\frac{2}{acx^2(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}} - \frac{4\mathrm{Int}\left(\frac{1}{x^3(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}},x\right)}{a} - 10a\mathrm{Int}\left(\frac{1}{x(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}},x\right)$$

output
```
-2/a/c/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2)-4*Defer(Int)(1/x^3/(a^2*c
*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)/a-10*a*Defer(Int)(1/x/(a^2*c*x^2+c)^(5/
2)/arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 13.53 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^2(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = \int \frac{1}{x^2(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[1/(x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[1/(x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \arctan(ax)^{3/2} \left(a^2cx^2+c\right)^{5/2}}dx$$

$$\Big\downarrow \text{5503}$$

$$-10a\int \frac{1}{x\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{4\int \frac{1}{x^3\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^2\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{5560}$$

$$-10a\int \frac{1}{x\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{4\int \frac{1}{x^3\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^2\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}$$

input
```
Int[1/(x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^{2} \left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}} \arctan \left(a x \right)^{\frac{3}{2}}}d x$$

input
```
int(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

output
```
int(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^2 (c+a^2cx^2)^{5/2} \arctan(ax)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="fricas
")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (c+a^2cx^2)^{5/2} \arctan(ax)^{3/2}}\,dx=\text{Timed out}$$

input
```
integrate(1/x**2/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(3/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^2 (c + a^2cx^2)^{5/2} \arctan(ax)^{3/2}} dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

## Giac [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^2 (c + a^2cx^2)^{5/2} \arctan(ax)^{3/2}} dx = \int \frac{1}{(a^2cx^2 + c)^{\frac{5}{2}} x^2 \arctan(ax)^{\frac{3}{2}}} dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(5/2)*x^2*arctan(a*x)^(3/2)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^2 (c + a^2cx^2)^{5/2} \arctan(ax)^{3/2}} dx = \int \frac{1}{x^2 \operatorname{atan}(a x)^{3/2} (c a^2 x^2 + c)^{5/2}} dx$$

input
```
int(1/(x^2*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(1/(x^2*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 78, normalized size of antiderivative = 3.00

$$\int \frac{1}{x^2 (c + a^2cx^2)^{5/2} \arctan(ax)^{3/2}} \, dx = \frac{\sqrt{c} \left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}}{atan(ax)^2 a^6x^8+3atan(ax)^2 a^4x^6+3atan(ax)^2 a^2x^4+atan(ax)^2 x^2}dx\right)}{c^3}$$

input
```
int(1/x^2/(a^2*c*x^2+c)^(5/2)/atan(a*x)^(3/2),x)
```

output
```
(sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(atan(a*x)**2*a**6*x**8
 + 3*atan(a*x)**2*a**4*x**6 + 3*atan(a*x)**2*a**2*x**4 + atan(a*x)**2*x**2
),x))/c**3
```

# 3.1039 $\int \frac{1}{x^3(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{1}{x^3(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = -\frac{2}{acx^3(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}} - \frac{6\text{Int}\left(\frac{1}{x^4(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}},x\right)}{a} - 12a\text{Int}\left(\frac{1}{x^2(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}},x\right)$$

output
```
-2/a/c/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2)-6*Defer(Int)(1/x^4/(a^2*c
*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)/a-12*a*Defer(Int)(1/x^2/(a^2*c*x^2+c)^(
5/2)/arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 14.93 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^3(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = \int \frac{1}{x^3(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[1/(x^3*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[1/(x^3*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \arctan(ax)^{3/2} \left(a^2cx^2+c\right)^{5/2}}dx$$

$$\Big\downarrow \ 5503$$

$$-12a\int \frac{1}{x^2 \left(a^2cx^2+c\right)^{5/2} \sqrt{\arctan(ax)}}dx-\frac{6\int \frac{1}{x^4\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^3\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \ 5560$$

$$-12a\int \frac{1}{x^2 \left(a^2cx^2+c\right)^{5/2} \sqrt{\arctan(ax)}}dx-\frac{6\int \frac{1}{x^4\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^3\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}$$

input
```
Int[1/(x^3*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^{3} \left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}} \arctan \left(a x \right)^{\frac{3}{2}}}d x$$

input
```
int(1/x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

output
```
int(1/x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^3 (c+a^2cx^2)^{5/2} \arctan(ax)^{3/2}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="fricas
")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (c+a^2cx^2)^{5/2} \arctan(ax)^{3/2}} \, dx = \text{Timed out}$$

input
```
integrate(1/x**3/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(3/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^3(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x^3(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^3(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = \int \frac{1}{x^3\,\mathrm{atan}\,(a\,x)^{3/2}\,(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int(1/(x^3*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(1/(x^3*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 78, normalized size of antiderivative = 3.00

$$\int \frac{1}{x^3 (c+a^2cx^2)^{5/2} \arctan(ax)^{3/2}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}}{atan(ax)^2a^6x^9+3atan(ax)^2a^4x^7+3atan(ax)^2a^2x^5+atan(ax)^2x^3}dx\right)}{c^3}$$

input
```
int(1/x^3/(a^2*c*x^2+c)^(5/2)/atan(a*x)^(3/2),x)
```

output
```
(sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(atan(a*x)**2*a**6*x**9
 + 3*atan(a*x)**2*a**4*x**7 + 3*atan(a*x)**2*a**2*x**5 + atan(a*x)**2*x**3
),x))/c**3
```

# 3.1040 $\int \frac{1}{x^4(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{1}{x^4(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = -\frac{2}{acx^4(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}} - \frac{8\text{Int}\left(\frac{1}{x^5(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}},x\right)}{a} - 14a\text{Int}\left(\frac{1}{x^3(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}},x\right)$$

output
```
-2/a/c/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2)-8*Defer(Int)(1/x^5/(a^2*c
*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)/a-14*a*Defer(Int)(1/x^3/(a^2*c*x^2+c)^(
5/2)/arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 17.90 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^4(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx = \int \frac{1}{x^4(c+a^2cx^2)^{5/2}\arctan(ax)^{3/2}}\,dx$$

input
```
Integrate[1/(x^4*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)),x]
```

output
```
Integrate[1/(x^4*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \arctan(ax)^{3/2} \left(a^2cx^2+c\right)^{5/2}}dx$$

$$\Big\downarrow \text{5503}$$

$$-\frac{8\int \frac{1}{x^5(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-14a\int \frac{1}{x^3\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{2}{acx^4\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{5560}$$

$$-\frac{8\int \frac{1}{x^5(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-14a\int \frac{1}{x^3\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{2}{acx^4\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}$$

input
```
Int[1/(x^4*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(3/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^4 \left(a^2 c\, x^2+c\right)^{\frac{5}{2}} \arctan \left(a x\right)^{\frac{3}{2}}}d x$$

input
```
int(1/x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

output
```
int(1/x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^4 (c+a^2cx^2)^{5/2} \arctan(ax)^{3/2}}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="fricas
")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (c+a^2cx^2)^{5/2} \arctan(ax)^{3/2}}\, dx = \text{Timed out}$$

input
```
integrate(1/x**4/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(3/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^4 (c + a^2 c x^2)^{5/2} \arctan(ax)^{3/2}} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^4 (c + a^2 c x^2)^{5/2} \arctan(ax)^{3/2}} \, dx = \int \frac{1}{(a^2 c x^2 + c)^{\frac{5}{2}} x^4 \arctan(ax)^{\frac{3}{2}}} \, dx$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(5/2)*x^4*arctan(a*x)^(3/2)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^4 (c + a^2 c x^2)^{5/2} \arctan(ax)^{3/2}} \, dx = \int \frac{1}{x^4 \operatorname{atan}(a x)^{3/2} (c a^2 x^2 + c)^{5/2}} \, dx$$

input
```
int(1/(x^4*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(1/(x^4*atan(a*x)^(3/2)*(c + a^2*c*x^2)^(5/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 78, normalized size of antiderivative = 3.00

$$\int \frac{1}{x^4 (c+a^2cx^2)^{5/2} \arctan(ax)^{3/2}} \, dx = \frac{\sqrt{c} \left( \int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}}{atan(ax)^2 a^6 x^{10}+3atan(ax)^2 a^4 x^8+3atan(ax)^2 a^2 x^6+atan(ax)^2 x^4} dx \right)}{c^3}$$

input
```
int(1/x^4/(a^2*c*x^2+c)^(5/2)/atan(a*x)^(3/2),x)
```

output
```
(sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(atan(a*x)**2*a**6*x**1
0 + 3*atan(a*x)**2*a**4*x**8 + 3*atan(a*x)**2*a**2*x**6 + atan(a*x)**2*x**
4),x))/c**3
```

# 3.1041 $\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)^{5/2}}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)^{5/2}}\,dx = \text{Int}\left(\frac{x^m(c+a^2cx^2)}{\arctan(ax)^{5/2}},x\right)$$

output
```
Defer(Int)(x^m*(a^2*c*x^2+c)/arctan(a*x)^(5/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.87 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)^{5/2}}\,dx = \int \frac{x^m(c+a^2cx^2)}{\arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[(x^m*(c + a^2*c*x^2))/ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[(x^m*(c + a^2*c*x^2))/ArcTan[a*x]^(5/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m (a^2 c x^2 + c)}{\arctan(ax)^{5/2}} \, dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{x^m (a^2 c x^2 + c)}{\arctan(ax)^{5/2}} dx$$

input
```
Int[(x^m*(c + a^2*c*x^2))/ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x^m (a^2 c\, x^2 + c)}{\arctan\left(a x\right)^{\frac{5}{2}}}dx$$

input
```
int(x^m*(a^2*c*x^2+c)/arctan(a*x)^(5/2),x)
```

output
```
int(x^m*(a^2*c*x^2+c)/arctan(a*x)^(5/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)^{5/2}}\,dx = \int \frac{(a^2cx^2+c)x^m}{\arctan{(ax)}^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)*x^m/arctan(a*x)^(5/2), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**m*(a**2*c*x**2+c)/atan(a*x)**(5/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)^{5/2}}\,dx = \int \frac{(a^2cx^2+c)x^m}{\arctan{(ax)}^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)*x^m/arctan(a*x)^(5/2), x)
```

## Mupad [N/A]

Not integrable

Time = 1.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)^{5/2}}\,dx = \int \frac{x^m\,\left(c\,a^2\,x^2+c\right)}{{\mathrm{atan}\left(a\,x\right)}^{5/2}}\,dx$$

input
```
int((x^m*(c + a^2*c*x^2))/atan(a*x)^(5/2),x)
```

output
```
int((x^m*(c + a^2*c*x^2))/atan(a*x)^(5/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.00

$$\int \frac{x^m(c+a^2cx^2)}{\arctan(ax)^{5/2}}\,dx = c\left(\left(\int \frac{x^m\sqrt{atan\,(ax)}\,x^2}{atan\,(ax)^3}dx\right)a^2+\int \frac{x^m\sqrt{atan\,(ax)}}{atan\,(ax)^3}dx\right)$$

input
```
int(x^m*(a^2*c*x^2+c)/atan(a*x)^(5/2),x)
```

output
```
c*(int((x**m*sqrt(atan(a*x))*x**2)/atan(a*x)**3,x)*a**2 + int((x**m*sqrt(a
tan(a*x)))/atan(a*x)**3,x))
```

# 3.1042 $\int \frac{x(c+a^2cx^2)}{\arctan(ax)^{5/2}}\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{x(c+a^2cx^2)}{\arctan(ax)^{5/2}}\,dx = \text{Int}\left(\frac{x(c+a^2cx^2)}{\arctan(ax)^{5/2}},x\right)$$

output
```
Defer(Int)(x*(a^2*c*x^2+c)/arctan(a*x)^(5/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x(c+a^2cx^2)}{\arctan(ax)^{5/2}}\,dx = \int \frac{x(c+a^2cx^2)}{\arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[(x*(c + a^2*c*x^2))/ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[(x*(c + a^2*c*x^2))/ArcTan[a*x]^(5/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a^2cx^2+c)}{\arctan(ax)^{5/2}}\,dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{x(a^2cx^2+c)}{\arctan(ax)^{5/2}}dx$$

input
```
Int[(x*(c + a^2*c*x^2))/ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{x(a^2c\,x^2+c)}{\arctan\left(ax\right)^{\frac{5}{2}}}dx$$

input
```
int(x*(a^2*c*x^2+c)/arctan(a*x)^(5/2),x)
```

output
```
int(x*(a^2*c*x^2+c)/arctan(a*x)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x(c+a^2cx^2)}{\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 5.40 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.45

$$\int \frac{x(c+a^2cx^2)}{\arctan(ax)^{5/2}}\,dx = c\left(\int \frac{x}{\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx + \int \frac{a^2x^3}{\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx\right)$$

input
```
integrate(x*(a**2*c*x**2+c)/atan(a*x)**(5/2),x)
```

output
```
c*(Integral(x/atan(a*x)**(5/2), x) + Integral(a**2*x**3/atan(a*x)**(5/2),
x))
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x(c+a^2cx^2)}{\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{x(c+a^2cx^2)}{\arctan(ax)^{5/2}}\,dx = \int \frac{(a^2cx^2+c)x}{\arctan{(ax)}^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)*x/arctan(a*x)^(5/2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{x(c+a^2cx^2)}{\arctan(ax)^{5/2}}\,dx = \int \frac{x\,\left(c\,a^2\,x^2+c\right)}{{\mathrm{atan}\left(a\,x\right)}^{5/2}}\,dx$$

input
```
int((x*(c + a^2*c*x^2))/atan(a*x)^(5/2),x)
```

output
```
int((x*(c + a^2*c*x^2))/atan(a*x)^(5/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.95

$$\int \frac{x(c+a^2cx^2)}{\arctan(ax)^{5/2}}\,dx = c\left(\left(\int \frac{\sqrt{atan(ax)}\,x^3}{atan(ax)^3}dx\right)a^2 + \int \frac{\sqrt{atan(ax)}\,x}{atan(ax)^3}dx\right)$$

input
```
int(x*(a^2*c*x^2+c)/atan(a*x)^(5/2),x)
```

output
```
c*(int((sqrt(atan(a*x))*x**3)/atan(a*x)**3,x)*a**2 + int((sqrt(atan(a*x))*
x)/atan(a*x)**3,x))
```

# 3.1043 $\int \frac{c+a^2cx^2}{\arctan(ax)^{5/2}}\,dx$



## Optimal result

Integrand size = 19, antiderivative size = 19

$$\int \frac{c+a^2cx^2}{\arctan(ax)^{5/2}}\,dx=\text{Int}\left(\frac{c+a^2cx^2}{\arctan(ax)^{5/2}},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)/arctan(a*x)^(5/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{c+a^2cx^2}{\arctan(ax)^{5/2}}\,dx=\int \frac{c+a^2cx^2}{\arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)/ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[(c + a^2*c*x^2)/ArcTan[a*x]^(5/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2cx^2+c}{\arctan(ax)^{5/2}}\,dx$$

$$\Big\downarrow \; 5560$$

$$\int \frac{a^2cx^2+c}{\arctan(ax)^{5/2}}dx$$

input
```
Int[(c + a^2*c*x^2)/ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{a^2c\,x^2+c}{\arctan\left(ax\right)^{\frac{5}{2}}}dx$$

input
```
int((a^2*c*x^2+c)/arctan(a*x)^(5/2),x)
```

output
```
int((a^2*c*x^2+c)/arctan(a*x)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{c+a^2cx^2}{\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 4.78 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.53

$$\int \frac{c+a^2cx^2}{\arctan(ax)^{5/2}}\,dx = c\left(\int \frac{a^2x^2}{\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx + \int \frac{1}{\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)/atan(a*x)**(5/2),x)
```

output
```
c*(Integral(a**2*x**2/atan(a*x)**(5/2), x) + Integral(atan(a*x)**(-5/2), x
))
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{c+a^2cx^2}{\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{c+a^2cx^2}{\arctan(ax)^{5/2}}\,dx=\int \frac{a^2cx^2+c}{\arctan{(ax)}^{\frac{5}{2}}}\,dx$$

input
```
integrate((a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)/arctan(a*x)^(5/2), x)
```

## Mupad [N/A]

Not integrable

Time = 1.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{c+a^2cx^2}{\arctan(ax)^{5/2}}\,dx=\int \frac{c\,a^2\,x^2+c}{{\mathrm{atan}\left(a\,x\right)}^{5/2}}\,dx$$

input
```
int((c + a^2*c*x^2)/atan(a*x)^(5/2),x)
```

output
```
int((c + a^2*c*x^2)/atan(a*x)^(5/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.00

$$\int \frac{c + a^2 c x^2}{\arctan(ax)^{5/2}}\,dx = c\left(\int \frac{\sqrt{atan\,(ax)}}{atan\,(ax)^3}dx + \left(\int \frac{\sqrt{atan\,(ax)}\,x^2}{atan\,(ax)^3}dx\right)a^2\right)$$

input
```
int((a^2*c*x^2+c)/atan(a*x)^(5/2),x)
```

output
```
c*(int(sqrt(atan(a*x))/atan(a*x)**3,x) + int((sqrt(atan(a*x))*x**2)/atan(a
*x)**3,x)*a**2)
```

# 3.1044 $\int \frac{c+a^2cx^2}{x\arctan(ax)^{5/2}}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{c+a^2cx^2}{x\arctan(ax)^{5/2}}\,dx = \text{Int}\left(\frac{c+a^2cx^2}{x\arctan(ax)^{5/2}},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)/x/arctan(a*x)^(5/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 2.63 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{c+a^2cx^2}{x\arctan(ax)^{5/2}}\,dx = \int \frac{c+a^2cx^2}{x\arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)/(x*ArcTan[a*x]^(5/2)),x]
```

output
```
Integrate[(c + a^2*c*x^2)/(x*ArcTan[a*x]^(5/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a^2cx^2+c}{x\arctan(ax)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int \frac{a^2cx^2+c}{x\arctan(ax)^{5/2}}dx$$

input
```
Int[(c + a^2*c*x^2)/(x*ArcTan[a*x]^(5/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{a^2c\,x^2+c}{x\arctan\left(ax\right)^{\frac{5}{2}}}dx$$

input
```
int((a^2*c*x^2+c)/x/arctan(a*x)^(5/2),x)
```

output
```
int((a^2*c*x^2+c)/x/arctan(a*x)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{c+a^2cx^2}{x\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)/x/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 6.39 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.32

$$\int \frac{c+a^2cx^2}{x\arctan(ax)^{5/2}}\,dx = c\left(\int \frac{1}{x\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx+\int \frac{a^2x}{\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)/x/atan(a*x)**(5/2),x)
```

output
```
c*(Integral(1/(x*atan(a*x)**(5/2)), x) + Integral(a**2*x/atan(a*x)**(5/2),
 x))
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{c+a^2cx^2}{x\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)/x/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{c+a^2cx^2}{x\arctan(ax)^{5/2}}\,dx = \int \frac{a^2cx^2+c}{x\arctan{(ax)^{\frac{5}{2}}}}\,dx$$

input
```
integrate((a^2*c*x^2+c)/x/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)/(x*arctan(a*x)^(5/2)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.97 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{c+a^2cx^2}{x\arctan(ax)^{5/2}}\,dx = \int \frac{c\,a^2\,x^2+c}{x\,\mathrm{atan}\left(a\,x\right)^{5/2}}\,dx$$

input
```
int((c + a^2*c*x^2)/(x*atan(a*x)^(5/2)),x)
```

output
```
int((c + a^2*c*x^2)/(x*atan(a*x)^(5/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.77

$$\int \frac{c + a^2 c x^2}{x \arctan(ax)^{5/2}} \, dx = c\left(\int \frac{\sqrt{atan\,(ax)}}{atan\,(ax)^3\,x} dx + \left(\int \frac{\sqrt{atan\,(ax)}\,x}{atan\,(ax)^3} dx\right) a^2\right)$$

input
```
int((a^2*c*x^2+c)/x/atan(a*x)^(5/2),x)
```

output
```
c*(int(sqrt(atan(a*x))/(atan(a*x)**3*x),x) + int((sqrt(atan(a*x))*x)/atan(
a*x)**3,x)*a**2)
```

# 3.1045 $\int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)^{5/2}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)^{5/2}}\,dx = \text{Int}\left(\frac{x^m(c+a^2cx^2)^2}{\arctan(ax)^{5/2}},x\right)$$

output
```
Defer(Int)(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)^{5/2}}\,dx = \int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[(x^m*(c + a^2*c*x^2)^2)/ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[(x^m*(c + a^2*c*x^2)^2)/ArcTan[a*x]^(5/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \left(a^2 c x^2+c\right)^2}{\arctan (a x)^{5/2}} \, dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x^m \left(a^2 c x^2+c\right)^2}{\arctan (a x)^{5/2}}dx$$

input
```
Int[(x^m*(c + a^2*c*x^2)^2)/ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^{m} \left(a^{2} c \,x^{2}+c \right)^{2}}{\arctan \left(a x \right)^{\frac{5}{2}}}d x$$

input
```
int(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)^{5/2}}\,dx = \int \frac{(a^2cx^2+c)^2x^m}{\arctan(ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*x^m/arctan(a*x)^(5/2), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**m*(a**2*c*x**2+c)**2/atan(a*x)**(5/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)^{5/2}}\,dx = \int \frac{(a^2cx^2+c)^2x^m}{\arctan{(ax)}^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^2*x^m/arctan(a*x)^(5/2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.93 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)^{5/2}}\,dx = \int \frac{x^m\,{\left(c\,a^2\,x^2+c\right)}^2}{{\mathrm{atan}\left(a\,x\right)}^{5/2}}\,dx$$

input
```
int((x^m*(c + a^2*c*x^2)^2)/atan(a*x)^(5/2),x)
```

output
```
int((x^m*(c + a^2*c*x^2)^2)/atan(a*x)^(5/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.96

$$\int \frac{x^m(c+a^2cx^2)^2}{\arctan(ax)^{5/2}}\,dx = c^2\left(\left(\int \frac{x^m\sqrt{atan\,(ax)}\,x^4}{atan\,(ax)^3}dx\right)a^4 + 2\left(\int \frac{x^m\sqrt{atan\,(ax)}\,x^2}{atan\,(ax)^3}dx\right)a^2 + \int \frac{x^m\sqrt{atan\,(ax)}}{atan\,(ax)^3}dx\right)$$

input
```
int(x^m*(a^2*c*x^2+c)^2/atan(a*x)^(5/2),x)
```

output
```
c**2*(int((x**m*sqrt(atan(a*x))*x**4)/atan(a*x)**3,x)*a**4 + 2*int((x**m*s
qrt(atan(a*x))*x**2)/atan(a*x)**3,x)*a**2 + int((x**m*sqrt(atan(a*x)))/ata
n(a*x)**3,x))
```

# 3.1046 $\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^{5/2}}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^{5/2}}\,dx = \text{Int}\left(\frac{x(c+a^2cx^2)^2}{\arctan(ax)^{5/2}},x\right)$$

output
```
Defer(Int)(x*(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^{5/2}}\,dx = \int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[(x*(c + a^2*c*x^2)^2)/ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[(x*(c + a^2*c*x^2)^2)/ArcTan[a*x]^(5/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a^2cx^2+c)^2}{\arctan(ax)^{5/2}}dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int \frac{x(a^2cx^2+c)^2}{\arctan(ax)^{5/2}}dx$$

input
```
Int[(x*(c + a^2*c*x^2)^2)/ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x(a^2c\,x^2+c)^2}{\arctan(ax)^{\frac{5}{2}}}dx$$

input
```
int(x*(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

output
```
int(x*(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 4.85 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.23

$$\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^{5/2}}\,dx = c^2\left(\int \frac{x}{\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx + \int \frac{2a^2x^3}{\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx + \int \frac{a^4x^5}{\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx\right)$$

input
```
integrate(x*(a**2*c*x**2+c)**2/atan(a*x)**(5/2),x)
```

output
```
c**2*(Integral(x/atan(a*x)**(5/2), x) + Integral(2*a**2*x**3/atan(a*x)**(5
/2), x) + Integral(a**4*x**5/atan(a*x)**(5/2), x))
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^{5/2}}\,dx = \int \frac{(a^2cx^2+c)^2x}{\arctan{(ax)}^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^2*x/arctan(a*x)^(5/2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.92 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^{5/2}}\,dx = \int \frac{x\,{\left(c\,a^2\,x^2+c\right)}^2}{{\mathrm{atan}\left(a\,x\right)}^{5/2}}\,dx$$

input
```
int((x*(c + a^2*c*x^2)^2)/atan(a*x)^(5/2),x)
```

output
```
int((x*(c + a^2*c*x^2)^2)/atan(a*x)^(5/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.86

$$\int \frac{x(c+a^2cx^2)^2}{\arctan(ax)^{5/2}}\,dx = c^2\left(\left(\int \frac{\sqrt{atan\,(ax)}\,x^5}{atan\,(ax)^3}dx\right)a^4 + 2\left(\int \frac{\sqrt{atan\,(ax)}\,x^3}{atan\,(ax)^3}dx\right)a^2 + \int \frac{\sqrt{atan\,(ax)}\,x}{atan\,(ax)^3}dx\right)$$

input
```
int(x*(a^2*c*x^2+c)^2/atan(a*x)^(5/2),x)
```

output
```
c**2*(int((sqrt(atan(a*x))*x**5)/atan(a*x)**3,x)*a**4 + 2*int((sqrt(atan(a
*x))*x**3)/atan(a*x)**3,x)*a**2 + int((sqrt(atan(a*x))*x)/atan(a*x)**3,x))
```

# 3.1047 $\int \frac{(c+a^2cx^2)^2}{\arctan(ax)^{5/2}}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{(c+a^2cx^2)^2}{\arctan(ax)^{5/2}}\,dx=\operatorname{Int}\left(\frac{(c+a^2cx^2)^2}{\arctan(ax)^{5/2}},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{(c+a^2cx^2)^2}{\arctan(ax)^{5/2}}\,dx=\int \frac{(c+a^2cx^2)^2}{\arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^2/ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[(c + a^2*c*x^2)^2/ArcTan[a*x]^(5/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^2}{\arctan(ax)^{5/2}}\,dx$$
$$\Big\downarrow \ 5560$$
$$\int \frac{(a^2cx^2+c)^2}{\arctan(ax)^{5/2}}dx$$

input
```
Int[(c + a^2*c*x^2)^2/ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{(a^2c\,x^2+c)^2}{\arctan\left(ax\right)^{\frac{5}{2}}}dx$$

input
```
int((a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

output
```
int((a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^2}{\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 5.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.33

$$\int \frac{(c+a^2cx^2)^2}{\arctan(ax)^{5/2}}\,dx = c^2\left(\int \frac{2a^2x^2}{\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx + \int \frac{a^4x^4}{\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx + \int \frac{1}{\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**2/atan(a*x)**(5/2),x)
```

output
```
c**2*(Integral(2*a**2*x**2/atan(a*x)**(5/2), x) + Integral(a**4*x**4/atan(
a*x)**(5/2), x) + Integral(atan(a*x)**(-5/2), x))
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^2}{\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^2}{\arctan(ax)^{5/2}}\,dx = \int \frac{(a^2cx^2+c)^2}{\arctan(ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^2/arctan(a*x)^(5/2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^2}{\arctan(ax)^{5/2}}\,dx = \int \frac{(c\,a^2\,x^2+c)^2}{\mathrm{atan}\left(a\,x\right)^{5/2}}\,dx$$

input
```
int((c + a^2*c*x^2)^2/atan(a*x)^(5/2),x)
```

output
```
int((c + a^2*c*x^2)^2/atan(a*x)^(5/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.95

$$\int \frac{(c+a^2cx^2)^2}{\arctan(ax)^{5/2}}\,dx = c^2\left(\int \frac{\sqrt{atan(ax)}}{atan(ax)^3}dx + \left(\int \frac{\sqrt{atan(ax)}\,x^4}{atan(ax)^3}dx\right)a^4 + 2\left(\int \frac{\sqrt{atan(ax)}\,x^2}{atan(ax)^3}dx\right)a^2\right)$$

input
```
int((a^2*c*x^2+c)^2/atan(a*x)^(5/2),x)
```

output
```
c**2*(int(sqrt(atan(a*x))/atan(a*x)**3,x) + int((sqrt(atan(a*x))*x**4)/ata
n(a*x)**3,x)*a**4 + 2*int((sqrt(atan(a*x))*x**2)/atan(a*x)**3,x)*a**2)
```

# 3.1048 $\int \frac{(c+a^2cx^2)^2}{x \arctan(ax)^{5/2}} \, dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{(c+a^2cx^2)^2}{x \arctan(ax)^{5/2}} \, dx = \text{Int}\left(\frac{(c+a^2cx^2)^2}{x \arctan(ax)^{5/2}}, x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^2/x/arctan(a*x)^(5/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.53 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(c+a^2cx^2)^2}{x \arctan(ax)^{5/2}} \, dx = \int \frac{(c+a^2cx^2)^2}{x \arctan(ax)^{5/2}} \, dx$$

input
```
Integrate[(c + a^2*c*x^2)^2/(x*ArcTan[a*x]^(5/2)),x]
```

output
```
Integrate[(c + a^2*c*x^2)^2/(x*ArcTan[a*x]^(5/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^2}{x\arctan(ax)^{5/2}}dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int \frac{(a^2cx^2+c)^2}{x\arctan(ax)^{5/2}}dx$$

input
```
Int[(c + a^2*c*x^2)^2/(x*ArcTan[a*x]^(5/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{(a^2c\,x^2+c)^2}{x\arctan\left(ax\right)^{\frac{5}{2}}}dx$$

input
```
int((a^2*c*x^2+c)^2/x/arctan(a*x)^(5/2),x)
```

output
```
int((a^2*c*x^2+c)^2/x/arctan(a*x)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^2/x/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 7.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.04

$$\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)^{5/2}}\,dx = c^2\left(\int \frac{1}{x\,\text{atan}^{\frac{5}{2}}(ax)}\,dx + \int \frac{2a^2x}{\text{atan}^{\frac{5}{2}}(ax)}\,dx + \int \frac{a^4x^3}{\text{atan}^{\frac{5}{2}}(ax)}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**2/x/atan(a*x)**(5/2),x)
```

output
```
c**2*(Integral(1/(x*atan(a*x)**(5/2)), x) + Integral(2*a**2*x/atan(a*x)**(
5/2), x) + Integral(a**4*x**3/atan(a*x)**(5/2), x))
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^2/x/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)^{5/2}}\,dx=\int \frac{(a^2cx^2+c)^2}{x\arctan(ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^2/x/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^2/(x*arctan(a*x)^(5/2)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)^{5/2}}\,dx=\int \frac{(c\,a^2\,x^2+c)^2}{x\,\mathrm{atan}\left(a\,x\right)^{5/2}}\,dx$$

input
```
int((c + a^2*c*x^2)^2/(x*atan(a*x)^(5/2)),x)
```

output
```
int((c + a^2*c*x^2)^2/(x*atan(a*x)^(5/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.62

$$\int \frac{(c+a^2cx^2)^2}{x\arctan(ax)^{5/2}}\,dx = c^2\left(\int \frac{\sqrt{atan(ax)}}{atan(ax)^3x}dx + \left(\int \frac{\sqrt{atan(ax)}\,x^3}{atan(ax)^3}dx\right)a^4 + 2\left(\int \frac{\sqrt{atan(ax)}\,x}{atan(ax)^3}dx\right)a^2\right)$$

input
```
int((a^2*c*x^2+c)^2/x/atan(a*x)^(5/2),x)
```

output
```
c**2*(int(sqrt(atan(a*x))/(atan(a*x)**3*x),x) + int((sqrt(atan(a*x))*x**3)
/atan(a*x)**3,x)*a**4 + 2*int((sqrt(atan(a*x))*x)/atan(a*x)**3,x)*a**2)
```

# 3.1049 $\int \frac{x^m \left(c+a^2 c x^2\right)^3}{\arctan(ax)^{5/2}} \, dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m (c+a^2 c x^2)^3}{\arctan(ax)^{5/2}} \, dx = \operatorname{Int}\left(\frac{x^m (c+a^2 c x^2)^3}{\arctan(ax)^{5/2}}, x\right)$$

output
```
Defer(Int)(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m (c+a^2 c x^2)^3}{\arctan(ax)^{5/2}} \, dx = \int \frac{x^m (c+a^2 c x^2)^3}{\arctan(ax)^{5/2}} \, dx$$

input
```
Integrate[(x^m*(c + a^2*c*x^2)^3)/ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[(x^m*(c + a^2*c*x^2)^3)/ArcTan[a*x]^(5/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m (a^2cx^2+c)^3}{\arctan(ax)^{5/2}} \, dx$$

$$\Big\downarrow \; 5560$$

$$\int \frac{x^m (a^2cx^2+c)^3}{\arctan(ax)^{5/2}} dx$$

input
```
Int[(x^m*(c + a^2*c*x^2)^3)/ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 1.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m (a^2c\,x^2+c)^3}{\arctan(ax)^{\frac{5}{2}}} dx$$

input
```
int(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.00

$$\int \frac{x^m (c+a^2 c x^2)^3}{\arctan(ax)^{5/2}}\,dx = \int \frac{(a^2 c x^2+c)^3 x^m}{\arctan(ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
integral((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*x^m/arctan(a*
x)^(5/2), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^m (c+a^2 c x^2)^3}{\arctan(ax)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**m*(a**2*c*x**2+c)**3/atan(a*x)**(5/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m (c+a^2 c x^2)^3}{\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m(c+a^2cx^2)^3}{\arctan(ax)^{5/2}}\,dx = \int \frac{(a^2cx^2+c)^3x^m}{\arctan{(ax)}^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^3*x^m/arctan(a*x)^(5/2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m(c+a^2cx^2)^3}{\arctan(ax)^{5/2}}\,dx = \int \frac{x^m\,{\left(c\,a^2\,x^2+c\right)}^3}{{\mathrm{atan}\left(a\,x\right)}^{5/2}}\,dx$$

input
```
int((x^m*(c + a^2*c*x^2)^3)/atan(a*x)^(5/2),x)
```

output
```
int((x^m*(c + a^2*c*x^2)^3)/atan(a*x)^(5/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 96, normalized size of antiderivative = 4.00

$$\int \frac{x^m(c+a^2cx^2)^3}{\arctan(ax)^{5/2}}\,dx = c^3\left(\left(\int \frac{x^m\sqrt{atan\left(ax\right)}\,x^6}{atan\left(ax\right)^3}dx\right)a^6 + 3\left(\int \frac{x^m\sqrt{atan\left(ax\right)}\,x^4}{atan\left(ax\right)^3}dx\right)a^4 + 3\left(\int \frac{x^m\sqrt{atan\left(ax\right)}\,x^2}{atan\left(ax\right)^3}dx\right)a^2 + \int \frac{x^m\sqrt{atan\left(ax\right)}}{atan\left(ax\right)^3}dx\right)$$

input
```
int(x^m*(a^2*c*x^2+c)^3/atan(a*x)^(5/2),x)
```

output
```
c**3*(int((x**m*sqrt(atan(a*x))*x**6)/atan(a*x)**3,x)*a**6 + 3*int((x**m*s
qrt(atan(a*x))*x**4)/atan(a*x)**3,x)*a**4 + 3*int((x**m*sqrt(atan(a*x))*x*
*2)/atan(a*x)**3,x)*a**2 + int((x**m*sqrt(atan(a*x)))/atan(a*x)**3,x))
```

# 3.1050 $\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)^{5/2}}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)^{5/2}}\,dx = \text{Int}\left(\frac{x(c+a^2cx^2)^3}{\arctan(ax)^{5/2}},x\right)$$

output
```
Defer(Int)(x*(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)^{5/2}}\,dx = \int \frac{x(c+a^2cx^2)^3}{\arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[(x*(c + a^2*c*x^2)^3)/ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[(x*(c + a^2*c*x^2)^3)/ArcTan[a*x]^(5/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a^2cx^2+c)^3}{\arctan(ax)^{5/2}}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{x(a^2cx^2+c)^3}{\arctan(ax)^{5/2}}dx$$

input
```
Int[(x*(c + a^2*c*x^2)^3)/ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x(a^2c\,x^2+c)^3}{\arctan(ax)^{\frac{5}{2}}}dx$$

input
```
int(x*(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

output
```
int(x*(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 7.92 (sec) , antiderivative size = 68, normalized size of antiderivative = 3.09

$$\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)^{5/2}}\,dx = c^3\left(\int \frac{x}{\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx + \int \frac{3a^2x^3}{\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx + \int \frac{3a^4x^5}{\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx + \int \frac{a^6x^7}{\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx\right)$$

input
```
integrate(x*(a**2*c*x**2+c)**3/atan(a*x)**(5/2),x)
```

output
```
c**3*(Integral(x/atan(a*x)**(5/2), x) + Integral(3*a**2*x**3/atan(a*x)**(5
/2), x) + Integral(3*a**4*x**5/atan(a*x)**(5/2), x) + Integral(a**6*x**7/a
tan(a*x)**(5/2), x))
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)^{5/2}}\,dx = \int \frac{(a^2cx^2+c)^3x}{\arctan(ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^3*x/arctan(a*x)^(5/2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)^{5/2}}\,dx = \int \frac{x\,(c\,a^2\,x^2+c)^3}{\mathrm{atan}\left(a\,x\right)^{5/2}}\,dx$$

input
```
int((x*(c + a^2*c*x^2)^3)/atan(a*x)^(5/2),x)
```

output
```
int((x*(c + a^2*c*x^2)^3)/atan(a*x)^(5/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 85, normalized size of antiderivative = 3.86

$$\int \frac{x(c+a^2cx^2)^3}{\arctan(ax)^{5/2}}\,dx = c^3\left(\left(\int \frac{\sqrt{atan\left(ax\right)}\,x^7}{atan\left(ax\right)^3}dx\right)a^6+3\left(\int \frac{\sqrt{atan\left(ax\right)}\,x^5}{atan\left(ax\right)^3}dx\right)a^4 + 3\left(\int \frac{\sqrt{atan\left(ax\right)}\,x^3}{atan\left(ax\right)^3}dx\right)a^2+\int \frac{\sqrt{atan\left(ax\right)}\,x}{atan\left(ax\right)^3}dx\right)$$

input
```
int(x*(a^2*c*x^2+c)^3/atan(a*x)^(5/2),x)
```

output
```
c**3*(int((sqrt(atan(a*x))*x**7)/atan(a*x)**3,x)*a**6 + 3*int((sqrt(atan(a
*x))*x**5)/atan(a*x)**3,x)*a**4 + 3*int((sqrt(atan(a*x))*x**3)/atan(a*x)**
3,x)*a**2 + int((sqrt(atan(a*x))*x)/atan(a*x)**3,x))
```

# 3.1051 $\int \frac{(c+a^2cx^2)^3}{\arctan(ax)^{5/2}}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{(c+a^2cx^2)^3}{\arctan(ax)^{5/2}}\,dx = \text{Int}\left(\frac{(c+a^2cx^2)^3}{\arctan(ax)^{5/2}},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{(c+a^2cx^2)^3}{\arctan(ax)^{5/2}}\,dx = \int \frac{(c+a^2cx^2)^3}{\arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^3/ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[(c + a^2*c*x^2)^3/ArcTan[a*x]^(5/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^3}{\arctan(ax)^{5/2}} \, dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int \frac{(a^2cx^2+c)^3}{\arctan(ax)^{5/2}}dx$$

input
```
Int[(c + a^2*c*x^2)^3/ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{(a^2c\,x^2+c)^3}{\arctan\,(ax)^{\frac{5}{2}}}dx$$

input
```
int((a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

output
```
int((a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^3}{\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 6.81 (sec) , antiderivative size = 68, normalized size of antiderivative = 3.24

$$\int \frac{(c+a^2cx^2)^3}{\arctan(ax)^{5/2}}\,dx = c^3\left(\int \frac{3a^2x^2}{\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx + \int \frac{3a^4x^4}{\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx + \int \frac{a^6x^6}{\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx + \int \frac{1}{\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**3/atan(a*x)**(5/2),x)
```

output
```
c**3*(Integral(3*a**2*x**2/atan(a*x)**(5/2), x) + Integral(3*a**4*x**4/ata
n(a*x)**(5/2), x) + Integral(a**6*x**6/atan(a*x)**(5/2), x) + Integral(ata
n(a*x)**(-5/2), x))
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^3}{\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^3}{\arctan(ax)^{5/2}}\,dx = \int \frac{(a^2cx^2+c)^3}{\arctan(ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^3/arctan(a*x)^(5/2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.77 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{(c+a^2cx^2)^3}{\arctan(ax)^{5/2}}\,dx = \int \frac{(c\,a^2\,x^2+c)^3}{\mathrm{atan}\left(a\,x\right)^{5/2}}\,dx$$

input
```
int((c + a^2*c*x^2)^3/atan(a*x)^(5/2),x)
```

output
```
int((c + a^2*c*x^2)^3/atan(a*x)^(5/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 84, normalized size of antiderivative = 4.00

$$\int \frac{(c+a^2cx^2)^3}{\arctan(ax)^{5/2}}\,dx = c^3\left(\int \frac{\sqrt{atan\left(ax\right)}}{atan\left(ax\right)^3}dx+\left(\int \frac{\sqrt{atan\left(ax\right)}\,x^6}{atan\left(ax\right)^3}dx\right)a^6+3\left(\int \frac{\sqrt{atan\left(ax\right)}\,x^4}{atan\left(ax\right)^3}dx\right)a^4+3\left(\int \frac{\sqrt{atan\left(ax\right)}\,x^2}{atan\left(ax\right)^3}dx\right)a^2\right)$$

input
```
int((a^2*c*x^2+c)^3/atan(a*x)^(5/2),x)
```

output
```
c**3*(int(sqrt(atan(a*x))/atan(a*x)**3,x) + int((sqrt(atan(a*x))*x**6)/ata
n(a*x)**3,x)*a**6 + 3*int((sqrt(atan(a*x))*x**4)/atan(a*x)**3,x)*a**4 + 3*
int((sqrt(atan(a*x))*x**2)/atan(a*x)**3,x)*a**2)
```

# 3.1052 $\int \frac{(c+a^2cx^2)^3}{x\arctan(ax)^{5/2}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{(c+a^2cx^2)^3}{x\arctan(ax)^{5/2}}\,dx=\text{Int}\left(\frac{(c+a^2cx^2)^3}{x\arctan(ax)^{5/2}},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^3/x/arctan(a*x)^(5/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.85 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(c+a^2cx^2)^3}{x\arctan(ax)^{5/2}}\,dx=\int \frac{(c+a^2cx^2)^3}{x\arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^3/(x*ArcTan[a*x]^(5/2)),x]
```

output
```
Integrate[(c + a^2*c*x^2)^3/(x*ArcTan[a*x]^(5/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a^2cx^2+c)^3}{x\arctan(ax)^{5/2}}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{(a^2cx^2+c)^3}{x\arctan(ax)^{5/2}}dx$$

input
```
Int[(c + a^2*c*x^2)^3/(x*ArcTan[a*x]^(5/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{(a^2c\,x^2+c)^3}{x\arctan\left(ax\right)^{\frac{5}{2}}}dx$$

input
```
int((a^2*c*x^2+c)^3/x/arctan(a*x)^(5/2),x)
```

output
```
int((a^2*c*x^2+c)^3/x/arctan(a*x)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^3}{x\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^3/x/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 8.09 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.83

$$\int \frac{(c+a^2cx^2)^3}{x\arctan(ax)^{5/2}}\,dx = c^3\left(\int \frac{1}{x\,\mathrm{atan}^{\frac{5}{2}}(ax)}\,dx + \int \frac{3a^2x}{\mathrm{atan}^{\frac{5}{2}}(ax)}\,dx + \int \frac{3a^4x^3}{\mathrm{atan}^{\frac{5}{2}}(ax)}\,dx + \int \frac{a^6x^5}{\mathrm{atan}^{\frac{5}{2}}(ax)}\,dx\right)$$

input
```
integrate((a**2*c*x**2+c)**3/x/atan(a*x)**(5/2),x)
```

output
```
c**3*(Integral(1/(x*atan(a*x)**(5/2)), x) + Integral(3*a**2*x/atan(a*x)**(
5/2), x) + Integral(3*a**4*x**3/atan(a*x)**(5/2), x) + Integral(a**6*x**5/
atan(a*x)**(5/2), x))
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(c + a^2cx^2)^3}{x \arctan(ax)^{5/2}} dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^3/x/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c + a^2cx^2)^3}{x \arctan(ax)^{5/2}} dx = \int \frac{(a^2cx^2 + c)^3}{x \arctan(ax)^{\frac{5}{2}}} dx$$

input
```
integrate((a^2*c*x^2+c)^3/x/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^3/(x*arctan(a*x)^(5/2)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.85 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(c + a^2cx^2)^3}{x \arctan(ax)^{5/2}} dx = \int \frac{(c a^2 x^2 + c)^3}{x \operatorname{atan}(a x)^{5/2}} dx$$

input
```
int((c + a^2*c*x^2)^3/(x*atan(a*x)^(5/2)),x)
```

output
```
int((c + a^2*c*x^2)^3/(x*atan(a*x)^(5/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 85, normalized size of antiderivative = 3.54

$$\int \frac{(c+a^2cx^2)^3}{x\arctan(ax)^{5/2}}\,dx = c^3\left(\int \frac{\sqrt{atan(ax)}}{atan(ax)^3 x}dx + \left(\int \frac{\sqrt{atan(ax)}\,x^5}{atan(ax)^3}dx\right)a^6 + 3\left(\int \frac{\sqrt{atan(ax)}\,x^3}{atan(ax)^3}dx\right)a^4 + 3\left(\int \frac{\sqrt{atan(ax)}\,x}{atan(ax)^3}dx\right)a^2\right)$$

input
```
int((a^2*c*x^2+c)^3/x/atan(a*x)^(5/2),x)
```

output
```
c**3*(int(sqrt(atan(a*x))/(atan(a*x)**3*x),x) + int((sqrt(atan(a*x))*x**5)
/atan(a*x)**3,x)*a**6 + 3*int((sqrt(atan(a*x))*x**3)/atan(a*x)**3,x)*a**4
+ 3*int((sqrt(atan(a*x))*x)/atan(a*x)**3,x)*a**2)
```

# 3.1053 $\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^{5/2}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^{5/2}}\,dx = -\frac{2x^m}{3ac\arctan(ax)^{3/2}} + \frac{2m\text{Int}\left(\frac{x^{-1+m}}{\arctan(ax)^{3/2}},x\right)}{3ac}$$

output
```
-2/3*x^m/a/c/arctan(a*x)^(3/2)+2/3*m*Defer(Int)(x^(-1+m)/arctan(a*x)^(3/2)
,x)/a/c
```

## Mathematica [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^{5/2}}\,dx = \int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[x^m/((c + a^2*c*x^2)*ArcTan[a*x]^(5/2)),x]
```

output
```
Integrate[x^m/((c + a^2*c*x^2)*ArcTan[a*x]^(5/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)}dx$$

$$\Big\downarrow \text{ 5461}$$

$$\frac{2m\int \frac{x^{m-1}}{\arctan(ax)^{3/2}}dx}{3ac}-\frac{2x^m}{3ac\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{ 5377}$$

$$\frac{2m\int \frac{x^{m-1}}{\arctan(ax)^{3/2}}dx}{3ac}-\frac{2x^m}{3ac\arctan(ax)^{3/2}}$$

input
```
Int[x^m/((c + a^2*c*x^2)*ArcTan[a*x]^(5/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{\left(a^2c\,x^2+c\right)\arctan\left(ax\right)^{\frac{5}{2}}}dx$$

input
```
int(x^m/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x)
```

output
```
int(x^m/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^{5/2}}\,dx = \int \frac{x^m}{(a^2cx^2+c)\arctan{(ax)}^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
integral(x^m/((a^2*c*x^2 + c)*arctan(a*x)^(5/2)), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**m/(a**2*c*x**2+c)/atan(a*x)**(5/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^{5/2}}\,dx = \int \frac{x^m}{(a^2cx^2+c)\arctan(ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(x^m/((a^2*c*x^2 + c)*arctan(a*x)^(5/2)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.90 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^{5/2}}\,dx = \int \frac{x^m}{\mathrm{atan}\left(a\,x\right)^{5/2}\,\left(c\,a^2\,x^2+c\right)}\,dx$$

input
```
int(x^m/(atan(a*x)^(5/2)*(c + a^2*c*x^2)),x)
```

output
```
int(x^m/(atan(a*x)^(5/2)*(c + a^2*c*x^2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{x^m}{(c+a^2cx^2)\arctan(ax)^{5/2}}\,dx = \frac{\int \frac{x^m\sqrt{atan(ax)}}{atan(ax)^3a^2x^2+atan(ax)^3}dx}{c}$$

input
```
int(x^m/(a^2*c*x^2+c)/atan(a*x)^(5/2),x)
```

output
```
int((x**m*sqrt(atan(a*x)))/(atan(a*x)**3*a**2*x**2 + atan(a*x)**3),x)/c
```

# 3.1054 $\int \frac{x}{(c+a^2cx^2)\arctan(ax)^{5/2}}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x}{(c+a^2cx^2)\arctan(ax)^{5/2}}\,dx = -\frac{2x}{3ac\arctan(ax)^{3/2}} + \frac{2\text{Int}\left(\frac{1}{\arctan(ax)^{3/2}},x\right)}{3ac}$$

output
```
-2/3*x/a/c/arctan(a*x)^(3/2)+2/3*Defer(Int)(1/arctan(a*x)^(3/2),x)/a/c
```

## Mathematica [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x}{(c+a^2cx^2)\arctan(ax)^{5/2}}\,dx = \int \frac{x}{(c+a^2cx^2)\arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[x/((c + a^2*c*x^2)*ArcTan[a*x]^(5/2)),x]
```

output
```
Integrate[x/((c + a^2*c*x^2)*ArcTan[a*x]^(5/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)}dx$$

$$\Big\downarrow \text{5457}$$

$$\frac{2\int \frac{1}{\arctan(ax)^{3/2}}dx}{3ac}-\frac{2x}{3ac\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{5353}$$

$$\frac{2\int \frac{1}{\arctan(ax)^{3/2}}dx}{3ac}-\frac{2x}{3ac\arctan(ax)^{3/2}}$$

input
```
Int[x/((c + a^2*c*x^2)*ArcTan[a*x]^(5/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x}{\left(a^{2} c \,x^{2}+c \right) \arctan \left(a x \right)^{\frac{5}{2}}}d x$$

input
```
int(x/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x)
```

output
```
int(x/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 3.49 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

$$\int \frac{x}{(c+a^2cx^2)\arctan(ax)^{5/2}}\,dx = \frac{\int \frac{x}{a^2x^2\operatorname{atan}^{\frac{5}{2}}(ax)+\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx}{c}$$

input
```
integrate(x/(a**2*c*x**2+c)/atan(a*x)**(5/2),x)
```

output
```
Integral(x/(a**2*x**2*atan(a*x)**(5/2) + atan(a*x)**(5/2)), x)/c
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x}{(c+a^2cx^2)\arctan(ax)^{5/2}}\,dx = \int \frac{x}{(a^2cx^2+c)\arctan(ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(x/((a^2*c*x^2 + c)*arctan(a*x)^(5/2)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x}{(c+a^2cx^2)\arctan(ax)^{5/2}}\,dx = \int \frac{x}{\mathrm{atan}(a\,x)^{5/2}\,(c\,a^2\,x^2+c)}\,dx$$

input
```
int(x/(atan(a*x)^(5/2)*(c + a^2*c*x^2)),x)
```

output
```
int(x/(atan(a*x)^(5/2)*(c + a^2*c*x^2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.59

$$\int \frac{x}{(c+a^2cx^2)\arctan(ax)^{5/2}}\,dx = \frac{\int \frac{\sqrt{atan(ax)}\,x}{atan(ax)^3a^2x^2+atan(ax)^3}dx}{c}$$

input
```
int(x/(a^2*c*x^2+c)/atan(a*x)^(5/2),x)
```

output
```
int((sqrt(atan(a*x))*x)/(atan(a*x)**3*a**2*x**2 + atan(a*x)**3),x)/c
```

# 3.1055 $\int \frac{1}{(c+a^2cx^2)\arctan(ax)^{5/2}}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 18

$$\int \frac{1}{(c+a^2cx^2)\arctan(ax)^{5/2}}\,dx = -\frac{2}{3ac\arctan(ax)^{3/2}}$$

output
```
-2/3/a/c/arctan(a*x)^(3/2)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{(c+a^2cx^2)\arctan(ax)^{5/2}}\,dx = -\frac{2}{3ac\arctan(ax)^{3/2}}$$

input
```
Integrate[1/((c + a^2*c*x^2)*ArcTan[a*x]^(5/2)),x]
```

output
```
-2/(3*a*c*ArcTan[a*x]^(3/2))
```

## Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)}dx$$

$$\Big\downarrow \text{ 5419}$$

$$-\frac{2}{3ac\arctan(ax)^{3/2}}$$

input
```
Int[1/((c + a^2*c*x^2)*ArcTan[a*x]^(5/2)),x]
```

output
```
-2/(3*a*c*ArcTan[a*x]^(3/2))
```

### Defintions of rubi rules used

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

## Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

| method | result | size |
|---|---|---|
| default | $-\frac{2}{3ac\arctan(ax)^{\frac{3}{2}}}$ | 15 |

input
```
int(1/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/a/c/arctan(a*x)^(3/2)
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{1}{(c+a^2cx^2)\arctan(ax)^{5/2}}\,dx = -\frac{2}{3\,ac\arctan{(ax)}^{\frac{3}{2}}}$$

input
```
integrate(1/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
-2/3/(a*c*arctan(a*x)^(3/2))
```

## Sympy [A] (verification not implemented)

Time = 3.55 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{1}{(c+a^2cx^2)\arctan(ax)^{5/2}}\,dx = -\frac{2}{3ac\operatorname{atan}^{\frac{3}{2}}{(ax)}}$$

input
```
integrate(1/(a**2*c*x**2+c)/atan(a*x)**(5/2),x)
```

output
```
-2/(3*a*c*atan(a*x)**(3/2))
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(c+a^2cx^2)\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{1}{(c+a^2cx^2)\arctan(ax)^{5/2}}\,dx = -\frac{2}{3\,ac\arctan{(ax)}^{\frac{3}{2}}}$$

input
```
integrate(1/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
-2/3/(a*c*arctan(a*x)^(3/2))
```

## Mupad [B] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

$$\int \frac{1}{(c+a^2cx^2)\arctan(ax)^{5/2}}\,dx = -\frac{2}{3\,a\,c\,\mathrm{atan}\left(a\,x\right)^{3/2}}$$

input
```
int(1/(atan(a*x)^(5/2)*(c + a^2*c*x^2)),x)
```

output
```
-2/(3*a*c*atan(a*x)^(3/2))
```

## Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{1}{(c+a^2cx^2)\arctan(ax)^{5/2}}\,dx = -\frac{2\sqrt{atan\left(ax\right)}}{3atan\left(ax\right)^{2}ac}$$

input
```
int(1/(a^2*c*x^2+c)/atan(a*x)^(5/2),x)
```

output
```
( - 2*sqrt(atan(a*x)))/(3*atan(a*x)**2*a*c)
```

# 3.1056 $\int \frac{1}{x(c+a^2cx^2)\arctan(ax)^{5/2}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x(c+a^2cx^2)\arctan(ax)^{5/2}}\,dx = -\frac{2}{3acx\arctan(ax)^{3/2}} - \frac{2\text{Int}\left(\frac{1}{x^2\arctan(ax)^{3/2}},x\right)}{3ac}$$

output
```
-2/3/a/c/x/arctan(a*x)^(3/2)-2/3*Defer(Int)(1/x^2/arctan(a*x)^(3/2),x)/a/c
```

## Mathematica [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x(c+a^2cx^2)\arctan(ax)^{5/2}}\,dx = \int \frac{1}{x(c+a^2cx^2)\arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[1/(x*(c + a^2*c*x^2)*ArcTan[a*x]^(5/2)),x]
```

output
```
Integrate[1/(x*(c + a^2*c*x^2)*ArcTan[a*x]^(5/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x \arctan(ax)^{5/2} (a^2cx^2 + c)} \, dx$$
$$\Big\downarrow \text{5461}$$
$$-\frac{2\int \frac{1}{x^2 \arctan(ax)^{3/2}}dx}{3ac} - \frac{2}{3acx \arctan(ax)^{3/2}}$$
$$\Big\downarrow \text{5377}$$
$$-\frac{2\int \frac{1}{x^2 \arctan(ax)^{3/2}}dx}{3ac} - \frac{2}{3acx \arctan(ax)^{3/2}}$$

input
```
Int[1/(x*(c + a^2*c*x^2)*ArcTan[a*x]^(5/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x \left(a^2 c\, x^2 + c\right) \arctan\left(a x\right)^{\frac{5}{2}}}dx$$

input
```
int(1/x/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x)
```

output
```
int(1/x/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x\,(c+a^2cx^2)\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 4.28 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int \frac{1}{x\,(c+a^2cx^2)\arctan(ax)^{5/2}}\,dx = \frac{\int \frac{1}{a^2x^3\operatorname{atan}^{\frac{5}{2}}(ax)+x\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx}{c}$$

input
```
integrate(1/x/(a**2*c*x**2+c)/atan(a*x)**(5/2),x)
```

output
```
Integral(1/(a**2*x**3*atan(a*x)**(5/2) + x*atan(a*x)**(5/2)), x)/c
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(c+a^2cx^2\right)\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)\arctan(ax)^{5/2}}\,dx = \int \frac{1}{(a^2cx^2+c)x\arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)*x*arctan(a*x)^(5/2)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)\arctan(ax)^{5/2}}\,dx = \int \frac{1}{x\,\mathrm{atan}\left(a\,x\right)^{5/2}\,\left(c\,a^2\,x^2+c\right)}\,dx$$

input
```
int(1/(x*atan(a*x)^(5/2)*(c + a^2*c*x^2)),x)
```

output
```
int(1/(x*atan(a*x)^(5/2)*(c + a^2*c*x^2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.50

$$\int \frac{1}{x(c+a^2cx^2)\arctan(ax)^{5/2}}\,dx = \frac{\int \frac{\sqrt{atan(ax)}}{atan(ax)^3a^2x^3+atan(ax)^3x}dx}{c}$$

input
```
int(1/x/(a^2*c*x^2+c)/atan(a*x)^(5/2),x)
```

output
```
int(sqrt(atan(a*x))/(atan(a*x)**3*a**2*x**3 + atan(a*x)**3*x),x)/c
```

# 3.1057 $\int \frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)^{5/2}} \, dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)^{5/2}} \, dx = \text{Int}\left(\frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)^{5/2}}, x\right)$$

output
```
Defer(Int)(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)^{5/2}} \, dx = \int \frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)^{5/2}} \, dx$$

input
```
Integrate[x^m/((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)),x]
```

output
```
Integrate[x^m/((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\arctan(ax)^{5/2} \left(a^2cx^2+c\right)^2} \, dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x^m}{\arctan(ax)^{5/2} \left(a^2cx^2+c\right)^2}dx$$

input
```
Int[x^m/((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{\left(a^2c\,x^2+c\right)^2 \arctan\left(ax\right)^{\frac{5}{2}}}dx$$

input
```
int(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

output
```
int(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.62

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^2 \arctan(ax)^{5/2}}\,dx = \int \frac{x^m}{\left(a^2cx^2+c\right)^2 \arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
integral(x^m/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arctan(a*x)^(5/2)), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^2 \arctan(ax)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**m/(a**2*c*x**2+c)**2/atan(a*x)**(5/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^2 \arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)^{5/2}} \, dx = \int \frac{x^m}{(a^2cx^2+c)^2 \arctan(ax)^{\frac{5}{2}}} \, dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(x^m/((a^2*c*x^2 + c)^2*arctan(a*x)^(5/2)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)^{5/2}} \, dx = \int \frac{x^m}{\mathrm{atan}(a\,x)^{5/2}\,(c\,a^2\,x^2+c)^2} \, dx$$

input
```
int(x^m/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^2),x)
```

output
```
int(x^m/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 859, normalized size of antiderivative = 35.79

$$\int \frac{x^m}{(c+a^2cx^2)^2 \arctan(ax)^{5/2}} \, dx = \text{Too large to display}$$

input
```
int(x^m/(a^2*c*x^2+c)^2/atan(a*x)^(5/2),x)
```

output
```
(2*(2*atan(a*x)**2*int((x**m*sqrt(atan(a*x))*x**2)/(atan(a*x)*a**4*x**4 +
2*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**5*m**2*x**2 - 6*atan(a*x)**2*int(
(x**m*sqrt(atan(a*x))*x**2)/(atan(a*x)*a**4*x**4 + 2*atan(a*x)*a**2*x**2 +
 atan(a*x)),x)*a**5*m*x**2 + 4*atan(a*x)**2*int((x**m*sqrt(atan(a*x))*x**2
)/(atan(a*x)*a**4*x**4 + 2*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**5*x**2 +
 2*atan(a*x)**2*int((x**m*sqrt(atan(a*x))*x**2)/(atan(a*x)*a**4*x**4 + 2*a
tan(a*x)*a**2*x**2 + atan(a*x)),x)*a**3*m**2 - 6*atan(a*x)**2*int((x**m*sq
rt(atan(a*x))*x**2)/(atan(a*x)*a**4*x**4 + 2*atan(a*x)*a**2*x**2 + atan(a*
x)),x)*a**3*m + 4*atan(a*x)**2*int((x**m*sqrt(atan(a*x))*x**2)/(atan(a*x)*
a**4*x**4 + 2*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**3 + 2*atan(a*x)**2*in
t((x**m*sqrt(atan(a*x)))/(atan(a*x)*a**4*x**4 + 2*atan(a*x)*a**2*x**2 + at
an(a*x)),x)*a**3*m**2*x**2 - 2*atan(a*x)**2*int((x**m*sqrt(atan(a*x)))/(at
an(a*x)*a**4*x**4 + 2*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**3*m*x**2 - 4*
atan(a*x)**2*int((x**m*sqrt(atan(a*x)))/(atan(a*x)*a**4*x**4 + 2*atan(a*x)
*a**2*x**2 + atan(a*x)),x)*a**3*x**2 + 2*atan(a*x)**2*int((x**m*sqrt(atan(
a*x)))/(atan(a*x)*a**4*x**4 + 2*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a*m**2
 - 2*atan(a*x)**2*int((x**m*sqrt(atan(a*x)))/(atan(a*x)*a**4*x**4 + 2*atan
(a*x)*a**2*x**2 + atan(a*x)),x)*a*m - 4*atan(a*x)**2*int((x**m*sqrt(atan(a
*x)))/(atan(a*x)*a**4*x**4 + 2*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a + ata
n(a*x)**2*int((x**m*sqrt(atan(a*x)))/(atan(a*x)**2*a**4*x**5 + 2*atan(a...
```

# 3.1058 $\int \frac{x^3}{(c+a^2cx^2)^2 \arctan(ax)^{5/2}} \, dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^3}{(c+a^2cx^2)^2 \arctan(ax)^{5/2}} \, dx = -\frac{2x^3}{3ac^2(1+a^2x^2)\arctan(ax)^{3/2}} - \frac{4x^2}{a^2c^2(1+a^2x^2)\sqrt{\arctan(ax)}} - \frac{4x^4}{3c^2(1+a^2x^2)\sqrt{\arctan(ax)}} + \frac{4\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{a^4c^2} + \frac{16}{3}\text{Int}\left(\frac{x^3}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}, x\right) + \frac{8}{3}a^2\text{Int}\left(\frac{x^5}{(c+a^2cx^2)^2\sqrt{\arctan(ax)}}, x\right)$$

output
```
-2/3*x^3/a/c^2/(a^2*x^2+1)/arctan(a*x)^(3/2)-4*x^2/a^2/c^2/(a^2*x^2+1)/arc
tan(a*x)^(1/2)-4/3*x^4/c^2/(a^2*x^2+1)/arctan(a*x)^(1/2)+4*Pi^(1/2)*Fresne
lS(2*arctan(a*x)^(1/2)/Pi^(1/2))/a^4/c^2+16/3*Defer(Int)(x^3/(a^2*c*x^2+c)
^2/arctan(a*x)^(1/2),x)+8/3*a^2*Defer(Int)(x^5/(a^2*c*x^2+c)^2/arctan(a*x)
^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 6.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^3}{(c+a^2cx^2)^2 \arctan(ax)^{5/2}}\,dx = \int \frac{x^3}{(c+a^2cx^2)^2 \arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[x^3/((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)),x]
```

output
```
Integrate[x^3/((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 1.78 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\arctan(ax)^{5/2}(a^2cx^2+c)^2}\,dx$$

$$\Big\downarrow \text{5503}$$

$$\frac{2\int \frac{x^2}{c^2(a^2x^2+1)^2\arctan(ax)^{3/2}}dx}{a}+\frac{2}{3}a\int \frac{x^4}{c^2(a^2x^2+1)^2\arctan(ax)^{3/2}}dx-\frac{2x^3}{3ac^2(a^2x^2+1)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\int \frac{x^2}{(a^2x^2+1)^2\arctan(ax)^{3/2}}dx}{ac^2}+\frac{2a\int \frac{x^4}{(a^2x^2+1)^2\arctan(ax)^{3/2}}dx}{3c^2}-\frac{2x^3}{3ac^2(a^2x^2+1)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{5477}$$

$$\frac{2\left(\frac{4\int \frac{x}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{a}-\frac{2x^2}{a(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{ac^2}+\frac{2a\int \frac{x^4}{\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}dx}{3c^2}-\frac{2x^3}{3ac^2\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{5503}$$

$$\frac{2\left(\frac{4\int \frac{x}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{a}-\frac{2x^2}{a(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{ac^2}+\frac{2a\left(4a\int \frac{x^5}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx+\frac{8\int \frac{x^3}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{a}-\frac{2x^4}{a(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{3c^2}-\frac{2x^3}{3ac^2\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{5505}$$

$$\frac{2a\left(4a\int \frac{x^5}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx+\frac{8\int \frac{x^3}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{a}-\frac{2x^4}{a(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{3c^2}+\frac{2\left(\frac{4\int \frac{ax}{\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3}-\frac{2x^2}{a(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{ac^2}-\frac{2x^3}{3ac^2\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{4906}$$

$$\frac{2a\left(4a\int \frac{x^5}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx+\frac{8\int \frac{x^3}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{a}-\frac{2x^4}{a(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{3c^2}+\frac{2\left(\frac{4\int \frac{\sin(2\arctan(ax))}{2\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3}-\frac{2x^2}{a(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{ac^2}-\frac{2x^3}{3ac^2\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2a\left(4a\int \frac{x^5}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx+\frac{8\int \frac{x^3}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{a}-\frac{2x^4}{a(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{3c^2}+\frac{2\left(\frac{2\int \frac{\sin(2\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3}-\frac{2x^2}{a(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{ac^2}-\frac{2x^3}{3ac^2\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2a\left(4a\int \frac{x^5}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx+\frac{8\int \frac{x^3}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{a}-\frac{2x^4}{a(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{3c^2}+\frac{2\left(\frac{2\int \frac{\sin(2\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3}-\frac{2x^2}{a(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{ac^2}-\frac{2x^3}{3ac^2\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{3786}$$

$$\frac{2a\left(4a\int \frac{x^5}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx+\frac{8\int \frac{x^3}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{a}-\frac{2x^4}{a(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{3c^2}+\frac{2\left(\frac{4\int \sin(2\arctan(ax))d\sqrt{\arctan(ax)}}{a^3}-\frac{2x^2}{a(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{ac^2}-\frac{2x^3}{3ac^2\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{3832}$$

$$\frac{2a\left(4a\int \frac{x^5}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx+\frac{8\int \frac{x^3}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{a}-\frac{2x^4}{a(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{3c^2}-\frac{2x^3}{3ac^2\left(a^2x^2+1\right)\arctan(ax)^{3/2}}+\frac{2\left(\frac{2\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{a^3}-\frac{2x^2}{a(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{ac^2}$$

$$\Big\downarrow \text{5560}$$

$$\frac{2a\left(4a\int\frac{x^5}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx+\frac{8\int\frac{x^3}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{a}-\frac{2x^4}{a(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{3c^2}-\frac{2x^3}{3ac^2\left(a^2x^2+1\right)\arctan(ax)^{3/2}}+\frac{2\left(\frac{2\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{a^3}-\frac{2x^2}{a(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{ac^2}$$

input
```
Int[x^3/((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int\frac{x^3}{\left(a^2c\,x^2+c\right)^2\arctan\left(ax\right)^{\frac{5}{2}}}dx$$

input
```
int(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

output
```
int(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int\frac{x^3}{\left(c+a^2cx^2\right)^2\arctan(ax)^{5/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 10.21 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.00

$$\int \frac{x^3}{(c+a^2cx^2)^2 \arctan(ax)^{5/2}}\,dx = \frac{\int \frac{x^3}{a^4x^4 \operatorname{atan}^{\frac{5}{2}}(ax)+2a^2x^2 \operatorname{atan}^{\frac{5}{2}}(ax)+\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx}{c^2}$$

input
```
integrate(x**3/(a**2*c*x**2+c)**2/atan(a*x)**(5/2),x)
```

output
```
Integral(x**3/(a**4*x**4*atan(a*x)**(5/2) + 2*a**2*x**2*atan(a*x)**(5/2) +
 atan(a*x)**(5/2)), x)/c**2
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(c+a^2cx^2)^2 \arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(c+a^2cx^2)^2 \arctan(ax)^{5/2}}\,dx = \int \frac{x^3}{(a^2cx^2+c)^2 \arctan(ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(x^3/((a^2*c*x^2 + c)^2*arctan(a*x)^(5/2)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{(c+a^2cx^2)^2 \arctan(ax)^{5/2}}\,dx = \int \frac{x^3}{\mathrm{atan}(a\,x)^{5/2}\,(c\,a^2\,x^2+c)^2}\,dx$$

input
```
int(x^3/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^2),x)
```

output
```
int(x^3/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.12

$$\int \frac{x^3}{(c+a^2cx^2)^2 \arctan(ax)^{5/2}}\,dx = \frac{\int \frac{\sqrt{atan(ax)}\,x^3}{atan(ax)^3a^4x^4+2atan(ax)^3a^2x^2+atan(ax)^3}dx}{c^2}$$

input
```
int(x^3/(a^2*c*x^2+c)^2/atan(a*x)^(5/2),x)
```

output
```
int((sqrt(atan(a*x))*x**3)/(atan(a*x)**3*a**4*x**4 + 2*atan(a*x)**3*a**2*x
**2 + atan(a*x)**3),x)/c**2
```

# 3.1059 $\int \frac{x^2}{(c+a^2cx^2)^2 \arctan(ax)^{5/2}} dx$



## Optimal result

Integrand size = 24, antiderivative size = 180

$$\int \frac{x^2}{(c+a^2cx^2)^2 \arctan(ax)^{5/2}} dx = -\frac{2x^2}{3ac^2(1+a^2x^2)\arctan(ax)^{3/2}} - \frac{8x}{3a^2c^2(1+a^2x^2)\sqrt{\arctan(ax)}} + \frac{16\sqrt{\arctan(ax)}}{3a^3c^2} - \frac{32\sqrt{\arctan(ax)}}{3a^3c^2(1+a^2x^2)} + \frac{16(1-a^2x^2)\sqrt{\arctan(ax)}}{3a^3c^2(1+a^2x^2)} + \frac{8\sqrt{\pi}\operatorname{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{3a^3c^2}$$

output
```
-2/3*x^2/a/c^2/(a^2*x^2+1)/arctan(a*x)^(3/2)-8/3*x/a^2/c^2/(a^2*x^2+1)/arc
tan(a*x)^(1/2)+16/3*arctan(a*x)^(1/2)/a^3/c^2-32/3*arctan(a*x)^(1/2)/a^3/c
^2/(a^2*x^2+1)+16/3*(-a^2*x^2+1)*arctan(a*x)^(1/2)/a^3/c^2/(a^2*x^2+1)+8/3
*Pi^(1/2)*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))/a^3/c^2
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.41 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.90

$$\int \frac{x^2}{(c+a^2cx^2)^2 \arctan(ax)^{5/2}}\,dx = \frac{-2ax(ax+4\arctan(ax))+4\sqrt{\pi}(1+a^2x^2)\arctan(ax)^{3/2}\,\text{FresnelC}\left(\frac{2\sqrt{}}{}\right.}{}$$

input
```
Integrate[x^2/((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)),x]
```

output
```
(-2*a*x*(a*x + 4*ArcTan[a*x]) + 4*Sqrt[Pi]*(1 + a^2*x^2)*ArcTan[a*x]^(3/2)
*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]] + Sqrt[2]*(1 + a^2*x^2)*((-I)*Ar
cTan[a*x])^(3/2)*Gamma[1/2, (-2*I)*ArcTan[a*x]] + Sqrt[2]*(1 + a^2*x^2)*(I
*ArcTan[a*x])^(3/2)*Gamma[1/2, (2*I)*ArcTan[a*x]])/(3*a^3*c^2*(1 + a^2*x^2
)*ArcTan[a*x]^(3/2))
```

## Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.96, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5477, 27, 5467, 5465, 5439, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\arctan(ax)^{5/2}(a^2cx^2+c)^2}dx$$

$$\Big\downarrow \text{5477}$$

$$\frac{4\int \frac{x}{c^2(a^2x^2+1)^2\arctan(ax)^{3/2}}dx}{3a}-\frac{2x^2}{3ac^2(a^2x^2+1)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{4\int \frac{x}{(a^2x^2+1)^2\arctan(ax)^{3/2}}dx}{3ac^2}-\frac{2x^2}{3ac^2(a^2x^2+1)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{5467}$$

$$\frac{4\left(16\int \frac{x\sqrt{\arctan(ax)}}{(a^2x^2+1)^2}dx-\frac{2x}{a(a^2x^2+1)\sqrt{\arctan(ax)}}+\frac{4(1-a^2x^2)\sqrt{\arctan(ax)}}{a^2(a^2x^2+1)}\right)}{3ac^2}-\frac{2x^2}{3ac^2(a^2x^2+1)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{5465}$$

$$\frac{4\left(16\left(\frac{\int \frac{1}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx}{4a}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)-\frac{2x}{a(a^2x^2+1)\sqrt{\arctan(ax)}}+\frac{4(1-a^2x^2)\sqrt{\arctan(ax)}}{a^2(a^2x^2+1)}\right)}{3ac^2}-\frac{2x^2}{3ac^2(a^2x^2+1)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{5439}$$

$$\frac{4\left(16\left(\frac{\int \frac{1}{(a^2x^2+1)\sqrt{\arctan(ax)}}d\arctan(ax)}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)-\frac{2x}{a(a^2x^2+1)\sqrt{\arctan(ax)}}+\frac{4(1-a^2x^2)\sqrt{\arctan(ax)}}{a^2(a^2x^2+1)}\right)}{3ac^2}-\frac{2x^2}{3ac^2(a^2x^2+1)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4\left(16\left(\frac{\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^2}{\sqrt{\arctan(ax)}}d\arctan(ax)}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)-\frac{2x}{a(a^2x^2+1)\sqrt{\arctan(ax)}}+\frac{4(1-a^2x^2)\sqrt{\arctan(ax)}}{a^2(a^2x^2+1)}\right)}{3ac^2}-\frac{2x^2}{3ac^2(a^2x^2+1)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{3793}$$

$$\frac{4\left(16\left(\frac{\int \left(\frac{\cos(2\arctan(ax))}{2\sqrt{\arctan(ax)}}+\frac{1}{2\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)-\frac{2x}{a(a^2x^2+1)\sqrt{\arctan(ax)}}+\frac{4(1-a^2x^2)\sqrt{\arctan(ax)}}{a^2(a^2x^2+1)}\right)}{3ac^2}-\frac{2x^2}{3ac^2(a^2x^2+1)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{4\left(16\left(\frac{\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\sqrt{\arctan(ax)}}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)-\frac{2x}{a(a^2x^2+1)\sqrt{\arctan(ax)}}+\frac{4(1-a^2x^2)\sqrt{\arctan(ax)}}{a^2(a^2x^2+1)}\right)}{3ac^2}-\frac{2x^2}{3ac^2\,(a^2x^2+1)\arctan(ax)^{3/2}}$$

input
```
Int[x^2/((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)),x]
```

output
```
(-2*x^2)/(3*a*c^2*(1 + a^2*x^2)*ArcTan[a*x]^(3/2)) + (4*((-2*x)/(a*(1 + a^
2*x^2)*Sqrt[ArcTan[a*x]]) + (4*(1 - a^2*x^2)*Sqrt[ArcTan[a*x]])/(a^2*(1 +
a^2*x^2)) + 16*(-1/2*Sqrt[ArcTan[a*x]]/(a^2*(1 + a^2*x^2)) + (Sqrt[ArcTan[
a*x]] + (Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/2)/(4*a^2))))/
(3*a*c^2)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5467
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*(x_))/((d_) + (e_.)*(x_)^2)^2,
 x_Symbol] :> Simp[x*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)*(d + e*x^2
))), x] + (-Simp[(1 - c^2*x^2)*((a + b*ArcTan[c*x])^(p + 2)/(b^2*e*(p + 1)*
(p + 2)*(d + e*x^2))), x] - Simp[4/(b^2*(p + 1)*(p + 2))   Int[x*((a + b*Ar
cTan[c*x])^(p + 2)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[e, c^2*d] && LtQ[p, -1] && NeQ[p, -2]
```

rule 5477
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*ArcT
an[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp[f*(m/(b*c*(p + 1)))   Int[(f*x
)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1
]
```

## Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.34

| method | result | size |
|---|---|---|
| default | $-\frac{-8\sqrt{\pi}\,\mathrm{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)\arctan(ax)^{\frac{3}{2}}+4\arctan(ax)\sin(2\arctan(ax))-\cos(2\arctan(ax))+1}{3a^{3}c^{2}\arctan(ax)^{\frac{3}{2}}}$ | 62 |

input
```
int(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/a^3/c^2*(-8*Pi^(1/2)*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))*arctan(a*
x)^(3/2)+4*arctan(a*x)*sin(2*arctan(a*x))-cos(2*arctan(a*x))+1)/arctan(a*x
)^(3/2)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2}{(c+a^2cx^2)^2\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{x^2}{(c+a^2cx^2)^2\arctan(ax)^{5/2}}\,dx = \frac{\int \frac{x^2}{a^4x^4\operatorname{atan}^{\frac{5}{2}}(ax)+2a^2x^2\operatorname{atan}^{\frac{5}{2}}(ax)+\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx}{c^2}$$

input
```
integrate(x**2/(a**2*c*x**2+c)**2/atan(a*x)**(5/2),x)
```

output
```
Integral(x**2/(a**4*x**4*atan(a*x)**(5/2) + 2*a**2*x**2*atan(a*x)**(5/2) +
 atan(a*x)**(5/2)), x)/c**2
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(c+a^2cx^2)^2\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F]

$$\int \frac{x^2}{(c+a^2cx^2)^2 \arctan(ax)^{5/2}}\,dx = \int \frac{x^2}{(a^2cx^2+c)^2 \arctan(ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(x^2/((a^2*c*x^2 + c)^2*arctan(a*x)^(5/2)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(c+a^2cx^2)^2 \arctan(ax)^{5/2}}\,dx = \int \frac{x^2}{\mathrm{atan}(a\,x)^{5/2}\,(c\,a^2\,x^2+c)^2}\,dx$$

input
```
int(x^2/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^2),x)
```

output
```
int(x^2/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^2), x)
```

## Reduce [F]

$$\int \frac{x^2}{(c+a^2cx^2)^2 \arctan(ax)^{5/2}}\,dx = \frac{\int \frac{\sqrt{atan(ax)}\,x^2}{atan(ax)^3a^4x^4+2atan(ax)^3a^2x^2+atan(ax)^3}dx}{c^2}$$

input
```
int(x^2/(a^2*c*x^2+c)^2/atan(a*x)^(5/2),x)
```

output
```
int((sqrt(atan(a*x))*x**2)/(atan(a*x)**3*a**4*x**4 + 2*atan(a*x)**3*a**2*x
**2 + atan(a*x)**3),x)/c**2
```

# 3.1060 $\int \frac{x}{(c+a^2cx^2)^2 \arctan(ax)^{5/2}} \, dx$



## Optimal result

Integrand size = 22, antiderivative size = 101

$$\int \frac{x}{(c+a^2cx^2)^2 \arctan(ax)^{5/2}} \, dx = -\frac{2x}{3ac^2(1+a^2x^2)\arctan(ax)^{3/2}} - \frac{4(1-a^2x^2)}{3a^2c^2(1+a^2x^2)\sqrt{\arctan(ax)}} - \frac{8\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{3a^2c^2}$$

output
```
-2/3*x/a/c^2/(a^2*x^2+1)/arctan(a*x)^(3/2)-4/3*(-a^2*x^2+1)/a^2/c^2/(a^2*x
^2+1)/arctan(a*x)^(1/2)-8/3*Pi^(1/2)*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2)
)/a^2/c^2
```

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.87

$$\int \frac{x}{(c+a^2cx^2)^2 \arctan(ax)^{5/2}} \, dx = -\frac{2\left(ax + (2-2a^2x^2)\arctan(ax) + 4\sqrt{\pi}(1+a^2x^2)\arctan(ax)^{3/2}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)\right)}{3a^2c^2(1+a^2x^2)\arctan(ax)^{3/2}}$$

input
```
Integrate[x/((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)),x]
```

output
```
(-2*(a*x + (2 - 2*a^2*x^2)*ArcTan[a*x] + 4*Sqrt[Pi]*(1 + a^2*x^2)*ArcTan[a
*x]^(3/2)*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]]))/(3*a^2*c^2*(1 + a^2*x
^2)*ArcTan[a*x]^(3/2))
```

## Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {5467, 27, 5505, 4906, 27, 3042, 3786, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^2}dx$$

$$\Big\downarrow \text{ 5467}$$

$$-\frac{16}{3}\int \frac{x}{c^2\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx-\frac{2x}{3ac^2\left(a^2x^2+1\right)\arctan(ax)^{3/2}}-\frac{4\left(1-a^2x^2\right)}{3a^2c^2\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{16\int \frac{x}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{3c^2}-\frac{2x}{3ac^2\left(a^2x^2+1\right)\arctan(ax)^{3/2}}-\frac{4\left(1-a^2x^2\right)}{3a^2c^2\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{ 5505}$$

$$-\frac{16\int \frac{ax}{\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}d\arctan(ax)}{3a^2c^2}-\frac{2x}{3ac^2\left(a^2x^2+1\right)\arctan(ax)^{3/2}}-\frac{4\left(1-a^2x^2\right)}{3a^2c^2\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}$$

$$\Big\downarrow \text{ 4906}$$

$$-\frac{16\int \frac{\sin(2\arctan(ax))}{2\sqrt{\arctan(ax)}}d\arctan(ax)}{3a^2c^2}-\frac{2x}{3ac^2\left(a^2x^2+1\right)\arctan(ax)^{3/2}}-\frac{4\left(1-a^2x^2\right)}{3a^2c^2\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}$$

$\big\downarrow$ 27

$$-\frac{8\int \frac{\sin(2\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)}{3a^2c^2}-\frac{2x}{3ac^2\left(a^2x^2+1\right)\arctan(ax)^{3/2}}-\frac{4\left(1-a^2x^2\right)}{3a^2c^2\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}$$

$\big\downarrow$ 3042

$$-\frac{8\int \frac{\sin(2\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)}{3a^2c^2}-\frac{2x}{3ac^2\left(a^2x^2+1\right)\arctan(ax)^{3/2}}-\frac{4\left(1-a^2x^2\right)}{3a^2c^2\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}$$

$\big\downarrow$ 3786

$$-\frac{16\int \sin(2\arctan(ax))d\sqrt{\arctan(ax)}}{3a^2c^2}-\frac{2x}{3ac^2\left(a^2x^2+1\right)\arctan(ax)^{3/2}}-\frac{4\left(1-a^2x^2\right)}{3a^2c^2\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}$$

$\big\downarrow$ 3832

$$-\frac{8\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{3a^2c^2}-\frac{2x}{3ac^2\left(a^2x^2+1\right)\arctan(ax)^{3/2}}-\frac{4\left(1-a^2x^2\right)}{3a^2c^2\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}$$

input
```
Int[x/((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)),x]
```

output
```
(-2*x)/(3*a*c^2*(1 + a^2*x^2)*ArcTan[a*x]^(3/2)) - (4*(1 - a^2*x^2))/(3*a^
2*c^2*(1 + a^2*x^2)*Sqrt[ArcTan[a*x]]) - (8*Sqrt[Pi]*FresnelS[(2*Sqrt[ArcT
an[a*x]])/Sqrt[Pi]])/(3*a^2*c^2)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5467
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*(x_))/((d_) + (e_.)*(x_)^2)^2,
 x_Symbol] :> Simp[x*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)*(d + e*x^2
))), x] + (-Simp[(1 - c^2*x^2)*((a + b*ArcTan[c*x])^(p + 2)/(b^2*e*(p + 1)*
(p + 2)*(d + e*x^2))), x] - Simp[4/(b^2*(p + 1)*(p + 2))   Int[x*((a + b*Ar
cTan[c*x])^(p + 2)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[e, c^2*d] && LtQ[p, -1] && NeQ[p, -2]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

## Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.58

| method | result | size |
|---|---|---|
| default | $-\frac{8\sqrt{\pi}\,\mathrm{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)\arctan(ax)^{\frac{3}{2}}+4\arctan(ax)\cos(2\arctan(ax))+\sin(2\arctan(ax))}{3a^2c^2\arctan(ax)^{\frac{3}{2}}}$ | 59 |

input
```
int(x/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3/a^2/c^2*(8*Pi^(1/2)*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2))*arctan(a*x
)^(3/2)+4*arctan(a*x)*cos(2*arctan(a*x))+sin(2*arctan(a*x)))/arctan(a*x)^(
3/2)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)^2\arctan(ax)^{5/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{x}{(c+a^2cx^2)^2\arctan(ax)^{5/2}}\,dx=\frac{\int \frac{x}{a^4x^4\,\mathrm{atan}^{\frac{5}{2}}(ax)+2a^2x^2\,\mathrm{atan}^{\frac{5}{2}}(ax)+\mathrm{atan}^{\frac{5}{2}}(ax)}\,dx}{c^2}$$

input
```
integrate(x/(a**2*c*x**2+c)**2/atan(a*x)**(5/2),x)
```

output
```
Integral(x/(a**4*x**4*atan(a*x)**(5/2) + 2*a**2*x**2*atan(a*x)**(5/2) + at
an(a*x)**(5/2)), x)/c**2
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)^2 \arctan(ax)^{5/2}} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F]

$$\int \frac{x}{(c+a^2cx^2)^2 \arctan(ax)^{5/2}} \, dx = \int \frac{x}{(a^2cx^2+c)^2 \arctan(ax)^{\frac{5}{2}}} \, dx$$

input
```
integrate(x/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(x/((a^2*c*x^2 + c)^2*arctan(a*x)^(5/2)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(c+a^2cx^2)^2 \arctan(ax)^{5/2}} \, dx = \int \frac{x}{\mathrm{atan}(a\,x)^{5/2}\,(c\,a^2\,x^2+c)^2} \, dx$$

input
```
int(x/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^2),x)
```

output
```
int(x/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^2), x)
```

## Reduce [F]

$$\int \frac{x}{(c+a^2cx^2)^2 \arctan(ax)^{5/2}}\,dx = \frac{-\frac{64\sqrt{atan(ax)}\,atan(ax)^3a^2x^2}{9} - \frac{64\sqrt{atan(ax)}\,atan(ax)^3}{9} - \frac{32\sqrt{atan(ax)}\,atan(ax)^2ax}{3} +$$

input
```
int(x/(a^2*c*x^2+c)^2/atan(a*x)^(5/2),x)
```

output
```
(2*( - 32*sqrt(atan(a*x))*atan(a*x)**3*a**2*x**2 - 32*sqrt(atan(a*x))*atan
(a*x)**3 - 48*sqrt(atan(a*x))*atan(a*x)**2*a*x + 96*atan(a*x)**2*int(sqrt(
atan(a*x))/(a**4*x**4 + 2*a**2*x**2 + 1),x)*a**3*x**2 + 96*atan(a*x)**2*in
t(sqrt(atan(a*x))/(a**4*x**4 + 2*a**2*x**2 + 1),x)*a + 6*sqrt(atan(a*x))*a
tan(a*x)*a**2*x**2 - 6*sqrt(atan(a*x))*atan(a*x) - 3*sqrt(atan(a*x))*a*x))
/(9*atan(a*x)**2*a**2*c**2*(a**2*x**2 + 1))
```

# 3.1061 $\int \frac{1}{\left(c+a^2cx^2\right)^2 \arctan(ax)^{5/2}}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 174

$$\int \frac{1}{(c+a^2cx^2)^2 \arctan(ax)^{5/2}}\,dx = -\frac{2}{3ac^2(1+a^2x^2)\arctan(ax)^{3/2}} + \frac{8x}{3c^2(1+a^2x^2)\sqrt{\arctan(ax)}} - \frac{16\sqrt{\arctan(ax)}}{3ac^2} + \frac{32\sqrt{\arctan(ax)}}{3ac^2(1+a^2x^2)} - \frac{16(1-a^2x^2)\sqrt{\arctan(ax)}}{3ac^2(1+a^2x^2)} - \frac{8\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{3ac^2}$$

output

```
-2/3/a/c^2/(a^2*x^2+1)/arctan(a*x)^(3/2)+8/3*x/c^2/(a^2*x^2+1)/arctan(a*x)
^(1/2)-16/3*arctan(a*x)^(1/2)/a/c^2+32/3*arctan(a*x)^(1/2)/a/c^2/(a^2*x^2+
1)-16/3*(-a^2*x^2+1)*arctan(a*x)^(1/2)/a/c^2/(a^2*x^2+1)-8/3*Pi^(1/2)*Fres
nelC(2*arctan(a*x)^(1/2)/Pi^(1/2))/a/c^2
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.98

$$\int \frac{1}{(c+a^2cx^2)^2 \arctan(ax)^{5/2}} \, dx = \frac{-2+8ax\arctan(ax)-4\sqrt{\pi}(1+a^2x^2)\arctan(ax)^{3/2}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right) \ldots}{\ldots}$$

input
```
Integrate[1/((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)),x]
```

output
```
(-2 + 8*a*x*ArcTan[a*x] - 4*Sqrt[Pi]*(1 + a^2*x^2)*ArcTan[a*x]^(3/2)*Fresn
elC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]] + Sqrt[2]*(1 + a^2*x^2)*Sqrt[I*ArcTan[
a*x]]*Sqrt[ArcTan[a*x]^2]*Gamma[1/2, (-2*I)*ArcTan[a*x]] + (Sqrt[2]*(1 + a
^2*x^2)*ArcTan[a*x]^2*Gamma[1/2, (2*I)*ArcTan[a*x]])/Sqrt[I*ArcTan[a*x]])/
(3*c^2*(a + a^3*x^2)*ArcTan[a*x]^(3/2))
```

## Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {5437, 27, 5467, 5465, 5439, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^2}dx$$

$$\Big\downarrow \text{ 5437}$$

$$-\frac{4}{3}a\int \frac{x}{c^2\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}dx-\frac{2}{3ac^2\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{4a\int \frac{x}{\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}dx}{3c^2}-\frac{2}{3ac^2\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{ 5467}$$

$$-\frac{4a\left(16\int \frac{x\sqrt{\arctan(ax)}}{\left(a^2x^2+1\right)^2}dx-\frac{2x}{a(a^2x^2+1)\sqrt{\arctan(ax)}}+\frac{4\left(1-a^2x^2\right)\sqrt{\arctan(ax)}}{a^2(a^2x^2+1)}\right)}{3c^2}-\frac{2}{3ac^2\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

$\Big\downarrow$ 5465

$$-\frac{4a\left(16\left(\frac{\int \frac{1}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{4a}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)-\frac{2x}{a(a^2x^2+1)\sqrt{\arctan(ax)}}+\frac{4\left(1-a^2x^2\right)\sqrt{\arctan(ax)}}{a^2(a^2x^2+1)}\right)}{3c^2}-\frac{2}{3ac^2\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

$\Big\downarrow$ 5439

$$-\frac{4a\left(16\left(\frac{\int \frac{1}{\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}d\arctan(ax)}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)-\frac{2x}{a(a^2x^2+1)\sqrt{\arctan(ax)}}+\frac{4\left(1-a^2x^2\right)\sqrt{\arctan(ax)}}{a^2(a^2x^2+1)}\right)}{3c^2}-\frac{2}{3ac^2\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

$\Big\downarrow$ 3042

$$-\frac{4a\left(16\left(\frac{\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^2}{\sqrt{\arctan(ax)}}d\arctan(ax)}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)-\frac{2x}{a(a^2x^2+1)\sqrt{\arctan(ax)}}+\frac{4\left(1-a^2x^2\right)\sqrt{\arctan(ax)}}{a^2(a^2x^2+1)}\right)}{3c^2}-\frac{2}{3ac^2\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

$\Big\downarrow$ 3793

$$-\frac{4a\left(16\left(\frac{\int \left(\frac{\cos(2\arctan(ax))}{2\sqrt{\arctan(ax)}}+\frac{1}{2\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)-\frac{2x}{a(a^2x^2+1)\sqrt{\arctan(ax)}}+\frac{4\left(1-a^2x^2\right)\sqrt{\arctan(ax)}}{a^2(a^2x^2+1)}\right)}{3c^2}-\frac{2}{3ac^2\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

$\Big\downarrow$ 2009

$$-\frac{4a\left(16\left(\frac{\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\sqrt{\arctan(ax)}}{4a^2}-\frac{\sqrt{\arctan(ax)}}{2a^2(a^2x^2+1)}\right)-\frac{2x}{a(a^2x^2+1)\sqrt{\arctan(ax)}}+\frac{4(1-a^2x^2)\sqrt{\arctan(ax)}}{a^2(a^2x^2+1)}\right)}{3c^2}-\frac{2}{3ac^2\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

input

```
Int[1/((c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)),x]
```

output

```
-2/(3*a*c^2*(1 + a^2*x^2)*ArcTan[a*x]^(3/2)) - (4*a*((-2*x)/(a*(1 + a^2*x^
2)*Sqrt[ArcTan[a*x]]) + (4*(1 - a^2*x^2)*Sqrt[ArcTan[a*x]])/(a^2*(1 + a^2*
x^2)) + 16*(-1/2*Sqrt[ArcTan[a*x]]/(a^2*(1 + a^2*x^2)) + (Sqrt[ArcTan[a*x]
] + (Sqrt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/2)/(4*a^2))))/(3*c
^2)
```

## Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5465
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)*((d_) + (e_.)*(x_)^2)^(q_
.), x_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^p/(2*e*(q +
1))), x] - Simp[b*(p/(2*c*(q + 1)))   Int[(d + e*x^2)^q*(a + b*ArcTan[c*x])
^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && EqQ[e, c^2*d] && GtQ[p,
 0] && NeQ[q, -1]
```

rule 5467
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*(x_))/((d_) + (e_.)*(x_)^2)^2,
 x_Symbol] :> Simp[x*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)*(d + e*x^2
))), x] + (-Simp[(1 - c^2*x^2)*((a + b*ArcTan[c*x])^(p + 2)/(b^2*e*(p + 1)*
(p + 2)*(d + e*x^2))), x] - Simp[4/(b^2*(p + 1)*(p + 2))   Int[x*((a + b*Ar
cTan[c*x])^(p + 2)/(d + e*x^2)^2), x], x]) /; FreeQ[{a, b, c, d, e}, x] &&
EqQ[e, c^2*d] && LtQ[p, -1] && NeQ[p, -2]
```

## Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.36

| method | result | size |
|---|---|---|
| default | $\frac{-8\sqrt{\pi}\,\mathrm{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)\arctan(ax)^{\frac{3}{2}}+4\arctan(ax)\sin(2\arctan(ax))-\cos(2\arctan(ax))-1}{3a\,c^{2}\arctan(ax)^{\frac{3}{2}}}$ | 62 |

input
```
int(1/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/a/c^2*(-8*Pi^(1/2)*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))*arctan(a*x)^
(3/2)+4*arctan(a*x)*sin(2*arctan(a*x))-cos(2*arctan(a*x))-1)/arctan(a*x)^(
3/2)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+a^2cx^2\right)^2\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{1}{\left(c+a^2cx^2\right)^2\arctan(ax)^{5/2}}\,dx = \frac{\int \frac{1}{a^4x^4\operatorname{atan}^{\frac{5}{2}}(ax)+2a^2x^2\operatorname{atan}^{\frac{5}{2}}(ax)+\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx}{c^2}$$

input
```
integrate(1/(a**2*c*x**2+c)**2/atan(a*x)**(5/2),x)
```

output
```
Integral(1/(a**4*x**4*atan(a*x)**(5/2) + 2*a**2*x**2*atan(a*x)**(5/2) + at
an(a*x)**(5/2)), x)/c**2
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(c+a^2cx^2)^2\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F]

$$\int \frac{1}{(c+a^2cx^2)^2\arctan(ax)^{5/2}}\,dx = \int \frac{1}{(a^2cx^2+c)^2\arctan(ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^2*arctan(a*x)^(5/2)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+a^2cx^2)^2\arctan(ax)^{5/2}}\,dx = \int \frac{1}{\mathrm{atan}(a\,x)^{5/2}\,(c\,a^2\,x^2+c)^2}\,dx$$

input
```
int(1/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^2),x)
```

output
```
int(1/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^2), x)
```

## Reduce [F]

$$\int \frac{1}{(c+a^2cx^2)^2 \arctan(ax)^{5/2}}\,dx = \frac{16\sqrt{atan(ax)}\,atan(ax)^2a^2x^2}{3} + \frac{16\sqrt{atan(ax)}\,atan(ax)^2}{3} - \frac{16atan(ax)^2\left(\int \frac{\sqrt{ata}}{atan(ax)a^4x^4+2ata}\right.}{3}$$

input
```
int(1/(a^2*c*x^2+c)^2/atan(a*x)^(5/2),x)
```

output
```
(2*(8*sqrt(atan(a*x))*atan(a*x)**2*a**2*x**2 + 8*sqrt(atan(a*x))*atan(a*x)
**2 - 8*atan(a*x)**2*int(sqrt(atan(a*x))/(atan(a*x)*a**4*x**4 + 2*atan(a*x
)*a**2*x**2 + atan(a*x)),x)*a**3*x**2 - 8*atan(a*x)**2*int(sqrt(atan(a*x))
/(atan(a*x)*a**4*x**4 + 2*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a + 4*sqrt(a
tan(a*x))*atan(a*x)*a*x - sqrt(atan(a*x))))/(3*atan(a*x)**2*a*c**2*(a**2*x
**2 + 1))
```

# 3.1062 $\int \frac{1}{x(c+a^2cx^2)^2 \arctan(ax)^{5/2}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2 \arctan(ax)^{5/2}}\,dx = -\frac{2}{3ac^2x\left(1+a^2x^2\right)\arctan(ax)^{3/2}} + \frac{4}{c^2\left(1+a^2x^2\right)\sqrt{\arctan(ax)}} + \frac{4}{3a^2c^2x^2\left(1+a^2x^2\right)\sqrt{\arctan(ax)}} + \frac{4\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{c^2} + \frac{8\text{Int}\left(\frac{1}{x^3(c+a^2cx^2)^2\sqrt{\arctan(ax)}},x\right)}{3a^2} + \frac{16}{3}\text{Int}\left(\frac{1}{x\left(c+a^2cx^2\right)^2\sqrt{\arctan(ax)}},x\right)$$

output

```
-2/3/a/c^2/x/(a^2*x^2+1)/arctan(a*x)^(3/2)+4/c^2/(a^2*x^2+1)/arctan(a*x)^(
1/2)+4/3/a^2/c^2/x^2/(a^2*x^2+1)/arctan(a*x)^(1/2)+4*Pi^(1/2)*FresnelS(2*a
rctan(a*x)^(1/2)/Pi^(1/2))/c^2+8/3*Defer(Int)(1/x^3/(a^2*c*x^2+c)^2/arctan
(a*x)^(1/2),x)/a^2+16/3*Defer(Int)(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x
)
```

## Mathematica [N/A]

Not integrable

Time = 2.81 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2 \arctan(ax)^{5/2}}\,dx=\int \frac{1}{x\left(c+a^2cx^2\right)^2 \arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[1/(x*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)),x]
```

output
```
Integrate[1/(x*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 1.83 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x \arctan(ax)^{5/2}\left(a^2cx^2+c\right)^2}\,dx$$

$$\Big\downarrow \text{5503}$$

$$-2a\int \frac{1}{c^2\left(a^2x^2+1\right)^2 \arctan(ax)^{3/2}}dx-\frac{2\int \frac{1}{c^2x^2\left(a^2x^2+1\right)^2 \arctan(ax)^{3/2}}dx}{3a}-\frac{2}{3ac^2x\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2a\int \frac{1}{\left(a^2x^2+1\right)^2 \arctan(ax)^{3/2}}dx}{c^2}-\frac{2\int \frac{1}{x^2\left(a^2x^2+1\right)^2 \arctan(ax)^{3/2}}dx}{3ac^2}-\frac{2}{3ac^2x\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{5437}$$

$$-\frac{2\int \frac{1}{x^2(a^2x^2+1)^2\arctan(ax)^{3/2}}dx}{3ac^2}-\frac{2a\left(-4a\int \frac{x}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx-\frac{2}{a(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{c^2}-\frac{2}{3ac^2x\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{5503}$$

$$-\frac{2a\left(-4a\int \frac{x}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx-\frac{2}{a(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{c^2}-\frac{2\left(-8a\int \frac{1}{x(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx-\frac{4\int \frac{1}{x^3\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ax^2(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{3ac^2}-\frac{2}{3ac^2x\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{5505}$$

$$-\frac{2a\left(-\frac{4\int \frac{ax}{\left(a^2x^2+1\right)\sqrt{\arctan(ax)}}d\arctan(ax)}{a}-\frac{2}{a(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{c^2}-\frac{2\left(-8a\int \frac{1}{x(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx-\frac{4\int \frac{1}{x^3\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ax^2(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{3ac^2}-\frac{2}{3ac^2x\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{4906}$$

$$-\frac{2a\left(-\frac{4\int \frac{\sin(2\arctan(ax))}{2\sqrt{\arctan(ax)}}d\arctan(ax)}{a}-\frac{2}{a(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{c^2}-\frac{2\left(-8a\int \frac{1}{x(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx-\frac{4\int \frac{1}{x^3\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ax^2(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{3ac^2}-\frac{2}{3ac^2x\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{2a\left(-\frac{2\int \frac{\sin(2\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)}{a}-\frac{2}{a(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{c^2}-\frac{2\left(-8a\int \frac{1}{x(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx-\frac{4\int \frac{1}{x^3\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ax^2(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{3ac^2}-\frac{2}{3ac^2x\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{2a\left(-\frac{2\int \frac{\sin(2\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)}{a}-\frac{2}{a(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{c^2}-\frac{2\left(-8a\int \frac{1}{x(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx-\frac{4\int \frac{1}{x^3\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ax^2(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{3ac^2}-\frac{2}{3ac^2x\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \; 3786$$

$$-\frac{2a\left(-\frac{4\int \sin(2\arctan(ax))d\sqrt{\arctan(ax)}}{a}-\frac{2}{a(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{c^2}-\frac{2\left(-8a\int \frac{1}{x(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx-\frac{4\int \frac{1}{x^3\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ax^2(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{3ac^2}-\frac{2}{3ac^2x\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \; 3832$$

$$-\frac{2\left(-8a\int \frac{1}{x(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx-\frac{4\int \frac{1}{x^3\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ax^2(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{3ac^2}-\frac{2a\left(-\frac{2}{a(a^2x^2+1)\sqrt{\arctan(ax)}}-\frac{2\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{a}\right)}{c^2}-\frac{2}{3ac^2x\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \; 5560$$

$$-\frac{2\left(-8a\int\frac{1}{x(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx-\frac{4\int\frac{1}{x^3\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ax^2(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{3ac^2}-\frac{2a\left(-\frac{2}{a(a^2x^2+1)\sqrt{\arctan(ax)}}-\frac{2\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{a}\right)}{c^2}-\frac{2}{3ac^2x\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

input
```
Int[1/(x*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int\frac{1}{x\left(a^2c\,x^2+c\right)^2\arctan\left(ax\right)^{\frac{5}{2}}}dx$$

input
```
int(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

output
```
int(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int\frac{1}{x\left(c+a^2cx^2\right)^2\arctan(ax)^{5/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 10.79 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\arctan(ax)^{5/2}}\,dx=\frac{\int \frac{1}{a^4x^5\operatorname{atan}^{\frac{5}{2}}(ax)+2a^2x^3\operatorname{atan}^{\frac{5}{2}}(ax)+x\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx}{c^2}$$

input
```
integrate(1/x/(a**2*c*x**2+c)**2/atan(a*x)**(5/2),x)
```

output
```
Integral(1/(a**4*x**5*atan(a*x)**(5/2) + 2*a**2*x**3*atan(a*x)**(5/2) + x*
atan(a*x)**(5/2)), x)/c**2
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\arctan(ax)^{5/2}}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\arctan(ax)^{5/2}}\,dx = \int \frac{1}{\left(a^2cx^2+c\right)^2x\arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^2*x*arctan(a*x)^(5/2)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.89 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\arctan(ax)^{5/2}}\,dx = \int \frac{1}{x\,\mathrm{atan}\left(a\,x\right)^{5/2}\,\left(c\,a^2\,x^2+c\right)^2}\,dx$$

input
```
int(1/(x*atan(a*x)^(5/2)*(c + a^2*c*x^2)^2),x)
```

output
```
int(1/(x*atan(a*x)^(5/2)*(c + a^2*c*x^2)^2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.08

$$\int \frac{1}{x\left(c+a^2cx^2\right)^2\arctan(ax)^{5/2}}\,dx = \frac{\int \frac{\sqrt{atan(ax)}}{atan(ax)^3a^4x^5+2atan(ax)^3a^2x^3+atan(ax)^3x}dx}{c^2}$$

input
```
int(1/x/(a^2*c*x^2+c)^2/atan(a*x)^(5/2),x)
```

output
```
int(sqrt(atan(a*x))/(atan(a*x)**3*a**4*x**5 + 2*atan(a*x)**3*a**2*x**3 + a
tan(a*x)**3*x),x)/c**2
```

# 3.1063 $\int \frac{1}{x^2(c+a^2cx^2)^2 \arctan(ax)^{5/2}} \, dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x^2 (c+a^2cx^2)^2 \arctan(ax)^{5/2}} \, dx = -\frac{2}{3ac^2x^2(1+a^2x^2)\arctan(ax)^{3/2}} + \frac{8}{3a^2c^2x^3(1+a^2x^2)\sqrt{\arctan(ax)}} + \frac{16}{3c^2x(1+a^2x^2)\sqrt{\arctan(ax)}} + \frac{16a\sqrt{\arctan(ax)}}{c^2} + \frac{8a\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{c^2} + \frac{8\text{Int}\left(\frac{1}{x^4(c+a^2cx^2)^2\sqrt{\arctan(ax)}}, x\right)}{a^2} + \frac{56}{3}\text{Int}\left(\frac{1}{x^2(c+a^2cx^2)^2\sqrt{\arctan(ax)}}, x\right)$$

output
```
-2/3/a/c^2/x^2/(a^2*x^2+1)/arctan(a*x)^(3/2)+8/3/a^2/c^2/x^3/(a^2*x^2+1)/a
rctan(a*x)^(1/2)+16/3/c^2/x/(a^2*x^2+1)/arctan(a*x)^(1/2)+16*a*arctan(a*x)
^(1/2)/c^2+8*a*Pi^(1/2)*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))/c^2+8*Defer
(Int)(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)/a^2+56/3*Defer(Int)(1/x^2
/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 5.10 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^2 (c+a^2cx^2)^2 \arctan(ax)^{5/2}}\,dx = \int \frac{1}{x^2 (c+a^2cx^2)^2 \arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[1/(x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)),x]
```

output
```
Integrate[1/(x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 1.57 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \arctan(ax)^{5/2} (a^2cx^2+c)^2}dx$$

$$\Big\downarrow \text{5503}$$

$$-\frac{8}{3}a\int \frac{1}{c^2x (a^2x^2+1)^2 \arctan(ax)^{3/2}}dx - \frac{4\int \frac{1}{c^2x^3(a^2x^2+1)^2\arctan(ax)^{3/2}}dx}{3a} - \frac{2}{3ac^2x^2 (a^2x^2+1)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{8a\int \frac{1}{x(a^2x^2+1)^2\arctan(ax)^{3/2}}dx}{3c^2} - \frac{4\int \frac{1}{x^3(a^2x^2+1)^2\arctan(ax)^{3/2}}dx}{3ac^2} - \frac{2}{3ac^2x^2 (a^2x^2+1)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{5503}$$

$$-\frac{8a\left(-6a\int \frac{1}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx-\frac{2\int \frac{1}{x^2\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ax(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{3c^2}-\frac{4\left(-10a\int \frac{1}{x^2(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx-\frac{6\int \frac{1}{x^4\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ax^3(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{3ac^2}-\frac{2}{3ac^2x^2\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{5439}$$

$$-\frac{8a\left(-\frac{2\int \frac{1}{x^2\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{a}-6\int \frac{1}{(a^2x^2+1)\sqrt{\arctan(ax)}}d\arctan(ax)-\frac{2}{ax(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{3c^2}-\frac{4\left(-10a\int \frac{1}{x^2(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx-\frac{6\int \frac{1}{x^4\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ax^3(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{3ac^2}-\frac{2}{3ac^2x^2\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{8a\left(-\frac{2\int \frac{1}{x^2\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{a}-6\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^2}{\sqrt{\arctan(ax)}}d\arctan(ax)-\frac{2}{ax(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{3c^2}-\frac{4\left(-10a\int \frac{1}{x^2(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx-\frac{6\int \frac{1}{x^4\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ax^3(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{3ac^2}-\frac{2}{3ac^2x^2\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{8a\left(-\frac{2\int \frac{1}{x^2\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{a}-6\int \left(\frac{\cos(2\arctan(ax))}{2\sqrt{\arctan(ax)}}+\frac{1}{2\sqrt{\arctan(ax)}}\right)d\arctan(ax)-\frac{2}{ax(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{3c^2}-\frac{\frac{4\left(-10a\int \frac{1}{x^2(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx-\frac{6\int \frac{1}{x^4\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ax^3(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{3ac^2}-\frac{2}{3ac^2x^2\left(a^2x^2+1\right)\arctan(ax)^{3/2}}}{}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{8a\left(-\frac{2\int \frac{1}{x^2\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ax(a^2x^2+1)\sqrt{\arctan(ax)}}-6\left(\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\sqrt{\arctan(ax)}\right)\right)}{3c^2}-\frac{4\left(-10a\int \frac{1}{x^2(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx-\frac{6\int \frac{1}{x^4\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ax^3(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{3ac^2}-\frac{2}{3ac^2x^2\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{5560}$$

$$-\frac{8a\left(-\frac{2\int \frac{1}{x^2\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ax(a^2x^2+1)\sqrt{\arctan(ax)}}-6\left(\frac{1}{2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\sqrt{\arctan(ax)}\right)\right)}{3c^2}-\frac{4\left(-10a\int \frac{1}{x^2(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx-\frac{6\int \frac{1}{x^4\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ax^3(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{3ac^2}-\frac{2}{3ac^2x^2\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

input
```
Int[1/(x^2*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^2 (a^2 c x^2 + c)^2 \arctan(ax)^{\frac{5}{2}}} dx$$

input
```
int(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

output
```
int(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^2 (c + a^2 c x^2)^2 \arctan(ax)^{5/2}} dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 14.82 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.04

$$\int \frac{1}{x^2 (c + a^2 c x^2)^2 \arctan(ax)^{5/2}} dx = \frac{\int \frac{1}{a^4 x^6 \operatorname{atan}^{\frac{5}{2}}(ax) + 2a^2 x^4 \operatorname{atan}^{\frac{5}{2}}(ax) + x^2 \operatorname{atan}^{\frac{5}{2}}(ax)} dx}{c^2}$$

input
```
integrate(1/x**2/(a**2*c*x**2+c)**2/atan(a*x)**(5/2),x)
```

output
```
Integral(1/(a**4*x**6*atan(a*x)**(5/2) + 2*a**2*x**4*atan(a*x)**(5/2) + x*
*2*atan(a*x)**(5/2)), x)/c**2
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^2 (c+a^2cx^2)^2 \arctan(ax)^{5/2}} dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (c+a^2cx^2)^2 \arctan(ax)^{5/2}} dx = \int \frac{1}{(a^2cx^2+c)^2 x^2 \arctan(ax)^{\frac{5}{2}}} dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^2*x^2*arctan(a*x)^(5/2)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.90 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (c+a^2cx^2)^2 \arctan(ax)^{5/2}} \, dx = \int \frac{1}{x^2 \operatorname{atan}(a\,x)^{5/2} \left(c\,a^2\,x^2+c\right)^2} \, dx$$

input
```
int(1/(x^2*atan(a*x)^(5/2)*(c + a^2*c*x^2)^2),x)
```

output
```
int(1/(x^2*atan(a*x)^(5/2)*(c + a^2*c*x^2)^2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.17

$$\int \frac{1}{x^2 (c+a^2cx^2)^2 \arctan(ax)^{5/2}} \, dx = \frac{\int \frac{\sqrt{atan(ax)}}{atan(ax)^3 a^4 x^6 + 2 atan(ax)^3 a^2 x^4 + atan(ax)^3 x^2} dx}{c^2}$$

input
```
int(1/x^2/(a^2*c*x^2+c)^2/atan(a*x)^(5/2),x)
```

output
```
int(sqrt(atan(a*x))/(atan(a*x)**3*a**4*x**6 + 2*atan(a*x)**3*a**2*x**4 + a
tan(a*x)**3*x**2),x)/c**2
```

# 3.1064 $\int \frac{1}{x^3(c+a^2cx^2)^2 \arctan(ax)^{5/2}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\begin{aligned}\int \frac{1}{x^3 (c+a^2cx^2)^2 \arctan(ax)^{5/2}}\,dx =& -\frac{2}{3ac^2x^3(1+a^2x^2)\arctan(ax)^{3/2}} \\ &+ \frac{4}{a^2c^2x^4(1+a^2x^2)\sqrt{\arctan(ax)}} + \frac{20}{3c^2x^2(1+a^2x^2)\sqrt{\arctan(ax)}} \\ &+ \frac{16\text{Int}\left(\frac{1}{x^5(c+a^2cx^2)^2\sqrt{\arctan(ax)}},x\right)}{a^2} + \frac{112}{3}\text{Int}\left(\frac{1}{x^3(c+a^2cx^2)^2\sqrt{\arctan(ax)}},x\right) \\ &+ \frac{80}{3}a^2\text{Int}\left(\frac{1}{x(c+a^2cx^2)^2\sqrt{\arctan(ax)}},x\right)\end{aligned}$$

output
```
-2/3/a/c^2/x^3/(a^2*x^2+1)/arctan(a*x)^(3/2)+4/a^2/c^2/x^4/(a^2*x^2+1)/arc
tan(a*x)^(1/2)+20/3/c^2/x^2/(a^2*x^2+1)/arctan(a*x)^(1/2)+16*Defer(Int)(1/
x^5/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)/a^2+112/3*Defer(Int)(1/x^3/(a^2*c
*x^2+c)^2/arctan(a*x)^(1/2),x)+80/3*a^2*Defer(Int)(1/x/(a^2*c*x^2+c)^2/arc
tan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 4.73 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^3 (c+a^2cx^2)^2 \arctan(ax)^{5/2}}\,dx = \int \frac{1}{x^3 (c+a^2cx^2)^2 \arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[1/(x^3*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)),x]
```

output
```
Integrate[1/(x^3*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.85 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \arctan(ax)^{5/2} (a^2cx^2+c)^2}\,dx$$

$$\Big\downarrow \text{5503}$$

$$-\frac{10}{3}a\int \frac{1}{c^2x^2 (a^2x^2+1)^2 \arctan(ax)^{3/2}}dx - \frac{2\int \frac{1}{c^2x^4(a^2x^2+1)^2 \arctan(ax)^{3/2}}dx}{a} - \frac{2}{3ac^2x^3 (a^2x^2+1) \arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{10a\int \frac{1}{x^2(a^2x^2+1)^2 \arctan(ax)^{3/2}}dx}{3c^2} - \frac{2\int \frac{1}{x^4(a^2x^2+1)^2 \arctan(ax)^{3/2}}dx}{ac^2} - \frac{2}{3ac^2x^3 (a^2x^2+1) \arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{5503}$$

$$-\frac{10a\left(-8a\int \frac{1}{x(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx-\frac{4\int \frac{1}{x^3\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ax^2(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{3c^2}-\frac{2\left(-\frac{8\int \frac{1}{x^5\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{a}-12a\int \frac{1}{x^3(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx-\frac{2}{ax^4(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{ac^2}-\frac{2}{3ac^2x^3\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{5560}$$

$$-\frac{10a\left(-8a\int \frac{1}{x(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx-\frac{4\int \frac{1}{x^3\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ax^2(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{3c^2}-\frac{2\left(-\frac{8\int \frac{1}{x^5\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{a}-12a\int \frac{1}{x^3(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx-\frac{2}{ax^4(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{ac^2}-\frac{2}{3ac^2x^3\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

input
```
Int[1/(x^3*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^3\left(a^2c\,x^2+c\right)^2\arctan\left(ax\right)^{\frac{5}{2}}}dx$$

input
```
int(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

output
```
int(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^3 (c+a^2cx^2)^2 \arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 18.76 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.04

$$\int \frac{1}{x^3 (c+a^2cx^2)^2 \arctan(ax)^{5/2}}\,dx = \frac{\int \frac{1}{a^4x^7 \operatorname{atan}^{\frac{5}{2}}(ax)+2a^2x^5 \operatorname{atan}^{\frac{5}{2}}(ax)+x^3 \operatorname{atan}^{\frac{5}{2}}(ax)}\,dx}{c^2}$$

input
```
integrate(1/x**3/(a**2*c*x**2+c)**2/atan(a*x)**(5/2),x)
```

output
```
Integral(1/(a**4*x**7*atan(a*x)**(5/2) + 2*a**2*x**5*atan(a*x)**(5/2) + x*
*3*atan(a*x)**(5/2)), x)/c**2
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^3 (c+a^2cx^2)^2 \arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3 (c+a^2cx^2)^2 \arctan(ax)^{5/2}}\,dx = \int \frac{1}{(a^2cx^2+c)^2 x^3 \arctan(ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^2*x^3*arctan(a*x)^(5/2)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.90 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3 (c+a^2cx^2)^2 \arctan(ax)^{5/2}}\,dx = \int \frac{1}{x^3 \operatorname{atan}(a\,x)^{5/2} (c\,a^2\,x^2+c)^2}\,dx$$

input
```
int(1/(x^3*atan(a*x)^(5/2)*(c + a^2*c*x^2)^2),x)
```

output
```
int(1/(x^3*atan(a*x)^(5/2)*(c + a^2*c*x^2)^2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.17

$$\int \frac{1}{x^3 (c+a^2cx^2)^2 \arctan(ax)^{5/2}}\,dx = \frac{\int \frac{\sqrt{atan(ax)}}{atan(ax)^3a^4x^7+2atan(ax)^3a^2x^5+atan(ax)^3x^3}dx}{c^2}$$

input
```
int(1/x^3/(a^2*c*x^2+c)^2/atan(a*x)^(5/2),x)
```

output
```
int(sqrt(atan(a*x))/(atan(a*x)**3*a**4*x**7 + 2*atan(a*x)**3*a**2*x**5 + a
tan(a*x)**3*x**3),x)/c**2
```

# 3.1065 $\int \frac{1}{x^4(c+a^2cx^2)^2\arctan(ax)^{5/2}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x^4(c+a^2cx^2)^2\arctan(ax)^{5/2}}\,dx = -\frac{2}{3ac^2x^4(1+a^2x^2)\arctan(ax)^{3/2}} + \frac{16}{3a^2c^2x^5(1+a^2x^2)\sqrt{\arctan(ax)}} + \frac{8}{c^2x^3(1+a^2x^2)\sqrt{\arctan(ax)}} + \frac{80\text{Int}\left(\frac{1}{x^6(c+a^2cx^2)^2\sqrt{\arctan(ax)}},x\right)}{3a^2} + \frac{184}{3}\text{Int}\left(\frac{1}{x^4(c+a^2cx^2)^2\sqrt{\arctan(ax)}},x\right) + 40a^2\text{Int}\left(\frac{1}{x^2(c+a^2cx^2)^2\sqrt{\arctan(ax)}},x\right)$$

output
```
-2/3/a/c^2/x^4/(a^2*x^2+1)/arctan(a*x)^(3/2)+16/3/a^2/c^2/x^5/(a^2*x^2+1)/
arctan(a*x)^(1/2)+8/c^2/x^3/(a^2*x^2+1)/arctan(a*x)^(1/2)+80/3*Defer(Int)(
1/x^6/(a^2*c*x^2+c)^2/arctan(a*x)^(1/2),x)/a^2+184/3*Defer(Int)(1/x^4/(a^2
*c*x^2+c)^2/arctan(a*x)^(1/2),x)+40*a^2*Defer(Int)(1/x^2/(a^2*c*x^2+c)^2/a
rctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 10.59 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^4 (c+a^2cx^2)^2 \arctan(ax)^{5/2}}\,dx = \int \frac{1}{x^4 (c+a^2cx^2)^2 \arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[1/(x^4*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)),x]
```

output
```
Integrate[1/(x^4*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.85 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \arctan(ax)^{5/2} (a^2cx^2+c)^2}\,dx$$

$$\Big\downarrow \text{5503}$$

$$-\frac{8\int \frac{1}{c^2x^5(a^2x^2+1)^2\arctan(ax)^{3/2}}dx}{3a} - 4a\int \frac{1}{c^2x^3(a^2x^2+1)^2\arctan(ax)^{3/2}}dx - \frac{2}{3ac^2x^4(a^2x^2+1)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{8\int \frac{1}{x^5(a^2x^2+1)^2\arctan(ax)^{3/2}}dx}{3ac^2} - \frac{4a\int \frac{1}{x^3(a^2x^2+1)^2\arctan(ax)^{3/2}}dx}{c^2} - \frac{2}{3ac^2x^4(a^2x^2+1)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{5503}$$

$$-\frac{4a\left(-10a\int \frac{1}{x^2(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx-\frac{6\int \frac{1}{x^4\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ax^3(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{c^2}-\frac{8\left(-\frac{10\int \frac{1}{x^6\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{a}-14a\int \frac{1}{x^4(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx-\frac{2}{ax^5(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{3ac^2}-\frac{2}{3ac^2x^4\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{5560}$$

$$-\frac{4a\left(-10a\int \frac{1}{x^2(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx-\frac{6\int \frac{1}{x^4\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ax^3(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{c^2}-\frac{8\left(-\frac{10\int \frac{1}{x^6\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}dx}{a}-14a\int \frac{1}{x^4(a^2x^2+1)^2\sqrt{\arctan(ax)}}dx-\frac{2}{ax^5(a^2x^2+1)\sqrt{\arctan(ax)}}\right)}{3ac^2}-\frac{2}{3ac^2x^4\left(a^2x^2+1\right)\arctan(ax)^{3/2}}$$

input
```
Int[1/(x^4*(c + a^2*c*x^2)^2*ArcTan[a*x]^(5/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^4\left(a^2c\,x^2+c\right)^2\arctan\left(ax\right)^{\frac{5}{2}}}dx$$

input
```
int(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

output
```
int(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^4 (c + a^2 c x^2)^2 \arctan(ax)^{5/2}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 24.64 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.04

$$\int \frac{1}{x^4 (c + a^2 c x^2)^2 \arctan(ax)^{5/2}} \, dx = \frac{\int \frac{1}{a^4 x^8 \operatorname{atan}^{\frac{5}{2}}(ax) + 2a^2 x^6 \operatorname{atan}^{\frac{5}{2}}(ax) + x^4 \operatorname{atan}^{\frac{5}{2}}(ax)} \, dx}{c^2}$$

input
```
integrate(1/x**4/(a**2*c*x**2+c)**2/atan(a*x)**(5/2),x)
```

output
```
Integral(1/(a**4*x**8*atan(a*x)**(5/2) + 2*a**2*x**6*atan(a*x)**(5/2) + x*
*4*atan(a*x)**(5/2)), x)/c**2
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^4 (c + a^2cx^2)^2 \arctan(ax)^{5/2}} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4 (c + a^2cx^2)^2 \arctan(ax)^{5/2}} \, dx = \int \frac{1}{(a^2cx^2 + c)^2 x^4 \arctan(ax)^{\frac{5}{2}}} \, dx$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^2/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^2*x^4*arctan(a*x)^(5/2)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.92 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4 (c + a^2cx^2)^2 \arctan(ax)^{5/2}} \, dx = \int \frac{1}{x^4 \operatorname{atan}(a\,x)^{5/2}\,(c\,a^2\,x^2 + c)^2} \, dx$$

input
```
int(1/(x^4*atan(a*x)^(5/2)*(c + a^2*c*x^2)^2),x)
```

output
```
int(1/(x^4*atan(a*x)^(5/2)*(c + a^2*c*x^2)^2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.17

$$\int \frac{1}{x^4 (c + a^2cx^2)^2 \arctan(ax)^{5/2}} \, dx = \frac{\int \frac{\sqrt{atan(ax)}}{atan(ax)^3 a^4 x^8 + 2atan(ax)^3 a^2 x^6 + atan(ax)^3 x^4} dx}{c^2}$$

input
```
int(1/x^4/(a^2*c*x^2+c)^2/atan(a*x)^(5/2),x)
```

output
```
int(sqrt(atan(a*x))/(atan(a*x)**3*a**4*x**8 + 2*atan(a*x)**3*a**2*x**6 + a
tan(a*x)**3*x**4),x)/c**2
```

# 3.1066 $\int \frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)^{5/2}} \, dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)^{5/2}} \, dx = \text{Int}\left(\frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)^{5/2}}, x\right)$$

output
```
Defer(Int)(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)^{5/2}} \, dx = \int \frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)^{5/2}} \, dx$$

input
```
Integrate[x^m/((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)),x]
```

output
```
Integrate[x^m/((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\arctan(ax)^{5/2} (a^2cx^2+c)^3} dx$$
$$\Big\downarrow \ 5560$$
$$\int \frac{x^m}{\arctan(ax)^{5/2} (a^2cx^2+c)^3} dx$$

input
```
Int[x^m/((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{(a^2c\,x^2+c)^3 \arctan(ax)^{\frac{5}{2}}} dx$$

input
```
int(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

output
```
int(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.08

$$\int \frac{x^m}{(c+a^2cx^2)^3\arctan(ax)^{5/2}}\,dx = \int \frac{x^m}{(a^2cx^2+c)^3\arctan(ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
integral(x^m/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 + c^3)*arctan(a
*x)^(5/2)), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^m}{(c+a^2cx^2)^3\arctan(ax)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**m/(a**2*c*x**2+c)**3/atan(a*x)**(5/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m}{(c+a^2cx^2)^3\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)^{5/2}} \, dx = \int \frac{x^m}{(a^2cx^2+c)^3 \arctan(ax)^{\frac{5}{2}}} \, dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(x^m/((a^2*c*x^2 + c)^3*arctan(a*x)^(5/2)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.97 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)^{5/2}} \, dx = \int \frac{x^m}{\mathrm{atan}\left(a\,x\right)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^3} \, dx$$

input
```
int(x^m/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^3),x)
```

output
```
int(x^m/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^3), x)
```

## Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 1536, normalized size of antiderivative = 64.00

$$\int \frac{x^m}{(c+a^2cx^2)^3 \arctan(ax)^{5/2}} \, dx = \text{Too large to display}$$

input
```
int(x^m/(a^2*c*x^2+c)^3/atan(a*x)^(5/2),x)
```

output
```
(2*(2*atan(a*x)**2*int((x**m*sqrt(atan(a*x))*x**2)/(atan(a*x)*a**6*x**6 +
3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**7*m**2*x*
*4 - 14*atan(a*x)**2*int((x**m*sqrt(atan(a*x))*x**2)/(atan(a*x)*a**6*x**6
+ 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**7*m*x**
4 + 24*atan(a*x)**2*int((x**m*sqrt(atan(a*x))*x**2)/(atan(a*x)*a**6*x**6 +
 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**7*x**4 +
 4*atan(a*x)**2*int((x**m*sqrt(atan(a*x))*x**2)/(atan(a*x)*a**6*x**6 + 3*a
tan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**5*m**2*x**2
- 28*atan(a*x)**2*int((x**m*sqrt(atan(a*x))*x**2)/(atan(a*x)*a**6*x**6 + 3
*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**5*m*x**2 +
 48*atan(a*x)**2*int((x**m*sqrt(atan(a*x))*x**2)/(atan(a*x)*a**6*x**6 + 3*
atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**5*x**2 + 2*
atan(a*x)**2*int((x**m*sqrt(atan(a*x))*x**2)/(atan(a*x)*a**6*x**6 + 3*atan
(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**3*m**2 - 14*ata
n(a*x)**2*int((x**m*sqrt(atan(a*x))*x**2)/(atan(a*x)*a**6*x**6 + 3*atan(a*
x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**3*m + 24*atan(a*x)
**2*int((x**m*sqrt(atan(a*x))*x**2)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**
4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**3 + 2*atan(a*x)**2*int((
x**m*sqrt(atan(a*x)))/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*ata
n(a*x)*a**2*x**2 + atan(a*x)),x)*a**5*m**2*x**4 - 6*atan(a*x)**2*int((x...
```

# 3.1067 $\int \frac{x^3}{(c+a^2cx^2)^3 \arctan(ax)^{5/2}} dx$



## Optimal result

Integrand size = 24, antiderivative size = 160

$$\int \frac{x^3}{(c+a^2cx^2)^3 \arctan(ax)^{5/2}} dx = -\frac{2x^3}{3ac^3(1+a^2x^2)^2 \arctan(ax)^{3/2}} - \frac{4x^2}{a^2c^3(1+a^2x^2)^2 \sqrt{\arctan(ax)}} + \frac{4x^4}{3c^3(1+a^2x^2)^2 \sqrt{\arctan(ax)}} + \frac{4\sqrt{2\pi}\,\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{3a^4c^3} - \frac{4\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{3a^4c^3}$$

output
```
-2/3*x^3/a/c^3/(a^2*x^2+1)^2/arctan(a*x)^(3/2)-4*x^2/a^2/c^3/(a^2*x^2+1)^2
/arctan(a*x)^(1/2)+4/3*x^4/c^3/(a^2*x^2+1)^2/arctan(a*x)^(1/2)+4/3*2^(1/2)
*Pi^(1/2)*FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))/a^4/c^3-4/3*Pi^(1
/2)*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2))/a^4/c^3
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.42

$$\int \frac{x^3}{(c+a^2cx^2)^3 \arctan(ax)^{5/2}}\,dx = \frac{i\sqrt{2}(1+a^2x^2)^2\,(-i\arctan(ax))^{3/2}\Gamma\left(\frac{1}{2},-2i\arctan(ax)\right)+\sqrt{2}(1+a^2x^2}{}$$

input
```
Integrate[x^3/((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)),x]
```

output
```
(I*Sqrt[2]*(1 + a^2*x^2)^2*((-I)*ArcTan[a*x])^(3/2)*Gamma[1/2, (-2*I)*ArcT
an[a*x]] + Sqrt[2]*(1 + a^2*x^2)^2*Sqrt[I*ArcTan[a*x]]*ArcTan[a*x]*Gamma[1
/2, (2*I)*ArcTan[a*x]] - 2*(a^2*x^2*(a*x + (6 - 2*a^2*x^2)*ArcTan[a*x]) +
I*(1 + a^2*x^2)^2*((-I)*ArcTan[a*x])^(3/2)*Gamma[1/2, (-4*I)*ArcTan[a*x]]
+ (1 + a^2*x^2)^2*Sqrt[I*ArcTan[a*x]]*ArcTan[a*x]*Gamma[1/2, (4*I)*ArcTan[
a*x]]))/(3*a^4*c^3*(1 + a^2*x^2)^2*ArcTan[a*x]^(3/2))
```

## Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.86, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {5503, 27, 5477, 5503, 5505, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\arctan(ax)^{5/2}\,(a^2cx^2+c)^3}\,dx$$

$$\Big\downarrow \text{5503}$$

$$\frac{2\int \frac{x^2}{c^3(a^2x^2+1)^3\arctan(ax)^{3/2}}dx}{a}-\frac{2}{3}a\int \frac{x^4}{c^3\,(a^2x^2+1)^3\arctan(ax)^{3/2}}dx-\frac{2x^3}{3ac^3\,(a^2x^2+1)^2\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\int \frac{x^2}{(a^2x^2+1)^3\arctan(ax)^{3/2}}dx}{ac^3}-\frac{2a\int \frac{x^4}{(a^2x^2+1)^3\arctan(ax)^{3/2}}dx}{3c^3}-\frac{2x^3}{3ac^3\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$\downarrow$ 5477

$$\frac{2\int \frac{x^2}{(a^2x^2+1)^3\arctan(ax)^{3/2}}dx}{ac^3}-\frac{2a\left(\frac{8\int\frac{x^3}{\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{a}-\frac{2x^4}{a(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3c^3}-\frac{2x^3}{3ac^3\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$\downarrow$ 5503

$$\frac{2\left(\frac{4\int\frac{x}{\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{a}-4a\int\frac{x^3}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx-\frac{2x^2}{a(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{ac^3}-\frac{2a\left(\frac{8\int\frac{x^3}{\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{a}-\frac{2x^4}{a(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3c^3}-\frac{2x^3}{3ac^3\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$\downarrow$ 5505

$$\frac{2\left(\frac{4\int\frac{ax}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3}-\frac{4\int\frac{a^3x^3}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3}-\frac{2x^2}{a(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{ac^3}-\frac{2a\left(\frac{8\int\frac{a^3x^3}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{a^5}-\frac{2x^4}{a(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3c^3}-\frac{2x^3}{3ac^3\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$\downarrow$ 4906

$$-\frac{2a\left(\frac{8\int\left(\frac{\sin(2\arctan(ax))}{4\sqrt{\arctan(ax)}}-\frac{\sin(4\arctan(ax))}{8\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^5}-\frac{2x^4}{a(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3c^3}+\frac{2\left(-\frac{4\int\left(\frac{\sin(2\arctan(ax))}{4\sqrt{\arctan(ax)}}-\frac{\sin(4\arctan(ax))}{8\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^3}+\frac{4\int\left(\frac{\sin(2\arctan(ax))}{4\sqrt{\arctan(ax)}}+\frac{\sin(4\arctan(ax))}{8\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^3}-\frac{2x^2}{a(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{ac^3}$$

$$\frac{2x^3}{3ac^3\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2x^3}{3ac^3\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}-\frac{2a\left(\frac{8\left(\frac{1}{4}\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)-\frac{1}{8}\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{a^5}-\frac{2x^4}{a(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3c^3}+\frac{2\left(-\frac{4\left(\frac{1}{4}\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)-\frac{1}{8}\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{a^3}+\frac{4\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{4}\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\cdots}}{}\right.\right.}{a^3}\cdots}{ac^3}$$

input
```
Int[x^3/((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)),x]
```

output
```
(-2*x^3)/(3*a*c^3*(1 + a^2*x^2)^2*ArcTan[a*x]^(3/2)) - (2*a*((-2*x^4)/(a*(
1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]]) + (8*(-1/8*(Sqrt[Pi/2]*FresnelS[2*Sqrt[2
/Pi]*Sqrt[ArcTan[a*x]]]) + (Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[P
i]])/4))/a^5))/(3*c^3) + (2*((-2*x^2)/(a*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]]
) - (4*(-1/8*(Sqrt[Pi/2]*FresnelS[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]]) + (Sqrt
[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/4))/a^3 + (4*((Sqrt[Pi/2]*F
resnelS[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/8 + (Sqrt[Pi]*FresnelS[(2*Sqrt[Ar
cTan[a*x]])/Sqrt[Pi]])/4))/a^3))/(a*c^3)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5477
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*ArcT
an[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp[f*(m/(b*c*(p + 1)))   Int[(f*x
)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1
]
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

## Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.70

| method | result |
|---|---|
| default | $-\frac{-16\sqrt{2}\,\sqrt{\pi}\ \mathrm{FresnelS}\left(\frac{2\sqrt{2}\,\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)\arctan(ax)^{\frac{3}{2}}+16\sqrt{\pi}\ \mathrm{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)\arctan(ax)^{\frac{3}{2}}+8\arctan(ax)\cos(2\arctan(}{12a^4c^3\arctan(ax)^{\frac{3}{2}}}$ |

input
```
int(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12/a^4/c^3*(-16*2^(1/2)*Pi^(1/2)*FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(a*x
)^(1/2))*arctan(a*x)^(3/2)+16*Pi^(1/2)*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/
2))*arctan(a*x)^(3/2)+8*arctan(a*x)*cos(2*arctan(a*x))-8*arctan(a*x)*cos(4
*arctan(a*x))+2*sin(2*arctan(a*x))-sin(4*arctan(a*x)))/arctan(a*x)^(3/2)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^3 \arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^3 \arctan(ax)^{5/2}}\,dx = \frac{\int \frac{x^3}{a^6x^6 \operatorname{atan}^{\frac{5}{2}}(ax)+3a^4x^4 \operatorname{atan}^{\frac{5}{2}}(ax)+3a^2x^2 \operatorname{atan}^{\frac{5}{2}}(ax)+\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx}{c^3}$$

input
```
integrate(x**3/(a**2*c*x**2+c)**3/atan(a*x)**(5/2),x)
```

output
```
Integral(x**3/(a**6*x**6*atan(a*x)**(5/2) + 3*a**4*x**4*atan(a*x)**(5/2) +
 3*a**2*x**2*atan(a*x)**(5/2) + atan(a*x)**(5/2)), x)/c**3
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^3 \arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F]

$$\int \frac{x^3}{(c+a^2cx^2)^3 \arctan(ax)^{5/2}}\,dx = \int \frac{x^3}{(a^2cx^2+c)^3 \arctan(ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(x^3/((a^2*c*x^2 + c)^3*arctan(a*x)^(5/2)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(c+a^2cx^2)^3 \arctan(ax)^{5/2}}\,dx = \int \frac{x^3}{\mathrm{atan}(a\,x)^{5/2}\,(c\,a^2\,x^2+c)^3}\,dx$$

input
```
int(x^3/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^3),x)
```

output
```
int(x^3/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^3), x)
```

## Reduce [F]

$$\int \frac{x^3}{(c+a^2cx^2)^3 \arctan(ax)^{5/2}}\,dx = \frac{\int \frac{\sqrt{atan(ax)}\,x^3}{atan(ax)^3a^6x^6+3atan(ax)^3a^4x^4+3atan(ax)^3a^2x^2+atan(ax)^3}dx}{c^3}$$

input
```
int(x^3/(a^2*c*x^2+c)^3/atan(a*x)^(5/2),x)
```

output
```
int((sqrt(atan(a*x))*x**3)/(atan(a*x)**3*a**6*x**6 + 3*atan(a*x)**3*a**4*x
**4 + 3*atan(a*x)**3*a**2*x**2 + atan(a*x)**3),x)/c**3
```

# 3.1068 $\int \frac{x^2}{\left(c+a^2cx^2\right)^3 \arctan(ax)^{5/2}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 129

$$\int \frac{x^2}{(c+a^2cx^2)^3 \arctan(ax)^{5/2}}\,dx = -\frac{2x^2}{3ac^3(1+a^2x^2)^2\arctan(ax)^{3/2}} - \frac{8x}{3a^2c^3(1+a^2x^2)^2\sqrt{\arctan(ax)}} + \frac{8x^3}{3c^3(1+a^2x^2)^2\sqrt{\arctan(ax)}} + \frac{4\sqrt{2\pi}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{3a^3c^3}$$

output
```
-2/3*x^2/a/c^3/(a^2*x^2+1)^2/arctan(a*x)^(3/2)-8/3*x/a^2/c^3/(a^2*x^2+1)^2
/arctan(a*x)^(1/2)+8/3*x^3/c^3/(a^2*x^2+1)^2/arctan(a*x)^(1/2)+4/3*2^(1/2)
*Pi^(1/2)*FresnelC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))/a^3/c^3
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.70 (sec) , antiderivative size = 259, normalized size of antiderivative = 2.01

$$\int \frac{x^2}{(c+a^2cx^2)^3 \arctan(ax)^{5/2}}\,dx = \frac{\frac{2\sqrt{2\pi}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{a^3} - \frac{16\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{a^3} + \frac{-\frac{8x^2}{a(1+a^2x^2)^2} + 32x \cdots}{\cdots}}{\cdots}$$

input
```
Integrate[x^2/((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)),x]
```

output
```
((2*Sqrt[2*Pi]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/a^3 - (16*Sqrt[Pi
]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/a^3 + ((-8*x^2)/(a*(1 + a^2*x^
2)^2) + (32*x^3*ArcTan[a*x])/(1 + a^2*x^2)^2 - (32*x*ArcTan[a*x])/(a + a^3
*x^2)^2 + (4*Sqrt[2]*((-I)*ArcTan[a*x])^(3/2)*Gamma[1/2, (-2*I)*ArcTan[a*x
]])/a^3 + (4*Sqrt[2]*(I*ArcTan[a*x])^(3/2)*Gamma[1/2, (2*I)*ArcTan[a*x]])/
a^3 + (7*((-I)*ArcTan[a*x])^(3/2)*Gamma[1/2, (-4*I)*ArcTan[a*x]])/a^3 + (7
*(I*ArcTan[a*x])^(3/2)*Gamma[1/2, (4*I)*ArcTan[a*x]])/a^3)/ArcTan[a*x]^(3/
2))/(12*c^3)
```

## Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 359 vs. 2(129) = 258.

Time = 2.32 (sec) , antiderivative size = 359, normalized size of antiderivative = 2.78, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.542$, Rules used = {5503, 27, 5503, 5439, 3042, 3793, 2009, 5505, 3042, 3793, 2009, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\arctan(ax)^{5/2}(a^2cx^2+c)^3}\,dx$$

$$\Big\downarrow \text{ 5503}$$

$$\frac{4\int \frac{x}{c^3(a^2x^2+1)^3\arctan(ax)^{3/2}}dx}{3a}-\frac{4}{3}a\int \frac{x^3}{c^3(a^2x^2+1)^3\arctan(ax)^{3/2}}dx-\frac{2x^2}{3ac^3(a^2x^2+1)^2\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{4\int \frac{x}{(a^2x^2+1)^3\arctan(ax)^{3/2}}dx}{3ac^3}-\frac{4a\int \frac{x^3}{(a^2x^2+1)^3\arctan(ax)^{3/2}}dx}{3c^3}-\frac{2x^2}{3ac^3(a^2x^2+1)^2\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{5503}$$

$$\frac{4\left(\frac{2\int \frac{1}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{a}-6a\int \frac{x^2}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx-\frac{2x}{a(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3ac^3}-\frac{4a\left(\frac{6\int \frac{x^2}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{a}-2a\int \frac{x^4}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx-\frac{2x^3}{a(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3c^3}-\frac{2x^2}{3ac^3(a^2x^2+1)^2\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{5439}$$

$$\frac{4\left(-6a\int \frac{x^2}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx+\frac{2\int \frac{1}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{a^2}-\frac{2x}{a(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3ac^3}-\frac{4a\left(\frac{6\int \frac{x^2}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{a}-2a\int \frac{x^4}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx-\frac{2x^3}{a(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3c^3}-\frac{2x^2}{3ac^3(a^2x^2+1)^2\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4\left(-6a\int \frac{x^2}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx+\frac{2\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^4}{\sqrt{\arctan(ax)}}d\arctan(ax)}{a^2}-\frac{2x}{a(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3ac^3}-\frac{4a\left(\frac{6\int \frac{x^2}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{a}-2a\int \frac{x^4}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx-\frac{2x^3}{a(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3c^3}-\frac{2x^2}{3ac^3\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$\Big\downarrow$ 3793

$$\frac{4\left(-6a\int \frac{x^2}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx+\frac{2\int \left(\frac{\cos(2\arctan(ax))}{2\sqrt{\arctan(ax)}}+\frac{\cos(4\arctan(ax))}{8\sqrt{\arctan(ax)}}+\frac{3}{8\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^2}-\frac{2x}{a(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3ac^3}-\frac{4a\left(\frac{6\int \frac{x^2}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{a}-2a\int \frac{x^4}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx-\frac{2x^3}{a(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3c^3}-\frac{2x^2}{3ac^3\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$\Big\downarrow$ 2009

$$\frac{4\left(-6a\int \frac{x^2}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx+\frac{2\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\arctan(ax)}\right)}{a^2}-\frac{2x}{a(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3ac^3}-\frac{4a\left(\frac{6\int \frac{x^2}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx}{a}-2a\int \frac{x^4}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx-\frac{2x^3}{a(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3c^3}-\frac{2x^2}{3ac^3\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$\Big\downarrow$ 5505

$$\frac{4\left(-\frac{6\int \frac{a^2x^2}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{a^2}+\frac{2\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\arctan(ax)}\right)}{a^2}-\right.}{3ac^3}$$

$$\frac{4a\left(\frac{6\int \frac{a^2x^2}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{a^4}-\frac{2\int \frac{a^4x^4}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{a^4}-\frac{2x^3}{a(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3c^3}-\frac{2x^2}{3ac^3\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{4\left(-\frac{6\int \frac{a^2x^2}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{a^2}+\frac{2\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\arctan(ax)}\right)}{a^2}-\right.}{3ac^3}$$

$$\frac{4a\left(-\frac{2\int \frac{\sin(\arctan(ax))^4}{\sqrt{\arctan(ax)}}d\arctan(ax)}{a^4}+\frac{6\int \frac{a^2x^2}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{a^4}-\frac{2x^3}{a(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3c^3}-\frac{2x^2}{3ac^3\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{ 3793}$$

$$\frac{4\left(-\frac{6\int \frac{a^2x^2}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{a^2}+\frac{2\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\arctan(ax)}\right)}{a^2}-\right.}{3ac^3}$$

$$\frac{4a\left(-\frac{2\int \left(-\frac{\cos(2\arctan(ax))}{2\sqrt{\arctan(ax)}}+\frac{\cos(4\arctan(ax))}{8\sqrt{\arctan(ax)}}+\frac{3}{8\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^4}+\frac{6\int \frac{a^2x^2}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{a^4}-\frac{2x^3}{a(a^2x^2+1)^2\sqrt{\text{arct}}}\right.}{3c^3}$$

$$\frac{2x^2}{3ac^3\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{4\left(-\frac{6\int \frac{a^2x^2}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{a^2}+\frac{2\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\arctan(ax)}\right)}{a^2}-\right.}{3ac^3}$$

$$\frac{4a\left(\frac{6\int \frac{a^2x^2}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{a^4}-\frac{2\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{2}\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\arctan(ax)}\right)}{a^4}-\right.}{3c^3}$$

$$\frac{2x^2}{3ac^3\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{4906}$$

$$\frac{4\left(-\frac{6\int \left(\frac{1}{8\sqrt{\arctan(ax)}}-\frac{\cos(4\arctan(ax))}{8\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^2}+\frac{2\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\text{arcta}}\right.}{a^2}\right.}{3ac^3}$$

$$\frac{4a\left(\frac{6\int \left(\frac{1}{8\sqrt{\arctan(ax)}}-\frac{\cos(4\arctan(ax))}{8\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^4}-\frac{2\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{2}\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\text{arcta}}\right.}{a^4}\right.}{3c^3}$$

$$\frac{2x^2}{3ac^3\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{4\left(-\frac{6\left(\frac{1}{4}\sqrt{\arctan(ax)}-\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{a^2}+\frac{2\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\right.}{a^2}\right.}{3ac^3}$$

$$\frac{2x^2}{3ac^3\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}-$$

$$\frac{4a\left(\frac{6\left(\frac{1}{4}\sqrt{\arctan(ax)}-\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{a^4}-\frac{2\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{2}\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\right.}{a^4}\right.}{3c^3}$$

input
```
Int[x^2/((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)),x]
```

output
```
(-2*x^2)/(3*a*c^3*(1 + a^2*x^2)^2*ArcTan[a*x]^(3/2)) - (4*a*((-2*x^3)/(a*(
1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]]) + (6*(Sqrt[ArcTan[a*x]]/4 - (Sqrt[Pi/2]*
FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/8))/a^4 - (2*((3*Sqrt[ArcTan[a*x
]])/4 + (Sqrt[Pi/2]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/8 - (Sqrt[Pi
]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/2))/a^4))/(3*c^3) + (4*((-2*x)
/(a*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]]) - (6*(Sqrt[ArcTan[a*x]]/4 - (Sqrt[P
i/2]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/8))/a^2 + (2*((3*Sqrt[ArcTa
n[a*x]])/4 + (Sqrt[Pi/2]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/8 + (Sq
rt[Pi]*FresnelC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/2))/a^2))/(3*a*c^3)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

## Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.53

| method | result | size |
|---|---|---|
| default | $-\frac{-16\sqrt{2}\,\sqrt{\pi}\,\operatorname{FresnelC}\left(\frac{2\sqrt{2}\,\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)\arctan(ax)^{\frac{3}{2}}+8\arctan(ax)\sin(4\arctan(ax))-\cos(4\arctan(ax))+1}{12a^{3}c^{3}\arctan(ax)^{\frac{3}{2}}}$ | 68 |

input
```
int(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12/a^3/c^3*(-16*2^(1/2)*Pi^(1/2)*FresnelC(2*2^(1/2)/Pi^(1/2)*arctan(a*x
)^(1/2))*arctan(a*x)^(3/2)+8*arctan(a*x)*sin(4*arctan(a*x))-cos(4*arctan(a
*x))+1)/arctan(a*x)^(3/2)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2}{\left(c+a^2cx^2\right)^3\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{x^2}{\left(c+a^2cx^2\right)^3\arctan(ax)^{5/2}}\,dx = \frac{\int \frac{x^2}{a^6x^6\operatorname{atan}^{\frac{5}{2}}(ax)+3a^4x^4\operatorname{atan}^{\frac{5}{2}}(ax)+3a^2x^2\operatorname{atan}^{\frac{5}{2}}(ax)+\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx}{c^3}$$

input
```
integrate(x**2/(a**2*c*x**2+c)**3/atan(a*x)**(5/2),x)
```

output
```
Integral(x**2/(a**6*x**6*atan(a*x)**(5/2) + 3*a**4*x**4*atan(a*x)**(5/2) +
 3*a**2*x**2*atan(a*x)**(5/2) + atan(a*x)**(5/2)), x)/c**3
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{\left(c+a^2cx^2\right)^3\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F]

$$\int \frac{x^2}{(c+a^2cx^2)^3 \arctan(ax)^{5/2}}\,dx = \int \frac{x^2}{(a^2cx^2+c)^3 \arctan(ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(x^2/((a^2*c*x^2 + c)^3*arctan(a*x)^(5/2)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(c+a^2cx^2)^3 \arctan(ax)^{5/2}}\,dx = \int \frac{x^2}{\mathrm{atan}(a\,x)^{5/2}\,(c\,a^2\,x^2+c)^3}\,dx$$

input
```
int(x^2/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^3),x)
```

output
```
int(x^2/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^3), x)
```

## Reduce [F]

$$\int \frac{x^2}{(c+a^2cx^2)^3 \arctan(ax)^{5/2}}\,dx = \frac{\int \frac{\sqrt{atan(ax)}\,x^2}{atan(ax)^3a^6x^6+3atan(ax)^3a^4x^4+3atan(ax)^3a^2x^2+atan(ax)^3}dx}{c^3}$$

input
```
int(x^2/(a^2*c*x^2+c)^3/atan(a*x)^(5/2),x)
```

output
```
int((sqrt(atan(a*x))*x**2)/(atan(a*x)**3*a**6*x**6 + 3*atan(a*x)**3*a**4*x
**4 + 3*atan(a*x)**3*a**2*x**2 + atan(a*x)**3),x)/c**3
```

# 3.1069 $\int \frac{x}{\left(c+a^2cx^2\right)^3 \arctan(ax)^{5/2}}\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 155

$$\int \frac{x}{\left(c+a^2cx^2\right)^3 \arctan(ax)^{5/2}}\,dx = -\frac{2x}{3ac^3\left(1+a^2x^2\right)^2\arctan(ax)^{3/2}} - \frac{4}{3a^2c^3\left(1+a^2x^2\right)^2\sqrt{\arctan(ax)}} + \frac{4x^2}{c^3\left(1+a^2x^2\right)^2\sqrt{\arctan(ax)}} - \frac{4\sqrt{2\pi}\,\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{3a^2c^3} - \frac{4\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{3a^2c^3}$$

output

```
-2/3*x/a/c^3/(a^2*x^2+1)^2/arctan(a*x)^(3/2)-4/3/a^2/c^3/(a^2*x^2+1)^2/arc
tan(a*x)^(1/2)+4*x^2/c^3/(a^2*x^2+1)^2/arctan(a*x)^(1/2)-4/3*2^(1/2)*Pi^(1
/2)*FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))/a^2/c^3-4/3*Pi^(1/2)*Fr
esnelS(2*arctan(a*x)^(1/2)/Pi^(1/2))/a^2/c^3
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.42

$$\int \frac{x}{(c+a^2cx^2)^3\arctan(ax)^{5/2}}\,dx = \frac{i\sqrt{2}(1+a^2x^2)^2\,(-i\arctan(ax))^{3/2}\Gamma\left(\frac{1}{2},-2i\arctan(ax)\right)+\sqrt{2}(1+a^2x^2 \cdots}{\cdots}$$

input
```
Integrate[x/((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)),x]
```

output
```
(I*Sqrt[2]*(1 + a^2*x^2)^2*((-I)*ArcTan[a*x])^(3/2)*Gamma[1/2, (-2*I)*ArcT
an[a*x]] + Sqrt[2]*(1 + a^2*x^2)^2*Sqrt[I*ArcTan[a*x]]*ArcTan[a*x]*Gamma[1
/2, (2*I)*ArcTan[a*x]] + 2*(-(a*x) - 2*ArcTan[a*x] + 6*a^2*x^2*ArcTan[a*x]
 + I*(1 + a^2*x^2)^2*((-I)*ArcTan[a*x])^(3/2)*Gamma[1/2, (-4*I)*ArcTan[a*x
]] + (1 + a^2*x^2)^2*Sqrt[I*ArcTan[a*x]]*ArcTan[a*x]*Gamma[1/2, (4*I)*ArcT
an[a*x]]))/(3*c^3*(a + a^3*x^2)^2*ArcTan[a*x]^(3/2))
```

## Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.88, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {5503, 27, 5437, 5503, 5505, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\arctan(ax)^{5/2}\,(a^2cx^2+c)^3}\,dx$$

$$\Big\downarrow \; 5503$$

$$\frac{2\int \frac{1}{c^3(a^2x^2+1)^3\arctan(ax)^{3/2}}dx}{3a} - 2a\int \frac{x^2}{c^3\,(a^2x^2+1)^3\arctan(ax)^{3/2}}dx - \frac{2x}{3ac^3\,(a^2x^2+1)^2\arctan(ax)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{2\int \frac{1}{(a^2x^2+1)^3 \arctan(ax)^{3/2}}dx}{3ac^3} - \frac{2a\int \frac{x^2}{(a^2x^2+1)^3 \arctan(ax)^{3/2}}dx}{c^3} - \frac{2x}{3ac^3\left(a^2x^2+1\right)^2 \arctan(ax)^{3/2}}$$

$$\Big\downarrow \ 5437$$

$$-\frac{2a\int \frac{x^2}{(a^2x^2+1)^3 \arctan(ax)^{3/2}}dx}{c^3} + \frac{2\left(-8a\int \frac{x}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx - \frac{2}{a(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3ac^3} - \frac{2x}{3ac^3\left(a^2x^2+1\right)^2 \arctan(ax)^{3/2}}$$

$$\Big\downarrow \ 5503$$

$$\frac{2\left(-8a\int \frac{x}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx - \frac{2}{a(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3ac^3} - \frac{2a\left(\frac{4\int \frac{x}{\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{a} - 4a\int \frac{x^3}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx - \frac{2x^2}{a(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{c^3} - \frac{2x}{3ac^3\left(a^2x^2+1\right)^2 \arctan(ax)^{3/2}}$$

$$\Big\downarrow \ 5505$$

$$\frac{2\left(-\frac{8\int \frac{ax}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{a} - \frac{2}{a(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3ac^3} - \frac{2a\left(\frac{4\int \frac{ax}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3} - \frac{4\int \frac{a^3x^3}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3} - \frac{2x^2}{a(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{c^3} - \frac{2x}{3ac^3\left(a^2x^2+1\right)^2 \arctan(ax)^{3/2}}$$

$$\Big\downarrow \ 4906$$

$$\frac{2\left(-\frac{8\int \left(\frac{\sin(2\arctan(ax))}{4\sqrt{\arctan(ax)}} + \frac{\sin(4\arctan(ax))}{8\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a} - \frac{2}{a(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3ac^3} - \frac{2a\left(-\frac{4\int \left(\frac{\sin(2\arctan(ax))}{4\sqrt{\arctan(ax)}} - \frac{\sin(4\arctan(ax))}{8\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^3} + \frac{4\int \left(\frac{\sin(2\arctan(ax))}{4\sqrt{\arctan(ax)}} + \frac{\sin(4\arctan(ax))}{8\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^3} - \frac{2x^2}{a(a^2x^2+1)^2\sqrt{\arctan(}}\right.}{c^3} - \frac{2x}{3ac^3\left(a^2x^2+1\right)^2 \arctan(ax)^{3/2}}$$

$$\Big\downarrow \ 2009$$

$$\frac{2\left(-\frac{2}{a(a^2x^2+1)^2\sqrt{\arctan(ax)}}-\frac{8\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{4}\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)\right)}{a}\right)}{3ac^3}-\frac{2x}{3ac^3\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}-\frac{2a\left(-\frac{4\left(\frac{1}{4}\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)-\frac{1}{8}\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{a^3}+\frac{4\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{4}\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\right.}{}\right.}{a^3}\right.}{c^3}$$

input
```
Int[x/((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)),x]
```

output
```
(-2*x)/(3*a*c^3*(1 + a^2*x^2)^2*ArcTan[a*x]^(3/2)) - (2*a*((-2*x^2)/(a*(1
+ a^2*x^2)^2*Sqrt[ArcTan[a*x]]) - (4*(-1/8*(Sqrt[Pi/2]*FresnelS[2*Sqrt[2/P
i]*Sqrt[ArcTan[a*x]]]) + (Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]
])/4))/a^3 + (4*((Sqrt[Pi/2]*FresnelS[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/8 +
 (Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/4))/a^3))/c^3 + (2*(-
2/(a*(1 + a^2*x^2)^2*Sqrt[ArcTan[a*x]]) - (8*((Sqrt[Pi/2]*FresnelS[2*Sqrt[
2/Pi]*Sqrt[ArcTan[a*x]]])/8 + (Sqrt[Pi]*FresnelS[(2*Sqrt[ArcTan[a*x]])/Sqr
t[Pi]])/4))/a))/(3*a*c^3)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

## Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.71

| method | result |
|---|---|
| default | $-\frac{16\sqrt{2}\,\sqrt{\pi}\,\mathrm{FresnelS}\left(\frac{2\sqrt{2}\,\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)\arctan(ax)^{\frac{3}{2}}+16\sqrt{\pi}\,\mathrm{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)\arctan(ax)^{\frac{3}{2}}+8\arctan(ax)\cos(2\arctan(a}{12a^2c^3\arctan(ax)^{\frac{3}{2}}}$ |

input
```
int(x/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12/a^2/c^3*(16*2^(1/2)*Pi^(1/2)*FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(a*x)
^(1/2))*arctan(a*x)^(3/2)+16*Pi^(1/2)*FresnelS(2*arctan(a*x)^(1/2)/Pi^(1/2
))*arctan(a*x)^(3/2)+8*arctan(a*x)*cos(2*arctan(a*x))+8*arctan(a*x)*cos(4*
arctan(a*x))+2*sin(2*arctan(a*x))+sin(4*arctan(a*x)))/arctan(a*x)^(3/2)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)^3 \arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{x}{(c+a^2cx^2)^3 \arctan(ax)^{5/2}}\,dx = \frac{\int \frac{x}{a^6x^6 \operatorname{atan}^{\frac{5}{2}}(ax)+3a^4x^4 \operatorname{atan}^{\frac{5}{2}}(ax)+3a^2x^2 \operatorname{atan}^{\frac{5}{2}}(ax)+\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx}{c^3}$$

input
```
integrate(x/(a**2*c*x**2+c)**3/atan(a*x)**(5/2),x)
```

output
```
Integral(x/(a**6*x**6*atan(a*x)**(5/2) + 3*a**4*x**4*atan(a*x)**(5/2) + 3*
a**2*x**2*atan(a*x)**(5/2) + atan(a*x)**(5/2)), x)/c**3
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)^3 \arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F]

$$\int \frac{x}{(c+a^2cx^2)^3 \arctan(ax)^{5/2}}\,dx = \int \frac{x}{(a^2cx^2+c)^3 \arctan(ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(x/((a^2*c*x^2 + c)^3*arctan(a*x)^(5/2)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(c+a^2cx^2)^3 \arctan(ax)^{5/2}}\,dx = \int \frac{x}{\operatorname{atan}(a\,x)^{5/2}\,(c\,a^2\,x^2+c)^3}\,dx$$

input
```
int(x/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^3),x)
```

output
```
int(x/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^3), x)
```

## Reduce [F]

$$\int \frac{x}{(c+a^2cx^2)^3 \arctan(ax)^{5/2}}\,dx = \text{Too large to display}$$

input
```
int(x/(a^2*c*x^2+c)^3/atan(a*x)^(5/2),x)
```

output
```
(2*( - 32*sqrt(atan(a*x))*atan(a*x)**3*a**4*x**4 - 64*sqrt(atan(a*x))*atan
(a*x)**3*a**2*x**2 - 32*sqrt(atan(a*x))*atan(a*x)**3 - 48*sqrt(atan(a*x))*
atan(a*x)**2*a**3*x**3 - 80*sqrt(atan(a*x))*atan(a*x)**2*a*x + 128*atan(a*
x)**2*int(sqrt(atan(a*x))/(a**6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1),x)*a
**5*x**4 + 256*atan(a*x)**2*int(sqrt(atan(a*x))/(a**6*x**6 + 3*a**4*x**4 +
 3*a**2*x**2 + 1),x)*a**3*x**2 + 128*atan(a*x)**2*int(sqrt(atan(a*x))/(a**
6*x**6 + 3*a**4*x**4 + 3*a**2*x**2 + 1),x)*a + 24*atan(a*x)**2*int((sqrt(a
tan(a*x))*x**3)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)
*a**2*x**2 + atan(a*x)),x)*a**8*x**4 + 48*atan(a*x)**2*int((sqrt(atan(a*x)
)*x**3)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x*
*2 + atan(a*x)),x)*a**6*x**2 + 24*atan(a*x)**2*int((sqrt(atan(a*x))*x**3)/
(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + ata
n(a*x)),x)*a**4 - 15*atan(a*x)**2*int((sqrt(atan(a*x))*x**2)/(atan(a*x)**2
*a**6*x**6 + 3*atan(a*x)**2*a**4*x**4 + 3*atan(a*x)**2*a**2*x**2 + atan(a*
x)**2),x)*a**7*x**4 - 30*atan(a*x)**2*int((sqrt(atan(a*x))*x**2)/(atan(a*x
)**2*a**6*x**6 + 3*atan(a*x)**2*a**4*x**4 + 3*atan(a*x)**2*a**2*x**2 + ata
n(a*x)**2),x)*a**5*x**2 - 15*atan(a*x)**2*int((sqrt(atan(a*x))*x**2)/(atan
(a*x)**2*a**6*x**6 + 3*atan(a*x)**2*a**4*x**4 + 3*atan(a*x)**2*a**2*x**2 +
 atan(a*x)**2),x)*a**3 - 10*sqrt(atan(a*x))*atan(a*x) - 5*sqrt(atan(a*x))*
a*x))/(15*atan(a*x)**2*a**2*c**3*(a**4*x**4 + 2*a**2*x**2 + 1))
```

# 3.1070 $\int \frac{1}{(c+a^2cx^2)^3 \arctan(ax)^{5/2}}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 125

$$\int \frac{1}{(c+a^2cx^2)^3 \arctan(ax)^{5/2}}\,dx = -\frac{2}{3ac^3(1+a^2x^2)^2\arctan(ax)^{3/2}}+\frac{16x}{3c^3(1+a^2x^2)^2\sqrt{\arctan(ax)}} - \frac{4\sqrt{2\pi}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{3ac^3} - \frac{8\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{3ac^3}$$

output
```
-2/3/a/c^3/(a^2*x^2+1)^2/arctan(a*x)^(3/2)+16/3*x/c^3/(a^2*x^2+1)^2/arctan
(a*x)^(1/2)-4/3*2^(1/2)*Pi^(1/2)*FresnelC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(
1/2))/a/c^3-8/3*Pi^(1/2)*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))/a/c^3
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.67 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.49

$$\int \frac{1}{(c+a^2cx^2)^3 \arctan(ax)^{5/2}}\,dx = \frac{2\left(-\frac{1}{a(1+a^2x^2)^2}+\frac{8x\arctan(ax)}{(1+a^2x^2)^2}-\frac{\sqrt{2}(-i\arctan(ax))^{3/2}\Gamma\left(\frac{1}{2},-2i\arctan(ax)\right)}{a}+\frac{\sqrt{2}\,\text{arct}}{}\right.}{3c^3}$$

input
```
Integrate[1/((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)),x]
```

output
```
(2*(-(1/(a*(1 + a^2*x^2)^2)) + (8*x*ArcTan[a*x])/(1 + a^2*x^2)^2 - (Sqrt[2
]*((-I)*ArcTan[a*x])^(3/2)*Gamma[1/2, (-2*I)*ArcTan[a*x]])/a + (Sqrt[2]*Ar
cTan[a*x]^2*Gamma[1/2, (2*I)*ArcTan[a*x]])/(a*Sqrt[I*ArcTan[a*x]]) - (((-I
)*ArcTan[a*x])^(3/2)*Gamma[1/2, (-4*I)*ArcTan[a*x]])/a + (ArcTan[a*x]^2*Ga
mma[1/2, (4*I)*ArcTan[a*x]])/(a*Sqrt[I*ArcTan[a*x]])))/(3*c^3*ArcTan[a*x]^
(3/2))
```

## Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.53, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {5437, 27, 5503, 5439, 3042, 3793, 2009, 5505, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arctan(ax)^{5/2} \left(a^2cx^2+c\right)^3}dx$$

$$\Big\downarrow \text{ 5437}$$

$$-\frac{8}{3}a\int \frac{x}{c^3 \left(a^2x^2+1\right)^3 \arctan(ax)^{3/2}}dx-\frac{2}{3ac^3 \left(a^2x^2+1\right)^2 \arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{8a\int \frac{x}{\left(a^2x^2+1\right)^3 \arctan(ax)^{3/2}}dx}{3c^3}-\frac{2}{3ac^3 \left(a^2x^2+1\right)^2 \arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{ 5503}$$

$$-\frac{8a\left(\frac{2\int \frac{1}{\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{a}-6a\int \frac{x^2}{\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx-\frac{2x}{a\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}\right)}{3c^3}-\frac{2}{3ac^3 \left(a^2x^2+1\right)^2 \arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{ 5439}$$

$$-\frac{8a\left(-6a\int \frac{x^2}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx+\frac{2\int \frac{1}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{a^2}-\frac{2x}{a(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3c^3}-\frac{2}{3ac^3\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$\big\downarrow$ 3042

$$-\frac{8a\left(-6a\int \frac{x^2}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx+\frac{2\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^4}{\sqrt{\arctan(ax)}}d\arctan(ax)}{a^2}-\frac{2x}{a(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3c^3}-\frac{2}{3ac^3\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$\big\downarrow$ 3793

$$-\frac{8a\left(-6a\int \frac{x^2}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx+\frac{2\int \left(\frac{\cos(2\arctan(ax))}{2\sqrt{\arctan(ax)}}+\frac{\cos(4\arctan(ax))}{8\sqrt{\arctan(ax)}}+\frac{3}{8\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^2}-\frac{2x}{a(a^2x^2+1)^2\sqrt{\arctan(a}}\right.}{3c^3}$$

$$\frac{2}{3ac^3\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$\big\downarrow$ 2009

$$-\frac{8a\left(-6a\int \frac{x^2}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx+\frac{2\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\pi}\operatorname{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\arctan(ax)}\right)}{a^2}-\right.}{3c^3}$$

$$\frac{2}{3ac^3\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$\big\downarrow$ 5505

$$-\frac{8a\left(-\frac{6\int \frac{a^2x^2}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{a^2}+\frac{2\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\pi}\operatorname{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{\arctan(ax)}\right.}{a^2}\right.}{3c^3}$$

$$\frac{2}{3ac^3\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$\big\downarrow$ 4906

$$-\frac{8a\left(-\frac{6\int\left(\frac{1}{8\sqrt{\arctan(ax)}}-\frac{\cos(4\arctan(ax))}{8\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^2}+\frac{2\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)+\frac{3}{4}\sqrt{a}\right.}{a^2}\right.}{3c^3}$$
$$\frac{2}{3ac^3\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$\big\downarrow$ 2009

$$-\frac{8a\left(-\frac{6\left(\frac{1}{4}\sqrt{\arctan(ax)}-\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{a^2}+\frac{2\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right.\right.}{a^2}\right.}{3c^3}$$
$$\frac{2}{3ac^3\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

input
```
Int[1/((c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)),x]
```

output
```
-2/(3*a*c^3*(1 + a^2*x^2)^2*ArcTan[a*x]^(3/2)) - (8*a*((-2*x)/(a*(1 + a^2*
x^2)^2*Sqrt[ArcTan[a*x]]) - (6*(Sqrt[ArcTan[a*x]]/4 - (Sqrt[Pi/2]*FresnelC
[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/8))/a^2 + (2*((3*Sqrt[ArcTan[a*x]])/4 +
(Sqrt[Pi/2]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/8 + (Sqrt[Pi]*Fresne
lC[(2*Sqrt[ArcTan[a*x]])/Sqrt[Pi]])/2))/a^2))/(3*c^3)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

## Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.90

| method | result |
|---|---|
| default | $\frac{-16\sqrt{2}\,\sqrt{\pi}\ \mathrm{FresnelC}\left(\frac{2\sqrt{2}\,\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)\arctan(ax)^{\frac{3}{2}}-32\sqrt{\pi}\ \mathrm{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)\arctan(ax)^{\frac{3}{2}}+16\arctan(ax)\sin(2\arctan(}{12a\,c^3\arctan(ax)^{\frac{3}{2}}}$ |

input
```
int(1/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/12/a/c^3*(-16*2^(1/2)*Pi^(1/2)*FresnelC(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(
1/2))*arctan(a*x)^(3/2)-32*Pi^(1/2)*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))
*arctan(a*x)^(3/2)+16*arctan(a*x)*sin(2*arctan(a*x))+8*arctan(a*x)*sin(4*a
rctan(a*x))-4*cos(2*arctan(a*x))-cos(4*arctan(a*x))-3)/arctan(a*x)^(3/2)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(c+a^2cx^2)^3\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F]

$$\int \frac{1}{(c+a^2cx^2)^3\arctan(ax)^{5/2}}\,dx = \frac{\int \frac{1}{a^6x^6\operatorname{atan}^{\frac{5}{2}}(ax)+3a^4x^4\operatorname{atan}^{\frac{5}{2}}(ax)+3a^2x^2\operatorname{atan}^{\frac{5}{2}}(ax)+\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx}{c^3}$$

input
```
integrate(1/(a**2*c*x**2+c)**3/atan(a*x)**(5/2),x)
```

output
```
Integral(1/(a**6*x**6*atan(a*x)**(5/2) + 3*a**4*x**4*atan(a*x)**(5/2) + 3*
a**2*x**2*atan(a*x)**(5/2) + atan(a*x)**(5/2)), x)/c**3
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(c+a^2cx^2)^3 \arctan(ax)^{5/2}} dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F]

$$\int \frac{1}{(c+a^2cx^2)^3 \arctan(ax)^{5/2}} dx = \int \frac{1}{(a^2cx^2+c)^3 \arctan(ax)^{\frac{5}{2}}} dx$$

input
```
integrate(1/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^3*arctan(a*x)^(5/2)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+a^2cx^2)^3 \arctan(ax)^{5/2}} dx = \int \frac{1}{\operatorname{atan}(a\,x)^{5/2}\,(c\,a^2\,x^2+c)^3} dx$$

input
```
int(1/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^3),x)
```

output
```
int(1/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^3), x)
```

## Reduce [F]

$$\int \frac{1}{(c+a^2cx^2)^3 \arctan(ax)^{5/2}}\,dx = \text{Too large to display}$$

input
```
int(1/(a^2*c*x^2+c)^3/atan(a*x)^(5/2),x)
```

output
```
(2*(48*sqrt(atan(a*x))*atan(a*x)**2*a**4*x**4 + 96*sqrt(atan(a*x))*atan(a*
x)**2*a**2*x**2 + 48*sqrt(atan(a*x))*atan(a*x)**2 - 64*atan(a*x)**2*int(sq
rt(atan(a*x))/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a
**2*x**2 + atan(a*x)),x)*a**5*x**4 - 128*atan(a*x)**2*int(sqrt(atan(a*x))/
(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + ata
n(a*x)),x)*a**3*x**2 - 64*atan(a*x)**2*int(sqrt(atan(a*x))/(atan(a*x)*a**6
*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a +
12*atan(a*x)**2*int((sqrt(atan(a*x))*x**3)/(atan(a*x)**2*a**6*x**6 + 3*ata
n(a*x)**2*a**4*x**4 + 3*atan(a*x)**2*a**2*x**2 + atan(a*x)**2),x)*a**8*x**
4 + 24*atan(a*x)**2*int((sqrt(atan(a*x))*x**3)/(atan(a*x)**2*a**6*x**6 + 3
*atan(a*x)**2*a**4*x**4 + 3*atan(a*x)**2*a**2*x**2 + atan(a*x)**2),x)*a**6
*x**2 + 12*atan(a*x)**2*int((sqrt(atan(a*x))*x**3)/(atan(a*x)**2*a**6*x**6
 + 3*atan(a*x)**2*a**4*x**4 + 3*atan(a*x)**2*a**2*x**2 + atan(a*x)**2),x)*
a**4 + 24*sqrt(atan(a*x))*atan(a*x)*a**3*x**3 + 40*sqrt(atan(a*x))*atan(a*
x)*a*x - 5*sqrt(atan(a*x))))/(15*atan(a*x)**2*a*c**3*(a**4*x**4 + 2*a**2*x
**2 + 1))
```

# 3.1071 $\int \frac{1}{x(c+a^2cx^2)^3 \arctan(ax)^{5/2}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x(c+a^2cx^2)^3 \arctan(ax)^{5/2}}\,dx = -\frac{2}{3ac^3x(1+a^2x^2)^2\arctan(ax)^{3/2}} + \frac{20}{3c^3(1+a^2x^2)^2\sqrt{\arctan(ax)}} + \frac{4}{3a^2c^3x^2(1+a^2x^2)^2\sqrt{\arctan(ax)}} + \frac{5\sqrt{2\pi}\,\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{3c^3} + \frac{20\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{3c^3} + \frac{8\text{Int}\left(\frac{1}{x^3(c+a^2cx^2)^3\sqrt{\arctan(ax)}},x\right)}{3a^2} + 8\text{Int}\left(\frac{1}{x(c+a^2cx^2)^3\sqrt{\arctan(ax)}},x\right)$$

output

```
-2/3/a/c^3/x/(a^2*x^2+1)^2/arctan(a*x)^(3/2)+20/3/c^3/(a^2*x^2+1)^2/arctan
(a*x)^(1/2)+4/3/a^2/c^3/x^2/(a^2*x^2+1)^2/arctan(a*x)^(1/2)+5/3*2^(1/2)*Pi
^(1/2)*FresnelS(2*2^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))/c^3+20/3*Pi^(1/2)*Fr
esnelS(2*arctan(a*x)^(1/2)/Pi^(1/2))/c^3+8/3*Defer(Int)(1/x^3/(a^2*c*x^2+c
)^3/arctan(a*x)^(1/2),x)/a^2+8*Defer(Int)(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^
(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 3.98 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\arctan(ax)^{5/2}}\,dx=\int \frac{1}{x\left(c+a^2cx^2\right)^3\arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[1/(x*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)),x]
```

output
```
Integrate[1/(x*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 1.39 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^3}dx$$

$$\Big\downarrow \text{5503}$$

$$-\frac{10}{3}a\int \frac{1}{c^3\left(a^2x^2+1\right)^3\arctan(ax)^{3/2}}dx-\frac{2\int \frac{1}{c^3x^2\left(a^2x^2+1\right)^3\arctan(ax)^{3/2}}dx}{3a}-\frac{2}{3ac^3x\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{10a\int \frac{1}{\left(a^2x^2+1\right)^3\arctan(ax)^{3/2}}dx}{3c^3}-\frac{2\int \frac{1}{x^2\left(a^2x^2+1\right)^3\arctan(ax)^{3/2}}dx}{3ac^3}-\frac{2}{3ac^3x\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$$\Big\downarrow \ 5437$$

$$-\frac{2\int \frac{1}{x^2(a^2x^2+1)^3\arctan(ax)^{3/2}}dx}{3ac^3}-\frac{10a\left(-8a\int \frac{x}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx-\frac{2}{a(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3c^3}-\frac{2}{3ac^3x\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$$\Big\downarrow \ 5503$$

$$-\frac{10a\left(-8a\int \frac{x}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx-\frac{2}{a(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3c^3}-\frac{2\left(-12a\int \frac{1}{x(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx-\frac{4\int \frac{1}{x^3\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ax^2(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3ac^3}-\frac{2}{3ac^3x\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$$\Big\downarrow \ 5505$$

$$-\frac{10a\left(-\frac{8\int \frac{ax}{\left(a^2x^2+1\right)^2\sqrt{\arctan(ax)}}d\arctan(ax)}{a}-\frac{2}{a(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3c^3}-\frac{2\left(-12a\int \frac{1}{x(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx-\frac{4\int \frac{1}{x^3\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ax^2(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3ac^3}-\frac{2}{3ac^3x\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$$\Big\downarrow \ 4906$$

$$-\frac{10a\left(-\frac{8\int \left(\frac{\sin(2\arctan(ax))}{4\sqrt{\arctan(ax)}}+\frac{\sin(4\arctan(ax))}{8\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a}-\frac{2}{a(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3c^3}-\frac{2\left(-12a\int \frac{1}{x(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx-\frac{4\int \frac{1}{x^3\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ax^2(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3ac^3}-\frac{2}{3ac^3x\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$$\Big\downarrow \ 2009$$

$$-\frac{2\left(-12a\int \frac{1}{x(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx-\frac{4\int \frac{1}{x^3\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ax^2(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3ac^3}-\frac{10a\left(-\frac{2}{a(a^2x^2+1)^2\sqrt{\arctan(ax)}}-\frac{8\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{4}\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)\right)}{a}\right)}{3c^3}-$$

$$\frac{2}{3ac^3x\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$$\Big\downarrow \; 5560$$

$$-\frac{2\left(-12a\int \frac{1}{x(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx-\frac{4\int \frac{1}{x^3\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ax^2(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3ac^3}-\frac{10a\left(-\frac{2}{a(a^2x^2+1)^2\sqrt{\arctan(ax)}}-\frac{8\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{4}\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)\right)}{a}\right)}{3c^3}-$$

$$\frac{2}{3ac^3x\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

input
```
Int[1/(x*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\left(a^{2} c\,x^{2}+c\right)^{3} \arctan\left(a x\right)^{\frac{5}{2}}}d x$$

input
```
int(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

output
```
int(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\arctan(ax)^{5/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 24.77 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.71

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\arctan(ax)^{5/2}}\,dx=\frac{\int \frac{1}{a^6x^7\operatorname{atan}^{\frac{5}{2}}(ax)+3a^4x^5\operatorname{atan}^{\frac{5}{2}}(ax)+3a^2x^3\operatorname{atan}^{\frac{5}{2}}(ax)+x\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx}{c^3}$$

input
```
integrate(1/x/(a**2*c*x**2+c)**3/atan(a*x)**(5/2),x)
```

output
```
Integral(1/(a**6*x**7*atan(a*x)**(5/2) + 3*a**4*x**5*atan(a*x)**(5/2) + 3*
a**2*x**3*atan(a*x)**(5/2) + x*atan(a*x)**(5/2)), x)/c**3
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\arctan(ax)^{5/2}}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\arctan(ax)^{5/2}}\,dx=\int \frac{1}{\left(a^2cx^2+c\right)^3x\arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^3*x*arctan(a*x)^(5/2)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.90 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\arctan(ax)^{5/2}}\,dx=\int \frac{1}{x\,\mathrm{atan}\left(a\,x\right)^{5/2}\,\left(c\,a^2\,x^2+c\right)^3}\,dx$$

input
```
int(1/(x*atan(a*x)^(5/2)*(c + a^2*c*x^2)^3),x)
```

output
```
int(1/(x*atan(a*x)^(5/2)*(c + a^2*c*x^2)^3), x)
```

## Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.67

$$\int \frac{1}{x\left(c+a^2cx^2\right)^3\arctan(ax)^{5/2}}\,dx=\frac{\int \frac{\sqrt{atan(ax)}}{atan(ax)^3a^6x^7+3atan(ax)^3a^4x^5+3atan(ax)^3a^2x^3+atan(ax)^3x}dx}{c^3}$$

input

```
int(1/x/(a^2*c*x^2+c)^3/atan(a*x)^(5/2),x)
```

output

```
int(sqrt(atan(a*x))/(atan(a*x)**3*a**6*x**7 + 3*atan(a*x)**3*a**4*x**5 + 3
*atan(a*x)**3*a**2*x**3 + atan(a*x)**3*x),x)/c**3
```

# 3.1072 $\int \frac{1}{x^2(c+a^2cx^2)^3 \arctan(ax)^{5/2}} dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\begin{aligned}\int \frac{1}{x^2 (c+a^2cx^2)^3 \arctan(ax)^{5/2}} dx =& -\frac{2}{3ac^3x^2 (1+a^2x^2)^2 \arctan(ax)^{3/2}} \\ &+ \frac{8}{3a^2c^3x^3 (1+a^2x^2)^2 \sqrt{\arctan(ax)}} + \frac{8}{c^3x (1+a^2x^2)^2 \sqrt{\arctan(ax)}} \\ &+ \frac{30a\sqrt{\arctan(ax)}}{c^3} + \frac{5a\sqrt{\frac{\pi}{2}}\, \text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{c^3} \\ &+ \frac{20a\sqrt{\pi}\, \text{FresnelC}\left(\frac{2\sqrt{\arctan(ax)}}{\sqrt{\pi}}\right)}{c^3} + \frac{8\text{Int}\left(\frac{1}{x^4(c+a^2cx^2)^3\sqrt{\arctan(ax)}}, x\right)}{a^2} \\ &+ \frac{80}{3}\text{Int}\left(\frac{1}{x^2 (c+a^2cx^2)^3 \sqrt{\arctan(ax)}}, x\right)\end{aligned}$$

output
```
-2/3/a/c^3/x^2/(a^2*x^2+1)^2/arctan(a*x)^(3/2)+8/3/a^2/c^3/x^3/(a^2*x^2+1)
^2/arctan(a*x)^(1/2)+8/c^3/x/(a^2*x^2+1)^2/arctan(a*x)^(1/2)+30*a*arctan(a
*x)^(1/2)/c^3+5/2*a*2^(1/2)*Pi^(1/2)*FresnelC(2*2^(1/2)/Pi^(1/2)*arctan(a*
x)^(1/2))/c^3+20*a*Pi^(1/2)*FresnelC(2*arctan(a*x)^(1/2)/Pi^(1/2))/c^3+8*D
efer(Int)(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)/a^2+80/3*Defer(Int)(1
/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 5.53 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^2 (c+a^2cx^2)^3 \arctan(ax)^{5/2}}\, dx = \int \frac{1}{x^2 (c+a^2cx^2)^3 \arctan(ax)^{5/2}}\, dx$$

input
```
Integrate[1/(x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)),x]
```

output
```
Integrate[1/(x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 1.59 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \arctan(ax)^{5/2} (a^2cx^2+c)^3}dx$$

$$\Big\downarrow \text{ 5503}$$

$$-4a\int \frac{1}{c^3x(a^2x^2+1)^3\arctan(ax)^{3/2}}dx-\frac{4\int \frac{1}{c^3x^3(a^2x^2+1)^3\arctan(ax)^{3/2}}dx}{3a}-\frac{2}{3ac^3x^2(a^2x^2+1)^2\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{4a\int \frac{1}{x(a^2x^2+1)^3\arctan(ax)^{3/2}}dx}{c^3}-\frac{4\int \frac{1}{x^3(a^2x^2+1)^3\arctan(ax)^{3/2}}dx}{3ac^3}-\frac{2}{3ac^3x^2(a^2x^2+1)^2\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{5503}$$

$$-\frac{4a\left(-10a\int \frac{1}{(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx-\frac{2\int \frac{1}{x^2\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ax(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{c^3}-\frac{4\left(-14a\int \frac{1}{x^2(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx-\frac{6\int \frac{1}{x^4\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ax^3(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3ac^3}-\frac{2}{3ac^3x^2\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{5439}$$

$$-\frac{4a\left(-\frac{2\int \frac{1}{x^2\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{a}-10\int \frac{1}{(a^2x^2+1)^2\sqrt{\arctan(ax)}}d\arctan(ax)-\frac{2}{ax(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{c^3}-\frac{4\left(-14a\int \frac{1}{x^2(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx-\frac{6\int \frac{1}{x^4\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ax^3(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3ac^3}-\frac{2}{3ac^3x^2\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{4a\left(-\frac{2\int \frac{1}{x^2\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{a}-10\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^4}{\sqrt{\arctan(ax)}}d\arctan(ax)-\frac{2}{ax(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{c^3}-\frac{4\left(-14a\int \frac{1}{x^2(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx-\frac{6\int \frac{1}{x^4\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ax^3(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3ac^3}-\frac{2}{3ac^3x^2\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{3793}$$

$$-\frac{4a\left(-\frac{2\int \frac{1}{x^2\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{a}-10\int \left(\frac{\cos(2\arctan(ax))}{2\sqrt{\arctan(ax)}}+\frac{\cos(4\arctan(ax))}{8\sqrt{\arctan(ax)}}+\frac{3}{8\sqrt{\arctan(ax)}}\right)d\arctan(ax)-\frac{}{ax(a^2x^2\cdots}\right.}{c^3}$$

$$\frac{\frac{4\left(-14a\int \frac{1}{x^2(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx-\frac{6\int \frac{1}{x^4\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ax^3(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3ac^3}-}{2}$$

$$\overline{3ac^3x^2\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$\downarrow$ 2009

$$-\frac{4a\left(-\frac{2\int \frac{1}{x^2\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ax(a^2x^2+1)^2\sqrt{\arctan(ax)}}-10\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\pi}\text{Fres}\cdots\right.\right.}{c^3}$$

$$\frac{\frac{4\left(-14a\int \frac{1}{x^2(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx-\frac{6\int \frac{1}{x^4\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ax^3(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3ac^3}-}{2}$$

$$\overline{3ac^3x^2\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$\downarrow$ 5560

$$-\frac{4a\left(-\frac{2\int \frac{1}{x^2\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ax(a^2x^2+1)^2\sqrt{\arctan(ax)}}-10\left(\frac{1}{8}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\pi}\text{Fres}\cdots\right.\right.}{c^3}$$

$$\frac{\frac{4\left(-14a\int \frac{1}{x^2(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx-\frac{6\int \frac{1}{x^4\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ax^3(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3ac^3}-}{2}$$

$$\overline{3ac^3x^2\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

input
```
Int[1/(x^2*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^{2} \left(a^{2} c \,x^{2}+c \right)^{3} \arctan \left(a x \right)^{\frac{5}{2}}}dx$$

input
```
int(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

output
```
int(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^2 (c+a^2cx^2)^3 \arctan(ax)^{5/2}}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 36.06 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.75

$$\int \frac{1}{x^2 (c+a^2cx^2)^3 \arctan(ax)^{5/2}}\, dx = \frac{\int \frac{1}{a^{6} x^{8} \operatorname{atan}^{\frac{5}{2}}{\left(a x \right)} + 3 a^{4} x^{6} \operatorname{atan}^{\frac{5}{2}}{\left(a x \right)} + 3 a^{2} x^{4} \operatorname{atan}^{\frac{5}{2}}{\left(a x \right)} + x^{2} \operatorname{atan}^{\frac{5}{2}}{\left(a x \right)}}\, dx}{c^{3}}$$

input
```
integrate(1/x**2/(a**2*c*x**2+c)**3/atan(a*x)**(5/2),x)
```

output
```
Integral(1/(a**6*x**8*atan(a*x)**(5/2) + 3*a**4*x**6*atan(a*x)**(5/2) + 3*
a**2*x**4*atan(a*x)**(5/2) + x**2*atan(a*x)**(5/2)), x)/c**3
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^2 (c + a^2cx^2)^3 \arctan(ax)^{5/2}} dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (c + a^2cx^2)^3 \arctan(ax)^{5/2}} dx = \int \frac{1}{(a^2cx^2 + c)^3 x^2 \arctan(ax)^{\frac{5}{2}}} dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^3*x^2*arctan(a*x)^(5/2)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.92 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 (c+a^2cx^2)^3 \arctan(ax)^{5/2}}\, dx = \int \frac{1}{x^2\, \mathrm{atan}\left(a\,x\right)^{5/2}\, {\left(c\,a^2\,x^2+c\right)}^3}\, dx$$

input
```
int(1/(x^2*atan(a*x)^(5/2)*(c + a^2*c*x^2)^3),x)
```

output
```
int(1/(x^2*atan(a*x)^(5/2)*(c + a^2*c*x^2)^3), x)
```

## Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.75

$$\int \frac{1}{x^2 (c+a^2cx^2)^3 \arctan(ax)^{5/2}}\, dx = \frac{\int \frac{\sqrt{atan(ax)}}{atan(ax)^3 a^6 x^8+3atan(ax)^3 a^4 x^6+3atan(ax)^3 a^2 x^4+atan(ax)^3 x^2}dx}{c^3}$$

input
```
int(1/x^2/(a^2*c*x^2+c)^3/atan(a*x)^(5/2),x)
```

output
```
int(sqrt(atan(a*x))/(atan(a*x)**3*a**6*x**8 + 3*atan(a*x)**3*a**4*x**6 + 3
*atan(a*x)**3*a**2*x**4 + atan(a*x)**3*x**2),x)/c**3
```

# 3.1073 $\int \frac{1}{x^3 (c+a^2 c x^2)^3 \arctan(ax)^{5/2}} \, dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x^3 (c+a^2 c x^2)^3 \arctan(ax)^{5/2}} \, dx = -\frac{2}{3 a c^3 x^3 (1+a^2 x^2)^2 \arctan(ax)^{3/2}} + \frac{4}{a^2 c^3 x^4 (1+a^2 x^2)^2 \sqrt{\arctan(ax)}} + \frac{28}{3 c^3 x^2 (1+a^2 x^2)^2 \sqrt{\arctan(ax)}} + \frac{16 \text{Int}\left(\frac{1}{x^5 (c+a^2 c x^2)^3 \sqrt{\arctan(ax)}}, x\right)}{a^2} + \frac{152}{3} \text{Int}\left(\frac{1}{x^3 (c+a^2 c x^2)^3 \sqrt{\arctan(ax)}}, x\right) + 56 a^2 \text{Int}\left(\frac{1}{x (c+a^2 c x^2)^3 \sqrt{\arctan(ax)}}, x\right)$$

output
```
-2/3/a/c^3/x^3/(a^2*x^2+1)^2/arctan(a*x)^(3/2)+4/a^2/c^3/x^4/(a^2*x^2+1)^2
/arctan(a*x)^(1/2)+28/3/c^3/x^2/(a^2*x^2+1)^2/arctan(a*x)^(1/2)+16*Defer(I
nt)(1/x^5/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)/a^2+152/3*Defer(Int)(1/x^3/
(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)+56*a^2*Defer(Int)(1/x/(a^2*c*x^2+c)^3
/arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 5.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^3 (c+a^2cx^2)^3 \arctan(ax)^{5/2}}\,dx=\int \frac{1}{x^3 (c+a^2cx^2)^3 \arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[1/(x^3*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)),x]
```

output
```
Integrate[1/(x^3*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.83 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \arctan(ax)^{5/2} (a^2cx^2+c)^3}dx$$

$$\Big\downarrow \text{5503}$$

$$-\frac{14}{3}a\int \frac{1}{c^3x^2 (a^2x^2+1)^3 \arctan(ax)^{3/2}}dx-\frac{2\int \frac{1}{c^3x^4(a^2x^2+1)^3 \arctan(ax)^{3/2}}dx}{a}-\frac{2}{3ac^3x^3 (a^2x^2+1)^2 \arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{14a\int \frac{1}{x^2(a^2x^2+1)^3 \arctan(ax)^{3/2}}dx}{3c^3}-\frac{2\int \frac{1}{x^4(a^2x^2+1)^3 \arctan(ax)^{3/2}}dx}{ac^3}-\frac{2}{3ac^3x^3 (a^2x^2+1)^2 \arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{5503}$$

$$-\frac{14a\left(-12a\int \frac{1}{x(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx-\frac{4\int \frac{1}{x^3\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ax^2(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3c^3}-\frac{2\left(-\frac{8\int \frac{1}{x^5\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{a}-16a\int \frac{1}{x^3(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx-\frac{2}{ax^4(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{ac^3}-\frac{2}{3ac^3x^3\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{5560}$$

$$-\frac{14a\left(-12a\int \frac{1}{x(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx-\frac{4\int \frac{1}{x^3\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ax^2(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3c^3}-\frac{2\left(-\frac{8\int \frac{1}{x^5\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{a}-16a\int \frac{1}{x^3(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx-\frac{2}{ax^4(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{ac^3}-\frac{2}{3ac^3x^3\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

input
```
Int[1/(x^3*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^3\left(a^2c\,x^2+c\right)^3\arctan\left(ax\right)^{\frac{5}{2}}}dx$$

input
```
int(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

output
```
int(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^3\arctan(ax)^{5/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 45.40 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.75

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^3\arctan(ax)^{5/2}}\,dx=\frac{\int \frac{1}{a^6x^9\operatorname{atan}^{\frac{5}{2}}(ax)+3a^4x^7\operatorname{atan}^{\frac{5}{2}}(ax)+3a^2x^5\operatorname{atan}^{\frac{5}{2}}(ax)+x^3\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx}{c^3}$$

input
```
integrate(1/x**3/(a**2*c*x**2+c)**3/atan(a*x)**(5/2),x)
```

output
```
Integral(1/(a**6*x**9*atan(a*x)**(5/2) + 3*a**4*x**7*atan(a*x)**(5/2) + 3*
a**2*x**5*atan(a*x)**(5/2) + x**3*atan(a*x)**(5/2)), x)/c**3
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^3 (c + a^2 c x^2)^3 \arctan(ax)^{5/2}}\, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3 (c + a^2 c x^2)^3 \arctan(ax)^{5/2}}\, dx = \int \frac{1}{(a^2 c x^2 + c)^3 x^3 \arctan(ax)^{\frac{5}{2}}}\, dx$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^3*x^3*arctan(a*x)^(5/2)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.92 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^3 (c + a^2 c x^2)^3 \arctan(ax)^{5/2}}\, dx = \int \frac{1}{x^3 \operatorname{atan}(a\, x)^{5/2} (c\, a^2\, x^2 + c)^3}\, dx$$

input
```
int(1/(x^3*atan(a*x)^(5/2)*(c + a^2*c*x^2)^3),x)
```

output
```
int(1/(x^3*atan(a*x)^(5/2)*(c + a^2*c*x^2)^3), x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.75

$$\int \frac{1}{x^3 (c + a^2cx^2)^3 \arctan(ax)^{5/2}}\,dx = \frac{\int \frac{\sqrt{atan(ax)}}{atan(ax)^3 a^6x^9+3atan(ax)^3 a^4x^7+3atan(ax)^3 a^2x^5+atan(ax)^3 x^3}dx}{c^3}$$

input
```
int(1/x^3/(a^2*c*x^2+c)^3/atan(a*x)^(5/2),x)
```

output
```
int(sqrt(atan(a*x))/(atan(a*x)**3*a**6*x**9 + 3*atan(a*x)**3*a**4*x**7 + 3
*atan(a*x)**3*a**2*x**5 + atan(a*x)**3*x**3),x)/c**3
```

# 3.1074 $\int \frac{1}{x^4(c+a^2cx^2)^3 \arctan(ax)^{5/2}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1}{x^4\left(c+a^2cx^2\right)^3\arctan(ax)^{5/2}}\,dx=-\frac{2}{3ac^3x^4\left(1+a^2x^2\right)^2\arctan(ax)^{3/2}}+\frac{16}{3a^2c^3x^5\left(1+a^2x^2\right)^2\sqrt{\arctan(ax)}}+\frac{32}{3c^3x^3\left(1+a^2x^2\right)^2\sqrt{\arctan(ax)}}+\frac{80\text{Int}\left(\frac{1}{x^6(c+a^2cx^2)^3\sqrt{\arctan(ax)}},x\right)}{3a^2}+80\text{Int}\left(\frac{1}{x^4\left(c+a^2cx^2\right)^3\sqrt{\arctan(ax)}},x\right)+\frac{224}{3}a^2\text{Int}\left(\frac{1}{x^2\left(c+a^2cx^2\right)^3\sqrt{\arctan(ax)}},x\right)$$

output
```
-2/3/a/c^3/x^4/(a^2*x^2+1)^2/arctan(a*x)^(3/2)+16/3/a^2/c^3/x^5/(a^2*x^2+1
)^2/arctan(a*x)^(1/2)+32/3/c^3/x^3/(a^2*x^2+1)^2/arctan(a*x)^(1/2)+80/3*De
fer(Int)(1/x^6/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)/a^2+80*Defer(Int)(1/x^
4/(a^2*c*x^2+c)^3/arctan(a*x)^(1/2),x)+224/3*a^2*Defer(Int)(1/x^2/(a^2*c*x
^2+c)^3/arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 11.71 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^4 (c + a^2cx^2)^3 \arctan(ax)^{5/2}} \, dx = \int \frac{1}{x^4 (c + a^2cx^2)^3 \arctan(ax)^{5/2}} \, dx$$

input
```
Integrate[1/(x^4*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)),x]
```

output
```
Integrate[1/(x^4*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.83 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \arctan(ax)^{5/2} (a^2cx^2 + c)^3} \, dx$$

$$\Big\downarrow \text{5503}$$

$$-\frac{8 \int \frac{1}{c^3x^5(a^2x^2+1)^3 \arctan(ax)^{3/2}}dx}{3a} - \frac{16}{3}a \int \frac{1}{c^3x^3 (a^2x^2 + 1)^3 \arctan(ax)^{3/2}}dx - \frac{2}{3ac^3x^4 (a^2x^2 + 1)^2 \arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{8 \int \frac{1}{x^5(a^2x^2+1)^3 \arctan(ax)^{3/2}}dx}{3ac^3} - \frac{16a \int \frac{1}{x^3(a^2x^2+1)^3 \arctan(ax)^{3/2}}dx}{3c^3} - \frac{2}{3ac^3x^4 (a^2x^2 + 1)^2 \arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{5503}$$

$$-\frac{16a\left(-14a\int \frac{1}{x^2(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx-\frac{6\int \frac{1}{x^4\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ax^3(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3c^3}-\frac{8\left(-\frac{10\int \frac{1}{x^6\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{a}-18a\int \frac{1}{x^4(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx-\frac{2}{ax^5(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3ac^3}-\frac{2}{3ac^3x^4\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{5560}$$

$$-\frac{16a\left(-14a\int \frac{1}{x^2(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx-\frac{6\int \frac{1}{x^4\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{ax^3(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3c^3}-\frac{8\left(-\frac{10\int \frac{1}{x^6\left(a^2x^2+1\right)^3\sqrt{\arctan(ax)}}dx}{a}-18a\int \frac{1}{x^4(a^2x^2+1)^3\sqrt{\arctan(ax)}}dx-\frac{2}{ax^5(a^2x^2+1)^2\sqrt{\arctan(ax)}}\right)}{3ac^3}-\frac{2}{3ac^3x^4\left(a^2x^2+1\right)^2\arctan(ax)^{3/2}}$$

input
```
Int[1/(x^4*(c + a^2*c*x^2)^3*ArcTan[a*x]^(5/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^4\left(a^2c\,x^2+c\right)^3\arctan\left(ax\right)^{\frac{5}{2}}}dx$$

input
```
int(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

output
```
int(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^4 (c+a^2cx^2)^3 \arctan(ax)^{5/2}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 58.20 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.75

$$\int \frac{1}{x^4 (c+a^2cx^2)^3 \arctan(ax)^{5/2}} \, dx = \frac{\int \frac{1}{a^6x^{10} \operatorname{atan}^{\frac{5}{2}}(ax)+3a^4x^8 \operatorname{atan}^{\frac{5}{2}}(ax)+3a^2x^6 \operatorname{atan}^{\frac{5}{2}}(ax)+x^4 \operatorname{atan}^{\frac{5}{2}}(ax)}\, dx}{c^3}$$

input
```
integrate(1/x**4/(a**2*c*x**2+c)**3/atan(a*x)**(5/2),x)
```

output
```
Integral(1/(a**6*x**10*atan(a*x)**(5/2) + 3*a**4*x**8*atan(a*x)**(5/2) + 3
*a**2*x**6*atan(a*x)**(5/2) + x**4*atan(a*x)**(5/2)), x)/c**3
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^4 \left(c+a^2 c x^2\right)^3 \arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4 \left(c+a^2 c x^2\right)^3 \arctan(ax)^{5/2}}\,dx = \int \frac{1}{\left(a^2 c x^2 + c\right)^3 x^4 \arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^3/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^3*x^4*arctan(a*x)^(5/2)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.93 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^4 \left(c+a^2 c x^2\right)^3 \arctan(ax)^{5/2}}\,dx = \int \frac{1}{x^4\,{\mathrm{atan}\left(a\,x\right)}^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^3}\,dx$$

input
```
int(1/(x^4*atan(a*x)^(5/2)*(c + a^2*c*x^2)^3),x)
```

output
```
int(1/(x^4*atan(a*x)^(5/2)*(c + a^2*c*x^2)^3), x)
```

## Reduce [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.75

$$\int \frac{1}{x^4 (c + a^2cx^2)^3 \arctan(ax)^{5/2}} \, dx = \frac{\int \frac{\sqrt{atan(ax)}}{atan(ax)^3 a^6 x^{10} + 3atan(ax)^3 a^4 x^8 + 3atan(ax)^3 a^2 x^6 + atan(ax)^3 x^4} dx}{c^3}$$

input
```
int(1/x^4/(a^2*c*x^2+c)^3/atan(a*x)^(5/2),x)
```

output
```
int(sqrt(atan(a*x))/(atan(a*x)**3*a**6*x**10 + 3*atan(a*x)**3*a**4*x**8 +
3*atan(a*x)**3*a**2*x**6 + atan(a*x)**3*x**4),x)/c**3
```

# 3.1075 $\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^{5/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^{5/2}}\,dx = \text{Int}\left(\frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^{5/2}},x\right)$$

output
```
Defer(Int)(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^{5/2}}\,dx = \int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[(x^m*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[(x^m*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^(5/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \sqrt{a^2 c x^2+c}}{\arctan(ax)^{5/2}} \, dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{x^m \sqrt{a^2 c x^2+c}}{\arctan(ax)^{5/2}}dx$$

input
```
Int[(x^m*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^m \sqrt{a^2 c\,x^2+c}}{\arctan\left(a x\right)^{\frac{5}{2}}}dx$$

input
```
int(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^{5/2}}\,dx = \int \frac{\sqrt{a^2cx^2+c}\,x^m}{\arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^m/arctan(a*x)^(5/2), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**m*(a**2*c*x**2+c)**(1/2)/atan(a*x)**(5/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^{5/2}}\,dx = \int \frac{x^m\,\sqrt{c\,a^2\,x^2+c}}{{\mathrm{atan}\left(a\,x\right)}^{5/2}}\,dx$$

input
```
int((x^m*(c + a^2*c*x^2)^(1/2))/atan(a*x)^(5/2),x)
```

output
```
int((x^m*(c + a^2*c*x^2)^(1/2))/atan(a*x)^(5/2), x)
```

## Reduce [N/A]

Not integrable

Time = 200.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m\sqrt{c+a^2cx^2}}{\arctan(ax)^{5/2}}\,dx = \int \frac{x^m\sqrt{a^2c\,x^2+c}}{atan\,(ax)^{\frac{5}{2}}}dx$$

input
```
int(x^m*(a^2*c*x^2+c)^(1/2)/atan(a*x)^(5/2),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^(1/2)/atan(a*x)^(5/2),x)
```

# 3.1076 $\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^{5/2}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^{5/2}}\,dx=\text{Int}\left(\frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^{5/2}},x\right)$$

output
```
Defer(Int)(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 2.67 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^{5/2}}\,dx=\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[(x*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[(x*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^(5/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{a^2cx^2+c}}{\arctan(ax)^{5/2}}dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int \frac{x\sqrt{a^2cx^2+c}}{\arctan(ax)^{5/2}}dx$$

input
```
Int[(x*Sqrt[c + a^2*c*x^2])/ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{x\sqrt{a^2c\,x^2+c}}{\arctan\left(ax\right)^{\frac{5}{2}}}dx$$

input
```
int(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x)
```

output
```
int(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 41.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^{5/2}}\,dx = \int \frac{x\sqrt{c\,(a^2x^2+1)}}{\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx$$

input
```
integrate(x*(a**2*c*x**2+c)**(1/2)/atan(a*x)**(5/2),x)
```

output
```
Integral(x*sqrt(c*(a**2*x**2 + 1))/atan(a*x)**(5/2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^{5/2}}\,dx = \int \frac{\sqrt{a^2cx^2+c}x}{\arctan(ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)*x/arctan(a*x)^(5/2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^{5/2}}\,dx = \int \frac{x\,\sqrt{c\,a^2\,x^2+c}}{\mathrm{atan}\left(a\,x\right)^{5/2}}\,dx$$

input
```
int((x*(c + a^2*c*x^2)^(1/2))/atan(a*x)^(5/2),x)
```

output
```
int((x*(c + a^2*c*x^2)^(1/2))/atan(a*x)^(5/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.17

$$\int \frac{x\sqrt{c+a^2cx^2}}{\arctan(ax)^{5/2}}\,dx = \sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,x}{atan(ax)^3}dx\right)$$

input
```
int(x*(a^2*c*x^2+c)^(1/2)/atan(a*x)^(5/2),x)
```

output
```
sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x)/atan(a*x)**3,x)
```

# 3.1077 $\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^{5/2}}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^{5/2}}\,dx=\text{Int}\left(\frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^{5/2}},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.72 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^{5/2}}\,dx=\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[Sqrt[c + a^2*c*x^2]/ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[Sqrt[c + a^2*c*x^2]/ArcTan[a*x]^(5/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2cx^2+c}}{\arctan(ax)^{5/2}}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{\sqrt{a^2cx^2+c}}{\arctan(ax)^{5/2}}dx$$

input
```
Int[Sqrt[c + a^2*c*x^2]/ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{a^2c\,x^2+c}}{\arctan\left(a x\right)^{\frac{5}{2}}}dx$$

input
```
int((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x)
```

output
```
int((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 39.63 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^{5/2}}\,dx = \int \frac{\sqrt{c\,(a^2x^2+1)}}{\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(1/2)/atan(a*x)**(5/2),x)
```

output
```
Integral(sqrt(c*(a**2*x**2 + 1))/atan(a*x)**(5/2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^{5/2}}\,dx = \int \frac{\sqrt{a^2cx^2+c}}{\arctan(ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)/arctan(a*x)^(5/2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^{5/2}}\,dx = \int \frac{\sqrt{c\,a^2\,x^2+c}}{\mathrm{atan}\left(a\,x\right)^{5/2}}\,dx$$

input
```
int((c + a^2*c*x^2)^(1/2)/atan(a*x)^(5/2),x)
```

output
```
int((c + a^2*c*x^2)^(1/2)/atan(a*x)^(5/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{c+a^2cx^2}}{\arctan(ax)^{5/2}}\,dx = \sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}}{atan(ax)^3}dx\right)$$

input
```
int((a^2*c*x^2+c)^(1/2)/atan(a*x)^(5/2),x)
```

output
```
sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/atan(a*x)**3,x)
```

# 3.1078 $\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^{5/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^{5/2}}\,dx = \text{Int}\left(\frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^{5/2}},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^(5/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 9.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^{5/2}}\,dx = \int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[Sqrt[c + a^2*c*x^2]/(x*ArcTan[a*x]^(5/2)),x]
```

output
```
Integrate[Sqrt[c + a^2*c*x^2]/(x*ArcTan[a*x]^(5/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2cx^2+c}}{x\arctan(ax)^{5/2}}dx$$
$$\Big\downarrow \ 5560$$
$$\int \frac{\sqrt{a^2cx^2+c}}{x\arctan(ax)^{5/2}}dx$$

input
```
Int[Sqrt[c + a^2*c*x^2]/(x*ArcTan[a*x]^(5/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{a^2c\,x^2+c}}{x\arctan\left(ax\right)^{\frac{5}{2}}}dx$$

input
```
int((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^(5/2),x)
```

output
```
int((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 54.91 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^{5/2}}\,dx = \int \frac{\sqrt{c\,(a^2x^2+1)}}{x\,\mathrm{atan}^{\frac{5}{2}}\,(ax)}\,dx$$

input
```
integrate((a**2*c*x**2+c)**(1/2)/x/atan(a*x)**(5/2),x)
```

output
```
Integral(sqrt(c*(a**2*x**2 + 1))/(x*atan(a*x)**(5/2)), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^{5/2}}\,dx = \int \frac{\sqrt{a^2cx^2+c}}{x\arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(1/2)/x/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a^2*c*x^2 + c)/(x*arctan(a*x)^(5/2)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^{5/2}}\,dx = \int \frac{\sqrt{c\,a^2\,x^2+c}}{x\,\mathrm{atan}\left(a\,x\right)^{5/2}}\,dx$$

input
```
int((c + a^2*c*x^2)^(1/2)/(x*atan(a*x)^(5/2)),x)
```

output
```
int((c + a^2*c*x^2)^(1/2)/(x*atan(a*x)^(5/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \frac{\sqrt{c+a^2cx^2}}{x\arctan(ax)^{5/2}}\,dx = \sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan\left(ax\right)}}{atan\left(ax\right)^3 x}dx\right)$$

input
```
int((a^2*c*x^2+c)^(1/2)/x/atan(a*x)^(5/2),x)
```

output
```
sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(atan(a*x)**3*x),x)
```

# 3.1079 $\int \frac{x^m\left(c+a^2cx^2\right)^{3/2}}{\arctan(ax)^{5/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)^{5/2}}\,dx=\operatorname{Int}\left(\frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)^{5/2}},x\right)$$

output
```
Defer(Int)(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)^{5/2}}\,dx=\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[(x^m*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[(x^m*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^(5/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \left(a^2 c x^2+c\right)^{3/2}}{\arctan(ax)^{5/2}} \, dx$$

$$\Big\downarrow \; 5560$$

$$\int \frac{x^m \left(a^2 c x^2+c\right)^{3/2}}{\arctan(ax)^{5/2}} dx$$

input
```
Int[(x^m*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^m \left(a^2 c\,x^2+c\right)^{\frac{3}{2}}}{\arctan\left(a x\right)^{\frac{5}{2}}}d x$$

input
```
int(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)^{5/2}}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}x^m}{\arctan{(ax)}^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
integral((a^2*c*x^2 + c)^(3/2)*x^m/arctan(a*x)^(5/2), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**m*(a**2*c*x**2+c)**(3/2)/atan(a*x)**(5/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)^{5/2}}\,dx = \int \frac{x^m\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}{{\mathrm{atan}\left(a\,x\right)}^{5/2}}\,dx$$

input
```
int((x^m*(c + a^2*c*x^2)^(3/2))/atan(a*x)^(5/2),x)
```

output
```
int((x^m*(c + a^2*c*x^2)^(3/2))/atan(a*x)^(5/2), x)
```

## Reduce [N/A]

Not integrable

Time = 200.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m(c+a^2cx^2)^{3/2}}{\arctan(ax)^{5/2}}\,dx = \int \frac{x^m(a^2c\,x^2+c)^{\frac{3}{2}}}{atan(ax)^{\frac{5}{2}}}dx$$

input
```
int(x^m*(a^2*c*x^2+c)^(3/2)/atan(a*x)^(5/2),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^(3/2)/atan(a*x)^(5/2),x)
```

# 3.1080 $\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^{5/2}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^{5/2}}\,dx=\text{Int}\left(\frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^{5/2}},x\right)$$

output
```
Defer(Int)(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 2.96 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^{5/2}}\,dx=\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[(x*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[(x*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^(5/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a^2cx^2+c)^{3/2}}{\arctan(ax)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int \frac{x(a^2cx^2+c)^{3/2}}{\arctan(ax)^{5/2}}dx$$

input
```
Int[(x*(c + a^2*c*x^2)^(3/2))/ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{x(a^2c\,x^2+c)^{\frac{3}{2}}}{\arctan(ax)^{\frac{5}{2}}}dx$$

input
```
int(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

output
```
int(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x*(a**2*c*x**2+c)**(3/2)/atan(a*x)**(5/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^{5/2}}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}x}{\arctan(ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)*x/arctan(a*x)^(5/2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^{5/2}}\,dx = \int \frac{x\,(c\,a^2\,x^2+c)^{3/2}}{\mathrm{atan}\,(a\,x)^{5/2}}\,dx$$

input
```
int((x*(c + a^2*c*x^2)^(3/2))/atan(a*x)^(5/2),x)
```

output
```
int((x*(c + a^2*c*x^2)^(3/2))/atan(a*x)^(5/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.54

$$\int \frac{x(c+a^2cx^2)^{3/2}}{\arctan(ax)^{5/2}}\,dx = \sqrt{c}\,c\left(\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}\,x^3}{atan\,(ax)^3}dx\right)a^2+\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}\,x}{atan\,(ax)^3}dx\right)$$

input
```
int(x*(a^2*c*x^2+c)^(3/2)/atan(a*x)^(5/2),x)
```

output
```
sqrt(c)*c*(int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**3)/atan(a*x)**3,x)*
a**2 + int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x)/atan(a*x)**3,x))
```

# 3.1081 $\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^{5/2}}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^{5/2}}\,dx=\text{Int}\left(\frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^{5/2}},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.76 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^{5/2}}\,dx=\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^(3/2)/ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[(c + a^2*c*x^2)^(3/2)/ArcTan[a*x]^(5/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2cx^2+c\right)^{3/2}}{\arctan(ax)^{5/2}}dx$$

$$\Big\downarrow \text{5560}$$

$$\int \frac{\left(a^2cx^2+c\right)^{3/2}}{\arctan(ax)^{5/2}}dx$$

input
```
Int[(c + a^2*c*x^2)^(3/2)/ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{\left(a^2c\,x^2+c\right)^{\frac{3}{2}}}{\arctan\left(ax\right)^{\frac{5}{2}}}dx$$

input
```
int((a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

output
```
int((a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a**2*c*x**2+c)**(3/2)/atan(a*x)**(5/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^{5/2}}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}}{\arctan(ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)/arctan(a*x)^(5/2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^{5/2}}\,dx = \int \frac{(c\,a^2\,x^2+c)^{3/2}}{\mathrm{atan}(a\,x)^{5/2}}\,dx$$

input
```
int((c + a^2*c*x^2)^(3/2)/atan(a*x)^(5/2),x)
```

output
```
int((c + a^2*c*x^2)^(3/2)/atan(a*x)^(5/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.61

$$\int \frac{(c+a^2cx^2)^{3/2}}{\arctan(ax)^{5/2}}\,dx = \sqrt{c}\,c\left(\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,x^2}{atan(ax)^3}dx\right)a^2+\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}}{atan(ax)^3}dx\right)$$

input
```
int((a^2*c*x^2+c)^(3/2)/atan(a*x)^(5/2),x)
```

output
```
sqrt(c)*c*(int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**2)/atan(a*x)**3,x)*
a**2 + int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/atan(a*x)**3,x))
```

# 3.1082 $\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)^{5/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)^{5/2}}\,dx = \operatorname{Int}\left(\frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)^{5/2}},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^(5/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 9.70 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)^{5/2}}\,dx = \int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^(3/2)/(x*ArcTan[a*x]^(5/2)),x]
```

output
```
Integrate[(c + a^2*c*x^2)^(3/2)/(x*ArcTan[a*x]^(5/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2 c x^2+c\right)^{3/2}}{x \arctan (a x)^{5/2}} \, dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{\left(a^2 c x^2+c\right)^{3/2}}{x \arctan (a x)^{5/2}}dx$$

input
```
Int[(c + a^2*c*x^2)^(3/2)/(x*ArcTan[a*x]^(5/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}}}{x \arctan \left(a x \right)^{\frac{5}{2}}}d x$$

input
```
int((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^(5/2),x)
```

output
```
int((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a**2*c*x**2+c)**(3/2)/x/atan(a*x)**(5/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)^{5/2}}\,dx = \int \frac{(a^2cx^2+c)^{\frac{3}{2}}}{x\arctan(ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(3/2)/x/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^(3/2)/(x*arctan(a*x)^(5/2)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)^{5/2}}\,dx = \int \frac{(c\,a^2\,x^2+c)^{3/2}}{x\,\mathrm{atan}\left(a\,x\right)^{5/2}}\,dx$$

input
```
int((c + a^2*c*x^2)^(3/2)/(x*atan(a*x)^(5/2)),x)
```

output
```
int((c + a^2*c*x^2)^(3/2)/(x*atan(a*x)^(5/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.35

$$\int \frac{(c+a^2cx^2)^{3/2}}{x\arctan(ax)^{5/2}}\,dx = \sqrt{c}\,c\left(\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,x}{atan(ax)^3}dx\right)a^2+\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}}{atan(ax)^3\,x}dx\right)$$

input
```
int((a^2*c*x^2+c)^(3/2)/x/atan(a*x)^(5/2),x)
```

output
```
sqrt(c)*c*(int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x)/atan(a*x)**3,x)*a**
2 + int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(atan(a*x)**3*x),x))
```

# 3.1083 $\int \frac{x^m (c+a^2 c x^2)^{5/2}}{\arctan(ax)^{5/2}} \, dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^m (c+a^2 c x^2)^{5/2}}{\arctan(ax)^{5/2}} \, dx = \text{Int}\left(\frac{x^m (c+a^2 c x^2)^{5/2}}{\arctan(ax)^{5/2}}, x\right)$$

output
```
Defer(Int)(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.44 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^m (c+a^2 c x^2)^{5/2}}{\arctan(ax)^{5/2}} \, dx = \int \frac{x^m (c+a^2 c x^2)^{5/2}}{\arctan(ax)^{5/2}} \, dx$$

input
```
Integrate[(x^m*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[(x^m*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^(5/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \left(a^2 c x^2+c\right)^{5/2}}{\arctan(ax)^{5/2}} \, dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x^m \left(a^2 c x^2+c\right)^{5/2}}{\arctan(ax)^{5/2}} dx$$

input
```
Int[(x^m*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 1.35 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^m (a^2 c\, x^2+c)^{\frac{5}{2}}}{\arctan\left(a x\right)^{\frac{5}{2}}}dx$$

input
```
int(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.88

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\arctan(ax)^{5/2}}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}x^m}{\arctan(ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
integral((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*sqrt(a^2*c*x^2 + c)*x^m/arcta
n(a*x)^(5/2), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\arctan(ax)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**m*(a**2*c*x**2+c)**(5/2)/atan(a*x)**(5/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^m*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\arctan(ax)^{5/2}}\,dx = \int \frac{x^m\,\left(c\,a^2\,x^2+c\right)^{5/2}}{\mathrm{atan}\left(a\,x\right)^{5/2}}\,dx$$

input
```
int((x^m*(c + a^2*c*x^2)^(5/2))/atan(a*x)^(5/2),x)
```

output
```
int((x^m*(c + a^2*c*x^2)^(5/2))/atan(a*x)^(5/2), x)
```

## Reduce [N/A]

Not integrable

Time = 200.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m(c+a^2cx^2)^{5/2}}{\arctan(ax)^{5/2}}\,dx = \int \frac{x^m(a^2c\,x^2+c)^{\frac{5}{2}}}{atan(ax)^{\frac{5}{2}}}dx$$

input
```
int(x^m*(a^2*c*x^2+c)^(5/2)/atan(a*x)^(5/2),x)
```

output
```
int(x^m*(a^2*c*x^2+c)^(5/2)/atan(a*x)^(5/2),x)
```

# 3.1084 $\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^{5/2}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^{5/2}}\,dx = \text{Int}\left(\frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^{5/2}},x\right)$$

output
```
Defer(Int)(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 3.59 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^{5/2}}\,dx = \int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[(x*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[(x*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^(5/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a^2cx^2+c)^{5/2}}{\arctan(ax)^{5/2}}\,dx$$
$$\Big\downarrow \; 5560$$
$$\int \frac{x(a^2cx^2+c)^{5/2}}{\arctan(ax)^{5/2}}dx$$

input
```
Int[(x*(c + a^2*c*x^2)^(5/2))/ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{x(a^2c\,x^2+c)^{\frac{5}{2}}}{\arctan\left(ax\right)^{\frac{5}{2}}}dx$$

input
```
int(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

output
```
int(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x(c + a^2cx^2)^{5/2}}{\arctan(ax)^{5/2}} dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x(c + a^2cx^2)^{5/2}}{\arctan(ax)^{5/2}} dx = \text{Timed out}$$

input
```
integrate(x*(a**2*c*x**2+c)**(5/2)/atan(a*x)**(5/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x(c + a^2cx^2)^{5/2}}{\arctan(ax)^{5/2}} dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^{5/2}}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}x}{\arctan(ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)*x/arctan(a*x)^(5/2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^{5/2}}\,dx = \int \frac{x\,(c\,a^2\,x^2+c)^{5/2}}{\mathrm{atan}\,(a\,x)^{5/2}}\,dx$$

input
```
int((x*(c + a^2*c*x^2)^(5/2))/atan(a*x)^(5/2),x)
```

output
```
int((x*(c + a^2*c*x^2)^(5/2))/atan(a*x)^(5/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 95, normalized size of antiderivative = 3.96

$$\int \frac{x(c+a^2cx^2)^{5/2}}{\arctan(ax)^{5/2}}\,dx = \sqrt{c}\,c^2\left(\left(\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}\,x^5}{atan\,(ax)^3}dx\right)a^4+2\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}\,x^3}{atan\,(ax)^3}dx\right.\right.\right.$$

input
```
int(x*(a^2*c*x^2+c)^(5/2)/atan(a*x)^(5/2),x)
```

output
```
sqrt(c)*c**2*(int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**5)/atan(a*x)**3,
x)*a**4 + 2*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**3)/atan(a*x)**3,x)
*a**2 + int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x)/atan(a*x)**3,x))
```

# 3.1085 $\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^{5/2}}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^{5/2}}\,dx = \text{Int}\left(\frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^{5/2}},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.86 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^{5/2}}\,dx = \int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^(5/2)/ArcTan[a*x]^(5/2),x]
```

output
```
Integrate[(c + a^2*c*x^2)^(5/2)/ArcTan[a*x]^(5/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2cx^2+c\right)^{5/2}}{\arctan(ax)^{5/2}}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{\left(a^2cx^2+c\right)^{5/2}}{\arctan(ax)^{5/2}}dx$$

input
```
Int[(c + a^2*c*x^2)^(5/2)/ArcTan[a*x]^(5/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{\left(a^2c\,x^2+c\right)^{\frac{5}{2}}}{\arctan\left(ax\right)^{\frac{5}{2}}}dx$$

input
```
int((a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

output
```
int((a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a**2*c*x**2+c)**(5/2)/atan(a*x)**(5/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^{5/2}}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}}{\arctan(ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)/arctan(a*x)^(5/2), x)
```

## Mupad [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^{5/2}}\,dx = \int \frac{(c\,a^2\,x^2+c)^{5/2}}{\mathrm{atan}(a\,x)^{5/2}}\,dx$$

input
```
int((c + a^2*c*x^2)^(5/2)/atan(a*x)^(5/2),x)
```

output
```
int((c + a^2*c*x^2)^(5/2)/atan(a*x)^(5/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 94, normalized size of antiderivative = 4.09

$$\int \frac{(c+a^2cx^2)^{5/2}}{\arctan(ax)^{5/2}}\,dx = \sqrt{c}\,c^2\left(\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,x^4}{atan(ax)^3}dx\right)a^4+2\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,x^2}{atan(ax)^3}dx\right)\right.$$

input
```
int((a^2*c*x^2+c)^(5/2)/atan(a*x)^(5/2),x)
```

output
```
sqrt(c)*c**2*(int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**4)/atan(a*x)**3,
x)*a**4 + 2*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**2)/atan(a*x)**3,x)
*a**2 + int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/atan(a*x)**3,x))
```

# 3.1086 $\int \frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)^{5/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)^{5/2}}\,dx=\text{Int}\left(\frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)^{5/2}},x\right)$$

output
```
Defer(Int)((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^(5/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 7.32 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)^{5/2}}\,dx=\int \frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[(c + a^2*c*x^2)^(5/2)/(x*ArcTan[a*x]^(5/2)),x]
```

output
```
Integrate[(c + a^2*c*x^2)^(5/2)/(x*ArcTan[a*x]^(5/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a^2cx^2+c\right)^{5/2}}{x \arctan(ax)^{5/2}} \, dx$$

$$\Big\downarrow \text{5560}$$

$$\int \frac{\left(a^2cx^2+c\right)^{5/2}}{x \arctan(ax)^{5/2}}dx$$

input
```
Int[(c + a^2*c*x^2)^(5/2)/(x*ArcTan[a*x]^(5/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{\left(a^2c\,x^2+c\right)^{\frac{5}{2}}}{x \arctan\left(ax\right)^{\frac{5}{2}}}dx$$

input
```
int((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^(5/2),x)
```

output
```
int((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{(c + a^2cx^2)^{5/2}}{x \arctan(ax)^{5/2}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(c + a^2cx^2)^{5/2}}{x \arctan(ax)^{5/2}} \, dx = \text{Timed out}$$

input
```
integrate((a**2*c*x**2+c)**(5/2)/x/atan(a*x)**(5/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(c + a^2cx^2)^{5/2}}{x \arctan(ax)^{5/2}} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)^{5/2}}\,dx = \int \frac{(a^2cx^2+c)^{\frac{5}{2}}}{x\arctan{(ax)}^{\frac{5}{2}}}\,dx$$

input
```
integrate((a^2*c*x^2+c)^(5/2)/x/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((a^2*c*x^2 + c)^(5/2)/(x*arctan(a*x)^(5/2)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)^{5/2}}\,dx = \int \frac{(c\,a^2\,x^2+c)^{5/2}}{x\,\mathrm{atan}\left(a\,x\right)^{5/2}}\,dx$$

input
```
int((c + a^2*c*x^2)^(5/2)/(x*atan(a*x)^(5/2)),x)
```

output
```
int((c + a^2*c*x^2)^(5/2)/(x*atan(a*x)^(5/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 95, normalized size of antiderivative = 3.65

$$\int \frac{(c+a^2cx^2)^{5/2}}{x\arctan(ax)^{5/2}}\,dx = \sqrt{c}\,c^2\left(\left(\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}\,x^3}{atan\,(ax)^3}dx\right)a^4+2\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax)}\,x}{atan\,(ax)^3}dx\right)\right.\right.$$

input
```
int((a^2*c*x^2+c)^(5/2)/x/atan(a*x)^(5/2),x)
```

output
```
sqrt(c)*c**2*(int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**3)/atan(a*x)**3,
x)*a**4 + 2*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x)/atan(a*x)**3,x)*a*
*2 + int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(atan(a*x)**3*x),x))
```

# 3.1087 $\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx = \mathrm{Int}\left(\frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}},x\right)$$

output
```
Defer(Int)(x^m/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx = \int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[x^m/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2)),x]
```

output
```
Integrate[x^m/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{ 5560}$$

$$\int \frac{x^m}{\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}dx$$

input
```
Int[x^m/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^m}{\sqrt{a^2c\,x^2+c}\,\arctan\left(ax\right)^{\frac{5}{2}}}dx$$

input
```
int(x^m/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x)
```

output
```
int(x^m/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx = \int \frac{x^m}{\sqrt{a^2cx^2+c}\arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
integral(x^m/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^(5/2)), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**m/(a**2*c*x**2+c)**(1/2)/atan(a*x)**(5/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx = \int \frac{x^m}{\sqrt{a^2cx^2+c}\arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(x^m/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^(5/2)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx = \int \frac{x^m}{\mathrm{atan}\left(a\,x\right)^{5/2}\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(x^m/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(x^m/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.88

$$\int \frac{x^m}{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx = \frac{\sqrt{c}\left(\int \frac{x^m\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}}{atan(ax)^3a^2x^2+atan(ax)^3}dx\right)}{c}$$

input
```
int(x^m/(a^2*c*x^2+c)^(1/2)/atan(a*x)^(5/2),x)
```

output
```
(sqrt(c)*int((x**m*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(atan(a*x)**3*a**2
*x**2 + atan(a*x)**3),x))/c
```

# 3.1088 $\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx = \text{Int}\left(\frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}},x\right)$$

output
```
Defer(Int)(x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.76 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx = \int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2)),x]
```

output
```
Integrate[x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x}{\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}dx$$

input
```
Int[x/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{x}{\sqrt{a^2c\,x^2+c}\,\arctan\left(ax\right)^{\frac{5}{2}}}dx$$

input
```
int(x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x)
```

output
```
int(x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 52.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx = \int \frac{x}{\sqrt{c\,(a^2x^2+1)}\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx$$

input
```
integrate(x/(a**2*c*x**2+c)**(1/2)/atan(a*x)**(5/2),x)
```

output
```
Integral(x/(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**(5/2)), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx=\int \frac{x}{\sqrt{a^2cx^2+c}\arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(x/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^(5/2)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx=\int \frac{x}{\mathrm{atan}\left(a\,x\right)^{5/2}\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(x/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(x/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.96

$$\int \frac{x}{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx=\frac{\sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,x}{atan(ax)^3a^2x^2+atan(ax)^3}dx\right)}{c}$$

input
```
int(x/(a^2*c*x^2+c)^(1/2)/atan(a*x)^(5/2),x)
```

output
```
(sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x)/(atan(a*x)**3*a**2*x*
*2 + atan(a*x)**3),x))/c
```

# 3.1089 $\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx=\text{Int}\left(\frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}},x\right)$$

output
```
Defer(Int)(1/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx=\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2)),x]
```

output
```
Integrate[1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}dx$$
$$\Big\downarrow \ 5560$$
$$\int \frac{1}{\arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}dx$$

input
```
Int[1/(Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{1}{\sqrt{a^2c\,x^2+c}\,\arctan\left(ax\right)^{\frac{5}{2}}}dx$$

input
```
int(1/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x)
```

output
```
int(1/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 51.68 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx = \int \frac{1}{\sqrt{c\,(a^2x^2+1)}\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx$$

input
```
integrate(1/(a**2*c*x**2+c)**(1/2)/atan(a*x)**(5/2),x)
```

output
```
Integral(1/(sqrt(c*(a**2*x**2 + 1))*atan(a*x)**(5/2)), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx = \int \frac{1}{\sqrt{a^2cx^2+c}\arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(a^2*c*x^2 + c)*arctan(a*x)^(5/2)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx = \int \frac{1}{\mathrm{atan}\left(a\,x\right)^{5/2}\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(1/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(1/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 85, normalized size of antiderivative = 3.70

$$\int \frac{1}{\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx = \frac{2\sqrt{c}\left(atan(ax)^2\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,x}{atan(ax)^2a^2x^2+atan(ax)^2}dx\right)a^2-\sqrt{a^2x^2+1}\,\sqrt{atan\,(ax}{3atan\left(ax\right)^2ac}$$

input
```
int(1/(a^2*c*x^2+c)^(1/2)/atan(a*x)^(5/2),x)
```

output
```
(2*sqrt(c)*(atan(a*x)**2*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x)/(atan
(a*x)**2*a**2*x**2 + atan(a*x)**2),x)*a**2 - sqrt(a**2*x**2 + 1)*sqrt(atan
(a*x))))/(3*atan(a*x)**2*a*c)
```

# 3.1090 $\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx=-\frac{2\sqrt{c+a^2cx^2}}{3acx\arctan(ax)^{3/2}}-\frac{2\mathrm{Int}\left(\frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}},x\right)}{3a}$$

output
```
-2/3*(a^2*c*x^2+c)^(1/2)/a/c/x/arctan(a*x)^(3/2)-2/3*Defer(Int)(1/x^2/(a^2
*c*x^2+c)^(1/2)/arctan(a*x)^(3/2),x)/a
```

## Mathematica [N/A]

Not integrable

Time = 4.06 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx=\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[1/(x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2)),x]
```

output
```
Integrate[1/(x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x \arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}dx$$

$$\Big\downarrow \text{5477}$$

$$-\frac{2\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^{3/2}}dx}{3a}-\frac{2\sqrt{a^2cx^2+c}}{3acx\arctan(ax)^{3/2}}$$

$$\Big\downarrow \text{5560}$$

$$-\frac{2\int \frac{1}{x^2\sqrt{a^2cx^2+c}\arctan(ax)^{3/2}}dx}{3a}-\frac{2\sqrt{a^2cx^2+c}}{3acx\arctan(ax)^{3/2}}$$

input
```
Int[1/(x*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{1}{x\sqrt{a^2c\,x^2+c}\,\arctan\left(ax\right)^{\frac{5}{2}}}dx$$

input
```
int(1/x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x)
```

output
```
int(1/x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [N/A]

Not integrable

Time = 109.34 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx = \int \frac{1}{x\sqrt{c\,(a^2x^2+1)}\operatorname{atan}^{\frac{5}{2}}(ax)}\,dx$$

input
```
integrate(1/x/(a**2*c*x**2+c)**(1/2)/atan(a*x)**(5/2),x)
```

output
```
Integral(1/(x*sqrt(c*(a**2*x**2 + 1))*atan(a*x)**(5/2)), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx = \int \frac{1}{\sqrt{a^2cx^2+c}x\arctan(ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(a^2*c*x^2 + c)*x*arctan(a*x)^(5/2)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx = \int \frac{1}{x\,\mathrm{atan}\left(a\,x\right)^{5/2}\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(1/(x*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(1/(x*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.85

$$\int \frac{1}{x\sqrt{c+a^2cx^2}\,\arctan(ax)^{5/2}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}}{atan(ax)^3a^2x^3+atan(ax)^3x}dx\right)}{c}$$

input
```
int(1/x/(a^2*c*x^2+c)^(1/2)/atan(a*x)^(5/2),x)
```

output
```
(sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(atan(a*x)**3*a**2*x**3
 + atan(a*x)**3*x),x))/c
```

# 3.1091 $\int \frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx=\text{Int}\left(\frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}},x\right)$$

output
```
Defer(Int)(1/x^2/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 9.97 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx=\int \frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[1/(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2)),x]
```

output
```
Integrate[1/(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}dx$$
$$\Big\downarrow \text{5560}$$
$$\int \frac{1}{x^2 \arctan(ax)^{5/2}\sqrt{a^2cx^2+c}}dx$$

input
```
Int[1/(x^2*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(5/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^2\sqrt{a^2c\,x^2+c}\,\arctan\left(ax\right)^{\frac{5}{2}}}dx$$

input
```
int(1/x^2/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x)
```

output
```
int(1/x^2/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x, algorithm="fricas
")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/x**2/(a**2*c*x**2+c)**(1/2)/atan(a*x)**(5/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx = \int \frac{1}{\sqrt{a^2cx^2+c}x^2\arctan{(ax)}^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(a^2*c*x^2 + c)*x^2*arctan(a*x)^(5/2)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx = \int \frac{1}{x^2\,\mathrm{atan}\left(a\,x\right)^{5/2}\,\sqrt{c\,a^2\,x^2+c}}\,dx$$

input
```
int(1/(x^2*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2)),x)
```

output
```
int(1/(x^2*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(1/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.92

$$\int \frac{1}{x^2\sqrt{c+a^2cx^2}\arctan(ax)^{5/2}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}}{atan(ax)^3a^2x^4+atan(ax)^3x^2}dx\right)}{c}$$

input
```
int(1/x^2/(a^2*c*x^2+c)^(1/2)/atan(a*x)^(5/2),x)
```

output
```
(sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(atan(a*x)**3*a**2*x**4
 + atan(a*x)**3*x**2),x))/c
```

# 3.1092 $\int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx = \text{Int}\left(\frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}},x\right)$$

output
```
Defer(Int)(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.43 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx = \int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[x^m/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)),x]
```

output
```
Integrate[x^m/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}}\,dx$$

$$\Big\downarrow \; 5560$$

$$\int \frac{x^m}{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}}dx$$

input
```
Int[x^m/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^m}{\left(a^2c\,x^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^{\frac{5}{2}}}dx$$

input
```
int(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

output
```
int(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.96

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx = \int \frac{x^m}{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^m/((a^4*c^2*x^4 + 2*a^2*c^2*x^2 + c^2)*arct
an(a*x)^(5/2)), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**m/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(5/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx = \int \frac{x^m}{(a^2cx^2+c)^{\frac{3}{2}}\arctan{(ax)}^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(x^m/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^(5/2)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx = \int \frac{x^m}{\mathrm{atan}\left(a\,x\right)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

input
```
int(x^m/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(x^m/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 3.79 (sec) , antiderivative size = 771, normalized size of antiderivative = 29.65

$$\int \frac{x^m}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx = \text{Too large to display}$$

input
```
int(x^m/(a^2*c*x^2+c)^(3/2)/atan(a*x)^(5/2),x)
```

output

```
(2*sqrt(c)*(2*atan(a*x)**2*int((x**m*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x
**2)/(atan(a*x)*a**4*x**4 + 2*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**5*m**
2*x**2 - 2*atan(a*x)**2*int((x**m*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**2
)/(atan(a*x)*a**4*x**4 + 2*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**5*m*x**2
 + 2*atan(a*x)**2*int((x**m*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**2)/(ata
n(a*x)*a**4*x**4 + 2*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**3*m**2 - 2*ata
n(a*x)**2*int((x**m*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**2)/(atan(a*x)*a
**4*x**4 + 2*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**3*m + 2*atan(a*x)**2*i
nt((x**m*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(atan(a*x)*a**4*x**4 + 2*ata
n(a*x)*a**2*x**2 + atan(a*x)),x)*a**3*m**2*x**2 - 2*atan(a*x)**2*int((x**m
*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(atan(a*x)*a**4*x**4 + 2*atan(a*x)*a
**2*x**2 + atan(a*x)),x)*a**3*x**2 + 2*atan(a*x)**2*int((x**m*sqrt(a**2*x*
*2 + 1)*sqrt(atan(a*x)))/(atan(a*x)*a**4*x**4 + 2*atan(a*x)*a**2*x**2 + at
an(a*x)),x)*a*m**2 - 2*atan(a*x)**2*int((x**m*sqrt(a**2*x**2 + 1)*sqrt(ata
n(a*x)))/(atan(a*x)*a**4*x**4 + 2*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a +
atan(a*x)**2*int((x**m*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(atan(a*x)**2*
a**4*x**5 + 2*atan(a*x)**2*a**2*x**3 + atan(a*x)**2*x),x)*a**2*m*x**2 + at
an(a*x)**2*int((x**m*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(atan(a*x)**2*a*
*4*x**5 + 2*atan(a*x)**2*a**2*x**3 + atan(a*x)**2*x),x)*m - 2*x**m*sqrt(a*
*2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)*a*m*x + 2*x**m*sqrt(a**2*x**2 + ...
```

# 3.1093 $\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx = -\frac{2x^3}{3ac\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}} - \frac{4x^2}{a^2c\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}} - \frac{8x^4}{3c\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}} + \frac{8\sqrt{2\pi}\sqrt{1+a^2x^2}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{a^4c\sqrt{c+a^2cx^2}} + \frac{44}{3}\text{Int}\left(\frac{x^3}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}},x\right) + 8a^2\text{Int}\left(\frac{x^5}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}},x\right)$$

output
```
-2/3*x^3/a/c/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2)-4*x^2/a^2/c/(a^2*c*x^2+
c)^(1/2)/arctan(a*x)^(1/2)-8/3*x^4/c/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2)
+8*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)*arctan(a*x
)^(1/2))/a^4/c/(a^2*c*x^2+c)^(1/2)+44/3*Defer(Int)(x^3/(a^2*c*x^2+c)^(3/2)
/arctan(a*x)^(1/2),x)+8*a^2*Defer(Int)(x^5/(a^2*c*x^2+c)^(3/2)/arctan(a*x)
^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 7.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx=\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[x^3/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)),x]
```

output
```
Integrate[x^3/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 3.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\arctan(ax)^{5/2}(a^2cx^2+c)^{3/2}}dx$$

$$\Big\downarrow \text{5503}$$

$$\frac{2\int \frac{x^2}{(a^2cx^2+c)^{3/2}\arctan(ax)^{3/2}}dx}{a}+\frac{4}{3}a\int \frac{x^4}{(a^2cx^2+c)^{3/2}\arctan(ax)^{3/2}}dx-\frac{2x^3}{3ac\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5503}$$

$$\frac{2\left(\frac{4\int \frac{x}{\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx}{a}+2a\int \frac{x^3}{(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{2x^2}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)}{a}+\frac{4}{3}a\left(6a\int \frac{x^5}{(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx+\frac{8\int \frac{x^3}{(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2x^4}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)-\frac{2x^3}{3ac\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \; 5506$$

$$\frac{2\left(\frac{4\sqrt{a^2x^2+1}\int \frac{x}{\left(a^2x^2+1\right)^{3/2}\sqrt{\arctan(ax)}}dx}{ac\sqrt{a^2cx^2+c}}+2a\int \frac{x^3}{(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{2x^2}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)}{a}+\frac{4}{3}a\left(6a\int \frac{x^5}{(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx+\frac{8\int \frac{x^3}{(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2x^4}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)-\frac{2x^3}{3ac\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \; 5505$$

$$\frac{4}{3}a\left(6a\int \frac{x^5}{(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx+\frac{8\int \frac{x^3}{(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2x^4}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)+\frac{2\left(2a\int \frac{x^3}{(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx+\frac{4\sqrt{a^2x^2+1}\int \frac{ax}{\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3c\sqrt{a^2cx^2+c}}-\frac{2x^2}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)}{a}-\frac{2x^3}{3ac\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \; 3042$$

$$\frac{4}{3}a\left(6a\int \frac{x^5}{(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx+\frac{8\int \frac{x^3}{(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2x^4}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)+\frac{2\left(2a\int \frac{x^3}{(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx+\frac{4\sqrt{a^2x^2+1}\int \frac{\sin(\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3c\sqrt{a^2cx^2+c}}-\frac{2x^2}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)}{a}-\frac{2x^3}{3ac\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3786}$$

$$\frac{4}{3}a\left(6a\int \frac{x^5}{\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx+\frac{8\int \frac{x^3}{\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2x^4}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)+\frac{2\left(2a\int \frac{x^3}{\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx+\frac{8\sqrt{a^2x^2+1}\int \frac{ax}{\sqrt{a^2x^2+1}}d\sqrt{\arctan(ax)}}{a^3c\sqrt{a^2cx^2+c}}-\frac{2x^2}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)}{a}-\frac{2x^3}{3ac\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3832}$$

$$\frac{4}{3}a\left(6a\int \frac{x^5}{\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx+\frac{8\int \frac{x^3}{\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2x^4}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)+\frac{2\left(2a\int \frac{x^3}{\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{2x^2}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}+\frac{4\sqrt{2\pi}\sqrt{a^2x^2+1}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{a^3c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{2x^3}{3ac\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5560}$$

$$\frac{4}{3}a\left(6a\int \frac{x^5}{\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx+\frac{8\int \frac{x^3}{\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2x^4}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)+\frac{2\left(2a\int \frac{x^3}{\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{2x^2}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}+\frac{4\sqrt{2\pi}\sqrt{a^2x^2+1}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{a^3c\sqrt{a^2cx^2+c}}\right)}{a}-\frac{2x^3}{3ac\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

input
```
Int[x^3/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^3}{\left(a^2 c\, x^2+c\right)^{\frac{3}{2}} \arctan\left(a x\right)^{\frac{5}{2}}}dx$$

input
```
int(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

output
```
int(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate(x**3/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(5/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx = \int \frac{x^3}{\mathrm{atan}\left(a\,x\right)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

input
```
int(x^3/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(x^3/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.42

$$\int \frac{x^3}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,x^3}{atan(ax)^3a^4x^4+2atan(ax)^3a^2x^2+atan(ax)^3}dx\right)}{c^2}$$

input
```
int(x^3/(a^2*c*x^2+c)^(3/2)/atan(a*x)^(5/2),x)
```

output
```
(sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**3)/(atan(a*x)**3*a**4
*x**4 + 2*atan(a*x)**3*a**2*x**2 + atan(a*x)**3),x))/c**2
```

# 3.1094 $\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\begin{aligned}\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx &= -\frac{2x^2}{3ac\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}\\ &-\frac{8x}{3a^2c\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}-\frac{4x^3}{3c\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}\\ &+\frac{8\sqrt{2\pi}\sqrt{1+a^2x^2}\,\mathrm{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{3a^3c\sqrt{c+a^2cx^2}}\\ &+4\mathrm{Int}\left(\frac{x^2}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}},x\right)+\frac{8}{3}a^2\mathrm{Int}\left(\frac{x^4}{(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}},x\right)\end{aligned}$$

output
```
-2/3*x^2/a/c/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2)-8/3*x/a^2/c/(a^2*c*x^2+
c)^(1/2)/arctan(a*x)^(1/2)-4/3*x^3/c/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2)
+8/3*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*arctan(a
*x)^(1/2))/a^3/c/(a^2*c*x^2+c)^(1/2)+4*Defer(Int)(x^2/(a^2*c*x^2+c)^(3/2)/
arctan(a*x)^(1/2),x)+8/3*a^2*Defer(Int)(x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x
)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 6.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx=\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[x^2/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)),x]
```

output
```
Integrate[x^2/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 2.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\arctan(ax)^{5/2}(a^2cx^2+c)^{3/2}}\,dx$$

$$\Big\downarrow \text{5503}$$

$$\frac{4\int \frac{x}{(a^2cx^2+c)^{3/2}\arctan(ax)^{3/2}}dx}{3a}+\frac{2}{3}a\int \frac{x^3}{(a^2cx^2+c)^{3/2}\arctan(ax)^{3/2}}dx-\frac{2x^2}{3ac\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5477}$$

$$\frac{4\left(\frac{2\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2x}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)}{3a}+\frac{2}{3}a\int \frac{x^3}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^{3/2}}dx-\frac{2x^2}{3ac\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$\Big\downarrow$ 5440

$$\frac{4\left(\frac{2\sqrt{a^2x^2+1}\int \frac{1}{\left(a^2x^2+1\right)^{3/2}\sqrt{\arctan(ax)}}dx}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)}{3a}+\frac{2}{3}a\int \frac{x^3}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^{3/2}}dx-\frac{2x^2}{3ac\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$\Big\downarrow$ 5439

$$\frac{4\left(\frac{2\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{2x}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)}{3a}+\frac{2}{3}a\int \frac{x^3}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^{3/2}}dx-\frac{2x^2}{3ac\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$\Big\downarrow$ 3042

$$\frac{4\left(\frac{2\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)}{\sqrt{\arctan(ax)}}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{2x}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)}{3a}+\frac{2}{3}a\int \frac{x^3}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^{3/2}}dx-\frac{2x^2}{3ac\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$\Big\downarrow$ 3785

$$\frac{4\left(\frac{4\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}}d\sqrt{\arctan(ax)}}{a^2c\sqrt{a^2cx^2+c}}-\frac{2x}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)}{3a}+\frac{2}{3}a\int \frac{x^3}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^{3/2}}dx-\frac{2x^2}{3ac\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$\Big\downarrow$ 3833

$$\frac{2}{3}a\int \frac{x^3}{(a^2cx^2+c)^{3/2}\arctan(ax)^{3/2}}dx+\frac{4\left(\frac{2\sqrt{2\pi}\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{a^2c\sqrt{a^2cx^2+c}}-\frac{2x}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)}{3a}-\frac{2x^2}{3ac\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5503}$$

$$\frac{2}{3}a\left(\frac{6\int \frac{x^2}{(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx}{a}+4a\int \frac{x^4}{(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{2x^3}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)+\frac{4\left(\frac{2\sqrt{2\pi}\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{a^2c\sqrt{a^2cx^2+c}}-\frac{2x}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)}{3a}-\frac{2x^2}{3ac\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5560}$$

$$\frac{2}{3}a\left(\frac{6\int \frac{x^2}{(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx}{a}+4a\int \frac{x^4}{(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{2x^3}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)+\frac{4\left(\frac{2\sqrt{2\pi}\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{a^2c\sqrt{a^2cx^2+c}}-\frac{2x}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)}{3a}-\frac{2x^2}{3ac\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

input

```
Int[x^2/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)),x]
```

output

```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^2}{\left(a^2 c\,x^2+c\right)^{\frac{3}{2}} \arctan\left(a x\right)^{\frac{5}{2}}}dx$$

input
```
int(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

output
```
int(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2}{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^2}{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**2/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(5/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx = \int \frac{x^2}{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(x^2/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^(5/2)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx = \int \frac{x^2}{\mathrm{atan}\left(a\,x\right)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

input
```
int(x^2/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(x^2/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.42

$$\int \frac{x^2}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,x^2}{atan(ax)^3a^4x^4+2atan(ax)^3a^2x^2+atan(ax)^3}dx\right)}{c^2}$$

input
```
int(x^2/(a^2*c*x^2+c)^(3/2)/atan(a*x)^(5/2),x)
```

output
```
(sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**2)/(atan(a*x)**3*a**4
*x**4 + 2*atan(a*x)**3*a**2*x**2 + atan(a*x)**3),x))/c**2
```

# 3.1095 $\int \frac{x}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 129

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx = -\frac{2x}{3ac\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}} - \frac{4}{3a^2c\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}} - \frac{4\sqrt{2\pi}\sqrt{1+a^2x^2}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{3a^2c\sqrt{c+a^2cx^2}}$$

output
```
-2/3*x/a/c/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2)-4/3/a^2/c/(a^2*c*x^2+c)^(
1/2)/arctan(a*x)^(1/2)-4/3*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)*FresnelS(2^(
1/2)/Pi^(1/2)*arctan(a*x)^(1/2))/a^2/c/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.96

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx = -\frac{2\left(ax+2\arctan(ax)-i\sqrt{1+a^2x^2}(-i\arctan(ax))^{3/2}\Gamma\left(\frac{1}{2},-i\arctan(ax)\right)+i\sqrt{1+a^2x^2}(i\arctan(ax))^{3/2}\cdots\right.}{3a^2c\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}$$

input
```
Integrate[x/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)),x]
```

output
```
(-2*(a*x + 2*ArcTan[a*x] - I*Sqrt[1 + a^2*x^2]*((-I)*ArcTan[a*x])^(3/2)*Ga
mma[1/2, (-I)*ArcTan[a*x]] + I*Sqrt[1 + a^2*x^2]*(I*ArcTan[a*x])^(3/2)*Gam
ma[1/2, I*ArcTan[a*x]]))/(3*a^2*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2))
```

## Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {5477, 5437, 5506, 5505, 3042, 3786, 3832}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 5477}$$

$$\frac{2\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^{3/2}}dx}{3a}-\frac{2x}{3ac\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 5437}$$

$$\frac{2\left(-2a\int \frac{x}{\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{2}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)}{3a}-\frac{2x}{3ac\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 5506}$$

$$\frac{2\left(-\frac{2a\sqrt{a^2x^2+1}\int \frac{x}{\left(a^2x^2+1\right)^{3/2}\sqrt{\arctan(ax)}}dx}{c\sqrt{a^2cx^2+c}}-\frac{2}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)}{3a}-\frac{2x}{3ac\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 5505}$$

$$\frac{2\left(-\frac{2\sqrt{a^2x^2+1}\int \frac{ax}{\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)}{3a}-\frac{2x}{3ac\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\left(-\frac{2\sqrt{a^2x^2+1}\int \frac{\sin(\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)}{3a}-\frac{2x}{3ac\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3786}$$

$$\frac{2\left(-\frac{4\sqrt{a^2x^2+1}\int \frac{ax}{\sqrt{a^2x^2+1}}d\sqrt{\arctan(ax)}}{ac\sqrt{a^2cx^2+c}}-\frac{2}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)}{3a}-\frac{2x}{3ac\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3832}$$

$$\frac{2\left(-\frac{2\sqrt{2\pi}\sqrt{a^2x^2+1}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{ac\sqrt{a^2cx^2+c}}-\frac{2}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)}{3a}-\frac{2x}{3ac\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

input
```
Int[x/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)),x]
```

output
```
(-2*x)/(3*a*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)) + (2*(-2/(a*c*Sqrt[c
+ a^2*c*x^2]*Sqrt[ArcTan[a*x]]) - (2*Sqrt[2*Pi]*Sqrt[1 + a^2*x^2]*FresnelS
[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(a*c*Sqrt[c + a^2*c*x^2])))/(3*a)
```

### Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3786
```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[2/d
   Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f
}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3832
```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5477
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*ArcT
an[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp[f*(m/(b*c*(p + 1)))   Int[(f*x
)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1
]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

## Maple [F]

$$\int \frac{x}{\left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} \arctan \left(a x \right)^{\frac{5}{2}}}d x$$

input
```
int(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

output
```
int(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x}{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate(x/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(5/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{5/2}}\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x}{\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{5/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx=\int \frac{x}{\mathrm{atan}\left(a\,x\right)^{5/2}\,\left(c\,a^2\,x^2+c\right)^{3/2}}\,dx$$

input
```
int(x/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(x/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2)), x)
```

## Reduce [F]

$$\int \frac{x}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx=\frac{2\sqrt{c}\left(-2atan(ax)^2\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,x}{atan(ax)a^4x^4+2atan(ax)a^2x^2+atan(ax)}dx\right)a^4x^2-2at$$

input
```
int(x/(a^2*c*x^2+c)^(3/2)/atan(a*x)^(5/2),x)
```

output
```
(2*sqrt(c)*( - 2*atan(a*x)**2*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x)/
(atan(a*x)*a**4*x**4 + 2*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**4*x**2 - 2
*atan(a*x)**2*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x)/(atan(a*x)*a**4*
x**4 + 2*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**2 - 2*sqrt(a**2*x**2 + 1)*
sqrt(atan(a*x))*atan(a*x) - sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*a*x))/(3*a
tan(a*x)**2*a**2*c**2*(a**2*x**2 + 1))
```

# 3.1096 $\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 126

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx = -\frac{2}{3ac\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}} + \frac{4x}{3c\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}} - \frac{4\sqrt{2\pi}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{3ac\sqrt{c+a^2cx^2}}$$

output
```
-2/3/a/c/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2)+4/3*x/c/(a^2*c*x^2+c)^(1/2)
/arctan(a*x)^(1/2)-4/3*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)*FresnelC(2^(1/2)
/Pi^(1/2)*arctan(a*x)^(1/2))/a/c/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.95

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx = \frac{-2+4ax\arctan(ax)-2\sqrt{1+a^2x^2}(-i\arctan(ax))^{3/2}\Gamma\left(\frac{1}{2},-i\arctan\right.}{3ac\sqrt{c+a^2cx^2}\,\text{arctar}}$$

input
```
Integrate[1/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)),x]
```

output
```
(-2 + 4*a*x*ArcTan[a*x] - 2*Sqrt[1 + a^2*x^2]*((-I)*ArcTan[a*x])^(3/2)*Gam
ma[1/2, (-I)*ArcTan[a*x]] - 2*Sqrt[1 + a^2*x^2]*(I*ArcTan[a*x])^(3/2)*Gamm
a[1/2, I*ArcTan[a*x]])/(3*a*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2))
```

## Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {5437, 5477, 5440, 5439, 3042, 3785, 3833}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 5437}$$

$$-\frac{2}{3}a\int \frac{x}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^{3/2}}dx-\frac{2}{3ac\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 5477}$$

$$-\frac{2}{3}a\left(\frac{2\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2x}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)-\frac{2}{3ac\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 5440}$$

$$-\frac{2}{3}a\left(\frac{2\sqrt{a^2x^2+1}\int \frac{1}{\left(a^2x^2+1\right)^{3/2}\sqrt{\arctan(ax)}}dx}{ac\sqrt{a^2cx^2+c}}-\frac{2x}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)-\frac{2}{3ac\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 5439}$$

$$-\frac{2}{3}a\left(\frac{2\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{2x}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)-\frac{2}{3ac\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2}{3}a\left(\frac{2\sqrt{a^2x^2+1}\int\frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)}{\sqrt{\arctan(ax)}}d\arctan(ax)}{a^2c\sqrt{a^2cx^2+c}}-\frac{2x}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)-\frac{2}{3ac\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3785}$$

$$-\frac{2}{3}a\left(\frac{4\sqrt{a^2x^2+1}\int\frac{1}{\sqrt{a^2x^2+1}}d\sqrt{\arctan(ax)}}{a^2c\sqrt{a^2cx^2+c}}-\frac{2x}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)-\frac{2}{3ac\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{3833}$$

$$-\frac{2}{3}a\left(\frac{2\sqrt{2\pi}\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{a^2c\sqrt{a^2cx^2+c}}-\frac{2x}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)-\frac{2}{3ac\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

input

```
Int[1/((c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)),x]
```

output

```
-2/(3*a*c*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2)) - (2*a*((-2*x)/(a*c*Sqrt[
c + a^2*c*x^2]*Sqrt[ArcTan[a*x]]) + (2*Sqrt[2*Pi]*Sqrt[1 + a^2*x^2]*Fresne
lC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/(a^2*c*Sqrt[c + a^2*c*x^2])))/3
```

## Defintions of rubi rules used

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3785

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> S
imp[2/d   Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c,
d, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

rule 3833
```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5440
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])   Int[(1 +
c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

rule 5477
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*ArcT
an[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp[f*(m/(b*c*(p + 1)))   Int[(f*x
)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1
]
```

## Maple [F]

$$\int \frac{1}{\left(a^{2} c \,x^{2}+c \right)^{\frac{3}{2}} \arctan \left(a x \right)^{\frac{5}{2}}}d x$$

input
```
int(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

output
```
int(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(5/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F]

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx = \int \frac{1}{(a^2cx^2+c)^{\frac{3}{2}}\arctan(ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(3/2)*arctan(a*x)^(5/2)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx = \int \frac{1}{\mathrm{atan}(a\,x)^{5/2}\,(c\,a^2\,x^2+c)^{3/2}}\,dx$$

input
```
int(1/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(1/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2)), x)
```

## Reduce [F]

$$\int \frac{1}{(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx = \frac{2\sqrt{c}\left(-2atan(ax)^2\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}}{atan(ax)a^4x^4+2atan(ax)a^2x^2+atan(ax)}dx\right)a^3x^2-2at\right.}{}$$

input
```
int(1/(a^2*c*x^2+c)^(3/2)/atan(a*x)^(5/2),x)
```

output
```
(2*sqrt(c)*( - 2*atan(a*x)**2*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(a
tan(a*x)*a**4*x**4 + 2*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**3*x**2 - 2*a
tan(a*x)**2*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(atan(a*x)*a**4*x**4
 + 2*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a + 2*sqrt(a**2*x**2 + 1)*sqrt(at
an(a*x))*atan(a*x)*a*x - sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))))/(3*atan(a*x
)**2*a*c**2*(a**2*x**2 + 1))
```

# 3.1097 $\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{5/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{5/2}}\,dx = -\frac{2}{3acx\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}} + \frac{8}{3c\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}} + \frac{4}{3a^2cx^2\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}} + \frac{8\sqrt{2\pi}\sqrt{1+a^2x^2}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{3c\sqrt{c+a^2cx^2}} + \frac{8\text{Int}\left(\frac{1}{x^3(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}},x\right)}{3a^2} + 4\text{Int}\left(\frac{1}{x\left(c+a^2cx^2\right)^{3/2}\sqrt{\arctan(ax)}},x\right)$$

output
```
-2/3/a/c/x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2)+8/3/c/(a^2*c*x^2+c)^(1/2)
/arctan(a*x)^(1/2)+4/3/a^2/c/x^2/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2)+8/3
*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)*arctan(a*x)^
(1/2))/c/(a^2*c*x^2+c)^(1/2)+8/3*Defer(Int)(1/x^3/(a^2*c*x^2+c)^(3/2)/arct
an(a*x)^(1/2),x)/a^2+4*Defer(Int)(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2
),x)
```

## Mathematica [N/A]

Not integrable

Time = 5.65 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{5/2}}\,dx=\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[1/(x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)),x]
```

output
```
Integrate[1/(x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 2.96 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 5503}$$

$$-\frac{4}{3}a\int \frac{1}{\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^{3/2}}dx-\frac{2\int \frac{1}{x^2\left(a^2cx^2+c\right)^{3/2}\arctan(ax)^{3/2}}dx}{3a}-\frac{2}{3acx\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{ 5437}$$

$$-\frac{2\int \frac{1}{x^2(a^2cx^2+c)^{3/2}\arctan(ax)^{3/2}}dx}{3a}-\frac{\frac{4}{3}a\left(-2a\int \frac{x}{(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{2}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)-}{}\frac{2}{3acx\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$\Big\downarrow$ 5503

$$-\frac{4}{3}a\left(-2a\int \frac{x}{(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{2}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)-\frac{2\left(-6a\int \frac{1}{x(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{4\int \frac{1}{x^3\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^2\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)}{3a}-\frac{2}{3acx\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$\Big\downarrow$ 5506

$$-\frac{4}{3}a\left(-\frac{2a\sqrt{a^2x^2+1}\int \frac{x}{(a^2x^2+1)^{3/2}\sqrt{\arctan(ax)}}dx}{c\sqrt{a^2cx^2+c}}-\frac{2}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)-\frac{2\left(-6a\int \frac{1}{x(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{4\int \frac{1}{x^3\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^2\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)}{3a}-\frac{2}{3acx\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$\Big\downarrow$ 5505

$$-\frac{4}{3}a\left(-\frac{2\sqrt{a^2x^2+1}\int \frac{ax}{\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)-\frac{2\left(-6a\int \frac{1}{x(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{4\int \frac{1}{x^3\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^2\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)}{3a}-\frac{2}{3acx\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$\Big\downarrow$ 3042

$$-\frac{4}{3}a\left(-\frac{2\sqrt{a^2x^2+1}\int \frac{\sin(\arctan(ax))}{\sqrt{\arctan(ax)}}d\arctan(ax)}{ac\sqrt{a^2cx^2+c}}-\frac{2}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)-\frac{2\left(-6a\int \frac{1}{x(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{4\int \frac{1}{x^3\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^2\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)}{3a}-\frac{2}{3acx\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$\big\downarrow$ 3786

$$-\frac{4}{3}a\left(-\frac{4\sqrt{a^2x^2+1}\int \frac{ax}{\sqrt{a^2x^2+1}}d\sqrt{\arctan(ax)}}{ac\sqrt{a^2cx^2+c}}-\frac{2}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)-\frac{2\left(-6a\int \frac{1}{x(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{4\int \frac{1}{x^3\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^2\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)}{3a}-\frac{2}{3acx\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$\big\downarrow$ 3832

$$-\frac{2\left(-6a\int \frac{1}{x(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{4\int \frac{1}{x^3\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^2\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)}{3a}-\frac{4}{3}a\left(-\frac{2\sqrt{2\pi}\sqrt{a^2x^2+1}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{ac\sqrt{a^2cx^2+c}}-\frac{2}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)-\frac{2}{3acx\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$\big\downarrow$ 5560

$$-\frac{2\left(-6a\int \frac{1}{x(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{4\int \frac{1}{x^3\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^2\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)}{3a}-\frac{4}{3}a\left(-\frac{2\sqrt{2\pi}\sqrt{a^2x^2+1}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{ac\sqrt{a^2cx^2+c}}-\frac{2}{ac\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)-\frac{2}{3acx\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

input
```
Int[1/(x*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{1}{x\,(a^2c\,x^2+c)^{\frac{3}{2}}\arctan\left(ax\right)^{\frac{5}{2}}}dx$$

input
```
int(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

output
```
int(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{5/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/x/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(5/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{5/2}}\,dx=\int \frac{1}{x\,\mathrm{atan}\left(a\,x\right)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^{3/2}}\,dx$$

input
```
int(1/(x*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(1/(x*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.38

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{5/2}}\,dx=\frac{\sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}}{atan(ax)^3a^4x^5+2atan(ax)^3a^2x^3+atan(ax)^3x}dx\right)}{c^2}$$

input
```
int(1/x/(a^2*c*x^2+c)^(3/2)/atan(a*x)^(5/2),x)
```

output
```
(sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(atan(a*x)**3*a**4*x**5
 + 2*atan(a*x)**3*a**2*x**3 + atan(a*x)**3*x),x))/c**2
```

# 3.1098 $\int \frac{1}{x^2(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{5/2}}\,dx = -\frac{2}{3acx^2\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}} + \frac{8}{3a^2cx^3\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}} + \frac{4}{cx\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}} + \frac{8a\sqrt{2\pi}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{c\sqrt{c+a^2cx^2}} + \frac{8\text{Int}\left(\frac{1}{x^4(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}},x\right)}{a^2} + \frac{44}{3}\text{Int}\left(\frac{1}{x^2\left(c+a^2cx^2\right)^{3/2}\sqrt{\arctan(ax)}},x\right)$$

output

```
-2/3/a/c/x^2/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2)+8/3/a^2/c/x^3/(a^2*c*x^
2+c)^(1/2)/arctan(a*x)^(1/2)+4/c/x/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2)+8
*a*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*arctan(a*x
)^(1/2))/c/(a^2*c*x^2+c)^(1/2)+8*Defer(Int)(1/x^4/(a^2*c*x^2+c)^(3/2)/arct
an(a*x)^(1/2),x)/a^2+44/3*Defer(Int)(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)
^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 9.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^2 (c+a^2cx^2)^{3/2} \arctan(ax)^{5/2}} \, dx = \int \frac{1}{x^2 (c+a^2cx^2)^{3/2} \arctan(ax)^{5/2}} \, dx$$

input
```
Integrate[1/(x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)),x]
```

output
```
Integrate[1/(x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 3.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \arctan(ax)^{5/2} (a^2cx^2+c)^{3/2}} \, dx$$

$$\Big\downarrow \text{5503}$$

$$-2a \int \frac{1}{x (a^2cx^2+c)^{3/2} \arctan(ax)^{3/2}} dx - \frac{4 \int \frac{1}{x^3 (a^2cx^2+c)^{3/2} \arctan(ax)^{3/2}} dx}{3a} - \frac{2}{3acx^2 \arctan(ax)^{3/2} \sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5503}$$

$$-2a\left(-4a\int \frac{1}{(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{2\int \frac{1}{x^2(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)-\frac{4\left(-8a\int \frac{1}{x^2(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{6\int \frac{1}{x^4\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^3\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)}{3a}-\frac{2}{3acx^2\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$\big\downarrow$ 5440

$$-2a\left(-\frac{4a\sqrt{a^2x^2+1}\int \frac{1}{(a^2x^2+1)^{3/2}\sqrt{\arctan(ax)}}dx}{c\sqrt{a^2cx^2+c}}-\frac{2\int \frac{1}{x^2(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)-\frac{4\left(-8a\int \frac{1}{x^2(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{6\int \frac{1}{x^4\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^3\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)}{3a}-\frac{2}{3acx^2\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$\big\downarrow$ 5439

$$-2a\left(-\frac{4\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}d\arctan(ax)}{c\sqrt{a^2cx^2+c}}-\frac{2\int \frac{1}{x^2(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)-\frac{4\left(-8a\int \frac{1}{x^2(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{6\int \frac{1}{x^4\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^3\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)}{3a}-\frac{2}{3acx^2\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$\big\downarrow$ 3042

$$\frac{-2a\left(-\frac{2\int \frac{1}{x^2(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{4\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)}{\sqrt{\arctan(ax)}}d\arctan(ax)}{c\sqrt{a^2cx^2+c}}-\frac{2}{acx\sqrt{\arctan(ax)}\sqrt{a^2cx^2}}\right.}{}$$

$$\frac{4\left(-8a\int \frac{1}{x^2(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{6\int \frac{1}{x^4\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^3\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)}{3a}-\frac{2}{3acx^2\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$\downarrow$ 3785

$$-2a\left(-\frac{8\sqrt{a^2x^2+1}\int \frac{1}{\sqrt{a^2x^2+1}}d\sqrt{\arctan(ax)}}{c\sqrt{a^2cx^2+c}}-\frac{2\int \frac{1}{x^2(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)$$

$$\frac{4\left(-8a\int \frac{1}{x^2(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{6\int \frac{1}{x^4\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^3\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)}{3a}-\frac{2}{3acx^2\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$\downarrow$ 3833

$$-2a\left(-\frac{2\int \frac{1}{x^2(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{4\sqrt{2\pi}\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{c\sqrt{a^2cx^2+c}}-\frac{2}{acx\sqrt{\arctan(ax)}\sqrt{a^2}}\right.$$

$$\frac{4\left(-8a\int \frac{1}{x^2(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{6\int \frac{1}{x^4\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^3\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)}{3a}-\frac{2}{3acx^2\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$\downarrow$ 5560

$$-2a\left(-\frac{2\int \frac{1}{x^2(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{4\sqrt{2\pi}\sqrt{a^2x^2+1}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{c\sqrt{a^2cx^2+c}}-\frac{2}{acx\sqrt{\arctan(ax)}\sqrt{a^2\dots}}\right.$$

$$\frac{4\left(-8a\int \frac{1}{x^2(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{6\int \frac{1}{x^4\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^3\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)}{3a}-\frac{2}{3acx^2\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

input
```
Int[1/(x^2*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^2\left(a^2c\,x^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^{\frac{5}{2}}}dx$$

input
```
int(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

output
```
int(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{5/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="fricas
")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2 \left(c+a^2cx^2\right)^{3/2} \arctan(ax)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/x**2/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(5/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^2 \left(c+a^2cx^2\right)^{3/2} \arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^2 (c+a^2cx^2)^{3/2} \arctan(ax)^{5/2}} \, dx = \int \frac{1}{(a^2cx^2+c)^{\frac{3}{2}} x^2 \arctan(ax)^{\frac{5}{2}}} \, dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(3/2)*x^2*arctan(a*x)^(5/2)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^2 (c+a^2cx^2)^{3/2} \arctan(ax)^{5/2}} \, dx = \int \frac{1}{x^2 \operatorname{atan}(a\,x)^{5/2} (c\,a^2\,x^2+c)^{3/2}} \, dx$$

input
```
int(1/(x^2*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(1/(x^2*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.46

$$\int \frac{1}{x^2 (c+a^2cx^2)^{3/2} \arctan(ax)^{5/2}} \, dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}}{atan(ax)^3a^4x^6+2atan(ax)^3a^2x^4+atan(ax)^3x^2}dx\right)}{c^2}$$

input
```
int(1/x^2/(a^2*c*x^2+c)^(3/2)/atan(a*x)^(5/2),x)
```

output
```
(sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(atan(a*x)**3*a**4*x**6
 + 2*atan(a*x)**3*a**2*x**4 + atan(a*x)**3*x**2),x))/c**2
```

# 3.1099 $\int \frac{1}{x^3(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{1}{x^3(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx = -\frac{2}{3acx^3\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}+\frac{4}{a^2cx^4\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}} +\frac{16}{3cx^2\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}+\frac{16\text{Int}\left(\frac{1}{x^5(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}},x\right)}{a^2} +\frac{92}{3}\text{Int}\left(\frac{1}{x^3(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}},x\right)+16a^2\text{Int}\left(\frac{1}{x(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}},x\right)$$

output

```
-2/3/a/c/x^3/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2)+4/a^2/c/x^4/(a^2*c*x^2+
c)^(1/2)/arctan(a*x)^(1/2)+16/3/c/x^2/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(1/2
)+16*Defer(Int)(1/x^5/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)/a^2+92/3*De
fer(Int)(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)+16*a^2*Defer(Int)(
1/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 11.75 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^3 (c+a^2cx^2)^{3/2} \arctan(ax)^{5/2}}\,dx = \int \frac{1}{x^3 (c+a^2cx^2)^{3/2} \arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[1/(x^3*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)),x]
```

output
```
Integrate[1/(x^3*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 1.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \arctan(ax)^{5/2} (a^2cx^2+c)^{3/2}}\,dx$$

$$\Big\downarrow \text{5503}$$

$$-\frac{8}{3}a\int \frac{1}{x^2 (a^2cx^2+c)^{3/2} \arctan(ax)^{3/2}}dx - \frac{2\int \frac{1}{x^4 (a^2cx^2+c)^{3/2} \arctan(ax)^{3/2}}dx}{a} - \frac{2}{3acx^3 \arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \text{5503}$$

$$-\frac{8}{3}a\left(-6a\int \frac{1}{x\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{4\int \frac{1}{x^3(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^2\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right.$$

$$\frac{2\left(-\frac{8\int \frac{1}{x^5\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-10a\int \frac{1}{x^3(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{2}{acx^4\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)}{a}-\frac{2}{3acx^3\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$\downarrow$ 5560

$$-\frac{8}{3}a\left(-6a\int \frac{1}{x\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{4\int \frac{1}{x^3(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^2\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right.$$

$$\frac{2\left(-\frac{8\int \frac{1}{x^5\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-10a\int \frac{1}{x^3(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{2}{acx^4\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)}{a}-\frac{2}{3acx^3\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

input
```
Int[1/(x^3*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^3\left(a^2c\,x^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^{\frac{5}{2}}}dx$$

input
```
int(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

output
```
int(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="fricas
")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/x**3/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(5/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x^3 (c + a^2cx^2)^{3/2} \arctan(ax)^{5/2}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^3 (c + a^2cx^2)^{3/2} \arctan(ax)^{5/2}} \, dx = \int \frac{1}{x^3 \operatorname{atan}(a\,x)^{5/2} \left(c\,a^2\,x^2 + c\right)^{3/2}} \,dx$$

input
```
int(1/(x^3*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(1/(x^3*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.46

$$\int \frac{1}{x^3 (c + a^2cx^2)^{3/2} \arctan(ax)^{5/2}} \, dx = \frac{\sqrt{c} \left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}}{atan(ax)^3 a^4x^7+2atan(ax)^3 a^2x^5+atan(ax)^3 x^3}dx\right)}{c^2}$$

input
```
int(1/x^3/(a^2*c*x^2+c)^(3/2)/atan(a*x)^(5/2),x)
```

output
```
(sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(atan(a*x)**3*a**4*x**7
 + 2*atan(a*x)**3*a**2*x**5 + atan(a*x)**3*x**3),x))/c**2
```

# 3.1100 $\int \frac{1}{x^4(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{1}{x^4(c+a^2cx^2)^{3/2}\arctan(ax)^{5/2}}\,dx = -\frac{2}{3acx^4\sqrt{c+a^2cx^2}\arctan(ax)^{3/2}}+\frac{16}{3a^2cx^5\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}+\frac{20}{3cx^3\sqrt{c+a^2cx^2}\sqrt{\arctan(ax)}}+\frac{80\text{Int}\left(\frac{1}{x^6(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}},x\right)}{3a^2}+52\text{Int}\left(\frac{1}{x^4(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}},x\right)+\frac{80}{3}a^2\text{Int}\left(\frac{1}{x^2(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}},x\right)$$

output
```
-2/3/a/c/x^4/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(3/2)+16/3/a^2/c/x^5/(a^2*c*x
^2+c)^(1/2)/arctan(a*x)^(1/2)+20/3/c/x^3/(a^2*c*x^2+c)^(1/2)/arctan(a*x)^(
1/2)+80/3*Defer(Int)(1/x^6/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)/a^2+52
*Defer(Int)(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)+80/3*a^2*Defer(
Int)(1/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 30.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^4 (c+a^2cx^2)^{3/2} \arctan(ax)^{5/2}}\,dx = \int \frac{1}{x^4 (c+a^2cx^2)^{3/2} \arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[1/(x^4*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)),x]
```

output
```
Integrate[1/(x^4*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 1.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \arctan(ax)^{5/2} (a^2cx^2+c)^{3/2}}\,dx$$

$$\Big\downarrow \ 5503$$

$$-\frac{8\int \frac{1}{x^5(a^2cx^2+c)^{3/2}\arctan(ax)^{3/2}}dx}{3a} - \frac{10}{3}a\int \frac{1}{x^3 (a^2cx^2+c)^{3/2}\arctan(ax)^{3/2}}dx - \frac{2}{3acx^4\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$$\Big\downarrow \ 5503$$

$$-\frac{10}{3}a\left(-8a\int \frac{1}{x^2\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{6\int \frac{1}{x^4(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^3\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right.$$
$$\frac{8\left(-\frac{10\int \frac{1}{x^6\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-12a\int \frac{1}{x^4(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{2}{acx^5\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)}{3a}-\frac{2}{3acx^4\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

$\big\downarrow$ 5560

$$-\frac{10}{3}a\left(-8a\int \frac{1}{x^2\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{6\int \frac{1}{x^4(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^3\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right.$$
$$\frac{8\left(-\frac{10\int \frac{1}{x^6\left(a^2cx^2+c\right)^{3/2}\sqrt{\arctan(ax)}}dx}{a}-12a\int \frac{1}{x^4(a^2cx^2+c)^{3/2}\sqrt{\arctan(ax)}}dx-\frac{2}{acx^5\sqrt{\arctan(ax)}\sqrt{a^2cx^2+c}}\right)}{3a}-\frac{2}{3acx^4\arctan(ax)^{3/2}\sqrt{a^2cx^2+c}}$$

input
```
Int[1/(x^4*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(5/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^4\left(a^2c\,x^2+c\right)^{\frac{3}{2}}\arctan\left(ax\right)^{\frac{5}{2}}}dx$$

input
```
int(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

output
```
int(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^4 (c + a^2cx^2)^{3/2} \arctan(ax)^{5/2}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="fricas
")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (c + a^2cx^2)^{3/2} \arctan(ax)^{5/2}} \, dx = \text{Timed out}$$

input
```
integrate(1/x**4/(a**2*c*x**2+c)**(3/2)/atan(a*x)**(5/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^4 (c + a^2cx^2)^{3/2} \arctan(ax)^{5/2}} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^4 (c+a^2cx^2)^{3/2} \arctan(ax)^{5/2}} \, dx = \int \frac{1}{(a^2cx^2+c)^{\frac{3}{2}} x^4 \arctan(ax)^{\frac{5}{2}}} \, dx$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(3/2)*x^4*arctan(a*x)^(5/2)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^4 (c+a^2cx^2)^{3/2} \arctan(ax)^{5/2}} \, dx = \int \frac{1}{x^4 \operatorname{atan}(a x)^{5/2} (c a^2 x^2 + c)^{3/2}} \, dx$$

input
```
int(1/(x^4*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2)),x)
```

output
```
int(1/(x^4*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(3/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.46

$$\int \frac{1}{x^4 (c+a^2cx^2)^{3/2} \arctan(ax)^{5/2}} \, dx = \frac{\sqrt{c} \left(\int \frac{\sqrt{a^2x^2+1}\, \sqrt{atan(ax)}}{atan(ax)^3 a^4x^8+2atan(ax)^3 a^2x^6+atan(ax)^3 x^4}dx\right)}{c^2}$$

input
```
int(1/x^4/(a^2*c*x^2+c)^(3/2)/atan(a*x)^(5/2),x)
```

output
```
(sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(atan(a*x)**3*a**4*x**8
 + 2*atan(a*x)**3*a**2*x**6 + atan(a*x)**3*x**4),x))/c**2
```

# 3.1101 $\int \frac{x^m}{\left(c+a^2cx^2\right)^{5/2} \arctan(ax)^{5/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^{5/2} \arctan(ax)^{5/2}}\,dx=\text{Int}\left(\frac{x^m}{\left(c+a^2cx^2\right)^{5/2} \arctan(ax)^{5/2}},x\right)$$

output
```
Defer(Int)(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 1.60 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{x^m}{\left(c+a^2cx^2\right)^{5/2} \arctan(ax)^{5/2}}\,dx=\int \frac{x^m}{\left(c+a^2cx^2\right)^{5/2} \arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[x^m/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)),x]
```

output
```
Integrate[x^m/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m}{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^{5/2}}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{x^m}{\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^{5/2}}dx$$

input
```
Int[x^m/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{x^m}{\left(a^2c\,x^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^{\frac{5}{2}}}dx$$

input
```
int(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

output
```
int(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.38

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx = \int \frac{x^m}{(a^2cx^2+c)^{\frac{5}{2}}\arctan{(ax)}^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a^2*c*x^2 + c)*x^m/((a^6*c^3*x^6 + 3*a^4*c^3*x^4 + 3*a^2*c^3
*x^2 + c^3)*arctan(a*x)^(5/2)), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**m/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(5/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx = \int \frac{x^m}{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^m/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(x^m/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^(5/2)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx = \int \frac{x^m}{\mathrm{atan}(a\,x)^{5/2}\,(c\,a^2\,x^2+c)^{5/2}}\,dx$$

input
```
int(x^m/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(x^m/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 1778, normalized size of antiderivative = 68.38

$$\int \frac{x^m}{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx = \text{Too large to display}$$

input
```
int(x^m/(a^2*c*x^2+c)^(5/2)/atan(a*x)^(5/2),x)
```

output

```
(2*sqrt(c)*(2*atan(a*x)**2*int((x**m*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x
**2)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2
+ atan(a*x)),x)*a**7*m**2*x**4 - 10*atan(a*x)**2*int((x**m*sqrt(a**2*x**2
+ 1)*sqrt(atan(a*x))*x**2)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 +
3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**7*m*x**4 + 12*atan(a*x)**2*int((x
**m*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**2)/(atan(a*x)*a**6*x**6 + 3*ata
n(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**7*x**4 + 4*ata
n(a*x)**2*int((x**m*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**2)/(atan(a*x)*a
**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a
**5*m**2*x**2 - 20*atan(a*x)**2*int((x**m*sqrt(a**2*x**2 + 1)*sqrt(atan(a*
x))*x**2)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*
x**2 + atan(a*x)),x)*a**5*m*x**2 + 24*atan(a*x)**2*int((x**m*sqrt(a**2*x**
2 + 1)*sqrt(atan(a*x))*x**2)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4
+ 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**5*x**2 + 2*atan(a*x)**2*int((x*
*m*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**2)/(atan(a*x)*a**6*x**6 + 3*atan
(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**3*m**2 - 10*ata
n(a*x)**2*int((x**m*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**2)/(atan(a*x)*a
**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a
**3*m + 12*atan(a*x)**2*int((x**m*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**2
)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 ...
```

# 3.1102 $\int \frac{x^3}{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{5/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 190

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{5/2}}\,dx = -\frac{2x^3}{3ac\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{3/2}} - \frac{4x^2}{a^2c\left(c+a^2cx^2\right)^{3/2}\sqrt{\arctan(ax)}} - \frac{\sqrt{2\pi}\sqrt{1+a^2x^2}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{a^4c^2\sqrt{c+a^2cx^2}} + \frac{\sqrt{6\pi}\sqrt{1+a^2x^2}\,\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)}{a^4c^2\sqrt{c+a^2cx^2}}$$

output
```
-2/3*x^3/a/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2)-4*x^2/a^2/c/(a^2*c*x^2+
c)^(3/2)/arctan(a*x)^(1/2)-2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)*FresnelS(2^(
1/2)/Pi^(1/2)*arctan(a*x)^(1/2))/a^4/c^2/(a^2*c*x^2+c)^(1/2)+6^(1/2)*Pi^(1
/2)*(a^2*x^2+1)^(1/2)*FresnelS(6^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))/a^4/c^2
/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.73 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.34

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx = \frac{-2a^2x^2(ax+6\arctan(ax))+\sqrt{6\pi}(1+a^2x^2)^{3/2}\arctan(ax)^{3/2}\left(-3\sqrt{\phantom{x}}\right.}{}$$

input
```
Integrate[x^3/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)),x]
```

output
```
(-2*a^2*x^2*(a*x + 6*ArcTan[a*x]) + Sqrt[6*Pi]*(1 + a^2*x^2)^(3/2)*ArcTan[
a*x]^(3/2)*(-3*Sqrt[3]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]] + FresnelS[S
qrt[6/Pi]*Sqrt[ArcTan[a*x]]]) - (1 + a^2*x^2)^(3/2)*ArcTan[a*x]*(3*Sqrt[(-
I)*ArcTan[a*x]]*Gamma[1/2, (-I)*ArcTan[a*x]] + 3*Sqrt[I*ArcTan[a*x]]*Gamma
[1/2, I*ArcTan[a*x]] + Sqrt[3]*(Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-3*I)*A
rcTan[a*x]] + Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (3*I)*ArcTan[a*x]])))/(3*a^4*
c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2))
```

## Rubi [A] (verified)

Time = 1.54 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.45, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {5477, 5503, 5506, 5505, 3042, 3793, 2009, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\arctan(ax)^{5/2}(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{5477}$$

$$\frac{2\int \frac{x^2}{(a^2cx^2+c)^{5/2}\arctan(ax)^{3/2}}dx}{a} - \frac{2x^3}{3ac\arctan(ax)^{3/2}(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5503}$$

$$\frac{2\left(\frac{4\int \frac{x}{\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-2a\int \frac{x^3}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{2x^2}{ac\sqrt{\arctan(ax)}(a^2cx^2+c)^{3/2}}\right)}{a}-\frac{2x^3}{3ac\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

$\big\downarrow$ 5506

$$\frac{2\left(\frac{4\sqrt{a^2x^2+1}\int \frac{x}{\left(a^2x^2+1\right)^{5/2}\sqrt{\arctan(ax)}}dx}{ac^2\sqrt{a^2cx^2+c}}-\frac{2a\sqrt{a^2x^2+1}\int \frac{x^3}{\left(a^2x^2+1\right)^{5/2}\sqrt{\arctan(ax)}}dx}{c^2\sqrt{a^2cx^2+c}}-\frac{2x^2}{ac\sqrt{\arctan(ax)}(a^2cx^2+c)^{3/2}}\right)}{a}-\frac{2x^3}{3ac\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

$\big\downarrow$ 5505

$$\frac{2\left(\frac{4\sqrt{a^2x^2+1}\int \frac{ax}{\left(a^2x^2+1\right)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3c^2\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}\int \frac{a^3x^3}{\left(a^2x^2+1\right)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3c^2\sqrt{a^2cx^2+c}}-\frac{2x^2}{ac\sqrt{\arctan(ax)}(a^2cx^2+c)^{3/2}}\right.}{a}$$
$$\frac{2x^3}{3ac\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

$\big\downarrow$ 3042

$$\frac{2\left(\frac{4\sqrt{a^2x^2+1}\int \frac{ax}{\left(a^2x^2+1\right)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3c^2\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}\int \frac{\sin(\arctan(ax))^3}{\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3c^2\sqrt{a^2cx^2+c}}-\frac{2x^2}{ac\sqrt{\arctan(ax)}(a^2cx^2+c)^{3/2}}\right)}{a}-\frac{2x^3}{3ac\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

$\big\downarrow$ 3793

$$\frac{2\left(\frac{4\sqrt{a^2x^2+1}\int \frac{ax}{\left(a^2x^2+1\right)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3c^2\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}\int \left(\frac{3ax}{4\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}-\frac{\sin(3\arctan(ax))}{4\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^3c^2\sqrt{a^2cx^2+c}}-\frac{}{ac\sqrt{\arctan(}}\right.}{a}$$
$$\frac{2x^3}{3ac\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

$\big\downarrow$ 2009

$$\frac{2\left(\frac{4\sqrt{a^2x^2+1}\int \frac{ax}{\left(a^2x^2+1\right)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3c^2\sqrt{a^2cx^2+c}}-\frac{2x^2}{ac\sqrt{\arctan(ax)}(a^2cx^2+c)^{3/2}}-\frac{2\sqrt{a^2x^2+1}\left(\frac{3}{2}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{2}\right.}{a^3c^2\sqrt{a^2cx^2+c}}\right.}{a}$$

$$\frac{2x^3}{3ac\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \; 4906$$

$$\frac{2\left(\frac{4\sqrt{a^2x^2+1}\int \left(\frac{ax}{4\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}+\frac{\sin(3\arctan(ax))}{4\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^3c^2\sqrt{a^2cx^2+c}}-\frac{2x^2}{ac\sqrt{\arctan(ax)}(a^2cx^2+c)^{3/2}}-\frac{2\sqrt{a^2x^2+1}\left(\frac{3}{2}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\right.\right.}{}\right.}{a}$$

$$\frac{2x^3}{3ac\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \; 2009$$

$$\frac{2\left(-\frac{2x^2}{ac\sqrt{\arctan(ax)}(a^2cx^2+c)^{3/2}}-\frac{2\sqrt{a^2x^2+1}\left(\frac{3}{2}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{2}\sqrt{\frac{\pi}{6}}\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{a^3c^2\sqrt{a^2cx^2+c}}+\frac{4\sqrt{a^2x^2+1}\left(\right.}{}\right.}{a}$$

$$\frac{2x^3}{3ac\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

input

```
Int[x^3/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)),x]
```

output

```
(-2*x^3)/(3*a*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)) + (2*((-2*x^2)/(a
*c*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]) - (2*Sqrt[1 + a^2*x^2]*((3*Sqr
t[Pi/2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/2 - (Sqrt[Pi/6]*FresnelS[S
qrt[6/Pi]*Sqrt[ArcTan[a*x]]])/2))/(a^3*c^2*Sqrt[c + a^2*c*x^2]) + (4*Sqrt[
1 + a^2*x^2]*((Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/2 + (Sqr
t[Pi/6]*FresnelS[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/2))/(a^3*c^2*Sqrt[c + a^2*
c*x^2])))/a
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5477
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*ArcT
an[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp[f*(m/(b*c*(p + 1)))   Int[(f*x
)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1
]
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

## Maple [F]

$$\int \frac{x^3}{(a^2c\,x^2+c)^{\frac{5}{2}} \arctan (ax)^{\frac{5}{2}}}dx$$

input
```
int(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

output
```
int(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**3/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(5/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{5/2}}\,dx=\int \frac{x^3}{\mathrm{atan}\left(a\,x\right)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

input
```
int(x^3/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(x^3/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2)), x)
```

## Reduce [F]

$$\int \frac{x^3}{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{5/2}}\,dx=\frac{\sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,x^3}{atan(ax)^3a^6x^6+3atan(ax)^3a^4x^4+3atan(ax)^3a^2x^2+atan(ax)^3}dx\right)}{c^3}$$

input
```
int(x^3/(a^2*c*x^2+c)^(5/2)/atan(a*x)^(5/2),x)
```

output
```
(sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**3)/(atan(a*x)**3*a**6
*x**6 + 3*atan(a*x)**3*a**4*x**4 + 3*atan(a*x)**3*a**2*x**2 + atan(a*x)**3
),x))/c**3
```

# 3.1103 $\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 224

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx=-\frac{2x^2}{3ac(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}-\frac{8x}{3a^2c(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}+\frac{4x^3}{3c(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}-\frac{\sqrt{2\pi}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{3a^3c^2\sqrt{c+a^2cx^2}}+\frac{\sqrt{6\pi}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)}{a^3c^2\sqrt{c+a^2cx^2}}$$

output
```
-2/3*x^2/a/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2)-8/3*x/a^2/c/(a^2*c*x^2+
c)^(3/2)/arctan(a*x)^(1/2)+4/3*x^3/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2)
-1/3*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*arctan(a
*x)^(1/2))/a^3/c^2/(a^2*c*x^2+c)^(1/2)+6^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)*
FresnelC(6^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))/a^3/c^2/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.62 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.39

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx = \frac{-(1+a^2x^2)^{3/2}(-i\arctan(ax))^{3/2}\Gamma\left(\frac{1}{2},-i\arctan(ax)\right)+\frac{-4a^2x^2\sqrt{i\arctan}}{}}{}$$

input
```
Integrate[x^2/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)),x]
```

output
```
(-((1 + a^2*x^2)^(3/2)*((-I)*ArcTan[a*x])^(3/2)*Gamma[1/2, (-I)*ArcTan[a*x
]]) + (-4*a^2*x^2*Sqrt[I*ArcTan[a*x]] + (16*I)*a*x*(I*ArcTan[a*x])^(3/2) -
 (8*I)*a^3*x^3*(I*ArcTan[a*x])^(3/2) + (1 + a^2*x^2)^(3/2)*ArcTan[a*x]^2*G
amma[1/2, I*ArcTan[a*x]] - (3*I)*Sqrt[3]*(1 + a^2*x^2)^(3/2)*ArcTan[a*x]*S
qrt[ArcTan[a*x]^2]*Gamma[1/2, (-3*I)*ArcTan[a*x]] - 3*Sqrt[3 + 3*a^2*x^2]*
ArcTan[a*x]^2*Gamma[1/2, (3*I)*ArcTan[a*x]] - 3*a^2*x^2*Sqrt[3 + 3*a^2*x^2
]*ArcTan[a*x]^2*Gamma[1/2, (3*I)*ArcTan[a*x]])/Sqrt[I*ArcTan[a*x]])/(6*a^3
*c^2*(1 + a^2*x^2)*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2))
```

## Rubi [A] (verified)

Time = 3.41 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.85, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {5503, 5477, 5503, 5440, 5439, 3042, 3793, 2009, 5506, 5505, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\arctan(ax)^{5/2}(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{5503}$$

$$\frac{4\int \frac{x}{(a^2cx^2+c)^{5/2}\arctan(ax)^{3/2}}dx}{3a}-\frac{2}{3}a\int \frac{x^3}{(a^2cx^2+c)^{5/2}\arctan(ax)^{3/2}}dx-\frac{2x^2}{3ac\arctan(ax)^{3/2}(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5477}$$

$$\frac{4\int \frac{x}{(a^2cx^2+c)^{5/2}\arctan(ax)^{3/2}}dx}{3a}-\frac{2}{3}a\left(\frac{6\int \frac{x^2}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2x^3}{ac\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}\right)-\frac{2x^2}{3ac\arctan(ax)^{3/2}\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5503}$$

$$\frac{4\left(\frac{2\int \frac{1}{\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-4a\int \frac{x^2}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{2x}{ac\sqrt{\arctan(ax)}(a^2cx^2+c)^{3/2}}\right)}{3a}-\frac{2}{3}a\left(\frac{6\int \frac{x^2}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2x^3}{ac\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}\right)-\frac{2x^2}{3ac\arctan(ax)^{3/2}\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5440}$$

$$\frac{4\left(\frac{2\sqrt{a^2x^2+1}\int \frac{1}{\left(a^2x^2+1\right)^{5/2}\sqrt{\arctan(ax)}}dx}{ac^2\sqrt{a^2cx^2+c}}-4a\int \frac{x^2}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{2x}{ac\sqrt{\arctan(ax)}(a^2cx^2+c)^{3/2}}\right)}{3a}-\frac{2}{3}a\left(\frac{6\int \frac{x^2}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2x^3}{ac\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}\right)-\frac{2x^2}{3ac\arctan(ax)^{3/2}\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5439}$$

$$\frac{4\left(\frac{2\sqrt{a^2x^2+1}\int \frac{1}{\left(a^2x^2+1\right)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}-4a\int \frac{x^2}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{2x}{ac\sqrt{\arctan(ax)}(a^2cx^2+c)^{3/2}}\right)}{3a}-\frac{2}{3}a\left(\frac{6\int \frac{x^2}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2x^3}{ac\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}\right)-\frac{2x^2}{3ac\arctan(ax)^{3/2}\,(a^2cx^2+c)^{3/2}}$$

$\Big\downarrow$ 3042

$$\frac{4\left(\frac{2\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^3}{\sqrt{\arctan(ax)}}d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}-4a\int \frac{x^2}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{2x}{ac\sqrt{\arctan(ax)}(a^2cx^2+c)^{3/2}}\right)}{3a}-\frac{2}{3}a\left(\frac{6\int \frac{x^2}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2x^3}{ac\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}\right)-\frac{2x^2}{3ac\arctan(ax)^{3/2}\,(a^2cx^2+c)^{3/2}}$$

$\Big\downarrow$ 3793

$$\frac{4\left(\frac{2\sqrt{a^2x^2+1}\int \left(\frac{\cos(3\arctan(ax))}{4\sqrt{\arctan(ax)}}+\frac{3}{4\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}-4a\int \frac{x^2}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{2x}{ac\sqrt{\arctan(ax)}(a^2cx^2+c}\right.}{3a}$$

$$\frac{2}{3}a\left(\frac{6\int \frac{x^2}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2x^3}{ac\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}\right)-\frac{2x^2}{3ac\arctan(ax)^{3/2}\,(a^2cx^2+c)^{3/2}}$$

$\Big\downarrow$ 2009

$$\frac{4\left(-4a\int \frac{x^2}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx+\frac{2\sqrt{a^2x^2+1}\left(\frac{3}{2}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{6}}\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{a^2c^2\sqrt{a^2cx^2+c}}-\frac{}{ac\sqrt{}}\right.}{3a}$$

$$\frac{2}{3}a\left(\frac{6\int \frac{x^2}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2x^3}{ac\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}\right)-\frac{2x^2}{3ac\arctan(ax)^{3/2}\,(a^2cx^2+c)^{3/2}}$$

$\Big\downarrow$ 5506

$$\frac{4\left(-\frac{4a\sqrt{a^2x^2+1}\int \frac{x^2}{\left(a^2x^2+1\right)^{5/2}\sqrt{\arctan(ax)}}dx}{c^2\sqrt{a^2cx^2+c}}+\frac{2\sqrt{a^2x^2+1}\left(\frac{3}{2}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{6}}\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{a^2c^2\sqrt{a^2cx^2+c}}-\right.}{3a}$$

$$\frac{2}{3}a\left(\frac{6\sqrt{a^2x^2+1}\int \frac{x^2}{(a^2x^2+1)^{5/2}\sqrt{\arctan(ax)}}dx}{ac^2\sqrt{a^2cx^2+c}}-\frac{2x^3}{ac\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}\right)-\frac{2x^2}{3ac\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{ 5505}$$

$$\frac{4\left(-\frac{4\sqrt{a^2x^2+1}\int \frac{a^2x^2}{\left(a^2x^2+1\right)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}+\frac{2\sqrt{a^2x^2+1}\left(\frac{3}{2}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{6}}\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(a\right.\right.}{a^2c^2\sqrt{a^2cx^2+c}}\right.}{3a}$$

$$\frac{2}{3}a\left(\frac{6\sqrt{a^2x^2+1}\int \frac{a^2x^2}{(a^2x^2+1)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{a^4c^2\sqrt{a^2cx^2+c}}-\frac{2x^3}{ac\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}\right)-\frac{2x^2}{3ac\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{ 4906}$$

$$\frac{4\left(-\frac{4\sqrt{a^2x^2+1}\int \left(\frac{1}{4\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}-\frac{\cos(3\arctan(ax))}{4\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}+\frac{2\sqrt{a^2x^2+1}\left(\frac{3}{2}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{6}}\text{FresnelC}\right.}{a^2c^2\sqrt{a^2cx^2+c}}\right.}{3a}$$

$$\frac{2}{3}a\left(\frac{6\sqrt{a^2x^2+1}\int \left(\frac{1}{4\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}-\frac{\cos(3\arctan(ax))}{4\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^4c^2\sqrt{a^2cx^2+c}}-\frac{2x^3}{ac\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}\right)-\frac{2x^2}{3ac\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{4\left(-\frac{4\sqrt{a^2x^2+1}\left(\frac{1}{2}\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{2}\sqrt{\frac{\pi}{6}}\operatorname{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{a^2c^2\sqrt{a^2cx^2+c}}+\frac{2\sqrt{a^2x^2+1}\left(\frac{3}{2}\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right.\right.}{a^2c^2\sqrt{a^2c}}\right.}{3a}$$

$$\frac{2x^2}{3ac\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}-$$

$$\frac{2}{3}a\left(\frac{6\sqrt{a^2x^2+1}\left(\frac{1}{2}\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{2}\sqrt{\frac{\pi}{6}}\operatorname{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{a^4c^2\sqrt{a^2cx^2+c}}-\frac{2x}{ac\sqrt{\arctan(ax)}}\right.$$

input
```
Int[x^2/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)),x]
```

output
```
(-2*x^2)/(3*a*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)) - (2*a*((-2*x^3)/
(a*c*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]) + (6*Sqrt[1 + a^2*x^2]*((Sqr
t[Pi/2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/2 - (Sqrt[Pi/6]*FresnelC[S
qrt[6/Pi]*Sqrt[ArcTan[a*x]]])/2))/(a^4*c^2*Sqrt[c + a^2*c*x^2])))/3 + (4*(
(-2*x)/(a*c*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]) - (4*Sqrt[1 + a^2*x^2
]*((Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/2 - (Sqrt[Pi/6]*Fre
snelC[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/2))/(a^2*c^2*Sqrt[c + a^2*c*x^2]) + (
2*Sqrt[1 + a^2*x^2]*((3*Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])
/2 + (Sqrt[Pi/6]*FresnelC[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/2))/(a^2*c^2*Sqrt
[c + a^2*c*x^2])))/(3*a)
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5440
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])   Int[(1 +
c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

rule 5477
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((f_.)*(x_))^(m_.)*((d_) + (e_.
)*(x_)^2)^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x^2)^(q + 1)*((a + b*ArcT
an[c*x])^(p + 1)/(b*c*d*(p + 1))), x] - Simp[f*(m/(b*c*(p + 1)))   Int[(f*x
)^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, e, f, m, q}, x] && EqQ[e, c^2*d] && EqQ[m + 2*q + 2, 0] && LtQ[p, -1
]
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

## Maple [F]

$$\int \frac{x^2}{(a^2c\,x^2+c)^{\frac{5}{2}} \arctan\left(ax\right)^{\frac{5}{2}}}dx$$

input
```
int(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

output
```
int(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2} \arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**2/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(5/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F]

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx = \int \frac{x^2}{(a^2cx^2+c)^{\frac{5}{2}}\arctan(ax)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(x^2/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^(5/2)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx = \int \frac{x^2}{\mathrm{atan}\left(a\,x\right)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

input
```
int(x^2/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(x^2/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2)), x)
```

## Reduce [F]

$$\int \frac{x^2}{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}\,x^2}{atan(ax)^3a^6x^6+3atan(ax)^3a^4x^4+3atan(ax)^3a^2x^2+atan(ax)^3}dx\right)}{c^3}$$

input
```
int(x^2/(a^2*c*x^2+c)^(5/2)/atan(a*x)^(5/2),x)
```

output
```
(sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**2)/(atan(a*x)**3*a**6
*x**6 + 3*atan(a*x)**3*a**4*x**4 + 3*atan(a*x)**3*a**2*x**2 + atan(a*x)**3
),x))/c**3
```

# 3.1104 $\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 222

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx = -\frac{2x}{3ac(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}} - \frac{4}{3a^2c(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}} + \frac{8x^2}{3c(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}} - \frac{\sqrt{2\pi}\sqrt{1+a^2x^2}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{3a^2c^2\sqrt{c+a^2cx^2}} - \frac{\sqrt{6\pi}\sqrt{1+a^2x^2}\,\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)}{a^2c^2\sqrt{c+a^2cx^2}}$$

output
```
-2/3*x/a/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2)-4/3/a^2/c/(a^2*c*x^2+c)^(
3/2)/arctan(a*x)^(1/2)+8/3*x^2/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2)-1/3
*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)*arctan(a*x)^
(1/2))/a^2/c^2/(a^2*c*x^2+c)^(1/2)-6^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)*Fres
nelS(6^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))/a^2/c^2/(a^2*c*x^2+c)^(1/2)
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.71 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.18

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx = \frac{-12(ax+(2-4a^2x^2)\arctan(ax))+4\sqrt{6\pi}(1+a^2x^2)^{3/2}\arctan(ax)^{3/}}{}$$

input
```
Integrate[x/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)),x]
```

output
```
(-12*(a*x + (2 - 4*a^2*x^2)*ArcTan[a*x]) + 4*Sqrt[6*Pi]*(1 + a^2*x^2)^(3/2
)*ArcTan[a*x]^(3/2)*(3*Sqrt[3]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]] - Fr
esnelS[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]]) + 7*(1 + a^2*x^2)^(3/2)*ArcTan[a*x]*
(3*Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2, (-I)*ArcTan[a*x]] + 3*Sqrt[I*ArcTan[a
*x]]*Gamma[1/2, I*ArcTan[a*x]] + Sqrt[3]*(Sqrt[(-I)*ArcTan[a*x]]*Gamma[1/2
, (-3*I)*ArcTan[a*x]] + Sqrt[I*ArcTan[a*x]]*Gamma[1/2, (3*I)*ArcTan[a*x]])
))/(18*a^2*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2))
```

## Rubi [A] (verified)

Time = 2.13 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.85, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5503, 5437, 5503, 5506, 5505, 3042, 3793, 2009, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\arctan(ax)^{5/2}(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \; 5503$$

$$\frac{2\int \frac{1}{(a^2cx^2+c)^{5/2}\arctan(ax)^{3/2}}dx}{3a}-\frac{4}{3}a\int \frac{x^2}{(a^2cx^2+c)^{5/2}\arctan(ax)^{3/2}}dx-\frac{2x}{3ac\arctan(ax)^{3/2}(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5437}$$

$$-\frac{4}{3}a\int \frac{x^2}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^{3/2}}dx+\frac{2\left(-6a\int \frac{x}{\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{2}{ac\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}\right)}{3a}-\frac{2x}{3ac\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{5503}$$

$$\frac{2\left(-6a\int \frac{x}{\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{2}{ac\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}\right)}{3a}-\frac{4}{3}a\left(\frac{4\int \frac{x}{\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-2a\int \frac{x^3}{\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{2x^2}{ac\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}\right)-\frac{2x}{3ac\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{5506}$$

$$\frac{2\left(-\frac{6a\sqrt{a^2x^2+1}\int \frac{x}{\left(a^2x^2+1\right)^{5/2}\sqrt{\arctan(ax)}}dx}{c^2\sqrt{a^2cx^2+c}}-\frac{2}{ac\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}\right)}{3a}-\frac{4}{3}a\left(\frac{4\sqrt{a^2x^2+1}\int \frac{x}{\left(a^2x^2+1\right)^{5/2}\sqrt{\arctan(ax)}}dx}{ac^2\sqrt{a^2cx^2+c}}-\frac{2a\sqrt{a^2x^2+1}\int \frac{x^3}{\left(a^2x^2+1\right)^{5/2}\sqrt{\arctan(ax)}}dx}{c^2\sqrt{a^2cx^2+c}}-\frac{2x^2}{ac\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}\right)-\frac{2x}{3ac\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{5505}$$

$$\frac{2\left(-\frac{6\sqrt{a^2x^2+1}\int \frac{ax}{\left(a^2x^2+1\right)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{2}{ac\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}\right)}{3a}-\frac{4}{3}a\left(\frac{4\sqrt{a^2x^2+1}\int \frac{ax}{\left(a^2x^2+1\right)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3c^2\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}\int \frac{a^3x^3}{\left(a^2x^2+1\right)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3c^2\sqrt{a^2cx^2+c}}-\frac{2x^2}{ac\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}\right)-\frac{2x}{3ac\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\left(-\frac{6\sqrt{a^2x^2+1}\int \frac{ax}{\left(a^2x^2+1\right)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{2}{ac\sqrt{\arctan(ax)}(a^2cx^2+c)^{3/2}}\right)}{3a}-\frac{4}{3}a\left(\frac{4\sqrt{a^2x^2+1}\int \frac{ax}{(a^2x^2+1)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3c^2\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}\int \frac{\sin(\arctan(ax))^3}{\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3c^2\sqrt{a^2cx^2+c}}-\frac{}{ac\sqrt{\arctan(}}\right.$$

$$\frac{2x}{3ac\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

$\big\downarrow$ 3793

$$\frac{2\left(-\frac{6\sqrt{a^2x^2+1}\int \frac{ax}{\left(a^2x^2+1\right)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{2}{ac\sqrt{\arctan(ax)}(a^2cx^2+c)^{3/2}}\right)}{3a}-\frac{4}{3}a\left(\frac{4\sqrt{a^2x^2+1}\int \frac{ax}{(a^2x^2+1)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3c^2\sqrt{a^2cx^2+c}}-\frac{2\sqrt{a^2x^2+1}\int \left(\frac{3ax}{4\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}-\frac{\sin(3\arctan(ax))}{4\sqrt{\arctan(ax)}}\right)d\,a}{a^3c^2\sqrt{a^2cx^2+c}}\right.$$

$$\frac{2x}{3ac\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

$\big\downarrow$ 2009

$$\frac{2\left(-\frac{6\sqrt{a^2x^2+1}\int \frac{ax}{\left(a^2x^2+1\right)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{2}{ac\sqrt{\arctan(ax)}(a^2cx^2+c)^{3/2}}\right)}{3a}-\frac{4}{3}a\left(\frac{4\sqrt{a^2x^2+1}\int \frac{ax}{(a^2x^2+1)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{a^3c^2\sqrt{a^2cx^2+c}}-\frac{2x^2}{ac\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}-\frac{2\sqrt{a^2x^2+1}\left(\frac{3}{2}\sqrt{\frac{\pi}{2}}\,\mathrm{Fre}\right.}{}\right.$$

$$\frac{2x}{3ac\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

$\big\downarrow$ 4906

$$\frac{2\left(-\frac{6\sqrt{a^2x^2+1}\int \left(\frac{ax}{4\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}+\frac{\sin(3\arctan(ax))}{4\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{2}{ac\sqrt{\arctan(ax)}(a^2cx^2+c)^{3/2}}\right)}{3a}-\frac{4}{3}a\left(\frac{4\sqrt{a^2x^2+1}\int \left(\frac{ax}{4\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}+\frac{\sin(3\arctan(ax))}{4\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^3c^2\sqrt{a^2cx^2+c}}-\frac{2x^2}{ac\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}-\frac{2\sqrt{a}}{}\right.$$

$$\frac{2x}{3ac\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

$\big\downarrow$ 2009

$$\frac{2\left(-\frac{6\sqrt{a^2x^2+1}\left(\frac{1}{2}\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{6}}\operatorname{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{ac^2\sqrt{a^2cx^2+c}}-\frac{2}{ac\sqrt{\arctan(ax)}(a^2cx^2+c)^{3/2}}\right)}{3a}-$$

$$\frac{2x}{3ac\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}-$$

$$\frac{4}{3}a\left(-\frac{2x^2}{ac\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}-\frac{2\sqrt{a^2x^2+1}\left(\frac{3}{2}\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{2}\sqrt{\frac{\pi}{6}}\operatorname{FresnelS}\left(\sqrt{\frac{6}{\pi}}\right.\right.}{a^3c^2\sqrt{a^2cx^2+c}}\right.$$

input
```
Int[x/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)),x]
```

output
```
(-2*x)/(3*a*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)) - (4*a*((-2*x^2)/(a
*c*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]) - (2*Sqrt[1 + a^2*x^2]*((3*Sqr
t[Pi/2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/2 - (Sqrt[Pi/6]*FresnelS[S
qrt[6/Pi]*Sqrt[ArcTan[a*x]]])/2))/(a^3*c^2*Sqrt[c + a^2*c*x^2]) + (4*Sqrt[
1 + a^2*x^2]*((Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/2 + (Sqr
t[Pi/6]*FresnelS[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/2))/(a^3*c^2*Sqrt[c + a^2*
c*x^2])))/3 + (2*(-2/(a*c*(c + a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]) - (6*Sq
rt[1 + a^2*x^2]*((Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/2 + (
Sqrt[Pi/6]*FresnelS[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/2))/(a*c^2*Sqrt[c + a^2
*c*x^2])))/(3*a)
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

## Maple [F]

$$\int \frac{x}{\left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}} \arctan \left(a x \right)^{\frac{5}{2}}}d x$$

input
```
int(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

output
```
int(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate(x/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(5/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx = \int \frac{x}{\mathrm{atan}\left(a\,x\right)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

input
```
int(x/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(x/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2)), x)
```

## Reduce [F]

$$\int \frac{x}{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx = \text{Too large to display}$$

input
```
int(x/(a^2*c*x^2+c)^(5/2)/atan(a*x)^(5/2),x)
```

output
```
(2*sqrt(c)*( - 2*atan(a*x)**2*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**
2)/(atan(a*x)**2*a**6*x**6 + 3*atan(a*x)**2*a**4*x**4 + 3*atan(a*x)**2*a**
2*x**2 + atan(a*x)**2),x)*a**7*x**4 - 4*atan(a*x)**2*int((sqrt(a**2*x**2 +
 1)*sqrt(atan(a*x))*x**2)/(atan(a*x)**2*a**6*x**6 + 3*atan(a*x)**2*a**4*x*
*4 + 3*atan(a*x)**2*a**2*x**2 + atan(a*x)**2),x)*a**5*x**2 - 2*atan(a*x)**
2*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**2)/(atan(a*x)**2*a**6*x**6 +
 3*atan(a*x)**2*a**4*x**4 + 3*atan(a*x)**2*a**2*x**2 + atan(a*x)**2),x)*a*
*3 - 6*atan(a*x)**2*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x)/(atan(a*x)
*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)
*a**6*x**4 - 12*atan(a*x)**2*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x)/(
atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan
(a*x)),x)*a**4*x**2 - 6*atan(a*x)**2*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*
x))*x)/(atan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**
2 + atan(a*x)),x)*a**2 - 2*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x) -
 sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*a*x))/(3*atan(a*x)**2*a**2*c**3*(a**4
*x**4 + 2*a**2*x**2 + 1))
```

# 3.1105 $\int \frac{1}{(c+a^2cx^2)^{5/2} \arctan(ax)^{5/2}} \, dx$



## Optimal result

Integrand size = 23, antiderivative size = 183

$$\int \frac{1}{(c+a^2cx^2)^{5/2} \arctan(ax)^{5/2}} \, dx = -\frac{2}{3ac(c+a^2cx^2)^{3/2} \arctan(ax)^{3/2}} + \frac{4x}{c(c+a^2cx^2)^{3/2} \sqrt{\arctan(ax)}} - \frac{\sqrt{2\pi}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{ac^2\sqrt{c+a^2cx^2}} - \frac{\sqrt{6\pi}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)}{ac^2\sqrt{c+a^2cx^2}}$$

output
```
-2/3/a/c/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2)+4*x/c/(a^2*c*x^2+c)^(3/2)/a
rctan(a*x)^(1/2)-2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)*FresnelC(2^(1/2)/Pi^(1
/2)*arctan(a*x)^(1/2))/a/c^2/(a^2*c*x^2+c)^(1/2)-6^(1/2)*Pi^(1/2)*(a^2*x^2
+1)^(1/2)*FresnelC(6^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))/a/c^2/(a^2*c*x^2+c)
^(1/2)
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.64

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx = \frac{-4+24ax\arctan(ax)-3(1+a^2x^2)^{3/2}\,(-i\arctan(ax))^{3/2}\Gamma\left(\frac{1}{2},-i\arctan(ax)\right)\ldots}{}$$

input
```
Integrate[1/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)),x]
```

output
```
(-4 + 24*a*x*ArcTan[a*x] - 3*(1 + a^2*x^2)^(3/2)*((-I)*ArcTan[a*x])^(3/2)*
Gamma[1/2, (-I)*ArcTan[a*x]] - 3*(1 + a^2*x^2)^(3/2)*(I*ArcTan[a*x])^(3/2)
*Gamma[1/2, I*ArcTan[a*x]] - 3*Sqrt[3 + 3*a^2*x^2]*((-I)*ArcTan[a*x])^(3/2
)*Gamma[1/2, (-3*I)*ArcTan[a*x]] - 3*a^2*x^2*Sqrt[3 + 3*a^2*x^2]*((-I)*Arc
Tan[a*x])^(3/2)*Gamma[1/2, (-3*I)*ArcTan[a*x]] - 3*Sqrt[3 + 3*a^2*x^2]*(I*
ArcTan[a*x])^(3/2)*Gamma[1/2, (3*I)*ArcTan[a*x]] - 3*a^2*x^2*Sqrt[3 + 3*a^
2*x^2]*(I*ArcTan[a*x])^(3/2)*Gamma[1/2, (3*I)*ArcTan[a*x]])/(6*c^2*(a + a^
3*x^2)*Sqrt[c + a^2*c*x^2]*ArcTan[a*x]^(3/2))
```

## Rubi [A] (verified)

Time = 1.86 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.46, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {5437, 5503, 5440, 5439, 3042, 3793, 2009, 5506, 5505, 4906, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\arctan(ax)^{5/2}\,(a^2cx^2+c)^{5/2}}dx$$

$$\Big\downarrow \text{5437}$$

$$-2a\int \frac{x}{(a^2cx^2+c)^{5/2}\arctan(ax)^{3/2}}dx-\frac{2}{3ac\arctan(ax)^{3/2}\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5503}$$

$$-2a\left(\frac{2\int \frac{1}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-4a\int \frac{x^2}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{2x}{ac\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}\right)-\frac{2}{3ac\arctan(ax)^{3/2}\,(a^2cx^2+c)^{3/2}}$$

$\downarrow$ 5440

$$-2a\left(\frac{2\sqrt{a^2x^2+1}\int \frac{1}{(a^2x^2+1)^{5/2}\sqrt{\arctan(ax)}}dx}{ac^2\sqrt{a^2cx^2+c}}-4a\int \frac{x^2}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{2x}{ac\sqrt{\arctan(ax)}\,(a^2cx^2+\right.$$
$$\frac{2}{3ac\arctan(ax)^{3/2}\,(a^2cx^2+c)^{3/2}}$$

$\downarrow$ 5439

$$-2a\left(\frac{2\sqrt{a^2x^2+1}\int \frac{1}{(a^2x^2+1)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}-4a\int \frac{x^2}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{2}{ac\sqrt{\arctan(ax}}\right.$$
$$\frac{2}{3ac\arctan(ax)^{3/2}\,(a^2cx^2+c)^{3/2}}$$

$\downarrow$ 3042

$$-2a\left(\frac{2\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^3}{\sqrt{\arctan(ax)}}d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}-4a\int \frac{x^2}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{2x}{ac\sqrt{\arctan(ax)}\,(a^2}\right.$$
$$\frac{2}{3ac\arctan(ax)^{3/2}\,(a^2cx^2+c)^{3/2}}$$

$\downarrow$ 3793

$$-2a\left(\frac{2\sqrt{a^2x^2+1}\int \left(\frac{\cos(3\arctan(ax))}{4\sqrt{\arctan(ax)}}+\frac{3}{4\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}-4a\int \frac{x^2}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx\right.$$
$$\frac{2}{3ac\arctan(ax)^{3/2}\,(a^2cx^2+c)^{3/2}}$$

$\downarrow$ 2009

$$-2a\left(-4a\int \frac{x^2}{\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx+\frac{2\sqrt{a^2x^2+1}\left(\frac{3}{2}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{6}}\text{Fresnel}\right.}{a^2c^2\sqrt{a^2cx^2+c}}\right.$$
$$\frac{2}{3ac\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

$\Big\downarrow$ 5506

$$-2a\left(-\frac{4a\sqrt{a^2x^2+1}\int \frac{x^2}{\left(a^2x^2+1\right)^{5/2}\sqrt{\arctan(ax)}}dx}{c^2\sqrt{a^2cx^2+c}}+\frac{2\sqrt{a^2x^2+1}\left(\frac{3}{2}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{6}}\text{Fres}\right.}{a^2c^2\sqrt{a^2cx^2+c}}\right.$$
$$\frac{2}{3ac\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

$\Big\downarrow$ 5505

$$-2a\left(-\frac{4\sqrt{a^2x^2+1}\int \frac{a^2x^2}{\left(a^2x^2+1\right)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}+\frac{2\sqrt{a^2x^2+1}\left(\frac{3}{2}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\right.}{a^2c^2\sqrt{a^2cx^2+c}}\right.$$
$$\frac{2}{3ac\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

$\Big\downarrow$ 4906

$$-2a\left(-\frac{4\sqrt{a^2x^2+1}\int \left(\frac{1}{4\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}-\frac{\cos(3\arctan(ax))}{4\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{a^2c^2\sqrt{a^2cx^2+c}}+\frac{2\sqrt{a^2x^2+1}\left(\frac{3}{2}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\right.\right.}{}\right.$$
$$\frac{2}{3ac\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

$\Big\downarrow$ 2009

$$-2a\left(-\frac{4\sqrt{a^2x^2+1}\left(\frac{1}{2}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)-\frac{1}{2}\sqrt{\frac{\pi}{6}}\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{a^2c^2\sqrt{a^2cx^2+c}}+\frac{2\sqrt{a^2x^2+1}\left(\right.}{}\right.$$
$$\frac{2}{3ac\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

input
```
Int[1/((c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)),x]
```

output
```
-2/(3*a*c*(c + a^2*c*x^2)^(3/2)*ArcTan[a*x]^(3/2)) - 2*a*((-2*x)/(a*c*(c +
 a^2*c*x^2)^(3/2)*Sqrt[ArcTan[a*x]]) - (4*Sqrt[1 + a^2*x^2]*((Sqrt[Pi/2]*F
resnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/2 - (Sqrt[Pi/6]*FresnelC[Sqrt[6/Pi]
*Sqrt[ArcTan[a*x]]])/2))/(a^2*c^2*Sqrt[c + a^2*c*x^2]) + (2*Sqrt[1 + a^2*x
^2]*((3*Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcTan[a*x]]])/2 + (Sqrt[Pi/6]
*FresnelC[Sqrt[6/Pi]*Sqrt[ArcTan[a*x]]])/2))/(a^2*c^2*Sqrt[c + a^2*c*x^2])
)
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 4906
```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

rule 5437
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_)^2)^(q_), x_S
ymbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p +
 1))), x] - Simp[2*c*((q + 1)/(b*(p + 1)))   Int[x*(d + e*x^2)^q*(a + b*Arc
Tan[c*x])^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[e, c^2*d] &&
LtQ[q, -1] && LtQ[p, -1]
```

rule 5439
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^q/c   Subst[Int[(a + b*x)^p/Cos[x]^(2*(q + 1)), x], x, Ar
cTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[e, c^2*d] && ILtQ[2*(
q + 1), 0] && (IntegerQ[q] || GtQ[d, 0])
```

rule 5440
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_), x_
Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])   Int[(1 +
c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
 EqQ[e, c^2*d] && ILtQ[2*(q + 1), 0] &&  !(IntegerQ[q] || GtQ[d, 0])
```

rule 5503
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[x^m*(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])^(p
+ 1)/(b*c*d*(p + 1))), x] + (-Simp[c*((m + 2*q + 2)/(b*(p + 1)))   Int[x^(m
 + 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x] - Simp[m/(b*c*(p +
1))   Int[x^(m - 1)*(d + e*x^2)^q*(a + b*ArcTan[c*x])^(p + 1), x], x]) /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[e, c^2*d] && IntegerQ[m] && LtQ[q, -1] &
& LtQ[p, -1] && NeQ[m + 2*q + 2, 0]
```

rule 5505
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^q/c^(m + 1)   Subst[Int[(a + b*x)^p*(Sin[x]^m/
Cos[x]^(m + 2*(q + 1))), x], x, ArcTan[c*x]], x] /; FreeQ[{a, b, c, d, e, p
}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] && (IntegerQ[q
] || GtQ[d, 0])
```

rule 5506
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(q_), x_Symbol] :> Simp[d^(q + 1/2)*(Sqrt[1 + c^2*x^2]/Sqrt[d + e*x^2])
  Int[x^m*(1 + c^2*x^2)^q*(a + b*ArcTan[c*x])^p, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[e, c^2*d] && IGtQ[m, 0] && ILtQ[m + 2*q + 1, 0] &&  !(I
ntegerQ[q] || GtQ[d, 0])
```

## Maple [F]

$$\int \frac{1}{\left(a^{2} c \,x^{2}+c \right)^{\frac{5}{2}} \arctan \left(a x \right)^{\frac{5}{2}}}d x$$

input
```
int(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

output
```
int(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(5/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F]

$$\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{5/2}}\,dx = \int \frac{1}{\left(a^2cx^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(5/2)*arctan(a*x)^(5/2)), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{5/2}}\,dx = \int \frac{1}{\mathrm{atan}\left(a\,x\right)^{5/2}\,\left(c\,a^2\,x^2+c\right)^{5/2}}\,dx$$

input
```
int(1/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(1/(atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2)), x)
```

## Reduce [F]

$$\int \frac{1}{(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx = \text{Too large to display}$$

input
```
int(1/(a^2*c*x^2+c)^(5/2)/atan(a*x)^(5/2),x)
```

output
```
(2*sqrt(c)*(2*atan(a*x)**2*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**3)/
(atan(a*x)**2*a**6*x**6 + 3*atan(a*x)**2*a**4*x**4 + 3*atan(a*x)**2*a**2*x
**2 + atan(a*x)**2),x)*a**8*x**4 + 4*atan(a*x)**2*int((sqrt(a**2*x**2 + 1)
*sqrt(atan(a*x))*x**3)/(atan(a*x)**2*a**6*x**6 + 3*atan(a*x)**2*a**4*x**4
+ 3*atan(a*x)**2*a**2*x**2 + atan(a*x)**2),x)*a**6*x**2 + 2*atan(a*x)**2*i
nt((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*x**3)/(atan(a*x)**2*a**6*x**6 + 3*
atan(a*x)**2*a**4*x**4 + 3*atan(a*x)**2*a**2*x**2 + atan(a*x)**2),x)*a**4
- 6*atan(a*x)**2*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(atan(a*x)*a**6
*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),x)*a**5
*x**4 - 12*atan(a*x)**2*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(atan(a*
x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(a*x)),
x)*a**3*x**2 - 6*atan(a*x)**2*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(a
tan(a*x)*a**6*x**6 + 3*atan(a*x)*a**4*x**4 + 3*atan(a*x)*a**2*x**2 + atan(
a*x)),x)*a + 4*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)*a**3*x**3 + 6
*sqrt(a**2*x**2 + 1)*sqrt(atan(a*x))*atan(a*x)*a*x - sqrt(a**2*x**2 + 1)*s
qrt(atan(a*x))))/(3*atan(a*x)**2*a*c**3*(a**4*x**4 + 2*a**2*x**2 + 1))
```

# 3.1106 $\int \frac{1}{x(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{1}{x(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx = -\frac{2}{3acx(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}} + \frac{16}{3c(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}} + \frac{4}{3a^2cx^2(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}} + \frac{4\sqrt{2\pi}\sqrt{1+a^2x^2}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{c^2\sqrt{c+a^2cx^2}} + \frac{4\sqrt{\frac{2\pi}{3}}\sqrt{1+a^2x^2}\,\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)}{c^2\sqrt{c+a^2cx^2}} + \frac{8\text{Int}\left(\frac{1}{x^3(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}},x\right)}{3a^2} + \frac{20}{3}\text{Int}\left(\frac{1}{x(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}},x\right)$$

output
```
-2/3/a/c/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2)+16/3/c/(a^2*c*x^2+c)^(3/2
)/arctan(a*x)^(1/2)+4/3/a^2/c/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2)+4*
2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)*arctan(a*x)^(
1/2))/c^2/(a^2*c*x^2+c)^(1/2)+4/3*6^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)*Fresn
elS(6^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))/c^2/(a^2*c*x^2+c)^(1/2)+8/3*Defer(
Int)(1/x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)/a^2+20/3*Defer(Int)(1/
x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 8.74 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{5/2}}\,dx=\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{5/2}}\,dx$$

input
```
Integrate[1/(x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)),x]
```

output
```
Integrate[1/(x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 2.36 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\arctan(ax)^{5/2}\left(a^2cx^2+c\right)^{5/2}}dx$$

$$\Big\downarrow \text{5503}$$

$$-\frac{8}{3}a\int \frac{1}{\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^{3/2}}dx-\frac{2\int \frac{1}{x^2\left(a^2cx^2+c\right)^{5/2}\arctan(ax)^{3/2}}dx}{3a}-\frac{2}{3acx\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

$$\Big\downarrow \text{5437}$$

$$-\frac{2\int \frac{1}{x^2(a^2cx^2+c)^{5/2}\arctan(ax)^{3/2}}dx}{3a}-$$

$$\frac{\frac{8}{3}a\left(-6a\int \frac{x}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{2}{ac\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}\right)-2}{3acx\arctan(ax)^{3/2}\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5503}$$

$$\frac{-\frac{8}{3}a\left(-6a\int \frac{x}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{2}{ac\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}\right)-2\left(-10a\int \frac{1}{x(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{4\int \frac{1}{x^3\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^2\sqrt{\arctan(ax)}(a^2cx^2+c)^{3/2}}\right)}{3a}-\frac{2}{3acx\arctan(ax)^{3/2}\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5506}$$

$$\frac{-\frac{8}{3}a\left(-\frac{6a\sqrt{a^2x^2+1}\int \frac{x}{(a^2x^2+1)^{5/2}\sqrt{\arctan(ax)}}dx}{c^2\sqrt{a^2cx^2+c}}-\frac{2}{ac\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}\right)-2\left(-10a\int \frac{1}{x(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{4\int \frac{1}{x^3\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^2\sqrt{\arctan(ax)}(a^2cx^2+c)^{3/2}}\right)}{3a}-\frac{2}{3acx\arctan(ax)^{3/2}\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5505}$$

$$\frac{-\frac{8}{3}a\left(-\frac{6\sqrt{a^2x^2+1}\int \frac{ax}{(a^2x^2+1)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{2}{ac\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3/2}}\right)-2\left(-10a\int \frac{1}{x(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{4\int \frac{1}{x^3\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^2\sqrt{\arctan(ax)}(a^2cx^2+c)^{3/2}}\right)}{3a}-\frac{2}{3acx\arctan(ax)^{3/2}\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{4906}$$

$$-\frac{8}{3}a\left(-\frac{6\sqrt{a^2x^2+1}\int\left(\frac{ax}{4\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}+\frac{\sin(3\arctan(ax))}{4\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{ac^2\sqrt{a^2cx^2+c}}-\frac{2}{ac\sqrt{\arctan(ax)}\left(a^2cx^2+c\right)^{3/2}}\right)-\frac{2\left(-10a\int\frac{1}{x(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{4\int\frac{1}{x^3\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^2\sqrt{\arctan(ax)}(a^2cx^2+c)^{3/2}}\right)}{3a}-\frac{2}{3acx\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

$\Big\downarrow$ 2009

$$-\frac{2\left(-10a\int\frac{1}{x(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{4\int\frac{1}{x^3\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^2\sqrt{\arctan(ax)}(a^2cx^2+c)^{3/2}}\right)}{3a}-\frac{8}{3}a\left(-\frac{6\sqrt{a^2x^2+1}\left(\frac{1}{2}\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{6}}\operatorname{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{ac^2\sqrt{a^2cx^2+c}}-\frac{}{ac\sqrt{\arctan(ax}}\right.\frac{2}{3acx\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

$\Big\downarrow$ 5560

$$-\frac{2\left(-10a\int\frac{1}{x(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{4\int\frac{1}{x^3\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^2\sqrt{\arctan(ax)}(a^2cx^2+c)^{3/2}}\right)}{3a}-\frac{8}{3}a\left(-\frac{6\sqrt{a^2x^2+1}\left(\frac{1}{2}\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{6}}\operatorname{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)\right)}{ac^2\sqrt{a^2cx^2+c}}-\frac{}{ac\sqrt{\arctan(ax}}\right.\frac{2}{3acx\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

input
```
Int[1/(x*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{1}{x\left(a^{2} c\,x^{2}+c\right)^{\frac{5}{2}} \arctan\left(a x\right)^{\frac{5}{2}}}d x$$

input
```
int(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

output
```
int(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(c+a^{2} c x^{2}\right)^{5/2} \arctan(a x)^{5/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x\left(c+a^{2} c x^{2}\right)^{5/2} \arctan(a x)^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate(1/x/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(5/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{x\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{5/2}}\,dx = \int \frac{1}{x\,\mathrm{atan}\left(a\,x\right)^{5/2}\,{\left(c\,a^2\,x^2+c\right)}^{5/2}}\,dx$$

input
```
int(1/(x*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(1/(x*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.92

$$\int \frac{1}{x(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}}{atan(ax)^3a^6x^7+3atan(ax)^3a^4x^5+3atan(ax)^3a^2x^3+atan(ax)^3x}dx\right)}{c^3}$$

input
```
int(1/x/(a^2*c*x^2+c)^(5/2)/atan(a*x)^(5/2),x)
```

output
```
(sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(atan(a*x)**3*a**6*x**7
 + 3*atan(a*x)**3*a**4*x**5 + 3*atan(a*x)**3*a**2*x**3 + atan(a*x)**3*x),x
))/c**3
```

# 3.1107 $\int \frac{1}{x^2(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\begin{aligned}\int \frac{1}{x^2(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx &= -\frac{2}{3acx^2(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}\\
&+\frac{8}{3a^2cx^3(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}+\frac{20}{3cx(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}\\
&+\frac{20a\sqrt{2\pi}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)}{c^2\sqrt{c+a^2cx^2}}\\
&+\frac{20a\sqrt{\frac{2\pi}{3}}\sqrt{1+a^2x^2}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arctan(ax)}\right)}{3c^2\sqrt{c+a^2cx^2}}\\
&+\frac{8\text{Int}\left(\frac{1}{x^4(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}},x\right)}{a^2}+\frac{68}{3}\text{Int}\left(\frac{1}{x^2(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}},x\right)\end{aligned}$$

output

```
-2/3/a/c/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2)+8/3/a^2/c/x^3/(a^2*c*x^
2+c)^(3/2)/arctan(a*x)^(1/2)+20/3/c/x/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2
)+20*a*2^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*arctan
(a*x)^(1/2))/c^2/(a^2*c*x^2+c)^(1/2)+20/9*a*6^(1/2)*Pi^(1/2)*(a^2*x^2+1)^(
1/2)*FresnelC(6^(1/2)/Pi^(1/2)*arctan(a*x)^(1/2))/c^2/(a^2*c*x^2+c)^(1/2)+
8*Defer(Int)(1/x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)/a^2+68/3*Defer
(Int)(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 8.68 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^2 (c+a^2cx^2)^{5/2} \arctan(ax)^{5/2}} \, dx = \int \frac{1}{x^2 (c+a^2cx^2)^{5/2} \arctan(ax)^{5/2}} \, dx$$

input
```
Integrate[1/(x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)),x]
```

output
```
Integrate[1/(x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 3.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 \arctan(ax)^{5/2} (a^2cx^2+c)^{5/2}} \, dx$$

$$\Big\downarrow \text{5503}$$

$$-\frac{10}{3}a \int \frac{1}{x (a^2cx^2+c)^{5/2} \arctan(ax)^{3/2}} dx - \frac{4 \int \frac{1}{x^3 (a^2cx^2+c)^{5/2} \arctan(ax)^{3/2}} dx}{3a} - \frac{2}{3acx^2 \arctan(ax)^{3/2} (a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5503}$$

$$-\frac{10}{3}a\left(-8a\int \frac{1}{(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{2\int \frac{1}{x^2(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx\sqrt{\arctan(ax)}\,(a^2cx^2+c)^{3}}\right.$$

$$\frac{4\left(-12a\int \frac{1}{x^2(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{6\int \frac{1}{x^4\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^3\sqrt{\arctan(ax)}(a^2cx^2+c)^{3/2}}\right)}{3a}-\frac{2}{3acx^2\arctan(ax)^{3/2}\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \; 5440$$

$$-\frac{10}{3}a\left(-\frac{8a\sqrt{a^2x^2+1}\int \frac{1}{(a^2x^2+1)^{5/2}\sqrt{\arctan(ax)}}dx}{c^2\sqrt{a^2cx^2+c}}-\frac{2\int \frac{1}{x^2(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx\sqrt{\arctan(ax)}\,(a^2cx^2+\right.$$

$$\frac{4\left(-12a\int \frac{1}{x^2(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{6\int \frac{1}{x^4\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^3\sqrt{\arctan(ax)}(a^2cx^2+c)^{3/2}}\right)}{3a}-\frac{2}{3acx^2\arctan(ax)^{3/2}\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \; 5439$$

$$-\frac{10}{3}a\left(-\frac{8\sqrt{a^2x^2+1}\int \frac{1}{(a^2x^2+1)^{3/2}\sqrt{\arctan(ax)}}d\arctan(ax)}{c^2\sqrt{a^2cx^2+c}}-\frac{2\int \frac{1}{x^2(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx\sqrt{\arctan(ax)}\,(\right.$$

$$\frac{4\left(-12a\int \frac{1}{x^2(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{6\int \frac{1}{x^4\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^3\sqrt{\arctan(ax)}(a^2cx^2+c)^{3/2}}\right)}{3a}-\frac{2}{3acx^2\arctan(ax)^{3/2}\,(a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{10}{3}a\left(-\frac{8\sqrt{a^2x^2+1}\int \frac{\sin\left(\arctan(ax)+\frac{\pi}{2}\right)^3}{\sqrt{\arctan(ax)}}d\arctan(ax)}{c^2\sqrt{a^2cx^2+c}}-\frac{2\int \frac{1}{x^2(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx\sqrt{\arctan(ax)}\,(a^2cx\right.$$

$$\frac{4\left(-12a\int \frac{1}{x^2(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{6\int \frac{1}{x^4\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^3\sqrt{\arctan(ax)}(a^2cx^2+c)^{3/2}}\right)}{3a}-$$

$$\frac{2}{3acx^2\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

$\downarrow$ 3793

$$-\frac{10}{3}a\left(-\frac{8\sqrt{a^2x^2+1}\int \left(\frac{\cos(3\arctan(ax))}{4\sqrt{\arctan(ax)}}+\frac{3}{4\sqrt{a^2x^2+1}\sqrt{\arctan(ax)}}\right)d\arctan(ax)}{c^2\sqrt{a^2cx^2+c}}-\frac{2\int \frac{1}{x^2(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\right.$$

$$\frac{4\left(-12a\int \frac{1}{x^2(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{6\int \frac{1}{x^4\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^3\sqrt{\arctan(ax)}(a^2cx^2+c)^{3/2}}\right)}{3a}-$$

$$\frac{2}{3acx^2\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

$\downarrow$ 2009

$$-\frac{10}{3}a\left(-\frac{2\int \frac{1}{x^2(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{8\sqrt{a^2x^2+1}\left(\frac{3}{2}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{6}}\text{FresnelC}\left(\right.\right.}{c^2\sqrt{a^2cx^2+c}}\right.$$

$$\frac{4\left(-12a\int \frac{1}{x^2(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{6\int \frac{1}{x^4\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^3\sqrt{\arctan(ax)}(a^2cx^2+c)^{3/2}}\right)}{3a}-$$

$$\frac{2}{3acx^2\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

$\downarrow$ 5560

$$-\frac{\frac{10}{3}a\left(-\frac{2\int \frac{1}{x^2(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{8\sqrt{a^2x^2+1}\left(\frac{3}{2}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arctan(ax)}\right)+\frac{1}{2}\sqrt{\frac{\pi}{6}}\,\text{FresnelC}\left(\sqrt{}\right.\right.}{c^2\sqrt{a^2cx^2+c}}\right.}{}$$

$$\frac{4\left(-12a\int \frac{1}{x^2(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{6\int \frac{1}{x^4\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^3\sqrt{\arctan(ax)}(a^2cx^2+c)^{3/2}}\right)}{3a}-\frac{2}{3acx^2\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

input
```
Int[1/(x^2*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^2\left(a^2c\,x^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^{\frac{5}{2}}}dx$$

input
```
int(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

output
```
int(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^2\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="fricas
")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (c + a^2 c x^2)^{5/2} \arctan(ax)^{5/2}} \, dx = \text{Timed out}$$

input
```
integrate(1/x**2/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(5/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^2 (c + a^2 c x^2)^{5/2} \arctan(ax)^{5/2}} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^2 (c+a^2cx^2)^{5/2} \arctan(ax)^{5/2}} \, dx = \int \frac{1}{(a^2cx^2+c)^{\frac{5}{2}} x^2 \arctan{(ax)}^{\frac{5}{2}}} \, dx$$

input
```
integrate(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(5/2)*x^2*arctan(a*x)^(5/2)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^2 (c+a^2cx^2)^{5/2} \arctan(ax)^{5/2}} \, dx = \int \frac{1}{x^2 \operatorname{atan}{(a\,x)}^{5/2} \,(c\,a^2\,x^2+c)^{5/2}} \, dx$$

input
```
int(1/(x^2*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(1/(x^2*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 78, normalized size of antiderivative = 3.00

$$\int \frac{1}{x^2 (c+a^2cx^2)^{5/2} \arctan(ax)^{5/2}} \, dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}}{atan(ax)^3a^6x^8+3atan(ax)^3a^4x^6+3atan(ax)^3a^2x^4+atan(ax)^3x^2}dx\right)}{c^3}$$

input
```
int(1/x^2/(a^2*c*x^2+c)^(5/2)/atan(a*x)^(5/2),x)
```

output
```
(sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(atan(a*x)**3*a**6*x**8
 + 3*atan(a*x)**3*a**4*x**6 + 3*atan(a*x)**3*a**2*x**4 + atan(a*x)**3*x**2
),x))/c**3
```

# 3.1108 $\int \frac{1}{x^3(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{1}{x^3\left(c+a^2cx^2\right)^{5/2}\arctan(ax)^{5/2}}\,dx = -\frac{2}{3acx^3\left(c+a^2cx^2\right)^{3/2}\arctan(ax)^{3/2}}+\frac{4}{a^2cx^4\left(c+a^2cx^2\right)^{3/2}\sqrt{\arctan(ax)}}+\frac{8}{cx^2\left(c+a^2cx^2\right)^{3/2}\sqrt{\arctan(ax)}}+\frac{16\text{Int}\left(\frac{1}{x^5(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}},x\right)}{a^2}+44\text{Int}\left(\frac{1}{x^3\left(c+a^2cx^2\right)^{5/2}\sqrt{\arctan(ax)}},x\right)+40a^2\text{Int}\left(\frac{1}{x\left(c+a^2cx^2\right)^{5/2}\sqrt{\arctan(ax)}},x\right)$$

output
```
-2/3/a/c/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2)+4/a^2/c/x^4/(a^2*c*x^2+
c)^(3/2)/arctan(a*x)^(1/2)+8/c/x^2/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(1/2)+1
6*Defer(Int)(1/x^5/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)/a^2+44*Defer(I
nt)(1/x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)+40*a^2*Defer(Int)(1/x/(
a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 14.13 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^3 (c+a^2cx^2)^{5/2} \arctan(ax)^{5/2}} \, dx = \int \frac{1}{x^3 (c+a^2cx^2)^{5/2} \arctan(ax)^{5/2}} \, dx$$

input
```
Integrate[1/(x^3*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)),x]
```

output
```
Integrate[1/(x^3*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 1.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 \arctan(ax)^{5/2} (a^2cx^2+c)^{5/2}} \, dx$$

$$\Big\downarrow \text{5503}$$

$$-4a \int \frac{1}{x^2 (a^2cx^2+c)^{5/2} \arctan(ax)^{3/2}} dx - \frac{2 \int \frac{1}{x^4 (a^2cx^2+c)^{5/2} \arctan(ax)^{3/2}} dx}{a} - \frac{2}{3acx^3 \arctan(ax)^{3/2} (a^2cx^2+c)^{3/2}}$$

$$\Big\downarrow \text{5503}$$

$$-4a\left(-10a\int \frac{1}{x\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{4\int \frac{1}{x^3(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^2\sqrt{\arctan(ax)}\left(a^2cx^2+c\right.}\right.$$
$$\frac{2\left(-\frac{8\int \frac{1}{x^5\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-14a\int \frac{1}{x^3(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{2}{acx^4\sqrt{\arctan(ax)}(a^2cx^2+c)^{3/2}}\right)}{a}-\frac{2}{3acx^3\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

$\downarrow$ 5560

$$-4a\left(-10a\int \frac{1}{x\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{4\int \frac{1}{x^3(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^2\sqrt{\arctan(ax)}\left(a^2cx^2+c\right.}\right.$$
$$\frac{2\left(-\frac{8\int \frac{1}{x^5\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-14a\int \frac{1}{x^3(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{2}{acx^4\sqrt{\arctan(ax)}(a^2cx^2+c)^{3/2}}\right)}{a}-\frac{2}{3acx^3\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

input
```
Int[1/(x^3*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \frac{1}{x^3\left(a^2c\,x^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^{\frac{5}{2}}}dx$$

input
```
int(1/x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

output
```
int(1/x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^3 (c + a^2cx^2)^{5/2} \arctan(ax)^{5/2}} dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="fricas
")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^3 (c + a^2cx^2)^{5/2} \arctan(ax)^{5/2}} dx = \text{Timed out}$$

input
```
integrate(1/x**3/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(5/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^3 (c + a^2cx^2)^{5/2} \arctan(ax)^{5/2}} dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x^3 (c+a^2cx^2)^{5/2} \arctan(ax)^{5/2}} dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^3/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^3 (c+a^2cx^2)^{5/2} \arctan(ax)^{5/2}} dx = \int \frac{1}{x^3 \operatorname{atan}(a\,x)^{5/2} (c\,a^2\,x^2+c)^{5/2}} dx$$

input
```
int(1/(x^3*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(1/(x^3*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 78, normalized size of antiderivative = 3.00

$$\int \frac{1}{x^3 (c+a^2cx^2)^{5/2} \arctan(ax)^{5/2}} dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}}{atan(ax)^3a^6x^9+3atan(ax)^3a^4x^7+3atan(ax)^3a^2x^5+atan(ax)^3x^3}dx\right)}{c^3}$$

input
```
int(1/x^3/(a^2*c*x^2+c)^(5/2)/atan(a*x)^(5/2),x)
```

output
```
(sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(atan(a*x)**3*a**6*x**9
 + 3*atan(a*x)**3*a**4*x**7 + 3*atan(a*x)**3*a**2*x**5 + atan(a*x)**3*x**3
),x))/c**3
```

# 3.1109 $\int \frac{1}{x^4(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 26

$$\int \frac{1}{x^4(c+a^2cx^2)^{5/2}\arctan(ax)^{5/2}}\,dx = -\frac{2}{3acx^4(c+a^2cx^2)^{3/2}\arctan(ax)^{3/2}}+\frac{16}{3a^2cx^5(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}+\frac{28}{3cx^3(c+a^2cx^2)^{3/2}\sqrt{\arctan(ax)}}+\frac{80\mathrm{Int}\left(\frac{1}{x^6(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}},x\right)}{3a^2}+\frac{212}{3}\mathrm{Int}\left(\frac{1}{x^4(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}},x\right)+56a^2\mathrm{Int}\left(\frac{1}{x^2(c+a^2cx^2)^{5/2}\sqrt{\arctan(ax)}},x\right)$$

output
```
-2/3/a/c/x^4/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(3/2)+16/3/a^2/c/x^5/(a^2*c*x
^2+c)^(3/2)/arctan(a*x)^(1/2)+28/3/c/x^3/(a^2*c*x^2+c)^(3/2)/arctan(a*x)^(
1/2)+80/3*Defer(Int)(1/x^6/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)/a^2+21
2/3*Defer(Int)(1/x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)+56*a^2*Defer
(Int)(1/x^2/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 36.71 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^4 (c + a^2 c x^2)^{5/2} \arctan(ax)^{5/2}} \, dx = \int \frac{1}{x^4 (c + a^2 c x^2)^{5/2} \arctan(ax)^{5/2}} \, dx$$

input
```
Integrate[1/(x^4*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)),x]
```

output
```
Integrate[1/(x^4*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 1.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4 \arctan(ax)^{5/2} (a^2 c x^2 + c)^{5/2}} \, dx$$

$$\Big\downarrow \text{5503}$$

$$-\frac{8 \int \frac{1}{x^5 (a^2 c x^2 + c)^{5/2} \arctan(ax)^{3/2}} dx}{3a} - \frac{14}{3} a \int \frac{1}{x^3 (a^2 c x^2 + c)^{5/2} \arctan(ax)^{3/2}} dx - \frac{2}{3 a c x^4 \arctan(ax)^{3/2} (a^2 c x^2 + c)^{3/2}}$$

$$\Big\downarrow \text{5503}$$

$$-\frac{14}{3}a\left(-12a\int\frac{1}{x^2\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{6\int\frac{1}{x^4(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^3\sqrt{\arctan(ax)}\left(a^2cx^2\right.}\right.$$
$$\frac{8\left(-\frac{10\int\frac{1}{x^6\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-16a\int\frac{1}{x^4(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{2}{acx^5\sqrt{\arctan(ax)}(a^2cx^2+c)^{3/2}}\right)}{3a}-$$
$$\frac{2}{3acx^4\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

$\downarrow$ 5560

$$-\frac{14}{3}a\left(-12a\int\frac{1}{x^2\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{6\int\frac{1}{x^4(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-\frac{2}{acx^3\sqrt{\arctan(ax)}\left(a^2cx^2\right.}\right.$$
$$\frac{8\left(-\frac{10\int\frac{1}{x^6\left(a^2cx^2+c\right)^{5/2}\sqrt{\arctan(ax)}}dx}{a}-16a\int\frac{1}{x^4(a^2cx^2+c)^{5/2}\sqrt{\arctan(ax)}}dx-\frac{2}{acx^5\sqrt{\arctan(ax)}(a^2cx^2+c)^{3/2}}\right)}{3a}-$$
$$\frac{2}{3acx^4\arctan(ax)^{3/2}\left(a^2cx^2+c\right)^{3/2}}$$

input
```
Int[1/(x^4*(c + a^2*c*x^2)^(5/2)*ArcTan[a*x]^(5/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int\frac{1}{x^4\left(a^2c\,x^2+c\right)^{\frac{5}{2}}\arctan\left(ax\right)^{\frac{5}{2}}}dx$$

input
```
int(1/x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

output
```
int(1/x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x)
```

## Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^4 (c + a^2cx^2)^{5/2} \arctan(ax)^{5/2}} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="fricas
")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^4 (c + a^2cx^2)^{5/2} \arctan(ax)^{5/2}} \, dx = \text{Timed out}$$

input
```
integrate(1/x**4/(a**2*c*x**2+c)**(5/2)/atan(a*x)**(5/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^4 (c + a^2cx^2)^{5/2} \arctan(ax)^{5/2}} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="maxima
")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

## Giac [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^4 (c+a^2cx^2)^{5/2} \arctan(ax)^{5/2}} \, dx = \int \frac{1}{(a^2cx^2+c)^{\frac{5}{2}} x^4 \arctan(ax)^{\frac{5}{2}}} \, dx$$

input
```
integrate(1/x^4/(a^2*c*x^2+c)^(5/2)/arctan(a*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((a^2*c*x^2 + c)^(5/2)*x^4*arctan(a*x)^(5/2)), x)
```

## Mupad [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^4 (c+a^2cx^2)^{5/2} \arctan(ax)^{5/2}} \, dx = \int \frac{1}{x^4 \operatorname{atan}(a\,x)^{5/2} (c\,a^2\,x^2+c)^{5/2}} \, dx$$

input
```
int(1/(x^4*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2)),x)
```

output
```
int(1/(x^4*atan(a*x)^(5/2)*(c + a^2*c*x^2)^(5/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 78, normalized size of antiderivative = 3.00

$$\int \frac{1}{x^4 (c+a^2cx^2)^{5/2} \arctan(ax)^{5/2}} \, dx = \frac{\sqrt{c}\left(\int \frac{\sqrt{a^2x^2+1}\,\sqrt{atan(ax)}}{atan(ax)^3 a^6 x^{10}+3atan(ax)^3 a^4 x^8+3atan(ax)^3 a^2 x^6+atan(ax)^3 x^4}dx\right)}{c^3}$$

input
```
int(1/x^4/(a^2*c*x^2+c)^(5/2)/atan(a*x)^(5/2),x)
```

output
```
(sqrt(c)*int((sqrt(a**2*x**2 + 1)*sqrt(atan(a*x)))/(atan(a*x)**3*a**6*x**1
0 + 3*atan(a*x)**3*a**4*x**8 + 3*atan(a*x)**3*a**2*x**6 + atan(a*x)**3*x**
4),x))/c**3
```

# 3.1110 $\int \frac{x \arctan(ax)^n}{c+a^2cx^2} dx$



## Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{x \arctan(ax)^n}{c+a^2cx^2} dx = \frac{x \arctan(ax)^{1+n}}{ac(1+n)} - \frac{\text{Int}(\arctan(ax)^{1+n}, x)}{ac(1+n)}$$

output
```
x*arctan(a*x)^(1+n)/a/c/(1+n)-Defer(Int)(arctan(a*x)^(1+n),x)/a/c/(1+n)
```

## Mathematica [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x \arctan(ax)^n}{c+a^2cx^2} dx = \int \frac{x \arctan(ax)^n}{c+a^2cx^2} dx$$

input
```
Integrate[(x*ArcTan[a*x]^n)/(c + a^2*c*x^2),x]
```

output
```
Integrate[(x*ArcTan[a*x]^n)/(c + a^2*c*x^2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \arctan(ax)^n}{a^2cx^2+c} \, dx$$
$$\Big\downarrow \text{5457}$$
$$\frac{x \arctan(ax)^{n+1}}{ac(n+1)}-\frac{\int \arctan(ax)^{n+1}dx}{ac(n+1)}$$
$$\Big\downarrow \text{5353}$$
$$\frac{x \arctan(ax)^{n+1}}{ac(n+1)}-\frac{\int \arctan(ax)^{n+1}dx}{ac(n+1)}$$

input
```
Int[(x*ArcTan[a*x]^n)/(c + a^2*c*x^2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{x \arctan(ax)^n}{a^2c\,x^2+c}dx$$

input
```
int(x*arctan(a*x)^n/(a^2*c*x^2+c),x)
```

output
```
int(x*arctan(a*x)^n/(a^2*c*x^2+c),x)
```

## Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x \arctan(ax)^n}{c + a^2cx^2} \, dx = \int \frac{x \arctan{(ax)}^n}{a^2cx^2 + c} \, dx$$

input
```
integrate(x*arctan(a*x)^n/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
integral(x*arctan(a*x)^n/(a^2*c*x^2 + c), x)
```

## Sympy [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{x \arctan(ax)^n}{c + a^2cx^2} \, dx = \frac{\int \frac{x \operatorname{atan}^n{(ax)}}{a^2x^2+1} \, dx}{c}$$

input
```
integrate(x*atan(a*x)**n/(a**2*c*x**2+c),x)
```

output
```
Integral(x*atan(a*x)**n/(a**2*x**2 + 1), x)/c
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x \arctan(ax)^n}{c + a^2cx^2} \, dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*arctan(a*x)^n/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x \arctan(ax)^n}{c+a^2cx^2} \, dx = \int \frac{x \arctan{(ax)}^n}{a^2cx^2+c} \, dx$$

input
```
integrate(x*arctan(a*x)^n/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
integrate(x*arctan(a*x)^n/(a^2*c*x^2 + c), x)
```

## Mupad [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{x \arctan(ax)^n}{c+a^2cx^2} \, dx = \int \frac{x \,\mathrm{atan}(a\,x)^n}{c\,a^2\,x^2+c} \, dx$$

input
```
int((x*atan(a*x)^n)/(c + a^2*c*x^2),x)
```

output
```
int((x*atan(a*x)^n)/(c + a^2*c*x^2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.25

$$\int \frac{x \arctan(ax)^n}{c + a^2 c x^2} \, dx = \frac{\int \frac{atan(ax)^n x}{a^2 x^2 + 1} dx}{c}$$

input
```
int(x*atan(a*x)^n/(a^2*c*x^2+c),x)
```

output
```
int((atan(a*x)**n*x)/(a**2*x**2 + 1),x)/c
```

# 3.1111 $\int \frac{\arctan(ax)^n}{c+a^2cx^2}\,dx$



## Optimal result

Integrand size = 19, antiderivative size = 20

$$\int \frac{\arctan(ax)^n}{c+a^2cx^2}\,dx=\frac{\arctan(ax)^{1+n}}{ac(1+n)}$$

output
```
arctan(a*x)^(1+n)/a/c/(1+n)
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax)^n}{c+a^2cx^2}\,dx=\frac{\arctan(ax)^{1+n}}{ac(1+n)}$$

input
```
Integrate[ArcTan[a*x]^n/(c + a^2*c*x^2),x]
```

output
```
ArcTan[a*x]^(1 + n)/(a*c*(1 + n))
```

## Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)^n}{a^2cx^2+c}\,dx$$
$$\Big\downarrow \text{ 5419}$$
$$\frac{\arctan(ax)^{n+1}}{ac(n+1)}$$

input
```
Int[ArcTan[a*x]^n/(c + a^2*c*x^2),x]
```

output
```
ArcTan[a*x]^(1 + n)/(a*c*(1 + n))
```

### Defintions of rubi rules used

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

## Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

| method | result | size |
|---|---|---|
| default | $\frac{\arctan(ax)^{1+n}}{ac(1+n)}$ | 21 |
| parallelrisch | $\frac{\arctan(ax)^{n}\arctan(ax)}{ca(1+n)}$ | 23 |
| risch | $\frac{i(\ln(-iax+1)-\ln(iax+1))\left(\frac{i(\ln(-iax+1)-\ln(iax+1))}{2}\right)^{n}}{2ca(1+n)}$ | 58 |

input
```
int(arctan(a*x)^n/(a^2*c*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
arctan(a*x)^(1+n)/a/c/(1+n)
```

## Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.05

$$\int \frac{\arctan(ax)^n}{c+a^2cx^2}\,dx=\frac{\arctan\left(ax\right)^n\arctan\left(ax\right)}{acn+ac}$$

input
```
integrate(arctan(a*x)^n/(a^2*c*x^2+c),x, algorithm="fricas")
```

output
```
arctan(a*x)^n*arctan(a*x)/(a*c*n + a*c)
```

## Sympy [F]

$$\int \frac{\arctan(ax)^n}{c+a^2cx^2}\,dx=\frac{\int \frac{\operatorname{atan}^n\left(ax\right)}{a^2x^2+1}\,dx}{c}$$

input
```
integrate(atan(a*x)**n/(a**2*c*x**2+c),x)
```

output
```
Integral(atan(a*x)**n/(a**2*x**2 + 1), x)/c
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)^n}{c+a^2cx^2}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(arctan(a*x)^n/(a^2*c*x^2+c),x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negati
ve exponent.
```

## Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax)^n}{c+a^2cx^2}\,dx = \frac{\arctan{(ax)}^{n+1}}{ac(n+1)}$$

input
```
integrate(arctan(a*x)^n/(a^2*c*x^2+c),x, algorithm="giac")
```

output
```
arctan(a*x)^(n + 1)/(a*c*(n + 1))
```

## Mupad [B] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(ax)^n}{c+a^2cx^2}\,dx = \frac{\mathrm{atan}(a\,x)^{n+1}}{a\,c\,(n+1)}$$

input
```
int(atan(a*x)^n/(c + a^2*c*x^2),x)
```

output
```
atan(a*x)^(n + 1)/(a*c*(n + 1))
```

## Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\arctan(ax)^n}{c + a^2 c x^2}\, dx = \frac{atan(ax)^n\, atan(ax)}{ac\,(n+1)}$$

input
```
int(atan(a*x)^n/(a^2*c*x^2+c),x)
```

output
```
(atan(a*x)**n*atan(a*x))/(a*c*(n + 1))
```

# 3.1112 $\int (fx)^m (d+c^2dx^2)^q (a+b\arctan(cx))^p\,dx$



## Optimal result

Integrand size = 28, antiderivative size = 28

$$\int (fx)^m (d+c^2dx^2)^q (a+b\arctan(cx))^p\,dx = \text{Int}\left((fx)^m (d+c^2dx^2)^q (a+b\arctan(cx))^p, x\right)$$

output
```
Defer(Int)((f*x)^m*(c^2*d*x^2+d)^q*(a+b*arctan(c*x))^p,x)
```

## Mathematica [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int (fx)^m (d+c^2dx^2)^q (a+b\arctan(cx))^p\,dx = \int (fx)^m (d+c^2dx^2)^q (a+b\arctan(cx))^p\,dx$$

input
```
Integrate[(f*x)^m*(d + c^2*d*x^2)^q*(a + b*ArcTan[c*x])^p,x]
```

output
```
Integrate[(f*x)^m*(d + c^2*d*x^2)^q*(a + b*ArcTan[c*x])^p, x]
```

## Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (fx)^m \left(c^2dx^2+d\right)^q (a+b\arctan(cx))^p \, dx$$

$$\Big\downarrow \text{5560}$$

$$\int (fx)^m \left(c^2dx^2+d\right)^q (a+b\arctan(cx))^p dx$$

input
```
Int[(f*x)^m*(d + c^2*d*x^2)^q*(a + b*ArcTan[c*x])^p,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 1.41 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int (fx)^m \left(c^2d\,x^2+d\right)^q (a+b\arctan(cx))^p \, dx$$

input
```
int((f*x)^m*(c^2*d*x^2+d)^q*(a+b*arctan(c*x))^p,x)
```

output
```
int((f*x)^m*(c^2*d*x^2+d)^q*(a+b*arctan(c*x))^p,x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int (fx)^m \left(d+c^2dx^2\right)^q (a+b\arctan(cx))^p\,dx = \int \left(c^2dx^2+d\right)^q (fx)^m\,(b\arctan(cx)+a)^p\,dx$$

input
```
integrate((f*x)^m*(c^2*d*x^2+d)^q*(a+b*arctan(c*x))^p,x, algorithm="fricas
")
```

output
```
integral((c^2*d*x^2 + d)^q*(f*x)^m*(b*arctan(c*x) + a)^p, x)
```

## Sympy [F(-1)]

Timed out.

$$\int (fx)^m \left(d+c^2dx^2\right)^q (a+b\arctan(cx))^p\,dx = \text{Timed out}$$

input
```
integrate((f*x)**m*(c**2*d*x**2+d)**q*(a+b*atan(c*x))**p,x)
```

output
```
Timed out
```

## Maxima [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int (fx)^m \left(d+c^2dx^2\right)^q (a+b\arctan(cx))^p\,dx = \int \left(c^2dx^2+d\right)^q (fx)^m\,(b\arctan(cx)+a)^p\,dx$$

input
```
integrate((f*x)^m*(c^2*d*x^2+d)^q*(a+b*arctan(c*x))^p,x, algorithm="maxima
")
```

output
```
integrate((c^2*d*x^2 + d)^q*(f*x)^m*(b*arctan(c*x) + a)^p, x)
```

## Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int (fx)^m \left(d+c^2dx^2\right)^q (a+b\arctan(cx))^p\,dx = \int \left(c^2dx^2+d\right)^q (fx)^m\,(b\arctan(cx)+a)^p\,dx$$

input
```
integrate((f*x)^m*(c^2*d*x^2+d)^q*(a+b*arctan(c*x))^p,x, algorithm="giac")
```

output
```
integrate((c^2*d*x^2 + d)^q*(f*x)^m*(b*arctan(c*x) + a)^p, x)
```

## Mupad [N/A]

Not integrable

Time = 0.95 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int (fx)^m \left(d+c^2dx^2\right)^q (a+b\arctan(cx))^p\,dx = \int (a+b\,\mathrm{atan}(c\,x))^p \left(d\,c^2\,x^2+d\right)^q (f\,x)^m\,dx$$

input
```
int((a + b*atan(c*x))^p*(d + c^2*d*x^2)^q*(f*x)^m,x)
```

output
```
int((a + b*atan(c*x))^p*(d + c^2*d*x^2)^q*(f*x)^m, x)
```

## Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.14

$$\int (fx)^m \left(d+c^2dx^2\right)^q (a+b\arctan(cx))^p\,dx$$
$$= f^m\left(\int x^m\left(c^2d\,x^2+d\right)^q(atan(cx)\,b+a)^p\,dx\right)$$

input
```
int((f*x)^m*(c^2*d*x^2+d)^q*(a+b*atan(c*x))^p,x)
```

output
```
f**m*int(x**m*(c**2*d*x**2 + d)**q*(atan(c*x)*b + a)**p,x)
```

# 3.1113 $\int x^3(d+ex^2)(a+b\arctan(cx))\,dx$



## Optimal result

Integrand size = 19, antiderivative size = 107

$$\int x^3(d+ex^2)(a+b\arctan(cx))\,dx = \frac{b(3c^2d-2e)\,x}{12c^5} - \frac{b(3c^2d-2e)\,x^3}{36c^3} - \frac{bex^5}{30c} - \frac{b(3c^2d-2e)\arctan(cx)}{12c^6} + \frac{1}{4}dx^4(a+b\arctan(cx)) + \frac{1}{6}ex^6(a+b\arctan(cx))$$

output
```
1/12*b*(3*c^2*d-2*e)*x/c^5-1/36*b*(3*c^2*d-2*e)*x^3/c^3-1/30*b*e*x^5/c-1/1
2*b*(3*c^2*d-2*e)*arctan(c*x)/c^6+1/4*d*x^4*(a+b*arctan(c*x))+1/6*e*x^6*(a
+b*arctan(c*x))
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.19

$$\int x^3(d+ex^2)(a+b\arctan(cx))\,dx = \frac{bdx}{4c^3}-\frac{bex}{6c^5}-\frac{bdx^3}{12c}+\frac{bex^3}{18c^3}+\frac{1}{4}adx^4-\frac{bex^5}{30c}+\frac{1}{6}aex^6-\frac{bd\arctan(cx)}{4c^4}+\frac{be\arctan(cx)}{6c^6}+\frac{1}{4}bdx^4\arctan(cx)+\frac{1}{6}bex^6\arctan(cx)$$

input
```
Integrate[x^3*(d + e*x^2)*(a + b*ArcTan[c*x]),x]
```

output
```
(b*d*x)/(4*c^3) - (b*e*x)/(6*c^5) - (b*d*x^3)/(12*c) + (b*e*x^3)/(18*c^3)
+ (a*d*x^4)/4 - (b*e*x^5)/(30*c) + (a*e*x^6)/6 - (b*d*ArcTan[c*x])/(4*c^4)
 + (b*e*ArcTan[c*x])/(6*c^6) + (b*d*x^4*ArcTan[c*x])/4 + (b*e*x^6*ArcTan[c
*x])/6
```

## Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.81, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {5511, 27, 363, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(d+ex^2)(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5511}$$

$$-bc\int \frac{x^4(2ex^2+3d)}{12(c^2x^2+1)}dx+\frac{1}{4}dx^4(a+b\arctan(cx))+\frac{1}{6}ex^6(a+b\arctan(cx))$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{12}bc\int \frac{x^4(2ex^2+3d)}{c^2x^2+1}dx+\frac{1}{4}dx^4(a+b\arctan(cx))+\frac{1}{6}ex^6(a+b\arctan(cx))$$

$$\Big\downarrow \text{363}$$

$$-\frac{1}{12}bc\left(\left(3d-\frac{2e}{c^2}\right)\int\frac{x^4}{c^2x^2+1}dx+\frac{2ex^5}{5c^2}\right)+\frac{1}{4}dx^4(a+b\arctan(cx))+\frac{1}{6}ex^6(a+b\arctan(cx))$$

$\Big\downarrow$ 254

$$-\frac{1}{12}bc\left(\left(3d-\frac{2e}{c^2}\right)\int\left(\frac{x^2}{c^2}+\frac{1}{c^4(c^2x^2+1)}-\frac{1}{c^4}\right)dx+\frac{2ex^5}{5c^2}\right)+\frac{1}{4}dx^4(a+b\arctan(cx))+\frac{1}{6}ex^6(a+b\arctan(cx))$$

$\Big\downarrow$ 2009

$$\frac{1}{4}dx^4(a+b\arctan(cx))+\frac{1}{6}ex^6(a+b\arctan(cx))-\frac{1}{12}bc\left(\left(\frac{\arctan(cx)}{c^5}-\frac{x}{c^4}+\frac{x^3}{3c^2}\right)\left(3d-\frac{2e}{c^2}\right)+\frac{2ex^5}{5c^2}\right)$$

input
```
Int[x^3*(d + e*x^2)*(a + b*ArcTan[c*x]),x]
```

output
```
(d*x^4*(a + b*ArcTan[c*x]))/4 + (e*x^6*(a + b*ArcTan[c*x]))/6 - (b*c*((2*e
*x^5)/(5*c^2) + (3*d - (2*e)/c^2)*(-(x/c^4) + x^3/(3*c^2) + ArcTan[c*x]/c^
5)))/12
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

## Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.01

| method | result |
|---|---|
| parts | $a\left(\frac{1}{6}e\,x^{6}+\frac{1}{4}d\,x^{4}\right)+\frac{b\left(\frac{c^{4}\arctan(cx)e\,x^{6}}{6}+\frac{\arctan(cx)d\,c^{4}x^{4}}{4}-\frac{\frac{2e\,c^{5}x^{5}}{5}+d\,c^{5}x^{3}-\frac{2e\,c^{3}x^{3}}{3}-3c^{3}xd+2ecx+\left(3c^{2}d-2e\right)\mathrm{arc}}{12c^{2}}\right.}{c^{4}}$ |
| derivativedivides | $\frac{\frac{a\left(\frac{1}{4}d\,c^{6}x^{4}+\frac{1}{6}e\,c^{6}x^{6}\right)}{c^{2}}+\frac{b\left(\frac{\arctan(cx)d\,c^{6}x^{4}}{4}+\frac{\arctan(cx)e\,c^{6}x^{6}}{6}-\frac{d\,c^{5}x^{3}}{12}-\frac{e\,c^{5}x^{5}}{30}+\frac{c^{3}xd}{4}+\frac{e\,c^{3}x^{3}}{18}-\frac{ecx}{6}-\frac{\left(3c^{2}d-2e\right)\arctan(cx)}{12}\right)}{c^{2}}}{c^{4}}$ |
| default | $\frac{\frac{a\left(\frac{1}{4}d\,c^{6}x^{4}+\frac{1}{6}e\,c^{6}x^{6}\right)}{c^{2}}+\frac{b\left(\frac{\arctan(cx)d\,c^{6}x^{4}}{4}+\frac{\arctan(cx)e\,c^{6}x^{6}}{6}-\frac{d\,c^{5}x^{3}}{12}-\frac{e\,c^{5}x^{5}}{30}+\frac{c^{3}xd}{4}+\frac{e\,c^{3}x^{3}}{18}-\frac{ecx}{6}-\frac{\left(3c^{2}d-2e\right)\arctan(cx)}{12}\right)}{c^{2}}}{c^{4}}$ |
| parallelrisch | $\frac{30x^{6}\arctan(cx)b\,c^{6}e+30a\,c^{6}e\,x^{6}+45db\arctan(cx)x^{4}c^{6}-6b\,c^{5}e\,x^{5}+45a\,c^{6}d\,x^{4}-15b\,c^{5}d\,x^{3}+10b\,c^{3}e\,x^{3}+45b\,c^{3}dx-45b}{180c^{6}}$ |
| risch | $-\frac{ib\left(2e\,x^{6}+3d\,x^{4}\right)\ln(icx+1)}{24}+\frac{x^{6}ea}{6}+\frac{ibe\,x^{6}\ln(-icx+1)}{12}+\frac{x^{4}da}{4}-\frac{be\,x^{5}}{30c}+\frac{ibd\,x^{4}\ln(-icx+1)}{8}-\frac{bd\,x^{3}}{12c}+$ |
| orering | $\frac{(30e^{2}c^{6}x^{8}+84e\,c^{6}d\,x^{6}+45c^{6}d^{2}x^{4}-10e^{2}c^{4}x^{6}-81e\,c^{4}d\,x^{4}-45c^{4}d^{2}x^{2}+50c^{2}e^{2}x^{4}-105e\,c^{2}d\,x^{2}-90c^{2}d^{2}+90e^{2}x^{2}+60de)}{90c^{6}(e\,x^{2}+d)}$ |

input
```
int(x^3*(e*x^2+d)*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/6*e*x^6+1/4*d*x^4)+b/c^4*(1/6*c^4*arctan(c*x)*e*x^6+1/4*arctan(c*x)*d
*c^4*x^4-1/12/c^2*(2/5*e*c^5*x^5+d*c^5*x^3-2/3*e*c^3*x^3-3*c^3*x*d+2*e*c*x
+(3*c^2*d-2*e)*arctan(c*x)))
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.03

$$\int x^3 (d + ex^2) (a + b \arctan(cx)) \, dx$$
$$= \frac{30\, ac^6 ex^6 + 45\, ac^6 dx^4 - 6\, bc^5 ex^5 - 5\,(3\, bc^5 d - 2\, bc^3 e) x^3 + 15\,(3\, bc^3 d - 2\, bce) x + 15\,(2\, bc^6 ex^6 + 3\, bc^6 dx^4}{180\, c^6}$$

input
```
integrate(x^3*(e*x^2+d)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
1/180*(30*a*c^6*e*x^6 + 45*a*c^6*d*x^4 - 6*b*c^5*e*x^5 - 5*(3*b*c^5*d - 2*
b*c^3*e)*x^3 + 15*(3*b*c^3*d - 2*b*c*e)*x + 15*(2*b*c^6*e*x^6 + 3*b*c^6*d*
x^4 - 3*b*c^2*d + 2*b*e)*arctan(c*x))/c^6
```

## Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.29

$$\int x^3 (d + ex^2) (a + b \arctan(cx)) \, dx$$
$$= \begin{cases} \frac{adx^4}{4} + \frac{aex^6}{6} + \frac{bdx^4 \operatorname{atan}(cx)}{4} + \frac{bex^6 \operatorname{atan}(cx)}{6} - \frac{bdx^3}{12c} - \frac{bex^5}{30c} + \frac{bdx}{4c^3} + \frac{bex^3}{18c^3} - \frac{bd \operatorname{atan}(cx)}{4c^4} - \frac{bex}{6c^5} + \frac{be \operatorname{atan}(cx)}{6c^6} & \text{for } c \\ a\left(\frac{dx^4}{4} + \frac{ex^6}{6}\right) & \text{other} \end{cases}$$

input
```
integrate(x**3*(e*x**2+d)*(a+b*atan(c*x)),x)
```

output
```
Piecewise((a*d*x**4/4 + a*e*x**6/6 + b*d*x**4*atan(c*x)/4 + b*e*x**6*atan(
c*x)/6 - b*d*x**3/(12*c) - b*e*x**5/(30*c) + b*d*x/(4*c**3) + b*e*x**3/(18
*c**3) - b*d*atan(c*x)/(4*c**4) - b*e*x/(6*c**5) + b*e*atan(c*x)/(6*c**6),
 Ne(c, 0)), (a*(d*x**4/4 + e*x**6/6), True))
```

## Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.01

$$\int x^3(d+ex^2)(a+b\arctan(cx))\,dx$$
$$=\frac{1}{6}aex^6+\frac{1}{4}adx^4+\frac{1}{12}\left(3x^4\arctan(cx)-c\left(\frac{c^2x^3-3x}{c^4}+\frac{3\arctan(cx)}{c^5}\right)\right)bd+\frac{1}{90}\left(15x^6\arctan(cx)-c\left(\frac{3c^4x^5-5c^2x^3+15x}{c^6}-\frac{15\arctan(cx)}{c^7}\right)\right)be$$

input
```
integrate(x^3*(e*x^2+d)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
1/6*a*e*x^6 + 1/4*a*d*x^4 + 1/12*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c
^4 + 3*arctan(c*x)/c^5))*b*d + 1/90*(15*x^6*arctan(c*x) - c*((3*c^4*x^5 -
5*c^2*x^3 + 15*x)/c^6 - 15*arctan(c*x)/c^7))*b*e
```

## Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.29

$$\int x^3(d+ex^2)(a+b\arctan(cx))\,dx$$
$$=\frac{30bc^6ex^6\arctan(cx)+30ac^6ex^6+45bc^6dx^4\arctan(cx)+45ac^6dx^4-6bc^5ex^5-15bc^5dx^3+10bc^3ex^3}{}$$

input
```
integrate(x^3*(e*x^2+d)*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
1/180*(30*b*c^6*e*x^6*arctan(c*x) + 30*a*c^6*e*x^6 + 45*b*c^6*d*x^4*arctan
(c*x) + 45*a*c^6*d*x^4 - 6*b*c^5*e*x^5 - 15*b*c^5*d*x^3 + 10*b*c^3*e*x^3 +
 45*pi*b*c^2*d*sgn(c)*sgn(x) + 45*b*c^3*d*x - 45*b*c^2*d*arctan(c*x) - 30*
pi*b*e*sgn(c)*sgn(x) - 30*b*c*e*x + 30*b*e*arctan(c*x))/c^6
```

## Mupad [B] (verification not implemented)

Time = 0.95 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.98

$$\int x^3(d+ex^2)\,(a+b\arctan(cx))\,dx = \frac{a\,d\,x^4}{4}+\frac{a\,e\,x^6}{6}+\frac{b\,d\,x}{4\,c^3}-\frac{b\,e\,x}{6\,c^5}-\frac{b\,d\,\mathrm{atan}(c\,x)}{4\,c^4}+\frac{b\,e\,\mathrm{atan}(c\,x)}{6\,c^6}+\frac{b\,d\,x^4\,\mathrm{atan}(c\,x)}{4}+\frac{b\,e\,x^6\,\mathrm{atan}(c\,x)}{6}-\frac{b\,d\,x^3}{12\,c}-\frac{b\,e\,x^5}{30\,c}+\frac{b\,e\,x^3}{18\,c^3}$$

input
```
int(x^3*(a + b*atan(c*x))*(d + e*x^2),x)
```

output
```
(a*d*x^4)/4 + (a*e*x^6)/6 + (b*d*x)/(4*c^3) - (b*e*x)/(6*c^5) - (b*d*atan(
c*x))/(4*c^4) + (b*e*atan(c*x))/(6*c^6) + (b*d*x^4*atan(c*x))/4 + (b*e*x^6
*atan(c*x))/6 - (b*d*x^3)/(12*c) - (b*e*x^5)/(30*c) + (b*e*x^3)/(18*c^3)
```

## Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.09

$$\int x^3(d+ex^2)\,(a+b\arctan(cx))\,dx = \frac{45atan(cx)\,b\,c^6d\,x^4+30atan(cx)\,b\,c^6e\,x^6-45atan(cx)\,b\,c^2d+30atan(cx)\,be+45a\,c^6d\,x^4+30a\,c^6e\,x^6-}{180c^6}$$

input
```
int(x^3*(e*x^2+d)*(a+b*atan(c*x)),x)
```

output
```
(45*atan(c*x)*b*c**6*d*x**4 + 30*atan(c*x)*b*c**6*e*x**6 - 45*atan(c*x)*b*
c**2*d + 30*atan(c*x)*b*e + 45*a*c**6*d*x**4 + 30*a*c**6*e*x**6 - 15*b*c**
5*d*x**3 - 6*b*c**5*e*x**5 + 45*b*c**3*d*x + 10*b*c**3*e*x**3 - 30*b*c*e*x
)/(180*c**6)
```

# 3.1114 $\int x^2(d+ex^2)(a+b\arctan(cx))\,dx$



## Optimal result

Integrand size = 19, antiderivative size = 94

$$\int x^2(d+ex^2)(a+b\arctan(cx))\,dx = -\frac{b(5c^2d-3e)\,x^2}{30c^3}-\frac{bex^4}{20c}+\frac{1}{3}dx^3(a+b\arctan(cx)) + \frac{1}{5}ex^5(a+b\arctan(cx)) + \frac{b(5c^2d-3e)\log(1+c^2x^2)}{30c^5}$$

output
```
-1/30*b*(5*c^2*d-3*e)*x^2/c^3-1/20*b*e*x^4/c+1/3*d*x^3*(a+b*arctan(c*x))+1
/5*e*x^5*(a+b*arctan(c*x))+1/30*b*(5*c^2*d-3*e)*ln(c^2*x^2+1)/c^5
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.27

$$\int x^2(d+ex^2)(a+b\arctan(cx))\,dx = -\frac{bdx^2}{6c}+\frac{bex^2}{10c^3}+\frac{1}{3}adx^3-\frac{bex^4}{20c}+\frac{1}{5}aex^5 + \frac{1}{3}bdx^3\arctan(cx)+\frac{1}{5}bex^5\arctan(cx) + \frac{bd\log(1+c^2x^2)}{6c^3}-\frac{be\log(1+c^2x^2)}{10c^5}$$

input
```
Integrate[x^2*(d + e*x^2)*(a + b*ArcTan[c*x]),x]
```

output
```
-1/6*(b*d*x^2)/c + (b*e*x^2)/(10*c^3) + (a*d*x^3)/3 - (b*e*x^4)/(20*c) + (
a*e*x^5)/5 + (b*d*x^3*ArcTan[c*x])/3 + (b*e*x^5*ArcTan[c*x])/5 + (b*d*Log[
1 + c^2*x^2])/(6*c^3) - (b*e*Log[1 + c^2*x^2])/(10*c^5)
```

## Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {5511, 27, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(d+ex^2\right)(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5511}$$

$$-bc\int \frac{x^3\left(3ex^2+5d\right)}{15\left(c^2x^2+1\right)}dx+\frac{1}{3}dx^3(a+b\arctan(cx))+\frac{1}{5}ex^5(a+b\arctan(cx))$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{15}bc\int \frac{x^3\left(3ex^2+5d\right)}{c^2x^2+1}dx+\frac{1}{3}dx^3(a+b\arctan(cx))+\frac{1}{5}ex^5(a+b\arctan(cx))$$

$$\Big\downarrow \text{354}$$

$$-\frac{1}{30}bc\int \frac{x^2\left(3ex^2+5d\right)}{c^2x^2+1}dx^2+\frac{1}{3}dx^3(a+b\arctan(cx))+\frac{1}{5}ex^5(a+b\arctan(cx))$$

$$\Big\downarrow \text{86}$$

$$-\frac{1}{30}bc\int \left(\frac{3ex^2}{c^2}+\frac{5c^2d-3e}{c^4}+\frac{3e-5c^2d}{c^4\left(c^2x^2+1\right)}\right)dx^2+\frac{1}{3}dx^3(a+b\arctan(cx))+\frac{1}{5}ex^5(a+b\arctan(cx))$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}dx^3(a+b\arctan(cx))+\frac{1}{5}ex^5(a+b\arctan(cx))-\frac{1}{30}bc\left(\frac{3ex^4}{2c^2}-\frac{(5c^2d-3e)\log(c^2x^2+1)}{c^6}+\frac{x^2(5c^2d-3e)}{c^4}\right)$$

input
```
Int[x^2*(d + e*x^2)*(a + b*ArcTan[c*x]),x]
```

output
```
(d*x^3*(a + b*ArcTan[c*x]))/3 + (e*x^5*(a + b*ArcTan[c*x]))/5 - (b*c*(((5*
c^2*d - 3*e)*x^2)/c^4 + (3*e*x^4)/(2*c^2) - ((5*c^2*d - 3*e)*Log[1 + c^2*x
^2])/c^6))/30
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

## Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.11

| method | result |
|---|---|
| parts | $a\left(\frac{1}{5}e\,x^5+\frac{1}{3}d\,x^3\right)+\frac{b\left(\frac{c^3\arctan(cx)e\,x^5}{5}+\frac{\arctan(cx)d\,c^3x^3}{3}-\frac{\frac{5c^4d\,x^2}{2}+\frac{3c^4e\,x^4}{4}-\frac{3c^2e\,x^2}{2}+\frac{\left(-5c^2d+3e\right)\ln\left(c^2x^2+1\right)}{2}}{15c^2}\right)}{c^3}$ |
| derivativedivides | $\frac{\frac{a\left(\frac{1}{3}d\,c^5x^3+\frac{1}{5}e\,c^5x^5\right)}{c^2}+\frac{b\left(\frac{\arctan(cx)d\,c^5x^3}{3}+\frac{\arctan(cx)e\,c^5x^5}{5}-\frac{c^4d\,x^2}{6}-\frac{c^4e\,x^4}{20}+\frac{c^2e\,x^2}{10}-\frac{\left(-5c^2d+3e\right)\ln\left(c^2x^2+1\right)}{30}\right)}{c^2}}{c^3}$ |
| default | $\frac{\frac{a\left(\frac{1}{3}d\,c^5x^3+\frac{1}{5}e\,c^5x^5\right)}{c^2}+\frac{b\left(\frac{\arctan(cx)d\,c^5x^3}{3}+\frac{\arctan(cx)e\,c^5x^5}{5}-\frac{c^4d\,x^2}{6}-\frac{c^4e\,x^4}{20}+\frac{c^2e\,x^2}{10}-\frac{\left(-5c^2d+3e\right)\ln\left(c^2x^2+1\right)}{30}\right)}{c^2}}{c^3}$ |
| parallelrisch | $\frac{12x^5\arctan(cx)b\,c^5e+12a\,c^5e\,x^5+20db\arctan(cx)x^3c^5-3b\,c^4e\,x^4+20a\,c^5d\,x^3-10b\,c^4d\,x^2+6b\,c^2e\,x^2+10\ln\left(c^2x^2+1\right)b}{60c^5}$ |
| risch | $-\frac{ib\left(3e\,x^5+5d\,x^3\right)\ln(icx+1)}{30}+\frac{ibe\,x^5\ln(-icx+1)}{10}+\frac{ibd\,x^3\ln(-icx+1)}{6}+\frac{ae\,x^5}{5}+\frac{x^3da}{3}-\frac{be\,x^4}{20c}-\frac{bd\,x^2}{6c}+$ |

input
```
int(x^2*(e*x^2+d)*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/5*e*x^5+1/3*d*x^3)+b/c^3*(1/5*c^3*arctan(c*x)*e*x^5+1/3*arctan(c*x)*d
*c^3*x^3-1/15/c^2*(5/2*c^4*d*x^2+3/4*c^4*e*x^4-3/2*c^2*e*x^2+1/2*(-5*c^2*d
+3*e)*ln(c^2*x^2+1)))
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.14

$$\int x^2(d+ex^2)(a+b\arctan(cx))\,dx$$
$$=\frac{12\,ac^5ex^5+20\,ac^5dx^3-3\,bc^4ex^4-2\,(5\,bc^4d-3\,bc^2e)x^2+4\,(3\,bc^5ex^5+5\,bc^5dx^3)\arctan(cx)+2\,(5\,bc^2d}{60\,c^5}$$

input
```
integrate(x^2*(e*x^2+d)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
1/60*(12*a*c^5*e*x^5 + 20*a*c^5*d*x^3 - 3*b*c^4*e*x^4 - 2*(5*b*c^4*d - 3*b
*c^2*e)*x^2 + 4*(3*b*c^5*e*x^5 + 5*b*c^5*d*x^3)*arctan(c*x) + 2*(5*b*c^2*d
 - 3*b*e)*log(c^2*x^2 + 1))/c^5
```

## Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.36

$$\int x^2(d+ex^2)(a+b\arctan(cx))\,dx$$
$$=\begin{cases}\frac{adx^3}{3}+\frac{aex^5}{5}+\frac{bdx^3\operatorname{atan}(cx)}{3}+\frac{bex^5\operatorname{atan}(cx)}{5}-\frac{bdx^2}{6c}-\frac{bex^4}{20c}+\frac{bd\log\left(x^2+\frac{1}{c^2}\right)}{6c^3}+\frac{bex^2}{10c^3}-\frac{be\log\left(x^2+\frac{1}{c^2}\right)}{10c^5} & \text{for } c\neq 0\\ a\left(\frac{dx^3}{3}+\frac{ex^5}{5}\right) & \text{otherwise}\end{cases}$$

input
```
integrate(x**2*(e*x**2+d)*(a+b*atan(c*x)),x)
```

output
```
Piecewise((a*d*x**3/3 + a*e*x**5/5 + b*d*x**3*atan(c*x)/3 + b*e*x**5*atan(
c*x)/5 - b*d*x**2/(6*c) - b*e*x**4/(20*c) + b*d*log(x**2 + c**(-2))/(6*c**
3) + b*e*x**2/(10*c**3) - b*e*log(x**2 + c**(-2))/(10*c**5), Ne(c, 0)), (a
*(d*x**3/3 + e*x**5/5), True))
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.12

$$\int x^2(d+ex^2)(a+b\arctan(cx))\,dx = \frac{1}{5}aex^5 + \frac{1}{3}adx^3 + \frac{1}{6}\left(2x^3\arctan(cx) - c\left(\frac{x^2}{c^2} - \frac{\log(c^2x^2+1)}{c^4}\right)\right)bd + \frac{1}{20}\left(4x^5\arctan(cx) - c\left(\frac{c^2x^4 - 2x^2}{c^4} + \frac{2\log(c^2x^2+1)}{c^6}\right)\right)be$$

input
```
integrate(x^2*(e*x^2+d)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
1/5*a*e*x^5 + 1/3*a*d*x^3 + 1/6*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*
x^2 + 1)/c^4))*b*d + 1/20*(4*x^5*arctan(c*x) - c*((c^2*x^4 - 2*x^2)/c^4 +
2*log(c^2*x^2 + 1)/c^6))*b*e
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.22

$$\int x^2(d+ex^2)(a+b\arctan(cx))\,dx = \frac{12bc^5ex^5\arctan(cx) + 12ac^5ex^5 + 20bc^5dx^3\arctan(cx) + 20ac^5dx^3 - 3bc^4ex^4 - 10bc^4dx^2 + 6bc^2ex^2}{60c^5}$$

input
```
integrate(x^2*(e*x^2+d)*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
1/60*(12*b*c^5*e*x^5*arctan(c*x) + 12*a*c^5*e*x^5 + 20*b*c^5*d*x^3*arctan(
c*x) + 20*a*c^5*d*x^3 - 3*b*c^4*e*x^4 - 10*b*c^4*d*x^2 + 6*b*c^2*e*x^2 + 1
0*b*c^2*d*log(c^2*x^2 + 1) - 6*b*e*log(c^2*x^2 + 1))/c^5
```

## Mupad [B] (verification not implemented)

Time = 0.91 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.07

$$\int x^2(d+ex^2)(a+b\arctan(cx))\,dx = \frac{a\,d\,x^3}{3}+\frac{a\,e\,x^5}{5}+\frac{b\,d\,x^3\,\mathrm{atan}(c\,x)}{3}+\frac{b\,e\,x^5\,\mathrm{atan}(c\,x)}{5}+\frac{b\,d\,\ln\left(c^2\,x^2+1\right)}{6\,c^3}-\frac{b\,e\,\ln\left(c^2\,x^2+1\right)}{10\,c^5}-\frac{b\,d\,x^2}{6\,c}-\frac{b\,e\,x^4}{20\,c}+\frac{b\,e\,x^2}{10\,c^3}$$

input
```
int(x^2*(a + b*atan(c*x))*(d + e*x^2),x)
```

output
```
(a*d*x^3)/3 + (a*e*x^5)/5 + (b*d*x^3*atan(c*x))/3 + (b*e*x^5*atan(c*x))/5
+ (b*d*log(c^2*x^2 + 1))/(6*c^3) - (b*e*log(c^2*x^2 + 1))/(10*c^5) - (b*d*
x^2)/(6*c) - (b*e*x^4)/(20*c) + (b*e*x^2)/(10*c^3)
```

## Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.22

$$\int x^2(d+ex^2)(a+b\arctan(cx))\,dx = \frac{20atan(cx)\,b\,c^5d\,x^3+12atan(cx)\,b\,c^5e\,x^5+10\log(c^2x^2+1)\,b\,c^2d-6\log(c^2x^2+1)\,be+20a\,c^5d\,x^3+12a}{60c^5}$$

input
```
int(x^2*(e*x^2+d)*(a+b*atan(c*x)),x)
```

output
```
(20*atan(c*x)*b*c**5*d*x**3 + 12*atan(c*x)*b*c**5*e*x**5 + 10*log(c**2*x**
2 + 1)*b*c**2*d - 6*log(c**2*x**2 + 1)*b*e + 20*a*c**5*d*x**3 + 12*a*c**5*
e*x**5 - 10*b*c**4*d*x**2 - 3*b*c**4*e*x**4 + 6*b*c**2*e*x**2)/(60*c**5)
```

# 3.1115 $\int x(d+ex^2)(a+b\arctan(cx))\,dx$



## Optimal result

Integrand size = 17, antiderivative size = 82

$$\int x(d+ex^2)(a+b\arctan(cx))\,dx = -\frac{b(2c^2d-e)\,x}{4c^3}-\frac{bex^3}{12c}-\frac{b(c^2d-e)^2\arctan(cx)}{4c^4e}+\frac{(d+ex^2)^2(a+b\arctan(cx))}{4e}$$

output
```
-1/4*b*(2*c^2*d-e)*x/c^3-1/12*b*e*x^3/c-1/4*b*(c^2*d-e)^2*arctan(c*x)/c^4/
e+1/4*(e*x^2+d)^2*(a+b*arctan(c*x))/e
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.26

$$\int x(d+ex^2)(a+b\arctan(cx))\,dx = -\frac{bdx}{2c}+\frac{bex}{4c^3}+\frac{1}{2}adx^2-\frac{bex^3}{12c}+\frac{1}{4}aex^4+\frac{bd\arctan(cx)}{2c^2}-\frac{be\arctan(cx)}{4c^4}+\frac{1}{2}bdx^2\arctan(cx)+\frac{1}{4}bex^4\arctan(cx)$$

input
```
Integrate[x*(d + e*x^2)*(a + b*ArcTan[c*x]),x]
```

output
```
-1/2*(b*d*x)/c + (b*e*x)/(4*c^3) + (a*d*x^2)/2 - (b*e*x^3)/(12*c) + (a*e*x
^4)/4 + (b*d*ArcTan[c*x])/(2*c^2) - (b*e*ArcTan[c*x])/(4*c^4) + (b*d*x^2*A
rcTan[c*x])/2 + (b*e*x^4*ArcTan[c*x])/4
```

## Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {5509, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(d+ex^2\right)(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \ 5509$$

$$\frac{\left(d+ex^2\right)^2(a+b\arctan(cx))}{4e}-\frac{bc\int \frac{\left(ex^2+d\right)^2}{c^2x^2+1}dx}{4e}$$

$$\Big\downarrow \ 300$$

$$\frac{\left(d+ex^2\right)^2(a+b\arctan(cx))}{4e}-\frac{bc\int \left(\frac{e^2x^2}{c^2}+\frac{(2c^2d-e)e}{c^4}+\frac{d^2c^4-2dec^2+e^2}{c^4(c^2x^2+1)}\right)dx}{4e}$$

$$\Big\downarrow \ 2009$$

$$\frac{\left(d+ex^2\right)^2(a+b\arctan(cx))}{4e}-\frac{bc\left(\frac{\arctan(cx)\left(c^2d-e\right)^2}{c^5}+\frac{e^2x^3}{3c^2}+\frac{ex(2c^2d-e)}{c^4}\right)}{4e}$$

input
```
Int[x*(d + e*x^2)*(a + b*ArcTan[c*x]),x]
```

output
```
((d + e*x^2)^2*(a + b*ArcTan[c*x]))/(4*e) - (b*c*(((2*c^2*d - e)*e*x)/c^4
+ (e^2*x^3)/(3*c^2) + ((c^2*d - e)^2*ArcTan[c*x])/c^5))/(4*e)
```

### Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5509
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*e*(q + 1))), x
] - Simp[b*(c/(2*e*(q + 1)))   Int[(d + e*x^2)^(q + 1)/(1 + c^2*x^2), x], x
] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]
```

## Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.06

| method | result |
|---|---|
| parts | $\frac{a(e x^2+d)^2}{4e}+\frac{b\arctan(cx)e x^4}{4}+\frac{b\arctan(cx)x^2 d}{2}-\frac{be x^3}{12c}-\frac{bdx}{2c}+\frac{bex}{4c^3}+\frac{bd\arctan(cx)}{2c^2}-\frac{be\arctan(cx)}{4c^4}$ |
| parallelrisch | $\frac{3x^4\arctan(cx)b c^4 e+3x^4 a c^4 e+6x^2\arctan(cx)b c^4 d-b c^3 e x^3+6x^2 a c^4 d-6b c^3 dx+6b c^2 d\arctan(cx)+3bcex-3eb\,\mathrm{arcta}}{12c^4}$ |
| derivativedivides | $\frac{\frac{a(c^2 e x^2+c^2 d)^2}{4c^2 e}+\frac{\arctan(cx)b c^2 d x^2}{2}+\frac{\arctan(cx)b c^2 e x^4}{4}-\frac{bcdx}{2}-\frac{bce x^3}{12}+\frac{bex}{4c}+\frac{\arctan(cx)bd}{2}-\frac{be\arctan(cx)}{4c^2}}{c^2}$ |
| default | $\frac{\frac{a(c^2 e x^2+c^2 d)^2}{4c^2 e}+\frac{\arctan(cx)b c^2 d x^2}{2}+\frac{\arctan(cx)b c^2 e x^4}{4}-\frac{bcdx}{2}-\frac{bce x^3}{12}+\frac{bex}{4c}+\frac{\arctan(cx)bd}{2}-\frac{be\arctan(cx)}{4c^2}}{c^2}$ |
| risch | $-\frac{i(e x^2+d)^2 b\ln(icx+1)}{8e}+\frac{ibd x^2\ln(-icx+1)}{4}+\frac{ib d^2\ln(c^2x^2+1)}{16e}+\frac{ae x^4}{4}+\frac{ieb x^4\ln(-icx+1)}{8}-\frac{b d^2\,\mathrm{arctan}}{8e}$ |
| orering | $\frac{(3e^2c^4x^6+14e c^4 d x^4+6c^4d^2x^2-3c^2e^2x^4+11e c^2 d x^2+6c^2d^2-6e^2x^2-3de)(a+b\arctan(cx))}{6c^4(e x^2+d)}-\frac{(c^2 e x^2+6c^2 d-3e)(c}{}$ |

input
```
int(x*(e*x^2+d)*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/4*a*(e*x^2+d)^2/e+1/4*b*arctan(c*x)*e*x^4+1/2*b*arctan(c*x)*x^2*d-1/12*b
*e*x^3/c-1/2*b*d*x/c+1/4*b*e*x/c^3+1/2*b*d*arctan(c*x)/c^2-1/4*b*e*arctan(
c*x)/c^4
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.09

$$\int x(d+ex^2)(a+b\arctan(cx))\,dx = \frac{3\,ac^4ex^4+6\,ac^4dx^2-bc^3ex^3-3\,(2\,bc^3d-bce)x+3\,(bc^4ex^4+2\,bc^4dx^2+2\,bc^2d-be)\arctan{(cx)}}{12\,c^4}$$

input
```
integrate(x*(e*x^2+d)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
1/12*(3*a*c^4*e*x^4 + 6*a*c^4*d*x^2 - b*c^3*e*x^3 - 3*(2*b*c^3*d - b*c*e)*
x + 3*(b*c^4*e*x^4 + 2*b*c^4*d*x^2 + 2*b*c^2*d - b*e)*arctan(c*x))/c^4
```

## Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.39

$$\int x(d+ex^2)(a+b\arctan(cx))\,dx = \begin{cases} \frac{adx^2}{2}+\frac{aex^4}{4}+\frac{bdx^2\,\mathrm{atan}\,(cx)}{2}+\frac{bex^4\,\mathrm{atan}\,(cx)}{4}-\frac{bdx}{2c}-\frac{bex^3}{12c}+\frac{bd\,\mathrm{atan}\,(cx)}{2c^2}+\frac{bex}{4c^3}-\frac{be\,\mathrm{atan}\,(cx)}{4c^4} & \text{for } c\neq 0 \\ a\left(\frac{dx^2}{2}+\frac{ex^4}{4}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(x*(e*x**2+d)*(a+b*atan(c*x)),x)
```

output
```
Piecewise((a*d*x**2/2 + a*e*x**4/4 + b*d*x**2*atan(c*x)/2 + b*e*x**4*atan(
c*x)/4 - b*d*x/(2*c) - b*e*x**3/(12*c) + b*d*atan(c*x)/(2*c**2) + b*e*x/(4
*c**3) - b*e*atan(c*x)/(4*c**4), Ne(c, 0)), (a*(d*x**2/2 + e*x**4/4), True
))
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.07

$$\int x(d+ex^2)(a+b\arctan(cx))\,dx$$
$$=\frac{1}{4}aex^4+\frac{1}{2}adx^2+\frac{1}{2}\left(x^2\arctan(cx)-c\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)\right)bd$$
$$+\frac{1}{12}\left(3x^4\arctan(cx)-c\left(\frac{c^2x^3-3x}{c^4}+\frac{3\arctan(cx)}{c^5}\right)\right)be$$

input
```
integrate(x*(e*x^2+d)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
1/4*a*e*x^4 + 1/2*a*d*x^2 + 1/2*(x^2*arctan(c*x) - c*(x/c^2 - arctan(c*x)/
c^3))*b*d + 1/12*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*
x)/c^5))*b*e
```

## Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.18

$$\int x(d+ex^2)(a+b\arctan(cx))\,dx$$
$$=\frac{3bc^4ex^4\arctan(cx)+3ac^4ex^4+6bc^4dx^2\arctan(cx)+6ac^4dx^2-bc^3ex^3-6bc^3dx+6bc^2d\arctan(cx)}{12c^4}$$

input
```
integrate(x*(e*x^2+d)*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
1/12*(3*b*c^4*e*x^4*arctan(c*x) + 3*a*c^4*e*x^4 + 6*b*c^4*d*x^2*arctan(c*x
) + 6*a*c^4*d*x^2 - b*c^3*e*x^3 - 6*b*c^3*d*x + 6*b*c^2*d*arctan(c*x) + 3*
b*c*e*x - 3*b*e*arctan(c*x))/c^4
```

## Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.04

$$\int x(d+ex^2)\,(a+b\arctan(cx))\,dx = \frac{a\,d\,x^2}{2}+\frac{a\,e\,x^4}{4}-\frac{b\,d\,x}{2\,c}+\frac{b\,e\,x}{4\,c^3}+\frac{b\,d\,\mathrm{atan}(c\,x)}{2\,c^2}-\frac{b\,e\,\mathrm{atan}(c\,x)}{4\,c^4}+\frac{b\,d\,x^2\,\mathrm{atan}(c\,x)}{2}+\frac{b\,e\,x^4\,\mathrm{atan}(c\,x)}{4}-\frac{b\,e\,x^3}{12\,c}$$

input
```
int(x*(a + b*atan(c*x))*(d + e*x^2),x)
```

output
```
(a*d*x^2)/2 + (a*e*x^4)/4 - (b*d*x)/(2*c) + (b*e*x)/(4*c^3) + (b*d*atan(c*
x))/(2*c^2) - (b*e*atan(c*x))/(4*c^4) + (b*d*x^2*atan(c*x))/2 + (b*e*x^4*a
tan(c*x))/4 - (b*e*x^3)/(12*c)
```

## Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.18

$$\int x(d+ex^2)\,(a+b\arctan(cx))\,dx = \frac{6atan(cx)\,b\,c^4d\,x^2+3atan(cx)\,b\,c^4e\,x^4+6atan(cx)\,b\,c^2d-3atan(cx)\,be+6a\,c^4d\,x^2+3a\,c^4e\,x^4-6b\,c^3d}{12c^4}$$

input
```
int(x*(e*x^2+d)*(a+b*atan(c*x)),x)
```

output
```
(6*atan(c*x)*b*c**4*d*x**2 + 3*atan(c*x)*b*c**4*e*x**4 + 6*atan(c*x)*b*c**
2*d - 3*atan(c*x)*b*e + 6*a*c**4*d*x**2 + 3*a*c**4*e*x**4 - 6*b*c**3*d*x -
 b*c**3*e*x**3 + 3*b*c*e*x)/(12*c**4)
```

# 3.1116 $\int (d+ex^2)(a+b\arctan(cx))\,dx$



## Optimal result

Integrand size = 16, antiderivative size = 68

$$\int (d+ex^2)(a+b\arctan(cx))\,dx = -\frac{bex^2}{6c} + dx(a+b\arctan(cx)) + \frac{1}{3}ex^3(a+b\arctan(cx)) - \frac{b(3c^2d-e)\log(1+c^2x^2)}{6c^3}$$

output
```
-1/6*b*e*x^2/c+d*x*(a+b*arctan(c*x))+1/3*e*x^3*(a+b*arctan(c*x))-1/6*b*(3*
c^2*d-e)*ln(c^2*x^2+1)/c^3
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.25

$$\int (d+ex^2)(a+b\arctan(cx))\,dx = adx - \frac{bex^2}{6c} + \frac{1}{3}aex^3 + bdx\arctan(cx) + \frac{1}{3}bex^3\arctan(cx) - \frac{bd\log(1+c^2x^2)}{2c} + \frac{be\log(1+c^2x^2)}{6c^3}$$

input
```
Integrate[(d + e*x^2)*(a + b*ArcTan[c*x]),x]
```

output
```
a*d*x - (b*e*x^2)/(6*c) + (a*e*x^3)/3 + b*d*x*ArcTan[c*x] + (b*e*x^3*ArcTa
n[c*x])/3 - (b*d*Log[1 + c^2*x^2])/(2*c) + (b*e*Log[1 + c^2*x^2])/(6*c^3)
```

## Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {5447, 27, 353, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5447}$$

$$-bc\int \frac{x\left(ex^2+3d\right)}{3\left(c^2x^2+1\right)}dx+dx(a+b\arctan(cx))+\frac{1}{3}ex^3(a+b\arctan(cx))$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{3}bc\int \frac{x\left(ex^2+3d\right)}{c^2x^2+1}dx+dx(a+b\arctan(cx))+\frac{1}{3}ex^3(a+b\arctan(cx))$$

$$\Big\downarrow \text{353}$$

$$-\frac{1}{6}bc\int \frac{ex^2+3d}{c^2x^2+1}dx^2+dx(a+b\arctan(cx))+\frac{1}{3}ex^3(a+b\arctan(cx))$$

$$\Big\downarrow \text{49}$$

$$-\frac{1}{6}bc\int \left(\frac{3c^2d-e}{c^2\left(c^2x^2+1\right)}+\frac{e}{c^2}\right)dx^2+dx(a+b\arctan(cx))+\frac{1}{3}ex^3(a+b\arctan(cx))$$

$$\Big\downarrow \text{2009}$$

$$dx(a+b\arctan(cx))+\frac{1}{3}ex^3(a+b\arctan(cx))-\frac{1}{6}bc\left(\frac{ex^2}{c^2}+\frac{\left(3c^2d-e\right)\log\left(c^2x^2+1\right)}{c^4}\right)$$

input
```
Int[(d + e*x^2)*(a + b*ArcTan[c*x]),x]
```

output
```
d*x*(a + b*ArcTan[c*x]) + (e*x^3*(a + b*ArcTan[c*x]))/3 - (b*c*((e*x^2)/c^
2 + ((3*c^2*d - e)*Log[1 + c^2*x^2])/c^4))/6
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5447
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Simp[(a + b*ArcTan[c*x])   u,
x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2), x], x], x]] /; FreeQ
[{a, b, c, d, e}, x] && (IntegerQ[q] || ILtQ[q + 1/2, 0])
```

## Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.12

| method | result |
|---|---|
| parts | $a\left(\frac{1}{3}e\,x^{3}+dx\right)+\frac{b\left(\frac{c\arctan(cx)x^{3}e}{3}+\arctan(cx)cxd-\frac{\frac{c^{2}e\,x^{2}}{2}+\frac{\left(3c^{2}d-e\right)\ln\left(c^{2}x^{2}+1\right)}{2}}{3c^{2}}\right)}{c}$ |
| derivativedivides | $\frac{\frac{a\left(c^{3}xd+\frac{1}{3}e\,c^{3}x^{3}\right)}{c^{2}}+\frac{b\left(\arctan(cx)c^{3}xd+\frac{\arctan(cx)e\,c^{3}x^{3}}{3}-\frac{c^{2}e\,x^{2}}{6}-\frac{\left(3c^{2}d-e\right)\ln\left(c^{2}x^{2}+1\right)}{6}\right)}{c^{2}}}{c}$ |
| default | $\frac{\frac{a\left(c^{3}xd+\frac{1}{3}e\,c^{3}x^{3}\right)}{c^{2}}+\frac{b\left(\arctan(cx)c^{3}xd+\frac{\arctan(cx)e\,c^{3}x^{3}}{3}-\frac{c^{2}e\,x^{2}}{6}-\frac{\left(3c^{2}d-e\right)\ln\left(c^{2}x^{2}+1\right)}{6}\right)}{c^{2}}}{c}$ |
| parallelrisch | $-\frac{-2x^{3}\arctan(cx)b\,c^{3}e-2a\,c^{3}e\,x^{3}-6x\arctan(cx)b\,c^{3}d+b\,c^{2}e\,x^{2}-6a\,c^{3}dx+3\ln(c^{2}x^{2}+1)b\,c^{2}d-\ln(c^{2}x^{2}+1)be}{6c^{3}}$ |
| risch | $-\frac{ib(e\,x^{3}+3dx)\ln(icx+1)}{6}+\frac{ibe\,x^{3}\ln(-icx+1)}{6}+\frac{ibdx\ln(-icx+1)}{2}+\frac{ae\,x^{3}}{3}+adx-\frac{be\,x^{2}}{6c}-\frac{\ln(-c^{2}x^{2}-1)}{2c}$ |

input
```
int((e*x^2+d)*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/3*e*x^3+d*x)+b/c*(1/3*c*arctan(c*x)*x^3*e+arctan(c*x)*c*x*d-1/3/c^2*(
1/2*c^2*e*x^2+1/2*(3*c^2*d-e)*ln(c^2*x^2+1)))
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.21

$$\int \left(d+ex^2\right)\left(a+b\arctan(cx)\right)dx$$
$$=\frac{2\,ac^3ex^3+6\,ac^3dx-bc^2ex^2+2\left(bc^3ex^3+3\,bc^3dx\right)\arctan\left(cx\right)-\left(3\,bc^2d-be\right)\log\left(c^2x^2+1\right)}{6\,c^3}$$

input
```
integrate((e*x^2+d)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
1/6*(2*a*c^3*e*x^3 + 6*a*c^3*d*x - b*c^2*e*x^2 + 2*(b*c^3*e*x^3 + 3*b*c^3*
d*x)*arctan(c*x) - (3*b*c^2*d - b*e)*log(c^2*x^2 + 1))/c^3
```

## Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.38

$$\int (d+ex^2)(a+b\arctan(cx))\,dx$$
$$=\begin{cases} adx+\frac{aex^3}{3}+bdx\,\mathrm{atan}\,(cx)+\frac{bex^3\,\mathrm{atan}\,(cx)}{3}-\frac{bd\log\left(x^2+\frac{1}{c^2}\right)}{2c}-\frac{bex^2}{6c}+\frac{be\log\left(x^2+\frac{1}{c^2}\right)}{6c^3} & \text{for } c\neq 0 \\ a\left(dx+\frac{ex^3}{3}\right) & \text{otherwise} \end{cases}$$

input
```
integrate((e*x**2+d)*(a+b*atan(c*x)),x)
```

output
```
Piecewise((a*d*x + a*e*x**3/3 + b*d*x*atan(c*x) + b*e*x**3*atan(c*x)/3 - b
*d*log(x**2 + c**(-2))/(2*c) - b*e*x**2/(6*c) + b*e*log(x**2 + c**(-2))/(6
*c**3), Ne(c, 0)), (a*(d*x + e*x**3/3), True))
```

## Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.18

$$\int (d+ex^2)(a+b\arctan(cx))\,dx$$
$$=\frac{1}{3}aex^3+\frac{1}{6}\left(2x^3\arctan(cx)-c\left(\frac{x^2}{c^2}-\frac{\log(c^2x^2+1)}{c^4}\right)\right)be+adx+\frac{(2cx\arctan(cx)-\log(c^2x^2+1))bd}{2c}$$

input
```
integrate((e*x^2+d)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
1/3*a*e*x^3 + 1/6*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))
*b*e + a*d*x + 1/2*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b*d/c
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.32

$$\int (d + ex^2) (a + b \arctan(cx)) \, dx$$
$$= \frac{2 bc^3ex^3 \arctan(cx) + 2 ac^3ex^3 + 6 bc^3dx \arctan(cx) + 6 ac^3dx - bc^2ex^2 - 3 bc^2d \log(c^2x^2 + 1) + be \log}{6 c^3}$$

input
```
integrate((e*x^2+d)*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
1/6*(2*b*c^3*e*x^3*arctan(c*x) + 2*a*c^3*e*x^3 + 6*b*c^3*d*x*arctan(c*x) +
 6*a*c^3*d*x - b*c^2*e*x^2 - 3*b*c^2*d*log(c^2*x^2 + 1) + b*e*log(c^2*x^2
+ 1))/c^3
```

## Mupad [B] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.10

$$\int (d + ex^2) (a + b \arctan(cx)) \, dx = a\,d\,x + \frac{a\,e\,x^3}{3} + b\,d\,x\,\mathrm{atan}(c\,x) + \frac{b\,e\,x^3\,\mathrm{atan}(c\,x)}{3} - \frac{b\,d\,\ln(c^2\,x^2 + 1)}{2\,c} + \frac{b\,e\,\ln(c^2\,x^2 + 1)}{6\,c^3} - \frac{b\,e\,x^2}{6\,c}$$

input
```
int((a + b*atan(c*x))*(d + e*x^2),x)
```

output
```
a*d*x + (a*e*x^3)/3 + b*d*x*atan(c*x) + (b*e*x^3*atan(c*x))/3 - (b*d*log(c
^2*x^2 + 1))/(2*c) + (b*e*log(c^2*x^2 + 1))/(6*c^3) - (b*e*x^2)/(6*c)
```

## Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.32

$$\int (d + ex^2) (a + b \arctan(cx)) dx$$
$$= \frac{6atan(cx) \, b \, c^3 dx + 2atan(cx) \, b \, c^3 e \, x^3 - 3 \log(c^2x^2 + 1) \, b \, c^2 d + \log(c^2x^2 + 1) \, be + 6a \, c^3 dx + 2a \, c^3 e \, x^3 - b}{6c^3}$$

input
```
int((e*x^2+d)*(a+b*atan(c*x)),x)
```

output
```
(6*atan(c*x)*b*c**3*d*x + 2*atan(c*x)*b*c**3*e*x**3 - 3*log(c**2*x**2 + 1)
*b*c**2*d + log(c**2*x**2 + 1)*b*e + 6*a*c**3*d*x + 2*a*c**3*e*x**3 - b*c*
*2*e*x**2)/(6*c**3)
```

# 3.1117 $\int \frac{(d+ex^2)(a+b\arctan(cx))}{x}\,dx$



## Optimal result

Integrand size = 19, antiderivative size = 77

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x}\,dx = -\frac{bex}{2c}+\frac{be\arctan(cx)}{2c^2}+\frac{1}{2}ex^2(a+b\arctan(cx)) + ad\log(x)+\frac{1}{2}ibd\,\text{PolyLog}(2,-icx) - \frac{1}{2}ibd\,\text{PolyLog}(2,icx)$$

output
```
-1/2*b*e*x/c+1/2*b*e*arctan(c*x)/c^2+1/2*e*x^2*(a+b*arctan(c*x))+a*d*ln(x)
+1/2*I*b*d*polylog(2,-I*c*x)-1/2*I*b*d*polylog(2,I*c*x)
```

## Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.08

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x}\,dx = -\frac{bex}{2c}+\frac{1}{2}aex^2+\frac{be\arctan(cx)}{2c^2} + \frac{1}{2}bex^2\arctan(cx)+ad\log(x) + \frac{1}{2}ibd\,\text{PolyLog}(2,-icx)-\frac{1}{2}ibd\,\text{PolyLog}(2,icx)$$

input
```
Integrate[((d + e*x^2)*(a + b*ArcTan[c*x]))/x,x]
```

output
```
-1/2*(b*e*x)/c + (a*e*x^2)/2 + (b*e*ArcTan[c*x])/(2*c^2) + (b*e*x^2*ArcTan
[c*x])/2 + a*d*Log[x] + (I/2)*b*d*PolyLog[2, (-I)*c*x] - (I/2)*b*d*PolyLog
[2, I*c*x]
```

## Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x}\,dx$$
$$\Big\downarrow \ 5515$$
$$\int \left(\frac{d(a+b\arctan(cx))}{x}+ex(a+b\arctan(cx))\right)dx$$
$$\Big\downarrow \ 2009$$
$$\frac{1}{2}ex^2(a+b\arctan(cx))+ad\log(x)+\frac{be\arctan(cx)}{2c^2}+\frac{1}{2}ibd\,\text{PolyLog}(2,-icx)-\frac{1}{2}ibd\,\text{PolyLog}(2,icx)-\frac{bex}{2c}$$

input
```
Int[((d + e*x^2)*(a + b*ArcTan[c*x]))/x,x]
```

output
```
-1/2*(b*e*x)/c + (b*e*ArcTan[c*x])/(2*c^2) + (e*x^2*(a + b*ArcTan[c*x]))/2
 + a*d*Log[x] + (I/2)*b*d*PolyLog[2, (-I)*c*x] - (I/2)*b*d*PolyLog[2, I*c*
x]
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

## Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.56

| method | result |
|---|---|
| derivativedivides | $\frac{ae\,x^2}{2}+ad\ln(cx)+\frac{b\left(\frac{\arctan(cx)e\,c^2x^2}{2}+\arctan(cx)d\,c^2\ln(cx)-\frac{e(cx-\arctan(cx))}{2}-d\,c^2\left(-\frac{i\ln(cx)\ln(icx+1)}{2}+\frac{i\ln(\cdots}{}\right.\right.}{c^2}$ |
| default | $\frac{ae\,x^2}{2}+ad\ln(cx)+\frac{b\left(\frac{\arctan(cx)e\,c^2x^2}{2}+\arctan(cx)d\,c^2\ln(cx)-\frac{e(cx-\arctan(cx))}{2}-d\,c^2\left(-\frac{i\ln(cx)\ln(icx+1)}{2}+\frac{i\ln(\cdots}{}\right.\right.}{c^2}$ |
| parts | $\frac{ae\,x^2}{2}+ad\ln(x)+b\left(\frac{\arctan(cx)x^2e}{2}+\arctan(cx)\,d\ln(cx)-\frac{e(cx-\arctan(cx))-ic^2d\ln(cx)\ln(icx+\cdots}{}\right.$ |
| risch | $-\frac{ibe\ln(icx+1)x^2}{4}+\frac{ibe\ln(c^2x^2+1)}{8c^2}+\frac{be\arctan(cx)}{4c^2}+\frac{ibe\ln(-icx+1)x^2}{4}-\frac{bex}{2c}+\frac{ibd\,\mathrm{dilog}(icx+1)}{2}-\frac{ibd\,\mathrm{d}\cdots}{}$ |

input
```
int((e*x^2+d)*(a+b*arctan(c*x))/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*a*e*x^2+a*d*ln(c*x)+b/c^2*(1/2*arctan(c*x)*e*c^2*x^2+arctan(c*x)*d*c^2
*ln(c*x)-1/2*e*(c*x-arctan(c*x))-d*c^2*(-1/2*I*ln(c*x)*ln(1+I*c*x)+1/2*I*l
n(c*x)*ln(1-I*c*x)-1/2*I*dilog(1+I*c*x)+1/2*I*dilog(1-I*c*x)))
```

## Fricas [F]

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x}\,dx = \int \frac{(ex^2+d)(b\arctan(cx)+a)}{x}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arctan(c*x))/x,x, algorithm="fricas")
```

output
```
integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arctan(c*x))/x, x)
```

## Sympy [F]

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x}\,dx = \int \frac{(a+b\operatorname{atan}(cx))(d+ex^2)}{x}\,dx$$

input
```
integrate((e*x**2+d)*(a+b*atan(c*x))/x,x)
```

output
```
Integral((a + b*atan(c*x))*(d + e*x**2)/x, x)
```

## Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.35

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x}\,dx = \frac{1}{2}aex^2 + ad\log(x) - \frac{\pi bc^2d\log(c^2x^2+1) - 4bc^2d\arctan(cx)\log(cx) + 2ibc^2d\mathrm{Li}_2(icx+1) - 2ibc^2d\mathrm{Li}_2(-icx+1) + 2bce}{4c^2}$$

input
```
integrate((e*x^2+d)*(a+b*arctan(c*x))/x,x, algorithm="maxima")
```

output
```
1/2*a*e*x^2 + a*d*log(x) - 1/4*(pi*b*c^2*d*log(c^2*x^2 + 1) - 4*b*c^2*d*ar
ctan(c*x)*log(c*x) + 2*I*b*c^2*d*dilog(I*c*x + 1) - 2*I*b*c^2*d*dilog(-I*c
*x + 1) + 2*b*c*e*x - 2*(b*c^2*e*x^2 + b*e)*arctan(c*x))/c^2
```

## Giac [F]

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x}\,dx = \int \frac{(ex^2+d)(b\arctan(cx)+a)}{x}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arctan(c*x))/x,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)*(b*arctan(c*x) + a)/x, x)
```

## Mupad [B] (verification not implemented)

Time = 1.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.14

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x}\,dx = \begin{cases} \frac{a\,(e\,x^2+2\,d\,\ln(x))}{2} & \text{if } c=0 \\ \frac{a\,(e\,x^2+2\,d\,\ln(x))}{2}-b\,e\left(\frac{x}{2\,c}-\operatorname{atan}(c\,x)\left(\frac{1}{2\,c^2}+\frac{x^2}{2}\right)\right)-\frac{b\,d\,(\operatorname{Li}_2(1-c\,x\,1\mathrm{i})-\operatorname{Li}_2(1+c\,x\,1\mathrm{i}))\,1\mathrm{i}}{2} & \text{if } c\neq 0 \end{cases}$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2))/x,x)
```

output
```
piecewise(c == 0, (a*(e*x^2 + 2*d*log(x)))/2, c ~= 0, (a*(e*x^2 + 2*d*log(
x)))/2 - b*e*(x/(2*c) - atan(c*x)*(1/(2*c^2) + x^2/2)) - (b*d*(dilog(- c*x
*1i + 1) - dilog(c*x*1i + 1))*1i)/2)
```

## Reduce [F]

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x}\,dx = \frac{atan(cx)\,b\,c^2e\,x^2+atan(cx)\,be+2\left(\int\frac{atan(cx)}{x}dx\right)b\,c^2d+2\log(x)\,a\,c^2d+a\,c^2e\,x^2-bcex}{2c^2}$$

input
```
int((e*x^2+d)*(a+b*atan(c*x))/x,x)
```

output
```
(atan(c*x)*b*c**2*e*x**2 + atan(c*x)*b*e + 2*int(atan(c*x)/x,x)*b*c**2*d +
 2*log(x)*a*c**2*d + a*c**2*e*x**2 - b*c*e*x)/(2*c**2)
```

# 3.1118 $\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^2}\,dx$



## Optimal result

Integrand size = 19, antiderivative size = 57

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^2}\,dx = -\frac{d(a+b\arctan(cx))}{x}+ex(a+b\arctan(cx)) + bcd\log(x)-\frac{b(c^2d+e)\log(1+c^2x^2)}{2c}$$

output
```
-d*(a+b*arctan(c*x))/x+e*x*(a+b*arctan(c*x))+b*c*d*ln(x)-1/2*b*(c^2*d+e)*l
n(c^2*x^2+1)/c
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.28

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^2}\,dx = -\frac{ad}{x}+aex-\frac{bd\arctan(cx)}{x} + bex\arctan(cx)+bcd\log(x)-\frac{1}{2}bcd\log\left(1+c^2x^2\right)-\frac{be\log(1+c^2x^2)}{2c}$$

input
```
Integrate[((d + e*x^2)*(a + b*ArcTan[c*x]))/x^2,x]
```

output
```
-((a*d)/x) + a*e*x - (b*d*ArcTan[c*x])/x + b*e*x*ArcTan[c*x] + b*c*d*Log[x
] - (b*c*d*Log[1 + c^2*x^2])/2 - (b*e*Log[1 + c^2*x^2])/(2*c)
```

## Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {5511, 25, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^2}dx$$

$$\Big\downarrow \text{5511}$$

$$-bc\int -\frac{d-ex^2}{x(c^2x^2+1)}dx-\frac{d(a+b\arctan(cx))}{x}+ex(a+b\arctan(cx))$$

$$\Big\downarrow \text{25}$$

$$bc\int \frac{d-ex^2}{x(c^2x^2+1)}dx-\frac{d(a+b\arctan(cx))}{x}+ex(a+b\arctan(cx))$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}bc\int \frac{d-ex^2}{x^2(c^2x^2+1)}dx^2-\frac{d(a+b\arctan(cx))}{x}+ex(a+b\arctan(cx))$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{2}bc\int \left(\frac{d}{x^2}+\frac{-dc^2-e}{c^2x^2+1}\right)dx^2-\frac{d(a+b\arctan(cx))}{x}+ex(a+b\arctan(cx))$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d(a+b\arctan(cx))}{x}+ex(a+b\arctan(cx))+\frac{1}{2}bc\left(d\log(x^2)-\frac{(c^2d+e)\log(c^2x^2+1)}{c^2}\right)$$

input
```
Int[((d + e*x^2)*(a + b*ArcTan[c*x]))/x^2,x]
```

output
```
-((d*(a + b*ArcTan[c*x]))/x) + e*x*(a + b*ArcTan[c*x]) + (b*c*(d*Log[x^2]
- ((c^2*d + e)*Log[1 + c^2*x^2])/c^2))/2
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

## Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.30

| method | result |
|---|---|
| derivativedivides | $c\left(\frac{a\left(ecx-\frac{dc}{x}\right)}{c^2}+\frac{b\left(\arctan(cx)cxe-\frac{\arctan(cx)dc}{x}+d\,c^2\ln(cx)-\frac{\left(c^2d+e\right)\ln\left(c^2x^2+1\right)}{2}\right)}{c^2}\right)$ |
| default | $c\left(\frac{a\left(ecx-\frac{dc}{x}\right)}{c^2}+\frac{b\left(\arctan(cx)cxe-\frac{\arctan(cx)dc}{x}+d\,c^2\ln(cx)-\frac{\left(c^2d+e\right)\ln\left(c^2x^2+1\right)}{2}\right)}{c^2}\right)$ |
| parts | $a\left(ex-\frac{d}{x}\right)+bc\left(\frac{\arctan(cx)xe}{c}-\frac{\arctan(cx)d}{cx}-\frac{\frac{\left(c^2d+e\right)\ln\left(c^2x^2+1\right)}{2}-d\,c^2\ln(cx)}{c^2}\right)$ |
| parallelrisch | $\frac{2b\,c^2d\ln(x)x-\ln(c^2x^2+1)b\,c^2dx+2x^2\arctan(cx)bce+2ae\,x^2c-\ln(c^2x^2+1)bex-2\arctan(cx)bcd-2adc}{2cx}$ |
| risch | $\frac{ib(-e\,x^2+d)\ln(icx+1)}{2x}+\frac{ibce\,x^2\ln(-icx+1)+2b\,c^2d\ln(x)x-\ln(c^2x^2+1)b\,c^2dx-ibcd\ln(-icx+1)+2ae\,x^2c-\ln(c^2x}{2cx}$ |

input
```
int((e*x^2+d)*(a+b*arctan(c*x))/x^2,x,method=_RETURNVERBOSE)
```

output
```
c*(a/c^2*(e*c*x-d*c/x)+b/c^2*(arctan(c*x)*c*x*e-arctan(c*x)*d*c/x+d*c^2*ln
(c*x)-1/2*(c^2*d+e)*ln(c^2*x^2+1)))
```

## Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.30

$$\int\frac{(d+ex^2)\,(a+b\arctan(cx))}{x^2}\,dx$$
$$=\frac{2\,bc^2dx\log(x)+2\,acex^2-2\,acd-(bc^2d+be)x\log(c^2x^2+1)+2\,(bcex^2-bcd)\arctan(cx)}{2\,cx}$$

input
```
integrate((e*x^2+d)*(a+b*arctan(c*x))/x^2,x, algorithm="fricas")
```

output
```
1/2*(2*b*c^2*d*x*log(x) + 2*a*c*e*x^2 - 2*a*c*d - (b*c^2*d + b*e)*x*log(c^
2*x^2 + 1) + 2*(b*c*e*x^2 - b*c*d)*arctan(c*x))/(c*x)
```

## Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.40

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^2}\,dx = \begin{cases} -\frac{ad}{x}+aex+bcd\log(x)-\frac{bcd\log\left(x^2+\frac{1}{c^2}\right)}{2}-\frac{bd\operatorname{atan}(cx)}{x}+bex\operatorname{atan}(cx)-\frac{be\log\left(x^2+\frac{1}{c^2}\right)}{2c} & \text{for } c\neq 0 \\ a\left(-\frac{d}{x}+ex\right) & \text{otherwise} \end{cases}$$

input
```
integrate((e*x**2+d)*(a+b*atan(c*x))/x**2,x)
```

output
```
Piecewise((-a*d/x + a*e*x + b*c*d*log(x) - b*c*d*log(x**2 + c**(-2))/2 - b
*d*atan(c*x)/x + b*e*x*atan(c*x) - b*e*log(x**2 + c**(-2))/(2*c), Ne(c, 0)
), (a*(-d/x + e*x), True))
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.28

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^2}\,dx = -\frac{1}{2}\left(c\left(\log\left(c^2x^2+1\right)-\log\left(x^2\right)\right)+\frac{2\arctan(cx)}{x}\right)bd + aex + \frac{(2\,cx\arctan(cx)-\log(c^2x^2+1))be}{2\,c} - \frac{ad}{x}$$

input
```
integrate((e*x^2+d)*(a+b*arctan(c*x))/x^2,x, algorithm="maxima")
```

output
```
-1/2*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b*d + a*e*x + 1/2
*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b*e/c - a*d/x
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.51

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^2}\,dx = \frac{2\,bcex^2\arctan(cx) - bc^2dx\log(c^2x^2+1) + 2\,bc^2dx\log(x) + 2\,acex^2 - 2\,bcd\arctan(cx) - bex\log(c^2x^2}{2\,cx}$$

input
```
integrate((e*x^2+d)*(a+b*arctan(c*x))/x^2,x, algorithm="giac")
```

output
```
1/2*(2*b*c*e*x^2*arctan(c*x) - b*c^2*d*x*log(c^2*x^2 + 1) + 2*b*c^2*d*x*lo
g(x) + 2*a*c*e*x^2 - 2*b*c*d*arctan(c*x) - b*e*x*log(c^2*x^2 + 1) - 2*a*c*
d)/(c*x)
```

## Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.21

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^2}\,dx = a\,e\,x - \frac{a\,d}{x} + b\,e\,x\,\mathrm{atan}(c\,x) - \frac{b\,c\,d\,\ln(c^2\,x^2+1)}{2} + b\,c\,d\,\ln(x) - \frac{b\,d\,\mathrm{atan}(c\,x)}{x} - \frac{b\,e\,\ln(c^2\,x^2+1)}{2\,c}$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2))/x^2,x)
```

output
```
a*e*x - (a*d)/x + b*e*x*atan(c*x) - (b*c*d*log(c^2*x^2 + 1))/2 + b*c*d*log
(x) - (b*d*atan(c*x))/x - (b*e*log(c^2*x^2 + 1))/(2*c)
```

## Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.51

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^2}\,dx$$
$$= \frac{-2atan(cx)\,bcd+2atan(cx)\,bce\,x^2-\log(c^2x^2+1)\,b\,c^2dx-\log(c^2x^2+1)\,bex+2\log(x)\,b\,c^2dx-2acd+}{2cx}$$

input
```
int((e*x^2+d)*(a+b*atan(c*x))/x^2,x)
```

output
```
( - 2*atan(c*x)*b*c*d + 2*atan(c*x)*b*c*e*x**2 - log(c**2*x**2 + 1)*b*c**2
*d*x - log(c**2*x**2 + 1)*b*e*x + 2*log(x)*b*c**2*d*x - 2*a*c*d + 2*a*c*e*
x**2)/(2*c*x)
```

# 3.1119 $\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^3}\,dx$



## Optimal result

Integrand size = 19, antiderivative size = 77

$$\int \frac{(d+ex^2)\,(a+b\arctan(cx))}{x^3}\,dx = -\frac{bcd}{2x}-\frac{1}{2}bc^2d\arctan(cx)-\frac{d(a+b\arctan(cx))}{2x^2}+ae\log(x)+\frac{1}{2}ibe\,\text{PolyLog}(2,-icx)-\frac{1}{2}ibe\,\text{PolyLog}(2,icx)$$

output
```
-1/2*b*c*d/x-1/2*b*c^2*d*arctan(c*x)-1/2*d*(a+b*arctan(c*x))/x^2+a*e*ln(x)
+1/2*I*b*e*polylog(2,-I*c*x)-1/2*I*b*e*polylog(2,I*c*x)
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.01 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.12

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^3}\,dx = -\frac{ad}{2x^2}-\frac{bd\arctan(cx)}{2x^2} - \frac{bcd\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-c^2x^2\right)}{2x} + ae\log(x)+\frac{1}{2}ibe\,\text{PolyLog}(2,-icx) - \frac{1}{2}ibe\,\text{PolyLog}(2,icx)$$

input
```
Integrate[((d + e*x^2)*(a + b*ArcTan[c*x]))/x^3,x]
```

output
```
-1/2*(a*d)/x^2 - (b*d*ArcTan[c*x])/(2*x^2) - (b*c*d*Hypergeometric2F1[-1/2
, 1, 1/2, -(c^2*x^2)])/(2*x) + a*e*Log[x] + (I/2)*b*e*PolyLog[2, (-I)*c*x]
 - (I/2)*b*e*PolyLog[2, I*c*x]
```

## Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^3}\,dx$$

$$\Big\downarrow \text{5515}$$

$$\int \left(\frac{d(a+b\arctan(cx))}{x^3}+\frac{e(a+b\arctan(cx))}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d(a+b\arctan(cx))}{2x^2}+ae\log(x)-\frac{1}{2}bc^2d\arctan(cx)-\frac{bcd}{2x}+\frac{1}{2}ibe\,\text{PolyLog}(2,-icx)-\frac{1}{2}ibe\,\text{PolyLog}(2,icx)$$

input
```
Int[((d + e*x^2)*(a + b*ArcTan[c*x]))/x^3,x]
```

output
```
-1/2*(b*c*d)/x - (b*c^2*d*ArcTan[c*x])/2 - (d*(a + b*ArcTan[c*x]))/(2*x^2)
 + a*e*Log[x] + (I/2)*b*e*PolyLog[2, (-I)*c*x] - (I/2)*b*e*PolyLog[2, I*c*
x]
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

## Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(63) = 126$.

Time = 0.26 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.68

| method | result |
|---|---|
| derivativedivides | $c^2\left(\frac{ae\ln(cx)}{c^2}-\frac{ad}{2c^2x^2}+\frac{b\left(\arctan(cx)e\ln(cx)-\frac{\arctan(cx)d}{2x^2}+\frac{ie\ln(cx)\ln(icx+1)}{2}-\frac{ie\ln(cx)\ln(-icx+1)}{2}+\frac{ie\,\mathrm{dilog}(icx+}{2}\right.}{c^2}\right.$ |
| default | $c^2\left(\frac{ae\ln(cx)}{c^2}-\frac{ad}{2c^2x^2}+\frac{b\left(\arctan(cx)e\ln(cx)-\frac{\arctan(cx)d}{2x^2}+\frac{ie\ln(cx)\ln(icx+1)}{2}-\frac{ie\ln(cx)\ln(-icx+1)}{2}+\frac{ie\,\mathrm{dilog}(icx+}{2}\right.}{c^2}\right.$ |
| parts | $-\frac{ad}{2x^2}+ae\ln(x)+b\,c^2\left(-\frac{\arctan(cx)d}{2c^2x^2}+\frac{\arctan(cx)\ln(cx)e}{c^2}-\frac{-ie\ln(cx)\ln(icx+1)+ie\ln(cx)\ln(-icx+1}{}\right.$ |
| risch | $\frac{ic^2bd\ln(-icx)}{4}-\frac{bcd}{2x}-\frac{ic^2bd\ln(c^2x^2+1)}{8}-\frac{b\,c^2d\arctan(cx)}{4}-\frac{ibd\ln(-icx+1)}{4x^2}-\frac{ibe\,\mathrm{dilog}(-icx+1)}{2}-\frac{ad}{2x^2}$ |

input
```
int((e*x^2+d)*(a+b*arctan(c*x))/x^3,x,method=_RETURNVERBOSE)
```

output
```
c^2*(a/c^2*e*ln(c*x)-1/2*a*d/c^2/x^2+b/c^2*(arctan(c*x)*e*ln(c*x)-1/2*arct
an(c*x)*d/x^2+1/2*I*e*ln(c*x)*ln(1+I*c*x)-1/2*I*e*ln(c*x)*ln(1-I*c*x)+1/2*
I*e*dilog(1+I*c*x)-1/2*I*e*dilog(1-I*c*x)+1/2*d*c^2*(-arctan(c*x)-1/c/x)))
```

## Fricas [F]

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^3}\,dx = \int \frac{(ex^2+d)(b\arctan(cx)+a)}{x^3}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arctan(c*x))/x^3,x, algorithm="fricas")
```

output
```
integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arctan(c*x))/x^3, x)
```

## Sympy [F]

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^3}\,dx = \int \frac{(a+b\,\mathrm{atan}(cx))(d+ex^2)}{x^3}\,dx$$

input
```
integrate((e*x**2+d)*(a+b*atan(c*x))/x**3,x)
```

output
```
Integral((a + b*atan(c*x))*(d + e*x**2)/x**3, x)
```

## Maxima [F]

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^3}\,dx = \int \frac{(ex^2+d)(b\arctan(cx)+a)}{x^3}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arctan(c*x))/x^3,x, algorithm="maxima")
```

output
```
-1/2*((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b*d + b*e*integrate(arcta
n(c*x)/x, x) + a*e*log(x) - 1/2*a*d/x^2
```

## Giac [F]

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^3}\,dx = \int \frac{(ex^2+d)(b\arctan(cx)+a)}{x^3}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arctan(c*x))/x^3,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)*(b*arctan(c*x) + a)/x^3, x)
```

## Mupad [B] (verification not implemented)

Time = 1.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.18

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^3}\,dx = \begin{cases} a\,e\,\ln(x)-\frac{a\,d}{2\,x^2} & \text{if } c=0 \\ a\,e\,\ln(x)-\frac{a\,d}{2\,x^2}-\frac{b\,d\,\mathrm{atan}(c\,x)}{2\,x^2}-\frac{b\,d\left(c^3\,\mathrm{atan}(c\,x)+\frac{c^2}{x}\right)}{2\,c}-\frac{b\,e\,(\mathrm{Li}_2(1-c\,x\,1\mathrm{i})-\mathrm{Li}_2(1+c\,x\,1\mathrm{i}))\,1\mathrm{i}}{2} & \text{if } c\neq 0 \end{cases}$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2))/x^3,x)
```

output
```
piecewise(c == 0, a*e*log(x) - (a*d)/(2*x^2), c ~= 0, a*e*log(x) - (b*e*(d
ilog(- c*x*1i + 1) - dilog(c*x*1i + 1))*1i)/2 - (a*d)/(2*x^2) - (b*d*atan(
c*x))/(2*x^2) - (b*d*(c^3*atan(c*x) + c^2/x))/(2*c))
```

## Reduce [F]

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^3}\,dx$$
$$= \frac{-atan(cx)\,b\,c^2d\,x^2 - atan(cx)\,bd + 2\left(\int \frac{atan(cx)}{x}dx\right)be\,x^2 + 2\log(x)\,ae\,x^2 - ad - bcdx}{2x^2}$$

input
```
int((e*x^2+d)*(a+b*atan(c*x))/x^3,x)
```

output
```
( - atan(c*x)*b*c**2*d*x**2 - atan(c*x)*b*d + 2*int(atan(c*x)/x,x)*b*e*x**
2 + 2*log(x)*a*e*x**2 - a*d - b*c*d*x)/(2*x**2)
```

# 3.1120 $\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^4} dx$



## Optimal result

Integrand size = 19, antiderivative size = 83

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^4} dx = -\frac{bcd}{6x^2} - \frac{d(a+b\arctan(cx))}{3x^3} - \frac{e(a+b\arctan(cx))}{x} - \frac{1}{3}bc(c^2d-3e)\log(x) + \frac{1}{6}bc(c^2d-3e)\log(1+c^2x^2)$$

output
```
-1/6*b*c*d/x^2-1/3*d*(a+b*arctan(c*x))/x^3-e*(a+b*arctan(c*x))/x-1/3*b*c*(
c^2*d-3*e)*ln(x)+1/6*b*c*(c^2*d-3*e)*ln(c^2*x^2+1)
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.18

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^4} dx = -\frac{ad}{3x^3} - \frac{ae}{x} - \frac{bd\arctan(cx)}{3x^3} - \frac{be\arctan(cx)}{x} + bce\log(x) - \frac{1}{2}bce\log(1+c^2x^2) + \frac{1}{6}bcd\left(-\frac{1}{x^2} - 2c^2\log(x) + c^2\log(1+c^2x^2)\right)$$

input
```
Integrate[((d + e*x^2)*(a + b*ArcTan[c*x]))/x^4,x]
```

output
```
-1/3*(a*d)/x^3 - (a*e)/x - (b*d*ArcTan[c*x])/(3*x^3) - (b*e*ArcTan[c*x])/x
 + b*c*e*Log[x] - (b*c*e*Log[1 + c^2*x^2])/2 + (b*c*d*(-x^(-2) - 2*c^2*Log
[x] + c^2*Log[1 + c^2*x^2]))/6
```

## Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {5511, 27, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^4}dx$$

$$\Big\downarrow \text{5511}$$

$$-bc\int -\frac{3ex^2+d}{3x^3(c^2x^2+1)}dx-\frac{d(a+b\arctan(cx))}{3x^3}-\frac{e(a+b\arctan(cx))}{x}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}bc\int \frac{3ex^2+d}{x^3(c^2x^2+1)}dx-\frac{d(a+b\arctan(cx))}{3x^3}-\frac{e(a+b\arctan(cx))}{x}$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{6}bc\int \frac{3ex^2+d}{x^4(c^2x^2+1)}dx^2-\frac{d(a+b\arctan(cx))}{3x^3}-\frac{e(a+b\arctan(cx))}{x}$$

$$\Big\downarrow \text{86}$$

$$\frac{1}{6}bc\int \left(\frac{d}{x^4}+\frac{c^4d-3c^2e}{c^2x^2+1}+\frac{3e-c^2d}{x^2}\right)dx^2-\frac{d(a+b\arctan(cx))}{3x^3}-\frac{e(a+b\arctan(cx))}{x}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d(a+b\arctan(cx))}{3x^3}-\frac{e(a+b\arctan(cx))}{x}+\frac{1}{6}bc\left(-\log(x^2)(c^2d-3e)+(c^2d-3e)\log(c^2x^2+1)-\frac{d}{x^2}\right)$$

input
```
Int[((d + e*x^2)*(a + b*ArcTan[c*x]))/x^4,x]
```

output
```
-1/3*(d*(a + b*ArcTan[c*x]))/x^3 - (e*(a + b*ArcTan[c*x]))/x + (b*c*(-(d/x
^2) - (c^2*d - 3*e)*Log[x^2] + (c^2*d - 3*e)*Log[1 + c^2*x^2]))/6
```

**Defintions of rubi rules used**

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

## Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.17

| method | result |
|---|---|
| parts | $a\left(-\frac{d}{3x^3}-\frac{e}{x}\right)+b\,c^3\left(-\frac{\arctan(cx)d}{3c^3x^3}-\frac{\arctan(cx)e}{c^3x}-\frac{\frac{\left(-c^2d+3e\right)\ln\left(c^2x^2+1\right)}{2}+\left(c^2d-3e\right)\ln(cx)+\frac{d}{2x^2}}{3c^2}\right)$ |
| derivativedivides | $c^3\left(\frac{a\left(-\frac{e}{cx}-\frac{d}{3c\,x^3}\right)}{c^2}+\frac{b\left(-\frac{\arctan(cx)e}{cx}-\frac{\arctan(cx)d}{3c\,x^3}-\frac{\left(c^2d-3e\right)\ln(cx)}{3}-\frac{d}{6x^2}-\frac{\left(-c^2d+3e\right)\ln\left(c^2x^2+1\right)}{6}\right)}{c^2}\right)$ |
| default | $c^3\left(\frac{a\left(-\frac{e}{cx}-\frac{d}{3c\,x^3}\right)}{c^2}+\frac{b\left(-\frac{\arctan(cx)e}{cx}-\frac{\arctan(cx)d}{3c\,x^3}-\frac{\left(c^2d-3e\right)\ln(cx)}{3}-\frac{d}{6x^2}-\frac{\left(-c^2d+3e\right)\ln\left(c^2x^2+1\right)}{6}\right)}{c^2}\right)$ |
| parallelrisch | $-\frac{2\ln(x)b\,c^3d\,x^3-\ln\left(c^2x^2+1\right)b\,c^3d\,x^3-b\,c^3d\,x^3-6\ln(x)bce\,x^3+3\ln\left(c^2x^2+1\right)bce\,x^3+6\arctan(cx)be\,x^2+6ae\,x^2+bcdx}{6x^3}$ |
| risch | $\frac{ib\left(3e\,x^2+d\right)\ln(icx+1)}{6x^3}-\frac{2\ln(x)b\,c^3d\,x^3-\ln\left(c^2x^2+1\right)b\,c^3d\,x^3-6\ln(x)bce\,x^3+3\ln\left(c^2x^2+1\right)bce\,x^3+3ibe\ln(-icx+1)}{6x^3}$ |

input
```
int((e*x^2+d)*(a+b*arctan(c*x))/x^4,x,method=_RETURNVERBOSE)
```

output
```
a*(-1/3*d/x^3-e/x)+b*c^3*(-1/3*arctan(c*x)*d/c^3/x^3-arctan(c*x)/c^3*e/x-1
/3/c^2*(1/2*(-c^2*d+3*e)*ln(c^2*x^2+1)+(c^2*d-3*e)*ln(c*x)+1/2*d/x^2))
```

## Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.02

$$\int\frac{(d+ex^2)\,(a+b\arctan(cx))}{x^4}\,dx$$
$$=\frac{(bc^3d-3\,bce)x^3\log\left(c^2x^2+1\right)-2\,(bc^3d-3\,bce)x^3\log\left(x\right)-bcdx-6\,aex^2-2\,ad-2\,(3\,bex^2+bd)\,\mathrm{arcta}}{6\,x^3}$$

input
```
integrate((e*x^2+d)*(a+b*arctan(c*x))/x^4,x, algorithm="fricas")
```

output
```
1/6*((b*c^3*d - 3*b*c*e)*x^3*log(c^2*x^2 + 1) - 2*(b*c^3*d - 3*b*c*e)*x^3*
log(x) - b*c*d*x - 6*a*e*x^2 - 2*a*d - 2*(3*b*e*x^2 + b*d)*arctan(c*x))/x^
3
```

## Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.40

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^4}\,dx = \begin{cases} -\frac{ad}{3x^3} - \frac{ae}{x} - \frac{bc^3d\log(x)}{3} + \frac{bc^3d\log\left(x^2+\frac{1}{c^2}\right)}{6} - \frac{bcd}{6x^2} + bce\log(x) - \frac{bce\log\left(x^2+\frac{1}{c^2}\right)}{2} - \frac{bd\,\mathrm{atan}(cx)}{3x^3} - \frac{be\,\mathrm{atan}(cx)}{x} & \text{for} \\ a\left(-\frac{d}{3x^3} - \frac{e}{x}\right) & \text{ot} \end{cases}$$

input
```
integrate((e*x**2+d)*(a+b*atan(c*x))/x**4,x)
```

output
```
Piecewise((-a*d/(3*x**3) - a*e/x - b*c**3*d*log(x)/3 + b*c**3*d*log(x**2 +
 c**(-2))/6 - b*c*d/(6*x**2) + b*c*e*log(x) - b*c*e*log(x**2 + c**(-2))/2
- b*d*atan(c*x)/(3*x**3) - b*e*atan(c*x)/x, Ne(c, 0)), (a*(-d/(3*x**3) - e
/x), True))
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.12

$$\begin{aligned}\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^4}\,dx &= \frac{1}{6}\left(\left(c^2\log\left(c^2x^2+1\right) - c^2\log\left(x^2\right) - \frac{1}{x^2}\right)c - \frac{2\arctan(cx)}{x^3}\right)bd \\ &\quad - \frac{1}{2}\left(c\left(\log\left(c^2x^2+1\right) - \log\left(x^2\right)\right) + \frac{2\arctan(cx)}{x}\right)be - \frac{ae}{x} - \frac{ad}{3x^3}\end{aligned}$$

input
```
integrate((e*x^2+d)*(a+b*arctan(c*x))/x^4,x, algorithm="maxima")
```

output
```
1/6*((c^2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*
b*d - 1/2*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b*e - a*e/x
- 1/3*a*d/x^3
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.22

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^4}\,dx = \frac{bc^3dx^3\log(c^2x^2+1) - 2\,bc^3dx^3\log(x) - 3\,bcex^3\log(c^2x^2+1) + 6\,bcex^3\log(x) - 6\,bex^2\arctan(cx) - b}{6\,x^3}$$

input
```
integrate((e*x^2+d)*(a+b*arctan(c*x))/x^4,x, algorithm="giac")
```

output
```
1/6*(b*c^3*d*x^3*log(c^2*x^2 + 1) - 2*b*c^3*d*x^3*log(x) - 3*b*c*e*x^3*log
(c^2*x^2 + 1) + 6*b*c*e*x^3*log(x) - 6*b*e*x^2*arctan(c*x) - b*c*d*x - 6*a
*e*x^2 - 2*b*d*arctan(c*x) - 2*a*d)/x^3
```

## Mupad [B] (verification not implemented)

Time = 0.90 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.11

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^4}\,dx = b\,c\,e\,\ln(x) - \frac{a\,e}{x} - \frac{b\,c\,e\,\ln(c^2\,x^2+1)}{2} - \frac{b\,c\,d}{6\,x^2} - \frac{a\,d}{3\,x^3} - \frac{b\,d\,\mathrm{atan}(c\,x)}{3\,x^3} - \frac{b\,e\,\mathrm{atan}(c\,x)}{x} + \frac{b\,c^3\,d\,\ln(c^2\,x^2+1)}{6} - \frac{b\,c^3\,d\,\ln(x)}{3}$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2))/x^4,x)
```

output
```
b*c*e*log(x) - (a*e)/x - (b*c*e*log(c^2*x^2 + 1))/2 - (b*c*d)/(6*x^2) - (a
*d)/(3*x^3) - (b*d*atan(c*x))/(3*x^3) - (b*e*atan(c*x))/x + (b*c^3*d*log(c
^2*x^2 + 1))/6 - (b*c^3*d*log(x))/3
```

## Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.22

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^4}\,dx$$
$$= \frac{-2atan(cx)\,bd-6atan(cx)\,be\,x^2+\log(c^2x^2+1)\,b\,c^3d\,x^3-3\log(c^2x^2+1)\,bce\,x^3-2\log(x)\,b\,c^3d\,x^3+6}{6x^3}$$

input
```
int((e*x^2+d)*(a+b*atan(c*x))/x^4,x)
```

output
```
( - 2*atan(c*x)*b*d - 6*atan(c*x)*b*e*x**2 + log(c**2*x**2 + 1)*b*c**3*d*x
**3 - 3*log(c**2*x**2 + 1)*b*c*e*x**3 - 2*log(x)*b*c**3*d*x**3 + 6*log(x)*
b*c*e*x**3 - 2*a*d - 6*a*e*x**2 - b*c*d*x)/(6*x**3)
```

# 3.1121 $\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^5}\,dx$



## Optimal result

Integrand size = 19, antiderivative size = 79

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^5}\,dx = -\frac{bcd}{12x^3}+\frac{bc(c^2d-2e)}{4x}+\frac{b(c^2d-e)^2\arctan(cx)}{4d} -\frac{(d+ex^2)^2(a+b\arctan(cx))}{4dx^4}$$

output
```
-1/12*b*c*d/x^3+1/4*b*c*(c^2*d-2*e)/x+1/4*b*(c^2*d-e)^2*arctan(c*x)/d-1/4*
(e*x^2+d)^2*(a+b*arctan(c*x))/d/x^4
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.23

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^5}\,dx = -\frac{ad}{4x^4}-\frac{ae}{2x^2}-\frac{bd\arctan(cx)}{4x^4}-\frac{be\arctan(cx)}{2x^2} -\frac{bcd\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-c^2x^2\right)}{12x^3} -\frac{bce\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-c^2x^2\right)}{2x}$$

input
```
Integrate[((d + e*x^2)*(a + b*ArcTan[c*x]))/x^5,x]
```

output
```
-1/4*(a*d)/x^4 - (a*e)/(2*x^2) - (b*d*ArcTan[c*x])/(4*x^4) - (b*e*ArcTan[c
*x])/(2*x^2) - (b*c*d*Hypergeometric2F1[-3/2, 1, -1/2, -(c^2*x^2)])/(12*x^
3) - (b*c*e*Hypergeometric2F1[-1/2, 1, 1/2, -(c^2*x^2)])/(2*x)
```

## Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {5511, 27, 359, 264, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^5}dx$$

$$\Big\downarrow \; 5511$$

$$-bc\int -\frac{2ex^2+d}{4x^4(c^2x^2+1)}dx-\frac{d(a+b\arctan(cx))}{4x^4}-\frac{e(a+b\arctan(cx))}{2x^2}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{4}bc\int \frac{2ex^2+d}{x^4(c^2x^2+1)}dx-\frac{d(a+b\arctan(cx))}{4x^4}-\frac{e(a+b\arctan(cx))}{2x^2}$$

$$\Big\downarrow \; 359$$

$$\frac{1}{4}bc\left(-(c^2d-2e)\int \frac{1}{x^2(c^2x^2+1)}dx-\frac{d}{3x^3}\right)-\frac{d(a+b\arctan(cx))}{4x^4}-\frac{e(a+b\arctan(cx))}{2x^2}$$

$$\Big\downarrow \; 264$$

$$\frac{1}{4}bc\left(-(c^2d-2e)\left(c^2\left(-\int \frac{1}{c^2x^2+1}dx\right)-\frac{1}{x}\right)-\frac{d}{3x^3}\right)-\frac{d(a+b\arctan(cx))}{4x^4}-\frac{e(a+b\arctan(cx))}{2x^2}$$

$$\Big\downarrow \; 216$$

$$-\frac{d(a+b\arctan(cx))}{4x^4}-\frac{e(a+b\arctan(cx))}{2x^2}+\frac{1}{4}bc\left(-\left(-c\arctan(cx)-\frac{1}{x}\right)(c^2d-2e)-\frac{d}{3x^3}\right)$$

input
```
Int[((d + e*x^2)*(a + b*ArcTan[c*x]))/x^5,x]
```

output
```
-1/4*(d*(a + b*ArcTan[c*x]))/x^4 - (e*(a + b*ArcTan[c*x]))/(2*x^2) + (b*c*
(-1/3*d/x^3 - (c^2*d - 2*e)*(-x^(-1) - c*ArcTan[c*x])))/4
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

## Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.22

| method | result |
|---|---|
| parts | $a\left(-\frac{d}{4x^4}-\frac{e}{2x^2}\right)+b\,c^4\left(-\frac{\arctan(cx)d}{4c^4x^4}-\frac{\arctan(cx)e}{2c^4x^2}-\frac{(-c^2d+2e)\arctan(cx)-\frac{c^2d-2e}{cx}+\frac{d}{3c\,x^3}}{4c^2}\right)$ |
| parallelrisch | $\frac{3x^4\arctan(cx)b\,c^4d-6\arctan(cx)b\,c^2e\,x^4+6a\,c^2e\,x^4+3b\,c^3d\,x^3-6bce\,x^3-6\arctan(cx)be\,x^2-6ae\,x^2-bcdx-3\arctan(c}{12x^4}$ |
| derivativedivides | $c^4\left(\frac{a\left(-\frac{e}{2c^2x^2}-\frac{d}{4c^2x^4}\right)}{c^2}+\frac{b\left(-\frac{\arctan(cx)e}{2c^2x^2}-\frac{\arctan(cx)d}{4c^2x^4}+\frac{c^2d-2e}{4cx}-\frac{d}{12c\,x^3}-\frac{(-c^2d+2e)\arctan(cx)}{4}\right)}{c^2}\right)$ |
| default | $c^4\left(\frac{a\left(-\frac{e}{2c^2x^2}-\frac{d}{4c^2x^4}\right)}{c^2}+\frac{b\left(-\frac{\arctan(cx)e}{2c^2x^2}-\frac{\arctan(cx)d}{4c^2x^4}+\frac{c^2d-2e}{4cx}-\frac{d}{12c\,x^3}-\frac{(-c^2d+2e)\arctan(cx)}{4}\right)}{c^2}\right)$ |
| risch | $\frac{ib(2e\,x^2+d)\ln(icx+1)}{8x^4}-\frac{3i\ln(-cx+i)b\,c^4d\,x^4-3i\ln(-cx-i)b\,c^4d\,x^4-6i\ln(-cx+i)b\,c^2e\,x^4+6i\ln(-cx-i)b\,c^2e\,x^4-}{24x^4}$ |
| orering | $\frac{(6c^4de\,x^6+9c^4d^2x^4-12c^2e^2x^6-15c^2de\,x^4+5c^2d^2x^2-12e^2x^4-21de\,x^2-4d^2)(a+b\arctan(cx))}{6x^4(e\,x^2+d)}+\frac{(3c^2d\,x^2-6e\,x^2-d}{}$ |

input
```
int((e*x^2+d)*(a+b*arctan(c*x))/x^5,x,method=_RETURNVERBOSE)
```

output
```
a*(-1/4*d/x^4-1/2*e/x^2)+b*c^4*(-1/4*arctan(c*x)*d/c^4/x^4-1/2*arctan(c*x)
/c^4*e/x^2-1/4/c^2*((-c^2*d+2*e)*arctan(c*x)-(c^2*d-2*e)/c/x+1/3*d/c/x^3))
```

## Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.95

$$\int\frac{(d+ex^2)\,(a+b\arctan(cx))}{x^5}\,dx=-\frac{bcdx+6\,aex^2-3\,(bc^3d-2\,bce)x^3+3\,ad-3\,((bc^4d-2\,bc^2e)x^4-2\,bex^2-bd)\arctan(cx)}{12\,x^4}$$

input
```
integrate((e*x^2+d)*(a+b*arctan(c*x))/x^5,x, algorithm="fricas")
```

output
```
-1/12*(b*c*d*x + 6*a*e*x^2 - 3*(b*c^3*d - 2*b*c*e)*x^3 + 3*a*d - 3*((b*c^4
*d - 2*b*c^2*e)*x^4 - 2*b*e*x^2 - b*d)*arctan(c*x))/x^4
```

## Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.25

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^5}\,dx = -\frac{ad}{4x^4}-\frac{ae}{2x^2}+\frac{bc^4d\,\mathrm{atan}\,(cx)}{4}+\frac{bc^3d}{4x}-\frac{bc^2e\,\mathrm{atan}\,(cx)}{2} - \frac{bcd}{12x^3}-\frac{bce}{2x}-\frac{bd\,\mathrm{atan}\,(cx)}{4x^4}-\frac{be\,\mathrm{atan}\,(cx)}{2x^2}$$

input
```
integrate((e*x**2+d)*(a+b*atan(c*x))/x**5,x)
```

output
```
-a*d/(4*x**4) - a*e/(2*x**2) + b*c**4*d*atan(c*x)/4 + b*c**3*d/(4*x) - b*c
**2*e*atan(c*x)/2 - b*c*d/(12*x**3) - b*c*e/(2*x) - b*d*atan(c*x)/(4*x**4)
 - b*e*atan(c*x)/(2*x**2)
```

## Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.01

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^5}\,dx = \frac{1}{12}\left(\left(3\,c^3\arctan(cx)+\frac{3\,c^2x^2-1}{x^3}\right)c-\frac{3\arctan(cx)}{x^4}\right)bd - \frac{1}{2}\left(\left(c\arctan(cx)+\frac{1}{x}\right)c+\frac{\arctan(cx)}{x^2}\right)be-\frac{ae}{2\,x^2}-\frac{ad}{4\,x^4}$$

input
```
integrate((e*x^2+d)*(a+b*arctan(c*x))/x^5,x, algorithm="maxima")
```

output
```
1/12*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan(c*x)/x^4)*b*d
 - 1/2*((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b*e - 1/2*a*e/x^2 - 1/4
*a*d/x^4
```

## Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.62

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^5}\,dx =$$
$$-\frac{3i\,bc^4dx^4\log(i\,cx+1)-3i\,bc^4dx^4\log(-i\,cx+1)-6i\,bc^2ex^4\log(i\,cx+1)+6i\,bc^2ex^4\log(-i\,cx+1)}{24\,x^4}$$

input
```
integrate((e*x^2+d)*(a+b*arctan(c*x))/x^5,x, algorithm="giac")
```

output
```
-1/24*(3*I*b*c^4*d*x^4*log(I*c*x + 1) - 3*I*b*c^4*d*x^4*log(-I*c*x + 1) -
6*I*b*c^2*e*x^4*log(I*c*x + 1) + 6*I*b*c^2*e*x^4*log(-I*c*x + 1) - 6*b*c^3
*d*x^3 + 12*b*c*e*x^3 + 12*b*e*x^2*arctan(c*x) + 2*b*c*d*x + 12*a*e*x^2 +
6*b*d*arctan(c*x) + 6*a*d)/x^4
```

## Mupad [B] (verification not implemented)

Time = 0.93 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.05

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^5}\,dx =$$
$$-\frac{\frac{a\,d}{4}+\frac{a\,x^2\,(d\,c^2+2\,e)}{4}+\frac{b\,d\,\mathrm{atan}(c\,x)}{4}+\frac{b\,c\,d\,x}{12}+\frac{b\,c^3\,x^5\,(2\,e-c^2\,d)}{4}+\frac{b\,c\,x^3\,(3\,e-c^2\,d)}{6}-\frac{a\,c^4\,e\,x^6}{2}+\frac{b\,x^2\,\mathrm{atan}(c\,x)\,(d\,c^2+2\,e)}{4}+}{c^2\,x^6+x^4}$$
$$-\frac{\mathrm{atan}\left(\frac{c^2\,x}{\sqrt{c^2}}\right)\,(2\,b\,e-b\,c^2\,d)\,(c^2)^{3/2}}{4\,c}$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2))/x^5,x)
```

output
```
- ((a*d)/4 + (a*x^2*(2*e + c^2*d))/4 + (b*d*atan(c*x))/4 + (b*c*d*x)/12 +
(b*c^3*x^5*(2*e - c^2*d))/4 + (b*c*x^3*(3*e - c^2*d))/6 - (a*c^4*e*x^6)/2
+ (b*x^2*atan(c*x)*(2*e + c^2*d))/4 + (b*c^2*e*x^4*atan(c*x))/2)/(x^4 + c^
2*x^6) - (atan((c^2*x)/(c^2)^(1/2))*(2*b*e - b*c^2*d)*(c^2)^(3/2))/(4*c)
```

## Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.11

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^5}\,dx$$
$$= \frac{3atan(cx)\,b\,c^4d\,x^4 - 6atan(cx)\,b\,c^2e\,x^4 - 3atan(cx)\,bd - 6atan(cx)\,be\,x^2 - 3ad - 6ae\,x^2 + 3b\,c^3d\,x^3 - bc}{12x^4}$$

input
```
int((e*x^2+d)*(a+b*atan(c*x))/x^5,x)
```

output
```
(3*atan(c*x)*b*c**4*d*x**4 - 6*atan(c*x)*b*c**2*e*x**4 - 3*atan(c*x)*b*d -
 6*atan(c*x)*b*e*x**2 - 3*a*d - 6*a*e*x**2 + 3*b*c**3*d*x**3 - b*c*d*x - 6
*b*c*e*x**3)/(12*x**4)
```

# 3.1122 $\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^6}\,dx$



## Optimal result

Integrand size = 19, antiderivative size = 110

$$\int \frac{(d+ex^2)\,(a+b\arctan(cx))}{x^6}\,dx = -\frac{bcd}{20x^4}+\frac{bc(3c^2d-5e)}{30x^2}-\frac{d(a+b\arctan(cx))}{5x^5}-\frac{e(a+b\arctan(cx))}{3x^3}+\frac{1}{15}bc^3\left(3c^2d-5e\right)\log(x)-\frac{1}{30}bc^3\left(3c^2d-5e\right)\log\left(1+c^2x^2\right)$$

output
```
-1/20*b*c*d/x^4+1/30*b*c*(3*c^2*d-5*e)/x^2-1/5*d*(a+b*arctan(c*x))/x^5-1/3
*e*(a+b*arctan(c*x))/x^3+1/15*b*c^3*(3*c^2*d-5*e)*ln(x)-1/30*b*c^3*(3*c^2*
d-5*e)*ln(c^2*x^2+1)
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.19

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^6}\,dx = -\frac{ad}{5x^5}-\frac{bcd}{20x^4}-\frac{ae}{3x^3}+\frac{bc^3d}{10x^2} -\frac{bd\arctan(cx)}{5x^5}-\frac{be\arctan(cx)}{3x^3} +\frac{1}{5}bc^5d\log(x)-\frac{1}{10}bc^5d\log\left(1+c^2x^2\right) +\frac{1}{6}bce\left(-\frac{1}{x^2}-2c^2\log(x)+c^2\log\left(1+c^2x^2\right)\right)$$

input
```
Integrate[((d + e*x^2)*(a + b*ArcTan[c*x]))/x^6,x]
```

output
```
-1/5*(a*d)/x^5 - (b*c*d)/(20*x^4) - (a*e)/(3*x^3) + (b*c^3*d)/(10*x^2) - (
b*d*ArcTan[c*x])/(5*x^5) - (b*e*ArcTan[c*x])/(3*x^3) + (b*c^5*d*Log[x])/5
- (b*c^5*d*Log[1 + c^2*x^2])/10 + (b*c*e*(-x^(-2) - 2*c^2*Log[x] + c^2*Log
[1 + c^2*x^2]))/6
```

## Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {5511, 27, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^6}dx$$

$$\Big\downarrow \text{5511}$$

$$-bc\int -\frac{5ex^2+3d}{15x^5(c^2x^2+1)}dx-\frac{d(a+b\arctan(cx))}{5x^5}-\frac{e(a+b\arctan(cx))}{3x^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{15}bc\int \frac{5ex^2+3d}{x^5\left(c^2x^2+1\right)}dx-\frac{d(a+b\arctan(cx))}{5x^5}-\frac{e(a+b\arctan(cx))}{3x^3}$$

$\big\downarrow$ 354

$$\frac{1}{30}bc\int \frac{5ex^2+3d}{x^6\left(c^2x^2+1\right)}dx^2-\frac{d(a+b\arctan(cx))}{5x^5}-\frac{e(a+b\arctan(cx))}{3x^3}$$

$\big\downarrow$ 86

$$\frac{1}{30}bc\int \left(\frac{3d}{x^6}+\frac{5c^4e-3c^6d}{c^2x^2+1}+\frac{3c^4d-5c^2e}{x^2}+\frac{5e-3c^2d}{x^4}\right)dx^2-\frac{d(a+b\arctan(cx))}{5x^5}-\frac{e(a+b\arctan(cx))}{3x^3}$$

$\big\downarrow$ 2009

$$-\frac{d(a+b\arctan(cx))}{5x^5}-\frac{e(a+b\arctan(cx))}{3x^3}+\frac{1}{30}bc\left(\frac{3c^2d-5e}{x^2}+c^2\log\left(x^2\right)\left(3c^2d-5e\right)-c^2\left(3c^2d-5e\right)\log\left(c^2x^2+1\right)-\frac{3d}{2x^4}\right)$$

input
```
Int[((d + e*x^2)*(a + b*ArcTan[c*x]))/x^6,x]
```

output
```
-1/5*(d*(a + b*ArcTan[c*x]))/x^5 - (e*(a + b*ArcTan[c*x]))/(3*x^3) + (b*c*
((-3*d)/(2*x^4) + (3*c^2*d - 5*e)/x^2 + c^2*(3*c^2*d - 5*e)*Log[x^2] - c^2
*(3*c^2*d - 5*e)*Log[1 + c^2*x^2]))/30
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

## Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.08

| method | result |
|---|---|
| parts | $a\left(-\frac{e}{3x^3}-\frac{d}{5x^5}\right)+b\,c^5\left(-\frac{\arctan(cx)e}{3c^5x^3}-\frac{\arctan(cx)d}{5c^5x^5}-\frac{\frac{(3c^2d-5e)\ln(c^2x^2+1)}{2}+(-3c^2d+5e)\ln(cx)-\frac{3c^2d\cdots}{2c^2\cdots}}{15c^2}\cdots\right.$ |
| derivativedivides | $c^5\left(\frac{a\left(-\frac{d}{5c^3x^5}-\frac{e}{3c^3x^3}\right)}{c^2}+\frac{b\left(-\frac{\arctan(cx)d}{5c^3x^5}-\frac{\arctan(cx)e}{3c^3x^3}-\frac{(3c^2d-5e)\ln(c^2x^2+1)}{30}-\frac{(-3c^2d+5e)\ln(cx)}{15}+\frac{3c^2d-5e}{30c^2x^2}-\frac{\cdots}{20\cdots}\cdots\right.}{c^2}\cdots\right.$ |
| default | $c^5\left(\frac{a\left(-\frac{d}{5c^3x^5}-\frac{e}{3c^3x^3}\right)}{c^2}+\frac{b\left(-\frac{\arctan(cx)d}{5c^3x^5}-\frac{\arctan(cx)e}{3c^3x^3}-\frac{(3c^2d-5e)\ln(c^2x^2+1)}{30}-\frac{(-3c^2d+5e)\ln(cx)}{15}+\frac{3c^2d-5e}{30c^2x^2}-\cdots\right.}{c^2}\cdots\right.$ |
| parallelrisch | $\frac{12\ln(x)bc^5dx^5-6\ln(c^2x^2+1)x^5bc^5d-6bc^5dx^5-20\ln(x)bc^3ex^5+10\ln(c^2x^2+1)x^5bc^3e+10bc^3ex^5+6bc^3dx^3-10b\cdots}{60x^5}$ |
| risch | $\frac{ib(5ex^2+3d)\ln(icx+1)}{30x^5}-\frac{-12\ln(x)bc^5dx^5+6\ln(-c^2x^2-1)bc^5dx^5+20\ln(x)bc^3ex^5-10\ln(-c^2x^2-1)bc^3ex^5-6b\cdots}{60x^5}$ |

input
```
int((e*x^2+d)*(a+b*arctan(c*x))/x^6,x,method=_RETURNVERBOSE)
```

output
```
a*(-1/3*e/x^3-1/5*d/x^5)+b*c^5*(-1/3*arctan(c*x)/c^5*e/x^3-1/5*arctan(c*x)
*d/c^5/x^5-1/15/c^2*(1/2*(3*c^2*d-5*e)*ln(c^2*x^2+1)+(-3*c^2*d+5*e)*ln(c*x
)-1/2*(3*c^2*d-5*e)/c^2/x^2+3/4*d/c^2/x^4))
```

## Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.01

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^6}\,dx = -\frac{2\,(3\,bc^5d-5\,bc^3e)x^5\log\left(c^2x^2+1\right)-4\,(3\,bc^5d-5\,bc^3e)x^5\log\left(x\right)+3\,bcdx+20\,aex^2-2\,(3\,bc^3d-5\,bc}{60\,x^5}$$

input
```
integrate((e*x^2+d)*(a+b*arctan(c*x))/x^6,x, algorithm="fricas")
```

output
```
-1/60*(2*(3*b*c^5*d - 5*b*c^3*e)*x^5*log(c^2*x^2 + 1) - 4*(3*b*c^5*d - 5*b
*c^3*e)*x^5*log(x) + 3*b*c*d*x + 20*a*e*x^2 - 2*(3*b*c^3*d - 5*b*c*e)*x^3
+ 12*a*d + 4*(5*b*e*x^2 + 3*b*d)*arctan(c*x))/x^5
```

## Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.39

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^6}\,dx = \begin{cases} -\frac{ad}{5x^5}-\frac{ae}{3x^3}+\frac{bc^5d\log\left(x\right)}{5}-\frac{bc^5d\log\left(x^2+\frac{1}{c^2}\right)}{10}+\frac{bc^3d}{10x^2}-\frac{bc^3e\log\left(x\right)}{3}+\frac{bc^3e\log\left(x^2+\frac{1}{c^2}\right)}{6}-\frac{bcd}{20x^4}-\frac{bce}{6x^2}-\frac{bd\,\mathrm{atan}\left(cx\right)}{5x^5}- \\ a\left(-\frac{d}{5x^5}-\frac{e}{3x^3}\right) \end{cases}$$

input
```
integrate((e*x**2+d)*(a+b*atan(c*x))/x**6,x)
```

output
```
Piecewise((-a*d/(5*x**5) - a*e/(3*x**3) + b*c**5*d*log(x)/5 - b*c**5*d*log
(x**2 + c**(-2))/10 + b*c**3*d/(10*x**2) - b*c**3*e*log(x)/3 + b*c**3*e*lo
g(x**2 + c**(-2))/6 - b*c*d/(20*x**4) - b*c*e/(6*x**2) - b*d*atan(c*x)/(5*
x**5) - b*e*atan(c*x)/(3*x**3), Ne(c, 0)), (a*(-d/(5*x**5) - e/(3*x**3)),
True))
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.05

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^6}\,dx = -\frac{1}{20}\left(\left(2\,c^4\log\left(c^2x^2+1\right)-2\,c^4\log\left(x^2\right)-\frac{2\,c^2x^2-1}{x^4}\right)c+\frac{4\,\arctan\left(cx\right)}{x^5}\right)bd + \frac{1}{6}\left(\left(c^2\log\left(c^2x^2+1\right)-c^2\log\left(x^2\right)-\frac{1}{x^2}\right)c-\frac{2\,\arctan\left(cx\right)}{x^3}\right)be-\frac{ae}{3\,x^3}-\frac{ad}{5\,x^5}$$

input
```
integrate((e*x^2+d)*(a+b*arctan(c*x))/x^6,x, algorithm="maxima")
```

output
```
-1/20*((2*c^4*log(c^2*x^2 + 1) - 2*c^4*log(x^2) - (2*c^2*x^2 - 1)/x^4)*c +
 4*arctan(c*x)/x^5)*b*d + 1/6*((c^2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^
2)*c - 2*arctan(c*x)/x^3)*b*e - 1/3*a*e/x^3 - 1/5*a*d/x^5
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.13

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^6}\,dx = -\frac{6\,bc^5dx^5\log\left(c^2x^2+1\right)-12\,bc^5dx^5\log\left(x\right)-10\,bc^3ex^5\log\left(c^2x^2+1\right)+20\,bc^3ex^5\log\left(x\right)-6\,bc^3dx^3+1}{60\,x^5}$$

input
```
integrate((e*x^2+d)*(a+b*arctan(c*x))/x^6,x, algorithm="giac")
```

output
```
-1/60*(6*b*c^5*d*x^5*log(c^2*x^2 + 1) - 12*b*c^5*d*x^5*log(x) - 10*b*c^3*e
*x^5*log(c^2*x^2 + 1) + 20*b*c^3*e*x^5*log(x) - 6*b*c^3*d*x^3 + 10*b*c*e*x
^3 + 20*b*e*x^2*arctan(c*x) + 3*b*c*d*x + 20*a*e*x^2 + 12*b*d*arctan(c*x)
+ 12*a*d)/x^5
```

## Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.01

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^6}\,dx = \frac{b\,c^3\,e\,\ln(c^2\,x^2+1)}{6}-\frac{b\,c^5\,d\,\ln(c^2\,x^2+1)}{10}-\frac{x^3\left(\frac{b\,c\,e}{6}-\frac{b\,c^3\,d}{10}\right)+\frac{a\,d}{5}+x^2\left(\frac{a\,e}{3}+\frac{b\,e\,\mathrm{atan}(c\,x)}{3}\right)+\frac{b\,d\,\mathrm{atan}(c\,x)}{5}+\frac{b\,c\,d\,x}{20}}{x^5}+\frac{b\,c^5\,d\,\ln(x)}{5}-\frac{b\,c^3\,e\,\ln(x)}{3}$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2))/x^6,x)
```

output
```
(b*c^3*e*log(c^2*x^2 + 1))/6 - (b*c^5*d*log(c^2*x^2 + 1))/10 - (x^3*((b*c*
e)/6 - (b*c^3*d)/10) + (a*d)/5 + x^2*((a*e)/3 + (b*e*atan(c*x))/3) + (b*d*
atan(c*x))/5 + (b*c*d*x)/20)/x^5 + (b*c^5*d*log(x))/5 - (b*c^3*e*log(x))/3
```

## Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.13

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^6}\,dx = \frac{-12atan(cx)\,bd-20atan(cx)\,be\,x^2-6\log(c^2x^2+1)\,b\,c^5d\,x^5+10\log(c^2x^2+1)\,b\,c^3e\,x^5+12\log(x)\,b\,c^5d}{60x^5}$$

input
```
int((e*x^2+d)*(a+b*atan(c*x))/x^6,x)
```

output
```
( - 12*atan(c*x)*b*d - 20*atan(c*x)*b*e*x**2 - 6*log(c**2*x**2 + 1)*b*c**5
*d*x**5 + 10*log(c**2*x**2 + 1)*b*c**3*e*x**5 + 12*log(x)*b*c**5*d*x**5 -
20*log(x)*b*c**3*e*x**5 - 12*a*d - 20*a*e*x**2 + 6*b*c**3*d*x**3 - 3*b*c*d
*x - 10*b*c*e*x**3)/(60*x**5)
```

# 3.1123 $\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^7}\,dx$



## Optimal result

Integrand size = 19, antiderivative size = 105

$$\int \frac{(d+ex^2)\,(a+b\arctan(cx))}{x^7}\,dx = -\frac{bcd}{30x^5}+\frac{bc(2c^2d-3e)}{36x^3}-\frac{bc^3(2c^2d-3e)}{12x} -\frac{1}{12}bc^4\left(2c^2d-3e\right)\arctan(cx) -\frac{d(a+b\arctan(cx))}{6x^6}-\frac{e(a+b\arctan(cx))}{4x^4}$$

output
```
-1/30*b*c*d/x^5+1/36*b*c*(2*c^2*d-3*e)/x^3-1/12*b*c^3*(2*c^2*d-3*e)/x-1/12
*b*c^4*(2*c^2*d-3*e)*arctan(c*x)-1/6*d*(a+b*arctan(c*x))/x^6-1/4*e*(a+b*ar
ctan(c*x))/x^4
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.92

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^7}\,dx = -\frac{ad}{6x^6}-\frac{ae}{4x^4}-\frac{bd\arctan(cx)}{6x^6}-\frac{be\arctan(cx)}{4x^4} - \frac{bcd\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},-c^2x^2\right)}{30x^5} - \frac{bce\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-c^2x^2\right)}{12x^3}$$

input
```
Integrate[((d + e*x^2)*(a + b*ArcTan[c*x]))/x^7,x]
```

output
```
-1/6*(a*d)/x^6 - (a*e)/(4*x^4) - (b*d*ArcTan[c*x])/(6*x^6) - (b*e*ArcTan[c
*x])/(4*x^4) - (b*c*d*Hypergeometric2F1[-5/2, 1, -3/2, -(c^2*x^2)])/(30*x^
5) - (b*c*e*Hypergeometric2F1[-3/2, 1, -1/2, -(c^2*x^2)])/(12*x^3)
```

## Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.82, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {5511, 27, 359, 264, 264, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^7}\,dx$$

$$\Big\downarrow \text{5511}$$

$$-bc\int -\frac{3ex^2+2d}{12x^6(c^2x^2+1)}dx - \frac{d(a+b\arctan(cx))}{6x^6} - \frac{e(a+b\arctan(cx))}{4x^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{12}bc\int \frac{3ex^2+2d}{x^6(c^2x^2+1)}dx - \frac{d(a+b\arctan(cx))}{6x^6} - \frac{e(a+b\arctan(cx))}{4x^4}$$

$$\Big\downarrow \text{359}$$

$$\frac{1}{12}bc\left(-(2c^2d-3e)\int \frac{1}{x^4(c^2x^2+1)}dx - \frac{2d}{5x^5}\right) - \frac{d(a+b\arctan(cx))}{6x^6} - \frac{e(a+b\arctan(cx))}{4x^4}$$

$$\Big\downarrow \text{264}$$

$$\frac{1}{12}bc\left(-(2c^2d-3e)\left(c^2\left(-\int\frac{1}{x^2(c^2x^2+1)}dx\right)-\frac{1}{3x^3}\right)-\frac{2d}{5x^5}\right)-\frac{d(a+b\arctan(cx))}{6x^6}-\frac{e(a+b\arctan(cx))}{4x^4}$$

$$\Big\downarrow \text{264}$$

$$\frac{1}{12}bc\left(-(2c^2d-3e)\left(-\left(c^2\left(c^2\left(-\int\frac{1}{c^2x^2+1}dx\right)-\frac{1}{x}\right)\right)-\frac{1}{3x^3}\right)-\frac{2d}{5x^5}\right)-\frac{d(a+b\arctan(cx))}{6x^6}-\frac{e(a+b\arctan(cx))}{4x^4}$$

$$\Big\downarrow \text{216}$$

$$-\frac{d(a+b\arctan(cx))}{6x^6}-\frac{e(a+b\arctan(cx))}{4x^4}+\frac{1}{12}bc\left(-\left(-\left(c^2\left(-c\arctan(cx)-\frac{1}{x}\right)\right)-\frac{1}{3x^3}\right)(2c^2d-3e)-\frac{2d}{5x^5}\right)$$

input
```
Int[((d + e*x^2)*(a + b*ArcTan[c*x]))/x^7,x]
```

output
```
-1/6*(d*(a + b*ArcTan[c*x]))/x^6 - (e*(a + b*ArcTan[c*x]))/(4*x^4) + (b*c*
((-2*d)/(5*x^5) - (2*c^2*d - 3*e)*(-1/3*1/x^3 - c^2*(-x^(-1) - c*ArcTan[c*
x]))))/12
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

## Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.04

| method | result |
|---|---|
| parallelrisch | $-\frac{30x^6\arctan(cx)bc^6d-45x^6\arctan(cx)bc^4e+30bc^5dx^5-45bc^3ex^5-10bc^3dx^3+15bcex^3+45\arctan(cx)bex^2+45a}{180x^6}$ |
| parts | $a\left(-\frac{e}{4x^4}-\frac{d}{6x^6}\right)+bc^6\left(-\frac{\arctan(cx)e}{4c^6x^4}-\frac{\arctan(cx)d}{6c^6x^6}-\frac{(2c^2d-3e)\arctan(cx)-\frac{-2c^2d+3e}{cx}-\frac{2c^2d-3e}{3c^3x^3}+\frac{2d}{5c^3x}}{12c^2}\right.$ |
| derivativedivides | $c^6\left(\frac{a\left(-\frac{e}{4c^4x^4}-\frac{d}{6c^4x^6}\right)}{c^2}+\frac{b\left(-\frac{\arctan(cx)e}{4c^4x^4}-\frac{\arctan(cx)d}{6c^4x^6}+\frac{-2c^2d+3e}{12cx}+\frac{2c^2d-3e}{36c^3x^3}-\frac{d}{30c^3x^5}-\frac{(2c^2d-3e)\arctan(cx)}{12}\right)}{c^2}\right)$ |
| default | $c^6\left(\frac{a\left(-\frac{e}{4c^4x^4}-\frac{d}{6c^4x^6}\right)}{c^2}+\frac{b\left(-\frac{\arctan(cx)e}{4c^4x^4}-\frac{\arctan(cx)d}{6c^4x^6}+\frac{-2c^2d+3e}{12cx}+\frac{2c^2d-3e}{36c^3x^3}-\frac{d}{30c^3x^5}-\frac{(2c^2d-3e)\arctan(cx)}{12}\right)}{c^2}\right)$ |
| risch | $\frac{ib(3ex^2+2d)\ln(icx+1)}{24x^6}-\frac{30i\ln(-cx-i)bc^6dx^6-30i\ln(-cx+i)bc^6dx^6-45i\ln(-cx-i)bc^4ex^6+45i\ln(-cx+i)bc^4}{}$ |
| orering | $-\frac{(90x^8ec^6d+120c^6d^2x^6-135c^4e^2x^8-130c^4dex^6+70c^4d^2x^4-75c^2e^2x^6-115c^2dex^4-14c^2d^2x^2+60e^2x^4+105dex^2}{90x^6(ex^2+d)}$ |

input
```
int((e*x^2+d)*(a+b*arctan(c*x))/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/180*(30*x^6*arctan(c*x)*b*c^6*d-45*x^6*arctan(c*x)*b*c^4*e+30*b*c^5*d*x
^5-45*b*c^3*e*x^5-10*b*c^3*d*x^3+15*b*c*e*x^3+45*arctan(c*x)*b*e*x^2+45*a*
e*x^2+6*b*c*d*x+30*arctan(c*x)*b*d+30*a*d)/x^6
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.93

$$\int\frac{(d+ex^2)(a+b\arctan(cx))}{x^7}\,dx=$$
$$-\frac{15\,(2\,bc^5d-3\,bc^3e)x^5+6\,bcdx+45\,aex^2-5\,(2\,bc^3d-3\,bce)x^3+30\,ad+15\,((2\,bc^6d-3\,bc^4e)x^6+3\,b}{180\,x^6}$$

input
```
integrate((e*x^2+d)*(a+b*arctan(c*x))/x^7,x, algorithm="fricas")
```

output
```
-1/180*(15*(2*b*c^5*d - 3*b*c^3*e)*x^5 + 6*b*c*d*x + 45*a*e*x^2 - 5*(2*b*c
^3*d - 3*b*c*e)*x^3 + 30*a*d + 15*((2*b*c^6*d - 3*b*c^4*e)*x^6 + 3*b*e*x^2
 + 2*b*d)*arctan(c*x))/x^6
```

## Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.16

$$\int \frac{(d+ex^2)\,(a+b\arctan(cx))}{x^7}\,dx = -\frac{ad}{6x^6} - \frac{ae}{4x^4} - \frac{bc^6 d\,\mathrm{atan}\,(cx)}{6} - \frac{bc^5 d}{6x} + \frac{bc^4 e\,\mathrm{atan}\,(cx)}{4} + \frac{bc^3 d}{18x^3} + \frac{bc^3 e}{4x} - \frac{bcd}{30x^5} - \frac{bce}{12x^3} - \frac{bd\,\mathrm{atan}\,(cx)}{6x^6} - \frac{be\,\mathrm{atan}\,(cx)}{4x^4}$$

input
```
integrate((e*x**2+d)*(a+b*atan(c*x))/x**7,x)
```

output
```
-a*d/(6*x**6) - a*e/(4*x**4) - b*c**6*d*atan(c*x)/6 - b*c**5*d/(6*x) + b*c
**4*e*atan(c*x)/4 + b*c**3*d/(18*x**3) + b*c**3*e/(4*x) - b*c*d/(30*x**5)
- b*c*e/(12*x**3) - b*d*atan(c*x)/(6*x**6) - b*e*atan(c*x)/(4*x**4)
```

## Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.98

$$\int \frac{(d+ex^2)\,(a+b\arctan(cx))}{x^7}\,dx = -\frac{1}{90}\left(\left(15\,c^5\arctan(cx) + \frac{15\,c^4x^4 - 5\,c^2x^2 + 3}{x^5}\right)c + \frac{15\,\arctan(cx)}{x^6}\right)bd + \frac{1}{12}\left(\left(3\,c^3\arctan(cx) + \frac{3\,c^2x^2 - 1}{x^3}\right)c - \frac{3\,\arctan(cx)}{x^4}\right)be - \frac{ae}{4\,x^4} - \frac{ad}{6\,x^6}$$

input
```
integrate((e*x^2+d)*(a+b*arctan(c*x))/x^7,x, algorithm="maxima")
```

output
```
-1/90*((15*c^5*arctan(c*x) + (15*c^4*x^4 - 5*c^2*x^2 + 3)/x^5)*c + 15*arct
an(c*x)/x^6)*b*d + 1/12*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*a
rctan(c*x)/x^4)*b*e - 1/4*a*e/x^4 - 1/6*a*d/x^6
```

## Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.41

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^7}\,dx = -\frac{30i\,bc^6dx^6\log(i\,cx-1)-30i\,bc^6dx^6\log(-i\,cx-1)-45i\,bc^4ex^6\log(i\,cx-1)+45i\,bc^4ex^6\log(-i\,cx$$

input
```
integrate((e*x^2+d)*(a+b*arctan(c*x))/x^7,x, algorithm="giac")
```

output
```
-1/360*(30*I*b*c^6*d*x^6*log(I*c*x - 1) - 30*I*b*c^6*d*x^6*log(-I*c*x - 1)
 - 45*I*b*c^4*e*x^6*log(I*c*x - 1) + 45*I*b*c^4*e*x^6*log(-I*c*x - 1) + 60
*b*c^5*d*x^5 - 90*b*c^3*e*x^5 - 20*b*c^3*d*x^3 + 30*b*c*e*x^3 + 90*b*e*x^2
*arctan(c*x) + 12*b*c*d*x + 90*a*e*x^2 + 60*b*d*arctan(c*x) + 60*a*d)/x^6
```

## Mupad [B] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.24

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^7}\,dx = \frac{b\,c^4\,\mathrm{atan}\left(\frac{b\,c^2\,x\,(3\,e-2\,c^2\,d)}{3\,b\,c\,e-2\,b\,c^3\,d}\right)\,(3\,e-2\,c^2\,d)}{12}-\frac{\mathrm{atan}(c\,x)\,\left(\frac{b\,e\,x^2}{4}+\frac{b\,d}{6}\right)}{x^6}-\frac{x^3\,\left(b\,c\,e-\frac{2\,b\,c^3\,d}{3}\right)+2\,a\,d-c^2\,x^5\,(3\,b\,c\,e-2\,b\,c^3\,d)+3\,a\,e\,x^2+\frac{2\,b\,c\,d\,x}{5}}{12\,x^6}$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2))/x^7,x)
```

output
```
(b*c^4*atan((b*c^2*x*(3*e - 2*c^2*d))/(3*b*c*e - 2*b*c^3*d))*(3*e - 2*c^2*
d))/12 - (atan(c*x)*((b*d)/6 + (b*e*x^2)/4))/x^6 - (x^3*(b*c*e - (2*b*c^3*
d)/3) + 2*a*d - c^2*x^5*(3*b*c*e - 2*b*c^3*d) + 3*a*e*x^2 + (2*b*c*d*x)/5)
/(12*x^6)
```

## Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.03

$$\int \frac{(d+ex^2)(a+b\arctan(cx))}{x^7}\,dx$$
$$= \frac{-30atan(cx)\,bc^6d\,x^6+45atan(cx)\,bc^4e\,x^6-30atan(cx)\,bd-45atan(cx)\,be\,x^2-30ad-45ae\,x^2-30b}{180x^6}$$

input
```
int((e*x^2+d)*(a+b*atan(c*x))/x^7,x)
```

output
```
( - 30*atan(c*x)*b*c**6*d*x**6 + 45*atan(c*x)*b*c**4*e*x**6 - 30*atan(c*x)
*b*d - 45*atan(c*x)*b*e*x**2 - 30*a*d - 45*a*e*x**2 - 30*b*c**5*d*x**5 + 1
0*b*c**3*d*x**3 + 45*b*c**3*e*x**5 - 6*b*c*d*x - 15*b*c*e*x**3)/(180*x**6)
```

# 3.1124 $\int x^3(d+ex^2)^2(a+b\arctan(cx))\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 185

$$\begin{aligned}\int x^3(d+ex^2)^2(a+b\arctan(cx))\,dx &= \frac{b(6c^4d^2-8c^2de+3e^2)\,x}{24c^7} \\ &\quad - \frac{b(6c^4d^2-8c^2de+3e^2)\,x^3}{72c^5} - \frac{b(8c^2d-3e)\,ex^5}{120c^3} \\ &\quad - \frac{be^2x^7}{56c} - \frac{b(6c^4d^2-8c^2de+3e^2)\arctan(cx)}{24c^8} \\ &\quad + \frac{1}{4}d^2x^4(a+b\arctan(cx)) \\ &\quad + \frac{1}{3}dex^6(a+b\arctan(cx)) \\ &\quad + \frac{1}{8}e^2x^8(a+b\arctan(cx))\end{aligned}$$

output
```
1/24*b*(6*c^4*d^2-8*c^2*d*e+3*e^2)*x/c^7-1/72*b*(6*c^4*d^2-8*c^2*d*e+3*e^2
)*x^3/c^5-1/120*b*(8*c^2*d-3*e)*e*x^5/c^3-1/56*b*e^2*x^7/c-1/24*b*(6*c^4*d
^2-8*c^2*d*e+3*e^2)*arctan(c*x)/c^8+1/4*d^2*x^4*(a+b*arctan(c*x))+1/3*d*e*
x^6*(a+b*arctan(c*x))+1/8*e^2*x^8*(a+b*arctan(c*x))
```

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.94

$$\int x^3\left(d+ex^2\right)^2(a+b\arctan(cx))\,dx=\frac{1}{4}d^2x^4(a+b\arctan(cx))+\frac{1}{3}dex^6(a+b\arctan(cx))+\frac{1}{8}e^2x^8(a+b\arctan(cx))+\frac{1}{840}be^2\left(\frac{105x}{c^7}-\frac{35x^3}{c^5}+\frac{21x^5}{c^3}-\frac{15x^7}{c}-\frac{105\arctan(cx)}{c^8}\right)-\frac{1}{45}bde\left(\frac{15x}{c^5}-\frac{5x^3}{c^3}+\frac{3x^5}{c}-\frac{15\arctan(cx)}{c^6}\right)+\frac{1}{12}bd^2\left(\frac{3x}{c^3}-\frac{x^3}{c}-\frac{3\arctan(cx)}{c^4}\right)$$

input
```
Integrate[x^3*(d + e*x^2)^2*(a + b*ArcTan[c*x]),x]
```

output
```
(d^2*x^4*(a + b*ArcTan[c*x]))/4 + (d*e*x^6*(a + b*ArcTan[c*x]))/3 + (e^2*x
^8*(a + b*ArcTan[c*x]))/8 + (b*e^2*((105*x)/c^7 - (35*x^3)/c^5 + (21*x^5)/
c^3 - (15*x^7)/c - (105*ArcTan[c*x])/c^8))/840 - (b*d*e*((15*x)/c^5 - (5*x
^3)/c^3 + (3*x^5)/c - (15*ArcTan[c*x])/c^6))/45 + (b*d^2*((3*x)/c^3 - x^3/
c - (3*ArcTan[c*x])/c^4))/12
```

## Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {5511, 27, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(d+ex^2\right)^2(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \; 5511$$

$$-bc\int \frac{x^4\left(3e^2x^4+8dex^2+6d^2\right)}{24\left(c^2x^2+1\right)}dx+\frac{1}{4}d^2x^4(a+b\arctan(cx))+\frac{1}{3}dex^6(a+b\arctan(cx))+\frac{1}{8}e^2x^8(a+b\arctan(cx))$$

$$\Big\downarrow \; 27$$

$$-\frac{1}{24}bc\int \frac{x^4\left(3e^2x^4+8dex^2+6d^2\right)}{c^2x^2+1}dx+\frac{1}{4}d^2x^4(a+b\arctan(cx))+\frac{1}{3}dex^6(a+b\arctan(cx))+\frac{1}{8}e^2x^8(a+b\arctan(cx))$$

$$\Big\downarrow \; 1584$$

$$-\frac{1}{24}bc\int \left(\frac{3e^2x^6}{c^2}+\frac{\left(8c^2d-3e\right)ex^4}{c^4}+\frac{\left(6d^2c^4-8dec^2+3e^2\right)x^2}{c^6}-\frac{6d^2c^4-8dec^2+3e^2}{c^8}+\frac{6d^2c^4-8dec^2+3e^2}{c^8\left(c^2x^2+1\right)}\right.$$
$$\frac{1}{4}d^2x^4(a+b\arctan(cx))+\frac{1}{3}dex^6(a+b\arctan(cx))+\frac{1}{8}e^2x^8(a+b\arctan(cx))$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{4}d^2x^4(a+b\arctan(cx))+\frac{1}{3}dex^6(a+b\arctan(cx))+\frac{1}{8}e^2x^8(a+b\arctan(cx))-$$
$$\frac{1}{24}bc\left(\frac{\arctan(cx)\left(6c^4d^2-8c^2de+3e^2\right)}{c^9}+\frac{3e^2x^7}{7c^2}+\frac{ex^5\left(8c^2d-3e\right)}{5c^4}-\frac{x\left(6c^4d^2-8c^2de+3e^2\right)}{c^8}+\frac{x^3\left(6c^4d^2-8c\right.}{3c^6}\right.$$

input
```
Int[x^3*(d + e*x^2)^2*(a + b*ArcTan[c*x]),x]
```

output
```
(d^2*x^4*(a + b*ArcTan[c*x]))/4 + (d*e*x^6*(a + b*ArcTan[c*x]))/3 + (e^2*x
^8*(a + b*ArcTan[c*x]))/8 - (b*c*(-(((6*c^4*d^2 - 8*c^2*d*e + 3*e^2)*x)/c^
8) + ((6*c^4*d^2 - 8*c^2*d*e + 3*e^2)*x^3)/(3*c^6) + ((8*c^2*d - 3*e)*e*x^
5)/(5*c^4) + (3*e^2*x^7)/(7*c^2) + ((6*c^4*d^2 - 8*c^2*d*e + 3*e^2)*ArcTan
[c*x])/c^9))/24
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

## Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.06

| method | result |
|---|---|
| parts | $a\left(\frac{1}{8}e^{2}x^{8}+\frac{1}{3}de\,x^{6}+\frac{1}{4}d^{2}x^{4}\right)+\frac{b\left(\frac{\arctan(cx)c^{4}e^{2}x^{8}}{8}+\frac{\arctan(cx)c^{4}de\,x^{6}}{3}+\frac{\arctan(cx)d^{2}c^{4}x^{4}}{4}-\frac{\frac{3e^{2}c^{7}x^{7}}{7}+\frac{8d\,c^{7}e\,x}{5}}{}\right.}{}$ |
| derivativedivides | $\frac{a\left(\frac{1}{4}d^{2}c^{8}x^{4}+\frac{1}{3}d\,c^{8}e\,x^{6}+\frac{1}{8}e^{2}c^{8}x^{8}\right)}{c^{4}}+\frac{b\left(\frac{\arctan(cx)d^{2}c^{8}x^{4}}{4}+\frac{\arctan(cx)d\,c^{8}e\,x^{6}}{3}+\frac{\arctan(cx)e^{2}c^{8}x^{8}}{8}-\frac{d^{2}c^{7}x^{3}}{12}-\frac{d\,c^{7}e\,x^{5}}{15}-\frac{e^{2}c^{7}x^{7}}{56}\right.}{c^{4}}$ |
| default | $\frac{a\left(\frac{1}{4}d^{2}c^{8}x^{4}+\frac{1}{3}d\,c^{8}e\,x^{6}+\frac{1}{8}e^{2}c^{8}x^{8}\right)}{c^{4}}+\frac{b\left(\frac{\arctan(cx)d^{2}c^{8}x^{4}}{4}+\frac{\arctan(cx)d\,c^{8}e\,x^{6}}{3}+\frac{\arctan(cx)e^{2}c^{8}x^{8}}{8}-\frac{d^{2}c^{7}x^{3}}{12}-\frac{d\,c^{7}e\,x^{5}}{15}-\frac{e^{2}c^{7}x^{7}}{56}\right.}{c^{4}}$ |
| parallelrisch | $315x^{8}\arctan(cx)b\,c^{8}e^{2}+315a\,c^{8}e^{2}x^{8}+840x^{6}\arctan(cx)b\,c^{8}de-45b\,c^{7}e^{2}x^{7}+840a\,c^{8}de\,x^{6}+630d^{2}b\arctan(cx)x^{4}c^{8}-16$ |
| risch | $\frac{ib\,e^{2}x^{8}\ln(-icx+1)}{16}-\frac{ib(3e^{2}x^{8}+8de\,x^{6}+6d^{2}x^{4})\ln(icx+1)}{48}+\frac{ib\,d^{2}x^{4}\ln(-icx+1)}{8}+\frac{x^{8}e^{2}a}{8}+\frac{ibde\,x^{6}\ln(-icx+1}{6}$ |
| orering | $\frac{(315c^{8}e^{3}x^{10}+1233c^{8}d\,e^{2}x^{8}+1722c^{8}d^{2}e\,x^{6}-63c^{6}e^{3}x^{8}+630c^{8}d^{3}x^{4}-419c^{6}d\,e^{2}x^{6}-1638c^{6}d^{2}e\,x^{4}+147c^{4}e^{3}x^{6}-630c^{6}d^{3}}{1260c}$ |

input
```
int(x^3*(e*x^2+d)^2*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/8*e^2*x^8+1/3*d*e*x^6+1/4*d^2*x^4)+b/c^4*(1/8*arctan(c*x)*c^4*e^2*x^8
+1/3*arctan(c*x)*c^4*d*e*x^6+1/4*arctan(c*x)*d^2*c^4*x^4-1/24/c^4*(3/7*e^2
*c^7*x^7+8/5*d*c^7*e*x^5+2*d^2*c^7*x^3-3/5*e^2*c^5*x^5-8/3*d*c^5*e*x^3-6*c
^5*x*d^2+e^2*c^3*x^3+8*d*c^3*e*x-3*c*x*e^2+(6*c^4*d^2-8*c^2*d*e+3*e^2)*arc
tan(c*x)))
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.09

$$\int x^{3}\left(d+ex^{2}\right)^{2}(a+b\arctan(cx))\,dx$$
$$=\frac{315\,ac^{8}e^{2}x^{8}+840\,ac^{8}dex^{6}-45\,bc^{7}e^{2}x^{7}+630\,ac^{8}d^{2}x^{4}-21\,(8\,bc^{7}de-3\,bc^{5}e^{2})x^{5}-35\,(6\,bc^{7}d^{2}-8\,bc^{5}de}{}$$

input
```
integrate(x^3*(e*x^2+d)^2*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
1/2520*(315*a*c^8*e^2*x^8 + 840*a*c^8*d*e*x^6 - 45*b*c^7*e^2*x^7 + 630*a*c
^8*d^2*x^4 - 21*(8*b*c^7*d*e - 3*b*c^5*e^2)*x^5 - 35*(6*b*c^7*d^2 - 8*b*c^
5*d*e + 3*b*c^3*e^2)*x^3 + 105*(6*b*c^5*d^2 - 8*b*c^3*d*e + 3*b*c*e^2)*x +
 105*(3*b*c^8*e^2*x^8 + 8*b*c^8*d*e*x^6 + 6*b*c^8*d^2*x^4 - 6*b*c^4*d^2 +
8*b*c^2*d*e - 3*b*e^2)*arctan(c*x))/c^8
```

## Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.41

$$\int x^3(d+ex^2)^2(a+b\arctan(cx))\,dx$$
$$= \begin{cases} \frac{ad^2x^4}{4}+\frac{adex^6}{3}+\frac{ae^2x^8}{8}+\frac{bd^2x^4\operatorname{atan}(cx)}{4}+\frac{bdex^6\operatorname{atan}(cx)}{3}+\frac{be^2x^8\operatorname{atan}(cx)}{8}-\frac{bd^2x^3}{12c}-\frac{bdex^5}{15c}-\frac{be^2x^7}{56c}+\frac{bd^2x}{4c^3}+\frac{bdex^3}{9c^3} \\ a\left(\frac{d^2x^4}{4}+\frac{dex^6}{3}+\frac{e^2x^8}{8}\right) \end{cases}$$

input
```
integrate(x**3*(e*x**2+d)**2*(a+b*atan(c*x)),x)
```

output
```
Piecewise((a*d**2*x**4/4 + a*d*e*x**6/3 + a*e**2*x**8/8 + b*d**2*x**4*atan
(c*x)/4 + b*d*e*x**6*atan(c*x)/3 + b*e**2*x**8*atan(c*x)/8 - b*d**2*x**3/(
12*c) - b*d*e*x**5/(15*c) - b*e**2*x**7/(56*c) + b*d**2*x/(4*c**3) + b*d*e
*x**3/(9*c**3) + b*e**2*x**5/(40*c**3) - b*d**2*atan(c*x)/(4*c**4) - b*d*e
*x/(3*c**5) - b*e**2*x**3/(24*c**5) + b*d*e*atan(c*x)/(3*c**6) + b*e**2*x/
(8*c**7) - b*e**2*atan(c*x)/(8*c**8), Ne(c, 0)), (a*(d**2*x**4/4 + d*e*x**
6/3 + e**2*x**8/8), True))
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.99

$$\int x^3(d+ex^2)^2(a+b\arctan(cx))\,dx = \frac{1}{8}ae^2x^8+\frac{1}{3}adex^6+\frac{1}{4}ad^2x^4$$
$$+\frac{1}{12}\left(3x^4\arctan(cx)-c\left(\frac{c^2x^3-3x}{c^4}+\frac{3\arctan(cx)}{c^5}\right)\right)bd^2$$
$$+\frac{1}{45}\left(15x^6\arctan(cx)-c\left(\frac{3c^4x^5-5c^2x^3+15x}{c^6}-\frac{15\arctan(cx)}{c^7}\right)\right)bde$$
$$+\frac{1}{840}\left(105x^8\arctan(cx)-c\left(\frac{15c^6x^7-21c^4x^5+35c^2x^3-105x}{c^8}+\frac{105\arctan(cx)}{c^9}\right)\right)be^2$$

input
```
integrate(x^3*(e*x^2+d)^2*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
1/8*a*e^2*x^8 + 1/3*a*d*e*x^6 + 1/4*a*d^2*x^4 + 1/12*(3*x^4*arctan(c*x) -
c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*b*d^2 + 1/45*(15*x^6*arctan(c
*x) - c*((3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan(c*x)/c^7))*b*d*e +
 1/840*(105*x^8*arctan(c*x) - c*((15*c^6*x^7 - 21*c^4*x^5 + 35*c^2*x^3 - 1
05*x)/c^8 + 105*arctan(c*x)/c^9))*b*e^2
```

## Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.19

$$\int x^3 (d + ex^2)^2 (a + b\arctan(cx))\,dx$$
$$= \frac{315\,bc^8e^2x^8\arctan(cx) + 315\,ac^8e^2x^8 + 840\,bc^8dex^6\arctan(cx) + 840\,ac^8dex^6 - 45\,bc^7e^2x^7 + 630\,bc^8d^2x}{}$$

input
```
integrate(x^3*(e*x^2+d)^2*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
1/2520*(315*b*c^8*e^2*x^8*arctan(c*x) + 315*a*c^8*e^2*x^8 + 840*b*c^8*d*e*
x^6*arctan(c*x) + 840*a*c^8*d*e*x^6 - 45*b*c^7*e^2*x^7 + 630*b*c^8*d^2*x^4
*arctan(c*x) + 630*a*c^8*d^2*x^4 - 168*b*c^7*d*e*x^5 - 210*b*c^7*d^2*x^3 +
 63*b*c^5*e^2*x^5 + 280*b*c^5*d*e*x^3 + 630*b*c^5*d^2*x - 105*b*c^3*e^2*x^
3 - 630*b*c^4*d^2*arctan(c*x) - 840*b*c^3*d*e*x + 840*b*c^2*d*e*arctan(c*x
) + 315*b*c*e^2*x - 315*b*e^2*arctan(c*x))/c^8
```

## Mupad [B] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 374, normalized size of antiderivative = 2.02

$$\int x^3 (d+ex^2)^2 (a+b\arctan(cx))\,dx$$
$$= x^4\left(\frac{\frac{a\,e^2}{c^2}-\frac{a\,e\,(2\,d\,c^2+e)}{c^2}}{4\,c^2}+\frac{a\,d\,(d\,c^2+2\,e)}{4\,c^2}\right)-x^6\left(\frac{a\,e^2}{6\,c^2}-\frac{a\,e\,(2\,d\,c^2+e)}{6\,c^2}\right)$$
$$+x^5\left(\frac{b\,e^2}{40\,c^3}-\frac{b\,d\,e}{15\,c}\right)+\mathrm{atan}(c\,x)\,\left(\frac{b\,d^2\,x^4}{4}+\frac{b\,d\,e\,x^6}{3}+\frac{b\,e^2\,x^8}{8}\right)$$
$$-x^2\left(\frac{\frac{\frac{a\,e^2}{c^2}-\frac{a\,e\,\left(2\,d\,c^2+e\right)}{c^2}}{c^2}+\frac{a\,d\,(d\,c^2+2\,e)}{c^2}}{2\,c^2}-\frac{a\,d^2}{2\,c^2}\right)$$
$$-x^3\left(\frac{\frac{b\,e^2}{8\,c^3}-\frac{b\,d\,e}{3\,c}}{3\,c^2}+\frac{b\,d^2}{12\,c}\right)+\frac{x\left(\frac{\frac{b\,e^2}{8\,c^3}-\frac{b\,d\,e}{3\,c}}{c^2}+\frac{b\,d^2}{4\,c}\right)}{c^2}+\frac{a\,e^2\,x^8}{8}$$
$$-\frac{b\,\mathrm{atan}\left(\frac{b\,c\,x\,(6\,c^4\,d^2-8\,c^2\,d\,e+3\,e^2)}{6\,b\,c^4\,d^2-8\,b\,c^2\,d\,e+3\,b\,e^2}\right)\,(6\,c^4\,d^2-8\,c^2\,d\,e+3\,e^2)}{24\,c^8}-\frac{b\,e^2\,x^7}{56\,c}$$

input
```
int(x^3*(a + b*atan(c*x))*(d + e*x^2)^2,x)
```

output
```
x^4*(((a*e^2)/c^2 - (a*e*(e + 2*c^2*d))/c^2)/(4*c^2) + (a*d*(2*e + c^2*d))
/(4*c^2)) - x^6*((a*e^2)/(6*c^2) - (a*e*(e + 2*c^2*d))/(6*c^2)) + x^5*((b*
e^2)/(40*c^3) - (b*d*e)/(15*c)) + atan(c*x)*((b*d^2*x^4)/4 + (b*e^2*x^8)/8
 + (b*d*e*x^6)/3) - x^2*((((a*e^2)/c^2 - (a*e*(e + 2*c^2*d))/c^2)/c^2 + (a
*d*(2*e + c^2*d))/c^2)/(2*c^2) - (a*d^2)/(2*c^2)) - x^3*(((b*e^2)/(8*c^3)
- (b*d*e)/(3*c))/(3*c^2) + (b*d^2)/(12*c)) + (x*(((b*e^2)/(8*c^3) - (b*d*e
)/(3*c))/c^2 + (b*d^2)/(4*c)))/c^2 + (a*e^2*x^8)/8 - (b*atan((b*c*x*(3*e^2
 + 6*c^4*d^2 - 8*c^2*d*e))/(3*b*e^2 + 6*b*c^4*d^2 - 8*b*c^2*d*e))*(3*e^2 +
 6*c^4*d^2 - 8*c^2*d*e))/(24*c^8) - (b*e^2*x^7)/(56*c)
```

## Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.19

$$\int x^3 (d + ex^2)^2 (a + b\arctan(cx))\, dx$$
$$= \frac{630atan(cx)\, b\, c^8 d^2 x^4 + 840atan(cx)\, b\, c^8 de\, x^6 + 315atan(cx)\, b\, c^8 e^2 x^8 - 630atan(cx)\, b\, c^4 d^2 + 840atan(cx}{}$$

input
```
int(x^3*(e*x^2+d)^2*(a+b*atan(c*x)),x)
```

output
```
(630*atan(c*x)*b*c**8*d**2*x**4 + 840*atan(c*x)*b*c**8*d*e*x**6 + 315*atan
(c*x)*b*c**8*e**2*x**8 - 630*atan(c*x)*b*c**4*d**2 + 840*atan(c*x)*b*c**2*
d*e - 315*atan(c*x)*b*e**2 + 630*a*c**8*d**2*x**4 + 840*a*c**8*d*e*x**6 +
315*a*c**8*e**2*x**8 - 210*b*c**7*d**2*x**3 - 168*b*c**7*d*e*x**5 - 45*b*c
**7*e**2*x**7 + 630*b*c**5*d**2*x + 280*b*c**5*d*e*x**3 + 63*b*c**5*e**2*x
**5 - 840*b*c**3*d*e*x - 105*b*c**3*e**2*x**3 + 315*b*c*e**2*x)/(2520*c**8
)
```

# 3.1125 $\int x^2(d+ex^2)^2 (a+b\arctan(cx))\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 161

$$\begin{aligned}\int x^2(d+ex^2)^2 (a+b\arctan(cx))\,dx &= -\frac{b(35c^4d^2-42c^2de+15e^2)\,x^2}{210c^5}-\frac{b(14c^2d-5e)\,ex^4}{140c^3}-\frac{be^2x^6}{42c}\\ &\quad+\frac{1}{3}d^2x^3(a+b\arctan(cx))+\frac{2}{5}dex^5(a+b\arctan(cx))\\ &\quad+\frac{1}{7}e^2x^7(a+b\arctan(cx))+\frac{b(35c^4d^2-42c^2de+15e^2)\log(1+c^2x^2)}{210c^7}\end{aligned}$$

output

```
-1/210*b*(35*c^4*d^2-42*c^2*d*e+15*e^2)*x^2/c^5-1/140*b*(14*c^2*d-5*e)*e*x
^4/c^3-1/42*b*e^2*x^6/c+1/3*d^2*x^3*(a+b*arctan(c*x))+2/5*d*e*x^5*(a+b*arc
tan(c*x))+1/7*e^2*x^7*(a+b*arctan(c*x))+1/210*b*(35*c^4*d^2-42*c^2*d*e+15*
e^2)*ln(c^2*x^2+1)/c^7
```

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.01

$$\int x^2(d+ex^2)^2(a+b\arctan(cx))\,dx$$
$$=\frac{c^2x^2(-30be^2+3bc^2e(28d+5ex^2)-2bc^4(35d^2+21dex^2+5e^2x^4)+4ac^5x(35d^2+42dex^2+15e^2x^4))+}{420c^7}$$

input
```
Integrate[x^2*(d + e*x^2)^2*(a + b*ArcTan[c*x]),x]
```

output
```
(c^2*x^2*(-30*b*e^2 + 3*b*c^2*e*(28*d + 5*e*x^2) - 2*b*c^4*(35*d^2 + 21*d*
e*x^2 + 5*e^2*x^4) + 4*a*c^5*x*(35*d^2 + 42*d*e*x^2 + 15*e^2*x^4)) + 4*b*c
^7*x^3*(35*d^2 + 42*d*e*x^2 + 15*e^2*x^4)*ArcTan[c*x] + 2*b*(35*c^4*d^2 -
42*c^2*d*e + 15*e^2)*Log[1 + c^2*x^2])/(420*c^7)
```

## Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5511, 27, 1578, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(d+ex^2)^2(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5511}$$

$$-bc\int \frac{x^3(15e^2x^4+42dex^2+35d^2)}{105(c^2x^2+1)}dx+\frac{1}{3}d^2x^3(a+b\arctan(cx))+\frac{2}{5}dex^5(a+b\arctan(cx))+\frac{1}{7}e^2x^7(a+b\arctan(cx))$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{105}bc\int \frac{x^3(15e^2x^4+42dex^2+35d^2)}{c^2x^2+1}dx+\frac{1}{3}d^2x^3(a+b\arctan(cx))+\frac{2}{5}dex^5(a+b\arctan(cx))+\frac{1}{7}e^2x^7(a+b\arctan(cx))$$

$$\Big\downarrow \text{1578}$$

$$-\frac{1}{210}bc\int \frac{x^2(15e^2x^4+42dex^2+35d^2)}{c^2x^2+1}dx^2+\frac{1}{3}d^2x^3(a+b\arctan(cx))+\frac{2}{5}dex^5(a+b\arctan(cx))+\frac{1}{7}e^2x^7(a+b\arctan(cx))$$

$$\Big\downarrow \text{1195}$$

$$-\frac{1}{210}bc\int \left(\frac{15e^2x^4}{c^2}+\frac{3(14c^2d-5e)\,ex^2}{c^4}+\frac{35d^2c^4-42dec^2+15e^2}{c^6}+\frac{-35d^2c^4+42dec^2-15e^2}{c^6(c^2x^2+1)}\right)dx^2+\frac{1}{3}d^2x^3(a+b\arctan(cx))+\frac{2}{5}dex^5(a+b\arctan(cx))+\frac{1}{7}e^2x^7(a+b\arctan(cx))$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}d^2x^3(a+b\arctan(cx))+\frac{2}{5}dex^5(a+b\arctan(cx))+\frac{1}{7}e^2x^7(a+b\arctan(cx))-\frac{1}{210}bc\left(\frac{5e^2x^6}{c^2}+\frac{3ex^4(14c^2d-5e)}{2c^4}-\frac{(35c^4d^2-42c^2de+15e^2)\log(c^2x^2+1)}{c^8}+\frac{x^2(35c^4d^2-42c^2de+15e^2)}{c^6}\right)$$

input
```
Int[x^2*(d + e*x^2)^2*(a + b*ArcTan[c*x]),x]
```

output
```
(d^2*x^3*(a + b*ArcTan[c*x]))/3 + (2*d*e*x^5*(a + b*ArcTan[c*x]))/5 + (e^2
*x^7*(a + b*ArcTan[c*x]))/7 - (b*c*(((35*c^4*d^2 - 42*c^2*d*e + 15*e^2)*x^
2)/c^6 + (3*(14*c^2*d - 5*e)*e*x^4)/(2*c^4) + (5*e^2*x^6)/c^2 - ((35*c^4*d
^2 - 42*c^2*d*e + 15*e^2)*Log[1 + c^2*x^2])/c^8))/210
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

## Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.12

| method | result |
|---|---|
| parts | $a\left(\frac{1}{7}e^2x^7+\frac{2}{5}de\,x^5+\frac{1}{3}d^2x^3\right)+\frac{b\left(\frac{\arctan(cx)c^3e^2x^7}{7}+\frac{2\arctan(cx)c^3de\,x^5}{5}+\frac{\arctan(cx)d^2c^3x^3}{3}-\frac{\frac{35c^6d^2x^2}{2}+21c^6 \cdots}{\cdots}\cdots\right)}{\cdots}$ |
| derivativedivides | $\frac{\frac{a\left(\frac{1}{3}d^2c^7x^3+\frac{2}{5}d\,c^7e\,x^5+\frac{1}{7}e^2c^7x^7\right)}{c^4}+\frac{b\left(\frac{\arctan(cx)d^2c^7x^3}{3}+\frac{2\arctan(cx)d\,c^7e\,x^5}{5}+\frac{\arctan(cx)e^2c^7x^7}{7}-\frac{c^6d^2x^2}{6}-\frac{c^6de\,x^4}{10}+\frac{c^4de \cdots}{5}\cdots\right)}{c^4}}{c^3}$ |
| default | $\frac{\frac{a\left(\frac{1}{3}d^2c^7x^3+\frac{2}{5}d\,c^7e\,x^5+\frac{1}{7}e^2c^7x^7\right)}{c^4}+\frac{b\left(\frac{\arctan(cx)d^2c^7x^3}{3}+\frac{2\arctan(cx)d\,c^7e\,x^5}{5}+\frac{\arctan(cx)e^2c^7x^7}{7}-\frac{c^6d^2x^2}{6}-\frac{c^6de\,x^4}{10}+\frac{c^4de \cdots}{5}\cdots\right)}{c^4}}{c^3}$ |
| parallelrisch | $\frac{60x^7\arctan(cx)b\,c^7e^2+60a\,c^7e^2x^7+168x^5\arctan(cx)b\,c^7de-10b\,c^6e^2x^6+168a\,c^7de\,x^5+140x^3\arctan(cx)b\,c^7d^2-42b \cdots}{\cdots}$ |
| risch | $\frac{ibde\,x^5\ln(-icx+1)}{5}-\frac{ib\left(15e^2x^7+42de\,x^5+35d^2x^3\right)\ln(icx+1)}{210}+\frac{ib\,d^2x^3\ln(-icx+1)}{6}+\frac{x^7e^2a}{7}+\frac{ib\,e^2x^7\ln(-ic \cdots}{14}\cdots$ |

input
```
int(x^2*(e*x^2+d)^2*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/7*e^2*x^7+2/5*d*e*x^5+1/3*d^2*x^3)+b/c^3*(1/7*arctan(c*x)*c^3*e^2*x^7
+2/5*arctan(c*x)*c^3*d*e*x^5+1/3*arctan(c*x)*d^2*c^3*x^3-1/105/c^4*(35/2*c
^6*d^2*x^2+21/2*c^6*d*e*x^4+5/2*c^6*e^2*x^6-21*c^4*d*e*x^2-15/4*c^4*e^2*x^
4+15/2*c^2*e^2*x^2+1/2*(-35*c^4*d^2+42*c^2*d*e-15*e^2)*ln(c^2*x^2+1)))
```

## Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.16

$$\int x^2 \left(d+ex^2\right)^2 (a+b\arctan(cx))\,dx = \frac{60\,ac^7e^2x^7+168\,ac^7dex^5-10\,bc^6e^2x^6+140\,ac^7d^2x^3-3\,(14\,bc^6de-5\,bc^4e^2)x^4-2\,(35\,bc^6d^2-42\,bc^4de}{}$$

input
```
integrate(x^2*(e*x^2+d)^2*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
1/420*(60*a*c^7*e^2*x^7 + 168*a*c^7*d*e*x^5 - 10*b*c^6*e^2*x^6 + 140*a*c^7
*d^2*x^3 - 3*(14*b*c^6*d*e - 5*b*c^4*e^2)*x^4 - 2*(35*b*c^6*d^2 - 42*b*c^4
*d*e + 15*b*c^2*e^2)*x^2 + 4*(15*b*c^7*e^2*x^7 + 42*b*c^7*d*e*x^5 + 35*b*c
^7*d^2*x^3)*arctan(c*x) + 2*(35*b*c^4*d^2 - 42*b*c^2*d*e + 15*b*e^2)*log(c
^2*x^2 + 1))/c^7
```

## Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.52

$$\int x^2 \left(d+ex^2\right)^2 (a+b\arctan(cx))\,dx = \begin{cases} \frac{ad^2x^3}{3}+\frac{2adex^5}{5}+\frac{ae^2x^7}{7}+\frac{bd^2x^3\operatorname{atan}(cx)}{3}+\frac{2bdex^5\operatorname{atan}(cx)}{5}+\frac{be^2x^7\operatorname{atan}(cx)}{7}-\frac{bd^2x^2}{6c}-\frac{bdex^4}{10c}-\frac{be^2x^6}{42c}+\frac{bd^2\log\left(x^2+\right.}{6c^3} \\ a\left(\frac{d^2x^3}{3}+\frac{2dex^5}{5}+\frac{e^2x^7}{7}\right) \end{cases}$$

input
```
integrate(x**2*(e*x**2+d)**2*(a+b*atan(c*x)),x)
```

output
```
Piecewise((a*d**2*x**3/3 + 2*a*d*e*x**5/5 + a*e**2*x**7/7 + b*d**2*x**3*at
an(c*x)/3 + 2*b*d*e*x**5*atan(c*x)/5 + b*e**2*x**7*atan(c*x)/7 - b*d**2*x*
*2/(6*c) - b*d*e*x**4/(10*c) - b*e**2*x**6/(42*c) + b*d**2*log(x**2 + c**(
-2))/(6*c**3) + b*d*e*x**2/(5*c**3) + b*e**2*x**4/(28*c**3) - b*d*e*log(x*
*2 + c**(-2))/(5*c**5) - b*e**2*x**2/(14*c**5) + b*e**2*log(x**2 + c**(-2)
)/(14*c**7), Ne(c, 0)), (a*(d**2*x**3/3 + 2*d*e*x**5/5 + e**2*x**7/7), Tru
e))
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.12

$$\int x^2(d+ex^2)^2(a+b\arctan(cx))\,dx$$
$$= \frac{1}{7}ae^2x^7 + \frac{2}{5}adex^5 + \frac{1}{3}ad^2x^3 + \frac{1}{6}\left(2x^3\arctan(cx) - c\left(\frac{x^2}{c^2} - \frac{\log(c^2x^2+1)}{c^4}\right)\right)bd^2 + \frac{1}{10}\left(4x^5\arctan(cx) - c\left(\frac{c^2x^4-2x^2}{c^4} + \frac{2\log(c^2x^2+1)}{c^6}\right)\right)bde + \frac{1}{84}\left(12x^7\arctan(cx) - c\left(\frac{2c^4x^6-3c^2x^4+6x^2}{c^6} - \frac{6\log(c^2x^2+1)}{c^8}\right)\right)be^2$$

input
```
integrate(x^2*(e*x^2+d)^2*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
1/7*a*e^2*x^7 + 2/5*a*d*e*x^5 + 1/3*a*d^2*x^3 + 1/6*(2*x^3*arctan(c*x) - c
*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*b*d^2 + 1/10*(4*x^5*arctan(c*x) - c*((c
^2*x^4 - 2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*b*d*e + 1/84*(12*x^7*arctan
(c*x) - c*((2*c^4*x^6 - 3*c^2*x^4 + 6*x^2)/c^6 - 6*log(c^2*x^2 + 1)/c^8))*
b*e^2
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.31

$$\int x^2(d+ex^2)^2(a+b\arctan(cx))\,dx$$
$$= \frac{60\,bc^7e^2x^7\arctan{(cx)}+60\,ac^7e^2x^7+168\,bc^7dex^5\arctan{(cx)}+168\,ac^7dex^5-10\,bc^6e^2x^6+140\,bc^7d^2x^3\,\varepsilon}{}$$

input
```
integrate(x^2*(e*x^2+d)^2*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
1/420*(60*b*c^7*e^2*x^7*arctan(c*x) + 60*a*c^7*e^2*x^7 + 168*b*c^7*d*e*x^5
*arctan(c*x) + 168*a*c^7*d*e*x^5 - 10*b*c^6*e^2*x^6 + 140*b*c^7*d^2*x^3*ar
ctan(c*x) + 140*a*c^7*d^2*x^3 - 42*b*c^6*d*e*x^4 - 70*b*c^6*d^2*x^2 + 15*b
*c^4*e^2*x^4 + 84*b*c^4*d*e*x^2 + 70*b*c^4*d^2*log(c^2*x^2 + 1) - 30*b*c^2
*e^2*x^2 - 84*b*c^2*d*e*log(c^2*x^2 + 1) + 30*b*e^2*log(c^2*x^2 + 1))/c^7
```

## Mupad [B] (verification not implemented)

Time = 1.16 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.19

$$\int x^2(d+ex^2)^2(a+b\arctan(cx))\,dx = \frac{a\,d^2\,x^3}{3}+\frac{a\,e^2\,x^7}{7}+\frac{b\,d^2\,\ln\left(c^2\,x^2+1\right)}{6\,c^3}+\frac{b\,e^2\,\ln\left(c^2\,x^2+1\right)}{14\,c^7}-\frac{b\,d^2\,x^2}{6\,c}-\frac{b\,e^2\,x^6}{42\,c}+\frac{b\,e^2\,x^4}{28\,c^3}-\frac{b\,e^2\,x^2}{14\,c^5}+\frac{2\,a\,d\,e\,x^5}{5}+\frac{b\,d^2\,x^3\,\mathrm{atan}(c\,x)}{3}+\frac{b\,e^2\,x^7\,\mathrm{atan}(c\,x)}{7}-\frac{b\,d\,e\,\ln\left(c^2\,x^2+1\right)}{5\,c^5}-\frac{b\,d\,e\,x^4}{10\,c}+\frac{b\,d\,e\,x^2}{5\,c^3}+\frac{2\,b\,d\,e\,x^5\,\mathrm{atan}(c\,x)}{5}$$

input
```
int(x^2*(a + b*atan(c*x))*(d + e*x^2)^2,x)
```

output
```
(a*d^2*x^3)/3 + (a*e^2*x^7)/7 + (b*d^2*log(c^2*x^2 + 1))/(6*c^3) + (b*e^2*
log(c^2*x^2 + 1))/(14*c^7) - (b*d^2*x^2)/(6*c) - (b*e^2*x^6)/(42*c) + (b*e
^2*x^4)/(28*c^3) - (b*e^2*x^2)/(14*c^5) + (2*a*d*e*x^5)/5 + (b*d^2*x^3*ata
n(c*x))/3 + (b*e^2*x^7*atan(c*x))/7 - (b*d*e*log(c^2*x^2 + 1))/(5*c^5) - (
b*d*e*x^4)/(10*c) + (b*d*e*x^2)/(5*c^3) + (2*b*d*e*x^5*atan(c*x))/5
```

## Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.31

$$\int x^2(d+ex^2)^2(a+b\arctan(cx))\,dx$$
$$= \frac{140atan(cx)\,bc^7d^2x^3 + 168atan(cx)\,bc^7de\,x^5 + 60atan(cx)\,bc^7e^2x^7 + 70\log(c^2x^2+1)\,bc^4d^2 - 84\log(c^2}{}$$

input
```
int(x^2*(e*x^2+d)^2*(a+b*atan(c*x)),x)
```

output
```
(140*atan(c*x)*b*c**7*d**2*x**3 + 168*atan(c*x)*b*c**7*d*e*x**5 + 60*atan(
c*x)*b*c**7*e**2*x**7 + 70*log(c**2*x**2 + 1)*b*c**4*d**2 - 84*log(c**2*x*
*2 + 1)*b*c**2*d*e + 30*log(c**2*x**2 + 1)*b*e**2 + 140*a*c**7*d**2*x**3 +
 168*a*c**7*d*e*x**5 + 60*a*c**7*e**2*x**7 - 70*b*c**6*d**2*x**2 - 42*b*c*
*6*d*e*x**4 - 10*b*c**6*e**2*x**6 + 84*b*c**4*d*e*x**2 + 15*b*c**4*e**2*x*
*4 - 30*b*c**2*e**2*x**2)/(420*c**7)
```

# 3.1126 $\int x(d+ex^2)^2(a+b\arctan(cx))\,dx$



## Optimal result

Integrand size = 19, antiderivative size = 115

$$\int x(d+ex^2)^2(a+b\arctan(cx))\,dx = -\frac{b(3c^4d^2-3c^2de+e^2)\,x}{6c^5} - \frac{b(3c^2d-e)\,ex^3}{18c^3} - \frac{be^2x^5}{30c} - \frac{b(c^2d-e)^3\arctan(cx)}{6c^6e} + \frac{(d+ex^2)^3\,(a+b\arctan(cx))}{6e}$$

output
```
-1/6*b*(3*c^4*d^2-3*c^2*d*e+e^2)*x/c^5-1/18*b*(3*c^2*d-e)*e*x^3/c^3-1/30*b
*e^2*x^5/c-1/6*b*(c^2*d-e)^3*arctan(c*x)/c^6/e+1/6*(e*x^2+d)^3*(a+b*arctan
(c*x))/e
```

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.22

$$\int x(d+ex^2)^2(a+b\arctan(cx))\,dx = \frac{cx(-15be^2+5bc^2e(9d+ex^2)+15ac^5x(3d^2+3dex^2+e^2x^4)-3bc^4(15d^2+5dex^2+e^2x^4))+15b(3c^4d^2}{90c^6}$$

input
```
Integrate[x*(d + e*x^2)^2*(a + b*ArcTan[c*x]),x]
```

output
```
(c*x*(-15*b*e^2 + 5*b*c^2*e*(9*d + e*x^2) + 15*a*c^5*x*(3*d^2 + 3*d*e*x^2
+ e^2*x^4) - 3*b*c^4*(15*d^2 + 5*d*e*x^2 + e^2*x^4)) + 15*b*(3*c^4*d^2 - 3
*c^2*d*e + e^2 + c^6*(3*d^2*x^2 + 3*d*e*x^4 + e^2*x^6))*ArcTan[c*x])/(90*c
^6)
```

## Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {5509, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(d+ex^2\right)^2(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{ 5509}$$

$$\frac{\left(d+ex^2\right)^3(a+b\arctan(cx))}{6e}-\frac{bc\int\frac{\left(ex^2+d\right)^3}{c^2x^2+1}dx}{6e}$$

$$\Big\downarrow \text{ 300}$$

$$\frac{\left(d+ex^2\right)^3(a+b\arctan(cx))}{6e}-\frac{bc\int\left(\frac{e^3x^4}{c^2}+\frac{(3c^2d-e)e^2x^2}{c^4}+\frac{e(3d^2c^4-3dec^2+e^2)}{c^6}+\frac{d^3c^6-3d^2ec^4+3de^2c^2-e^3}{c^6(c^2x^2+1)}\right)dx}{6e}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\left(d+ex^2\right)^3(a+b\arctan(cx))}{6e}-\frac{bc\left(\frac{\arctan(cx)(c^2d-e)^3}{c^7}+\frac{e^3x^5}{5c^2}+\frac{e^2x^3(3c^2d-e)}{3c^4}+\frac{ex(3c^4d^2-3c^2de+e^2)}{c^6}\right)}{6e}$$

input
```
Int[x*(d + e*x^2)^2*(a + b*ArcTan[c*x]),x]
```

output
```
((d + e*x^2)^3*(a + b*ArcTan[c*x]))/(6*e) - (b*c*((e*(3*c^4*d^2 - 3*c^2*d*
e + e^2)*x)/c^6 + ((3*c^2*d - e)*e^2*x^3)/(3*c^4) + (e^3*x^5)/(5*c^2) + ((
c^2*d - e)^3*ArcTan[c*x])/c^7))/(6*e)
```

**Defintions of rubi rules used**

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5509
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*e*(q + 1))), x
] - Simp[b*(c/(2*e*(q + 1)))   Int[(d + e*x^2)^(q + 1)/(1 + c^2*x^2), x], x
] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]
```

## Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.62

| method | result |
|---|---|
| parallelrisch | $\frac{15x^6 \arctan(cx) b c^6 e^2+15x^6 a c^6 e^2+45x^4 \arctan(cx) b c^6 d e-3b c^5 e^2 x^5+45x^4 a c^6 d e+45x^2 \arctan(cx) b c^6 d^2-15b c^5 d e x}{9}$ |
| parts | $\frac{a(e x^2+d)^3}{6e}+\frac{b\left(\frac{\arctan(cx)c^2e^2x^6}{6}+\frac{\arctan(cx)c^2e x^4 d}{2}+\frac{\arctan(cx)c^2x^2d^2}{2}+\frac{\arctan(cx)c^2d^3}{6e}-\frac{3c^5d^2ex+c^5d e^2x^3+\frac{e^3c^5x^5}{5}}{}\right)}{c^2}$ |
| derivativedivides | $\frac{\frac{a(c^2e x^2+c^2d)^3}{6c^4e}+\frac{b\left(\frac{\arctan(cx)c^6d^3}{6e}+\frac{\arctan(cx)c^6d^2x^2}{2}+\frac{\arctan(cx)e c^6 d x^4}{2}+\frac{\arctan(cx)e^2c^6x^6}{6}-\frac{3c^5d^2ex+c^5d e^2x^3+\frac{e^3c^5x^5}{5}}{}\right)}{c^4}}{c^2}$ |
| default | $\frac{\frac{a(c^2e x^2+c^2d)^3}{6c^4e}+\frac{b\left(\frac{\arctan(cx)c^6d^3}{6e}+\frac{\arctan(cx)c^6d^2x^2}{2}+\frac{\arctan(cx)e c^6 d x^4}{2}+\frac{\arctan(cx)e^2c^6x^6}{6}-\frac{3c^5d^2ex+c^5d e^2x^3+\frac{e^3c^5x^5}{5}}{}\right)}{c^4}}{c^2}$ |
| orering | $\frac{(15c^6e^3x^8+69c^6d e^2x^6+165c^6d^2e x^4-5c^4e^3x^6+45c^6d^3x^2-76c^4d e^2x^4+120c^4d^2e x^2+25c^2e^3x^4+45c^4d^3-130c^2d e^2x}{45c^6(e x^2+d)}$ |
| risch | $\frac{ie^2b x^6\ln(-icx+1)}{12}-\frac{i(e x^2+d)^3 b\ln(icx+1)}{12e}-\frac{ebd\arctan\left(\frac{(c^7d^3-6c^5d^2e+6c^3d e^2-2c e^3)x}{c^6d^3-6d^2e c^4+6e^2d c^2-2e^3}\right)}{4c^4}+\frac{b d^2\arctan\left(\frac{(c^7}{c^6}\right)}{}$ |

input
```
int(x*(e*x^2+d)^2*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/90*(15*x^6*arctan(c*x)*b*c^6*e^2+15*x^6*a*c^6*e^2+45*x^4*arctan(c*x)*b*c
^6*d*e-3*b*c^5*e^2*x^5+45*x^4*a*c^6*d*e+45*x^2*arctan(c*x)*b*c^6*d^2-15*b*
c^5*d*e*x^3+45*x^2*a*c^6*d^2+5*b*c^3*e^2*x^3-45*b*c^5*d^2*x+45*b*c^4*d^2*a
rctan(c*x)+45*b*c^3*d*e*x-45*b*c^2*d*e*arctan(c*x)-15*b*c*e^2*x+15*b*e^2*a
rctan(c*x))/c^6
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.44

$$\int x\left(d+ex^2\right)^2\left(a+b\arctan(cx)\right)dx = \frac{15\,ac^6e^2x^6+45\,ac^6dex^4-3\,bc^5e^2x^5+45\,ac^6d^2x^2-5\,(3\,bc^5de-bc^3e^2)x^3-15\,(3\,bc^5d^2-3\,bc^3de+bce^2)}{90\,c^6}$$

input
```
integrate(x*(e*x^2+d)^2*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
1/90*(15*a*c^6*e^2*x^6 + 45*a*c^6*d*e*x^4 - 3*b*c^5*e^2*x^5 + 45*a*c^6*d^2
*x^2 - 5*(3*b*c^5*d*e - b*c^3*e^2)*x^3 - 15*(3*b*c^5*d^2 - 3*b*c^3*d*e + b
*c*e^2)*x + 15*(b*c^6*e^2*x^6 + 3*b*c^6*d*e*x^4 + 3*b*c^6*d^2*x^2 + 3*b*c^
4*d^2 - 3*b*c^2*d*e + b*e^2)*arctan(c*x))/c^6
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(102) = 204$.

Time = 0.40 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.90

$$\int x(d+ex^2)^2(a+b\arctan(cx))\,dx$$
$$= \begin{cases} \frac{ad^2x^2}{2} + \frac{adex^4}{2} + \frac{ae^2x^6}{6} + \frac{bd^2x^2\operatorname{atan}(cx)}{2} + \frac{bdex^4\operatorname{atan}(cx)}{2} + \frac{be^2x^6\operatorname{atan}(cx)}{6} - \frac{bd^2x}{2c} - \frac{bdex^3}{6c} - \frac{be^2x^5}{30c} + \frac{bd^2\operatorname{atan}(cx)}{2c^2} + \\ a\left(\frac{d^2x^2}{2} + \frac{dex^4}{2} + \frac{e^2x^6}{6}\right) \end{cases}$$

input
```
integrate(x*(e*x**2+d)**2*(a+b*atan(c*x)),x)
```

output
```
Piecewise((a*d**2*x**2/2 + a*d*e*x**4/2 + a*e**2*x**6/6 + b*d**2*x**2*atan
(c*x)/2 + b*d*e*x**4*atan(c*x)/2 + b*e**2*x**6*atan(c*x)/6 - b*d**2*x/(2*c
) - b*d*e*x**3/(6*c) - b*e**2*x**5/(30*c) + b*d**2*atan(c*x)/(2*c**2) + b*
d*e*x/(2*c**3) + b*e**2*x**3/(18*c**3) - b*d*e*atan(c*x)/(2*c**4) - b*e**2
*x/(6*c**5) + b*e**2*atan(c*x)/(6*c**6), Ne(c, 0)), (a*(d**2*x**2/2 + d*e*
x**4/2 + e**2*x**6/6), True))
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.36

$$\int x(d+ex^2)^2(a+b\arctan(cx))\,dx$$
$$= \frac{1}{6}ae^2x^6 + \frac{1}{2}adex^4 + \frac{1}{2}ad^2x^2 + \frac{1}{2}\left(x^2\arctan(cx) - c\left(\frac{x}{c^2} - \frac{\arctan(cx)}{c^3}\right)\right)bd^2$$
$$+ \frac{1}{6}\left(3x^4\arctan(cx) - c\left(\frac{c^2x^3 - 3x}{c^4} + \frac{3\arctan(cx)}{c^5}\right)\right)bde$$
$$+ \frac{1}{90}\left(15x^6\arctan(cx) - c\left(\frac{3c^4x^5 - 5c^2x^3 + 15x}{c^6} - \frac{15\arctan(cx)}{c^7}\right)\right)be^2$$

input
```
integrate(x*(e*x^2+d)^2*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
1/6*a*e^2*x^6 + 1/2*a*d*e*x^4 + 1/2*a*d^2*x^2 + 1/2*(x^2*arctan(c*x) - c*(
x/c^2 - arctan(c*x)/c^3))*b*d^2 + 1/6*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3
*x)/c^4 + 3*arctan(c*x)/c^5))*b*d*e + 1/90*(15*x^6*arctan(c*x) - c*((3*c^4
*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan(c*x)/c^7))*b*e^2
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(105) = 210$.

Time = 0.24 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.94

$$\int x(d+ex^2)^2(a+b\arctan(cx))\,dx$$
$$= \frac{15\,bc^6e^2x^6\arctan(cx) + 15\,ac^6e^2x^6 + 45\,bc^6dex^4\arctan(cx) + 45\,ac^6dex^4 - 3\,bc^5e^2x^5 + 45\,bc^6d^2x^2\operatorname{arcta}}{}$$

input
```
integrate(x*(e*x^2+d)^2*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
1/90*(15*b*c^6*e^2*x^6*arctan(c*x) + 15*a*c^6*e^2*x^6 + 45*b*c^6*d*e*x^4*a
rctan(c*x) + 45*a*c^6*d*e*x^4 - 3*b*c^5*e^2*x^5 + 45*b*c^6*d^2*x^2*arctan(
c*x) + 45*a*c^6*d^2*x^2 - 15*b*c^5*d*e*x^3 - 45*pi*b*c^4*d^2*sgn(c)*sgn(x)
 - 45*b*c^5*d^2*x + 5*b*c^3*e^2*x^3 + 45*b*c^4*d^2*arctan(c*x) + 45*pi*b*c
^2*d*e*sgn(c)*sgn(x) + 45*b*c^3*d*e*x - 45*b*c^2*d*e*arctan(c*x) - 15*pi*b
*e^2*sgn(c)*sgn(x) - 15*b*c*e^2*x + 15*b*e^2*arctan(c*x))/c^6
```

## Mupad [B] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.45

$$\int x(d+ex^2)^2(a+b\arctan(cx))\,dx = \frac{a\,d^2\,x^2}{2}+\frac{a\,e^2\,x^6}{6}-\frac{b\,e^2\,x^5}{30\,c}+\frac{b\,e^2\,x^3}{18\,c^3}+\frac{a\,d\,e\,x^4}{2}-\frac{b\,d^2\,x}{2\,c}-\frac{b\,e^2\,x}{6\,c^5}+\frac{b\,d^2\,\mathrm{atan}(c\,x)}{2\,c^2}+\frac{b\,e^2\,\mathrm{atan}(c\,x)}{6\,c^6}+\frac{b\,d^2\,x^2\,\mathrm{atan}(c\,x)}{2}+\frac{b\,e^2\,x^6\,\mathrm{atan}(c\,x)}{6}-\frac{b\,d\,e\,x^3}{6\,c}+\frac{b\,d\,e\,x}{2\,c^3}-\frac{b\,d\,e\,\mathrm{atan}(c\,x)}{2\,c^4}+\frac{b\,d\,e\,x^4\,\mathrm{atan}(c\,x)}{2}$$

input
```
int(x*(a + b*atan(c*x))*(d + e*x^2)^2,x)
```

output
```
(a*d^2*x^2)/2 + (a*e^2*x^6)/6 - (b*e^2*x^5)/(30*c) + (b*e^2*x^3)/(18*c^3)
+ (a*d*e*x^4)/2 - (b*d^2*x)/(2*c) - (b*e^2*x)/(6*c^5) + (b*d^2*atan(c*x))/
(2*c^2) + (b*e^2*atan(c*x))/(6*c^6) + (b*d^2*x^2*atan(c*x))/2 + (b*e^2*x^6
*atan(c*x))/6 - (b*d*e*x^3)/(6*c) + (b*d*e*x)/(2*c^3) - (b*d*e*atan(c*x))/
(2*c^4) + (b*d*e*x^4*atan(c*x))/2
```

## Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.61

$$\int x(d+ex^2)^2(a+b\arctan(cx))\,dx = \frac{45atan(cx)\,b\,c^6d^2x^2+45atan(cx)\,b\,c^6de\,x^4+15atan(cx)\,b\,c^6e^2x^6+45atan(cx)\,b\,c^4d^2-45atan(cx)\,b\,c^2d}{}$$

input
```
int(x*(e*x^2+d)^2*(a+b*atan(c*x)),x)
```

output
```
(45*atan(c*x)*b*c**6*d**2*x**2 + 45*atan(c*x)*b*c**6*d*e*x**4 + 15*atan(c*
x)*b*c**6*e**2*x**6 + 45*atan(c*x)*b*c**4*d**2 - 45*atan(c*x)*b*c**2*d*e +
 15*atan(c*x)*b*e**2 + 45*a*c**6*d**2*x**2 + 45*a*c**6*d*e*x**4 + 15*a*c**
6*e**2*x**6 - 45*b*c**5*d**2*x - 15*b*c**5*d*e*x**3 - 3*b*c**5*e**2*x**5 +
 45*b*c**3*d*e*x + 5*b*c**3*e**2*x**3 - 15*b*c*e**2*x)/(90*c**6)
```

# 3.1127 $\int (d+ex^2)^2 (a+b\arctan(cx))\,dx$



## Optimal result

Integrand size = 18, antiderivative size = 124

$$\begin{aligned}\int (d+ex^2)^2 (a+b\arctan(cx))\,dx = &-\frac{b(10c^2d-3e)\,ex^2}{30c^3}-\frac{be^2x^4}{20c}\\&+d^2x(a+b\arctan(cx))+\frac{2}{3}dex^3(a+b\arctan(cx))\\&+\frac{1}{5}e^2x^5(a+b\arctan(cx))\\&-\frac{b(15c^4d^2-10c^2de+3e^2)\log(1+c^2x^2)}{30c^5}\end{aligned}$$

output
```
-1/30*b*(10*c^2*d-3*e)*e*x^2/c^3-1/20*b*e^2*x^4/c+d^2*x*(a+b*arctan(c*x))+
2/3*d*e*x^3*(a+b*arctan(c*x))+1/5*e^2*x^5*(a+b*arctan(c*x))-1/30*b*(15*c^4
*d^2-10*c^2*d*e+3*e^2)*ln(c^2*x^2+1)/c^5
```

## Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.05

$$\int \left(d+ex^2\right)^2 (a+b\arctan(cx))\,dx = \frac{c^2x(4ac^3(15d^2+10dex^2+3e^2x^4)+bex(6e-c^2(20d+3ex^2)))+4bc^5x(15d^2+10dex^2+3e^2x^4)\arctan(}{60c^5}$$

input
```
Integrate[(d + e*x^2)^2*(a + b*ArcTan[c*x]),x]
```

output
```
(c^2*x*(4*a*c^3*(15*d^2 + 10*d*e*x^2 + 3*e^2*x^4) + b*e*x*(6*e - c^2*(20*d
 + 3*e*x^2))) + 4*b*c^5*x*(15*d^2 + 10*d*e*x^2 + 3*e^2*x^4)*ArcTan[c*x] -
2*b*(15*c^4*d^2 - 10*c^2*d*e + 3*e^2)*Log[1 + c^2*x^2])/(60*c^5)
```

## Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {5447, 27, 1576, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)^2 (a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5447}$$

$$-bc\int \frac{x\left(3e^2x^4+10dex^2+15d^2\right)}{15\left(c^2x^2+1\right)}dx+d^2x(a+b\arctan(cx))+\frac{2}{3}dex^3(a+b\arctan(cx))+\frac{1}{5}e^2x^5(a+b\arctan(cx))$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{15}bc\int \frac{x\left(3e^2x^4+10dex^2+15d^2\right)}{c^2x^2+1}dx+d^2x(a+b\arctan(cx))+\frac{2}{3}dex^3(a+b\arctan(cx))+\frac{1}{5}e^2x^5(a+b\arctan(cx))$$

$$\Big\downarrow \text{1576}$$

$$-\frac{1}{30}bc\int\frac{3e^2x^4+10dex^2+15d^2}{c^2x^2+1}dx^2+d^2x(a+b\arctan(cx))+\frac{2}{3}dex^3(a+b\arctan(cx))+\frac{1}{5}e^2x^5(a+b\arctan(cx))$$

$$\Big\downarrow \text{1140}$$

$$-\frac{1}{30}bc\int\left(\frac{3e^2x^2}{c^2}+\frac{(10c^2d-3e)\,e}{c^4}+\frac{15d^2c^4-10dec^2+3e^2}{c^4\,(c^2x^2+1)}\right)dx^2+d^2x(a+b\arctan(cx))+\frac{2}{3}dex^3(a+b\arctan(cx))+\frac{1}{5}e^2x^5(a+b\arctan(cx))$$

$$\Big\downarrow \text{2009}$$

$$d^2x(a+b\arctan(cx))+\frac{2}{3}dex^3(a+b\arctan(cx))+\frac{1}{5}e^2x^5(a+b\arctan(cx))-\frac{1}{30}bc\left(\frac{3e^2x^4}{2c^2}+\frac{ex^2(10c^2d-3e)}{c^4}+\frac{(15c^4d^2-10c^2de+3e^2)\log\left(c^2x^2+1\right)}{c^6}\right)$$

input
```
Int[(d + e*x^2)^2*(a + b*ArcTan[c*x]),x]
```

output
```
d^2*x*(a + b*ArcTan[c*x]) + (2*d*e*x^3*(a + b*ArcTan[c*x]))/3 + (e^2*x^5*(
a + b*ArcTan[c*x]))/5 - (b*c*(((10*c^2*d - 3*e)*e*x^2)/c^4 + (3*e^2*x^4)/(
2*c^2) + ((15*c^4*d^2 - 10*c^2*d*e + 3*e^2)*Log[1 + c^2*x^2])/c^6))/30
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 1576
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x]
, x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5447
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Simp[(a + b*ArcTan[c*x])   u,
x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2), x], x], x]] /; FreeQ
[{a, b, c, d, e}, x] && (IntegerQ[q] || ILtQ[q + 1/2, 0])
```

## Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.10

| method | result |
|---|---|
| parts | $a\left(\frac{1}{5}e^2x^5+\frac{2}{3}de\,x^3+d^2x\right)+\frac{b\left(\frac{\arctan(cx)c\,e^2x^5}{5}+\frac{2\arctan(cx)cde\,x^3}{3}+\arctan(cx)cx\,d^2-\frac{5c^4de\,x^2+\frac{3c^4e^2x^4}{4}-3c}{}\right.}{c}$ |
| derivativedivides | $\frac{\frac{a\left(c^5x\,d^2+\frac{2}{3}d\,c^5e\,x^3+\frac{1}{5}e^2c^5x^5\right)}{c^4}+\frac{b\left(\arctan(cx)c^5x\,d^2+\frac{2\arctan(cx)d\,c^5e\,x^3}{3}+\frac{\arctan(cx)e^2c^5x^5}{5}-\frac{c^4de\,x^2}{3}-\frac{c^4e^2x^4}{20}+\frac{c^2e^2x^2}{10}-\right.}{c^4}}{c}$ |
| default | $\frac{\frac{a\left(c^5x\,d^2+\frac{2}{3}d\,c^5e\,x^3+\frac{1}{5}e^2c^5x^5\right)}{c^4}+\frac{b\left(\arctan(cx)c^5x\,d^2+\frac{2\arctan(cx)d\,c^5e\,x^3}{3}+\frac{\arctan(cx)e^2c^5x^5}{5}-\frac{c^4de\,x^2}{3}-\frac{c^4e^2x^4}{20}+\frac{c^2e^2x^2}{10}-\right.}{c^4}}{c}$ |
| parallelrisch | $-\frac{-12x^5\arctan(cx)b\,c^5e^2-12a\,c^5e^2x^5-40x^3\arctan(cx)b\,c^5de+3b\,c^4e^2x^4-40a\,c^5de\,x^3-60x\arctan(cx)b\,c^5d^2+20b\,c^4}{60c^5}$ |
| risch | $-\frac{ib(3e^2x^5+10de\,x^3+15d^2x)\ln(icx+1)}{30}+\frac{ib\,e^2x^5\ln(-icx+1)}{10}+\frac{ibde\,x^3\ln(-icx+1)}{3}+\frac{a\,e^2x^5}{5}+\frac{ib\,d^2x\ln(-icx}{2}$ |

input
```
int((e*x^2+d)^2*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/5*e^2*x^5+2/3*d*e*x^3+d^2*x)+b/c*(1/5*arctan(c*x)*c*e^2*x^5+2/3*arcta
n(c*x)*c*d*e*x^3+arctan(c*x)*c*x*d^2-1/15/c^4*(5*c^4*d*e*x^2+3/4*c^4*e^2*x
^4-3/2*c^2*e^2*x^2+1/2*(15*c^4*d^2-10*c^2*d*e+3*e^2)*ln(c^2*x^2+1)))
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.21

$$\int (d+ex^2)^2 (a+b\arctan(cx))\,dx = \frac{12\,ac^5e^2x^5 + 40\,ac^5dex^3 - 3\,bc^4e^2x^4 + 60\,ac^5d^2x - 2\,(10\,bc^4de - 3\,bc^2e^2)x^2 + 4\,(3\,bc^5e^2x^5 + 10\,bc^5dex^3 +}{60\,c^5}$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
1/60*(12*a*c^5*e^2*x^5 + 40*a*c^5*d*e*x^3 - 3*b*c^4*e^2*x^4 + 60*a*c^5*d^2
*x - 2*(10*b*c^4*d*e - 3*b*c^2*e^2)*x^2 + 4*(3*b*c^5*e^2*x^5 + 10*b*c^5*d*
e*x^3 + 15*b*c^5*d^2*x)*arctan(c*x) - 2*(15*b*c^4*d^2 - 10*b*c^2*d*e + 3*b
*e^2)*log(c^2*x^2 + 1))/c^5
```

## Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.56

$$\int (d+ex^2)^2 (a+b\arctan(cx))\,dx = \begin{cases} ad^2x + \frac{2adex^3}{3} + \frac{ae^2x^5}{5} + bd^2x\,\mathrm{atan}\,(cx) + \frac{2bdex^3\,\mathrm{atan}\,(cx)}{3} + \frac{be^2x^5\,\mathrm{atan}\,(cx)}{5} - \frac{bd^2\log\left(x^2+\frac{1}{c^2}\right)}{2c} - \frac{bdex^2}{3c} - \frac{be^2x^4}{20c} + \\ a\left(d^2x + \frac{2dex^3}{3} + \frac{e^2x^5}{5}\right) \end{cases}$$

input
```
integrate((e*x**2+d)**2*(a+b*atan(c*x)),x)
```

output
```
Piecewise((a*d**2*x + 2*a*d*e*x**3/3 + a*e**2*x**5/5 + b*d**2*x*atan(c*x)
+ 2*b*d*e*x**3*atan(c*x)/3 + b*e**2*x**5*atan(c*x)/5 - b*d**2*log(x**2 + c
**(-2))/(2*c) - b*d*e*x**2/(3*c) - b*e**2*x**4/(20*c) + b*d*e*log(x**2 + c
**(-2))/(3*c**3) + b*e**2*x**2/(10*c**3) - b*e**2*log(x**2 + c**(-2))/(10*
c**5), Ne(c, 0)), (a*(d**2*x + 2*d*e*x**3/3 + e**2*x**5/5), True))
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.19

$$\int \left(d+ex^2\right)^2 (a+b\arctan(cx))\,dx$$
$$=\frac{1}{5}\,ae^2x^5+\frac{2}{3}\,adex^3+\frac{1}{3}\left(2\,x^3\arctan\left(cx\right)-c\left(\frac{x^2}{c^2}-\frac{\log\left(c^2x^2+1\right)}{c^4}\right)\right)bde$$
$$+\frac{1}{20}\left(4\,x^5\arctan\left(cx\right)-c\left(\frac{c^2x^4-2\,x^2}{c^4}+\frac{2\,\log\left(c^2x^2+1\right)}{c^6}\right)\right)be^2$$
$$+ad^2x+\frac{\left(2\,cx\arctan\left(cx\right)-\log\left(c^2x^2+1\right)\right)bd^2}{2\,c}$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
1/5*a*e^2*x^5 + 2/3*a*d*e*x^3 + 1/3*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(
c^2*x^2 + 1)/c^4))*b*d*e + 1/20*(4*x^5*arctan(c*x) - c*((c^2*x^4 - 2*x^2)/
c^4 + 2*log(c^2*x^2 + 1)/c^6))*b*e^2 + a*d^2*x + 1/2*(2*c*x*arctan(c*x) -
log(c^2*x^2 + 1))*b*d^2/c
```

## Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.39

$$\int \left(d+ex^2\right)^2 (a+b\arctan(cx))\,dx$$
$$=\frac{12\,bc^5e^2x^5\arctan\left(cx\right)+12\,ac^5e^2x^5+40\,bc^5dex^3\arctan\left(cx\right)+40\,ac^5dex^3-3\,bc^4e^2x^4+60\,bc^5d^2x\arctan}{}$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
1/60*(12*b*c^5*e^2*x^5*arctan(c*x) + 12*a*c^5*e^2*x^5 + 40*b*c^5*d*e*x^3*a
rctan(c*x) + 40*a*c^5*d*e*x^3 - 3*b*c^4*e^2*x^4 + 60*b*c^5*d^2*x*arctan(c*
x) + 60*a*c^5*d^2*x - 20*b*c^4*d*e*x^2 - 30*b*c^4*d^2*log(c^2*x^2 + 1) + 6
*b*c^2*e^2*x^2 + 20*b*c^2*d*e*log(c^2*x^2 + 1) - 6*b*e^2*log(c^2*x^2 + 1))
/c^5
```

## Mupad [B] (verification not implemented)

Time = 1.02 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.21

$$\int (d+ex^2)^2 (a+b\arctan(cx))\,dx = \frac{a\,e^2\,x^5}{5}+a\,d^2\,x-\frac{b\,d^2\,\ln\left(c^2\,x^2+1\right)}{2\,c} - \frac{b\,e^2\,\ln\left(c^2\,x^2+1\right)}{10\,c^5}-\frac{b\,e^2\,x^4}{20\,c} + \frac{b\,e^2\,x^2}{10\,c^3}+\frac{2\,a\,d\,e\,x^3}{3}+b\,d^2\,x\,\mathrm{atan}(c\,x) + \frac{b\,e^2\,x^5\,\mathrm{atan}(c\,x)}{5}+\frac{b\,d\,e\,\ln\left(c^2\,x^2+1\right)}{3\,c^3} - \frac{b\,d\,e\,x^2}{3\,c}+\frac{2\,b\,d\,e\,x^3\,\mathrm{atan}(c\,x)}{3}$$

input
```
int((a + b*atan(c*x))*(d + e*x^2)^2,x)
```

output
```
(a*e^2*x^5)/5 + a*d^2*x - (b*d^2*log(c^2*x^2 + 1))/(2*c) - (b*e^2*log(c^2*
x^2 + 1))/(10*c^5) - (b*e^2*x^4)/(20*c) + (b*e^2*x^2)/(10*c^3) + (2*a*d*e*
x^3)/3 + b*d^2*x*atan(c*x) + (b*e^2*x^5*atan(c*x))/5 + (b*d*e*log(c^2*x^2
+ 1))/(3*c^3) - (b*d*e*x^2)/(3*c) + (2*b*d*e*x^3*atan(c*x))/3
```

## Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.39

$$\int (d+ex^2)^2 (a+b\arctan(cx))\,dx = \frac{60atan(cx)\,b\,c^5d^2x+40atan(cx)\,b\,c^5de\,x^3+12atan(cx)\,b\,c^5e^2x^5-30\log(c^2x^2+1)\,b\,c^4d^2+20\log(c^2x^2+\dots}{}$$

input
```
int((e*x^2+d)^2*(a+b*atan(c*x)),x)
```

output
```
(60*atan(c*x)*b*c**5*d**2*x + 40*atan(c*x)*b*c**5*d*e*x**3 + 12*atan(c*x)*
b*c**5*e**2*x**5 - 30*log(c**2*x**2 + 1)*b*c**4*d**2 + 20*log(c**2*x**2 +
1)*b*c**2*d*e - 6*log(c**2*x**2 + 1)*b*e**2 + 60*a*c**5*d**2*x + 40*a*c**5
*d*e*x**3 + 12*a*c**5*e**2*x**5 - 20*b*c**4*d*e*x**2 - 3*b*c**4*e**2*x**4
+ 6*b*c**2*e**2*x**2)/(60*c**5)
```

# 3.1128 $\int \frac{(d+ex^2)^2(a+b\arctan(cx))}{x}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 137

$$\int \frac{(d+ex^2)^2(a+b\arctan(cx))}{x}\,dx = -\frac{bdex}{c}+\frac{be^2x}{4c^3}-\frac{be^2x^3}{12c}+\frac{bde\arctan(cx)}{c^2} - \frac{be^2\arctan(cx)}{4c^4}+dex^2(a+b\arctan(cx)) + \frac{1}{4}e^2x^4(a+b\arctan(cx))+ad^2\log(x) + \frac{1}{2}ibd^2\,\text{PolyLog}(2,-icx)-\frac{1}{2}ibd^2\,\text{PolyLog}(2,icx)$$

output
```
-b*d*e*x/c+1/4*b*e^2*x/c^3-1/12*b*e^2*x^3/c+b*d*e*arctan(c*x)/c^2-1/4*b*e^
2*arctan(c*x)/c^4+d*e*x^2*(a+b*arctan(c*x))+1/4*e^2*x^4*(a+b*arctan(c*x))+
a*d^2*ln(x)+1/2*I*b*d^2*polylog(2,-I*c*x)-1/2*I*b*d^2*polylog(2,I*c*x)
```

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.90

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x}\,dx = -\frac{bde(cx-\arctan(cx))}{c^2} - \frac{be^2(-3cx+c^3x^3+3\arctan(cx))}{12c^4} + dex^2(a+b\arctan(cx)) + \frac{1}{4}e^2x^4(a+b\arctan(cx)) + ad^2\log(x) + \frac{1}{2}ibd^2\operatorname{PolyLog}(2,-icx) - \frac{1}{2}ibd^2\operatorname{PolyLog}(2,icx)$$

input
```
Integrate[((d + e*x^2)^2*(a + b*ArcTan[c*x]))/x,x]
```

output
```
-((b*d*e*(c*x - ArcTan[c*x]))/c^2) - (b*e^2*(-3*c*x + c^3*x^3 + 3*ArcTan[c
*x]))/(12*c^4) + d*e*x^2*(a + b*ArcTan[c*x]) + (e^2*x^4*(a + b*ArcTan[c*x]
))/4 + a*d^2*Log[x] + (I/2)*b*d^2*PolyLog[2, (-I)*c*x] - (I/2)*b*d^2*PolyL
og[2, I*c*x]
```

## Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x}\,dx$$

$$\Big\downarrow \text{5515}$$

$$\int \left(\frac{d^2(a+b\arctan(cx))}{x} + 2dex(a+b\arctan(cx)) + e^2x^3(a+b\arctan(cx))\right)dx$$

$$\Big\downarrow \text{2009}$$

$$dex^2(a+b\arctan(cx))+\frac{1}{4}e^2x^4(a+b\arctan(cx))+ad^2\log(x)-\frac{be^2\arctan(cx)}{4c^4}+\frac{bde\arctan(cx)}{c^2}+\frac{be^2x}{4c^3}+\frac{1}{2}ibd^2\operatorname{PolyLog}(2,-icx)-\frac{1}{2}ibd^2\operatorname{PolyLog}(2,icx)-\frac{bdex}{c}-\frac{be^2x^3}{12c}$$

input
```
Int[((d + e*x^2)^2*(a + b*ArcTan[c*x]))/x,x]
```

output
```
-((b*d*e*x)/c) + (b*e^2*x)/(4*c^3) - (b*e^2*x^3)/(12*c) + (b*d*e*ArcTan[c*
x])/c^2 - (b*e^2*ArcTan[c*x])/(4*c^4) + d*e*x^2*(a + b*ArcTan[c*x]) + (e^2
*x^4*(a + b*ArcTan[c*x]))/4 + a*d^2*Log[x] + (I/2)*b*d^2*PolyLog[2, (-I)*c
*x] - (I/2)*b*d^2*PolyLog[2, I*c*x]
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

## Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.28

| method | result |
|---|---|
| derivativedivides | $a d e \,x^{2}+\frac{a \,e^{2} x^{4}}{4}+a \,d^{2} \ln \left(c x \right)+\frac{b \left(\arctan \left(c x \right) c^{4} d e \,x^{2}+\frac{\arctan \left(c x \right) e^{2} c^{4} x^{4}}{4}+\arctan \left(c x \right) c^{4} d^{2} \ln \left(c x \right)-\frac{e \left(4 c^{3} x d +\right.}{}\right.}{}$ |
| default | $a d e \,x^{2}+\frac{a \,e^{2} x^{4}}{4}+a \,d^{2} \ln \left(c x \right)+\frac{b \left(\arctan \left(c x \right) c^{4} d e \,x^{2}+\frac{\arctan \left(c x \right) e^{2} c^{4} x^{4}}{4}+\arctan \left(c x \right) c^{4} d^{2} \ln \left(c x \right)-\frac{e \left(4 c^{3} x d +\right.}{}\right.}{}$ |
| parts | $a \left(\frac{e^{2} x^{4}}{4}+d e \,x^{2}+d^{2} \ln \left(x \right)\right)+b \left(\frac{\arctan \left(c x \right) e^{2} x^{4}}{4}+\arctan \left(c x \right) d e \,x^{2}+\arctan \left(c x \right) d^{2} \ln \left(c\right.\right.$ |
| risch | $-\frac{a \,e^{2}}{4 c^{4}}+\frac{b \,e^{2} x}{4 c^{3}}-\frac{b \,e^{2} x^{3}}{12 c}+\frac{i b d e \ln \left(-i c x +1\right) x^{2}}{2}-\frac{i b e d \ln \left(i c x +1\right) x^{2}}{2}-\frac{b \,e^{2} \arctan \left(c x \right)}{4 c^{4}}+\frac{i b \,e^{2} \ln \left(-i c x +1\right) x^{4}}{8}$ |

input
```
int((e*x^2+d)^2*(a+b*arctan(c*x))/x,x,method=_RETURNVERBOSE)
```

output
```
a*d*e*x^2+1/4*a*e^2*x^4+a*d^2*ln(c*x)+b/c^4*(arctan(c*x)*c^4*d*e*x^2+1/4*a
rctan(c*x)*e^2*c^4*x^4+arctan(c*x)*c^4*d^2*ln(c*x)-1/4*e*(4*c^3*x*d+1/3*e*
c^3*x^3-e*c*x+(-4*c^2*d+e)*arctan(c*x))-c^4*d^2*(-1/2*I*ln(c*x)*ln(1+I*c*x
)+1/2*I*ln(c*x)*ln(1-I*c*x)-1/2*I*dilog(1+I*c*x)+1/2*I*dilog(1-I*c*x)))
```

## Fricas [F]

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x}\,dx = \int \frac{(ex^2+d)^2(b\arctan(cx)+a)}{x}\,dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x,x, algorithm="fricas")
```

output
```
integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d
^2)*arctan(c*x))/x, x)
```

## Sympy [F]

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x}\,dx=\int \frac{(a+b\operatorname{atan}(cx))\,(d+ex^2)^2}{x}\,dx$$

input
```
integrate((e*x**2+d)**2*(a+b*atan(c*x))/x,x)
```

output
```
Integral((a + b*atan(c*x))*(d + e*x**2)**2/x, x)
```

## Maxima [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.26

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x}\,dx=\frac{1}{4}\,ae^2x^4+adex^2+ad^2\log(x) - \frac{bc^3e^2x^3+3\,\pi bc^4d^2\log(c^2x^2+1)-12\,bc^4d^2\arctan(cx)\log(cx)+6i\,bc^4d^2\mathrm{Li}_2(i\,cx+1)-6i\,bc^4d^2\mathrm{Li}_2(-}{12\,c^4}$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x,x, algorithm="maxima")
```

output
```
1/4*a*e^2*x^4 + a*d*e*x^2 + a*d^2*log(x) - 1/12*(b*c^3*e^2*x^3 + 3*pi*b*c^
4*d^2*log(c^2*x^2 + 1) - 12*b*c^4*d^2*arctan(c*x)*log(c*x) + 6*I*b*c^4*d^2
*dilog(I*c*x + 1) - 6*I*b*c^4*d^2*dilog(-I*c*x + 1) + 3*(4*b*c^3*d*e - b*c
*e^2)*x - 3*(b*c^4*e^2*x^4 + 4*b*c^4*d*e*x^2 + 4*b*c^2*d*e - b*e^2)*arctan
(c*x))/c^4
```

## Giac [F]

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x}\,dx=\int \frac{(ex^2+d)^2(b\arctan(cx)+a)}{x}\,dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^2*(b*arctan(c*x) + a)/x, x)
```

## Mupad [B] (verification not implemented)

Time = 1.08 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.15

$$\int \frac{(d+ex^2)^2(a+b\arctan(cx))}{x}\,dx = \begin{cases} \frac{a\,(4\,d^2\,\ln(x)+e^2\,x^4+4\,d\,e\,x^2)}{4} \\ \frac{a\,(4\,d^2\,\ln(x)+e^2\,x^4+4\,d\,e\,x^2)}{4} - 2\,b\,d\,e\left(\frac{x}{2\,c} - \mathrm{atan}(c\,x)\left(\frac{1}{2\,c^2}+\frac{x^2}{2}\right)\right) - \frac{b\,e^2\,(3\,\mathrm{atan}(c\,x)-3\,c\,x+c^3\,x^3)}{12\,c^4} + \frac{b\,e^2\,x^4\,\mathrm{atan}(c\,x)}{4} \end{cases}$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^2)/x,x)
```

output
```
piecewise(c == 0, (a*(4*d^2*log(x) + e^2*x^4 + 4*d*e*x^2))/4, c ~= 0, (a*(
4*d^2*log(x) + e^2*x^4 + 4*d*e*x^2))/4 - (b*d^2*dilog(- c*x*1i + 1)*1i)/2
+ (b*d^2*dilog(c*x*1i + 1)*1i)/2 - 2*b*d*e*(x/(2*c) - atan(c*x)*(1/(2*c^2)
 + x^2/2)) - (b*e^2*(3*atan(c*x) - 3*c*x + c^3*x^3))/(12*c^4) + (b*e^2*x^4
*atan(c*x))/4)
```

## Reduce [F]

$$\int \frac{(d+ex^2)^2(a+b\arctan(cx))}{x}\,dx = \frac{12atan(cx)\,b\,c^4de\,x^2 + 3atan(cx)\,b\,c^4e^2x^4 + 12atan(cx)\,b\,c^2de - 3atan(cx)\,b\,e^2 + 12\left(\int \frac{atan(cx)}{x}dx\right)b\,c^4d^2}{12c^4}$$

input
```
int((e*x^2+d)^2*(a+b*atan(c*x))/x,x)
```

output
```
(12*atan(c*x)*b*c**4*d*e*x**2 + 3*atan(c*x)*b*c**4*e**2*x**4 + 12*atan(c*x
)*b*c**2*d*e - 3*atan(c*x)*b*e**2 + 12*int(atan(c*x)/x,x)*b*c**4*d**2 + 12
*log(x)*a*c**4*d**2 + 12*a*c**4*d*e*x**2 + 3*a*c**4*e**2*x**4 - 12*b*c**3*
d*e*x - b*c**3*e**2*x**3 + 3*b*c*e**2*x)/(12*c**4)
```

# 3.1129 $\int \frac{(d+ex^2)^2(a+b\arctan(cx))}{x^2}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 109

$$\int \frac{(d+ex^2)^2(a+b\arctan(cx))}{x^2}\,dx = -\frac{be^2x^2}{6c} - \frac{d^2(a+b\arctan(cx))}{x} + 2dex(a+b\arctan(cx)) + \frac{1}{3}e^2x^3(a+b\arctan(cx)) + bcd^2\log(x) - \frac{b(3c^4d^2+6c^2de-e^2)\log(1+c^2x^2)}{6c^3}$$

output
```
-1/6*b*e^2*x^2/c-d^2*(a+b*arctan(c*x))/x+2*d*e*x*(a+b*arctan(c*x))+1/3*e^2
*x^3*(a+b*arctan(c*x))+b*c*d^2*ln(x)-1/6*b*(3*c^4*d^2+6*c^2*d*e-e^2)*ln(c^
2*x^2+1)/c^3
```

## Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.05

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^2} dx = \frac{1}{6}\left(-\frac{6ad^2}{x} + 12adex - \frac{be^2x^2}{c} + 2ae^2x^3 + \frac{2b(-3d^2+6dex^2+e^2x^4)\arctan(cx)}{x} + 6bcd^2\log(x) + \frac{b(-3c^4d^2-6c^2de+e^2)\log(1+c^2x^2)}{c^3}\right)$$

input
```
Integrate[((d + e*x^2)^2*(a + b*ArcTan[c*x]))/x^2,x]
```

output
```
((-6*a*d^2)/x + 12*a*d*e*x - (b*e^2*x^2)/c + 2*a*e^2*x^3 + (2*b*(-3*d^2 +
6*d*e*x^2 + e^2*x^4)*ArcTan[c*x])/x + 6*b*c*d^2*Log[x] + (b*(-3*c^4*d^2 -
6*c^2*d*e + e^2)*Log[1 + c^2*x^2])/c^3)/6
```

## Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5511, 27, 1578, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^2} dx$$

$$\Big\downarrow \text{5511}$$

$$-bc\int -\frac{-e^2x^4-6dex^2+3d^2}{3x(c^2x^2+1)}dx - \frac{d^2(a+b\arctan(cx))}{x} + 2dex(a+b\arctan(cx)) + \frac{1}{3}e^2x^3(a+b\arctan(cx))$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}bc\int \frac{-e^2x^4-6dex^2+3d^2}{x\left(c^2x^2+1\right)}dx-\frac{d^2(a+b\arctan(cx))}{x}+2dex(a+b\arctan(cx))+\frac{1}{3}e^2x^3(a+b\arctan(cx))$$

$$\Big\downarrow \text{1578}$$

$$\frac{1}{6}bc\int \frac{-e^2x^4-6dex^2+3d^2}{x^2\left(c^2x^2+1\right)}dx^2-\frac{d^2(a+b\arctan(cx))}{x}+2dex(a+b\arctan(cx))+\frac{1}{3}e^2x^3(a+b\arctan(cx))$$

$$\Big\downarrow \text{1195}$$

$$\frac{1}{6}bc\int \left(\frac{3d^2}{x^2}-\frac{e^2}{c^2}+\frac{-3d^2c^4-6dec^2+e^2}{c^2\left(c^2x^2+1\right)}\right)dx^2-\frac{d^2(a+b\arctan(cx))}{x}+2dex(a+b\arctan(cx))+\frac{1}{3}e^2x^3(a+b\arctan(cx))$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d^2(a+b\arctan(cx))}{x}+2dex(a+b\arctan(cx))+\frac{1}{3}e^2x^3(a+b\arctan(cx))+\frac{1}{6}bc\left(-\frac{e^2x^2}{c^2}-\frac{\left(3c^4d^2+6c^2de-e^2\right)\log\left(c^2x^2+1\right)}{c^4}+3d^2\log\left(x^2\right)\right)$$

input

```
Int[((d + e*x^2)^2*(a + b*ArcTan[c*x]))/x^2,x]
```

output

```
-((d^2*(a + b*ArcTan[c*x]))/x) + 2*d*e*x*(a + b*ArcTan[c*x]) + (e^2*x^3*(a
 + b*ArcTan[c*x]))/3 + (b*c*(-((e^2*x^2)/c^2) + 3*d^2*Log[x^2] - ((3*c^4*d
^2 + 6*c^2*d*e - e^2)*Log[1 + c^2*x^2])/c^4))/6
```

## Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1195

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

## Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.23

| method | result |
|---|---|
| parts | $a\left(\frac{e^{2}x^{3}}{3}+2dex-\frac{d^{2}}{x}\right)+bc\left(\frac{\arctan(cx)e^{2}x^{3}}{3c}+\frac{2\arctan(cx)xde}{c}-\frac{\arctan(cx)d^{2}}{cx}-\frac{\frac{c^{2}e^{2}x^{2}}{2}+\frac{\left(3c^{4}d^{2}+6c^{2}\right.}{}}{}\right.$ |
| derivativedivides | $c\left(\frac{a\left(2d\,c^{3}ex+\frac{e^{2}c^{3}x^{3}}{3}-\frac{c^{3}d^{2}}{x}\right)}{c^{4}}+\frac{b\left(2\arctan(cx)c^{3}xde+\frac{\arctan(cx)e^{2}c^{3}x^{3}}{3}-\frac{\arctan(cx)c^{3}d^{2}}{x}-\frac{c^{2}e^{2}x^{2}}{6}-\frac{\left(3c^{4}d^{2}+6c^{2}de-\right.}{}\right.}{c^{4}}\right.$ |
| default | $c\left(\frac{a\left(2d\,c^{3}ex+\frac{e^{2}c^{3}x^{3}}{3}-\frac{c^{3}d^{2}}{x}\right)}{c^{4}}+\frac{b\left(2\arctan(cx)c^{3}xde+\frac{\arctan(cx)e^{2}c^{3}x^{3}}{3}-\frac{\arctan(cx)c^{3}d^{2}}{x}-\frac{c^{2}e^{2}x^{2}}{6}-\frac{\left(3c^{4}d^{2}+6c^{2}de-\right.}{}\right.}{c^{4}}\right.$ |
| parallelrisch | $\frac{2x^{4}\arctan(cx)b\,c^{3}e^{2}+2a\,c^{3}e^{2}x^{4}+6b\,c^{4}d^{2}\ln(x)x-3\ln\left(c^{2}x^{2}+1\right)b\,c^{4}d^{2}x+12x^{2}\arctan(cx)b\,c^{3}de-b\,c^{2}e^{2}x^{3}+12a\,c^{3}de\,x^{2}}{6x\,c^{3}}$ |
| risch | $\frac{ib\left(-e^{2}x^{4}-6de\,x^{2}+3d^{2}\right)\ln(icx+1)}{6x}+\frac{ib\,c^{3}e^{2}x^{4}\ln(-icx+1)+6ib\,c^{3}de\,x^{2}\ln(-icx+1)+2a\,c^{3}e^{2}x^{4}+6b\,c^{4}d^{2}\ln(x)x-3\ln}{}$ |

input
```
int((e*x^2+d)^2*(a+b*arctan(c*x))/x^2,x,method=_RETURNVERBOSE)
```

output
```
a*(1/3*e^2*x^3+2*d*e*x-d^2/x)+b*c*(1/3*arctan(c*x)/c*e^2*x^3+2*arctan(c*x)
/c*x*d*e-arctan(c*x)*d^2/c/x-1/3/c^4*(1/2*c^2*e^2*x^2+1/2*(3*c^4*d^2+6*c^2
*d*e-e^2)*ln(c^2*x^2+1)-3*c^4*d^2*ln(c*x)))
```

## Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.28

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^2} dx = \frac{2ac^3e^2x^4 + 6bc^4d^2x\log(x) + 12ac^3dex^2 - bc^2e^2x^3 - 6ac^3d^2 - (3bc^4d^2 + 6bc^2de - be^2)x\log(c^2x^2+1)}{6c^3x}$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^2,x, algorithm="fricas")
```

output
```
1/6*(2*a*c^3*e^2*x^4 + 6*b*c^4*d^2*x*log(x) + 12*a*c^3*d*e*x^2 - b*c^2*e^2
*x^3 - 6*a*c^3*d^2 - (3*b*c^4*d^2 + 6*b*c^2*d*e - b*e^2)*x*log(c^2*x^2 + 1
) + 2*(b*c^3*e^2*x^4 + 6*b*c^3*d*e*x^2 - 3*b*c^3*d^2)*arctan(c*x))/(c^3*x)
```

## Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.51

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^2} dx = \begin{cases} -\frac{ad^2}{x} + 2adex + \frac{ae^2x^3}{3} + bcd^2\log(x) - \frac{bcd^2\log\left(x^2+\frac{1}{c^2}\right)}{2} - \frac{bd^2\operatorname{atan}(cx)}{x} + 2bdex\operatorname{atan}(cx) + \frac{be^2x^3\operatorname{atan}(cx)}{3} - \\ a\left(-\frac{d^2}{x} + 2dex + \frac{e^2x^3}{3}\right) \end{cases}$$

input
```
integrate((e*x**2+d)**2*(a+b*atan(c*x))/x**2,x)
```

output
```
Piecewise((-a*d**2/x + 2*a*d*e*x + a*e**2*x**3/3 + b*c*d**2*log(x) - b*c*d
**2*log(x**2 + c**(-2))/2 - b*d**2*atan(c*x)/x + 2*b*d*e*x*atan(c*x) + b*e
**2*x**3*atan(c*x)/3 - b*d*e*log(x**2 + c**(-2))/c - b*e**2*x**2/(6*c) + b
*e**2*log(x**2 + c**(-2))/(6*c**3), Ne(c, 0)), (a*(-d**2/x + 2*d*e*x + e**
2*x**3/3), True))
```

## Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.19

$$\begin{aligned}\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^2}\,dx &= \frac{1}{3}\,ae^2x^3 - \frac{1}{2}\left(c\left(\log\left(c^2x^2+1\right)-\log\left(x^2\right)\right)+\frac{2\,\arctan\left(cx\right)}{x}\right)bd^2 \\ &\quad + \frac{1}{6}\left(2\,x^3\arctan\left(cx\right)-c\left(\frac{x^2}{c^2}-\frac{\log\left(c^2x^2+1\right)}{c^4}\right)\right)be^2 \\ &\quad + 2\,adex + \frac{(2\,cx\arctan\left(cx\right)-\log\left(c^2x^2+1\right))bde}{c} - \frac{ad^2}{x}\end{aligned}$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^2,x, algorithm="maxima")
```

output
```
1/3*a*e^2*x^3 - 1/2*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b*
d^2 + 1/6*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*b*e^2 +
 2*a*d*e*x + (2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b*d*e/c - a*d^2/x
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.50

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^2}\,dx = \frac{2\,bc^3e^2x^4\arctan\left(cx\right)+2\,ac^3e^2x^4+12\,bc^3dex^2\arctan\left(cx\right)-3\,bc^4d^2x\log\left(c^2x^2+1\right)+6\,bc^4d^2x\log\left(x\right)+}{6\,c^3x}$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^2,x, algorithm="giac")
```

output
```
1/6*(2*b*c^3*e^2*x^4*arctan(c*x) + 2*a*c^3*e^2*x^4 + 12*b*c^3*d*e*x^2*arct
an(c*x) - 3*b*c^4*d^2*x*log(c^2*x^2 + 1) + 6*b*c^4*d^2*x*log(x) + 12*a*c^3
*d*e*x^2 - b*c^2*e^2*x^3 - 6*b*c^3*d^2*arctan(c*x) - 6*b*c^2*d*e*x*log(c^2
*x^2 + 1) - 6*a*c^3*d^2 + b*e^2*x*log(c^2*x^2 + 1))/(c^3*x)
```

## Mupad [B] (verification not implemented)

Time = 1.03 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.24

$$\int \frac{(d+ex^2)^2(a+b\arctan(cx))}{x^2}\,dx = \frac{a\,e^2\,x^3}{3}-\frac{a\,d^2}{x}+2\,a\,d\,e\,x+\frac{b\,e^2\,\ln(c^2\,x^2+1)}{6\,c^3} - \frac{b\,e^2\,x^2}{6\,c}-\frac{b\,c\,d^2\,\ln(c^2\,x^2+1)}{2}+b\,c\,d^2\,\ln(x) - \frac{b\,d^2\,\mathrm{atan}(c\,x)}{x}+\frac{b\,e^2\,x^3\,\mathrm{atan}(c\,x)}{3} - \frac{b\,d\,e\,\ln(c^2\,x^2+1)}{c}+2\,b\,d\,e\,x\,\mathrm{atan}(c\,x)$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^2)/x^2,x)
```

output
```
(a*e^2*x^3)/3 - (a*d^2)/x + 2*a*d*e*x + (b*e^2*log(c^2*x^2 + 1))/(6*c^3) -
 (b*e^2*x^2)/(6*c) - (b*c*d^2*log(c^2*x^2 + 1))/2 + b*c*d^2*log(x) - (b*d^
2*atan(c*x))/x + (b*e^2*x^3*atan(c*x))/3 - (b*d*e*log(c^2*x^2 + 1))/c + 2*
b*d*e*x*atan(c*x)
```

## Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.50

$$\int \frac{(d+ex^2)^2(a+b\arctan(cx))}{x^2}\,dx = \frac{-6atan(cx)\,b\,c^3d^2+12atan(cx)\,b\,c^3de\,x^2+2atan(cx)\,b\,c^3e^2x^4-3\log(c^2x^2+1)\,b\,c^4d^2x-6\log(c^2x^2+1}{6c^3x}$$

input
```
int((e*x^2+d)^2*(a+b*atan(c*x))/x^2,x)
```

output
```
( - 6*atan(c*x)*b*c**3*d**2 + 12*atan(c*x)*b*c**3*d*e*x**2 + 2*atan(c*x)*b
*c**3*e**2*x**4 - 3*log(c**2*x**2 + 1)*b*c**4*d**2*x - 6*log(c**2*x**2 + 1
)*b*c**2*d*e*x + log(c**2*x**2 + 1)*b*e**2*x + 6*log(x)*b*c**4*d**2*x - 6*
a*c**3*d**2 + 12*a*c**3*d*e*x**2 + 2*a*c**3*e**2*x**4 - b*c**2*e**2*x**3)/
(6*c**3*x)
```

# 3.1130 $\int \frac{(d+ex^2)^2(a+b\arctan(cx))}{x^3}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 128

$$
\begin{aligned}
\int \frac{(d+ex^2)^2(a+b\arctan(cx))}{x^3}\,dx = &-\frac{bcd^2}{2x}-\frac{be^2x}{2c}-\frac{1}{2}bc^2d^2\arctan(cx)\\
&+\frac{be^2\arctan(cx)}{2c^2}-\frac{d^2(a+b\arctan(cx))}{2x^2}\\
&+\frac{1}{2}e^2x^2(a+b\arctan(cx))+2ade\log(x)\\
&+ibde\operatorname{PolyLog}(2,-icx)-ibde\operatorname{PolyLog}(2,icx)
\end{aligned}
$$

output

```
-1/2*b*c*d^2/x-1/2*b*e^2*x/c-1/2*b*c^2*d^2*arctan(c*x)+1/2*b*e^2*arctan(c*
x)/c^2-1/2*d^2*(a+b*arctan(c*x))/x^2+1/2*e^2*x^2*(a+b*arctan(c*x))+2*a*d*e
*ln(x)+I*b*d*e*polylog(2,-I*c*x)-I*b*d*e*polylog(2,I*c*x)
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.08 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.92

$$\int \frac{(d+ex^2)^2(a+b\arctan(cx))}{x^3}\,dx = \frac{1}{2}\left(-\frac{be^2(cx-\arctan(cx))}{c^2}-\frac{d^2(a+b\arctan(cx))}{x^2}+e^2x^2(a+b\arctan(cx))-\frac{bcd^2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-c^2x^2\right)}{x}+4ade\log(x)+2ibde\,\text{PolyLog}(2,-icx)-2ibde\,\text{PolyLog}(2,icx)\right)$$

input
```
Integrate[((d + e*x^2)^2*(a + b*ArcTan[c*x]))/x^3,x]
```

output
```
(-((b*e^2*(c*x - ArcTan[c*x]))/c^2) - (d^2*(a + b*ArcTan[c*x]))/x^2 + e^2*
x^2*(a + b*ArcTan[c*x]) - (b*c*d^2*Hypergeometric2F1[-1/2, 1, 1/2, -(c^2*x
^2)])/x + 4*a*d*e*Log[x] + (2*I)*b*d*e*PolyLog[2, (-I)*c*x] - (2*I)*b*d*e*
PolyLog[2, I*c*x])/2
```

## Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2(a+b\arctan(cx))}{x^3}\,dx$$

$$\Big\downarrow \text{5515}$$

$$\int \left(\frac{d^2(a+b\arctan(cx))}{x^3}+\frac{2de(a+b\arctan(cx))}{x}+e^2x(a+b\arctan(cx))\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d^2(a+b\arctan(cx))}{2x^2}+\frac{1}{2}e^2x^2(a+b\arctan(cx))+2ade\log(x)-\frac{1}{2}bc^2d^2\arctan(cx)+\frac{be^2\arctan(cx)}{2c^2}-\frac{bcd^2}{2x}+ibde\operatorname{PolyLog}(2,-icx)-ibde\operatorname{PolyLog}(2,icx)-\frac{be^2x}{2c}$$

input
```
Int[((d + e*x^2)^2*(a + b*ArcTan[c*x]))/x^3,x]
```

output
```
-1/2*(b*c*d^2)/x - (b*e^2*x)/(2*c) - (b*c^2*d^2*ArcTan[c*x])/2 + (b*e^2*Ar
cTan[c*x])/(2*c^2) - (d^2*(a + b*ArcTan[c*x]))/(2*x^2) + (e^2*x^2*(a + b*A
rcTan[c*x]))/2 + 2*a*d*e*Log[x] + I*b*d*e*PolyLog[2, (-I)*c*x] - I*b*d*e*P
olyLog[2, I*c*x]
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

## Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.39

| method | result |
|---|---|
| parts | $a\left(\frac{e^2x^2}{2}-\frac{d^2}{2x^2}+2de\ln\left(x\right)\right)+b\,c^2\left(\frac{\arctan(cx)e^2x^2}{2c^2}-\frac{\arctan(cx)d^2}{2c^2x^2}+\frac{2\arctan(cx)de\ln(cx)}{c^2}-\frac{cx\,e^2}{}\right.$ |
| derivativedivides | $c^2\left(\frac{2ade\ln(cx)}{c^2}-\frac{a\,d^2}{2c^2x^2}+\frac{a\,x^2e^2}{2c^2}+\frac{b\left(2\arctan(cx)c^2de\ln(cx)-\frac{\arctan(cx)c^2d^2}{2x^2}+\frac{\arctan(cx)c^2x^2e^2}{2}-\frac{cx\,e^2}{2}+\frac{(-c}{}\right.}{}\right.$ |
| default | $c^2\left(\frac{2ade\ln(cx)}{c^2}-\frac{a\,d^2}{2c^2x^2}+\frac{a\,x^2e^2}{2c^2}+\frac{b\left(2\arctan(cx)c^2de\ln(cx)-\frac{\arctan(cx)c^2d^2}{2x^2}+\frac{\arctan(cx)c^2x^2e^2}{2}-\frac{cx\,e^2}{2}+\frac{(-c}{}\right.}{}\right.$ |
| risch | $\frac{a\,e^2x^2}{2}-\frac{b\,e^2x}{2c}+\frac{b\,e^2\arctan(cx)}{4c^2}-\frac{bc\,d^2}{2x}-\frac{b\,c^2d^2\arctan(cx)}{4}+\frac{ib\,e^2\ln(c^2x^2+1)}{8c^2}+\frac{ib\,e^2\ln(-icx+1)x^2}{4}-$ |

input
```
int((e*x^2+d)^2*(a+b*arctan(c*x))/x^3,x,method=_RETURNVERBOSE)
```

output
```
a*(1/2*e^2*x^2-1/2*d^2/x^2+2*d*e*ln(x))+b*c^2*(1/2*arctan(c*x)/c^2*e^2*x^2
-1/2*arctan(c*x)*d^2/c^2/x^2+2*arctan(c*x)/c^2*d*e*ln(c*x)-1/2/c^4*(c*x*e^
2+(c^4*d^2-e^2)*arctan(c*x)+c^3*d^2/x+4*c^2*d*e*(-1/2*I*ln(c*x)*ln(1+I*c*x
)+1/2*I*ln(c*x)*ln(1-I*c*x)-1/2*I*dilog(1+I*c*x)+1/2*I*dilog(1-I*c*x))))
```

## Fricas [F]

$$\int\frac{\left(d+ex^2\right)^2\left(a+b\arctan(cx)\right)}{x^3}\,dx=\int\frac{\left(ex^2+d\right)^2\left(b\arctan\left(cx\right)+a\right)}{x^3}\,dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^3,x, algorithm="fricas")
```

output
```
integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d
^2)*arctan(c*x))/x^3, x)
```

## Sympy [F]

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^3}\,dx = \int \frac{(a+b\,\mathrm{atan}\,(cx))\,(d+ex^2)^2}{x^3}\,dx$$

input
```
integrate((e*x**2+d)**2*(a+b*atan(c*x))/x**3,x)
```

output
```
Integral((a + b*atan(c*x))*(d + e*x**2)**2/x**3, x)
```

## Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.20

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^3}\,dx$$
$$= \frac{1}{2}\,ae^2x^2 - \frac{1}{2}\left(\left(c\arctan(cx) + \frac{1}{x}\right)c + \frac{\arctan(cx)}{x^2}\right)bd^2 + 2\,ade\log(x) - \frac{ad^2}{2\,x^2}$$
$$- \frac{\pi bc^2de\log(c^2x^2+1) - 4\,bc^2de\arctan(cx)\log(cx) + 2i\,bc^2de\mathrm{Li}_2(i\,cx+1) - 2i\,bc^2de\mathrm{Li}_2(-i\,cx+1) + }{2\,c^2}$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^3,x, algorithm="maxima")
```

output
```
1/2*a*e^2*x^2 - 1/2*((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b*d^2 + 2*
a*d*e*log(x) - 1/2*a*d^2/x^2 - 1/2*(pi*b*c^2*d*e*log(c^2*x^2 + 1) - 4*b*c^
2*d*e*arctan(c*x)*log(c*x) + 2*I*b*c^2*d*e*dilog(I*c*x + 1) - 2*I*b*c^2*d*
e*dilog(-I*c*x + 1) + b*c*e^2*x - (b*c^2*e^2*x^2 + b*e^2)*arctan(c*x))/c^2
```

## Giac [F]

$$\int \frac{(d+ex^2)^2(a+b\arctan(cx))}{x^3}\,dx = \int \frac{(ex^2+d)^2(b\arctan(cx)+a)}{x^3}\,dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^3,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^2*(b*arctan(c*x) + a)/x^3, x)
```

## Mupad [B] (verification not implemented)

Time = 1.02 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.23

$$\int \frac{(d+ex^2)^2(a+b\arctan(cx))}{x^3}\,dx = \begin{cases} \frac{a\,(e^2\,x^4-d^2+4\,d\,e\,x^2\,\ln(x))}{2\,x^2} \\ \frac{a\,(e^2\,x^4-d^2+4\,d\,e\,x^2\,\ln(x))}{2\,x^2} - b\,e^2\left(\frac{x}{2\,c} - \mathrm{atan}(c\,x)\left(\frac{1}{2\,c^2}+\frac{x^2}{2}\right)\right) - \frac{b\,d^2\left(c^3\,\mathrm{atan}(c\,x)+\frac{c^2}{x}\right)}{2\,c} - \frac{b\,d^2\,\mathrm{atan}(c\,x)}{2\,x^2} - b\,d\,e\,(\mathrm{Li}\end{cases}$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^2)/x^3,x)
```

output
```
piecewise(c == 0, (a*(- d^2 + e^2*x^4 + 4*d*e*x^2*log(x)))/(2*x^2), c ~= 0
, - b*e^2*(x/(2*c) - atan(c*x)*(1/(2*c^2) + x^2/2)) + (a*(- d^2 + e^2*x^4
+ 4*d*e*x^2*log(x)))/(2*x^2) - b*d*e*(dilog(- c*x*1i + 1) - dilog(c*x*1i +
 1))*1i - (b*d^2*(c^3*atan(c*x) + c^2/x))/(2*c) - (b*d^2*atan(c*x))/(2*x^2
))
```

## Reduce [F]

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^3}\,dx$$
$$= \frac{-atan(cx)\,b\,c^4d^2x^2 - atan(cx)\,b\,c^2d^2 + atan(cx)\,b\,c^2e^2x^4 + atan(cx)\,b\,e^2x^2 + 4\left(\int \frac{atan(cx)}{x}dx\right) b\,c^2de\,x^2 +}{2c^2x^2}$$

input
```
int((e*x^2+d)^2*(a+b*atan(c*x))/x^3,x)
```

output
```
( - atan(c*x)*b*c**4*d**2*x**2 - atan(c*x)*b*c**2*d**2 + atan(c*x)*b*c**2*
e**2*x**4 + atan(c*x)*b*e**2*x**2 + 4*int(atan(c*x)/x,x)*b*c**2*d*e*x**2 +
 4*log(x)*a*c**2*d*e*x**2 - a*c**2*d**2 + a*c**2*e**2*x**4 - b*c**3*d**2*x
 - b*c*e**2*x**3)/(2*c**2*x**2)
```

# 3.1131 $\int \frac{(d+ex^2)^2(a+b\arctan(cx))}{x^4}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 115

$$\int \frac{(d+ex^2)^2(a+b\arctan(cx))}{x^4}\,dx = -\frac{bcd^2}{6x^2} - \frac{d^2(a+b\arctan(cx))}{3x^3} - \frac{2de(a+b\arctan(cx))}{x} + e^2x(a+b\arctan(cx)) - \frac{1}{3}bcd(c^2d-6e)\log(x) + \frac{b(c^4d^2-6c^2de-3e^2)\log(1+c^2x^2)}{6c}$$

output
```
-1/6*b*c*d^2/x^2-1/3*d^2*(a+b*arctan(c*x))/x^3-2*d*e*(a+b*arctan(c*x))/x+e
^2*x*(a+b*arctan(c*x))-1/3*b*c*d*(c^2*d-6*e)*ln(x)+1/6*b*(c^4*d^2-6*c^2*d*
e-3*e^2)*ln(c^2*x^2+1)/c
```

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.03

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^4}\,dx = \frac{1}{6}\left(-\frac{2ad^2}{x^3}-\frac{bcd^2}{x^2}-\frac{12ade}{x}+6ae^2x-\frac{2b(d^2+6dex^2-3e^2x^4)\arctan(cx)}{x^3}-2bcd(c^2d-6e)\log(x)+\frac{b(c^4d^2-6c^2de-3e^2)\log(1+c^2x^2)}{c}\right)$$

input
```
Integrate[((d + e*x^2)^2*(a + b*ArcTan[c*x]))/x^4,x]
```

output
```
((-2*a*d^2)/x^3 - (b*c*d^2)/x^2 - (12*a*d*e)/x + 6*a*e^2*x - (2*b*(d^2 + 6
*d*e*x^2 - 3*e^2*x^4)*ArcTan[c*x])/x^3 - 2*b*c*d*(c^2*d - 6*e)*Log[x] + (b
*(c^4*d^2 - 6*c^2*d*e - 3*e^2)*Log[1 + c^2*x^2])/c)/6
```

## Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5511, 27, 1578, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^4}\,dx$$

$$\Big\downarrow \text{5511}$$

$$-bc\int -\frac{-3e^2x^4+6dex^2+d^2}{3x^3(c^2x^2+1)}dx-\frac{d^2(a+b\arctan(cx))}{3x^3}-\frac{2de(a+b\arctan(cx))}{x}+e^2x(a+b\arctan(cx))$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}bc\int \frac{-3e^2x^4+6dex^2+d^2}{x^3\left(c^2x^2+1\right)}dx-\frac{d^2(a+b\arctan(cx))}{3x^3}-\frac{2de(a+b\arctan(cx))}{x}+e^2x(a+b\arctan(cx))$$

$\Big\downarrow$ 1578

$$\frac{1}{6}bc\int \frac{-3e^2x^4+6dex^2+d^2}{x^4\left(c^2x^2+1\right)}dx^2-\frac{d^2(a+b\arctan(cx))}{3x^3}-\frac{2de(a+b\arctan(cx))}{x}+e^2x(a+b\arctan(cx))$$

$\Big\downarrow$ 1195

$$\frac{1}{6}bc\int \left(\frac{d^2}{x^4}-\frac{\left(c^2d-6e\right)d}{x^2}+\frac{d^2c^4-6dec^2-3e^2}{c^2x^2+1}\right)dx^2-\frac{d^2(a+b\arctan(cx))}{3x^3}-\frac{2de(a+b\arctan(cx))}{x}+e^2x(a+b\arctan(cx))$$

$\Big\downarrow$ 2009

$$-\frac{d^2(a+b\arctan(cx))}{3x^3}-\frac{2de(a+b\arctan(cx))}{x}+e^2x(a+b\arctan(cx))+\frac{1}{6}bc\left(-d\log\left(x^2\right)\left(c^2d-6e\right)+\frac{\left(c^4d^2-6c^2de-3e^2\right)\log\left(c^2x^2+1\right)}{c^2}-\frac{d^2}{x^2}\right)$$

input
```
Int[((d + e*x^2)^2*(a + b*ArcTan[c*x]))/x^4,x]
```

output
```
-1/3*(d^2*(a + b*ArcTan[c*x]))/x^3 - (2*d*e*(a + b*ArcTan[c*x]))/x + e^2*x
*(a + b*ArcTan[c*x]) + (b*c*(-(d^2/x^2) - d*(c^2*d - 6*e)*Log[x^2] + ((c^4
*d^2 - 6*c^2*d*e - 3*e^2)*Log[1 + c^2*x^2])/c^2))/6
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

## Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.21

| method | result |
|---|---|
| derivativedivides | $c^3\left(\frac{a\left(cx\,e^2-\frac{c\,d^2}{3x^3}-\frac{2cde}{x}\right)}{c^4}+\frac{b\left(\arctan(cx)cx\,e^2-\frac{\arctan(cx)c\,d^2}{3x^3}-\frac{2\arctan(cx)cde}{x}-\frac{d\,c^2\left(c^2d-6e\right)\ln(cx)}{3}-\frac{c^2d^2}{6x^2}-\frac{\left(-c^4\right.}{}\right.}{c^4}\right.$ |
| default | $c^3\left(\frac{a\left(cx\,e^2-\frac{c\,d^2}{3x^3}-\frac{2cde}{x}\right)}{c^4}+\frac{b\left(\arctan(cx)cx\,e^2-\frac{\arctan(cx)c\,d^2}{3x^3}-\frac{2\arctan(cx)cde}{x}-\frac{d\,c^2\left(c^2d-6e\right)\ln(cx)}{3}-\frac{c^2d^2}{6x^2}-\frac{\left(-c^4\right.}{}\right.}{c^4}\right.$ |
| parts | $a\left(e^2x-\frac{d^2}{3x^3}-\frac{2de}{x}\right)+b\,c^3\left(\frac{\arctan(cx)x\,e^2}{c^3}-\frac{\arctan(cx)d^2}{3c^3x^3}-\frac{2\arctan(cx)de}{c^3x}-\frac{\frac{\left(-c^4d^2+6c^2de+3e^2\right)\ln}{2}}{}\right.$ |
| parallelrisch | $-\frac{2\ln(x)b\,c^4d^2x^3-\ln(c^2x^2+1)x^3b\,c^4d^2-b\,c^4d^2x^3-12\ln(x)b\,c^2de\,x^3+6\ln(c^2x^2+1)x^3b\,c^2de-6x^4\arctan(cx)bc\,e^2-6x}{6c\,x^3}$ |
| risch | $\frac{ib\left(-3e^2x^4+6de\,x^2+d^2\right)\ln(icx+1)}{6x^3}-\frac{2\ln(x)b\,c^4d^2x^3-\ln(-c^2x^2-1)b\,c^4d^2x^3-3ibc\,e^2x^4\ln(-icx+1)-12\ln(x)b\,c^2de}{}$ |

input
```
int((e*x^2+d)^2*(a+b*arctan(c*x))/x^4,x,method=_RETURNVERBOSE)
```

output
```
c^3*(a/c^4*(c*x*e^2-1/3*c*d^2/x^3-2*c*d*e/x)+b/c^4*(arctan(c*x)*c*x*e^2-1/
3*arctan(c*x)*c*d^2/x^3-2*arctan(c*x)*c*d*e/x-1/3*d*c^2*(c^2*d-6*e)*ln(c*x
)-1/6*c^2*d^2/x^2-1/6*(-c^4*d^2+6*c^2*d*e+3*e^2)*ln(c^2*x^2+1)))
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.21

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^4}\,dx$$
$$= \frac{6\,ace^2x^4 - bc^2d^2x - 12\,acdex^2 + (bc^4d^2 - 6\,bc^2de - 3\,be^2)x^3\log(c^2x^2+1) - 2\,(bc^4d^2 - 6\,bc^2de)x^3\log(x}{6\,cx^3}$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^4,x, algorithm="fricas")
```

output
```
1/6*(6*a*c*e^2*x^4 - b*c^2*d^2*x - 12*a*c*d*e*x^2 + (b*c^4*d^2 - 6*b*c^2*d
*e - 3*b*e^2)*x^3*log(c^2*x^2 + 1) - 2*(b*c^4*d^2 - 6*b*c^2*d*e)*x^3*log(x
) - 2*a*c*d^2 + 2*(3*b*c*e^2*x^4 - 6*b*c*d*e*x^2 - b*c*d^2)*arctan(c*x))/(
c*x^3)
```

## Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.57

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^4}\,dx$$
$$= \begin{cases} -\frac{ad^2}{3x^3} - \frac{2ade}{x} + ae^2x - \frac{bc^3d^2\log(x)}{3} + \frac{bc^3d^2\log\left(x^2+\frac{1}{c^2}\right)}{6} - \frac{bcd^2}{6x^2} + 2bcde\log(x) - bcde\log\left(x^2+\frac{1}{c^2}\right) - \frac{bd^2\operatorname{atan}}{3x^3} \\ a\left(-\frac{d^2}{3x^3} - \frac{2de}{x} + e^2x\right) \end{cases}$$

input
```
integrate((e*x**2+d)**2*(a+b*atan(c*x))/x**4,x)
```

output
```
Piecewise((-a*d**2/(3*x**3) - 2*a*d*e/x + a*e**2*x - b*c**3*d**2*log(x)/3
+ b*c**3*d**2*log(x**2 + c**(-2))/6 - b*c*d**2/(6*x**2) + 2*b*c*d*e*log(x)
 - b*c*d*e*log(x**2 + c**(-2)) - b*d**2*atan(c*x)/(3*x**3) - 2*b*d*e*atan(
c*x)/x + b*e**2*x*atan(c*x) - b*e**2*log(x**2 + c**(-2))/(2*c), Ne(c, 0)),
 (a*(-d**2/(3*x**3) - 2*d*e/x + e**2*x), True))
```

## Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^4}\,dx = \frac{1}{6}\left(\left(c^2\log\left(c^2x^2+1\right)-c^2\log\left(x^2\right)-\frac{1}{x^2}\right)c-\frac{2\arctan\left(cx\right)}{x^3}\right)bd^2 - \left(c\left(\log\left(c^2x^2+1\right)-\log\left(x^2\right)\right)+\frac{2\arctan\left(cx\right)}{x}\right)bde+ae^2x + \frac{\left(2\,cx\arctan\left(cx\right)-\log\left(c^2x^2+1\right)\right)be^2}{2\,c}-\frac{2\,ade}{x}-\frac{ad^2}{3\,x^3}$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^4,x, algorithm="maxima")
```

output
```
1/6*((c^2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*
b*d^2 - (c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b*d*e + a*e^2*
x + 1/2*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b*e^2/c - 2*a*d*e/x - 1/3*a
*d^2/x^3
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.49

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^4}\,dx = \frac{bc^4d^2x^3\log\left(c^2x^2+1\right)-2\,bc^4d^2x^3\log\left(x\right)+6\,bce^2x^4\arctan\left(cx\right)-6\,bc^2dex^3\log\left(c^2x^2+1\right)+12\,bc^2dex^3\,l}{}$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^4,x, algorithm="giac")
```

output
```
1/6*(b*c^4*d^2*x^3*log(c^2*x^2 + 1) - 2*b*c^4*d^2*x^3*log(x) + 6*b*c*e^2*x
^4*arctan(c*x) - 6*b*c^2*d*e*x^3*log(c^2*x^2 + 1) + 12*b*c^2*d*e*x^3*log(x
) + 6*a*c*e^2*x^4 - 12*b*c*d*e*x^2*arctan(c*x) - 3*b*e^2*x^3*log(c^2*x^2 +
 1) - b*c^2*d^2*x - 12*a*c*d*e*x^2 - 2*b*c*d^2*arctan(c*x) - 2*a*c*d^2)/(c
*x^3)
```

## Mupad [B] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.23

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^4}\,dx = a\,e^2\,x-\frac{a\,d^2}{3\,x^3}+\frac{b\,c^3\,d^2\,\ln\left(c^2\,x^2+1\right)}{6}-\frac{b\,e^2\,\ln\left(c^2\,x^2+1\right)}{2\,c}-\frac{b\,c^3\,d^2\,\ln\left(x\right)}{3}-\frac{2\,a\,d\,e}{x}+b\,e^2\,x\,\mathrm{atan}(c\,x)-\frac{b\,c\,d^2}{6\,x^2}-\frac{b\,d^2\,\mathrm{atan}(c\,x)}{3\,x^3}-b\,c\,d\,e\,\ln\left(c^2\,x^2+1\right)+2\,b\,c\,d\,e\,\ln\left(x\right)-\frac{2\,b\,d\,e\,\mathrm{atan}(c\,x)}{x}$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^2)/x^4,x)
```

output
```
a*e^2*x - (a*d^2)/(3*x^3) + (b*c^3*d^2*log(c^2*x^2 + 1))/6 - (b*e^2*log(c^
2*x^2 + 1))/(2*c) - (b*c^3*d^2*log(x))/3 - (2*a*d*e)/x + b*e^2*x*atan(c*x)
 - (b*c*d^2)/(6*x^2) - (b*d^2*atan(c*x))/(3*x^3) - b*c*d*e*log(c^2*x^2 + 1
) + 2*b*c*d*e*log(x) - (2*b*d*e*atan(c*x))/x
```

## Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.49

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^4}\,dx = \frac{-2atan(cx)\,bc\,d^2-12atan(cx)\,bcde\,x^2+6atan(cx)\,bc\,e^2x^4+\log(c^2x^2+1)\,b\,c^4d^2x^3-6\log(c^2x^2+1)\,b\,c}{}$$

input
```
int((e*x^2+d)^2*(a+b*atan(c*x))/x^4,x)
```

output
```
( - 2*atan(c*x)*b*c*d**2 - 12*atan(c*x)*b*c*d*e*x**2 + 6*atan(c*x)*b*c*e**
2*x**4 + log(c**2*x**2 + 1)*b*c**4*d**2*x**3 - 6*log(c**2*x**2 + 1)*b*c**2
*d*e*x**3 - 3*log(c**2*x**2 + 1)*b*e**2*x**3 - 2*log(x)*b*c**4*d**2*x**3 +
 12*log(x)*b*c**2*d*e*x**3 - 2*a*c*d**2 - 12*a*c*d*e*x**2 + 6*a*c*e**2*x**
4 - b*c**2*d**2*x)/(6*c*x**3)
```

# 3.1132 $\int \frac{(d+ex^2)^2(a+b\arctan(cx))}{x^5}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 139

$$\int \frac{(d+ex^2)^2(a+b\arctan(cx))}{x^5}\,dx = -\frac{bcd^2}{12x^3}+\frac{bc^3d^2}{4x}-\frac{bcde}{x}+\frac{1}{4}bc^4d^2\arctan(cx) - bc^2de\arctan(cx)-\frac{d^2(a+b\arctan(cx))}{4x^4} - \frac{de(a+b\arctan(cx))}{x^2}+ae^2\log(x) + \frac{1}{2}ibe^2\operatorname{PolyLog}(2,-icx)-\frac{1}{2}ibe^2\operatorname{PolyLog}(2,icx)$$

output
```
-1/12*b*c*d^2/x^3+1/4*b*c^3*d^2/x-b*c*d*e/x+1/4*b*c^4*d^2*arctan(c*x)-b*c^
2*d*e*arctan(c*x)-1/4*d^2*(a+b*arctan(c*x))/x^4-d*e*(a+b*arctan(c*x))/x^2+
a*e^2*ln(x)+1/2*I*b*e^2*polylog(2,-I*c*x)-1/2*I*b*e^2*polylog(2,I*c*x)
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.07 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.94

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^5}\,dx = -\frac{d^2(a+b\arctan(cx))}{4x^4} - \frac{de(a+b\arctan(cx))}{x^2} - \frac{bcd^2\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-c^2x^2\right)}{12x^3} - \frac{bcde\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-c^2x^2\right)}{x} + ae^2\log(x) + \frac{1}{2}ibe^2\,\text{PolyLog}(2,-icx) - \frac{1}{2}ibe^2\,\text{PolyLog}(2,icx)$$

input
```
Integrate[((d + e*x^2)^2*(a + b*ArcTan[c*x]))/x^5,x]
```

output
```
-1/4*(d^2*(a + b*ArcTan[c*x]))/x^4 - (d*e*(a + b*ArcTan[c*x]))/x^2 - (b*c*
d^2*Hypergeometric2F1[-3/2, 1, -1/2, -(c^2*x^2)])/(12*x^3) - (b*c*d*e*Hype
rgeometric2F1[-1/2, 1, 1/2, -(c^2*x^2)])/x + a*e^2*Log[x] + (I/2)*b*e^2*Po
lyLog[2, (-I)*c*x] - (I/2)*b*e^2*PolyLog[2, I*c*x]
```

## Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^5}\,dx$$

$$\Big\downarrow \; 5515$$

$$\int \left(\frac{d^2(a+b\arctan(cx))}{x^5} + \frac{2de(a+b\arctan(cx))}{x^3} + \frac{e^2(a+b\arctan(cx))}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d^2(a+b\arctan(cx))}{4x^4}-\frac{de(a+b\arctan(cx))}{x^2}+ae^2\log(x)+\frac{1}{4}bc^4d^2\arctan(cx)-bc^2de\arctan(cx)+\frac{bc^3d^2}{4x}-\frac{bcd^2}{12x^3}-\frac{bcde}{x}+\frac{1}{2}ibe^2\operatorname{PolyLog}(2,-icx)-\frac{1}{2}ibe^2\operatorname{PolyLog}(2,icx)$$

input
```
Int[((d + e*x^2)^2*(a + b*ArcTan[c*x]))/x^5,x]
```

output
```
-1/12*(b*c*d^2)/x^3 + (b*c^3*d^2)/(4*x) - (b*c*d*e)/x + (b*c^4*d^2*ArcTan[
c*x])/4 - b*c^2*d*e*ArcTan[c*x] - (d^2*(a + b*ArcTan[c*x]))/(4*x^4) - (d*e
*(a + b*ArcTan[c*x]))/x^2 + a*e^2*Log[x] + (I/2)*b*e^2*PolyLog[2, (-I)*c*x
] - (I/2)*b*e^2*PolyLog[2, I*c*x]
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

## Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.40

| method | result |
|---|---|
| derivativedivides | $c^4\left(-\frac{a\,d^2}{4c^4x^4}+\frac{a\,e^2\ln(cx)}{c^4}-\frac{ade}{c^4x^2}+\frac{b\left(-\frac{\arctan(cx)d^2}{4x^4}+\arctan(cx)e^2\ln(cx)-\frac{\arctan(cx)de}{x^2}+\frac{ie^2\ln(cx)\ln(icx+1)}{2}-\right.}{}\right.$ |
| default | $c^4\left(-\frac{a\,d^2}{4c^4x^4}+\frac{a\,e^2\ln(cx)}{c^4}-\frac{ade}{c^4x^2}+\frac{b\left(-\frac{\arctan(cx)d^2}{4x^4}+\arctan(cx)e^2\ln(cx)-\frac{\arctan(cx)de}{x^2}+\frac{ie^2\ln(cx)\ln(icx+1)}{2}-\right.}{}\right.$ |
| parts | $a\left(-\frac{d^2}{4x^4}-\frac{de}{x^2}+e^2\ln(x)\right)+b\,c^4\left(-\frac{\arctan(cx)d^2}{4c^4x^4}-\frac{\arctan(cx)de}{c^4x^2}+\frac{\arctan(cx)\ln(cx)e^2}{c^4}-\frac{-2ie^2\ln}{}\right.$ |
| risch | $-\frac{ib\,e^2\,\mathrm{dilog}(-icx+1)}{2}+\frac{b\,c^4d^2\arctan(cx)}{8}-\frac{bc\,d^2}{12x^3}+\frac{b\,c^3d^2}{4x}+\frac{ib\,c^2de\ln(icx+1)}{2}-\frac{ib\,d^2\ln(-icx+1)}{8x^4}+\frac{ib\,c^4}{}$ |

input
```
int((e*x^2+d)^2*(a+b*arctan(c*x))/x^5,x,method=_RETURNVERBOSE)
```

output
```
c^4*(-1/4*a*d^2/c^4/x^4+a/c^4*e^2*ln(c*x)-a/c^4*d*e/x^2+b/c^4*(-1/4*arctan
(c*x)*d^2/x^4+arctan(c*x)*e^2*ln(c*x)-arctan(c*x)*d*e/x^2+1/2*I*e^2*ln(c*x
)*ln(1+I*c*x)-1/2*I*e^2*ln(c*x)*ln(1-I*c*x)+1/2*I*e^2*dilog(1+I*c*x)-1/2*I
*e^2*dilog(1-I*c*x)+1/4*d*c^2*(-(-c^2*d+4*e)/c/x-1/3*d/c/x^3+(c^2*d-4*e)*a
rctan(c*x))))
```

## Fricas [F]

$$\int\frac{(d+ex^2)^2\,(a+b\arctan(cx))}{x^5}\,dx=\int\frac{(ex^2+d)^2(b\arctan{(cx)}+a)}{x^5}\,dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^5,x, algorithm="fricas")
```

output
```
integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d
^2)*arctan(c*x))/x^5, x)
```

## Sympy [F]

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^5} \, dx = \int \frac{(a+b\,\mathrm{atan}\,(cx))\,(d+ex^2)^2}{x^5} \, dx$$

input
```
integrate((e*x**2+d)**2*(a+b*atan(c*x))/x**5,x)
```

output
```
Integral((a + b*atan(c*x))*(d + e*x**2)**2/x**5, x)
```

## Maxima [F]

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^5} \, dx = \int \frac{(ex^2+d)^2 (b\arctan(cx)+a)}{x^5} \, dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^5,x, algorithm="maxima")
```

output
```
1/12*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan(c*x)/x^4)*b*d
^2 - ((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b*d*e + b*e^2*integrate(a
rctan(c*x)/x, x) + a*e^2*log(x) - a*d*e/x^2 - 1/4*a*d^2/x^4
```

## Giac [F]

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^5} \, dx = \int \frac{(ex^2+d)^2 (b\arctan(cx)+a)}{x^5} \, dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^5,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^2*(b*arctan(c*x) + a)/x^5, x)
```

## Mupad [B] (verification not implemented)

Time = 1.13 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.27

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^5}\,dx$$
$$= \begin{cases} a e^2 \ln(x) - \frac{\frac{a d^2}{4} + a e d x^2}{x^4} \\ a e^2 \ln(x) - \frac{\frac{a d^2}{4} + a e d x^2}{x^4} - \frac{b d^2 \left(\frac{\frac{c^2}{3} - c^4 x^2}{x^3} - c^5 \operatorname{atan}(c x)\right)}{4 c} - 2 b d e \left(\frac{c^3 \operatorname{atan}(c x) + \frac{c^2}{x}}{2 c} + \frac{\operatorname{atan}(c x)}{2 x^2}\right) - \frac{b d^2 \operatorname{atan}(c x)}{4 x^4} - \end{cases}$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^2)/x^5,x)
```

output
```
piecewise(c == 0, - ((a*d^2)/4 + a*d*e*x^2)/x^4 + a*e^2*log(x), c ~= 0, -
((a*d^2)/4 + a*d*e*x^2)/x^4 + a*e^2*log(x) - (b*e^2*dilog(- c*x*1i + 1)*1i
)/2 + (b*e^2*dilog(c*x*1i + 1)*1i)/2 - (b*d^2*((c^2/3 - c^4*x^2)/x^3 - c^5
*atan(c*x)))/(4*c) - 2*b*d*e*((c^3*atan(c*x) + c^2/x)/(2*c) + atan(c*x)/(2
*x^2)) - (b*d^2*atan(c*x))/(4*x^4))
```

## Reduce [F]

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^5}\,dx$$
$$= \frac{3atan(cx)\,b\,c^4d^2x^4 - 12atan(cx)\,b\,c^2de\,x^4 - 3atan(cx)\,b\,d^2 - 12atan(cx)\,bde\,x^2 + 12\left(\int \frac{atan(cx)}{x}dx\right) b\,e^2x}{12x^4}$$

input
```
int((e*x^2+d)^2*(a+b*atan(c*x))/x^5,x)
```

output
```
(3*atan(c*x)*b*c**4*d**2*x**4 - 12*atan(c*x)*b*c**2*d*e*x**4 - 3*atan(c*x)
*b*d**2 - 12*atan(c*x)*b*d*e*x**2 + 12*int(atan(c*x)/x,x)*b*e**2*x**4 + 12
*log(x)*a*e**2*x**4 - 3*a*d**2 - 12*a*d*e*x**2 + 3*b*c**3*d**2*x**3 - b*c*
d**2*x - 12*b*c*d*e*x**3)/(12*x**4)
```

# 3.1133 $\int \frac{\left(d+ex^2\right)^2(a+b\arctan(cx))}{x^6}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 150

$$\int \frac{\left(d+ex^2\right)^2(a+b\arctan(cx))}{x^6}\,dx = -\frac{bcd^2}{20x^4}+\frac{bcd(3c^2d-10e)}{30x^2}-\frac{d^2(a+b\arctan(cx))}{5x^5} -\frac{2de(a+b\arctan(cx))}{3x^3}-\frac{e^2(a+b\arctan(cx))}{x} +\frac{1}{15}bc\left(3c^4d^2-10c^2de+15e^2\right)\log(x) -\frac{1}{30}bc\left(3c^4d^2-10c^2de+15e^2\right)\log\left(1+c^2x^2\right)$$

output
```
-1/20*b*c*d^2/x^4+1/30*b*c*d*(3*c^2*d-10*e)/x^2-1/5*d^2*(a+b*arctan(c*x))/
x^5-2/3*d*e*(a+b*arctan(c*x))/x^3-e^2*(a+b*arctan(c*x))/x+1/15*b*c*(3*c^4*
d^2-10*c^2*d*e+15*e^2)*ln(x)-1/30*b*c*(3*c^4*d^2-10*c^2*d*e+15*e^2)*ln(c^2
*x^2+1)
```

## Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.99

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^6} dx = \frac{1}{60}\left(-\frac{12d^2(a+b\arctan(cx))}{x^5} - \frac{40de(a+b\arctan(cx))}{x^3} - \frac{60e^2(a+b\arctan(cx))}{x} + 30bce^2(2\log(x)-\log(1+c^2x^2)) - 20bcde\left(\frac{1}{x^2}+2c^2\log(x)-c^2\log(1+c^2x^2)\right) - 3bcd^2\left(\frac{1}{x^4}-\frac{2c^2}{x^2}-4c^4\log(x) + 2c^4\log(1+c^2x^2)\right)\right)$$

input
```
Integrate[((d + e*x^2)^2*(a + b*ArcTan[c*x]))/x^6,x]
```

output
```
((-12*d^2*(a + b*ArcTan[c*x]))/x^5 - (40*d*e*(a + b*ArcTan[c*x]))/x^3 - (6
0*e^2*(a + b*ArcTan[c*x]))/x + 30*b*c*e^2*(2*Log[x] - Log[1 + c^2*x^2]) -
20*b*c*d*e*(x^(-2) + 2*c^2*Log[x] - c^2*Log[1 + c^2*x^2]) - 3*b*c*d^2*(x^(
-4) - (2*c^2)/x^2 - 4*c^4*Log[x] + 2*c^4*Log[1 + c^2*x^2]))/60
```

## Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5511, 27, 1578, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^6} dx$$

$$\Big\downarrow \text{5511}$$

$$-bc\int -\frac{15e^2x^4+10dex^2+3d^2}{15x^5(c^2x^2+1)}dx-\frac{d^2(a+b\arctan(cx))}{5x^5}-\frac{2de(a+b\arctan(cx))}{3x^3}-\frac{e^2(a+b\arctan(cx))}{x}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{15}bc\int \frac{15e^2x^4+10dex^2+3d^2}{x^5(c^2x^2+1)}dx-\frac{d^2(a+b\arctan(cx))}{5x^5}-\frac{2de(a+b\arctan(cx))}{3x^3}-\frac{e^2(a+b\arctan(cx))}{x}$$

$$\Big\downarrow \text{1578}$$

$$\frac{1}{30}bc\int \frac{15e^2x^4+10dex^2+3d^2}{x^6(c^2x^2+1)}dx^2-\frac{d^2(a+b\arctan(cx))}{5x^5}-\frac{2de(a+b\arctan(cx))}{3x^3}-\frac{e^2(a+b\arctan(cx))}{x}$$

$$\Big\downarrow \text{1195}$$

$$\frac{1}{30}bc\int \left(\frac{3d^2}{x^6}-\frac{(3c^2d-10e)\,d}{x^4}+\frac{-3d^2c^6+10dec^4-15e^2c^2}{c^2x^2+1}+\frac{3d^2c^4-10dec^2+15e^2}{x^2}\right)dx^2-\frac{d^2(a+b\arctan(cx))}{5x^5}-\frac{2de(a+b\arctan(cx))}{3x^3}-\frac{e^2(a+b\arctan(cx))}{x}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d^2(a+b\arctan(cx))}{5x^5}-\frac{2de(a+b\arctan(cx))}{3x^3}-\frac{e^2(a+b\arctan(cx))}{x}+\frac{1}{30}bc\left(\frac{d(3c^2d-10e)}{x^2}+\log\left(x^2\right)\left(3c^4d^2-10c^2de+15e^2\right)-\left(3c^4d^2-10c^2de+15e^2\right)\log\left(c^2x^2+1\right)-\frac{3d^2}{2x^4}\right)$$

input
```
Int[((d + e*x^2)^2*(a + b*ArcTan[c*x]))/x^6,x]
```

output
```
-1/5*(d^2*(a + b*ArcTan[c*x]))/x^5 - (2*d*e*(a + b*ArcTan[c*x]))/(3*x^3) -
 (e^2*(a + b*ArcTan[c*x]))/x + (b*c*((-3*d^2)/(2*x^4) + (d*(3*c^2*d - 10*e
))/x^2 + (3*c^4*d^2 - 10*c^2*d*e + 15*e^2)*Log[x^2] - (3*c^4*d^2 - 10*c^2*
d*e + 15*e^2)*Log[1 + c^2*x^2]))/30
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

## Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.11

| method | result |
|---|---|
| parts | $a\left(-\frac{2de}{3x^3}-\frac{e^2}{x}-\frac{d^2}{5x^5}\right)+b\,c^5\left(-\frac{2\arctan(cx)de}{3c^5x^3}-\frac{\arctan(cx)e^2}{c^5x}-\frac{\arctan(cx)d^2}{5c^5x^5}-\frac{\left(3c^4d^2-10c^2de+15e^2\right.}{2}\right.$ |
| derivativedivides | $c^5\left(\frac{a\left(-\frac{2de}{3c\,x^3}-\frac{e^2}{cx}-\frac{d^2}{5c\,x^5}\right)}{c^4}+\frac{b\left(-\frac{2\arctan(cx)de}{3c\,x^3}-\frac{\arctan(cx)e^2}{cx}-\frac{\arctan(cx)d^2}{5c\,x^5}-\frac{\left(-3c^4d^2+10c^2de-15e^2\right)\ln(cx)}{15}+\frac{d(3c}{3}\right.}{c^4}\right.$ |
| default | $c^5\left(\frac{a\left(-\frac{2de}{3c\,x^3}-\frac{e^2}{cx}-\frac{d^2}{5c\,x^5}\right)}{c^4}+\frac{b\left(-\frac{2\arctan(cx)de}{3c\,x^3}-\frac{\arctan(cx)e^2}{cx}-\frac{\arctan(cx)d^2}{5c\,x^5}-\frac{\left(-3c^4d^2+10c^2de-15e^2\right)\ln(cx)}{15}+\frac{d(3c}{3}\right.}{c^4}\right.$ |
| parallelrisch | $12\ln(x)b\,c^5d^2x^5-6\ln(c^2x^2+1)x^5b\,c^5d^2-6b\,c^5d^2x^5-40\ln(x)b\,c^3de\,x^5+20\ln(c^2x^2+1)x^5b\,c^3de+20b\,c^3de\,x^5+60\ln(x)$ |
| risch | $\frac{ib(15e^2x^4+10de\,x^2+3d^2)\ln(icx+1)}{30x^5}-\frac{-12\ln(x)b\,c^5d^2x^5+6\ln(-c^2x^2-1)b\,c^5d^2x^5+40\ln(x)b\,c^3de\,x^5-20\ln(-c^2x^2}{}$ |

input
```
int((e*x^2+d)^2*(a+b*arctan(c*x))/x^6,x,method=_RETURNVERBOSE)
```

output
```
a*(-2/3*d*e/x^3-e^2/x-1/5*d^2/x^5)+b*c^5*(-2/3*arctan(c*x)/c^5*d*e/x^3-arc
tan(c*x)/c^5*e^2/x-1/5*arctan(c*x)*d^2/c^5/x^5-1/15/c^4*(1/2*(3*c^4*d^2-10
*c^2*d*e+15*e^2)*ln(c^2*x^2+1)+(-3*c^4*d^2+10*c^2*d*e-15*e^2)*ln(c*x)-1/2*
d*(3*c^2*d-10*e)/x^2+3/4*d^2/x^4))
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.07

$$\int \frac{(d+ex^2)^2\,(a+b\arctan(cx))}{x^6}\,dx = -\frac{60\,ae^2x^4+2\,(3\,bc^5d^2-10\,bc^3de+15\,bce^2)x^5\log\left(c^2x^2+1\right)-4\,(3\,bc^5d^2-10\,bc^3de+15\,bce^2)x^5\log\left(x\right.}{60}$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^6,x, algorithm="fricas")
```

output
```
-1/60*(60*a*e^2*x^4 + 2*(3*b*c^5*d^2 - 10*b*c^3*d*e + 15*b*c*e^2)*x^5*log(
c^2*x^2 + 1) - 4*(3*b*c^5*d^2 - 10*b*c^3*d*e + 15*b*c*e^2)*x^5*log(x) + 3*
b*c*d^2*x + 40*a*d*e*x^2 - 2*(3*b*c^3*d^2 - 10*b*c*d*e)*x^3 + 12*a*d^2 + 4
*(15*b*e^2*x^4 + 10*b*d*e*x^2 + 3*b*d^2)*arctan(c*x))/x^5
```

## Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.57

$$\int \frac{(d+ex^2)^2(a+b\arctan(cx))}{x^6}\,dx = \begin{cases} -\frac{ad^2}{5x^5} - \frac{2ade}{3x^3} - \frac{ae^2}{x} + \frac{bc^5d^2\log(x)}{5} - \frac{bc^5d^2\log\left(x^2+\frac{1}{c^2}\right)}{10} + \frac{bc^3d^2}{10x^2} - \frac{2bc^3de\log(x)}{3} + \frac{bc^3de\log\left(x^2+\frac{1}{c^2}\right)}{3} - \frac{bcd^2}{20x^4} - \frac{bcde}{3x^2} + \\ a\left(-\frac{d^2}{5x^5} - \frac{2de}{3x^3} - \frac{e^2}{x}\right) \end{cases}$$

input
```
integrate((e*x**2+d)**2*(a+b*atan(c*x))/x**6,x)
```

output
```
Piecewise((-a*d**2/(5*x**5) - 2*a*d*e/(3*x**3) - a*e**2/x + b*c**5*d**2*lo
g(x)/5 - b*c**5*d**2*log(x**2 + c**(-2))/10 + b*c**3*d**2/(10*x**2) - 2*b*
c**3*d*e*log(x)/3 + b*c**3*d*e*log(x**2 + c**(-2))/3 - b*c*d**2/(20*x**4)
- b*c*d*e/(3*x**2) + b*c*e**2*log(x) - b*c*e**2*log(x**2 + c**(-2))/2 - b*
d**2*atan(c*x)/(5*x**5) - 2*b*d*e*atan(c*x)/(3*x**3) - b*e**2*atan(c*x)/x,
 Ne(c, 0)), (a*(-d**2/(5*x**5) - 2*d*e/(3*x**3) - e**2/x), True))
```

## Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.11

$$\begin{aligned}\int \frac{(d+ex^2)^2(a+b\arctan(cx))}{x^6}\,dx &= -\frac{1}{20}\left(\left(2c^4\log(c^2x^2+1) - 2c^4\log(x^2) - \frac{2c^2x^2-1}{x^4}\right)c + \frac{4\arctan(cx)}{x^5}\right)bd^2 \\ &\quad + \frac{1}{3}\left(\left(c^2\log(c^2x^2+1) - c^2\log(x^2) - \frac{1}{x^2}\right)c - \frac{2\arctan(cx)}{x^3}\right)bde \\ &\quad - \frac{1}{2}\left(c(\log(c^2x^2+1) - \log(x^2)) + \frac{2\arctan(cx)}{x}\right)be^2 - \frac{ae^2}{x} - \frac{2ade}{3x^3} - \frac{ad^2}{5x^5}\end{aligned}$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^6,x, algorithm="maxima")
```

output
```
-1/20*((2*c^4*log(c^2*x^2 + 1) - 2*c^4*log(x^2) - (2*c^2*x^2 - 1)/x^4)*c +
 4*arctan(c*x)/x^5)*b*d^2 + 1/3*((c^2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/
x^2)*c - 2*arctan(c*x)/x^3)*b*d*e - 1/2*(c*(log(c^2*x^2 + 1) - log(x^2)) +
 2*arctan(c*x)/x)*b*e^2 - a*e^2/x - 2/3*a*d*e/x^3 - 1/5*a*d^2/x^5
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.30

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^6} dx =$$
$$-\frac{6\,bc^5d^2x^5\log(c^2x^2+1) - 12\,bc^5d^2x^5\log(x) - 20\,bc^3dex^5\log(c^2x^2+1) + 40\,bc^3dex^5\log(x) + 30\,bce^2}{}$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^6,x, algorithm="giac")
```

output
```
-1/60*(6*b*c^5*d^2*x^5*log(c^2*x^2 + 1) - 12*b*c^5*d^2*x^5*log(x) - 20*b*c
^3*d*e*x^5*log(c^2*x^2 + 1) + 40*b*c^3*d*e*x^5*log(x) + 30*b*c*e^2*x^5*log
(c^2*x^2 + 1) - 60*b*c*e^2*x^5*log(x) - 6*b*c^3*d^2*x^3 + 60*b*e^2*x^4*arc
tan(c*x) + 20*b*c*d*e*x^3 + 60*a*e^2*x^4 + 40*b*d*e*x^2*arctan(c*x) + 3*b*
c*d^2*x + 40*a*d*e*x^2 + 12*b*d^2*arctan(c*x) + 12*a*d^2)/x^5
```

## Mupad [B] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.19

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^6} dx = \frac{b c^3 d^2}{10 x^2} - \frac{a e^2}{x} - \frac{b c^5 d^2 \ln(c^2 x^2+1)}{10} - \frac{a d^2}{5 x^5} + \frac{b c^5 d^2 \ln(x)}{5} - \frac{2 a d e}{3 x^3} - \frac{b c e^2 \ln(c^2 x^2+1)}{2} - \frac{b c d^2}{20 x^4} + b c e^2 \ln(x) - \frac{b d^2 \operatorname{atan}(c x)}{5 x^5} - \frac{b e^2 \operatorname{atan}(c x)}{x} + \frac{b c^3 d e \ln(c^2 x^2+1)}{3} - \frac{2 b c^3 d e \ln(x)}{3} - \frac{b c d e}{3 x^2} - \frac{2 b d e \operatorname{atan}(c x)}{3 x^3}$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^2)/x^6,x)
```

output
```
(b*c^3*d^2)/(10*x^2) - (a*e^2)/x - (b*c^5*d^2*log(c^2*x^2 + 1))/10 - (a*d^
2)/(5*x^5) + (b*c^5*d^2*log(x))/5 - (2*a*d*e)/(3*x^3) - (b*c*e^2*log(c^2*x
^2 + 1))/2 - (b*c*d^2)/(20*x^4) + b*c*e^2*log(x) - (b*d^2*atan(c*x))/(5*x^
5) - (b*e^2*atan(c*x))/x + (b*c^3*d*e*log(c^2*x^2 + 1))/3 - (2*b*c^3*d*e*l
og(x))/3 - (b*c*d*e)/(3*x^2) - (2*b*d*e*atan(c*x))/(3*x^3)
```

## Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.30

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^6} dx = \frac{-12 atan(cx)\, b d^2 - 40 atan(cx)\, bde\, x^2 - 60 atan(cx)\, b e^2 x^4 - 6 \log(c^2x^2+1)\, b c^5 d^2 x^5 + 20 \log(c^2x^2+1)\, b}{}$$

input
```
int((e*x^2+d)^2*(a+b*atan(c*x))/x^6,x)
```

output
```
( - 12*atan(c*x)*b*d**2 - 40*atan(c*x)*b*d*e*x**2 - 60*atan(c*x)*b*e**2*x*
*4 - 6*log(c**2*x**2 + 1)*b*c**5*d**2*x**5 + 20*log(c**2*x**2 + 1)*b*c**3*
d*e*x**5 - 30*log(c**2*x**2 + 1)*b*c*e**2*x**5 + 12*log(x)*b*c**5*d**2*x**
5 - 40*log(x)*b*c**3*d*e*x**5 + 60*log(x)*b*c*e**2*x**5 - 12*a*d**2 - 40*a
*d*e*x**2 - 60*a*e**2*x**4 + 6*b*c**3*d**2*x**3 - 3*b*c*d**2*x - 20*b*c*d*
e*x**3)/(60*x**5)
```

# 3.1134 $\int \frac{(d+ex^2)^2(a+b\arctan(cx))}{x^7}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 111

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^7}\,dx = -\frac{bcd^2}{30x^5}+\frac{bcd(c^2d-3e)}{18x^3}-\frac{bc(c^4d^2-3c^2de+3e^2)}{6x} - \frac{b(c^2d-e)^3\arctan(cx)}{6d} - \frac{(d+ex^2)^3(a+b\arctan(cx))}{6dx^6}$$

output
```
-1/30*b*c*d^2/x^5+1/18*b*c*d*(c^2*d-3*e)/x^3-1/6*b*c*(c^4*d^2-3*c^2*d*e+3*
e^2)/x-1/6*b*(c^2*d-e)^3*arctan(c*x)/d-1/6*(e*x^2+d)^3*(a+b*arctan(c*x))/d
/x^6
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.01

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^7}\,dx = -\frac{bcd^2x\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},-c^2x^2\right)+5\left((d^2+3dex^2+3e^2x^4)(a+b\arctan(cx))+bcdex^3\,\text{I}\right.}{30x^6}$$

input
```
Integrate[((d + e*x^2)^2*(a + b*ArcTan[c*x]))/x^7,x]
```

output
```
-1/30*(b*c*d^2*x*Hypergeometric2F1[-5/2, 1, -3/2, -(c^2*x^2)] + 5*((d^2 +
3*d*e*x^2 + 3*e^2*x^4)*(a + b*ArcTan[c*x]) + b*c*d*e*x^3*Hypergeometric2F1
[-3/2, 1, -1/2, -(c^2*x^2)] + 3*b*c*e^2*x^5*Hypergeometric2F1[-1/2, 1, 1/2
, -(c^2*x^2)]))/x^6
```

## Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {5511, 27, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^7} \, dx$$

$$\Big\downarrow \text{5511}$$

$$-bc\int -\frac{(ex^2+d)^3}{6dx^6(c^2x^2+1)}dx-\frac{(d+ex^2)^3 (a+b\arctan(cx))}{6dx^6}$$

$$\Big\downarrow \text{27}$$

$$\frac{bc\int \frac{(ex^2+d)^3}{x^6(c^2x^2+1)}dx}{6d}-\frac{(d+ex^2)^3 (a+b\arctan(cx))}{6dx^6}$$

$$\Big\downarrow \text{364}$$

$$\frac{bc\int \left(\frac{d^3}{x^6}-\frac{(c^2d-3e)d^2}{x^4}+\frac{(d^2c^4-3dec^2+3e^2)d}{x^2}-\frac{(c^2d-e)^3}{c^2x^2+1}\right)dx}{6d}-\frac{(d+ex^2)^3 (a+b\arctan(cx))}{6dx^6}$$

$$\Big\downarrow \text{2009}$$

$$\frac{bc\left(-\frac{\arctan(cx)(c^2d-e)^3}{c}+\frac{d^2(c^2d-3e)}{3x^3}-\frac{d(c^4d^2-3c^2de+3e^2)}{x}-\frac{d^3}{5x^5}\right)}{6d}-\frac{(d+ex^2)^3 (a+b\arctan(cx))}{6dx^6}$$

input
```
Int[((d + e*x^2)^2*(a + b*ArcTan[c*x]))/x^7,x]
```

output
```
-1/6*((d + e*x^2)^3*(a + b*ArcTan[c*x]))/(d*x^6) + (b*c*(-1/5*d^3/x^5 + (d
^2*(c^2*d - 3*e))/(3*x^3) - (d*(c^4*d^2 - 3*c^2*d*e + 3*e^2))/x - ((c^2*d
- e)^3*ArcTan[c*x])/c))/(6*d)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

## Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.50

| method | result |
|---|---|
| parts | $a\left(-\frac{de}{2x^4}-\frac{e^2}{2x^2}-\frac{d^2}{6x^6}\right)+b\,c^6\left(-\frac{\arctan(cx)de}{2c^6x^4}-\frac{\arctan(cx)e^2}{2c^6x^2}-\frac{\arctan(cx)d^2}{6c^6x^6}-\frac{(c^4d^2-3c^2de+3e^2)\,a}{}\right.$ |
| derivativedivides | $c^6\left(\frac{a\left(-\frac{de}{2c^2x^4}-\frac{e^2}{2c^2x^2}-\frac{d^2}{6c^2x^6}\right)}{c^4}+\frac{b\left(-\frac{\arctan(cx)de}{2c^2x^4}-\frac{\arctan(cx)e^2}{2c^2x^2}-\frac{\arctan(cx)d^2}{6c^2x^6}+\frac{-c^4d^2+3c^2de-3e^2}{6cx}+\frac{d\left(c^2d-3e\right)}{18c\,x^3}-\right.}{c^4}\right.$ |
| default | $c^6\left(\frac{a\left(-\frac{de}{2c^2x^4}-\frac{e^2}{2c^2x^2}-\frac{d^2}{6c^2x^6}\right)}{c^4}+\frac{b\left(-\frac{\arctan(cx)de}{2c^2x^4}-\frac{\arctan(cx)e^2}{2c^2x^2}-\frac{\arctan(cx)d^2}{6c^2x^6}+\frac{-c^4d^2+3c^2de-3e^2}{6cx}+\frac{d\left(c^2d-3e\right)}{18c\,x^3}-\right.}{c^4}\right.$ |
| parallelrisch | $-\frac{15x^6\arctan(cx)b\,c^6d^2-45x^6\arctan(cx)b\,c^4de+15b\,c^5d^2x^5+45x^6\arctan(cx)b\,c^2e^2-45a\,c^2e^2x^6-45b\,c^3de\,x^5+45bc}{}$ |
| risch | $\frac{ib(3e^2x^4+3de\,x^2+d^2)\ln(icx+1)}{12x^6}-\frac{45ib\,e^2\ln(-icx+1)x^4-45i\ln(-cx+i)b\,c^2e^2x^6+45i\ln(-cx-i)b\,c^2e^2x^6+15i\ln(-}{}$ |
| orering | $-\frac{(30x^8e\,c^6d^2+60c^6d^3x^6-90x^8e^2c^4d-165x^6e\,c^4d^2+90x^8e^3c^2+35c^4d^3x^4+135x^6e^2c^2d-108x^4e\,c^2d^2+90x^6e^3-7c^2d}{45x^6(e\,x^2+d)}$ |

input
```
int((e*x^2+d)^2*(a+b*arctan(c*x))/x^7,x,method=_RETURNVERBOSE)
```

output
```
a*(-1/2*d*e/x^4-1/2*e^2/x^2-1/6*d^2/x^6)+b*c^6*(-1/2*arctan(c*x)/c^6*d*e/x
^4-1/2*arctan(c*x)/c^6*e^2/x^2-1/6*arctan(c*x)*d^2/c^6/x^6-1/6/c^4*((c^4*d
^2-3*c^2*d*e+3*e^2)*arctan(c*x)-(-c^4*d^2+3*c^2*d*e-3*e^2)/c/x-1/3*d/c*(c^
2*d-3*e)/x^3+1/5/c*d^2/x^5))
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.31

$$\int\frac{\left(d+ex^2\right)^2\left(a+b\arctan(cx)\right)}{x^7}\,dx=$$
$$-\frac{45\,ae^2x^4+15\,(bc^5d^2-3\,bc^3de+3\,bce^2)x^5+3\,bcd^2x+45\,adex^2-5\,(bc^3d^2-3\,bcde)x^3+15\,ad^2+15\,(}{90\,x^6}$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^7,x, algorithm="fricas")
```

output
```
-1/90*(45*a*e^2*x^4 + 15*(b*c^5*d^2 - 3*b*c^3*d*e + 3*b*c*e^2)*x^5 + 3*b*c
*d^2*x + 45*a*d*e*x^2 - 5*(b*c^3*d^2 - 3*b*c*d*e)*x^3 + 15*a*d^2 + 15*(3*b
*e^2*x^4 + (b*c^6*d^2 - 3*b*c^4*d*e + 3*b*c^2*e^2)*x^6 + 3*b*d*e*x^2 + b*d
^2)*arctan(c*x))/x^6
```

## Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.73

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^7}\,dx = -\frac{ad^2}{6x^6} - \frac{ade}{2x^4} - \frac{ae^2}{2x^2} - \frac{bc^6d^2\operatorname{atan}(cx)}{6} - \frac{bc^5d^2}{6x} + \frac{bc^4de\operatorname{atan}(cx)}{2} + \frac{bc^3d^2}{18x^3} + \frac{bc^3de}{2x} - \frac{bc^2e^2\operatorname{atan}(cx)}{2} - \frac{bcd^2}{30x^5} - \frac{bcde}{6x^3} - \frac{bce^2}{2x} - \frac{bd^2\operatorname{atan}(cx)}{6x^6} - \frac{bde\operatorname{atan}(cx)}{2x^4} - \frac{be^2\operatorname{atan}(cx)}{2x^2}$$

input
```
integrate((e*x**2+d)**2*(a+b*atan(c*x))/x**7,x)
```

output
```
-a*d**2/(6*x**6) - a*d*e/(2*x**4) - a*e**2/(2*x**2) - b*c**6*d**2*atan(c*x
)/6 - b*c**5*d**2/(6*x) + b*c**4*d*e*atan(c*x)/2 + b*c**3*d**2/(18*x**3) +
 b*c**3*d*e/(2*x) - b*c**2*e**2*atan(c*x)/2 - b*c*d**2/(30*x**5) - b*c*d*e
/(6*x**3) - b*c*e**2/(2*x) - b*d**2*atan(c*x)/(6*x**6) - b*d*e*atan(c*x)/(
2*x**4) - b*e**2*atan(c*x)/(2*x**2)
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.31

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^7}\,dx$$
$$= -\frac{1}{90}\left(\left(15\,c^5\arctan(cx) + \frac{15\,c^4x^4 - 5\,c^2x^2 + 3}{x^5}\right)c + \frac{15\arctan(cx)}{x^6}\right)bd^2 + \frac{1}{6}\left(\left(3\,c^3\arctan(cx) + \frac{3\,c^2x^2 - 1}{x^3}\right)c - \frac{3\arctan(cx)}{x^4}\right)bde - \frac{1}{2}\left(\left(c\arctan(cx) + \frac{1}{x}\right)c + \frac{\arctan(cx)}{x^2}\right)be^2 - \frac{ae^2}{2\,x^2} - \frac{ade}{2\,x^4} - \frac{ad^2}{6\,x^6}$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^7,x, algorithm="maxima")
```

output
```
-1/90*((15*c^5*arctan(c*x) + (15*c^4*x^4 - 5*c^2*x^2 + 3)/x^5)*c + 15*arct
an(c*x)/x^6)*b*d^2 + 1/6*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*
arctan(c*x)/x^4)*b*d*e - 1/2*((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b
*e^2 - 1/2*a*e^2/x^2 - 1/2*a*d*e/x^4 - 1/6*a*d^2/x^6
```

## Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.87 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.14

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^7}\,dx =$$
$$-\frac{-15i\,bc^6d^2x^6\log(i\,cx+1) + 15i\,bc^6d^2x^6\log(-i\,cx+1) + 45i\,bc^4dex^6\log(i\,cx+1) - 45i\,bc^4dex^6\log(}{}$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^7,x, algorithm="giac")
```

output
```
-1/180*(-15*I*b*c^6*d^2*x^6*log(I*c*x + 1) + 15*I*b*c^6*d^2*x^6*log(-I*c*x
 + 1) + 45*I*b*c^4*d*e*x^6*log(I*c*x + 1) - 45*I*b*c^4*d*e*x^6*log(-I*c*x
+ 1) + 30*b*c^5*d^2*x^5 - 45*I*b*c^2*e^2*x^6*log(I*c*x + 1) + 45*I*b*c^2*e
^2*x^6*log(-I*c*x + 1) - 90*b*c^3*d*e*x^5 - 10*b*c^3*d^2*x^3 + 90*b*c*e^2*
x^5 + 90*b*e^2*x^4*arctan(c*x) + 30*b*c*d*e*x^3 + 90*a*e^2*x^4 + 90*b*d*e*
x^2*arctan(c*x) + 6*b*c*d^2*x + 90*a*d*e*x^2 + 30*b*d^2*arctan(c*x) + 30*a
*d^2)/x^6
```

## Mupad [B] (verification not implemented)

Time = 1.18 (sec) , antiderivative size = 256, normalized size of antiderivative = 2.31

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^7}\,dx = -\frac{\frac{a\,d^2}{6}+\frac{b\,d^2\,\mathrm{atan}(c\,x)}{6}-\frac{a\,c^4\,e^2\,x^8}{2}+\frac{a\,e\,x^4\,(d\,c^2+e)}{2}+\frac{b\,c\,x^5\,(2\,c^4\,d^2-6\,c^2\,d\,e+9\,e^2)}{18}+\frac{b\,c\,d^2\,x}{30}+\frac{a\,d\,x^2\,(d\,c^2+3\,e)}{6}+\frac{b\,c^3\,x^7\,(c^4\,d^2}{c^2\,x^8+x^6} - \frac{\mathrm{atan}\left(\frac{c^2\,x}{\sqrt{c^2}}\right)\,{\left(c^2\right)}^{5/2}\,(b\,c^4\,d^2-3\,b\,c^2\,d\,e+3\,b\,e^2)}{6\,c^3}$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^2)/x^7,x)
```

output
```
- ((a*d^2)/6 + (b*d^2*atan(c*x))/6 - (a*c^4*e^2*x^8)/2 + (a*e*x^4*(e + c^2
*d))/2 + (b*c*x^5*(9*e^2 + 2*c^4*d^2 - 6*c^2*d*e))/18 + (b*c*d^2*x)/30 + (
a*d*x^2*(3*e + c^2*d))/6 + (b*c^3*x^7*(3*e^2 + c^4*d^2 - 3*c^2*d*e))/6 + (
b*c*d*x^3*(15*e - 2*c^2*d))/90 + (b*d*x^2*atan(c*x)*(3*e + c^2*d))/6 + (b*
c^2*e^2*x^6*atan(c*x))/2 + (b*e*x^4*atan(c*x)*(e + c^2*d))/2)/(x^6 + c^2*x
^8) - (atan((c^2*x)/(c^2)^(1/2))*(c^2)^(5/2)*(3*b*e^2 + b*c^4*d^2 - 3*b*c^
2*d*e))/(6*c^3)
```

## Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.56

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^7}\,dx$$
$$= \frac{-15atan(cx)\,b\,c^6d^2x^6+45atan(cx)\,b\,c^4de\,x^6-45atan(cx)\,b\,c^2e^2x^6-15atan(cx)\,b\,d^2-45atan(cx)\,bde\,x}{}$$

input
```
int((e*x^2+d)^2*(a+b*atan(c*x))/x^7,x)
```

output
```
( - 15*atan(c*x)*b*c**6*d**2*x**6 + 45*atan(c*x)*b*c**4*d*e*x**6 - 45*atan
(c*x)*b*c**2*e**2*x**6 - 15*atan(c*x)*b*d**2 - 45*atan(c*x)*b*d*e*x**2 - 4
5*atan(c*x)*b*e**2*x**4 - 15*a*d**2 - 45*a*d*e*x**2 - 45*a*e**2*x**4 - 15*
b*c**5*d**2*x**5 + 5*b*c**3*d**2*x**3 + 45*b*c**3*d*e*x**5 - 3*b*c*d**2*x
- 15*b*c*d*e*x**3 - 45*b*c*e**2*x**5)/(90*x**6)
```

# 3.1135 $\int \frac{(d+ex^2)^2(a+b\arctan(cx))}{x^8}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 186

$$\begin{aligned}\int \frac{(d+ex^2)^2(a+b\arctan(cx))}{x^8}\,dx =& -\frac{bcd^2}{42x^6}+\frac{bcd(5c^2d-14e)}{140x^4}\\&-\frac{bc(15c^4d^2-42c^2de+35e^2)}{210x^2}\\&-\frac{d^2(a+b\arctan(cx))}{7x^7}\\&-\frac{2de(a+b\arctan(cx))}{5x^5}-\frac{e^2(a+b\arctan(cx))}{3x^3}\\&-\frac{1}{105}bc^3(15c^4d^2-42c^2de+35e^2)\log(x)\\&+\frac{1}{210}bc^3(15c^4d^2-42c^2de+35e^2)\log(1+c^2x^2)\end{aligned}$$

output
```
-1/42*b*c*d^2/x^6+1/140*b*c*d*(5*c^2*d-14*e)/x^4-1/210*b*c*(15*c^4*d^2-42*
c^2*d*e+35*e^2)/x^2-1/7*d^2*(a+b*arctan(c*x))/x^7-2/5*d*e*(a+b*arctan(c*x)
)/x^5-1/3*e^2*(a+b*arctan(c*x))/x^3-1/105*b*c^3*(15*c^4*d^2-42*c^2*d*e+35*
e^2)*ln(x)+1/210*b*c^3*(15*c^4*d^2-42*c^2*d*e+35*e^2)*ln(c^2*x^2+1)
```

## Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.95

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^8} dx = \frac{1}{420}\left(-\frac{60d^2(a+b\arctan(cx))}{x^7} - \frac{168de(a+b\arctan(cx))}{x^5} - \frac{140e^2(a+b\arctan(cx))}{x^3} - 70bce^2\left(\frac{1}{x^2}+2c^2\log(x)-c^2\log\left(1+c^2x^2\right)\right) - 42bcde\left(\frac{1}{x^4}-\frac{2c^2}{x^2}-4c^4\log(x) + 2c^4\log\left(1+c^2x^2\right)\right) - 5bcd^2\left(\frac{2-3c^2x^2+6c^4x^4}{x^6} + 12c^6\log(x)-6c^6\log\left(1+c^2x^2\right)\right)\right)$$

input
```
Integrate[((d + e*x^2)^2*(a + b*ArcTan[c*x]))/x^8,x]
```

output
```
((-60*d^2*(a + b*ArcTan[c*x]))/x^7 - (168*d*e*(a + b*ArcTan[c*x]))/x^5 - (
140*e^2*(a + b*ArcTan[c*x]))/x^3 - 70*b*c*e^2*(x^(-2) + 2*c^2*Log[x] - c^2
*Log[1 + c^2*x^2]) - 42*b*c*d*e*(x^(-4) - (2*c^2)/x^2 - 4*c^4*Log[x] + 2*c
^4*Log[1 + c^2*x^2]) - 5*b*c*d^2*((2 - 3*c^2*x^2 + 6*c^4*x^4)/x^6 + 12*c^6
*Log[x] - 6*c^6*Log[1 + c^2*x^2]))/420
```

## Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5511, 27, 1578, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^8} \, dx$$

$\Big\downarrow$ 5511

$$-bc\int -\frac{35e^2x^4+42dex^2+15d^2}{105x^7(c^2x^2+1)}dx-\frac{d^2(a+b\arctan(cx))}{7x^7}-\frac{2de(a+b\arctan(cx))}{5x^5}-\frac{e^2(a+b\arctan(cx))}{3x^3}$$

$\Big\downarrow$ 27

$$\frac{1}{105}bc\int \frac{35e^2x^4+42dex^2+15d^2}{x^7(c^2x^2+1)}dx-\frac{d^2(a+b\arctan(cx))}{7x^7}-\frac{2de(a+b\arctan(cx))}{5x^5}-\frac{e^2(a+b\arctan(cx))}{3x^3}$$

$\Big\downarrow$ 1578

$$\frac{1}{210}bc\int \frac{35e^2x^4+42dex^2+15d^2}{x^8(c^2x^2+1)}dx^2-\frac{d^2(a+b\arctan(cx))}{7x^7}-\frac{2de(a+b\arctan(cx))}{5x^5}-\frac{e^2(a+b\arctan(cx))}{3x^3}$$

$\Big\downarrow$ 1195

$$\frac{1}{210}bc\int \left(\frac{15d^2}{x^8}-\frac{3(5c^2d-14e)d}{x^6}+\frac{15d^2c^8-42dec^6+35e^2c^4}{c^2x^2+1}+\frac{-15d^2c^6+42dec^4-35e^2c^2}{x^2}+\frac{15d^2c^4-42de}{x^4}\right.$$
$$\frac{d^2(a+b\arctan(cx))}{7x^7}-\frac{2de(a+b\arctan(cx))}{5x^5}-\frac{e^2(a+b\arctan(cx))}{3x^3}$$

$\Big\downarrow$ 2009

$$-\frac{d^2(a+b\arctan(cx))}{7x^7}-\frac{2de(a+b\arctan(cx))}{5x^5}-\frac{e^2(a+b\arctan(cx))}{3x^3}+$$
$$\frac{1}{210}bc\left(\frac{3d(5c^2d-14e)}{2x^4}-\frac{15c^4d^2-42c^2de+35e^2}{x^2}-\left(c^2\log\left(x^2\right)\left(15c^4d^2-42c^2de+35e^2\right)\right)+c^2\left(15c^4d^2-42c^2\right.\right.$$

input
```
Int[((d + e*x^2)^2*(a + b*ArcTan[c*x]))/x^8,x]
```

output
```
-1/7*(d^2*(a + b*ArcTan[c*x]))/x^7 - (2*d*e*(a + b*ArcTan[c*x]))/(5*x^5) -
 (e^2*(a + b*ArcTan[c*x]))/(3*x^3) + (b*c*((-5*d^2)/x^6 + (3*d*(5*c^2*d -
14*e))/(2*x^4) - (15*c^4*d^2 - 42*c^2*d*e + 35*e^2)/x^2 - c^2*(15*c^4*d^2
- 42*c^2*d*e + 35*e^2)*Log[x^2] + c^2*(15*c^4*d^2 - 42*c^2*d*e + 35*e^2)*L
og[1 + c^2*x^2]))/210
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 1578
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a
+ b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Int
egerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

## Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.09

| method | result |
|---|---|
| parts | $a\left(-\frac{d^2}{7x^7}-\frac{e^2}{3x^3}-\frac{2de}{5x^5}\right)+b\,c^7\left(-\frac{\arctan(cx)d^2}{7c^7x^7}-\frac{\arctan(cx)e^2}{3c^7x^3}-\frac{2\arctan(cx)de}{5c^7x^5}-\frac{\left(-15c^4d^2+42c^2de-\right.}{2}\right.$ |
| derivativedivides | $c^7\left(\frac{a\left(-\frac{e^2}{3c^3x^3}-\frac{2de}{5c^3x^5}-\frac{d^2}{7c^3x^7}\right)}{c^4}+\frac{b\left(-\frac{\arctan(cx)e^2}{3c^3x^3}-\frac{2\arctan(cx)de}{5c^3x^5}-\frac{\arctan(cx)d^2}{7c^3x^7}-\frac{\left(-15c^4d^2+42c^2de-35e^2\right)\ln\left(c^2x^2\right.}{210}\right.}{}\right.$ |
| default | $c^7\left(\frac{a\left(-\frac{e^2}{3c^3x^3}-\frac{2de}{5c^3x^5}-\frac{d^2}{7c^3x^7}\right)}{c^4}+\frac{b\left(-\frac{\arctan(cx)e^2}{3c^3x^3}-\frac{2\arctan(cx)de}{5c^3x^5}-\frac{\arctan(cx)d^2}{7c^3x^7}-\frac{\left(-15c^4d^2+42c^2de-35e^2\right)\ln\left(c^2x^2\right.}{210}\right.}{}\right.$ |
| parallelrisch | $-\frac{60\ln(x)b\,c^7d^2x^7-30\ln(c^2x^2+1)b\,c^7d^2x^7-30b\,c^7d^2x^7-168\ln(x)b\,c^5de\,x^7+84\ln(c^2x^2+1)b\,c^5de\,x^7+84b\,c^5de\,x^7+1}{}$ |
| risch | $\frac{ib(35e^2x^4+42de\,x^2+15d^2)\ln(icx+1)}{210x^7}-\frac{60\ln(x)b\,c^7d^2x^7-30\ln(c^2x^2+1)b\,c^7d^2x^7-168\ln(x)b\,c^5de\,x^7+84\ln(c^2x^2+}{}$ |

input
```
int((e*x^2+d)^2*(a+b*arctan(c*x))/x^8,x,method=_RETURNVERBOSE)
```

output
```
a*(-1/7*d^2/x^7-1/3*e^2/x^3-2/5*d*e/x^5)+b*c^7*(-1/7*arctan(c*x)*d^2/c^7/x
^7-1/3*arctan(c*x)/c^7*e^2/x^3-2/5*arctan(c*x)/c^7*d*e/x^5-1/105/c^4*(1/2*
(-15*c^4*d^2+42*c^2*d*e-35*e^2)*ln(c^2*x^2+1)-1/2*(-15*c^4*d^2+42*c^2*d*e-
35*e^2)/c^2/x^2+(15*c^4*d^2-42*c^2*d*e+35*e^2)*ln(c*x)+5/2/c^2*d^2/x^6-3/4
*d/c^2*(5*c^2*d-14*e)/x^4))
```

## Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.04

$$\int \frac{(d+ex^2)^2\,(a+b\arctan(cx))}{x^8}\,dx$$
$$=\frac{2\,(15\,bc^7d^2-42\,bc^5de+35\,bc^3e^2)x^7\log\left(c^2x^2+1\right)-4\,(15\,bc^7d^2-42\,bc^5de+35\,bc^3e^2)x^7\log\left(x\right)-140\,ae}{}$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^8,x, algorithm="fricas")
```

output
```
1/420*(2*(15*b*c^7*d^2 - 42*b*c^5*d*e + 35*b*c^3*e^2)*x^7*log(c^2*x^2 + 1)
 - 4*(15*b*c^7*d^2 - 42*b*c^5*d*e + 35*b*c^3*e^2)*x^7*log(x) - 140*a*e^2*x
^4 - 2*(15*b*c^5*d^2 - 42*b*c^3*d*e + 35*b*c*e^2)*x^5 - 10*b*c*d^2*x - 168
*a*d*e*x^2 + 3*(5*b*c^3*d^2 - 14*b*c*d*e)*x^3 - 60*a*d^2 - 4*(35*b*e^2*x^4
 + 42*b*d*e*x^2 + 15*b*d^2)*arctan(c*x))/x^7
```

## Sympy [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.55

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^8}\,dx$$
$$= \begin{cases} -\frac{ad^2}{7x^7} - \frac{2ade}{5x^5} - \frac{ae^2}{3x^3} - \frac{bc^7d^2\log{(x)}}{7} + \frac{bc^7d^2\log{\left(x^2+\frac{1}{c^2}\right)}}{14} - \frac{bc^5d^2}{14x^2} + \frac{2bc^5de\log{(x)}}{5} - \frac{bc^5de\log{\left(x^2+\frac{1}{c^2}\right)}}{5} + \frac{bc^3d^2}{28x^4} + \frac{bc^3de}{5x^2} \\ a\left(-\frac{d^2}{7x^7} - \frac{2de}{5x^5} - \frac{e^2}{3x^3}\right) \end{cases}$$

input
```
integrate((e*x**2+d)**2*(a+b*atan(c*x))/x**8,x)
```

output
```
Piecewise((-a*d**2/(7*x**7) - 2*a*d*e/(5*x**5) - a*e**2/(3*x**3) - b*c**7*
d**2*log(x)/7 + b*c**7*d**2*log(x**2 + c**(-2))/14 - b*c**5*d**2/(14*x**2)
 + 2*b*c**5*d*e*log(x)/5 - b*c**5*d*e*log(x**2 + c**(-2))/5 + b*c**3*d**2/
(28*x**4) + b*c**3*d*e/(5*x**2) - b*c**3*e**2*log(x)/3 + b*c**3*e**2*log(x
**2 + c**(-2))/6 - b*c*d**2/(42*x**6) - b*c*d*e/(10*x**4) - b*c*e**2/(6*x*
*2) - b*d**2*atan(c*x)/(7*x**7) - 2*b*d*e*atan(c*x)/(5*x**5) - b*e**2*atan
(c*x)/(3*x**3), Ne(c, 0)), (a*(-d**2/(7*x**7) - 2*d*e/(5*x**5) - e**2/(3*x
**3)), True))
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.06

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^8}\,dx$$
$$= \frac{1}{84}\left(\left(6\,c^6\log\left(c^2x^2+1\right)-6\,c^6\log\left(x^2\right)-\frac{6\,c^4x^4-3\,c^2x^2+2}{x^6}\right)c-\frac{12\,\arctan\left(cx\right)}{x^7}\right)bd^2 - \frac{1}{10}\left(\left(2\,c^4\log\left(c^2x^2+1\right)-2\,c^4\log\left(x^2\right)-\frac{2\,c^2x^2-1}{x^4}\right)c+\frac{4\,\arctan\left(cx\right)}{x^5}\right)bde + \frac{1}{6}\left(\left(c^2\log\left(c^2x^2+1\right)-c^2\log\left(x^2\right)-\frac{1}{x^2}\right)c-\frac{2\,\arctan\left(cx\right)}{x^3}\right)be^2 - \frac{ae^2}{3\,x^3}-\frac{2\,ade}{5\,x^5}-\frac{ad^2}{7\,x^7}$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^8,x, algorithm="maxima")
```

output
```
1/84*((6*c^6*log(c^2*x^2 + 1) - 6*c^6*log(x^2) - (6*c^4*x^4 - 3*c^2*x^2 +
2)/x^6)*c - 12*arctan(c*x)/x^7)*b*d^2 - 1/10*((2*c^4*log(c^2*x^2 + 1) - 2*
c^4*log(x^2) - (2*c^2*x^2 - 1)/x^4)*c + 4*arctan(c*x)/x^5)*b*d*e + 1/6*((c
^2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*b*e^2 -
 1/3*a*e^2/x^3 - 2/5*a*d*e/x^5 - 1/7*a*d^2/x^7
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.25

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^8}\,dx$$
$$= \frac{30\,bc^7d^2x^7\log\left(c^2x^2+1\right)-60\,bc^7d^2x^7\log\left(x\right)-84\,bc^5dex^7\log\left(c^2x^2+1\right)+168\,bc^5dex^7\log\left(x\right)+70\,bc^3e^2}{}$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x))/x^8,x, algorithm="giac")
```

output
```
1/420*(30*b*c^7*d^2*x^7*log(c^2*x^2 + 1) - 60*b*c^7*d^2*x^7*log(x) - 84*b*
c^5*d*e*x^7*log(c^2*x^2 + 1) + 168*b*c^5*d*e*x^7*log(x) + 70*b*c^3*e^2*x^7
*log(c^2*x^2 + 1) - 140*b*c^3*e^2*x^7*log(x) - 30*b*c^5*d^2*x^5 + 84*b*c^3
*d*e*x^5 + 15*b*c^3*d^2*x^3 - 70*b*c*e^2*x^5 - 140*b*e^2*x^4*arctan(c*x) -
 42*b*c*d*e*x^3 - 140*a*e^2*x^4 - 168*b*d*e*x^2*arctan(c*x) - 10*b*c*d^2*x
 - 168*a*d*e*x^2 - 60*b*d^2*arctan(c*x) - 60*a*d^2)/x^7
```

## Mupad [B] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.25

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^8} dx = -\frac{60\,a\,d^2+60\,b\,d^2\,\mathrm{atan}(c\,x)+140\,a\,e^2\,x^4-15\,b\,c^3\,d^2\,x^3+30\,b\,c^5\,d^2\,x^5+10\,b\,c\,d^2\,x+168\,a\,d\,e\,x^2+70\,b\,\ldots}{\ldots}$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^2)/x^8,x)
```

output
```
-(60*a*d^2 + 60*b*d^2*atan(c*x) + 140*a*e^2*x^4 - 15*b*c^3*d^2*x^3 + 30*b*
c^5*d^2*x^5 + 10*b*c*d^2*x + 168*a*d*e*x^2 + 70*b*c*e^2*x^5 + 140*b*e^2*x^
4*atan(c*x) + 60*b*c^7*d^2*x^7*log(x) + 140*b*c^3*e^2*x^7*log(x) - 84*b*c^
3*d*e*x^5 + 42*b*c*d*e*x^3 - 30*b*c^7*d^2*x^7*log(c^2*x^2 + 1) - 70*b*c^3*
e^2*x^7*log(c^2*x^2 + 1) + 168*b*d*e*x^2*atan(c*x) - 168*b*c^5*d*e*x^7*log
(x) + 84*b*c^5*d*e*x^7*log(c^2*x^2 + 1))/(420*x^7)
```

## Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.25

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))}{x^8} dx = \frac{-60 atan(cx)\, b\, d^2-168 atan(cx)\, bde\, x^2-140 atan(cx)\, b\, e^2x^4+30\log(c^2x^2+1)\, b\, c^7d^2x^7-84\log(c^2x^2+\ldots}{\ldots}$$

input
```
int((e*x^2+d)^2*(a+b*atan(c*x))/x^8,x)
```

output
```
( - 60*atan(c*x)*b*d**2 - 168*atan(c*x)*b*d*e*x**2 - 140*atan(c*x)*b*e**2*
x**4 + 30*log(c**2*x**2 + 1)*b*c**7*d**2*x**7 - 84*log(c**2*x**2 + 1)*b*c*
*5*d*e*x**7 + 70*log(c**2*x**2 + 1)*b*c**3*e**2*x**7 - 60*log(x)*b*c**7*d*
*2*x**7 + 168*log(x)*b*c**5*d*e*x**7 - 140*log(x)*b*c**3*e**2*x**7 - 60*a*
d**2 - 168*a*d*e*x**2 - 140*a*e**2*x**4 - 30*b*c**5*d**2*x**5 + 15*b*c**3*
d**2*x**3 + 84*b*c**3*d*e*x**5 - 10*b*c*d**2*x - 42*b*c*d*e*x**3 - 70*b*c*
e**2*x**5)/(420*x**7)
```

# 3.1136 $\int x^3(d+ex^2)^3 (a+b\arctan(cx))\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 240

$$\int x^3(d+ex^2)^3 (a+b\arctan(cx))\,dx = \frac{b(10c^6d^3-20c^4d^2e+15c^2de^2-4e^3)\,x}{40c^9} - \frac{b(10c^6d^3-20c^4d^2e+15c^2de^2-4e^3)\,x^3}{120c^7} - \frac{be(20c^4d^2-15c^2de+4e^2)\,x^5}{200c^5} - \frac{b(15c^2d-4e)\,e^2x^7}{280c^3} - \frac{be^3x^9}{90c} + \frac{b(c^2d-e)^4\,(c^2d+4e)\arctan(cx)}{40c^{10}e^2} - \frac{d(d+ex^2)^4\,(a+b\arctan(cx))}{8e^2} + \frac{(d+ex^2)^5\,(a+b\arctan(cx))}{10e^2}$$

output
```
1/40*b*(10*c^6*d^3-20*c^4*d^2*e+15*c^2*d*e^2-4*e^3)*x/c^9-1/120*b*(10*c^6*
d^3-20*c^4*d^2*e+15*c^2*d*e^2-4*e^3)*x^3/c^7-1/200*b*e*(20*c^4*d^2-15*c^2*
d*e+4*e^2)*x^5/c^5-1/280*b*(15*c^2*d-4*e)*e^2*x^7/c^3-1/90*b*e^3*x^9/c+1/4
0*b*(c^2*d-e)^4*(c^2*d+4*e)*arctan(c*x)/c^10/e^2-1/8*d*(e*x^2+d)^4*(a+b*ar
ctan(c*x))/e^2+1/10*(e*x^2+d)^5*(a+b*arctan(c*x))/e^2
```

## Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.02

$$\int x^3(d+ex^2)^3(a+b\arctan(cx))\,dx = -\frac{be^3(315cx-105c^3x^3+63c^5x^5-45c^7x^7+35c^9x^9-315\arctan(cx))}{3150c^{10}} - \frac{bd^2e(15cx-5c^3x^3+3c^5x^5-15\arctan(cx))}{30c^6} - \frac{bd^3(-3cx+c^3x^3+3\arctan(cx))}{12c^4} - \frac{bde^2(-105cx+35c^3x^3-21c^5x^5+15c^7x^7+105\arctan(cx))}{280c^8} + \frac{1}{4}d^3x^4(a+b\arctan(cx)) + \frac{1}{2}d^2ex^6(a+b\arctan(cx)) + \frac{3}{8}de^2x^8(a+b\arctan(cx)) + \frac{1}{10}e^3x^{10}(a+b\arctan(cx))$$

input
```
Integrate[x^3*(d + e*x^2)^3*(a + b*ArcTan[c*x]),x]
```

output
```
-1/3150*(b*e^3*(315*c*x - 105*c^3*x^3 + 63*c^5*x^5 - 45*c^7*x^7 + 35*c^9*x
^9 - 315*ArcTan[c*x]))/c^10 - (b*d^2*e*(15*c*x - 5*c^3*x^3 + 3*c^5*x^5 - 1
5*ArcTan[c*x]))/(30*c^6) - (b*d^3*(-3*c*x + c^3*x^3 + 3*ArcTan[c*x]))/(12*
c^4) - (b*d*e^2*(-105*c*x + 35*c^3*x^3 - 21*c^5*x^5 + 15*c^7*x^7 + 105*Arc
Tan[c*x]))/(280*c^8) + (d^3*x^4*(a + b*ArcTan[c*x]))/4 + (d^2*e*x^6*(a + b
*ArcTan[c*x]))/2 + (3*d*e^2*x^8*(a + b*ArcTan[c*x]))/8 + (e^3*x^10*(a + b*
ArcTan[c*x]))/10
```

## Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.26, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5511, 27, 403, 403, 27, 403, 403, 299, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(d+ex^2)^3(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5511}$$

$$-bc\int -\frac{(d-4ex^2)(ex^2+d)^4}{40e^2(c^2x^2+1)}dx+\frac{(d+ex^2)^5(a+b\arctan(cx))}{10e^2}-\frac{d(d+ex^2)^4(a+b\arctan(cx))}{8e^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{bc\int\frac{(d-4ex^2)(ex^2+d)^4}{c^2x^2+1}dx}{40e^2}+\frac{(d+ex^2)^5(a+b\arctan(cx))}{10e^2}-\frac{d(d+ex^2)^4(a+b\arctan(cx))}{8e^2}$$

$$\Big\downarrow \text{403}$$

$$\frac{bc\left(\frac{\int\frac{(ex^2+d)^3\left(d(9dc^2+4e)-(23c^2d-36e)ex^2\right)}{c^2x^2+1}dx}{9c^2}-\frac{4ex(d+ex^2)^4}{9c^2}\right)}{40e^2}+\frac{(d+ex^2)^5(a+b\arctan(cx))}{10e^2}-\frac{d(d+ex^2)^4(a+b\arctan(cx))}{8e^2}$$

$$\Big\downarrow \text{403}$$

$$\frac{bc\left(\frac{\frac{\int\frac{3(ex^2+d)^2\left(d(21d^2c^4+17dec^2-12e^2)-e(25d^2c^4-135dec^2+84e^2)x^2\right)}{c^2x^2+1}dx}{7c^2}-\frac{ex(23c^2d-36e)(d+ex^2)^3}{7c^2}}{9c^2}-\frac{4ex(d+ex^2)^4}{9c^2}\right)}{40e^2}+\frac{(d+ex^2)^5(a+b\arctan(cx))}{10e^2}-\frac{d(d+ex^2)^4(a+b\arctan(cx))}{8e^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{bc\left(\frac{\frac{3\int\frac{(ex^2+d)^2\left(d(21d^2c^4+17dec^2-12e^2)-e(25d^2c^4-135dec^2+84e^2)x^2\right)}{c^2x^2+1}dx}{7c^2}-\frac{ex(23c^2d-36e)(d+ex^2)^3}{7c^2}}{9c^2}-\frac{4ex(d+ex^2)^4}{9c^2}\right)}{40e^2}+\frac{(d+ex^2)^5(a+b\arctan(cx))}{10e^2}-\frac{d(d+ex^2)^4(a+b\arctan(cx))}{8e^2}$$

$$\Big\downarrow \text{403}$$

$$\frac{bc\left(\frac{\frac{3\left(\frac{\int \frac{\left(ex^2+d\right)\left(e\left(5d^3c^6+750d^2ec^4-1071de^2c^2+420e^3\right)x^2+d\left(105d^3c^6+110d^2ec^4-195de^2c^2+84e^3\right)\right)}{c^2x^2+1}dx}{5c^2}-\frac{ex\left(25c^4d^2-135c^2de+84e^2\right)\left(d+ex^2\right)^2}{5c^2}\right)}{7c^2}-\frac{ex}{}}{9c^2}\right.}{40e^2}$$

$$\frac{\left(d+ex^2\right)^5(a+b\arctan(cx))}{10e^2}-\frac{d\left(d+ex^2\right)^4(a+b\arctan(cx))}{8e^2}$$

$\downarrow$ 403

$$\frac{bc\left(\frac{\frac{3\left(\frac{\frac{\int \frac{e\left(325d^4c^8+1815d^3ec^6-4977d^2e^2c^4+4305de^3c^2-1260e^4\right)x^2+d\left(315d^4c^8+325d^3ec^6-1335d^2e^2c^4+1323de^3c^2-420e^4\right)}{c^2x^2+1}dx}{3c^2}+\frac{ex\left(5c^6d^3+750c^4d^2e-10\right.}{}}{5c^2}\right.}{7c^2}}{9c^2}\right.}{40e^2}$$

$$\frac{\left(d+ex^2\right)^5(a+b\arctan(cx))}{10e^2}-\frac{d\left(d+ex^2\right)^4(a+b\arctan(cx))}{8e^2}$$

$\downarrow$ 299

$$\frac{bc\left(\frac{\frac{3\left(\frac{\frac{\frac{315\left(c^2d+4e\right)\left(c^2d-e\right)^4\int\frac{1}{c^2x^2+1}dx}{c^2}+\frac{ex\left(325c^8d^4+1815c^6d^3e-4977c^4d^2e^2+4305c^2de^3-1260e^4\right)}{c^2}}{3c^2}+\frac{ex\left(5c^6d^3+750c^4d^2e-1071c^2de^2+420e^3\right)\left(d+ex\right.}{3c^2}}{5c^2}\right.}{7c^2}}{9c^2}\right.}{40e^2}$$

$$\frac{\left(d+ex^2\right)^5(a+b\arctan(cx))}{10e^2}-\frac{d\left(d+ex^2\right)^4(a+b\arctan(cx))}{8e^2}$$

$\downarrow$ 216

$$\frac{(d+ex^2)^5(a+b\arctan(cx))}{10e^2}-\frac{d(d+ex^2)^4(a+b\arctan(cx))}{8e^2}+\frac{bc\left(\frac{3\left(\frac{\frac{\frac{315\arctan(cx)(c^2d+4e)(c^2d-e)^4}{c^3}+\frac{ex(325c^8d^4+1815c^6d^3e-4977c^4d^2e^2+4305c^2de^3-1260e^4)}{c^2}}{3c^2}+\frac{ex(5c^6d^3+750c^4d^2e-1071c^2de^2+420e^3)(d+ex^2)}{3c^2}}{5c^2}\right.}{7c^2}}{9c^2}\right)}{40e^2}$$

input
```
Int[x^3*(d + e*x^2)^3*(a + b*ArcTan[c*x]),x]
```

output
```
-1/8*(d*(d + e*x^2)^4*(a + b*ArcTan[c*x]))/e^2 + ((d + e*x^2)^5*(a + b*Arc
Tan[c*x]))/(10*e^2) + (b*c*((-4*e*x*(d + e*x^2)^4)/(9*c^2) + (-1/7*((23*c^
2*d - 36*e)*e*x*(d + e*x^2)^3)/c^2 + (3*(-1/5*(e*(25*c^4*d^2 - 135*c^2*d*e
 + 84*e^2)*x*(d + e*x^2)^2)/c^2 + ((e*(5*c^6*d^3 + 750*c^4*d^2*e - 1071*c^
2*d*e^2 + 420*e^3)*x*(d + e*x^2))/(3*c^2) + ((e*(325*c^8*d^4 + 1815*c^6*d^
3*e - 4977*c^4*d^2*e^2 + 4305*c^2*d*e^3 - 1260*e^4)*x)/c^2 + (315*(c^2*d -
 e)^4*(c^2*d + 4*e)*ArcTan[c*x])/c^3)/(3*c^2))/(5*c^2)))/(7*c^2))/(9*c^2))
)/(40*e^2)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

## Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.25

| method | result |
|---|---|
| parts | $a\left(\frac{1}{10}e^3x^{10}+\frac{3}{8}e^2d\,x^8+\frac{1}{2}e\,d^2x^6+\frac{1}{4}d^3x^4\right)+\frac{b\left(\frac{\arctan(cx)c^4e^3x^{10}}{10}+\frac{3\arctan(cx)c^4e^2d\,x^8}{8}+\frac{\arctan(cx)c^4d^2}{2}\right.}{}$ |
| derivativedivides | $\frac{\frac{a\left(\frac{1}{4}d^3c^{10}x^4+\frac{1}{2}d^2c^{10}e\,x^6+\frac{3}{8}d\,c^{10}e^2x^8+\frac{1}{10}e^3c^{10}x^{10}\right)}{c^6}+\frac{b\left(\frac{\arctan(cx)d^3c^{10}x^4}{4}+\frac{\arctan(cx)d^2c^{10}e\,x^6}{2}+\frac{3\arctan(cx)d\,c^{10}e^2x^8}{8}+\right.}{}}{}$ |
| default | $\frac{\frac{a\left(\frac{1}{4}d^3c^{10}x^4+\frac{1}{2}d^2c^{10}e\,x^6+\frac{3}{8}d\,c^{10}e^2x^8+\frac{1}{10}e^3c^{10}x^{10}\right)}{c^6}+\frac{b\left(\frac{\arctan(cx)d^3c^{10}x^4}{4}+\frac{\arctan(cx)d^2c^{10}e\,x^6}{2}+\frac{3\arctan(cx)d\,c^{10}e^2x^8}{8}+\right.}{}}{}$ |
| parallelrisch | $\frac{4725x^8\arctan(cx)b\,c^{10}d\,e^2+6300b\,c^4d^2e\arctan(cx)-4725b\,c^2d\,e^2\arctan(cx)-3150b\,c^6d^3\arctan(cx)+6300x^6\arctan(}{}$ |
| risch | $-\frac{3bd\,e^2x^7}{56c}-\frac{b\,d^2e\,x^5}{10c}+\frac{3bd\,e^2x^5}{40c^3}+\frac{b\,d^2e\,x^3}{6c^3}-\frac{bd\,e^2x^3}{8c^5}-\frac{b\,d^2ex}{2c^5}+\frac{3bd\,e^2x}{8c^7}+\frac{b\,d^2e\arctan(cx)}{2c^6}-\frac{3bd\,e^2\,a}{8}$ |
| orering | $\frac{(420c^{10}e^4x^{12}+2080c^{10}d\,e^3x^{10}+4065c^{10}d^2e^2x^8-60c^8e^4x^{10}+3780c^{10}d^3e\,x^6-425c^8d\,e^3x^8+1050c^{10}d^4x^4-1395c^8d^2e^2}{}$ |

input
```
int(x^3*(e*x^2+d)^3*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/10*e^3*x^10+3/8*e^2*d*x^8+1/2*e*d^2*x^6+1/4*d^3*x^4)+b/c^4*(1/10*arct
an(c*x)*c^4*e^3*x^10+3/8*arctan(c*x)*c^4*e^2*d*x^8+1/2*arctan(c*x)*c^4*d^2
*e*x^6+1/4*arctan(c*x)*d^3*c^4*x^4-1/40/c^6*(4/9*e^3*c^9*x^9+15/7*d*c^9*e^
2*x^7+4*d^2*c^9*e*x^5+10/3*d^3*c^9*x^3-4/7*e^3*c^7*x^7-3*d*c^7*e^2*x^5-20/
3*d^2*c^7*e*x^3-10*c^7*x*d^3+4/5*e^3*c^5*x^5+5*c^5*d*e^2*x^3+20*c^5*d^2*e*
x-4/3*e^3*c^3*x^3-15*c^3*x*d*e^2+4*c*x*e^3+(10*c^6*d^3-20*c^4*d^2*e+15*c^2
*d*e^2-4*e^3)*arctan(c*x)))
```

## Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.27

$$\int x^3(d+ex^2)^3(a+b\arctan(cx))\,dx = \frac{1260\,ac^{10}e^3x^{10}+4725\,ac^{10}de^2x^8-140\,bc^9e^3x^9+6300\,ac^{10}d^2ex^6+3150\,ac^{10}d^3x^4-45\,(15\,bc^9de^2-4\,bc^7}{}$$

input
```
integrate(x^3*(e*x^2+d)^3*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
1/12600*(1260*a*c^10*e^3*x^10 + 4725*a*c^10*d*e^2*x^8 - 140*b*c^9*e^3*x^9
+ 6300*a*c^10*d^2*e*x^6 + 3150*a*c^10*d^3*x^4 - 45*(15*b*c^9*d*e^2 - 4*b*c
^7*e^3)*x^7 - 63*(20*b*c^9*d^2*e - 15*b*c^7*d*e^2 + 4*b*c^5*e^3)*x^5 - 105
*(10*b*c^9*d^3 - 20*b*c^7*d^2*e + 15*b*c^5*d*e^2 - 4*b*c^3*e^3)*x^3 + 315*
(10*b*c^7*d^3 - 20*b*c^5*d^2*e + 15*b*c^3*d*e^2 - 4*b*c*e^3)*x + 315*(4*b*
c^10*e^3*x^10 + 15*b*c^10*d*e^2*x^8 + 20*b*c^10*d^2*e*x^6 + 10*b*c^10*d^3*
x^4 - 10*b*c^6*d^3 + 20*b*c^4*d^2*e - 15*b*c^2*d*e^2 + 4*b*e^3)*arctan(c*x
))/c^10
```

## Sympy [A] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.71

$$\int x^3(d+ex^2)^3(a+b\arctan(cx))\,dx = \begin{cases} \frac{ad^3x^4}{4}+\frac{ad^2ex^6}{2}+\frac{3ade^2x^8}{8}+\frac{ae^3x^{10}}{10}+\frac{bd^3x^4\operatorname{atan}(cx)}{4}+\frac{bd^2ex^6\operatorname{atan}(cx)}{2}+\frac{3bde^2x^8\operatorname{atan}(cx)}{8}+\frac{be^3x^{10}\operatorname{atan}(cx)}{10}-\frac{bd^3x^3}{12c} \\ a\left(\frac{d^3x^4}{4}+\frac{d^2ex^6}{2}+\frac{3de^2x^8}{8}+\frac{e^3x^{10}}{10}\right) \end{cases}$$

input
```
integrate(x**3*(e*x**2+d)**3*(a+b*atan(c*x)),x)
```

output
```
Piecewise((a*d**3*x**4/4 + a*d**2*e*x**6/2 + 3*a*d*e**2*x**8/8 + a*e**3*x*
*10/10 + b*d**3*x**4*atan(c*x)/4 + b*d**2*e*x**6*atan(c*x)/2 + 3*b*d*e**2*
x**8*atan(c*x)/8 + b*e**3*x**10*atan(c*x)/10 - b*d**3*x**3/(12*c) - b*d**2
*e*x**5/(10*c) - 3*b*d*e**2*x**7/(56*c) - b*e**3*x**9/(90*c) + b*d**3*x/(4
*c**3) + b*d**2*e*x**3/(6*c**3) + 3*b*d*e**2*x**5/(40*c**3) + b*e**3*x**7/
(70*c**3) - b*d**3*atan(c*x)/(4*c**4) - b*d**2*e*x/(2*c**5) - b*d*e**2*x**
3/(8*c**5) - b*e**3*x**5/(50*c**5) + b*d**2*e*atan(c*x)/(2*c**6) + 3*b*d*e
**2*x/(8*c**7) + b*e**3*x**3/(30*c**7) - 3*b*d*e**2*atan(c*x)/(8*c**8) - b
*e**3*x/(10*c**9) + b*e**3*atan(c*x)/(10*c**10), Ne(c, 0)), (a*(d**3*x**4/
4 + d**2*e*x**6/2 + 3*d*e**2*x**8/8 + e**3*x**10/10), True))
```

## Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.12

$$\int x^3 (d+ex^2)^3 (a+b\arctan(cx))\,dx = \frac{1}{10}ae^3x^{10} + \frac{3}{8}ade^2x^8 + \frac{1}{2}ad^2ex^6 + \frac{1}{4}ad^3x^4 + \frac{1}{12}\left(3x^4\arctan(cx) - c\left(\frac{c^2x^3-3x}{c^4} + \frac{3\arctan(cx)}{c^5}\right)\right)bd^3 + \frac{1}{30}\left(15x^6\arctan(cx) - c\left(\frac{3c^4x^5-5c^2x^3+15x}{c^6} - \frac{15\arctan(cx)}{c^7}\right)\right)bd^2e + \frac{1}{280}\left(105x^8\arctan(cx) - c\left(\frac{15c^6x^7-21c^4x^5+35c^2x^3-105x}{c^8} + \frac{105\arctan(cx)}{c^9}\right)\right)bde^2 + \frac{1}{3150}\left(315x^{10}\arctan(cx) - c\left(\frac{35c^8x^9-45c^6x^7+63c^4x^5-105c^2x^3+315x}{c^{10}} - \frac{315\arctan(cx)}{c^{11}}\right)\right)b$$

input
```
integrate(x^3*(e*x^2+d)^3*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
1/10*a*e^3*x^10 + 3/8*a*d*e^2*x^8 + 1/2*a*d^2*e*x^6 + 1/4*a*d^3*x^4 + 1/12
*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*b*d^3 +
 1/30*(15*x^6*arctan(c*x) - c*((3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arc
tan(c*x)/c^7))*b*d^2*e + 1/280*(105*x^8*arctan(c*x) - c*((15*c^6*x^7 - 21*
c^4*x^5 + 35*c^2*x^3 - 105*x)/c^8 + 105*arctan(c*x)/c^9))*b*d*e^2 + 1/3150
*(315*x^10*arctan(c*x) - c*((35*c^8*x^9 - 45*c^6*x^7 + 63*c^4*x^5 - 105*c^
2*x^3 + 315*x)/c^10 - 315*arctan(c*x)/c^11))*b*e^3
```

## Giac [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.64

$$\int x^3(d+ex^2)^3(a+b\arctan(cx))\,dx$$
$$= \frac{1260\,bc^{10}e^3x^{10}\arctan(cx) + 1260\,ac^{10}e^3x^{10} + 4725\,bc^{10}de^2x^8\arctan(cx) + 4725\,ac^{10}de^2x^8 - 140\,bc^9e^3x^9}{}$$

input
```
integrate(x^3*(e*x^2+d)^3*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
1/12600*(1260*b*c^10*e^3*x^10*arctan(c*x) + 1260*a*c^10*e^3*x^10 + 4725*b*
c^10*d*e^2*x^8*arctan(c*x) + 4725*a*c^10*d*e^2*x^8 - 140*b*c^9*e^3*x^9 + 6
300*b*c^10*d^2*e*x^6*arctan(c*x) + 6300*a*c^10*d^2*e*x^6 - 675*b*c^9*d*e^2
*x^7 + 3150*b*c^10*d^3*x^4*arctan(c*x) + 3150*a*c^10*d^3*x^4 - 1260*b*c^9*
d^2*e*x^5 + 180*b*c^7*e^3*x^7 - 1050*b*c^9*d^3*x^3 + 945*b*c^7*d*e^2*x^5 +
 2100*b*c^7*d^2*e*x^3 - 252*b*c^5*e^3*x^5 + 3150*pi*b*c^6*d^3*sgn(c)*sgn(x
) + 3150*b*c^7*d^3*x - 1575*b*c^5*d*e^2*x^3 - 3150*b*c^6*d^3*arctan(c*x) -
 6300*pi*b*c^4*d^2*e*sgn(c)*sgn(x) - 6300*b*c^5*d^2*e*x + 420*b*c^3*e^3*x^
3 + 6300*b*c^4*d^2*e*arctan(c*x) + 4725*pi*b*c^2*d*e^2*sgn(c)*sgn(x) + 472
5*b*c^3*d*e^2*x - 4725*b*c^2*d*e^2*arctan(c*x) - 1260*pi*b*e^3*sgn(c)*sgn(
x) - 1260*b*c*e^3*x + 1260*b*e^3*arctan(c*x))/c^10
```

## Mupad [B] (verification not implemented)

Time = 0.95 (sec) , antiderivative size = 599, normalized size of antiderivative = 2.50

$$\int x^3(d+ex^2)^3(a+b\arctan(cx))\,dx$$
$$= x^3\left(\frac{\frac{\frac{b\,e^3}{10\,c^3}-\frac{3\,b\,d\,e^2}{8\,c}}{c^2}+\frac{b\,d^2\,e}{2\,c}}{3\,c^2}-\frac{b\,d^3}{12\,c}\right)-x^8\left(\frac{a\,e^3}{8\,c^2}-\frac{a\,e^2\,(3\,d\,c^2+e)}{8\,c^2}\right)$$
$$+x^6\left(\frac{\frac{a\,e^3}{c^2}-\frac{a\,e^2\,(3\,d\,c^2+e)}{c^2}}{6\,c^2}+\frac{a\,d\,e\,(d\,c^2+e)}{2\,c^2}\right)+x^7\left(\frac{b\,e^3}{70\,c^3}-\frac{3\,b\,d\,e^2}{56\,c}\right)$$
$$+\mathrm{atan}(c\,x)\left(\frac{b\,d^3\,x^4}{4}+\frac{b\,d^2\,e\,x^6}{2}+\frac{3\,b\,d\,e^2\,x^8}{8}+\frac{b\,e^3\,x^{10}}{10}\right)-x^5\left(\frac{\frac{b\,e^3}{10\,c^3}-\frac{3\,b\,d\,e^2}{8\,c}}{5\,c^2}+\frac{b\,d^2\,e}{10\,c}\right)$$
$$+x^2\left(\frac{\frac{\frac{\frac{a\,e^3}{c^2}-\frac{a\,e^2\,(3\,d\,c^2+e)}{c^2}}{c^2}+\frac{3\,a\,d\,e\,(d\,c^2+e)}{c^2}}{c^2}-\frac{a\,d^2\,(d\,c^2+3\,e)}{c^2}}{2\,c^2}+\frac{a\,d^3}{2\,c^2}\right)$$
$$-x^4\left(\frac{\frac{\frac{a\,e^3}{c^2}-\frac{a\,e^2\,(3\,d\,c^2+e)}{c^2}}{c^2}+\frac{3\,a\,d\,e\,(d\,c^2+e)}{c^2}}{4\,c^2}-\frac{a\,d^2\,(d\,c^2+3\,e)}{4\,c^2}\right)$$
$$+\frac{a\,e^3\,x^{10}}{10}-\frac{x\left(\frac{\frac{\frac{b\,e^3}{10\,c^3}-\frac{3\,b\,d\,e^2}{8\,c}}{c^2}+\frac{b\,d^2\,e}{2\,c}}{c^2}-\frac{b\,d^3}{4\,c}\right)}{c^2}-\frac{b\,e^3\,x^9}{90\,c}$$
$$+\frac{b\,\mathrm{atan}\left(\frac{b\,c\,x\,(-10\,c^6\,d^3+20\,c^4\,d^2\,e-15\,c^2\,d\,e^2+4\,e^3)}{-10\,b\,c^6\,d^3+20\,b\,c^4\,d^2\,e-15\,b\,c^2\,d\,e^2+4\,b\,e^3}\right)(-10\,c^6\,d^3+20\,c^4\,d^2\,e-15\,c^2\,d\,e^2+4\,e^3)}{40\,c^{10}}$$

input
```
int(x^3*(a + b*atan(c*x))*(d + e*x^2)^3,x)
```

output
```
x^3*((((b*e^3)/(10*c^3) - (3*b*d*e^2)/(8*c))/c^2 + (b*d^2*e)/(2*c))/(3*c^2
) - (b*d^3)/(12*c)) - x^8*((a*e^3)/(8*c^2) - (a*e^2*(e + 3*c^2*d))/(8*c^2)
) + x^6*(((a*e^3)/c^2 - (a*e^2*(e + 3*c^2*d))/c^2)/(6*c^2) + (a*d*e*(e + c
^2*d))/(2*c^2)) + x^7*((b*e^3)/(70*c^3) - (3*b*d*e^2)/(56*c)) + atan(c*x)*
((b*d^3*x^4)/4 + (b*e^3*x^10)/10 + (b*d^2*e*x^6)/2 + (3*b*d*e^2*x^8)/8) -
x^5*(((b*e^3)/(10*c^3) - (3*b*d*e^2)/(8*c))/(5*c^2) + (b*d^2*e)/(10*c)) +
x^2*(((((a*e^3)/c^2 - (a*e^2*(e + 3*c^2*d))/c^2)/c^2 + (3*a*d*e*(e + c^2*d
))/c^2)/c^2 - (a*d^2*(3*e + c^2*d))/c^2)/(2*c^2) + (a*d^3)/(2*c^2)) - x^4*
((((a*e^3)/c^2 - (a*e^2*(e + 3*c^2*d))/c^2)/c^2 + (3*a*d*e*(e + c^2*d))/c^
2)/(4*c^2) - (a*d^2*(3*e + c^2*d))/(4*c^2)) + (a*e^3*x^10)/10 - (x*((((b*e
^3)/(10*c^3) - (3*b*d*e^2)/(8*c))/c^2 + (b*d^2*e)/(2*c))/c^2 - (b*d^3)/(4*
c)))/c^2 - (b*e^3*x^9)/(90*c) + (b*atan((b*c*x*(4*e^3 - 10*c^6*d^3 - 15*c^
2*d*e^2 + 20*c^4*d^2*e))/(4*b*e^3 - 10*b*c^6*d^3 - 15*b*c^2*d*e^2 + 20*b*c
^4*d^2*e))*(4*e^3 - 10*c^6*d^3 - 15*c^2*d*e^2 + 20*c^4*d^2*e))/(40*c^10)
```

## Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.41

$$\int x^3 (d+ex^2)^3 (a+b\arctan(cx))\,dx$$
$$= \frac{1260atan(cx)\,b\,e^3 - 3150atan(cx)\,b\,c^6d^3 + 3150a\,c^{10}d^3x^4 + 1260a\,c^{10}e^3x^{10} - 1050b\,c^9d^3x^3 - 140b\,c^9e^3x^9}{}$$

input
```
int(x^3*(e*x^2+d)^3*(a+b*atan(c*x)),x)
```

output
```
(3150*atan(c*x)*b*c**10*d**3*x**4 + 6300*atan(c*x)*b*c**10*d**2*e*x**6 + 4
725*atan(c*x)*b*c**10*d*e**2*x**8 + 1260*atan(c*x)*b*c**10*e**3*x**10 - 31
50*atan(c*x)*b*c**6*d**3 + 6300*atan(c*x)*b*c**4*d**2*e - 4725*atan(c*x)*b
*c**2*d*e**2 + 1260*atan(c*x)*b*e**3 + 3150*a*c**10*d**3*x**4 + 6300*a*c**
10*d**2*e*x**6 + 4725*a*c**10*d*e**2*x**8 + 1260*a*c**10*e**3*x**10 - 1050
*b*c**9*d**3*x**3 - 1260*b*c**9*d**2*e*x**5 - 675*b*c**9*d*e**2*x**7 - 140
*b*c**9*e**3*x**9 + 3150*b*c**7*d**3*x + 2100*b*c**7*d**2*e*x**3 + 945*b*c
**7*d*e**2*x**5 + 180*b*c**7*e**3*x**7 - 6300*b*c**5*d**2*e*x - 1575*b*c**
5*d*e**2*x**3 - 252*b*c**5*e**3*x**5 + 4725*b*c**3*d*e**2*x + 420*b*c**3*e
**3*x**3 - 1260*b*c*e**3*x)/(12600*c**10)
```

# 3.1137 $\int x^2(d+ex^2)^3(a+b\arctan(cx))\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 239

$$\begin{aligned}\int x^2(d+ex^2)^3(a+b\arctan(cx))\,dx &= -\frac{b(105c^6d^3-189c^4d^2e+135c^2de^2-35e^3)\,x^2}{630c^7}-\frac{be(189c^4d^2-135c^2de+35e^2)\,x^4}{1260c^5}\\&\quad-\frac{b(27c^2d-7e)\,e^2x^6}{378c^3}-\frac{be^3x^8}{72c}+\frac{1}{3}d^3x^3(a+b\arctan(cx))\\&\quad+\frac{3}{5}d^2ex^5(a+b\arctan(cx))+\frac{3}{7}de^2x^7(a+b\arctan(cx))+\frac{1}{9}e^3x^9(a+b\arctan(cx))\\&\quad+\frac{b(105c^6d^3-189c^4d^2e+135c^2de^2-35e^3)\log(1+c^2x^2)}{630c^9}\end{aligned}$$

output

```
-1/630*b*(105*c^6*d^3-189*c^4*d^2*e+135*c^2*d*e^2-35*e^3)*x^2/c^7-1/1260*b
*e*(189*c^4*d^2-135*c^2*d*e+35*e^2)*x^4/c^5-1/378*b*(27*c^2*d-7*e)*e^2*x^6
/c^3-1/72*b*e^3*x^8/c+1/3*d^3*x^3*(a+b*arctan(c*x))+3/5*d^2*e*x^5*(a+b*arc
tan(c*x))+3/7*d*e^2*x^7*(a+b*arctan(c*x))+1/9*e^3*x^9*(a+b*arctan(c*x))+1/
630*b*(105*c^6*d^3-189*c^4*d^2*e+135*c^2*d*e^2-35*e^3)*ln(c^2*x^2+1)/c^9
```

## Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.05

$$\int x^2 (d+ex^2)^3 (a+b\arctan(cx))\,dx = \frac{1}{3}d^3x^3(a+b\arctan(cx)) + \frac{3}{5}d^2ex^5(a+b\arctan(cx)) + \frac{3}{7}de^2x^7(a+b\arctan(cx)) + \frac{1}{9}e^3x^9(a+b\arctan(cx)) + \frac{1}{216}be^3\left(\frac{12x^2}{c^7} - \frac{6x^4}{c^5} + \frac{4x^6}{c^3} - \frac{3x^8}{c} - \frac{12\log(1+c^2x^2)}{c^9}\right) - \frac{1}{28}bde^2\left(\frac{6x^2}{c^5} - \frac{3x^4}{c^3} + \frac{2x^6}{c} - \frac{6\log(1+c^2x^2)}{c^7}\right) + \frac{3}{20}bd^2e\left(\frac{2x^2}{c^3} - \frac{x^4}{c} - \frac{2\log(1+c^2x^2)}{c^5}\right) - \frac{1}{6}bd^3\left(\frac{x^2}{c} - \frac{\log(1+c^2x^2)}{c^3}\right)$$

input

```
Integrate[x^2*(d + e*x^2)^3*(a + b*ArcTan[c*x]),x]
```

output

```
(d^3*x^3*(a + b*ArcTan[c*x]))/3 + (3*d^2*e*x^5*(a + b*ArcTan[c*x]))/5 + (3
*d*e^2*x^7*(a + b*ArcTan[c*x]))/7 + (e^3*x^9*(a + b*ArcTan[c*x]))/9 + (b*e
^3*((12*x^2)/c^7 - (6*x^4)/c^5 + (4*x^6)/c^3 - (3*x^8)/c - (12*Log[1 + c^2
*x^2])/c^9))/216 - (b*d*e^2*((6*x^2)/c^5 - (3*x^4)/c^3 + (2*x^6)/c - (6*Lo
g[1 + c^2*x^2])/c^7))/28 + (3*b*d^2*e*((2*x^2)/c^3 - x^4/c - (2*Log[1 + c^
2*x^2])/c^5))/20 - (b*d^3*(x^2/c - Log[1 + c^2*x^2]/c^3))/6
```

## Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5511, 27, 2331, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(d+ex^2\right)^3 (a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5511}$$

$$-bc\int \frac{x^3\left(35e^3x^6+135de^2x^4+189d^2ex^2+105d^3\right)}{315\left(c^2x^2+1\right)}dx+\frac{1}{3}d^3x^3(a+b\arctan(cx))+\frac{3}{5}d^2ex^5(a+b\arctan(cx))+\frac{3}{7}de^2x^7(a+b\arctan(cx))+\frac{1}{9}e^3x^9(a+b\arctan(cx))$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{315}bc\int \frac{x^3\left(35e^3x^6+135de^2x^4+189d^2ex^2+105d^3\right)}{c^2x^2+1}dx+\frac{1}{3}d^3x^3(a+b\arctan(cx))+\frac{3}{5}d^2ex^5(a+b\arctan(cx))+\frac{3}{7}de^2x^7(a+b\arctan(cx))+\frac{1}{9}e^3x^9(a+b\arctan(cx))$$

$$\Big\downarrow \text{2331}$$

$$-\frac{1}{630}bc\int \frac{x^2\left(35e^3x^6+135de^2x^4+189d^2ex^2+105d^3\right)}{c^2x^2+1}dx^2+\frac{1}{3}d^3x^3(a+b\arctan(cx))+\frac{3}{5}d^2ex^5(a+b\arctan(cx))+\frac{3}{7}de^2x^7(a+b\arctan(cx))+\frac{1}{9}e^3x^9(a+b\arctan(cx))$$

$$\Big\downarrow \text{2123}$$

$$-\frac{1}{630}bc\int \left(\frac{35e^3x^6}{c^2}+\frac{5\left(27c^2d-7e\right)e^2x^4}{c^4}+\frac{e\left(189d^2c^4-135dec^2+35e^2\right)x^2}{c^6}+\frac{105d^3c^6-189d^2ec^4+135de^2c^2}{c^8}\right.$$
$$\frac{1}{3}d^3x^3(a+b\arctan(cx))+\frac{3}{5}d^2ex^5(a+b\arctan(cx))+\frac{3}{7}de^2x^7(a+b\arctan(cx))+\frac{1}{9}e^3x^9(a+b\arctan(cx))$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}d^3x^3(a+b\arctan(cx))+\frac{3}{5}d^2ex^5(a+b\arctan(cx))+\frac{3}{7}de^2x^7(a+b\arctan(cx))+\frac{1}{9}e^3x^9(a+b\arctan(cx))-\frac{1}{630}bc\left(\frac{35e^3x^8}{4c^2}+\frac{5e^2x^6(27c^2d-7e)}{3c^4}+\frac{ex^4(189c^4d^2-135c^2de+35e^2)}{2c^6}-\frac{(105c^6d^3-189c^4d^2e+135c^2de^2-3}{c^{10}}\right.$$

input
```
Int[x^2*(d + e*x^2)^3*(a + b*ArcTan[c*x]),x]
```

output
```
(d^3*x^3*(a + b*ArcTan[c*x]))/3 + (3*d^2*e*x^5*(a + b*ArcTan[c*x]))/5 + (3
*d*e^2*x^7*(a + b*ArcTan[c*x]))/7 + (e^3*x^9*(a + b*ArcTan[c*x]))/9 - (b*c
*(((105*c^6*d^3 - 189*c^4*d^2*e + 135*c^2*d*e^2 - 35*e^3)*x^2)/c^8 + (e*(1
89*c^4*d^2 - 135*c^2*d*e + 35*e^2)*x^4)/(2*c^6) + (5*(27*c^2*d - 7*e)*e^2*
x^6)/(3*c^4) + (35*e^3*x^8)/(4*c^2) - ((105*c^6*d^3 - 189*c^4*d^2*e + 135*
c^2*d*e^2 - 35*e^3)*Log[1 + c^2*x^2])/c^10))/630
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

## Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.14

| method | result |
|---|---|
| parts | $a\left(\frac{1}{9}e^3x^9+\frac{3}{7}e^2d\,x^7+\frac{3}{5}d^2e\,x^5+\frac{1}{3}d^3x^3\right)+\frac{b\left(\frac{\arctan(cx)c^3e^3x^9}{9}+\frac{3\arctan(cx)c^3e^2d\,x^7}{7}+\frac{3\arctan(cx)c^3d^2e\,x}{5}\right.}{}$ |
| derivativedivides | $\frac{a\left(\frac{1}{3}d^3c^9x^3+\frac{3}{5}d^2c^9e\,x^5+\frac{3}{7}d\,c^9e^2x^7+\frac{1}{9}e^3c^9x^9\right)}{c^6}+\frac{b\left(\frac{\arctan(cx)d^3c^9x^3}{3}+\frac{3\arctan(cx)d^2c^9e\,x^5}{5}+\frac{3\arctan(cx)d\,c^9e^2x^7}{7}+\arctan(c\right.}{}$ |
| default | $\frac{a\left(\frac{1}{3}d^3c^9x^3+\frac{3}{5}d^2c^9e\,x^5+\frac{3}{7}d\,c^9e^2x^7+\frac{1}{9}e^3c^9x^9\right)}{c^6}+\frac{b\left(\frac{\arctan(cx)d^3c^9x^3}{3}+\frac{3\arctan(cx)d^2c^9e\,x^5}{5}+\frac{3\arctan(cx)d\,c^9e^2x^7}{7}+\arctan(c\right.}{}$ |
| parallelrisch | $840x^9\arctan(cx)b\,c^9e^3+840a\,c^9e^3x^9+3240x^7\arctan(cx)b\,c^9d\,e^2-105b\,c^8e^3x^8+3240a\,c^9d\,e^2x^7+4536x^5\arctan(cx)b\,c$ |
| risch | $-\frac{ib(35e^3x^9+135e^2d\,x^7+189d^2e\,x^5+105d^3x^3)\ln(icx+1)}{630}+\frac{3ib\,d^2e\,x^5\ln(-icx+1)}{10}+\frac{3ibd\,e^2x^7\ln(-icx+1)}{14}+x^9$ |

input
```
int(x^2*(e*x^2+d)^3*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/9*e^3*x^9+3/7*e^2*d*x^7+3/5*d^2*e*x^5+1/3*d^3*x^3)+b/c^3*(1/9*arctan(
c*x)*c^3*e^3*x^9+3/7*arctan(c*x)*c^3*e^2*d*x^7+3/5*arctan(c*x)*c^3*d^2*e*x
^5+1/3*arctan(c*x)*d^3*c^3*x^3-1/315/c^6*(105/2*c^8*d^3*x^2+189/4*c^8*d^2*
e*x^4+45/2*c^8*d*e^2*x^6-189/2*c^6*d^2*e*x^2+35/8*c^8*e^3*x^8-135/4*c^6*d*
e^2*x^4-35/6*c^6*e^3*x^6+135/2*c^4*d*e^2*x^2+35/4*c^4*e^3*x^4-35/2*c^2*e^3
*x^2+1/2*(-105*c^6*d^3+189*c^4*d^2*e-135*c^2*d*e^2+35*e^3)*ln(c^2*x^2+1)))
```

## Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.16

$$\int x^2 (d+ex^2)^3 (a+b\arctan(cx))\,dx$$
$$= \frac{840\,ac^9e^3x^9 + 3240\,ac^9de^2x^7 - 105\,bc^8e^3x^8 + 4536\,ac^9d^2ex^5 + 2520\,ac^9d^3x^3 - 20\,(27\,bc^8de^2 - 7\,bc^6e^3)x^6}{}$$

input
```
integrate(x^2*(e*x^2+d)^3*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
1/7560*(840*a*c^9*e^3*x^9 + 3240*a*c^9*d*e^2*x^7 - 105*b*c^8*e^3*x^8 + 453
6*a*c^9*d^2*e*x^5 + 2520*a*c^9*d^3*x^3 - 20*(27*b*c^8*d*e^2 - 7*b*c^6*e^3)
*x^6 - 6*(189*b*c^8*d^2*e - 135*b*c^6*d*e^2 + 35*b*c^4*e^3)*x^4 - 12*(105*
b*c^8*d^3 - 189*b*c^6*d^2*e + 135*b*c^4*d*e^2 - 35*b*c^2*e^3)*x^2 + 24*(35
*b*c^9*e^3*x^9 + 135*b*c^9*d*e^2*x^7 + 189*b*c^9*d^2*e*x^5 + 105*b*c^9*d^3
*x^3)*arctan(c*x) + 12*(105*b*c^6*d^3 - 189*b*c^4*d^2*e + 135*b*c^2*d*e^2
- 35*b*e^3)*log(c^2*x^2 + 1))/c^9
```

## Sympy [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.63

$$\int x^2 (d+ex^2)^3 (a+b\arctan(cx))\,dx$$
$$= \begin{cases} \frac{ad^3x^3}{3} + \frac{3ad^2ex^5}{5} + \frac{3ade^2x^7}{7} + \frac{ae^3x^9}{9} + \frac{bd^3x^3\operatorname{atan}(cx)}{3} + \frac{3bd^2ex^5\operatorname{atan}(cx)}{5} + \frac{3bde^2x^7\operatorname{atan}(cx)}{7} + \frac{be^3x^9\operatorname{atan}(cx)}{9} - \frac{bd^3x^2}{6c} \\ a\left(\frac{d^3x^3}{3} + \frac{3d^2ex^5}{5} + \frac{3de^2x^7}{7} + \frac{e^3x^9}{9}\right) \end{cases}$$

input
```
integrate(x**2*(e*x**2+d)**3*(a+b*atan(c*x)),x)
```

output
```
Piecewise((a*d**3*x**3/3 + 3*a*d**2*e*x**5/5 + 3*a*d*e**2*x**7/7 + a*e**3*
x**9/9 + b*d**3*x**3*atan(c*x)/3 + 3*b*d**2*e*x**5*atan(c*x)/5 + 3*b*d*e**
2*x**7*atan(c*x)/7 + b*e**3*x**9*atan(c*x)/9 - b*d**3*x**2/(6*c) - 3*b*d**
2*e*x**4/(20*c) - b*d*e**2*x**6/(14*c) - b*e**3*x**8/(72*c) + b*d**3*log(x
**2 + c**(-2))/(6*c**3) + 3*b*d**2*e*x**2/(10*c**3) + 3*b*d*e**2*x**4/(28*
c**3) + b*e**3*x**6/(54*c**3) - 3*b*d**2*e*log(x**2 + c**(-2))/(10*c**5) -
 3*b*d*e**2*x**2/(14*c**5) - b*e**3*x**4/(36*c**5) + 3*b*d*e**2*log(x**2 +
 c**(-2))/(14*c**7) + b*e**3*x**2/(18*c**7) - b*e**3*log(x**2 + c**(-2))/(
18*c**9), Ne(c, 0)), (a*(d**3*x**3/3 + 3*d**2*e*x**5/5 + 3*d*e**2*x**7/7 +
 e**3*x**9/9), True))
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.11

$$\int x^2(d+ex^2)^3(a+b\arctan(cx))\,dx = \frac{1}{9}ae^3x^9 + \frac{3}{7}ade^2x^7 + \frac{3}{5}ad^2ex^5 + \frac{1}{3}ad^3x^3 + \frac{1}{6}\left(2x^3\arctan(cx) - c\left(\frac{x^2}{c^2} - \frac{\log(c^2x^2+1)}{c^4}\right)\right)bd^3 + \frac{3}{20}\left(4x^5\arctan(cx) - c\left(\frac{c^2x^4-2x^2}{c^4} + \frac{2\log(c^2x^2+1)}{c^6}\right)\right)bd^2e + \frac{1}{28}\left(12x^7\arctan(cx) - c\left(\frac{2c^4x^6-3c^2x^4+6x^2}{c^6} - \frac{6\log(c^2x^2+1)}{c^8}\right)\right)bde^2 + \frac{1}{216}\left(24x^9\arctan(cx) - c\left(\frac{3c^6x^8-4c^4x^6+6c^2x^4-12x^2}{c^8} + \frac{12\log(c^2x^2+1)}{c^{10}}\right)\right)be^3$$

input
```
integrate(x^2*(e*x^2+d)^3*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
1/9*a*e^3*x^9 + 3/7*a*d*e^2*x^7 + 3/5*a*d^2*e*x^5 + 1/3*a*d^3*x^3 + 1/6*(2
*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*b*d^3 + 3/20*(4*x^5
*arctan(c*x) - c*((c^2*x^4 - 2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*b*d^2*e
 + 1/28*(12*x^7*arctan(c*x) - c*((2*c^4*x^6 - 3*c^2*x^4 + 6*x^2)/c^6 - 6*l
og(c^2*x^2 + 1)/c^8))*b*d*e^2 + 1/216*(24*x^9*arctan(c*x) - c*((3*c^6*x^8
- 4*c^4*x^6 + 6*c^2*x^4 - 12*x^2)/c^8 + 12*log(c^2*x^2 + 1)/c^10))*b*e^3
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.35

$$\int x^2(d+ex^2)^3(a+b\arctan(cx))\,dx$$
$$= \frac{840\,bc^9e^3x^9\arctan(cx)+840\,ac^9e^3x^9+3240\,bc^9de^2x^7\arctan(cx)+3240\,ac^9de^2x^7-105\,bc^8e^3x^8+4536}{}$$

input
```
integrate(x^2*(e*x^2+d)^3*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
1/7560*(840*b*c^9*e^3*x^9*arctan(c*x) + 840*a*c^9*e^3*x^9 + 3240*b*c^9*d*e
^2*x^7*arctan(c*x) + 3240*a*c^9*d*e^2*x^7 - 105*b*c^8*e^3*x^8 + 4536*b*c^9
*d^2*e*x^5*arctan(c*x) + 4536*a*c^9*d^2*e*x^5 - 540*b*c^8*d*e^2*x^6 + 2520
*b*c^9*d^3*x^3*arctan(c*x) + 2520*a*c^9*d^3*x^3 - 1134*b*c^8*d^2*e*x^4 + 1
40*b*c^6*e^3*x^6 - 1260*b*c^8*d^3*x^2 + 810*b*c^6*d*e^2*x^4 + 2268*b*c^6*d
^2*e*x^2 - 210*b*c^4*e^3*x^4 + 1260*b*c^6*d^3*log(c^2*x^2 + 1) - 1620*b*c^
4*d*e^2*x^2 - 2268*b*c^4*d^2*e*log(c^2*x^2 + 1) + 420*b*c^2*e^3*x^2 + 1620
*b*c^2*d*e^2*log(c^2*x^2 + 1) - 420*b*e^3*log(c^2*x^2 + 1))/c^9
```

## Mupad [B] (verification not implemented)

Time = 1.31 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.24

$$\begin{aligned}\int x^2(d+ex^2)^3(a+b\arctan(cx))\,dx &= \frac{a\,d^3\,x^3}{3}+\frac{a\,e^3\,x^9}{9}+\frac{b\,d^3\,\ln\left(c^2\,x^2+1\right)}{6\,c^3}\\
&-\frac{b\,e^3\,\ln\left(c^2\,x^2+1\right)}{18\,c^9}-\frac{b\,d^3\,x^2}{6\,c}-\frac{b\,e^3\,x^8}{72\,c}\\
&+\frac{b\,e^3\,x^6}{54\,c^3}-\frac{b\,e^3\,x^4}{36\,c^5}+\frac{b\,e^3\,x^2}{18\,c^7}+\frac{3\,a\,d^2\,e\,x^5}{5}\\
&+\frac{3\,a\,d\,e^2\,x^7}{7}+\frac{b\,d^3\,x^3\,\mathrm{atan}(c\,x)}{3}\\
&+\frac{b\,e^3\,x^9\,\mathrm{atan}(c\,x)}{9}+\frac{3\,b\,d^2\,e\,x^5\,\mathrm{atan}(c\,x)}{5}\\
&+\frac{3\,b\,d\,e^2\,x^7\,\mathrm{atan}(c\,x)}{7}-\frac{3\,b\,d^2\,e\,\ln\left(c^2\,x^2+1\right)}{10\,c^5}\\
&+\frac{3\,b\,d\,e^2\,\ln\left(c^2\,x^2+1\right)}{14\,c^7}-\frac{3\,b\,d^2\,e\,x^4}{20\,c}\\
&+\frac{3\,b\,d^2\,e\,x^2}{10\,c^3}-\frac{b\,d\,e^2\,x^6}{14\,c}+\frac{3\,b\,d\,e^2\,x^4}{28\,c^3}-\frac{3\,b\,d\,e^2\,x^2}{14\,c^5}\end{aligned}$$

input
```
int(x^2*(a + b*atan(c*x))*(d + e*x^2)^3,x)
```

output
```
(a*d^3*x^3)/3 + (a*e^3*x^9)/9 + (b*d^3*log(c^2*x^2 + 1))/(6*c^3) - (b*e^3*
log(c^2*x^2 + 1))/(18*c^9) - (b*d^3*x^2)/(6*c) - (b*e^3*x^8)/(72*c) + (b*e
^3*x^6)/(54*c^3) - (b*e^3*x^4)/(36*c^5) + (b*e^3*x^2)/(18*c^7) + (3*a*d^2*
e*x^5)/5 + (3*a*d*e^2*x^7)/7 + (b*d^3*x^3*atan(c*x))/3 + (b*e^3*x^9*atan(c
*x))/9 + (3*b*d^2*e*x^5*atan(c*x))/5 + (3*b*d*e^2*x^7*atan(c*x))/7 - (3*b*
d^2*e*log(c^2*x^2 + 1))/(10*c^5) + (3*b*d*e^2*log(c^2*x^2 + 1))/(14*c^7) -
 (3*b*d^2*e*x^4)/(20*c) + (3*b*d^2*e*x^2)/(10*c^3) - (b*d*e^2*x^6)/(14*c)
+ (3*b*d*e^2*x^4)/(28*c^3) - (3*b*d*e^2*x^2)/(14*c^5)
```

## Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.35

$$\int x^2(d+ex^2)^3(a+b\arctan(cx))\,dx = \frac{2520atan(cx)\,b\,c^9d^3x^3+4536atan(cx)\,b\,c^9d^2e\,x^5+3240atan(cx)\,b\,c^9d\,e^2x^7+840atan(cx)\,b\,c^9e^3x^9+126}{}$$

input
```
int(x^2*(e*x^2+d)^3*(a+b*atan(c*x)),x)
```

output
```
(2520*atan(c*x)*b*c**9*d**3*x**3 + 4536*atan(c*x)*b*c**9*d**2*e*x**5 + 324
0*atan(c*x)*b*c**9*d*e**2*x**7 + 840*atan(c*x)*b*c**9*e**3*x**9 + 1260*log
(c**2*x**2 + 1)*b*c**6*d**3 - 2268*log(c**2*x**2 + 1)*b*c**4*d**2*e + 1620
*log(c**2*x**2 + 1)*b*c**2*d*e**2 - 420*log(c**2*x**2 + 1)*b*e**3 + 2520*a
*c**9*d**3*x**3 + 4536*a*c**9*d**2*e*x**5 + 3240*a*c**9*d*e**2*x**7 + 840*
a*c**9*e**3*x**9 - 1260*b*c**8*d**3*x**2 - 1134*b*c**8*d**2*e*x**4 - 540*b
*c**8*d*e**2*x**6 - 105*b*c**8*e**3*x**8 + 2268*b*c**6*d**2*e*x**2 + 810*b
*c**6*d*e**2*x**4 + 140*b*c**6*e**3*x**6 - 1620*b*c**4*d*e**2*x**2 - 210*b
*c**4*e**3*x**4 + 420*b*c**2*e**3*x**2)/(7560*c**9)
```

# 3.1138 $\int x(d+ex^2)^3 (a+b\arctan(cx))\,dx$



## Optimal result

Integrand size = 19, antiderivative size = 158

$$\int x\left(d+ex^2\right)^3 (a+b\arctan(cx))\,dx = -\frac{b(2c^2d-e)\left(2c^4d^2-2c^2de+e^2\right)x}{8c^7} - \frac{be(6c^4d^2-4c^2de+e^2)\,x^3}{24c^5} - \frac{b(4c^2d-e)\,e^2x^5}{40c^3} - \frac{be^3x^7}{56c} - \frac{b(c^2d-e)^4\arctan(cx)}{8c^8e} + \frac{\left(d+ex^2\right)^4(a+b\arctan(cx))}{8e}$$

output
```
-1/8*b*(2*c^2*d-e)*(2*c^4*d^2-2*c^2*d*e+e^2)*x/c^7-1/24*b*e*(6*c^4*d^2-4*c
^2*d*e+e^2)*x^3/c^5-1/40*b*(4*c^2*d-e)*e^2*x^5/c^3-1/56*b*e^3*x^7/c-1/8*b*
(c^2*d-e)^4*arctan(c*x)/c^8/e+1/8*(e*x^2+d)^4*(a+b*arctan(c*x))/e
```

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.37

$$\int x\left(d+ex^2\right)^3(a+b\arctan(cx))\,dx$$
$$=\frac{cx(105be^3-35bc^2e^2(12d+ex^2)+7bc^4e(90d^2+20dex^2+3e^2x^4)+105ac^7x(4d^3+6d^2ex^2+4de^2x^4+e}{}$$

input
```
Integrate[x*(d + e*x^2)^3*(a + b*ArcTan[c*x]),x]
```

output
```
(c*x*(105*b*e^3 - 35*b*c^2*e^2*(12*d + e*x^2) + 7*b*c^4*e*(90*d^2 + 20*d*e
*x^2 + 3*e^2*x^4) + 105*a*c^7*x*(4*d^3 + 6*d^2*e*x^2 + 4*d*e^2*x^4 + e^3*x
^6) - 3*b*c^6*(140*d^3 + 70*d^2*e*x^2 + 28*d*e^2*x^4 + 5*e^3*x^6)) + 105*b
*(4*c^6*d^3 - 6*c^4*d^2*e + 4*c^2*d*e^2 - e^3 + c^8*(4*d^3*x^2 + 6*d^2*e*x
^4 + 4*d*e^2*x^6 + e^3*x^8))*ArcTan[c*x])/(840*c^8)
```

## Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {5509, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(d+ex^2\right)^3(a+b\arctan(cx))\,dx$$
$$\Big\downarrow \text{5509}$$
$$\frac{\left(d+ex^2\right)^4(a+b\arctan(cx))}{8e}-\frac{bc\int\frac{\left(ex^2+d\right)^4}{c^2x^2+1}dx}{8e}$$
$$\Big\downarrow \text{300}$$
$$\frac{\left(d+ex^2\right)^4(a+b\arctan(cx))}{8e}-\frac{bc\int\left(\frac{e^4x^6}{c^2}+\frac{(4c^2d-e)e^3x^4}{c^4}+\frac{e^2(6d^2c^4-4dec^2+e^2)x^2}{c^6}+\frac{(2c^2d-e)e(2d^2c^4-2dec^2+e^2)}{c^8}+\frac{d^4c^8-4d^3ec^6+6d^2e^2c^4-4de^3c^2+e^4}{c^8(c^2x^2+1)}\right)dx}{8e}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(d+ex^2)^4(a+b\arctan(cx))}{8e} - \frac{bc\left(\frac{\arctan(cx)(c^2d-e)^4}{c^9}+\frac{e^4x^7}{7c^2}+\frac{e^3x^5(4c^2d-e)}{5c^4}+\frac{ex(2c^2d-e)(2c^4d^2-2c^2de+e^2)}{c^8}+\frac{e^2x^3(6c^4d^2-4c^2de+e^2)}{3c^6}\right)}{8e}$$

input
```
Int[x*(d + e*x^2)^3*(a + b*ArcTan[c*x]),x]
```

output
```
((d + e*x^2)^4*(a + b*ArcTan[c*x]))/(8*e) - (b*c*(((2*c^2*d - e)*e*(2*c^4*
d^2 - 2*c^2*d*e + e^2)*x)/c^8 + (e^2*(6*c^4*d^2 - 4*c^2*d*e + e^2)*x^3)/(3
*c^6) + ((4*c^2*d - e)*e^3*x^5)/(5*c^4) + (e^4*x^7)/(7*c^2) + ((c^2*d - e)
^4*ArcTan[c*x])/c^9))/(8*e)
```

### Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5509
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*e*(q + 1))), x
] - Simp[b*(c/(2*e*(q + 1)))   Int[(d + e*x^2)^(q + 1)/(1 + c^2*x^2), x], x
] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]
```

## Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.67

| method | result |
|---|---|
| parts | $\frac{a(e x^2+d)^4}{8e} + \frac{b\left(\frac{\arctan(cx)c^2e^3x^8}{8} + \frac{\arctan(cx)c^2e^2x^6 d}{2} + \frac{3\arctan(cx)c^2 e x^4 d^2}{4} + \frac{\arctan(cx)c^2x^2d^3}{2} + \frac{\arctan(cx)c^2d^4}{8e} - \frac{4c^7 \cdots}{}\right.}{}$ |
| derivativedivides | $\frac{\frac{a\left(c^2 e x^2+c^2 d\right)^4}{8c^6 e} + \frac{b\left(\frac{\arctan(cx)c^8d^4}{8e} + \frac{\arctan(cx)c^8d^3x^2}{2} + \frac{3\arctan(cx)e c^8d^2x^4}{4} + \frac{\arctan(cx)e^2c^8 d x^6}{2} + \frac{\arctan(cx)e^3c^8x^8}{8} - \frac{4c^7 \cdots}{}\right.}{}}{}$ |
| default | $\frac{\frac{a\left(c^2 e x^2+c^2 d\right)^4}{8c^6 e} + \frac{b\left(\frac{\arctan(cx)c^8d^4}{8e} + \frac{\arctan(cx)c^8d^3x^2}{2} + \frac{3\arctan(cx)e c^8d^2x^4}{4} + \frac{\arctan(cx)e^2c^8 d x^6}{2} + \frac{\arctan(cx)e^3c^8x^8}{8} - \frac{4c^7 \cdots}{}\right.}{}}{}$ |
| parallelrisch | $\frac{105x^8\arctan(cx)b c^8e^3+105x^8a c^8e^3+420x^6\arctan(cx)b c^8 d e^2-15b c^7e^3x^7+420x^6 a c^8 d e^2+630x^4\arctan(cx)b c^8d^2e \cdots}{}$ |
| orering | $\frac{(105c^8e^4x^{10}+564c^8 d e^3x^8+1302c^8d^2e^2x^6-21c^6e^4x^8+2100c^8d^3 e x^4-199c^6 d e^3x^6+420c^8d^4x^2-1498c^6d^2e^2x^4+49c^4 \cdots}{}$ |
| risch | Expression too large to display |

input
```
int(x*(e*x^2+d)^3*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/8*a*(e*x^2+d)^4/e+b/c^2*(1/8*arctan(c*x)*c^2*e^3*x^8+1/2*arctan(c*x)*c^2
*e^2*x^6*d+3/4*arctan(c*x)*c^2*e*x^4*d^2+1/2*arctan(c*x)*c^2*x^2*d^3+1/8*a
rctan(c*x)*c^2/e*d^4-1/8/c^6/e*(4*c^7*d^3*e*x+2*c^7*d^2*e^2*x^3+4/5*c^7*d*
e^3*x^5+1/7*e^4*c^7*x^7-6*c^5*x*d^2*e^2-4/3*c^5*d*e^3*x^3-1/5*e^4*c^5*x^5+
4*c^3*x*d*e^3+1/3*e^4*c^3*x^3-c*x*e^4+(c^8*d^4-4*c^6*d^3*e+6*c^4*d^2*e^2-4
*c^2*d*e^3+e^4)*arctan(c*x)))
```

## Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.63

$$\int x(d+ex^2)^3(a+b\arctan(cx))\,dx$$
$$= \frac{105\,ac^8e^3x^8+420\,ac^8de^2x^6-15\,bc^7e^3x^7+630\,ac^8d^2ex^4+420\,ac^8d^3x^2-21\,(4\,bc^7de^2-bc^5e^3)x^5-35\,(6 \cdots}{}$$

input
```
integrate(x*(e*x^2+d)^3*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
1/840*(105*a*c^8*e^3*x^8 + 420*a*c^8*d*e^2*x^6 - 15*b*c^7*e^3*x^7 + 630*a*
c^8*d^2*e*x^4 + 420*a*c^8*d^3*x^2 - 21*(4*b*c^7*d*e^2 - b*c^5*e^3)*x^5 - 3
5*(6*b*c^7*d^2*e - 4*b*c^5*d*e^2 + b*c^3*e^3)*x^3 - 105*(4*b*c^7*d^3 - 6*b
*c^5*d^2*e + 4*b*c^3*d*e^2 - b*c*e^3)*x + 105*(b*c^8*e^3*x^8 + 4*b*c^8*d*e
^2*x^6 + 6*b*c^8*d^2*e*x^4 + 4*b*c^8*d^3*x^2 + 4*b*c^6*d^3 - 6*b*c^4*d^2*e
 + 4*b*c^2*d*e^2 - b*e^3)*arctan(c*x))/c^8
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 350 vs. $2(144) = 288$.

Time = 0.53 (sec) , antiderivative size = 350, normalized size of antiderivative = 2.22

$$\int x(d+ex^2)^3(a+b\arctan(cx))\,dx$$
$$= \begin{cases} \frac{ad^3x^2}{2} + \frac{3ad^2ex^4}{4} + \frac{ade^2x^6}{2} + \frac{ae^3x^8}{8} + \frac{bd^3x^2\operatorname{atan}(cx)}{2} + \frac{3bd^2ex^4\operatorname{atan}(cx)}{4} + \frac{bde^2x^6\operatorname{atan}(cx)}{2} + \frac{be^3x^8\operatorname{atan}(cx)}{8} - \frac{bd^3x}{2c} - \\ a\left(\frac{d^3x^2}{2} + \frac{3d^2ex^4}{4} + \frac{de^2x^6}{2} + \frac{e^3x^8}{8}\right) \end{cases}$$

input
```
integrate(x*(e*x**2+d)**3*(a+b*atan(c*x)),x)
```

output
```
Piecewise((a*d**3*x**2/2 + 3*a*d**2*e*x**4/4 + a*d*e**2*x**6/2 + a*e**3*x*
*8/8 + b*d**3*x**2*atan(c*x)/2 + 3*b*d**2*e*x**4*atan(c*x)/4 + b*d*e**2*x*
*6*atan(c*x)/2 + b*e**3*x**8*atan(c*x)/8 - b*d**3*x/(2*c) - b*d**2*e*x**3/
(4*c) - b*d*e**2*x**5/(10*c) - b*e**3*x**7/(56*c) + b*d**3*atan(c*x)/(2*c*
*2) + 3*b*d**2*e*x/(4*c**3) + b*d*e**2*x**3/(6*c**3) + b*e**3*x**5/(40*c**
3) - 3*b*d**2*e*atan(c*x)/(4*c**4) - b*d*e**2*x/(2*c**5) - b*e**3*x**3/(24
*c**5) + b*d*e**2*atan(c*x)/(2*c**6) + b*e**3*x/(8*c**7) - b*e**3*atan(c*x
)/(8*c**8), Ne(c, 0)), (a*(d**3*x**2/2 + 3*d**2*e*x**4/4 + d*e**2*x**6/2 +
 e**3*x**8/8), True))
```

## Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.47

$$\int x(d+ex^2)^3(a+b\arctan(cx))\,dx = \frac{1}{8}ae^3x^8 + \frac{1}{2}ade^2x^6 + \frac{3}{4}ad^2ex^4 + \frac{1}{2}ad^3x^2 + \frac{1}{2}\left(x^2\arctan(cx) - c\left(\frac{x}{c^2} - \frac{\arctan(cx)}{c^3}\right)\right)bd^3 + \frac{1}{4}\left(3x^4\arctan(cx) - c\left(\frac{c^2x^3-3x}{c^4} + \frac{3\arctan(cx)}{c^5}\right)\right)bd^2e + \frac{1}{30}\left(15x^6\arctan(cx) - c\left(\frac{3c^4x^5-5c^2x^3+15x}{c^6} - \frac{15\arctan(cx)}{c^7}\right)\right)bde^2 + \frac{1}{840}\left(105x^8\arctan(cx) - c\left(\frac{15c^6x^7-21c^4x^5+35c^2x^3-105x}{c^8} + \frac{105\arctan(cx)}{c^9}\right)\right)be^3$$

input
```
integrate(x*(e*x^2+d)^3*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
1/8*a*e^3*x^8 + 1/2*a*d*e^2*x^6 + 3/4*a*d^2*e*x^4 + 1/2*a*d^3*x^2 + 1/2*(x
^2*arctan(c*x) - c*(x/c^2 - arctan(c*x)/c^3))*b*d^3 + 1/4*(3*x^4*arctan(c*
x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*b*d^2*e + 1/30*(15*x^6*a
rctan(c*x) - c*((3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan(c*x)/c^7))*
b*d*e^2 + 1/840*(105*x^8*arctan(c*x) - c*((15*c^6*x^7 - 21*c^4*x^5 + 35*c^
2*x^3 - 105*x)/c^8 + 105*arctan(c*x)/c^9))*b*e^3
```

## Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.82

$$\int x(d+ex^2)^3(a+b\arctan(cx))\,dx = \frac{105\,bc^8e^3x^8\arctan(cx) + 105\,ac^8e^3x^8 + 420\,bc^8de^2x^6\arctan(cx) + 420\,ac^8de^2x^6 - 15\,bc^7e^3x^7 + 630\,bc^8d}{}$$

input
```
integrate(x*(e*x^2+d)^3*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
1/840*(105*b*c^8*e^3*x^8*arctan(c*x) + 105*a*c^8*e^3*x^8 + 420*b*c^8*d*e^2
*x^6*arctan(c*x) + 420*a*c^8*d*e^2*x^6 - 15*b*c^7*e^3*x^7 + 630*b*c^8*d^2*
e*x^4*arctan(c*x) + 630*a*c^8*d^2*e*x^4 - 84*b*c^7*d*e^2*x^5 + 420*b*c^8*d
^3*x^2*arctan(c*x) + 420*a*c^8*d^3*x^2 - 210*b*c^7*d^2*e*x^3 + 21*b*c^5*e^
3*x^5 - 420*b*c^7*d^3*x + 140*b*c^5*d*e^2*x^3 + 420*b*c^6*d^3*arctan(c*x)
+ 630*b*c^5*d^2*e*x - 35*b*c^3*e^3*x^3 - 630*b*c^4*d^2*e*arctan(c*x) - 420
*b*c^3*d*e^2*x + 420*b*c^2*d*e^2*arctan(c*x) + 105*b*c*e^3*x - 105*b*e^3*a
rctan(c*x))/c^8
```

## Mupad [B] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 442, normalized size of antiderivative = 2.80

$$\int x(d+ex^2)^3(a+b\arctan(cx))\,dx$$
$$= x\left(\frac{\frac{\frac{b\,e^3}{8\,c^3}-\frac{b\,d\,e^2}{2\,c}}{c^2}+\frac{3\,b\,d^2\,e}{4\,c}}{c^2}-\frac{b\,d^3}{2\,c}\right)-x^6\left(\frac{a\,e^3}{6\,c^2}-\frac{a\,e^2\,(3\,d\,c^2+e)}{6\,c^2}\right)$$
$$+x^4\left(\frac{\frac{a\,e^3}{c^2}-\frac{a\,e^2\,(3\,d\,c^2+e)}{c^2}}{4\,c^2}+\frac{3\,a\,d\,e\,(d\,c^2+e)}{4\,c^2}\right)+x^5\left(\frac{b\,e^3}{40\,c^3}-\frac{b\,d\,e^2}{10\,c}\right)$$
$$+\mathrm{atan}(c\,x)\left(\frac{b\,d^3\,x^2}{2}+\frac{3\,b\,d^2\,e\,x^4}{4}+\frac{b\,d\,e^2\,x^6}{2}+\frac{b\,e^3\,x^8}{8}\right)-x^3\left(\frac{\frac{b\,e^3}{8\,c^3}-\frac{b\,d\,e^2}{2\,c}}{3\,c^2}+\frac{b\,d^2\,e}{4\,c}\right)$$
$$-x^2\left(\frac{\frac{\frac{a\,e^3}{c^2}-\frac{a\,e^2\,(3\,d\,c^2+e)}{c^2}}{c^2}+\frac{3\,a\,d\,e\,(d\,c^2+e)}{c^2}}{2\,c^2}-\frac{a\,d^2\,(d\,c^2+3\,e)}{2\,c^2}\right)+\frac{a\,e^3\,x^8}{8}-\frac{b\,e^3\,x^7}{56\,c}$$
$$-\frac{b\,\mathrm{atan}\left(\frac{b\,c\,x\,(e-2\,c^2\,d)\,(2\,c^4\,d^2-2\,c^2\,d\,e+e^2)}{-4\,b\,c^6\,d^3+6\,b\,c^4\,d^2\,e-4\,b\,c^2\,d\,e^2+b\,e^3}\right)\,(e-2\,c^2\,d)\,(2\,c^4\,d^2-2\,c^2\,d\,e+e^2)}{8\,c^8}$$

input
```
int(x*(a + b*atan(c*x))*(d + e*x^2)^3,x)
```

output
```
x*((((b*e^3)/(8*c^3) - (b*d*e^2)/(2*c))/c^2 + (3*b*d^2*e)/(4*c))/c^2 - (b*
d^3)/(2*c)) - x^6*((a*e^3)/(6*c^2) - (a*e^2*(e + 3*c^2*d))/(6*c^2)) + x^4*
(((a*e^3)/c^2 - (a*e^2*(e + 3*c^2*d))/c^2)/(4*c^2) + (3*a*d*e*(e + c^2*d))
/(4*c^2)) + x^5*((b*e^3)/(40*c^3) - (b*d*e^2)/(10*c)) + atan(c*x)*((b*d^3*
x^2)/2 + (b*e^3*x^8)/8 + (3*b*d^2*e*x^4)/4 + (b*d*e^2*x^6)/2) - x^3*(((b*e
^3)/(8*c^3) - (b*d*e^2)/(2*c))/(3*c^2) + (b*d^2*e)/(4*c)) - x^2*((((a*e^3)
/c^2 - (a*e^2*(e + 3*c^2*d))/c^2)/c^2 + (3*a*d*e*(e + c^2*d))/c^2)/(2*c^2)
 - (a*d^2*(3*e + c^2*d))/(2*c^2)) + (a*e^3*x^8)/8 - (b*e^3*x^7)/(56*c) - (
b*atan((b*c*x*(e - 2*c^2*d)*(e^2 + 2*c^4*d^2 - 2*c^2*d*e))/(b*e^3 - 4*b*c^
6*d^3 - 4*b*c^2*d*e^2 + 6*b*c^4*d^2*e))*(e - 2*c^2*d)*(e^2 + 2*c^4*d^2 - 2
*c^2*d*e))/(8*c^8)
```

## Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.82

$$\int x(d+ex^2)^3(a+b\arctan(cx))\,dx$$
$$= \frac{420atan(cx)\,b\,c^8d^3x^2+630atan(cx)\,b\,c^8d^2e\,x^4+420atan(cx)\,b\,c^8d\,e^2x^6+105atan(cx)\,b\,c^8e^3x^8+420ata}{}$$

input
```
int(x*(e*x^2+d)^3*(a+b*atan(c*x)),x)
```

output
```
(420*atan(c*x)*b*c**8*d**3*x**2 + 630*atan(c*x)*b*c**8*d**2*e*x**4 + 420*a
tan(c*x)*b*c**8*d*e**2*x**6 + 105*atan(c*x)*b*c**8*e**3*x**8 + 420*atan(c*
x)*b*c**6*d**3 - 630*atan(c*x)*b*c**4*d**2*e + 420*atan(c*x)*b*c**2*d*e**2
 - 105*atan(c*x)*b*e**3 + 420*a*c**8*d**3*x**2 + 630*a*c**8*d**2*e*x**4 +
420*a*c**8*d*e**2*x**6 + 105*a*c**8*e**3*x**8 - 420*b*c**7*d**3*x - 210*b*
c**7*d**2*e*x**3 - 84*b*c**7*d*e**2*x**5 - 15*b*c**7*e**3*x**7 + 630*b*c**
5*d**2*e*x + 140*b*c**5*d*e**2*x**3 + 21*b*c**5*e**3*x**5 - 420*b*c**3*d*e
**2*x - 35*b*c**3*e**3*x**3 + 105*b*c*e**3*x)/(840*c**8)
```

# 3.1139 $\int (d+ex^2)^3 (a+b\arctan(cx))\,dx$



## Optimal result

Integrand size = 18, antiderivative size = 188

$$\begin{aligned}
&\int (d+ex^2)^3 (a+b\arctan(cx))\,dx \\
&= -\frac{be(35c^4d^2-21c^2de+5e^2)\,x^2}{70c^5} - \frac{b(21c^2d-5e)\,e^2x^4}{140c^3} - \frac{be^3x^6}{42c} \\
&\quad + d^3x(a+b\arctan(cx)) + d^2ex^3(a+b\arctan(cx)) + \frac{3}{5}de^2x^5(a+b\arctan(cx)) \\
&\quad + \frac{1}{7}e^3x^7(a+b\arctan(cx)) - \frac{b(35c^6d^3-35c^4d^2e+21c^2de^2-5e^3)\log(1+c^2x^2)}{70c^7}
\end{aligned}$$

output
```
-1/70*b*e*(35*c^4*d^2-21*c^2*d*e+5*e^2)*x^2/c^5-1/140*b*(21*c^2*d-5*e)*e^2
*x^4/c^3-1/42*b*e^3*x^6/c+d^3*x*(a+b*arctan(c*x))+d^2*e*x^3*(a+b*arctan(c*
x))+3/5*d*e^2*x^5*(a+b*arctan(c*x))+1/7*e^3*x^7*(a+b*arctan(c*x))-1/70*b*(
35*c^6*d^3-35*c^4*d^2*e+21*c^2*d*e^2-5*e^3)*ln(c^2*x^2+1)/c^7
```

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.02

$$\int (d+ex^2)^3 (a+b\arctan(cx))\,dx$$
$$=\frac{c^2x(12ac^5(35d^3+35d^2ex^2+21de^2x^4+5e^3x^6)-bex(30e^2-3c^2e(42d+5ex^2)+c^4(210d^2+63dex^2+1}{}$$

input
```
Integrate[(d + e*x^2)^3*(a + b*ArcTan[c*x]),x]
```

output
```
(c^2*x*(12*a*c^5*(35*d^3 + 35*d^2*e*x^2 + 21*d*e^2*x^4 + 5*e^3*x^6) - b*e*
x*(30*e^2 - 3*c^2*e*(42*d + 5*e*x^2) + c^4*(210*d^2 + 63*d*e*x^2 + 10*e^2*
x^4))) + 12*b*c^7*x*(35*d^3 + 35*d^2*e*x^2 + 21*d*e^2*x^4 + 5*e^3*x^6)*Arc
Tan[c*x] - 6*b*(35*c^6*d^3 - 35*c^4*d^2*e + 21*c^2*d*e^2 - 5*e^3)*Log[1 +
c^2*x^2])/(420*c^7)
```

## Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {5447, 27, 2331, 2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex^2)^3 (a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5447}$$

$$-bc\int \frac{x(5e^3x^6+21de^2x^4+35d^2ex^2+35d^3)}{35(c^2x^2+1)}dx+d^3x(a+b\arctan(cx))+d^2ex^3(a+b\arctan(cx))+\frac{3}{5}de^2x^5(a+b\arctan(cx))+\frac{1}{7}e^3x^7(a+b\arctan(cx))$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{35}bc\int \frac{x(5e^3x^6+21de^2x^4+35d^2ex^2+35d^3)}{c^2x^2+1}dx+d^3x(a+b\arctan(cx))+d^2ex^3(a+b\arctan(cx))+\frac{3}{5}de^2x^5(a+b\arctan(cx))+\frac{1}{7}e^3x^7(a+b\arctan(cx))$$

$\Big\downarrow$ 2331

$$-\frac{1}{70}bc\int \frac{5e^3x^6+21de^2x^4+35d^2ex^2+35d^3}{c^2x^2+1}dx^2+d^3x(a+b\arctan(cx))+d^2ex^3(a+b\arctan(cx))+\frac{3}{5}de^2x^5(a+b\arctan(cx))+\frac{1}{7}e^3x^7(a+b\arctan(cx))$$

$\Big\downarrow$ 2389

$$-\frac{1}{70}bc\int \left(\frac{5e^3x^4}{c^2}+\frac{(21c^2d-5e)e^2x^2}{c^4}+\frac{e(35d^2c^4-21dec^2+5e^2)}{c^6}+\frac{35d^3c^6-35d^2ec^4+21de^2c^2-5e^3}{c^6(c^2x^2+1)}\right)dx^2+d^3x(a+b\arctan(cx))+d^2ex^3(a+b\arctan(cx))+\frac{3}{5}de^2x^5(a+b\arctan(cx))+\frac{1}{7}e^3x^7(a+b\arctan(cx))$$

$\Big\downarrow$ 2009

$$d^3x(a+b\arctan(cx))+d^2ex^3(a+b\arctan(cx))+\frac{3}{5}de^2x^5(a+b\arctan(cx))+\frac{1}{7}e^3x^7(a+b\arctan(cx))-\frac{1}{70}bc\left(\frac{5e^3x^6}{3c^2}+\frac{e^2x^4(21c^2d-5e)}{2c^4}+\frac{ex^2(35c^4d^2-21c^2de+5e^2)}{c^6}+\frac{(35c^6d^3-35c^4d^2e+21c^2de^2-5e^3)\log(c^2x}{c^8}\right.$$

input

```
Int[(d + e*x^2)^3*(a + b*ArcTan[c*x]),x]
```

output

```
d^3*x*(a + b*ArcTan[c*x]) + d^2*e*x^3*(a + b*ArcTan[c*x]) + (3*d*e^2*x^5*(
a + b*ArcTan[c*x]))/5 + (e^3*x^7*(a + b*ArcTan[c*x]))/7 - (b*c*((e*(35*c^4
*d^2 - 21*c^2*d*e + 5*e^2)*x^2)/c^6 + ((21*c^2*d - 5*e)*e^2*x^4)/(2*c^4) +
 (5*e^3*x^6)/(3*c^2) + ((35*c^6*d^3 - 35*c^4*d^2*e + 21*c^2*d*e^2 - 5*e^3)
*Log[1 + c^2*x^2])/c^8))/70
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

rule 5447
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Simp[(a + b*ArcTan[c*x])   u,
x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2), x], x], x]] /; FreeQ
[{a, b, c, d, e}, x] && (IntegerQ[q] || ILtQ[q + 1/2, 0])
```

## Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.12

| method | result |
|---|---|
| parts | $a\left(\frac{1}{7}x^7e^3+\frac{3}{5}x^5e^2d+x^3e\,d^2+x\,d^3\right)+\frac{b\left(\frac{\arctan(cx)c\,e^3x^7}{7}+\frac{3\arctan(cx)c\,e^2d\,x^5}{5}+\arctan(cx)c\,d^2e\,x^3+\text{arct}\right.}{}$ |
| derivativedivides | $\frac{\frac{a\left(c^7x\,d^3+d^2c^7e\,x^3+\frac{3}{5}d\,c^7e^2x^5+\frac{1}{7}e^3c^7x^7\right)}{c^6}+\frac{b\left(\arctan(cx)c^7x\,d^3+\arctan(cx)d^2c^7e\,x^3+\frac{3\arctan(cx)d\,c^7e^2x^5}{5}+\frac{\arctan(cx)e^3c^7x^7}{7}\right.}{}}{c}$ |
| default | $\frac{\frac{a\left(c^7x\,d^3+d^2c^7e\,x^3+\frac{3}{5}d\,c^7e^2x^5+\frac{1}{7}e^3c^7x^7\right)}{c^6}+\frac{b\left(\arctan(cx)c^7x\,d^3+\arctan(cx)d^2c^7e\,x^3+\frac{3\arctan(cx)d\,c^7e^2x^5}{5}+\frac{\arctan(cx)e^3c^7x^7}{7}\right.}{}}{c}$ |
| parallelrisch | $-\frac{-60x^7\arctan(cx)b\,c^7e^3-60a\,c^7e^3x^7-252x^5\arctan(cx)b\,c^7d\,e^2+10b\,c^6e^3x^6-252a\,c^7d\,e^2x^5-420x^3\arctan(cx)b\,c^7d}{}$ |
| risch | $\frac{ib\,d^3x\ln(-icx+1)}{2}-\frac{ib(5x^7e^3+21x^5e^2d+35x^3e\,d^2+35x\,d^3)\ln(icx+1)}{70}+\frac{3ibd\,e^2x^5\ln(-icx+1)}{10}+\frac{a\,e^3x^7}{7}+\frac{ib\,d}{}$ |

input
```
int((e*x^2+d)^3*(a+b*arctan(c*x)),x,method=_RETURNVERBOSE)
```

output
```
a*(1/7*x^7*e^3+3/5*x^5*e^2*d+x^3*e*d^2+x*d^3)+b/c*(1/7*arctan(c*x)*c*e^3*x
^7+3/5*arctan(c*x)*c*e^2*d*x^5+arctan(c*x)*c*d^2*e*x^3+arctan(c*x)*c*x*d^3
-1/35/c^6*(35/2*c^6*d^2*e*x^2+21/4*c^6*d*e^2*x^4+5/6*c^6*e^3*x^6-21/2*c^4*
d*e^2*x^2-5/4*c^4*e^3*x^4+5/2*c^2*e^3*x^2+1/2*(35*c^6*d^3-35*c^4*d^2*e+21*
c^2*d*e^2-5*e^3)*ln(c^2*x^2+1)))
```

## Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.22

$$\int \left(d+ex^2\right)^3(a+b\arctan(cx))\,dx$$
$$=\frac{60\,ac^7e^3x^7+252\,ac^7de^2x^5-10\,bc^6e^3x^6+420\,ac^7d^2ex^3+420\,ac^7d^3x-3\,(21\,bc^6de^2-5\,bc^4e^3)x^4-6\,(35}{}$$

input
```
integrate((e*x^2+d)^3*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
1/420*(60*a*c^7*e^3*x^7 + 252*a*c^7*d*e^2*x^5 - 10*b*c^6*e^3*x^6 + 420*a*c
^7*d^2*e*x^3 + 420*a*c^7*d^3*x - 3*(21*b*c^6*d*e^2 - 5*b*c^4*e^3)*x^4 - 6*
(35*b*c^6*d^2*e - 21*b*c^4*d*e^2 + 5*b*c^2*e^3)*x^2 + 12*(5*b*c^7*e^3*x^7
+ 21*b*c^7*d*e^2*x^5 + 35*b*c^7*d^2*e*x^3 + 35*b*c^7*d^3*x)*arctan(c*x) -
6*(35*b*c^6*d^3 - 35*b*c^4*d^2*e + 21*b*c^2*d*e^2 - 5*b*e^3)*log(c^2*x^2 +
 1))/c^7
```

## Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.63

$$\int (d+ex^2)^3 (a+b\arctan(cx))\,dx$$
$$=\begin{cases} ad^3x + ad^2ex^3 + \frac{3ade^2x^5}{5} + \frac{ae^3x^7}{7} + bd^3x\operatorname{atan}(cx) + bd^2ex^3\operatorname{atan}(cx) + \frac{3bde^2x^5\operatorname{atan}(cx)}{5} + \frac{be^3x^7\operatorname{atan}(cx)}{7} - \\ a\left(d^3x + d^2ex^3 + \frac{3de^2x^5}{5} + \frac{e^3x^7}{7}\right) \end{cases}$$

input
```
integrate((e*x**2+d)**3*(a+b*atan(c*x)),x)
```

output
```
Piecewise((a*d**3*x + a*d**2*e*x**3 + 3*a*d*e**2*x**5/5 + a*e**3*x**7/7 +
b*d**3*x*atan(c*x) + b*d**2*e*x**3*atan(c*x) + 3*b*d*e**2*x**5*atan(c*x)/5
 + b*e**3*x**7*atan(c*x)/7 - b*d**3*log(x**2 + c**(-2))/(2*c) - b*d**2*e*x
**2/(2*c) - 3*b*d*e**2*x**4/(20*c) - b*e**3*x**6/(42*c) + b*d**2*e*log(x**
2 + c**(-2))/(2*c**3) + 3*b*d*e**2*x**2/(10*c**3) + b*e**3*x**4/(28*c**3)
- 3*b*d*e**2*log(x**2 + c**(-2))/(10*c**5) - b*e**3*x**2/(14*c**5) + b*e**
3*log(x**2 + c**(-2))/(14*c**7), Ne(c, 0)), (a*(d**3*x + d**2*e*x**3 + 3*d
*e**2*x**5/5 + e**3*x**7/7), True))
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.18

$$\int (d + ex^2)^3 (a + b \arctan(cx)) \, dx$$
$$= \frac{1}{7} ae^3x^7 + \frac{3}{5} ade^2x^5 + ad^2ex^3 + \frac{1}{2} \left(2 x^3 \arctan(cx) - c\left(\frac{x^2}{c^2} - \frac{\log(c^2x^2+1)}{c^4}\right)\right)bd^2e + \frac{3}{20} \left(4 x^5 \arctan(cx) - c\left(\frac{c^2x^4 - 2x^2}{c^4} + \frac{2 \log(c^2x^2+1)}{c^6}\right)\right)bde^2 + \frac{1}{84} \left(12 x^7 \arctan(cx) - c\left(\frac{2 c^4x^6 - 3 c^2x^4 + 6 x^2}{c^6} - \frac{6 \log(c^2x^2+1)}{c^8}\right)\right)be^3 + ad^3x + \frac{(2 cx \arctan(cx) - \log(c^2x^2+1))bd^3}{2 c}$$

input
```
integrate((e*x^2+d)^3*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
1/7*a*e^3*x^7 + 3/5*a*d*e^2*x^5 + a*d^2*e*x^3 + 1/2*(2*x^3*arctan(c*x) - c
*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*b*d^2*e + 3/20*(4*x^5*arctan(c*x) - c*(
(c^2*x^4 - 2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*b*d*e^2 + 1/84*(12*x^7*ar
ctan(c*x) - c*((2*c^4*x^6 - 3*c^2*x^4 + 6*x^2)/c^6 - 6*log(c^2*x^2 + 1)/c^
8))*b*e^3 + a*d^3*x + 1/2*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b*d^3/c
```

## Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.43

$$\int (d + ex^2)^3 (a + b \arctan(cx)) \, dx$$
$$= \frac{60 bc^7e^3x^7 \arctan(cx) + 60 ac^7e^3x^7 + 252 bc^7de^2x^5 \arctan(cx) + 252 ac^7de^2x^5 - 10 bc^6e^3x^6 + 420 bc^7d^2e}{}$$

input
```
integrate((e*x^2+d)^3*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
1/420*(60*b*c^7*e^3*x^7*arctan(c*x) + 60*a*c^7*e^3*x^7 + 252*b*c^7*d*e^2*x
^5*arctan(c*x) + 252*a*c^7*d*e^2*x^5 - 10*b*c^6*e^3*x^6 + 420*b*c^7*d^2*e*
x^3*arctan(c*x) + 420*a*c^7*d^2*e*x^3 - 63*b*c^6*d*e^2*x^4 + 420*b*c^7*d^3
*x*arctan(c*x) + 420*a*c^7*d^3*x - 210*b*c^6*d^2*e*x^2 + 15*b*c^4*e^3*x^4
- 210*b*c^6*d^3*log(c^2*x^2 + 1) + 126*b*c^4*d*e^2*x^2 + 210*b*c^4*d^2*e*l
og(c^2*x^2 + 1) - 30*b*c^2*e^3*x^2 - 126*b*c^2*d*e^2*log(c^2*x^2 + 1) + 30
*b*e^3*log(c^2*x^2 + 1))/c^7
```

## Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.27

$$\int (d+ex^2)^3 (a+b\arctan(cx))\,dx = \frac{a\,e^3\,x^7}{7} + a\,d^3\,x - \frac{b\,d^3\,\ln(c^2\,x^2+1)}{2\,c} + \frac{b\,e^3\,\ln(c^2\,x^2+1)}{14\,c^7} - \frac{b\,e^3\,x^6}{42\,c} + \frac{b\,e^3\,x^4}{28\,c^3} - \frac{b\,e^3\,x^2}{14\,c^5} + b\,d^3\,x\,\mathrm{atan}(c\,x) + a\,d^2\,e\,x^3 + \frac{3\,a\,d\,e^2\,x^5}{5} + \frac{b\,e^3\,x^7\,\mathrm{atan}(c\,x)}{7} + b\,d^2\,e\,x^3\,\mathrm{atan}(c\,x) + \frac{3\,b\,d\,e^2\,x^5\,\mathrm{atan}(c\,x)}{5} + \frac{b\,d^2\,e\,\ln(c^2\,x^2+1)}{2\,c^3} - \frac{3\,b\,d\,e^2\,\ln(c^2\,x^2+1)}{10\,c^5} - \frac{b\,d^2\,e\,x^2}{2\,c} - \frac{3\,b\,d\,e^2\,x^4}{20\,c} + \frac{3\,b\,d\,e^2\,x^2}{10\,c^3}$$

input
```
int((a + b*atan(c*x))*(d + e*x^2)^3,x)
```

output
```
(a*e^3*x^7)/7 + a*d^3*x - (b*d^3*log(c^2*x^2 + 1))/(2*c) + (b*e^3*log(c^2*
x^2 + 1))/(14*c^7) - (b*e^3*x^6)/(42*c) + (b*e^3*x^4)/(28*c^3) - (b*e^3*x^
2)/(14*c^5) + b*d^3*x*atan(c*x) + a*d^2*e*x^3 + (3*a*d*e^2*x^5)/5 + (b*e^3
*x^7*atan(c*x))/7 + b*d^2*e*x^3*atan(c*x) + (3*b*d*e^2*x^5*atan(c*x))/5 +
(b*d^2*e*log(c^2*x^2 + 1))/(2*c^3) - (3*b*d*e^2*log(c^2*x^2 + 1))/(10*c^5)
 - (b*d^2*e*x^2)/(2*c) - (3*b*d*e^2*x^4)/(20*c) + (3*b*d*e^2*x^2)/(10*c^3)
```

## Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.43

$$\int (d+ex^2)^3 (a+b\arctan(cx))\,dx = \frac{420atan(cx)\,b\,c^7d^3x+420atan(cx)\,b\,c^7d^2e\,x^3+252atan(cx)\,b\,c^7d\,e^2x^5+60atan(cx)\,b\,c^7e^3x^7-210\log(c$$

input
```
int((e*x^2+d)^3*(a+b*atan(c*x)),x)
```

output
```
(420*atan(c*x)*b*c**7*d**3*x + 420*atan(c*x)*b*c**7*d**2*e*x**3 + 252*atan
(c*x)*b*c**7*d*e**2*x**5 + 60*atan(c*x)*b*c**7*e**3*x**7 - 210*log(c**2*x*
*2 + 1)*b*c**6*d**3 + 210*log(c**2*x**2 + 1)*b*c**4*d**2*e - 126*log(c**2*
x**2 + 1)*b*c**2*d*e**2 + 30*log(c**2*x**2 + 1)*b*e**3 + 420*a*c**7*d**3*x
 + 420*a*c**7*d**2*e*x**3 + 252*a*c**7*d*e**2*x**5 + 60*a*c**7*e**3*x**7 -
 210*b*c**6*d**2*e*x**2 - 63*b*c**6*d*e**2*x**4 - 10*b*c**6*e**3*x**6 + 12
6*b*c**4*d*e**2*x**2 + 15*b*c**4*e**3*x**4 - 30*b*c**2*e**3*x**2)/(420*c**
7)
```

# 3.1140 $\int \frac{(d+ex^2)^3(a+b\arctan(cx))}{x}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 228

$$\int \frac{(d+ex^2)^3(a+b\arctan(cx))}{x}\,dx = -\frac{3bd^2ex}{2c}+\frac{3bde^2x}{4c^3}-\frac{be^3x}{6c^5}-\frac{bde^2x^3}{4c}+\frac{be^3x^3}{18c^3} -\frac{be^3x^5}{30c}+\frac{3bd^2e\arctan(cx)}{2c^2}-\frac{3bde^2\arctan(cx)}{4c^4} +\frac{be^3\arctan(cx)}{6c^6}+\frac{3}{2}d^2ex^2(a+b\arctan(cx)) +\frac{3}{4}de^2x^4(a+b\arctan(cx)) +\frac{1}{6}e^3x^6(a+b\arctan(cx))+ad^3\log(x) +\frac{1}{2}ibd^3\operatorname{PolyLog}(2,-icx)-\frac{1}{2}ibd^3\operatorname{PolyLog}(2,icx)$$

output
```
-3/2*b*d^2*e*x/c+3/4*b*d*e^2*x/c^3-1/6*b*e^3*x/c^5-1/4*b*d*e^2*x^3/c+1/18*
b*e^3*x^3/c^3-1/30*b*e^3*x^5/c+3/2*b*d^2*e*arctan(c*x)/c^2-3/4*b*d*e^2*arc
tan(c*x)/c^4+1/6*b*e^3*arctan(c*x)/c^6+3/2*d^2*e*x^2*(a+b*arctan(c*x))+3/4
*d*e^2*x^4*(a+b*arctan(c*x))+1/6*e^3*x^6*(a+b*arctan(c*x))+a*d^3*ln(x)+1/2
*I*b*d^3*polylog(2,-I*c*x)-1/2*I*b*d^3*polylog(2,I*c*x)
```

## Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.83

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x}\,dx = -\frac{be^3(15cx-5c^3x^3+3c^5x^5-15\arctan(cx))}{90c^6} - \frac{3bd^2e(cx-\arctan(cx))}{2c^2} - \frac{bde^2(-3cx+c^3x^3+3\arctan(cx))}{4c^4} + \frac{3}{2}d^2ex^2(a+b\arctan(cx)) + \frac{3}{4}de^2x^4(a+b\arctan(cx)) + \frac{1}{6}e^3x^6(a+b\arctan(cx)) + ad^3\log(x) + \frac{1}{2}ibd^3\operatorname{PolyLog}(2,-icx) - \frac{1}{2}ibd^3\operatorname{PolyLog}(2,icx)$$

input
```
Integrate[((d + e*x^2)^3*(a + b*ArcTan[c*x]))/x,x]
```

output
```
-1/90*(b*e^3*(15*c*x - 5*c^3*x^3 + 3*c^5*x^5 - 15*ArcTan[c*x]))/c^6 - (3*b
*d^2*e*(c*x - ArcTan[c*x]))/(2*c^2) - (b*d*e^2*(-3*c*x + c^3*x^3 + 3*ArcTa
n[c*x]))/(4*c^4) + (3*d^2*e*x^2*(a + b*ArcTan[c*x]))/2 + (3*d*e^2*x^4*(a +
 b*ArcTan[c*x]))/4 + (e^3*x^6*(a + b*ArcTan[c*x]))/6 + a*d^3*Log[x] + (I/2
)*b*d^3*PolyLog[2, (-I)*c*x] - (I/2)*b*d^3*PolyLog[2, I*c*x]
```

## Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x}\,dx$$

$$\Big\downarrow \text{5515}$$

$$\int \left(\frac{d^3(a+b\arctan(cx))}{x}+3d^2ex(a+b\arctan(cx))+3de^2x^3(a+b\arctan(cx))+e^3x^5(a+b\arctan(cx))\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3}{2}d^2ex^2(a+b\arctan(cx))+\frac{3}{4}de^2x^4(a+b\arctan(cx))+\frac{1}{6}e^3x^6(a+b\arctan(cx))+ad^3\log(x)+\frac{be^3\arctan(cx)}{6c^6}-\frac{3bde^2\arctan(cx)}{4c^4}+\frac{3bd^2e\arctan(cx)}{2c^2}-\frac{be^3x}{6c^5}+\frac{3bde^2x}{4c^3}+\frac{be^3x^3}{18c^3}+\frac{1}{2}ibd^3\operatorname{PolyLog}(2,-icx)-\frac{1}{2}ibd^3\operatorname{PolyLog}(2,icx)-\frac{3bd^2ex}{2c}-\frac{bde^2x^3}{4c}-\frac{be^3x^5}{30c}$$

input
```
Int[((d + e*x^2)^3*(a + b*ArcTan[c*x]))/x,x]
```

output
```
(-3*b*d^2*e*x)/(2*c) + (3*b*d*e^2*x)/(4*c^3) - (b*e^3*x)/(6*c^5) - (b*d*e^
2*x^3)/(4*c) + (b*e^3*x^3)/(18*c^3) - (b*e^3*x^5)/(30*c) + (3*b*d^2*e*ArcT
an[c*x])/(2*c^2) - (3*b*d*e^2*ArcTan[c*x])/(4*c^4) + (b*e^3*ArcTan[c*x])/(
6*c^6) + (3*d^2*e*x^2*(a + b*ArcTan[c*x]))/2 + (3*d*e^2*x^4*(a + b*ArcTan[
c*x]))/4 + (e^3*x^6*(a + b*ArcTan[c*x]))/6 + a*d^3*Log[x] + (I/2)*b*d^3*Po
lyLog[2, (-I)*c*x] - (I/2)*b*d^3*PolyLog[2, I*c*x]
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

## Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.13

| method | result |
|---|---|
| parts | $a\left(\frac{x^6 e^3}{6}+\frac{3x^4 e^2 d}{4}+\frac{3x^2 e\, d^2}{2}+d^3\ln\left(x\right)\right)+b\left(\frac{\arctan(cx)e^3x^6}{6}+\frac{3\arctan(cx)e^2d\,x^4}{4}+\frac{3\arctan(cx)d^2e}{2}\right.$ |
| derivativedivides | $\frac{a\left(\frac{3c^6d^2e\,x^2}{2}+\frac{3c^6d\,e^2x^4}{4}+\frac{c^6e^3x^6}{6}+c^6d^3\ln(cx)\right)}{c^6}+\frac{b\left(\frac{3\arctan(cx)c^6d^2e\,x^2}{2}+\frac{3\arctan(cx)c^6d\,e^2x^4}{4}+\frac{\arctan(cx)e^3c^6x^6}{6}+a\right.}{}$ |
| default | $\frac{a\left(\frac{3c^6d^2e\,x^2}{2}+\frac{3c^6d\,e^2x^4}{4}+\frac{c^6e^3x^6}{6}+c^6d^3\ln(cx)\right)}{c^6}+\frac{b\left(\frac{3\arctan(cx)c^6d^2e\,x^2}{2}+\frac{3\arctan(cx)c^6d\,e^2x^4}{4}+\frac{\arctan(cx)e^3c^6x^6}{6}+a\right.}{}$ |
| risch | $\frac{3a\,d^2e\,x^2}{2}+\frac{3ad\,e^2x^4}{4}-\frac{b\,e^3x}{6c^5}+\frac{b\,e^3x^3}{18c^3}-\frac{b\,e^3x^5}{30c}+\frac{b\,e^3\arctan(cx)}{6c^6}-\frac{3b\,d^2ex}{2c}+\frac{3bd\,e^2x}{4c^3}-\frac{bd\,e^2x^3}{4c}+3b$ |

input
```
int((e*x^2+d)^3*(a+b*arctan(c*x))/x,x,method=_RETURNVERBOSE)
```

output
```
a*(1/6*x^6*e^3+3/4*x^4*e^2*d+3/2*x^2*e*d^2+d^3*ln(x))+b*(1/6*arctan(c*x)*e
^3*x^6+3/4*arctan(c*x)*e^2*d*x^4+3/2*arctan(c*x)*d^2*e*x^2+arctan(c*x)*d^3
*ln(c*x)-1/12/c^6*(e*(18*c^5*x*d^2+3*d*c^5*e*x^3+2/5*e^2*c^5*x^5-9*d*c^3*e
*x-2/3*e^2*c^3*x^3+2*c*x*e^2+(-18*c^4*d^2+9*c^2*d*e-2*e^2)*arctan(c*x))-6*
I*c^6*d^3*ln(c*x)*ln(1+I*c*x)+6*I*c^6*d^3*ln(c*x)*ln(1-I*c*x)-6*I*c^6*d^3*
dilog(1+I*c*x)+6*I*c^6*d^3*dilog(1-I*c*x)))
```

## Fricas [F]

$$\int \frac{(d+ex^2)^3\,(a+b\arctan(cx))}{x}\,dx=\int \frac{(ex^2+d)^3(b\arctan\left(cx\right)+a)}{x}\,dx$$

input
```
integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x,x, algorithm="fricas")
```

output
```
integral((a*e^3*x^6 + 3*a*d*e^2*x^4 + 3*a*d^2*e*x^2 + a*d^3 + (b*e^3*x^6 +
 3*b*d*e^2*x^4 + 3*b*d^2*e*x^2 + b*d^3)*arctan(c*x))/x, x)
```

## Sympy [F]

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x}\,dx = \int \frac{(a+b\,\mathrm{atan}\,(cx))\,(d+ex^2)^3}{x}\,dx$$

input
```
integrate((e*x**2+d)**3*(a+b*atan(c*x))/x,x)
```

output
```
Integral((a + b*atan(c*x))*(d + e*x**2)**3/x, x)
```

## Maxima [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.10

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x}\,dx = \frac{1}{6}\,ae^3x^6 + \frac{3}{4}\,ade^2x^4 + \frac{3}{2}\,ad^2ex^2 + ad^3\log(x) - \frac{6\,bc^5e^3x^5 + 45\,\pi bc^6d^3\log(c^2x^2+1) - 180\,bc^6d^3\arctan(cx)\log(cx) + 90i\,bc^6d^3\mathrm{Li}_2(i\,cx+1) - 90i\,bc^6 d$$

input
```
integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x,x, algorithm="maxima")
```

output
```
1/6*a*e^3*x^6 + 3/4*a*d*e^2*x^4 + 3/2*a*d^2*e*x^2 + a*d^3*log(x) - 1/180*(
6*b*c^5*e^3*x^5 + 45*pi*b*c^6*d^3*log(c^2*x^2 + 1) - 180*b*c^6*d^3*arctan(
c*x)*log(c*x) + 90*I*b*c^6*d^3*dilog(I*c*x + 1) - 90*I*b*c^6*d^3*dilog(-I*
c*x + 1) + 5*(9*b*c^5*d*e^2 - 2*b*c^3*e^3)*x^3 + 15*(18*b*c^5*d^2*e - 9*b*
c^3*d*e^2 + 2*b*c*e^3)*x - 15*(2*b*c^6*e^3*x^6 + 9*b*c^6*d*e^2*x^4 + 18*b*
c^6*d^2*e*x^2 + 18*b*c^4*d^2*e - 9*b*c^2*d*e^2 + 2*b*e^3)*arctan(c*x))/c^6
```

## Giac [F]

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x}\,dx = \int \frac{(ex^2+d)^3(b\arctan(cx)+a)}{x}\,dx$$

input
```
integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^3*(b*arctan(c*x) + a)/x, x)
```

## Mupad [B] (verification not implemented)

Time = 1.16 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.02

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x}\,dx = \begin{cases} \frac{a e^3 x^6}{6} + a d^3 \ln(x) + \\ \frac{a e^3 x^6}{6} + a d^3 \ln(x) - \frac{b e^3 \left(\frac{x}{c^4} - \frac{\operatorname{atan}(c x)}{c^5} + \frac{x^5}{5} - \frac{x^3}{3 c^2}\right)}{6 c} - 3 b d^2 e \left(\frac{x}{2 c} - \operatorname{atan}(c x)\left(\frac{1}{2 c^2} + \frac{x^2}{2}\right)\right) + \frac{3 a d^2 e x^2}{2} + \frac{3 a d}{} \end{cases}$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^3)/x,x)
```

output
```
piecewise(c == 0, (a*e^3*x^6)/6 + a*d^3*log(x) + (3*a*d^2*e*x^2)/2 + (3*a*
d*e^2*x^4)/4, c ~= 0, (a*e^3*x^6)/6 + a*d^3*log(x) - (b*d^3*dilog(- c*x*1i
 + 1)*1i)/2 + (b*d^3*dilog(c*x*1i + 1)*1i)/2 - (b*e^3*(x/c^4 - atan(c*x)/c
^5 + x^5/5 - x^3/(3*c^2)))/(6*c) - 3*b*d^2*e*(x/(2*c) - atan(c*x)*(1/(2*c^
2) + x^2/2)) + (3*a*d^2*e*x^2)/2 + (3*a*d*e^2*x^4)/4 - 3*b*d*e^2*((3*atan(
c*x) - 3*c*x + c^3*x^3)/(12*c^4) - (x^4*atan(c*x))/4) + (b*e^3*x^6*atan(c*
x))/6)
```

## Reduce [F]

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x}\,dx$$
$$= \frac{270atan(cx)\,b\,c^6d^2e\,x^2 + 135atan(cx)\,b\,c^6d\,e^2x^4 + 30atan(cx)\,b\,c^6e^3x^6 + 270atan(cx)\,b\,c^4d^2e - 135atan(}{}$$

input
```
int((e*x^2+d)^3*(a+b*atan(c*x))/x,x)
```

output
```
(270*atan(c*x)*b*c**6*d**2*e*x**2 + 135*atan(c*x)*b*c**6*d*e**2*x**4 + 30*
atan(c*x)*b*c**6*e**3*x**6 + 270*atan(c*x)*b*c**4*d**2*e - 135*atan(c*x)*b
*c**2*d*e**2 + 30*atan(c*x)*b*e**3 + 180*int(atan(c*x)/x,x)*b*c**6*d**3 +
180*log(x)*a*c**6*d**3 + 270*a*c**6*d**2*e*x**2 + 135*a*c**6*d*e**2*x**4 +
 30*a*c**6*e**3*x**6 - 270*b*c**5*d**2*e*x - 45*b*c**5*d*e**2*x**3 - 6*b*c
**5*e**3*x**5 + 135*b*c**3*d*e**2*x + 10*b*c**3*e**3*x**3 - 30*b*c*e**3*x)
/(180*c**6)
```

# 3.1141 $\int \frac{(d+ex^2)^3(a+b\arctan(cx))}{x^2}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 160

$$\int \frac{(d+ex^2)^3(a+b\arctan(cx))}{x^2}\,dx = -\frac{b(5c^2d-e)e^2x^2}{10c^3}-\frac{be^3x^4}{20c}-\frac{d^3(a+b\arctan(cx))}{x}+3d^2ex(a+b\arctan(cx))+de^2x^3(a+b\arctan(cx))+\frac{1}{5}e^3x^5(a+b\arctan(cx))+bcd^3\log(x)-\frac{b(5c^6d^3+15c^4d^2e-5c^2de^2+e^3)\log(1+c^2x^2)}{10c^5}$$

output
```
-1/10*b*(5*c^2*d-e)*e^2*x^2/c^3-1/20*b*e^3*x^4/c-d^3*(a+b*arctan(c*x))/x+3
*d^2*e*x*(a+b*arctan(c*x))+d*e^2*x^3*(a+b*arctan(c*x))+1/5*e^3*x^5*(a+b*ar
ctan(c*x))+b*c*d^3*ln(x)-1/10*b*(5*c^6*d^3+15*c^4*d^2*e-5*c^2*d*e^2+e^3)*l
n(c^2*x^2+1)/c^5
```

## Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.06

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^2}\,dx$$
$$=\frac{1}{20}\left(-\frac{20ad^3}{x}+60ad^2ex+\frac{2be^2(-5c^2d+e)\,x^2}{c^3}+20ade^2x^3-\frac{be^3x^4}{c}+4ae^3x^5+\frac{4b(-5d^3+15d^2ex^2+5de^2x^4+e^3x^6)\arctan(cx)}{x}+20bcd^3\log(x)-\frac{2b(5c^6d^3+15c^4d^2e-5c^2de^2+e^3)\log(1+c^2x^2)}{c^5}\right)$$

input
```
Integrate[((d + e*x^2)^3*(a + b*ArcTan[c*x]))/x^2,x]
```

output
```
((-20*a*d^3)/x + 60*a*d^2*e*x + (2*b*e^2*(-5*c^2*d + e)*x^2)/c^3 + 20*a*d*
e^2*x^3 - (b*e^3*x^4)/c + 4*a*e^3*x^5 + (4*b*(-5*d^3 + 15*d^2*e*x^2 + 5*d*
e^2*x^4 + e^3*x^6)*ArcTan[c*x])/x + 20*b*c*d^3*Log[x] - (2*b*(5*c^6*d^3 +
15*c^4*d^2*e - 5*c^2*d*e^2 + e^3)*Log[1 + c^2*x^2])/c^5)/20
```

## Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {5511, 27, 2331, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^2}\,dx$$
$$\Big\downarrow \text{ 5511}$$
$$-bc\int -\frac{-e^3x^6-5de^2x^4-15d^2ex^2+5d^3}{5x\,(c^2x^2+1)}dx-\frac{d^3(a+b\arctan(cx))}{x}+3d^2ex(a+b\arctan(cx))+de^2x^3(a+b\arctan(cx))+\frac{1}{5}e^3x^5(a+b\arctan(cx))$$

$$\Big\downarrow \; 27$$

$$\frac{1}{5}bc\int \frac{-e^3x^6-5de^2x^4-15d^2ex^2+5d^3}{x\left(c^2x^2+1\right)}dx-\frac{d^3(a+b\arctan(cx))}{x}+3d^2ex(a+b\arctan(cx))+de^2x^3(a+b\arctan(cx))+\frac{1}{5}e^3x^5(a+b\arctan(cx))$$

$$\Big\downarrow \; 2331$$

$$\frac{1}{10}bc\int \frac{-e^3x^6-5de^2x^4-15d^2ex^2+5d^3}{x^2\left(c^2x^2+1\right)}dx^2-\frac{d^3(a+b\arctan(cx))}{x}+3d^2ex(a+b\arctan(cx))+de^2x^3(a+b\arctan(cx))+\frac{1}{5}e^3x^5(a+b\arctan(cx))$$

$$\Big\downarrow \; 2123$$

$$\frac{1}{10}bc\int \left(\frac{5d^3}{x^2}-\frac{\left(5c^2d-e\right)e^2}{c^4}-\frac{e^3x^2}{c^2}+\frac{-5d^3c^6-15d^2ec^4+5de^2c^2-e^3}{c^4\left(c^2x^2+1\right)}\right)dx^2-\frac{d^3(a+b\arctan(cx))}{x}+3d^2ex(a+b\arctan(cx))+de^2x^3(a+b\arctan(cx))+\frac{1}{5}e^3x^5(a+b\arctan(cx))$$

$$\Big\downarrow \; 2009$$

$$-\frac{d^3(a+b\arctan(cx))}{x}+3d^2ex(a+b\arctan(cx))+de^2x^3(a+b\arctan(cx))+\frac{1}{5}e^3x^5(a+b\arctan(cx))+\frac{1}{10}bc\left(-\frac{e^3x^4}{2c^2}-\frac{e^2x^2\left(5c^2d-e\right)}{c^4}-\frac{\left(5c^6d^3+15c^4d^2e-5c^2de^2+e^3\right)\log\left(c^2x^2+1\right)}{c^6}+5d^3\log\left(x^2\right)\right)$$

input
```
Int[((d + e*x^2)^3*(a + b*ArcTan[c*x]))/x^2,x]
```

output
```
-((d^3*(a + b*ArcTan[c*x]))/x) + 3*d^2*e*x*(a + b*ArcTan[c*x]) + d*e^2*x^3
*(a + b*ArcTan[c*x]) + (e^3*x^5*(a + b*ArcTan[c*x]))/5 + (b*c*(-(((5*c^2*d
 - e)*e^2*x^2)/c^4) - (e^3*x^4)/(2*c^2) + 5*d^3*Log[x^2] - ((5*c^6*d^3 + 1
5*c^4*d^2*e - 5*c^2*d*e^2 + e^3)*Log[1 + c^2*x^2])/c^6))/10
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

## Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.21

| method | result |
|---|---|
| parts | $a\left(\frac{e^3x^5}{5}+d\,e^2x^3+3d^2ex-\frac{d^3}{x}\right)+bc\left(\frac{\arctan(cx)e^3x^5}{5c}+\frac{\arctan(cx)x^3d\,e^2}{c}+\frac{3\arctan(cx)x\,d^2e}{c}-\right.$ |
| derivativedivides | $c\left(\frac{a\left(3c^5d^2ex+c^5d\,e^2x^3+\frac{e^3c^5x^5}{5}-\frac{c^5d^3}{x}\right)}{c^6}+\frac{b\left(3\arctan(cx)c^5x\,d^2e+\arctan(cx)c^5d\,e^2x^3+\frac{\arctan(cx)e^3c^5x^5}{5}-\right.}{}\right.$ |
| default | $c\left(\frac{a\left(3c^5d^2ex+c^5d\,e^2x^3+\frac{e^3c^5x^5}{5}-\frac{c^5d^3}{x}\right)}{c^6}+\frac{b\left(3\arctan(cx)c^5x\,d^2e+\arctan(cx)c^5d\,e^2x^3+\frac{\arctan(cx)e^3c^5x^5}{5}-\right.}{}\right.$ |
| parallelrisch | $\frac{4x^6\arctan(cx)b\,c^5e^3+4a\,c^5e^3x^6+20x^4\arctan(cx)b\,c^5d\,e^2-b\,c^4e^3x^5+20a\,c^5d\,e^2x^4+20b\,c^6d^3\ln(x)x-10\ln(c^2x^2+1)b}{}$ |
| risch | $\frac{ib\left(-x^6e^3-5x^4e^2d-15x^2e\,d^2+5d^3\right)\ln(icx+1)}{10x}+\frac{10ib\,c^5d\,e^2x^4\ln(-icx+1)+2ib\,c^5e^3x^6\ln(-icx+1)+4a\,c^5e^3x^6-10i}{}$ |

input
```
int((e*x^2+d)^3*(a+b*arctan(c*x))/x^2,x,method=_RETURNVERBOSE)
```

output
```
a*(1/5*e^3*x^5+d*e^2*x^3+3*d^2*e*x-d^3/x)+b*c*(1/5*arctan(c*x)/c*e^3*x^5+a
rctan(c*x)/c*x^3*d*e^2+3*arctan(c*x)/c*x*d^2*e-arctan(c*x)*d^3/c/x-1/5/c^6
*(1/4*c^4*e^3*x^4+5/2*c^4*d*e^2*x^2-1/2*c^2*e^3*x^2+1/2*(5*c^6*d^3+15*c^4*
d^2*e-5*c^2*d*e^2+e^3)*ln(c^2*x^2+1)-5*c^6*d^3*ln(c*x)))
```

## Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.29

$$\int \frac{(d+ex^2)^3\,(a+b\arctan(cx))}{x^2}\,dx$$
$$=\frac{4\,ac^5e^3x^6+20\,ac^5de^2x^4-bc^4e^3x^5+20\,bc^6d^3x\log(x)+60\,ac^5d^2ex^2-20\,ac^5d^3-2\,(5\,bc^4de^2-bc^2e^3)x^3}{}$$

input
```
integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^2,x, algorithm="fricas")
```

output
```
1/20*(4*a*c^5*e^3*x^6 + 20*a*c^5*d*e^2*x^4 - b*c^4*e^3*x^5 + 20*b*c^6*d^3*
x*log(x) + 60*a*c^5*d^2*e*x^2 - 20*a*c^5*d^3 - 2*(5*b*c^4*d*e^2 - b*c^2*e^
3)*x^3 - 2*(5*b*c^6*d^3 + 15*b*c^4*d^2*e - 5*b*c^2*d*e^2 + b*e^3)*x*log(c^
2*x^2 + 1) + 4*(b*c^5*e^3*x^6 + 5*b*c^5*d*e^2*x^4 + 15*b*c^5*d^2*e*x^2 - 5
*b*c^5*d^3)*arctan(c*x))/(c^5*x)
```

## Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.61

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^2}\,dx$$
$$= \begin{cases} -\frac{ad^3}{x} + 3ad^2ex + ade^2x^3 + \frac{ae^3x^5}{5} + bcd^3\log(x) - \frac{bcd^3\log\left(x^2+\frac{1}{c^2}\right)}{2} - \frac{bd^3\operatorname{atan}(cx)}{x} + 3bd^2ex\operatorname{atan}(cx) + bde^2 \\ a\left(-\frac{d^3}{x} + 3d^2ex + de^2x^3 + \frac{e^3x^5}{5}\right) \end{cases}$$

input
```
integrate((e*x**2+d)**3*(a+b*atan(c*x))/x**2,x)
```

output
```
Piecewise((-a*d**3/x + 3*a*d**2*e*x + a*d*e**2*x**3 + a*e**3*x**5/5 + b*c*
d**3*log(x) - b*c*d**3*log(x**2 + c**(-2))/2 - b*d**3*atan(c*x)/x + 3*b*d*
*2*e*x*atan(c*x) + b*d*e**2*x**3*atan(c*x) + b*e**3*x**5*atan(c*x)/5 - 3*b
*d**2*e*log(x**2 + c**(-2))/(2*c) - b*d*e**2*x**2/(2*c) - b*e**3*x**4/(20*
c) + b*d*e**2*log(x**2 + c**(-2))/(2*c**3) + b*e**3*x**2/(10*c**3) - b*e**
3*log(x**2 + c**(-2))/(10*c**5), Ne(c, 0)), (a*(-d**3/x + 3*d**2*e*x + d*e
**2*x**3 + e**3*x**5/5), True))
```

## Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.23

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^2}\,dx$$
$$= \frac{1}{5}ae^3x^5 + ade^2x^3 - \frac{1}{2}\left(c\left(\log\left(c^2x^2+1\right) - \log\left(x^2\right)\right) + \frac{2\arctan(cx)}{x}\right)bd^3$$
$$+ \frac{1}{2}\left(2x^3\arctan(cx) - c\left(\frac{x^2}{c^2} - \frac{\log\left(c^2x^2+1\right)}{c^4}\right)\right)bde^2$$
$$+ \frac{1}{20}\left(4x^5\arctan(cx) - c\left(\frac{c^2x^4-2x^2}{c^4} + \frac{2\log\left(c^2x^2+1\right)}{c^6}\right)\right)be^3$$
$$+ 3ad^2ex + \frac{3\left(2cx\arctan(cx) - \log\left(c^2x^2+1\right)\right)bd^2e}{2c} - \frac{ad^3}{x}$$

input
```
integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^2,x, algorithm="maxima")
```

output
```
1/5*a*e^3*x^5 + a*d*e^2*x^3 - 1/2*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arc
tan(c*x)/x)*b*d^3 + 1/2*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)
/c^4))*b*d*e^2 + 1/20*(4*x^5*arctan(c*x) - c*((c^2*x^4 - 2*x^2)/c^4 + 2*lo
g(c^2*x^2 + 1)/c^6))*b*e^3 + 3*a*d^2*e*x + 3/2*(2*c*x*arctan(c*x) - log(c^
2*x^2 + 1))*b*d^2*e/c - a*d^3/x
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.54

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^2}\,dx$$
$$= \frac{4bc^5e^3x^6\arctan(cx) + 4ac^5e^3x^6 + 20bc^5de^2x^4\arctan(cx) + 20ac^5de^2x^4 - bc^4e^3x^5 + 60bc^5d^2ex^2\arctan}{}$$

input
```
integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^2,x, algorithm="giac")
```

output
```
1/20*(4*b*c^5*e^3*x^6*arctan(c*x) + 4*a*c^5*e^3*x^6 + 20*b*c^5*d*e^2*x^4*a
rctan(c*x) + 20*a*c^5*d*e^2*x^4 - b*c^4*e^3*x^5 + 60*b*c^5*d^2*e*x^2*arcta
n(c*x) - 10*b*c^6*d^3*x*log(c^2*x^2 + 1) + 20*b*c^6*d^3*x*log(x) + 60*a*c^
5*d^2*e*x^2 - 10*b*c^4*d*e^2*x^3 - 20*b*c^5*d^3*arctan(c*x) - 30*b*c^4*d^2
*e*x*log(c^2*x^2 + 1) - 20*a*c^5*d^3 + 2*b*c^2*e^3*x^3 + 10*b*c^2*d*e^2*x*
log(c^2*x^2 + 1) - 2*b*e^3*x*log(c^2*x^2 + 1))/(c^5*x)
```

## Mupad [B] (verification not implemented)

Time = 1.00 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.48

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^2}\,dx = x\left(\frac{\frac{a\,e^3}{c^2}-\frac{a\,e^2\,(3\,d\,c^2+e)}{c^2}}{c^2}+\frac{3\,a\,d\,e\,(d\,c^2+e)}{c^2}\right) - x^3\left(\frac{a\,e^3}{3\,c^2}-\frac{a\,e^2\,(3\,d\,c^2+e)}{3\,c^2}\right)+x^2\left(\frac{b\,e^3}{10\,c^3}-\frac{b\,d\,e^2}{2\,c}\right)-\frac{a\,d^3}{x} + \frac{a\,e^3\,x^5}{5}-\frac{\ln\left(c^2\,x^2+1\right)\,\left(5\,b\,c^6\,d^3+15\,b\,c^4\,d^2\,e-5\,b\,c^2\,d\,e^2+b\,e^3\right)}{10\,c^5} + \frac{\mathrm{atan}(c\,x)\,\left(-b\,d^3+3\,b\,d^2\,e\,x^2+b\,d\,e^2\,x^4+\frac{b\,e^3\,x^6}{5}\right)}{x}-\frac{b\,e^3\,x^4}{20\,c}+b\,c\,d^3\,\ln\left(x\right)$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^3)/x^2,x)
```

output
```
x*(((a*e^3)/c^2 - (a*e^2*(e + 3*c^2*d))/c^2)/c^2 + (3*a*d*e*(e + c^2*d))/c
^2) - x^3*((a*e^3)/(3*c^2) - (a*e^2*(e + 3*c^2*d))/(3*c^2)) + x^2*((b*e^3)
/(10*c^3) - (b*d*e^2)/(2*c)) - (a*d^3)/x + (a*e^3*x^5)/5 - (log(c^2*x^2 +
1)*(b*e^3 + 5*b*c^6*d^3 - 5*b*c^2*d*e^2 + 15*b*c^4*d^2*e))/(10*c^5) + (ata
n(c*x)*((b*e^3*x^6)/5 - b*d^3 + 3*b*d^2*e*x^2 + b*d*e^2*x^4))/x - (b*e^3*x
^4)/(20*c) + b*c*d^3*log(x)
```

## Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.54

$$\int \frac{(d+ex^2)^3(a+b\arctan(cx))}{x^2}\,dx$$
$$= \frac{-20atan(cx)\,b\,c^5d^3+60atan(cx)\,b\,c^5d^2e\,x^2+20atan(cx)\,b\,c^5d\,e^2x^4+4atan(cx)\,b\,c^5e^3x^6-10\log(c^2x^2+}{}$$

input
```
int((e*x^2+d)^3*(a+b*atan(c*x))/x^2,x)
```

output
```
( - 20*atan(c*x)*b*c**5*d**3 + 60*atan(c*x)*b*c**5*d**2*e*x**2 + 20*atan(c
*x)*b*c**5*d*e**2*x**4 + 4*atan(c*x)*b*c**5*e**3*x**6 - 10*log(c**2*x**2 +
 1)*b*c**6*d**3*x - 30*log(c**2*x**2 + 1)*b*c**4*d**2*e*x + 10*log(c**2*x*
*2 + 1)*b*c**2*d*e**2*x - 2*log(c**2*x**2 + 1)*b*e**3*x + 20*log(x)*b*c**6
*d**3*x - 20*a*c**5*d**3 + 60*a*c**5*d**2*e*x**2 + 20*a*c**5*d*e**2*x**4 +
 4*a*c**5*e**3*x**6 - 10*b*c**4*d*e**2*x**3 - b*c**4*e**3*x**5 + 2*b*c**2*
e**3*x**3)/(20*c**5*x)
```

# 3.1142 $\int \frac{(d+ex^2)^3(a+b\arctan(cx))}{x^3}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 200

$$\int \frac{(d+ex^2)^3(a+b\arctan(cx))}{x^3}\,dx = -\frac{bcd^3}{2x}-\frac{3bde^2x}{2c}+\frac{be^3x}{4c^3}-\frac{be^3x^3}{12c}-\frac{1}{2}bc^2d^3\arctan(cx)+\frac{3bde^2\arctan(cx)}{2c^2}-\frac{be^3\arctan(cx)}{4c^4}-\frac{d^3(a+b\arctan(cx))}{2x^2}+\frac{3}{2}de^2x^2(a+b\arctan(cx))+\frac{1}{4}e^3x^4(a+b\arctan(cx))+3ad^2e\log(x)+\frac{3}{2}ibd^2e\,\mathrm{PolyLog}(2,-icx)-\frac{3}{2}ibd^2e\,\mathrm{PolyLog}(2,icx)$$

output
```
-1/2*b*c*d^3/x-3/2*b*d*e^2*x/c+1/4*b*e^3*x/c^3-1/12*b*e^3*x^3/c-1/2*b*c^2*
d^3*arctan(c*x)+3/2*b*d*e^2*arctan(c*x)/c^2-1/4*b*e^3*arctan(c*x)/c^4-1/2*
d^3*(a+b*arctan(c*x))/x^2+3/2*d*e^2*x^2*(a+b*arctan(c*x))+1/4*e^3*x^4*(a+b
*arctan(c*x))+3*a*d^2*e*ln(x)+3/2*I*b*d^2*e*polylog(2,-I*c*x)-3/2*I*b*d^2*
e*polylog(2,I*c*x)
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.11 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.85

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^3} dx = \frac{1}{12}\left(-\frac{18bde^2(cx-\arctan(cx))}{c^2} - \frac{be^3(-3cx+c^3x^3+3\arctan(cx))}{c^4} - \frac{6d^3(a+b\arctan(cx))}{x^2} + 18de^2x^2(a+b\arctan(cx)) + 3e^3x^4(a+b\arctan(cx)) - \frac{6bcd^3\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-c^2x^2\right)}{x} + 36ad^2e\log(x) + 18ibd^2e\,\text{PolyLog}(2,-icx) - 18ibd^2e\,\text{PolyLog}(2,icx)\right)$$

input
```
Integrate[((d + e*x^2)^3*(a + b*ArcTan[c*x]))/x^3,x]
```

output
```
((-18*b*d*e^2*(c*x - ArcTan[c*x]))/c^2 - (b*e^3*(-3*c*x + c^3*x^3 + 3*ArcT
an[c*x]))/c^4 - (6*d^3*(a + b*ArcTan[c*x]))/x^2 + 18*d*e^2*x^2*(a + b*ArcT
an[c*x]) + 3*e^3*x^4*(a + b*ArcTan[c*x]) - (6*b*c*d^3*Hypergeometric2F1[-1
/2, 1, 1/2, -(c^2*x^2)])/x + 36*a*d^2*e*Log[x] + (18*I)*b*d^2*e*PolyLog[2,
 (-I)*c*x] - (18*I)*b*d^2*e*PolyLog[2, I*c*x])/12
```

## Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^3} \, dx$$

$$\Big\downarrow \text{5515}$$

$$\int \left(\frac{d^3(a+b\arctan(cx))}{x^3}+\frac{3d^2e(a+b\arctan(cx))}{x}+3de^2x(a+b\arctan(cx))+e^3x^3(a+b\arctan(cx))\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d^3(a+b\arctan(cx))}{2x^2}+\frac{3}{2}de^2x^2(a+b\arctan(cx))+\frac{1}{4}e^3x^4(a+b\arctan(cx))+3ad^2e\log(x)-\frac{be^3\arctan(cx)}{4c^4}-\frac{1}{2}bc^2d^3\arctan(cx)+\frac{3bde^2\arctan(cx)}{2c^2}+\frac{be^3x}{4c^3}-\frac{bcd^3}{2x}+\frac{3}{2}ibd^2e\,\text{PolyLog}(2,-icx)-\frac{3}{2}ibd^2e\,\text{PolyLog}(2,icx)-\frac{3bde^2x}{2c}-\frac{be^3x^3}{12c}$$

input
```
Int[((d + e*x^2)^3*(a + b*ArcTan[c*x]))/x^3,x]
```

output
```
-1/2*(b*c*d^3)/x - (3*b*d*e^2*x)/(2*c) + (b*e^3*x)/(4*c^3) - (b*e^3*x^3)/(
12*c) - (b*c^2*d^3*ArcTan[c*x])/2 + (3*b*d*e^2*ArcTan[c*x])/(2*c^2) - (b*e
^3*ArcTan[c*x])/(4*c^4) - (d^3*(a + b*ArcTan[c*x]))/(2*x^2) + (3*d*e^2*x^2
*(a + b*ArcTan[c*x]))/2 + (e^3*x^4*(a + b*ArcTan[c*x]))/4 + 3*a*d^2*e*Log[
x] + ((3*I)/2)*b*d^2*e*PolyLog[2, (-I)*c*x] - ((3*I)/2)*b*d^2*e*PolyLog[2,
 I*c*x]
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

## Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.20

| method | result |
|---|---|
| parts | $a\left(\frac{e^{3}x^{4}}{4}+\frac{3d\,e^{2}x^{2}}{2}-\frac{d^{3}}{2x^{2}}+3e\,d^{2}\ln\left(x\right)\right)+b\,c^{2}\left(\frac{\arctan(cx)e^{3}x^{4}}{4c^{2}}+\frac{3\arctan(cx)e^{2}d\,x^{2}}{2c^{2}}-\frac{\arctan(cx)d}{2c^{2}x^{2}}\right.$ |
| derivativedivides | $c^{2}\left(\frac{a\left(\frac{3c^{4}d\,e^{2}x^{2}}{2}+\frac{c^{4}e^{3}x^{4}}{4}+3c^{4}d^{2}e\ln(cx)-\frac{c^{4}d^{3}}{2x^{2}}\right)}{c^{6}}+\frac{b\left(\frac{3\arctan(cx)c^{4}d\,e^{2}x^{2}}{2}+\frac{\arctan(cx)e^{3}c^{4}x^{4}}{4}+3\arctan(cx)c^{4}d^{2}e\ln\right.}{}\right.$ |
| default | $c^{2}\left(\frac{a\left(\frac{3c^{4}d\,e^{2}x^{2}}{2}+\frac{c^{4}e^{3}x^{4}}{4}+3c^{4}d^{2}e\ln(cx)-\frac{c^{4}d^{3}}{2x^{2}}\right)}{c^{6}}+\frac{b\left(\frac{3\arctan(cx)c^{4}d\,e^{2}x^{2}}{2}+\frac{\arctan(cx)e^{3}c^{4}x^{4}}{4}+3\arctan(cx)c^{4}d^{2}e\ln\right.}{}\right.$ |
| risch | $\frac{b\,e^{3}x}{4c^{3}}-\frac{b\,e^{3}x^{3}}{12c}-\frac{b\,e^{3}\arctan(cx)}{4c^{4}}-\frac{3bd\,e^{2}x}{2c}+\frac{3bd\,e^{2}\arctan(cx)}{2c^{2}}-\frac{ib\,d^{3}\ln(-icx+1)}{4x^{2}}-\frac{ib\,e^{3}\ln(icx+1)x^{4}}{8}-$ |

input
```
int((e*x^2+d)^3*(a+b*arctan(c*x))/x^3,x,method=_RETURNVERBOSE)
```

output
```
a*(1/4*e^3*x^4+3/2*d*e^2*x^2-1/2*d^3/x^2+3*e*d^2*ln(x))+b*c^2*(1/4*arctan(
c*x)/c^2*e^3*x^4+3/2*arctan(c*x)/c^2*e^2*d*x^2-1/2*arctan(c*x)*d^3/c^2/x^2
+3*arctan(c*x)/c^2*d^2*e*ln(c*x)-1/4/c^6*(1/3*e^3*c^3*x^3+6*c^3*x*d*e^2-c*
x*e^3+(2*c^6*d^3-6*c^2*d*e^2+e^3)*arctan(c*x)+2*c^5*d^3/x+12*c^4*d^2*e*(-1
/2*I*ln(c*x)*ln(1+I*c*x)+1/2*I*ln(c*x)*ln(1-I*c*x)-1/2*I*dilog(1+I*c*x)+1/
2*I*dilog(1-I*c*x))))
```

## Fricas [F]

$$\int \frac{(d+ex^2)^3(a+b\arctan(cx))}{x^3}\,dx=\int \frac{(ex^2+d)^3(b\arctan(cx)+a)}{x^3}\,dx$$

input
```
integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^3,x, algorithm="fricas")
```

output
```
integral((a*e^3*x^6 + 3*a*d*e^2*x^4 + 3*a*d^2*e*x^2 + a*d^3 + (b*e^3*x^6 +
 3*b*d*e^2*x^4 + 3*b*d^2*e*x^2 + b*d^3)*arctan(c*x))/x^3, x)
```

## Sympy [F]

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^3}\,dx = \int \frac{(a+b\operatorname{atan}(cx))\,(d+ex^2)^3}{x^3}\,dx$$

input
```
integrate((e*x**2+d)**3*(a+b*atan(c*x))/x**3,x)
```

output
```
Integral((a + b*atan(c*x))*(d + e*x**2)**3/x**3, x)
```

## Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.12

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^3}\,dx = \frac{1}{4}\,ae^3x^4 + \frac{3}{2}\,ade^2x^2 - \frac{1}{2}\left(\left(c\arctan(cx) + \frac{1}{x}\right)c + \frac{\arctan(cx)}{x^2}\right)bd^3 + 3\,ad^2e\log(x) - \frac{ad^3}{2\,x^2} - \frac{bc^3e^3x^3 + 9\,\pi bc^4d^2e\log(c^2x^2+1) - 36\,bc^4d^2e\arctan(cx)\log(cx) + 18i\,bc^4d^2e\mathrm{Li}_2(i\,cx+1) - 18i\,bc^4d}{12\,c^4}$$

input
```
integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^3,x, algorithm="maxima")
```

output
```
1/4*a*e^3*x^4 + 3/2*a*d*e^2*x^2 - 1/2*((c*arctan(c*x) + 1/x)*c + arctan(c*
x)/x^2)*b*d^3 + 3*a*d^2*e*log(x) - 1/2*a*d^3/x^2 - 1/12*(b*c^3*e^3*x^3 + 9
*pi*b*c^4*d^2*e*log(c^2*x^2 + 1) - 36*b*c^4*d^2*e*arctan(c*x)*log(c*x) + 1
8*I*b*c^4*d^2*e*dilog(I*c*x + 1) - 18*I*b*c^4*d^2*e*dilog(-I*c*x + 1) + 3*
(6*b*c^3*d*e^2 - b*c*e^3)*x - 3*(b*c^4*e^3*x^4 + 6*b*c^4*d*e^2*x^2 + 6*b*c
^2*d*e^2 - b*e^3)*arctan(c*x))/c^4
```

## Giac [F]

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^3}\,dx = \int \frac{(ex^2+d)^3(b\arctan(cx)+a)}{x^3}\,dx$$

input
```
integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^3,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^3*(b*arctan(c*x) + a)/x^3, x)
```

## Mupad [B] (verification not implemented)

Time = 1.09 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.12

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^3}\,dx = \begin{cases} \frac{a\,e^3\,x^4}{4} - \frac{a\,d^3}{2\,x^2} + \frac{3\,a\,d\,e^2\,x^2}{2} + 3 \\ \frac{a\,e^3\,x^4}{4} - \frac{a\,d^3}{2\,x^2} - \frac{b\,d^3\,\left(c^3\,\mathrm{atan}(c\,x)+\frac{c^2}{x}\right)}{2\,c} - 3\,b\,d\,e^2\,\left(\frac{x}{2\,c} - \mathrm{atan}(c\,x)\,\left(\frac{1}{2\,c^2}+\frac{x^2}{2}\right)\right) + \frac{3\,a\,d\,e^2\,x^2}{2} + 3\,a\,d^2\,e\,\ln(x) - \end{cases}$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^3)/x^3,x)
```

output
```
piecewise(c == 0, - (a*d^3)/(2*x^2) + (a*e^3*x^4)/4 + (3*a*d*e^2*x^2)/2 +
3*a*d^2*e*log(x), c ~= 0, - (a*d^3)/(2*x^2) + (a*e^3*x^4)/4 - (b*d^3*(c^3*
atan(c*x) + c^2/x))/(2*c) - 3*b*d*e^2*(x/(2*c) - atan(c*x)*(1/(2*c^2) + x^
2/2)) + (3*a*d*e^2*x^2)/2 + 3*a*d^2*e*log(x) - (b*e^3*(3*atan(c*x) - 3*c*x
 + c^3*x^3))/(12*c^4) - (b*d^2*e*dilog(- c*x*1i + 1)*3i)/2 + (b*d^2*e*dilo
g(c*x*1i + 1)*3i)/2 - (b*d^3*atan(c*x))/(2*x^2) + (b*e^3*x^4*atan(c*x))/4)
```

## Reduce [F]

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^3}\,dx$$
$$= \frac{-6atan(cx)\,b\,c^6d^3x^2 - 6atan(cx)\,b\,c^4d^3 + 18atan(cx)\,b\,c^4d\,e^2x^4 + 3atan(cx)\,b\,c^4e^3x^6 + 18atan(cx)\,b\,c^2d}{}$$

input
```
int((e*x^2+d)^3*(a+b*atan(c*x))/x^3,x)
```

output
```
( - 6*atan(c*x)*b*c**6*d**3*x**2 - 6*atan(c*x)*b*c**4*d**3 + 18*atan(c*x)*
b*c**4*d*e**2*x**4 + 3*atan(c*x)*b*c**4*e**3*x**6 + 18*atan(c*x)*b*c**2*d*
e**2*x**2 - 3*atan(c*x)*b*e**3*x**2 + 36*int(atan(c*x)/x,x)*b*c**4*d**2*e*
x**2 + 36*log(x)*a*c**4*d**2*e*x**2 - 6*a*c**4*d**3 + 18*a*c**4*d*e**2*x**
4 + 3*a*c**4*e**3*x**6 - 6*b*c**5*d**3*x - 18*b*c**3*d*e**2*x**3 - b*c**3*
e**3*x**5 + 3*b*c*e**3*x**3)/(12*c**4*x**2)
```

# 3.1143 $\int \frac{(d+ex^2)^3(a+b\arctan(cx))}{x^4}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 158

$$\int \frac{(d+ex^2)^3(a+b\arctan(cx))}{x^4}\,dx = -\frac{bcd^3}{6x^2}-\frac{be^3x^2}{6c}-\frac{d^3(a+b\arctan(cx))}{3x^3} - \frac{3d^2e(a+b\arctan(cx))}{x}+3de^2x(a+b\arctan(cx)) + \frac{1}{3}e^3x^3(a+b\arctan(cx))-\frac{1}{3}bcd^2\left(c^2d-9e\right)\log(x) + \frac{b(c^2d+e)\left(c^4d^2-10c^2de+e^2\right)\log\left(1+c^2x^2\right)}{6c^3}$$

output
```
-1/6*b*c*d^3/x^2-1/6*b*e^3*x^2/c-1/3*d^3*(a+b*arctan(c*x))/x^3-3*d^2*e*(a+
b*arctan(c*x))/x+3*d*e^2*x*(a+b*arctan(c*x))+1/3*e^3*x^3*(a+b*arctan(c*x))
-1/3*b*c*d^2*(c^2*d-9*e)*ln(x)+1/6*b*(c^2*d+e)*(c^4*d^2-10*c^2*d*e+e^2)*ln
(c^2*x^2+1)/c^3
```

## Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.05

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^4}\,dx = \frac{1}{6}\left(-\frac{2ad^3}{x^3}-\frac{bcd^3}{x^2}-\frac{18ad^2e}{x}+18ade^2x-\frac{be^3x^2}{c}+2ae^3x^3+\frac{2b(-d^3-9d^2ex^2+9de^2x^4+e^3x^6)\arctan(cx)}{x^3}-2bcd^2(c^2d-9e)\log(x)+\frac{b(c^6d^3-9c^4d^2e-9c^2de^2+e^3)\log(1+c^2x^2)}{c^3}\right)$$

input
```
Integrate[((d + e*x^2)^3*(a + b*ArcTan[c*x]))/x^4,x]
```

output
```
((-2*a*d^3)/x^3 - (b*c*d^3)/x^2 - (18*a*d^2*e)/x + 18*a*d*e^2*x - (b*e^3*x
^2)/c + 2*a*e^3*x^3 + (2*b*(-d^3 - 9*d^2*e*x^2 + 9*d*e^2*x^4 + e^3*x^6)*Ar
cTan[c*x])/x^3 - 2*b*c*d^2*(c^2*d - 9*e)*Log[x] + (b*(c^6*d^3 - 9*c^4*d^2*
e - 9*c^2*d*e^2 + e^3)*Log[1 + c^2*x^2])/c^3)/6
```

## Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5511, 27, 2331, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^4}\,dx$$

$$\Big\downarrow \text{ 5511}$$

$$-bc\int -\frac{-e^3x^6-9de^2x^4+9d^2ex^2+d^3}{3x^3(c^2x^2+1)}dx-\frac{d^3(a+b\arctan(cx))}{3x^3}-\frac{3d^2e(a+b\arctan(cx))}{x}+3de^2x(a+b\arctan(cx))+\frac{1}{3}e^3x^3(a+b\arctan(cx))$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}bc\int \frac{-e^3x^6-9de^2x^4+9d^2ex^2+d^3}{x^3\left(c^2x^2+1\right)}dx-\frac{d^3(a+b\arctan(cx))}{3x^3}-\frac{3d^2e(a+b\arctan(cx))}{x}+3de^2x(a+b\arctan(cx))+\frac{1}{3}e^3x^3(a+b\arctan(cx))$$

$$\Big\downarrow \text{2331}$$

$$\frac{1}{6}bc\int \frac{-e^3x^6-9de^2x^4+9d^2ex^2+d^3}{x^4\left(c^2x^2+1\right)}dx^2-\frac{d^3(a+b\arctan(cx))}{3x^3}-\frac{3d^2e(a+b\arctan(cx))}{x}+3de^2x(a+b\arctan(cx))+\frac{1}{3}e^3x^3(a+b\arctan(cx))$$

$$\Big\downarrow \text{2123}$$

$$\frac{1}{6}bc\int \left(\frac{d^3}{x^4}-\frac{\left(c^2d-9e\right)d^2}{x^2}-\frac{e^3}{c^2}+\frac{\left(dc^2+e\right)\left(d^2c^4-10dec^2+e^2\right)}{c^2\left(c^2x^2+1\right)}\right)dx^2-\frac{d^3(a+b\arctan(cx))}{3x^3}-\frac{3d^2e(a+b\arctan(cx))}{x}+3de^2x(a+b\arctan(cx))+\frac{1}{3}e^3x^3(a+b\arctan(cx))$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d^3(a+b\arctan(cx))}{3x^3}-\frac{3d^2e(a+b\arctan(cx))}{x}+3de^2x(a+b\arctan(cx))+\frac{1}{3}e^3x^3(a+b\arctan(cx))+\frac{1}{6}bc\left(-d^2\log\left(x^2\right)\left(c^2d-9e\right)-\frac{e^3x^2}{c^2}+\frac{\left(c^2d+e\right)\left(c^4d^2-10c^2de+e^2\right)\log\left(c^2x^2+1\right)}{c^4}-\frac{d^3}{x^2}\right)$$

input
```
Int[((d + e*x^2)^3*(a + b*ArcTan[c*x]))/x^4,x]
```

output
```
-1/3*(d^3*(a + b*ArcTan[c*x]))/x^3 - (3*d^2*e*(a + b*ArcTan[c*x]))/x + 3*d
*e^2*x*(a + b*ArcTan[c*x]) + (e^3*x^3*(a + b*ArcTan[c*x]))/3 + (b*c*(-(d^3
/x^2) - (e^3*x^2)/c^2 - d^2*(c^2*d - 9*e)*Log[x^2] + ((c^2*d + e)*(c^4*d^2
 - 10*c^2*d*e + e^2)*Log[1 + c^2*x^2])/c^4))/6
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

## Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.23

| method | result |
|---|---|
| parts | $a\left(\frac{e^3x^3}{3}+3d\,e^2x-\frac{d^3}{3x^3}-\frac{3e\,d^2}{x}\right)+b\,c^3\left(\frac{\arctan(cx)x^3e^3}{3c^3}+\frac{3\arctan(cx)x\,e^2d}{c^3}-\frac{\arctan(cx)d^3}{3c^3x^3}-\frac{3\,\text{ar}}{}\right.$ |
| derivativedivides | $c^3\left(\frac{a\left(3c^3xd\,e^2+\frac{e^3c^3x^3}{3}-\frac{3c^3d^2e}{x}-\frac{c^3d^3}{3x^3}\right)}{c^6}+\frac{b\left(3\arctan(cx)c^3xd\,e^2+\frac{\arctan(cx)e^3c^3x^3}{3}-\frac{3\arctan(cx)c^3d^2e}{x}-\frac{\arctan(c}{3x^3}\right.}{}\right.$ |
| default | $c^3\left(\frac{a\left(3c^3xd\,e^2+\frac{e^3c^3x^3}{3}-\frac{3c^3d^2e}{x}-\frac{c^3d^3}{3x^3}\right)}{c^6}+\frac{b\left(3\arctan(cx)c^3xd\,e^2+\frac{\arctan(cx)e^3c^3x^3}{3}-\frac{3\arctan(cx)c^3d^2e}{x}-\frac{\arctan(c}{3x^3}\right.}{}\right.$ |
| parallelrisch | $-\frac{2\ln(x)b\,c^6d^3x^3-\ln(c^2x^2+1)x^3b\,c^6d^3-2x^6\arctan(cx)b\,c^3e^3-2a\,c^3e^3x^6-b\,c^6d^3x^3-18\ln(x)b\,c^4d^2e\,x^3+9\ln(c^2x^2+1}{}$ |
| risch | $\frac{ib\left(-x^6e^3-9x^4e^2d+9x^2e\,d^2+d^3\right)\ln(icx+1)}{6x^3}-\frac{-ib\,c^3e^3x^6\ln(-icx+1)+2\ln(x)b\,c^6d^3x^3-\ln(-c^2x^2-1)b\,c^6d^3x^3-9ib}{}$ |

input
```
int((e*x^2+d)^3*(a+b*arctan(c*x))/x^4,x,method=_RETURNVERBOSE)
```

output
```
a*(1/3*e^3*x^3+3*d*e^2*x-1/3*d^3/x^3-3*e*d^2/x)+b*c^3*(1/3*arctan(c*x)/c^3
*x^3*e^3+3*arctan(c*x)/c^3*x*e^2*d-1/3*arctan(c*x)*d^3/c^3/x^3-3*arctan(c*
x)/c^3*d^2*e/x-1/3/c^6*(1/2*c^2*e^3*x^2+1/2*(-c^6*d^3+9*c^4*d^2*e+9*c^2*d*
e^2-e^3)*ln(c^2*x^2+1)+c^4*d^2*(c^2*d-9*e)*ln(c*x)+1/2*c^4*d^3/x^2))
```

## Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.30

$$\int \frac{(d+ex^2)^3\,(a+b\arctan(cx))}{x^4}\,dx$$
$$=\frac{2\,ac^3e^3x^6+18\,ac^3de^2x^4-bc^2e^3x^5-bc^4d^3x-18\,ac^3d^2ex^2-2\,ac^3d^3+(bc^6d^3-9\,bc^4d^2e-9\,bc^2de^2+be^3}{}$$

input
```
integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^4,x, algorithm="fricas")
```

output
```
1/6*(2*a*c^3*e^3*x^6 + 18*a*c^3*d*e^2*x^4 - b*c^2*e^3*x^5 - b*c^4*d^3*x -
18*a*c^3*d^2*e*x^2 - 2*a*c^3*d^3 + (b*c^6*d^3 - 9*b*c^4*d^2*e - 9*b*c^2*d*
e^2 + b*e^3)*x^3*log(c^2*x^2 + 1) - 2*(b*c^6*d^3 - 9*b*c^4*d^2*e)*x^3*log(
x) + 2*(b*c^3*e^3*x^6 + 9*b*c^3*d*e^2*x^4 - 9*b*c^3*d^2*e*x^2 - b*c^3*d^3)
*arctan(c*x))/(c^3*x^3)
```

## Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.72

$$\int \frac{(d+ex^2)^3(a+b\arctan(cx))}{x^4}\,dx = \begin{cases} -\frac{ad^3}{3x^3} - \frac{3ad^2e}{x} + 3ade^2x + \frac{ae^3x^3}{3} - \frac{bc^3d^3\log(x)}{3} + \frac{bc^3d^3\log\left(x^2+\frac{1}{c^2}\right)}{6} - \frac{bcd^3}{6x^2} + 3bcd^2e\log(x) - \frac{3bcd^2e\log\left(x^2+\frac{1}{c^2}\right)}{2} \\ a\left(-\frac{d^3}{3x^3} - \frac{3d^2e}{x} + 3de^2x + \frac{e^3x^3}{3}\right) \end{cases}$$

input
```
integrate((e*x**2+d)**3*(a+b*atan(c*x))/x**4,x)
```

output
```
Piecewise((-a*d**3/(3*x**3) - 3*a*d**2*e/x + 3*a*d*e**2*x + a*e**3*x**3/3
- b*c**3*d**3*log(x)/3 + b*c**3*d**3*log(x**2 + c**(-2))/6 - b*c*d**3/(6*x
**2) + 3*b*c*d**2*e*log(x) - 3*b*c*d**2*e*log(x**2 + c**(-2))/2 - b*d**3*a
tan(c*x)/(3*x**3) - 3*b*d**2*e*atan(c*x)/x + 3*b*d*e**2*x*atan(c*x) + b*e*
*3*x**3*atan(c*x)/3 - 3*b*d*e**2*log(x**2 + c**(-2))/(2*c) - b*e**3*x**2/(
6*c) + b*e**3*log(x**2 + c**(-2))/(6*c**3), Ne(c, 0)), (a*(-d**3/(3*x**3)
- 3*d**2*e/x + 3*d*e**2*x + e**3*x**3/3), True))
```

## Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.22

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^4}\,dx$$
$$= \frac{1}{3}ae^3x^3 + \frac{1}{6}\left(\left(c^2\log\left(c^2x^2+1\right) - c^2\log\left(x^2\right) - \frac{1}{x^2}\right)c - \frac{2\arctan(cx)}{x^3}\right)bd^3 - \frac{3}{2}\left(c\left(\log\left(c^2x^2+1\right) - \log\left(x^2\right)\right) + \frac{2\arctan(cx)}{x}\right)bd^2e + \frac{1}{6}\left(2x^3\arctan(cx) - c\left(\frac{x^2}{c^2} - \frac{\log\left(c^2x^2+1\right)}{c^4}\right)\right)be^3 + 3ade^2x + \frac{3\left(2cx\arctan(cx) - \log\left(c^2x^2+1\right)\right)bde^2}{2c} - \frac{3ad^2e}{x} - \frac{ad^3}{3x^3}$$

input
```
integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^4,x, algorithm="maxima")
```

output
```
1/3*a*e^3*x^3 + 1/6*((c^2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*a
rctan(c*x)/x^3)*b*d^3 - 3/2*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*
x)/x)*b*d^2*e + 1/6*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4
))*b*e^3 + 3*a*d*e^2*x + 3/2*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b*d*e^
2/c - 3*a*d^2*e/x - 1/3*a*d^3/x^3
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.61

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^4}\,dx$$
$$= \frac{2bc^3e^3x^6\arctan(cx) + bc^6d^3x^3\log\left(c^2x^2+1\right) - 2bc^6d^3x^3\log(x) + 2ac^3e^3x^6 + 18bc^3de^2x^4\arctan(cx) - \cdots}{\cdots}$$

input
```
integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^4,x, algorithm="giac")
```

output
```
1/6*(2*b*c^3*e^3*x^6*arctan(c*x) + b*c^6*d^3*x^3*log(c^2*x^2 + 1) - 2*b*c^
6*d^3*x^3*log(x) + 2*a*c^3*e^3*x^6 + 18*b*c^3*d*e^2*x^4*arctan(c*x) - 9*b*
c^4*d^2*e*x^3*log(c^2*x^2 + 1) + 18*b*c^4*d^2*e*x^3*log(x) + 18*a*c^3*d*e^
2*x^4 - b*c^2*e^3*x^5 - 18*b*c^3*d^2*e*x^2*arctan(c*x) - 9*b*c^2*d*e^2*x^3
*log(c^2*x^2 + 1) - b*c^4*d^3*x - 18*a*c^3*d^2*e*x^2 - 2*b*c^3*d^3*arctan(
c*x) + b*e^3*x^3*log(c^2*x^2 + 1) - 2*a*c^3*d^3)/(c^3*x^3)
```

## Mupad [B] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.28

$$\int \frac{(d+ex^2)^3\,(a+b\arctan(cx))}{x^4}\,dx$$
$$= \frac{a\,e^3\,x^3}{3} - \ln(x)\,\left(\frac{b\,c^3\,d^3}{3} - 3\,b\,c\,d^2\,e\right) - \frac{\frac{b\,c^2\,d^3\,x}{2} + a\,c\,d^3 + 9\,a\,e\,c\,d^2\,x^2}{3\,c\,x^3}$$
$$- x\left(\frac{a\,e^3}{c^2} - \frac{a\,e^2\,(3\,d\,c^2+e)}{c^2}\right) + \frac{\ln(c^2\,x^2+1)\,\left(b\,c^6\,d^3 - 9\,b\,c^4\,d^2\,e - 9\,b\,c^2\,d\,e^2 + b\,e^3\right)}{6\,c^3}$$
$$- \frac{\mathrm{atan}(c\,x)\,\left(\frac{b\,d^3}{3} + 3\,b\,d^2\,e\,x^2 - 3\,b\,d\,e^2\,x^4 - \frac{b\,e^3\,x^6}{3}\right)}{x^3} - \frac{b\,e^3\,x^2}{6\,c}$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^3)/x^4,x)
```

output
```
(a*e^3*x^3)/3 - log(x)*((b*c^3*d^3)/3 - 3*b*c*d^2*e) - (a*c*d^3 + (b*c^2*d
^3*x)/2 + 9*a*c*d^2*e*x^2)/(3*c*x^3) - x*((a*e^3)/c^2 - (a*e^2*(e + 3*c^2*
d))/c^2) + (log(c^2*x^2 + 1)*(b*e^3 + b*c^6*d^3 - 9*b*c^2*d*e^2 - 9*b*c^4*
d^2*e))/(6*c^3) - (atan(c*x)*((b*d^3)/3 - (b*e^3*x^6)/3 + 3*b*d^2*e*x^2 -
3*b*d*e^2*x^4))/x^3 - (b*e^3*x^2)/(6*c)
```

## Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.61

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^4}\,dx$$
$$= \frac{-2atan(cx)\,b\,c^3d^3 - 18atan(cx)\,b\,c^3d^2e\,x^2 + 18atan(cx)\,b\,c^3d\,e^2x^4 + 2atan(cx)\,b\,c^3e^3x^6 + \log(c^2x^2+1)\,b}{}$$

input
```
int((e*x^2+d)^3*(a+b*atan(c*x))/x^4,x)
```

output
```
( - 2*atan(c*x)*b*c**3*d**3 - 18*atan(c*x)*b*c**3*d**2*e*x**2 + 18*atan(c*
x)*b*c**3*d*e**2*x**4 + 2*atan(c*x)*b*c**3*e**3*x**6 + log(c**2*x**2 + 1)*
b*c**6*d**3*x**3 - 9*log(c**2*x**2 + 1)*b*c**4*d**2*e*x**3 - 9*log(c**2*x*
*2 + 1)*b*c**2*d*e**2*x**3 + log(c**2*x**2 + 1)*b*e**3*x**3 - 2*log(x)*b*c
**6*d**3*x**3 + 18*log(x)*b*c**4*d**2*e*x**3 - 2*a*c**3*d**3 - 18*a*c**3*d
**2*e*x**2 + 18*a*c**3*d*e**2*x**4 + 2*a*c**3*e**3*x**6 - b*c**4*d**3*x -
b*c**2*e**3*x**5)/(6*c**3*x**3)
```

# 3.1144 $\int \frac{(d+ex^2)^3(a+b\arctan(cx))}{x^5}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 200

$$\int \frac{(d+ex^2)^3(a+b\arctan(cx))}{x^5}\,dx = -\frac{bcd^3}{12x^3}+\frac{bc^3d^3}{4x}-\frac{3bcd^2e}{2x}-\frac{be^3x}{2c}+\frac{1}{4}bc^4d^3\arctan(cx) - \frac{3}{2}bc^2d^2e\arctan(cx)+\frac{be^3\arctan(cx)}{2c^2} - \frac{d^3(a+b\arctan(cx))}{4x^4}-\frac{3d^2e(a+b\arctan(cx))}{2x^2} + \frac{1}{2}e^3x^2(a+b\arctan(cx))+3ade^2\log(x) + \frac{3}{2}ibde^2\,\mathrm{PolyLog}(2,-icx) - \frac{3}{2}ibde^2\,\mathrm{PolyLog}(2,icx)$$

output

```
-1/12*b*c*d^3/x^3+1/4*b*c^3*d^3/x-3/2*b*c*d^2*e/x-1/2*b*e^3*x/c+1/4*b*c^4*
d^3*arctan(c*x)-3/2*b*c^2*d^2*e*arctan(c*x)+1/2*b*e^3*arctan(c*x)/c^2-1/4*
d^3*(a+b*arctan(c*x))/x^4-3/2*d^2*e*(a+b*arctan(c*x))/x^2+1/2*e^3*x^2*(a+b
*arctan(c*x))+3*a*d*e^2*ln(x)+3/2*I*b*d*e^2*polylog(2,-I*c*x)-3/2*I*b*d*e^
2*polylog(2,I*c*x)
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.15 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.84

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^5}\,dx = \frac{1}{12}\left(-\frac{6be^3(cx-\arctan(cx))}{c^2} - \frac{3d^3(a+b\arctan(cx))}{x^4} - \frac{18d^2e(a+b\arctan(cx))}{x^2} + 6e^3x^2(a+b\arctan(cx)) - \frac{bcd^3\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-c^2x^2\right)}{x^3} - \frac{18bcd^2e\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-c^2x^2\right)}{x} + 36ade^2\log(x) + 18ibde^2\,\text{PolyLog}(2,-icx) - 18ibde^2\,\text{PolyLog}(2,icx)\right)$$

input
```
Integrate[((d + e*x^2)^3*(a + b*ArcTan[c*x]))/x^5,x]
```

output
```
((-6*b*e^3*(c*x - ArcTan[c*x]))/c^2 - (3*d^3*(a + b*ArcTan[c*x]))/x^4 - (1
8*d^2*e*(a + b*ArcTan[c*x]))/x^2 + 6*e^3*x^2*(a + b*ArcTan[c*x]) - (b*c*d^
3*Hypergeometric2F1[-3/2, 1, -1/2, -(c^2*x^2)])/x^3 - (18*b*c*d^2*e*Hyperg
eometric2F1[-1/2, 1, 1/2, -(c^2*x^2)])/x + 36*a*d*e^2*Log[x] + (18*I)*b*d*
e^2*PolyLog[2, (-I)*c*x] - (18*I)*b*d*e^2*PolyLog[2, I*c*x])/12
```

## Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^5} \, dx$$

$\Big\downarrow$ 5515

$$\int \left(\frac{d^3(a+b\arctan(cx))}{x^5}+\frac{3d^2e(a+b\arctan(cx))}{x^3}+\frac{3de^2(a+b\arctan(cx))}{x}+e^3x(a+b\arctan(cx))\right) dx$$

$\Big\downarrow$ 2009

$$-\frac{d^3(a+b\arctan(cx))}{4x^4}-\frac{3d^2e(a+b\arctan(cx))}{2x^2}+\frac{1}{2}e^3x^2(a+b\arctan(cx))+3ade^2\log(x)+\frac{1}{4}bc^4d^3\arctan(cx)-\frac{3}{2}bc^2d^2e\arctan(cx)+\frac{be^3\arctan(cx)}{2c^2}+\frac{bc^3d^3}{4x}-\frac{bcd^3}{12x^3}-\frac{3bcd^2e}{2x}+\frac{3}{2}ibde^2\operatorname{PolyLog}(2,-icx)-\frac{3}{2}ibde^2\operatorname{PolyLog}(2,icx)-\frac{be^3x}{2c}$$

input
```
Int[((d + e*x^2)^3*(a + b*ArcTan[c*x]))/x^5,x]
```

output
```
-1/12*(b*c*d^3)/x^3 + (b*c^3*d^3)/(4*x) - (3*b*c*d^2*e)/(2*x) - (b*e^3*x)/
(2*c) + (b*c^4*d^3*ArcTan[c*x])/4 - (3*b*c^2*d^2*e*ArcTan[c*x])/2 + (b*e^3
*ArcTan[c*x])/(2*c^2) - (d^3*(a + b*ArcTan[c*x]))/(4*x^4) - (3*d^2*e*(a +
b*ArcTan[c*x]))/(2*x^2) + (e^3*x^2*(a + b*ArcTan[c*x]))/2 + 3*a*d*e^2*Log[
x] + ((3*I)/2)*b*d*e^2*PolyLog[2, (-I)*c*x] - ((3*I)/2)*b*d*e^2*PolyLog[2,
 I*c*x]
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

## Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.20

| method | result |
|---|---|
| parts | $a\left(\frac{e^3x^2}{2}-\frac{d^3}{4x^4}-\frac{3e\,d^2}{2x^2}+3e^2d\ln\left(x\right)\right)+b\,c^4\left(\frac{\arctan(cx)x^2e^3}{2c^4}-\frac{\arctan(cx)d^3}{4c^4x^4}-\frac{3\arctan(cx)d^2e}{2c^4x^2}+\right.$ |
| derivativedivides | $c^4\left(\frac{a\left(\frac{c^2e^3x^2}{2}-\frac{3c^2d^2e}{2x^2}+3c^2d\,e^2\ln(cx)-\frac{c^2d^3}{4x^4}\right)}{c^6}+\frac{b\left(\frac{\arctan(cx)c^2x^2e^3}{2}-\frac{3\arctan(cx)c^2d^2e}{2x^2}+3\arctan(cx)c^2d\,e^2\ln(cx)\right.}{}\right.$ |
| default | $c^4\left(\frac{a\left(\frac{c^2e^3x^2}{2}-\frac{3c^2d^2e}{2x^2}+3c^2d\,e^2\ln(cx)-\frac{c^2d^3}{4x^4}\right)}{c^6}+\frac{b\left(\frac{\arctan(cx)c^2x^2e^3}{2}-\frac{3\arctan(cx)c^2d^2e}{2x^2}+3\arctan(cx)c^2d\,e^2\ln(cx)\right.}{}\right.$ |
| risch | $-\frac{b\,e^3x}{2c}+\frac{b\,c^4d^3\arctan(cx)}{8}+\frac{b\,e^3\arctan(cx)}{4c^2}-\frac{3bc\,d^2e}{2x}-\frac{3b\,c^2d^2e\arctan(cx)}{4}-\frac{bc\,d^3}{12x^3}+\frac{b\,c^3d^3}{4x}+\frac{3ib\,c^2e}{}$ |

input
```
int((e*x^2+d)^3*(a+b*arctan(c*x))/x^5,x,method=_RETURNVERBOSE)
```

output
```
a*(1/2*e^3*x^2-1/4*d^3/x^4-3/2*e*d^2/x^2+3*e^2*d*ln(x))+b*c^4*(1/2*arctan(
c*x)/c^4*x^2*e^3-1/4*arctan(c*x)*d^3/c^4/x^4-3/2*arctan(c*x)/c^4*d^2*e/x^2
+3*arctan(c*x)/c^4*e^2*d*ln(c*x)-1/4/c^6*(2*c*x*e^3+(-c^6*d^3+6*c^4*d^2*e-
2*e^3)*arctan(c*x)-c^3*d^2*(c^2*d-6*e)/x+1/3*c^3*d^3/x^3+12*c^2*d*e^2*(-1/
2*I*ln(c*x)*ln(1+I*c*x)+1/2*I*ln(c*x)*ln(1-I*c*x)-1/2*I*dilog(1+I*c*x)+1/2
*I*dilog(1-I*c*x))))
```

## Fricas [F]

$$\int\frac{(d+ex^2)^3\,(a+b\arctan(cx))}{x^5}\,dx=\int\frac{(ex^2+d)^3(b\arctan\left(cx\right)+a)}{x^5}\,dx$$

input
```
integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^5,x, algorithm="fricas")
```

output
```
integral((a*e^3*x^6 + 3*a*d*e^2*x^4 + 3*a*d^2*e*x^2 + a*d^3 + (b*e^3*x^6 +
 3*b*d*e^2*x^4 + 3*b*d^2*e*x^2 + b*d^3)*arctan(c*x))/x^5, x)
```

## Sympy [F]

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^5}\,dx = \int \frac{(a+b\operatorname{atan}(cx))\,(d+ex^2)^3}{x^5}\,dx$$

input
```
integrate((e*x**2+d)**3*(a+b*atan(c*x))/x**5,x)
```

output
```
Integral((a + b*atan(c*x))*(d + e*x**2)**3/x**5, x)
```

## Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.09

$$\begin{aligned}
&\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^5}\,dx \\
&= \frac{1}{2}ae^3x^2 + \frac{1}{12}\left(\left(3\,c^3\arctan(cx) + \frac{3\,c^2x^2-1}{x^3}\right)c - \frac{3\,\arctan(cx)}{x^4}\right)bd^3 \\
&\quad - \frac{3}{2}\left(\left(c\arctan(cx) + \frac{1}{x}\right)c + \frac{\arctan(cx)}{x^2}\right)bd^2e + 3\,ade^2\log(x) - \frac{3\,ad^2e}{2\,x^2} - \frac{ad^3}{4\,x^4} \\
&\quad - \frac{3\,\pi bc^2de^2\log(c^2x^2+1) - 12\,bc^2de^2\arctan(cx)\log(cx) + 6i\,bc^2de^2\mathrm{Li}_2(i\,cx+1) - 6i\,bc^2de^2\mathrm{Li}_2(-i\,cx + \dots}{4\,c^2}
\end{aligned}$$

input
```
integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^5,x, algorithm="maxima")
```

output
```
1/2*a*e^3*x^2 + 1/12*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arct
an(c*x)/x^4)*b*d^3 - 3/2*((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b*d^2
*e + 3*a*d*e^2*log(x) - 3/2*a*d^2*e/x^2 - 1/4*a*d^3/x^4 - 1/4*(3*pi*b*c^2*
d*e^2*log(c^2*x^2 + 1) - 12*b*c^2*d*e^2*arctan(c*x)*log(c*x) + 6*I*b*c^2*d
*e^2*dilog(I*c*x + 1) - 6*I*b*c^2*d*e^2*dilog(-I*c*x + 1) + 2*b*c*e^3*x -
2*(b*c^2*e^3*x^2 + b*e^3)*arctan(c*x))/c^2
```

## Giac [F]

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^5}\,dx = \int \frac{(ex^2+d)^3(b\arctan(cx)+a)}{x^5}\,dx$$

input
```
integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^5,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^3*(b*arctan(c*x) + a)/x^5, x)
```

## Mupad [B] (verification not implemented)

Time = 1.07 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^5}\,dx$$
$$= \begin{cases} -\frac{a\,(d^3-2\,e^3\,x^6+6\,d^2\,e\,x^2-12\,d\,e^2\,x}{4\,x^4} \\ -b\,e^3\left(\frac{x}{2\,c}-\mathrm{atan}(c\,x)\left(\frac{1}{2\,c^2}+\frac{x^2}{2}\right)\right)-\frac{a\,(d^3-2\,e^3\,x^6+6\,d^2\,e\,x^2-12\,d\,e^2\,x^4\,\ln(x))}{4\,x^4}-\frac{b\,d^3\left(\frac{\frac{c^2}{3}-c^4\,x^2}{x^3}-c^5\,\mathrm{atan}(c\,x)\right)}{4\,c}-3\,b \end{cases}$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^3)/x^5,x)
```

output
```
piecewise(c == 0, -(a*(d^3 - 2*e^3*x^6 + 6*d^2*e*x^2 - 12*d*e^2*x^4*log(x)
))/(4*x^4), c ~= 0, - b*e^3*(x/(2*c) - atan(c*x)*(1/(2*c^2) + x^2/2)) - (a
*(d^3 - 2*e^3*x^6 + 6*d^2*e*x^2 - 12*d*e^2*x^4*log(x)))/(4*x^4) - (b*d^3*(
(c^2/3 - c^4*x^2)/x^3 - c^5*atan(c*x)))/(4*c) - 3*b*d^2*e*((c^3*atan(c*x)
+ c^2/x)/(2*c) + atan(c*x)/(2*x^2)) - (b*d*e^2*dilog(- c*x*1i + 1)*3i)/2 +
 (b*d*e^2*dilog(c*x*1i + 1)*3i)/2 - (b*d^3*atan(c*x))/(4*x^4))
```

## Reduce [F]

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^5} dx$$
$$= \frac{3atan(cx)\, b\, c^6 d^3 x^4 - 18atan(cx)\, b\, c^4 d^2 e\, x^4 - 3atan(cx)\, b\, c^2 d^3 - 18atan(cx)\, b\, c^2 d^2 e\, x^2 + 6atan(cx)\, b\, c^2 e^3}{}$$

input
```
int((e*x^2+d)^3*(a+b*atan(c*x))/x^5,x)
```

output
```
(3*atan(c*x)*b*c**6*d**3*x**4 - 18*atan(c*x)*b*c**4*d**2*e*x**4 - 3*atan(c
*x)*b*c**2*d**3 - 18*atan(c*x)*b*c**2*d**2*e*x**2 + 6*atan(c*x)*b*c**2*e**
3*x**6 + 6*atan(c*x)*b*e**3*x**4 + 36*int(atan(c*x)/x,x)*b*c**2*d*e**2*x**
4 + 36*log(x)*a*c**2*d*e**2*x**4 - 3*a*c**2*d**3 - 18*a*c**2*d**2*e*x**2 +
 6*a*c**2*e**3*x**6 + 3*b*c**5*d**3*x**3 - b*c**3*d**3*x - 18*b*c**3*d**2*
e*x**3 - 6*b*c*e**3*x**5)/(12*c**2*x**4)
```

# 3.1145 $\int \frac{(d+ex^2)^3(a+b\arctan(cx))}{x^6}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 177

$$\int \frac{(d+ex^2)^3(a+b\arctan(cx))}{x^6}\,dx = -\frac{bcd^3}{20x^4}+\frac{bcd^2(c^2d-5e)}{10x^2} - \frac{d^3(a+b\arctan(cx))}{5x^5}-\frac{d^2e(a+b\arctan(cx))}{x^3} - \frac{3de^2(a+b\arctan(cx))}{x}+e^3x(a+b\arctan(cx)) + \frac{1}{5}bcd(c^4d^2-5c^2de+15e^2)\log(x) - \frac{b(c^6d^3-5c^4d^2e+15c^2de^2+5e^3)\log(1+c^2x^2)}{10c}$$

output
```
-1/20*b*c*d^3/x^4+1/10*b*c*d^2*(c^2*d-5*e)/x^2-1/5*d^3*(a+b*arctan(c*x))/x
^5-d^2*e*(a+b*arctan(c*x))/x^3-3*d*e^2*(a+b*arctan(c*x))/x+e^3*x*(a+b*arct
an(c*x))+1/5*b*c*d*(c^4*d^2-5*c^2*d*e+15*e^2)*ln(x)-1/10*b*(c^6*d^3-5*c^4*
d^2*e+15*c^2*d*e^2+5*e^3)*ln(c^2*x^2+1)/c
```

## Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.04

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^6} dx$$
$$= \frac{1}{20}\left(-\frac{4ad^3}{x^5}-\frac{bcd^3}{x^4}-\frac{20ad^2e}{x^3}+\frac{2bcd^2(c^2d-5e)}{x^2}-\frac{60ade^2}{x}+20ae^3x\right.$$
$$-\frac{4b(d^3+5d^2ex^2+15de^2x^4-5e^3x^6)\arctan(cx)}{x^5}+4bcd\left(c^4d^2-5c^2de+15e^2\right)\log(x)$$
$$\left.-\frac{2b(c^6d^3-5c^4d^2e+15c^2de^2+5e^3)\log(1+c^2x^2)}{c}\right)$$

input
```
Integrate[((d + e*x^2)^3*(a + b*ArcTan[c*x]))/x^6,x]
```

output
```
((-4*a*d^3)/x^5 - (b*c*d^3)/x^4 - (20*a*d^2*e)/x^3 + (2*b*c*d^2*(c^2*d - 5
*e))/x^2 - (60*a*d*e^2)/x + 20*a*e^3*x - (4*b*(d^3 + 5*d^2*e*x^2 + 15*d*e^
2*x^4 - 5*e^3*x^6)*ArcTan[c*x])/x^5 + 4*b*c*d*(c^4*d^2 - 5*c^2*d*e + 15*e^
2)*Log[x] - (2*b*(c^6*d^3 - 5*c^4*d^2*e + 15*c^2*d*e^2 + 5*e^3)*Log[1 + c^
2*x^2])/c)/20
```

## Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5511, 27, 2331, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^6} dx$$
$$\Big\downarrow \text{5511}$$
$$-bc\int -\frac{-5e^3x^6+15de^2x^4+5d^2ex^2+d^3}{5x^5(c^2x^2+1)}dx-\frac{d^3(a+b\arctan(cx))}{5x^5}-\frac{d^2e(a+b\arctan(cx))}{x^3}-\frac{3de^2(a+b\arctan(cx))}{x}+e^3x(a+b\arctan(cx))$$

$$\Big\downarrow \; 27$$

$$\frac{1}{5}bc\int \frac{-5e^3x^6+15de^2x^4+5d^2ex^2+d^3}{x^5\left(c^2x^2+1\right)}dx-\frac{d^3(a+b\arctan(cx))}{5x^5}-\frac{d^2e(a+b\arctan(cx))}{x^3}-\frac{3de^2(a+b\arctan(cx))}{x}+e^3x(a+b\arctan(cx))$$

$$\Big\downarrow \; 2331$$

$$\frac{1}{10}bc\int \frac{-5e^3x^6+15de^2x^4+5d^2ex^2+d^3}{x^6\left(c^2x^2+1\right)}dx^2-\frac{d^3(a+b\arctan(cx))}{5x^5}-\frac{d^2e(a+b\arctan(cx))}{x^3}-\frac{3de^2(a+b\arctan(cx))}{x}+e^3x(a+b\arctan(cx))$$

$$\Big\downarrow \; 2123$$

$$\frac{1}{10}bc\int \left(\frac{d^3}{x^6}-\frac{\left(c^2d-5e\right)d^2}{x^4}+\frac{\left(d^2c^4-5dec^2+15e^2\right)d}{x^2}+\frac{-d^3c^6+5d^2ec^4-15de^2c^2-5e^3}{c^2x^2+1}\right)dx^2-\frac{d^3(a+b\arctan(cx))}{5x^5}-\frac{d^2e(a+b\arctan(cx))}{x^3}-\frac{3de^2(a+b\arctan(cx))}{x}+e^3x(a+b\arctan(cx))$$

$$\Big\downarrow \; 2009$$

$$-\frac{d^3(a+b\arctan(cx))}{5x^5}-\frac{d^2e(a+b\arctan(cx))}{x^3}-\frac{3de^2(a+b\arctan(cx))}{x}+e^3x(a+b\arctan(cx))+\frac{1}{10}bc\left(\frac{d^2\left(c^2d-5e\right)}{x^2}+d\log\left(x^2\right)\left(c^4d^2-5c^2de+15e^2\right)-\frac{\left(c^6d^3-5c^4d^2e+15c^2de^2+5e^3\right)\log\left(c^2x^2+1\right)}{c^2}-\frac{d^3}{2x^4}\right.$$

input
```
Int[((d + e*x^2)^3*(a + b*ArcTan[c*x]))/x^6,x]
```

output
```
-1/5*(d^3*(a + b*ArcTan[c*x]))/x^5 - (d^2*e*(a + b*ArcTan[c*x]))/x^3 - (3*
d*e^2*(a + b*ArcTan[c*x]))/x + e^3*x*(a + b*ArcTan[c*x]) + (b*c*(-1/2*d^3/
x^4 + (d^2*(c^2*d - 5*e))/x^2 + d*(c^4*d^2 - 5*c^2*d*e + 15*e^2)*Log[x^2]
- ((c^6*d^3 - 5*c^4*d^2*e + 15*c^2*d*e^2 + 5*e^3)*Log[1 + c^2*x^2])/c^2))/
10
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

## Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.18

| method | result |
|---|---|
| derivativedivides | $c^5\left(\frac{a\left(cx\,e^3-\frac{c\,d^3}{5x^5}-\frac{3cd\,e^2}{x}-\frac{c\,d^2e}{x^3}\right)}{c^6}+\frac{b\left(\arctan(cx)cx\,e^3-\frac{\arctan(cx)c\,d^3}{5x^5}-\frac{3\arctan(cx)cd\,e^2}{x}-\frac{\arctan(cx)c\,d^2e}{x^3}+\frac{c^2d^2(c}{10}\right.}{}\right.$ |
| default | $c^5\left(\frac{a\left(cx\,e^3-\frac{c\,d^3}{5x^5}-\frac{3cd\,e^2}{x}-\frac{c\,d^2e}{x^3}\right)}{c^6}+\frac{b\left(\arctan(cx)cx\,e^3-\frac{\arctan(cx)c\,d^3}{5x^5}-\frac{3\arctan(cx)cd\,e^2}{x}-\frac{\arctan(cx)c\,d^2e}{x^3}+\frac{c^2d^2(c}{10}\right.}{}\right.$ |
| parts | $a\left(e^3x-\frac{e\,d^2}{x^3}-\frac{3e^2d}{x}-\frac{d^3}{5x^5}\right)+b\,c^5\left(\frac{\arctan(cx)x\,e^3}{c^5}-\frac{\arctan(cx)d^2e}{c^5x^3}-\frac{3\arctan(cx)e^2d}{c^5x}-\frac{\arctan(cx)}{5c^5x^5}\right.$ |
| parallelrisch | $\frac{4\ln(x)b\,c^6d^3x^5-2\ln(c^2x^2+1)b\,c^6d^3x^5-2b\,c^6d^3x^5-20\ln(x)b\,c^4d^2e\,x^5+10\ln(c^2x^2+1)b\,c^4d^2e\,x^5+10b\,c^4d^2e\,x^5+60\ln(}{}$ |
| risch | $\frac{ib(-5x^6e^3+15x^4e^2d+5x^2e\,d^2+d^3)\ln(icx+1)}{10x^5}+\frac{4\ln(x)b\,c^6d^3x^5-2\ln(c^2x^2+1)b\,c^6d^3x^5-20\ln(x)b\,c^4d^2e\,x^5+10\ln(c}{}$ |

input
```
int((e*x^2+d)^3*(a+b*arctan(c*x))/x^6,x,method=_RETURNVERBOSE)
```

output
```
c^5*(a/c^6*(c*x*e^3-1/5*c*d^3/x^5-3*c*d*e^2/x-c*d^2*e/x^3)+b/c^6*(arctan(c
*x)*c*x*e^3-1/5*arctan(c*x)*c*d^3/x^5-3*arctan(c*x)*c*d*e^2/x-arctan(c*x)*
c*d^2*e/x^3+1/10*c^2*d^2*(c^2*d-5*e)/x^2-1/20*c^2*d^3/x^4+1/5*d*c^2*(c^4*d
^2-5*c^2*d*e+15*e^2)*ln(c*x)-1/10*(c^6*d^3-5*c^4*d^2*e+15*c^2*d*e^2+5*e^3)
*ln(c^2*x^2+1)))
```

## Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.21

$$\int \frac{(d+ex^2)^3\,(a+b\arctan(cx))}{x^6}\,dx$$
$$=\frac{20\,ace^3x^6-60\,acde^2x^4-bc^2d^3x-20\,acd^2ex^2-2\,(bc^6d^3-5\,bc^4d^2e+15\,bc^2de^2+5\,be^3)x^5\log\left(c^2x^2+1\right)}{}$$

input
```
integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^6,x, algorithm="fricas")
```

output
```
1/20*(20*a*c*e^3*x^6 - 60*a*c*d*e^2*x^4 - b*c^2*d^3*x - 20*a*c*d^2*e*x^2 -
 2*(b*c^6*d^3 - 5*b*c^4*d^2*e + 15*b*c^2*d*e^2 + 5*b*e^3)*x^5*log(c^2*x^2
+ 1) + 4*(b*c^6*d^3 - 5*b*c^4*d^2*e + 15*b*c^2*d*e^2)*x^5*log(x) - 4*a*c*d
^3 + 2*(b*c^4*d^3 - 5*b*c^2*d^2*e)*x^3 + 4*(5*b*c*e^3*x^6 - 15*b*c*d*e^2*x
^4 - 5*b*c*d^2*e*x^2 - b*c*d^3)*arctan(c*x))/(c*x^5)
```

## Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.63

$$\int \frac{(d+ex^2)^3(a+b\arctan(cx))}{x^6}\,dx$$

$$= \begin{cases} -\frac{ad^3}{5x^5} - \frac{ad^2e}{x^3} - \frac{3ade^2}{x} + ae^3x + \frac{bc^5d^3\log(x)}{5} - \frac{bc^5d^3\log\left(x^2+\frac{1}{c^2}\right)}{10} + \frac{bc^3d^3}{10x^2} - bc^3d^2e\log(x) + \frac{bc^3d^2e\log\left(x^2+\frac{1}{c^2}\right)}{2} - \\ a\left(-\frac{d^3}{5x^5} - \frac{d^2e}{x^3} - \frac{3de^2}{x} + e^3x\right) \end{cases}$$

input
```
integrate((e*x**2+d)**3*(a+b*atan(c*x))/x**6,x)
```

output
```
Piecewise((-a*d**3/(5*x**5) - a*d**2*e/x**3 - 3*a*d*e**2/x + a*e**3*x + b*
c**5*d**3*log(x)/5 - b*c**5*d**3*log(x**2 + c**(-2))/10 + b*c**3*d**3/(10*
x**2) - b*c**3*d**2*e*log(x) + b*c**3*d**2*e*log(x**2 + c**(-2))/2 - b*c*d
**3/(20*x**4) - b*c*d**2*e/(2*x**2) + 3*b*c*d*e**2*log(x) - 3*b*c*d*e**2*l
og(x**2 + c**(-2))/2 - b*d**3*atan(c*x)/(5*x**5) - b*d**2*e*atan(c*x)/x**3
 - 3*b*d*e**2*atan(c*x)/x + b*e**3*x*atan(c*x) - b*e**3*log(x**2 + c**(-2)
)/(2*c), Ne(c, 0)), (a*(-d**3/(5*x**5) - d**2*e/x**3 - 3*d*e**2/x + e**3*x
), True))
```

## Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.18

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^6}\,dx$$
$$= -\frac{1}{20}\left(\left(2\,c^4\log\left(c^2x^2+1\right)-2\,c^4\log\left(x^2\right)-\frac{2\,c^2x^2-1}{x^4}\right)c+\frac{4\,\arctan\left(cx\right)}{x^5}\right)bd^3$$
$$+\frac{1}{2}\left(\left(c^2\log\left(c^2x^2+1\right)-c^2\log\left(x^2\right)-\frac{1}{x^2}\right)c-\frac{2\,\arctan\left(cx\right)}{x^3}\right)bd^2e$$
$$-\frac{3}{2}\left(c\left(\log\left(c^2x^2+1\right)-\log\left(x^2\right)\right)+\frac{2\,\arctan\left(cx\right)}{x}\right)bde^2+ae^3x$$
$$+\frac{\left(2\,cx\arctan\left(cx\right)-\log\left(c^2x^2+1\right)\right)be^3}{2\,c}-\frac{3\,ade^2}{x}-\frac{ad^2e}{x^3}-\frac{ad^3}{5\,x^5}$$

input
```
integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^6,x, algorithm="maxima")
```

output
```
-1/20*((2*c^4*log(c^2*x^2 + 1) - 2*c^4*log(x^2) - (2*c^2*x^2 - 1)/x^4)*c +
 4*arctan(c*x)/x^5)*b*d^3 + 1/2*((c^2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/
x^2)*c - 2*arctan(c*x)/x^3)*b*d^2*e - 3/2*(c*(log(c^2*x^2 + 1) - log(x^2))
 + 2*arctan(c*x)/x)*b*d*e^2 + a*e^3*x + 1/2*(2*c*x*arctan(c*x) - log(c^2*x
^2 + 1))*b*e^3/c - 3*a*d*e^2/x - a*d^2*e/x^3 - 1/5*a*d^3/x^5
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.51

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^6}\,dx =$$
$$-\frac{2\,bc^6d^3x^5\log\left(c^2x^2+1\right)-4\,bc^6d^3x^5\log\left(x\right)-10\,bc^4d^2ex^5\log\left(c^2x^2+1\right)+20\,bc^4d^2ex^5\log\left(x\right)-20\,bce^3}{}$$

input
```
integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^6,x, algorithm="giac")
```

output
```
-1/20*(2*b*c^6*d^3*x^5*log(c^2*x^2 + 1) - 4*b*c^6*d^3*x^5*log(x) - 10*b*c^
4*d^2*e*x^5*log(c^2*x^2 + 1) + 20*b*c^4*d^2*e*x^5*log(x) - 20*b*c*e^3*x^6*
arctan(c*x) + 30*b*c^2*d*e^2*x^5*log(c^2*x^2 + 1) - 60*b*c^2*d*e^2*x^5*log
(x) - 2*b*c^4*d^3*x^3 - 20*a*c*e^3*x^6 + 60*b*c*d*e^2*x^4*arctan(c*x) + 10
*b*e^3*x^5*log(c^2*x^2 + 1) + 10*b*c^2*d^2*e*x^3 + 60*a*c*d*e^2*x^4 + 20*b
*c*d^2*e*x^2*arctan(c*x) + b*c^2*d^3*x + 20*a*c*d^2*e*x^2 + 4*b*c*d^3*arct
an(c*x) + 4*a*c*d^3)/(c*x^5)
```

## Mupad [B] (verification not implemented)

Time = 0.96 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.10

$$\int \frac{(d+ex^2)^3\,(a+b\arctan(cx))}{x^6}\,dx$$
$$= \ln(x)\left(\frac{b\,c^5\,d^3}{5}-b\,c^3\,d^2\,e+3\,b\,c\,d\,e^2\right) - \frac{a\,d^3-x^3\left(\frac{b\,c^3\,d^3}{2}-\frac{5\,b\,c\,d^2\,e}{2}\right)+\frac{b\,c\,d^3\,x}{4}+5\,a\,d^2\,e\,x^2+15\,a\,d\,e^2\,x^4}{5\,x^5} - \frac{\ln(c^2\,x^2+1)\,\left(b\,c^6\,d^3-5\,b\,c^4\,d^2\,e+15\,b\,c^2\,d\,e^2+5\,b\,e^3\right)}{10\,c} - \frac{\mathrm{atan}(c\,x)\,\left(\frac{b\,d^3}{5}+b\,d^2\,e\,x^2+3\,b\,d\,e^2\,x^4-b\,e^3\,x^6\right)}{x^5}+a\,e^3\,x$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^3)/x^6,x)
```

output
```
log(x)*((b*c^5*d^3)/5 + 3*b*c*d*e^2 - b*c^3*d^2*e) - (a*d^3 - x^3*((b*c^3*
d^3)/2 - (5*b*c*d^2*e)/2) + (b*c*d^3*x)/4 + 5*a*d^2*e*x^2 + 15*a*d*e^2*x^4
)/(5*x^5) - (log(c^2*x^2 + 1)*(5*b*e^3 + b*c^6*d^3 + 15*b*c^2*d*e^2 - 5*b*
c^4*d^2*e))/(10*c) - (atan(c*x)*((b*d^3)/5 - b*e^3*x^6 + b*d^2*e*x^2 + 3*b
*d*e^2*x^4))/x^5 + a*e^3*x
```

## Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.52

$$\int \frac{(d+ex^2)^3(a+b\arctan(cx))}{x^6}\,dx$$
$$= \frac{-4atan(cx)\,bc\,d^3-20atan(cx)\,bc\,d^2e\,x^2-60atan(cx)\,bcd\,e^2x^4+20atan(cx)\,bc\,e^3x^6-2\log(c^2x^2+1)\,b\,c}{}$$

input
```
int((e*x^2+d)^3*(a+b*atan(c*x))/x^6,x)
```

output
```
( - 4*atan(c*x)*b*c*d**3 - 20*atan(c*x)*b*c*d**2*e*x**2 - 60*atan(c*x)*b*c
*d*e**2*x**4 + 20*atan(c*x)*b*c*e**3*x**6 - 2*log(c**2*x**2 + 1)*b*c**6*d*
*3*x**5 + 10*log(c**2*x**2 + 1)*b*c**4*d**2*e*x**5 - 30*log(c**2*x**2 + 1)
*b*c**2*d*e**2*x**5 - 10*log(c**2*x**2 + 1)*b*e**3*x**5 + 4*log(x)*b*c**6*
d**3*x**5 - 20*log(x)*b*c**4*d**2*e*x**5 + 60*log(x)*b*c**2*d*e**2*x**5 -
4*a*c*d**3 - 20*a*c*d**2*e*x**2 - 60*a*c*d*e**2*x**4 + 20*a*c*e**3*x**6 +
2*b*c**4*d**3*x**3 - b*c**2*d**3*x - 10*b*c**2*d**2*e*x**3)/(20*c*x**5)
```

# 3.1146 $\int \frac{(d+ex^2)^3(a+b\arctan(cx))}{x^7}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 228

$$\begin{aligned}\int \frac{(d+ex^2)^3(a+b\arctan(cx))}{x^7}\,dx = & -\frac{bcd^3}{30x^5}+\frac{bc^3d^3}{18x^3}-\frac{bcd^2e}{4x^3}-\frac{bc^5d^3}{6x} \\ & +\frac{3bc^3d^2e}{4x}-\frac{3bcde^2}{2x}-\frac{1}{6}bc^6d^3\arctan(cx) \\ & +\frac{3}{4}bc^4d^2e\arctan(cx)-\frac{3}{2}bc^2de^2\arctan(cx) \\ & -\frac{d^3(a+b\arctan(cx))}{6x^6}-\frac{3d^2e(a+b\arctan(cx))}{4x^4} \\ & -\frac{3de^2(a+b\arctan(cx))}{2x^2}+ae^3\log(x) \\ & +\frac{1}{2}ibe^3\operatorname{PolyLog}(2,-icx)-\frac{1}{2}ibe^3\operatorname{PolyLog}(2,icx)\end{aligned}$$

output
```
-1/30*b*c*d^3/x^5+1/18*b*c^3*d^3/x^3-1/4*b*c*d^2*e/x^3-1/6*b*c^5*d^3/x+3/4
*b*c^3*d^2*e/x-3/2*b*c*d*e^2/x-1/6*b*c^6*d^3*arctan(c*x)+3/4*b*c^4*d^2*e*a
rctan(c*x)-3/2*b*c^2*d*e^2*arctan(c*x)-1/6*d^3*(a+b*arctan(c*x))/x^6-3/4*d
^2*e*(a+b*arctan(c*x))/x^4-3/2*d*e^2*(a+b*arctan(c*x))/x^2+a*e^3*ln(x)+1/2
*I*b*e^3*polylog(2,-I*c*x)-1/2*I*b*e^3*polylog(2,I*c*x)
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.10 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.77

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^7}\,dx = \frac{1}{60}\left(-\frac{10d^3(a+b\arctan(cx))}{x^6} - \frac{45d^2e(a+b\arctan(cx))}{x^4} - \frac{90de^2(a+b\arctan(cx))}{x^2} - \frac{2bcd^3\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},-c^2x^2\right)}{x^5} - \frac{15bcd^2e\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-c^2x^2\right)}{x^3} - \frac{90bcde^2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-c^2x^2\right)}{x} + 60ae^3\log(x) + 30ibe^3\,\text{PolyLog}(2,-icx) - 30ibe^3\,\text{PolyLog}(2,icx)\right)$$

input
```
Integrate[((d + e*x^2)^3*(a + b*ArcTan[c*x]))/x^7,x]
```

output
```
((-10*d^3*(a + b*ArcTan[c*x]))/x^6 - (45*d^2*e*(a + b*ArcTan[c*x]))/x^4 -
(90*d*e^2*(a + b*ArcTan[c*x]))/x^2 - (2*b*c*d^3*Hypergeometric2F1[-5/2, 1,
 -3/2, -(c^2*x^2)]])/x^5 - (15*b*c*d^2*e*Hypergeometric2F1[-3/2, 1, -1/2, -
(c^2*x^2)]])/x^3 - (90*b*c*d*e^2*Hypergeometric2F1[-1/2, 1, 1/2, -(c^2*x^2)
]])/x + 60*a*e^3*Log[x] + (30*I)*b*e^3*PolyLog[2, (-I)*c*x] - (30*I)*b*e^3*
PolyLog[2, I*c*x])/60
```

## Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^7} \, dx$$

$$\Big\downarrow \text{5515}$$

$$\int \left(\frac{d^3(a+b\arctan(cx))}{x^7}+\frac{3d^2e(a+b\arctan(cx))}{x^5}+\frac{3de^2(a+b\arctan(cx))}{x^3}+\frac{e^3(a+b\arctan(cx))}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d^3(a+b\arctan(cx))}{6x^6}-\frac{3d^2e(a+b\arctan(cx))}{4x^4}-\frac{3de^2(a+b\arctan(cx))}{2x^2}+ae^3\log(x)-\frac{1}{6}bc^6d^3\arctan(cx)+\frac{3}{4}bc^4d^2e\arctan(cx)-\frac{3}{2}bc^2de^2\arctan(cx)-\frac{bc^5d^3}{6x}+\frac{bc^3d^3}{18x^3}+\frac{3bc^3d^2e}{4x}-\frac{bcd^3}{30x^5}-\frac{bcd^2e}{4x^3}-\frac{3bcde^2}{2x}+\frac{1}{2}ibe^3\operatorname{PolyLog}(2,-icx)-\frac{1}{2}ibe^3\operatorname{PolyLog}(2,icx)$$

input
```
Int[((d + e*x^2)^3*(a + b*ArcTan[c*x]))/x^7,x]
```

output
```
-1/30*(b*c*d^3)/x^5 + (b*c^3*d^3)/(18*x^3) - (b*c*d^2*e)/(4*x^3) - (b*c^5*
d^3)/(6*x) + (3*b*c^3*d^2*e)/(4*x) - (3*b*c*d*e^2)/(2*x) - (b*c^6*d^3*ArcT
an[c*x])/6 + (3*b*c^4*d^2*e*ArcTan[c*x])/4 - (3*b*c^2*d*e^2*ArcTan[c*x])/2
 - (d^3*(a + b*ArcTan[c*x]))/(6*x^6) - (3*d^2*e*(a + b*ArcTan[c*x]))/(4*x^
4) - (3*d*e^2*(a + b*ArcTan[c*x]))/(2*x^2) + a*e^3*Log[x] + (I/2)*b*e^3*Po
lyLog[2, (-I)*c*x] - (I/2)*b*e^3*PolyLog[2, I*c*x]
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

## Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.13

| method | result |
|---|---|
| derivativedivides | $c^6\left(\frac{a\left(-\frac{3d\,e^2}{2x^2}+e^3\ln(cx)-\frac{3d^2e}{4x^4}-\frac{d^3}{6x^6}\right)}{c^6}+\frac{b\left(-\frac{3\arctan(cx)d\,e^2}{2x^2}+\arctan(cx)e^3\ln(cx)-\frac{3\arctan(cx)d^2e}{4x^4}-\frac{\arctan(cx)d^3}{6x^6}-\right.}{}\right.$ |
| default | $c^6\left(\frac{a\left(-\frac{3d\,e^2}{2x^2}+e^3\ln(cx)-\frac{3d^2e}{4x^4}-\frac{d^3}{6x^6}\right)}{c^6}+\frac{b\left(-\frac{3\arctan(cx)d\,e^2}{2x^2}+\arctan(cx)e^3\ln(cx)-\frac{3\arctan(cx)d^2e}{4x^4}-\frac{\arctan(cx)d^3}{6x^6}-\right.}{}\right.$ |
| parts | $a\left(-\frac{3d^2e}{4x^4}-\frac{3d\,e^2}{2x^2}-\frac{d^3}{6x^6}+e^3\ln(x)\right)+b\,c^6\left(-\frac{3\arctan(cx)d^2e}{4c^6x^4}-\frac{3\arctan(cx)e^2d}{2c^6x^2}-\frac{\arctan(cx)d^3}{6c^6x^6}-\right.$ |
| risch | $-\frac{bc\,d^2e}{4x^3}+\frac{3b\,c^3d^2e}{4x}-\frac{3bcd\,e^2}{2x}+\frac{3b\,c^4d^2e\arctan(cx)}{4}-\frac{3b\,c^2d\,e^2\arctan(cx)}{2}-\frac{bc\,d^3}{30x^5}+\frac{b\,c^3d^3}{18x^3}-\frac{b\,c^5d^3}{6x}-$ |

input
```
int((e*x^2+d)^3*(a+b*arctan(c*x))/x^7,x,method=_RETURNVERBOSE)
```

output
```
c^6*(a/c^6*(-3/2*d*e^2/x^2+e^3*ln(c*x)-3/4*d^2*e/x^4-1/6*d^3/x^6)+b/c^6*(-
3/2*arctan(c*x)*d*e^2/x^2+arctan(c*x)*e^3*ln(c*x)-3/4*arctan(c*x)*d^2*e/x^
4-1/6*arctan(c*x)*d^3/x^6+1/2*I*e^3*ln(c*x)*ln(1+I*c*x)-1/2*I*e^3*ln(c*x)*
ln(1-I*c*x)+1/2*I*e^3*dilog(1+I*c*x)-1/2*I*e^3*dilog(1-I*c*x)+1/12*d*c^2*(
(-2*c^4*d^2+9*c^2*d*e-18*e^2)*arctan(c*x)-(2*c^4*d^2-9*c^2*d*e+18*e^2)/c/x
-2/5/c*d^2/x^5+1/3*d/c*(2*c^2*d-9*e)/x^3)))
```

## Fricas [F]

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^7} \, dx = \int \frac{(ex^2+d)^3 (b\arctan(cx)+a)}{x^7} \, dx$$

input
```
integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^7,x, algorithm="fricas")
```

output
```
integral((a*e^3*x^6 + 3*a*d*e^2*x^4 + 3*a*d^2*e*x^2 + a*d^3 + (b*e^3*x^6 +
 3*b*d*e^2*x^4 + 3*b*d^2*e*x^2 + b*d^3)*arctan(c*x))/x^7, x)
```

## Sympy [F]

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^7} \, dx = \int \frac{(a+b\operatorname{atan}(cx)) (d+ex^2)^3}{x^7} \, dx$$

input
```
integrate((e*x**2+d)**3*(a+b*atan(c*x))/x**7,x)
```

output
```
Integral((a + b*atan(c*x))*(d + e*x**2)**3/x**7, x)
```

## Maxima [F]

$$\int \frac{(d+ex^2)^3(a+b\arctan(cx))}{x^7}\,dx = \int \frac{(ex^2+d)^3(b\arctan(cx)+a)}{x^7}\,dx$$

input
```
integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^7,x, algorithm="maxima")
```

output
```
-1/90*((15*c^5*arctan(c*x) + (15*c^4*x^4 - 5*c^2*x^2 + 3)/x^5)*c + 15*arct
an(c*x)/x^6)*b*d^3 + 1/4*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*
arctan(c*x)/x^4)*b*d^2*e - 3/2*((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)
*b*d*e^2 + b*e^3*integrate(arctan(c*x)/x, x) + a*e^3*log(x) - 3/2*a*d*e^2/
x^2 - 3/4*a*d^2*e/x^4 - 1/6*a*d^3/x^6
```

## Giac [F]

$$\int \frac{(d+ex^2)^3(a+b\arctan(cx))}{x^7}\,dx = \int \frac{(ex^2+d)^3(b\arctan(cx)+a)}{x^7}\,dx$$

input
```
integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^7,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^3*(b*arctan(c*x) + a)/x^7, x)
```

## Mupad [B] (verification not implemented)

Time = 1.19 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.14

$$\int \frac{(d+ex^2)^3(a+b\arctan(cx))}{x^7}\,dx$$
$$= \begin{cases} a\,e^3\,\ln(x) - \frac{\frac{a\,d^3}{6}+\frac{3\,a\,d^2\,e}{4}\cdots}{\cdots} \\ a\,e^3\,\ln(x) - \frac{\frac{a\,d^3}{6}+\frac{3\,a\,d^2\,e\,x^2}{4}+\frac{3\,a\,d\,e^2\,x^4}{2}}{x^6} - 3\,b\,d^2\,e\left(\frac{\mathrm{atan}(c\,x)}{4\,x^4}+\frac{\frac{\frac{c^2}{3}-c^4\,x^2}{x^3}-c^5\,\mathrm{atan}(c\,x)}{4\,c}\right) - \frac{b\,d^3\left(\frac{c^6\,x^4-\frac{c^4\,x^2}{3}+\frac{c^2}{5}}{x^5}+c^7\,\mathrm{atan}\cdots\right.}{6\,c} \end{cases}$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^3)/x^7,x)
```

output
```
piecewise(c == 0, - ((a*d^3)/6 + (3*a*d^2*e*x^2)/4 + (3*a*d*e^2*x^4)/2)/x^
6 + a*e^3*log(x), c ~= 0, - ((a*d^3)/6 + (3*a*d^2*e*x^2)/4 + (3*a*d*e^2*x^
4)/2)/x^6 + a*e^3*log(x) - (b*e^3*dilog(- c*x*1i + 1)*1i)/2 + (b*e^3*dilog
(c*x*1i + 1)*1i)/2 - 3*b*d^2*e*(atan(c*x)/(4*x^4) + ((c^2/3 - c^4*x^2)/x^3
 - c^5*atan(c*x))/(4*c)) - (b*d^3*((c^2/5 - (c^4*x^2)/3 + c^6*x^4)/x^5 + c
^7*atan(c*x)))/(6*c) - 3*b*d*e^2*((c^3*atan(c*x) + c^2/x)/(2*c) + atan(c*x
)/(2*x^2)) - (b*d^3*atan(c*x))/(6*x^6))
```

## Reduce [F]

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^7}\,dx$$
$$= \frac{-30atan(cx)\,b\,c^6d^3x^6+135atan(cx)\,b\,c^4d^2e\,x^6-270atan(cx)\,b\,c^2d\,e^2x^6-30atan(cx)\,b\,d^3-135atan(ca}{}$$

input
```
int((e*x^2+d)^3*(a+b*atan(c*x))/x^7,x)
```

output
```
( - 30*atan(c*x)*b*c**6*d**3*x**6 + 135*atan(c*x)*b*c**4*d**2*e*x**6 - 270
*atan(c*x)*b*c**2*d*e**2*x**6 - 30*atan(c*x)*b*d**3 - 135*atan(c*x)*b*d**2
*e*x**2 - 270*atan(c*x)*b*d*e**2*x**4 + 180*int(atan(c*x)/x,x)*b*e**3*x**6
 + 180*log(x)*a*e**3*x**6 - 30*a*d**3 - 135*a*d**2*e*x**2 - 270*a*d*e**2*x
**4 - 30*b*c**5*d**3*x**5 + 10*b*c**3*d**3*x**3 + 135*b*c**3*d**2*e*x**5 -
 6*b*c*d**3*x - 45*b*c*d**2*e*x**3 - 270*b*c*d*e**2*x**5)/(180*x**6)
```

# 3.1147 $\int \frac{(d+ex^2)^3(a+b\arctan(cx))}{x^8} dx$



## Optimal result

Integrand size = 21, antiderivative size = 224

$$\int \frac{(d+ex^2)^3(a+b\arctan(cx))}{x^8} dx = -\frac{bcd^3}{42x^6} + \frac{bcd^2(5c^2d-21e)}{140x^4} - \frac{bcd(5c^4d^2-21c^2de+35e^2)}{70x^2} - \frac{d^3(a+b\arctan(cx))}{7x^7} - \frac{3d^2e(a+b\arctan(cx))}{5x^5} - \frac{de^2(a+b\arctan(cx))}{x^3} - \frac{e^3(a+b\arctan(cx))}{x} - \frac{1}{35}bc(5c^6d^3-21c^4d^2e+35c^2de^2-35e^3)\log(x) + \frac{1}{70}bc(5c^6d^3-21c^4d^2e+35c^2de^2-35e^3)\log(1+c^2x^2)$$

output
```
-1/42*b*c*d^3/x^6+1/140*b*c*d^2*(5*c^2*d-21*e)/x^4-1/70*b*c*d*(5*c^4*d^2-2
1*c^2*d*e+35*e^2)/x^2-1/7*d^3*(a+b*arctan(c*x))/x^7-3/5*d^2*e*(a+b*arctan(
c*x))/x^5-d*e^2*(a+b*arctan(c*x))/x^3-e^3*(a+b*arctan(c*x))/x-1/35*b*c*(5*
c^6*d^3-21*c^4*d^2*e+35*c^2*d*e^2-35*e^3)*ln(x)+1/70*b*c*(5*c^6*d^3-21*c^4
*d^2*e+35*c^2*d*e^2-35*e^3)*ln(c^2*x^2+1)
```

## Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.03

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^8}\,dx = -\frac{d^3(a+b\arctan(cx))}{7x^7} - \frac{3d^2e(a+b\arctan(cx))}{5x^5} - \frac{de^2(a+b\arctan(cx))}{x^3} - \frac{e^3(a+b\arctan(cx))}{x} + \frac{1}{2}bce^3\left(2\log(x)-\log\left(1+c^2x^2\right)\right) - \frac{1}{2}bcde^2\left(\frac{1}{x^2}+2c^2\log(x)-c^2\log\left(1+c^2x^2\right)\right) - \frac{3}{20}bcd^2e\left(\frac{1}{x^4}-\frac{2c^2}{x^2}-4c^4\log(x)+2c^4\log\left(1+c^2x^2\right)\right) - \frac{1}{84}bcd^3\left(\frac{2}{x^6}-\frac{3c^2}{x^4}+\frac{6c^4}{x^2}+12c^6\log(x)-6c^6\log\left(1+c^2x^2\right)\right)$$

input
```
Integrate[((d + e*x^2)^3*(a + b*ArcTan[c*x]))/x^8,x]
```

output
```
-1/7*(d^3*(a + b*ArcTan[c*x]))/x^7 - (3*d^2*e*(a + b*ArcTan[c*x]))/(5*x^5)
 - (d*e^2*(a + b*ArcTan[c*x]))/x^3 - (e^3*(a + b*ArcTan[c*x]))/x + (b*c*e^
3*(2*Log[x] - Log[1 + c^2*x^2]))/2 - (b*c*d*e^2*(x^(-2) + 2*c^2*Log[x] - c
^2*Log[1 + c^2*x^2]))/2 - (3*b*c*d^2*e*(x^(-4) - (2*c^2)/x^2 - 4*c^4*Log[x
] + 2*c^4*Log[1 + c^2*x^2]))/20 - (b*c*d^3*(2/x^6 - (3*c^2)/x^4 + (6*c^4)/
x^2 + 12*c^6*Log[x] - 6*c^6*Log[1 + c^2*x^2]))/84
```

## Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {5511, 27, 2331, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^8} dx$$

$\Big\downarrow$ 5511

$$-bc\int -\frac{35e^3x^6+35de^2x^4+21d^2ex^2+5d^3}{35x^7(c^2x^2+1)}dx-\frac{d^3(a+b\arctan(cx))}{7x^7}-\frac{3d^2e(a+b\arctan(cx))}{5x^5}-\frac{de^2(a+b\arctan(cx))}{x^3}-\frac{e^3(a+b\arctan(cx))}{x}$$

$\Big\downarrow$ 27

$$\frac{1}{35}bc\int \frac{35e^3x^6+35de^2x^4+21d^2ex^2+5d^3}{x^7(c^2x^2+1)}dx-\frac{d^3(a+b\arctan(cx))}{7x^7}-\frac{3d^2e(a+b\arctan(cx))}{5x^5}-\frac{de^2(a+b\arctan(cx))}{x^3}-\frac{e^3(a+b\arctan(cx))}{x}$$

$\Big\downarrow$ 2331

$$\frac{1}{70}bc\int \frac{35e^3x^6+35de^2x^4+21d^2ex^2+5d^3}{x^8(c^2x^2+1)}dx^2-\frac{d^3(a+b\arctan(cx))}{7x^7}-\frac{3d^2e(a+b\arctan(cx))}{5x^5}-\frac{de^2(a+b\arctan(cx))}{x^3}-\frac{e^3(a+b\arctan(cx))}{x}$$

$\Big\downarrow$ 2123

$$\frac{1}{70}bc\int \left(\frac{5d^3}{x^8}-\frac{(5c^2d-21e)d^2}{x^6}+\frac{(5d^2c^4-21dec^2+35e^2)d}{x^4}+\frac{5d^3c^8-21d^2ec^6+35de^2c^4-35e^3c^2}{c^2x^2+1}+\frac{-5d^3c^6\right.$$
$$\frac{d^3(a+b\arctan(cx))}{7x^7}-\frac{3d^2e(a+b\arctan(cx))}{5x^5}-\frac{de^2(a+b\arctan(cx))}{x^3}-\frac{e^3(a+b\arctan(cx))}{x}$$

$\Big\downarrow$ 2009

$$-\frac{d^3(a+b\arctan(cx))}{7x^7}-\frac{3d^2e(a+b\arctan(cx))}{5x^5}-\frac{de^2(a+b\arctan(cx))}{x^3}-\frac{e^3(a+b\arctan(cx))}{x}+$$
$$\frac{1}{70}bc\left(\frac{d^2(5c^2d-21e)}{2x^4}-\frac{d(5c^4d^2-21c^2de+35e^2)}{x^2}-\log(x^2)(5c^6d^3-21c^4d^2e+35c^2de^2-35e^3)+(5c^6d^3-2\right.$$

input
```
Int[((d + e*x^2)^3*(a + b*ArcTan[c*x]))/x^8,x]
```

output
```
-1/7*(d^3*(a + b*ArcTan[c*x]))/x^7 - (3*d^2*e*(a + b*ArcTan[c*x]))/(5*x^5)
 - (d*e^2*(a + b*ArcTan[c*x]))/x^3 - (e^3*(a + b*ArcTan[c*x]))/x + (b*c*((
-5*d^3)/(3*x^6) + (d^2*(5*c^2*d - 21*e))/(2*x^4) - (d*(5*c^4*d^2 - 21*c^2*
d*e + 35*e^2))/x^2 - (5*c^6*d^3 - 21*c^4*d^2*e + 35*c^2*d*e^2 - 35*e^3)*Lo
g[x^2] + (5*c^6*d^3 - 21*c^4*d^2*e + 35*c^2*d*e^2 - 35*e^3)*Log[1 + c^2*x^
2]))/70
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

## Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.10

| method | result |
|---|---|
| parts | $a\left(-\frac{d^3}{7x^7}-\frac{d\,e^2}{x^3}-\frac{e^3}{x}-\frac{3d^2e}{5x^5}\right)+b\,c^7\left(-\frac{\arctan(cx)d^3}{7c^7x^7}-\frac{\arctan(cx)e^2d}{c^7x^3}-\frac{\arctan(cx)e^3}{c^7x}-\frac{3\arctan(cx}{5c^7x^5}\right.$ |
| derivativedivides | $c^7\left(\frac{a\left(-\frac{d\,e^2}{c\,x^3}-\frac{e^3}{cx}-\frac{3d^2e}{5c\,x^5}-\frac{d^3}{7c\,x^7}\right)}{c^6}+\frac{b\left(-\frac{\arctan(cx)d\,e^2}{c\,x^3}-\frac{\arctan(cx)e^3}{cx}-\frac{3\arctan(cx)d^2e}{5c\,x^5}-\frac{\arctan(cx)d^3}{7c\,x^7}-\frac{(5c^6d^3-21c^4}{}\right.}{}\right.$ |
| default | $c^7\left(\frac{a\left(-\frac{d\,e^2}{c\,x^3}-\frac{e^3}{cx}-\frac{3d^2e}{5c\,x^5}-\frac{d^3}{7c\,x^7}\right)}{c^6}+\frac{b\left(-\frac{\arctan(cx)d\,e^2}{c\,x^3}-\frac{\arctan(cx)e^3}{cx}-\frac{3\arctan(cx)d^2e}{5c\,x^5}-\frac{\arctan(cx)d^3}{7c\,x^7}-\frac{(5c^6d^3-21c^4}{}\right.}{}\right.$ |
| parallelrisch | $-\frac{252a\,d^2e\,x^2+420ad\,e^2x^4+126b\,c^5d^2e\,x^7-210b\,c^3d\,e^2x^7-252\ln(x)b\,c^5d^2e\,x^7+126\ln(c^2x^2+1)b\,c^5d^2e\,x^7+420\ln(x)}{}$ |
| risch | $\frac{ib(35x^6e^3+35x^4e^2d+21x^2e\,d^2+5d^3)\ln(icx+1)}{70x^7}-\frac{60\ln(x)b\,c^7d^3x^7-30\ln(c^2x^2+1)b\,c^7d^3x^7-252\ln(x)b\,c^5d^2e\,x^7+1}{}$ |

input
```
int((e*x^2+d)^3*(a+b*arctan(c*x))/x^8,x,method=_RETURNVERBOSE)
```

output
```
a*(-1/7*d^3/x^7-d*e^2/x^3-e^3/x-3/5*d^2*e/x^5)+b*c^7*(-1/7*arctan(c*x)*d^3
/c^7/x^7-arctan(c*x)/c^7*e^2*d/x^3-arctan(c*x)/c^7*e^3/x-3/5*arctan(c*x)/c
^7*d^2*e/x^5-1/35/c^6*(1/2*(-5*c^6*d^3+21*c^4*d^2*e-35*c^2*d*e^2+35*e^3)*l
n(c^2*x^2+1)+(5*c^6*d^3-21*c^4*d^2*e+35*c^2*d*e^2-35*e^3)*ln(c*x)-1/4*d^2*
(5*c^2*d-21*e)/x^4+5/6*d^3/x^6+1/2*d*(5*c^4*d^2-21*c^2*d*e+35*e^2)/x^2))
```

## Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.08

$$\int\frac{(d+ex^2)^3\,(a+b\arctan(cx))}{x^8}\,dx=$$
$$-\frac{420\,ae^3x^6-6\,(5\,bc^7d^3-21\,bc^5d^2e+35\,bc^3de^2-35\,bce^3)x^7\log\left(c^2x^2+1\right)+12\,(5\,bc^7d^3-21\,bc^5d^2e+}{}$$

input
```
integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^8,x, algorithm="fricas")
```

output
```
-1/420*(420*a*e^3*x^6 - 6*(5*b*c^7*d^3 - 21*b*c^5*d^2*e + 35*b*c^3*d*e^2 -
 35*b*c*e^3)*x^7*log(c^2*x^2 + 1) + 12*(5*b*c^7*d^3 - 21*b*c^5*d^2*e + 35*
b*c^3*d*e^2 - 35*b*c*e^3)*x^7*log(x) + 420*a*d*e^2*x^4 + 10*b*c*d^3*x + 25
2*a*d^2*e*x^2 + 6*(5*b*c^5*d^3 - 21*b*c^3*d^2*e + 35*b*c*d*e^2)*x^5 + 60*a
*d^3 - 3*(5*b*c^3*d^3 - 21*b*c*d^2*e)*x^3 + 12*(35*b*e^3*x^6 + 35*b*d*e^2*
x^4 + 21*b*d^2*e*x^2 + 5*b*d^3)*arctan(c*x))/x^7
```

## Sympy [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.62

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^8} dx$$
$$= \begin{cases} -\frac{ad^3}{7x^7} - \frac{3ad^2e}{5x^5} - \frac{ade^2}{x^3} - \frac{ae^3}{x} - \frac{bc^7d^3\log(x)}{7} + \frac{bc^7d^3\log\left(x^2+\frac{1}{c^2}\right)}{14} - \frac{bc^5d^3}{14x^2} + \frac{3bc^5d^2e\log(x)}{5} - \frac{3bc^5d^2e\log\left(x^2+\frac{1}{c^2}\right)}{10} + \frac{bc^3}{28x} \\ a\left(-\frac{d^3}{7x^7} - \frac{3d^2e}{5x^5} - \frac{de^2}{x^3} - \frac{e^3}{x}\right) \end{cases}$$

input
```
integrate((e*x**2+d)**3*(a+b*atan(c*x))/x**8,x)
```

output
```
Piecewise((-a*d**3/(7*x**7) - 3*a*d**2*e/(5*x**5) - a*d*e**2/x**3 - a*e**3
/x - b*c**7*d**3*log(x)/7 + b*c**7*d**3*log(x**2 + c**(-2))/14 - b*c**5*d*
*3/(14*x**2) + 3*b*c**5*d**2*e*log(x)/5 - 3*b*c**5*d**2*e*log(x**2 + c**(-
2))/10 + b*c**3*d**3/(28*x**4) + 3*b*c**3*d**2*e/(10*x**2) - b*c**3*d*e**2
*log(x) + b*c**3*d*e**2*log(x**2 + c**(-2))/2 - b*c*d**3/(42*x**6) - 3*b*c
*d**2*e/(20*x**4) - b*c*d*e**2/(2*x**2) + b*c*e**3*log(x) - b*c*e**3*log(x
**2 + c**(-2))/2 - b*d**3*atan(c*x)/(7*x**7) - 3*b*d**2*e*atan(c*x)/(5*x**
5) - b*d*e**2*atan(c*x)/x**3 - b*e**3*atan(c*x)/x, Ne(c, 0)), (a*(-d**3/(7
*x**7) - 3*d**2*e/(5*x**5) - d*e**2/x**3 - e**3/x), True))
```

## Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.10

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^8}\,dx$$
$$= \frac{1}{84}\left(\left(6\,c^6\log\left(c^2x^2+1\right)-6\,c^6\log\left(x^2\right)-\frac{6\,c^4x^4-3\,c^2x^2+2}{x^6}\right)c-\frac{12\arctan\left(cx\right)}{x^7}\right)bd^3$$
$$-\frac{3}{20}\left(\left(2\,c^4\log\left(c^2x^2+1\right)-2\,c^4\log\left(x^2\right)-\frac{2\,c^2x^2-1}{x^4}\right)c+\frac{4\arctan\left(cx\right)}{x^5}\right)bd^2e$$
$$+\frac{1}{2}\left(\left(c^2\log\left(c^2x^2+1\right)-c^2\log\left(x^2\right)-\frac{1}{x^2}\right)c-\frac{2\arctan\left(cx\right)}{x^3}\right)bde^2$$
$$-\frac{1}{2}\left(c\left(\log\left(c^2x^2+1\right)-\log\left(x^2\right)\right)+\frac{2\arctan\left(cx\right)}{x}\right)be^3-\frac{ae^3}{x}-\frac{ade^2}{x^3}-\frac{3\,ad^2e}{5\,x^5}-\frac{ad^3}{7\,x^7}$$

input
```
integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^8,x, algorithm="maxima")
```

output
```
1/84*((6*c^6*log(c^2*x^2 + 1) - 6*c^6*log(x^2) - (6*c^4*x^4 - 3*c^2*x^2 +
2)/x^6)*c - 12*arctan(c*x)/x^7)*b*d^3 - 3/20*((2*c^4*log(c^2*x^2 + 1) - 2*
c^4*log(x^2) - (2*c^2*x^2 - 1)/x^4)*c + 4*arctan(c*x)/x^5)*b*d^2*e + 1/2*(
(c^2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*b*d*e
^2 - 1/2*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b*e^3 - a*e^3
/x - a*d*e^2/x^3 - 3/5*a*d^2*e/x^5 - 1/7*a*d^3/x^7
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.35

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^8}\,dx$$
$$= \frac{30\,bc^7d^3x^7\log\left(c^2x^2+1\right)-60\,bc^7d^3x^7\log\left(x\right)-126\,bc^5d^2ex^7\log\left(c^2x^2+1\right)+252\,bc^5d^2ex^7\log\left(x\right)+210\,b}{}$$

input
```
integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^8,x, algorithm="giac")
```

output
```
1/420*(30*b*c^7*d^3*x^7*log(c^2*x^2 + 1) - 60*b*c^7*d^3*x^7*log(x) - 126*b
*c^5*d^2*e*x^7*log(c^2*x^2 + 1) + 252*b*c^5*d^2*e*x^7*log(x) + 210*b*c^3*d
*e^2*x^7*log(c^2*x^2 + 1) - 420*b*c^3*d*e^2*x^7*log(x) - 30*b*c^5*d^3*x^5
- 210*b*c*e^3*x^7*log(c^2*x^2 + 1) + 420*b*c*e^3*x^7*log(x) + 126*b*c^3*d^
2*e*x^5 - 420*b*e^3*x^6*arctan(c*x) + 15*b*c^3*d^3*x^3 - 210*b*c*d*e^2*x^5
 - 420*a*e^3*x^6 - 420*b*d*e^2*x^4*arctan(c*x) - 63*b*c*d^2*e*x^3 - 420*a*
d*e^2*x^4 - 252*b*d^2*e*x^2*arctan(c*x) - 10*b*c*d^3*x - 252*a*d^2*e*x^2 -
 60*b*d^3*arctan(c*x) - 60*a*d^3)/x^7
```

## Mupad [B] (verification not implemented)

Time = 1.01 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.05

$$\int \frac{(d+ex^2)^3\,(a+b\arctan(cx))}{x^8}\,dx$$
$$= \ln\left(c^2\,x^2+1\right)\,\left(\frac{b\,c^7\,d^3}{14}-\frac{3\,b\,c^5\,d^2\,e}{10}+\frac{b\,c^3\,d\,e^2}{2}-\frac{b\,c\,e^3}{2}\right)$$
$$-\ln\left(x\right)\,\left(\frac{b\,c^7\,d^3}{7}-\frac{3\,b\,c^5\,d^2\,e}{5}+b\,c^3\,d\,e^2-b\,c\,e^3\right)$$
$$-\frac{5\,a\,d^3-x^3\,\left(\frac{5\,b\,c^3\,d^3}{4}-\frac{21\,b\,c\,d^2\,e}{4}\right)+x^5\,\left(\frac{5\,b\,c^5\,d^3}{2}-\frac{21\,b\,c^3\,d^2\,e}{2}+\frac{35\,b\,c\,d\,e^2}{2}\right)+35\,a\,e^3\,x^6+\frac{5\,b\,c\,d^3\,x}{6}+21\,a\,d^2\,e\,x^2}{35\,x^7}$$
$$-\frac{\mathrm{atan}(c\,x)\,\left(\frac{b\,d^3}{7}+\frac{3\,b\,d^2\,e\,x^2}{5}+b\,d\,e^2\,x^4+b\,e^3\,x^6\right)}{x^7}$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^3)/x^8,x)
```

output
```
log(c^2*x^2 + 1)*((b*c^7*d^3)/14 - (b*c*e^3)/2 + (b*c^3*d*e^2)/2 - (3*b*c^
5*d^2*e)/10) - log(x)*((b*c^7*d^3)/7 - b*c*e^3 + b*c^3*d*e^2 - (3*b*c^5*d^
2*e)/5) - (5*a*d^3 - x^3*((5*b*c^3*d^3)/4 - (21*b*c*d^2*e)/4) + x^5*((5*b*
c^5*d^3)/2 + (35*b*c*d*e^2)/2 - (21*b*c^3*d^2*e)/2) + 35*a*e^3*x^6 + (5*b*
c*d^3*x)/6 + 21*a*d^2*e*x^2 + 35*a*d*e^2*x^4)/(35*x^7) - (atan(c*x)*((b*d^
3)/7 + b*e^3*x^6 + (3*b*d^2*e*x^2)/5 + b*d*e^2*x^4))/x^7
```

## Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.35

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^8}\,dx$$
$$= \frac{-60atan(cx)\,b\,d^3 - 252atan(cx)\,b\,d^2e\,x^2 - 420atan(cx)\,bd\,e^2x^4 - 420atan(cx)\,b\,e^3x^6 + 30\log(c^2x^2+1)}{}$$

input
```
int((e*x^2+d)^3*(a+b*atan(c*x))/x^8,x)
```

output
```
( - 60*atan(c*x)*b*d**3 - 252*atan(c*x)*b*d**2*e*x**2 - 420*atan(c*x)*b*d*
e**2*x**4 - 420*atan(c*x)*b*e**3*x**6 + 30*log(c**2*x**2 + 1)*b*c**7*d**3*
x**7 - 126*log(c**2*x**2 + 1)*b*c**5*d**2*e*x**7 + 210*log(c**2*x**2 + 1)*
b*c**3*d*e**2*x**7 - 210*log(c**2*x**2 + 1)*b*c*e**3*x**7 - 60*log(x)*b*c*
*7*d**3*x**7 + 252*log(x)*b*c**5*d**2*e*x**7 - 420*log(x)*b*c**3*d*e**2*x*
*7 + 420*log(x)*b*c*e**3*x**7 - 60*a*d**3 - 252*a*d**2*e*x**2 - 420*a*d*e*
*2*x**4 - 420*a*e**3*x**6 - 30*b*c**5*d**3*x**5 + 15*b*c**3*d**3*x**3 + 12
6*b*c**3*d**2*e*x**5 - 10*b*c*d**3*x - 63*b*c*d**2*e*x**3 - 210*b*c*d*e**2
*x**5)/(420*x**7)
```

# 3.1148 $\int \frac{(d+ex^2)^3(a+b\arctan(cx))}{x^9}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 152

$$\int \frac{(d+ex^2)^3(a+b\arctan(cx))}{x^9}\,dx = -\frac{bcd^3}{56x^7}+\frac{bcd^2(c^2d-4e)}{40x^5}-\frac{bcd(c^4d^2-4c^2de+6e^2)}{24x^3} + \frac{bc(c^2d-2e)(c^4d^2-2c^2de+2e^2)}{8x} + \frac{b(c^2d-e)^4\arctan(cx)}{8d} - \frac{(d+ex^2)^4(a+b\arctan(cx))}{8dx^8}$$

output

```
-1/56*b*c*d^3/x^7+1/40*b*c*d^2*(c^2*d-4*e)/x^5-1/24*b*c*d*(c^4*d^2-4*c^2*d
*e+6*e^2)/x^3+1/8*b*c*(c^2*d-2*e)*(c^4*d^2-2*c^2*d*e+2*e^2)/x+1/8*b*(c^2*d
-e)^4*arctan(c*x)/d-1/8*(e*x^2+d)^4*(a+b*arctan(c*x))/d/x^8
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.12 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.01

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^9}\,dx = -\frac{5bcd^3x\,\text{Hypergeometric2F1}\left(-\frac{7}{2},1,-\frac{5}{2},-c^2x^2\right)+28bcd^2ex^3\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},-c^2x^2\right)+\cdots}{\cdots}$$

input
```
Integrate[((d + e*x^2)^3*(a + b*ArcTan[c*x]))/x^9,x]
```

output
```
-1/280*(5*b*c*d^3*x*Hypergeometric2F1[-7/2, 1, -5/2, -(c^2*x^2)] + 28*b*c*
d^2*e*x^3*Hypergeometric2F1[-5/2, 1, -3/2, -(c^2*x^2)] + 35*((d^3 + 4*d^2*
e*x^2 + 6*d*e^2*x^4 + 4*e^3*x^6)*(a + b*ArcTan[c*x]) + 2*b*c*d*e^2*x^5*Hyp
ergeometric2F1[-3/2, 1, -1/2, -(c^2*x^2)] + 4*b*c*e^3*x^7*Hypergeometric2F
1[-1/2, 1, 1/2, -(c^2*x^2)]))/x^8
```

## Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {5511, 27, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^9}\,dx$$

$$\Big\downarrow \text{5511}$$

$$-bc\int -\frac{(ex^2+d)^4}{8dx^8 (c^2x^2+1)}dx-\frac{(d+ex^2)^4 (a+b\arctan(cx))}{8dx^8}$$

$$\Big\downarrow \text{27}$$

$$\frac{bc\int \frac{(ex^2+d)^4}{x^8(c^2x^2+1)}dx}{8d}-\frac{(d+ex^2)^4 (a+b\arctan(cx))}{8dx^8}$$

$$\Big\downarrow \; 364$$

$$\frac{bc\int\left(\frac{d^4}{x^8}-\frac{(c^2d-4e)d^3}{x^6}+\frac{(d^2c^4-4dec^2+6e^2)d^2}{x^4}+\frac{(c^2d-2e)(-d^2c^4+2dec^2-2e^2)d}{x^2}+\frac{(c^2d-e)^4}{c^2x^2+1}\right)dx}{8d}-\frac{(d+ex^2)^4(a+b\arctan(cx))}{8dx^8}$$

$$\Big\downarrow \; 2009$$

$$\frac{bc\left(\frac{\arctan(cx)(c^2d-e)^4}{c}+\frac{d^3(c^2d-4e)}{5x^5}-\frac{d^2(c^4d^2-4c^2de+6e^2)}{3x^3}+\frac{d(c^2d-2e)(c^4d^2-2c^2de+2e^2)}{x}-\frac{d^4}{7x^7}\right)}{8d}-\frac{(d+ex^2)^4(a+b\arctan(cx))}{8dx^8}$$

input
```
Int[((d + e*x^2)^3*(a + b*ArcTan[c*x]))/x^9,x]
```

output
```
-1/8*((d + e*x^2)^4*(a + b*ArcTan[c*x]))/(d*x^8) + (b*c*(-1/7*d^4/x^7 + (d
^3*(c^2*d - 4*e))/(5*x^5) - (d^2*(c^4*d^2 - 4*c^2*d*e + 6*e^2))/(3*x^3) +
(d*(c^2*d - 2*e)*(c^4*d^2 - 2*c^2*d*e + 2*e^2))/x + ((c^2*d - e)^4*ArcTan[
c*x])/c))/(8*d)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

## Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.64

| method | result |
|---|---|
| parts | $a\left(-\frac{3d\,e^{2}}{4x^{4}}-\frac{e^{3}}{2x^{2}}-\frac{d^{2}e}{2x^{6}}-\frac{d^{3}}{8x^{8}}\right)+b\,c^{8}\left(-\frac{3\arctan(cx)e^{2}d}{4c^{8}x^{4}}-\frac{\arctan(cx)e^{3}}{2c^{8}x^{2}}-\frac{\arctan(cx)d^{2}e}{2c^{8}x^{6}}-\frac{\arctan(}{8c^{8}}\right.$ |
| derivativedivides | $c^{8}\left(\frac{a\left(-\frac{d^{2}e}{2c^{2}x^{6}}-\frac{e^{3}}{2c^{2}x^{2}}-\frac{d^{3}}{8c^{2}x^{8}}-\frac{3d\,e^{2}}{4c^{2}x^{4}}\right)}{c^{6}}+\frac{b\left(-\frac{\arctan(cx)d^{2}e}{2c^{2}x^{6}}-\frac{\arctan(cx)e^{3}}{2c^{2}x^{2}}-\frac{\arctan(cx)d^{3}}{8c^{2}x^{8}}-\frac{3\arctan(cx)d\,e^{2}}{4c^{2}x^{4}}+\frac{c^{6}d^{3}-}{}\right.}{}\right.$ |
| default | $c^{8}\left(\frac{a\left(-\frac{d^{2}e}{2c^{2}x^{6}}-\frac{e^{3}}{2c^{2}x^{2}}-\frac{d^{3}}{8c^{2}x^{8}}-\frac{3d\,e^{2}}{4c^{2}x^{4}}\right)}{c^{6}}+\frac{b\left(-\frac{\arctan(cx)d^{2}e}{2c^{2}x^{6}}-\frac{\arctan(cx)e^{3}}{2c^{2}x^{2}}-\frac{\arctan(cx)d^{3}}{8c^{2}x^{8}}-\frac{3\arctan(cx)d\,e^{2}}{4c^{2}x^{4}}+\frac{c^{6}d^{3}-}{}\right.}{}\right.$ |
| parallelrisch | $\frac{105x^{8}\arctan(cx)b\,c^{8}d^{3}-420x^{8}\arctan(cx)b\,c^{6}d^{2}e+105b\,c^{7}d^{3}x^{7}+630x^{8}\arctan(cx)b\,c^{4}d\,e^{2}-420b\,c^{5}d^{2}e\,x^{7}-420x^{8}\arctan}{}$ |
| orering | $\frac{(70c^{8}d^{3}e\,x^{10}+175c^{8}d^{4}x^{8}-280x^{10}e^{2}c^{6}d^{2}-665c^{6}d^{3}e\,x^{8}+420x^{10}e^{3}c^{4}d+105c^{6}d^{4}x^{6}+910x^{8}e^{2}c^{4}d^{2}-280x^{10}e^{4}c^{2}-427c^{4}}{}$ |
| risch | $\frac{ib\left(4x^{6}e^{3}+6x^{4}e^{2}d+4x^{2}e\,d^{2}+d^{3}\right)\ln(icx+1)}{16x^{8}}-\frac{840a\,d^{2}e\,x^{2}+1260ad\,e^{2}x^{4}+840bc\,e^{3}x^{7}+840b\,c^{5}d^{2}e\,x^{7}-1260b\,c^{3}d\,e^{2}x^{7}}{}$ |

input
```
int((e*x^2+d)^3*(a+b*arctan(c*x))/x^9,x,method=_RETURNVERBOSE)
```

output
```
a*(-3/4*d*e^2/x^4-1/2*e^3/x^2-1/2*d^2*e/x^6-1/8*d^3/x^8)+b*c^8*(-3/4*arcta
n(c*x)/c^8*e^2*d/x^4-1/2*arctan(c*x)/c^8*e^3/x^2-1/2*arctan(c*x)/c^8*d^2*e
/x^6-1/8*arctan(c*x)*d^3/c^8/x^8-1/8/c^6*((-c^6*d^3+4*c^4*d^2*e-6*c^2*d*e^
2+4*e^3)*arctan(c*x)-(c^6*d^3-4*c^4*d^2*e+6*c^2*d*e^2-4*e^3)/c/x-1/5*d^2/c
*(c^2*d-4*e)/x^5+1/7/c*d^3/x^7+1/3*d/c*(c^4*d^2-4*c^2*d*e+6*e^2)/x^3))
```

## Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.50

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^9}\,dx = -\frac{420\,ae^3x^6 + 630\,ade^2x^4 - 105\,(bc^7d^3 - 4\,bc^5d^2e + 6\,bc^3de^2 - 4\,bce^3)x^7 + 15\,bcd^3x + 420\,ad^2ex^2 + 35\,(}{}$$

input
```
integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^9,x, algorithm="fricas")
```

output
```
-1/840*(420*a*e^3*x^6 + 630*a*d*e^2*x^4 - 105*(b*c^7*d^3 - 4*b*c^5*d^2*e +
 6*b*c^3*d*e^2 - 4*b*c*e^3)*x^7 + 15*b*c*d^3*x + 420*a*d^2*e*x^2 + 35*(b*c
^5*d^3 - 4*b*c^3*d^2*e + 6*b*c*d*e^2)*x^5 + 105*a*d^3 - 21*(b*c^3*d^3 - 4*
b*c*d^2*e)*x^3 + 105*(4*b*e^3*x^6 - (b*c^8*d^3 - 4*b*c^6*d^2*e + 6*b*c^4*d
*e^2 - 4*b*c^2*e^3)*x^8 + 6*b*d*e^2*x^4 + 4*b*d^2*e*x^2 + b*d^3)*arctan(c*
x))/x^8
```

## Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 309 vs. 2(139) = 278.

Time = 0.52 (sec) , antiderivative size = 309, normalized size of antiderivative = 2.03

$$\begin{aligned}\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^9}\,dx = &-\frac{ad^3}{8x^8} - \frac{ad^2e}{2x^6} - \frac{3ade^2}{4x^4} - \frac{ae^3}{2x^2} + \frac{bc^8d^3\operatorname{atan}(cx)}{8} \\ &+ \frac{bc^7d^3}{8x} - \frac{bc^6d^2e\operatorname{atan}(cx)}{2} - \frac{bc^5d^3}{24x^3} - \frac{bc^5d^2e}{2x} \\ &+ \frac{3bc^4de^2\operatorname{atan}(cx)}{4} + \frac{bc^3d^3}{40x^5} + \frac{bc^3d^2e}{6x^3} + \frac{3bc^3de^2}{4x} \\ &- \frac{bc^2e^3\operatorname{atan}(cx)}{2} - \frac{bcd^3}{56x^7} - \frac{bcd^2e}{10x^5} - \frac{bcde^2}{4x^3} \\ &- \frac{bce^3}{2x} - \frac{bd^3\operatorname{atan}(cx)}{8x^8} - \frac{bd^2e\operatorname{atan}(cx)}{2x^6} \\ &- \frac{3bde^2\operatorname{atan}(cx)}{4x^4} - \frac{be^3\operatorname{atan}(cx)}{2x^2}\end{aligned}$$

input
```
integrate((e*x**2+d)**3*(a+b*atan(c*x))/x**9,x)
```

output
```
-a*d**3/(8*x**8) - a*d**2*e/(2*x**6) - 3*a*d*e**2/(4*x**4) - a*e**3/(2*x**
2) + b*c**8*d**3*atan(c*x)/8 + b*c**7*d**3/(8*x) - b*c**6*d**2*e*atan(c*x)
/2 - b*c**5*d**3/(24*x**3) - b*c**5*d**2*e/(2*x) + 3*b*c**4*d*e**2*atan(c*
x)/4 + b*c**3*d**3/(40*x**5) + b*c**3*d**2*e/(6*x**3) + 3*b*c**3*d*e**2/(4
*x) - b*c**2*e**3*atan(c*x)/2 - b*c*d**3/(56*x**7) - b*c*d**2*e/(10*x**5)
- b*c*d*e**2/(4*x**3) - b*c*e**3/(2*x) - b*d**3*atan(c*x)/(8*x**8) - b*d**
2*e*atan(c*x)/(2*x**6) - 3*b*d*e**2*atan(c*x)/(4*x**4) - b*e**3*atan(c*x)/
(2*x**2)
```

## Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.43

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^9}\,dx$$
$$= \frac{1}{840}\left(\left(105\,c^7\arctan(cx) + \frac{105\,c^6x^6 - 35\,c^4x^4 + 21\,c^2x^2 - 15}{x^7}\right)c - \frac{105\arctan(cx)}{x^8}\right)bd^3$$
$$- \frac{1}{30}\left(\left(15\,c^5\arctan(cx) + \frac{15\,c^4x^4 - 5\,c^2x^2 + 3}{x^5}\right)c + \frac{15\arctan(cx)}{x^6}\right)bd^2e$$
$$+ \frac{1}{4}\left(\left(3\,c^3\arctan(cx) + \frac{3\,c^2x^2 - 1}{x^3}\right)c - \frac{3\arctan(cx)}{x^4}\right)bde^2$$
$$- \frac{1}{2}\left(\left(c\arctan(cx) + \frac{1}{x}\right)c + \frac{\arctan(cx)}{x^2}\right)be^3 - \frac{ae^3}{2\,x^2} - \frac{3\,ade^2}{4\,x^4} - \frac{ad^2e}{2\,x^6} - \frac{ad^3}{8\,x^8}$$

input
```
integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^9,x, algorithm="maxima")
```

output
```
1/840*((105*c^7*arctan(c*x) + (105*c^6*x^6 - 35*c^4*x^4 + 21*c^2*x^2 - 15)
/x^7)*c - 105*arctan(c*x)/x^8)*b*d^3 - 1/30*((15*c^5*arctan(c*x) + (15*c^4
*x^4 - 5*c^2*x^2 + 3)/x^5)*c + 15*arctan(c*x)/x^6)*b*d^2*e + 1/4*((3*c^3*a
rctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan(c*x)/x^4)*b*d*e^2 - 1/2*((c
*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b*e^3 - 1/2*a*e^3/x^2 - 3/4*a*d*e
^2/x^4 - 1/2*a*d^2*e/x^6 - 1/8*a*d^3/x^8
```

## Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 5.63 (sec) , antiderivative size = 363, normalized size of antiderivative = 2.39

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^9}\,dx = -\frac{105i\,bc^8d^3x^8\log(i\,cx+1) - 105i\,bc^8d^3x^8\log(-i\,cx+1) - 420i\,bc^6d^2ex^8\log(i\,cx+1) + 420i\,bc^6d^2ex^8}{}$$

input
```
integrate((e*x^2+d)^3*(a+b*arctan(c*x))/x^9,x, algorithm="giac")
```

output
```
-1/1680*(105*I*b*c^8*d^3*x^8*log(I*c*x + 1) - 105*I*b*c^8*d^3*x^8*log(-I*c
*x + 1) - 420*I*b*c^6*d^2*e*x^8*log(I*c*x + 1) + 420*I*b*c^6*d^2*e*x^8*log
(-I*c*x + 1) - 210*b*c^7*d^3*x^7 + 630*I*b*c^4*d*e^2*x^8*log(I*c*x + 1) -
630*I*b*c^4*d*e^2*x^8*log(-I*c*x + 1) + 840*b*c^5*d^2*e*x^7 - 420*I*b*c^2*
e^3*x^8*log(I*c*x + 1) + 420*I*b*c^2*e^3*x^8*log(-I*c*x + 1) + 70*b*c^5*d^
3*x^5 - 1260*b*c^3*d*e^2*x^7 - 280*b*c^3*d^2*e*x^5 + 840*b*c*e^3*x^7 + 840
*b*e^3*x^6*arctan(c*x) - 42*b*c^3*d^3*x^3 + 420*b*c*d*e^2*x^5 + 840*a*e^3*
x^6 + 1260*b*d*e^2*x^4*arctan(c*x) + 168*b*c*d^2*e*x^3 + 1260*a*d*e^2*x^4
+ 840*b*d^2*e*x^2*arctan(c*x) + 30*b*c*d^3*x + 840*a*d^2*e*x^2 + 210*b*d^3
*arctan(c*x) + 210*a*d^3)/x^8
```

## Mupad [B] (verification not implemented)

Time = 0.96 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.98

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^9}\,dx$$
$$= \frac{b\,c^2\,\mathrm{atan}\left(\frac{b\,c^2\,x\,(2\,e-c^2\,d)\,(c^4\,d^2-2\,c^2\,d\,e+2\,e^2)}{b\,c^7\,d^3-4\,b\,c^5\,d^2\,e+6\,b\,c^3\,d\,e^2-4\,b\,c\,e^3}\right)\,(2\,e-c^2\,d)\,(c^4\,d^2-2\,c^2\,d\,e+2\,e^2)}{8}$$
$$-\frac{\mathrm{atan}(c\,x)\,\left(\frac{b\,d^3}{8}+\frac{b\,d^2\,e\,x^2}{2}+\frac{3\,b\,d\,e^2\,x^4}{4}+\frac{b\,e^3\,x^6}{2}\right)}{x^8}$$
$$-\frac{a\,d^3-x^3\,\left(\frac{b\,c^3\,d^3}{5}-\frac{4\,b\,c\,d^2\,e}{5}\right)-x^7\,(b\,c^7\,d^3-4\,b\,c^5\,d^2\,e+6\,b\,c^3\,d\,e^2-4\,b\,c\,e^3)+x^5\,\left(\frac{b\,c^5\,d^3}{3}-\frac{4\,b\,c^3\,d^2\,e}{3}+2\,b\right.}{8\,x^8}$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^3)/x^9,x)
```

output
```
(b*c^2*atan((b*c^2*x*(2*e - c^2*d)*(2*e^2 + c^4*d^2 - 2*c^2*d*e))/(b*c^7*d
^3 - 4*b*c*e^3 + 6*b*c^3*d*e^2 - 4*b*c^5*d^2*e))*(2*e - c^2*d)*(2*e^2 + c^
4*d^2 - 2*c^2*d*e))/8 - (atan(c*x)*((b*d^3)/8 + (b*e^3*x^6)/2 + (b*d^2*e*x
^2)/2 + (3*b*d*e^2*x^4)/4))/x^8 - (a*d^3 - x^3*((b*c^3*d^3)/5 - (4*b*c*d^2
*e)/5) - x^7*(b*c^7*d^3 - 4*b*c*e^3 + 6*b*c^3*d*e^2 - 4*b*c^5*d^2*e) + x^5
*((b*c^5*d^3)/3 + 2*b*c*d*e^2 - (4*b*c^3*d^2*e)/3) + 4*a*e^3*x^6 + (b*c*d^
3*x)/7 + 4*a*d^2*e*x^2 + 6*a*d*e^2*x^4)/(8*x^8)
```

## Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.80

$$\int \frac{(d+ex^2)^3 (a+b\arctan(cx))}{x^9}\,dx = \frac{105atan(cx)\,b\,c^8d^3x^8 - 420atan(cx)\,b\,c^6d^2e\,x^8 + 630atan(cx)\,b\,c^4d\,e^2x^8 - 420atan(cx)\,b\,c^2e^3x^8 - 105ata}{}$$

input
```
int((e*x^2+d)^3*(a+b*atan(c*x))/x^9,x)
```

output
```
(105*atan(c*x)*b*c**8*d**3*x**8 - 420*atan(c*x)*b*c**6*d**2*e*x**8 + 630*a
tan(c*x)*b*c**4*d*e**2*x**8 - 420*atan(c*x)*b*c**2*e**3*x**8 - 105*atan(c*
x)*b*d**3 - 420*atan(c*x)*b*d**2*e*x**2 - 630*atan(c*x)*b*d*e**2*x**4 - 42
0*atan(c*x)*b*e**3*x**6 - 105*a*d**3 - 420*a*d**2*e*x**2 - 630*a*d*e**2*x*
*4 - 420*a*e**3*x**6 + 105*b*c**7*d**3*x**7 - 35*b*c**5*d**3*x**5 - 420*b*
c**5*d**2*e*x**7 + 21*b*c**3*d**3*x**3 + 140*b*c**3*d**2*e*x**5 + 630*b*c*
*3*d*e**2*x**7 - 15*b*c*d**3*x - 84*b*c*d**2*e*x**3 - 210*b*c*d*e**2*x**5
- 420*b*c*e**3*x**7)/(840*x**8)
```

# 3.1149 $\int (c+dx^2)^4 \arctan(ax)\,dx$



## Optimal result

Integrand size = 14, antiderivative size = 244

$$\begin{aligned}
\int (c+dx^2)^4 \arctan(ax)\,dx &= -\frac{d(420a^6c^3-378a^4c^2d+180a^2cd^2-35d^3)\,x^2}{630a^7}-\frac{d^2(378a^4c^2-180a^2cd+35d^2)\,x^4}{1260a^5}\\
&\quad-\frac{(36a^2c-7d)\,d^3x^6}{378a^3}-\frac{d^4x^8}{72a}+c^4x\arctan(ax)+\frac{4}{3}c^3dx^3\arctan(ax)\\
&\quad+\frac{6}{5}c^2d^2x^5\arctan(ax)+\frac{4}{7}cd^3x^7\arctan(ax)+\frac{1}{9}d^4x^9\arctan(ax)\\
&\quad-\frac{(315a^8c^4-420a^6c^3d+378a^4c^2d^2-180a^2cd^3+35d^4)\log(1+a^2x^2)}{630a^9}
\end{aligned}$$

output
```
-1/630*d*(420*a^6*c^3-378*a^4*c^2*d+180*a^2*c*d^2-35*d^3)*x^2/a^7-1/1260*d
^2*(378*a^4*c^2-180*a^2*c*d+35*d^2)*x^4/a^5-1/378*(36*a^2*c-7*d)*d^3*x^6/a
^3-1/72*d^4*x^8/a+c^4*x*arctan(a*x)+4/3*c^3*d*x^3*arctan(a*x)+6/5*c^2*d^2*
x^5*arctan(a*x)+4/7*c*d^3*x^7*arctan(a*x)+1/9*d^4*x^9*arctan(a*x)-1/630*(3
15*a^8*c^4-420*a^6*c^3*d+378*a^4*c^2*d^2-180*a^2*c*d^3+35*d^4)*ln(a^2*x^2+
1)/a^9
```

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.87

$$\int (c+dx^2)^4 \arctan(ax)\,dx = -\frac{a^2dx^2(-420d^3+30a^2d^2(72c+7dx^2)-4a^4d(1134c^2+270cdx^2+35d^2x^4)+3a^6(1680c^3+756c^2dx^2+\cdots}{}$$

input
```
Integrate[(c + d*x^2)^4*ArcTan[a*x],x]
```

output
```
-1/7560*(a^2*d*x^2*(-420*d^3 + 30*a^2*d^2*(72*c + 7*d*x^2) - 4*a^4*d*(1134
*c^2 + 270*c*d*x^2 + 35*d^2*x^4) + 3*a^6*(1680*c^3 + 756*c^2*d*x^2 + 240*c
*d^2*x^4 + 35*d^3*x^6)) - 24*a^9*x*(315*c^4 + 420*c^3*d*x^2 + 378*c^2*d^2*
x^4 + 180*c*d^3*x^6 + 35*d^4*x^8)*ArcTan[a*x] + 12*(315*a^8*c^4 - 420*a^6*
c^3*d + 378*a^4*c^2*d^2 - 180*a^2*c*d^3 + 35*d^4)*Log[1 + a^2*x^2])/a^9
```

## Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {5447, 27, 2331, 2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(ax)\left(c+dx^2\right)^4\,dx$$

$$\Big\downarrow \; 5447$$

$$-a\int \frac{x\left(35d^4x^8+180cd^3x^6+378c^2d^2x^4+420c^3dx^2+315c^4\right)}{315\left(a^2x^2+1\right)}dx+c^4x\arctan(ax)+\frac{4}{3}c^3dx^3\arctan(ax)+\frac{6}{5}c^2d^2x^5\arctan(ax)+\frac{4}{7}cd^3x^7\arctan(ax)+\frac{1}{9}d^4x^9\arctan(ax)$$

$$\Big\downarrow \; 27$$

$$-\frac{1}{315}a\int \frac{x(35d^4x^8+180cd^3x^6+378c^2d^2x^4+420c^3dx^2+315c^4)}{a^2x^2+1}dx+c^4x\arctan(ax)+\frac{4}{3}c^3dx^3\arctan(ax)+\frac{6}{5}c^2d^2x^5\arctan(ax)+\frac{4}{7}cd^3x^7\arctan(ax)+\frac{1}{9}d^4x^9\arctan(ax)$$

$\Big\downarrow$ 2331

$$-\frac{1}{630}a\int \frac{35d^4x^8+180cd^3x^6+378c^2d^2x^4+420c^3dx^2+315c^4}{a^2x^2+1}dx^2+c^4x\arctan(ax)+\frac{4}{3}c^3dx^3\arctan(ax)+\frac{6}{5}c^2d^2x^5\arctan(ax)+\frac{4}{7}cd^3x^7\arctan(ax)+\frac{1}{9}d^4x^9\arctan(ax)$$

$\Big\downarrow$ 2389

$$-\frac{1}{630}a\int \left(\frac{35d^4x^6}{a^2}+\frac{5(36a^2c-7d)d^3x^4}{a^4}+\frac{d^2(378c^2a^4-180cda^2+35d^2)x^2}{a^6}+\frac{d(420c^3a^6-378c^2da^4+180cd\right.$$
$$c^4x\arctan(ax)+\frac{4}{3}c^3dx^3\arctan(ax)+\frac{6}{5}c^2d^2x^5\arctan(ax)+\frac{4}{7}cd^3x^7\arctan(ax)+\frac{1}{9}d^4x^9\arctan(ax)$$

$\Big\downarrow$ 2009

$$-\frac{1}{630}a\left(\frac{35d^4x^8}{4a^2}+\frac{5d^3x^6(36a^2c-7d)}{3a^4}+\frac{d^2x^4(378a^4c^2-180a^2cd+35d^2)}{2a^6}+\frac{dx^2(420a^6c^3-378a^4c^2d+180a^2c\right.$$
$$c^4x\arctan(ax)+\frac{4}{3}c^3dx^3\arctan(ax)+\frac{6}{5}c^2d^2x^5\arctan(ax)+\frac{4}{7}cd^3x^7\arctan(ax)+\frac{1}{9}d^4x^9\arctan(ax)$$

input
```
Int[(c + d*x^2)^4*ArcTan[a*x],x]
```

output
```
c^4*x*ArcTan[a*x] + (4*c^3*d*x^3*ArcTan[a*x])/3 + (6*c^2*d^2*x^5*ArcTan[a*
x])/5 + (4*c*d^3*x^7*ArcTan[a*x])/7 + (d^4*x^9*ArcTan[a*x])/9 - (a*((d*(42
0*a^6*c^3 - 378*a^4*c^2*d + 180*a^2*c*d^2 - 35*d^3)*x^2)/a^8 + (d^2*(378*a
^4*c^2 - 180*a^2*c*d + 35*d^2)*x^4)/(2*a^6) + (5*(36*a^2*c - 7*d)*d^3*x^6)
/(3*a^4) + (35*d^4*x^8)/(4*a^2) + ((315*a^8*c^4 - 420*a^6*c^3*d + 378*a^4*
c^2*d^2 - 180*a^2*c*d^3 + 35*d^4)*Log[1 + a^2*x^2])/a^10))/630
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

rule 5447
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Simp[(a + b*ArcTan[c*x])   u,
x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2), x], x], x]] /; FreeQ
[{a, b, c, d, e}, x] && (IntegerQ[q] || ILtQ[q + 1/2, 0])
```

## Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.00

| method | result |
|---|---|
| parts | $\frac{d^4x^9\arctan(ax)}{9}+\frac{4c\,d^3x^7\arctan(ax)}{7}+\frac{6c^2d^2x^5\arctan(ax)}{5}+\frac{4c^3d\,x^3\arctan(ax)}{3}+c^4x\arctan(ax)-$ |
| derivativedivides | $\frac{\arctan(ax)ax\,c^4+\frac{4\arctan(ax)a\,c^3d\,x^3}{3}+\frac{6\arctan(ax)a\,c^2d^2x^5}{5}+\frac{4\arctan(ax)ac\,d^3x^7}{7}+\frac{\arctan(ax)a\,d^4x^9}{9}-\frac{210c^3a^8d\,x^2+\frac{189c^2}{}}{}}{}$ |
| default | $\frac{\arctan(ax)ax\,c^4+\frac{4\arctan(ax)a\,c^3d\,x^3}{3}+\frac{6\arctan(ax)a\,c^2d^2x^5}{5}+\frac{4\arctan(ax)ac\,d^3x^7}{7}+\frac{\arctan(ax)a\,d^4x^9}{9}-\frac{210c^3a^8d\,x^2+\frac{189c^2}{}}{}}{}$ |
| parallelrisch | $-\frac{-840x^9\arctan(ax)a^9d^4-4320x^7\arctan(ax)a^9c\,d^3+105d^4a^8x^8-9072x^5\arctan(ax)a^9c^2d^2+720c\,a^8d^3x^6-10080x^3}{}$ |
| meijerg | $\frac{d^4\left(\frac{x^2a^2\left(-15a^6x^6+20x^4a^4-30a^2x^2+60\right)}{270}+\frac{4x^{10}a^{10}\arctan\left(\sqrt{a^2x^2}\right)}{9\sqrt{a^2x^2}}-\frac{2\ln\left(a^2x^2+1\right)}{9}\right)}{4a^9}+\frac{d^3c\left(-\frac{a^2x^2\left(4x^4a^4-6a^2x^2+12\right)}{42}\right.}{}$ |
| risch | $\frac{2i\ln(-iax+1)d^3c\,x^7}{7}+\frac{i\left(-\frac{1}{9}d^4x^9-\frac{4}{7}d^3c\,x^7-\frac{6}{5}c^2d^2x^5-\frac{4}{3}d\,c^3x^3-x\,c^4\right)\ln(iax+1)}{2}+\frac{i\ln(-iax+1)x\,c^4}{2}-\frac{d^4x^8}{72a}+$ |

input
```
int((d*x^2+c)^4*arctan(a*x),x,method=_RETURNVERBOSE)
```

output
```
1/9*d^4*x^9*arctan(a*x)+4/7*c*d^3*x^7*arctan(a*x)+6/5*c^2*d^2*x^5*arctan(a
*x)+4/3*c^3*d*x^3*arctan(a*x)+c^4*x*arctan(a*x)-1/315*a*(1/2*d/a^8*(35/4*a
^6*d^3*x^8+60*a^6*c*d^2*x^6+189*a^6*c^2*d*x^4+420*a^6*c^3*x^2-35/3*a^4*d^3
*x^6-90*a^4*c*d^2*x^4-378*a^4*c^2*d*x^2+35/2*a^2*d^3*x^4+180*a^2*c*d^2*x^2
-35*d^3*x^2)+1/2*(315*a^8*c^4-420*a^6*c^3*d+378*a^4*c^2*d^2-180*a^2*c*d^3+
35*d^4)/a^10*ln(a^2*x^2+1))
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.97

$$\int \left(c+dx^2\right)^4\arctan(ax)\,dx =$$
$$-\frac{105\,a^8d^4x^8+20\,(36\,a^8cd^3-7\,a^6d^4)x^6+6\,(378\,a^8c^2d^2-180\,a^6cd^3+35\,a^4d^4)x^4+12\,(420\,a^8c^3d-378}{}$$

input
```
integrate((d*x^2+c)^4*arctan(a*x),x, algorithm="fricas")
```

output
```
-1/7560*(105*a^8*d^4*x^8 + 20*(36*a^8*c*d^3 - 7*a^6*d^4)*x^6 + 6*(378*a^8*
c^2*d^2 - 180*a^6*c*d^3 + 35*a^4*d^4)*x^4 + 12*(420*a^8*c^3*d - 378*a^6*c^
2*d^2 + 180*a^4*c*d^3 - 35*a^2*d^4)*x^2 - 24*(35*a^9*d^4*x^9 + 180*a^9*c*d
^3*x^7 + 378*a^9*c^2*d^2*x^5 + 420*a^9*c^3*d*x^3 + 315*a^9*c^4*x)*arctan(a
*x) + 12*(315*a^8*c^4 - 420*a^6*c^3*d + 378*a^4*c^2*d^2 - 180*a^2*c*d^3 +
35*d^4)*log(a^2*x^2 + 1))/a^9
```

## Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.29

$$\int (c+dx^2)^4 \arctan(ax)\,dx$$
$$= \begin{cases} c^4x\,\mathrm{atan}\,(ax) + \frac{4c^3dx^3\,\mathrm{atan}\,(ax)}{3} + \frac{6c^2d^2x^5\,\mathrm{atan}\,(ax)}{5} + \frac{4cd^3x^7\,\mathrm{atan}\,(ax)}{7} + \frac{d^4x^9\,\mathrm{atan}\,(ax)}{9} - \frac{c^4\log\left(x^2+\frac{1}{a^2}\right)}{2a} - \frac{2c^3dx^2}{3a} - \\ 0 \end{cases}$$

input
```
integrate((d*x**2+c)**4*atan(a*x),x)
```

output
```
Piecewise((c**4*x*atan(a*x) + 4*c**3*d*x**3*atan(a*x)/3 + 6*c**2*d**2*x**5
*atan(a*x)/5 + 4*c*d**3*x**7*atan(a*x)/7 + d**4*x**9*atan(a*x)/9 - c**4*lo
g(x**2 + a**(-2))/(2*a) - 2*c**3*d*x**2/(3*a) - 3*c**2*d**2*x**4/(10*a) -
2*c*d**3*x**6/(21*a) - d**4*x**8/(72*a) + 2*c**3*d*log(x**2 + a**(-2))/(3*
a**3) + 3*c**2*d**2*x**2/(5*a**3) + c*d**3*x**4/(7*a**3) + d**4*x**6/(54*a
**3) - 3*c**2*d**2*log(x**2 + a**(-2))/(5*a**5) - 2*c*d**3*x**2/(7*a**5) -
 d**4*x**4/(36*a**5) + 2*c*d**3*log(x**2 + a**(-2))/(7*a**7) + d**4*x**2/(
18*a**7) - d**4*log(x**2 + a**(-2))/(18*a**9), Ne(a, 0)), (0, True))
```

## Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.93

$$\int (c+dx^2)^4 \arctan(ax)\,dx =$$
$$-\frac{1}{7560}a\left(\frac{105\,a^6d^4x^8 + 20\,(36\,a^6cd^3 - 7\,a^4d^4)x^6 + 6\,(378\,a^6c^2d^2 - 180\,a^4cd^3 + 35\,a^2d^4)x^4 + 12\,(420\,a^6c}{a^8}\right.$$
$$+\frac{1}{315}\left(35\,d^4x^9 + 180\,cd^3x^7 + 378\,c^2d^2x^5 + 420\,c^3dx^3 + 315\,c^4x\right)\arctan(ax)$$

input
```
integrate((d*x^2+c)^4*arctan(a*x),x, algorithm="maxima")
```

output
```
-1/7560*a*((105*a^6*d^4*x^8 + 20*(36*a^6*c*d^3 - 7*a^4*d^4)*x^6 + 6*(378*a
^6*c^2*d^2 - 180*a^4*c*d^3 + 35*a^2*d^4)*x^4 + 12*(420*a^6*c^3*d - 378*a^4
*c^2*d^2 + 180*a^2*c*d^3 - 35*d^4)*x^2)/a^8 + 12*(315*a^8*c^4 - 420*a^6*c^
3*d + 378*a^4*c^2*d^2 - 180*a^2*c*d^3 + 35*d^4)*log(a^2*x^2 + 1)/a^10) + 1
/315*(35*d^4*x^9 + 180*c*d^3*x^7 + 378*c^2*d^2*x^5 + 420*c^3*d*x^3 + 315*c
^4*x)*arctan(a*x)
```

## Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.95

$$\int \left(c+dx^2\right)^4 \arctan(ax)\,dx$$
$$= \frac{1}{315}\left(35\,d^4x^9+180\,cd^3x^7+378\,c^2d^2x^5+420\,c^3dx^3+315\,c^4x\right)\arctan\left(ax\right)$$
$$-\frac{105\,a^7d^4x^8+720\,a^7cd^3x^6+2268\,a^7c^2d^2x^4-140\,a^5d^4x^6+5040\,a^7c^3dx^2-1080\,a^5cd^3x^4-4536\,a^5c^2d^2}{7560\,a^8}$$
$$-\frac{\left(315\,a^8c^4-420\,a^6c^3d+378\,a^4c^2d^2-180\,a^2cd^3+35\,d^4\right)\log\left(a^2x^2+1\right)}{630\,a^9}$$

input
```
integrate((d*x^2+c)^4*arctan(a*x),x, algorithm="giac")
```

output
```
1/315*(35*d^4*x^9 + 180*c*d^3*x^7 + 378*c^2*d^2*x^5 + 420*c^3*d*x^3 + 315*
c^4*x)*arctan(a*x) - 1/7560*(105*a^7*d^4*x^8 + 720*a^7*c*d^3*x^6 + 2268*a^
7*c^2*d^2*x^4 - 140*a^5*d^4*x^6 + 5040*a^7*c^3*d*x^2 - 1080*a^5*c*d^3*x^4
- 4536*a^5*c^2*d^2*x^2 + 210*a^3*d^4*x^4 + 2160*a^3*c*d^3*x^2 - 420*a*d^4*
x^2)/a^8 - 1/630*(315*a^8*c^4 - 420*a^6*c^3*d + 378*a^4*c^2*d^2 - 180*a^2*
c*d^3 + 35*d^4)*log(a^2*x^2 + 1)/a^9
```

## Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.95

$$\int \left(c+dx^2\right)^4 \arctan(ax)\, dx$$
$$= \operatorname{atan}(a\,x)\,\left(c^4\,x+\frac{4\,c^3\,d\,x^3}{3}+\frac{6\,c^2\,d^2\,x^5}{5}+\frac{4\,c\,d^3\,x^7}{7}+\frac{d^4\,x^9}{9}\right)$$
$$+x^2\left(\frac{\frac{\frac{d^4}{9\,a^3}-\frac{4\,c\,d^3}{7\,a}}{a^2}+\frac{6\,c^2\,d^2}{5\,a}}{2\,a^2}-\frac{2\,c^3\,d}{3\,a}\right)+x^6\left(\frac{d^4}{54\,a^3}-\frac{2\,c\,d^3}{21\,a}\right)-x^4\left(\frac{\frac{d^4}{9\,a^3}-\frac{4\,c\,d^3}{7\,a}}{4\,a^2}+\frac{3\,c^2\,d^2}{10\,a}\right)$$
$$-\frac{\ln\left(a^2\,x^2+1\right)\,\left(315\,a^8\,c^4-420\,a^6\,c^3\,d+378\,a^4\,c^2\,d^2-180\,a^2\,c\,d^3+35\,d^4\right)}{630\,a^9}-\frac{d^4\,x^8}{72\,a}$$

input
```
int(atan(a*x)*(c + d*x^2)^4,x)
```

output
```
atan(a*x)*(c^4*x + (d^4*x^9)/9 + (4*c^3*d*x^3)/3 + (4*c*d^3*x^7)/7 + (6*c^
2*d^2*x^5)/5) + x^2*(((d^4/(9*a^3) - (4*c*d^3)/(7*a))/a^2 + (6*c^2*d^2)/(5
*a))/(2*a^2) - (2*c^3*d)/(3*a)) + x^6*(d^4/(54*a^3) - (2*c*d^3)/(21*a)) -
x^4*((d^4/(9*a^3) - (4*c*d^3)/(7*a))/(4*a^2) + (3*c^2*d^2)/(10*a)) - (log(
a^2*x^2 + 1)*(35*d^4 + 315*a^8*c^4 - 180*a^2*c*d^3 - 420*a^6*c^3*d + 378*a
^4*c^2*d^2))/(630*a^9) - (d^4*x^8)/(72*a)
```

## Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.21

$$\int \left(c+dx^2\right)^4 \arctan(ax)\, dx$$
$$= \frac{7560 atan(ax)\, a^9 c^4 x + 10080 atan(ax)\, a^9 c^3 d\, x^3 + 9072 atan(ax)\, a^9 c^2 d^2 x^5 + 4320 atan(ax)\, a^9 c\, d^3 x^7 + 840}{}$$

input
```
int((d*x^2+c)^4*atan(a*x),x)
```

output
```
(7560*atan(a*x)*a**9*c**4*x + 10080*atan(a*x)*a**9*c**3*d*x**3 + 9072*atan
(a*x)*a**9*c**2*d**2*x**5 + 4320*atan(a*x)*a**9*c*d**3*x**7 + 840*atan(a*x
)*a**9*d**4*x**9 - 3780*log(a**2*x**2 + 1)*a**8*c**4 + 5040*log(a**2*x**2
+ 1)*a**6*c**3*d - 4536*log(a**2*x**2 + 1)*a**4*c**2*d**2 + 2160*log(a**2*
x**2 + 1)*a**2*c*d**3 - 420*log(a**2*x**2 + 1)*d**4 - 5040*a**8*c**3*d*x**
2 - 2268*a**8*c**2*d**2*x**4 - 720*a**8*c*d**3*x**6 - 105*a**8*d**4*x**8 +
 4536*a**6*c**2*d**2*x**2 + 1080*a**6*c*d**3*x**4 + 140*a**6*d**4*x**6 - 2
160*a**4*c*d**3*x**2 - 210*a**4*d**4*x**4 + 420*a**2*d**4*x**2)/(7560*a**9
)
```

# 3.1150 $\int \frac{x^3(a+b\arctan(cx))}{d+ex^2}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 361

$$\begin{aligned}\int \frac{x^3(a+b\arctan(cx))}{d+ex^2}\,dx = &-\frac{bx}{2ce}+\frac{b\arctan(cx)}{2c^2e}+\frac{x^2(a+b\arctan(cx))}{2e}\\ &+\frac{d(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)}{e^2}\\ &-\frac{d(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2e^2}\\ &-\frac{d(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{2e^2}\\ &-\frac{ibd\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2e^2}\\ &+\frac{ibd\,\mathrm{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{4e^2}\\ &+\frac{ibd\,\mathrm{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{4e^2}\end{aligned}$$

output
```
-1/2*b*x/c/e+1/2*b*arctan(c*x)/c^2/e+1/2*x^2*(a+b*arctan(c*x))/e+d*(a+b*ar
ctan(c*x))*ln(2/(1-I*c*x))/e^2-1/2*d*(a+b*arctan(c*x))*ln(2*c*((-d)^(1/2)-
e^(1/2)*x)/(c*(-d)^(1/2)-I*e^(1/2))/(1-I*c*x))/e^2-1/2*d*(a+b*arctan(c*x))
*ln(2*c*((-d)^(1/2)+e^(1/2)*x)/(c*(-d)^(1/2)+I*e^(1/2))/(1-I*c*x))/e^2-1/2
*I*b*d*polylog(2,1-2/(1-I*c*x))/e^2+1/4*I*b*d*polylog(2,1-2*c*((-d)^(1/2)-
e^(1/2)*x)/(c*(-d)^(1/2)-I*e^(1/2))/(1-I*c*x))/e^2+1/4*I*b*d*polylog(2,1-2
*c*((-d)^(1/2)+e^(1/2)*x)/(c*(-d)^(1/2)+I*e^(1/2))/(1-I*c*x))/e^2
```

## Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 475, normalized size of antiderivative = 1.32

$$\int \frac{x^3(a+b\arctan(cx))}{d+ex^2}\,dx$$
$$= \frac{-2bcex+2ac^2ex^2+2be\arctan(cx)+2bc^2ex^2\arctan(cx)+ibc^2d\log(1+icx)\log\left(\frac{c(\sqrt{-d}-\sqrt{e}x)}{c\sqrt{-d}-i\sqrt{e}}\right)-ibc^2d}{}$$

input
```
Integrate[(x^3*(a + b*ArcTan[c*x]))/(d + e*x^2),x]
```

output
```
(-2*b*c*e*x + 2*a*c^2*e*x^2 + 2*b*e*ArcTan[c*x] + 2*b*c^2*e*x^2*ArcTan[c*x
] + I*b*c^2*d*Log[1 + I*c*x]*Log[(c*(Sqrt[-d] - Sqrt[e]*x))/(c*Sqrt[-d] -
I*Sqrt[e])] - I*b*c^2*d*Log[1 - I*c*x]*Log[(c*(Sqrt[-d] - Sqrt[e]*x))/(c*S
qrt[-d] + I*Sqrt[e])] - I*b*c^2*d*Log[1 - I*c*x]*Log[(c*(Sqrt[-d] + Sqrt[e
]*x))/(c*Sqrt[-d] - I*Sqrt[e])] + I*b*c^2*d*Log[1 + I*c*x]*Log[(c*(Sqrt[-d
] + Sqrt[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])] - 2*a*c^2*d*Log[d + e*x^2] + I*b
*c^2*d*PolyLog[2, (Sqrt[e]*(I - c*x))/(c*Sqrt[-d] + I*Sqrt[e])] - I*b*c^2*
d*PolyLog[2, (Sqrt[e]*(1 - I*c*x))/(I*c*Sqrt[-d] + Sqrt[e])] + I*b*c^2*d*P
olyLog[2, (Sqrt[e]*(1 + I*c*x))/(I*c*Sqrt[-d] + Sqrt[e])] - I*b*c^2*d*Poly
Log[2, (Sqrt[e]*(I + c*x))/(c*Sqrt[-d] + I*Sqrt[e])])/(4*c^2*e^2)
```

## Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 359, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5451, 5361, 262, 216, 5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\arctan(cx))}{d+ex^2}dx$$

$$\Big\downarrow \text{5451}$$

$$\frac{\int x(a+b\arctan(cx))dx}{e}-\frac{d\int \frac{x(a+b\arctan(cx))}{ex^2+d}dx}{e}$$

$$\Big\downarrow \text{5361}$$

$$\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\int \frac{x^2}{c^2x^2+1}dx}{e}-\frac{d\int \frac{x(a+b\arctan(cx))}{ex^2+d}dx}{e}$$

$$\Big\downarrow \text{262}$$

$$\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\int \frac{1}{c^2x^2+1}dx}{c^2}\right)}{e}-\frac{d\int \frac{x(a+b\arctan(cx))}{ex^2+d}dx}{e}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)}{e}-\frac{d\int \frac{x(a+b\arctan(cx))}{ex^2+d}dx}{e}$$

$$\Big\downarrow \text{5515}$$

$$\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)}{e}-\frac{d\int \left(\frac{a+b\arctan(cx)}{2\sqrt{e}(\sqrt{e}x+\sqrt{-d})}-\frac{a+b\arctan(cx)}{2\sqrt{e}(\sqrt{-d}-\sqrt{e}x)}\right)dx}{e}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{2}x^2(a+b\arctan(cx))-\frac{1}{2}bc\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)}{e}-$$

$$\frac{d\left(\frac{(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(1-icx)(c\sqrt{-d}-i\sqrt{e})}\right)}{2e}+\frac{(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(1-icx)(c\sqrt{-d}+i\sqrt{e})}\right)}{2e}-\frac{\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))}{e}-\frac{ib\,\text{PolyLog}(}{}\right.}{e}$$

input
```
Int[(x^3*(a + b*ArcTan[c*x]))/(d + e*x^2),x]
```

output
```
((x^2*(a + b*ArcTan[c*x]))/2 - (b*c*(x/c^2 - ArcTan[c*x]/c^3))/2)/e - (d*(
-(((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e) + ((a + b*ArcTan[c*x])*Log[(
2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(2*e)
 + ((a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*
Sqrt[e])*(1 - I*c*x))])/(2*e) + ((I/2)*b*PolyLog[2, 1 - 2/(1 - I*c*x)])/e
- ((I/4)*b*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sq
rt[e])*(1 - I*c*x))])/e - ((I/4)*b*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sqrt[e]
*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/e))/e
```

## Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

## Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 503, normalized size of antiderivative = 1.39

| method | result |
| --- | --- |
| risch | $\frac{ibd\,\mathrm{dilog}\left(\frac{c\sqrt{de}-(icx+1)e+e}{c\sqrt{de}+e}\right)}{4e^2} - \frac{ib\ln(icx+1)}{4c^2e} + \frac{ibd\,\mathrm{dilog}\left(\frac{c\sqrt{de}+(icx+1)e-e}{c\sqrt{de}-e}\right)}{4e^2} - \frac{bx}{2ce} + \frac{ib\ln(-icx+1)x^2}{4e} - \frac{ibd\ln}{}$ |
| parts | $\frac{a\,x^2}{2e} - \frac{ad\ln(e\,x^2+d)}{2e^2} + \frac{b\left(\frac{\arctan(cx)c^4x^2}{2e} - \frac{\arctan(cx)c^4d\ln\left(c^2e\,x^2+c^2d\right)}{2e^2} - \frac{c^2\left(\frac{cx-\arctan(cx)}{e} - \frac{dc^2\left(-\frac{i\left(\ln(cx-i)\ln\left(\right.\right.}{}\right.}{}\right.}{}\right.}{}$ |
| | $\ln(cx-i)\ln$ |

input
```
int(x^3*(a+b*arctan(c*x))/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
1/4*I*b*d/e^2*dilog((c*(d*e)^(1/2)-(1+I*c*x)*e+e)/(c*(d*e)^(1/2)+e))-1/4*I
*b/c^2/e*ln(1+I*c*x)+1/4*I*b*d/e^2*dilog((c*(d*e)^(1/2)+(1+I*c*x)*e-e)/(c*
(d*e)^(1/2)-e))-1/2*b*x/c/e+1/4*I*b/e*ln(1-I*c*x)*x^2-1/4*I*b*d/e^2*ln(1-I
*c*x)*ln((c*(d*e)^(1/2)+(1-I*c*x)*e-e)/(c*(d*e)^(1/2)-e))+1/4*I/c^2*b/e*ln
(1-I*c*x)-1/4*I*b*d/e^2*dilog((c*(d*e)^(1/2)-(1-I*c*x)*e+e)/(c*(d*e)^(1/2)
+e))-1/4*I*b*d/e^2*dilog((c*(d*e)^(1/2)+(1-I*c*x)*e-e)/(c*(d*e)^(1/2)-e))+
1/2*a*x^2/e+1/2/c^2*a/e-1/2*a*d/e^2*ln((1-I*c*x)^2*e-c^2*d-2*(1-I*c*x)*e+e
)+1/4*I*b*d/e^2*ln(1+I*c*x)*ln((c*(d*e)^(1/2)-(1+I*c*x)*e+e)/(c*(d*e)^(1/2
)+e))+1/4*I*b*d/e^2*ln(1+I*c*x)*ln((c*(d*e)^(1/2)+(1+I*c*x)*e-e)/(c*(d*e)^
(1/2)-e))-1/4*I*b*d/e^2*ln(1-I*c*x)*ln((c*(d*e)^(1/2)-(1-I*c*x)*e+e)/(c*(d
*e)^(1/2)+e))-1/4*I*b/e*ln(1+I*c*x)*x^2
```

## Fricas [F]

$$\int \frac{x^3(a+b\arctan(cx))}{d+ex^2}\,dx = \int \frac{(b\arctan(cx)+a)x^3}{ex^2+d}\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*x^3*arctan(c*x) + a*x^3)/(e*x^2 + d), x)
```

## Sympy [F]

$$\int \frac{x^3(a+b\arctan(cx))}{d+ex^2}\,dx = \int \frac{x^3(a+b\operatorname{atan}(cx))}{d+ex^2}\,dx$$

input
```
integrate(x**3*(a+b*atan(c*x))/(e*x**2+d),x)
```

output
```
Integral(x**3*(a + b*atan(c*x))/(d + e*x**2), x)
```

## Maxima [F]

$$\int \frac{x^3(a+b\arctan(cx))}{d+ex^2}\,dx = \int \frac{(b\arctan(cx)+a)x^3}{ex^2+d}\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))/(e*x^2+d),x, algorithm="maxima")
```

output
```
1/2*a*(x^2/e - d*log(e*x^2 + d)/e^2) + 2*b*integrate(1/2*x^3*arctan(c*x)/(
e*x^2 + d), x)
```

## Giac [F]

$$\int \frac{x^3(a+b\arctan(cx))}{d+ex^2}\,dx = \int \frac{(b\arctan(cx)+a)x^3}{ex^2+d}\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))/(e*x^2+d),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)*x^3/(e*x^2 + d), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\arctan(cx))}{d+ex^2}\,dx = \int \frac{x^3\,(a+b\,\mathrm{atan}(c\,x))}{e\,x^2+d}\,dx$$

input
```
int((x^3*(a + b*atan(c*x)))/(d + e*x^2),x)
```

output
```
int((x^3*(a + b*atan(c*x)))/(d + e*x^2), x)
```

## Reduce [F]

$$\int \frac{x^3(a+b\arctan(cx))}{d+ex^2}\,dx = \frac{2\left(\int \frac{atan(cx)x^3}{e\,x^2+d}dx\right)b\,e^2-\log(e\,x^2+d)\,ad+ae\,x^2}{2e^2}$$

input
```
int(x^3*(a+b*atan(c*x))/(e*x^2+d),x)
```

output
```
(2*int((atan(c*x)*x**3)/(d + e*x**2),x)*b*e**2 - log(d + e*x**2)*a*d + a*e
*x**2)/(2*e**2)
```

# 3.1151 $\int \frac{x(a+b\arctan(cx))}{d+ex^2}\,dx$



## Optimal result

Integrand size = 19, antiderivative size = 311

$$\int \frac{x(a+b\arctan(cx))}{d+ex^2}\,dx = -\frac{(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)}{e} + \frac{(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2e} + \frac{(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{2e} + \frac{ib\,\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2e} - \frac{ib\,\text{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{4e} - \frac{ib\,\text{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{4e}$$

output

```
-(a+b*arctan(c*x))*ln(2/(1-I*c*x))/e+1/2*(a+b*arctan(c*x))*ln(2*c*((-d)^(1
/2)-e^(1/2)*x)/(c*(-d)^(1/2)-I*e^(1/2))/(1-I*c*x))/e+1/2*(a+b*arctan(c*x))
*ln(2*c*((-d)^(1/2)+e^(1/2)*x)/(c*(-d)^(1/2)+I*e^(1/2))/(1-I*c*x))/e+1/2*I
*b*polylog(2,1-2/(1-I*c*x))/e-1/4*I*b*polylog(2,1-2*c*((-d)^(1/2)-e^(1/2)*
x)/(c*(-d)^(1/2)-I*e^(1/2))/(1-I*c*x))/e-1/4*I*b*polylog(2,1-2*c*((-d)^(1/
2)+e^(1/2)*x)/(c*(-d)^(1/2)+I*e^(1/2))/(1-I*c*x))/e
```

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.42

$$\int \frac{x(a+b\arctan(cx))}{d+ex^2}\,dx = -\frac{ib\log(1+icx)\log\left(\frac{c(\sqrt{-d}-\sqrt{e}x)}{c\sqrt{-d}-i\sqrt{e}}\right)}{4e} + \frac{ib\log(1-icx)\log\left(\frac{c(\sqrt{-d}-\sqrt{e}x)}{c\sqrt{-d}+i\sqrt{e}}\right)}{4e} + \frac{ib\log(1-icx)\log\left(\frac{c(\sqrt{-d}+\sqrt{e}x)}{c\sqrt{-d}-i\sqrt{e}}\right)}{4e} - \frac{ib\log(1+icx)\log\left(\frac{c(\sqrt{-d}+\sqrt{e}x)}{c\sqrt{-d}+i\sqrt{e}}\right)}{4e} + \frac{a\log(d+ex^2)}{2e} + \frac{ib\,\text{PolyLog}\left(2,-\frac{\sqrt{e}(1-icx)}{ic\sqrt{-d}-\sqrt{e}}\right)}{4e} + \frac{ib\,\text{PolyLog}\left(2,\frac{\sqrt{e}(1-icx)}{ic\sqrt{-d}+\sqrt{e}}\right)}{4e} - \frac{ib\,\text{PolyLog}\left(2,-\frac{\sqrt{e}(1+icx)}{ic\sqrt{-d}-\sqrt{e}}\right)}{4e} - \frac{ib\,\text{PolyLog}\left(2,\frac{\sqrt{e}(1+icx)}{ic\sqrt{-d}+\sqrt{e}}\right)}{4e}$$

input
```
Integrate[(x*(a + b*ArcTan[c*x]))/(d + e*x^2),x]
```

output
```
((-1/4*I)*b*Log[1 + I*c*x]*Log[(c*(Sqrt[-d] - Sqrt[e]*x))/(c*Sqrt[-d] - I*
Sqrt[e])])/e + ((I/4)*b*Log[1 - I*c*x]*Log[(c*(Sqrt[-d] - Sqrt[e]*x))/(c*S
qrt[-d] + I*Sqrt[e])])/e + ((I/4)*b*Log[1 - I*c*x]*Log[(c*(Sqrt[-d] + Sqrt
[e]*x))/(c*Sqrt[-d] - I*Sqrt[e])])/e - ((I/4)*b*Log[1 + I*c*x]*Log[(c*(Sqr
t[-d] + Sqrt[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])])/e + (a*Log[d + e*x^2])/(2*e
) + ((I/4)*b*PolyLog[2, -((Sqrt[e]*(1 - I*c*x))/(I*c*Sqrt[-d] - Sqrt[e]))]
)/e + ((I/4)*b*PolyLog[2, (Sqrt[e]*(1 - I*c*x))/(I*c*Sqrt[-d] + Sqrt[e])])
/e - ((I/4)*b*PolyLog[2, -((Sqrt[e]*(1 + I*c*x))/(I*c*Sqrt[-d] - Sqrt[e]))
])/e - ((I/4)*b*PolyLog[2, (Sqrt[e]*(1 + I*c*x))/(I*c*Sqrt[-d] + Sqrt[e])]
)/e
```

## Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\arctan(cx))}{d+ex^2}\,dx$$

$$\Big\downarrow \text{ 5515}$$

$$\int \left(\frac{a+b\arctan(cx)}{2\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)}-\frac{a+b\arctan(cx)}{2\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(1-icx)(c\sqrt{-d}-i\sqrt{e})}\right)}{2e}+\frac{(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(1-icx)(c\sqrt{-d}+i\sqrt{e})}\right)}{2e}-\frac{\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))}{e}-\frac{ib\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{4e}-\frac{ib\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)}{4e}+\frac{ib\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2e}$$

input
```
Int[(x*(a + b*ArcTan[c*x]))/(d + e*x^2),x]
```

output
```
-(((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e) + ((a + b*ArcTan[c*x])*Log[(
2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(2*e)
 + ((a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*
Sqrt[e])*(1 - I*c*x))])/(2*e) + ((I/2)*b*PolyLog[2, 1 - 2/(1 - I*c*x)])/e
- ((I/4)*b*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sq
rt[e])*(1 - I*c*x))])/e - ((I/4)*b*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sqrt[e]
*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/e
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

## Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.27

| method | result |
|---|---|
| risch | $\frac{ib\ln(-icx+1)\ln\left(\frac{c\sqrt{de}-(-icx+1)e+e}{c\sqrt{de}+e}\right)}{4e}+\frac{ib\ln(-icx+1)\ln\left(\frac{c\sqrt{de}+(-icx+1)e-e}{c\sqrt{de}-e}\right)}{4e}+\frac{ib\,\mathrm{dilog}\left(\frac{c\sqrt{de}-(-icx+1)e+e}{c\sqrt{de}+e}\right)}{4e}-$ |
| parts | $\frac{a\ln(e\,x^2+d)}{2e}+\frac{b\left(\frac{\arctan(cx)c^2\ln\left(c^2e\,x^2+c^2d\right)}{2e}-\frac{c^2\left(-\frac{i\left(\ln(cx-i)\ln\left(c^2e\,x^2+c^2d\right)-2e\left(\frac{\ln(cx-i)\left(\ln\left(\frac{\mathrm{RootOf}\left(e\_Z^2+2\right.}{\mathrm{RootOf}\left(e\_Z^2\right.}\right.\right.}{}\right.\right.}{}\right.\right.}{}\right.}{}$ |
| derivativedivides | $\frac{\frac{a\,c^2\ln\left(c^2e\,x^2+c^2d\right)}{2e}+b\,c^2\left(\frac{\arctan(cx)\ln\left(c^2e\,x^2+c^2d\right)}{2e}-\frac{-\frac{i\left(\ln(cx-i)\ln\left(c^2e\,x^2+c^2d\right)-2e\left(\frac{\ln(cx-i)\left(\ln\left(\frac{\mathrm{RootOf}\left(e\_Z^2+2\right.}{\mathrm{RootOf}\left(e\_Z^2\right.}\right.\right.}{}\right.\right.}{}}{}\right.}{}$ |
| default | $\frac{\frac{a\,c^2\ln\left(c^2e\,x^2+c^2d\right)}{2e}+b\,c^2\left(\frac{\arctan(cx)\ln\left(c^2e\,x^2+c^2d\right)}{2e}-\frac{-\frac{i\left(\ln(cx-i)\ln\left(c^2e\,x^2+c^2d\right)-2e\left(\frac{\ln(cx-i)\left(\ln\left(\frac{\mathrm{RootOf}\left(e\_Z^2+2\right.}{\mathrm{RootOf}\left(e\_Z^2\right.}\right.\right.}{}\right.\right.}{}}{}\right.}{}$ |

input
```
int(x*(a+b*arctan(c*x))/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
1/4*I*b*ln(1-I*c*x)/e*ln((c*(d*e)^(1/2)-(1-I*c*x)*e+e)/(c*(d*e)^(1/2)+e))+
1/4*I*b*ln(1-I*c*x)/e*ln((c*(d*e)^(1/2)+(1-I*c*x)*e-e)/(c*(d*e)^(1/2)-e))+
1/4*I*b/e*dilog((c*(d*e)^(1/2)-(1-I*c*x)*e+e)/(c*(d*e)^(1/2)+e))+1/4*I*b/e
*dilog((c*(d*e)^(1/2)+(1-I*c*x)*e-e)/(c*(d*e)^(1/2)-e))+1/2*a/e*ln((1-I*c*
x)^2*e-c^2*d-2*(1-I*c*x)*e+e)-1/4*I*b*ln(1+I*c*x)/e*ln((c*(d*e)^(1/2)-(1+I
*c*x)*e+e)/(c*(d*e)^(1/2)+e))-1/4*I*b*ln(1+I*c*x)/e*ln((c*(d*e)^(1/2)+(1+I
*c*x)*e-e)/(c*(d*e)^(1/2)-e))-1/4*I*b/e*dilog((c*(d*e)^(1/2)-(1+I*c*x)*e+e
)/(c*(d*e)^(1/2)+e))-1/4*I*b/e*dilog((c*(d*e)^(1/2)+(1+I*c*x)*e-e)/(c*(d*e
)^(1/2)-e))
```

## Fricas [F]

$$\int \frac{x(a+b\arctan(cx))}{d+ex^2}\,dx = \int \frac{(b\arctan(cx)+a)x}{ex^2+d}\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*x*arctan(c*x) + a*x)/(e*x^2 + d), x)
```

## Sympy [F]

$$\int \frac{x(a+b\arctan(cx))}{d+ex^2}\,dx = \int \frac{x(a+b\operatorname{atan}(cx))}{d+ex^2}\,dx$$

input
```
integrate(x*(a+b*atan(c*x))/(e*x**2+d),x)
```

output
```
Integral(x*(a + b*atan(c*x))/(d + e*x**2), x)
```

## Maxima [F]

$$\int \frac{x(a+b\arctan(cx))}{d+ex^2}\,dx=\int \frac{(b\arctan{(cx)}+a)x}{ex^2+d}\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))/(e*x^2+d),x, algorithm="maxima")
```

output
```
2*b*integrate(1/2*x*arctan(c*x)/(e*x^2 + d), x) + 1/2*a*log(e*x^2 + d)/e
```

## Giac [F]

$$\int \frac{x(a+b\arctan(cx))}{d+ex^2}\,dx=\int \frac{(b\arctan{(cx)}+a)x}{ex^2+d}\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))/(e*x^2+d),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)*x/(e*x^2 + d), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\arctan(cx))}{d+ex^2}\,dx=\int \frac{x\,(a+b\,\mathrm{atan}(c\,x))}{e\,x^2+d}\,dx$$

input
```
int((x*(a + b*atan(c*x)))/(d + e*x^2),x)
```

output
```
int((x*(a + b*atan(c*x)))/(d + e*x^2), x)
```

## Reduce [F]

$$\int \frac{x(a+b\arctan(cx))}{d+ex^2}\,dx = \frac{2\left(\int \frac{atan(cx)x}{e\,x^2+d}dx\right)be+\log(e\,x^2+d)\,a}{2e}$$

input
```
int(x*(a+b*atan(c*x))/(e*x^2+d),x)
```

output
```
(2*int((atan(c*x)*x)/(d + e*x**2),x)*b*e + log(d + e*x**2)*a)/(2*e)
```

# 3.1152 $\int \frac{a+b\arctan(cx)}{x(d+ex^2)}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 353

$$\begin{aligned}\int \frac{a+b\arctan(cx)}{x(d+ex^2)}\,dx &= \frac{a\log(x)}{d}+\frac{(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)}{d}\\
&-\frac{(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2d}\\
&-\frac{(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{2d}\\
&+\frac{ib\,\mathrm{PolyLog}(2,-icx)}{2d}-\frac{ib\,\mathrm{PolyLog}(2,icx)}{2d}\\
&-\frac{ib\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2d}\\
&+\frac{ib\,\mathrm{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{4d}\\
&+\frac{ib\,\mathrm{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{4d}\end{aligned}$$

output
```
a*ln(x)/d+(a+b*arctan(c*x))*ln(2/(1-I*c*x))/d-1/2*(a+b*arctan(c*x))*ln(2*c
*((-d)^(1/2)-e^(1/2)*x)/(c*(-d)^(1/2)-I*e^(1/2))/(1-I*c*x))/d-1/2*(a+b*arc
tan(c*x))*ln(2*c*((-d)^(1/2)+e^(1/2)*x)/(c*(-d)^(1/2)+I*e^(1/2))/(1-I*c*x)
)/d+1/2*I*b*polylog(2,-I*c*x)/d-1/2*I*b*polylog(2,I*c*x)/d-1/2*I*b*polylog
(2,1-2/(1-I*c*x))/d+1/4*I*b*polylog(2,1-2*c*((-d)^(1/2)-e^(1/2)*x)/(c*(-d)
^(1/2)-I*e^(1/2))/(1-I*c*x))/d+1/4*I*b*polylog(2,1-2*c*((-d)^(1/2)+e^(1/2)
*x)/(c*(-d)^(1/2)+I*e^(1/2))/(1-I*c*x))/d
```

## Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 484, normalized size of antiderivative = 1.37

$$\begin{aligned}\int \frac{a+b\arctan(cx)}{x(d+ex^2)}\,dx =& \frac{a\log(x)}{d}+\frac{ib\log(1+icx)\log\left(\frac{c(\sqrt{-d}-\sqrt{e}x)}{c\sqrt{-d}-i\sqrt{e}}\right)}{4d}\\&-\frac{ib\log(1-icx)\log\left(\frac{c(\sqrt{-d}-\sqrt{e}x)}{c\sqrt{-d}+i\sqrt{e}}\right)}{4d}\\&-\frac{ib\log(1-icx)\log\left(\frac{c(\sqrt{-d}+\sqrt{e}x)}{c\sqrt{-d}-i\sqrt{e}}\right)}{4d}\\&+\frac{ib\log(1+icx)\log\left(\frac{c(\sqrt{-d}+\sqrt{e}x)}{c\sqrt{-d}+i\sqrt{e}}\right)}{4d}-\frac{a\log(d+ex^2)}{2d}\\&+\frac{ib\,\mathrm{PolyLog}(2,-icx)}{2d}-\frac{ib\,\mathrm{PolyLog}(2,icx)}{2d}\\&-\frac{ib\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{e}(1-icx)}{ic\sqrt{-d}-\sqrt{e}}\right)}{4d}-\frac{ib\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}(1-icx)}{ic\sqrt{-d}+\sqrt{e}}\right)}{4d}\\&+\frac{ib\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{e}(1+icx)}{ic\sqrt{-d}-\sqrt{e}}\right)}{4d}+\frac{ib\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}(1+icx)}{ic\sqrt{-d}+\sqrt{e}}\right)}{4d}\end{aligned}$$

input
```
Integrate[(a + b*ArcTan[c*x])/(x*(d + e*x^2)),x]
```

output
```
(a*Log[x])/d + ((I/4)*b*Log[1 + I*c*x]*Log[(c*(Sqrt[-d] - Sqrt[e]*x))/(c*S
qrt[-d] - I*Sqrt[e])])/d - ((I/4)*b*Log[1 - I*c*x]*Log[(c*(Sqrt[-d] - Sqrt
[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])])/d - ((I/4)*b*Log[1 - I*c*x]*Log[(c*(Sqr
t[-d] + Sqrt[e]*x))/(c*Sqrt[-d] - I*Sqrt[e])])/d + ((I/4)*b*Log[1 + I*c*x]
*Log[(c*(Sqrt[-d] + Sqrt[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])])/d - (a*Log[d +
e*x^2])/(2*d) + ((I/2)*b*PolyLog[2, (-I)*c*x])/d - ((I/2)*b*PolyLog[2, I*c
*x])/d - ((I/4)*b*PolyLog[2, -((Sqrt[e]*(1 - I*c*x))/(I*c*Sqrt[-d] - Sqrt[
e]))])/d - ((I/4)*b*PolyLog[2, (Sqrt[e]*(1 - I*c*x))/(I*c*Sqrt[-d] + Sqrt[
e])])/d + ((I/4)*b*PolyLog[2, -((Sqrt[e]*(1 + I*c*x))/(I*c*Sqrt[-d] - Sqrt
[e]))])/d + ((I/4)*b*PolyLog[2, (Sqrt[e]*(1 + I*c*x))/(I*c*Sqrt[-d] + Sqrt
[e])])/d
```

## Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5463, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x(d+ex^2)}\,dx$$

$$\Big\downarrow \text{5463}$$

$$\int \left(\frac{a+b\arctan(cx)}{dx}-\frac{ex(a+b\arctan(cx))}{d(d+ex^2)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(1-icx)(c\sqrt{-d}-i\sqrt{e})}\right)}{2d}-\frac{(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(1-icx)(c\sqrt{-d}+i\sqrt{e})}\right)}{2d}+\frac{\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))}{d}+\frac{a\log(x)}{d}+\frac{ib\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{4d}+\frac{ib\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)}{4d}+\frac{ib\operatorname{PolyLog}(2,-icx)}{2d}-\frac{ib\operatorname{PolyLog}(2,icx)}{2d}-\frac{ib\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2d}$$

input
```
Int[(a + b*ArcTan[c*x])/(x*(d + e*x^2)),x]
```

output
```
(a*Log[x])/d + ((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/d - ((a + b*ArcTan
[c*x])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c
*x))])/(2*d) - ((a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*S
qrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(2*d) + ((I/2)*b*PolyLog[2, (-I)*c*x])
/d - ((I/2)*b*PolyLog[2, I*c*x])/d - ((I/2)*b*PolyLog[2, 1 - 2/(1 - I*c*x)
])/d + ((I/4)*b*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] -
 I*Sqrt[e])*(1 - I*c*x))])/d + ((I/4)*b*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sq
rt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/d
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5463
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] && !(EqQ[m, 1] && NeQ[a, 0])
```

## Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.23

| method | result |
|---|---|
| risch | $-\frac{ib\,\mathrm{dilog}(-icx+1)}{2d}-\frac{ib\ln(-icx+1)\ln\left(\frac{c\sqrt{de}-(-icx+1)e+e}{c\sqrt{de}+e}\right)}{4d}-\frac{ib\ln(-icx+1)\ln\left(\frac{c\sqrt{de}+(-icx+1)e-e}{c\sqrt{de}-e}\right)}{4d}-\frac{ib\,\mathrm{dilog}}{}$ |
| derivativedivides | $-\frac{a\ln(c^2e\,x^2+c^2d)}{2d}+\frac{a\ln(cx)}{d}+b\,c^2\left(-\frac{\arctan(cx)\ln(c^2e\,x^2+c^2d)}{2d\,c^2}+\frac{\arctan(cx)\ln(cx)}{d\,c^2}-\frac{-\frac{i\ln(cx)\ln(icx+1)}{2}}{}\right.$ |
| default | $-\frac{a\ln(c^2e\,x^2+c^2d)}{2d}+\frac{a\ln(cx)}{d}+b\,c^2\left(-\frac{\arctan(cx)\ln(c^2e\,x^2+c^2d)}{2d\,c^2}+\frac{\arctan(cx)\ln(cx)}{d\,c^2}-\frac{-\frac{i\ln(cx)\ln(icx+1)}{2}}{}\right.$ |
| parts | $-\frac{a\ln(e\,x^2+d)}{2d}+\frac{a\ln(x)}{d}+b\left(-\frac{\arctan(cx)\ln(c^2e\,x^2+c^2d)}{2d}+\frac{\arctan(cx)\ln(cx)}{d}-\frac{c^2\left(-\frac{i\ln(cx)\ln(icx+1)}{c^2d}+\right.}{}\right.$ |

input
```
int((a+b*arctan(c*x))/x/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*b/d*dilog(1-I*c*x)-1/4*I*b/d*ln(1-I*c*x)*ln((c*(d*e)^(1/2)-(1-I*c*x
)*e+e)/(c*(d*e)^(1/2)+e))-1/4*I*b/d*ln(1-I*c*x)*ln((c*(d*e)^(1/2)+(1-I*c*x
)*e-e)/(c*(d*e)^(1/2)-e))-1/4*I*b/d*dilog((c*(d*e)^(1/2)-(1-I*c*x)*e+e)/(c
*(d*e)^(1/2)+e))-1/4*I*b/d*dilog((c*(d*e)^(1/2)+(1-I*c*x)*e-e)/(c*(d*e)^(1
/2)-e))+a/d*ln(-I*c*x)-1/2*a/d*ln((1-I*c*x)^2*e-c^2*d-2*(1-I*c*x)*e+e)+1/2
*I*b/d*dilog(1+I*c*x)+1/4*I*b/d*ln(1+I*c*x)*ln((c*(d*e)^(1/2)-(1+I*c*x)*e+
e)/(c*(d*e)^(1/2)+e))+1/4*I*b/d*ln(1+I*c*x)*ln((c*(d*e)^(1/2)+(1+I*c*x)*e-
e)/(c*(d*e)^(1/2)-e))+1/4*I*b/d*dilog((c*(d*e)^(1/2)-(1+I*c*x)*e+e)/(c*(d*
e)^(1/2)+e))+1/4*I*b/d*dilog((c*(d*e)^(1/2)+(1+I*c*x)*e-e)/(c*(d*e)^(1/2)-
e))
```

## Fricas [F]

$$\int \frac{a + b\arctan(cx)}{x\,(d + ex^2)}\,dx = \int \frac{b\arctan\left(cx\right) + a}{(ex^2 + d)x}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*arctan(c*x) + a)/(e*x^3 + d*x), x)
```

## Sympy [F]

$$\int \frac{a + b\arctan(cx)}{x\,(d + ex^2)}\,dx = \int \frac{a + b\,\mathrm{atan}\left(cx\right)}{x\left(d + ex^2\right)}\,dx$$

input
```
integrate((a+b*atan(c*x))/x/(e*x**2+d),x)
```

output
```
Integral((a + b*atan(c*x))/(x*(d + e*x**2)), x)
```

## Maxima [F]

$$\int \frac{a+b\arctan(cx)}{x\,(d+ex^2)}\,dx = \int \frac{b\arctan\left(cx\right)+a}{(ex^2+d)x}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x/(e*x^2+d),x, algorithm="maxima")
```

output
```
-1/2*a*(log(e*x^2 + d)/d - 2*log(x)/d) + 2*b*integrate(1/2*arctan(c*x)/(e*
x^3 + d*x), x)
```

## Giac [F]

$$\int \frac{a+b\arctan(cx)}{x\,(d+ex^2)}\,dx = \int \frac{b\arctan\left(cx\right)+a}{(ex^2+d)x}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x/(e*x^2+d),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)/((e*x^2 + d)*x), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{x\,(d+ex^2)}\,dx = \int \frac{a+b\,\mathrm{atan}(c\,x)}{x\,\left(e\,x^2+d\right)}\,dx$$

input
```
int((a + b*atan(c*x))/(x*(d + e*x^2)),x)
```

output
```
int((a + b*atan(c*x))/(x*(d + e*x^2)), x)
```

## Reduce [F]

$$\int \frac{a + b \arctan(cx)}{x\,(d + ex^2)}\,dx = \frac{2\left(\int \frac{atan(cx)}{e\,x^3+dx}dx\right)bd - \log(e\,x^2 + d)\,a + 2\log(x)\,a}{2d}$$

input
```
int((a+b*atan(c*x))/x/(e*x^2+d),x)
```

output
```
(2*int(atan(c*x)/(d*x + e*x**3),x)*b*d - log(d + e*x**2)*a + 2*log(x)*a)/(
2*d)
```

# 3.1153 $\int \frac{a+b\arctan(cx)}{x^3(d+ex^2)}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 409

$$\begin{aligned}\int \frac{a+b\arctan(cx)}{x^3(d+ex^2)}\,dx = &-\frac{bc}{2dx}-\frac{bc^2\arctan(cx)}{2d}-\frac{a+b\arctan(cx)}{2dx^2}\\
&-\frac{ae\log(x)}{d^2}-\frac{e(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)}{d^2}\\
&+\frac{e(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2d^2}\\
&+\frac{e(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{2d^2}\\
&-\frac{ibe\,\mathrm{PolyLog}(2,-icx)}{2d^2}+\frac{ibe\,\mathrm{PolyLog}(2,icx)}{2d^2}\\
&+\frac{ibe\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2d^2}\\
&-\frac{ibe\,\mathrm{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{4d^2}\\
&-\frac{ibe\,\mathrm{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{4d^2}\end{aligned}$$

output
```
-1/2*b*c/d/x-1/2*b*c^2*arctan(c*x)/d-1/2*(a+b*arctan(c*x))/d/x^2-a*e*ln(x)
/d^2-e*(a+b*arctan(c*x))*ln(2/(1-I*c*x))/d^2+1/2*e*(a+b*arctan(c*x))*ln(2*
c*((-d)^(1/2)-e^(1/2)*x)/(c*(-d)^(1/2)-I*e^(1/2))/(1-I*c*x))/d^2+1/2*e*(a+
b*arctan(c*x))*ln(2*c*((-d)^(1/2)+e^(1/2)*x)/(c*(-d)^(1/2)+I*e^(1/2))/(1-I
*c*x))/d^2-1/2*I*b*e*polylog(2,-I*c*x)/d^2+1/2*I*b*e*polylog(2,I*c*x)/d^2+
1/2*I*b*e*polylog(2,1-2/(1-I*c*x))/d^2-1/4*I*b*e*polylog(2,1-2*c*((-d)^(1/
2)-e^(1/2)*x)/(c*(-d)^(1/2)-I*e^(1/2))/(1-I*c*x))/d^2-1/4*I*b*e*polylog(2,
1-2*c*((-d)^(1/2)+e^(1/2)*x)/(c*(-d)^(1/2)+I*e^(1/2))/(1-I*c*x))/d^2
```

## Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.23 (sec) , antiderivative size = 514, normalized size of antiderivative = 1.26

$$\int \frac{a+b\arctan(cx)}{x^3(d+ex^2)}\,dx =$$

$$-\frac{2ad+2bd\arctan(cx)+2bcdx\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-c^2x^2\right)+4aex^2\log(x)+ibex^2\log(1+ic}{}$$

input
```
Integrate[(a + b*ArcTan[c*x])/(x^3*(d + e*x^2)),x]
```

output
```
-1/4*(2*a*d + 2*b*d*ArcTan[c*x] + 2*b*c*d*x*Hypergeometric2F1[-1/2, 1, 1/2
, -(c^2*x^2)] + 4*a*e*x^2*Log[x] + I*b*e*x^2*Log[1 + I*c*x]*Log[(c*(Sqrt[-
d] - Sqrt[e]*x))/(c*Sqrt[-d] - I*Sqrt[e])] - I*b*e*x^2*Log[1 - I*c*x]*Log[
(c*(Sqrt[-d] - Sqrt[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])] - I*b*e*x^2*Log[1 - I
*c*x]*Log[(c*(Sqrt[-d] + Sqrt[e]*x))/(c*Sqrt[-d] - I*Sqrt[e])] + I*b*e*x^2
*Log[1 + I*c*x]*Log[(c*(Sqrt[-d] + Sqrt[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])] -
 2*a*e*x^2*Log[d + e*x^2] + (2*I)*b*e*x^2*PolyLog[2, (-I)*c*x] - (2*I)*b*e
*x^2*PolyLog[2, I*c*x] + I*b*e*x^2*PolyLog[2, (Sqrt[e]*(I - c*x))/(c*Sqrt[
-d] + I*Sqrt[e])] - I*b*e*x^2*PolyLog[2, (Sqrt[e]*(1 - I*c*x))/(I*c*Sqrt[-
d] + Sqrt[e])] + I*b*e*x^2*PolyLog[2, (Sqrt[e]*(1 + I*c*x))/(I*c*Sqrt[-d]
+ Sqrt[e])] - I*b*e*x^2*PolyLog[2, (Sqrt[e]*(I + c*x))/(c*Sqrt[-d] + I*Sqr
t[e])])/(d^2*x^2)
```

## Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 399, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5453, 5361, 264, 216, 5463, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x^3\left(d+ex^2\right)}dx$$

$$\Big\downarrow \text{ 5453}$$

$$\frac{\int \frac{a+b\arctan(cx)}{x^3}dx}{d}-\frac{e\int \frac{a+b\arctan(cx)}{x(ex^2+d)}dx}{d}$$

$$\Big\downarrow \text{ 5361}$$

$$\frac{\frac{1}{2}bc\int \frac{1}{x^2(c^2x^2+1)}dx-\frac{a+b\arctan(cx)}{2x^2}}{d}-\frac{e\int \frac{a+b\arctan(cx)}{x(ex^2+d)}dx}{d}$$

$$\Big\downarrow \text{ 264}$$

$$\frac{\frac{1}{2}bc\left(c^2\left(-\int \frac{1}{c^2x^2+1}dx\right)-\frac{1}{x}\right)-\frac{a+b\arctan(cx)}{2x^2}}{d}-\frac{e\int \frac{a+b\arctan(cx)}{x(ex^2+d)}dx}{d}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{\frac{1}{2}bc\left(-c\arctan(cx)-\frac{1}{x}\right)-\frac{a+b\arctan(cx)}{2x^2}}{d}-\frac{e\int \frac{a+b\arctan(cx)}{x(ex^2+d)}dx}{d}$$

$$\Big\downarrow \text{ 5463}$$

$$\frac{\frac{1}{2}bc\left(-c\arctan(cx)-\frac{1}{x}\right)-\frac{a+b\arctan(cx)}{2x^2}}{d}-\frac{e\int \left(\frac{a+b\arctan(cx)}{dx}-\frac{ex(a+b\arctan(cx))}{d(ex^2+d)}\right)dx}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{1}{2}bc\left(-c\arctan(cx)-\frac{1}{x}\right)-\frac{a+b\arctan(cx)}{2x^2}}{d}-$$

$$e\left(-\frac{(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(1-icx)(c\sqrt{-d}-i\sqrt{e})}\right)}{2d}-\frac{(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(1-icx)(c\sqrt{-d}+i\sqrt{e})}\right)}{2d}+\frac{\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))}{d}+\frac{a\log(x)}{d}-\right.$$

input
```
Int[(a + b*ArcTan[c*x])/(x^3*(d + e*x^2)),x]
```

output
```
(-1/2*(a + b*ArcTan[c*x])/x^2 + (b*c*(-x^(-1) - c*ArcTan[c*x]))/2)/d - (e*
((a*Log[x])/d + ((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/d - ((a + b*ArcTa
n[c*x])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*
c*x))])/(2*d) - ((a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*
Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(2*d) + ((I/2)*b*PolyLog[2, (-I)*c*x]
)/d - ((I/2)*b*PolyLog[2, I*c*x])/d - ((I/2)*b*PolyLog[2, 1 - 2/(1 - I*c*x
)])/d + ((I/4)*b*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d]
- I*Sqrt[e])*(1 - I*c*x))])/d + ((I/4)*b*PolyLog[2, 1 - (2*c*(Sqrt[-d] + S
qrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/d))/d
```

### Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5463
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)^(m_.))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[a + b*ArcTan[c*x], x^m/(d + e*x^2), x], x]
 /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[m] &&  !(EqQ[m, 1] && NeQ[a, 0])
```

## Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 571, normalized size of antiderivative = 1.40

| method | result |
|---|---|
| risch | $\frac{ibe\,\mathrm{dilog}(-icx+1)}{2d^2}-\frac{ibe\,\mathrm{dilog}(icx+1)}{2d^2}+\frac{ibe\,\mathrm{dilog}\left(\frac{c\sqrt{de}-(-icx+1)e+e}{c\sqrt{de}+e}\right)}{4d^2}+\frac{ibe\,\mathrm{dilog}\left(\frac{c\sqrt{de}+(-icx+1)e-e}{c\sqrt{de}-e}\right)}{4d^2}-\frac{ibe}{}$ |
| parts | $a\left(\frac{e\ln(e\,x^2+d)}{2d^2}-\frac{1}{2d\,x^2}-\frac{e\ln(x)}{d^2}\right)+b\,c^2\left(\frac{\arctan(cx)e\ln(c^2e\,x^2+c^2d)}{2c^2d^2}-\frac{\arctan(cx)}{2d\,c^2x^2}-\frac{\arctan(cx)e\ln(cx}{c^2d^2}\right.$ |

input
```
int((a+b*arctan(c*x))/x^3/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
1/2*I*b/d^2*e*dilog(1-I*c*x)-1/2*I*b*e/d^2*dilog(1+I*c*x)+1/4*I*b*e/d^2*di
log((c*(d*e)^(1/2)-(1-I*c*x)*e+e)/(c*(d*e)^(1/2)+e))+1/4*I*b*e/d^2*dilog((
c*(d*e)^(1/2)+(1-I*c*x)*e-e)/(c*(d*e)^(1/2)-e))-1/4*I*b*e/d^2*ln(1+I*c*x)*
ln((c*(d*e)^(1/2)-(1+I*c*x)*e+e)/(c*(d*e)^(1/2)+e))+1/4*I*b*e/d^2*ln(1-I*c
*x)*ln((c*(d*e)^(1/2)-(1-I*c*x)*e+e)/(c*(d*e)^(1/2)+e))-1/2*b*c/d/x+1/4*I*
b/d*ln(1+I*c*x)/x^2-1/4*I*b*e/d^2*ln(1+I*c*x)*ln((c*(d*e)^(1/2)+(1+I*c*x)*
e-e)/(c*(d*e)^(1/2)-e))-1/2*a/d/x^2-a/d^2*e*ln(-I*c*x)+1/2*a*e/d^2*ln((1-I
*c*x)^2*e-c^2*d-2*(1-I*c*x)*e+e)+1/4*I*b*e/d^2*ln(1-I*c*x)*ln((c*(d*e)^(1/
2)+(1-I*c*x)*e-e)/(c*(d*e)^(1/2)-e))+1/4*I*c^2*b/d*ln(-I*c*x)-1/4*I*b/d*ln
(1-I*c*x)/x^2-1/4*I*b*e/d^2*dilog((c*(d*e)^(1/2)-(1+I*c*x)*e+e)/(c*(d*e)^(
1/2)+e))-1/4*I*c^2*b/d*ln(I*c*x)+1/4*I*c^2*b/d*ln(1+I*c*x)-1/4*I*b*e/d^2*d
ilog((c*(d*e)^(1/2)+(1+I*c*x)*e-e)/(c*(d*e)^(1/2)-e))-1/4*I*c^2*b/d*ln(1-I
*c*x)
```

## Fricas [F]

$$\int \frac{a + b \arctan(cx)}{x^3 (d + ex^2)} \, dx = \int \frac{b \arctan(cx) + a}{(ex^2 + d)x^3} \, dx$$

input
```
integrate((a+b*arctan(c*x))/x^3/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*arctan(c*x) + a)/(e*x^5 + d*x^3), x)
```

## Sympy [F]

$$\int \frac{a + b \arctan(cx)}{x^3 (d + ex^2)} \, dx = \int \frac{a + b \operatorname{atan}(cx)}{x^3 (d + ex^2)} \, dx$$

input
```
integrate((a+b*atan(c*x))/x**3/(e*x**2+d),x)
```

output
```
Integral((a + b*atan(c*x))/(x**3*(d + e*x**2)), x)
```

## Maxima [F]

$$\int \frac{a+b\arctan(cx)}{x^3(d+ex^2)}\,dx = \int \frac{b\arctan(cx)+a}{(ex^2+d)x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^3/(e*x^2+d),x, algorithm="maxima")
```

output
```
1/2*a*(e*log(e*x^2 + d)/d^2 - 2*e*log(x)/d^2 - 1/(d*x^2)) + 2*b*integrate(
1/2*arctan(c*x)/(e*x^5 + d*x^3), x)
```

## Giac [F]

$$\int \frac{a+b\arctan(cx)}{x^3(d+ex^2)}\,dx = \int \frac{b\arctan(cx)+a}{(ex^2+d)x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^3/(e*x^2+d),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)/((e*x^2 + d)*x^3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{x^3(d+ex^2)}\,dx = \int \frac{a+b\,\mathrm{atan}(c\,x)}{x^3\,(e\,x^2+d)}\,dx$$

input
```
int((a + b*atan(c*x))/(x^3*(d + e*x^2)),x)
```

output
```
int((a + b*atan(c*x))/(x^3*(d + e*x^2)), x)
```

## Reduce [F]

$$\int \frac{a+b\arctan(cx)}{x^3(d+ex^2)}\,dx$$
$$= \frac{2\left(\int \frac{atan(cx)}{e\,x^5+d\,x^3}dx\right)b\,d^2x^2+\log(e\,x^2+d)\,ae\,x^2-2\log(x)\,ae\,x^2-ad}{2d^2x^2}$$

input
```
int((a+b*atan(c*x))/x^3/(e*x^2+d),x)
```

output
```
(2*int(atan(c*x)/(d*x**3 + e*x**5),x)*b*d**2*x**2 + log(d + e*x**2)*a*e*x*
*2 - 2*log(x)*a*e*x**2 - a*d)/(2*d**2*x**2)
```

# 3.1154 $\int \frac{x^2(a+b\arctan(cx))}{d+ex^2}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 555

$$\begin{aligned}
\int \frac{x^2(a+b\arctan(cx))}{d+ex^2}\,dx &= \frac{ax}{e}+\frac{bx\arctan(cx)}{e}-\frac{a\sqrt{d}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{3/2}} \\
&\quad -\frac{ib\sqrt{-d}\log(1+icx)\log\left(\frac{c(\sqrt{-d}-\sqrt{e}x)}{c\sqrt{-d}-i\sqrt{e}}\right)}{4e^{3/2}} \\
&\quad +\frac{ib\sqrt{-d}\log(1-icx)\log\left(\frac{c(\sqrt{-d}-\sqrt{e}x)}{c\sqrt{-d}+i\sqrt{e}}\right)}{4e^{3/2}} \\
&\quad -\frac{ib\sqrt{-d}\log(1-icx)\log\left(\frac{c(\sqrt{-d}+\sqrt{e}x)}{c\sqrt{-d}-i\sqrt{e}}\right)}{4e^{3/2}} \\
&\quad +\frac{ib\sqrt{-d}\log(1+icx)\log\left(\frac{c(\sqrt{-d}+\sqrt{e}x)}{c\sqrt{-d}+i\sqrt{e}}\right)}{4e^{3/2}} \\
&\quad -\frac{b\log(1+c^2x^2)}{2ce}+\frac{ib\sqrt{-d}\,\text{PolyLog}\left(2,\frac{\sqrt{e}(i-cx)}{c\sqrt{-d}+i\sqrt{e}}\right)}{4e^{3/2}} \\
&\quad -\frac{ib\sqrt{-d}\,\text{PolyLog}\left(2,\frac{\sqrt{e}(1-icx)}{ic\sqrt{-d}+\sqrt{e}}\right)}{4e^{3/2}} \\
&\quad -\frac{ib\sqrt{-d}\,\text{PolyLog}\left(2,\frac{\sqrt{e}(1+icx)}{ic\sqrt{-d}+\sqrt{e}}\right)}{4e^{3/2}} \\
&\quad +\frac{ib\sqrt{-d}\,\text{PolyLog}\left(2,\frac{\sqrt{e}(i+cx)}{c\sqrt{-d}+i\sqrt{e}}\right)}{4e^{3/2}}
\end{aligned}$$

output
```
a*x/e+b*x*arctan(c*x)/e-a*d^(1/2)*arctan(e^(1/2)*x/d^(1/2))/e^(3/2)-1/4*I*
b*(-d)^(1/2)*ln(1+I*c*x)*ln(c*((-d)^(1/2)-e^(1/2)*x)/(c*(-d)^(1/2)-I*e^(1/
2)))/e^(3/2)+1/4*I*b*(-d)^(1/2)*ln(1-I*c*x)*ln(c*((-d)^(1/2)-e^(1/2)*x)/(c
*(-d)^(1/2)+I*e^(1/2)))/e^(3/2)-1/4*I*b*(-d)^(1/2)*ln(1-I*c*x)*ln(c*((-d)^
(1/2)+e^(1/2)*x)/(c*(-d)^(1/2)-I*e^(1/2)))/e^(3/2)+1/4*I*b*(-d)^(1/2)*ln(1
+I*c*x)*ln(c*((-d)^(1/2)+e^(1/2)*x)/(c*(-d)^(1/2)+I*e^(1/2)))/e^(3/2)-1/2*
b*ln(c^2*x^2+1)/c/e+1/4*I*b*(-d)^(1/2)*polylog(2,e^(1/2)*(I-c*x)/(c*(-d)^(
1/2)+I*e^(1/2)))/e^(3/2)-1/4*I*b*(-d)^(1/2)*polylog(2,e^(1/2)*(1-I*c*x)/(I
*c*(-d)^(1/2)+e^(1/2)))/e^(3/2)-1/4*I*b*(-d)^(1/2)*polylog(2,e^(1/2)*(1+I*
c*x)/(I*c*(-d)^(1/2)+e^(1/2)))/e^(3/2)+1/4*I*b*(-d)^(1/2)*polylog(2,e^(1/2
)*(I+c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/e^(3/2)
```

## Mathematica [A] (warning: unable to verify)

Time = 2.34 (sec) , antiderivative size = 766, normalized size of antiderivative = 1.38

$$\int \frac{x^2(a+b\arctan(cx))}{d+ex^2}\,dx = \frac{ax}{e} - \frac{a\sqrt{d}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{3/2}}$$
$$+ \frac{b\left(4cx\arctan(cx) - 2\log\left(1+c^2x^2\right) + \frac{c^2d\left(-4\arctan(cx)\operatorname{arctanh}\left(\frac{cd}{\sqrt{-c^2de}x}\right) - 2\arccos\left(\frac{c^2d+e}{-c^2d+e}\right)\operatorname{arctanh}\left(\frac{cex}{\sqrt{-c^2de}}\right)\right.}{\phantom{x}}\right.}{\phantom{x}}$$

input
```
Integrate[(x^2*(a + b*ArcTan[c*x]))/(d + e*x^2),x]
```

output
```
(a*x)/e - (a*Sqrt[d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/e^(3/2) + (b*(4*c*x*ArcT
an[c*x] - 2*Log[1 + c^2*x^2] + (c^2*d*(-4*ArcTan[c*x]*ArcTanh[(c*d)/(Sqrt[
-(c^2*d*e)]*x)] - 2*ArcCos[(c^2*d + e)/(-(c^2*d) + e)]*ArcTanh[(c*e*x)/Sqr
t[-(c^2*d*e)]] - (ArcCos[(c^2*d + e)/(-(c^2*d) + e)] - (2*I)*ArcTanh[(c*e*
x)/Sqrt[-(c^2*d*e)]])*Log[(2*c*d*(I*e + Sqrt[-(c^2*d*e)])*(-I + c*x))/((c^
2*d - e)*(-(c*d) + Sqrt[-(c^2*d*e)]*x))] - (ArcCos[(c^2*d + e)/(-(c^2*d) +
 e)] + (2*I)*ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]])*Log[(2*c*d*((-I)*e + Sqrt[
-(c^2*d*e)])*(I + c*x))/((c^2*d - e)*(-(c*d) + Sqrt[-(c^2*d*e)]*x))] + (Ar
cCos[(c^2*d + e)/(-(c^2*d) + e)] + (2*I)*ArcTanh[(c*d)/(Sqrt[-(c^2*d*e)]*x
)] + (2*I)*ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]])*Log[(Sqrt[2]*Sqrt[-(c^2*d*e)
])/(Sqrt[-(c^2*d) + e]*E^(I*ArcTan[c*x])*Sqrt[-(c^2*d) - e + (-(c^2*d) + e
)*Cos[2*ArcTan[c*x]]])] + (ArcCos[(c^2*d + e)/(-(c^2*d) + e)] - (2*I)*ArcT
anh[(c*d)/(Sqrt[-(c^2*d*e)]*x)] - (2*I)*ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]])
*Log[(Sqrt[2]*Sqrt[-(c^2*d*e)]*E^(I*ArcTan[c*x]))/(Sqrt[-(c^2*d) + e]*Sqrt
[-(c^2*d) - e + (-(c^2*d) + e)*Cos[2*ArcTan[c*x]]])] + I*(-PolyLog[2, ((c^
2*d + e - (2*I)*Sqrt[-(c^2*d*e)])*(c*d + Sqrt[-(c^2*d*e)]*x))/((c^2*d - e)
*(c*d - Sqrt[-(c^2*d*e)]*x))] + PolyLog[2, ((c^2*d + e + (2*I)*Sqrt[-(c^2*
d*e)])*(c*d + Sqrt[-(c^2*d*e)]*x))/((c^2*d - e)*(c*d - Sqrt[-(c^2*d*e)]*x)
)])))/Sqrt[-(c^2*d*e)]))/(4*c*e)
```

## Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 550, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5451, 2009, 5445, 218, 5443, 2856, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\arctan(cx))}{d+ex^2}dx$$

$$\Big\downarrow \text{5451}$$

$$\frac{\int (a+b\arctan(cx))dx}{e}-\frac{d\int \frac{a+b\arctan(cx)}{ex^2+d}dx}{e}$$

$$\Big\downarrow \text{2009}$$

$$\frac{ax+bx\arctan(cx)-\frac{b\log(c^2x^2+1)}{2c}}{e}-\frac{d\int \frac{a+b\arctan(cx)}{ex^2+d}dx}{e}$$

$$\Big\downarrow \text{ 5445}$$

$$\frac{ax+bx\arctan(cx)-\frac{b\log(c^2x^2+1)}{2c}}{e}-\frac{d\left(a\int\frac{1}{ex^2+d}dx+b\int\frac{\arctan(cx)}{ex^2+d}dx\right)}{e}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{ax+bx\arctan(cx)-\frac{b\log(c^2x^2+1)}{2c}}{e}-\frac{d\left(b\int\frac{\arctan(cx)}{ex^2+d}dx+\frac{a\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}}\right)}{e}$$

$$\Big\downarrow \text{ 5443}$$

$$\frac{ax+bx\arctan(cx)-\frac{b\log(c^2x^2+1)}{2c}}{e}-\frac{d\left(\frac{a\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}}+b\left(\frac{1}{2}i\int\frac{\log(1-icx)}{ex^2+d}dx-\frac{1}{2}i\int\frac{\log(icx+1)}{ex^2+d}dx\right)\right)}{e}$$

$$\Big\downarrow \text{ 2856}$$

$$\frac{ax+bx\arctan(cx)-\frac{b\log(c^2x^2+1)}{2c}}{e}-\frac{d\left(\frac{a\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}}+b\left(\frac{1}{2}i\int\left(\frac{\sqrt{-d}\log(1-icx)}{2d(\sqrt{-d}-\sqrt{e}x)}+\frac{\sqrt{-d}\log(1-icx)}{2d(\sqrt{e}x+\sqrt{-d})}\right)dx-\frac{1}{2}i\int\left(\frac{\sqrt{-d}\log(icx+1)}{2d(\sqrt{-d}-\sqrt{e}x)}+\frac{\sqrt{-d}\log(icx+1)}{2d(\sqrt{e}x+\sqrt{-d})}\right)dx\right)\right)}{e}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{ax+bx\arctan(cx)-\frac{b\log(c^2x^2+1)}{2c}}{e}-\frac{d\left(\frac{a\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}}+b\left(\frac{1}{2}i\left(-\frac{\text{PolyLog}\left(2,\frac{\sqrt{e}(1-icx)}{i\sqrt{-d}c+\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{\text{PolyLog}\left(2,\frac{\sqrt{e}(cx+i)}{\sqrt{-d}c+i\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{\log(1-icx)\log\left(\frac{c(\sqrt{-d}-\sqrt{e}x)}{c\sqrt{-d}+i\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}}-\frac{\log(1-icx)\log\left(\frac{c}{\cdots}\right.}{2\sqrt{-d}\cdots}\right.\right.\right.}{}$$

input
```
Int[(x^2*(a + b*ArcTan[c*x]))/(d + e*x^2),x]
```

output
```
(a*x + b*x*ArcTan[c*x] - (b*Log[1 + c^2*x^2])/(2*c))/e - (d*((a*ArcTan[(Sq
rt[e]*x)/Sqrt[d]])/(Sqrt[d]*Sqrt[e]) + b*((-1/2*I)*((Log[1 + I*c*x]*Log[(c
*(Sqrt[-d] - Sqrt[e]*x))/(c*Sqrt[-d] - I*Sqrt[e])])/(2*Sqrt[-d]*Sqrt[e]) -
 (Log[1 + I*c*x]*Log[(c*(Sqrt[-d] + Sqrt[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])])
/(2*Sqrt[-d]*Sqrt[e]) - PolyLog[2, (Sqrt[e]*(I - c*x))/(c*Sqrt[-d] + I*Sqr
t[e])]/(2*Sqrt[-d]*Sqrt[e]) + PolyLog[2, (Sqrt[e]*(1 + I*c*x))/(I*c*Sqrt[-
d] + Sqrt[e])]/(2*Sqrt[-d]*Sqrt[e])) + (I/2)*((Log[1 - I*c*x]*Log[(c*(Sqrt
[-d] - Sqrt[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])])/(2*Sqrt[-d]*Sqrt[e]) - (Log[
1 - I*c*x]*Log[(c*(Sqrt[-d] + Sqrt[e]*x))/(c*Sqrt[-d] - I*Sqrt[e])])/(2*Sq
rt[-d]*Sqrt[e]) - PolyLog[2, (Sqrt[e]*(1 - I*c*x))/(I*c*Sqrt[-d] + Sqrt[e]
)]/(2*Sqrt[-d]*Sqrt[e]) + PolyLog[2, (Sqrt[e]*(I + c*x))/(c*Sqrt[-d] + I*S
qrt[e])]/(2*Sqrt[-d]*Sqrt[e])))))/e
```

### Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2856
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

rule 5443
```
Int[ArcTan[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Simp[I/2   Int[
Log[1 - I*c*x]/(d + e*x^2), x], x] - Simp[I/2   Int[Log[1 + I*c*x]/(d + e*x
^2), x], x] /; FreeQ[{c, d, e}, x]
```

rule 5445
```
Int[(ArcTan[(c_.)*(x_)]*(b_.) + (a_))/((d_.) + (e_.)*(x_)^2), x_Symbol] :>
Simp[a   Int[1/(d + e*x^2), x], x] + Simp[b   Int[ArcTan[c*x]/(d + e*x^2),
x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

## Maple [A] (verified)

Time = 1.19 (sec) , antiderivative size = 526, normalized size of antiderivative = 0.95

| method | result |
|---|---|
| risch | $\frac{ib\ln(-icx+1)x}{2e} - \frac{iad\,\operatorname{arctanh}\left(\frac{2(-icx+1)e-2e}{2c\sqrt{de}}\right)}{e\sqrt{de}} - \frac{b\ln(-icx+1)}{2ce} + \frac{b}{ce} - \frac{bd\ln(-icx+1)\ln\left(\frac{c\sqrt{de}-(-icx+1)e+e}{c\sqrt{de}+e}\right)}{4e\sqrt{de}}$ |
| derivativedivides | Expression too large to display |
| default | Expression too large to display |
| parts | Expression too large to display |

input
```
int(x^2*(a+b*arctan(c*x))/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
1/2*I*b/e*ln(1-I*c*x)*x-I*a*d/e/(d*e)^(1/2)*arctanh(1/2*(2*(1-I*c*x)*e-2*e
)/c/(d*e)^(1/2))-1/2/c*b/e*ln(1-I*c*x)+1/c*b/e-1/4*b*d/e*ln(1-I*c*x)/(d*e)
^(1/2)*ln((c*(d*e)^(1/2)-(1-I*c*x)*e+e)/(c*(d*e)^(1/2)+e))+1/4*b*d/e*ln(1-
I*c*x)/(d*e)^(1/2)*ln((c*(d*e)^(1/2)+(1-I*c*x)*e-e)/(c*(d*e)^(1/2)-e))-1/4
*b*d/e/(d*e)^(1/2)*dilog((c*(d*e)^(1/2)-(1-I*c*x)*e+e)/(c*(d*e)^(1/2)+e))+
1/4*b*d/e/(d*e)^(1/2)*dilog((c*(d*e)^(1/2)+(1-I*c*x)*e-e)/(c*(d*e)^(1/2)-e
))+a*x/e+I/c*a/e-1/2*I*b/e*ln(1+I*c*x)*x-1/2*b/c/e*ln(1+I*c*x)-1/4*b*d/e*l
n(1+I*c*x)/(d*e)^(1/2)*ln((c*(d*e)^(1/2)-(1+I*c*x)*e+e)/(c*(d*e)^(1/2)+e))
+1/4*b*d/e*ln(1+I*c*x)/(d*e)^(1/2)*ln((c*(d*e)^(1/2)+(1+I*c*x)*e-e)/(c*(d*
e)^(1/2)-e))-1/4*b*d/e/(d*e)^(1/2)*dilog((c*(d*e)^(1/2)-(1+I*c*x)*e+e)/(c*
(d*e)^(1/2)+e))+1/4*b*d/e/(d*e)^(1/2)*dilog((c*(d*e)^(1/2)+(1+I*c*x)*e-e)/
(c*(d*e)^(1/2)-e))
```

## Fricas [F]

$$\int \frac{x^2(a+b\arctan(cx))}{d+ex^2}\,dx = \int \frac{(b\arctan(cx)+a)x^2}{ex^2+d}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*x^2*arctan(c*x) + a*x^2)/(e*x^2 + d), x)
```

## Sympy [F]

$$\int \frac{x^2(a+b\arctan(cx))}{d+ex^2}\,dx = \int \frac{x^2(a+b\,\mathrm{atan}\,(cx))}{d+ex^2}\,dx$$

input
```
integrate(x**2*(a+b*atan(c*x))/(e*x**2+d),x)
```

output
```
Integral(x**2*(a + b*atan(c*x))/(d + e*x**2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a+b\arctan(cx))}{d+ex^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(a+b*arctan(c*x))/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

## Giac [F]

$$\int \frac{x^2(a+b\arctan(cx))}{d+ex^2}\,dx=\int \frac{(b\arctan(cx)+a)x^2}{ex^2+d}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))/(e*x^2+d),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)*x^2/(e*x^2 + d), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\arctan(cx))}{d+ex^2}\,dx=\int \frac{x^2\,(a+b\,\mathrm{atan}(c\,x))}{e\,x^2+d}\,dx$$

input
```
int((x^2*(a + b*atan(c*x)))/(d + e*x^2),x)
```

output
```
int((x^2*(a + b*atan(c*x)))/(d + e*x^2), x)
```

## Reduce [F]

$$\int \frac{x^2(a+b\arctan(cx))}{d+ex^2}\,dx = \frac{-2\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)ac-atan(cx)^2\,b\,c^2d+2atan(cx)\,bcex+2\left(\int \frac{atan(cx)}{c^2e\,x^4+c^2d\,x^2+e\,x^2+d}dx\right)b\,c^3d^2-2\left(\int\right.}{2c\,e^2}$$

input
```
int(x^2*(a+b*atan(c*x))/(e*x^2+d),x)
```

output
```
( - 2*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*c - atan(c*x)**2*b*c
**2*d + 2*atan(c*x)*b*c*e*x + 2*int(atan(c*x)/(c**2*d*x**2 + c**2*e*x**4 +
 d + e*x**2),x)*b*c**3*d**2 - 2*int(atan(c*x)/(c**2*d*x**2 + c**2*e*x**4 +
 d + e*x**2),x)*b*c*d*e - log(c**2*x**2 + 1)*b*e + 2*a*c*e*x)/(2*c*e**2)
```

# 3.1155 $\int \frac{a+b\arctan(cx)}{d+ex^2}\,dx$



## Optimal result

Integrand size = 18, antiderivative size = 517

$$
\begin{aligned}
\int \frac{a+b\arctan(cx)}{d+ex^2}\,dx = {} & \frac{a\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}} - \frac{ib\log(1+icx)\log\left(\frac{c(\sqrt{-d}-\sqrt{e}x)}{c\sqrt{-d}-i\sqrt{e}}\right)}{4\sqrt{-d}\sqrt{e}} \\
& + \frac{ib\log(1-icx)\log\left(\frac{c(\sqrt{-d}-\sqrt{e}x)}{c\sqrt{-d}+i\sqrt{e}}\right)}{4\sqrt{-d}\sqrt{e}} \\
& - \frac{ib\log(1-icx)\log\left(\frac{c(\sqrt{-d}+\sqrt{e}x)}{c\sqrt{-d}-i\sqrt{e}}\right)}{4\sqrt{-d}\sqrt{e}} \\
& + \frac{ib\log(1+icx)\log\left(\frac{c(\sqrt{-d}+\sqrt{e}x)}{c\sqrt{-d}+i\sqrt{e}}\right)}{4\sqrt{-d}\sqrt{e}} \\
& + \frac{ib\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}(i-cx)}{c\sqrt{-d}+i\sqrt{e}}\right)}{4\sqrt{-d}\sqrt{e}} - \frac{ib\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}(1-icx)}{ic\sqrt{-d}+\sqrt{e}}\right)}{4\sqrt{-d}\sqrt{e}} \\
& - \frac{ib\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}(1+icx)}{ic\sqrt{-d}+\sqrt{e}}\right)}{4\sqrt{-d}\sqrt{e}} + \frac{ib\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}(i+cx)}{c\sqrt{-d}+i\sqrt{e}}\right)}{4\sqrt{-d}\sqrt{e}}
\end{aligned}
$$

output
```
a*arctan(e^(1/2)*x/d^(1/2))/d^(1/2)/e^(1/2)-1/4*I*b*ln(1+I*c*x)*ln(c*((-d)
^(1/2)-e^(1/2)*x)/(c*(-d)^(1/2)-I*e^(1/2)))/(-d)^(1/2)/e^(1/2)+1/4*I*b*ln(
1-I*c*x)*ln(c*((-d)^(1/2)-e^(1/2)*x)/(c*(-d)^(1/2)+I*e^(1/2)))/(-d)^(1/2)/
e^(1/2)-1/4*I*b*ln(1-I*c*x)*ln(c*((-d)^(1/2)+e^(1/2)*x)/(c*(-d)^(1/2)-I*e^
(1/2)))/(-d)^(1/2)/e^(1/2)+1/4*I*b*ln(1+I*c*x)*ln(c*((-d)^(1/2)+e^(1/2)*x)
/(c*(-d)^(1/2)+I*e^(1/2)))/(-d)^(1/2)/e^(1/2)+1/4*I*b*polylog(2,e^(1/2)*(I
-c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/(-d)^(1/2)/e^(1/2)-1/4*I*b*polylog(2,e^(1/
2)*(1-I*c*x)/(I*c*(-d)^(1/2)+e^(1/2)))/(-d)^(1/2)/e^(1/2)-1/4*I*b*polylog(
2,e^(1/2)*(1+I*c*x)/(I*c*(-d)^(1/2)+e^(1/2)))/(-d)^(1/2)/e^(1/2)+1/4*I*b*p
olylog(2,e^(1/2)*(I+c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/(-d)^(1/2)/e^(1/2)
```

## Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 461, normalized size of antiderivative = 0.89

$$\int \frac{a+b\arctan(cx)}{d+ex^2}\,dx = \frac{4a\sqrt{-d}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)-ib\sqrt{d}\log(1+icx)\log\left(\frac{c(\sqrt{-d}-\sqrt{e}x)}{c\sqrt{-d}-i\sqrt{e}}\right)+ib\sqrt{d}\log(1-icx)\log\left(\frac{c(\sqrt{-d}-\sqrt{e}x)}{c\sqrt{-d}+i\sqrt{e}}\right)-ib\sqrt{}$$

input
```
Integrate[(a + b*ArcTan[c*x])/(d + e*x^2),x]
```

output
```
(4*a*Sqrt[-d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]] - I*b*Sqrt[d]*Log[1 + I*c*x]*Log
[(c*(Sqrt[-d] - Sqrt[e]*x))/(c*Sqrt[-d] - I*Sqrt[e])] + I*b*Sqrt[d]*Log[1
- I*c*x]*Log[(c*(Sqrt[-d] - Sqrt[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])] - I*b*Sq
rt[d]*Log[1 - I*c*x]*Log[(c*(Sqrt[-d] + Sqrt[e]*x))/(c*Sqrt[-d] - I*Sqrt[e
])] + I*b*Sqrt[d]*Log[1 + I*c*x]*Log[(c*(Sqrt[-d] + Sqrt[e]*x))/(c*Sqrt[-d
] + I*Sqrt[e])] + I*b*Sqrt[d]*PolyLog[2, (Sqrt[e]*(I - c*x))/(c*Sqrt[-d] +
 I*Sqrt[e])] - I*b*Sqrt[d]*PolyLog[2, (Sqrt[e]*(1 - I*c*x))/(I*c*Sqrt[-d]
+ Sqrt[e])] - I*b*Sqrt[d]*PolyLog[2, (Sqrt[e]*(1 + I*c*x))/(I*c*Sqrt[-d] +
 Sqrt[e])] + I*b*Sqrt[d]*PolyLog[2, (Sqrt[e]*(I + c*x))/(c*Sqrt[-d] + I*Sq
rt[e])])/(4*Sqrt[-d^2]*Sqrt[e])
```

## Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 510, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {5445, 218, 5443, 2856, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{d+ex^2}\,dx$$

$$\Big\downarrow \text{5445}$$

$$a\int \frac{1}{ex^2+d}dx+b\int \frac{\arctan(cx)}{ex^2+d}dx$$

$$\Big\downarrow \text{218}$$

$$b\int \frac{\arctan(cx)}{ex^2+d}dx+\frac{a\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}}$$

$$\Big\downarrow \text{5443}$$

$$\frac{a\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}}+b\left(\frac{1}{2}i\int \frac{\log(1-icx)}{ex^2+d}dx-\frac{1}{2}i\int \frac{\log(icx+1)}{ex^2+d}dx\right)$$

$$\Big\downarrow \text{2856}$$

$$\frac{a\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}}+$$
$$b\left(\frac{1}{2}i\int \left(\frac{\sqrt{-d}\log(1-icx)}{2d\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{\sqrt{-d}\log(1-icx)}{2d\left(\sqrt{e}x+\sqrt{-d}\right)}\right)dx-\frac{1}{2}i\int \left(\frac{\sqrt{-d}\log(icx+1)}{2d\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{\sqrt{-d}\log(icx+1)}{2d\left(\sqrt{e}x+\sqrt{-d}\right)}\right)dx\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{a\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}}+$$
$$b\left(\frac{1}{2}i\left(-\frac{\text{PolyLog}\left(2,\frac{\sqrt{e}(1-icx)}{i\sqrt{-d}c+\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{\text{PolyLog}\left(2,\frac{\sqrt{e}(cx+i)}{\sqrt{-d}c+i\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{\log(1-icx)\log\left(\frac{c\left(\sqrt{-d}-\sqrt{e}x\right)}{c\sqrt{-d}+i\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}}-\frac{\log(1-icx)\log}{2\sqrt{-}}\right.\right.$$

input
```
Int[(a + b*ArcTan[c*x])/(d + e*x^2),x]
```

output
```
(a*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*Sqrt[e]) + b*((-1/2*I)*((Log[1 +
I*c*x]*Log[(c*(Sqrt[-d] - Sqrt[e]*x))/(c*Sqrt[-d] - I*Sqrt[e])])/(2*Sqrt[-
d]*Sqrt[e]) - (Log[1 + I*c*x]*Log[(c*(Sqrt[-d] + Sqrt[e]*x))/(c*Sqrt[-d] +
 I*Sqrt[e])])/(2*Sqrt[-d]*Sqrt[e]) - PolyLog[2, (Sqrt[e]*(I - c*x))/(c*Sqr
t[-d] + I*Sqrt[e])]/(2*Sqrt[-d]*Sqrt[e]) + PolyLog[2, (Sqrt[e]*(1 + I*c*x)
)/(I*c*Sqrt[-d] + Sqrt[e])]/(2*Sqrt[-d]*Sqrt[e])) + (I/2)*((Log[1 - I*c*x]
*Log[(c*(Sqrt[-d] - Sqrt[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])])/(2*Sqrt[-d]*Sqr
t[e]) - (Log[1 - I*c*x]*Log[(c*(Sqrt[-d] + Sqrt[e]*x))/(c*Sqrt[-d] - I*Sqr
t[e])])/(2*Sqrt[-d]*Sqrt[e]) - PolyLog[2, (Sqrt[e]*(1 - I*c*x))/(I*c*Sqrt[
-d] + Sqrt[e])]/(2*Sqrt[-d]*Sqrt[e]) + PolyLog[2, (Sqrt[e]*(I + c*x))/(c*S
qrt[-d] + I*Sqrt[e])]/(2*Sqrt[-d]*Sqrt[e])))
```

### Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2856
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

rule 5443
```
Int[ArcTan[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Simp[I/2   Int[
Log[1 - I*c*x]/(d + e*x^2), x], x] - Simp[I/2   Int[Log[1 + I*c*x]/(d + e*x
^2), x], x] /; FreeQ[{c, d, e}, x]
```

rule 5445
```
Int[(ArcTan[(c_.)*(x_)]*(b_.) + (a_))/((d_.) + (e_.)*(x_)^2), x_Symbol] :>
Simp[a   Int[1/(d + e*x^2), x], x] + Simp[b   Int[ArcTan[c*x]/(d + e*x^2),
x], x] /; FreeQ[{a, b, c, d, e}, x]
```

## Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 400, normalized size of antiderivative = 0.77

| method | result |
|---|---|
| risch | $\frac{b\ln(-icx+1)\ln\left(\frac{c\sqrt{de}-(-icx+1)e+e}{c\sqrt{de}+e}\right)}{4\sqrt{de}}-\frac{b\ln(-icx+1)\ln\left(\frac{c\sqrt{de}+(-icx+1)e-e}{c\sqrt{de}-e}\right)}{4\sqrt{de}}+\frac{b\,\mathrm{dilog}\left(\frac{c\sqrt{de}-(-icx+1)e+e}{c\sqrt{de}+e}\right)}{4\sqrt{de}}-$ |
| derivativedivides | $\frac{\frac{ac\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}}-\frac{ib\,c^2\ln\left(1-\frac{\left(c^2d-e\right)(icx+1)^2}{\left(c^2x^2+1\right)\left(-c^2d-2\sqrt{c^2de}-e\right)}\right)\arctan(cx)\sqrt{c^2de}}{c^4d^2-2c^2de+e^2}+\frac{ib\ln\left(1-\frac{\left(c^2d-e\right)(icx+1)^2}{\left(c^2x^2+1\right)\left(-c^2d-2\sqrt{c^2de}-e\right)}\right)\mathrm{arc}}{2d\left(c^4d^2-2c^2de+e^2\right)}}{}$ |
| default | $\frac{\frac{ac\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}}-\frac{ib\,c^2\ln\left(1-\frac{\left(c^2d-e\right)(icx+1)^2}{\left(c^2x^2+1\right)\left(-c^2d-2\sqrt{c^2de}-e\right)}\right)\arctan(cx)\sqrt{c^2de}}{c^4d^2-2c^2de+e^2}+\frac{ib\ln\left(1-\frac{\left(c^2d-e\right)(icx+1)^2}{\left(c^2x^2+1\right)\left(-c^2d-2\sqrt{c^2de}-e\right)}\right)\mathrm{arc}}{2d\left(c^4d^2-2c^2de+e^2\right)}}{}$ |
| parts | $\frac{a\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}}-\frac{ibc\ln\left(1-\frac{\left(c^2d-e\right)(icx+1)^2}{\left(c^2x^2+1\right)\left(-c^2d-2\sqrt{c^2de}-e\right)}\right)\arctan(cx)\sqrt{c^2de}}{c^4d^2-2c^2de+e^2}-\frac{b\sqrt{c^2de}\,\arctan(cx)^2}{2cde}-\frac{b\sqrt{c^2de}\ \mathrm{p}}{}$ |

input
```
int((a+b*arctan(c*x))/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
1/4*b*ln(1-I*c*x)/(d*e)^(1/2)*ln((c*(d*e)^(1/2)-(1-I*c*x)*e+e)/(c*(d*e)^(1
/2)+e))-1/4*b*ln(1-I*c*x)/(d*e)^(1/2)*ln((c*(d*e)^(1/2)+(1-I*c*x)*e-e)/(c*
(d*e)^(1/2)-e))+1/4*b/(d*e)^(1/2)*dilog((c*(d*e)^(1/2)-(1-I*c*x)*e+e)/(c*(
d*e)^(1/2)+e))-1/4*b/(d*e)^(1/2)*dilog((c*(d*e)^(1/2)+(1-I*c*x)*e-e)/(c*(d
*e)^(1/2)-e))+I*a/(d*e)^(1/2)*arctanh(1/2*(2*(1-I*c*x)*e-2*e)/c/(d*e)^(1/2
))+1/4*b*ln(1+I*c*x)/(d*e)^(1/2)*ln((c*(d*e)^(1/2)-(1+I*c*x)*e+e)/(c*(d*e)
^(1/2)+e))-1/4*b*ln(1+I*c*x)/(d*e)^(1/2)*ln((c*(d*e)^(1/2)+(1+I*c*x)*e-e)/
(c*(d*e)^(1/2)-e))+1/4*b/(d*e)^(1/2)*dilog((c*(d*e)^(1/2)-(1+I*c*x)*e+e)/(
c*(d*e)^(1/2)+e))-1/4*b/(d*e)^(1/2)*dilog((c*(d*e)^(1/2)+(1+I*c*x)*e-e)/(c
*(d*e)^(1/2)-e))
```

## Fricas [F]

$$\int \frac{a+b\arctan(cx)}{d+ex^2}\,dx=\int \frac{b\arctan\left(cx\right)+a}{ex^2+d}\,dx$$

input
```
integrate((a+b*arctan(c*x))/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*arctan(c*x) + a)/(e*x^2 + d), x)
```

## Sympy [F]

$$\int \frac{a+b\arctan(cx)}{d+ex^2}\,dx = \int \frac{a+b\operatorname{atan}(cx)}{d+ex^2}\,dx$$

input
```
integrate((a+b*atan(c*x))/(e*x**2+d),x)
```

output
```
Integral((a + b*atan(c*x))/(d + e*x**2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\arctan(cx)}{d+ex^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arctan(c*x))/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

## Giac [F]

$$\int \frac{a+b\arctan(cx)}{d+ex^2}\,dx=\int \frac{b\arctan(cx)+a}{ex^2+d}\,dx$$

input
```
integrate((a+b*arctan(c*x))/(e*x^2+d),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)/(e*x^2 + d), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{d+ex^2}\,dx=\int \frac{a+b\,\mathrm{atan}(c\,x)}{e\,x^2+d}\,dx$$

input
```
int((a + b*atan(c*x))/(d + e*x^2),x)
```

output
```
int((a + b*atan(c*x))/(d + e*x^2), x)
```

## Reduce [F]

$$\int \frac{a+b\arctan(cx)}{d+ex^2}\,dx$$
$$=\frac{2\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)a+atan(cx)^2\,bcd-2\left(\int\frac{atan(cx)}{c^2e\,x^4+c^2d\,x^2+e\,x^2+d}dx\right)b\,c^2d^2+2\left(\int\frac{atan(cx)}{c^2e\,x^4+c^2d\,x^2+e\,x^2+d}dx\right)b}{2de}$$

input
```
int((a+b*atan(c*x))/(e*x^2+d),x)
```

output
```
(2*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a + atan(c*x)**2*b*c*d -
2*int(atan(c*x)/(c**2*d*x**2 + c**2*e*x**4 + d + e*x**2),x)*b*c**2*d**2 +
2*int(atan(c*x)/(c**2*d*x**2 + c**2*e*x**4 + d + e*x**2),x)*b*d*e)/(2*d*e)
```

# 3.1156 $\int \frac{a+b\arctan(cx)}{x^2(d+ex^2)}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 561

$$\begin{aligned}\int \frac{a+b\arctan(cx)}{x^2(d+ex^2)}\,dx =& -\frac{a+b\arctan(cx)}{dx}-\frac{a\sqrt{e}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}} \\ &+\frac{bc\log(x)}{d}-\frac{ib\sqrt{e}\log(1+icx)\log\left(\frac{c(\sqrt{-d}-\sqrt{e}x)}{c\sqrt{-d}-i\sqrt{e}}\right)}{4(-d)^{3/2}} \\ &+\frac{ib\sqrt{e}\log(1-icx)\log\left(\frac{c(\sqrt{-d}-\sqrt{e}x)}{c\sqrt{-d}+i\sqrt{e}}\right)}{4(-d)^{3/2}} \\ &-\frac{ib\sqrt{e}\log(1-icx)\log\left(\frac{c(\sqrt{-d}+\sqrt{e}x)}{c\sqrt{-d}-i\sqrt{e}}\right)}{4(-d)^{3/2}} \\ &+\frac{ib\sqrt{e}\log(1+icx)\log\left(\frac{c(\sqrt{-d}+\sqrt{e}x)}{c\sqrt{-d}+i\sqrt{e}}\right)}{4(-d)^{3/2}}-\frac{bc\log(1+c^2x^2)}{2d} \\ &+\frac{ib\sqrt{e}\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}(i-cx)}{c\sqrt{-d}+i\sqrt{e}}\right)}{4(-d)^{3/2}}-\frac{ib\sqrt{e}\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}(1-icx)}{ic\sqrt{-d}+\sqrt{e}}\right)}{4(-d)^{3/2}} \\ &-\frac{ib\sqrt{e}\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}(1+icx)}{ic\sqrt{-d}+\sqrt{e}}\right)}{4(-d)^{3/2}}+\frac{ib\sqrt{e}\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}(i+cx)}{c\sqrt{-d}+i\sqrt{e}}\right)}{4(-d)^{3/2}}\end{aligned}$$

output
```
-(a+b*arctan(c*x))/d/x-a*e^(1/2)*arctan(e^(1/2)*x/d^(1/2))/d^(3/2)+b*c*ln(
x)/d-1/4*I*b*e^(1/2)*ln(1+I*c*x)*ln(c*((-d)^(1/2)-e^(1/2)*x)/(c*(-d)^(1/2)
-I*e^(1/2)))/(-d)^(3/2)+1/4*I*b*e^(1/2)*ln(1-I*c*x)*ln(c*((-d)^(1/2)-e^(1/
2)*x)/(c*(-d)^(1/2)+I*e^(1/2)))/(-d)^(3/2)-1/4*I*b*e^(1/2)*ln(1-I*c*x)*ln(
c*((-d)^(1/2)+e^(1/2)*x)/(c*(-d)^(1/2)-I*e^(1/2)))/(-d)^(3/2)+1/4*I*b*e^(1
/2)*ln(1+I*c*x)*ln(c*((-d)^(1/2)+e^(1/2)*x)/(c*(-d)^(1/2)+I*e^(1/2)))/(-d)
^(3/2)-1/2*b*c*ln(c^2*x^2+1)/d+1/4*I*b*e^(1/2)*polylog(2,e^(1/2)*(I-c*x)/(
c*(-d)^(1/2)+I*e^(1/2)))/(-d)^(3/2)-1/4*I*b*e^(1/2)*polylog(2,e^(1/2)*(1-I
*c*x)/(I*c*(-d)^(1/2)+e^(1/2)))/(-d)^(3/2)-1/4*I*b*e^(1/2)*polylog(2,e^(1/
2)*(1+I*c*x)/(I*c*(-d)^(1/2)+e^(1/2)))/(-d)^(3/2)+1/4*I*b*e^(1/2)*polylog(
2,e^(1/2)*(I+c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/(-d)^(3/2)
```

## Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 467, normalized size of antiderivative = 0.83

$$\int \frac{a+b\arctan(cx)}{x^2\left(d+ex^2\right)}\,dx$$

$$= \frac{-\frac{a+b\arctan(cx)}{x}+bc\left(\log(x)-\frac{1}{2}\log\left(1+c^2x^2\right)\right)-\frac{\sqrt{e}\left(4a\sqrt{-d}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)+ib\sqrt{d}\left(\log(1+icx)\log\left(\frac{c\left(\sqrt{-d}+\sqrt{e}x\right)}{c\sqrt{-d}+i\sqrt{e}}\right)+\text{PolyLog}\left(\right.\right.\right.}{}}{}$$

input
```
Integrate[(a + b*ArcTan[c*x])/(x^2*(d + e*x^2)),x]
```

output
```
(-((a + b*ArcTan[c*x])/x) + b*c*(Log[x] - Log[1 + c^2*x^2]/2) - (Sqrt[e]*(
4*a*Sqrt[-d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + I*b*Sqrt[d]*(Log[1 + I*c*x]*Log
[(c*(Sqrt[-d] + Sqrt[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])] + PolyLog[2, (Sqrt[e
]*(I - c*x))/(c*Sqrt[-d] + I*Sqrt[e])]) - I*b*Sqrt[d]*(Log[1 - I*c*x]*Log[
(c*(Sqrt[-d] + Sqrt[e]*x))/(c*Sqrt[-d] - I*Sqrt[e])] + PolyLog[2, (Sqrt[e]
*(1 - I*c*x))/(I*c*Sqrt[-d] + Sqrt[e])]) - I*b*Sqrt[d]*(Log[1 + I*c*x]*Log
[(c*(Sqrt[-d] - Sqrt[e]*x))/(c*Sqrt[-d] - I*Sqrt[e])] + PolyLog[2, (Sqrt[e
]*(1 + I*c*x))/(I*c*Sqrt[-d] + Sqrt[e])]) + I*b*Sqrt[d]*(Log[1 - I*c*x]*Lo
g[(c*(Sqrt[-d] - Sqrt[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])] + PolyLog[2, (Sqrt[
e]*(I + c*x))/(c*Sqrt[-d] + I*Sqrt[e])])))/(4*Sqrt[-d^2]))/d
```

## Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 558, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {5453, 5361, 243, 47, 14, 16, 5445, 218, 5443, 2856, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x^2(d+ex^2)}dx$$

$$\Big\downarrow \text{5453}$$

$$\frac{\int \frac{a+b\arctan(cx)}{x^2}dx}{d}-\frac{e\int \frac{a+b\arctan(cx)}{ex^2+d}dx}{d}$$

$$\Big\downarrow \text{5361}$$

$$\frac{bc\int \frac{1}{x(c^2x^2+1)}dx-\frac{a+b\arctan(cx)}{x}}{d}-\frac{e\int \frac{a+b\arctan(cx)}{ex^2+d}dx}{d}$$

$$\Big\downarrow \text{243}$$

$$\frac{\frac{1}{2}bc\int \frac{1}{x^2(c^2x^2+1)}dx^2-\frac{a+b\arctan(cx)}{x}}{d}-\frac{e\int \frac{a+b\arctan(cx)}{ex^2+d}dx}{d}$$

$$\Big\downarrow \text{47}$$

$$\frac{\frac{1}{2}bc\left(\int \frac{1}{x^2}dx^2-c^2\int \frac{1}{c^2x^2+1}dx^2\right)-\frac{a+b\arctan(cx)}{x}}{d}-\frac{e\int \frac{a+b\arctan(cx)}{ex^2+d}dx}{d}$$

$$\Big\downarrow \text{14}$$

$$\frac{\frac{1}{2}bc\left(\log\left(x^2\right)-c^2\int \frac{1}{c^2x^2+1}dx^2\right)-\frac{a+b\arctan(cx)}{x}}{d}-\frac{e\int \frac{a+b\arctan(cx)}{ex^2+d}dx}{d}$$

$$\Big\downarrow \text{16}$$

$$\frac{\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right)\right)-\frac{a+b\arctan(cx)}{x}}{d}-\frac{e\int \frac{a+b\arctan(cx)}{ex^2+d}dx}{d}$$

$$\Big\downarrow \text{5445}$$

$$\frac{\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right)\right)-\frac{a+b\arctan(cx)}{x}}{d}-\frac{e\left(a\int \frac{1}{ex^2+d}dx+b\int \frac{\arctan(cx)}{ex^2+d}dx\right)}{d}$$

$\big\downarrow$ 218

$$\frac{\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right)\right)-\frac{a+b\arctan(cx)}{x}}{d}-\frac{e\left(b\int\frac{\arctan(cx)}{ex^2+d}dx+\frac{a\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}}\right)}{d}$$

$\big\downarrow$ 5443

$$\frac{\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right)\right)-\frac{a+b\arctan(cx)}{x}}{d}-\frac{e\left(\frac{a\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}}+b\left(\frac{1}{2}i\int\frac{\log(1-icx)}{ex^2+d}dx-\frac{1}{2}i\int\frac{\log(icx+1)}{ex^2+d}dx\right)\right)}{d}$$

$\big\downarrow$ 2856

$$\frac{\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right)\right)-\frac{a+b\arctan(cx)}{x}}{d}-\frac{e\left(\frac{a\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}}+b\left(\frac{1}{2}i\int\left(\frac{\sqrt{-d}\log(1-icx)}{2d\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{\sqrt{-d}\log(1-icx)}{2d\left(\sqrt{e}x+\sqrt{-d}\right)}\right)dx-\frac{1}{2}i\int\left(\frac{\sqrt{-d}\log(icx+1)}{2d\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{\sqrt{-d}\log(icx+1)}{2d\left(\sqrt{e}x+\sqrt{-d}\right)}\right)dx\right)\right)}{d}$$

$\big\downarrow$ 2009

$$\frac{\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right)\right)-\frac{a+b\arctan(cx)}{x}}{d}-$$

$$e\left(\frac{a\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}\sqrt{e}}+b\left(\frac{1}{2}i\left(-\frac{\text{PolyLog}\left(2,\frac{\sqrt{e}(1-icx)}{i\sqrt{-d}c+\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{\text{PolyLog}\left(2,\frac{\sqrt{e}(cx+i)}{\sqrt{-d}c+i\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{\log(1-icx)\log\left(\frac{c\left(\sqrt{-d}-\sqrt{e}x\right)}{c\sqrt{-d}+i\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}}-\frac{\log(1-icx)\log\left(\frac{c}{\ }\right.}{2\sqrt{-d}}\right.\right.\right.$$

input
```
Int[(a + b*ArcTan[c*x])/(x^2*(d + e*x^2)),x]
```

output
```
(-((a + b*ArcTan[c*x])/x) + (b*c*(Log[x^2] - Log[1 + c^2*x^2]))/2)/d - (e*
((a*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*Sqrt[e]) + b*((-1/2*I)*((Log[1 +
 I*c*x]*Log[(c*(Sqrt[-d] - Sqrt[e]*x))/(c*Sqrt[-d] - I*Sqrt[e])])/(2*Sqrt[
-d]*Sqrt[e]) - (Log[1 + I*c*x]*Log[(c*(Sqrt[-d] + Sqrt[e]*x))/(c*Sqrt[-d]
+ I*Sqrt[e])])/(2*Sqrt[-d]*Sqrt[e]) - PolyLog[2, (Sqrt[e]*(I - c*x))/(c*Sq
rt[-d] + I*Sqrt[e])]/(2*Sqrt[-d]*Sqrt[e]) + PolyLog[2, (Sqrt[e]*(1 + I*c*x
))/(I*c*Sqrt[-d] + Sqrt[e])]/(2*Sqrt[-d]*Sqrt[e])) + (I/2)*((Log[1 - I*c*x
]*Log[(c*(Sqrt[-d] - Sqrt[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])])/(2*Sqrt[-d]*Sq
rt[e]) - (Log[1 - I*c*x]*Log[(c*(Sqrt[-d] + Sqrt[e]*x))/(c*Sqrt[-d] - I*Sq
rt[e])])/(2*Sqrt[-d]*Sqrt[e]) - PolyLog[2, (Sqrt[e]*(1 - I*c*x))/(I*c*Sqrt
[-d] + Sqrt[e])]/(2*Sqrt[-d]*Sqrt[e]) + PolyLog[2, (Sqrt[e]*(I + c*x))/(c*
Sqrt[-d] + I*Sqrt[e])]/(2*Sqrt[-d]*Sqrt[e])))))/d
```

## Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2856
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5443
```
Int[ArcTan[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Simp[I/2   Int[
Log[1 - I*c*x]/(d + e*x^2), x], x] - Simp[I/2   Int[Log[1 + I*c*x]/(d + e*x
^2), x], x] /; FreeQ[{c, d, e}, x]
```

rule 5445
```
Int[(ArcTan[(c_.)*(x_)]*(b_.) + (a_))/((d_.) + (e_.)*(x_)^2), x_Symbol] :>
Simp[a   Int[1/(d + e*x^2), x], x] + Simp[b   Int[ArcTan[c*x]/(d + e*x^2),
x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

## Maple [A] (verified)

Time = 1.25 (sec) , antiderivative size = 537, normalized size of antiderivative = 0.96

| method | result |
|---|---|
| risch | $-\frac{be\ln(-icx+1)\ln\left(\frac{c\sqrt{de}-(-icx+1)e+e}{c\sqrt{de}+e}\right)}{4d\sqrt{de}}+\frac{be\ln(-icx+1)\ln\left(\frac{c\sqrt{de}+(-icx+1)e-e}{c\sqrt{de}-e}\right)}{4d\sqrt{de}}-\frac{be\,\text{dilog}\left(\frac{c\sqrt{de}-(-icx+1)e+e}{c\sqrt{de}+e}\right)}{4d\sqrt{de}}$ |
| parts | Expression too large to display |
| derivativedivides | Expression too large to display |
| default | Expression too large to display |

input
```
int((a+b*arctan(c*x))/x^2/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
-1/4*b*e/d*ln(1-I*c*x)/(d*e)^(1/2)*ln((c*(d*e)^(1/2)-(1-I*c*x)*e+e)/(c*(d*
e)^(1/2)+e))+1/4*b*e/d*ln(1-I*c*x)/(d*e)^(1/2)*ln((c*(d*e)^(1/2)+(1-I*c*x)
*e-e)/(c*(d*e)^(1/2)-e))-1/4*b*e/d/(d*e)^(1/2)*dilog((c*(d*e)^(1/2)-(1-I*c
*x)*e+e)/(c*(d*e)^(1/2)+e))+1/4*b*e/d/(d*e)^(1/2)*dilog((c*(d*e)^(1/2)+(1-
I*c*x)*e-e)/(c*(d*e)^(1/2)-e))+1/2*c*b/d*ln(-I*c*x)-1/2*c*b/d*ln(1-I*c*x)+
1/2*I*b/d*ln(1+I*c*x)/x-1/2*I*b/d*ln(1-I*c*x)/x-a/d/x-1/4*b*e/d*ln(1+I*c*x
)/(d*e)^(1/2)*ln((c*(d*e)^(1/2)-(1+I*c*x)*e+e)/(c*(d*e)^(1/2)+e))+1/4*b*e/
d*ln(1+I*c*x)/(d*e)^(1/2)*ln((c*(d*e)^(1/2)+(1+I*c*x)*e-e)/(c*(d*e)^(1/2)-
e))-1/4*b*e/d/(d*e)^(1/2)*dilog((c*(d*e)^(1/2)-(1+I*c*x)*e+e)/(c*(d*e)^(1/
2)+e))+1/4*b*e/d/(d*e)^(1/2)*dilog((c*(d*e)^(1/2)+(1+I*c*x)*e-e)/(c*(d*e)^
(1/2)-e))+1/2*b*c/d*ln(I*c*x)-1/2*b*c/d*ln(1+I*c*x)-I*a*e/d/(d*e)^(1/2)*ar
ctanh(1/2*(2*(1-I*c*x)*e-2*e)/c/(d*e)^(1/2))
```

## Fricas [F]

$$\int \frac{a + b \arctan(cx)}{x^2 (d + ex^2)} \, dx = \int \frac{b \arctan(cx) + a}{(ex^2 + d)x^2} \, dx$$

input
```
integrate((a+b*arctan(c*x))/x^2/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*arctan(c*x) + a)/(e*x^4 + d*x^2), x)
```

## Sympy [F]

$$\int \frac{a + b \arctan(cx)}{x^2 (d + ex^2)} \, dx = \int \frac{a + b \operatorname{atan}(cx)}{x^2 (d + ex^2)} \, dx$$

input
```
integrate((a+b*atan(c*x))/x**2/(e*x**2+d),x)
```

output
```
Integral((a + b*atan(c*x))/(x**2*(d + e*x**2)), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\arctan(cx)}{x^2 (d + ex^2)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arctan(c*x))/x^2/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

## Giac [F]

$$\int \frac{a + b\arctan(cx)}{x^2 (d + ex^2)}\,dx = \int \frac{b\arctan(cx) + a}{(ex^2 + d)x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^2/(e*x^2+d),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)/((e*x^2 + d)*x^2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\arctan(cx)}{x^2 (d + ex^2)}\,dx = \int \frac{a + b\,\mathrm{atan}(c\,x)}{x^2\,(e\,x^2 + d)}\,dx$$

input
```
int((a + b*atan(c*x))/(x^2*(d + e*x^2)),x)
```

output
```
int((a + b*atan(c*x))/(x^2*(d + e*x^2)), x)
```

## Reduce [F]

$$\int \frac{a+b\arctan(cx)}{x^2\left(d+ex^2\right)}\,dx$$
$$= \frac{-2\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)ax - atan(cx)^2\,bcdx - 2atan(cx)\,bd + 2\left(\int \frac{atan(cx)}{c^2e\,x^4+c^2d\,x^2+e\,x^2+d}dx\right)b\,c^2d^2x - 2\left(\int \right.}{2d^2x}$$

input
```
int((a+b*atan(c*x))/x^2/(e*x^2+d),x)
```

output
```
( - 2*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*x - atan(c*x)**2*b*c
*d*x - 2*atan(c*x)*b*d + 2*int(atan(c*x)/(c**2*d*x**2 + c**2*e*x**4 + d +
e*x**2),x)*b*c**2*d**2*x - 2*int(atan(c*x)/(c**2*d*x**2 + c**2*e*x**4 + d
+ e*x**2),x)*b*d*e*x - log(c**2*x**2 + 1)*b*c*d*x + 2*log(x)*b*c*d*x - 2*a
*d)/(2*d**2*x)
```

# 3.1157 $\int \frac{x^3(a+b\arctan(cx))}{(d+ex^2)^2}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 403

$$
\begin{aligned}
\int \frac{x^3(a+b\arctan(cx))}{(d+ex^2)^2}\,dx = & -\frac{bc^2d\arctan(cx)}{2(c^2d-e)e^2}+\frac{d(a+b\arctan(cx))}{2e^2(d+ex^2)} \\
& +\frac{bc\sqrt{d}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2(c^2d-e)e^{3/2}}-\frac{(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)}{e^2} \\
& +\frac{(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2e^2} \\
& +\frac{(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{2e^2} \\
& +\frac{ib\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2e^2} \\
& -\frac{ib\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{4e^2} \\
& -\frac{ib\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{4e^2}
\end{aligned}
$$

output
```
-1/2*b*c^2*d*arctan(c*x)/(c^2*d-e)/e^2+1/2*d*(a+b*arctan(c*x))/e^2/(e*x^2+
d)+1/2*b*c*d^(1/2)*arctan(e^(1/2)*x/d^(1/2))/(c^2*d-e)/e^(3/2)-(a+b*arctan
(c*x))*ln(2/(1-I*c*x))/e^2+1/2*(a+b*arctan(c*x))*ln(2*c*((-d)^(1/2)-e^(1/2
)*x)/(c*(-d)^(1/2)-I*e^(1/2))/(1-I*c*x))/e^2+1/2*(a+b*arctan(c*x))*ln(2*c*
((-d)^(1/2)+e^(1/2)*x)/(c*(-d)^(1/2)+I*e^(1/2))/(1-I*c*x))/e^2+1/2*I*b*pol
ylog(2,1-2/(1-I*c*x))/e^2-1/4*I*b*polylog(2,1-2*c*((-d)^(1/2)-e^(1/2)*x)/(
c*(-d)^(1/2)-I*e^(1/2))/(1-I*c*x))/e^2-1/4*I*b*polylog(2,1-2*c*((-d)^(1/2)
+e^(1/2)*x)/(c*(-d)^(1/2)+I*e^(1/2))/(1-I*c*x))/e^2
```

## Mathematica [A] (verified)

Time = 6.75 (sec) , antiderivative size = 522, normalized size of antiderivative = 1.30

$$\int \frac{x^3(a+b\arctan(cx))}{(d+ex^2)^2}\,dx = \frac{2a\left(\frac{d}{d+ex^2}+\log\left(d+ex^2\right)\right)+b\left(-\frac{2c^2d\arctan(cx)}{c^2d-e}+\frac{2d\arctan(cx)}{d+ex^2}+\frac{2c\sqrt{d}\sqrt{e}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{c^2d-e}+2\arctan(cx)\log\left(-\frac{i\sqrt{d}}{\sqrt{e}}\right.\right.}{}$$

input
```
Integrate[(x^3*(a + b*ArcTan[c*x]))/(d + e*x^2)^2,x]
```

output
```
(2*a*(d/(d + e*x^2) + Log[d + e*x^2]) + b*((-2*c^2*d*ArcTan[c*x])/(c^2*d -
 e) + (2*d*ArcTan[c*x])/(d + e*x^2) + (2*c*Sqrt[d]*Sqrt[e]*ArcTan[(Sqrt[e]
*x)/Sqrt[d]])/(c^2*d - e) + 2*ArcTan[c*x]*Log[((-I)*Sqrt[d])/Sqrt[e] + x]
+ 2*ArcTan[c*x]*Log[(I*Sqrt[d])/Sqrt[e] + x] + I*Log[((-I)*Sqrt[d])/Sqrt[e
] + x]*Log[(Sqrt[e]*(-1 - I*c*x))/(c*Sqrt[d] - Sqrt[e])] - I*Log[((-I)*Sqr
t[d])/Sqrt[e] + x]*Log[(Sqrt[e]*(1 - I*c*x))/(c*Sqrt[d] + Sqrt[e])] - I*Lo
g[(I*Sqrt[d])/Sqrt[e] + x]*Log[(Sqrt[e]*(-1 + I*c*x))/(c*Sqrt[d] - Sqrt[e]
)] + I*Log[(I*Sqrt[d])/Sqrt[e] + x]*Log[(Sqrt[e]*(1 + I*c*x))/(c*Sqrt[d] +
 Sqrt[e])] - I*PolyLog[2, (c*(Sqrt[d] - I*Sqrt[e]*x))/(c*Sqrt[d] - Sqrt[e]
)] + I*PolyLog[2, (c*(Sqrt[d] - I*Sqrt[e]*x))/(c*Sqrt[d] + Sqrt[e])] + I*P
olyLog[2, (c*(Sqrt[d] + I*Sqrt[e]*x))/(c*Sqrt[d] - Sqrt[e])] - I*PolyLog[2
, (c*(Sqrt[d] + I*Sqrt[e]*x))/(c*Sqrt[d] + Sqrt[e])]))/(4*e^2)
```

## Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\arctan(cx))}{(d+ex^2)^2}dx$$

$$\Big\downarrow \text{ 5515}$$

$$\int \left(\frac{x(a+b\arctan(cx))}{e(d+ex^2)}-\frac{dx(a+b\arctan(cx))}{e(d+ex^2)^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{d(a+b\arctan(cx))}{2e^2(d+ex^2)}+\frac{(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(1-icx)(c\sqrt{-d}-i\sqrt{e})}\right)}{2e^2}+\frac{(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(1-icx)(c\sqrt{-d}+i\sqrt{e})}\right)}{2e^2}-\frac{\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))}{e^2}+\frac{bc\sqrt{d}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2e^{3/2}(c^2d-e)}-\frac{bc^2d\arctan(cx)}{2e^2(c^2d-e)}-\frac{ib\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{4e^2}-\frac{ib\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)}{4e^2}+\frac{ib\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2e^2}$$

input
```
Int[(x^3*(a + b*ArcTan[c*x]))/(d + e*x^2)^2,x]
```

output
```
-1/2*(b*c^2*d*ArcTan[c*x])/((c^2*d - e)*e^2) + (d*(a + b*ArcTan[c*x]))/(2*
e^2*(d + e*x^2)) + (b*c*Sqrt[d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*(c^2*d - e
)*e^(3/2)) - ((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e^2 + ((a + b*ArcTan
[c*x])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c
*x))])/(2*e^2) + ((a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c
*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(2*e^2) + ((I/2)*b*PolyLog[2, 1 - 2/
(1 - I*c*x)])/e^2 - ((I/4)*b*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/(
(c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/e^2 - ((I/4)*b*PolyLog[2, 1 - (2*c
*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/e^2
```

**Defintions of rubi rules used**

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.59 (sec) , antiderivative size = 716, normalized size of antiderivative = 1.78

| method | result |
|---|---|
| parts | $\frac{ad}{2e^2(e\,x^2+d)}+\frac{a\ln(e\,x^2+d)}{2e^2}+\frac{b\left(\frac{\arctan(cx)c^4\ln\left(c^2e\,x^2+c^2d\right)}{2e^2}+\frac{\arctan(cx)c^6d}{2e^2\left(c^2e\,x^2+c^2d\right)}-\frac{c^4\left(-\frac{i\left(\ln(cx-i)\ln\left(c^2e\,x^2+c^2d\right)-\right.}{}\right.}{}\right.}{}$ |
| derivativedivides | $\frac{\frac{a\,c^6d}{2e^2\left(c^2e\,x^2+c^2d\right)}+\frac{a\,c^4\ln\left(c^2e\,x^2+c^2d\right)}{2e^2}+b\,c^4\left(\frac{\arctan(cx)c^2d}{2e^2\left(c^2e\,x^2+c^2d\right)}+\frac{\arctan(cx)\ln\left(c^2e\,x^2+c^2d\right)}{2e^2}-\frac{-\frac{i\left(\ln(cx-i)\ln\left(c^2e\,x^2+c^2d\right)\right.}{}}{}\right.}{}$ |
| default | $\frac{\frac{a\,c^6d}{2e^2\left(c^2e\,x^2+c^2d\right)}+\frac{a\,c^4\ln\left(c^2e\,x^2+c^2d\right)}{2e^2}+b\,c^4\left(\frac{\arctan(cx)c^2d}{2e^2\left(c^2e\,x^2+c^2d\right)}+\frac{\arctan(cx)\ln\left(c^2e\,x^2+c^2d\right)}{2e^2}-\frac{-\frac{i\left(\ln(cx-i)\ln\left(c^2e\,x^2+c^2d\right)\right.}{}}{}\right.}{}$ |

input
```
int(x^3*(a+b*arctan(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*a/e^2*d/(e*x^2+d)+1/2*a/e^2*ln(e*x^2+d)+b/c^4*(1/2*arctan(c*x)*c^4/e^2
*ln(c^2*e*x^2+c^2*d)+1/2*arctan(c*x)*c^6*d/e^2/(c^2*e*x^2+c^2*d)-1/2*c^4*(
1/e^2*(-1/2*I*(ln(c*x-I)*ln(c^2*e*x^2+c^2*d)-2*e*(1/2*ln(c*x-I)*(ln((RootO
f(e*_Z^2+2*I*e*_Z+c^2*d-e,index=1)-c*x+I)/RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,i
ndex=1))+ln((RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=2)-c*x+I)/RootOf(e*_Z^2+
2*I*e*_Z+c^2*d-e,index=2)))/e+1/2*(dilog((RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,i
ndex=1)-c*x+I)/RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=1))+dilog((RootOf(e*_Z
^2+2*I*e*_Z+c^2*d-e,index=2)-c*x+I)/RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=2
)))/e))+1/2*I*(ln(I+c*x)*ln(c^2*e*x^2+c^2*d)-2*e*(1/2*ln(I+c*x)*(ln((RootO
f(e*_Z^2-2*I*e*_Z+c^2*d-e,index=1)-c*x-I)/RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,i
ndex=1))+ln((RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=2)-c*x-I)/RootOf(e*_Z^2-
2*I*e*_Z+c^2*d-e,index=2)))/e+1/2*(dilog((RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,i
ndex=1)-c*x-I)/RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=1))+dilog((RootOf(e*_Z
^2-2*I*e*_Z+c^2*d-e,index=2)-c*x-I)/RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=2
)))/e)))+c^2*d/e^2/(c^2*d-e)*arctan(c*x)-c*d/e/(c^2*d-e)/(d*e)^(1/2)*arcta
n(e*x/(d*e)^(1/2))))
```

## Fricas [F]

$$\int \frac{x^3(a+b\arctan(cx))}{(d+ex^2)^2}\,dx=\int \frac{(b\arctan(cx)+a)x^3}{(ex^2+d)^2}\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*x^3*arctan(c*x) + a*x^3)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\arctan(cx))}{(d+ex^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(a+b*atan(c*x))/(e*x**2+d)**2,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{x^3(a+b\arctan(cx))}{(d+ex^2)^2}\,dx = \int \frac{(b\arctan(cx)+a)x^3}{(ex^2+d)^2}\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
1/2*a*(d/(e^3*x^2 + d*e^2) + log(e*x^2 + d)/e^2) + 2*b*integrate(1/2*x^3*a
rctan(c*x)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

## Giac [F]

$$\int \frac{x^3(a+b\arctan(cx))}{(d+ex^2)^2}\,dx = \int \frac{(b\arctan(cx)+a)x^3}{(ex^2+d)^2}\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)*x^3/(e*x^2 + d)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\arctan(cx))}{(d+ex^2)^2}\,dx=\int \frac{x^3\,(a+b\,\mathrm{atan}(c\,x))}{(e\,x^2+d)^2}\,dx$$

input
```
int((x^3*(a + b*atan(c*x)))/(d + e*x^2)^2,x)
```

output
```
int((x^3*(a + b*atan(c*x)))/(d + e*x^2)^2, x)
```

## Reduce [F]

$$\int \frac{x^3(a+b\arctan(cx))}{(d+ex^2)^2}\,dx$$
$$=\frac{2\left(\int \frac{atan(cx)x^3}{e^2x^4+2de\,x^2+d^2}dx\right)bd\,e^2+2\left(\int \frac{atan(cx)x^3}{e^2x^4+2de\,x^2+d^2}dx\right)b\,e^3x^2+\log(e\,x^2+d)\,ad+\log(e\,x^2+d)\,ae\,x^2-ae\,x}{2e^2\,(e\,x^2+d)}$$

input
```
int(x^3*(a+b*atan(c*x))/(e*x^2+d)^2,x)
```

output
```
(2*int((atan(c*x)*x**3)/(d**2 + 2*d*e*x**2 + e**2*x**4),x)*b*d*e**2 + 2*in
t((atan(c*x)*x**3)/(d**2 + 2*d*e*x**2 + e**2*x**4),x)*b*e**3*x**2 + log(d
+ e*x**2)*a*d + log(d + e*x**2)*a*e*x**2 - a*e*x**2)/(2*e**2*(d + e*x**2))
```

# 3.1158 $\int \frac{x(a+b\arctan(cx))}{(d+ex^2)^2}\,dx$



## Optimal result

Integrand size = 19, antiderivative size = 91

$$\int \frac{x(a+b\arctan(cx))}{(d+ex^2)^2}\,dx = \frac{bc^2\arctan(cx)}{2(c^2d-e)\,e} - \frac{a+b\arctan(cx)}{2e\,(d+ex^2)} - \frac{bc\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2\sqrt{d}\,(c^2d-e)\,\sqrt{e}}$$

output
```
1/2*b*c^2*arctan(c*x)/(c^2*d-e)/e-1/2*(a+b*arctan(c*x))/e/(e*x^2+d)-1/2*b*
c*arctan(e^(1/2)*x/d^(1/2))/d^(1/2)/(c^2*d-e)/e^(1/2)
```

## Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.08

$$\int \frac{x(a+b\arctan(cx))}{(d+ex^2)^2}\,dx = \frac{a\sqrt{d}(c^2d-e) - b\sqrt{d}e(1+c^2x^2)\arctan(cx) + bc\sqrt{e}(d+ex^2)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2\sqrt{d}e\,(-c^2d+e)\,(d+ex^2)}$$

input
```
Integrate[(x*(a + b*ArcTan[c*x]))/(d + e*x^2)^2,x]
```

output
```
(a*Sqrt[d]*(c^2*d - e) - b*Sqrt[d]*e*(1 + c^2*x^2)*ArcTan[c*x] + b*c*Sqrt[
e]*(d + e*x^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*Sqrt[d]*e*(-(c^2*d) + e)*(d
 + e*x^2))
```

## Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {5509, 303, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\arctan(cx))}{(d+ex^2)^2}dx$$

$$\Big\downarrow \text{5509}$$

$$\frac{bc\int \frac{1}{(c^2x^2+1)(ex^2+d)}dx}{2e}-\frac{a+b\arctan(cx)}{2e(d+ex^2)}$$

$$\Big\downarrow \text{303}$$

$$\frac{bc\left(\frac{c^2\int \frac{1}{c^2x^2+1}dx}{c^2d-e}-\frac{e\int \frac{1}{ex^2+d}dx}{c^2d-e}\right)}{2e}-\frac{a+b\arctan(cx)}{2e(d+ex^2)}$$

$$\Big\downarrow \text{216}$$

$$\frac{bc\left(\frac{c\arctan(cx)}{c^2d-e}-\frac{e\int \frac{1}{ex^2+d}dx}{c^2d-e}\right)}{2e}-\frac{a+b\arctan(cx)}{2e(d+ex^2)}$$

$$\Big\downarrow \text{218}$$

$$\frac{bc\left(\frac{c\arctan(cx)}{c^2d-e}-\frac{\sqrt{e}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}(c^2d-e)}\right)}{2e}-\frac{a+b\arctan(cx)}{2e(d+ex^2)}$$

input
```
Int[(x*(a + b*ArcTan[c*x]))/(d + e*x^2)^2,x]
```

output
```
-1/2*(a + b*ArcTan[c*x])/(e*(d + e*x^2)) + (b*c*((c*ArcTan[c*x])/(c^2*d -
e) - (Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c^2*d - e))))/(2*e)
```

### Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 303
```
Int[1/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Simp[b/(b
*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x
^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 5509
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*e*(q + 1))), x
] - Simp[b*(c/(2*e*(q + 1)))   Int[(d + e*x^2)^(q + 1)/(1 + c^2*x^2), x], x
] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]
```

## Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.09

| method | result |
|---|---|
| parts | $-\frac{a}{2e(e\,x^2+d)}-\frac{b\,c^2\arctan(cx)}{2e(c^2e\,x^2+c^2d)}+\frac{b\,c^2\arctan(cx)}{2(c^2d-e)e}-\frac{bc\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2(c^2d-e)\sqrt{de}}$ |
| derivativedivides | $\frac{-\frac{a\,c^4}{2e\left(c^2e\,x^2+c^2d\right)}+b\,c^4\left(-\frac{\arctan(cx)}{2e\left(c^2e\,x^2+c^2d\right)}+\frac{-\frac{e\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\left(c^2d-e\right)c\sqrt{de}}+\frac{\arctan(cx)}{c^2d-e}}{2e}\right)}{c^2}$ |
| default | $\frac{-\frac{a\,c^4}{2e\left(c^2e\,x^2+c^2d\right)}+b\,c^4\left(-\frac{\arctan(cx)}{2e\left(c^2e\,x^2+c^2d\right)}+\frac{-\frac{e\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\left(c^2d-e\right)c\sqrt{de}}+\frac{\arctan(cx)}{c^2d-e}}{2e}\right)}{c^2}$ |
| risch | $\frac{ib\ln(icx+1)}{4e(e\,x^2+d)}-\frac{ic^2b\ln\left((-icx+1)^2e-c^2d-2(-icx+1)e+e\right)}{8(c^2d-e)e}-\frac{icb\,\mathrm{arctanh}\left(\frac{2(-icx+1)e-2e}{2c\sqrt{de}}\right)}{4(c^2d-e)\sqrt{de}}-\frac{ic^4b\ln(-icx+1}{4(c^2d-e)(-c^2e\,x^2}$ |

input
```
int(x*(a+b*arctan(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/e/(e*x^2+d)-1/2*b*c^2*arctan(c*x)/e/(c^2*e*x^2+c^2*d)+1/2*b*c^2*arc
tan(c*x)/(c^2*d-e)/e-1/2*b*c/(c^2*d-e)/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))
```

## Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.57

$$\int \frac{x(a+b\arctan(cx))}{(d+ex^2)^2}\,dx$$
$$=\left[-\frac{2\,ac^2d^2-2\,ade-(bcex^2+bcd)\sqrt{-de}\log\left(\frac{ex^2-2\sqrt{-de}x-d}{ex^2+d}\right)-2\left(bc^2dex^2+bde\right)\arctan(cx)}{4\left(c^2d^3e-d^2e^2+(c^2d^2e^2-de^3)x^2\right)},\right.$$
$$\left.-\frac{ac^2d^2-ade+(bcex^2+bcd)\sqrt{de}\arctan\left(\frac{\sqrt{de}x}{d}\right)-\left(bc^2dex^2+bde\right)\arctan(cx)}{2\left(c^2d^3e-d^2e^2+(c^2d^2e^2-de^3)x^2\right)}\right]$$

input
```
integrate(x*(a+b*arctan(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
[-1/4*(2*a*c^2*d^2 - 2*a*d*e - (b*c*e*x^2 + b*c*d)*sqrt(-d*e)*log((e*x^2 -
 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) - 2*(b*c^2*d*e*x^2 + b*d*e)*arctan(c*x))
/(c^2*d^3*e - d^2*e^2 + (c^2*d^2*e^2 - d*e^3)*x^2), -1/2*(a*c^2*d^2 - a*d*
e + (b*c*e*x^2 + b*c*d)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) - (b*c^2*d*e*x^2 +
 b*d*e)*arctan(c*x))/(c^2*d^3*e - d^2*e^2 + (c^2*d^2*e^2 - d*e^3)*x^2)]
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x(a+b\arctan(cx))}{(d+ex^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x*(a+b*atan(c*x))/(e*x**2+d)**2,x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x(a+b\arctan(cx))}{(d+ex^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(a+b*arctan(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

## Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 192, normalized size of antiderivative = 2.11

$$\int \frac{x(a+b\arctan(cx))}{(d+ex^2)^2}\,dx = -\frac{bc\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\left(c^2d-e\right)\sqrt{de}} + \frac{-i\,bc^2ex^2\log\left(i\,cx+1\right)+i\,bc^2ex^2\log\left(-i\,cx+1\right)-2\,bc^2d\arctan\left(cx\right)-i\,bc^2d\log\left(i\,cx+1\right)+i\,bc^2d\log}{4\left(c^2de^2x^2+c^2d^2e-e^3x^2-de^2\right)} - \frac{a}{2\left(e^2x^2+de\right)} - \frac{a}{2\left(ex^2+d\right)e}$$

input

```
integrate(x*(a+b*arctan(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

output

```
-1/2*b*c*arctan(e*x/sqrt(d*e))/((c^2*d - e)*sqrt(d*e)) + 1/4*(-I*b*c^2*e*x
^2*log(I*c*x + 1) + I*b*c^2*e*x^2*log(-I*c*x + 1) - 2*b*c^2*d*arctan(c*x)
- I*b*c^2*d*log(I*c*x + 1) + I*b*c^2*d*log(-I*c*x + 1) - 2*a*c^2*d + 2*b*e
*arctan(c*x) + 2*a*e)/(c^2*d*e^2*x^2 + c^2*d^2*e - e^3*x^2 - d*e^2) - 1/2*
a/(e^2*x^2 + d*e) - 1/2*a/((e*x^2 + d)*e)
```

## Mupad [B] (verification not implemented)

Time = 1.22 (sec) , antiderivative size = 696, normalized size of antiderivative = 7.65

$$\int \frac{x(a+b\arctan(cx))}{(d+ex^2)^2}\,dx = \frac{b\,c\,\ln\left(e\,x+\sqrt{-d\,e}\right)\,\sqrt{-d\,e}}{4\,d\,e^2-4\,c^2\,d^2\,e} - \frac{2\,b\,c^2\,\mathrm{atan}\left(-\frac{\frac{c^2\left(c^8\,e\,x+\frac{c^2\left(2\,c^5\,e^3-4\,c^7\,d\,e^2+2\,c^9\,d^2\,e+\frac{c^2\,x\left(8\,c^{10}\,d^3\,e^2-8\,c^8\,d^2\,e^3-8\,c^6\,d\,e^4+8\,c^4\,e^5\right)1i}{4\,e^2-4\,c^2\,d\,e}\right)1i}{4\,e^2-4\,c^2\,d\,e}\right)}{4\,e^2-4\,c^2\,d\,e}-\frac{c^2\left(-c^8\,e\,x+\frac{c^2\left(2\,c^5\,e^3-\right.}{}\right.}{}}{\frac{c^2\left(c^8\,e\,x+\frac{c^2\left(2\,c^5\,e^3-4\,c^7\,d\,e^2+2\,c^9\,d^2\,e+\frac{c^2\,x\left(8\,c^{10}\,d^3\,e^2-8\,c^8\,d^2\,e^3-8\,c^6\,d\,e^4+8\,c^4\,e^5\right)1i}{4\,e^2-4\,c^2\,d\,e}\right)1i}{4\,e^2-4\,c^2\,d\,e}\right)1i}{4\,e^2-4\,c^2\,d\,e}+\frac{c^2\left(-c^8\,e\,x+\frac{c^2\left(2\,c^5\,e^3-\right.}{}\right.}{}}\right)}{4\,e^2-4\,c^2\,d\,e} - \frac{b\,\mathrm{atan}(c\,x)}{2\,e\,\left(e\,x^2+d\right)} - \frac{b\,c\,\ln\left(e\,x-\sqrt{-d\,e}\right)\,\sqrt{-d\,e}}{4\,\left(d\,e^2-c^2\,d^2\,e\right)} - \frac{a}{2\,e^2\,x^2+2\,d\,e}$$

input
```
int((x*(a + b*atan(c*x)))/(d + e*x^2)^2,x)
```

output
```
(b*c*log(e*x + (-d*e)^(1/2))*(-d*e)^(1/2))/(4*d*e^2 - 4*c^2*d^2*e) - (2*b*
c^2*atan(-((c^2*((c^2*(2*c^5*e^3 - 4*c^7*d*e^2 + 2*c^9*d^2*e + (c^2*x*(8*c
^4*e^5 - 8*c^6*d*e^4 - 8*c^8*d^2*e^3 + 8*c^10*d^3*e^2)*1i)/(4*e^2 - 4*c^2*
d*e))*1i)/(4*e^2 - 4*c^2*d*e) + c^8*e*x))/(4*e^2 - 4*c^2*d*e) - (c^2*((c^2
*(2*c^5*e^3 - 4*c^7*d*e^2 + 2*c^9*d^2*e - (c^2*x*(8*c^4*e^5 - 8*c^6*d*e^4
- 8*c^8*d^2*e^3 + 8*c^10*d^3*e^2)*1i)/(4*e^2 - 4*c^2*d*e))*1i)/(4*e^2 - 4*
c^2*d*e) - c^8*e*x))/(4*e^2 - 4*c^2*d*e))/((c^2*((c^2*(2*c^5*e^3 - 4*c^7*d
*e^2 + 2*c^9*d^2*e + (c^2*x*(8*c^4*e^5 - 8*c^6*d*e^4 - 8*c^8*d^2*e^3 + 8*c
^10*d^3*e^2)*1i)/(4*e^2 - 4*c^2*d*e))*1i)/(4*e^2 - 4*c^2*d*e) + c^8*e*x)*1
i)/(4*e^2 - 4*c^2*d*e) + (c^2*((c^2*(2*c^5*e^3 - 4*c^7*d*e^2 + 2*c^9*d^2*e
 - (c^2*x*(8*c^4*e^5 - 8*c^6*d*e^4 - 8*c^8*d^2*e^3 + 8*c^10*d^3*e^2)*1i)/(
4*e^2 - 4*c^2*d*e))*1i)/(4*e^2 - 4*c^2*d*e) - c^8*e*x)*1i)/(4*e^2 - 4*c^2*
d*e))))/(4*e^2 - 4*c^2*d*e) - (b*atan(c*x))/(2*e*(d + e*x^2)) - (b*c*log(e
*x - (-d*e)^(1/2))*(-d*e)^(1/2))/(4*(d*e^2 - c^2*d^2*e)) - a/(2*d*e + 2*e^
2*x^2)
```

## Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.38

$$\int \frac{x(a + b\arctan(cx))}{(d + ex^2)^2}\,dx$$
$$= \frac{-\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right) bcd - \sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right) bce\,x^2 + atan(cx)\,b\,c^2de\,x^2 + atan(cx)\,bde + a\,c^2de\,x^2 -}{2de\,(c^2de\,x^2 + c^2d^2 - e^2x^2 - de)}$$

input
```
int(x*(a+b*atan(c*x))/(e*x^2+d)^2,x)
```

output
```
( - sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*b*c*d - sqrt(e)*sqrt(d)*
atan((e*x)/(sqrt(e)*sqrt(d)))*b*c*e*x**2 + atan(c*x)*b*c**2*d*e*x**2 + ata
n(c*x)*b*d*e + a*c**2*d*e*x**2 - a*e**2*x**2)/(2*d*e*(c**2*d**2 + c**2*d*e
*x**2 - d*e - e**2*x**2))
```

# 3.1159 $\int \frac{a+b\arctan(cx)}{x(d+ex^2)^2}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 443

$$\begin{aligned}\int \frac{a+b\arctan(cx)}{x\left(d+ex^2\right)^2}\,dx &= -\frac{bc^2\arctan(cx)}{2d\left(c^2d-e\right)}+\frac{a+b\arctan(cx)}{2d\left(d+ex^2\right)}+\frac{bc\sqrt{e}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}\left(c^2d-e\right)}\\
&\quad+\frac{a\log(x)}{d^2}+\frac{(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)}{d^2}\\
&\quad-\frac{(a+b\arctan(cx))\log\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)}{2d^2}\\
&\quad-\frac{(a+b\arctan(cx))\log\left(\frac{2c\left(\sqrt{-d}+\sqrt{e}x\right)}{\left(c\sqrt{-d}+i\sqrt{e}\right)(1-icx)}\right)}{2d^2}\\
&\quad+\frac{ib\operatorname{PolyLog}(2,-icx)}{2d^2}-\frac{ib\operatorname{PolyLog}(2,icx)}{2d^2}\\
&\quad-\frac{ib\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2d^2}\\
&\quad+\frac{ib\operatorname{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)}{4d^2}\\
&\quad+\frac{ib\operatorname{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}+\sqrt{e}x\right)}{\left(c\sqrt{-d}+i\sqrt{e}\right)(1-icx)}\right)}{4d^2}\end{aligned}$$

output
```
-1/2*b*c^2*arctan(c*x)/d/(c^2*d-e)+1/2*(a+b*arctan(c*x))/d/(e*x^2+d)+1/2*b
*c*e^(1/2)*arctan(e^(1/2)*x/d^(1/2))/d^(3/2)/(c^2*d-e)+a*ln(x)/d^2+(a+b*ar
ctan(c*x))*ln(2/(1-I*c*x))/d^2-1/2*(a+b*arctan(c*x))*ln(2*c*((-d)^(1/2)-e^
(1/2)*x)/(c*(-d)^(1/2)-I*e^(1/2))/(1-I*c*x))/d^2-1/2*(a+b*arctan(c*x))*ln(
2*c*((-d)^(1/2)+e^(1/2)*x)/(c*(-d)^(1/2)+I*e^(1/2))/(1-I*c*x))/d^2+1/2*I*b
*polylog(2,-I*c*x)/d^2-1/2*I*b*polylog(2,I*c*x)/d^2-1/2*I*b*polylog(2,1-2/
(1-I*c*x))/d^2+1/4*I*b*polylog(2,1-2*c*((-d)^(1/2)-e^(1/2)*x)/(c*(-d)^(1/2
)-I*e^(1/2))/(1-I*c*x))/d^2+1/4*I*b*polylog(2,1-2*c*((-d)^(1/2)+e^(1/2)*x)
/(c*(-d)^(1/2)+I*e^(1/2))/(1-I*c*x))/d^2
```

## Mathematica [A] (verified)

Time = 4.66 (sec) , antiderivative size = 590, normalized size of antiderivative = 1.33

$$\int \frac{a+b\arctan(cx)}{x\left(d+ex^2\right)^2}\,dx = \frac{2a\left(\frac{d}{d+ex^2}+2\log(x)-\log\left(d+ex^2\right)\right)+b\left(-\frac{2c^2d\arctan(cx)}{c^2d-e}+\frac{2d\arctan(cx)}{d+ex^2}+\frac{2c\sqrt{d}\sqrt{e}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{c^2d-e}+4\arctan(cx)\right.}{}$$

input
```
Integrate[(a + b*ArcTan[c*x])/(x*(d + e*x^2)^2),x]
```

output
```
(2*a*(d/(d + e*x^2) + 2*Log[x] - Log[d + e*x^2]) + b*((-2*c^2*d*ArcTan[c*x
])/(c^2*d - e) + (2*d*ArcTan[c*x])/(d + e*x^2) + (2*c*Sqrt[d]*Sqrt[e]*ArcT
an[(Sqrt[e]*x)/Sqrt[d]])/(c^2*d - e) + 4*ArcTan[c*x]*Log[x] - 2*ArcTan[c*x
]*Log[((-I)*Sqrt[d])/Sqrt[e] + x] - 2*ArcTan[c*x]*Log[(I*Sqrt[d])/Sqrt[e]
+ x] - I*Log[((-I)*Sqrt[d])/Sqrt[e] + x]*Log[(Sqrt[e]*(-1 - I*c*x))/(c*Sqr
t[d] - Sqrt[e])] - (2*I)*Log[x]*Log[1 - I*c*x] + I*Log[((-I)*Sqrt[d])/Sqrt
[e] + x]*Log[(Sqrt[e]*(1 - I*c*x))/(c*Sqrt[d] + Sqrt[e])] + I*Log[(I*Sqrt[
d])/Sqrt[e] + x]*Log[(Sqrt[e]*(-1 + I*c*x))/(c*Sqrt[d] - Sqrt[e])] + (2*I)
*Log[x]*Log[1 + I*c*x] - I*Log[(I*Sqrt[d])/Sqrt[e] + x]*Log[(Sqrt[e]*(1 +
I*c*x))/(c*Sqrt[d] + Sqrt[e])] + (2*I)*PolyLog[2, (-I)*c*x] - (2*I)*PolyLo
g[2, I*c*x] + I*PolyLog[2, (c*(Sqrt[d] - I*Sqrt[e]*x))/(c*Sqrt[d] - Sqrt[e
])] - I*PolyLog[2, (c*(Sqrt[d] - I*Sqrt[e]*x))/(c*Sqrt[d] + Sqrt[e])] - I*
PolyLog[2, (c*(Sqrt[d] + I*Sqrt[e]*x))/(c*Sqrt[d] - Sqrt[e])] + I*PolyLog[
2, (c*(Sqrt[d] + I*Sqrt[e]*x))/(c*Sqrt[d] + Sqrt[e])]))/(4*d^2)
```

## Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x\left(d+ex^2\right)^2}dx$$

$$\Big\downarrow \text{ 5515}$$

$$\int \left(-\frac{ex(a+b\arctan(cx))}{d^2\left(d+ex^2\right)}+\frac{a+b\arctan(cx)}{d^2x}-\frac{ex(a+b\arctan(cx))}{d\left(d+ex^2\right)^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{(a+b\arctan(cx))\log\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{(1-icx)\left(c\sqrt{-d}-i\sqrt{e}\right)}\right)}{2d^2}-\frac{(a+b\arctan(cx))\log\left(\frac{2c\left(\sqrt{-d}+\sqrt{e}x\right)}{(1-icx)\left(c\sqrt{-d}+i\sqrt{e}\right)}\right)}{2d^2}+\frac{\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))}{d^2}+\frac{a+b\arctan(cx)}{2d\left(d+ex^2\right)}+\frac{a\log(x)}{d^2}+\frac{bc\sqrt{e}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}\left(c^2d-e\right)}-\frac{bc^2\arctan(cx)}{2d\left(c^2d-e\right)}+\frac{ib\,\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)}{4d^2}+\frac{ib\,\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{e}x+\sqrt{-d}\right)}{\left(\sqrt{-d}c+i\sqrt{e}\right)(1-icx)}\right)}{4d^2}+\frac{ib\,\text{PolyLog}(2,-icx)}{2d^2}-\frac{ib\,\text{PolyLog}(2,icx)}{2d^2}-\frac{ib\,\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2d^2}$$

input
```
Int[(a + b*ArcTan[c*x])/(x*(d + e*x^2)^2),x]
```

output
```
-1/2*(b*c^2*ArcTan[c*x])/(d*(c^2*d - e)) + (a + b*ArcTan[c*x])/(2*d*(d + e
*x^2)) + (b*c*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*(c^2*d - e))
 + (a*Log[x])/d^2 + ((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/d^2 - ((a + b
*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(
1 - I*c*x))])/(2*d^2) - ((a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] + Sqrt[e]*
x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(2*d^2) + ((I/2)*b*PolyLog[2,
 (-I)*c*x])/d^2 - ((I/2)*b*PolyLog[2, I*c*x])/d^2 - ((I/2)*b*PolyLog[2, 1
- 2/(1 - I*c*x)])/d^2 + ((I/4)*b*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x
))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/d^2 + ((I/4)*b*PolyLog[2, 1 -
(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/d^2
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.60 (sec) , antiderivative size = 808, normalized size of antiderivative = 1.82

| method | result |
|---|---|
| parts | Expression too large to display |
| derivativedivides | Expression too large to display |
| default | Expression too large to display |
| risch | $-\frac{ib\,c^{2} e \ln\left(icx+1\right)}{4d\left(c^{2}d-e\right)\left(-c^{2}e\,x^{2}-c^{2}d\right)}+\frac{ic^{2}be\ln\left(-icx+1\right)}{4d\left(c^{2}d-e\right)\left(-c^{2}e\,x^{2}-c^{2}d\right)}+\frac{ib\ln\left(icx+1\right)\ln\left(\frac{c\sqrt{de}-\left(icx+1\right)e+e}{c\sqrt{de}+e}\right)}{4d^{2}}+\frac{ib\,\mathrm{dilog}\left(icx+1\right.}{2d^{2}}$ |

input
```
int((a+b*arctan(c*x))/x/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/d^2*ln(e*x^2+d)+1/2*a/d/(e*x^2+d)+a*ln(x)/d^2+b*(1/2*c^2*arctan(c*x
)/d/(c^2*e*x^2+c^2*d)-1/2*arctan(c*x)/d^2*ln(c^2*e*x^2+c^2*d)+arctan(c*x)/
d^2*ln(c*x)-1/2*c^4*(1/d/c^2/(c^2*d-e)*arctan(c*x)-1/d/c^3*e/(c^2*d-e)/(d*
e)^(1/2)*arctan(e*x/(d*e)^(1/2))-I/c^4/d^2*ln(c*x)*ln(1+I*c*x)+I/c^4/d^2*l
n(c*x)*ln(1-I*c*x)-I/c^4/d^2*dilog(1+I*c*x)+I/c^4/d^2*dilog(1-I*c*x)-1/d^2
/c^4*(-1/2*I*(ln(c*x-I)*ln(c^2*e*x^2+c^2*d)-2*e*(1/2*ln(c*x-I)*(ln((RootOf
(e*_Z^2+2*I*e*_Z+c^2*d-e,index=1)-c*x+I)/RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,in
dex=1))+ln((RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=2)-c*x+I)/RootOf(e*_Z^2+2
*I*e*_Z+c^2*d-e,index=2)))/e+1/2*(dilog((RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,in
dex=1)-c*x+I)/RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=1))+dilog((RootOf(e*_Z^
2+2*I*e*_Z+c^2*d-e,index=2)-c*x+I)/RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=2)
))/e))+1/2*I*(ln(I+c*x)*ln(c^2*e*x^2+c^2*d)-2*e*(1/2*ln(I+c*x)*(ln((RootOf
(e*_Z^2-2*I*e*_Z+c^2*d-e,index=1)-c*x-I)/RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,in
dex=1))+ln((RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=2)-c*x-I)/RootOf(e*_Z^2-2
*I*e*_Z+c^2*d-e,index=2)))/e+1/2*(dilog((RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,in
dex=1)-c*x-I)/RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=1))+dilog((RootOf(e*_Z^
2-2*I*e*_Z+c^2*d-e,index=2)-c*x-I)/RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=2)
))/e)))))
```

## Fricas [F]

$$\int \frac{a+b\arctan(cx)}{x\left(d+ex^2\right)^2}\,dx=\int \frac{b\arctan\left(cx\right)+a}{\left(ex^2+d\right)^2x}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*arctan(c*x) + a)/(e^2*x^5 + 2*d*e*x^3 + d^2*x), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{x\left(d+ex^2\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*atan(c*x))/x/(e*x**2+d)**2,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{a+b\arctan(cx)}{x\left(d+ex^2\right)^2}\,dx = \int \frac{b\arctan\left(cx\right)+a}{\left(ex^2+d\right)^2x}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
1/2*a*(1/(d*e*x^2 + d^2) - log(e*x^2 + d)/d^2 + 2*log(x)/d^2) + 2*b*integr
ate(1/2*arctan(c*x)/(e^2*x^5 + 2*d*e*x^3 + d^2*x), x)
```

## Giac [F]

$$\int \frac{a+b\arctan(cx)}{x\left(d+ex^2\right)^2}\,dx = \int \frac{b\arctan\left(cx\right)+a}{\left(ex^2+d\right)^2x}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)/((e*x^2 + d)^2*x), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{x\left(d+ex^2\right)^2}\,dx=\int \frac{a+b\,\mathrm{atan}(c\,x)}{x\,\left(e\,x^2+d\right)^2}\,dx$$

input
```
int((a + b*atan(c*x))/(x*(d + e*x^2)^2),x)
```

output
```
int((a + b*atan(c*x))/(x*(d + e*x^2)^2), x)
```

## Reduce [F]

$$\int \frac{a+b\arctan(cx)}{x\left(d+ex^2\right)^2}\,dx = \frac{2\left(\int \frac{atan(cx)}{e^2x^5+2de\,x^3+d^2x}dx\right)b\,d^3+2\left(\int \frac{atan(cx)}{e^2x^5+2de\,x^3+d^2x}dx\right)b\,d^2e\,x^2-\log(e\,x^2+d)\,ad-\log(e\,x^2+d)\,ae\,x^2+2}{2d^2\left(e\,x^2+d\right)}$$

input
```
int((a+b*atan(c*x))/x/(e*x^2+d)^2,x)
```

output
```
(2*int(atan(c*x)/(d**2*x + 2*d*e*x**3 + e**2*x**5),x)*b*d**3 + 2*int(atan(
c*x)/(d**2*x + 2*d*e*x**3 + e**2*x**5),x)*b*d**2*e*x**2 - log(d + e*x**2)*
a*d - log(d + e*x**2)*a*e*x**2 + 2*log(x)*a*d + 2*log(x)*a*e*x**2 - a*e*x*
*2)/(2*d**2*(d + e*x**2))
```

# 3.1160 $\int \frac{a+b\arctan(cx)}{x^3\left(d+ex^2\right)^2}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 489

$$\begin{aligned}\int \frac{a+b\arctan(cx)}{x^3\left(d+ex^2\right)^2}\,dx = & -\frac{bc}{2d^2x}-\frac{bc^2\arctan(cx)}{2d^2}+\frac{bc^2e\arctan(cx)}{2d^2\left(c^2d-e\right)}-\frac{a+b\arctan(cx)}{2d^2x^2}\\ & -\frac{e(a+b\arctan(cx))}{2d^2\left(d+ex^2\right)}-\frac{bce^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{5/2}\left(c^2d-e\right)}\\ & -\frac{2ae\log(x)}{d^3}-\frac{2e(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)}{d^3}\\ & +\frac{e(a+b\arctan(cx))\log\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)}{d^3}\\ & +\frac{e(a+b\arctan(cx))\log\left(\frac{2c\left(\sqrt{-d}+\sqrt{e}x\right)}{\left(c\sqrt{-d}+i\sqrt{e}\right)(1-icx)}\right)}{d^3}\\ & -\frac{ibe\operatorname{PolyLog}(2,-icx)}{d^3}+\frac{ibe\operatorname{PolyLog}(2,icx)}{d^3}\\ & +\frac{ibe\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{d^3}\\ & -\frac{ibe\operatorname{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)}{2d^3}\\ & -\frac{ibe\operatorname{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}+\sqrt{e}x\right)}{\left(c\sqrt{-d}+i\sqrt{e}\right)(1-icx)}\right)}{2d^3}\end{aligned}$$

output
```
-1/2*b*c/d^2/x-1/2*b*c^2*arctan(c*x)/d^2+1/2*b*c^2*e*arctan(c*x)/d^2/(c^2*
d-e)-1/2*(a+b*arctan(c*x))/d^2/x^2-1/2*e*(a+b*arctan(c*x))/d^2/(e*x^2+d)-1
/2*b*c*e^(3/2)*arctan(e^(1/2)*x/d^(1/2))/d^(5/2)/(c^2*d-e)-2*a*e*ln(x)/d^3
-2*e*(a+b*arctan(c*x))*ln(2/(1-I*c*x))/d^3+e*(a+b*arctan(c*x))*ln(2*c*((-d
)^(1/2)-e^(1/2)*x)/(c*(-d)^(1/2)-I*e^(1/2))/(1-I*c*x))/d^3+e*(a+b*arctan(c
*x))*ln(2*c*((-d)^(1/2)+e^(1/2)*x)/(c*(-d)^(1/2)+I*e^(1/2))/(1-I*c*x))/d^3
-I*b*e*polylog(2,-I*c*x)/d^3+I*b*e*polylog(2,I*c*x)/d^3+I*b*e*polylog(2,1-
2/(1-I*c*x))/d^3-1/2*I*b*e*polylog(2,1-2*c*((-d)^(1/2)-e^(1/2)*x)/(c*(-d)^
(1/2)-I*e^(1/2))/(1-I*c*x))/d^3-1/2*I*b*e*polylog(2,1-2*c*((-d)^(1/2)+e^(1
/2)*x)/(c*(-d)^(1/2)+I*e^(1/2))/(1-I*c*x))/d^3
```

## Mathematica [A] (verified)

Time = 9.02 (sec) , antiderivative size = 643, normalized size of antiderivative = 1.31

$$\int \frac{a+b\arctan(cx)}{x^3\left(d+ex^2\right)^2}\,dx =$$

$$-\frac{a\left(d\left(\frac{1}{x^2}+\frac{e}{d+ex^2}\right)+4e\log(x)-2e\log\left(d+ex^2\right)\right)+b\left(\frac{cd}{x}+\frac{c^2d(c^2d-2e)\arctan(cx)}{c^2d-e}+d\left(\frac{1}{x^2}+\frac{e}{d+ex^2}\right)\arctan(\right.}{}$$

input
```
Integrate[(a + b*ArcTan[c*x])/(x^3*(d + e*x^2)^2),x]
```

output
```
-1/2*(a*(d*(x^(-2) + e/(d + e*x^2)) + 4*e*Log[x] - 2*e*Log[d + e*x^2]) + b
*((c*d)/x + (c^2*d*(c^2*d - 2*e)*ArcTan[c*x])/(c^2*d - e) + d*(x^(-2) + e/
(d + e*x^2))*ArcTan[c*x] + (c*Sqrt[d]*e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])
/(c^2*d - e) + 4*e*ArcTan[c*x]*Log[x] - 2*e*ArcTan[c*x]*Log[d + e*x^2] - (
2*I)*e*(Log[x]*(Log[1 - I*c*x] - Log[1 + I*c*x]) - PolyLog[2, (-I)*c*x] +
PolyLog[2, I*c*x]) - e*(2*ArcTan[c*x]*Log[((-I)*Sqrt[d])/Sqrt[e] + x] + 2*
ArcTan[c*x]*Log[(I*Sqrt[d])/Sqrt[e] + x] + I*Log[((-I)*Sqrt[d])/Sqrt[e] +
x]*Log[(Sqrt[e]*(-1 - I*c*x))/(c*Sqrt[d] - Sqrt[e])] - I*Log[((-I)*Sqrt[d]
)/Sqrt[e] + x]*Log[(Sqrt[e]*(1 - I*c*x))/(c*Sqrt[d] + Sqrt[e])] - I*Log[(I
*Sqrt[d])/Sqrt[e] + x]*Log[(Sqrt[e]*(-1 + I*c*x))/(c*Sqrt[d] - Sqrt[e])] +
 I*Log[(I*Sqrt[d])/Sqrt[e] + x]*Log[(Sqrt[e]*(1 + I*c*x))/(c*Sqrt[d] + Sqr
t[e])] - 2*ArcTan[c*x]*Log[d + e*x^2] - I*PolyLog[2, (c*(Sqrt[d] - I*Sqrt[
e]*x))/(c*Sqrt[d] - Sqrt[e])] + I*PolyLog[2, (c*(Sqrt[d] - I*Sqrt[e]*x))/(
c*Sqrt[d] + Sqrt[e])] + I*PolyLog[2, (c*(Sqrt[d] + I*Sqrt[e]*x))/(c*Sqrt[d
] - Sqrt[e])] - I*PolyLog[2, (c*(Sqrt[d] + I*Sqrt[e]*x))/(c*Sqrt[d] + Sqrt
[e])])))/d^3
```

## Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 489, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x^3\left(d+ex^2\right)^2}\,dx$$

$$\Big\downarrow \text{5515}$$

$$\int \left(\frac{2e^2x(a+b\arctan(cx))}{d^3\left(d+ex^2\right)}-\frac{2e(a+b\arctan(cx))}{d^3x}+\frac{e^2x(a+b\arctan(cx))}{d^2\left(d+ex^2\right)^2}+\frac{a+b\arctan(cx)}{d^2x^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2e\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))}{d^3}+\frac{e(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(1-icx)(c\sqrt{-d}-i\sqrt{e})}\right)}{d^3}+\frac{e(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(1-icx)(c\sqrt{-d}+i\sqrt{e})}\right)}{d^3}-\frac{e(a+b\arctan(cx))}{2d^2(d+ex^2)}-\frac{a+b\arctan(cx)}{2d^2x^2}-\frac{2ae\log(x)}{d^3}-\frac{bce^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{5/2}(c^2d-e)}+\frac{bc^2e\arctan(cx)}{2d^2(c^2d-e)}-\frac{bc^2\arctan(cx)}{2d^2}-\frac{ibe\operatorname{PolyLog}(2,-icx)}{d^3}+\frac{ibe\operatorname{PolyLog}(2,icx)}{d^3}+\frac{ibe\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{d^3}-\frac{ibe\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2d^3}-\frac{ibe\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)}{2d^3}-\frac{bc}{2d^2x}$$

input
```
Int[(a + b*ArcTan[c*x])/(x^3*(d + e*x^2)^2),x]
```

output
```
-1/2*(b*c)/(d^2*x) - (b*c^2*ArcTan[c*x])/(2*d^2) + (b*c^2*e*ArcTan[c*x])/(
2*d^2*(c^2*d - e)) - (a + b*ArcTan[c*x])/(2*d^2*x^2) - (e*(a + b*ArcTan[c*
x]))/(2*d^2*(d + e*x^2)) - (b*c*e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^
(5/2)*(c^2*d - e)) - (2*a*e*Log[x])/d^3 - (2*e*(a + b*ArcTan[c*x])*Log[2/(
1 - I*c*x)])/d^3 + (e*(a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))
/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/d^3 + (e*(a + b*ArcTan[c*x])*Log
[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/d^3
 - (I*b*e*PolyLog[2, (-I)*c*x])/d^3 + (I*b*e*PolyLog[2, I*c*x])/d^3 + (I*b
*e*PolyLog[2, 1 - 2/(1 - I*c*x)])/d^3 - ((I/2)*b*e*PolyLog[2, 1 - (2*c*(Sq
rt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/d^3 - ((I/2)
*b*e*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])
*(1 - I*c*x))])/d^3
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.80 (sec) , antiderivative size = 851, normalized size of antiderivative = 1.74

| method | result |
|---|---|
| parts | Expression too large to display |
| derivativedivides | Expression too large to display |
| default | Expression too large to display |
| risch | $\frac{c^2 a e}{2d^2(-c^2 e\,x^2-c^2 d)}-\frac{bc}{2d^2 x}-\frac{ibe\,\mathrm{dilog}(icx+1)}{d^3}-\frac{ibe\,\mathrm{dilog}\left(\frac{c\sqrt{de}-(icx+1)e+e}{c\sqrt{de}+e}\right)}{2d^3}-\frac{ibe\,\mathrm{dilog}\left(\frac{c\sqrt{de}+(icx+1)e-e}{c\sqrt{de}-e}\right)}{2d^3}$ |

input
```
int((a+b*arctan(c*x))/x^3/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
a*e/d^3*ln(e*x^2+d)-1/2*a*e/d^2/(e*x^2+d)-1/2*a/d^2/x^2-2*a*e*ln(x)/d^3+b*
c^2*(-1/2*arctan(c*x)*e/d^2/(c^2*e*x^2+c^2*d)+1/c^2*arctan(c*x)*e/d^3*ln(c
^2*e*x^2+c^2*d)-1/2*arctan(c*x)/d^2/c^2/x^2-2/c^2*arctan(c*x)/d^3*e*ln(c*x
)-1/2*c^4*(-4/d^3/c^6*e*(-1/2*I*ln(c*x)*ln(1+I*c*x)+1/2*I*ln(c*x)*ln(1-I*c
*x)-1/2*I*dilog(1+I*c*x)+1/2*I*dilog(1-I*c*x))+2/d^3/c^6*e*(-1/2*I*(ln(c*x
-I)*ln(c^2*e*x^2+c^2*d)-2*e*(1/2*ln(c*x-I)*(ln((RootOf(e*_Z^2+2*I*e*_Z+c^2
*d-e,index=1)-c*x+I)/RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=1))+ln((RootOf(e
*_Z^2+2*I*e*_Z+c^2*d-e,index=2)-c*x+I)/RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,inde
x=2)))/e+1/2*(dilog((RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=1)-c*x+I)/RootOf
(e*_Z^2+2*I*e*_Z+c^2*d-e,index=1))+dilog((RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,i
ndex=2)-c*x+I)/RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=2)))/e))+1/2*I*(ln(I+c
*x)*ln(c^2*e*x^2+c^2*d)-2*e*(1/2*ln(I+c*x)*(ln((RootOf(e*_Z^2-2*I*e*_Z+c^2
*d-e,index=1)-c*x-I)/RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=1))+ln((RootOf(e
*_Z^2-2*I*e*_Z+c^2*d-e,index=2)-c*x-I)/RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,inde
x=2)))/e+1/2*(dilog((RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=1)-c*x-I)/RootOf
(e*_Z^2-2*I*e*_Z+c^2*d-e,index=1))+dilog((RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,i
ndex=2)-c*x-I)/RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=2)))/e)))-1/d^2/c^4*(1
/(c^2*d-e)*(-c^2*d+2*e)*arctan(c*x)-1/c/x-e^2/(c^2*d-e)/c/(d*e)^(1/2)*arct
an(e*x/(d*e)^(1/2)))))
```

## Fricas [F]

$$\int \frac{a+b\arctan(cx)}{x^3\left(d+ex^2\right)^2}\,dx = \int \frac{b\arctan\left(cx\right)+a}{\left(ex^2+d\right)^2x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^3/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*arctan(c*x) + a)/(e^2*x^7 + 2*d*e*x^5 + d^2*x^3), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{x^3\left(d+ex^2\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*atan(c*x))/x**3/(e*x**2+d)**2,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{a+b\arctan(cx)}{x^3\left(d+ex^2\right)^2}\,dx = \int \frac{b\arctan\left(cx\right)+a}{\left(ex^2+d\right)^2x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^3/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
-1/2*a*((2*e*x^2 + d)/(d^2*e*x^4 + d^3*x^2) - 2*e*log(e*x^2 + d)/d^3 + 4*e
*log(x)/d^3) + 2*b*integrate(1/2*arctan(c*x)/(e^2*x^7 + 2*d*e*x^5 + d^2*x^
3), x)
```

## Giac [F]

$$\int \frac{a+b\arctan(cx)}{x^3\left(d+ex^2\right)^2}\,dx = \int \frac{b\arctan\left(cx\right)+a}{\left(ex^2+d\right)^2x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^3/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)/((e*x^2 + d)^2*x^3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{x^3\left(d+ex^2\right)^2}\,dx = \int \frac{a+b\,\mathrm{atan}(c\,x)}{x^3\left(e\,x^2+d\right)^2}\,dx$$

input
```
int((a + b*atan(c*x))/(x^3*(d + e*x^2)^2),x)
```

output
```
int((a + b*atan(c*x))/(x^3*(d + e*x^2)^2), x)
```

## Reduce [F]

$$\int \frac{a+b\arctan(cx)}{x^3\left(d+ex^2\right)^2}\,dx = \frac{2\left(\int \frac{atan(cx)}{e^2x^7+2de\,x^5+d^2x^3}dx\right)b\,d^4x^2+2\left(\int \frac{atan(cx)}{e^2x^7+2de\,x^5+d^2x^3}dx\right)b\,d^3e\,x^4+2\log(e\,x^2+d)\,ade\,x^2+2\log(e\,x^2+a}{2d^3x^2\left(e\,x^2+d\right)}$$

input
```
int((a+b*atan(c*x))/x^3/(e*x^2+d)^2,x)
```

output
```
(2*int(atan(c*x)/(d**2*x**3 + 2*d*e*x**5 + e**2*x**7),x)*b*d**4*x**2 + 2*i
nt(atan(c*x)/(d**2*x**3 + 2*d*e*x**5 + e**2*x**7),x)*b*d**3*e*x**4 + 2*log
(d + e*x**2)*a*d*e*x**2 + 2*log(d + e*x**2)*a*e**2*x**4 - 4*log(x)*a*d*e*x
**2 - 4*log(x)*a*e**2*x**4 - a*d**2 + 2*a*e**2*x**4)/(2*d**3*x**2*(d + e*x
**2))
```

# 3.1161 $\int \frac{x^2(a+b\arctan(cx))}{(d+ex^2)^2}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 1335

$$\int \frac{x^2(a+b\arctan(cx))}{(d+ex^2)^2}\,dx = \text{Too large to display}$$

output
```
-1/2*x*(a+b*arctan(c*x))/e/(e*x^2+d)+a*arctan(e^(1/2)*x/d^(1/2))/d^(1/2)/e
^(3/2)-1/2*(a+b*arctan(c*x))*arctan(e^(1/2)*x/d^(1/2))/d^(1/2)/e^(3/2)+1/8
*I*b*c*ln(-e^(1/2)*(1+(-c^2)^(1/2)*x)/(I*(-c^2)^(1/2)*d^(1/2)-e^(1/2)))*ln
(1-I*e^(1/2)*x/d^(1/2))/(-c^2)^(1/2)/d^(1/2)/e^(3/2)-1/4*I*b*ln(1+I*c*x)*l
n(c*((-d)^(1/2)-e^(1/2)*x)/(c*(-d)^(1/2)-I*e^(1/2)))/(-d)^(1/2)/e^(3/2)-1/
8*I*b*c*polylog(2,(-c^2)^(1/2)*(d^(1/2)+I*e^(1/2)*x)/((-c^2)^(1/2)*d^(1/2)
-I*e^(1/2)))/(-c^2)^(1/2)/d^(1/2)/e^(3/2)-1/4*I*b*polylog(2,e^(1/2)*(1+I*c
*x)/(I*c*(-d)^(1/2)+e^(1/2)))/(-d)^(1/2)/e^(3/2)+1/4*I*b*polylog(2,e^(1/2)
*(I+c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/(-d)^(1/2)/e^(3/2)+1/4*I*b*ln(1-I*c*x)*
ln(c*((-d)^(1/2)-e^(1/2)*x)/(c*(-d)^(1/2)+I*e^(1/2)))/(-d)^(1/2)/e^(3/2)-1
/4*I*b*polylog(2,e^(1/2)*(1-I*c*x)/(I*c*(-d)^(1/2)+e^(1/2)))/(-d)^(1/2)/e^
(3/2)-1/8*I*b*c*polylog(2,(-c^2)^(1/2)*(d^(1/2)-I*e^(1/2)*x)/((-c^2)^(1/2)
*d^(1/2)-I*e^(1/2)))/(-c^2)^(1/2)/d^(1/2)/e^(3/2)+1/4*b*c*ln(c^2*x^2+1)/(c
^2*d-e)/e-1/4*b*c*ln(e*x^2+d)/(c^2*d-e)/e+1/4*I*b*ln(1+I*c*x)*ln(c*((-d)^(
1/2)+e^(1/2)*x)/(c*(-d)^(1/2)+I*e^(1/2)))/(-d)^(1/2)/e^(3/2)-1/8*I*b*c*ln(
e^(1/2)*(1+(-c^2)^(1/2)*x)/(I*(-c^2)^(1/2)*d^(1/2)+e^(1/2)))*ln(1+I*e^(1/2
)*x/d^(1/2))/(-c^2)^(1/2)/d^(1/2)/e^(3/2)+1/8*I*b*c*polylog(2,(-c^2)^(1/2)
*(d^(1/2)+I*e^(1/2)*x)/((-c^2)^(1/2)*d^(1/2)+I*e^(1/2)))/(-c^2)^(1/2)/d^(1
/2)/e^(3/2)+1/8*I*b*c*ln(-e^(1/2)*(1-(-c^2)^(1/2)*x)/(I*(-c^2)^(1/2)*d^(1/
2)-e^(1/2)))*ln(1+I*e^(1/2)*x/d^(1/2))/(-c^2)^(1/2)/d^(1/2)/e^(3/2)-1/8...
```

## Mathematica [A] (warning: unable to verify)

Time = 8.94 (sec) , antiderivative size = 877, normalized size of antiderivative = 0.66

$$\int \frac{x^2(a+b\arctan(cx))}{(d+ex^2)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(x^2*(a + b*ArcTan[c*x]))/(d + e*x^2)^2,x]
```

output
```
-1/2*(a*x)/(e*(d + e*x^2)) + (a*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*Sqrt[d]*e^
(3/2)) + (b*c*((-2*Log[(c^2*d + e + (c^2*d - e)*Cos[2*ArcTan[c*x]])/(c^2*d
 + e)])/(c^2*d - e) + (-4*ArcTan[c*x]*ArcTanh[Sqrt[-(c^2*d*e)]/(c*e*x)] +
2*ArcCos[(c^2*d + e)/(-(c^2*d) + e)]*ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]] + (
ArcCos[(c^2*d + e)/(-(c^2*d) + e)] - (2*I)*ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)
]])*Log[(-2*c^2*d*(I*e + Sqrt[-(c^2*d*e)])*(-I + c*x))/((c^2*d - e)*(c^2*d
 - c*Sqrt[-(c^2*d*e)]*x))] + (ArcCos[(c^2*d + e)/(-(c^2*d) + e)] + (2*I)*A
rcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]])*Log[((2*I)*c^2*d*(e + I*Sqrt[-(c^2*d*e)]
)*(I + c*x))/((c^2*d - e)*(c^2*d - c*Sqrt[-(c^2*d*e)]*x))] - (ArcCos[(c^2*
d + e)/(-(c^2*d) + e)] - (2*I)*ArcTanh[Sqrt[-(c^2*d*e)]/(c*e*x)] + (2*I)*A
rcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]])*Log[(Sqrt[2]*Sqrt[-(c^2*d*e)])/(Sqrt[-(c
^2*d) + e]*E^(I*ArcTan[c*x])*Sqrt[-(c^2*d) - e + (-(c^2*d) + e)*Cos[2*ArcT
an[c*x]]])] - (ArcCos[(c^2*d + e)/(-(c^2*d) + e)] + (2*I)*ArcTanh[Sqrt[-(c
^2*d*e)]/(c*e*x)] - (2*I)*ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]])*Log[(Sqrt[2]*
Sqrt[-(c^2*d*e)]*E^(I*ArcTan[c*x]))/(Sqrt[-(c^2*d) + e]*Sqrt[-(c^2*d) - e
+ (-(c^2*d) + e)*Cos[2*ArcTan[c*x]]])] + I*(PolyLog[2, ((c^2*d + e - (2*I)
*Sqrt[-(c^2*d*e)])*(c^2*d + c*Sqrt[-(c^2*d*e)]*x))/((c^2*d - e)*(c^2*d - c
*Sqrt[-(c^2*d*e)]*x))] - PolyLog[2, ((c^2*d + e + (2*I)*Sqrt[-(c^2*d*e)])*
(c^2*d + c*Sqrt[-(c^2*d*e)]*x))/((c^2*d - e)*(c^2*d - c*Sqrt[-(c^2*d*e)]*x
))]))/Sqrt[-(c^2*d*e)] - (4*ArcTan[c*x]*Sin[2*ArcTan[c*x]])/(c^2*d + e ...
```

## Rubi [A] (verified)

Time = 2.27 (sec) , antiderivative size = 1335, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\arctan(cx))}{(d+ex^2)^2}\,dx$$

$$\Big\downarrow \text{ 5515}$$

$$\int \left(\frac{a+b\arctan(cx)}{e\,(d+ex^2)}-\frac{d(a+b\arctan(cx))}{e\,(d+ex^2)^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\arctan(cx))}{2\sqrt{d}e^{3/2}}-\frac{x(a+b\arctan(cx))}{2e(ex^2+d)}+\frac{a\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}e^{3/2}}-\frac{ib\log(icx+1)\log\left(\frac{c(\sqrt{-d}-\sqrt{e}x)}{c\sqrt{-d}-i\sqrt{e}}\right)}{4\sqrt{-d}e^{3/2}}+\frac{ib\log(1-icx)\log\left(\frac{c(\sqrt{-d}-\sqrt{e}x)}{\sqrt{-d}c+i\sqrt{e}}\right)}{4\sqrt{-d}e^{3/2}}-\frac{ib\log(1-icx)\log\left(\frac{c(\sqrt{e}x+\sqrt{-d})}{c\sqrt{-d}-i\sqrt{e}}\right)}{4\sqrt{-d}e^{3/2}}+\frac{ib\log(icx+1)\log\left(\frac{c(\sqrt{e}x+\sqrt{-d})}{\sqrt{-d}c+i\sqrt{e}}\right)}{4\sqrt{-d}e^{3/2}}-\frac{ibc\log\left(\frac{\sqrt{e}\left(1-\sqrt{-c^2}x\right)}{i\sqrt{-c^2}\sqrt{d}+\sqrt{e}}\right)\log\left(1-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{8\sqrt{-c^2}\sqrt{d}e^{3/2}}+\frac{ibc\log\left(-\frac{\sqrt{e}\left(\sqrt{-c^2}x+1\right)}{i\sqrt{-c^2}\sqrt{d}-\sqrt{e}}\right)\log\left(1-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{8\sqrt{-c^2}\sqrt{d}e^{3/2}}+\frac{ibc\log\left(-\frac{\sqrt{e}\left(1-\sqrt{-c^2}x\right)}{i\sqrt{-c^2}\sqrt{d}-\sqrt{e}}\right)\log\left(\frac{i\sqrt{e}x}{\sqrt{d}}+1\right)}{8\sqrt{-c^2}\sqrt{d}e^{3/2}}-\frac{ibc\log\left(\frac{\sqrt{e}\left(\sqrt{-c^2}x+1\right)}{i\sqrt{-c^2}\sqrt{d}+\sqrt{e}}\right)\log\left(\frac{i\sqrt{e}x}{\sqrt{d}}+1\right)}{8\sqrt{-c^2}\sqrt{d}e^{3/2}}+\frac{bc\log(c^2x^2+1)}{4(c^2d-e)e}-\frac{bc\log(ex^2+d)}{4(c^2d-e)e}+\frac{ib\,\text{PolyLog}\left(2,\frac{\sqrt{e}(i-cx)}{\sqrt{-d}c+i\sqrt{e}}\right)}{4\sqrt{-d}e^{3/2}}-\frac{ib\,\text{PolyLog}\left(2,\frac{\sqrt{e}(1-icx)}{i\sqrt{-d}c+\sqrt{e}}\right)}{4\sqrt{-d}e^{3/2}}-\frac{ib\,\text{PolyLog}\left(2,\frac{\sqrt{e}(icx+1)}{i\sqrt{-d}c+\sqrt{e}}\right)}{4\sqrt{-d}e^{3/2}}+\frac{ib\,\text{PolyLog}\left(2,\frac{\sqrt{e}(cx+i)}{\sqrt{-d}c+i\sqrt{e}}\right)}{4\sqrt{-d}e^{3/2}}-\frac{ibc\,\text{PolyLog}\left(2,\frac{\sqrt{-c^2}\left(\sqrt{d}-i\sqrt{e}x\right)}{\sqrt{-c^2}\sqrt{d}-i\sqrt{e}}\right)}{8\sqrt{-c^2}\sqrt{d}e^{3/2}}+\frac{ibc\,\text{PolyLog}\left(2,\frac{\sqrt{-c^2}\left(\sqrt{d}-i\sqrt{e}x\right)}{\sqrt{-c^2}\sqrt{d}+i\sqrt{e}}\right)}{8\sqrt{-c^2}\sqrt{d}e^{3/2}}-\frac{ibc\,\text{PolyLog}\left(2,\frac{\sqrt{-c^2}\left(i\sqrt{e}x+\sqrt{d}\right)}{\sqrt{-c^2}\sqrt{d}-i\sqrt{e}}\right)}{8\sqrt{-c^2}\sqrt{d}e^{3/2}}+\frac{ibc\,\text{PolyLog}\left(2,\frac{\sqrt{-c^2}\left(i\sqrt{e}x+\sqrt{d}\right)}{\sqrt{-c^2}\sqrt{d}+i\sqrt{e}}\right)}{8\sqrt{-c^2}\sqrt{d}e^{3/2}}$$

input
```
Int[(x^2*(a + b*ArcTan[c*x]))/(d + e*x^2)^2,x]
```

output
```
-1/2*(x*(a + b*ArcTan[c*x]))/(e*(d + e*x^2)) + (a*ArcTan[(Sqrt[e]*x)/Sqrt[
d]])/(Sqrt[d]*e^(3/2)) - ((a + b*ArcTan[c*x])*ArcTan[(Sqrt[e]*x)/Sqrt[d]])
/(2*Sqrt[d]*e^(3/2)) - ((I/4)*b*Log[1 + I*c*x]*Log[(c*(Sqrt[-d] - Sqrt[e]*
x))/(c*Sqrt[-d] - I*Sqrt[e])])/(Sqrt[-d]*e^(3/2)) + ((I/4)*b*Log[1 - I*c*x
]*Log[(c*(Sqrt[-d] - Sqrt[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])])/(Sqrt[-d]*e^(3
/2)) - ((I/4)*b*Log[1 - I*c*x]*Log[(c*(Sqrt[-d] + Sqrt[e]*x))/(c*Sqrt[-d]
- I*Sqrt[e])])/(Sqrt[-d]*e^(3/2)) + ((I/4)*b*Log[1 + I*c*x]*Log[(c*(Sqrt[-
d] + Sqrt[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])])/(Sqrt[-d]*e^(3/2)) - ((I/8)*b*
c*Log[(Sqrt[e]*(1 - Sqrt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] + Sqrt[e])]*Log[1
 - (I*Sqrt[e]*x)/Sqrt[d]])/(Sqrt[-c^2]*Sqrt[d]*e^(3/2)) + ((I/8)*b*c*Log[-
((Sqrt[e]*(1 + Sqrt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] - Sqrt[e]))]*Log[1 - (
I*Sqrt[e]*x)/Sqrt[d]])/(Sqrt[-c^2]*Sqrt[d]*e^(3/2)) + ((I/8)*b*c*Log[-((Sq
rt[e]*(1 - Sqrt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] - Sqrt[e]))]*Log[1 + (I*Sq
rt[e]*x)/Sqrt[d]])/(Sqrt[-c^2]*Sqrt[d]*e^(3/2)) - ((I/8)*b*c*Log[(Sqrt[e]*
(1 + Sqrt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] + Sqrt[e])]*Log[1 + (I*Sqrt[e]*x
)/Sqrt[d]])/(Sqrt[-c^2]*Sqrt[d]*e^(3/2)) + (b*c*Log[1 + c^2*x^2])/(4*(c^2*
d - e)*e) - (b*c*Log[d + e*x^2])/(4*(c^2*d - e)*e) + ((I/4)*b*PolyLog[2, (
Sqrt[e]*(I - c*x))/(c*Sqrt[-d] + I*Sqrt[e])])/(Sqrt[-d]*e^(3/2)) - ((I/4)*
b*PolyLog[2, (Sqrt[e]*(1 - I*c*x))/(I*c*Sqrt[-d] + Sqrt[e])])/(Sqrt[-d]*e^
(3/2)) - ((I/4)*b*PolyLog[2, (Sqrt[e]*(1 + I*c*x))/(I*c*Sqrt[-d] + Sqrt...
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

## Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2304 vs. 2(991) = 1982.

Time = 3.13 (sec) , antiderivative size = 2305, normalized size of antiderivative = 1.73

| method | result | size |
|---|---|---|
| parts | Expression too large to display | 2305 |
| derivativedivides | Expression too large to display | 2344 |
| default | Expression too large to display | 2344 |
| risch | Expression too large to display | 2391 |

input
```
int(x^2*(a+b*arctan(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
3/4*I*b*c^3*d*ln(1-(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*d*e)^(
1/2)-e))*arctan(c*x)/(c^2*d-e)/e/(c^4*d^2-2*c^2*d*e+e^2)*(c^2*d*e)^(1/2)+1
/4*I*b/c*ln(1-(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*d*e)^(1/2)-
e))*arctan(c*x)/d/(c^2*d-e)/(c^4*d^2-2*c^2*d*e+e^2)*e*(c^2*d*e)^(1/2)-1/4*
I*b*c^5*d^2*ln(1-(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*d*e)^(1/
2)-e))*arctan(c*x)/(c^2*d-e)/e^2/(c^4*d^2-2*c^2*d*e+e^2)*(c^2*d*e)^(1/2)-3
/8*b*c*polylog(2,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*d*e)^(1/
2)-e))/(c^4*d^2-2*c^2*d*e+e^2)/(c^2*d-e)*(c^2*d*e)^(1/2)-1/4*b*(d*e)^(1/2)
/d/e*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)
^(1/2))/(c^2*d-e)+1/2*b*c^2*arctan(c*x)/(c^2*d-e)/(c^2*e*x^2+c^2*d)*x-1/4*
b*c^2*(d*e)^(1/2)/e^2*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c
^2*d+2*e)/c/(d*e)^(1/2))/(c^2*d-e)+b*c^3/(c^2*d-e)^2/e*d*ln((1+I*c*x)/(c^2
*x^2+1)^(1/2))-3/4*b*c*arctan(c*x)^2/(c^4*d^2-2*c^2*d*e+e^2)/(c^2*d-e)*(c^
2*d*e)^(1/2)+1/4*b*c*(c^2*d*e)^(1/2)/(c^2*d-e)/e^2*arctan(c*x)^2+1/8*b*c*(
c^2*d*e)^(1/2)/(c^2*d-e)/e^2*polylog(2,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(
-c^2*d-2*(c^2*d*e)^(1/2)-e))-1/4*b*c^3/(c^2*d-e)^2/e*d*ln(c^2*d*(1+I*c*x)^
4/(c^2*x^2+1)^2+2*c^2*d*(1+I*c*x)^2/(c^2*x^2+1)-e*(1+I*c*x)^4/(c^2*x^2+1)^
2+c^2*d+2*e*(1+I*c*x)^2/(c^2*x^2+1)-e)+1/4*b*c/(c^2*d-e)^2*ln(c^2*d*(1+I*c
*x)^4/(c^2*x^2+1)^2+2*c^2*d*(1+I*c*x)^2/(c^2*x^2+1)-e*(1+I*c*x)^4/(c^2*x^2
+1)^2+c^2*d+2*e*(1+I*c*x)^2/(c^2*x^2+1)-e)-1/4*b*(d*e)^(1/2)/d*arctanh(...
```

## Fricas [F]

$$\int \frac{x^2(a+b\arctan(cx))}{(d+ex^2)^2}\,dx = \int \frac{(b\arctan(cx)+a)x^2}{(ex^2+d)^2}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*x^2*arctan(c*x) + a*x^2)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\arctan(cx))}{(d+ex^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(a+b*atan(c*x))/(e*x**2+d)**2,x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a+b\arctan(cx))}{(d+ex^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(a+b*arctan(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

## Giac [F]

$$\int \frac{x^2(a+b\arctan(cx))}{(d+ex^2)^2}\,dx = \int \frac{(b\arctan(cx)+a)x^2}{(ex^2+d)^2}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)*x^2/(e*x^2 + d)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\arctan(cx))}{(d+ex^2)^2}\,dx = \int \frac{x^2\,(a+b\,\mathrm{atan}(c\,x))}{(e\,x^2+d)^2}\,dx$$

input
```
int((x^2*(a + b*atan(c*x)))/(d + e*x^2)^2,x)
```

output
```
int((x^2*(a + b*atan(c*x)))/(d + e*x^2)^2, x)
```

## Reduce [F]

$$\int \frac{x^2(a+b\arctan(cx))}{(d+ex^2)^2}\,dx = \text{Too large to display}$$

input
```
int(x^2*(a+b*atan(c*x))/(e*x^2+d)^2,x)
```

output

```
(sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*c**4*d**3 + sqrt(e)*sqrt(
d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*c**4*d**2*e*x**2 - sqrt(e)*sqrt(d)*atan
((e*x)/(sqrt(e)*sqrt(d)))*a*d*e**2 - sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*s
qrt(d)))*a*e**3*x**2 + atan(c*x)**2*b*c**3*d**3*e + atan(c*x)**2*b*c**3*d*
*2*e**2*x**2 - atan(c*x)**2*b*c*d**2*e**2 - atan(c*x)**2*b*c*d*e**3*x**2 -
 2*atan(c*x)*b*c**4*d**3*e*x + 2*atan(c*x)*b*c**2*d**2*e**2*x + 2*int((ata
n(c*x)*x**2)/(c**4*d**3*x**2 + 2*c**4*d**2*e*x**4 + c**4*d*e**2*x**6 + c**
2*d**3 + 3*c**2*d**2*e*x**2 + 3*c**2*d*e**2*x**4 + c**2*e**3*x**6 + d**2*e
 + 2*d*e**2*x**2 + e**3*x**4),x)*b*c**8*d**6*e + 2*int((atan(c*x)*x**2)/(c
**4*d**3*x**2 + 2*c**4*d**2*e*x**4 + c**4*d*e**2*x**6 + c**2*d**3 + 3*c**2
*d**2*e*x**2 + 3*c**2*d*e**2*x**4 + c**2*e**3*x**6 + d**2*e + 2*d*e**2*x**
2 + e**3*x**4),x)*b*c**8*d**5*e**2*x**2 - 4*int((atan(c*x)*x**2)/(c**4*d**
3*x**2 + 2*c**4*d**2*e*x**4 + c**4*d*e**2*x**6 + c**2*d**3 + 3*c**2*d**2*e
*x**2 + 3*c**2*d*e**2*x**4 + c**2*e**3*x**6 + d**2*e + 2*d*e**2*x**2 + e**
3*x**4),x)*b*c**6*d**5*e**2 - 4*int((atan(c*x)*x**2)/(c**4*d**3*x**2 + 2*c
**4*d**2*e*x**4 + c**4*d*e**2*x**6 + c**2*d**3 + 3*c**2*d**2*e*x**2 + 3*c*
*2*d*e**2*x**4 + c**2*e**3*x**6 + d**2*e + 2*d*e**2*x**2 + e**3*x**4),x)*b
*c**6*d**4*e**3*x**2 + 4*int((atan(c*x)*x**2)/(c**4*d**3*x**2 + 2*c**4*d**
2*e*x**4 + c**4*d*e**2*x**6 + c**2*d**3 + 3*c**2*d**2*e*x**2 + 3*c**2*d*e*
*2*x**4 + c**2*e**3*x**6 + d**2*e + 2*d*e**2*x**2 + e**3*x**4),x)*b*c**...
```

# 3.1162 $\int \frac{a+b\arctan(cx)}{(d+ex^2)^2}\,dx$

## Optimal result

Integrand size = 18, antiderivative size = 819

$$\int \frac{a+b\arctan(cx)}{(d+ex^2)^2}\,dx = \frac{x(a+b\arctan(cx))}{2d(d+ex^2)} + \frac{(a+b\arctan(cx))\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{3/2}\sqrt{e}}$$
$$+ \frac{ibc\log\left(\frac{\sqrt{e}\left(1-\sqrt{-c^2}x\right)}{i\sqrt{-c^2}\sqrt{d}+\sqrt{e}}\right)\log\left(1-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{8\sqrt{-c^2}d^{3/2}\sqrt{e}}$$
$$- \frac{ibc\log\left(-\frac{\sqrt{e}\left(1+\sqrt{-c^2}x\right)}{i\sqrt{-c^2}\sqrt{d}-\sqrt{e}}\right)\log\left(1-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{8\sqrt{-c^2}d^{3/2}\sqrt{e}}$$
$$- \frac{ibc\log\left(-\frac{\sqrt{e}\left(1-\sqrt{-c^2}x\right)}{i\sqrt{-c^2}\sqrt{d}-\sqrt{e}}\right)\log\left(1+\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{8\sqrt{-c^2}d^{3/2}\sqrt{e}}$$
$$+ \frac{ibc\log\left(\frac{\sqrt{e}\left(1+\sqrt{-c^2}x\right)}{i\sqrt{-c^2}\sqrt{d}+\sqrt{e}}\right)\log\left(1+\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{8\sqrt{-c^2}d^{3/2}\sqrt{e}} - \frac{bc\log(1+c^2x^2)}{4d(c^2d-e)}$$
$$+ \frac{bc\log(d+ex^2)}{4d(c^2d-e)} + \frac{ibc\,\text{PolyLog}\left(2,\frac{\sqrt{-c^2}\left(\sqrt{d}-i\sqrt{e}x\right)}{\sqrt{-c^2}\sqrt{d}-i\sqrt{e}}\right)}{8\sqrt{-c^2}d^{3/2}\sqrt{e}}$$
$$- \frac{ibc\,\text{PolyLog}\left(2,\frac{\sqrt{-c^2}\left(\sqrt{d}-i\sqrt{e}x\right)}{\sqrt{-c^2}\sqrt{d}+i\sqrt{e}}\right)}{8\sqrt{-c^2}d^{3/2}\sqrt{e}}$$
$$+ \frac{ibc\,\text{PolyLog}\left(2,\frac{\sqrt{-c^2}\left(\sqrt{d}+i\sqrt{e}x\right)}{\sqrt{-c^2}\sqrt{d}-i\sqrt{e}}\right)}{8\sqrt{-c^2}d^{3/2}\sqrt{e}}$$
$$- \frac{ibc\,\text{PolyLog}\left(2,\frac{\sqrt{-c^2}\left(\sqrt{d}+i\sqrt{e}x\right)}{\sqrt{-c^2}\sqrt{d}+i\sqrt{e}}\right)}{8\sqrt{-c^2}d^{3/2}\sqrt{e}}$$

output
```
1/2*x*(a+b*arctan(c*x))/d/(e*x^2+d)+1/2*(a+b*arctan(c*x))*arctan(e^(1/2)*x
/d^(1/2))/d^(3/2)/e^(1/2)+1/8*I*b*c*ln(e^(1/2)*(1-(-c^2)^(1/2)*x)/(I*(-c^2
)^(1/2)*d^(1/2)+e^(1/2)))*ln(1-I*e^(1/2)*x/d^(1/2))/(-c^2)^(1/2)/d^(3/2)/e
^(1/2)-1/8*I*b*c*ln(-e^(1/2)*(1+(-c^2)^(1/2)*x)/(I*(-c^2)^(1/2)*d^(1/2)-e^
(1/2)))*ln(1-I*e^(1/2)*x/d^(1/2))/(-c^2)^(1/2)/d^(3/2)/e^(1/2)-1/8*I*b*c*l
n(-e^(1/2)*(1-(-c^2)^(1/2)*x)/(I*(-c^2)^(1/2)*d^(1/2)-e^(1/2)))*ln(1+I*e^(
1/2)*x/d^(1/2))/(-c^2)^(1/2)/d^(3/2)/e^(1/2)+1/8*I*b*c*ln(e^(1/2)*(1+(-c^2
)^(1/2)*x)/(I*(-c^2)^(1/2)*d^(1/2)+e^(1/2)))*ln(1+I*e^(1/2)*x/d^(1/2))/(-c
^2)^(1/2)/d^(3/2)/e^(1/2)-1/4*b*c*ln(c^2*x^2+1)/d/(c^2*d-e)+1/4*b*c*ln(e*x
^2+d)/d/(c^2*d-e)+1/8*I*b*c*polylog(2,(-c^2)^(1/2)*(d^(1/2)-I*e^(1/2)*x)/(
(-c^2)^(1/2)*d^(1/2)-I*e^(1/2)))/(-c^2)^(1/2)/d^(3/2)/e^(1/2)-1/8*I*b*c*po
lylog(2,(-c^2)^(1/2)*(d^(1/2)-I*e^(1/2)*x)/((-c^2)^(1/2)*d^(1/2)+I*e^(1/2)
))/(-c^2)^(1/2)/d^(3/2)/e^(1/2)+1/8*I*b*c*polylog(2,(-c^2)^(1/2)*(d^(1/2)+
I*e^(1/2)*x)/((-c^2)^(1/2)*d^(1/2)-I*e^(1/2)))/(-c^2)^(1/2)/d^(3/2)/e^(1/2
)-1/8*I*b*c*polylog(2,(-c^2)^(1/2)*(d^(1/2)+I*e^(1/2)*x)/((-c^2)^(1/2)*d^(
1/2)+I*e^(1/2)))/(-c^2)^(1/2)/d^(3/2)/e^(1/2)
```

## Mathematica [A] (warning: unable to verify)

Time = 8.99 (sec) , antiderivative size = 861, normalized size of antiderivative = 1.05

$$\int \frac{a+b\arctan(cx)}{(d+ex^2)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*ArcTan[c*x])/(d + e*x^2)^2,x]
```

output
```
(a*x)/(2*d*(d + e*x^2)) + (a*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*Sqrt[
e]) + (b*c*((2*Log[1 + ((c^2*d - e)*Cos[2*ArcTan[c*x]])/(c^2*d + e)])/(c^2
*d - e) + (-4*ArcTan[c*x]*ArcTanh[Sqrt[-(c^2*d*e)]/(c*e*x)] + 2*ArcCos[-((
c^2*d + e)/(c^2*d - e))]*ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]] - (ArcCos[-((c^
2*d + e)/(c^2*d - e))] + (2*I)*ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]])*Log[(2*c
^2*d*((-I)*e + Sqrt[-(c^2*d*e)])*(-I + c*x))/((c^2*d - e)*(c^2*d + c*Sqrt[
-(c^2*d*e)]*x))] - (ArcCos[-((c^2*d + e)/(c^2*d - e))] - (2*I)*ArcTanh[(c*
e*x)/Sqrt[-(c^2*d*e)]])*Log[(2*c^2*d*(I*e + Sqrt[-(c^2*d*e)])*(I + c*x))/(
(c^2*d - e)*(c^2*d + c*Sqrt[-(c^2*d*e)]*x))] + (ArcCos[-((c^2*d + e)/(c^2*
d - e))] - (2*I)*(ArcTanh[(c*d)/(Sqrt[-(c^2*d*e)]*x)] + ArcTanh[(c*e*x)/Sq
rt[-(c^2*d*e)]]))*Log[(Sqrt[2]*Sqrt[-(c^2*d*e)])/(Sqrt[c^2*d - e]*E^(I*Arc
Tan[c*x])*Sqrt[c^2*d + e + (c^2*d - e)*Cos[2*ArcTan[c*x]]])] + (ArcCos[-((
c^2*d + e)/(c^2*d - e))] + (2*I)*(ArcTanh[(c*d)/(Sqrt[-(c^2*d*e)]*x)] + Ar
cTanh[(c*e*x)/Sqrt[-(c^2*d*e)]]))*Log[(Sqrt[2]*Sqrt[-(c^2*d*e)]*E^(I*ArcTa
n[c*x]))/(Sqrt[c^2*d - e]*Sqrt[c^2*d + e + (c^2*d - e)*Cos[2*ArcTan[c*x]]]
)] + I*(PolyLog[2, ((c^2*d + e - (2*I)*Sqrt[-(c^2*d*e)])*(c^2*d - c*Sqrt[-
(c^2*d*e)]*x))/((c^2*d - e)*(c^2*d + c*Sqrt[-(c^2*d*e)]*x))] - PolyLog[2,
((c^2*d + e + (2*I)*Sqrt[-(c^2*d*e)])*(c^2*d - c*Sqrt[-(c^2*d*e)]*x))/((c^
2*d - e)*(c^2*d + c*Sqrt[-(c^2*d*e)]*x))]))/Sqrt[-(c^2*d*e)] + (4*ArcTan[c
*x]*Sin[2*ArcTan[c*x]])/(c^2*d + e + (c^2*d - e)*Cos[2*ArcTan[c*x]])))/...
```

## Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 806, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5447, 27, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{(d+ex^2)^2}\,dx$$

$$\Big\downarrow \text{5447}$$

$$-bc\int \frac{\frac{x}{d(ex^2+d)}+\frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}\sqrt{e}}}{2\left(c^2x^2+1\right)}dx+\frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\arctan(cx))}{2d^{3/2}\sqrt{e}}+\frac{x(a+b\arctan(cx))}{2d\left(d+ex^2\right)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{2}bc\int \frac{\frac{x}{d(ex^2+d)}+\frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}\sqrt{e}}}{c^2x^2+1}dx+\frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\arctan(cx))}{2d^{3/2}\sqrt{e}}+\frac{x(a+b\arctan(cx))}{2d(d+ex^2)}$$

$$\Big\downarrow \text{7276}$$

$$-\frac{1}{2}bc\int \left(\frac{x}{d(c^2x^2+1)(ex^2+d)}+\frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}\sqrt{e}(c^2x^2+1)}\right)dx+\frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\arctan(cx))}{2d^{3/2}\sqrt{e}}+\frac{x(a+b\arctan(cx))}{2d(d+ex^2)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\arctan(cx))}{2d^{3/2}\sqrt{e}}+\frac{x(a+b\arctan(cx))}{2d(ex^2+d)}-\frac{1}{2}bc\left(-\frac{i\log\left(\frac{\sqrt{e}(1-\sqrt{-c^2}x)}{i\sqrt{-c^2}\sqrt{d}+\sqrt{e}}\right)\log\left(1-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{4\sqrt{-c^2}d^{3/2}\sqrt{e}}+\frac{i\log\left(-\frac{\sqrt{e}(\sqrt{-c^2}x+1)}{i\sqrt{-c^2}\sqrt{d}-\sqrt{e}}\right)\log\left(1-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{4\sqrt{-c^2}d^{3/2}\sqrt{e}}+\frac{i\log\left(-\frac{\sqrt{e}(1-\sqrt{-c^2}x)}{i\sqrt{-c^2}\sqrt{d}-\sqrt{e}}\right)}{4\sqrt{-c^2}d^{3/2}}\right.$$

input
```
Int[(a + b*ArcTan[c*x])/(d + e*x^2)^2,x]
```

output
```
(x*(a + b*ArcTan[c*x]))/(2*d*(d + e*x^2)) + ((a + b*ArcTan[c*x])*ArcTan[(S
qrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*Sqrt[e]) - (b*c*(((-1/4*I)*Log[(Sqrt[e]*(1
 - Sqrt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] + Sqrt[e])]*Log[1 - (I*Sqrt[e]*x)/S
qrt[d]])/(Sqrt[-c^2]*d^(3/2)*Sqrt[e]) + ((I/4)*Log[-((Sqrt[e]*(1 + Sqrt[-c
^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] - Sqrt[e]))]*Log[1 - (I*Sqrt[e]*x)/Sqrt[d]])
/(Sqrt[-c^2]*d^(3/2)*Sqrt[e]) + ((I/4)*Log[-((Sqrt[e]*(1 - Sqrt[-c^2]*x))/
(I*Sqrt[-c^2]*Sqrt[d] - Sqrt[e]))]*Log[1 + (I*Sqrt[e]*x)/Sqrt[d]])/(Sqrt[-
c^2]*d^(3/2)*Sqrt[e]) - ((I/4)*Log[(Sqrt[e]*(1 + Sqrt[-c^2]*x))/(I*Sqrt[-c
^2]*Sqrt[d] + Sqrt[e])]*Log[1 + (I*Sqrt[e]*x)/Sqrt[d]])/(Sqrt[-c^2]*d^(3/2
)*Sqrt[e]) + Log[1 + c^2*x^2]/(2*d*(c^2*d - e)) - Log[d + e*x^2]/(2*d*(c^2
*d - e)) - ((I/4)*PolyLog[2, (Sqrt[-c^2]*(Sqrt[d] - I*Sqrt[e]*x))/(Sqrt[-c
^2]*Sqrt[d] - I*Sqrt[e])])/(Sqrt[-c^2]*d^(3/2)*Sqrt[e]) + ((I/4)*PolyLog[2
, (Sqrt[-c^2]*(Sqrt[d] - I*Sqrt[e]*x))/(Sqrt[-c^2]*Sqrt[d] + I*Sqrt[e])])/
(Sqrt[-c^2]*d^(3/2)*Sqrt[e]) - ((I/4)*PolyLog[2, (Sqrt[-c^2]*(Sqrt[d] + I*
Sqrt[e]*x))/(Sqrt[-c^2]*Sqrt[d] - I*Sqrt[e])])/(Sqrt[-c^2]*d^(3/2)*Sqrt[e]
) + ((I/4)*PolyLog[2, (Sqrt[-c^2]*(Sqrt[d] + I*Sqrt[e]*x))/(Sqrt[-c^2]*Sqr
t[d] + I*Sqrt[e])])/(Sqrt[-c^2]*d^(3/2)*Sqrt[e])))/2
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5447
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Simp[(a + b*ArcTan[c*x])   u,
x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2), x], x], x]] /; FreeQ
[{a, b, c, d, e}, x] && (IntegerQ[q] || ILtQ[q + 1/2, 0])
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

## Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2172 vs. 2(611) = 1222.

Time = 2.15 (sec) , antiderivative size = 2173, normalized size of antiderivative = 2.65

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 2173 |
| parts | Expression too large to display | 2305 |
| derivativedivides | Expression too large to display | 2320 |
| default | Expression too large to display | 2320 |

input
```
int((a+b*arctan(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*I*c^4*b*ln(1-I*c*x)/(c^2*d-e)/(-c^2*e*x^2-c^2*d)*x+1/8*b/d/(d*e)^(1/2
)*dilog((c*(d*e)^(1/2)-(1-I*c*x)*e+e)/(c*(d*e)^(1/2)+e))-1/8*b/d/(d*e)^(1/
2)*dilog((c*(d*e)^(1/2)+(1-I*c*x)*e-e)/(c*(d*e)^(1/2)-e))+1/8*b*c/d/(c^2*d
-e)*ln((1+I*c*x)^2*e-c^2*d-2*(1+I*c*x)*e+e)+1/4*b*c^2/(c^2*d-e)/(d*e)^(1/2
)*arctanh(1/2*(2*(1+I*c*x)*e-2*e)/c/(d*e)^(1/2))+1/4*b*c^3*ln(1+I*c*x)/(c^
2*d-e)/(-c^2*e*x^2-c^2*d)+1/4*I*b*c^4*ln(1+I*c*x)/(c^2*d-e)/(-c^2*e*x^2-c^
2*d)*x+1/4*c^3*b*ln(1-I*c*x)/(c^2*d-e)/(-c^2*e*x^2-c^2*d)-1/2*c^2*a/d/(-c^
2*e*x^2-c^2*d)*x+1/8*c*b/d/(c^2*d-e)*ln((1-I*c*x)^2*e-c^2*d-2*(1-I*c*x)*e+
e)+1/4*c^2*b/(c^2*d-e)/(d*e)^(1/2)*arctanh(1/2*(2*(1-I*c*x)*e-2*e)/c/(d*e)
^(1/2))+1/2*I*a/d/(d*e)^(1/2)*arctanh(1/2*(2*(1-I*c*x)*e-2*e)/c/(d*e)^(1/2
))+1/8*b*c^2*ln(1+I*c*x)/d/(c^2*d-e)/(-c^2*e*x^2-c^2*d)/(d*e)^(1/2)*ln((c*
(d*e)^(1/2)-(1+I*c*x)*e+e)/(c*(d*e)^(1/2)+e))*e^2*x^2-1/8*b*c^2*ln(1+I*c*x
)/d/(c^2*d-e)/(-c^2*e*x^2-c^2*d)/(d*e)^(1/2)*ln((c*(d*e)^(1/2)+(1+I*c*x)*e
-e)/(c*(d*e)^(1/2)-e))*e^2*x^2+1/4*b*c^3*ln(1+I*c*x)/d/(c^2*d-e)/(-c^2*e*x
^2-c^2*d)*e*x^2-1/8*b*c^4*ln(1+I*c*x)*d/(c^2*d-e)/(-c^2*e*x^2-c^2*d)/(d*e)
^(1/2)*ln((c*(d*e)^(1/2)-(1+I*c*x)*e+e)/(c*(d*e)^(1/2)+e))+1/8*b*c^4*ln(1+
I*c*x)*d/(c^2*d-e)/(-c^2*e*x^2-c^2*d)/(d*e)^(1/2)*ln((c*(d*e)^(1/2)+(1+I*c
*x)*e-e)/(c*(d*e)^(1/2)-e))+1/8*b*c^2*ln(1+I*c*x)/(c^2*d-e)/(-c^2*e*x^2-c^
2*d)/(d*e)^(1/2)*ln((c*(d*e)^(1/2)-(1+I*c*x)*e+e)/(c*(d*e)^(1/2)+e))*e-1/8
*b*c^2*ln(1+I*c*x)/(c^2*d-e)/(-c^2*e*x^2-c^2*d)/(d*e)^(1/2)*ln((c*(d*e)...
```

## Fricas [F]

$$\int \frac{a+b\arctan(cx)}{(d+ex^2)^2}\,dx=\int \frac{b\arctan(cx)+a}{(ex^2+d)^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*arctan(c*x) + a)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{\left(d+ex^2\right)^2}\,dx=\text{Timed out}$$

input
```
integrate((a+b*atan(c*x))/(e*x**2+d)**2,x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\arctan(cx)}{\left(d+ex^2\right)^2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*arctan(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

## Giac [F]

$$\int \frac{a+b\arctan(cx)}{\left(d+ex^2\right)^2}\,dx=\int \frac{b\arctan\left(cx\right)+a}{\left(ex^2+d\right)^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)/(e*x^2 + d)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{(d+ex^2)^2}\,dx = \int \frac{a+b\,\mathrm{atan}(c\,x)}{(e\,x^2+d)^2}\,dx$$

input
```
int((a + b*atan(c*x))/(d + e*x^2)^2,x)
```

output
```
int((a + b*atan(c*x))/(d + e*x^2)^2, x)
```

## Reduce [F]

$$\int \frac{a+b\arctan(cx)}{(d+ex^2)^2}\,dx = \text{Too large to display}$$

input
```
int((a+b*atan(c*x))/(e*x^2+d)^2,x)
```

output
```
(sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*c**4*d**3 + sqrt(e)*sqrt(
d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*c**4*d**2*e*x**2 - sqrt(e)*sqrt(d)*atan
((e*x)/(sqrt(e)*sqrt(d)))*a*d*e**2 - sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*s
qrt(d)))*a*e**3*x**2 + atan(c*x)**2*b*c**3*d**3*e + atan(c*x)**2*b*c**3*d*
*2*e**2*x**2 - atan(c*x)**2*b*c*d**2*e**2 - atan(c*x)**2*b*c*d*e**3*x**2 +
 2*atan(c*x)*b*c**2*d**2*e**2*x - 2*atan(c*x)*b*d*e**3*x + 2*int((atan(c*x
)*x**2)/(c**4*d**3*x**2 + 2*c**4*d**2*e*x**4 + c**4*d*e**2*x**6 + c**2*d**
3 + 3*c**2*d**2*e*x**2 + 3*c**2*d*e**2*x**4 + c**2*e**3*x**6 + d**2*e + 2*
d*e**2*x**2 + e**3*x**4),x)*b*c**8*d**6*e + 2*int((atan(c*x)*x**2)/(c**4*d
**3*x**2 + 2*c**4*d**2*e*x**4 + c**4*d*e**2*x**6 + c**2*d**3 + 3*c**2*d**2
*e*x**2 + 3*c**2*d*e**2*x**4 + c**2*e**3*x**6 + d**2*e + 2*d*e**2*x**2 + e
**3*x**4),x)*b*c**8*d**5*e**2*x**2 - 4*int((atan(c*x)*x**2)/(c**4*d**3*x**
2 + 2*c**4*d**2*e*x**4 + c**4*d*e**2*x**6 + c**2*d**3 + 3*c**2*d**2*e*x**2
 + 3*c**2*d*e**2*x**4 + c**2*e**3*x**6 + d**2*e + 2*d*e**2*x**2 + e**3*x**
4),x)*b*c**6*d**5*e**2 - 4*int((atan(c*x)*x**2)/(c**4*d**3*x**2 + 2*c**4*d
**2*e*x**4 + c**4*d*e**2*x**6 + c**2*d**3 + 3*c**2*d**2*e*x**2 + 3*c**2*d*
e**2*x**4 + c**2*e**3*x**6 + d**2*e + 2*d*e**2*x**2 + e**3*x**4),x)*b*c**6
*d**4*e**3*x**2 + 4*int((atan(c*x)*x**2)/(c**4*d**3*x**2 + 2*c**4*d**2*e*x
**4 + c**4*d*e**2*x**6 + c**2*d**3 + 3*c**2*d**2*e*x**2 + 3*c**2*d*e**2*x*
*4 + c**2*e**3*x**6 + d**2*e + 2*d*e**2*x**2 + e**3*x**4),x)*b*c**2*d**...
```

# 3.1163 $\int \frac{a+b\arctan(cx)}{x^2(d+ex^2)^2}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 1382

$$\int \frac{a+b\arctan(cx)}{x^2(d+ex^2)^2}\,dx = \text{Too large to display}$$

output
```
-(a+b*arctan(c*x))/d^2/x-1/2*e*x*(a+b*arctan(c*x))/d^2/(e*x^2+d)-a*e^(1/2)
*arctan(e^(1/2)*x/d^(1/2))/d^(5/2)-1/2*e^(1/2)*(a+b*arctan(c*x))*arctan(e^
(1/2)*x/d^(1/2))/d^(5/2)+b*c*ln(x)/d^2+1/8*I*b*c*e^(1/2)*polylog(2,(-c^2)^
(1/2)*(d^(1/2)+I*e^(1/2)*x)/((-c^2)^(1/2)*d^(1/2)+I*e^(1/2)))/(-c^2)^(1/2)
/d^(5/2)-1/8*I*b*c*e^(1/2)*polylog(2,(-c^2)^(1/2)*(d^(1/2)+I*e^(1/2)*x)/((
-c^2)^(1/2)*d^(1/2)-I*e^(1/2)))/(-c^2)^(1/2)/d^(5/2)+1/4*I*b*e^(1/2)*ln(1-
I*c*x)*ln(c*((-d)^(1/2)+e^(1/2)*x)/(c*(-d)^(1/2)-I*e^(1/2)))/(-d)^(5/2)+1/
8*I*b*c*e^(1/2)*ln(-e^(1/2)*(1+(-c^2)^(1/2)*x)/(I*(-c^2)^(1/2)*d^(1/2)-e^(
1/2)))*ln(1-I*e^(1/2)*x/d^(1/2))/(-c^2)^(1/2)/d^(5/2)-1/8*I*b*c*e^(1/2)*po
lylog(2,(-c^2)^(1/2)*(d^(1/2)-I*e^(1/2)*x)/((-c^2)^(1/2)*d^(1/2)-I*e^(1/2)
))/(-c^2)^(1/2)/d^(5/2)+1/8*I*b*c*e^(1/2)*polylog(2,(-c^2)^(1/2)*(d^(1/2)-
I*e^(1/2)*x)/((-c^2)^(1/2)*d^(1/2)+I*e^(1/2)))/(-c^2)^(1/2)/d^(5/2)-1/8*I*
b*c*e^(1/2)*ln(e^(1/2)*(1-(-c^2)^(1/2)*x)/(I*(-c^2)^(1/2)*d^(1/2)+e^(1/2))
)*ln(1-I*e^(1/2)*x/d^(1/2))/(-c^2)^(1/2)/d^(5/2)+1/4*I*b*e^(1/2)*ln(1+I*c*
x)*ln(c*((-d)^(1/2)-e^(1/2)*x)/(c*(-d)^(1/2)-I*e^(1/2)))/(-d)^(5/2)-1/2*b*
c*ln(c^2*x^2+1)/d^2+1/4*b*c*e*ln(c^2*x^2+1)/d^2/(c^2*d-e)-1/4*b*c*e*ln(e*x
^2+d)/d^2/(c^2*d-e)+1/8*I*b*c*e^(1/2)*ln(-e^(1/2)*(1-(-c^2)^(1/2)*x)/(I*(-
c^2)^(1/2)*d^(1/2)-e^(1/2)))*ln(1+I*e^(1/2)*x/d^(1/2))/(-c^2)^(1/2)/d^(5/2
)-1/4*I*b*e^(1/2)*ln(1-I*c*x)*ln(c*((-d)^(1/2)-e^(1/2)*x)/(c*(-d)^(1/2)+I*
e^(1/2)))/(-d)^(5/2)+1/4*I*b*e^(1/2)*polylog(2,e^(1/2)*(1-I*c*x)/(I*c*(...
```

## Mathematica [A] (verified)

Time = 12.68 (sec) , antiderivative size = 992, normalized size of antiderivative = 0.72

$$\int \frac{a+b\arctan(cx)}{x^2(d+ex^2)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*ArcTan[c*x])/(x^2*(d + e*x^2)^2),x]
```

output
```
-(a/(d^2*x)) - (a*e*x)/(2*d^2*(d + e*x^2)) - (3*a*Sqrt[e]*ArcTan[(Sqrt[e]*
x)/Sqrt[d]])/(2*d^(5/2)) + b*c^5*(-(ArcTan[c*x]/(c^5*d^2*x)) + Log[(c*x)/S
qrt[1 + c^2*x^2]]/(c^4*d^2) - (e*Log[1 - ((-(c^2*d) + e)*Cos[2*ArcTan[c*x]
])/(c^2*d + e)])/(4*c^4*d^2*(c^2*d - e)) - (3*e*(4*ArcTan[c*x]*ArcTanh[(c*
d)/(Sqrt[-(c^2*d*e)]*x)] + 2*ArcCos[(-(c^2*d) - e)/(c^2*d - e)]*ArcTanh[(c
*e*x)/Sqrt[-(c^2*d*e)]] - (ArcCos[(-(c^2*d) - e)/(c^2*d - e)] - (2*I)*ArcT
anh[(c*e*x)/Sqrt[-(c^2*d*e)]])*Log[1 - ((c^2*d + e - (2*I)*Sqrt[-(c^2*d*e)
])*(2*c^2*d - 2*c*Sqrt[-(c^2*d*e)]*x))/((c^2*d - e)*(2*c^2*d + 2*c*Sqrt[-(
c^2*d*e)]*x))] + (-ArcCos[(-(c^2*d) - e)/(c^2*d - e)] - (2*I)*ArcTanh[(c*e
*x)/Sqrt[-(c^2*d*e)]])*Log[1 - ((c^2*d + e + (2*I)*Sqrt[-(c^2*d*e)])*(2*c^
2*d - 2*c*Sqrt[-(c^2*d*e)]*x))/((c^2*d - e)*(2*c^2*d + 2*c*Sqrt[-(c^2*d*e)
]*x))] + (ArcCos[(-(c^2*d) - e)/(c^2*d - e)] - (2*I)*(ArcTanh[(c*d)/(Sqrt[
-(c^2*d*e)]*x)] + ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]]))*Log[(Sqrt[2]*Sqrt[-(
c^2*d*e)])/(Sqrt[c^2*d - e]*E^(I*ArcTan[c*x])*Sqrt[c^2*d + e + (c^2*d - e)
*Cos[2*ArcTan[c*x]]])] + (ArcCos[(-(c^2*d) - e)/(c^2*d - e)] + (2*I)*(ArcT
anh[(c*d)/(Sqrt[-(c^2*d*e)]*x)] + ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]]))*Log[
(Sqrt[2]*Sqrt[-(c^2*d*e)]*E^(I*ArcTan[c*x]))/(Sqrt[c^2*d - e]*Sqrt[c^2*d +
 e + (c^2*d - e)*Cos[2*ArcTan[c*x]]])] + I*(PolyLog[2, ((c^2*d + e - (2*I)
*Sqrt[-(c^2*d*e)])*(2*c^2*d - 2*c*Sqrt[-(c^2*d*e)]*x))/((c^2*d - e)*(2*c^2
*d + 2*c*Sqrt[-(c^2*d*e)]*x))] - PolyLog[2, ((c^2*d + e + (2*I)*Sqrt[-(...
```

## Rubi [A] (verified)

Time = 2.06 (sec) , antiderivative size = 1382, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x^2\,(d+ex^2)^2}\,dx$$

$$\Big\downarrow \; 5515$$

$$\int \left(-\frac{e(a+b\arctan(cx))}{d^2\,(d+ex^2)}+\frac{a+b\arctan(cx)}{d^2x^2}-\frac{e(a+b\arctan(cx))}{d\,(d+ex^2)^2}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{\sqrt{e}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\arctan(cx))}{2d^{5/2}}-\frac{a+b\arctan(cx)}{d^2x}-\frac{ex(a+b\arctan(cx))}{2d^2\left(ex^2+d\right)}-\frac{a\sqrt{e}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2}}+\frac{bc\log(x)}{d^2}+\frac{ib\sqrt{e}\log(icx+1)\log\left(\frac{c\left(\sqrt{-d}-\sqrt{e}x\right)}{c\sqrt{-d}-i\sqrt{e}}\right)}{4(-d)^{5/2}}-\frac{ib\sqrt{e}\log(1-icx)\log\left(\frac{c\left(\sqrt{-d}-\sqrt{e}x\right)}{\sqrt{-d}c+i\sqrt{e}}\right)}{4(-d)^{5/2}}+\frac{ib\sqrt{e}\log(1-icx)\log\left(\frac{c\left(\sqrt{e}x+\sqrt{-d}\right)}{c\sqrt{-d}-i\sqrt{e}}\right)}{4(-d)^{5/2}}-\frac{ib\sqrt{e}\log(icx+1)\log\left(\frac{c\left(\sqrt{e}x+\sqrt{-d}\right)}{\sqrt{-d}c+i\sqrt{e}}\right)}{4(-d)^{5/2}}-\frac{ibc\sqrt{e}\log\left(\frac{\sqrt{e}\left(1-\sqrt{-c^2}x\right)}{i\sqrt{-c^2}\sqrt{d}+\sqrt{e}}\right)\log\left(1-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{8\sqrt{-c^2}d^{5/2}}+\frac{ibc\sqrt{e}\log\left(-\frac{\sqrt{e}\left(\sqrt{-c^2}x+1\right)}{i\sqrt{-c^2}\sqrt{d}-\sqrt{e}}\right)\log\left(1-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{8\sqrt{-c^2}d^{5/2}}+\frac{ibc\sqrt{e}\log\left(-\frac{\sqrt{e}\left(1-\sqrt{-c^2}x\right)}{i\sqrt{-c^2}\sqrt{d}-\sqrt{e}}\right)\log\left(\frac{i\sqrt{e}x}{\sqrt{d}}+1\right)}{8\sqrt{-c^2}d^{5/2}}-\frac{ibc\sqrt{e}\log\left(\frac{\sqrt{e}\left(\sqrt{-c^2}x+1\right)}{i\sqrt{-c^2}\sqrt{d}+\sqrt{e}}\right)\log\left(\frac{i\sqrt{e}x}{\sqrt{d}}+1\right)}{8\sqrt{-c^2}d^{5/2}}+\frac{bce\log\left(c^2x^2+1\right)}{4d^2\left(c^2d-e\right)}-\frac{bc\log\left(c^2x^2+1\right)}{2d^2}-\frac{bce\log\left(ex^2+d\right)}{4d^2\left(c^2d-e\right)}-\frac{ib\sqrt{e}\operatorname{PolyLog}\left(2,\frac{\sqrt{e}(i-cx)}{\sqrt{-d}c+i\sqrt{e}}\right)}{4(-d)^{5/2}}+\frac{ib\sqrt{e}\operatorname{PolyLog}\left(2,\frac{\sqrt{e}(1-icx)}{i\sqrt{-d}c+\sqrt{e}}\right)}{4(-d)^{5/2}}+\frac{ib\sqrt{e}\operatorname{PolyLog}\left(2,\frac{\sqrt{e}(icx+1)}{i\sqrt{-d}c+\sqrt{e}}\right)}{4(-d)^{5/2}}-\frac{ib\sqrt{e}\operatorname{PolyLog}\left(2,\frac{\sqrt{e}(cx+i)}{\sqrt{-d}c+i\sqrt{e}}\right)}{4(-d)^{5/2}}-\frac{ibc\sqrt{e}\operatorname{PolyLog}\left(2,\frac{\sqrt{-c^2}\left(\sqrt{d}-i\sqrt{e}x\right)}{\sqrt{-c^2}\sqrt{d}-i\sqrt{e}}\right)}{8\sqrt{-c^2}d^{5/2}}+\frac{ibc\sqrt{e}\operatorname{PolyLog}\left(2,\frac{\sqrt{-c^2}\left(\sqrt{d}-i\sqrt{e}x\right)}{\sqrt{-c^2}\sqrt{d}+i\sqrt{e}}\right)}{8\sqrt{-c^2}d^{5/2}}-\frac{ibc\sqrt{e}\operatorname{PolyLog}\left(2,\frac{\sqrt{-c^2}\left(i\sqrt{e}x+\sqrt{d}\right)}{\sqrt{-c^2}\sqrt{d}-i\sqrt{e}}\right)}{8\sqrt{-c^2}d^{5/2}}+\frac{ibc\sqrt{e}\operatorname{PolyLog}\left(2,\frac{\sqrt{-c^2}\left(i\sqrt{e}x+\sqrt{d}\right)}{\sqrt{-c^2}\sqrt{d}+i\sqrt{e}}\right)}{8\sqrt{-c^2}d^{5/2}}$$

input
```
Int[(a + b*ArcTan[c*x])/(x^2*(d + e*x^2)^2),x]
```

output

```
-((a + b*ArcTan[c*x])/(d^2*x)) - (e*x*(a + b*ArcTan[c*x]))/(2*d^2*(d + e*x
^2)) - (a*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/d^(5/2) - (Sqrt[e]*(a + b*A
rcTan[c*x])*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(5/2)) + (b*c*Log[x])/d^2 +
((I/4)*b*Sqrt[e]*Log[1 + I*c*x]*Log[(c*(Sqrt[-d] - Sqrt[e]*x))/(c*Sqrt[-d]
 - I*Sqrt[e])])/(-d)^(5/2) - ((I/4)*b*Sqrt[e]*Log[1 - I*c*x]*Log[(c*(Sqrt[
-d] - Sqrt[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])])/(-d)^(5/2) + ((I/4)*b*Sqrt[e]
*Log[1 - I*c*x]*Log[(c*(Sqrt[-d] + Sqrt[e]*x))/(c*Sqrt[-d] - I*Sqrt[e])])/
(-d)^(5/2) - ((I/4)*b*Sqrt[e]*Log[1 + I*c*x]*Log[(c*(Sqrt[-d] + Sqrt[e]*x)
)/(c*Sqrt[-d] + I*Sqrt[e])])/(-d)^(5/2) - ((I/8)*b*c*Sqrt[e]*Log[(Sqrt[e]*
(1 - Sqrt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] + Sqrt[e])]*Log[1 - (I*Sqrt[e]*x
)/Sqrt[d]])/(Sqrt[-c^2]*d^(5/2)) + ((I/8)*b*c*Sqrt[e]*Log[-((Sqrt[e]*(1 +
Sqrt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] - Sqrt[e]))]*Log[1 - (I*Sqrt[e]*x)/Sq
rt[d]])/(Sqrt[-c^2]*d^(5/2)) + ((I/8)*b*c*Sqrt[e]*Log[-((Sqrt[e]*(1 - Sqrt
[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] - Sqrt[e]))]*Log[1 + (I*Sqrt[e]*x)/Sqrt[d
]])/(Sqrt[-c^2]*d^(5/2)) - ((I/8)*b*c*Sqrt[e]*Log[(Sqrt[e]*(1 + Sqrt[-c^2]
*x))/(I*Sqrt[-c^2]*Sqrt[d] + Sqrt[e])]*Log[1 + (I*Sqrt[e]*x)/Sqrt[d]])/(Sq
rt[-c^2]*d^(5/2)) - (b*c*Log[1 + c^2*x^2])/(2*d^2) + (b*c*e*Log[1 + c^2*x^
2])/(4*d^2*(c^2*d - e)) - (b*c*e*Log[d + e*x^2])/(4*d^2*(c^2*d - e)) - ((I
/4)*b*Sqrt[e]*PolyLog[2, (Sqrt[e]*(I - c*x))/(c*Sqrt[-d] + I*Sqrt[e])])/(-
d)^(5/2) + ((I/4)*b*Sqrt[e]*PolyLog[2, (Sqrt[e]*(1 - I*c*x))/(I*c*Sqrt[...
```

### Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

## Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2567 vs. $2(1036) = 2072$.

Time = 1.86 (sec) , antiderivative size = 2568, normalized size of antiderivative = 1.86

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 2568 |
| parts | Expression too large to display | 3558 |
| derivativedivides | Expression too large to display | 3591 |
| default | Expression too large to display | 3591 |

input
```
int((a+b*arctan(c*x))/x^2/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-3/8*b/d^2*e/(d*e)^(1/2)*dilog((c*(d*e)^(1/2)-(1+I*c*x)*e+e)/(c*(d*e)^(1/2
)+e))+3/8*b/d^2*e/(d*e)^(1/2)*dilog((c*(d*e)^(1/2)+(1+I*c*x)*e-e)/(c*(d*e)
^(1/2)-e))+1/2*I*b/d^2*ln(1+I*c*x)/x-1/4*c^3*b/d*e*ln(1-I*c*x)/(c^2*d-e)/(
-c^2*e*x^2-c^2*d)-1/4*c^2*b/d*e/(c^2*d-e)/(d*e)^(1/2)*arctanh(1/2*(2*(1-I*
c*x)*e-2*e)/c/(d*e)^(1/2))+1/8*c^2*b/d*e^2*ln(1-I*c*x)/(c^2*d-e)/(-c^2*e*x
^2-c^2*d)/(d*e)^(1/2)*ln((c*(d*e)^(1/2)+(1-I*c*x)*e-e)/(c*(d*e)^(1/2)-e))+
1/4*I*c^4*b*ln(1-I*c*x)/d/(c^2*d-e)/(-c^2*e*x^2-c^2*d)*e*x-1/4*I*c^2*b/d^2
*e^2*ln(1-I*c*x)/(c^2*d-e)/(-c^2*e*x^2-c^2*d)*x-1/8*c^2*b/d*e^2*ln(1-I*c*x
)/(c^2*d-e)/(-c^2*e*x^2-c^2*d)/(d*e)^(1/2)*ln((c*(d*e)^(1/2)-(1-I*c*x)*e+e
)/(c*(d*e)^(1/2)+e))-1/8*b*c^2/d*e^2*ln(1+I*c*x)/(c^2*d-e)/(-c^2*e*x^2-c^2
*d)/(d*e)^(1/2)*ln((c*(d*e)^(1/2)-(1+I*c*x)*e+e)/(c*(d*e)^(1/2)+e))-1/4*c^
3*b/d^2*e^2*ln(1-I*c*x)/(c^2*d-e)/(-c^2*e*x^2-c^2*d)*x^2+1/8*c^4*b*ln(1-I*
c*x)/(c^2*d-e)/(-c^2*e*x^2-c^2*d)/(d*e)^(1/2)*ln((c*(d*e)^(1/2)-(1-I*c*x)*
e+e)/(c*(d*e)^(1/2)+e))*e-1/8*c^4*b*ln(1-I*c*x)/(c^2*d-e)/(-c^2*e*x^2-c^2*
d)/(d*e)^(1/2)*ln((c*(d*e)^(1/2)+(1-I*c*x)*e-e)/(c*(d*e)^(1/2)-e))*e-1/4*b
*c^3/d^2*e^2*ln(1+I*c*x)/(c^2*d-e)/(-c^2*e*x^2-c^2*d)*x^2+1/8*b*c^4*e*ln(1
+I*c*x)/(c^2*d-e)/(-c^2*e*x^2-c^2*d)/(d*e)^(1/2)*ln((c*(d*e)^(1/2)-(1+I*c*
x)*e+e)/(c*(d*e)^(1/2)+e))-1/8*b*c^4*e*ln(1+I*c*x)/(c^2*d-e)/(-c^2*e*x^2-c
^2*d)/(d*e)^(1/2)*ln((c*(d*e)^(1/2)+(1+I*c*x)*e-e)/(c*(d*e)^(1/2)-e))-3/2*
I*a/d^2*e/(d*e)^(1/2)*arctanh(1/2*(2*(1-I*c*x)*e-2*e)/c/(d*e)^(1/2))-1/...
```

## Fricas [F]

$$\int \frac{a + b \arctan(cx)}{x^2 (d + ex^2)^2} \, dx = \int \frac{b \arctan(cx) + a}{(ex^2 + d)^2 x^2} \, dx$$

input
```
integrate((a+b*arctan(c*x))/x^2/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*arctan(c*x) + a)/(e^2*x^6 + 2*d*e*x^4 + d^2*x^2), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{a + b \arctan(cx)}{x^2 (d + ex^2)^2} \, dx = \text{Timed out}$$

input
```
integrate((a+b*atan(c*x))/x**2/(e*x**2+d)**2,x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b \arctan(cx)}{x^2 (d + ex^2)^2} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arctan(c*x))/x^2/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

## Giac [F]

$$\int \frac{a+b\arctan(cx)}{x^2(d+ex^2)^2}\,dx=\int \frac{b\arctan(cx)+a}{(ex^2+d)^2x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^2/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)/((e*x^2 + d)^2*x^2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{x^2(d+ex^2)^2}\,dx=\int \frac{a+b\,\mathrm{atan}(c\,x)}{x^2\,(e\,x^2+d)^2}\,dx$$

input
```
int((a + b*atan(c*x))/(x^2*(d + e*x^2)^2),x)
```

output
```
int((a + b*atan(c*x))/(x^2*(d + e*x^2)^2), x)
```

## Reduce [F]

$$\int \frac{a+b\arctan(cx)}{x^2(d+ex^2)^2}\,dx=\text{too large to display}$$

input
```
int((a+b*atan(c*x))/x^2/(e*x^2+d)^2,x)
```

output

```
( - 9*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*c**4*d**3*x - 9*sqrt
(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*c**4*d**2*e*x**3 + 12*sqrt(e)*
sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*c**2*d**2*e*x + 12*sqrt(e)*sqrt(d)
*atan((e*x)/(sqrt(e)*sqrt(d)))*a*c**2*d*e**2*x**3 - 3*sqrt(e)*sqrt(d)*atan
((e*x)/(sqrt(e)*sqrt(d)))*a*d*e**2*x - 3*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(
e)*sqrt(d)))*a*e**3*x**3 - 3*atan(c*x)**2*b*c**5*d**4*x - 3*atan(c*x)**2*b
*c**5*d**3*e*x**3 + 3*atan(c*x)**2*b*c**3*d**3*e*x + 3*atan(c*x)**2*b*c**3
*d**2*e**2*x**3 - 6*atan(c*x)*b*c**4*d**4 - 6*atan(c*x)*b*c**4*d**3*e*x**2
 + 8*atan(c*x)*b*c**2*d**3*e + 6*atan(c*x)*b*c**2*d**2*e**2*x**2 - 2*atan(
c*x)*b*d**2*e**2 + 18*int(atan(c*x)/(3*c**4*d**3*x**2 + 6*c**4*d**2*e*x**4
 + 3*c**4*d*e**2*x**6 + 3*c**2*d**3 + 5*c**2*d**2*e*x**2 + c**2*d*e**2*x**
4 - c**2*e**3*x**6 - d**2*e - 2*d*e**2*x**2 - e**3*x**4),x)*b*c**8*d**7*x
+ 18*int(atan(c*x)/(3*c**4*d**3*x**2 + 6*c**4*d**2*e*x**4 + 3*c**4*d*e**2*
x**6 + 3*c**2*d**3 + 5*c**2*d**2*e*x**2 + c**2*d*e**2*x**4 - c**2*e**3*x**
6 - d**2*e - 2*d*e**2*x**2 - e**3*x**4),x)*b*c**8*d**6*e*x**3 - 60*int(ata
n(c*x)/(3*c**4*d**3*x**2 + 6*c**4*d**2*e*x**4 + 3*c**4*d*e**2*x**6 + 3*c**
2*d**3 + 5*c**2*d**2*e*x**2 + c**2*d*e**2*x**4 - c**2*e**3*x**6 - d**2*e -
 2*d*e**2*x**2 - e**3*x**4),x)*b*c**6*d**6*e*x - 60*int(atan(c*x)/(3*c**4*
d**3*x**2 + 6*c**4*d**2*e*x**4 + 3*c**4*d*e**2*x**6 + 3*c**2*d**3 + 5*c**2
*d**2*e*x**2 + c**2*d*e**2*x**4 - c**2*e**3*x**6 - d**2*e - 2*d*e**2*x*...
```

# 3.1164 $\int \frac{x^5(a+b\arctan(cx))}{(d+ex^2)^3}\,dx$

## Optimal result

Integrand size = 21, antiderivative size = 532

$$\int \frac{x^5(a+b\arctan(cx))}{(d+ex^2)^3}\,dx = -\frac{bcdx}{8(c^2d-e)e^2(d+ex^2)}+\frac{bc^4d^2\arctan(cx)}{4(c^2d-e)^2e^3}-\frac{bc^2d\arctan(cx)}{(c^2d-e)e^3}-\frac{d^2(a+b\arctan(cx))}{4e^3(d+ex^2)^2}+\frac{d(a+b\arctan(cx))}{e^3(d+ex^2)}+\frac{bc\sqrt{d}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{(c^2d-e)e^{5/2}}-\frac{bc\sqrt{d}(3c^2d-e)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8(c^2d-e)^2e^{5/2}}-\frac{(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)}{e^3}+\frac{(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2e^3}+\frac{(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{2e^3}+\frac{ib\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2e^3}-\frac{ib\,\mathrm{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{4e^3}-\frac{ib\,\mathrm{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{4e^3}$$

output
```
-1/8*b*c*d*x/(c^2*d-e)/e^2/(e*x^2+d)+1/4*b*c^4*d^2*arctan(c*x)/(c^2*d-e)^2
/e^3-b*c^2*d*arctan(c*x)/(c^2*d-e)/e^3-1/4*d^2*(a+b*arctan(c*x))/e^3/(e*x^
2+d)^2+d*(a+b*arctan(c*x))/e^3/(e*x^2+d)+b*c*d^(1/2)*arctan(e^(1/2)*x/d^(1
/2))/(c^2*d-e)/e^(5/2)-1/8*b*c*d^(1/2)*(3*c^2*d-e)*arctan(e^(1/2)*x/d^(1/2
))/(c^2*d-e)^2/e^(5/2)-(a+b*arctan(c*x))*ln(2/(1-I*c*x))/e^3+1/2*(a+b*arct
an(c*x))*ln(2*c*((-d)^(1/2)-e^(1/2)*x)/(c*(-d)^(1/2)-I*e^(1/2))/(1-I*c*x))
/e^3+1/2*(a+b*arctan(c*x))*ln(2*c*((-d)^(1/2)+e^(1/2)*x)/(c*(-d)^(1/2)+I*e
^(1/2))/(1-I*c*x))/e^3+1/2*I*b*polylog(2,1-2/(1-I*c*x))/e^3-1/4*I*b*polylo
g(2,1-2*c*((-d)^(1/2)-e^(1/2)*x)/(c*(-d)^(1/2)-I*e^(1/2))/(1-I*c*x))/e^3-1
/4*I*b*polylog(2,1-2*c*((-d)^(1/2)+e^(1/2)*x)/(c*(-d)^(1/2)+I*e^(1/2))/(1-
I*c*x))/e^3
```

## Mathematica [A] (verified)

Time = 8.93 (sec) , antiderivative size = 589, normalized size of antiderivative = 1.11

$$\int \frac{x^5(a+b\arctan(cx))}{(d+ex^2)^3}\,dx$$

$$= \frac{a\left(\frac{d(3d+4ex^2)}{(d+ex^2)^2}+2\log(d+ex^2)\right)+b\left(-\frac{cdex}{2(c^2d-e)(d+ex^2)}+\frac{c^2d(-3c^2d+4e)\arctan(cx)}{(-c^2d+e)^2}+\frac{d(3d+4ex^2)\arctan(cx)}{(d+ex^2)^2}+\frac{c\sqrt{d}(5c}{}\right.}{}$$

input
```
Integrate[(x^5*(a + b*ArcTan[c*x]))/(d + e*x^2)^3,x]
```

output
```
(a*((d*(3*d + 4*e*x^2))/(d + e*x^2)^2 + 2*Log[d + e*x^2]) + b*(-1/2*(c*d*e
*x)/((c^2*d - e)*(d + e*x^2)) + (c^2*d*(-3*c^2*d + 4*e)*ArcTan[c*x])/(-(c^
2*d) + e)^2 + (d*(3*d + 4*e*x^2)*ArcTan[c*x])/(d + e*x^2)^2 + (c*Sqrt[d]*(
5*c^2*d - 7*e)*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*(-(c^2*d) + e)^2) +
 2*ArcTan[c*x]*Log[((-I)*Sqrt[d])/Sqrt[e] + x] + 2*ArcTan[c*x]*Log[(I*Sqrt
[d])/Sqrt[e] + x] + I*Log[((-I)*Sqrt[d])/Sqrt[e] + x]*Log[(Sqrt[e]*(-1 - I
*c*x))/(c*Sqrt[d] - Sqrt[e])] - I*Log[((-I)*Sqrt[d])/Sqrt[e] + x]*Log[(Sqr
t[e]*(1 - I*c*x))/(c*Sqrt[d] + Sqrt[e])] - I*Log[(I*Sqrt[d])/Sqrt[e] + x]*
Log[(Sqrt[e]*(-1 + I*c*x))/(c*Sqrt[d] - Sqrt[e])] + I*Log[(I*Sqrt[d])/Sqrt
[e] + x]*Log[(Sqrt[e]*(1 + I*c*x))/(c*Sqrt[d] + Sqrt[e])] - I*PolyLog[2, (
c*(Sqrt[d] - I*Sqrt[e]*x))/(c*Sqrt[d] - Sqrt[e])] + I*PolyLog[2, (c*(Sqrt[
d] - I*Sqrt[e]*x))/(c*Sqrt[d] + Sqrt[e])] + I*PolyLog[2, (c*(Sqrt[d] + I*S
qrt[e]*x))/(c*Sqrt[d] - Sqrt[e])] - I*PolyLog[2, (c*(Sqrt[d] + I*Sqrt[e]*x
))/(c*Sqrt[d] + Sqrt[e])]))/(4*e^3)
```

## Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 532, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(a+b\arctan(cx))}{(d+ex^2)^3}\,dx$$

$$\Big\downarrow \text{5515}$$

$$\int \left(\frac{d^2x(a+b\arctan(cx))}{e^2(d+ex^2)^3}-\frac{2dx(a+b\arctan(cx))}{e^2(d+ex^2)^2}+\frac{x(a+b\arctan(cx))}{e^2(d+ex^2)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d^2(a+b\arctan(cx))}{4e^3(d+ex^2)^2}+\frac{d(a+b\arctan(cx))}{e^3(d+ex^2)}+\frac{(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(1-icx)(c\sqrt{-d}-i\sqrt{e})}\right)}{2e^3}+\frac{(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(1-icx)(c\sqrt{-d}+i\sqrt{e})}\right)}{2e^3}-\frac{\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))}{e^3}-\frac{bc\sqrt{d}(3c^2d-e)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8e^{5/2}(c^2d-e)^2}+\frac{bc\sqrt{d}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{e^{5/2}(c^2d-e)}-\frac{bc^2d\arctan(cx)}{e^3(c^2d-e)}+\frac{bc^4d^2\arctan(cx)}{4e^3(c^2d-e)^2}-\frac{bcdx}{8e^2(c^2d-e)(d+ex^2)}-\frac{ib\,\text{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{4e^3}-\frac{ib\,\text{PolyLog}\left(2,1-\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)}{4e^3}+\frac{ib\,\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2e^3}$$

input
```
Int[(x^5*(a + b*ArcTan[c*x]))/(d + e*x^2)^3,x]
```

output
```
-1/8*(b*c*d*x)/((c^2*d - e)*e^2*(d + e*x^2)) + (b*c^4*d^2*ArcTan[c*x])/(4*
(c^2*d - e)^2*e^3) - (b*c^2*d*ArcTan[c*x])/((c^2*d - e)*e^3) - (d^2*(a + b
*ArcTan[c*x]))/(4*e^3*(d + e*x^2)^2) + (d*(a + b*ArcTan[c*x]))/(e^3*(d + e
*x^2)) + (b*c*Sqrt[d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/((c^2*d - e)*e^(5/2)) -
 (b*c*Sqrt[d]*(3*c^2*d - e)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*(c^2*d - e)^2*
e^(5/2)) - ((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/e^3 + ((a + b*ArcTan[c
*x])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x
))])/(2*e^3) + ((a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*S
qrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(2*e^3) + ((I/2)*b*PolyLog[2, 1 - 2/(1
 - I*c*x)])/e^3 - ((I/4)*b*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c
*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/e^3 - ((I/4)*b*PolyLog[2, 1 - (2*c*(
Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/e^3
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.77 (sec) , antiderivative size = 817, normalized size of antiderivative = 1.54

| method | result | size |
|---|---|---|
| parts | Expression too large to display | 817 |
| derivativedivides | Expression too large to display | 843 |
| default | Expression too large to display | 843 |
| risch | Expression too large to display | 1666 |

input
```
int(x^5*(a+b*arctan(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
a*(-1/4*d^2/e^3/(e*x^2+d)^2+1/e^3*d/(e*x^2+d)+1/2/e^3*ln(e*x^2+d))+b/c^6*(
arctan(c*x)*c^8*d/e^3/(c^2*e*x^2+c^2*d)+1/2*arctan(c*x)*c^6/e^3*ln(c^2*e*x
^2+c^2*d)-1/4*arctan(c*x)*c^10*d^2/e^3/(c^2*e*x^2+c^2*d)^2-1/4*c^6*(c^2*d/
e^3*((3*c^2*d-4*e)/(c^2*d-e)^2*arctan(c*x)-1/(c^2*d-e)^2*e*((-1/2*c^2*d+1/
2*e)*c*x/(c^2*e*x^2+c^2*d)+1/2*(5*c^2*d-7*e)/c/(d*e)^(1/2)*arctan(e*x/(d*e
)^(1/2))))+2/e^3*(-1/2*I*(ln(c*x-I)*ln(c^2*e*x^2+c^2*d)-2*e*(1/2*ln(c*x-I)
*(ln((RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=1)-c*x+I)/RootOf(e*_Z^2+2*I*e*_
Z+c^2*d-e,index=1))+ln((RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=2)-c*x+I)/Roo
tOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=2)))/e+1/2*(dilog((RootOf(e*_Z^2+2*I*e*_
Z+c^2*d-e,index=1)-c*x+I)/RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=1))+dilog((
RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=2)-c*x+I)/RootOf(e*_Z^2+2*I*e*_Z+c^2*
d-e,index=2)))/e))+1/2*I*(ln(I+c*x)*ln(c^2*e*x^2+c^2*d)-2*e*(1/2*ln(I+c*x)
*(ln((RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=1)-c*x-I)/RootOf(e*_Z^2-2*I*e*_
Z+c^2*d-e,index=1))+ln((RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=2)-c*x-I)/Roo
tOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=2)))/e+1/2*(dilog((RootOf(e*_Z^2-2*I*e*_
Z+c^2*d-e,index=1)-c*x-I)/RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=1))+dilog((
RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=2)-c*x-I)/RootOf(e*_Z^2-2*I*e*_Z+c^2*
d-e,index=2)))/e)))))
```

## Fricas [F]

$$\int \frac{x^5(a+b\arctan(cx))}{(d+ex^2)^3}\,dx = \int \frac{(b\arctan{(cx)}+a)x^5}{(ex^2+d)^3}\,dx$$

input
```
integrate(x^5*(a+b*arctan(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral((b*x^5*arctan(c*x) + a*x^5)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2
+ d^3), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\arctan(cx))}{(d+ex^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**5*(a+b*atan(c*x))/(e*x**2+d)**3,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{x^5(a+b\arctan(cx))}{(d+ex^2)^3}\,dx = \int \frac{(b\arctan(cx)+a)x^5}{(ex^2+d)^3}\,dx$$

input
```
integrate(x^5*(a+b*arctan(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
1/4*a*((4*d*e*x^2 + 3*d^2)/(e^5*x^4 + 2*d*e^4*x^2 + d^2*e^3) + 2*log(e*x^2
 + d)/e^3) + 2*b*integrate(1/2*x^5*arctan(c*x)/(e^3*x^6 + 3*d*e^2*x^4 + 3*
d^2*e*x^2 + d^3), x)
```

## Giac [F]

$$\int \frac{x^5(a+b\arctan(cx))}{(d+ex^2)^3}\,dx = \int \frac{(b\arctan(cx)+a)x^5}{(ex^2+d)^3}\,dx$$

input
```
integrate(x^5*(a+b*arctan(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)*x^5/(e*x^2 + d)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(a+b\arctan(cx))}{(d+ex^2)^3}\,dx=\int \frac{x^5\,(a+b\,\mathrm{atan}(c\,x))}{{(e\,x^2+d)}^3}\,dx$$

input
```
int((x^5*(a + b*atan(c*x)))/(d + e*x^2)^3,x)
```

output
```
int((x^5*(a + b*atan(c*x)))/(d + e*x^2)^3, x)
```

## Reduce [F]

$$\int \frac{x^5(a+b\arctan(cx))}{(d+ex^2)^3}\,dx$$
$$=\frac{4\left(\int \frac{atan(cx)x^5}{e^3x^6+3d\,e^2x^4+3d^2e\,x^2+d^3}dx\right)b\,d^2e^3+8\left(\int \frac{atan(cx)x^5}{e^3x^6+3d\,e^2x^4+3d^2e\,x^2+d^3}dx\right)bd\,e^4x^2+4\left(\int \frac{atan(cx)x^5}{e^3x^6+3d\,e^2x^4+3d^2e\,x^2+d^3}d\right.}{4e^3\left(e^2x^4+2de\right.}$$

input
```
int(x^5*(a+b*atan(c*x))/(e*x^2+d)^3,x)
```

output
```
(4*int((atan(c*x)*x**5)/(d**3 + 3*d**2*e*x**2 + 3*d*e**2*x**4 + e**3*x**6)
,x)*b*d**2*e**3 + 8*int((atan(c*x)*x**5)/(d**3 + 3*d**2*e*x**2 + 3*d*e**2*
x**4 + e**3*x**6),x)*b*d*e**4*x**2 + 4*int((atan(c*x)*x**5)/(d**3 + 3*d**2
*e*x**2 + 3*d*e**2*x**4 + e**3*x**6),x)*b*e**5*x**4 + 2*log(d + e*x**2)*a*
d**2 + 4*log(d + e*x**2)*a*d*e*x**2 + 2*log(d + e*x**2)*a*e**2*x**4 + a*d*
*2 - 2*a*e**2*x**4)/(4*e**3*(d**2 + 2*d*e*x**2 + e**2*x**4))
```

# 3.1165 $\int \frac{x^3(a+b\arctan(cx))}{(d+ex^2)^3}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 130

$$\int \frac{x^3(a+b\arctan(cx))}{(d+ex^2)^3}\,dx = \frac{bcx}{8(c^2d-e)\,e\,(d+ex^2)} - \frac{b\arctan(cx)}{4d\,(c^2d-e)^2} + \frac{x^4(a+b\arctan(cx))}{4d\,(d+ex^2)^2} - \frac{bc(c^2d-3e)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8\sqrt{d}\,(c^2d-e)^2\,e^{3/2}}$$

output
```
1/8*b*c*x/(c^2*d-e)/e/(e*x^2+d)-1/4*b*arctan(c*x)/d/(c^2*d-e)^2+1/4*x^4*(a
+b*arctan(c*x))/d/(e*x^2+d)^2-1/8*b*c*(c^2*d-3*e)*arctan(e^(1/2)*x/d^(1/2)
)/d^(1/2)/(c^2*d-e)^2/e^(3/2)
```

## Mathematica [A] (verified)

Time = 3.12 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.22

$$\int \frac{x^3(a+b\arctan(cx))}{(d+ex^2)^3}\,dx = \frac{\frac{2ad}{(d+ex^2)^2} + \frac{-4ac^2d+4ae+bcex}{(c^2d-e)(d+ex^2)} + \frac{2bc^2(c^2d-2e)\arctan(cx)}{(-c^2d+e)^2} - \frac{2b(d+2ex^2)\arctan(cx)}{(d+ex^2)^2} - \frac{bc(c^2d-3e)\sqrt{e}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}(-c^2d+e)^2}}{8e^2}$$

input
```
Integrate[(x^3*(a + b*ArcTan[c*x]))/(d + e*x^2)^3,x]
```

output
```
((2*a*d)/(d + e*x^2)^2 + (-4*a*c^2*d + 4*a*e + b*c*e*x)/((c^2*d - e)*(d +
e*x^2)) + (2*b*c^2*(c^2*d - 2*e)*ArcTan[c*x])/(-(c^2*d) + e)^2 - (2*b*(d +
 2*e*x^2)*ArcTan[c*x])/(d + e*x^2)^2 - (b*c*(c^2*d - 3*e)*Sqrt[e]*ArcTan[(
Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(-(c^2*d) + e)^2))/(8*e^2)
```

## Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5511, 27, 372, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\arctan(cx))}{(d+ex^2)^3}\,dx$$

$$\Big\downarrow \; 5511$$

$$\frac{x^4(a+b\arctan(cx))}{4d\,(d+ex^2)^2}-bc\int \frac{x^4}{4d\,(c^2x^2+1)\,(ex^2+d)^2}dx$$

$$\Big\downarrow \; 27$$

$$\frac{x^4(a+b\arctan(cx))}{4d\,(d+ex^2)^2}-\frac{bc\int \frac{x^4}{(c^2x^2+1)(ex^2+d)^2}dx}{4d}$$

$$\Big\downarrow \; 372$$

$$\frac{x^4(a+b\arctan(cx))}{4d\,(d+ex^2)^2}-\frac{bc\left(\frac{\int \frac{(c^2d-2e)x^2+d}{(c^2x^2+1)(ex^2+d)}dx}{2e(c^2d-e)}-\frac{dx}{2e(c^2d-e)(d+ex^2)}\right)}{4d}$$

$$\Big\downarrow \; 397$$

$$\frac{x^4(a+b\arctan(cx))}{4d\left(d+ex^2\right)^2}-\frac{bc\left(\frac{\frac{2e\int\frac{1}{c^2x^2+1}dx}{c^2d-e}+\frac{d\left(c^2d-3e\right)\int\frac{1}{ex^2+d}dx}{c^2d-e}}{2e(c^2d-e)}-\frac{dx}{2e(c^2d-e)(d+ex^2)}\right)}{4d}$$

$$\Big\downarrow \text{216}$$

$$\frac{x^4(a+b\arctan(cx))}{4d\left(d+ex^2\right)^2}-\frac{bc\left(\frac{\frac{d\left(c^2d-3e\right)\int\frac{1}{ex^2+d}dx}{c^2d-e}+\frac{2e\arctan(cx)}{c\left(c^2d-e\right)}}{2e(c^2d-e)}-\frac{dx}{2e(c^2d-e)(d+ex^2)}\right)}{4d}$$

$$\Big\downarrow \text{218}$$

$$\frac{x^4(a+b\arctan(cx))}{4d\left(d+ex^2\right)^2}-\frac{bc\left(\frac{\frac{2e\arctan(cx)}{c\left(c^2d-e\right)}+\frac{\sqrt{d}\left(c^2d-3e\right)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{e}\left(c^2d-e\right)}}{2e(c^2d-e)}-\frac{dx}{2e(c^2d-e)(d+ex^2)}\right)}{4d}$$

input
```
Int[(x^3*(a + b*ArcTan[c*x]))/(d + e*x^2)^3,x]
```

output
```
(x^4*(a + b*ArcTan[c*x]))/(4*d*(d + e*x^2)^2) - (b*c*(-1/2*(d*x)/((c^2*d -
 e)*e*(d + e*x^2)) + ((2*e*ArcTan[c*x])/(c*(c^2*d - e)) + (Sqrt[d]*(c^2*d
- 3*e)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/((c^2*d - e)*Sqrt[e]))/(2*(c^2*d - e)*
e)))/(4*d)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

## Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.62

| method | result |
|---|---|
| parts | $a\left(\frac{d}{4e^2(e\,x^2+d)^2}-\frac{1}{2e^2(e\,x^2+d)}\right)+\frac{b\left(-\frac{\arctan(cx)c^6}{2e^2\left(c^2e\,x^2+c^2d\right)}+\frac{\arctan(cx)c^8d}{4e^2\left(c^2e\,x^2+c^2d\right)^2}-\frac{c^6\left(\frac{\left(-c^2d+2e\right)\arctan(cx)}{\left(c^2d-e\right)^2}+\frac{e^2\left(-\ldots\right.}{\ldots}\right.}{\ldots}\right.}{c^4}$ |
| derivativedivides | $\frac{a\,c^6\left(\frac{d\,c^2}{4e^2\left(c^2e\,x^2+c^2d\right)^2}-\frac{1}{2e^2\left(c^2e\,x^2+c^2d\right)}\right)+b\,c^6\left(\frac{\arctan(cx)d\,c^2}{4e^2\left(c^2e\,x^2+c^2d\right)^2}-\frac{\arctan(cx)}{2e^2\left(c^2e\,x^2+c^2d\right)}-\frac{\frac{\left(-c^2d+2e\right)\arctan(cx)}{\left(c^2d-e\right)^2}+\frac{e^2\left(-\ldots\right.}{\ldots}}{\ldots}\right.}{c^4}$ |
| default | $\frac{a\,c^6\left(\frac{d\,c^2}{4e^2\left(c^2e\,x^2+c^2d\right)^2}-\frac{1}{2e^2\left(c^2e\,x^2+c^2d\right)}\right)+b\,c^6\left(\frac{\arctan(cx)d\,c^2}{4e^2\left(c^2e\,x^2+c^2d\right)^2}-\frac{\arctan(cx)}{2e^2\left(c^2e\,x^2+c^2d\right)}-\frac{\frac{\left(-c^2d+2e\right)\arctan(cx)}{\left(c^2d-e\right)^2}+\frac{e^2\left(-\ldots\right.}{\ldots}}{\ldots}\right.}{c^4}$ |
| risch | Expression too large to display |

input
```
int(x^3*(a+b*arctan(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
a*(1/4*d/e^2/(e*x^2+d)^2-1/2/e^2/(e*x^2+d))+b/c^4*(-1/2*arctan(c*x)*c^6/e^
2/(c^2*e*x^2+c^2*d)+1/4*arctan(c*x)*c^8*d/e^2/(c^2*e*x^2+c^2*d)^2-1/4*c^6/
e^2*((-c^2*d+2*e)/(c^2*d-e)^2*arctan(c*x)+e^2/(c^2*d-e)^2*(-1/2*(c^2*d-e)/
e*c*x/(c^2*e*x^2+c^2*d)+1/2*(c^2*d-3*e)/e/c/(d*e)^(1/2)*arctan(e*x/(d*e)^(
1/2)))))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 338 vs. $2(114) = 228$.

Time = 0.20 (sec) , antiderivative size = 697, normalized size of antiderivative = 5.36

$$\int \frac{x^3(a + b\arctan(cx))}{(d + ex^2)^3}\,dx$$
$$= \left[-\frac{4\,ac^4d^4 - 8\,ac^2d^3e + 4\,ad^2e^2 - 2\,(bc^3d^2e^2 - bcde^3)x^3 + 8\,(ac^4d^3e - 2\,ac^2d^2e^2 + ade^3)x^2 - (bc^3d^3 - 3}{16\,(c^4d^5e^2 - 2\,c}\right.$$
$$-\frac{2\,ac^4d^4 - 4\,ac^2d^3e + 2\,ad^2e^2 - (bc^3d^2e^2 - bcde^3)x^3 + 4\,(ac^4d^3e - 2\,ac^2d^2e^2 + ade^3)x^2 + (bc^3d^3 - 3\,bc}{8\,(c^4d^5e^2 - 2\,c^2d^4e^3}$$

input
```
integrate(x^3*(a+b*arctan(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
[-1/16*(4*a*c^4*d^4 - 8*a*c^2*d^3*e + 4*a*d^2*e^2 - 2*(b*c^3*d^2*e^2 - b*c
*d*e^3)*x^3 + 8*(a*c^4*d^3*e - 2*a*c^2*d^2*e^2 + a*d*e^3)*x^2 - (b*c^3*d^3
 - 3*b*c*d^2*e + (b*c^3*d*e^2 - 3*b*c*e^3)*x^4 + 2*(b*c^3*d^2*e - 3*b*c*d*
e^2)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x - d)/(e*x^2 + d)) - 2*(b*
c^3*d^3*e - b*c*d^2*e^2)*x + 4*(2*b*d*e^3*x^2 + b*d^2*e^2 - (b*c^4*d^2*e^2
 - 2*b*c^2*d*e^3)*x^4)*arctan(c*x))/(c^4*d^5*e^2 - 2*c^2*d^4*e^3 + d^3*e^4
 + (c^4*d^3*e^4 - 2*c^2*d^2*e^5 + d*e^6)*x^4 + 2*(c^4*d^4*e^3 - 2*c^2*d^3*
e^4 + d^2*e^5)*x^2), -1/8*(2*a*c^4*d^4 - 4*a*c^2*d^3*e + 2*a*d^2*e^2 - (b*
c^3*d^2*e^2 - b*c*d*e^3)*x^3 + 4*(a*c^4*d^3*e - 2*a*c^2*d^2*e^2 + a*d*e^3)
*x^2 + (b*c^3*d^3 - 3*b*c*d^2*e + (b*c^3*d*e^2 - 3*b*c*e^3)*x^4 + 2*(b*c^3
*d^2*e - 3*b*c*d*e^2)*x^2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) - (b*c^3*d^3*e
- b*c*d^2*e^2)*x + 2*(2*b*d*e^3*x^2 + b*d^2*e^2 - (b*c^4*d^2*e^2 - 2*b*c^2
*d*e^3)*x^4)*arctan(c*x))/(c^4*d^5*e^2 - 2*c^2*d^4*e^3 + d^3*e^4 + (c^4*d^
3*e^4 - 2*c^2*d^2*e^5 + d*e^6)*x^4 + 2*(c^4*d^4*e^3 - 2*c^2*d^3*e^4 + d^2*
e^5)*x^2)]
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\arctan(cx))}{(d+ex^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(a+b*atan(c*x))/(e*x**2+d)**3,x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(a+b\arctan(cx))}{(d+ex^2)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(a+b*arctan(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

## Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.27 (sec) , antiderivative size = 762, normalized size of antiderivative = 5.86

$$\int \frac{x^3(a+b\arctan(cx))}{(d+ex^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(a+b*arctan(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
-1/8*(b*c^3*d - 3*b*c*e)*arctan(e*x/sqrt(d*e))/((c^4*d^2*e - 2*c^2*d*e^2 +
 e^3)*sqrt(d*e)) + 1/8*(-I*b*c^4*d*e^2*x^4*log(I*c*x + 1) + I*b*c^4*d*e^2*
x^4*log(-I*c*x + 1) - 4*b*c^4*d^2*e*x^2*arctan(c*x) - 2*I*b*c^4*d^2*e*x^2*
log(I*c*x + 1) + 2*I*b*c^2*e^3*x^4*log(I*c*x + 1) + 2*I*b*c^4*d^2*e*x^2*lo
g(-I*c*x + 1) - 2*I*b*c^2*e^3*x^4*log(-I*c*x + 1) - 4*a*c^4*d^2*e*x^2 + b*
c^3*d*e^2*x^3 - 2*b*c^4*d^3*arctan(c*x) + 8*b*c^2*d*e^2*x^2*arctan(c*x) -
I*b*c^4*d^3*log(I*c*x + 1) + 4*I*b*c^2*d*e^2*x^2*log(I*c*x + 1) + I*b*c^4*
d^3*log(-I*c*x + 1) - 4*I*b*c^2*d*e^2*x^2*log(-I*c*x + 1) - 2*a*c^4*d^3 +
b*c^3*d^2*e*x + 8*a*c^2*d*e^2*x^2 - b*c*e^3*x^3 + 4*b*c^2*d^2*e*arctan(c*x
) - 4*b*e^3*x^2*arctan(c*x) + 2*I*b*c^2*d^2*e*log(I*c*x + 1) - 2*I*b*c^2*d
^2*e*log(-I*c*x + 1) + 4*a*c^2*d^2*e - b*c*d*e^2*x - 4*a*e^3*x^2 - 2*b*d*e
^2*arctan(c*x) - 2*a*d*e^2)/(c^4*d^2*e^4*x^4 + 2*c^4*d^3*e^3*x^2 - 2*c^2*d
*e^5*x^4 + c^4*d^4*e^2 - 4*c^2*d^2*e^4*x^2 + e^6*x^4 - 2*c^2*d^3*e^3 + 2*d
*e^5*x^2 + d^2*e^4) - 1/8*(4*a*c^2*d*e*x^2 - b*c*e^2*x^3 + 2*a*c^2*d^2 - b
*c*d*e*x - 4*a*e^2*x^2 - 2*a*d*e)/(c^2*d*e^4*x^4 + 2*c^2*d^2*e^3*x^2 - e^5
*x^4 + c^2*d^3*e^2 - 2*d*e^4*x^2 - d^2*e^3) - 1/8*(4*a*c^2*d*e*x^2 - b*c*e
^2*x^3 + 2*a*c^2*d^2 - b*c*d*e*x - 4*a*e^2*x^2 - 2*a*d*e)/((c^2*d*e^2 - e^
3)*(e*x^2 + d)^2)
```

## Mupad [B] (verification not implemented)

Time = 3.92 (sec) , antiderivative size = 273, normalized size of antiderivative = 2.10

$$\int \frac{x^3(a+b\arctan(cx))}{(d+ex^2)^3}\,dx = \frac{b\,c^4\,d\,\mathrm{atan}(c\,x)}{4\,e^2\,(e-c^2\,d)^2} - \frac{a\,d}{4\,e^2\,(e\,x^2+d)^2} - \frac{b\,d\,\mathrm{atan}(c\,x)}{4\,e^2\,(e\,x^2+d)^2} - \frac{b\,c\,x^3}{8\,(e-c^2\,d)\,(e\,x^2+d)^2} - \frac{b\,c^2\,\mathrm{atan}(c\,x)}{2\,e\,(e-c^2\,d)^2} - \frac{b\,x^2\,\mathrm{atan}(c\,x)}{2\,e\,(e\,x^2+d)^2} - \frac{b\,c^3\,\mathrm{atan}\left(\frac{x\,\sqrt{-d\,e^3}\,1\mathrm{i}}{d\,e}\right)\,\sqrt{-d\,e^3}\,1\mathrm{i}}{8\,e^3\,(e-c^2\,d)^2} - \frac{a\,x^2}{2\,e\,(e\,x^2+d)^2} - \frac{b\,c\,d\,x}{8\,e\,(e-c^2\,d)\,(e\,x^2+d)^2} + \frac{b\,c\,\mathrm{atan}\left(\frac{x\,\sqrt{-d\,e^3}\,1\mathrm{i}}{d\,e}\right)\,\sqrt{-d\,e^3}\,3\mathrm{i}}{8\,d\,e^2\,(e-c^2\,d)^2}$$

input
```
int((x^3*(a + b*atan(c*x)))/(d + e*x^2)^3,x)
```

output
```
(b*c^4*d*atan(c*x))/(4*e^2*(e - c^2*d)^2) - (a*d)/(4*e^2*(d + e*x^2)^2) -
(b*d*atan(c*x))/(4*e^2*(d + e*x^2)^2) - (b*c*x^3)/(8*(e - c^2*d)*(d + e*x^
2)^2) - (b*c^2*atan(c*x))/(2*e*(e - c^2*d)^2) - (b*x^2*atan(c*x))/(2*e*(d
+ e*x^2)^2) - (b*c^3*atan((x*(-d*e^3)^(1/2)*1i)/(d*e))*(-d*e^3)^(1/2)*1i)/
(8*e^3*(e - c^2*d)^2) - (a*x^2)/(2*e*(d + e*x^2)^2) - (b*c*d*x)/(8*e*(e -
c^2*d)*(d + e*x^2)^2) + (b*c*atan((x*(-d*e^3)^(1/2)*1i)/(d*e))*(-d*e^3)^(1
/2)*3i)/(8*d*e^2*(e - c^2*d)^2)
```

## Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 408, normalized size of antiderivative = 3.14

$$\int \frac{x^3(a+b\arctan(cx))}{(d+ex^2)^3}\,dx = \frac{-\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)b\,c^3d^3-2\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)b\,c^3d^2e\,x^2-\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)b\,c^3d\,e^2x^4+3\sqrt{e}\,\cdots}{\cdots}$$

input
```
int(x^3*(a+b*atan(c*x))/(e*x^2+d)^3,x)
```

output
```
( - sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*b*c**3*d**3 - 2*sqrt(e)*
sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*b*c**3*d**2*e*x**2 - sqrt(e)*sqrt(d)
*atan((e*x)/(sqrt(e)*sqrt(d)))*b*c**3*d*e**2*x**4 + 3*sqrt(e)*sqrt(d)*atan
((e*x)/(sqrt(e)*sqrt(d)))*b*c*d**2*e + 6*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(
e)*sqrt(d)))*b*c*d*e**2*x**2 + 3*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(
d)))*b*c*e**3*x**4 + 2*atan(c*x)*b*c**4*d**2*e**2*x**4 - 4*atan(c*x)*b*c**
2*d*e**3*x**4 - 2*atan(c*x)*b*d**2*e**2 - 4*atan(c*x)*b*d*e**3*x**2 + 2*a*
c**4*d**2*e**2*x**4 - 4*a*c**2*d*e**3*x**4 + 2*a*e**4*x**4 + b*c**3*d**3*e
*x + b*c**3*d**2*e**2*x**3 - b*c*d**2*e**2*x - b*c*d*e**3*x**3)/(8*d*e**2*
(c**4*d**4 + 2*c**4*d**3*e*x**2 + c**4*d**2*e**2*x**4 - 2*c**2*d**3*e - 4*
c**2*d**2*e**2*x**2 - 2*c**2*d*e**3*x**4 + d**2*e**2 + 2*d*e**3*x**2 + e**
4*x**4))
```

# 3.1166 $\int \frac{x(a+b\arctan(cx))}{(d+ex^2)^3}\,dx$



## Optimal result

Integrand size = 19, antiderivative size = 131

$$\int \frac{x(a+b\arctan(cx))}{(d+ex^2)^3}\,dx = -\frac{bcx}{8d(c^2d-e)(d+ex^2)}+\frac{bc^4\arctan(cx)}{4(c^2d-e)^2e} -\frac{a+b\arctan(cx)}{4e(d+ex^2)^2}-\frac{bc(3c^2d-e)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{3/2}(c^2d-e)^2\sqrt{e}}$$

output
```
-1/8*b*c*x/d/(c^2*d-e)/(e*x^2+d)+1/4*b*c^4*arctan(c*x)/(c^2*d-e)^2/e-1/4*(
a+b*arctan(c*x))/e/(e*x^2+d)^2-1/8*b*c*(3*c^2*d-e)*arctan(e^(1/2)*x/d^(1/2
))/d^(3/2)/(c^2*d-e)^2/e^(1/2)
```

## Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00

$$\int \frac{x(a+b\arctan(cx))}{(d+ex^2)^3}\,dx = \frac{1}{8}\left(-\frac{\frac{2a}{e}+\frac{bcx(d+ex^2)}{d(c^2d-e)}}{(d+ex^2)^2}+\frac{2b\left(\frac{c^4}{(-c^2d+e)^2}-\frac{1}{(d+ex^2)^2}\right)\arctan(cx)}{e} -\frac{bc(3c^2d-e)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}\sqrt{e}(-c^2d+e)^2}\right)$$

input
```
Integrate[(x*(a + b*ArcTan[c*x]))/(d + e*x^2)^3,x]
```

output
```
(-(((2*a)/e + (b*c*x*(d + e*x^2))/(d*(c^2*d - e)))/(d + e*x^2)^2) + (2*b*(
c^4/(-(c^2*d) + e)^2 - (d + e*x^2)^(-2))*ArcTan[c*x])/e - (b*c*(3*c^2*d -
e)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(3/2)*Sqrt[e]*(-(c^2*d) + e)^2))/8
```

## Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {5509, 316, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\arctan(cx))}{(d+ex^2)^3} \, dx$$

$$\Big\downarrow \text{5509}$$

$$\frac{bc\int \frac{1}{(c^2x^2+1)(ex^2+d)^2}dx}{4e}-\frac{a+b\arctan(cx)}{4e(d+ex^2)^2}$$

$$\Big\downarrow \text{316}$$

$$\frac{bc\left(\frac{\int \frac{-ex^2c^2+2dc^2-e}{\left(c^2x^2+1\right)\left(ex^2+d\right)}dx}{2d(c^2d-e)}-\frac{ex}{2d(c^2d-e)(d+ex^2)}\right)}{4e}-\frac{a+b\arctan(cx)}{4e(d+ex^2)^2}$$

$$\Big\downarrow \text{397}$$

$$\frac{bc\left(\frac{\frac{2c^4d\int \frac{1}{c^2x^2+1}dx}{c^2d-e}-\frac{e\left(3c^2d-e\right)\int \frac{1}{ex^2+d}dx}{c^2d-e}}{2d(c^2d-e)}-\frac{ex}{2d(c^2d-e)(d+ex^2)}\right)}{4e}-\frac{a+b\arctan(cx)}{4e(d+ex^2)^2}$$

$$\Big\downarrow \text{216}$$

$$\frac{bc\left(\frac{\frac{2c^3d\arctan(cx)}{c^2d-e}-\frac{e(3c^2d-e)\int\frac{1}{ex^2+d}dx}{c^2d-e}}{2d(c^2d-e)}-\frac{ex}{2d(c^2d-e)(d+ex^2)}\right)}{4e}-\frac{a+b\arctan(cx)}{4e\left(d+ex^2\right)^2}$$

$\Big\downarrow$ 218

$$\frac{bc\left(\frac{\frac{2c^3d\arctan(cx)}{c^2d-e}-\frac{\sqrt{e}(3c^2d-e)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{\sqrt{d}(c^2d-e)}}{2d(c^2d-e)}-\frac{ex}{2d(c^2d-e)(d+ex^2)}\right)}{4e}-\frac{a+b\arctan(cx)}{4e\left(d+ex^2\right)^2}$$

input
```
Int[(x*(a + b*ArcTan[c*x]))/(d + e*x^2)^3,x]
```

output
```
-1/4*(a + b*ArcTan[c*x])/(e*(d + e*x^2)^2) + (b*c*(-1/2*(e*x)/(d*(c^2*d -
e)*(d + e*x^2)) + ((2*c^3*d*ArcTan[c*x])/(c^2*d - e) - ((3*c^2*d - e)*Sqrt
[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c^2*d - e)))/(2*d*(c^2*d - e)))
)/(4*e)
```

## Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 5509
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*e*(q + 1))), x
] - Simp[b*(c/(2*e*(q + 1)))   Int[(d + e*x^2)^(q + 1)/(1 + c^2*x^2), x], x
] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]
```

## Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.20

| method | result | siz |
|---|---|---|
| parts | $-\frac{a}{4e(e\,x^2+d)^2}+\frac{b\left(-\frac{\arctan(cx)c^6}{4e\left(c^2e\,x^2+c^2d\right)^2}+\frac{c^6\left(-\frac{e\left(\frac{\left(c^2d-e\right)x}{2dc\left(c^2e\,x^2+c^2d\right)}+\frac{\left(3c^2d-e\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2d\,c^3\sqrt{de}}\right)}{\left(c^2d-e\right)^2}+\frac{\arctan(cx)}{\left(c^2d-e\right)^2}\right)}{4e}\right)}{c^2}$ | 15 |
| derivativedivides | $\frac{-\frac{a\,c^6}{4e\left(c^2e\,x^2+c^2d\right)^2}+b\,c^6\left(-\frac{\arctan(cx)}{4e\left(c^2e\,x^2+c^2d\right)^2}+\frac{-\frac{e\left(\frac{\left(c^2d-e\right)x}{2dc\left(c^2e\,x^2+c^2d\right)}+\frac{\left(3c^2d-e\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2d\,c^3\sqrt{de}}\right)}{\left(c^2d-e\right)^2}+\frac{\arctan(cx)}{\left(c^2d-e\right)^2}}{4e}\right)}{c^2}$ | 16 |
| default | $\frac{-\frac{a\,c^6}{4e\left(c^2e\,x^2+c^2d\right)^2}+b\,c^6\left(-\frac{\arctan(cx)}{4e\left(c^2e\,x^2+c^2d\right)^2}+\frac{-\frac{e\left(\frac{\left(c^2d-e\right)x}{2dc\left(c^2e\,x^2+c^2d\right)}+\frac{\left(3c^2d-e\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2d\,c^3\sqrt{de}}\right)}{\left(c^2d-e\right)^2}+\frac{\arctan(cx)}{\left(c^2d-e\right)^2}}{4e}\right)}{c^2}$ | 16 |
| risch | Expression too large to display | 11 |

input
```
int(x*(a+b*arctan(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a/e/(e*x^2+d)^2+b/c^2*(-1/4*arctan(c*x)*c^6/e/(c^2*e*x^2+c^2*d)^2+1/4
*c^6/e*(-1/(c^2*d-e)^2*e*(1/2*(c^2*d-e)/d/c*x/(c^2*e*x^2+c^2*d)+1/2*(3*c^2
*d-e)/d/c^3/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2)))+1/(c^2*d-e)^2*arctan(c*x)
))
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 307 vs. 2(115) = 230.

Time = 0.18 (sec) , antiderivative size = 637, normalized size of antiderivative = 4.86

$$\int \frac{x(a+b\arctan(cx))}{(d+ex^2)^3}\,dx$$
$$= \Bigg[-\frac{4\,ac^4d^4-8\,ac^2d^3e+4\,ad^2e^2+2\,(bc^3d^2e^2-bcde^3)x^3-(3\,bc^3d^3-bcd^2e+(3\,bc^3de^2-bce^3)x^4+2\,(3\,b}{16\,(c^4d^6e-2\,c^2d^5e^2+d^4e^3+(c^4}$$
$$-\frac{2\,ac^4d^4-4\,ac^2d^3e+2\,ad^2e^2+(bc^3d^2e^2-bcde^3)x^3+(3\,bc^3d^3-bcd^2e+(3\,bc^3de^2-bce^3)x^4+2\,(3\,bc^3}{8\,(c^4d^6e-2\,c^2d^5e^2+d^4e^3+(c^4d^4e^3-}$$

input
```
integrate(x*(a+b*arctan(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
[-1/16*(4*a*c^4*d^4 - 8*a*c^2*d^3*e + 4*a*d^2*e^2 + 2*(b*c^3*d^2*e^2 - b*c
*d*e^3)*x^3 - (3*b*c^3*d^3 - b*c*d^2*e + (3*b*c^3*d*e^2 - b*c*e^3)*x^4 + 2
*(3*b*c^3*d^2*e - b*c*d*e^2)*x^2)*sqrt(-d*e)*log((e*x^2 - 2*sqrt(-d*e)*x -
 d)/(e*x^2 + d)) + 2*(b*c^3*d^3*e - b*c*d^2*e^2)*x - 4*(b*c^4*d^2*e^2*x^4
+ 2*b*c^4*d^3*e*x^2 + 2*b*c^2*d^3*e - b*d^2*e^2)*arctan(c*x))/(c^4*d^6*e -
 2*c^2*d^5*e^2 + d^4*e^3 + (c^4*d^4*e^3 - 2*c^2*d^3*e^4 + d^2*e^5)*x^4 + 2
*(c^4*d^5*e^2 - 2*c^2*d^4*e^3 + d^3*e^4)*x^2), -1/8*(2*a*c^4*d^4 - 4*a*c^2
*d^3*e + 2*a*d^2*e^2 + (b*c^3*d^2*e^2 - b*c*d*e^3)*x^3 + (3*b*c^3*d^3 - b*
c*d^2*e + (3*b*c^3*d*e^2 - b*c*e^3)*x^4 + 2*(3*b*c^3*d^2*e - b*c*d*e^2)*x^
2)*sqrt(d*e)*arctan(sqrt(d*e)*x/d) + (b*c^3*d^3*e - b*c*d^2*e^2)*x - 2*(b*
c^4*d^2*e^2*x^4 + 2*b*c^4*d^3*e*x^2 + 2*b*c^2*d^3*e - b*d^2*e^2)*arctan(c*
x))/(c^4*d^6*e - 2*c^2*d^5*e^2 + d^4*e^3 + (c^4*d^4*e^3 - 2*c^2*d^3*e^4 +
d^2*e^5)*x^4 + 2*(c^4*d^5*e^2 - 2*c^2*d^4*e^3 + d^3*e^4)*x^2)]
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x(a+b\arctan(cx))}{(d+ex^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x*(a+b*atan(c*x))/(e*x**2+d)**3,x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x(a+b\arctan(cx))}{(d+ex^2)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(a+b*arctan(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

## Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.09 (sec) , antiderivative size = 533, normalized size of antiderivative = 4.07

$$\int \frac{x(a+b\arctan(cx))}{(d+ex^2)^3}\,dx = -\frac{(3\,bc^3d-bce)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\,(c^4d^3-2\,c^2d^2e+de^2)\sqrt{de}}$$
$$+\frac{i\,bc^4de^2x^4\log(i\,cx-1)-i\,bc^4de^2x^4\log(-i\,cx-1)+2i\,bc^4d^2ex^2\log(i\,cx-1)-2i\,bc^4d^2ex^2\log(-i\,c}{8\,(c^4}$$
$$-\frac{bce^2x^3+2\,ac^2d^2+bcdex-2\,ade}{8\,(c^2d^2e^3x^4+2\,c^2d^3e^2x^2-de^4x^4+c^2d^4e-2\,d^2e^3x^2-d^3e^2)}$$
$$-\frac{bce^2x^3+2\,ac^2d^2+bcdex-2\,ade}{8\,(c^2d^2e-de^2)(ex^2+d)^2}$$

input
```
integrate(x*(a+b*arctan(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
-1/8*(3*b*c^3*d - b*c*e)*arctan(e*x/sqrt(d*e))/((c^4*d^3 - 2*c^2*d^2*e + d
*e^2)*sqrt(d*e)) + 1/8*(I*b*c^4*d*e^2*x^4*log(I*c*x - 1) - I*b*c^4*d*e^2*x
^4*log(-I*c*x - 1) + 2*I*b*c^4*d^2*e*x^2*log(I*c*x - 1) - 2*I*b*c^4*d^2*e*
x^2*log(-I*c*x - 1) - b*c^3*d*e^2*x^3 - 2*b*c^4*d^3*arctan(c*x) + I*b*c^4*
d^3*log(I*c*x - 1) - I*b*c^4*d^3*log(-I*c*x - 1) - 2*a*c^4*d^3 - b*c^3*d^2
*e*x + b*c*e^3*x^3 + 4*b*c^2*d^2*e*arctan(c*x) + 4*a*c^2*d^2*e + b*c*d*e^2
*x - 2*b*d*e^2*arctan(c*x) - 2*a*d*e^2)/(c^4*d^3*e^3*x^4 + 2*c^4*d^4*e^2*x
^2 - 2*c^2*d^2*e^4*x^4 + c^4*d^5*e - 4*c^2*d^3*e^3*x^2 + d*e^5*x^4 - 2*c^2
*d^4*e^2 + 2*d^2*e^4*x^2 + d^3*e^3) - 1/8*(b*c*e^2*x^3 + 2*a*c^2*d^2 + b*c
*d*e*x - 2*a*d*e)/(c^2*d^2*e^3*x^4 + 2*c^2*d^3*e^2*x^2 - d*e^4*x^4 + c^2*d
^4*e - 2*d^2*e^3*x^2 - d^3*e^2) - 1/8*(b*c*e^2*x^3 + 2*a*c^2*d^2 + b*c*d*e
*x - 2*a*d*e)/((c^2*d^2*e - d*e^2)*(e*x^2 + d)^2)
```

## Mupad [B] (verification not implemented)

Time = 3.18 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.53

$$\int \frac{x(a+b\arctan(cx))}{(d+ex^2)^3}\,dx = \frac{b\,c\,x}{8\,\left(e-c^2\,d\right)\,\left(e\,x^2+d\right)^2}-\frac{b\,\mathrm{atan}(c\,x)}{4\,e\,\left(e\,x^2+d\right)^2}-\frac{a}{4\,e\,\left(e\,x^2+d\right)^2}+\frac{b\,c^4\,\mathrm{atan}(c\,x)}{4\,e\,\left(e-c^2\,d\right)^2}+\frac{b\,c\,e\,x^3}{8\,d\,\left(e-c^2\,d\right)\,\left(e\,x^2+d\right)^2}+\frac{b\,c\,\mathrm{atan}\left(\frac{x\,\sqrt{-d^3\,e}\,1\mathrm{i}}{d^2}\right)\,\sqrt{-d^3\,e}\,1\mathrm{i}}{8\,d^3\,\left(e-c^2\,d\right)^2}-\frac{b\,c^3\,\mathrm{atan}\left(\frac{x\,\sqrt{-d^3\,e}\,1\mathrm{i}}{d^2}\right)\,\sqrt{-d^3\,e}\,3\mathrm{i}}{8\,d^2\,e\,\left(e-c^2\,d\right)^2}$$

input
```
int((x*(a + b*atan(c*x)))/(d + e*x^2)^3,x)
```

output
```
(b*c*x)/(8*(e - c^2*d)*(d + e*x^2)^2) - (b*atan(c*x))/(4*e*(d + e*x^2)^2)
- a/(4*e*(d + e*x^2)^2) + (b*c^4*atan(c*x))/(4*e*(e - c^2*d)^2) + (b*c*ata
n((x*(-d^3*e)^(1/2)*1i)/d^2)*(-d^3*e)^(1/2)*1i)/(8*d^3*(e - c^2*d)^2) - (b
*c^3*atan((x*(-d^3*e)^(1/2)*1i)/d^2)*(-d^3*e)^(1/2)*3i)/(8*d^2*e*(e - c^2*
d)^2) + (b*c*e*x^3)/(8*d*(e - c^2*d)*(d + e*x^2)^2)
```

## Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 397, normalized size of antiderivative = 3.03

$$\int \frac{x(a+b\arctan(cx))}{(d+ex^2)^3}\,dx = \frac{-3\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)b\,c^3d^3-6\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)b\,c^3d^2e\,x^2-3\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)b\,c^3d\,e^2x^4+\sqrt{e}\cdots}{}$$

input
```
int(x*(a+b*atan(c*x))/(e*x^2+d)^3,x)
```

output
```
( - 3*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*b*c**3*d**3 - 6*sqrt(e
)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*b*c**3*d**2*e*x**2 - 3*sqrt(e)*sqr
t(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*b*c**3*d*e**2*x**4 + sqrt(e)*sqrt(d)*at
an((e*x)/(sqrt(e)*sqrt(d)))*b*c*d**2*e + 2*sqrt(e)*sqrt(d)*atan((e*x)/(sqr
t(e)*sqrt(d)))*b*c*d*e**2*x**2 + sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(
d)))*b*c*e**3*x**4 + 4*atan(c*x)*b*c**4*d**3*e*x**2 + 2*atan(c*x)*b*c**4*d
**2*e**2*x**4 + 4*atan(c*x)*b*c**2*d**3*e - 2*atan(c*x)*b*d**2*e**2 - 2*a*
c**4*d**4 + 4*a*c**2*d**3*e - 2*a*d**2*e**2 - b*c**3*d**3*e*x - b*c**3*d**
2*e**2*x**3 + b*c*d**2*e**2*x + b*c*d*e**3*x**3)/(8*d**2*e*(c**4*d**4 + 2*
c**4*d**3*e*x**2 + c**4*d**2*e**2*x**4 - 2*c**2*d**3*e - 4*c**2*d**2*e**2*
x**2 - 2*c**2*d*e**3*x**4 + d**2*e**2 + 2*d*e**3*x**2 + e**4*x**4))
```

# 3.1167 $\int \frac{a+b\arctan(cx)}{x(d+ex^2)^3}\,dx$

## Optimal result

Integrand size = 21, antiderivative size = 574

$$\int \frac{a+b\arctan(cx)}{x(d+ex^2)^3}\,dx = \frac{bcex}{8d^2(c^2d-e)(d+ex^2)} - \frac{bc^4\arctan(cx)}{4d(c^2d-e)^2} - \frac{bc^2\arctan(cx)}{2d^2(c^2d-e)} + \frac{a+b\arctan(cx)}{4d(d+ex^2)^2} + \frac{a+b\arctan(cx)}{2d^2(d+ex^2)} + \frac{bc\sqrt{e}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{5/2}(c^2d-e)} + \frac{bc(3c^2d-e)\sqrt{e}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}(c^2d-e)^2} + \frac{a\log(x)}{d^3} + \frac{(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)}{d^3} - \frac{(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2d^3} - \frac{(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{2d^3} + \frac{ib\,\text{PolyLog}(2,-icx)}{2d^3} - \frac{ib\,\text{PolyLog}(2,icx)}{2d^3} - \frac{ib\,\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2d^3} + \frac{ib\,\text{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{4d^3} + \frac{ib\,\text{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{4d^3}$$

output

```
1/8*b*c*e*x/d^2/(c^2*d-e)/(e*x^2+d)-1/4*b*c^4*arctan(c*x)/d/(c^2*d-e)^2-1/
2*b*c^2*arctan(c*x)/d^2/(c^2*d-e)+1/4*(a+b*arctan(c*x))/d/(e*x^2+d)^2+1/2*
(a+b*arctan(c*x))/d^2/(e*x^2+d)+1/2*b*c*e^(1/2)*arctan(e^(1/2)*x/d^(1/2))/
d^(5/2)/(c^2*d-e)+1/8*b*c*(3*c^2*d-e)*e^(1/2)*arctan(e^(1/2)*x/d^(1/2))/d^
(5/2)/(c^2*d-e)^2+a*ln(x)/d^3+(a+b*arctan(c*x))*ln(2/(1-I*c*x))/d^3-1/2*(a
+b*arctan(c*x))*ln(2*c*((-d)^(1/2)-e^(1/2)*x)/(c*(-d)^(1/2)-I*e^(1/2))/(1-
I*c*x))/d^3-1/2*(a+b*arctan(c*x))*ln(2*c*((-d)^(1/2)+e^(1/2)*x)/(c*(-d)^(1
/2)+I*e^(1/2))/(1-I*c*x))/d^3+1/4*I*b*polylog(2,1-2*c*((-d)^(1/2)+e^(1/2)*
x)/(c*(-d)^(1/2)+I*e^(1/2))/(1-I*c*x))/d^3-1/2*I*b*polylog(2,I*c*x)/d^3-1/
2*I*b*polylog(2,1-2/(1-I*c*x))/d^3+1/2*I*b*polylog(2,-I*c*x)/d^3+1/4*I*b*p
olylog(2,1-2*c*((-d)^(1/2)-e^(1/2)*x)/(c*(-d)^(1/2)-I*e^(1/2))/(1-I*c*x))/
d^3
```

## Mathematica [A] (verified)

Time = 8.49 (sec) , antiderivative size = 645, normalized size of antiderivative = 1.12

$$\int \frac{a+b\arctan(cx)}{x\left(d+ex^2\right)^3}\,dx$$
$$= \frac{2a\left(\frac{d(3d+2ex^2)}{(d+ex^2)^2}+4\log(x)-2\log\left(d+ex^2\right)\right)+b\left(\frac{cdex}{(c^2d-e)(d+ex^2)}+\frac{2c^2d(-3c^2d+2e)\arctan(cx)}{(-c^2d+e)^2}+\frac{2d(3d+2ex^2)\arctan(}{(d+ex^2)^2}\right.}{}$$

input
```
Integrate[(a + b*ArcTan[c*x])/(x*(d + e*x^2)^3),x]
```

output
```
(2*a*((d*(3*d + 2*e*x^2))/(d + e*x^2)^2 + 4*Log[x] - 2*Log[d + e*x^2]) + b
*((c*d*e*x)/((c^2*d - e)*(d + e*x^2)) + (2*c^2*d*(-3*c^2*d + 2*e)*ArcTan[c
*x])/(-(c^2*d) + e)^2 + (2*d*(3*d + 2*e*x^2)*ArcTan[c*x])/(d + e*x^2)^2 +
(c*Sqrt[d]*(7*c^2*d - 5*e)*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(-(c^2*d)
+ e)^2 + 8*ArcTan[c*x]*Log[x] - 4*ArcTan[c*x]*Log[((-I)*Sqrt[d])/Sqrt[e] +
 x] - 4*ArcTan[c*x]*Log[(I*Sqrt[d])/Sqrt[e] + x] - (2*I)*Log[((-I)*Sqrt[d]
)/Sqrt[e] + x]*Log[(Sqrt[e]*(-1 - I*c*x))/(c*Sqrt[d] - Sqrt[e])] + (2*I)*L
og[((-I)*Sqrt[d])/Sqrt[e] + x]*Log[(Sqrt[e]*(1 - I*c*x))/(c*Sqrt[d] + Sqrt
[e])] + (2*I)*Log[(I*Sqrt[d])/Sqrt[e] + x]*Log[(Sqrt[e]*(-1 + I*c*x))/(c*S
qrt[d] - Sqrt[e])] - (2*I)*Log[(I*Sqrt[d])/Sqrt[e] + x]*Log[(Sqrt[e]*(1 +
I*c*x))/(c*Sqrt[d] + Sqrt[e])] - (4*I)*(Log[x]*(Log[1 - I*c*x] - Log[1 + I
*c*x]) - PolyLog[2, (-I)*c*x] + PolyLog[2, I*c*x]) + (2*I)*PolyLog[2, (c*(
Sqrt[d] - I*Sqrt[e]*x))/(c*Sqrt[d] - Sqrt[e])] - (2*I)*PolyLog[2, (c*(Sqrt
[d] - I*Sqrt[e]*x))/(c*Sqrt[d] + Sqrt[e])] - (2*I)*PolyLog[2, (c*(Sqrt[d]
+ I*Sqrt[e]*x))/(c*Sqrt[d] - Sqrt[e])] + (2*I)*PolyLog[2, (c*(Sqrt[d] + I*
Sqrt[e]*x))/(c*Sqrt[d] + Sqrt[e])]))/(8*d^3)
```

## Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 574, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x\left(d+ex^2\right)^3}\,dx$$

$$\Big\downarrow \text{5515}$$

$$\int \left(-\frac{ex(a+b\arctan(cx))}{d^3\left(d+ex^2\right)}+\frac{a+b\arctan(cx)}{d^3x}-\frac{ex(a+b\arctan(cx))}{d^2\left(d+ex^2\right)^2}-\frac{ex(a+b\arctan(cx))}{d\left(d+ex^2\right)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(a+b\arctan(cx))\log\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{(1-icx)\left(c\sqrt{-d}-i\sqrt{e}\right)}\right)}{2d^3}-\frac{(a+b\arctan(cx))\log\left(\frac{2c\left(\sqrt{-d}+\sqrt{e}x\right)}{(1-icx)\left(c\sqrt{-d}+i\sqrt{e}\right)}\right)}{2d^3}+\frac{\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))}{d^3}+\frac{a+b\arctan(cx)}{2d^2\left(d+ex^2\right)}+\frac{a+b\arctan(cx)}{4d\left(d+ex^2\right)^2}+\frac{a\log(x)}{d^3}+\frac{bc\sqrt{e}\left(3c^2d-e\right)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}\left(c^2d-e\right)^2}+\frac{bc\sqrt{e}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2d^{5/2}\left(c^2d-e\right)}-\frac{bc^2\arctan(cx)}{2d^2\left(c^2d-e\right)}-\frac{bc^4\arctan(cx)}{4d\left(c^2d-e\right)^2}+\frac{bcex}{8d^2\left(c^2d-e\right)\left(d+ex^2\right)}+\frac{ib\,\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)}{4d^3}+\frac{ib\,\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{e}x+\sqrt{-d}\right)}{\left(\sqrt{-d}c+i\sqrt{e}\right)(1-icx)}\right)}{4d^3}+\frac{ib\,\text{PolyLog}(2,-icx)}{2d^3}-\frac{ib\,\text{PolyLog}(2,icx)}{2d^3}-\frac{ib\,\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2d^3}$$

input
```
Int[(a + b*ArcTan[c*x])/(x*(d + e*x^2)^3),x]
```

output
```
(b*c*e*x)/(8*d^2*(c^2*d - e)*(d + e*x^2)) - (b*c^4*ArcTan[c*x])/(4*d*(c^2*
d - e)^2) - (b*c^2*ArcTan[c*x])/(2*d^2*(c^2*d - e)) + (a + b*ArcTan[c*x])/
(4*d*(d + e*x^2)^2) + (a + b*ArcTan[c*x])/(2*d^2*(d + e*x^2)) + (b*c*Sqrt[
e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(5/2)*(c^2*d - e)) + (b*c*(3*c^2*d -
e)*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(5/2)*(c^2*d - e)^2) + (a*Log
[x])/d^3 + ((a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/d^3 - ((a + b*ArcTan[c
*x])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x
))])/(2*d^3) - ((a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*S
qrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(2*d^3) + ((I/2)*b*PolyLog[2, (-I)*c*x
])/d^3 - ((I/2)*b*PolyLog[2, I*c*x])/d^3 - ((I/2)*b*PolyLog[2, 1 - 2/(1 -
I*c*x)])/d^3 + ((I/4)*b*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sq
rt[-d] - I*Sqrt[e])*(1 - I*c*x))])/d^3 + ((I/4)*b*PolyLog[2, 1 - (2*c*(Sqr
t[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/d^3
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.76 (sec) , antiderivative size = 903, normalized size of antiderivative = 1.57

| method | result | size |
|---|---|---|
| parts | Expression too large to display | 903 |
| derivativedivides | Expression too large to display | 920 |
| default | Expression too large to display | 920 |
| risch | Expression too large to display | 1677 |

input
```
int((a+b*arctan(c*x))/x/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*a/d/(e*x^2+d)^2+1/2*a/d^2/(e*x^2+d)-1/2*a/d^3*ln(e*x^2+d)+a*ln(x)/d^3+
b*(-1/2*arctan(c*x)/d^3*ln(c^2*e*x^2+c^2*d)+1/4*c^4*arctan(c*x)/d/(c^2*e*x
^2+c^2*d)^2+1/2*c^2*arctan(c*x)/d^2/(c^2*e*x^2+c^2*d)+arctan(c*x)/d^3*ln(c
*x)-1/2*c^6*(-I/c^6/d^3*ln(c*x)*ln(1+I*c*x)+I/c^6/d^3*ln(c*x)*ln(1-I*c*x)-
I/c^6/d^3*dilog(1+I*c*x)+I/c^6/d^3*dilog(1-I*c*x)-1/d^3/c^6*(-1/2*I*(ln(c*
x-I)*ln(c^2*e*x^2+c^2*d)-2*e*(1/2*ln(c*x-I)*(ln((RootOf(e*_Z^2+2*I*e*_Z+c^
2*d-e,index=1)-c*x+I)/RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=1))+ln((RootOf(
e*_Z^2+2*I*e*_Z+c^2*d-e,index=2)-c*x+I)/RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,ind
ex=2)))/e+1/2*(dilog((RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=1)-c*x+I)/RootO
f(e*_Z^2+2*I*e*_Z+c^2*d-e,index=1))+dilog((RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,
index=2)-c*x+I)/RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=2)))/e))+1/2*I*(ln(I+
c*x)*ln(c^2*e*x^2+c^2*d)-2*e*(1/2*ln(I+c*x)*(ln((RootOf(e*_Z^2-2*I*e*_Z+c^
2*d-e,index=1)-c*x-I)/RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=1))+ln((RootOf(
e*_Z^2-2*I*e*_Z+c^2*d-e,index=2)-c*x-I)/RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,ind
ex=2)))/e+1/2*(dilog((RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=1)-c*x-I)/RootO
f(e*_Z^2-2*I*e*_Z+c^2*d-e,index=1))+dilog((RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,
index=2)-c*x-I)/RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=2)))/e)))+1/2/d^2/c^4
*(-1/(c^2*d-e)^2*e*((1/2*c^2*d-1/2*e)*c*x/(c^2*e*x^2+c^2*d)+1/2*(7*c^2*d-5
*e)/c/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2)))+(3*c^2*d-2*e)/(c^2*d-e)^2*arcta
n(c*x))))
```

## Fricas [F]

$$\int \frac{a+b\arctan(cx)}{x\left(d+ex^2\right)^3}\,dx=\int \frac{b\arctan\left(cx\right)+a}{\left(ex^2+d\right)^3x}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral((b*arctan(c*x) + a)/(e^3*x^7 + 3*d*e^2*x^5 + 3*d^2*e*x^3 + d^3*x)
, x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{x\left(d+ex^2\right)^3}\,dx=\text{Timed out}$$

input
```
integrate((a+b*atan(c*x))/x/(e*x**2+d)**3,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{a+b\arctan(cx)}{x\left(d+ex^2\right)^3}\,dx=\int \frac{b\arctan\left(cx\right)+a}{\left(ex^2+d\right)^3x}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
1/4*a*((2*e*x^2 + 3*d)/(d^2*e^2*x^4 + 2*d^3*e*x^2 + d^4) - 2*log(e*x^2 + d
)/d^3 + 4*log(x)/d^3) + 2*b*integrate(1/2*arctan(c*x)/(e^3*x^7 + 3*d*e^2*x
^5 + 3*d^2*e*x^3 + d^3*x), x)
```

## Giac [F]

$$\int \frac{a+b\arctan(cx)}{x\left(d+ex^2\right)^3}\,dx=\int \frac{b\arctan\left(cx\right)+a}{\left(ex^2+d\right)^3x}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)/((e*x^2 + d)^3*x), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{x\left(d+ex^2\right)^3}\,dx=\int \frac{a+b\,\mathrm{atan}(c\,x)}{x\,\left(e\,x^2+d\right)^3}\,dx$$

input
```
int((a + b*atan(c*x))/(x*(d + e*x^2)^3),x)
```

output
```
int((a + b*atan(c*x))/(x*(d + e*x^2)^3), x)
```

## Reduce [F]

$$\int \frac{a+b\arctan(cx)}{x\left(d+ex^2\right)^3}\,dx = \frac{4\left(\int \frac{atan(cx)}{e^3x^7+3d\,e^2x^5+3d^2e\,x^3+d^3x}dx\right)b\,d^5+8\left(\int \frac{atan(cx)}{e^3x^7+3d\,e^2x^5+3d^2e\,x^3+d^3x}dx\right)b\,d^4e\,x^2+4\left(\int \frac{atan(cx)}{e^3x^7+3d\,e^2x^5+3d^2e\,x^3+d^3x}\right.}{}$$

input
```
int((a+b*atan(c*x))/x/(e*x^2+d)^3,x)
```

output
```
(4*int(atan(c*x)/(d**3*x + 3*d**2*e*x**3 + 3*d*e**2*x**5 + e**3*x**7),x)*b
*d**5 + 8*int(atan(c*x)/(d**3*x + 3*d**2*e*x**3 + 3*d*e**2*x**5 + e**3*x**
7),x)*b*d**4*e*x**2 + 4*int(atan(c*x)/(d**3*x + 3*d**2*e*x**3 + 3*d*e**2*x
**5 + e**3*x**7),x)*b*d**3*e**2*x**4 - 2*log(d + e*x**2)*a*d**2 - 4*log(d
+ e*x**2)*a*d*e*x**2 - 2*log(d + e*x**2)*a*e**2*x**4 + 4*log(x)*a*d**2 + 8
*log(x)*a*d*e*x**2 + 4*log(x)*a*e**2*x**4 + 2*a*d**2 - a*e**2*x**4)/(4*d**
3*(d**2 + 2*d*e*x**2 + e**2*x**4))
```

# 3.1168 $\int \frac{a+b\arctan(cx)}{x^3(d+ex^2)^3}\,dx$

## Optimal result

Integrand size = 21, antiderivative size = 629

$$\int \frac{a+b\arctan(cx)}{x^3(d+ex^2)^3}\,dx = -\frac{bc}{2d^3x} - \frac{bce^2x}{8d^3(c^2d-e)(d+ex^2)} - \frac{bc^2\arctan(cx)}{2d^3} + \frac{bc^4e\arctan(cx)}{4d^2(c^2d-e)^2} + \frac{bc^2e\arctan(cx)}{d^3(c^2d-e)} - \frac{a+b\arctan(cx)}{2d^3x^2} - \frac{e(a+b\arctan(cx))}{4d^2(d+ex^2)^2} - \frac{e(a+b\arctan(cx))}{d^3(d+ex^2)} - \frac{bce^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{7/2}(c^2d-e)} - \frac{bc(3c^2d-e)e^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{7/2}(c^2d-e)^2} - \frac{3ae\log(x)}{d^4} - \frac{3e(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)}{d^4} + \frac{3e(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2d^4} + \frac{3e(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{2d^4} - \frac{3ibe\,\text{PolyLog}(2,-icx)}{2d^4} + \frac{3ibe\,\text{PolyLog}(2,icx)}{2d^4} + \frac{3ibe\,\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2d^4} - \frac{3ibe\,\text{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{4d^4} - \frac{3ibe\,\text{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{4d^4}$$

output
```
-1/2*b*c/d^3/x-1/8*b*c*e^2*x/d^3/(c^2*d-e)/(e*x^2+d)-1/2*b*c^2*arctan(c*x)
/d^3+1/4*b*c^4*e*arctan(c*x)/d^2/(c^2*d-e)^2+b*c^2*e*arctan(c*x)/d^3/(c^2*
d-e)-1/2*(a+b*arctan(c*x))/d^3/x^2-1/4*e*(a+b*arctan(c*x))/d^2/(e*x^2+d)^2
-e*(a+b*arctan(c*x))/d^3/(e*x^2+d)-b*c*e^(3/2)*arctan(e^(1/2)*x/d^(1/2))/d
^(7/2)/(c^2*d-e)-1/8*b*c*(3*c^2*d-e)*e^(3/2)*arctan(e^(1/2)*x/d^(1/2))/d^(
7/2)/(c^2*d-e)^2-3*a*e*ln(x)/d^4-3*e*(a+b*arctan(c*x))*ln(2/(1-I*c*x))/d^4
+3/2*e*(a+b*arctan(c*x))*ln(2*c*((-d)^(1/2)-e^(1/2)*x)/(c*(-d)^(1/2)-I*e^(
1/2))/(1-I*c*x))/d^4+3/2*e*(a+b*arctan(c*x))*ln(2*c*((-d)^(1/2)+e^(1/2)*x)
/(c*(-d)^(1/2)+I*e^(1/2))/(1-I*c*x))/d^4-3/2*I*b*e*polylog(2,-I*c*x)/d^4+3
/2*I*b*e*polylog(2,I*c*x)/d^4-3/4*I*b*e*polylog(2,1-2*c*((-d)^(1/2)+e^(1/2
)*x)/(c*(-d)^(1/2)+I*e^(1/2))/(1-I*c*x))/d^4-3/4*I*b*e*polylog(2,1-2*c*((-
d)^(1/2)-e^(1/2)*x)/(c*(-d)^(1/2)-I*e^(1/2))/(1-I*c*x))/d^4+3/2*I*b*e*poly
log(2,1-2/(1-I*c*x))/d^4
```

## Mathematica [A] (verified)

Time = 11.30 (sec) , antiderivative size = 723, normalized size of antiderivative = 1.15

$$\int \frac{a+b\arctan(cx)}{x^3(d+ex^2)^3}\,dx$$

$$= \frac{-a\left(\frac{d(2d^2+9dex^2+6e^2x^4)}{x^2(d+ex^2)^2}+12e\log(x)-6e\log(d+ex^2)\right)+b\left(-\frac{2cd}{x}-\frac{cde^2x}{2(c^2d-e)(d+ex^2)}+\frac{c^2d(-2c^4d^2+9c^2de-6e^2)}{(-c^2d+e)^2}\right.}{}$$

input
```
Integrate[(a + b*ArcTan[c*x])/(x^3*(d + e*x^2)^3),x]
```

output
```
(-(a*((d*(2*d^2 + 9*d*e*x^2 + 6*e^2*x^4))/(x^2*(d + e*x^2)^2) + 12*e*Log[x
] - 6*e*Log[d + e*x^2])) + b*((-2*c*d)/x - (c*d*e^2*x)/(2*(c^2*d - e)*(d +
 e*x^2)) + (c^2*d*(-2*c^4*d^2 + 9*c^2*d*e - 6*e^2)*ArcTan[c*x])/(-(c^2*d)
+ e)^2 - (d*(2*d^2 + 9*d*e*x^2 + 6*e^2*x^4)*ArcTan[c*x])/(x^2*(d + e*x^2)^
2) + (c*Sqrt[d]*e^(3/2)*(-11*c^2*d + 9*e)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*
(-(c^2*d) + e)^2) - 12*e*ArcTan[c*x]*Log[x] + 6*e*ArcTan[c*x]*(Log[((-I)*S
qrt[d])/Sqrt[e] + x] + Log[(I*Sqrt[d])/Sqrt[e] + x] - Log[d + e*x^2]) + 6*
e*ArcTan[c*x]*Log[d + e*x^2] - (6*I)*e*(Log[x]*Log[1 + I*c*x] + PolyLog[2,
 (-I)*c*x]) + (6*I)*e*(Log[x]*Log[1 - I*c*x] + PolyLog[2, I*c*x]) - (3*I)*
e*(Log[(I*Sqrt[d])/Sqrt[e] + x]*Log[(Sqrt[e]*(-1 + I*c*x))/(c*Sqrt[d] - Sq
rt[e])] + PolyLog[2, (c*(Sqrt[d] - I*Sqrt[e]*x))/(c*Sqrt[d] - Sqrt[e])]) +
 (3*I)*e*(Log[(I*Sqrt[d])/Sqrt[e] + x]*Log[(Sqrt[e]*(1 + I*c*x))/(c*Sqrt[d
] + Sqrt[e])] + PolyLog[2, (c*(Sqrt[d] - I*Sqrt[e]*x))/(c*Sqrt[d] + Sqrt[e
])]) + (3*I)*e*(Log[((-I)*Sqrt[d])/Sqrt[e] + x]*Log[(Sqrt[e]*(-1 - I*c*x))
/(c*Sqrt[d] - Sqrt[e])] + PolyLog[2, (c*(Sqrt[d] + I*Sqrt[e]*x))/(c*Sqrt[d
] - Sqrt[e])]) - (3*I)*e*(Log[((-I)*Sqrt[d])/Sqrt[e] + x]*Log[(Sqrt[e]*(1
- I*c*x))/(c*Sqrt[d] + Sqrt[e])] + PolyLog[2, (c*(Sqrt[d] + I*Sqrt[e]*x))/
(c*Sqrt[d] + Sqrt[e])]))))/(4*d^4)
```

## Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 629, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x^3(d+ex^2)^3}\,dx$$

$$\Big\downarrow \text{5515}$$

$$\int \left(\frac{3e^2x(a+b\arctan(cx))}{d^4(d+ex^2)}-\frac{3e(a+b\arctan(cx))}{d^4x}+\frac{2e^2x(a+b\arctan(cx))}{d^3(d+ex^2)^2}+\frac{a+b\arctan(cx)}{d^3x^3}+\frac{e^2x(a+b\arctan(cx))}{d^2(d+\ldots}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3e\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))}{d^4}+\frac{3e(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(1-icx)(c\sqrt{-d}-i\sqrt{e})}\right)}{2d^4}+\frac{3e(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(1-icx)(c\sqrt{-d}+i\sqrt{e})}\right)}{2d^4}-\frac{e(a+b\arctan(cx))}{d^3(d+ex^2)}-\frac{a+b\arctan(cx)}{2d^3x^2}-\frac{e(a+b\arctan(cx))}{4d^2(d+ex^2)^2}-\frac{3ae\log(x)}{d^4}-\frac{bce^{3/2}(3c^2d-e)\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{7/2}(c^2d-e)^2}-\frac{bce^{3/2}\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{7/2}(c^2d-e)}+\frac{bc^2e\arctan(cx)}{d^3(c^2d-e)}-\frac{bc^2\arctan(cx)}{2d^3}+\frac{bc^4e\arctan(cx)}{4d^2(c^2d-e)^2}-\frac{bce^2x}{8d^3(c^2d-e)(d+ex^2)}-\frac{3ibe\operatorname{PolyLog}(2,-icx)}{2d^4}+\frac{3ibe\operatorname{PolyLog}(2,icx)}{2d^4}+\frac{3ibe\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2d^4}-\frac{3ibe\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{4d^4}-\frac{3ibe\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)}{4d^4}-\frac{bc}{2d^3x}$$

input
```
Int[(a + b*ArcTan[c*x])/(x^3*(d + e*x^2)^3),x]
```

output
```
-1/2*(b*c)/(d^3*x) - (b*c*e^2*x)/(8*d^3*(c^2*d - e)*(d + e*x^2)) - (b*c^2*
ArcTan[c*x])/(2*d^3) + (b*c^4*e*ArcTan[c*x])/(4*d^2*(c^2*d - e)^2) + (b*c^
2*e*ArcTan[c*x])/(d^3*(c^2*d - e)) - (a + b*ArcTan[c*x])/(2*d^3*x^2) - (e*
(a + b*ArcTan[c*x]))/(4*d^2*(d + e*x^2)^2) - (e*(a + b*ArcTan[c*x]))/(d^3*
(d + e*x^2)) - (b*c*e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(7/2)*(c^2*d -
 e)) - (b*c*(3*c^2*d - e)*e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(7/2)*
(c^2*d - e)^2) - (3*a*e*Log[x])/d^4 - (3*e*(a + b*ArcTan[c*x])*Log[2/(1 -
I*c*x)])/d^4 + (3*e*(a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/(
(c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(2*d^4) + (3*e*(a + b*ArcTan[c*x])
*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])
/(2*d^4) - (((3*I)/2)*b*e*PolyLog[2, (-I)*c*x])/d^4 + (((3*I)/2)*b*e*PolyL
og[2, I*c*x])/d^4 + (((3*I)/2)*b*e*PolyLog[2, 1 - 2/(1 - I*c*x)])/d^4 - ((
(3*I)/4)*b*e*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*
Sqrt[e])*(1 - I*c*x))])/d^4 - (((3*I)/4)*b*e*PolyLog[2, 1 - (2*c*(Sqrt[-d]
 + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/d^4
```

**Defintions of rubi rules used**

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.14 (sec) , antiderivative size = 951, normalized size of antiderivative = 1.51

| method | result | size |
|---|---|---|
| parts | Expression too large to display | 951 |
| derivativedivides | Expression too large to display | 987 |
| default | Expression too large to display | 987 |
| risch | Expression too large to display | 1858 |

input
```
int((a+b*arctan(c*x))/x^3/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-a*e/d^3/(e*x^2+d)+3/2*a*e/d^4*ln(e*x^2+d)-1/4*a*e/d^2/(e*x^2+d)^2-1/2*a/d
^3/x^2-3*a*e*ln(x)/d^4+b*c^2*(3/2/c^2*arctan(c*x)*e/d^4*ln(c^2*e*x^2+c^2*d
)-1/4*c^2*arctan(c*x)*e/d^2/(c^2*e*x^2+c^2*d)^2-arctan(c*x)*e/d^3/(c^2*e*x
^2+c^2*d)-1/2*arctan(c*x)/d^3/c^2/x^2-3/c^2*arctan(c*x)/d^4*e*ln(c*x)-1/2*
c^6*(-6/d^4/c^8*e*(-1/2*I*ln(c*x)*ln(1+I*c*x)+1/2*I*ln(c*x)*ln(1-I*c*x)-1/
2*I*dilog(1+I*c*x)+1/2*I*dilog(1-I*c*x))+3/d^4/c^8*e*(-1/2*I*(ln(c*x-I)*ln
(c^2*e*x^2+c^2*d)-2*e*(1/2*ln(c*x-I)*(ln((RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,i
ndex=1)-c*x+I)/RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=1))+ln((RootOf(e*_Z^2+
2*I*e*_Z+c^2*d-e,index=2)-c*x+I)/RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=2)))
/e+1/2*(dilog((RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=1)-c*x+I)/RootOf(e*_Z^
2+2*I*e*_Z+c^2*d-e,index=1))+dilog((RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=2
)-c*x+I)/RootOf(e*_Z^2+2*I*e*_Z+c^2*d-e,index=2)))/e))+1/2*I*(ln(I+c*x)*ln
(c^2*e*x^2+c^2*d)-2*e*(1/2*ln(I+c*x)*(ln((RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,i
ndex=1)-c*x-I)/RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=1))+ln((RootOf(e*_Z^2-
2*I*e*_Z+c^2*d-e,index=2)-c*x-I)/RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=2)))
/e+1/2*(dilog((RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=1)-c*x-I)/RootOf(e*_Z^
2-2*I*e*_Z+c^2*d-e,index=1))+dilog((RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=2
)-c*x-I)/RootOf(e*_Z^2-2*I*e*_Z+c^2*d-e,index=2)))/e)))-1/2/d^3/c^6*((-2*c
^4*d^2+9*c^2*d*e-6*e^2)/(c^2*d-e)^2*arctan(c*x)-e^2/(c^2*d-e)^2*((1/2*c^2*
d-1/2*e)*c*x/(c^2*e*x^2+c^2*d)+1/2*(11*c^2*d-9*e)/c/(d*e)^(1/2)*arctan(...
```

## Fricas [F]

$$\int \frac{a+b\arctan(cx)}{x^3\left(d+ex^2\right)^3}\,dx=\int \frac{b\arctan\left(cx\right)+a}{\left(ex^2+d\right)^3x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^3/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral((b*arctan(c*x) + a)/(e^3*x^9 + 3*d*e^2*x^7 + 3*d^2*e*x^5 + d^3*x^
3), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{x^3\left(d+ex^2\right)^3}\,dx=\text{Timed out}$$

input
```
integrate((a+b*atan(c*x))/x**3/(e*x**2+d)**3,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{a+b\arctan(cx)}{x^3\left(d+ex^2\right)^3}\,dx=\int \frac{b\arctan\left(cx\right)+a}{\left(ex^2+d\right)^3x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^3/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
-1/4*a*((6*e^2*x^4 + 9*d*e*x^2 + 2*d^2)/(d^3*e^2*x^6 + 2*d^4*e*x^4 + d^5*x
^2) - 6*e*log(e*x^2 + d)/d^4 + 12*e*log(x)/d^4) + 2*b*integrate(1/2*arctan
(c*x)/(e^3*x^9 + 3*d*e^2*x^7 + 3*d^2*e*x^5 + d^3*x^3), x)
```

## Giac [F]

$$\int \frac{a+b\arctan(cx)}{x^3\left(d+ex^2\right)^3}\,dx=\int \frac{b\arctan\left(cx\right)+a}{\left(ex^2+d\right)^3x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^3/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)/((e*x^2 + d)^3*x^3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{x^3\left(d+ex^2\right)^3}\,dx = \int \frac{a+b\,\mathrm{atan}(c\,x)}{x^3\,\left(e\,x^2+d\right)^3}\,dx$$

input
```
int((a + b*atan(c*x))/(x^3*(d + e*x^2)^3),x)
```

output
```
int((a + b*atan(c*x))/(x^3*(d + e*x^2)^3), x)
```

## Reduce [F]

$$\int \frac{a+b\arctan(cx)}{x^3\left(d+ex^2\right)^3}\,dx = \frac{4\left(\int \frac{atan(cx)}{e^3x^9+3d\,e^2x^7+3d^2e\,x^5+d^3x^3}dx\right)b\,d^6x^2+8\left(\int \frac{atan(cx)}{e^3x^9+3d\,e^2x^7+3d^2e\,x^5+d^3x^3}dx\right)b\,d^5e\,x^4+4\left(\int \frac{atan(cx)}{e^3x^9+3d\,e^2x^7+3d^2e\,x^5}\right.}{}$$

input
```
int((a+b*atan(c*x))/x^3/(e*x^2+d)^3,x)
```

output
```
(4*int(atan(c*x)/(d**3*x**3 + 3*d**2*e*x**5 + 3*d*e**2*x**7 + e**3*x**9),x
)*b*d**6*x**2 + 8*int(atan(c*x)/(d**3*x**3 + 3*d**2*e*x**5 + 3*d*e**2*x**7
 + e**3*x**9),x)*b*d**5*e*x**4 + 4*int(atan(c*x)/(d**3*x**3 + 3*d**2*e*x**
5 + 3*d*e**2*x**7 + e**3*x**9),x)*b*d**4*e**2*x**6 + 6*log(d + e*x**2)*a*d
**2*e*x**2 + 12*log(d + e*x**2)*a*d*e**2*x**4 + 6*log(d + e*x**2)*a*e**3*x
**6 - 12*log(x)*a*d**2*e*x**2 - 24*log(x)*a*d*e**2*x**4 - 12*log(x)*a*e**3
*x**6 - 2*a*d**3 - 6*a*d**2*e*x**2 + 3*a*e**3*x**6)/(4*d**4*x**2*(d**2 + 2
*d*e*x**2 + e**2*x**4))
```

# 3.1169 $\int \frac{x^2(a+b\arctan(cx))}{(d+ex^2)^3}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 966

$$\int \frac{x^2(a+b\arctan(cx))}{(d+ex^2)^3}\,dx = \text{Too large to display}$$

output
```
1/8*b*c/(c^2*d-e)/e/(e*x^2+d)-1/4*x*(a+b*arctan(c*x))/e/(e*x^2+d)^2+1/8*x*
(a+b*arctan(c*x))/d/e/(e*x^2+d)+1/8*(a+b*arctan(c*x))*arctan(e^(1/2)*x/d^(
1/2))/d^(3/2)/e^(3/2)+1/32*I*b*c*ln(e^(1/2)*(1+(-c^2)^(1/2)*x)/(I*(-c^2)^(
1/2)*d^(1/2)+e^(1/2)))*ln(1+I*e^(1/2)*x/d^(1/2))/(-c^2)^(1/2)/d^(3/2)/e^(3
/2)-1/32*I*b*c*ln(-e^(1/2)*(1+(-c^2)^(1/2)*x)/(I*(-c^2)^(1/2)*d^(1/2)-e^(1
/2)))*ln(1-I*e^(1/2)*x/d^(1/2))/(-c^2)^(1/2)/d^(3/2)/e^(3/2)-1/32*I*b*c*ln
(-e^(1/2)*(1-(-c^2)^(1/2)*x)/(I*(-c^2)^(1/2)*d^(1/2)-e^(1/2)))*ln(1+I*e^(1
/2)*x/d^(1/2))/(-c^2)^(1/2)/d^(3/2)/e^(3/2)+1/32*I*b*c*polylog(2,(-c^2)^(1
/2)*(d^(1/2)-I*e^(1/2)*x)/((-c^2)^(1/2)*d^(1/2)-I*e^(1/2)))/(-c^2)^(1/2)/d
^(3/2)/e^(3/2)+1/16*b*c*(5*c^2*d-3*e)*ln(c^2*x^2+1)/d/(c^2*d-e)^2/e-1/4*b*
c*ln(c^2*x^2+1)/d/(c^2*d-e)/e-1/16*b*c*(5*c^2*d-3*e)*ln(e*x^2+d)/d/(c^2*d-
e)^2/e+1/4*b*c*ln(e*x^2+d)/d/(c^2*d-e)/e+1/32*I*b*c*polylog(2,(-c^2)^(1/2)
*(d^(1/2)+I*e^(1/2)*x)/((-c^2)^(1/2)*d^(1/2)-I*e^(1/2)))/(-c^2)^(1/2)/d^(3
/2)/e^(3/2)+1/32*I*b*c*ln(e^(1/2)*(1-(-c^2)^(1/2)*x)/(I*(-c^2)^(1/2)*d^(1/
2)+e^(1/2)))*ln(1-I*e^(1/2)*x/d^(1/2))/(-c^2)^(1/2)/d^(3/2)/e^(3/2)-1/32*I
*b*c*polylog(2,(-c^2)^(1/2)*(d^(1/2)-I*e^(1/2)*x)/((-c^2)^(1/2)*d^(1/2)+I*
e^(1/2)))/(-c^2)^(1/2)/d^(3/2)/e^(3/2)-1/32*I*b*c*polylog(2,(-c^2)^(1/2)*(
d^(1/2)+I*e^(1/2)*x)/((-c^2)^(1/2)*d^(1/2)+I*e^(1/2)))/(-c^2)^(1/2)/d^(3/2
)/e^(3/2)
```

## Mathematica [A] (warning: unable to verify)

Time = 12.06 (sec) , antiderivative size = 1744, normalized size of antiderivative = 1.81

$$\int \frac{x^2(a+b\arctan(cx))}{(d+ex^2)^3}\,dx = \text{Too large to display}$$

input
```
Integrate[(x^2*(a + b*ArcTan[c*x]))/(d + e*x^2)^3,x]
```

output
```
-1/4*(a*x)/(e*(d + e*x^2)^2) + (a*x)/(8*d*e*(d + e*x^2)) + (a*ArcTan[(Sqrt
[e]*x)/Sqrt[d]])/(8*d^(3/2)*e^(3/2)) + (b*c^3*((-2*Log[1 + ((c^2*d - e)*Co
s[2*ArcTan[c*x]])/(c^2*d + e)])/(c^2*d) - (2*Log[1 + ((c^2*d - e)*Cos[2*Ar
cTan[c*x]])/(c^2*d + e)])/e + ((c^2*d - e)*e*(-4*ArcTan[c*x]*ArcTanh[Sqrt[
-(c^2*d*e)]/(c*e*x)] + 2*ArcCos[-((c^2*d + e)/(c^2*d - e))]*ArcTanh[(c*e*x
)/Sqrt[-(c^2*d*e)]] - (ArcCos[-((c^2*d + e)/(c^2*d - e))] + (2*I)*ArcTanh[
(c*e*x)/Sqrt[-(c^2*d*e)]])*Log[(2*c^2*d*((-I)*e + Sqrt[-(c^2*d*e)])*(-I +
c*x))/((c^2*d - e)*(c^2*d + c*Sqrt[-(c^2*d*e)]*x))] - (ArcCos[-((c^2*d + e
)/(c^2*d - e))] - (2*I)*ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]])*Log[(2*c^2*d*(I
*e + Sqrt[-(c^2*d*e)])*(I + c*x))/((c^2*d - e)*(c^2*d + c*Sqrt[-(c^2*d*e)]
*x))] + (ArcCos[-((c^2*d + e)/(c^2*d - e))] - (2*I)*(ArcTanh[(c*d)/(Sqrt[-
(c^2*d*e)]*x)] + ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]]))*Log[(Sqrt[2]*Sqrt[-(c
^2*d*e)])/(Sqrt[c^2*d - e]*E^(I*ArcTan[c*x])*Sqrt[c^2*d + e + (c^2*d - e)*
Cos[2*ArcTan[c*x]]])] + (ArcCos[-((c^2*d + e)/(c^2*d - e))] + (2*I)*(ArcTa
nh[(c*d)/(Sqrt[-(c^2*d*e)]*x)] + ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]]))*Log[(
Sqrt[2]*Sqrt[-(c^2*d*e)]*E^(I*ArcTan[c*x]))/(Sqrt[c^2*d - e]*Sqrt[c^2*d +
e + (c^2*d - e)*Cos[2*ArcTan[c*x]]])] + I*(PolyLog[2, ((c^2*d + e - (2*I)*
Sqrt[-(c^2*d*e)])*(c^2*d - c*Sqrt[-(c^2*d*e)]*x))/((c^2*d - e)*(c^2*d + c*
Sqrt[-(c^2*d*e)]*x))] - PolyLog[2, ((c^2*d + e + (2*I)*Sqrt[-(c^2*d*e)])*(
c^2*d - c*Sqrt[-(c^2*d*e)]*x))/((c^2*d - e)*(c^2*d + c*Sqrt[-(c^2*d*e)]...
```

## Rubi [A] (verified)

Time = 2.21 (sec) , antiderivative size = 966, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\arctan(cx))}{(d+ex^2)^3}\,dx$$

$$\Big\downarrow \text{5515}$$

$$\int \left(\frac{a+b\arctan(cx)}{e\left(d+ex^2\right)^2}-\frac{d(a+b\arctan(cx))}{e\left(d+ex^2\right)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{ib\log\left(\frac{\sqrt{e}\left(1-\sqrt{-c^2}x\right)}{i\sqrt{-c^2}\sqrt{d}+\sqrt{e}}\right)\log\left(1-\frac{i\sqrt{e}x}{\sqrt{d}}\right)c}{32\sqrt{-c^2}d^{3/2}e^{3/2}}-\frac{ib\log\left(-\frac{\sqrt{e}\left(\sqrt{-c^2}x+1\right)}{i\sqrt{-c^2}\sqrt{d}-\sqrt{e}}\right)\log\left(1-\frac{i\sqrt{e}x}{\sqrt{d}}\right)c}{32\sqrt{-c^2}d^{3/2}e^{3/2}}-\frac{ib\log\left(-\frac{\sqrt{e}\left(1-\sqrt{-c^2}x\right)}{i\sqrt{-c^2}\sqrt{d}-\sqrt{e}}\right)\log\left(\frac{i\sqrt{e}x}{\sqrt{d}}+1\right)c}{32\sqrt{-c^2}d^{3/2}e^{3/2}}+\frac{ib\log\left(\frac{\sqrt{e}\left(\sqrt{-c^2}x+1\right)}{i\sqrt{-c^2}\sqrt{d}+\sqrt{e}}\right)\log\left(\frac{i\sqrt{e}x}{\sqrt{d}}+1\right)c}{32\sqrt{-c^2}d^{3/2}e^{3/2}}-\frac{b\log\left(c^2x^2+1\right)c}{4d\left(c^2d-e\right)e}+\frac{b\left(5c^2d-3e\right)\log\left(c^2x^2+1\right)c}{16d\left(c^2d-e\right)^2e}+\frac{b\log\left(ex^2+d\right)c}{4d\left(c^2d-e\right)e}-\frac{b\left(5c^2d-3e\right)\log\left(ex^2+d\right)c}{16d\left(c^2d-e\right)^2e}+\frac{ib\operatorname{PolyLog}\left(2,\frac{\sqrt{-c^2}\left(\sqrt{d}-i\sqrt{e}x\right)}{\sqrt{-c^2}\sqrt{d}-i\sqrt{e}}\right)c}{32\sqrt{-c^2}d^{3/2}e^{3/2}}-\frac{ib\operatorname{PolyLog}\left(2,\frac{\sqrt{-c^2}\left(\sqrt{d}-i\sqrt{e}x\right)}{\sqrt{-c^2}\sqrt{d}+i\sqrt{e}}\right)c}{32\sqrt{-c^2}d^{3/2}e^{3/2}}+\frac{ib\operatorname{PolyLog}\left(2,\frac{\sqrt{-c^2}\left(i\sqrt{e}x+\sqrt{d}\right)}{\sqrt{-c^2}\sqrt{d}-i\sqrt{e}}\right)c}{32\sqrt{-c^2}d^{3/2}e^{3/2}}-\frac{ib\operatorname{PolyLog}\left(2,\frac{\sqrt{-c^2}\left(i\sqrt{e}x+\sqrt{d}\right)}{\sqrt{-c^2}\sqrt{d}+i\sqrt{e}}\right)c}{32\sqrt{-c^2}d^{3/2}e^{3/2}}+\frac{bc}{8\left(c^2d-e\right)e\left(ex^2+d\right)}+\frac{x(a+b\arctan(cx))}{8de\left(ex^2+d\right)}-\frac{x(a+b\arctan(cx))}{4e\left(ex^2+d\right)^2}+\frac{(a+b\arctan(cx))\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{3/2}e^{3/2}}$$

input
```
Int[(x^2*(a + b*ArcTan[c*x]))/(d + e*x^2)^3,x]
```

output
```
(b*c)/(8*(c^2*d - e)*e*(d + e*x^2)) - (x*(a + b*ArcTan[c*x]))/(4*e*(d + e*
x^2)^2) + (x*(a + b*ArcTan[c*x]))/(8*d*e*(d + e*x^2)) + ((a + b*ArcTan[c*x
])*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(3/2)*e^(3/2)) + ((I/32)*b*c*Log[(Sqr
t[e]*(1 - Sqrt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] + Sqrt[e])]*Log[1 - (I*Sqrt
[e]*x)/Sqrt[d]])/(Sqrt[-c^2]*d^(3/2)*e^(3/2)) - ((I/32)*b*c*Log[-((Sqrt[e]
*(1 + Sqrt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] - Sqrt[e]))]*Log[1 - (I*Sqrt[e]
*x)/Sqrt[d]])/(Sqrt[-c^2]*d^(3/2)*e^(3/2)) - ((I/32)*b*c*Log[-((Sqrt[e]*(1
 - Sqrt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] - Sqrt[e]))]*Log[1 + (I*Sqrt[e]*x)
/Sqrt[d]])/(Sqrt[-c^2]*d^(3/2)*e^(3/2)) + ((I/32)*b*c*Log[(Sqrt[e]*(1 + Sq
rt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] + Sqrt[e])]*Log[1 + (I*Sqrt[e]*x)/Sqrt[
d]])/(Sqrt[-c^2]*d^(3/2)*e^(3/2)) + (b*c*(5*c^2*d - 3*e)*Log[1 + c^2*x^2])
/(16*d*(c^2*d - e)^2*e) - (b*c*Log[1 + c^2*x^2])/(4*d*(c^2*d - e)*e) - (b*
c*(5*c^2*d - 3*e)*Log[d + e*x^2])/(16*d*(c^2*d - e)^2*e) + (b*c*Log[d + e*
x^2])/(4*d*(c^2*d - e)*e) + ((I/32)*b*c*PolyLog[2, (Sqrt[-c^2]*(Sqrt[d] -
I*Sqrt[e]*x))/(Sqrt[-c^2]*Sqrt[d] - I*Sqrt[e])])/(Sqrt[-c^2]*d^(3/2)*e^(3/
2)) - ((I/32)*b*c*PolyLog[2, (Sqrt[-c^2]*(Sqrt[d] - I*Sqrt[e]*x))/(Sqrt[-c
^2]*Sqrt[d] + I*Sqrt[e])])/(Sqrt[-c^2]*d^(3/2)*e^(3/2)) + ((I/32)*b*c*Poly
Log[2, (Sqrt[-c^2]*(Sqrt[d] + I*Sqrt[e]*x))/(Sqrt[-c^2]*Sqrt[d] - I*Sqrt[e
])])/(Sqrt[-c^2]*d^(3/2)*e^(3/2)) - ((I/32)*b*c*PolyLog[2, (Sqrt[-c^2]*(Sq
rt[d] + I*Sqrt[e]*x))/(Sqrt[-c^2]*Sqrt[d] + I*Sqrt[e])])/(Sqrt[-c^2]*d^...
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

## Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 3773 vs. 2(750) = 1500.

Time = 3.33 (sec) , antiderivative size = 3774, normalized size of antiderivative = 3.91

| method | result | size |
|---|---|---|
| parts | Expression too large to display | 3774 |
| derivativedivides | Expression too large to display | 3815 |
| default | Expression too large to display | 3815 |
| risch | Expression too large to display | 6982 |

input
```
int(x^2*(a+b*arctan(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-3/8*b*c^3*arctan(c*x)^2/(c^4*d^2-2*c^2*d*e+e^2)^2*(c^2*d*e)^(1/2)-3/16*b*
c^3*polylog(2,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*d*e)^(1/2)-
e))/(c^4*d^2-2*c^2*d*e+e^2)^2*(c^2*d*e)^(1/2)-1/16*b*(d*e)^(1/2)/d^2*arcta
nh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2))/(c
^4*d^2-2*c^2*d*e+e^2)-1/8*b*c^5/(c^4*d^2-2*c^2*d*e+e^2)/(c^2*e*x^2+c^2*d)^
2*d-1/8*b*c*(c^2*d*e)^(1/2)/d/e/(c^4*d^2-2*c^2*d*e+e^2)*arctan(c*x)^2+1/8*
b*c^8/(c^4*d^2-2*c^2*d*e+e^2)/(c^2*e*x^2+c^2*d)^2*d*arctan(c*x)*x^3-1/4*b*
c^6/(c^4*d^2-2*c^2*d*e+e^2)/(c^2*e*x^2+c^2*d)^2*e*arctan(c*x)*x^3-1/16*b*(
d*e)^(1/2)/d^2*e*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)+2*c^2*d+
2*e)/c/(d*e)^(1/2))/(c^4*d^2-2*c^2*d*e+e^2)/(c^2*d-e)-1/8*b*c^4/(c^4*d^2-2
*c^2*d*e+e^2)/(c^2*e*x^2+c^2*d)^2*e*arctan(c*x)*x-3/8*I*b*c^3*ln(1-(c^2*d-
e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*d*e)^(1/2)-e))*arctan(c*x)/(c^4*
d^2-2*c^2*d*e+e^2)^2*(c^2*d*e)^(1/2)-1/8*I*b*c^5/(c^4*d^2-2*c^2*d*e+e^2)/(
c^2*e*x^2+c^2*d)^2*d*arctan(c*x)-1/16*I*b/c*e^2*ln(1-(c^2*d-e)*(1+I*c*x)^2
/(c^2*x^2+1)/(-c^2*d+2*(c^2*d*e)^(1/2)-e))*arctan(c*x)/d^2/(c^4*d^2-2*c^2*
d*e+e^2)^2*(c^2*d*e)^(1/2)-1/16*I*b*c^7*d^2*ln(1-(c^2*d-e)*(1+I*c*x)^2/(c^
2*x^2+1)/(-c^2*d+2*(c^2*d*e)^(1/2)-e))*arctan(c*x)/(c^4*d^2-2*c^2*d*e+e^2)
^2/e^2*(c^2*d*e)^(1/2)-1/8*I*b*c*(c^2*d*e)^(1/2)/d/e/(c^4*d^2-2*c^2*d*e+e^
2)*arctan(c*x)*ln(1-(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d-2*(c^2*d*e)^
(1/2)-e))-1/8*I*b*c^5/(c^4*d^2-2*c^2*d*e+e^2)/(c^2*e*x^2+c^2*d)^2/d*e^2...
```

## Fricas [F]

$$\int \frac{x^2(a+b\arctan(cx))}{(d+ex^2)^3}\,dx=\int \frac{(b\arctan(cx)+a)x^2}{(ex^2+d)^3}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral((b*x^2*arctan(c*x) + a*x^2)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2
+ d^3), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\arctan(cx))}{(d+ex^2)^3}\,dx=\text{Timed out}$$

input
```
integrate(x**2*(a+b*atan(c*x))/(e*x**2+d)**3,x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a+b\arctan(cx))}{(d+ex^2)^3}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^2*(a+b*arctan(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

## Giac [F]

$$\int \frac{x^2(a+b\arctan(cx))}{(d+ex^2)^3}\,dx=\int \frac{(b\arctan(cx)+a)x^2}{(ex^2+d)^3}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)*x^2/(e*x^2 + d)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\arctan(cx))}{(d+ex^2)^3}\,dx=\int \frac{x^2\,(a+b\,\mathrm{atan}(c\,x))}{(e\,x^2+d)^3}\,dx$$

input
```
int((x^2*(a + b*atan(c*x)))/(d + e*x^2)^3,x)
```

output
```
int((x^2*(a + b*atan(c*x)))/(d + e*x^2)^3, x)
```

## Reduce [F]

$$\int \frac{x^2(a+b\arctan(cx))}{(d+ex^2)^3}\,dx=\text{too large to display}$$

input
```
int(x^2*(a+b*atan(c*x))/(e*x^2+d)^3,x)
```

output
```
(3*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*c**8*d**6 + 6*sqrt(e)*s
qrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*c**8*d**5*e*x**2 + 3*sqrt(e)*sqrt(d
)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*c**8*d**4*e**2*x**4 - 10*sqrt(e)*sqrt(d)
*atan((e*x)/(sqrt(e)*sqrt(d)))*a*c**4*d**4*e**2 - 20*sqrt(e)*sqrt(d)*atan(
(e*x)/(sqrt(e)*sqrt(d)))*a*c**4*d**3*e**3*x**2 - 10*sqrt(e)*sqrt(d)*atan((
e*x)/(sqrt(e)*sqrt(d)))*a*c**4*d**2*e**4*x**4 + 8*sqrt(e)*sqrt(d)*atan((e*
x)/(sqrt(e)*sqrt(d)))*a*c**2*d**3*e**3 + 16*sqrt(e)*sqrt(d)*atan((e*x)/(sq
rt(e)*sqrt(d)))*a*c**2*d**2*e**4*x**2 + 8*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt
(e)*sqrt(d)))*a*c**2*d*e**5*x**4 - sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqr
t(d)))*a*d**2*e**4 - 2*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*d*e
**5*x**2 - sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*e**6*x**4 + 4*a
tan(c*x)**2*b*c**7*d**6*e + 8*atan(c*x)**2*b*c**7*d**5*e**2*x**2 + 4*atan(
c*x)**2*b*c**7*d**4*e**3*x**4 - 8*atan(c*x)**2*b*c**5*d**5*e**2 - 16*atan(
c*x)**2*b*c**5*d**4*e**3*x**2 - 8*atan(c*x)**2*b*c**5*d**3*e**4*x**4 + 4*a
tan(c*x)**2*b*c**3*d**4*e**3 + 8*atan(c*x)**2*b*c**3*d**3*e**4*x**2 + 4*at
an(c*x)**2*b*c**3*d**2*e**5*x**4 - 8*atan(c*x)*b*c**8*d**6*e*x + 16*atan(c
*x)*b*c**6*d**5*e**2*x + 8*atan(c*x)*b*c**6*d**4*e**3*x**3 - 8*atan(c*x)*b
*c**4*d**4*e**3*x - 16*atan(c*x)*b*c**4*d**3*e**4*x**3 + 8*atan(c*x)*b*c**
2*d**2*e**5*x**3 + 24*int((atan(c*x)*x**2)/(3*c**6*d**5*x**2 + 9*c**6*d**4
*e*x**4 + 9*c**6*d**3*e**2*x**6 + 3*c**6*d**2*e**3*x**8 + 3*c**4*d**5 +...
```

# 3.1170 $\int \frac{a+b\arctan(cx)}{(d+ex^2)^3}\,dx$

## Optimal result

Integrand size = 18, antiderivative size = 893

$$\int \frac{a+b\arctan(cx)}{(d+ex^2)^3}\,dx = -\frac{bc}{8d(c^2d-e)(d+ex^2)} + \frac{x(a+b\arctan(cx))}{4d(d+ex^2)^2} + \frac{3x(a+b\arctan(cx))}{8d^2(d+ex^2)} + \frac{3(a+b\arctan(cx))\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}\sqrt{e}} + \frac{3ibc\log\left(\frac{\sqrt{e}\left(1-\sqrt{-c^2}x\right)}{i\sqrt{-c^2}\sqrt{d}+\sqrt{e}}\right)\log\left(1-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{32\sqrt{-c^2}d^{5/2}\sqrt{e}} - \frac{3ibc\log\left(-\frac{\sqrt{e}\left(1+\sqrt{-c^2}x\right)}{i\sqrt{-c^2}\sqrt{d}-\sqrt{e}}\right)\log\left(1-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{32\sqrt{-c^2}d^{5/2}\sqrt{e}} - \frac{3ibc\log\left(-\frac{\sqrt{e}\left(1-\sqrt{-c^2}x\right)}{i\sqrt{-c^2}\sqrt{d}-\sqrt{e}}\right)\log\left(1+\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{32\sqrt{-c^2}d^{5/2}\sqrt{e}} + \frac{3ibc\log\left(\frac{\sqrt{e}\left(1+\sqrt{-c^2}x\right)}{i\sqrt{-c^2}\sqrt{d}+\sqrt{e}}\right)\log\left(1+\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{32\sqrt{-c^2}d^{5/2}\sqrt{e}} - \frac{bc(5c^2d-3e)\log(1+c^2x^2)}{16d^2(c^2d-e)^2} + \frac{bc(5c^2d-3e)\log(d+ex^2)}{16d^2(c^2d-e)^2} + \frac{3ibc\,\text{PolyLog}\left(2,\frac{\sqrt{-c^2}\left(\sqrt{d}-i\sqrt{e}x\right)}{\sqrt{-c^2}\sqrt{d}-i\sqrt{e}}\right)}{32\sqrt{-c^2}d^{5/2}\sqrt{e}} - \frac{3ibc\,\text{PolyLog}\left(2,\frac{\sqrt{-c^2}\left(\sqrt{d}-i\sqrt{e}x\right)}{\sqrt{-c^2}\sqrt{d}+i\sqrt{e}}\right)}{32\sqrt{-c^2}d^{5/2}\sqrt{e}} + \frac{3ibc\,\text{PolyLog}\left(2,\frac{\sqrt{-c^2}\left(\sqrt{d}+i\sqrt{e}x\right)}{\sqrt{-c^2}\sqrt{d}-i\sqrt{e}}\right)}{32\sqrt{-c^2}d^{5/2}\sqrt{e}} - \frac{3ibc\,\text{PolyLog}\left(2,\frac{\sqrt{-c^2}\left(\sqrt{d}+i\sqrt{e}x\right)}{\sqrt{-c^2}\sqrt{d}+i\sqrt{e}}\right)}{32\sqrt{-c^2}d^{5/2}\sqrt{e}}$$

output
```
-1/8*b*c/d/(c^2*d-e)/(e*x^2+d)+1/4*x*(a+b*arctan(c*x))/d/(e*x^2+d)^2+3/8*x
*(a+b*arctan(c*x))/d^2/(e*x^2+d)+3/8*(a+b*arctan(c*x))*arctan(e^(1/2)*x/d^
(1/2))/d^(5/2)/e^(1/2)+3/32*I*b*c*ln(e^(1/2)*(1-(-c^2)^(1/2)*x)/(I*(-c^2)^
(1/2)*d^(1/2)+e^(1/2)))*ln(1-I*e^(1/2)*x/d^(1/2))/(-c^2)^(1/2)/d^(5/2)/e^(
1/2)-3/32*I*b*c*ln(-e^(1/2)*(1+(-c^2)^(1/2)*x)/(I*(-c^2)^(1/2)*d^(1/2)-e^(
1/2)))*ln(1-I*e^(1/2)*x/d^(1/2))/(-c^2)^(1/2)/d^(5/2)/e^(1/2)-3/32*I*b*c*l
n(-e^(1/2)*(1-(-c^2)^(1/2)*x)/(I*(-c^2)^(1/2)*d^(1/2)-e^(1/2)))*ln(1+I*e^(
1/2)*x/d^(1/2))/(-c^2)^(1/2)/d^(5/2)/e^(1/2)+3/32*I*b*c*ln(e^(1/2)*(1+(-c^
2)^(1/2)*x)/(I*(-c^2)^(1/2)*d^(1/2)+e^(1/2)))*ln(1+I*e^(1/2)*x/d^(1/2))/(-
c^2)^(1/2)/d^(5/2)/e^(1/2)-1/16*b*c*(5*c^2*d-3*e)*ln(c^2*x^2+1)/d^2/(c^2*d
-e)^2+1/16*b*c*(5*c^2*d-3*e)*ln(e*x^2+d)/d^2/(c^2*d-e)^2+3/32*I*b*c*polylo
g(2,(-c^2)^(1/2)*(d^(1/2)-I*e^(1/2)*x)/((-c^2)^(1/2)*d^(1/2)-I*e^(1/2)))/(
-c^2)^(1/2)/d^(5/2)/e^(1/2)-3/32*I*b*c*polylog(2,(-c^2)^(1/2)*(d^(1/2)-I*e
^(1/2)*x)/((-c^2)^(1/2)*d^(1/2)+I*e^(1/2)))/(-c^2)^(1/2)/d^(5/2)/e^(1/2)+3
/32*I*b*c*polylog(2,(-c^2)^(1/2)*(d^(1/2)+I*e^(1/2)*x)/((-c^2)^(1/2)*d^(1/
2)-I*e^(1/2)))/(-c^2)^(1/2)/d^(5/2)/e^(1/2)-3/32*I*b*c*polylog(2,(-c^2)^(1
/2)*(d^(1/2)+I*e^(1/2)*x)/((-c^2)^(1/2)*d^(1/2)+I*e^(1/2)))/(-c^2)^(1/2)/d
^(5/2)/e^(1/2)
```

## Mathematica [A] (warning: unable to verify)

Time = 10.39 (sec) , antiderivative size = 1745, normalized size of antiderivative = 1.95

$$\int \frac{a+b\arctan(cx)}{(d+ex^2)^3}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*ArcTan[c*x])/(d + e*x^2)^3,x]
```

output
```
(a*x)/(4*d*(d + e*x^2)^2) + (3*a*x)/(8*d^2*(d + e*x^2)) + (3*a*ArcTan[(Sqr
t[e]*x)/Sqrt[d]])/(8*d^(5/2)*Sqrt[e]) + (b*c*(10*c^2*d*Log[1 + ((c^2*d - e
)*Cos[2*ArcTan[c*x]])/(c^2*d + e)] - 6*e*Log[1 + ((c^2*d - e)*Cos[2*ArcTan
[c*x]])/(c^2*d + e)] + (3*c^2*d*(c^2*d - e)*(-4*ArcTan[c*x]*ArcTanh[Sqrt[-
(c^2*d*e)]/(c*e*x)] + 2*ArcCos[-((c^2*d + e)/(c^2*d - e))]*ArcTanh[(c*e*x)
/Sqrt[-(c^2*d*e)]] - (ArcCos[-((c^2*d + e)/(c^2*d - e))] + (2*I)*ArcTanh[(
c*e*x)/Sqrt[-(c^2*d*e)]])*Log[(2*c^2*d*((-I)*e + Sqrt[-(c^2*d*e)])*(-I + c
*x))/((c^2*d - e)*(c^2*d + c*Sqrt[-(c^2*d*e)]*x))] - (ArcCos[-((c^2*d + e)
/(c^2*d - e))] - (2*I)*ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]])*Log[(2*c^2*d*(I*
e + Sqrt[-(c^2*d*e)])*(I + c*x))/((c^2*d - e)*(c^2*d + c*Sqrt[-(c^2*d*e)]*
x))] + (ArcCos[-((c^2*d + e)/(c^2*d - e))] - (2*I)*(ArcTanh[(c*d)/(Sqrt[-(
c^2*d*e)]*x)] + ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]]))*Log[(Sqrt[2]*Sqrt[-(c^
2*d*e)])/(Sqrt[c^2*d - e]*E^(I*ArcTan[c*x])*Sqrt[c^2*d + e + (c^2*d - e)*C
os[2*ArcTan[c*x]]])] + (ArcCos[-((c^2*d + e)/(c^2*d - e))] + (2*I)*(ArcTan
h[(c*d)/(Sqrt[-(c^2*d*e)]*x)] + ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]]))*Log[(S
qrt[2]*Sqrt[-(c^2*d*e)]*E^(I*ArcTan[c*x]))/(Sqrt[c^2*d - e]*Sqrt[c^2*d + e
 + (c^2*d - e)*Cos[2*ArcTan[c*x]]])] + I*(PolyLog[2, ((c^2*d + e - (2*I)*S
qrt[-(c^2*d*e)])*(c^2*d - c*Sqrt[-(c^2*d*e)]*x))/((c^2*d - e)*(c^2*d + c*S
qrt[-(c^2*d*e)]*x))] - PolyLog[2, ((c^2*d + e + (2*I)*Sqrt[-(c^2*d*e)])*(c
^2*d - c*Sqrt[-(c^2*d*e)]*x))/((c^2*d - e)*(c^2*d + c*Sqrt[-(c^2*d*e)]*...
```

## Rubi [A] (verified)

Time = 1.39 (sec) , antiderivative size = 875, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5447, 27, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{(d+ex^2)^3}dx$$

$$\Big\downarrow \text{5447}$$

$$-bc\int \frac{\frac{3ex^3+5dx}{d^2(ex^2+d)^2}+\frac{3\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2}\sqrt{e}}}{8(c^2x^2+1)}dx+\frac{3\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\arctan(cx))}{8d^{5/2}\sqrt{e}}+\frac{3x(a+b\arctan(cx))}{8d^2(d+ex^2)}+\frac{x(a+b\arctan(cx))}{4d(d+ex^2)^2}$$

$$\Big\downarrow \; 27$$

$$-\frac{1}{8}bc\int \frac{\frac{3ex^3+5dx}{d^2(ex^2+d)^2}+\frac{3\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2}\sqrt{e}}}{c^2x^2+1}dx+\frac{3\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\arctan(cx))}{8d^{5/2}\sqrt{e}}+\frac{3x(a+b\arctan(cx))}{8d^2(d+ex^2)}+\frac{x(a+b\arctan(cx))}{4d(d+ex^2)^2}$$

$$\Big\downarrow \; 7276$$

$$-\frac{1}{8}bc\int \left(\frac{x(3ex^2+5d)}{d^2(c^2x^2+1)(ex^2+d)^2}+\frac{3\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{5/2}\sqrt{e}(c^2x^2+1)}\right)dx+\frac{3\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\arctan(cx))}{8d^{5/2}\sqrt{e}}+\frac{3x(a+b\arctan(cx))}{8d^2(d+ex^2)}+\frac{x(a+b\arctan(cx))}{4d(d+ex^2)^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{3\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(a+b\arctan(cx))}{8d^{5/2}\sqrt{e}}+\frac{3x(a+b\arctan(cx))}{8d^2(ex^2+d)}+\frac{x(a+b\arctan(cx))}{4d(ex^2+d)^2}-\frac{1}{8}bc\left(-\frac{3i\log\left(\frac{\sqrt{e}\left(1-\sqrt{-c^2}x\right)}{i\sqrt{-c^2}\sqrt{d}+\sqrt{e}}\right)\log\left(1-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{4\sqrt{-c^2}d^{5/2}\sqrt{e}}+\frac{3i\log\left(-\frac{\sqrt{e}\left(\sqrt{-c^2}x+1\right)}{i\sqrt{-c^2}\sqrt{d}-\sqrt{e}}\right)\log\left(1-\frac{i\sqrt{e}x}{\sqrt{d}}\right)}{4\sqrt{-c^2}d^{5/2}\sqrt{e}}+\frac{3i\log\left(-\frac{\sqrt{e}\left(1-\sqrt{-c^2}\right.}{i\sqrt{-c^2}\sqrt{d}-\sqrt{}}\right.}{4\sqrt{-c^2}}\right.$$

input
```
Int[(a + b*ArcTan[c*x])/(d + e*x^2)^3,x]
```

output
```
(x*(a + b*ArcTan[c*x]))/(4*d*(d + e*x^2)^2) + (3*x*(a + b*ArcTan[c*x]))/(8
*d^2*(d + e*x^2)) + (3*(a + b*ArcTan[c*x])*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8
*d^(5/2)*Sqrt[e]) - (b*c*(1/(d*(c^2*d - e)*(d + e*x^2)) - (((3*I)/4)*Log[(
Sqrt[e]*(1 - Sqrt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] + Sqrt[e])]*Log[1 - (I*S
qrt[e]*x)/Sqrt[d]])/(Sqrt[-c^2]*d^(5/2)*Sqrt[e]) + (((3*I)/4)*Log[-((Sqrt[
e]*(1 + Sqrt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] - Sqrt[e]))]*Log[1 - (I*Sqrt[
e]*x)/Sqrt[d]])/(Sqrt[-c^2]*d^(5/2)*Sqrt[e]) + (((3*I)/4)*Log[-((Sqrt[e]*(
1 - Sqrt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] - Sqrt[e]))]*Log[1 + (I*Sqrt[e]*x
)/Sqrt[d]])/(Sqrt[-c^2]*d^(5/2)*Sqrt[e]) - (((3*I)/4)*Log[(Sqrt[e]*(1 + Sq
rt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] + Sqrt[e])]*Log[1 + (I*Sqrt[e]*x)/Sqrt[
d]])/(Sqrt[-c^2]*d^(5/2)*Sqrt[e]) + ((5*c^2*d - 3*e)*Log[1 + c^2*x^2])/(2*
d^2*(c^2*d - e)^2) - ((5*c^2*d - 3*e)*Log[d + e*x^2])/(2*d^2*(c^2*d - e)^2
) - (((3*I)/4)*PolyLog[2, (Sqrt[-c^2]*(Sqrt[d] - I*Sqrt[e]*x))/(Sqrt[-c^2]
*Sqrt[d] - I*Sqrt[e])]])/(Sqrt[-c^2]*d^(5/2)*Sqrt[e]) + (((3*I)/4)*PolyLog[
2, (Sqrt[-c^2]*(Sqrt[d] - I*Sqrt[e]*x))/(Sqrt[-c^2]*Sqrt[d] + I*Sqrt[e])])
/(Sqrt[-c^2]*d^(5/2)*Sqrt[e]) - (((3*I)/4)*PolyLog[2, (Sqrt[-c^2]*(Sqrt[d]
 + I*Sqrt[e]*x))/(Sqrt[-c^2]*Sqrt[d] - I*Sqrt[e])])/(Sqrt[-c^2]*d^(5/2)*Sq
rt[e]) + (((3*I)/4)*PolyLog[2, (Sqrt[-c^2]*(Sqrt[d] + I*Sqrt[e]*x))/(Sqrt[
-c^2]*Sqrt[d] + I*Sqrt[e])])/(Sqrt[-c^2]*d^(5/2)*Sqrt[e])))/8
```

**Defintions of rubi rules used**

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5447
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Simp[(a + b*ArcTan[c*x])   u,
x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2), x], x], x]] /; FreeQ
[{a, b, c, d, e}, x] && (IntegerQ[q] || ILtQ[q + 1/2, 0])
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

## Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4006 vs. 2(681) = 1362.

Time = 3.43 (sec) , antiderivative size = 4007, normalized size of antiderivative = 4.49

| method | result | size |
|---|---|---|
| parts | Expression too large to display | 4007 |
| derivativedivides | Expression too large to display | 4032 |
| default | Expression too large to display | 4032 |
| risch | Expression too large to display | 5059 |

input
```
int((a+b*arctan(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
3/8*b*c^4/(c^4*d^2-2*c^2*d*e+e^2)/(c^2*e*x^2+c^2*d)^2/d^2*arctan(c*x)*x^3*
e^3+5/8*b*c^4/(c^4*d^2-2*c^2*d*e+e^2)/(c^2*e*x^2+c^2*d)^2/d*arctan(c*x)*x*
e^2+5/16*b*c^2*(d*e)^(1/2)/d^2*e*arctanh(1/4*(2*(c^2*d-e)*(1+I*c*x)^2/(c^2
*x^2+1)+2*c^2*d+2*e)/c/(d*e)^(1/2))/(c^4*d^2-2*c^2*d*e+e^2)/(c^2*d-e)-3/8*
I*b*c*(c^2*d*e)^(1/2)/d^2/(c^4*d^2-2*c^2*d*e+e^2)*arctan(c*x)*ln(1-(c^2*d-
e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d-2*(c^2*d*e)^(1/2)-e))+5/4*I*b*c^7/(c^4*
d^2-2*c^2*d*e+e^2)/(c^2*e*x^2+c^2*d)^2*arctan(c*x)*x^2*e-3/4*b*c^6/(c^4*d^
2-2*c^2*d*e+e^2)/(c^2*e*x^2+c^2*d)^2/d*arctan(c*x)*x^3*e^2+3/4*b*c^5*arcta
n(c*x)^2/(c^4*d^2-2*c^2*d*e+e^2)^2*(c^2*d*e)^(1/2)+3/8*b*c^5*polylog(2,(c^
2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*d*e)^(1/2)-e))/(c^4*d^2-2*c^
2*d*e+e^2)^2*(c^2*d*e)^(1/2)+5/16*b*c^5/(c^4*d^2-2*c^2*d*e+e^2)/(c^2*d-e)*
ln(c^2*d*(1+I*c*x)^4/(c^2*x^2+1)^2+2*c^2*d*(1+I*c*x)^2/(c^2*x^2+1)-e*(1+I*
c*x)^4/(c^2*x^2+1)^2+c^2*d+2*e*(1+I*c*x)^2/(c^2*x^2+1)-e)-5/4*b*c^5/(c^4*d
^2-2*c^2*d*e+e^2)/(c^2*d-e)*ln((1+I*c*x)/(c^2*x^2+1)^(1/2))-3/32*b*c^7*pol
ylog(2,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*d*e)^(1/2)-e))/e/(
c^4*d^2-2*c^2*d*e+e^2)^2*(c^2*d*e)^(1/2)*d+3/8*b*c*e^2*polylog(2,(c^2*d-e)
*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*d*e)^(1/2)-e))/(c^4*d^2-2*c^2*d*e+
e^2)^2/d^2*(c^2*d*e)^(1/2)+3/4*b*c*e^2*arctan(c*x)^2/(c^4*d^2-2*c^2*d*e+e^
2)^2/d^2*(c^2*d*e)^(1/2)-3/32*b/c*e^3*polylog(2,(c^2*d-e)*(1+I*c*x)^2/(c^2
*x^2+1)/(-c^2*d+2*(c^2*d*e)^(1/2)-e))/(c^4*d^2-2*c^2*d*e+e^2)^2/d^3*(c^...
```

## Fricas [F]

$$\int \frac{a + b\arctan(cx)}{(d + ex^2)^3} \, dx = \int \frac{b\arctan(cx) + a}{(ex^2 + d)^3} \, dx$$

input
```
integrate((a+b*arctan(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral((b*arctan(c*x) + a)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3),
x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{a + b\arctan(cx)}{(d + ex^2)^3} \, dx = \text{Timed out}$$

input
```
integrate((a+b*atan(c*x))/(e*x**2+d)**3,x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\arctan(cx)}{(d + ex^2)^3} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arctan(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

## Giac [F]

$$\int \frac{a+b\arctan(cx)}{(d+ex^2)^3}\,dx = \int \frac{b\arctan(cx)+a}{(ex^2+d)^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)/(e*x^2 + d)^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{(d+ex^2)^3}\,dx = \int \frac{a+b\,\mathrm{atan}(c\,x)}{(e\,x^2+d)^3}\,dx$$

input
```
int((a + b*atan(c*x))/(d + e*x^2)^3,x)
```

output
```
int((a + b*atan(c*x))/(d + e*x^2)^3, x)
```

## Reduce [F]

$$\int \frac{a+b\arctan(cx)}{(d+ex^2)^3}\,dx = \text{too large to display}$$

input
```
int((a+b*atan(c*x))/(e*x^2+d)^3,x)
```

output
```
(9*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*c**8*d**6 + 18*sqrt(e)*
sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*c**8*d**5*e*x**2 + 9*sqrt(e)*sqrt(
d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*c**8*d**4*e**2*x**4 - 30*sqrt(e)*sqrt(d
)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*c**4*d**4*e**2 - 60*sqrt(e)*sqrt(d)*atan
((e*x)/(sqrt(e)*sqrt(d)))*a*c**4*d**3*e**3*x**2 - 30*sqrt(e)*sqrt(d)*atan(
(e*x)/(sqrt(e)*sqrt(d)))*a*c**4*d**2*e**4*x**4 + 24*sqrt(e)*sqrt(d)*atan((
e*x)/(sqrt(e)*sqrt(d)))*a*c**2*d**3*e**3 + 48*sqrt(e)*sqrt(d)*atan((e*x)/(
sqrt(e)*sqrt(d)))*a*c**2*d**2*e**4*x**2 + 24*sqrt(e)*sqrt(d)*atan((e*x)/(s
qrt(e)*sqrt(d)))*a*c**2*d*e**5*x**4 - 3*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e
)*sqrt(d)))*a*d**2*e**4 - 6*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*
a*d*e**5*x**2 - 3*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*e**6*x**
4 + 12*atan(c*x)**2*b*c**7*d**6*e + 24*atan(c*x)**2*b*c**7*d**5*e**2*x**2
+ 12*atan(c*x)**2*b*c**7*d**4*e**3*x**4 - 24*atan(c*x)**2*b*c**5*d**5*e**2
 - 48*atan(c*x)**2*b*c**5*d**4*e**3*x**2 - 24*atan(c*x)**2*b*c**5*d**3*e**
4*x**4 + 12*atan(c*x)**2*b*c**3*d**4*e**3 + 24*atan(c*x)**2*b*c**3*d**3*e*
*4*x**2 + 12*atan(c*x)**2*b*c**3*d**2*e**5*x**4 + 48*atan(c*x)*b*c**6*d**5
*e**2*x + 24*atan(c*x)*b*c**6*d**4*e**3*x**3 - 104*atan(c*x)*b*c**4*d**4*e
**3*x - 48*atan(c*x)*b*c**4*d**3*e**4*x**3 + 64*atan(c*x)*b*c**2*d**3*e**4
*x + 24*atan(c*x)*b*c**2*d**2*e**5*x**3 - 8*atan(c*x)*b*d**2*e**5*x + 72*i
nt((atan(c*x)*x**2)/(3*c**6*d**5*x**2 + 9*c**6*d**4*e*x**4 + 9*c**6*d**...
```

# 3.1171 $\int \frac{a+b\arctan(cx)}{x^2\left(d+ex^2\right)^3}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 1518

$$\int \frac{a+b\arctan(cx)}{x^2\left(d+ex^2\right)^3}\,dx = \text{Too large to display}$$

output
```
1/8*b*c*e/d^2/(c^2*d-e)/(e*x^2+d)-1/4*b*c*e*ln(e*x^2+d)/d^3/(c^2*d-e)+1/4*
b*c*e*ln(c^2*x^2+1)/d^3/(c^2*d-e)+7/32*I*b*c*e^(1/2)*ln(-e^(1/2)*(1-(-c^2)
^(1/2)*x)/(I*(-c^2)^(1/2)*d^(1/2)-e^(1/2)))*ln(1+I*e^(1/2)*x/d^(1/2))/(-c^
2)^(1/2)/d^(7/2)+7/32*I*b*c*e^(1/2)*ln(-e^(1/2)*(1+(-c^2)^(1/2)*x)/(I*(-c^
2)^(1/2)*d^(1/2)-e^(1/2)))*ln(1-I*e^(1/2)*x/d^(1/2))/(-c^2)^(1/2)/d^(7/2)-
7/32*I*b*c*e^(1/2)*ln(e^(1/2)*(1+(-c^2)^(1/2)*x)/(I*(-c^2)^(1/2)*d^(1/2)+e
^(1/2)))*ln(1+I*e^(1/2)*x/d^(1/2))/(-c^2)^(1/2)/d^(7/2)-7/32*I*b*c*e^(1/2)
*ln(e^(1/2)*(1-(-c^2)^(1/2)*x)/(I*(-c^2)^(1/2)*d^(1/2)+e^(1/2)))*ln(1-I*e^
(1/2)*x/d^(1/2))/(-c^2)^(1/2)/d^(7/2)+7/32*I*b*c*e^(1/2)*polylog(2,(-c^2)^
(1/2)*(d^(1/2)+I*e^(1/2)*x)/((-c^2)^(1/2)*d^(1/2)+I*e^(1/2)))/(-c^2)^(1/2)
/d^(7/2)+7/32*I*b*c*e^(1/2)*polylog(2,(-c^2)^(1/2)*(d^(1/2)-I*e^(1/2)*x)/(
(-c^2)^(1/2)*d^(1/2)+I*e^(1/2)))/(-c^2)^(1/2)/d^(7/2)-7/32*I*b*c*e^(1/2)*p
olylog(2,(-c^2)^(1/2)*(d^(1/2)+I*e^(1/2)*x)/((-c^2)^(1/2)*d^(1/2)-I*e^(1/2
)))/(-c^2)^(1/2)/d^(7/2)-7/32*I*b*c*e^(1/2)*polylog(2,(-c^2)^(1/2)*(d^(1/2
)-I*e^(1/2)*x)/((-c^2)^(1/2)*d^(1/2)-I*e^(1/2)))/(-c^2)^(1/2)/d^(7/2)-1/16
*b*c*(5*c^2*d-3*e)*e*ln(e*x^2+d)/d^3/(c^2*d-e)^2+1/16*b*c*(5*c^2*d-3*e)*e*
ln(c^2*x^2+1)/d^3/(c^2*d-e)^2-1/2*b*c*ln(c^2*x^2+1)/d^3-a*e^(1/2)*arctan(e
^(1/2)*x/d^(1/2))/d^(7/2)-7/8*e^(1/2)*(a+b*arctan(c*x))*arctan(e^(1/2)*x/d
^(1/2))/d^(7/2)+b*c*ln(x)/d^3-(a+b*arctan(c*x))/d^3/x+1/4*I*b*e^(1/2)*poly
log(2,e^(1/2)*(I+c*x)/(c*(-d)^(1/2)+I*e^(1/2)))/(-d)^(7/2)+1/4*I*b*e^(1...
```

## Mathematica [A] (verified)

Time = 12.89 (sec) , antiderivative size = 2005, normalized size of antiderivative = 1.32

$$\int \frac{a+b\arctan(cx)}{x^2(d+ex^2)^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*ArcTan[c*x])/(x^2*(d + e*x^2)^3),x]
```

output
```
-(a/(d^3*x)) - (a*e*x)/(4*d^2*(d + e*x^2)^2) - (7*a*e*x)/(8*d^3*(d + e*x^2
)) - (15*a*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(7/2)) + b*c^7*(-(Arc
Tan[c*x]/(c^7*d^3*x)) + Log[(c*x)/Sqrt[1 + c^2*x^2]]/(c^6*d^3) - (9*e*Log[
1 - ((-(c^2*d) + e)*Cos[2*ArcTan[c*x]])/(c^2*d + e)])/(16*c^4*d^2*(c^2*d -
 e)^2) + (7*e^2*Log[1 - ((-(c^2*d) + e)*Cos[2*ArcTan[c*x]])/(c^2*d + e)])/
(16*c^6*d^3*(c^2*d - e)^2) - (15*e*(4*ArcTan[c*x]*ArcTanh[(c*d)/(Sqrt[-(c^
2*d*e)]*x)] + 2*ArcCos[(-(c^2*d) - e)/(c^2*d - e)]*ArcTanh[(c*e*x)/Sqrt[-(
c^2*d*e)]] - (ArcCos[(-(c^2*d) - e)/(c^2*d - e)] - (2*I)*ArcTanh[(c*e*x)/S
qrt[-(c^2*d*e)]])*Log[1 - ((c^2*d + e - (2*I)*Sqrt[-(c^2*d*e)])*(2*c^2*d -
 2*c*Sqrt[-(c^2*d*e)]*x))/((c^2*d - e)*(2*c^2*d + 2*c*Sqrt[-(c^2*d*e)]*x))
] + (-ArcCos[(-(c^2*d) - e)/(c^2*d - e)] - (2*I)*ArcTanh[(c*e*x)/Sqrt[-(c^
2*d*e)]])*Log[1 - ((c^2*d + e + (2*I)*Sqrt[-(c^2*d*e)])*(2*c^2*d - 2*c*Sqr
t[-(c^2*d*e)]*x))/((c^2*d - e)*(2*c^2*d + 2*c*Sqrt[-(c^2*d*e)]*x))] + (Arc
Cos[(-(c^2*d) - e)/(c^2*d - e)] - (2*I)*(ArcTanh[(c*d)/(Sqrt[-(c^2*d*e)]*x
)] + ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]]))*Log[(Sqrt[2]*Sqrt[-(c^2*d*e)])/(S
qrt[c^2*d - e]*E^(I*ArcTan[c*x])*Sqrt[c^2*d + e + (c^2*d - e)*Cos[2*ArcTan
[c*x]]])] + (ArcCos[(-(c^2*d) - e)/(c^2*d - e)] + (2*I)*(ArcTanh[(c*d)/(Sq
rt[-(c^2*d*e)]*x)] + ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]]))*Log[(Sqrt[2]*Sqrt
[-(c^2*d*e)]*E^(I*ArcTan[c*x]))/(Sqrt[c^2*d - e]*Sqrt[c^2*d + e + (c^2*d -
 e)*Cos[2*ArcTan[c*x]]])] + I*(PolyLog[2, ((c^2*d + e - (2*I)*Sqrt[-(c^...
```

## Rubi [A] (verified)

Time = 2.84 (sec) , antiderivative size = 1518, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x^2\left(d+ex^2\right)^3}\,dx$$

$$\Big\downarrow \text{5515}$$

$$\int \left(-\frac{e(a+b\arctan(cx))}{d^3\left(d+ex^2\right)}+\frac{a+b\arctan(cx)}{d^3x^2}-\frac{e(a+b\arctan(cx))}{d^2\left(d+ex^2\right)^2}-\frac{e(a+b\arctan(cx))}{d\left(d+ex^2\right)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{7x(a+b\arctan(cx))e}{8d^3(ex^2+d)}-\frac{x(a+b\arctan(cx))e}{4d^2(ex^2+d)^2}+\frac{bc\log(c^2x^2+1)e}{4d^3(c^2d-e)}+\frac{bc(5c^2d-3e)\log(c^2x^2+1)e}{16d^3(c^2d-e)^2}-\frac{bc\log(ex^2+d)e}{4d^3(c^2d-e)}-\frac{bc(5c^2d-3e)\log(ex^2+d)e}{16d^3(c^2d-e)^2}+\frac{bce}{8d^2(c^2d-e)(ex^2+d)}-\frac{7(a+b\arctan(cx))\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\sqrt{e}}{8d^{7/2}}-\frac{a\arctan\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)\sqrt{e}}{d^{7/2}}-\frac{ib\log(icx+1)\log\left(\frac{c(\sqrt{-d}-\sqrt{e}x)}{c\sqrt{-d}-i\sqrt{e}}\right)\sqrt{e}}{4(-d)^{7/2}}+\frac{ib\log(1-icx)\log\left(\frac{c(\sqrt{-d}-\sqrt{e}x)}{\sqrt{-d}c+i\sqrt{e}}\right)\sqrt{e}}{4(-d)^{7/2}}-\frac{ib\log(1-icx)\log\left(\frac{c(\sqrt{e}x+\sqrt{-d})}{c\sqrt{-d}-i\sqrt{e}}\right)\sqrt{e}}{4(-d)^{7/2}}+\frac{ib\log(icx+1)\log\left(\frac{c(\sqrt{e}x+\sqrt{-d})}{\sqrt{-d}c+i\sqrt{e}}\right)\sqrt{e}}{4(-d)^{7/2}}-\frac{7ibc\log\left(\frac{\sqrt{e}\left(1-\sqrt{-c^2}x\right)}{i\sqrt{-c^2}\sqrt{d}+\sqrt{e}}\right)\log\left(1-\frac{i\sqrt{e}x}{\sqrt{d}}\right)\sqrt{e}}{32\sqrt{-c^2}d^{7/2}}+\frac{7ibc\log\left(-\frac{\sqrt{e}\left(\sqrt{-c^2}x+1\right)}{i\sqrt{-c^2}\sqrt{d}-\sqrt{e}}\right)\log\left(1-\frac{i\sqrt{e}x}{\sqrt{d}}\right)\sqrt{e}}{32\sqrt{-c^2}d^{7/2}}+\frac{7ibc\log\left(-\frac{\sqrt{e}\left(1-\sqrt{-c^2}x\right)}{i\sqrt{-c^2}\sqrt{d}-\sqrt{e}}\right)\log\left(\frac{i\sqrt{e}x}{\sqrt{d}}+1\right)\sqrt{e}}{32\sqrt{-c^2}d^{7/2}}-\frac{7ibc\log\left(\frac{\sqrt{e}\left(\sqrt{-c^2}x+1\right)}{i\sqrt{-c^2}\sqrt{d}+\sqrt{e}}\right)\log\left(\frac{i\sqrt{e}x}{\sqrt{d}}+1\right)\sqrt{e}}{32\sqrt{-c^2}d^{7/2}}+\frac{ib\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}(i-cx)}{\sqrt{-d}c+i\sqrt{e}}\right)\sqrt{e}}{4(-d)^{7/2}}-\frac{ib\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}(1-icx)}{i\sqrt{-d}c+\sqrt{e}}\right)\sqrt{e}}{4(-d)^{7/2}}-\frac{ib\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}(icx+1)}{i\sqrt{-d}c+\sqrt{e}}\right)\sqrt{e}}{4(-d)^{7/2}}+\frac{ib\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}(cx+i)}{\sqrt{-d}c+i\sqrt{e}}\right)\sqrt{e}}{4(-d)^{7/2}}-\frac{7ibc\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-c^2}\left(\sqrt{d}-i\sqrt{e}x\right)}{\sqrt{-c^2}\sqrt{d}-i\sqrt{e}}\right)\sqrt{e}}{32\sqrt{-c^2}d^{7/2}}+\frac{7ibc\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-c^2}\left(\sqrt{d}-i\sqrt{e}x\right)}{\sqrt{-c^2}\sqrt{d}+i\sqrt{e}}\right)\sqrt{e}}{32\sqrt{-c^2}d^{7/2}}-\frac{7ibc\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-c^2}\left(i\sqrt{e}x+\sqrt{d}\right)}{\sqrt{-c^2}\sqrt{d}-i\sqrt{e}}\right)\sqrt{e}}{32\sqrt{-c^2}d^{7/2}}+\frac{7ibc\,\mathrm{PolyLog}\left(2,\frac{\sqrt{-c^2}\left(i\sqrt{e}x+\sqrt{d}\right)}{\sqrt{-c^2}\sqrt{d}+i\sqrt{e}}\right)\sqrt{e}}{32\sqrt{-c^2}d^{7/2}}-\frac{a+b\arctan(cx)}{d^3x}+\frac{bc\log(x)}{d^3}-\frac{bc\log(c^2x^2+1)}{2d^3}$$

input
```
Int[(a + b*ArcTan[c*x])/(x^2*(d + e*x^2)^3),x]
```

output
```
(b*c*e)/(8*d^2*(c^2*d - e)*(d + e*x^2)) - (a + b*ArcTan[c*x])/(d^3*x) - (e
*x*(a + b*ArcTan[c*x]))/(4*d^2*(d + e*x^2)^2) - (7*e*x*(a + b*ArcTan[c*x])
)/(8*d^3*(d + e*x^2)) - (a*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/d^(7/2) -
(7*Sqrt[e]*(a + b*ArcTan[c*x])*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(7/2)) +
(b*c*Log[x])/d^3 - ((I/4)*b*Sqrt[e]*Log[1 + I*c*x]*Log[(c*(Sqrt[-d] - Sqrt
[e]*x))/(c*Sqrt[-d] - I*Sqrt[e])])/(-d)^(7/2) + ((I/4)*b*Sqrt[e]*Log[1 - I
*c*x]*Log[(c*(Sqrt[-d] - Sqrt[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])])/(-d)^(7/2)
 - ((I/4)*b*Sqrt[e]*Log[1 - I*c*x]*Log[(c*(Sqrt[-d] + Sqrt[e]*x))/(c*Sqrt[
-d] - I*Sqrt[e])])/(-d)^(7/2) + ((I/4)*b*Sqrt[e]*Log[1 + I*c*x]*Log[(c*(Sq
rt[-d] + Sqrt[e]*x))/(c*Sqrt[-d] + I*Sqrt[e])])/(-d)^(7/2) - (((7*I)/32)*b
*c*Sqrt[e]*Log[(Sqrt[e]*(1 - Sqrt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] + Sqrt[e
])]*Log[1 - (I*Sqrt[e]*x)/Sqrt[d]])/(Sqrt[-c^2]*d^(7/2)) + (((7*I)/32)*b*c
*Sqrt[e]*Log[-((Sqrt[e]*(1 + Sqrt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] - Sqrt[e
]))]*Log[1 - (I*Sqrt[e]*x)/Sqrt[d]])/(Sqrt[-c^2]*d^(7/2)) + (((7*I)/32)*b*
c*Sqrt[e]*Log[-((Sqrt[e]*(1 - Sqrt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] - Sqrt[
e]))]*Log[1 + (I*Sqrt[e]*x)/Sqrt[d]])/(Sqrt[-c^2]*d^(7/2)) - (((7*I)/32)*b
*c*Sqrt[e]*Log[(Sqrt[e]*(1 + Sqrt[-c^2]*x))/(I*Sqrt[-c^2]*Sqrt[d] + Sqrt[e
])]*Log[1 + (I*Sqrt[e]*x)/Sqrt[d]])/(Sqrt[-c^2]*d^(7/2)) - (b*c*Log[1 + c^
2*x^2])/(2*d^3) + (b*c*(5*c^2*d - 3*e)*e*Log[1 + c^2*x^2])/(16*d^3*(c^2*d
- e)^2) + (b*c*e*Log[1 + c^2*x^2])/(4*d^3*(c^2*d - e)) - (b*c*(5*c^2*d ...
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.14 (sec) , antiderivative size = 5730, normalized size of antiderivative = 3.77

| method | result | size |
|---|---|---|
| parts | Expression too large to display | 5730 |
| derivativedivides | Expression too large to display | 5776 |
| default | Expression too large to display | 5776 |
| risch | Expression too large to display | 7503 |

input
```
int((a+b*arctan(c*x))/x^2/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

## Fricas [F]

$$\int \frac{a+b\arctan(cx)}{x^2\,(d+ex^2)^3}\,dx = \int \frac{b\arctan(cx)+a}{(ex^2+d)^3x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^2/(e*x^2+d)^3,x, algorithm="fricas")
```

output
```
integral((b*arctan(c*x) + a)/(e^3*x^8 + 3*d*e^2*x^6 + 3*d^2*e*x^4 + d^3*x^
2), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{x^2\,(d+ex^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((a+b*atan(c*x))/x**2/(e*x**2+d)**3,x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\arctan(cx)}{x^2\left(d+ex^2\right)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arctan(c*x))/x^2/(e*x^2+d)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

## Giac [F]

$$\int \frac{a+b\arctan(cx)}{x^2\left(d+ex^2\right)^3}\,dx = \int \frac{b\arctan\left(cx\right)+a}{\left(ex^2+d\right)^3x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^2/(e*x^2+d)^3,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)/((e*x^2 + d)^3*x^2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{x^2\left(d+ex^2\right)^3}\,dx = \int \frac{a+b\,\mathrm{atan}(c\,x)}{x^2\,\left(e\,x^2+d\right)^3}\,dx$$

input
```
int((a + b*atan(c*x))/(x^2*(d + e*x^2)^3),x)
```

output
```
int((a + b*atan(c*x))/(x^2*(d + e*x^2)^3), x)
```

## Reduce [F]

$$\int \frac{a+b\arctan(cx)}{x^2\left(d+ex^2\right)^3}\,dx = \text{too large to display}$$

input
```
int((a+b*atan(c*x))/x^2/(e*x^2+d)^3,x)
```

output
```
( - 225*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*c**8*d**6*x - 450*
sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*c**8*d**5*e*x**3 - 225*sqr
t(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*c**8*d**4*e**2*x**5 + 600*sqr
t(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*c**6*d**5*e*x + 1200*sqrt(e)*
sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*c**6*d**4*e**2*x**3 + 600*sqrt(e)*
sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*c**6*d**3*e**3*x**5 - 570*sqrt(e)*
sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*c**4*d**4*e**2*x - 1140*sqrt(e)*sq
rt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*c**4*d**3*e**3*x**3 - 570*sqrt(e)*sq
rt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*c**4*d**2*e**4*x**5 + 240*sqrt(e)*sq
rt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*c**2*d**3*e**3*x + 480*sqrt(e)*sqrt(
d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*c**2*d**2*e**4*x**3 + 240*sqrt(e)*sqrt(
d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*c**2*d*e**5*x**5 - 45*sqrt(e)*sqrt(d)*a
tan((e*x)/(sqrt(e)*sqrt(d)))*a*d**2*e**4*x - 90*sqrt(e)*sqrt(d)*atan((e*x)
/(sqrt(e)*sqrt(d)))*a*d*e**5*x**3 - 45*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)
*sqrt(d)))*a*e**6*x**5 - 60*atan(c*x)**2*b*c**9*d**7*x - 120*atan(c*x)**2*
b*c**9*d**6*e*x**3 - 60*atan(c*x)**2*b*c**9*d**5*e**2*x**5 + 120*atan(c*x)
**2*b*c**7*d**6*e*x + 240*atan(c*x)**2*b*c**7*d**5*e**2*x**3 + 120*atan(c*
x)**2*b*c**7*d**4*e**3*x**5 - 60*atan(c*x)**2*b*c**5*d**5*e**2*x - 120*ata
n(c*x)**2*b*c**5*d**4*e**3*x**3 - 60*atan(c*x)**2*b*c**5*d**3*e**4*x**5 -
120*atan(c*x)*b*c**8*d**7 - 240*atan(c*x)*b*c**8*d**6*e*x**2 - 120*atan...
```

# 3.1172 $\int x^3\sqrt{d+ex^2}(a+b\arctan(cx))\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 223

$$\begin{aligned}\int x^3\sqrt{d+ex^2}(a+b\arctan(cx))\,dx=&-\frac{b(c^2d-12e)\,x\sqrt{d+ex^2}}{120c^3e}-\frac{bx(d+ex^2)^{3/2}}{20ce}\\&-\frac{d(d+ex^2)^{3/2}(a+b\arctan(cx))}{3e^2}\\&+\frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{5e^2}\\&+\frac{b(c^2d-e)^{3/2}(2c^2d+3e)\arctan\left(\frac{\sqrt{c^2d-e}x}{\sqrt{d+ex^2}}\right)}{15c^5e^2}\\&+\frac{b(15c^4d^2+20c^2de-24e^2)\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{120c^5e^{3/2}}\end{aligned}$$

output
```
-1/120*b*(c^2*d-12*e)*x*(e*x^2+d)^(1/2)/c^3/e-1/20*b*x*(e*x^2+d)^(3/2)/c/e
-1/3*d*(e*x^2+d)^(3/2)*(a+b*arctan(c*x))/e^2+1/5*(e*x^2+d)^(5/2)*(a+b*arct
an(c*x))/e^2+1/15*b*(c^2*d-e)^(3/2)*(2*c^2*d+3*e)*arctan((c^2*d-e)^(1/2)*x
/(e*x^2+d)^(1/2))/c^5/e^2+1/120*b*(15*c^4*d^2+20*c^2*d*e-24*e^2)*arctanh(e
^(1/2)*x/(e*x^2+d)^(1/2))/c^5/e^(3/2)
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.75

$$\int x^3\sqrt{d+ex^2}(a+b\arctan(cx))\,dx$$

$$= \frac{-c^2\sqrt{d+ex^2}(8ac^3(2d^2-dex^2-3e^2x^4)+bex(-12e+c^2(7d+6ex^2)))-8bc^5\sqrt{d+ex^2}(2d^2-dex^2-3}{}$$

input
```
Integrate[x^3*Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]),x]
```

output
```
(-(c^2*Sqrt[d + e*x^2]*(8*a*c^3*(2*d^2 - d*e*x^2 - 3*e^2*x^4) + b*e*x*(-12
*e + c^2*(7*d + 6*e*x^2)))) - 8*b*c^5*Sqrt[d + e*x^2]*(2*d^2 - d*e*x^2 - 3
*e^2*x^4)*ArcTan[c*x] - (4*I)*b*(c^2*d - e)^(3/2)*(2*c^2*d + 3*e)*Log[((-6
0*I)*c^6*e^2*(c*d - I*e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(c^2*d -
e)^(5/2)*(2*c^2*d + 3*e)*(I + c*x))] + (4*I)*b*(c^2*d - e)^(3/2)*(2*c^2*d
+ 3*e)*Log[((60*I)*c^6*e^2*(c*d + I*e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2])
)/(b*(c^2*d - e)^(5/2)*(2*c^2*d + 3*e)*(-I + c*x))] + b*Sqrt[e]*(15*c^4*d^
2 + 20*c^2*d*e - 24*e^2)*Log[e*x + Sqrt[e]*Sqrt[d + e*x^2]])/(120*c^5*e^2)
```

## Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {5511, 27, 403, 403, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{d+ex^2}(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{ 5511}$$

$$-bc\int -\frac{(2d-3ex^2)\left(ex^2+d\right)^{3/2}}{15e^2\left(c^2x^2+1\right)}dx+\frac{\left(d+ex^2\right)^{5/2}(a+b\arctan(cx))}{5e^2}-\frac{d\left(d+ex^2\right)^{3/2}(a+b\arctan(cx))}{3e^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{bc\int \frac{(2d-3ex^2)(ex^2+d)^{3/2}}{c^2x^2+1}dx}{15e^2}+\frac{\left(d+ex^2\right)^{5/2}(a+b\arctan(cx))}{5e^2}-\frac{d\left(d+ex^2\right)^{3/2}(a+b\arctan(cx))}{3e^2}$$

$$\Big\downarrow \text{403}$$

$$\frac{bc\left(\frac{\int \frac{\sqrt{ex^2+d}\left(d\left(8dc^2+3e\right)-\left(c^2d-12e\right)ex^2\right)}{c^2x^2+1}dx}{4c^2}-\frac{3ex(d+ex^2)^{3/2}}{4c^2}\right)}{15e^2}+\frac{\left(d+ex^2\right)^{5/2}(a+b\arctan(cx))}{5e^2}-\frac{d\left(d+ex^2\right)^{3/2}(a+b\arctan(cx))}{3e^2}$$

$$\Big\downarrow \text{403}$$

$$\frac{bc\left(\frac{\frac{\int \frac{e\left(15d^2c^4+20dec^2-24e^2\right)x^2+d\left(16d^2c^4+7dec^2-12e^2\right)}{\left(c^2x^2+1\right)\sqrt{ex^2+d}}dx}{2c^2}-\frac{ex\left(c^2d-12e\right)\sqrt{d+ex^2}}{2c^2}}{4c^2}-\frac{3ex(d+ex^2)^{3/2}}{4c^2}\right)}{15e^2}+\frac{\left(d+ex^2\right)^{5/2}(a+b\arctan(cx))}{5e^2}-\frac{d\left(d+ex^2\right)^{3/2}(a+b\arctan(cx))}{3e^2}$$

$$\Big\downarrow \text{398}$$

$$\frac{bc\left(\frac{\frac{\frac{8\left(2c^2d+3e\right)\left(c^2d-e\right)^2\int \frac{1}{\left(c^2x^2+1\right)\sqrt{ex^2+d}}dx}{c^2}+\frac{e\left(15c^4d^2+20c^2de-24e^2\right)\int \frac{1}{\sqrt{ex^2+d}}dx}{c^2}}{2c^2}-\frac{ex\left(c^2d-12e\right)\sqrt{d+ex^2}}{2c^2}}{4c^2}-\frac{3ex(d+ex^2)^{3/2}}{4c^2}\right)}{15e^2}+\frac{\left(d+ex^2\right)^{5/2}(a+b\arctan(cx))}{5e^2}-\frac{d\left(d+ex^2\right)^{3/2}(a+b\arctan(cx))}{3e^2}$$

$$\Big\downarrow \text{224}$$

$$\frac{bc\left(\frac{\frac{\frac{8(2c^2d+3e)(c^2d-e)^2\int\frac{1}{(c^2x^2+1)\sqrt{ex^2+d}}dx}{c^2}+\frac{e(15c^4d^2+20c^2de-24e^2)\int\frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{c^2}}{2c^2}-\frac{ex(c^2d-12e)\sqrt{d+ex^2}}{2c^2}}{4c^2}-\frac{3ex(d+ex^2)^{3/2}}{4c^2}\right)}{15e^2}+\frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{5e^2}-\frac{d(d+ex^2)^{3/2}(a+b\arctan(cx))}{3e^2}$$

$$\Big\downarrow \; 219$$

$$\frac{bc\left(\frac{\frac{\frac{8(2c^2d+3e)(c^2d-e)^2\int\frac{1}{(c^2x^2+1)\sqrt{ex^2+d}}dx}{c^2}+\frac{\sqrt{e}(15c^4d^2+20c^2de-24e^2)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2}}{2c^2}-\frac{ex(c^2d-12e)\sqrt{d+ex^2}}{2c^2}}{4c^2}-\frac{3ex(d+ex^2)^{3/2}}{4c^2}\right)}{15e^2}+\frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{5e^2}-\frac{d(d+ex^2)^{3/2}(a+b\arctan(cx))}{3e^2}$$

$$\Big\downarrow \; 291$$

$$\frac{bc\left(\frac{\frac{\frac{8(2c^2d+3e)(c^2d-e)^2\int\frac{1}{1-\frac{(e-c^2d)x^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{c^2}+\frac{\sqrt{e}(15c^4d^2+20c^2de-24e^2)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2}}{2c^2}-\frac{ex(c^2d-12e)\sqrt{d+ex^2}}{2c^2}}{4c^2}-\frac{3ex(d+ex^2)^{3/2}}{4c^2}\right)}{15e^2}+\frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{5e^2}-\frac{d(d+ex^2)^{3/2}(a+b\arctan(cx))}{3e^2}$$

$$\Big\downarrow \; 216$$

$$\frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{5e^2}-\frac{d(d+ex^2)^{3/2}(a+b\arctan(cx))}{3e^2}+\frac{bc\left(\frac{\frac{\frac{8(2c^2d+3e)(c^2d-e)^{3/2}\arctan\left(\frac{x\sqrt{c^2d-e}}{\sqrt{d+ex^2}}\right)}{c^2}+\frac{\sqrt{e}(15c^4d^2+20c^2de-24e^2)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2}}{2c^2}-\frac{ex(c^2d-12e)\sqrt{d+ex^2}}{2c^2}}{4c^2}-\frac{3ex(d+ex^2)^{3/2}}{4c^2}\right)}{15e^2}$$

input
```
Int[x^3*Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]),x]
```

output
```
-1/3*(d*(d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/e^2 + ((d + e*x^2)^(5/2)*(a
 + b*ArcTan[c*x]))/(5*e^2) + (b*c*((-3*e*x*(d + e*x^2)^(3/2))/(4*c^2) + (-
1/2*((c^2*d - 12*e)*e*x*Sqrt[d + e*x^2])/c^2 + ((8*(c^2*d - e)^(3/2)*(2*c^
2*d + 3*e)*ArcTan[(Sqrt[c^2*d - e]*x)/Sqrt[d + e*x^2]])/c^2 + (Sqrt[e]*(15
*c^4*d^2 + 20*c^2*d*e - 24*e^2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/c^2)
/(2*c^2))/(4*c^2)))/(15*e^2)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

## Maple [F]

$$\int x^{3} \sqrt{e\,x^{2}+d}\,(a+b\arctan (c x))\,dx$$

input
```
int(x^3*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x)
```

output
```
int(x^3*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x)
```

## Fricas [A] (verification not implemented)

Time = 2.29 (sec) , antiderivative size = 1200, normalized size of antiderivative = 5.38

$$\int x^3\sqrt{d+ex^2}(a+b\arctan(cx))\,dx=\text{Too large to display}$$

input
```
integrate(x^3*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
[-1/240*((15*b*c^4*d^2 + 20*b*c^2*d*e - 24*b*e^2)*sqrt(e)*log(-2*e*x^2 + 2
*sqrt(e*x^2 + d)*sqrt(e)*x - d) + 4*(2*b*c^4*d^2 + b*c^2*d*e - 3*b*e^2)*sq
rt(-c^2*d + e)*log((((c^4*d^2 - 8*c^2*d*e + 8*e^2)*x^4 - 2*(3*c^2*d^2 - 4*d
*e)*x^2 - 4*((c^2*d - 2*e)*x^3 - d*x)*sqrt(-c^2*d + e)*sqrt(e*x^2 + d) + d
^2)/(c^4*x^4 + 2*c^2*x^2 + 1)) - 2*(24*a*c^5*e^2*x^4 + 8*a*c^5*d*e*x^2 - 6
*b*c^4*e^2*x^3 - 16*a*c^5*d^2 - (7*b*c^4*d*e - 12*b*c^2*e^2)*x + 8*(3*b*c^
5*e^2*x^4 + b*c^5*d*e*x^2 - 2*b*c^5*d^2)*arctan(c*x))*sqrt(e*x^2 + d))/(c^
5*e^2), 1/240*(8*(2*b*c^4*d^2 + b*c^2*d*e - 3*b*e^2)*sqrt(c^2*d - e)*arcta
n(1/2*sqrt(c^2*d - e)*((c^2*d - 2*e)*x^2 - d)*sqrt(e*x^2 + d)/((c^2*d*e -
e^2)*x^3 + (c^2*d^2 - d*e)*x)) - (15*b*c^4*d^2 + 20*b*c^2*d*e - 24*b*e^2)*
sqrt(e)*log(-2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x - d) + 2*(24*a*c^5*e^2*
x^4 + 8*a*c^5*d*e*x^2 - 6*b*c^4*e^2*x^3 - 16*a*c^5*d^2 - (7*b*c^4*d*e - 12
*b*c^2*e^2)*x + 8*(3*b*c^5*e^2*x^4 + b*c^5*d*e*x^2 - 2*b*c^5*d^2)*arctan(c
*x))*sqrt(e*x^2 + d))/(c^5*e^2), -1/120*((15*b*c^4*d^2 + 20*b*c^2*d*e - 24
*b*e^2)*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) + 2*(2*b*c^4*d^2 + b*c
^2*d*e - 3*b*e^2)*sqrt(-c^2*d + e)*log((((c^4*d^2 - 8*c^2*d*e + 8*e^2)*x^4
 - 2*(3*c^2*d^2 - 4*d*e)*x^2 - 4*((c^2*d - 2*e)*x^3 - d*x)*sqrt(-c^2*d + e)
*sqrt(e*x^2 + d) + d^2)/(c^4*x^4 + 2*c^2*x^2 + 1)) - (24*a*c^5*e^2*x^4 + 8
*a*c^5*d*e*x^2 - 6*b*c^4*e^2*x^3 - 16*a*c^5*d^2 - (7*b*c^4*d*e - 12*b*c^2*
e^2)*x + 8*(3*b*c^5*e^2*x^4 + b*c^5*d*e*x^2 - 2*b*c^5*d^2)*arctan(c*x))...
```

## Sympy [F]

$$\int x^3\sqrt{d+ex^2}(a+b\arctan(cx))\,dx=\int x^3(a+b\,\mathrm{atan}\,(cx))\,\sqrt{d+ex^2}\,dx$$

input
```
integrate(x**3*(e*x**2+d)**(1/2)*(a+b*atan(c*x)),x)
```

output
```
Integral(x**3*(a + b*atan(c*x))*sqrt(d + e*x**2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int x^3\sqrt{d+ex^2}(a+b\arctan(cx))\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

## Giac [F(-2)]

Exception generated.

$$\int x^3\sqrt{d+ex^2}(a+b\arctan(cx))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:index.cc index_m i_lex_is_greater Error: Bad Argument V
alue
```

## Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{d+ex^2}(a+b\arctan(cx))\,dx = \int x^3\,(a+b\,\mathrm{atan}(c\,x))\,\sqrt{e\,x^2+d}\,dx$$

input
```
int(x^3*(a + b*atan(c*x))*(d + e*x^2)^(1/2),x)
```

output
```
int(x^3*(a + b*atan(c*x))*(d + e*x^2)^(1/2), x)
```

## Reduce [F]

$$\int x^3\sqrt{d+ex^2}(a+b\arctan(cx))\,dx$$
$$=\frac{-2\sqrt{e\,x^2+d}\,a\,d^2+\sqrt{e\,x^2+d}\,ade\,x^2+3\sqrt{e\,x^2+d}\,a\,e^2x^4+15\left(\int\sqrt{e\,x^2+d}\,atan(cx)\,x^3dx\right)b\,e^2}{15e^2}$$

input
```
int(x^3*(e*x^2+d)^(1/2)*(a+b*atan(c*x)),x)
```

output
```
( - 2*sqrt(d + e*x**2)*a*d**2 + sqrt(d + e*x**2)*a*d*e*x**2 + 3*sqrt(d + e
*x**2)*a*e**2*x**4 + 15*int(sqrt(d + e*x**2)*atan(c*x)*x**3,x)*b*e**2)/(15
*e**2)
```

# 3.1173 $\int x^2\sqrt{d+ex^2}(a+b\arctan(cx))\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 23

$$\int x^2\sqrt{d+ex^2}(a+b\arctan(cx))\,dx = \frac{adx\sqrt{d+ex^2}}{8e}+\frac{1}{4}ax^3\sqrt{d+ex^2} - \frac{ad^2\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{8e^{3/2}} + b\text{Int}\left(x^2\sqrt{d+ex^2}\arctan(cx),x\right)$$

output
```
1/8*a*d*x*(e*x^2+d)^(1/2)/e+1/4*a*x^3*(e*x^2+d)^(1/2)-1/8*a*d^2*arctanh(e^
(1/2)*x/(e*x^2+d)^(1/2))/e^(3/2)+b*Defer(Int)(x^2*(e*x^2+d)^(1/2)*arctan(c
*x),x)
```

## Mathematica [N/A]

Not integrable

Time = 12.32 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int x^2\sqrt{d+ex^2}(a+b\arctan(cx))\,dx = \int x^2\sqrt{d+ex^2}(a+b\arctan(cx))\,dx$$

input
```
Integrate[x^2*Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]),x]
```

output
```
Integrate[x^2*Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{d+ex^2}(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5517}$$

$$a\int x^2\sqrt{ex^2+d}dx+b\int x^2\sqrt{ex^2+d}\arctan(cx)dx$$

$$\Big\downarrow \text{248}$$

$$a\left(\frac{1}{4}d\int \frac{x^2}{\sqrt{ex^2+d}}dx+\frac{1}{4}x^3\sqrt{d+ex^2}\right)+b\int x^2\sqrt{ex^2+d}\arctan(cx)dx$$

$$\Big\downarrow \text{262}$$

$$a\left(\frac{1}{4}d\left(\frac{x\sqrt{d+ex^2}}{2e}-\frac{d\int \frac{1}{\sqrt{ex^2+d}}dx}{2e}\right)+\frac{1}{4}x^3\sqrt{d+ex^2}\right)+b\int x^2\sqrt{ex^2+d}\arctan(cx)dx$$

$$\Big\downarrow \text{224}$$

$$a\left(\frac{1}{4}d\left(\frac{x\sqrt{d+ex^2}}{2e}-\frac{d\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{2e}\right)+\frac{1}{4}x^3\sqrt{d+ex^2}\right)+ b\int x^2\sqrt{ex^2+d}\arctan(cx)dx$$

$$\Big\downarrow \text{219}$$

$$b\int x^2\sqrt{ex^2+d}\arctan(cx)dx+$$
$$a\left(\frac{1}{4}d\left(\frac{x\sqrt{d+ex^2}}{2e}-\frac{d\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2e^{3/2}}\right)+\frac{1}{4}x^3\sqrt{d+ex^2}\right)$$
$$\Big\downarrow \ 5560$$
$$b\int x^2\sqrt{ex^2+d}\arctan(cx)dx+$$
$$a\left(\frac{1}{4}d\left(\frac{x\sqrt{d+ex^2}}{2e}-\frac{d\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2e^{3/2}}\right)+\frac{1}{4}x^3\sqrt{d+ex^2}\right)$$

input
```
Int[x^2*Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int x^2\sqrt{e\,x^2+d}\,(a+b\arctan(cx))\,dx$$

input
```
int(x^2*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x)
```

output
```
int(x^2*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x)
```

## Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int x^2\sqrt{d+ex^2}(a+b\arctan(cx))\,dx=\int \sqrt{ex^2+d}(b\arctan(cx)+a)x^2\,dx$$

input
```
integrate(x^2*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
integral((b*x^2*arctan(c*x) + a*x^2)*sqrt(e*x^2 + d), x)
```

## Sympy [N/A]

Not integrable

Time = 21.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int x^2\sqrt{d+ex^2}(a+b\arctan(cx))\,dx=\int x^2(a+b\,\mathrm{atan}\,(cx))\sqrt{d+ex^2}\,dx$$

input
```
integrate(x**2*(e*x**2+d)**(1/2)*(a+b*atan(c*x)),x)
```

output
```
Integral(x**2*(a + b*atan(c*x))*sqrt(d + e*x**2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int x^2\sqrt{d+ex^2}(a+b\arctan(cx))\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^2*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

## Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x^2\sqrt{d+ex^2}(a+b\arctan(cx))\,dx=\int\sqrt{ex^2+d}(b\arctan{(cx)}+a)x^2\,dx$$

input
```
integrate(x^2*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arctan(c*x) + a)*x^2, x)
```

## Mupad [N/A]

Not integrable

Time = 0.96 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x^2\sqrt{d+ex^2}(a+b\arctan(cx))\,dx=\int x^2\,\left(a+b\,\mathrm{atan}(c\,x)\right)\,\sqrt{e\,x^2+d}\,dx$$

input
```
int(x^2*(a + b*atan(c*x))*(d + e*x^2)^(1/2),x)
```

output
```
int(x^2*(a + b*atan(c*x))*(d + e*x^2)^(1/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 87, normalized size of antiderivative = 3.78

$$\int x^2\sqrt{d+ex^2}(a+b\arctan(cx))\,dx$$
$$= \frac{\sqrt{e\,x^2+d}\,adex+2\sqrt{e\,x^2+d}\,a\,e^2x^3-\sqrt{e}\log\left(\frac{\sqrt{e\,x^2+d}+\sqrt{e}\,x}{\sqrt{d}}\right)a\,d^2+8\left(\int\sqrt{e\,x^2+d}\,atan(cx)\,x^2dx\right)b\,e^2}{8e^2}$$

input
```
int(x^2*(e*x^2+d)^(1/2)*(a+b*atan(c*x)),x)
```

output
```
(sqrt(d + e*x**2)*a*d*e*x + 2*sqrt(d + e*x**2)*a*e**2*x**3 - sqrt(e)*log((
sqrt(d + e*x**2) + sqrt(e)*x)/sqrt(d))*a*d**2 + 8*int(sqrt(d + e*x**2)*ata
n(c*x)*x**2,x)*b*e**2)/(8*e**2)
```

# 3.1174 $\int x\sqrt{d+ex^2}(a+b\arctan(cx))\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 140

$$\int x\sqrt{d+ex^2}(a+b\arctan(cx))\,dx = -\frac{bx\sqrt{d+ex^2}}{6c}+\frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{3e} - \frac{b(c^2d-e)^{3/2}\arctan\left(\frac{\sqrt{c^2d-e}x}{\sqrt{d+ex^2}}\right)}{3c^3e} - \frac{b(3c^2d-2e)\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{6c^3\sqrt{e}}$$

output
```
-1/6*b*x*(e*x^2+d)^(1/2)/c+1/3*(e*x^2+d)^(3/2)*(a+b*arctan(c*x))/e-1/3*b*(
c^2*d-e)^(3/2)*arctan((c^2*d-e)^(1/2)*x/(e*x^2+d)^(1/2))/c^3/e-1/6*b*(3*c^
2*d-2*e)*arctanh(e^(1/2)*x/(e*x^2+d)^(1/2))/c^3/e^(1/2)
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.99

$$\int x\sqrt{d+ex^2}(a+b\arctan(cx))\,dx$$
$$= \frac{c^2\sqrt{d+ex^2}(-bex+2ac(d+ex^2))+2bc^3(d+ex^2)^{3/2}\arctan(cx)-ib(c^2d-e)^{3/2}\log\left(\frac{12c^4e\left(-icd+ex-i\sqrt{c^2}\right.}{b(c^2d-e)^{5/2}(-}\right.}{6}$$

input
```
Integrate[x*Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]),x]
```

output
```
(c^2*Sqrt[d + e*x^2]*(-(b*e*x) + 2*a*c*(d + e*x^2)) + 2*b*c^3*(d + e*x^2)^
(3/2)*ArcTan[c*x] - I*b*(c^2*d - e)^(3/2)*Log[(12*c^4*e*((-I)*c*d + e*x -
I*Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(c^2*d - e)^(5/2)*(-I + c*x))] + I*
b*(c^2*d - e)^(3/2)*Log[(12*c^4*e*(I*c*d + e*x + I*Sqrt[c^2*d - e]*Sqrt[d
+ e*x^2]))/(b*(c^2*d - e)^(5/2)*(I + c*x))] + b*Sqrt[e]*(-3*c^2*d + 2*e)*L
og[e*x + Sqrt[e]*Sqrt[d + e*x^2]])/(6*c^3*e)
```

## Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5509, 318, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{d+ex^2}(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5509}$$

$$\frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{3e}-\frac{bc\int\frac{(ex^2+d)^{3/2}}{c^2x^2+1}dx}{3e}$$

$$\Big\downarrow \text{318}$$

$$\frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{3e}-\frac{bc\left(\frac{\int \frac{(3c^2d-2e)ex^2+d(2c^2d-e)}{(c^2x^2+1)\sqrt{ex^2+d}}dx}{2c^2}+\frac{ex\sqrt{d+ex^2}}{2c^2}\right)}{3e}$$

$\downarrow$ 398

$$\frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{3e}-\frac{bc\left(\frac{\frac{2(c^2d-e)^2\int \frac{1}{(c^2x^2+1)\sqrt{ex^2+d}}dx}{c^2}+\frac{e(3c^2d-2e)\int \frac{1}{\sqrt{ex^2+d}}dx}{c^2}}{2c^2}+\frac{ex\sqrt{d+ex^2}}{2c^2}\right)}{3e}$$

$\downarrow$ 224

$$\frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{3e}-\frac{bc\left(\frac{\frac{2(c^2d-e)^2\int \frac{1}{(c^2x^2+1)\sqrt{ex^2+d}}dx}{c^2}+\frac{e(3c^2d-2e)\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{c^2}}{2c^2}+\frac{ex\sqrt{d+ex^2}}{2c^2}\right)}{3e}$$

$\downarrow$ 219

$$\frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{3e}-\frac{bc\left(\frac{\frac{2(c^2d-e)^2\int \frac{1}{(c^2x^2+1)\sqrt{ex^2+d}}dx}{c^2}+\frac{\sqrt{e}(3c^2d-2e)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2}}{2c^2}+\frac{ex\sqrt{d+ex^2}}{2c^2}\right)}{3e}$$

$\downarrow$ 291

$$\frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{3e}-\frac{bc\left(\frac{\frac{2(c^2d-e)^2\int \frac{1}{1-\frac{(e-c^2d)x^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{c^2}+\frac{\sqrt{e}(3c^2d-2e)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2}}{2c^2}+\frac{ex\sqrt{d+ex^2}}{2c^2}\right)}{3e}$$

$\downarrow$ 216

$$\frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{3e}-\frac{bc\left(\frac{\frac{2(c^2d-e)^{3/2}\arctan\left(\frac{x\sqrt{c^2d-e}}{\sqrt{d+ex^2}}\right)}{c^2}+\frac{\sqrt{e}(3c^2d-2e)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2}}{2c^2}+\frac{ex\sqrt{d+ex^2}}{2c^2}\right)}{3e}$$

input
```
Int[x*Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]),x]
```

output
```
((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/(3*e) - (b*c*((e*x*Sqrt[d + e*x^2]
)/(2*c^2) + ((2*(c^2*d - e)^(3/2)*ArcTan[(Sqrt[c^2*d - e]*x)/Sqrt[d + e*x^
2]])/c^2 + ((3*c^2*d - 2*e)*Sqrt[e]*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/
c^2)/(2*c^2)))/(3*e)
```

### Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 5509
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*e*(q + 1))), x
] - Simp[b*(c/(2*e*(q + 1)))   Int[(d + e*x^2)^(q + 1)/(1 + c^2*x^2), x], x
] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]
```

## Maple [F]

$$\int x\sqrt{e\,x^{2}+d}\,(a+b\arctan\left(cx\right))\,dx$$

input
```
int(x*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x)
```

output
```
int(x*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x)
```

## Fricas [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 879, normalized size of antiderivative = 6.28

$$\int x\sqrt{d+ex^2}(a+b\arctan(cx))\,dx = \text{Too large to display}$$

input
```
integrate(x*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
[-1/12*((3*b*c^2*d - 2*b*e)*sqrt(e)*log(-2*e*x^2 - 2*sqrt(e*x^2 + d)*sqrt(
e)*x - d) + (b*c^2*d - b*e)*sqrt(-c^2*d + e)*log(((c^4*d^2 - 8*c^2*d*e + 8
*e^2)*x^4 - 2*(3*c^2*d^2 - 4*d*e)*x^2 + 4*((c^2*d - 2*e)*x^3 - d*x)*sqrt(-
c^2*d + e)*sqrt(e*x^2 + d) + d^2)/(c^4*x^4 + 2*c^2*x^2 + 1)) - 2*(2*a*c^3*
e*x^2 + 2*a*c^3*d - b*c^2*e*x + 2*(b*c^3*e*x^2 + b*c^3*d)*arctan(c*x))*sqr
t(e*x^2 + d))/(c^3*e), -1/12*(2*(b*c^2*d - b*e)*sqrt(c^2*d - e)*arctan(1/2
*sqrt(c^2*d - e)*((c^2*d - 2*e)*x^2 - d)*sqrt(e*x^2 + d)/((c^2*d*e - e^2)*
x^3 + (c^2*d^2 - d*e)*x)) + (3*b*c^2*d - 2*b*e)*sqrt(e)*log(-2*e*x^2 - 2*s
qrt(e*x^2 + d)*sqrt(e)*x - d) - 2*(2*a*c^3*e*x^2 + 2*a*c^3*d - b*c^2*e*x +
 2*(b*c^3*e*x^2 + b*c^3*d)*arctan(c*x))*sqrt(e*x^2 + d))/(c^3*e), 1/12*(2*
(3*b*c^2*d - 2*b*e)*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - (b*c^2*d
 - b*e)*sqrt(-c^2*d + e)*log(((c^4*d^2 - 8*c^2*d*e + 8*e^2)*x^4 - 2*(3*c^2
*d^2 - 4*d*e)*x^2 + 4*((c^2*d - 2*e)*x^3 - d*x)*sqrt(-c^2*d + e)*sqrt(e*x^
2 + d) + d^2)/(c^4*x^4 + 2*c^2*x^2 + 1)) + 2*(2*a*c^3*e*x^2 + 2*a*c^3*d -
b*c^2*e*x + 2*(b*c^3*e*x^2 + b*c^3*d)*arctan(c*x))*sqrt(e*x^2 + d))/(c^3*e
), -1/6*((b*c^2*d - b*e)*sqrt(c^2*d - e)*arctan(1/2*sqrt(c^2*d - e)*((c^2*
d - 2*e)*x^2 - d)*sqrt(e*x^2 + d)/((c^2*d*e - e^2)*x^3 + (c^2*d^2 - d*e)*x
)) - (3*b*c^2*d - 2*b*e)*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - (2*
a*c^3*e*x^2 + 2*a*c^3*d - b*c^2*e*x + 2*(b*c^3*e*x^2 + b*c^3*d)*arctan(c*x
))*sqrt(e*x^2 + d))/(c^3*e)]
```

## Sympy [F]

$$\int x\sqrt{d+ex^2}(a+b\arctan(cx))\,dx=\int x(a+b\,\mathrm{atan}\,(cx))\,\sqrt{d+ex^2}\,dx$$

input
```
integrate(x*(e*x**2+d)**(1/2)*(a+b*atan(c*x)),x)
```

output
```
Integral(x*(a + b*atan(c*x))*sqrt(d + e*x**2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int x\sqrt{d+ex^2}(a+b\arctan(cx))\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e-c^2*d>0)', see `assume?` for m
ore detail
```

## Giac [F(-2)]

Exception generated.

$$\int x\sqrt{d+ex^2}(a+b\arctan(cx))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:index.cc index_m i_lex_is_greater Error: Bad Argument V
alue
```

## Mupad [F(-1)]

Timed out.

$$\int x\sqrt{d+ex^2}(a+b\arctan(cx))\,dx = \int x\,(a+b\,\mathrm{atan}(c\,x))\,\sqrt{e\,x^2+d}\,dx$$

input
```
int(x*(a + b*atan(c*x))*(d + e*x^2)^(1/2),x)
```

output
```
int(x*(a + b*atan(c*x))*(d + e*x^2)^(1/2), x)
```

## Reduce [F]

$$\int x\sqrt{d+ex^2}(a+b\arctan(cx))\,dx$$
$$=\frac{\sqrt{e\,x^2+d}\,ad+\sqrt{e\,x^2+d}\,ae\,x^2+3\left(\int\sqrt{e\,x^2+d}\,atan(cx)\,xdx\right)be}{3e}$$

input
```
int(x*(e*x^2+d)^(1/2)*(a+b*atan(c*x)),x)
```

output
```
(sqrt(d + e*x**2)*a*d + sqrt(d + e*x**2)*a*e*x**2 + 3*int(sqrt(d + e*x**2)
*atan(c*x)*x,x)*b*e)/(3*e)
```

# 3.1175 $\int \sqrt{d+ex^2}(a+b\arctan(cx))\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \sqrt{d+ex^2}(a+b\arctan(cx))\,dx = \text{Int}\left(\sqrt{d+ex^2}(a+b\arctan(cx)),x\right)$$

output
```
Defer(Int)((e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x)
```

## Mathematica [N/A]

Not integrable

Time = 5.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \sqrt{d+ex^2}(a+b\arctan(cx))\,dx = \int \sqrt{d+ex^2}(a+b\arctan(cx))\,dx$$

input
```
Integrate[Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]),x]
```

output
```
Integrate[Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d+ex^2}(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \ 5560$$

$$\int \sqrt{d+ex^2}(a+b\arctan(cx))dx$$

input
```
Int[Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \sqrt{e\,x^2+d}\,(a+b\arctan(cx))\,dx$$

input
```
int((e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x)
```

output
```
int((e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x)
```

## Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \sqrt{d+ex^2}(a+b\arctan(cx))\,dx = \int \sqrt{ex^2+d}(b\arctan(cx)+a)\,dx$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arctan(c*x) + a), x)
```

## Sympy [N/A]

Not integrable

Time = 6.44 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \sqrt{d+ex^2}(a+b\arctan(cx))\,dx = \int (a+b\,\mathrm{atan}(cx))\sqrt{d+ex^2}\,dx$$

input
```
integrate((e*x**2+d)**(1/2)*(a+b*atan(c*x)),x)
```

output
```
Integral((a + b*atan(c*x))*sqrt(d + e*x**2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \sqrt{d+ex^2}(a+b\arctan(cx))\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

## Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \sqrt{d+ex^2}(a+b\arctan(cx))\,dx = \int \sqrt{ex^2+d}(b\arctan(cx)+a)\,dx$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arctan(c*x) + a), x)
```

## Mupad [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \sqrt{d+ex^2}(a+b\arctan(cx))\,dx = \int (a+b\,\mathrm{atan}(c\,x))\,\sqrt{e\,x^2+d}\,dx$$

input
```
int((a + b*atan(c*x))*(d + e*x^2)^(1/2),x)
```

output
```
int((a + b*atan(c*x))*(d + e*x^2)^(1/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 61, normalized size of antiderivative = 3.05

$$\int \sqrt{d+ex^2}(a+b\arctan(cx))\,dx = \frac{\sqrt{e\,x^2+d}\,aex+\sqrt{e}\log\left(\frac{\sqrt{e\,x^2+d}+\sqrt{e}\,x}{\sqrt{d}}\right)ad+2\left(\int \sqrt{e\,x^2+d}\,atan(cx)\,dx\right)be}{2e}$$

input
```
int((e*x^2+d)^(1/2)*(a+b*atan(c*x)),x)
```

output
```
(sqrt(d + e*x**2)*a*e*x + sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)*x)/sqrt(
d))*a*d + 2*int(sqrt(d + e*x**2)*atan(c*x),x)*b*e)/(2*e)
```

# 3.1176 $\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x}\,dx = a\sqrt{d+ex^2} - a\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right) + b\operatorname{Int}\left(\frac{\sqrt{d+ex^2}\arctan(cx)}{x},x\right)$$

output
```
a*(e*x^2+d)^(1/2)-a*d^(1/2)*arctanh((e*x^2+d)^(1/2)/d^(1/2))+b*Defer(Int)(
(e*x^2+d)^(1/2)*arctan(c*x)/x,x)
```

## Mathematica [N/A]

Not integrable

Time = 9.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x}\,dx = \int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x}\,dx$$

input
```
Integrate[(Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/x,x]
```

output
```
Integrate[(Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/x, x]
```

## Rubi [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x}\,dx$$

$$\Big\downarrow \ 5517$$

$$a\int \frac{\sqrt{ex^2+d}}{x}dx+b\int \frac{\sqrt{ex^2+d}\arctan(cx)}{x}dx$$

$$\Big\downarrow \ 243$$

$$\frac{1}{2}a\int \frac{\sqrt{ex^2+d}}{x^2}dx^2+b\int \frac{\sqrt{ex^2+d}\arctan(cx)}{x}dx$$

$$\Big\downarrow \ 60$$

$$\frac{1}{2}a\left(d\int \frac{1}{x^2\sqrt{ex^2+d}}dx^2+2\sqrt{d+ex^2}\right)+b\int \frac{\sqrt{ex^2+d}\arctan(cx)}{x}dx$$

$$\Big\downarrow \ 73$$

$$\frac{1}{2}a\left(\frac{2d\int \frac{1}{\frac{x^4}{e}-\frac{d}{e}}d\sqrt{ex^2+d}}{e}+2\sqrt{d+ex^2}\right)+b\int \frac{\sqrt{ex^2+d}\arctan(cx)}{x}dx$$

$$\Big\downarrow \ 221$$

$$b\int \frac{\sqrt{ex^2+d}\arctan(cx)}{x}dx+\frac{1}{2}a\left(2\sqrt{d+ex^2}-2\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\right)$$

$$\Big\downarrow \ 5560$$

$$b\int \frac{\sqrt{ex^2+d}\arctan(cx)}{x}dx+\frac{1}{2}a\left(2\sqrt{d+ex^2}-2\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\right)$$

input
```
Int[(Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/x,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{e\,x^2+d}\,(a+b\arctan(cx))}{x}dx$$

input
```
int((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x,x)
```

output
```
int((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x,x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x}\,dx=\int \frac{\sqrt{ex^2+d}(b\arctan(cx)+a)}{x}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x,x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arctan(c*x) + a)/x, x)
```

## Sympy [N/A]

Not integrable

Time = 5.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x}\,dx = \int \frac{(a+b\,\mathrm{atan}\,(cx))\,\sqrt{d+ex^2}}{x}\,dx$$

input
```
integrate((e*x**2+d)**(1/2)*(a+b*atan(c*x))/x,x)
```

output
```
Integral((a + b*atan(c*x))*sqrt(d + e*x**2)/x, x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

## Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x}\,dx = \int \frac{\sqrt{ex^2+d}(b\arctan(cx)+a)}{x}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x,x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arctan(c*x) + a)/x, x)
```

## Mupad [N/A]

Not integrable

Time = 1.10 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))\,\sqrt{e\,x^2+d}}{x}\,dx$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^(1/2))/x,x)
```

output
```
int(((a + b*atan(c*x))*(d + e*x^2)^(1/2))/x, x)
```

## Reduce [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 84, normalized size of antiderivative = 3.65

$$\begin{aligned}\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x}\,dx &= \sqrt{e\,x^2+d}\,a+\sqrt{d}\log\left(\frac{\sqrt{e\,x^2+d}-\sqrt{d}+\sqrt{e}\,x}{\sqrt{d}}\right)a\\ &\quad -\sqrt{d}\log\left(\frac{\sqrt{e\,x^2+d}+\sqrt{d}+\sqrt{e}\,x}{\sqrt{d}}\right)a\\ &\quad +\left(\int \frac{\sqrt{e\,x^2+d}\,atan(cx)}{x}dx\right)b\end{aligned}$$

input
```
int((e*x^2+d)^(1/2)*(a+b*atan(c*x))/x,x)
```

output
```
sqrt(d + e*x**2)*a + sqrt(d)*log((sqrt(d + e*x**2) - sqrt(d) + sqrt(e)*x)/
sqrt(d))*a - sqrt(d)*log((sqrt(d + e*x**2) + sqrt(d) + sqrt(e)*x)/sqrt(d))
*a + int((sqrt(d + e*x**2)*atan(c*x))/x,x)*b
```

# 3.1177 $\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^2}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^2}\,dx = -\frac{a\sqrt{d+ex^2}}{x}+a\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) + b\text{Int}\left(\frac{\sqrt{d+ex^2}\arctan(cx)}{x^2},x\right)$$

output
```
-a*(e*x^2+d)^(1/2)/x+a*e^(1/2)*arctanh(e^(1/2)*x/(e*x^2+d)^(1/2))+b*Defer(
Int)((e*x^2+d)^(1/2)*arctan(c*x)/x^2,x)
```

## Mathematica [N/A]

Not integrable

Time = 9.08 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^2}\,dx = \int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^2}\,dx$$

input
```
Integrate[(Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/x^2,x]
```

output
```
Integrate[(Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/x^2, x]
```

## Rubi [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^2}dx$$

$$\Big\downarrow \text{5517}$$

$$a\int \frac{\sqrt{ex^2+d}}{x^2}dx+b\int \frac{\sqrt{ex^2+d}\arctan(cx)}{x^2}dx$$

$$\Big\downarrow \text{247}$$

$$a\left(e\int \frac{1}{\sqrt{ex^2+d}}dx-\frac{\sqrt{d+ex^2}}{x}\right)+b\int \frac{\sqrt{ex^2+d}\arctan(cx)}{x^2}dx$$

$$\Big\downarrow \text{224}$$

$$a\left(e\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}-\frac{\sqrt{d+ex^2}}{x}\right)+b\int \frac{\sqrt{ex^2+d}\arctan(cx)}{x^2}dx$$

$$\Big\downarrow \text{219}$$

$$b\int \frac{\sqrt{ex^2+d}\arctan(cx)}{x^2}dx+a\left(\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{\sqrt{d+ex^2}}{x}\right)$$

$$\Big\downarrow \text{5560}$$

$$b\int \frac{\sqrt{ex^2+d}\arctan(cx)}{x^2}dx+a\left(\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{\sqrt{d+ex^2}}{x}\right)$$

input
```
Int[(Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/x^2,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{e\,x^{2}+d}\,(a+b\arctan{(c x)})}{x^{2}}dx$$

input
```
int((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^2,x)
```

output
```
int((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^2}\,dx = \int \frac{\sqrt{ex^2+d}(b\arctan{(cx)}+a)}{x^2}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^2,x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arctan(c*x) + a)/x^2, x)
```

## Sympy [N/A]

Not integrable

Time = 2.89 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^2}\,dx = \int \frac{(a+b\,\mathrm{atan}\,(cx))\,\sqrt{d+ex^2}}{x^2}\,dx$$

input
```
integrate((e*x**2+d)**(1/2)*(a+b*atan(c*x))/x**2,x)
```

output
```
Integral((a + b*atan(c*x))*sqrt(d + e*x**2)/x**2, x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

## Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^2}\,dx = \int \frac{\sqrt{ex^2+d}(b\arctan(cx)+a)}{x^2}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^2,x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arctan(c*x) + a)/x^2, x)
```

## Mupad [N/A]

Not integrable

Time = 1.33 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^2}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))\,\sqrt{e\,x^2+d}}{x^2}\,dx$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^(1/2))/x^2,x)
```

output
```
int(((a + b*atan(c*x))*(d + e*x^2)^(1/2))/x^2, x)
```

## Reduce [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.91

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^2}\,dx = \frac{-\sqrt{e\,x^2+d}\,a+\sqrt{e}\log\left(\frac{\sqrt{e\,x^2+d}+\sqrt{e}\,x}{\sqrt{d}}\right)ax-\sqrt{e}\,ax+\left(\int \frac{\sqrt{e\,x^2+d}\,atan(cx)}{x^2}dx\right)bx}{x}$$

input
```
int((e*x^2+d)^(1/2)*(a+b*atan(c*x))/x^2,x)
```

output
```
( - sqrt(d + e*x**2)*a + sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)*x)/sqrt(d
))*a*x - sqrt(e)*a*x + int((sqrt(d + e*x**2)*atan(c*x))/x**2,x)*b*x)/x
```

# 3.1178 $\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^3}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^3}\,dx = -\frac{a\sqrt{d+ex^2}}{2x^2} - \frac{ae\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{2\sqrt{d}} + b\text{Int}\left(\frac{\sqrt{d+ex^2}\arctan(cx)}{x^3},x\right)$$

output
```
-1/2*a*(e*x^2+d)^(1/2)/x^2-1/2*a*e*arctanh((e*x^2+d)^(1/2)/d^(1/2))/d^(1/2
)+b*Defer(Int)((e*x^2+d)^(1/2)*arctan(c*x)/x^3,x)
```

## Mathematica [N/A]

Not integrable

Time = 10.54 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^3}\,dx = \int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^3}\,dx$$

input
```
Integrate[(Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/x^3,x]
```

output
```
Integrate[(Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/x^3, x]
```

## Rubi [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^3}dx$$

$$\Big\downarrow \text{ 5517}$$

$$a\int \frac{\sqrt{ex^2+d}}{x^3}dx+b\int \frac{\sqrt{ex^2+d}\arctan(cx)}{x^3}dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}a\int \frac{\sqrt{ex^2+d}}{x^4}dx^2+b\int \frac{\sqrt{ex^2+d}\arctan(cx)}{x^3}dx$$

$$\Big\downarrow \text{ 51}$$

$$\frac{1}{2}a\left(\frac{1}{2}e\int \frac{1}{x^2\sqrt{ex^2+d}}dx^2-\frac{\sqrt{d+ex^2}}{x^2}\right)+b\int \frac{\sqrt{ex^2+d}\arctan(cx)}{x^3}dx$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{2}a\left(\int \frac{1}{\frac{x^4}{e}-\frac{d}{e}}d\sqrt{ex^2+d}-\frac{\sqrt{d+ex^2}}{x^2}\right)+b\int \frac{\sqrt{ex^2+d}\arctan(cx)}{x^3}dx$$

$$\Big\downarrow \text{ 221}$$

$$b\int \frac{\sqrt{ex^2+d}\arctan(cx)}{x^3}dx+\frac{1}{2}a\left(-\frac{e\,\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{d+ex^2}}{x^2}\right)$$

$$\Big\downarrow \text{ 5560}$$

$$b\int \frac{\sqrt{ex^2+d}\arctan(cx)}{x^3}dx+\frac{1}{2}a\left(-\frac{e\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{d+ex^2}}{x^2}\right)$$

input
```
Int[(Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/x^3,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{e\,x^2+d}\,(a+b\arctan(cx))}{x^3}dx$$

input
```
int((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^3,x)
```

output
```
int((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^3}\,dx=\int \frac{\sqrt{ex^2+d}(b\arctan(cx)+a)}{x^3}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^3,x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arctan(c*x) + a)/x^3, x)
```

## Sympy [N/A]

Not integrable

Time = 3.85 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^3}\,dx = \int \frac{(a+b\operatorname{atan}(cx))\sqrt{d+ex^2}}{x^3}\,dx$$

input
```
integrate((e*x**2+d)**(1/2)*(a+b*atan(c*x))/x**3,x)
```

output
```
Integral((a + b*atan(c*x))*sqrt(d + e*x**2)/x**3, x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

## Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^3}\,dx = \int \frac{\sqrt{ex^2+d}(b\arctan(cx)+a)}{x^3}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^3,x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arctan(c*x) + a)/x^3, x)
```

## Mupad [N/A]

Not integrable

Time = 1.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^3}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))\,\sqrt{e\,x^2+d}}{x^3}\,dx$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^(1/2))/x^3,x)
```

output
```
int(((a + b*atan(c*x))*(d + e*x^2)^(1/2))/x^3, x)
```

## Reduce [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 107, normalized size of antiderivative = 4.65

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^3}\,dx = \frac{-\sqrt{e\,x^2+d}\,ad+\sqrt{d}\log\left(\frac{\sqrt{e\,x^2+d}-\sqrt{d}+\sqrt{e}\,x}{\sqrt{d}}\right)ae\,x^2-\sqrt{d}\log\left(\frac{\sqrt{e\,x^2+d}+\sqrt{d}+\sqrt{e}\,x}{\sqrt{d}}\right)ae\,x^2+2\left(\int\frac{\sqrt{e\,x^2+d}\,atan(cx)}{x^3}d\right.}{2d\,x^2}$$

input
```
int((e*x^2+d)^(1/2)*(a+b*atan(c*x))/x^3,x)
```

output
```
( - sqrt(d + e*x**2)*a*d + sqrt(d)*log((sqrt(d + e*x**2) - sqrt(d) + sqrt(
e)*x)/sqrt(d))*a*e*x**2 - sqrt(d)*log((sqrt(d + e*x**2) + sqrt(d) + sqrt(e
)*x)/sqrt(d))*a*e*x**2 + 2*int((sqrt(d + e*x**2)*atan(c*x))/x**3,x)*b*d*x**
*2)/(2*d*x**2)
```

# 3.1179 $\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^4}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 137

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^4}\,dx = -\frac{bc\sqrt{d+ex^2}}{6x^2} - \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{3dx^3} + \frac{bc(2c^2d-3e)\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{6\sqrt{d}} - \frac{b(c^2d-e)^{3/2}\operatorname{arctanh}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{3d}$$

output
```
-1/6*b*c*(e*x^2+d)^(1/2)/x^2-1/3*(e*x^2+d)^(3/2)*(a+b*arctan(c*x))/d/x^3+1
/6*b*c*(2*c^2*d-3*e)*arctanh((e*x^2+d)^(1/2)/d^(1/2))/d^(1/2)-1/3*b*(c^2*d
-e)^(3/2)*arctanh(c*(e*x^2+d)^(1/2)/(c^2*d-e)^(1/2))/d
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 288, normalized size of antiderivative = 2.10

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^4}\,dx =$$

$$-\frac{\sqrt{d+ex^2}(bcdx+2a(d+ex^2))+2b(d+ex^2)^{3/2}\arctan(cx)+bc\sqrt{d}(2c^2d-3e)\,x^3\log(x)-bc\sqrt{d}(2c^2d}{}$$

input
```
Integrate[(Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/x^4,x]
```

output
```
-1/6*(Sqrt[d + e*x^2]*(b*c*d*x + 2*a*(d + e*x^2)) + 2*b*(d + e*x^2)^(3/2)*
ArcTan[c*x] + b*c*Sqrt[d]*(2*c^2*d - 3*e)*x^3*Log[x] - b*c*Sqrt[d]*(2*c^2*
d - 3*e)*x^3*Log[d + Sqrt[d]*Sqrt[d + e*x^2]] + b*(c^2*d - e)^(3/2)*x^3*Lo
g[(12*c*d*(c*d - I*e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(c^2*d - e)^
(5/2)*(I + c*x))] + b*(c^2*d - e)^(3/2)*x^3*Log[(12*c*d*(c*d + I*e*x + Sqr
t[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(c^2*d - e)^(5/2)*(-I + c*x))])/(d*x^3)
```

## Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {5511, 27, 354, 109, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^4}\,dx$$

$$\Big\downarrow \text{5511}$$

$$-bc\int -\frac{(ex^2+d)^{3/2}}{3dx^3(c^2x^2+1)}dx-\frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{3dx^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{bc\int \frac{(ex^2+d)^{3/2}}{x^3(c^2x^2+1)}dx}{3d}-\frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{3dx^3}$$

$$\Big\downarrow \text{354}$$

$$\frac{bc\int \frac{(ex^2+d)^{3/2}}{x^4(c^2x^2+1)}dx^2}{6d}-\frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{3dx^3}$$

$$\Big\downarrow \text{109}$$

$$\frac{bc\left(-\int \frac{(c^2d-2e)ex^2+d(2c^2d-3e)}{2x^2(c^2x^2+1)\sqrt{ex^2+d}}dx^2-\frac{d\sqrt{d+ex^2}}{x^2}\right)}{6d}-\frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{3dx^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{bc\left(-\frac{1}{2}\int \frac{(c^2d-2e)ex^2+d(2c^2d-3e)}{x^2(c^2x^2+1)\sqrt{ex^2+d}}dx^2-\frac{d\sqrt{d+ex^2}}{x^2}\right)}{6d}-\frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{3dx^3}$$

$$\Big\downarrow \text{174}$$

$$\frac{bc\left(\frac{1}{2}\left(2(c^2d-e)^2\int \frac{1}{(c^2x^2+1)\sqrt{ex^2+d}}dx^2-d(2c^2d-3e)\int \frac{1}{x^2\sqrt{ex^2+d}}dx^2\right)-\frac{d\sqrt{d+ex^2}}{x^2}\right)}{6d}-\frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{3dx^3}$$

$$\Big\downarrow \text{73}$$

$$\frac{bc\left(\frac{1}{2}\left(\frac{4(c^2d-e)^2\int \frac{1}{\frac{c^2x^4}{e}-\frac{c^2d}{e}+1}d\sqrt{ex^2+d}}{e}-\frac{2d(2c^2d-3e)\int \frac{1}{\frac{x^4}{e}-\frac{d}{e}}d\sqrt{ex^2+d}}{e}\right)-\frac{d\sqrt{d+ex^2}}{x^2}\right)}{6d}-\frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{3dx^3}$$

$$\Big\downarrow \text{221}$$

$$\frac{bc\left(\frac{1}{2}\left(2\sqrt{d}(2c^2d-3e)\,\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)-\frac{4(c^2d-e)^{3/2}\text{arctanh}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{c}\right)-\frac{d\sqrt{d+ex^2}}{x^2}\right)}{6d}-\frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{3dx^3}$$

input
```
Int[(Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/x^4,x]
```

output
```
-1/3*((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/(d*x^3) + (b*c*(-((d*Sqrt[d +
 e*x^2])/x^2) + (2*Sqrt[d]*(2*c^2*d - 3*e)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]
] - (4*(c^2*d - e)^(3/2)*ArcTanh[(c*Sqrt[d + e*x^2])/Sqrt[c^2*d - e]])/c)/
2))/(6*d)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

## Maple [F]

$$\int \frac{\sqrt{e\,x^{2}+d}\,(a+b\arctan(cx))}{x^{4}}dx$$

input
```
int((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^4,x)
```

output
```
int((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^4,x)
```

## Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 864, normalized size of antiderivative = 6.31

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^4}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^4,x, algorithm="fricas")
```

output
```
[-1/12*((b*c^2*d - b*e)*sqrt(c^2*d - e)*x^3*log((c^4*e^2*x^4 + 8*c^4*d^2 -
 8*c^2*d*e + 2*(4*c^4*d*e - 3*c^2*e^2)*x^2 + 4*(c^3*e*x^2 + 2*c^3*d - c*e)
*sqrt(c^2*d - e)*sqrt(e*x^2 + d) + e^2)/(c^4*x^4 + 2*c^2*x^2 + 1)) + (2*b*
c^3*d - 3*b*c*e)*sqrt(d)*x^3*log(-(e*x^2 - 2*sqrt(e*x^2 + d)*sqrt(d) + 2*d
)/x^2) + 2*(b*c*d*x + 2*a*e*x^2 + 2*a*d + 2*(b*e*x^2 + b*d)*arctan(c*x))*s
qrt(e*x^2 + d))/(d*x^3), -1/12*(2*(b*c^2*d - b*e)*sqrt(-c^2*d + e)*x^3*arc
tan(-1/2*(c^2*e*x^2 + 2*c^2*d - e)*sqrt(-c^2*d + e)*sqrt(e*x^2 + d)/(c^3*d
^2 - c*d*e + (c^3*d*e - c*e^2)*x^2)) + (2*b*c^3*d - 3*b*c*e)*sqrt(d)*x^3*l
og(-(e*x^2 - 2*sqrt(e*x^2 + d)*sqrt(d) + 2*d)/x^2) + 2*(b*c*d*x + 2*a*e*x^
2 + 2*a*d + 2*(b*e*x^2 + b*d)*arctan(c*x))*sqrt(e*x^2 + d))/(d*x^3), -1/12
*(2*(2*b*c^3*d - 3*b*c*e)*sqrt(-d)*x^3*arctan(sqrt(e*x^2 + d)*sqrt(-d)/d)
+ (b*c^2*d - b*e)*sqrt(c^2*d - e)*x^3*log((c^4*e^2*x^4 + 8*c^4*d^2 - 8*c^2
*d*e + 2*(4*c^4*d*e - 3*c^2*e^2)*x^2 + 4*(c^3*e*x^2 + 2*c^3*d - c*e)*sqrt(
c^2*d - e)*sqrt(e*x^2 + d) + e^2)/(c^4*x^4 + 2*c^2*x^2 + 1)) + 2*(b*c*d*x
+ 2*a*e*x^2 + 2*a*d + 2*(b*e*x^2 + b*d)*arctan(c*x))*sqrt(e*x^2 + d))/(d*x
^3), -1/6*((b*c^2*d - b*e)*sqrt(-c^2*d + e)*x^3*arctan(-1/2*(c^2*e*x^2 + 2
*c^2*d - e)*sqrt(-c^2*d + e)*sqrt(e*x^2 + d)/(c^3*d^2 - c*d*e + (c^3*d*e -
 c*e^2)*x^2)) + (2*b*c^3*d - 3*b*c*e)*sqrt(-d)*x^3*arctan(sqrt(e*x^2 + d)*
sqrt(-d)/d) + (b*c*d*x + 2*a*e*x^2 + 2*a*d + 2*(b*e*x^2 + b*d)*arctan(c*x)
)*sqrt(e*x^2 + d))/(d*x^3)]
```

## Sympy [F]

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^4}\,dx = \int \frac{(a+b\,\mathrm{atan}\,(cx))\,\sqrt{d+ex^2}}{x^4}\,dx$$

input
```
integrate((e*x**2+d)**(1/2)*(a+b*atan(c*x))/x**4,x)
```

output
```
Integral((a + b*atan(c*x))*sqrt(d + e*x**2)/x**4, x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

## Giac [F]

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^4}\,dx = \int \frac{\sqrt{ex^2+d}(b\arctan(cx)+a)}{x^4}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^4,x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arctan(c*x) + a)/x^4, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^4}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))\,\sqrt{e\,x^2+d}}{x^4}\,dx$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^(1/2))/x^4,x)
```

output
```
int(((a + b*atan(c*x))*(d + e*x^2)^(1/2))/x^4, x)
```

## Reduce [F]

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^4}\,dx$$
$$=\frac{-\sqrt{e\,x^2+d}\,ad-\sqrt{e\,x^2+d}\,ae\,x^2-\sqrt{e}\,ae\,x^3+3\left(\int\frac{\sqrt{e\,x^2+d}\,atan(cx)}{x^4}dx\right)bd\,x^3}{3d\,x^3}$$

input
```
int((e*x^2+d)^(1/2)*(a+b*atan(c*x))/x^4,x)
```

output
```
( - sqrt(d + e*x**2)*a*d - sqrt(d + e*x**2)*a*e*x**2 - sqrt(e)*a*e*x**3 +
3*int((sqrt(d + e*x**2)*atan(c*x))/x**4,x)*b*d*x**3)/(3*d*x**3)
```

# 3.1180 $\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^5}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^5}\,dx=-\frac{a\sqrt{d+ex^2}}{4x^4}-\frac{ae\sqrt{d+ex^2}}{8dx^2}+\frac{ae^2\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{8d^{3/2}}+b\text{Int}\left(\frac{\sqrt{d+ex^2}\arctan(cx)}{x^5},x\right)$$

output
```
-1/4*a*(e*x^2+d)^(1/2)/x^4-1/8*a*e*(e*x^2+d)^(1/2)/d/x^2+1/8*a*e^2*arctanh
((e*x^2+d)^(1/2)/d^(1/2))/d^(3/2)+b*Defer(Int)((e*x^2+d)^(1/2)*arctan(c*x)
/x^5,x)
```

## Mathematica [N/A]

Not integrable

Time = 12.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^5}\,dx=\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^5}\,dx$$

input
```
Integrate[(Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/x^5,x]
```

output
```
Integrate[(Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/x^5, x]
```

## Rubi [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^5}\,dx$$

$$\Big\downarrow \text{5517}$$

$$a\int \frac{\sqrt{ex^2+d}}{x^5}dx+b\int \frac{\sqrt{ex^2+d}\arctan(cx)}{x^5}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}a\int \frac{\sqrt{ex^2+d}}{x^6}dx^2+b\int \frac{\sqrt{ex^2+d}\arctan(cx)}{x^5}dx$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}a\left(\frac{1}{4}e\int \frac{1}{x^4\sqrt{ex^2+d}}dx^2-\frac{\sqrt{d+ex^2}}{2x^4}\right)+b\int \frac{\sqrt{ex^2+d}\arctan(cx)}{x^5}dx$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{2}a\left(\frac{1}{4}e\left(-\frac{e\int \frac{1}{x^2\sqrt{ex^2+d}}dx^2}{2d}-\frac{\sqrt{d+ex^2}}{dx^2}\right)-\frac{\sqrt{d+ex^2}}{2x^4}\right)+b\int \frac{\sqrt{ex^2+d}\arctan(cx)}{x^5}dx$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}a\left(\frac{1}{4}e\left(-\frac{\int \frac{1}{\frac{x^4}{e}-\frac{d}{e}}d\sqrt{ex^2+d}}{d}-\frac{\sqrt{d+ex^2}}{dx^2}\right)-\frac{\sqrt{d+ex^2}}{2x^4}\right)+b\int \frac{\sqrt{ex^2+d}\arctan(cx)}{x^5}dx$$

$$\Big\downarrow \text{221}$$

$$b\int \frac{\sqrt{ex^2+d}\arctan(cx)}{x^5}dx+\frac{1}{2}a\left(\frac{1}{4}e\left(\frac{\text{earctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{\sqrt{d+ex^2}}{dx^2}\right)-\frac{\sqrt{d+ex^2}}{2x^4}\right)$$

$$\Big\downarrow \text{5560}$$

$$b\int \frac{\sqrt{ex^2+d}\arctan(cx)}{x^5}dx+\frac{1}{2}a\left(\frac{1}{4}e\left(\frac{\text{earctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{\sqrt{d+ex^2}}{dx^2}\right)-\frac{\sqrt{d+ex^2}}{2x^4}\right)$$

input
```
Int[(Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/x^5,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{e\,x^2+d}\,(a+b\arctan(cx))}{x^5}dx$$

input
```
int((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^5,x)
```

output
```
int((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^5,x)
```

## Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^5}\,dx=\int \frac{\sqrt{ex^2+d}(b\arctan(cx)+a)}{x^5}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^5,x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arctan(c*x) + a)/x^5, x)
```

## Sympy [N/A]

Not integrable

Time = 9.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^5}\,dx = \int \frac{(a+b\,\mathrm{atan}\,(cx))\,\sqrt{d+ex^2}}{x^5}\,dx$$

input
```
integrate((e*x**2+d)**(1/2)*(a+b*atan(c*x))/x**5,x)
```

output
```
Integral((a + b*atan(c*x))*sqrt(d + e*x**2)/x**5, x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^5}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^5,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

## Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^5}\,dx = \int \frac{\sqrt{ex^2+d}(b\arctan(cx)+a)}{x^5}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^5,x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arctan(c*x) + a)/x^5, x)
```

## Mupad [N/A]

Not integrable

Time = 1.34 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^5}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))\,\sqrt{e\,x^2+d}}{x^5}\,dx$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^(1/2))/x^5,x)
```

output
```
int(((a + b*atan(c*x))*(d + e*x^2)^(1/2))/x^5, x)
```

## Reduce [N/A]

Not integrable

Time = 1.13 (sec) , antiderivative size = 131, normalized size of antiderivative = 5.70

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^5}\,dx$$
$$= \frac{-2\sqrt{e\,x^2+d}\,a\,d^2-\sqrt{e\,x^2+d}\,ade\,x^2-\sqrt{d}\log\left(\frac{\sqrt{e\,x^2+d}-\sqrt{d}+\sqrt{e}\,x}{\sqrt{d}}\right)a\,e^2x^4+\sqrt{d}\log\left(\frac{\sqrt{e\,x^2+d}+\sqrt{d}+\sqrt{e}\,x}{\sqrt{d}}\right)a\,e^2x}{8d^2x^4}$$

input
```
int((e*x^2+d)^(1/2)*(a+b*atan(c*x))/x^5,x)
```

output
```
( - 2*sqrt(d + e*x**2)*a*d**2 - sqrt(d + e*x**2)*a*d*e*x**2 - sqrt(d)*log(
(sqrt(d + e*x**2) - sqrt(d) + sqrt(e)*x)/sqrt(d))*a*e**2*x**4 + sqrt(d)*lo
g((sqrt(d + e*x**2) + sqrt(d) + sqrt(e)*x)/sqrt(d))*a*e**2*x**4 + 8*int((s
qrt(d + e*x**2)*atan(c*x))/x**5,x)*b*d**2*x**4)/(8*d**2*x**4)
```

# 3.1181 $\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^6}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 224

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^6}\,dx = \frac{bc(12c^2d-e)\sqrt{d+ex^2}}{120dx^2} - \frac{bc(d+ex^2)^{3/2}}{20dx^4} - \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{5dx^5} + \frac{2e(d+ex^2)^{3/2}(a+b\arctan(cx))}{15d^2x^3} - \frac{bc(24c^4d^2-20c^2de-15e^2)\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{120d^{3/2}} + \frac{b(c^2d-e)^{3/2}(3c^2d+2e)\operatorname{arctanh}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{15d^2}$$

output

```
1/120*b*c*(12*c^2*d-e)*(e*x^2+d)^(1/2)/d/x^2-1/20*b*c*(e*x^2+d)^(3/2)/d/x^
4-1/5*(e*x^2+d)^(3/2)*(a+b*arctan(c*x))/d/x^5+2/15*e*(e*x^2+d)^(3/2)*(a+b*
arctan(c*x))/d^2/x^3-1/120*b*c*(24*c^4*d^2-20*c^2*d*e-15*e^2)*arctanh((e*x
^2+d)^(1/2)/d^(1/2))/d^(3/2)+1/15*b*(c^2*d-e)^(3/2)*(3*c^2*d+2*e)*arctanh(
c*(e*x^2+d)^(1/2)/(c^2*d-e)^(1/2))/d^2
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.84

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^6}\,dx$$

$$= \frac{-\sqrt{d+ex^2}(8a(3d^2+dex^2-2e^2x^4)+bcdx(7ex^2+d(6-12c^2x^2)))-8b\sqrt{d+ex^2}(3d^2+dex^2-2e^2x^4)}{}$$

input
```
Integrate[(Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/x^6,x]
```

output
```
(-(Sqrt[d + e*x^2]*(8*a*(3*d^2 + d*e*x^2 - 2*e^2*x^4) + b*c*d*x*(7*e*x^2 +
 d*(6 - 12*c^2*x^2)))) - 8*b*Sqrt[d + e*x^2]*(3*d^2 + d*e*x^2 - 2*e^2*x^4)
*ArcTan[c*x] + b*c*Sqrt[d]*(24*c^4*d^2 - 20*c^2*d*e - 15*e^2)*x^5*Log[x] -
 b*c*Sqrt[d]*(24*c^4*d^2 - 20*c^2*d*e - 15*e^2)*x^5*Log[d + Sqrt[d]*Sqrt[d
 + e*x^2]] + 4*b*(c^2*d - e)^(3/2)*(3*c^2*d + 2*e)*x^5*Log[(-60*c*d^2*(c*d
 - I*e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(c^2*d - e)^(5/2)*(3*c^2*d
 + 2*e)*(I + c*x))] + 4*b*(c^2*d - e)^(3/2)*(3*c^2*d + 2*e)*x^5*Log[(-60*c
*d^2*(c*d + I*e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(c^2*d - e)^(5/2)
*(3*c^2*d + 2*e)*(-I + c*x))])/(120*d^2*x^5)
```

## Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {5511, 27, 435, 166, 27, 166, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^6}\,dx$$

$$\Big\downarrow \; 5511$$

$$-bc\int -\frac{\left(3d-2ex^2\right)\left(ex^2+d\right)^{3/2}}{15d^2x^5\left(c^2x^2+1\right)}dx+\frac{2e\left(d+ex^2\right)^{3/2}(a+b\arctan(cx))}{15d^2x^3}-\frac{\left(d+ex^2\right)^{3/2}(a+b\arctan(cx))}{5dx^5}$$

$$\Big\downarrow \; 27$$

$$\frac{bc\int \frac{\left(3d-2ex^2\right)\left(ex^2+d\right)^{3/2}}{x^5\left(c^2x^2+1\right)}dx}{15d^2}+\frac{2e\left(d+ex^2\right)^{3/2}(a+b\arctan(cx))}{15d^2x^3}-\frac{\left(d+ex^2\right)^{3/2}(a+b\arctan(cx))}{5dx^5}$$

$$\Big\downarrow \; 435$$

$$\frac{bc\int \frac{\left(3d-2ex^2\right)\left(ex^2+d\right)^{3/2}}{x^6\left(c^2x^2+1\right)}dx^2}{30d^2}+\frac{2e\left(d+ex^2\right)^{3/2}(a+b\arctan(cx))}{15d^2x^3}-\frac{\left(d+ex^2\right)^{3/2}(a+b\arctan(cx))}{5dx^5}$$

$$\Big\downarrow \; 166$$

$$\frac{bc\left(\frac{1}{2}\int -\frac{\sqrt{ex^2+d}\left(e\left(3dc^2+8e\right)x^2+d\left(12c^2d-e\right)\right)}{2x^4\left(c^2x^2+1\right)}dx^2-\frac{3d\left(d+ex^2\right)^{3/2}}{2x^4}\right)}{30d^2}+\frac{2e\left(d+ex^2\right)^{3/2}(a+b\arctan(cx))}{15d^2x^3}-\frac{\left(d+ex^2\right)^{3/2}(a+b\arctan(cx))}{5dx^5}$$

$$\Big\downarrow \; 27$$

$$\frac{bc\left(-\frac{1}{4}\int \frac{\sqrt{ex^2+d}\left(e\left(3dc^2+8e\right)x^2+d\left(12c^2d-e\right)\right)}{x^4\left(c^2x^2+1\right)}dx^2-\frac{3d\left(d+ex^2\right)^{3/2}}{2x^4}\right)}{30d^2}+\frac{2e\left(d+ex^2\right)^{3/2}(a+b\arctan(cx))}{15d^2x^3}-\frac{\left(d+ex^2\right)^{3/2}(a+b\arctan(cx))}{5dx^5}$$

$$\Big\downarrow \; 166$$

$$\frac{bc\left(\frac{1}{4}\left(\frac{d\left(12c^2d-e\right)\sqrt{d+ex^2}}{x^2}-\int -\frac{e\left(12d^2c^4-7dec^2-16e^2\right)x^2+d\left(24d^2c^4-20dec^2-15e^2\right)}{2x^2\left(c^2x^2+1\right)\sqrt{ex^2+d}}dx^2\right)-\frac{3d\left(d+ex^2\right)^{3/2}}{2x^4}\right)}{30d^2}+\frac{2e\left(d+ex^2\right)^{3/2}(a+b\arctan(cx))}{15d^2x^3}-\frac{\left(d+ex^2\right)^{3/2}(a+b\arctan(cx))}{5dx^5}$$

$$\Big\downarrow \; 27$$

$$\frac{bc\left(\frac{1}{4}\left(\frac{1}{2}\int\frac{e(12d^2c^4-7dec^2-16e^2)x^2+d(24d^2c^4-20dec^2-15e^2)}{x^2(c^2x^2+1)\sqrt{ex^2+d}}dx^2+\frac{d(12c^2d-e)\sqrt{d+ex^2}}{x^2}\right)-\frac{3d(d+ex^2)^{3/2}}{2x^4}\right)}{30d^2}+\frac{2e(d+ex^2)^{3/2}(a+b\arctan(cx))}{15d^2x^3}-\frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{5dx^5}$$

$\Big\downarrow$ 174

$$\frac{bc\left(\frac{1}{4}\left(\frac{1}{2}\left(d(24c^4d^2-20c^2de-15e^2)\int\frac{1}{x^2\sqrt{ex^2+d}}dx^2-8(c^2d-e)^2(3c^2d+2e)\int\frac{1}{(c^2x^2+1)\sqrt{ex^2+d}}dx^2\right)+\frac{d(12c^2d-e)}{x^2}\right.\right.}{30d^2}\frac{2e(d+ex^2)^{3/2}(a+b\arctan(cx))}{15d^2x^3}-\frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{5dx^5}$$

$\Big\downarrow$ 73

$$\frac{bc\left(\frac{1}{4}\left(\frac{1}{2}\left(\frac{2d(24c^4d^2-20c^2de-15e^2)\int\frac{1}{\frac{x^4}{e}-\frac{d}{e}}d\sqrt{ex^2+d}}{e}-\frac{16(c^2d-e)^2(3c^2d+2e)\int\frac{1}{\frac{c^2x^4}{e}-\frac{c^2d}{e}+1}d\sqrt{ex^2+d}}{e}\right)+\frac{d(12c^2d-e)\sqrt{d+ex^2}}{x^2}\right)-\right.}{30d^2}\frac{2e(d+ex^2)^{3/2}(a+b\arctan(cx))}{15d^2x^3}-\frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{5dx^5}$$

$\Big\downarrow$ 221

$$\frac{2e(d+ex^2)^{3/2}(a+b\arctan(cx))}{15d^2x^3}-\frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{5dx^5}+\frac{bc\left(\frac{1}{4}\left(\frac{1}{2}\left(\frac{16(c^2d-e)^{3/2}(3c^2d+2e)\text{arctanh}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{c}-2\sqrt{d}(24c^4d^2-20c^2de-15e^2)\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\right)+\frac{d(12c^2d-\right.\right.}{30d^2}$$

input
```
Int[(Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/x^6,x]
```

output
```
-1/5*((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/(d*x^5) + (2*e*(d + e*x^2)^(3
/2)*(a + b*ArcTan[c*x]))/(15*d^2*x^3) + (b*c*((-3*d*(d + e*x^2)^(3/2))/(2*
x^4) + ((d*(12*c^2*d - e)*Sqrt[d + e*x^2])/x^2 + (-2*Sqrt[d]*(24*c^4*d^2 -
 20*c^2*d*e - 15*e^2)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]] + (16*(c^2*d - e)^(
3/2)*(3*c^2*d + 2*e)*ArcTanh[(c*Sqrt[d + e*x^2])/Sqrt[c^2*d - e]])/c)/2)/4
))/(30*d^2)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

## Maple [F]

$$\int \frac{\sqrt{e\,x^{2}+d}\,(a+b\arctan\left(cx\right))}{x^{6}}dx$$

input
```
int((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^6,x)
```

output
```
int((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^6,x)
```

## Fricas [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 1162, normalized size of antiderivative = 5.19

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^6}\,dx=\text{Too large to display}$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^6,x, algorithm="fricas")
```

output
```
[-1/240*(4*(3*b*c^4*d^2 - b*c^2*d*e - 2*b*e^2)*sqrt(c^2*d - e)*x^5*log((c^
4*e^2*x^4 + 8*c^4*d^2 - 8*c^2*d*e + 2*(4*c^4*d*e - 3*c^2*e^2)*x^2 - 4*(c^3
*e*x^2 + 2*c^3*d - c*e)*sqrt(c^2*d - e)*sqrt(e*x^2 + d) + e^2)/(c^4*x^4 +
2*c^2*x^2 + 1)) + (24*b*c^5*d^2 - 20*b*c^3*d*e - 15*b*c*e^2)*sqrt(d)*x^5*l
og(-(e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(d) + 2*d)/x^2) - 2*(16*a*e^2*x^4 - 6*b
*c*d^2*x - 8*a*d*e*x^2 + (12*b*c^3*d^2 - 7*b*c*d*e)*x^3 - 24*a*d^2 + 8*(2*
b*e^2*x^4 - b*d*e*x^2 - 3*b*d^2)*arctan(c*x))*sqrt(e*x^2 + d))/(d^2*x^5),
1/240*(8*(3*b*c^4*d^2 - b*c^2*d*e - 2*b*e^2)*sqrt(-c^2*d + e)*x^5*arctan(-
1/2*(c^2*e*x^2 + 2*c^2*d - e)*sqrt(-c^2*d + e)*sqrt(e*x^2 + d)/(c^3*d^2 -
c*d*e + (c^3*d*e - c*e^2)*x^2)) - (24*b*c^5*d^2 - 20*b*c^3*d*e - 15*b*c*e^
2)*sqrt(d)*x^5*log(-(e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(d) + 2*d)/x^2) + 2*(16
*a*e^2*x^4 - 6*b*c*d^2*x - 8*a*d*e*x^2 + (12*b*c^3*d^2 - 7*b*c*d*e)*x^3 -
24*a*d^2 + 8*(2*b*e^2*x^4 - b*d*e*x^2 - 3*b*d^2)*arctan(c*x))*sqrt(e*x^2 +
 d))/(d^2*x^5), 1/120*((24*b*c^5*d^2 - 20*b*c^3*d*e - 15*b*c*e^2)*sqrt(-d)
*x^5*arctan(sqrt(e*x^2 + d)*sqrt(-d)/d) - 2*(3*b*c^4*d^2 - b*c^2*d*e - 2*b
*e^2)*sqrt(c^2*d - e)*x^5*log((c^4*e^2*x^4 + 8*c^4*d^2 - 8*c^2*d*e + 2*(4*
c^4*d*e - 3*c^2*e^2)*x^2 - 4*(c^3*e*x^2 + 2*c^3*d - c*e)*sqrt(c^2*d - e)*s
qrt(e*x^2 + d) + e^2)/(c^4*x^4 + 2*c^2*x^2 + 1)) + (16*a*e^2*x^4 - 6*b*c*d
^2*x - 8*a*d*e*x^2 + (12*b*c^3*d^2 - 7*b*c*d*e)*x^3 - 24*a*d^2 + 8*(2*b*e^
2*x^4 - b*d*e*x^2 - 3*b*d^2)*arctan(c*x))*sqrt(e*x^2 + d))/(d^2*x^5), 1...
```

## Sympy [F]

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^6}\,dx=\int \frac{(a+b\operatorname{atan}(cx))\sqrt{d+ex^2}}{x^6}\,dx$$

input
```
integrate((e*x**2+d)**(1/2)*(a+b*atan(c*x))/x**6,x)
```

output
```
Integral((a + b*atan(c*x))*sqrt(d + e*x**2)/x**6, x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^6}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^6,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

## Giac [F]

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^6}\,dx = \int \frac{\sqrt{ex^2+d}(b\arctan(cx)+a)}{x^6}\,dx$$

input
```
integrate((e*x^2+d)^(1/2)*(a+b*arctan(c*x))/x^6,x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arctan(c*x) + a)/x^6, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^6}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))\,\sqrt{e\,x^2+d}}{x^6}\,dx$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^(1/2))/x^6,x)
```

output
```
int(((a + b*atan(c*x))*(d + e*x^2)^(1/2))/x^6, x)
```

## Reduce [F]

$$\int \frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{x^6}\,dx$$
$$= \frac{-3\sqrt{e\,x^2+d}\,a\,d^2-\sqrt{e\,x^2+d}\,ade\,x^2+2\sqrt{e\,x^2+d}\,a\,e^2x^4-2\sqrt{e}\,a\,e^2x^5+15\left(\int\frac{\sqrt{e\,x^2+d}\,atan(cx)}{x^6}dx\right)b\,d^2x^5}{15d^2x^5}$$

input
```
int((e*x^2+d)^(1/2)*(a+b*atan(c*x))/x^6,x)
```

output
```
( - 3*sqrt(d + e*x**2)*a*d**2 - sqrt(d + e*x**2)*a*d*e*x**2 + 2*sqrt(d + e
*x**2)*a*e**2*x**4 - 2*sqrt(e)*a*e**2*x**5 + 15*int((sqrt(d + e*x**2)*atan
(c*x))/x**6,x)*b*d**2*x**5)/(15*d**2*x**5)
```

# 3.1182 $\int x^3(d+ex^2)^{3/2}(a+b\arctan(cx))\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 279

$$\int x^3(d+ex^2)^{3/2}(a+b\arctan(cx))\,dx=\frac{b(3c^4d^2+54c^2de-40e^2)\,x\sqrt{d+ex^2}}{560c^5e}-\frac{b(13c^2d-30e)\,x(d+ex^2)^{3/2}}{840c^3e}-\frac{bx(d+ex^2)^{5/2}}{42ce}-\frac{d(d+ex^2)^{5/2}(a+b\arctan(cx))}{5e^2}+\frac{(d+ex^2)^{7/2}(a+b\arctan(cx))}{7e^2}+\frac{b(c^2d-e)^{5/2}(2c^2d+5e)\arctan\left(\frac{\sqrt{c^2d-e}\,x}{\sqrt{d+ex^2}}\right)}{35c^7e^2}+\frac{b(35c^6d^3+70c^4d^2e-168c^2de^2+80e^3)\,\text{arctanh}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{560c^7e^{3/2}}$$

output

```
1/560*b*(3*c^4*d^2+54*c^2*d*e-40*e^2)*x*(e*x^2+d)^(1/2)/c^5/e-1/840*b*(13*
c^2*d-30*e)*x*(e*x^2+d)^(3/2)/c^3/e-1/42*b*x*(e*x^2+d)^(5/2)/c/e-1/5*d*(e*
x^2+d)^(5/2)*(a+b*arctan(c*x))/e^2+1/7*(e*x^2+d)^(7/2)*(a+b*arctan(c*x))/e
^2+1/35*b*(c^2*d-e)^(5/2)*(2*c^2*d+5*e)*arctan((c^2*d-e)^(1/2)*x/(e*x^2+d)
^(1/2))/c^7/e^2+1/560*b*(35*c^6*d^3+70*c^4*d^2*e-168*c^2*d*e^2+80*e^3)*arc
tanh(e^(1/2)*x/(e*x^2+d)^(1/2))/c^7/e^(3/2)
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.50

$$\int x^3(d+ex^2)^{3/2}(a+b\arctan(cx))\,dx =$$

$$-\frac{c^2\sqrt{d+ex^2}\Big(48ac^5(2d-5ex^2)(d+ex^2)^2+bex(120e^2-6c^2e(37d+10ex^2)+c^4(57d^2+106dex^2+40e^2}{}$$

input
```
Integrate[x^3*(d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]),x]
```

output
```
-1/1680*(c^2*Sqrt[d + e*x^2]*(48*a*c^5*(2*d - 5*e*x^2)*(d + e*x^2)^2 + b*e
*x*(120*e^2 - 6*c^2*e*(37*d + 10*e*x^2) + c^4*(57*d^2 + 106*d*e*x^2 + 40*e
^2*x^4))) + 48*b*c^7*(2*d - 5*e*x^2)*(d + e*x^2)^(5/2)*ArcTan[c*x] + (24*I
)*b*(c^2*d - e)^(5/2)*(2*c^2*d + 5*e)*Log[((-140*I)*c^8*e^2*(c*d - I*e*x +
 Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(c^2*d - e)^(7/2)*(2*c^2*d + 5*e)*(I
 + c*x))] - (24*I)*b*(c^2*d - e)^(5/2)*(2*c^2*d + 5*e)*Log[((140*I)*c^8*e^
2*(c*d + I*e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(c^2*d - e)^(7/2)*(2
*c^2*d + 5*e)*(-I + c*x))] - 3*b*Sqrt[e]*(35*c^6*d^3 + 70*c^4*d^2*e - 168*
c^2*d*e^2 + 80*e^3)*Log[e*x + Sqrt[e]*Sqrt[d + e*x^2]])/(c^7*e^2)
```

## Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {5511, 27, 403, 403, 27, 403, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(d+ex^2)^{3/2}(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5511}$$

$$-bc\int -\frac{(2d-5ex^2)(ex^2+d)^{5/2}}{35e^2(c^2x^2+1)}dx+\frac{(d+ex^2)^{7/2}(a+b\arctan(cx))}{7e^2}-\frac{d(d+ex^2)^{5/2}(a+b\arctan(cx))}{5e^2}$$

$$\Big\downarrow 27$$

$$\frac{bc\int \frac{(2d-5ex^2)(ex^2+d)^{5/2}}{c^2x^2+1}dx}{35e^2}+\frac{(d+ex^2)^{7/2}(a+b\arctan(cx))}{7e^2}-\frac{d(d+ex^2)^{5/2}(a+b\arctan(cx))}{5e^2}$$

$$\Big\downarrow 403$$

$$\frac{bc\left(\frac{\int \frac{(ex^2+d)^{3/2}\left(d(12dc^2+5e)-(13c^2d-30e)ex^2\right)}{c^2x^2+1}dx}{6c^2}-\frac{5ex(d+ex^2)^{5/2}}{6c^2}\right)}{35e^2}+\frac{(d+ex^2)^{7/2}(a+b\arctan(cx))}{7e^2}-\frac{d(d+ex^2)^{5/2}(a+b\arctan(cx))}{5e^2}$$

$$\Big\downarrow 403$$

$$\frac{bc\left(\frac{\frac{\int \frac{3\sqrt{ex^2+d}\left(e(3d^2c^4+54dec^2-40e^2)x^2+d(16d^2c^4+11dec^2-10e^2)\right)}{c^2x^2+1}dx}{4c^2}-\frac{ex(13c^2d-30e)(d+ex^2)^{3/2}}{4c^2}}{6c^2}-\frac{5ex(d+ex^2)^{5/2}}{6c^2}\right)}{35e^2}+\frac{(d+ex^2)^{7/2}(a+b\arctan(cx))}{7e^2}-\frac{d(d+ex^2)^{5/2}(a+b\arctan(cx))}{5e^2}$$

$$\Big\downarrow 27$$

$$\frac{bc\left(\frac{\frac{3\int \frac{\sqrt{ex^2+d}\left(e(3d^2c^4+54dec^2-40e^2)x^2+d(16d^2c^4+11dec^2-10e^2)\right)}{c^2x^2+1}dx}{4c^2}-\frac{ex(13c^2d-30e)(d+ex^2)^{3/2}}{4c^2}}{6c^2}-\frac{5ex(d+ex^2)^{5/2}}{6c^2}\right)}{35e^2}+\frac{(d+ex^2)^{7/2}(a+b\arctan(cx))}{7e^2}-\frac{d(d+ex^2)^{5/2}(a+b\arctan(cx))}{5e^2}$$

$$\Big\downarrow 403$$

$$bc\left(\frac{\frac{3\left(\frac{\int\frac{e\left(35d^3c^6+70d^2ec^4-168de^2c^2+80e^3\right)x^2+d\left(32d^3c^6+19d^2ec^4-74de^2c^2+40e^3\right)}{\left(c^2x^2+1\right)\sqrt{ex^2+d}}dx}{2c^2}+\frac{ex\left(3c^4d^2+54c^2de-40e^2\right)\sqrt{d+ex^2}}{2c^2}\right)}{4c^2}-\frac{ex\left(13c^2d-30e\right)\left(d+ex^2\right.}{4c^2}}{6c^2}\right.$$

$$35e^2$$

$$\frac{\left(d+ex^2\right)^{7/2}(a+b\arctan(cx))}{7e^2}-\frac{d\left(d+ex^2\right)^{5/2}(a+b\arctan(cx))}{5e^2}$$

$$\Big\downarrow \text{398}$$

$$bc\left(\frac{\frac{3\left(\frac{\frac{16\left(2c^2d+5e\right)\left(c^2d-e\right)^3\int\frac{1}{\left(c^2x^2+1\right)\sqrt{ex^2+d}}dx}{c^2}+\frac{e\left(35c^6d^3+70c^4d^2e-168c^2de^2+80e^3\right)\int\frac{1}{\sqrt{ex^2+d}}dx}{c^2}}{2c^2}+\frac{ex\left(3c^4d^2+54c^2de-40e^2\right)\sqrt{d+ex^2}}{2c^2}\right)}{4c^2}-\frac{ex\left(13\right.}{}}{6c^2}\right.$$

$$35e^2$$

$$\frac{\left(d+ex^2\right)^{7/2}(a+b\arctan(cx))}{7e^2}-\frac{d\left(d+ex^2\right)^{5/2}(a+b\arctan(cx))}{5e^2}$$

$$\Big\downarrow \text{224}$$

$$bc\left(\frac{\frac{3\left(\frac{\frac{16\left(2c^2d+5e\right)\left(c^2d-e\right)^3\int\frac{1}{\left(c^2x^2+1\right)\sqrt{ex^2+d}}dx}{c^2}+\frac{e\left(35c^6d^3+70c^4d^2e-168c^2de^2+80e^3\right)\int\frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{c^2}}{2c^2}+\frac{ex\left(3c^4d^2+54c^2de-40e^2\right)\sqrt{d+ex^2}}{2c^2}\right.}{4c^2}}{6c^2}\right.$$

$$35e^2$$

$$\frac{\left(d+ex^2\right)^{7/2}(a+b\arctan(cx))}{7e^2}-\frac{d\left(d+ex^2\right)^{5/2}(a+b\arctan(cx))}{5e^2}$$

$$\Big\downarrow \text{219}$$

$$bc\left(\frac{\frac{3\left(\frac{\frac{16\left(2c^2d+5e\right)\left(c^2d-e\right)^3\int\frac{1}{\left(c^2x^2+1\right)\sqrt{ex^2+d}}dx}{c^2}+\frac{\sqrt{e}\left(35c^6d^3+70c^4d^2e-168c^2de^2+80e^3\right)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2}}{2c^2}+\frac{ex\left(3c^4d^2+54c^2de-40e^2\right)\sqrt{d+ex}}{2c^2}\right.}{4c^2}}{6c^2}\right.$$

$$\overline{\qquad 35e^2 \qquad}$$

$$\frac{\left(d+ex^2\right)^{7/2}(a+b\arctan(cx))}{7e^2}-\frac{d\left(d+ex^2\right)^{5/2}(a+b\arctan(cx))}{5e^2}$$

$$\Big\downarrow \text{291}$$

$$bc\left(\frac{\frac{3\left(\frac{\frac{16\left(2c^2d+5e\right)\left(c^2d-e\right)^3\int\frac{1}{1-\frac{\left(e-c^2d\right)x^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{c^2}+\frac{\sqrt{e}\left(35c^6d^3+70c^4d^2e-168c^2de^2+80e^3\right)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2}}{2c^2}+\frac{ex\left(3c^4d^2+54c^2de-40e^2\right)\sqrt{d}}{2c^2}\right.}{4c^2}}{6c^2}\right.$$

$$\overline{\qquad 35e^2 \qquad}$$

$$\frac{\left(d+ex^2\right)^{7/2}(a+b\arctan(cx))}{7e^2}-\frac{d\left(d+ex^2\right)^{5/2}(a+b\arctan(cx))}{5e^2}$$

$$\Big\downarrow \text{216}$$

$$\frac{\left(d+ex^2\right)^{7/2}(a+b\arctan(cx))}{7e^2}-\frac{d\left(d+ex^2\right)^{5/2}(a+b\arctan(cx))}{5e^2}+$$

$$bc\left(\frac{\frac{3\left(\frac{\frac{16\left(2c^2d+5e\right)\left(c^2d-e\right)^{5/2}\arctan\left(\frac{x\sqrt{c^2d-e}}{\sqrt{d+ex^2}}\right)}{c^2}+\frac{\sqrt{e}\left(35c^6d^3+70c^4d^2e-168c^2de^2+80e^3\right)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2}}{2c^2}+\frac{ex\left(3c^4d^2+54c^2de-40e^2\right)\sqrt{d+ex^2}}{2c^2}\right.}{4c^2}}{6c^2}\right.$$

$$\overline{\qquad 35e^2 \qquad}$$

input
```
Int[x^3*(d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]),x]
```

output
```
-1/5*(d*(d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]))/e^2 + ((d + e*x^2)^(7/2)*(a
 + b*ArcTan[c*x]))/(7*e^2) + (b*c*((-5*e*x*(d + e*x^2)^(5/2))/(6*c^2) + (-
1/4*((13*c^2*d - 30*e)*e*x*(d + e*x^2)^(3/2))/c^2 + (3*((e*(3*c^4*d^2 + 54
*c^2*d*e - 40*e^2)*x*Sqrt[d + e*x^2])/(2*c^2) + ((16*(c^2*d - e)^(5/2)*(2*
c^2*d + 5*e)*ArcTan[(Sqrt[c^2*d - e]*x)/Sqrt[d + e*x^2]])/c^2 + (Sqrt[e]*(
35*c^6*d^3 + 70*c^4*d^2*e - 168*c^2*d*e^2 + 80*e^3)*ArcTanh[(Sqrt[e]*x)/Sq
rt[d + e*x^2]])/c^2)/(2*c^2)))/(4*c^2))/(6*c^2)))/(35*e^2)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

## Maple [F]

$$\int x^{3} \left(e\,x^{2}+d \right)^{\frac{3}{2}} \left(a +b \arctan \left(c x \right)\right)dx$$

input
```
int(x^3*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x)
```

output
```
int(x^3*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x)
```

## Fricas [A] (verification not implemented)

Time = 8.72 (sec) , antiderivative size = 1566, normalized size of antiderivative = 5.61

$$\int x^3 \left(d+ex^2\right)^{3/2} (a+b\arctan(cx))\,dx=\text{Too large to display}$$

input
```
integrate(x^3*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
[1/3360*(3*(35*b*c^6*d^3 + 70*b*c^4*d^2*e - 168*b*c^2*d*e^2 + 80*b*e^3)*sq
rt(e)*log(-2*e*x^2 - 2*sqrt(e*x^2 + d)*sqrt(e)*x - d) + 24*(2*b*c^6*d^3 +
b*c^4*d^2*e - 8*b*c^2*d*e^2 + 5*b*e^3)*sqrt(-c^2*d + e)*log(((c^4*d^2 - 8*
c^2*d*e + 8*e^2)*x^4 - 2*(3*c^2*d^2 - 4*d*e)*x^2 + 4*((c^2*d - 2*e)*x^3 -
d*x)*sqrt(-c^2*d + e)*sqrt(e*x^2 + d) + d^2)/(c^4*x^4 + 2*c^2*x^2 + 1)) +
2*(240*a*c^7*e^3*x^6 + 384*a*c^7*d*e^2*x^4 - 40*b*c^6*e^3*x^5 + 48*a*c^7*d
^2*e*x^2 - 96*a*c^7*d^3 - 2*(53*b*c^6*d*e^2 - 30*b*c^4*e^3)*x^3 - 3*(19*b*
c^6*d^2*e - 74*b*c^4*d*e^2 + 40*b*c^2*e^3)*x + 48*(5*b*c^7*e^3*x^6 + 8*b*c
^7*d*e^2*x^4 + b*c^7*d^2*e*x^2 - 2*b*c^7*d^3)*arctan(c*x))*sqrt(e*x^2 + d)
)/(c^7*e^2), 1/3360*(48*(2*b*c^6*d^3 + b*c^4*d^2*e - 8*b*c^2*d*e^2 + 5*b*e
^3)*sqrt(c^2*d - e)*arctan(1/2*sqrt(c^2*d - e)*((c^2*d - 2*e)*x^2 - d)*sqr
t(e*x^2 + d)/((c^2*d*e - e^2)*x^3 + (c^2*d^2 - d*e)*x)) + 3*(35*b*c^6*d^3
+ 70*b*c^4*d^2*e - 168*b*c^2*d*e^2 + 80*b*e^3)*sqrt(e)*log(-2*e*x^2 - 2*sq
rt(e*x^2 + d)*sqrt(e)*x - d) + 2*(240*a*c^7*e^3*x^6 + 384*a*c^7*d*e^2*x^4
- 40*b*c^6*e^3*x^5 + 48*a*c^7*d^2*e*x^2 - 96*a*c^7*d^3 - 2*(53*b*c^6*d*e^2
 - 30*b*c^4*e^3)*x^3 - 3*(19*b*c^6*d^2*e - 74*b*c^4*d*e^2 + 40*b*c^2*e^3)*
x + 48*(5*b*c^7*e^3*x^6 + 8*b*c^7*d*e^2*x^4 + b*c^7*d^2*e*x^2 - 2*b*c^7*d^
3)*arctan(c*x))*sqrt(e*x^2 + d))/(c^7*e^2), -1/1680*(3*(35*b*c^6*d^3 + 70*
b*c^4*d^2*e - 168*b*c^2*d*e^2 + 80*b*e^3)*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(
e*x^2 + d)) - 12*(2*b*c^6*d^3 + b*c^4*d^2*e - 8*b*c^2*d*e^2 + 5*b*e^3)*...
```

## Sympy [F]

$$\int x^3 \left(d+ex^2\right)^{3/2} (a+b\arctan(cx))\,dx = \int x^3 (a+b\,\mathrm{atan}\,(cx)) \left(d+ex^2\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x**3*(e*x**2+d)**(3/2)*(a+b*atan(c*x)),x)
```

output
```
Integral(x**3*(a + b*atan(c*x))*(d + e*x**2)**(3/2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int x^3(d+ex^2)^{3/2}(a+b\arctan(cx))\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

## Giac [F(-2)]

Exception generated.

$$\int x^3(d+ex^2)^{3/2}(a+b\arctan(cx))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:index.cc index_m i_lex_is_greater Error: Bad Argument V
alue
```

## Mupad [F(-1)]

Timed out.

$$\int x^3(d+ex^2)^{3/2}(a+b\arctan(cx))\,dx = \int x^3\,(a+b\,\mathrm{atan}(c\,x))\,\left(e\,x^2+d\right)^{3/2}\,dx$$

input
```
int(x^3*(a + b*atan(c*x))*(d + e*x^2)^(3/2),x)
```

output
```
int(x^3*(a + b*atan(c*x))*(d + e*x^2)^(3/2), x)
```

## Reduce [F]

$$\int x^3(d+ex^2)^{3/2}(a+b\arctan(cx))\,dx = \frac{-2\sqrt{e\,x^2+d}\,a\,d^3+\sqrt{e\,x^2+d}\,a\,d^2e\,x^2+8\sqrt{e\,x^2+d}\,ad\,e^2x^4+5\sqrt{e\,x^2+d}\,a\,e^3x^6+35}{35e^2}$$

input
```
int(x^3*(e*x^2+d)^(3/2)*(a+b*atan(c*x)),x)
```

output
```
( - 2*sqrt(d + e*x**2)*a*d**3 + sqrt(d + e*x**2)*a*d**2*e*x**2 + 8*sqrt(d
+ e*x**2)*a*d*e**2*x**4 + 5*sqrt(d + e*x**2)*a*e**3*x**6 + 35*int(sqrt(d +
 e*x**2)*atan(c*x)*x**5,x)*b*e**3 + 35*int(sqrt(d + e*x**2)*atan(c*x)*x**3
,x)*b*d*e**2)/(35*e**2)
```

# 3.1183 $\int x^2(d+ex^2)^{3/2}(a+b\arctan(cx))\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 23

$$\int x^2\left(d+ex^2\right)^{3/2}(a+b\arctan(cx))\,dx=\frac{ad^2x\sqrt{d+ex^2}}{16e}+\frac{1}{8}adx^3\sqrt{d+ex^2}+\frac{1}{6}ax^3\left(d+ex^2\right)^{3/2}-\frac{ad^3\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{16e^{3/2}}+b\text{Int}\left(x^2\left(d+ex^2\right)^{3/2}\arctan(cx),x\right)$$

output
```
1/16*a*d^2*x*(e*x^2+d)^(1/2)/e+1/8*a*d*x^3*(e*x^2+d)^(1/2)+1/6*a*x^3*(e*x^
2+d)^(3/2)-1/16*a*d^3*arctanh(e^(1/2)*x/(e*x^2+d)^(1/2))/e^(3/2)+b*Defer(I
nt)(x^2*(e*x^2+d)^(3/2)*arctan(c*x),x)
```

## Mathematica [N/A]

Not integrable

Time = 12.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int x^2\left(d+ex^2\right)^{3/2}(a+b\arctan(cx))\,dx=\int x^2\left(d+ex^2\right)^{3/2}(a+b\arctan(cx))\,dx$$

input
```
Integrate[x^2*(d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]),x]
```

output
```
Integrate[x^2*(d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(d+ex^2\right)^{3/2} (a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5517}$$

$$a\int x^2 \left(ex^2+d\right)^{3/2}dx+b\int x^2 \left(ex^2+d\right)^{3/2}\arctan(cx)dx$$

$$\Big\downarrow \text{248}$$

$$a\left(\frac{1}{2}d\int x^2\sqrt{ex^2+d}dx+\frac{1}{6}x^3\left(d+ex^2\right)^{3/2}\right)+b\int x^2 \left(ex^2+d\right)^{3/2}\arctan(cx)dx$$

$$\Big\downarrow \text{248}$$

$$a\left(\frac{1}{2}d\left(\frac{1}{4}d\int \frac{x^2}{\sqrt{ex^2+d}}dx+\frac{1}{4}x^3\sqrt{d+ex^2}\right)+\frac{1}{6}x^3\left(d+ex^2\right)^{3/2}\right)+ b\int x^2 \left(ex^2+d\right)^{3/2}\arctan(cx)dx$$

$$\Big\downarrow \text{262}$$

$$a\left(\frac{1}{2}d\left(\frac{1}{4}d\left(\frac{x\sqrt{d+ex^2}}{2e}-\frac{d\int \frac{1}{\sqrt{ex^2+d}}dx}{2e}\right)+\frac{1}{4}x^3\sqrt{d+ex^2}\right)+\frac{1}{6}x^3\left(d+ex^2\right)^{3/2}\right)+ b\int x^2 \left(ex^2+d\right)^{3/2}\arctan(cx)dx$$

$$\Big\downarrow \text{224}$$

$$a\left(\frac{1}{2}d\left(\frac{1}{4}d\left(\frac{x\sqrt{d+ex^2}}{2e}-\frac{d\int\frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{2e}\right)+\frac{1}{4}x^3\sqrt{d+ex^2}\right)+\frac{1}{6}x^3\left(d+ex^2\right)^{3/2}\right)+b\int x^2\left(ex^2+d\right)^{3/2}\arctan(cx)dx$$

$$\Big\downarrow \text{219}$$

$$b\int x^2\left(ex^2+d\right)^{3/2}\arctan(cx)dx+a\left(\frac{1}{2}d\left(\frac{1}{4}d\left(\frac{x\sqrt{d+ex^2}}{2e}-\frac{d\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2e^{3/2}}\right)+\frac{1}{4}x^3\sqrt{d+ex^2}\right)+\frac{1}{6}x^3\left(d+ex^2\right)^{3/2}\right)$$

$$\Big\downarrow \text{5560}$$

$$b\int x^2\left(ex^2+d\right)^{3/2}\arctan(cx)dx+a\left(\frac{1}{2}d\left(\frac{1}{4}d\left(\frac{x\sqrt{d+ex^2}}{2e}-\frac{d\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2e^{3/2}}\right)+\frac{1}{4}x^3\sqrt{d+ex^2}\right)+\frac{1}{6}x^3\left(d+ex^2\right)^{3/2}\right)$$

input
```
Int[x^2*(d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int x^2\left(e\,x^2+d\right)^{\frac{3}{2}}\left(a+b\arctan\left(cx\right)\right)dx$$

input
```
int(x^2*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x)
```

output
```
int(x^2*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x)
```

## Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.87

$$\int x^2 \left(d+e x^2\right)^{3/2} (a+b \arctan(cx))\, dx = \int \left(e x^2+d\right)^{\frac{3}{2}} (b \arctan(cx)+a) x^2\, dx$$

input
```
integrate(x^2*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
integral((a*e*x^4 + a*d*x^2 + (b*e*x^4 + b*d*x^2)*arctan(c*x))*sqrt(e*x^2
+ d), x)
```

## Sympy [N/A]

Not integrable

Time = 59.80 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int x^2 \left(d+e x^2\right)^{3/2} (a+b \arctan(cx))\, dx = \int x^2 (a+b \operatorname{atan}(cx)) \left(d+e x^2\right)^{\frac{3}{2}}\, dx$$

input
```
integrate(x**2*(e*x**2+d)**(3/2)*(a+b*atan(c*x)),x)
```

output
```
Integral(x**2*(a + b*atan(c*x))*(d + e*x**2)**(3/2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int x^2 \left(d+e x^2\right)^{3/2} (a+b \arctan(cx))\, dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

## Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x^2 (d+ex^2)^{3/2} (a+b\arctan(cx))\,dx = \int (ex^2+d)^{\frac{3}{2}} (b\arctan(cx)+a)x^2\,dx$$

input
```
integrate(x^2*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(3/2)*(b*arctan(c*x) + a)*x^2, x)
```

## Mupad [N/A]

Not integrable

Time = 1.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x^2 (d+ex^2)^{3/2} (a+b\arctan(cx))\,dx = \int x^2\,(a+b\,\mathrm{atan}(c\,x))\,\left(e\,x^2+d\right)^{3/2}\,dx$$

input
```
int(x^2*(a + b*atan(c*x))*(d + e*x^2)^(3/2),x)
```

output
```
int(x^2*(a + b*atan(c*x))*(d + e*x^2)^(3/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 133, normalized size of antiderivative = 5.78

$$\int x^2 (d+ex^2)^{3/2} (a+b\arctan(cx))\,dx = \frac{3\sqrt{e\,x^2+d}\,a\,d^2ex+14\sqrt{e\,x^2+d}\,ad\,e^2x^3+8\sqrt{e\,x^2+d}\,a\,e^3x^5-3\sqrt{e}\log\left(\frac{\sqrt{e\,x^2+d}+\sqrt{e}\,x}{\sqrt{d}}\right)}{48e^2}$$

input
```
int(x^2*(e*x^2+d)^(3/2)*(a+b*atan(c*x)),x)
```

output
```
(3*sqrt(d + e*x**2)*a*d**2*e*x + 14*sqrt(d + e*x**2)*a*d*e**2*x**3 + 8*sqr
t(d + e*x**2)*a*e**3*x**5 - 3*sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)*x)/s
qrt(d))*a*d**3 + 48*int(sqrt(d + e*x**2)*atan(c*x)*x**4,x)*b*e**3 + 48*int
(sqrt(d + e*x**2)*atan(c*x)*x**2,x)*b*d*e**2)/(48*e**2)
```

# 3.1184 $\int x(d+ex^2)^{3/2}(a+b\arctan(cx))\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 181

$$\int x\left(d+ex^2\right)^{3/2}(a+b\arctan(cx))\,dx=-\frac{b(7c^2d-4e)\,x\sqrt{d+ex^2}}{40c^3}-\frac{bx(d+ex^2)^{3/2}}{20c}+\frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{5e}-\frac{b(c^2d-e)^{5/2}\arctan\left(\frac{\sqrt{c^2d-e}\,x}{\sqrt{d+ex^2}}\right)}{5c^5e}-\frac{b(15c^4d^2-20c^2de+8e^2)\operatorname{arctanh}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{40c^5\sqrt{e}}$$

output

```
-1/40*b*(7*c^2*d-4*e)*x*(e*x^2+d)^(1/2)/c^3-1/20*b*x*(e*x^2+d)^(3/2)/c+1/5
*(e*x^2+d)^(5/2)*(a+b*arctan(c*x))/e-1/5*b*(c^2*d-e)^(5/2)*arctan((c^2*d-e
)^(1/2)*x/(e*x^2+d)^(1/2))/c^5/e-1/40*b*(15*c^4*d^2-20*c^2*d*e+8*e^2)*arct
anh(e^(1/2)*x/(e*x^2+d)^(1/2))/c^5/e^(1/2)
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.73

$$\int x\left(d+ex^2\right)^{3/2}(a+b\arctan(cx))\,dx=\frac{c^2\sqrt{d+ex^2}\left(8ac^3(d+ex^2)^2+bex(4e-c^2(9d+2ex^2))\right)+8bc^5(d+ex^2)^{5/2}\arctan(cx)}{}$$

input
```
Integrate[x*(d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]),x]
```

output
```
(c^2*Sqrt[d + e*x^2]*(8*a*c^3*(d + e*x^2)^2 + b*e*x*(4*e - c^2*(9*d + 2*e*
x^2))) + 8*b*c^5*(d + e*x^2)^(5/2)*ArcTan[c*x] - (4*I)*b*(c^2*d - e)^(5/2)
*Log[(20*c^6*e*((-I)*c*d + e*x - I*Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(c
^2*d - e)^(7/2)*(-I + c*x))] + (4*I)*b*(c^2*d - e)^(5/2)*Log[(20*c^6*e*(I*
c*d + e*x + I*Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(c^2*d - e)^(7/2)*(I +
c*x))] - b*Sqrt[e]*(15*c^4*d^2 - 20*c^2*d*e + 8*e^2)*Log[e*x + Sqrt[e]*Sqr
t[d + e*x^2]])/(40*c^5*e)
```

## Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}=0.381$, Rules used = {5509, 318, 403, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(d+ex^2\right)^{3/2}(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5509}$$

$$\frac{\left(d+ex^2\right)^{5/2}(a+b\arctan(cx))}{5e}-\frac{bc\int\frac{(ex^2+d)^{5/2}}{c^2x^2+1}dx}{5e}$$

$$\Big\downarrow \text{318}$$

$$\frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{5e}-\frac{bc\left(\frac{\int\frac{\sqrt{ex^2+d}\left((7c^2d-4e)ex^2+d(4c^2d-e)\right)}{c^2x^2+1}dx}{4c^2}+\frac{ex(d+ex^2)^{3/2}}{4c^2}\right)}{5e}$$

$$\Big\downarrow \text{403}$$

$$\frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{5e}-\frac{bc\left(\frac{\frac{\int\frac{e(15d^2c^4-20dec^2+8e^2)x^2+d(8d^2c^4-9dec^2+4e^2)}{(c^2x^2+1)\sqrt{ex^2+d}}dx}{2c^2}+\frac{ex(7c^2d-4e)\sqrt{d+ex^2}}{2c^2}}{4c^2}+\frac{ex(d+ex^2)^{3/2}}{4c^2}\right)}{5e}$$

$$\Big\downarrow \text{398}$$

$$\frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{5e}-\frac{bc\left(\frac{\frac{\frac{8(c^2d-e)^3\int\frac{1}{(c^2x^2+1)\sqrt{ex^2+d}}dx}{c^2}+\frac{e(15c^4d^2-20c^2de+8e^2)\int\frac{1}{\sqrt{ex^2+d}}dx}{c^2}}{2c^2}+\frac{ex(7c^2d-4e)\sqrt{d+ex^2}}{2c^2}}{4c^2}+\frac{ex(d+ex^2)^{3/2}}{4c^2}\right)}{5e}$$

$$\Big\downarrow \text{224}$$

$$\frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{5e}-\frac{bc\left(\frac{\frac{\frac{8(c^2d-e)^3\int\frac{1}{(c^2x^2+1)\sqrt{ex^2+d}}dx}{c^2}+\frac{e(15c^4d^2-20c^2de+8e^2)\int\frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{c^2}}{2c^2}+\frac{ex(7c^2d-4e)\sqrt{d+ex^2}}{2c^2}}{4c^2}+\frac{ex(d+ex^2)^{3/2}}{4c^2}\right)}{5e}$$

$$\Big\downarrow \text{219}$$

$$\frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{5e}-\frac{bc\left(\frac{\frac{\frac{8(c^2d-e)^3\int\frac{1}{(c^2x^2+1)\sqrt{ex^2+d}}dx}{c^2}+\frac{\sqrt{e}(15c^4d^2-20c^2de+8e^2)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2}}{2c^2}+\frac{ex(7c^2d-4e)\sqrt{d+ex^2}}{2c^2}}{4c^2}+\frac{ex(d+ex^2)^{3/2}}{4c^2}\right)}{5e}$$

$$\Big\downarrow \text{291}$$

$$\frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{5e}-\frac{bc\left(\frac{\frac{\frac{8(c^2d-e)^3\int\frac{1}{1-\frac{(e-c^2d)x^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{c^2}+\frac{\sqrt{e}(15c^4d^2-20c^2de+8e^2)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2}}{2c^2}+\frac{ex(7c^2d-4e)\sqrt{d+ex^2}}{2c^2}}{4c^2}+\frac{ex(d+ex^2)^{3/2}}{4c^2}\right)}{5e}$$

$\Big\downarrow$ 216

$$\frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{5e}-\frac{bc\left(\frac{\frac{\frac{8(c^2d-e)^{5/2}\arctan\left(\frac{x\sqrt{c^2d-e}}{\sqrt{d+ex^2}}\right)}{c^2}+\frac{\sqrt{e}(15c^4d^2-20c^2de+8e^2)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2}}{2c^2}+\frac{ex(7c^2d-4e)\sqrt{d+ex^2}}{2c^2}}{4c^2}+\frac{ex(d+ex^2)^{3/2}}{4c^2}\right)}{5e}$$

input
```
Int[x*(d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]),x]
```

output
```
((d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]))/(5*e) - (b*c*((e*x*(d + e*x^2)^(3/
2))/(4*c^2) + (((7*c^2*d - 4*e)*e*x*Sqrt[d + e*x^2])/(2*c^2) + ((8*(c^2*d
- e)^(5/2)*ArcTan[(Sqrt[c^2*d - e]*x)/Sqrt[d + e*x^2]])/c^2 + (Sqrt[e]*(15
*c^4*d^2 - 20*c^2*d*e + 8*e^2)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/c^2)/
(2*c^2))/(4*c^2)))/(5*e)
```

## Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 5509
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*e*(q + 1))), x
] - Simp[b*(c/(2*e*(q + 1)))   Int[(d + e*x^2)^(q + 1)/(1 + c^2*x^2), x], x
] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]
```

## Maple [F]

$$\int x\left(e\,x^{2}+d\right)^{\frac{3}{2}}\left(a+b\arctan\left(cx\right)\right)dx$$

input
```
int(x*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x)
```

output
```
int(x*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x)
```

## Fricas [A] (verification not implemented)

Time = 2.08 (sec) , antiderivative size = 1192, normalized size of antiderivative = 6.59

$$\int x\left(d+ex^2\right)^{3/2}(a+b\arctan(cx))\,dx = \text{Too large to display}$$

input
```
integrate(x*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
[1/80*((15*b*c^4*d^2 - 20*b*c^2*d*e + 8*b*e^2)*sqrt(e)*log(-2*e*x^2 + 2*sq
rt(e*x^2 + d)*sqrt(e)*x - d) + 4*(b*c^4*d^2 - 2*b*c^2*d*e + b*e^2)*sqrt(-c
^2*d + e)*log(((c^4*d^2 - 8*c^2*d*e + 8*e^2)*x^4 - 2*(3*c^2*d^2 - 4*d*e)*x
^2 - 4*((c^2*d - 2*e)*x^3 - d*x)*sqrt(-c^2*d + e)*sqrt(e*x^2 + d) + d^2)/(
c^4*x^4 + 2*c^2*x^2 + 1)) + 2*(8*a*c^5*e^2*x^4 + 16*a*c^5*d*e*x^2 - 2*b*c^
4*e^2*x^3 + 8*a*c^5*d^2 - (9*b*c^4*d*e - 4*b*c^2*e^2)*x + 8*(b*c^5*e^2*x^4
 + 2*b*c^5*d*e*x^2 + b*c^5*d^2)*arctan(c*x))*sqrt(e*x^2 + d))/(c^5*e), -1/
80*(8*(b*c^4*d^2 - 2*b*c^2*d*e + b*e^2)*sqrt(c^2*d - e)*arctan(1/2*sqrt(c^
2*d - e)*((c^2*d - 2*e)*x^2 - d)*sqrt(e*x^2 + d)/((c^2*d*e - e^2)*x^3 + (c
^2*d^2 - d*e)*x)) - (15*b*c^4*d^2 - 20*b*c^2*d*e + 8*b*e^2)*sqrt(e)*log(-2
*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x - d) - 2*(8*a*c^5*e^2*x^4 + 16*a*c^5*
d*e*x^2 - 2*b*c^4*e^2*x^3 + 8*a*c^5*d^2 - (9*b*c^4*d*e - 4*b*c^2*e^2)*x +
8*(b*c^5*e^2*x^4 + 2*b*c^5*d*e*x^2 + b*c^5*d^2)*arctan(c*x))*sqrt(e*x^2 +
d))/(c^5*e), 1/40*((15*b*c^4*d^2 - 20*b*c^2*d*e + 8*b*e^2)*sqrt(-e)*arctan
(sqrt(-e)*x/sqrt(e*x^2 + d)) + 2*(b*c^4*d^2 - 2*b*c^2*d*e + b*e^2)*sqrt(-c
^2*d + e)*log(((c^4*d^2 - 8*c^2*d*e + 8*e^2)*x^4 - 2*(3*c^2*d^2 - 4*d*e)*x
^2 - 4*((c^2*d - 2*e)*x^3 - d*x)*sqrt(-c^2*d + e)*sqrt(e*x^2 + d) + d^2)/(
c^4*x^4 + 2*c^2*x^2 + 1)) + (8*a*c^5*e^2*x^4 + 16*a*c^5*d*e*x^2 - 2*b*c^4*
e^2*x^3 + 8*a*c^5*d^2 - (9*b*c^4*d*e - 4*b*c^2*e^2)*x + 8*(b*c^5*e^2*x^4 +
 2*b*c^5*d*e*x^2 + b*c^5*d^2)*arctan(c*x))*sqrt(e*x^2 + d))/(c^5*e), -1...
```

## Sympy [F]

$$\int x(d+ex^2)^{3/2}(a+b\arctan(cx))\,dx=\int x(a+b\,\mathrm{atan}\,(cx))\left(d+ex^2\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(x*(e*x**2+d)**(3/2)*(a+b*atan(c*x)),x)
```

output
```
Integral(x*(a + b*atan(c*x))*(d + e*x**2)**(3/2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int x(d+ex^2)^{3/2}(a+b\arctan(cx))\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e-c^2*d>0)', see `assume?` for m
ore detail
```

## Giac [F(-2)]

Exception generated.

$$\int x(d+ex^2)^{3/2}(a+b\arctan(cx))\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate(x*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:index.cc index_m i_lex_is_greater Error: Bad Argument V
alue
```

## Mupad [F(-1)]

Timed out.

$$\int x(d+ex^2)^{3/2}(a+b\arctan(cx))\,dx=\int x\,(a+b\,\mathrm{atan}(c\,x))\,\left(e\,x^2+d\right)^{3/2}\,dx$$

input
```
int(x*(a + b*atan(c*x))*(d + e*x^2)^(3/2),x)
```

output
```
int(x*(a + b*atan(c*x))*(d + e*x^2)^(3/2), x)
```

## Reduce [F]

$$\int x(d+ex^2)^{3/2}(a+b\arctan(cx))\,dx=\frac{\sqrt{e\,x^2+d}\,a\,d^2+2\sqrt{e\,x^2+d}\,ade\,x^2+\sqrt{e\,x^2+d}\,a\,e^2x^4+5\left(\int\sqrt{e\,x^2+d}\,atan(cx)\,x^3dx\right.}{5e}$$

input
```
int(x*(e*x^2+d)^(3/2)*(a+b*atan(c*x)),x)
```

output
```
(sqrt(d + e*x**2)*a*d**2 + 2*sqrt(d + e*x**2)*a*d*e*x**2 + sqrt(d + e*x**2
)*a*e**2*x**4 + 5*int(sqrt(d + e*x**2)*atan(c*x)*x**3,x)*b*e**2 + 5*int(sq
rt(d + e*x**2)*atan(c*x)*x,x)*b*d*e)/(5*e)
```

# 3.1185 $\int (d+ex^2)^{3/2} (a+b\arctan(cx))\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \left(d+ex^2\right)^{3/2} (a+b\arctan(cx))\,dx = \text{Int}\left(\left(d+ex^2\right)^{3/2} (a+b\arctan(cx)),x\right)$$

output
```
Defer(Int)((e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x)
```

## Mathematica [N/A]

Not integrable

Time = 5.49 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \left(d+ex^2\right)^{3/2} (a+b\arctan(cx))\,dx = \int \left(d+ex^2\right)^{3/2} (a+b\arctan(cx))\,dx$$

input
```
Integrate[(d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]),x]
```

output
```
Integrate[(d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)^{3/2} (a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5560}$$

$$\int \left(d+ex^2\right)^{3/2} (a+b\arctan(cx))dx$$

input
```
Int[(d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \left(e\,x^{2}+d\right)^{\frac{3}{2}} \left(a+b\arctan\left(cx\right)\right)dx$$

input
```
int((e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x)
```

output
```
int((e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x)
```

## Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.85

$$\int \left(d+ex^2\right)^{3/2} (a+b\arctan(cx))\,dx = \int \left(ex^2+d\right)^{\frac{3}{2}} (b\arctan(cx)+a)\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arctan(c*x))*sqrt(e*x^2 + d), x)
```

## Sympy [N/A]

Not integrable

Time = 27.82 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \left(d+ex^2\right)^{3/2} (a+b\arctan(cx))\,dx = \int (a+b\operatorname{atan}(cx)) \left(d+ex^2\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((e*x**2+d)**(3/2)*(a+b*atan(c*x)),x)
```

output
```
Integral((a + b*atan(c*x))*(d + e*x**2)**(3/2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \left(d+ex^2\right)^{3/2} (a+b\arctan(cx))\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

## Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \left(d+ex^2\right)^{3/2}(a+b\arctan(cx))\,dx=\int (ex^2+d)^{\frac{3}{2}}(b\arctan(cx)+a)\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(3/2)*(b*arctan(c*x) + a), x)
```

## Mupad [N/A]

Not integrable

Time = 0.95 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \left(d+ex^2\right)^{3/2}(a+b\arctan(cx))\,dx=\int (a+b\,\mathrm{atan}(c\,x))\,\left(e\,x^2+d\right)^{3/2}\,dx$$

input
```
int((a + b*atan(c*x))*(d + e*x^2)^(3/2),x)
```

output
```
int((a + b*atan(c*x))*(d + e*x^2)^(3/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 108, normalized size of antiderivative = 5.40

$$\int (d+ex^2)^{3/2}(a+b\arctan(cx))\,dx = \frac{5\sqrt{e\,x^2+d}\,adex + 2\sqrt{e\,x^2+d}\,a\,e^2x^3 + 3\sqrt{e}\log\left(\frac{\sqrt{e\,x^2+d}+\sqrt{e}\,x}{\sqrt{d}}\right)a\,d^2 + 8\left(\int \sqrt{e\,x^2+d}\,at}{8e}$$

input
```
int((e*x^2+d)^(3/2)*(a+b*atan(c*x)),x)
```

output
```
(5*sqrt(d + e*x**2)*a*d*e*x + 2*sqrt(d + e*x**2)*a*e**2*x**3 + 3*sqrt(e)*l
og((sqrt(d + e*x**2) + sqrt(e)*x)/sqrt(d))*a*d**2 + 8*int(sqrt(d + e*x**2)
*atan(c*x)*x**2,x)*b*e**2 + 8*int(sqrt(d + e*x**2)*atan(c*x),x)*b*d*e)/(8*
e)
```

# 3.1186 $\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x}\,dx = ad\sqrt{d+ex^2} + \frac{1}{3}a(d+ex^2)^{3/2} - ad^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right) + b\text{Int}\left(\frac{(d+ex^2)^{3/2}\arctan(cx)}{x},x\right)$$

output
```
a*d*(e*x^2+d)^(1/2)+1/3*a*(e*x^2+d)^(3/2)-a*d^(3/2)*arctanh((e*x^2+d)^(1/2
)/d^(1/2))+b*Defer(Int)((e*x^2+d)^(3/2)*arctan(c*x)/x,x)
```

## Mathematica [N/A]

Not integrable

Time = 9.45 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x}\,dx = \int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x}\,dx$$

input
```
Integrate[((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/x,x]
```

output
```
Integrate[((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/x, x]
```

## Rubi [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d+ex^2\right)^{3/2}(a+b\arctan(cx))}{x}dx$$

$$\Big\downarrow \text{5517}$$

$$a\int \frac{\left(ex^2+d\right)^{3/2}}{x}dx+b\int \frac{\left(ex^2+d\right)^{3/2}\arctan(cx)}{x}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}a\int \frac{\left(ex^2+d\right)^{3/2}}{x^2}dx^2+b\int \frac{\left(ex^2+d\right)^{3/2}\arctan(cx)}{x}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}a\left(d\int \frac{\sqrt{ex^2+d}}{x^2}dx^2+\frac{2}{3}\left(d+ex^2\right)^{3/2}\right)+b\int \frac{\left(ex^2+d\right)^{3/2}\arctan(cx)}{x}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}a\left(d\left(d\int \frac{1}{x^2\sqrt{ex^2+d}}dx^2+2\sqrt{d+ex^2}\right)+\frac{2}{3}\left(d+ex^2\right)^{3/2}\right)+ b\int \frac{\left(ex^2+d\right)^{3/2}\arctan(cx)}{x}dx$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}a\left(d\left(\frac{2d\int \frac{1}{\frac{x^4}{e}-\frac{d}{e}}d\sqrt{ex^2+d}}{e}+2\sqrt{d+ex^2}\right)+\frac{2}{3}\left(d+ex^2\right)^{3/2}\right)+$$
$$b\int \frac{\left(ex^2+d\right)^{3/2}\arctan(cx)}{x}dx$$

$$\Big\downarrow \ 221$$

$$b\int \frac{\left(ex^2+d\right)^{3/2}\arctan(cx)}{x}dx+$$
$$\frac{1}{2}a\left(d\left(2\sqrt{d+ex^2}-2\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\right)+\frac{2}{3}\left(d+ex^2\right)^{3/2}\right)$$

$$\Big\downarrow \ 5560$$

$$b\int \frac{\left(ex^2+d\right)^{3/2}\arctan(cx)}{x}dx+$$
$$\frac{1}{2}a\left(d\left(2\sqrt{d+ex^2}-2\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\right)+\frac{2}{3}\left(d+ex^2\right)^{3/2}\right)$$

input
```
Int[((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/x,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\left(e\,x^{2}+d\right)^{\frac{3}{2}}\left(a+b\arctan\left(cx\right)\right)}{x}dx$$

input
```
int((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x,x)
```

output
```
int((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x,x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.74

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}(b\arctan(cx)+a)}{x}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x,x, algorithm="fricas")
```

output
```
integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arctan(c*x))*sqrt(e*x^2 + d)/x,
x)
```

## Sympy [N/A]

Not integrable

Time = 21.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x}\,dx = \int \frac{(a+b\,\mathrm{atan}(cx))(d+ex^2)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((e*x**2+d)**(3/2)*(a+b*atan(c*x))/x,x)
```

output
```
Integral((a + b*atan(c*x))*(d + e*x**2)**(3/2)/x, x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

## Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x}\,dx=\int \frac{(ex^2+d)^{\frac{3}{2}}(b\arctan(cx)+a)}{x}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(3/2)*(b*arctan(c*x) + a)/x, x)
```

## Mupad [N/A]

Not integrable

Time = 1.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x}\,dx=\int \frac{(a+b\,\mathrm{atan}(c\,x))\,(e\,x^2+d)^{3/2}}{x}\,dx$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^(3/2))/x,x)
```

output
```
int(((a + b*atan(c*x))*(d + e*x^2)^(3/2))/x, x)
```

## Reduce [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 123, normalized size of antiderivative = 5.35

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\arctan(cx))}{x}\,dx = \frac{4\sqrt{e\,x^2+d}\,ad}{3} + \frac{\sqrt{e\,x^2+d}\,ae\,x^2}{3} + \sqrt{d}\log\left(\frac{\sqrt{e\,x^2+d}-\sqrt{d}+\sqrt{e}\,x}{\sqrt{d}}\right)ad - \sqrt{d}\log\left(\frac{\sqrt{e\,x^2+d}+\sqrt{d}+\sqrt{e}\,x}{\sqrt{d}}\right)ad + \left(\int \frac{\sqrt{e\,x^2+d}\,atan(cx)}{x}dx\right)bd + \left(\int \sqrt{e\,x^2+d}\,atan(cx)\,xdx\right)be$$

input

```
int((e*x^2+d)^(3/2)*(a+b*atan(c*x))/x,x)
```

output

```
(4*sqrt(d + e*x**2)*a*d + sqrt(d + e*x**2)*a*e*x**2 + 3*sqrt(d)*log((sqrt(
d + e*x**2) - sqrt(d) + sqrt(e)*x)/sqrt(d))*a*d - 3*sqrt(d)*log((sqrt(d +
e*x**2) + sqrt(d) + sqrt(e)*x)/sqrt(d))*a*d + 3*int((sqrt(d + e*x**2)*atan
(c*x))/x,x)*b*d + 3*int(sqrt(d + e*x**2)*atan(c*x)*x,x)*b*e)/3
```

# 3.1187 $\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^2}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^2}\,dx = \frac{3}{2}aex\sqrt{d+ex^2} - \frac{a(d+ex^2)^{3/2}}{x} + \frac{3}{2}ad\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) + b\text{Int}\left(\frac{(d+ex^2)^{3/2}\arctan(cx)}{x^2},x\right)$$

output
```
3/2*a*e*x*(e*x^2+d)^(1/2)-a*(e*x^2+d)^(3/2)/x+3/2*a*d*e^(1/2)*arctanh(e^(1
/2)*x/(e*x^2+d)^(1/2))+b*Defer(Int)((e*x^2+d)^(3/2)*arctan(c*x)/x^2,x)
```

## Mathematica [N/A]

Not integrable

Time = 10.20 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^2}\,dx = \int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^2}\,dx$$

input
```
Integrate[((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/x^2,x]
```

output
```
Integrate[((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/x^2, x]
```

## Rubi [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^2}dx$$

$$\Big\downarrow \text{5517}$$

$$a\int \frac{(ex^2+d)^{3/2}}{x^2}dx+b\int \frac{(ex^2+d)^{3/2}\arctan(cx)}{x^2}dx$$

$$\Big\downarrow \text{247}$$

$$a\left(3e\int \sqrt{ex^2+d}dx-\frac{(d+ex^2)^{3/2}}{x}\right)+b\int \frac{(ex^2+d)^{3/2}\arctan(cx)}{x^2}dx$$

$$\Big\downarrow \text{211}$$

$$a\left(3e\left(\frac{1}{2}d\int \frac{1}{\sqrt{ex^2+d}}dx+\frac{1}{2}x\sqrt{d+ex^2}\right)-\frac{(d+ex^2)^{3/2}}{x}\right)+b\int \frac{(ex^2+d)^{3/2}\arctan(cx)}{x^2}dx$$

$$\Big\downarrow \text{224}$$

$$a\left(3e\left(\frac{1}{2}d\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}+\frac{1}{2}x\sqrt{d+ex^2}\right)-\frac{(d+ex^2)^{3/2}}{x}\right)+ b\int \frac{(ex^2+d)^{3/2}\arctan(cx)}{x^2}dx$$

$$\Big\downarrow \text{219}$$

$$b\int \frac{\left(ex^2+d\right)^{3/2}\arctan(cx)}{x^2}dx+$$
$$a\left(3e\left(\frac{d\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{e}}+\frac{1}{2}x\sqrt{d+ex^2}\right)-\frac{\left(d+ex^2\right)^{3/2}}{x}\right)$$

$$\Big\downarrow \; 5560$$

$$b\int \frac{\left(ex^2+d\right)^{3/2}\arctan(cx)}{x^2}dx+$$
$$a\left(3e\left(\frac{d\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{e}}+\frac{1}{2}x\sqrt{d+ex^2}\right)-\frac{\left(d+ex^2\right)^{3/2}}{x}\right)$$

input
```
Int[((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/x^2,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\left(e\,x^2+d\right)^{\frac{3}{2}}\left(a+b\arctan\left(cx\right)\right)}{x^2}dx$$

input
```
int((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^2,x)
```

output
```
int((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.74

$$\int \frac{\left(d+ex^2\right)^{3/2}(a+b\arctan(cx))}{x^2}\,dx=\int \frac{\left(ex^2+d\right)^{\frac{3}{2}}(b\arctan(cx)+a)}{x^2}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^2,x, algorithm="fricas")
```

output
```
integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arctan(c*x))*sqrt(e*x^2 + d)/x^2
, x)
```

## Sympy [N/A]

Not integrable

Time = 14.82 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{\left(d+ex^2\right)^{3/2}(a+b\arctan(cx))}{x^2}\,dx=\int \frac{(a+b\,\mathrm{atan}\,(cx))\left(d+ex^2\right)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((e*x**2+d)**(3/2)*(a+b*atan(c*x))/x**2,x)
```

output
```
Integral((a + b*atan(c*x))*(d + e*x**2)**(3/2)/x**2, x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\left(d+ex^2\right)^{3/2}(a+b\arctan(cx))}{x^2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

## Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^2}\,dx=\int \frac{(ex^2+d)^{\frac{3}{2}}(b\arctan(cx)+a)}{x^2}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^2,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(3/2)*(b*arctan(c*x) + a)/x^2, x)
```

## Mupad [N/A]

Not integrable

Time = 1.61 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^2}\,dx=\int \frac{(a+b\,\mathrm{atan}(c\,x))\,(e\,x^2+d)^{3/2}}{x^2}\,dx$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^(3/2))/x^2,x)
```

output
```
int(((a + b*atan(c*x))*(d + e*x^2)^(3/2))/x^2, x)
```

## Reduce [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 109, normalized size of antiderivative = 4.74

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^2}\,dx = \frac{-8\sqrt{e\,x^2+d}\,ad+4\sqrt{e\,x^2+d}\,ae\,x^2+12\sqrt{e}\log\left(\frac{\sqrt{e\,x^2+d}+\sqrt{e}\,x}{\sqrt{d}}\right)adx$$

input
```
int((e*x^2+d)^(3/2)*(a+b*atan(c*x))/x^2,x)
```

output
```
( - 8*sqrt(d + e*x**2)*a*d + 4*sqrt(d + e*x**2)*a*e*x**2 + 12*sqrt(e)*log(
(sqrt(d + e*x**2) + sqrt(e)*x)/sqrt(d))*a*d*x - 9*sqrt(e)*a*d*x + 8*int((s
qrt(d + e*x**2)*atan(c*x))/x**2,x)*b*d*x + 8*int(sqrt(d + e*x**2)*atan(c*x
),x)*b*e*x)/(8*x)
```

# 3.1188 $\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^3}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^3}\,dx = \frac{3}{2}ae\sqrt{d+ex^2}-\frac{a(d+ex^2)^{3/2}}{2x^2} - \frac{3}{2}a\sqrt{d}e\,\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)+b\text{Int}\left(\frac{(d+ex^2)^{3/2}\arctan(cx)}{x^3},x\right)$$

output
```
3/2*a*e*(e*x^2+d)^(1/2)-1/2*a*(e*x^2+d)^(3/2)/x^2-3/2*a*d^(1/2)*e*arctanh(
(e*x^2+d)^(1/2)/d^(1/2))+b*Defer(Int)((e*x^2+d)^(3/2)*arctan(c*x)/x^3,x)
```

## Mathematica [N/A]

Not integrable

Time = 10.93 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^3}\,dx = \int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^3}\,dx$$

input
```
Integrate[((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/x^3,x]
```

output
```
Integrate[((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/x^3, x]
```

## Rubi [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^3}dx$$

$$\Big\downarrow \text{5517}$$

$$a\int \frac{(ex^2+d)^{3/2}}{x^3}dx+b\int \frac{(ex^2+d)^{3/2}\arctan(cx)}{x^3}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}a\int \frac{(ex^2+d)^{3/2}}{x^4}dx^2+b\int \frac{(ex^2+d)^{3/2}\arctan(cx)}{x^3}dx$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}a\left(\frac{3}{2}e\int \frac{\sqrt{ex^2+d}}{x^2}dx^2-\frac{(d+ex^2)^{3/2}}{x^2}\right)+b\int \frac{(ex^2+d)^{3/2}\arctan(cx)}{x^3}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}a\left(\frac{3}{2}e\left(d\int \frac{1}{x^2\sqrt{ex^2+d}}dx^2+2\sqrt{d+ex^2}\right)-\frac{(d+ex^2)^{3/2}}{x^2}\right)+$$
$$b\int \frac{(ex^2+d)^{3/2}\arctan(cx)}{x^3}dx$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}a\left(\frac{3}{2}e\left(\frac{2d\int \frac{1}{\frac{x^4}{e}-\frac{d}{e}}d\sqrt{ex^2+d}}{e}+2\sqrt{d+ex^2}\right)-\frac{\left(d+ex^2\right)^{3/2}}{x^2}\right)+ b\int \frac{\left(ex^2+d\right)^{3/2}\arctan(cx)}{x^3}dx$$

$$\Big\downarrow \text{221}$$

$$b\int \frac{\left(ex^2+d\right)^{3/2}\arctan(cx)}{x^3}dx+ \frac{1}{2}a\left(\frac{3}{2}e\left(2\sqrt{d+ex^2}-2\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\right)-\frac{\left(d+ex^2\right)^{3/2}}{x^2}\right)$$

$$\Big\downarrow \text{5560}$$

$$b\int \frac{\left(ex^2+d\right)^{3/2}\arctan(cx)}{x^3}dx+ \frac{1}{2}a\left(\frac{3}{2}e\left(2\sqrt{d+ex^2}-2\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\right)-\frac{\left(d+ex^2\right)^{3/2}}{x^2}\right)$$

input
```
Int[((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/x^3,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\left(e\,x^{2}+d\right)^{\frac{3}{2}}\left(a+b\arctan\left(cx\right)\right)}{x^{3}}dx$$

input
```
int((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^3,x)
```

output
```
int((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.74

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^3}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}(b\arctan(cx)+a)}{x^3}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^3,x, algorithm="fricas")
```

output
```
integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arctan(c*x))*sqrt(e*x^2 + d)/x^3
, x)
```

## Sympy [N/A]

Not integrable

Time = 11.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^3}\,dx = \int \frac{(a+b\,\mathrm{atan}\,(cx))\,(d+ex^2)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((e*x**2+d)**(3/2)*(a+b*atan(c*x))/x**3,x)
```

output
```
Integral((a + b*atan(c*x))*(d + e*x**2)**(3/2)/x**3, x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

## Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^3}\,dx=\int \frac{(ex^2+d)^{\frac{3}{2}}(b\arctan(cx)+a)}{x^3}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^3,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(3/2)*(b*arctan(c*x) + a)/x^3, x)
```

## Mupad [N/A]

Not integrable

Time = 1.43 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^3}\,dx=\int \frac{(a+b\,\mathrm{atan}(c\,x))\,(e\,x^2+d)^{3/2}}{x^3}\,dx$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^(3/2))/x^3,x)
```

output
```
int(((a + b*atan(c*x))*(d + e*x^2)^(3/2))/x^3, x)
```

## Reduce [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 145, normalized size of antiderivative = 6.30

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^3}\,dx = \frac{-\sqrt{e\,x^2+d}\,ad+2\sqrt{e\,x^2+d}\,ae\,x^2+3\sqrt{d}\log\left(\frac{\sqrt{e\,x^2+d}-\sqrt{d}+\sqrt{e}\,x}{\sqrt{d}}\right)ae}{}$$

input
```
int((e*x^2+d)^(3/2)*(a+b*atan(c*x))/x^3,x)
```

output
```
( - sqrt(d + e*x**2)*a*d + 2*sqrt(d + e*x**2)*a*e*x**2 + 3*sqrt(d)*log((sq
rt(d + e*x**2) - sqrt(d) + sqrt(e)*x)/sqrt(d))*a*e*x**2 - 3*sqrt(d)*log((s
qrt(d + e*x**2) + sqrt(d) + sqrt(e)*x)/sqrt(d))*a*e*x**2 + 2*int((sqrt(d +
 e*x**2)*atan(c*x))/x**3,x)*b*d*x**2 + 2*int((sqrt(d + e*x**2)*atan(c*x))/
x,x)*b*e*x**2)/(2*x**2)
```

# 3.1189 $\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^4}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^4}\,dx = -\frac{ae\sqrt{d+ex^2}}{x} - \frac{a(d+ex^2)^{3/2}}{3x^3} + ae^{3/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) + b\text{Int}\left(\frac{(d+ex^2)^{3/2}\arctan(cx)}{x^4},x\right)$$

output
```
-a*e*(e*x^2+d)^(1/2)/x-1/3*a*(e*x^2+d)^(3/2)/x^3+a*e^(3/2)*arctanh(e^(1/2)
*x/(e*x^2+d)^(1/2))+b*Defer(Int)((e*x^2+d)^(3/2)*arctan(c*x)/x^4,x)
```

## Mathematica [N/A]

Not integrable

Time = 32.69 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^4}\,dx = \int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^4}\,dx$$

input
```
Integrate[((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/x^4,x]
```

output
```
Integrate[((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/x^4, x]
```

## Rubi [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^4}dx$$

$$\Big\downarrow \text{ 5517}$$

$$a\int \frac{(ex^2+d)^{3/2}}{x^4}dx+b\int \frac{(ex^2+d)^{3/2}\arctan(cx)}{x^4}dx$$

$$\Big\downarrow \text{ 247}$$

$$a\left(e\int \frac{\sqrt{ex^2+d}}{x^2}dx-\frac{(d+ex^2)^{3/2}}{3x^3}\right)+b\int \frac{(ex^2+d)^{3/2}\arctan(cx)}{x^4}dx$$

$$\Big\downarrow \text{ 247}$$

$$a\left(e\left(e\int \frac{1}{\sqrt{ex^2+d}}dx-\frac{\sqrt{d+ex^2}}{x}\right)-\frac{(d+ex^2)^{3/2}}{3x^3}\right)+b\int \frac{(ex^2+d)^{3/2}\arctan(cx)}{x^4}dx$$

$$\Big\downarrow \text{ 224}$$

$$a\left(e\left(e\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}-\frac{\sqrt{d+ex^2}}{x}\right)-\frac{(d+ex^2)^{3/2}}{3x^3}\right)+ b\int \frac{(ex^2+d)^{3/2}\arctan(cx)}{x^4}dx$$

$$\Big\downarrow \text{ 219}$$

$$b\int \frac{\left(ex^2+d\right)^{3/2}\arctan(cx)}{x^4}dx+ a\left(e\left(\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{\sqrt{d+ex^2}}{x}\right)-\frac{\left(d+ex^2\right)^{3/2}}{3x^3}\right)$$

$$\Big\downarrow \text{ 5560}$$

$$b\int \frac{\left(ex^2+d\right)^{3/2}\arctan(cx)}{x^4}dx+ a\left(e\left(\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)-\frac{\sqrt{d+ex^2}}{x}\right)-\frac{\left(d+ex^2\right)^{3/2}}{3x^3}\right)$$

input
```
Int[((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/x^4,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\left(e\,x^2+d\right)^{\frac{3}{2}}\left(a+b\arctan\left(cx\right)\right)}{x^4}dx$$

input
```
int((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^4,x)
```

output
```
int((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^4,x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.74

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^4}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}(b\arctan(cx)+a)}{x^4}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^4,x, algorithm="fricas")
```

output
```
integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arctan(c*x))*sqrt(e*x^2 + d)/x^4
, x)
```

## Sympy [N/A]

Not integrable

Time = 11.43 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^4}\,dx = \int \frac{(a+b\,\mathrm{atan}\,(cx))\,(d+ex^2)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((e*x**2+d)**(3/2)*(a+b*atan(c*x))/x**4,x)
```

output
```
Integral((a + b*atan(c*x))*(d + e*x**2)**(3/2)/x**4, x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

## Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^4}\,dx=\int \frac{(ex^2+d)^{\frac{3}{2}}(b\arctan(cx)+a)}{x^4}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^4,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(3/2)*(b*arctan(c*x) + a)/x^4, x)
```

## Mupad [N/A]

Not integrable

Time = 1.35 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^4}\,dx=\int \frac{(a+b\,\mathrm{atan}(c\,x))\,\left(e\,x^2+d\right)^{3/2}}{x^4}\,dx$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^(3/2))/x^4,x)
```

output
```
int(((a + b*atan(c*x))*(d + e*x^2)^(3/2))/x^4, x)
```

## Reduce [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 111, normalized size of antiderivative = 4.83

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^4}\,dx = \frac{-\sqrt{e\,x^2+d}\,ad-4\sqrt{e\,x^2+d}\,ae\,x^2+3\sqrt{e}\log\left(\frac{\sqrt{e\,x^2+d}+\sqrt{e}\,x}{\sqrt{d}}\right)ae\,x^3 -}{3x^3}$$

input
```
int((e*x^2+d)^(3/2)*(a+b*atan(c*x))/x^4,x)
```

output
```
( - sqrt(d + e*x**2)*a*d - 4*sqrt(d + e*x**2)*a*e*x**2 + 3*sqrt(e)*log((sq
rt(d + e*x**2) + sqrt(e)*x)/sqrt(d))*a*e*x**3 + 3*int((sqrt(d + e*x**2)*at
an(c*x))/x**4,x)*b*d*x**3 + 3*int((sqrt(d + e*x**2)*atan(c*x))/x**2,x)*b*e
*x**3)/(3*x**3)
```

# 3.1190 $\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^5}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^5}\,dx = -\frac{3ae\sqrt{d+ex^2}}{8x^2}-\frac{a(d+ex^2)^{3/2}}{4x^4}-\frac{3ae^2\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{8\sqrt{d}}+b\text{Int}\left(\frac{(d+ex^2)^{3/2}\arctan(cx)}{x^5},x\right)$$

output
```
-3/8*a*e*(e*x^2+d)^(1/2)/x^2-1/4*a*(e*x^2+d)^(3/2)/x^4-3/8*a*e^2*arctanh((
e*x^2+d)^(1/2)/d^(1/2))/d^(1/2)+b*Defer(Int)((e*x^2+d)^(3/2)*arctan(c*x)/x
^5,x)
```

## Mathematica [N/A]

Not integrable

Time = 11.88 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^5}\,dx = \int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^5}\,dx$$

input
```
Integrate[((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/x^5,x]
```

output
```
Integrate[((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/x^5, x]
```

## Rubi [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d+ex^2\right)^{3/2}(a+b\arctan(cx))}{x^5}dx$$

$$\Big\downarrow \text{5517}$$

$$a\int \frac{\left(ex^2+d\right)^{3/2}}{x^5}dx+b\int \frac{\left(ex^2+d\right)^{3/2}\arctan(cx)}{x^5}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}a\int \frac{\left(ex^2+d\right)^{3/2}}{x^6}dx^2+b\int \frac{\left(ex^2+d\right)^{3/2}\arctan(cx)}{x^5}dx$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}a\left(\frac{3}{4}e\int \frac{\sqrt{ex^2+d}}{x^4}dx^2-\frac{\left(d+ex^2\right)^{3/2}}{2x^4}\right)+b\int \frac{\left(ex^2+d\right)^{3/2}\arctan(cx)}{x^5}dx$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}a\left(\frac{3}{4}e\left(\frac{1}{2}e\int \frac{1}{x^2\sqrt{ex^2+d}}dx^2-\frac{\sqrt{d+ex^2}}{x^2}\right)-\frac{\left(d+ex^2\right)^{3/2}}{2x^4}\right)+ b\int \frac{\left(ex^2+d\right)^{3/2}\arctan(cx)}{x^5}dx$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}a\left(\frac{3}{4}e\left(\int \frac{1}{\frac{x^4}{e}-\frac{d}{e}}d\sqrt{ex^2+d}-\frac{\sqrt{d+ex^2}}{x^2}\right)-\frac{(d+ex^2)^{3/2}}{2x^4}\right)+ b\int \frac{(ex^2+d)^{3/2}\arctan(cx)}{x^5}dx$$

$\Big\downarrow$ 221

$$b\int \frac{(ex^2+d)^{3/2}\arctan(cx)}{x^5}dx+ \frac{1}{2}a\left(\frac{3}{4}e\left(-\frac{e\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{d+ex^2}}{x^2}\right)-\frac{(d+ex^2)^{3/2}}{2x^4}\right)$$

$\Big\downarrow$ 5560

$$b\int \frac{(ex^2+d)^{3/2}\arctan(cx)}{x^5}dx+ \frac{1}{2}a\left(\frac{3}{4}e\left(-\frac{e\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{\sqrt{d}}-\frac{\sqrt{d+ex^2}}{x^2}\right)-\frac{(d+ex^2)^{3/2}}{2x^4}\right)$$

input
```
Int[((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/x^5,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{(e\,x^2+d)^{\frac{3}{2}}\,(a+b\arctan(cx))}{x^5}dx$$

input
```
int((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^5,x)
```

output
```
int((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^5,x)
```

## Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.74

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^5}\,dx=\int \frac{(ex^2+d)^{\frac{3}{2}}(b\arctan(cx)+a)}{x^5}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^5,x, algorithm="fricas")
```

output
```
integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arctan(c*x))*sqrt(e*x^2 + d)/x^5
, x)
```

## Sympy [N/A]

Not integrable

Time = 19.94 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^5}\,dx=\int \frac{(a+b\,\mathrm{atan}\,(cx))\,(d+ex^2)^{\frac{3}{2}}}{x^5}\,dx$$

input
```
integrate((e*x**2+d)**(3/2)*(a+b*atan(c*x))/x**5,x)
```

output
```
Integral((a + b*atan(c*x))*(d + e*x**2)**(3/2)/x**5, x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^5}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^5,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

## Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^5}\,dx=\int \frac{(ex^2+d)^{\frac{3}{2}}(b\arctan(cx)+a)}{x^5}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^5,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(3/2)*(b*arctan(c*x) + a)/x^5, x)
```

## Mupad [N/A]

Not integrable

Time = 1.52 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^5}\,dx=\int \frac{(a+b\,\mathrm{atan}(c\,x))\,(e\,x^2+d)^{3/2}}{x^5}\,dx$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^(3/2))/x^5,x)
```

output
```
int(((a + b*atan(c*x))*(d + e*x^2)^(3/2))/x^5, x)
```

## Reduce [N/A]

Not integrable

Time = 1.40 (sec) , antiderivative size = 158, normalized size of antiderivative = 6.87

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^5}\,dx = \frac{-2\sqrt{e\,x^2+d}\,a\,d^2-5\sqrt{e\,x^2+d}\,ade\,x^2+3\sqrt{d}\log\left(\frac{\sqrt{e\,x^2+d}-\sqrt{d}+\sqrt{e}\,x}{\sqrt{d}}\right)}{}$$

input
```
int((e*x^2+d)^(3/2)*(a+b*atan(c*x))/x^5,x)
```

output
```
( - 2*sqrt(d + e*x**2)*a*d**2 - 5*sqrt(d + e*x**2)*a*d*e*x**2 + 3*sqrt(d)*
log((sqrt(d + e*x**2) - sqrt(d) + sqrt(e)*x)/sqrt(d))*a*e**2*x**4 - 3*sqrt
(d)*log((sqrt(d + e*x**2) + sqrt(d) + sqrt(e)*x)/sqrt(d))*a*e**2*x**4 + 8*
int((sqrt(d + e*x**2)*atan(c*x))/x**5,x)*b*d**2*x**4 + 8*int((sqrt(d + e*x
**2)*atan(c*x))/x**3,x)*b*d*e*x**4)/(8*d*x**4)
```

# 3.1191 $\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^6}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 178

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^6}\,dx = \frac{bc(4c^2d-7e)\sqrt{d+ex^2}}{40x^2} - \frac{bc(d+ex^2)^{3/2}}{20x^4} - \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{5dx^5} - \frac{bc(8c^4d^2-20c^2de+15e^2)\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{40\sqrt{d}} + \frac{b(c^2d-e)^{5/2}\operatorname{arctanh}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{5d}$$

output

```
1/40*b*c*(4*c^2*d-7*e)*(e*x^2+d)^(1/2)/x^2-1/20*b*c*(e*x^2+d)^(3/2)/x^4-1/
5*(e*x^2+d)^(5/2)*(a+b*arctan(c*x))/d/x^5-1/40*b*c*(8*c^4*d^2-20*c^2*d*e+1
5*e^2)*arctanh((e*x^2+d)^(1/2)/d^(1/2))/d^(1/2)+1/5*b*(c^2*d-e)^(5/2)*arct
anh(c*(e*x^2+d)^(1/2)/(c^2*d-e)^(1/2))/d
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.88

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^6}\,dx = \frac{-\sqrt{d+ex^2}\Big(8a(d+ex^2)^2+bcdx(9ex^2+d(2-4c^2x^2))\Big)-8b(d+$$

input
```
Integrate[((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/x^6,x]
```

output
```
(-(Sqrt[d + e*x^2]*(8*a*(d + e*x^2)^2 + b*c*d*x*(9*e*x^2 + d*(2 - 4*c^2*x^
2)))) - 8*b*(d + e*x^2)^(5/2)*ArcTan[c*x] + b*c*Sqrt[d]*(8*c^4*d^2 - 20*c^
2*d*e + 15*e^2)*x^5*Log[x] - b*c*Sqrt[d]*(8*c^4*d^2 - 20*c^2*d*e + 15*e^2)
*x^5*Log[d + Sqrt[d]*Sqrt[d + e*x^2]] + 4*b*(c^2*d - e)^(5/2)*x^5*Log[(-20
*c*d*(c*d - I*e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(c^2*d - e)^(7/2)
*(I + c*x))] + 4*b*(c^2*d - e)^(5/2)*x^5*Log[(-20*c*d*(c*d + I*e*x + Sqrt[
c^2*d - e]*Sqrt[d + e*x^2]))/(b*(c^2*d - e)^(7/2)*(-I + c*x))])/(40*d*x^5)
```

## Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {5511, 27, 354, 109, 27, 166, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^6}\,dx$$

$$\Big\downarrow \text{5511}$$

$$-bc\int -\frac{(ex^2+d)^{5/2}}{5dx^5(c^2x^2+1)}dx-\frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{5dx^5}$$

$$\Big\downarrow \text{27}$$

$$\frac{bc\int \frac{\left(ex^2+d\right)^{5/2}}{x^5\left(c^2x^2+1\right)}dx}{5d}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\arctan(cx)\right)}{5dx^5}$$

$$\Big\downarrow \text{354}$$

$$\frac{bc\int \frac{\left(ex^2+d\right)^{5/2}}{x^6\left(c^2x^2+1\right)}dx^2}{10d}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\arctan(cx)\right)}{5dx^5}$$

$$\Big\downarrow \text{109}$$

$$\frac{bc\left(-\frac{1}{2}\int \frac{\sqrt{ex^2+d}\left(\left(c^2d-4e\right)ex^2+d\left(4c^2d-7e\right)\right)}{2x^4\left(c^2x^2+1\right)}dx^2-\frac{d\left(d+ex^2\right)^{3/2}}{2x^4}\right)}{10d}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\arctan(cx)\right)}{5dx^5}$$

$$\Big\downarrow \text{27}$$

$$\frac{bc\left(-\frac{1}{4}\int \frac{\sqrt{ex^2+d}\left(\left(c^2d-4e\right)ex^2+d\left(4c^2d-7e\right)\right)}{x^4\left(c^2x^2+1\right)}dx^2-\frac{d\left(d+ex^2\right)^{3/2}}{2x^4}\right)}{10d}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\arctan(cx)\right)}{5dx^5}$$

$$\Big\downarrow \text{166}$$

$$\frac{bc\left(\frac{1}{4}\left(\frac{d\left(4c^2d-7e\right)\sqrt{d+ex^2}}{x^2}-\int -\frac{e\left(4d^2c^4-9dec^2+8e^2\right)x^2+d\left(8d^2c^4-20dec^2+15e^2\right)}{2x^2\left(c^2x^2+1\right)\sqrt{ex^2+d}}dx^2\right)-\frac{d\left(d+ex^2\right)^{3/2}}{2x^4}\right)}{10d}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\arctan(cx)\right)}{5dx^5}$$

$$\Big\downarrow \text{27}$$

$$\frac{bc\left(\frac{1}{4}\left(\frac{1}{2}\int \frac{e\left(4d^2c^4-9dec^2+8e^2\right)x^2+d\left(8d^2c^4-20dec^2+15e^2\right)}{x^2\left(c^2x^2+1\right)\sqrt{ex^2+d}}dx^2+\frac{d\left(4c^2d-7e\right)\sqrt{d+ex^2}}{x^2}\right)-\frac{d\left(d+ex^2\right)^{3/2}}{2x^4}\right)}{10d}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\arctan(cx)\right)}{5dx^5}$$

$$\Big\downarrow \text{174}$$

$$\frac{bc\left(\frac{1}{4}\left(\frac{1}{2}\left(d\left(8c^4d^2-20c^2de+15e^2\right)\int \frac{1}{x^2\sqrt{ex^2+d}}dx^2-8\left(c^2d-e\right)^3\int \frac{1}{\left(c^2x^2+1\right)\sqrt{ex^2+d}}dx^2\right)+\frac{d\left(4c^2d-7e\right)\sqrt{d+ex^2}}{x^2}\right)-\frac{d(}{}\right.}{10d}-\frac{\left(d+ex^2\right)^{5/2}\left(a+b\arctan(cx)\right)}{5dx^5}$$

$$\Big\downarrow \text{73}$$

$$\frac{bc\left(\frac{1}{4}\left(\frac{1}{2}\left(\frac{2d(8c^4d^2-20c^2de+15e^2)\int\frac{1}{\frac{x^4}{e}-\frac{d}{e}}d\sqrt{ex^2+d}}{e}-\frac{16(c^2d-e)^3\int\frac{1}{\frac{c^2x^4}{e}-\frac{c^2d}{e}+1}d\sqrt{ex^2+d}}{e}\right)+\frac{d(4c^2d-7e)\sqrt{d+ex^2}}{x^2}\right)-\frac{d(d+ex^2)}{2x^4}}{10d}$$
$$\frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{5dx^5}$$

$$\Big\downarrow \text{221}$$

$$\frac{bc\left(\frac{1}{4}\left(\frac{1}{2}\left(\frac{16(c^2d-e)^{5/2}\text{arctanh}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{c}-2\sqrt{d}(8c^4d^2-20c^2de+15e^2)\,\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\right)+\frac{d(4c^2d-7e)\sqrt{d+ex^2}}{x^2}\right.}{10d}$$
$$\frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{5dx^5}$$

input
```
Int[((d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]))/x^6,x]
```

output
```
-1/5*((d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]))/(d*x^5) + (b*c*(-1/2*(d*(d +
e*x^2)^(3/2))/x^4 + ((d*(4*c^2*d - 7*e)*Sqrt[d + e*x^2])/x^2 + (-2*Sqrt[d]
*(8*c^4*d^2 - 20*c^2*d*e + 15*e^2)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]] + (16*
(c^2*d - e)^(5/2)*ArcTanh[(c*Sqrt[d + e*x^2])/Sqrt[c^2*d - e]])/c)/2)/4))/
(10*d)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 109
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*((e + f
*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] + Simp[1/(b*(b*e - a*f)*(m + 1))
 Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1)
+ c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p)
 + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a, b, c,
d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] ||
IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

## Maple [F]

$$\int \frac{(e\,x^2+d)^{\frac{3}{2}}\,(a+b\arctan(cx))}{x^6}dx$$

input
```
int((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^6,x)
```

output
```
int((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^6,x)
```

## Fricas [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 1151, normalized size of antiderivative = 6.47

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\arctan(cx))}{x^6}\,dx = \text{Too large to display}$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^6,x, algorithm="fricas")
```

output
```
[1/80*(4*(b*c^4*d^2 - 2*b*c^2*d*e + b*e^2)*sqrt(c^2*d - e)*x^5*log((c^4*e^
2*x^4 + 8*c^4*d^2 - 8*c^2*d*e + 2*(4*c^4*d*e - 3*c^2*e^2)*x^2 + 4*(c^3*e*x
^2 + 2*c^3*d - c*e)*sqrt(c^2*d - e)*sqrt(e*x^2 + d) + e^2)/(c^4*x^4 + 2*c^
2*x^2 + 1)) + (8*b*c^5*d^2 - 20*b*c^3*d*e + 15*b*c*e^2)*sqrt(d)*x^5*log(-(
e*x^2 - 2*sqrt(e*x^2 + d)*sqrt(d) + 2*d)/x^2) - 2*(8*a*e^2*x^4 + 2*b*c*d^2
*x + 16*a*d*e*x^2 - (4*b*c^3*d^2 - 9*b*c*d*e)*x^3 + 8*a*d^2 + 8*(b*e^2*x^4
 + 2*b*d*e*x^2 + b*d^2)*arctan(c*x))*sqrt(e*x^2 + d))/(d*x^5), 1/80*(8*(b*
c^4*d^2 - 2*b*c^2*d*e + b*e^2)*sqrt(-c^2*d + e)*x^5*arctan(-1/2*(c^2*e*x^2
 + 2*c^2*d - e)*sqrt(-c^2*d + e)*sqrt(e*x^2 + d)/(c^3*d^2 - c*d*e + (c^3*d
*e - c*e^2)*x^2)) + (8*b*c^5*d^2 - 20*b*c^3*d*e + 15*b*c*e^2)*sqrt(d)*x^5*
log(-(e*x^2 - 2*sqrt(e*x^2 + d)*sqrt(d) + 2*d)/x^2) - 2*(8*a*e^2*x^4 + 2*b
*c*d^2*x + 16*a*d*e*x^2 - (4*b*c^3*d^2 - 9*b*c*d*e)*x^3 + 8*a*d^2 + 8*(b*e
^2*x^4 + 2*b*d*e*x^2 + b*d^2)*arctan(c*x))*sqrt(e*x^2 + d))/(d*x^5), 1/40*
((8*b*c^5*d^2 - 20*b*c^3*d*e + 15*b*c*e^2)*sqrt(-d)*x^5*arctan(sqrt(e*x^2
+ d)*sqrt(-d)/d) + 2*(b*c^4*d^2 - 2*b*c^2*d*e + b*e^2)*sqrt(c^2*d - e)*x^5
*log((c^4*e^2*x^4 + 8*c^4*d^2 - 8*c^2*d*e + 2*(4*c^4*d*e - 3*c^2*e^2)*x^2
+ 4*(c^3*e*x^2 + 2*c^3*d - c*e)*sqrt(c^2*d - e)*sqrt(e*x^2 + d) + e^2)/(c^
4*x^4 + 2*c^2*x^2 + 1)) - (8*a*e^2*x^4 + 2*b*c*d^2*x + 16*a*d*e*x^2 - (4*b
*c^3*d^2 - 9*b*c*d*e)*x^3 + 8*a*d^2 + 8*(b*e^2*x^4 + 2*b*d*e*x^2 + b*d^2)*
arctan(c*x))*sqrt(e*x^2 + d))/(d*x^5), 1/40*(4*(b*c^4*d^2 - 2*b*c^2*d*e...
```

## Sympy [F]

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^6}\,dx=\int \frac{(a+b\,\mathrm{atan}\,(cx))\,(d+ex^2)^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((e*x**2+d)**(3/2)*(a+b*atan(c*x))/x**6,x)
```

output
```
Integral((a + b*atan(c*x))*(d + e*x**2)**(3/2)/x**6, x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^6}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^6,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

## Giac [F]

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^6}\,dx = \int \frac{(ex^2+d)^{\frac{3}{2}}(b\arctan(cx)+a)}{x^6}\,dx$$

input
```
integrate((e*x^2+d)^(3/2)*(a+b*arctan(c*x))/x^6,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(3/2)*(b*arctan(c*x) + a)/x^6, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^6}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))\,(e\,x^2+d)^{3/2}}{x^6}\,dx$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^(3/2))/x^6,x)
```

output
```
int(((a + b*atan(c*x))*(d + e*x^2)^(3/2))/x^6, x)
```

## Reduce [F]

$$\int \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{x^6}\,dx = \frac{-\sqrt{e\,x^2+d}\,a\,d^2-2\sqrt{e\,x^2+d}\,ade\,x^2-\sqrt{e\,x^2+d}\,a\,e^2x^4-\sqrt{e}\,a\,e^2}{5d}$$

input
```
int((e*x^2+d)^(3/2)*(a+b*atan(c*x))/x^6,x)
```

output
```
( - sqrt(d + e*x**2)*a*d**2 - 2*sqrt(d + e*x**2)*a*d*e*x**2 - sqrt(d + e*x
**2)*a*e**2*x**4 - sqrt(e)*a*e**2*x**5 + 5*int((sqrt(d + e*x**2)*atan(c*x)
)/x**6,x)*b*d**2*x**5 + 5*int((sqrt(d + e*x**2)*atan(c*x))/x**4,x)*b*d*e*x
**5)/(5*d*x**5)
```

# 3.1192 $\int x^3(d+ex^2)^{5/2}(a+b\arctan(cx))\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 345

$$\int x^3(d+ex^2)^{5/2}(a+b\arctan(cx))\,dx = \frac{b(59c^6d^3+712c^4d^2e-1104c^2de^2+448e^3)x\sqrt{d+ex^2}}{8064c^7e} - \frac{b(69c^4d^2-520c^2de+336e^2)x(d+ex^2)^{3/2}}{12096c^5e} - \frac{b(33c^2d-56e)x(d+ex^2)^{5/2}}{3024c^3e} - \frac{bx(d+ex^2)^{7/2}}{72ce} - \frac{d(d+ex^2)^{7/2}(a+b\arctan(cx))}{7e^2} + \frac{(d+ex^2)^{9/2}(a+b\arctan(cx))}{9e^2} + \frac{b(c^2d-e)^{7/2}(2c^2d+7e)\arctan\left(\frac{\sqrt{c^2d-ex}}{\sqrt{d+ex^2}}\right)}{63c^9e^2} + \frac{b(315c^8d^4+840c^6d^3e-3024c^4d^2e^2+2880c^2de^3-896e^4)\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{8064c^9e^{3/2}}$$

output
```
1/8064*b*(59*c^6*d^3+712*c^4*d^2*e-1104*c^2*d*e^2+448*e^3)*x*(e*x^2+d)^(1/
2)/c^7/e-1/12096*b*(69*c^4*d^2-520*c^2*d*e+336*e^2)*x*(e*x^2+d)^(3/2)/c^5/
e-1/3024*b*(33*c^2*d-56*e)*x*(e*x^2+d)^(5/2)/c^3/e-1/72*b*x*(e*x^2+d)^(7/2
)/c/e-1/7*d*(e*x^2+d)^(7/2)*(a+b*arctan(c*x))/e^2+1/9*(e*x^2+d)^(9/2)*(a+b
*arctan(c*x))/e^2+1/63*b*(c^2*d-e)^(7/2)*(2*c^2*d+7*e)*arctan((c^2*d-e)^(1
/2)*x/(e*x^2+d)^(1/2))/c^9/e^2+1/8064*b*(315*c^8*d^4+840*c^6*d^3*e-3024*c^
4*d^2*e^2+2880*c^2*d*e^3-896*e^4)*arctanh(e^(1/2)*x/(e*x^2+d)^(1/2))/c^9/e
^(3/2)
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.59 (sec) , antiderivative size = 470, normalized size of antiderivative = 1.36

$$\int x^3\left(d+ex^2\right)^{5/2}(a+b\arctan(cx))\,dx =$$

$$-\frac{c^2\sqrt{d+ex^2}\Big(384ac^7(2d-7ex^2)\left(d+ex^2\right)^3+bex(-1344e^3+48c^2e^2(83d+14ex^2)-8c^4e(453d^2+242de}{}$$

input
```
Integrate[x^3*(d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]),x]
```

output
```
-1/24192*(c^2*Sqrt[d + e*x^2]*(384*a*c^7*(2*d - 7*e*x^2)*(d + e*x^2)^3 + b
*e*x*(-1344*e^3 + 48*c^2*e^2*(83*d + 14*e*x^2) - 8*c^4*e*(453*d^2 + 242*d*
e*x^2 + 56*e^2*x^4) + 3*c^6*(187*d^3 + 558*d^2*e*x^2 + 424*d*e^2*x^4 + 112
*e^3*x^6))) + 384*b*c^9*(2*d - 7*e*x^2)*(d + e*x^2)^(7/2)*ArcTan[c*x] + (1
92*I)*b*(c^2*d - e)^(7/2)*(2*c^2*d + 7*e)*Log[((-252*I)*c^10*e^2*(c*d - I*
e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(c^2*d - e)^(9/2)*(2*c^2*d + 7*
e)*(I + c*x))] - (192*I)*b*(c^2*d - e)^(7/2)*(2*c^2*d + 7*e)*Log[((252*I)*
c^10*e^2*(c*d + I*e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(c^2*d - e)^(
9/2)*(2*c^2*d + 7*e)*(-I + c*x))] + 3*b*Sqrt[e]*(-315*c^8*d^4 - 840*c^6*d^
3*e + 3024*c^4*d^2*e^2 - 2880*c^2*d*e^3 + 896*e^4)*Log[e*x + Sqrt[e]*Sqrt[
d + e*x^2]])/(c^9*e^2)
```

## Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.06, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {5511, 27, 403, 403, 403, 27, 403, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(d+ex^2\right)^{5/2} (a+b\arctan(cx)) \, dx$$

$$\Big\downarrow \text{5511}$$

$$-bc\int -\frac{\left(2d-7ex^2\right)\left(ex^2+d\right)^{7/2}}{63e^2\left(c^2x^2+1\right)}dx+\frac{\left(d+ex^2\right)^{9/2}(a+b\arctan(cx))}{9e^2}-\frac{d\left(d+ex^2\right)^{7/2}(a+b\arctan(cx))}{7e^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{bc\int \frac{\left(2d-7ex^2\right)\left(ex^2+d\right)^{7/2}}{c^2x^2+1}dx}{63e^2}+\frac{\left(d+ex^2\right)^{9/2}(a+b\arctan(cx))}{9e^2}-\frac{d\left(d+ex^2\right)^{7/2}(a+b\arctan(cx))}{7e^2}$$

$$\Big\downarrow \text{403}$$

$$\frac{bc\left(\frac{\int \frac{\left(ex^2+d\right)^{5/2}\left(d\left(16dc^2+7e\right)-\left(33c^2d-56e\right)ex^2\right)}{c^2x^2+1}dx}{8c^2}-\frac{7ex\left(d+ex^2\right)^{7/2}}{8c^2}\right)}{63e^2}+\frac{\left(d+ex^2\right)^{9/2}(a+b\arctan(cx))}{9e^2}-\frac{d\left(d+ex^2\right)^{7/2}(a+b\arctan(cx))}{7e^2}$$

$$\Big\downarrow \text{403}$$

$$\frac{bc\left(\frac{\frac{\int \frac{\left(ex^2+d\right)^{3/2}\left(d\left(96d^2c^4+75dec^2-56e^2\right)-e\left(69d^2c^4-520dec^2+336e^2\right)x^2\right)}{c^2x^2+1}dx}{6c^2}-\frac{ex\left(33c^2d-56e\right)\left(d+ex^2\right)^{5/2}}{6c^2}}{8c^2}-\frac{7ex\left(d+ex^2\right)^{7/2}}{8c^2}\right)}{63e^2}+\frac{\left(d+ex^2\right)^{9/2}(a+b\arctan(cx))}{9e^2}-\frac{d\left(d+ex^2\right)^{7/2}(a+b\arctan(cx))}{7e^2}$$

$$\Big\downarrow \text{403}$$

$$\frac{bc\left(\frac{\frac{\frac{\int \frac{3\sqrt{ex^2+d}\left(e\left(59d^3c^6+712d^2ec^4-1104de^2c^2+448e^3\right)x^2+d\left(128d^3c^6+123d^2ec^4-248de^2c^2+112e^3\right)\right)}{c^2x^2+1}dx}{4c^2}-\frac{ex\left(69c^4d^2-520c^2de+336e^2\right)\left(d+ex^2\right)^{3/2}}{4c^2}}{6c^2}-\frac{e}{}}{8c^2}\right.}{63e^2}$$

$$\frac{\left(d+ex^2\right)^{9/2}(a+b\arctan(cx))}{9e^2}-\frac{d\left(d+ex^2\right)^{7/2}(a+b\arctan(cx))}{7e^2}$$

$\downarrow$ 27

$$\frac{bc\left(\frac{\frac{\frac{3\int \frac{\sqrt{ex^2+d}\left(e\left(59d^3c^6+712d^2ec^4-1104de^2c^2+448e^3\right)x^2+d\left(128d^3c^6+123d^2ec^4-248de^2c^2+112e^3\right)\right)}{c^2x^2+1}dx}{4c^2}-\frac{ex\left(69c^4d^2-520c^2de+336e^2\right)\left(d+ex^2\right)^{3/2}}{4c^2}}{6c^2}-\frac{e}{}}{8c^2}\right.}{63e^2}$$

$$\frac{\left(d+ex^2\right)^{9/2}(a+b\arctan(cx))}{9e^2}-\frac{d\left(d+ex^2\right)^{7/2}(a+b\arctan(cx))}{7e^2}$$

$\downarrow$ 403

$$\frac{bc\left(\frac{\frac{3\left(\frac{\int \frac{e\left(315d^4c^8+840d^3ec^6-3024d^2e^2c^4+2880de^3c^2-896e^4\right)x^2+d\left(256d^4c^8+187d^3ec^6-1208d^2e^2c^4+1328de^3c^2-448e^4\right)}{\left(c^2x^2+1\right)\sqrt{ex^2+d}}dx}{2c^2}+\frac{ex\left(59c^6d^3+712c^4d^2e-110\right.}{2c}\right.}{4c^2}}{6c^2}}{8c^2}\right.}{63e^2}$$

$$\frac{\left(d+ex^2\right)^{9/2}(a+b\arctan(cx))}{9e^2}-\frac{d\left(d+ex^2\right)^{7/2}(a+b\arctan(cx))}{7e^2}$$

$\downarrow$ 398

$$bc\left(\frac{\frac{\frac{3\left(\frac{128\left(2c^2d+7e\right)\left(c^2d-e\right)^4\int\frac{1}{\left(c^2x^2+1\right)\sqrt{ex^2+d}}dx}{c^2}+\frac{\frac{e\left(315c^8d^4+840c^6d^3e-3024c^4d^2e^2+2880c^2de^3-896e^4\right)\int\frac{1}{\sqrt{ex^2+d}}dx}{c^2}}{2c^2}+\frac{ex\left(59c^6d^3+712c^4d^2e-11\cdots\right.}{2\cdots}\right.}{4c^2}}{6c^2}}{8c^2}\right.$$

$$\overline{\qquad\qquad\qquad 63e^2}$$

$$\frac{\left(d+ex^2\right)^{9/2}(a+b\arctan(cx))}{9e^2}-\frac{d\left(d+ex^2\right)^{7/2}(a+b\arctan(cx))}{7e^2}$$

$\downarrow$ 224

$$bc\left(\frac{\frac{\frac{3\left(\frac{128\left(2c^2d+7e\right)\left(c^2d-e\right)^4\int\frac{1}{\left(c^2x^2+1\right)\sqrt{ex^2+d}}dx}{c^2}+\frac{\frac{e\left(315c^8d^4+840c^6d^3e-3024c^4d^2e^2+2880c^2de^3-896e^4\right)\int\frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{c^2}}{2c^2}+\frac{ex\left(59c^6d^3+712\cdots\right.}{\cdots}\right.}{4c^2}}{6c^2}}{8c^2}\right.$$

$$\overline{\qquad\qquad\qquad 63e^2}$$

$$\frac{\left(d+ex^2\right)^{9/2}(a+b\arctan(cx))}{9e^2}-\frac{d\left(d+ex^2\right)^{7/2}(a+b\arctan(cx))}{7e^2}$$

$\downarrow$ 219

$$bc\left(\frac{\frac{\frac{3\left(\frac{128\left(2c^2d+7e\right)\left(c^2d-e\right)^4\int\frac{1}{\left(c^2x^2+1\right)\sqrt{ex^2+d}}dx}{c^2}+\frac{\frac{\sqrt{e}\left(315c^8d^4+840c^6d^3e-3024c^4d^2e^2+2880c^2de^3-896e^4\right)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2}}{2c^2}+\frac{ex\left(59c^6d^3+7\cdots\right.}{\cdots}\right.}{4c^2}}{6c^2}}{8c^2}\right.$$

$$\overline{\qquad\qquad\qquad 63e^2}$$

$$\frac{\left(d+ex^2\right)^{9/2}(a+b\arctan(cx))}{9e^2}-\frac{d\left(d+ex^2\right)^{7/2}(a+b\arctan(cx))}{7e^2}$$

$\downarrow$ 291

$$bc\left(\frac{\frac{\frac{3\left(\frac{\frac{128\left(2c^2d+7e\right)\left(c^2d-e\right)^4\int\frac{1}{1-\frac{\left(e-c^2d\right)x^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{c^2}+\frac{\sqrt{e}\left(315c^8d^4+840c^6d^3e-3024c^4d^2e^2+2880c^2de^3-896e^4\right)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2}}{2c^2}+\frac{ex\left(59c^6d\right.}{}\right.}{4c^2}}{6c^2}}{8c^2}\right.$$

$$\overline{\qquad\qquad\qquad\qquad\qquad\qquad 63e^2}$$

$$\frac{\left(d+ex^2\right)^{9/2}(a+b\arctan(cx))}{9e^2}-\frac{d\left(d+ex^2\right)^{7/2}(a+b\arctan(cx))}{7e^2}$$

$\downarrow$ 216

$$\frac{\left(d+ex^2\right)^{9/2}(a+b\arctan(cx))}{9e^2}-\frac{d\left(d+ex^2\right)^{7/2}(a+b\arctan(cx))}{7e^2}+$$

$$bc\left(\frac{\frac{\frac{3\left(\frac{\frac{128\left(2c^2d+7e\right)\left(c^2d-e\right)^{7/2}\arctan\left(\frac{x\sqrt{c^2d-e}}{\sqrt{d+ex^2}}\right)}{c^2}+\frac{\sqrt{e}\left(315c^8d^4+840c^6d^3e-3024c^4d^2e^2+2880c^2de^3-896e^4\right)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2}}{2c^2}+\frac{ex\left(59c^6d^3+712\right.}{}\right.}{4c^2}}{6c^2}}{8c^2}\right.$$

$$\overline{\qquad\qquad\qquad\qquad\qquad\qquad 63e^2}$$

input
```
Int[x^3*(d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]),x]
```

output
```
-1/7*(d*(d + e*x^2)^(7/2)*(a + b*ArcTan[c*x]))/e^2 + ((d + e*x^2)^(9/2)*(a
 + b*ArcTan[c*x]))/(9*e^2) + (b*c*((-7*e*x*(d + e*x^2)^(7/2))/(8*c^2) + (-
1/6*((33*c^2*d - 56*e)*e*x*(d + e*x^2)^(5/2))/c^2 + (-1/4*(e*(69*c^4*d^2 -
 520*c^2*d*e + 336*e^2)*x*(d + e*x^2)^(3/2))/c^2 + (3*((e*(59*c^6*d^3 + 71
2*c^4*d^2*e - 1104*c^2*d*e^2 + 448*e^3)*x*Sqrt[d + e*x^2])/(2*c^2) + ((128
*(c^2*d - e)^(7/2)*(2*c^2*d + 7*e)*ArcTan[(Sqrt[c^2*d - e]*x)/Sqrt[d + e*x
^2]])/c^2 + (Sqrt[e]*(315*c^8*d^4 + 840*c^6*d^3*e - 3024*c^4*d^2*e^2 + 288
0*c^2*d*e^3 - 896*e^4)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/c^2)/(2*c^2))
)/(4*c^2))/(6*c^2))/(8*c^2)))/(63*e^2)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

## Maple [F]

$$\int x^3 \left(e\,x^2+d\right)^{\frac{5}{2}} \left(a+b\arctan\left(cx\right)\right) dx$$

input
```
int(x^3*(e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x)
```

output
```
int(x^3*(e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x)
```

## Fricas [A] (verification not implemented)

Time = 29.10 (sec) , antiderivative size = 1978, normalized size of antiderivative = 5.73

$$\int x^3 (d + ex^2)^{5/2} (a + b \arctan(cx)) \, dx = \text{Too large to display}$$

input
```
integrate(x^3*(e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
[-1/48384*(3*(315*b*c^8*d^4 + 840*b*c^6*d^3*e - 3024*b*c^4*d^2*e^2 + 2880*
b*c^2*d*e^3 - 896*b*e^4)*sqrt(e)*log(-2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*
x - d) + 192*(2*b*c^8*d^4 + b*c^6*d^3*e - 15*b*c^4*d^2*e^2 + 19*b*c^2*d*e^
3 - 7*b*e^4)*sqrt(-c^2*d + e)*log(((c^4*d^2 - 8*c^2*d*e + 8*e^2)*x^4 - 2*(
3*c^2*d^2 - 4*d*e)*x^2 - 4*((c^2*d - 2*e)*x^3 - d*x)*sqrt(-c^2*d + e)*sqrt
(e*x^2 + d) + d^2)/(c^4*x^4 + 2*c^2*x^2 + 1)) - 2*(2688*a*c^9*e^4*x^8 + 72
96*a*c^9*d*e^3*x^6 - 336*b*c^8*e^4*x^7 + 5760*a*c^9*d^2*e^2*x^4 + 384*a*c^
9*d^3*e*x^2 - 768*a*c^9*d^4 - 8*(159*b*c^8*d*e^3 - 56*b*c^6*e^4)*x^5 - 2*(
837*b*c^8*d^2*e^2 - 968*b*c^6*d*e^3 + 336*b*c^4*e^4)*x^3 - 3*(187*b*c^8*d^
3*e - 1208*b*c^6*d^2*e^2 + 1328*b*c^4*d*e^3 - 448*b*c^2*e^4)*x + 384*(7*b*
c^9*e^4*x^8 + 19*b*c^9*d*e^3*x^6 + 15*b*c^9*d^2*e^2*x^4 + b*c^9*d^3*e*x^2
- 2*b*c^9*d^4)*arctan(c*x))*sqrt(e*x^2 + d))/(c^9*e^2), 1/48384*(384*(2*b*
c^8*d^4 + b*c^6*d^3*e - 15*b*c^4*d^2*e^2 + 19*b*c^2*d*e^3 - 7*b*e^4)*sqrt(
c^2*d - e)*arctan(1/2*sqrt(c^2*d - e)*((c^2*d - 2*e)*x^2 - d)*sqrt(e*x^2 +
 d)/((c^2*d*e - e^2)*x^3 + (c^2*d^2 - d*e)*x)) - 3*(315*b*c^8*d^4 + 840*b*
c^6*d^3*e - 3024*b*c^4*d^2*e^2 + 2880*b*c^2*d*e^3 - 896*b*e^4)*sqrt(e)*log
(-2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x - d) + 2*(2688*a*c^9*e^4*x^8 + 729
6*a*c^9*d*e^3*x^6 - 336*b*c^8*e^4*x^7 + 5760*a*c^9*d^2*e^2*x^4 + 384*a*c^9
*d^3*e*x^2 - 768*a*c^9*d^4 - 8*(159*b*c^8*d*e^3 - 56*b*c^6*e^4)*x^5 - 2*(8
37*b*c^8*d^2*e^2 - 968*b*c^6*d*e^3 + 336*b*c^4*e^4)*x^3 - 3*(187*b*c^8*...
```

## Sympy [F]

$$\int x^3 (d + ex^2)^{5/2} (a + b \arctan(cx)) \, dx = \int x^3 (a + b \operatorname{atan}(cx)) (d + ex^2)^{\frac{5}{2}} \, dx$$

input
```
integrate(x**3*(e*x**2+d)**(5/2)*(a+b*atan(c*x)),x)
```

output
```
Integral(x**3*(a + b*atan(c*x))*(d + e*x**2)**(5/2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int x^3(d+ex^2)^{5/2}(a+b\arctan(cx))\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

## Giac [F(-2)]

Exception generated.

$$\int x^3(d+ex^2)^{5/2}(a+b\arctan(cx))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x^3*(e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:index.cc index_m i_lex_is_greater Error: Bad Argument V
alue
```

## Mupad [F(-1)]

Timed out.

$$\int x^3 \left(d+ex^2\right)^{5/2} (a+b\arctan(cx))\,dx=\int x^3\,(a+b\,\mathrm{atan}(c\,x))\,\left(e\,x^2+d\right)^{5/2}dx$$

input
```
int(x^3*(a + b*atan(c*x))*(d + e*x^2)^(5/2),x)
```

output
```
int(x^3*(a + b*atan(c*x))*(d + e*x^2)^(5/2), x)
```

## Reduce [F]

$$\int x^3 \left(d+ex^2\right)^{5/2} (a+b\arctan(cx))\,dx=\frac{-2\sqrt{e\,x^2+d}\,a\,d^4+\sqrt{e\,x^2+d}\,a\,d^3e\,x^2+15\sqrt{e\,x^2+d}\,a\,d^2e^2x^4+19\sqrt{e\,x^2+d}\,ad\,e^3x^6}{}$$

input
```
int(x^3*(e*x^2+d)^(5/2)*(a+b*atan(c*x)),x)
```

output
```
( - 2*sqrt(d + e*x**2)*a*d**4 + sqrt(d + e*x**2)*a*d**3*e*x**2 + 15*sqrt(d
 + e*x**2)*a*d**2*e**2*x**4 + 19*sqrt(d + e*x**2)*a*d*e**3*x**6 + 7*sqrt(d
 + e*x**2)*a*e**4*x**8 + 63*int(sqrt(d + e*x**2)*atan(c*x)*x**7,x)*b*e**4
+ 126*int(sqrt(d + e*x**2)*atan(c*x)*x**5,x)*b*d*e**3 + 63*int(sqrt(d + e*
x**2)*atan(c*x)*x**3,x)*b*d**2*e**2)/(63*e**2)
```

# 3.1193 $\int x^2(d+ex^2)^{5/2}(a+b\arctan(cx))\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 23

$$\int x^2(d+ex^2)^{5/2}(a+b\arctan(cx))\,dx = \frac{5ad^3x\sqrt{d+ex^2}}{128e} + \frac{5}{64}ad^2x^3\sqrt{d+ex^2} + \frac{5}{48}adx^3(d+ex^2)^{3/2} + \frac{1}{8}ax^3(d+ex^2)^{5/2} - \frac{5ad^4\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{128e^{3/2}} + b\mathrm{Int}\left(x^2(d+ex^2)^{5/2}\arctan(cx),x\right)$$

output
```
5/128*a*d^3*x*(e*x^2+d)^(1/2)/e+5/64*a*d^2*x^3*(e*x^2+d)^(1/2)+5/48*a*d*x^
3*(e*x^2+d)^(3/2)+1/8*a*x^3*(e*x^2+d)^(5/2)-5/128*a*d^4*arctanh(e^(1/2)*x/
(e*x^2+d)^(1/2))/e^(3/2)+b*Defer(Int)(x^2*(e*x^2+d)^(5/2)*arctan(c*x),x)
```

## Mathematica [N/A]

Not integrable

Time = 12.67 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int x^2(d+ex^2)^{5/2}(a+b\arctan(cx))\,dx = \int x^2(d+ex^2)^{5/2}(a+b\arctan(cx))\,dx$$

input
```
Integrate[x^2*(d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]),x]
```

output
```
Integrate[x^2*(d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(d+ex^2\right)^{5/2}(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5517}$$

$$a\int x^2\left(ex^2+d\right)^{5/2}dx+b\int x^2\left(ex^2+d\right)^{5/2}\arctan(cx)dx$$

$$\Big\downarrow \text{248}$$

$$a\left(\frac{5}{8}d\int x^2\left(ex^2+d\right)^{3/2}dx+\frac{1}{8}x^3\left(d+ex^2\right)^{5/2}\right)+b\int x^2\left(ex^2+d\right)^{5/2}\arctan(cx)dx$$

$$\Big\downarrow \text{248}$$

$$a\left(\frac{5}{8}d\left(\frac{1}{2}d\int x^2\sqrt{ex^2+d}dx+\frac{1}{6}x^3\left(d+ex^2\right)^{3/2}\right)+\frac{1}{8}x^3\left(d+ex^2\right)^{5/2}\right)+ b\int x^2\left(ex^2+d\right)^{5/2}\arctan(cx)dx$$

$$\Big\downarrow \text{248}$$

$$a\left(\frac{5}{8}d\left(\frac{1}{2}d\left(\frac{1}{4}d\int\frac{x^2}{\sqrt{ex^2+d}}dx+\frac{1}{4}x^3\sqrt{d+ex^2}\right)+\frac{1}{6}x^3\left(d+ex^2\right)^{3/2}\right)+\frac{1}{8}x^3\left(d+ex^2\right)^{5/2}\right)+ b\int x^2\left(ex^2+d\right)^{5/2}\arctan(cx)dx$$

$$\Big\downarrow \text{262}$$

$$a\left(\frac{5}{8}d\left(\frac{1}{2}d\left(\frac{1}{4}d\left(\frac{x\sqrt{d+ex^2}}{2e}-\frac{d\int \frac{1}{\sqrt{ex^2+d}}dx}{2e}\right)+\frac{1}{4}x^3\sqrt{d+ex^2}\right)+\frac{1}{6}x^3\left(d+ex^2\right)^{3/2}\right)+\frac{1}{8}x^3\left(d+ex^2\right)^{5/2}\right)+ b\int x^2\left(ex^2+d\right)^{5/2}\arctan(cx)dx$$

$$\Big\downarrow \text{224}$$

$$a\left(\frac{5}{8}d\left(\frac{1}{2}d\left(\frac{1}{4}d\left(\frac{x\sqrt{d+ex^2}}{2e}-\frac{d\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{2e}\right)+\frac{1}{4}x^3\sqrt{d+ex^2}\right)+\frac{1}{6}x^3\left(d+ex^2\right)^{3/2}\right)+\frac{1}{8}x^3\left(d+ex^2\right)\right. b\int x^2\left(ex^2+d\right)^{5/2}\arctan(cx)dx$$

$$\Big\downarrow \text{219}$$

$$b\int x^2\left(ex^2+d\right)^{5/2}\arctan(cx)dx+ a\left(\frac{5}{8}d\left(\frac{1}{2}d\left(\frac{1}{4}d\left(\frac{x\sqrt{d+ex^2}}{2e}-\frac{d\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2e^{3/2}}\right)+\frac{1}{4}x^3\sqrt{d+ex^2}\right)+\frac{1}{6}x^3\left(d+ex^2\right)^{3/2}\right)+\frac{1}{8}x^3\left(d+ex^2\right)^{5}\right.$$

$$\Big\downarrow \text{5560}$$

$$b\int x^2\left(ex^2+d\right)^{5/2}\arctan(cx)dx+ a\left(\frac{5}{8}d\left(\frac{1}{2}d\left(\frac{1}{4}d\left(\frac{x\sqrt{d+ex^2}}{2e}-\frac{d\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2e^{3/2}}\right)+\frac{1}{4}x^3\sqrt{d+ex^2}\right)+\frac{1}{6}x^3\left(d+ex^2\right)^{3/2}\right)+\frac{1}{8}x^3\left(d+ex^2\right)^{5}\right.$$

input
```
Int[x^2*(d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.72 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int x^{2} \left(e \,x^{2}+d \right)^{\frac{5}{2}} \left(a +b \arctan \left(c x \right)\right)d x$$

input
```
int(x^2*(e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x)
```

output
```
int(x^2*(e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x)
```

## Fricas [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.91

$$\int x^2 \left(d+e x^2\right)^{5/2} (a+b \arctan (c x)) \, dx=\int { {\left(e x^{2} + d\right)}^{\frac{5}{2}} {\left(b \arctan \left(c x\right) + a\right)} x^{2} \,d x }$$

input
```
integrate(x^2*(e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
integral((a*e^2*x^6 + 2*a*d*e*x^4 + a*d^2*x^2 + (b*e^2*x^6 + 2*b*d*e*x^4 +
 b*d^2*x^2)*arctan(c*x))*sqrt(e*x^2 + d), x)
```

## Sympy [F(-1)]

Timed out.

$$\int x^2 \left(d+e x^2\right)^{5/2} (a+b \arctan (c x)) \, dx=\text{Timed out}$$

input
```
integrate(x**2*(e*x**2+d)**(5/2)*(a+b*atan(c*x)),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int x^2 (d+ex^2)^{5/2} (a+b\arctan(cx))\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

## Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x^2 (d+ex^2)^{5/2} (a+b\arctan(cx))\,dx = \int (ex^2+d)^{\frac{5}{2}}(b\arctan(cx)+a)x^2\,dx$$

input
```
integrate(x^2*(e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(5/2)*(b*arctan(c*x) + a)*x^2, x)
```

## Mupad [N/A]

Not integrable

Time = 1.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x^2 (d+ex^2)^{5/2} (a+b\arctan(cx))\,dx = \int x^2\,(a+b\,\mathrm{atan}(c\,x))\,\left(e\,x^2+d\right)^{5/2}\,dx$$

input
```
int(x^2*(a + b*atan(c*x))*(d + e*x^2)^(5/2),x)
```

output
```
int(x^2*(a + b*atan(c*x))*(d + e*x^2)^(5/2), x)
```

## Reduce [N/A]

Not integrable

Time = 1.17 (sec) , antiderivative size = 180, normalized size of antiderivative = 7.83

$$\int x^2 (d+ex^2)^{5/2} (a+b\arctan(cx))\,dx = \frac{15\sqrt{e\,x^2+d}\,a\,d^3ex+118\sqrt{e\,x^2+d}\,a\,d^2e^2x^3+136\sqrt{e\,x^2+d}\,ad\,e^3x^5+48\sqrt{e\,x^2+d}\,a\,e}{}$$

input
```
int(x^2*(e*x^2+d)^(5/2)*(a+b*atan(c*x)),x)
```

output
```
(15*sqrt(d + e*x**2)*a*d**3*e*x + 118*sqrt(d + e*x**2)*a*d**2*e**2*x**3 +
136*sqrt(d + e*x**2)*a*d*e**3*x**5 + 48*sqrt(d + e*x**2)*a*e**4*x**7 - 15*
sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)*x)/sqrt(d))*a*d**4 + 384*int(sqrt(
d + e*x**2)*atan(c*x)*x**6,x)*b*e**4 + 768*int(sqrt(d + e*x**2)*atan(c*x)*
x**4,x)*b*d*e**3 + 384*int(sqrt(d + e*x**2)*atan(c*x)*x**2,x)*b*d**2*e**2)
/(384*e**2)
```

# 3.1194 $\int x(d+ex^2)^{5/2}(a+b\arctan(cx))\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 233

$$\int x(d+ex^2)^{5/2}(a+b\arctan(cx))\,dx = -\frac{b(19c^4d^2-22c^2de+8e^2)\,x\sqrt{d+ex^2}}{112c^5} - \frac{b(11c^2d-6e)\,x(d+ex^2)^{3/2}}{168c^3} - \frac{bx(d+ex^2)^{5/2}}{42c} + \frac{(d+ex^2)^{7/2}(a+b\arctan(cx))}{7e} - \frac{b(c^2d-e)^{7/2}\arctan\left(\frac{\sqrt{c^2d-e}x}{\sqrt{d+ex^2}}\right)}{7c^7e} - \frac{b(35c^6d^3-70c^4d^2e+56c^2de^2-16e^3)\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{112c^7\sqrt{e}}$$

output

```
-1/112*b*(19*c^4*d^2-22*c^2*d*e+8*e^2)*x*(e*x^2+d)^(1/2)/c^5-1/168*b*(11*c
^2*d-6*e)*x*(e*x^2+d)^(3/2)/c^3-1/42*b*x*(e*x^2+d)^(5/2)/c+1/7*(e*x^2+d)^(
7/2)*(a+b*arctan(c*x))/e-1/7*b*(c^2*d-e)^(7/2)*arctan((c^2*d-e)^(1/2)*x/(e
*x^2+d)^(1/2))/c^7/e-1/112*b*(35*c^6*d^3-70*c^4*d^2*e+56*c^2*d*e^2-16*e^3)
*arctanh(e^(1/2)*x/(e*x^2+d)^(1/2))/c^7/e^(1/2)
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.52

$$\int x(d+ex^2)^{5/2}(a+b\arctan(cx))\,dx = \frac{c^2\sqrt{d+ex^2}\left(48ac^5(d+ex^2)^3-bex(24e^2-6c^2e(13d+2ex^2)+c^4(87d^2+38dex^2+8e\right.}{}$$

input
```
Integrate[x*(d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]),x]
```

output
```
(c^2*Sqrt[d + e*x^2]*(48*a*c^5*(d + e*x^2)^3 - b*e*x*(24*e^2 - 6*c^2*e*(13
*d + 2*e*x^2) + c^4*(87*d^2 + 38*d*e*x^2 + 8*e^2*x^4))) + 48*b*c^7*(d + e*
x^2)^(7/2)*ArcTan[c*x] - (24*I)*b*(c^2*d - e)^(7/2)*Log[(28*c^8*e*((-I)*c*
d + e*x - I*Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(c^2*d - e)^(9/2)*(-I + c
*x))] + (24*I)*b*(c^2*d - e)^(7/2)*Log[(28*c^8*e*(I*c*d + e*x + I*Sqrt[c^2
*d - e]*Sqrt[d + e*x^2]))/(b*(c^2*d - e)^(9/2)*(I + c*x))] + 3*b*Sqrt[e]*(
-35*c^6*d^3 + 70*c^4*d^2*e - 56*c^2*d*e^2 + 16*e^3)*Log[e*x + Sqrt[e]*Sqrt
[d + e*x^2]])/(336*c^7*e)
```

## Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {5509, 318, 403, 27, 403, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(d+ex^2)^{5/2}(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{ 5509}$$

$$\frac{(d+ex^2)^{7/2}(a+b\arctan(cx))}{7e}-\frac{bc\int\frac{(ex^2+d)^{7/2}}{c^2x^2+1}dx}{7e}$$

$\Big\downarrow$ 318

$$\frac{(d+ex^2)^{7/2}(a+b\arctan(cx))}{7e}-\frac{bc\left(\frac{\int \frac{\left(ex^2+d\right)^{3/2}\left(\left(11c^2d-6e\right)ex^2+d\left(6c^2d-e\right)\right)}{c^2x^2+1}dx}{6c^2}+\frac{ex(d+ex^2)^{5/2}}{6c^2}\right)}{7e}$$

$\Big\downarrow$ 403

$$\frac{(d+ex^2)^{7/2}(a+b\arctan(cx))}{7e}-\frac{bc\left(\frac{\frac{\int \frac{3\sqrt{ex^2+d}\left(e\left(19d^2c^4-22dec^2+8e^2\right)x^2+d\left(8d^2c^4-5dec^2+2e^2\right)\right)}{c^2x^2+1}dx}{4c^2}+\frac{ex\left(11c^2d-6e\right)\left(d+ex^2\right)^{3/2}}{4c^2}}{6c^2}+\frac{ex(d+ex^2)^{5/2}}{6c^2}\right)}{7e}$$

$\Big\downarrow$ 27

$$\frac{(d+ex^2)^{7/2}(a+b\arctan(cx))}{7e}-\frac{bc\left(\frac{\frac{3\int \frac{\sqrt{ex^2+d}\left(e\left(19d^2c^4-22dec^2+8e^2\right)x^2+d\left(8d^2c^4-5dec^2+2e^2\right)\right)}{c^2x^2+1}dx}{4c^2}+\frac{ex\left(11c^2d-6e\right)\left(d+ex^2\right)^{3/2}}{4c^2}}{6c^2}+\frac{ex(d+ex^2)^{5/2}}{6c^2}\right)}{7e}$$

$\Big\downarrow$ 403

$$\frac{(d+ex^2)^{7/2}(a+b\arctan(cx))}{7e}-\frac{bc\left(\frac{\frac{3\left(\frac{\int \frac{e\left(35d^3c^6-70d^2ec^4+56de^2c^2-16e^3\right)x^2+d\left(16d^3c^6-29d^2ec^4+26de^2c^2-8e^3\right)}{\left(c^2x^2+1\right)\sqrt{ex^2+d}}dx}{2c^2}+\frac{ex\left(19c^4d^2-22c^2de+8e^2\right)\sqrt{d+ex^2}}{2c^2}\right)}{4c^2}+\frac{ex\left(11c^2d-6e\right)\left(d+ex^2\right)^{3/}}{4c^2}}{6c^2}}{7e}$$

$\Big\downarrow$ 398

$$\frac{\left(d+ex^2\right)^{7/2}(a+b\arctan(cx))}{7e}-\frac{bc\left(\frac{\frac{3\left(\frac{\frac{16\left(c^2d-e\right)^4\int\frac{1}{\left(c^2x^2+1\right)\sqrt{ex^2+d}}dx}{c^2}+\frac{e\left(35c^6d^3-70c^4d^2e+56c^2de^2-16e^3\right)\int\frac{1}{\sqrt{ex^2+d}}dx}{c^2}}{2c^2}+\frac{ex\left(19c^4d^2-22c^2de+8e^2\right)\sqrt{d+ex^2}}{2c^2}\right)}{4c^2}+\frac{ex\left(11c^2d-6e\right)\left(d\right.}{4c^2}}{6c^2}\right.}{7e}$$

$\Big\downarrow$ 224

$$\frac{\left(d+ex^2\right)^{7/2}(a+b\arctan(cx))}{7e}-\frac{bc\left(\frac{\frac{3\left(\frac{\frac{16\left(c^2d-e\right)^4\int\frac{1}{\left(c^2x^2+1\right)\sqrt{ex^2+d}}dx}{c^2}+\frac{e\left(35c^6d^3-70c^4d^2e+56c^2de^2-16e^3\right)\int\frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{c^2}}{2c^2}+\frac{ex\left(19c^4d^2-22c^2de+8e^2\right)\sqrt{d+ex^2}}{2c^2}\right)}{4c^2}+\frac{ex\left(11c^2\right.}{}}{6c^2}\right.}{7e}$$

$\Big\downarrow$ 219

$$\frac{\left(d+ex^2\right)^{7/2}(a+b\arctan(cx))}{7e}-\frac{bc\left(\frac{\frac{3\left(\frac{\frac{16\left(c^2d-e\right)^4\int\frac{1}{\left(c^2x^2+1\right)\sqrt{ex^2+d}}dx}{c^2}+\frac{\sqrt{e}\left(35c^6d^3-70c^4d^2e+56c^2de^2-16e^3\right)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2}}{2c^2}+\frac{ex\left(19c^4d^2-22c^2de+8e^2\right)\sqrt{d+ex^2}}{2c^2}\right)}{4c^2}+\frac{ex\left(11\right.}{}}{6c^2}\right.}{7e}$$

$\Big\downarrow$ 291

$$\frac{(d+ex^2)^{7/2}(a+b\arctan(cx))}{7e} - \frac{bc\left(\frac{3\left(\frac{\frac{16(c^2d-e)^4\int \frac{1}{1-\frac{(e-c^2d)x^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{c^2}+\frac{\sqrt{e}(35c^6d^3-70c^4d^2e+56c^2de^2-16e^3)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2}}{2c^2}+\frac{ex(19c^4d^2-22c^2de+8e^2)\sqrt{d+ex^2}}{2c^2}\right)}{4c^2}+\frac{ex \ldots}{\ldots}}{6c^2} \ldots\right)}{7e}$$

$\downarrow$ 216

$$\frac{(d+ex^2)^{7/2}(a+b\arctan(cx))}{7e} - \frac{bc\left(\frac{3\left(\frac{\frac{16(c^2d-e)^{7/2}\arctan\left(\frac{x\sqrt{c^2d-e}}{\sqrt{d+ex^2}}\right)}{c^2}+\frac{\sqrt{e}(35c^6d^3-70c^4d^2e+56c^2de^2-16e^3)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2}}{2c^2}+\frac{ex(19c^4d^2-22c^2de+8e^2)\sqrt{d+ex^2}}{2c^2}\right)}{4c^2}+\frac{ex(11c^2 \ldots}{\ldots}}{6c^2} \ldots\right)}{7e}$$

input
```
Int[x*(d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]),x]
```

output
```
((d + e*x^2)^(7/2)*(a + b*ArcTan[c*x]))/(7*e) - (b*c*((e*x*(d + e*x^2)^(5/
2))/(6*c^2) + (((11*c^2*d - 6*e)*e*x*(d + e*x^2)^(3/2))/(4*c^2) + (3*((e*(
19*c^4*d^2 - 22*c^2*d*e + 8*e^2)*x*Sqrt[d + e*x^2])/(2*c^2) + ((16*(c^2*d
- e)^(7/2)*ArcTan[(Sqrt[c^2*d - e]*x)/Sqrt[d + e*x^2]])/c^2 + (Sqrt[e]*(35
*c^6*d^3 - 70*c^4*d^2*e + 56*c^2*d*e^2 - 16*e^3)*ArcTanh[(Sqrt[e]*x)/Sqrt[
d + e*x^2]])/c^2)/(2*c^2)))/(4*c^2))/(6*c^2)))/(7*e)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 5509
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*e*(q + 1))), x
] - Simp[b*(c/(2*e*(q + 1)))   Int[(d + e*x^2)^(q + 1)/(1 + c^2*x^2), x], x
] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]
```

## Maple [F]

$$\int x(e\,x^2+d)^{\frac{5}{2}}\,(a+b\arctan\,(cx))\,dx$$

input
```
int(x*(e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x)
```

output
```
int(x*(e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x)
```

## Fricas [A] (verification not implemented)

Time = 8.56 (sec) , antiderivative size = 1562, normalized size of antiderivative = 6.70

$$\int x(d+ex^2)^{5/2}\,(a+b\arctan(cx))\,dx = \text{Too large to display}$$

input
```
integrate(x*(e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
[-1/672*(3*(35*b*c^6*d^3 - 70*b*c^4*d^2*e + 56*b*c^2*d*e^2 - 16*b*e^3)*sqr
t(e)*log(-2*e*x^2 - 2*sqrt(e*x^2 + d)*sqrt(e)*x - d) + 24*(b*c^6*d^3 - 3*b
*c^4*d^2*e + 3*b*c^2*d*e^2 - b*e^3)*sqrt(-c^2*d + e)*log(((c^4*d^2 - 8*c^2
*d*e + 8*e^2)*x^4 - 2*(3*c^2*d^2 - 4*d*e)*x^2 + 4*((c^2*d - 2*e)*x^3 - d*x
)*sqrt(-c^2*d + e)*sqrt(e*x^2 + d) + d^2)/(c^4*x^4 + 2*c^2*x^2 + 1)) - 2*(
48*a*c^7*e^3*x^6 + 144*a*c^7*d*e^2*x^4 - 8*b*c^6*e^3*x^5 + 144*a*c^7*d^2*e
*x^2 + 48*a*c^7*d^3 - 2*(19*b*c^6*d*e^2 - 6*b*c^4*e^3)*x^3 - 3*(29*b*c^6*d
^2*e - 26*b*c^4*d*e^2 + 8*b*c^2*e^3)*x + 48*(b*c^7*e^3*x^6 + 3*b*c^7*d*e^2
*x^4 + 3*b*c^7*d^2*e*x^2 + b*c^7*d^3)*arctan(c*x))*sqrt(e*x^2 + d))/(c^7*e
), -1/672*(48*(b*c^6*d^3 - 3*b*c^4*d^2*e + 3*b*c^2*d*e^2 - b*e^3)*sqrt(c^2
*d - e)*arctan(1/2*sqrt(c^2*d - e)*((c^2*d - 2*e)*x^2 - d)*sqrt(e*x^2 + d)
/((c^2*d*e - e^2)*x^3 + (c^2*d^2 - d*e)*x)) + 3*(35*b*c^6*d^3 - 70*b*c^4*d
^2*e + 56*b*c^2*d*e^2 - 16*b*e^3)*sqrt(e)*log(-2*e*x^2 - 2*sqrt(e*x^2 + d)
*sqrt(e)*x - d) - 2*(48*a*c^7*e^3*x^6 + 144*a*c^7*d*e^2*x^4 - 8*b*c^6*e^3*
x^5 + 144*a*c^7*d^2*e*x^2 + 48*a*c^7*d^3 - 2*(19*b*c^6*d*e^2 - 6*b*c^4*e^3
)*x^3 - 3*(29*b*c^6*d^2*e - 26*b*c^4*d*e^2 + 8*b*c^2*e^3)*x + 48*(b*c^7*e^
3*x^6 + 3*b*c^7*d*e^2*x^4 + 3*b*c^7*d^2*e*x^2 + b*c^7*d^3)*arctan(c*x))*sq
rt(e*x^2 + d))/(c^7*e), 1/336*(3*(35*b*c^6*d^3 - 70*b*c^4*d^2*e + 56*b*c^2
*d*e^2 - 16*b*e^3)*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - 12*(b*c^6
*d^3 - 3*b*c^4*d^2*e + 3*b*c^2*d*e^2 - b*e^3)*sqrt(-c^2*d + e)*log(((c^...
```

## Sympy [F]

$$\int x(d+ex^2)^{5/2}(a+b\arctan(cx))\,dx=\int x(a+b\,\mathrm{atan}\,(cx))\left(d+ex^2\right)^{\frac{5}{2}}\,dx$$

input
```
integrate(x*(e*x**2+d)**(5/2)*(a+b*atan(c*x)),x)
```

output
```
Integral(x*(a + b*atan(c*x))*(d + e*x**2)**(5/2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int x(d+ex^2)^{5/2}(a+b\arctan(cx))\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e-c^2*d>0)', see `assume?` for m
ore detail
```

## Giac [F(-2)]

Exception generated.

$$\int x(d+ex^2)^{5/2}(a+b\arctan(cx))\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(x*(e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:index.cc index_m i_lex_is_greater Error: Bad Argument V
alue
```

## Mupad [F(-1)]

Timed out.

$$\int x(d+ex^2)^{5/2}(a+b\arctan(cx))\,dx = \int x\,(a+b\,\mathrm{atan}(c\,x))\,\left(e\,x^2+d\right)^{5/2}\,dx$$

input
```
int(x*(a + b*atan(c*x))*(d + e*x^2)^(5/2),x)
```

output
```
int(x*(a + b*atan(c*x))*(d + e*x^2)^(5/2), x)
```

## Reduce [F]

$$\int x(d+ex^2)^{5/2}(a+b\arctan(cx))\,dx = \frac{\sqrt{e\,x^2+d}\,a\,d^3+3\sqrt{e\,x^2+d}\,a\,d^2e\,x^2+3\sqrt{e\,x^2+d}\,ad\,e^2x^4+\sqrt{e\,x^2+d}\,a\,e^3x^6+7\left(\int \sqrt{}\right.}{}$$

input
```
int(x*(e*x^2+d)^(5/2)*(a+b*atan(c*x)),x)
```

output
```
(sqrt(d + e*x**2)*a*d**3 + 3*sqrt(d + e*x**2)*a*d**2*e*x**2 + 3*sqrt(d + e
*x**2)*a*d*e**2*x**4 + sqrt(d + e*x**2)*a*e**3*x**6 + 7*int(sqrt(d + e*x**
2)*atan(c*x)*x**5,x)*b*e**3 + 14*int(sqrt(d + e*x**2)*atan(c*x)*x**3,x)*b*
d*e**2 + 7*int(sqrt(d + e*x**2)*atan(c*x)*x,x)*b*d**2*e)/(7*e)
```

# 3.1195 $\int (d+ex^2)^{5/2} (a+b\arctan(cx))\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 20

$$\int (d+ex^2)^{5/2} (a+b\arctan(cx))\,dx = \text{Int}\left((d+ex^2)^{5/2} (a+b\arctan(cx)),x\right)$$

output
```
Defer(Int)((e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x)
```

## Mathematica [N/A]

Not integrable

Time = 5.77 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (d+ex^2)^{5/2} (a+b\arctan(cx))\,dx = \int (d+ex^2)^{5/2} (a+b\arctan(cx))\,dx$$

input
```
Integrate[(d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]),x]
```

output
```
Integrate[(d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)^{5/2} (a+b\arctan(cx))\,dx$$
$$\Big\downarrow \; 5560$$
$$\int \left(d+ex^2\right)^{5/2} (a+b\arctan(cx))dx$$

input
```
Int[(d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \left(e\,x^2+d\right)^{\frac{5}{2}} (a+b\arctan(cx))\,dx$$

input
```
int((e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x)
```

output
```
int((e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 3.05

$$\int \left(d+ex^2\right)^{5/2}(a+b\arctan(cx))\,dx=\int (ex^2+d)^{\frac{5}{2}}(b\arctan(cx)+a)\,dx$$

input
```
integrate((e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d
^2)*arctan(c*x))*sqrt(e*x^2 + d), x)
```

## Sympy [N/A]

Not integrable

Time = 91.84 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \left(d+ex^2\right)^{5/2}(a+b\arctan(cx))\,dx=\int (a+b\,\mathrm{atan}(cx))\left(d+ex^2\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((e*x**2+d)**(5/2)*(a+b*atan(c*x)),x)
```

output
```
Integral((a + b*atan(c*x))*(d + e*x**2)**(5/2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \left(d+ex^2\right)^{5/2}(a+b\arctan(cx))\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

## Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \left(d+ex^2\right)^{5/2}(a+b\arctan(cx))\,dx=\int \left(ex^2+d\right)^{\frac{5}{2}}(b\arctan\left(cx\right)+a)\,dx$$

input
```
integrate((e*x^2+d)^(5/2)*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(5/2)*(b*arctan(c*x) + a), x)
```

## Mupad [N/A]

Not integrable

Time = 0.99 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \left(d+ex^2\right)^{5/2}(a+b\arctan(cx))\,dx=\int (a+b\,\mathrm{atan}(c\,x))\,\left(e\,x^2+d\right)^{5/2}\,dx$$

input
```
int((a + b*atan(c*x))*(d + e*x^2)^(5/2),x)
```

output
```
int((a + b*atan(c*x))*(d + e*x^2)^(5/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 155, normalized size of antiderivative = 7.75

$$\int \left(d+ex^2\right)^{5/2}(a+b\arctan(cx))\,dx = \frac{33\sqrt{e\,x^2+d}\,a\,d^2ex+26\sqrt{e\,x^2+d}\,ad\,e^2x^3+8\sqrt{e\,x^2+d}\,a\,e^3x^5+15\sqrt{e}\log\left(\frac{\sqrt{e\,x^2+d}+\sqrt{e}}{\sqrt{d}}\right.}{}$$

input
```
int((e*x^2+d)^(5/2)*(a+b*atan(c*x)),x)
```

output
```
(33*sqrt(d + e*x**2)*a*d**2*e*x + 26*sqrt(d + e*x**2)*a*d*e**2*x**3 + 8*sq
rt(d + e*x**2)*a*e**3*x**5 + 15*sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)*x)
/sqrt(d))*a*d**3 + 48*int(sqrt(d + e*x**2)*atan(c*x)*x**4,x)*b*e**3 + 96*i
nt(sqrt(d + e*x**2)*atan(c*x)*x**2,x)*b*d*e**2 + 48*int(sqrt(d + e*x**2)*a
tan(c*x),x)*b*d**2*e)/(48*e)
```

# 3.1196 $\int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x}\,dx = ad^2\sqrt{d+ex^2}+\frac{1}{3}ad(d+ex^2)^{3/2} +\frac{1}{5}a(d+ex^2)^{5/2}-ad^{5/2}\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)+b\text{Int}\left(\frac{(d+ex^2)^{5/2}\arctan(cx)}{x},x\right)$$

output
```
a*d^2*(e*x^2+d)^(1/2)+1/3*a*d*(e*x^2+d)^(3/2)+1/5*a*(e*x^2+d)^(5/2)-a*d^(5
/2)*arctanh((e*x^2+d)^(1/2)/d^(1/2))+b*Defer(Int)((e*x^2+d)^(5/2)*arctan(c
*x)/x,x)
```

## Mathematica [N/A]

Not integrable

Time = 9.43 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x}\,dx = \int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x}\,dx$$

input
```
Integrate[((d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]))/x,x]
```

output
```
Integrate[((d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]))/x, x]
```

## Rubi [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x}dx$$

$$\Big\downarrow \text{5517}$$

$$a\int \frac{(ex^2+d)^{5/2}}{x}dx+b\int \frac{(ex^2+d)^{5/2}\arctan(cx)}{x}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}a\int \frac{(ex^2+d)^{5/2}}{x^2}dx^2+b\int \frac{(ex^2+d)^{5/2}\arctan(cx)}{x}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}a\left(d\int \frac{(ex^2+d)^{3/2}}{x^2}dx^2+\frac{2}{5}(d+ex^2)^{5/2}\right)+b\int \frac{(ex^2+d)^{5/2}\arctan(cx)}{x}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}a\left(d\left(d\int \frac{\sqrt{ex^2+d}}{x^2}dx^2+\frac{2}{3}(d+ex^2)^{3/2}\right)+\frac{2}{5}(d+ex^2)^{5/2}\right)+ b\int \frac{(ex^2+d)^{5/2}\arctan(cx)}{x}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}a\left(d\left(d\left(d\int \frac{1}{x^2\sqrt{ex^2+d}}dx^2+2\sqrt{d+ex^2}\right)+\frac{2}{3}(d+ex^2)^{3/2}\right)+\frac{2}{5}(d+ex^2)^{5/2}\right)+ b\int \frac{(ex^2+d)^{5/2}\arctan(cx)}{x}dx$$

$$\Big\downarrow \; 73$$

$$\frac{1}{2}a\left(d\left(d\left(\frac{2d\int \frac{1}{\frac{x^4}{e}-\frac{d}{e}}d\sqrt{ex^2+d}}{e}+2\sqrt{d+ex^2}\right)+\frac{2}{3}\left(d+ex^2\right)^{3/2}\right)+\frac{2}{5}\left(d+ex^2\right)^{5/2}\right)+ b\int \frac{\left(ex^2+d\right)^{5/2}\arctan(cx)}{x}dx$$

$$\Big\downarrow \; 221$$

$$b\int \frac{\left(ex^2+d\right)^{5/2}\arctan(cx)}{x}dx+ \frac{1}{2}a\left(d\left(d\left(2\sqrt{d+ex^2}-2\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\right)+\frac{2}{3}\left(d+ex^2\right)^{3/2}\right)+\frac{2}{5}\left(d+ex^2\right)^{5/2}\right)$$

$$\Big\downarrow \; 5560$$

$$b\int \frac{\left(ex^2+d\right)^{5/2}\arctan(cx)}{x}dx+ \frac{1}{2}a\left(d\left(d\left(2\sqrt{d+ex^2}-2\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\right)+\frac{2}{3}\left(d+ex^2\right)^{3/2}\right)+\frac{2}{5}\left(d+ex^2\right)^{5/2}\right)$$

input
```
Int[((d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]))/x,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\left(e\,x^{2}+d\right)^{\frac{5}{2}}\left(a+b\arctan\left(cx\right)\right)}{x}dx$$

input
```
int((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x,x)
```

output
```
int((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x,x)
```

## Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.78

$$\int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x}\,dx = \int \frac{(ex^2+d)^{\frac{5}{2}}(b\arctan(cx)+a)}{x}\,dx$$

input
```
integrate((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x,x, algorithm="fricas")
```

output
```
integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d
^2)*arctan(c*x))*sqrt(e*x^2 + d)/x, x)
```

## Sympy [N/A]

Not integrable

Time = 48.88 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x}\,dx = \int \frac{(a+b\,\mathrm{atan}(cx))(d+ex^2)^{\frac{5}{2}}}{x}\,dx$$

input
```
integrate((e*x**2+d)**(5/2)*(a+b*atan(c*x))/x,x)
```

output
```
Integral((a + b*atan(c*x))*(d + e*x**2)**(5/2)/x, x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

## Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x}\,dx = \int \frac{(ex^2+d)^{\frac{5}{2}}(b\arctan(cx)+a)}{x}\,dx$$

input
```
integrate((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(5/2)*(b*arctan(c*x) + a)/x, x)
```

## Mupad [N/A]

Not integrable

Time = 1.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))\,(e\,x^2+d)^{5/2}}{x}\,dx$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^(5/2))/x,x)
```

output
```
int(((a + b*atan(c*x))*(d + e*x^2)^(5/2))/x, x)
```

## Reduce [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 174, normalized size of antiderivative = 7.57

$$\int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x}\,dx = \frac{23\sqrt{e\,x^2+d}\,a\,d^2}{15}+\frac{11\sqrt{e\,x^2+d}\,ade\,x^2}{15}+\frac{\sqrt{e\,x^2+d}\,a\,e^2x^4}{5}+\sqrt{d}\log\left(\frac{\sqrt{e\,x^2+d}-\sqrt{d}+\sqrt{e}\,x}{\sqrt{d}}\right)a\,d^2-\sqrt{d}\log\left(\frac{\sqrt{e\,x^2+d}+\sqrt{d}+\sqrt{e}\,x}{\sqrt{d}}\right)a\,d^2+\left(\int\frac{\sqrt{e\,x^2+d}\,atan(cx)}{x}dx\right)b\,d^2+\left(\int\sqrt{e\,x^2+d}\,atan(cx)\,x^3dx\right)b\,e^2+2\left(\int\sqrt{e\,x^2+d}\,atan(cx)\,xdx\right)bde$$

input
```
int((e*x^2+d)^(5/2)*(a+b*atan(c*x))/x,x)
```

output
```
(23*sqrt(d + e*x**2)*a*d**2 + 11*sqrt(d + e*x**2)*a*d*e*x**2 + 3*sqrt(d +
e*x**2)*a*e**2*x**4 + 15*sqrt(d)*log((sqrt(d + e*x**2) - sqrt(d) + sqrt(e)
*x)/sqrt(d))*a*d**2 - 15*sqrt(d)*log((sqrt(d + e*x**2) + sqrt(d) + sqrt(e)
*x)/sqrt(d))*a*d**2 + 15*int((sqrt(d + e*x**2)*atan(c*x))/x,x)*b*d**2 + 15
*int(sqrt(d + e*x**2)*atan(c*x)*x**3,x)*b*e**2 + 30*int(sqrt(d + e*x**2)*a
tan(c*x)*x,x)*b*d*e)/15
```

# 3.1197 $\int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x^2}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x^2}\,dx = \frac{15}{8}adex\sqrt{d+ex^2}+\frac{5}{4}aex(d+ex^2)^{3/2} - \frac{a(d+ex^2)^{5/2}}{x}+\frac{15}{8}ad^2\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)+b\text{Int}\left(\frac{(d+ex^2)^{5/2}\arctan(cx)}{x^2},x\right)$$

output
```
15/8*a*d*e*x*(e*x^2+d)^(1/2)+5/4*a*e*x*(e*x^2+d)^(3/2)-a*(e*x^2+d)^(5/2)/x
+15/8*a*d^2*e^(1/2)*arctanh(e^(1/2)*x/(e*x^2+d)^(1/2))+b*Defer(Int)((e*x^2
+d)^(5/2)*arctan(c*x)/x^2,x)
```

## Mathematica [N/A]

Not integrable

Time = 10.33 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x^2}\,dx = \int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x^2}\,dx$$

input
```
Integrate[((d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]))/x^2,x]
```

output
```
Integrate[((d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]))/x^2, x]
```

## Rubi [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(d+ex^2\right)^{5/2} (a+b\arctan(cx))}{x^2} dx$$

$$\Big\downarrow \text{5517}$$

$$a\int \frac{\left(ex^2+d\right)^{5/2}}{x^2}dx+b\int \frac{\left(ex^2+d\right)^{5/2}\arctan(cx)}{x^2}dx$$

$$\Big\downarrow \text{247}$$

$$a\left(5e\int \left(ex^2+d\right)^{3/2}dx-\frac{\left(d+ex^2\right)^{5/2}}{x}\right)+b\int \frac{\left(ex^2+d\right)^{5/2}\arctan(cx)}{x^2}dx$$

$$\Big\downarrow \text{211}$$

$$a\left(5e\left(\frac{3}{4}d\int \sqrt{ex^2+d}dx+\frac{1}{4}x\left(d+ex^2\right)^{3/2}\right)-\frac{\left(d+ex^2\right)^{5/2}}{x}\right)+ b\int \frac{\left(ex^2+d\right)^{5/2}\arctan(cx)}{x^2}dx$$

$$\Big\downarrow \text{211}$$

$$a\left(5e\left(\frac{3}{4}d\left(\frac{1}{2}d\int \frac{1}{\sqrt{ex^2+d}}dx+\frac{1}{2}x\sqrt{d+ex^2}\right)+\frac{1}{4}x\left(d+ex^2\right)^{3/2}\right)-\frac{\left(d+ex^2\right)^{5/2}}{x}\right)+ b\int \frac{\left(ex^2+d\right)^{5/2}\arctan(cx)}{x^2}dx$$

$$\Big\downarrow \text{224}$$

$$a\left(5e\left(\frac{3}{4}d\left(\frac{1}{2}d\int\frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}+\frac{1}{2}x\sqrt{d+ex^2}\right)+\frac{1}{4}x\left(d+ex^2\right)^{3/2}\right)-\frac{\left(d+ex^2\right)^{5/2}}{x}\right)+b\int\frac{\left(ex^2+d\right)^{5/2}\arctan(cx)}{x^2}dx$$

$\Big\downarrow$ 219

$$b\int\frac{\left(ex^2+d\right)^{5/2}\arctan(cx)}{x^2}dx+a\left(5e\left(\frac{3}{4}d\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{e}}+\frac{1}{2}x\sqrt{d+ex^2}\right)+\frac{1}{4}x\left(d+ex^2\right)^{3/2}\right)-\frac{\left(d+ex^2\right)^{5/2}}{x}\right)$$

$\Big\downarrow$ 5560

$$b\int\frac{\left(ex^2+d\right)^{5/2}\arctan(cx)}{x^2}dx+a\left(5e\left(\frac{3}{4}d\left(\frac{d\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{e}}+\frac{1}{2}x\sqrt{d+ex^2}\right)+\frac{1}{4}x\left(d+ex^2\right)^{3/2}\right)-\frac{\left(d+ex^2\right)^{5/2}}{x}\right)$$

input
```
Int[((d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]))/x^2,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int\frac{\left(e\,x^2+d\right)^{\frac{5}{2}}\left(a+b\arctan\left(cx\right)\right)}{x^2}dx$$

input
```
int((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x^2,x)
```

output
```
int((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.78

$$\int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x^2}\,dx = \int \frac{(ex^2+d)^{\frac{5}{2}}(b\arctan(cx)+a)}{x^2}\,dx$$

input
```
integrate((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x^2,x, algorithm="fricas")
```

output
```
integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d
^2)*arctan(c*x))*sqrt(e*x^2 + d)/x^2, x)
```

## Sympy [N/A]

Not integrable

Time = 69.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x^2}\,dx = \int \frac{(a+b\,\mathrm{atan}(cx))(d+ex^2)^{\frac{5}{2}}}{x^2}\,dx$$

input
```
integrate((e*x**2+d)**(5/2)*(a+b*atan(c*x))/x**2,x)
```

output
```
Integral((a + b*atan(c*x))*(d + e*x**2)**(5/2)/x**2, x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

## Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x^2}\,dx = \int \frac{(ex^2+d)^{\frac{5}{2}}(b\arctan(cx)+a)}{x^2}\,dx$$

input
```
integrate((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x^2,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(5/2)*(b*arctan(c*x) + a)/x^2, x)
```

## Mupad [N/A]

Not integrable

Time = 1.85 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x^2}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))\,(e\,x^2+d)^{5/2}}{x^2}\,dx$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^(5/2))/x^2,x)
```

output
```
int(((a + b*atan(c*x))*(d + e*x^2)^(5/2))/x^2, x)
```

## Reduce [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 161, normalized size of antiderivative = 7.00

$$\int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x^2}\,dx = \frac{-8\sqrt{e\,x^2+d}\,a\,d^2+9\sqrt{e\,x^2+d}\,ade\,x^2+2\sqrt{e\,x^2+d}\,a\,e^2x^4+15\sqrt{e}}{}$$

input
```
int((e*x^2+d)^(5/2)*(a+b*atan(c*x))/x^2,x)
```

output
```
( - 8*sqrt(d + e*x**2)*a*d**2 + 9*sqrt(d + e*x**2)*a*d*e*x**2 + 2*sqrt(d +
 e*x**2)*a*e**2*x**4 + 15*sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)*x)/sqrt(
d))*a*d**2*x - 10*sqrt(e)*a*d**2*x + 8*int((sqrt(d + e*x**2)*atan(c*x))/x*
*2,x)*b*d**2*x + 8*int(sqrt(d + e*x**2)*atan(c*x)*x**2,x)*b*e**2*x + 16*in
t(sqrt(d + e*x**2)*atan(c*x),x)*b*d*e*x)/(8*x)
```

# 3.1198 $\int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x^3}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x^3}\,dx = \frac{5}{2}ade\sqrt{d+ex^2}+\frac{5}{6}ae(d+ex^2)^{3/2} - \frac{a(d+ex^2)^{5/2}}{2x^2} - \frac{5}{2}ad^{3/2}e\,\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right) + b\text{Int}\left(\frac{(d+ex^2)^{5/2}\arctan(cx)}{x^3},x\right)$$

output
```
5/2*a*d*e*(e*x^2+d)^(1/2)+5/6*a*e*(e*x^2+d)^(3/2)-1/2*a*(e*x^2+d)^(5/2)/x^
2-5/2*a*d^(3/2)*e*arctanh((e*x^2+d)^(1/2)/d^(1/2))+b*Defer(Int)((e*x^2+d)^
(5/2)*arctan(c*x)/x^3,x)
```

## Mathematica [N/A]

Not integrable

Time = 11.35 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x^3}\,dx = \int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x^3}\,dx$$

input
```
Integrate[((d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]))/x^3,x]
```

output
```
Integrate[((d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]))/x^3, x]
```

## Rubi [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x^3}dx$$

$$\Big\downarrow \text{5517}$$

$$a\int \frac{(ex^2+d)^{5/2}}{x^3}dx+b\int \frac{(ex^2+d)^{5/2}\arctan(cx)}{x^3}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}a\int \frac{(ex^2+d)^{5/2}}{x^4}dx^2+b\int \frac{(ex^2+d)^{5/2}\arctan(cx)}{x^3}dx$$

$$\Big\downarrow \text{51}$$

$$\frac{1}{2}a\left(\frac{5}{2}e\int \frac{(ex^2+d)^{3/2}}{x^2}dx^2-\frac{(d+ex^2)^{5/2}}{x^2}\right)+b\int \frac{(ex^2+d)^{5/2}\arctan(cx)}{x^3}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}a\left(\frac{5}{2}e\left(d\int \frac{\sqrt{ex^2+d}}{x^2}dx^2+\frac{2}{3}(d+ex^2)^{3/2}\right)-\frac{(d+ex^2)^{5/2}}{x^2}\right)+ b\int \frac{(ex^2+d)^{5/2}\arctan(cx)}{x^3}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}a\left(\frac{5}{2}e\left(d\left(d\int \frac{1}{x^2\sqrt{ex^2+d}}dx^2+2\sqrt{d+ex^2}\right)+\frac{2}{3}\left(d+ex^2\right)^{3/2}\right)-\frac{\left(d+ex^2\right)^{5/2}}{x^2}\right)+ b\int \frac{\left(ex^2+d\right)^{5/2}\arctan(cx)}{x^3}dx$$

$\Big\downarrow$ 73

$$\frac{1}{2}a\left(\frac{5}{2}e\left(d\left(\frac{2d\int \frac{1}{\frac{x^4}{e}-\frac{d}{e}}d\sqrt{ex^2+d}}{e}+2\sqrt{d+ex^2}\right)+\frac{2}{3}\left(d+ex^2\right)^{3/2}\right)-\frac{\left(d+ex^2\right)^{5/2}}{x^2}\right)+ b\int \frac{\left(ex^2+d\right)^{5/2}\arctan(cx)}{x^3}dx$$

$\Big\downarrow$ 221

$$b\int \frac{\left(ex^2+d\right)^{5/2}\arctan(cx)}{x^3}dx+ \frac{1}{2}a\left(\frac{5}{2}e\left(d\left(2\sqrt{d+ex^2}-2\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\right)+\frac{2}{3}\left(d+ex^2\right)^{3/2}\right)-\frac{\left(d+ex^2\right)^{5/2}}{x^2}\right)$$

$\Big\downarrow$ 5560

$$b\int \frac{\left(ex^2+d\right)^{5/2}\arctan(cx)}{x^3}dx+ \frac{1}{2}a\left(\frac{5}{2}e\left(d\left(2\sqrt{d+ex^2}-2\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\right)+\frac{2}{3}\left(d+ex^2\right)^{3/2}\right)-\frac{\left(d+ex^2\right)^{5/2}}{x^2}\right)$$

input
```
Int[((d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]))/x^3,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\left(e\,x^{2}+d\right)^{\frac{5}{2}}\left(a+b\arctan\left(cx\right)\right)}{x^{3}}dx$$

input
```
int((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x^3,x)
```

output
```
int((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x^3,x)
```

## Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.78

$$\int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x^3}\,dx = \int \frac{(ex^2+d)^{\frac{5}{2}}(b\arctan(cx)+a)}{x^3}\,dx$$

input
```
integrate((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x^3,x, algorithm="fricas")
```

output
```
integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d
^2)*arctan(c*x))*sqrt(e*x^2 + d)/x^3, x)
```

## Sympy [N/A]

Not integrable

Time = 47.82 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x^3}\,dx = \int \frac{(a+b\,\mathrm{atan}(cx))(d+ex^2)^{\frac{5}{2}}}{x^3}\,dx$$

input
```
integrate((e*x**2+d)**(5/2)*(a+b*atan(c*x))/x**3,x)
```

output
```
Integral((a + b*atan(c*x))*(d + e*x**2)**(5/2)/x**3, x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

## Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x^3}\,dx = \int \frac{(ex^2+d)^{\frac{5}{2}}(b\arctan(cx)+a)}{x^3}\,dx$$

input
```
integrate((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x^3,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(5/2)*(b*arctan(c*x) + a)/x^3, x)
```

## Mupad [N/A]

Not integrable

Time = 1.54 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x^3}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))\,(e\,x^2+d)^{5/2}}{x^3}\,dx$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^(5/2))/x^3,x)
```

output
```
int(((a + b*atan(c*x))*(d + e*x^2)^(5/2))/x^3, x)
```

## Reduce [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 195, normalized size of antiderivative = 8.48

$$\int \frac{(d+ex^2)^{5/2}\,(a+b\arctan(cx))}{x^3}\,dx = \frac{-3\sqrt{e\,x^2+d}\,a\,d^2+14\sqrt{e\,x^2+d}\,ade\,x^2+2\sqrt{e\,x^2+d}\,a\,e^2x^4+15\sqrt{}}{}$$

input
```
int((e*x^2+d)^(5/2)*(a+b*atan(c*x))/x^3,x)
```

output
```
( - 3*sqrt(d + e*x**2)*a*d**2 + 14*sqrt(d + e*x**2)*a*d*e*x**2 + 2*sqrt(d
+ e*x**2)*a*e**2*x**4 + 15*sqrt(d)*log((sqrt(d + e*x**2) - sqrt(d) + sqrt(
e)*x)/sqrt(d))*a*d*e*x**2 - 15*sqrt(d)*log((sqrt(d + e*x**2) + sqrt(d) + s
qrt(e)*x)/sqrt(d))*a*d*e*x**2 + 6*int((sqrt(d + e*x**2)*atan(c*x))/x**3,x)
*b*d**2*x**2 + 12*int((sqrt(d + e*x**2)*atan(c*x))/x,x)*b*d*e*x**2 + 6*int
(sqrt(d + e*x**2)*atan(c*x)*x,x)*b*e**2*x**2)/(6*x**2)
```

# 3.1199 $\int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x^4}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x^4}\,dx = \frac{5}{2}ae^2x\sqrt{d+ex^2} - \frac{5ae(d+ex^2)^{3/2}}{3x} - \frac{a(d+ex^2)^{5/2}}{3x^3} + \frac{5}{2}ade^{3/2}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right) + b\text{Int}\left(\frac{(d+ex^2)^{5/2}\arctan(cx)}{x^4},x\right)$$

output
```
5/2*a*e^2*x*(e*x^2+d)^(1/2)-5/3*a*e*(e*x^2+d)^(3/2)/x-1/3*a*(e*x^2+d)^(5/2
)/x^3+5/2*a*d*e^(3/2)*arctanh(e^(1/2)*x/(e*x^2+d)^(1/2))+b*Defer(Int)((e*x
^2+d)^(5/2)*arctan(c*x)/x^4,x)
```

## Mathematica [N/A]

Not integrable

Time = 10.38 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x^4}\,dx = \int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x^4}\,dx$$

input
```
Integrate[((d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]))/x^4,x]
```

output
```
Integrate[((d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]))/x^4, x]
```

## Rubi [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x^4}dx$$

$$\Big\downarrow \text{5517}$$

$$a\int \frac{(ex^2+d)^{5/2}}{x^4}dx+b\int \frac{(ex^2+d)^{5/2}\arctan(cx)}{x^4}dx$$

$$\Big\downarrow \text{247}$$

$$a\left(\frac{5}{3}e\int \frac{(ex^2+d)^{3/2}}{x^2}dx-\frac{(d+ex^2)^{5/2}}{3x^3}\right)+b\int \frac{(ex^2+d)^{5/2}\arctan(cx)}{x^4}dx$$

$$\Big\downarrow \text{247}$$

$$a\left(\frac{5}{3}e\left(3e\int \sqrt{ex^2+d}dx-\frac{(d+ex^2)^{3/2}}{x}\right)-\frac{(d+ex^2)^{5/2}}{3x^3}\right)+b\int \frac{(ex^2+d)^{5/2}\arctan(cx)}{x^4}dx$$

$$\Big\downarrow \text{211}$$

$$a\left(\frac{5}{3}e\left(3e\left(\frac{1}{2}d\int \frac{1}{\sqrt{ex^2+d}}dx+\frac{1}{2}x\sqrt{d+ex^2}\right)-\frac{(d+ex^2)^{3/2}}{x}\right)-\frac{(d+ex^2)^{5/2}}{3x^3}\right)+ b\int \frac{(ex^2+d)^{5/2}\arctan(cx)}{x^4}dx$$

$$\Big\downarrow \text{224}$$

$$a\left(\frac{5}{3}e\left(3e\left(\frac{1}{2}d\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}+\frac{1}{2}x\sqrt{d+ex^2}\right)-\frac{\left(d+ex^2\right)^{3/2}}{x}\right)-\frac{\left(d+ex^2\right)^{5/2}}{3x^3}\right)+b\int \frac{\left(ex^2+d\right)^{5/2}\arctan(cx)}{x^4}dx$$

$$\Big\downarrow \text{219}$$

$$b\int \frac{\left(ex^2+d\right)^{5/2}\arctan(cx)}{x^4}dx+a\left(\frac{5}{3}e\left(3e\left(\frac{d\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{e}}+\frac{1}{2}x\sqrt{d+ex^2}\right)-\frac{\left(d+ex^2\right)^{3/2}}{x}\right)-\frac{\left(d+ex^2\right)^{5/2}}{3x^3}\right)$$

$$\Big\downarrow \text{5560}$$

$$b\int \frac{\left(ex^2+d\right)^{5/2}\arctan(cx)}{x^4}dx+a\left(\frac{5}{3}e\left(3e\left(\frac{d\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2\sqrt{e}}+\frac{1}{2}x\sqrt{d+ex^2}\right)-\frac{\left(d+ex^2\right)^{3/2}}{x}\right)-\frac{\left(d+ex^2\right)^{5/2}}{3x^3}\right)$$

input
```
Int[((d + e*x^2)^(5/2)*(a + b*ArcTan[c*x]))/x^4,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.77 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{\left(e\,x^{2}+d\right)^{\frac{5}{2}}\left(a+b\arctan\left(cx\right)\right)}{x^{4}}dx$$

input
```
int((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x^4,x)
```

output
```
int((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x^4,x)
```

## Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.78

$$\int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x^4}\,dx = \int \frac{(ex^2+d)^{\frac{5}{2}}(b\arctan(cx)+a)}{x^4}\,dx$$

input
```
integrate((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x^4,x, algorithm="fricas")
```

output
```
integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d
^2)*arctan(c*x))*sqrt(e*x^2 + d)/x^4, x)
```

## Sympy [N/A]

Not integrable

Time = 47.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x^4}\,dx = \int \frac{(a+b\,\mathrm{atan}\,(cx))\,(d+ex^2)^{\frac{5}{2}}}{x^4}\,dx$$

input
```
integrate((e*x**2+d)**(5/2)*(a+b*atan(c*x))/x**4,x)
```

output
```
Integral((a + b*atan(c*x))*(d + e*x**2)**(5/2)/x**4, x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

## Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x^4}\,dx=\int \frac{(ex^2+d)^{\frac{5}{2}}(b\arctan(cx)+a)}{x^4}\,dx$$

input
```
integrate((e*x^2+d)^(5/2)*(a+b*arctan(c*x))/x^4,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(5/2)*(b*arctan(c*x) + a)/x^4, x)
```

## Mupad [N/A]

Not integrable

Time = 1.61 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x^4}\,dx=\int \frac{(a+b\,\mathrm{atan}(c\,x))\,(e\,x^2+d)^{5/2}}{x^4}\,dx$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^(5/2))/x^4,x)
```

output
```
int(((a + b*atan(c*x))*(d + e*x^2)^(5/2))/x^4, x)
```

## Reduce [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 169, normalized size of antiderivative = 7.35

$$\int \frac{(d+ex^2)^{5/2}(a+b\arctan(cx))}{x^4}\,dx = \frac{-4\sqrt{e\,x^2+d}\,a\,d^2-28\sqrt{e\,x^2+d}\,ade\,x^2+6\sqrt{e\,x^2+d}\,a\,e^2x^4+30\sqrt{}}{}$$

input
```
int((e*x^2+d)^(5/2)*(a+b*atan(c*x))/x^4,x)
```

output
```
( - 4*sqrt(d + e*x**2)*a*d**2 - 28*sqrt(d + e*x**2)*a*d*e*x**2 + 6*sqrt(d
+ e*x**2)*a*e**2*x**4 + 30*sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)*x)/sqrt
(d))*a*d*e*x**3 + 5*sqrt(e)*a*d*e*x**3 + 12*int((sqrt(d + e*x**2)*atan(c*x
))/x**4,x)*b*d**2*x**3 + 24*int((sqrt(d + e*x**2)*atan(c*x))/x**2,x)*b*d*e
*x**3 + 12*int(sqrt(d + e*x**2)*atan(c*x),x)*b*e**2*x**3)/(12*x**3)
```

# 3.1200 $\int \frac{x^3(a+b\arctan(cx))}{\sqrt{d+ex^2}}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 176

$$\int \frac{x^3(a+b\arctan(cx))}{\sqrt{d+ex^2}}\,dx = -\frac{bx\sqrt{d+ex^2}}{6ce} - \frac{d\sqrt{d+ex^2}(a+b\arctan(cx))}{e^2} + \frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{3e^2} + \frac{b\sqrt{c^2d-e}(2c^2d+e)\arctan\left(\frac{\sqrt{c^2d-e}x}{\sqrt{d+ex^2}}\right)}{3c^3e^2} + \frac{b(3c^2d+2e)\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{6c^3e^{3/2}}$$

output
```
-1/6*b*x*(e*x^2+d)^(1/2)/c/e-d*(e*x^2+d)^(1/2)*(a+b*arctan(c*x))/e^2+1/3*(
e*x^2+d)^(3/2)*(a+b*arctan(c*x))/e^2+1/3*b*(c^2*d-e)^(1/2)*(2*c^2*d+e)*arc
tan((c^2*d-e)^(1/2)*x/(e*x^2+d)^(1/2))/c^3/e^2+1/6*b*(3*c^2*d+2*e)*arctanh
(e^(1/2)*x/(e*x^2+d)^(1/2))/c^3/e^(3/2)
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 377, normalized size of antiderivative = 2.14

$$\int \frac{x^3(a+b\arctan(cx))}{\sqrt{d+ex^2}}\,dx$$
$$= \frac{-\frac{\sqrt{d+ex^2}(bex+ac(4d-2ex^2))}{c}+2b(-2d+ex^2)\sqrt{d+ex^2}\arctan(cx)-\frac{ib(2c^4d^2-c^2de-e^2)\log\left(\frac{12ic^4e^2\left(cd-iex+\sqrt{c^2d-e}\sqrt{d\cdots}\right.}{b\sqrt{c^2d-e}\left(-2c^4d^2+c^2de+e^2\right)\cdots}\right)}{c^3\sqrt{c^2d-e}}\cdots}{6e^2}$$

input
```
Integrate[(x^3*(a + b*ArcTan[c*x]))/Sqrt[d + e*x^2],x]
```

output
```
(-((Sqrt[d + e*x^2]*(b*e*x + a*c*(4*d - 2*e*x^2)))/c) + 2*b*(-2*d + e*x^2)
*Sqrt[d + e*x^2]*ArcTan[c*x] - (I*b*(2*c^4*d^2 - c^2*d*e - e^2)*Log[((12*I
)*c^4*e^2*(c*d - I*e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*Sqrt[c^2*d -
 e]*(-2*c^4*d^2 + c^2*d*e + e^2)*(I + c*x))])/(c^3*Sqrt[c^2*d - e]) + (I*b
*(2*c^4*d^2 - c^2*d*e - e^2)*Log[((-12*I)*c^4*e^2*(c*d + I*e*x + Sqrt[c^2*
d - e]*Sqrt[d + e*x^2]))/(b*Sqrt[c^2*d - e]*(-2*c^4*d^2 + c^2*d*e + e^2)*(
-I + c*x))])/(c^3*Sqrt[c^2*d - e]) + (b*Sqrt[e]*(3*c^2*d + 2*e)*Log[e*x +
Sqrt[e]*Sqrt[d + e*x^2]])/c^3)/(6*e^2)
```

## Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {5511, 27, 403, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\arctan(cx))}{\sqrt{d+ex^2}}\,dx$$
$$\Big\downarrow \; 5511$$

$$-bc\int -\frac{(2d-ex^2)\sqrt{ex^2+d}}{3e^2(c^2x^2+1)}dx+\frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{3e^2}-\frac{d\sqrt{d+ex^2}(a+b\arctan(cx))}{e^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{bc\int \frac{(2d-ex^2)\sqrt{ex^2+d}}{c^2x^2+1}dx}{3e^2}+\frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{3e^2}-\frac{d\sqrt{d+ex^2}(a+b\arctan(cx))}{e^2}$$

$$\Big\downarrow \text{ 403}$$

$$\frac{bc\left(\frac{\int \frac{e(3dc^2+2e)x^2+d(4dc^2+e)}{(c^2x^2+1)\sqrt{ex^2+d}}dx}{2c^2}-\frac{ex\sqrt{d+ex^2}}{2c^2}\right)}{3e^2}+\frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{3e^2}-\frac{d\sqrt{d+ex^2}(a+b\arctan(cx))}{e^2}$$

$$\Big\downarrow \text{ 398}$$

$$\frac{bc\left(\frac{\frac{e(3c^2d+2e)\int \frac{1}{\sqrt{ex^2+d}}dx}{c^2}+\frac{2(c^2d-e)(2c^2d+e)\int \frac{1}{(c^2x^2+1)\sqrt{ex^2+d}}dx}{c^2}}{2c^2}-\frac{ex\sqrt{d+ex^2}}{2c^2}\right)}{3e^2}+\frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{3e^2}-\frac{d\sqrt{d+ex^2}(a+b\arctan(cx))}{e^2}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{bc\left(\frac{\frac{2(c^2d-e)(2c^2d+e)\int \frac{1}{(c^2x^2+1)\sqrt{ex^2+d}}dx}{c^2}+\frac{e(3c^2d+2e)\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{c^2}}{2c^2}-\frac{ex\sqrt{d+ex^2}}{2c^2}\right)}{3e^2}+\frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{3e^2}-\frac{d\sqrt{d+ex^2}(a+b\arctan(cx))}{e^2}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{bc\left(\frac{\frac{2(c^2d-e)(2c^2d+e)\int \frac{1}{(c^2x^2+1)\sqrt{ex^2+d}}dx}{c^2}+\frac{\sqrt{e}(3c^2d+2e)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2}}{2c^2}-\frac{ex\sqrt{d+ex^2}}{2c^2}\right)}{3e^2}+\frac{(d+ex^2)^{3/2}(a+b\arctan(cx))}{3e^2}-\frac{d\sqrt{d+ex^2}(a+b\arctan(cx))}{e^2}$$

$$\Big\downarrow \ 291$$

$$\frac{bc\left(\frac{\frac{2\left(c^2d-e\right)\left(2c^2d+e\right)\int\frac{1}{1-\frac{\left(e-c^2d\right)x^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{c^2}+\frac{\sqrt{e}\left(3c^2d+2e\right)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2}}{2c^2}-\frac{ex\sqrt{d+ex^2}}{2c^2}\right)}{3e^2}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\arctan(cx)\right)}{3e^2}-\frac{d\sqrt{d+ex^2}(a+b\arctan(cx))}{e^2}$$

$$\Big\downarrow \ 216$$

$$\frac{\left(d+ex^2\right)^{3/2}\left(a+b\arctan(cx)\right)}{3e^2}-\frac{d\sqrt{d+ex^2}(a+b\arctan(cx))}{e^2}+\frac{bc\left(\frac{\frac{2\sqrt{c^2d-e}\left(2c^2d+e\right)\arctan\left(\frac{x\sqrt{c^2d-e}}{\sqrt{d+ex^2}}\right)}{c^2}+\frac{\sqrt{e}\left(3c^2d+2e\right)\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2}}{2c^2}-\frac{ex\sqrt{d+ex^2}}{2c^2}\right)}{3e^2}$$

input
```
Int[(x^3*(a + b*ArcTan[c*x]))/Sqrt[d + e*x^2],x]
```

output
```
-((d*Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/e^2) + ((d + e*x^2)^(3/2)*(a + b
*ArcTan[c*x]))/(3*e^2) + (b*c*(-1/2*(e*x*Sqrt[d + e*x^2])/c^2 + ((2*Sqrt[c
^2*d - e]*(2*c^2*d + e)*ArcTan[(Sqrt[c^2*d - e]*x)/Sqrt[d + e*x^2]])/c^2 +
 (Sqrt[e]*(3*c^2*d + 2*e)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/c^2)/(2*c^
2)))/(3*e^2)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

## Maple [F]

$$\int \frac{x^3(a + b \arctan(cx))}{\sqrt{e\,x^2 + d}}dx$$

input
```
int(x^3*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x)
```

output
```
int(x^3*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x)
```

## Fricas [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 882, normalized size of antiderivative = 5.01

$$\int \frac{x^3(a + b \arctan(cx))}{\sqrt{d + ex^2}}\, dx = \text{Too large to display}$$

input
```
integrate(x^3*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
[1/12*((3*b*c^2*d + 2*b*e)*sqrt(e)*log(-2*e*x^2 - 2*sqrt(e*x^2 + d)*sqrt(e
)*x - d) + (2*b*c^2*d + b*e)*sqrt(-c^2*d + e)*log(((c^4*d^2 - 8*c^2*d*e +
8*e^2)*x^4 - 2*(3*c^2*d^2 - 4*d*e)*x^2 + 4*((c^2*d - 2*e)*x^3 - d*x)*sqrt(
-c^2*d + e)*sqrt(e*x^2 + d) + d^2)/(c^4*x^4 + 2*c^2*x^2 + 1)) + 2*(2*a*c^3
*e*x^2 - 4*a*c^3*d - b*c^2*e*x + 2*(b*c^3*e*x^2 - 2*b*c^3*d)*arctan(c*x))*
sqrt(e*x^2 + d))/(c^3*e^2), 1/12*(2*(2*b*c^2*d + b*e)*sqrt(c^2*d - e)*arct
an(1/2*sqrt(c^2*d - e)*((c^2*d - 2*e)*x^2 - d)*sqrt(e*x^2 + d)/((c^2*d*e -
 e^2)*x^3 + (c^2*d^2 - d*e)*x)) + (3*b*c^2*d + 2*b*e)*sqrt(e)*log(-2*e*x^2
 - 2*sqrt(e*x^2 + d)*sqrt(e)*x - d) + 2*(2*a*c^3*e*x^2 - 4*a*c^3*d - b*c^2
*e*x + 2*(b*c^3*e*x^2 - 2*b*c^3*d)*arctan(c*x))*sqrt(e*x^2 + d))/(c^3*e^2)
, -1/12*(2*(3*b*c^2*d + 2*b*e)*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d))
 - (2*b*c^2*d + b*e)*sqrt(-c^2*d + e)*log(((c^4*d^2 - 8*c^2*d*e + 8*e^2)*x
^4 - 2*(3*c^2*d^2 - 4*d*e)*x^2 + 4*((c^2*d - 2*e)*x^3 - d*x)*sqrt(-c^2*d +
 e)*sqrt(e*x^2 + d) + d^2)/(c^4*x^4 + 2*c^2*x^2 + 1)) - 2*(2*a*c^3*e*x^2 -
 4*a*c^3*d - b*c^2*e*x + 2*(b*c^3*e*x^2 - 2*b*c^3*d)*arctan(c*x))*sqrt(e*x
^2 + d))/(c^3*e^2), 1/6*((2*b*c^2*d + b*e)*sqrt(c^2*d - e)*arctan(1/2*sqrt
(c^2*d - e)*((c^2*d - 2*e)*x^2 - d)*sqrt(e*x^2 + d)/((c^2*d*e - e^2)*x^3 +
 (c^2*d^2 - d*e)*x)) - (3*b*c^2*d + 2*b*e)*sqrt(-e)*arctan(sqrt(-e)*x/sqrt
(e*x^2 + d)) + (2*a*c^3*e*x^2 - 4*a*c^3*d - b*c^2*e*x + 2*(b*c^3*e*x^2 - 2
*b*c^3*d)*arctan(c*x))*sqrt(e*x^2 + d))/(c^3*e^2)]
```

## Sympy [F]

$$\int \frac{x^3(a + b\arctan(cx))}{\sqrt{d + ex^2}}\,dx = \int \frac{x^3(a + b\operatorname{atan}(cx))}{\sqrt{d + ex^2}}\,dx$$

input
```
integrate(x**3*(a+b*atan(c*x))/(e*x**2+d)**(1/2),x)
```

output
```
Integral(x**3*(a + b*atan(c*x))/sqrt(d + e*x**2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(a+b\arctan(cx))}{\sqrt{d+ex^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

## Giac [F]

$$\int \frac{x^3(a+b\arctan(cx))}{\sqrt{d+ex^2}}\,dx = \int \frac{(b\arctan(cx)+a)x^3}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)*x^3/sqrt(e*x^2 + d), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\arctan(cx))}{\sqrt{d+ex^2}}\,dx = \int \frac{x^3\,(a+b\,\mathrm{atan}(c\,x))}{\sqrt{e\,x^2+d}}\,dx$$

input
```
int((x^3*(a + b*atan(c*x)))/(d + e*x^2)^(1/2),x)
```

output
```
int((x^3*(a + b*atan(c*x)))/(d + e*x^2)^(1/2), x)
```

## Reduce [F]

$$\int \frac{x^3(a+b\arctan(cx))}{\sqrt{d+ex^2}}\,dx = \frac{-2\sqrt{e\,x^2+d}\,ad+\sqrt{e\,x^2+d}\,ae\,x^2+3\left(\int \frac{atan(cx)x^3}{\sqrt{e\,x^2+d}}dx\right)b\,e^2}{3e^2}$$

input
```
int(x^3*(a+b*atan(c*x))/(e*x^2+d)^(1/2),x)
```

output
```
( - 2*sqrt(d + e*x**2)*a*d + sqrt(d + e*x**2)*a*e*x**2 + 3*int((atan(c*x)*
x**3)/sqrt(d + e*x**2),x)*b*e**2)/(3*e**2)
```

# 3.1201 $\int \frac{x^2(a+b\arctan(cx))}{\sqrt{d+ex^2}}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^2(a+b\arctan(cx))}{\sqrt{d+ex^2}}\,dx = \frac{ax\sqrt{d+ex^2}}{2e} - \frac{ad\,\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2e^{3/2}} + b\text{Int}\left(\frac{x^2\arctan(cx)}{\sqrt{d+ex^2}},x\right)$$

output
```
1/2*a*x*(e*x^2+d)^(1/2)/e-1/2*a*d*arctanh(e^(1/2)*x/(e*x^2+d)^(1/2))/e^(3/
2)+b*Defer(Int)(x^2*arctan(c*x)/(e*x^2+d)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 11.15 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^2(a+b\arctan(cx))}{\sqrt{d+ex^2}}\,dx = \int \frac{x^2(a+b\arctan(cx))}{\sqrt{d+ex^2}}\,dx$$

input
```
Integrate[(x^2*(a + b*ArcTan[c*x]))/Sqrt[d + e*x^2],x]
```

output
```
Integrate[(x^2*(a + b*ArcTan[c*x]))/Sqrt[d + e*x^2], x]
```

## Rubi [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\arctan(cx))}{\sqrt{d+ex^2}}\,dx$$

$$\Big\downarrow \ 5517$$

$$a\int \frac{x^2}{\sqrt{ex^2+d}}dx+b\int \frac{x^2\arctan(cx)}{\sqrt{ex^2+d}}dx$$

$$\Big\downarrow \ 262$$

$$a\left(\frac{x\sqrt{d+ex^2}}{2e}-\frac{d\int \frac{1}{\sqrt{ex^2+d}}dx}{2e}\right)+b\int \frac{x^2\arctan(cx)}{\sqrt{ex^2+d}}dx$$

$$\Big\downarrow \ 224$$

$$a\left(\frac{x\sqrt{d+ex^2}}{2e}-\frac{d\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{2e}\right)+b\int \frac{x^2\arctan(cx)}{\sqrt{ex^2+d}}dx$$

$$\Big\downarrow \ 219$$

$$b\int \frac{x^2\arctan(cx)}{\sqrt{ex^2+d}}dx+a\left(\frac{x\sqrt{d+ex^2}}{2e}-\frac{d\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2e^{3/2}}\right)$$

$$\Big\downarrow \ 5560$$

$$b\int \frac{x^2\arctan(cx)}{\sqrt{ex^2+d}}dx+a\left(\frac{x\sqrt{d+ex^2}}{2e}-\frac{d\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2e^{3/2}}\right)$$

input
```
Int[(x^2*(a + b*ArcTan[c*x]))/Sqrt[d + e*x^2],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x^2(a+b\arctan(cx))}{\sqrt{e\,x^2+d}}dx$$

input
```
int(x^2*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x)
```

output
```
int(x^2*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{x^2(a+b\arctan(cx))}{\sqrt{d+ex^2}}\,dx = \int \frac{(b\arctan(cx)+a)x^2}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
integral((b*x^2*arctan(c*x) + a*x^2)/sqrt(e*x^2 + d), x)
```

## Sympy [N/A]

Not integrable

Time = 10.64 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{x^2(a+b\arctan(cx))}{\sqrt{d+ex^2}}\,dx=\int \frac{x^2(a+b\,\mathrm{atan}\,(cx))}{\sqrt{d+ex^2}}\,dx$$

input
```
integrate(x**2*(a+b*atan(c*x))/(e*x**2+d)**(1/2),x)
```

output
```
Integral(x**2*(a + b*atan(c*x))/sqrt(d + e*x**2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a+b\arctan(cx))}{\sqrt{d+ex^2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^2*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

## Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^2(a+b\arctan(cx))}{\sqrt{d+ex^2}}\,dx=\int \frac{(b\arctan(cx)+a)x^2}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)*x^2/sqrt(e*x^2 + d), x)
```

## Mupad [N/A]

Not integrable

Time = 1.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^2(a+b\arctan(cx))}{\sqrt{d+ex^2}}\,dx=\int \frac{x^2\,(a+b\,\mathrm{atan}(c\,x))}{\sqrt{e\,x^2+d}}\,dx$$

input
```
int((x^2*(a + b*atan(c*x)))/(d + e*x^2)^(1/2),x)
```

output
```
int((x^2*(a + b*atan(c*x)))/(d + e*x^2)^(1/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 69, normalized size of antiderivative = 3.00

$$\int \frac{x^2(a+b\arctan(cx))}{\sqrt{d+ex^2}}\,dx$$
$$=\frac{\sqrt{e\,x^2+d}\,aex-\sqrt{e}\log\left(\frac{\sqrt{e\,x^2+d}+\sqrt{e}\,x}{\sqrt{d}}\right)ad+2\left(\int\frac{atan(cx)\,x^2}{\sqrt{e\,x^2+d}}dx\right)b\,e^2}{2e^2}$$

input
```
int(x^2*(a+b*atan(c*x))/(e*x^2+d)^(1/2),x)
```

output
```
(sqrt(d + e*x**2)*a*e*x - sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)*x)/sqrt(
d))*a*d + 2*int((atan(c*x)*x**2)/sqrt(d + e*x**2),x)*b*e**2)/(2*e**2)
```

# 3.1202 $\int \frac{x(a+b\arctan(cx))}{\sqrt{d+ex^2}}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 103

$$\int \frac{x(a+b\arctan(cx))}{\sqrt{d+ex^2}}\,dx=\frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{e}-\frac{b\sqrt{c^2d-e}\arctan\left(\frac{\sqrt{c^2d-e}x}{\sqrt{d+ex^2}}\right)}{ce}-\frac{b\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}}$$

output
```
(e*x^2+d)^(1/2)*(a+b*arctan(c*x))/e-b*(c^2*d-e)^(1/2)*arctan((c^2*d-e)^(1/
2)*x/(e*x^2+d)^(1/2))/c/e-b*arctanh(e^(1/2)*x/(e*x^2+d)^(1/2))/c/e^(1/2)
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 251, normalized size of antiderivative = 2.44

$$\int \frac{x(a+b\arctan(cx))}{\sqrt{d+ex^2}}\,dx = \frac{2ac\sqrt{d+ex^2}+2bc\sqrt{d+ex^2}\arctan(cx)-ib\sqrt{c^2d-e}\log\left(\frac{4c^2e\left(-icd+ex-i\sqrt{c^2d-e}\sqrt{d+ex^2}\right)}{b(c^2d-e)^{3/2}(-i+cx)}\right)+ib\sqrt{c^2d-e}\,\mathrm{lo}}{2ce}$$

input
```
Integrate[(x*(a + b*ArcTan[c*x]))/Sqrt[d + e*x^2],x]
```

output
```
(2*a*c*Sqrt[d + e*x^2] + 2*b*c*Sqrt[d + e*x^2]*ArcTan[c*x] - I*b*Sqrt[c^2*
d - e]*Log[(4*c^2*e*((-I)*c*d + e*x - I*Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/
(b*(c^2*d - e)^(3/2)*(-I + c*x))] + I*b*Sqrt[c^2*d - e]*Log[(4*c^2*e*(I*c*
d + e*x + I*Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(c^2*d - e)^(3/2)*(I + c*
x))] - 2*b*Sqrt[e]*Log[e*x + Sqrt[e]*Sqrt[d + e*x^2]])/(2*c*e)
```

## Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5509, 301, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\arctan(cx))}{\sqrt{d+ex^2}}dx$$

$$\Big\downarrow \text{5509}$$

$$\frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{e}-\frac{bc\int \frac{\sqrt{ex^2+d}}{c^2x^2+1}dx}{e}$$

$$\Big\downarrow \text{301}$$

$$\frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{e}-\frac{bc\left(\frac{e\int \frac{1}{\sqrt{ex^2+d}}dx}{c^2}+\frac{(c^2d-e)\int \frac{1}{(c^2x^2+1)\sqrt{ex^2+d}}dx}{c^2}\right)}{e}$$

$$\Big\downarrow \text{224}$$

$$\frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{e}-\frac{bc\left(\frac{(c^2d-e)\int \frac{1}{(c^2x^2+1)\sqrt{ex^2+d}}dx}{c^2}+\frac{e\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{c^2}\right)}{e}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{e}-\frac{bc\left(\frac{(c^2d-e)\int\frac{1}{(c^2x^2+1)\sqrt{ex^2+d}}dx}{c^2}+\frac{\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2}\right)}{e}$$

$$\Big\downarrow \text{291}$$

$$\frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{e}-\frac{bc\left(\frac{(c^2d-e)\int\frac{1}{1-\frac{(e-c^2d)x^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{c^2}+\frac{\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2}\right)}{e}$$

$$\Big\downarrow \text{216}$$

$$\frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{e}-\frac{bc\left(\frac{\sqrt{c^2d-e}\arctan\left(\frac{x\sqrt{c^2d-e}}{\sqrt{d+ex^2}}\right)}{c^2}+\frac{\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2}\right)}{e}$$

input
```
Int[(x*(a + b*ArcTan[c*x]))/Sqrt[d + e*x^2],x]
```

output
```
(Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/e - (b*c*((Sqrt[c^2*d - e]*ArcTan[(S
qrt[c^2*d - e]*x)/Sqrt[d + e*x^2]])/c^2 + (Sqrt[e]*ArcTanh[(Sqrt[e]*x)/Sqr
t[d + e*x^2]])/c^2))/e
```

## Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

rule 5509
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*e*(q + 1))), x
] - Simp[b*(c/(2*e*(q + 1)))   Int[(d + e*x^2)^(q + 1)/(1 + c^2*x^2), x], x
] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]
```

## Maple [F]

$$\int \frac{x(a+b\arctan(cx))}{\sqrt{e\,x^2+d}}dx$$

input
```
int(x*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x)
```

output
```
int(x*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x)
```

## Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 647, normalized size of antiderivative = 6.28

$$\int \frac{x(a+b\arctan(cx))}{\sqrt{d+ex^2}}\,dx$$
$$= \left[\frac{2\,b\sqrt{e}\log\left(-2\,ex^2+2\,\sqrt{ex^2+d}\sqrt{e}x-d\right)+\sqrt{-c^2d+e}b\log\left(\frac{(c^4d^2-8\,c^2de+8\,e^2)x^4-2\,(3\,c^2d^2-4\,de)x^2-4\,((c^2d-2\,e}{c^4x^4+2\,c^2x^2+1}\right.}{4\,ce}\right.$$
$$-\frac{\sqrt{c^2d-e}b\arctan\left(\frac{\sqrt{c^2d-e}((c^2d-2\,e)x^2-d)\sqrt{ex^2+d}}{2\,((c^2de-e^2)x^3+(c^2d^2-de)x)}\right)-b\sqrt{e}\log\left(-2\,ex^2+2\,\sqrt{ex^2+d}\sqrt{e}x-d\right)-2\,\sqrt{ex^2+}}{2\,ce}$$
$$-\frac{\sqrt{c^2d-e}b\arctan\left(\frac{\sqrt{c^2d-e}((c^2d-2\,e)x^2-d)\sqrt{ex^2+d}}{2\,((c^2de-e^2)x^3+(c^2d^2-de)x)}\right)-2\,b\sqrt{-e}\arctan\left(\frac{\sqrt{-e}x}{\sqrt{ex^2+d}}\right)-2\,\sqrt{ex^2+d}(bc\arctan(cx}{2\,ce}$$

input
```
integrate(x*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(2*b*sqrt(e)*log(-2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x - d) + sqrt(-
c^2*d + e)*b*log(((c^4*d^2 - 8*c^2*d*e + 8*e^2)*x^4 - 2*(3*c^2*d^2 - 4*d*e
)*x^2 - 4*((c^2*d - 2*e)*x^3 - d*x)*sqrt(-c^2*d + e)*sqrt(e*x^2 + d) + d^2
)/(c^4*x^4 + 2*c^2*x^2 + 1)) + 4*sqrt(e*x^2 + d)*(b*c*arctan(c*x) + a*c))/
(c*e), -1/2*(sqrt(c^2*d - e)*b*arctan(1/2*sqrt(c^2*d - e)*((c^2*d - 2*e)*x
^2 - d)*sqrt(e*x^2 + d)/((c^2*d*e - e^2)*x^3 + (c^2*d^2 - d*e)*x)) - b*sqr
t(e)*log(-2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e)*x - d) - 2*sqrt(e*x^2 + d)*(
b*c*arctan(c*x) + a*c))/(c*e), 1/4*(4*b*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*
x^2 + d)) + sqrt(-c^2*d + e)*b*log(((c^4*d^2 - 8*c^2*d*e + 8*e^2)*x^4 - 2*
(3*c^2*d^2 - 4*d*e)*x^2 - 4*((c^2*d - 2*e)*x^3 - d*x)*sqrt(-c^2*d + e)*sqr
t(e*x^2 + d) + d^2)/(c^4*x^4 + 2*c^2*x^2 + 1)) + 4*sqrt(e*x^2 + d)*(b*c*ar
ctan(c*x) + a*c))/(c*e), -1/2*(sqrt(c^2*d - e)*b*arctan(1/2*sqrt(c^2*d - e
)*((c^2*d - 2*e)*x^2 - d)*sqrt(e*x^2 + d)/((c^2*d*e - e^2)*x^3 + (c^2*d^2
- d*e)*x)) - 2*b*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 + d)) - 2*sqrt(e*x^
2 + d)*(b*c*arctan(c*x) + a*c))/(c*e)]
```

## Sympy [F]

$$\int \frac{x(a+b\arctan(cx))}{\sqrt{d+ex^2}}\,dx = \int \frac{x(a+b\,\mathrm{atan}\,(cx))}{\sqrt{d+ex^2}}\,dx$$

input
```
integrate(x*(a+b*atan(c*x))/(e*x**2+d)**(1/2),x)
```

output
```
Integral(x*(a + b*atan(c*x))/sqrt(d + e*x**2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x(a+b\arctan(cx))}{\sqrt{d+ex^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e-c^2*d>0)', see `assume?` for m
ore detail
```

## Giac [F]

$$\int \frac{x(a+b\arctan(cx))}{\sqrt{d+ex^2}}\,dx = \int \frac{(b\arctan(cx)+a)x}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)*x/sqrt(e*x^2 + d), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\arctan(cx))}{\sqrt{d+ex^2}}\,dx = \int \frac{x\,(a+b\,\mathrm{atan}(c\,x))}{\sqrt{e\,x^2+d}}\,dx$$

input
```
int((x*(a + b*atan(c*x)))/(d + e*x^2)^(1/2),x)
```

output
```
int((x*(a + b*atan(c*x)))/(d + e*x^2)^(1/2), x)
```

## Reduce [F]

$$\int \frac{x(a+b\arctan(cx))}{\sqrt{d+ex^2}}\,dx = \frac{\sqrt{e\,x^2+d}\,a+\left(\int \frac{atan(cx)x}{\sqrt{e\,x^2+d}}dx\right)be}{e}$$

input
```
int(x*(a+b*atan(c*x))/(e*x^2+d)^(1/2),x)
```

output
```
(sqrt(d + e*x**2)*a + int((atan(c*x)*x)/sqrt(d + e*x**2),x)*b*e)/e
```

# 3.1203 $\int \frac{a+b\arctan(cx)}{\sqrt{d+ex^2}}\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{a+b\arctan(cx)}{\sqrt{d+ex^2}}\,dx = \operatorname{Int}\left(\frac{a+b\arctan(cx)}{\sqrt{d+ex^2}},x\right)$$

output
```
Defer(Int)((a+b*arctan(c*x))/(e*x^2+d)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 2.46 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{a+b\arctan(cx)}{\sqrt{d+ex^2}}\,dx = \int \frac{a+b\arctan(cx)}{\sqrt{d+ex^2}}\,dx$$

input
```
Integrate[(a + b*ArcTan[c*x])/Sqrt[d + e*x^2],x]
```

output
```
Integrate[(a + b*ArcTan[c*x])/Sqrt[d + e*x^2], x]
```

## Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{\sqrt{d+ex^2}}dx$$
$$\Big\downarrow \text{ 5560}$$
$$\int \frac{a+b\arctan(cx)}{\sqrt{d+ex^2}}dx$$

input
```
Int[(a + b*ArcTan[c*x])/Sqrt[d + e*x^2],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{a+b\arctan(cx)}{\sqrt{e\,x^2+d}}dx$$

input
```
int((a+b*arctan(c*x))/(e*x^2+d)^(1/2),x)
```

output
```
int((a+b*arctan(c*x))/(e*x^2+d)^(1/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{a + b \arctan(cx)}{\sqrt{d + ex^2}} \, dx = \int \frac{b \arctan(cx) + a}{\sqrt{ex^2 + d}} \, dx$$

input
```
integrate((a+b*arctan(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
integral((b*arctan(c*x) + a)/sqrt(e*x^2 + d), x)
```

## Sympy [N/A]

Not integrable

Time = 3.00 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{a + b \arctan(cx)}{\sqrt{d + ex^2}} \, dx = \int \frac{a + b \operatorname{atan}(cx)}{\sqrt{d + ex^2}} \, dx$$

input
```
integrate((a+b*atan(c*x))/(e*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*atan(c*x))/sqrt(d + e*x**2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b \arctan(cx)}{\sqrt{d + ex^2}} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arctan(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

## Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{a+b\arctan(cx)}{\sqrt{d+ex^2}}\,dx=\int \frac{b\arctan\left(cx\right)+a}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate((a+b*arctan(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)/sqrt(e*x^2 + d), x)
```

## Mupad [N/A]

Not integrable

Time = 0.98 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{a+b\arctan(cx)}{\sqrt{d+ex^2}}\,dx=\int \frac{a+b\,\mathrm{atan}(c\,x)}{\sqrt{e\,x^2+d}}\,dx$$

input
```
int((a + b*atan(c*x))/(d + e*x^2)^(1/2),x)
```

output
```
int((a + b*atan(c*x))/(d + e*x^2)^(1/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.40

$$\int \frac{a+b\arctan(cx)}{\sqrt{d+ex^2}}\,dx = \frac{\sqrt{e}\log\left(\frac{\sqrt{e\,x^2+d}+\sqrt{e}\,x}{\sqrt{d}}\right)a+\left(\int\frac{atan(cx)}{\sqrt{e\,x^2+d}}dx\right)be}{e}$$

input
```
int((a+b*atan(c*x))/(e*x^2+d)^(1/2),x)
```

output
```
(sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)*x)/sqrt(d))*a + int(atan(c*x)/sqr
t(d + e*x**2),x)*b*e)/e
```

# 3.1204 $\int \frac{a+b\arctan(cx)}{x\sqrt{d+ex^2}}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{a+b\arctan(cx)}{x\sqrt{d+ex^2}}\,dx=-\frac{a\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{\sqrt{d}}+b\text{Int}\left(\frac{\arctan(cx)}{x\sqrt{d+ex^2}},x\right)$$

output
```
-a*arctanh((e*x^2+d)^(1/2)/d^(1/2))/d^(1/2)+b*Defer(Int)(arctan(c*x)/x/(e*
x^2+d)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 3.90 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\arctan(cx)}{x\sqrt{d+ex^2}}\,dx=\int \frac{a+b\arctan(cx)}{x\sqrt{d+ex^2}}\,dx$$

input
```
Integrate[(a + b*ArcTan[c*x])/(x*Sqrt[d + e*x^2]),x]
```

output
```
Integrate[(a + b*ArcTan[c*x])/(x*Sqrt[d + e*x^2]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x\sqrt{d+ex^2}}\,dx$$

$$\Big\downarrow \text{ 5517}$$

$$a\int \frac{1}{x\sqrt{ex^2+d}}dx+b\int \frac{\arctan(cx)}{x\sqrt{ex^2+d}}dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}a\int \frac{1}{x^2\sqrt{ex^2+d}}dx^2+b\int \frac{\arctan(cx)}{x\sqrt{ex^2+d}}dx$$

$$\Big\downarrow \text{ 73}$$

$$\frac{a\int \frac{1}{\frac{x^4}{e}-\frac{d}{e}}d\sqrt{ex^2+d}}{e}+b\int \frac{\arctan(cx)}{x\sqrt{ex^2+d}}dx$$

$$\Big\downarrow \text{ 221}$$

$$b\int \frac{\arctan(cx)}{x\sqrt{ex^2+d}}dx-\frac{a\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{\sqrt{d}}$$

$$\Big\downarrow \text{ 5560}$$

$$b\int \frac{\arctan(cx)}{x\sqrt{ex^2+d}}dx-\frac{a\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{\sqrt{d}}$$

input
```
Int[(a + b*ArcTan[c*x])/(x*Sqrt[d + e*x^2]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{a+b\arctan\left(cx\right)}{x\sqrt{e\,x^{2}+d}}dx$$

input
```
int((a+b*arctan(c*x))/x/(e*x^2+d)^(1/2),x)
```

output
```
int((a+b*arctan(c*x))/x/(e*x^2+d)^(1/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.35

$$\int \frac{a+b\arctan(cx)}{x\sqrt{d+ex^2}}\,dx = \int \frac{b\arctan\left(cx\right)+a}{\sqrt{ex^2+d}x}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arctan(c*x) + a)/(e*x^3 + d*x), x)
```

## Sympy [N/A]

Not integrable

Time = 3.67 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{a+b\arctan(cx)}{x\sqrt{d+ex^2}}\,dx = \int \frac{a+b\,\mathrm{atan}\left(cx\right)}{x\sqrt{d+ex^2}}\,dx$$

input
```
integrate((a+b*atan(c*x))/x/(e*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*atan(c*x))/(x*sqrt(d + e*x**2)), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\arctan(cx)}{x\sqrt{d + ex^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arctan(c*x))/x/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

## Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a + b\arctan(cx)}{x\sqrt{d + ex^2}}\,dx = \int \frac{b\arctan(cx) + a}{\sqrt{ex^2 + d}x}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)/(sqrt(e*x^2 + d)*x), x)
```

## Mupad [N/A]

Not integrable

Time = 1.32 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+b\arctan(cx)}{x\sqrt{d+ex^2}}\,dx=\int \frac{a+b\,\mathrm{atan}(c\,x)}{x\,\sqrt{e\,x^2+d}}\,dx$$

input
```
int((a + b*atan(c*x))/(x*(d + e*x^2)^(1/2)),x)
```

output
```
int((a + b*atan(c*x))/(x*(d + e*x^2)^(1/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 81, normalized size of antiderivative = 3.52

$$\int \frac{a+b\arctan(cx)}{x\sqrt{d+ex^2}}\,dx = \frac{\sqrt{d}\,\log\left(\frac{\sqrt{e\,x^2+d}-\sqrt{d}+\sqrt{e}\,x}{\sqrt{d}}\right)a-\sqrt{d}\,\log\left(\frac{\sqrt{e\,x^2+d}+\sqrt{d}+\sqrt{e}\,x}{\sqrt{d}}\right)a+\left(\int \frac{atan(cx)}{\sqrt{e\,x^2+d}\,x}dx\right)bd}{d}$$

input
```
int((a+b*atan(c*x))/x/(e*x^2+d)^(1/2),x)
```

output
```
(sqrt(d)*log((sqrt(d + e*x**2) - sqrt(d) + sqrt(e)*x)/sqrt(d))*a - sqrt(d)
*log((sqrt(d + e*x**2) + sqrt(d) + sqrt(e)*x)/sqrt(d))*a + int(atan(c*x)/(
sqrt(d + e*x**2)*x),x)*b*d)/d
```

# 3.1205 $\int \frac{a+b\arctan(cx)}{x^2\sqrt{d+ex^2}}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 100

$$\int \frac{a+b\arctan(cx)}{x^2\sqrt{d+ex^2}}\,dx = -\frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{dx}-\frac{b\text{carctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{\sqrt{d}}+\frac{b\sqrt{c^2d-e}\text{arctanh}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{d}$$

output
```
-(e*x^2+d)^(1/2)*(a+b*arctan(c*x))/d/x-b*c*arctanh((e*x^2+d)^(1/2)/d^(1/2)
)/d^(1/2)+b*(c^2*d-e)^(1/2)*arctanh(c*(e*x^2+d)^(1/2)/(c^2*d-e)^(1/2))/d
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.47

$$\int \frac{a+b\arctan(cx)}{x^2\sqrt{d+ex^2}}\,dx = \frac{-2a\sqrt{d+ex^2}-2b\sqrt{d+ex^2}\arctan(cx)+2bc\sqrt{d}x\log(x)-2bc\sqrt{d}x\log\left(d+\sqrt{d}\sqrt{d+ex^2}\right)+b\sqrt{c^2d-}}{2dx}$$

input
```
Integrate[(a + b*ArcTan[c*x])/(x^2*Sqrt[d + e*x^2]),x]
```

output
```
(-2*a*Sqrt[d + e*x^2] - 2*b*Sqrt[d + e*x^2]*ArcTan[c*x] + 2*b*c*Sqrt[d]*x*
Log[x] - 2*b*c*Sqrt[d]*x*Log[d + Sqrt[d]*Sqrt[d + e*x^2]] + b*Sqrt[c^2*d -
 e]*x*Log[(-4*c*d*(c*d - I*e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(c^2
*d - e)^(3/2)*(I + c*x))] + b*Sqrt[c^2*d - e]*x*Log[(-4*c*d*(c*d + I*e*x +
 Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(c^2*d - e)^(3/2)*(-I + c*x))])/(2*d
*x)
```

## Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {5511, 25, 27, 354, 94, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x^2\sqrt{d+ex^2}}dx$$

$$\Big\downarrow \text{5511}$$

$$-bc\int -\frac{\sqrt{ex^2+d}}{dx\left(c^2x^2+1\right)}dx-\frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{dx}$$

$$\Big\downarrow \text{25}$$

$$bc\int \frac{\sqrt{ex^2+d}}{dx\left(c^2x^2+1\right)}dx-\frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{dx}$$

$$\Big\downarrow \text{27}$$

$$\frac{bc\int \frac{\sqrt{ex^2+d}}{x(c^2x^2+1)}dx}{d}-\frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{dx}$$

$$\Big\downarrow \text{354}$$

$$\frac{bc\int \frac{\sqrt{ex^2+d}}{x^2(c^2x^2+1)}dx^2}{2d}-\frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{dx}$$

$$\Big\downarrow \text{94}$$

$$\frac{bc\left(d\int \frac{1}{x^2\sqrt{ex^2+d}}dx^2-(c^2d-e)\int \frac{1}{(c^2x^2+1)\sqrt{ex^2+d}}dx^2\right)}{2d}-\frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{dx}$$

$$\Big\downarrow \text{73}$$

$$\frac{bc\left(\frac{2d\int \frac{1}{\frac{x^4}{e}-\frac{d}{e}}d\sqrt{ex^2+d}}{e}-\frac{2(c^2d-e)\int \frac{1}{\frac{c^2x^4}{e}-\frac{c^2d}{e}+1}d\sqrt{ex^2+d}}{e}\right)}{2d}-\frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{dx}$$

$$\Big\downarrow \text{221}$$

$$\frac{bc\left(\frac{2\sqrt{c^2d-e}\text{arctanh}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{c}-2\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\right)}{2d}-\frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{dx}$$

input
```
Int[(a + b*ArcTan[c*x])/(x^2*Sqrt[d + e*x^2]),x]
```

output
```
-((Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/(d*x)) + (b*c*(-2*Sqrt[d]*ArcTanh[
Sqrt[d + e*x^2]/Sqrt[d]] + (2*Sqrt[c^2*d - e]*ArcTanh[(c*Sqrt[d + e*x^2])/
Sqrt[c^2*d - e]])/c))/(2*d)
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 94
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[(e + f*x)^(p - 1)/(a + b*x), x],
x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[(e + f*x)^(p - 1)/(c + d*x), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

## Maple [F]

$$\int \frac{a + b\arctan(cx)}{x^2\sqrt{e\,x^2 + d}}dx$$

input
```
int((a+b*arctan(c*x))/x^2/(e*x^2+d)^(1/2),x)
```

output
```
int((a+b*arctan(c*x))/x^2/(e*x^2+d)^(1/2),x)
```

## Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 666, normalized size of antiderivative = 6.66

$$\int \frac{a + b\arctan(cx)}{x^2\sqrt{d + ex^2}}\,dx$$
$$= \left[ \frac{2\,bc\sqrt{d}x\log\left(-\frac{ex^2-2\,\sqrt{ex^2+d}\sqrt{d}+2\,d}{x^2}\right) + \sqrt{c^2d-e}bx\log\left(\frac{c^4e^2x^4+8\,c^4d^2-8\,c^2de+2\,(4\,c^4de-3\,c^2e^2)x^2+4\,(c^3ex^2+2\,c^3d-ce}{c^4x^4+2\,c^2x^2+1}\right.}{4\,dx} \right.$$

input
```
integrate((a+b*arctan(c*x))/x^2/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(2*b*c*sqrt(d)*x*log(-(e*x^2 - 2*sqrt(e*x^2 + d)*sqrt(d) + 2*d)/x^2)
+ sqrt(c^2*d - e)*b*x*log((c^4*e^2*x^4 + 8*c^4*d^2 - 8*c^2*d*e + 2*(4*c^4*
d*e - 3*c^2*e^2)*x^2 + 4*(c^3*e*x^2 + 2*c^3*d - c*e)*sqrt(c^2*d - e)*sqrt(
e*x^2 + d) + e^2)/(c^4*x^4 + 2*c^2*x^2 + 1)) - 4*sqrt(e*x^2 + d)*(b*arctan
(c*x) + a))/(d*x), 1/2*(b*c*sqrt(d)*x*log(-(e*x^2 - 2*sqrt(e*x^2 + d)*sqrt
(d) + 2*d)/x^2) + sqrt(-c^2*d + e)*b*x*arctan(-1/2*(c^2*e*x^2 + 2*c^2*d -
e)*sqrt(-c^2*d + e)*sqrt(e*x^2 + d)/(c^3*d^2 - c*d*e + (c^3*d*e - c*e^2)*x
^2)) - 2*sqrt(e*x^2 + d)*(b*arctan(c*x) + a))/(d*x), 1/4*(4*b*c*sqrt(-d)*x
*arctan(sqrt(e*x^2 + d)*sqrt(-d)/d) + sqrt(c^2*d - e)*b*x*log((c^4*e^2*x^4
 + 8*c^4*d^2 - 8*c^2*d*e + 2*(4*c^4*d*e - 3*c^2*e^2)*x^2 + 4*(c^3*e*x^2 +
2*c^3*d - c*e)*sqrt(c^2*d - e)*sqrt(e*x^2 + d) + e^2)/(c^4*x^4 + 2*c^2*x^2
 + 1)) - 4*sqrt(e*x^2 + d)*(b*arctan(c*x) + a))/(d*x), 1/2*(2*b*c*sqrt(-d)
*x*arctan(sqrt(e*x^2 + d)*sqrt(-d)/d) + sqrt(-c^2*d + e)*b*x*arctan(-1/2*(
c^2*e*x^2 + 2*c^2*d - e)*sqrt(-c^2*d + e)*sqrt(e*x^2 + d)/(c^3*d^2 - c*d*e
 + (c^3*d*e - c*e^2)*x^2)) - 2*sqrt(e*x^2 + d)*(b*arctan(c*x) + a))/(d*x)]
```

## Sympy [F]

$$\int \frac{a + b\arctan(cx)}{x^2\sqrt{d + ex^2}}\,dx = \int \frac{a + b\operatorname{atan}(cx)}{x^2\sqrt{d + ex^2}}\,dx$$

input
```
integrate((a+b*atan(c*x))/x**2/(e*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*atan(c*x))/(x**2*sqrt(d + e*x**2)), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\arctan(cx)}{x^2\sqrt{d+ex^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arctan(c*x))/x^2/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

## Giac [F]

$$\int \frac{a+b\arctan(cx)}{x^2\sqrt{d+ex^2}}\,dx = \int \frac{b\arctan(cx)+a}{\sqrt{ex^2+d}x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^2/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)/(sqrt(e*x^2 + d)*x^2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{x^2\sqrt{d+ex^2}}\,dx = \int \frac{a+b\,\mathrm{atan}(c\,x)}{x^2\,\sqrt{e\,x^2+d}}\,dx$$

input
```
int((a + b*atan(c*x))/(x^2*(d + e*x^2)^(1/2)),x)
```

output
```
int((a + b*atan(c*x))/(x^2*(d + e*x^2)^(1/2)), x)
```

## Reduce [F]

$$\int \frac{a+b\arctan(cx)}{x^2\sqrt{d+ex^2}}\,dx = \frac{-\sqrt{e\,x^2+d}\,a-\sqrt{e}\,ax+\left(\int \frac{atan(cx)}{\sqrt{e\,x^2+d}\,x^2}dx\right)bdx}{dx}$$

input
```
int((a+b*atan(c*x))/x^2/(e*x^2+d)^(1/2),x)
```

output
```
( - sqrt(d + e*x**2)*a - sqrt(e)*a*x + int(atan(c*x)/(sqrt(d + e*x**2)*x**
2),x)*b*d*x)/(d*x)
```

# 3.1206 $\int \frac{a+b\arctan(cx)}{x^3\sqrt{d+ex^2}}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{a+b\arctan(cx)}{x^3\sqrt{d+ex^2}}\,dx = -\frac{a\sqrt{d+ex^2}}{2dx^2}+\frac{ae\,\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{2d^{3/2}}+b\text{Int}\left(\frac{\arctan(cx)}{x^3\sqrt{d+ex^2}},x\right)$$

output
```
-1/2*a*(e*x^2+d)^(1/2)/d/x^2+1/2*a*e*arctanh((e*x^2+d)^(1/2)/d^(1/2))/d^(3
/2)+b*Defer(Int)(arctan(c*x)/x^3/(e*x^2+d)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 10.85 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\arctan(cx)}{x^3\sqrt{d+ex^2}}\,dx = \int \frac{a+b\arctan(cx)}{x^3\sqrt{d+ex^2}}\,dx$$

input
```
Integrate[(a + b*ArcTan[c*x])/(x^3*Sqrt[d + e*x^2]),x]
```

output
```
Integrate[(a + b*ArcTan[c*x])/(x^3*Sqrt[d + e*x^2]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x^3\sqrt{d+ex^2}}\,dx$$

$$\Big\downarrow \text{5517}$$

$$a\int \frac{1}{x^3\sqrt{ex^2+d}}dx+b\int \frac{\arctan(cx)}{x^3\sqrt{ex^2+d}}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}a\int \frac{1}{x^4\sqrt{ex^2+d}}dx^2+b\int \frac{\arctan(cx)}{x^3\sqrt{ex^2+d}}dx$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{2}a\left(-\frac{e\int \frac{1}{x^2\sqrt{ex^2+d}}dx^2}{2d}-\frac{\sqrt{d+ex^2}}{dx^2}\right)+b\int \frac{\arctan(cx)}{x^3\sqrt{ex^2+d}}dx$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}a\left(-\frac{\int \frac{1}{\frac{x^4}{e}-\frac{d}{e}}d\sqrt{ex^2+d}}{d}-\frac{\sqrt{d+ex^2}}{dx^2}\right)+b\int \frac{\arctan(cx)}{x^3\sqrt{ex^2+d}}dx$$

$$\Big\downarrow \text{221}$$

$$b\int \frac{\arctan(cx)}{x^3\sqrt{ex^2+d}}dx+\frac{1}{2}a\left(\frac{e\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{\sqrt{d+ex^2}}{dx^2}\right)$$

$$\Big\downarrow \text{5560}$$

$$b\int \frac{\arctan(cx)}{x^3\sqrt{ex^2+d}}dx+\frac{1}{2}a\left(\frac{e\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{\sqrt{d+ex^2}}{dx^2}\right)$$

input
```
Int[(a + b*ArcTan[c*x])/(x^3*Sqrt[d + e*x^2]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{a+b\arctan\left(cx\right)}{x^{3}\sqrt{e\,x^{2}+d}}dx$$

input
```
int((a+b*arctan(c*x))/x^3/(e*x^2+d)^(1/2),x)
```

output
```
int((a+b*arctan(c*x))/x^3/(e*x^2+d)^(1/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.43

$$\int \frac{a+b\arctan(cx)}{x^3\sqrt{d+ex^2}}\,dx=\int \frac{b\arctan\left(cx\right)+a}{\sqrt{ex^2+dx^3}}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^3/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arctan(c*x) + a)/(e*x^5 + d*x^3), x)
```

## Sympy [N/A]

Not integrable

Time = 5.73 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{a+b\arctan(cx)}{x^3\sqrt{d+ex^2}}\,dx = \int \frac{a+b\,\mathrm{atan}\,(cx)}{x^3\sqrt{d+ex^2}}\,dx$$

input
```
integrate((a+b*atan(c*x))/x**3/(e*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*atan(c*x))/(x**3*sqrt(d + e*x**2)), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\arctan(cx)}{x^3\sqrt{d+ex^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arctan(c*x))/x^3/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

## Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+b\arctan(cx)}{x^3\sqrt{d+ex^2}}\,dx = \int \frac{b\arctan\,(cx)+a}{\sqrt{ex^2+d}x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^3/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)/(sqrt(e*x^2 + d)*x^3), x)
```

## Mupad [N/A]

Not integrable

Time = 1.36 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+b\arctan(cx)}{x^3\sqrt{d+ex^2}}\,dx=\int \frac{a+b\,\mathrm{atan}(c\,x)}{x^3\,\sqrt{e\,x^2+d}}\,dx$$

input
```
int((a + b*atan(c*x))/(x^3*(d + e*x^2)^(1/2)),x)
```

output
```
int((a + b*atan(c*x))/(x^3*(d + e*x^2)^(1/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 111, normalized size of antiderivative = 4.83

$$\int \frac{a+b\arctan(cx)}{x^3\sqrt{d+ex^2}}\,dx = \frac{-\sqrt{e\,x^2+d}\,ad-\sqrt{d}\log\left(\frac{\sqrt{e\,x^2+d}-\sqrt{d}+\sqrt{e}\,x}{\sqrt{d}}\right)ae\,x^2+\sqrt{d}\log\left(\frac{\sqrt{e\,x^2+d}+\sqrt{d}+\sqrt{e}\,x}{\sqrt{d}}\right)ae\,x^2+2\left(\int \frac{atan(cx)}{\sqrt{e\,x^2+d}\,x^3}dx\right)b}{2d^2x^2}$$

input
```
int((a+b*atan(c*x))/x^3/(e*x^2+d)^(1/2),x)
```

output
```
( - sqrt(d + e*x**2)*a*d - sqrt(d)*log((sqrt(d + e*x**2) - sqrt(d) + sqrt(
e)*x)/sqrt(d))*a*e*x**2 + sqrt(d)*log((sqrt(d + e*x**2) + sqrt(d) + sqrt(e
)*x)/sqrt(d))*a*e*x**2 + 2*int(atan(c*x)/(sqrt(d + e*x**2)*x**3),x)*b*d**2
*x**2)/(2*d**2*x**2)
```

# 3.1207 $\int \frac{a+b\arctan(cx)}{x^4\sqrt{d+ex^2}}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 179

$$\int \frac{a+b\arctan(cx)}{x^4\sqrt{d+ex^2}}\,dx = -\frac{bc\sqrt{d+ex^2}}{6dx^2}-\frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{3dx^3}+\frac{2e\sqrt{d+ex^2}(a+b\arctan(cx))}{3d^2x}+\frac{bc(2c^2d+3e)\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{6d^{3/2}}-\frac{b\sqrt{c^2d-e}(c^2d+2e)\operatorname{arctanh}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{3d^2}$$

output
```
-1/6*b*c*(e*x^2+d)^(1/2)/d/x^2-1/3*(e*x^2+d)^(1/2)*(a+b*arctan(c*x))/d/x^3
+2/3*e*(e*x^2+d)^(1/2)*(a+b*arctan(c*x))/d^2/x+1/6*b*c*(2*c^2*d+3*e)*arcta
nh((e*x^2+d)^(1/2)/d^(1/2))/d^(3/2)-1/3*b*(c^2*d-e)^(1/2)*(c^2*d+2*e)*arct
anh(c*(e*x^2+d)^(1/2)/(c^2*d-e)^(1/2))/d^2
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 372, normalized size of antiderivative = 2.08

$$\int \frac{a+b\arctan(cx)}{x^4\sqrt{d+ex^2}}\,dx =$$

$$-\frac{\frac{\sqrt{d+ex^2}(bcdx+2a(d-2ex^2))}{x^3}+\frac{2b(d-2ex^2)\sqrt{d+ex^2}\arctan(cx)}{x^3}+bc\sqrt{d}(2c^2d+3e)\log(x)-bc\sqrt{d}(2c^2d+3e)\log\left(d\right.}{}$$

input
```
Integrate[(a + b*ArcTan[c*x])/(x^4*Sqrt[d + e*x^2]),x]
```

output
```
-1/6*((Sqrt[d + e*x^2]*(b*c*d*x + 2*a*(d - 2*e*x^2)))/x^3 + (2*b*(d - 2*e*
x^2)*Sqrt[d + e*x^2]*ArcTan[c*x])/x^3 + b*c*Sqrt[d]*(2*c^2*d + 3*e)*Log[x]
 - b*c*Sqrt[d]*(2*c^2*d + 3*e)*Log[d + Sqrt[d]*Sqrt[d + e*x^2]] + (b*(c^4*
d^2 + c^2*d*e - 2*e^2)*Log[(12*c*d^2*(c*d - I*e*x + Sqrt[c^2*d - e]*Sqrt[d
 + e*x^2]))/(b*Sqrt[c^2*d - e]*(c^4*d^2 + c^2*d*e - 2*e^2)*(I + c*x))]])/Sq
rt[c^2*d - e] + (b*(c^4*d^2 + c^2*d*e - 2*e^2)*Log[(12*c*d^2*(c*d + I*e*x
+ Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*Sqrt[c^2*d - e]*(c^4*d^2 + c^2*d*e
- 2*e^2)*(-I + c*x))]])/Sqrt[c^2*d - e])/d^2
```

## Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {5511, 27, 435, 166, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x^4\sqrt{d+ex^2}}\,dx$$

$$\Big\downarrow \text{5511}$$

$$-bc\int -\frac{\left(d-2ex^2\right)\sqrt{ex^2+d}}{3d^2x^3\left(c^2x^2+1\right)}dx+\frac{2e\sqrt{d+ex^2}(a+b\arctan(cx))}{3d^2x}-\frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{3dx^3}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{bc\int \frac{\left(d-2ex^2\right)\sqrt{ex^2+d}}{x^3\left(c^2x^2+1\right)}dx}{3d^2}+\frac{2e\sqrt{d+ex^2}(a+b\arctan(cx))}{3d^2x}-\frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{3dx^3}$$

$$\Big\downarrow \text{ 435}$$

$$\frac{bc\int \frac{\left(d-2ex^2\right)\sqrt{ex^2+d}}{x^4\left(c^2x^2+1\right)}dx^2}{6d^2}+\frac{2e\sqrt{d+ex^2}(a+b\arctan(cx))}{3d^2x}-\frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{3dx^3}$$

$$\Big\downarrow \text{ 166}$$

$$\frac{bc\left(\int -\frac{e\left(dc^2+4e\right)x^2+d\left(2dc^2+3e\right)}{2x^2\left(c^2x^2+1\right)\sqrt{ex^2+d}}dx^2-\frac{d\sqrt{d+ex^2}}{x^2}\right)}{6d^2}+\frac{2e\sqrt{d+ex^2}(a+b\arctan(cx))}{3d^2x}-\frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{3dx^3}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{bc\left(-\frac{1}{2}\int \frac{e\left(dc^2+4e\right)x^2+d\left(2dc^2+3e\right)}{x^2\left(c^2x^2+1\right)\sqrt{ex^2+d}}dx^2-\frac{d\sqrt{d+ex^2}}{x^2}\right)}{6d^2}+\frac{2e\sqrt{d+ex^2}(a+b\arctan(cx))}{3d^2x}-\frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{3dx^3}$$

$$\Big\downarrow \text{ 174}$$

$$\frac{bc\left(\frac{1}{2}\left(2\left(c^2d-e\right)\left(c^2d+2e\right)\int \frac{1}{\left(c^2x^2+1\right)\sqrt{ex^2+d}}dx^2-d\left(2c^2d+3e\right)\int \frac{1}{x^2\sqrt{ex^2+d}}dx^2\right)-\frac{d\sqrt{d+ex^2}}{x^2}\right)}{6d^2}+\frac{2e\sqrt{d+ex^2}(a+b\arctan(cx))}{3d^2x}-\frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{3dx^3}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{bc\left(\frac{1}{2}\left(\frac{4\left(c^2d-e\right)\left(c^2d+2e\right)\int \frac{1}{\frac{c^2x^4}{e}-\frac{c^2d}{e}+1}d\sqrt{ex^2+d}}{e}-\frac{2d\left(2c^2d+3e\right)\int \frac{1}{\frac{x^4}{e}-\frac{d}{e}}d\sqrt{ex^2+d}}{e}\right)-\frac{d\sqrt{d+ex^2}}{x^2}\right)}{6d^2}+\frac{2e\sqrt{d+ex^2}(a+b\arctan(cx))}{3d^2x}-\frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{3dx^3}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{2e\sqrt{d+ex^2}(a+b\arctan(cx))}{3d^2x}-\frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{3dx^3}+\frac{bc\left(\frac{1}{2}\left(2\sqrt{d}(2c^2d+3e)\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)-\frac{4\sqrt{c^2d-e}(c^2d+2e)\operatorname{arctanh}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{c}\right)-\frac{d\sqrt{d+ex^2}}{x^2}\right)}{6d^2}$$

input
```
Int[(a + b*ArcTan[c*x])/(x^4*Sqrt[d + e*x^2]),x]
```

output
```
-1/3*(Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]))/(d*x^3) + (2*e*Sqrt[d + e*x^2]*
(a + b*ArcTan[c*x]))/(3*d^2*x) + (b*c*(-((d*Sqrt[d + e*x^2])/x^2) + (2*Sqr
t[d]*(2*c^2*d + 3*e)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]] - (4*Sqrt[c^2*d - e]
*(c^2*d + 2*e)*ArcTanh[(c*Sqrt[d + e*x^2])/Sqrt[c^2*d - e]])/c)/2))/(6*d^2
)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 166
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^n*((e + f*x)^(p + 1)/(b*(b*e - a*f)*(m + 1))), x] - Simp[1/(b*(b*e -
a*f)*(m + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[b*
c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g - e*h
)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c, d,
e, f, g, h, p}, x] && ILtQ[m, -1] && GtQ[n, 0]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

## Maple [F]

$$\int \frac{a + b\arctan(cx)}{x^4\sqrt{e\,x^2 + d}}dx$$

input
```
int((a+b*arctan(c*x))/x^4/(e*x^2+d)^(1/2),x)
```

output
```
int((a+b*arctan(c*x))/x^4/(e*x^2+d)^(1/2),x)
```

## Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 874, normalized size of antiderivative = 4.88

$$\int \frac{a+b\arctan(cx)}{x^4\sqrt{d+ex^2}}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*arctan(c*x))/x^4/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
[1/12*((b*c^2*d + 2*b*e)*sqrt(c^2*d - e)*x^3*log((c^4*e^2*x^4 + 8*c^4*d^2
 - 8*c^2*d*e + 2*(4*c^4*d*e - 3*c^2*e^2)*x^2 - 4*(c^3*e*x^2 + 2*c^3*d - c*e
)*sqrt(c^2*d - e)*sqrt(e*x^2 + d) + e^2)/(c^4*x^4 + 2*c^2*x^2 + 1)) + (2*b
*c^3*d + 3*b*c*e)*sqrt(d)*x^3*log(-(e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(d) + 2*
d)/x^2) - 2*(b*c*d*x - 4*a*e*x^2 + 2*a*d - 2*(2*b*e*x^2 - b*d)*arctan(c*x)
)*sqrt(e*x^2 + d))/(d^2*x^3), -1/12*(2*(b*c^2*d + 2*b*e)*sqrt(-c^2*d + e)*
x^3*arctan(-1/2*(c^2*e*x^2 + 2*c^2*d - e)*sqrt(-c^2*d + e)*sqrt(e*x^2 + d)
/(c^3*d^2 - c*d*e + (c^3*d*e - c*e^2)*x^2)) - (2*b*c^3*d + 3*b*c*e)*sqrt(d
)*x^3*log(-(e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(d) + 2*d)/x^2) + 2*(b*c*d*x - 4
*a*e*x^2 + 2*a*d - 2*(2*b*e*x^2 - b*d)*arctan(c*x))*sqrt(e*x^2 + d))/(d^2*
x^3), -1/12*(2*(2*b*c^3*d + 3*b*c*e)*sqrt(-d)*x^3*arctan(sqrt(e*x^2 + d)*s
qrt(-d)/d) - (b*c^2*d + 2*b*e)*sqrt(c^2*d - e)*x^3*log((c^4*e^2*x^4 + 8*c^
4*d^2 - 8*c^2*d*e + 2*(4*c^4*d*e - 3*c^2*e^2)*x^2 - 4*(c^3*e*x^2 + 2*c^3*d
 - c*e)*sqrt(c^2*d - e)*sqrt(e*x^2 + d) + e^2)/(c^4*x^4 + 2*c^2*x^2 + 1))
+ 2*(b*c*d*x - 4*a*e*x^2 + 2*a*d - 2*(2*b*e*x^2 - b*d)*arctan(c*x))*sqrt(e
*x^2 + d))/(d^2*x^3), -1/6*((b*c^2*d + 2*b*e)*sqrt(-c^2*d + e)*x^3*arctan(
-1/2*(c^2*e*x^2 + 2*c^2*d - e)*sqrt(-c^2*d + e)*sqrt(e*x^2 + d)/(c^3*d^2 -
 c*d*e + (c^3*d*e - c*e^2)*x^2)) + (2*b*c^3*d + 3*b*c*e)*sqrt(-d)*x^3*arct
an(sqrt(e*x^2 + d)*sqrt(-d)/d) + (b*c*d*x - 4*a*e*x^2 + 2*a*d - 2*(2*b*e*x
^2 - b*d)*arctan(c*x))*sqrt(e*x^2 + d))/(d^2*x^3)]
```

## Sympy [F]

$$\int \frac{a+b\arctan(cx)}{x^4\sqrt{d+ex^2}}\,dx = \int \frac{a+b\operatorname{atan}(cx)}{x^4\sqrt{d+ex^2}}\,dx$$

input
```
integrate((a+b*atan(c*x))/x**4/(e*x**2+d)**(1/2),x)
```

output
```
Integral((a + b*atan(c*x))/(x**4*sqrt(d + e*x**2)), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\arctan(cx)}{x^4\sqrt{d + ex^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arctan(c*x))/x^4/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

## Giac [F]

$$\int \frac{a + b\arctan(cx)}{x^4\sqrt{d + ex^2}}\,dx = \int \frac{b\arctan{(cx)} + a}{\sqrt{ex^2 + d}x^4}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^4/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)/(sqrt(e*x^2 + d)*x^4), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{x^4\sqrt{d+ex^2}}\,dx = \int \frac{a+b\,\mathrm{atan}(c\,x)}{x^4\,\sqrt{e\,x^2+d}}\,dx$$

input
```
int((a + b*atan(c*x))/(x^4*(d + e*x^2)^(1/2)),x)
```

output
```
int((a + b*atan(c*x))/(x^4*(d + e*x^2)^(1/2)), x)
```

## Reduce [F]

$$\int \frac{a+b\arctan(cx)}{x^4\sqrt{d+ex^2}}\,dx = \frac{-\sqrt{e\,x^2+d}\,ad+2\sqrt{e\,x^2+d}\,ae\,x^2-2\sqrt{e}\,ae\,x^3+3\left(\int \frac{atan(cx)}{\sqrt{e\,x^2+d}\,x^4}dx\right)b\,d^2x^3}{3d^2x^3}$$

input
```
int((a+b*atan(c*x))/x^4/(e*x^2+d)^(1/2),x)
```

output
```
( - sqrt(d + e*x**2)*a*d + 2*sqrt(d + e*x**2)*a*e*x**2 - 2*sqrt(e)*a*e*x**
3 + 3*int(atan(c*x)/(sqrt(d + e*x**2)*x**4),x)*b*d**2*x**3)/(3*d**2*x**3)
```

# 3.1208 $\int \frac{x^3(a+b\arctan(cx))}{(d+ex^2)^{3/2}}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 137

$$\int \frac{x^3(a+b\arctan(cx))}{(d+ex^2)^{3/2}}\,dx = \frac{d(a+b\arctan(cx))}{e^2\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{e^2} - \frac{b(2c^2d-e)\arctan\left(\frac{\sqrt{c^2d-e}x}{\sqrt{d+ex^2}}\right)}{c\sqrt{c^2d-e}e^2}-\frac{b\,\mathrm{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{ce^{3/2}}$$

output
```
d*(a+b*arctan(c*x))/e^2/(e*x^2+d)^(1/2)+(e*x^2+d)^(1/2)*(a+b*arctan(c*x))/
e^2-b*(2*c^2*d-e)*arctan((c^2*d-e)^(1/2)*x/(e*x^2+d)^(1/2))/c/(c^2*d-e)^(1
/2)/e^2-b*arctanh(e^(1/2)*x/(e*x^2+d)^(1/2))/c/e^(3/2)
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.46 (sec) , antiderivative size = 321, normalized size of antiderivative = 2.34

$$\int \frac{x^3(a+b\arctan(cx))}{(d+ex^2)^{3/2}}\,dx = \frac{\frac{2a(2d+ex^2)}{\sqrt{d+ex^2}}+\frac{2b(2d+ex^2)\arctan(cx)}{\sqrt{d+ex^2}}-\frac{ib(2c^2d-e)\log\left(\frac{4c^2e^2\left(-icd+ex-i\sqrt{c^2d-e}\sqrt{d+ex^2}\right)}{b\sqrt{c^2d-e}\left(2c^2d-e\right)(-i+cx)}\right)}{c\sqrt{c^2d-e}}+\frac{ib(}{}}{2e^2}$$

input
```
Integrate[(x^3*(a + b*ArcTan[c*x]))/(d + e*x^2)^(3/2),x]
```

output
```
((2*a*(2*d + e*x^2))/Sqrt[d + e*x^2] + (2*b*(2*d + e*x^2)*ArcTan[c*x])/Sqr
t[d + e*x^2] - (I*b*(2*c^2*d - e)*Log[(4*c^2*e^2*((-I)*c*d + e*x - I*Sqrt[
c^2*d - e]*Sqrt[d + e*x^2]))/(b*Sqrt[c^2*d - e]*(2*c^2*d - e)*(-I + c*x))]
)/(c*Sqrt[c^2*d - e]) + (I*b*(2*c^2*d - e)*Log[(4*c^2*e^2*(I*c*d + e*x + I
*Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*Sqrt[c^2*d - e]*(2*c^2*d - e)*(I + c
*x))])/(c*Sqrt[c^2*d - e]) - (2*b*Sqrt[e]*Log[e*x + Sqrt[e]*Sqrt[d + e*x^2
]])/c)/(2*e^2)
```

## Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {5511, 27, 398, 224, 219, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\arctan(cx))}{(d+ex^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{5511}$$

$$-bc\int \frac{ex^2+2d}{e^2(c^2x^2+1)\sqrt{ex^2+d}}dx+\frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{e^2}+\frac{d(a+b\arctan(cx))}{e^2\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{bc\int \frac{ex^2+2d}{(c^2x^2+1)\sqrt{ex^2+d}}dx}{e^2}+\frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{e^2}+\frac{d(a+b\arctan(cx))}{e^2\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{398}$$

$$-\frac{bc\left(\frac{e\int \frac{1}{\sqrt{ex^2+d}}dx}{c^2}+\left(2d-\frac{e}{c^2}\right)\int \frac{1}{(c^2x^2+1)\sqrt{ex^2+d}}dx\right)}{e^2}+\frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{e^2}+\frac{d(a+b\arctan(cx))}{e^2\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{224}$$

$$-\frac{bc\left(\left(2d-\frac{e}{c^2}\right)\int \frac{1}{(c^2x^2+1)\sqrt{ex^2+d}}dx+\frac{e\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{c^2}\right)}{e^2}+\frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{e^2}+\frac{d(a+b\arctan(cx))}{e^2\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{bc\left(\left(2d-\frac{e}{c^2}\right)\int \frac{1}{(c^2x^2+1)\sqrt{ex^2+d}}dx+\frac{\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2}\right)}{e^2}+\frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{e^2}+\frac{d(a+b\arctan(cx))}{e^2\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 291}$$

$$-\frac{bc\left(\left(2d-\frac{e}{c^2}\right)\int \frac{1}{1-\frac{(e-c^2d)x^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}+\frac{\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2}\right)}{e^2}+\frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{e^2}+\frac{d(a+b\arctan(cx))}{e^2\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{\sqrt{d+ex^2}(a+b\arctan(cx))}{e^2}+\frac{d(a+b\arctan(cx))}{e^2\sqrt{d+ex^2}}-\frac{bc\left(\frac{\left(2d-\frac{e}{c^2}\right)\arctan\left(\frac{x\sqrt{c^2d-e}}{\sqrt{d+ex^2}}\right)}{\sqrt{c^2d-e}}+\frac{\sqrt{e}\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c^2}\right)}{e^2}$$

input
```
Int[(x^3*(a + b*ArcTan[c*x]))/(d + e*x^2)^(3/2),x]
```

output
```
(d*(a + b*ArcTan[c*x]))/(e^2*Sqrt[d + e*x^2]) + (Sqrt[d + e*x^2]*(a + b*Ar
cTan[c*x]))/e^2 - (b*c*(((2*d - e/c^2)*ArcTan[(Sqrt[c^2*d - e]*x)/Sqrt[d +
 e*x^2]])/Sqrt[c^2*d - e] + (Sqrt[e]*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])
/c^2))/e^2
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

## Maple [F]

$$\int \frac{x^3(a + b \arctan(cx))}{(e x^2 + d)^{\frac{3}{2}}} dx$$

input
```
int(x^3*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x)
```

output
```
int(x^3*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. 2(121) = 242.

Time = 0.43 (sec) , antiderivative size = 1291, normalized size of antiderivative = 9.42

$$\int \frac{x^3(a + b \arctan(cx))}{(d + ex^2)^{3/2}} dx = \text{Too large to display}$$

input
```
integrate(x^3*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(2*(b*c^2*d^2 - b*d*e + (b*c^2*d*e - b*e^2)*x^2)*sqrt(e)*log(-2*e*x^2
 + 2*sqrt(e*x^2 + d)*sqrt(e)*x - d) + (2*b*c^2*d^2 - b*d*e + (2*b*c^2*d*e
- b*e^2)*x^2)*sqrt(-c^2*d + e)*log(((c^4*d^2 - 8*c^2*d*e + 8*e^2)*x^4 - 2*
(3*c^2*d^2 - 4*d*e)*x^2 - 4*((c^2*d - 2*e)*x^3 - d*x)*sqrt(-c^2*d + e)*sqr
t(e*x^2 + d) + d^2)/(c^4*x^4 + 2*c^2*x^2 + 1)) + 4*(2*a*c^3*d^2 - 2*a*c*d*
e + (a*c^3*d*e - a*c*e^2)*x^2 + (2*b*c^3*d^2 - 2*b*c*d*e + (b*c^3*d*e - b*
c*e^2)*x^2)*arctan(c*x))*sqrt(e*x^2 + d))/(c^3*d^2*e^2 - c*d*e^3 + (c^3*d*
e^3 - c*e^4)*x^2), -1/2*((2*b*c^2*d^2 - b*d*e + (2*b*c^2*d*e - b*e^2)*x^2)
*sqrt(c^2*d - e)*arctan(1/2*sqrt(c^2*d - e)*((c^2*d - 2*e)*x^2 - d)*sqrt(e
*x^2 + d)/((c^2*d*e - e^2)*x^3 + (c^2*d^2 - d*e)*x)) - (b*c^2*d^2 - b*d*e
+ (b*c^2*d*e - b*e^2)*x^2)*sqrt(e)*log(-2*e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(e
)*x - d) - 2*(2*a*c^3*d^2 - 2*a*c*d*e + (a*c^3*d*e - a*c*e^2)*x^2 + (2*b*c
^3*d^2 - 2*b*c*d*e + (b*c^3*d*e - b*c*e^2)*x^2)*arctan(c*x))*sqrt(e*x^2 +
d))/(c^3*d^2*e^2 - c*d*e^3 + (c^3*d*e^3 - c*e^4)*x^2), 1/4*(4*(b*c^2*d^2 -
 b*d*e + (b*c^2*d*e - b*e^2)*x^2)*sqrt(-e)*arctan(sqrt(-e)*x/sqrt(e*x^2 +
d)) + (2*b*c^2*d^2 - b*d*e + (2*b*c^2*d*e - b*e^2)*x^2)*sqrt(-c^2*d + e)*l
og(((c^4*d^2 - 8*c^2*d*e + 8*e^2)*x^4 - 2*(3*c^2*d^2 - 4*d*e)*x^2 - 4*((c^
2*d - 2*e)*x^3 - d*x)*sqrt(-c^2*d + e)*sqrt(e*x^2 + d) + d^2)/(c^4*x^4 + 2
*c^2*x^2 + 1)) + 4*(2*a*c^3*d^2 - 2*a*c*d*e + (a*c^3*d*e - a*c*e^2)*x^2 +
(2*b*c^3*d^2 - 2*b*c*d*e + (b*c^3*d*e - b*c*e^2)*x^2)*arctan(c*x))*sqrt...
```

## Sympy [F]

$$\int \frac{x^3(a+b\arctan(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{x^3(a+b\operatorname{atan}(cx))}{(d+ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3*(a+b*atan(c*x))/(e*x**2+d)**(3/2),x)
```

output
```
Integral(x**3*(a + b*atan(c*x))/(d + e*x**2)**(3/2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(a+b\arctan(cx))}{(d+ex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

## Giac [F]

$$\int \frac{x^3(a+b\arctan(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{(b\arctan(cx)+a)x^3}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)*x^3/(e*x^2 + d)^(3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\arctan(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{x^3\,(a+b\,\mathrm{atan}(c\,x))}{(e\,x^2+d)^{3/2}}\,dx$$

input
```
int((x^3*(a + b*atan(c*x)))/(d + e*x^2)^(3/2),x)
```

output
```
int((x^3*(a + b*atan(c*x)))/(d + e*x^2)^(3/2), x)
```

## Reduce [F]

$$\int \frac{x^3(a+b\arctan(cx))}{(d+ex^2)^{3/2}}\,dx = \frac{2\sqrt{e\,x^2+d}\,ad+\sqrt{e\,x^2+d}\,ae\,x^2+\left(\int \frac{atan(cx)x^3}{\sqrt{e\,x^2+d}\,d+\sqrt{e\,x^2+d}\,e\,x^2}dx\right)bd\,e^2+\left(\int \frac{}{\sqrt{e\,x}}\right.}{e^2\,(e\,x^2+d)}$$

input
```
int(x^3*(a+b*atan(c*x))/(e*x^2+d)^(3/2),x)
```

output
```
(2*sqrt(d + e*x**2)*a*d + sqrt(d + e*x**2)*a*e*x**2 + int((atan(c*x)*x**3)
/(sqrt(d + e*x**2)*d + sqrt(d + e*x**2)*e*x**2),x)*b*d*e**2 + int((atan(c*
x)*x**3)/(sqrt(d + e*x**2)*d + sqrt(d + e*x**2)*e*x**2),x)*b*e**3*x**2)/(e
**2*(d + e*x**2))
```

# 3.1209 $\int \frac{x^2(a+b\arctan(cx))}{(d+ex^2)^{3/2}}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^2(a+b\arctan(cx))}{(d+ex^2)^{3/2}}\,dx=-\frac{ax}{e\sqrt{d+ex^2}}+\frac{a\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{e^{3/2}}+b\text{Int}\left(\frac{x^2\arctan(cx)}{(d+ex^2)^{3/2}},x\right)$$

output
```
-a*x/e/(e*x^2+d)^(1/2)+a*arctanh(e^(1/2)*x/(e*x^2+d)^(1/2))/e^(3/2)+b*Defe
r(Int)(x^2*arctan(c*x)/(e*x^2+d)^(3/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 20.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^2(a+b\arctan(cx))}{(d+ex^2)^{3/2}}\,dx=\int \frac{x^2(a+b\arctan(cx))}{(d+ex^2)^{3/2}}\,dx$$

input
```
Integrate[(x^2*(a + b*ArcTan[c*x]))/(d + e*x^2)^(3/2),x]
```

output
```
Integrate[(x^2*(a + b*ArcTan[c*x]))/(d + e*x^2)^(3/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\arctan(cx))}{(d+ex^2)^{3/2}}dx$$

$$\Big\downarrow \text{5517}$$

$$a\int \frac{x^2}{(ex^2+d)^{3/2}}dx+b\int \frac{x^2\arctan(cx)}{(ex^2+d)^{3/2}}dx$$

$$\Big\downarrow \text{252}$$

$$a\left(\frac{\int \frac{1}{\sqrt{ex^2+d}}dx}{e}-\frac{x}{e\sqrt{d+ex^2}}\right)+b\int \frac{x^2\arctan(cx)}{(ex^2+d)^{3/2}}dx$$

$$\Big\downarrow \text{224}$$

$$a\left(\frac{\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{e}-\frac{x}{e\sqrt{d+ex^2}}\right)+b\int \frac{x^2\arctan(cx)}{(ex^2+d)^{3/2}}dx$$

$$\Big\downarrow \text{219}$$

$$b\int \frac{x^2\arctan(cx)}{(ex^2+d)^{3/2}}dx+a\left(\frac{\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{e^{3/2}}-\frac{x}{e\sqrt{d+ex^2}}\right)$$

$$\Big\downarrow \text{5560}$$

$$b\int \frac{x^2\arctan(cx)}{(ex^2+d)^{3/2}}dx+a\left(\frac{\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{e^{3/2}}-\frac{x}{e\sqrt{d+ex^2}}\right)$$

input
```
Int[(x^2*(a + b*ArcTan[c*x]))/(d + e*x^2)^(3/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x^2(a + b\arctan(cx))}{(e\,x^2 + d)^{\frac{3}{2}}}dx$$

input
```
int(x^2*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x)
```

output
```
int(x^2*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.04

$$\int \frac{x^2(a + b\arctan(cx))}{(d + ex^2)^{3/2}}\,dx = \int \frac{(b\arctan(cx) + a)x^2}{(ex^2 + d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
integral((b*x^2*arctan(c*x) + a*x^2)*sqrt(e*x^2 + d)/(e^2*x^4 + 2*d*e*x^2
+ d^2), x)
```

## Sympy [N/A]

Not integrable

Time = 50.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{x^2(a+b\arctan(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{x^2(a+b\,\mathrm{atan}\,(cx))}{(d+ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2*(a+b*atan(c*x))/(e*x**2+d)**(3/2),x)
```

output
```
Integral(x**2*(a + b*atan(c*x))/(d + e*x**2)**(3/2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a+b\arctan(cx))}{(d+ex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

## Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^2(a+b\arctan(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{(b\arctan(cx)+a)x^2}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)*x^2/(e*x^2 + d)^(3/2), x)
```

## Mupad [N/A]

Not integrable

Time = 1.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^2(a+b\arctan(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{x^2\,(a+b\,\mathrm{atan}(c\,x))}{{(e\,x^2+d)}^{3/2}}\,dx$$

input
```
int((x^2*(a + b*atan(c*x)))/(d + e*x^2)^(3/2),x)
```

output
```
int((x^2*(a + b*atan(c*x)))/(d + e*x^2)^(3/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 179, normalized size of antiderivative = 7.78

$$\int \frac{x^2(a+b\arctan(cx))}{(d+ex^2)^{3/2}}\,dx = \frac{-\sqrt{e\,x^2+d}\,aex+\sqrt{e}\log\left(\frac{\sqrt{e\,x^2+d}+\sqrt{e}\,x}{\sqrt{d}}\right)ad+\sqrt{e}\log\left(\frac{\sqrt{e\,x^2+d}+\sqrt{e}\,x}{\sqrt{d}}\right)ae\,x^2-\cdots}{e\cdots}$$

input
```
int(x^2*(a+b*atan(c*x))/(e*x^2+d)^(3/2),x)
```

output
```
( - sqrt(d + e*x**2)*a*e*x + sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)*x)/sq
rt(d))*a*d + sqrt(e)*log((sqrt(d + e*x**2) + sqrt(e)*x)/sqrt(d))*a*e*x**2
- sqrt(e)*a*d - sqrt(e)*a*e*x**2 + int((atan(c*x)*x**2)/(sqrt(d + e*x**2)*
d + sqrt(d + e*x**2)*e*x**2),x)*b*d*e**2 + int((atan(c*x)*x**2)/(sqrt(d +
e*x**2)*d + sqrt(d + e*x**2)*e*x**2),x)*b*e**3*x**2)/(e**2*(d + e*x**2))
```

# 3.1210 $\int \frac{x(a+b\arctan(cx))}{(d+ex^2)^{3/2}}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 71

$$\int \frac{x(a+b\arctan(cx))}{(d+ex^2)^{3/2}}\,dx = -\frac{a+b\arctan(cx)}{e\sqrt{d+ex^2}} + \frac{bc\arctan\left(\frac{\sqrt{c^2d-e}x}{\sqrt{d+ex^2}}\right)}{\sqrt{c^2d-e}e}$$

output
```
-(a+b*arctan(c*x))/e/(e*x^2+d)^(1/2)+b*c*arctan((c^2*d-e)^(1/2)*x/(e*x^2+d
)^(1/2))/(c^2*d-e)^(1/2)/e
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 210, normalized size of antiderivative = 2.96

$$\int \frac{x(a+b\arctan(cx))}{(d+ex^2)^{3/2}}\,dx = -\frac{\frac{2a}{\sqrt{d+ex^2}} + \frac{2b\arctan(cx)}{\sqrt{d+ex^2}} + \frac{ibc\log\left(-\frac{4ie\left(cd-iex+\sqrt{c^2d-e}\sqrt{d+ex^2}\right)}{b\sqrt{c^2d-e}(i+cx)}\right)}{\sqrt{c^2d-e}} - \frac{ibc\log\left(\frac{4ie\left(cd+iex+\sqrt{c^2d-e}\sqrt{d+ex^2}\right)}{b\sqrt{c^2d-e}(-i+cx)}\right)}{\sqrt{c^2d-e}}}{2e}$$

input
```
Integrate[(x*(a + b*ArcTan[c*x]))/(d + e*x^2)^(3/2),x]
```

output
```
-1/2*((2*a)/Sqrt[d + e*x^2] + (2*b*ArcTan[c*x])/Sqrt[d + e*x^2] + (I*b*c*L
og[((-4*I)*e*(c*d - I*e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*Sqrt[c^2*
d - e]*(I + c*x))])/Sqrt[c^2*d - e] - (I*b*c*Log[((4*I)*e*(c*d + I*e*x + S
qrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*Sqrt[c^2*d - e]*(-I + c*x))])/Sqrt[c^2
*d - e])/e
```

## Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5509, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\arctan(cx))}{(d+ex^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{5509}$$

$$\frac{bc\int \frac{1}{(c^2x^2+1)\sqrt{ex^2+d}}dx}{e}-\frac{a+b\arctan(cx)}{e\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{291}$$

$$\frac{bc\int \frac{1}{1-\frac{(e-c^2d)x^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{e}-\frac{a+b\arctan(cx)}{e\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{216}$$

$$\frac{bc\arctan\left(\frac{x\sqrt{c^2d-e}}{\sqrt{d+ex^2}}\right)}{e\sqrt{c^2d-e}}-\frac{a+b\arctan(cx)}{e\sqrt{d+ex^2}}$$

input
```
Int[(x*(a + b*ArcTan[c*x]))/(d + e*x^2)^(3/2),x]
```

output
```
-((a + b*ArcTan[c*x])/(e*Sqrt[d + e*x^2])) + (b*c*ArcTan[(Sqrt[c^2*d - e]*
x)/Sqrt[d + e*x^2]])/(Sqrt[c^2*d - e]*e)
```

### Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 5509
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*e*(q + 1))), x
] - Simp[b*(c/(2*e*(q + 1)))   Int[(d + e*x^2)^(q + 1)/(1 + c^2*x^2), x], x
] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]
```

## Maple [F]

$$\int \frac{x(a+b\arctan(cx))}{(e\,x^2+d)^{\frac{3}{2}}}dx$$

input
```
int(x*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x)
```

output
```
int(x*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(63) = 126$.

Time = 0.18 (sec) , antiderivative size = 379, normalized size of antiderivative = 5.34

$$\int \frac{x(a+b\arctan(cx))}{(d+ex^2)^{3/2}}\,dx = \left[-\frac{(bcex^2+bcd)\sqrt{-c^2d+e}\log\left(\frac{(c^4d^2-8\,c^2de+8\,e^2)x^4-2\,(3\,c^2d^2-4\,de)x^2-4\,((c^2d-2\,e)x^3-}{c^4x^4+2\,c^2x^2+1}\right.}{4\,(c^2d^2e-de^2+}\right.$$

input
```
integrate(x*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
[-1/4*((b*c*e*x^2 + b*c*d)*sqrt(-c^2*d + e)*log(((c^4*d^2 - 8*c^2*d*e + 8*
e^2)*x^4 - 2*(3*c^2*d^2 - 4*d*e)*x^2 - 4*((c^2*d - 2*e)*x^3 - d*x)*sqrt(-c
^2*d + e)*sqrt(e*x^2 + d) + d^2)/(c^4*x^4 + 2*c^2*x^2 + 1)) + 4*(a*c^2*d -
 a*e + (b*c^2*d - b*e)*arctan(c*x))*sqrt(e*x^2 + d))/(c^2*d^2*e - d*e^2 +
(c^2*d*e^2 - e^3)*x^2), 1/2*((b*c*e*x^2 + b*c*d)*sqrt(c^2*d - e)*arctan(1/
2*sqrt(c^2*d - e)*((c^2*d - 2*e)*x^2 - d)*sqrt(e*x^2 + d)/((c^2*d*e - e^2)
*x^3 + (c^2*d^2 - d*e)*x)) - 2*(a*c^2*d - a*e + (b*c^2*d - b*e)*arctan(c*x
))*sqrt(e*x^2 + d))/(c^2*d^2*e - d*e^2 + (c^2*d*e^2 - e^3)*x^2)]
```

## Sympy [F]

$$\int \frac{x(a + b\arctan(cx))}{(d + ex^2)^{3/2}}\,dx = \int \frac{x(a + b\operatorname{atan}(cx))}{(d + ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(a+b*atan(c*x))/(e*x**2+d)**(3/2),x)
```

output
```
Integral(x*(a + b*atan(c*x))/(d + e*x**2)**(3/2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x(a + b\arctan(cx))}{(d + ex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e-c^2*d>0)', see `assume?` for m
ore detail
```

## Giac [F]

$$\int \frac{x(a+b\arctan(cx))}{(d+ex^2)^{3/2}}\,dx=\int \frac{(b\arctan(cx)+a)x}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)*x/(e*x^2 + d)^(3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\arctan(cx))}{(d+ex^2)^{3/2}}\,dx=\int \frac{x\,(a+b\,\mathrm{atan}(c\,x))}{(e\,x^2+d)^{3/2}}\,dx$$

input
```
int((x*(a + b*atan(c*x)))/(d + e*x^2)^(3/2),x)
```

output
```
int((x*(a + b*atan(c*x)))/(d + e*x^2)^(3/2), x)
```

## Reduce [F]

$$\int \frac{x(a+b\arctan(cx))}{(d+ex^2)^{3/2}}\,dx=\frac{-\sqrt{e\,x^2+d}\,a+\left(\int \frac{atan(cx)x}{\sqrt{e\,x^2+d}\,d+\sqrt{e\,x^2+d}\,e\,x^2}dx\right)bde+\left(\int \frac{atan(cx)x}{\sqrt{e\,x^2+d}\,d+\sqrt{e\,x^2+d}\,e\,x^2}dx\right)b}{e\,(e\,x^2+d)}$$

input
```
int(x*(a+b*atan(c*x))/(e*x^2+d)^(3/2),x)
```

output
```
( - sqrt(d + e*x**2)*a + int((atan(c*x)*x)/(sqrt(d + e*x**2)*d + sqrt(d +
e*x**2)*e*x**2),x)*b*d*e + int((atan(c*x)*x)/(sqrt(d + e*x**2)*d + sqrt(d
+ e*x**2)*e*x**2),x)*b*e**2*x**2)/(e*(d + e*x**2))
```

# 3.1211 $\int \frac{a+b\arctan(cx)}{(d+ex^2)^{3/2}}\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 70

$$\int \frac{a+b\arctan(cx)}{(d+ex^2)^{3/2}}\,dx = \frac{x(a+b\arctan(cx))}{d\sqrt{d+ex^2}} + \frac{b\,\text{arctanh}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{d\sqrt{c^2d-e}}$$

output
```
x*(a+b*arctan(c*x))/d/(e*x^2+d)^(1/2)+b*arctanh(c*(e*x^2+d)^(1/2)/(c^2*d-e
)^(1/2))/d/(c^2*d-e)^(1/2)
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 202, normalized size of antiderivative = 2.89

$$\int \frac{a+b\arctan(cx)}{(d+ex^2)^{3/2}}\,dx = \frac{\frac{2ax}{\sqrt{d+ex^2}} + \frac{2bx\arctan(cx)}{\sqrt{d+ex^2}} + \frac{b\log\left(-\frac{4cd\left(cd-iex+\sqrt{c^2d-e}\sqrt{d+ex^2}\right)}{b\sqrt{c^2d-e}(i+cx)}\right)}{\sqrt{c^2d-e}} + \frac{b\log\left(-\frac{4cd\left(cd+iex+\sqrt{c^2d-e}\sqrt{d+e\cdots}\right)}{b\sqrt{c^2d-e}(-i+cx)}\right)}{\sqrt{c^2d-e}}}{2d}$$

input
```
Integrate[(a + b*ArcTan[c*x])/(d + e*x^2)^(3/2),x]
```

output
```
((2*a*x)/Sqrt[d + e*x^2] + (2*b*x*ArcTan[c*x])/Sqrt[d + e*x^2] + (b*Log[(-
4*c*d*(c*d - I*e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*Sqrt[c^2*d - e]*
(I + c*x))])/Sqrt[c^2*d - e] + (b*Log[(-4*c*d*(c*d + I*e*x + Sqrt[c^2*d -
e]*Sqrt[d + e*x^2]))/(b*Sqrt[c^2*d - e]*(-I + c*x))])/Sqrt[c^2*d - e])/(2*
d)
```

## Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5447, 27, 353, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{(d+ex^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{5447}$$

$$\frac{x(a+b\arctan(cx))}{d\sqrt{d+ex^2}}-bc\int \frac{x}{d\left(c^2x^2+1\right)\sqrt{ex^2+d}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{x(a+b\arctan(cx))}{d\sqrt{d+ex^2}}-\frac{bc\int \frac{x}{(c^2x^2+1)\sqrt{ex^2+d}}dx}{d}$$

$$\Big\downarrow \text{353}$$

$$\frac{x(a+b\arctan(cx))}{d\sqrt{d+ex^2}}-\frac{bc\int \frac{1}{(c^2x^2+1)\sqrt{ex^2+d}}dx^2}{2d}$$

$$\Big\downarrow \text{73}$$

$$\frac{x(a+b\arctan(cx))}{d\sqrt{d+ex^2}}-\frac{bc\int \frac{1}{\frac{c^2x^4}{e}-\frac{c^2d}{e}+1}d\sqrt{ex^2+d}}{de}$$

$$\Big\downarrow \text{221}$$

$$\frac{x(a+b\arctan(cx))}{d\sqrt{d+ex^2}}+\frac{b\text{arctanh}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{d\sqrt{c^2d-e}}$$

input
```
Int[(a + b*ArcTan[c*x])/(d + e*x^2)^(3/2),x]
```

output
```
(x*(a + b*ArcTan[c*x]))/(d*Sqrt[d + e*x^2]) + (b*ArcTanh[(c*Sqrt[d + e*x^2
])/Sqrt[c^2*d - e]])/(d*Sqrt[c^2*d - e])
```

**Defintions of rubi rules used**

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 5447
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Simp[(a + b*ArcTan[c*x])   u,
x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2), x], x], x]] /; FreeQ
[{a, b, c, d, e}, x] && (IntegerQ[q] || ILtQ[q + 1/2, 0])
```

## Maple [F]

$$\int \frac{a+b\arctan(cx)}{(e\,x^2+d)^{\frac{3}{2}}}dx$$

input
```
int((a+b*arctan(c*x))/(e*x^2+d)^(3/2),x)
```

output
```
int((a+b*arctan(c*x))/(e*x^2+d)^(3/2),x)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. 2(62) = 124.

Time = 0.16 (sec) , antiderivative size = 388, normalized size of antiderivative = 5.54

$$\int \frac{a+b\arctan(cx)}{(d+ex^2)^{3/2}}\,dx = \left[\frac{(bex^2+bd)\sqrt{c^2d-e}\log\left(\frac{c^4e^2x^4+8\,c^4d^2-8\,c^2de+2\,(4\,c^4de-3\,c^2e^2)x^2+4\,(c^3ex^2+2\,c^3d-ce)\sqrt{c^2d-}}{c^4x^4+2\,c^2x^2+1}\right.}{4\,(c^2d^3-d^2e+(c^2d^2e}\right.$$

input
```
integrate((a+b*arctan(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*((b*e*x^2 + b*d)*sqrt(c^2*d - e)*log((c^4*e^2*x^4 + 8*c^4*d^2 - 8*c^2
*d*e + 2*(4*c^4*d*e - 3*c^2*e^2)*x^2 + 4*(c^3*e*x^2 + 2*c^3*d - c*e)*sqrt(
c^2*d - e)*sqrt(e*x^2 + d) + e^2)/(c^4*x^4 + 2*c^2*x^2 + 1)) + 4*sqrt(e*x^
2 + d)*((b*c^2*d - b*e)*x*arctan(c*x) + (a*c^2*d - a*e)*x))/(c^2*d^3 - d^2
*e + (c^2*d^2*e - d*e^2)*x^2), 1/2*((b*e*x^2 + b*d)*sqrt(-c^2*d + e)*arcta
n(-1/2*(c^2*e*x^2 + 2*c^2*d - e)*sqrt(-c^2*d + e)*sqrt(e*x^2 + d)/(c^3*d^2
 - c*d*e + (c^3*d*e - c*e^2)*x^2)) + 2*sqrt(e*x^2 + d)*((b*c^2*d - b*e)*x*
arctan(c*x) + (a*c^2*d - a*e)*x))/(c^2*d^3 - d^2*e + (c^2*d^2*e - d*e^2)*x
^2)]
```

## Sympy [F]

$$\int \frac{a + b\arctan(cx)}{(d + ex^2)^{3/2}}\, dx = \int \frac{a + b\operatorname{atan}(cx)}{(d + ex^2)^{\frac{3}{2}}}\, dx$$

input
```
integrate((a+b*atan(c*x))/(e*x**2+d)**(3/2),x)
```

output
```
Integral((a + b*atan(c*x))/(d + e*x**2)**(3/2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\arctan(cx)}{(d + ex^2)^{3/2}}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arctan(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e-c^2*d>0)', see `assume?` for m
ore detail
```

## Giac [F]

$$\int \frac{a + b\arctan(cx)}{(d + ex^2)^{3/2}}\, dx = \int \frac{b\arctan(cx) + a}{(ex^2 + d)^{\frac{3}{2}}}\, dx$$

input
```
integrate((a+b*arctan(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)/(e*x^2 + d)^(3/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{(d+ex^2)^{3/2}}\,dx=\int \frac{a+b\,\mathrm{atan}(c\,x)}{(e\,x^2+d)^{3/2}}\,dx$$

input
```
int((a + b*atan(c*x))/(d + e*x^2)^(3/2),x)
```

output
```
int((a + b*atan(c*x))/(d + e*x^2)^(3/2), x)
```

## Reduce [F]

$$\int \frac{a+b\arctan(cx)}{(d+ex^2)^{3/2}}\,dx=\frac{\sqrt{e\,x^2+d}\,aex+\sqrt{e}\,ad+\sqrt{e}\,ae\,x^2+\left(\int \frac{atan(cx)}{\sqrt{e\,x^2+d}\,d+\sqrt{e\,x^2+d}\,e\,x^2}dx\right)b\,d^2e+\left(\int \frac{}{\sqrt{e\,x^2+}}\right.}{de\,(e\,x^2+d)}$$

input
```
int((a+b*atan(c*x))/(e*x^2+d)^(3/2),x)
```

output
```
(sqrt(d + e*x**2)*a*e*x + sqrt(e)*a*d + sqrt(e)*a*e*x**2 + int(atan(c*x)/(
sqrt(d + e*x**2)*d + sqrt(d + e*x**2)*e*x**2),x)*b*d**2*e + int(atan(c*x)/
(sqrt(d + e*x**2)*d + sqrt(d + e*x**2)*e*x**2),x)*b*d*e**2*x**2)/(d*e*(d +
 e*x**2))
```

# 3.1212 $\int \frac{a+b\arctan(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{a+b\arctan(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx = \frac{a}{d\sqrt{d+ex^2}} - \frac{a\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{3/2}} + b\text{Int}\left(\frac{\arctan(cx)}{x\left(d+ex^2\right)^{3/2}},x\right)$$

output
```
a/d/(e*x^2+d)^(1/2)-a*arctanh((e*x^2+d)^(1/2)/d^(1/2))/d^(3/2)+b*Defer(Int
)(arctan(c*x)/x/(e*x^2+d)^(3/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 8.96 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\arctan(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx = \int \frac{a+b\arctan(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx$$

input
```
Integrate[(a + b*ArcTan[c*x])/(x*(d + e*x^2)^(3/2)),x]
```

output
```
Integrate[(a + b*ArcTan[c*x])/(x*(d + e*x^2)^(3/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x(d+ex^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{5517}$$

$$a\int \frac{1}{x(ex^2+d)^{3/2}}dx+b\int \frac{\arctan(cx)}{x(ex^2+d)^{3/2}}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}a\int \frac{1}{x^2(ex^2+d)^{3/2}}dx^2+b\int \frac{\arctan(cx)}{x(ex^2+d)^{3/2}}dx$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{2}a\left(\frac{\int \frac{1}{x^2\sqrt{ex^2+d}}dx^2}{d}+\frac{2}{d\sqrt{d+ex^2}}\right)+b\int \frac{\arctan(cx)}{x(ex^2+d)^{3/2}}dx$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}a\left(\frac{2\int \frac{1}{\frac{x^4}{e}-\frac{d}{e}}d\sqrt{ex^2+d}}{de}+\frac{2}{d\sqrt{d+ex^2}}\right)+b\int \frac{\arctan(cx)}{x(ex^2+d)^{3/2}}dx$$

$$\Big\downarrow \text{221}$$

$$b\int \frac{\arctan(cx)}{x(ex^2+d)^{3/2}}dx+\frac{1}{2}a\left(\frac{2}{d\sqrt{d+ex^2}}-\frac{2\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{3/2}}\right)$$

$$\Big\downarrow \text{5560}$$

$$b\int \frac{\arctan(cx)}{x(ex^2+d)^{3/2}}dx+\frac{1}{2}a\left(\frac{2}{d\sqrt{d+ex^2}}-\frac{2\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{3/2}}\right)$$

input
```
Int[(a + b*ArcTan[c*x])/(x*(d + e*x^2)^(3/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{a+b \arctan (c x)}{x\left(e\,x^{2}+d\right)^{\frac{3}{2}}}dx$$

input
```
int((a+b*arctan(c*x))/x/(e*x^2+d)^(3/2),x)
```

output
```
int((a+b*arctan(c*x))/x/(e*x^2+d)^(3/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.83

$$\int \frac{a+b\arctan(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{b\arctan (cx)+a}{\left(ex^2+d\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arctan(c*x) + a)/(e^2*x^5 + 2*d*e*x^3 + d^2*x)
, x)
```

## Sympy [N/A]

Not integrable

Time = 37.68 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{a+b\arctan(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{a+b\,\mathrm{atan}\,(cx)}{x\left(d+ex^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*atan(c*x))/x/(e*x**2+d)**(3/2),x)
```

output
```
Integral((a + b*atan(c*x))/(x*(d + e*x**2)**(3/2)), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\arctan(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*arctan(c*x))/x/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

## Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+b\arctan(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{b\arctan\left(cx\right)+a}{\left(ex^2+d\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)/((e*x^2 + d)^(3/2)*x), x)
```

## Mupad [N/A]

Not integrable

Time = 1.42 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+b\arctan(cx)}{x\,(d+ex^2)^{3/2}}\,dx = \int \frac{a+b\,\mathrm{atan}(c\,x)}{x\,(e\,x^2+d)^{3/2}}\,dx$$

input
```
int((a + b*atan(c*x))/(x*(d + e*x^2)^(3/2)),x)
```

output
```
int((a + b*atan(c*x))/(x*(d + e*x^2)^(3/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 223, normalized size of antiderivative = 9.70

$$\int \frac{a+b\arctan(cx)}{x\,(d+ex^2)^{3/2}}\,dx = \frac{\sqrt{e\,x^2+d}\,ad+\sqrt{d}\log\left(\frac{\sqrt{e\,x^2+d}-\sqrt{d}+\sqrt{e}\,x}{\sqrt{d}}\right)ad+\sqrt{d}\log\left(\frac{\sqrt{e\,x^2+d}-\sqrt{d}+\sqrt{e}\,x}{\sqrt{d}}\right)ae\,x^2-\cdots}{\cdots}$$

input
```
int((a+b*atan(c*x))/x/(e*x^2+d)^(3/2),x)
```

output
```
(sqrt(d + e*x**2)*a*d + sqrt(d)*log((sqrt(d + e*x**2) - sqrt(d) + sqrt(e)*
x)/sqrt(d))*a*d + sqrt(d)*log((sqrt(d + e*x**2) - sqrt(d) + sqrt(e)*x)/sqr
t(d))*a*e*x**2 - sqrt(d)*log((sqrt(d + e*x**2) + sqrt(d) + sqrt(e)*x)/sqrt
(d))*a*d - sqrt(d)*log((sqrt(d + e*x**2) + sqrt(d) + sqrt(e)*x)/sqrt(d))*a
*e*x**2 + int(atan(c*x)/(sqrt(d + e*x**2)*d*x + sqrt(d + e*x**2)*e*x**3),x
)*b*d**3 + int(atan(c*x)/(sqrt(d + e*x**2)*d*x + sqrt(d + e*x**2)*e*x**3),
x)*b*d**2*e*x**2)/(d**2*(d + e*x**2))
```

# 3.1213 $\int \frac{a+b\arctan(cx)}{x^2(d+ex^2)^{3/2}}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 135

$$\int \frac{a+b\arctan(cx)}{x^2(d+ex^2)^{3/2}}\,dx = \frac{a+b\arctan(cx)}{dx\sqrt{d+ex^2}} - \frac{2\sqrt{d+ex^2}(a+b\arctan(cx))}{d^2x} - \frac{bc\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{b(c^2d-2e)\operatorname{arctanh}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{d^2\sqrt{c^2d-e}}$$

output
```
(a+b*arctan(c*x))/d/x/(e*x^2+d)^(1/2)-2*(e*x^2+d)^(1/2)*(a+b*arctan(c*x))/
d^2/x-b*c*arctanh((e*x^2+d)^(1/2)/d^(1/2))/d^(3/2)+b*(c^2*d-2*e)*arctanh(c
*(e*x^2+d)^(1/2)/(c^2*d-e)^(1/2))/d^2/(c^2*d-e)^(1/2)
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 306, normalized size of antiderivative = 2.27

$$\int \frac{a+b\arctan(cx)}{x^2(d+ex^2)^{3/2}}\,dx = \frac{-\frac{2a(d+2ex^2)}{x\sqrt{d+ex^2}} - \frac{2b(d+2ex^2)\arctan(cx)}{x\sqrt{d+ex^2}} + 2bc\sqrt{d}\log(x) - 2bc\sqrt{d}\log\left(d+\sqrt{d}\sqrt{d+ex^2}\right)}{2d}$$

input
```
Integrate[(a + b*ArcTan[c*x])/(x^2*(d + e*x^2)^(3/2)),x]
```

output
```
((-2*a*(d + 2*e*x^2))/(x*Sqrt[d + e*x^2]) - (2*b*(d + 2*e*x^2)*ArcTan[c*x]
)/(x*Sqrt[d + e*x^2]) + 2*b*c*Sqrt[d]*Log[x] - 2*b*c*Sqrt[d]*Log[d + Sqrt[
d]*Sqrt[d + e*x^2]] + (b*(c^2*d - 2*e)*Log[(-4*c*d^2*(c*d - I*e*x + Sqrt[c
^2*d - e]*Sqrt[d + e*x^2]))/(b*(c^2*d - 2*e)*Sqrt[c^2*d - e]*(I + c*x))])/
Sqrt[c^2*d - e] + (b*(c^2*d - 2*e)*Log[(-4*c*d^2*(c*d + I*e*x + Sqrt[c^2*d
 - e]*Sqrt[d + e*x^2]))/(b*(c^2*d - 2*e)*Sqrt[c^2*d - e]*(-I + c*x))])/Sqr
t[c^2*d - e])/(2*d^2)
```

## Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {5511, 25, 27, 435, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x^2(d+ex^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{5511}$$

$$-bc\int -\frac{2ex^2+d}{d^2x(c^2x^2+1)\sqrt{ex^2+d}}dx-\frac{2ex(a+b\arctan(cx))}{d^2\sqrt{d+ex^2}}-\frac{a+b\arctan(cx)}{dx\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{25}$$

$$bc\int \frac{2ex^2+d}{d^2x(c^2x^2+1)\sqrt{ex^2+d}}dx-\frac{2ex(a+b\arctan(cx))}{d^2\sqrt{d+ex^2}}-\frac{a+b\arctan(cx)}{dx\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{bc\int \frac{2ex^2+d}{x(c^2x^2+1)\sqrt{ex^2+d}}dx}{d^2}-\frac{2ex(a+b\arctan(cx))}{d^2\sqrt{d+ex^2}}-\frac{a+b\arctan(cx)}{dx\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{435}$$

$$\frac{bc\int \frac{2ex^2+d}{x^2(c^2x^2+1)\sqrt{ex^2+d}}dx^2}{2d^2}-\frac{2ex(a+b\arctan(cx))}{d^2\sqrt{d+ex^2}}-\frac{a+b\arctan(cx)}{dx\sqrt{d+ex^2}}$$

$$\Big\downarrow 174$$

$$\frac{bc\left(d\int \frac{1}{x^2\sqrt{ex^2+d}}dx^2-(c^2d-2e)\int \frac{1}{(c^2x^2+1)\sqrt{ex^2+d}}dx^2\right)}{2d^2}-\frac{2ex(a+b\arctan(cx))}{d^2\sqrt{d+ex^2}}-\frac{a+b\arctan(cx)}{dx\sqrt{d+ex^2}}$$

$$\Big\downarrow 73$$

$$\frac{bc\left(\frac{2d\int \frac{1}{\frac{x^4}{e}-\frac{d}{e}}d\sqrt{ex^2+d}}{e}-\frac{2(c^2d-2e)\int \frac{1}{\frac{c^2x^4}{e}-\frac{c^2d}{e}+1}d\sqrt{ex^2+d}}{e}\right)}{2d^2}-\frac{2ex(a+b\arctan(cx))}{d^2\sqrt{d+ex^2}}-\frac{a+b\arctan(cx)}{dx\sqrt{d+ex^2}}$$

$$\Big\downarrow 221$$

$$-\frac{2ex(a+b\arctan(cx))}{d^2\sqrt{d+ex^2}}-\frac{a+b\arctan(cx)}{dx\sqrt{d+ex^2}}+\frac{bc\left(\frac{2(c^2d-2e)\text{arctanh}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{c\sqrt{c^2d-e}}-2\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)\right)}{2d^2}$$

input
```
Int[(a + b*ArcTan[c*x])/(x^2*(d + e*x^2)^(3/2)),x]
```

output
```
-((a + b*ArcTan[c*x])/(d*x*Sqrt[d + e*x^2])) - (2*e*x*(a + b*ArcTan[c*x]))
/(d^2*Sqrt[d + e*x^2]) + (b*c*(-2*Sqrt[d]*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]]
 + (2*(c^2*d - 2*e)*ArcTanh[(c*Sqrt[d + e*x^2])/Sqrt[c^2*d - e]])/(c*Sqrt[
c^2*d - e])))/(2*d^2)
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

## Maple [F]

$$\int \frac{a+b\arctan\left(cx\right)}{x^{2}\left(e\,x^{2}+d\right)^{\frac{3}{2}}}dx$$

input
```
int((a+b*arctan(c*x))/x^2/(e*x^2+d)^(3/2),x)
```

output
```
int((a+b*arctan(c*x))/x^2/(e*x^2+d)^(3/2),x)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 306 vs. 2(119) = 238.

Time = 0.28 (sec) , antiderivative size = 1323, normalized size of antiderivative = 9.80

$$\int \frac{a + b\arctan(cx)}{x^2 (d + ex^2)^{3/2}} \, dx = \text{Too large to display}$$

input
```
integrate((a+b*arctan(c*x))/x^2/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
[-1/4*(((b*c^2*d*e - 2*b*e^2)*x^3 + (b*c^2*d^2 - 2*b*d*e)*x)*sqrt(c^2*d -
e)*log((c^4*e^2*x^4 + 8*c^4*d^2 - 8*c^2*d*e + 2*(4*c^4*d*e - 3*c^2*e^2)*x^
2 - 4*(c^3*e*x^2 + 2*c^3*d - c*e)*sqrt(c^2*d - e)*sqrt(e*x^2 + d) + e^2)/(
c^4*x^4 + 2*c^2*x^2 + 1)) - 2*((b*c^3*d*e - b*c*e^2)*x^3 + (b*c^3*d^2 - b*
c*d*e)*x)*sqrt(d)*log(-(e*x^2 - 2*sqrt(e*x^2 + d)*sqrt(d) + 2*d)/x^2) + 4*
(a*c^2*d^2 - a*d*e + 2*(a*c^2*d*e - a*e^2)*x^2 + (b*c^2*d^2 - b*d*e + 2*(b
*c^2*d*e - b*e^2)*x^2)*arctan(c*x))*sqrt(e*x^2 + d))/((c^2*d^3*e - d^2*e^2
)*x^3 + (c^2*d^4 - d^3*e)*x), 1/2*(((b*c^2*d*e - 2*b*e^2)*x^3 + (b*c^2*d^2
 - 2*b*d*e)*x)*sqrt(-c^2*d + e)*arctan(-1/2*(c^2*e*x^2 + 2*c^2*d - e)*sqrt
(-c^2*d + e)*sqrt(e*x^2 + d)/(c^3*d^2 - c*d*e + (c^3*d*e - c*e^2)*x^2)) +
((b*c^3*d*e - b*c*e^2)*x^3 + (b*c^3*d^2 - b*c*d*e)*x)*sqrt(d)*log(-(e*x^2
- 2*sqrt(e*x^2 + d)*sqrt(d) + 2*d)/x^2) - 2*(a*c^2*d^2 - a*d*e + 2*(a*c^2*
d*e - a*e^2)*x^2 + (b*c^2*d^2 - b*d*e + 2*(b*c^2*d*e - b*e^2)*x^2)*arctan(
c*x))*sqrt(e*x^2 + d))/((c^2*d^3*e - d^2*e^2)*x^3 + (c^2*d^4 - d^3*e)*x),
1/4*(4*((b*c^3*d*e - b*c*e^2)*x^3 + (b*c^3*d^2 - b*c*d*e)*x)*sqrt(-d)*arct
an(sqrt(e*x^2 + d)*sqrt(-d)/d) - ((b*c^2*d*e - 2*b*e^2)*x^3 + (b*c^2*d^2 -
 2*b*d*e)*x)*sqrt(c^2*d - e)*log((c^4*e^2*x^4 + 8*c^4*d^2 - 8*c^2*d*e + 2*
(4*c^4*d*e - 3*c^2*e^2)*x^2 - 4*(c^3*e*x^2 + 2*c^3*d - c*e)*sqrt(c^2*d - e
)*sqrt(e*x^2 + d) + e^2)/(c^4*x^4 + 2*c^2*x^2 + 1)) - 4*(a*c^2*d^2 - a*d*e
 + 2*(a*c^2*d*e - a*e^2)*x^2 + (b*c^2*d^2 - b*d*e + 2*(b*c^2*d*e - b*e^...
```

## Sympy [F]

$$\int \frac{a+b\arctan(cx)}{x^2\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{a+b\,\mathrm{atan}\,(cx)}{x^2\left(d+ex^2\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*atan(c*x))/x**2/(e*x**2+d)**(3/2),x)
```

output
```
Integral((a + b*atan(c*x))/(x**2*(d + e*x**2)**(3/2)), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\arctan(cx)}{x^2\left(d+ex^2\right)^{3/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*arctan(c*x))/x^2/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

## Giac [F]

$$\int \frac{a+b\arctan(cx)}{x^2\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{b\arctan(cx)+a}{\left(ex^2+d\right)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^2/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)/((e*x^2 + d)^(3/2)*x^2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{x^2\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{a+b\,\mathrm{atan}(c\,x)}{x^2\,\left(e\,x^2+d\right)^{3/2}}\,dx$$

input
```
int((a + b*atan(c*x))/(x^2*(d + e*x^2)^(3/2)),x)
```

output
```
int((a + b*atan(c*x))/(x^2*(d + e*x^2)^(3/2)), x)
```

## Reduce [F]

$$\int \frac{a+b\arctan(cx)}{x^2\left(d+ex^2\right)^{3/2}}\,dx=\frac{-\sqrt{e\,x^2+d}\,ad-2\sqrt{e\,x^2+d}\,ae\,x^2-2\sqrt{e}\,adx-2\sqrt{e}\,ae\,x^3+\left(\int \frac{atan(cx)}{\sqrt{e\,x^2+d}\,d\,x^2+\sqrt{e\,x^2}}\right.}{d^2x\left(e\,x^2+d\right)}$$

input
```
int((a+b*atan(c*x))/x^2/(e*x^2+d)^(3/2),x)
```

output
```
( - sqrt(d + e*x**2)*a*d - 2*sqrt(d + e*x**2)*a*e*x**2 - 2*sqrt(e)*a*d*x -
 2*sqrt(e)*a*e*x**3 + int(atan(c*x)/(sqrt(d + e*x**2)*d*x**2 + sqrt(d + e*
x**2)*e*x**4),x)*b*d**3*x + int(atan(c*x)/(sqrt(d + e*x**2)*d*x**2 + sqrt(
d + e*x**2)*e*x**4),x)*b*d**2*e*x**3)/(d**2*x*(d + e*x**2))
```

# 3.1214 $\int \frac{a+b\arctan(cx)}{x^3(d+ex^2)^{3/2}}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{a+b\arctan(cx)}{x^3(d+ex^2)^{3/2}}\,dx = -\frac{3ae}{2d^2\sqrt{d+ex^2}}-\frac{a}{2dx^2\sqrt{d+ex^2}} + \frac{3ae\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{2d^{5/2}}+b\text{Int}\left(\frac{\arctan(cx)}{x^3(d+ex^2)^{3/2}},x\right)$$

output
```
-3/2*a*e/d^2/(e*x^2+d)^(1/2)-1/2*a/d/x^2/(e*x^2+d)^(1/2)+3/2*a*e*arctanh((
e*x^2+d)^(1/2)/d^(1/2))/d^(5/2)+b*Defer(Int)(arctan(c*x)/x^3/(e*x^2+d)^(3/
2),x)
```

## Mathematica [N/A]

Not integrable

Time = 12.74 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\arctan(cx)}{x^3(d+ex^2)^{3/2}}\,dx = \int \frac{a+b\arctan(cx)}{x^3(d+ex^2)^{3/2}}\,dx$$

input
```
Integrate[(a + b*ArcTan[c*x])/(x^3*(d + e*x^2)^(3/2)),x]
```

output
```
Integrate[(a + b*ArcTan[c*x])/(x^3*(d + e*x^2)^(3/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{5517}$$

$$a\int \frac{1}{x^3\left(ex^2+d\right)^{3/2}}dx+b\int \frac{\arctan(cx)}{x^3\left(ex^2+d\right)^{3/2}}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}a\int \frac{1}{x^4\left(ex^2+d\right)^{3/2}}dx^2+b\int \frac{\arctan(cx)}{x^3\left(ex^2+d\right)^{3/2}}dx$$

$$\Big\downarrow \text{52}$$

$$\frac{1}{2}a\left(-\frac{3e\int \frac{1}{x^2\left(ex^2+d\right)^{3/2}}dx^2}{2d}-\frac{1}{dx^2\sqrt{d+ex^2}}\right)+b\int \frac{\arctan(cx)}{x^3\left(ex^2+d\right)^{3/2}}dx$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{2}a\left(-\frac{3e\left(\frac{\int \frac{1}{x^2\sqrt{ex^2+d}}dx^2}{d}+\frac{2}{d\sqrt{d+ex^2}}\right)}{2d}-\frac{1}{dx^2\sqrt{d+ex^2}}\right)+b\int \frac{\arctan(cx)}{x^3\left(ex^2+d\right)^{3/2}}dx$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}a\left(-\frac{3e\left(\frac{2\int\frac{1}{\frac{x^4}{e}-\frac{d}{e}}d\sqrt{ex^2+d}}{de}+\frac{2}{d\sqrt{d+ex^2}}\right)}{2d}-\frac{1}{dx^2\sqrt{d+ex^2}}\right)+b\int\frac{\arctan(cx)}{x^3\left(ex^2+d\right)^{3/2}}dx$$

$$\Big\downarrow \text{221}$$

$$b\int\frac{\arctan(cx)}{x^3\left(ex^2+d\right)^{3/2}}dx+\frac{1}{2}a\left(-\frac{3e\left(\frac{2}{d\sqrt{d+ex^2}}-\frac{2\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{3/2}}\right)}{2d}-\frac{1}{dx^2\sqrt{d+ex^2}}\right)$$

$$\Big\downarrow \text{5560}$$

$$b\int\frac{\arctan(cx)}{x^3\left(ex^2+d\right)^{3/2}}dx+\frac{1}{2}a\left(-\frac{3e\left(\frac{2}{d\sqrt{d+ex^2}}-\frac{2\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{3/2}}\right)}{2d}-\frac{1}{dx^2\sqrt{d+ex^2}}\right)$$

input
```
Int[(a + b*ArcTan[c*x])/(x^3*(d + e*x^2)^(3/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int\frac{a+b\arctan\left(cx\right)}{x^3\left(e\,x^2+d\right)^{\frac{3}{2}}}dx$$

input
```
int((a+b*arctan(c*x))/x^3/(e*x^2+d)^(3/2),x)
```

output
```
int((a+b*arctan(c*x))/x^3/(e*x^2+d)^(3/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.91

$$\int \frac{a+b\arctan(cx)}{x^3(d+ex^2)^{3/2}}\,dx = \int \frac{b\arctan(cx)+a}{(ex^2+d)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^3/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arctan(c*x) + a)/(e^2*x^7 + 2*d*e*x^5 + d^2*x^
3), x)
```

## Sympy [N/A]

Not integrable

Time = 67.79 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{a+b\arctan(cx)}{x^3(d+ex^2)^{3/2}}\,dx = \int \frac{a+b\operatorname{atan}(cx)}{x^3(d+ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*atan(c*x))/x**3/(e*x**2+d)**(3/2),x)
```

output
```
Integral((a + b*atan(c*x))/(x**3*(d + e*x**2)**(3/2)), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\arctan(cx)}{x^3(d+ex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arctan(c*x))/x^3/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

## Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+b\arctan(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{b\arctan\left(cx\right)+a}{\left(ex^2+d\right)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^3/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)/((e*x^2 + d)^(3/2)*x^3), x)
```

## Mupad [N/A]

Not integrable

Time = 1.54 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+b\arctan(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{a+b\,\mathrm{atan}(c\,x)}{x^3\,{\left(e\,x^2+d\right)}^{3/2}}\,dx$$

input
```
int((a + b*atan(c*x))/(x^3*(d + e*x^2)^(3/2)),x)
```

output
```
int((a + b*atan(c*x))/(x^3*(d + e*x^2)^(3/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 269, normalized size of antiderivative = 11.70

$$\int \frac{a+b\arctan(cx)}{x^3\,(d+ex^2)^{3/2}}\,dx = \frac{-\sqrt{e\,x^2+d}\,a\,d^2-3\sqrt{e\,x^2+d}\,ade\,x^2-3\sqrt{d}\,\log\left(\frac{\sqrt{e\,x^2+d}-\sqrt{d}+\sqrt{e}\,x}{\sqrt{d}}\right)ade\,x^2-3\sqrt{d}\,\mathrm{l}}{}$$

input
```
int((a+b*atan(c*x))/x^3/(e*x^2+d)^(3/2),x)
```

output
```
( - sqrt(d + e*x**2)*a*d**2 - 3*sqrt(d + e*x**2)*a*d*e*x**2 - 3*sqrt(d)*lo
g((sqrt(d + e*x**2) - sqrt(d) + sqrt(e)*x)/sqrt(d))*a*d*e*x**2 - 3*sqrt(d)
*log((sqrt(d + e*x**2) - sqrt(d) + sqrt(e)*x)/sqrt(d))*a*e**2*x**4 + 3*sqr
t(d)*log((sqrt(d + e*x**2) + sqrt(d) + sqrt(e)*x)/sqrt(d))*a*d*e*x**2 + 3*
sqrt(d)*log((sqrt(d + e*x**2) + sqrt(d) + sqrt(e)*x)/sqrt(d))*a*e**2*x**4
+ 2*int(atan(c*x)/(sqrt(d + e*x**2)*d*x**3 + sqrt(d + e*x**2)*e*x**5),x)*b
*d**4*x**2 + 2*int(atan(c*x)/(sqrt(d + e*x**2)*d*x**3 + sqrt(d + e*x**2)*e
*x**5),x)*b*d**3*e*x**4)/(2*d**3*x**2*(d + e*x**2))
```

# 3.1215 $\int \frac{a+b\arctan(cx)}{x^4(d+ex^2)^{3/2}}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 216

$$\int \frac{a+b\arctan(cx)}{x^4\left(d+ex^2\right)^{3/2}}\,dx = -\frac{bc\sqrt{d+ex^2}}{6d^2x^2}+\frac{a+b\arctan(cx)}{dx^3\sqrt{d+ex^2}} - \frac{4\sqrt{d+ex^2}(a+b\arctan(cx))}{3d^2x^3}+\frac{8e\sqrt{d+ex^2}(a+b\arctan(cx))}{3d^3x} + \frac{bc(2c^2d+9e)\,\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{6d^{5/2}}-\frac{b(c^4d^2+4c^2de-8e^2)\,\text{arctanh}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{3d^3\sqrt{c^2d-e}}$$

output

```
-1/6*b*c*(e*x^2+d)^(1/2)/d^2/x^2+(a+b*arctan(c*x))/d/x^3/(e*x^2+d)^(1/2)-4
/3*(e*x^2+d)^(1/2)*(a+b*arctan(c*x))/d^2/x^3+8/3*e*(e*x^2+d)^(1/2)*(a+b*ar
ctan(c*x))/d^3/x+1/6*b*c*(2*c^2*d+9*e)*arctanh((e*x^2+d)^(1/2)/d^(1/2))/d^
(5/2)-1/3*b*(c^4*d^2+4*c^2*d*e-8*e^2)*arctanh(c*(e*x^2+d)^(1/2)/(c^2*d-e)^
(1/2))/d^3/(c^2*d-e)^(1/2)
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.88

$$\int \frac{a+b\arctan(cx)}{x^4\left(d+ex^2\right)^{3/2}}\,dx =$$

$$-\frac{\frac{bcdx(d+ex^2)+2a(d^2-4dex^2-8e^2x^4)}{x^3\sqrt{d+ex^2}}+\frac{2b(d^2-4dex^2-8e^2x^4)\arctan(cx)}{x^3\sqrt{d+ex^2}}+bc\sqrt{d}(2c^2d+9e)\log(x)-bc\sqrt{d}(2c^2d+9e)\log\cdots}{\cdots}$$

input
```
Integrate[(a + b*ArcTan[c*x])/(x^4*(d + e*x^2)^(3/2)),x]
```

output
```
-1/6*((b*c*d*x*(d + e*x^2) + 2*a*(d^2 - 4*d*e*x^2 - 8*e^2*x^4))/(x^3*Sqrt[
d + e*x^2]) + (2*b*(d^2 - 4*d*e*x^2 - 8*e^2*x^4)*ArcTan[c*x])/(x^3*Sqrt[d
+ e*x^2]) + b*c*Sqrt[d]*(2*c^2*d + 9*e)*Log[x] - b*c*Sqrt[d]*(2*c^2*d + 9*
e)*Log[d + Sqrt[d]*Sqrt[d + e*x^2]] + (b*(c^4*d^2 + 4*c^2*d*e - 8*e^2)*Log
[(12*c*d^3*(c*d - I*e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*Sqrt[c^2*d
- e]*(c^4*d^2 + 4*c^2*d*e - 8*e^2)*(I + c*x))]])/Sqrt[c^2*d - e] + (b*(c^4*
d^2 + 4*c^2*d*e - 8*e^2)*Log[(12*c*d^3*(c*d + I*e*x + Sqrt[c^2*d - e]*Sqrt
[d + e*x^2]))/(b*Sqrt[c^2*d - e]*(c^4*d^2 + 4*c^2*d*e - 8*e^2)*(-I + c*x))
])/Sqrt[c^2*d - e])/d^3
```

## Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {5511, 27, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x^4\left(d+ex^2\right)^{3/2}}\,dx$$

$$\Big\downarrow \; 5511$$

$$-bc\int -\frac{-8e^2x^4-4dex^2+d^2}{3d^3x^3\left(c^2x^2+1\right)\sqrt{ex^2+d}}dx+\frac{8e^2x(a+b\arctan(cx))}{3d^3\sqrt{d+ex^2}}+\frac{4e(a+b\arctan(cx))}{3d^2x\sqrt{d+ex^2}}-\frac{a+b\arctan(cx)}{3dx^3\sqrt{d+ex^2}}$$

$\Big\downarrow$ 27

$$\frac{bc\int \frac{-8e^2x^4-4dex^2+d^2}{x^3(c^2x^2+1)\sqrt{ex^2+d}}dx}{3d^3}+\frac{8e^2x(a+b\arctan(cx))}{3d^3\sqrt{d+ex^2}}+\frac{4e(a+b\arctan(cx))}{3d^2x\sqrt{d+ex^2}}-\frac{a+b\arctan(cx)}{3dx^3\sqrt{d+ex^2}}$$

$\Big\downarrow$ 7276

$$\frac{bc\int \left(\frac{d^2}{x^3\sqrt{ex^2+d}}-\frac{(dc^2+4e)d}{x\sqrt{ex^2+d}}+\frac{(d^2c^4+4dec^2-8e^2)x}{(c^2x^2+1)\sqrt{ex^2+d}}\right)dx}{3d^3}+\frac{8e^2x(a+b\arctan(cx))}{3d^3\sqrt{d+ex^2}}+\frac{4e(a+b\arctan(cx))}{3d^2x\sqrt{d+ex^2}}-\frac{a+b\arctan(cx)}{3dx^3\sqrt{d+ex^2}}$$

$\Big\downarrow$ 2009

$$\frac{8e^2x(a+b\arctan(cx))}{3d^3\sqrt{d+ex^2}}+\frac{4e(a+b\arctan(cx))}{3d^2x\sqrt{d+ex^2}}-\frac{a+b\arctan(cx)}{3dx^3\sqrt{d+ex^2}}+\frac{bc\left(\sqrt{d}(c^2d+4e)\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)-\frac{(c^4d^2+4c^2de-8e^2)\operatorname{arctanh}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{c\sqrt{c^2d-e}}+\frac{1}{2}\sqrt{d}e\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)-\frac{d\sqrt{d+ex^2}}{2x^2}\right)}{3d^3}$$

input
```
Int[(a + b*ArcTan[c*x])/(x^4*(d + e*x^2)^(3/2)),x]
```

output
```
-1/3*(a + b*ArcTan[c*x])/(d*x^3*Sqrt[d + e*x^2]) + (4*e*(a + b*ArcTan[c*x]
))/(3*d^2*x*Sqrt[d + e*x^2]) + (8*e^2*x*(a + b*ArcTan[c*x]))/(3*d^3*Sqrt[d
 + e*x^2]) + (b*c*(-1/2*(d*Sqrt[d + e*x^2])/x^2 + (Sqrt[d]*e*ArcTanh[Sqrt[
d + e*x^2]/Sqrt[d]])/2 + Sqrt[d]*(c^2*d + 4*e)*ArcTanh[Sqrt[d + e*x^2]/Sqr
t[d]] - ((c^4*d^2 + 4*c^2*d*e - 8*e^2)*ArcTanh[(c*Sqrt[d + e*x^2])/Sqrt[c^
2*d - e]])/(c*Sqrt[c^2*d - e])))/(3*d^3)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

## Maple [F]

$$\int \frac{a+b\arctan\left(cx\right)}{x^{4}\left(e\,x^{2}+d\right)^{\frac{3}{2}}}dx$$

input
```
int((a+b*arctan(c*x))/x^4/(e*x^2+d)^(3/2),x)
```

output
```
int((a+b*arctan(c*x))/x^4/(e*x^2+d)^(3/2),x)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 455 vs. $2(186) = 372$.

Time = 0.45 (sec) , antiderivative size = 1926, normalized size of antiderivative = 8.92

$$\int \frac{a + b \arctan(cx)}{x^4 (d + ex^2)^{3/2}} \, dx = \text{Too large to display}$$

input
```
integrate((a+b*arctan(c*x))/x^4/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
[-1/12*(((b*c^4*d^2*e + 4*b*c^2*d*e^2 - 8*b*e^3)*x^5 + (b*c^4*d^3 + 4*b*c^
2*d^2*e - 8*b*d*e^2)*x^3)*sqrt(c^2*d - e)*log((c^4*e^2*x^4 + 8*c^4*d^2 - 8
*c^2*d*e + 2*(4*c^4*d*e - 3*c^2*e^2)*x^2 + 4*(c^3*e*x^2 + 2*c^3*d - c*e)*s
qrt(c^2*d - e)*sqrt(e*x^2 + d) + e^2)/(c^4*x^4 + 2*c^2*x^2 + 1)) - ((2*b*c
^5*d^2*e + 7*b*c^3*d*e^2 - 9*b*c*e^3)*x^5 + (2*b*c^5*d^3 + 7*b*c^3*d^2*e -
 9*b*c*d*e^2)*x^3)*sqrt(d)*log(-(e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(d) + 2*d)/
x^2) + 2*(2*a*c^2*d^3 - 16*(a*c^2*d*e^2 - a*e^3)*x^4 - 2*a*d^2*e + (b*c^3*
d^2*e - b*c*d*e^2)*x^3 - 8*(a*c^2*d^2*e - a*d*e^2)*x^2 + (b*c^3*d^3 - b*c*
d^2*e)*x + 2*(b*c^2*d^3 - 8*(b*c^2*d*e^2 - b*e^3)*x^4 - b*d^2*e - 4*(b*c^2
*d^2*e - b*d*e^2)*x^2)*arctan(c*x))*sqrt(e*x^2 + d))/((c^2*d^4*e - d^3*e^2
)*x^5 + (c^2*d^5 - d^4*e)*x^3), -1/12*(2*((b*c^4*d^2*e + 4*b*c^2*d*e^2 - 8
*b*e^3)*x^5 + (b*c^4*d^3 + 4*b*c^2*d^2*e - 8*b*d*e^2)*x^3)*sqrt(-c^2*d + e
)*arctan(-1/2*(c^2*e*x^2 + 2*c^2*d - e)*sqrt(-c^2*d + e)*sqrt(e*x^2 + d)/(
c^3*d^2 - c*d*e + (c^3*d*e - c*e^2)*x^2)) - ((2*b*c^5*d^2*e + 7*b*c^3*d*e^
2 - 9*b*c*e^3)*x^5 + (2*b*c^5*d^3 + 7*b*c^3*d^2*e - 9*b*c*d*e^2)*x^3)*sqrt
(d)*log(-(e*x^2 + 2*sqrt(e*x^2 + d)*sqrt(d) + 2*d)/x^2) + 2*(2*a*c^2*d^3 -
 16*(a*c^2*d*e^2 - a*e^3)*x^4 - 2*a*d^2*e + (b*c^3*d^2*e - b*c*d*e^2)*x^3
- 8*(a*c^2*d^2*e - a*d*e^2)*x^2 + (b*c^3*d^3 - b*c*d^2*e)*x + 2*(b*c^2*d^3
 - 8*(b*c^2*d*e^2 - b*e^3)*x^4 - b*d^2*e - 4*(b*c^2*d^2*e - b*d*e^2)*x^2)*
arctan(c*x))*sqrt(e*x^2 + d))/((c^2*d^4*e - d^3*e^2)*x^5 + (c^2*d^5 - d...
```

## Sympy [F]

$$\int \frac{a+b\arctan(cx)}{x^4(d+ex^2)^{3/2}}\,dx = \int \frac{a+b\,\mathrm{atan}\,(cx)}{x^4(d+ex^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*atan(c*x))/x**4/(e*x**2+d)**(3/2),x)
```

output
```
Integral((a + b*atan(c*x))/(x**4*(d + e*x**2)**(3/2)), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\arctan(cx)}{x^4(d+ex^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arctan(c*x))/x^4/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

## Giac [F]

$$\int \frac{a+b\arctan(cx)}{x^4(d+ex^2)^{3/2}}\,dx = \int \frac{b\arctan(cx)+a}{(ex^2+d)^{\frac{3}{2}}x^4}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^4/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)/((e*x^2 + d)^(3/2)*x^4), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{x^4\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{a+b\,\mathrm{atan}(c\,x)}{x^4\,{\left(e\,x^2+d\right)}^{3/2}}\,dx$$

input
```
int((a + b*atan(c*x))/(x^4*(d + e*x^2)^(3/2)),x)
```

output
```
int((a + b*atan(c*x))/(x^4*(d + e*x^2)^(3/2)), x)
```

## Reduce [F]

$$\int \frac{a+b\arctan(cx)}{x^4\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{atan(cx)\,b+a}{x^4\,(e\,x^2+d)^{\frac{3}{2}}}dx$$

input
```
int((a+b*atan(c*x))/x^4/(e*x^2+d)^(3/2),x)
```

output
```
int((a+b*atan(c*x))/x^4/(e*x^2+d)^(3/2),x)
```

# 3.1216 $\int \frac{x^4(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^4(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx = -\frac{ax^3}{3e\,(d+ex^2)^{3/2}} - \frac{ax}{e^2\sqrt{d+ex^2}} + \frac{a\text{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{e^{5/2}} + b\text{Int}\left(\frac{x^4\arctan(cx)}{(d+ex^2)^{5/2}},x\right)$$

output
```
-1/3*a*x^3/e/(e*x^2+d)^(3/2)-a*x/e^2/(e*x^2+d)^(1/2)+a*arctanh(e^(1/2)*x/(
e*x^2+d)^(1/2))/e^(5/2)+b*Defer(Int)(x^4*arctan(c*x)/(e*x^2+d)^(5/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 16.43 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^4(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{x^4(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx$$

input
```
Integrate[(x^4*(a + b*ArcTan[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
Integrate[(x^4*(a + b*ArcTan[c*x]))/(d + e*x^2)^(5/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(a+b\arctan(cx))}{(d+ex^2)^{5/2}}dx$$

$$\Big\downarrow \text{5517}$$

$$a\int \frac{x^4}{(ex^2+d)^{5/2}}dx+b\int \frac{x^4\arctan(cx)}{(ex^2+d)^{5/2}}dx$$

$$\Big\downarrow \text{252}$$

$$a\left(\frac{\int \frac{x^2}{(ex^2+d)^{3/2}}dx}{e}-\frac{x^3}{3e(d+ex^2)^{3/2}}\right)+b\int \frac{x^4\arctan(cx)}{(ex^2+d)^{5/2}}dx$$

$$\Big\downarrow \text{252}$$

$$a\left(\frac{\frac{\int \frac{1}{\sqrt{ex^2+d}}dx}{e}-\frac{x}{e\sqrt{d+ex^2}}}{e}-\frac{x^3}{3e(d+ex^2)^{3/2}}\right)+b\int \frac{x^4\arctan(cx)}{(ex^2+d)^{5/2}}dx$$

$$\Big\downarrow \text{224}$$

$$a\left(\frac{\frac{\int \frac{1}{1-\frac{ex^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{e}-\frac{x}{e\sqrt{d+ex^2}}}{e}-\frac{x^3}{3e(d+ex^2)^{3/2}}\right)+b\int \frac{x^4\arctan(cx)}{(ex^2+d)^{5/2}}dx$$

$$\Big\downarrow \text{219}$$

$$b\int \frac{x^4 \arctan(cx)}{(ex^2+d)^{5/2}}dx + a\left(\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{e^{3/2}} - \frac{x}{e\sqrt{d+ex^2}}}{e} - \frac{x^3}{3e\left(d+ex^2\right)^{3/2}}\right)$$

$$\Big\downarrow \; 5560$$

$$b\int \frac{x^4 \arctan(cx)}{(ex^2+d)^{5/2}}dx + a\left(\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{e^{3/2}} - \frac{x}{e\sqrt{d+ex^2}}}{e} - \frac{x^3}{3e\left(d+ex^2\right)^{3/2}}\right)$$

input
```
Int[(x^4*(a + b*ArcTan[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x^4(a+b\arctan(cx))}{(e\,x^2+d)^{\frac{5}{2}}}dx$$

input
```
int(x^4*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x)
```

output
```
int(x^4*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.52

$$\int \frac{x^4(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{(b\arctan(cx)+a)x^4}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
integral((b*x^4*arctan(c*x) + a*x^4)*sqrt(e*x^2 + d)/(e^3*x^6 + 3*d*e^2*x^
4 + 3*d^2*e*x^2 + d^3), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^4(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**4*(a+b*atan(c*x))/(e*x**2+d)**(5/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^4*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

## Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^4(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx=\int \frac{(b\arctan(cx)+a)x^4}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)*x^4/(e*x^2 + d)^(5/2), x)
```

## Mupad [N/A]

Not integrable

Time = 1.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^4(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx=\int \frac{x^4\,(a+b\,\mathrm{atan}(c\,x))}{(e\,x^2+d)^{5/2}}\,dx$$

input
```
int((x^4*(a + b*atan(c*x)))/(d + e*x^2)^(5/2),x)
```

output
```
int((x^4*(a + b*atan(c*x)))/(d + e*x^2)^(5/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 336, normalized size of antiderivative = 14.61

$$\int \frac{x^4(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx = \frac{-3\sqrt{e\,x^2+d}\,adex-4\sqrt{e\,x^2+d}\,a\,e^2x^3+3\sqrt{e}\log\left(\frac{\sqrt{e\,x^2+d}+\sqrt{e}\,x}{\sqrt{d}}\right)a\,d^2+6\sqrt{e}\,l}{}$$

input
```
int(x^4*(a+b*atan(c*x))/(e*x^2+d)^(5/2),x)
```

output
```
( - 3*sqrt(d + e*x**2)*a*d*e*x - 4*sqrt(d + e*x**2)*a*e**2*x**3 + 3*sqrt(e
)*log((sqrt(d + e*x**2) + sqrt(e)*x)/sqrt(d))*a*d**2 + 6*sqrt(e)*log((sqrt
(d + e*x**2) + sqrt(e)*x)/sqrt(d))*a*d*e*x**2 + 3*sqrt(e)*log((sqrt(d + e*
x**2) + sqrt(e)*x)/sqrt(d))*a*e**2*x**4 + 3*int((atan(c*x)*x**4)/(sqrt(d +
 e*x**2)*d**2 + 2*sqrt(d + e*x**2)*d*e*x**2 + sqrt(d + e*x**2)*e**2*x**4),
x)*b*d**2*e**3 + 6*int((atan(c*x)*x**4)/(sqrt(d + e*x**2)*d**2 + 2*sqrt(d
+ e*x**2)*d*e*x**2 + sqrt(d + e*x**2)*e**2*x**4),x)*b*d*e**4*x**2 + 3*int(
(atan(c*x)*x**4)/(sqrt(d + e*x**2)*d**2 + 2*sqrt(d + e*x**2)*d*e*x**2 + sq
rt(d + e*x**2)*e**2*x**4),x)*b*e**5*x**4)/(3*e**3*(d**2 + 2*d*e*x**2 + e**
2*x**4))
```

# 3.1217 $\int \frac{x^3(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 143

$$\int \frac{x^3(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx = \frac{bcx}{3(c^2d-e)e\sqrt{d+ex^2}}+\frac{d(a+b\arctan(cx))}{3e^2(d+ex^2)^{3/2}} - \frac{a+b\arctan(cx)}{e^2\sqrt{d+ex^2}}+\frac{bc(2c^2d-3e)\arctan\left(\frac{\sqrt{c^2d-e}x}{\sqrt{d+ex^2}}\right)}{3(c^2d-e)^{3/2}e^2}$$

output
```
1/3*b*c*x/(c^2*d-e)/e/(e*x^2+d)^(1/2)+1/3*d*(a+b*arctan(c*x))/e^2/(e*x^2+d
)^(3/2)-(a+b*arctan(c*x))/e^2/(e*x^2+d)^(1/2)+1/3*b*c*(2*c^2*d-3*e)*arctan
((c^2*d-e)^(1/2)*x/(e*x^2+d)^(1/2))/(c^2*d-e)^(3/2)/e^2
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.42 (sec) , antiderivative size = 326, normalized size of antiderivative = 2.28

$$\int \frac{x^3(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx = \frac{2\sqrt{c^2d-e}(bcex(d+ex^2)-a(c^2d-e)(2d+3ex^2))-2b(c^2d-e)^{3/2}(2d+3e}{}$$

input
```
Integrate[(x^3*(a + b*ArcTan[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
(2*Sqrt[c^2*d - e]*(b*c*e*x*(d + e*x^2) - a*(c^2*d - e)*(2*d + 3*e*x^2)) -
 2*b*(c^2*d - e)^(3/2)*(2*d + 3*e*x^2)*ArcTan[c*x] - I*b*c*(2*c^2*d - 3*e)
*(d + e*x^2)^(3/2)*Log[((-12*I)*Sqrt[c^2*d - e]*e^2*(c*d - I*e*x + Sqrt[c^
2*d - e]*Sqrt[d + e*x^2]))/(b*(2*c^2*d - 3*e)*(I + c*x))] + I*b*c*(2*c^2*d
 - 3*e)*(d + e*x^2)^(3/2)*Log[((12*I)*Sqrt[c^2*d - e]*e^2*(c*d + I*e*x + S
qrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(2*c^2*d - 3*e)*(-I + c*x))])/(6*(c^2*
d - e)^(3/2)*e^2*(d + e*x^2)^(3/2))
```

## Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {5511, 27, 402, 27, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{5511}$$

$$-bc\int -\frac{3ex^2+2d}{3e^2(c^2x^2+1)(ex^2+d)^{3/2}}dx-\frac{a+b\arctan(cx)}{e^2\sqrt{d+ex^2}}+\frac{d(a+b\arctan(cx))}{3e^2(d+ex^2)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{bc\int \frac{3ex^2+2d}{(c^2x^2+1)(ex^2+d)^{3/2}}dx}{3e^2}-\frac{a+b\arctan(cx)}{e^2\sqrt{d+ex^2}}+\frac{d(a+b\arctan(cx))}{3e^2(d+ex^2)^{3/2}}$$

$$\Big\downarrow \text{402}$$

$$\frac{bc\left(\frac{\int \frac{d(2c^2d-3e)}{(c^2x^2+1)\sqrt{ex^2+d}}dx}{d(c^2d-e)}+\frac{ex}{(c^2d-e)\sqrt{d+ex^2}}\right)}{3e^2}-\frac{a+b\arctan(cx)}{e^2\sqrt{d+ex^2}}+\frac{d(a+b\arctan(cx))}{3e^2(d+ex^2)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{bc\left(\frac{(2c^2d-3e)\int \frac{1}{(c^2x^2+1)\sqrt{ex^2+d}}dx}{c^2d-e}+\frac{ex}{(c^2d-e)\sqrt{d+ex^2}}\right)}{3e^2}-\frac{a+b\arctan(cx)}{e^2\sqrt{d+ex^2}}+\frac{d(a+b\arctan(cx))}{3e^2(d+ex^2)^{3/2}}$$

$$\Big\downarrow \text{ 291}$$

$$\frac{bc\left(\frac{(2c^2d-3e)\int \frac{1}{1-\frac{(e-c^2d)x^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{c^2d-e}+\frac{ex}{(c^2d-e)\sqrt{d+ex^2}}\right)}{3e^2}-\frac{a+b\arctan(cx)}{e^2\sqrt{d+ex^2}}+\frac{d(a+b\arctan(cx))}{3e^2(d+ex^2)^{3/2}}$$

$$\Big\downarrow \text{ 216}$$

$$-\frac{a+b\arctan(cx)}{e^2\sqrt{d+ex^2}}+\frac{d(a+b\arctan(cx))}{3e^2(d+ex^2)^{3/2}}+\frac{bc\left(\frac{(2c^2d-3e)\arctan\left(\frac{x\sqrt{c^2d-e}}{\sqrt{d+ex^2}}\right)}{(c^2d-e)^{3/2}}+\frac{ex}{(c^2d-e)\sqrt{d+ex^2}}\right)}{3e^2}$$

input
```
Int[(x^3*(a + b*ArcTan[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
(d*(a + b*ArcTan[c*x]))/(3*e^2*(d + e*x^2)^(3/2)) - (a + b*ArcTan[c*x])/(e
^2*Sqrt[d + e*x^2]) + (b*c*((e*x)/((c^2*d - e)*Sqrt[d + e*x^2]) + ((2*c^2*
d - 3*e)*ArcTan[(Sqrt[c^2*d - e]*x)/Sqrt[d + e*x^2]])/(c^2*d - e)^(3/2)))/
(3*e^2)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

## Maple [F]

$$\int \frac{x^3(a+b\arctan(cx))}{(e\,x^2+d)^{\frac{5}{2}}}dx$$

input
```
int(x^3*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x)
```

output
```
int(x^3*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 415 vs. $2(125) = 250$.

Time = 0.66 (sec) , antiderivative size = 863, normalized size of antiderivative = 6.03

$$\int \frac{x^3(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
[-1/12*((2*b*c^3*d^3 - 3*b*c*d^2*e + (2*b*c^3*d*e^2 - 3*b*c*e^3)*x^4 + 2*(
2*b*c^3*d^2*e - 3*b*c*d*e^2)*x^2)*sqrt(-c^2*d + e)*log(((c^4*d^2 - 8*c^2*d
*e + 8*e^2)*x^4 - 2*(3*c^2*d^2 - 4*d*e)*x^2 - 4*((c^2*d - 2*e)*x^3 - d*x)*
sqrt(-c^2*d + e)*sqrt(e*x^2 + d) + d^2)/(c^4*x^4 + 2*c^2*x^2 + 1)) + 4*(2*
a*c^4*d^3 - 4*a*c^2*d^2*e + 2*a*d*e^2 - (b*c^3*d*e^2 - b*c*e^3)*x^3 + 3*(a
*c^4*d^2*e - 2*a*c^2*d*e^2 + a*e^3)*x^2 - (b*c^3*d^2*e - b*c*d*e^2)*x + (2
*b*c^4*d^3 - 4*b*c^2*d^2*e + 2*b*d*e^2 + 3*(b*c^4*d^2*e - 2*b*c^2*d*e^2 +
b*e^3)*x^2)*arctan(c*x))*sqrt(e*x^2 + d))/(c^4*d^4*e^2 - 2*c^2*d^3*e^3 + d
^2*e^4 + (c^4*d^2*e^4 - 2*c^2*d*e^5 + e^6)*x^4 + 2*(c^4*d^3*e^3 - 2*c^2*d^
2*e^4 + d*e^5)*x^2), 1/6*((2*b*c^3*d^3 - 3*b*c*d^2*e + (2*b*c^3*d*e^2 - 3*
b*c*e^3)*x^4 + 2*(2*b*c^3*d^2*e - 3*b*c*d*e^2)*x^2)*sqrt(c^2*d - e)*arctan
(1/2*sqrt(c^2*d - e)*((c^2*d - 2*e)*x^2 - d)*sqrt(e*x^2 + d)/((c^2*d*e - e
^2)*x^3 + (c^2*d^2 - d*e)*x)) - 2*(2*a*c^4*d^3 - 4*a*c^2*d^2*e + 2*a*d*e^2
 - (b*c^3*d*e^2 - b*c*e^3)*x^3 + 3*(a*c^4*d^2*e - 2*a*c^2*d*e^2 + a*e^3)*x
^2 - (b*c^3*d^2*e - b*c*d*e^2)*x + (2*b*c^4*d^3 - 4*b*c^2*d^2*e + 2*b*d*e^
2 + 3*(b*c^4*d^2*e - 2*b*c^2*d*e^2 + b*e^3)*x^2)*arctan(c*x))*sqrt(e*x^2 +
 d))/(c^4*d^4*e^2 - 2*c^2*d^3*e^3 + d^2*e^4 + (c^4*d^2*e^4 - 2*c^2*d*e^5 +
 e^6)*x^4 + 2*(c^4*d^3*e^3 - 2*c^2*d^2*e^4 + d*e^5)*x^2)]
```

## Sympy [F]

$$\int \frac{x^3(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{x^3(a+b\,\mathrm{atan}\,(cx))}{(d+ex^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**3*(a+b*atan(c*x))/(e*x**2+d)**(5/2),x)
```

output
```
Integral(x**3*(a + b*atan(c*x))/(d + e*x**2)**(5/2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(a + b\arctan(cx))}{(d + ex^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

## Giac [F]

$$\int \frac{x^3(a + b\arctan(cx))}{(d + ex^2)^{5/2}}\,dx = \int \frac{(b\arctan(cx) + a)x^3}{(ex^2 + d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)*x^3/(e*x^2 + d)^(5/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{x^3\,(a+b\,\mathrm{atan}(c\,x))}{(e\,x^2+d)^{5/2}}\,dx$$

input
```
int((x^3*(a + b*atan(c*x)))/(d + e*x^2)^(5/2),x)
```

output
```
int((x^3*(a + b*atan(c*x)))/(d + e*x^2)^(5/2), x)
```

## Reduce [F]

$$\int \frac{x^3(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx = \frac{-2\sqrt{e\,x^2+d}\,ad-3\sqrt{e\,x^2+d}\,ae\,x^2+3\left(\int \frac{atan(cx)x^3}{\sqrt{e\,x^2+d}\,d^2+2\sqrt{e\,x^2+d}\,de\,x^2+\sqrt{e\,x^2+d}\,e^2x^4}\right.}{}$$

input
```
int(x^3*(a+b*atan(c*x))/(e*x^2+d)^(5/2),x)
```

output
```
( - 2*sqrt(d + e*x**2)*a*d - 3*sqrt(d + e*x**2)*a*e*x**2 + 3*int((atan(c*x
)*x**3)/(sqrt(d + e*x**2)*d**2 + 2*sqrt(d + e*x**2)*d*e*x**2 + sqrt(d + e*
x**2)*e**2*x**4),x)*b*d**2*e**2 + 6*int((atan(c*x)*x**3)/(sqrt(d + e*x**2)
*d**2 + 2*sqrt(d + e*x**2)*d*e*x**2 + sqrt(d + e*x**2)*e**2*x**4),x)*b*d*e
**3*x**2 + 3*int((atan(c*x)*x**3)/(sqrt(d + e*x**2)*d**2 + 2*sqrt(d + e*x*
*2)*d*e*x**2 + sqrt(d + e*x**2)*e**2*x**4),x)*b*e**4*x**4)/(3*e**2*(d**2 +
 2*d*e*x**2 + e**2*x**4))
```

# 3.1218 $\int \frac{x^2(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 109

$$\int \frac{x^2(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx=\frac{bc}{3(c^2d-e)e\sqrt{d+ex^2}}+\frac{x^3(a+b\arctan(cx))}{3d(d+ex^2)^{3/2}}-\frac{b\operatorname{arctanh}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{3d(c^2d-e)^{3/2}}$$

output
```
1/3*b*c/(c^2*d-e)/e/(e*x^2+d)^(1/2)+1/3*x^3*(a+b*arctan(c*x))/d/(e*x^2+d)^
(3/2)-1/3*b*arctanh(c*(e*x^2+d)^(1/2)/(c^2*d-e)^(1/2))/d/(c^2*d-e)^(3/2)
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.68 (sec) , antiderivative size = 252, normalized size of antiderivative = 2.31

$$\int \frac{x^2(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx=$$

$$-\frac{\frac{2adx}{e(d+ex^2)^{3/2}}-\frac{2(bcd+a(c^2d-e)x)}{(c^2d-e)e\sqrt{d+ex^2}}-\frac{2bx^3\arctan(cx)}{(d+ex^2)^{3/2}}+\frac{b\log\left(\frac{12cd\sqrt{c^2d-e}\left(cd-iex+\sqrt{c^2d-e}\sqrt{d+ex^2}\right)}{b(i+cx)}\right)}{(c^2d-e)^{3/2}}+\frac{b\log\left(\frac{12cd\sqrt{c^2d-e}\left(cd+iex+\sqrt{c\right.}}{b(-i+cx)}\right.}{(c^2d-e)^{3/2}}}{6d}$$

input
```
Integrate[(x^2*(a + b*ArcTan[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
-1/6*((2*a*d*x)/(e*(d + e*x^2)^(3/2)) - (2*(b*c*d + a*(c^2*d - e)*x))/((c^
2*d - e)*e*Sqrt[d + e*x^2]) - (2*b*x^3*ArcTan[c*x])/(d + e*x^2)^(3/2) + (b
*Log[(12*c*d*Sqrt[c^2*d - e]*(c*d - I*e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2
]))/(b*(I + c*x))])/(c^2*d - e)^(3/2) + (b*Log[(12*c*d*Sqrt[c^2*d - e]*(c*
d + I*e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(-I + c*x))])/(c^2*d - e)
^(3/2))/d
```

## Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {5511, 27, 354, 87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\arctan(cx))}{(d+ex^2)^{5/2}}dx$$

$$\Big\downarrow \text{ 5511}$$

$$\frac{x^3(a+b\arctan(cx))}{3d\,(d+ex^2)^{3/2}}-bc\int \frac{x^3}{3d\,(c^2x^2+1)\,(ex^2+d)^{3/2}}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{x^3(a+b\arctan(cx))}{3d\,(d+ex^2)^{3/2}}-\frac{bc\int \frac{x^3}{(c^2x^2+1)(ex^2+d)^{3/2}}dx}{3d}$$

$$\Big\downarrow \text{ 354}$$

$$\frac{x^3(a+b\arctan(cx))}{3d\,(d+ex^2)^{3/2}}-\frac{bc\int \frac{x^2}{(c^2x^2+1)(ex^2+d)^{3/2}}dx^2}{6d}$$

$$\Big\downarrow \text{ 87}$$

$$\frac{x^3(a+b\arctan(cx))}{3d\left(d+ex^2\right)^{3/2}}-\frac{bc\left(-\frac{\int\frac{1}{\left(c^2x^2+1\right)\sqrt{ex^2+d}}dx^2}{c^2d-e}-\frac{2d}{e(c^2d-e)\sqrt{d+ex^2}}\right)}{6d}$$

$\Big\downarrow$ 73

$$\frac{x^3(a+b\arctan(cx))}{3d\left(d+ex^2\right)^{3/2}}-\frac{bc\left(-\frac{2\int\frac{1}{\frac{c^2x^4}{e}-\frac{c^2d}{e}+1}d\sqrt{ex^2+d}}{e(c^2d-e)}-\frac{2d}{e(c^2d-e)\sqrt{d+ex^2}}\right)}{6d}$$

$\Big\downarrow$ 221

$$\frac{x^3(a+b\arctan(cx))}{3d\left(d+ex^2\right)^{3/2}}-\frac{bc\left(\frac{2\text{arctanh}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{c(c^2d-e)^{3/2}}-\frac{2d}{e(c^2d-e)\sqrt{d+ex^2}}\right)}{6d}$$

input
```
Int[(x^2*(a + b*ArcTan[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
(x^3*(a + b*ArcTan[c*x]))/(3*d*(d + e*x^2)^(3/2)) - (b*c*((-2*d)/((c^2*d -
 e)*e*Sqrt[d + e*x^2]) + (2*ArcTanh[(c*Sqrt[d + e*x^2])/Sqrt[c^2*d - e]])/
(c*(c^2*d - e)^(3/2))))/(6*d)
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

## Maple [F]

$$\int \frac{x^2(a + b\arctan(cx))}{(e\,x^2 + d)^{\frac{5}{2}}}dx$$

input
```
int(x^2*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x)
```

output
```
int(x^2*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 317 vs. 2(93) = 186.

Time = 0.29 (sec) , antiderivative size = 676, normalized size of antiderivative = 6.20

$$\int \frac{x^2(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx = \left[-\frac{(be^3x^4+2\,bde^2x^2+bd^2e)\sqrt{c^2d-e}\log\left(\frac{c^4e^2x^4+8\,c^4d^2-8\,c^2de+2\,(4\,c^4de-3\,c^2e^2)x^2\cdots}{c^4x^4+2\,c\cdots}\right)\cdots}{12\,(\cdots}\right.$$

$$-\frac{(be^3x^4+2\,bde^2x^2+bd^2e)\sqrt{-c^2d+e}\arctan\left(-\frac{(c^2ex^2+2\,c^2d-e)\sqrt{-c^2d+e}\sqrt{ex^2+d}}{2\,(c^3d^2-cde+(c^3de-ce^2)x^2)}\right)-2\,(bc^3d^3-bcd^2e+(bc^4d^2e\cdots}{6\,(c^4d^5e-2\,c^2d^4e^2+d^3e^3+(c^4d^3e^3-2\,c^2d^2e^4+\cdots}$$

input
```
integrate(x^2*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
[-1/12*((b*e^3*x^4 + 2*b*d*e^2*x^2 + b*d^2*e)*sqrt(c^2*d - e)*log((c^4*e^2
*x^4 + 8*c^4*d^2 - 8*c^2*d*e + 2*(4*c^4*d*e - 3*c^2*e^2)*x^2 + 4*(c^3*e*x^
2 + 2*c^3*d - c*e)*sqrt(c^2*d - e)*sqrt(e*x^2 + d) + e^2)/(c^4*x^4 + 2*c^2
*x^2 + 1)) - 4*(b*c^3*d^3 - b*c*d^2*e + (b*c^4*d^2*e - 2*b*c^2*d*e^2 + b*e
^3)*x^3*arctan(c*x) + (a*c^4*d^2*e - 2*a*c^2*d*e^2 + a*e^3)*x^3 + (b*c^3*d
^2*e - b*c*d*e^2)*x^2)*sqrt(e*x^2 + d))/(c^4*d^5*e - 2*c^2*d^4*e^2 + d^3*e
^3 + (c^4*d^3*e^3 - 2*c^2*d^2*e^4 + d*e^5)*x^4 + 2*(c^4*d^4*e^2 - 2*c^2*d^
3*e^3 + d^2*e^4)*x^2), -1/6*((b*e^3*x^4 + 2*b*d*e^2*x^2 + b*d^2*e)*sqrt(-c
^2*d + e)*arctan(-1/2*(c^2*e*x^2 + 2*c^2*d - e)*sqrt(-c^2*d + e)*sqrt(e*x^
2 + d)/(c^3*d^2 - c*d*e + (c^3*d*e - c*e^2)*x^2)) - 2*(b*c^3*d^3 - b*c*d^2
*e + (b*c^4*d^2*e - 2*b*c^2*d*e^2 + b*e^3)*x^3*arctan(c*x) + (a*c^4*d^2*e
- 2*a*c^2*d*e^2 + a*e^3)*x^3 + (b*c^3*d^2*e - b*c*d*e^2)*x^2)*sqrt(e*x^2 +
 d))/(c^4*d^5*e - 2*c^2*d^4*e^2 + d^3*e^3 + (c^4*d^3*e^3 - 2*c^2*d^2*e^4 +
 d*e^5)*x^4 + 2*(c^4*d^4*e^2 - 2*c^2*d^3*e^3 + d^2*e^4)*x^2)]
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(a+b*atan(c*x))/(e*x**2+d)**(5/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{x^2(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{(b\arctan(cx)+a)x^2}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*a*(x/((e*x^2 + d)^(3/2)*e) - x/(sqrt(e*x^2 + d)*d*e)) + 2*b*integrate
(1/2*x^2*arctan(c*x)/((e^2*x^4 + 2*d*e*x^2 + d^2)*sqrt(e*x^2 + d)), x)
```

## Giac [F]

$$\int \frac{x^2(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{(b\arctan(cx)+a)x^2}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)*x^2/(e*x^2 + d)^(5/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{x^2\,(a+b\,\mathrm{atan}(c\,x))}{{(e\,x^2+d)}^{5/2}}\,dx$$

input
```
int((x^2*(a + b*atan(c*x)))/(d + e*x^2)^(5/2),x)
```

output
```
int((x^2*(a + b*atan(c*x)))/(d + e*x^2)^(5/2), x)
```

## Reduce [F]

$$\int \frac{x^2(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx = \frac{\sqrt{e\,x^2+d}\,a\,e^2x^3+\sqrt{e}\,a\,d^2+2\sqrt{e}\,ade\,x^2+\sqrt{e}\,a\,e^2x^4+3\Big(\int \frac{atan}{\sqrt{e\,x^2+d}\,d^2+2\sqrt{e\,x^2+}}}{}$$

input
```
int(x^2*(a+b*atan(c*x))/(e*x^2+d)^(5/2),x)
```

output
```
(sqrt(d + e*x**2)*a*e**2*x**3 + sqrt(e)*a*d**2 + 2*sqrt(e)*a*d*e*x**2 + sq
rt(e)*a*e**2*x**4 + 3*int((atan(c*x)*x**2)/(sqrt(d + e*x**2)*d**2 + 2*sqrt
(d + e*x**2)*d*e*x**2 + sqrt(d + e*x**2)*e**2*x**4),x)*b*d**3*e**2 + 6*int
((atan(c*x)*x**2)/(sqrt(d + e*x**2)*d**2 + 2*sqrt(d + e*x**2)*d*e*x**2 + s
qrt(d + e*x**2)*e**2*x**4),x)*b*d**2*e**3*x**2 + 3*int((atan(c*x)*x**2)/(s
qrt(d + e*x**2)*d**2 + 2*sqrt(d + e*x**2)*d*e*x**2 + sqrt(d + e*x**2)*e**2
*x**4),x)*b*d*e**4*x**4)/(3*d*e**2*(d**2 + 2*d*e*x**2 + e**2*x**4))
```

# 3.1219 $\int \frac{x(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 110

$$\int \frac{x(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx = -\frac{bcx}{3d(c^2d-e)\sqrt{d+ex^2}} - \frac{a+b\arctan(cx)}{3e(d+ex^2)^{3/2}} + \frac{bc^3\arctan\left(\frac{\sqrt{c^2d-e}x}{\sqrt{d+ex^2}}\right)}{3(c^2d-e)^{3/2}e}$$

output
```
-1/3*b*c*x/d/(c^2*d-e)/(e*x^2+d)^(1/2)-1/3*(a+b*arctan(c*x))/e/(e*x^2+d)^(
3/2)+1/3*b*c^3*arctan((c^2*d-e)^(1/2)*x/(e*x^2+d)^(1/2))/(c^2*d-e)^(3/2)/e
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 259, normalized size of antiderivative = 2.35

$$\int \frac{x(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx = \frac{1}{6}\left(-\frac{2a}{e(d+ex^2)^{3/2}} - \frac{2bcx}{(c^2d^2-de)\sqrt{d+ex^2}} - \frac{2b\arctan(cx)}{e(d+ex^2)^{3/2}} - \frac{ibc^3\log\left(-\frac{12i\sqrt{c^2d-e}e\left(cd-iex+\sqrt{c^2d-e}\sqrt{d+ex^2}\right)}{bc^2(i+cx)}\right)}{(c^2d-e)^{3/2}e} + \frac{ibc^3\log\left(\frac{12i\sqrt{c^2d-e}e\left(cd+iex+\sqrt{c^2d-e}\sqrt{d+ex^2}\right)}{bc^2(-i+cx)}\right)}{(c^2d-e)^{3/2}e}\right)$$

input
```
Integrate[(x*(a + b*ArcTan[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
((-2*a)/(e*(d + e*x^2)^(3/2)) - (2*b*c*x)/((c^2*d^2 - d*e)*Sqrt[d + e*x^2]
) - (2*b*ArcTan[c*x])/(e*(d + e*x^2)^(3/2)) - (I*b*c^3*Log[((-12*I)*Sqrt[c
^2*d - e]*e*(c*d - I*e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*c^2*(I + c
*x))])/((c^2*d - e)^(3/2)*e) + (I*b*c^3*Log[((12*I)*Sqrt[c^2*d - e]*e*(c*d
 + I*e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*c^2*(-I + c*x))])/((c^2*d
- e)^(3/2)*e))/6
```

## Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {5509, 296, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\arctan(cx))}{(d+ex^2)^{5/2}}dx$$

$$\Big\downarrow \text{ 5509}$$

$$\frac{bc\int \frac{1}{(c^2x^2+1)(ex^2+d)^{3/2}}dx}{3e}-\frac{a+b\arctan(cx)}{3e\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 296}$$

$$\frac{bc\left(\frac{c^2\int \frac{1}{\left(c^2x^2+1\right)\sqrt{ex^2+d}}dx}{c^2d-e}-\frac{ex}{d(c^2d-e)\sqrt{d+ex^2}}\right)}{3e}-\frac{a+b\arctan(cx)}{3e\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 291}$$

$$\frac{bc\left(\frac{c^2\int \frac{1}{1-\frac{\left(e-c^2d\right)x^2}{ex^2+d}}d\frac{x}{\sqrt{ex^2+d}}}{c^2d-e}-\frac{ex}{d(c^2d-e)\sqrt{d+ex^2}}\right)}{3e}-\frac{a+b\arctan(cx)}{3e\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{bc\left(\frac{c^2\arctan\left(\frac{x\sqrt{c^2d-e}}{\sqrt{d+ex^2}}\right)}{(c^2d-e)^{3/2}}-\frac{ex}{d(c^2d-e)\sqrt{d+ex^2}}\right)}{3e}-\frac{a+b\arctan(cx)}{3e\left(d+ex^2\right)^{3/2}}$$

input
```
Int[(x*(a + b*ArcTan[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
-1/3*(a + b*ArcTan[c*x])/(e*(d + e*x^2)^(3/2)) + (b*c*(-((e*x)/(d*(c^2*d -
 e)*Sqrt[d + e*x^2])) + (c^2*ArcTan[(Sqrt[c^2*d - e]*x)/Sqrt[d + e*x^2]])/
(c^2*d - e)^(3/2)))/(3*e)
```

### Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

rule 5509
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Simp[(d + e*x^2)^(q + 1)*((a + b*ArcTan[c*x])/(2*e*(q + 1))), x
] - Simp[b*(c/(2*e*(q + 1)))   Int[(d + e*x^2)^(q + 1)/(1 + c^2*x^2), x], x
] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[q, -1]
```

## Maple [F]

$$\int \frac{x(a+b\arctan(cx))}{(e\,x^2+d)^{\frac{5}{2}}}dx$$

input
```
int(x*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x)
```

output
```
int(x*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(94) = 188$.

Time = 0.40 (sec) , antiderivative size = 679, normalized size of antiderivative = 6.17

$$\int \frac{x(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx = \left[\frac{(bc^3de^2x^4+2\,bc^3d^2ex^2+bc^3d^3)\sqrt{-c^2d+e}\log\left(\frac{(c^4d^2-8\,c^2de+8\,e^2)x^4-2\,(3\,c^2d^2-4\,d\right.}{c^4}}{12\,(c}\right.$$

input
```
integrate(x*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
[1/12*((b*c^3*d*e^2*x^4 + 2*b*c^3*d^2*e*x^2 + b*c^3*d^3)*sqrt(-c^2*d + e)*
log(((c^4*d^2 - 8*c^2*d*e + 8*e^2)*x^4 - 2*(3*c^2*d^2 - 4*d*e)*x^2 + 4*((c
^2*d - 2*e)*x^3 - d*x)*sqrt(-c^2*d + e)*sqrt(e*x^2 + d) + d^2)/(c^4*x^4 +
2*c^2*x^2 + 1)) - 4*(a*c^4*d^3 - 2*a*c^2*d^2*e + a*d*e^2 + (b*c^3*d*e^2 -
b*c*e^3)*x^3 + (b*c^3*d^2*e - b*c*d*e^2)*x + (b*c^4*d^3 - 2*b*c^2*d^2*e +
b*d*e^2)*arctan(c*x))*sqrt(e*x^2 + d))/(c^4*d^5*e - 2*c^2*d^4*e^2 + d^3*e^
3 + (c^4*d^3*e^3 - 2*c^2*d^2*e^4 + d*e^5)*x^4 + 2*(c^4*d^4*e^2 - 2*c^2*d^3
*e^3 + d^2*e^4)*x^2), 1/6*((b*c^3*d*e^2*x^4 + 2*b*c^3*d^2*e*x^2 + b*c^3*d^
3)*sqrt(c^2*d - e)*arctan(1/2*sqrt(c^2*d - e)*((c^2*d - 2*e)*x^2 - d)*sqrt
(e*x^2 + d)/((c^2*d*e - e^2)*x^3 + (c^2*d^2 - d*e)*x)) - 2*(a*c^4*d^3 - 2*
a*c^2*d^2*e + a*d*e^2 + (b*c^3*d*e^2 - b*c*e^3)*x^3 + (b*c^3*d^2*e - b*c*d
*e^2)*x + (b*c^4*d^3 - 2*b*c^2*d^2*e + b*d*e^2)*arctan(c*x))*sqrt(e*x^2 +
d))/(c^4*d^5*e - 2*c^2*d^4*e^2 + d^3*e^3 + (c^4*d^3*e^3 - 2*c^2*d^2*e^4 +
d*e^5)*x^4 + 2*(c^4*d^4*e^2 - 2*c^2*d^3*e^3 + d^2*e^4)*x^2)]
```

## Sympy [F]

$$\int \frac{x(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{x(a+b\,\mathrm{atan}\,(cx))}{(d+ex^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*(a+b*atan(c*x))/(e*x**2+d)**(5/2),x)
```

output
```
Integral(x*(a + b*atan(c*x))/(d + e*x**2)**(5/2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e-c^2*d>0)', see `assume?` for m
ore detail
```

## Giac [F]

$$\int \frac{x(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{(b\arctan(cx)+a)x}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)*x/(e*x^2 + d)^(5/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{x\,(a+b\,\mathrm{atan}(c\,x))}{(e\,x^2+d)^{5/2}}\,dx$$

input
```
int((x*(a + b*atan(c*x)))/(d + e*x^2)^(5/2),x)
```

output
```
int((x*(a + b*atan(c*x)))/(d + e*x^2)^(5/2), x)
```

## Reduce [F]

$$\int \frac{x(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx = \frac{-\sqrt{e\,x^2+d}\,a+3\left(\int \frac{atan(cx)x}{\sqrt{e\,x^2+d}\,d^2+2\sqrt{e\,x^2+d}\,de\,x^2+\sqrt{e\,x^2+d}\,e^2x^4}dx\right)b\,d^2e+6\left(\int \frac{}{\sqrt{e\,x^2+d}}\right.}{3e\,(e^2}$$

input
```
int(x*(a+b*atan(c*x))/(e*x^2+d)^(5/2),x)
```

output
```
( - sqrt(d + e*x**2)*a + 3*int((atan(c*x)*x)/(sqrt(d + e*x**2)*d**2 + 2*sq
rt(d + e*x**2)*d*e*x**2 + sqrt(d + e*x**2)*e**2*x**4),x)*b*d**2*e + 6*int(
(atan(c*x)*x)/(sqrt(d + e*x**2)*d**2 + 2*sqrt(d + e*x**2)*d*e*x**2 + sqrt(
d + e*x**2)*e**2*x**4),x)*b*d*e**2*x**2 + 3*int((atan(c*x)*x)/(sqrt(d + e*
x**2)*d**2 + 2*sqrt(d + e*x**2)*d*e*x**2 + sqrt(d + e*x**2)*e**2*x**4),x)*
b*e**3*x**4)/(3*e*(d**2 + 2*d*e*x**2 + e**2*x**4))
```

# 3.1220 $\int \frac{a+b\arctan(cx)}{(d+ex^2)^{5/2}}\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 144

$$\int \frac{a+b\arctan(cx)}{(d+ex^2)^{5/2}}\,dx = -\frac{bc}{3d(c^2d-e)\sqrt{d+ex^2}} + \frac{x(a+b\arctan(cx))}{3d(d+ex^2)^{3/2}} + \frac{2x(a+b\arctan(cx))}{3d^2\sqrt{d+ex^2}} + \frac{b(3c^2d-2e)\,\text{arctanh}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{3d^2(c^2d-e)^{3/2}}$$

output
```
-1/3*b*c/d/(c^2*d-e)/(e*x^2+d)^(1/2)+1/3*x*(a+b*arctan(c*x))/d/(e*x^2+d)^(
3/2)+2/3*x*(a+b*arctan(c*x))/d^2/(e*x^2+d)^(1/2)+1/3*b*(3*c^2*d-2*e)*arcta
nh(c*(e*x^2+d)^(1/2)/(c^2*d-e)^(1/2))/d^2/(c^2*d-e)^(3/2)
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 317, normalized size of antiderivative = 2.20

$$\int \frac{a+b\arctan(cx)}{(d+ex^2)^{5/2}}\,dx = \frac{2\sqrt{c^2d-e}(-bcd(d+ex^2)+a(c^2d-e)\,x(3d+2ex^2))+2b(c^2d-e)^{3/2}\,x(3d+2e}{}$$

input
```
Integrate[(a + b*ArcTan[c*x])/(d + e*x^2)^(5/2),x]
```

output
```
(2*Sqrt[c^2*d - e]*(-(b*c*d*(d + e*x^2)) + a*(c^2*d - e)*x*(3*d + 2*e*x^2)
) + 2*b*(c^2*d - e)^(3/2)*x*(3*d + 2*e*x^2)*ArcTan[c*x] + b*(3*c^2*d - 2*e
)*(d + e*x^2)^(3/2)*Log[(-12*c*d^2*Sqrt[c^2*d - e]*(c*d - I*e*x + Sqrt[c^2
*d - e]*Sqrt[d + e*x^2]))/(b*(3*c^2*d - 2*e)*(I + c*x))] + b*(3*c^2*d - 2*
e)*(d + e*x^2)^(3/2)*Log[(-12*c*d^2*Sqrt[c^2*d - e]*(c*d + I*e*x + Sqrt[c^
2*d - e]*Sqrt[d + e*x^2]))/(b*(3*c^2*d - 2*e)*(-I + c*x))])/(6*d^2*(c^2*d
- e)^(3/2)*(d + e*x^2)^(3/2))
```

## Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5447, 27, 435, 87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{(d+ex^2)^{5/2}}dx$$

$$\Big\downarrow \text{5447}$$

$$-bc\int \frac{x(2ex^2+3d)}{3d^2(c^2x^2+1)(ex^2+d)^{3/2}}dx+\frac{2x(a+b\arctan(cx))}{3d^2\sqrt{d+ex^2}}+\frac{x(a+b\arctan(cx))}{3d(d+ex^2)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{bc\int \frac{x(2ex^2+3d)}{(c^2x^2+1)(ex^2+d)^{3/2}}dx}{3d^2}+\frac{2x(a+b\arctan(cx))}{3d^2\sqrt{d+ex^2}}+\frac{x(a+b\arctan(cx))}{3d(d+ex^2)^{3/2}}$$

$$\Big\downarrow \text{435}$$

$$-\frac{bc\int \frac{2ex^2+3d}{(c^2x^2+1)(ex^2+d)^{3/2}}dx^2}{6d^2}+\frac{2x(a+b\arctan(cx))}{3d^2\sqrt{d+ex^2}}+\frac{x(a+b\arctan(cx))}{3d(d+ex^2)^{3/2}}$$

$$\Big\downarrow \text{87}$$

$$-\frac{bc\left(\frac{(3c^2d-2e)\int \frac{1}{(c^2x^2+1)\sqrt{ex^2+d}}dx^2}{c^2d-e}+\frac{2d}{(c^2d-e)\sqrt{d+ex^2}}\right)}{6d^2}+\frac{2x(a+b\arctan(cx))}{3d^2\sqrt{d+ex^2}}+\frac{x(a+b\arctan(cx))}{3d\,(d+ex^2)^{3/2}}$$

$$\Big\downarrow \textbf{73}$$

$$-\frac{bc\left(\frac{2(3c^2d-2e)\int \frac{1}{\frac{c^2x^4}{e}-\frac{c^2d}{e}+1}d\sqrt{ex^2+d}}{e(c^2d-e)}+\frac{2d}{(c^2d-e)\sqrt{d+ex^2}}\right)}{6d^2}+\frac{2x(a+b\arctan(cx))}{3d^2\sqrt{d+ex^2}}+\frac{x(a+b\arctan(cx))}{3d\,(d+ex^2)^{3/2}}$$

$$\Big\downarrow \textbf{221}$$

$$\frac{2x(a+b\arctan(cx))}{3d^2\sqrt{d+ex^2}}+\frac{x(a+b\arctan(cx))}{3d\,(d+ex^2)^{3/2}}-\frac{bc\left(\frac{2d}{(c^2d-e)\sqrt{d+ex^2}}-\frac{2(3c^2d-2e)\text{arctanh}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{c(c^2d-e)^{3/2}}\right)}{6d^2}$$

input
```
Int[(a + b*ArcTan[c*x])/(d + e*x^2)^(5/2),x]
```

output
```
(x*(a + b*ArcTan[c*x]))/(3*d*(d + e*x^2)^(3/2)) + (2*x*(a + b*ArcTan[c*x])
)/(3*d^2*Sqrt[d + e*x^2]) - (b*c*((2*d)/((c^2*d - e)*Sqrt[d + e*x^2]) - (2
*(3*c^2*d - 2*e)*ArcTanh[(c*Sqrt[d + e*x^2])/Sqrt[c^2*d - e]])/(c*(c^2*d -
 e)^(3/2))))/(6*d^2)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 5447
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Simp[(a + b*ArcTan[c*x])   u,
x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2), x], x], x]] /; FreeQ
[{a, b, c, d, e}, x] && (IntegerQ[q] || ILtQ[q + 1/2, 0])
```

## Maple [F]

$$\int \frac{a+b\arctan(cx)}{(e\,x^2+d)^{\frac{5}{2}}}dx$$

input
```
int((a+b*arctan(c*x))/(e*x^2+d)^(5/2),x)
```

output
```
int((a+b*arctan(c*x))/(e*x^2+d)^(5/2),x)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 411 vs. $2(124) = 248$.

Time = 0.53 (sec) , antiderivative size = 864, normalized size of antiderivative = 6.00

$$\int \frac{a+b\arctan(cx)}{(d+ex^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*arctan(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
[1/12*((3*b*c^2*d^3 + (3*b*c^2*d*e^2 - 2*b*e^3)*x^4 - 2*b*d^2*e + 2*(3*b*c
^2*d^2*e - 2*b*d*e^2)*x^2)*sqrt(c^2*d - e)*log((c^4*e^2*x^4 + 8*c^4*d^2 -
8*c^2*d*e + 2*(4*c^4*d*e - 3*c^2*e^2)*x^2 + 4*(c^3*e*x^2 + 2*c^3*d - c*e)*
sqrt(c^2*d - e)*sqrt(e*x^2 + d) + e^2)/(c^4*x^4 + 2*c^2*x^2 + 1)) - 4*(b*c
^3*d^3 - b*c*d^2*e - 2*(a*c^4*d^2*e - 2*a*c^2*d*e^2 + a*e^3)*x^3 + (b*c^3*
d^2*e - b*c*d*e^2)*x^2 - 3*(a*c^4*d^3 - 2*a*c^2*d^2*e + a*d*e^2)*x - (2*(b
*c^4*d^2*e - 2*b*c^2*d*e^2 + b*e^3)*x^3 + 3*(b*c^4*d^3 - 2*b*c^2*d^2*e + b
*d*e^2)*x)*arctan(c*x))*sqrt(e*x^2 + d))/(c^4*d^6 - 2*c^2*d^5*e + d^4*e^2
+ (c^4*d^4*e^2 - 2*c^2*d^3*e^3 + d^2*e^4)*x^4 + 2*(c^4*d^5*e - 2*c^2*d^4*e
^2 + d^3*e^3)*x^2), 1/6*((3*b*c^2*d^3 + (3*b*c^2*d*e^2 - 2*b*e^3)*x^4 - 2*
b*d^2*e + 2*(3*b*c^2*d^2*e - 2*b*d*e^2)*x^2)*sqrt(-c^2*d + e)*arctan(-1/2*
(c^2*e*x^2 + 2*c^2*d - e)*sqrt(-c^2*d + e)*sqrt(e*x^2 + d)/(c^3*d^2 - c*d*
e + (c^3*d*e - c*e^2)*x^2)) - 2*(b*c^3*d^3 - b*c*d^2*e - 2*(a*c^4*d^2*e -
2*a*c^2*d*e^2 + a*e^3)*x^3 + (b*c^3*d^2*e - b*c*d*e^2)*x^2 - 3*(a*c^4*d^3
- 2*a*c^2*d^2*e + a*d*e^2)*x - (2*(b*c^4*d^2*e - 2*b*c^2*d*e^2 + b*e^3)*x^
3 + 3*(b*c^4*d^3 - 2*b*c^2*d^2*e + b*d*e^2)*x)*arctan(c*x))*sqrt(e*x^2 + d
))/(c^4*d^6 - 2*c^2*d^5*e + d^4*e^2 + (c^4*d^4*e^2 - 2*c^2*d^3*e^3 + d^2*e
^4)*x^4 + 2*(c^4*d^5*e - 2*c^2*d^4*e^2 + d^3*e^3)*x^2)]
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{a + b \arctan(cx)}{(d + ex^2)^{5/2}} \, dx = \text{Timed out}$$

input
```
integrate((a+b*atan(c*x))/(e*x**2+d)**(5/2),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{a + b \arctan(cx)}{(d + ex^2)^{5/2}} \, dx = \int \frac{b \arctan(cx) + a}{(ex^2 + d)^{\frac{5}{2}}} \, dx$$

input
```
integrate((a+b*arctan(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
1/3*a*(2*x/(sqrt(e*x^2 + d)*d^2) + x/((e*x^2 + d)^(3/2)*d)) + 2*b*integrat
e(1/2*arctan(c*x)/((e^2*x^4 + 2*d*e*x^2 + d^2)*sqrt(e*x^2 + d)), x)
```

## Giac [F]

$$\int \frac{a + b \arctan(cx)}{(d + ex^2)^{5/2}} \, dx = \int \frac{b \arctan(cx) + a}{(ex^2 + d)^{\frac{5}{2}}} \, dx$$

input
```
integrate((a+b*arctan(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)/(e*x^2 + d)^(5/2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{(d+ex^2)^{5/2}}\,dx = \int \frac{a+b\,\mathrm{atan}(c\,x)}{(e\,x^2+d)^{5/2}}\,dx$$

input
```
int((a + b*atan(c*x))/(d + e*x^2)^(5/2),x)
```

output
```
int((a + b*atan(c*x))/(d + e*x^2)^(5/2), x)
```

## Reduce [F]

$$\int \frac{a+b\arctan(cx)}{(d+ex^2)^{5/2}}\,dx = \frac{3\sqrt{e\,x^2+d}\,adex+2\sqrt{e\,x^2+d}\,a\,e^2x^3-2\sqrt{e}\,a\,d^2-4\sqrt{e}\,ade\,x^2-2\sqrt{e}\,a\,e^2x^4+3(}{}$$

input
```
int((a+b*atan(c*x))/(e*x^2+d)^(5/2),x)
```

output
```
(3*sqrt(d + e*x**2)*a*d*e*x + 2*sqrt(d + e*x**2)*a*e**2*x**3 - 2*sqrt(e)*a
*d**2 - 4*sqrt(e)*a*d*e*x**2 - 2*sqrt(e)*a*e**2*x**4 + 3*int(atan(c*x)/(sq
rt(d + e*x**2)*d**2 + 2*sqrt(d + e*x**2)*d*e*x**2 + sqrt(d + e*x**2)*e**2*
x**4),x)*b*d**4*e + 6*int(atan(c*x)/(sqrt(d + e*x**2)*d**2 + 2*sqrt(d + e*
x**2)*d*e*x**2 + sqrt(d + e*x**2)*e**2*x**4),x)*b*d**3*e**2*x**2 + 3*int(a
tan(c*x)/(sqrt(d + e*x**2)*d**2 + 2*sqrt(d + e*x**2)*d*e*x**2 + sqrt(d + e
*x**2)*e**2*x**4),x)*b*d**2*e**3*x**4)/(3*d**2*e*(d**2 + 2*d*e*x**2 + e**2
*x**4))
```

# 3.1221 $\int \frac{a+b\arctan(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{a+b\arctan(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx = \frac{a}{3d\left(d+ex^2\right)^{3/2}}+\frac{a}{d^2\sqrt{d+ex^2}}-\frac{a\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{5/2}}+b\text{Int}\left(\frac{\arctan(cx)}{x\left(d+ex^2\right)^{5/2}},x\right)$$

output
```
1/3*a/d/(e*x^2+d)^(3/2)+a/d^2/(e*x^2+d)^(1/2)-a*arctanh((e*x^2+d)^(1/2)/d^
(1/2))/d^(5/2)+b*Defer(Int)(arctan(c*x)/x/(e*x^2+d)^(5/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 13.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\arctan(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx = \int \frac{a+b\arctan(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx$$

input
```
Integrate[(a + b*ArcTan[c*x])/(x*(d + e*x^2)^(5/2)),x]
```

output
```
Integrate[(a + b*ArcTan[c*x])/(x*(d + e*x^2)^(5/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x\left(d+ex^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{5517}$$

$$a\int \frac{1}{x\left(ex^2+d\right)^{5/2}}dx+b\int \frac{\arctan(cx)}{x\left(ex^2+d\right)^{5/2}}dx$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}a\int \frac{1}{x^2\left(ex^2+d\right)^{5/2}}dx^2+b\int \frac{\arctan(cx)}{x\left(ex^2+d\right)^{5/2}}dx$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{2}a\left(\frac{\int \frac{1}{x^2(ex^2+d)^{3/2}}dx^2}{d}+\frac{2}{3d\left(d+ex^2\right)^{3/2}}\right)+b\int \frac{\arctan(cx)}{x\left(ex^2+d\right)^{5/2}}dx$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{2}a\left(\frac{\frac{\int \frac{1}{x^2\sqrt{ex^2+d}}dx^2}{d}+\frac{2}{d\sqrt{d+ex^2}}}{d}+\frac{2}{3d\left(d+ex^2\right)^{3/2}}\right)+b\int \frac{\arctan(cx)}{x\left(ex^2+d\right)^{5/2}}dx$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}a\left(\frac{\frac{2\int \frac{1}{\frac{x^4}{e}-\frac{d}{e}}d\sqrt{ex^2+d}}{de}+\frac{2}{d\sqrt{d+ex^2}}}{d}+\frac{2}{3d\left(d+ex^2\right)^{3/2}}\right)+b\int \frac{\arctan(cx)}{x\left(ex^2+d\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 221}$$

$$b\int \frac{\arctan(cx)}{x\left(ex^2+d\right)^{5/2}}dx+\frac{1}{2}a\left(\frac{\frac{2}{d\sqrt{d+ex^2}}-\frac{2\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{3/2}}}{d}+\frac{2}{3d\left(d+ex^2\right)^{3/2}}\right)$$

$$\Big\downarrow \text{ 5560}$$

$$b\int \frac{\arctan(cx)}{x\left(ex^2+d\right)^{5/2}}dx+\frac{1}{2}a\left(\frac{\frac{2}{d\sqrt{d+ex^2}}-\frac{2\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{3/2}}}{d}+\frac{2}{3d\left(d+ex^2\right)^{3/2}}\right)$$

input
```
Int[(a + b*ArcTan[c*x])/(x*(d + e*x^2)^(5/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{a+b\arctan\left(cx\right)}{x\left(e\,x^{2}+d\right)^{\frac{5}{2}}}dx$$

input
```
int((a+b*arctan(c*x))/x/(e*x^2+d)^(5/2),x)
```

output
```
int((a+b*arctan(c*x))/x/(e*x^2+d)^(5/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.30

$$\int \frac{a+b\arctan(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{b\arctan\left(cx\right)+a}{\left(ex^2+d\right)^{\frac{5}{2}}x}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arctan(c*x) + a)/(e^3*x^7 + 3*d*e^2*x^5 + 3*d^
2*e*x^3 + d^3*x), x)
```

## Sympy [N/A]

Not integrable

Time = 75.66 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{a+b\arctan(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{a+b\,\mathrm{atan}\left(cx\right)}{x\left(d+ex^2\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*atan(c*x))/x/(e*x**2+d)**(5/2),x)
```

output
```
Integral((a + b*atan(c*x))/(x*(d + e*x**2)**(5/2)), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\arctan(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((a+b*arctan(c*x))/x/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

## Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+b\arctan(cx)}{x\,(d+ex^2)^{5/2}}\,dx = \int \frac{b\arctan(cx)+a}{(ex^2+d)^{\frac{5}{2}}x}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)/((e*x^2 + d)^(5/2)*x), x)
```

## Mupad [N/A]

Not integrable

Time = 1.47 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+b\arctan(cx)}{x\,(d+ex^2)^{5/2}}\,dx = \int \frac{a+b\,\mathrm{atan}(c\,x)}{x\,(e\,x^2+d)^{5/2}}\,dx$$

input
```
int((a + b*atan(c*x))/(x*(d + e*x^2)^(5/2)),x)
```

output
```
int((a + b*atan(c*x))/(x*(d + e*x^2)^(5/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 0.83 (sec) , antiderivative size = 433, normalized size of antiderivative = 18.83

$$\int \frac{a+b\arctan(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx = \frac{4\sqrt{e\,x^2+d}\,a\,d^2+3\sqrt{e\,x^2+d}\,ade\,x^2+3\sqrt{d}\log\left(\frac{\sqrt{e\,x^2+d}-\sqrt{d}+\sqrt{e}\,x}{\sqrt{d}}\right)a\,d^2+6\sqrt{d}\log(}{}$$

input
```
int((a+b*atan(c*x))/x/(e*x^2+d)^(5/2),x)
```

output
```
(4*sqrt(d + e*x**2)*a*d**2 + 3*sqrt(d + e*x**2)*a*d*e*x**2 + 3*sqrt(d)*log
((sqrt(d + e*x**2) - sqrt(d) + sqrt(e)*x)/sqrt(d))*a*d**2 + 6*sqrt(d)*log(
(sqrt(d + e*x**2) - sqrt(d) + sqrt(e)*x)/sqrt(d))*a*d*e*x**2 + 3*sqrt(d)*l
og((sqrt(d + e*x**2) - sqrt(d) + sqrt(e)*x)/sqrt(d))*a*e**2*x**4 - 3*sqrt(
d)*log((sqrt(d + e*x**2) + sqrt(d) + sqrt(e)*x)/sqrt(d))*a*d**2 - 6*sqrt(d
)*log((sqrt(d + e*x**2) + sqrt(d) + sqrt(e)*x)/sqrt(d))*a*d*e*x**2 - 3*sqr
t(d)*log((sqrt(d + e*x**2) + sqrt(d) + sqrt(e)*x)/sqrt(d))*a*e**2*x**4 + 3
*int(atan(c*x)/(sqrt(d + e*x**2)*d**2*x + 2*sqrt(d + e*x**2)*d*e*x**3 + sq
rt(d + e*x**2)*e**2*x**5),x)*b*d**5 + 6*int(atan(c*x)/(sqrt(d + e*x**2)*d*
*2*x + 2*sqrt(d + e*x**2)*d*e*x**3 + sqrt(d + e*x**2)*e**2*x**5),x)*b*d**4
*e*x**2 + 3*int(atan(c*x)/(sqrt(d + e*x**2)*d**2*x + 2*sqrt(d + e*x**2)*d*
e*x**3 + sqrt(d + e*x**2)*e**2*x**5),x)*b*d**3*e**2*x**4)/(3*d**3*(d**2 +
2*d*e*x**2 + e**2*x**4))
```

# 3.1222 $\int \frac{a+b\arctan(cx)}{x^2(d+ex^2)^{5/2}}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 216

$$\int \frac{a+b\arctan(cx)}{x^2(d+ex^2)^{5/2}}\,dx = \frac{bce}{3d^2(c^2d-e)\sqrt{d+ex^2}} + \frac{a+b\arctan(cx)}{3dx(d+ex^2)^{3/2}} + \frac{4(a+b\arctan(cx))}{3d^2x\sqrt{d+ex^2}} - \frac{8\sqrt{d+ex^2}(a+b\arctan(cx))}{3d^3x} - \frac{bc\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{5/2}} + \frac{b(3c^4d^2-12c^2de+8e^2)\operatorname{arctanh}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{3d^3(c^2d-e)^{3/2}}$$

output
```
1/3*b*c*e/d^2/(c^2*d-e)/(e*x^2+d)^(1/2)+1/3*(a+b*arctan(c*x))/d/x/(e*x^2+d
)^(3/2)+4/3*(a+b*arctan(c*x))/d^2/x/(e*x^2+d)^(1/2)-8/3*(e*x^2+d)^(1/2)*(a
+b*arctan(c*x))/d^3/x-b*c*arctanh((e*x^2+d)^(1/2)/d^(1/2))/d^(5/2)+1/3*b*(
3*c^4*d^2-12*c^2*d*e+8*e^2)*arctanh(c*(e*x^2+d)^(1/2)/(c^2*d-e)^(1/2))/d^3
/(c^2*d-e)^(3/2)
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.71 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.94

$$\int \frac{a+b\arctan(cx)}{x^2(d+ex^2)^{5/2}}\,dx = \frac{-\frac{2adex}{(d+ex^2)^{3/2}}+\frac{2e(bcd+5a(-c^2d+e)x)}{(c^2d-e)\sqrt{d+ex^2}}-\frac{6a\sqrt{d+ex^2}}{x}-\frac{2b(3d^2+12dex^2+8e^2x^4)\arctan(cx)}{x(d+ex^2)^{3/2}}+6bc\sqrt{\cdots}}{\cdots}$$

input
```
Integrate[(a + b*ArcTan[c*x])/(x^2*(d + e*x^2)^(5/2)),x]
```

output
```
((-2*a*d*e*x)/(d + e*x^2)^(3/2) + (2*e*(b*c*d + 5*a*(-(c^2*d) + e)*x))/((c
^2*d - e)*Sqrt[d + e*x^2]) - (6*a*Sqrt[d + e*x^2])/x - (2*b*(3*d^2 + 12*d*
e*x^2 + 8*e^2*x^4)*ArcTan[c*x])/(x*(d + e*x^2)^(3/2)) + 6*b*c*Sqrt[d]*Log[
x] - 6*b*c*Sqrt[d]*Log[d + Sqrt[d]*Sqrt[d + e*x^2]] + (b*(3*c^4*d^2 - 12*c
^2*d*e + 8*e^2)*Log[(-12*c*d^3*Sqrt[c^2*d - e]*(c*d - I*e*x + Sqrt[c^2*d -
 e]*Sqrt[d + e*x^2]))/(b*(3*c^4*d^2 - 12*c^2*d*e + 8*e^2)*(I + c*x))])/(c^
2*d - e)^(3/2) + (b*(3*c^4*d^2 - 12*c^2*d*e + 8*e^2)*Log[(-12*c*d^3*Sqrt[c
^2*d - e]*(c*d + I*e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(3*c^4*d^2 -
 12*c^2*d*e + 8*e^2)*(-I + c*x))])/(c^2*d - e)^(3/2))/(6*d^3)
```

## Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.22, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {5511, 27, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x^2(d+ex^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 5511}$$

$$-bc\int -\frac{8e^2x^4+12dex^2+3d^2}{3d^3x\left(c^2x^2+1\right)\left(ex^2+d\right)^{3/2}}dx-\frac{8ex(a+b\arctan(cx))}{3d^3\sqrt{d+ex^2}}-\frac{4ex(a+b\arctan(cx))}{3d^2\left(d+ex^2\right)^{3/2}}-\frac{a+b\arctan(cx)}{dx\left(d+ex^2\right)^{3/2}}$$

$\Big\downarrow$ 27

$$\frac{bc\int \frac{8e^2x^4+12dex^2+3d^2}{x(c^2x^2+1)(ex^2+d)^{3/2}}dx}{3d^3}-\frac{8ex(a+b\arctan(cx))}{3d^3\sqrt{d+ex^2}}-\frac{4ex(a+b\arctan(cx))}{3d^2\left(d+ex^2\right)^{3/2}}-\frac{a+b\arctan(cx)}{dx\left(d+ex^2\right)^{3/2}}$$

$\Big\downarrow$ 7276

$$\frac{bc\int \left(\frac{3d^2}{x(ex^2+d)^{3/2}}+\frac{8e^2x}{c^2(ex^2+d)^{3/2}}-\frac{(3d^2c^4-12dec^2+8e^2)x}{c^2(c^2x^2+1)(ex^2+d)^{3/2}}\right)dx}{3d^3}-\frac{8ex(a+b\arctan(cx))}{3d^3\sqrt{d+ex^2}}-\frac{4ex(a+b\arctan(cx))}{3d^2\left(d+ex^2\right)^{3/2}}-\frac{a+b\arctan(cx)}{dx\left(d+ex^2\right)^{3/2}}$$

$\Big\downarrow$ 2009

$$-\frac{8ex(a+b\arctan(cx))}{3d^3\sqrt{d+ex^2}}-\frac{4ex(a+b\arctan(cx))}{3d^2\left(d+ex^2\right)^{3/2}}-\frac{a+b\arctan(cx)}{dx\left(d+ex^2\right)^{3/2}}+\frac{bc\left(\frac{(3c^4d^2-12c^2de+8e^2)\text{arctanh}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{c(c^2d-e)^{3/2}}-3\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)-\frac{8e}{c^2\sqrt{d+ex^2}}-\frac{3c^4d^2-12c^2de+8e^2}{c^2(c^2d-e)\sqrt{d+ex^2}}+\frac{3d}{\sqrt{d+ex^2}}\right)}{3d^3}$$

input

```
Int[(a + b*ArcTan[c*x])/(x^2*(d + e*x^2)^(5/2)),x]
```

output

```
-((a + b*ArcTan[c*x])/(d*x*(d + e*x^2)^(3/2))) - (4*e*x*(a + b*ArcTan[c*x]
))/(3*d^2*(d + e*x^2)^(3/2)) - (8*e*x*(a + b*ArcTan[c*x]))/(3*d^3*Sqrt[d +
 e*x^2]) + (b*c*((3*d)/Sqrt[d + e*x^2] - (8*e)/(c^2*Sqrt[d + e*x^2]) - (3*
c^4*d^2 - 12*c^2*d*e + 8*e^2)/(c^2*(c^2*d - e)*Sqrt[d + e*x^2]) - 3*Sqrt[d
]*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]] + ((3*c^4*d^2 - 12*c^2*d*e + 8*e^2)*Arc
Tanh[(c*Sqrt[d + e*x^2])/Sqrt[c^2*d - e]])/(c*(c^2*d - e)^(3/2))))/(3*d^3)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

## Maple [F]

$$\int \frac{a+b\arctan(cx)}{x^2\left(e\,x^2+d\right)^{\frac{5}{2}}}dx$$

input
```
int((a+b*arctan(c*x))/x^2/(e*x^2+d)^(5/2),x)
```

output
```
int((a+b*arctan(c*x))/x^2/(e*x^2+d)^(5/2),x)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 655 vs. $2(186) = 372$.

Time = 1.02 (sec) , antiderivative size = 2720, normalized size of antiderivative = 12.59

$$\int \frac{a + b\arctan(cx)}{x^2 (d + ex^2)^{5/2}}\, dx = \text{Too large to display}$$

input
```
integrate((a+b*arctan(c*x))/x^2/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
[-1/12*(((3*b*c^4*d^2*e^2 - 12*b*c^2*d*e^3 + 8*b*e^4)*x^5 + 2*(3*b*c^4*d^3
*e - 12*b*c^2*d^2*e^2 + 8*b*d*e^3)*x^3 + (3*b*c^4*d^4 - 12*b*c^2*d^3*e + 8
*b*d^2*e^2)*x)*sqrt(c^2*d - e)*log((c^4*e^2*x^4 + 8*c^4*d^2 - 8*c^2*d*e +
2*(4*c^4*d*e - 3*c^2*e^2)*x^2 - 4*(c^3*e*x^2 + 2*c^3*d - c*e)*sqrt(c^2*d -
 e)*sqrt(e*x^2 + d) + e^2)/(c^4*x^4 + 2*c^2*x^2 + 1)) - 6*((b*c^5*d^2*e^2
- 2*b*c^3*d*e^3 + b*c*e^4)*x^5 + 2*(b*c^5*d^3*e - 2*b*c^3*d^2*e^2 + b*c*d*
e^3)*x^3 + (b*c^5*d^4 - 2*b*c^3*d^3*e + b*c*d^2*e^2)*x)*sqrt(d)*log(-(e*x^
2 - 2*sqrt(e*x^2 + d)*sqrt(d) + 2*d)/x^2) + 4*(3*a*c^4*d^4 - 6*a*c^2*d^3*e
 + 3*a*d^2*e^2 + 8*(a*c^4*d^2*e^2 - 2*a*c^2*d*e^3 + a*e^4)*x^4 - (b*c^3*d^
2*e^2 - b*c*d*e^3)*x^3 + 12*(a*c^4*d^3*e - 2*a*c^2*d^2*e^2 + a*d*e^3)*x^2
- (b*c^3*d^3*e - b*c*d^2*e^2)*x + (3*b*c^4*d^4 - 6*b*c^2*d^3*e + 3*b*d^2*e
^2 + 8*(b*c^4*d^2*e^2 - 2*b*c^2*d*e^3 + b*e^4)*x^4 + 12*(b*c^4*d^3*e - 2*b
*c^2*d^2*e^2 + b*d*e^3)*x^2)*arctan(c*x))*sqrt(e*x^2 + d))/((c^4*d^5*e^2 -
 2*c^2*d^4*e^3 + d^3*e^4)*x^5 + 2*(c^4*d^6*e - 2*c^2*d^5*e^2 + d^4*e^3)*x^
3 + (c^4*d^7 - 2*c^2*d^6*e + d^5*e^2)*x), 1/6*(((3*b*c^4*d^2*e^2 - 12*b*c^
2*d*e^3 + 8*b*e^4)*x^5 + 2*(3*b*c^4*d^3*e - 12*b*c^2*d^2*e^2 + 8*b*d*e^3)*
x^3 + (3*b*c^4*d^4 - 12*b*c^2*d^3*e + 8*b*d^2*e^2)*x)*sqrt(-c^2*d + e)*arc
tan(-1/2*(c^2*e*x^2 + 2*c^2*d - e)*sqrt(-c^2*d + e)*sqrt(e*x^2 + d)/(c^3*d
^2 - c*d*e + (c^3*d*e - c*e^2)*x^2)) + 3*((b*c^5*d^2*e^2 - 2*b*c^3*d*e^3 +
 b*c*e^4)*x^5 + 2*(b*c^5*d^3*e - 2*b*c^3*d^2*e^2 + b*c*d*e^3)*x^3 + (b*...
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{x^2\left(d+ex^2\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*atan(c*x))/x**2/(e*x**2+d)**(5/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\arctan(cx)}{x^2\left(d+ex^2\right)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arctan(c*x))/x^2/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

## Giac [F]

$$\int \frac{a+b\arctan(cx)}{x^2\left(d+ex^2\right)^{5/2}}\,dx = \int \frac{b\arctan(cx)+a}{\left(ex^2+d\right)^{\frac{5}{2}}x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^2/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)/((e*x^2 + d)^(5/2)*x^2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{x^2\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{a+b\,\mathrm{atan}(c\,x)}{x^2\,{\left(e\,x^2+d\right)}^{5/2}}\,dx$$

input
```
int((a + b*atan(c*x))/(x^2*(d + e*x^2)^(5/2)),x)
```

output
```
int((a + b*atan(c*x))/(x^2*(d + e*x^2)^(5/2)), x)
```

## Reduce [F]

$$\int \frac{a+b\arctan(cx)}{x^2\left(d+ex^2\right)^{5/2}}\,dx=\frac{-3\sqrt{e\,x^2+d}\,a\,d^2-12\sqrt{e\,x^2+d}\,ade\,x^2-8\sqrt{e\,x^2+d}\,a\,e^2x^4+8\sqrt{e}\,a\,d^2x+16\sqrt{e}}{}$$

input
```
int((a+b*atan(c*x))/x^2/(e*x^2+d)^(5/2),x)
```

output
```
( - 3*sqrt(d + e*x**2)*a*d**2 - 12*sqrt(d + e*x**2)*a*d*e*x**2 - 8*sqrt(d
+ e*x**2)*a*e**2*x**4 + 8*sqrt(e)*a*d**2*x + 16*sqrt(e)*a*d*e*x**3 + 8*sqr
t(e)*a*e**2*x**5 + 3*int(atan(c*x)/(sqrt(d + e*x**2)*d**2*x**2 + 2*sqrt(d
+ e*x**2)*d*e*x**4 + sqrt(d + e*x**2)*e**2*x**6),x)*b*d**5*x + 6*int(atan(
c*x)/(sqrt(d + e*x**2)*d**2*x**2 + 2*sqrt(d + e*x**2)*d*e*x**4 + sqrt(d +
e*x**2)*e**2*x**6),x)*b*d**4*e*x**3 + 3*int(atan(c*x)/(sqrt(d + e*x**2)*d*
*2*x**2 + 2*sqrt(d + e*x**2)*d*e*x**4 + sqrt(d + e*x**2)*e**2*x**6),x)*b*d
**3*e**2*x**5)/(3*d**3*x*(d**2 + 2*d*e*x**2 + e**2*x**4))
```

# 3.1223 $\int \frac{a+b\arctan(cx)}{x^3(d+ex^2)^{5/2}}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{a+b\arctan(cx)}{x^3(d+ex^2)^{5/2}}\,dx = -\frac{5ae}{6d^2(d+ex^2)^{3/2}} - \frac{a}{2dx^2(d+ex^2)^{3/2}} - \frac{5ae}{2d^3\sqrt{d+ex^2}} + \frac{5ae\,\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{2d^{7/2}} + b\text{Int}\left(\frac{\arctan(cx)}{x^3(d+ex^2)^{5/2}},x\right)$$

output
```
-5/6*a*e/d^2/(e*x^2+d)^(3/2)-1/2*a/d/x^2/(e*x^2+d)^(3/2)-5/2*a*e/d^3/(e*x^
2+d)^(1/2)+5/2*a*e*arctanh((e*x^2+d)^(1/2)/d^(1/2))/d^(7/2)+b*Defer(Int)(a
rctan(c*x)/x^3/(e*x^2+d)^(5/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 15.86 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{a+b\arctan(cx)}{x^3(d+ex^2)^{5/2}}\,dx = \int \frac{a+b\arctan(cx)}{x^3(d+ex^2)^{5/2}}\,dx$$

input
```
Integrate[(a + b*ArcTan[c*x])/(x^3*(d + e*x^2)^(5/2)),x]
```

output
```
Integrate[(a + b*ArcTan[c*x])/(x^3*(d + e*x^2)^(5/2)), x]
```

## Rubi [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx$$

$$\Big\downarrow \ 5517$$

$$a\int \frac{1}{x^3\left(ex^2+d\right)^{5/2}}dx+b\int \frac{\arctan(cx)}{x^3\left(ex^2+d\right)^{5/2}}dx$$

$$\Big\downarrow \ 243$$

$$\frac{1}{2}a\int \frac{1}{x^4\left(ex^2+d\right)^{5/2}}dx^2+b\int \frac{\arctan(cx)}{x^3\left(ex^2+d\right)^{5/2}}dx$$

$$\Big\downarrow \ 52$$

$$\frac{1}{2}a\left(-\frac{5e\int \frac{1}{x^2\left(ex^2+d\right)^{5/2}}dx^2}{2d}-\frac{1}{dx^2\left(d+ex^2\right)^{3/2}}\right)+b\int \frac{\arctan(cx)}{x^3\left(ex^2+d\right)^{5/2}}dx$$

$$\Big\downarrow \ 61$$

$$\frac{1}{2}a\left(-\frac{5e\left(\frac{\int \frac{1}{x^2\left(ex^2+d\right)^{3/2}}dx^2}{d}+\frac{2}{3d\left(d+ex^2\right)^{3/2}}\right)}{2d}-\frac{1}{dx^2\left(d+ex^2\right)^{3/2}}\right)+b\int \frac{\arctan(cx)}{x^3\left(ex^2+d\right)^{5/2}}dx$$

$$\Big\downarrow \ 61$$

$$\frac{1}{2}a\left(-\frac{5e\left(\frac{\frac{\int \frac{1}{x^2\sqrt{ex^2+d}}dx^2}{d}+\frac{2}{d\sqrt{d+ex^2}}}{d}+\frac{2}{3d(d+ex^2)^{3/2}}\right)}{2d}-\frac{1}{dx^2\left(d+ex^2\right)^{3/2}}\right)+b\int \frac{\arctan(cx)}{x^3\left(ex^2+d\right)^{5/2}}dx$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}a\left(-\frac{5e\left(\frac{\frac{2\int \frac{1}{\frac{x^4}{e}-\frac{d}{e}}d\sqrt{ex^2+d}}{de}+\frac{2}{d\sqrt{d+ex^2}}}{d}+\frac{2}{3d(d+ex^2)^{3/2}}\right)}{2d}-\frac{1}{dx^2\left(d+ex^2\right)^{3/2}}\right)+b\int \frac{\arctan(cx)}{x^3\left(ex^2+d\right)^{5/2}}dx$$

$$\Big\downarrow \text{221}$$

$$b\int \frac{\arctan(cx)}{x^3\left(ex^2+d\right)^{5/2}}dx+\frac{1}{2}a\left(-\frac{5e\left(\frac{\frac{2}{d\sqrt{d+ex^2}}-\frac{2\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{3/2}}}{d}+\frac{2}{3d(d+ex^2)^{3/2}}\right)}{2d}-\frac{1}{dx^2\left(d+ex^2\right)^{3/2}}\right)$$

$$\Big\downarrow \text{5560}$$

$$b\int \frac{\arctan(cx)}{x^3\left(ex^2+d\right)^{5/2}}dx+\frac{1}{2}a\left(-\frac{5e\left(\frac{\frac{2}{d\sqrt{d+ex^2}}-\frac{2\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{3/2}}}{d}+\frac{2}{3d(d+ex^2)^{3/2}}\right)}{2d}-\frac{1}{dx^2\left(d+ex^2\right)^{3/2}}\right)$$

input
```
Int[(a + b*ArcTan[c*x])/(x^3*(d + e*x^2)^(5/2)),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{a+b\arctan (cx)}{x^{3} (e\,x^{2}+d)^{\frac{5}{2}}}dx$$

input
```
int((a+b*arctan(c*x))/x^3/(e*x^2+d)^(5/2),x)
```

output
```
int((a+b*arctan(c*x))/x^3/(e*x^2+d)^(5/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.39

$$\int \frac{a+b\arctan(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{b\arctan (cx)+a}{(ex^2+d)^{\frac{5}{2}}x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^3/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arctan(c*x) + a)/(e^3*x^9 + 3*d*e^2*x^7 + 3*d^
2*e*x^5 + d^3*x^3), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{x^3(d+ex^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*atan(c*x))/x**3/(e*x**2+d)**(5/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\arctan(cx)}{x^3(d+ex^2)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arctan(c*x))/x^3/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

## Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+b\arctan(cx)}{x^3(d+ex^2)^{5/2}}\,dx = \int \frac{b\arctan(cx)+a}{(ex^2+d)^{\frac{5}{2}}x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^3/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)/((e*x^2 + d)^(5/2)*x^3), x)
```

## Mupad [N/A]

Not integrable

Time = 1.52 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+b\arctan(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{a+b\,\mathrm{atan}(c\,x)}{x^3\,{\left(e\,x^2+d\right)}^{5/2}}\,dx$$

input
```
int((a + b*atan(c*x))/(x^3*(d + e*x^2)^(5/2)),x)
```

output
```
int((a + b*atan(c*x))/(x^3*(d + e*x^2)^(5/2)), x)
```

## Reduce [N/A]

Not integrable

Time = 200.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{a+b\arctan(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{atan(cx)\,b+a}{x^3\left(e\,x^2+d\right)^{\frac{5}{2}}}dx$$

input
```
int((a+b*atan(c*x))/x^3/(e*x^2+d)^(5/2),x)
```

output
```
int((a+b*atan(c*x))/x^3/(e*x^2+d)^(5/2),x)
```

# 3.1224 $\int \frac{a+b\arctan(cx)}{x^4(d+ex^2)^{5/2}}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 294

$$\begin{aligned}\int \frac{a+b\arctan(cx)}{x^4(d+ex^2)^{5/2}}\,dx &= -\frac{bce(c^2d+e)}{6d^3(c^2d-e)\sqrt{d+ex^2}}-\frac{bc}{6d^2x^2\sqrt{d+ex^2}}\\ &+\frac{a+b\arctan(cx)}{3dx^3(d+ex^2)^{3/2}}+\frac{2(a+b\arctan(cx))}{d^2x^3\sqrt{d+ex^2}}-\frac{8\sqrt{d+ex^2}(a+b\arctan(cx))}{3d^3x^3}\\ &+\frac{16e\sqrt{d+ex^2}(a+b\arctan(cx))}{3d^4x}+\frac{bc(2c^2d+15e)\,\text{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{6d^{7/2}}\\ &-\frac{b(c^2d-2e)(c^4d^2+8c^2de-8e^2)\,\text{arctanh}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{3d^4(c^2d-e)^{3/2}}\end{aligned}$$

output
```
-1/6*b*c*e*(c^2*d+e)/d^3/(c^2*d-e)/(e*x^2+d)^(1/2)-1/6*b*c/d^2/x^2/(e*x^2+
d)^(1/2)+1/3*(a+b*arctan(c*x))/d/x^3/(e*x^2+d)^(3/2)+2*(a+b*arctan(c*x))/d
^2/x^3/(e*x^2+d)^(1/2)-8/3*(e*x^2+d)^(1/2)*(a+b*arctan(c*x))/d^3/x^3+16/3*
e*(e*x^2+d)^(1/2)*(a+b*arctan(c*x))/d^4/x+1/6*b*c*(2*c^2*d+15*e)*arctanh((
e*x^2+d)^(1/2)/d^(1/2))/d^(7/2)-1/3*b*(c^2*d-2*e)*(c^4*d^2+8*c^2*d*e-8*e^2
)*arctanh(c*(e*x^2+d)^(1/2)/(c^2*d-e)^(1/2))/d^4/(c^2*d-e)^(3/2)
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.18 (sec) , antiderivative size = 510, normalized size of antiderivative = 1.73

$$\int \frac{a+b\arctan(cx)}{x^4(d+ex^2)^{5/2}}\,dx =$$

$$-\frac{\frac{2a(d^3-6d^2ex^2-24de^2x^4-16e^3x^6)}{x^3(d+ex^2)^{3/2}}+\frac{bcd(e(-d+ex^2)+c^2d(d+ex^2))}{(c^2d-e)x^2\sqrt{d+ex^2}}+\frac{2b(d^3-6d^2ex^2-24de^2x^4-16e^3x^6)\arctan(cx)}{x^3(d+ex^2)^{3/2}}+bc\sqrt{d}(2c^2d+$$

input
```
Integrate[(a + b*ArcTan[c*x])/(x^4*(d + e*x^2)^(5/2)),x]
```

output
```
-1/6*((2*a*(d^3 - 6*d^2*e*x^2 - 24*d*e^2*x^4 - 16*e^3*x^6))/(x^3*(d + e*x^
2)^(3/2)) + (b*c*d*(e*(-d + e*x^2) + c^2*d*(d + e*x^2)))/((c^2*d - e)*x^2*
Sqrt[d + e*x^2]) + (2*b*(d^3 - 6*d^2*e*x^2 - 24*d*e^2*x^4 - 16*e^3*x^6)*Ar
cTan[c*x])/(x^3*(d + e*x^2)^(3/2)) + b*c*Sqrt[d]*(2*c^2*d + 15*e)*Log[x] -
 b*c*Sqrt[d]*(2*c^2*d + 15*e)*Log[d + Sqrt[d]*Sqrt[d + e*x^2]] + (b*(c^6*d
^3 + 6*c^4*d^2*e - 24*c^2*d*e^2 + 16*e^3)*Log[(12*c*d^4*Sqrt[c^2*d - e]*(c
*d - I*e*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(c^6*d^3 + 6*c^4*d^2*e -
 24*c^2*d*e^2 + 16*e^3)*(I + c*x))])/(c^2*d - e)^(3/2) + (b*(c^6*d^3 + 6*c
^4*d^2*e - 24*c^2*d*e^2 + 16*e^3)*Log[(12*c*d^4*Sqrt[c^2*d - e]*(c*d + I*e
*x + Sqrt[c^2*d - e]*Sqrt[d + e*x^2]))/(b*(c^6*d^3 + 6*c^4*d^2*e - 24*c^2*
d*e^2 + 16*e^3)*(-I + c*x))])/(c^2*d - e)^(3/2))/d^4
```

## Rubi [A] (verified)

Time = 1.42 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.35, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {5511, 27, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\arctan(cx)}{x^4(d+ex^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{5511}$$

$$-bc\int -\frac{-16e^3x^6-24de^2x^4-6d^2ex^2+d^3}{3d^4x^3\left(c^2x^2+1\right)\left(ex^2+d\right)^{3/2}}dx+\frac{16e^2x(a+b\arctan(cx))}{3d^4\sqrt{d+ex^2}}+\frac{8e^2x(a+b\arctan(cx))}{3d^3\left(d+ex^2\right)^{3/2}}+\frac{2e(a+b\arctan(cx))}{d^2x\left(d+ex^2\right)^{3/2}}-\frac{a+b\arctan(cx)}{3dx^3\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{bc\int \frac{-16e^3x^6-24de^2x^4-6d^2ex^2+d^3}{x^3(c^2x^2+1)(ex^2+d)^{3/2}}dx}{3d^4}+\frac{16e^2x(a+b\arctan(cx))}{3d^4\sqrt{d+ex^2}}+\frac{8e^2x(a+b\arctan(cx))}{3d^3\left(d+ex^2\right)^{3/2}}+\frac{2e(a+b\arctan(cx))}{d^2x\left(d+ex^2\right)^{3/2}}-\frac{a+b\arctan(cx)}{3dx^3\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \text{7276}$$

$$\frac{bc\int \left(\frac{d^3}{x^3(ex^2+d)^{3/2}}-\frac{(dc^2+6e)d^2}{x(ex^2+d)^{3/2}}-\frac{16e^3x}{c^2(ex^2+d)^{3/2}}+\frac{(c^2d-2e)(d^2c^4+8dec^2-8e^2)x}{c^2(c^2x^2+1)(ex^2+d)^{3/2}}\right)dx}{3d^4}+\frac{16e^2x(a+b\arctan(cx))}{3d^4\sqrt{d+ex^2}}+\frac{8e^2x(a+b\arctan(cx))}{3d^3\left(d+ex^2\right)^{3/2}}+\frac{2e(a+b\arctan(cx))}{d^2x\left(d+ex^2\right)^{3/2}}-\frac{a+b\arctan(cx)}{3dx^3\left(d+ex^2\right)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{16e^2x(a+b\arctan(cx))}{3d^4\sqrt{d+ex^2}}+\frac{8e^2x(a+b\arctan(cx))}{3d^3\left(d+ex^2\right)^{3/2}}+\frac{2e(a+b\arctan(cx))}{d^2x\left(d+ex^2\right)^{3/2}}-\frac{a+b\arctan(cx)}{3dx^3\left(d+ex^2\right)^{3/2}}+\frac{bc\left(\sqrt{d}(c^2d+6e)\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)-\frac{(c^2d-2e)(c^4d^2+8c^2de-8e^2)\operatorname{arctanh}\left(\frac{c\sqrt{d+ex^2}}{\sqrt{c^2d-e}}\right)}{c(c^2d-e)^{3/2}}+\frac{3}{2}\sqrt{d}e\operatorname{arctanh}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)+\frac{}{c^2\sqrt{}}\right.}{3d^4}$$

input
```
Int[(a + b*ArcTan[c*x])/(x^4*(d + e*x^2)^(5/2)),x]
```

output
```
-1/3*(a + b*ArcTan[c*x])/(d*x^3*(d + e*x^2)^(3/2)) + (2*e*(a + b*ArcTan[c*
x]))/(d^2*x*(d + e*x^2)^(3/2)) + (8*e^2*x*(a + b*ArcTan[c*x]))/(3*d^3*(d +
 e*x^2)^(3/2)) + (16*e^2*x*(a + b*ArcTan[c*x]))/(3*d^4*Sqrt[d + e*x^2]) +
(b*c*((-3*d*e)/(2*Sqrt[d + e*x^2]) + (16*e^2)/(c^2*Sqrt[d + e*x^2]) - (d*(
c^2*d + 6*e))/Sqrt[d + e*x^2] + ((c^2*d - 2*e)*(c^4*d^2 + 8*c^2*d*e - 8*e^
2))/(c^2*(c^2*d - e)*Sqrt[d + e*x^2]) - d^2/(2*x^2*Sqrt[d + e*x^2]) + (3*S
qrt[d]*e*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/2 + Sqrt[d]*(c^2*d + 6*e)*ArcTa
nh[Sqrt[d + e*x^2]/Sqrt[d]] - ((c^2*d - 2*e)*(c^4*d^2 + 8*c^2*d*e - 8*e^2)
*ArcTanh[(c*Sqrt[d + e*x^2])/Sqrt[c^2*d - e]])/(c*(c^2*d - e)^(3/2))))/(3*
d^4)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

## Maple [F]

$$\int \frac{a+b\arctan(cx)}{x^4 (e\,x^2+d)^{\frac{5}{2}}}dx$$

input
```
int((a+b*arctan(c*x))/x^4/(e*x^2+d)^(5/2),x)
```

output
```
int((a+b*arctan(c*x))/x^4/(e*x^2+d)^(5/2),x)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 840 vs. $2(256) = 512$.

Time = 1.72 (sec) , antiderivative size = 3466, normalized size of antiderivative = 11.79

$$\int \frac{a+b\arctan(cx)}{x^4 (d+ex^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*arctan(c*x))/x^4/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
Too large to include
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{x^4 (d+ex^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*atan(c*x))/x**4/(e*x**2+d)**(5/2),x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{a+b\arctan(cx)}{x^4\left(d+ex^2\right)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arctan(c*x))/x^4/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

## Giac [F]

$$\int \frac{a+b\arctan(cx)}{x^4\left(d+ex^2\right)^{5/2}}\,dx = \int \frac{b\arctan(cx)+a}{(ex^2+d)^{\frac{5}{2}}x^4}\,dx$$

input
```
integrate((a+b*arctan(c*x))/x^4/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)/((e*x^2 + d)^(5/2)*x^4), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arctan(cx)}{x^4\left(d+ex^2\right)^{5/2}}\,dx = \int \frac{a+b\,\mathrm{atan}(c\,x)}{x^4\,\left(e\,x^2+d\right)^{5/2}}\,dx$$

input
```
int((a + b*atan(c*x))/(x^4*(d + e*x^2)^(5/2)),x)
```

output
```
int((a + b*atan(c*x))/(x^4*(d + e*x^2)^(5/2)), x)
```

## Reduce [F]

$$\int \frac{a + b\arctan(cx)}{x^4 (d + ex^2)^{5/2}}\,dx = \int \frac{atan(cx)\,b + a}{x^4 \left(e\,x^2 + d\right)^{\frac{5}{2}}}dx$$

input
```
int((a+b*atan(c*x))/x^4/(e*x^2+d)^(5/2),x)
```

output
```
int((a+b*atan(c*x))/x^4/(e*x^2+d)^(5/2),x)
```

# 3.1225 $\int \frac{\arctan(ax)}{(c+dx^2)^{7/2}}\,dx$



## Optimal result

Integrand size = 16, antiderivative size = 208

$$\int \frac{\arctan(ax)}{(c+dx^2)^{7/2}}\,dx = -\frac{a}{15c(a^2c-d)(c+dx^2)^{3/2}} - \frac{a(7a^2c-4d)}{15c^2(a^2c-d)^2\sqrt{c+dx^2}} + \frac{x\arctan(ax)}{5c(c+dx^2)^{5/2}} + \frac{4x\arctan(ax)}{15c^2(c+dx^2)^{3/2}} + \frac{8x\arctan(ax)}{15c^3\sqrt{c+dx^2}} + \frac{(15a^4c^2-20a^2cd+8d^2)\,\mathrm{arctanh}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c-d}}\right)}{15c^3(a^2c-d)^{5/2}}$$

output
```
-1/15*a/c/(a^2*c-d)/(d*x^2+c)^(3/2)-1/15*a*(7*a^2*c-4*d)/c^2/(a^2*c-d)^2/(
d*x^2+c)^(1/2)+1/5*x*arctan(a*x)/c/(d*x^2+c)^(5/2)+4/15*x*arctan(a*x)/c^2/
(d*x^2+c)^(3/2)+8/15*x*arctan(a*x)/c^3/(d*x^2+c)^(1/2)+1/15*(15*a^4*c^2-20
*a^2*c*d+8*d^2)*arctanh(a*(d*x^2+c)^(1/2)/(a^2*c-d)^(1/2))/c^3/(a^2*c-d)^(
5/2)
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.66

$$\int \frac{\arctan(ax)}{(c+dx^2)^{7/2}}\,dx = \frac{-\frac{2ac(-d(5c+4dx^2)+a^2c(8c+7dx^2))}{(-a^2c+d)^2(c+dx^2)^{3/2}} + \frac{2x(15c^2+20cdx^2+8d^2x^4)\arctan(ax)}{(c+dx^2)^{5/2}} + \frac{(15a^4c^2-20a^2cd+8d^2)\log\left(-\cdots\right.}{\cdots}}{\cdots}$$

input
```
Integrate[ArcTan[a*x]/(c + d*x^2)^(7/2),x]
```

output
```
((-2*a*c*(-(d*(5*c + 4*d*x^2)) + a^2*c*(8*c + 7*d*x^2)))/((-(a^2*c) + d)^2
*(c + d*x^2)^(3/2)) + (2*x*(15*c^2 + 20*c*d*x^2 + 8*d^2*x^4)*ArcTan[a*x])/
(c + d*x^2)^(5/2) + ((15*a^4*c^2 - 20*a^2*c*d + 8*d^2)*Log[(-60*a*c^3*(a^2
*c - d)^(3/2)*(a*c - I*d*x + Sqrt[a^2*c - d]*Sqrt[c + d*x^2]))/((15*a^4*c^
2 - 20*a^2*c*d + 8*d^2)*(I + a*x))])/(a^2*c - d)^(5/2) + ((15*a^4*c^2 - 20
*a^2*c*d + 8*d^2)*Log[(-60*a*c^3*(a^2*c - d)^(3/2)*(a*c + I*d*x + Sqrt[a^2
*c - d]*Sqrt[c + d*x^2]))/((15*a^4*c^2 - 20*a^2*c*d + 8*d^2)*(-I + a*x))])
/(a^2*c - d)^(5/2))/(30*c^3)
```

## Rubi [A] (warning: unable to verify)

Time = 1.07 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {5447, 27, 7266, 1192, 25, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)}{(c+dx^2)^{7/2}}\,dx$$

$$\Big\downarrow \text{5447}$$

$$-a\int \frac{x(8d^2x^4+20cdx^2+15c^2)}{15c^3(a^2x^2+1)(dx^2+c)^{5/2}}dx + \frac{8x\arctan(ax)}{15c^3\sqrt{c+dx^2}} + \frac{4x\arctan(ax)}{15c^2(c+dx^2)^{3/2}} + \frac{x\arctan(ax)}{5c(c+dx^2)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\int \frac{x(8d^2x^4+20cdx^2+15c^2)}{(a^2x^2+1)(dx^2+c)^{5/2}}dx}{15c^3}+\frac{8x\arctan(ax)}{15c^3\sqrt{c+dx^2}}+\frac{4x\arctan(ax)}{15c^2\left(c+dx^2\right)^{3/2}}+\frac{x\arctan(ax)}{5c\left(c+dx^2\right)^{5/2}}$$

$$\Big\downarrow \text{7266}$$

$$-\frac{a\int \frac{8d^2x^4+20cdx^2+15c^2}{(a^2x^2+1)(dx^2+c)^{5/2}}dx^2}{30c^3}+\frac{8x\arctan(ax)}{15c^3\sqrt{c+dx^2}}+\frac{4x\arctan(ax)}{15c^2\left(c+dx^2\right)^{3/2}}+\frac{x\arctan(ax)}{5c\left(c+dx^2\right)^{5/2}}$$

$$\Big\downarrow \text{1192}$$

$$-\frac{a\int -\frac{8d^2x^8+4cd^2x^4+3c^2d^2}{x^8(-a^2x^4+a^2c-d)}d\sqrt{dx^2+c}}{15c^3d^2}+\frac{8x\arctan(ax)}{15c^3\sqrt{c+dx^2}}+\frac{4x\arctan(ax)}{15c^2\left(c+dx^2\right)^{3/2}}+\frac{x\arctan(ax)}{5c\left(c+dx^2\right)^{5/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{a\int \frac{8d^2x^8+4cd^2x^4+3c^2d^2}{x^8(-a^2x^4+a^2c-d)}d\sqrt{dx^2+c}}{15c^3d^2}+\frac{8x\arctan(ax)}{15c^3\sqrt{c+dx^2}}+\frac{4x\arctan(ax)}{15c^2\left(c+dx^2\right)^{3/2}}+\frac{x\arctan(ax)}{5c\left(c+dx^2\right)^{5/2}}$$

$$\Big\downarrow \text{1584}$$

$$\frac{a\int \left(-\frac{(15c^2a^4-20cda^2+8d^2)d^2}{(d-a^2c)^2(a^2x^4-a^2c+d)}+\frac{c(7a^2c-4d)d^2}{(a^2c-d)^2x^4}+\frac{3c^2d^2}{(a^2c-d)x^8}\right)d\sqrt{dx^2+c}}{15c^3d^2}+\frac{8x\arctan(ax)}{15c^3\sqrt{c+dx^2}}+\frac{4x\arctan(ax)}{15c^2\left(c+dx^2\right)^{3/2}}+\frac{x\arctan(ax)}{5c\left(c+dx^2\right)^{5/2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a\left(\frac{c^2d^2}{x^6(a^2c-d)}+\frac{cd^2(7a^2c-4d)}{x^2(a^2c-d)^2}-\frac{d^2(15a^4c^2-20a^2cd+8d^2)\text{arctanh}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c-d}}\right)}{a(a^2c-d)^{5/2}}\right)}{15c^3d^2}+\frac{8x\arctan(ax)}{15c^3\sqrt{c+dx^2}}+\frac{4x\arctan(ax)}{15c^2\left(c+dx^2\right)^{3/2}}+\frac{x\arctan(ax)}{5c\left(c+dx^2\right)^{5/2}}$$

input
```
Int[ArcTan[a*x]/(c + d*x^2)^(7/2),x]
```

output
```
(x*ArcTan[a*x])/(5*c*(c + d*x^2)^(5/2)) + (4*x*ArcTan[a*x])/(15*c^2*(c + d
*x^2)^(3/2)) + (8*x*ArcTan[a*x])/(15*c^3*Sqrt[c + d*x^2]) - (a*((c^2*d^2)/
((a^2*c - d)*x^6) + (c*(7*a^2*c - 4*d)*d^2)/((a^2*c - d)^2*x^2) - (d^2*(15
*a^4*c^2 - 20*a^2*c*d + 8*d^2)*ArcTanh[(a*Sqrt[c + d*x^2])/Sqrt[a^2*c - d]
])/(a*(a^2*c - d)^(5/2))))/(15*c^3*d^2)
```

## Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5447
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Simp[(a + b*ArcTan[c*x])   u,
x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2), x], x], x]] /; FreeQ
[{a, b, c, d, e}, x] && (IntegerQ[q] || ILtQ[q + 1/2, 0])
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

## Maple [F]

$$\int \frac{\arctan (ax)}{(d\,x^{2}+c)^{\frac{7}{2}}}dx$$

input
```
int(arctan(a*x)/(d*x^2+c)^(7/2),x)
```

output
```
int(arctan(a*x)/(d*x^2+c)^(7/2),x)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 619 vs. $2(180) = 360$.

Time = 0.20 (sec) , antiderivative size = 1280, normalized size of antiderivative = 6.15

$$\int \frac{\arctan(ax)}{(c+dx^2)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate(arctan(a*x)/(d*x^2+c)^(7/2),x, algorithm="fricas")
```

output
```
[1/60*((15*a^4*c^5 - 20*a^2*c^4*d + (15*a^4*c^2*d^3 - 20*a^2*c*d^4 + 8*d^5
)*x^6 + 8*c^3*d^2 + 3*(15*a^4*c^3*d^2 - 20*a^2*c^2*d^3 + 8*c*d^4)*x^4 + 3*
(15*a^4*c^4*d - 20*a^2*c^3*d^2 + 8*c^2*d^3)*x^2)*sqrt(a^2*c - d)*log((a^4*
d^2*x^4 + 8*a^4*c^2 - 8*a^2*c*d + 2*(4*a^4*c*d - 3*a^2*d^2)*x^2 + 4*(a^3*d
*x^2 + 2*a^3*c - a*d)*sqrt(a^2*c - d)*sqrt(d*x^2 + c) + d^2)/(a^4*x^4 + 2*
a^2*x^2 + 1)) - 4*(8*a^5*c^5 - 13*a^3*c^4*d + 5*a*c^3*d^2 + (7*a^5*c^3*d^2
 - 11*a^3*c^2*d^3 + 4*a*c*d^4)*x^4 + 3*(5*a^5*c^4*d - 8*a^3*c^3*d^2 + 3*a*
c^2*d^3)*x^2 - (8*(a^6*c^3*d^2 - 3*a^4*c^2*d^3 + 3*a^2*c*d^4 - d^5)*x^5 +
20*(a^6*c^4*d - 3*a^4*c^3*d^2 + 3*a^2*c^2*d^3 - c*d^4)*x^3 + 15*(a^6*c^5 -
 3*a^4*c^4*d + 3*a^2*c^3*d^2 - c^2*d^3)*x)*arctan(a*x))*sqrt(d*x^2 + c))/(
a^6*c^9 - 3*a^4*c^8*d + 3*a^2*c^7*d^2 - c^6*d^3 + (a^6*c^6*d^3 - 3*a^4*c^5
*d^4 + 3*a^2*c^4*d^5 - c^3*d^6)*x^6 + 3*(a^6*c^7*d^2 - 3*a^4*c^6*d^3 + 3*a
^2*c^5*d^4 - c^4*d^5)*x^4 + 3*(a^6*c^8*d - 3*a^4*c^7*d^2 + 3*a^2*c^6*d^3 -
 c^5*d^4)*x^2), 1/30*((15*a^4*c^5 - 20*a^2*c^4*d + (15*a^4*c^2*d^3 - 20*a^
2*c*d^4 + 8*d^5)*x^6 + 8*c^3*d^2 + 3*(15*a^4*c^3*d^2 - 20*a^2*c^2*d^3 + 8*
c*d^4)*x^4 + 3*(15*a^4*c^4*d - 20*a^2*c^3*d^2 + 8*c^2*d^3)*x^2)*sqrt(-a^2*
c + d)*arctan(-1/2*(a^2*d*x^2 + 2*a^2*c - d)*sqrt(-a^2*c + d)*sqrt(d*x^2 +
 c)/(a^3*c^2 - a*c*d + (a^3*c*d - a*d^2)*x^2)) - 2*(8*a^5*c^5 - 13*a^3*c^4
*d + 5*a*c^3*d^2 + (7*a^5*c^3*d^2 - 11*a^3*c^2*d^3 + 4*a*c*d^4)*x^4 + 3*(5
*a^5*c^4*d - 8*a^3*c^3*d^2 + 3*a*c^2*d^3)*x^2 - (8*(a^6*c^3*d^2 - 3*a^4...
```

## Sympy [F]

$$\int \frac{\arctan(ax)}{(c+dx^2)^{7/2}}\,dx = \int \frac{\operatorname{atan}(ax)}{(c+dx^2)^{\frac{7}{2}}}\,dx$$

input
```
integrate(atan(a*x)/(d*x**2+c)**(7/2),x)
```

output
```
Integral(atan(a*x)/(c + d*x**2)**(7/2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)}{(c+dx^2)^{7/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(arctan(a*x)/(d*x^2+c)^(7/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(d-a^2*c>0)', see `assume?` for m
ore detail
```

## Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.98

$$\int \frac{\arctan(ax)}{(c+dx^2)^{7/2}}\,dx = -\frac{1}{15}a\left(\frac{(15\,a^4c^2-20\,a^2cd+8\,d^2)\arctan\left(\frac{\sqrt{dx^2+c}a}{\sqrt{-a^2c+d}}\right)}{(a^4c^5-2\,a^2c^4d+c^3d^2)\sqrt{-a^2c+d}a}+\frac{7\,(dx^2+c)a^2c+a^2c^2-4\,(dx^2+c)d-cd}{(a^4c^4-2\,a^2c^3d+c^2d^2)(dx^2+c)^{\frac{3}{2}}}\right)+\frac{\left(4\,x^2\left(\frac{2\,d^2x^2}{c^3}+\frac{5\,d}{c^2}\right)+\frac{15}{c}\right)x\arctan\left(ax\right)}{15\,(dx^2+c)^{\frac{5}{2}}}$$

input
```
integrate(arctan(a*x)/(d*x^2+c)^(7/2),x, algorithm="giac")
```

output
```
-1/15*a*((15*a^4*c^2 - 20*a^2*c*d + 8*d^2)*arctan(sqrt(d*x^2 + c)*a/sqrt(-
a^2*c + d))/((a^4*c^5 - 2*a^2*c^4*d + c^3*d^2)*sqrt(-a^2*c + d)*a) + (7*(d
*x^2 + c)*a^2*c + a^2*c^2 - 4*(d*x^2 + c)*d - c*d)/((a^4*c^4 - 2*a^2*c^3*d
 + c^2*d^2)*(d*x^2 + c)^(3/2))) + 1/15*(4*x^2*(2*d^2*x^2/c^3 + 5*d/c^2) +
15/c)*x*arctan(a*x)/(d*x^2 + c)^(5/2)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)}{(c+dx^2)^{7/2}}\,dx = \int \frac{\mathrm{atan}(a\,x)}{(d\,x^2+c)^{7/2}}\,dx$$

input
```
int(atan(a*x)/(c + d*x^2)^(7/2),x)
```

output
```
int(atan(a*x)/(c + d*x^2)^(7/2), x)
```

## Reduce [F]

$$\int \frac{\arctan(ax)}{(c+dx^2)^{7/2}}\,dx = \int \frac{atan(ax)}{\sqrt{d\,x^2+c}\,c^3+3\sqrt{d\,x^2+c}\,c^2d\,x^2+3\sqrt{d\,x^2+c}\,c\,d^2x^4+\sqrt{d\,x^2+c}\,d^3x^6}dx$$

input
```
int(atan(a*x)/(d*x^2+c)^(7/2),x)
```

output
```
int(atan(a*x)/(sqrt(c + d*x**2)*c**3 + 3*sqrt(c + d*x**2)*c**2*d*x**2 + 3*
sqrt(c + d*x**2)*c*d**2*x**4 + sqrt(c + d*x**2)*d**3*x**6),x)
```

# 3.1226 $\int \frac{\arctan(ax)}{(c+dx^2)^{9/2}}\,dx$



## Optimal result

Integrand size = 16, antiderivative size = 293

$$\int \frac{\arctan(ax)}{(c+dx^2)^{9/2}}\,dx = -\frac{a}{35c(a^2c-d)(c+dx^2)^{5/2}} - \frac{a(11a^2c-6d)}{105c^2(a^2c-d)^2(c+dx^2)^{3/2}} - \frac{a(19a^4c^2-22a^2cd+8d^2)}{35c^3(a^2c-d)^3\sqrt{c+dx^2}} + \frac{x\arctan(ax)}{7c(c+dx^2)^{7/2}} + \frac{6x\arctan(ax)}{35c^2(c+dx^2)^{5/2}} + \frac{8x\arctan(ax)}{35c^3(c+dx^2)^{3/2}} + \frac{16x\arctan(ax)}{35c^4\sqrt{c+dx^2}} + \frac{(35a^6c^3-70a^4c^2d+56a^2cd^2-16d^3)\,\mathrm{arctanh}\left(\frac{a\sqrt{c+dx^2}}{\sqrt{a^2c-d}}\right)}{35c^4(a^2c-d)^{7/2}}$$

output
```
-1/35*a/c/(a^2*c-d)/(d*x^2+c)^(5/2)-1/105*a*(11*a^2*c-6*d)/c^2/(a^2*c-d)^2
/(d*x^2+c)^(3/2)-1/35*a*(19*a^4*c^2-22*a^2*c*d+8*d^2)/c^3/(a^2*c-d)^3/(d*x
^2+c)^(1/2)+1/7*x*arctan(a*x)/c/(d*x^2+c)^(7/2)+6/35*x*arctan(a*x)/c^2/(d*
x^2+c)^(5/2)+8/35*x*arctan(a*x)/c^3/(d*x^2+c)^(3/2)+16/35*x*arctan(a*x)/c^
4/(d*x^2+c)^(1/2)+1/35*(35*a^6*c^3-70*a^4*c^2*d+56*a^2*c*d^2-16*d^3)*arcta
nh(a*(d*x^2+c)^(1/2)/(a^2*c-d)^(1/2))/c^4/(a^2*c-d)^(7/2)
```

## Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.79 (sec) , antiderivative size = 450, normalized size of antiderivative = 1.54

$$\int \frac{\arctan(ax)}{(c+dx^2)^{9/2}}\,dx = \frac{-\frac{2ac\left(3c^2(-a^2c+d)^2+c(11a^2c-6d)(a^2c-d)(c+dx^2)+3(19a^4c^2-22a^2cd+8d^2)(c+dx^2)^2\right)}{(a^2c-d)^3(c+dx^2)^{5/2}}+\frac{6x(35c^3+70c^2dx^2+\cdots}{(\cdots}}{\cdots}$$

input
```
Integrate[ArcTan[a*x]/(c + d*x^2)^(9/2),x]
```

output
```
((-2*a*c*(3*c^2*(-(a^2*c) + d)^2 + c*(11*a^2*c - 6*d)*(a^2*c - d)*(c + d*x
^2) + 3*(19*a^4*c^2 - 22*a^2*c*d + 8*d^2)*(c + d*x^2)^2))/((a^2*c - d)^3*(
c + d*x^2)^(5/2)) + (6*x*(35*c^3 + 70*c^2*d*x^2 + 56*c*d^2*x^4 + 16*d^3*x^
6)*ArcTan[a*x])/(c + d*x^2)^(7/2) + (3*(35*a^6*c^3 - 70*a^4*c^2*d + 56*a^2
*c*d^2 - 16*d^3)*Log[(-140*a*c^4*(a^2*c - d)^(5/2)*(a*c - I*d*x + Sqrt[a^2
*c - d]*Sqrt[c + d*x^2]))/((35*a^6*c^3 - 70*a^4*c^2*d + 56*a^2*c*d^2 - 16*
d^3)*(I + a*x))])/(a^2*c - d)^(7/2) + (3*(35*a^6*c^3 - 70*a^4*c^2*d + 56*a
^2*c*d^2 - 16*d^3)*Log[(-140*a*c^4*(a^2*c - d)^(5/2)*(a*c + I*d*x + Sqrt[a
^2*c - d]*Sqrt[c + d*x^2]))/((35*a^6*c^3 - 70*a^4*c^2*d + 56*a^2*c*d^2 - 1
6*d^3)*(-I + a*x))])/(a^2*c - d)^(7/2))/(210*c^4)
```

## Rubi [A] (verified)

Time = 1.28 (sec) , antiderivative size = 291, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {5447, 27, 7266, 2122, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(ax)}{(c+dx^2)^{9/2}}\,dx$$

$$\Big\downarrow \; 5447$$

$$-a\int \frac{x\left(16d^3x^6+56cd^2x^4+70c^2dx^2+35c^3\right)}{35c^4\left(a^2x^2+1\right)\left(dx^2+c\right)^{7/2}}dx+\frac{16x\arctan(ax)}{35c^4\sqrt{c+dx^2}}+\frac{8x\arctan(ax)}{35c^3\left(c+dx^2\right)^{3/2}}+\frac{6x\arctan(ax)}{35c^2\left(c+dx^2\right)^{5/2}}+\frac{x\arctan(ax)}{7c\left(c+dx^2\right)^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{a\int \frac{x\left(16d^3x^6+56cd^2x^4+70c^2dx^2+35c^3\right)}{\left(a^2x^2+1\right)\left(dx^2+c\right)^{7/2}}dx}{35c^4}+\frac{16x\arctan(ax)}{35c^4\sqrt{c+dx^2}}+\frac{8x\arctan(ax)}{35c^3\left(c+dx^2\right)^{3/2}}+\frac{6x\arctan(ax)}{35c^2\left(c+dx^2\right)^{5/2}}+\frac{x\arctan(ax)}{7c\left(c+dx^2\right)^{7/2}}$$

$$\Big\downarrow \text{7266}$$

$$-\frac{a\int \frac{16d^3x^6+56cd^2x^4+70c^2dx^2+35c^3}{\left(a^2x^2+1\right)\left(dx^2+c\right)^{7/2}}dx^2}{70c^4}+\frac{16x\arctan(ax)}{35c^4\sqrt{c+dx^2}}+\frac{8x\arctan(ax)}{35c^3\left(c+dx^2\right)^{3/2}}+\frac{6x\arctan(ax)}{35c^2\left(c+dx^2\right)^{5/2}}+\frac{x\arctan(ax)}{7c\left(c+dx^2\right)^{7/2}}$$

$$\Big\downarrow \text{2122}$$

$$-\frac{a\int \left(-\frac{5dc^3}{\left(a^2c-d\right)\left(dx^2+c\right)^{7/2}}-\frac{\left(11a^2c-6d\right)dc^2}{\left(d-a^2c\right)^2\left(dx^2+c\right)^{5/2}}+\frac{d\left(19c^2a^4-22cda^2+8d^2\right)c}{\left(d-a^2c\right)^3\left(dx^2+c\right)^{3/2}}+\frac{35c^3a^6-70c^2da^4+56cd^2a^2-16d^3}{\left(a^2c-d\right)^3\left(a^2x^2+1\right)\sqrt{dx^2+c}}\right)dx^2}{70c^4}+\frac{16x\arctan(ax)}{35c^4\sqrt{c+dx^2}}+\frac{8x\arctan(ax)}{35c^3\left(c+dx^2\right)^{3/2}}+\frac{6x\arctan(ax)}{35c^2\left(c+dx^2\right)^{5/2}}+\frac{x\arctan(ax)}{7c\left(c+dx^2\right)^{7/2}}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a\left(\frac{2c^3}{\left(a^2c-d\right)\left(c+dx^2\right)^{5/2}}+\frac{2c^2\left(11a^2c-6d\right)}{3\left(a^2c-d\right)^2\left(c+dx^2\right)^{3/2}}+\frac{2c\left(19a^4c^2-22a^2cd+8d^2\right)}{\left(a^2c-d\right)^3\sqrt{c+dx^2}}-\frac{2\left(35a^6c^3-70a^4c^2d+56a^2cd^2-16d^3\right)\text{arctanh}\left(\frac{a\sqrt{c+dx}}{\sqrt{a^2c-}}\right.}{a\left(a^2c-d\right)^{7/2}}}{70c^4}$$

$$\frac{16x\arctan(ax)}{35c^4\sqrt{c+dx^2}}+\frac{8x\arctan(ax)}{35c^3\left(c+dx^2\right)^{3/2}}+\frac{6x\arctan(ax)}{35c^2\left(c+dx^2\right)^{5/2}}+\frac{x\arctan(ax)}{7c\left(c+dx^2\right)^{7/2}}$$

input

```
Int[ArcTan[a*x]/(c + d*x^2)^(9/2),x]
```

output
```
(x*ArcTan[a*x])/(7*c*(c + d*x^2)^(7/2)) + (6*x*ArcTan[a*x])/(35*c^2*(c + d
*x^2)^(5/2)) + (8*x*ArcTan[a*x])/(35*c^3*(c + d*x^2)^(3/2)) + (16*x*ArcTan
[a*x])/(35*c^4*Sqrt[c + d*x^2]) - (a*((2*c^3)/((a^2*c - d)*(c + d*x^2)^(5/
2)) + (2*c^2*(11*a^2*c - 6*d))/(3*(a^2*c - d)^2*(c + d*x^2)^(3/2)) + (2*c*
(19*a^4*c^2 - 22*a^2*c*d + 8*d^2))/((a^2*c - d)^3*Sqrt[c + d*x^2]) - (2*(3
5*a^6*c^3 - 70*a^4*c^2*d + 56*a^2*c*d^2 - 16*d^3)*ArcTanh[(a*Sqrt[c + d*x^
2])/Sqrt[a^2*c - d]])/(a*(a^2*c - d)^(7/2))))/(70*c^4)
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2122
```
Int[((Px_)*((c_.) + (d_.)*(x_))^(n_))/((a_.) + (b_.)*(x_)), x_Symbol] :> In
t[ExpandIntegrand[1/Sqrt[c + d*x], Px*((c + d*x)^(n + 1/2)/(a + b*x)), x],
x] /; FreeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && ILtQ[n + 1/2, 0]
```

rule 5447
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(q_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^q, x]}, Simp[(a + b*ArcTan[c*x])   u,
x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2*x^2), x], x], x]] /; FreeQ
[{a, b, c, d, e}, x] && (IntegerQ[q] || ILtQ[q + 1/2, 0])
```

rule 7266
```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Simp[1/(m + 1)   Subst[Int[SubstFor[x^(m
+ 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && Function
OfQ[x^(m + 1), u, x]
```

## Maple [F]

$$\int \frac{\arctan (ax)}{\left(d\,x^{2}+c \right)^{\frac{9}{2}}}dx$$

input
```
int(arctan(a*x)/(d*x^2+c)^(9/2),x)
```

output
```
int(arctan(a*x)/(d*x^2+c)^(9/2),x)
```

## Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 972 vs. $2(257) = 514$.

Time = 0.51 (sec) , antiderivative size = 1986, normalized size of antiderivative = 6.78

$$\int \frac{\arctan(ax)}{(c+dx^2)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate(arctan(a*x)/(d*x^2+c)^(9/2),x, algorithm="fricas")
```

output
```
[1/420*(3*(35*a^6*c^7 - 70*a^4*c^6*d + 56*a^2*c^5*d^2 + (35*a^6*c^3*d^4 -
70*a^4*c^2*d^5 + 56*a^2*c*d^6 - 16*d^7)*x^8 - 16*c^4*d^3 + 4*(35*a^6*c^4*d
^3 - 70*a^4*c^3*d^4 + 56*a^2*c^2*d^5 - 16*c*d^6)*x^6 + 6*(35*a^6*c^5*d^2 -
 70*a^4*c^4*d^3 + 56*a^2*c^3*d^4 - 16*c^2*d^5)*x^4 + 4*(35*a^6*c^6*d - 70*
a^4*c^5*d^2 + 56*a^2*c^4*d^3 - 16*c^3*d^4)*x^2)*sqrt(a^2*c - d)*log((a^4*d
^2*x^4 + 8*a^4*c^2 - 8*a^2*c*d + 2*(4*a^4*c*d - 3*a^2*d^2)*x^2 + 4*(a^3*d*
x^2 + 2*a^3*c - a*d)*sqrt(a^2*c - d)*sqrt(d*x^2 + c) + d^2)/(a^4*x^4 + 2*a
^2*x^2 + 1)) - 4*(71*a^7*c^7 - 160*a^5*c^6*d + 122*a^3*c^5*d^2 - 33*a*c^4*
d^3 + 3*(19*a^7*c^4*d^3 - 41*a^5*c^3*d^4 + 30*a^3*c^2*d^5 - 8*a*c*d^6)*x^6
 + (182*a^7*c^5*d^2 - 397*a^5*c^4*d^3 + 293*a^3*c^3*d^4 - 78*a*c^2*d^5)*x^
4 + (196*a^7*c^6*d - 434*a^5*c^5*d^2 + 325*a^3*c^4*d^3 - 87*a*c^3*d^4)*x^2
 - 3*(16*(a^8*c^4*d^3 - 4*a^6*c^3*d^4 + 6*a^4*c^2*d^5 - 4*a^2*c*d^6 + d^7)
*x^7 + 56*(a^8*c^5*d^2 - 4*a^6*c^4*d^3 + 6*a^4*c^3*d^4 - 4*a^2*c^2*d^5 + c
*d^6)*x^5 + 70*(a^8*c^6*d - 4*a^6*c^5*d^2 + 6*a^4*c^4*d^3 - 4*a^2*c^3*d^4
+ c^2*d^5)*x^3 + 35*(a^8*c^7 - 4*a^6*c^6*d + 6*a^4*c^5*d^2 - 4*a^2*c^4*d^3
 + c^3*d^4)*x)*arctan(a*x))*sqrt(d*x^2 + c))/(a^8*c^12 - 4*a^6*c^11*d + 6*
a^4*c^10*d^2 - 4*a^2*c^9*d^3 + c^8*d^4 + (a^8*c^8*d^4 - 4*a^6*c^7*d^5 + 6*
a^4*c^6*d^6 - 4*a^2*c^5*d^7 + c^4*d^8)*x^8 + 4*(a^8*c^9*d^3 - 4*a^6*c^8*d^
4 + 6*a^4*c^7*d^5 - 4*a^2*c^6*d^6 + c^5*d^7)*x^6 + 6*(a^8*c^10*d^2 - 4*a^6
*c^9*d^3 + 6*a^4*c^8*d^4 - 4*a^2*c^7*d^5 + c^6*d^6)*x^4 + 4*(a^8*c^11*d...
```

## Sympy [F]

$$\int \frac{\arctan(ax)}{(c+dx^2)^{9/2}}\,dx = \int \frac{\operatorname{atan}(ax)}{(c+dx^2)^{\frac{9}{2}}}\,dx$$

input
```
integrate(atan(a*x)/(d*x**2+c)**(9/2),x)
```

output
```
Integral(atan(a*x)/(c + d*x**2)**(9/2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{\arctan(ax)}{(c+dx^2)^{9/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(arctan(a*x)/(d*x^2+c)^(9/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(d-a^2*c>0)', see `assume?` for m
ore detail
```

## Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.15

$$\int \frac{\arctan(ax)}{(c+dx^2)^{9/2}}\,dx =$$

$$-\frac{1}{105}a\left(\frac{3\,(35\,a^6c^3-70\,a^4c^2d+56\,a^2cd^2-16\,d^3)\arctan\left(\frac{\sqrt{dx^2+c}a}{\sqrt{-a^2c+d}}\right)}{(a^6c^7-3\,a^4c^6d+3\,a^2c^5d^2-c^4d^3)\sqrt{-a^2c+d}a}+\frac{57\,(dx^2+c)^2a^4c^2+11\,(dx^2+c)a^4\right.$$

$$+\frac{\left(2\left(4\,x^2\left(\frac{2\,d^3x^2}{c^4}+\frac{7\,d^2}{c^3}\right)+\frac{35\,d}{c^2}\right)x^2+\frac{35}{c}\right)x\arctan\left(ax\right)}{35\,(dx^2+c)^{\frac{7}{2}}}$$

input
```
integrate(arctan(a*x)/(d*x^2+c)^(9/2),x, algorithm="giac")
```

output
```
-1/105*a*(3*(35*a^6*c^3 - 70*a^4*c^2*d + 56*a^2*c*d^2 - 16*d^3)*arctan(sqr
t(d*x^2 + c)*a/sqrt(-a^2*c + d))/((a^6*c^7 - 3*a^4*c^6*d + 3*a^2*c^5*d^2 -
 c^4*d^3)*sqrt(-a^2*c + d)*a) + (57*(d*x^2 + c)^2*a^4*c^2 + 11*(d*x^2 + c)
*a^4*c^3 + 3*a^4*c^4 - 66*(d*x^2 + c)^2*a^2*c*d - 17*(d*x^2 + c)*a^2*c^2*d
 - 6*a^2*c^3*d + 24*(d*x^2 + c)^2*d^2 + 6*(d*x^2 + c)*c*d^2 + 3*c^2*d^2)/(
(a^6*c^6 - 3*a^4*c^5*d + 3*a^2*c^4*d^2 - c^3*d^3)*(d*x^2 + c)^(5/2))) + 1/
35*(2*(4*x^2*(2*d^3*x^2/c^4 + 7*d^2/c^3) + 35*d/c^2)*x^2 + 35/c)*x*arctan(
a*x)/(d*x^2 + c)^(7/2)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(ax)}{(c+dx^2)^{9/2}}\,dx = \int \frac{\mathrm{atan}(a\,x)}{(d\,x^2+c)^{9/2}}\,dx$$

input
```
int(atan(a*x)/(c + d*x^2)^(9/2),x)
```

output
```
int(atan(a*x)/(c + d*x^2)^(9/2), x)
```

## Reduce [F]

$$\int \frac{\arctan(ax)}{(c+dx^2)^{9/2}}\,dx = \int \frac{atan(ax)}{(d\,x^2+c)^{\frac{9}{2}}}dx$$

input
```
int(atan(a*x)/(d*x^2+c)^(9/2),x)
```

output
```
int(atan(a*x)/(d*x^2+c)^(9/2),x)
```

# 3.1227 $\int x^m (d+ex^2)^3 (a+b\arctan(cx))\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 351

$$
\begin{aligned}
&\int x^m (d+ex^2)^3 (a+b\arctan(cx))\,dx \\
&= -\frac{be(e^2(15+8m+m^2)-3c^2de(21+10m+m^2)+3c^4d^2(35+12m+m^2))\,x^{2+m}}{c^5(6+5m+m^2)(35+12m+m^2)} \\
&\quad+\frac{be^2(e(5+m)-3c^2d(7+m))\,x^{4+m}}{c^3(4+m)(35+12m+m^2)}-\frac{be^3x^{6+m}}{c(6+m)(7+m)} \\
&\quad+\frac{d^3x^{1+m}(a+b\arctan(cx))}{1+m}+\frac{3d^2ex^{3+m}(a+b\arctan(cx))}{3+m} \\
&\quad+\frac{3de^2x^{5+m}(a+b\arctan(cx))}{5+m}+\frac{e^3x^{7+m}(a+b\arctan(cx))}{7+m} \\
&\quad-\frac{b\left(\frac{c^6d^3}{1+m}-\frac{e(e^2(15+8m+m^2)-3c^2de(21+10m+m^2)+3c^4d^2(35+12m+m^2))}{(3+m)(35+12m+m^2)}\right)x^{2+m}\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},-\right.}{c^5(2+m)}
\end{aligned}
$$

output

```
-b*e*(e^2*(m^2+8*m+15)-3*c^2*d*e*(m^2+10*m+21)+3*c^4*d^2*(m^2+12*m+35))*x^
(2+m)/c^5/(m^2+5*m+6)/(m^2+12*m+35)+b*e^2*(e*(5+m)-3*c^2*d*(7+m))*x^(4+m)/
c^3/(4+m)/(m^2+12*m+35)-b*e^3*x^(6+m)/c/(6+m)/(7+m)+d^3*x^(1+m)*(a+b*arcta
n(c*x))/(1+m)+3*d^2*e*x^(3+m)*(a+b*arctan(c*x))/(3+m)+3*d*e^2*x^(5+m)*(a+b
*arctan(c*x))/(5+m)+e^3*x^(7+m)*(a+b*arctan(c*x))/(7+m)-b*(c^6*d^3/(1+m)-e
*(e^2*(m^2+8*m+15)-3*c^2*d*e*(m^2+10*m+21)+3*c^4*d^2*(m^2+12*m+35))/(3+m)/
(m^2+12*m+35))*x^(2+m)*hypergeom([1, 1+1/2*m],[2+1/2*m],-c^2*x^2)/c^5/(2+m
)
```

## Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.75

$$\int x^m (d+ex^2)^3 (a+b\arctan(cx))\,dx$$
$$= x^{1+m}\left(\frac{d^3(a+b\arctan(cx))}{1+m}+\frac{3d^2ex^2(a+b\arctan(cx))}{3+m}+\frac{3de^2x^4(a+b\arctan(cx))}{5+m}+\frac{e^3x^6(a+b\arctan(cx))}{7+m}-\frac{bce^3x^7\,\text{Hypergeometric2F1}\left(1,4+\frac{m}{2},5+\frac{m}{2},-c^2x^2\right)}{(7+m)(8+m)}-\frac{bcd^3x\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},-c^2x^2\right)}{2+3m+m^2}-\frac{3bcd^2ex^3\,\text{Hypergeometric2F1}\left(1,\frac{4+m}{2},\frac{6+m}{2},-c^2x^2\right)}{12+7m+m^2}-\frac{3bcde^2x^5\,\text{Hypergeometric2F1}\left(1,\frac{6+m}{2},\frac{8+m}{2},-c^2x^2\right)}{(5+m)(6+m)}\right)$$

input
```
Integrate[x^m*(d + e*x^2)^3*(a + b*ArcTan[c*x]),x]
```

output
```
x^(1 + m)*((d^3*(a + b*ArcTan[c*x]))/(1 + m) + (3*d^2*e*x^2*(a + b*ArcTan[
c*x]))/(3 + m) + (3*d*e^2*x^4*(a + b*ArcTan[c*x]))/(5 + m) + (e^3*x^6*(a +
 b*ArcTan[c*x]))/(7 + m) - (b*c*e^3*x^7*Hypergeometric2F1[1, 4 + m/2, 5 +
m/2, -(c^2*x^2)])/((7 + m)*(8 + m)) - (b*c*d^3*x*Hypergeometric2F1[1, (2 +
 m)/2, (4 + m)/2, -(c^2*x^2)])/(2 + 3*m + m^2) - (3*b*c*d^2*e*x^3*Hypergeo
metric2F1[1, (4 + m)/2, (6 + m)/2, -(c^2*x^2)])/(12 + 7*m + m^2) - (3*b*c*
d*e^2*x^5*Hypergeometric2F1[1, (6 + m)/2, (8 + m)/2, -(c^2*x^2)])/((5 + m)
*(6 + m)))
```

## Rubi [A] (verified)

Time = 1.88 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5511, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \left(d+ex^2\right)^3 (a+b\arctan(cx))\,dx$$

$\big\downarrow$ 5511

$$-bc\int \frac{x^{m+1}\left(\frac{e^3x^6}{m+7}+\frac{3de^2x^4}{m+5}+\frac{3d^2ex^2}{m+3}+\frac{d^3}{m+1}\right)}{c^2x^2+1}dx+\frac{d^3x^{m+1}(a+b\arctan(cx))}{m+1}+\frac{3d^2ex^{m+3}(a+b\arctan(cx))}{m+3}+\frac{3de^2x^{m+5}(a+b\arctan(cx))}{m+5}+\frac{e^3x^{m+7}(a+b\arctan(cx))}{m+7}$$

$\big\downarrow$ 2333

$$-bc\int \left(\frac{e\left(3d^2\left(m^2+12m+35\right)c^4-3de\left(m^2+10m+21\right)c^2+e^2\left(m^2+8m+15\right)\right)x^{m+1}}{c^6(m+3)(m+5)(m+7)}+\frac{\left(105d^3c^6+d^3m^3c^6\right.}{}\right.$$
$$\frac{d^3x^{m+1}(a+b\arctan(cx))}{m+1}+\frac{3d^2ex^{m+3}(a+b\arctan(cx))}{m+3}+\frac{3de^2x^{m+5}(a+b\arctan(cx))}{m+5}+\frac{e^3x^{m+7}(a+b\arctan(cx))}{m+7}$$

$\big\downarrow$ 2009

$$\frac{d^3x^{m+1}(a+b\arctan(cx))}{m+1}+\frac{3d^2ex^{m+3}(a+b\arctan(cx))}{m+3}+\frac{3de^2x^{m+5}(a+b\arctan(cx))}{m+5}+\frac{e^3x^{m+7}(a+b\arctan(cx))}{m+7}-$$
$$bc\left(\frac{e^3x^{m+6}}{c^2(m+6)(m+7)}+\frac{e^2x^{m+4}\left(\frac{3c^2d}{m+5}-\frac{e}{m+7}\right)}{c^4(m+4)}+\frac{ex^{m+2}\left(3c^4d^2\left(m^2+12m+35\right)-3c^2de\left(m^2+10m+21\right)+e\right.}{c^6(m+2)(m+3)(m+5)(m+7)}\right.$$

input
```
Int[x^m*(d + e*x^2)^3*(a + b*ArcTan[c*x]),x]
```

output
```
(d^3*x^(1 + m)*(a + b*ArcTan[c*x]))/(1 + m) + (3*d^2*e*x^(3 + m)*(a + b*Ar
cTan[c*x]))/(3 + m) + (3*d*e^2*x^(5 + m)*(a + b*ArcTan[c*x]))/(5 + m) + (e
^3*x^(7 + m)*(a + b*ArcTan[c*x]))/(7 + m) - b*c*((e*(e^2*(15 + 8*m + m^2)
- 3*c^2*d*e*(21 + 10*m + m^2) + 3*c^4*d^2*(35 + 12*m + m^2))*x^(2 + m))/(c
^6*(2 + m)*(3 + m)*(5 + m)*(7 + m)) + (e^2*((3*c^2*d)/(5 + m) - e/(7 + m))
*x^(4 + m))/(c^4*(4 + m)) + (e^3*x^(6 + m))/(c^2*(6 + m)*(7 + m)) - ((e^3*
(15 + 23*m + 9*m^2 + m^3) - 3*c^2*d*e^2*(21 + 31*m + 11*m^2 + m^3) + 3*c^4
*d^2*e*(35 + 47*m + 13*m^2 + m^3) - c^6*d^3*(105 + 71*m + 15*m^2 + m^3))*x
^(2 + m)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, -(c^2*x^2)])/(c^6*(1 +
 m)*(2 + m)*(3 + m)*(5 + m)*(7 + m)))
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

## Maple [F]

$$\int x^m (e\,x^2+d)^3 \left(a+b\arctan\left(cx\right)\right) dx$$

input
```
int(x^m*(e*x^2+d)^3*(a+b*arctan(c*x)),x)
```

output
```
int(x^m*(e*x^2+d)^3*(a+b*arctan(c*x)),x)
```

## Fricas [F]

$$\int x^m \left(d+ex^2\right)^3 \left(a+b\arctan(cx)\right) dx = \int \left(ex^2+d\right)^3 (b\arctan\left(cx\right)+a)x^m\, dx$$

input
```
integrate(x^m*(e*x^2+d)^3*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
integral((a*e^3*x^6 + 3*a*d*e^2*x^4 + 3*a*d^2*e*x^2 + a*d^3 + (b*e^3*x^6 +
 3*b*d*e^2*x^4 + 3*b*d^2*e*x^2 + b*d^3)*arctan(c*x))*x^m, x)
```

## Sympy [F]

$$\int x^m (d+ex^2)^3 (a+b\arctan(cx))\,dx = \int x^m(a+b\operatorname{atan}(cx))\left(d+ex^2\right)^3\,dx$$

input
```
integrate(x**m*(e*x**2+d)**3*(a+b*atan(c*x)),x)
```

output
```
Integral(x**m*(a + b*atan(c*x))*(d + e*x**2)**3, x)
```

## Maxima [F]

$$\int x^m (d+ex^2)^3 (a+b\arctan(cx))\,dx = \int (ex^2+d)^3(b\arctan(cx)+a)x^m\,dx$$

input
```
integrate(x^m*(e*x^2+d)^3*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
a*e^3*x^(m + 7)/(m + 7) + 3*a*d*e^2*x^(m + 5)/(m + 5) + 3*a*d^2*e*x^(m + 3
)/(m + 3) + a*d^3*x^(m + 1)/(m + 1) + (((b*e^3*m^3 + 9*b*e^3*m^2 + 23*b*e^
3*m + 15*b*e^3)*x^7 + 3*(b*d*e^2*m^3 + 11*b*d*e^2*m^2 + 31*b*d*e^2*m + 21*
b*d*e^2)*x^5 + 3*(b*d^2*e*m^3 + 13*b*d^2*e*m^2 + 47*b*d^2*e*m + 35*b*d^2*e
)*x^3 + (b*d^3*m^3 + 15*b*d^3*m^2 + 71*b*d^3*m + 105*b*d^3)*x)*x^m*arctan(
c*x) - (m^4 + 16*m^3 + 86*m^2 + 176*m + 105)*integrate(((b*c*e^3*m^3 + 9*b
*c*e^3*m^2 + 23*b*c*e^3*m + 15*b*c*e^3)*x^7 + 3*(b*c*d*e^2*m^3 + 11*b*c*d*
e^2*m^2 + 31*b*c*d*e^2*m + 21*b*c*d*e^2)*x^5 + 3*(b*c*d^2*e*m^3 + 13*b*c*d
^2*e*m^2 + 47*b*c*d^2*e*m + 35*b*c*d^2*e)*x^3 + (b*c*d^3*m^3 + 15*b*c*d^3*
m^2 + 71*b*c*d^3*m + 105*b*c*d^3)*x)*x^m/(m^4 + 16*m^3 + (c^2*m^4 + 16*c^2
*m^3 + 86*c^2*m^2 + 176*c^2*m + 105*c^2)*x^2 + 86*m^2 + 176*m + 105), x))/
(m^4 + 16*m^3 + 86*m^2 + 176*m + 105)
```

## Giac [F]

$$\int x^m \left(d+ex^2\right)^3 (a+b\arctan(cx))\,dx = \int \left(ex^2+d\right)^3 (b\arctan(cx)+a)x^m\,dx$$

input
```
integrate(x^m*(e*x^2+d)^3*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^3*(b*arctan(c*x) + a)*x^m, x)
```

## Mupad [F(-1)]

Timed out.

$$\int x^m \left(d+ex^2\right)^3 (a+b\arctan(cx))\,dx = \int x^m\,(a+b\,\mathrm{atan}(c\,x))\,\left(e\,x^2+d\right)^3\,dx$$

input
```
int(x^m*(a + b*atan(c*x))*(d + e*x^2)^3,x)
```

output
```
int(x^m*(a + b*atan(c*x))*(d + e*x^2)^3, x)
```

## Reduce [F]

$$\int x^m \left(d+ex^2\right)^3 (a+b\arctan(cx))\,dx = \text{too large to display}$$

input
```
int(x^m*(e*x^2+d)^3*(a+b*atan(c*x)),x)
```

output

```
(x**m*atan(c*x)*b*c**7*d**3*m**7*x + 27*x**m*atan(c*x)*b*c**7*d**3*m**6*x
+ 295*x**m*atan(c*x)*b*c**7*d**3*m**5*x + 1665*x**m*atan(c*x)*b*c**7*d**3*
m**4*x + 5104*x**m*atan(c*x)*b*c**7*d**3*m**3*x + 8028*x**m*atan(c*x)*b*c*
*7*d**3*m**2*x + 5040*x**m*atan(c*x)*b*c**7*d**3*m*x + 3*x**m*atan(c*x)*b*
c**7*d**2*e*m**7*x**3 + 75*x**m*atan(c*x)*b*c**7*d**2*e*m**6*x**3 + 741*x*
*m*atan(c*x)*b*c**7*d**2*e*m**5*x**3 + 3657*x**m*atan(c*x)*b*c**7*d**2*e*m
**4*x**3 + 9336*x**m*atan(c*x)*b*c**7*d**2*e*m**3*x**3 + 11388*x**m*atan(c
*x)*b*c**7*d**2*e*m**2*x**3 + 5040*x**m*atan(c*x)*b*c**7*d**2*e*m*x**3 + 3
*x**m*atan(c*x)*b*c**7*d*e**2*m**7*x**5 + 69*x**m*atan(c*x)*b*c**7*d*e**2*
m**6*x**5 + 621*x**m*atan(c*x)*b*c**7*d*e**2*m**5*x**5 + 2775*x**m*atan(c*
x)*b*c**7*d*e**2*m**4*x**5 + 6432*x**m*atan(c*x)*b*c**7*d*e**2*m**3*x**5 +
 7236*x**m*atan(c*x)*b*c**7*d*e**2*m**2*x**5 + 3024*x**m*atan(c*x)*b*c**7*
d*e**2*m*x**5 + x**m*atan(c*x)*b*c**7*e**3*m**7*x**7 + 21*x**m*atan(c*x)*b
*c**7*e**3*m**6*x**7 + 175*x**m*atan(c*x)*b*c**7*e**3*m**5*x**7 + 735*x**m
*atan(c*x)*b*c**7*e**3*m**4*x**7 + 1624*x**m*atan(c*x)*b*c**7*e**3*m**3*x*
*7 + 1764*x**m*atan(c*x)*b*c**7*e**3*m**2*x**7 + 720*x**m*atan(c*x)*b*c**7
*e**3*m*x**7 + x**m*a*c**7*d**3*m**7*x + 27*x**m*a*c**7*d**3*m**6*x + 295*
x**m*a*c**7*d**3*m**5*x + 1665*x**m*a*c**7*d**3*m**4*x + 5104*x**m*a*c**7*
d**3*m**3*x + 8028*x**m*a*c**7*d**3*m**2*x + 5040*x**m*a*c**7*d**3*m*x + 3
*x**m*a*c**7*d**2*e*m**7*x**3 + 75*x**m*a*c**7*d**2*e*m**6*x**3 + 741*x...
```

# 3.1228 $\int x^m (d+ex^2)^2 (a+b\arctan(cx))\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 202

$$\int x^m (d+ex^2)^2 (a+b\arctan(cx))\,dx = -\frac{be\left(\frac{2c^2d}{3+m}-\frac{e}{5+m}\right)x^{2+m}}{c^3(2+m)} - \frac{be^2x^{4+m}}{c(4+m)(5+m)} + \frac{d^2x^{1+m}(a+b\arctan(cx))}{1+m} + \frac{2dex^{3+m}(a+b\arctan(cx))}{3+m} + \frac{e^2x^{5+m}(a+b\arctan(cx))}{5+m} - \frac{b\left(\frac{c^4d^2}{1+m}-\frac{2c^2de}{3+m}+\frac{e^2}{5+m}\right)x^{2+m}\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},-c^2x^2\right)}{c^3(2+m)}$$

output

```
-b*e*(2*c^2*d/(3+m)-e/(5+m))*x^(2+m)/c^3/(2+m)-b*e^2*x^(4+m)/c/(4+m)/(5+m)
+d^2*x^(1+m)*(a+b*arctan(c*x))/(1+m)+2*d*e*x^(3+m)*(a+b*arctan(c*x))/(3+m)
+e^2*x^(5+m)*(a+b*arctan(c*x))/(5+m)-b*(c^4*d^2/(1+m)-2*c^2*d*e/(3+m)+e^2/
(5+m))*x^(2+m)*hypergeom([1, 1+1/2*m],[2+1/2*m],-c^2*x^2)/c^3/(2+m)
```

## Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.96

$$\int x^m \left(d+ex^2\right)^2 (a+b\arctan(cx))\,dx = x^{1+m}\left(\frac{d^2(a+b\arctan(cx))}{1+m}+\frac{2dex^2(a+b\arctan(cx))}{3+m}+\frac{e^2x^4(a+b\arctan(cx))}{5+m} - \frac{bcd^2x\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},-c^2x^2\right)}{2+3m+m^2} - \frac{2bcdex^3\,\text{Hypergeometric2F1}\left(1,\frac{4+m}{2},\frac{6+m}{2},-c^2x^2\right)}{12+7m+m^2} - \frac{bce^2x^5\,\text{Hypergeometric2F1}\left(1,\frac{6+m}{2},\frac{8+m}{2},-c^2x^2\right)}{(5+m)(6+m)}\right)$$

input
```
Integrate[x^m*(d + e*x^2)^2*(a + b*ArcTan[c*x]),x]
```

output
```
x^(1 + m)*((d^2*(a + b*ArcTan[c*x]))/(1 + m) + (2*d*e*x^2*(a + b*ArcTan[c*
x]))/(3 + m) + (e^2*x^4*(a + b*ArcTan[c*x]))/(5 + m) - (b*c*d^2*x*Hypergeo
metric2F1[1, (2 + m)/2, (4 + m)/2, -(c^2*x^2)])/(2 + 3*m + m^2) - (2*b*c*d
*e*x^3*Hypergeometric2F1[1, (4 + m)/2, (6 + m)/2, -(c^2*x^2)])/(12 + 7*m +
 m^2) - (b*c*e^2*x^5*Hypergeometric2F1[1, (6 + m)/2, (8 + m)/2, -(c^2*x^2)
])/((5 + m)*(6 + m)))
```

## Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5511, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \left(d+ex^2\right)^2 (a+b\arctan(cx))\,dx$$

$$\Big\downarrow \; 5511$$

$$-bc\int \frac{x^{m+1}\left(\frac{e^2x^4}{m+5}+\frac{2dex^2}{m+3}+\frac{d^2}{m+1}\right)}{c^2x^2+1}dx+\frac{d^2x^{m+1}(a+b\arctan(cx))}{m+1}+\frac{2dex^{m+3}(a+b\arctan(cx))}{m+3}+\frac{e^2x^{m+5}(a+b\arctan(cx))}{m+5}$$

$\Big\downarrow$ 1584

$$-bc\int \left(\frac{e\left(\frac{2c^2d}{m+3}-\frac{e}{m+5}\right)x^{m+1}}{c^4}+\frac{(15d^2c^4+d^2m^2c^4+8d^2mc^4-2dem^2c^2-10dec^2-12demc^2+3e^2+e^2m^2+}{c^4(m+1)(m+3)(m+5)(c^2x^2+1)}\right.$$
$$\frac{d^2x^{m+1}(a+b\arctan(cx))}{m+1}+\frac{2dex^{m+3}(a+b\arctan(cx))}{m+3}+\frac{e^2x^{m+5}(a+b\arctan(cx))}{m+5}$$

$\Big\downarrow$ 2009

$$\frac{d^2x^{m+1}(a+b\arctan(cx))}{m+1}+\frac{2dex^{m+3}(a+b\arctan(cx))}{m+3}+\frac{e^2x^{m+5}(a+b\arctan(cx))}{m+5}-$$
$$bc\left(\frac{e^2x^{m+4}}{c^2(m+4)(m+5)}+\frac{x^{m+2}\left(c^4d^2(m^2+8m+15)-2c^2de(m^2+6m+5)+e^2(m^2+4m+3)\right)\text{Hypergeomet}}{c^4(m+1)(m+2)(m+3)(m+5)}\right.$$

input
```
Int[x^m*(d + e*x^2)^2*(a + b*ArcTan[c*x]),x]
```

output
```
(d^2*x^(1 + m)*(a + b*ArcTan[c*x]))/(1 + m) + (2*d*e*x^(3 + m)*(a + b*ArcT
an[c*x]))/(3 + m) + (e^2*x^(5 + m)*(a + b*ArcTan[c*x]))/(5 + m) - b*c*((e*
((2*c^2*d)/(3 + m) - e/(5 + m))*x^(2 + m))/(c^4*(2 + m)) + (e^2*x^(4 + m))
/(c^2*(4 + m)*(5 + m)) + ((e^2*(3 + 4*m + m^2) - 2*c^2*d*e*(5 + 6*m + m^2)
 + c^4*d^2*(15 + 8*m + m^2))*x^(2 + m)*Hypergeometric2F1[1, (2 + m)/2, (4
+ m)/2, -(c^2*x^2)])/(c^4*(1 + m)*(2 + m)*(3 + m)*(5 + m)))
```

## Defintions of rubi rules used

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

## Maple [F]

$$\int x^m (e x^2 + d)^2 (a + b \arctan(cx))\, dx$$

input
```
int(x^m*(e*x^2+d)^2*(a+b*arctan(c*x)),x)
```

output
```
int(x^m*(e*x^2+d)^2*(a+b*arctan(c*x)),x)
```

## Fricas [F]

$$\int x^m (d + ex^2)^2 (a + b\arctan(cx))\, dx = \int (ex^2 + d)^2 (b\arctan(cx) + a) x^m\, dx$$

input
```
integrate(x^m*(e*x^2+d)^2*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d
^2)*arctan(c*x))*x^m, x)
```

## Sympy [F]

$$\int x^m \left(d+ex^2\right)^2 (a+b\arctan(cx))\,dx=\int x^m(a+b\,\mathrm{atan}\,(cx))\left(d+ex^2\right)^2\,dx$$

input
```
integrate(x**m*(e*x**2+d)**2*(a+b*atan(c*x)),x)
```

output
```
Integral(x**m*(a + b*atan(c*x))*(d + e*x**2)**2, x)
```

## Maxima [F]

$$\int x^m \left(d+ex^2\right)^2 (a+b\arctan(cx))\,dx=\int \left(ex^2+d\right)^2(b\arctan(cx)+a)x^m\,dx$$

input
```
integrate(x^m*(e*x^2+d)^2*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
a*e^2*x^(m + 5)/(m + 5) + 2*a*d*e*x^(m + 3)/(m + 3) + a*d^2*x^(m + 1)/(m +
 1) + (((b*e^2*m^2 + 4*b*e^2*m + 3*b*e^2)*x^5 + 2*(b*d*e*m^2 + 6*b*d*e*m +
 5*b*d*e)*x^3 + (b*d^2*m^2 + 8*b*d^2*m + 15*b*d^2)*x)*x^m*arctan(c*x) - (m
^3 + 9*m^2 + 23*m + 15)*integrate(((b*c*e^2*m^2 + 4*b*c*e^2*m + 3*b*c*e^2)
*x^5 + 2*(b*c*d*e*m^2 + 6*b*c*d*e*m + 5*b*c*d*e)*x^3 + (b*c*d^2*m^2 + 8*b*
c*d^2*m + 15*b*c*d^2)*x)*x^m/(m^3 + (c^2*m^3 + 9*c^2*m^2 + 23*c^2*m + 15*c
^2)*x^2 + 9*m^2 + 23*m + 15), x))/(m^3 + 9*m^2 + 23*m + 15)
```

## Giac [F]

$$\int x^m \left(d+ex^2\right)^2 (a+b\arctan(cx))\,dx=\int \left(ex^2+d\right)^2(b\arctan(cx)+a)x^m\,dx$$

input
```
integrate(x^m*(e*x^2+d)^2*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^2*(b*arctan(c*x) + a)*x^m, x)
```

## Mupad [F(-1)]

Timed out.

$$\int x^m \left(d+ex^2\right)^2 (a+b\arctan(cx))\,dx=\int x^m\,\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)\,{\left(e\,x^2+d\right)}^2\,dx$$

input
```
int(x^m*(a + b*atan(c*x))*(d + e*x^2)^2,x)
```

output
```
int(x^m*(a + b*atan(c*x))*(d + e*x^2)^2, x)
```

## Reduce [F]

$$\int x^m \left(d+ex^2\right)^2 (a+b\arctan(cx))\,dx=\text{too large to display}$$

input
```
int(x^m*(e*x^2+d)^2*(a+b*atan(c*x)),x)
```

output
```
(x**m*atan(c*x)*b*c**5*d**2*m**5*x + 14*x**m*atan(c*x)*b*c**5*d**2*m**4*x
+ 71*x**m*atan(c*x)*b*c**5*d**2*m**3*x + 154*x**m*atan(c*x)*b*c**5*d**2*m*
*2*x + 120*x**m*atan(c*x)*b*c**5*d**2*m*x + 2*x**m*atan(c*x)*b*c**5*d*e*m*
*5*x**3 + 24*x**m*atan(c*x)*b*c**5*d*e*m**4*x**3 + 98*x**m*atan(c*x)*b*c**
5*d*e*m**3*x**3 + 156*x**m*atan(c*x)*b*c**5*d*e*m**2*x**3 + 80*x**m*atan(c
*x)*b*c**5*d*e*m*x**3 + x**m*atan(c*x)*b*c**5*e**2*m**5*x**5 + 10*x**m*ata
n(c*x)*b*c**5*e**2*m**4*x**5 + 35*x**m*atan(c*x)*b*c**5*e**2*m**3*x**5 + 5
0*x**m*atan(c*x)*b*c**5*e**2*m**2*x**5 + 24*x**m*atan(c*x)*b*c**5*e**2*m*x
**5 + x**m*a*c**5*d**2*m**5*x + 14*x**m*a*c**5*d**2*m**4*x + 71*x**m*a*c**
5*d**2*m**3*x + 154*x**m*a*c**5*d**2*m**2*x + 120*x**m*a*c**5*d**2*m*x + 2
*x**m*a*c**5*d*e*m**5*x**3 + 24*x**m*a*c**5*d*e*m**4*x**3 + 98*x**m*a*c**5
*d*e*m**3*x**3 + 156*x**m*a*c**5*d*e*m**2*x**3 + 80*x**m*a*c**5*d*e*m*x**3
 + x**m*a*c**5*e**2*m**5*x**5 + 10*x**m*a*c**5*e**2*m**4*x**5 + 35*x**m*a*
c**5*e**2*m**3*x**5 + 50*x**m*a*c**5*e**2*m**2*x**5 + 24*x**m*a*c**5*e**2*
m*x**5 - x**m*b*c**4*d**2*m**4 - 14*x**m*b*c**4*d**2*m**3 - 71*x**m*b*c**4
*d**2*m**2 - 154*x**m*b*c**4*d**2*m - 120*x**m*b*c**4*d**2 - 2*x**m*b*c**4
*d*e*m**4*x**2 - 20*x**m*b*c**4*d*e*m**3*x**2 - 58*x**m*b*c**4*d*e*m**2*x*
*2 - 40*x**m*b*c**4*d*e*m*x**2 - x**m*b*c**4*e**2*m**4*x**4 - 6*x**m*b*c**
4*e**2*m**3*x**4 - 11*x**m*b*c**4*e**2*m**2*x**4 - 6*x**m*b*c**4*e**2*m*x*
*4 + 2*x**m*b*c**2*d*e*m**4 + 24*x**m*b*c**2*d*e*m**3 + 98*x**m*b*c**2*...
```

# 3.1229 $\int x^m(d+ex^2)(a+b\arctan(cx))\,dx$



## Optimal result

Integrand size = 19, antiderivative size = 122

$$\int x^m(d+ex^2)(a+b\arctan(cx))\,dx = -\frac{bex^{2+m}}{c(6+5m+m^2)}+\frac{dx^{1+m}(a+b\arctan(cx))}{1+m}+\frac{ex^{3+m}(a+b\arctan(cx))}{3+m} - \frac{b\left(\frac{c^2d}{1+m}-\frac{e}{3+m}\right)x^{2+m}\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},-c^2x^2\right)}{c(2+m)}$$

output

```
-b*e*x^(2+m)/c/(m^2+5*m+6)+d*x^(1+m)*(a+b*arctan(c*x))/(1+m)+e*x^(3+m)*(a+
b*arctan(c*x))/(3+m)-b*(c^2*d/(1+m)-e/(3+m))*x^(2+m)*hypergeom([1, 1+1/2*m
],[2+1/2*m],-c^2*x^2)/c/(2+m)
```

## Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.98

$$\int x^m (d+ex^2) (a+b\arctan(cx))\,dx$$
$$= x^{1+m}\left(-\frac{bcdx\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},-c^2x^2\right)}{2+3m+m^2} + \frac{\frac{(d(3+m)+e(1+m)x^2)(a+b\arctan(cx))}{1+m}-\frac{bcex^3\,\text{Hypergeometric2F1}\left(1,\frac{4+m}{2},\frac{6+m}{2},-c^2x^2\right)}{4+m}}{3+m}\right)$$

input
```
Integrate[x^m*(d + e*x^2)*(a + b*ArcTan[c*x]),x]
```

output
```
x^(1 + m)*(-((b*c*d*x*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, -(c^2*x^2
)])/(2 + 3*m + m^2)) + (((d*(3 + m) + e*(1 + m)*x^2)*(a + b*ArcTan[c*x]))/
(1 + m) - (b*c*e*x^3*Hypergeometric2F1[1, (4 + m)/2, (6 + m)/2, -(c^2*x^2)
])/(4 + m))/(3 + m))
```

## Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {5511, 363, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m (d+ex^2) (a+b\arctan(cx))\,dx$$
$$\Big\downarrow \text{5511}$$
$$-bc\int \frac{x^{m+1}\left(\frac{ex^2}{m+3}+\frac{d}{m+1}\right)}{c^2x^2+1}dx+\frac{dx^{m+1}(a+b\arctan(cx))}{m+1}+\frac{ex^{m+3}(a+b\arctan(cx))}{m+3}$$
$$\Big\downarrow \text{363}$$

$$-bc\left(\left(\frac{d}{m+1}-\frac{e}{c^2(m+3)}\right)\int \frac{x^{m+1}}{c^2x^2+1}dx+\frac{ex^{m+2}}{c^2(m+2)(m+3)}\right)+\frac{dx^{m+1}(a+b\arctan(cx))}{m+1}+\frac{ex^{m+3}(a+b\arctan(cx))}{m+3}$$

$\downarrow$ 278

$$\frac{dx^{m+1}(a+b\arctan(cx))}{m+1}+\frac{ex^{m+3}(a+b\arctan(cx))}{m+3}-bc\left(\frac{x^{m+2}\left(\frac{d}{m+1}-\frac{e}{c^2(m+3)}\right)\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},-c^2x^2\right)}{m+2}+\frac{ex^{m+2}}{c^2(m+2)(m+3)}\right)$$

input
```
Int[x^m*(d + e*x^2)*(a + b*ArcTan[c*x]),x]
```

output
```
(d*x^(1 + m)*(a + b*ArcTan[c*x]))/(1 + m) + (e*x^(3 + m)*(a + b*ArcTan[c*x
]))/(3 + m) - b*c*((e*x^(2 + m))/(c^2*(2 + m)*(3 + m)) + ((d/(1 + m) - e/(
c^2*(3 + m)))*x^(2 + m)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, -(c^2*x
^2)])/(2 + m))
```

## Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

## Maple [F]

$$\int x^m (e\,x^2 + d)\,(a + b\arctan(cx))\,dx$$

input
```
int(x^m*(e*x^2+d)*(a+b*arctan(c*x)),x)
```

output
```
int(x^m*(e*x^2+d)*(a+b*arctan(c*x)),x)
```

## Fricas [F]

$$\int x^m (d + ex^2)\,(a + b\arctan(cx))\,dx = \int (ex^2 + d)(b\arctan(cx) + a)x^m\,dx$$

input
```
integrate(x^m*(e*x^2+d)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arctan(c*x))*x^m, x)
```

## Sympy [F]

$$\int x^m(d+ex^2)(a+b\arctan(cx))\,dx=\int x^m(a+b\operatorname{atan}(cx))\left(d+ex^2\right)\,dx$$

input
```
integrate(x**m*(e*x**2+d)*(a+b*atan(c*x)),x)
```

output
```
Integral(x**m*(a + b*atan(c*x))*(d + e*x**2), x)
```

## Maxima [F]

$$\int x^m(d+ex^2)(a+b\arctan(cx))\,dx=\int (ex^2+d)(b\arctan(cx)+a)x^m\,dx$$

input
```
integrate(x^m*(e*x^2+d)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
a*e*x^(m + 3)/(m + 3) + a*d*x^(m + 1)/(m + 1) + (((b*e*m + b*e)*x^3 + (b*d
*m + 3*b*d)*x)*x^m*arctan(c*x) - (m^2 + 4*m + 3)*integrate(((b*c*e*m + b*c
*e)*x^3 + (b*c*d*m + 3*b*c*d)*x)*x^m/((c^2*m^2 + 4*c^2*m + 3*c^2)*x^2 + m^
2 + 4*m + 3), x))/(m^2 + 4*m + 3)
```

## Giac [F]

$$\int x^m(d+ex^2)(a+b\arctan(cx))\,dx=\int (ex^2+d)(b\arctan(cx)+a)x^m\,dx$$

input
```
integrate(x^m*(e*x^2+d)*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)*(b*arctan(c*x) + a)*x^m, x)
```

## Mupad [F(-1)]

Timed out.

$$\int x^m(d+ex^2)(a+b\arctan(cx))\,dx=\int x^m\,(a+b\,\mathrm{atan}(c\,x))\,\left(e\,x^2+d\right)\,dx$$

input
```
int(x^m*(a + b*atan(c*x))*(d + e*x^2),x)
```

output
```
int(x^m*(a + b*atan(c*x))*(d + e*x^2), x)
```

## Reduce [F]

$$\int x^m(d+ex^2)(a+b\arctan(cx))\,dx=\text{Too large to display}$$

input
```
int(x^m*(e*x^2+d)*(a+b*atan(c*x)),x)
```

output

```
(x**m*atan(c*x)*b*c**3*d*m**3*x + 5*x**m*atan(c*x)*b*c**3*d*m**2*x + 6*x**
m*atan(c*x)*b*c**3*d*m*x + x**m*atan(c*x)*b*c**3*e*m**3*x**3 + 3*x**m*atan
(c*x)*b*c**3*e*m**2*x**3 + 2*x**m*atan(c*x)*b*c**3*e*m*x**3 + x**m*a*c**3*
d*m**3*x + 5*x**m*a*c**3*d*m**2*x + 6*x**m*a*c**3*d*m*x + x**m*a*c**3*e*m*
*3*x**3 + 3*x**m*a*c**3*e*m**2*x**3 + 2*x**m*a*c**3*e*m*x**3 - x**m*b*c**2
*d*m**2 - 5*x**m*b*c**2*d*m - 6*x**m*b*c**2*d - x**m*b*c**2*e*m**2*x**2 -
x**m*b*c**2*e*m*x**2 + x**m*b*e*m**2 + 3*x**m*b*e*m + 2*x**m*b*e + int(x**
m/(c**2*m**2*x**3 + 4*c**2*m*x**3 + 3*c**2*x**3 + m**2*x + 4*m*x + 3*x),x)
*b*c**2*d*m**5 + 9*int(x**m/(c**2*m**2*x**3 + 4*c**2*m*x**3 + 3*c**2*x**3
+ m**2*x + 4*m*x + 3*x),x)*b*c**2*d*m**4 + 29*int(x**m/(c**2*m**2*x**3 + 4
*c**2*m*x**3 + 3*c**2*x**3 + m**2*x + 4*m*x + 3*x),x)*b*c**2*d*m**3 + 39*i
nt(x**m/(c**2*m**2*x**3 + 4*c**2*m*x**3 + 3*c**2*x**3 + m**2*x + 4*m*x + 3
*x),x)*b*c**2*d*m**2 + 18*int(x**m/(c**2*m**2*x**3 + 4*c**2*m*x**3 + 3*c**
2*x**3 + m**2*x + 4*m*x + 3*x),x)*b*c**2*d*m - int(x**m/(c**2*m**2*x**3 +
4*c**2*m*x**3 + 3*c**2*x**3 + m**2*x + 4*m*x + 3*x),x)*b*e*m**5 - 7*int(x*
*m/(c**2*m**2*x**3 + 4*c**2*m*x**3 + 3*c**2*x**3 + m**2*x + 4*m*x + 3*x),x
)*b*e*m**4 - 17*int(x**m/(c**2*m**2*x**3 + 4*c**2*m*x**3 + 3*c**2*x**3 + m
**2*x + 4*m*x + 3*x),x)*b*e*m**3 - 17*int(x**m/(c**2*m**2*x**3 + 4*c**2*m*
x**3 + 3*c**2*x**3 + m**2*x + 4*m*x + 3*x),x)*b*e*m**2 - 6*int(x**m/(c**2*
m**2*x**3 + 4*c**2*m*x**3 + 3*c**2*x**3 + m**2*x + 4*m*x + 3*x),x)*b*e*...
```

# 3.1230 $\int \frac{x^m(a+b\arctan(cx))}{d+ex^2}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{x^m(a+b\arctan(cx))}{d+ex^2}\,dx = \frac{ax^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{ex^2}{d}\right)}{d(1+m)} + b\text{Int}\left(\frac{x^m\arctan(cx)}{d+ex^2},x\right)$$

output
```
a*x^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-e*x^2/d)/d/(1+m)+b*Defer(I
nt)(x^m*arctan(c*x)/(e*x^2+d),x)
```

## Mathematica [N/A]

Not integrable

Time = 2.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{x^m(a+b\arctan(cx))}{d+ex^2}\,dx = \int \frac{x^m(a+b\arctan(cx))}{d+ex^2}\,dx$$

input
```
Integrate[(x^m*(a + b*ArcTan[c*x]))/(d + e*x^2),x]
```

output
```
Integrate[(x^m*(a + b*ArcTan[c*x]))/(d + e*x^2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m(a+b\arctan(cx))}{d+ex^2}\,dx$$

$$\Big\downarrow \text{5517}$$

$$a\int \frac{x^m}{ex^2+d}dx+b\int \frac{x^m\arctan(cx)}{ex^2+d}dx$$

$$\Big\downarrow \text{278}$$

$$b\int \frac{x^m\arctan(cx)}{ex^2+d}dx+\frac{ax^{m+1}\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{ex^2}{d}\right)}{d(m+1)}$$

$$\Big\downarrow \text{5560}$$

$$b\int \frac{x^m\arctan(cx)}{ex^2+d}dx+\frac{ax^{m+1}\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{ex^2}{d}\right)}{d(m+1)}$$

input
```
Int[(x^m*(a + b*ArcTan[c*x]))/(d + e*x^2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 1.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{x^m(a+b\arctan(cx))}{e\,x^2+d}dx$$

input
```
int(x^m*(a+b*arctan(c*x))/(e*x^2+d),x)
```

output
```
int(x^m*(a+b*arctan(c*x))/(e*x^2+d),x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{x^m(a+b\arctan(cx))}{d+ex^2}\,dx = \int \frac{(b\arctan(cx)+a)x^m}{ex^2+d}\,dx$$

input
```
integrate(x^m*(a+b*arctan(c*x))/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b*arctan(c*x) + a)*x^m/(e*x^2 + d), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^m(a+b\arctan(cx))}{d+ex^2}\,dx = \text{Timed out}$$

input
```
integrate(x**m*(a+b*atan(c*x))/(e*x**2+d),x)
```

output
```
Timed out
```

## Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{x^m(a+b\arctan(cx))}{d+ex^2}\,dx=\int \frac{(b\arctan(cx)+a)x^m}{ex^2+d}\,dx$$

input
```
integrate(x^m*(a+b*arctan(c*x))/(e*x^2+d),x, algorithm="maxima")
```

output
```
integrate((b*arctan(c*x) + a)*x^m/(e*x^2 + d), x)
```

## Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{x^m(a+b\arctan(cx))}{d+ex^2}\,dx=\int \frac{(b\arctan(cx)+a)x^m}{ex^2+d}\,dx$$

input
```
integrate(x^m*(a+b*arctan(c*x))/(e*x^2+d),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)*x^m/(e*x^2 + d), x)
```

## Mupad [N/A]

Not integrable

Time = 0.95 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{x^m(a+b\arctan(cx))}{d+ex^2}\,dx=\int \frac{x^m\,(a+b\,\mathrm{atan}(c\,x))}{e\,x^2+d}\,dx$$

input
```
int((x^m*(a + b*atan(c*x)))/(d + e*x^2),x)
```

output
```
int((x^m*(a + b*atan(c*x)))/(d + e*x^2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.86

$$\int \frac{x^m(a+b\arctan(cx))}{d+ex^2}\,dx = \left(\int \frac{x^m}{e\,x^2+d}dx\right)a + \left(\int \frac{x^m\,atan(cx)}{e\,x^2+d}dx\right)b$$

input
```
int(x^m*(a+b*atan(c*x))/(e*x^2+d),x)
```

output
```
int(x**m/(d + e*x**2),x)*a + int((x**m*atan(c*x))/(d + e*x**2),x)*b
```

# 3.1231 $\int \frac{x^m(a+b\arctan(cx))}{(d+ex^2)^2}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{x^m(a+b\arctan(cx))}{(d+ex^2)^2}\,dx = \frac{ax^{1+m}\,\text{Hypergeometric2F1}\left(2,\frac{1+m}{2},\frac{3+m}{2},-\frac{ex^2}{d}\right)}{d^2(1+m)} + b\text{Int}\left(\frac{x^m\arctan(cx)}{(d+ex^2)^2},x\right)$$

output
```
a*x^(1+m)*hypergeom([2, 1/2+1/2*m],[3/2+1/2*m],-e*x^2/d)/d^2/(1+m)+b*Defer
(Int)(x^m*arctan(c*x)/(e*x^2+d)^2,x)
```

## Mathematica [N/A]

Not integrable

Time = 3.55 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{x^m(a+b\arctan(cx))}{(d+ex^2)^2}\,dx = \int \frac{x^m(a+b\arctan(cx))}{(d+ex^2)^2}\,dx$$

input
```
Integrate[(x^m*(a + b*ArcTan[c*x]))/(d + e*x^2)^2,x]
```

output
```
Integrate[(x^m*(a + b*ArcTan[c*x]))/(d + e*x^2)^2, x]
```

## Rubi [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m(a+b\arctan(cx))}{(d+ex^2)^2}\,dx$$

$$\Big\downarrow \text{ 5517}$$

$$a\int \frac{x^m}{(ex^2+d)^2}dx+b\int \frac{x^m\arctan(cx)}{(ex^2+d)^2}dx$$

$$\Big\downarrow \text{ 278}$$

$$b\int \frac{x^m\arctan(cx)}{(ex^2+d)^2}dx+\frac{ax^{m+1}\text{Hypergeometric2F1}\left(2,\frac{m+1}{2},\frac{m+3}{2},-\frac{ex^2}{d}\right)}{d^2(m+1)}$$

$$\Big\downarrow \text{ 5560}$$

$$b\int \frac{x^m\arctan(cx)}{(ex^2+d)^2}dx+\frac{ax^{m+1}\text{Hypergeometric2F1}\left(2,\frac{m+1}{2},\frac{m+3}{2},-\frac{ex^2}{d}\right)}{d^2(m+1)}$$

input
```
Int[(x^m*(a + b*ArcTan[c*x]))/(d + e*x^2)^2,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 3.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{x^m(a+b\arctan(cx))}{(e\,x^2+d)^2}dx$$

input
```
int(x^m*(a+b*arctan(c*x))/(e*x^2+d)^2,x)
```

output
```
int(x^m*(a+b*arctan(c*x))/(e*x^2+d)^2,x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.62

$$\int \frac{x^m(a+b\arctan(cx))}{(d+ex^2)^2}\,dx = \int \frac{(b\arctan(cx)+a)x^m}{(ex^2+d)^2}\,dx$$

input
```
integrate(x^m*(a+b*arctan(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b*arctan(c*x) + a)*x^m/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^m(a+b\arctan(cx))}{(d+ex^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**m*(a+b*atan(c*x))/(e*x**2+d)**2,x)
```

output
```
Timed out
```

## Maxima [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{x^m(a+b\arctan(cx))}{(d+ex^2)^2}\,dx = \int \frac{(b\arctan(cx)+a)x^m}{(ex^2+d)^2}\,dx$$

input
```
integrate(x^m*(a+b*arctan(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
integrate((b*arctan(c*x) + a)*x^m/(e*x^2 + d)^2, x)
```

## Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{x^m(a+b\arctan(cx))}{(d+ex^2)^2}\,dx = \int \frac{(b\arctan(cx)+a)x^m}{(ex^2+d)^2}\,dx$$

input
```
integrate(x^m*(a+b*arctan(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)*x^m/(e*x^2 + d)^2, x)
```

## Mupad [N/A]

Not integrable

Time = 0.92 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int \frac{x^m(a+b\arctan(cx))}{(d+ex^2)^2}\,dx = \int \frac{x^m\,(a+b\,\mathrm{atan}(c\,x))}{(e\,x^2+d)^2}\,dx$$

input
```
int((x^m*(a + b*atan(c*x)))/(d + e*x^2)^2,x)
```

output
```
int((x^m*(a + b*atan(c*x)))/(d + e*x^2)^2, x)
```

## Reduce [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 48567, normalized size of antiderivative = 2312.71

$$\int \frac{x^m(a+b\arctan(cx))}{(d+ex^2)^2}\,dx = \text{Too large to display}$$

input
```
int(x^m*(a+b*atan(c*x))/(e*x^2+d)^2,x)
```

output
```
(x**m*atan(c*x)*b*c**2*m*x - 2*x**m*atan(c*x)*b*c**2*x - x**m*b*c + int(x*
*m/(c**4*d**3*m**2*x**3 - c**4*d**3*m*x**3 - 2*c**4*d**3*x**3 + 2*c**4*d**
2*e*m**2*x**5 - 2*c**4*d**2*e*m*x**5 - 4*c**4*d**2*e*x**5 + c**4*d*e**2*m*
*2*x**7 - c**4*d*e**2*m*x**7 - 2*c**4*d*e**2*x**7 + c**2*d**3*m**2*x - c**
2*d**3*m*x - 2*c**2*d**3*x + 3*c**2*d**2*e*m**2*x**3 - 5*c**2*d**2*e*m*x**
3 - 2*c**2*d**2*e*x**3 + 3*c**2*d*e**2*m**2*x**5 - 7*c**2*d*e**2*m*x**5 +
2*c**2*d*e**2*x**5 + c**2*e**3*m**2*x**7 - 3*c**2*e**3*m*x**7 + 2*c**2*e**
3*x**7 + d**2*e*m**2*x - 3*d**2*e*m*x + 2*d**2*e*x + 2*d*e**2*m**2*x**3 -
6*d*e**2*m*x**3 + 4*d*e**2*x**3 + e**3*m**2*x**5 - 3*e**3*m*x**5 + 2*e**3*
x**5),x)*b*c**3*d**3*m**3 - int(x**m/(c**4*d**3*m**2*x**3 - c**4*d**3*m*x*
*3 - 2*c**4*d**3*x**3 + 2*c**4*d**2*e*m**2*x**5 - 2*c**4*d**2*e*m*x**5 - 4
*c**4*d**2*e*x**5 + c**4*d*e**2*m**2*x**7 - c**4*d*e**2*m*x**7 - 2*c**4*d*
e**2*x**7 + c**2*d**3*m**2*x - c**2*d**3*m*x - 2*c**2*d**3*x + 3*c**2*d**2
*e*m**2*x**3 - 5*c**2*d**2*e*m*x**3 - 2*c**2*d**2*e*x**3 + 3*c**2*d*e**2*m
**2*x**5 - 7*c**2*d*e**2*m*x**5 + 2*c**2*d*e**2*x**5 + c**2*e**3*m**2*x**7
 - 3*c**2*e**3*m*x**7 + 2*c**2*e**3*x**7 + d**2*e*m**2*x - 3*d**2*e*m*x +
2*d**2*e*x + 2*d*e**2*m**2*x**3 - 6*d*e**2*m*x**3 + 4*d*e**2*x**3 + e**3*m
**2*x**5 - 3*e**3*m*x**5 + 2*e**3*x**5),x)*b*c**3*d**3*m**2 - 2*int(x**m/(
c**4*d**3*m**2*x**3 - c**4*d**3*m*x**3 - 2*c**4*d**3*x**3 + 2*c**4*d**2*e*
m**2*x**5 - 2*c**4*d**2*e*m*x**5 - 4*c**4*d**2*e*x**5 + c**4*d*e**2*m**...
```

# 3.1232 $\int x^m(d+ex^2)^{3/2}(a+b\arctan(cx))\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 23

$$\int x^m\left(d+ex^2\right)^{3/2}(a+b\arctan(cx))\,dx=\frac{adx^{1+m}\sqrt{d+ex^2}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{1+m}{2},\frac{3+m}{2},-\frac{ex^2}{d}\right)}{(1+m)\sqrt{1+\frac{ex^2}{d}}}+b\text{Int}\left(x^m\left(d+ex^2\right)^{3/2}\arctan(cx),x\right)$$

output
```
a*d*x^(1+m)*(e*x^2+d)^(1/2)*hypergeom([-3/2, 1/2+1/2*m],[3/2+1/2*m],-e*x^2
/d)/(1+m)/(1+e*x^2/d)^(1/2)+b*Defer(Int)(x^m*(e*x^2+d)^(3/2)*arctan(c*x),x
)
```

## Mathematica [N/A]

Not integrable

Time = 2.76 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int x^m\left(d+ex^2\right)^{3/2}(a+b\arctan(cx))\,dx=\int x^m\left(d+ex^2\right)^{3/2}(a+b\arctan(cx))\,dx$$

input
```
Integrate[x^m*(d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]),x]
```

output
```
Integrate[x^m*(d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \left(d+ex^2\right)^{3/2} (a+b\arctan(cx))\, dx$$

$$\Big\downarrow \text{5517}$$

$$a\int x^m \left(ex^2+d\right)^{3/2}dx+b\int x^m \left(ex^2+d\right)^{3/2}\arctan(cx)dx$$

$$\Big\downarrow \text{279}$$

$$\frac{ad\sqrt{d+ex^2}\int x^m \left(\frac{ex^2}{d}+1\right)^{3/2}dx}{\sqrt{\frac{ex^2}{d}+1}}+b\int x^m \left(ex^2+d\right)^{3/2}\arctan(cx)dx$$

$$\Big\downarrow \text{278}$$

$$b\int x^m \left(ex^2+d\right)^{3/2}\arctan(cx)dx+\frac{adx^{m+1}\sqrt{d+ex^2}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{m+1}{2},\frac{m+3}{2},-\frac{ex^2}{d}\right)}{(m+1)\sqrt{\frac{ex^2}{d}+1}}$$

$$\Big\downarrow \text{5560}$$

$$b\int x^m(ex^2+d)^{3/2}\arctan(cx)dx+\frac{adx^{m+1}\sqrt{d+ex^2}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{m+1}{2},\frac{m+3}{2},-\frac{ex^2}{d}\right)}{(m+1)\sqrt{\frac{ex^2}{d}+1}}$$

input
```
Int[x^m*(d + e*x^2)^(3/2)*(a + b*ArcTan[c*x]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int x^m(e\,x^2+d)^{\frac{3}{2}}\,(a+b\arctan\,(cx))\,dx$$

input
```
int(x^m*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x)
```

output
```
int(x^m*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x)
```

## Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.74

$$\int x^m\left(d+ex^2\right)^{3/2}(a+b\arctan(cx))\,dx=\int (ex^2+d)^{\frac{3}{2}}(b\arctan\,(cx)+a)x^m\,dx$$

input
```
integrate(x^m*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arctan(c*x))*sqrt(e*x^2 + d)*x^m
, x)
```

## Sympy [F(-1)]

Timed out.

$$\int x^m \left(d+ex^2\right)^{3/2} (a+b\arctan(cx))\,dx = \text{Timed out}$$

input
```
integrate(x**m*(e*x**2+d)**(3/2)*(a+b*atan(c*x)),x)
```

output
```
Timed out
```

## Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x^m \left(d+ex^2\right)^{3/2} (a+b\arctan(cx))\,dx = \int (ex^2+d)^{\frac{3}{2}}(b\arctan(cx)+a)x^m\,dx$$

input
```
integrate(x^m*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
integrate((e*x^2 + d)^(3/2)*(b*arctan(c*x) + a)*x^m, x)
```

## Giac [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x^m \left(d+ex^2\right)^{3/2} (a+b\arctan(cx))\,dx = \int (ex^2+d)^{\frac{3}{2}}(b\arctan(cx)+a)x^m\,dx$$

input
```
integrate(x^m*(e*x^2+d)^(3/2)*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^(3/2)*(b*arctan(c*x) + a)*x^m, x)
```

## Mupad [N/A]

Not integrable

Time = 1.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x^m (d+ex^2)^{3/2} (a+b\arctan(cx))\,dx = \int x^m\,(a+b\,\mathrm{atan}(c\,x))\,\left(e\,x^2+d\right)^{3/2}\,dx$$

input
```
int(x^m*(a + b*atan(c*x))*(d + e*x^2)^(3/2),x)
```

output
```
int(x^m*(a + b*atan(c*x))*(d + e*x^2)^(3/2), x)
```

## Reduce [N/A]

Not integrable

Time = 200.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x^m (d+ex^2)^{3/2} (a+b\arctan(cx))\,dx = \int x^m \left(e\,x^2+d\right)^{\frac{3}{2}} (atan(cx)\,b+a)\,dx$$

input
```
int(x^m*(e*x^2+d)^(3/2)*(a+b*atan(c*x)),x)
```

output
```
int(x^m*(e*x^2+d)^(3/2)*(a+b*atan(c*x)),x)
```

# 3.1233 $\int x^m\sqrt{d+ex^2}(a+b\arctan(cx))\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 23

$$\int x^m\sqrt{d+ex^2}(a+b\arctan(cx))\,dx = \frac{ax^{1+m}\sqrt{d+ex^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},-\frac{ex^2}{d}\right)}{(1+m)\sqrt{1+\frac{ex^2}{d}}} + b\text{Int}\left(x^m\sqrt{d+ex^2}\arctan(cx),x\right)$$

output
```
a*x^(1+m)*(e*x^2+d)^(1/2)*hypergeom([-1/2, 1/2+1/2*m],[3/2+1/2*m],-e*x^2/d
)/(1+m)/(1+e*x^2/d)^(1/2)+b*Defer(Int)(x^m*(e*x^2+d)^(1/2)*arctan(c*x),x)
```

## Mathematica [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int x^m\sqrt{d+ex^2}(a+b\arctan(cx))\,dx = \int x^m\sqrt{d+ex^2}(a+b\arctan(cx))\,dx$$

input
```
Integrate[x^m*Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]),x]
```

output
```
Integrate[x^m*Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \sqrt{d+ex^2}(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5517}$$

$$a\int x^m\sqrt{ex^2+d}dx+b\int x^m\sqrt{ex^2+d}\arctan(cx)dx$$

$$\Big\downarrow \text{279}$$

$$\frac{a\sqrt{d+ex^2}\int x^m\sqrt{\frac{ex^2}{d}+1}dx}{\sqrt{\frac{ex^2}{d}+1}}+b\int x^m\sqrt{ex^2+d}\arctan(cx)dx$$

$$\Big\downarrow \text{278}$$

$$b\int x^m\sqrt{ex^2+d}\arctan(cx)dx+\frac{ax^{m+1}\sqrt{d+ex^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},-\frac{ex^2}{d}\right)}{(m+1)\sqrt{\frac{ex^2}{d}+1}}$$

$$\Big\downarrow \text{5560}$$

$$b\int x^m\sqrt{ex^2+d}\arctan(cx)dx+\frac{ax^{m+1}\sqrt{d+ex^2}\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},-\frac{ex^2}{d}\right)}{(m+1)\sqrt{\frac{ex^2}{d}+1}}$$

input
```
Int[x^m*Sqrt[d + e*x^2]*(a + b*ArcTan[c*x]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int x^m\sqrt{e\,x^2+d}\,(a+b\arctan(cx))\,dx$$

input
```
int(x^m*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x)
```

output
```
int(x^m*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x^m\sqrt{d+ex^2}(a+b\arctan(cx))\,dx=\int \sqrt{ex^2+d}(b\arctan(cx)+a)x^m\,dx$$

input
```
integrate(x^m*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arctan(c*x) + a)*x^m, x)
```

## Sympy [N/A]

Not integrable

Time = 41.60 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int x^m\sqrt{d+ex^2}(a+b\arctan(cx))\,dx=\int x^m(a+b\,\mathrm{atan}(cx))\sqrt{d+ex^2}\,dx$$

input
```
integrate(x**m*(e*x**2+d)**(1/2)*(a+b*atan(c*x)),x)
```

output
```
Integral(x**m*(a + b*atan(c*x))*sqrt(d + e*x**2), x)
```

## Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x^m\sqrt{d+ex^2}(a+b\arctan(cx))\,dx=\int\sqrt{ex^2+d}(b\arctan{(cx)}+a)x^m\,dx$$

input
```
integrate(x^m*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arctan(c*x) + a)*x^m, x)
```

## Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x^m\sqrt{d+ex^2}(a+b\arctan(cx))\,dx=\int\sqrt{ex^2+d}(b\arctan{(cx)}+a)x^m\,dx$$

input
```
integrate(x^m*(e*x^2+d)^(1/2)*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x^2 + d)*(b*arctan(c*x) + a)*x^m, x)
```

## Mupad [N/A]

Not integrable

Time = 1.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x^m\sqrt{d+ex^2}(a+b\arctan(cx))\,dx=\int x^m\,\left(a+b\,\mathrm{atan}(c\,x)\right)\,\sqrt{e\,x^2+d}\,dx$$

input
```
int(x^m*(a + b*atan(c*x))*(d + e*x^2)^(1/2),x)
```

output
```
int(x^m*(a + b*atan(c*x))*(d + e*x^2)^(1/2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.61

$$\int x^m\sqrt{d+ex^2}(a+b\arctan(cx))\,dx = \left(\int x^m\sqrt{e\,x^2+d}\,atan(cx)\,dx\right)b + \left(\int x^m\sqrt{e\,x^2+d}dx\right)a$$

input
```
int(x^m*(e*x^2+d)^(1/2)*(a+b*atan(c*x)),x)
```

output
```
int(x**m*sqrt(d + e*x**2)*atan(c*x),x)*b + int(x**m*sqrt(d + e*x**2),x)*a
```

# 3.1234 $\int \frac{x^m(a+b\arctan(cx))}{\sqrt{d+ex^2}}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^m(a+b\arctan(cx))}{\sqrt{d+ex^2}}\,dx = \frac{ax^{1+m}\sqrt{1+\frac{ex^2}{d}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+m}{2},\frac{3+m}{2},-\frac{ex^2}{d}\right)}{(1+m)\sqrt{d+ex^2}} + b\text{Int}\left(\frac{x^m\arctan(cx)}{\sqrt{d+ex^2}},x\right)$$

output
```
a*x^(1+m)*(1+e*x^2/d)^(1/2)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],-e*x^2/
d)/(1+m)/(e*x^2+d)^(1/2)+b*Defer(Int)(x^m*arctan(c*x)/(e*x^2+d)^(1/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 2.67 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^m(a+b\arctan(cx))}{\sqrt{d+ex^2}}\,dx = \int \frac{x^m(a+b\arctan(cx))}{\sqrt{d+ex^2}}\,dx$$

input
```
Integrate[(x^m*(a + b*ArcTan[c*x]))/Sqrt[d + e*x^2],x]
```

output
```
Integrate[(x^m*(a + b*ArcTan[c*x]))/Sqrt[d + e*x^2], x]
```

## Rubi [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m(a+b\arctan(cx))}{\sqrt{d+ex^2}}dx$$

$$\Big\downarrow \text{ 5517}$$

$$a\int \frac{x^m}{\sqrt{ex^2+d}}dx+b\int \frac{x^m\arctan(cx)}{\sqrt{ex^2+d}}dx$$

$$\Big\downarrow \text{ 279}$$

$$\frac{a\sqrt{\frac{ex^2}{d}+1}\int \frac{x^m}{\sqrt{\frac{ex^2}{d}+1}}dx}{\sqrt{d+ex^2}}+b\int \frac{x^m\arctan(cx)}{\sqrt{ex^2+d}}dx$$

$$\Big\downarrow \text{ 278}$$

$$b\int \frac{x^m\arctan(cx)}{\sqrt{ex^2+d}}dx+\frac{ax^{m+1}\sqrt{\frac{ex^2}{d}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},-\frac{ex^2}{d}\right)}{(m+1)\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 5560}$$

$$b\int \frac{x^m\arctan(cx)}{\sqrt{ex^2+d}}dx+\frac{ax^{m+1}\sqrt{\frac{ex^2}{d}+1}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},-\frac{ex^2}{d}\right)}{(m+1)\sqrt{d+ex^2}}$$

input
```
Int[(x^m*(a + b*ArcTan[c*x]))/Sqrt[d + e*x^2],x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x^m(a + b\arctan(cx))}{\sqrt{e\,x^2 + d}}dx$$

input
```
int(x^m*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x)
```

output
```
int(x^m*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^m(a + b\arctan(cx))}{\sqrt{d + ex^2}}\,dx = \int \frac{(b\arctan(cx) + a)x^m}{\sqrt{ex^2 + d}}\,dx$$

input
```
integrate(x^m*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
integral((b*arctan(c*x) + a)*x^m/sqrt(e*x^2 + d), x)
```

## Sympy [N/A]

Not integrable

Time = 25.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{x^m(a + b\arctan(cx))}{\sqrt{d + ex^2}}\,dx = \int \frac{x^m(a + b\,\mathrm{atan}(cx))}{\sqrt{d + ex^2}}\,dx$$

input
```
integrate(x**m*(a+b*atan(c*x))/(e*x**2+d)**(1/2),x)
```

output
```
Integral(x**m*(a + b*atan(c*x))/sqrt(d + e*x**2), x)
```

## Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^m(a+b\arctan(cx))}{\sqrt{d+ex^2}}\,dx=\int \frac{(b\arctan{(cx)}+a)x^m}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate(x^m*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*arctan(c*x) + a)*x^m/sqrt(e*x^2 + d), x)
```

## Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^m(a+b\arctan(cx))}{\sqrt{d+ex^2}}\,dx=\int \frac{(b\arctan{(cx)}+a)x^m}{\sqrt{ex^2+d}}\,dx$$

input
```
integrate(x^m*(a+b*arctan(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)*x^m/sqrt(e*x^2 + d), x)
```

## Mupad [N/A]

Not integrable

Time = 1.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^m(a+b\arctan(cx))}{\sqrt{d+ex^2}}\,dx=\int \frac{x^m\,\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}{\sqrt{e\,x^2+d}}\,dx$$

input
```
int((x^m*(a + b*atan(c*x)))/(d + e*x^2)^(1/2),x)
```

output
```
int((x^m*(a + b*atan(c*x)))/(d + e*x^2)^(1/2), x)
```

## Reduce [N/A]

Not integrable

Time = 200.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^m(a+b\arctan(cx))}{\sqrt{d+ex^2}}\,dx=\int \frac{x^m(atan(cx)\,b+a)}{\sqrt{e\,x^2+d}}dx$$

input
```
int(x^m*(a+b*atan(c*x))/(e*x^2+d)^(1/2),x)
```

output
```
int(x^m*(a+b*atan(c*x))/(e*x^2+d)^(1/2),x)
```

# 3.1235 $\int \frac{x^m(a+b\arctan(cx))}{(d+ex^2)^{3/2}}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^m(a+b\arctan(cx))}{(d+ex^2)^{3/2}}\,dx = \frac{ax^{1+m}\sqrt{1+\frac{ex^2}{d}}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{1+m}{2},\frac{3+m}{2},-\frac{ex^2}{d}\right)}{d(1+m)\sqrt{d+ex^2}} + b\text{Int}\left(\frac{x^m\arctan(cx)}{(d+ex^2)^{3/2}},x\right)$$

output
```
a*x^(1+m)*(1+e*x^2/d)^(1/2)*hypergeom([3/2, 1/2+1/2*m],[3/2+1/2*m],-e*x^2/
d)/d/(1+m)/(e*x^2+d)^(1/2)+b*Defer(Int)(x^m*arctan(c*x)/(e*x^2+d)^(3/2),x)
```

## Mathematica [N/A]

Not integrable

Time = 3.64 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^m(a+b\arctan(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{x^m(a+b\arctan(cx))}{(d+ex^2)^{3/2}}\,dx$$

input
```
Integrate[(x^m*(a + b*ArcTan[c*x]))/(d + e*x^2)^(3/2),x]
```

output
```
Integrate[(x^m*(a + b*ArcTan[c*x]))/(d + e*x^2)^(3/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m(a+b\arctan(cx))}{(d+ex^2)^{3/2}}dx$$

$$\Big\downarrow \text{ 5517}$$

$$a\int \frac{x^m}{(ex^2+d)^{3/2}}dx+b\int \frac{x^m\arctan(cx)}{(ex^2+d)^{3/2}}dx$$

$$\Big\downarrow \text{ 279}$$

$$\frac{a\sqrt{\frac{ex^2}{d}+1}\int \frac{x^m}{\left(\frac{ex^2}{d}+1\right)^{3/2}}dx}{d\sqrt{d+ex^2}}+b\int \frac{x^m\arctan(cx)}{(ex^2+d)^{3/2}}dx$$

$$\Big\downarrow \text{ 278}$$

$$b\int \frac{x^m\arctan(cx)}{(ex^2+d)^{3/2}}dx+\frac{ax^{m+1}\sqrt{\frac{ex^2}{d}+1}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m+1}{2},\frac{m+3}{2},-\frac{ex^2}{d}\right)}{d(m+1)\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{ 5560}$$

$$b\int \frac{x^m\arctan(cx)}{(ex^2+d)^{3/2}}dx+\frac{ax^{m+1}\sqrt{\frac{ex^2}{d}+1}\,\text{Hypergeometric2F1}\left(\frac{3}{2},\frac{m+1}{2},\frac{m+3}{2},-\frac{ex^2}{d}\right)}{d(m+1)\sqrt{d+ex^2}}$$

input
```
Int[(x^m*(a + b*ArcTan[c*x]))/(d + e*x^2)^(3/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x^m(a+b\arctan(cx))}{(e\,x^2+d)^{\frac{3}{2}}}dx$$

input
```
int(x^m*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x)
```

output
```
int(x^m*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.87

$$\int \frac{x^m(a+b\arctan(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{(b\arctan(cx)+a)x^m}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arctan(c*x) + a)*x^m/(e^2*x^4 + 2*d*e*x^2 + d^
2), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^m(a+b\arctan(cx))}{(d+ex^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**m*(a+b*atan(c*x))/(e*x**2+d)**(3/2),x)
```

output
```
Timed out
```

## Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^m(a+b\arctan(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{(b\arctan(cx)+a)x^m}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*arctan(c*x) + a)*x^m/(e*x^2 + d)^(3/2), x)
```

## Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^m(a+b\arctan(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{(b\arctan(cx)+a)x^m}{(ex^2+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^m*(a+b*arctan(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)*x^m/(e*x^2 + d)^(3/2), x)
```

## Mupad [N/A]

Not integrable

Time = 1.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^m(a+b\arctan(cx))}{(d+ex^2)^{3/2}}\,dx = \int \frac{x^m\,(a+b\,\mathrm{atan}(c\,x))}{(e\,x^2+d)^{3/2}}\,dx$$

input
```
int((x^m*(a + b*atan(c*x)))/(d + e*x^2)^(3/2),x)
```

output
```
int((x^m*(a + b*atan(c*x)))/(d + e*x^2)^(3/2), x)
```

## Reduce [N/A]

Not integrable

Time = 200.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^m(a+b\arctan(cx))}{(d+ex^2)^{3/2}}\,dx=\int \frac{x^m(atan(cx)\,b+a)}{(e\,x^2+d)^{\frac{3}{2}}}dx$$

input
```
int(x^m*(a+b*atan(c*x))/(e*x^2+d)^(3/2),x)
```

output
```
int(x^m*(a+b*atan(c*x))/(e*x^2+d)^(3/2),x)
```

# 3.1236 $\int \frac{x^m(a+b\arctan(cx))}{\left(d+ex^2\right)^{5/2}}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{x^m(a+b\arctan(cx))}{\left(d+ex^2\right)^{5/2}}\,dx=\frac{ax^{1+m}\sqrt{1+\frac{ex^2}{d}}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{1+m}{2},\frac{3+m}{2},-\frac{ex^2}{d}\right)}{d^2(1+m)\sqrt{d+ex^2}}+b\text{Int}\left(\frac{x^m\arctan(cx)}{\left(d+ex^2\right)^{5/2}},x\right)$$

output
```
a*x^(1+m)*(1+e*x^2/d)^(1/2)*hypergeom([5/2, 1/2+1/2*m],[3/2+1/2*m],-e*x^2/
d)/d^2/(1+m)/(e*x^2+d)^(1/2)+b*Defer(Int)(x^m*arctan(c*x)/(e*x^2+d)^(5/2),
x)
```

## Mathematica [N/A]

Not integrable

Time = 5.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int \frac{x^m(a+b\arctan(cx))}{\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{x^m(a+b\arctan(cx))}{\left(d+ex^2\right)^{5/2}}\,dx$$

input
```
Integrate[(x^m*(a + b*ArcTan[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
Integrate[(x^m*(a + b*ArcTan[c*x]))/(d + e*x^2)^(5/2), x]
```

## Rubi [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{5517}$$

$$a\int \frac{x^m}{(ex^2+d)^{5/2}}dx+b\int \frac{x^m\arctan(cx)}{(ex^2+d)^{5/2}}dx$$

$$\Big\downarrow \text{279}$$

$$\frac{a\sqrt{\frac{ex^2}{d}+1}\int \frac{x^m}{\left(\frac{ex^2}{d}+1\right)^{5/2}}dx}{d^2\sqrt{d+ex^2}}+b\int \frac{x^m\arctan(cx)}{(ex^2+d)^{5/2}}dx$$

$$\Big\downarrow \text{278}$$

$$b\int \frac{x^m\arctan(cx)}{(ex^2+d)^{5/2}}dx+\frac{ax^{m+1}\sqrt{\frac{ex^2}{d}+1}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{m+1}{2},\frac{m+3}{2},-\frac{ex^2}{d}\right)}{d^2(m+1)\sqrt{d+ex^2}}$$

$$\Big\downarrow \text{5560}$$

$$b\int \frac{x^m\arctan(cx)}{(ex^2+d)^{5/2}}dx+\frac{ax^{m+1}\sqrt{\frac{ex^2}{d}+1}\,\text{Hypergeometric2F1}\left(\frac{5}{2},\frac{m+1}{2},\frac{m+3}{2},-\frac{ex^2}{d}\right)}{d^2(m+1)\sqrt{d+ex^2}}$$

input
```
Int[(x^m*(a + b*ArcTan[c*x]))/(d + e*x^2)^(5/2),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{x^m (a + b \arctan (cx))}{(e\,x^2 + d)^{\frac{5}{2}}} dx$$

input
```
int(x^m*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x)
```

output
```
int(x^m*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x)
```

## Fricas [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.35

$$\int \frac{x^m (a + b \arctan(cx))}{(d + ex^2)^{5/2}} \, dx = \int \frac{(b \arctan (cx) + a) x^m}{(ex^2 + d)^{\frac{5}{2}}} \, dx$$

input
```
integrate(x^m*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x^2 + d)*(b*arctan(c*x) + a)*x^m/(e^3*x^6 + 3*d*e^2*x^4 +
3*d^2*e*x^2 + d^3), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^m (a + b \arctan(cx))}{(d + ex^2)^{5/2}} \, dx = \text{Timed out}$$

input
```
integrate(x**m*(a+b*atan(c*x))/(e*x**2+d)**(5/2),x)
```

output
```
Timed out
```

## Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^m(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{(b\arctan(cx)+a)x^m}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^m*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*arctan(c*x) + a)*x^m/(e*x^2 + d)^(5/2), x)
```

## Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^m(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{(b\arctan(cx)+a)x^m}{(ex^2+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^m*(a+b*arctan(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)*x^m/(e*x^2 + d)^(5/2), x)
```

## Mupad [N/A]

Not integrable

Time = 1.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^m(a+b\arctan(cx))}{(d+ex^2)^{5/2}}\,dx = \int \frac{x^m\,(a+b\,\mathrm{atan}(c\,x))}{(e\,x^2+d)^{5/2}}\,dx$$

input
```
int((x^m*(a + b*atan(c*x)))/(d + e*x^2)^(5/2),x)
```

output
```
int((x^m*(a + b*atan(c*x)))/(d + e*x^2)^(5/2), x)
```

## Reduce [N/A]

Not integrable

Time = 200.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{x^m(a + b\arctan(cx))}{(d + ex^2)^{5/2}}\,dx = \int \frac{x^m(atan(cx)\,b + a)}{(e\,x^2 + d)^{\frac{5}{2}}}dx$$

input
```
int(x^m*(a+b*atan(c*x))/(e*x^2+d)^(5/2),x)
```

output
```
int(x^m*(a+b*atan(c*x))/(e*x^2+d)^(5/2),x)
```

# 3.1237 $\int x^m(d+ex^2)^p(a+b\arctan(cx))\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 21

$$\int x^m(d+ex^2)^p(a+b\arctan(cx))\,dx = \frac{ax^{1+m}(d+ex^2)^p\left(1+\frac{ex^2}{d}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1+m}{2},-p,\frac{3+m}{2},-\frac{ex^2}{d}\right)}{1+m} + b\text{Int}\left(x^m(d+ex^2)^p\arctan(cx),x\right)$$

output
```
a*x^(1+m)*(e*x^2+d)^p*hypergeom([-p, 1/2+1/2*m],[3/2+1/2*m],-e*x^2/d)/(1+m
)/((1+e*x^2/d)^p)+b*Defer(Int)(x^m*(e*x^2+d)^p*arctan(c*x),x)
```

## Mathematica [N/A]

Not integrable

Time = 2.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int x^m(d+ex^2)^p(a+b\arctan(cx))\,dx = \int x^m(d+ex^2)^p(a+b\arctan(cx))\,dx$$

input
```
Integrate[x^m*(d + e*x^2)^p*(a + b*ArcTan[c*x]),x]
```

output
```
Integrate[x^m*(d + e*x^2)^p*(a + b*ArcTan[c*x]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \left(d+ex^2\right)^p (a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5517}$$

$$a\int x^m \left(ex^2+d\right)^p dx+b\int x^m \left(ex^2+d\right)^p \arctan(cx)dx$$

$$\Big\downarrow \text{279}$$

$$a\left(d+ex^2\right)^p \left(\frac{ex^2}{d}+1\right)^{-p}\int x^m \left(\frac{ex^2}{d}+1\right)^p dx+b\int x^m \left(ex^2+d\right)^p \arctan(cx)dx$$

$$\Big\downarrow \text{278}$$

$$b\int x^m \left(ex^2+d\right)^p \arctan(cx)dx+\frac{ax^{m+1}\left(d+ex^2\right)^p \left(\frac{ex^2}{d}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{m+1}{2},-p,\frac{m+3}{2},-\frac{ex^2}{d}\right)}{m+1}$$

$$\Big\downarrow \text{5560}$$

$$b\int x^m \left(ex^2+d\right)^p \arctan(cx)dx+\frac{ax^{m+1}\left(d+ex^2\right)^p \left(\frac{ex^2}{d}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{m+1}{2},-p,\frac{m+3}{2},-\frac{ex^2}{d}\right)}{m+1}$$

input
```
Int[x^m*(d + e*x^2)^p*(a + b*ArcTan[c*x]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 1.66 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int x^{m} \left(e\,x^{2}+d\right)^{p} \left(a+b\arctan\left(c x\right)\right)dx$$

input
```
int(x^m*(e*x^2+d)^p*(a+b*arctan(c*x)),x)
```

output
```
int(x^m*(e*x^2+d)^p*(a+b*arctan(c*x)),x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int x^m(d+ex^2)^p(a+b\arctan(cx))\,dx=\int (b\arctan\left(cx\right)+a)\left(ex^2+d\right)^px^m\,dx$$

input
```
integrate(x^m*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
integral((b*arctan(c*x) + a)*(e*x^2 + d)^p*x^m, x)
```

## Sympy [F(-1)]

Timed out.

$$\int x^m (d+ex^2)^p (a+b\arctan(cx))\,dx = \text{Timed out}$$

input
```
integrate(x**m*(e*x**2+d)**p*(a+b*atan(c*x)),x)
```

output
```
Timed out
```

## Maxima [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int x^m (d+ex^2)^p (a+b\arctan(cx))\,dx = \int (b\arctan(cx)+a)(ex^2+d)^p x^m\,dx$$

input
```
integrate(x^m*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
integrate((b*arctan(c*x) + a)*(e*x^2 + d)^p*x^m, x)
```

## Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int x^m (d+ex^2)^p (a+b\arctan(cx))\,dx = \int (b\arctan(cx)+a)(ex^2+d)^p x^m\,dx$$

input
```
integrate(x^m*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)*(e*x^2 + d)^p*x^m, x)
```

## Mupad [N/A]

Not integrable

Time = 1.47 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.10

$$\int x^m (d+ex^2)^p (a+b\arctan(cx))\,dx = \int x^m\,(a+b\,\mathrm{atan}(c\,x))\,\left(e\,x^2+d\right)^p\,dx$$

input
```
int(x^m*(a + b*atan(c*x))*(d + e*x^2)^p,x)
```

output
```
int(x^m*(a + b*atan(c*x))*(d + e*x^2)^p, x)
```

## Reduce [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 833, normalized size of antiderivative = 39.67

$$\int x^m (d+ex^2)^p (a+b\arctan(cx))\,dx = \text{Too large to display}$$

input
```
int(x^m*(e*x^2+d)^p*(a+b*atan(c*x)),x)
```

output

```
(x**m*(d + e*x**2)**p*a*x + int((x**m*(d + e*x**2)**p*atan(c*x)*x**2)/(d*m
 + 2*d*p + d + e*m*x**2 + 2*e*p*x**2 + e*x**2),x)*b*e*m**2 + 4*int((x**m*(
d + e*x**2)**p*atan(c*x)*x**2)/(d*m + 2*d*p + d + e*m*x**2 + 2*e*p*x**2 +
e*x**2),x)*b*e*m*p + 2*int((x**m*(d + e*x**2)**p*atan(c*x)*x**2)/(d*m + 2*
d*p + d + e*m*x**2 + 2*e*p*x**2 + e*x**2),x)*b*e*m + 4*int((x**m*(d + e*x*
*2)**p*atan(c*x)*x**2)/(d*m + 2*d*p + d + e*m*x**2 + 2*e*p*x**2 + e*x**2),
x)*b*e*p**2 + 4*int((x**m*(d + e*x**2)**p*atan(c*x)*x**2)/(d*m + 2*d*p + d
 + e*m*x**2 + 2*e*p*x**2 + e*x**2),x)*b*e*p + int((x**m*(d + e*x**2)**p*at
an(c*x)*x**2)/(d*m + 2*d*p + d + e*m*x**2 + 2*e*p*x**2 + e*x**2),x)*b*e +
int((x**m*(d + e*x**2)**p*atan(c*x))/(d*m + 2*d*p + d + e*m*x**2 + 2*e*p*x
**2 + e*x**2),x)*b*d*m**2 + 4*int((x**m*(d + e*x**2)**p*atan(c*x))/(d*m +
2*d*p + d + e*m*x**2 + 2*e*p*x**2 + e*x**2),x)*b*d*m*p + 2*int((x**m*(d +
e*x**2)**p*atan(c*x))/(d*m + 2*d*p + d + e*m*x**2 + 2*e*p*x**2 + e*x**2),x
)*b*d*m + 4*int((x**m*(d + e*x**2)**p*atan(c*x))/(d*m + 2*d*p + d + e*m*x*
*2 + 2*e*p*x**2 + e*x**2),x)*b*d*p**2 + 4*int((x**m*(d + e*x**2)**p*atan(c
*x))/(d*m + 2*d*p + d + e*m*x**2 + 2*e*p*x**2 + e*x**2),x)*b*d*p + int((x*
*m*(d + e*x**2)**p*atan(c*x))/(d*m + 2*d*p + d + e*m*x**2 + 2*e*p*x**2 + e
*x**2),x)*b*d + 2*int((x**m*(d + e*x**2)**p)/(d*m + 2*d*p + d + e*m*x**2 +
 2*e*p*x**2 + e*x**2),x)*a*d*m*p + 4*int((x**m*(d + e*x**2)**p)/(d*m + 2*d
*p + d + e*m*x**2 + 2*e*p*x**2 + e*x**2),x)*a*d*p**2 + 2*int((x**m*(d +...
```

# 3.1238 $\int x^{-2-2p}(d+ex^2)^p (a+b\arctan(cx))\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 25

$$\int x^{-2-2p}(d+ex^2)^p (a+b\arctan(cx))\,dx = -\frac{ax^{-1-2p}(d+ex^2)^p \left(1+\frac{ex^2}{d}\right)^{-p} \text{Hypergeometric2F1}\left(\frac{1}{2}(-1-2p),-p,\frac{1}{2}(1-2p),-\frac{ex^2}{d}\right)}{1+2p} + b\text{Int}\left(x^{-2-2p}(d+ex^2)^p \arctan(cx),x\right)$$

output
```
-a*x^(-1-2*p)*(e*x^2+d)^p*hypergeom([-p, -1/2-p],[1/2-p],-e*x^2/d)/(1+2*p)
/((1+e*x^2/d)^p)+b*Defer(Int)(x^(-2-2*p)*(e*x^2+d)^p*arctan(c*x),x)
```

## Mathematica [N/A]

Not integrable

Time = 2.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int x^{-2-2p}(d+ex^2)^p (a+b\arctan(cx))\,dx = \int x^{-2-2p}(d+ex^2)^p (a+b\arctan(cx))\,dx$$

input
```
Integrate[x^(-2 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]),x]
```

output
```
Integrate[x^(-2 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-2p-2}\left(d+ex^2\right)^p(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5517}$$

$$a\int x^{-2(p+1)}\left(ex^2+d\right)^p dx+b\int x^{-2(p+1)}\left(ex^2+d\right)^p\arctan(cx)dx$$

$$\Big\downarrow \text{279}$$

$$a\left(d+ex^2\right)^p\left(\frac{ex^2}{d}+1\right)^{-p}\int x^{-2(p+1)}\left(\frac{ex^2}{d}+1\right)^p dx+b\int x^{-2(p+1)}\left(ex^2+d\right)^p\arctan(cx)dx$$

$$\Big\downarrow \text{278}$$

$$b\int x^{-2(p+1)}\left(ex^2+d\right)^p\arctan(cx)dx-\frac{ax^{-2p-1}\left(d+ex^2\right)^p\left(\frac{ex^2}{d}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2}(-2p-1),-p,\frac{1}{2}(1-2p),-\frac{ex^2}{d}\right)}{2p+1}$$

$$\Big\downarrow \text{5560}$$

$$b\int x^{-2(p+1)}\left(ex^2+d\right)^p\arctan(cx)dx-\frac{ax^{-2p-1}\left(d+ex^2\right)^p\left(\frac{ex^2}{d}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2}(-2p-1),-p,\frac{1}{2}(1-2p),-\frac{ex^2}{d}\right)}{2p+1}$$

input
```
Int[x^(-2 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 1.00 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int x^{-2-2p} \left(e\,x^{2}+d\right)^{p} \left(a+b\arctan\left(cx\right)\right)dx$$

input
```
int(x^(-2-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x)
```

output
```
int(x^(-2-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int x^{-2-2p} \left(d+ex^2\right)^p (a+b\arctan(cx))\,dx = \int \left(b\arctan\left(cx\right)+a\right)\left(ex^2+d\right)^p x^{-2\,p-2}\,dx$$

input
```
integrate(x^(-2-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
integral((b*arctan(c*x) + a)*(e*x^2 + d)^p*x^(-2*p - 2), x)
```

## Sympy [F(-1)]

Timed out.

$$\int x^{-2-2p}\left(d+ex^2\right)^p\left(a+b\arctan(cx)\right)dx=\text{Timed out}$$

input
```
integrate(x**(-2-2*p)*(e*x**2+d)**p*(a+b*atan(c*x)),x)
```

output
```
Timed out
```

## Maxima [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int x^{-2-2p}\left(d+ex^2\right)^p\left(a+b\arctan(cx)\right)dx=\int (b\arctan{(cx)}+a)\left(ex^2+d\right)^p x^{-2\,p-2}\,dx$$

input
```
integrate(x^(-2-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
integrate((b*arctan(c*x) + a)*(e*x^2 + d)^p*x^(-2*p - 2), x)
```

## Giac [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int x^{-2-2p}\left(d+ex^2\right)^p\left(a+b\arctan(cx)\right)dx=\int (b\arctan{(cx)}+a)\left(ex^2+d\right)^p x^{-2\,p-2}\,dx$$

input
```
integrate(x^(-2-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)*(e*x^2 + d)^p*x^(-2*p - 2), x)
```

## Mupad [N/A]

Not integrable

Time = 1.14 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int x^{-2-2p}\left(d+ex^2\right)^p\left(a+b\arctan(cx)\right)dx=\int\frac{\left(a+b\,\mathrm{atan}(c\,x)\right)\,\left(e\,x^2+d\right)^p}{x^{2\,p+2}}\,dx$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^p)/x^(2*p + 2),x)
```

output
```
int(((a + b*atan(c*x))*(d + e*x^2)^p)/x^(2*p + 2), x)
```

## Reduce [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 3643, normalized size of antiderivative = 145.72

$$\int x^{-2-2p}\left(d+ex^2\right)^p\left(a+b\arctan(cx)\right)dx=\text{Too large to display}$$

input
```
int(x^(-2-2*p)*(e*x^2+d)^p*(a+b*atan(c*x)),x)
```

output
```
( - 2*(d + e*x**2)**p*atan(c*x)*b*c**3*d**2*p - 2*(d + e*x**2)**p*atan(c*x
)*b*c*d*e*p - 2*(d + e*x**2)**p*a*c**3*d**2*p - 2*(d + e*x**2)**p*a*c*d*e*
p - (d + e*x**2)**p*b*c**2*d*e*x - (d + e*x**2)**p*b*e**2*x - 8*x**(2*p)*i
nt((d + e*x**2)**p/(2*x**(2*p)*c**4*d**2*p*x**4 + x**(2*p)*c**4*d**2*x**4
+ 2*x**(2*p)*c**4*d*e*p*x**6 + x**(2*p)*c**4*d*e*x**6 + 2*x**(2*p)*c**2*d*
*2*p*x**2 + x**(2*p)*c**2*d**2*x**2 + 2*x**(2*p)*c**2*d*e*p*x**4 + 2*x**(2
*p)*c**2*d*e*x**4 + x**(2*p)*c**2*e**2*x**6 + x**(2*p)*d*e*x**2 + x**(2*p)
*e**2*x**4),x)*a*c**3*d**3*e*p**3*x - 4*x**(2*p)*int((d + e*x**2)**p/(2*x*
*(2*p)*c**4*d**2*p*x**4 + x**(2*p)*c**4*d**2*x**4 + 2*x**(2*p)*c**4*d*e*p*
x**6 + x**(2*p)*c**4*d*e*x**6 + 2*x**(2*p)*c**2*d**2*p*x**2 + x**(2*p)*c**
2*d**2*x**2 + 2*x**(2*p)*c**2*d*e*p*x**4 + 2*x**(2*p)*c**2*d*e*x**4 + x**(
2*p)*c**2*e**2*x**6 + x**(2*p)*d*e*x**2 + x**(2*p)*e**2*x**4),x)*a*c**3*d*
*3*e*p**2*x - 4*x**(2*p)*int((d + e*x**2)**p/(2*x**(2*p)*c**4*d**2*p*x**4
+ x**(2*p)*c**4*d**2*x**4 + 2*x**(2*p)*c**4*d*e*p*x**6 + x**(2*p)*c**4*d*e
*x**6 + 2*x**(2*p)*c**2*d**2*p*x**2 + x**(2*p)*c**2*d**2*x**2 + 2*x**(2*p)
*c**2*d*e*p*x**4 + 2*x**(2*p)*c**2*d*e*x**4 + x**(2*p)*c**2*e**2*x**6 + x*
*(2*p)*d*e*x**2 + x**(2*p)*e**2*x**4),x)*a*c*d**2*e**2*p**2*x + 4*x**(2*p)
*int((d + e*x**2)**p/(2*x**(2*p)*c**4*d**2*p*x**3 + x**(2*p)*c**4*d**2*x**
3 + 2*x**(2*p)*c**4*d*e*p*x**5 + x**(2*p)*c**4*d*e*x**5 + 2*x**(2*p)*c**2*
d**2*p*x + x**(2*p)*c**2*d**2*x + 2*x**(2*p)*c**2*d*e*p*x**3 + 2*x**(2*...
```

# 3.1239 $\int x^{-3-2p}(d+ex^2)^p\,(a+b\arctan(cx))\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 129

$$\int x^{-3-2p}(d+ex^2)^p\,(a+b\arctan(cx))\,dx = -\frac{bcx^{-1-2p}(d+ex^2)^p\left(1+\frac{ex^2}{d}\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{2}(-1-2p),1,-1-p,\frac{1}{2}(1-2p),-c^2x^2,-\frac{ex^2}{d}\right)}{2\,(1+3p+2p^2)} - \frac{x^{-2(1+p)}(d+ex^2)^{1+p}\,(a+b\arctan(cx))}{2d(1+p)}$$

output
```
-1/2*b*c*x^(-1-2*p)*(e*x^2+d)^p*AppellF1(-1/2-p,1,-1-p,1/2-p,-c^2*x^2,-e*x
^2/d)/(2*p^2+3*p+1)/((1+e*x^2/d)^p)-1/2*(e*x^2+d)^(p+1)*(a+b*arctan(c*x))/
d/(p+1)/(x^(2*p+2))
```

## Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.29

$$\int x^{-3-2p}(d+ex^2)^p\,(a+b\arctan(cx))\,dx = -\frac{x^{-2(1+p)}(d+ex^2)^p\left(1+\frac{ex^2}{d}\right)^{-p}\left(b(c^2d-e)\,x\operatorname{AppellF1}\left(-\frac{1}{2}-p,-p,1,\frac{1}{2}-p,-\frac{ex^2}{d},-c^2x^2\right)+c(1+2\right.}{2cd(1+p)(}$$

input
```
Integrate[x^(-3 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]),x]
```

output
```
-1/2*((d + e*x^2)^p*(b*(c^2*d - e)*x*AppellF1[-1/2 - p, -p, 1, 1/2 - p, -(
(e*x^2)/d), -(c^2*x^2)] + c*(1 + 2*p)*(d + e*x^2)*(1 + (e*x^2)/d)^p*(a + b
*ArcTan[c*x]) + b*e*x*Hypergeometric2F1[-1/2 - p, -p, 1/2 - p, -((e*x^2)/d
)]))/(c*d*(1 + p)*(1 + 2*p)*x^(2*(1 + p))*(1 + (e*x^2)/d)^p)
```

## Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5511, 27, 393, 152, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-2p-3}\left(d+ex^2\right)^p(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5511}$$

$$-bc\int -\frac{x^{-2(p+1)}\left(ex^2+d\right)^{p+1}}{2d(p+1)\left(c^2x^2+1\right)}dx-\frac{x^{-2(p+1)}\left(d+ex^2\right)^{p+1}(a+b\arctan(cx))}{2d(p+1)}$$

$$\Big\downarrow \text{27}$$

$$\frac{bc\int \frac{x^{-2(p+1)}\left(ex^2+d\right)^{p+1}}{c^2x^2+1}dx}{2d(p+1)}-\frac{x^{-2(p+1)}\left(d+ex^2\right)^{p+1}(a+b\arctan(cx))}{2d(p+1)}$$

$$\Big\downarrow \text{393}$$

$$\frac{bcx^{-2p-3}\left(x^2\right)^{p+\frac{3}{2}}\int \frac{\left(x^2\right)^{-p-\frac{3}{2}}\left(ex^2+d\right)^{p+1}}{c^2x^2+1}dx^2}{4d(p+1)}-\frac{x^{-2(p+1)}\left(d+ex^2\right)^{p+1}(a+b\arctan(cx))}{2d(p+1)}$$

$$\Big\downarrow \text{152}$$

$$\frac{bcx^{-2p-3}\left(x^2\right)^{p+\frac{3}{2}}\left(d+ex^2\right)^p\left(\frac{ex^2}{d}+1\right)^{-p}\int \frac{\left(x^2\right)^{-p-\frac{3}{2}}\left(\frac{ex^2}{d}+1\right)^{p+1}}{c^2x^2+1}dx^2}{4(p+1)}-\frac{x^{-2(p+1)}\left(d+ex^2\right)^{p+1}(a+b\arctan(cx))}{2d(p+1)}$$

$$\Big\downarrow 150$$

$$-\frac{x^{-2(p+1)}\left(d+ex^2\right)^{p+1}(a+b\arctan(cx))}{2d(p+1)}-\frac{bcx^{-2p-1}\left(d+ex^2\right)^p\left(\frac{ex^2}{d}+1\right)^{-p}\text{AppellF1}\left(-p-\frac{1}{2},-p-1,1,\frac{1}{2}-p,-\frac{ex^2}{d},-c^2x^2\right)}{2(p+1)(2p+1)}$$

input
```
Int[x^(-3 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]),x]
```

output
```
-1/2*(b*c*x^(-1 - 2*p)*(d + e*x^2)^p*AppellF1[-1/2 - p, -1 - p, 1, 1/2 - p
, -((e*x^2)/d), -(c^2*x^2)])/((1 + p)*(1 + 2*p)*(1 + (e*x^2)/d)^p) - ((d +
 e*x^2)^(1 + p)*(a + b*ArcTan[c*x]))/(2*d*(1 + p)*x^(2*(1 + p)))
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 152
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])
Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e, f, m,
 n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

rule 393
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Simp[(e*x)^m/(2*x*(x^2)^(Simplify[(m + 1)/2] - 1))   Subs
t[Int[x^(Simplify[(m + 1)/2] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^2], x]
/; FreeQ[{a, b, c, d, e, m, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simp
lify[m + 2*p]] &&  !IntegerQ[m]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

## Maple [F]

$$\int x^{-3-2p}\left(e\,x^{2}+d\right)^{p}\left(a+b\arctan\left(cx\right)\right)dx$$

input
```
int(x^(-3-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x)
```

output
```
int(x^(-3-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x)
```

## Fricas [F]

$$\int x^{-3-2p}(d+ex^2)^p(a+b\arctan(cx))\,dx=\int(b\arctan(cx)+a)\left(ex^2+d\right)^p x^{-2p-3}\,dx$$

input
```
integrate(x^(-3-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
integral((b*arctan(c*x) + a)*(e*x^2 + d)^p*x^(-2*p - 3), x)
```

## Sympy [F(-1)]

Timed out.

$$\int x^{-3-2p}\left(d+ex^2\right)^p(a+b\arctan(cx))\,dx=\text{Timed out}$$

input
```
integrate(x**(-3-2*p)*(e*x**2+d)**p*(a+b*atan(c*x)),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int x^{-3-2p}\left(d+ex^2\right)^p(a+b\arctan(cx))\,dx=\int(b\arctan(cx)+a)\left(ex^2+d\right)^p x^{-2p-3}\,dx$$

input
```
integrate(x^(-3-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
b*integrate(arctan(c*x)*e^(p*log(e*x^2 + d) - 2*p*log(x))/x^3, x) - 1/2*(e
*x^2 + d)*a*e^(p*log(e*x^2 + d) - 2*p*log(x))/(d*(p + 1)*x^2)
```

## Giac [F]

$$\int x^{-3-2p}\left(d+ex^2\right)^p(a+b\arctan(cx))\,dx=\int(b\arctan(cx)+a)\left(ex^2+d\right)^p x^{-2p-3}\,dx$$

input
```
integrate(x^(-3-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)*(e*x^2 + d)^p*x^(-2*p - 3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int x^{-3-2p}\left(d+ex^2\right)^p(a+b\arctan(cx))\,dx=\int\frac{(a+b\,\mathrm{atan}(c\,x))\,\left(e\,x^2+d\right)^p}{x^{2\,p+3}}\,dx$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^p)/x^(2*p + 3),x)
```

output
```
int(((a + b*atan(c*x))*(d + e*x^2)^p)/x^(2*p + 3), x)
```

## Reduce [F]

$$\int x^{-3-2p}\left(d+ex^2\right)^p(a+b\arctan(cx))\,dx=\text{Too large to display}$$

input
```
int(x^(-3-2*p)*(e*x^2+d)^p*(a+b*atan(c*x)),x)
```

output
```
( - (d + e*x**2)**p*atan(c*x)*b*c*d - (d + e*x**2)**p*atan(c*x)*b*c*e*x**2
 - (d + e*x**2)**p*a*c*d - (d + e*x**2)**p*a*c*e*x**2 - (d + e*x**2)**p*b*
e*x + x**(2*p)*int((d + e*x**2)**p/(x**(2*p)*c**2*d*p*x**4 + x**(2*p)*c**2
*d*x**4 + x**(2*p)*c**2*e*p*x**6 + x**(2*p)*c**2*e*x**6 + x**(2*p)*d*p*x**
2 + x**(2*p)*d*x**2 + x**(2*p)*e*p*x**4 + x**(2*p)*e*x**4),x)*b*c**2*d**2*
p*x**2 + x**(2*p)*int((d + e*x**2)**p/(x**(2*p)*c**2*d*p*x**4 + x**(2*p)*c
**2*d*x**4 + x**(2*p)*c**2*e*p*x**6 + x**(2*p)*c**2*e*x**6 + x**(2*p)*d*p*
x**2 + x**(2*p)*d*x**2 + x**(2*p)*e*p*x**4 + x**(2*p)*e*x**4),x)*b*c**2*d*
*2*x**2 - 2*x**(2*p)*int((d + e*x**2)**p/(x**(2*p)*c**2*d*p*x**4 + x**(2*p
)*c**2*d*x**4 + x**(2*p)*c**2*e*p*x**6 + x**(2*p)*c**2*e*x**6 + x**(2*p)*d
*p*x**2 + x**(2*p)*d*x**2 + x**(2*p)*e*p*x**4 + x**(2*p)*e*x**4),x)*b*d*e*
p**2*x**2 - 3*x**(2*p)*int((d + e*x**2)**p/(x**(2*p)*c**2*d*p*x**4 + x**(2
*p)*c**2*d*x**4 + x**(2*p)*c**2*e*p*x**6 + x**(2*p)*c**2*e*x**6 + x**(2*p)
*d*p*x**2 + x**(2*p)*d*x**2 + x**(2*p)*e*p*x**4 + x**(2*p)*e*x**4),x)*b*d*
e*p*x**2 - x**(2*p)*int((d + e*x**2)**p/(x**(2*p)*c**2*d*p*x**4 + x**(2*p)
*c**2*d*x**4 + x**(2*p)*c**2*e*p*x**6 + x**(2*p)*c**2*e*x**6 + x**(2*p)*d*
p*x**2 + x**(2*p)*d*x**2 + x**(2*p)*e*p*x**4 + x**(2*p)*e*x**4),x)*b*d*e*x
**2 - 2*x**(2*p)*int((d + e*x**2)**p/(x**(2*p)*c**2*d*p*x**2 + x**(2*p)*c*
*2*d*x**2 + x**(2*p)*c**2*e*p*x**4 + x**(2*p)*c**2*e*x**4 + x**(2*p)*d*p +
 x**(2*p)*d + x**(2*p)*e*p*x**2 + x**(2*p)*e*x**2),x)*b*c**2*d*e*p**2*x...
```

# 3.1240 $\int x^{-4-2p}(d+ex^2)^p (a+b\arctan(cx))\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 25

$$\int x^{-4-2p}(d+ex^2)^p (a+b\arctan(cx))\,dx = -\frac{ax^{-3-2p}(d+ex^2)^p\left(1+\frac{ex^2}{d}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2}(-3-2p),-p,\frac{1}{2}(-1-2p),-\frac{ex^2}{d}\right)}{3+2p} + b\text{Int}\left(x^{-4-2p}(d+ex^2)^p\arctan(cx),x\right)$$

output
```
-a*x^(-3-2*p)*(e*x^2+d)^p*hypergeom([-p, -3/2-p],[-1/2-p],-e*x^2/d)/(3+2*p
)/((1+e*x^2/d)^p)+b*Defer(Int)(x^(-4-2*p)*(e*x^2+d)^p*arctan(c*x),x)
```

## Mathematica [N/A]

Not integrable

Time = 2.59 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int x^{-4-2p}(d+ex^2)^p (a+b\arctan(cx))\,dx = \int x^{-4-2p}(d+ex^2)^p (a+b\arctan(cx))\,dx$$

input
```
Integrate[x^(-4 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]),x]
```

output
```
Integrate[x^(-4 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-2p-4}\left(d+ex^2\right)^p(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \ 5517$$

$$a\int x^{-2(p+2)}\left(ex^2+d\right)^p dx+b\int x^{-2(p+2)}\left(ex^2+d\right)^p\arctan(cx)dx$$

$$\Big\downarrow \ 279$$

$$a\left(d+ex^2\right)^p\left(\frac{ex^2}{d}+1\right)^{-p}\int x^{-2(p+2)}\left(\frac{ex^2}{d}+1\right)^p dx+b\int x^{-2(p+2)}\left(ex^2+d\right)^p\arctan(cx)dx$$

$$\Big\downarrow \ 278$$

$$b\int x^{-2(p+2)}\left(ex^2+d\right)^p\arctan(cx)dx-\frac{ax^{-2p-3}\left(d+ex^2\right)^p\left(\frac{ex^2}{d}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2}(-2p-3),-p,\frac{1}{2}(-2p-1),-\frac{ex^2}{d}\right)}{2p+3}$$

$$\Big\downarrow \ 5560$$

$$b\int x^{-2(p+2)}\left(ex^2+d\right)^p\arctan(cx)dx-\frac{ax^{-2p-3}\left(d+ex^2\right)^p\left(\frac{ex^2}{d}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2}(-2p-3),-p,\frac{1}{2}(-2p-1),-\frac{ex^2}{d}\right)}{2p+3}$$

input
```
Int[x^(-4 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 1.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int x^{-4-2p} \left(e\,x^2+d\right)^p \left(a+b\arctan\left(cx\right)\right)dx$$

input
```
int(x^(-4-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x)
```

output
```
int(x^(-4-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x)
```

## Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int x^{-4-2p} \left(d+ex^2\right)^p \left(a+b\arctan(cx)\right)dx = \int \left(b\arctan\left(cx\right)+a\right)\left(ex^2+d\right)^p x^{-2\,p-4}\,dx$$

input
```
integrate(x^(-4-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
integral((b*arctan(c*x) + a)*(e*x^2 + d)^p*x^(-2*p - 4), x)
```

## Sympy [F(-1)]

Timed out.

$$\int x^{-4-2p}\left(d+ex^2\right)^p\left(a+b\arctan(cx)\right)dx=\text{Timed out}$$

input
```
integrate(x**(-4-2*p)*(e*x**2+d)**p*(a+b*atan(c*x)),x)
```

output
```
Timed out
```

## Maxima [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int x^{-4-2p}\left(d+ex^2\right)^p\left(a+b\arctan(cx)\right)dx=\int\left(b\arctan\left(cx\right)+a\right)\left(ex^2+d\right)^px^{-2\,p-4}\,dx$$

input
```
integrate(x^(-4-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
integrate((b*arctan(c*x) + a)*(e*x^2 + d)^p*x^(-2*p - 4), x)
```

## Giac [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int x^{-4-2p}\left(d+ex^2\right)^p\left(a+b\arctan(cx)\right)dx=\int\left(b\arctan\left(cx\right)+a\right)\left(ex^2+d\right)^px^{-2\,p-4}\,dx$$

input
```
integrate(x^(-4-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)*(e*x^2 + d)^p*x^(-2*p - 4), x)
```

## Mupad [N/A]

Not integrable

Time = 1.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int x^{-4-2p}\left(d+ex^2\right)^p(a+b\arctan(cx))\,dx=\int\frac{(a+b\,\mathrm{atan}(c\,x))\,\left(e\,x^2+d\right)^p}{x^{2\,p+4}}\,dx$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^p)/x^(2*p + 4),x)
```

output
```
int(((a + b*atan(c*x))*(d + e*x^2)^p)/x^(2*p + 4), x)
```

## Reduce [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 21533, normalized size of antiderivative = 861.32

$$\int x^{-4-2p}\left(d+ex^2\right)^p(a+b\arctan(cx))\,dx=\text{Too large to display}$$

input
```
int(x^(-4-2*p)*(e*x^2+d)^p*(a+b*atan(c*x)),x)
```

output
```
( - 4*(d + e*x**2)**p*atan(c*x)*b*c**3*d**3*p**3 - 10*(d + e*x**2)**p*atan
(c*x)*b*c**3*d**3*p**2 - 6*(d + e*x**2)**p*atan(c*x)*b*c**3*d**3*p - 6*(d
+ e*x**2)**p*atan(c*x)*b*c*d**2*e*p**2 - 6*(d + e*x**2)**p*atan(c*x)*b*c*d
**2*e*p - 4*(d + e*x**2)**p*a*c**3*d**3*p**3 - 10*(d + e*x**2)**p*a*c**3*d
**3*p**2 - 6*(d + e*x**2)**p*a*c**3*d**3*p - 6*(d + e*x**2)**p*a*c*d**2*e*
p**2 - 6*(d + e*x**2)**p*a*c*d**2*e*p + 2*(d + e*x**2)**p*b*c**6*d**3*p**2
*x**3 + 5*(d + e*x**2)**p*b*c**6*d**3*p*x**3 + 3*(d + e*x**2)**p*b*c**6*d*
*3*x**3 - 2*(d + e*x**2)**p*b*c**4*d**3*p**2*x - 3*(d + e*x**2)**p*b*c**4*
d**3*p*x + 2*(d + e*x**2)**p*b*c**4*d**2*e*p*x**3 + 3*(d + e*x**2)**p*b*c*
*4*d**2*e*x**3 - 2*(d + e*x**2)**p*b*c**2*d**2*e*p**2*x - 3*(d + e*x**2)**
p*b*c**2*d**2*e*p*x - 2*(d + e*x**2)**p*b*c**2*d*e**2*p**2*x**3 - 3*(d + e
*x**2)**p*b*c**2*d*e**2*p*x**3 + 2*(d + e*x**2)**p*b*d*e**2*p**2*x - 2*(d
+ e*x**2)**p*b*e**3*p*x**3 - 16*x**(2*p)*int((d + e*x**2)**p/(4*x**(2*p)*c
**4*d**2*p**2*x**5 + 12*x**(2*p)*c**4*d**2*p*x**5 + 9*x**(2*p)*c**4*d**2*x
**5 + 4*x**(2*p)*c**4*d*e*p**2*x**7 + 12*x**(2*p)*c**4*d*e*p*x**7 + 9*x**(
2*p)*c**4*d*e*x**7 + 4*x**(2*p)*c**2*d**2*p**2*x**3 + 12*x**(2*p)*c**2*d**
2*p*x**3 + 9*x**(2*p)*c**2*d**2*x**3 + 4*x**(2*p)*c**2*d*e*p**2*x**5 + 18*
x**(2*p)*c**2*d*e*p*x**5 + 18*x**(2*p)*c**2*d*e*x**5 + 6*x**(2*p)*c**2*e**
2*p*x**7 + 9*x**(2*p)*c**2*e**2*x**7 + 6*x**(2*p)*d*e*p*x**3 + 9*x**(2*p)*
d*e*x**3 + 6*x**(2*p)*e**2*p*x**5 + 9*x**(2*p)*e**2*x**5),x)*b*c**4*d**...
```

# 3.1241 $\int x^{-5-2p}(d+ex^2)^p\,(a+b\arctan(cx))\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 285

$$\int x^{-5-2p}(d+ex^2)^p\,(a+b\arctan(cx))\,dx = -\frac{b(e+c^2d(1+p))\,x^{-3-2p}(d+ex^2)^p\left(1+\frac{ex^2}{d}\right)^{-p}\operatorname{AppellF1}\left(\frac{1}{2}(-3-2p),1,-1-p,\frac{1}{2}(-1-2p),-c^2x^2\right.}{2cd(1+p)(2+p)(3+2p)} + \frac{ex^{-2(1+p)}(d+ex^2)^{1+p}\,(a+b\arctan(cx))}{2d^2(1+p)(2+p)} - \frac{x^{-2(2+p)}(d+ex^2)^{1+p}\,(a+b\arctan(cx))}{2d(2+p)} + \frac{bex^{-3-2p}(d+ex^2)^p\left(1+\frac{ex^2}{d}\right)^{-p}\operatorname{Hypergeometric2F1}\left(\frac{1}{2}(-3-2p),-1-p,\frac{1}{2}(-1-2p),-\frac{ex^2}{d}\right)}{2cd\left(6+13p+9p^2+2p^3\right)}$$

output

```
-1/2*b*(e+c^2*d*(p+1))*x^(-3-2*p)*(e*x^2+d)^p*AppellF1(-3/2-p,1,-1-p,-1/2-
p,-c^2*x^2,-e*x^2/d)/c/d/(p+1)/(2+p)/(3+2*p)/((1+e*x^2/d)^p)+1/2*e*(e*x^2+
d)^(p+1)*(a+b*arctan(c*x))/d^2/(p+1)/(2+p)/(x^(2*p+2))-1/2*(e*x^2+d)^(p+1)
*(a+b*arctan(c*x))/d/(2+p)/(x^(4+2*p))+1/2*b*e*x^(-3-2*p)*(e*x^2+d)^p*hype
rgeom([-1-p, -3/2-p],[-1/2-p],-e*x^2/d)/c/d/(2*p^3+9*p^2+13*p+6)/((1+e*x^2
/d)^p)
```

## Mathematica [F]

$$\int x^{-5-2p}\left(d+ex^2\right)^p(a+b\arctan(cx))\,dx=\int x^{-5-2p}\left(d+ex^2\right)^p(a+b\arctan(cx))\,dx$$

input
```
Integrate[x^(-5 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]),x]
```

output
```
Integrate[x^(-5 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]), x]
```

## Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {5511, 27, 446, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-2p-5}\left(d+ex^2\right)^p(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \; 5511$$

$$-bc\int -\frac{x^{-2(p+2)}\left(d(p+1)-ex^2\right)\left(ex^2+d\right)^{p+1}}{2d^2(p+1)(p+2)\left(c^2x^2+1\right)}dx+\frac{ex^{-2(p+1)}\left(d+ex^2\right)^{p+1}(a+b\arctan(cx))}{2d^2(p+1)(p+2)}-\frac{x^{-2(p+2)}\left(d+ex^2\right)^{p+1}(a+b\arctan(cx))}{2d(p+2)}$$

$$\Big\downarrow \; 27$$

$$\frac{bc\int \frac{x^{-2(p+2)}\left(d(p+1)-ex^2\right)\left(ex^2+d\right)^{p+1}}{c^2x^2+1}dx}{2d^2(p+1)(p+2)}+\frac{ex^{-2(p+1)}\left(d+ex^2\right)^{p+1}(a+b\arctan(cx))}{2d^2(p+1)(p+2)}-\frac{x^{-2(p+2)}\left(d+ex^2\right)^{p+1}(a+b\arctan(cx))}{2d(p+2)}$$

$$\Big\downarrow \; 446$$

$$\frac{bc\int \left(\frac{(d(p+1)c^2+e)x^{-2(p+2)}(ex^2+d)^{p+1}}{c^2(c^2x^2+1)}-\frac{ex^{-2(p+2)}(ex^2+d)^{p+1}}{c^2}\right)dx}{2d^2(p+1)(p+2)}+$$
$$\frac{ex^{-2(p+1)}(d+ex^2)^{p+1}(a+b\arctan(cx))}{2d^2(p+1)(p+2)}-\frac{x^{-2(p+2)}(d+ex^2)^{p+1}(a+b\arctan(cx))}{2d(p+2)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{ex^{-2(p+1)}(d+ex^2)^{p+1}(a+b\arctan(cx))}{2d^2(p+1)(p+2)}-\frac{x^{-2(p+2)}(d+ex^2)^{p+1}(a+b\arctan(cx))}{2d(p+2)}+$$
$$\frac{bc\left(\frac{dex^{-2p-3}(d+ex^2)^p\left(\frac{ex^2}{d}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2}(-2p-3),-p-1,\frac{1}{2}(-2p-1),-\frac{ex^2}{d}\right)}{c^2(2p+3)}-\frac{dx^{-2p-3}(c^2d(p+1)+e)(d+ex^2)^p\left(\frac{ex^2}{d}+1\right)}{c}\right.}{2d^2(p+1)(p+2)}$$

input
```
Int[x^(-5 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]),x]
```

output
```
(e*(d + e*x^2)^(1 + p)*(a + b*ArcTan[c*x]))/(2*d^2*(1 + p)*(2 + p)*x^(2*(1
 + p))) - ((d + e*x^2)^(1 + p)*(a + b*ArcTan[c*x]))/(2*d*(2 + p)*x^(2*(2 +
 p))) + (b*c*(-((d*(e + c^2*d*(1 + p))*x^(-3 - 2*p)*(d + e*x^2)^p*AppellF1
[-3/2 - p, -1 - p, 1, -1/2 - p, -((e*x^2)/d), -(c^2*x^2)])/(c^2*(3 + 2*p)*
(1 + (e*x^2)/d)^p)) + (d*e*x^(-3 - 2*p)*(d + e*x^2)^p*Hypergeometric2F1[(-
3 - 2*p)/2, -1 - p, (-1 - 2*p)/2, -((e*x^2)/d)])/(c^2*(3 + 2*p)*(1 + (e*x^
2)/d)^p)))/(2*d^2*(1 + p)*(2 + p))
```

## Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 446
```
Int[(((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((e_) + (f_.)*(x_)^2))/(
(c_) + (d_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(a + b*x^2)^
p*((e + f*x^2)/(c + d*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

## Maple [F]

$$\int x^{-5-2p}\left(e\,x^{2}+d\right)^{p}\left(a+b\arctan\left(cx\right)\right)dx$$

input
```
int(x^(-5-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x)
```

output
```
int(x^(-5-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x)
```

## Fricas [F]

$$\int x^{-5-2p}\left(d+ex^2\right)^p(a+b\arctan(cx))\,dx=\int\left(b\arctan\left(cx\right)+a\right)\left(ex^2+d\right)^{p}x^{-2\,p-5}\,dx$$

input
```
integrate(x^(-5-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
integral((b*arctan(c*x) + a)*(e*x^2 + d)^p*x^(-2*p - 5), x)
```

## Sympy [F(-1)]

Timed out.

$$\int x^{-5-2p}\left(d+ex^2\right)^p\left(a+b\arctan(cx)\right)dx=\text{Timed out}$$

input
```
integrate(x**(-5-2*p)*(e*x**2+d)**p*(a+b*atan(c*x)),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int x^{-5-2p}\left(d+ex^2\right)^p\left(a+b\arctan(cx)\right)dx=\int\left(b\arctan\left(cx\right)+a\right)\left(ex^2+d\right)^px^{-2\,p-5}\,dx$$

input
```
integrate(x^(-5-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
b*integrate(arctan(c*x)*e^(p*log(e*x^2 + d) - 2*p*log(x))/x^5, x) + 1/2*(e
^2*x^4 - d*e*p*x^2 - d^2*(p + 1))*a*e^(p*log(e*x^2 + d) - 2*p*log(x))/((p^
2 + 3*p + 2)*d^2*x^4)
```

## Giac [F]

$$\int x^{-5-2p}\left(d+ex^2\right)^p\left(a+b\arctan(cx)\right)dx=\int\left(b\arctan\left(cx\right)+a\right)\left(ex^2+d\right)^px^{-2\,p-5}\,dx$$

input
```
integrate(x^(-5-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)*(e*x^2 + d)^p*x^(-2*p - 5), x)
```

## Mupad [F(-1)]

Timed out.

$$\int x^{-5-2p}\left(d+ex^2\right)^p(a+b\arctan(cx))\,dx=\int \frac{(a+b\,\mathrm{atan}(c\,x))\,\left(e\,x^2+d\right)^p}{x^{2\,p+5}}\,dx$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^p)/x^(2*p + 5),x)
```

output
```
int(((a + b*atan(c*x))*(d + e*x^2)^p)/x^(2*p + 5), x)
```

## Reduce [F]

$$\int x^{-5-2p}\left(d+ex^2\right)^p(a+b\arctan(cx))\,dx=\text{too large to display}$$

input
```
int(x^(-5-2*p)*(e*x^2+d)^p*(a+b*atan(c*x)),x)
```

output
```
( - 12*(d + e*x**2)**p*atan(c*x)*b*c**3*d**3*p**3 - 36*(d + e*x**2)**p*ata
n(c*x)*b*c**3*d**3*p**2 - 33*(d + e*x**2)**p*atan(c*x)*b*c**3*d**3*p - 9*(
d + e*x**2)**p*atan(c*x)*b*c**3*d**3 - 12*(d + e*x**2)**p*atan(c*x)*b*c**3
*d**2*e*p**3*x**2 - 24*(d + e*x**2)**p*atan(c*x)*b*c**3*d**2*e*p**2*x**2 -
 9*(d + e*x**2)**p*atan(c*x)*b*c**3*d**2*e*p*x**2 + 12*(d + e*x**2)**p*ata
n(c*x)*b*c**3*d*e**2*p**2*x**4 + 24*(d + e*x**2)**p*atan(c*x)*b*c**3*d*e**
2*p*x**4 + 9*(d + e*x**2)**p*atan(c*x)*b*c**3*d*e**2*x**4 - 12*(d + e*x**2
)**p*a*c**3*d**3*p**3 - 36*(d + e*x**2)**p*a*c**3*d**3*p**2 - 33*(d + e*x*
*2)**p*a*c**3*d**3*p - 9*(d + e*x**2)**p*a*c**3*d**3 - 12*(d + e*x**2)**p*
a*c**3*d**2*e*p**3*x**2 - 24*(d + e*x**2)**p*a*c**3*d**2*e*p**2*x**2 - 9*(
d + e*x**2)**p*a*c**3*d**2*e*p*x**2 + 12*(d + e*x**2)**p*a*c**3*d*e**2*p**
2*x**4 + 24*(d + e*x**2)**p*a*c**3*d*e**2*p*x**4 + 9*(d + e*x**2)**p*a*c**
3*d*e**2*x**4 - 6*(d + e*x**2)**p*b*c**4*d**3*p**2*x - 9*(d + e*x**2)**p*b
*c**4*d**3*p*x - 3*(d + e*x**2)**p*b*c**4*d**3*x + 12*(d + e*x**2)**p*b*c*
*4*d**2*e*p**3*x**3 + 24*(d + e*x**2)**p*b*c**4*d**2*e*p**2*x**3 + 3*(d +
e*x**2)**p*b*c**4*d**2*e*p*x**3 - 9*(d + e*x**2)**p*b*c**4*d**2*e*x**3 + 2
2*(d + e*x**2)**p*b*c**2*d*e**2*p**2*x**3 + 39*(d + e*x**2)**p*b*c**2*d*e*
*2*p*x**3 + 9*(d + e*x**2)**p*b*c**2*d*e**2*x**3 + 6*(d + e*x**2)**p*b*e**
3*p*x**3 + 9*(d + e*x**2)**p*b*e**3*x**3 - 48*x**(2*p)*int((d + e*x**2)**p
/(4*x**(2*p)*c**2*d*p**4*x**4 + 20*x**(2*p)*c**2*d*p**3*x**4 + 35*x**(2...
```

# 3.1242 $\int x^{-6-2p}(d+ex^2)^p\,(a+b\arctan(cx))\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 25

$$\int x^{-6-2p}\left(d+ex^2\right)^p(a+b\arctan(cx))\,dx = -\frac{ax^{-5-2p}(d+ex^2)^p\left(1+\frac{ex^2}{d}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2}(-5-2p),-p,\frac{1}{2}(-3-2p),-\frac{ex^2}{d}\right)}{5+2p} + b\text{Int}\left(x^{-6-2p}\left(d+ex^2\right)^p\arctan(cx),x\right)$$

output
```
-a*x^(-5-2*p)*(e*x^2+d)^p*hypergeom([-p, -5/2-p],[-3/2-p],-e*x^2/d)/(5+2*p
)/((1+e*x^2/d)^p)+b*Defer(Int)(x^(-6-2*p)*(e*x^2+d)^p*arctan(c*x),x)
```

## Mathematica [N/A]

Not integrable

Time = 2.77 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int x^{-6-2p}\left(d+ex^2\right)^p(a+b\arctan(cx))\,dx = \int x^{-6-2p}\left(d+ex^2\right)^p(a+b\arctan(cx))\,dx$$

input
```
Integrate[x^(-6 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]),x]
```

output
```
Integrate[x^(-6 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-2p-6}\left(d+ex^2\right)^p(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5517}$$

$$a\int x^{-2(p+3)}\left(ex^2+d\right)^p dx+b\int x^{-2(p+3)}\left(ex^2+d\right)^p\arctan(cx)dx$$

$$\Big\downarrow \text{279}$$

$$a\left(d+ex^2\right)^p\left(\frac{ex^2}{d}+1\right)^{-p}\int x^{-2(p+3)}\left(\frac{ex^2}{d}+1\right)^p dx+b\int x^{-2(p+3)}\left(ex^2+d\right)^p\arctan(cx)dx$$

$$\Big\downarrow \text{278}$$

$$b\int x^{-2(p+3)}\left(ex^2+d\right)^p\arctan(cx)dx-\frac{ax^{-2p-5}\left(d+ex^2\right)^p\left(\frac{ex^2}{d}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2}(-2p-5),-p,\frac{1}{2}(-2p-3),-\frac{ex^2}{d}\right)}{2p+5}$$

$$\Big\downarrow \text{5560}$$

$$b\int x^{-2(p+3)}\left(ex^2+d\right)^p\arctan(cx)dx-\frac{ax^{-2p-5}\left(d+ex^2\right)^p\left(\frac{ex^2}{d}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2}(-2p-5),-p,\frac{1}{2}(-2p-3),-\frac{ex^2}{d}\right)}{2p+5}$$

input
```
Int[x^(-6 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 1.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int x^{-6-2p} \left(e\,x^{2}+d\right)^{p} \left(a+b\arctan\left(cx\right)\right)dx$$

input
```
int(x^(-6-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x)
```

output
```
int(x^(-6-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x)
```

## Fricas [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int x^{-6-2p}(d+ex^2)^p\,(a+b\arctan(cx))\,dx=\int (b\arctan\left(cx\right)+a)\left(ex^2+d\right)^p x^{-2\,p-6}\,dx$$

input
```
integrate(x^(-6-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
integral((b*arctan(c*x) + a)*(e*x^2 + d)^p*x^(-2*p - 6), x)
```

## Sympy [F(-1)]

Timed out.

$$\int x^{-6-2p}\left(d+ex^2\right)^p\left(a+b\arctan(cx)\right)dx=\text{Timed out}$$

input
```
integrate(x**(-6-2*p)*(e*x**2+d)**p*(a+b*atan(c*x)),x)
```

output
```
Timed out
```

## Maxima [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int x^{-6-2p}\left(d+ex^2\right)^p\left(a+b\arctan(cx)\right)dx=\int (b\arctan(cx)+a)\left(ex^2+d\right)^p x^{-2p-6}\,dx$$

input
```
integrate(x^(-6-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
integrate((b*arctan(c*x) + a)*(e*x^2 + d)^p*x^(-2*p - 6), x)
```

## Giac [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int x^{-6-2p}\left(d+ex^2\right)^p\left(a+b\arctan(cx)\right)dx=\int (b\arctan(cx)+a)\left(ex^2+d\right)^p x^{-2p-6}\,dx$$

input
```
integrate(x^(-6-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)*(e*x^2 + d)^p*x^(-2*p - 6), x)
```

## Mupad [N/A]

Not integrable

Time = 1.10 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int x^{-6-2p}\left(d+ex^2\right)^p (a+b\arctan(cx))\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))\,\left(e\,x^2+d\right)^p}{x^{2\,p+6}}\,dx$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^p)/x^(2*p + 6),x)
```

output
```
int(((a + b*atan(c*x))*(d + e*x^2)^p)/x^(2*p + 6), x)
```

## Reduce [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 37894, normalized size of antiderivative = 1515.76

$$\int x^{-6-2p}\left(d+ex^2\right)^p (a+b\arctan(cx))\,dx = \text{Too large to display}$$

input
```
int(x^(-6-2*p)*(e*x^2+d)^p*(a+b*atan(c*x)),x)
```

output
```
( - 4*(d + e*x**2)**p*atan(c*x)*b*c**3*d**4*p**4 - 22*(d + e*x**2)**p*atan
(c*x)*b*c**3*d**4*p**3 - 38*(d + e*x**2)**p*atan(c*x)*b*c**3*d**4*p**2 - 2
0*(d + e*x**2)**p*atan(c*x)*b*c**3*d**4*p - 10*(d + e*x**2)**p*atan(c*x)*b
*c*d**3*e*p**3 - 30*(d + e*x**2)**p*atan(c*x)*b*c*d**3*e*p**2 - 20*(d + e*
x**2)**p*atan(c*x)*b*c*d**3*e*p - 4*(d + e*x**2)**p*a*c**3*d**4*p**4 - 22*
(d + e*x**2)**p*a*c**3*d**4*p**3 - 38*(d + e*x**2)**p*a*c**3*d**4*p**2 - 2
0*(d + e*x**2)**p*a*c**3*d**4*p - 10*(d + e*x**2)**p*a*c*d**3*e*p**3 - 30*
(d + e*x**2)**p*a*c*d**3*e*p**2 - 20*(d + e*x**2)**p*a*c*d**3*e*p + 2*(d +
 e*x**2)**p*b*c**6*d**4*p**3*x**3 + 9*(d + e*x**2)**p*b*c**6*d**4*p**2*x**
3 + 10*(d + e*x**2)**p*b*c**6*d**4*p*x**3 - 2*(d + e*x**2)**p*b*c**4*d**4*
p**3*x - 7*(d + e*x**2)**p*b*c**4*d**4*p**2*x - 5*(d + e*x**2)**p*b*c**4*d
**4*p*x + 4*(d + e*x**2)**p*b*c**4*d**3*e*p**2*x**3 + 10*(d + e*x**2)**p*b
*c**4*d**3*e*p*x**3 - 4*(d + e*x**2)**p*b*c**4*d**2*e**2*p*x**5 - 10*(d +
e*x**2)**p*b*c**4*d**2*e**2*x**5 - 2*(d + e*x**2)**p*b*c**2*d**3*e*p**3*x
- 7*(d + e*x**2)**p*b*c**2*d**3*e*p**2*x - 5*(d + e*x**2)**p*b*c**2*d**3*e
*p*x - 2*(d + e*x**2)**p*b*c**2*d**2*e**2*p**3*x**3 - 5*(d + e*x**2)**p*b*
c**2*d**2*e**2*p**2*x**3 - 4*(d + e*x**2)**p*b*c**2*d*e**3*p*x**5 - 10*(d
+ e*x**2)**p*b*c**2*d*e**3*x**5 + 2*(d + e*x**2)**p*b*d**2*e**2*p**3*x + 2
*(d + e*x**2)**p*b*d**2*e**2*p**2*x - 4*(d + e*x**2)**p*b*d*e**3*p**2*x**3
 + 4*(d + e*x**2)**p*b*e**4*p*x**5 - 16*x**(2*p)*int((d + e*x**2)**p/(4...
```

# 3.1243 $\int x^{-7-2p}(d+ex^2)^p (a+b\arctan(cx))\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 466

$$\int x^{-7-2p}(d+ex^2)^p (a+b\arctan(cx))\,dx =$$

$$-\frac{b(2e^2+2c^2de(1+p)+c^4d^2(2+3p+p^2))\,x^{-5-2p}(d+ex^2)^p\left(1+\frac{ex^2}{d}\right)^{-p}\text{AppellF1}\left(\frac{1}{2}(-5-2p),1,-\right.}{2c^3d^2(1+p)(2+p)(3+p)(5+2p)}$$

$$-\frac{e^2x^{-2(1+p)}(d+ex^2)^{1+p}(a+b\arctan(cx))}{d^3(1+p)(2+p)(3+p)}$$

$$+\frac{ex^{-2(2+p)}(d+ex^2)^{1+p}(a+b\arctan(cx))}{d^2(2+p)(3+p)}-\frac{x^{-2(3+p)}(d+ex^2)^{1+p}(a+b\arctan(cx))}{2d(3+p)}$$

$$+\frac{be(e+c^2d(1+p))\,x^{-5-2p}(d+ex^2)^p\left(1+\frac{ex^2}{d}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2}(-5-2p),-1-p,\frac{1}{2}(-3-2\right.}{c^3d^2(1+p)(2+p)(3+p)(5+2p)}$$

$$-\frac{be^2x^{-3-2p}(d+ex^2)^p\left(1+\frac{ex^2}{d}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2}(-3-2p),-1-p,\frac{1}{2}(-1-2p),-\frac{ex^2}{d}\right)}{cd^2(1+p)(2+p)(3+p)(3+2p)}$$

output
```
-1/2*b*(2*e^2+2*c^2*d*e*(p+1)+c^4*d^2*(p^2+3*p+2))*x^(-5-2*p)*(e*x^2+d)^p*
AppellF1(-5/2-p,1,-1-p,-3/2-p,-c^2*x^2,-e*x^2/d)/c^3/d^2/(p+1)/(2+p)/(3+p)
/(5+2*p)/((1+e*x^2/d)^p)-e^2*(e*x^2+d)^(p+1)*(a+b*arctan(c*x))/d^3/(p+1)/(
2+p)/(3+p)/(x^(2*p+2))+e*(e*x^2+d)^(p+1)*(a+b*arctan(c*x))/d^2/(2+p)/(3+p)
/(x^(4+2*p))-1/2*(e*x^2+d)^(p+1)*(a+b*arctan(c*x))/d/(3+p)/(x^(6+2*p))+b*e
*(e+c^2*d*(p+1))*x^(-5-2*p)*(e*x^2+d)^p*hypergeom([-1-p, -5/2-p],[-3/2-p],
-e*x^2/d)/c^3/d^2/(p+1)/(2+p)/(3+p)/(5+2*p)/((1+e*x^2/d)^p)-b*e^2*x^(-3-2*
p)*(e*x^2+d)^p*hypergeom([-1-p, -3/2-p],[-1/2-p],-e*x^2/d)/c/d^2/(p+1)/(2+
p)/(3+p)/(3+2*p)/((1+e*x^2/d)^p)
```

## Mathematica [F]

$$\int x^{-7-2p}\left(d+ex^2\right)^p(a+b\arctan(cx))\,dx=\int x^{-7-2p}\left(d+ex^2\right)^p(a+b\arctan(cx))\,dx$$

input
```
Integrate[x^(-7 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]),x]
```

output
```
Integrate[x^(-7 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]), x]
```

## Rubi [A] (warning: unable to verify)

Time = 1.71 (sec) , antiderivative size = 432, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}=0.160$, Rules used = {5511, 27, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-2p-7}\left(d+ex^2\right)^p(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5511}$$

$$-bc\int -\frac{x^{-2(p+3)}\left(ex^2+d\right)^{p+1}\left(2e^2x^4-2de(p+1)x^2+d^2(p+1)(p+2)\right)}{2d^3(p+1)(p+2)(p+3)\left(c^2x^2+1\right)}dx-\frac{e^2x^{-2(p+1)}\left(d+ex^2\right)^{p+1}(a+b\arctan(cx))}{d^3(p+1)(p+2)(p+3)}+\frac{ex^{-2(p+2)}\left(d+ex^2\right)^{p+1}(a+b\arctan(cx))}{d^2(p+2)(p+3)}-\frac{x^{-2(p+3)}\left(d+ex^2\right)^{p+1}(a+b\arctan(cx))}{2d(p+3)}$$

$\Big\downarrow$ 27

$$\frac{bc\int \frac{x^{-2(p+3)}\left(ex^2+d\right)^{p+1}\left(2e^2x^4-2de(p+1)x^2+d^2(p+1)(p+2)\right)}{c^2x^2+1}dx}{2d^3(p+1)(p+2)(p+3)}-\frac{e^2x^{-2(p+1)}\left(d+ex^2\right)^{p+1}(a+b\arctan(cx))}{d^3(p+1)(p+2)(p+3)}+\frac{ex^{-2(p+2)}\left(d+ex^2\right)^{p+1}(a+b\arctan(cx))}{d^2(p+2)(p+3)}-\frac{x^{-2(p+3)}\left(d+ex^2\right)^{p+1}(a+b\arctan(cx))}{2d(p+3)}$$

$\Big\downarrow$ 7276

$$\frac{bc\int \left(-\frac{2e\left(d(p+1)c^2+e\right)x^{-2(p+3)}\left(ex^2+d\right)^{p+1}}{c^4}+\frac{2e^2x^{2-2(p+3)}\left(ex^2+d\right)^{p+1}}{c^2}+\frac{\left(2d^2c^4+d^2p^2c^4+3d^2pc^4+2dec^2+2depc^2+2e^2\right)x^{-2(p+3)}\left(ex^2\right.}{c^4\left(c^2x^2+1\right)}\right.}{2d^3(p+1)(p+2)(p+3)}-\frac{e^2x^{-2(p+1)}\left(d+ex^2\right)^{p+1}(a+b\arctan(cx))}{d^3(p+1)(p+2)(p+3)}+\frac{ex^{-2(p+2)}\left(d+ex^2\right)^{p+1}(a+b\arctan(cx))}{d^2(p+2)(p+3)}-\frac{x^{-2(p+3)}\left(d+ex^2\right)^{p+1}(a+b\arctan(cx))}{2d(p+3)}$$

$\Big\downarrow$ 2009

$$-\frac{e^2x^{-2(p+1)}\left(d+ex^2\right)^{p+1}(a+b\arctan(cx))}{d^3(p+1)(p+2)(p+3)}+\frac{ex^{-2(p+2)}\left(d+ex^2\right)^{p+1}(a+b\arctan(cx))}{d^2(p+2)(p+3)}-\frac{x^{-2(p+3)}\left(d+ex^2\right)^{p+1}(a+b\arctan(cx))}{2d(p+3)}+\frac{bc\left(-\frac{dx^{-2p-5}\left(c^4d^2\left(p^2+3p+2\right)+2c^2de(p+1)+2e^2\right)\left(d+ex^2\right)^p\left(\frac{ex^2}{d}+1\right)^{-p}\operatorname{AppellF1}\left(-p-\frac{5}{2},-p-1,1,-p-\frac{3}{2},-\frac{ex^2}{d},-c^2x^2\right)}{c^4(2p+5)}-\frac{2de^2x^{-2p-3}(d\right.}{}$$

input
```
Int[x^(-7 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]),x]
```

output
```
-((e^2*(d + e*x^2)^(1 + p)*(a + b*ArcTan[c*x]))/(d^3*(1 + p)*(2 + p)*(3 +
p)*x^(2*(1 + p)))) + (e*(d + e*x^2)^(1 + p)*(a + b*ArcTan[c*x]))/(d^2*(2 +
 p)*(3 + p)*x^(2*(2 + p))) - ((d + e*x^2)^(1 + p)*(a + b*ArcTan[c*x]))/(2*
d*(3 + p)*x^(2*(3 + p))) + (b*c*(-((d*(2*e^2 + 2*c^2*d*e*(1 + p) + c^4*d^2
*(2 + 3*p + p^2))*x^(-5 - 2*p)*(d + e*x^2)^p*AppellF1[-5/2 - p, -1 - p, 1,
 -3/2 - p, -((e*x^2)/d), -(c^2*x^2)])/(c^4*(5 + 2*p)*(1 + (e*x^2)/d)^p)) +
 (2*d*e*(e + c^2*d*(1 + p))*x^(-5 - 2*p)*(d + e*x^2)^p*Hypergeometric2F1[(
-5 - 2*p)/2, -1 - p, (-3 - 2*p)/2, -((e*x^2)/d)])/(c^4*(5 + 2*p)*(1 + (e*x
^2)/d)^p) - (2*d*e^2*x^(-3 - 2*p)*(d + e*x^2)^p*Hypergeometric2F1[(-3 - 2*
p)/2, -1 - p, (-1 - 2*p)/2, -((e*x^2)/d)])/(c^2*(3 + 2*p)*(1 + (e*x^2)/d)^
p)))/(2*d^3*(1 + p)*(2 + p)*(3 + p))
```

### Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5511
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(q_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^q, x]}, Sim
p[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[SimplifyIntegrand[u/(1 + c^2
*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && ((IGtQ[q, 0] &&
  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*q + 3, 0])) || (IGtQ[(m + 1)/2, 0] &&
!(ILtQ[q, 0] && GtQ[m + 2*q + 3, 0])) || (ILtQ[(m + 2*q + 1)/2, 0] &&  !ILt
Q[(m - 1)/2, 0]))
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

## Maple [F]

$$\int x^{-7-2p}\left(e\,x^{2}+d\right)^{p}\left(a+b\arctan\left(cx\right)\right)dx$$

input
```
int(x^(-7-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x)
```

output
```
int(x^(-7-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x)
```

## Fricas [F]

$$\int x^{-7-2p}\left(d+ex^{2}\right)^{p}\left(a+b\arctan(cx)\right)dx=\int\left(b\arctan\left(cx\right)+a\right)\left(ex^{2}+d\right)^{p}x^{-2\,p-7}\,dx$$

input
```
integrate(x^(-7-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
integral((b*arctan(c*x) + a)*(e*x^2 + d)^p*x^(-2*p - 7), x)
```

## Sympy [F(-1)]

Timed out.

$$\int x^{-7-2p}\left(d+ex^{2}\right)^{p}\left(a+b\arctan(cx)\right)dx=\text{Timed out}$$

input
```
integrate(x**(-7-2*p)*(e*x**2+d)**p*(a+b*atan(c*x)),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int x^{-7-2p}\left(d+ex^2\right)^p\left(a+b\arctan(cx)\right)dx=\int(b\arctan(cx)+a)\left(ex^2+d\right)^px^{-2p-7}\,dx$$

input
```
integrate(x^(-7-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
b*integrate(arctan(c*x)*e^(p*log(e*x^2 + d) - 2*p*log(x))/x^7, x) - 1/2*(2
*e^3*x^6 - 2*d*e^2*p*x^4 + (p^2 + p)*d^2*e*x^2 + (p^2 + 3*p + 2)*d^3)*a*e^
(p*log(e*x^2 + d) - 2*p*log(x))/((p^3 + 6*p^2 + 11*p + 6)*d^3*x^6)
```

## Giac [F]

$$\int x^{-7-2p}\left(d+ex^2\right)^p\left(a+b\arctan(cx)\right)dx=\int(b\arctan(cx)+a)\left(ex^2+d\right)^px^{-2p-7}\,dx$$

input
```
integrate(x^(-7-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)*(e*x^2 + d)^p*x^(-2*p - 7), x)
```

## Mupad [F(-1)]

Timed out.

$$\int x^{-7-2p}\left(d+ex^2\right)^p\left(a+b\arctan(cx)\right)dx=\int\frac{(a+b\,\mathrm{atan}(c\,x))\,\left(e\,x^2+d\right)^p}{x^{2\,p+7}}\,dx$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^p)/x^(2*p + 7),x)
```

output
```
int(((a + b*atan(c*x))*(d + e*x^2)^p)/x^(2*p + 7), x)
```

## Reduce [F]

$$\int x^{-7-2p}(d+ex^2)^p(a+b\arctan(cx))\,dx = \text{too large to display}$$

input
```
int(x^(-7-2*p)*(e*x^2+d)^p*(a+b*atan(c*x)),x)
```

output
```
( - 6*(d + e*x**2)**p*atan(c*x)*b*c**3*d**3*p**3 - 33*(d + e*x**2)**p*atan
(c*x)*b*c**3*d**3*p**2 - 57*(d + e*x**2)**p*atan(c*x)*b*c**3*d**3*p - 30*(
d + e*x**2)**p*atan(c*x)*b*c**3*d**3 - 6*(d + e*x**2)**p*atan(c*x)*b*c**3*
d**2*e*p**3*x**2 - 21*(d + e*x**2)**p*atan(c*x)*b*c**3*d**2*e*p**2*x**2 -
15*(d + e*x**2)**p*atan(c*x)*b*c**3*d**2*e*p*x**2 + 12*(d + e*x**2)**p*ata
n(c*x)*b*c**3*d*e**2*p**2*x**4 + 30*(d + e*x**2)**p*atan(c*x)*b*c**3*d*e**
2*p*x**4 - 12*(d + e*x**2)**p*atan(c*x)*b*c**3*e**3*p*x**6 - 30*(d + e*x**
2)**p*atan(c*x)*b*c**3*e**3*x**6 - 6*(d + e*x**2)**p*a*c**3*d**3*p**3 - 33
*(d + e*x**2)**p*a*c**3*d**3*p**2 - 57*(d + e*x**2)**p*a*c**3*d**3*p - 30*
(d + e*x**2)**p*a*c**3*d**3 - 6*(d + e*x**2)**p*a*c**3*d**2*e*p**3*x**2 -
21*(d + e*x**2)**p*a*c**3*d**2*e*p**2*x**2 - 15*(d + e*x**2)**p*a*c**3*d**
2*e*p*x**2 + 12*(d + e*x**2)**p*a*c**3*d*e**2*p**2*x**4 + 30*(d + e*x**2)*
*p*a*c**3*d*e**2*p*x**4 - 12*(d + e*x**2)**p*a*c**3*e**3*p*x**6 - 30*(d +
e*x**2)**p*a*c**3*e**3*x**6 - 3*(d + e*x**2)**p*b*c**4*d**3*p**2*x - 9*(d
+ e*x**2)**p*b*c**4*d**3*p*x - 6*(d + e*x**2)**p*b*c**4*d**3*x + 12*(d + e
*x**2)**p*b*c**2*d*e**2*p**2*x**3 + 30*(d + e*x**2)**p*b*c**2*d*e**2*p*x**
3 - 12*(d + e*x**2)**p*b*c**2*e**3*p*x**5 - 30*(d + e*x**2)**p*b*c**2*e**3
*x**5 + 4*(d + e*x**2)**p*b*e**3*p*x**3 + 10*(d + e*x**2)**p*b*e**3*x**3 -
 24*x**(2*p)*int((d + e*x**2)**p/(4*x**(2*p)*c**4*d**2*p**5*x**6 + 44*x**(
2*p)*c**4*d**2*p**4*x**6 + 189*x**(2*p)*c**4*d**2*p**3*x**6 + 394*x**(2...
```

# 3.1244 $\int x^{-8-2p}(d+ex^2)^p(a+b\arctan(cx))\,dx$



## Optimal result

Integrand size = 25, antiderivative size = 25

$$\int x^{-8-2p}(d+ex^2)^p(a+b\arctan(cx))\,dx = -\frac{ax^{-7-2p}(d+ex^2)^p\left(1+\frac{ex^2}{d}\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2}(-7-2p),-p,\frac{1}{2}(-5-2p),-\frac{ex^2}{d}\right)}{7+2p} + b\text{Int}\left(x^{-8-2p}(d+ex^2)^p\arctan(cx),x\right)$$

output
```
-a*x^(-7-2*p)*(e*x^2+d)^p*hypergeom([-p, -7/2-p],[-5/2-p],-e*x^2/d)/(7+2*p
)/((1+e*x^2/d)^p)+b*Defer(Int)(x^(-8-2*p)*(e*x^2+d)^p*arctan(c*x),x)
```

## Mathematica [N/A]

Not integrable

Time = 2.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int x^{-8-2p}(d+ex^2)^p(a+b\arctan(cx))\,dx = \int x^{-8-2p}(d+ex^2)^p(a+b\arctan(cx))\,dx$$

input
```
Integrate[x^(-8 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]),x]
```

output
```
Integrate[x^(-8 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]), x]
```

## Rubi [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{-2p-8}\left(d+ex^2\right)^p(a+b\arctan(cx))\,dx$$

$$\Big\downarrow \text{5517}$$

$$a\int x^{-2(p+4)}\left(ex^2+d\right)^p\,dx+b\int x^{-2(p+4)}\left(ex^2+d\right)^p\arctan(cx)dx$$

$$\Big\downarrow \text{279}$$

$$a\left(d+ex^2\right)^p\left(\frac{ex^2}{d}+1\right)^{-p}\int x^{-2(p+4)}\left(\frac{ex^2}{d}+1\right)^p dx+b\int x^{-2(p+4)}\left(ex^2+d\right)^p\arctan(cx)dx$$

$$\Big\downarrow \text{278}$$

$$b\int x^{-2(p+4)}\left(ex^2+d\right)^p\arctan(cx)dx-\frac{ax^{-2p-7}\left(d+ex^2\right)^p\left(\frac{ex^2}{d}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2}(-2p-7),-p,\frac{1}{2}(-2p-5),-\frac{ex^2}{d}\right)}{2p+7}$$

$$\Big\downarrow \text{5560}$$

$$b\int x^{-2(p+4)}\left(ex^2+d\right)^p\arctan(cx)dx-\frac{ax^{-2p-7}\left(d+ex^2\right)^p\left(\frac{ex^2}{d}+1\right)^{-p}\text{Hypergeometric2F1}\left(\frac{1}{2}(-2p-7),-p,\frac{1}{2}(-2p-5),-\frac{ex^2}{d}\right)}{2p+7}$$

input
```
Int[x^(-8 - 2*p)*(d + e*x^2)^p*(a + b*ArcTan[c*x]),x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 1.76 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int x^{-8-2p}\left(e\,x^{2}+d\right)^{p}\left(a+b\arctan\left(cx\right)\right)dx$$

input
```
int(x^(-8-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x)
```

output
```
int(x^(-8-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x)
```

## Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int x^{-8-2p}\left(d+ex^2\right)^p\left(a+b\arctan(cx)\right)dx=\int\left(b\arctan\left(cx\right)+a\right)\left(ex^2+d\right)^{p}x^{-2\,p-8}\,dx$$

input
```
integrate(x^(-8-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="fricas")
```

output
```
integral((b*arctan(c*x) + a)*(e*x^2 + d)^p*x^(-2*p - 8), x)
```

## Sympy [F(-1)]

Timed out.

$$\int x^{-8-2p}(d+ex^2)^p\,(a+b\arctan(cx))\,dx = \text{Timed out}$$

input
```
integrate(x**(-8-2*p)*(e*x**2+d)**p*(a+b*atan(c*x)),x)
```

output
```
Timed out
```

## Maxima [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int x^{-8-2p}(d+ex^2)^p\,(a+b\arctan(cx))\,dx = \int (b\arctan(cx)+a)\left(ex^2+d\right)^p x^{-2\,p-8}\,dx$$

input
```
integrate(x^(-8-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="maxima")
```

output
```
integrate((b*arctan(c*x) + a)*(e*x^2 + d)^p*x^(-2*p - 8), x)
```

## Giac [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int x^{-8-2p}(d+ex^2)^p\,(a+b\arctan(cx))\,dx = \int (b\arctan(cx)+a)\left(ex^2+d\right)^p x^{-2\,p-8}\,dx$$

input
```
integrate(x^(-8-2*p)*(e*x^2+d)^p*(a+b*arctan(c*x)),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)*(e*x^2 + d)^p*x^(-2*p - 8), x)
```

## Mupad [N/A]

Not integrable

Time = 1.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int x^{-8-2p}\left(d+ex^2\right)^p(a+b\arctan(cx))\,dx=\int\frac{(a+b\,\mathrm{atan}(c\,x))\,\left(e\,x^2+d\right)^p}{x^{2\,p+8}}\,dx$$

input
```
int(((a + b*atan(c*x))*(d + e*x^2)^p)/x^(2*p + 8),x)
```

output
```
int(((a + b*atan(c*x))*(d + e*x^2)^p)/x^(2*p + 8), x)
```

## Reduce [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 77868, normalized size of antiderivative = 3114.72

$$\int x^{-8-2p}\left(d+ex^2\right)^p(a+b\arctan(cx))\,dx=\text{Too large to display}$$

input
```
int(x^(-8-2*p)*(e*x^2+d)^p*(a+b*atan(c*x)),x)
```

output

```
( - 8*(d + e*x**2)**p*atan(c*x)*b*c**5*d**4*e*p**6*x**4 - 76*(d + e*x**2)*
*p*atan(c*x)*b*c**5*d**4*e*p**5*x**4 - 256*(d + e*x**2)**p*atan(c*x)*b*c**
5*d**4*e*p**4*x**4 - 356*(d + e*x**2)**p*atan(c*x)*b*c**5*d**4*e*p**3*x**4
 - 168*(d + e*x**2)**p*atan(c*x)*b*c**5*d**4*e*p**2*x**4 - 8*(d + e*x**2)*
*p*atan(c*x)*b*c**3*d**5*p**6 - 88*(d + e*x**2)**p*atan(c*x)*b*c**3*d**5*p
**5 - 370*(d + e*x**2)**p*atan(c*x)*b*c**3*d**5*p**4 - 740*(d + e*x**2)**p
*atan(c*x)*b*c**3*d**5*p**3 - 702*(d + e*x**2)**p*atan(c*x)*b*c**3*d**5*p*
*2 - 252*(d + e*x**2)**p*atan(c*x)*b*c**3*d**5*p - 28*(d + e*x**2)**p*atan
(c*x)*b*c*d**4*e*p**5 - 210*(d + e*x**2)**p*atan(c*x)*b*c*d**4*e*p**4 - 56
0*(d + e*x**2)**p*atan(c*x)*b*c*d**4*e*p**3 - 630*(d + e*x**2)**p*atan(c*x
)*b*c*d**4*e*p**2 - 252*(d + e*x**2)**p*atan(c*x)*b*c*d**4*e*p - 8*(d + e*
x**2)**p*a*c**5*d**4*e*p**6*x**4 - 76*(d + e*x**2)**p*a*c**5*d**4*e*p**5*x
**4 - 256*(d + e*x**2)**p*a*c**5*d**4*e*p**4*x**4 - 356*(d + e*x**2)**p*a*
c**5*d**4*e*p**3*x**4 - 168*(d + e*x**2)**p*a*c**5*d**4*e*p**2*x**4 - 8*(d
 + e*x**2)**p*a*c**3*d**5*p**6 - 88*(d + e*x**2)**p*a*c**3*d**5*p**5 - 370
*(d + e*x**2)**p*a*c**3*d**5*p**4 - 740*(d + e*x**2)**p*a*c**3*d**5*p**3 -
 702*(d + e*x**2)**p*a*c**3*d**5*p**2 - 252*(d + e*x**2)**p*a*c**3*d**5*p
- 28*(d + e*x**2)**p*a*c*d**4*e*p**5 - 210*(d + e*x**2)**p*a*c*d**4*e*p**4
 - 560*(d + e*x**2)**p*a*c*d**4*e*p**3 - 630*(d + e*x**2)**p*a*c*d**4*e*p*
*2 - 252*(d + e*x**2)**p*a*c*d**4*e*p + 4*(d + e*x**2)**p*b*c**6*d**5*p...
```

# 3.1245 $\int x^3(d+ex^2)(a+b\arctan(cx))^2\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 271

$$\begin{aligned}\int x^3(d+ex^2)(a+b\arctan(cx))^2\,dx &= \frac{abdx}{2c^3}-\frac{abex}{3c^5}+\frac{b^2dx^2}{12c^2}-\frac{4b^2ex^2}{45c^4}\\&+\frac{b^2ex^4}{60c^2}+\frac{b^2dx\arctan(cx)}{2c^3}\\&-\frac{b^2ex\arctan(cx)}{3c^5}-\frac{bdx^3(a+b\arctan(cx))}{6c}\\&+\frac{bex^3(a+b\arctan(cx))}{9c^3}\\&-\frac{bex^5(a+b\arctan(cx))}{15c}\\&-\frac{d(a+b\arctan(cx))^2}{4c^4}+\frac{e(a+b\arctan(cx))^2}{6c^6}\\&+\frac{1}{4}dx^4(a+b\arctan(cx))^2\\&+\frac{1}{6}ex^6(a+b\arctan(cx))^2\\&-\frac{b^2d\log(1+c^2x^2)}{3c^4}+\frac{23b^2e\log(1+c^2x^2)}{90c^6}\end{aligned}$$

output
```
1/2*a*b*d*x/c^3-1/3*a*b*e*x/c^5+1/12*b^2*d*x^2/c^2-4/45*b^2*e*x^2/c^4+1/60
*b^2*e*x^4/c^2+1/2*b^2*d*x*arctan(c*x)/c^3-1/3*b^2*e*x*arctan(c*x)/c^5-1/6
*b*d*x^3*(a+b*arctan(c*x))/c+1/9*b*e*x^3*(a+b*arctan(c*x))/c^3-1/15*b*e*x^
5*(a+b*arctan(c*x))/c-1/4*d*(a+b*arctan(c*x))^2/c^4+1/6*e*(a+b*arctan(c*x)
)^2/c^6+1/4*d*x^4*(a+b*arctan(c*x))^2+1/6*e*x^6*(a+b*arctan(c*x))^2-1/3*b^
2*d*ln(c^2*x^2+1)/c^4+23/90*b^2*e*ln(c^2*x^2+1)/c^6
```

## Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.89

$$\int x^3(d+ex^2)(a+b\arctan(cx))^2\,dx$$
$$= \frac{cx(15a^2c^5x^3(3d+2ex^2)+b^2cx(-16e+3c^2(5d+ex^2))-2ab(30e-5c^2(9d+2ex^2)+3c^4(5dx^2+2ex^4)}{}$$

input
```
Integrate[x^3*(d + e*x^2)*(a + b*ArcTan[c*x])^2,x]
```

output
```
(c*x*(15*a^2*c^5*x^3*(3*d + 2*e*x^2) + b^2*c*x*(-16*e + 3*c^2*(5*d + e*x^2
)) - 2*a*b*(30*e - 5*c^2*(9*d + 2*e*x^2) + 3*c^4*(5*d*x^2 + 2*e*x^4))) + 2
*b*(b*c*x*(-30*e + 5*c^2*(9*d + 2*e*x^2) - 3*c^4*(5*d*x^2 + 2*e*x^4)) + 15
*a*(-3*c^2*d + 2*e + c^6*(3*d*x^4 + 2*e*x^6)))*ArcTan[c*x] + 15*b^2*(-3*c^
2*d + 2*e + c^6*(3*d*x^4 + 2*e*x^6))*ArcTan[c*x]^2 + 2*b^2*(-30*c^2*d + 23
*e)*Log[1 + c^2*x^2])/(180*c^6)
```

## Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(d+ex^2)(a+b\arctan(cx))^2\,dx$$

$$\Big\downarrow \ 5515$$

$$\int \left(dx^3(a+b\arctan(cx))^2+ex^5(a+b\arctan(cx))^2\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{e(a+b\arctan(cx))^2}{6c^6}-\frac{d(a+b\arctan(cx))^2}{4c^4}+\frac{bex^3(a+b\arctan(cx))}{9c^3}+\frac{1}{4}dx^4(a+b\arctan(cx))^2-\frac{bdx^3(a+b\arctan(cx))}{6c}+\frac{1}{6}ex^6(a+b\arctan(cx))^2-\frac{bex^5(a+b\arctan(cx))}{15c}-\frac{abex}{3c^5}+\frac{abdx}{2c^3}-\frac{b^2ex\arctan(cx)}{3c^5}+\frac{b^2dx\arctan(cx)}{2c^3}-\frac{4b^2ex^2}{45c^4}+\frac{b^2dx^2}{12c^2}+\frac{b^2ex^4}{60c^2}+\frac{23b^2e\log\left(c^2x^2+1\right)}{90c^6}-\frac{b^2d\log\left(c^2x^2+1\right)}{3c^4}$$

input
```
Int[x^3*(d + e*x^2)*(a + b*ArcTan[c*x])^2,x]
```

output
```
(a*b*d*x)/(2*c^3) - (a*b*e*x)/(3*c^5) + (b^2*d*x^2)/(12*c^2) - (4*b^2*e*x^
2)/(45*c^4) + (b^2*e*x^4)/(60*c^2) + (b^2*d*x*ArcTan[c*x])/(2*c^3) - (b^2*
e*x*ArcTan[c*x])/(3*c^5) - (b*d*x^3*(a + b*ArcTan[c*x]))/(6*c) + (b*e*x^3*
(a + b*ArcTan[c*x]))/(9*c^3) - (b*e*x^5*(a + b*ArcTan[c*x]))/(15*c) - (d*(
a + b*ArcTan[c*x])^2)/(4*c^4) + (e*(a + b*ArcTan[c*x])^2)/(6*c^6) + (d*x^4
*(a + b*ArcTan[c*x])^2)/4 + (e*x^6*(a + b*ArcTan[c*x])^2)/6 - (b^2*d*Log[1
 + c^2*x^2])/(3*c^4) + (23*b^2*e*Log[1 + c^2*x^2])/(90*c^6)
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

## Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.11

| method | result |
|---|---|
| parts | $a^2\left(\frac{1}{6}e\,x^6+\frac{1}{4}d\,x^4\right)+\frac{b^2\left(\frac{\arctan(cx)^2c^4e\,x^6}{6}+\frac{\arctan(cx)^2c^4x^4d}{4}-\frac{\frac{2\arctan(cx)e\,c^5x^5}{5}+\arctan(cx)d\,c^5x^3-\frac{2\arctan(cx)}{3}}{}\right.}{}$ |
| derivativedivides | $\frac{\frac{a^2\left(\frac{1}{4}d\,c^6x^4+\frac{1}{6}e\,c^6x^6\right)}{c^2}+\frac{b^2\left(\frac{\arctan(cx)^2d\,c^6x^4}{4}+\frac{\arctan(cx)^2e\,c^6x^6}{6}-\frac{\arctan(cx)d\,c^5x^3}{6}-\frac{\arctan(cx)e\,c^5x^5}{15}+\frac{\arctan(cx)c^3xd}{2}+\right.}{}}{}$ |
| default | $\frac{\frac{a^2\left(\frac{1}{4}d\,c^6x^4+\frac{1}{6}e\,c^6x^6\right)}{c^2}+\frac{b^2\left(\frac{\arctan(cx)^2d\,c^6x^4}{4}+\frac{\arctan(cx)^2e\,c^6x^6}{6}-\frac{\arctan(cx)d\,c^5x^3}{6}-\frac{\arctan(cx)e\,c^5x^5}{15}+\frac{\arctan(cx)c^3xd}{2}+\right.}{}}{}$ |
| parallelrisch | $-\frac{12ab\,c^5e\,x^5+30ab\,c^5d\,x^3+60b^2c^2d\ln(c^2x^2+1)-90adb\arctan(cx)x^4c^6-60x^6\arctan(cx)ab\,c^6e-60abe\arctan(cx)-}{}$ |
| risch | $-\frac{4b^2e\,x^2}{45c^4}+\frac{b^2e\,x^4}{60c^2}+\frac{23b^2e\ln(c^2x^2+1)}{90c^6}-\frac{abex}{3c^5}+\frac{b^2d\,x^2}{12c^2}-\frac{b^2d\ln(c^2x^2+1)}{3c^4}+\frac{abdx}{2c^3}-\frac{ib(60a\,c^6e\,x^6+30ib}{}$ |

input
```
int(x^3*(e*x^2+d)*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*(1/6*e*x^6+1/4*d*x^4)+b^2/c^4*(1/6*arctan(c*x)^2*c^4*e*x^6+1/4*arctan(
c*x)^2*c^4*x^4*d-1/6/c^2*(2/5*arctan(c*x)*e*c^5*x^5+arctan(c*x)*d*c^5*x^3-
2/3*arctan(c*x)*e*c^3*x^3-3*arctan(c*x)*c^3*x*d+2*arctan(c*x)*c*x*e+3*arct
an(c*x)^2*c^2*d-2*arctan(c*x)^2*e-1/2*c^4*d*x^2-1/10*c^4*e*x^4+8/15*c^2*e*
x^2-1/30*(-60*c^2*d+46*e)*ln(c^2*x^2+1)-1/30*(45*c^2*d-30*e)*arctan(c*x)^2
))+2*a*b/c^4*(1/6*arctan(c*x)*c^4*e*x^6+1/4*arctan(c*x)*c^4*x^4*d-1/12/c^2
*(2/5*e*c^5*x^5+d*c^5*x^3-2/3*e*c^3*x^3-3*c^3*x*d+2*e*c*x+(3*c^2*d-2*e)*ar
ctan(c*x)))
```

## Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.07

$$\int x^3(d+ex^2)\,(a+b\arctan(cx))^2\,dx$$

$$=\frac{30\,a^2c^6ex^6-12\,abc^5ex^5+3\,(15\,a^2c^6d+b^2c^4e)x^4-10\,(3\,abc^5d-2\,abc^3e)x^3+(15\,b^2c^4d-16\,b^2c^2e)x^2+}{}$$

input
```
integrate(x^3*(e*x^2+d)*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

output
```
1/180*(30*a^2*c^6*e*x^6 - 12*a*b*c^5*e*x^5 + 3*(15*a^2*c^6*d + b^2*c^4*e)*
x^4 - 10*(3*a*b*c^5*d - 2*a*b*c^3*e)*x^3 + (15*b^2*c^4*d - 16*b^2*c^2*e)*x
^2 + 15*(2*b^2*c^6*e*x^6 + 3*b^2*c^6*d*x^4 - 3*b^2*c^2*d + 2*b^2*e)*arctan
(c*x)^2 + 30*(3*a*b*c^3*d - 2*a*b*c*e)*x + 2*(30*a*b*c^6*e*x^6 + 45*a*b*c^
6*d*x^4 - 6*b^2*c^5*e*x^5 - 45*a*b*c^2*d - 5*(3*b^2*c^5*d - 2*b^2*c^3*e)*x
^3 + 30*a*b*e + 15*(3*b^2*c^3*d - 2*b^2*c*e)*x)*arctan(c*x) - 2*(30*b^2*c^
2*d - 23*b^2*e)*log(c^2*x^2 + 1))/c^6
```

## Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.47

$$\int x^3(d+ex^2)(a+b\arctan(cx))^2\,dx$$
$$=\begin{cases} \frac{a^2dx^4}{4}+\frac{a^2ex^6}{6}+\frac{abdx^4\operatorname{atan}(cx)}{2}+\frac{abex^6\operatorname{atan}(cx)}{3}-\frac{abdx^3}{6c}-\frac{abex^5}{15c}+\frac{abdx}{2c^3}+\frac{abex^3}{9c^3}-\frac{abd\operatorname{atan}(cx)}{2c^4}-\frac{abex}{3c^5}+\frac{abe\operatorname{atan}(\text{}}{3c^6} \\ a^2\left(\frac{dx^4}{4}+\frac{ex^6}{6}\right) \end{cases}$$

input
```
integrate(x**3*(e*x**2+d)*(a+b*atan(c*x))**2,x)
```

output
```
Piecewise((a**2*d*x**4/4 + a**2*e*x**6/6 + a*b*d*x**4*atan(c*x)/2 + a*b*e*
x**6*atan(c*x)/3 - a*b*d*x**3/(6*c) - a*b*e*x**5/(15*c) + a*b*d*x/(2*c**3)
 + a*b*e*x**3/(9*c**3) - a*b*d*atan(c*x)/(2*c**4) - a*b*e*x/(3*c**5) + a*b
*e*atan(c*x)/(3*c**6) + b**2*d*x**4*atan(c*x)**2/4 + b**2*e*x**6*atan(c*x)
**2/6 - b**2*d*x**3*atan(c*x)/(6*c) - b**2*e*x**5*atan(c*x)/(15*c) + b**2*
d*x**2/(12*c**2) + b**2*e*x**4/(60*c**2) + b**2*d*x*atan(c*x)/(2*c**3) + b
**2*e*x**3*atan(c*x)/(9*c**3) - b**2*d*log(x**2 + c**(-2))/(3*c**4) - b**2
*d*atan(c*x)**2/(4*c**4) - 4*b**2*e*x**2/(45*c**4) - b**2*e*x*atan(c*x)/(3
*c**5) + 23*b**2*e*log(x**2 + c**(-2))/(90*c**6) + b**2*e*atan(c*x)**2/(6*
c**6), Ne(c, 0)), (a**2*(d*x**4/4 + e*x**6/6), True))
```

## Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.13

$$\int x^3(d+ex^2)(a+b\arctan(cx))^2\,dx$$
$$= \frac{1}{6}b^2ex^6\arctan(cx)^2 + \frac{1}{6}a^2ex^6 + \frac{1}{4}b^2dx^4\arctan(cx)^2 + \frac{1}{4}a^2dx^4 + \frac{1}{6}\left(3x^4\arctan(cx) - c\left(\frac{c^2x^3-3x}{c^4} + \frac{3\arctan(cx)}{c^5}\right)\right)abd - \frac{1}{12}\left(2c\left(\frac{c^2x^3-3x}{c^4} + \frac{3\arctan(cx)}{c^5}\right)\arctan(cx) - \frac{c^2x^2+3\arctan(cx)^2-4\log(c^2x^2+1)}{c^4}\right)b^2d + \frac{1}{45}\left(15x^6\arctan(cx) - c\left(\frac{3c^4x^5-5c^2x^3+15x}{c^6} - \frac{15\arctan(cx)}{c^7}\right)\right)abe - \frac{1}{180}\left(4c\left(\frac{3c^4x^5-5c^2x^3+15x}{c^6} - \frac{15\arctan(cx)}{c^7}\right)\arctan(cx) - \frac{3c^4x^4-16c^2x^2-30\arctan(cx)^2}{c^6}\right.$$

input
```
integrate(x^3*(e*x^2+d)*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

output
```
1/6*b^2*e*x^6*arctan(c*x)^2 + 1/6*a^2*e*x^6 + 1/4*b^2*d*x^4*arctan(c*x)^2
+ 1/4*a^2*d*x^4 + 1/6*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arct
an(c*x)/c^5))*a*b*d - 1/12*(2*c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5)*
arctan(c*x) - (c^2*x^2 + 3*arctan(c*x)^2 - 4*log(c^2*x^2 + 1))/c^4)*b^2*d
+ 1/45*(15*x^6*arctan(c*x) - c*((3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*ar
ctan(c*x)/c^7))*a*b*e - 1/180*(4*c*((3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 1
5*arctan(c*x)/c^7)*arctan(c*x) - (3*c^4*x^4 - 16*c^2*x^2 - 30*arctan(c*x)^
2 + 46*log(c^2*x^2 + 1))/c^6)*b^2*e
```

## Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.33

$$\int x^3(d+ex^2)(a+b\arctan(cx))^2\,dx$$
$$= \frac{30b^2c^6ex^6\arctan(cx)^2 + 60abc^6ex^6\arctan(cx) + 30a^2c^6ex^6 + 45b^2c^6dx^4\arctan(cx)^2 + 90abc^6dx^4\,\mathrm{arc}}{}$$

input
```
integrate(x^3*(e*x^2+d)*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

output
```
1/180*(30*b^2*c^6*e*x^6*arctan(c*x)^2 + 60*a*b*c^6*e*x^6*arctan(c*x) + 30*
a^2*c^6*e*x^6 + 45*b^2*c^6*d*x^4*arctan(c*x)^2 + 90*a*b*c^6*d*x^4*arctan(c
*x) - 12*b^2*c^5*e*x^5*arctan(c*x) + 45*a^2*c^6*d*x^4 - 12*a*b*c^5*e*x^5 -
 30*b^2*c^5*d*x^3*arctan(c*x) - 30*a*b*c^5*d*x^3 + 3*b^2*c^4*e*x^4 + 20*b^
2*c^3*e*x^3*arctan(c*x) + 15*b^2*c^4*d*x^2 + 20*a*b*c^3*e*x^3 + 90*b^2*c^3
*d*x*arctan(c*x) + 90*pi*a*b*c^2*d*sgn(c)*sgn(x) + 90*a*b*c^3*d*x - 16*b^2
*c^2*e*x^2 - 45*b^2*c^2*d*arctan(c*x)^2 - 90*a*b*c^2*d*arctan(c*x) - 60*b^
2*c*e*x*arctan(c*x) - 60*b^2*c^2*d*log(c^2*x^2 + 1) - 60*pi*a*b*e*sgn(c)*s
gn(x) - 60*a*b*c*e*x + 30*b^2*e*arctan(c*x)^2 + 60*a*b*e*arctan(c*x) + 46*
b^2*e*log(c^2*x^2 + 1))/c^6
```

## Mupad [B] (verification not implemented)

Time = 1.97 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.25

$$\int x^3(d+ex^2)(a+b\arctan(cx))^2\,dx$$
$$= \frac{46\,b^2\,e\,\ln\left(c^2\,x^2+1\right)+30\,b^2\,e\,\mathrm{atan}(c\,x)^2-60\,b^2\,c^2\,d\,\ln\left(c^2\,x^2+1\right)+45\,a^2\,c^6\,d\,x^4+15\,b^2\,c^4\,d\,x^2+30\,a^2\,c^6}{}$$

input
```
int(x^3*(a + b*atan(c*x))^2*(d + e*x^2),x)
```

output
```
(46*b^2*e*log(c^2*x^2 + 1) + 30*b^2*e*atan(c*x)^2 - 60*b^2*c^2*d*log(c^2*x
^2 + 1) + 45*a^2*c^6*d*x^4 + 15*b^2*c^4*d*x^2 + 30*a^2*c^6*e*x^6 - 16*b^2*
c^2*e*x^2 + 3*b^2*c^4*e*x^4 + 60*a*b*e*atan(c*x) - 45*b^2*c^2*d*atan(c*x)^
2 + 45*b^2*c^6*d*x^4*atan(c*x)^2 + 30*b^2*c^6*e*x^6*atan(c*x)^2 - 30*a*b*c
^5*d*x^3 + 20*a*b*c^3*e*x^3 - 12*a*b*c^5*e*x^5 + 90*b^2*c^3*d*x*atan(c*x)
- 60*a*b*c*e*x - 30*b^2*c^5*d*x^3*atan(c*x) + 20*b^2*c^3*e*x^3*atan(c*x) -
 12*b^2*c^5*e*x^5*atan(c*x) + 90*a*b*c^3*d*x - 90*a*b*c^2*d*atan(c*x) - 60
*b^2*c*e*x*atan(c*x) + 90*a*b*c^6*d*x^4*atan(c*x) + 60*a*b*c^6*e*x^6*atan(
c*x))/(180*c^6)
```

## Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.25

$$\int x^3(d+ex^2)(a+b\arctan(cx))^2\,dx$$
$$=\frac{90atan(cx)\,ab\,c^6d\,x^4+60atan(cx)\,ab\,c^6e\,x^6-45atan(cx)^2\,b^2c^2d+60atan(cx)\,abe-60\log(c^2x^2+1)\,b^2c}{}$$

input
```
int(x^3*(e*x^2+d)*(a+b*atan(c*x))^2,x)
```

output
```
(45*atan(c*x)**2*b**2*c**6*d*x**4 + 30*atan(c*x)**2*b**2*c**6*e*x**6 - 45*
atan(c*x)**2*b**2*c**2*d + 30*atan(c*x)**2*b**2*e + 90*atan(c*x)*a*b*c**6*
d*x**4 + 60*atan(c*x)*a*b*c**6*e*x**6 - 90*atan(c*x)*a*b*c**2*d + 60*atan(
c*x)*a*b*e - 30*atan(c*x)*b**2*c**5*d*x**3 - 12*atan(c*x)*b**2*c**5*e*x**5
 + 90*atan(c*x)*b**2*c**3*d*x + 20*atan(c*x)*b**2*c**3*e*x**3 - 60*atan(c*
x)*b**2*c*e*x - 60*log(c**2*x**2 + 1)*b**2*c**2*d + 46*log(c**2*x**2 + 1)*
b**2*e + 45*a**2*c**6*d*x**4 + 30*a**2*c**6*e*x**6 - 30*a*b*c**5*d*x**3 -
12*a*b*c**5*e*x**5 + 90*a*b*c**3*d*x + 20*a*b*c**3*e*x**3 - 60*a*b*c*e*x +
 15*b**2*c**4*d*x**2 + 3*b**2*c**4*e*x**4 - 16*b**2*c**2*e*x**2)/(180*c**6
)
```

# 3.1246 $\int x^2(d+ex^2)(a+b\arctan(cx))^2\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 323

$$\begin{aligned}\int x^2(d+ex^2)(a+b\arctan(cx))^2\,dx &= \frac{b^2dx}{3c^2}-\frac{3b^2ex}{10c^4}+\frac{b^2ex^3}{30c^2}-\frac{b^2d\arctan(cx)}{3c^3}\\&+\frac{3b^2e\arctan(cx)}{10c^5}-\frac{bdx^2(a+b\arctan(cx))}{3c}\\&+\frac{bex^2(a+b\arctan(cx))}{5c^3}\\&-\frac{bex^4(a+b\arctan(cx))}{10c}\\&-\frac{id(a+b\arctan(cx))^2}{3c^3}+\frac{ie(a+b\arctan(cx))^2}{5c^5}\\&+\frac{1}{3}dx^3(a+b\arctan(cx))^2\\&+\frac{1}{5}ex^5(a+b\arctan(cx))^2\\&-\frac{2bd(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{3c^3}\\&+\frac{2be(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{5c^5}\\&-\frac{ib^2d\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{3c^3}\\&+\frac{ib^2e\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{5c^5}\end{aligned}$$

output
```
1/3*b^2*d*x/c^2-3/10*b^2*e*x/c^4+1/30*b^2*e*x^3/c^2-1/3*b^2*d*arctan(c*x)/
c^3+3/10*b^2*e*arctan(c*x)/c^5-1/3*b*d*x^2*(a+b*arctan(c*x))/c+1/5*b*e*x^2
*(a+b*arctan(c*x))/c^3-1/10*b*e*x^4*(a+b*arctan(c*x))/c-1/3*I*b^2*d*polylo
g(2,1-2/(1+I*c*x))/c^3+1/5*I*e*(a+b*arctan(c*x))^2/c^5+1/3*d*x^3*(a+b*arct
an(c*x))^2+1/5*e*x^5*(a+b*arctan(c*x))^2-2/3*b*d*(a+b*arctan(c*x))*ln(2/(1
+I*c*x))/c^3+2/5*b*e*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^5-1/3*I*d*(a+b*ar
ctan(c*x))^2/c^3+1/5*I*b^2*e*polylog(2,1-2/(1+I*c*x))/c^5
```

## Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.89

$$\int x^2(d+ex^2)\,(a+b\arctan(cx))^2\,dx$$
$$= \frac{9abe+10b^2c^3dx-9b^2cex-10abc^4dx^2+6abc^2ex^2+10a^2c^5dx^3+b^2c^3ex^3-3abc^4ex^4+6a^2c^5ex^5+2b^2(}{}$$

input
```
Integrate[x^2*(d + e*x^2)*(a + b*ArcTan[c*x])^2,x]
```

output
```
(9*a*b*e + 10*b^2*c^3*d*x - 9*b^2*c*e*x - 10*a*b*c^4*d*x^2 + 6*a*b*c^2*e*x
^2 + 10*a^2*c^5*d*x^3 + b^2*c^3*e*x^3 - 3*a*b*c^4*e*x^4 + 6*a^2*c^5*e*x^5
+ 2*b^2*((5*I)*c^2*d - (3*I)*e + c^5*(5*d*x^3 + 3*e*x^5))*ArcTan[c*x]^2 -
b*ArcTan[c*x]*(-4*a*c^5*x^3*(5*d + 3*e*x^2) + b*(1 + c^2*x^2)*(-9*e + c^2*
(10*d + 3*e*x^2)) + 4*b*(5*c^2*d - 3*e)*Log[1 + E^((2*I)*ArcTan[c*x])]) +
10*a*b*c^2*d*Log[1 + c^2*x^2] - 6*a*b*e*Log[1 + c^2*x^2] + (2*I)*b^2*(5*c^
2*d - 3*e)*PolyLog[2, -E^((2*I)*ArcTan[c*x])])/(30*c^5)
```

## Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 (d+ex^2) (a+b\arctan(cx))^2 \, dx$$

$$\Big\downarrow \text{5515}$$

$$\int \left(dx^2(a+b\arctan(cx))^2+ex^4(a+b\arctan(cx))^2\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{ie(a+b\arctan(cx))^2}{5c^5}+\frac{2be\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{5c^5}-\frac{id(a+b\arctan(cx))^2}{3c^3}-\frac{2bd\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{3c^3}+\frac{bex^2(a+b\arctan(cx))}{5c^3}+\frac{1}{3}dx^3(a+b\arctan(cx))^2-\frac{bdx^2(a+b\arctan(cx))}{3c}+\frac{1}{5}ex^5(a+b\arctan(cx))^2-\frac{bex^4(a+b\arctan(cx))}{10c}+\frac{3b^2e\arctan(cx)}{10c^5}-\frac{b^2d\arctan(cx)}{3c^3}+\frac{ib^2e\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{5c^5}-\frac{3b^2ex}{10c^4}-\frac{ib^2d\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{3c^3}+\frac{b^2dx}{3c^2}+\frac{b^2ex^3}{30c^2}$$

input

```
Int[x^2*(d + e*x^2)*(a + b*ArcTan[c*x])^2,x]
```

output

```
(b^2*d*x)/(3*c^2) - (3*b^2*e*x)/(10*c^4) + (b^2*e*x^3)/(30*c^2) - (b^2*d*A
rcTan[c*x])/(3*c^3) + (3*b^2*e*ArcTan[c*x])/(10*c^5) - (b*d*x^2*(a + b*Arc
Tan[c*x]))/(3*c) + (b*e*x^2*(a + b*ArcTan[c*x]))/(5*c^3) - (b*e*x^4*(a + b
*ArcTan[c*x]))/(10*c) - ((I/3)*d*(a + b*ArcTan[c*x])^2)/c^3 + ((I/5)*e*(a
+ b*ArcTan[c*x])^2)/c^5 + (d*x^3*(a + b*ArcTan[c*x])^2)/3 + (e*x^5*(a + b*
ArcTan[c*x])^2)/5 - (2*b*d*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(3*c^3)
 + (2*b*e*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(5*c^5) - ((I/3)*b^2*d*P
olyLog[2, 1 - 2/(1 + I*c*x)])/c^3 + ((I/5)*b^2*e*PolyLog[2, 1 - 2/(1 + I*c
*x)])/c^5
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

## Maple [A] (verified)

Time = 1.70 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.26

| method | result |
|---|---|
| parts | $a^2\left(\frac{1}{5}e\,x^5+\frac{1}{3}d\,x^3\right)+\frac{b^2\left(\frac{\arctan(cx)^2c^3e\,x^5}{5}+\frac{\arctan(cx)^2d\,c^3x^3}{3}-\frac{2\left(\frac{5\arctan(cx)c^4d\,x^2}{2}+\frac{3\arctan(cx)c^4e\,x^4}{4}-\frac{3\arctan}{}\right.}{}\right.}{}$ |
| derivativedivides | $\frac{\frac{a^2\left(\frac{1}{3}d\,c^5x^3+\frac{1}{5}e\,c^5x^5\right)}{c^2}+\frac{b^2\left(\frac{\arctan(cx)^2d\,c^5x^3}{3}+\frac{\arctan(cx)^2e\,c^5x^5}{5}-\frac{\arctan(cx)c^4d\,x^2}{3}-\frac{\arctan(cx)c^4e\,x^4}{10}+\frac{\arctan(cx)c^2e\,x^2}{5}+\right.}{}}{}$ |
| default | $\frac{\frac{a^2\left(\frac{1}{3}d\,c^5x^3+\frac{1}{5}e\,c^5x^5\right)}{c^2}+\frac{b^2\left(\frac{\arctan(cx)^2d\,c^5x^3}{3}+\frac{\arctan(cx)^2e\,c^5x^5}{5}-\frac{\arctan(cx)c^4d\,x^2}{3}-\frac{\arctan(cx)c^4e\,x^4}{10}+\frac{\arctan(cx)c^2e\,x^2}{5}+\right.}{}}{}$ |
| risch | $-\frac{3b^2ex}{10c^4}+\frac{b^2e\,x^3}{30c^2}+\frac{3b^2e\arctan(cx)}{20c^5}+\frac{b^2dx}{3c^2}-\frac{b^2d\arctan(cx)}{6c^3}-\frac{eba\,x^4}{10c}+\frac{eba\,x^2}{5c^3}-\frac{abd\,x^2}{3c}-\frac{ibad\ln(icx-}{3}$ |

input
```
int(x^2*(e*x^2+d)*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*(1/5*e*x^5+1/3*d*x^3)+b^2/c^3*(1/5*arctan(c*x)^2*c^3*e*x^5+1/3*arctan(
c*x)^2*d*c^3*x^3-2/15/c^2*(5/2*arctan(c*x)*c^4*d*x^2+3/4*arctan(c*x)*c^4*e
*x^4-3/2*arctan(c*x)*c^2*e*x^2-5/2*arctan(c*x)*ln(c^2*x^2+1)*c^2*d+3/2*arc
tan(c*x)*ln(c^2*x^2+1)*e-1/4*e*c^3*x^3-5/2*c^3*x*d+9/4*e*c*x-1/4*(-10*c^2*
d+9*e)*arctan(c*x)-1/4*(-10*c^2*d+6*e)*(-1/2*I*(ln(c*x-I)*ln(c^2*x^2+1)-di
log(-1/2*I*(I+c*x))-ln(c*x-I)*ln(-1/2*I*(I+c*x))-1/2*ln(c*x-I)^2)+1/2*I*(l
n(I+c*x)*ln(c^2*x^2+1)-dilog(1/2*I*(c*x-I))-ln(I+c*x)*ln(1/2*I*(c*x-I))-1/
2*ln(I+c*x)^2))))+2*a*b/c^3*(1/5*arctan(c*x)*c^3*e*x^5+1/3*arctan(c*x)*d*c
^3*x^3-1/15/c^2*(5/2*c^4*d*x^2+3/4*c^4*e*x^4-3/2*c^2*e*x^2+1/2*(-5*c^2*d+3
*e)*ln(c^2*x^2+1)))
```

## Fricas [F]

$$\int x^2 (d + ex^2) (a + b \arctan(cx))^2 dx = \int (ex^2 + d)(b \arctan(cx) + a)^2 x^2 dx$$

input
```
integrate(x^2*(e*x^2+d)*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

output
```
integral(a^2*e*x^4 + a^2*d*x^2 + (b^2*e*x^4 + b^2*d*x^2)*arctan(c*x)^2 + 2
*(a*b*e*x^4 + a*b*d*x^2)*arctan(c*x), x)
```

## Sympy [F]

$$\int x^2 (d + ex^2) (a + b \arctan(cx))^2 dx = \int x^2 (a + b \operatorname{atan}(cx))^2 (d + ex^2) dx$$

input
```
integrate(x**2*(e*x**2+d)*(a+b*atan(c*x))**2,x)
```

output
```
Integral(x**2*(a + b*atan(c*x))**2*(d + e*x**2), x)
```

## Maxima [F]

$$\int x^2 (d + ex^2) (a + b \arctan(cx))^2 dx = \int (ex^2 + d)(b \arctan(cx) + a)^2 x^2 dx$$

input
```
integrate(x^2*(e*x^2+d)*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

output
```
1/5*a^2*e*x^5 + 1/3*a^2*d*x^3 + 1/3*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(
c^2*x^2 + 1)/c^4))*a*b*d + 1/10*(4*x^5*arctan(c*x) - c*((c^2*x^4 - 2*x^2)/
c^4 + 2*log(c^2*x^2 + 1)/c^6))*a*b*e + 1/60*(3*b^2*e*x^5 + 5*b^2*d*x^3)*ar
ctan(c*x)^2 - 1/240*(3*b^2*e*x^5 + 5*b^2*d*x^3)*log(c^2*x^2 + 1)^2 + integ
rate(1/240*(180*(b^2*c^2*e*x^6 + b^2*d*x^2 + (b^2*c^2*d + b^2*e)*x^4)*arct
an(c*x)^2 + 15*(b^2*c^2*e*x^6 + b^2*d*x^2 + (b^2*c^2*d + b^2*e)*x^4)*log(c
^2*x^2 + 1)^2 - 8*(3*b^2*c*e*x^5 + 5*b^2*c*d*x^3)*arctan(c*x) + 4*(3*b^2*c
^2*e*x^6 + 5*b^2*c^2*d*x^4)*log(c^2*x^2 + 1))/(c^2*x^2 + 1), x)
```

## Giac [F]

$$\int x^2(d+ex^2)(a+b\arctan(cx))^2\,dx=\int (ex^2+d)(b\arctan(cx)+a)^2x^2\,dx$$

input
```
integrate(x^2*(e*x^2+d)*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)*(b*arctan(c*x) + a)^2*x^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int x^2(d+ex^2)(a+b\arctan(cx))^2\,dx=\int x^2\,(a+b\,\mathrm{atan}(c\,x))^2\,\left(e\,x^2+d\right)\,dx$$

input
```
int(x^2*(a + b*atan(c*x))^2*(d + e*x^2),x)
```

output
```
int(x^2*(a + b*atan(c*x))^2*(d + e*x^2), x)
```

## Reduce [F]

$$\int x^2(d+ex^2)(a+b\arctan(cx))^2\,dx$$
$$= \frac{10atan(cx)^2 b^2c^5d\,x^3+6atan(cx)^2 b^2c^5e\,x^5+10atan(cx)^2 b^2c^3dx-6atan(cx)^2 b^2cex+20atan(cx)\,ab\,c^5 \ldots}{}$$

input
```
int(x^2*(e*x^2+d)*(a+b*atan(c*x))^2,x)
```

output
```
(10*atan(c*x)**2*b**2*c**5*d*x**3 + 6*atan(c*x)**2*b**2*c**5*e*x**5 + 10*a
tan(c*x)**2*b**2*c**3*d*x - 6*atan(c*x)**2*b**2*c*e*x + 20*atan(c*x)*a*b*c
**5*d*x**3 + 12*atan(c*x)*a*b*c**5*e*x**5 - 10*atan(c*x)*b**2*c**4*d*x**2
- 3*atan(c*x)*b**2*c**4*e*x**4 - 10*atan(c*x)*b**2*c**2*d + 6*atan(c*x)*b*
*2*c**2*e*x**2 + 9*atan(c*x)*b**2*e - 10*int(atan(c*x)**2,x)*b**2*c**3*d +
 6*int(atan(c*x)**2,x)*b**2*c*e + 10*log(c**2*x**2 + 1)*a*b*c**2*d - 6*log
(c**2*x**2 + 1)*a*b*e + 10*a**2*c**5*d*x**3 + 6*a**2*c**5*e*x**5 - 10*a*b*
c**4*d*x**2 - 3*a*b*c**4*e*x**4 + 6*a*b*c**2*e*x**2 + 10*b**2*c**3*d*x + b
**2*c**3*e*x**3 - 9*b**2*c*e*x)/(30*c**5)
```

# 3.1247 $\int x(d+ex^2)\left(a+b\arctan(cx)\right)^2\,dx$



## Optimal result

Integrand size = 19, antiderivative size = 199

$$\begin{aligned}\int x(d+ex^2)\left(a+b\arctan(cx)\right)^2\,dx = & -\frac{abdx}{c}+\frac{abex}{2c^3}+\frac{b^2ex^2}{12c^2}-\frac{b^2dx\arctan(cx)}{c} \\ & +\frac{b^2ex\arctan(cx)}{2c^3}-\frac{bex^3(a+b\arctan(cx))}{6c} \\ & +\frac{d(a+b\arctan(cx))^2}{2c^2}-\frac{e(a+b\arctan(cx))^2}{4c^4} \\ & +\frac{1}{2}dx^2(a+b\arctan(cx))^2 \\ & +\frac{1}{4}ex^4(a+b\arctan(cx))^2 \\ & +\frac{b^2d\log\left(1+c^2x^2\right)}{2c^2}-\frac{b^2e\log\left(1+c^2x^2\right)}{3c^4}\end{aligned}$$

output
```
-a*b*d*x/c+1/2*a*b*e*x/c^3+1/12*b^2*e*x^2/c^2-b^2*d*x*arctan(c*x)/c+1/2*b^
2*e*x*arctan(c*x)/c^3-1/6*b*e*x^3*(a+b*arctan(c*x))/c+1/2*d*(a+b*arctan(c*
x))^2/c^2-1/4*e*(a+b*arctan(c*x))^2/c^4+1/2*d*x^2*(a+b*arctan(c*x))^2+1/4*
e*x^4*(a+b*arctan(c*x))^2+1/2*b^2*d*ln(c^2*x^2+1)/c^2-1/3*b^2*e*ln(c^2*x^2
+1)/c^4
```

## Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.90

$$\int x(d+ex^2)(a+b\arctan(cx))^2\,dx = \frac{cx(6abe+b^2cex+3a^2c^3x(2d+ex^2)-2abc^2(6d+ex^2))+2b(6ac^2d-3ae+3bcex-bc^3x(6d+ex^2)+\ldots}{1\ldots}$$

input

```
Integrate[x*(d + e*x^2)*(a + b*ArcTan[c*x])^2,x]
```

output

```
(c*x*(6*a*b*e + b^2*c*e*x + 3*a^2*c^3*x*(2*d + e*x^2) - 2*a*b*c^2*(6*d + e
*x^2)) + 2*b*(6*a*c^2*d - 3*a*e + 3*b*c*e*x - b*c^3*x*(6*d + e*x^2) + 3*a*
c^4*(2*d*x^2 + e*x^4))*ArcTan[c*x] + 3*b^2*(2*c^2*d - e + c^4*(2*d*x^2 + e
*x^4))*ArcTan[c*x]^2 + 2*b^2*(3*c^2*d - 2*e)*Log[1 + c^2*x^2])/(12*c^4)
```

## Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(d+ex^2)(a+b\arctan(cx))^2\,dx$$

$$\Big\downarrow \text{5515}$$

$$\int \left(dx(a+b\arctan(cx))^2+ex^3(a+b\arctan(cx))^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{e(a+b\arctan(cx))^2}{4c^4}+\frac{d(a+b\arctan(cx))^2}{2c^2}+\frac{1}{2}dx^2(a+b\arctan(cx))^2+\frac{1}{4}ex^4(a+b\arctan(cx))^2-\frac{bex^3(a+b\arctan(cx))}{6c}+\frac{abex}{2c^3}-\frac{abdx}{c}+\frac{b^2ex\arctan(cx)}{2c^3}-\frac{b^2dx\arctan(cx)}{c}+\frac{b^2d\log(c^2x^2+1)}{2c^2}+\frac{b^2ex^2}{12c^2}-\frac{b^2e\log(c^2x^2+1)}{3c^4}$$

input
```
Int[x*(d + e*x^2)*(a + b*ArcTan[c*x])^2,x]
```

output
```
-((a*b*d*x)/c) + (a*b*e*x)/(2*c^3) + (b^2*e*x^2)/(12*c^2) - (b^2*d*x*ArcTa
n[c*x])/c + (b^2*e*x*ArcTan[c*x])/(2*c^3) - (b*e*x^3*(a + b*ArcTan[c*x]))/
(6*c) + (d*(a + b*ArcTan[c*x])^2)/(2*c^2) - (e*(a + b*ArcTan[c*x])^2)/(4*c
^4) + (d*x^2*(a + b*ArcTan[c*x])^2)/2 + (e*x^4*(a + b*ArcTan[c*x])^2)/4 +
(b^2*d*Log[1 + c^2*x^2])/(2*c^2) - (b^2*e*Log[1 + c^2*x^2])/(3*c^4)
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

## Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.25

| method | result |
|---|---|
| parts | $\frac{a^{2} \left(e \,x^{2}+d \right)^{2}}{4 e}+\frac{\arctan \left(c x \right)^{2} b^{2} e \,x^{4}}{4}+\frac{b^{2} \arctan \left(c x \right)^{2} d \,x^{2}}{2}-\frac{b^{2} e \arctan \left(c x \right) x^{3}}{6 c}-\frac{b^{2} d x \arctan \left(c x \right)}{c}+\frac{b^{2} e x \arctan}{2 c^{3}}$ |
| derivativedivides | $\frac{a^{2} \left(c^{2} e \,x^{2}+c^{2} d \right)^{2}}{4 c^{2} e}+\frac{b^{2} \arctan \left(c x \right)^{2} d \,c^{2} x^{2}}{2}+\frac{b^{2} c^{2} \arctan \left(c x \right)^{2} e \,x^{4}}{4}-b^{2} \arctan \left(c x \right) d c x -\frac{b^{2} c e \arctan \left(c x \right) x^{3}}{6}+\frac{b^{2} e x \arctan \left(c x \right)}{2 c}+b^{2}$ |
| default | $\frac{a^{2} \left(c^{2} e \,x^{2}+c^{2} d \right)^{2}}{4 c^{2} e}+\frac{b^{2} \arctan \left(c x \right)^{2} d \,c^{2} x^{2}}{2}+\frac{b^{2} c^{2} \arctan \left(c x \right)^{2} e \,x^{4}}{4}-b^{2} \arctan \left(c x \right) d c x -\frac{b^{2} c e \arctan \left(c x \right) x^{3}}{6}+\frac{b^{2} e x \arctan \left(c x \right)}{2 c}+b^{2}$ |
| parallelrisch | $3 x^{4} \arctan \left(c x \right)^{2} b^{2} c^{4} e +6 x^{4} \arctan \left(c x \right) a b \,c^{4} e +3 c^{4} a^{2} e \,x^{4}+6 x^{2} \arctan \left(c x \right)^{2} b^{2} c^{4} d -2 x^{3} \arctan \left(c x \right) b^{2} c^{3} e +12 x^{2} \arctan (c x$ |
| risch | $-\frac{b^{2} \left(c^{4} e \,x^{4}+2 c^{4} d \,x^{2}+2 c^{2} d -e \right) \ln \left(i c x +1\right)^{2}}{16 c^{4}}-\frac{i b \left(6 a \,c^{4} e \,x^{4}+3 i b \,c^{4} e \,x^{4} \ln \left(-i c x +1\right)+12 a \,c^{4} d \,x^{2}-2 b \,c^{3} e \,x^{3}+6 i b \,c^{4} d \,x^{2}\right.}{}$ |

input
```
int(x*(e*x^2+d)*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*a^2*(e*x^2+d)^2/e+1/4*arctan(c*x)^2*b^2*e*x^4+1/2*b^2*arctan(c*x)^2*d*
x^2-1/6*b^2/c*e*arctan(c*x)*x^3-b^2*d*x*arctan(c*x)/c+1/2*b^2*e*x*arctan(c
*x)/c^3+1/2*b^2/c^2*arctan(c*x)^2*d-1/4*b^2/c^4*e*arctan(c*x)^2+1/12*b^2*e
*x^2/c^2+1/2*b^2*d*ln(c^2*x^2+1)/c^2-1/3*b^2*e*ln(c^2*x^2+1)/c^4+1/2*arcta
n(c*x)*a*b*e*x^4+a*b*arctan(c*x)*d*x^2-1/6/c*a*b*e*x^3-a*b*d*x/c+1/2*a*b*e
*x/c^3+1/c^2*a*b*d*arctan(c*x)-1/2/c^4*a*b*e*arctan(c*x)
```

## Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.10

$$\int x(d+ex^2)(a+b\arctan(cx))^2\,dx$$
$$= \frac{3a^2c^4ex^4 - 2abc^3ex^3 + (6a^2c^4d + b^2c^2e)x^2 + 3(b^2c^4ex^4 + 2b^2c^4dx^2 + 2b^2c^2d - b^2e)\arctan(cx)^2 - 6(2\cdots}{}$$

input
```
integrate(x*(e*x^2+d)*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

output
```
1/12*(3*a^2*c^4*e*x^4 - 2*a*b*c^3*e*x^3 + (6*a^2*c^4*d + b^2*c^2*e)*x^2 +
3*(b^2*c^4*e*x^4 + 2*b^2*c^4*d*x^2 + 2*b^2*c^2*d - b^2*e)*arctan(c*x)^2 -
6*(2*a*b*c^3*d - a*b*c*e)*x + 2*(3*a*b*c^4*e*x^4 + 6*a*b*c^4*d*x^2 - b^2*c
^3*e*x^3 + 6*a*b*c^2*d - 3*a*b*e - 3*(2*b^2*c^3*d - b^2*c*e)*x)*arctan(c*x
) + 2*(3*b^2*c^2*d - 2*b^2*e)*log(c^2*x^2 + 1))/c^4
```

## Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.49

$$\int x(d+ex^2)(a+b\arctan(cx))^2\,dx$$
$$= \begin{cases} \frac{a^2dx^2}{2} + \frac{a^2ex^4}{4} + abdx^2\operatorname{atan}(cx) + \frac{abex^4\operatorname{atan}(cx)}{2} - \frac{abdx}{c} - \frac{abex^3}{6c} + \frac{abd\operatorname{atan}(cx)}{c^2} + \frac{abex}{2c^3} - \frac{abe\operatorname{atan}(cx)}{2c^4} + \frac{b^2dx^2\operatorname{at}\cdots}{2} \\ a^2\left(\frac{dx^2}{2} + \frac{ex^4}{4}\right) \end{cases}$$

input
```
integrate(x*(e*x**2+d)*(a+b*atan(c*x))**2,x)
```

output
```
Piecewise((a**2*d*x**2/2 + a**2*e*x**4/4 + a*b*d*x**2*atan(c*x) + a*b*e*x*
*4*atan(c*x)/2 - a*b*d*x/c - a*b*e*x**3/(6*c) + a*b*d*atan(c*x)/c**2 + a*b
*e*x/(2*c**3) - a*b*e*atan(c*x)/(2*c**4) + b**2*d*x**2*atan(c*x)**2/2 + b*
*2*e*x**4*atan(c*x)**2/4 - b**2*d*x*atan(c*x)/c - b**2*e*x**3*atan(c*x)/(6
*c) + b**2*d*log(x**2 + c**(-2))/(2*c**2) + b**2*d*atan(c*x)**2/(2*c**2) +
 b**2*e*x**2/(12*c**2) + b**2*e*x*atan(c*x)/(2*c**3) - b**2*e*log(x**2 + c
**(-2))/(3*c**4) - b**2*e*atan(c*x)**2/(4*c**4), Ne(c, 0)), (a**2*(d*x**2/
2 + e*x**4/4), True))
```

## Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.24

$$\int x(d+ex^2)(a+b\arctan(cx))^2\,dx = \frac{1}{4}b^2ex^4\arctan(cx)^2 + \frac{1}{4}a^2ex^4 + \frac{1}{2}b^2dx^2\arctan(cx)^2 + \frac{1}{2}a^2dx^2 + \left(x^2\arctan(cx) - c\left(\frac{x}{c^2} - \frac{\arctan(cx)}{c^3}\right)\right)abd - \frac{1}{2}\left(2c\left(\frac{x}{c^2} - \frac{\arctan(cx)}{c^3}\right)\arctan(cx) + \frac{\arctan(cx)^2 - \log(c^2x^2+1)}{c^2}\right)b^2d + \frac{1}{6}\left(3x^4\arctan(cx) - c\left(\frac{c^2x^3-3x}{c^4} + \frac{3\arctan(cx)}{c^5}\right)\right)abe - \frac{1}{12}\left(2c\left(\frac{c^2x^3-3x}{c^4} + \frac{3\arctan(cx)}{c^5}\right)\arctan(cx) - \frac{c^2x^2 + 3\arctan(cx)^2 - 4\log(c^2x^2+1)}{c^4}\right)b^2e$$

input
```
integrate(x*(e*x^2+d)*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

output
```
1/4*b^2*e*x^4*arctan(c*x)^2 + 1/4*a^2*e*x^4 + 1/2*b^2*d*x^2*arctan(c*x)^2
+ 1/2*a^2*d*x^2 + (x^2*arctan(c*x) - c*(x/c^2 - arctan(c*x)/c^3))*a*b*d -
1/2*(2*c*(x/c^2 - arctan(c*x)/c^3)*arctan(c*x) + (arctan(c*x)^2 - log(c^2*
x^2 + 1))/c^2)*b^2*d + 1/6*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3
*arctan(c*x)/c^5))*a*b*e - 1/12*(2*c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/
c^5)*arctan(c*x) - (c^2*x^2 + 3*arctan(c*x)^2 - 4*log(c^2*x^2 + 1))/c^4)*b
^2*e
```

## Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.30

$$\int x(d+ex^2)\,(a+b\arctan(cx))^2\,dx$$
$$=\frac{3\,b^2c^4ex^4\arctan{(cx)}^2+6\,abc^4ex^4\arctan{(cx)}+3\,a^2c^4ex^4+6\,b^2c^4dx^2\arctan{(cx)}^2+12\,abc^4dx^2\arctan{(}$$

input
```
integrate(x*(e*x^2+d)*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

output
```
1/12*(3*b^2*c^4*e*x^4*arctan(c*x)^2 + 6*a*b*c^4*e*x^4*arctan(c*x) + 3*a^2*
c^4*e*x^4 + 6*b^2*c^4*d*x^2*arctan(c*x)^2 + 12*a*b*c^4*d*x^2*arctan(c*x) -
 2*b^2*c^3*e*x^3*arctan(c*x) + 6*a^2*c^4*d*x^2 - 2*a*b*c^3*e*x^3 - 12*b^2*
c^3*d*x*arctan(c*x) - 12*a*b*c^3*d*x + b^2*c^2*e*x^2 + 6*b^2*c^2*d*arctan(
c*x)^2 + 12*a*b*c^2*d*arctan(c*x) + 6*b^2*c*e*x*arctan(c*x) + 6*b^2*c^2*d*
log(c^2*x^2 + 1) + 6*a*b*c*e*x - 3*b^2*e*arctan(c*x)^2 - 6*a*b*e*arctan(c*
x) - 4*b^2*e*log(c^2*x^2 + 1))/c^4
```

## Mupad [B] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.25

$$\begin{aligned}\int x(d+ex^2)\,(a+b\arctan(cx))^2\,dx &= \frac{a^2\,d\,x^2}{2}+\frac{a^2\,e\,x^4}{4}+\frac{b^2\,d\,\ln\left(c^2\,x^2+1\right)}{2\,c^2}\\ &\quad-\frac{b^2\,e\,\ln\left(c^2\,x^2+1\right)}{3\,c^4}+\frac{b^2\,e\,x^2}{12\,c^2}+\frac{b^2\,d\,\mathrm{atan}(c\,x)^2}{2\,c^2}\\ &\quad-\frac{b^2\,e\,\mathrm{atan}(c\,x)^2}{4\,c^4}+\frac{b^2\,d\,x^2\,\mathrm{atan}(c\,x)^2}{2}\\ &\quad+\frac{b^2\,e\,x^4\,\mathrm{atan}(c\,x)^2}{4}-\frac{a\,b\,e\,x^3}{6\,c}-\frac{b^2\,d\,x\,\mathrm{atan}(c\,x)}{c}\\ &\quad+\frac{b^2\,e\,x\,\mathrm{atan}(c\,x)}{2\,c^3}-\frac{b^2\,e\,x^3\,\mathrm{atan}(c\,x)}{6\,c}-\frac{a\,b\,d\,x}{c}\\ &\quad+\frac{a\,b\,e\,x}{2\,c^3}+\frac{a\,b\,d\,\mathrm{atan}(c\,x)}{c^2}-\frac{a\,b\,e\,\mathrm{atan}(c\,x)}{2\,c^4}\\ &\quad+a\,b\,d\,x^2\,\mathrm{atan}(c\,x)+\frac{a\,b\,e\,x^4\,\mathrm{atan}(c\,x)}{2}\end{aligned}$$

input
```
int(x*(a + b*atan(c*x))^2*(d + e*x^2),x)
```

output
```
(a^2*d*x^2)/2 + (a^2*e*x^4)/4 + (b^2*d*log(c^2*x^2 + 1))/(2*c^2) - (b^2*e*
log(c^2*x^2 + 1))/(3*c^4) + (b^2*e*x^2)/(12*c^2) + (b^2*d*atan(c*x)^2)/(2*
c^2) - (b^2*e*atan(c*x)^2)/(4*c^4) + (b^2*d*x^2*atan(c*x)^2)/2 + (b^2*e*x^
4*atan(c*x)^2)/4 - (a*b*e*x^3)/(6*c) - (b^2*d*x*atan(c*x))/c + (b^2*e*x*at
an(c*x))/(2*c^3) - (b^2*e*x^3*atan(c*x))/(6*c) - (a*b*d*x)/c + (a*b*e*x)/(
2*c^3) + (a*b*d*atan(c*x))/c^2 - (a*b*e*atan(c*x))/(2*c^4) + a*b*d*x^2*ata
n(c*x) + (a*b*e*x^4*atan(c*x))/2
```

## Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.30

$$\int x(d+ex^2)(a+b\arctan(cx))^2\,dx = \frac{6atan(cx)^2 b^2c^4d\,x^2+3atan(cx)^2 b^2c^4e\,x^4+6atan(cx)^2 b^2c^2d-3atan(cx)^2 b^2e+12atan(cx)\,ab\,c^4d\,x^2+}{}$$

input
```
int(x*(e*x^2+d)*(a+b*atan(c*x))^2,x)
```

output
```
(6*atan(c*x)**2*b**2*c**4*d*x**2 + 3*atan(c*x)**2*b**2*c**4*e*x**4 + 6*ata
n(c*x)**2*b**2*c**2*d - 3*atan(c*x)**2*b**2*e + 12*atan(c*x)*a*b*c**4*d*x*
*2 + 6*atan(c*x)*a*b*c**4*e*x**4 + 12*atan(c*x)*a*b*c**2*d - 6*atan(c*x)*a
*b*e - 12*atan(c*x)*b**2*c**3*d*x - 2*atan(c*x)*b**2*c**3*e*x**3 + 6*atan(
c*x)*b**2*c*e*x + 6*log(c**2*x**2 + 1)*b**2*c**2*d - 4*log(c**2*x**2 + 1)*
b**2*e + 6*a**2*c**4*d*x**2 + 3*a**2*c**4*e*x**4 - 12*a*b*c**3*d*x - 2*a*b
*c**3*e*x**3 + 6*a*b*c*e*x + b**2*c**2*e*x**2)/(12*c**4)
```

# 3.1248 $\int (d+ex^2) (a+b\arctan(cx))^2 dx$



## Optimal result

Integrand size = 18, antiderivative size = 231

$$\int (d+ex^2) (a+b\arctan(cx))^2 dx = \frac{b^2ex}{3c^2} - \frac{b^2e\arctan(cx)}{3c^3} - \frac{bex^2(a+b\arctan(cx))}{3c} + \frac{id(a+b\arctan(cx))^2}{c} - \frac{ie(a+b\arctan(cx))^2}{3c^3} + dx(a+b\arctan(cx))^2 + \frac{1}{3}ex^3(a+b\arctan(cx))^2 + \frac{2bd(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c} - \frac{2be(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{3c^3} + \frac{ib^2d\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c} - \frac{ib^2e\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{3c^3}$$

output
```
1/3*b^2*e*x/c^2-1/3*b^2*e*arctan(c*x)/c^3-1/3*b*e*x^2*(a+b*arctan(c*x))/c+
I*d*(a+b*arctan(c*x))^2/c-1/3*I*e*(a+b*arctan(c*x))^2/c^3+d*x*(a+b*arctan(
c*x))^2+1/3*e*x^3*(a+b*arctan(c*x))^2+2*b*d*(a+b*arctan(c*x))*ln(2/(1+I*c*
x))/c-2/3*b*e*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^3+I*b^2*d*polylog(2,1-2/
(1+I*c*x))/c-1/3*I*b^2*e*polylog(2,1-2/(1+I*c*x))/c^3
```

## Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.90

$$\int (d + ex^2) (a + b \arctan(cx))^2 \, dx$$
$$= \frac{3a^2c^3dx + b^2cex - abc^2ex^2 + a^2c^3ex^3 + b^2(-3ic^2d + ie + c^3(3dx + ex^3)) \arctan(cx)^2 - b \arctan(cx) (-2}{}$$

input
```
Integrate[(d + e*x^2)*(a + b*ArcTan[c*x])^2,x]
```

output
```
(3*a^2*c^3*d*x + b^2*c*e*x - a*b*c^2*e*x^2 + a^2*c^3*e*x^3 + b^2*((-3*I)*c
^2*d + I*e + c^3*(3*d*x + e*x^3))*ArcTan[c*x]^2 - b*ArcTan[c*x]*(-2*a*c^3*
x*(3*d + e*x^2) + b*(e + c^2*e*x^2) + 2*b*(-3*c^2*d + e)*Log[1 + E^((2*I)*
ArcTan[c*x])]) - 3*a*b*c^2*d*Log[1 + c^2*x^2] + a*b*e*Log[1 + c^2*x^2] - I
*b^2*(3*c^2*d - e)*PolyLog[2, -E^((2*I)*ArcTan[c*x])])/(3*c^3)
```

## Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {5449, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d + ex^2) (a + b \arctan(cx))^2 \, dx$$

$$\Big\downarrow \text{5449}$$

$$\int \left(d(a + b \arctan(cx))^2 + ex^2(a + b \arctan(cx))^2\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ie(a+b\arctan(cx))^2}{3c^3}-\frac{2be\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{3c^3}+dx(a+b\arctan(cx))^2+\frac{id(a+b\arctan(cx))^2}{c}+\frac{2bd\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}+\frac{1}{3}ex^3(a+b\arctan(cx))^2-\frac{bex^2(a+b\arctan(cx))}{3c}-\frac{b^2e\arctan(cx)}{3c^3}-\frac{ib^2e\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{3c^3}+\frac{b^2ex}{3c^2}+\frac{ib^2d\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{c}$$

input
```
Int[(d + e*x^2)*(a + b*ArcTan[c*x])^2,x]
```

output
```
(b^2*e*x)/(3*c^2) - (b^2*e*ArcTan[c*x])/(3*c^3) - (b*e*x^2*(a + b*ArcTan[c
*x]))/(3*c) + (I*d*(a + b*ArcTan[c*x])^2)/c - ((I/3)*e*(a + b*ArcTan[c*x])
^2)/c^3 + d*x*(a + b*ArcTan[c*x])^2 + (e*x^3*(a + b*ArcTan[c*x])^2)/3 + (2
*b*d*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/c - (2*b*e*(a + b*ArcTan[c*x]
)*Log[2/(1 + I*c*x)])/(3*c^3) + (I*b^2*d*PolyLog[2, 1 - 2/(1 + I*c*x)])/c
- ((I/3)*b^2*e*PolyLog[2, 1 - 2/(1 + I*c*x)])/c^3
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5449
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (d + e*x^2)^q, x], x
] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[q] && IGtQ[p, 0]
```

## Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.39

| method | result |
|---|---|
| parts | $a^2\left(\frac{1}{3}e\,x^3+dx\right)+\frac{b^2\left(\frac{\arctan(cx)^2ce\,x^3}{3}+\arctan(cx)^2cxd-\frac{2\left(\frac{\arctan(cx)c^2e\,x^2}{2}+\frac{3\arctan(cx)\ln\left(c^2x^2+1\right)c^2d}{2}-\frac{\arctan}{}\right.}{}\right.}{}$ |
| derivativedivides | $\frac{\frac{a^2\left(c^3xd+\frac{1}{3}e\,c^3x^3\right)}{c^2}+\frac{b^2\left(\arctan(cx)^2c^3xd+\frac{\arctan(cx)^2e\,c^3x^3}{3}-\frac{\arctan(cx)c^2e\,x^2}{3}-\arctan(cx)\ln\left(c^2x^2+1\right)c^2d+\frac{\arctan(cx)\ln\left(c^2\right.}{3}\right.}{}}{}$ |
| default | $\frac{\frac{a^2\left(c^3xd+\frac{1}{3}e\,c^3x^3\right)}{c^2}+\frac{b^2\left(\arctan(cx)^2c^3xd+\frac{\arctan(cx)^2e\,c^3x^3}{3}-\frac{\arctan(cx)c^2e\,x^2}{3}-\arctan(cx)\ln\left(c^2x^2+1\right)c^2d+\frac{\arctan(cx)\ln\left(c^2\right.}{3}\right.}{}}{}$ |
| risch | $\frac{ib^2d\ln\left(\frac{1}{2}+\frac{icx}{2}\right)\ln\left(\frac{1}{2}-\frac{icx}{2}\right)}{c}+\frac{ib^2d\ln(icx+1)\ln(-icx+1)}{2c}-\frac{ib^2d\ln\left(\frac{1}{2}+\frac{icx}{2}\right)\ln(-icx+1)}{c}+\frac{ib^2e\ln\left(\frac{1}{2}+\frac{icx}{2}\right)\ln(-}{3c^3}$ |

input
```
int((e*x^2+d)*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*(1/3*e*x^3+d*x)+b^2/c*(1/3*arctan(c*x)^2*c*e*x^3+arctan(c*x)^2*c*x*d-2
/3/c^2*(1/2*arctan(c*x)*c^2*e*x^2+3/2*arctan(c*x)*ln(c^2*x^2+1)*c^2*d-1/2*
arctan(c*x)*ln(c^2*x^2+1)*e-1/2*e*(c*x-arctan(c*x))-1/2*(3*c^2*d-e)*(-1/2*
I*(ln(c*x-I)*ln(c^2*x^2+1)-dilog(-1/2*I*(I+c*x))-ln(c*x-I)*ln(-1/2*I*(I+c*
x))-1/2*ln(c*x-I)^2)+1/2*I*(ln(I+c*x)*ln(c^2*x^2+1)-dilog(1/2*I*(c*x-I))-l
n(I+c*x)*ln(1/2*I*(c*x-I))-1/2*ln(I+c*x)^2))))+2*a*b/c*(1/3*arctan(c*x)*c*
e*x^3+arctan(c*x)*c*x*d-1/3/c^2*(1/2*c^2*e*x^2+1/2*(3*c^2*d-e)*ln(c^2*x^2+
1)))
```

## Fricas [F]

$$\int (d+ex^2)(a+b\arctan(cx))^2\,dx = \int (ex^2+d)(b\arctan(cx)+a)^2\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

output
```
integral(a^2*e*x^2 + a^2*d + (b^2*e*x^2 + b^2*d)*arctan(c*x)^2 + 2*(a*b*e*
x^2 + a*b*d)*arctan(c*x), x)
```

## Sympy [F]

$$\int (d+ex^2)(a+b\arctan(cx))^2\,dx = \int (a+b\operatorname{atan}(cx))^2\,(d+ex^2)\,dx$$

input
```
integrate((e*x**2+d)*(a+b*atan(c*x))**2,x)
```

output
```
Integral((a + b*atan(c*x))**2*(d + e*x**2), x)
```

## Maxima [F]

$$\int (d+ex^2)(a+b\arctan(cx))^2\,dx = \int (ex^2+d)(b\arctan(cx)+a)^2\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

output
```
1/3*a^2*e*x^3 + 36*b^2*c^2*e*integrate(1/48*x^4*arctan(c*x)^2/(c^2*x^2 + 1
), x) + 3*b^2*c^2*e*integrate(1/48*x^4*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x
) + 4*b^2*c^2*e*integrate(1/48*x^4*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) + 36
*b^2*c^2*d*integrate(1/48*x^2*arctan(c*x)^2/(c^2*x^2 + 1), x) + 3*b^2*c^2*
d*integrate(1/48*x^2*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + 12*b^2*c^2*d*i
ntegrate(1/48*x^2*log(c^2*x^2 + 1)/(c^2*x^2 + 1), x) + 1/4*b^2*d*arctan(c*
x)^3/c - 8*b^2*c*e*integrate(1/48*x^3*arctan(c*x)/(c^2*x^2 + 1), x) - 24*b
^2*c*d*integrate(1/48*x*arctan(c*x)/(c^2*x^2 + 1), x) + 1/3*(2*x^3*arctan(
c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*a*b*e + a^2*d*x + 36*b^2*e*inte
grate(1/48*x^2*arctan(c*x)^2/(c^2*x^2 + 1), x) + 3*b^2*e*integrate(1/48*x^
2*log(c^2*x^2 + 1)^2/(c^2*x^2 + 1), x) + 3*b^2*d*integrate(1/48*log(c^2*x^
2 + 1)^2/(c^2*x^2 + 1), x) + (2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*a*b*d/
c + 1/12*(b^2*e*x^3 + 3*b^2*d*x)*arctan(c*x)^2 - 1/48*(b^2*e*x^3 + 3*b^2*d
*x)*log(c^2*x^2 + 1)^2
```

## Giac [F]

$$\int (d+ex^2)(a+b\arctan(cx))^2\,dx=\int (ex^2+d)(b\arctan(cx)+a)^2\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)*(b*arctan(c*x) + a)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int (d+ex^2)(a+b\arctan(cx))^2\,dx=\int (a+b\,\mathrm{atan}(c\,x))^2\,(e\,x^2+d)\,dx$$

input
```
int((a + b*atan(c*x))^2*(d + e*x^2),x)
```

output
```
int((a + b*atan(c*x))^2*(d + e*x^2), x)
```

## Reduce [F]

$$\int (d+ex^2)(a+b\arctan(cx))^2\,dx$$
$$= \frac{3atan(cx)^2 b^2c^3dx + atan(cx)^2 b^2c^3e\,x^3 + 6atan(cx)\,ab\,c^3dx + 2atan(cx)\,ab\,c^3e\,x^3 - atan(cx)\,b^2c^2e\,x^2 -}{}$$

input
```
int((e*x^2+d)*(a+b*atan(c*x))^2,x)
```

output
```
(3*atan(c*x)**2*b**2*c**3*d*x + atan(c*x)**2*b**2*c**3*e*x**3 + 6*atan(c*x
)*a*b*c**3*d*x + 2*atan(c*x)*a*b*c**3*e*x**3 - atan(c*x)*b**2*c**2*e*x**2
- atan(c*x)*b**2*e - 6*int((atan(c*x)*x)/(c**2*x**2 + 1),x)*b**2*c**4*d +
2*int((atan(c*x)*x)/(c**2*x**2 + 1),x)*b**2*c**2*e - 3*log(c**2*x**2 + 1)*
a*b*c**2*d + log(c**2*x**2 + 1)*a*b*e + 3*a**2*c**3*d*x + a**2*c**3*e*x**3
 - a*b*c**2*e*x**2 + b**2*c*e*x)/(3*c**3)
```

# 3.1249 $\int \frac{(d+ex^2)(a+b\arctan(cx))^2}{x}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 217

$$\int \frac{(d+ex^2)(a+b\arctan(cx))^2}{x}\,dx = -\frac{abex}{c} - \frac{b^2ex\arctan(cx)}{c} + \frac{e(a+b\arctan(cx))^2}{2c^2} + \frac{1}{2}ex^2(a+b\arctan(cx))^2 + 2d(a+b\arctan(cx))^2\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right) + \frac{b^2e\log(1+c^2x^2)}{2c^2} - ibd(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right) + ibd(a+b\arctan(cx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx}\right) - \frac{1}{2}b^2d\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right) + \frac{1}{2}b^2d\operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)$$

output
```
-a*b*e*x/c-b^2*e*x*arctan(c*x)/c+1/2*e*(a+b*arctan(c*x))^2/c^2+1/2*e*x^2*(
a+b*arctan(c*x))^2-2*d*(a+b*arctan(c*x))^2*arctanh(-1+2/(1+I*c*x))+1/2*b^2
*e*ln(c^2*x^2+1)/c^2-I*b*d*(a+b*arctan(c*x))*polylog(2,1-2/(1+I*c*x))+I*b*
d*(a+b*arctan(c*x))*polylog(2,-1+2/(1+I*c*x))-1/2*b^2*d*polylog(3,1-2/(1+I
*c*x))+1/2*b^2*d*polylog(3,-1+2/(1+I*c*x))
```

## Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.21

$$\int \frac{(d+ex^2)(a+b\arctan(cx))^2}{x}\,dx$$
$$= \frac{1}{2}a^2ex^2 + \frac{abe(-cx+(1+c^2x^2)\arctan(cx))}{c^2} + a^2d\log(x)$$
$$+ \frac{b^2e(-2cx\arctan(cx)+(1+c^2x^2)\arctan(cx)^2+\log(1+c^2x^2))}{2c^2}$$
$$+ iabd(\text{PolyLog}(2,-icx)-\text{PolyLog}(2,icx)) + b^2d\bigg(-\frac{i\pi^3}{24}+\frac{2}{3}i\arctan(cx)^3$$
$$+\arctan(cx)^2\log\left(1-e^{-2i\arctan(cx)}\right)-\arctan(cx)^2\log\left(1+e^{2i\arctan(cx)}\right)$$
$$+i\arctan(cx)\,\text{PolyLog}\left(2,e^{-2i\arctan(cx)}\right)+i\arctan(cx)\,\text{PolyLog}\left(2,-e^{2i\arctan(cx)}\right)$$
$$+\frac{1}{2}\text{PolyLog}\left(3,e^{-2i\arctan(cx)}\right)-\frac{1}{2}\text{PolyLog}\left(3,-e^{2i\arctan(cx)}\right)\bigg)$$

input
```
Integrate[((d + e*x^2)*(a + b*ArcTan[c*x])^2)/x,x]
```

output
```
(a^2*e*x^2)/2 + (a*b*e*(-(c*x) + (1 + c^2*x^2)*ArcTan[c*x]))/c^2 + a^2*d*L
og[x] + (b^2*e*(-2*c*x*ArcTan[c*x] + (1 + c^2*x^2)*ArcTan[c*x]^2 + Log[1 +
 c^2*x^2]))/(2*c^2) + I*a*b*d*(PolyLog[2, (-I)*c*x] - PolyLog[2, I*c*x]) +
 b^2*d*((-1/24*I)*Pi^3 + ((2*I)/3)*ArcTan[c*x]^3 + ArcTan[c*x]^2*Log[1 - E
^((-2*I)*ArcTan[c*x])] - ArcTan[c*x]^2*Log[1 + E^((2*I)*ArcTan[c*x])] + I*
ArcTan[c*x]*PolyLog[2, E^((-2*I)*ArcTan[c*x])] + I*ArcTan[c*x]*PolyLog[2,
-E^((2*I)*ArcTan[c*x])] + PolyLog[3, E^((-2*I)*ArcTan[c*x])]/2 - PolyLog[3
, -E^((2*I)*ArcTan[c*x])]/2)
```

## Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)(a+b\arctan(cx))^2}{x}dx$$

$$\Big\downarrow \text{5515}$$

$$\int \left(\frac{d(a+b\arctan(cx))^2}{x}+ex(a+b\arctan(cx))^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$2d\text{arctanh}\left(1-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2+\frac{e(a+b\arctan(cx))^2}{2c^2}-ibd\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))+ibd\,\text{PolyLog}\left(2,\frac{2}{icx+1}-1\right)(a+b\arctan(cx))+\frac{1}{2}ex^2(a+b\arctan(cx))^2-\frac{abex}{c}-\frac{b^2ex\arctan(cx)}{c}+\frac{b^2e\log(c^2x^2+1)}{2c^2}-\frac{1}{2}b^2d\,\text{PolyLog}\left(3,1-\frac{2}{icx+1}\right)+\frac{1}{2}b^2d\,\text{PolyLog}\left(3,\frac{2}{icx+1}-1\right)$$

input
```
Int[((d + e*x^2)*(a + b*ArcTan[c*x])^2)/x,x]
```

output
```
-((a*b*e*x)/c) - (b^2*e*x*ArcTan[c*x])/c + (e*(a + b*ArcTan[c*x])^2)/(2*c^
2) + (e*x^2*(a + b*ArcTan[c*x])^2)/2 + 2*d*(a + b*ArcTan[c*x])^2*ArcTanh[1
 - 2/(1 + I*c*x)] + (b^2*e*Log[1 + c^2*x^2])/(2*c^2) - I*b*d*(a + b*ArcTan
[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)] + I*b*d*(a + b*ArcTan[c*x])*PolyLog[2
, -1 + 2/(1 + I*c*x)] - (b^2*d*PolyLog[3, 1 - 2/(1 + I*c*x)])/2 + (b^2*d*P
olyLog[3, -1 + 2/(1 + I*c*x)])/2
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 14.89 (sec) , antiderivative size = 1262, normalized size of antiderivative = 5.82

| method | result | size |
|---|---|---|
| parts | Expression too large to display | 1262 |
| derivativedivides | Expression too large to display | 1263 |
| default | Expression too large to display | 1263 |

input
```
int((e*x^2+d)*(a+b*arctan(c*x))^2/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*a^2*e*x^2+a^2*d*ln(x)+b^2*(1/2*arctan(c*x)^2*x^2*e+arctan(c*x)^2*d*ln(
c*x)-1/c^2*(2*I*c^2*d*arctan(c*x)*polylog(2,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+
1/2*I*c^2*d*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^
2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2-1/2*I*c^2*d*Pi*cs
gn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x
)^2+1/2*I*c^2*d*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(I*((1+I*c*x)^2
/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2-1/2*I*c^2*d*P
i*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c
*x)^2+1/2*I*c^2*d*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^
2+1)+1))^2*arctan(c*x)^2+e*arctan(c*x)*(c*x-I)-1/2*I*c^2*d*Pi*csgn(I*((1+I
*c*x)^2/(c^2*x^2+1)-1))*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((1+I*c
*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2+e*ln((1+I*
c*x)^2/(c^2*x^2+1)+1)+2*I*c^2*d*arctan(c*x)*polylog(2,(1+I*c*x)/(c^2*x^2+1
)^(1/2))-1/2*I*c^2*d*Pi*arctan(c*x)^2-1/2*arctan(c*x)^2*e+ln((1+I*c*x)^2/(
c^2*x^2+1)-1)*c^2*d*arctan(c*x)^2-ln((1+I*c*x)/(c^2*x^2+1)^(1/2)+1)*c^2*d*
arctan(c*x)^2-1/2*I*c^2*d*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)
^2/(c^2*x^2+1)+1))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+
1)+1))*arctan(c*x)^2-2*polylog(3,-(1+I*c*x)/(c^2*x^2+1)^(1/2))*c^2*d-ln(1-
(1+I*c*x)/(c^2*x^2+1)^(1/2))*c^2*d*arctan(c*x)^2-I*c^2*d*arctan(c*x)*polyl
og(2,-(1+I*c*x)^2/(c^2*x^2+1))-2*polylog(3,(1+I*c*x)/(c^2*x^2+1)^(1/2))...
```

## Fricas [F]

$$\int \frac{(d+ex^2)(a+b\arctan(cx))^2}{x}\,dx = \int \frac{(ex^2+d)(b\arctan(cx)+a)^2}{x}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arctan(c*x))^2/x,x, algorithm="fricas")
```

output
```
integral((a^2*e*x^2 + a^2*d + (b^2*e*x^2 + b^2*d)*arctan(c*x)^2 + 2*(a*b*e
*x^2 + a*b*d)*arctan(c*x))/x, x)
```

## Sympy [F]

$$\int \frac{(d+ex^2)(a+b\arctan(cx))^2}{x}\,dx = \int \frac{(a+b\,\mathrm{atan}\,(cx))^2\,(d+ex^2)}{x}\,dx$$

input
```
integrate((e*x**2+d)*(a+b*atan(c*x))**2/x,x)
```

output
```
Integral((a + b*atan(c*x))**2*(d + e*x**2)/x, x)
```

## Maxima [F]

$$\int \frac{(d+ex^2)(a+b\arctan(cx))^2}{x}\,dx = \int \frac{(ex^2+d)(b\arctan(cx)+a)^2}{x}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arctan(c*x))^2/x,x, algorithm="maxima")
```

output
```
1/8*b^2*e*x^2*arctan(c*x)^2 - 1/32*b^2*e*x^2*log(c^2*x^2 + 1)^2 + 12*b^2*c
^2*e*integrate(1/16*x^4*arctan(c*x)^2/(c^2*x^3 + x), x) + b^2*c^2*e*integr
ate(1/16*x^4*log(c^2*x^2 + 1)^2/(c^2*x^3 + x), x) + 32*a*b*c^2*e*integrate
(1/16*x^4*arctan(c*x)/(c^2*x^3 + x), x) + 2*b^2*c^2*e*integrate(1/16*x^4*l
og(c^2*x^2 + 1)/(c^2*x^3 + x), x) + 12*b^2*c^2*d*integrate(1/16*x^2*arctan
(c*x)^2/(c^2*x^3 + x), x) + 32*a*b*c^2*d*integrate(1/16*x^2*arctan(c*x)/(c
^2*x^3 + x), x) + 1/96*b^2*d*log(c^2*x^2 + 1)^3 + 1/2*a^2*e*x^2 - 4*b^2*c*
e*integrate(1/16*x^3*arctan(c*x)/(c^2*x^3 + x), x) + 12*b^2*e*integrate(1/
16*x^2*arctan(c*x)^2/(c^2*x^3 + x), x) + 32*a*b*e*integrate(1/16*x^2*arcta
n(c*x)/(c^2*x^3 + x), x) + 12*b^2*d*integrate(1/16*arctan(c*x)^2/(c^2*x^3
+ x), x) + b^2*d*integrate(1/16*log(c^2*x^2 + 1)^2/(c^2*x^3 + x), x) + 32*
a*b*d*integrate(1/16*arctan(c*x)/(c^2*x^3 + x), x) + 1/96*b^2*e*log(c^2*x^
2 + 1)^3/c^2 + a^2*d*log(x)
```

## Giac [F]

$$\int \frac{(d+ex^2)(a+b\arctan(cx))^2}{x}\,dx = \int \frac{(ex^2+d)(b\arctan(cx)+a)^2}{x}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arctan(c*x))^2/x,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)*(b*arctan(c*x) + a)^2/x, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)(a+b\arctan(cx))^2}{x}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^2\,(e\,x^2+d)}{x}\,dx$$

input
```
int(((a + b*atan(c*x))^2*(d + e*x^2))/x,x)
```

output
```
int(((a + b*atan(c*x))^2*(d + e*x^2))/x, x)
```

## Reduce [F]

$$\int \frac{(d+ex^2)(a+b\arctan(cx))^2}{x}\,dx$$
$$= \frac{atan(cx)^2\,b^2c^2e\,x^2+atan(cx)^2\,b^2e+2atan(cx)\,ab\,c^2e\,x^2+2atan(cx)\,abe-2atan(cx)\,b^2cex+4\left(\int \frac{atan(c}{x}\right.}{2c^2}$$

input
```
int((e*x^2+d)*(a+b*atan(c*x))^2/x,x)
```

output
```
(atan(c*x)**2*b**2*c**2*e*x**2 + atan(c*x)**2*b**2*e + 2*atan(c*x)*a*b*c**
2*e*x**2 + 2*atan(c*x)*a*b*e - 2*atan(c*x)*b**2*c*e*x + 4*int(atan(c*x)/x,
x)*a*b*c**2*d + 2*int(atan(c*x)**2/x,x)*b**2*c**2*d + log(c**2*x**2 + 1)*b
**2*e + 2*log(x)*a**2*c**2*d + a**2*c**2*e*x**2 - 2*a*b*c*e*x)/(2*c**2)
```

# 3.1250 $\int \frac{(d+ex^2)(a+b\arctan(cx))^2}{x^2}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 172

$$\begin{aligned}\int \frac{(d+ex^2)(a+b\arctan(cx))^2}{x^2}\,dx &= -icd(a+b\arctan(cx))^2+\frac{ie(a+b\arctan(cx))^2}{c}\\&\quad-\frac{d(a+b\arctan(cx))^2}{x}+ex(a+b\arctan(cx))^2\\&\quad+\frac{2be(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c}\\&\quad+2bcd(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right)\\&\quad-ib^2cd\,\mathrm{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)\\&\quad+\frac{ib^2e\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c}\end{aligned}$$

output
```
-I*c*d*(a+b*arctan(c*x))^2+I*e*(a+b*arctan(c*x))^2/c-d*(a+b*arctan(c*x))^2
/x+e*x*(a+b*arctan(c*x))^2+2*b*e*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c+2*b*c
*d*(a+b*arctan(c*x))*ln(2-2/(1-I*c*x))-I*b^2*c*d*polylog(2,-1+2/(1-I*c*x))
+I*b^2*e*polylog(2,1-2/(1+I*c*x))/c
```

## Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.19

$$\int \frac{(d+ex^2)(a+b\arctan(cx))^2}{x^2}\,dx$$
$$= \frac{-a^2cd + a^2cex^2 + abcd(-2\arctan(cx) + cx(2\log(cx) - \log(1+c^2x^2))) + abex(2cx\arctan(cx) - \log(1$$

input
```
Integrate[((d + e*x^2)*(a + b*ArcTan[c*x])^2)/x^2,x]
```

output
```
(-(a^2*c*d) + a^2*c*e*x^2 + a*b*c*d*(-2*ArcTan[c*x] + c*x*(2*Log[c*x] - Lo
g[1 + c^2*x^2])) + a*b*e*x*(2*c*x*ArcTan[c*x] - Log[1 + c^2*x^2]) + b^2*e*
x*(ArcTan[c*x]*((-I + c*x)*ArcTan[c*x] + 2*Log[1 + E^((2*I)*ArcTan[c*x])])
 - I*PolyLog[2, -E^((2*I)*ArcTan[c*x])]) - b^2*c*d*(ArcTan[c*x]^2 - 2*c*x*
ArcTan[c*x]*Log[1 - E^((2*I)*ArcTan[c*x])] + I*c*x*(ArcTan[c*x]^2 + PolyLo
g[2, E^((2*I)*ArcTan[c*x])])))/(c*x)
```

## Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)(a+b\arctan(cx))^2}{x^2}\,dx$$
$$\Big\downarrow \text{5515}$$
$$\int \left(\frac{d(a+b\arctan(cx))^2}{x^2} + e(a+b\arctan(cx))^2\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-icd(a+b\arctan(cx))^2-\frac{d(a+b\arctan(cx))^2}{x}+2bcd\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))+\frac{ie(a+b\arctan(cx))^2}{c}+ex(a+b\arctan(cx))^2+\frac{2be\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}-ib^2cd\,\mathrm{PolyLog}\left(2,\frac{2}{1-icx}-1\right)+\frac{ib^2e\,\mathrm{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{c}$$

input
```
Int[((d + e*x^2)*(a + b*ArcTan[c*x])^2)/x^2,x]
```

output
```
(-I)*c*d*(a + b*ArcTan[c*x])^2 + (I*e*(a + b*ArcTan[c*x])^2)/c - (d*(a + b
*ArcTan[c*x])^2)/x + e*x*(a + b*ArcTan[c*x])^2 + (2*b*e*(a + b*ArcTan[c*x]
)*Log[2/(1 + I*c*x)])/c + 2*b*c*d*(a + b*ArcTan[c*x])*Log[2 - 2/(1 - I*c*x
)] - I*b^2*c*d*PolyLog[2, -1 + 2/(1 - I*c*x)] + (I*b^2*e*PolyLog[2, 1 - 2/
(1 + I*c*x)])/c
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

## Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 355 vs. $2(164) = 328$.

Time = 1.54 (sec) , antiderivative size = 356, normalized size of antiderivative = 2.07

| method | result |
|---|---|
| derivativedivides | $c\left(\frac{a^2\left(ecx-\frac{dc}{x}\right)}{c^2}+\frac{b^2\left(\arctan(cx)^2cxe-\frac{\arctan(cx)^2dc}{x}+2\arctan(cx)dc^2\ln(cx)-\arctan(cx)\ln\left(c^2x^2+1\right)c^2d-\arctan\right.}{}\right.$ |
| default | $c\left(\frac{a^2\left(ecx-\frac{dc}{x}\right)}{c^2}+\frac{b^2\left(\arctan(cx)^2cxe-\frac{\arctan(cx)^2dc}{x}+2\arctan(cx)dc^2\ln(cx)-\arctan(cx)\ln\left(c^2x^2+1\right)c^2d-\arctan\right.}{}\right.$ |
| parts | $a^2\left(ex-\frac{d}{x}\right)+b^2c\left(\frac{\arctan(cx)^2xe}{c}-\frac{\arctan(cx)^2d}{cx}-\frac{2\left(\frac{\arctan(cx)\ln\left(c^2x^2+1\right)c^2d}{2}+\frac{\arctan(cx)\ln\left(c^2x^2+1\right)e}{2}-\right.}{}\right.$ |

input

```
int((e*x^2+d)*(a+b*arctan(c*x))^2/x^2,x,method=_RETURNVERBOSE)
```

output

```
c*(a^2/c^2*(e*c*x-d*c/x)+b^2/c^2*(arctan(c*x)^2*c*x*e-arctan(c*x)^2*d*c/x+
2*arctan(c*x)*d*c^2*ln(c*x)-arctan(c*x)*ln(c^2*x^2+1)*c^2*d-arctan(c*x)*ln
(c^2*x^2+1)*e+(c^2*d+e)*(-1/2*I*(ln(c*x-I)*ln(c^2*x^2+1)-dilog(-1/2*I*(I+c
*x))-ln(c*x-I)*ln(-1/2*I*(I+c*x))-1/2*ln(c*x-I)^2)+1/2*I*(ln(I+c*x)*ln(c^2
*x^2+1)-dilog(1/2*I*(c*x-I))-ln(I+c*x)*ln(1/2*I*(c*x-I))-1/2*ln(I+c*x)^2))
-2*d*c^2*(-1/2*I*ln(c*x)*ln(1+I*c*x)+1/2*I*ln(c*x)*ln(1-I*c*x)-1/2*I*dilog
(1+I*c*x)+1/2*I*dilog(1-I*c*x)))+2*a*b/c^2*(arctan(c*x)*c*x*e-arctan(c*x)*
d*c/x+d*c^2*ln(c*x)-1/2*(c^2*d+e)*ln(c^2*x^2+1)))
```

## Fricas [F]

$$\int \frac{(d+ex^2)(a+b\arctan(cx))^2}{x^2}\,dx = \int \frac{(ex^2+d)(b\arctan(cx)+a)^2}{x^2}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arctan(c*x))^2/x^2,x, algorithm="fricas")
```

output
```
integral((a^2*e*x^2 + a^2*d + (b^2*e*x^2 + b^2*d)*arctan(c*x)^2 + 2*(a*b*e
*x^2 + a*b*d)*arctan(c*x))/x^2, x)
```

## Sympy [F]

$$\int \frac{(d+ex^2)(a+b\arctan(cx))^2}{x^2}\,dx = \int \frac{(a+b\,\mathrm{atan}(cx))^2(d+ex^2)}{x^2}\,dx$$

input
```
integrate((e*x**2+d)*(a+b*atan(c*x))**2/x**2,x)
```

output
```
Integral((a + b*atan(c*x))**2*(d + e*x**2)/x**2, x)
```

## Maxima [F]

$$\int \frac{(d+ex^2)(a+b\arctan(cx))^2}{x^2}\,dx = \int \frac{(ex^2+d)(b\arctan(cx)+a)^2}{x^2}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arctan(c*x))^2/x^2,x, algorithm="maxima")
```

output
```
-(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*a*b*d + a^2*e*x + (2*
c*x*arctan(c*x) - log(c^2*x^2 + 1))*a*b*e/c - a^2*d/x + 1/16*(4*(b^2*e*x^2
 - b^2*d)*arctan(c*x)^2 - (b^2*e*x^2 - b^2*d)*log(c^2*x^2 + 1)^2 + 4*(b^2*
c*d*arctan(c*x)^3 + 48*b^2*c^2*e*integrate(1/16*x^4*arctan(c*x)^2/(c^2*x^4
 + x^2), x) + 4*b^2*c^2*e*integrate(1/16*x^4*log(c^2*x^2 + 1)^2/(c^2*x^4 +
 x^2), x) + 16*b^2*c^2*e*integrate(1/16*x^4*log(c^2*x^2 + 1)/(c^2*x^4 + x^
2), x) + 4*b^2*c^2*d*integrate(1/16*x^2*log(c^2*x^2 + 1)^2/(c^2*x^4 + x^2)
, x) - 16*b^2*c^2*d*integrate(1/16*x^2*log(c^2*x^2 + 1)/(c^2*x^4 + x^2), x
) + b^2*e*arctan(c*x)^3/c - 32*b^2*c*e*integrate(1/16*x^3*arctan(c*x)/(c^2
*x^4 + x^2), x) + 32*b^2*c*d*integrate(1/16*x*arctan(c*x)/(c^2*x^4 + x^2),
 x) + 4*b^2*e*integrate(1/16*x^2*log(c^2*x^2 + 1)^2/(c^2*x^4 + x^2), x) +
48*b^2*d*integrate(1/16*arctan(c*x)^2/(c^2*x^4 + x^2), x) + 4*b^2*d*integr
ate(1/16*log(c^2*x^2 + 1)^2/(c^2*x^4 + x^2), x))*x)/x
```

## Giac [F]

$$\int \frac{(d+ex^2)\,(a+b\arctan(cx))^2}{x^2}\,dx = \int \frac{(ex^2+d)(b\arctan(cx)+a)^2}{x^2}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arctan(c*x))^2/x^2,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)*(b*arctan(c*x) + a)^2/x^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)\,(a+b\arctan(cx))^2}{x^2}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^2\,(e\,x^2+d)}{x^2}\,dx$$

input
```
int(((a + b*atan(c*x))^2*(d + e*x^2))/x^2,x)
```

output
```
int(((a + b*atan(c*x))^2*(d + e*x^2))/x^2, x)
```

## Reduce [F]

$$\int \frac{(d+ex^2)\,(a+b\arctan(cx))^2}{x^2}\,dx$$
$$= \frac{-atan(cx)^2\,b^2cd - 2atan(cx)\,abcd + 2atan(cx)\,abce\,x^2 + \left(\int atan(cx)^2\,dx\right) b^2cex + 2\left(\int \frac{atan(cx)}{c^2x^3+x}dx\right) b^2c^2}{cx}$$

input
```
int((e*x^2+d)*(a+b*atan(c*x))^2/x^2,x)
```

output
```
( - atan(c*x)**2*b**2*c*d - 2*atan(c*x)*a*b*c*d + 2*atan(c*x)*a*b*c*e*x**2
 + int(atan(c*x)**2,x)*b**2*c*e*x + 2*int(atan(c*x)/(c**2*x**3 + x),x)*b**
2*c**2*d*x - log(c**2*x**2 + 1)*a*b*c**2*d*x - log(c**2*x**2 + 1)*a*b*e*x
+ 2*log(x)*a*b*c**2*d*x - a**2*c*d + a**2*c*e*x**2)/(c*x)
```

# 3.1251 $\int \frac{(d+ex^2)(a+b\arctan(cx))^2}{x^3}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 220

$$\begin{aligned}\int \frac{(d+ex^2)(a+b\arctan(cx))^2}{x^3}\,dx = &-\frac{bcd(a+b\arctan(cx))}{x}\\ &-\frac{1}{2}c^2d(a+b\arctan(cx))^2-\frac{d(a+b\arctan(cx))^2}{2x^2}\\ &+2e(a+b\arctan(cx))^2\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right)\\ &+b^2c^2d\log(x)-\frac{1}{2}b^2c^2d\log\left(1+c^2x^2\right)\\ &-ibe(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)\\ &+ibe(a+b\arctan(cx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)\\ &-\frac{1}{2}b^2e\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)\\ &+\frac{1}{2}b^2e\operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)\end{aligned}$$

output
```
-b*c*d*(a+b*arctan(c*x))/x-1/2*c^2*d*(a+b*arctan(c*x))^2-1/2*d*(a+b*arctan
(c*x))^2/x^2-2*e*(a+b*arctan(c*x))^2*arctanh(-1+2/(1+I*c*x))+b^2*c^2*d*ln(
x)-1/2*b^2*c^2*d*ln(c^2*x^2+1)-I*b*e*(a+b*arctan(c*x))*polylog(2,1-2/(1+I*
c*x))+I*b*e*(a+b*arctan(c*x))*polylog(2,-1+2/(1+I*c*x))-1/2*b^2*e*polylog(
3,1-2/(1+I*c*x))+1/2*b^2*e*polylog(3,-1+2/(1+I*c*x))
```

## Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.24

$$\begin{aligned}
&\int \frac{(d+ex^2)\,(a+b\arctan(cx))^2}{x^3}\,dx \\
&= -\frac{a^2d}{2x^2} - \frac{abd(\arctan(cx)+cx(1+cx\arctan(cx)))}{x^2} + a^2e\log(x) \\
&\quad - \frac{b^2d\left(2cx\arctan(cx)+(1+c^2x^2)\arctan(cx)^2-2c^2x^2\log\left(\frac{cx}{\sqrt{1+c^2x^2}}\right)\right)}{2x^2} \\
&\quad + iabe(\text{PolyLog}(2,-icx)-\text{PolyLog}(2,icx)) \\
&\quad + \frac{1}{24}b^2e\big(-i\pi^3+16i\arctan(cx)^3+24\arctan(cx)^2\log\left(1-e^{-2i\arctan(cx)}\right) \\
&\qquad - 24\arctan(cx)^2\log\left(1+e^{2i\arctan(cx)}\right)+24i\arctan(cx)\,\text{PolyLog}\left(2,e^{-2i\arctan(cx)}\right) \\
&\qquad + 24i\arctan(cx)\,\text{PolyLog}\left(2,-e^{2i\arctan(cx)}\right)+12\,\text{PolyLog}\left(3,e^{-2i\arctan(cx)}\right) \\
&\qquad - 12\,\text{PolyLog}\left(3,-e^{2i\arctan(cx)}\right)\big)
\end{aligned}$$

input
```
Integrate[((d + e*x^2)*(a + b*ArcTan[c*x])^2)/x^3,x]
```

output
```
-1/2*(a^2*d)/x^2 - (a*b*d*(ArcTan[c*x] + c*x*(1 + c*x*ArcTan[c*x])))/x^2 +
 a^2*e*Log[x] - (b^2*d*(2*c*x*ArcTan[c*x] + (1 + c^2*x^2)*ArcTan[c*x]^2 -
2*c^2*x^2*Log[(c*x)/Sqrt[1 + c^2*x^2]]))/(2*x^2) + I*a*b*e*(PolyLog[2, (-I
)*c*x] - PolyLog[2, I*c*x]) + (b^2*e*((-I)*Pi^3 + (16*I)*ArcTan[c*x]^3 + 2
4*ArcTan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] - 24*ArcTan[c*x]^2*Log[1 +
 E^((2*I)*ArcTan[c*x])] + (24*I)*ArcTan[c*x]*PolyLog[2, E^((-2*I)*ArcTan[c
*x])] + (24*I)*ArcTan[c*x]*PolyLog[2, -E^((2*I)*ArcTan[c*x])] + 12*PolyLog
[3, E^((-2*I)*ArcTan[c*x])] - 12*PolyLog[3, -E^((2*I)*ArcTan[c*x])]))/24
```

## Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)(a+b\arctan(cx))^2}{x^3}\,dx$$

$$\Big\downarrow \; 5515$$

$$\int \left(\frac{d(a+b\arctan(cx))^2}{x^3}+\frac{e(a+b\arctan(cx))^2}{x}\right)dx$$

$$\Big\downarrow \; 2009$$

$$2e\,\text{arctanh}\left(1-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2-\frac{1}{2}c^2d(a+b\arctan(cx))^2-\frac{d(a+b\arctan(cx))^2}{2x^2}-\frac{bcd(a+b\arctan(cx))}{x}-ibe\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))+ibe\,\text{PolyLog}\left(2,\frac{2}{icx+1}-1\right)(a+b\arctan(cx))-\frac{1}{2}b^2c^2d\log\left(c^2x^2+1\right)+b^2c^2d\log(x)-\frac{1}{2}b^2e\,\text{PolyLog}\left(3,1-\frac{2}{icx+1}\right)+\frac{1}{2}b^2e\,\text{PolyLog}\left(3,\frac{2}{icx+1}-1\right)$$

input
```
Int[((d + e*x^2)*(a + b*ArcTan[c*x])^2)/x^3,x]
```

output
```
-((b*c*d*(a + b*ArcTan[c*x]))/x) - (c^2*d*(a + b*ArcTan[c*x])^2)/2 - (d*(a
 + b*ArcTan[c*x])^2)/(2*x^2) + 2*e*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1
+ I*c*x)] + b^2*c^2*d*Log[x] - (b^2*c^2*d*Log[1 + c^2*x^2])/2 - I*b*e*(a +
 b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)] + I*b*e*(a + b*ArcTan[c*x])*
PolyLog[2, -1 + 2/(1 + I*c*x)] - (b^2*e*PolyLog[3, 1 - 2/(1 + I*c*x)])/2 +
 (b^2*e*PolyLog[3, -1 + 2/(1 + I*c*x)])/2
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 25.94 (sec) , antiderivative size = 1289, normalized size of antiderivative = 5.86

| method | result | size |
|---|---|---|
| derivativedivides | Expression too large to display | 1289 |
| default | Expression too large to display | 1289 |
| parts | Expression too large to display | 1318 |

input
```
int((e*x^2+d)*(a+b*arctan(c*x))^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
c^2*(-1/2*a^2*d/c^2/x^2+a^2/c^2*e*ln(c*x)+b^2/c^2*(-1/2*arctan(c*x)^2*c^2*
d+1/2*I*e*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1
))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*
x)^2-1/2*arctan(c*x)^2*d/x^2+1/2*I*e*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)
/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2+1/2*I*e*Pi*csgn(((1+I*c*x)^2
/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan(c*x)^2-1/2*I*Pi*e*cs
gn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^
2-1/2*c*d*arctan(c*x)*(I*c*x+(c^2*x^2+1)^(1/2)+1)/x-1/2*c*d*arctan(c*x)*(I
*c*x-(c^2*x^2+1)^(1/2)+1)/x-1/2*I*Pi*e*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))
*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(
c*x)^2-1/2*I*Pi*e*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/
(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2-1/2*I*e*Pi*csg
n(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(((1+I*c*
x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+1/2*I*e*P
i*arctan(c*x)^2-2*I*e*arctan(c*x)*polylog(2,(1+I*c*x)/(c^2*x^2+1)^(1/2))-2
*I*e*arctan(c*x)*polylog(2,-(1+I*c*x)/(c^2*x^2+1)^(1/2))+I*e*arctan(c*x)*p
olylog(2,-(1+I*c*x)^2/(c^2*x^2+1))+arctan(c*x)^2*e*ln(c*x)-e*ln((1+I*c*x)^
2/(c^2*x^2+1)-1)*arctan(c*x)^2+e*arctan(c*x)^2*ln((1+I*c*x)/(c^2*x^2+1)^(1
/2)+1)+e*arctan(c*x)^2*ln(1-(1+I*c*x)/(c^2*x^2+1)^(1/2))+2*e*polylog(3,-(1
+I*c*x)/(c^2*x^2+1)^(1/2))+2*e*polylog(3,(1+I*c*x)/(c^2*x^2+1)^(1/2))-1...
```

## Fricas [F]

$$\int \frac{(d+ex^2)\,(a+b\arctan(cx))^2}{x^3}\,dx = \int \frac{(ex^2+d)(b\arctan(cx)+a)^2}{x^3}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arctan(c*x))^2/x^3,x, algorithm="fricas")
```

output
```
integral((a^2*e*x^2 + a^2*d + (b^2*e*x^2 + b^2*d)*arctan(c*x)^2 + 2*(a*b*e
*x^2 + a*b*d)*arctan(c*x))/x^3, x)
```

## Sympy [F]

$$\int \frac{(d+ex^2)(a+b\arctan(cx))^2}{x^3}\,dx = \int \frac{(a+b\,\mathrm{atan}\,(cx))^2\,(d+ex^2)}{x^3}\,dx$$

input
```
integrate((e*x**2+d)*(a+b*atan(c*x))**2/x**3,x)
```

output
```
Integral((a + b*atan(c*x))**2*(d + e*x**2)/x**3, x)
```

## Maxima [F]

$$\int \frac{(d+ex^2)(a+b\arctan(cx))^2}{x^3}\,dx = \int \frac{(ex^2+d)(b\arctan(cx)+a)^2}{x^3}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arctan(c*x))^2/x^3,x, algorithm="maxima")
```

output
```
-((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*a*b*d + a^2*e*log(x) - 1/2*a^
2*d/x^2 - 1/96*(12*b^2*d*arctan(c*x)^2 - 3*b^2*d*log(c^2*x^2 + 1)^2 - (115
2*b^2*c^2*e*integrate(1/16*x^4*arctan(c*x)^2/(c^2*x^5 + x^3), x) + 3072*a*
b*c^2*e*integrate(1/16*x^4*arctan(c*x)/(c^2*x^5 + x^3), x) + 1152*b^2*c^2*
d*integrate(1/16*x^2*arctan(c*x)^2/(c^2*x^5 + x^3), x) + 96*b^2*c^2*d*inte
grate(1/16*x^2*log(c^2*x^2 + 1)^2/(c^2*x^5 + x^3), x) - 192*b^2*c^2*d*inte
grate(1/16*x^2*log(c^2*x^2 + 1)/(c^2*x^5 + x^3), x) + b^2*e*log(c^2*x^2 +
1)^3 + 384*b^2*c*d*integrate(1/16*x*arctan(c*x)/(c^2*x^5 + x^3), x) + 1152
*b^2*e*integrate(1/16*x^2*arctan(c*x)^2/(c^2*x^5 + x^3), x) + 96*b^2*e*int
egrate(1/16*x^2*log(c^2*x^2 + 1)^2/(c^2*x^5 + x^3), x) + 3072*a*b*e*integr
ate(1/16*x^2*arctan(c*x)/(c^2*x^5 + x^3), x) + 1152*b^2*d*integrate(1/16*a
rctan(c*x)^2/(c^2*x^5 + x^3), x) + 96*b^2*d*integrate(1/16*log(c^2*x^2 + 1
)^2/(c^2*x^5 + x^3), x))*x^2)/x^2
```

## Giac [F]

$$\int \frac{(d+ex^2)(a+b\arctan(cx))^2}{x^3}\,dx = \int \frac{(ex^2+d)(b\arctan(cx)+a)^2}{x^3}\,dx$$

input
```
integrate((e*x^2+d)*(a+b*arctan(c*x))^2/x^3,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)*(b*arctan(c*x) + a)^2/x^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)(a+b\arctan(cx))^2}{x^3}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^2\,(e\,x^2+d)}{x^3}\,dx$$

input
```
int(((a + b*atan(c*x))^2*(d + e*x^2))/x^3,x)
```

output
```
int(((a + b*atan(c*x))^2*(d + e*x^2))/x^3, x)
```

## Reduce [F]

$$\int \frac{(d+ex^2)(a+b\arctan(cx))^2}{x^3}\,dx = \frac{-atan(cx)^2\,b^2c^2d\,x^2-atan(cx)^2\,b^2d-2atan(cx)\,ab\,c^2d\,x^2-2atan(cx)\,abd-2atan(cx)\,b^2cdx+4\Big(\int \frac{ata}{}$$

input
```
int((e*x^2+d)*(a+b*atan(c*x))^2/x^3,x)
```

output
```
( - atan(c*x)**2*b**2*c**2*d*x**2 - atan(c*x)**2*b**2*d - 2*atan(c*x)*a*b*
c**2*d*x**2 - 2*atan(c*x)*a*b*d - 2*atan(c*x)*b**2*c*d*x + 4*int(atan(c*x)
/x,x)*a*b*e*x**2 + 2*int(atan(c*x)**2/x,x)*b**2*e*x**2 - log(c**2*x**2 + 1
)*b**2*c**2*d*x**2 + 2*log(x)*a**2*e*x**2 + 2*log(x)*b**2*c**2*d*x**2 - a*
*2*d - 2*a*b*c*d*x)/(2*x**2)
```

# 3.1252 $\int x^3(d+ex^2)^2(a+b\arctan(cx))^2\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 502

$$\begin{aligned}
\int x^3(d+ex^2)^2(a+b\arctan(cx))^2\,dx
&= \frac{abd^2x}{2c^3}-\frac{2abdex}{3c^5}+\frac{abe^2x}{4c^7}+\frac{b^2d^2x^2}{12c^2}-\frac{8b^2dex^2}{45c^4}+\frac{71b^2e^2x^2}{840c^6}+\frac{b^2dex^4}{30c^2}-\frac{3b^2e^2x^4}{140c^4}\\
&+\frac{b^2e^2x^6}{168c^2}+\frac{b^2d^2x\arctan(cx)}{2c^3}-\frac{2b^2dex\arctan(cx)}{3c^5}+\frac{b^2e^2x\arctan(cx)}{4c^7}\\
&-\frac{bd^2x^3(a+b\arctan(cx))}{6c}+\frac{2bdex^3(a+b\arctan(cx))}{9c^3}-\frac{be^2x^3(a+b\arctan(cx))}{12c^5}\\
&-\frac{2bdex^5(a+b\arctan(cx))}{15c}+\frac{be^2x^5(a+b\arctan(cx))}{20c^3}-\frac{be^2x^7(a+b\arctan(cx))}{28c}\\
&-\frac{d^2(a+b\arctan(cx))^2}{4c^4}+\frac{de(a+b\arctan(cx))^2}{3c^6}-\frac{e^2(a+b\arctan(cx))^2}{8c^8}\\
&+\frac{1}{4}d^2x^4(a+b\arctan(cx))^2+\frac{1}{3}dex^6(a+b\arctan(cx))^2+\frac{1}{8}e^2x^8(a+b\arctan(cx))^2\\
&-\frac{b^2d^2\log(1+c^2x^2)}{3c^4}+\frac{23b^2de\log(1+c^2x^2)}{45c^6}-\frac{22b^2e^2\log(1+c^2x^2)}{105c^8}
\end{aligned}$$

output
```
-2/15*b*d*e*x^5*(a+b*arctan(c*x))/c-2/3*a*b*d*e*x/c^5-2/3*b^2*d*e*x*arctan
(c*x)/c^5+2/9*b*d*e*x^3*(a+b*arctan(c*x))/c^3+1/2*a*b*d^2*x/c^3+1/2*b^2*d^
2*x*arctan(c*x)/c^3-1/6*b*d^2*x^3*(a+b*arctan(c*x))/c-1/4*d^2*(a+b*arctan(
c*x))^2/c^4+1/4*d^2*x^4*(a+b*arctan(c*x))^2+1/8*e^2*x^8*(a+b*arctan(c*x))^
2-1/8*e^2*(a+b*arctan(c*x))^2/c^8-22/105*b^2*e^2*ln(c^2*x^2+1)/c^8+1/3*d*e
*(a+b*arctan(c*x))^2/c^6+1/3*d*e*x^6*(a+b*arctan(c*x))^2+71/840*b^2*e^2*x^
2/c^6-3/140*b^2*e^2*x^4/c^4+1/168*b^2*e^2*x^6/c^2+1/4*a*b*e^2*x/c^7-8/45*b
^2*d*e*x^2/c^4+1/30*b^2*d*e*x^4/c^2+23/45*b^2*d*e*ln(c^2*x^2+1)/c^6+1/4*b^
2*e^2*x*arctan(c*x)/c^7-1/12*b*e^2*x^3*(a+b*arctan(c*x))/c^5+1/20*b*e^2*x^
5*(a+b*arctan(c*x))/c^3-1/28*b*e^2*x^7*(a+b*arctan(c*x))/c+1/12*b^2*d^2*x^
2/c^2-1/3*b^2*d^2*ln(c^2*x^2+1)/c^4
```

## Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 414, normalized size of antiderivative = 0.82

$$\int x^3(d+ex^2)^2(a+b\arctan(cx))^2\,dx$$
$$= \frac{cx(105a^2c^7x^3(6d^2+8dex^2+3e^2x^4)+b^2cx(213e^2-2c^2e(224d+27ex^2)+3c^4(70d^2+28dex^2+5e^2x^4))}{}$$

input
```
Integrate[x^3*(d + e*x^2)^2*(a + b*ArcTan[c*x])^2,x]
```

output
```
(c*x*(105*a^2*c^7*x^3*(6*d^2 + 8*d*e*x^2 + 3*e^2*x^4) + b^2*c*x*(213*e^2 -
 2*c^2*e*(224*d + 27*e*x^2) + 3*c^4*(70*d^2 + 28*d*e*x^2 + 5*e^2*x^4)) - 2
*a*b*(-315*e^2 + 105*c^2*e*(8*d + e*x^2) - 7*c^4*(90*d^2 + 40*d*e*x^2 + 9*
e^2*x^4) + 3*c^6*(70*d^2*x^2 + 56*d*e*x^4 + 15*e^2*x^6))) + 2*b*(b*c*x*(31
5*e^2 - 105*c^2*e*(8*d + e*x^2) + 7*c^4*(90*d^2 + 40*d*e*x^2 + 9*e^2*x^4)
- 3*c^6*(70*d^2*x^2 + 56*d*e*x^4 + 15*e^2*x^6)) + 105*a*(-6*c^4*d^2 + 8*c^
2*d*e - 3*e^2 + c^8*(6*d^2*x^4 + 8*d*e*x^6 + 3*e^2*x^8)))*ArcTan[c*x] + 10
5*b^2*(-6*c^4*d^2 + 8*c^2*d*e - 3*e^2 + c^8*(6*d^2*x^4 + 8*d*e*x^6 + 3*e^2
*x^8))*ArcTan[c*x]^2 - 8*b^2*(105*c^4*d^2 - 161*c^2*d*e + 66*e^2)*Log[1 +
c^2*x^2])/(2520*c^8)
```

## Rubi [A] (verified)

Time = 1.45 (sec) , antiderivative size = 502, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(d+ex^2\right)^2 (a+b\arctan(cx))^2\,dx$$

$$\Big\downarrow \text{5515}$$

$$\int \left(d^2x^3(a+b\arctan(cx))^2+2dex^5(a+b\arctan(cx))^2+e^2x^7(a+b\arctan(cx))^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{e^2(a+b\arctan(cx))^2}{8c^8}+\frac{de(a+b\arctan(cx))^2}{3c^6}-\frac{be^2x^3(a+b\arctan(cx))}{12c^5}-\frac{d^2(a+b\arctan(cx))^2}{4c^4}+\frac{2bdex^3(a+b\arctan(cx))}{9c^3}+\frac{be^2x^5(a+b\arctan(cx))}{20c^3}+\frac{1}{4}d^2x^4(a+b\arctan(cx))^2-\frac{bd^2x^3(a+b\arctan(cx))}{6c}+\frac{1}{3}dex^6(a+b\arctan(cx))^2-\frac{2bdex^5(a+b\arctan(cx))}{15c}+\frac{1}{8}e^2x^8(a+b\arctan(cx))^2-\frac{be^2x^7(a+b\arctan(cx))}{28c}+\frac{abe^2x}{4c^7}-\frac{2abdex}{3c^5}+\frac{abd^2x}{2c^3}+\frac{b^2e^2x\arctan(cx)}{4c^7}-\frac{2b^2dex\arctan(cx)}{3c^5}+\frac{b^2d^2x\arctan(cx)}{2c^3}+\frac{71b^2e^2x^2}{840c^6}-\frac{8b^2dex^2}{45c^4}-\frac{3b^2e^2x^4}{140c^4}+\frac{b^2d^2x^2}{12c^2}+\frac{b^2dex^4}{30c^2}+\frac{b^2e^2x^6}{168c^2}-\frac{22b^2e^2\log\left(c^2x^2+1\right)}{105c^8}+\frac{23b^2de\log\left(c^2x^2+1\right)}{45c^6}-\frac{b^2d^2\log\left(c^2x^2+1\right)}{3c^4}$$

input
```
Int[x^3*(d + e*x^2)^2*(a + b*ArcTan[c*x])^2,x]
```

output
```
(a*b*d^2*x)/(2*c^3) - (2*a*b*d*e*x)/(3*c^5) + (a*b*e^2*x)/(4*c^7) + (b^2*d
^2*x^2)/(12*c^2) - (8*b^2*d*e*x^2)/(45*c^4) + (71*b^2*e^2*x^2)/(840*c^6) +
 (b^2*d*e*x^4)/(30*c^2) - (3*b^2*e^2*x^4)/(140*c^4) + (b^2*e^2*x^6)/(168*c
^2) + (b^2*d^2*x*ArcTan[c*x])/(2*c^3) - (2*b^2*d*e*x*ArcTan[c*x])/(3*c^5)
+ (b^2*e^2*x*ArcTan[c*x])/(4*c^7) - (b*d^2*x^3*(a + b*ArcTan[c*x]))/(6*c)
+ (2*b*d*e*x^3*(a + b*ArcTan[c*x]))/(9*c^3) - (b*e^2*x^3*(a + b*ArcTan[c*x
]))/(12*c^5) - (2*b*d*e*x^5*(a + b*ArcTan[c*x]))/(15*c) + (b*e^2*x^5*(a +
b*ArcTan[c*x]))/(20*c^3) - (b*e^2*x^7*(a + b*ArcTan[c*x]))/(28*c) - (d^2*(
a + b*ArcTan[c*x])^2)/(4*c^4) + (d*e*(a + b*ArcTan[c*x])^2)/(3*c^6) - (e^2
*(a + b*ArcTan[c*x])^2)/(8*c^8) + (d^2*x^4*(a + b*ArcTan[c*x])^2)/4 + (d*e
*x^6*(a + b*ArcTan[c*x])^2)/3 + (e^2*x^8*(a + b*ArcTan[c*x])^2)/8 - (b^2*d
^2*Log[1 + c^2*x^2])/(3*c^4) + (23*b^2*d*e*Log[1 + c^2*x^2])/(45*c^6) - (2
2*b^2*e^2*Log[1 + c^2*x^2])/(105*c^8)
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

## Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 552, normalized size of antiderivative = 1.10

| method | result |
|---|---|
| parts | $a^2\left(\frac{1}{8}e^2x^8+\frac{1}{3}de\,x^6+\frac{1}{4}d^2x^4\right)+\frac{b^2\left(\frac{\arctan(cx)^2c^4e^2x^8}{8}+\frac{\arctan(cx)^2c^4de\,x^6}{3}+\frac{\arctan(cx)^2d^2c^4x^4}{4}-\frac{3\arctan(cx)}{7}\right.}{}$ |
| derivativedivides | $\frac{a^2\left(\frac{1}{4}d^2c^8x^4+\frac{1}{3}d\,c^8e\,x^6+\frac{1}{8}e^2c^8x^8\right)}{c^4}+\frac{b^2\left(\frac{\arctan(cx)^2d^2c^8x^4}{4}+\frac{\arctan(cx)^2d\,c^8e\,x^6}{3}+\frac{\arctan(cx)^2e^2c^8x^8}{8}-\frac{\arctan(cx)d^2c^7x^3}{6}-\right.}{}$ |
| default | $\frac{a^2\left(\frac{1}{4}d^2c^8x^4+\frac{1}{3}d\,c^8e\,x^6+\frac{1}{8}e^2c^8x^8\right)}{c^4}+\frac{b^2\left(\frac{\arctan(cx)^2d^2c^8x^4}{4}+\frac{\arctan(cx)^2d\,c^8e\,x^6}{3}+\frac{\arctan(cx)^2e^2c^8x^8}{8}-\frac{\arctan(cx)d^2c^7x^3}{6}-\right.}{}$ |
| parallelrisch | $-\frac{90ab\,c^7e^2x^7+420ab\,c^7d^2x^3-1680ab\,c^2de\arctan(cx)+336ab\,c^7de\,x^5-15b^2c^6e^2x^6-210b^2c^6d^2x^2+54b^2c^4e^2x^4-213b}{}$ |
| risch | $-\frac{2abdex}{3c^5}-\frac{22b^2e^2\ln(c^2x^2+1)}{105c^8}+\frac{71b^2e^2x^2}{840c^6}-\frac{3b^2e^2x^4}{140c^4}+\frac{b^2e^2x^6}{168c^2}+\frac{ab\,e^2x}{4c^7}-\frac{8b^2de\,x^2}{45c^4}+\frac{b^2de\,x^4}{30c^2}+\frac{23b^2}{}$ |

input

```
int(x^3*(e*x^2+d)^2*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

output

```
a^2*(1/8*e^2*x^8+1/3*d*e*x^6+1/4*d^2*x^4)+b^2/c^4*(1/8*arctan(c*x)^2*c^4*e
^2*x^8+1/3*arctan(c*x)^2*c^4*d*e*x^6+1/4*arctan(c*x)^2*d^2*c^4*x^4-1/12/c^
4*(3/7*arctan(c*x)*e^2*c^7*x^7+8/5*arctan(c*x)*d*c^7*e*x^5+2*arctan(c*x)*d
^2*c^7*x^3-3/5*arctan(c*x)*e^2*c^5*x^5-8/3*arctan(c*x)*d*c^5*e*x^3-6*arcta
n(c*x)*c^5*x*d^2+arctan(c*x)*e^2*c^3*x^3+8*arctan(c*x)*d*c^3*e*x-3*arctan(
c*x)*c*x*e^2+6*arctan(c*x)^2*c^4*d^2-8*arctan(c*x)^2*c^2*d*e+3*arctan(c*x)
^2*e^2-c^6*d^2*x^2-2/5*c^6*d*e*x^4-1/14*e^2*c^6*x^6+32/15*d*c^4*e*x^2+9/35
*e^2*c^4*x^4-71/70*e^2*c^2*x^2-1/210*(-840*c^4*d^2+1288*c^2*d*e-528*e^2)*l
n(c^2*x^2+1)-1/210*(630*c^4*d^2-840*c^2*d*e+315*e^2)*arctan(c*x)^2))+2*a*b
/c^4*(1/8*arctan(c*x)*c^4*e^2*x^8+1/3*arctan(c*x)*c^4*d*e*x^6+1/4*arctan(c
*x)*d^2*c^4*x^4-1/24/c^4*(3/7*e^2*c^7*x^7+8/5*d*c^7*e*x^5+2*d^2*c^7*x^3-3/
5*e^2*c^5*x^5-8/3*d*c^5*e*x^3-6*c^5*x*d^2+e^2*c^3*x^3+8*d*c^3*e*x-3*c*x*e^
2+(6*c^4*d^2-8*c^2*d*e+3*e^2)*arctan(c*x)))
```

## Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 530, normalized size of antiderivative = 1.06

$$\int x^3(d+ex^2)^2(a+b\arctan(cx))^2\,dx$$

$$=\frac{315\,a^2c^8e^2x^8-90\,abc^7e^2x^7+15\,(56\,a^2c^8de+b^2c^6e^2)x^6-42\,(8\,abc^7de-3\,abc^5e^2)x^5+6\,(105\,a^2c^8d^2+14}{}$$

input
```
integrate(x^3*(e*x^2+d)^2*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

output
```
1/2520*(315*a^2*c^8*e^2*x^8 - 90*a*b*c^7*e^2*x^7 + 15*(56*a^2*c^8*d*e + b^
2*c^6*e^2)*x^6 - 42*(8*a*b*c^7*d*e - 3*a*b*c^5*e^2)*x^5 + 6*(105*a^2*c^8*d
^2 + 14*b^2*c^6*d*e - 9*b^2*c^4*e^2)*x^4 - 70*(6*a*b*c^7*d^2 - 8*a*b*c^5*d
*e + 3*a*b*c^3*e^2)*x^3 + (210*b^2*c^6*d^2 - 448*b^2*c^4*d*e + 213*b^2*c^2
*e^2)*x^2 + 105*(3*b^2*c^8*e^2*x^8 + 8*b^2*c^8*d*e*x^6 + 6*b^2*c^8*d^2*x^4
 - 6*b^2*c^4*d^2 + 8*b^2*c^2*d*e - 3*b^2*e^2)*arctan(c*x)^2 + 210*(6*a*b*c
^5*d^2 - 8*a*b*c^3*d*e + 3*a*b*c*e^2)*x + 2*(315*a*b*c^8*e^2*x^8 + 840*a*b
*c^8*d*e*x^6 - 45*b^2*c^7*e^2*x^7 + 630*a*b*c^8*d^2*x^4 - 630*a*b*c^4*d^2
+ 840*a*b*c^2*d*e - 21*(8*b^2*c^7*d*e - 3*b^2*c^5*e^2)*x^5 - 315*a*b*e^2 -
 35*(6*b^2*c^7*d^2 - 8*b^2*c^5*d*e + 3*b^2*c^3*e^2)*x^3 + 105*(6*b^2*c^5*d
^2 - 8*b^2*c^3*d*e + 3*b^2*c*e^2)*x)*arctan(c*x) - 8*(105*b^2*c^4*d^2 - 16
1*b^2*c^2*d*e + 66*b^2*e^2)*log(c^2*x^2 + 1))/c^8
```

## Sympy [A] (verification not implemented)

Time = 0.88 (sec) , antiderivative size = 758, normalized size of antiderivative = 1.51

$$\int x^3(d+ex^2)^2(a+b\arctan(cx))^2\,dx = \text{Too large to display}$$

input
```
integrate(x**3*(e*x**2+d)**2*(a+b*atan(c*x))**2,x)
```

output
```
Piecewise((a**2*d**2*x**4/4 + a**2*d*e*x**6/3 + a**2*e**2*x**8/8 + a*b*d**
2*x**4*atan(c*x)/2 + 2*a*b*d*e*x**6*atan(c*x)/3 + a*b*e**2*x**8*atan(c*x)/
4 - a*b*d**2*x**3/(6*c) - 2*a*b*d*e*x**5/(15*c) - a*b*e**2*x**7/(28*c) + a
*b*d**2*x/(2*c**3) + 2*a*b*d*e*x**3/(9*c**3) + a*b*e**2*x**5/(20*c**3) - a
*b*d**2*atan(c*x)/(2*c**4) - 2*a*b*d*e*x/(3*c**5) - a*b*e**2*x**3/(12*c**5
) + 2*a*b*d*e*atan(c*x)/(3*c**6) + a*b*e**2*x/(4*c**7) - a*b*e**2*atan(c*x
)/(4*c**8) + b**2*d**2*x**4*atan(c*x)**2/4 + b**2*d*e*x**6*atan(c*x)**2/3
+ b**2*e**2*x**8*atan(c*x)**2/8 - b**2*d**2*x**3*atan(c*x)/(6*c) - 2*b**2*
d*e*x**5*atan(c*x)/(15*c) - b**2*e**2*x**7*atan(c*x)/(28*c) + b**2*d**2*x*
*2/(12*c**2) + b**2*d*e*x**4/(30*c**2) + b**2*e**2*x**6/(168*c**2) + b**2*
d**2*x*atan(c*x)/(2*c**3) + 2*b**2*d*e*x**3*atan(c*x)/(9*c**3) + b**2*e**2
*x**5*atan(c*x)/(20*c**3) - b**2*d**2*log(x**2 + c**(-2))/(3*c**4) - b**2*
d**2*atan(c*x)**2/(4*c**4) - 8*b**2*d*e*x**2/(45*c**4) - 3*b**2*e**2*x**4/
(140*c**4) - 2*b**2*d*e*x*atan(c*x)/(3*c**5) - b**2*e**2*x**3*atan(c*x)/(1
2*c**5) + 23*b**2*d*e*log(x**2 + c**(-2))/(45*c**6) + b**2*d*e*atan(c*x)**
2/(3*c**6) + 71*b**2*e**2*x**2/(840*c**6) + b**2*e**2*x*atan(c*x)/(4*c**7)
 - 22*b**2*e**2*log(x**2 + c**(-2))/(105*c**8) - b**2*e**2*atan(c*x)**2/(8
*c**8), Ne(c, 0)), (a**2*(d**2*x**4/4 + d*e*x**6/3 + e**2*x**8/8), True))
```

## Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 516, normalized size of antiderivative = 1.03

$$\int x^3(d+ex^2)^2(a+b\arctan(cx))^2\,dx = \frac{1}{8}b^2e^2x^8\arctan(cx)^2 + \frac{1}{8}a^2e^2x^8 + \frac{1}{3}b^2dex^6\arctan(cx)^2 + \frac{1}{3}a^2dex^6 + \frac{1}{4}b^2d^2x^4\arctan(cx)^2 + \frac{1}{4}a^2d^2x^4 + \frac{1}{6}\left(3x^4\arctan(cx) - c\left(\frac{c^2x^3-3x}{c^4} + \frac{3\arctan(cx)}{c^5}\right)\right)abd^2 - \frac{1}{12}\left(2c\left(\frac{c^2x^3-3x}{c^4} + \frac{3\arctan(cx)}{c^5}\right)\arctan(cx) - \frac{c^2x^2+3\arctan(cx)^2-4\log(c^2x^2+1)}{c^4}\right)b^2d^2 + \frac{2}{45}\left(15x^6\arctan(cx) - c\left(\frac{3c^4x^5-5c^2x^3+15x}{c^6} - \frac{15\arctan(cx)}{c^7}\right)\right)abde - \frac{1}{90}\left(4c\left(\frac{3c^4x^5-5c^2x^3+15x}{c^6} - \frac{15\arctan(cx)}{c^7}\right)\arctan(cx) - \frac{3c^4x^4-16c^2x^2-30\arctan(cx)^2+}{c^6}\right. + \frac{1}{420}\left(105x^8\arctan(cx) - c\left(\frac{15c^6x^7-21c^4x^5+35c^2x^3-105x}{c^8} + \frac{105\arctan(cx)}{c^9}\right)\right)abe^2 - \frac{1}{840}\left(2c\left(\frac{15c^6x^7-21c^4x^5+35c^2x^3-105x}{c^8} + \frac{105\arctan(cx)}{c^9}\right)\arctan(cx) - \frac{5c^6x^6-18c^4x^4+7}{}\right.$$

input
```
integrate(x^3*(e*x^2+d)^2*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

output
```
1/8*b^2*e^2*x^8*arctan(c*x)^2 + 1/8*a^2*e^2*x^8 + 1/3*b^2*d*e*x^6*arctan(c
*x)^2 + 1/3*a^2*d*e*x^6 + 1/4*b^2*d^2*x^4*arctan(c*x)^2 + 1/4*a^2*d^2*x^4
+ 1/6*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*a*
b*d^2 - 1/12*(2*c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5)*arctan(c*x) -
(c^2*x^2 + 3*arctan(c*x)^2 - 4*log(c^2*x^2 + 1))/c^4)*b^2*d^2 + 2/45*(15*x
^6*arctan(c*x) - c*((3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan(c*x)/c^
7))*a*b*d*e - 1/90*(4*c*((3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan(c*
x)/c^7)*arctan(c*x) - (3*c^4*x^4 - 16*c^2*x^2 - 30*arctan(c*x)^2 + 46*log(
c^2*x^2 + 1))/c^6)*b^2*d*e + 1/420*(105*x^8*arctan(c*x) - c*((15*c^6*x^7 -
 21*c^4*x^5 + 35*c^2*x^3 - 105*x)/c^8 + 105*arctan(c*x)/c^9))*a*b*e^2 - 1/
840*(2*c*((15*c^6*x^7 - 21*c^4*x^5 + 35*c^2*x^3 - 105*x)/c^8 + 105*arctan(
c*x)/c^9)*arctan(c*x) - (5*c^6*x^6 - 18*c^4*x^4 + 71*c^2*x^2 + 105*arctan(
c*x)^2 - 176*log(c^2*x^2 + 1))/c^8)*b^2*e^2
```

## Giac [A] (verification not implemented)

Time = 1.03 (sec) , antiderivative size = 639, normalized size of antiderivative = 1.27

$$\int x^3 \left(d+ex^2\right)^2 (a+b\arctan(cx))^2\,dx$$
$$= \frac{315\,b^2c^8e^2x^8\arctan\left(cx\right)^2+630\,abc^8e^2x^8\arctan\left(cx\right)+315\,a^2c^8e^2x^8+840\,b^2c^8dex^6\arctan\left(cx\right)^2+1680\,a}{}$$

input
```
integrate(x^3*(e*x^2+d)^2*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

output
```
1/2520*(315*b^2*c^8*e^2*x^8*arctan(c*x)^2 + 630*a*b*c^8*e^2*x^8*arctan(c*x
) + 315*a^2*c^8*e^2*x^8 + 840*b^2*c^8*d*e*x^6*arctan(c*x)^2 + 1680*a*b*c^8
*d*e*x^6*arctan(c*x) - 90*b^2*c^7*e^2*x^7*arctan(c*x) + 840*a^2*c^8*d*e*x^
6 - 90*a*b*c^7*e^2*x^7 + 630*b^2*c^8*d^2*x^4*arctan(c*x)^2 + 1260*a*b*c^8*
d^2*x^4*arctan(c*x) - 336*b^2*c^7*d*e*x^5*arctan(c*x) + 630*a^2*c^8*d^2*x^
4 - 336*a*b*c^7*d*e*x^5 + 15*b^2*c^6*e^2*x^6 - 420*b^2*c^7*d^2*x^3*arctan(
c*x) + 126*b^2*c^5*e^2*x^5*arctan(c*x) - 420*a*b*c^7*d^2*x^3 + 84*b^2*c^6*
d*e*x^4 + 126*a*b*c^5*e^2*x^5 + 560*b^2*c^5*d*e*x^3*arctan(c*x) + 210*b^2*
c^6*d^2*x^2 + 560*a*b*c^5*d*e*x^3 - 54*b^2*c^4*e^2*x^4 + 1260*b^2*c^5*d^2*
x*arctan(c*x) - 210*b^2*c^3*e^2*x^3*arctan(c*x) + 1260*a*b*c^5*d^2*x - 448
*b^2*c^4*d*e*x^2 - 210*a*b*c^3*e^2*x^3 - 630*b^2*c^4*d^2*arctan(c*x)^2 - 1
260*a*b*c^4*d^2*arctan(c*x) - 1680*b^2*c^3*d*e*x*arctan(c*x) - 840*b^2*c^4
*d^2*log(c^2*x^2 + 1) - 1680*a*b*c^3*d*e*x + 213*b^2*c^2*e^2*x^2 + 840*b^2
*c^2*d*e*arctan(c*x)^2 + 1680*a*b*c^2*d*e*arctan(c*x) + 630*b^2*c*e^2*x*ar
ctan(c*x) + 1288*b^2*c^2*d*e*log(c^2*x^2 + 1) + 630*a*b*c*e^2*x - 315*b^2*
e^2*arctan(c*x)^2 - 630*a*b*e^2*arctan(c*x) - 528*b^2*e^2*log(c^2*x^2 + 1)
)/c^8
```

## Mupad [B] (verification not implemented)

Time = 7.52 (sec) , antiderivative size = 929, normalized size of antiderivative = 1.85

$$\int x^3 \left(d+ex^2\right)^2 (a+b\arctan(cx))^2\,dx = \text{Too large to display}$$

input
```
int(x^3*(a + b*atan(c*x))^2*(d + e*x^2)^2,x)
```

output
```
(a^2*d^2*x^4)/4 + (a^2*e^2*x^8)/8 - (b^2*d^2*log(c^2*x^2 + 1))/(3*c^4) - (
22*b^2*e^2*log(c^2*x^2 + 1))/(105*c^8) + (b^2*d^2*x^2)/(12*c^2) + (b^2*e^2
*x^6)/(168*c^2) - (3*b^2*e^2*x^4)/(140*c^4) + (71*b^2*e^2*x^2)/(840*c^6) -
 (b^2*d^2*atan(c*x)^2)/(4*c^4) - (b^2*e^2*atan(c*x)^2)/(8*c^8) + (b^2*d^2*
x^4*atan(c*x)^2)/4 + (b^2*e^2*x^8*atan(c*x)^2)/8 + (a^2*d*e*x^6)/3 - (b^2*
d^2*x^3*atan(c*x))/(6*c) - (b^2*e^2*x^7*atan(c*x))/(28*c) + (b^2*e^2*x^5*a
tan(c*x))/(20*c^3) - (b^2*e^2*x^3*atan(c*x))/(12*c^5) + (a*b*d^2*x)/(2*c^3
) + (a*b*e^2*x)/(4*c^7) + (a*b*d^2*x^4*atan(c*x))/2 + (a*b*e^2*x^8*atan(c*
x))/4 + (23*b^2*d*e*log(c^2*x^2 + 1))/(45*c^6) - (a*b*d^2*x^3)/(6*c) - (a*
b*e^2*x^7)/(28*c) + (a*b*e^2*x^5)/(20*c^3) - (a*b*e^2*x^3)/(12*c^5) + (b^2
*d*e*x^4)/(30*c^2) - (8*b^2*d*e*x^2)/(45*c^4) + (b^2*d*e*atan(c*x)^2)/(3*c
^6) + (b^2*d^2*x*atan(c*x))/(2*c^3) + (b^2*e^2*x*atan(c*x))/(4*c^7) + (b^2
*d*e*x^6*atan(c*x)^2)/3 - (a*b*d^2*atan((3*b*c*e^2*x)/(3*b*e^2 + 6*b*c^4*d
^2 - 8*b*c^2*d*e) + (6*b*c^5*d^2*x)/(3*b*e^2 + 6*b*c^4*d^2 - 8*b*c^2*d*e)
- (8*b*c^3*d*e*x)/(3*b*e^2 + 6*b*c^4*d^2 - 8*b*c^2*d*e)))/(2*c^4) - (a*b*e
^2*atan((3*b*c*e^2*x)/(3*b*e^2 + 6*b*c^4*d^2 - 8*b*c^2*d*e) + (6*b*c^5*d^2
*x)/(3*b*e^2 + 6*b*c^4*d^2 - 8*b*c^2*d*e) - (8*b*c^3*d*e*x)/(3*b*e^2 + 6*b
*c^4*d^2 - 8*b*c^2*d*e)))/(4*c^8) - (2*b^2*d*e*x^5*atan(c*x))/(15*c) + (2*
b^2*d*e*x^3*atan(c*x))/(9*c^3) - (2*a*b*d*e*x)/(3*c^5) + (2*a*b*d*e*x^6*at
an(c*x))/3 - (2*a*b*d*e*x^5)/(15*c) + (2*a*b*d*e*x^3)/(9*c^3) - (2*b^2*...
```

## Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 639, normalized size of antiderivative = 1.27

$$\int x^3(d+ex^2)^2(a+b\arctan(cx))^2\,dx$$
$$= \frac{-315atan(cx)^2 b^2e^2 - 528\log(c^2x^2+1)\,b^2e^2 - 630atan(cx)^2 b^2c^4d^2 - 630atan(cx)\,ab\,e^2 - 840\log(c^2x^2+\cdots}{}$$

input
```
int(x^3*(e*x^2+d)^2*(a+b*atan(c*x))^2,x)
```

output
```
(630*atan(c*x)**2*b**2*c**8*d**2*x**4 + 840*atan(c*x)**2*b**2*c**8*d*e*x**
6 + 315*atan(c*x)**2*b**2*c**8*e**2*x**8 - 630*atan(c*x)**2*b**2*c**4*d**2
 + 840*atan(c*x)**2*b**2*c**2*d*e - 315*atan(c*x)**2*b**2*e**2 + 1260*atan
(c*x)*a*b*c**8*d**2*x**4 + 1680*atan(c*x)*a*b*c**8*d*e*x**6 + 630*atan(c*x
)*a*b*c**8*e**2*x**8 - 1260*atan(c*x)*a*b*c**4*d**2 + 1680*atan(c*x)*a*b*c
**2*d*e - 630*atan(c*x)*a*b*e**2 - 420*atan(c*x)*b**2*c**7*d**2*x**3 - 336
*atan(c*x)*b**2*c**7*d*e*x**5 - 90*atan(c*x)*b**2*c**7*e**2*x**7 + 1260*at
an(c*x)*b**2*c**5*d**2*x + 560*atan(c*x)*b**2*c**5*d*e*x**3 + 126*atan(c*x
)*b**2*c**5*e**2*x**5 - 1680*atan(c*x)*b**2*c**3*d*e*x - 210*atan(c*x)*b**
2*c**3*e**2*x**3 + 630*atan(c*x)*b**2*c*e**2*x - 840*log(c**2*x**2 + 1)*b*
*2*c**4*d**2 + 1288*log(c**2*x**2 + 1)*b**2*c**2*d*e - 528*log(c**2*x**2 +
 1)*b**2*e**2 + 630*a**2*c**8*d**2*x**4 + 840*a**2*c**8*d*e*x**6 + 315*a**
2*c**8*e**2*x**8 - 420*a*b*c**7*d**2*x**3 - 336*a*b*c**7*d*e*x**5 - 90*a*b
*c**7*e**2*x**7 + 1260*a*b*c**5*d**2*x + 560*a*b*c**5*d*e*x**3 + 126*a*b*c
**5*e**2*x**5 - 1680*a*b*c**3*d*e*x - 210*a*b*c**3*e**2*x**3 + 630*a*b*c*e
**2*x + 210*b**2*c**6*d**2*x**2 + 84*b**2*c**6*d*e*x**4 + 15*b**2*c**6*e**
2*x**6 - 448*b**2*c**4*d*e*x**2 - 54*b**2*c**4*e**2*x**4 + 213*b**2*c**2*e
**2*x**2)/(2520*c**8)
```

# 3.1253 $\int x^2(d+ex^2)^2 (a+b\arctan(cx))^2\,dx$

## Optimal result

Integrand size = 23, antiderivative size = 580

$$\int x^2(d+ex^2)^2(a+b\arctan(cx))^2\,dx = \frac{b^2d^2x}{3c^2}-\frac{3b^2dex}{5c^4}+\frac{11b^2e^2x}{42c^6}+\frac{b^2dex^3}{15c^2}-\frac{5b^2e^2x^3}{126c^4}+\frac{b^2e^2x^5}{105c^2}-\frac{b^2d^2\arctan(cx)}{3c^3}+\frac{3b^2de\arctan(cx)}{5c^5}-\frac{11b^2e^2\arctan(cx)}{42c^7}-\frac{bd^2x^2(a+b\arctan(cx))}{3c}+\frac{2bdex^2(a+b\arctan(cx))}{5c^3}-\frac{be^2x^2(a+b\arctan(cx))}{7c^5}-\frac{bdex^4(a+b\arctan(cx))}{5c}+\frac{be^2x^4(a+b\arctan(cx))}{14c^3}-\frac{be^2x^6(a+b\arctan(cx))}{21c}-\frac{id^2(a+b\arctan(cx))^2}{3c^3}+\frac{2ide(a+b\arctan(cx))^2}{5c^5}-\frac{ie^2(a+b\arctan(cx))^2}{7c^7}+\frac{1}{3}d^2x^3(a+b\arctan(cx))^2+\frac{2}{5}dex^5(a+b\arctan(cx))^2+\frac{1}{7}e^2x^7(a+b\arctan(cx))^2-\frac{2bd^2(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{3c^3}+\frac{4bde(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{5c^5}-\frac{2be^2(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{7c^7}-\frac{ib^2d^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{3c^3}+\frac{2ib^2de\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{5c^5}-\frac{ib^2e^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{7c^7}$$

output
```
2/5*b*d*e*x^2*(a+b*arctan(c*x))/c^3-1/5*b*d*e*x^4*(a+b*arctan(c*x))/c+4/5*
b*d*e*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^5-2/3*b*d^2*(a+b*arctan(c*x))*ln
(2/(1+I*c*x))/c^3-1/3*b*d^2*x^2*(a+b*arctan(c*x))/c+1/3*d^2*x^3*(a+b*arcta
n(c*x))^2+1/7*e^2*x^7*(a+b*arctan(c*x))^2+2/5*d*e*x^5*(a+b*arctan(c*x))^2-
3/5*b^2*d*e*x/c^4+1/15*b^2*d*e*x^3/c^2+3/5*b^2*d*e*arctan(c*x)/c^5-1/7*b*e
^2*x^2*(a+b*arctan(c*x))/c^5+1/14*b*e^2*x^4*(a+b*arctan(c*x))/c^3-1/21*b*e
^2*x^6*(a+b*arctan(c*x))/c-2/7*b*e^2*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^7
-1/3*I*b^2*d^2*polylog(2,1-2/(1+I*c*x))/c^3-1/7*I*b^2*e^2*polylog(2,1-2/(1
+I*c*x))/c^7+1/3*b^2*d^2*x/c^2-1/3*b^2*d^2*arctan(c*x)/c^3-11/42*b^2*e^2*a
rctan(c*x)/c^7+11/42*b^2*e^2*x/c^6-5/126*b^2*e^2*x^3/c^4+1/105*b^2*e^2*x^5
/c^2-1/3*I*d^2*(a+b*arctan(c*x))^2/c^3-1/7*I*e^2*(a+b*arctan(c*x))^2/c^7+2
/5*I*b^2*d*e*polylog(2,1-2/(1+I*c*x))/c^5+2/5*I*d*e*(a+b*arctan(c*x))^2/c^
5
```

## Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 513, normalized size of antiderivative = 0.88

$$\int x^2(d+ex^2)^2(a+b\arctan(cx))^2\,dx$$
$$= \frac{378abc^2de-165abe^2+210b^2c^5d^2x-378b^2c^3dex+165b^2ce^2x-210abc^6d^2x^2+252abc^4dex^2-90abc^2e^2}{}$$

input
```
Integrate[x^2*(d + e*x^2)^2*(a + b*ArcTan[c*x])^2,x]
```

output
```
(378*a*b*c^2*d*e - 165*a*b*e^2 + 210*b^2*c^5*d^2*x - 378*b^2*c^3*d*e*x + 1
65*b^2*c*e^2*x - 210*a*b*c^6*d^2*x^2 + 252*a*b*c^4*d*e*x^2 - 90*a*b*c^2*e^
2*x^2 + 210*a^2*c^7*d^2*x^3 + 42*b^2*c^5*d*e*x^3 - 25*b^2*c^3*e^2*x^3 - 12
6*a*b*c^6*d*e*x^4 + 45*a*b*c^4*e^2*x^4 + 252*a^2*c^7*d*e*x^5 + 6*b^2*c^5*e
^2*x^5 - 30*a*b*c^6*e^2*x^6 + 90*a^2*c^7*e^2*x^7 + 6*b^2*((35*I)*c^4*d^2 -
 (42*I)*c^2*d*e + (15*I)*e^2 + c^7*(35*d^2*x^3 + 42*d*e*x^5 + 15*e^2*x^7))
*ArcTan[c*x]^2 - 3*b*ArcTan[c*x]*(-4*a*c^7*x^3*(35*d^2 + 42*d*e*x^2 + 15*e
^2*x^4) + b*(1 + c^2*x^2)*(55*e^2 - c^2*e*(126*d + 25*e*x^2) + 2*c^4*(35*d
^2 + 21*d*e*x^2 + 5*e^2*x^4)) + 4*b*(35*c^4*d^2 - 42*c^2*d*e + 15*e^2)*Log
[1 + E^((2*I)*ArcTan[c*x])]) + 210*a*b*c^4*d^2*Log[1 + c^2*x^2] - 252*a*b*
c^2*d*e*Log[1 + c^2*x^2] + 90*a*b*e^2*Log[1 + c^2*x^2] + (6*I)*b^2*(35*c^4
*d^2 - 42*c^2*d*e + 15*e^2)*PolyLog[2, -E^((2*I)*ArcTan[c*x])])/(630*c^7)
```

## Rubi [A] (verified)

Time = 1.39 (sec) , antiderivative size = 580, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(d+e x^2\right)^2 (a+b \arctan (c x))^2 \, dx$$

$$\Big\downarrow \text{5515}$$

$$\int \left(d^2 x^2 (a+b \arctan (c x))^2+2 d e x^4 (a+b \arctan (c x))^2+e^2 x^6 (a+b \arctan (c x))^2\right) \, dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{i e^2 (a+b \arctan (c x))^2}{7 c^7}-\frac{2 b e^2 \log \left(\frac{2}{1+i c x}\right) (a+b \arctan (c x))}{7 c^7}+\frac{2 i d e (a+b \arctan (c x))^2}{5 c^5}+\frac{4 b d e \log \left(\frac{2}{1+i c x}\right) (a+b \arctan (c x))}{5 c^5}-\frac{b e^2 x^2 (a+b \arctan (c x))}{7 c^5}-\frac{i d^2 (a+b \arctan (c x))^2}{3 c^3}-\frac{2 b d^2 \log \left(\frac{2}{1+i c x}\right) (a+b \arctan (c x))}{3 c^3}+\frac{2 b d e x^2 (a+b \arctan (c x))}{5 c^3}+\frac{b e^2 x^4 (a+b \arctan (c x))}{14 c^3}+\frac{1}{3} d^2 x^3 (a+b \arctan (c x))^2-\frac{b d^2 x^2 (a+b \arctan (c x))}{3 c}+\frac{2}{5} d e x^5 (a+b \arctan (c x))^2-\frac{b d e x^4 (a+b \arctan (c x))}{5 c}+\frac{1}{7} e^2 x^7 (a+b \arctan (c x))^2-\frac{b e^2 x^6 (a+b \arctan (c x))}{21 c}-\frac{11 b^2 e^2 \arctan (c x)}{42 c^7}+\frac{3 b^2 d e \arctan (c x)}{5 c^5}-\frac{b^2 d^2 \arctan (c x)}{3 c^3}-\frac{i b^2 e^2 \text{PolyLog}\left(2,1-\frac{2}{i c x+1}\right)}{7 c^7}+\frac{11 b^2 e^2 x}{42 c^6}+\frac{2 i b^2 d e \text{PolyLog}\left(2,1-\frac{2}{i c x+1}\right)}{5 c^5}-\frac{3 b^2 d e x}{5 c^4}-\frac{5 b^2 e^2 x^3}{126 c^4}-\frac{i b^2 d^2 \text{PolyLog}\left(2,1-\frac{2}{i c x+1}\right)}{3 c^3}+\frac{b^2 d^2 x}{3 c^2}+\frac{b^2 d e x^3}{15 c^2}+\frac{b^2 e^2 x^5}{105 c^2}$$

input
```
Int[x^2*(d + e*x^2)^2*(a + b*ArcTan[c*x])^2,x]
```

output
```
(b^2*d^2*x)/(3*c^2) - (3*b^2*d*e*x)/(5*c^4) + (11*b^2*e^2*x)/(42*c^6) + (b
^2*d*e*x^3)/(15*c^2) - (5*b^2*e^2*x^3)/(126*c^4) + (b^2*e^2*x^5)/(105*c^2)
 - (b^2*d^2*ArcTan[c*x])/(3*c^3) + (3*b^2*d*e*ArcTan[c*x])/(5*c^5) - (11*b
^2*e^2*ArcTan[c*x])/(42*c^7) - (b*d^2*x^2*(a + b*ArcTan[c*x]))/(3*c) + (2*
b*d*e*x^2*(a + b*ArcTan[c*x]))/(5*c^3) - (b*e^2*x^2*(a + b*ArcTan[c*x]))/(
7*c^5) - (b*d*e*x^4*(a + b*ArcTan[c*x]))/(5*c) + (b*e^2*x^4*(a + b*ArcTan[
c*x]))/(14*c^3) - (b*e^2*x^6*(a + b*ArcTan[c*x]))/(21*c) - ((I/3)*d^2*(a +
 b*ArcTan[c*x])^2)/c^3 + (((2*I)/5)*d*e*(a + b*ArcTan[c*x])^2)/c^5 - ((I/7
)*e^2*(a + b*ArcTan[c*x])^2)/c^7 + (d^2*x^3*(a + b*ArcTan[c*x])^2)/3 + (2*
d*e*x^5*(a + b*ArcTan[c*x])^2)/5 + (e^2*x^7*(a + b*ArcTan[c*x])^2)/7 - (2*
b*d^2*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(3*c^3) + (4*b*d*e*(a + b*Ar
cTan[c*x])*Log[2/(1 + I*c*x)])/(5*c^5) - (2*b*e^2*(a + b*ArcTan[c*x])*Log[
2/(1 + I*c*x)])/(7*c^7) - ((I/3)*b^2*d^2*PolyLog[2, 1 - 2/(1 + I*c*x)])/c^
3 + (((2*I)/5)*b^2*d*e*PolyLog[2, 1 - 2/(1 + I*c*x)])/c^5 - ((I/7)*b^2*e^2
*PolyLog[2, 1 - 2/(1 + I*c*x)])/c^7
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

## Maple [A] (verified)

Time = 2.23 (sec) , antiderivative size = 634, normalized size of antiderivative = 1.09

| method | result |
|---|---|
| parts | $a^2\left(\frac{1}{7}e^2x^7+\frac{2}{5}de\,x^5+\frac{1}{3}d^2x^3\right)+\frac{b^2\left(\frac{\arctan(cx)^2c^3e^2x^7}{7}+\frac{2\arctan(cx)^2c^3de\,x^5}{5}+\frac{\arctan(cx)^2d^2c^3x^3}{3}-\frac{2\left(\frac{35\arctan\ldots}{}\right)}{}\right)}{}$ |
| derivativedivides | $\frac{\frac{a^2\left(\frac{1}{3}d^2c^7x^3+\frac{2}{5}dc^7e\,x^5+\frac{1}{7}e^2c^7x^7\right)}{c^4}+\frac{b^2\left(\frac{\arctan(cx)^2d^2c^7x^3}{3}+\frac{2\arctan(cx)^2dc^7e\,x^5}{5}+\frac{\arctan(cx)^2e^2c^7x^7}{7}-\frac{\arctan(cx)c^6d^2x^2}{3}\ldots\right)}{}}{}$ |
| default | $\frac{\frac{a^2\left(\frac{1}{3}d^2c^7x^3+\frac{2}{5}dc^7e\,x^5+\frac{1}{7}e^2c^7x^7\right)}{c^4}+\frac{b^2\left(\frac{\arctan(cx)^2d^2c^7x^3}{3}+\frac{2\arctan(cx)^2dc^7e\,x^5}{5}+\frac{\arctan(cx)^2e^2c^7x^7}{7}-\frac{\arctan(cx)c^6d^2x^2}{3}\ldots\right)}{}}{}$ |
| risch | Expression too large to display |

input
```
int(x^2*(e*x^2+d)^2*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*(1/7*e^2*x^7+2/5*d*e*x^5+1/3*d^2*x^3)+b^2/c^3*(1/7*arctan(c*x)^2*c^3*e
^2*x^7+2/5*arctan(c*x)^2*c^3*d*e*x^5+1/3*arctan(c*x)^2*d^2*c^3*x^3-2/105/c
^4*(35/2*arctan(c*x)*c^6*d^2*x^2+21/2*arctan(c*x)*e*c^6*d*x^4+5/2*arctan(c
*x)*e^2*c^6*x^6-21*arctan(c*x)*d*c^4*e*x^2-15/4*arctan(c*x)*e^2*c^4*x^4+15
/2*arctan(c*x)*e^2*c^2*x^2-35/2*arctan(c*x)*ln(c^2*x^2+1)*c^4*d^2+21*arcta
n(c*x)*ln(c^2*x^2+1)*c^2*d*e-15/2*arctan(c*x)*ln(c^2*x^2+1)*e^2-1/2*e^2*c^
5*x^5-7/2*d*c^5*e*x^3-35/2*c^5*x*d^2+25/12*e^2*c^3*x^3+63/2*d*c^3*e*x-55/4
*c*x*e^2-1/4*(-70*c^4*d^2+126*c^2*d*e-55*e^2)*arctan(c*x)-1/4*(-70*c^4*d^2
+84*c^2*d*e-30*e^2)*(-1/2*I*(ln(c*x-I)*ln(c^2*x^2+1)-dilog(-1/2*I*(I+c*x))
-ln(c*x-I)*ln(-1/2*I*(I+c*x))-1/2*ln(c*x-I)^2)+1/2*I*(ln(I+c*x)*ln(c^2*x^2
+1)-dilog(1/2*I*(c*x-I))-ln(I+c*x)*ln(1/2*I*(c*x-I))-1/2*ln(I+c*x)^2))))+2
*a*b/c^3*(1/7*arctan(c*x)*c^3*e^2*x^7+2/5*arctan(c*x)*c^3*d*e*x^5+1/3*arct
an(c*x)*d^2*c^3*x^3-1/105/c^4*(35/2*c^6*d^2*x^2+21/2*c^6*d*e*x^4+5/2*e^2*c
^6*x^6-21*d*c^4*e*x^2-15/4*e^2*c^4*x^4+15/2*e^2*c^2*x^2+1/2*(-35*c^4*d^2+4
2*c^2*d*e-15*e^2)*ln(c^2*x^2+1)))
```

## Fricas [F]

$$\int x^2 \left(d+ex^2\right)^2 (a+b\arctan(cx))^2 \, dx = \int \left(ex^2+d\right)^2 (b\arctan(cx)+a)^2 x^2 \, dx$$

input
```
integrate(x^2*(e*x^2+d)^2*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

output
```
integral(a^2*e^2*x^6 + 2*a^2*d*e*x^4 + a^2*d^2*x^2 + (b^2*e^2*x^6 + 2*b^2*
d*e*x^4 + b^2*d^2*x^2)*arctan(c*x)^2 + 2*(a*b*e^2*x^6 + 2*a*b*d*e*x^4 + a*
b*d^2*x^2)*arctan(c*x), x)
```

## Sympy [F]

$$\int x^2 \left(d+ex^2\right)^2 (a+b\arctan(cx))^2 \, dx = \int x^2 (a+b\operatorname{atan}(cx))^2 \left(d+ex^2\right)^2 \, dx$$

input
```
integrate(x**2*(e*x**2+d)**2*(a+b*atan(c*x))**2,x)
```

output
```
Integral(x**2*(a + b*atan(c*x))**2*(d + e*x**2)**2, x)
```

## Maxima [F]

$$\int x^2 (d+ex^2)^2 (a+b\arctan(cx))^2 \, dx = \int (ex^2+d)^2 (b\arctan(cx)+a)^2 x^2 \, dx$$

input
```
integrate(x^2*(e*x^2+d)^2*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

output
```
1/7*a^2*e^2*x^7 + 2/5*a^2*d*e*x^5 + 1/3*a^2*d^2*x^3 + 1/3*(2*x^3*arctan(c*
x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*a*b*d^2 + 1/5*(4*x^5*arctan(c*x)
- c*((c^2*x^4 - 2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*a*b*d*e + 1/42*(12*x
^7*arctan(c*x) - c*((2*c^4*x^6 - 3*c^2*x^4 + 6*x^2)/c^6 - 6*log(c^2*x^2 +
1)/c^8))*a*b*e^2 + 1/420*(15*b^2*e^2*x^7 + 42*b^2*d*e*x^5 + 35*b^2*d^2*x^3
)*arctan(c*x)^2 - 1/1680*(15*b^2*e^2*x^7 + 42*b^2*d*e*x^5 + 35*b^2*d^2*x^3
)*log(c^2*x^2 + 1)^2 + integrate(1/1680*(1260*(b^2*c^2*e^2*x^8 + (2*b^2*c^
2*d*e + b^2*e^2)*x^6 + b^2*d^2*x^2 + (b^2*c^2*d^2 + 2*b^2*d*e)*x^4)*arctan
(c*x)^2 + 105*(b^2*c^2*e^2*x^8 + (2*b^2*c^2*d*e + b^2*e^2)*x^6 + b^2*d^2*x
^2 + (b^2*c^2*d^2 + 2*b^2*d*e)*x^4)*log(c^2*x^2 + 1)^2 - 8*(15*b^2*c*e^2*x
^7 + 42*b^2*c*d*e*x^5 + 35*b^2*c*d^2*x^3)*arctan(c*x) + 4*(15*b^2*c^2*e^2*
x^8 + 42*b^2*c^2*d*e*x^6 + 35*b^2*c^2*d^2*x^4)*log(c^2*x^2 + 1))/(c^2*x^2
+ 1), x)
```

## Giac [F]

$$\int x^2 (d+ex^2)^2 (a+b\arctan(cx))^2 \, dx = \int (ex^2+d)^2 (b\arctan(cx)+a)^2 x^2 \, dx$$

input
```
integrate(x^2*(e*x^2+d)^2*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^2*(b*arctan(c*x) + a)^2*x^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int x^2 \left(d+ex^2\right)^2 (a+b\arctan(cx))^2\,dx=\int x^2\,(a+b\,\mathrm{atan}(c\,x))^2\,\left(e\,x^2+d\right)^2\,dx$$

input
```
int(x^2*(a + b*atan(c*x))^2*(d + e*x^2)^2,x)
```

output
```
int(x^2*(a + b*atan(c*x))^2*(d + e*x^2)^2, x)
```

## Reduce [F]

$$\int x^2 \left(d+ex^2\right)^2 (a+b\arctan(cx))^2\,dx$$
$$=\frac{252atan(cx)^2\,b^2c^7de\,x^5-252atan(cx)^2\,b^2c^3dex+420atan(cx)\,ab\,c^7d^2x^3+180atan(cx)\,ab\,c^7e^2x^7-126a}{}$$

input
```
int(x^2*(e*x^2+d)^2*(a+b*atan(c*x))^2,x)
```

output
```
(210*atan(c*x)**2*b**2*c**7*d**2*x**3 + 252*atan(c*x)**2*b**2*c**7*d*e*x**
5 + 90*atan(c*x)**2*b**2*c**7*e**2*x**7 + 210*atan(c*x)**2*b**2*c**5*d**2*
x - 252*atan(c*x)**2*b**2*c**3*d*e*x + 90*atan(c*x)**2*b**2*c*e**2*x + 420
*atan(c*x)*a*b*c**7*d**2*x**3 + 504*atan(c*x)*a*b*c**7*d*e*x**5 + 180*atan
(c*x)*a*b*c**7*e**2*x**7 - 210*atan(c*x)*b**2*c**6*d**2*x**2 - 126*atan(c*
x)*b**2*c**6*d*e*x**4 - 30*atan(c*x)*b**2*c**6*e**2*x**6 - 210*atan(c*x)*b
**2*c**4*d**2 + 252*atan(c*x)*b**2*c**4*d*e*x**2 + 45*atan(c*x)*b**2*c**4*
e**2*x**4 + 378*atan(c*x)*b**2*c**2*d*e - 90*atan(c*x)*b**2*c**2*e**2*x**2
 - 165*atan(c*x)*b**2*e**2 - 210*int(atan(c*x)**2,x)*b**2*c**5*d**2 + 252*
int(atan(c*x)**2,x)*b**2*c**3*d*e - 90*int(atan(c*x)**2,x)*b**2*c*e**2 + 2
10*log(c**2*x**2 + 1)*a*b*c**4*d**2 - 252*log(c**2*x**2 + 1)*a*b*c**2*d*e
+ 90*log(c**2*x**2 + 1)*a*b*e**2 + 210*a**2*c**7*d**2*x**3 + 252*a**2*c**7
*d*e*x**5 + 90*a**2*c**7*e**2*x**7 - 210*a*b*c**6*d**2*x**2 - 126*a*b*c**6
*d*e*x**4 - 30*a*b*c**6*e**2*x**6 + 252*a*b*c**4*d*e*x**2 + 45*a*b*c**4*e*
*2*x**4 - 90*a*b*c**2*e**2*x**2 + 210*b**2*c**5*d**2*x + 42*b**2*c**5*d*e*
x**3 + 6*b**2*c**5*e**2*x**5 - 378*b**2*c**3*d*e*x - 25*b**2*c**3*e**2*x**
3 + 165*b**2*c*e**2*x)/(630*c**7)
```

# 3.1254 $\int x(d+ex^2)^2 (a+b\arctan(cx))^2 \, dx$



## Optimal result

Integrand size = 21, antiderivative size = 380

$$\begin{aligned}\int x(d+ex^2)^2 (a+b\arctan(cx))^2 \, dx = &-\frac{abd^2x}{c}+\frac{abdex}{c^3}-\frac{abe^2x}{3c^5}+\frac{b^2dex^2}{6c^2}-\frac{4b^2e^2x^2}{45c^4} \\ &+\frac{b^2e^2x^4}{60c^2}-\frac{b^2d^2x\arctan(cx)}{c}+\frac{b^2dex\arctan(cx)}{c^3} \\ &-\frac{b^2e^2x\arctan(cx)}{3c^5}-\frac{bdex^3(a+b\arctan(cx))}{3c} \\ &+\frac{be^2x^3(a+b\arctan(cx))}{9c^3} \\ &-\frac{be^2x^5(a+b\arctan(cx))}{15c} \\ &+\frac{d^2(a+b\arctan(cx))^2}{2c^2} \\ &-\frac{de(a+b\arctan(cx))^2}{2c^4}+\frac{e^2(a+b\arctan(cx))^2}{6c^6} \\ &+\frac{1}{2}d^2x^2(a+b\arctan(cx))^2 \\ &+\frac{1}{2}dex^4(a+b\arctan(cx))^2 \\ &+\frac{1}{6}e^2x^6(a+b\arctan(cx))^2+\frac{b^2d^2\log(1+c^2x^2)}{2c^2} \\ &-\frac{2b^2de\log(1+c^2x^2)}{3c^4}+\frac{23b^2e^2\log(1+c^2x^2)}{90c^6}\end{aligned}$$

output
```
-a*b*d^2*x/c+a*b*d*e*x/c^3-1/3*a*b*e^2*x/c^5+1/6*b^2*d*e*x^2/c^2-4/45*b^2*
e^2*x^2/c^4+1/60*b^2*e^2*x^4/c^2-b^2*d^2*x*arctan(c*x)/c+b^2*d*e*x*arctan(
c*x)/c^3-1/3*b^2*e^2*x*arctan(c*x)/c^5-1/3*b*d*e*x^3*(a+b*arctan(c*x))/c+1
/9*b*e^2*x^3*(a+b*arctan(c*x))/c^3-1/15*b*e^2*x^5*(a+b*arctan(c*x))/c+1/2*
d^2*(a+b*arctan(c*x))^2/c^2-1/2*d*e*(a+b*arctan(c*x))^2/c^4+1/6*e^2*(a+b*a
rctan(c*x))^2/c^6+1/2*d^2*x^2*(a+b*arctan(c*x))^2+1/2*d*e*x^4*(a+b*arctan(
c*x))^2+1/6*e^2*x^6*(a+b*arctan(c*x))^2+1/2*b^2*d^2*ln(c^2*x^2+1)/c^2-2/3*
b^2*d*e*ln(c^2*x^2+1)/c^4+23/90*b^2*e^2*ln(c^2*x^2+1)/c^6
```

## Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 317, normalized size of antiderivative = 0.83

$$\int x(d+ex^2)^2 (a+b\arctan(cx))^2 \, dx$$
$$= \frac{cx(30a^2c^5x(3d^2+3dex^2+e^2x^4)+b^2cex(-16e+3c^2(10d+ex^2))-4ab(15e^2-5c^2e(9d+ex^2)+3c^4(15\ldots}{}$$

input
```
Integrate[x*(d + e*x^2)^2*(a + b*ArcTan[c*x])^2,x]
```

output
```
(c*x*(30*a^2*c^5*x*(3*d^2 + 3*d*e*x^2 + e^2*x^4) + b^2*c*e*x*(-16*e + 3*c^
2*(10*d + e*x^2)) - 4*a*b*(15*e^2 - 5*c^2*e*(9*d + e*x^2) + 3*c^4*(15*d^2
+ 5*d*e*x^2 + e^2*x^4))) + 4*b*(-(b*c*x*(15*e^2 - 5*c^2*e*(9*d + e*x^2) +
3*c^4*(15*d^2 + 5*d*e*x^2 + e^2*x^4))) + 15*a*(3*c^4*d^2 - 3*c^2*d*e + e^2
 + c^6*(3*d^2*x^2 + 3*d*e*x^4 + e^2*x^6)))*ArcTan[c*x] + 30*b^2*(3*c^4*d^2
 - 3*c^2*d*e + e^2 + c^6*(3*d^2*x^2 + 3*d*e*x^4 + e^2*x^6))*ArcTan[c*x]^2
+ 2*b^2*(45*c^4*d^2 - 60*c^2*d*e + 23*e^2)*Log[1 + c^2*x^2])/(180*c^6)
```

## Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(d+ex^2\right)^2(a+b\arctan(cx))^2\,dx$$

$$\Big\downarrow \text{5515}$$

$$\int \left(d^2x(a+b\arctan(cx))^2+2dex^3(a+b\arctan(cx))^2+e^2x^5(a+b\arctan(cx))^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^2(a+b\arctan(cx))^2}{6c^6}-\frac{de(a+b\arctan(cx))^2}{2c^4}+\frac{be^2x^3(a+b\arctan(cx))}{9c^3}+\frac{d^2(a+b\arctan(cx))^2}{2c^2}+\frac{1}{2}d^2x^2(a+b\arctan(cx))^2+\frac{1}{2}dex^4(a+b\arctan(cx))^2-\frac{bdex^3(a+b\arctan(cx))}{3c}+\frac{1}{6}e^2x^6(a+b\arctan(cx))^2-\frac{be^2x^5(a+b\arctan(cx))}{15c}-\frac{abe^2x}{3c^5}+\frac{abdex}{c^3}-\frac{abd^2x}{c}-\frac{b^2e^2x\arctan(cx)}{3c^5}+\frac{b^2dex\arctan(cx)}{c^3}-\frac{b^2d^2x\arctan(cx)}{c}-\frac{4b^2e^2x^2}{45c^4}+\frac{b^2d^2\log\left(c^2x^2+1\right)}{2c^2}+\frac{b^2dex^2}{6c^2}+\frac{b^2e^2x^4}{60c^2}+\frac{23b^2e^2\log\left(c^2x^2+1\right)}{90c^6}-\frac{2b^2de\log\left(c^2x^2+1\right)}{3c^4}$$

input
```
Int[x*(d + e*x^2)^2*(a + b*ArcTan[c*x])^2,x]
```

output
```
-((a*b*d^2*x)/c) + (a*b*d*e*x)/c^3 - (a*b*e^2*x)/(3*c^5) + (b^2*d*e*x^2)/(
6*c^2) - (4*b^2*e^2*x^2)/(45*c^4) + (b^2*e^2*x^4)/(60*c^2) - (b^2*d^2*x*Ar
cTan[c*x])/c + (b^2*d*e*x*ArcTan[c*x])/c^3 - (b^2*e^2*x*ArcTan[c*x])/(3*c^
5) - (b*d*e*x^3*(a + b*ArcTan[c*x]))/(3*c) + (b*e^2*x^3*(a + b*ArcTan[c*x]
))/(9*c^3) - (b*e^2*x^5*(a + b*ArcTan[c*x]))/(15*c) + (d^2*(a + b*ArcTan[c
*x])^2)/(2*c^2) - (d*e*(a + b*ArcTan[c*x])^2)/(2*c^4) + (e^2*(a + b*ArcTan
[c*x])^2)/(6*c^6) + (d^2*x^2*(a + b*ArcTan[c*x])^2)/2 + (d*e*x^4*(a + b*Ar
cTan[c*x])^2)/2 + (e^2*x^6*(a + b*ArcTan[c*x])^2)/6 + (b^2*d^2*Log[1 + c^2
*x^2])/(2*c^2) - (2*b^2*d*e*Log[1 + c^2*x^2])/(3*c^4) + (23*b^2*e^2*Log[1
+ c^2*x^2])/(90*c^6)
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

## Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 492, normalized size of antiderivative = 1.29

| method | result |
|---|---|
| parts | $\frac{a^2(e\,x^2+d)^3}{6e}+\frac{b^2\arctan(cx)^2e^2x^6}{6}+\frac{b^2\arctan(cx)^2ed\,x^4}{2}+\frac{b^2\arctan(cx)^2d^2x^2}{2}-\frac{b^2e^2\arctan(cx)x^5}{15c}-$ |
| derivativedivides | $\frac{a^2\left(c^2e\,x^2+c^2d\right)^3}{6c^4e}+\frac{b^2\arctan(cx)^2d^2c^2x^2}{2}+\frac{b^2c^2\arctan(cx)^2ed\,x^4}{2}+\frac{b^2c^2\arctan(cx)^2e^2x^6}{6}-b^2c\,d^2x\arctan(cx)-$ |
| default | $\frac{a^2\left(c^2e\,x^2+c^2d\right)^3}{6c^4e}+\frac{b^2\arctan(cx)^2d^2c^2x^2}{2}+\frac{b^2c^2\arctan(cx)^2ed\,x^4}{2}+\frac{b^2c^2\arctan(cx)^2e^2x^6}{6}-b^2c\,d^2x\arctan(cx)-$ |
| parallelrisch | $-180ab\,c^2de\arctan(cx)+3b^2c^4e^2x^4-16b^2c^2e^2x^2-30b^2c^2de+16e^2b^2+30b^2c^4de\,x^2+90c^6a^2de\,x^4+90x^2\arctan(cx)^2b^2$ |
| risch | $-\frac{b^2\left(e^2c^6x^6+3c^6de\,x^4+3c^6d^2x^2+3c^4d^2-3c^2de+e^2\right)\ln(icx+1)^2}{24c^6}-\frac{b^2e^2x^6\ln(-icx+1)^2}{24}-\frac{4b^2e^2x^2}{45c^4}+\frac{b^2e^2x^4}{60c^2}+$ |

input
```
int(x*(e*x^2+d)^2*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*a^2*(e*x^2+d)^3/e+1/6*b^2*arctan(c*x)^2*e^2*x^6+1/2*b^2*arctan(c*x)^2*
e*d*x^4+1/2*b^2*arctan(c*x)^2*d^2*x^2-1/15*b^2/c*e^2*arctan(c*x)*x^5-1/3*b
^2/c*e*arctan(c*x)*x^3*d-b^2*d^2*x*arctan(c*x)/c+1/9*b^2/c^3*e^2*arctan(c*
x)*x^3+b^2*d*e*x*arctan(c*x)/c^3-1/3*b^2*e^2*x*arctan(c*x)/c^5+1/2*b^2/c^2
*arctan(c*x)^2*d^2-1/2*b^2/c^4*e*arctan(c*x)^2*d+1/6*b^2/c^6*e^2*arctan(c*
x)^2+1/6*b^2*d*e*x^2/c^2+1/60*b^2*e^2*x^4/c^2-4/45*b^2*e^2*x^2/c^4+1/2*b^2
*d^2*ln(c^2*x^2+1)/c^2-2/3*b^2*d*e*ln(c^2*x^2+1)/c^4+23/90*b^2*e^2*ln(c^2*
x^2+1)/c^6+2*a*b/c^2*(1/6*arctan(c*x)*c^2*e^2*x^6+1/2*arctan(c*x)*c^2*e*d*
x^4+1/2*arctan(c*x)*d^2*c^2*x^2+1/6*arctan(c*x)*c^2/e*d^3-1/6/c^4/e*(3*c^5
*d^2*e*x+c^5*d*e^2*x^3+1/5*e^3*c^5*x^5-3*c^3*x*d*e^2-1/3*e^3*c^3*x^3+c*x*e
^3+(c^6*d^3-3*c^4*d^2*e+3*c^2*d*e^2-e^3)*arctan(c*x)))
```

## Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.09

$$\int x(d+ex^2)^2(a+b\arctan(cx))^2\,dx = \frac{30\,a^2c^6e^2x^6-12\,abc^5e^2x^5+3\,(30\,a^2c^6de+b^2c^4e^2)x^4-20\,(3\,abc^5de-abc^3e^2)x^3+2\,(45\,a^2c^6d^2+15\,b^2c^4}{}$$

input
```
integrate(x*(e*x^2+d)^2*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

output
```
1/180*(30*a^2*c^6*e^2*x^6 - 12*a*b*c^5*e^2*x^5 + 3*(30*a^2*c^6*d*e + b^2*c
^4*e^2)*x^4 - 20*(3*a*b*c^5*d*e - a*b*c^3*e^2)*x^3 + 2*(45*a^2*c^6*d^2 + 1
5*b^2*c^4*d*e - 8*b^2*c^2*e^2)*x^2 + 30*(b^2*c^6*e^2*x^6 + 3*b^2*c^6*d*e*x
^4 + 3*b^2*c^6*d^2*x^2 + 3*b^2*c^4*d^2 - 3*b^2*c^2*d*e + b^2*e^2)*arctan(c
*x)^2 - 60*(3*a*b*c^5*d^2 - 3*a*b*c^3*d*e + a*b*c*e^2)*x + 4*(15*a*b*c^6*e
^2*x^6 + 45*a*b*c^6*d*e*x^4 - 3*b^2*c^5*e^2*x^5 + 45*a*b*c^6*d^2*x^2 + 45*
a*b*c^4*d^2 - 45*a*b*c^2*d*e + 15*a*b*e^2 - 5*(3*b^2*c^5*d*e - b^2*c^3*e^2
)*x^3 - 15*(3*b^2*c^5*d^2 - 3*b^2*c^3*d*e + b^2*c*e^2)*x)*arctan(c*x) + 2*
(45*b^2*c^4*d^2 - 60*b^2*c^2*d*e + 23*b^2*e^2)*log(c^2*x^2 + 1))/c^6
```

## Sympy [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 575, normalized size of antiderivative = 1.51

$$\int x(d+ex^2)^2 (a+b\arctan(cx))^2 \, dx$$
$$= \begin{cases} \frac{a^2d^2x^2}{2} + \frac{a^2dex^4}{2} + \frac{a^2e^2x^6}{6} + abd^2x^2 \operatorname{atan}(cx) + abdex^4 \operatorname{atan}(cx) + \frac{abe^2x^6 \operatorname{atan}(cx)}{3} - \frac{abd^2x}{c} - \frac{abdex^3}{3c} - \frac{abe^2x^5}{15c} \\ a^2\left(\frac{d^2x^2}{2} + \frac{dex^4}{2} + \frac{e^2x^6}{6}\right) \end{cases}$$

input
```
integrate(x*(e*x**2+d)**2*(a+b*atan(c*x))**2,x)
```

output
```
Piecewise((a**2*d**2*x**2/2 + a**2*d*e*x**4/2 + a**2*e**2*x**6/6 + a*b*d**
2*x**2*atan(c*x) + a*b*d*e*x**4*atan(c*x) + a*b*e**2*x**6*atan(c*x)/3 - a*
b*d**2*x/c - a*b*d*e*x**3/(3*c) - a*b*e**2*x**5/(15*c) + a*b*d**2*atan(c*x
)/c**2 + a*b*d*e*x/c**3 + a*b*e**2*x**3/(9*c**3) - a*b*d*e*atan(c*x)/c**4
- a*b*e**2*x/(3*c**5) + a*b*e**2*atan(c*x)/(3*c**6) + b**2*d**2*x**2*atan(
c*x)**2/2 + b**2*d*e*x**4*atan(c*x)**2/2 + b**2*e**2*x**6*atan(c*x)**2/6 -
 b**2*d**2*x*atan(c*x)/c - b**2*d*e*x**3*atan(c*x)/(3*c) - b**2*e**2*x**5*
atan(c*x)/(15*c) + b**2*d**2*log(x**2 + c**(-2))/(2*c**2) + b**2*d**2*atan
(c*x)**2/(2*c**2) + b**2*d*e*x**2/(6*c**2) + b**2*e**2*x**4/(60*c**2) + b*
*2*d*e*x*atan(c*x)/c**3 + b**2*e**2*x**3*atan(c*x)/(9*c**3) - 2*b**2*d*e*l
og(x**2 + c**(-2))/(3*c**4) - b**2*d*e*atan(c*x)**2/(2*c**4) - 4*b**2*e**2
*x**2/(45*c**4) - b**2*e**2*x*atan(c*x)/(3*c**5) + 23*b**2*e**2*log(x**2 +
 c**(-2))/(90*c**6) + b**2*e**2*atan(c*x)**2/(6*c**6), Ne(c, 0)), (a**2*(d
**2*x**2/2 + d*e*x**4/2 + e**2*x**6/6), True))
```

## Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 433, normalized size of antiderivative = 1.14

$$\int x(d+ex^2)^2(a+b\arctan(cx))^2\,dx$$
$$= \frac{1}{6}b^2e^2x^6\arctan(cx)^2 + \frac{1}{6}a^2e^2x^6 + \frac{1}{2}b^2dex^4\arctan(cx)^2 + \frac{1}{2}a^2dex^4 + \frac{1}{2}b^2d^2x^2\arctan(cx)^2 + \frac{1}{2}a^2d^2x^2 + \left(x^2\arctan(cx) - c\left(\frac{x}{c^2} - \frac{\arctan(cx)}{c^3}\right)\right)abd^2 - \frac{1}{2}\left(2c\left(\frac{x}{c^2} - \frac{\arctan(cx)}{c^3}\right)\arctan(cx) + \frac{\arctan(cx)^2 - \log(c^2x^2+1)}{c^2}\right)b^2d^2 + \frac{1}{3}\left(3x^4\arctan(cx) - c\left(\frac{c^2x^3-3x}{c^4} + \frac{3\arctan(cx)}{c^5}\right)\right)abde - \frac{1}{6}\left(2c\left(\frac{c^2x^3-3x}{c^4} + \frac{3\arctan(cx)}{c^5}\right)\arctan(cx) - \frac{c^2x^2 + 3\arctan(cx)^2 - 4\log(c^2x^2+1)}{c^4}\right)b^2de + \frac{1}{45}\left(15x^6\arctan(cx) - c\left(\frac{3c^4x^5 - 5c^2x^3 + 15x}{c^6} - \frac{15\arctan(cx)}{c^7}\right)\right)abe^2 - \frac{1}{180}\left(4c\left(\frac{3c^4x^5 - 5c^2x^3 + 15x}{c^6} - \frac{15\arctan(cx)}{c^7}\right)\arctan(cx) - \frac{3c^4x^4 - 16c^2x^2 - 30\arctan(cx)^2}{c^6}\right.$$

input
```
integrate(x*(e*x^2+d)^2*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

output
```
1/6*b^2*e^2*x^6*arctan(c*x)^2 + 1/6*a^2*e^2*x^6 + 1/2*b^2*d*e*x^4*arctan(c
*x)^2 + 1/2*a^2*d*e*x^4 + 1/2*b^2*d^2*x^2*arctan(c*x)^2 + 1/2*a^2*d^2*x^2
+ (x^2*arctan(c*x) - c*(x/c^2 - arctan(c*x)/c^3))*a*b*d^2 - 1/2*(2*c*(x/c^
2 - arctan(c*x)/c^3)*arctan(c*x) + (arctan(c*x)^2 - log(c^2*x^2 + 1))/c^2)
*b^2*d^2 + 1/3*(3*x^4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)
/c^5))*a*b*d*e - 1/6*(2*c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5)*arctan
(c*x) - (c^2*x^2 + 3*arctan(c*x)^2 - 4*log(c^2*x^2 + 1))/c^4)*b^2*d*e + 1/
45*(15*x^6*arctan(c*x) - c*((3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arctan
(c*x)/c^7))*a*b*e^2 - 1/180*(4*c*((3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*
arctan(c*x)/c^7)*arctan(c*x) - (3*c^4*x^4 - 16*c^2*x^2 - 30*arctan(c*x)^2
+ 46*log(c^2*x^2 + 1))/c^6)*b^2*e^2
```

## Giac [A] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 548, normalized size of antiderivative = 1.44

$$\int x(d+ex^2)^2 (a+b\arctan(cx))^2\,dx$$
$$= \frac{30\,b^2c^6e^2x^6\arctan{(cx)}^2 + 60\,abc^6e^2x^6\arctan{(cx)} + 30\,a^2c^6e^2x^6 + 90\,b^2c^6dex^4\arctan{(cx)}^2 + 180\,abc^6de}{}$$

input
```
integrate(x*(e*x^2+d)^2*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

output
```
1/180*(30*b^2*c^6*e^2*x^6*arctan(c*x)^2 + 60*a*b*c^6*e^2*x^6*arctan(c*x) +
 30*a^2*c^6*e^2*x^6 + 90*b^2*c^6*d*e*x^4*arctan(c*x)^2 + 180*a*b*c^6*d*e*x
^4*arctan(c*x) - 12*b^2*c^5*e^2*x^5*arctan(c*x) + 90*a^2*c^6*d*e*x^4 - 12*
a*b*c^5*e^2*x^5 + 90*b^2*c^6*d^2*x^2*arctan(c*x)^2 + 180*a*b*c^6*d^2*x^2*a
rctan(c*x) - 60*b^2*c^5*d*e*x^3*arctan(c*x) + 90*a^2*c^6*d^2*x^2 - 60*a*b*
c^5*d*e*x^3 + 3*b^2*c^4*e^2*x^4 - 180*b^2*c^5*d^2*x*arctan(c*x) + 20*b^2*c
^3*e^2*x^3*arctan(c*x) - 180*pi*a*b*c^4*d^2*sgn(c)*sgn(x) - 180*a*b*c^5*d^
2*x + 30*b^2*c^4*d*e*x^2 + 20*a*b*c^3*e^2*x^3 + 90*b^2*c^4*d^2*arctan(c*x)
^2 + 180*a*b*c^4*d^2*arctan(c*x) + 180*b^2*c^3*d*e*x*arctan(c*x) + 90*b^2*
c^4*d^2*log(c^2*x^2 + 1) + 180*pi*a*b*c^2*d*e*sgn(c)*sgn(x) + 180*a*b*c^3*
d*e*x - 16*b^2*c^2*e^2*x^2 - 90*b^2*c^2*d*e*arctan(c*x)^2 - 180*a*b*c^2*d*
e*arctan(c*x) - 60*b^2*c*e^2*x*arctan(c*x) - 120*b^2*c^2*d*e*log(c^2*x^2 +
 1) - 60*pi*a*b*e^2*sgn(c)*sgn(x) - 60*a*b*c*e^2*x + 30*b^2*e^2*arctan(c*x
)^2 + 60*a*b*e^2*arctan(c*x) + 46*b^2*e^2*log(c^2*x^2 + 1))/c^6
```

## Mupad [B] (verification not implemented)

Time = 6.00 (sec) , antiderivative size = 780, normalized size of antiderivative = 2.05

$$\int x(d+ex^2)^2 (a+b\arctan(cx))^2\,dx = \text{Too large to display}$$

input
```
int(x*(a + b*atan(c*x))^2*(d + e*x^2)^2,x)
```

output
```
(a^2*d^2*x^2)/2 + (a^2*e^2*x^6)/6 + (b^2*d^2*log(c^2*x^2 + 1))/(2*c^2) + (
23*b^2*e^2*log(c^2*x^2 + 1))/(90*c^6) + (b^2*e^2*x^4)/(60*c^2) - (4*b^2*e^
2*x^2)/(45*c^4) + (b^2*d^2*atan(c*x)^2)/(2*c^2) + (b^2*e^2*atan(c*x)^2)/(6
*c^6) + (b^2*d^2*x^2*atan(c*x)^2)/2 + (b^2*e^2*x^6*atan(c*x)^2)/6 + (a^2*d
*e*x^4)/2 - (b^2*e^2*x^5*atan(c*x))/(15*c) + (b^2*e^2*x^3*atan(c*x))/(9*c^
3) - (a*b*d^2*x)/c - (a*b*e^2*x)/(3*c^5) + a*b*d^2*x^2*atan(c*x) + (a*b*e^
2*x^6*atan(c*x))/3 - (2*b^2*d*e*log(c^2*x^2 + 1))/(3*c^4) - (a*b*e^2*x^5)/
(15*c) + (a*b*e^2*x^3)/(9*c^3) + (b^2*d*e*x^2)/(6*c^2) - (b^2*d*e*atan(c*x
)^2)/(2*c^4) - (b^2*d^2*x*atan(c*x))/c - (b^2*e^2*x*atan(c*x))/(3*c^5) + (
b^2*d*e*x^4*atan(c*x)^2)/2 + (a*b*d^2*atan((b*c*e^2*x)/(b*e^2 + 3*b*c^4*d^
2 - 3*b*c^2*d*e) + (3*b*c^5*d^2*x)/(b*e^2 + 3*b*c^4*d^2 - 3*b*c^2*d*e) - (
3*b*c^3*d*e*x)/(b*e^2 + 3*b*c^4*d^2 - 3*b*c^2*d*e)))/c^2 + (a*b*e^2*atan((
b*c*e^2*x)/(b*e^2 + 3*b*c^4*d^2 - 3*b*c^2*d*e) + (3*b*c^5*d^2*x)/(b*e^2 +
3*b*c^4*d^2 - 3*b*c^2*d*e) - (3*b*c^3*d*e*x)/(b*e^2 + 3*b*c^4*d^2 - 3*b*c^
2*d*e)))/(3*c^6) - (b^2*d*e*x^3*atan(c*x))/(3*c) + (a*b*d*e*x)/c^3 + a*b*d
*e*x^4*atan(c*x) - (a*b*d*e*x^3)/(3*c) + (b^2*d*e*x*atan(c*x))/c^3 - (a*b*
d*e*atan((b*c*e^2*x)/(b*e^2 + 3*b*c^4*d^2 - 3*b*c^2*d*e) + (3*b*c^5*d^2*x)
/(b*e^2 + 3*b*c^4*d^2 - 3*b*c^2*d*e) - (3*b*c^3*d*e*x)/(b*e^2 + 3*b*c^4*d^
2 - 3*b*c^2*d*e)))/c^4
```

## Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 507, normalized size of antiderivative = 1.33

$$\int x(d+ex^2)^2 (a+b\arctan(cx))^2\,dx$$
$$= \frac{30a^2c^6e^2x^6 + 30atan(cx)^2 b^2e^2 + 46\log(c^2x^2+1)\,b^2e^2 + 90atan(cx)^2 b^2c^6d^2x^2 + 30atan(cx)^2 b^2c^6e^2x^6 +}{}$$

input
```
int(x*(e*x^2+d)^2*(a+b*atan(c*x))^2,x)
```

output
```
(90*atan(c*x)**2*b**2*c**6*d**2*x**2 + 90*atan(c*x)**2*b**2*c**6*d*e*x**4
+ 30*atan(c*x)**2*b**2*c**6*e**2*x**6 + 90*atan(c*x)**2*b**2*c**4*d**2 - 9
0*atan(c*x)**2*b**2*c**2*d*e + 30*atan(c*x)**2*b**2*e**2 + 180*atan(c*x)*a
*b*c**6*d**2*x**2 + 180*atan(c*x)*a*b*c**6*d*e*x**4 + 60*atan(c*x)*a*b*c**
6*e**2*x**6 + 180*atan(c*x)*a*b*c**4*d**2 - 180*atan(c*x)*a*b*c**2*d*e + 6
0*atan(c*x)*a*b*e**2 - 180*atan(c*x)*b**2*c**5*d**2*x - 60*atan(c*x)*b**2*
c**5*d*e*x**3 - 12*atan(c*x)*b**2*c**5*e**2*x**5 + 180*atan(c*x)*b**2*c**3
*d*e*x + 20*atan(c*x)*b**2*c**3*e**2*x**3 - 60*atan(c*x)*b**2*c*e**2*x + 9
0*log(c**2*x**2 + 1)*b**2*c**4*d**2 - 120*log(c**2*x**2 + 1)*b**2*c**2*d*e
 + 46*log(c**2*x**2 + 1)*b**2*e**2 + 90*a**2*c**6*d**2*x**2 + 90*a**2*c**6
*d*e*x**4 + 30*a**2*c**6*e**2*x**6 - 180*a*b*c**5*d**2*x - 60*a*b*c**5*d*e
*x**3 - 12*a*b*c**5*e**2*x**5 + 180*a*b*c**3*d*e*x + 20*a*b*c**3*e**2*x**3
 - 60*a*b*c*e**2*x + 30*b**2*c**4*d*e*x**2 + 3*b**2*c**4*e**2*x**4 - 16*b*
*2*c**2*e**2*x**2)/(180*c**6)
```

# 3.1255 $\int (d+ex^2)^2 (a+b\arctan(cx))^2\,dx$

## Optimal result

Integrand size = 20, antiderivative size = 442

$$\int (d+ex^2)^2 (a+b\arctan(cx))^2\,dx = \frac{2b^2dex}{3c^2} - \frac{3b^2e^2x}{10c^4} + \frac{b^2e^2x^3}{30c^2} - \frac{2b^2de\arctan(cx)}{3c^3} + \frac{3b^2e^2\arctan(cx)}{10c^5} - \frac{2bdex^2(a+b\arctan(cx))}{3c} + \frac{be^2x^2(a+b\arctan(cx))}{5c^3} - \frac{be^2x^4(a+b\arctan(cx))}{10c} + \frac{id^2(a+b\arctan(cx))^2}{c} - \frac{2ide(a+b\arctan(cx))^2}{3c^3} + \frac{ie^2(a+b\arctan(cx))^2}{5c^5} + d^2x(a+b\arctan(cx))^2 + \frac{2}{3}dex^3(a+b\arctan(cx))^2 + \frac{1}{5}e^2x^5(a+b\arctan(cx))^2 + \frac{2bd^2(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c} - \frac{4bde(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{3c^3} + \frac{2be^2(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{5c^5} + \frac{ib^2d^2\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c} - \frac{2ib^2de\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{3c^3} + \frac{ib^2e^2\,\mathrm{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{5c^5}$$

output
```
2/3*b^2*d*e*x/c^2-3/10*b^2*e^2*x/c^4+1/30*b^2*e^2*x^3/c^2-2/3*b^2*d*e*arct
an(c*x)/c^3+3/10*b^2*e^2*arctan(c*x)/c^5-2/3*b*d*e*x^2*(a+b*arctan(c*x))/c
+1/5*b*e^2*x^2*(a+b*arctan(c*x))/c^3-1/10*b*e^2*x^4*(a+b*arctan(c*x))/c+1/
5*I*e^2*(a+b*arctan(c*x))^2/c^5+I*b^2*d^2*polylog(2,1-2/(1+I*c*x))/c-2/3*I
*d*e*(a+b*arctan(c*x))^2/c^3+d^2*x*(a+b*arctan(c*x))^2+2/3*d*e*x^3*(a+b*ar
ctan(c*x))^2+1/5*e^2*x^5*(a+b*arctan(c*x))^2+2*b*d^2*(a+b*arctan(c*x))*ln(
2/(1+I*c*x))/c-4/3*b*d*e*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^3+2/5*b*e^2*(
a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^5+I*d^2*(a+b*arctan(c*x))^2/c-2/3*I*b^2
*d*e*polylog(2,1-2/(1+I*c*x))/c^3+1/5*I*b^2*e^2*polylog(2,1-2/(1+I*c*x))/c
^5
```

## Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 391, normalized size of antiderivative = 0.88

$$\int \left(d+ex^2\right)^2 (a+b\arctan(cx))^2\,dx$$
$$= \frac{9abe^2+30a^2c^5d^2x+20b^2c^3dex-9b^2ce^2x-20abc^4dex^2+6abc^2e^2x^2+20a^2c^5dex^3+b^2c^3e^2x^3-3abc^4e^2}{}$$

input
```
Integrate[(d + e*x^2)^2*(a + b*ArcTan[c*x])^2,x]
```

output
```
(9*a*b*e^2 + 30*a^2*c^5*d^2*x + 20*b^2*c^3*d*e*x - 9*b^2*c*e^2*x - 20*a*b*
c^4*d*e*x^2 + 6*a*b*c^2*e^2*x^2 + 20*a^2*c^5*d*e*x^3 + b^2*c^3*e^2*x^3 - 3
*a*b*c^4*e^2*x^4 + 6*a^2*c^5*e^2*x^5 + 2*b^2*((-15*I)*c^4*d^2 + (10*I)*c^2
*d*e - (3*I)*e^2 + c^5*(15*d^2*x + 10*d*e*x^3 + 3*e^2*x^5))*ArcTan[c*x]^2
+ b*ArcTan[c*x]*(4*a*c^5*x*(15*d^2 + 10*d*e*x^2 + 3*e^2*x^4) - b*e*(1 + c^
2*x^2)*(-9*e + c^2*(20*d + 3*e*x^2)) + 4*b*(15*c^4*d^2 - 10*c^2*d*e + 3*e^
2)*Log[1 + E^((2*I)*ArcTan[c*x])]) - 30*a*b*c^4*d^2*Log[1 + c^2*x^2] + 20*
a*b*c^2*d*e*Log[1 + c^2*x^2] - 6*a*b*e^2*Log[1 + c^2*x^2] - (2*I)*b^2*(15*
c^4*d^2 - 10*c^2*d*e + 3*e^2)*PolyLog[2, -E^((2*I)*ArcTan[c*x])])/(30*c^5)
```

## Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 442, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5449, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(d+ex^2\right)^2 (a+b\arctan(cx))^2 dx$$

$$\Big\downarrow \text{5449}$$

$$\int \left(d^2(a+b\arctan(cx))^2+2dex^2(a+b\arctan(cx))^2+e^2x^4(a+b\arctan(cx))^2\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{ie^2(a+b\arctan(cx))^2}{5c^5}+\frac{2be^2\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{5c^5}-\frac{2ide(a+b\arctan(cx))^2}{3c^3}-\frac{4bde\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{3c^3}+\frac{be^2x^2(a+b\arctan(cx))}{5c^3}+d^2x(a+b\arctan(cx))^2+\frac{id^2(a+b\arctan(cx))^2}{c}+\frac{2bd^2\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}+\frac{2}{3}dex^3(a+b\arctan(cx))^2-\frac{2bdex^2(a+b\arctan(cx))}{3c}+\frac{1}{5}e^2x^5(a+b\arctan(cx))^2-\frac{be^2x^4(a+b\arctan(cx))}{10c}+\frac{3b^2e^2\arctan(cx)}{10c^5}-\frac{2b^2de\arctan(cx)}{3c^3}+\frac{ib^2e^2\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{5c^5}-\frac{3b^2e^2x}{10c^4}-\frac{2ib^2de\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{3c^3}+\frac{2b^2dex}{3c^2}+\frac{b^2e^2x^3}{30c^2}+\frac{ib^2d^2\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{c}$$

input
```
Int[(d + e*x^2)^2*(a + b*ArcTan[c*x])^2,x]
```

output
```
(2*b^2*d*e*x)/(3*c^2) - (3*b^2*e^2*x)/(10*c^4) + (b^2*e^2*x^3)/(30*c^2) -
(2*b^2*d*e*ArcTan[c*x])/(3*c^3) + (3*b^2*e^2*ArcTan[c*x])/(10*c^5) - (2*b*
d*e*x^2*(a + b*ArcTan[c*x]))/(3*c) + (b*e^2*x^2*(a + b*ArcTan[c*x]))/(5*c^
3) - (b*e^2*x^4*(a + b*ArcTan[c*x]))/(10*c) + (I*d^2*(a + b*ArcTan[c*x])^2
)/c - (((2*I)/3)*d*e*(a + b*ArcTan[c*x])^2)/c^3 + ((I/5)*e^2*(a + b*ArcTan
[c*x])^2)/c^5 + d^2*x*(a + b*ArcTan[c*x])^2 + (2*d*e*x^3*(a + b*ArcTan[c*x
])^2)/3 + (e^2*x^5*(a + b*ArcTan[c*x])^2)/5 + (2*b*d^2*(a + b*ArcTan[c*x])
*Log[2/(1 + I*c*x)])/c - (4*b*d*e*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/
(3*c^3) + (2*b*e^2*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(5*c^5) + (I*b^
2*d^2*PolyLog[2, 1 - 2/(1 + I*c*x)])/c - (((2*I)/3)*b^2*d*e*PolyLog[2, 1 -
 2/(1 + I*c*x)])/c^3 + ((I/5)*b^2*e^2*PolyLog[2, 1 - 2/(1 + I*c*x)])/c^5
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5449
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (d + e*x^2)^q, x], x
] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[q] && IGtQ[p, 0]
```

## Maple [A] (verified)

Time = 1.73 (sec) , antiderivative size = 493, normalized size of antiderivative = 1.12

| method | result |
|---|---|
| parts | $a^2\left(\frac{1}{5}e^2x^5+\frac{2}{3}de\,x^3+d^2x\right)+\frac{b^2\left(\frac{\arctan(cx)^2 c\,e^2x^5}{5}+\frac{2\arctan(cx)^2 cde\,x^3}{3}+\arctan(cx)^2 cx\,d^2-\frac{2\left(5\arctan(cx)d\,c\right.}{}\right.}{}$ |
| derivativedivides | $\frac{\frac{a^2\left(c^5x\,d^2+\frac{2}{3}d\,c^5e\,x^3+\frac{1}{5}e^2c^5x^5\right)}{c^4}+\frac{b^2\left(\arctan(cx)^2c^5x\,d^2+\frac{2\arctan(cx)^2d\,c^5e\,x^3}{3}+\frac{\arctan(cx)^2e^2c^5x^5}{5}-\frac{2\arctan(cx)d\,c^4e\,x^2}{3}-\right.}{}}{}$ |
| default | $\frac{\frac{a^2\left(c^5x\,d^2+\frac{2}{3}d\,c^5e\,x^3+\frac{1}{5}e^2c^5x^5\right)}{c^4}+\frac{b^2\left(\arctan(cx)^2c^5x\,d^2+\frac{2\arctan(cx)^2d\,c^5e\,x^3}{3}+\frac{\arctan(cx)^2e^2c^5x^5}{5}-\frac{2\arctan(cx)d\,c^4e\,x^2}{3}-\right.}{}}{}$ |
| risch | Expression too large to display |

input
```
int((e*x^2+d)^2*(a+b*arctan(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*(1/5*e^2*x^5+2/3*d*e*x^3+d^2*x)+b^2/c*(1/5*arctan(c*x)^2*c*e^2*x^5+2/3
*arctan(c*x)^2*c*d*e*x^3+arctan(c*x)^2*c*x*d^2-2/15/c^4*(5*arctan(c*x)*d*c
^4*e*x^2+3/4*arctan(c*x)*e^2*c^4*x^4-3/2*arctan(c*x)*e^2*c^2*x^2+15/2*arct
an(c*x)*ln(c^2*x^2+1)*c^4*d^2-5*arctan(c*x)*ln(c^2*x^2+1)*c^2*d*e+3/2*arct
an(c*x)*ln(c^2*x^2+1)*e^2-1/4*e*(20*c^3*x*d+e*c^3*x^3-9*e*c*x+(-20*c^2*d+9
*e)*arctan(c*x))-1/4*(30*c^4*d^2-20*c^2*d*e+6*e^2)*(-1/2*I*(ln(c*x-I)*ln(c
^2*x^2+1)-dilog(-1/2*I*(I+c*x))-ln(c*x-I)*ln(-1/2*I*(I+c*x))-1/2*ln(c*x-I)
^2)+1/2*I*(ln(I+c*x)*ln(c^2*x^2+1)-dilog(1/2*I*(c*x-I))-ln(I+c*x)*ln(1/2*I
*(c*x-I))-1/2*ln(I+c*x)^2))))+2*a*b/c*(1/5*arctan(c*x)*c*e^2*x^5+2/3*arcta
n(c*x)*c*d*e*x^3+arctan(c*x)*c*x*d^2-1/15/c^4*(5*d*c^4*e*x^2+3/4*e^2*c^4*x
^4-3/2*e^2*c^2*x^2+1/2*(15*c^4*d^2-10*c^2*d*e+3*e^2)*ln(c^2*x^2+1)))
```

## Fricas [F]

$$\int \left(d+ex^2\right)^2 (a+b\arctan(cx))^2\,dx=\int \left(ex^2+d\right)^2(b\arctan\left(cx\right)+a)^2\,dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x))^2,x, algorithm="fricas")
```

output
```
integral(a^2*e^2*x^4 + 2*a^2*d*e*x^2 + a^2*d^2 + (b^2*e^2*x^4 + 2*b^2*d*e*
x^2 + b^2*d^2)*arctan(c*x)^2 + 2*(a*b*e^2*x^4 + 2*a*b*d*e*x^2 + a*b*d^2)*a
rctan(c*x), x)
```

## Sympy [F]

$$\int \left(d+ex^2\right)^2 (a+b\arctan(cx))^2\,dx=\int \left(a+b\operatorname{atan}\left(cx\right)\right)^2\left(d+ex^2\right)^2\,dx$$

input
```
integrate((e*x**2+d)**2*(a+b*atan(c*x))**2,x)
```

output
```
Integral((a + b*atan(c*x))**2*(d + e*x**2)**2, x)
```

## Maxima [F]

$$\int (d+ex^2)^2 (a+b\arctan(cx))^2 \, dx = \int (ex^2+d)^2 (b\arctan(cx)+a)^2 \, dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x))^2,x, algorithm="maxima")
```

output
```
1/5*a^2*e^2*x^5 + 2/3*a^2*d*e*x^3 + 180*b^2*c^2*e^2*integrate(1/240*x^6*ar
ctan(c*x)^2/(c^2*x^2 + 1), x) + 15*b^2*c^2*e^2*integrate(1/240*x^6*log(c^2
*x^2 + 1)^2/(c^2*x^2 + 1), x) + 12*b^2*c^2*e^2*integrate(1/240*x^6*log(c^2
*x^2 + 1)/(c^2*x^2 + 1), x) + 360*b^2*c^2*d*e*integrate(1/240*x^4*arctan(c
*x)^2/(c^2*x^2 + 1), x) + 30*b^2*c^2*d*e*integrate(1/240*x^4*log(c^2*x^2 +
 1)^2/(c^2*x^2 + 1), x) + 40*b^2*c^2*d*e*integrate(1/240*x^4*log(c^2*x^2 +
 1)/(c^2*x^2 + 1), x) + 180*b^2*c^2*d^2*integrate(1/240*x^2*arctan(c*x)^2/
(c^2*x^2 + 1), x) + 15*b^2*c^2*d^2*integrate(1/240*x^2*log(c^2*x^2 + 1)^2/
(c^2*x^2 + 1), x) + 60*b^2*c^2*d^2*integrate(1/240*x^2*log(c^2*x^2 + 1)/(c
^2*x^2 + 1), x) + 1/4*b^2*d^2*arctan(c*x)^3/c - 24*b^2*c*e^2*integrate(1/2
40*x^5*arctan(c*x)/(c^2*x^2 + 1), x) - 80*b^2*c*d*e*integrate(1/240*x^3*ar
ctan(c*x)/(c^2*x^2 + 1), x) - 120*b^2*c*d^2*integrate(1/240*x*arctan(c*x)/
(c^2*x^2 + 1), x) + 2/3*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)
/c^4))*a*b*d*e + 1/10*(4*x^5*arctan(c*x) - c*((c^2*x^4 - 2*x^2)/c^4 + 2*lo
g(c^2*x^2 + 1)/c^6))*a*b*e^2 + a^2*d^2*x + 180*b^2*e^2*integrate(1/240*x^4
*arctan(c*x)^2/(c^2*x^2 + 1), x) + 15*b^2*e^2*integrate(1/240*x^4*log(c^2*
x^2 + 1)^2/(c^2*x^2 + 1), x) + 360*b^2*d*e*integrate(1/240*x^2*arctan(c*x)
^2/(c^2*x^2 + 1), x) + 30*b^2*d*e*integrate(1/240*x^2*log(c^2*x^2 + 1)^2/(
c^2*x^2 + 1), x) + 15*b^2*d^2*integrate(1/240*log(c^2*x^2 + 1)^2/(c^2*x^2
+ 1), x) + (2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*a*b*d^2/c + 1/60*(3*b...
```

## Giac [F]

$$\int (d+ex^2)^2 (a+b\arctan(cx))^2 \, dx = \int (ex^2+d)^2 (b\arctan(cx)+a)^2 \, dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x))^2,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^2*(b*arctan(c*x) + a)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \left(d+ex^2\right)^2 (a+b\arctan(cx))^2\,dx=\int (a+b\,\mathrm{atan}(c\,x))^2\,\left(e\,x^2+d\right)^2\,dx$$

input
```
int((a + b*atan(c*x))^2*(d + e*x^2)^2,x)
```

output
```
int((a + b*atan(c*x))^2*(d + e*x^2)^2, x)
```

## Reduce [F]

$$\int \left(d+ex^2\right)^2 (a+b\arctan(cx))^2\,dx$$
$$=\frac{-20atan(cx)\,b^2c^4de\,x^2+20\log(c^2x^2+1)\,ab\,c^2de-20ab\,c^4de\,x^2-60\left(\int \frac{atan(cx)x}{c^2x^2+1}dx\right)b^2c^6d^2-12\left(\int \frac{atan(}{c^2x^2}\right.}{}$$

input
```
int((e*x^2+d)^2*(a+b*atan(c*x))^2,x)
```

output
```
(30*atan(c*x)**2*b**2*c**5*d**2*x + 20*atan(c*x)**2*b**2*c**5*d*e*x**3 + 6
*atan(c*x)**2*b**2*c**5*e**2*x**5 + 60*atan(c*x)*a*b*c**5*d**2*x + 40*atan
(c*x)*a*b*c**5*d*e*x**3 + 12*atan(c*x)*a*b*c**5*e**2*x**5 - 20*atan(c*x)*b
**2*c**4*d*e*x**2 - 3*atan(c*x)*b**2*c**4*e**2*x**4 - 20*atan(c*x)*b**2*c*
*2*d*e + 6*atan(c*x)*b**2*c**2*e**2*x**2 + 9*atan(c*x)*b**2*e**2 - 60*int(
(atan(c*x)*x)/(c**2*x**2 + 1),x)*b**2*c**6*d**2 + 40*int((atan(c*x)*x)/(c*
*2*x**2 + 1),x)*b**2*c**4*d*e - 12*int((atan(c*x)*x)/(c**2*x**2 + 1),x)*b*
*2*c**2*e**2 - 30*log(c**2*x**2 + 1)*a*b*c**4*d**2 + 20*log(c**2*x**2 + 1)
*a*b*c**2*d*e - 6*log(c**2*x**2 + 1)*a*b*e**2 + 30*a**2*c**5*d**2*x + 20*a
**2*c**5*d*e*x**3 + 6*a**2*c**5*e**2*x**5 - 20*a*b*c**4*d*e*x**2 - 3*a*b*c
**4*e**2*x**4 + 6*a*b*c**2*e**2*x**2 + 20*b**2*c**3*d*e*x + b**2*c**3*e**2
*x**3 - 9*b**2*c*e**2*x)/(30*c**5)
```

# 3.1256 $\int \frac{(d+ex^2)^2(a+b\arctan(cx))^2}{x}\,dx$

## Optimal result

Integrand size = 23, antiderivative size = 355

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))^2}{x}\,dx = -\frac{2abdex}{c}+\frac{abe^2x}{2c^3}+\frac{b^2e^2x^2}{12c^2}-\frac{2b^2dex\arctan(cx)}{c}+\frac{b^2e^2x\arctan(cx)}{2c^3}-\frac{be^2x^3(a+b\arctan(cx))}{6c}+\frac{de(a+b\arctan(cx))^2}{c^2}-\frac{e^2(a+b\arctan(cx))^2}{4c^4}+dex^2(a+b\arctan(cx))^2+\frac{1}{4}e^2x^4(a+b\arctan(cx))^2+2d^2(a+b\arctan(cx))^2\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right)+\frac{b^2de\log(1+c^2x^2)}{c^2}-\frac{b^2e^2\log(1+c^2x^2)}{3c^4}-ibd^2(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)+ibd^2(a+b\arctan(cx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)-\frac{1}{2}b^2d^2\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)+\frac{1}{2}b^2d^2\operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)$$

output
```
-2*a*b*d*e*x/c+1/2*a*b*e^2*x/c^3+1/12*b^2*e^2*x^2/c^2-2*b^2*d*e*x*arctan(c
*x)/c+1/2*b^2*e^2*x*arctan(c*x)/c^3-1/6*b*e^2*x^3*(a+b*arctan(c*x))/c+d*e*
(a+b*arctan(c*x))^2/c^2-1/4*e^2*(a+b*arctan(c*x))^2/c^4+d*e*x^2*(a+b*arcta
n(c*x))^2+1/4*e^2*x^4*(a+b*arctan(c*x))^2-2*d^2*(a+b*arctan(c*x))^2*arctan
h(-1+2/(1+I*c*x))+b^2*d*e*ln(c^2*x^2+1)/c^2-1/3*b^2*e^2*ln(c^2*x^2+1)/c^4-
I*b*d^2*(a+b*arctan(c*x))*polylog(2,1-2/(1+I*c*x))+I*b*d^2*(a+b*arctan(c*x
))*polylog(2,-1+2/(1+I*c*x))-1/2*b^2*d^2*polylog(3,1-2/(1+I*c*x))+1/2*b^2*
d^2*polylog(3,-1+2/(1+I*c*x))
```

## Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.10

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))^2}{x}\,dx$$
$$= a^2dex^2 + \frac{1}{4}a^2e^2x^4 + \frac{2abde(-cx+(1+c^2x^2)\arctan(cx))}{c^2} + \frac{abe^2(3cx-c^3x^3+3(-1+c^4x^4)\arctan(cx))}{6c^4} + a^2d^2\log(x) + \frac{b^2e^2(1+c^2x^2+(6cx-2c^3x^3)\arctan(cx)+3(-1+c^4x^4)\arctan(cx)^2-4\log(1+c^2x^2))}{12c^4} + \frac{b^2de(-2cx\arctan(cx)+(1+c^2x^2)\arctan(cx)^2+\log(1+c^2x^2))}{c^2} + iabd^2(\text{PolyLog}(2,-icx)-\text{PolyLog}(2,icx)) + b^2d^2\left(-\frac{i\pi^3}{24}+\frac{2}{3}i\arctan(cx)^3 + \arctan(cx)^2\log\left(1-e^{-2i\arctan(cx)}\right) - \arctan(cx)^2\log\left(1+e^{2i\arctan(cx)}\right) + i\arctan(cx)\,\text{PolyLog}\left(2,e^{-2i\arctan(cx)}\right) + i\arctan(cx)\,\text{PolyLog}\left(2,-e^{2i\arctan(cx)}\right) + \frac{1}{2}\text{PolyLog}\left(3,e^{-2i\arctan(cx)}\right) - \frac{1}{2}\text{PolyLog}\left(3,-e^{2i\arctan(cx)}\right)\right)$$

input
```
Integrate[((d + e*x^2)^2*(a + b*ArcTan[c*x])^2)/x,x]
```

output
```
a^2*d*e*x^2 + (a^2*e^2*x^4)/4 + (2*a*b*d*e*(-(c*x) + (1 + c^2*x^2)*ArcTan[
c*x]))/c^2 + (a*b*e^2*(3*c*x - c^3*x^3 + 3*(-1 + c^4*x^4)*ArcTan[c*x]))/(6
*c^4) + a^2*d^2*Log[x] + (b^2*e^2*(1 + c^2*x^2 + (6*c*x - 2*c^3*x^3)*ArcTa
n[c*x] + 3*(-1 + c^4*x^4)*ArcTan[c*x]^2 - 4*Log[1 + c^2*x^2]))/(12*c^4) +
(b^2*d*e*(-2*c*x*ArcTan[c*x] + (1 + c^2*x^2)*ArcTan[c*x]^2 + Log[1 + c^2*x
^2]))/c^2 + I*a*b*d^2*(PolyLog[2, (-I)*c*x] - PolyLog[2, I*c*x]) + b^2*d^2
*((-1/24*I)*Pi^3 + ((2*I)/3)*ArcTan[c*x]^3 + ArcTan[c*x]^2*Log[1 - E^((-2*
I)*ArcTan[c*x])] - ArcTan[c*x]^2*Log[1 + E^((2*I)*ArcTan[c*x])] + I*ArcTan
[c*x]*PolyLog[2, E^((-2*I)*ArcTan[c*x])] + I*ArcTan[c*x]*PolyLog[2, -E^((2
*I)*ArcTan[c*x])] + PolyLog[3, E^((-2*I)*ArcTan[c*x])]/2 - PolyLog[3, -E^(
(2*I)*ArcTan[c*x])]/2)
```

## Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))^2}{x} \, dx$$

$$\Big\downarrow \text{5515}$$

$$\int \left(\frac{d^2(a+b\arctan(cx))^2}{x}+2dex(a+b\arctan(cx))^2+e^2x^3(a+b\arctan(cx))^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$2d^2\text{arctanh}\left(1-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2-\frac{e^2(a+b\arctan(cx))^2}{4c^4}+\frac{de(a+b\arctan(cx))^2}{c^2}-ibd^2\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))+ibd^2\,\text{PolyLog}\left(2,\frac{2}{icx+1}-1\right)(a+b\arctan(cx))+dex^2(a+b\arctan(cx))^2+\frac{1}{4}e^2x^4(a+b\arctan(cx))^2-\frac{be^2x^3(a+b\arctan(cx))}{6c}+\frac{abe^2x}{2c^3}-\frac{2abdex}{c}+\frac{b^2e^2x\arctan(cx)}{2c^3}-\frac{2b^2dex\arctan(cx)}{c}+\frac{b^2de\log(c^2x^2+1)}{c^2}+\frac{b^2e^2x^2}{12c^2}-\frac{b^2e^2\log(c^2x^2+1)}{3c^4}-\frac{1}{2}b^2d^2\,\text{PolyLog}\left(3,1-\frac{2}{icx+1}\right)+\frac{1}{2}b^2d^2\,\text{PolyLog}\left(3,\frac{2}{icx+1}-1\right)$$

input
```
Int[((d + e*x^2)^2*(a + b*ArcTan[c*x])^2)/x,x]
```

output
```
(-2*a*b*d*e*x)/c + (a*b*e^2*x)/(2*c^3) + (b^2*e^2*x^2)/(12*c^2) - (2*b^2*d
*e*x*ArcTan[c*x])/c + (b^2*e^2*x*ArcTan[c*x])/(2*c^3) - (b*e^2*x^3*(a + b*
ArcTan[c*x]))/(6*c) + (d*e*(a + b*ArcTan[c*x])^2)/c^2 - (e^2*(a + b*ArcTan
[c*x])^2)/(4*c^4) + d*e*x^2*(a + b*ArcTan[c*x])^2 + (e^2*x^4*(a + b*ArcTan
[c*x])^2)/4 + 2*d^2*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)] + (b^
2*d*e*Log[1 + c^2*x^2])/c^2 - (b^2*e^2*Log[1 + c^2*x^2])/(3*c^4) - I*b*d^2
*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)] + I*b*d^2*(a + b*ArcTan
[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)] - (b^2*d^2*PolyLog[3, 1 - 2/(1 + I*c
*x)])/2 + (b^2*d^2*PolyLog[3, -1 + 2/(1 + I*c*x)])/2
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 29.38 (sec) , antiderivative size = 1524, normalized size of antiderivative = 4.29

| method | result | size |
|---|---|---|
| derivativedivides | Expression too large to display | 1524 |
| default | Expression too large to display | 1524 |
| parts | Expression too large to display | 1526 |

input
```
int((e*x^2+d)^2*(a+b*arctan(c*x))^2/x,x,method=_RETURNVERBOSE)
```

output
```
a^2*d*e*x^2-2*b^2*d*e*x*arctan(c*x)/c+1/12*b^2*e^2*x^2/c^2+1/2*b^2*e^2*x*a
rctan(c*x)/c^3-1/4*b^2/c^4*e^2*arctan(c*x)^2-1/6*b^2/c*e^2*arctan(c*x)*x^3
+a^2*d^2*ln(c*x)+b^2/c^2*e*arctan(c*x)^2*d+1/12*b^2/c^4*e^2+2*b^2*polylog(
3,-(1+I*c*x)/(c^2*x^2+1)^(1/2))*d^2+2*b^2*polylog(3,(1+I*c*x)/(c^2*x^2+1)^
(1/2))*d^2-1/2*b^2*d^2*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))+1/2*I*b^2*d^2*P
i*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arctan
(c*x)^2+2*I*b^2/c^2*arctan(c*x)*d*e+1/4*b^2*arctan(c*x)^2*e^2*x^4-1/2*I*b^
2*d^2*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*a
rctan(c*x)^2+1/2*I*b^2*d^2*Pi*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^
2/(c^2*x^2+1)+1))^3*arctan(c*x)^2+1/4*a^2*e^2*x^4-1/2*I*b^2*d^2*Pi*csgn(I/
((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)
^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+1/2*I*b^2*d^2*Pi*csgn(I*((1+I*c*x)^2/(c
^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)
/((1+I*c*x)^2/(c^2*x^2+1)+1))*arctan(c*x)^2-1/2*I*b^2*d^2*Pi*csgn(I*((1+I*
c*x)^2/(c^2*x^2+1)-1))*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^
2*x^2+1)+1))^2*arctan(c*x)^2+b^2*arctan(c*x)^2*d*e*x^2-1/2*I*b^2*d^2*Pi*cs
gn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(((1+I*c
*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+I*b^2*d^
2*arctan(c*x)*polylog(2,-(1+I*c*x)^2/(c^2*x^2+1))-2*b^2/c^2*d*e*ln((1+I*c*
x)^2/(c^2*x^2+1)+1)-2*I*b^2*polylog(2,(1+I*c*x)/(c^2*x^2+1)^(1/2))*d^2*...
```

## Fricas [F]

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))^2}{x}\,dx = \int \frac{(ex^2+d)^2(b\arctan(cx)+a)^2}{x}\,dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x))^2/x,x, algorithm="fricas")
```

output
```
integral((a^2*e^2*x^4 + 2*a^2*d*e*x^2 + a^2*d^2 + (b^2*e^2*x^4 + 2*b^2*d*e
*x^2 + b^2*d^2)*arctan(c*x)^2 + 2*(a*b*e^2*x^4 + 2*a*b*d*e*x^2 + a*b*d^2)*
arctan(c*x))/x, x)
```

## Sympy [F]

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))^2}{x} dx = \int \frac{(a+b\,\mathrm{atan}\,(cx))^2 (d+ex^2)^2}{x} dx$$

input
```
integrate((e*x**2+d)**2*(a+b*atan(c*x))**2/x,x)
```

output
```
Integral((a + b*atan(c*x))**2*(d + e*x**2)**2/x, x)
```

## Maxima [F]

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))^2}{x} dx = \int \frac{(ex^2+d)^2(b\arctan(cx)+a)^2}{x} dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x))^2/x,x, algorithm="maxima")
```

output
```
1/4*a^2*e^2*x^4 + 12*b^2*c^2*e^2*integrate(1/16*x^6*arctan(c*x)^2/(c^2*x^3
 + x), x) + b^2*c^2*e^2*integrate(1/16*x^6*log(c^2*x^2 + 1)^2/(c^2*x^3 + x
), x) + 32*a*b*c^2*e^2*integrate(1/16*x^6*arctan(c*x)/(c^2*x^3 + x), x) +
b^2*c^2*e^2*integrate(1/16*x^6*log(c^2*x^2 + 1)/(c^2*x^3 + x), x) + 24*b^2
*c^2*d*e*integrate(1/16*x^4*arctan(c*x)^2/(c^2*x^3 + x), x) + 2*b^2*c^2*d*
e*integrate(1/16*x^4*log(c^2*x^2 + 1)^2/(c^2*x^3 + x), x) + 64*a*b*c^2*d*e
*integrate(1/16*x^4*arctan(c*x)/(c^2*x^3 + x), x) + 4*b^2*c^2*d*e*integrat
e(1/16*x^4*log(c^2*x^2 + 1)/(c^2*x^3 + x), x) + 12*b^2*c^2*d^2*integrate(1
/16*x^2*arctan(c*x)^2/(c^2*x^3 + x), x) + 32*a*b*c^2*d^2*integrate(1/16*x^
2*arctan(c*x)/(c^2*x^3 + x), x) + 1/96*b^2*d^2*log(c^2*x^2 + 1)^3 + a^2*d*
e*x^2 - 2*b^2*c*e^2*integrate(1/16*x^5*arctan(c*x)/(c^2*x^3 + x), x) - 8*b
^2*c*d*e*integrate(1/16*x^3*arctan(c*x)/(c^2*x^3 + x), x) + 12*b^2*e^2*int
egrate(1/16*x^4*arctan(c*x)^2/(c^2*x^3 + x), x) + b^2*e^2*integrate(1/16*x
^4*log(c^2*x^2 + 1)^2/(c^2*x^3 + x), x) + 32*a*b*e^2*integrate(1/16*x^4*ar
ctan(c*x)/(c^2*x^3 + x), x) + 24*b^2*d*e*integrate(1/16*x^2*arctan(c*x)^2/
(c^2*x^3 + x), x) + 64*a*b*d*e*integrate(1/16*x^2*arctan(c*x)/(c^2*x^3 + x
), x) + 12*b^2*d^2*integrate(1/16*arctan(c*x)^2/(c^2*x^3 + x), x) + b^2*d^
2*integrate(1/16*log(c^2*x^2 + 1)^2/(c^2*x^3 + x), x) + 32*a*b*d^2*integra
te(1/16*arctan(c*x)/(c^2*x^3 + x), x) + 1/48*b^2*d*e*log(c^2*x^2 + 1)^3/c^
2 + a^2*d^2*log(x) + 1/16*(b^2*e^2*x^4 + 4*b^2*d*e*x^2)*arctan(c*x)^2 -...
```

## Giac [F]

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))^2}{x} dx = \int \frac{(ex^2+d)^2(b\arctan(cx)+a)^2}{x} dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x))^2/x,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^2*(b*arctan(c*x) + a)^2/x, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))^2}{x} dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^2 (e\,x^2+d)^2}{x} dx$$

input
```
int(((a + b*atan(c*x))^2*(d + e*x^2)^2)/x,x)
```

output
```
int(((a + b*atan(c*x))^2*(d + e*x^2)^2)/x, x)
```

## Reduce [F]

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))^2}{x} dx$$
$$= \frac{12atan(cx)^2 b^2c^4de\,x^2 + 3atan(cx)^2 b^2c^4e^2x^4 + 12atan(cx)^2 b^2c^2de - 3atan(cx)^2 b^2e^2 + 24atan(cx)\,ab\,c^4}{}$$

input
```
int((e*x^2+d)^2*(a+b*atan(c*x))^2/x,x)
```

output
```
(12*atan(c*x)**2*b**2*c**4*d*e*x**2 + 3*atan(c*x)**2*b**2*c**4*e**2*x**4 +
 12*atan(c*x)**2*b**2*c**2*d*e - 3*atan(c*x)**2*b**2*e**2 + 24*atan(c*x)*a
*b*c**4*d*e*x**2 + 6*atan(c*x)*a*b*c**4*e**2*x**4 + 24*atan(c*x)*a*b*c**2*
d*e - 6*atan(c*x)*a*b*e**2 - 24*atan(c*x)*b**2*c**3*d*e*x - 2*atan(c*x)*b*
*2*c**3*e**2*x**3 + 6*atan(c*x)*b**2*c*e**2*x + 24*int(atan(c*x)/x,x)*a*b*
c**4*d**2 + 12*int(atan(c*x)**2/x,x)*b**2*c**4*d**2 + 12*log(c**2*x**2 + 1
)*b**2*c**2*d*e - 4*log(c**2*x**2 + 1)*b**2*e**2 + 12*log(x)*a**2*c**4*d**
2 + 12*a**2*c**4*d*e*x**2 + 3*a**2*c**4*e**2*x**4 - 24*a*b*c**3*d*e*x - 2*
a*b*c**3*e**2*x**3 + 6*a*b*c*e**2*x + b**2*c**2*e**2*x**2)/(12*c**4)
```

# 3.1257 $\int \frac{(d+ex^2)^2(a+b\arctan(cx))^2}{x^2}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 343

$$\begin{aligned}\int \frac{(d+ex^2)^2(a+b\arctan(cx))^2}{x^2}\,dx &= \frac{b^2e^2x}{3c^2}-\frac{b^2e^2\arctan(cx)}{3c^3}-\frac{be^2x^2(a+b\arctan(cx))}{3c}\\
&\quad-icd^2(a+b\arctan(cx))^2\\
&\quad+\frac{2ide(a+b\arctan(cx))^2}{c}\\
&\quad-\frac{ie^2(a+b\arctan(cx))^2}{3c^3}-\frac{d^2(a+b\arctan(cx))^2}{x}\\
&\quad+2dex(a+b\arctan(cx))^2\\
&\quad+\frac{1}{3}e^2x^3(a+b\arctan(cx))^2\\
&\quad+\frac{4bde(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{c}\\
&\quad-\frac{2be^2(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{3c^3}\\
&\quad+2bcd^2(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right)\\
&\quad-ib^2cd^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)\\
&\quad+\frac{2ib^2de\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{c}\\
&\quad-\frac{ib^2e^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{3c^3}\end{aligned}$$

output
```
1/3*b^2*e^2*x/c^2-1/3*b^2*e^2*arctan(c*x)/c^3-1/3*b*e^2*x^2*(a+b*arctan(c*
x))/c-I*c*d^2*(a+b*arctan(c*x))^2+2*I*d*e*(a+b*arctan(c*x))^2/c-1/3*I*e^2*
(a+b*arctan(c*x))^2/c^3-d^2*(a+b*arctan(c*x))^2/x+2*d*e*x*(a+b*arctan(c*x)
)^2+1/3*e^2*x^3*(a+b*arctan(c*x))^2+4*b*d*e*(a+b*arctan(c*x))*ln(2/(1+I*c*
x))/c-2/3*b*e^2*(a+b*arctan(c*x))*ln(2/(1+I*c*x))/c^3+2*b*c*d^2*(a+b*arcta
n(c*x))*ln(2-2/(1-I*c*x))-I*b^2*c*d^2*polylog(2,-1+2/(1-I*c*x))+2*I*b^2*d*
e*polylog(2,1-2/(1+I*c*x))/c-1/3*I*b^2*e^2*polylog(2,1-2/(1+I*c*x))/c^3
```

## Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.02

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))^2}{x^2}\,dx$$
$$= \frac{1}{3}\left(-\frac{3a^2d^2}{x} + 6a^2dex + a^2e^2x^3 + \frac{6abde(2cx\arctan(cx) - \log(1+c^2x^2))}{c} + \frac{abe^2(-c^2x^2 + 2c^3x^3\arctan(cx) + \log(1+c^2x^2))}{c^3} - \frac{3abd^2(2\arctan(cx) + cx(-2\log(cx) + \log(1+c^2x^2)))}{x} + \frac{6b^2de\left(\arctan(cx)\left((-i+cx)\arctan(cx) + 2\log\left(1+e^{2i\arctan(cx)}\right)\right) - i\,\text{PolyLog}\left(2,-e^{2i\arctan(cx)}\right)\right)}{c} + \frac{b^2e^2\left(cx + (i+c^3x^3)\arctan(cx)^2 - \arctan(cx)\left(1+c^2x^2+2\log\left(1+e^{2i\arctan(cx)}\right)\right) + i\,\text{PolyLog}\left(2,-e\right.\right.}{c^3} + 3b^2cd^2\left(\arctan(cx)\left(\left(-i-\frac{1}{cx}\right)\arctan(cx) + 2\log\left(1-e^{2i\arctan(cx)}\right)\right) - i\,\text{PolyLog}\left(2,e^{2i\arctan(cx)}\right)\right)\right)$$

input
```
Integrate[((d + e*x^2)^2*(a + b*ArcTan[c*x])^2)/x^2,x]
```

output
```
((-3*a^2*d^2)/x + 6*a^2*d*e*x + a^2*e^2*x^3 + (6*a*b*d*e*(2*c*x*ArcTan[c*x
] - Log[1 + c^2*x^2]))/c + (a*b*e^2*(-(c^2*x^2) + 2*c^3*x^3*ArcTan[c*x] +
Log[1 + c^2*x^2]))/c^3 - (3*a*b*d^2*(2*ArcTan[c*x] + c*x*(-2*Log[c*x] + Lo
g[1 + c^2*x^2])))/x + (6*b^2*d*e*(ArcTan[c*x]*((-I + c*x)*ArcTan[c*x] + 2*
Log[1 + E^((2*I)*ArcTan[c*x])]) - I*PolyLog[2, -E^((2*I)*ArcTan[c*x])]))/c
 + (b^2*e^2*(c*x + (I + c^3*x^3)*ArcTan[c*x]^2 - ArcTan[c*x]*(1 + c^2*x^2
+ 2*Log[1 + E^((2*I)*ArcTan[c*x])]) + I*PolyLog[2, -E^((2*I)*ArcTan[c*x])]
))/c^3 + 3*b^2*c*d^2*(ArcTan[c*x]*((-I - 1/(c*x))*ArcTan[c*x] + 2*Log[1 -
E^((2*I)*ArcTan[c*x])]) - I*PolyLog[2, E^((2*I)*ArcTan[c*x])]))/3
```

## Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))^2}{x^2} \, dx$$

$$\Big\downarrow \text{5515}$$

$$\int \left(\frac{d^2(a+b\arctan(cx))^2}{x^2}+2de(a+b\arctan(cx))^2+e^2x^2(a+b\arctan(cx))^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ie^2(a+b\arctan(cx))^2}{3c^3}-\frac{2be^2\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{3c^3}-icd^2(a+b\arctan(cx))^2-\frac{d^2(a+b\arctan(cx))^2}{x}+2bcd^2\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))+2dex(a+b\arctan(cx))^2+\frac{2ide(a+b\arctan(cx))^2}{c}+\frac{4bde\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}+\frac{1}{3}e^2x^3(a+b\arctan(cx))^2-\frac{be^2x^2(a+b\arctan(cx))}{3c}-\frac{b^2e^2\arctan(cx)}{3c^3}-\frac{ib^2e^2\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{3c^3}+\frac{b^2e^2x}{3c^2}-ib^2cd^2\operatorname{PolyLog}\left(2,\frac{2}{1-icx}-1\right)+\frac{2ib^2de\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{c}$$

input
```
Int[((d + e*x^2)^2*(a + b*ArcTan[c*x])^2)/x^2,x]
```

output
```
(b^2*e^2*x)/(3*c^2) - (b^2*e^2*ArcTan[c*x])/(3*c^3) - (b*e^2*x^2*(a + b*Ar
cTan[c*x]))/(3*c) - I*c*d^2*(a + b*ArcTan[c*x])^2 + ((2*I)*d*e*(a + b*ArcT
an[c*x])^2)/c - ((I/3)*e^2*(a + b*ArcTan[c*x])^2)/c^3 - (d^2*(a + b*ArcTan
[c*x])^2)/x + 2*d*e*x*(a + b*ArcTan[c*x])^2 + (e^2*x^3*(a + b*ArcTan[c*x])
^2)/3 + (4*b*d*e*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/c - (2*b*e^2*(a +
 b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(3*c^3) + 2*b*c*d^2*(a + b*ArcTan[c*x]
)*Log[2 - 2/(1 - I*c*x)] - I*b^2*c*d^2*PolyLog[2, -1 + 2/(1 - I*c*x)] + ((
2*I)*b^2*d*e*PolyLog[2, 1 - 2/(1 + I*c*x)])/c - ((I/3)*b^2*e^2*PolyLog[2,
1 - 2/(1 + I*c*x)])/c^3
```

**Defintions of rubi rules used**

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

## Maple [A] (verified)

Time = 1.67 (sec) , antiderivative size = 524, normalized size of antiderivative = 1.53

| method | result |
|---|---|
| derivativedivides | $c\left(\frac{a^2\left(2d\,c^3ex+\frac{e^2c^3x^3}{3}-\frac{c^3d^2}{x}\right)}{c^4}+\frac{b^2\left(2\arctan(cx)^2c^3xde+\frac{\arctan(cx)^2e^2c^3x^3}{3}-\frac{\arctan(cx)^2c^3d^2}{x}-\frac{\arctan(cx)e^2c^2x^2}{3}-\right.}{}\right.$ |
| default | $c\left(\frac{a^2\left(2d\,c^3ex+\frac{e^2c^3x^3}{3}-\frac{c^3d^2}{x}\right)}{c^4}+\frac{b^2\left(2\arctan(cx)^2c^3xde+\frac{\arctan(cx)^2e^2c^3x^3}{3}-\frac{\arctan(cx)^2c^3d^2}{x}-\frac{\arctan(cx)e^2c^2x^2}{3}-\right.}{}\right.$ |
| parts | $a^2\left(\frac{e^2x^3}{3}+2dxe-\frac{d^2}{x}\right)+b^2c\left(\frac{\arctan(cx)^2e^2x^3}{3c}+\frac{2\arctan(cx)^2dex}{c}-\frac{\arctan(cx)^2d^2}{cx}-\frac{2\left(\frac{\arctan(cx)e}{2}\right.}{}\right.$ |

input
```
int((e*x^2+d)^2*(a+b*arctan(c*x))^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
c*(a^2/c^4*(2*d*c^3*e*x+1/3*e^2*c^3*x^3-c^3*d^2/x)+b^2/c^4*(2*arctan(c*x)^
2*c^3*x*d*e+1/3*arctan(c*x)^2*e^2*c^3*x^3-arctan(c*x)^2*c^3*d^2/x-1/3*arct
an(c*x)*e^2*c^2*x^2-arctan(c*x)*ln(c^2*x^2+1)*c^4*d^2-2*arctan(c*x)*ln(c^2
*x^2+1)*c^2*d*e+1/3*arctan(c*x)*ln(c^2*x^2+1)*e^2+2*arctan(c*x)*c^4*d^2*ln
(c*x)+1/3*e^2*(c*x-arctan(c*x))+1/3*(3*c^4*d^2+6*c^2*d*e-e^2)*(-1/2*I*(ln(
c*x-I)*ln(c^2*x^2+1)-dilog(-1/2*I*(I+c*x))-ln(c*x-I)*ln(-1/2*I*(I+c*x))-1/
2*ln(c*x-I)^2)+1/2*I*(ln(I+c*x)*ln(c^2*x^2+1)-dilog(1/2*I*(c*x-I))-ln(I+c*
x)*ln(1/2*I*(c*x-I))-1/2*ln(I+c*x)^2))-2*c^4*d^2*(-1/2*I*ln(c*x)*ln(1+I*c*
x)+1/2*I*ln(c*x)*ln(1-I*c*x)-1/2*I*dilog(1+I*c*x)+1/2*I*dilog(1-I*c*x)))+2
*a*b/c^4*(2*arctan(c*x)*d*c^3*e*x+1/3*arctan(c*x)*e^2*c^3*x^3-arctan(c*x)*
c^3*d^2/x-1/6*e^2*c^2*x^2-1/6*(3*c^4*d^2+6*c^2*d*e-e^2)*ln(c^2*x^2+1)+c^4*
d^2*ln(c*x)))
```

## Fricas [F]

$$\int \frac{(d+ex^2)^2\,(a+b\arctan(cx))^2}{x^2}\,dx = \int \frac{(ex^2+d)^2(b\arctan(cx)+a)^2}{x^2}\,dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x))^2/x^2,x, algorithm="fricas")
```

output
```
integral((a^2*e^2*x^4 + 2*a^2*d*e*x^2 + a^2*d^2 + (b^2*e^2*x^4 + 2*b^2*d*e
*x^2 + b^2*d^2)*arctan(c*x)^2 + 2*(a*b*e^2*x^4 + 2*a*b*d*e*x^2 + a*b*d^2)*
arctan(c*x))/x^2, x)
```

## Sympy [F]

$$\int \frac{(d+ex^2)^2\,(a+b\arctan(cx))^2}{x^2}\,dx = \int \frac{(a+b\operatorname{atan}(cx))^2\,(d+ex^2)^2}{x^2}\,dx$$

input
```
integrate((e*x**2+d)**2*(a+b*atan(c*x))**2/x**2,x)
```

output
```
Integral((a + b*atan(c*x))**2*(d + e*x**2)**2/x**2, x)
```

## Maxima [F]

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))^2}{x^2} \, dx = \int \frac{(ex^2+d)^2 (b\arctan(cx)+a)^2}{x^2} \, dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x))^2/x^2,x, algorithm="maxima")
```

output
```
1/3*a^2*e^2*x^3 - (c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*a*b*
d^2 + 1/3*(2*x^3*arctan(c*x) - c*(x^2/c^2 - log(c^2*x^2 + 1)/c^4))*a*b*e^2
 + 2*a^2*d*e*x + 2*(2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*a*b*d*e/c - a^2*
d^2/x + 1/48*(4*(b^2*e^2*x^4 + 6*b^2*d*e*x^2 - 3*b^2*d^2)*arctan(c*x)^2 -
(b^2*e^2*x^4 + 6*b^2*d*e*x^2 - 3*b^2*d^2)*log(c^2*x^2 + 1)^2 + 12*(b^2*c*d
^2*arctan(c*x)^3 + 144*b^2*c^2*e^2*integrate(1/48*x^6*arctan(c*x)^2/(c^2*x
^4 + x^2), x) + 12*b^2*c^2*e^2*integrate(1/48*x^6*log(c^2*x^2 + 1)^2/(c^2*
x^4 + x^2), x) + 16*b^2*c^2*e^2*integrate(1/48*x^6*log(c^2*x^2 + 1)/(c^2*x
^4 + x^2), x) + 288*b^2*c^2*d*e*integrate(1/48*x^4*arctan(c*x)^2/(c^2*x^4
+ x^2), x) + 24*b^2*c^2*d*e*integrate(1/48*x^4*log(c^2*x^2 + 1)^2/(c^2*x^4
 + x^2), x) + 96*b^2*c^2*d*e*integrate(1/48*x^4*log(c^2*x^2 + 1)/(c^2*x^4
+ x^2), x) + 12*b^2*c^2*d^2*integrate(1/48*x^2*log(c^2*x^2 + 1)^2/(c^2*x^4
 + x^2), x) - 48*b^2*c^2*d^2*integrate(1/48*x^2*log(c^2*x^2 + 1)/(c^2*x^4
+ x^2), x) + 2*b^2*d*e*arctan(c*x)^3/c - 32*b^2*c*e^2*integrate(1/48*x^5*a
rctan(c*x)/(c^2*x^4 + x^2), x) - 192*b^2*c*d*e*integrate(1/48*x^3*arctan(c
*x)/(c^2*x^4 + x^2), x) + 96*b^2*c*d^2*integrate(1/48*x*arctan(c*x)/(c^2*x
^4 + x^2), x) + 144*b^2*e^2*integrate(1/48*x^4*arctan(c*x)^2/(c^2*x^4 + x^
2), x) + 12*b^2*e^2*integrate(1/48*x^4*log(c^2*x^2 + 1)^2/(c^2*x^4 + x^2),
 x) + 24*b^2*d*e*integrate(1/48*x^2*log(c^2*x^2 + 1)^2/(c^2*x^4 + x^2), x)
 + 144*b^2*d^2*integrate(1/48*arctan(c*x)^2/(c^2*x^4 + x^2), x) + 12*b^...
```

## Giac [F]

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))^2}{x^2} \, dx = \int \frac{(ex^2+d)^2 (b\arctan(cx)+a)^2}{x^2} \, dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x))^2/x^2,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^2*(b*arctan(c*x) + a)^2/x^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))^2}{x^2}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^2\,(e\,x^2+d)^2}{x^2}\,dx$$

input
```
int(((a + b*atan(c*x))^2*(d + e*x^2)^2)/x^2,x)
```

output
```
int(((a + b*atan(c*x))^2*(d + e*x^2)^2)/x^2, x)
```

## Reduce [F]

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))^2}{x^2}\,dx$$
$$= \frac{-3atan(cx)^2\,b^2c^3d^2 + 6atan(cx)^2\,b^2c^3de\,x^2 + atan(cx)^2\,b^2c^3e^2x^4 - 6atan(cx)\,ab\,c^3d^2 + 12atan(cx)\,ab\,c^3\ldots}{}$$

input
```
int((e*x^2+d)^2*(a+b*atan(c*x))^2/x^2,x)
```

output
```
( - 3*atan(c*x)**2*b**2*c**3*d**2 + 6*atan(c*x)**2*b**2*c**3*d*e*x**2 + at
an(c*x)**2*b**2*c**3*e**2*x**4 - 6*atan(c*x)*a*b*c**3*d**2 + 12*atan(c*x)*
a*b*c**3*d*e*x**2 + 2*atan(c*x)*a*b*c**3*e**2*x**4 - atan(c*x)*b**2*c**2*e
**2*x**3 - atan(c*x)*b**2*e**2*x + 6*int(atan(c*x)/(c**2*x**3 + x),x)*b**2
*c**4*d**2*x - 12*int((atan(c*x)*x)/(c**2*x**2 + 1),x)*b**2*c**4*d*e*x + 2
*int((atan(c*x)*x)/(c**2*x**2 + 1),x)*b**2*c**2*e**2*x - 3*log(c**2*x**2 +
 1)*a*b*c**4*d**2*x - 6*log(c**2*x**2 + 1)*a*b*c**2*d*e*x + log(c**2*x**2
+ 1)*a*b*e**2*x + 6*log(x)*a*b*c**4*d**2*x - 3*a**2*c**3*d**2 + 6*a**2*c**
3*d*e*x**2 + a**2*c**3*e**2*x**4 - a*b*c**2*e**2*x**3 + b**2*c*e**2*x**2)/
(3*c**3*x)
```

# 3.1258 $\int \frac{(d+ex^2)^2(a+b\arctan(cx))^2}{x^3} dx$

## Optimal result

Integrand size = 23, antiderivative size = 320

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))^2}{x^3}\,dx = -\frac{abe^2x}{c} - \frac{b^2e^2x\arctan(cx)}{c} - \frac{bcd^2(a+b\arctan(cx))}{x} - \frac{1}{2}c^2d^2(a+b\arctan(cx))^2 + \frac{e^2(a+b\arctan(cx))^2}{2c^2} - \frac{d^2(a+b\arctan(cx))^2}{2x^2} + \frac{1}{2}e^2x^2(a+b\arctan(cx))^2 + 4de(a+b\arctan(cx))^2\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right) + b^2c^2d^2\log(x) - \frac{1}{2}b^2c^2d^2\log\left(1+c^2x^2\right) + \frac{b^2e^2\log\left(1+c^2x^2\right)}{2c^2} - 2ibde(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right) + 2ibde(a+b\arctan(cx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx}\right) - b^2de\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right) + b^2de\operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)$$

output

```
-a*b*e^2*x/c-b^2*e^2*x*arctan(c*x)/c-b*c*d^2*(a+b*arctan(c*x))/x-1/2*c^2*d
^2*(a+b*arctan(c*x))^2+1/2*e^2*(a+b*arctan(c*x))^2/c^2-1/2*d^2*(a+b*arctan
(c*x))^2/x^2+1/2*e^2*x^2*(a+b*arctan(c*x))^2-4*d*e*(a+b*arctan(c*x))^2*arc
tanh(-1+2/(1+I*c*x))+b^2*c^2*d^2*ln(x)-1/2*b^2*c^2*d^2*ln(c^2*x^2+1)+1/2*b
^2*e^2*ln(c^2*x^2+1)/c^2-2*I*b*d*e*(a+b*arctan(c*x))*polylog(2,1-2/(1+I*c*
x))+2*I*b*d*e*(a+b*arctan(c*x))*polylog(2,-1+2/(1+I*c*x))-b^2*d*e*polylog(
3,1-2/(1+I*c*x))+b^2*d*e*polylog(3,-1+2/(1+I*c*x))
```

## Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.15

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))^2}{x^3}\,dx$$
$$= \frac{1}{2}\left(-\frac{a^2d^2}{x^2}+a^2e^2x^2+\frac{2abe^2(-cx+(1+c^2x^2)\arctan(cx))}{c^2} - \frac{2abd^2(\arctan(cx)+cx(1+cx\arctan(cx)))}{x^2}+4a^2de\log(x) - \frac{b^2d^2\left(2cx\arctan(cx)+(1+c^2x^2)\arctan(cx)^2-2c^2x^2\log\left(\frac{cx}{\sqrt{1+c^2x^2}}\right)\right)}{x^2} + \frac{b^2e^2(-2cx\arctan(cx)+(1+c^2x^2)\arctan(cx)^2+\log(1+c^2x^2))}{c^2} + 4iabde(\text{PolyLog}(2,-icx)-\text{PolyLog}(2,icx)) + \frac{1}{6}b^2de\left(-i\pi^3+16i\arctan(cx)^3+24\arctan(cx)^2\log\left(1-e^{-2i\arctan(cx)}\right) - 24\arctan(cx)^2\log\left(1+e^{2i\arctan(cx)}\right)+24i\arctan(cx)\text{PolyLog}\left(2,e^{-2i\arctan(cx)}\right) + 24i\arctan(cx)\text{PolyLog}\left(2,-e^{2i\arctan(cx)}\right)+12\text{PolyLog}\left(3,e^{-2i\arctan(cx)}\right) - 12\text{PolyLog}\left(3,-e^{2i\arctan(cx)}\right)\right)\right)$$

input
```
Integrate[((d + e*x^2)^2*(a + b*ArcTan[c*x])^2)/x^3,x]
```

output
```
(-((a^2*d^2)/x^2) + a^2*e^2*x^2 + (2*a*b*e^2*(-(c*x) + (1 + c^2*x^2)*ArcTa
n[c*x]))/c^2 - (2*a*b*d^2*(ArcTan[c*x] + c*x*(1 + c*x*ArcTan[c*x])))/x^2 +
 4*a^2*d*e*Log[x] - (b^2*d^2*(2*c*x*ArcTan[c*x] + (1 + c^2*x^2)*ArcTan[c*x
]^2 - 2*c^2*x^2*Log[(c*x)/Sqrt[1 + c^2*x^2]]))/x^2 + (b^2*e^2*(-2*c*x*ArcT
an[c*x] + (1 + c^2*x^2)*ArcTan[c*x]^2 + Log[1 + c^2*x^2]))/c^2 + (4*I)*a*b
*d*e*(PolyLog[2, (-I)*c*x] - PolyLog[2, I*c*x]) + (b^2*d*e*((-I)*Pi^3 + (1
6*I)*ArcTan[c*x]^3 + 24*ArcTan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] - 24
*ArcTan[c*x]^2*Log[1 + E^((2*I)*ArcTan[c*x])] + (24*I)*ArcTan[c*x]*PolyLog
[2, E^((-2*I)*ArcTan[c*x])] + (24*I)*ArcTan[c*x]*PolyLog[2, -E^((2*I)*ArcT
an[c*x])] + 12*PolyLog[3, E^((-2*I)*ArcTan[c*x])] - 12*PolyLog[3, -E^((2*I
)*ArcTan[c*x])]))/6)/2
```

## Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))^2}{x^3} \, dx$$

$$\Big\downarrow \; 5515$$

$$\int \left(\frac{d^2(a+b\arctan(cx))^2}{x^3}+\frac{2de(a+b\arctan(cx))^2}{x}+e^2x(a+b\arctan(cx))^2\right)dx$$

$$\Big\downarrow \; 2009$$

$$4de\text{arctanh}\left(1-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2-\frac{1}{2}c^2d^2(a+b\arctan(cx))^2+\frac{e^2(a+b\arctan(cx))^2}{2c^2}-\frac{d^2(a+b\arctan(cx))^2}{2x^2}-\frac{bcd^2(a+b\arctan(cx))}{x}-2ibde\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))+2ibde\,\text{PolyLog}\left(2,\frac{2}{icx+1}-1\right)(a+b\arctan(cx))+\frac{1}{2}e^2x^2(a+b\arctan(cx))^2-\frac{abe^2x}{c}-\frac{b^2e^2x\arctan(cx)}{c}-\frac{1}{2}b^2c^2d^2\log\left(c^2x^2+1\right)+b^2c^2d^2\log(x)+\frac{b^2e^2\log\left(c^2x^2+1\right)}{2c^2}-b^2de\,\text{PolyLog}\left(3,1-\frac{2}{icx+1}\right)+b^2de\,\text{PolyLog}\left(3,\frac{2}{icx+1}-1\right)$$

input

```
Int[((d + e*x^2)^2*(a + b*ArcTan[c*x])^2)/x^3,x]
```

output

```
-((a*b*e^2*x)/c) - (b^2*e^2*x*ArcTan[c*x])/c - (b*c*d^2*(a + b*ArcTan[c*x]
))/x - (c^2*d^2*(a + b*ArcTan[c*x])^2)/2 + (e^2*(a + b*ArcTan[c*x])^2)/(2*
c^2) - (d^2*(a + b*ArcTan[c*x])^2)/(2*x^2) + (e^2*x^2*(a + b*ArcTan[c*x])^
2)/2 + 4*d*e*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)] + b^2*c^2*d^
2*Log[x] - (b^2*c^2*d^2*Log[1 + c^2*x^2])/2 + (b^2*e^2*Log[1 + c^2*x^2])/(
2*c^2) - (2*I)*b*d*e*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)] + (
2*I)*b*d*e*(a + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)] - b^2*d*e*Po
lyLog[3, 1 - 2/(1 + I*c*x)] + b^2*d*e*PolyLog[3, -1 + 2/(1 + I*c*x)]
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 29.32 (sec) , antiderivative size = 1500, normalized size of antiderivative = 4.69

| method | result | size |
|---|---|---|
| parts | Expression too large to display | 1500 |
| derivativedivides | Expression too large to display | 1521 |
| default | Expression too large to display | 1521 |

input
```
int((e*x^2+d)^2*(a+b*arctan(c*x))^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
I*b^2*d*e*Pi*arctan(c*x)^2+2*I*b^2*d*e*arctan(c*x)*polylog(2,-(1+I*c*x)^2/
(c^2*x^2+1))-4*I*b^2*polylog(2,-(1+I*c*x)/(c^2*x^2+1)^(1/2))*arctan(c*x)*d
*e-4*I*b^2*polylog(2,(1+I*c*x)/(c^2*x^2+1)^(1/2))*arctan(c*x)*d*e-b^2*e^2*
x*arctan(c*x)/c+4*b^2*polylog(3,-(1+I*c*x)/(c^2*x^2+1)^(1/2))*d*e+4*b^2*po
lylog(3,(1+I*c*x)/(c^2*x^2+1)^(1/2))*d*e+b^2*c^2*ln((1+I*c*x)/(c^2*x^2+1)^
(1/2)-1)*d^2-1/2*b^2*c^2*arctan(c*x)^2*d^2-b^2/c^2*e^2*ln((1+I*c*x)^2/(c^2
*x^2+1)+1)+1/2*b^2/c^2*arctan(c*x)^2*e^2+b^2*c^2*ln((1+I*c*x)/(c^2*x^2+1)^
(1/2)+1)*d^2-1/2*b^2*arctan(c*x)^2*d^2/x^2+1/2*b^2*arctan(c*x)^2*e^2*x^2-b
^2*d*e*polylog(3,-(1+I*c*x)^2/(c^2*x^2+1))+2*b^2*ln(1-(1+I*c*x)/(c^2*x^2+1
)^(1/2))*arctan(c*x)^2*d*e+2*b^2*arctan(c*x)^2*d*e*ln(c*x)+I*b^2/c^2*arcta
n(c*x)*e^2-b^2*c*d^2*arctan(c*x)/x-2*b^2*ln((1+I*c*x)^2/(c^2*x^2+1)-1)*arc
tan(c*x)^2*d*e+I*b^2*d*e*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^
2/(c^2*x^2+1)+1))*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1
)+1))*arctan(c*x)^2-I*b^2*d*e*Pi*csgn(I*((1+I*c*x)^2/(c^2*x^2+1)-1))*csgn(
I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2
-I*b^2*d*e*Pi*csgn(I/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(I*((1+I*c*x)^2/(c^2
*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2-I*b^2*d*e*Pi*csgn(
I*((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))*csgn(((1+I*c*x)
^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^2*arctan(c*x)^2+I*b^2*d*e*P
i*csgn(((1+I*c*x)^2/(c^2*x^2+1)-1)/((1+I*c*x)^2/(c^2*x^2+1)+1))^3*arcta...
```

## Fricas [F]

$$\int \frac{(d+ex^2)^2(a+b\arctan(cx))^2}{x^3}\,dx = \int \frac{(ex^2+d)^2(b\arctan(cx)+a)^2}{x^3}\,dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x))^2/x^3,x, algorithm="fricas")
```

output
```
integral((a^2*e^2*x^4 + 2*a^2*d*e*x^2 + a^2*d^2 + (b^2*e^2*x^4 + 2*b^2*d*e
*x^2 + b^2*d^2)*arctan(c*x)^2 + 2*(a*b*e^2*x^4 + 2*a*b*d*e*x^2 + a*b*d^2)*
arctan(c*x))/x^3, x)
```

## Sympy [F]

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))^2}{x^3}\,dx = \int \frac{(a+b\operatorname{atan}(cx))^2 (d+ex^2)^2}{x^3}\,dx$$

input
```
integrate((e*x**2+d)**2*(a+b*atan(c*x))**2/x**3,x)
```

output
```
Integral((a + b*atan(c*x))**2*(d + e*x**2)**2/x**3, x)
```

## Maxima [F]

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))^2}{x^3}\,dx = \int \frac{(ex^2+d)^2 (b\arctan(cx)+a)^2}{x^3}\,dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x))^2/x^3,x, algorithm="maxima")
```

output
```
1/2*a^2*e^2*x^2 - ((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*a*b*d^2 + 2*
a^2*d*e*log(x) - 1/2*a^2*d^2/x^2 + 1/96*((1152*b^2*c^2*e^2*integrate(1/16*
x^6*arctan(c*x)^2/(c^2*x^5 + x^3), x) + 96*b^2*c^2*e^2*integrate(1/16*x^6*
log(c^2*x^2 + 1)^2/(c^2*x^5 + x^3), x) + 3072*a*b*c^2*e^2*integrate(1/16*x
^6*arctan(c*x)/(c^2*x^5 + x^3), x) + 192*b^2*c^2*e^2*integrate(1/16*x^6*lo
g(c^2*x^2 + 1)/(c^2*x^5 + x^3), x) + 2304*b^2*c^2*d*e*integrate(1/16*x^4*a
rctan(c*x)^2/(c^2*x^5 + x^3), x) + 6144*a*b*c^2*d*e*integrate(1/16*x^4*arc
tan(c*x)/(c^2*x^5 + x^3), x) + 1152*b^2*c^2*d^2*integrate(1/16*x^2*arctan(
c*x)^2/(c^2*x^5 + x^3), x) + 96*b^2*c^2*d^2*integrate(1/16*x^2*log(c^2*x^2
 + 1)^2/(c^2*x^5 + x^3), x) - 192*b^2*c^2*d^2*integrate(1/16*x^2*log(c^2*x
^2 + 1)/(c^2*x^5 + x^3), x) + 2*b^2*d*e*log(c^2*x^2 + 1)^3 - 384*b^2*c*e^2
*integrate(1/16*x^5*arctan(c*x)/(c^2*x^5 + x^3), x) + 384*b^2*c*d^2*integr
ate(1/16*x*arctan(c*x)/(c^2*x^5 + x^3), x) + 1152*b^2*e^2*integrate(1/16*x
^4*arctan(c*x)^2/(c^2*x^5 + x^3), x) + 3072*a*b*e^2*integrate(1/16*x^4*arc
tan(c*x)/(c^2*x^5 + x^3), x) + 2304*b^2*d*e*integrate(1/16*x^2*arctan(c*x)
^2/(c^2*x^5 + x^3), x) + 192*b^2*d*e*integrate(1/16*x^2*log(c^2*x^2 + 1)^2
/(c^2*x^5 + x^3), x) + 6144*a*b*d*e*integrate(1/16*x^2*arctan(c*x)/(c^2*x^
5 + x^3), x) + 1152*b^2*d^2*integrate(1/16*arctan(c*x)^2/(c^2*x^5 + x^3),
x) + 96*b^2*d^2*integrate(1/16*log(c^2*x^2 + 1)^2/(c^2*x^5 + x^3), x) + b^
2*e^2*log(c^2*x^2 + 1)^3/c^2)*x^2 + 12*(b^2*e^2*x^4 - b^2*d^2)*arctan(c...
```

## Giac [F]

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))^2}{x^3} \, dx = \int \frac{(ex^2+d)^2(b\arctan(cx)+a)^2}{x^3} \, dx$$

input
```
integrate((e*x^2+d)^2*(a+b*arctan(c*x))^2/x^3,x, algorithm="giac")
```

output
```
integrate((e*x^2 + d)^2*(b*arctan(c*x) + a)^2/x^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))^2}{x^3}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^2\,(e\,x^2+d)^2}{x^3}\,dx$$

input
```
int(((a + b*atan(c*x))^2*(d + e*x^2)^2)/x^3,x)
```

output
```
int(((a + b*atan(c*x))^2*(d + e*x^2)^2)/x^3, x)
```

## Reduce [F]

$$\int \frac{(d+ex^2)^2 (a+b\arctan(cx))^2}{x^3}\,dx = \frac{-atan(cx)^2 b^2c^4d^2x^2 - atan(cx)^2 b^2c^2d^2 + atan(cx)^2 b^2c^2e^2x^4 + atan(cx)^2 b^2e^2x^2 - 2atan(cx)\,ab\,c^4d^2x^2}{}$$

input
```
int((e*x^2+d)^2*(a+b*atan(c*x))^2/x^3,x)
```

output
```
( - atan(c*x)**2*b**2*c**4*d**2*x**2 - atan(c*x)**2*b**2*c**2*d**2 + atan(
c*x)**2*b**2*c**2*e**2*x**4 + atan(c*x)**2*b**2*e**2*x**2 - 2*atan(c*x)*a*
b*c**4*d**2*x**2 - 2*atan(c*x)*a*b*c**2*d**2 + 2*atan(c*x)*a*b*c**2*e**2*x
**4 + 2*atan(c*x)*a*b*e**2*x**2 - 2*atan(c*x)*b**2*c**3*d**2*x - 2*atan(c*
x)*b**2*c*e**2*x**3 + 8*int(atan(c*x)/x,x)*a*b*c**2*d*e*x**2 + 4*int(atan(
c*x)**2/x,x)*b**2*c**2*d*e*x**2 - log(c**2*x**2 + 1)*b**2*c**4*d**2*x**2 +
 log(c**2*x**2 + 1)*b**2*e**2*x**2 + 4*log(x)*a**2*c**2*d*e*x**2 + 2*log(x
)*b**2*c**4*d**2*x**2 - a**2*c**2*d**2 + a**2*c**2*e**2*x**4 - 2*a*b*c**3*
d**2*x - 2*a*b*c*e**2*x**3)/(2*c**2*x**2)
```

# 3.1259 $\int \frac{x^3(a+b\arctan(cx))^2}{d+ex^2}\,dx$

## Optimal result

Integrand size = 23, antiderivative size = 590

$$\int \frac{x^3(a+b\arctan(cx))^2}{d+ex^2}\,dx = -\frac{abx}{ce} - \frac{b^2x\arctan(cx)}{ce} + \frac{(a+b\arctan(cx))^2}{2c^2e} + \frac{x^2(a+b\arctan(cx))^2}{2e} + \frac{d(a+b\arctan(cx))^2\log\left(\frac{2}{1-icx}\right)}{e^2} - \frac{d(a+b\arctan(cx))^2\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2e^2} - \frac{d(a+b\arctan(cx))^2\log\left(\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{2e^2} + \frac{b^2\log(1+c^2x^2)}{2c^2e} - \frac{ibd(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{e^2} + \frac{ibd(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2e^2} + \frac{ibd(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{2e^2} + \frac{b^2d\operatorname{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{2e^2} - \frac{b^2d\operatorname{PolyLog}\left(3,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{4e^2} - \frac{b^2d\operatorname{PolyLog}\left(3,1-\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{4e^2}$$

output
```
-a*b*x/c/e-b^2*x*arctan(c*x)/c/e+1/2*(a+b*arctan(c*x))^2/c^2/e+1/2*x^2*(a+
b*arctan(c*x))^2/e+d*(a+b*arctan(c*x))^2*ln(2/(1-I*c*x))/e^2-1/2*d*(a+b*ar
ctan(c*x))^2*ln(2*c*((-d)^(1/2)-e^(1/2)*x)/(c*(-d)^(1/2)-I*e^(1/2))/(1-I*c
*x))/e^2-1/2*d*(a+b*arctan(c*x))^2*ln(2*c*((-d)^(1/2)+e^(1/2)*x)/(c*(-d)^(
1/2)+I*e^(1/2))/(1-I*c*x))/e^2+1/2*b^2*ln(c^2*x^2+1)/c^2/e-I*b*d*(a+b*arct
an(c*x))*polylog(2,1-2/(1-I*c*x))/e^2+1/2*I*b*d*(a+b*arctan(c*x))*polylog(
2,1-2*c*((-d)^(1/2)-e^(1/2)*x)/(c*(-d)^(1/2)-I*e^(1/2))/(1-I*c*x))/e^2+1/2
*I*b*d*(a+b*arctan(c*x))*polylog(2,1-2*c*((-d)^(1/2)+e^(1/2)*x)/(c*(-d)^(1
/2)+I*e^(1/2))/(1-I*c*x))/e^2+1/2*b^2*d*polylog(3,1-2/(1-I*c*x))/e^2-1/4*b
^2*d*polylog(3,1-2*c*((-d)^(1/2)-e^(1/2)*x)/(c*(-d)^(1/2)-I*e^(1/2))/(1-I*
c*x))/e^2-1/4*b^2*d*polylog(3,1-2*c*((-d)^(1/2)+e^(1/2)*x)/(c*(-d)^(1/2)+I
*e^(1/2))/(1-I*c*x))/e^2
```

## Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1520 vs. 2(590) = 1180.

Time = 11.49 (sec) , antiderivative size = 1520, normalized size of antiderivative = 2.58

$$\int \frac{x^3(a+b\arctan(cx))^2}{d+ex^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(x^3*(a + b*ArcTan[c*x])^2)/(d + e*x^2),x]
```

output
```
(-4*a*b*c*e*x + 2*a^2*c^2*e*x^2 + 4*a*b*e*ArcTan[c*x] - 4*b^2*c*e*x*ArcTan
[c*x] + 4*a*b*c^2*e*x^2*ArcTan[c*x] + 2*b^2*e*ArcTan[c*x]^2 + 2*b^2*c^2*e*
x^2*ArcTan[c*x]^2 - (8*I)*a*b*c^2*d*ArcSin[Sqrt[(c^2*d)/(c^2*d - e)]]*ArcT
an[(c*e*x)/Sqrt[c^2*d*e]] + 8*a*b*c^2*d*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTa
n[c*x])] + 4*b^2*c^2*d*ArcTan[c*x]^2*Log[1 + E^((2*I)*ArcTan[c*x])] + 4*a*
b*c^2*d*ArcSin[Sqrt[(c^2*d)/(c^2*d - e)]]*Log[1 + ((c^2*d + e + 2*Sqrt[c^2
*d*e])*E^((2*I)*ArcTan[c*x]))/(c^2*d - e)] - 4*a*b*c^2*d*ArcTan[c*x]*Log[1
 + ((c^2*d + e + 2*Sqrt[c^2*d*e])*E^((2*I)*ArcTan[c*x]))/(c^2*d - e)] + 4*
b^2*c^2*d*ArcSin[Sqrt[(c^2*d)/(c^2*d - e)]]*ArcTan[c*x]*Log[1 + ((c^2*d +
e + 2*Sqrt[c^2*d*e])*E^((2*I)*ArcTan[c*x]))/(c^2*d - e)] - 4*b^2*c^2*d*Arc
Tan[c*x]^2*Log[1 + ((c^2*d + e + 2*Sqrt[c^2*d*e])*E^((2*I)*ArcTan[c*x]))/(
c^2*d - e)] - 4*a*b*c^2*d*ArcSin[Sqrt[(c^2*d)/(c^2*d - e)]]*Log[(-2*Sqrt[c
^2*d*e]*E^((2*I)*ArcTan[c*x]) + e*(-1 + E^((2*I)*ArcTan[c*x])) + c^2*d*(1
+ E^((2*I)*ArcTan[c*x])))/(c^2*d - e)] - 4*a*b*c^2*d*ArcTan[c*x]*Log[(-2*S
qrt[c^2*d*e]*E^((2*I)*ArcTan[c*x]) + e*(-1 + E^((2*I)*ArcTan[c*x])) + c^2*
d*(1 + E^((2*I)*ArcTan[c*x])))/(c^2*d - e)] - 4*b^2*c^2*d*ArcSin[Sqrt[(c^2
*d)/(c^2*d - e)]]*ArcTan[c*x]*Log[(-2*Sqrt[c^2*d*e]*E^((2*I)*ArcTan[c*x])
+ e*(-1 + E^((2*I)*ArcTan[c*x])) + c^2*d*(1 + E^((2*I)*ArcTan[c*x])))/(c^2
*d - e)] - 4*b^2*c^2*d*ArcTan[c*x]^2*Log[(-2*Sqrt[c^2*d*e]*E^((2*I)*ArcTan
[c*x]) + e*(-1 + E^((2*I)*ArcTan[c*x])) + c^2*d*(1 + E^((2*I)*ArcTan[c*...
```

## Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 579, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {5451, 5361, 5451, 2009, 5419, 5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\arctan(cx))^2}{d+ex^2}dx$$

$$\Big\downarrow \text{5451}$$

$$\frac{\int x(a+b\arctan(cx))^2dx}{e}-\frac{d\int \frac{x(a+b\arctan(cx))^2}{ex^2+d}dx}{e}$$

$$\Big\downarrow \text{5361}$$

$$\frac{\frac{1}{2}x^2(a+b\arctan(cx))^2-bc\int\frac{x^2(a+b\arctan(cx))}{c^2x^2+1}dx}{e}-\frac{d\int\frac{x(a+b\arctan(cx))^2}{ex^2+d}dx}{e}$$

$\downarrow$ 5451

$$\frac{\frac{1}{2}x^2(a+b\arctan(cx))^2-bc\left(\frac{\int(a+b\arctan(cx))dx}{c^2}-\frac{\int\frac{a+b\arctan(cx)}{c^2x^2+1}dx}{c^2}\right)}{e}-\frac{d\int\frac{x(a+b\arctan(cx))^2}{ex^2+d}dx}{e}$$

$\downarrow$ 2009

$$\frac{\frac{1}{2}x^2(a+b\arctan(cx))^2-bc\left(\frac{ax+bx\arctan(cx)-\frac{b\log\left(c^2x^2+1\right)}{2c}}{c^2}-\frac{\int\frac{a+b\arctan(cx)}{c^2x^2+1}dx}{c^2}\right)}{e}-\frac{d\int\frac{x(a+b\arctan(cx))^2}{ex^2+d}dx}{e}$$

$\downarrow$ 5419

$$\frac{\frac{1}{2}x^2(a+b\arctan(cx))^2-bc\left(\frac{ax+bx\arctan(cx)-\frac{b\log\left(c^2x^2+1\right)}{2c}}{c^2}-\frac{(a+b\arctan(cx))^2}{2bc^3}\right)}{e}-\frac{d\int\frac{x(a+b\arctan(cx))^2}{ex^2+d}dx}{e}$$

$\downarrow$ 5515

$$\frac{\frac{1}{2}x^2(a+b\arctan(cx))^2-bc\left(\frac{ax+bx\arctan(cx)-\frac{b\log\left(c^2x^2+1\right)}{2c}}{c^2}-\frac{(a+b\arctan(cx))^2}{2bc^3}\right)}{e}-\frac{d\int\left(\frac{(a+b\arctan(cx))^2}{2\sqrt{e}\left(\sqrt{e}x+\sqrt{-d}\right)}-\frac{(a+b\arctan(cx))^2}{2\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)}\right)dx}{e}$$

$\downarrow$ 2009

$$\frac{\frac{1}{2}x^2(a+b\arctan(cx))^2-bc\left(\frac{ax+bx\arctan(cx)-\frac{b\log\left(c^2x^2+1\right)}{2c}}{c^2}-\frac{(a+b\arctan(cx))^2}{2bc^3}\right)}{e}-\frac{d\left(-\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)}{2e}-\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c\left(\sqrt{e}x+\sqrt{-d}\right)}{\left(\sqrt{-d}c+i\sqrt{e}\right)(1-icx)}\right)}{2e}+\frac{(a+b\arctan(cx))^2}{\cdots}\right.}{}$$

input
```
Int[(x^3*(a + b*ArcTan[c*x])^2)/(d + e*x^2),x]
```

output
```
((x^2*(a + b*ArcTan[c*x])^2)/2 - b*c*(-1/2*(a + b*ArcTan[c*x])^2/(b*c^3) +
 (a*x + b*x*ArcTan[c*x] - (b*Log[1 + c^2*x^2])/(2*c))/c^2))/e - (d*(-(((a
+ b*ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/e) + ((a + b*ArcTan[c*x])^2*Log[(2*
c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(2*e) +
 ((a + b*ArcTan[c*x])^2*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*
Sqrt[e])*(1 - I*c*x))])/(2*e) + (I*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/
(1 - I*c*x)])/e - ((I/2)*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(Sqrt[-
d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/e - ((I/2)*b*(a
+ b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d]
+ I*Sqrt[e])*(1 - I*c*x))])/e - (b^2*PolyLog[3, 1 - 2/(1 - I*c*x)])/(2*e)
+ (b^2*PolyLog[3, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e
])*(1 - I*c*x))])/(4*e) + (b^2*PolyLog[3, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))
/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(4*e)))/e
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

## Maple [F]

$$\int \frac{x^3(a+b\arctan(cx))^2}{e\,x^2+d}dx$$

input
```
int(x^3*(a+b*arctan(c*x))^2/(e*x^2+d),x)
```

output
```
int(x^3*(a+b*arctan(c*x))^2/(e*x^2+d),x)
```

## Fricas [F]

$$\int \frac{x^3(a+b\arctan(cx))^2}{d+ex^2}\,dx = \int \frac{(b\arctan(cx)+a)^2x^3}{ex^2+d}\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))^2/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b^2*x^3*arctan(c*x)^2 + 2*a*b*x^3*arctan(c*x) + a^2*x^3)/(e*x^2
+ d), x)
```

## Sympy [F]

$$\int \frac{x^3(a+b\arctan(cx))^2}{d+ex^2}\,dx = \int \frac{x^3(a+b\,\mathrm{atan}\,(cx))^2}{d+ex^2}\,dx$$

input
```
integrate(x**3*(a+b*atan(c*x))**2/(e*x**2+d),x)
```

output
```
Integral(x**3*(a + b*atan(c*x))**2/(d + e*x**2), x)
```

## Maxima [F]

$$\int \frac{x^3(a+b\arctan(cx))^2}{d+ex^2}\,dx = \int \frac{(b\arctan(cx)+a)^2x^3}{ex^2+d}\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))^2/(e*x^2+d),x, algorithm="maxima")
```

output
```
1/2*a^2*(x^2/e - d*log(e*x^2 + d)/e^2) + integrate((b^2*x^3*arctan(c*x)^2
+ 2*a*b*x^3*arctan(c*x))/(e*x^2 + d), x)
```

## Giac [F]

$$\int \frac{x^3(a+b\arctan(cx))^2}{d+ex^2}\,dx = \int \frac{(b\arctan(cx)+a)^2x^3}{ex^2+d}\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))^2/(e*x^2+d),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)^2*x^3/(e*x^2 + d), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\arctan(cx))^2}{d+ex^2}\,dx=\int \frac{x^3\,{\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}^2}{e\,x^2+d}\,dx$$

input
```
int((x^3*(a + b*atan(c*x))^2)/(d + e*x^2),x)
```

output
```
int((x^3*(a + b*atan(c*x))^2)/(d + e*x^2), x)
```

## Reduce [F]

$$\int \frac{x^3(a+b\arctan(cx))^2}{d+ex^2}\,dx = \frac{4\left(\int \frac{atan(cx)\,x^3}{e\,x^2+d}dx\right)ab\,e^2+2\left(\int \frac{atan(cx)^2\,x^3}{e\,x^2+d}dx\right)b^2e^2-\log\left(e\,x^2+d\right)a^2d+a^2e\,x^2}{2e^2}$$

input
```
int(x^3*(a+b*atan(c*x))^2/(e*x^2+d),x)
```

output
```
(4*int((atan(c*x)*x**3)/(d + e*x**2),x)*a*b*e**2 + 2*int((atan(c*x)**2*x**
3)/(d + e*x**2),x)*b**2*e**2 - log(d + e*x**2)*a**2*d + a**2*e*x**2)/(2*e*
*2)
```

# 3.1260 $\int \frac{x^2(a+b\arctan(cx))^2}{d+ex^2}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 554

$$\begin{aligned}\int \frac{x^2(a+b\arctan(cx))^2}{d+ex^2}\,dx &= \frac{i(a+b\arctan(cx))^2}{ce}+\frac{x(a+b\arctan(cx))^2}{e}+\frac{2b(a+b\arctan(cx))\log\left(\frac{2}{1+icx}\right)}{ce}\\
&+\frac{\sqrt{-d}(a+b\arctan(cx))^2\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2e^{3/2}}\\
&-\frac{\sqrt{-d}(a+b\arctan(cx))^2\log\left(\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{2e^{3/2}}+\frac{ib^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{ce}\\
&-\frac{ib\sqrt{-d}(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2e^{3/2}}\\
&+\frac{ib\sqrt{-d}(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{2e^{3/2}}\\
&+\frac{b^2\sqrt{-d}\operatorname{PolyLog}\left(3,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{4e^{3/2}}\\
&-\frac{b^2\sqrt{-d}\operatorname{PolyLog}\left(3,1-\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{4e^{3/2}}\end{aligned}$$

output
```
I*(a+b*arctan(c*x))^2/c/e+x*(a+b*arctan(c*x))^2/e+2*b*(a+b*arctan(c*x))*ln
(2/(1+I*c*x))/c/e+1/2*(-d)^(1/2)*(a+b*arctan(c*x))^2*ln(2*c*((-d)^(1/2)-e^
(1/2)*x)/(c*(-d)^(1/2)-I*e^(1/2))/(1-I*c*x))/e^(3/2)-1/2*(-d)^(1/2)*(a+b*a
rctan(c*x))^2*ln(2*c*((-d)^(1/2)+e^(1/2)*x)/(c*(-d)^(1/2)+I*e^(1/2))/(1-I*
c*x))/e^(3/2)+I*b^2*polylog(2,1-2/(1+I*c*x))/c/e-1/2*I*b*(-d)^(1/2)*(a+b*a
rctan(c*x))*polylog(2,1-2*c*((-d)^(1/2)-e^(1/2)*x)/(c*(-d)^(1/2)-I*e^(1/2)
)/(1-I*c*x))/e^(3/2)+1/2*I*b*(-d)^(1/2)*(a+b*arctan(c*x))*polylog(2,1-2*c*
((-d)^(1/2)+e^(1/2)*x)/(c*(-d)^(1/2)+I*e^(1/2))/(1-I*c*x))/e^(3/2)+1/4*b^2
*(-d)^(1/2)*polylog(3,1-2*c*((-d)^(1/2)-e^(1/2)*x)/(c*(-d)^(1/2)-I*e^(1/2)
)/(1-I*c*x))/e^(3/2)-1/4*b^2*(-d)^(1/2)*polylog(3,1-2*c*((-d)^(1/2)+e^(1/2
)*x)/(c*(-d)^(1/2)+I*e^(1/2))/(1-I*c*x))/e^(3/2)
```

## Mathematica [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\arctan(cx))^2}{d+ex^2}\,dx = \$\text{Aborted}$$

input
```
Integrate[(x^2*(a + b*ArcTan[c*x])^2)/(d + e*x^2),x]
```

output
```
$Aborted
```

## Rubi [A] (verified)

Time = 1.68 (sec) , antiderivative size = 568, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {5451, 5345, 5449, 2009, 5455, 5379, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\arctan(cx))^2}{d+ex^2}\,dx$$
$$\Big\downarrow \; 5451$$

$$\frac{\int (a+b\arctan(cx))^2dx}{e}-\frac{d\int \frac{(a+b\arctan(cx))^2}{ex^2+d}dx}{e}$$

$\downarrow$ 5345

$$\frac{x(a+b\arctan(cx))^2-2bc\int \frac{x(a+b\arctan(cx))}{c^2x^2+1}dx}{e}-\frac{d\int \frac{(a+b\arctan(cx))^2}{ex^2+d}dx}{e}$$

$\downarrow$ 5449

$$\frac{x(a+b\arctan(cx))^2-2bc\int \frac{x(a+b\arctan(cx))}{c^2x^2+1}dx}{e}-\frac{d\int \left(\frac{\sqrt{-d}(a+b\arctan(cx))^2}{2d(\sqrt{-d}-\sqrt{e}x)}+\frac{\sqrt{-d}(a+b\arctan(cx))^2}{2d(\sqrt{e}x+\sqrt{-d})}\right)dx}{e}$$

$\downarrow$ 2009

$$\frac{x(a+b\arctan(cx))^2-2bc\int \frac{x(a+b\arctan(cx))}{c^2x^2+1}dx}{e}-\frac{d\left(-\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{(a+b\arctan(cx))^2}{2}\right.}{}$$

$\downarrow$ 5455

$$\frac{x(a+b\arctan(cx))^2-2bc\left(-\frac{\int \frac{a+b\arctan(cx)}{i-cx}dx}{c}-\frac{i(a+b\arctan(cx))^2}{2bc^2}\right)}{e}-\frac{d\left(-\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{(a+b\arctan(cx))^2}{2}\right.}{}$$

$\downarrow$ 5379

$$\frac{x(a+b\arctan(cx))^2-2bc\left(-\frac{\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}-b\int \frac{\log\left(\frac{2}{icx+1}\right)}{c^2x^2+1}dx}{c}-\frac{i(a+b\arctan(cx))^2}{2bc^2}\right)}{e}-\frac{d\left(-\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{(a+b\arctan(cx))^2}{2}\right.}{}$$

$\downarrow$ 2849

$$\frac{x(a+b\arctan(cx))^2-2bc\left(-\frac{\frac{ib\int\frac{\log\left(\frac{2}{icx+1}\right)}{1-\frac{2}{icx+1}}d\frac{1}{icx+1}}{c}+\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}}{c}-\frac{i(a+b\arctan(cx))^2}{2bc^2}\right)}{e}-\frac{d\left(-\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{(a+b\arctan(cx))^2}{2\ldots}\right.}{}$$

$\downarrow$ 2752

$$\frac{x(a+b\arctan(cx))^2-2bc\left(-\frac{i(a+b\arctan(cx))^2}{2bc^2}-\frac{\frac{\log\left(\frac{2}{1+icx}\right)(a+b\arctan(cx))}{c}+\frac{ib\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)}{2c}}{c}\right)}{e}-\frac{d\left(-\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{(a+b\arctan(cx))^2}{2\ldots}\right.}{}$$

input
```
Int[(x^2*(a + b*ArcTan[c*x])^2)/(d + e*x^2),x]
```

output
```
(x*(a + b*ArcTan[c*x])^2 - 2*b*c*(((-1/2*I)*(a + b*ArcTan[c*x])^2)/(b*c^2)
 - (((a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/c + ((I/2)*b*PolyLog[2, 1 - 2
/(1 + I*c*x)])/c)/c))/e - (d*(((a + b*ArcTan[c*x])^2*Log[(2*c*(Sqrt[-d] -
Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(2*Sqrt[-d]*Sqrt[e])
- ((a + b*ArcTan[c*x])^2*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I
*Sqrt[e])*(1 - I*c*x))])/(2*Sqrt[-d]*Sqrt[e]) - ((I/2)*b*(a + b*ArcTan[c*x
])*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(
1 - I*c*x))])/(Sqrt[-d]*Sqrt[e]) + ((I/2)*b*(a + b*ArcTan[c*x])*PolyLog[2,
 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])
/(Sqrt[-d]*Sqrt[e]) + (b^2*PolyLog[3, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c
*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(4*Sqrt[-d]*Sqrt[e]) - (b^2*PolyLog[
3, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))
])/(4*Sqrt[-d]*Sqrt[e])))/e
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5379
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol]
 :> Simp[(-(a + b*ArcTan[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c*(
p/e)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 + c^2*x^2))
, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 + e^2, 0
]
```

rule 5449
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (d + e*x^2)^q, x], x
] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[q] && IGtQ[p, 0]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5455
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*e*(p + 1))), x] - Si
mp[1/(c*d)   Int[(a + b*ArcTan[c*x])^p/(I - c*x), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && IGtQ[p, 0]
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 158.36 (sec) , antiderivative size = 92148, normalized size of antiderivative = 166.33

| method | result | size |
|---|---|---|
| derivativedivides | Expression too large to display | 92148 |
| default | Expression too large to display | 92148 |

input
```
int(x^2*(a+b*arctan(c*x))^2/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

## Fricas [F]

$$\int \frac{x^2(a+b\arctan(cx))^2}{d+ex^2}\,dx=\int \frac{(b\arctan{(cx)}+a)^2x^2}{ex^2+d}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))^2/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b^2*x^2*arctan(c*x)^2 + 2*a*b*x^2*arctan(c*x) + a^2*x^2)/(e*x^2
+ d), x)
```

## Sympy [F]

$$\int \frac{x^2(a+b\arctan(cx))^2}{d+ex^2}\,dx = \int \frac{x^2(a+b\,\mathrm{atan}\,(cx))^2}{d+ex^2}\,dx$$

input
```
integrate(x**2*(a+b*atan(c*x))**2/(e*x**2+d),x)
```

output
```
Integral(x**2*(a + b*atan(c*x))**2/(d + e*x**2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a+b\arctan(cx))^2}{d+ex^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(a+b*arctan(c*x))^2/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

## Giac [F]

$$\int \frac{x^2(a+b\arctan(cx))^2}{d+ex^2}\,dx = \int \frac{(b\arctan(cx)+a)^2x^2}{ex^2+d}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))^2/(e*x^2+d),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)^2*x^2/(e*x^2 + d), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\arctan(cx))^2}{d+ex^2}\,dx = \int \frac{x^2\,(a+b\,\mathrm{atan}(c\,x))^2}{e\,x^2+d}\,dx$$

input
```
int((x^2*(a + b*atan(c*x))^2)/(d + e*x^2),x)
```

output
```
int((x^2*(a + b*atan(c*x))^2)/(d + e*x^2), x)
```

## Reduce [F]

$$\int \frac{x^2(a+b\arctan(cx))^2}{d+ex^2}\,dx = \frac{-\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)a^2c - atan(cx)^2\,ab\,c^2d + 2atan(cx)\,abcex + 2\left(\int \frac{atan(cx)}{c^2e\,x^4+c^2d\,x^2+e\,x^2+d}dx\right)ab\,c^3d^2 - 2}{c\,e^2}$$

input
```
int(x^2*(a+b*atan(c*x))^2/(e*x^2+d),x)
```

output
```
( - sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a**2*c - atan(c*x)**2*a*
b*c**2*d + 2*atan(c*x)*a*b*c*e*x + 2*int(atan(c*x)/(c**2*d*x**2 + c**2*e*x
**4 + d + e*x**2),x)*a*b*c**3*d**2 - 2*int(atan(c*x)/(c**2*d*x**2 + c**2*e
*x**4 + d + e*x**2),x)*a*b*c*d*e + int((atan(c*x)**2*x**2)/(d + e*x**2),x)
*b**2*c*e**2 - log(c**2*x**2 + 1)*a*b*e + a**2*c*e*x)/(c*e**2)
```

# 3.1261 $\int \frac{x(a+b\arctan(cx))^2}{d+ex^2}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 492

$$\begin{aligned}\int \frac{x(a+b\arctan(cx))^2}{d+ex^2}\,dx =& -\frac{(a+b\arctan(cx))^2\log\left(\frac{2}{1-icx}\right)}{e} \\ &+\frac{(a+b\arctan(cx))^2\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2e} \\ &+\frac{(a+b\arctan(cx))^2\log\left(\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{2e} \\ &+\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{e} \\ &-\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2e} \\ &-\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{2e} \\ &-\frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{2e} \\ &+\frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{4e} \\ &+\frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{4e}\end{aligned}$$

output
```
-(a+b*arctan(c*x))^2*ln(2/(1-I*c*x))/e+1/2*(a+b*arctan(c*x))^2*ln(2*c*((-d
)^(1/2)-e^(1/2)*x)/(c*(-d)^(1/2)-I*e^(1/2))/(1-I*c*x))/e+1/2*(a+b*arctan(c
*x))^2*ln(2*c*((-d)^(1/2)+e^(1/2)*x)/(c*(-d)^(1/2)+I*e^(1/2))/(1-I*c*x))/e
+I*b*(a+b*arctan(c*x))*polylog(2,1-2/(1-I*c*x))/e-1/2*I*b*(a+b*arctan(c*x)
)*polylog(2,1-2*c*((-d)^(1/2)-e^(1/2)*x)/(c*(-d)^(1/2)-I*e^(1/2))/(1-I*c*x
))/e-1/2*I*b*(a+b*arctan(c*x))*polylog(2,1-2*c*((-d)^(1/2)+e^(1/2)*x)/(c*(
-d)^(1/2)+I*e^(1/2))/(1-I*c*x))/e-1/2*b^2*polylog(3,1-2/(1-I*c*x))/e+1/4*b
^2*polylog(3,1-2*c*((-d)^(1/2)-e^(1/2)*x)/(c*(-d)^(1/2)-I*e^(1/2))/(1-I*c*
x))/e+1/4*b^2*polylog(3,1-2*c*((-d)^(1/2)+e^(1/2)*x)/(c*(-d)^(1/2)+I*e^(1/
2))/(1-I*c*x))/e
```

## Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1322 vs. $2(492) = 984$.

Time = 11.29 (sec) , antiderivative size = 1322, normalized size of antiderivative = 2.69

$$\int \frac{x(a+b\arctan(cx))^2}{d+ex^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(x*(a + b*ArcTan[c*x])^2)/(d + e*x^2),x]
```

output
```
((8*I)*a*b*ArcSin[Sqrt[(c^2*d)/(c^2*d - e)]]*ArcTan[(c*e*x)/Sqrt[c^2*d*e]]
 - 8*a*b*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] - 4*b^2*ArcTan[c*x]^2*
Log[1 + E^((2*I)*ArcTan[c*x])] - 4*a*b*ArcSin[Sqrt[(c^2*d)/(c^2*d - e)]]*L
og[1 + ((c^2*d + e + 2*Sqrt[c^2*d*e])*E^((2*I)*ArcTan[c*x]))/(c^2*d - e)]
+ 4*a*b*ArcTan[c*x]*Log[1 + ((c^2*d + e + 2*Sqrt[c^2*d*e])*E^((2*I)*ArcTan
[c*x]))/(c^2*d - e)] - 4*b^2*ArcSin[Sqrt[(c^2*d)/(c^2*d - e)]]*ArcTan[c*x]
*Log[1 + ((c^2*d + e + 2*Sqrt[c^2*d*e])*E^((2*I)*ArcTan[c*x]))/(c^2*d - e)
] + 4*b^2*ArcTan[c*x]^2Log[1 + ((c^2*d + e + 2*Sqrt[c^2*d*e])*E^((2*I)*Ar
cTan[c*x]))/(c^2*d - e)] + 4*a*b*ArcSin[Sqrt[(c^2*d)/(c^2*d - e)]]*Log[(-2
*Sqrt[c^2*d*e]*E^((2*I)*ArcTan[c*x]) + e*(-1 + E^((2*I)*ArcTan[c*x])) + c^
2*d*(1 + E^((2*I)*ArcTan[c*x])))/(c^2*d - e)] + 4*a*b*ArcTan[c*x]*Log[(-2*
Sqrt[c^2*d*e]*E^((2*I)*ArcTan[c*x]) + e*(-1 + E^((2*I)*ArcTan[c*x])) + c^2
*d*(1 + E^((2*I)*ArcTan[c*x])))/(c^2*d - e)] + 4*b^2*ArcSin[Sqrt[(c^2*d)/(
c^2*d - e)]]*ArcTan[c*x]*Log[(-2*Sqrt[c^2*d*e]*E^((2*I)*ArcTan[c*x]) + e*(
-1 + E^((2*I)*ArcTan[c*x])) + c^2*d*(1 + E^((2*I)*ArcTan[c*x])))/(c^2*d -
e)] + 4*b^2*ArcTan[c*x]^2*Log[(-2*Sqrt[c^2*d*e]*E^((2*I)*ArcTan[c*x]) + e*
(-1 + E^((2*I)*ArcTan[c*x])) + c^2*d*(1 + E^((2*I)*ArcTan[c*x])))/(c^2*d -
 e)] - 4*b^2*ArcSin[Sqrt[(c^2*d)/(c^2*d - e)]]*ArcTan[c*x]*Log[((2*I)*c^2*
d - (2*I)*Sqrt[c^2*d*e] + 2*c*(-e + Sqrt[c^2*d*e])*x)/((c^2*d - e)*(I + c*
x))] - 2*b^2*ArcTan[c*x]^2*Log[((2*I)*c^2*d - (2*I)*Sqrt[c^2*d*e] + 2*c...
```

## Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 492, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\arctan(cx))^2}{d+ex^2}\,dx$$

$$\Big\downarrow \text{5515}$$

$$\int \left(\frac{(a+b\arctan(cx))^2}{2\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)}-\frac{(a+b\arctan(cx))^2}{2\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2e}-\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)}{2e}+\frac{(a+b\arctan(cx))^2\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(1-icx)(c\sqrt{-d}-i\sqrt{e})}\right)}{2e}+\frac{(a+b\arctan(cx))^2\log\left(\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(1-icx)(c\sqrt{-d}+i\sqrt{e})}\right)}{2e}+\frac{ib\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)(a+b\arctan(cx))}{e}-\frac{\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))^2}{e}+\frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{4e}+\frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)}{4e}-\frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{2e}$$

input
```
Int[(x*(a + b*ArcTan[c*x])^2)/(d + e*x^2),x]
```

output
```
-(((a + b*ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/e) + ((a + b*ArcTan[c*x])^2*L
og[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(
2*e) + ((a + b*ArcTan[c*x])^2*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d
] + I*Sqrt[e])*(1 - I*c*x))])/(2*e) + (I*b*(a + b*ArcTan[c*x])*PolyLog[2,
1 - 2/(1 - I*c*x)])/e - ((I/2)*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(
Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/e - ((I/2)
*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqr
t[-d] + I*Sqrt[e])*(1 - I*c*x))])/e - (b^2*PolyLog[3, 1 - 2/(1 - I*c*x)])/
(2*e) + (b^2*PolyLog[3, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*
Sqrt[e])*(1 - I*c*x))])/(4*e) + (b^2*PolyLog[3, 1 - (2*c*(Sqrt[-d] + Sqrt[
e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(4*e)
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

## Maple [F]

$$\int \frac{x(a+b\arctan(cx))^2}{e\,x^2+d}dx$$

input
```
int(x*(a+b*arctan(c*x))^2/(e*x^2+d),x)
```

output
```
int(x*(a+b*arctan(c*x))^2/(e*x^2+d),x)
```

## Fricas [F]

$$\int \frac{x(a+b\arctan(cx))^2}{d+ex^2}\,dx = \int \frac{(b\arctan(cx)+a)^2x}{ex^2+d}\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))^2/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b^2*x*arctan(c*x)^2 + 2*a*b*x*arctan(c*x) + a^2*x)/(e*x^2 + d),
x)
```

## Sympy [F]

$$\int \frac{x(a+b\arctan(cx))^2}{d+ex^2}\,dx = \int \frac{x(a+b\,\mathrm{atan}\,(cx))^2}{d+ex^2}\,dx$$

input
```
integrate(x*(a+b*atan(c*x))**2/(e*x**2+d),x)
```

output
```
Integral(x*(a + b*atan(c*x))**2/(d + e*x**2), x)
```

## Maxima [F]

$$\int \frac{x(a+b\arctan(cx))^2}{d+ex^2}\,dx = \int \frac{(b\arctan(cx)+a)^2 x}{ex^2+d}\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))^2/(e*x^2+d),x, algorithm="maxima")
```

output
```
1/2*a^2*log(e*x^2 + d)/e + integrate((b^2*x*arctan(c*x)^2 + 2*a*b*x*arctan
(c*x))/(e*x^2 + d), x)
```

## Giac [F]

$$\int \frac{x(a+b\arctan(cx))^2}{d+ex^2}\,dx = \int \frac{(b\arctan(cx)+a)^2 x}{ex^2+d}\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))^2/(e*x^2+d),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)^2*x/(e*x^2 + d), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\arctan(cx))^2}{d+ex^2}\,dx=\int \frac{x\,{\left(a+b\,\mathrm{atan}\left(c\,x\right)\right)}^2}{e\,x^2+d}\,dx$$

input
```
int((x*(a + b*atan(c*x))^2)/(d + e*x^2),x)
```

output
```
int((x*(a + b*atan(c*x))^2)/(d + e*x^2), x)
```

## Reduce [F]

$$\int \frac{x(a+b\arctan(cx))^2}{d+ex^2}\,dx=\frac{4\left(\int \frac{atan(cx)x}{e\,x^2+d}dx\right)abe+2\left(\int \frac{atan(cx)^2x}{e\,x^2+d}dx\right)b^2e+\log\left(e\,x^2+d\right)a^2}{2e}$$

input
```
int(x*(a+b*atan(c*x))^2/(e*x^2+d),x)
```

output
```
(4*int((atan(c*x)*x)/(d + e*x**2),x)*a*b*e + 2*int((atan(c*x)**2*x)/(d + e
*x**2),x)*b**2*e + log(d + e*x**2)*a**2)/(2*e)
```

# 3.1262 $\int \frac{(a+b\arctan(cx))^2}{d+ex^2}\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 460

$$\begin{aligned}\int \frac{(a+b\arctan(cx))^2}{d+ex^2}\,dx &= \frac{(a+b\arctan(cx))^2\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2\sqrt{-d}\sqrt{e}} \\ &\quad - \frac{(a+b\arctan(cx))^2\log\left(\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{2\sqrt{-d}\sqrt{e}} \\ &\quad - \frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2\sqrt{-d}\sqrt{e}} \\ &\quad + \frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{2\sqrt{-d}\sqrt{e}} \\ &\quad + \frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{4\sqrt{-d}\sqrt{e}} \\ &\quad - \frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{4\sqrt{-d}\sqrt{e}}\end{aligned}$$

output
```
1/2*(a+b*arctan(c*x))^2*ln(2*c*((-d)^(1/2)-e^(1/2)*x)/(c*(-d)^(1/2)-I*e^(1
/2))/(1-I*c*x))/(-d)^(1/2)/e^(1/2)-1/2*(a+b*arctan(c*x))^2*ln(2*c*((-d)^(1
/2)+e^(1/2)*x)/(c*(-d)^(1/2)+I*e^(1/2))/(1-I*c*x))/(-d)^(1/2)/e^(1/2)-1/2*
I*b*(a+b*arctan(c*x))*polylog(2,1-2*c*((-d)^(1/2)-e^(1/2)*x)/(c*(-d)^(1/2)
-I*e^(1/2))/(1-I*c*x))/(-d)^(1/2)/e^(1/2)+1/2*I*b*(a+b*arctan(c*x))*polylo
g(2,1-2*c*((-d)^(1/2)+e^(1/2)*x)/(c*(-d)^(1/2)+I*e^(1/2))/(1-I*c*x))/(-d)^
(1/2)/e^(1/2)+1/4*b^2*polylog(3,1-2*c*((-d)^(1/2)-e^(1/2)*x)/(c*(-d)^(1/2)
-I*e^(1/2))/(1-I*c*x))/(-d)^(1/2)/e^(1/2)-1/4*b^2*polylog(3,1-2*c*((-d)^(1
/2)+e^(1/2)*x)/(c*(-d)^(1/2)+I*e^(1/2))/(1-I*c*x))/(-d)^(1/2)/e^(1/2)
```

## Mathematica [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{d+ex^2}\,dx = \$\text{Aborted}$$

input
```
Integrate[(a + b*ArcTan[c*x])^2/(d + e*x^2),x]
```

output
```
$Aborted
```

## Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 460, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {5449, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^2}{d+ex^2}\,dx$$

$$\Big\downarrow \text{5449}$$

$$\int \left(\frac{\sqrt{-d}(a+b\arctan(cx))^2}{2d\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{\sqrt{-d}(a+b\arctan(cx))^2}{2d\left(\sqrt{-d}+\sqrt{e}x\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{(a+b\arctan(cx))^2\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(1-icx)(c\sqrt{-d}-i\sqrt{e})}\right)}{2\sqrt{-d}\sqrt{e}}-\frac{(a+b\arctan(cx))^2\log\left(\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(1-icx)(c\sqrt{-d}+i\sqrt{e})}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{4\sqrt{-d}\sqrt{e}}-\frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)}{4\sqrt{-d}\sqrt{e}}$$

input
```
Int[(a + b*ArcTan[c*x])^2/(d + e*x^2),x]
```

output
```
((a + b*ArcTan[c*x])^2*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*S
qrt[e])*(1 - I*c*x))])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcTan[c*x])^2*Log[(
2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(2*Sq
rt[-d]*Sqrt[e]) - ((I/2)*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(Sqrt[-
d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(Sqrt[-d]*Sqrt[e
]) + ((I/2)*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*
x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(Sqrt[-d]*Sqrt[e]) + (b^2*Pol
yLog[3, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*
c*x))])/(4*Sqrt[-d]*Sqrt[e]) - (b^2*PolyLog[3, 1 - (2*c*(Sqrt[-d] + Sqrt[e
]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(4*Sqrt[-d]*Sqrt[e])
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5449
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (d + e*x^2)^q, x], x
] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[q] && IGtQ[p, 0]
```

## Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2343 vs. $2(362) = 724$.

Time = 39.49 (sec) , antiderivative size = 2344, normalized size of antiderivative = 5.10

| method | result | size |
|---|---|---|
| parts | Expression too large to display | 2344 |
| derivativedivides | Expression too large to display | 5222 |
| default | Expression too large to display | 5222 |

input
```
int((a+b*arctan(c*x))^2/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
a^2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))+b^2/c*(-1/2*(-(c^2*d*e)^(1/2)*c^2*
d+2*c^2*d*e-(c^2*d*e)^(1/2)*e)/d/(c^4*d^2-2*c^2*d*e+e^2)*polylog(2,(c^2*d-
e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d-2*(c^2*d*e)^(1/2)-e))*arctan(c*x)-1/2*I
*(-(c^2*d*e)^(1/2)*c^2*d+2*c^2*d*e-(c^2*d*e)^(1/2)*e)*c^2*ln(1-(c^2*d-e)*(
1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d-2*(c^2*d*e)^(1/2)-e))*arctan(c*x)^2/e/(c^4*
d^2-2*c^2*d*e+e^2)-1/3*(-(c^2*d*e)^(1/2)*c^2*d+2*c^2*d*e-(c^2*d*e)^(1/2)*e
)*c^2/e/(c^4*d^2-2*c^2*d*e+e^2)*arctan(c*x)^3-1/2*I*(-(c^2*d*e)^(1/2)*c^2*
d+2*c^2*d*e-(c^2*d*e)^(1/2)*e)*ln(1-(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^
2*d-2*(c^2*d*e)^(1/2)-e))*arctan(c*x)^2/d/(c^4*d^2-2*c^2*d*e+e^2)-1/2*(-(c
^2*d*e)^(1/2)*c^2*d+2*c^2*d*e-(c^2*d*e)^(1/2)*e)*c^2/e/(c^4*d^2-2*c^2*d*e+
e^2)*polylog(2,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d-2*(c^2*d*e)^(1/2)
-e))*arctan(c*x)-1/3*(c^2*d*e)^(1/2)/d/e*arctan(c*x)^3-1/3*(-(c^2*d*e)^(1/
2)*c^2*d+2*c^2*d*e-(c^2*d*e)^(1/2)*e)/d/(c^4*d^2-2*c^2*d*e+e^2)*arctan(c*x
)^3+I*(c^2*d-2*(c^2*d*e)^(1/2)+e)*ln(1-(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(
-c^2*d-2*(c^2*d*e)^(1/2)-e))*arctan(c*x)^2*c^2/(c^4*d^2-2*c^2*d*e+e^2)-1/4
*I*(c^2*d*e)^(1/2)/d/e*polylog(3,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d
+2*(c^2*d*e)^(1/2)-e))+I*(c^2*d-2*(c^2*d*e)^(1/2)+e)*polylog(3,(c^2*d-e)*(
1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d-2*(c^2*d*e)^(1/2)-e))*c^2/(2*c^4*d^2-4*c^2*
d*e+2*e^2)-1/2*(c^2*d*e)^(1/2)/d/e*arctan(c*x)*polylog(2,(c^2*d-e)*(1+I*c*
x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*d*e)^(1/2)-e))+(c^2*d-2*(c^2*d*e)^(1/2)...
```

## Fricas [F]

$$\int \frac{(a+b\arctan(cx))^2}{d+ex^2}\,dx = \int \frac{(b\arctan(cx)+a)^2}{ex^2+d}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)/(e*x^2 + d), x)
```

## Sympy [F]

$$\int \frac{(a+b\arctan(cx))^2}{d+ex^2}\,dx = \int \frac{(a+b\operatorname{atan}(cx))^2}{d+ex^2}\,dx$$

input
```
integrate((a+b*atan(c*x))**2/(e*x**2+d),x)
```

output
```
Integral((a + b*atan(c*x))**2/(d + e*x**2), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\arctan(cx))^2}{d+ex^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arctan(c*x))^2/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

## Giac [F]

$$\int \frac{(a+b\arctan(cx))^2}{d+ex^2}\,dx = \int \frac{(b\arctan(cx)+a)^2}{ex^2+d}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/(e*x^2+d),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)^2/(e*x^2 + d), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{d+ex^2}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^2}{e\,x^2+d}\,dx$$

input
```
int((a + b*atan(c*x))^2/(d + e*x^2),x)
```

output
```
int((a + b*atan(c*x))^2/(d + e*x^2), x)
```

## Reduce [F]

$$\int \frac{(a+b\arctan(cx))^2}{d+ex^2}\,dx = \frac{3\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)a^2+atan(cx)^3\,b^2cd+3atan(cx)^2\,abcd-6\left(\int \frac{atan(cx)}{c^2e\,x^4+c^2d\,x^2+e\,x^2+d}dx\right)ab\,c^2d^2+6\left(\int\right.}{3de}$$

input
```
int((a+b*atan(c*x))^2/(e*x^2+d),x)
```

output

```
(3*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a**2 + atan(c*x)**3*b**2*
c*d + 3*atan(c*x)**2*a*b*c*d - 6*int(atan(c*x)/(c**2*d*x**2 + c**2*e*x**4
+ d + e*x**2),x)*a*b*c**2*d**2 + 6*int(atan(c*x)/(c**2*d*x**2 + c**2*e*x**
4 + d + e*x**2),x)*a*b*d*e - 3*int(atan(c*x)**2/(c**2*d*x**2 + c**2*e*x**4
 + d + e*x**2),x)*b**2*c**2*d**2 + 3*int(atan(c*x)**2/(c**2*d*x**2 + c**2*
e*x**4 + d + e*x**2),x)*b**2*d*e)/(3*d*e)
```

# 3.1263 $\int \frac{(a+b\arctan(cx))^2}{x(d+ex^2)}\,dx$

## Optimal result

Integrand size = 23, antiderivative size = 637

$$\int \frac{(a+b\arctan(cx))^2}{x\,(d+ex^2)}\,dx = \frac{2(a+b\arctan(cx))^2\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right)}{d} + \frac{(a+b\arctan(cx))^2\log\left(\frac{2}{1-icx}\right)}{d} - \frac{(a+b\arctan(cx))^2\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2d} - \frac{(a+b\arctan(cx))^2\log\left(\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{2d} - \frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{d} - \frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{d} + \frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)}{d} + \frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2d} + \frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{2d} + \frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{2d} - \frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2d} + \frac{b^2\operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)}{2d} - \frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{4d} - \frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{4d}$$

output
```
-2*(a+b*arctan(c*x))^2*arctanh(-1+2/(1+I*c*x))/d+(a+b*arctan(c*x))^2*ln(2/
(1-I*c*x))/d-1/2*(a+b*arctan(c*x))^2*ln(2*c*((-d)^(1/2)-e^(1/2)*x)/(c*(-d)
^(1/2)-I*e^(1/2))/(1-I*c*x))/d-1/2*(a+b*arctan(c*x))^2*ln(2*c*((-d)^(1/2)+
e^(1/2)*x)/(c*(-d)^(1/2)+I*e^(1/2))/(1-I*c*x))/d-I*b*(a+b*arctan(c*x))*pol
ylog(2,1-2/(1-I*c*x))/d-I*b*(a+b*arctan(c*x))*polylog(2,1-2/(1+I*c*x))/d+I
*b*(a+b*arctan(c*x))*polylog(2,-1+2/(1+I*c*x))/d+1/2*I*b*(a+b*arctan(c*x))
*polylog(2,1-2*c*((-d)^(1/2)-e^(1/2)*x)/(c*(-d)^(1/2)-I*e^(1/2))/(1-I*c*x)
)/d+1/2*I*b*(a+b*arctan(c*x))*polylog(2,1-2*c*((-d)^(1/2)+e^(1/2)*x)/(c*(-
d)^(1/2)+I*e^(1/2))/(1-I*c*x))/d+1/2*b^2*polylog(3,1-2/(1-I*c*x))/d-1/2*b^
2*polylog(3,1-2/(1+I*c*x))/d+1/2*b^2*polylog(3,-1+2/(1+I*c*x))/d-1/4*b^2*p
olylog(3,1-2*c*((-d)^(1/2)-e^(1/2)*x)/(c*(-d)^(1/2)-I*e^(1/2))/(1-I*c*x))/
d-1/4*b^2*polylog(3,1-2*c*((-d)^(1/2)+e^(1/2)*x)/(c*(-d)^(1/2)+I*e^(1/2))/
(1-I*c*x))/d
```

## Mathematica [A] (warning: unable to verify)

Time = 9.92 (sec) , antiderivative size = 1264, normalized size of antiderivative = 1.98

$$\int \frac{(a+b\arctan(cx))^2}{x\,(d+ex^2)}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*ArcTan[c*x])^2/(x*(d + e*x^2)),x]
```

output
```
(24*a^2*Log[x] - 12*a^2*Log[d + e*x^2] - 24*a*b*((-I)*ArcTan[c*x]^2 + (2*I
)*ArcSin[Sqrt[(c^2*d)/(c^2*d - e)]]*ArcTan[(c*e*x)/Sqrt[c^2*d*e]] - 2*ArcT
an[c*x]*Log[1 - E^((2*I)*ArcTan[c*x])] + (-ArcSin[Sqrt[(c^2*d)/(c^2*d - e)
]] + ArcTan[c*x])*Log[1 + ((c^2*d + e + 2*Sqrt[c^2*d*e])*E^((2*I)*ArcTan[c
*x]))/(c^2*d - e)] + (ArcSin[Sqrt[(c^2*d)/(c^2*d - e)]] + ArcTan[c*x])*Log
[(-2*Sqrt[c^2*d*e]*E^((2*I)*ArcTan[c*x]) + e*(-1 + E^((2*I)*ArcTan[c*x]))
+ c^2*d*(1 + E^((2*I)*ArcTan[c*x])))/(c^2*d - e)] + I*(ArcTan[c*x]^2 + Pol
yLog[2, E^((2*I)*ArcTan[c*x])]) - (I/2)*(PolyLog[2, ((-(c^2*d) - e + 2*Sqr
t[c^2*d*e])*E^((2*I)*ArcTan[c*x]))/(c^2*d - e)] + PolyLog[2, -(((c^2*d + e
 + 2*Sqrt[c^2*d*e])*E^((2*I)*ArcTan[c*x]))/(c^2*d - e))])) + b^2*((-I)*Pi^
3 + (16*I)*ArcTan[c*x]^3 + 24*ArcTan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])
] + 24*ArcSin[Sqrt[(c^2*d)/(c^2*d - e)]]*ArcTan[c*x]*Log[1 + ((c^2*d + e +
 2*Sqrt[c^2*d*e])*E^((2*I)*ArcTan[c*x]))/(c^2*d - e)] - 24*ArcTan[c*x]^2*L
og[1 + ((c^2*d + e + 2*Sqrt[c^2*d*e])*E^((2*I)*ArcTan[c*x]))/(c^2*d - e)]
- 24*ArcSin[Sqrt[(c^2*d)/(c^2*d - e)]]*ArcTan[c*x]*Log[(-2*Sqrt[c^2*d*e]*E
^((2*I)*ArcTan[c*x]) + e*(-1 + E^((2*I)*ArcTan[c*x])) + c^2*d*(1 + E^((2*I
)*ArcTan[c*x])))/(c^2*d - e)] - 24*ArcTan[c*x]^2*Log[(-2*Sqrt[c^2*d*e]*E^(
(2*I)*ArcTan[c*x]) + e*(-1 + E^((2*I)*ArcTan[c*x])) + c^2*d*(1 + E^((2*I)*
ArcTan[c*x])))/(c^2*d - e)] + 24*ArcSin[Sqrt[(c^2*d)/(c^2*d - e)]]*ArcTan[
c*x]*Log[((2*I)*c^2*d - (2*I)*Sqrt[c^2*d*e] + 2*c*(-e + Sqrt[c^2*d*e])*...
```

## Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 637, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^2}{x(d+ex^2)}dx$$

$$\Big\downarrow \text{5515}$$

$$\int \left(\frac{(a+b\arctan(cx))^2}{dx}-\frac{ex(a+b\arctan(cx))^2}{d(d+ex^2)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{d}+\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2d}+\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)}{2d}-\frac{(a+b\arctan(cx))^2\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(1-icx)(c\sqrt{-d}-i\sqrt{e})}\right)}{2d}-\frac{(a+b\arctan(cx))^2\log\left(\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(1-icx)(c\sqrt{-d}+i\sqrt{e})}\right)}{2d}-\frac{ib\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)(a+b\arctan(cx))}{d}-\frac{ib\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)(a+b\arctan(cx))}{d}+\frac{ib\operatorname{PolyLog}\left(2,\frac{2}{icx+1}-1\right)(a+b\arctan(cx))}{d}+\frac{\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))^2}{d}-\frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{4d}-\frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)}{4d}+\frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{2d}-\frac{b^2\operatorname{PolyLog}\left(3,1-\frac{2}{icx+1}\right)}{2d}+\frac{b^2\operatorname{PolyLog}\left(3,\frac{2}{icx+1}-1\right)}{2d}$$

input
```
Int[(a + b*ArcTan[c*x])^2/(x*(d + e*x^2)),x]
```

output
```
(2*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)])/d + ((a + b*ArcTan[c*
x])^2*Log[2/(1 - I*c*x)])/d - ((a + b*ArcTan[c*x])^2*Log[(2*c*(Sqrt[-d] -
Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(2*d) - ((a + b*ArcTa
n[c*x])^2*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 -
I*c*x))])/(2*d) - (I*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 - I*c*x)])/
d - (I*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)])/d + (I*b*(a +
b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)])/d + ((I/2)*b*(a + b*ArcTan[
c*x])*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e]
)*(1 - I*c*x))])/d + ((I/2)*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(Sqr
t[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/d + (b^2*Poly
Log[3, 1 - 2/(1 - I*c*x)])/(2*d) - (b^2*PolyLog[3, 1 - 2/(1 + I*c*x)])/(2*
d) + (b^2*PolyLog[3, -1 + 2/(1 + I*c*x)])/(2*d) - (b^2*PolyLog[3, 1 - (2*c
*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(4*d) -
(b^2*PolyLog[3, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])
*(1 - I*c*x))])/(4*d)
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

## Maple [F]

$$\int \frac{(a+b\arctan(cx))^2}{x\,(e\,x^2+d)}dx$$

input
```
int((a+b*arctan(c*x))^2/x/(e*x^2+d),x)
```

output
```
int((a+b*arctan(c*x))^2/x/(e*x^2+d),x)
```

## Fricas [F]

$$\int \frac{(a+b\arctan(cx))^2}{x\,(d+ex^2)}\,dx = \int \frac{(b\arctan(cx)+a)^2}{(ex^2+d)x}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)/(e*x^3 + d*x), x)
```

## Sympy [F]

$$\int \frac{(a+b\arctan(cx))^2}{x\,(d+ex^2)}\,dx = \int \frac{(a+b\,\mathrm{atan}\,(cx))^2}{x\,(d+ex^2)}\,dx$$

input
```
integrate((a+b*atan(c*x))**2/x/(e*x**2+d),x)
```

output
```
Integral((a + b*atan(c*x))**2/(x*(d + e*x**2)), x)
```

## Maxima [F]

$$\int \frac{(a+b\arctan(cx))^2}{x\,(d+ex^2)}\,dx = \int \frac{(b\arctan\,(cx)+a)^2}{(ex^2+d)x}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x/(e*x^2+d),x, algorithm="maxima")
```

output
```
-1/2*a^2*(log(e*x^2 + d)/d - 2*log(x)/d) + integrate((b^2*arctan(c*x)^2 +
2*a*b*arctan(c*x))/(e*x^3 + d*x), x)
```

## Giac [F]

$$\int \frac{(a+b\arctan(cx))^2}{x\,(d+ex^2)}\,dx = \int \frac{(b\arctan\,(cx)+a)^2}{(ex^2+d)x}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x/(e*x^2+d),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)^2/((e*x^2 + d)*x), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{x(d+ex^2)}\,dx = \int \frac{\left(a+b\,\mathrm{atan}(c\,x)\right)^2}{x\,\left(e\,x^2+d\right)}\,dx$$

input
```
int((a + b*atan(c*x))^2/(x*(d + e*x^2)),x)
```

output
```
int((a + b*atan(c*x))^2/(x*(d + e*x^2)), x)
```

## Reduce [F]

$$\int \frac{(a+b\arctan(cx))^2}{x(d+ex^2)}\,dx = \frac{4\left(\int \frac{atan(cx)}{e\,x^3+dx}dx\right)abd+2\left(\int \frac{atan(cx)^2}{e\,x^3+dx}dx\right)b^2d-\log(e\,x^2+d)\,a^2+2\log(x)\,a^2}{2d}$$

input
```
int((a+b*atan(c*x))^2/x/(e*x^2+d),x)
```

output
```
(4*int(atan(c*x)/(d*x + e*x**3),x)*a*b*d + 2*int(atan(c*x)**2/(d*x + e*x**
3),x)*b**2*d - log(d + e*x**2)*a**2 + 2*log(x)*a**2)/(2*d)
```

# 3.1264 $\int \frac{(a+b\arctan(cx))^2}{x^2(d+ex^2)}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 553

$$\begin{aligned}\int \frac{(a+b\arctan(cx))^2}{x^2(d+ex^2)}\,dx = &-\frac{ic(a+b\arctan(cx))^2}{d}-\frac{(a+b\arctan(cx))^2}{dx}\\
&+\frac{\sqrt{e}(a+b\arctan(cx))^2\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2(-d)^{3/2}}\\
&-\frac{\sqrt{e}(a+b\arctan(cx))^2\log\left(\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{2(-d)^{3/2}}\\
&+\frac{2bc(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right)}{d}\\
&-\frac{ib^2c\operatorname{PolyLog}\left(2,-1+\frac{2}{1-icx}\right)}{d}\\
&-\frac{ib\sqrt{e}(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2(-d)^{3/2}}\\
&+\frac{ib\sqrt{e}(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{2(-d)^{3/2}}\\
&+\frac{b^2\sqrt{e}\operatorname{PolyLog}\left(3,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{4(-d)^{3/2}}\\
&-\frac{b^2\sqrt{e}\operatorname{PolyLog}\left(3,1-\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{4(-d)^{3/2}}\end{aligned}$$

output
```
-I*c*(a+b*arctan(c*x))^2/d-(a+b*arctan(c*x))^2/d/x+1/2*e^(1/2)*(a+b*arctan
(c*x))^2*ln(2*c*((-d)^(1/2)-e^(1/2)*x)/(c*(-d)^(1/2)-I*e^(1/2))/(1-I*c*x))
/(-d)^(3/2)-1/2*e^(1/2)*(a+b*arctan(c*x))^2*ln(2*c*((-d)^(1/2)+e^(1/2)*x)/
(c*(-d)^(1/2)+I*e^(1/2))/(1-I*c*x))/(-d)^(3/2)+2*b*c*(a+b*arctan(c*x))*ln(
2-2/(1-I*c*x))/d-I*b^2*c*polylog(2,-1+2/(1-I*c*x))/d-1/2*I*b*e^(1/2)*(a+b*
arctan(c*x))*polylog(2,1-2*c*((-d)^(1/2)-e^(1/2)*x)/(c*(-d)^(1/2)-I*e^(1/2
))/(1-I*c*x))/(-d)^(3/2)+1/2*I*b*e^(1/2)*(a+b*arctan(c*x))*polylog(2,1-2*c
*((-d)^(1/2)+e^(1/2)*x)/(c*(-d)^(1/2)+I*e^(1/2))/(1-I*c*x))/(-d)^(3/2)+1/4
*b^2*e^(1/2)*polylog(3,1-2*c*((-d)^(1/2)-e^(1/2)*x)/(c*(-d)^(1/2)-I*e^(1/2
))/(1-I*c*x))/(-d)^(3/2)-1/4*b^2*e^(1/2)*polylog(3,1-2*c*((-d)^(1/2)+e^(1/
2)*x)/(c*(-d)^(1/2)+I*e^(1/2))/(1-I*c*x))/(-d)^(3/2)
```

## Mathematica [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+ex^2)}\,dx = \$\text{Aborted}$$

input
```
Integrate[(a + b*ArcTan[c*x])^2/(x^2*(d + e*x^2)),x]
```

output
```
$Aborted
```

## Rubi [A] (verified)

Time = 1.52 (sec) , antiderivative size = 564, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {5453, 5361, 5449, 2009, 5459, 5403, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+ex^2)}\,dx$$

$$\Big\downarrow \text{5453}$$

$$\frac{\int \frac{(a+b\arctan(cx))^2}{x^2}dx}{d}-\frac{e\int \frac{(a+b\arctan(cx))^2}{ex^2+d}dx}{d}$$

$$\Big\downarrow \text{ 5361}$$

$$\frac{2bc\int \frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx-\frac{(a+b\arctan(cx))^2}{x}}{d}-\frac{e\int \frac{(a+b\arctan(cx))^2}{ex^2+d}dx}{d}$$

$$\Big\downarrow \text{ 5449}$$

$$\frac{2bc\int \frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx-\frac{(a+b\arctan(cx))^2}{x}}{d}-\frac{e\int \left(\frac{\sqrt{-d}(a+b\arctan(cx))^2}{2d(\sqrt{-d}-\sqrt{e}x)}+\frac{\sqrt{-d}(a+b\arctan(cx))^2}{2d(\sqrt{e}x+\sqrt{-d})}\right)dx}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2bc\int \frac{a+b\arctan(cx)}{x(c^2x^2+1)}dx-\frac{(a+b\arctan(cx))^2}{x}}{d}-$$
$$\frac{e\left(-\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{(a+b\arctan(cx))^2}{2}\right.}{}$$

$$\Big\downarrow \text{ 5459}$$

$$\frac{-\frac{(a+b\arctan(cx))^2}{x}+2bc\left(i\int \frac{a+b\arctan(cx)}{x(cx+i)}dx-\frac{i(a+b\arctan(cx))^2}{2b}\right)}{d}-$$
$$\frac{e\left(-\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{(a+b\arctan(cx))^2}{2}\right.}{}$$

$$\Big\downarrow \text{ 5403}$$

$$\frac{-\frac{(a+b\arctan(cx))^2}{x}+2bc\left(i\left(ibc\int \frac{\log\left(2-\frac{2}{1-icx}\right)}{c^2x^2+1}dx-i\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))\right)-\frac{i(a+b\arctan(cx))^2}{2b}\right)}{d}-$$
$$\frac{e\left(-\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{(a+b\arctan(cx))^2}{2}\right.}{}$$

$$\Big\downarrow \text{ 2897}$$

$$\frac{-\frac{(a+b\arctan(cx))^2}{x}+2bc\left(i\left(-i\log\left(2-\frac{2}{1-icx}\right)(a+b\arctan(cx))-\frac{1}{2}b\operatorname{PolyLog}\left(2,\frac{2}{1-icx}-1\right)\right)-\frac{i(a+b\arctan(cx))^2}{2b}\right.}{d}$$

$$e\left(-\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)}{2\sqrt{-d}\sqrt{e}}+\frac{(a+b\arctan(cx))^2}{2}\right.$$

input
```
Int[(a + b*ArcTan[c*x])^2/(x^2*(d + e*x^2)),x]
```

output
```
(-((a + b*ArcTan[c*x])^2/x) + 2*b*c*(((-1/2*I)*(a + b*ArcTan[c*x])^2)/b +
I*((-I)*(a + b*ArcTan[c*x])*Log[2 - 2/(1 - I*c*x)] - (b*PolyLog[2, -1 + 2/
(1 - I*c*x)])/2)))/d - (e*(((a + b*ArcTan[c*x])^2*Log[(2*c*(Sqrt[-d] - Sqr
t[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(2*Sqrt[-d]*Sqrt[e]) - (
(a + b*ArcTan[c*x])^2*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sq
rt[e])*(1 - I*c*x))])/(2*Sqrt[-d]*Sqrt[e]) - ((I/2)*b*(a + b*ArcTan[c*x])*
PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 -
 I*c*x))])/(Sqrt[-d]*Sqrt[e]) + ((I/2)*b*(a + b*ArcTan[c*x])*PolyLog[2, 1
- (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(S
qrt[-d]*Sqrt[e]) + (b^2*PolyLog[3, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sq
rt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(4*Sqrt[-d]*Sqrt[e]) - (b^2*PolyLog[3,
1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/
(4*Sqrt[-d]*Sqrt[e])))/d
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5403
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x_
Symbol] :> Simp[(a + b*ArcTan[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] - Si
mp[b*c*(p/d)   Int[(a + b*ArcTan[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]/(1
 + c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*
d^2 + e^2, 0]
```

rule 5449
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (d + e*x^2)^q, x], x
] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[q] && IGtQ[p, 0]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5459
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
x_Symbol] :> Simp[(-I)*((a + b*ArcTan[c*x])^(p + 1)/(b*d*(p + 1))), x] + Si
mp[I/d   Int[(a + b*ArcTan[c*x])^p/(x*(I + c*x)), x], x] /; FreeQ[{a, b, c,
 d, e}, x] && EqQ[e, c^2*d] && GtQ[p, 0]
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 74.88 (sec) , antiderivative size = 101616, normalized size of antiderivative = 183.75

| method | result | size |
|---|---|---|
| derivativedivides | Expression too large to display | 101616 |
| default | Expression too large to display | 101616 |

input
```
int((a+b*arctan(c*x))^2/x^2/(e*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

## Fricas [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+ex^2)}\,dx = \int \frac{(b\arctan(cx)+a)^2}{(ex^2+d)x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^2/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)/(e*x^4 + d*x^2), x)
```

## Sympy [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+ex^2)}\,dx = \int \frac{(a+b\,\mathrm{atan}(cx))^2}{x^2(d+ex^2)}\,dx$$

input
```
integrate((a+b*atan(c*x))**2/x**2/(e*x**2+d),x)
```

output
```
Integral((a + b*atan(c*x))**2/(x**2*(d + e*x**2)), x)
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+ex^2)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arctan(c*x))^2/x^2/(e*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

## Giac [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+ex^2)}\,dx = \int \frac{(b\arctan(cx)+a)^2}{(ex^2+d)x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^2/(e*x^2+d),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)^2/((e*x^2 + d)*x^2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+ex^2)}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^2}{x^2\,(e\,x^2+d)}\,dx$$

input
```
int((a + b*atan(c*x))^2/(x^2*(d + e*x^2)),x)
```

output
```
int((a + b*atan(c*x))^2/(x^2*(d + e*x^2)), x)
```

## Reduce [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+ex^2)}\,dx$$
$$= \frac{-\sqrt{e}\,\sqrt{d}\,atan\left(\frac{ex}{\sqrt{e}\,\sqrt{d}}\right)a^2x - atan(cx)^2\,abcdx - 2atan(cx)\,abd + 2\left(\int \frac{atan(cx)}{c^2e\,x^4+c^2d\,x^2+e\,x^2+d}dx\right)ab\,c^2d^2x - 2(}{d^2x}$$

input
```
int((a+b*atan(c*x))^2/x^2/(e*x^2+d),x)
```

output
```
( - sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a**2*x - atan(c*x)**2*a*
b*c*d*x - 2*atan(c*x)*a*b*d + 2*int(atan(c*x)/(c**2*d*x**2 + c**2*e*x**4 +
 d + e*x**2),x)*a*b*c**2*d**2*x - 2*int(atan(c*x)/(c**2*d*x**2 + c**2*e*x*
*4 + d + e*x**2),x)*a*b*d*e*x + int(atan(c*x)**2/(d*x**2 + e*x**4),x)*b**2
*d**2*x - log(c**2*x**2 + 1)*a*b*c*d*x + 2*log(x)*a*b*c*d*x - a**2*d)/(d**
2*x)
```

# 3.1265 $\int \frac{(a+b\arctan(cx))^2}{x^3(d+ex^2)}\,dx$

## Optimal result

Integrand size = 23, antiderivative size = 745

$$\int \frac{(a+b\arctan(cx))^2}{x^3(d+ex^2)}\,dx = -\frac{bc(a+b\arctan(cx))}{dx} - \frac{c^2(a+b\arctan(cx))^2}{2d} - \frac{(a+b\arctan(cx))^2}{2dx^2} - \frac{2e(a+b\arctan(cx))^2\operatorname{arctanh}\left(1-\frac{2}{1+icx}\right)}{d^2} + \frac{b^2c^2\log(x)}{d} - \frac{e(a+b\arctan(cx))^2\log\left(\frac{2}{1-icx}\right)}{d^2} + \frac{e(a+b\arctan(cx))^2\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2d^2} + \frac{e(a+b\arctan(cx))^2\log\left(\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{2d^2} - \frac{b^2c^2\log(1+c^2x^2)}{2d} + \frac{ibe(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{d^2} + \frac{ibe(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1+icx}\right)}{d^2} - \frac{ibe(a+b\arctan(cx))\operatorname{PolyLog}\left(2,-1+\frac{2}{1+icx}\right)}{d^2} - \frac{ibe(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2d^2} - \frac{ibe(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{2d^2} - \frac{b^2e\operatorname{PolyLog}\left(3,1-\frac{2}{1-icx}\right)}{2d^2} + \frac{b^2e\operatorname{PolyLog}\left(3,1-\frac{2}{1+icx}\right)}{2d^2} - \frac{b^2e\operatorname{PolyLog}\left(3,-1+\frac{2}{1+icx}\right)}{2d^2} + \frac{b^2e\operatorname{PolyLog}\left(3,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{4d^2} + \frac{b^2e\operatorname{PolyLog}\left(3,1-\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{4d^2}$$

output
```
-b*c*(a+b*arctan(c*x))/d/x-1/2*c^2*(a+b*arctan(c*x))^2/d-1/2*(a+b*arctan(c
*x))^2/d/x^2+2*e*(a+b*arctan(c*x))^2*arctanh(-1+2/(1+I*c*x))/d^2+b^2*c^2*l
n(x)/d-e*(a+b*arctan(c*x))^2*ln(2/(1-I*c*x))/d^2+1/2*e*(a+b*arctan(c*x))^2
*ln(2*c*((-d)^(1/2)-e^(1/2)*x)/(c*(-d)^(1/2)-I*e^(1/2))/(1-I*c*x))/d^2+1/2
*e*(a+b*arctan(c*x))^2*ln(2*c*((-d)^(1/2)+e^(1/2)*x)/(c*(-d)^(1/2)+I*e^(1/
2))/(1-I*c*x))/d^2-1/2*b^2*c^2*ln(c^2*x^2+1)/d+I*b*e*(a+b*arctan(c*x))*pol
ylog(2,1-2/(1+I*c*x))/d^2-I*b*e*(a+b*arctan(c*x))*polylog(2,-1+2/(1+I*c*x)
)/d^2-1/2*I*b*e*(a+b*arctan(c*x))*polylog(2,1-2*c*((-d)^(1/2)+e^(1/2)*x)/(
c*(-d)^(1/2)+I*e^(1/2))/(1-I*c*x))/d^2+I*b*e*(a+b*arctan(c*x))*polylog(2,1
-2/(1-I*c*x))/d^2-1/2*I*b*e*(a+b*arctan(c*x))*polylog(2,1-2*c*((-d)^(1/2)-
e^(1/2)*x)/(c*(-d)^(1/2)-I*e^(1/2))/(1-I*c*x))/d^2-1/2*b^2*e*polylog(3,1-2
/(1-I*c*x))/d^2+1/2*b^2*e*polylog(3,1-2/(1+I*c*x))/d^2-1/2*b^2*e*polylog(3
,-1+2/(1+I*c*x))/d^2+1/4*b^2*e*polylog(3,1-2*c*((-d)^(1/2)-e^(1/2)*x)/(c*(
-d)^(1/2)-I*e^(1/2))/(1-I*c*x))/d^2+1/4*b^2*e*polylog(3,1-2*c*((-d)^(1/2)+
e^(1/2)*x)/(c*(-d)^(1/2)+I*e^(1/2))/(1-I*c*x))/d^2
```

## Mathematica [A] (warning: unable to verify)

Time = 11.97 (sec) , antiderivative size = 1409, normalized size of antiderivative = 1.89

$$\int \frac{(a+b\arctan(cx))^2}{x^3(d+ex^2)}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*ArcTan[c*x])^2/(x^3*(d + e*x^2)),x]
```

output
```
-1/24*((12*a^2*d)/x^2 + (24*a*b*c*d)/x + (24*a*b*d*(1 + c^2*x^2)*ArcTan[c*
x])/x^2 + 24*a^2*e*Log[x] - 12*a^2*e*Log[d + e*x^2] - (24*I)*a*b*e*(ArcTan
[c*x]*(ArcTan[c*x] + (2*I)*Log[1 - E^((2*I)*ArcTan[c*x])]) + PolyLog[2, E^
((2*I)*ArcTan[c*x])]) - (48*a*b*(c^2*d - e)*e*((-I)*ArcTan[c*x]^2 + (2*I)*
ArcSin[Sqrt[(c^2*d)/(c^2*d - e)]]*ArcTan[(c*e*x)/Sqrt[c^2*d*e]] + (-ArcSin
[Sqrt[(c^2*d)/(c^2*d - e)]] + ArcTan[c*x])*Log[1 + ((c^2*d + e + 2*Sqrt[c^
2*d*e])*E^((2*I)*ArcTan[c*x]))/(c^2*d - e)] + (ArcSin[Sqrt[(c^2*d)/(c^2*d
- e)]] + ArcTan[c*x])*Log[(-2*Sqrt[c^2*d*e]*E^((2*I)*ArcTan[c*x]) + e*(-1
+ E^((2*I)*ArcTan[c*x])) + c^2*d*(1 + E^((2*I)*ArcTan[c*x])))/(c^2*d - e)]
 - (I/2)*(PolyLog[2, ((-(c^2*d) - e + 2*Sqrt[c^2*d*e])*E^((2*I)*ArcTan[c*x
]))/(c^2*d - e)] + PolyLog[2, -(((c^2*d + e + 2*Sqrt[c^2*d*e])*E^((2*I)*Ar
cTan[c*x]))/(c^2*d - e))])))/(2*c^2*d - 2*e) + b^2*((-I)*e*Pi^3 + (24*c*d*
ArcTan[c*x])/x + (12*d*(1 + c^2*x^2)*ArcTan[c*x]^2)/x^2 + (8*I)*e*ArcTan[c
*x]^3 + 24*e*ArcTan[c*x]^2*Log[1 - E^((-2*I)*ArcTan[c*x])] - 24*c^2*d*Log[
(c*x)/Sqrt[1 + c^2*x^2]] + (24*I)*e*ArcTan[c*x]*PolyLog[2, E^((-2*I)*ArcTa
n[c*x])] + 12*e*PolyLog[3, E^((-2*I)*ArcTan[c*x])]) + 2*b^2*e*((4*I)*ArcTa
n[c*x]^3 + 12*ArcSin[Sqrt[(c^2*d)/(c^2*d - e)]]*ArcTan[c*x]*Log[1 + ((c^2*
d + e + 2*Sqrt[c^2*d*e])*E^((2*I)*ArcTan[c*x]))/(c^2*d - e)] - 12*ArcTan[c
*x]^2*Log[1 + ((c^2*d + e + 2*Sqrt[c^2*d*e])*E^((2*I)*ArcTan[c*x]))/(c^2*d
 - e)] - 12*ArcSin[Sqrt[(c^2*d)/(c^2*d - e)]]*ArcTan[c*x]*Log[(-2*Sqrt[...
```

## Rubi [A] (verified)

Time = 1.81 (sec) , antiderivative size = 724, normalized size of antiderivative = 0.97, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {5453, 5361, 5453, 5361, 243, 47, 14, 16, 5419, 5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^2}{x^3(d+ex^2)}dx$$

$$\Big\downarrow \text{5453}$$

$$\frac{\int \frac{(a+b\arctan(cx))^2}{x^3}dx}{d}-\frac{e\int \frac{(a+b\arctan(cx))^2}{x(ex^2+d)}dx}{d}$$

$$\Big\downarrow \text{5361}$$

$$\frac{bc\int \frac{a+b\arctan(cx)}{x^2(c^2x^2+1)}dx-\frac{(a+b\arctan(cx))^2}{2x^2}}{d}-\frac{e\int \frac{(a+b\arctan(cx))^2}{x(ex^2+d)}dx}{d}$$

$\Big\downarrow$ 5453

$$\frac{bc\left(\int \frac{a+b\arctan(cx)}{x^2}dx-c^2\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)-\frac{(a+b\arctan(cx))^2}{2x^2}}{d}-\frac{e\int \frac{(a+b\arctan(cx))^2}{x(ex^2+d)}dx}{d}$$

$\Big\downarrow$ 5361

$$\frac{bc\left(c^2\left(-\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)+bc\int \frac{1}{x(c^2x^2+1)}dx-\frac{a+b\arctan(cx)}{x}\right)-\frac{(a+b\arctan(cx))^2}{2x^2}}{d}-\frac{e\int \frac{(a+b\arctan(cx))^2}{x(ex^2+d)}dx}{d}$$

$\Big\downarrow$ 243

$$\frac{bc\left(c^2\left(-\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)+\frac{1}{2}bc\int \frac{1}{x^2(c^2x^2+1)}dx^2-\frac{a+b\arctan(cx)}{x}\right)-\frac{(a+b\arctan(cx))^2}{2x^2}}{d}-\frac{e\int \frac{(a+b\arctan(cx))^2}{x(ex^2+d)}dx}{d}$$

$\Big\downarrow$ 47

$$\frac{bc\left(c^2\left(-\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)+\frac{1}{2}bc\left(\int \frac{1}{x^2}dx^2-c^2\int \frac{1}{c^2x^2+1}dx^2\right)-\frac{a+b\arctan(cx)}{x}\right)-\frac{(a+b\arctan(cx))^2}{2x^2}}{d}-\frac{e\int \frac{(a+b\arctan(cx))^2}{x(ex^2+d)}dx}{d}$$

$\Big\downarrow$ 14

$$\frac{bc\left(c^2\left(-\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)+\frac{1}{2}bc\left(\log\left(x^2\right)-c^2\int \frac{1}{c^2x^2+1}dx^2\right)-\frac{a+b\arctan(cx)}{x}\right)-\frac{(a+b\arctan(cx))^2}{2x^2}}{d}-\frac{e\int \frac{(a+b\arctan(cx))^2}{x(ex^2+d)}dx}{d}$$

$\Big\downarrow$ 16

$$\frac{bc\left(c^2\left(-\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)-\frac{a+b\arctan(cx)}{x}+\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right)\right)\right)-\frac{(a+b\arctan(cx))^2}{2x^2}}{d}-\frac{e\int \frac{(a+b\arctan(cx))^2}{x(ex^2+d)}dx}{d}$$

$$\Big\downarrow \text{5419}$$

$$\frac{bc\left(-\frac{c(a+b\arctan(cx))^2}{2b}-\frac{a+b\arctan(cx)}{x}+\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right)\right)\right)-\frac{(a+b\arctan(cx))^2}{2x^2}}{d}-\frac{e\int\frac{(a+b\arctan(cx))^2}{x(ex^2+d)}dx}{d}$$

$$\Big\downarrow \text{5515}$$

$$\frac{bc\left(-\frac{c(a+b\arctan(cx))^2}{2b}-\frac{a+b\arctan(cx)}{x}+\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right)\right)\right)-\frac{(a+b\arctan(cx))^2}{2x^2}}{d}-\frac{e\int\left(\frac{(a+b\arctan(cx))^2}{dx}-\frac{ex(a+b\arctan(cx))^2}{d(ex^2+d)}\right)dx}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{bc\left(-\frac{c(a+b\arctan(cx))^2}{2b}-\frac{a+b\arctan(cx)}{x}+\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right)\right)\right)-\frac{(a+b\arctan(cx))^2}{2x^2}}{d}-$$

$$e\left(\frac{2\text{arctanh}\left(1-\frac{2}{1+icx}\right)(a+b\arctan(cx))^2}{d}+\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2d}+\frac{ib(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\right.}{2d}\right.$$

input
```
Int[(a + b*ArcTan[c*x])^2/(x^3*(d + e*x^2)),x]
```

output
```
(-1/2*(a + b*ArcTan[c*x])^2/x^2 + b*c*(-((a + b*ArcTan[c*x])/x) - (c*(a +
b*ArcTan[c*x])^2)/(2*b) + (b*c*(Log[x^2] - Log[1 + c^2*x^2]))/2))/d - (e*(
(2*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)])/d + ((a + b*ArcTan[c*
x])^2*Log[2/(1 - I*c*x)])/d - ((a + b*ArcTan[c*x])^2*Log[(2*c*(Sqrt[-d] -
Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(2*d) - ((a + b*ArcTa
n[c*x])^2*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 -
I*c*x))])/(2*d) - (I*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 - I*c*x)])/
d - (I*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)])/d + (I*b*(a +
b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)])/d + ((I/2)*b*(a + b*ArcTan[
c*x])*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e]
)*(1 - I*c*x))])/d + ((I/2)*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(Sqr
t[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/d + (b^2*Poly
Log[3, 1 - 2/(1 - I*c*x)])/(2*d) - (b^2*PolyLog[3, 1 - 2/(1 + I*c*x)])/(2*
d) + (b^2*PolyLog[3, -1 + 2/(1 + I*c*x)])/(2*d) - (b^2*PolyLog[3, 1 - (2*c
*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(4*d) -
(b^2*PolyLog[3, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])
*(1 - I*c*x))])/(4*d)))/d
```

**Defintions of rubi rules used**

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

## Maple [F]

$$\int \frac{(a + b\arctan(cx))^2}{x^3(e\,x^2 + d)}dx$$

input
```
int((a+b*arctan(c*x))^2/x^3/(e*x^2+d),x)
```

output
```
int((a+b*arctan(c*x))^2/x^3/(e*x^2+d),x)
```

## Fricas [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^3\,(d+ex^2)}\,dx = \int \frac{(b\arctan(cx)+a)^2}{(ex^2+d)x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^3/(e*x^2+d),x, algorithm="fricas")
```

output
```
integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)/(e*x^5 + d*x^3), x)
```

## Sympy [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^3\,(d+ex^2)}\,dx = \int \frac{(a+b\,\mathrm{atan}\,(cx))^2}{x^3\,(d+ex^2)}\,dx$$

input
```
integrate((a+b*atan(c*x))**2/x**3/(e*x**2+d),x)
```

output
```
Integral((a + b*atan(c*x))**2/(x**3*(d + e*x**2)), x)
```

## Maxima [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^3\,(d+ex^2)}\,dx = \int \frac{(b\arctan(cx)+a)^2}{(ex^2+d)x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^3/(e*x^2+d),x, algorithm="maxima")
```

output
```
1/2*a^2*(e*log(e*x^2 + d)/d^2 - 2*e*log(x)/d^2 - 1/(d*x^2)) + integrate((b
^2*arctan(c*x)^2 + 2*a*b*arctan(c*x))/(e*x^5 + d*x^3), x)
```

## Giac [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^3\left(d+ex^2\right)}\,dx=\int \frac{(b\arctan(cx)+a)^2}{(ex^2+d)x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^3/(e*x^2+d),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)^2/((e*x^2 + d)*x^3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{x^3\left(d+ex^2\right)}\,dx=\int \frac{\left(a+b\,\mathrm{atan}(c\,x)\right)^2}{x^3\,\left(e\,x^2+d\right)}\,dx$$

input
```
int((a + b*atan(c*x))^2/(x^3*(d + e*x^2)),x)
```

output
```
int((a + b*atan(c*x))^2/(x^3*(d + e*x^2)), x)
```

## Reduce [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^3\left(d+ex^2\right)}\,dx=\frac{4\left(\int \frac{\mathrm{atan}(cx)}{e\,x^5+d\,x^3}dx\right)ab\,d^2x^2+2\left(\int \frac{\mathrm{atan}(cx)^2}{e\,x^5+d\,x^3}dx\right)b^2d^2x^2+\log(e\,x^2+d)\,a^2e\,x^2-2\log(x)\,a^2e\,x^2-a^2d}{2d^2x^2}$$

input
```
int((a+b*atan(c*x))^2/x^3/(e*x^2+d),x)
```

output
```
(4*int(atan(c*x)/(d*x**3 + e*x**5),x)*a*b*d**2*x**2 + 2*int(atan(c*x)**2/(
d*x**3 + e*x**5),x)*b**2*d**2*x**2 + log(d + e*x**2)*a**2*e*x**2 - 2*log(x
)*a**2*e*x**2 - a**2*d)/(2*d**2*x**2)
```

# 3.1266 $\int \frac{x^3(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 943

$$\int \frac{x^3(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx = \text{Too large to display}$$

output
```
-1/2*c^2*d*(a+b*arctan(c*x))^2/(c^2*d-e)/e^2+1/4*(a+b*arctan(c*x))^2/e^2/(
1-e^(1/2)*x/(-d)^(1/2))+1/4*(a+b*arctan(c*x))^2/e^2/(1+e^(1/2)*x/(-d)^(1/2
))-(a+b*arctan(c*x))^2*ln(2/(1-I*c*x))/e^2-1/2*b*c*(-d)^(1/2)*(a+b*arctan(
c*x))*ln(2*c*((-d)^(1/2)-e^(1/2)*x)/(c*(-d)^(1/2)-I*e^(1/2))/(1-I*c*x))/(c
^2*d-e)/e^(3/2)+1/2*(a+b*arctan(c*x))^2*ln(2*c*((-d)^(1/2)-e^(1/2)*x)/(c*(
-d)^(1/2)-I*e^(1/2))/(1-I*c*x))/e^2+1/2*b*c*(-d)^(1/2)*(a+b*arctan(c*x))*l
n(2*c*((-d)^(1/2)+e^(1/2)*x)/(c*(-d)^(1/2)+I*e^(1/2))/(1-I*c*x))/(c^2*d-e)
/e^(3/2)+1/2*(a+b*arctan(c*x))^2*ln(2*c*((-d)^(1/2)+e^(1/2)*x)/(c*(-d)^(1/
2)+I*e^(1/2))/(1-I*c*x))/e^2-1/2*I*b*(a+b*arctan(c*x))*polylog(2,1-2*c*((-
d)^(1/2)-e^(1/2)*x)/(c*(-d)^(1/2)-I*e^(1/2))/(1-I*c*x))/e^2+1/4*I*b^2*c*(-
d)^(1/2)*polylog(2,1-2*c*((-d)^(1/2)-e^(1/2)*x)/(c*(-d)^(1/2)-I*e^(1/2))/(
1-I*c*x))/(c^2*d-e)/e^(3/2)+I*b*(a+b*arctan(c*x))*polylog(2,1-2/(1-I*c*x))
/e^2-1/2*I*b*(a+b*arctan(c*x))*polylog(2,1-2*c*((-d)^(1/2)+e^(1/2)*x)/(c*(
-d)^(1/2)+I*e^(1/2))/(1-I*c*x))/e^2-1/4*I*b^2*c*(-d)^(1/2)*polylog(2,1-2*c
*((-d)^(1/2)+e^(1/2)*x)/(c*(-d)^(1/2)+I*e^(1/2))/(1-I*c*x))/(c^2*d-e)/e^(3
/2)-1/2*b^2*polylog(3,1-2/(1-I*c*x))/e^2+1/4*b^2*polylog(3,1-2*c*((-d)^(1/
2)-e^(1/2)*x)/(c*(-d)^(1/2)-I*e^(1/2))/(1-I*c*x))/e^2+1/4*b^2*polylog(3,1-
2*c*((-d)^(1/2)+e^(1/2)*x)/(c*(-d)^(1/2)+I*e^(1/2))/(1-I*c*x))/e^2
```

## Mathematica [F]

$$\int \frac{x^3(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx=\int \frac{x^3(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx$$

input
```
Integrate[(x^3*(a + b*ArcTan[c*x])^2)/(d + e*x^2)^2,x]
```

output
```
Integrate[(x^3*(a + b*ArcTan[c*x])^2)/(d + e*x^2)^2, x]
```

## Rubi [A] (verified)

Time = 1.92 (sec) , antiderivative size = 943, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx$$

$$\Big\downarrow \text{5515}$$

$$\int \left(\frac{x(a+b\arctan(cx))^2}{e\,(d+ex^2)}-\frac{dx(a+b\arctan(cx))^2}{e\,(d+ex^2)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{ic\sqrt{-d}\,\text{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)b^2}{4\left(c^2d-e\right)e^{3/2}}-\frac{ic\sqrt{-d}\,\text{PolyLog}\left(2,1-\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)b^2}{4\left(c^2d-e\right)e^{3/2}}-\frac{\text{PolyLog}\left(3,1-\frac{2}{1-icx}\right)b^2}{2e^2}+\frac{\text{PolyLog}\left(3,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)b^2}{4e^2}+\frac{\text{PolyLog}\left(3,1-\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)b^2}{4e^2}-\frac{c\sqrt{-d}(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)b}{2\left(c^2d-e\right)e^{3/2}}+\frac{c\sqrt{-d}(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)b}{2\left(c^2d-e\right)e^{3/2}}+\frac{i(a+b\arctan(cx))\,\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)b}{e^2}-\frac{i(a+b\arctan(cx))\,\text{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)b}{2e^2}-\frac{i(a+b\arctan(cx))\,\text{PolyLog}\left(2,1-\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)b}{2e^2}+\frac{(a+b\arctan(cx))^2}{4e^2\left(1-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}+\frac{(a+b\arctan(cx))^2}{4e^2\left(\frac{\sqrt{e}x}{\sqrt{-d}}+1\right)}-\frac{c^2d(a+b\arctan(cx))^2}{2\left(c^2d-e\right)e^2}-\frac{(a+b\arctan(cx))^2\log\left(\frac{2}{1-icx}\right)}{e^2}+\frac{(a+b\arctan(cx))^2\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2e^2}+\frac{(a+b\arctan(cx))^2\log\left(\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)}{2e^2}$$

input
```
Int[(x^3*(a + b*ArcTan[c*x])^2)/(d + e*x^2)^2,x]
```

output
```
-1/2*(c^2*d*(a + b*ArcTan[c*x])^2)/((c^2*d - e)*e^2) + (a + b*ArcTan[c*x])
^2/(4*e^2*(1 - (Sqrt[e]*x)/Sqrt[-d])) + (a + b*ArcTan[c*x])^2/(4*e^2*(1 +
(Sqrt[e]*x)/Sqrt[-d])) - ((a + b*ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/e^2 -
(b*c*Sqrt[-d]*(a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqr
t[-d] - I*Sqrt[e])*(1 - I*c*x))]])/(2*(c^2*d - e)*e^(3/2)) + ((a + b*ArcTan
[c*x])^2*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I
*c*x))]])/(2*e^2) + (b*c*Sqrt[-d]*(a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] +
Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))]])/(2*(c^2*d - e)*e^(3/2
)) + ((a + b*ArcTan[c*x])^2*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d]
+ I*Sqrt[e])*(1 - I*c*x))]])/(2*e^2) + (I*b*(a + b*ArcTan[c*x])*PolyLog[2,
1 - 2/(1 - I*c*x)])/e^2 + ((I/4)*b^2*c*Sqrt[-d]*PolyLog[2, 1 - (2*c*(Sqrt[
-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))]])/((c^2*d - e)*e^
(3/2)) - ((I/2)*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt
[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))]])/e^2 - ((I/4)*b^2*c*Sqrt[-
d]*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(
1 - I*c*x))]])/((c^2*d - e)*e^(3/2)) - ((I/2)*b*(a + b*ArcTan[c*x])*PolyLog
[2, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x)
)]])/e^2 - (b^2*PolyLog[3, 1 - 2/(1 - I*c*x)])/(2*e^2) + (b^2*PolyLog[3, 1
- (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))]])/(4
*e^2) + (b^2*PolyLog[3, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] +...
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

## Maple [F]

$$\int \frac{x^3 (a+b\arctan(cx))^2}{(e\,x^2+d)^2}dx$$

input
```
int(x^3*(a+b*arctan(c*x))^2/(e*x^2+d)^2,x)
```

output
```
int(x^3*(a+b*arctan(c*x))^2/(e*x^2+d)^2,x)
```

## Fricas [F]

$$\int \frac{x^3 (a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx = \int \frac{(b\arctan(cx)+a)^2 x^3}{(ex^2+d)^2}\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))^2/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b^2*x^3*arctan(c*x)^2 + 2*a*b*x^3*arctan(c*x) + a^2*x^3)/(e^2*x^
4 + 2*d*e*x^2 + d^2), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^3 (a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(a+b*atan(c*x))**2/(e*x**2+d)**2,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{x^3(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx = \int \frac{(b\arctan(cx)+a)^2x^3}{(ex^2+d)^2}\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))^2/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
1/2*a^2*(d/(e^3*x^2 + d*e^2) + log(e*x^2 + d)/e^2) + integrate((b^2*x^3*ar
ctan(c*x)^2 + 2*a*b*x^3*arctan(c*x))/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

## Giac [F]

$$\int \frac{x^3(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx = \int \frac{(b\arctan(cx)+a)^2x^3}{(ex^2+d)^2}\,dx$$

input
```
integrate(x^3*(a+b*arctan(c*x))^2/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)^2*x^3/(e*x^2 + d)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx = \int \frac{x^3\,(a+b\,\mathrm{atan}(c\,x))^2}{(e\,x^2+d)^2}\,dx$$

input
```
int((x^3*(a + b*atan(c*x))^2)/(d + e*x^2)^2,x)
```

output
```
int((x^3*(a + b*atan(c*x))^2)/(d + e*x^2)^2, x)
```

## Reduce [F]

$$\int \frac{x^3(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx$$
$$= \frac{4\left(\int \frac{atan(cx)x^3}{e^2x^4+2de\,x^2+d^2}dx\right)abd\,e^2+4\left(\int \frac{atan(cx)x^3}{e^2x^4+2de\,x^2+d^2}dx\right)ab\,e^3x^2+2\left(\int \frac{atan(cx)^2x^3}{e^2x^4+2de\,x^2+d^2}dx\right)b^2d\,e^2+2\left(\int \frac{atan(}{e^2x^4+2}\right.}{2e^2\left(e\,x^2+d\right)}$$

input
```
int(x^3*(a+b*atan(c*x))^2/(e*x^2+d)^2,x)
```

output
```
(4*int((atan(c*x)*x**3)/(d**2 + 2*d*e*x**2 + e**2*x**4),x)*a*b*d*e**2 + 4*
int((atan(c*x)*x**3)/(d**2 + 2*d*e*x**2 + e**2*x**4),x)*a*b*e**3*x**2 + 2*
int((atan(c*x)**2*x**3)/(d**2 + 2*d*e*x**2 + e**2*x**4),x)*b**2*d*e**2 + 2
*int((atan(c*x)**2*x**3)/(d**2 + 2*d*e*x**2 + e**2*x**4),x)*b**2*e**3*x**2
 + log(d + e*x**2)*a**2*d + log(d + e*x**2)*a**2*e*x**2 - a**2*e*x**2)/(2*
e**2*(d + e*x**2))
```

# 3.1267 $\int \frac{x^2(a+b\arctan(cx))^2}{(d+ex^2)^2} dx$



## Optimal result

Integrand size = 23, antiderivative size = 1033

$$\int \frac{x^2(a+b\arctan(cx))^2}{(d+ex^2)^2} dx = \text{Too large to display}$$

output
```
-1/2*I*b^2*c*polylog(2,1-2/(1+I*c*x))/(c^2*d-e)/e+1/4*(a+b*arctan(c*x))^2/
e^(3/2)/((-d)^(1/2)-e^(1/2)*x)-1/4*(a+b*arctan(c*x))^2/e^(3/2)/((-d)^(1/2)
+e^(1/2)*x)+b*c*(a+b*arctan(c*x))*ln(2/(1-I*c*x))/(c^2*d-e)/e-b*c*(a+b*arc
tan(c*x))*ln(2/(1+I*c*x))/(c^2*d-e)/e-1/2*b*c*(a+b*arctan(c*x))*ln(2*c*((-
d)^(1/2)-e^(1/2)*x)/(c*(-d)^(1/2)-I*e^(1/2))/(1-I*c*x))/(c^2*d-e)/e+1/4*(a
+b*arctan(c*x))^2*ln(2*c*((-d)^(1/2)-e^(1/2)*x)/(c*(-d)^(1/2)-I*e^(1/2))/(
1-I*c*x))/(-d)^(1/2)/e^(3/2)-1/2*b*c*(a+b*arctan(c*x))*ln(2*c*((-d)^(1/2)+
e^(1/2)*x)/(c*(-d)^(1/2)+I*e^(1/2))/(1-I*c*x))/(c^2*d-e)/e-1/4*(a+b*arctan
(c*x))^2*ln(2*c*((-d)^(1/2)+e^(1/2)*x)/(c*(-d)^(1/2)+I*e^(1/2))/(1-I*c*x))
/(-d)^(1/2)/e^(3/2)+1/4*I*b^2*c*polylog(2,1-2*c*((-d)^(1/2)+e^(1/2)*x)/(c*
(-d)^(1/2)+I*e^(1/2))/(1-I*c*x))/(c^2*d-e)/e+1/4*I*b^2*c*polylog(2,1-2*c*(
(-d)^(1/2)-e^(1/2)*x)/(c*(-d)^(1/2)-I*e^(1/2))/(1-I*c*x))/(c^2*d-e)/e+1/4*
I*b*(a+b*arctan(c*x))*polylog(2,1-2*c*((-d)^(1/2)+e^(1/2)*x)/(c*(-d)^(1/2)
+I*e^(1/2))/(1-I*c*x))/(-d)^(1/2)/e^(3/2)-1/4*I*b*(a+b*arctan(c*x))*polylo
g(2,1-2*c*((-d)^(1/2)-e^(1/2)*x)/(c*(-d)^(1/2)-I*e^(1/2))/(1-I*c*x))/(-d)^
(1/2)/e^(3/2)-1/2*I*b^2*c*polylog(2,1-2/(1-I*c*x))/(c^2*d-e)/e-1/2*I*c*(a+
b*arctan(c*x))^2/(c^2*d-e)/e+1/8*b^2*polylog(3,1-2*c*((-d)^(1/2)-e^(1/2)*x
)/(c*(-d)^(1/2)-I*e^(1/2))/(1-I*c*x))/(-d)^(1/2)/e^(3/2)-1/8*b^2*polylog(3
,1-2*c*((-d)^(1/2)+e^(1/2)*x)/(c*(-d)^(1/2)+I*e^(1/2))/(1-I*c*x))/(-d)^(1/
2)/e^(3/2)
```

## Mathematica [F]

$$\int \frac{x^2(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx=\int \frac{x^2(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx$$

input
```
Integrate[(x^2*(a + b*ArcTan[c*x])^2)/(d + e*x^2)^2,x]
```

output
```
Integrate[(x^2*(a + b*ArcTan[c*x])^2)/(d + e*x^2)^2, x]
```

## Rubi [A] (verified)

Time = 2.17 (sec) , antiderivative size = 1033, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx$$

$$\Big\downarrow \text{5515}$$

$$\int \left(\frac{(a+b\arctan(cx))^2}{e(d+ex^2)}-\frac{d(a+b\arctan(cx))^2}{e(d+ex^2)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ic\,\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)b^2}{2(c^2d-e)e}-\frac{ic\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)b^2}{2(c^2d-e)e}+\frac{ic\,\text{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)b^2}{4(c^2d-e)e}+\frac{ic\,\text{PolyLog}\left(2,1-\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)b^2}{4(c^2d-e)e}+\frac{\text{PolyLog}\left(3,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)b^2}{8\sqrt{-d}e^{3/2}}-\frac{\text{PolyLog}\left(3,1-\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)b^2}{8\sqrt{-d}e^{3/2}}+\frac{c(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)b}{(c^2d-e)e}-\frac{c(a+b\arctan(cx))\log\left(\frac{2}{icx+1}\right)b}{(c^2d-e)e}-\frac{c(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)b}{2(c^2d-e)e}-\frac{c(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)b}{2(c^2d-e)e}-\frac{i(a+b\arctan(cx))\,\text{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)b}{4\sqrt{-d}e^{3/2}}+\frac{i(a+b\arctan(cx))\,\text{PolyLog}\left(2,1-\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)b}{4\sqrt{-d}e^{3/2}}-\frac{ic(a+b\arctan(cx))^2}{2(c^2d-e)e}+\frac{(a+b\arctan(cx))^2}{4e^{3/2}(\sqrt{-d}-\sqrt{e}x)}-\frac{(a+b\arctan(cx))^2}{4e^{3/2}(\sqrt{e}x+\sqrt{-d})}+\frac{(a+b\arctan(cx))^2\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{4\sqrt{-d}e^{3/2}}-\frac{(a+b\arctan(cx))^2\log\left(\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)}{4\sqrt{-d}e^{3/2}}$$

input
```
Int[(x^2*(a + b*ArcTan[c*x])^2)/(d + e*x^2)^2,x]
```

output
```
((-1/2*I)*c*(a + b*ArcTan[c*x])^2)/((c^2*d - e)*e) + (a + b*ArcTan[c*x])^2
/(4*e^(3/2)*(Sqrt[-d] - Sqrt[e]*x)) - (a + b*ArcTan[c*x])^2/(4*e^(3/2)*(Sq
rt[-d] + Sqrt[e]*x)) + (b*c*(a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/((c^2*
d - e)*e) - (b*c*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/((c^2*d - e)*e) -
 (b*c*(a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] -
I*Sqrt[e])*(1 - I*c*x))])/(2*(c^2*d - e)*e) + ((a + b*ArcTan[c*x])^2*Log[(
2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(4*Sq
rt[-d]*e^(3/2)) - (b*c*(a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x)
)/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(2*(c^2*d - e)*e) - ((a + b*Arc
Tan[c*x])^2*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1
- I*c*x))])/(4*Sqrt[-d]*e^(3/2)) - ((I/2)*b^2*c*PolyLog[2, 1 - 2/(1 - I*c*
x)])/((c^2*d - e)*e) - ((I/2)*b^2*c*PolyLog[2, 1 - 2/(1 + I*c*x)])/((c^2*d
 - e)*e) + ((I/4)*b^2*c*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sq
rt[-d] - I*Sqrt[e])*(1 - I*c*x))])/((c^2*d - e)*e) - ((I/4)*b*(a + b*ArcTa
n[c*x])*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[
e])*(1 - I*c*x))])/(Sqrt[-d]*e^(3/2)) + ((I/4)*b^2*c*PolyLog[2, 1 - (2*c*(
Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/((c^2*d -
e)*e) + ((I/4)*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sqrt[
e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(Sqrt[-d]*e^(3/2)) + (b^2*
PolyLog[3, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(...
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 29.83 (sec) , antiderivative size = 6565, normalized size of antiderivative = 6.36

| method | result | size |
|---|---|---|
| parts | Expression too large to display | 6565 |
| derivativedivides | Expression too large to display | 6636 |
| default | Expression too large to display | 6636 |

input
```
int(x^2*(a+b*arctan(c*x))^2/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

## Fricas [F]

$$\int \frac{x^2(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx=\int \frac{(b\arctan(cx)+a)^2x^2}{(ex^2+d)^2}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))^2/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b^2*x^2*arctan(c*x)^2 + 2*a*b*x^2*arctan(c*x) + a^2*x^2)/(e^2*x^
4 + 2*d*e*x^2 + d^2), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx=\text{Timed out}$$

input
```
integrate(x**2*(a+b*atan(c*x))**2/(e*x**2+d)**2,x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(a+b*arctan(c*x))^2/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

## Giac [F]

$$\int \frac{x^2(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx = \int \frac{(b\arctan(cx)+a)^2x^2}{(ex^2+d)^2}\,dx$$

input
```
integrate(x^2*(a+b*arctan(c*x))^2/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)^2*x^2/(e*x^2 + d)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx = \int \frac{x^2\,(a+b\,\mathrm{atan}(c\,x))^2}{(e\,x^2+d)^2}\,dx$$

input
```
int((x^2*(a + b*atan(c*x))^2)/(d + e*x^2)^2,x)
```

output
```
int((x^2*(a + b*atan(c*x))^2)/(d + e*x^2)^2, x)
```

## Reduce [F]

$$\int \frac{x^2(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx = \text{too large to display}$$

input
```
int(x^2*(a+b*atan(c*x))^2/(e*x^2+d)^2,x)
```

output
```
(3*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a**2*c**4*d**3 + 3*sqrt(e
)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a**2*c**4*d**2*e*x**2 - 3*sqrt(e)*
sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a**2*d*e**2 - 3*sqrt(e)*sqrt(d)*atan
((e*x)/(sqrt(e)*sqrt(d)))*a**2*e**3*x**2 - 6*sqrt(e)*sqrt(d)*atan((e*x)/(s
qrt(e)*sqrt(d)))*b**2*c**2*d**2*e - 6*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*
sqrt(d)))*b**2*c**2*d*e**2*x**2 + 2*atan(c*x)**3*b**2*c**3*d**3*e + 2*atan
(c*x)**3*b**2*c**3*d**2*e**2*x**2 - 2*atan(c*x)**3*b**2*c*d**2*e**2 - 2*at
an(c*x)**3*b**2*c*d*e**3*x**2 + 6*atan(c*x)**2*a*b*c**3*d**3*e + 6*atan(c*
x)**2*a*b*c**3*d**2*e**2*x**2 - 6*atan(c*x)**2*a*b*c*d**2*e**2 - 6*atan(c*
x)**2*a*b*c*d*e**3*x**2 - 6*atan(c*x)**2*b**2*c**4*d**3*e*x + 6*atan(c*x)*
*2*b**2*c**2*d**2*e**2*x - 12*atan(c*x)*a*b*c**4*d**3*e*x + 12*atan(c*x)*a
*b*c**2*d**2*e**2*x + 6*atan(c*x)*b**2*c**3*d**2*e**2*x**2 + 6*atan(c*x)*b
**2*c*d**2*e**2 + 12*int((atan(c*x)*x**2)/(c**4*d**3*x**2 + 2*c**4*d**2*e*
x**4 + c**4*d*e**2*x**6 + c**2*d**3 + 3*c**2*d**2*e*x**2 + 3*c**2*d*e**2*x
**4 + c**2*e**3*x**6 + d**2*e + 2*d*e**2*x**2 + e**3*x**4),x)*a*b*c**8*d**
6*e + 12*int((atan(c*x)*x**2)/(c**4*d**3*x**2 + 2*c**4*d**2*e*x**4 + c**4*
d*e**2*x**6 + c**2*d**3 + 3*c**2*d**2*e*x**2 + 3*c**2*d*e**2*x**4 + c**2*e
**3*x**6 + d**2*e + 2*d*e**2*x**2 + e**3*x**4),x)*a*b*c**8*d**5*e**2*x**2
- 24*int((atan(c*x)*x**2)/(c**4*d**3*x**2 + 2*c**4*d**2*e*x**4 + c**4*d*e*
*2*x**6 + c**2*d**3 + 3*c**2*d**2*e*x**2 + 3*c**2*d*e**2*x**4 + c**2*e*...
```

# 3.1268 $\int \frac{x(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 457

$$\begin{aligned}\int \frac{x(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx &= \frac{c^2(a+b\arctan(cx))^2}{2(c^2d-e)e} - \frac{(a+b\arctan(cx))^2}{4de\left(1-\frac{\sqrt{e}x}{\sqrt{-d}}\right)} \\ &\quad - \frac{(a+b\arctan(cx))^2}{4de\left(1+\frac{\sqrt{e}x}{\sqrt{-d}}\right)} \\ &\quad - \frac{bc(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{2\sqrt{-d}(c^2d-e)\sqrt{e}} \\ &\quad + \frac{bc(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{2\sqrt{-d}(c^2d-e)\sqrt{e}} \\ &\quad + \frac{ib^2c\,\mathrm{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{4\sqrt{-d}(c^2d-e)\sqrt{e}} \\ &\quad - \frac{ib^2c\,\mathrm{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}+\sqrt{e}x)}{(c\sqrt{-d}+i\sqrt{e})(1-icx)}\right)}{4\sqrt{-d}(c^2d-e)\sqrt{e}}\end{aligned}$$

output
```
1/2*c^2*(a+b*arctan(c*x))^2/(c^2*d-e)/e-1/4*(a+b*arctan(c*x))^2/d/e/(1-e^(
1/2)*x/(-d)^(1/2))-1/4*(a+b*arctan(c*x))^2/d/e/(1+e^(1/2)*x/(-d)^(1/2))-1/
2*b*c*(a+b*arctan(c*x))*ln(2*c*((-d)^(1/2)-e^(1/2)*x)/(c*(-d)^(1/2)-I*e^(1
/2))/(1-I*c*x))/(-d)^(1/2)/(c^2*d-e)/e^(1/2)+1/2*b*c*(a+b*arctan(c*x))*ln(
2*c*((-d)^(1/2)+e^(1/2)*x)/(c*(-d)^(1/2)+I*e^(1/2))/(1-I*c*x))/(-d)^(1/2)/
(c^2*d-e)/e^(1/2)+1/4*I*b^2*c*polylog(2,1-2*c*((-d)^(1/2)-e^(1/2)*x)/(c*(-
d)^(1/2)-I*e^(1/2))/(1-I*c*x))/(-d)^(1/2)/(c^2*d-e)/e^(1/2)-1/4*I*b^2*c*po
lylog(2,1-2*c*((-d)^(1/2)+e^(1/2)*x)/(c*(-d)^(1/2)+I*e^(1/2))/(1-I*c*x))/(
-d)^(1/2)/(c^2*d-e)/e^(1/2)
```

## Mathematica [A] (warning: unable to verify)

Time = 6.42 (sec) , antiderivative size = 836, normalized size of antiderivative = 1.83

$$\int \frac{x(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(x*(a + b*ArcTan[c*x])^2)/(d + e*x^2)^2,x]
```

output
```
((-2*a^2)/(e*(d + e*x^2)) + (4*a*b*(-(((1 + c^2*x^2)*ArcTan[c*x])/(d + e*x
^2)) + (c*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*Sqrt[e])))/(-(c^2*d) + e)
+ (b^2*c^2*((4*ArcTan[c*x]^2)/(c^2*d + e + (c^2*d - e)*Cos[2*ArcTan[c*x]])
 + (-4*ArcTan[c*x]*ArcTanh[(c*d)/(Sqrt[-(c^2*d*e)]*x)] - 2*ArcCos[-((c^2*d
 + e)/(c^2*d - e))]*ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]] + (ArcCos[-((c^2*d +
 e)/(c^2*d - e))] + (2*I)*ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]])*Log[(2*c*d*((
-I)*e + Sqrt[-(c^2*d*e)])*(-I + c*x))/((c^2*d - e)*(c*d + Sqrt[-(c^2*d*e)]
*x))] + (ArcCos[-((c^2*d + e)/(c^2*d - e))] - (2*I)*ArcTanh[(c*e*x)/Sqrt[-
(c^2*d*e)]])*Log[(2*c*d*(I*e + Sqrt[-(c^2*d*e)])*(I + c*x))/((c^2*d - e)*(
c*d + Sqrt[-(c^2*d*e)]*x))] - (ArcCos[-((c^2*d + e)/(c^2*d - e))] - (2*I)*
(ArcTanh[(c*d)/(Sqrt[-(c^2*d*e)]*x)] + ArcTanh[(c*e*x)/Sqrt[-(c^2*d*e)]]))
*Log[(Sqrt[2]*Sqrt[-(c^2*d*e)])/(Sqrt[c^2*d - e]*E^(I*ArcTan[c*x])*Sqrt[c^
2*d + e + (c^2*d - e)*Cos[2*ArcTan[c*x]]])] - (ArcCos[-((c^2*d + e)/(c^2*d
 - e))] + (2*I)*(ArcTanh[(c*d)/(Sqrt[-(c^2*d*e)]*x)] + ArcTanh[(c*e*x)/Sqr
t[-(c^2*d*e)]]))*Log[(Sqrt[2]*Sqrt[-(c^2*d*e)]*E^(I*ArcTan[c*x]))/(Sqrt[c^
2*d - e]*Sqrt[c^2*d + e + (c^2*d - e)*Cos[2*ArcTan[c*x]]])] - I*(PolyLog[2
, ((c^2*d + e - (2*I)*Sqrt[-(c^2*d*e)])*(c*d - Sqrt[-(c^2*d*e)]*x))/((c^2*
d - e)*(c*d + Sqrt[-(c^2*d*e)]*x))] - PolyLog[2, ((c^2*d + e + (2*I)*Sqrt[
-(c^2*d*e)])*(c*d - Sqrt[-(c^2*d*e)]*x))/((c^2*d - e)*(c*d + Sqrt[-(c^2*d*
e)]*x))]))/Sqrt[-(c^2*d*e)]))/(c^2*d - e))/4
```

## Rubi [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 996 vs. $2(457) = 914$.

Time = 1.56 (sec) , antiderivative size = 996, normalized size of antiderivative = 2.18, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {5513, 5389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx$$

$$\Big\downarrow \text{5513}$$

$$\frac{\int \frac{(a+b\arctan(cx))^2}{\left(1-\frac{\sqrt{e}x}{\sqrt{-d}}\right)^2}dx}{4(-d)^{3/2}\sqrt{e}}-\frac{\int \frac{(a+b\arctan(cx))^2}{\left(\frac{\sqrt{e}x}{\sqrt{-d}}+1\right)^2}dx}{4(-d)^{3/2}\sqrt{e}}$$

$$\Big\downarrow \text{5389}$$

$$\frac{\frac{\sqrt{-d}(a+b\arctan(cx))^2}{\sqrt{e}\left(1-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}-\frac{2bc\sqrt{-d}\int\left(-\frac{\sqrt{-d}(\sqrt{e}x+\sqrt{-d})(a+b\arctan(cx))c^2}{(c^2d-e)(c^2x^2+1)}-\frac{\sqrt{-d}e(a+b\arctan(cx))}{(c^2d-e)(\sqrt{-d}-\sqrt{e}x)}\right)dx}{\sqrt{e}}}{4(-d)^{3/2}\sqrt{e}}-\frac{\frac{2bc\sqrt{-d}\int\left(\frac{c^2(d+\sqrt{-d}\sqrt{e}x)(a+b\arctan(cx))}{(c^2d-e)(c^2x^2+1)}-\frac{\sqrt{-d}e(a+b\arctan(cx))}{(c^2d-e)(\sqrt{e}x+\sqrt{-d})}\right)dx}{\sqrt{e}}-\frac{\sqrt{-d}(a+b\arctan(cx))^2}{\sqrt{e}\left(\frac{\sqrt{e}x}{\sqrt{-d}}+1\right)}}{4(-d)^{3/2}\sqrt{e}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\sqrt{-d}(a+b\arctan(cx))^2}{\sqrt{e}\left(1-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}-\frac{2bc\sqrt{-d}\left(\frac{i\sqrt{-d}\sqrt{e}(a+b\arctan(cx))^2}{2b(c^2d-e)}+\frac{cd(a+b\arctan(cx))^2}{2b(c^2d-e)}-\frac{\sqrt{-d}\sqrt{e}\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))}{c^2d-e}+\frac{\sqrt{-d}\sqrt{e}\log\left(\frac{2}{icx+1}\right)(a+b}{c^2d-e}\right.}{}}{}$$

$$\frac{2bc\sqrt{-d}\left(-\frac{i\sqrt{-d}\sqrt{e}(a+b\arctan(cx))^2}{2b(c^2d-e)}+\frac{cd(a+b\arctan(cx))^2}{2b(c^2d-e)}+\frac{\sqrt{-d}\sqrt{e}\log\left(\frac{2}{1-icx}\right)(a+b\arctan(cx))}{c^2d-e}-\frac{\sqrt{-d}\sqrt{e}\log\left(\frac{2}{icx+1}\right)(a+b\arctan(cx))}{c^2d-e}-\frac{\sqrt{-d}\sqrt{e}\log\left(\right.}{}\right.}{}$$

input
```
Int[(x*(a + b*ArcTan[c*x])^2)/(d + e*x^2)^2,x]
```

output
```
((Sqrt[-d]*(a + b*ArcTan[c*x])^2)/(Sqrt[e]*(1 - (Sqrt[e]*x)/Sqrt[-d])) - (
2*b*c*Sqrt[-d]*((c*d*(a + b*ArcTan[c*x])^2)/(2*b*(c^2*d - e)) + ((I/2)*Sqr
t[-d]*Sqrt[e]*(a + b*ArcTan[c*x])^2)/(b*(c^2*d - e)) - (Sqrt[-d]*Sqrt[e]*(
a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/(c^2*d - e) + (Sqrt[-d]*Sqrt[e]*(a
+ b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(c^2*d - e) + (Sqrt[-d]*Sqrt[e]*(a +
b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*
(1 - I*c*x))])/(c^2*d - e) + ((I/2)*b*Sqrt[-d]*Sqrt[e]*PolyLog[2, 1 - 2/(1
 - I*c*x)])/(c^2*d - e) + ((I/2)*b*Sqrt[-d]*Sqrt[e]*PolyLog[2, 1 - 2/(1 +
I*c*x)])/(c^2*d - e) - ((I/2)*b*Sqrt[-d]*Sqrt[e]*PolyLog[2, 1 - (2*c*(Sqrt
[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(c^2*d - e)))/
Sqrt[e])/(4*(-d)^(3/2)*Sqrt[e]) - (-((Sqrt[-d]*(a + b*ArcTan[c*x])^2)/(Sqr
t[e]*(1 + (Sqrt[e]*x)/Sqrt[-d]))) + (2*b*c*Sqrt[-d]*((c*d*(a + b*ArcTan[c*
x])^2)/(2*b*(c^2*d - e)) - ((I/2)*Sqrt[-d]*Sqrt[e]*(a + b*ArcTan[c*x])^2)/
(b*(c^2*d - e)) + (Sqrt[-d]*Sqrt[e]*(a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)]
)/(c^2*d - e) - (Sqrt[-d]*Sqrt[e]*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/
(c^2*d - e) - (Sqrt[-d]*Sqrt[e]*(a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] + S
qrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(c^2*d - e) - ((I/2)*b
*Sqrt[-d]*Sqrt[e]*PolyLog[2, 1 - 2/(1 - I*c*x)])/(c^2*d - e) - ((I/2)*b*Sq
rt[-d]*Sqrt[e]*PolyLog[2, 1 - 2/(1 + I*c*x)])/(c^2*d - e) + ((I/2)*b*Sqrt[
-d]*Sqrt[e]*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + ...
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5389
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_)*((d_) + (e_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(d + e*x)^(q + 1)*((a + b*ArcTan[c*x])^p/(e*(q + 1))), x] - S
imp[b*c*(p/(e*(q + 1)))   Int[ExpandIntegrand[(a + b*ArcTan[c*x])^(p - 1),
(d + e*x)^(q + 1)/(1 + c^2*x^2), x], x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 IGtQ[p, 1] && IntegerQ[q] && NeQ[q, -1]
```

rule 5513
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2)^2
, x_Symbol] :> Simp[1/(4*d^2*Rt[-e/d, 2])   Int[(a + b*ArcTan[c*x])^p/(1 -
Rt[-e/d, 2]*x)^2, x], x] - Simp[1/(4*d^2*Rt[-e/d, 2])   Int[(a + b*ArcTan[c
*x])^p/(1 + Rt[-e/d, 2]*x)^2, x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p
, 0]
```

## Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1174 vs. $2(377) = 754$.

Time = 3.71 (sec) , antiderivative size = 1175, normalized size of antiderivative = 2.57

| method | result | size |
|---|---|---|
| parts | Expression too large to display | 1175 |
| derivativedivides | Expression too large to display | 1209 |
| default | Expression too large to display | 1209 |

input
```
int(x*(a+b*arctan(c*x))^2/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^2/e/(e*x^2+d)-1/2*b^2*c^2*arctan(c*x)^2/e/(c^2*e*x^2+c^2*d)+1/2*b^2
*c^2/(c^2*d-e)/(c^4*d^2-2*c^2*d*e+e^2)*polylog(2,(c^2*d-e)*(1+I*c*x)^2/(c^
2*x^2+1)/(-c^2*d-2*(c^2*d*e)^(1/2)-e))*(c^2*d*e)^(1/2)-1/2*b^2*c^4/e/(c^2*
d-e)/(c^4*d^2-2*c^2*d*e+e^2)*arctan(c*x)^2*(c^2*d*e)^(1/2)*d-1/4*b^2*c^4/e
/(c^2*d-e)/(c^4*d^2-2*c^2*d*e+e^2)*polylog(2,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^
2+1)/(-c^2*d-2*(c^2*d*e)^(1/2)-e))*(c^2*d*e)^(1/2)*d-1/2*I*b^2*c^4/e*ln(1-
(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d-2*(c^2*d*e)^(1/2)-e))*arctan(c*x
)/(c^2*d-e)/(c^4*d^2-2*c^2*d*e+e^2)*(c^2*d*e)^(1/2)*d-1/2*I*b^2*e*ln(1-(c^
2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d-2*(c^2*d*e)^(1/2)-e))*arctan(c*x)/d
/(c^2*d-e)/(c^4*d^2-2*c^2*d*e+e^2)*(c^2*d*e)^(1/2)-1/2*b^2*e/d/(c^2*d-e)/(
c^4*d^2-2*c^2*d*e+e^2)*arctan(c*x)^2*(c^2*d*e)^(1/2)-1/4*b^2*e/d/(c^2*d-e)
/(c^4*d^2-2*c^2*d*e+e^2)*polylog(2,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2
*d-2*(c^2*d*e)^(1/2)-e))*(c^2*d*e)^(1/2)+1/2*I*b^2/e*(c^2*d*e)^(1/2)/d/(c^
2*d-e)*arctan(c*x)*ln(1-(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*d
*e)^(1/2)-e))+1/2*b^2/e*(c^2*d*e)^(1/2)/d/(c^2*d-e)*arctan(c*x)^2+1/2*b^2*
c^2/e/(c^2*d-e)*arctan(c*x)^2+b^2*c^2/(c^2*d-e)/(c^4*d^2-2*c^2*d*e+e^2)*ar
ctan(c*x)^2*(c^2*d*e)^(1/2)+1/4*b^2/e*(c^2*d*e)^(1/2)/d/(c^2*d-e)*polylog(
2,(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d+2*(c^2*d*e)^(1/2)-e))+I*b^2*c^
2*ln(1-(c^2*d-e)*(1+I*c*x)^2/(c^2*x^2+1)/(-c^2*d-2*(c^2*d*e)^(1/2)-e))*arc
tan(c*x)/(c^2*d-e)/(c^4*d^2-2*c^2*d*e+e^2)*(c^2*d*e)^(1/2)-a*b*c^2*arct...
```

## Fricas [F]

$$\int \frac{x(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx = \int \frac{(b\arctan(cx)+a)^2x}{(ex^2+d)^2}\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))^2/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b^2*x*arctan(c*x)^2 + 2*a*b*x*arctan(c*x) + a^2*x)/(e^2*x^4 + 2*
d*e*x^2 + d^2), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{x(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(x*(a+b*atan(c*x))**2/(e*x**2+d)**2,x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{x(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(a+b*arctan(c*x))^2/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

## Giac [F]

$$\int \frac{x(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx = \int \frac{(b\arctan(cx)+a)^2x}{(ex^2+d)^2}\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))^2/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)^2*x/(e*x^2 + d)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx = \int \frac{x\,(a+b\,\mathrm{atan}(c\,x))^2}{(e\,x^2+d)^2}\,dx$$

input
```
int((x*(a + b*atan(c*x))^2)/(d + e*x^2)^2,x)
```

output
```
int((x*(a + b*atan(c*x))^2)/(d + e*x^2)^2, x)
```

## Reduce [F]

$$\int \frac{x(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx = \text{Too large to display}$$

input
```
int(x*(a+b*atan(c*x))^2/(e*x^2+d)^2,x)
```

output
```
( - 2*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*b*c**3*d**2 - 2*sqrt
(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*b*c**3*d*e*x**2 - 2*sqrt(e)*sq
rt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a*b*c*d*e - 2*sqrt(e)*sqrt(d)*atan((e*
x)/(sqrt(e)*sqrt(d)))*a*b*c*e**2*x**2 + atan(c*x)**2*b**2*c**4*d**2*e*x**2
 - atan(c*x)**2*b**2*c**2*d**2*e - atan(c*x)**2*b**2*c**2*d*e**2*x**2 + at
an(c*x)**2*b**2*d*e**2 + 2*atan(c*x)*a*b*c**4*d**2*e*x**2 + 2*atan(c*x)*a*
b*c**2*d**2*e + 2*atan(c*x)*a*b*c**2*d*e**2*x**2 + 2*atan(c*x)*a*b*d*e**2
+ 2*atan(c*x)*b**2*c**3*d**2*e*x - 2*atan(c*x)*b**2*c*d*e**2*x - 4*int((at
an(c*x)*x**2)/(c**4*d**3*x**2 + 2*c**4*d**2*e*x**4 + c**4*d*e**2*x**6 + c*
*2*d**3 + 3*c**2*d**2*e*x**2 + 3*c**2*d*e**2*x**4 + c**2*e**3*x**6 + d**2*
e + 2*d*e**2*x**2 + e**3*x**4),x)*b**2*c**7*d**5*e - 4*int((atan(c*x)*x**2
)/(c**4*d**3*x**2 + 2*c**4*d**2*e*x**4 + c**4*d*e**2*x**6 + c**2*d**3 + 3*
c**2*d**2*e*x**2 + 3*c**2*d*e**2*x**4 + c**2*e**3*x**6 + d**2*e + 2*d*e**2
*x**2 + e**3*x**4),x)*b**2*c**7*d**4*e**2*x**2 + 4*int((atan(c*x)*x**2)/(c
**4*d**3*x**2 + 2*c**4*d**2*e*x**4 + c**4*d*e**2*x**6 + c**2*d**3 + 3*c**2
*d**2*e*x**2 + 3*c**2*d*e**2*x**4 + c**2*e**3*x**6 + d**2*e + 2*d*e**2*x**
2 + e**3*x**4),x)*b**2*c**5*d**4*e**2 + 4*int((atan(c*x)*x**2)/(c**4*d**3*
x**2 + 2*c**4*d**2*e*x**4 + c**4*d*e**2*x**6 + c**2*d**3 + 3*c**2*d**2*e*x
**2 + 3*c**2*d*e**2*x**4 + c**2*e**3*x**6 + d**2*e + 2*d*e**2*x**2 + e**3*
x**4),x)*b**2*c**5*d**3*e**3*x**2 + 4*int((atan(c*x)*x**2)/(c**4*d**3*x...
```

# 3.1269 $\int \frac{(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx$



## Optimal result

Integrand size = 20, antiderivative size = 1039

$$\int \frac{(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx = \text{Too large to display}$$

output
```
1/4*I*b*(a+b*arctan(c*x))*polylog(2,1-2*c*((-d)^(1/2)-e^(1/2)*x)/(c*(-d)^(
1/2)-I*e^(1/2))/(1-I*c*x))/(-d)^(3/2)/e^(1/2)-1/4*(a+b*arctan(c*x))^2/d/e^
(1/2)/((-d)^(1/2)-e^(1/2)*x)+1/4*(a+b*arctan(c*x))^2/d/e^(1/2)/((-d)^(1/2)
+e^(1/2)*x)-b*c*(a+b*arctan(c*x))*ln(2/(1-I*c*x))/d/(c^2*d-e)+b*c*(a+b*arc
tan(c*x))*ln(2/(1+I*c*x))/d/(c^2*d-e)+1/2*b*c*(a+b*arctan(c*x))*ln(2*c*((-
d)^(1/2)-e^(1/2)*x)/(c*(-d)^(1/2)-I*e^(1/2))/(1-I*c*x))/d/(c^2*d-e)-1/4*(a
+b*arctan(c*x))^2*ln(2*c*((-d)^(1/2)-e^(1/2)*x)/(c*(-d)^(1/2)-I*e^(1/2))/(
1-I*c*x))/(-d)^(3/2)/e^(1/2)+1/2*b*c*(a+b*arctan(c*x))*ln(2*c*((-d)^(1/2)+
e^(1/2)*x)/(c*(-d)^(1/2)+I*e^(1/2))/(1-I*c*x))/d/(c^2*d-e)+1/4*(a+b*arctan
(c*x))^2*ln(2*c*((-d)^(1/2)+e^(1/2)*x)/(c*(-d)^(1/2)+I*e^(1/2))/(1-I*c*x))
/(-d)^(3/2)/e^(1/2)-1/4*I*b^2*c*polylog(2,1-2*c*((-d)^(1/2)-e^(1/2)*x)/(c*
(-d)^(1/2)-I*e^(1/2))/(1-I*c*x))/d/(c^2*d-e)+1/2*I*b^2*c*polylog(2,1-2/(1-
I*c*x))/d/(c^2*d-e)+1/2*I*c*(a+b*arctan(c*x))^2/d/(c^2*d-e)-1/4*I*b*(a+b*a
rctan(c*x))*polylog(2,1-2*c*((-d)^(1/2)+e^(1/2)*x)/(c*(-d)^(1/2)+I*e^(1/2)
)/(1-I*c*x))/(-d)^(3/2)/e^(1/2)+1/2*I*b^2*c*polylog(2,1-2/(1+I*c*x))/d/(c^
2*d-e)-1/4*I*b^2*c*polylog(2,1-2*c*((-d)^(1/2)+e^(1/2)*x)/(c*(-d)^(1/2)+I*
e^(1/2))/(1-I*c*x))/d/(c^2*d-e)-1/8*b^2*polylog(3,1-2*c*((-d)^(1/2)-e^(1/2
)*x)/(c*(-d)^(1/2)-I*e^(1/2))/(1-I*c*x))/(-d)^(3/2)/e^(1/2)+1/8*b^2*polylo
g(3,1-2*c*((-d)^(1/2)+e^(1/2)*x)/(c*(-d)^(1/2)+I*e^(1/2))/(1-I*c*x))/(-d)^
(3/2)/e^(1/2)
```

## Mathematica [F]

$$\int \frac{(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx = \int \frac{(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx$$

input
```
Integrate[(a + b*ArcTan[c*x])^2/(d + e*x^2)^2,x]
```

output
```
Integrate[(a + b*ArcTan[c*x])^2/(d + e*x^2)^2, x]
```

## Rubi [A] (verified)

Time = 1.76 (sec) , antiderivative size = 1039, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5449, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx$$

$$\Big\downarrow \text{5449}$$

$$\int \left(-\frac{e(a+b\arctan(cx))^2}{2d(-de-e^2x^2)}-\frac{e(a+b\arctan(cx))^2}{4d\left(\sqrt{-d}\sqrt{e}-ex\right)^2}-\frac{e(a+b\arctan(cx))^2}{4d\left(\sqrt{-d}\sqrt{e}+ex\right)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{ic\,\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)b^2}{2d(c^2d-e)}+\frac{ic\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)b^2}{2d(c^2d-e)}-\frac{ic\,\text{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)b^2}{4d(c^2d-e)}-\frac{ic\,\text{PolyLog}\left(2,1-\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)b^2}{4d(c^2d-e)}-\frac{\text{PolyLog}\left(3,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)b^2}{8(-d)^{3/2}\sqrt{e}}+\frac{\text{PolyLog}\left(3,1-\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)b^2}{8(-d)^{3/2}\sqrt{e}}-\frac{c(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)b}{d(c^2d-e)}+\frac{c(a+b\arctan(cx))\log\left(\frac{2}{icx+1}\right)b}{d(c^2d-e)}+\frac{c(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)b}{2d(c^2d-e)}+\frac{c(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)b}{2d(c^2d-e)}+\frac{i(a+b\arctan(cx))\,\text{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)b}{4(-d)^{3/2}\sqrt{e}}-\frac{i(a+b\arctan(cx))\,\text{PolyLog}\left(2,1-\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)b}{4(-d)^{3/2}\sqrt{e}}+\frac{ic(a+b\arctan(cx))^2}{2d(c^2d-e)}-\frac{(a+b\arctan(cx))^2}{4d\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{(a+b\arctan(cx))^2}{4d\sqrt{e}\left(\sqrt{e}x+\sqrt{-d}\right)}-\frac{(a+b\arctan(cx))^2\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{4(-d)^{3/2}\sqrt{e}}+\frac{(a+b\arctan(cx))^2\log\left(\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)}{4(-d)^{3/2}\sqrt{e}}$$

input
```
Int[(a + b*ArcTan[c*x])^2/(d + e*x^2)^2,x]
```

output
```
((I/2)*c*(a + b*ArcTan[c*x])^2)/(d*(c^2*d - e)) - (a + b*ArcTan[c*x])^2/(4
*d*Sqrt[e]*(Sqrt[-d] - Sqrt[e]*x)) + (a + b*ArcTan[c*x])^2/(4*d*Sqrt[e]*(S
qrt[-d] + Sqrt[e]*x)) - (b*c*(a + b*ArcTan[c*x])*Log[2/(1 - I*c*x)])/(d*(c
^2*d - e)) + (b*c*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(d*(c^2*d - e))
+ (b*c*(a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] -
 I*Sqrt[e])*(1 - I*c*x))])/(2*d*(c^2*d - e)) - ((a + b*ArcTan[c*x])^2*Log[
(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(4*(
-d)^(3/2)*Sqrt[e]) + (b*c*(a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] + Sqrt[e]
*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(2*d*(c^2*d - e)) + ((a + b*
ArcTan[c*x])^2*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*
(1 - I*c*x))])/(4*(-d)^(3/2)*Sqrt[e]) + ((I/2)*b^2*c*PolyLog[2, 1 - 2/(1 -
 I*c*x)])/(d*(c^2*d - e)) + ((I/2)*b^2*c*PolyLog[2, 1 - 2/(1 + I*c*x)])/(d
*(c^2*d - e)) - ((I/4)*b^2*c*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/(
(c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(d*(c^2*d - e)) + ((I/4)*b*(a + b*
ArcTan[c*x])*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*
Sqrt[e])*(1 - I*c*x))])/((-d)^(3/2)*Sqrt[e]) - ((I/4)*b^2*c*PolyLog[2, 1 -
 (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(d*
(c^2*d - e)) - ((I/4)*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(Sqrt[-d]
+ Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/((-d)^(3/2)*Sqrt[e]
) - (b^2*PolyLog[3, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*S...
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5449
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^2)^(q_.), x
_Symbol] :> Int[ExpandIntegrand[(a + b*ArcTan[c*x])^p, (d + e*x^2)^q, x], x
] /; FreeQ[{a, b, c, d, e}, x] && IntegerQ[q] && IGtQ[p, 0]
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 27.81 (sec) , antiderivative size = 6565, normalized size of antiderivative = 6.32

| method | result | size |
|---|---|---|
| parts | Expression too large to display | 6565 |
| derivativedivides | Expression too large to display | 6570 |
| default | Expression too large to display | 6570 |

input
```
int((a+b*arctan(c*x))^2/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

## Fricas [F]

$$\int \frac{(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx = \int \frac{(b\arctan(cx)+a)^2}{(ex^2+d)^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)/(e^2*x^4 + 2*d*e*x^
2 + d^2), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*atan(c*x))**2/(e*x**2+d)**2,x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arctan(c*x))^2/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

## Giac [F]

$$\int \frac{(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx = \int \frac{(b\arctan(cx)+a)^2}{(ex^2+d)^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)^2/(e*x^2 + d)^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^2}{(e\,x^2+d)^2}\,dx$$

input
```
int((a + b*atan(c*x))^2/(d + e*x^2)^2,x)
```

output
```
int((a + b*atan(c*x))^2/(d + e*x^2)^2, x)
```

## Reduce [F]

$$\int \frac{(a+b\arctan(cx))^2}{(d+ex^2)^2}\,dx = \text{too large to display}$$

input
```
int((a+b*atan(c*x))^2/(e*x^2+d)^2,x)
```

output
```
(3*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a**2*c**5*d**3 + 3*sqrt(e
)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a**2*c**5*d**2*e*x**2 - 3*sqrt(e)*
sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a**2*c*d*e**2 - 3*sqrt(e)*sqrt(d)*at
an((e*x)/(sqrt(e)*sqrt(d)))*a**2*c*e**3*x**2 + 6*sqrt(e)*sqrt(d)*atan((e*x
)/(sqrt(e)*sqrt(d)))*b**2*c*d*e**2 + 6*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)
*sqrt(d)))*b**2*c*e**3*x**2 + 2*atan(c*x)**3*b**2*c**4*d**3*e + 2*atan(c*x
)**3*b**2*c**4*d**2*e**2*x**2 - 2*atan(c*x)**3*b**2*c**2*d**2*e**2 - 2*ata
n(c*x)**3*b**2*c**2*d*e**3*x**2 + 6*atan(c*x)**2*a*b*c**4*d**3*e + 6*atan(
c*x)**2*a*b*c**4*d**2*e**2*x**2 - 6*atan(c*x)**2*a*b*c**2*d**2*e**2 - 6*at
an(c*x)**2*a*b*c**2*d*e**3*x**2 + 6*atan(c*x)**2*b**2*c**3*d**2*e**2*x - 6
*atan(c*x)**2*b**2*c*d*e**3*x + 12*atan(c*x)*a*b*c**3*d**2*e**2*x - 12*ata
n(c*x)*a*b*c*d*e**3*x - 6*atan(c*x)*b**2*c**2*d*e**3*x**2 - 6*atan(c*x)*b*
*2*d*e**3 + 12*int((atan(c*x)*x**2)/(c**4*d**3*x**2 + 2*c**4*d**2*e*x**4 +
 c**4*d*e**2*x**6 + c**2*d**3 + 3*c**2*d**2*e*x**2 + 3*c**2*d*e**2*x**4 +
c**2*e**3*x**6 + d**2*e + 2*d*e**2*x**2 + e**3*x**4),x)*a*b*c**9*d**6*e +
12*int((atan(c*x)*x**2)/(c**4*d**3*x**2 + 2*c**4*d**2*e*x**4 + c**4*d*e**2
*x**6 + c**2*d**3 + 3*c**2*d**2*e*x**2 + 3*c**2*d*e**2*x**4 + c**2*e**3*x*
*6 + d**2*e + 2*d*e**2*x**2 + e**3*x**4),x)*a*b*c**9*d**5*e**2*x**2 - 24*i
nt((atan(c*x)*x**2)/(c**4*d**3*x**2 + 2*c**4*d**2*e*x**4 + c**4*d*e**2*x**
6 + c**2*d**3 + 3*c**2*d**2*e*x**2 + 3*c**2*d*e**2*x**4 + c**2*e**3*x**...
```

# 3.1270 $\int \frac{(a+b\arctan(cx))^2}{x(d+ex^2)^2}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 1087

$$\int \frac{(a+b\arctan(cx))^2}{x\left(d+ex^2\right)^2}\,dx = \text{Too large to display}$$

output
```
-1/2*c^2*(a+b*arctan(c*x))^2/d/(c^2*d-e)+1/4*(a+b*arctan(c*x))^2/d^2/(1-e^
(1/2)*x/(-d)^(1/2))+1/4*(a+b*arctan(c*x))^2/d^2/(1+e^(1/2)*x/(-d)^(1/2))-2
*(a+b*arctan(c*x))^2*arctanh(-1+2/(1+I*c*x))/d^2+(a+b*arctan(c*x))^2*ln(2/
(1-I*c*x))/d^2-1/2*b*c*e^(1/2)*(a+b*arctan(c*x))*ln(2*c*((-d)^(1/2)-e^(1/2
)*x)/(c*(-d)^(1/2)-I*e^(1/2))/(1-I*c*x))/(-d)^(3/2)/(c^2*d-e)-1/2*(a+b*arc
tan(c*x))^2*ln(2*c*((-d)^(1/2)-e^(1/2)*x)/(c*(-d)^(1/2)-I*e^(1/2))/(1-I*c*
x))/d^2+1/2*b*c*e^(1/2)*(a+b*arctan(c*x))*ln(2*c*((-d)^(1/2)+e^(1/2)*x)/(c
*(-d)^(1/2)+I*e^(1/2))/(1-I*c*x))/(-d)^(3/2)/(c^2*d-e)-1/2*(a+b*arctan(c*x
))^2*ln(2*c*((-d)^(1/2)+e^(1/2)*x)/(c*(-d)^(1/2)+I*e^(1/2))/(1-I*c*x))/d^2
-1/4*I*b^2*c*e^(1/2)*polylog(2,1-2*c*((-d)^(1/2)+e^(1/2)*x)/(c*(-d)^(1/2)+
I*e^(1/2))/(1-I*c*x))/(-d)^(3/2)/(c^2*d-e)+1/2*I*b*(a+b*arctan(c*x))*polyl
og(2,1-2*c*((-d)^(1/2)-e^(1/2)*x)/(c*(-d)^(1/2)-I*e^(1/2))/(1-I*c*x))/d^2+
1/4*I*b^2*c*e^(1/2)*polylog(2,1-2*c*((-d)^(1/2)-e^(1/2)*x)/(c*(-d)^(1/2)-I
*e^(1/2))/(1-I*c*x))/(-d)^(3/2)/(c^2*d-e)+1/2*I*b*(a+b*arctan(c*x))*polylo
g(2,1-2*c*((-d)^(1/2)+e^(1/2)*x)/(c*(-d)^(1/2)+I*e^(1/2))/(1-I*c*x))/d^2+I
*b*(a+b*arctan(c*x))*polylog(2,-1+2/(1+I*c*x))/d^2-I*b*(a+b*arctan(c*x))*p
olylog(2,1-2/(1-I*c*x))/d^2-I*b*(a+b*arctan(c*x))*polylog(2,1-2/(1+I*c*x))
/d^2+1/2*b^2*polylog(3,1-2/(1-I*c*x))/d^2-1/2*b^2*polylog(3,1-2/(1+I*c*x))
/d^2+1/2*b^2*polylog(3,-1+2/(1+I*c*x))/d^2-1/4*b^2*polylog(3,1-2*c*((-d)^(
1/2)-e^(1/2)*x)/(c*(-d)^(1/2)-I*e^(1/2))/(1-I*c*x))/d^2-1/4*b^2*polylog...
```

## Mathematica [F]

$$\int \frac{(a+b\arctan(cx))^2}{x(d+ex^2)^2}\,dx=\int \frac{(a+b\arctan(cx))^2}{x(d+ex^2)^2}\,dx$$

input
```
Integrate[(a + b*ArcTan[c*x])^2/(x*(d + e*x^2)^2),x]
```

output
```
Integrate[(a + b*ArcTan[c*x])^2/(x*(d + e*x^2)^2), x]
```

## Rubi [A] (verified)

Time = 2.21 (sec) , antiderivative size = 1087, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^2}{x(d+ex^2)^2}\,dx$$

$$\Big\downarrow \text{5515}$$

$$\int \left(-\frac{ex(a+b\arctan(cx))^2}{d^2(d+ex^2)}+\frac{(a+b\arctan(cx))^2}{d^2x}-\frac{ex(a+b\arctan(cx))^2}{d(d+ex^2)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{ic\sqrt{e}\,\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)b^2}{4(-d)^{3/2}\left(c^2d-e\right)}-\frac{ic\sqrt{e}\,\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{e}x+\sqrt{-d}\right)}{\left(\sqrt{-d}c+i\sqrt{e}\right)(1-icx)}\right)b^2}{4(-d)^{3/2}\left(c^2d-e\right)}+\frac{\text{PolyLog}\left(3,1-\frac{2}{1-icx}\right)b^2}{2d^2}-\frac{\text{PolyLog}\left(3,1-\frac{2}{icx+1}\right)b^2}{2d^2}+\frac{\text{PolyLog}\left(3,\frac{2}{icx+1}-1\right)b^2}{2d^2}-\frac{\text{PolyLog}\left(3,1-\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)b^2}{4d^2}-\frac{\text{PolyLog}\left(3,1-\frac{2c\left(\sqrt{e}x+\sqrt{-d}\right)}{\left(\sqrt{-d}c+i\sqrt{e}\right)(1-icx)}\right)b^2}{4d^2}-\frac{c\sqrt{e}(a+b\arctan(cx))\log\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)b}{2(-d)^{3/2}\left(c^2d-e\right)}+\frac{c\sqrt{e}(a+b\arctan(cx))\log\left(\frac{2c\left(\sqrt{e}x+\sqrt{-d}\right)}{\left(\sqrt{-d}c+i\sqrt{e}\right)(1-icx)}\right)b}{2(-d)^{3/2}\left(c^2d-e\right)}-\frac{i(a+b\arctan(cx))\,\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)b}{d^2}-\frac{i(a+b\arctan(cx))\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)b}{d^2}+\frac{i(a+b\arctan(cx))\,\text{PolyLog}\left(2,\frac{2}{icx+1}-1\right)b}{d^2}+\frac{i(a+b\arctan(cx))\,\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)b}{2d^2}+\frac{i(a+b\arctan(cx))\,\text{PolyLog}\left(2,1-\frac{2c\left(\sqrt{e}x+\sqrt{-d}\right)}{\left(\sqrt{-d}c+i\sqrt{e}\right)(1-icx)}\right)b}{2d^2}-\frac{c^2(a+b\arctan(cx))^2}{2d\left(c^2d-e\right)}+\frac{(a+b\arctan(cx))^2}{4d^2\left(1-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}+\frac{(a+b\arctan(cx))^2}{4d^2\left(\frac{\sqrt{e}x}{\sqrt{-d}}+1\right)}+\frac{2(a+b\arctan(cx))^2\text{arctanh}\left(1-\frac{2}{icx+1}\right)}{d^2}+\frac{(a+b\arctan(cx))^2\log\left(\frac{2}{1-icx}\right)}{d^2}-\frac{(a+b\arctan(cx))^2\log\left(\frac{2c\left(\sqrt{-d}-\sqrt{e}x\right)}{\left(c\sqrt{-d}-i\sqrt{e}\right)(1-icx)}\right)}{2d^2}-\frac{(a+b\arctan(cx))^2\log\left(\frac{2c\left(\sqrt{e}x+\sqrt{-d}\right)}{\left(\sqrt{-d}c+i\sqrt{e}\right)(1-icx)}\right)}{2d^2}$$

input
```
Int[(a + b*ArcTan[c*x])^2/(x*(d + e*x^2)^2),x]
```

output
```
-1/2*(c^2*(a + b*ArcTan[c*x])^2)/(d*(c^2*d - e)) + (a + b*ArcTan[c*x])^2/(
4*d^2*(1 - (Sqrt[e]*x)/Sqrt[-d])) + (a + b*ArcTan[c*x])^2/(4*d^2*(1 + (Sqr
t[e]*x)/Sqrt[-d])) + (2*(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)])/
d^2 + ((a + b*ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/d^2 - (b*c*Sqrt[e]*(a + b
*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(
1 - I*c*x))])/(2*(-d)^(3/2)*(c^2*d - e)) - ((a + b*ArcTan[c*x])^2*Log[(2*c
*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(2*d^2)
+ (b*c*Sqrt[e]*(a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sq
rt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(2*(-d)^(3/2)*(c^2*d - e)) - ((a + b*Ar
cTan[c*x])^2*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1
 - I*c*x))])/(2*d^2) - (I*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 - I*c*
x)])/d^2 - (I*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)])/d^2 + (
I*b*(a + b*ArcTan[c*x])*PolyLog[2, -1 + 2/(1 + I*c*x)])/d^2 + ((I/4)*b^2*c
*Sqrt[e]*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt
[e])*(1 - I*c*x))])/((-d)^(3/2)*(c^2*d - e)) + ((I/2)*b*(a + b*ArcTan[c*x]
)*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1
 - I*c*x))])/d^2 - ((I/4)*b^2*c*Sqrt[e]*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sq
rt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/((-d)^(3/2)*(c^2*d - e)
) + ((I/2)*b*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x
))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/d^2 + (b^2*PolyLog[3, 1 - 2...
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

## Maple [F]

$$\int \frac{(a+b\arctan(cx))^2}{x(e\,x^2+d)^2}dx$$

input
```
int((a+b*arctan(c*x))^2/x/(e*x^2+d)^2,x)
```

output
```
int((a+b*arctan(c*x))^2/x/(e*x^2+d)^2,x)
```

## Fricas [F]

$$\int \frac{(a+b\arctan(cx))^2}{x(d+ex^2)^2}\,dx = \int \frac{(b\arctan(cx)+a)^2}{(ex^2+d)^2x}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)/(e^2*x^5 + 2*d*e*x^
3 + d^2*x), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{x(d+ex^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*atan(c*x))**2/x/(e*x**2+d)**2,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(a+b\arctan(cx))^2}{x\,(d+ex^2)^2}\,dx = \int \frac{(b\arctan(cx)+a)^2}{(ex^2+d)^2x}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
1/2*a^2*(1/(d*e*x^2 + d^2) - log(e*x^2 + d)/d^2 + 2*log(x)/d^2) + integrat
e((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x))/(e^2*x^5 + 2*d*e*x^3 + d^2*x), x
)
```

## Giac [F]

$$\int \frac{(a+b\arctan(cx))^2}{x\,(d+ex^2)^2}\,dx = \int \frac{(b\arctan(cx)+a)^2}{(ex^2+d)^2x}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)^2/((e*x^2 + d)^2*x), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{x\,(d+ex^2)^2}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^2}{x\,(e\,x^2+d)^2}\,dx$$

input
```
int((a + b*atan(c*x))^2/(x*(d + e*x^2)^2),x)
```

output
```
int((a + b*atan(c*x))^2/(x*(d + e*x^2)^2), x)
```

## Reduce [F]

$$\int \frac{(a+b\arctan(cx))^2}{x\left(d+ex^2\right)^2}\,dx$$
$$= \frac{4\left(\int \frac{atan(cx)}{e^2x^5+2de\,x^3+d^2x}dx\right)ab\,d^3+4\left(\int \frac{atan(cx)}{e^2x^5+2de\,x^3+d^2x}dx\right)ab\,d^2e\,x^2+2\left(\int \frac{atan(cx)^2}{e^2x^5+2de\,x^3+d^2x}dx\right)b^2d^3+2\left(\int \frac{a}{e^2x^5}\right.}{2d^2\left(e\right.}$$

input
```
int((a+b*atan(c*x))^2/x/(e*x^2+d)^2,x)
```

output
```
(4*int(atan(c*x)/(d**2*x + 2*d*e*x**3 + e**2*x**5),x)*a*b*d**3 + 4*int(ata
n(c*x)/(d**2*x + 2*d*e*x**3 + e**2*x**5),x)*a*b*d**2*e*x**2 + 2*int(atan(c
*x)**2/(d**2*x + 2*d*e*x**3 + e**2*x**5),x)*b**2*d**3 + 2*int(atan(c*x)**2
/(d**2*x + 2*d*e*x**3 + e**2*x**5),x)*b**2*d**2*e*x**2 - log(d + e*x**2)*a
**2*d - log(d + e*x**2)*a**2*e*x**2 + 2*log(x)*a**2*d + 2*log(x)*a**2*e*x*
*2 - a**2*e*x**2)/(2*d**2*(d + e*x**2))
```

# 3.1271 $\int \frac{(a+b\arctan(cx))^2}{x^2(d+ex^2)^2}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 1141

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+ex^2)^2}\,dx = \text{Too large to display}$$

output
```
-1/2*I*b^2*c*e*polylog(2,1-2/(1-I*c*x))/d^2/(c^2*d-e)-I*c*(a+b*arctan(c*x)
)^2/d^2-(a+b*arctan(c*x))^2/d^2/x+1/4*e^(1/2)*(a+b*arctan(c*x))^2/d^2/((-d
)^(1/2)-e^(1/2)*x)-1/4*e^(1/2)*(a+b*arctan(c*x))^2/d^2/((-d)^(1/2)+e^(1/2)
*x)+b*c*e*(a+b*arctan(c*x))*ln(2/(1-I*c*x))/d^2/(c^2*d-e)-b*c*e*(a+b*arcta
n(c*x))*ln(2/(1+I*c*x))/d^2/(c^2*d-e)-1/2*b*c*e*(a+b*arctan(c*x))*ln(2*c*(
(-d)^(1/2)-e^(1/2)*x)/(c*(-d)^(1/2)-I*e^(1/2))/(1-I*c*x))/d^2/(c^2*d-e)-3/
4*e^(1/2)*(a+b*arctan(c*x))^2*ln(2*c*((-d)^(1/2)-e^(1/2)*x)/(c*(-d)^(1/2)-
I*e^(1/2))/(1-I*c*x))/(-d)^(5/2)-1/2*b*c*e*(a+b*arctan(c*x))*ln(2*c*((-d)^
(1/2)+e^(1/2)*x)/(c*(-d)^(1/2)+I*e^(1/2))/(1-I*c*x))/d^2/(c^2*d-e)+3/4*e^(
1/2)*(a+b*arctan(c*x))^2*ln(2*c*((-d)^(1/2)+e^(1/2)*x)/(c*(-d)^(1/2)+I*e^(
1/2))/(1-I*c*x))/(-d)^(5/2)+2*b*c*(a+b*arctan(c*x))*ln(2-2/(1-I*c*x))/d^2+
1/4*I*b^2*c*e*polylog(2,1-2*c*((-d)^(1/2)-e^(1/2)*x)/(c*(-d)^(1/2)-I*e^(1/
2))/(1-I*c*x))/d^2/(c^2*d-e)-1/2*I*b^2*c*e*polylog(2,1-2/(1+I*c*x))/d^2/(c
^2*d-e)+3/4*I*b*e^(1/2)*(a+b*arctan(c*x))*polylog(2,1-2*c*((-d)^(1/2)-e^(1
/2)*x)/(c*(-d)^(1/2)-I*e^(1/2))/(1-I*c*x))/(-d)^(5/2)-1/2*I*c*e*(a+b*arcta
n(c*x))^2/d^2/(c^2*d-e)+1/4*I*b^2*c*e*polylog(2,1-2*c*((-d)^(1/2)+e^(1/2)*
x)/(c*(-d)^(1/2)+I*e^(1/2))/(1-I*c*x))/d^2/(c^2*d-e)-I*b^2*c*polylog(2,-1+
2/(1-I*c*x))/d^2-3/4*I*b*e^(1/2)*(a+b*arctan(c*x))*polylog(2,1-2*c*((-d)^(
1/2)+e^(1/2)*x)/(c*(-d)^(1/2)+I*e^(1/2))/(1-I*c*x))/(-d)^(5/2)-3/8*b^2*e^(
1/2)*polylog(3,1-2*c*((-d)^(1/2)-e^(1/2)*x)/(c*(-d)^(1/2)-I*e^(1/2))/(1...
```

## Mathematica [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+ex^2)^2}\,dx = \$\text{Aborted}$$

input
```
Integrate[(a + b*ArcTan[c*x])^2/(x^2*(d + e*x^2)^2),x]
```

output
```
$Aborted
```

## Rubi [A] (verified)

Time = 2.21 (sec) , antiderivative size = 1141, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^2}{x^2\left(d+ex^2\right)^2}dx$$

$$\Big\downarrow \text{5515}$$

$$\int \left(-\frac{e(a+b\arctan(cx))^2}{d^2\left(d+ex^2\right)}+\frac{(a+b\arctan(cx))^2}{d^2x^2}-\frac{e(a+b\arctan(cx))^2}{d\left(d+ex^2\right)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$
-\frac{ice\,\text{PolyLog}\left(2,1-\frac{2}{1-icx}\right)b^2}{2d^2(c^2d-e)}-\frac{ic\,\text{PolyLog}\left(2,\frac{2}{1-icx}-1\right)b^2}{d^2}-\frac{ice\,\text{PolyLog}\left(2,1-\frac{2}{icx+1}\right)b^2}{2d^2(c^2d-e)}+\frac{ice\,\text{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)b^2}{4d^2(c^2d-e)}+\frac{ice\,\text{PolyLog}\left(2,1-\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)b^2}{4d^2(c^2d-e)}-\frac{3\sqrt{e}\,\text{PolyLog}\left(3,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)b^2}{8(-d)^{5/2}}+\frac{3\sqrt{e}\,\text{PolyLog}\left(3,1-\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)b^2}{8(-d)^{5/2}}+\frac{ce(a+b\arctan(cx))\log\left(\frac{2}{1-icx}\right)b}{d^2(c^2d-e)}-\frac{ce(a+b\arctan(cx))\log\left(\frac{2}{icx+1}\right)b}{d^2(c^2d-e)}-\frac{ce(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)b}{2d^2(c^2d-e)}-\frac{ce(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)b}{2d^2(c^2d-e)}+\frac{2c(a+b\arctan(cx))\log\left(2-\frac{2}{1-icx}\right)b}{d^2}+\frac{3i\sqrt{e}(a+b\arctan(cx))\,\text{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)b}{4(-d)^{5/2}}-\frac{3i\sqrt{e}(a+b\arctan(cx))\,\text{PolyLog}\left(2,1-\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)b}{4(-d)^{5/2}}-\frac{ice(a+b\arctan(cx))^2}{2d^2(c^2d-e)}-\frac{(a+b\arctan(cx))^2}{d^2x}+\frac{\sqrt{e}(a+b\arctan(cx))^2}{4d^2(\sqrt{-d}-\sqrt{e}x)}-\frac{\sqrt{e}(a+b\arctan(cx))^2}{4d^2(\sqrt{e}x+\sqrt{-d})}-\frac{ic(a+b\arctan(cx))^2}{d^2}-\frac{3\sqrt{e}(a+b\arctan(cx))^2\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{4(-d)^{5/2}}+\frac{3\sqrt{e}(a+b\arctan(cx))^2\log\left(\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)}{4(-d)^{5/2}}
$$

input
```
Int[(a + b*ArcTan[c*x])^2/(x^2*(d + e*x^2)^2),x]
```

output
```
((-I)*c*(a + b*ArcTan[c*x])^2)/d^2 - ((I/2)*c*e*(a + b*ArcTan[c*x])^2)/(d^
2*(c^2*d - e)) - (a + b*ArcTan[c*x])^2/(d^2*x) + (Sqrt[e]*(a + b*ArcTan[c*
x])^2)/(4*d^2*(Sqrt[-d] - Sqrt[e]*x)) - (Sqrt[e]*(a + b*ArcTan[c*x])^2)/(4
*d^2*(Sqrt[-d] + Sqrt[e]*x)) + (b*c*e*(a + b*ArcTan[c*x])*Log[2/(1 - I*c*x
)])/(d^2*(c^2*d - e)) - (b*c*e*(a + b*ArcTan[c*x])*Log[2/(1 + I*c*x)])/(d^
2*(c^2*d - e)) - (b*c*e*(a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x
))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/(2*d^2*(c^2*d - e)) - (3*Sqrt[
e]*(a + b*ArcTan[c*x])^2*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I
*Sqrt[e])*(1 - I*c*x))])/(4*(-d)^(5/2)) - (b*c*e*(a + b*ArcTan[c*x])*Log[(
2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(2*d^
2*(c^2*d - e)) + (3*Sqrt[e]*(a + b*ArcTan[c*x])^2*Log[(2*c*(Sqrt[-d] + Sqr
t[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(4*(-d)^(5/2)) + (2*b*c*
(a + b*ArcTan[c*x])*Log[2 - 2/(1 - I*c*x)])/d^2 - ((I/2)*b^2*c*e*PolyLog[2
, 1 - 2/(1 - I*c*x)])/(d^2*(c^2*d - e)) - (I*b^2*c*PolyLog[2, -1 + 2/(1 -
I*c*x)])/d^2 - ((I/2)*b^2*c*e*PolyLog[2, 1 - 2/(1 + I*c*x)])/(d^2*(c^2*d -
 e)) + ((I/4)*b^2*c*e*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt
[-d] - I*Sqrt[e])*(1 - I*c*x))])/(d^2*(c^2*d - e)) + (((3*I)/4)*b*Sqrt[e]*
(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-
d] - I*Sqrt[e])*(1 - I*c*x))])/(-d)^(5/2) + ((I/4)*b^2*c*e*PolyLog[2, 1 -
(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(...
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

## Maple [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^2(e\,x^2+d)^2}dx$$

input
```
int((a+b*arctan(c*x))^2/x^2/(e*x^2+d)^2,x)
```

output
```
int((a+b*arctan(c*x))^2/x^2/(e*x^2+d)^2,x)
```

## Fricas [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+ex^2)^2}\,dx = \int \frac{(b\arctan(cx)+a)^2}{(ex^2+d)^2x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^2/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)/(e^2*x^6 + 2*d*e*x^
4 + d^2*x^2), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+ex^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*atan(c*x))**2/x**2/(e*x**2+d)**2,x)
```

output
```
Timed out
```

## Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+ex^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*arctan(c*x))^2/x^2/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

## Giac [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+ex^2)^2}\,dx = \int \frac{(b\arctan(cx)+a)^2}{(ex^2+d)^2x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^2/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)^2/((e*x^2 + d)^2*x^2), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+ex^2)^2}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))^2}{x^2\,(e\,x^2+d)^2}\,dx$$

input
```
int((a + b*atan(c*x))^2/(x^2*(d + e*x^2)^2),x)
```

output
```
int((a + b*atan(c*x))^2/(x^2*(d + e*x^2)^2), x)
```

## Reduce [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^2(d+ex^2)^2}\,dx = \text{too large to display}$$

input
```
int((a+b*atan(c*x))^2/x^2/(e*x^2+d)^2,x)
```

output
```
( - 9*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a**2*c**4*d**3*x - 9*s
qrt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a**2*c**4*d**2*e*x**3 + 12*sq
rt(e)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a**2*c**2*d**2*e*x + 12*sqrt(e
)*sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a**2*c**2*d*e**2*x**3 - 3*sqrt(e)*
sqrt(d)*atan((e*x)/(sqrt(e)*sqrt(d)))*a**2*d*e**2*x - 3*sqrt(e)*sqrt(d)*at
an((e*x)/(sqrt(e)*sqrt(d)))*a**2*e**3*x**3 - 6*sqrt(e)*sqrt(d)*atan((e*x)/
(sqrt(e)*sqrt(d)))*b**2*c**2*d**2*e*x - 6*sqrt(e)*sqrt(d)*atan((e*x)/(sqrt
(e)*sqrt(d)))*b**2*c**2*d*e**2*x**3 - 2*atan(c*x)**3*b**2*c**5*d**4*x - 2*
atan(c*x)**3*b**2*c**5*d**3*e*x**3 + 2*atan(c*x)**3*b**2*c**3*d**3*e*x + 2
*atan(c*x)**3*b**2*c**3*d**2*e**2*x**3 - 6*atan(c*x)**2*a*b*c**5*d**4*x -
6*atan(c*x)**2*a*b*c**5*d**3*e*x**3 + 6*atan(c*x)**2*a*b*c**3*d**3*e*x + 6
*atan(c*x)**2*a*b*c**3*d**2*e**2*x**3 - 6*atan(c*x)**2*b**2*c**4*d**4 - 6*
atan(c*x)**2*b**2*c**4*d**3*e*x**2 + 8*atan(c*x)**2*b**2*c**2*d**3*e + 6*a
tan(c*x)**2*b**2*c**2*d**2*e**2*x**2 - 2*atan(c*x)**2*b**2*d**2*e**2 - 12*
atan(c*x)*a*b*c**4*d**4 - 12*atan(c*x)*a*b*c**4*d**3*e*x**2 + 16*atan(c*x)
*a*b*c**2*d**3*e + 12*atan(c*x)*a*b*c**2*d**2*e**2*x**2 - 4*atan(c*x)*a*b*
d**2*e**2 + 6*atan(c*x)*b**2*c**3*d**2*e**2*x**3 + 6*atan(c*x)*b**2*c*d**2
*e**2*x + 36*int(atan(c*x)/(3*c**4*d**3*x**3 + 6*c**4*d**2*e*x**5 + 3*c**4
*d*e**2*x**7 + 3*c**2*d**3*x + 5*c**2*d**2*e*x**3 + c**2*d*e**2*x**5 - c**
2*e**3*x**7 - d**2*e*x - 2*d*e**2*x**3 - e**3*x**5),x)*b**2*c**7*d**7*x...
```

# 3.1272 $\int \frac{(a+b\arctan(cx))^2}{x^3(d+ex^2)^2}\,dx$



## Optimal result

Integrand size = 23, antiderivative size = 1181

$$\int \frac{(a+b\arctan(cx))^2}{x^3(d+ex^2)^2}\,dx = \text{Too large to display}$$

output
```
4*e*(a+b*arctan(c*x))^2*arctanh(-1+2/(1+I*c*x))/d^3+1/2*c^2*e*(a+b*arctan(
c*x))^2/d^2/(c^2*d-e)-1/2*c^2*(a+b*arctan(c*x))^2/d^2-1/2*(a+b*arctan(c*x)
)^2/d^2/x^2-1/2*b^2*c^2*ln(c^2*x^2+1)/d^2+e*(a+b*arctan(c*x))^2*ln(2*c*((-
d)^(1/2)+e^(1/2)*x)/(c*(-d)^(1/2)+I*e^(1/2))/(1-I*c*x))/d^3+e*(a+b*arctan(
c*x))^2*ln(2*c*((-d)^(1/2)-e^(1/2)*x)/(c*(-d)^(1/2)-I*e^(1/2))/(1-I*c*x))/
d^3+1/2*b^2*e*polylog(3,1-2*c*((-d)^(1/2)+e^(1/2)*x)/(c*(-d)^(1/2)+I*e^(1/
2))/(1-I*c*x))/d^3+1/2*b^2*e*polylog(3,1-2*c*((-d)^(1/2)-e^(1/2)*x)/(c*(-d
)^(1/2)-I*e^(1/2))/(1-I*c*x))/d^3-1/4*e*(a+b*arctan(c*x))^2/d^3/(1-e^(1/2)
*x/(-d)^(1/2))-1/4*e*(a+b*arctan(c*x))^2/d^3/(1+e^(1/2)*x/(-d)^(1/2))+1/4*
I*b^2*c*e^(3/2)*polylog(2,1-2*c*((-d)^(1/2)-e^(1/2)*x)/(c*(-d)^(1/2)-I*e^(
1/2))/(1-I*c*x))/(-d)^(5/2)/(c^2*d-e)-I*b*e*(a+b*arctan(c*x))*polylog(2,1-
2*c*((-d)^(1/2)+e^(1/2)*x)/(c*(-d)^(1/2)+I*e^(1/2))/(1-I*c*x))/d^3-I*b*e*(
a+b*arctan(c*x))*polylog(2,1-2*c*((-d)^(1/2)-e^(1/2)*x)/(c*(-d)^(1/2)-I*e^
(1/2))/(1-I*c*x))/d^3+2*I*b*e*(a+b*arctan(c*x))*polylog(2,1-2/(1+I*c*x))/d
^3+2*I*b*e*(a+b*arctan(c*x))*polylog(2,1-2/(1-I*c*x))/d^3-b*c*(a+b*arctan(
c*x))/d^2/x+1/2*b*c*e^(3/2)*(a+b*arctan(c*x))*ln(2*c*((-d)^(1/2)+e^(1/2)*x
)/(c*(-d)^(1/2)+I*e^(1/2))/(1-I*c*x))/(-d)^(5/2)/(c^2*d-e)-1/2*b*c*e^(3/2)
*(a+b*arctan(c*x))*ln(2*c*((-d)^(1/2)-e^(1/2)*x)/(c*(-d)^(1/2)-I*e^(1/2))/
(1-I*c*x))/(-d)^(5/2)/(c^2*d-e)-1/4*I*b^2*c*e^(3/2)*polylog(2,1-2*c*((-d)^
(1/2)+e^(1/2)*x)/(c*(-d)^(1/2)+I*e^(1/2))/(1-I*c*x))/(-d)^(5/2)/(c^2*d-...
```

## Mathematica [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^3(d+ex^2)^2}\,dx = \int \frac{(a+b\arctan(cx))^2}{x^3(d+ex^2)^2}\,dx$$

input
```
Integrate[(a + b*ArcTan[c*x])^2/(x^3*(d + e*x^2)^2),x]
```

output
```
Integrate[(a + b*ArcTan[c*x])^2/(x^3*(d + e*x^2)^2), x]
```

## Rubi [A] (verified)

Time = 2.47 (sec) , antiderivative size = 1181, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {5515, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))^2}{x^3\left(d+ex^2\right)^2}dx$$

$$\Big\downarrow \text{ 5515}$$

$$\int \left(\frac{2e^2x(a+b\arctan(cx))^2}{d^3\left(d+ex^2\right)}-\frac{2e(a+b\arctan(cx))^2}{d^3x}+\frac{e^2x(a+b\arctan(cx))^2}{d^2\left(d+ex^2\right)^2}+\frac{(a+b\arctan(cx))^2}{d^2x^3}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{c^2\log(x)b^2}{d^2}-\frac{c^2\log(c^2x^2+1)\,b^2}{2d^2}+\frac{ice^{3/2}\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)b^2}{4(-d)^{5/2}(c^2d-e)}-\frac{ice^{3/2}\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)b^2}{4(-d)^{5/2}(c^2d-e)}-\frac{e\operatorname{PolyLog}\left(3,1-\frac{2}{1-icx}\right)b^2}{d^3}+\frac{e\operatorname{PolyLog}\left(3,1-\frac{2}{icx+1}\right)b^2}{d^3}-\frac{e\operatorname{PolyLog}\left(3,\frac{2}{icx+1}-1\right)b^2}{d^3}+\frac{e\operatorname{PolyLog}\left(3,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)b^2}{2d^3}+\frac{e\operatorname{PolyLog}\left(3,1-\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)b^2}{2d^3}-\frac{c(a+b\arctan(cx))b}{d^2x}-\frac{ce^{3/2}(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)b}{2(-d)^{5/2}(c^2d-e)}+\frac{ce^{3/2}(a+b\arctan(cx))\log\left(\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)b}{2(-d)^{5/2}(c^2d-e)}+\frac{2ie(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)b}{d^3}+\frac{2ie(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{icx+1}\right)b}{d^3}-\frac{2ie(a+b\arctan(cx))\operatorname{PolyLog}\left(2,\frac{2}{icx+1}-1\right)b}{d^3}-\frac{ie(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)b}{d^3}-\frac{ie(a+b\arctan(cx))\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)b}{d^3}+\frac{c^2e(a+b\arctan(cx))^2}{2d^2(c^2d-e)}-\frac{e(a+b\arctan(cx))^2}{4d^3\left(1-\frac{\sqrt{e}x}{\sqrt{-d}}\right)}-\frac{e(a+b\arctan(cx))^2}{4d^3\left(\frac{\sqrt{e}x}{\sqrt{-d}}+1\right)}-\frac{c^2(a+b\arctan(cx))^2}{2d^2}-\frac{(a+b\arctan(cx))^2}{2d^2x^2}-\frac{4e(a+b\arctan(cx))^2\operatorname{arctanh}\left(1-\frac{2}{icx+1}\right)}{d^3}-\frac{2e(a+b\arctan(cx))^2\log\left(\frac{2}{1-icx}\right)}{d^3}+\frac{e(a+b\arctan(cx))^2\log\left(\frac{2c(\sqrt{-d}-\sqrt{e}x)}{(c\sqrt{-d}-i\sqrt{e})(1-icx)}\right)}{d^3}+\frac{e(a+b\arctan(cx))^2\log\left(\frac{2c(\sqrt{e}x+\sqrt{-d})}{(\sqrt{-d}c+i\sqrt{e})(1-icx)}\right)}{d^3}$$

input
```
Int[(a + b*ArcTan[c*x])^2/(x^3*(d + e*x^2)^2),x]
```

output
```
-((b*c*(a + b*ArcTan[c*x]))/(d^2*x)) - (c^2*(a + b*ArcTan[c*x])^2)/(2*d^2)
 + (c^2*e*(a + b*ArcTan[c*x])^2)/(2*d^2*(c^2*d - e)) - (a + b*ArcTan[c*x])
^2/(2*d^2*x^2) - (e*(a + b*ArcTan[c*x])^2)/(4*d^3*(1 - (Sqrt[e]*x)/Sqrt[-d
])) - (e*(a + b*ArcTan[c*x])^2)/(4*d^3*(1 + (Sqrt[e]*x)/Sqrt[-d])) - (4*e*
(a + b*ArcTan[c*x])^2*ArcTanh[1 - 2/(1 + I*c*x)])/d^3 + (b^2*c^2*Log[x])/d
^2 - (2*e*(a + b*ArcTan[c*x])^2*Log[2/(1 - I*c*x)])/d^3 - (b*c*e^(3/2)*(a
+ b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e]
)*(1 - I*c*x))])/(2*(-d)^(5/2)*(c^2*d - e)) + (e*(a + b*ArcTan[c*x])^2*Log
[(2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/d^3
 + (b*c*e^(3/2)*(a + b*ArcTan[c*x])*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*S
qrt[-d] + I*Sqrt[e])*(1 - I*c*x))])/(2*(-d)^(5/2)*(c^2*d - e)) + (e*(a + b
*ArcTan[c*x])^2*Log[(2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[-d] + I*Sqrt[e])
*(1 - I*c*x))])/d^3 - (b^2*c^2*Log[1 + c^2*x^2])/(2*d^2) + ((2*I)*b*e*(a +
 b*ArcTan[c*x])*PolyLog[2, 1 - 2/(1 - I*c*x)])/d^3 + ((2*I)*b*e*(a + b*Arc
Tan[c*x])*PolyLog[2, 1 - 2/(1 + I*c*x)])/d^3 - ((2*I)*b*e*(a + b*ArcTan[c*
x])*PolyLog[2, -1 + 2/(1 + I*c*x)])/d^3 + ((I/4)*b^2*c*e^(3/2)*PolyLog[2,
1 - (2*c*(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/
((-d)^(5/2)*(c^2*d - e)) - (I*b*e*(a + b*ArcTan[c*x])*PolyLog[2, 1 - (2*c*
(Sqrt[-d] - Sqrt[e]*x))/((c*Sqrt[-d] - I*Sqrt[e])*(1 - I*c*x))])/d^3 - ((I
/4)*b^2*c*e^(3/2)*PolyLog[2, 1 - (2*c*(Sqrt[-d] + Sqrt[e]*x))/((c*Sqrt[...
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5515
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*ArcTan[c*x]
)^p, (f*x)^m*(d + e*x^2)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d
, e, f, m}, x] && IntegerQ[q] && IGtQ[p, 0] && ((EqQ[p, 1] && GtQ[q, 0]) ||
 IntegerQ[m])
```

## Maple [F]

$$\int \frac{(a+b\arctan (cx))^2}{x^3 (e\,x^2+d)^2}dx$$

input
```
int((a+b*arctan(c*x))^2/x^3/(e*x^2+d)^2,x)
```

output
```
int((a+b*arctan(c*x))^2/x^3/(e*x^2+d)^2,x)
```

## Fricas [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^3 (d+ex^2)^2} dx = \int \frac{(b\arctan (cx)+a)^2}{(ex^2+d)^2 x^3} dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^3/(e*x^2+d)^2,x, algorithm="fricas")
```

output
```
integral((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x) + a^2)/(e^2*x^7 + 2*d*e*x^
5 + d^2*x^3), x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{x^3 (d+ex^2)^2} dx = \text{Timed out}$$

input
```
integrate((a+b*atan(c*x))**2/x**3/(e*x**2+d)**2,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^3\left(d+ex^2\right)^2}\,dx = \int \frac{\left(b\arctan\left(cx\right)+a\right)^2}{\left(ex^2+d\right)^2x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^3/(e*x^2+d)^2,x, algorithm="maxima")
```

output
```
-1/2*a^2*((2*e*x^2 + d)/(d^2*e*x^4 + d^3*x^2) - 2*e*log(e*x^2 + d)/d^3 + 4
*e*log(x)/d^3) + integrate((b^2*arctan(c*x)^2 + 2*a*b*arctan(c*x))/(e^2*x^
7 + 2*d*e*x^5 + d^2*x^3), x)
```

## Giac [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^3\left(d+ex^2\right)^2}\,dx = \int \frac{\left(b\arctan\left(cx\right)+a\right)^2}{\left(ex^2+d\right)^2x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))^2/x^3/(e*x^2+d)^2,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)^2/((e*x^2 + d)^2*x^3), x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))^2}{x^3\left(d+ex^2\right)^2}\,dx = \int \frac{\left(a+b\,\mathrm{atan}(c\,x)\right)^2}{x^3\left(e\,x^2+d\right)^2}\,dx$$

input
```
int((a + b*atan(c*x))^2/(x^3*(d + e*x^2)^2),x)
```

output
```
int((a + b*atan(c*x))^2/(x^3*(d + e*x^2)^2), x)
```

## Reduce [F]

$$\int \frac{(a+b\arctan(cx))^2}{x^3(d+ex^2)^2}\,dx$$
$$= \frac{4\left(\int \frac{atan(cx)}{e^2x^7+2de\,x^5+d^2x^3}dx\right)ab\,d^4x^2+4\left(\int \frac{atan(cx)}{e^2x^7+2de\,x^5+d^2x^3}dx\right)ab\,d^3e\,x^4+2\left(\int \frac{atan(cx)^2}{e^2x^7+2de\,x^5+d^2x^3}dx\right)b^2d^4x^2+2}{}$$

input
```
int((a+b*atan(c*x))^2/x^3/(e*x^2+d)^2,x)
```

output
```
(4*int(atan(c*x)/(d**2*x**3 + 2*d*e*x**5 + e**2*x**7),x)*a*b*d**4*x**2 + 4
*int(atan(c*x)/(d**2*x**3 + 2*d*e*x**5 + e**2*x**7),x)*a*b*d**3*e*x**4 + 2
*int(atan(c*x)**2/(d**2*x**3 + 2*d*e*x**5 + e**2*x**7),x)*b**2*d**4*x**2 +
 2*int(atan(c*x)**2/(d**2*x**3 + 2*d*e*x**5 + e**2*x**7),x)*b**2*d**3*e*x*
*4 + 2*log(d + e*x**2)*a**2*d*e*x**2 + 2*log(d + e*x**2)*a**2*e**2*x**4 -
4*log(x)*a**2*d*e*x**2 - 4*log(x)*a**2*e**2*x**4 - a**2*d**2 + 2*a**2*e**2
*x**4)/(2*d**3*x**2*(d + e*x**2))
```

# 3.1273 $\int x^4 \arctan(x) \log(1+x^2)\, dx$



## Optimal result

Integrand size = 12, antiderivative size = 111

$$\int x^4 \arctan(x) \log(1+x^2)\, dx = -\frac{77x^2}{300} + \frac{9x^4}{200} - \frac{2}{5}x \arctan(x) + \frac{2}{15}x^3 \arctan(x) - \frac{2}{25}x^5 \arctan(x) + \frac{\arctan(x)^2}{5} + \frac{137}{300}\log(1+x^2) + \frac{1}{10}x^2 \log(1+x^2) - \frac{1}{20}x^4 \log(1+x^2) + \frac{1}{5}x^5 \arctan(x) \log(1+x^2) - \frac{1}{20}\log^2(1+x^2)$$

output
```
-77/300*x^2+9/200*x^4-2/5*x*arctan(x)+2/15*x^3*arctan(x)-2/25*x^5*arctan(x
)+1/5*arctan(x)^2+137/300*ln(x^2+1)+1/10*x^2*ln(x^2+1)-1/20*x^4*ln(x^2+1)+
1/5*x^5*arctan(x)*ln(x^2+1)-1/20*ln(x^2+1)^2
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.71

$$\int x^4 \arctan(x) \log\left(1+x^2\right) dx = \frac{1}{600}\left(x^2\left(-154+27x^2\right)+120\arctan(x)^2 + \left(274+60x^2-30x^4\right)\log\left(1+x^2\right)-30\log^2\left(1+x^2\right) + 8x\arctan(x)\left(-30+10x^2-6x^4+15x^4\log\left(1+x^2\right)\right)\right)$$

input
```
Integrate[x^4*ArcTan[x]*Log[1 + x^2],x]
```

output
```
(x^2*(-154 + 27*x^2) + 120*ArcTan[x]^2 + (274 + 60*x^2 - 30*x^4)*Log[1 + x
^2] - 30*Log[1 + x^2]^2 + 8*x*ArcTan[x]*(-30 + 10*x^2 - 6*x^4 + 15*x^4*Log
[1 + x^2]))/600
```

## Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.14, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5556, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \arctan(x) \log\left(x^2+1\right) dx$$

$$\Big\downarrow \ 5556$$

$$-2\int \left(\frac{x^3\left(4\arctan(x)x^3-x^2+2\right)}{20\left(x^2+1\right)}-\frac{x\log\left(x^2+1\right)}{10\left(x^2+1\right)}\right)dx+\frac{1}{5}x^5\arctan(x)\log\left(x^2+1\right)- \frac{1}{10}\log^2\left(x^2+1\right)+\frac{1}{10}x^2\log\left(x^2+1\right)-\frac{1}{20}x^4\log\left(x^2+1\right)$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{5}x^5\arctan(x)\log\left(x^2+1\right)-2\left(\frac{1}{25}x^5\arctan(x)-\frac{1}{15}x^3\arctan(x)+\frac{1}{5}x\arctan(x)-\frac{\arctan(x)^2}{10}-\frac{9x^4}{400}+\frac{77x^2}{600}-\frac{1}{40}\log^2\left(x^2+1\right)-\frac{137}{600}\log\left(\right.\right.$$
$$\frac{1}{10}\log^2\left(x^2+1\right)+\frac{1}{10}x^2\log\left(x^2+1\right)-\frac{1}{20}x^4\log\left(x^2+1\right)$$

input
```
Int[x^4*ArcTan[x]*Log[1 + x^2],x]
```

output
```
(x^2*Log[1 + x^2])/10 - (x^4*Log[1 + x^2])/20 + (x^5*ArcTan[x]*Log[1 + x^2
])/5 - Log[1 + x^2]^2/10 - 2*((77*x^2)/600 - (9*x^4)/400 + (x*ArcTan[x])/5
 - (x^3*ArcTan[x])/15 + (x^5*ArcTan[x])/25 - ArcTan[x]^2/10 - (137*Log[1 +
 x^2])/600 - Log[1 + x^2]^2/40)
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5556
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcTan[c*x]), x
]}, Simp[(d + e*Log[f + g*x^2])   u, x] - Simp[2*e*g   Int[ExpandIntegrand[
x*(u/(f + g*x^2)), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && Intege
rQ[m] && NeQ[m, -1]
```

## Maple [A] (verified)

Time = 3.46 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.82

| method | result |
|---|---|
| parallelrisch | $\frac{x^5\arctan(x)\ln(x^2+1)}{5}-\frac{2x^5\arctan(x)}{25}-\frac{x^4\ln(x^2+1)}{20}+\frac{9x^4}{200}+\frac{2x^3\arctan(x)}{15}+\frac{x^2\ln(x^2+1)}{10}-\frac{77x^2}{300}-\frac{2x\arctan}{5}$ |
| default | Expression too large to display |
| risch | Expression too large to display |

input
```
int(x^4*arctan(x)*ln(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/5*x^5*arctan(x)*ln(x^2+1)-2/25*x^5*arctan(x)-1/20*x^4*ln(x^2+1)+9/200*x^
4+2/15*x^3*arctan(x)+1/10*x^2*ln(x^2+1)-77/300*x^2-2/5*x*arctan(x)+1/5*arc
tan(x)^2-1/20*ln(x^2+1)^2+137/300*ln(x^2+1)+77/300
```

## Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.65

$$\int x^4 \arctan(x) \log\left(1+x^2\right) dx = \frac{9}{200}x^4 - \frac{77}{300}x^2 - \frac{2}{75}\left(3x^5 - 5x^3 + 15x\right)\arctan(x) + \frac{1}{5}\arctan(x)^2 + \frac{1}{300}\left(60x^5\arctan(x) - 15x^4 + 30x^2 + 137\right)\log\left(x^2 + 1\right) - \frac{1}{20}\log\left(x^2+1\right)^2$$

input
```
integrate(x^4*arctan(x)*log(x^2+1),x, algorithm="fricas")
```

output
```
9/200*x^4 - 77/300*x^2 - 2/75*(3*x^5 - 5*x^3 + 15*x)*arctan(x) + 1/5*arcta
n(x)^2 + 1/300*(60*x^5*arctan(x) - 15*x^4 + 30*x^2 + 137)*log(x^2 + 1) - 1
/20*log(x^2 + 1)^2
```

## Sympy [A] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.96

$$\int x^4 \arctan(x) \log\left(1+x^2\right) dx = \frac{x^5\log\left(x^2+1\right)\operatorname{atan}(x)}{5} - \frac{2x^5\operatorname{atan}(x)}{25} - \frac{x^4\log\left(x^2+1\right)}{20} + \frac{9x^4}{200} + \frac{2x^3\operatorname{atan}(x)}{15} + \frac{x^2\log\left(x^2+1\right)}{10} - \frac{77x^2}{300} - \frac{2x\operatorname{atan}(x)}{5} - \frac{\log\left(x^2+1\right)^2}{20} + \frac{137\log\left(x^2+1\right)}{300} + \frac{\operatorname{atan}^2(x)}{5}$$

input
```
integrate(x**4*atan(x)*ln(x**2+1),x)
```

output
```
x**5*log(x**2 + 1)*atan(x)/5 - 2*x**5*atan(x)/25 - x**4*log(x**2 + 1)/20 +
 9*x**4/200 + 2*x**3*atan(x)/15 + x**2*log(x**2 + 1)/10 - 77*x**2/300 - 2*
x*atan(x)/5 - log(x**2 + 1)**2/20 + 137*log(x**2 + 1)/300 + atan(x)**2/5
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.72

$$\int x^4 \arctan(x) \log\left(1+x^2\right) \, dx$$
$$= \frac{9}{200}\,x^4 - \frac{77}{300}\,x^2 + \frac{1}{75}\left(15\,x^5 \log\left(x^2+1\right) - 6\,x^5 + 10\,x^3 - 30\,x + 30\,\arctan\left(x\right)\right)\arctan\left(x\right) - \frac{1}{5}\,\arctan\left(x\right)^2 - \frac{1}{300}\left(15\,x^4 - 30\,x^2 - 137\right)\log\left(x^2+1\right) - \frac{1}{20}\,\log\left(x^2+1\right)^2$$

input
```
integrate(x^4*arctan(x)*log(x^2+1),x, algorithm="maxima")
```

output
```
9/200*x^4 - 77/300*x^2 + 1/75*(15*x^5*log(x^2 + 1) - 6*x^5 + 10*x^3 - 30*x
 + 30*arctan(x))*arctan(x) - 1/5*arctan(x)^2 - 1/300*(15*x^4 - 30*x^2 - 13
7)*log(x^2 + 1) - 1/20*log(x^2 + 1)^2
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.51

$$\int x^4 \arctan(x) \log(1+x^2)\,dx = \frac{1}{10}\pi x^5 \log(x^2+1)\operatorname{sgn}(x) - \frac{1}{5}x^5 \arctan\left(\frac{1}{x}\right)\log(x^2+1) - \frac{1}{25}\pi x^5 \operatorname{sgn}(x) + \frac{2}{25}x^5 \arctan\left(\frac{1}{x}\right) - \frac{1}{20}x^4 \log(x^2+1) + \frac{1}{15}\pi x^3 \operatorname{sgn}(x) + \frac{9}{200}x^4 - \frac{2}{15}x^3 \arctan\left(\frac{1}{x}\right) + \frac{1}{10}x^2 \log(x^2+1) - \frac{3}{10}\pi^2 \operatorname{sgn}(x) - \frac{1}{5}\pi x \operatorname{sgn}(x) - \frac{1}{5}\pi \arctan\left(\frac{1}{x}\right)\operatorname{sgn}(x) + \frac{1}{10}\pi^2 - \frac{77}{300}x^2 + \frac{1}{5}\pi \arctan(x) + \frac{1}{5}\pi \arctan\left(\frac{1}{x}\right) + \frac{2}{5}x \arctan\left(\frac{1}{x}\right) + \frac{1}{5}\arctan\left(\frac{1}{x}\right)^2 - \frac{1}{20}\log(x^2+1)^2 + \frac{137}{300}\log(x^2+1)$$

input
```
integrate(x^4*arctan(x)*log(x^2+1),x, algorithm="giac")
```

output
```
1/10*pi*x^5*log(x^2 + 1)*sgn(x) - 1/5*x^5*arctan(1/x)*log(x^2 + 1) - 1/25*
pi*x^5*sgn(x) + 2/25*x^5*arctan(1/x) - 1/20*x^4*log(x^2 + 1) + 1/15*pi*x^3
*sgn(x) + 9/200*x^4 - 2/15*x^3*arctan(1/x) + 1/10*x^2*log(x^2 + 1) - 3/10*
pi^2*sgn(x) - 1/5*pi*x*sgn(x) - 1/5*pi*arctan(1/x)*sgn(x) + 1/10*pi^2 - 77
/300*x^2 + 1/5*pi*arctan(x) + 1/5*pi*arctan(1/x) + 2/5*x*arctan(1/x) + 1/5
*arctan(1/x)^2 - 1/20*log(x^2 + 1)^2 + 137/300*log(x^2 + 1)
```

## Mupad [B] (verification not implemented)

Time = 0.89 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.74

$$\int x^4 \arctan(x) \log(1+x^2)\, dx = \frac{137\ln(x^2+1)}{300} - \frac{\ln(x^2+1)^2}{20} + \frac{\operatorname{atan}(x)^2}{5} - \operatorname{atan}(x)\left(\frac{2x}{5} - \frac{2x^3}{15} + \frac{2x^5}{25} - \frac{x^5\ln(x^2+1)}{5}\right) + \ln(x^2+1)\left(\frac{x^2}{10} - \frac{x^4}{20}\right) - \frac{77x^2}{300} + \frac{9x^4}{200}$$

input
```
int(x^4*log(x^2 + 1)*atan(x),x)
```

output
```
(137*log(x^2 + 1))/300 - log(x^2 + 1)^2/20 + atan(x)^2/5 - atan(x)*((2*x)/
5 - (2*x^3)/15 + (2*x^5)/25 - (x^5*log(x^2 + 1))/5) + log(x^2 + 1)*(x^2/10
 - x^4/20) - (77*x^2)/300 + (9*x^4)/200
```

## Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.80

$$\int x^4 \arctan(x) \log(1+x^2)\, dx = \frac{atan(x)^2}{5} + \frac{atan(x)\log(x^2+1)x^5}{5} - \frac{2atan(x)x^5}{25} + \frac{2atan(x)x^3}{15} - \frac{2atan(x)x}{5} - \frac{\log(x^2+1)^2}{20} - \frac{\log(x^2+1)x^4}{20} + \frac{\log(x^2+1)x^2}{10} + \frac{137\log(x^2+1)}{300} + \frac{9x^4}{200} - \frac{77x^2}{300}$$

input
```
int(x^4*atan(x)*log(x^2+1),x)
```

output
```
(120*atan(x)**2 + 120*atan(x)*log(x**2 + 1)*x**5 - 48*atan(x)*x**5 + 80*at
an(x)*x**3 - 240*atan(x)*x - 30*log(x**2 + 1)**2 - 30*log(x**2 + 1)*x**4 +
 60*log(x**2 + 1)*x**2 + 274*log(x**2 + 1) + 27*x**4 - 154*x**2)/600
```

# 3.1274 $\int x^3 \arctan(x) \log(1+x^2) \, dx$



## Optimal result

Integrand size = 12, antiderivative size = 88

$$\int x^3 \arctan(x) \log(1+x^2) \, dx = -\frac{25x}{24} + \frac{7x^3}{72} + \frac{25\arctan(x)}{24} + \frac{1}{4}x^2 \arctan(x) - \frac{1}{8}x^4 \arctan(x) + \frac{1}{4}x \log(1+x^2) - \frac{1}{12}x^3 \log(1+x^2) - \frac{1}{4}\arctan(x) \log(1+x^2) + \frac{1}{4}x^4 \arctan(x) \log(1+x^2)$$

output
```
-25/24*x+7/72*x^3+25/24*arctan(x)+1/4*x^2*arctan(x)-1/8*x^4*arctan(x)+1/4*
x*ln(x^2+1)-1/12*x^3*ln(x^2+1)-1/4*arctan(x)*ln(x^2+1)+1/4*x^4*arctan(x)*l
n(x^2+1)
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.64

$$\int x^3 \arctan(x) \log(1+x^2) \, dx = \frac{1}{72}\left(x\left(-75+7x^2-6(-3+x^2)\log(1+x^2)\right) + 3\arctan(x)\left(25+6x^2-3x^4 + 6(-1+x^4)\log(1+x^2)\right)\right)$$

input
```
Integrate[x^3*ArcTan[x]*Log[1 + x^2],x]
```

output
```
(x*(-75 + 7*x^2 - 6*(-3 + x^2)*Log[1 + x^2]) + 3*ArcTan[x]*(25 + 6*x^2 - 3
*x^4 + 6*(-1 + x^4)*Log[1 + x^2]))/72
```

## Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5554, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \arctan(x) \log\left(x^2+1\right) \, dx$$

$$\Big\downarrow \text{5554}$$

$$-\int \left(\frac{x^2\left(2-x^2\right)}{8\left(x^2+1\right)}-\frac{1}{4}\left(1-x^2\right)\log\left(x^2+1\right)\right)dx-\frac{1}{8}x^4\arctan(x)+\frac{1}{4}x^2\arctan(x)-\frac{1}{4}\arctan(x)\log\left(x^2+1\right)+\frac{1}{4}x^4\arctan(x)\log\left(x^2+1\right)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{8}x^4\arctan(x)+\frac{1}{4}x^2\arctan(x)-\frac{1}{4}\arctan(x)\log\left(x^2+1\right)+\frac{1}{4}x^4\arctan(x)\log\left(x^2+1\right)+\frac{25\arctan(x)}{24}+\frac{7x^3}{72}+\frac{1}{4}x\log\left(x^2+1\right)-\frac{1}{12}x^3\log\left(x^2+1\right)-\frac{25x}{24}$$

input
```
Int[x^3*ArcTan[x]*Log[1 + x^2],x]
```

output
```
(-25*x)/24 + (7*x^3)/72 + (25*ArcTan[x])/24 + (x^2*ArcTan[x])/4 - (x^4*Arc
Tan[x])/8 + (x*Log[1 + x^2])/4 - (x^3*Log[1 + x^2])/12 - (ArcTan[x]*Log[1
+ x^2])/4 + (x^4*ArcTan[x]*Log[1 + x^2])/4
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5554
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*Log[f + g*x^2])
, x]}, Simp[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[ExpandIntegrand[u/
(1 + c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[(m +
1)/2, 0]
```

## Maple [A] (verified)

Time = 2.50 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.81

| method | result |
|---|---|
| parallelrisch | $-\frac{25x}{24}+\frac{7x^3}{72}+\frac{25\arctan(x)}{24}+\frac{x^2\arctan(x)}{4}-\frac{x^4\arctan(x)}{8}+\frac{x\ln(x^2+1)}{4}-\frac{x^3\ln(x^2+1)}{12}-\frac{\arctan(x)\ln(x^2+1)}{4}$ |
| orering | $\frac{(9x^8-76x^6-497x^4-1570x^2-1350)\arctan(x)\ln(x^2+1)}{12x^2(x^2-3)}-\frac{(19x^8-274x^6-2162x^4-6210x^2-4725)\left(3x^2\arctan(x)\ln(x^2+\right.}{72x^4(x^2-3)}$ |
| default | Expression too large to display |
| risch | Expression too large to display |

input
```
int(x^3*arctan(x)*ln(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-25/24*x+7/72*x^3+25/24*arctan(x)+1/4*x^2*arctan(x)-1/8*x^4*arctan(x)+1/4*
x*ln(x^2+1)-1/12*x^3*ln(x^2+1)-1/4*arctan(x)*ln(x^2+1)+1/4*x^4*arctan(x)*l
n(x^2+1)
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.56

$$\int x^3 \arctan(x) \log\left(1+x^2\right) \, dx = \frac{7}{72} \, x^{3} - \frac{1}{24} \, \left(3 \, x^{4} - 6 \, x^{2} - 25\right) \arctan\left(x\right) - \frac{1}{12} \, \left(x^{3} - 3 \, \left(x^{4} - 1\right) \arctan\left(x\right) - 3 \, x\right) \log\left(x^{2} + 1\right) - \frac{25}{24} \, x$$

input

```
integrate(x^3*arctan(x)*log(x^2+1),x, algorithm="fricas")
```

output

```
7/72*x^3 - 1/24*(3*x^4 - 6*x^2 - 25)*arctan(x) - 1/12*(x^3 - 3*(x^4 - 1)*a
rctan(x) - 3*x)*log(x^2 + 1) - 25/24*x
```

## Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.94

$$\int x^3 \arctan(x) \log\left(1+x^2\right) \, dx = \frac{x^{4} \log\left(x^{2} + 1\right) \operatorname{atan}\left(x\right)}{4} - \frac{x^{4} \operatorname{atan}\left(x\right)}{8} - \frac{x^{3} \log\left(x^{2} + 1\right)}{12} + \frac{7x^{3}}{72} + \frac{x^{2} \operatorname{atan}\left(x\right)}{4} + \frac{x \log\left(x^{2} + 1\right)}{4} - \frac{25x}{24} - \frac{\log\left(x^{2} + 1\right) \operatorname{atan}\left(x\right)}{4} + \frac{25 \operatorname{atan}\left(x\right)}{24}$$

input

```
integrate(x**3*atan(x)*ln(x**2+1),x)
```

output

```
x**4*log(x**2 + 1)*atan(x)/4 - x**4*atan(x)/8 - x**3*log(x**2 + 1)/12 + 7*
x**3/72 + x**2*atan(x)/4 + x*log(x**2 + 1)/4 - 25*x/24 - log(x**2 + 1)*ata
n(x)/4 + 25*atan(x)/24
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.70

$$\int x^3 \arctan(x)\log\left(1+x^2\right)\,dx = \frac{7}{72}\,x^3 + \frac{1}{8}\,\left(2\,x^4\log\left(x^2+1\right) - x^4 + 2\,x^2 - 2\,\log\left(x^2+1\right)\right)\arctan\left(x\right) - \frac{1}{12}\,\left(x^3 - 3\,x\right)\log\left(x^2+1\right) - \frac{25}{24}\,x + \frac{25}{24}\,\arctan\left(x\right)$$

input
```
integrate(x^3*arctan(x)*log(x^2+1),x, algorithm="maxima")
```

output
```
7/72*x^3 + 1/8*(2*x^4*log(x^2 + 1) - x^4 + 2*x^2 - 2*log(x^2 + 1))*arctan(
x) - 1/12*(x^3 - 3*x)*log(x^2 + 1) - 25/24*x + 25/24*arctan(x)
```

## Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.41

$$\int x^3 \arctan(x)\log\left(1+x^2\right)\,dx = \frac{1}{8}\,\pi x^4\log\left(x^2+1\right)\mathrm{sgn}(x) - \frac{1}{4}\,x^4\arctan\left(\frac{1}{x}\right)\log\left(x^2+1\right) - \frac{1}{16}\,\pi x^4\mathrm{sgn}(x) + \frac{1}{8}\,x^4\arctan\left(\frac{1}{x}\right) - \frac{1}{12}\,x^3\log\left(x^2+1\right) + \frac{1}{8}\,\pi x^2\mathrm{sgn}(x) + \frac{7}{72}\,x^3 - \frac{1}{4}\,x^2\arctan\left(\frac{1}{x}\right) - \frac{1}{8}\,\pi\log\left(x^2+1\right)\mathrm{sgn}(x) + \frac{1}{4}\,x\log\left(x^2+1\right) + \frac{1}{4}\,\arctan\left(\frac{1}{x}\right)\log\left(x^2+1\right) - \frac{25}{24}\,\pi\mathrm{sgn}(x) - \frac{25}{24}\,x + \frac{25}{24}\,\arctan\left(x\right)$$

input
```
integrate(x^3*arctan(x)*log(x^2+1),x, algorithm="giac")
```

output
```
1/8*pi*x^4*log(x^2 + 1)*sgn(x) - 1/4*x^4*arctan(1/x)*log(x^2 + 1) - 1/16*p
i*x^4*sgn(x) + 1/8*x^4*arctan(1/x) - 1/12*x^3*log(x^2 + 1) + 1/8*pi*x^2*sg
n(x) + 7/72*x^3 - 1/4*x^2*arctan(1/x) - 1/8*pi*log(x^2 + 1)*sgn(x) + 1/4*x
*log(x^2 + 1) + 1/4*arctan(1/x)*log(x^2 + 1) - 25/24*pi*sgn(x) - 25/24*x +
 25/24*arctan(x)
```

## Mupad [B] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.78

$$\int x^3 \arctan(x) \log\left(1+x^2\right)\,dx = \frac{25\,\mathrm{atan}(x)}{24} + \frac{x^2\,\mathrm{atan}(x)}{4} + x\left(\frac{\ln\left(x^2+1\right)}{4} - \frac{25}{24}\right) - x^3\left(\frac{\ln\left(x^2+1\right)}{12} - \frac{7}{72}\right) - x^4\left(\frac{\mathrm{atan}(x)}{8} - \frac{\ln\left(x^2+1\right)\,\mathrm{atan}(x)}{4}\right) - \frac{\ln\left(x^2+1\right)\,\mathrm{atan}(x)}{4}$$

input
```
int(x^3*log(x^2 + 1)*atan(x),x)
```

output
```
(25*atan(x))/24 + (x^2*atan(x))/4 + x*(log(x^2 + 1)/4 - 25/24) - x^3*(log(
x^2 + 1)/12 - 7/72) - x^4*(atan(x)/8 - (log(x^2 + 1)*atan(x))/4) - (log(x^
2 + 1)*atan(x))/4
```

## Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.80

$$\int x^3 \arctan(x) \log\left(1+x^2\right)\,dx = \frac{atan(x)\log(x^2+1)\,x^4}{4} - \frac{atan(x)\log(x^2+1)}{4} - \frac{atan(x)\,x^4}{8} + \frac{atan(x)\,x^2}{4} + \frac{25atan(x)}{24} - \frac{\log(x^2+1)\,x^3}{12} + \frac{\log(x^2+1)\,x}{4} + \frac{7x^3}{72} - \frac{25x}{24}$$

input
```
int(x^3*atan(x)*log(x^2+1),x)
```

output
```
(18*atan(x)*log(x**2 + 1)*x**4 - 18*atan(x)*log(x**2 + 1) - 9*atan(x)*x**4
 + 18*atan(x)*x**2 + 75*atan(x) - 6*log(x**2 + 1)*x**3 + 18*log(x**2 + 1)*
x + 7*x**3 - 75*x)/72
```

# 3.1275 $\int x^2 \arctan(x) \log(1+x^2)\,dx$



## Optimal result

Integrand size = 12, antiderivative size = 82

$$\int x^2 \arctan(x) \log(1+x^2)\,dx = \frac{5x^2}{18} + \frac{2}{3}x\arctan(x) - \frac{2}{9}x^3\arctan(x) - \frac{\arctan(x)^2}{3} - \frac{11}{18}\log(1+x^2) - \frac{1}{6}x^2\log(1+x^2) + \frac{1}{3}x^3\arctan(x)\log(1+x^2) + \frac{1}{12}\log^2(1+x^2)$$

output
```
5/18*x^2+2/3*x*arctan(x)-2/9*x^3*arctan(x)-1/3*arctan(x)^2-11/18*ln(x^2+1)
-1/6*x^2*ln(x^2+1)+1/3*x^3*arctan(x)*ln(x^2+1)+1/12*ln(x^2+1)^2
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.78

$$\int x^2 \arctan(x) \log(1+x^2)\,dx = \frac{1}{36}\left(10x^2 - 12\arctan(x)^2 - 2(11+3x^2)\log(1+x^2) + 3\log^2(1+x^2) + 4x\arctan(x)\left(6-2x^2+3x^2\log(1+x^2)\right)\right)$$

input
```
Integrate[x^2*ArcTan[x]*Log[1 + x^2],x]
```

output
```
(10*x^2 - 12*ArcTan[x]^2 - 2*(11 + 3*x^2)*Log[1 + x^2] + 3*Log[1 + x^2]^2
+ 4*x*ArcTan[x]*(6 - 2*x^2 + 3*x^2*Log[1 + x^2]))/36
```

## Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.18, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5556, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \arctan(x) \log\left(x^2+1\right) dx$$

$$\Big\downarrow \text{5556}$$

$$-2\int \left(\frac{x\log\left(x^2+1\right)}{6\left(x^2+1\right)}-\frac{x^3(1-2x\arctan(x))}{6\left(x^2+1\right)}\right)dx+\frac{1}{3}x^3\arctan(x)\log\left(x^2+1\right)+\frac{1}{6}\log^2\left(x^2+1\right)-\frac{1}{6}x^2\log\left(x^2+1\right)$$

$$\Big\downarrow \text{2009}$$

$$-2\left(\frac{1}{9}x^3\arctan(x)-\frac{1}{3}x\arctan(x)+\frac{\arctan(x)^2}{6}-\frac{5x^2}{36}+\frac{1}{24}\log^2\left(x^2+1\right)+\frac{11}{36}\log\left(x^2+1\right)\right)+\frac{1}{3}x^3\arctan(x)\log\left(x^2+1\right)+\frac{1}{6}\log^2\left(x^2+1\right)-\frac{1}{6}x^2\log\left(x^2+1\right)$$

input
```
Int[x^2*ArcTan[x]*Log[1 + x^2],x]
```

output
```
-1/6*(x^2*Log[1 + x^2]) + (x^3*ArcTan[x]*Log[1 + x^2])/3 + Log[1 + x^2]^2/
6 - 2*((-5*x^2)/36 - (x*ArcTan[x])/3 + (x^3*ArcTan[x])/9 + ArcTan[x]^2/6 +
 (11*Log[1 + x^2])/36 + Log[1 + x^2]^2/24)
```

**Defintions of rubi rules used**

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5556
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcTan[c*x]), x
]}, Simp[(d + e*Log[f + g*x^2])   u, x] - Simp[2*e*g   Int[ExpandIntegrand[
x*(u/(f + g*x^2)), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && Intege
rQ[m] && NeQ[m, -1]
```

## Maple [A] (verified)

Time = 2.58 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.83

| method | result |
|---|---|
| parallelrisch | $\frac{x^3 \arctan(x)\ln(x^2+1)}{3} - \frac{2x^3\arctan(x)}{9} - \frac{x^2\ln(x^2+1)}{6} + \frac{5x^2}{18} + \frac{2x\arctan(x)}{3} - \frac{\arctan(x)^2}{3} + \frac{\ln(x^2+1)^2}{12} - \frac{11\ln}{}$ |
| default | Expression too large to display |
| risch | Expression too large to display |

input
```
int(x^2*arctan(x)*ln(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/3*x^3*arctan(x)*ln(x^2+1)-2/9*x^3*arctan(x)-1/6*x^2*ln(x^2+1)+5/18*x^2+2
/3*x*arctan(x)-1/3*arctan(x)^2+1/12*ln(x^2+1)^2-11/18*ln(x^2+1)-5/18
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.67

$$\int x^2\arctan(x)\log\left(1+x^2\right)\,dx = \frac{5}{18}x^2 - \frac{2}{9}\left(x^3-3x\right)\arctan(x) - \frac{1}{3}\arctan(x)^2 + \frac{1}{18}\left(6x^3\arctan(x) - 3x^2 - 11\right)\log\left(x^2+1\right) + \frac{1}{12}\log\left(x^2+1\right)^2$$

input
```
integrate(x^2*arctan(x)*log(x^2+1),x, algorithm="fricas")
```

output
```
5/18*x^2 - 2/9*(x^3 - 3*x)*arctan(x) - 1/3*arctan(x)^2 + 1/18*(6*x^3*arcta
n(x) - 3*x^2 - 11)*log(x^2 + 1) + 1/12*log(x^2 + 1)^2
```

## Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.95

$$\int x^2 \arctan(x) \log\left(1+x^2\right) \, dx = \frac{x^3 \log\left(x^2+1\right) \operatorname{atan}\left(x\right)}{3} - \frac{2x^3 \operatorname{atan}\left(x\right)}{9} - \frac{x^2 \log\left(x^2+1\right)}{6} + \frac{5x^2}{18} + \frac{2x \operatorname{atan}\left(x\right)}{3} + \frac{\log\left(x^2+1\right)^2}{12} - \frac{11 \log\left(x^2+1\right)}{18} - \frac{\operatorname{atan}^2\left(x\right)}{3}$$

input
```
integrate(x**2*atan(x)*ln(x**2+1),x)
```

output
```
x**3*log(x**2 + 1)*atan(x)/3 - 2*x**3*atan(x)/9 - x**2*log(x**2 + 1)/6 + 5
*x**2/18 + 2*x*atan(x)/3 + log(x**2 + 1)**2/12 - 11*log(x**2 + 1)/18 - ata
n(x)**2/3
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.79

$$\int x^2 \arctan(x) \log\left(1+x^2\right) \, dx = \frac{5}{18} x^2 + \frac{1}{9} \left(3 x^3 \log\left(x^2+1\right) - 2 x^3 + 6 x - 6 \arctan\left(x\right)\right) \arctan\left(x\right) + \frac{1}{3} \arctan\left(x\right)^2 - \frac{1}{18} \left(3 x^2 + 11\right) \log\left(x^2+1\right) + \frac{1}{12} \log\left(x^2+1\right)^2$$

input
```
integrate(x^2*arctan(x)*log(x^2+1),x, algorithm="maxima")
```

output
```
5/18*x^2 + 1/9*(3*x^3*log(x^2 + 1) - 2*x^3 + 6*x - 6*arctan(x))*arctan(x)
+ 1/3*arctan(x)^2 - 1/18*(3*x^2 + 11)*log(x^2 + 1) + 1/12*log(x^2 + 1)^2
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(66) = 132$.

Time = 0.12 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.65

$$\int x^2 \arctan(x) \log\left(1+x^2\right)\,dx = \frac{1}{6}\,\pi x^3 \log\left(x^2+1\right)\mathrm{sgn}(x) - \frac{1}{3}\,x^3 \arctan\left(\frac{1}{x}\right)\log\left(x^2+1\right) - \frac{1}{9}\,\pi x^3\mathrm{sgn}(x) + \frac{2}{9}\,x^3\arctan\left(\frac{1}{x}\right) - \frac{1}{6}\,x^2\log\left(x^2+1\right) + \frac{1}{6}\,\pi^2\mathrm{sgn}(x) + \frac{1}{3}\,\pi x\mathrm{sgn}(x) + \frac{1}{3}\,\pi\arctan\left(\frac{1}{x}\right)\mathrm{sgn}(x) - \frac{1}{6}\,\pi^2 + \frac{5}{18}\,x^2 - \frac{1}{3}\,\pi\arctan\left(x\right) - \frac{1}{3}\,\pi\arctan\left(\frac{1}{x}\right) - \frac{2}{3}\,x\arctan\left(\frac{1}{x}\right) - \frac{1}{3}\,\arctan\left(\frac{1}{x}\right)^2 + \frac{1}{12}\,\log\left(x^2+1\right)^2 - \frac{11}{18}\,\log\left(x^2+1\right)$$

input
```
integrate(x^2*arctan(x)*log(x^2+1),x, algorithm="giac")
```

output
```
1/6*pi*x^3*log(x^2 + 1)*sgn(x) - 1/3*x^3*arctan(1/x)*log(x^2 + 1) - 1/9*pi
*x^3*sgn(x) + 2/9*x^3*arctan(1/x) - 1/6*x^2*log(x^2 + 1) + 1/6*pi^2*sgn(x)
 + 1/3*pi*x*sgn(x) + 1/3*pi*arctan(1/x)*sgn(x) - 1/6*pi^2 + 5/18*x^2 - 1/3
*pi*arctan(x) - 1/3*pi*arctan(1/x) - 2/3*x*arctan(1/x) - 1/3*arctan(1/x)^2
 + 1/12*log(x^2 + 1)^2 - 11/18*log(x^2 + 1)
```

## Mupad [B] (verification not implemented)

Time = 0.89 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.79

$$\int x^2 \arctan(x) \log\left(1+x^2\right) dx = \frac{\ln\left(x^2+1\right)^2}{12} - \frac{11 \ln\left(x^2+1\right)}{18} - \frac{\mathrm{atan}(x)^2}{3} - x^2 \left(\frac{\ln\left(x^2+1\right)}{6} - \frac{5}{18}\right) - x^3 \left(\frac{2\,\mathrm{atan}(x)}{9} - \frac{\ln\left(x^2+1\right)\,\mathrm{atan}(x)}{3}\right) + \frac{2\,x\,\mathrm{atan}(x)}{3}$$

input
```
int(x^2*log(x^2 + 1)*atan(x),x)
```

output
```
log(x^2 + 1)^2/12 - (11*log(x^2 + 1))/18 - atan(x)^2/3 - x^2*(log(x^2 + 1)
/6 - 5/18) - x^3*((2*atan(x))/9 - (log(x^2 + 1)*atan(x))/3) + (2*x*atan(x)
)/3
```

## Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.80

$$\int x^2 \arctan(x) \log\left(1+x^2\right) dx = -\frac{atan(x)^2}{3} + \frac{atan(x)\log(x^2+1)\,x^3}{3} - \frac{2atan(x)\,x^3}{9} + \frac{2atan(x)\,x}{3} + \frac{\log(x^2+1)^2}{12} - \frac{\log(x^2+1)\,x^2}{6} - \frac{11\log(x^2+1)}{18} + \frac{5x^2}{18}$$

input
```
int(x^2*atan(x)*log(x^2+1),x)
```

output
```
( - 12*atan(x)**2 + 12*atan(x)*log(x**2 + 1)*x**3 - 8*atan(x)*x**3 + 24*at
an(x)*x + 3*log(x**2 + 1)**2 - 6*log(x**2 + 1)*x**2 - 22*log(x**2 + 1) + 1
0*x**2)/36
```

# 3.1276 $\int x \arctan(x) \log(1+x^2)\, dx$



## Optimal result

Integrand size = 10, antiderivative size = 49

$$\int x \arctan(x) \log(1+x^2)\, dx = \frac{3x}{2} - \frac{3\arctan(x)}{2} - \frac{1}{2}x^2\arctan(x) - \frac{1}{2}x\log(1+x^2) + \frac{1}{2}(1+x^2)\arctan(x)\log(1+x^2)$$

output
```
3/2*x-3/2*arctan(x)-1/2*x^2*arctan(x)-1/2*x*ln(x^2+1)+1/2*(x^2+1)*arctan(x
)*ln(x^2+1)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.78

$$\int x \arctan(x) \log(1+x^2)\, dx = \frac{1}{2}\left(3x - 3\arctan(x) - x^2\arctan(x) + \left(-x + (1+x^2)\arctan(x)\right)\log(1+x^2)\right)$$

input
```
Integrate[x*ArcTan[x]*Log[1 + x^2],x]
```

output
```
(3*x - 3*ArcTan[x] - x^2*ArcTan[x] + (-x + (1 + x^2)*ArcTan[x])*Log[1 + x^
2])/2
```

## Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5554, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\arctan(x)\log\left(x^2+1\right)\,dx$$

$$\Big\downarrow \text{5554}$$

$$-\int\left(\frac{1}{2}\log\left(x^2+1\right)-\frac{x^2}{2\left(x^2+1\right)}\right)dx-\frac{1}{2}x^2\arctan(x)+\frac{1}{2}\left(x^2+1\right)\arctan(x)\log\left(x^2+1\right)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{2}x^2\arctan(x)+\frac{1}{2}\left(x^2+1\right)\arctan(x)\log\left(x^2+1\right)-\frac{3\arctan(x)}{2}-\frac{1}{2}x\log\left(x^2+1\right)+\frac{3x}{2}$$

input
```
Int[x*ArcTan[x]*Log[1 + x^2],x]
```

output
```
(3*x)/2 - (3*ArcTan[x])/2 - (x^2*ArcTan[x])/2 - (x*Log[1 + x^2])/2 + ((1 +
 x^2)*ArcTan[x]*Log[1 + x^2])/2
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5554
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*Log[f + g*x^2])
, x]}, Simp[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[ExpandIntegrand[u/
(1 + c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[(m +
1)/2, 0]
```

## Maple [A] (verified)

Time = 2.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

| method | result |
|---|---|
| parallelrisch | $\frac{x^2\arctan(x)\ln(x^2+1)}{2}-\frac{x^2\arctan(x)}{2}-\frac{x\ln(x^2+1)}{2}+\frac{\arctan(x)\ln(x^2+1)}{2}+\frac{3x}{2}-\frac{3\arctan(x)}{2}$ |
| orering | $\frac{(x^6+9x^2+18)\arctan(x)\ln(x^2+1)}{x^2(x^2-3)}-\frac{(x^6+2x^4+21x^2+36)\left(\arctan(x)\ln(x^2+1)+\frac{\ln(x^2+1)x}{x^2+1}+\frac{2x^2\arctan(x)}{x^2+1}\right)}{2x^2(x^2-3)}-\frac{3(x^2+1)}{}$ |
| default | Expression too large to display |
| risch | Expression too large to display |

input
```
int(x*arctan(x)*ln(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2*arctan(x)*ln(x^2+1)-1/2*x^2*arctan(x)-1/2*x*ln(x^2+1)+1/2*arctan(x
)*ln(x^2+1)+3/2*x-3/2*arctan(x)
```

## Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.67

$$\int x\arctan(x)\log\left(1+x^2\right)\,dx=-\frac{1}{2}\left(x^2+3\right)\arctan\left(x\right)+\frac{1}{2}\left(\left(x^2+1\right)\arctan\left(x\right)-x\right)\log\left(x^2+1\right)+\frac{3}{2}\,x$$

input
```
integrate(x*arctan(x)*log(x^2+1),x, algorithm="fricas")
```

output
```
-1/2*(x^2 + 3)*arctan(x) + 1/2*((x^2 + 1)*arctan(x) - x)*log(x^2 + 1) + 3/
2*x
```

## Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.14

$$\int x\arctan(x)\log\left(1+x^2\right)\,dx=\frac{x^2\log\left(x^2+1\right)\operatorname{atan}(x)}{2}-\frac{x^2\operatorname{atan}(x)}{2}-\frac{x\log\left(x^2+1\right)}{2}+\frac{3x}{2}+\frac{\log\left(x^2+1\right)\operatorname{atan}(x)}{2}-\frac{3\operatorname{atan}(x)}{2}$$

input
```
integrate(x*atan(x)*ln(x**2+1),x)
```

output
```
x**2*log(x**2 + 1)*atan(x)/2 - x**2*atan(x)/2 - x*log(x**2 + 1)/2 + 3*x/2
+ log(x**2 + 1)*atan(x)/2 - 3*atan(x)/2
```

## Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

$$\int x\arctan(x)\log\left(1+x^2\right)\,dx=-\frac{1}{2}\left(x^2-\left(x^2+1\right)\log\left(x^2+1\right)+1\right)\arctan(x)-\frac{1}{2}x\log\left(x^2+1\right)+\frac{3}{2}x-\arctan(x)$$

input
```
integrate(x*arctan(x)*log(x^2+1),x, algorithm="maxima")
```

output
```
-1/2*(x^2 - (x^2 + 1)*log(x^2 + 1) + 1)*arctan(x) - 1/2*x*log(x^2 + 1) + 3
/2*x - arctan(x)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(39) = 78$.

Time = 0.13 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.76

$$\int x\arctan(x)\log\left(1+x^2\right)\,dx = \frac{1}{4}\pi x^2\log\left(x^2+1\right)\operatorname{sgn}(x) - \frac{1}{2}x^2\arctan\left(\frac{1}{x}\right)\log\left(x^2+1\right) - \frac{1}{4}\pi x^2\operatorname{sgn}(x) + \frac{1}{2}x^2\arctan\left(\frac{1}{x}\right) + \frac{1}{4}\pi\log\left(x^2+1\right)\operatorname{sgn}(x) - \frac{1}{2}x\log\left(x^2+1\right) - \frac{1}{2}\arctan\left(\frac{1}{x}\right)\log\left(x^2+1\right) + \frac{3}{2}x - \frac{3}{2}\arctan(x)$$

input
```
integrate(x*arctan(x)*log(x^2+1),x, algorithm="giac")
```

output
```
1/4*pi*x^2*log(x^2 + 1)*sgn(x) - 1/2*x^2*arctan(1/x)*log(x^2 + 1) - 1/4*pi
*x^2*sgn(x) + 1/2*x^2*arctan(1/x) + 1/4*pi*log(x^2 + 1)*sgn(x) - 1/2*x*log
(x^2 + 1) - 1/2*arctan(1/x)*log(x^2 + 1) + 3/2*x - 3/2*arctan(x)
```

## Mupad [B] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int x\arctan(x)\log\left(1+x^2\right)\,dx = \frac{\ln\left(x^2+1\right)\,\mathrm{atan}(x)}{2} - x\left(\frac{\ln\left(x^2+1\right)}{2} - \frac{3}{2}\right) - x^2\left(\frac{\mathrm{atan}(x)}{2} - \frac{\ln\left(x^2+1\right)\,\mathrm{atan}(x)}{2}\right) - \frac{3\,\mathrm{atan}(x)}{2}$$

input
```
int(x*log(x^2 + 1)*atan(x),x)
```

output
```
(log(x^2 + 1)*atan(x))/2 - x*(log(x^2 + 1)/2 - 3/2) - x^2*(atan(x)/2 - (lo
g(x^2 + 1)*atan(x))/2) - (3*atan(x))/2
```

## Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.96

$$\int x\arctan(x)\log\left(1+x^2\right)\,dx=\frac{atan(x)\log(x^2+1)\,x^2}{2}+\frac{atan(x)\log(x^2+1)}{2}-\frac{atan(x)\,x^2}{2}-\frac{3atan(x)}{2}-\frac{\log(x^2+1)\,x}{2}+\frac{3x}{2}$$

input

```
int(x*atan(x)*log(x^2+1),x)
```

output

```
(atan(x)*log(x**2 + 1)*x**2 + atan(x)*log(x**2 + 1) - atan(x)*x**2 - 3*ata
n(x) - log(x**2 + 1)*x + 3*x)/2
```

# 3.1277 $\int \arctan(x) \log(1+x^2)\, dx$



## Optimal result

Integrand size = 9, antiderivative size = 38

$$\int \arctan(x) \log(1+x^2)\, dx = -2x\arctan(x) + \arctan(x)^2 + \log(1+x^2) + x\arctan(x)\log(1+x^2) - \frac{1}{4}\log^2(1+x^2)$$

output
```
-2*x*arctan(x)+arctan(x)^2+ln(x^2+1)+x*arctan(x)*ln(x^2+1)-1/4*ln(x^2+1)^2
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \arctan(x) \log(1+x^2)\, dx = -2x\arctan(x) + \arctan(x)^2 + \log(1+x^2) + x\arctan(x)\log(1+x^2) - \frac{1}{4}\log^2(1+x^2)$$

input
```
Integrate[ArcTan[x]*Log[1 + x^2],x]
```

output
```
-2*x*ArcTan[x] + ArcTan[x]^2 + Log[1 + x^2] + x*ArcTan[x]*Log[1 + x^2] - L
og[1 + x^2]^2/4
```

## Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.26, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.889$, Rules used = {5544, 2925, 2837, 2738, 5451, 5345, 240, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \arctan(x)\log\left(x^2+1\right)\,dx$$

$$\Big\downarrow \ 5544$$

$$-2\int \frac{x^2\arctan(x)}{x^2+1}dx-\int \frac{x\log\left(x^2+1\right)}{x^2+1}dx+x\arctan(x)\log\left(x^2+1\right)$$

$$\Big\downarrow \ 2925$$

$$-2\int \frac{x^2\arctan(x)}{x^2+1}dx-\frac{1}{2}\int \frac{\log\left(x^2+1\right)}{x^2+1}dx^2+x\arctan(x)\log\left(x^2+1\right)$$

$$\Big\downarrow \ 2837$$

$$-2\int \frac{x^2\arctan(x)}{x^2+1}dx-\frac{1}{2}\int \frac{\log\left(x^2+1\right)}{x^2}d\left(x^2+1\right)+x\arctan(x)\log\left(x^2+1\right)$$

$$\Big\downarrow \ 2738$$

$$-2\int \frac{x^2\arctan(x)}{x^2+1}dx+x\arctan(x)\log\left(x^2+1\right)-\frac{1}{4}\log^2\left(x^2+1\right)$$

$$\Big\downarrow \ 5451$$

$$-2\left(\int \arctan(x)dx-\int \frac{\arctan(x)}{x^2+1}dx\right)+x\arctan(x)\log\left(x^2+1\right)-\frac{1}{4}\log^2\left(x^2+1\right)$$

$$\Big\downarrow \ 5345$$

$$-2\left(-\int \frac{\arctan(x)}{x^2+1}dx-\int \frac{x}{x^2+1}dx+x\arctan(x)\right)+x\arctan(x)\log\left(x^2+1\right)-\frac{1}{4}\log^2\left(x^2+1\right)$$

$$\Big\downarrow \ 240$$

$$-2\left(-\int \frac{\arctan(x)}{x^2+1}dx+x\arctan(x)-\frac{1}{2}\log\left(x^2+1\right)\right)+x\arctan(x)\log\left(x^2+1\right)-\frac{1}{4}\log^2\left(x^2+1\right)$$

$$\Big\downarrow \text{5419}$$

$$x\arctan(x)\log\left(x^2+1\right)-2\left(-\frac{1}{2}\arctan(x)^2+x\arctan(x)-\frac{1}{2}\log\left(x^2+1\right)\right)-\frac{1}{4}\log^2\left(x^2+1\right)$$

input
```
Int[ArcTan[x]*Log[1 + x^2],x]
```

output
```
-2*(x*ArcTan[x] - ArcTan[x]^2/2 - Log[1 + x^2]/2) + x*ArcTan[x]*Log[1 + x^
2] - Log[1 + x^2]^2/4
```

### Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 2738
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

rule 2837
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[1/e   Subst[Int[(f*(x/d))^q*(a + b*Log[c*x
^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] &&
EqQ[e*f - d*g, 0]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

rule 5345
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
+ b*ArcTan[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTan[c*x^n])^(p
 - 1)/(1 + c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0] &&
 (EqQ[n, 1] || EqQ[p, 1])
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5544
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.)), x_Symbol] :> Simp[x*(d + e*Log[f + g*x^2])*(a + b*ArcTan[c*x]), x] +
 (-Simp[b*c   Int[x*((d + e*Log[f + g*x^2])/(1 + c^2*x^2)), x], x] - Simp[2
*e*g   Int[x^2*((a + b*ArcTan[c*x])/(f + g*x^2)), x], x]) /; FreeQ[{a, b, c
, d, e, f, g}, x]
```

## Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

| method | result | size |
|---|---|---|
| parallelrisch | $-2x\arctan\left(x\right)+\arctan\left(x\right)^{2}+\ln\left(x^{2}+1\right)+x\arctan\left(x\right)\ln\left(x^{2}+1\right)-\frac{\ln\left(x^{2}+1\right)^{2}}{4}$ | 37 |
| default | Expression too large to display | 1913 |
| risch | Expression too large to display | 4618 |

input
```
int(arctan(x)*ln(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-2*x*arctan(x)+arctan(x)^2+ln(x^2+1)+x*arctan(x)*ln(x^2+1)-1/4*ln(x^2+1)^2
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.87

$$\int \arctan(x)\log\left(1+x^2\right)\,dx = -2\,x\arctan\left(x\right)+\arctan\left(x\right)^2 + \left(x\arctan\left(x\right)+1\right)\log\left(x^2+1\right)-\frac{1}{4}\,\log\left(x^2+1\right)^2$$

input
```
integrate(arctan(x)*log(x^2+1),x, algorithm="fricas")
```

output
```
-2*x*arctan(x) + arctan(x)^2 + (x*arctan(x) + 1)*log(x^2 + 1) - 1/4*log(x^
2 + 1)^2
```

## Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03

$$\int \arctan(x)\log\left(1+x^2\right)\,dx = x\log\left(x^2+1\right)\operatorname{atan}\left(x\right)-2x\operatorname{atan}\left(x\right) - \frac{\log\left(x^2+1\right)^2}{4}+\log\left(x^2+1\right)+\operatorname{atan}^2\left(x\right)$$

input
```
integrate(atan(x)*ln(x**2+1),x)
```

output
```
x*log(x**2 + 1)*atan(x) - 2*x*atan(x) - log(x**2 + 1)**2/4 + log(x**2 + 1)
 + atan(x)**2
```

## Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.11

$$\int \arctan(x)\log(1+x^2)\,dx = (x\log(x^2+1) - 2x + 2\arctan(x))\arctan(x) - \arctan(x)^2 - \frac{1}{4}\log(x^2+1)^2 + \log(x^2+1)$$

input
```
integrate(arctan(x)*log(x^2+1),x, algorithm="maxima")
```

output
```
(x*log(x^2 + 1) - 2*x + 2*arctan(x))*arctan(x) - arctan(x)^2 - 1/4*log(x^2
 + 1)^2 + log(x^2 + 1)
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. 2(36) = 72.

Time = 0.13 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.42

$$\int \arctan(x)\log(1+x^2)\,dx = \frac{1}{2}\pi x\log(x^2+1)\operatorname{sgn}(x) - x\arctan\left(\frac{1}{x}\right)\log(x^2+1) - \frac{3}{2}\pi^2\operatorname{sgn}(x) - \pi x\operatorname{sgn}(x) - \pi\arctan\left(\frac{1}{x}\right)\operatorname{sgn}(x) + \frac{1}{2}\pi^2 + \pi\arctan(x) + \pi\arctan\left(\frac{1}{x}\right) + 2x\arctan\left(\frac{1}{x}\right) + \arctan\left(\frac{1}{x}\right)^2 - \frac{1}{4}\log(x^2+1)^2 + \log(x^2+1)$$

input
```
integrate(arctan(x)*log(x^2+1),x, algorithm="giac")
```

output
```
1/2*pi*x*log(x^2 + 1)*sgn(x) - x*arctan(1/x)*log(x^2 + 1) - 3/2*pi^2*sgn(x
) - pi*x*sgn(x) - pi*arctan(1/x)*sgn(x) + 1/2*pi^2 + pi*arctan(x) + pi*arc
tan(1/x) + 2*x*arctan(1/x) + arctan(1/x)^2 - 1/4*log(x^2 + 1)^2 + log(x^2
+ 1)
```

## Mupad [B] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03

$$\int \arctan(x)\log\left(1+x^2\right) dx = \ln\left(x^2+1\right) - \frac{\ln\left(x^2+1\right)^2}{4} + \operatorname{atan}(x)^2 - x\left(2\operatorname{atan}(x) - \ln\left(x^2+1\right)\operatorname{atan}(x)\right)$$

input
```
int(log(x^2 + 1)*atan(x),x)
```

output
```
log(x^2 + 1) - log(x^2 + 1)^2/4 + atan(x)^2 - x*(2*atan(x) - log(x^2 + 1)*
atan(x))
```

## Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

$$\int \arctan(x)\log\left(1+x^2\right) dx = atan(x)^2 + atan(x)\log\left(x^2+1\right)x - 2atan(x)\,x - \frac{\log\left(x^2+1\right)^2}{4} + \log\left(x^2+1\right)$$

input
```
int(atan(x)*log(x^2+1),x)
```

output
```
(4*atan(x)**2 + 4*atan(x)*log(x**2 + 1)*x - 8*atan(x)*x - log(x**2 + 1)**2
 + 4*log(x**2 + 1))/4
```

# 3.1278 $\int \frac{\arctan(x)\log(1+x^2)}{x}\,dx$



## Optimal result

Integrand size = 12, antiderivative size = 189

$$\int \frac{\arctan(x)\log(1+x^2)}{x}\,dx = -\frac{1}{2}i\log^2(1+ix)\log(-ix)+\frac{1}{2}i\log^2(1-ix)\log(ix) + i\log(1-ix)\,\mathrm{PolyLog}(2,1-ix) - i\log(1+ix)\,\mathrm{PolyLog}(2,1+ix) - \frac{1}{2}i\left(\log(1-ix)+\log(1+ix)-\log\left(1+x^2\right)\right)\mathrm{PolyLog}(2,-ix)+\frac{1}{2}i\left(\log(1-ix)+\log(1+ix)-\log\left(1+x^2\right)\right)\mathrm{PolyLog}(2,ix) - i\,\mathrm{PolyLog}(3,1-ix)+i\,\mathrm{PolyLog}(3,1+ix)$$

output
```
-1/2*I*ln(1+I*x)^2*ln(-I*x)+1/2*I*ln(1-I*x)^2*ln(I*x)+I*ln(1-I*x)*polylog(
2,1-I*x)-I*ln(1+I*x)*polylog(2,1+I*x)-1/2*I*(ln(1-I*x)+ln(1+I*x)-ln(x^2+1)
)*polylog(2,-I*x)+1/2*I*(ln(1-I*x)+ln(1+I*x)-ln(x^2+1))*polylog(2,I*x)-I*p
olylog(3,1-I*x)+I*polylog(3,1+I*x)
```

## Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.61

$$\int \frac{\arctan(x)\log(1+x^2)}{x}\,dx = \frac{1}{2}i\left(-\log^2(1-ix)\log(x)+\log^2(1+ix)\log(x) - 2\log(1+ix)\log(x)\log(-i+x) - \log(-ix)\log^2(-i+x)+\log(x)\log^2(-i+x) + 2\log(1-ix)\log(x)\log(i+x)+\log(ix)\log^2(i+x) - \log(x)\log^2(i+x)+2\log(i+x)\operatorname{PolyLog}(2,1-ix) - 2\log(-i+x)\operatorname{PolyLog}(2,1+ix) - \log(1-ix)\operatorname{PolyLog}(2,-ix) + \log(1+ix)\operatorname{PolyLog}(2,-ix) - 2\log(-i+x)\operatorname{PolyLog}(2,-ix) + \log\left(1+x^2\right)\operatorname{PolyLog}(2,-ix) - \log(1-ix)\operatorname{PolyLog}(2,ix)+\log(1+ix)\operatorname{PolyLog}(2,ix) + 2\log(i+x)\operatorname{PolyLog}(2,ix)-\log\left(1+x^2\right)\operatorname{PolyLog}(2,ix) - 2\operatorname{PolyLog}(3,1-ix)+2\operatorname{PolyLog}(3,1+ix)\right)$$

input
```
Integrate[(ArcTan[x]*Log[1 + x^2])/x,x]
```

output
```
(I/2)*(-(Log[1 - I*x]^2*Log[x]) + Log[1 + I*x]^2*Log[x] - 2*Log[1 + I*x]*L
og[x]*Log[-I + x] - Log[(-I)*x]*Log[-I + x]^2 + Log[x]*Log[-I + x]^2 + 2*L
og[1 - I*x]*Log[x]*Log[I + x] + Log[I*x]*Log[I + x]^2 - Log[x]*Log[I + x]^
2 + 2*Log[I + x]*PolyLog[2, 1 - I*x] - 2*Log[-I + x]*PolyLog[2, 1 + I*x] -
 Log[1 - I*x]*PolyLog[2, (-I)*x] + Log[1 + I*x]*PolyLog[2, (-I)*x] - 2*Log
[-I + x]*PolyLog[2, (-I)*x] + Log[1 + x^2]*PolyLog[2, (-I)*x] - Log[1 - I*
x]*PolyLog[2, I*x] + Log[1 + I*x]*PolyLog[2, I*x] + 2*Log[I + x]*PolyLog[2
, I*x] - Log[1 + x^2]*PolyLog[2, I*x] - 2*PolyLog[3, 1 - I*x] + 2*PolyLog[
3, 1 + I*x])
```

## Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.87, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5546, 2843, 2881, 2821, 5355, 2838, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(x)\log(x^2+1)}{x}\,dx$$

$$\Big\downarrow \text{5546}$$

$$-\left(\left(-\log(x^2+1)+\log(1-ix)+\log(1+ix)\right)\int \frac{\arctan(x)}{x}dx\right)+\frac{1}{2}i\int \frac{\log^2(1-ix)}{x}dx-\frac{1}{2}i\int \frac{\log^2(ix+1)}{x}dx$$

$$\Big\downarrow \text{2843}$$

$$-\left(\left(-\log(x^2+1)+\log(1-ix)+\log(1+ix)\right)\int \frac{\arctan(x)}{x}dx\right)-\frac{1}{2}i\left(\log^2(1+ix)\log(-ix)-2i\int \frac{\log(ix+1)\log(-ix)}{ix+1}dx\right)+\frac{1}{2}i\left(2i\int \frac{\log(1-ix)\log(ix)}{1-ix}dx+\log(ix)\log^2(1-ix)\right)$$

$$\Big\downarrow \text{2881}$$

$$-\left(\left(-\log(x^2+1)+\log(1-ix)+\log(1+ix)\right)\int \frac{\arctan(x)}{x}dx\right)-\frac{1}{2}i\left(\log^2(1+ix)\log(-ix)-2\int \frac{\log(ix+1)\log(-ix)}{ix+1}d(ix+1)\right)+\frac{1}{2}i\left(\log^2(1-ix)\log(ix)-2\int \frac{\log(1-ix)\log(ix)}{1-ix}d(1-ix)\right)$$

$$\Big\downarrow \text{2821}$$

$$-\left(\left(-\log(x^2+1)+\log(1-ix)+\log(1+ix)\right)\int \frac{\arctan(x)}{x}dx\right)+\frac{1}{2}i\left(\log^2(1-ix)\log(ix)-2\left(\int \frac{\text{PolyLog}(2,1-ix)}{1-ix}d(1-ix)-\text{PolyLog}(2,1-ix)\log(1-ix)\right)\right)-\frac{1}{2}i\left(\log^2(1+ix)\log(-ix)-2\left(\int \frac{\text{PolyLog}(2,ix+1)}{ix+1}d(ix+1)-\text{PolyLog}(2,ix+1)\log(1+ix)\right)\right)$$

$$\Big\downarrow \text{5355}$$

$$\frac{1}{2}i\left(\log^2(1-ix)\log(ix)-2\left(\int\frac{\text{PolyLog}(2,1-ix)}{1-ix}d(1-ix)-\text{PolyLog}(2,1-ix)\log(1-ix)\right)\right)-\frac{1}{2}i\left(\log^2(1+ix)\log(-ix)-2\left(\int\frac{\text{PolyLog}(2,ix+1)}{ix+1}d(ix+1)-\text{PolyLog}(2,ix+1)\log(1+ix)\right)\right)-\left(\left(-\log\left(x^2+1\right)+\log(1-ix)+\log(1+ix)\right)\left(\frac{1}{2}i\int\frac{\log(1-ix)}{x}dx-\frac{1}{2}i\int\frac{\log(ix+1)}{x}dx\right)\right)$$

$$\Big\downarrow \text{2838}$$

$$\frac{1}{2}i\left(\log^2(1-ix)\log(ix)-2\left(\int\frac{\text{PolyLog}(2,1-ix)}{1-ix}d(1-ix)-\text{PolyLog}(2,1-ix)\log(1-ix)\right)\right)-\frac{1}{2}i\left(\log^2(1+ix)\log(-ix)-2\left(\int\frac{\text{PolyLog}(2,ix+1)}{ix+1}d(ix+1)-\text{PolyLog}(2,ix+1)\log(1+ix)\right)\right)-\left(\left(\frac{1}{2}i\,\text{PolyLog}(2,-ix)-\frac{1}{2}i\,\text{PolyLog}(2,ix)\right)\left(-\log\left(x^2+1\right)+\log(1-ix)+\log(1+ix)\right)\right)$$

$$\Big\downarrow \text{7143}$$

$$-\left(\left(\frac{1}{2}i\,\text{PolyLog}(2,-ix)-\frac{1}{2}i\,\text{PolyLog}(2,ix)\right)\left(-\log\left(x^2+1\right)+\log(1-ix)+\log(1+ix)\right)\right)+\frac{1}{2}i(\log^2(1-ix)\log(ix)-2(\text{PolyLog}(3,1-ix)-\text{PolyLog}(2,1-ix)\log(1-ix)))-\frac{1}{2}i(\log^2(1+ix)\log(-ix)-2(\text{PolyLog}(3,ix+1)-\text{PolyLog}(2,ix+1)\log(1+ix)))$$

input
```
Int[(ArcTan[x]*Log[1 + x^2])/x,x]
```

output
```
-((Log[1 - I*x] + Log[1 + I*x] - Log[1 + x^2])*((I/2)*PolyLog[2, (-I)*x] -
 (I/2)*PolyLog[2, I*x])) + (I/2)*(Log[1 - I*x]^2*Log[I*x] - 2*(-(Log[1 - I
*x]*PolyLog[2, 1 - I*x]) + PolyLog[3, 1 - I*x])) - (I/2)*(Log[1 + I*x]^2*L
og[(-I)*x] - 2*(-(Log[1 + I*x]*PolyLog[2, 1 + I*x]) + PolyLog[3, 1 + I*x])
)
```

### Defintions of rubi rules used

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2843
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Simp[b*e*n*(p/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

rule 2881
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Simp[1/e   Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*
((e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

rule 5355
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Simp[I*(b/2)   Int[Log[1 - I*c*x]/x, x], x] - Simp[I*(b/2)   Int[Log[1
+ I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

rule 5546
```
Int[(ArcTan[(c_.)*(x_)]*Log[(f_.) + (g_.)*(x_)^2])/(x_), x_Symbol] :> Simp[
(Log[f + g*x^2] - Log[1 - I*c*x] - Log[1 + I*c*x])   Int[ArcTan[c*x]/x, x],
 x] + (Simp[I/2   Int[Log[1 - I*c*x]^2/x, x], x] - Simp[I/2   Int[Log[1 + I
*c*x]^2/x, x], x]) /; FreeQ[{c, f, g}, x] && EqQ[g, c^2*f]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.86 (sec) , antiderivative size = 2966, normalized size of antiderivative = 15.69

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 2966 |

input
```
int(arctan(x)*ln(x^2+1)/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*I*ln(x+I)^2*ln(1+I*(x+I))+I*ln(x+I)*polylog(2,-I*(x+I))-I*polylog(3,-I
*(x+I))-1/2*I*ln(x-I)^2*ln(1-I*(x-I))-I*ln(x-I)*polylog(2,I*(x-I))+I*polyl
og(3,I*(x-I))-1/8*I*Pi^2*csgn(I*(x-RootOf(_Z^2+1,index=1))*(x+RootOf(_Z^2+
1,index=1)))*(csgn(I*(x+RootOf(_Z^2+1,index=1)))*csgn(x+RootOf(_Z^2+1,inde
x=1))*csgn(I*(x-RootOf(_Z^2+1,index=1))*(x+RootOf(_Z^2+1,index=1)))^2+csgn
(I*(x-RootOf(_Z^2+1,index=1)))^2*csgn(I*(x+RootOf(_Z^2+1,index=1)))*csgn(x
-RootOf(_Z^2+1,index=1))^2-csgn(I*(x-RootOf(_Z^2+1,index=1)))^2*csgn(I*(x-
RootOf(_Z^2+1,index=1))*(x+RootOf(_Z^2+1,index=1)))*csgn(x-RootOf(_Z^2+1,i
ndex=1))^2+csgn(I*(x-RootOf(_Z^2+1,index=1)))*csgn(I*(x+RootOf(_Z^2+1,inde
x=1)))*csgn(x+RootOf(_Z^2+1,index=1))^2*csgn(I*(x-RootOf(_Z^2+1,index=1))*
(x+RootOf(_Z^2+1,index=1)))+csgn(I*(x-RootOf(_Z^2+1,index=1)))*csgn(I*(x-R
ootOf(_Z^2+1,index=1))*(x+RootOf(_Z^2+1,index=1)))^2*csgn(x-RootOf(_Z^2+1,
index=1))^2+csgn(I*(x-RootOf(_Z^2+1,index=1)))*csgn(I*(x+RootOf(_Z^2+1,ind
ex=1)))*csgn(I*(x-RootOf(_Z^2+1,index=1))*(x+RootOf(_Z^2+1,index=1)))*csgn
(x-RootOf(_Z^2+1,index=1))-csgn(I*(x-RootOf(_Z^2+1,index=1)))*csgn(I*(x+Ro
otOf(_Z^2+1,index=1)))*csgn(x-RootOf(_Z^2+1,index=1))^2+csgn(I*(x-RootOf(_
Z^2+1,index=1)))*csgn(I*(x-RootOf(_Z^2+1,index=1))*(x+RootOf(_Z^2+1,index=
1)))*csgn(x-RootOf(_Z^2+1,index=1))^2+csgn(I*(x+RootOf(_Z^2+1,index=1)))*c
sgn(I*(x-RootOf(_Z^2+1,index=1))*(x+RootOf(_Z^2+1,index=1)))*csgn(x-RootOf
(_Z^2+1,index=1))^2-csgn(I*(x-RootOf(_Z^2+1,index=1)))*csgn(I*(x+RootOf...
```

## Fricas [F]

$$\int \frac{\arctan(x)\log(1+x^2)}{x}\,dx = \int \frac{\arctan(x)\log(x^2+1)}{x}\,dx$$

input
```
integrate(arctan(x)*log(x^2+1)/x,x, algorithm="fricas")
```

output
```
integral(arctan(x)*log(x^2 + 1)/x, x)
```

## Sympy [F]

$$\int \frac{\arctan(x)\log(1+x^2)}{x}\,dx = \int \frac{\log(x^2+1)\,\mathrm{atan}(x)}{x}\,dx$$

input
```
integrate(atan(x)*ln(x**2+1)/x,x)
```

output
```
Integral(log(x**2 + 1)*atan(x)/x, x)
```

## Maxima [F]

$$\int \frac{\arctan(x)\log(1+x^2)}{x}\,dx = \int \frac{\arctan(x)\log(x^2+1)}{x}\,dx$$

input
```
integrate(arctan(x)*log(x^2+1)/x,x, algorithm="maxima")
```

output
```
integrate(arctan(x)*log(x^2 + 1)/x, x)
```

## Giac [F]

$$\int \frac{\arctan(x)\log(1+x^2)}{x}\,dx = \int \frac{\arctan(x)\log(x^2+1)}{x}\,dx$$

input
```
integrate(arctan(x)*log(x^2+1)/x,x, algorithm="giac")
```

output
```
integrate(arctan(x)*log(x^2 + 1)/x, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(x)\log(1+x^2)}{x}\,dx = \int \frac{\ln(x^2+1)\,\mathrm{atan}(x)}{x}\,dx$$

input
```
int((log(x^2 + 1)*atan(x))/x,x)
```

output
```
int((log(x^2 + 1)*atan(x))/x, x)
```

## Reduce [F]

$$\int \frac{\arctan(x)\log(1+x^2)}{x}\,dx = \int \frac{atan(x)\log(x^2+1)}{x}dx$$

input
```
int(atan(x)*log(x^2+1)/x,x)
```

output
```
int((atan(x)*log(x**2 + 1))/x,x)
```

# 3.1279 $\int \frac{\arctan(x)\log(1+x^2)}{x^2}\,dx$



## Optimal result

Integrand size = 12, antiderivative size = 41

$$\int \frac{\arctan(x)\log(1+x^2)}{x^2}\,dx = \arctan(x)^2 - \frac{\arctan(x)\log(1+x^2)}{x} - \frac{1}{4}\log^2(1+x^2) - \frac{\operatorname{PolyLog}(2,-x^2)}{2}$$

output
```
arctan(x)^2-arctan(x)*ln(x^2+1)/x-1/4*ln(x^2+1)^2-1/2*polylog(2,-x^2)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(x)\log(1+x^2)}{x^2}\,dx = \arctan(x)^2 - \frac{\arctan(x)\log(1+x^2)}{x} - \frac{1}{4}\log^2(1+x^2) - \frac{\operatorname{PolyLog}(2,-x^2)}{2}$$

input
```
Integrate[(ArcTan[x]*Log[1 + x^2])/x^2,x]
```

output
```
ArcTan[x]^2 - (ArcTan[x]*Log[1 + x^2])/x - Log[1 + x^2]^2/4 - PolyLog[2, -
x^2]/2
```

## Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5552, 2925, 2857, 2009, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(x)\log\left(x^2+1\right)}{x^2}dx$$

$$\Big\downarrow \text{5552}$$

$$2\int \frac{\arctan(x)}{x^2+1}dx+\int \frac{\log\left(x^2+1\right)}{x\left(x^2+1\right)}dx-\frac{\arctan(x)\log\left(x^2+1\right)}{x}$$

$$\Big\downarrow \text{2925}$$

$$2\int \frac{\arctan(x)}{x^2+1}dx+\frac{1}{2}\int \frac{\log\left(x^2+1\right)}{x^2\left(x^2+1\right)}dx^2-\frac{\arctan(x)\log\left(x^2+1\right)}{x}$$

$$\Big\downarrow \text{2857}$$

$$2\int \frac{\arctan(x)}{x^2+1}dx+\frac{1}{2}\int \left(\frac{\log\left(x^2+1\right)}{-x^2-1}+\frac{\log\left(x^2+1\right)}{x^2}\right)dx^2-\frac{\arctan(x)\log\left(x^2+1\right)}{x}$$

$$\Big\downarrow \text{2009}$$

$$2\int \frac{\arctan(x)}{x^2+1}dx-\frac{\arctan(x)\log\left(x^2+1\right)}{x}+\frac{1}{2}\left(-\text{PolyLog}\left(2,-x^2\right)-\frac{1}{2}\log^2\left(x^2+1\right)\right)$$

$$\Big\downarrow \text{5419}$$

$$-\frac{\arctan(x)\log\left(x^2+1\right)}{x}+\arctan(x)^2+\frac{1}{2}\left(-\text{PolyLog}\left(2,-x^2\right)-\frac{1}{2}\log^2\left(x^2+1\right)\right)$$

input
```
Int[(ArcTan[x]*Log[1 + x^2])/x^2,x]
```

output
```
ArcTan[x]^2 - (ArcTan[x]*Log[1 + x^2])/x + (-1/2*Log[1 + x^2]^2 - PolyLog[
2, -x^2])/2
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2857
```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5552
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(d + e*Log[f + g*x^2])*((a +
b*ArcTan[c*x])/(m + 1)), x] + (-Simp[b*(c/(m + 1))   Int[x^(m + 1)*((d + e*
Log[f + g*x^2])/(1 + c^2*x^2)), x], x] - Simp[2*e*(g/(m + 1))   Int[x^(m +
2)*((a + b*ArcTan[c*x])/(f + g*x^2)), x], x]) /; FreeQ[{a, b, c, d, e, f, g
}, x] && ILtQ[m/2, 0]
```

## Maple [F]

$$\int \frac{\arctan(x)\ln(x^2+1)}{x^2}dx$$

input
```
int(arctan(x)*ln(x^2+1)/x^2,x)
```

output
```
int(arctan(x)*ln(x^2+1)/x^2,x)
```

## Fricas [F]

$$\int \frac{\arctan(x)\log(1+x^2)}{x^2}\,dx = \int \frac{\arctan(x)\log(x^2+1)}{x^2}\,dx$$

input
```
integrate(arctan(x)*log(x^2+1)/x^2,x, algorithm="fricas")
```

output
```
integral(arctan(x)*log(x^2 + 1)/x^2, x)
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 38.81 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

$$\int \frac{\arctan(x)\log(1+x^2)}{x^2}\,dx = -\frac{\log(x^2+1)^2}{4} + \operatorname{atan}^2(x) - \frac{\operatorname{Li}_2(x^2e^{i\pi})}{2} - \frac{\log(x^2+1)\operatorname{atan}(x)}{x}$$

input
```
integrate(atan(x)*ln(x**2+1)/x**2,x)
```

output
```
-log(x**2 + 1)**2/4 + atan(x)**2 - polylog(2, x**2*exp_polar(I*pi))/2 - lo
g(x**2 + 1)*atan(x)/x
```

## Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.41

$$\int \frac{\arctan(x)\log(1+x^2)}{x^2}\,dx = -\left(\frac{\log(x^2+1)}{x} - 2\arctan(x)\right)\arctan(x) - \arctan(x)^2 + \frac{1}{2}\log(-x^2)\log(x^2+1) - \frac{1}{4}\log(x^2+1)^2 + \frac{1}{2}\mathrm{Li}_2(x^2+1)$$

input
```
integrate(arctan(x)*log(x^2+1)/x^2,x, algorithm="maxima")
```

output
```
-(log(x^2 + 1)/x - 2*arctan(x))*arctan(x) - arctan(x)^2 + 1/2*log(-x^2)*lo
g(x^2 + 1) - 1/4*log(x^2 + 1)^2 + 1/2*dilog(x^2 + 1)
```

## Giac [F]

$$\int \frac{\arctan(x)\log(1+x^2)}{x^2}\,dx = \int \frac{\arctan(x)\log(x^2+1)}{x^2}\,dx$$

input
```
integrate(arctan(x)*log(x^2+1)/x^2,x, algorithm="giac")
```

output
```
integrate(arctan(x)*log(x^2 + 1)/x^2, x)
```

## Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.88

$$\int \frac{\arctan(x)\log(1+x^2)}{x^2}\,dx = \mathrm{atan}(x)^2 - \frac{\ln(x^2+1)^2}{4} - \frac{\mathrm{Li}_2(x^2+1)}{2} - \frac{\ln(x^2+1)\,\mathrm{atan}(x)}{x}$$

input
```
int((log(x^2 + 1)*atan(x))/x^2,x)
```

output
```
atan(x)^2 - log(x^2 + 1)^2/4 - dilog(x^2 + 1)/2 - (log(x^2 + 1)*atan(x))/x
```

## Reduce [F]

$$\int \frac{\arctan(x)\log(1+x^2)}{x^2}\,dx = \frac{atan(x)^2 x - atan(x)\log(x^2+1) + \left(\int \frac{\log(x^2+1)}{x^3+x}dx\right)x}{x}$$

input

```
int(atan(x)*log(x^2+1)/x^2,x)
```

output

```
(atan(x)**2*x - atan(x)*log(x**2 + 1) + int(log(x**2 + 1)/(x**3 + x),x)*x)
/x
```

# 3.1280 $\int \frac{\arctan(x)\log(1+x^2)}{x^3}\,dx$



## Optimal result

Integrand size = 12, antiderivative size = 69

$$\int \frac{\arctan(x)\log(1+x^2)}{x^3}\,dx = \arctan(x) - \frac{\log(1+x^2)}{2x} - \frac{1}{2}\arctan(x)\log(1+x^2) - \frac{\arctan(x)\log(1+x^2)}{2x^2} + \frac{1}{2}i\,\text{PolyLog}(2,-ix) - \frac{1}{2}i\,\text{PolyLog}(2,ix)$$

output
```
arctan(x)-1/2*ln(x^2+1)/x-1/2*arctan(x)*ln(x^2+1)-1/2*arctan(x)*ln(x^2+1)/
x^2+1/2*I*polylog(2,-I*x)-1/2*I*polylog(2,I*x)
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.71

$$\int \frac{\arctan(x)\log(1+x^2)}{x^3}\,dx = \arctan(x) - \frac{(x+\arctan(x)+x^2\arctan(x))\log(1+x^2)}{2x^2} + \frac{1}{2}i(\text{PolyLog}(2,-ix) - \text{PolyLog}(2,ix))$$

input
```
Integrate[(ArcTan[x]*Log[1 + x^2])/x^3,x]
```

output
```
ArcTan[x] - ((x + ArcTan[x] + x^2*ArcTan[x])*Log[1 + x^2])/(2*x^2) + (I/2)
*(PolyLog[2, (-I)*x] - PolyLog[2, I*x])
```

## Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.10, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5556, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(x)\log\left(x^2+1\right)}{x^3}\,dx$$

$$\Big\downarrow \text{5556}$$

$$-2\int\left(-\frac{\arctan(x)}{2x}-\frac{1}{2\left(x^2+1\right)}\right)dx-\frac{\arctan(x)\log\left(x^2+1\right)}{2x^2}-\frac{1}{2}\arctan(x)\log\left(x^2+1\right)-\frac{\log\left(x^2+1\right)}{2x}$$

$$\Big\downarrow \text{2009}$$

$$-2\left(-\frac{\arctan(x)}{2}-\frac{1}{4}i\,\text{PolyLog}(2,-ix)+\frac{1}{4}i\,\text{PolyLog}(2,ix)\right)-\frac{\arctan(x)\log\left(x^2+1\right)}{2x^2}-\frac{1}{2}\arctan(x)\log\left(x^2+1\right)-\frac{\log\left(x^2+1\right)}{2x}$$

input
```
Int[(ArcTan[x]*Log[1 + x^2])/x^3,x]
```

output
```
-1/2*Log[1 + x^2]/x - (ArcTan[x]*Log[1 + x^2])/2 - (ArcTan[x]*Log[1 + x^2]
)/(2*x^2) - 2*(-1/2*ArcTan[x] - (I/4)*PolyLog[2, (-I)*x] + (I/4)*PolyLog[2
, I*x])
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5556
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcTan[c*x]), x
]}, Simp[(d + e*Log[f + g*x^2])   u, x] - Simp[2*e*g   Int[ExpandIntegrand[
x*(u/(f + g*x^2)), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && Intege
rQ[m] && NeQ[m, -1]
```

## Maple [F]

$$\int \frac{\arctan(x)\ln(x^2+1)}{x^3}dx$$

input
```
int(arctan(x)*ln(x^2+1)/x^3,x)
```

output
```
int(arctan(x)*ln(x^2+1)/x^3,x)
```

## Fricas [F]

$$\int \frac{\arctan(x)\log(1+x^2)}{x^3}\,dx = \int \frac{\arctan(x)\log(x^2+1)}{x^3}\,dx$$

input
```
integrate(arctan(x)*log(x^2+1)/x^3,x, algorithm="fricas")
```

output
```
integral(arctan(x)*log(x^2 + 1)/x^3, x)
```

## Sympy [F]

$$\int \frac{\arctan(x)\log\left(1+x^2\right)}{x^3}\,dx=\int \frac{\log\left(x^{2}+1\right)\operatorname{atan}\left(x\right)}{x^{3}}\,dx$$

input
```
integrate(atan(x)*ln(x**2+1)/x**3,x)
```

output
```
Integral(log(x**2 + 1)*atan(x)/x**3, x)
```

## Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.01

$$\int \frac{\arctan(x)\log\left(1+x^2\right)}{x^3}\,dx$$
$$=\frac{4\,x^{2}\arctan\left(x\right)\log\left(x\right)+4\,x^{2}\arctan\left(x\right)-2i\,x^{2}\mathrm{Li}_{2}(i\,x+1)+2i\,x^{2}\mathrm{Li}_{2}(-i\,x+1)-\left(\pi x^{2}+2\left(x^{2}+1\right)\mathrm{arct}\right.}{4\,x^{2}}$$

input
```
integrate(arctan(x)*log(x^2+1)/x^3,x, algorithm="maxima")
```

output
```
1/4*(4*x^2*arctan(x)*log(x) + 4*x^2*arctan(x) - 2*I*x^2*dilog(I*x + 1) + 2
*I*x^2*dilog(-I*x + 1) - (pi*x^2 + 2*(x^2 + 1)*arctan(x) + 2*x)*log(x^2 +
1))/x^2
```

## Giac [F]

$$\int \frac{\arctan(x)\log\left(1+x^2\right)}{x^3}\,dx=\int \frac{\arctan\left(x\right)\log\left(x^{2}+1\right)}{x^{3}}\,dx$$

input
```
integrate(arctan(x)*log(x^2+1)/x^3,x, algorithm="giac")
```

output
```
integrate(arctan(x)*log(x^2 + 1)/x^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(x)\log(1+x^2)}{x^3}\,dx = \int \frac{\ln(x^2+1)\,\mathrm{atan}(x)}{x^3}\,dx$$

input
```
int((log(x^2 + 1)*atan(x))/x^3,x)
```

output
```
int((log(x^2 + 1)*atan(x))/x^3, x)
```

## Reduce [F]

$$\int \frac{\arctan(x)\log(1+x^2)}{x^3}\,dx = \int \frac{atan(x)\log(x^2+1)}{x^3}dx$$

input
```
int(atan(x)*log(x^2+1)/x^3,x)
```

output
```
int((atan(x)*log(x**2 + 1))/x**3,x)
```

# 3.1281 $\int \frac{\arctan(x)\log(1+x^2)}{x^4}\,dx$



## Optimal result

Integrand size = 12, antiderivative size = 81

$$\int \frac{\arctan(x)\log(1+x^2)}{x^4}\,dx = -\frac{2\arctan(x)}{3x}-\frac{\arctan(x)^2}{3}+\log(x)-\frac{1}{2}\log\left(1+x^2\right)-\frac{\log(1+x^2)}{6x^2}-\frac{\arctan(x)\log(1+x^2)}{3x^3}+\frac{1}{12}\log^2\left(1+x^2\right)+\frac{\text{PolyLog}(2,-x^2)}{6}$$

output
```
-2/3*arctan(x)/x-1/3*arctan(x)^2+ln(x)-1/2*ln(x^2+1)-1/6*ln(x^2+1)/x^2-1/3
*arctan(x)*ln(x^2+1)/x^3+1/12*ln(x^2+1)^2+1/6*polylog(2,-x^2)
```

## Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(x)\log(1+x^2)}{x^4}\,dx = -\frac{2\arctan(x)}{3x}-\frac{\arctan(x)^2}{3}+\log(x)-\frac{1}{2}\log\left(1+x^2\right)-\frac{\log(1+x^2)}{6x^2}-\frac{\arctan(x)\log(1+x^2)}{3x^3}+\frac{1}{12}\log^2\left(1+x^2\right)+\frac{\text{PolyLog}(2,-x^2)}{6}$$

input
```
Integrate[(ArcTan[x]*Log[1 + x^2])/x^4,x]
```

output
```
(-2*ArcTan[x])/(3*x) - ArcTan[x]^2/3 + Log[x] - Log[1 + x^2]/2 - Log[1 + x
^2]/(6*x^2) - (ArcTan[x]*Log[1 + x^2])/(3*x^3) + Log[1 + x^2]^2/12 + PolyL
og[2, -x^2]/6
```

## Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.23, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.917$, Rules used = {5552, 2925, 2857, 2009, 5453, 5361, 243, 47, 14, 16, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(x)\log(x^2+1)}{x^4}dx$$

$$\Big\downarrow \text{5552}$$

$$\frac{2}{3}\int \frac{\arctan(x)}{x^2(x^2+1)}dx+\frac{1}{3}\int \frac{\log(x^2+1)}{x^3(x^2+1)}dx-\frac{\arctan(x)\log(x^2+1)}{3x^3}$$

$$\Big\downarrow \text{2925}$$

$$\frac{2}{3}\int \frac{\arctan(x)}{x^2(x^2+1)}dx+\frac{1}{6}\int \frac{\log(x^2+1)}{x^4(x^2+1)}dx^2-\frac{\arctan(x)\log(x^2+1)}{3x^3}$$

$$\Big\downarrow \text{2857}$$

$$\frac{2}{3}\int \frac{\arctan(x)}{x^2(x^2+1)}dx+\frac{1}{6}\int \left(\frac{\log(x^2+1)}{x^2+1}-\frac{\log(x^2+1)}{x^2}+\frac{\log(x^2+1)}{x^4}\right)dx^2-\frac{\arctan(x)\log(x^2+1)}{3x^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{3}\int \frac{\arctan(x)}{x^2(x^2+1)}dx-\frac{\arctan(x)\log(x^2+1)}{3x^3}+\frac{1}{6}\left(\text{PolyLog}(2,-x^2)+\frac{1}{2}\log^2(x^2+1)-\frac{\log(x^2+1)}{x^2}-\log(x^2+1)+\log(x^2)\right)$$

$$\Big\downarrow \; 5453$$

$$\frac{2}{3}\left(\int \frac{\arctan(x)}{x^2}dx-\int \frac{\arctan(x)}{x^2+1}dx\right)-\frac{\arctan(x)\log\left(x^2+1\right)}{3x^3}+\frac{1}{6}\left(\text{PolyLog}\left(2,-x^2\right)+\frac{1}{2}\log^2\left(x^2+1\right)-\frac{\log\left(x^2+1\right)}{x^2}-\log\left(x^2+1\right)+\log\left(x^2\right)\right)$$

$$\Big\downarrow \; 5361$$

$$\frac{2}{3}\left(-\int \frac{\arctan(x)}{x^2+1}dx+\int \frac{1}{x\left(x^2+1\right)}dx-\frac{\arctan(x)}{x}\right)-\frac{\arctan(x)\log\left(x^2+1\right)}{3x^3}+\frac{1}{6}\left(\text{PolyLog}\left(2,-x^2\right)+\frac{1}{2}\log^2\left(x^2+1\right)-\frac{\log\left(x^2+1\right)}{x^2}-\log\left(x^2+1\right)+\log\left(x^2\right)\right)$$

$$\Big\downarrow \; 243$$

$$\frac{2}{3}\left(-\int \frac{\arctan(x)}{x^2+1}dx+\frac{1}{2}\int \frac{1}{x^2\left(x^2+1\right)}dx^2-\frac{\arctan(x)}{x}\right)-\frac{\arctan(x)\log\left(x^2+1\right)}{3x^3}+\frac{1}{6}\left(\text{PolyLog}\left(2,-x^2\right)+\frac{1}{2}\log^2\left(x^2+1\right)-\frac{\log\left(x^2+1\right)}{x^2}-\log\left(x^2+1\right)+\log\left(x^2\right)\right)$$

$$\Big\downarrow \; 47$$

$$\frac{2}{3}\left(-\int \frac{\arctan(x)}{x^2+1}dx+\frac{1}{2}\left(\int \frac{1}{x^2}dx^2-\int \frac{1}{x^2+1}dx^2\right)-\frac{\arctan(x)}{x}\right)-\frac{\arctan(x)\log\left(x^2+1\right)}{3x^3}+\frac{1}{6}\left(\text{PolyLog}\left(2,-x^2\right)+\frac{1}{2}\log^2\left(x^2+1\right)-\frac{\log\left(x^2+1\right)}{x^2}-\log\left(x^2+1\right)+\log\left(x^2\right)\right)$$

$$\Big\downarrow \; 14$$

$$\frac{2}{3}\left(-\int \frac{\arctan(x)}{x^2+1}dx+\frac{1}{2}\left(\log\left(x^2\right)-\int \frac{1}{x^2+1}dx^2\right)-\frac{\arctan(x)}{x}\right)-\frac{\arctan(x)\log\left(x^2+1\right)}{3x^3}+\frac{1}{6}\left(\text{PolyLog}\left(2,-x^2\right)+\frac{1}{2}\log^2\left(x^2+1\right)-\frac{\log\left(x^2+1\right)}{x^2}-\log\left(x^2+1\right)+\log\left(x^2\right)\right)$$

$$\Big\downarrow \; 16$$

$$\frac{2}{3}\left(-\int \frac{\arctan(x)}{x^2+1}dx-\frac{\arctan(x)}{x}+\frac{1}{2}\left(\log\left(x^2\right)-\log\left(x^2+1\right)\right)\right)-\frac{\arctan(x)\log\left(x^2+1\right)}{3x^3}+\frac{1}{6}\left(\text{PolyLog}\left(2,-x^2\right)+\frac{1}{2}\log^2\left(x^2+1\right)-\frac{\log\left(x^2+1\right)}{x^2}-\log\left(x^2+1\right)+\log\left(x^2\right)\right)$$

$$\Big\downarrow \; 5419$$

$$\frac{2}{3}\left(-\frac{1}{2}\arctan(x)^2-\frac{\arctan(x)}{x}+\frac{1}{2}\left(\log\left(x^2\right)-\log\left(x^2+1\right)\right)\right)-\frac{\arctan(x)\log\left(x^2+1\right)}{3x^3}+\frac{1}{6}\left(\operatorname{PolyLog}\left(2,-x^2\right)+\frac{1}{2}\log^2\left(x^2+1\right)-\frac{\log\left(x^2+1\right)}{x^2}-\log\left(x^2+1\right)+\log\left(x^2\right)\right)$$

input
```
Int[(ArcTan[x]*Log[1 + x^2])/x^4,x]
```

output
```
(2*(-(ArcTan[x]/x) - ArcTan[x]^2/2 + (Log[x^2] - Log[1 + x^2])/2))/3 - (Ar
cTan[x]*Log[1 + x^2])/(3*x^3) + (Log[x^2] - Log[1 + x^2] - Log[1 + x^2]/x^
2 + Log[1 + x^2]^2/2 + PolyLog[2, -x^2])/6
```

**Defintions of rubi rules used**

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2857
```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5552
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(d + e*Log[f + g*x^2])*((a +
b*ArcTan[c*x])/(m + 1)), x] + (-Simp[b*(c/(m + 1))   Int[x^(m + 1)*((d + e*
Log[f + g*x^2])/(1 + c^2*x^2)), x], x] - Simp[2*e*(g/(m + 1))   Int[x^(m +
2)*((a + b*ArcTan[c*x])/(f + g*x^2)), x], x]) /; FreeQ[{a, b, c, d, e, f, g
}, x] && ILtQ[m/2, 0]
```

## Maple [F]

$$\int \frac{\arctan(x)\ln(x^2+1)}{x^4}dx$$

input
```
int(arctan(x)*ln(x^2+1)/x^4,x)
```

output
```
int(arctan(x)*ln(x^2+1)/x^4,x)
```

## Fricas [F]

$$\int \frac{\arctan(x)\log(1+x^2)}{x^4}\,dx = \int \frac{\arctan(x)\log(x^2+1)}{x^4}\,dx$$

input
```
integrate(arctan(x)*log(x^2+1)/x^4,x, algorithm="fricas")
```

output
```
integral(arctan(x)*log(x^2 + 1)/x^4, x)
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 12.87 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.20

$$\begin{aligned}\int \frac{\arctan(x)\log(1+x^2)}{x^4}\,dx &= \frac{2\log(x)}{3} + \frac{\log(2x^2)}{6} + \frac{\log(x^2+1)^2}{12} - \frac{\log(x^2+1)}{3} \\ &\quad - \frac{\log(2x^2+2)}{6} - \frac{\operatorname{atan}^2(x)}{3} + \frac{\operatorname{Li}_2(x^2e^{i\pi})}{6} \\ &\quad - \frac{2\operatorname{atan}(x)}{3x} - \frac{\log(x^2+1)}{6x^2} - \frac{\log(x^2+1)\operatorname{atan}(x)}{3x^3}\end{aligned}$$

input
```
integrate(atan(x)*ln(x**2+1)/x**4,x)
```

output
```
2*log(x)/3 + log(2*x**2)/6 + log(x**2 + 1)**2/12 - log(x**2 + 1)/3 - log(2
*x**2 + 2)/6 - atan(x)**2/3 + polylog(2, x**2*exp_polar(I*pi))/6 - 2*atan(
x)/(3*x) - log(x**2 + 1)/(6*x**2) - log(x**2 + 1)*atan(x)/(3*x**3)
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.17

$$\int \frac{\arctan(x)\log(1+x^2)}{x^4}\,dx = -\frac{1}{3}\left(\frac{2}{x} + \frac{\log(x^2+1)}{x^3} + 2\arctan(x)\right)\arctan(x) + \frac{4x^2\arctan(x)^2 + x^2\log(x^2+1)^2 - 2x^2\mathrm{Li}_2(x^2+1) + 12x^2\log(x) - 2(x^2\log(-x^2) + 3x^2 + 1)\log(x^2+1)}{12x^2}$$

input
```
integrate(arctan(x)*log(x^2+1)/x^4,x, algorithm="maxima")
```

output
```
-1/3*(2/x + log(x^2 + 1)/x^3 + 2*arctan(x))*arctan(x) + 1/12*(4*x^2*arctan
(x)^2 + x^2*log(x^2 + 1)^2 - 2*x^2*dilog(x^2 + 1) + 12*x^2*log(x) - 2*(x^2
*log(-x^2) + 3*x^2 + 1)*log(x^2 + 1))/x^2
```

## Giac [F]

$$\int \frac{\arctan(x)\log(1+x^2)}{x^4}\,dx = \int \frac{\arctan(x)\log(x^2+1)}{x^4}\,dx$$

input
```
integrate(arctan(x)*log(x^2+1)/x^4,x, algorithm="giac")
```

output
```
integrate(arctan(x)*log(x^2 + 1)/x^4, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(x)\log(1+x^2)}{x^4}\,dx = \int \frac{\ln(x^2+1)\,\operatorname{atan}(x)}{x^4}\,dx$$

input
```
int((log(x^2 + 1)*atan(x))/x^4,x)
```

output
```
int((log(x^2 + 1)*atan(x))/x^4, x)
```

## Reduce [F]

$$\int \frac{\arctan(x)\log(1+x^2)}{x^4}\,dx = \int \frac{atan(x)\log(x^2+1)}{x^4}dx$$

input
```
int(atan(x)*log(x^2+1)/x^4,x)
```

output
```
int((atan(x)*log(x**2 + 1))/x**4,x)
```

# 3.1282 $\int \frac{\arctan(x)\log(1+x^2)}{x^5}\,dx$



## Optimal result

Integrand size = 12, antiderivative size = 102

$$\begin{aligned}\int \frac{\arctan(x)\log(1+x^2)}{x^5}\,dx = &-\frac{5}{12x}-\frac{11\arctan(x)}{12}-\frac{\arctan(x)}{4x^2}\\&-\frac{\log(1+x^2)}{12x^3}+\frac{\log(1+x^2)}{4x}\\&+\frac{1}{4}\arctan(x)\log(1+x^2)-\frac{\arctan(x)\log(1+x^2)}{4x^4}\\&-\frac{1}{4}i\,\mathrm{PolyLog}(2,-ix)+\frac{1}{4}i\,\mathrm{PolyLog}(2,ix)\end{aligned}$$

output
```
-5/12/x-11/12*arctan(x)-1/4*arctan(x)/x^2-1/12*ln(x^2+1)/x^3+1/4*ln(x^2+1)
/x+1/4*arctan(x)*ln(x^2+1)-1/4*arctan(x)*ln(x^2+1)/x^4-1/4*I*polylog(2,-I*
x)+1/4*I*polylog(2,I*x)
```

## Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.96

$$\int \frac{\arctan(x)\log(1+x^2)}{x^5}\,dx = -\frac{1}{6x}-\frac{2\arctan(x)}{3}+\frac{1}{2}\left(\frac{1}{2}\left(-\frac{1}{x}-\arctan(x)\right)-\frac{\arctan(x)}{2x^2}\right)+\frac{(-x+3x^3-3\arctan(x)+3x^4\arctan(x))\log(1+x^2)}{12x^4}-\frac{1}{4}i(\text{PolyLog}(2,-ix)-\text{PolyLog}(2,ix))$$

input
```
Integrate[(ArcTan[x]*Log[1 + x^2])/x^5,x]
```

output
```
-1/6*1/x - (2*ArcTan[x])/3 + ((-x^(-1) - ArcTan[x])/2 - ArcTan[x]/(2*x^2))
/2 + ((-x + 3*x^3 - 3*ArcTan[x] + 3*x^4*ArcTan[x])*Log[1 + x^2])/(12*x^4)
- (I/4)*(PolyLog[2, (-I)*x] - PolyLog[2, I*x])
```

## Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.03, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5556, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(x)\log(x^2+1)}{x^5}\,dx$$

$$\Big\downarrow \; 5556$$

$$-2\int \left(-\frac{1-3x^2}{12x^2(x^2+1)}-\frac{(1-x^2)\arctan(x)}{4x^3}\right)dx+\frac{1}{4}\arctan(x)\log(x^2+1)-\frac{\arctan(x)\log(x^2+1)}{4x^4}+\frac{\log(x^2+1)}{4x}-\frac{\log(x^2+1)}{12x^3}$$

$$\Big\downarrow \; 2009$$

$$-2\left(\frac{\arctan(x)}{8x^2}+\frac{11\arctan(x)}{24}+\frac{1}{8}i\,\text{PolyLog}(2,-ix)-\frac{1}{8}i\,\text{PolyLog}(2,ix)+\frac{5}{24x}\right)+\frac{1}{4}\arctan(x)\log\left(x^2+1\right)-\frac{\arctan(x)\log\left(x^2+1\right)}{4x^4}+\frac{\log\left(x^2+1\right)}{4x}-\frac{\log\left(x^2+1\right)}{12x^3}$$

input
```
Int[(ArcTan[x]*Log[1 + x^2])/x^5,x]
```

output
```
-1/12*Log[1 + x^2]/x^3 + Log[1 + x^2]/(4*x) + (ArcTan[x]*Log[1 + x^2])/4 -
 (ArcTan[x]*Log[1 + x^2])/(4*x^4) - 2*(5/(24*x) + (11*ArcTan[x])/24 + ArcT
an[x]/(8*x^2) + (I/8)*PolyLog[2, (-I)*x] - (I/8)*PolyLog[2, I*x])
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5556
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcTan[c*x]), x
]}, Simp[(d + e*Log[f + g*x^2])   u, x] - Simp[2*e*g   Int[ExpandIntegrand[
x*(u/(f + g*x^2)), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && Intege
rQ[m] && NeQ[m, -1]
```

## Maple [F]

$$\int \frac{\arctan(x)\ln\left(x^2+1\right)}{x^5}dx$$

input
```
int(arctan(x)*ln(x^2+1)/x^5,x)
```

output
```
int(arctan(x)*ln(x^2+1)/x^5,x)
```

## Fricas [F]

$$\int \frac{\arctan(x)\log(1+x^2)}{x^5}\,dx = \int \frac{\arctan(x)\log(x^2+1)}{x^5}\,dx$$

input
```
integrate(arctan(x)*log(x^2+1)/x^5,x, algorithm="fricas")
```

output
```
integral(arctan(x)*log(x^2 + 1)/x^5, x)
```

## Sympy [F]

$$\int \frac{\arctan(x)\log(1+x^2)}{x^5}\,dx = \int \frac{\log(x^2+1)\,\mathrm{atan}(x)}{x^5}\,dx$$

input
```
integrate(atan(x)*ln(x**2+1)/x**5,x)
```

output
```
Integral(log(x**2 + 1)*atan(x)/x**5, x)
```

## Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.87

$$\int \frac{\arctan(x)\log(1+x^2)}{x^5}\,dx = -\frac{12\,x^4\arctan(x)\log(x) - 6i\,x^4\mathrm{Li}_2(i\,x+1) + 6i\,x^4\mathrm{Li}_2(-i\,x+1) + 10\,x^3 + 2\,(11\,x^4+3\,x^2)\arctan(x) -}{24\,x^4}$$

input
```
integrate(arctan(x)*log(x^2+1)/x^5,x, algorithm="maxima")
```

output
```
-1/24*(12*x^4*arctan(x)*log(x) - 6*I*x^4*dilog(I*x + 1) + 6*I*x^4*dilog(-I
*x + 1) + 10*x^3 + 2*(11*x^4 + 3*x^2)*arctan(x) - (3*pi*x^4 + 6*x^3 + 6*(x
^4 - 1)*arctan(x) - 2*x)*log(x^2 + 1))/x^4
```

## Giac [F]

$$\int \frac{\arctan(x)\log(1+x^2)}{x^5}\,dx = \int \frac{\arctan(x)\log(x^2+1)}{x^5}\,dx$$

input
```
integrate(arctan(x)*log(x^2+1)/x^5,x, algorithm="giac")
```

output
```
integrate(arctan(x)*log(x^2 + 1)/x^5, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(x)\log(1+x^2)}{x^5}\,dx = \int \frac{\ln(x^2+1)\,\mathrm{atan}(x)}{x^5}\,dx$$

input
```
int((log(x^2 + 1)*atan(x))/x^5,x)
```

output
```
int((log(x^2 + 1)*atan(x))/x^5, x)
```

## Reduce [F]

$$\int \frac{\arctan(x)\log(1+x^2)}{x^5}\,dx = \int \frac{atan(x)\log(x^2+1)}{x^5}dx$$

input
```
int(atan(x)*log(x^2+1)/x^5,x)
```

output
```
int((atan(x)*log(x**2 + 1))/x**5,x)
```

# 3.1283 $\int \frac{\arctan(x)\log(1+x^2)}{x^6}\,dx$



## Optimal result

Integrand size = 12, antiderivative size = 114

$$\int \frac{\arctan(x)\log(1+x^2)}{x^6}\,dx = -\frac{7}{60x^2} - \frac{2\arctan(x)}{15x^3} + \frac{2\arctan(x)}{5x} + \frac{\arctan(x)^2}{5} - \frac{5\log(x)}{6} + \frac{5}{12}\log\left(1+x^2\right) - \frac{\log\left(1+x^2\right)}{20x^4} + \frac{\log\left(1+x^2\right)}{10x^2} - \frac{\arctan(x)\log\left(1+x^2\right)}{5x^5} - \frac{1}{20}\log^2\left(1+x^2\right) - \frac{\text{PolyLog}\left(2,-x^2\right)}{10}$$

output
```
-7/60/x^2-2/15*arctan(x)/x^3+2/5*arctan(x)/x+1/5*arctan(x)^2-5/6*ln(x)+5/1
2*ln(x^2+1)-1/20*ln(x^2+1)/x^4+1/10*ln(x^2+1)/x^2-1/5*arctan(x)*ln(x^2+1)/
x^5-1/20*ln(x^2+1)^2-1/10*polylog(2,-x^2)
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00

$$\int \frac{\arctan(x)\log(1+x^2)}{x^6}\,dx = -\frac{7}{60x^2} - \frac{2\arctan(x)}{15x^3} + \frac{2\arctan(x)}{5x} + \frac{\arctan(x)^2}{5} - \frac{5\log(x)}{6} + \frac{5}{12}\log(1+x^2) - \frac{\log(1+x^2)}{20x^4} + \frac{\log(1+x^2)}{10x^2} - \frac{\arctan(x)\log(1+x^2)}{5x^5} - \frac{1}{20}\log^2(1+x^2) - \frac{\text{PolyLog}(2,-x^2)}{10}$$

input
```
Integrate[(ArcTan[x]*Log[1 + x^2])/x^6,x]
```

output
```
-7/(60*x^2) - (2*ArcTan[x])/(15*x^3) + (2*ArcTan[x])/(5*x) + ArcTan[x]^2/5
 - (5*Log[x])/6 + (5*Log[1 + x^2])/12 - Log[1 + x^2]/(20*x^4) + Log[1 + x^
2]/(10*x^2) - (ArcTan[x]*Log[1 + x^2])/(5*x^5) - Log[1 + x^2]^2/20 - PolyL
og[2, -x^2]/10
```

## Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.38, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 1.333$, Rules used = {5552, 2925, 2857, 2009, 5453, 5361, 243, 54, 2009, 5453, 5361, 243, 47, 14, 16, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\arctan(x)\log(x^2+1)}{x^6}dx$$

$$\Big\downarrow \text{5552}$$

$$\frac{2}{5}\int \frac{\arctan(x)}{x^4(x^2+1)}dx + \frac{1}{5}\int \frac{\log(x^2+1)}{x^5(x^2+1)}dx - \frac{\arctan(x)\log(x^2+1)}{5x^5}$$

$$\Big\downarrow \text{2925}$$

$$\frac{2}{5}\int \frac{\arctan(x)}{x^4(x^2+1)}dx+\frac{1}{10}\int \frac{\log(x^2+1)}{x^6(x^2+1)}dx^2-\frac{\arctan(x)\log(x^2+1)}{5x^5}$$

$$\Big\downarrow \text{2857}$$

$$\frac{2}{5}\int \frac{\arctan(x)}{x^4(x^2+1)}dx+\frac{1}{10}\int \left(\frac{\log(x^2+1)}{-x^2-1}+\frac{\log(x^2+1)}{x^2}-\frac{\log(x^2+1)}{x^4}+\frac{\log(x^2+1)}{x^6}\right)dx^2-\frac{\arctan(x)\log(x^2+1)}{5x^5}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{5}\int \frac{\arctan(x)}{x^4(x^2+1)}dx-\frac{\arctan(x)\log(x^2+1)}{5x^5}+\frac{1}{10}\left(-\text{PolyLog}(2,-x^2)-\frac{1}{2x^2}-\frac{1}{2}\log^2(x^2+1)+\frac{\log(x^2+1)}{x^2}+\frac{3}{2}\log(x^2+1)-\frac{3\log(x^2)}{2}-\frac{\log(x^2+1)}{2x^4}\right)$$

$$\Big\downarrow \text{5453}$$

$$\frac{2}{5}\left(\int \frac{\arctan(x)}{x^4}dx-\int \frac{\arctan(x)}{x^2(x^2+1)}dx\right)-\frac{\arctan(x)\log(x^2+1)}{5x^5}+\frac{1}{10}\left(-\text{PolyLog}(2,-x^2)-\frac{1}{2x^2}-\frac{1}{2}\log^2(x^2+1)+\frac{\log(x^2+1)}{x^2}+\frac{3}{2}\log(x^2+1)-\frac{3\log(x^2)}{2}-\frac{\log(x^2+1)}{2x^4}\right)$$

$$\Big\downarrow \text{5361}$$

$$\frac{2}{5}\left(-\int \frac{\arctan(x)}{x^2(x^2+1)}dx+\frac{1}{3}\int \frac{1}{x^3(x^2+1)}dx-\frac{\arctan(x)}{3x^3}\right)-\frac{\arctan(x)\log(x^2+1)}{5x^5}+\frac{1}{10}\left(-\text{PolyLog}(2,-x^2)-\frac{1}{2x^2}-\frac{1}{2}\log^2(x^2+1)+\frac{\log(x^2+1)}{x^2}+\frac{3}{2}\log(x^2+1)-\frac{3\log(x^2)}{2}-\frac{\log(x^2+1)}{2x^4}\right)$$

$$\Big\downarrow \text{243}$$

$$\frac{2}{5}\left(-\int \frac{\arctan(x)}{x^2(x^2+1)}dx+\frac{1}{6}\int \frac{1}{x^4(x^2+1)}dx^2-\frac{\arctan(x)}{3x^3}\right)-\frac{\arctan(x)\log(x^2+1)}{5x^5}+\frac{1}{10}\left(-\text{PolyLog}(2,-x^2)-\frac{1}{2x^2}-\frac{1}{2}\log^2(x^2+1)+\frac{\log(x^2+1)}{x^2}+\frac{3}{2}\log(x^2+1)-\frac{3\log(x^2)}{2}-\frac{\log(x^2+1)}{2x^4}\right)$$

$$\Big\downarrow \text{54}$$

$$\frac{2}{5}\left(-\int \frac{\arctan(x)}{x^2(x^2+1)}dx+\frac{1}{6}\int \left(-\frac{1}{x^2}+\frac{1}{x^4}+\frac{1}{x^2+1}\right)dx^2-\frac{\arctan(x)}{3x^3}\right)-\frac{\arctan(x)\log(x^2+1)}{5x^5}+\frac{1}{10}\left(-\text{PolyLog}(2,-x^2)-\frac{1}{2x^2}-\frac{1}{2}\log^2(x^2+1)+\frac{\log(x^2+1)}{x^2}+\frac{3}{2}\log(x^2+1)-\frac{3\log(x^2)}{2}-\frac{\log(x^2+1)}{2x^4}\right)$$

$\big\downarrow$ 2009

$$\frac{2}{5}\left(-\int \frac{\arctan(x)}{x^2(x^2+1)}dx-\frac{\arctan(x)}{3x^3}+\frac{1}{6}\left(-\frac{1}{x^2}-\log(x^2)+\log(x^2+1)\right)\right)-\frac{\arctan(x)\log(x^2+1)}{5x^5}+\frac{1}{10}\left(-\text{PolyLog}(2,-x^2)-\frac{1}{2x^2}-\frac{1}{2}\log^2(x^2+1)+\frac{\log(x^2+1)}{x^2}+\frac{3}{2}\log(x^2+1)-\frac{3\log(x^2)}{2}-\frac{\log(x^2+1)}{2x^4}\right)$$

$\big\downarrow$ 5453

$$\frac{2}{5}\left(-\int \frac{\arctan(x)}{x^2}dx+\int \frac{\arctan(x)}{x^2+1}dx-\frac{\arctan(x)}{3x^3}+\frac{1}{6}\left(-\frac{1}{x^2}-\log(x^2)+\log(x^2+1)\right)\right)-\frac{\arctan(x)\log(x^2+1)}{5x^5}+\frac{1}{10}\left(-\text{PolyLog}(2,-x^2)-\frac{1}{2x^2}-\frac{1}{2}\log^2(x^2+1)+\frac{\log(x^2+1)}{x^2}+\frac{3}{2}\log(x^2+1)-\frac{3\log(x^2)}{2}-\frac{\log(x^2+1)}{2x^4}\right)$$

$\big\downarrow$ 5361

$$\frac{2}{5}\left(\int \frac{\arctan(x)}{x^2+1}dx-\int \frac{1}{x(x^2+1)}dx-\frac{\arctan(x)}{3x^3}+\frac{\arctan(x)}{x}+\frac{1}{6}\left(-\frac{1}{x^2}-\log(x^2)+\log(x^2+1)\right)\right)-\frac{\arctan(x)\log(x^2+1)}{5x^5}+\frac{1}{10}\left(-\text{PolyLog}(2,-x^2)-\frac{1}{2x^2}-\frac{1}{2}\log^2(x^2+1)+\frac{\log(x^2+1)}{x^2}+\frac{3}{2}\log(x^2+1)-\frac{3\log(x^2)}{2}-\frac{\log(x^2+1)}{2x^4}\right)$$

$\big\downarrow$ 243

$$\frac{2}{5}\left(\int \frac{\arctan(x)}{x^2+1}dx-\frac{1}{2}\int \frac{1}{x^2(x^2+1)}dx^2-\frac{\arctan(x)}{3x^3}+\frac{\arctan(x)}{x}+\frac{1}{6}\left(-\frac{1}{x^2}-\log(x^2)+\log(x^2+1)\right)\right)-\frac{\arctan(x)\log(x^2+1)}{5x^5}+\frac{1}{10}\left(-\text{PolyLog}(2,-x^2)-\frac{1}{2x^2}-\frac{1}{2}\log^2(x^2+1)+\frac{\log(x^2+1)}{x^2}+\frac{3}{2}\log(x^2+1)-\frac{3\log(x^2)}{2}-\frac{\log(x^2+1)}{2x^4}\right)$$

$\big\downarrow$ 47

$$\frac{2}{5}\left(\int \frac{\arctan(x)}{x^2+1}dx + \frac{1}{2}\left(\int \frac{1}{x^2+1}dx^2 - \int \frac{1}{x^2}dx^2\right) - \frac{\arctan(x)}{3x^3} + \frac{\arctan(x)}{x} + \frac{1}{6}\left(-\frac{1}{x^2} - \log\left(x^2\right) + \log\left(x^2 + \right.\right.\right.$$
$$\frac{\arctan(x)\log\left(x^2+1\right)}{5x^5} +$$
$$\frac{1}{10}\left(-\operatorname{PolyLog}\left(2,-x^2\right) - \frac{1}{2x^2} - \frac{1}{2}\log^2\left(x^2+1\right) + \frac{\log\left(x^2+1\right)}{x^2} + \frac{3}{2}\log\left(x^2+1\right) - \frac{3\log\left(x^2\right)}{2} - \frac{\log\left(x^2+1\right)}{2x^4}\right)$$

$\downarrow$ 14

$$\frac{2}{5}\left(\int \frac{\arctan(x)}{x^2+1}dx + \frac{1}{2}\left(\int \frac{1}{x^2+1}dx^2 - \log\left(x^2\right)\right) - \frac{\arctan(x)}{3x^3} + \frac{\arctan(x)}{x} + \frac{1}{6}\left(-\frac{1}{x^2} - \log\left(x^2\right) + \log\left(x^2 + \right.\right.\right.$$
$$\frac{\arctan(x)\log\left(x^2+1\right)}{5x^5} +$$
$$\frac{1}{10}\left(-\operatorname{PolyLog}\left(2,-x^2\right) - \frac{1}{2x^2} - \frac{1}{2}\log^2\left(x^2+1\right) + \frac{\log\left(x^2+1\right)}{x^2} + \frac{3}{2}\log\left(x^2+1\right) - \frac{3\log\left(x^2\right)}{2} - \frac{\log\left(x^2+1\right)}{2x^4}\right)$$

$\downarrow$ 16

$$\frac{2}{5}\left(\int \frac{\arctan(x)}{x^2+1}dx - \frac{\arctan(x)}{3x^3} + \frac{\arctan(x)}{x} + \frac{1}{2}\left(\log\left(x^2+1\right) - \log\left(x^2\right)\right) + \frac{1}{6}\left(-\frac{1}{x^2} - \log\left(x^2\right) + \log\left(x^2+1\right)\right.\right.$$
$$\frac{\arctan(x)\log\left(x^2+1\right)}{5x^5} +$$
$$\frac{1}{10}\left(-\operatorname{PolyLog}\left(2,-x^2\right) - \frac{1}{2x^2} - \frac{1}{2}\log^2\left(x^2+1\right) + \frac{\log\left(x^2+1\right)}{x^2} + \frac{3}{2}\log\left(x^2+1\right) - \frac{3\log\left(x^2\right)}{2} - \frac{\log\left(x^2+1\right)}{2x^4}\right)$$

$\downarrow$ 5419

$$-\frac{\arctan(x)\log\left(x^2+1\right)}{5x^5} +$$
$$\frac{2}{5}\left(-\frac{\arctan(x)}{3x^3} + \frac{\arctan(x)^2}{2} + \frac{\arctan(x)}{x} + \frac{1}{2}\left(\log\left(x^2+1\right) - \log\left(x^2\right)\right) + \frac{1}{6}\left(-\frac{1}{x^2} - \log\left(x^2\right) + \log\left(x^2+1\right)\right)\right)$$
$$\frac{1}{10}\left(-\operatorname{PolyLog}\left(2,-x^2\right) - \frac{1}{2x^2} - \frac{1}{2}\log^2\left(x^2+1\right) + \frac{\log\left(x^2+1\right)}{x^2} + \frac{3}{2}\log\left(x^2+1\right) - \frac{3\log\left(x^2\right)}{2} - \frac{\log\left(x^2+1\right)}{2x^4}\right)$$

input
```
Int[(ArcTan[x]*Log[1 + x^2])/x^6,x]
```

output
```
-1/5*(ArcTan[x]*Log[1 + x^2])/x^5 + (2*(-1/3*ArcTan[x]/x^3 + ArcTan[x]/x +
 ArcTan[x]^2/2 + (-Log[x^2] + Log[1 + x^2])/2 + (-x^(-2) - Log[x^2] + Log[
1 + x^2])/6))/5 + (-1/2*1/x^2 - (3*Log[x^2])/2 + (3*Log[1 + x^2])/2 - Log[
1 + x^2]/(2*x^4) + Log[1 + x^2]/x^2 - Log[1 + x^2]^2/2 - PolyLog[2, -x^2])
/10
```

## Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2857
```
Int[(Log[(c_.)*((d_) + (e_.)*(x_))]*(x_)^(m_.))/((f_) + (g_.)*(x_)), x_Symb
ol] :> Int[ExpandIntegrand[Log[c*(d + e*x)], x^m/(f + g*x), x], x] /; FreeQ
[{c, d, e, f, g}, x] && EqQ[e*f - d*g, 0] && EqQ[c*d, 1] && IntegerQ[m]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5552
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(d + e*Log[f + g*x^2])*((a +
b*ArcTan[c*x])/(m + 1)), x] + (-Simp[b*(c/(m + 1))   Int[x^(m + 1)*((d + e*
Log[f + g*x^2])/(1 + c^2*x^2)), x], x] - Simp[2*e*(g/(m + 1))   Int[x^(m +
2)*((a + b*ArcTan[c*x])/(f + g*x^2)), x], x]) /; FreeQ[{a, b, c, d, e, f, g
}, x] && ILtQ[m/2, 0]
```

## Maple [F]

$$\int \frac{\arctan(x)\ln(x^2+1)}{x^6}dx$$

input
```
int(arctan(x)*ln(x^2+1)/x^6,x)
```

output
```
int(arctan(x)*ln(x^2+1)/x^6,x)
```

## Fricas [F]

$$\int \frac{\arctan(x)\log(1+x^2)}{x^6}\,dx = \int \frac{\arctan(x)\log(x^2+1)}{x^6}\,dx$$

input
```
integrate(arctan(x)*log(x^2+1)/x^6,x, algorithm="fricas")
```

output
```
integral(arctan(x)*log(x^2 + 1)/x^6, x)
```

## Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 16.83 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.18

$$\begin{aligned}\int \frac{\arctan(x)\log(1+x^2)}{x^6}\,dx = &-\frac{8\log(x)}{15} - \frac{\log(x^2)}{20} - \frac{\log(2x^2)}{10} - \frac{\log(x^2+1)^2}{20} \\ &+ \frac{19\log(x^2+1)}{60} + \frac{\log(2x^2+2)}{10} + \frac{\operatorname{atan}^2(x)}{5} \\ &- \frac{\operatorname{Li}_2(x^2e^{i\pi})}{10} + \frac{2\operatorname{atan}(x)}{5x} + \frac{\log(x^2+1)}{10x^2} - \frac{7}{60x^2} \\ &- \frac{2\operatorname{atan}(x)}{15x^3} - \frac{\log(x^2+1)}{20x^4} - \frac{\log(x^2+1)\operatorname{atan}(x)}{5x^5}\end{aligned}$$

input
```
integrate(atan(x)*ln(x**2+1)/x**6,x)
```

output
```
-8*log(x)/15 - log(x**2)/20 - log(2*x**2)/10 - log(x**2 + 1)**2/20 + 19*lo
g(x**2 + 1)/60 + log(2*x**2 + 2)/10 + atan(x)**2/5 - polylog(2, x**2*exp_p
olar(I*pi))/10 + 2*atan(x)/(5*x) + log(x**2 + 1)/(10*x**2) - 7/(60*x**2) -
 2*atan(x)/(15*x**3) - log(x**2 + 1)/(20*x**4) - log(x**2 + 1)*atan(x)/(5*
x**5)
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.01

$$\int \frac{\arctan(x)\log(1+x^2)}{x^6}\,dx = \frac{1}{15}\left(\frac{2\,(3\,x^2-1)}{x^3} - \frac{3\,\log(x^2+1)}{x^5} + 6\,\arctan(x)\right)\arctan(x) - \frac{12\,x^4\arctan(x)^2 + 3\,x^4\log(x^2+1)^2 - 6\,x^4\mathrm{Li}_2(x^2+1) + 50\,x^4\log(x) + 7\,x^2 - (6\,x^4\log(-x^2) + 25\,x^4}{60\,x^4}$$

input
```
integrate(arctan(x)*log(x^2+1)/x^6,x, algorithm="maxima")
```

output
```
1/15*(2*(3*x^2 - 1)/x^3 - 3*log(x^2 + 1)/x^5 + 6*arctan(x))*arctan(x) - 1/
60*(12*x^4*arctan(x)^2 + 3*x^4*log(x^2 + 1)^2 - 6*x^4*dilog(x^2 + 1) + 50*
x^4*log(x) + 7*x^2 - (6*x^4*log(-x^2) + 25*x^4 + 6*x^2 - 3)*log(x^2 + 1))/
x^4
```

## Giac [F]

$$\int \frac{\arctan(x)\log(1+x^2)}{x^6}\,dx = \int \frac{\arctan(x)\log(x^2+1)}{x^6}\,dx$$

input
```
integrate(arctan(x)*log(x^2+1)/x^6,x, algorithm="giac")
```

output
```
integrate(arctan(x)*log(x^2 + 1)/x^6, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{\arctan(x)\log(1+x^2)}{x^6}\,dx = \int \frac{\ln(x^2+1)\,\mathrm{atan}(x)}{x^6}\,dx$$

input
```
int((log(x^2 + 1)*atan(x))/x^6,x)
```

output
```
int((log(x^2 + 1)*atan(x))/x^6, x)
```

## Reduce [F]

$$\int \frac{\arctan(x)\log(1+x^2)}{x^6}\,dx$$
$$= \frac{12atan(x)^2x^5 - 12atan(x)\log(x^2+1) + 24atan(x)x^4 - 8atan(x)x^2 + 12\left(\int \frac{\log(x^2+1)}{x^3+x}dx\right)x^5 + 25\log(x}{60x^5}$$

input
```
int(atan(x)*log(x^2+1)/x^6,x)
```

output
```
(12*atan(x)**2*x**5 - 12*atan(x)*log(x**2 + 1) + 24*atan(x)*x**4 - 8*atan(
x)*x**2 + 12*int(log(x**2 + 1)/(x**3 + x),x)*x**5 + 25*log(x**2 + 1)*x**5
+ 6*log(x**2 + 1)*x**3 - 3*log(x**2 + 1)*x - 50*log(x)*x**5 - 7*x**3)/(60*
x**5)
```

# 3.1284 $\int x^4(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 278

$$\int x^4(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx = -\frac{2aex}{5c^4}-\frac{77bex^2}{300c^3}+\frac{2aex^3}{15c^2}+\frac{9bex^4}{200c}-\frac{2}{25}aex^5+\frac{2ae\arctan(cx)}{5c^5}-\frac{2bex\arctan(cx)}{5c^4}+\frac{2bex^3\arctan(cx)}{15c^2}-\frac{2}{25}bex^5\arctan(cx)+\frac{be\arctan(cx)^2}{5c^5}+\frac{137be\log\left(1+c^2x^2\right)}{300c^5}+\frac{be\log^2\left(1+c^2x^2\right)}{20c^5}+\frac{bx^2\left(d+e\log\left(1+c^2x^2\right)\right)}{10c^3}-\frac{bx^4\left(d+e\log\left(1+c^2x^2\right)\right)}{20c}+\frac{1}{5}x^5(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)-\frac{b\log\left(1+c^2x^2\right)\left(d+e\log\left(1+c^2x^2\right)\right)}{10c^5}$$

output
```
-2/5*a*e*x/c^4-77/300*b*e*x^2/c^3+2/15*a*e*x^3/c^2+9/200*b*e*x^4/c-2/25*a*
e*x^5+2/5*a*e*arctan(c*x)/c^5-2/5*b*e*x*arctan(c*x)/c^4+2/15*b*e*x^3*arcta
n(c*x)/c^2-2/25*b*e*x^5*arctan(c*x)+1/5*b*e*arctan(c*x)^2/c^5+137/300*b*e*
ln(c^2*x^2+1)/c^5+1/20*b*e*ln(c^2*x^2+1)^2/c^5+1/10*b*x^2*(d+e*ln(c^2*x^2+
1))/c^3-1/20*b*x^4*(d+e*ln(c^2*x^2+1))/c+1/5*x^5*(a+b*arctan(c*x))*(d+e*ln
(c^2*x^2+1))-1/10*b*ln(c^2*x^2+1)*(d+e*ln(c^2*x^2+1))/c^5
```

## Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.77

$$\int x^4(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx$$
$$=\frac{cx(bcx(-30d(-2+c^2x^2)+e(-154+27c^2x^2))+8a(15c^4dx^4-2e(15-5c^2x^2+3c^4x^4)))+120be\arctan$$

input
```
Integrate[x^4*(a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]),x]
```

output
```
(c*x*(b*c*x*(-30*d*(-2 + c^2*x^2) + e*(-154 + 27*c^2*x^2)) + 8*a*(15*c^4*d
*x^4 - 2*e*(15 - 5*c^2*x^2 + 3*c^4*x^4))) + 120*b*e*ArcTan[c*x]^2 + (-60*b
*d + 120*a*c^5*e*x^5 + 2*b*e*(137 + 30*c^2*x^2 - 15*c^4*x^4))*Log[1 + c^2*
x^2] - 30*b*e*Log[1 + c^2*x^2]^2 + 8*ArcTan[c*x]*(30*a*e + 15*b*c^5*d*x^5
- 2*b*c*e*x*(15 - 5*c^2*x^2 + 3*c^4*x^4) + 15*b*c^5*e*x^5*Log[1 + c^2*x^2]
))/(600*c^5)
```

## Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}=0.077$, Rules used = {5556, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)\,dx$$
$$\Big\downarrow \text{5556}$$
$$-2c^2e\int\left(\frac{4ac^3x^6+4bc^3\arctan(cx)x^6-bc^2x^5+2bx^3}{20c^3\left(c^2x^2+1\right)}-\frac{bx\log\left(c^2x^2+1\right)}{10c^5\left(c^2x^2+1\right)}\right)dx+\frac{1}{5}x^5(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)-\frac{bx^4\left(e\log\left(c^2x^2+1\right)+d\right)}{20c}-\frac{b\log\left(c^2x^2+1\right)\left(e\log\left(c^2x^2+1\right)+d\right)}{10c^5}+\frac{bx^2\left(e\log\left(c^2x^2+1\right)+d\right)}{10c^3}$$
$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}x^5(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)-$$
$$2c^2e\left(-\frac{a\arctan(cx)}{5c^7}+\frac{ax}{5c^6}-\frac{ax^3}{15c^4}+\frac{ax^5}{25c^2}-\frac{b\arctan(cx)^2}{10c^7}+\frac{bx\arctan(cx)}{5c^6}-\frac{bx^3\arctan(cx)}{15c^4}+\frac{bx^5\arctan(cx)}{25c^2}\right.$$
$$\frac{bx^4\left(e\log\left(c^2x^2+1\right)+d\right)}{20c}-\frac{b\log\left(c^2x^2+1\right)\left(e\log\left(c^2x^2+1\right)+d\right)}{10c^5}+\frac{bx^2\left(e\log\left(c^2x^2+1\right)+d\right)}{10c^3}$$

input
```
Int[x^4*(a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]),x]
```

output
```
(b*x^2*(d + e*Log[1 + c^2*x^2]))/(10*c^3) - (b*x^4*(d + e*Log[1 + c^2*x^2]
))/(20*c) + (x^5*(a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]))/5 - (b*Log[
1 + c^2*x^2]*(d + e*Log[1 + c^2*x^2]))/(10*c^5) - 2*c^2*e*((a*x)/(5*c^6) +
 (77*b*x^2)/(600*c^5) - (a*x^3)/(15*c^4) - (9*b*x^4)/(400*c^3) + (a*x^5)/(
25*c^2) - (a*ArcTan[c*x])/(5*c^7) + (b*x*ArcTan[c*x])/(5*c^6) - (b*x^3*Arc
Tan[c*x])/(15*c^4) + (b*x^5*ArcTan[c*x])/(25*c^2) - (b*ArcTan[c*x]^2)/(10*
c^7) - (137*b*Log[1 + c^2*x^2])/(600*c^7) - (b*Log[1 + c^2*x^2]^2)/(40*c^7
))
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5556
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcTan[c*x]), x
]}, Simp[(d + e*Log[f + g*x^2])   u, x] - Simp[2*e*g   Int[ExpandIntegrand[
x*(u/(f + g*x^2)), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && Intege
rQ[m] && NeQ[m, -1]
```

## Maple [A] (verified)

Time = 4.43 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.04

| method | result |
|---|---|
| parallelrisch | $\frac{-240xace+154eb+80a\,c^3e\,x^3+274\ln(c^2x^2+1)be-60bd+27b\,c^4e\,x^4-154b\,c^2e\,x^2-48a\,c^5e\,x^5+80x^3\arctan(cx)b\,c^3e+120a\,c^5}{}$ |
| default | Expression too large to display |
| parts | Expression too large to display |
| risch | Expression too large to display |

input
```
int(x^4*(a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1)),x,method=_RETURNVERBOSE)
```

output
```
1/600*(-240*x*a*c*e+154*e*b+80*a*c^3*e*x^3+274*ln(c^2*x^2+1)*b*e-60*b*d+27
*b*c^4*e*x^4-154*b*c^2*e*x^2-48*a*c^5*e*x^5+80*x^3*arctan(c*x)*b*c^3*e+120
*a*c^5*d*x^5-48*x^5*arctan(c*x)*b*c^5*e-30*b*c^4*d*x^4+120*e*b*ln(c^2*x^2+
1)*arctan(c*x)*x^5*c^5+60*c^2*x^2*b*d-60*ln(c^2*x^2+1)*b*d-30*e*b*ln(c^2*x
^2+1)^2+240*a*e*arctan(c*x)+120*e*b*arctan(c*x)^2-240*e*b*arctan(c*x)*x*c-
30*e*b*ln(c^2*x^2+1)*x^4*c^4+120*b*arctan(c*x)*x^5*c^5*d+60*x^2*ln(c^2*x^2
+1)*b*c^2*e+120*a*e*ln(c^2*x^2+1)*x^5*c^5)/c^5
```

## Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.79

$$\int x^4(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx$$
$$=\frac{80\,ac^3ex^3+24\,(5\,ac^5d-2\,ac^5e)x^5-3\,(10\,bc^4d-9\,bc^4e)x^4-240\,acex+120\,be\arctan(cx)^2-30\,be\log(}{}$$

input
```
integrate(x^4*(a+b*arctan(c*x))*(d+e*log(c^2*x^2+1)),x, algorithm="fricas"
)
```

output
```
1/600*(80*a*c^3*e*x^3 + 24*(5*a*c^5*d - 2*a*c^5*e)*x^5 - 3*(10*b*c^4*d - 9
*b*c^4*e)*x^4 - 240*a*c*e*x + 120*b*e*arctan(c*x)^2 - 30*b*e*log(c^2*x^2 +
 1)^2 + 2*(30*b*c^2*d - 77*b*c^2*e)*x^2 + 8*(10*b*c^3*e*x^3 + 3*(5*b*c^5*d
 - 2*b*c^5*e)*x^5 - 30*b*c*e*x + 30*a*e)*arctan(c*x) + 2*(60*b*c^5*e*x^5*a
rctan(c*x) + 60*a*c^5*e*x^5 - 15*b*c^4*e*x^4 + 30*b*c^2*e*x^2 - 30*b*d + 1
37*b*e)*log(c^2*x^2 + 1))/c^5
```

## Sympy [A] (verification not implemented)

Time = 1.74 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.22

$$\int x^4(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx$$

$$=\begin{cases}\frac{adx^5}{5}+\frac{aex^5\log\left(c^2x^2+1\right)}{5}-\frac{2aex^5}{25}+\frac{2aex^3}{15c^2}-\frac{2aex}{5c^4}+\frac{2ae\operatorname{atan}(cx)}{5c^5}+\frac{bdx^5\operatorname{atan}(cx)}{5}+\frac{bex^5\log\left(c^2x^2+1\right)\operatorname{atan}(cx)}{5}-\frac{2bex^5\,\mathrm{a}}{2}\\ \frac{adx^5}{5}\end{cases}$$

input
```
integrate(x**4*(a+b*atan(c*x))*(d+e*ln(c**2*x**2+1)),x)
```

output
```
Piecewise((a*d*x**5/5 + a*e*x**5*log(c**2*x**2 + 1)/5 - 2*a*e*x**5/25 + 2*
a*e*x**3/(15*c**2) - 2*a*e*x/(5*c**4) + 2*a*e*atan(c*x)/(5*c**5) + b*d*x**
5*atan(c*x)/5 + b*e*x**5*log(c**2*x**2 + 1)*atan(c*x)/5 - 2*b*e*x**5*atan(
c*x)/25 - b*d*x**4/(20*c) - b*e*x**4*log(c**2*x**2 + 1)/(20*c) + 9*b*e*x**
4/(200*c) + 2*b*e*x**3*atan(c*x)/(15*c**2) + b*d*x**2/(10*c**3) + b*e*x**2
*log(c**2*x**2 + 1)/(10*c**3) - 77*b*e*x**2/(300*c**3) - 2*b*e*x*atan(c*x)
/(5*c**4) - b*d*log(c**2*x**2 + 1)/(10*c**5) - b*e*log(c**2*x**2 + 1)**2/(
20*c**5) + 137*b*e*log(c**2*x**2 + 1)/(300*c**5) + b*e*atan(c*x)**2/(5*c**
5), Ne(c, 0)), (a*d*x**5/5, True))
```

## Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.92

$$\int x^4(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx=\frac{1}{5}adx^5$$
$$+\frac{1}{75}\left(15\,x^5\log\left(c^2x^2+1\right)-2\,c^2\left(\frac{3\,c^4x^5-5\,c^2x^3+15\,x}{c^6}-\frac{15\,\arctan\left(cx\right)}{c^7}\right)\right)be\arctan\left(cx\right)$$
$$+\frac{1}{20}\left(4\,x^5\arctan\left(cx\right)-c\left(\frac{c^2x^4-2\,x^2}{c^4}+\frac{2\,\log\left(c^2x^2+1\right)}{c^6}\right)\right)bd$$
$$+\frac{1}{75}\left(15\,x^5\log\left(c^2x^2+1\right)-2\,c^2\left(\frac{3\,c^4x^5-5\,c^2x^3+15\,x}{c^6}-\frac{15\,\arctan\left(cx\right)}{c^7}\right)\right)ae$$
$$+\frac{\left(27\,c^4x^4-154\,c^2x^2-120\,\arctan\left(cx\right)^2-2\left(15\,c^4x^4-30\,c^2x^2-137\right)\log\left(c^2x^2+1\right)-30\,\log\left(c^2x^2+1\right.\right.}{600\,c^5}$$

input
```
integrate(x^4*(a+b*arctan(c*x))*(d+e*log(c^2*x^2+1)),x, algorithm="maxima"
)
```

output
```
1/5*a*d*x^5 + 1/75*(15*x^5*log(c^2*x^2 + 1) - 2*c^2*((3*c^4*x^5 - 5*c^2*x^
3 + 15*x)/c^6 - 15*arctan(c*x)/c^7))*b*e*arctan(c*x) + 1/20*(4*x^5*arctan(
c*x) - c*((c^2*x^4 - 2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*b*d + 1/75*(15*
x^5*log(c^2*x^2 + 1) - 2*c^2*((3*c^4*x^5 - 5*c^2*x^3 + 15*x)/c^6 - 15*arct
an(c*x)/c^7))*a*e + 1/600*(27*c^4*x^4 - 154*c^2*x^2 - 120*arctan(c*x)^2 -
2*(15*c^4*x^4 - 30*c^2*x^2 - 137)*log(c^2*x^2 + 1) - 30*log(c^2*x^2 + 1)^2
)*b*e/c^5
```

## Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 451, normalized size of antiderivative = 1.62

$$\int x^4(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx$$
$$=\frac{60\,\pi bc^5ex^5\log\left(c^2x^2+1\right)\mathrm{sgn}(c)\,\mathrm{sgn}(x)+60\,\pi bc^5dx^5\mathrm{sgn}(c)\,\mathrm{sgn}(x)-24\,\pi bc^5ex^5\mathrm{sgn}(c)\,\mathrm{sgn}(x)-120\,bc^5ex}{}$$

input
```
integrate(x^4*(a+b*arctan(c*x))*(d+e*log(c^2*x^2+1)),x, algorithm="giac")
```

output
```
1/600*(60*pi*b*c^5*e*x^5*log(c^2*x^2 + 1)*sgn(c)*sgn(x) + 60*pi*b*c^5*d*x^
5*sgn(c)*sgn(x) - 24*pi*b*c^5*e*x^5*sgn(c)*sgn(x) - 120*b*c^5*e*x^5*arctan
(1/(c*x))*log(c^2*x^2 + 1) - 120*b*c^5*d*x^5*arctan(1/(c*x)) + 48*b*c^5*e*
x^5*arctan(1/(c*x)) + 120*a*c^5*e*x^5*log(c^2*x^2 + 1) + 120*a*c^5*d*x^5 -
 48*a*c^5*e*x^5 - 30*b*c^4*e*x^4*log(c^2*x^2 + 1) + 40*pi*b*c^3*e*x^3*sgn(
c)*sgn(x) - 30*b*c^4*d*x^4 + 27*b*c^4*e*x^4 - 80*b*c^3*e*x^3*arctan(1/(c*x
)) + 80*a*c^3*e*x^3 + 60*b*c^2*e*x^2*log(c^2*x^2 + 1) - 120*pi*b*c*e*x*sgn
(c)*sgn(x) + 60*b*c^2*d*x^2 - 154*b*c^2*e*x^2 - 180*pi^2*b*e*sgn(c)*sgn(x)
 - 120*pi*b*e*arctan(1/(c*x))*sgn(c)*sgn(x) + 240*b*c*e*x*arctan(1/(c*x))
- 240*pi*a*e*sgn(c)*sgn(x) + 60*pi^2*b*e - 240*a*c*e*x + 120*pi*b*e*arctan
(c*x) + 120*pi*b*e*arctan(1/(c*x)) + 120*b*e*arctan(1/(c*x))^2 - 30*b*e*lo
g(c^2*x^2 + 1)^2 + 240*a*e*arctan(c*x) - 60*b*d*log(c^2*x^2 + 1) + 274*b*e
*log(c^2*x^2 + 1))/c^5
```

## Mupad [B] (verification not implemented)

Time = 4.13 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.99

$$\int x^4(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx = \frac{a\,d\,x^5}{5}-\frac{2\,a\,e\,x^5}{25}-\frac{b\,e\ln\left(c^2\,x^2+1\right)^2}{20\,c^5}-\frac{2\,a\,e\,x}{5\,c^4}+\frac{2\,a\,e\,\mathrm{atan}(c\,x)}{5\,c^5}+\frac{b\,d\,x^5\,\mathrm{atan}(c\,x)}{5}-\frac{2\,b\,e\,x^5\,\mathrm{atan}(c\,x)}{25}-\frac{b\,d\,\ln\left(c^2\,x^2+1\right)}{10\,c^5}+\frac{137\,b\,e\,\ln\left(c^2\,x^2+1\right)}{300\,c^5}+\frac{2\,a\,e\,x^3}{15\,c^2}-\frac{b\,d\,x^4}{20\,c}+\frac{b\,d\,x^2}{10\,c^3}+\frac{9\,b\,e\,x^4}{200\,c}-\frac{77\,b\,e\,x^2}{300\,c^3}+\frac{a\,e\,x^5\,\ln\left(c^2\,x^2+1\right)}{5}+\frac{b\,e\,\mathrm{atan}(c\,x)^2}{5\,c^5}+\frac{2\,b\,e\,x^3\,\mathrm{atan}(c\,x)}{15\,c^2}+\frac{b\,e\,x^5\,\mathrm{atan}(c\,x)\,\ln\left(c^2\,x^2+1\right)}{5}-\frac{b\,e\,x^4\,\ln\left(c^2\,x^2+1\right)}{20\,c}+\frac{b\,e\,x^2\,\ln\left(c^2\,x^2+1\right)}{10\,c^3}-\frac{2\,b\,e\,x\,\mathrm{atan}(c\,x)}{5\,c^4}$$

input
```
int(x^4*(a + b*atan(c*x))*(d + e*log(c^2*x^2 + 1)),x)
```

output
```
(a*d*x^5)/5 - (2*a*e*x^5)/25 - (b*e*log(c^2*x^2 + 1)^2)/(20*c^5) - (2*a*e*
x)/(5*c^4) + (2*a*e*atan(c*x))/(5*c^5) + (b*d*x^5*atan(c*x))/5 - (2*b*e*x^
5*atan(c*x))/25 - (b*d*log(c^2*x^2 + 1))/(10*c^5) + (137*b*e*log(c^2*x^2 +
 1))/(300*c^5) + (2*a*e*x^3)/(15*c^2) - (b*d*x^4)/(20*c) + (b*d*x^2)/(10*c
^3) + (9*b*e*x^4)/(200*c) - (77*b*e*x^2)/(300*c^3) + (a*e*x^5*log(c^2*x^2
+ 1))/5 + (b*e*atan(c*x)^2)/(5*c^5) + (2*b*e*x^3*atan(c*x))/(15*c^2) + (b*
e*x^5*atan(c*x)*log(c^2*x^2 + 1))/5 - (b*e*x^4*log(c^2*x^2 + 1))/(20*c) +
(b*e*x^2*log(c^2*x^2 + 1))/(10*c^3) - (2*b*e*x*atan(c*x))/(5*c^4)
```

## Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.01

$$\int x^4(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx$$
$$=\frac{120atan(cx)^2\,be+120atan(cx)\log(c^2x^2+1)\,b\,c^5e\,x^5+240atan(cx)\,ae+120atan(cx)\,b\,c^5d\,x^5-48atan\cdots}{\cdots}$$

input
```
int(x^4*(a+b*atan(c*x))*(d+e*log(c^2*x^2+1)),x)
```

output
```
(120*atan(c*x)**2*b*e + 120*atan(c*x)*log(c**2*x**2 + 1)*b*c**5*e*x**5 + 2
40*atan(c*x)*a*e + 120*atan(c*x)*b*c**5*d*x**5 - 48*atan(c*x)*b*c**5*e*x**
5 + 80*atan(c*x)*b*c**3*e*x**3 - 240*atan(c*x)*b*c*e*x - 30*log(c**2*x**2
+ 1)**2*b*e + 120*log(c**2*x**2 + 1)*a*c**5*e*x**5 - 30*log(c**2*x**2 + 1)
*b*c**4*e*x**4 + 60*log(c**2*x**2 + 1)*b*c**2*e*x**2 - 60*log(c**2*x**2 +
1)*b*d + 274*log(c**2*x**2 + 1)*b*e + 120*a*c**5*d*x**5 - 48*a*c**5*e*x**5
 + 80*a*c**3*e*x**3 - 240*a*c*e*x - 30*b*c**4*d*x**4 + 27*b*c**4*e*x**4 +
60*b*c**2*d*x**2 - 154*b*c**2*e*x**2)/(600*c**5)
```

# 3.1285 $\int x^3(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 221

$$\begin{aligned}\int x^3(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx\\ &=\frac{b(2d-3e)x}{8c^3}-\frac{2bex}{3c^3}-\frac{b(2d-e)x^3}{24c}+\frac{bex^3}{18c}-\frac{b(2d-3e)\arctan(cx)}{8c^4}\\ &+\frac{2be\arctan(cx)}{3c^4}+\frac{ex^2(a+b\arctan(cx))}{4c^2}-\frac{1}{8}ex^4(a+b\arctan(cx))\\ &+\frac{bex\log\left(1+c^2x^2\right)}{4c^3}-\frac{bex^3\log\left(1+c^2x^2\right)}{12c}-\frac{e(a+b\arctan(cx))\log\left(1+c^2x^2\right)}{4c^4}\\ &+\frac{1}{4}x^4(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\end{aligned}$$

output

```
1/8*b*(2*d-3*e)*x/c^3-2/3*b*e*x/c^3-1/24*b*(2*d-e)*x^3/c+1/18*b*e*x^3/c-1/
8*b*(2*d-3*e)*arctan(c*x)/c^4+2/3*b*e*arctan(c*x)/c^4+1/4*e*x^2*(a+b*arcta
n(c*x))/c^2-1/8*e*x^4*(a+b*arctan(c*x))+1/4*b*e*x*ln(c^2*x^2+1)/c^3-1/12*b
*e*x^3*ln(c^2*x^2+1)/c-1/4*e*(a+b*arctan(c*x))*ln(c^2*x^2+1)/c^4+1/4*x^4*(
a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1))
```

## Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.74

$$\int x^3(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx$$
$$=\frac{cx(18ac^3dx^3-6bd(-3+c^2x^2)-9acex(-2+c^2x^2)+be(-75+7c^2x^2))-6e(bcx(-3+c^2x^2)+a(3-3$$

input
```
Integrate[x^3*(a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]),x]
```

output
```
(c*x*(18*a*c^3*d*x^3 - 6*b*d*(-3 + c^2*x^2) - 9*a*c*e*x*(-2 + c^2*x^2) + b
*e*(-75 + 7*c^2*x^2)) - 6*e*(b*c*x*(-3 + c^2*x^2) + a*(3 - 3*c^4*x^4))*Log
[1 + c^2*x^2] + 3*b*ArcTan[c*x]*(e*(25 + 6*c^2*x^2 - 3*c^4*x^4) + 6*d*(-1
+ c^4*x^4) + 6*e*(-1 + c^4*x^4)*Log[1 + c^2*x^2]))/(72*c^4)
```

## Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {5554, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)\,dx$$
$$\Big\downarrow \; 5554$$
$$-bc\int\left(\frac{x^2\left(c^2(2d-e)x^2+2e\right)}{8c^2\left(c^2x^2+1\right)}-\frac{e\left(1-c^2x^2\right)\log\left(c^2x^2+1\right)}{4c^4}\right)dx+\frac{1}{4}x^4(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)+\frac{ex^2(a+b\arctan(cx))}{4c^2}-\frac{e\log\left(c^2x^2+1\right)(a+b\arctan(cx))}{4c^4}-\frac{1}{8}ex^4(a+b\arctan(cx))$$
$$\Big\downarrow \; 2009$$

$$\frac{1}{4}x^4(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)+\frac{ex^2(a+b\arctan(cx))}{4c^2}-\frac{e\log\left(c^2x^2+1\right)(a+b\arctan(cx))}{4c^4}-\frac{1}{8}ex^4(a+b\arctan(cx))-bc\left(\frac{(2d-3e)\arctan(cx)}{8c^5}-\frac{2e\arctan(cx)}{3c^5}-\frac{x(2d-3e)}{8c^4}+\frac{2ex}{3c^4}+\frac{x^3(2d-e)}{24c^2}-\frac{ex^3}{18c^2}+\frac{ex^3\log\left(c^2x^2+1\right)}{12c^2}-\frac{ex\right.$$

input
```
Int[x^3*(a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]),x]
```

output
```
(e*x^2*(a + b*ArcTan[c*x]))/(4*c^2) - (e*x^4*(a + b*ArcTan[c*x]))/8 - (e*(
a + b*ArcTan[c*x])*Log[1 + c^2*x^2])/(4*c^4) + (x^4*(a + b*ArcTan[c*x])*(d
 + e*Log[1 + c^2*x^2]))/4 - b*c*(-1/8*((2*d - 3*e)*x)/c^4 + (2*e*x)/(3*c^4
) + ((2*d - e)*x^3)/(24*c^2) - (e*x^3)/(18*c^2) + ((2*d - 3*e)*ArcTan[c*x]
)/(8*c^5) - (2*e*ArcTan[c*x])/(3*c^5) - (e*x*Log[1 + c^2*x^2])/(4*c^4) + (
e*x^3*Log[1 + c^2*x^2])/(12*c^2))
```

## Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5554
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*Log[f + g*x^2])
, x]}, Simp[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[ExpandIntegrand[u/
(1 + c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[(m +
1)/2, 0]
```

## Maple [A] (verified)

Time = 2.74 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.10

| method | result |
|---|---|
| parallelrisch | $\frac{18eb\ln(c^2x^2+1)\arctan(cx)x^4c^4+18b\arctan(cx)x^4c^4d-9b\arctan(cx)x^4c^4e+18ae\ln(c^2x^2+1)x^4c^4+18c^4adx^4-9ac^4ex^4-}{}$ |
| orering | $\frac{(9x^8c^8-76c^6x^6-497c^4x^4-1570c^2x^2-1350)(a+b\arctan(cx))(d+e\ln(c^2x^2+1))}{12c^6x^2(c^2x^2-3)}-\frac{(19x^8c^8-274c^6x^6-2162c^4x^4-6210c^2}{}$ |
| default | Expression too large to display |
| parts | Expression too large to display |
| risch | Expression too large to display |

input
```
int(x^3*(a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1)),x,method=_RETURNVERBOSE)
```

output
```
1/72*(18*e*b*ln(c^2*x^2+1)*arctan(c*x)*x^4*c^4+18*b*arctan(c*x)*x^4*c^4*d-
9*b*arctan(c*x)*x^4*c^4*e+18*a*e*ln(c^2*x^2+1)*x^4*c^4+18*c^4*a*d*x^4-9*a*
c^4*e*x^4-6*e*b*ln(c^2*x^2+1)*x^3*c^3-6*b*c^3*d*x^3+7*b*c^3*e*x^3+18*arcta
n(c*x)*b*c^2*e*x^2+18*a*c^2*e*x^2+18*ln(c^2*x^2+1)*b*c*e*x+18*b*c*d*x-75*b
*c*e*x-18*arctan(c*x)*ln(c^2*x^2+1)*b*e-18*arctan(c*x)*b*d+75*b*arctan(c*x
)*e-18*ln(c^2*x^2+1)*a*e-18*a*e)/c^4
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.81

$$\int x^3(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx$$
$$=\frac{18\,ac^2ex^2+9\,(2\,ac^4d-ac^4e)x^4-(6\,bc^3d-7\,bc^3e)x^3+3\,(6\,bcd-25\,bce)x+3\,(6\,bc^2ex^2+3\,(2\,bc^4d-bc}{}$$

input
```
integrate(x^3*(a+b*arctan(c*x))*(d+e*log(c^2*x^2+1)),x, algorithm="fricas"
)
```

output
```
1/72*(18*a*c^2*e*x^2 + 9*(2*a*c^4*d - a*c^4*e)*x^4 - (6*b*c^3*d - 7*b*c^3*
e)*x^3 + 3*(6*b*c*d - 25*b*c*e)*x + 3*(6*b*c^2*e*x^2 + 3*(2*b*c^4*d - b*c^
4*e)*x^4 - 6*b*d + 25*b*e)*arctan(c*x) + 6*(3*a*c^4*e*x^4 - b*c^3*e*x^3 +
3*b*c*e*x - 3*a*e + 3*(b*c^4*e*x^4 - b*e)*arctan(c*x))*log(c^2*x^2 + 1))/c
^4
```

## Sympy [A] (verification not implemented)

Time = 1.22 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.26

$$\int x^3(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx$$
$$=\begin{cases}\frac{adx^4}{4}+\frac{aex^4\log\left(c^2x^2+1\right)}{4}-\frac{aex^4}{8}+\frac{aex^2}{4c^2}-\frac{ae\log\left(c^2x^2+1\right)}{4c^4}+\frac{bdx^4\operatorname{atan}(cx)}{4}+\frac{bex^4\log\left(c^2x^2+1\right)\operatorname{atan}(cx)}{4}-\frac{bex^4\operatorname{atan}(cx)}{8} \\ \frac{adx^4}{4}\end{cases}$$

input
```
integrate(x**3*(a+b*atan(c*x))*(d+e*ln(c**2*x**2+1)),x)
```

output
```
Piecewise((a*d*x**4/4 + a*e*x**4*log(c**2*x**2 + 1)/4 - a*e*x**4/8 + a*e*x
**2/(4*c**2) - a*e*log(c**2*x**2 + 1)/(4*c**4) + b*d*x**4*atan(c*x)/4 + b*
e*x**4*log(c**2*x**2 + 1)*atan(c*x)/4 - b*e*x**4*atan(c*x)/8 - b*d*x**3/(1
2*c) - b*e*x**3*log(c**2*x**2 + 1)/(12*c) + 7*b*e*x**3/(72*c) + b*e*x**2*a
tan(c*x)/(4*c**2) + b*d*x/(4*c**3) + b*e*x*log(c**2*x**2 + 1)/(4*c**3) - 2
5*b*e*x/(24*c**3) - b*d*atan(c*x)/(4*c**4) - b*e*log(c**2*x**2 + 1)*atan(c
*x)/(4*c**4) + 25*b*e*atan(c*x)/(24*c**4), Ne(c, 0)), (a*d*x**4/4, True))
```

## Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.01

$$\int x^3(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx$$
$$=\frac{1}{4}adx^4+\frac{1}{72}bce\left(\frac{7\,c^2x^3-6\,(c^2x^3-3\,x)\log\left(c^2x^2+1\right)-75\,x}{c^4}+\frac{75\arctan\left(cx\right)}{c^5}\right)$$
$$+\frac{1}{8}\left(2\,x^4\log\left(c^2x^2+1\right)-c^2\left(\frac{c^2x^4-2\,x^2}{c^4}+\frac{2\,\log\left(c^2x^2+1\right)}{c^6}\right)\right)be\arctan\left(cx\right)$$
$$+\frac{1}{12}\left(3\,x^4\arctan\left(cx\right)-c\left(\frac{c^2x^3-3\,x}{c^4}+\frac{3\,\arctan\left(cx\right)}{c^5}\right)\right)bd$$
$$+\frac{1}{8}\left(2\,x^4\log\left(c^2x^2+1\right)-c^2\left(\frac{c^2x^4-2\,x^2}{c^4}+\frac{2\,\log\left(c^2x^2+1\right)}{c^6}\right)\right)ae$$

input
```
integrate(x^3*(a+b*arctan(c*x))*(d+e*log(c^2*x^2+1)),x, algorithm="maxima"
)
```

output
```
1/4*a*d*x^4 + 1/72*b*c*e*((7*c^2*x^3 - 6*(c^2*x^3 - 3*x)*log(c^2*x^2 + 1)
- 75*x)/c^4 + 75*arctan(c*x)/c^5) + 1/8*(2*x^4*log(c^2*x^2 + 1) - c^2*((c^
2*x^4 - 2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6))*b*e*arctan(c*x) + 1/12*(3*x^
4*arctan(c*x) - c*((c^2*x^3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*b*d + 1/8*(2*
x^4*log(c^2*x^2 + 1) - c^2*((c^2*x^4 - 2*x^2)/c^4 + 2*log(c^2*x^2 + 1)/c^6
))*a*e
```

## Giac [F(-2)]

Exception generated.

$$\int x^3(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a+b*arctan(c*x))*(d+e*log(c^2*x^2+1)),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

## Mupad [B] (verification not implemented)

Time = 2.38 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.34

$$\int x^3(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx$$
$$=\frac{a\,d\,x^4}{4}-\frac{a\,e\,x^4}{8}+\frac{b\,d\,x}{4\,c^3}-\frac{25\,b\,e\,x}{24\,c^3}+\frac{b\,d\,x^4\,\mathrm{atan}(c\,x)}{4}-\frac{b\,e\,x^4\,\mathrm{atan}(c\,x)}{8}-\frac{a\,e\,\ln\left(c^2\,x^2+1\right)}{4\,c^4}+\frac{a\,e\,x^2}{4\,c^2}-\frac{b\,d\,x^3}{12\,c}-\frac{b\,d\,\mathrm{atan}\left(\frac{6\,b\,c\,d\,x}{6\,b\,d-25\,b\,e}-\frac{25\,b\,c\,e\,x}{6\,b\,d-25\,b\,e}\right)}{4\,c^4}+\frac{7\,b\,e\,x^3}{72\,c}+\frac{25\,b\,e\,\mathrm{atan}\left(\frac{6\,b\,c\,d\,x}{6\,b\,d-25\,b\,e}-\frac{25\,b\,c\,e\,x}{6\,b\,d-25\,b\,e}\right)}{24\,c^4}+\frac{a\,e\,x^4\,\ln\left(c^2\,x^2+1\right)}{4}+\frac{b\,e\,x\,\ln\left(c^2\,x^2+1\right)}{4\,c^3}-\frac{b\,e\,\mathrm{atan}(c\,x)\,\ln\left(c^2\,x^2+1\right)}{4\,c^4}+\frac{b\,e\,x^2\,\mathrm{atan}(c\,x)}{4\,c^2}+\frac{b\,e\,x^4\,\mathrm{atan}(c\,x)\,\ln\left(c^2\,x^2+1\right)}{4}-\frac{b\,e\,x^3\,\ln\left(c^2\,x^2+1\right)}{12\,c}$$

input
```
int(x^3*(a + b*atan(c*x))*(d + e*log(c^2*x^2 + 1)),x)
```

output
```
(a*d*x^4)/4 - (a*e*x^4)/8 + (b*d*x)/(4*c^3) - (25*b*e*x)/(24*c^3) + (b*d*x
^4*atan(c*x))/4 - (b*e*x^4*atan(c*x))/8 - (a*e*log(c^2*x^2 + 1))/(4*c^4) +
 (a*e*x^2)/(4*c^2) - (b*d*x^3)/(12*c) - (b*d*atan((6*b*c*d*x)/(6*b*d - 25*
b*e) - (25*b*c*e*x)/(6*b*d - 25*b*e)))/(4*c^4) + (7*b*e*x^3)/(72*c) + (25*
b*e*atan((6*b*c*d*x)/(6*b*d - 25*b*e) - (25*b*c*e*x)/(6*b*d - 25*b*e)))/(2
4*c^4) + (a*e*x^4*log(c^2*x^2 + 1))/4 + (b*e*x*log(c^2*x^2 + 1))/(4*c^3) -
 (b*e*atan(c*x)*log(c^2*x^2 + 1))/(4*c^4) + (b*e*x^2*atan(c*x))/(4*c^2) +
(b*e*x^4*atan(c*x)*log(c^2*x^2 + 1))/4 - (b*e*x^3*log(c^2*x^2 + 1))/(12*c)
```

## Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.08

$$\int x^3(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx$$
$$=\frac{18atan(cx)\log(c^2x^2+1)\,b\,c^4e\,x^4-18atan(cx)\log(c^2x^2+1)\,be+18atan(cx)\,b\,c^4d\,x^4-9atan(cx)\,b\,c^4e\,x}{}$$

input
```
int(x^3*(a+b*atan(c*x))*(d+e*log(c^2*x^2+1)),x)
```

output

```
(18*atan(c*x)*log(c**2*x**2 + 1)*b*c**4*e*x**4 - 18*atan(c*x)*log(c**2*x**
2 + 1)*b*e + 18*atan(c*x)*b*c**4*d*x**4 - 9*atan(c*x)*b*c**4*e*x**4 + 18*a
tan(c*x)*b*c**2*e*x**2 - 18*atan(c*x)*b*d + 75*atan(c*x)*b*e + 18*log(c**2
*x**2 + 1)*a*c**4*e*x**4 - 18*log(c**2*x**2 + 1)*a*e - 6*log(c**2*x**2 + 1
)*b*c**3*e*x**3 + 18*log(c**2*x**2 + 1)*b*c*e*x + 18*a*c**4*d*x**4 - 9*a*c
**4*e*x**4 + 18*a*c**2*e*x**2 - 6*b*c**3*d*x**3 + 7*b*c**3*e*x**3 + 18*b*c
*d*x - 75*b*c*e*x)/(72*c**4)
```

# 3.1286 $\int x^2(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 213

$$\begin{aligned}\int x^2(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx &= \frac{2aex}{3c^2}+\frac{5bex^2}{18c}-\frac{2}{9}aex^3-\frac{2ae\arctan(cx)}{3c^3}+\frac{2bex\arctan(cx)}{3c^2}\\ &\quad-\frac{2}{9}bex^3\arctan(cx)-\frac{be\arctan(cx)^2}{3c^3}-\frac{11be\log\left(1+c^2x^2\right)}{18c^3}\\ &\quad-\frac{be\log^2\left(1+c^2x^2\right)}{12c^3}-\frac{bx^2\left(d+e\log\left(1+c^2x^2\right)\right)}{6c}\\ &\quad+\frac{1}{3}x^3(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)+\frac{b\log\left(1+c^2x^2\right)\left(d+e\log\left(1+c^2x^2\right)\right)}{6c^3}\end{aligned}$$

output
```
2/3*a*e*x/c^2+5/18*b*e*x^2/c-2/9*a*e*x^3-2/3*a*e*arctan(c*x)/c^3+2/3*b*e*x
*arctan(c*x)/c^2-2/9*b*e*x^3*arctan(c*x)-1/3*b*e*arctan(c*x)^2/c^3-11/18*b
*e*ln(c^2*x^2+1)/c^3-1/12*b*e*ln(c^2*x^2+1)^2/c^3-1/6*b*x^2*(d+e*ln(c^2*x^
2+1))/c+1/3*x^3*(a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1))+1/6*b*ln(c^2*x^2+1)*
(d+e*ln(c^2*x^2+1))/c^3
```

## Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.80

$$\int x^2(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx$$
$$=\frac{2cx(bc(-3d+5e)x+6ac^2dx^2-4ae(-3+c^2x^2))-12be\arctan(cx)^2+2(3bd+6ac^3ex^3-be(11+3c^2x}{}$$

input
```
Integrate[x^2*(a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]),x]
```

output
```
(2*c*x*(b*c*(-3*d + 5*e)*x + 6*a*c^2*d*x^2 - 4*a*e*(-3 + c^2*x^2)) - 12*b*
e*ArcTan[c*x]^2 + 2*(3*b*d + 6*a*c^3*e*x^3 - b*e*(11 + 3*c^2*x^2))*Log[1 +
 c^2*x^2] + 3*b*e*Log[1 + c^2*x^2]^2 - 4*ArcTan[c*x]*(6*a*e + b*c*x*(-6*e
- 3*c^2*d*x^2 + 2*c^2*e*x^2) - 3*b*c^3*e*x^3*Log[1 + c^2*x^2]))/(36*c^3)
```

## Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {5556, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)\,dx$$
$$\Big\downarrow \text{5556}$$
$$-2c^2e\int\left(\frac{bx\log\left(c^2x^2+1\right)}{6c^3\left(c^2x^2+1\right)}-\frac{x^3(-2cx\arctan(cx)b+b-2acx)}{6c\left(c^2x^2+1\right)}\right)dx+\frac{1}{3}x^3(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)-\frac{bx^2\left(e\log\left(c^2x^2+1\right)+d\right)}{6c}+\frac{b\log\left(c^2x^2+1\right)\left(e\log\left(c^2x^2+1\right)+d\right)}{6c^3}$$
$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}x^3(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)-$$
$$2c^2e\left(\frac{a\arctan(cx)}{3c^5}-\frac{ax}{3c^4}+\frac{ax^3}{9c^2}+\frac{b\arctan(cx)^2}{6c^5}-\frac{bx\arctan(cx)}{3c^4}+\frac{bx^3\arctan(cx)}{9c^2}-\frac{5bx^2}{36c^3}+\frac{b\log^2\left(c^2x^2+1\right)}{24c^5}\right.$$
$$\frac{bx^2\left(e\log\left(c^2x^2+1\right)+d\right)}{6c}+\frac{b\log\left(c^2x^2+1\right)\left(e\log\left(c^2x^2+1\right)+d\right)}{6c^3}$$

input
```
Int[x^2*(a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]),x]
```

output
```
-1/6*(b*x^2*(d + e*Log[1 + c^2*x^2]))/c + (x^3*(a + b*ArcTan[c*x])*(d + e*
Log[1 + c^2*x^2]))/3 + (b*Log[1 + c^2*x^2]*(d + e*Log[1 + c^2*x^2]))/(6*c^
3) - 2*c^2*e*(-1/3*(a*x)/c^4 - (5*b*x^2)/(36*c^3) + (a*x^3)/(9*c^2) + (a*A
rcTan[c*x])/(3*c^5) - (b*x*ArcTan[c*x])/(3*c^4) + (b*x^3*ArcTan[c*x])/(9*c
^2) + (b*ArcTan[c*x]^2)/(6*c^5) + (11*b*Log[1 + c^2*x^2])/(36*c^5) + (b*Lo
g[1 + c^2*x^2]^2)/(24*c^5))
```

**Defintions of rubi rules used**

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5556
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcTan[c*x]), x
]}, Simp[(d + e*Log[f + g*x^2])   u, x] - Simp[2*e*g   Int[ExpandIntegrand[
x*(u/(f + g*x^2)), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && Intege
rQ[m] && NeQ[m, -1]
```

## Maple [A] (verified)

Time = 2.88 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.02

| method | result |
|---|---|
| parallelrisch | $\frac{12eb\ln(c^2x^2+1)\arctan(cx)x^3c^3+12b\arctan(cx)x^3c^3d-8x^3\arctan(cx)bc^3e+12ae\ln(c^2x^2+1)x^3c^3+12ac^3dx^3-8ac^3ex^3-}{}$ |
| default | Expression too large to display |
| parts | Expression too large to display |
| risch | Expression too large to display |

input
```
int(x^2*(a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1)),x,method=_RETURNVERBOSE)
```

output
```
1/36*(12*e*b*ln(c^2*x^2+1)*arctan(c*x)*x^3*c^3+12*b*arctan(c*x)*x^3*c^3*d-
8*x^3*arctan(c*x)*b*c^3*e+12*a*e*ln(c^2*x^2+1)*x^3*c^3+12*a*c^3*d*x^3-8*a*
c^3*e*x^3-6*x^2*ln(c^2*x^2+1)*b*c^2*e-6*c^2*x^2*b*d+10*b*c^2*e*x^2+24*e*b*
arctan(c*x)*x*c+24*x*a*c*e-12*e*b*arctan(c*x)^2+3*e*b*ln(c^2*x^2+1)^2-24*a
*e*arctan(c*x)+6*ln(c^2*x^2+1)*b*d-22*ln(c^2*x^2+1)*b*e)/c^3
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.79

$$\int x^2(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx$$
$$=\frac{24\,acex+4\,(3\,ac^3d-2\,ac^3e)x^3-12\,be\arctan\left(cx\right)^2+3\,be\log\left(c^2x^2+1\right)^2-2\,(3\,bc^2d-5\,bc^2e)x^2+4\,(6\,b}{}$$

input
```
integrate(x^2*(a+b*arctan(c*x))*(d+e*log(c^2*x^2+1)),x, algorithm="fricas"
)
```

output
```
1/36*(24*a*c*e*x + 4*(3*a*c^3*d - 2*a*c^3*e)*x^3 - 12*b*e*arctan(c*x)^2 +
3*b*e*log(c^2*x^2 + 1)^2 - 2*(3*b*c^2*d - 5*b*c^2*e)*x^2 + 4*(6*b*c*e*x +
(3*b*c^3*d - 2*b*c^3*e)*x^3 - 6*a*e)*arctan(c*x) + 2*(6*b*c^3*e*x^3*arctan
(c*x) + 6*a*c^3*e*x^3 - 3*b*c^2*e*x^2 + 3*b*d - 11*b*e)*log(c^2*x^2 + 1))/
c^3
```

## Sympy [A] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.21

$$\int x^2(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx$$
$$=\begin{cases}\frac{adx^3}{3}+\frac{aex^3\log\left(c^2x^2+1\right)}{3}-\frac{2aex^3}{9}+\frac{2aex}{3c^2}-\frac{2ae\,\mathrm{atan}\,(cx)}{3c^3}+\frac{bdx^3\,\mathrm{atan}\,(cx)}{3}+\frac{bex^3\log\left(c^2x^2+1\right)\mathrm{atan}\,(cx)}{3}-\frac{2bex^3\,\mathrm{atan}\,(cx)}{9}-\\\frac{adx^3}{3}\end{cases}$$

input
```
integrate(x**2*(a+b*atan(c*x))*(d+e*ln(c**2*x**2+1)),x)
```

output
```
Piecewise((a*d*x**3/3 + a*e*x**3*log(c**2*x**2 + 1)/3 - 2*a*e*x**3/9 + 2*a
*e*x/(3*c**2) - 2*a*e*atan(c*x)/(3*c**3) + b*d*x**3*atan(c*x)/3 + b*e*x**3
*log(c**2*x**2 + 1)*atan(c*x)/3 - 2*b*e*x**3*atan(c*x)/9 - b*d*x**2/(6*c)
- b*e*x**2*log(c**2*x**2 + 1)/(6*c) + 5*b*e*x**2/(18*c) + 2*b*e*x*atan(c*x
)/(3*c**2) + b*d*log(c**2*x**2 + 1)/(6*c**3) + b*e*log(c**2*x**2 + 1)**2/(
12*c**3) - 11*b*e*log(c**2*x**2 + 1)/(18*c**3) - b*e*atan(c*x)**2/(3*c**3)
, Ne(c, 0)), (a*d*x**3/3, True))
```

## Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00

$$\int x^2(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx$$
$$=\frac{1}{3}adx^3+\frac{1}{9}\left(3x^3\log\left(c^2x^2+1\right)-2c^2\left(\frac{c^2x^3-3x}{c^4}+\frac{3\arctan(cx)}{c^5}\right)\right)be\arctan(cx)$$
$$+\frac{1}{6}\left(2x^3\arctan(cx)-c\left(\frac{x^2}{c^2}-\frac{\log\left(c^2x^2+1\right)}{c^4}\right)\right)bd$$
$$+\frac{1}{9}\left(3x^3\log\left(c^2x^2+1\right)-2c^2\left(\frac{c^2x^3-3x}{c^4}+\frac{3\arctan(cx)}{c^5}\right)\right)ae$$
$$+\frac{\left(10c^2x^2+12\arctan(cx)^2-2\left(3c^2x^2+11\right)\log\left(c^2x^2+1\right)+3\log\left(c^2x^2+1\right)^2\right)be}{36c^3}$$

input
```
integrate(x^2*(a+b*arctan(c*x))*(d+e*log(c^2*x^2+1)),x, algorithm="maxima"
)
```

output
```
1/3*a*d*x^3 + 1/9*(3*x^3*log(c^2*x^2 + 1) - 2*c^2*((c^2*x^3 - 3*x)/c^4 + 3
*arctan(c*x)/c^5))*b*e*arctan(c*x) + 1/6*(2*x^3*arctan(c*x) - c*(x^2/c^2 -
 log(c^2*x^2 + 1)/c^4))*b*d + 1/9*(3*x^3*log(c^2*x^2 + 1) - 2*c^2*((c^2*x^
3 - 3*x)/c^4 + 3*arctan(c*x)/c^5))*a*e + 1/36*(10*c^2*x^2 + 12*arctan(c*x)
^2 - 2*(3*c^2*x^2 + 11)*log(c^2*x^2 + 1) + 3*log(c^2*x^2 + 1)^2)*b*e/c^3
```

## Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.69

$$\int x^2(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx$$
$$=\frac{6\,\pi bc^3ex^3\log\left(c^2x^2+1\right)\operatorname{sgn}(c)\operatorname{sgn}(x)+6\,\pi bc^3dx^3\operatorname{sgn}(c)\operatorname{sgn}(x)-4\,\pi bc^3ex^3\operatorname{sgn}(c)\operatorname{sgn}(x)-12\,bc^3ex^3\operatorname{arc}}{}$$

input
```
integrate(x^2*(a+b*arctan(c*x))*(d+e*log(c^2*x^2+1)),x, algorithm="giac")
```

output
```
1/36*(6*pi*b*c^3*e*x^3*log(c^2*x^2 + 1)*sgn(c)*sgn(x) + 6*pi*b*c^3*d*x^3*s
gn(c)*sgn(x) - 4*pi*b*c^3*e*x^3*sgn(c)*sgn(x) - 12*b*c^3*e*x^3*arctan(1/(c
*x))*log(c^2*x^2 + 1) - 12*b*c^3*d*x^3*arctan(1/(c*x)) + 8*b*c^3*e*x^3*arc
tan(1/(c*x)) + 12*a*c^3*e*x^3*log(c^2*x^2 + 1) + 12*a*c^3*d*x^3 - 8*a*c^3*
e*x^3 - 6*b*c^2*e*x^2*log(c^2*x^2 + 1) + 12*pi*b*c*e*x*sgn(c)*sgn(x) - 6*b
*c^2*d*x^2 + 10*b*c^2*e*x^2 + 6*pi^2*b*e*sgn(c)*sgn(x) + 12*pi*b*e*arctan(
1/(c*x))*sgn(c)*sgn(x) - 24*b*c*e*x*arctan(1/(c*x)) - 6*pi^2*b*e + 24*a*c*
e*x - 12*pi*b*e*arctan(c*x) - 12*pi*b*e*arctan(1/(c*x)) - 12*b*e*arctan(1/
(c*x))^2 + 3*b*e*log(c^2*x^2 + 1)^2 - 24*a*e*arctan(c*x) + 6*b*d*log(c^2*x
^2 + 1) - 22*b*e*log(c^2*x^2 + 1))/c^3
```

## Mupad [B] (verification not implemented)

Time = 3.22 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00

$$\int x^2(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx$$
$$=\frac{a\,d\,x^3}{3}-\frac{2\,a\,e\,x^3}{9}+\frac{b\,e\ln\left(c^2\,x^2+1\right)^2}{12\,c^3}+\frac{2\,a\,e\,x}{3\,c^2}-\frac{2\,a\,e\,\mathrm{atan}(c\,x)}{3\,c^3}$$
$$+\frac{b\,d\,x^3\,\mathrm{atan}(c\,x)}{3}-\frac{2\,b\,e\,x^3\,\mathrm{atan}(c\,x)}{9}+\frac{b\,d\,\ln\left(c^2\,x^2+1\right)}{6\,c^3}-\frac{11\,b\,e\,\ln\left(c^2\,x^2+1\right)}{18\,c^3}$$
$$-\frac{b\,d\,x^2}{6\,c}+\frac{5\,b\,e\,x^2}{18\,c}+\frac{a\,e\,x^3\,\ln\left(c^2\,x^2+1\right)}{3}-\frac{b\,e\,\mathrm{atan}(c\,x)^2}{3\,c^3}$$
$$+\frac{b\,e\,x^3\,\mathrm{atan}(c\,x)\,\ln\left(c^2\,x^2+1\right)}{3}-\frac{b\,e\,x^2\,\ln\left(c^2\,x^2+1\right)}{6\,c}+\frac{2\,b\,e\,x\,\mathrm{atan}(c\,x)}{3\,c^2}$$

input
```
int(x^2*(a + b*atan(c*x))*(d + e*log(c^2*x^2 + 1)),x)
```

output
```
(a*d*x^3)/3 - (2*a*e*x^3)/9 + (b*e*log(c^2*x^2 + 1)^2)/(12*c^3) + (2*a*e*x
)/(3*c^2) - (2*a*e*atan(c*x))/(3*c^3) + (b*d*x^3*atan(c*x))/3 - (2*b*e*x^3
*atan(c*x))/9 + (b*d*log(c^2*x^2 + 1))/(6*c^3) - (11*b*e*log(c^2*x^2 + 1))
/(18*c^3) - (b*d*x^2)/(6*c) + (5*b*e*x^2)/(18*c) + (a*e*x^3*log(c^2*x^2 +
1))/3 - (b*e*atan(c*x)^2)/(3*c^3) + (b*e*x^3*atan(c*x)*log(c^2*x^2 + 1))/3
 - (b*e*x^2*log(c^2*x^2 + 1))/(6*c) + (2*b*e*x*atan(c*x))/(3*c^2)
```

## Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.01

$$\int x^2(a + b\arctan(cx)) \left(d + e\log\left(1 + c^2x^2\right)\right) dx$$

$$= \frac{-12atan(cx)^2 be + 12atan(cx)\log(c^2x^2+1)\, b\, c^3 e\, x^3 - 24atan(cx)\, ae + 12atan(cx)\, b\, c^3 d\, x^3 - 8atan(cx)}{}$$

input
```
int(x^2*(a+b*atan(c*x))*(d+e*log(c^2*x^2+1)),x)
```

output
```
( - 12*atan(c*x)**2*b*e + 12*atan(c*x)*log(c**2*x**2 + 1)*b*c**3*e*x**3 -
24*atan(c*x)*a*e + 12*atan(c*x)*b*c**3*d*x**3 - 8*atan(c*x)*b*c**3*e*x**3
+ 24*atan(c*x)*b*c*e*x + 3*log(c**2*x**2 + 1)**2*b*e + 12*log(c**2*x**2 +
1)*a*c**3*e*x**3 - 6*log(c**2*x**2 + 1)*b*c**2*e*x**2 + 6*log(c**2*x**2 +
1)*b*d - 22*log(c**2*x**2 + 1)*b*e + 12*a*c**3*d*x**3 - 8*a*c**3*e*x**3 +
24*a*c*e*x - 6*b*c**2*d*x**2 + 10*b*c**2*e*x**2)/(36*c**3)
```

# 3.1287 $\int x(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 137

$$\begin{aligned}&\int x(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx\\&=-\frac{b(d-e)x}{2c}+\frac{bex}{c}+\frac{b(d-e)\arctan(cx)}{2c^2}-\frac{be\arctan(cx)}{c^2}\\&\quad+\frac{1}{2}dx^2(a+b\arctan(cx))-\frac{1}{2}ex^2(a+b\arctan(cx))\\&\quad-\frac{bex\log\left(1+c^2x^2\right)}{2c}+\frac{e\left(1+c^2x^2\right)(a+b\arctan(cx))\log\left(1+c^2x^2\right)}{2c^2}\end{aligned}$$

output
```
-1/2*b*(d-e)*x/c+b*e*x/c+1/2*b*(d-e)*arctan(c*x)/c^2-b*e*arctan(c*x)/c^2+1
/2*d*x^2*(a+b*arctan(c*x))-1/2*e*x^2*(a+b*arctan(c*x))-1/2*b*e*x*ln(c^2*x^
2+1)/c+1/2*e*(c^2*x^2+1)*(a+b*arctan(c*x))*ln(c^2*x^2+1)/c^2
```

## Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.77

$$\int x(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx$$
$$=\frac{cx(-b(d-3e)+ac(d-e)x)+e(a-bcx+ac^2x^2)\log\left(1+c^2x^2\right)+b\arctan(cx)\left(d+c^2dx^2-e(3+c^2x\right.}{2c^2}$$

input
```
Integrate[x*(a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]),x]
```

output
```
(c*x*(-(b*(d - 3*e)) + a*c*(d - e)*x) + e*(a - b*c*x + a*c^2*x^2)*Log[1 +
c^2*x^2] + b*ArcTan[c*x]*(d + c^2*d*x^2 - e*(3 + c^2*x^2) + (e + c^2*e*x^2
)*Log[1 + c^2*x^2]))/(2*c^2)
```

## Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5554, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)\,dx$$

$$\Big\downarrow \text{5554}$$

$$-bc\int\left(\frac{(d-e)x^2}{2\left(c^2x^2+1\right)}+\frac{e\log\left(c^2x^2+1\right)}{2c^2}\right)dx+\frac{e\left(c^2x^2+1\right)\log\left(c^2x^2+1\right)(a+b\arctan(cx))}{2c^2}+\frac{1}{2}dx^2(a+b\arctan(cx))-\frac{1}{2}ex^2(a+b\arctan(cx))$$

$$\Big\downarrow \text{2009}$$

$$\frac{e\left(c^2x^2+1\right)\log\left(c^2x^2+1\right)(a+b\arctan(cx))}{2c^2}+\frac{1}{2}dx^2(a+b\arctan(cx))-\frac{1}{2}ex^2(a+b\arctan(cx))-bc\left(-\frac{(d-e)\arctan(cx)}{2c^3}+\frac{e\arctan(cx)}{c^3}+\frac{x(d-e)}{2c^2}+\frac{ex\log\left(c^2x^2+1\right)}{2c^2}-\frac{ex}{c^2}\right)$$

input
```
Int[x*(a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]),x]
```

output
```
(d*x^2*(a + b*ArcTan[c*x]))/2 - (e*x^2*(a + b*ArcTan[c*x]))/2 + (e*(1 + c^
2*x^2)*(a + b*ArcTan[c*x])*Log[1 + c^2*x^2])/(2*c^2) - b*c*(((d - e)*x)/(2
*c^2) - (e*x)/c^2 - ((d - e)*ArcTan[c*x])/(2*c^3) + (e*ArcTan[c*x])/c^3 +
(e*x*Log[1 + c^2*x^2])/(2*c^2))
```

**Defintions of rubi rules used**

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5554
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*Log[f + g*x^2])
, x]}, Simp[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[ExpandIntegrand[u/
(1 + c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[(m +
1)/2, 0]
```

## Maple [A] (verified)

Time = 2.06 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.23

| method | result |
|---|---|
| parallelrisch | $\frac{\arctan(cx)\ln(c^2x^2+1)b\,c^2e\,x^2+\arctan(cx)b\,c^2d\,x^2-\arctan(cx)b\,c^2e\,x^2+\ln(c^2x^2+1)a\,c^2e\,x^2+a\,c^2d\,x^2-a\,c^2e\,x^2-\ln(c^2x^2+1}{2c^2}$ |
| orering | $\frac{(c^6x^6+9c^2x^2+18)(a+b\arctan(cx))(d+e\ln(c^2x^2+1))}{c^4x^2(c^2x^2-3)}-\frac{(c^6x^6+2c^4x^4+21c^2x^2+36)\left((a+b\arctan(cx))(d+e\ln(c^2x^2+1))\right.}{2c^4x^2(c^2x^2-3)}$ |
| default | Expression too large to display |
| parts | Expression too large to display |
| risch | Expression too large to display |

input
```
int(x*(a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1)),x,method=_RETURNVERBOSE)
```

output
```
1/2*(arctan(c*x)*ln(c^2*x^2+1)*b*c^2*e*x^2+arctan(c*x)*b*c^2*d*x^2-arctan(
c*x)*b*c^2*e*x^2+ln(c^2*x^2+1)*a*c^2*e*x^2+a*c^2*d*x^2-a*c^2*e*x^2-ln(c^2*
x^2+1)*b*c*e*x-b*c*d*x+3*b*c*e*x+arctan(c*x)*ln(c^2*x^2+1)*b*e+arctan(c*x)
*b*d-3*b*arctan(c*x)*e+ln(c^2*x^2+1)*a*e)/c^2
```

## Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.85

$$\int x(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx$$
$$=\frac{(ac^2d-ac^2e)x^2-(bcd-3\,bce)x+((bc^2d-bc^2e)x^2+bd-3\,be)\arctan{(cx)}+(ac^2ex^2-bcex+ae+(b}{2\,c^2}$$

input
```
integrate(x*(a+b*arctan(c*x))*(d+e*log(c^2*x^2+1)),x, algorithm="fricas")
```

output
```
1/2*((a*c^2*d - a*c^2*e)*x^2 - (b*c*d - 3*b*c*e)*x + ((b*c^2*d - b*c^2*e)*
x^2 + b*d - 3*b*e)*arctan(c*x) + (a*c^2*e*x^2 - b*c*e*x + a*e + (b*c^2*e*x
^2 + b*e)*arctan(c*x))*log(c^2*x^2 + 1))/c^2
```

## Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.47

$$\int x(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx$$
$$=\begin{cases}\frac{adx^2}{2}+\frac{aex^2\log{(c^2x^2+1)}}{2}-\frac{aex^2}{2}+\frac{ae\log{(c^2x^2+1)}}{2c^2}+\frac{bdx^2\operatorname{atan}{(cx)}}{2}+\frac{bex^2\log{(c^2x^2+1)}\operatorname{atan}{(cx)}}{2}-\frac{bex^2\operatorname{atan}{(cx)}}{2}-\frac{bdx}{2c}-\\ \frac{adx^2}{2}\end{cases}$$

input
```
integrate(x*(a+b*atan(c*x))*(d+e*ln(c**2*x**2+1)),x)
```

output
```
Piecewise((a*d*x**2/2 + a*e*x**2*log(c**2*x**2 + 1)/2 - a*e*x**2/2 + a*e*l
og(c**2*x**2 + 1)/(2*c**2) + b*d*x**2*atan(c*x)/2 + b*e*x**2*log(c**2*x**2
 + 1)*atan(c*x)/2 - b*e*x**2*atan(c*x)/2 - b*d*x/(2*c) - b*e*x*log(c**2*x*
*2 + 1)/(2*c) + 3*b*e*x/(2*c) + b*d*atan(c*x)/(2*c**2) + b*e*log(c**2*x**2
 + 1)*atan(c*x)/(2*c**2) - 3*b*e*atan(c*x)/(2*c**2), Ne(c, 0)), (a*d*x**2/
2, True))
```

## Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.09

$$\int x(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx = \frac{1}{2}adx^2+\frac{1}{2}\left(x^2\arctan(cx)-c\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)\right)bd - \frac{\left(x\log\left(c^2x^2+1\right)-3x+\frac{2\arctan(cx)}{c}\right)be}{2c} - \frac{\left(c^2x^2-\left(c^2x^2+1\right)\log\left(c^2x^2+1\right)+1\right)be\arctan(cx)}{2c^2} - \frac{\left(c^2x^2-\left(c^2x^2+1\right)\log\left(c^2x^2+1\right)+1\right)ae}{2c^2}$$

input
```
integrate(x*(a+b*arctan(c*x))*(d+e*log(c^2*x^2+1)),x, algorithm="maxima")
```

output
```
1/2*a*d*x^2 + 1/2*(x^2*arctan(c*x) - c*(x/c^2 - arctan(c*x)/c^3))*b*d - 1/
2*(x*log(c^2*x^2 + 1) - 3*x + 2*arctan(c*x)/c)*b*e/c - 1/2*(c^2*x^2 - (c^2
*x^2 + 1)*log(c^2*x^2 + 1) + 1)*b*e*arctan(c*x)/c^2 - 1/2*(c^2*x^2 - (c^2*
x^2 + 1)*log(c^2*x^2 + 1) + 1)*a*e/c^2
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 261 vs. $2(125) = 250$.

Time = 0.27 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.91

$$\int x(a + b\arctan(cx))\left(d + e\log\left(1 + c^2x^2\right)\right)\,dx$$
$$= \frac{\pi bc^2ex^2\log\left(c^2x^2+1\right)\mathrm{sgn}(c)\,\mathrm{sgn}(x) + \pi bc^2dx^2\mathrm{sgn}(c)\,\mathrm{sgn}(x) - \pi bc^2ex^2\mathrm{sgn}(c)\,\mathrm{sgn}(x) - 2\,bc^2ex^2\arctan\left(\right.}{}$$

input
```
integrate(x*(a+b*arctan(c*x))*(d+e*log(c^2*x^2+1)),x, algorithm="giac")
```

output
```
1/4*(pi*b*c^2*e*x^2*log(c^2*x^2 + 1)*sgn(c)*sgn(x) + pi*b*c^2*d*x^2*sgn(c)
*sgn(x) - pi*b*c^2*e*x^2*sgn(c)*sgn(x) - 2*b*c^2*e*x^2*arctan(1/(c*x))*log
(c^2*x^2 + 1) - 2*b*c^2*d*x^2*arctan(1/(c*x)) + 2*b*c^2*e*x^2*arctan(1/(c*
x)) + 2*a*c^2*e*x^2*log(c^2*x^2 + 1) + 2*a*c^2*d*x^2 - 2*a*c^2*e*x^2 + pi*
b*e*log(c^2*x^2 + 1)*sgn(c)*sgn(x) - 2*b*c*e*x*log(c^2*x^2 + 1) - 2*b*c*d*
x + 6*b*c*e*x - 2*b*e*arctan(1/(c*x))*log(c^2*x^2 + 1) + 2*b*d*arctan(c*x)
 - 6*b*e*arctan(c*x) + 2*a*e*log(c^2*x^2 + 1))/c^2
```

## Mupad [B] (verification not implemented)

Time = 1.78 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.66

$$\int x(a + b\arctan(cx))\left(d + e\log\left(1 + c^2x^2\right)\right)\,dx$$
$$= \frac{a\,d\,x^2}{2} - \frac{a\,e\,x^2}{2} - \frac{b\,d\,x}{2\,c} + \frac{3\,b\,e\,x}{2\,c} + \frac{b\,d\,x^2\,\mathrm{atan}(c\,x)}{2} - \frac{b\,e\,x^2\,\mathrm{atan}(c\,x)}{2} + \frac{a\,e\,\ln\left(c^2\,x^2+1\right)}{2\,c^2} + \frac{b\,d\,\mathrm{atan}\left(\frac{b\,c\,d\,x}{b\,d-3\,b\,e} - \frac{3\,b\,c\,e\,x}{b\,d-3\,b\,e}\right)}{2\,c^2} - \frac{3\,b\,e\,\mathrm{atan}\left(\frac{b\,c\,d\,x}{b\,d-3\,b\,e} - \frac{3\,b\,c\,e\,x}{b\,d-3\,b\,e}\right)}{2\,c^2} + \frac{a\,e\,x^2\,\ln\left(c^2\,x^2+1\right)}{2} - \frac{b\,e\,x\,\ln\left(c^2\,x^2+1\right)}{2\,c} + \frac{b\,e\,\mathrm{atan}(c\,x)\,\ln\left(c^2\,x^2+1\right)}{2\,c^2} + \frac{b\,e\,x^2\,\mathrm{atan}(c\,x)\,\ln\left(c^2\,x^2+1\right)}{2}$$

input
```
int(x*(a + b*atan(c*x))*(d + e*log(c^2*x^2 + 1)),x)
```

output
```
(a*d*x^2)/2 - (a*e*x^2)/2 - (b*d*x)/(2*c) + (3*b*e*x)/(2*c) + (b*d*x^2*ata
n(c*x))/2 - (b*e*x^2*atan(c*x))/2 + (a*e*log(c^2*x^2 + 1))/(2*c^2) + (b*d*
atan((b*c*d*x)/(b*d - 3*b*e) - (3*b*c*e*x)/(b*d - 3*b*e)))/(2*c^2) - (3*b*
e*atan((b*c*d*x)/(b*d - 3*b*e) - (3*b*c*e*x)/(b*d - 3*b*e)))/(2*c^2) + (a*
e*x^2*log(c^2*x^2 + 1))/2 - (b*e*x*log(c^2*x^2 + 1))/(2*c) + (b*e*atan(c*x
)*log(c^2*x^2 + 1))/(2*c^2) + (b*e*x^2*atan(c*x)*log(c^2*x^2 + 1))/2
```

## Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.22

$$\int x(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx$$
$$= \frac{atan(cx)\log(c^2x^2+1)\,b\,c^2e\,x^2+atan(cx)\log(c^2x^2+1)\,be+atan(cx)\,b\,c^2d\,x^2-atan(cx)\,b\,c^2e\,x^2+atan}{}$$

input
```
int(x*(a+b*atan(c*x))*(d+e*log(c^2*x^2+1)),x)
```

output
```
(atan(c*x)*log(c**2*x**2 + 1)*b*c**2*e*x**2 + atan(c*x)*log(c**2*x**2 + 1)
*b*e + atan(c*x)*b*c**2*d*x**2 - atan(c*x)*b*c**2*e*x**2 + atan(c*x)*b*d -
 3*atan(c*x)*b*e + log(c**2*x**2 + 1)*a*c**2*e*x**2 + log(c**2*x**2 + 1)*a
*e - log(c**2*x**2 + 1)*b*c*e*x + a*c**2*d*x**2 - a*c**2*e*x**2 - b*c*d*x
+ 3*b*c*e*x)/(2*c**2)
```

# 3.1288 $\int (a+b \arctan (c x)) \left(d+e \log \left(1+c^2 x^2\right)\right) \, dx$



## Optimal result

Integrand size = 23, antiderivative size = 100

$$\begin{aligned}\int (a+b \arctan (c x)) \left(d+e \log \left(1+c^2 x^2\right)\right) \, dx &= -2 a e x-2 b e x \arctan (c x)+\frac{e (a+b \arctan (c x))^2}{b c}+\frac{b e \log \left(1+c^2 x^2\right)}{c} \\ &\quad +x (a+b \arctan (c x)) \left(d+e \log \left(1+c^2 x^2\right)\right)-\frac{b \left(d+e \log \left(1+c^2 x^2\right)\right)^2}{4 c e}\end{aligned}$$

output

```
-2*a*e*x-2*b*e*x*arctan(c*x)+e*(a+b*arctan(c*x))^2/b/c+b*e*ln(c^2*x^2+1)/c
+x*(a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1))-1/4*b*(d+e*ln(c^2*x^2+1))^2/c/e
```

## Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.38

$$\begin{aligned}\int (a+b \arctan (c x)) \left(d+e \log \left(1+c^2 x^2\right)\right) \, dx &= a d x-2 a e x+\frac{2 a e \arctan (c x)}{c}+b d x \arctan (c x)-2 b e x \arctan (c x) \\ &\quad +\frac{b e \arctan (c x)^2}{c}-\frac{b d \log \left(1+c^2 x^2\right)}{2 c}+\frac{b e \log \left(1+c^2 x^2\right)}{c} \\ &\quad +a e x \log \left(1+c^2 x^2\right)+b e x \arctan (c x) \log \left(1+c^2 x^2\right)-\frac{b e \log ^2\left(1+c^2 x^2\right)}{4 c}\end{aligned}$$

input
```
Integrate[(a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]),x]
```

output
```
a*d*x - 2*a*e*x + (2*a*e*ArcTan[c*x])/c + b*d*x*ArcTan[c*x] - 2*b*e*x*ArcT
an[c*x] + (b*e*ArcTan[c*x]^2)/c - (b*d*Log[1 + c^2*x^2])/(2*c) + (b*e*Log[
1 + c^2*x^2])/c + a*e*x*Log[1 + c^2*x^2] + b*e*x*ArcTan[c*x]*Log[1 + c^2*x
^2] - (b*e*Log[1 + c^2*x^2]^2)/(4*c)
```

## Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {5544, 2925, 2837, 2738, 5451, 2009, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)dx$$

$$\Big\downarrow \text{5544}$$

$$-2c^2e\int \frac{x^2(a+b\arctan(cx))}{c^2x^2+1}dx-bc\int \frac{x\left(d+e\log\left(c^2x^2+1\right)\right)}{c^2x^2+1}dx+x(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)$$

$$\Big\downarrow \text{2925}$$

$$-2c^2e\int \frac{x^2(a+b\arctan(cx))}{c^2x^2+1}dx-\frac{1}{2}bc\int \frac{d+e\log\left(c^2x^2+1\right)}{c^2x^2+1}dx^2+x(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)$$

$$\Big\downarrow \text{2837}$$

$$-2c^2e\int \frac{x^2(a+b\arctan(cx))}{c^2x^2+1}dx-\frac{b\int \frac{d+e\log\left(c^2x^2+1\right)}{x^2}d\left(c^2x^2+1\right)}{2c}+x(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)$$

$$\Big\downarrow \text{2738}$$

$$-2c^2e\int \frac{x^2(a+b\arctan(cx))}{c^2x^2+1}dx+x(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)-\frac{b\left(e\log\left(c^2x^2+1\right)+d\right)^2}{4ce}$$

$$\Big\downarrow \text{5451}$$

$$-2c^2e\left(\frac{\int (a+b\arctan(cx))dx}{c^2}-\frac{\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx}{c^2}\right)+x(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)-\frac{b\left(e\log\left(c^2x^2+1\right)+d\right)^2}{4ce}$$

$$\Big\downarrow \text{2009}$$

$$-2c^2e\left(\frac{ax+bx\arctan(cx)-\frac{b\log(c^2x^2+1)}{2c}}{c^2}-\frac{\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx}{c^2}\right)+x(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)-\frac{b\left(e\log\left(c^2x^2+1\right)+d\right)^2}{4ce}$$

$$\Big\downarrow \text{5419}$$

$$x(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)-2c^2e\left(\frac{ax+bx\arctan(cx)-\frac{b\log(c^2x^2+1)}{2c}}{c^2}-\frac{(a+b\arctan(cx))^2}{2bc^3}\right)-\frac{b\left(e\log\left(c^2x^2+1\right)+d\right)^2}{4ce}$$

input

```
Int[(a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]),x]
```

output

```
x*(a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]) - (b*(d + e*Log[1 + c^2*x^2
])^2)/(4*c*e) - 2*c^2*e*(-1/2*(a + b*ArcTan[c*x])^2/(b*c^3) + (a*x + b*x*A
rcTan[c*x] - (b*Log[1 + c^2*x^2])/(2*c))/c^2)
```

## Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2738

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

rule 2837
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Simp[1/e   Subst[Int[(f*(x/d))^q*(a + b*Log[c*x
^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] &&
EqQ[e*f - d*g, 0]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5544
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.)), x_Symbol] :> Simp[x*(d + e*Log[f + g*x^2])*(a + b*ArcTan[c*x]), x] +
 (-Simp[b*c   Int[x*((d + e*Log[f + g*x^2])/(1 + c^2*x^2)), x], x] - Simp[2
*e*g   Int[x^2*((a + b*ArcTan[c*x])/(f + g*x^2)), x], x]) /; FreeQ[{a, b, c
, d, e, f, g}, x]
```

## Maple [A] (verified)

Time = 1.79 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.37

| method | result |
|---|---|
| parallelrisch | $\frac{4eb\ln(c^2x^2+1)\arctan(cx)xc+4b\arctan(cx)xcd-8eb\arctan(cx)xc+4ae\ln(c^2x^2+1)xc+4acdx-8xace+4eb\arctan(cx)^2-eb\ln}{4c}$ |
| default | Expression too large to display |
| parts | Expression too large to display |
| risch | Expression too large to display |

input
```
int((a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1)),x,method=_RETURNVERBOSE)
```

output
```
1/4*(4*e*b*ln(c^2*x^2+1)*arctan(c*x)*x*c+4*b*arctan(c*x)*x*c*d-8*e*b*arcta
n(c*x)*x*c+4*a*e*ln(c^2*x^2+1)*x*c+4*a*c*d*x-8*x*a*c*e+4*e*b*arctan(c*x)^2
-e*b*ln(c^2*x^2+1)^2+8*a*e*arctan(c*x)-2*ln(c^2*x^2+1)*b*d+4*ln(c^2*x^2+1)
*b*e)/c
```

## Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.05

$$\int (a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx$$
$$=\frac{4\,be\arctan\left(cx\right)^2-be\log\left(c^2x^2+1\right)^2+4\left(acd-2\,ace\right)x+4\left(2\,ae+\left(bcd-2\,bce\right)x\right)\arctan\left(cx\right)+2\left(2\,bce\right.}{4\,c}$$

input
```
integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1)),x, algorithm="fricas")
```

output
```
1/4*(4*b*e*arctan(c*x)^2 - b*e*log(c^2*x^2 + 1)^2 + 4*(a*c*d - 2*a*c*e)*x
+ 4*(2*a*e + (b*c*d - 2*b*c*e)*x)*arctan(c*x) + 2*(2*b*c*e*x*arctan(c*x) +
 2*a*c*e*x - b*d + 2*b*e)*log(c^2*x^2 + 1))/c
```

## Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.48

$$\int (a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx$$
$$=\begin{cases} adx + aex\log\left(c^2x^2+1\right) - 2aex + \frac{2ae\,\mathrm{atan}\,(cx)}{c} + bdx\,\mathrm{atan}\,(cx) + bex\log\left(c^2x^2+1\right)\mathrm{atan}\,(cx) - 2bex\,\mathrm{at}\\ adx \end{cases}$$

input
```
integrate((a+b*atan(c*x))*(d+e*ln(c**2*x**2+1)),x)
```

output
```
Piecewise((a*d*x + a*e*x*log(c**2*x**2 + 1) - 2*a*e*x + 2*a*e*atan(c*x)/c
+ b*d*x*atan(c*x) + b*e*x*log(c**2*x**2 + 1)*atan(c*x) - 2*b*e*x*atan(c*x)
 - b*d*log(c**2*x**2 + 1)/(2*c) - b*e*log(c**2*x**2 + 1)**2/(4*c) + b*e*lo
g(c**2*x**2 + 1)/c + b*e*atan(c*x)**2/c, Ne(c, 0)), (a*d*x, True))
```

## Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.53

$$\begin{aligned}\int (a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx\\ = -\left(2\,c^2\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)-x\log\left(c^2x^2+1\right)\right)be\arctan(cx)\\ -\left(2\,c^2\left(\frac{x}{c^2}-\frac{\arctan(cx)}{c^3}\right)-x\log\left(c^2x^2+1\right)\right)ae\\ +adx+\frac{(2\,cx\arctan(cx)-\log(c^2x^2+1))bd}{2\,c}\\ -\frac{\left(4\,\arctan(cx)^2+\log\left(c^2x^2+1\right)^2-4\,\log\left(c^2x^2+1\right)\right)be}{4\,c}\end{aligned}$$

input
```
integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1)),x, algorithm="maxima")
```

output
```
-(2*c^2*(x/c^2 - arctan(c*x)/c^3) - x*log(c^2*x^2 + 1))*b*e*arctan(c*x) -
(2*c^2*(x/c^2 - arctan(c*x)/c^3) - x*log(c^2*x^2 + 1))*a*e + a*d*x + 1/2*(
2*c*x*arctan(c*x) - log(c^2*x^2 + 1))*b*d/c - 1/4*(4*arctan(c*x)^2 + log(c
^2*x^2 + 1)^2 - 4*log(c^2*x^2 + 1))*b*e/c
```

## Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 261 vs. 2(98) = 196.

Time = 0.20 (sec) , antiderivative size = 261, normalized size of antiderivative = 2.61

$$\int (a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx$$
$$= \frac{2\,\pi bcex\log\left(c^2x^2+1\right)\operatorname{sgn}(c)\operatorname{sgn}(x)+2\,\pi bcdx\operatorname{sgn}(c)\operatorname{sgn}(x)-4\,\pi bcex\operatorname{sgn}(c)\operatorname{sgn}(x)-4\,bcex\arctan\left(\frac{1}{cx}\right)}{}$$

input
```
integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1)),x, algorithm="giac")
```

output
```
1/4*(2*pi*b*c*e*x*log(c^2*x^2 + 1)*sgn(c)*sgn(x) + 2*pi*b*c*d*x*sgn(c)*sgn
(x) - 4*pi*b*c*e*x*sgn(c)*sgn(x) - 4*b*c*e*x*arctan(1/(c*x))*log(c^2*x^2 +
 1) - 6*pi^2*b*e*sgn(c)*sgn(x) - 4*pi*b*e*arctan(1/(c*x))*sgn(c)*sgn(x) -
4*b*c*d*x*arctan(1/(c*x)) + 8*b*c*e*x*arctan(1/(c*x)) + 4*a*c*e*x*log(c^2*
x^2 + 1) - 8*pi*a*e*sgn(c)*sgn(x) + 2*pi^2*b*e + 4*a*c*d*x - 8*a*c*e*x + 4
*pi*b*e*arctan(c*x) + 4*pi*b*e*arctan(1/(c*x)) + 4*b*e*arctan(1/(c*x))^2 -
 b*e*log(c^2*x^2 + 1)^2 + 8*a*e*arctan(c*x) - 2*b*d*log(c^2*x^2 + 1) + 4*b
*e*log(c^2*x^2 + 1))/c
```

## Mupad [B] (verification not implemented)

Time = 1.12 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.34

$$\int (a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx$$
$$= a\,d\,x-2\,a\,e\,x-\frac{b\,e\ln\left(c^2\,x^2+1\right)^2}{4\,c}+b\,d\,x\,\mathrm{atan}(c\,x)-2\,b\,e\,x\,\mathrm{atan}(c\,x)$$
$$+a\,e\,x\,\ln\left(c^2\,x^2+1\right)+\frac{2\,a\,e\,\mathrm{atan}(c\,x)}{c}-\frac{b\,d\,\ln\left(c^2\,x^2+1\right)}{2\,c}$$
$$+\frac{b\,e\,\ln\left(c^2\,x^2+1\right)}{c}+\frac{b\,e\,\mathrm{atan}(c\,x)^2}{c}+b\,e\,x\,\mathrm{atan}(c\,x)\,\ln\left(c^2\,x^2+1\right)$$

input
```
int((a + b*atan(c*x))*(d + e*log(c^2*x^2 + 1)),x)
```

output
```
a*d*x - 2*a*e*x - (b*e*log(c^2*x^2 + 1)^2)/(4*c) + b*d*x*atan(c*x) - 2*b*e
*x*atan(c*x) + a*e*x*log(c^2*x^2 + 1) + (2*a*e*atan(c*x))/c - (b*d*log(c^2
*x^2 + 1))/(2*c) + (b*e*log(c^2*x^2 + 1))/c + (b*e*atan(c*x)^2)/c + b*e*x*
atan(c*x)*log(c^2*x^2 + 1)
```

## Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.36

$$\int (a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)\,dx$$
$$=\frac{4atan(cx)^2be+4atan(cx)\log(c^2x^2+1)bcex+8atan(cx)ae+4atan(cx)bcdx-8atan(cx)bcex-\log(c}{4c}$$

input
```
int((a+b*atan(c*x))*(d+e*log(c^2*x^2+1)),x)
```

output
```
(4*atan(c*x)**2*b*e + 4*atan(c*x)*log(c**2*x**2 + 1)*b*c*e*x + 8*atan(c*x)
*a*e + 4*atan(c*x)*b*c*d*x - 8*atan(c*x)*b*c*e*x - log(c**2*x**2 + 1)**2*b
*e + 4*log(c**2*x**2 + 1)*a*c*e*x - 2*log(c**2*x**2 + 1)*b*d + 4*log(c**2*
x**2 + 1)*b*e + 4*a*c*d*x - 8*a*c*e*x)/(4*c)
```

# 3.1289 $\int \frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 282

$$\begin{aligned}\int \frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x}\,dx &= ad\log(x)+\frac{1}{2}ibe\log(icx)\log^2(1-icx)\\ &\quad-\frac{1}{2}ibe\log(-icx)\log^2(1+icx)+\frac{1}{2}ibd\operatorname{PolyLog}(2,-icx)\\ &\quad-\frac{1}{2}ibe\left(\log(1-icx)+\log(1+icx)-\log\left(1+c^2x^2\right)\right)\operatorname{PolyLog}(2,-icx)\\ &\quad-\frac{1}{2}ibd\operatorname{PolyLog}(2,icx)\\ &\quad+\frac{1}{2}ibe\left(\log(1-icx)+\log(1+icx)-\log\left(1+c^2x^2\right)\right)\operatorname{PolyLog}(2,icx)\\ &\quad-\frac{1}{2}ae\operatorname{PolyLog}\left(2,-c^2x^2\right)+ibe\log(1-icx)\operatorname{PolyLog}(2,1-icx)\\ &\quad-ibe\log(1+icx)\operatorname{PolyLog}(2,1+icx)\\ &\quad-ibe\operatorname{PolyLog}(3,1-icx)+ibe\operatorname{PolyLog}(3,1+icx)\end{aligned}$$

output
```
a*d*ln(x)+1/2*I*b*e*ln(I*c*x)*ln(1-I*c*x)^2-1/2*I*b*e*ln(-I*c*x)*ln(1+I*c*
x)^2+1/2*I*b*d*polylog(2,-I*c*x)-1/2*I*b*e*(ln(1-I*c*x)+ln(1+I*c*x)-ln(c^2
*x^2+1))*polylog(2,-I*c*x)-1/2*I*b*d*polylog(2,I*c*x)+1/2*I*b*e*(ln(1-I*c*
x)+ln(1+I*c*x)-ln(c^2*x^2+1))*polylog(2,I*c*x)-1/2*a*e*polylog(2,-c^2*x^2)
+I*b*e*ln(1-I*c*x)*polylog(2,1-I*c*x)-I*b*e*ln(1+I*c*x)*polylog(2,1+I*c*x)
-I*b*e*polylog(3,1-I*c*x)+I*b*e*polylog(3,1+I*c*x)
```

## Mathematica [F]

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x}\,dx$$
$$=\int \frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x}\,dx$$

input
```
Integrate[((a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]))/x,x]
```

output
```
Integrate[((a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]))/x, x]
```

## Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.85, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {5550, 5355, 2838, 5548, 2838, 5546, 2843, 2881, 2821, 5355, 2838, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{x}\,dx$$
$$\Big\downarrow \text{5550}$$
$$e\int \frac{(a+b\arctan(cx))\log\left(c^2x^2+1\right)}{x}dx+d\int \frac{a+b\arctan(cx)}{x}dx$$
$$\Big\downarrow \text{5355}$$

$$e\int \frac{(a+b\arctan(cx))\log\left(c^2x^2+1\right)}{x}dx+$$
$$d\left(\frac{1}{2}ib\int \frac{\log(1-icx)}{x}dx-\frac{1}{2}ib\int \frac{\log(icx+1)}{x}dx+a\log(x)\right)$$

$\Big\downarrow$ 2838

$$e\int \frac{(a+b\arctan(cx))\log\left(c^2x^2+1\right)}{x}dx+$$
$$d\left(a\log(x)+\frac{1}{2}ib\,\text{PolyLog}(2,-icx)-\frac{1}{2}ib\,\text{PolyLog}(2,icx)\right)$$

$\Big\downarrow$ 5548

$$e\left(a\int \frac{\log\left(c^2x^2+1\right)}{x}dx+b\int \frac{\arctan(cx)\log\left(c^2x^2+1\right)}{x}dx\right)+$$
$$d\left(a\log(x)+\frac{1}{2}ib\,\text{PolyLog}(2,-icx)-\frac{1}{2}ib\,\text{PolyLog}(2,icx)\right)$$

$\Big\downarrow$ 2838

$$e\left(b\int \frac{\arctan(cx)\log\left(c^2x^2+1\right)}{x}dx-\frac{1}{2}a\,\text{PolyLog}\left(2,-c^2x^2\right)\right)+$$
$$d\left(a\log(x)+\frac{1}{2}ib\,\text{PolyLog}(2,-icx)-\frac{1}{2}ib\,\text{PolyLog}(2,icx)\right)$$

$\Big\downarrow$ 5546

$$e\left(-\frac{1}{2}a\,\text{PolyLog}\left(2,-c^2x^2\right)+b\left(-\left(\left(-\log\left(c^2x^2+1\right)+\log(1-icx)+\log(1+icx)\right)\int \frac{\arctan(cx)}{x}dx\right)+\frac{1}{2}i\int\right.\right.$$
$$d\left(a\log(x)+\frac{1}{2}ib\,\text{PolyLog}(2,-icx)-\frac{1}{2}ib\,\text{PolyLog}(2,icx)\right)$$

$\Big\downarrow$ 2843

$$e\left(-\frac{1}{2}a\,\text{PolyLog}\left(2,-c^2x^2\right)+b\left(-\left(\left(-\log\left(c^2x^2+1\right)+\log(1-icx)+\log(1+icx)\right)\int \frac{\arctan(cx)}{x}dx\right)+\frac{1}{2}i\left(2\right.\right.\right.$$
$$d\left(a\log(x)+\frac{1}{2}ib\,\text{PolyLog}(2,-icx)-\frac{1}{2}ib\,\text{PolyLog}(2,icx)\right)$$

$\Big\downarrow$ 2881

$$e\left(-\frac{1}{2}a\,\text{PolyLog}\left(2,-c^2x^2\right)+b\left(-\left(\left(-\log\left(c^2x^2+1\right)+\log(1-icx)+\log(1+icx)\right)\int \frac{\arctan(cx)}{x}dx\right)+\frac{1}{2}i\left(\text{lo}\right.\right.\right.$$
$$d\left(a\log(x)+\frac{1}{2}ib\,\text{PolyLog}(2,-icx)-\frac{1}{2}ib\,\text{PolyLog}(2,icx)\right)$$

$\Big\downarrow$ 2821

$$e\left(-\frac{1}{2}a\,\text{PolyLog}\left(2,-c^2x^2\right)+b\left(-\left(\left(-\log\left(c^2x^2+1\right)+\log(1-icx)+\log(1+icx)\right)\int\frac{\arctan(cx)}{x}dx\right)+\frac{1}{2}i\left(\text{lo}\right.\right.\right.$$
$$d\left(a\log(x)+\frac{1}{2}ib\,\text{PolyLog}(2,-icx)-\frac{1}{2}ib\,\text{PolyLog}(2,icx)\right)$$

$\Big\downarrow$ 5355

$$e\left(-\frac{1}{2}a\,\text{PolyLog}\left(2,-c^2x^2\right)+b\left(-\left(\left(-\log\left(c^2x^2+1\right)+\log(1-icx)+\log(1+icx)\right)\left(\frac{1}{2}i\int\frac{\log(1-icx)}{x}dx-\frac{1}{2}\right.\right.\right.\right.$$
$$d\left(a\log(x)+\frac{1}{2}ib\,\text{PolyLog}(2,-icx)-\frac{1}{2}ib\,\text{PolyLog}(2,icx)\right)$$

$\Big\downarrow$ 2838

$$e\left(-\frac{1}{2}a\,\text{PolyLog}\left(2,-c^2x^2\right)+b\left(\frac{1}{2}i\left(\log(icx)\log^2(1-icx)-2\left(\int\frac{\text{PolyLog}(2,1-icx)}{1-icx}d(1-icx)-\text{PolyLog}(2,\right.\right.\right.\right.$$
$$d\left(a\log(x)+\frac{1}{2}ib\,\text{PolyLog}(2,-icx)-\frac{1}{2}ib\,\text{PolyLog}(2,icx)\right)$$

$\Big\downarrow$ 7143

$$e\left(-\frac{1}{2}a\,\text{PolyLog}\left(2,-c^2x^2\right)+b\left(-\left(\left(\frac{1}{2}i\,\text{PolyLog}(2,-icx)-\frac{1}{2}i\,\text{PolyLog}(2,icx)\right)\left(-\log\left(c^2x^2+1\right)+\log(1-ic\right.\right.\right.\right.$$
$$d\left(a\log(x)+\frac{1}{2}ib\,\text{PolyLog}(2,-icx)-\frac{1}{2}ib\,\text{PolyLog}(2,icx)\right)$$

input
```
Int[((a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]))/x,x]
```

output
```
d*(a*Log[x] + (I/2)*b*PolyLog[2, (-I)*c*x] - (I/2)*b*PolyLog[2, I*c*x]) +
e*(-1/2*(a*PolyLog[2, -(c^2*x^2)]) + b*(-((Log[1 - I*c*x] + Log[1 + I*c*x]
 - Log[1 + c^2*x^2])*((I/2)*PolyLog[2, (-I)*c*x] - (I/2)*PolyLog[2, I*c*x]
)) + (I/2)*(Log[I*c*x]*Log[1 - I*c*x]^2 - 2*(-(Log[1 - I*c*x]*PolyLog[2, 1
 - I*c*x]) + PolyLog[3, 1 - I*c*x])) - (I/2)*(Log[(-I)*c*x]*Log[1 + I*c*x]
^2 - 2*(-(Log[1 + I*c*x]*PolyLog[2, 1 + I*c*x]) + PolyLog[3, 1 + I*c*x])))
)
```

### Defintions of rubi rules used

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2843
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d
+ e*x)^n])^p/g), x] - Simp[b*e*n*(p/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]
*((a + b*Log[c*(d + e*x)^n])^(p - 1)/(d + e*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

rule 2881
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] :> Simp[1/e   Subst[Int[(k*(x/d))^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*
((e*i - d*j)/e + j*(x/e))^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
 f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

rule 5355
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x]
 + (Simp[I*(b/2)   Int[Log[1 - I*c*x]/x, x], x] - Simp[I*(b/2)   Int[Log[1
+ I*c*x]/x, x], x]) /; FreeQ[{a, b, c}, x]
```

rule 5546
```
Int[(ArcTan[(c_.)*(x_)]*Log[(f_.) + (g_.)*(x_)^2])/(x_), x_Symbol] :> Simp[
(Log[f + g*x^2] - Log[1 - I*c*x] - Log[1 + I*c*x])   Int[ArcTan[c*x]/x, x],
 x] + (Simp[I/2   Int[Log[1 - I*c*x]^2/x, x], x] - Simp[I/2   Int[Log[1 + I
*c*x]^2/x, x], x]) /; FreeQ[{c, f, g}, x] && EqQ[g, c^2*f]
```

rule 5548
```
Int[(Log[(f_.) + (g_.)*(x_)^2]*(ArcTan[(c_.)*(x_)]*(b_.) + (a_)))/(x_), x_S
ymbol] :> Simp[a   Int[Log[f + g*x^2]/x, x], x] + Simp[b   Int[Log[f + g*x^
2]*(ArcTan[c*x]/x), x], x] /; FreeQ[{a, b, c, f, g}, x]
```

rule 5550
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*(Log[(f_.) + (g_.)*(x_)^2]*(e_.) +
(d_)))/(x_), x_Symbol] :> Simp[d   Int[(a + b*ArcTan[c*x])/x, x], x] + Simp
[e   Int[Log[f + g*x^2]*((a + b*ArcTan[c*x])/x), x], x] /; FreeQ[{a, b, c,
d, e, f, g}, x]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

## Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 8.94 (sec) , antiderivative size = 5420, normalized size of antiderivative = 19.22

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 5420 |

input
```
int((a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1))/x,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

## Fricas [F]

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x}\,dx = \int \frac{(b\arctan\left(cx\right)+a)(e\log\left(c^2x^2+1\right)+d)}{x}\,dx$$

input
```
integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1))/x,x, algorithm="fricas")
```

output
```
integral((b*d*arctan(c*x) + a*d + (b*e*arctan(c*x) + a*e)*log(c^2*x^2 + 1)
)/x, x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(a + b\arctan(cx))\,(d + e\log(1 + c^2x^2))}{x}\,dx = \text{Timed out}$$

input
```
integrate((a+b*atan(c*x))*(d+e*ln(c**2*x**2+1))/x,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(a + b\arctan(cx))\,(d + e\log(1 + c^2x^2))}{x}\,dx = \int \frac{(b\arctan(cx) + a)(e\log(c^2x^2 + 1) + d)}{x}\,dx$$

input
```
integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1))/x,x, algorithm="maxima")
```

output
```
a*d*log(x) + 1/2*integrate(2*(b*d*arctan(c*x) + (b*e*arctan(c*x) + a*e)*lo
g(c^2*x^2 + 1))/x, x)
```

## Giac [F]

$$\int \frac{(a+b\arctan(cx))\,(d+e\log(1+c^2x^2))}{x}\,dx = \int \frac{(b\arctan(cx)+a)(e\log(c^2x^2+1)+d)}{x}\,dx$$

input
```
integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1))/x,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)*(e*log(c^2*x^2 + 1) + d)/x, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))\,(d+e\log(1+c^2x^2))}{x}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))\,\left(d+e\,\ln\left(c^2\,x^2+1\right)\right)}{x}\,dx$$

input
```
int(((a + b*atan(c*x))*(d + e*log(c^2*x^2 + 1)))/x,x)
```

output
```
int(((a + b*atan(c*x))*(d + e*log(c^2*x^2 + 1)))/x, x)
```

## Reduce [F]

$$\int \frac{(a+b\arctan(cx))\,(d+e\log(1+c^2x^2))}{x}\,dx = \left(\int \frac{atan(cx)}{c^2x^3+x}dx\right)bd + \left(\int \frac{\log(c^2x^2+1)}{c^2x^3+x}dx\right)ae + \left(\int \frac{atan(cx)\log(c^2x^2+1)\,x}{c^2x^2+1}dx\right)b\,c^2e + \left(\int \frac{atan(cx)\log(c^2x^2+1)}{c^2x^3+x}dx\right)be + \left(\int \frac{atan(cx)\,x}{c^2x^2+1}dx\right)b\,c^2d + \frac{\log(c^2x^2+1)^2\,ae}{4} + \log(x)\,ad$$

input
```
int((a+b*atan(c*x))*(d+e*log(c^2*x^2+1))/x,x)
```

output
```
(4*int(atan(c*x)/(c**2*x**3 + x),x)*b*d + 4*int(log(c**2*x**2 + 1)/(c**2*x
**3 + x),x)*a*e + 4*int((atan(c*x)*log(c**2*x**2 + 1)*x)/(c**2*x**2 + 1),x
)*b*c**2*e + 4*int((atan(c*x)*log(c**2*x**2 + 1))/(c**2*x**3 + x),x)*b*e +
 4*int((atan(c*x)*x)/(c**2*x**2 + 1),x)*b*c**2*d + log(c**2*x**2 + 1)**2*a
*e + 4*log(x)*a*d)/4
```

# 3.1290 $\int \frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x^2}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 100

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x^2}\,dx = \frac{ce(a+b\arctan(cx))^2}{b}-\frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x} + \frac{1}{2}bc\left(d+e\log\left(1+c^2x^2\right)\right)\log\left(1-\frac{1}{1+c^2x^2}\right)-\frac{1}{2}bce\,\mathrm{PolyLog}\left(2,\frac{1}{1+c^2x^2}\right)$$

output
```
c*e*(a+b*arctan(c*x))^2/b-(a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1))/x+1/2*b*c*
(d+e*ln(c^2*x^2+1))*ln(1-1/(c^2*x^2+1))-1/2*b*c*e*polylog(2,1/(c^2*x^2+1))
```

## Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.11

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x^2}\,dx$$
$$= \frac{ce(a+b\arctan(cx))^2}{b} - \frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x} + bc\left(-\frac{\left(d+e\log\left(1+c^2x^2\right)\right)\left(d-2e\log\left(-c^2x^2\right)+e\log\left(1+c^2x^2\right)\right)}{4e} + \frac{1}{2}e\,\text{PolyLog}\left(2,1+c^2x^2\right)\right)$$

input
```
Integrate[((a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]))/x^2,x]
```

output
```
(c*e*(a + b*ArcTan[c*x])^2)/b - ((a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^
2]))/x + b*c*(-1/4*((d + e*Log[1 + c^2*x^2])*(d - 2*e*Log[-(c^2*x^2)] + e*
Log[1 + c^2*x^2]))/e + (e*PolyLog[2, 1 + c^2*x^2])/2)
```

## Rubi [A] (warning: unable to verify)

Time = 0.69 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.82, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {5552, 2925, 2858, 25, 27, 2779, 2838, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{x^2}\,dx$$

$$\Big\downarrow \text{5552}$$

$$2c^2e\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx + bc\int \frac{d+e\log\left(c^2x^2+1\right)}{x\left(c^2x^2+1\right)}dx - \frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{x}$$

$$\Big\downarrow \text{2925}$$

$$2c^2e\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx+\frac{1}{2}bc\int \frac{d+e\log\left(c^2x^2+1\right)}{x^2\left(c^2x^2+1\right)}dx^2-\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{x}$$

$$\Big\downarrow \; 2858$$

$$2c^2e\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx+\frac{b\int \frac{d+e\log\left(c^2x^2+1\right)}{x^4}d(c^2x^2+1)}{2c}-\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{x}$$

$$\Big\downarrow \; 25$$

$$2c^2e\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx-\frac{b\int -\frac{d+e\log\left(c^2x^2+1\right)}{x^4}d(c^2x^2+1)}{2c}-\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{x}$$

$$\Big\downarrow \; 27$$

$$2c^2e\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx-\frac{1}{2}bc\int -\frac{d+e\log\left(c^2x^2+1\right)}{c^2x^4}d(c^2x^2+1)-\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{x}$$

$$\Big\downarrow \; 2779$$

$$2c^2e\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx-\frac{1}{2}bc\left(e\int \frac{\log\left(1-\frac{1}{x^2}\right)}{x^2}d(c^2x^2+1)-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(c^2x^2+1\right)+d\right)\right)-\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{x}$$

$$\Big\downarrow \; 2838$$

$$2c^2e\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx-\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{x}-\frac{1}{2}bc\left(e\,\text{PolyLog}\left(2,\frac{1}{x^2}\right)-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(c^2x^2+1\right)+d\right)\right)$$

$$\Big\downarrow \; 5419$$

$$-\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{x}+\frac{ce(a+b\arctan(cx))^2}{b}-\frac{1}{2}bc\left(e\,\text{PolyLog}\left(2,\frac{1}{x^2}\right)-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(c^2x^2+1\right)+d\right)\right)$$

input
```
Int[((a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]))/x^2,x]
```

output
```
(c*e*(a + b*ArcTan[c*x])^2)/b - ((a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^
2]))/x - (b*c*(-(Log[1 - x^(-2)]*(d + e*Log[1 + c^2*x^2])) + e*PolyLog[2,
x^(-2)]))/2
```

**Defintions of rubi rules used**

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5552
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(d + e*Log[f + g*x^2])*((a +
b*ArcTan[c*x])/(m + 1)), x] + (-Simp[b*(c/(m + 1))   Int[x^(m + 1)*((d + e*
Log[f + g*x^2])/(1 + c^2*x^2)), x], x] - Simp[2*e*(g/(m + 1))   Int[x^(m +
2)*((a + b*ArcTan[c*x])/(f + g*x^2)), x], x]) /; FreeQ[{a, b, c, d, e, f, g
}, x] && ILtQ[m/2, 0]
```

## Maple [F]

$$\int \frac{(a+b\arctan(cx))\left(d+e\ln\left(c^2x^2+1\right)\right)}{x^2}dx$$

input
```
int((a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1))/x^2,x)
```

output
```
int((a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1))/x^2,x)
```

## Fricas [F]

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x^2}\,dx = \int \frac{(b\arctan\left(cx\right)+a)(e\log\left(c^2x^2+1\right)+d)}{x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1))/x^2,x, algorithm="fricas"
)
```

output
```
integral((b*d*arctan(c*x) + a*d + (b*e*arctan(c*x) + a*e)*log(c^2*x^2 + 1)
)/x^2, x)
```

## Sympy [F(-2)]

Exception generated.

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*atan(c*x))*(d+e*ln(c**2*x**2+1))/x**2,x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real zoo
```

## Maxima [F]

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x^2}\,dx = \int \frac{(b\arctan\left(cx\right)+a)(e\log\left(c^2x^2+1\right)+d)}{x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1))/x^2,x, algorithm="maxima"
)
```

output
```
-1/2*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b*d + (2*c*arctan
(c*x) - log(c^2*x^2 + 1)/x)*a*e + b*e*integrate(arctan(c*x)*log(c^2*x^2 +
1)/x^2, x) - a*d/x
```

## Giac [F]

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x^2}\,dx = \int \frac{(b\arctan\left(cx\right)+a)(e\log\left(c^2x^2+1\right)+d)}{x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1))/x^2,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)*(e*log(c^2*x^2 + 1) + d)/x^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x^2}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))\,\left(d+e\,\ln\left(c^2\,x^2+1\right)\right)}{x^2}\,dx$$

input
```
int(((a + b*atan(c*x))*(d + e*log(c^2*x^2 + 1)))/x^2,x)
```

output
```
int(((a + b*atan(c*x))*(d + e*log(c^2*x^2 + 1)))/x^2, x)
```

## Reduce [F]

$$\int \frac{(a + b\arctan(cx))\,(d + e\log(1 + c^2x^2))}{x^2}\,dx$$
$$= \frac{2atan(cx)^2\,bcex - 2atan(cx)\log(c^2x^2 + 1)\,be + 4atan(cx)\,acex - 2atan(cx)\,bd + 2\left(\int \frac{\log(c^2x^2+1)}{c^2x^3+x}dx\right) bce\ldots}{2x}$$

input
```
int((a+b*atan(c*x))*(d+e*log(c^2*x^2+1))/x^2,x)
```

output
```
(2*atan(c*x)**2*b*c*e*x - 2*atan(c*x)*log(c**2*x**2 + 1)*b*e + 4*atan(c*x)
*a*c*e*x - 2*atan(c*x)*b*d + 2*int(log(c**2*x**2 + 1)/(c**2*x**3 + x),x)*b
*c*e*x - 2*log(c**2*x**2 + 1)*a*e - log(c**2*x**2 + 1)*b*c*d*x + 2*log(x)*
b*c*d*x - 2*a*d)/(2*x)
```

# 3.1291 $\int \frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x^3}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 154

$$\begin{aligned}\int \frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x^3}\,dx &= bc^2e\arctan(cx)+ac^2e\log(x)-\frac{1}{2}ac^2e\log\left(1+c^2x^2\right)-\frac{bc(d+e\log(1+c^2x^2))}{2x}\\&\quad-\frac{1}{2}bc^2\arctan(cx)\left(d+e\log\left(1+c^2x^2\right)\right)-\frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{2x^2}\\&\quad+\frac{1}{2}ibc^2e\,\text{PolyLog}(2,-icx)-\frac{1}{2}ibc^2e\,\text{PolyLog}(2,icx)\end{aligned}$$

output
```
b*c^2*e*arctan(c*x)+a*c^2*e*ln(x)-1/2*a*c^2*e*ln(c^2*x^2+1)-1/2*b*c*(d+e*l
n(c^2*x^2+1))/x-1/2*b*c^2*arctan(c*x)*(d+e*ln(c^2*x^2+1))-1/2*(a+b*arctan(
c*x))*(d+e*ln(c^2*x^2+1))/x^2+1/2*I*b*c^2*e*polylog(2,-I*c*x)-1/2*I*b*c^2*
e*polylog(2,I*c*x)
```

## Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.23

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x^3}\,dx =$$
$$-\frac{ad+bcdx+bd\arctan(cx)+bc^2dx^2\arctan(cx)-2bc^2ex^2\arctan(cx)-2ac^2ex^2\log(x)+ae\log\left(1+c\right.}{}$$

input
```
Integrate[((a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]))/x^3,x]
```

output
```
-1/2*(a*d + b*c*d*x + b*d*ArcTan[c*x] + b*c^2*d*x^2*ArcTan[c*x] - 2*b*c^2*
e*x^2*ArcTan[c*x] - 2*a*c^2*e*x^2*Log[x] + a*e*Log[1 + c^2*x^2] + b*c*e*x*
Log[1 + c^2*x^2] + a*c^2*e*x^2*Log[1 + c^2*x^2] + b*e*ArcTan[c*x]*Log[1 +
c^2*x^2] + b*c^2*e*x^2*ArcTan[c*x]*Log[1 + c^2*x^2] - I*b*c^2*e*x^2*PolyLo
g[2, (-I)*c*x] + I*b*c^2*e*x^2*PolyLog[2, I*c*x])/x^2
```

## Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.95, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {5556, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{x^3}\,dx$$

$$\Big\downarrow \text{5556}$$

$$-2c^2e\int \left(-\frac{a+bcx}{2x\left(c^2x^2+1\right)}-\frac{b\arctan(cx)}{2x}\right)dx-\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{2x^2}-\frac{1}{2}bc^2\arctan(cx)\left(e\log\left(c^2x^2+1\right)+d\right)-\frac{bc\left(e\log\left(c^2x^2+1\right)+d\right)}{2x}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{2x^2}-2c^2e\left(\frac{1}{4}a\log\left(c^2x^2+1\right)-\frac{1}{2}a\log(x)-\frac{1}{2}b\arctan(cx)-\frac{1}{4}ib\operatorname{PolyLog}(2,-icx)+\frac{1}{4}ib\operatorname{PolyLog}(2,icx)\right)-\frac{1}{2}bc^2\arctan(cx)\left(e\log\left(c^2x^2+1\right)+d\right)-\frac{bc(e\log\left(c^2x^2+1\right)+d)}{2x}$$

input
```
Int[((a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]))/x^3,x]
```

output
```
-1/2*(b*c*(d + e*Log[1 + c^2*x^2]))/x - (b*c^2*ArcTan[c*x]*(d + e*Log[1 +
c^2*x^2]))/2 - ((a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]))/(2*x^2) - 2*
c^2*e*(-1/2*(b*ArcTan[c*x]) - (a*Log[x])/2 + (a*Log[1 + c^2*x^2])/4 - (I/4
)*b*PolyLog[2, (-I)*c*x] + (I/4)*b*PolyLog[2, I*c*x])
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5556
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcTan[c*x]), x
]}, Simp[(d + e*Log[f + g*x^2])   u, x] - Simp[2*e*g   Int[ExpandIntegrand[
x*(u/(f + g*x^2)), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && Intege
rQ[m] && NeQ[m, -1]
```

## Maple [F]

$$\int \frac{(a+b\arctan(cx))\left(d+e\ln\left(c^2x^2+1\right)\right)}{x^3}dx$$

input
```
int((a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1))/x^3,x)
```

output
```
int((a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1))/x^3,x)
```

## Fricas [F]

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x^3}\,dx = \int \frac{(b\arctan\left(cx\right)+a)(e\log\left(c^2x^2+1\right)+d)}{x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1))/x^3,x, algorithm="fricas"
)
```

output
```
integral((b*d*arctan(c*x) + a*d + (b*e*arctan(c*x) + a*e)*log(c^2*x^2 + 1)
)/x^3, x)
```

## Sympy [F]

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x^3}\,dx = \int \frac{(a+b\,\mathrm{atan}\left(cx\right))\left(d+e\log\left(c^2x^2+1\right)\right)}{x^3}\,dx$$

input
```
integrate((a+b*atan(c*x))*(d+e*ln(c**2*x**2+1))/x**3,x)
```

output
```
Integral((a + b*atan(c*x))*(d + e*log(c**2*x**2 + 1))/x**3, x)
```

## Maxima [F]

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x^3}\,dx = \int \frac{(b\arctan\left(cx\right)+a)(e\log\left(c^2x^2+1\right)+d)}{x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1))/x^3,x, algorithm="maxima"
)
```

output
```
-1/2*((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b*d - 1/2*(c^2*(log(c^2*x
^2 + 1) - log(x^2)) + log(c^2*x^2 + 1)/x^2)*a*e + 1/2*(4*c^4*x^2*integrate
(1/2*x*arctan(c*x)/(c^2*x^2 + 1), x) + 2*c^2*x^2*arctan(c*x) + 4*c^2*x^2*i
ntegrate(1/2*arctan(c*x)/(c^2*x^3 + x), x) - (c*x + (c^2*x^2 + 1)*arctan(c
*x))*log(c^2*x^2 + 1))*b*e/x^2 - 1/2*a*d/x^2
```

## Giac [F]

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x^3}\,dx = \int \frac{(b\arctan(cx)+a)(e\log(c^2x^2+1)+d)}{x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1))/x^3,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)*(e*log(c^2*x^2 + 1) + d)/x^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x^3}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))\,\left(d+e\,\ln\left(c^2\,x^2+1\right)\right)}{x^3}\,dx$$

input
```
int(((a + b*atan(c*x))*(d + e*log(c^2*x^2 + 1)))/x^3,x)
```

output
```
int(((a + b*atan(c*x))*(d + e*log(c^2*x^2 + 1)))/x^3, x)
```

## Reduce [F]

$$\int \frac{(a+b\arctan(cx))\,(d+e\log(1+c^2x^2))}{x^3}\,dx$$
$$= \frac{-atan(cx)\,b\,c^2d\,x^2 - atan(cx)\,bd + 2\left(\int \frac{atan(cx)\log(c^2x^2+1)}{x^3}dx\right)be\,x^2 - \log(c^2x^2+1)\,a\,c^2e\,x^2 - \log(c^2x^2+}{2x^2}$$

input
```
int((a+b*atan(c*x))*(d+e*log(c^2*x^2+1))/x^3,x)
```

output
```
( - atan(c*x)*b*c**2*d*x**2 - atan(c*x)*b*d + 2*int((atan(c*x)*log(c**2*x*
*2 + 1))/x**3,x)*b*e*x**2 - log(c**2*x**2 + 1)*a*c**2*e*x**2 - log(c**2*x*
*2 + 1)*a*e + 2*log(x)*a*c**2*e*x**2 - a*d - b*c*d*x)/(2*x**2)
```

# 3.1292 $\int \frac{(a+b\arctan(cx))(d+e\log(1+c^2x^2))}{x^4}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 189

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x^4}\,dx = -\frac{2c^2e(a+b\arctan(cx))}{3x}-\frac{c^3e(a+b\arctan(cx))^2}{3b}+bc^3e\log(x)-\frac{1}{3}bc^3e\log\left(1+c^2x^2\right) -\frac{bc(1+c^2x^2)\left(d+e\log\left(1+c^2x^2\right)\right)}{6x^2}-\frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{3x^3} -\frac{1}{6}bc^3\left(d+e\log\left(1+c^2x^2\right)\right)\log\left(1-\frac{1}{1+c^2x^2}\right)+\frac{1}{6}bc^3e\,\text{PolyLog}\left(2,\frac{1}{1+c^2x^2}\right)$$

output
```
-2/3*c^2*e*(a+b*arctan(c*x))/x-1/3*c^3*e*(a+b*arctan(c*x))^2/b+b*c^3*e*ln(
x)-1/3*b*c^3*e*ln(c^2*x^2+1)-1/6*b*c*(c^2*x^2+1)*(d+e*ln(c^2*x^2+1))/x^2-1
/3*(a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1))/x^3-1/6*b*c^3*(d+e*ln(c^2*x^2+1))
*ln(1-1/(c^2*x^2+1))+1/6*b*c^3*e*polylog(2,1/(c^2*x^2+1))
```

## Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.96

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x^4}\,dx$$
$$=\frac{1}{12}\left(-\frac{8c^2e(a+b\arctan(cx))}{x}-\frac{4c^3e(a+b\arctan(cx))^2}{b}+6bc^3e\left(2\log(x)-\log\left(1+c^2x^2\right)\right)-\frac{2bc(d+e\log(1+c^2x^2))}{x^2}-\frac{4(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x^3}+\frac{bc^3\left(d+e\log\left(1+c^2x^2\right)\right)^2}{e}-2bc^3\left(\log\left(-c^2x^2\right)\left(d+e\log\left(1+c^2x^2\right)\right)+e\,\text{PolyLog}\left(2,1+c^2x^2\right)\right)\right)$$

input
```
Integrate[((a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]))/x^4,x]
```

output
```
((-8*c^2*e*(a + b*ArcTan[c*x]))/x - (4*c^3*e*(a + b*ArcTan[c*x])^2)/b + 6*
b*c^3*e*(2*Log[x] - Log[1 + c^2*x^2]) - (2*b*c*(d + e*Log[1 + c^2*x^2]))/x
^2 - (4*(a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]))/x^3 + (b*c^3*(d + e*
Log[1 + c^2*x^2])^2)/e - 2*b*c^3*(Log[-(c^2*x^2)]*(d + e*Log[1 + c^2*x^2])
 + e*PolyLog[2, 1 + c^2*x^2]))/12
```

## Rubi [A] (warning: unable to verify)

Time = 1.36 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.93, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {5552, 2925, 2858, 27, 2789, 2751, 16, 2779, 2838, 5453, 5361, 243, 47, 14, 16, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{x^4}\,dx$$

$$\Big\downarrow \text{5552}$$

$$\frac{2}{3}c^2e\int \frac{a+b\arctan(cx)}{x^2\left(c^2x^2+1\right)}dx+\frac{1}{3}bc\int \frac{d+e\log\left(c^2x^2+1\right)}{x^3\left(c^2x^2+1\right)}dx-\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{3x^3}$$

$\Big\downarrow$ 2925

$$\frac{2}{3}c^2e\int \frac{a+b\arctan(cx)}{x^2\left(c^2x^2+1\right)}dx+\frac{1}{6}bc\int \frac{d+e\log\left(c^2x^2+1\right)}{x^4\left(c^2x^2+1\right)}dx^2-\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{3x^3}$$

$\Big\downarrow$ 2858

$$\frac{2}{3}c^2e\int \frac{a+b\arctan(cx)}{x^2\left(c^2x^2+1\right)}dx+\frac{b\int \frac{d+e\log\left(c^2x^2+1\right)}{x^6}d\left(c^2x^2+1\right)}{6c}-\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{3x^3}$$

$\Big\downarrow$ 27

$$\frac{2}{3}c^2e\int \frac{a+b\arctan(cx)}{x^2\left(c^2x^2+1\right)}dx+\frac{1}{6}bc^3\int \frac{d+e\log\left(c^2x^2+1\right)}{c^4x^6}d\left(c^2x^2+1\right)-\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{3x^3}$$

$\Big\downarrow$ 2789

$$\frac{2}{3}c^2e\int \frac{a+b\arctan(cx)}{x^2\left(c^2x^2+1\right)}dx+\frac{1}{6}bc^3\left(\int -\frac{d+e\log\left(c^2x^2+1\right)}{c^2x^4}d\left(c^2x^2+1\right)+\int \frac{d+e\log\left(c^2x^2+1\right)}{c^4x^4}d\left(c^2x^2+1\right)\right)-\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{3x^3}$$

$\Big\downarrow$ 2751

$$\frac{2}{3}c^2e\int \frac{a+b\arctan(cx)}{x^2\left(c^2x^2+1\right)}dx+\frac{1}{6}bc^3\left(\int -\frac{d+e\log\left(c^2x^2+1\right)}{c^2x^4}d\left(c^2x^2+1\right)-e\int -\frac{1}{c^2x^2}d\left(c^2x^2+1\right)-\frac{\left(c^2x^2+1\right)\left(e\log\left(c^2x^2+1\right)+d\right)}{c^2x^2}\right)-\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{3x^3}$$

$\Big\downarrow$ 16

$$\frac{2}{3}c^2e\int \frac{a+b\arctan(cx)}{x^2\left(c^2x^2+1\right)}dx+$$
$$\frac{1}{6}bc^3\left(\int -\frac{d+e\log\left(c^2x^2+1\right)}{c^2x^4}d(c^2x^2+1)-\frac{\left(c^2x^2+1\right)\left(e\log\left(c^2x^2+1\right)+d\right)}{c^2x^2}+e\log\left(-c^2x^2\right)\right)-$$
$$\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{3x^3}$$

$$\Big\downarrow \text{ 2779}$$

$$\frac{2}{3}c^2e\int \frac{a+b\arctan(cx)}{x^2\left(c^2x^2+1\right)}dx+$$
$$\frac{1}{6}bc^3\left(e\int \frac{\log\left(1-\frac{1}{x^2}\right)}{x^2}d(c^2x^2+1)-\frac{\left(c^2x^2+1\right)\left(e\log\left(c^2x^2+1\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(c^2x^2+1\right)+d\right)+\right.$$
$$\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{3x^3}$$

$$\Big\downarrow \text{ 2838}$$

$$\frac{2}{3}c^2e\int \frac{a+b\arctan(cx)}{x^2\left(c^2x^2+1\right)}dx-\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{3x^3}+$$
$$\frac{1}{6}bc^3\left(-\frac{\left(c^2x^2+1\right)\left(e\log\left(c^2x^2+1\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(c^2x^2+1\right)+d\right)+e\log\left(-c^2x^2\right)+e\,\text{PolyLog}\left(\right.\right.$$

$$\Big\downarrow \text{ 5453}$$

$$\frac{2}{3}c^2e\left(\int \frac{a+b\arctan(cx)}{x^2}dx-c^2\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)-$$
$$\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{3x^3}+$$
$$\frac{1}{6}bc^3\left(-\frac{\left(c^2x^2+1\right)\left(e\log\left(c^2x^2+1\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(c^2x^2+1\right)+d\right)+e\log\left(-c^2x^2\right)+e\,\text{PolyLog}\left(\right.\right.$$

$$\Big\downarrow \text{ 5361}$$

$$\frac{2}{3}c^2e\left(c^2\left(-\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)+bc\int \frac{1}{x\left(c^2x^2+1\right)}dx-\frac{a+b\arctan(cx)}{x}\right)-$$
$$\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{3x^3}+$$
$$\frac{1}{6}bc^3\left(-\frac{\left(c^2x^2+1\right)\left(e\log\left(c^2x^2+1\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(c^2x^2+1\right)+d\right)+e\log\left(-c^2x^2\right)+e\,\text{PolyLog}\left(\right.\right.$$

$$\Big\downarrow \text{ 243}$$

$$\frac{2}{3}c^2e\left(c^2\left(-\int\frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)+\frac{1}{2}bc\int\frac{1}{x^2\left(c^2x^2+1\right)}dx^2-\frac{a+b\arctan(cx)}{x}\right)-\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{3x^3}+\frac{1}{6}bc^3\left(-\frac{\left(c^2x^2+1\right)\left(e\log\left(c^2x^2+1\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(c^2x^2+1\right)+d\right)+e\log\left(-c^2x^2\right)+e\operatorname{PolyLog}\left(\right.\right.$$

$$\Big\downarrow \; 47$$

$$\frac{2}{3}c^2e\left(c^2\left(-\int\frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)+\frac{1}{2}bc\left(\int\frac{1}{x^2}dx^2-c^2\int\frac{1}{c^2x^2+1}dx^2\right)-\frac{a+b\arctan(cx)}{x}\right)-\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{3x^3}+\frac{1}{6}bc^3\left(-\frac{\left(c^2x^2+1\right)\left(e\log\left(c^2x^2+1\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(c^2x^2+1\right)+d\right)+e\log\left(-c^2x^2\right)+e\operatorname{PolyLog}\left(\right.\right.$$

$$\Big\downarrow \; 14$$

$$\frac{2}{3}c^2e\left(c^2\left(-\int\frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)+\frac{1}{2}bc\left(\log\left(x^2\right)-c^2\int\frac{1}{c^2x^2+1}dx^2\right)-\frac{a+b\arctan(cx)}{x}\right)-\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{3x^3}+\frac{1}{6}bc^3\left(-\frac{\left(c^2x^2+1\right)\left(e\log\left(c^2x^2+1\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(c^2x^2+1\right)+d\right)+e\log\left(-c^2x^2\right)+e\operatorname{PolyLog}\left(\right.\right.$$

$$\Big\downarrow \; 16$$

$$\frac{2}{3}c^2e\left(c^2\left(-\int\frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)-\frac{a+b\arctan(cx)}{x}+\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right)\right)\right)-\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{3x^3}+\frac{1}{6}bc^3\left(-\frac{\left(c^2x^2+1\right)\left(e\log\left(c^2x^2+1\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(c^2x^2+1\right)+d\right)+e\log\left(-c^2x^2\right)+e\operatorname{PolyLog}\left(\right.\right.$$

$$\Big\downarrow \; 5419$$

$$-\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{3x^3}+\frac{2}{3}c^2e\left(-\frac{c(a+b\arctan(cx))^2}{2b}-\frac{a+b\arctan(cx)}{x}+\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right)\right)\right)+\frac{1}{6}bc^3\left(-\frac{\left(c^2x^2+1\right)\left(e\log\left(c^2x^2+1\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(c^2x^2+1\right)+d\right)+e\log\left(-c^2x^2\right)+e\operatorname{PolyLog}\left(\right.\right.$$

input
```
Int[((a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]))/x^4,x]
```

output
```
(2*c^2*e*(-((a + b*ArcTan[c*x])/x) - (c*(a + b*ArcTan[c*x])^2)/(2*b) + (b*
c*(Log[x^2] - Log[1 + c^2*x^2]))/2))/3 - ((a + b*ArcTan[c*x])*(d + e*Log[1
 + c^2*x^2]))/(3*x^3) + (b*c^3*(e*Log[-(c^2*x^2)] - ((1 + c^2*x^2)*(d + e*
Log[1 + c^2*x^2]))/(c^2*x^2) - Log[1 - x^(-2)]*(d + e*Log[1 + c^2*x^2]) +
e*PolyLog[2, x^(-2)]))/6
```

### Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5552
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(d + e*Log[f + g*x^2])*((a +
b*ArcTan[c*x])/(m + 1)), x] + (-Simp[b*(c/(m + 1))   Int[x^(m + 1)*((d + e*
Log[f + g*x^2])/(1 + c^2*x^2)), x], x] - Simp[2*e*(g/(m + 1))   Int[x^(m +
2)*((a + b*ArcTan[c*x])/(f + g*x^2)), x], x]) /; FreeQ[{a, b, c, d, e, f, g
}, x] && ILtQ[m/2, 0]
```

## Maple [F]

$$\int \frac{(a+b\arctan(cx))\,(d+e\ln(c^2x^2+1))}{x^4}dx$$

input
```
int((a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1))/x^4,x)
```

output
```
int((a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1))/x^4,x)
```

## Fricas [F]

$$\int \frac{(a+b\arctan(cx))\,(d+e\log(1+c^2x^2))}{x^4}\,dx = \int \frac{(b\arctan(cx)+a)(e\log(c^2x^2+1)+d)}{x^4}\,dx$$

input
```
integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1))/x^4,x, algorithm="fricas"
)
```

output
```
integral((b*d*arctan(c*x) + a*d + (b*e*arctan(c*x) + a*e)*log(c^2*x^2 + 1)
)/x^4, x)
```

## Sympy [F(-2)]

Exception generated.

$$\int \frac{(a+b\arctan(cx))\,(d+e\log(1+c^2x^2))}{x^4}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*atan(c*x))*(d+e*ln(c**2*x**2+1))/x**4,x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real zoo
```

## Maxima [F]

$$\int \frac{(a+b\arctan(cx))\,(d+e\log(1+c^2x^2))}{x^4}\,dx = \int \frac{(b\arctan(cx)+a)(e\log(c^2x^2+1)+d)}{x^4}\,dx$$

input
```
integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1))/x^4,x, algorithm="maxima"
)
```

output
```
1/6*((c^2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c - 2*arctan(c*x)/x^3)*
b*d - 1/3*(2*(c*arctan(c*x) + 1/x)*c^2 + log(c^2*x^2 + 1)/x^3)*a*e + b*e*i
ntegrate(arctan(c*x)*log(c^2*x^2 + 1)/x^4, x) - 1/3*a*d/x^3
```

## Giac [F]

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x^4}\,dx = \int \frac{(b\arctan\left(cx\right)+a)(e\log\left(c^2x^2+1\right)+d)}{x^4}\,dx$$

input
```
integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1))/x^4,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)*(e*log(c^2*x^2 + 1) + d)/x^4, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x^4}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))\,\left(d+e\,\ln\left(c^2\,x^2+1\right)\right)}{x^4}\,dx$$

input
```
int(((a + b*atan(c*x))*(d + e*log(c^2*x^2 + 1)))/x^4,x)
```

output
```
int(((a + b*atan(c*x))*(d + e*log(c^2*x^2 + 1)))/x^4, x)
```

## Reduce [F]

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x^4}\,dx = \frac{-4atan(cx)\,a\,c^3e\,x^3-2atan(cx)\,bd+6\left(\int \frac{atan(cx)\log(c^2x^2+1)}{x^4}dx\right)be\,x^3-2\log(c^2x^2+1)\,ae+\log(c^2x^2+1}{6x^3}$$

input
```
int((a+b*atan(c*x))*(d+e*log(c^2*x^2+1))/x^4,x)
```

output
```
( - 4*atan(c*x)*a*c**3*e*x**3 - 2*atan(c*x)*b*d + 6*int((atan(c*x)*log(c**
2*x**2 + 1))/x**4,x)*b*e*x**3 - 2*log(c**2*x**2 + 1)*a*e + log(c**2*x**2 +
 1)*b*c**3*d*x**3 - 2*log(x)*b*c**3*d*x**3 - 4*a*c**2*e*x**2 - 2*a*d - b*c
*d*x)/(6*x**3)
```

# 3.1293 $\int \frac{(a+b\arctan(cx))(d+e\log(1+c^2x^2))}{x^5}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 225

$$\begin{aligned}\int \frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x^5}\,dx &= -\frac{ac^2e}{4x^2}-\frac{5bc^3e}{12x}-\frac{11}{12}bc^4e\arctan(cx)-\frac{bc^2e\arctan(cx)}{4x^2}-\frac{1}{2}ac^4e\log(x)\\ &\quad+\frac{1}{4}ac^4e\log\left(1+c^2x^2\right)-\frac{bc(d+e\log\left(1+c^2x^2\right))}{12x^3}+\frac{bc^3(d+e\log\left(1+c^2x^2\right))}{4x}\\ &\quad+\frac{1}{4}bc^4\arctan(cx)\left(d+e\log\left(1+c^2x^2\right)\right)-\frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{4x^4}\\ &\quad-\frac{1}{4}ibc^4e\,\mathrm{PolyLog}(2,-icx)+\frac{1}{4}ibc^4e\,\mathrm{PolyLog}(2,icx)\end{aligned}$$

output
```
-1/4*a*c^2*e/x^2-5/12*b*c^3*e/x-11/12*b*c^4*e*arctan(c*x)-1/4*b*c^2*e*arct
an(c*x)/x^2-1/2*a*c^4*e*ln(x)+1/4*a*c^4*e*ln(c^2*x^2+1)-1/12*b*c*(d+e*ln(c
^2*x^2+1))/x^3+1/4*b*c^3*(d+e*ln(c^2*x^2+1))/x+1/4*b*c^4*arctan(c*x)*(d+e*
ln(c^2*x^2+1))-1/4*(a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1))/x^4-1/4*I*b*c^4*e
*polylog(2,-I*c*x)+1/4*I*b*c^4*e*polylog(2,I*c*x)
```

## Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.16

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x^5}\,dx = -\frac{3ad+bcdx+3ac^2ex^2-3bc^3dx^3+5bc^3ex^3+3bd\arctan(cx)+3bc^2ex^2\arctan(cx)-3bc^4dx^4\arctan(c$$

input
```
Integrate[((a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]))/x^5,x]
```

output
```
-1/12*(3*a*d + b*c*d*x + 3*a*c^2*e*x^2 - 3*b*c^3*d*x^3 + 5*b*c^3*e*x^3 + 3
*b*d*ArcTan[c*x] + 3*b*c^2*e*x^2*ArcTan[c*x] - 3*b*c^4*d*x^4*ArcTan[c*x] +
 11*b*c^4*e*x^4*ArcTan[c*x] + 6*a*c^4*e*x^4*Log[x] + 3*a*e*Log[1 + c^2*x^2
] + b*c*e*x*Log[1 + c^2*x^2] - 3*b*c^3*e*x^3*Log[1 + c^2*x^2] - 3*a*c^4*e*
x^4*Log[1 + c^2*x^2] + 3*b*e*ArcTan[c*x]*Log[1 + c^2*x^2] - 3*b*c^4*e*x^4*
ArcTan[c*x]*Log[1 + c^2*x^2] + (3*I)*b*c^4*e*x^4*PolyLog[2, (-I)*c*x] - (3
*I)*b*c^4*e*x^4*PolyLog[2, I*c*x])/x^4
```

## Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.96, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {5556, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{x^5}\,dx$$

$$\Big\downarrow \text{5556}$$

$$-2c^2e\int \left(-\frac{-3bc^3x^3+bcx+3a}{12x^3\left(c^2x^2+1\right)}-\frac{b\left(1-c^2x^2\right)\arctan(cx)}{4x^3}\right)dx-\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{4x^4}+\frac{1}{4}bc^4\arctan(cx)\left(e\log\left(c^2x^2+1\right)+d\right)-\frac{bc\left(e\log\left(c^2x^2+1\right)+d\right)}{12x^3}+\frac{bc^3\left(e\log\left(c^2x^2+1\right)+d\right)}{4x}$$

$$\downarrow \text{2009}$$

$$-\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{4x^4}-$$
$$2c^2e\left(-\frac{1}{8}ac^2\log\left(c^2x^2+1\right)+\frac{1}{4}ac^2\log(x)+\frac{a}{8x^2}+\frac{11}{24}bc^2\arctan(cx)+\frac{b\arctan(cx)}{8x^2}+\frac{1}{8}ibc^2\,\text{PolyLog}(2,-icx)-\right.$$
$$\frac{1}{4}bc^4\arctan(cx)\left(e\log\left(c^2x^2+1\right)+d\right)-\frac{bc\left(e\log\left(c^2x^2+1\right)+d\right)}{12x^3}+\frac{bc^3\left(e\log\left(c^2x^2+1\right)+d\right)}{4x}$$

input
```
Int[((a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]))/x^5,x]
```

output
```
-1/12*(b*c*(d + e*Log[1 + c^2*x^2]))/x^3 + (b*c^3*(d + e*Log[1 + c^2*x^2])
)/(4*x) + (b*c^4*ArcTan[c*x]*(d + e*Log[1 + c^2*x^2]))/4 - ((a + b*ArcTan[
c*x])*(d + e*Log[1 + c^2*x^2]))/(4*x^4) - 2*c^2*e*(a/(8*x^2) + (5*b*c)/(24
*x) + (11*b*c^2*ArcTan[c*x])/24 + (b*ArcTan[c*x])/(8*x^2) + (a*c^2*Log[x])
/4 - (a*c^2*Log[1 + c^2*x^2])/8 + (I/8)*b*c^2*PolyLog[2, (-I)*c*x] - (I/8)
*b*c^2*PolyLog[2, I*c*x])
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5556
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcTan[c*x]), x
]}, Simp[(d + e*Log[f + g*x^2])   u, x] - Simp[2*e*g   Int[ExpandIntegrand[
x*(u/(f + g*x^2)), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && Intege
rQ[m] && NeQ[m, -1]
```

## Maple [F]

$$\int \frac{(a+b\arctan(cx))\left(d+e\ln\left(c^2x^2+1\right)\right)}{x^5}dx$$

input
```
int((a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1))/x^5,x)
```

output
```
int((a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1))/x^5,x)
```

## Fricas [F]

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x^5}\,dx = \int \frac{(b\arctan\left(cx\right)+a)(e\log\left(c^2x^2+1\right)+d)}{x^5}\,dx$$

input
```
integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1))/x^5,x, algorithm="fricas"
)
```

output
```
integral((b*d*arctan(c*x) + a*d + (b*e*arctan(c*x) + a*e)*log(c^2*x^2 + 1)
)/x^5, x)
```

## Sympy [F]

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x^5}\,dx = \int \frac{(a+b\operatorname{atan}\left(cx\right))\left(d+e\log\left(c^2x^2+1\right)\right)}{x^5}\,dx$$

input
```
integrate((a+b*atan(c*x))*(d+e*ln(c**2*x**2+1))/x**5,x)
```

output
```
Integral((a + b*atan(c*x))*(d + e*log(c**2*x**2 + 1))/x**5, x)
```

## Maxima [F]

$$\int \frac{(a+b\arctan(cx))\,(d+e\log(1+c^2x^2))}{x^5}\,dx = \int \frac{(b\arctan(cx)+a)(e\log(c^2x^2+1)+d)}{x^5}\,dx$$

input
```
integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1))/x^5,x, algorithm="maxima"
)
```

output
```
1/12*((3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3)*c - 3*arctan(c*x)/x^4)*b*d
 + 1/4*((c^2*log(c^2*x^2 + 1) - c^2*log(x^2) - 1/x^2)*c^2 - log(c^2*x^2 +
1)/x^4)*a*e - 1/12*(72*c^6*x^4*integrate(1/12*x*arctan(c*x)/(c^2*x^2 + 1),
 x) + 8*c^4*x^4*arctan(c*x) - 72*c^2*x^4*integrate(1/12*arctan(c*x)/(c^2*x
^5 + x^3), x) + 2*c^3*x^3 - (3*c^3*x^3 - c*x + 3*(c^4*x^4 - 1)*arctan(c*x)
)*log(c^2*x^2 + 1))*b*e/x^4 - 1/4*a*d/x^4
```

## Giac [F]

$$\int \frac{(a+b\arctan(cx))\,(d+e\log(1+c^2x^2))}{x^5}\,dx = \int \frac{(b\arctan(cx)+a)(e\log(c^2x^2+1)+d)}{x^5}\,dx$$

input
```
integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1))/x^5,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)*(e*log(c^2*x^2 + 1) + d)/x^5, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x^5}\,dx = \int \frac{\left(a+b\,\mathrm{atan}(c\,x)\right)\,\left(d+e\,\ln\left(c^2\,x^2+1\right)\right)}{x^5}\,dx$$

input
```
int(((a + b*atan(c*x))*(d + e*log(c^2*x^2 + 1)))/x^5,x)
```

output
```
int(((a + b*atan(c*x))*(d + e*log(c^2*x^2 + 1)))/x^5, x)
```

## Reduce [F]

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x^5}\,dx = \frac{3atan(cx)\,b\,c^4d\,x^4-3atan(cx)\,bd+12\left(\int \frac{atan(cx)\log(c^2x^2+1)}{x^5}dx\right)be\,x^4+3\log(c^2x^2+1)\,a\,c^4e\,x^4-3\log(c^2}{12x^4}$$

input
```
int((a+b*atan(c*x))*(d+e*log(c^2*x^2+1))/x^5,x)
```

output
```
(3*atan(c*x)*b*c**4*d*x**4 - 3*atan(c*x)*b*d + 12*int((atan(c*x)*log(c**2*
x**2 + 1))/x**5,x)*b*e*x**4 + 3*log(c**2*x**2 + 1)*a*c**4*e*x**4 - 3*log(c
**2*x**2 + 1)*a*e - 6*log(x)*a*c**4*e*x**4 - 3*a*c**2*e*x**2 - 3*a*d + 3*b
*c**3*d*x**3 - b*c*d*x)/(12*x**4)
```

# 3.1294 $\int \frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x^6}\,dx$



## Optimal result

Integrand size = 26, antiderivative size = 248

$$\begin{aligned}
\int \frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x^6}\,dx
&= -\frac{7bc^3e}{60x^2}-\frac{2c^2e(a+b\arctan(cx))}{15x^3}+\frac{2c^4e(a+b\arctan(cx))}{5x}+\frac{c^5e(a+b\arctan(cx))^2}{5b}\\
&\quad -\frac{5}{6}bc^5e\log(x)+\frac{19}{60}bc^5e\log\left(1+c^2x^2\right)-\frac{bc\left(d+e\log\left(1+c^2x^2\right)\right)}{20x^4}\\
&\quad +\frac{bc^3\left(1+c^2x^2\right)\left(d+e\log\left(1+c^2x^2\right)\right)}{10x^2}-\frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{5x^5}\\
&\quad +\frac{1}{10}bc^5\left(d+e\log\left(1+c^2x^2\right)\right)\log\left(1-\frac{1}{1+c^2x^2}\right)-\frac{1}{10}bc^5e\,\text{PolyLog}\left(2,\frac{1}{1+c^2x^2}\right)
\end{aligned}$$

output
```
-7/60*b*c^3*e/x^2-2/15*c^2*e*(a+b*arctan(c*x))/x^3+2/5*c^4*e*(a+b*arctan(c
*x))/x+1/5*c^5*e*(a+b*arctan(c*x))^2/b-5/6*b*c^5*e*ln(x)+19/60*b*c^5*e*ln(
c^2*x^2+1)-1/20*b*c*(d+e*ln(c^2*x^2+1))/x^4+1/10*b*c^3*(c^2*x^2+1)*(d+e*ln
(c^2*x^2+1))/x^2-1/5*(a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1))/x^5+1/10*b*c^5*
(d+e*ln(c^2*x^2+1))*ln(1-1/(c^2*x^2+1))-1/10*b*c^5*e*polylog(2,1/(c^2*x^2+
1))
```

## Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.12

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x^6}\,dx$$
$$= \frac{1}{60}\left(-\frac{8c^2e(a+b\arctan(cx))}{x^3} - 24c^4e\left(-\frac{a+b\arctan(cx)}{x} - \frac{c(a+b\arctan(cx))^2}{2b} + bc\left(\log(x) - \frac{1}{2}\log\left(1+c^2x^2\right)\right)\right) - 6bc^5e\left(2\log(x) - \log\left(1+c^2x^2\right)\right) + 7bc^3e\left(-\frac{1}{x^2} - 2c^2\log(x) + c^2\log\left(1+c^2x^2\right)\right) - \frac{3bc\left(d+e\log\left(1+c^2x^2\right)\right)}{x^4} + \frac{6bc^3\left(d+e\log\left(1+c^2x^2\right)\right)}{x^2} - \frac{12(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x^5} - \frac{3bc^5\left(d+e\log\left(1+c^2x^2\right)\right)^2}{e} + 6bc^5\left(\log\left(-c^2x^2\right)\left(d+e\log\left(1+c^2x^2\right)\right) + e\,\mathrm{PolyLog}\left(2,1+c^2x^2\right)\right)\right)$$

input
```
Integrate[((a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]))/x^6,x]
```

output
```
((-8*c^2*e*(a + b*ArcTan[c*x]))/x^3 - 24*c^4*e*(-((a + b*ArcTan[c*x])/x) -
 (c*(a + b*ArcTan[c*x])^2)/(2*b) + b*c*(Log[x] - Log[1 + c^2*x^2]/2)) - 6*
b*c^5*e*(2*Log[x] - Log[1 + c^2*x^2]) + 7*b*c^3*e*(-x^(-2) - 2*c^2*Log[x]
+ c^2*Log[1 + c^2*x^2]) - (3*b*c*(d + e*Log[1 + c^2*x^2]))/x^4 + (6*b*c^3*
(d + e*Log[1 + c^2*x^2]))/x^2 - (12*(a + b*ArcTan[c*x])*(d + e*Log[1 + c^2
*x^2]))/x^5 - (3*b*c^5*(d + e*Log[1 + c^2*x^2])^2)/e + 6*b*c^5*(Log[-(c^2*
x^2)]*(d + e*Log[1 + c^2*x^2]) + e*PolyLog[2, 1 + c^2*x^2]))/60
```

## Rubi [A] (warning: unable to verify)

Time = 2.46 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.17, number of steps used = 27, number of rules used = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {5552, 2925, 2858, 25, 27, 2789, 2756, 54, 2009, 2789, 2751, 16, 2779, 2838, 5453, 5361, 243, 54, 2009, 5453, 5361, 243, 47, 14, 16, 5419}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{x^6}dx$$

$$\Big\downarrow \text{5552}$$

$$\frac{2}{5}c^2e\int \frac{a+b\arctan(cx)}{x^4\left(c^2x^2+1\right)}dx+\frac{1}{5}bc\int \frac{d+e\log\left(c^2x^2+1\right)}{x^5\left(c^2x^2+1\right)}dx-\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{5x^5}$$

$$\Big\downarrow \text{2925}$$

$$\frac{2}{5}c^2e\int \frac{a+b\arctan(cx)}{x^4\left(c^2x^2+1\right)}dx+\frac{1}{10}bc\int \frac{d+e\log\left(c^2x^2+1\right)}{x^6\left(c^2x^2+1\right)}dx^2-\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{5x^5}$$

$$\Big\downarrow \text{2858}$$

$$\frac{2}{5}c^2e\int \frac{a+b\arctan(cx)}{x^4\left(c^2x^2+1\right)}dx+\frac{b\int \frac{d+e\log\left(c^2x^2+1\right)}{x^8}d\left(c^2x^2+1\right)}{10c}-\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{5x^5}$$

$$\Big\downarrow \text{25}$$

$$\frac{2}{5}c^2e\int \frac{a+b\arctan(cx)}{x^4\left(c^2x^2+1\right)}dx-\frac{b\int -\frac{d+e\log\left(c^2x^2+1\right)}{x^8}d\left(c^2x^2+1\right)}{10c}-\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{5x^5}$$

$$\Big\downarrow \text{27}$$

$$\frac{2}{5}c^2e\int \frac{a+b\arctan(cx)}{x^4\left(c^2x^2+1\right)}dx-\frac{1}{10}bc^5\int -\frac{d+e\log\left(c^2x^2+1\right)}{c^6x^8}d\left(c^2x^2+1\right)-\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{5x^5}$$

$$\Big\downarrow \text{2789}$$

$$\frac{2}{5}c^2e\int \frac{a+b\arctan(cx)}{x^4\left(c^2x^2+1\right)}dx-\frac{1}{10}bc^5\left(\int -\frac{d+e\log\left(c^2x^2+1\right)}{c^6x^6}d(c^2x^2+1)+\int \frac{d+e\log\left(c^2x^2+1\right)}{c^4x^6}d(c^2x^2+1)\right)-\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{5x^5}$$

$$\Big\downarrow \text{2756}$$

$$\frac{2}{5}c^2e\int \frac{a+b\arctan(cx)}{x^4\left(c^2x^2+1\right)}dx-\frac{1}{10}bc^5\left(\int \frac{d+e\log\left(c^2x^2+1\right)}{c^4x^6}d(c^2x^2+1)-\frac{1}{2}e\int \frac{1}{c^4x^6}d(c^2x^2+1)+\frac{e\log\left(c^2x^2+1\right)+d}{2c^4x^4}\right)-\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{5x^5}$$

$$\Big\downarrow \text{54}$$

$$\frac{2}{5}c^2e\int \frac{a+b\arctan(cx)}{x^4\left(c^2x^2+1\right)}dx-\frac{1}{10}bc^5\left(\int \frac{d+e\log\left(c^2x^2+1\right)}{c^4x^6}d(c^2x^2+1)-\frac{1}{2}e\int \left(-\frac{1}{c^2x^2}+\frac{1}{x^2}+\frac{1}{c^4x^4}\right)d(c^2x^2+1)+\frac{e\log\left(c^2x^2+1\right)+d}{2c^4x^4}\right)-\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{5x^5}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{5}c^2e\int \frac{a+b\arctan(cx)}{x^4\left(c^2x^2+1\right)}dx-\frac{1}{10}bc^5\left(\int \frac{d+e\log\left(c^2x^2+1\right)}{c^4x^6}d(c^2x^2+1)-\frac{1}{2}e\left(-\frac{1}{c^2x^2}-\log\left(-c^2x^2\right)+\log\left(c^2x^2+1\right)\right)+\frac{e\log\left(c^2x^2+1\right)+d}{2c^4x^4}\right)-\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{5x^5}$$

$$\Big\downarrow \text{2789}$$

$$\frac{2}{5}c^2e\int \frac{a+b\arctan(cx)}{x^4\left(c^2x^2+1\right)}dx-\frac{1}{10}bc^5\left(\int -\frac{d+e\log\left(c^2x^2+1\right)}{c^2x^4}d(c^2x^2+1)+\int \frac{d+e\log\left(c^2x^2+1\right)}{c^4x^4}d(c^2x^2+1)-\frac{1}{2}e\left(-\frac{1}{c^2x^2}-\log\left(-c^2x^2\right)-\right.\right.$$

$$\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{5x^5}$$

$$\Big\downarrow \text{2751}$$

$$\frac{2}{5}c^2e\int \frac{a+b\arctan(cx)}{x^4\left(c^2x^2+1\right)}dx-$$
$$\frac{1}{10}bc^5\left(\int -\frac{d+e\log\left(c^2x^2+1\right)}{c^2x^4}d(c^2x^2+1)-e\int -\frac{1}{c^2x^2}d(c^2x^2+1)-\frac{\left(c^2x^2+1\right)\left(e\log\left(c^2x^2+1\right)+d\right)}{c^2x^2}-\frac{1}{2}e\right.$$
$$\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{5x^5}$$

$\downarrow$ 16

$$\frac{2}{5}c^2e\int \frac{a+b\arctan(cx)}{x^4\left(c^2x^2+1\right)}dx-$$
$$\frac{1}{10}bc^5\left(\int -\frac{d+e\log\left(c^2x^2+1\right)}{c^2x^4}d(c^2x^2+1)-\frac{\left(c^2x^2+1\right)\left(e\log\left(c^2x^2+1\right)+d\right)}{c^2x^2}+e\log\left(-c^2x^2\right)-\frac{1}{2}e\left(-\frac{1}{c^2x^2}\right.\right.$$
$$\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{5x^5}$$

$\downarrow$ 2779

$$\frac{2}{5}c^2e\int \frac{a+b\arctan(cx)}{x^4\left(c^2x^2+1\right)}dx-$$
$$\frac{1}{10}bc^5\left(e\int \frac{\log\left(1-\frac{1}{x^2}\right)}{x^2}d(c^2x^2+1)-\frac{\left(c^2x^2+1\right)\left(e\log\left(c^2x^2+1\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(c^2x^2+1\right)+d\right)\right.$$
$$\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{5x^5}$$

$\downarrow$ 2838

$$\frac{2}{5}c^2e\int \frac{a+b\arctan(cx)}{x^4\left(c^2x^2+1\right)}dx-\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{5x^5}-$$
$$\frac{1}{10}bc^5\left(-\frac{\left(c^2x^2+1\right)\left(e\log\left(c^2x^2+1\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(c^2x^2+1\right)+d\right)+e\log\left(-c^2x^2\right)-\frac{1}{2}e\left(-\frac{1}{c^2x^2}\right.\right.$$

$\downarrow$ 5453

$$\frac{2}{5}c^2e\left(\int \frac{a+b\arctan(cx)}{x^4}dx-c^2\int \frac{a+b\arctan(cx)}{x^2\left(c^2x^2+1\right)}dx\right)-$$
$$\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{5x^5}-$$
$$\frac{1}{10}bc^5\left(-\frac{\left(c^2x^2+1\right)\left(e\log\left(c^2x^2+1\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(c^2x^2+1\right)+d\right)+e\log\left(-c^2x^2\right)-\frac{1}{2}e\left(-\frac{1}{c^2x^2}\right.\right.$$

$\downarrow$ 5361

$$\frac{2}{5}c^2e\left(c^2\left(-\int\frac{a+b\arctan(cx)}{x^2\left(c^2x^2+1\right)}dx\right)+\frac{1}{3}bc\int\frac{1}{x^3\left(c^2x^2+1\right)}dx-\frac{a+b\arctan(cx)}{3x^3}\right)-\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{5x^5}-\frac{1}{10}bc^5\left(-\frac{\left(c^2x^2+1\right)\left(e\log\left(c^2x^2+1\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(c^2x^2+1\right)+d\right)+e\log\left(-c^2x^2\right)-\frac{1}{2}e\left(-\frac{1}{c^2x^2}\right.\right.$$

$$\Big\downarrow \text{243}$$

$$\frac{2}{5}c^2e\left(c^2\left(-\int\frac{a+b\arctan(cx)}{x^2\left(c^2x^2+1\right)}dx\right)+\frac{1}{6}bc\int\frac{1}{x^4\left(c^2x^2+1\right)}dx^2-\frac{a+b\arctan(cx)}{3x^3}\right)-\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{5x^5}-\frac{1}{10}bc^5\left(-\frac{\left(c^2x^2+1\right)\left(e\log\left(c^2x^2+1\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(c^2x^2+1\right)+d\right)+e\log\left(-c^2x^2\right)-\frac{1}{2}e\left(-\frac{1}{c^2x^2}\right.\right.$$

$$\Big\downarrow \text{54}$$

$$\frac{2}{5}c^2e\left(c^2\left(-\int\frac{a+b\arctan(cx)}{x^2\left(c^2x^2+1\right)}dx\right)+\frac{1}{6}bc\int\left(\frac{c^4}{c^2x^2+1}-\frac{c^2}{x^2}+\frac{1}{x^4}\right)dx^2-\frac{a+b\arctan(cx)}{3x^3}\right)-\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{5x^5}-\frac{1}{10}bc^5\left(-\frac{\left(c^2x^2+1\right)\left(e\log\left(c^2x^2+1\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(c^2x^2+1\right)+d\right)+e\log\left(-c^2x^2\right)-\frac{1}{2}e\left(-\frac{1}{c^2x^2}\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{5}c^2e\left(c^2\left(-\int\frac{a+b\arctan(cx)}{x^2\left(c^2x^2+1\right)}dx\right)-\frac{a+b\arctan(cx)}{3x^3}+\frac{1}{6}bc\left(c^2\left(-\log\left(x^2\right)\right)+c^2\log\left(c^2x^2+1\right)-\frac{1}{x^2}\right)\right)-\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{5x^5}-\frac{1}{10}bc^5\left(-\frac{\left(c^2x^2+1\right)\left(e\log\left(c^2x^2+1\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(c^2x^2+1\right)+d\right)+e\log\left(-c^2x^2\right)-\frac{1}{2}e\left(-\frac{1}{c^2x^2}\right.\right.$$

$$\Big\downarrow \text{5453}$$

$$\frac{2}{5}c^2e\left(-\left(c^2\left(\int\frac{a+b\arctan(cx)}{x^2}dx-c^2\int\frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)\right)-\frac{a+b\arctan(cx)}{3x^3}+\frac{1}{6}bc\left(c^2\left(-\log\left(x^2\right)\right)+\right.\right.\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{5x^5}-\frac{1}{10}bc^5\left(-\frac{\left(c^2x^2+1\right)\left(e\log\left(c^2x^2+1\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(c^2x^2+1\right)+d\right)+e\log\left(-c^2x^2\right)-\frac{1}{2}e\left(-\frac{1}{c^2x^2}\right.\right.$$

$$\Big\downarrow \text{5361}$$

$$\frac{2}{5}c^2e\left(-\left(c^2\left(c^2\left(-\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)+bc\int \frac{1}{x\left(c^2x^2+1\right)}dx-\frac{a+b\arctan(cx)}{x}\right)\right)-\frac{a+b\arctan(cx)}{3x^3}+\right.$$
$$\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{5x^5}-$$
$$\frac{1}{10}bc^5\left(-\frac{\left(c^2x^2+1\right)\left(e\log\left(c^2x^2+1\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(c^2x^2+1\right)+d\right)+e\log\left(-c^2x^2\right)-\frac{1}{2}e\left(-\frac{1}{c^2x^2}\right.\right.$$

$\downarrow$ 243

$$\frac{2}{5}c^2e\left(-\left(c^2\left(c^2\left(-\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)+\frac{1}{2}bc\int \frac{1}{x^2\left(c^2x^2+1\right)}dx^2-\frac{a+b\arctan(cx)}{x}\right)\right)-\frac{a+b\arctan(cx}{3x^3}\right.$$
$$\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{5x^5}-$$
$$\frac{1}{10}bc^5\left(-\frac{\left(c^2x^2+1\right)\left(e\log\left(c^2x^2+1\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(c^2x^2+1\right)+d\right)+e\log\left(-c^2x^2\right)-\frac{1}{2}e\left(-\frac{1}{c^2x^2}\right.\right.$$

$\downarrow$ 47

$$\frac{2}{5}c^2e\left(-\left(c^2\left(c^2\left(-\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)+\frac{1}{2}bc\left(\int \frac{1}{x^2}dx^2-c^2\int \frac{1}{c^2x^2+1}dx^2\right)-\frac{a+b\arctan(cx)}{x}\right)\right)-\frac{a}{}\right.$$
$$\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{5x^5}-$$
$$\frac{1}{10}bc^5\left(-\frac{\left(c^2x^2+1\right)\left(e\log\left(c^2x^2+1\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(c^2x^2+1\right)+d\right)+e\log\left(-c^2x^2\right)-\frac{1}{2}e\left(-\frac{1}{c^2x^2}\right.\right.$$

$\downarrow$ 14

$$\frac{2}{5}c^2e\left(-\left(c^2\left(c^2\left(-\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)+\frac{1}{2}bc\left(\log\left(x^2\right)-c^2\int \frac{1}{c^2x^2+1}dx^2\right)-\frac{a+b\arctan(cx)}{x}\right)\right)-\frac{a+}{}\right.$$
$$\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{5x^5}-$$
$$\frac{1}{10}bc^5\left(-\frac{\left(c^2x^2+1\right)\left(e\log\left(c^2x^2+1\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(c^2x^2+1\right)+d\right)+e\log\left(-c^2x^2\right)-\frac{1}{2}e\left(-\frac{1}{c^2x^2}\right.\right.$$

$\downarrow$ 16

$$\frac{2}{5}c^2e\left(-\left(c^2\left(c^2\left(-\int \frac{a+b\arctan(cx)}{c^2x^2+1}dx\right)-\frac{a+b\arctan(cx)}{x}+\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right)\right)\right)\right)-\frac{a+b\,\mathrm{arc}}{3x}\right.$$
$$\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{5x^5}-$$
$$\frac{1}{10}bc^5\left(-\frac{\left(c^2x^2+1\right)\left(e\log\left(c^2x^2+1\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(c^2x^2+1\right)+d\right)+e\log\left(-c^2x^2\right)-\frac{1}{2}e\left(-\frac{1}{c^2x^2}\right.\right.$$

$$\Big\downarrow \text{5419}$$

$$-\frac{(a+b\arctan(cx))\left(e\log\left(c^2x^2+1\right)+d\right)}{5x^5}+$$
$$\frac{2}{5}c^2e\left(-\left(c^2\left(-\frac{c(a+b\arctan(cx))^2}{2b}-\frac{a+b\arctan(cx)}{x}+\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(c^2x^2+1\right)\right)\right)\right)-\frac{a+b\arctan(cx)}{3x^3}$$
$$\frac{1}{10}bc^5\left(-\frac{\left(c^2x^2+1\right)\left(e\log\left(c^2x^2+1\right)+d\right)}{c^2x^2}-\log\left(1-\frac{1}{x^2}\right)\left(e\log\left(c^2x^2+1\right)+d\right)+e\log\left(-c^2x^2\right)-\frac{1}{2}e\left(-\frac{1}{c^2x^2}\right.\right.$$

input
```
Int[((a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]))/x^6,x]
```

output
```
-1/5*((a + b*ArcTan[c*x])*(d + e*Log[1 + c^2*x^2]))/x^5 + (2*c^2*e*(-1/3*(
a + b*ArcTan[c*x])/x^3 - c^2*(-((a + b*ArcTan[c*x])/x) - (c*(a + b*ArcTan[
c*x])^2)/(2*b) + (b*c*(Log[x^2] - Log[1 + c^2*x^2]))/2) + (b*c*(-x^(-2) -
c^2*Log[x^2] + c^2*Log[1 + c^2*x^2]))/6))/5 - (b*c^5*(e*Log[-(c^2*x^2)] -
(e*(-(1/(c^2*x^2)) - Log[-(c^2*x^2)] + Log[1 + c^2*x^2]))/2 + (d + e*Log[1
 + c^2*x^2])/(2*c^4*x^4) - ((1 + c^2*x^2)*(d + e*Log[1 + c^2*x^2]))/(c^2*x
^2) - Log[1 - x^(-2)]*(d + e*Log[1 + c^2*x^2]) + e*PolyLog[2, x^(-2)]))/10
```

## Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

rule 5361
```
Int[((a_.) + ArcTan[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[x^(m + 1)*((a + b*ArcTan[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m +
1))   Int[x^(m + n)*((a + b*ArcTan[c*x^n])^(p - 1)/(1 + c^2*x^(2*n))), x],
x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1] &
& IntegerQ[m])) && NeQ[m, -1]
```

rule 5419
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symbo
l] :> Simp[(a + b*ArcTan[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b,
c, d, e, p}, x] && EqQ[e, c^2*d] && NeQ[p, -1]
```

rule 5453
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTan[c*x])^p, x],
 x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTan[c*x])^p/(d + e*x^2)
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 5552
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(d + e*Log[f + g*x^2])*((a +
b*ArcTan[c*x])/(m + 1)), x] + (-Simp[b*(c/(m + 1))   Int[x^(m + 1)*((d + e*
Log[f + g*x^2])/(1 + c^2*x^2)), x], x] - Simp[2*e*(g/(m + 1))   Int[x^(m +
2)*((a + b*ArcTan[c*x])/(f + g*x^2)), x], x]) /; FreeQ[{a, b, c, d, e, f, g
}, x] && ILtQ[m/2, 0]
```

## Maple [F]

$$\int \frac{(a+b\arctan(cx))\left(d+e\ln\left(c^2x^2+1\right)\right)}{x^6}dx$$

input
```
int((a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1))/x^6,x)
```

output
```
int((a+b*arctan(c*x))*(d+e*ln(c^2*x^2+1))/x^6,x)
```

## Fricas [F]

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(1+c^2x^2\right)\right)}{x^6}\,dx = \int \frac{(b\arctan(cx)+a)(e\log\left(c^2x^2+1\right)+d)}{x^6}\,dx$$

input
```
integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1))/x^6,x, algorithm="fricas"
)
```

output
```
integral((b*d*arctan(c*x) + a*d + (b*e*arctan(c*x) + a*e)*log(c^2*x^2 + 1)
)/x^6, x)
```

## Sympy [F(-2)]

Exception generated.

$$\int \frac{(a + b\arctan(cx))\left(d + e\log\left(1 + c^2x^2\right)\right)}{x^6}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+b*atan(c*x))*(d+e*ln(c**2*x**2+1))/x**6,x)
```

output
```
Exception raised: TypeError >> Invalid comparison of non-real zoo
```

## Maxima [F]

$$\int \frac{(a + b\arctan(cx))\left(d + e\log\left(1 + c^2x^2\right)\right)}{x^6}\,dx = \int \frac{(b\arctan(cx) + a)(e\log(c^2x^2 + 1) + d)}{x^6}\,dx$$

input
```
integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1))/x^6,x, algorithm="maxima"
)
```

output
```
-1/20*((2*c^4*log(c^2*x^2 + 1) - 2*c^4*log(x^2) - (2*c^2*x^2 - 1)/x^4)*c +
 4*arctan(c*x)/x^5)*b*d + 1/15*(2*(3*c^3*arctan(c*x) + (3*c^2*x^2 - 1)/x^3
)*c^2 - 3*log(c^2*x^2 + 1)/x^5)*a*e + b*e*integrate(arctan(c*x)*log(c^2*x^
2 + 1)/x^6, x) - 1/5*a*d/x^5
```

## Giac [F]

$$\int \frac{(a + b\arctan(cx))\left(d + e\log\left(1 + c^2x^2\right)\right)}{x^6}\,dx = \int \frac{(b\arctan(cx) + a)(e\log(c^2x^2 + 1) + d)}{x^6}\,dx$$

input
```
integrate((a+b*arctan(c*x))*(d+e*log(c^2*x^2+1))/x^6,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)*(e*log(c^2*x^2 + 1) + d)/x^6, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\arctan(cx))\,(d + e\log(1 + c^2x^2))}{x^6}\,dx = \int \frac{(a + b\,\mathrm{atan}(c\,x))\;(d + e\;\ln(c^2\,x^2 + 1))}{x^6}\,dx$$

input
```
int(((a + b*atan(c*x))*(d + e*log(c^2*x^2 + 1)))/x^6,x)
```

output
```
int(((a + b*atan(c*x))*(d + e*log(c^2*x^2 + 1)))/x^6, x)
```

## Reduce [F]

$$\int \frac{(a + b\arctan(cx))\,(d + e\log(1 + c^2x^2))}{x^6}\,dx = \frac{12atan(cx)^2\,b\,c^5e\,x^5 - 12atan(cx)\log(c^2x^2+1)\,be + 24atan(cx)\,a\,c^5e\,x^5 + 24atan(cx)\,b\,c^4e\,x^4 - 8atan(}{}$$

input
```
int((a+b*atan(c*x))*(d+e*log(c^2*x^2+1))/x^6,x)
```

output
```
(12*atan(c*x)**2*b*c**5*e*x**5 - 12*atan(c*x)*log(c**2*x**2 + 1)*b*e + 24*
atan(c*x)*a*c**5*e*x**5 + 24*atan(c*x)*b*c**4*e*x**4 - 8*atan(c*x)*b*c**2*
e*x**2 - 12*atan(c*x)*b*d + 12*int(log(c**2*x**2 + 1)/(c**2*x**3 + x),x)*b
*c**5*e*x**5 - 12*log(c**2*x**2 + 1)*a*e - 6*log(c**2*x**2 + 1)*b*c**5*d*x
**5 + 25*log(c**2*x**2 + 1)*b*c**5*e*x**5 + 6*log(c**2*x**2 + 1)*b*c**3*e*
x**3 - 3*log(c**2*x**2 + 1)*b*c*e*x + 12*log(x)*b*c**5*d*x**5 - 50*log(x)*
b*c**5*e*x**5 + 24*a*c**4*e*x**4 - 8*a*c**2*e*x**2 - 12*a*d + 6*b*c**3*d*x
**3 - 7*b*c**3*e*x**3 - 3*b*c*d*x)/(60*x**5)
```

# 3.1295 $\int x(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)\,dx$



## Optimal result

Integrand size = 22, antiderivative size = 562

$$\begin{aligned}
&\int x(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)\,dx\\
&=-\frac{b(d-e)x}{2c}+\frac{bex}{c}+\frac{b(d-e)\arctan(cx)}{2c^2}+\frac{1}{2}dx^2(a+b\arctan(cx))\\
&\quad-\frac{1}{2}ex^2(a+b\arctan(cx))-\frac{be\sqrt{f}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{c\sqrt{g}}-\frac{be(c^2f-g)\arctan(cx)\log\left(\frac{2}{1-icx}\right)}{c^2g}\\
&\quad+\frac{be(c^2f-g)\arctan(cx)\log\left(\frac{2c(\sqrt{-f}-\sqrt{g}x)}{(c\sqrt{-f}-i\sqrt{g})(1-icx)}\right)}{2c^2g}\\
&\quad+\frac{be(c^2f-g)\arctan(cx)\log\left(\frac{2c(\sqrt{-f}+\sqrt{g}x)}{(c\sqrt{-f}+i\sqrt{g})(1-icx)}\right)}{2c^2g}\\
&\quad-\frac{bex\log\left(f+gx^2\right)}{2c}-\frac{be(c^2f-g)\arctan(cx)\log\left(f+gx^2\right)}{2c^2g}\\
&\quad+\frac{e(f+gx^2)(a+b\arctan(cx))\log\left(f+gx^2\right)}{2g}+\frac{ibe(c^2f-g)\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2c^2g}\\
&\quad-\frac{ibe(c^2f-g)\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-f}-\sqrt{g}x)}{(c\sqrt{-f}-i\sqrt{g})(1-icx)}\right)}{4c^2g}\\
&\quad-\frac{ibe(c^2f-g)\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-f}+\sqrt{g}x)}{(c\sqrt{-f}+i\sqrt{g})(1-icx)}\right)}{4c^2g}
\end{aligned}$$

output
```
-1/2*b*(d-e)*x/c+b*e*x/c+1/2*b*(d-e)*arctan(c*x)/c^2+1/2*d*x^2*(a+b*arctan
(c*x))-1/2*e*x^2*(a+b*arctan(c*x))-b*e*f^(1/2)*arctan(g^(1/2)*x/f^(1/2))/c
/g^(1/2)-b*e*(c^2*f-g)*arctan(c*x)*ln(2/(1-I*c*x))/c^2/g+1/2*b*e*(c^2*f-g)
*arctan(c*x)*ln(2*c*((-f)^(1/2)-g^(1/2)*x)/(c*(-f)^(1/2)-I*g^(1/2))/(1-I*c
*x))/c^2/g+1/2*b*e*(c^2*f-g)*arctan(c*x)*ln(2*c*((-f)^(1/2)+g^(1/2)*x)/(c*
(-f)^(1/2)+I*g^(1/2))/(1-I*c*x))/c^2/g-1/2*b*e*x*ln(g*x^2+f)/c-1/2*b*e*(c^
2*f-g)*arctan(c*x)*ln(g*x^2+f)/c^2/g+1/2*e*(g*x^2+f)*(a+b*arctan(c*x))*ln(
g*x^2+f)/g+1/2*I*b*e*(c^2*f-g)*polylog(2,1-2/(1-I*c*x))/c^2/g-1/4*I*b*e*(c
^2*f-g)*polylog(2,1-2*c*((-f)^(1/2)-g^(1/2)*x)/(c*(-f)^(1/2)-I*g^(1/2))/(1
-I*c*x))/c^2/g-1/4*I*b*e*(c^2*f-g)*polylog(2,1-2*c*((-f)^(1/2)+g^(1/2)*x)/
(c*(-f)^(1/2)+I*g^(1/2))/(1-I*c*x))/c^2/g
```

## Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1140 vs. $2(562) = 1124$.

Time = 6.14 (sec) , antiderivative size = 1140, normalized size of antiderivative = 2.03

$$\int x(a + b\arctan(cx)) \left(d + e\log\left(f + gx^2\right)\right) dx = \text{Too large to display}$$

input
```
Integrate[x*(a + b*ArcTan[c*x])*(d + e*Log[f + g*x^2]),x]
```

output
```
(-2*b*c*d*g*x + 6*b*c*e*g*x + 2*a*c^2*d*g*x^2 - 2*a*c^2*e*g*x^2 + 2*b*d*g*
ArcTan[c*x] - 2*b*e*g*ArcTan[c*x] + 2*b*c^2*d*g*x^2*ArcTan[c*x] - 2*b*c^2*
e*g*x^2*ArcTan[c*x] - 4*b*c*e*Sqrt[f]*Sqrt[g]*ArcTan[(Sqrt[g]*x)/Sqrt[f]]
+ (4*I)*b*c^2*e*f*ArcSin[Sqrt[(c^2*f)/(c^2*f - g)]]*ArcTan[(c*g*x)/Sqrt[c^
2*f*g]] - (4*I)*b*e*g*ArcSin[Sqrt[(c^2*f)/(c^2*f - g)]]*ArcTan[(c*g*x)/Sqr
t[c^2*f*g]] - 4*b*c^2*e*f*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] + 4*b
*e*g*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] + 2*b*c^2*e*f*ArcSin[Sqrt[
(c^2*f)/(c^2*f - g)]]*Log[(c^2*(1 + E^((2*I)*ArcTan[c*x]))*f + (-1 + E^((2
*I)*ArcTan[c*x]))*g - 2*E^((2*I)*ArcTan[c*x])*Sqrt[c^2*f*g])/(c^2*f - g)]
- 2*b*e*g*ArcSin[Sqrt[(c^2*f)/(c^2*f - g)]]*Log[(c^2*(1 + E^((2*I)*ArcTan[
c*x]))*f + (-1 + E^((2*I)*ArcTan[c*x]))*g - 2*E^((2*I)*ArcTan[c*x])*Sqrt[c
^2*f*g])/(c^2*f - g)] + 2*b*c^2*e*f*ArcTan[c*x]*Log[(c^2*(1 + E^((2*I)*Arc
Tan[c*x]))*f + (-1 + E^((2*I)*ArcTan[c*x]))*g - 2*E^((2*I)*ArcTan[c*x])*Sq
rt[c^2*f*g])/(c^2*f - g)] - 2*b*e*g*ArcTan[c*x]*Log[(c^2*(1 + E^((2*I)*Arc
Tan[c*x]))*f + (-1 + E^((2*I)*ArcTan[c*x]))*g - 2*E^((2*I)*ArcTan[c*x])*Sq
rt[c^2*f*g])/(c^2*f - g)] - 2*b*c^2*e*f*ArcSin[Sqrt[(c^2*f)/(c^2*f - g)]]*
Log[1 + (E^((2*I)*ArcTan[c*x])*(c^2*f + g + 2*Sqrt[c^2*f*g]))/(c^2*f - g)]
 + 2*b*e*g*ArcSin[Sqrt[(c^2*f)/(c^2*f - g)]]*Log[1 + (E^((2*I)*ArcTan[c*x]
)*(c^2*f + g + 2*Sqrt[c^2*f*g]))/(c^2*f - g)] + 2*b*c^2*e*f*ArcTan[c*x]*Lo
g[1 + (E^((2*I)*ArcTan[c*x])*(c^2*f + g + 2*Sqrt[c^2*f*g]))/(c^2*f - g)...
```

## Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 554, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{5554, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)\,dx$$

$$\Big\downarrow \text{ 5554}$$

$$-bc\int\left(\frac{(d-e)x^2}{2\left(c^2x^2+1\right)}+\frac{e\left(gx^2+f\right)\log\left(gx^2+f\right)}{2g\left(c^2x^2+1\right)}\right)dx+\frac{1}{2}dx^2(a+b\arctan(cx))+\frac{e\left(f+gx^2\right)\log\left(f+gx^2\right)(a+b\arctan(cx))}{2g}-\frac{1}{2}ex^2(a+b\arctan(cx))$$

$$\downarrow \text{2009}$$

$$\frac{1}{2}dx^2(a+b\arctan(cx))+\frac{e(f+gx^2)\log(f+gx^2)(a+b\arctan(cx))}{2g}-\frac{1}{2}ex^2(a+b\arctan(cx))-bc\left(-\frac{(d-e)\arctan(cx)}{2c^3}+\frac{e\sqrt{f}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{c^2\sqrt{g}}+\frac{e\arctan(cx)(c^2f-g)\log(f+gx^2)}{2c^3g}+\frac{e\arctan(cx)(c^2f-g)}{c^3g}\right.$$

input

```
Int[x*(a + b*ArcTan[c*x])*(d + e*Log[f + g*x^2]),x]
```

output

```
(d*x^2*(a + b*ArcTan[c*x]))/2 - (e*x^2*(a + b*ArcTan[c*x]))/2 + (e*(f + g*
x^2)*(a + b*ArcTan[c*x])*Log[f + g*x^2])/(2*g) - b*c*(((d - e)*x)/(2*c^2)
- (e*x)/c^2 - ((d - e)*ArcTan[c*x])/(2*c^3) + (e*Sqrt[f]*ArcTan[(Sqrt[g]*x
)/Sqrt[f]])/(c^2*Sqrt[g]) + (e*(c^2*f - g)*ArcTan[c*x]*Log[2/(1 - I*c*x)])
/(c^3*g) - (e*(c^2*f - g)*ArcTan[c*x]*Log[(2*c*(Sqrt[-f] - Sqrt[g]*x))/((c
*Sqrt[-f] - I*Sqrt[g])*(1 - I*c*x))])/(2*c^3*g) - (e*(c^2*f - g)*ArcTan[c*
x]*Log[(2*c*(Sqrt[-f] + Sqrt[g]*x))/((c*Sqrt[-f] + I*Sqrt[g])*(1 - I*c*x))
])/(2*c^3*g) + (e*x*Log[f + g*x^2])/(2*c^2) + (e*(c^2*f - g)*ArcTan[c*x]*L
og[f + g*x^2])/(2*c^3*g) - ((I/2)*e*(c^2*f - g)*PolyLog[2, 1 - 2/(1 - I*c*
x)])/(c^3*g) + ((I/4)*e*(c^2*f - g)*PolyLog[2, 1 - (2*c*(Sqrt[-f] - Sqrt[g
]*x))/((c*Sqrt[-f] - I*Sqrt[g])*(1 - I*c*x))])/(c^3*g) + ((I/4)*e*(c^2*f -
 g)*PolyLog[2, 1 - (2*c*(Sqrt[-f] + Sqrt[g]*x))/((c*Sqrt[-f] + I*Sqrt[g])*
(1 - I*c*x))])/(c^3*g))
```

### Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5554

```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*Log[f + g*x^2])
, x]}, Simp[(a + b*ArcTan[c*x])   u, x] - Simp[b*c   Int[ExpandIntegrand[u/
(1 + c^2*x^2), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && IGtQ[(m +
1)/2, 0]
```

## Maple [F]

$$\int x(a+b\arctan(cx))\left(d+e\ln\left(g\,x^{2}+f\right)\right)dx$$

input
```
int(x*(a+b*arctan(c*x))*(d+e*ln(g*x^2+f)),x)
```

output
```
int(x*(a+b*arctan(c*x))*(d+e*ln(g*x^2+f)),x)
```

## Fricas [F]

$$\int x(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)\,dx$$
$$=\int(b\arctan(cx)+a)\left(e\log\left(gx^2+f\right)+d\right)x\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))*(d+e*log(g*x^2+f)),x, algorithm="fricas")
```

output
```
integral(b*d*x*arctan(c*x) + a*d*x + (b*e*x*arctan(c*x) + a*e*x)*log(g*x^2
 + f), x)
```

## Sympy [F(-1)]

Timed out.

$$\int x(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)\,dx=\text{Timed out}$$

input
```
integrate(x*(a+b*atan(c*x))*(d+e*ln(g*x**2+f)),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int x(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)\,dx$$
$$=\int(b\arctan(cx)+a)\left(e\log\left(gx^2+f\right)+d\right)x\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))*(d+e*log(g*x^2+f)),x, algorithm="maxima")
```

output
```
1/2*a*d*x^2 + 1/2*(x^2*arctan(c*x) - c*(x/c^2 - arctan(c*x)/c^3))*b*d - 1/
2*(g*x^2 - (g*x^2 + f)*log(g*x^2 + f) + f)*a*e/g - 1/2*(2*c*f*arctan(g*x/s
qrt(f*g)) + (4*c^4*g*integrate(1/2*x^3*arctan(c*x)/(c^2*g*x^2 + c^2*f), x)
 + 4*c^2*g*integrate(1/2*x*arctan(c*x)/(c^2*g*x^2 + c^2*f), x) - 2*c*x + (
c*x - (c^2*x^2 + 1)*arctan(c*x))*log(g*x^2 + f))*sqrt(f*g))*b*e/(sqrt(f*g)
*c^2)
```

## Giac [F]

$$\int x(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)\,dx$$
$$=\int(b\arctan(cx)+a)\left(e\log\left(gx^2+f\right)+d\right)x\,dx$$

input
```
integrate(x*(a+b*arctan(c*x))*(d+e*log(g*x^2+f)),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)*(e*log(g*x^2 + f) + d)*x, x)
```

## Mupad [F(-1)]

Timed out.

$$\int x(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)\,dx$$
$$=\int x\,(a+b\,\mathrm{atan}(c\,x))\,\left(d+e\,\ln\left(g\,x^2+f\right)\right)\,dx$$

input
```
int(x*(a + b*atan(c*x))*(d + e*log(f + g*x^2)),x)
```

output
```
int(x*(a + b*atan(c*x))*(d + e*log(f + g*x^2)), x)
```

## Reduce [F]

$$\int x(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)\,dx$$
$$=\frac{-2\sqrt{g}\,\sqrt{f}\,atan\left(\frac{gx}{\sqrt{g}\,\sqrt{f}}\right)bce+atan(cx)\log(g\,x^2+f)\,b\,c^2ef+atan(cx)\log(g\,x^2+f)\,b\,c^2eg\,x^2+atan(cx}{}$$

input
```
int(x*(a+b*atan(c*x))*(d+e*log(g*x^2+f)),x)
```

output
```
( - 2*sqrt(g)*sqrt(f)*atan((g*x)/(sqrt(g)*sqrt(f)))*b*c*e + atan(c*x)*log(
f + g*x**2)*b*c**2*e*f + atan(c*x)*log(f + g*x**2)*b*c**2*e*g*x**2 + atan(
c*x)*b*c**2*d*g*x**2 - atan(c*x)*b*c**2*e*g*x**2 + atan(c*x)*b*d*g - atan(
c*x)*b*e*g - int(log(f + g*x**2)/(c**2*x**2 + 1),x)*b*c**3*e*f + int(log(f
 + g*x**2)/(c**2*x**2 + 1),x)*b*c*e*g + log(f + g*x**2)*a*c**2*e*f + log(f
 + g*x**2)*a*c**2*e*g*x**2 - log(f + g*x**2)*b*c*e*g*x + a*c**2*d*g*x**2 -
 a*c**2*e*g*x**2 - b*c*d*g*x + 3*b*c*e*g*x)/(2*c**2*g)
```

# 3.1296 $\int (a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)\,dx$



## Optimal result

Integrand size = 21, antiderivative size = 656

$$\begin{aligned}
&\int (a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)\,dx\\
&= -2aex-2bex\arctan(cx)+\frac{2ae\sqrt{f}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{g}}\\
&\quad+\frac{ibe\sqrt{-f}\log(1+icx)\log\left(\frac{c\left(\sqrt{-f}-\sqrt{g}x\right)}{c\sqrt{-f}-i\sqrt{g}}\right)}{2\sqrt{g}}-\frac{ibe\sqrt{-f}\log(1-icx)\log\left(\frac{c\left(\sqrt{-f}-\sqrt{g}x\right)}{c\sqrt{-f}+i\sqrt{g}}\right)}{2\sqrt{g}}\\
&\quad+\frac{ibe\sqrt{-f}\log(1-icx)\log\left(\frac{c\left(\sqrt{-f}+\sqrt{g}x\right)}{c\sqrt{-f}-i\sqrt{g}}\right)}{2\sqrt{g}}-\frac{ibe\sqrt{-f}\log(1+icx)\log\left(\frac{c\left(\sqrt{-f}+\sqrt{g}x\right)}{c\sqrt{-f}+i\sqrt{g}}\right)}{2\sqrt{g}}\\
&\quad+\frac{be\log\left(1+c^2x^2\right)}{c}+x(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)\\
&\quad-\frac{b\log\left(-\frac{g\left(1+c^2x^2\right)}{c^2f-g}\right)\left(d+e\log\left(f+gx^2\right)\right)}{2c}-\frac{ibe\sqrt{-f}\,\mathrm{PolyLog}\left(2,\frac{\sqrt{g}(i-cx)}{c\sqrt{-f}+i\sqrt{g}}\right)}{2\sqrt{g}}\\
&\quad+\frac{ibe\sqrt{-f}\,\mathrm{PolyLog}\left(2,\frac{\sqrt{g}(1-icx)}{ic\sqrt{-f}+\sqrt{g}}\right)}{2\sqrt{g}}+\frac{ibe\sqrt{-f}\,\mathrm{PolyLog}\left(2,\frac{\sqrt{g}(1+icx)}{ic\sqrt{-f}+\sqrt{g}}\right)}{2\sqrt{g}}\\
&\quad-\frac{ibe\sqrt{-f}\,\mathrm{PolyLog}\left(2,\frac{\sqrt{g}(i+cx)}{c\sqrt{-f}+i\sqrt{g}}\right)}{2\sqrt{g}}-\frac{be\,\mathrm{PolyLog}\left(2,\frac{c^2\left(f+gx^2\right)}{c^2f-g}\right)}{2c}
\end{aligned}$$

output
```
-2*a*e*x-2*b*e*x*arctan(c*x)+2*a*e*f^(1/2)*arctan(g^(1/2)*x/f^(1/2))/g^(1/
2)+1/2*I*b*e*(-f)^(1/2)*ln(1+I*c*x)*ln(c*((-f)^(1/2)-g^(1/2)*x)/(c*(-f)^(1
/2)-I*g^(1/2)))/g^(1/2)-1/2*I*b*e*(-f)^(1/2)*ln(1-I*c*x)*ln(c*((-f)^(1/2)-
g^(1/2)*x)/(c*(-f)^(1/2)+I*g^(1/2)))/g^(1/2)+1/2*I*b*e*(-f)^(1/2)*ln(1-I*c
*x)*ln(c*((-f)^(1/2)+g^(1/2)*x)/(c*(-f)^(1/2)-I*g^(1/2)))/g^(1/2)-1/2*I*b*
e*(-f)^(1/2)*ln(1+I*c*x)*ln(c*((-f)^(1/2)+g^(1/2)*x)/(c*(-f)^(1/2)+I*g^(1/
2)))/g^(1/2)+b*e*ln(c^2*x^2+1)/c+x*(a+b*arctan(c*x))*(d+e*ln(g*x^2+f))-1/2
*b*ln(-g*(c^2*x^2+1)/(c^2*f-g))*(d+e*ln(g*x^2+f))/c-1/2*I*b*e*(-f)^(1/2)*p
olylog(2,g^(1/2)*(I-c*x)/(c*(-f)^(1/2)+I*g^(1/2)))/g^(1/2)+1/2*I*b*e*(-f)^
(1/2)*polylog(2,g^(1/2)*(1-I*c*x)/(I*c*(-f)^(1/2)+g^(1/2)))/g^(1/2)+1/2*I*
b*e*(-f)^(1/2)*polylog(2,g^(1/2)*(1+I*c*x)/(I*c*(-f)^(1/2)+g^(1/2)))/g^(1/
2)-1/2*I*b*e*(-f)^(1/2)*polylog(2,g^(1/2)*(I+c*x)/(c*(-f)^(1/2)+I*g^(1/2))
)/g^(1/2)-1/2*b*e*polylog(2,c^2*(g*x^2+f)/(c^2*f-g))/c
```

## Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1352 vs. 2(656) = 1312.

Time = 3.00 (sec) , antiderivative size = 1352, normalized size of antiderivative = 2.06

$$\int (a + b \arctan(cx)) \left(d + e \log\left(f + gx^2\right)\right) dx = \text{Too large to display}$$

input
```
Integrate[(a + b*ArcTan[c*x])*(d + e*Log[f + g*x^2]),x]
```

output
```
a*d*x - 2*a*e*x + b*d*x*ArcTan[c*x] + (2*a*e*Sqrt[f]*ArcTan[(Sqrt[g]*x)/Sq
rt[f]])/Sqrt[g] - (b*d*Log[1 + c^2*x^2])/(2*c) + a*e*x*Log[f + g*x^2] + b*
e*(x*ArcTan[c*x] - Log[1 + c^2*x^2]/(2*c))*Log[f + g*x^2] + (b*e*g*(((-Log
[(-I)/c + x] - Log[I/c + x] + Log[1 + c^2*x^2])*Log[f + g*x^2])/(2*g) + (L
og[(-I)/c + x]*Log[1 - (Sqrt[g]*((-I)/c + x))/((-I)*Sqrt[f] - (I*Sqrt[g])/
c)] + PolyLog[2, (Sqrt[g]*((-I)/c + x))/((-I)*Sqrt[f] - (I*Sqrt[g])/c)])/(
2*g) + (Log[(-I)/c + x]*Log[1 - (Sqrt[g]*((-I)/c + x))/(I*Sqrt[f] - (I*Sqr
t[g])/c)] + PolyLog[2, (Sqrt[g]*((-I)/c + x))/(I*Sqrt[f] - (I*Sqrt[g])/c)]
)/(2*g) + (Log[I/c + x]*Log[1 - (Sqrt[g]*(I/c + x))/((-I)*Sqrt[f] + (I*Sqr
t[g])/c)] + PolyLog[2, (Sqrt[g]*(I/c + x))/((-I)*Sqrt[f] + (I*Sqrt[g])/c)]
)/(2*g) + (Log[I/c + x]*Log[1 - (Sqrt[g]*(I/c + x))/(I*Sqrt[f] + (I*Sqrt[g
])/c)] + PolyLog[2, (Sqrt[g]*(I/c + x))/(I*Sqrt[f] + (I*Sqrt[g])/c)])/(2*g
)))/c - (b*e*(4*c*x*ArcTan[c*x] + 4*Log[1/Sqrt[1 + c^2*x^2]] + (c^2*f*(4*A
rcTan[c*x]*ArcTanh[Sqrt[-(c^2*f*g)]/(c*g*x)] - 2*ArcCos[(c^2*f + g)/(-(c^2
*f) + g)]*ArcTanh[(c*g*x)/Sqrt[-(c^2*f*g)]] - (ArcCos[(c^2*f + g)/(-(c^2*f
) + g)] - (2*I)*ArcTanh[(c*g*x)/Sqrt[-(c^2*f*g)]])*Log[(-2*c^2*f*(I*g + Sq
rt[-(c^2*f*g)])*(-I + c*x))/((c^2*f - g)*(c^2*f - c*Sqrt[-(c^2*f*g)]*x))]
- (ArcCos[(c^2*f + g)/(-(c^2*f) + g)] + (2*I)*ArcTanh[(c*g*x)/Sqrt[-(c^2*f
*g)]])*Log[((2*I)*c^2*f*(g + I*Sqrt[-(c^2*f*g)])*(I + c*x))/((c^2*f - g)*(
c^2*f - c*Sqrt[-(c^2*f*g)]*x))] + (ArcCos[(c^2*f + g)/(-(c^2*f) + g)] -...
```

## Rubi [A] (verified)

Time = 1.77 (sec) , antiderivative size = 653, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {5544, 2925, 2841, 2840, 2838, 5451, 2009, 5445, 218, 5443, 2856, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)dx$$

$$\Big\downarrow \text{5544}$$

$$-2eg\int \frac{x^2(a+b\arctan(cx))}{gx^2+f}dx-bc\int \frac{x\left(d+e\log\left(gx^2+f\right)\right)}{c^2x^2+1}dx+x(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)$$

$$\Big\downarrow \text{2925}$$

$$-2eg\int \frac{x^2(a+b\arctan(cx))}{gx^2+f}dx-\frac{1}{2}bc\int \frac{d+e\log\left(gx^2+f\right)}{c^2x^2+1}dx^2+x(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)$$

$$\Big\downarrow \ 2841$$

$$-2eg\int \frac{x^2(a+b\arctan(cx))}{gx^2+f}dx-\frac{1}{2}bc\left(\frac{\log\left(-\frac{g\left(c^2x^2+1\right)}{c^2f-g}\right)\left(d+e\log\left(f+gx^2\right)\right)}{c^2}-\frac{eg\int \frac{\log\left(-\frac{g\left(c^2x^2+1\right)}{c^2f-g}\right)}{gx^2+f}dx^2}{c^2}\right)+x(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)$$

$$\Big\downarrow \ 2840$$

$$-2eg\int \frac{x^2(a+b\arctan(cx))}{gx^2+f}dx-\frac{1}{2}bc\left(\frac{\log\left(-\frac{g\left(c^2x^2+1\right)}{c^2f-g}\right)\left(d+e\log\left(f+gx^2\right)\right)}{c^2}-\frac{e\int \frac{\log\left(1-\frac{c^2\left(gx^2+f\right)}{c^2f-g}\right)}{x^2}d\left(gx^2+f\right)}{c^2}\right)+x(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)$$

$$\Big\downarrow \ 2838$$

$$-2eg\int \frac{x^2(a+b\arctan(cx))}{gx^2+f}dx+x(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)-\frac{1}{2}bc\left(\frac{\log\left(-\frac{g\left(c^2x^2+1\right)}{c^2f-g}\right)\left(d+e\log\left(f+gx^2\right)\right)}{c^2}+\frac{e\,\text{PolyLog}\left(2,\frac{c^2\left(gx^2+f\right)}{c^2f-g}\right)}{c^2}\right)$$

$$\Big\downarrow \ 5451$$

$$-2eg\left(\frac{\int (a+b\arctan(cx))dx}{g}-\frac{f\int \frac{a+b\arctan(cx)}{gx^2+f}dx}{g}\right)+x(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)-\frac{1}{2}bc\left(\frac{\log\left(-\frac{g\left(c^2x^2+1\right)}{c^2f-g}\right)\left(d+e\log\left(f+gx^2\right)\right)}{c^2}+\frac{e\,\text{PolyLog}\left(2,\frac{c^2\left(gx^2+f\right)}{c^2f-g}\right)}{c^2}\right)$$

$$\Big\downarrow \ 2009$$

$$-2eg\left(\frac{ax+bx\arctan(cx)-\frac{b\log(c^2x^2+1)}{2c}}{g}-\frac{f\int\frac{a+b\arctan(cx)}{gx^2+f}dx}{g}\right)+x(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)-\frac{1}{2}bc\left(\frac{\log\left(-\frac{g(c^2x^2+1)}{c^2f-g}\right)\left(d+e\log\left(f+gx^2\right)\right)}{c^2}+\frac{e\,\text{PolyLog}\left(2,\frac{c^2(gx^2+f)}{c^2f-g}\right)}{c^2}\right)$$

$$\Big\downarrow \; 5445$$

$$-2eg\left(\frac{ax+bx\arctan(cx)-\frac{b\log(c^2x^2+1)}{2c}}{g}-\frac{f\left(a\int\frac{1}{gx^2+f}dx+b\int\frac{\arctan(cx)}{gx^2+f}dx\right)}{g}\right)+x(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)-\frac{1}{2}bc\left(\frac{\log\left(-\frac{g(c^2x^2+1)}{c^2f-g}\right)\left(d+e\log\left(f+gx^2\right)\right)}{c^2}+\frac{e\,\text{PolyLog}\left(2,\frac{c^2(gx^2+f)}{c^2f-g}\right)}{c^2}\right)$$

$$\Big\downarrow \; 218$$

$$-2eg\left(\frac{ax+bx\arctan(cx)-\frac{b\log(c^2x^2+1)}{2c}}{g}-\frac{f\left(b\int\frac{\arctan(cx)}{gx^2+f}dx+\frac{a\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}}\right)}{g}\right)+x(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)-\frac{1}{2}bc\left(\frac{\log\left(-\frac{g(c^2x^2+1)}{c^2f-g}\right)\left(d+e\log\left(f+gx^2\right)\right)}{c^2}+\frac{e\,\text{PolyLog}\left(2,\frac{c^2(gx^2+f)}{c^2f-g}\right)}{c^2}\right)$$

$$\Big\downarrow \; 5443$$

$$-2eg\left(\frac{ax+bx\arctan(cx)-\frac{b\log(c^2x^2+1)}{2c}}{g}-\frac{f\left(\frac{a\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}}+b\left(\frac{1}{2}i\int\frac{\log(1-icx)}{gx^2+f}dx-\frac{1}{2}i\int\frac{\log(icx+1)}{gx^2+f}dx\right)\right)}{g}\right)+x(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)-\frac{1}{2}bc\left(\frac{\log\left(-\frac{g(c^2x^2+1)}{c^2f-g}\right)\left(d+e\log\left(f+gx^2\right)\right)}{c^2}+\frac{e\,\text{PolyLog}\left(2,\frac{c^2(gx^2+f)}{c^2f-g}\right)}{c^2}\right)$$

$$\Big\downarrow \; 2856$$

$$-2eg\left(\frac{ax+bx\arctan(cx)-\frac{b\log(c^2x^2+1)}{2c}}{g}-\frac{f\left(\frac{a\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}}+b\left(\frac{1}{2}i\int\left(\frac{\sqrt{-f}\log(1-icx)}{2f(\sqrt{-f}-\sqrt{g}x)}+\frac{\sqrt{-f}\log(1-icx)}{2f(\sqrt{g}x+\sqrt{-f})}\right)dx-\frac{1}{2}i\int\right.}{g}\right.$$

$$x(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)-\frac{1}{2}bc\left(\frac{\log\left(-\frac{g(c^2x^2+1)}{c^2f-g}\right)\left(d+e\log\left(f+gx^2\right)\right)}{c^2}+\frac{e\,\text{PolyLog}\left(2,\frac{c^2(gx^2+f)}{c^2f-g}\right)}{c^2}\right)$$

$$\Big\downarrow \text{2009}$$

$$-2eg\left(\frac{ax+bx\arctan(cx)-\frac{b\log(c^2x^2+1)}{2c}}{g}-\frac{f\left(\frac{a\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}}+b\left(\frac{1}{2}i\left(-\frac{\text{PolyLog}\left(2,\frac{\sqrt{g}(1-icx)}{i\sqrt{-f}c+\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}}+\frac{\text{PolyLog}\left(2,\frac{\sqrt{g}(cx+i)}{\sqrt{-f}c+i\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}}\right.\right.\right.}{}\right.$$

$$x(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)-\frac{1}{2}bc\left(\frac{\log\left(-\frac{g(c^2x^2+1)}{c^2f-g}\right)\left(d+e\log\left(f+gx^2\right)\right)}{c^2}+\frac{e\,\text{PolyLog}\left(2,\frac{c^2(gx^2+f)}{c^2f-g}\right)}{c^2}\right)$$

input
```
Int[(a + b*ArcTan[c*x])*(d + e*Log[f + g*x^2]),x]
```

output
```
x*(a + b*ArcTan[c*x])*(d + e*Log[f + g*x^2]) - 2*e*g*((a*x + b*x*ArcTan[c*
x] - (b*Log[1 + c^2*x^2])/(2*c))/g - (f*((a*ArcTan[(Sqrt[g]*x)/Sqrt[f]])/(
Sqrt[f]*Sqrt[g]) + b*((-1/2*I)*((Log[1 + I*c*x]*Log[(c*(Sqrt[-f] - Sqrt[g]
*x))/(c*Sqrt[-f] - I*Sqrt[g])])/(2*Sqrt[-f]*Sqrt[g]) - (Log[1 + I*c*x]*Log
[(c*(Sqrt[-f] + Sqrt[g]*x))/(c*Sqrt[-f] + I*Sqrt[g])])/(2*Sqrt[-f]*Sqrt[g]
) - PolyLog[2, (Sqrt[g]*(I - c*x))/(c*Sqrt[-f] + I*Sqrt[g])]/(2*Sqrt[-f]*S
qrt[g]) + PolyLog[2, (Sqrt[g]*(1 + I*c*x))/(I*c*Sqrt[-f] + Sqrt[g])]/(2*Sq
rt[-f]*Sqrt[g])) + (I/2)*((Log[1 - I*c*x]*Log[(c*(Sqrt[-f] - Sqrt[g]*x))/(
c*Sqrt[-f] + I*Sqrt[g])])/(2*Sqrt[-f]*Sqrt[g]) - (Log[1 - I*c*x]*Log[(c*(S
qrt[-f] + Sqrt[g]*x))/(c*Sqrt[-f] - I*Sqrt[g])])/(2*Sqrt[-f]*Sqrt[g]) - Po
lyLog[2, (Sqrt[g]*(1 - I*c*x))/(I*c*Sqrt[-f] + Sqrt[g])]/(2*Sqrt[-f]*Sqrt[
g]) + PolyLog[2, (Sqrt[g]*(I + c*x))/(c*Sqrt[-f] + I*Sqrt[g])]/(2*Sqrt[-f]
*Sqrt[g])))))/g) - (b*c*((Log[-((g*(1 + c^2*x^2))/(c^2*f - g))]*(d + e*Log
[f + g*x^2]))/c^2 + (e*PolyLog[2, (c^2*(f + g*x^2))/(c^2*f - g)])/c^2))/2
```

### Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2840
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Simp[1/g   Subst[Int[(a + b*Log[1 + c*e*(x/g)])/x, x], x, f + g*
x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c
*(e*f - d*g), 0]
```

rule 2841
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[Log[e*((f + g*x)/(e*f - d*g))]*((a + b*Log[c*(d + e*x
)^n])/g), x] - Simp[b*e*(n/g)   Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

rule 2856
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

rule 5443
```
Int[ArcTan[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Simp[I/2   Int[
Log[1 - I*c*x]/(d + e*x^2), x], x] - Simp[I/2   Int[Log[1 + I*c*x]/(d + e*x
^2), x], x] /; FreeQ[{c, d, e}, x]
```

rule 5445
```
Int[(ArcTan[(c_.)*(x_)]*(b_.) + (a_))/((d_.) + (e_.)*(x_)^2), x_Symbol] :>
Simp[a   Int[1/(d + e*x^2), x], x] + Simp[b   Int[ArcTan[c*x]/(d + e*x^2),
x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 5451
```
Int[(((a_.) + ArcTan[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (e
_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTan[c*x]
)^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTan[c*x])^p/(d
+ e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m, 1]
```

rule 5544
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.)), x_Symbol] :> Simp[x*(d + e*Log[f + g*x^2])*(a + b*ArcTan[c*x]), x] +
 (-Simp[b*c   Int[x*((d + e*Log[f + g*x^2])/(1 + c^2*x^2)), x], x] - Simp[2
*e*g   Int[x^2*((a + b*ArcTan[c*x])/(f + g*x^2)), x], x]) /; FreeQ[{a, b, c
, d, e, f, g}, x]
```

## Maple [F]

$$\int (a+b\arctan(cx))\left(d+e\ln\left(g\,x^2+f\right)\right)dx$$

input
```
int((a+b*arctan(c*x))*(d+e*ln(g*x^2+f)),x)
```

output
```
int((a+b*arctan(c*x))*(d+e*ln(g*x^2+f)),x)
```

## Fricas [F]

$$\int (a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)\,dx$$
$$=\int (b\arctan(cx)+a)\left(e\log\left(gx^2+f\right)+d\right)dx$$

input
```
integrate((a+b*arctan(c*x))*(d+e*log(g*x^2+f)),x, algorithm="fricas")
```

output
```
integral(b*d*arctan(c*x) + a*d + (b*e*arctan(c*x) + a*e)*log(g*x^2 + f), x
)
```

## Sympy [F(-1)]

Timed out.

$$\int (a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)\,dx=\text{Timed out}$$

input
```
integrate((a+b*atan(c*x))*(d+e*ln(g*x**2+f)),x)
```

output
```
Timed out
```

## Maxima [F]

$$\int (a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)\,dx$$
$$=\int (b\arctan(cx)+a)\left(e\log\left(gx^2+f\right)+d\right)dx$$

input
```
integrate((a+b*arctan(c*x))*(d+e*log(g*x^2+f)),x, algorithm="maxima")
```

output
```
(2*g*(f*arctan(g*x/sqrt(f*g))/(sqrt(f*g)*g) - x/g) + x*log(g*x^2 + f))*a*e
 + a*d*x + b*e*integrate(arctan(c*x)*log(g*x^2 + f), x) + 1/2*(2*c*x*arcta
n(c*x) - log(c^2*x^2 + 1))*b*d/c
```

## Giac [F]

$$\int (a + b\arctan(cx)) \left(d + e\log\left(f + gx^2\right)\right) dx = \int (b\arctan(cx) + a)\left(e\log\left(gx^2 + f\right) + d\right) dx$$

input
```
integrate((a+b*arctan(c*x))*(d+e*log(g*x^2+f)),x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)*(e*log(g*x^2 + f) + d), x)
```

## Mupad [F(-1)]

Timed out.

$$\int (a + b\arctan(cx)) \left(d + e\log\left(f + gx^2\right)\right) dx = \int (a + b\,\mathrm{atan}(c\,x)) \left(d + e\,\ln\left(g\,x^2 + f\right)\right) dx$$

input
```
int((a + b*atan(c*x))*(d + e*log(f + g*x^2)),x)
```

output
```
int((a + b*atan(c*x))*(d + e*log(f + g*x^2)), x)
```

## Reduce [F]

$$\int (a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)\,dx$$
$$=\frac{8\sqrt{g}\,\sqrt{f}\,atan\left(\frac{gx}{\sqrt{g}\,\sqrt{f}}\right)ace+4atan(cx)^2\,b\,c^2ef+4atan(cx)\log(g\,x^2+f)\,bcegx+4atan(cx)\,bcdgx-8at$$

input
```
int((a+b*atan(c*x))*(d+e*log(g*x^2+f)),x)
```

output
```
(8*sqrt(g)*sqrt(f)*atan((g*x)/(sqrt(g)*sqrt(f)))*a*c*e + 4*atan(c*x)**2*b*
c**2*e*f + 4*atan(c*x)*log(f + g*x**2)*b*c*e*g*x + 4*atan(c*x)*b*c*d*g*x -
 8*atan(c*x)*b*c*e*g*x - 8*int(atan(c*x)/(c**2*f*x**2 + c**2*g*x**4 + f +
g*x**2),x)*b*c**3*e*f**2 + 8*int(atan(c*x)/(c**2*f*x**2 + c**2*g*x**4 + f
+ g*x**2),x)*b*c*e*f*g - 4*int((log(f + g*x**2)*x)/(c**2*f*x**2 + c**2*g*x
**4 + f + g*x**2),x)*b*c**2*e*f*g + 4*int((log(f + g*x**2)*x)/(c**2*f*x**2
 + c**2*g*x**4 + f + g*x**2),x)*b*e*g**2 - 2*log(c**2*x**2 + 1)*b*d*g + 4*
log(c**2*x**2 + 1)*b*e*g - log(f + g*x**2)**2*b*e*g + 4*log(f + g*x**2)*a*
c*e*g*x + 4*a*c*d*g*x - 8*a*c*e*g*x)/(4*c*g)
```

# 3.1297 $\int \frac{(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 24

$$\begin{aligned}
&\int \frac{(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x}\,dx \\
&= ad\log(x)+\frac{1}{2}ae\log\left(-\frac{gx^2}{f}\right)\log\left(f+gx^2\right) \\
&\quad+\frac{1}{2}ibd\,\text{PolyLog}(2,-icx)-\frac{1}{2}ibd\,\text{PolyLog}(2,icx) \\
&\quad+\frac{1}{2}ae\,\text{PolyLog}\left(2,1+\frac{gx^2}{f}\right)+be\text{Int}\left(\frac{\arctan(cx)\log\left(f+gx^2\right)}{x},x\right)
\end{aligned}$$

output
```
a*d*ln(x)+1/2*a*e*ln(-g*x^2/f)*ln(g*x^2+f)+1/2*I*b*d*polylog(2,-I*c*x)-1/2
*I*b*d*polylog(2,I*c*x)+1/2*a*e*polylog(2,1+g*x^2/f)+b*e*Defer(Int)(arctan
(c*x)*ln(g*x^2+f)/x,x)
```

## Mathematica [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x}\,dx = \int \frac{(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x}\,dx$$

input
```
Integrate[((a + b*ArcTan[c*x])*(d + e*Log[f + g*x^2]))/x,x]
```

output
```
Integrate[((a + b*ArcTan[c*x])*(d + e*Log[f + g*x^2]))/x, x]
```

## Rubi [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x}\,dx$$

$$\Big\downarrow \text{5550}$$

$$d\int \frac{a+b\arctan(cx)}{x}dx+e\int \frac{(a+b\arctan(cx))\log\left(gx^2+f\right)}{x}dx$$

$$\Big\downarrow \text{5355}$$

$$e\int \frac{(a+b\arctan(cx))\log\left(gx^2+f\right)}{x}dx+ d\left(\frac{1}{2}ib\int \frac{\log(1-icx)}{x}dx-\frac{1}{2}ib\int \frac{\log(icx+1)}{x}dx+a\log(x)\right)$$

$$\Big\downarrow \text{2838}$$

$$e\int \frac{(a+b\arctan(cx))\log\left(gx^2+f\right)}{x}dx+d\left(a\log(x)+\frac{1}{2}ib\,\text{PolyLog}(2,-icx)-\frac{1}{2}ib\,\text{PolyLog}(2,icx)\right)$$

$$\Big\downarrow \text{5548}$$

$$e\left(a\int \frac{\log\left(gx^2+f\right)}{x}dx+b\int \frac{\arctan(cx)\log\left(gx^2+f\right)}{x}dx\right)+d\left(a\log(x)+\frac{1}{2}ib\,\text{PolyLog}(2,-icx)-\frac{1}{2}ib\,\text{PolyLog}(2,icx)\right)$$

$$\Big\downarrow \text{2904}$$

$$e\left(\frac{1}{2}a\int \frac{\log\left(gx^2+f\right)}{x^2}dx^2+b\int \frac{\arctan(cx)\log\left(gx^2+f\right)}{x}dx\right)+d\left(a\log(x)+\frac{1}{2}ib\,\text{PolyLog}(2,-icx)-\frac{1}{2}ib\,\text{PolyLog}(2,icx)\right)$$

$$\Big\downarrow \text{2841}$$

$$e\left(\frac{1}{2}a\left(\log\left(-\frac{gx^2}{f}\right)\log\left(f+gx^2\right)-g\int \frac{\log\left(-\frac{gx^2}{f}\right)}{gx^2+f}dx^2\right)+b\int \frac{\arctan(cx)\log\left(gx^2+f\right)}{x}dx\right)+d\left(a\log(x)+\frac{1}{2}ib\,\text{PolyLog}(2,-icx)-\frac{1}{2}ib\,\text{PolyLog}(2,icx)\right)$$

$$\Big\downarrow \text{2752}$$

$$e\left(b\int \frac{\arctan(cx)\log\left(gx^2+f\right)}{x}dx+\frac{1}{2}a\left(\text{PolyLog}\left(2,\frac{gx^2}{f}+1\right)+\log\left(-\frac{gx^2}{f}\right)\log\left(f+gx^2\right)\right)\right)+d\left(a\log(x)+\frac{1}{2}ib\,\text{PolyLog}(2,-icx)-\frac{1}{2}ib\,\text{PolyLog}(2,icx)\right)$$

$$\Big\downarrow \text{7299}$$

$$e\left(b\int \frac{\arctan(cx)\log\left(gx^2+f\right)}{x}dx+\frac{1}{2}a\left(\text{PolyLog}\left(2,\frac{gx^2}{f}+1\right)+\log\left(-\frac{gx^2}{f}\right)\log\left(f+gx^2\right)\right)\right)+d\left(a\log(x)+\frac{1}{2}ib\,\text{PolyLog}(2,-icx)-\frac{1}{2}ib\,\text{PolyLog}(2,icx)\right)$$

input
```
Int[((a + b*ArcTan[c*x])*(d + e*Log[f + g*x^2]))/x,x]
```

output
```
$Aborted
```

## Maple [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(a + b \arctan (cx)) (d + e \ln (g x^2 + f))}{x} dx$$

input
```
int((a+b*arctan(c*x))*(d+e*ln(g*x^2+f))/x,x)
```

output
```
int((a+b*arctan(c*x))*(d+e*ln(g*x^2+f))/x,x)
```

## Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{(a + b \arctan(cx)) (d + e \log (f + gx^2))}{x} dx = \int \frac{(b \arctan (cx) + a)(e \log (gx^2 + f) + d)}{x} dx$$

input
```
integrate((a+b*arctan(c*x))*(d+e*log(g*x^2+f))/x,x, algorithm="fricas")
```

output
```
integral((b*d*arctan(c*x) + a*d + (b*e*arctan(c*x) + a*e)*log(g*x^2 + f))/
x, x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x}\,dx=\text{Timed out}$$

input
```
integrate((a+b*atan(c*x))*(d+e*ln(g*x**2+f))/x,x)
```

output
```
Timed out
```

## Maxima [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.79

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x}\,dx=\int \frac{(b\arctan(cx)+a)(e\log(gx^2+f)+d)}{x}\,dx$$

input
```
integrate((a+b*arctan(c*x))*(d+e*log(g*x^2+f))/x,x, algorithm="maxima")
```

output
```
a*d*log(x) + 1/2*integrate(2*(b*d*arctan(c*x) + (b*e*arctan(c*x) + a*e)*lo
g(g*x^2 + f))/x, x)
```

## Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x}\,dx=\int \frac{(b\arctan(cx)+a)(e\log(gx^2+f)+d)}{x}\,dx$$

input
```
integrate((a+b*arctan(c*x))*(d+e*log(g*x^2+f))/x,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)*(e*log(g*x^2 + f) + d)/x, x)
```

## Mupad [N/A]

Not integrable

Time = 1.39 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x}\,dx = \int \frac{\left(a+b\,\mathrm{atan}(c\,x)\right)\,\left(d+e\,\ln\left(g\,x^2+f\right)\right)}{x}\,dx$$

input
```
int(((a + b*atan(c*x))*(d + e*log(f + g*x^2)))/x,x)
```

output
```
int(((a + b*atan(c*x))*(d + e*log(f + g*x^2)))/x, x)
```

## Reduce [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 148, normalized size of antiderivative = 6.17

$$\begin{aligned}\int \frac{(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x}\,dx &= \left(\int \frac{atan(cx)}{g\,x^3+fx}dx\right)bdf+\left(\int \frac{\log(g\,x^2+f)}{g\,x^3+fx}dx\right)aef \\ &\quad+\left(\int \frac{atan(cx)\log(g\,x^2+f)\,x}{g\,x^2+f}dx\right)beg+\left(\int \frac{atan(cx)\log(g\,x^2+f)}{g\,x^3+fx}dx\right)bef \\ &\quad+\left(\int \frac{atan(cx)\,x}{g\,x^2+f}dx\right)bdg+\frac{\log(g\,x^2+f)^2\,ae}{4}+\log(x)\,ad\end{aligned}$$

input
```
int((a+b*atan(c*x))*(d+e*log(g*x^2+f))/x,x)
```

output

```
(4*int(atan(c*x)/(f*x + g*x**3),x)*b*d*f + 4*int(log(f + g*x**2)/(f*x + g*
x**3),x)*a*e*f + 4*int((atan(c*x)*log(f + g*x**2)*x)/(f + g*x**2),x)*b*e*g
 + 4*int((atan(c*x)*log(f + g*x**2))/(f*x + g*x**3),x)*b*e*f + 4*int((atan
(c*x)*x)/(f + g*x**2),x)*b*d*g + log(f + g*x**2)**2*a*e + 4*log(x)*a*d)/4
```

# 3.1298 $\int \frac{(a+b\arctan(cx))(d+e\log(f+gx^2))}{x^2}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 672

$$\begin{aligned}
&\int \frac{(a+b\arctan(cx))\,(d+e\log(f+gx^2))}{x^2}\,dx \\
&= \frac{2ae\sqrt{g}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}} - \frac{ibe\sqrt{g}\log(1+icx)\log\left(\frac{c(\sqrt{-f}-\sqrt{g}x)}{c\sqrt{-f}-i\sqrt{g}}\right)}{2\sqrt{-f}} \\
&\quad + \frac{ibe\sqrt{g}\log(1-icx)\log\left(\frac{c(\sqrt{-f}-\sqrt{g}x)}{c\sqrt{-f}+i\sqrt{g}}\right)}{2\sqrt{-f}} - \frac{ibe\sqrt{g}\log(1-icx)\log\left(\frac{c(\sqrt{-f}+\sqrt{g}x)}{c\sqrt{-f}-i\sqrt{g}}\right)}{2\sqrt{-f}} \\
&\quad + \frac{ibe\sqrt{g}\log(1+icx)\log\left(\frac{c(\sqrt{-f}+\sqrt{g}x)}{c\sqrt{-f}+i\sqrt{g}}\right)}{2\sqrt{-f}} - \frac{(a+b\arctan(cx))\,(d+e\log(f+gx^2))}{x} \\
&\quad + \frac{1}{2}bc\log\left(-\frac{gx^2}{f}\right)(d+e\log(f+gx^2)) \\
&\quad - \frac{1}{2}bc\log\left(-\frac{g(1+c^2x^2)}{c^2f-g}\right)(d+e\log(f+gx^2)) \\
&\quad + \frac{ibe\sqrt{g}\,\mathrm{PolyLog}\left(2,\frac{\sqrt{g}(i-cx)}{c\sqrt{-f}+i\sqrt{g}}\right)}{2\sqrt{-f}} - \frac{ibe\sqrt{g}\,\mathrm{PolyLog}\left(2,\frac{\sqrt{g}(1-icx)}{ic\sqrt{-f}+\sqrt{g}}\right)}{2\sqrt{-f}} \\
&\quad - \frac{ibe\sqrt{g}\,\mathrm{PolyLog}\left(2,\frac{\sqrt{g}(1+icx)}{ic\sqrt{-f}+\sqrt{g}}\right)}{2\sqrt{-f}} + \frac{ibe\sqrt{g}\,\mathrm{PolyLog}\left(2,\frac{\sqrt{g}(i+cx)}{c\sqrt{-f}+i\sqrt{g}}\right)}{2\sqrt{-f}} \\
&\quad - \frac{1}{2}bce\,\mathrm{PolyLog}\left(2,\frac{c^2(f+gx^2)}{c^2f-g}\right) + \frac{1}{2}bce\,\mathrm{PolyLog}\left(2,1+\frac{gx^2}{f}\right)
\end{aligned}$$

output
```
2*a*e*g^(1/2)*arctan(g^(1/2)*x/f^(1/2))/f^(1/2)-1/2*I*b*e*g^(1/2)*ln(1+I*c
*x)*ln(c*((-f)^(1/2)-g^(1/2)*x)/(c*(-f)^(1/2)-I*g^(1/2)))/(-f)^(1/2)+1/2*I
*b*e*g^(1/2)*ln(1-I*c*x)*ln(c*((-f)^(1/2)-g^(1/2)*x)/(c*(-f)^(1/2)+I*g^(1/
2)))/(-f)^(1/2)-1/2*I*b*e*g^(1/2)*ln(1-I*c*x)*ln(c*((-f)^(1/2)+g^(1/2)*x)/
(c*(-f)^(1/2)-I*g^(1/2)))/(-f)^(1/2)+1/2*I*b*e*g^(1/2)*ln(1+I*c*x)*ln(c*((
-f)^(1/2)+g^(1/2)*x)/(c*(-f)^(1/2)+I*g^(1/2)))/(-f)^(1/2)-(a+b*arctan(c*x)
)*(d+e*ln(g*x^2+f))/x+1/2*b*c*ln(-g*x^2/f)*(d+e*ln(g*x^2+f))-1/2*b*c*ln(-g
*(c^2*x^2+1)/(c^2*f-g))*(d+e*ln(g*x^2+f))+1/2*I*b*e*g^(1/2)*polylog(2,g^(1
/2)*(I-c*x)/(c*(-f)^(1/2)+I*g^(1/2)))/(-f)^(1/2)-1/2*I*b*e*g^(1/2)*polylog
(2,g^(1/2)*(1-I*c*x)/(I*c*(-f)^(1/2)+g^(1/2)))/(-f)^(1/2)-1/2*I*b*e*g^(1/2
)*polylog(2,g^(1/2)*(1+I*c*x)/(I*c*(-f)^(1/2)+g^(1/2)))/(-f)^(1/2)+1/2*I*b
*e*g^(1/2)*polylog(2,g^(1/2)*(I+c*x)/(c*(-f)^(1/2)+I*g^(1/2)))/(-f)^(1/2)-
1/2*b*c*e*polylog(2,c^2*(g*x^2+f)/(c^2*f-g))+1/2*b*c*e*polylog(2,1+g*x^2/f
)
```

## Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 552, normalized size of antiderivative = 0.82

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x^2}\,dx$$

$$= \frac{1}{2}\left(-\frac{2(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x} + \frac{e\sqrt{g}\left(4a\sqrt{-f}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)+ib\sqrt{f}\left(\log(1+icx)\log\left(\frac{c(\sqrt{-f}+\sqrt{g}x)}{c\sqrt{-f}+i\sqrt{g}}\right)+\operatorname{PolyLog}\left(2,\frac{\sqrt{g}(i-cx)}{c\sqrt{-f}+i\sqrt{g}}\right)\right)-ib\sqrt{\cdots}\right.}{\cdots} + bc\left(\left(\log\left(-\frac{gx^2}{f}\right)-\log\left(-\frac{g(1+c^2x^2)}{c^2f-g}\right)\right)\left(d+e\log\left(f+gx^2\right)\right) - e\operatorname{PolyLog}\left(2,\frac{c^2(f+gx^2)}{c^2f-g}\right)+e\operatorname{PolyLog}\left(2,1+\frac{gx^2}{f}\right)\right)\right)$$

input
```
Integrate[((a + b*ArcTan[c*x])*(d + e*Log[f + g*x^2]))/x^2,x]
```

output
```
((-2*(a + b*ArcTan[c*x])*(d + e*Log[f + g*x^2]))/x + (e*Sqrt[g]*(4*a*Sqrt[
-f]*ArcTan[(Sqrt[g]*x)/Sqrt[f]] + I*b*Sqrt[f]*(Log[1 + I*c*x]*Log[(c*(Sqrt
[-f] + Sqrt[g]*x))/(c*Sqrt[-f] + I*Sqrt[g])] + PolyLog[2, (Sqrt[g]*(I - c*
x))/(c*Sqrt[-f] + I*Sqrt[g])]) - I*b*Sqrt[f]*(Log[1 - I*c*x]*Log[(c*(Sqrt[
-f] + Sqrt[g]*x))/(c*Sqrt[-f] - I*Sqrt[g])] + PolyLog[2, (Sqrt[g]*(1 - I*c
*x))/(I*c*Sqrt[-f] + Sqrt[g])]) - I*b*Sqrt[f]*(Log[1 + I*c*x]*Log[(c*(Sqrt
[-f] - Sqrt[g]*x))/(c*Sqrt[-f] - I*Sqrt[g])] + PolyLog[2, (Sqrt[g]*(1 + I*
c*x))/(I*c*Sqrt[-f] + Sqrt[g])]) + I*b*Sqrt[f]*(Log[1 - I*c*x]*Log[(c*(Sqr
t[-f] - Sqrt[g]*x))/(c*Sqrt[-f] + I*Sqrt[g])] + PolyLog[2, (Sqrt[g]*(I + c
*x))/(c*Sqrt[-f] + I*Sqrt[g])])))/Sqrt[-f^2] + b*c*((Log[-((g*x^2)/f)] - L
og[-((g*(1 + c^2*x^2))/(c^2*f - g))])*(d + e*Log[f + g*x^2]) - e*PolyLog[2
, (c^2*(f + g*x^2))/(c^2*f - g)] + e*PolyLog[2, 1 + (g*x^2)/f]))/2
```

## Rubi [A] (verified)

Time = 1.44 (sec) , antiderivative size = 649, normalized size of antiderivative = 0.97, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5552, 2925, 2863, 2009, 5445, 218, 5443, 2856, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x^2}\,dx$$

$$\Big\downarrow \text{5552}$$

$$2eg\int \frac{a+b\arctan(cx)}{gx^2+f}dx+bc\int \frac{d+e\log\left(gx^2+f\right)}{x\left(c^2x^2+1\right)}dx-\frac{(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x}$$

$$\Big\downarrow \text{2925}$$

$$2eg\int \frac{a+b\arctan(cx)}{gx^2+f}dx+\frac{1}{2}bc\int \frac{d+e\log\left(gx^2+f\right)}{x^2\left(c^2x^2+1\right)}dx^2-\frac{(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x}$$

$$\Big\downarrow \text{2863}$$

$$2eg\int \frac{a+b\arctan(cx)}{gx^2+f}dx+\frac{1}{2}bc\int \left(\frac{d+e\log\left(gx^2+f\right)}{x^2}-\frac{c^2\left(d+e\log\left(gx^2+f\right)\right)}{c^2x^2+1}\right)dx^2-\frac{(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x}$$

$\downarrow$ 2009

$$2eg\int \frac{a+b\arctan(cx)}{gx^2+f}dx-\frac{(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x}+\frac{1}{2}bc\left(-\log\left(-\frac{g\left(c^2x^2+1\right)}{c^2f-g}\right)\left(d+e\log\left(f+gx^2\right)\right)-e\,\text{PolyLog}\left(2,\frac{c^2\left(gx^2+f\right)}{c^2f-g}\right)+\log\left(-\frac{gx^2}{f}\right)\left(d+e\log\left(f+\right.\right.\right.$$

$\downarrow$ 5445

$$2eg\left(a\int \frac{1}{gx^2+f}dx+b\int \frac{\arctan(cx)}{gx^2+f}dx\right)-\frac{(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x}+\frac{1}{2}bc\left(-\log\left(-\frac{g\left(c^2x^2+1\right)}{c^2f-g}\right)\left(d+e\log\left(f+gx^2\right)\right)-e\,\text{PolyLog}\left(2,\frac{c^2\left(gx^2+f\right)}{c^2f-g}\right)+\log\left(-\frac{gx^2}{f}\right)\left(d+e\log\left(f+\right.\right.\right.$$

$\downarrow$ 218

$$2eg\left(b\int \frac{\arctan(cx)}{gx^2+f}dx+\frac{a\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}}\right)-\frac{(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x}+\frac{1}{2}bc\left(-\log\left(-\frac{g\left(c^2x^2+1\right)}{c^2f-g}\right)\left(d+e\log\left(f+gx^2\right)\right)-e\,\text{PolyLog}\left(2,\frac{c^2\left(gx^2+f\right)}{c^2f-g}\right)+\log\left(-\frac{gx^2}{f}\right)\left(d+e\log\left(f+\right.\right.\right.$$

$\downarrow$ 5443

$$2eg\left(\frac{a\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}}+b\left(\frac{1}{2}i\int \frac{\log(1-icx)}{gx^2+f}dx-\frac{1}{2}i\int \frac{\log(icx+1)}{gx^2+f}dx\right)\right)-\frac{(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x}+\frac{1}{2}bc\left(-\log\left(-\frac{g\left(c^2x^2+1\right)}{c^2f-g}\right)\left(d+e\log\left(f+gx^2\right)\right)-e\,\text{PolyLog}\left(2,\frac{c^2\left(gx^2+f\right)}{c^2f-g}\right)+\log\left(-\frac{gx^2}{f}\right)\left(d+e\log\left(f+\right.\right.\right.$$

$\downarrow$ 2856

$$2eg\left(\frac{a\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}}+b\left(\frac{1}{2}i\int\left(\frac{\sqrt{-f}\log(1-icx)}{2f\left(\sqrt{-f}-\sqrt{g}x\right)}+\frac{\sqrt{-f}\log(1-icx)}{2f\left(\sqrt{g}x+\sqrt{-f}\right)}\right)dx-\frac{1}{2}i\int\left(\frac{\sqrt{-f}\log(icx+1)}{2f\left(\sqrt{-f}-\sqrt{g}x\right)}+\frac{\sqrt{}}{2}\right.\right.\right.$$

$$\frac{(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x}+$$

$$\frac{1}{2}bc\left(-\log\left(-\frac{g(c^2x^2+1)}{c^2f-g}\right)\left(d+e\log\left(f+gx^2\right)\right)-e\,\text{PolyLog}\left(2,\frac{c^2(gx^2+f)}{c^2f-g}\right)+\log\left(-\frac{gx^2}{f}\right)\left(d+e\log\left(f\right.\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x}+$$

$$2eg\left(\frac{a\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}\sqrt{g}}+b\left(\frac{1}{2}i\left(-\frac{\text{PolyLog}\left(2,\frac{\sqrt{g}(1-icx)}{i\sqrt{-f}c+\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}}+\frac{\text{PolyLog}\left(2,\frac{\sqrt{g}(cx+i)}{\sqrt{-f}c+i\sqrt{g}}\right)}{2\sqrt{-f}\sqrt{g}}+\frac{\log(1-icx)\log\left(\frac{c(\sqrt{-f}-}{c\sqrt{-f}-}\right.}{2\sqrt{-f}\sqrt{g}}\right.\right.\right.$$

$$\frac{1}{2}bc\left(-\log\left(-\frac{g(c^2x^2+1)}{c^2f-g}\right)\left(d+e\log\left(f+gx^2\right)\right)-e\,\text{PolyLog}\left(2,\frac{c^2(gx^2+f)}{c^2f-g}\right)+\log\left(-\frac{gx^2}{f}\right)\left(d+e\log\left(f\right.\right.\right.$$

input
```
Int[((a + b*ArcTan[c*x])*(d + e*Log[f + g*x^2]))/x^2,x]
```

output
```
-(((a + b*ArcTan[c*x])*(d + e*Log[f + g*x^2]))/x) + 2*e*g*((a*ArcTan[(Sqrt
[g]*x)/Sqrt[f]])/(Sqrt[f]*Sqrt[g]) + b*((-1/2*I)*((Log[1 + I*c*x]*Log[(c*(
Sqrt[-f] - Sqrt[g]*x))/(c*Sqrt[-f] - I*Sqrt[g])])/(2*Sqrt[-f]*Sqrt[g]) - (
Log[1 + I*c*x]*Log[(c*(Sqrt[-f] + Sqrt[g]*x))/(c*Sqrt[-f] + I*Sqrt[g])])/(
2*Sqrt[-f]*Sqrt[g]) - PolyLog[2, (Sqrt[g]*(I - c*x))/(c*Sqrt[-f] + I*Sqrt[
g])]/(2*Sqrt[-f]*Sqrt[g]) + PolyLog[2, (Sqrt[g]*(1 + I*c*x))/(I*c*Sqrt[-f]
 + Sqrt[g])]/(2*Sqrt[-f]*Sqrt[g])) + (I/2)*((Log[1 - I*c*x]*Log[(c*(Sqrt[-
f] - Sqrt[g]*x))/(c*Sqrt[-f] + I*Sqrt[g])])/(2*Sqrt[-f]*Sqrt[g]) - (Log[1
- I*c*x]*Log[(c*(Sqrt[-f] + Sqrt[g]*x))/(c*Sqrt[-f] - I*Sqrt[g])])/(2*Sqrt
[-f]*Sqrt[g]) - PolyLog[2, (Sqrt[g]*(1 - I*c*x))/(I*c*Sqrt[-f] + Sqrt[g])]
/(2*Sqrt[-f]*Sqrt[g]) + PolyLog[2, (Sqrt[g]*(I + c*x))/(c*Sqrt[-f] + I*Sqr
t[g])]/(2*Sqrt[-f]*Sqrt[g])))) + (b*c*(Log[-((g*x^2)/f)]*(d + e*Log[f + g*
x^2]) - Log[-((g*(1 + c^2*x^2))/(c^2*f - g))]*(d + e*Log[f + g*x^2]) - e*P
olyLog[2, (c^2*(f + g*x^2))/(c^2*f - g)] + e*PolyLog[2, 1 + (g*x^2)/f]))/2
```

### Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2856
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(r_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*Log[c*(d + e*x)
^n])^p, (f + g*x^r)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g, n, r}, x] && I
GtQ[p, 0] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[r] && NeQ[r, 1]))
```

rule 2863
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((h_.)*(x_))
^(m_.)*((f_) + (g_.)*(x_)^(r_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a
 + b*Log[c*(d + e*x)^n])^p, (h*x)^m*(f + g*x^r)^q, x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, m, n, p, q, r}, x] && IntegerQ[m] && IntegerQ[q]
```

rule 2925
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_))^(p_.)]*(b_.))^(q_.)*(x_)^(m
_.)*((f_) + (g_.)*(x_)^(s_))^(r_.), x_Symbol] :> Simp[1/n   Subst[Int[x^(Si
mplify[(m + 1)/n] - 1)*(f + g*x^(s/n))^r*(a + b*Log[c*(d + e*x)^p])^q, x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p, q, r, s}, x] && Integer
Q[r] && IntegerQ[s/n] && IntegerQ[Simplify[(m + 1)/n]] && (GtQ[(m + 1)/n, 0
] || IGtQ[q, 0])
```

rule 5443
```
Int[ArcTan[(c_.)*(x_)]/((d_.) + (e_.)*(x_)^2), x_Symbol] :> Simp[I/2   Int[
Log[1 - I*c*x]/(d + e*x^2), x], x] - Simp[I/2   Int[Log[1 + I*c*x]/(d + e*x
^2), x], x] /; FreeQ[{c, d, e}, x]
```

rule 5445
```
Int[(ArcTan[(c_.)*(x_)]*(b_.) + (a_))/((d_.) + (e_.)*(x_)^2), x_Symbol] :>
Simp[a   Int[1/(d + e*x^2), x], x] + Simp[b   Int[ArcTan[c*x]/(d + e*x^2),
x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 5552
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)*(d + e*Log[f + g*x^2])*((a +
b*ArcTan[c*x])/(m + 1)), x] + (-Simp[b*(c/(m + 1))   Int[x^(m + 1)*((d + e*
Log[f + g*x^2])/(1 + c^2*x^2)), x], x] - Simp[2*e*(g/(m + 1))   Int[x^(m +
2)*((a + b*ArcTan[c*x])/(f + g*x^2)), x], x]) /; FreeQ[{a, b, c, d, e, f, g
}, x] && ILtQ[m/2, 0]
```

## Maple [F]

$$\int \frac{(a+b\arctan(cx))\left(d+e\ln\left(g\,x^{2}+f\right)\right)}{x^{2}}dx$$

input
```
int((a+b*arctan(c*x))*(d+e*ln(g*x^2+f))/x^2,x)
```

output
```
int((a+b*arctan(c*x))*(d+e*ln(g*x^2+f))/x^2,x)
```

## Fricas [F]

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(f+gx^{2}\right)\right)}{x^{2}}\,dx = \int \frac{(b\arctan(cx)+a)(e\log\left(gx^{2}+f\right)+d)}{x^{2}}\,dx$$

input
```
integrate((a+b*arctan(c*x))*(d+e*log(g*x^2+f))/x^2,x, algorithm="fricas")
```

output
```
integral((b*d*arctan(c*x) + a*d + (b*e*arctan(c*x) + a*e)*log(g*x^2 + f))/
x^2, x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))\,(d+e\log{(f+gx^2)})}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*atan(c*x))*(d+e*ln(g*x**2+f))/x**2,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(a+b\arctan(cx))\,(d+e\log{(f+gx^2)})}{x^2}\,dx = \int \frac{(b\arctan{(cx)}+a)(e\log{(gx^2+f)}+d)}{x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))*(d+e*log(g*x^2+f))/x^2,x, algorithm="maxima")
```

output
```
-1/2*(c*(log(c^2*x^2 + 1) - log(x^2)) + 2*arctan(c*x)/x)*b*d + (2*g*arctan
(g*x/sqrt(f*g))/sqrt(f*g) - log(g*x^2 + f)/x)*a*e + b*e*integrate(arctan(c
*x)*log(g*x^2 + f)/x^2, x) - a*d/x
```

## Giac [F]

$$\int \frac{(a+b\arctan(cx))\,(d+e\log{(f+gx^2)})}{x^2}\,dx = \int \frac{(b\arctan{(cx)}+a)(e\log{(gx^2+f)}+d)}{x^2}\,dx$$

input
```
integrate((a+b*arctan(c*x))*(d+e*log(g*x^2+f))/x^2,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)*(e*log(g*x^2 + f) + d)/x^2, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x^2}\,dx = \int \frac{(a+b\,\mathrm{atan}(c\,x))\,\left(d+e\,\ln\left(g\,x^2+f\right)\right)}{x^2}\,dx$$

input
```
int(((a + b*atan(c*x))*(d + e*log(f + g*x^2)))/x^2,x)
```

output
```
int(((a + b*atan(c*x))*(d + e*log(f + g*x^2)))/x^2, x)
```

## Reduce [F]

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x^2}\,dx = \frac{4\sqrt{g}\,\sqrt{f}\,atan\left(\frac{gx}{\sqrt{g}\,\sqrt{f}}\right)aex-2atan(cx)\,bdf+2\left(\int\frac{atan(cx)\log\left(g\,x^2+f\right)}{x^2}dx\right)befx-\log(c^2x^2+1)\,bcdfx-2\,\mathrm{lo}}{2fx}$$

input
```
int((a+b*atan(c*x))*(d+e*log(g*x^2+f))/x^2,x)
```

output
```
(4*sqrt(g)*sqrt(f)*atan((g*x)/(sqrt(g)*sqrt(f)))*a*e*x - 2*atan(c*x)*b*d*f
 + 2*int((atan(c*x)*log(f + g*x**2))/x**2,x)*b*e*f*x - log(c**2*x**2 + 1)*
b*c*d*f*x - 2*log(f + g*x**2)*a*e*f + 2*log(x)*b*c*d*f*x - 2*a*d*f)/(2*f*x
)
```

# 3.1299 $\int \frac{(a+b\arctan(cx))(d+e\log(f+gx^2))}{x^3}\,dx$



## Optimal result

Integrand size = 24, antiderivative size = 528

$$\begin{aligned}
&\int \frac{(a+b\arctan(cx))(d+e\log(f+gx^2))}{x^3}\,dx\\
&= \frac{bce\sqrt{g}\arctan\left(\frac{\sqrt{g}x}{\sqrt{f}}\right)}{\sqrt{f}}+\frac{aeg\log(x)}{f}-\frac{be(c^2f-g)\arctan(cx)\log\left(\frac{2}{1-icx}\right)}{f}\\
&\quad+\frac{be(c^2f-g)\arctan(cx)\log\left(\frac{2c(\sqrt{-f}-\sqrt{g}x)}{(c\sqrt{-f}-i\sqrt{g})(1-icx)}\right)}{2f}\\
&\quad+\frac{be(c^2f-g)\arctan(cx)\log\left(\frac{2c(\sqrt{-f}+\sqrt{g}x)}{(c\sqrt{-f}+i\sqrt{g})(1-icx)}\right)}{2f}-\frac{aeg\log(f+gx^2)}{2f}\\
&\quad-\frac{bc(d+e\log(f+gx^2))}{2x}-\frac{1}{2}bc^2\arctan(cx)\left(d+e\log\left(f+gx^2\right)\right)\\
&\quad-\frac{(a+b\arctan(cx))(d+e\log(f+gx^2))}{2x^2}+\frac{ibeg\operatorname{PolyLog}(2,-icx)}{2f}\\
&\quad-\frac{ibeg\operatorname{PolyLog}(2,icx)}{2f}+\frac{ibe(c^2f-g)\operatorname{PolyLog}\left(2,1-\frac{2}{1-icx}\right)}{2f}\\
&\quad-\frac{ibe(c^2f-g)\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-f}-\sqrt{g}x)}{(c\sqrt{-f}-i\sqrt{g})(1-icx)}\right)}{4f}\\
&\quad-\frac{ibe(c^2f-g)\operatorname{PolyLog}\left(2,1-\frac{2c(\sqrt{-f}+\sqrt{g}x)}{(c\sqrt{-f}+i\sqrt{g})(1-icx)}\right)}{4f}
\end{aligned}$$

output
```
b*c*e*g^(1/2)*arctan(g^(1/2)*x/f^(1/2))/f^(1/2)+a*e*g*ln(x)/f-b*e*(c^2*f-g
)*arctan(c*x)*ln(2/(1-I*c*x))/f+1/2*b*e*(c^2*f-g)*arctan(c*x)*ln(2*c*((-f)
^(1/2)-g^(1/2)*x)/(c*(-f)^(1/2)-I*g^(1/2))/(1-I*c*x))/f+1/2*b*e*(c^2*f-g)*
arctan(c*x)*ln(2*c*((-f)^(1/2)+g^(1/2)*x)/(c*(-f)^(1/2)+I*g^(1/2))/(1-I*c*
x))/f-1/2*a*e*g*ln(g*x^2+f)/f-1/2*b*c*(d+e*ln(g*x^2+f))/x-1/2*b*c^2*arctan
(c*x)*(d+e*ln(g*x^2+f))-1/2*(a+b*arctan(c*x))*(d+e*ln(g*x^2+f))/x^2+1/2*I*
b*e*g*polylog(2,-I*c*x)/f-1/2*I*b*e*g*polylog(2,I*c*x)/f+1/2*I*b*e*(c^2*f-
g)*polylog(2,1-2/(1-I*c*x))/f-1/4*I*b*e*(c^2*f-g)*polylog(2,1-2*c*((-f)^(1
/2)-g^(1/2)*x)/(c*(-f)^(1/2)-I*g^(1/2))/(1-I*c*x))/f-1/4*I*b*e*(c^2*f-g)*p
olylog(2,1-2*c*((-f)^(1/2)+g^(1/2)*x)/(c*(-f)^(1/2)+I*g^(1/2))/(1-I*c*x))/
f
```

## Mathematica [B] (warning: unable to verify)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1217 vs. 2(528) = 1056.

Time = 4.61 (sec) , antiderivative size = 1217, normalized size of antiderivative = 2.30

$$\int \frac{(a + b \arctan(cx)) (d + e \log (f + gx^2))}{x^3} \, dx = \text{Too large to display}$$

input
```
Integrate[((a + b*ArcTan[c*x])*(d + e*Log[f + g*x^2]))/x^3,x]
```

output
```
-1/4*(2*a*d*f + 2*b*c*d*f*x + 2*b*d*f*ArcTan[c*x] + 2*b*c^2*d*f*x^2*ArcTan
[c*x] - 4*b*c*e*Sqrt[f]*Sqrt[g]*x^2*ArcTan[(Sqrt[g]*x)/Sqrt[f]] - (4*I)*b*
c^2*e*f*x^2*ArcSin[Sqrt[(c^2*f)/(c^2*f - g)]]*ArcTan[(c*g*x)/Sqrt[c^2*f*g]
] + (4*I)*b*e*g*x^2*ArcSin[Sqrt[(c^2*f)/(c^2*f - g)]]*ArcTan[(c*g*x)/Sqrt[
c^2*f*g]] - 4*b*e*g*x^2*ArcTan[c*x]*Log[1 - E^((2*I)*ArcTan[c*x])] + 4*b*c
^2*e*f*x^2*ArcTan[c*x]*Log[1 + E^((2*I)*ArcTan[c*x])] - 2*b*c^2*e*f*x^2*Ar
cSin[Sqrt[(c^2*f)/(c^2*f - g)]]*Log[(c^2*(1 + E^((2*I)*ArcTan[c*x]))*f + (
-1 + E^((2*I)*ArcTan[c*x]))*g - 2*E^((2*I)*ArcTan[c*x])*Sqrt[c^2*f*g])/(c^
2*f - g)] + 2*b*e*g*x^2*ArcSin[Sqrt[(c^2*f)/(c^2*f - g)]]*Log[(c^2*(1 + E^
((2*I)*ArcTan[c*x]))*f + (-1 + E^((2*I)*ArcTan[c*x]))*g - 2*E^((2*I)*ArcTa
n[c*x])*Sqrt[c^2*f*g])/(c^2*f - g)] - 2*b*c^2*e*f*x^2*ArcTan[c*x]*Log[(c^2
*(1 + E^((2*I)*ArcTan[c*x]))*f + (-1 + E^((2*I)*ArcTan[c*x]))*g - 2*E^((2*
I)*ArcTan[c*x])*Sqrt[c^2*f*g])/(c^2*f - g)] + 2*b*e*g*x^2*ArcTan[c*x]*Log[
(c^2*(1 + E^((2*I)*ArcTan[c*x]))*f + (-1 + E^((2*I)*ArcTan[c*x]))*g - 2*E^
((2*I)*ArcTan[c*x])*Sqrt[c^2*f*g])/(c^2*f - g)] + 2*b*c^2*e*f*x^2*ArcSin[S
qrt[(c^2*f)/(c^2*f - g)]]*Log[1 + (E^((2*I)*ArcTan[c*x])*(c^2*f + g + 2*Sq
rt[c^2*f*g]))/(c^2*f - g)] - 2*b*e*g*x^2*ArcSin[Sqrt[(c^2*f)/(c^2*f - g)]]
*Log[1 + (E^((2*I)*ArcTan[c*x])*(c^2*f + g + 2*Sqrt[c^2*f*g]))/(c^2*f - g)
] - 2*b*c^2*e*f*x^2*ArcTan[c*x]*Log[1 + (E^((2*I)*ArcTan[c*x])*(c^2*f + g
+ 2*Sqrt[c^2*f*g]))/(c^2*f - g)] + 2*b*e*g*x^2*ArcTan[c*x]*Log[1 + (E^(...
```

## Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 544, normalized size of antiderivative = 1.03, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = $\{5556, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)}{x^3}\,dx$$

$$\Big\downarrow \; 5556$$

$$-2eg\int\left(-\frac{a+bcx}{2x\left(gx^2+f\right)}-\frac{b(c^2x^2+1)\arctan(cx)}{2x\left(gx^2+f\right)}\right)dx-\frac{(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)}{2x^2}-\frac{1}{2}bc^2\arctan(cx)\left(d+e\log\left(f+gx^2\right)\right)-\frac{bc\left(d+e\log\left(f+gx^2\right)\right)}{2x}$$

$$\Big\downarrow \textbf{2009}$$

$$-2eg\left(\frac{a\log\left(f+gx^2\right)}{4f}-\frac{a\log(x)}{2f}+\frac{b\arctan(cx)\left(c^2f-g\right)\log\left(\frac{2}{1-icx}\right)}{2fg}-\frac{b\arctan(cx)\left(c^2f-g\right)\log\left(\frac{2c(\sqrt{-f}-}{(1-icx)(c\sqrt{}}\right.}{4fg}\right.-\frac{(a+b\arctan(cx))\left(d+e\log\left(f+gx^2\right)\right)}{2x^2}-\frac{1}{2}bc^2\arctan(cx)\left(d+e\log\left(f+gx^2\right)\right)-\frac{bc\left(d+e\log\left(f+gx^2\right)\right)}{2x}$$

input
```
Int[((a + b*ArcTan[c*x])*(d + e*Log[f + g*x^2]))/x^3,x]
```

output
```
-1/2*(b*c*(d + e*Log[f + g*x^2]))/x - (b*c^2*ArcTan[c*x]*(d + e*Log[f + g*
x^2]))/2 - ((a + b*ArcTan[c*x])*(d + e*Log[f + g*x^2]))/(2*x^2) - 2*e*g*(-
1/2*(b*c*ArcTan[(Sqrt[g]*x)/Sqrt[f]])/(Sqrt[f]*Sqrt[g]) - (a*Log[x])/(2*f)
 + (b*(c^2*f - g)*ArcTan[c*x]*Log[2/(1 - I*c*x)])/(2*f*g) - (b*(c^2*f - g)
*ArcTan[c*x]*Log[(2*c*(Sqrt[-f] - Sqrt[g]*x))/((c*Sqrt[-f] - I*Sqrt[g])*(1
 - I*c*x))])/(4*f*g) - (b*(c^2*f - g)*ArcTan[c*x]*Log[(2*c*(Sqrt[-f] + Sqr
t[g]*x))/((c*Sqrt[-f] + I*Sqrt[g])*(1 - I*c*x))])/(4*f*g) + (a*Log[f + g*x
^2])/(4*f) - ((I/4)*b*PolyLog[2, (-I)*c*x])/f + ((I/4)*b*PolyLog[2, I*c*x]
)/f - ((I/4)*b*(c^2*f - g)*PolyLog[2, 1 - 2/(1 - I*c*x)])/(f*g) + ((I/8)*b
*(c^2*f - g)*PolyLog[2, 1 - (2*c*(Sqrt[-f] - Sqrt[g]*x))/((c*Sqrt[-f] - I*
Sqrt[g])*(1 - I*c*x))])/(f*g) + ((I/8)*b*(c^2*f - g)*PolyLog[2, 1 - (2*c*(
Sqrt[-f] + Sqrt[g]*x))/((c*Sqrt[-f] + I*Sqrt[g])*(1 - I*c*x))])/(f*g))
```

### Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5556
```
Int[((a_.) + ArcTan[(c_.)*(x_)]*(b_.))*((d_.) + Log[(f_.) + (g_.)*(x_)^2]*(
e_.))*(x_)^(m_.), x_Symbol] :> With[{u = IntHide[x^m*(a + b*ArcTan[c*x]), x
]}, Simp[(d + e*Log[f + g*x^2])   u, x] - Simp[2*e*g   Int[ExpandIntegrand[
x*(u/(f + g*x^2)), x], x], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && Intege
rQ[m] && NeQ[m, -1]
```

## Maple [F]

$$\int \frac{(a+b\arctan(cx))\,(d+e\ln(g\,x^2+f))}{x^3}dx$$

input
```
int((a+b*arctan(c*x))*(d+e*ln(g*x^2+f))/x^3,x)
```

output
```
int((a+b*arctan(c*x))*(d+e*ln(g*x^2+f))/x^3,x)
```

## Fricas [F]

$$\int \frac{(a+b\arctan(cx))\,(d+e\log(f+gx^2))}{x^3}\,dx = \int \frac{(b\arctan(cx)+a)(e\log(gx^2+f)+d)}{x^3}\,dx$$

input
```
integrate((a+b*arctan(c*x))*(d+e*log(g*x^2+f))/x^3,x, algorithm="fricas")
```

output
```
integral((b*d*arctan(c*x) + a*d + (b*e*arctan(c*x) + a*e)*log(g*x^2 + f))/
x^3, x)
```

## Sympy [F(-1)]

Timed out.

$$\int \frac{(a + b\arctan(cx)) (d + e\log(f + gx^2))}{x^3} dx = \text{Timed out}$$

input
```
integrate((a+b*atan(c*x))*(d+e*ln(g*x**2+f))/x**3,x)
```

output
```
Timed out
```

## Maxima [F]

$$\int \frac{(a + b\arctan(cx)) (d + e\log(f + gx^2))}{x^3} dx = \int \frac{(b\arctan(cx) + a)(e\log(gx^2 + f) + d)}{x^3} dx$$

input
```
integrate((a+b*arctan(c*x))*(d+e*log(g*x^2+f))/x^3,x, algorithm="maxima")
```

output
```
-1/2*((c*arctan(c*x) + 1/x)*c + arctan(c*x)/x^2)*b*d - 1/2*(g*(log(g*x^2 +
 f)/f - log(x^2)/f) + log(g*x^2 + f)/x^2)*a*e + 1/2*(2*c*g*x^2*arctan(g*x/
sqrt(f*g)) + (4*c^2*g*x^2*integrate(1/2*x*arctan(c*x)/(g*x^2 + f), x) + 4*
g*x^2*integrate(1/2*arctan(c*x)/(g*x^3 + f*x), x) - (c*x + (c^2*x^2 + 1)*a
rctan(c*x))*log(g*x^2 + f))*sqrt(f*g))*b*e/(sqrt(f*g)*x^2) - 1/2*a*d/x^2
```

## Giac [F]

$$\int \frac{(a + b\arctan(cx)) (d + e\log(f + gx^2))}{x^3} dx = \int \frac{(b\arctan(cx) + a)(e\log(gx^2 + f) + d)}{x^3} dx$$

input
```
integrate((a+b*arctan(c*x))*(d+e*log(g*x^2+f))/x^3,x, algorithm="giac")
```

output
```
integrate((b*arctan(c*x) + a)*(e*log(g*x^2 + f) + d)/x^3, x)
```

## Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b \arctan(cx)) (d + e \log(f + gx^2))}{x^3} dx = \int \frac{(a + b \operatorname{atan}(c x)) (d + e \ln(g x^2 + f))}{x^3} dx$$

input
```
int(((a + b*atan(c*x))*(d + e*log(f + g*x^2)))/x^3,x)
```

output
```
int(((a + b*atan(c*x))*(d + e*log(f + g*x^2)))/x^3, x)
```

## Reduce [F]

$$\int \frac{(a + b \arctan(cx)) (d + e \log(f + gx^2))}{x^3} dx = \text{Too large to display}$$

input
```
int((a+b*atan(c*x))*(d+e*log(g*x^2+f))/x^3,x)
```

output
```
(2*sqrt(g)*sqrt(f)*atan((g*x)/(sqrt(g)*sqrt(f)))*b*c*e*g*x**2 - atan(c*x)*
log(f + g*x**2)*b*c**2*e*f**2 - atan(c*x)*log(f + g*x**2)*b*e*f*g - atan(c
*x)*b*c**4*d*f**2*x**2 - atan(c*x)*b*c**4*e*f**2*x**2 - atan(c*x)*b*c**2*d
*f**2 - atan(c*x)*b*c**2*d*f*g*x**2 - atan(c*x)*b*c**2*e*f**2 - atan(c*x)*
b*c**2*e*f*g*x**2 - atan(c*x)*b*d*f*g - atan(c*x)*b*e*f*g - 2*int(atan(c*x
)/(c**4*f**2*x**5 + c**4*f*g*x**7 + c**2*f**2*x**3 + 2*c**2*f*g*x**5 + c**
2*g**2*x**7 + f*g*x**3 + g**2*x**5),x)*b*c**4*e*f**4*x**2 - 4*int(atan(c*x
)/(c**4*f**2*x**5 + c**4*f*g*x**7 + c**2*f**2*x**3 + 2*c**2*f*g*x**5 + c**
2*g**2*x**7 + f*g*x**3 + g**2*x**5),x)*b*c**2*e*f**3*g*x**2 - 2*int(atan(c
*x)/(c**4*f**2*x**5 + c**4*f*g*x**7 + c**2*f**2*x**3 + 2*c**2*f*g*x**5 + c
**2*g**2*x**7 + f*g*x**3 + g**2*x**5),x)*b*e*f**2*g**2*x**2 - 2*int(atan(c
*x)/(c**4*f**2*x**3 + c**4*f*g*x**5 + c**2*f**2*x + 2*c**2*f*g*x**3 + c**2
*g**2*x**5 + f*g*x + g**2*x**3),x)*b*c**6*e*f**4*x**2 - 4*int(atan(c*x)/(c
**4*f**2*x**3 + c**4*f*g*x**5 + c**2*f**2*x + 2*c**2*f*g*x**3 + c**2*g**2*
x**5 + f*g*x + g**2*x**3),x)*b*c**4*e*f**3*g*x**2 - 2*int(atan(c*x)/(c**4*
f**2*x**3 + c**4*f*g*x**5 + c**2*f**2*x + 2*c**2*f*g*x**3 + c**2*g**2*x**5
 + f*g*x + g**2*x**3),x)*b*c**2*e*f**2*g**2*x**2 + int(log(f + g*x**2)/(c*
*4*f**2*x**4 + c**4*f*g*x**6 + c**2*f**2*x**2 + 2*c**2*f*g*x**4 + c**2*g**
2*x**6 + f*g*x**2 + g**2*x**4),x)*b*c**5*e*f**4*x**2 + int(log(f + g*x**2)
/(c**4*f**2*x**4 + c**4*f*g*x**6 + c**2*f**2*x**2 + 2*c**2*f*g*x**4 + c...
```

# CHAPTER 4

## Appendix



## 4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

### Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

## Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                    convert(leaf_count_result,string)," vs. $2 (",
                                    convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

## Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

## SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
   r"""
   Return the tree size of this expression.
   """
   #print("Enter tree_size, expr is ",expr)

   if expr not in SR:
      # deal with lists, tuples, vectors
      return 1 + sum(tree_size(a) for a in expr)
   expr = SR(expr)
   x, aa = expr.operator(), expr.operands()
   if x is None:
      return 1
   else:
      return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
   if expr.operator() == operator.pow:   #isinstance(expr,Pow):
      if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
         if debug: print ("expr is sqrt")
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   #debug=False
   m = func.name() in ['exp','log','ln',
         'sin','cos','tan','cot','sec','csc',
         'arcsin','arccos','arctan','arccot','arcsec','arccsc',
         'sinh','cosh','tanh','coth','sech','csch',
         'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
      'arctan2','floor','abs'
      ]
   if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral',
           'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
           'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:    #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
```

```
        return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
        if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
          return 1
        else:
          return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
      else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
  elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif is_elementary_function(expn.operator()):   #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
  elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(4,m1)    #max(4,m1)
  elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
    return max(5,m1)    #max(5,m1)
  elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
    return max(6,m1)    #max(6,m1)
  elif str(expn).find("Integral") != -1: #this will never happen, since it
          #is checked before calling the grading function that is passed.
          #but kept it here.
    m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
    return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
  else:
    return 9

#main function
def grade_antiderivative(result,optimal):


  if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

## 4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file